

ARTICLE 53

The Ten-fold Division within Five Sacred Geometries & its Manifestation in the Ten Whorls of the UPA, the Subquark State of the $E_8 \times E_8$ Heterotic Superstring

by

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Abstract

Five sacred geometries — the inner form of the Tree of Life, the first three Platonic solids, the 2-dimensional Sri Yantra, the disdyakis triacontahedron & the 1-tree — are shown to possess 240 structural components or geometrical elements. They correspond to the 240 roots of the rank-8 Lie group E_8 because in each case they divide into 72 components or elements of one kind and 168 of another kind, in analogy to the 72 roots of E_6 , the rank-6 exceptional subgroup of E_8 , and to the remaining 168 roots of E_8 . Furthermore, the 72 components are proved to form three sets of 24 and the 168 components are shown to form seven sets of 24, so that all 240 components form ten sets of 24. Each sacred geometry therefore has a ten-fold division, indicating a similar division in the holistic systems that they represent. The best known example of this is the Kabbalistic Tree of Life with ten Sephiroth that comprise the Supernal Triad and the seven Sephiroth of Construction. A less well-known example is the “ultimate physical atom,” or UPA, the basic unit of matter paranormally described over a century ago by the Theosophists Annie Besant and C.W. Leadbeater. This has been identified by the author as the $E_8 \times E_8$ heterotic superstring constituent of up and down quarks. Its ten whorls correspond to the ten sets of structural components making up each sacred geometry. The analogy suggests that 24 E_8 gauge charges are spread along each whorl as the counterpart of each set of 24 components. The ten-fold composition of the $E_8 \times E_8$ heterotic superstring predicted by this analogy with sacred geometries is a consequence of the ten-fold nature of God, which is most explicitly expressed by the ten Sephiroth of the Tree of Life.

Table 1. Gematria number values of the ten Sephiroth in the four Worlds.

	SEPHIRAH	GODNAME	ARCHANGEL	ORDER OF ANGELS	MUNDANE CHAKRA
1	Kether (Crown) 620	EHYEH (I am) 21	Metatron (Angel of the Presence) 314	Chaioth ha Qadesh (Holy Living Creatures) 833	Rashith ha Gilgalim First Swirlings. (Primum Mobile) 636
2	Chokmah (Wisdom) 73	YAHWEH, YAH (The Lord) 26, 15	Raziel (Herald of the Deity) 248	Auphanim (Wheels) 187	Masloth (The Sphere of the Zodiac) 140
3	Binah (Understanding) 67	ELOHIM (God in multiplicity) 50	Tzaphkiel (Contemplation of God) 311	Aralim (Thrones) 282	Shabathai Rest. (Saturn) 317
	Daath (Knowledge) 474				
4	Chesed (Mercy) 72	EL (God) 31	Tzadkiel (Benevolence of God) 62	Chasmalim (Shining Ones) 428	Tzadekh Righteousness. (Jupiter) 194
5	Geburah (Severity) 216	ELOHA (The Almighty) 36	Samael (Severity of God) 131	Seraphim (Fiery Serpents) 630	Madim Vehement Strength. (Mars) 95
6	Tiphareth (Beauty) 1081	YAHWEH ELOHIM (God the Creator) 76	Michael (Like unto God) 101	Malachim (Kings) 140	Shemesh The Solar Light. (Sun) 640
7	Netzach (Victory) 148	YAHWEH SABAOTH (Lord of Hosts) 129	Haniel (Grace of God) 97	Tarshishim or Elohim 1260	Nogah Glittering Splendour. (Venus) 64
8	Hod (Glory) 15	ELOHIM SABAOTH (God of Hosts) 153	Raphael (Divine Physician) 311	Beni Elohim (Sons of God) 112	Kokab The Stellar Light. (Mercury) 48
9	Yesod (Foundation) 80	SHADDAI EL CHAI (Almighty Living God) 49, 363	Gabriel (Strong Man of God) 246	Cherubim (The Strong) 272	Levanah The Lunar Flame. (Moon) 87
10	Malkuth (Kingdom) 496	ADONAI MELEKH (The Lord and King) 65, 155	Sandalphon (Manifest Messiah) 280	Ashim (Souls of Fire) 351	Cholem Yesodeth The Breaker of the Foundations. The Elements. (Earth) 168

(All numbers in this table that are referred to in the article are written in **boldface**).

The Sephiroth exist in the four Worlds of Atziluth, Beriah, Yetzirah and Assiyah. Corresponding to them are the Godnames, Archangels, Order of Angels and Mundane Chakras (their physical manifestation, traditionally symbolised by celestial bodies). This table gives their number values obtained by the ancient practice of gematria, wherein a number is assigned to each letter of the alphabet, thereby giving to a word a number value that is the sum of the numbers of its letters.

1. The inner Tree of Life

The 10-fold UPA/superstring as the 10-fold division of the (7+7) polygons

The 240 roots of the Lie group E_8 can be regarded as consisting of the 72 roots that are also roots of its exceptional subgroup E_6 and 168 other roots (Fig. 1). When their 47 sectors are tetractyses, the seven enfolded polygons of the inner Tree of Life have 120 yods on their 42 sides, where

$$120 = 2^2 + 4^2 + 6^2 + 8^2.$$

They comprise 36 ($= 6^2$) black yods at the corners of polygons and 84 ($= 2^2 + 4^2 + 8^2$) red hexagonal yods. The two separate sets of seven enfolded polygons have ($2 \times 36 = 72$) black yods at their corners and ($2 \times 84 = 168$) red hexagonal yods. The distinction between corners and hexagonal yods corresponds to E_6 being a subgroup of E_8 . This is the sacred geometrical counterpart of the symmetry-breakdown of E_8 into

The 72 roots of E_6 and the 168 extra roots of E_8

In terms of the eight orthonormal unit vectors $\{u_i\}$, the 240 roots of E_8 are:

$$\pm u_i \pm u_j \quad (i, j = 1, 2, \dots, 8)$$

and

$$\frac{1}{2}(\pm u_1, \pm u_2, \dots, \pm u_8) \quad (\text{even number of '+'s})$$

Their explicit forms as 8-tuples and their numbers are listed below:

- (1, 1, 0, 0, 0, 0, 0, 0) and all permutations. Number = $\binom{8}{2} = 28$;
 - (-1, -1, 0, 0, 0, 0, 0, 0) and all permutations. Number = $\binom{8}{2} = 28$;
 - (1, -1, 0, 0, 0, 0, 0, 0) and all permutations. Number = $2 \times \binom{8}{2} = 56$;
 - ($-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}$) and all permutations. Number = $\binom{8}{2} = 28$;
 - ($-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$) and all permutations. Number = $\binom{8}{2} = 28$;
 - ($-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}$) and all permutations. Number = $\binom{8}{4} = 70$;
 - ($\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}$). Number = 1;
 - ($-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}$). Number = 1.
- } 168
} 72

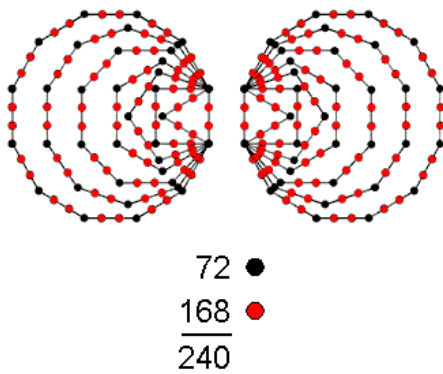


Figure 1

E_6 that has been considered by superstring theorists in order to reproduce the physics of the Standard Model. Each yod on the boundaries of the polygons symbolizes a root of E_8 to which corresponds a gauge charge and its associated gauge boson that mediates the unified force between superstrings.

The group E_6 contains the exceptional subgroup F_4 with dimension 52. It has four simple roots and 48 roots. As the first six enfolded polygons have 24 corners outside their shared root edge, the counterpart of these 48 roots are the ($24+24=48$) black yods at the corners of the two sets of the first (6+6) enfolded polygons that are outside their root edges. The two separate dodecagons have ($12+12=24$) corners. We see that the 72 corners divide naturally into three groups of 24 corners. Do the 168 red hexagonal yods also split up naturally into seven groups of 24? Indeed, they do! Below are shown the numbers of hexagonal yods on the sides of one set of seven enfolded polygons:

triangle	square	pentagon	hexagon	octagon	decagon	dodecagon
4	6	8	10	14	18	24

THE SUPERSTRING IS THE MICRO-PHYSICAL
MANIFESTATION OF THE TREE OF LIFE BLUEPRINT

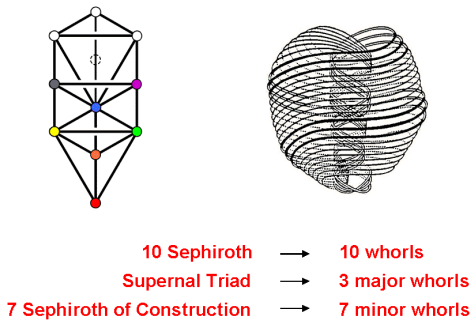


Figure 2

(the root edge must be associated with the dodecagon because this then generates its 24 hexagonal yods). The 18 hexagonal yods on the sides of the decagon can only be grouped with the six hexagonal yods of the square in order to form a set of 24, the 14 hexagonal yods of the octagon can only be grouped with the ten hexagonal yods of the hexagon and the eight hexagonal yods of the pentagon can only be grouped with the four hexagonal yods of the remaining triangle to generate 24 hexagonal yods in both pairs. Hence, there are two sets of 24 (dodecagons), two sets of 24 (decagon & square), two sets of 24 (octagon & hexagon) and one set of 24 (triangle & pentagon). No other combinations are possible. Amazingly, both the 72 corners and the 168 hexagonal yods group naturally into ten sets of 24 yods. The significance of this for superstrings is as follows: in some of

his research articles on his website, the author identified the UPA described by Besant & Leadbeater as the subquark state of the $E_8 \times E_8$ heterotic superstring. Its ten whorls "carry" the 240 gauge charges of E_8 corresponding to the 240 roots of this group. In the spirit of heterotic string theory, 24 gauge charges are regarded as spread out along each whorl. The three major whorls carry 72 gauge charges and the seven minor whorls carry 168 gauge charges. This 72:168 division is *precisely* what we see in the distinction between the 72 corners and the 168 hexagonal yods on the sides of the two sets of seven enfolded polygons! The corners of a tetractys are analogous to the three major whorls, which are the micro-physical manifestation of the three Sephiroth of the Supernal Triad, and its hexagonal yods correspond to the seven minor whorls, which are the micro-physical manifestation of the seven Sephiroth of Construction (Fig. 2). Remarkably, this analogy extends even to *individual* whorls, for the ten sets of 24 gauge charges of E_8 carried by the ten whorls correspond to the ten sets of corners and hexagonal yods.

The counterpart in the first six enfolded polygons (which constitute a holistic set in themselves) of each set of 24 gauge charges are the 24 intrinsic corners associated^{*} with them. The 60 polygons enfolded in ten overlapping Trees of Life, each mapping a whorl, have 240 associated corners that correspond to the

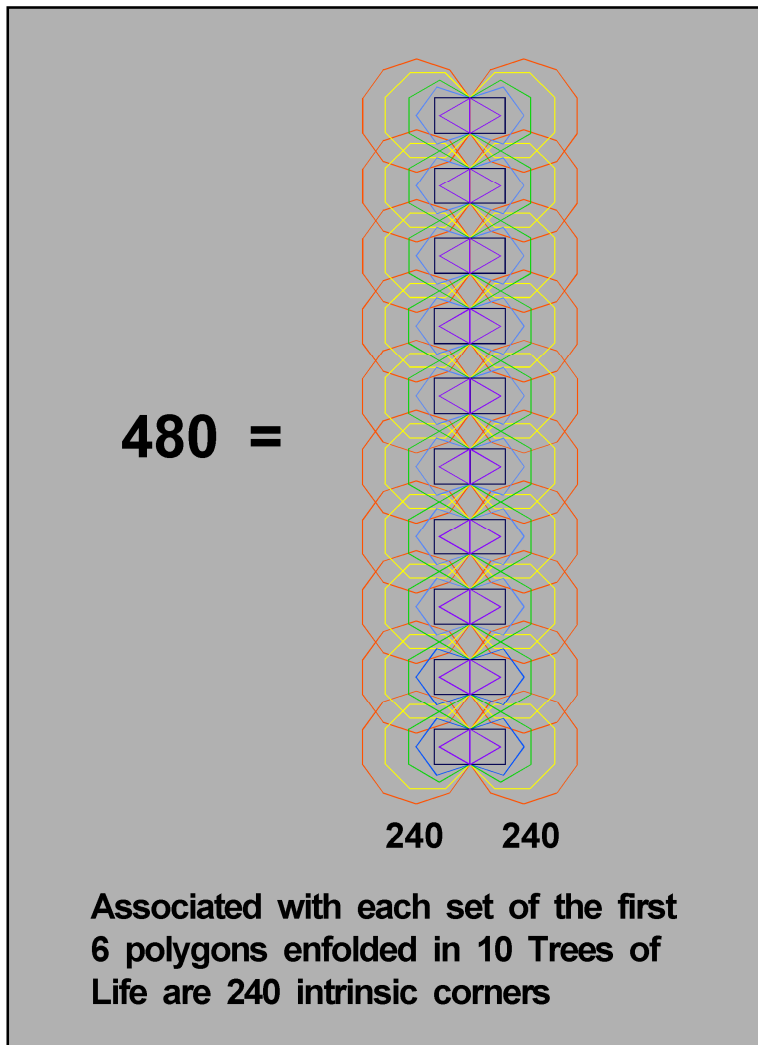


Figure 3. The (240+240) intrinsic corners of the first (6+6) polygons enfolded in 10 overlapping Trees of Life denote the (240+240) roots of $E_8 \times E_8$. Each Tree represents one of the 10 whorls of the subquark state of the $E_8 \times E_8$ heterotic superstring.

240 roots of E_8 and their associated gauge charges (Fig. 3). The (240+240) intrinsic corners associated with both sets of 60 polygons enfolded in ten overlapping Trees of Life denote the (240+240=480) roots of $E_8 \times E_8$, i.e., the mirror symmetry of the inner Tree of Life is responsible for the direct product nature of the anomaly-free gauge symmetry group of the unified force between $E_8 \times E_8$ heterotic superstrings.

^{*} The two endpoints of the root edge are shared by both sets of seven polygons. One can be associated with one set and one with the other set. The topmost corner of each hexagon coincides with the lowest corner of a hexagon enfolded in the next higher Tree. Both sets of the first six polygons have 50 corners. 25 of them are associated with each set but only 24 belong exclusively to each one in the sense of being unshared with polygons enfolded in the next higher Tree. Only 24 corners are *intrinsic* to a set.

$$\text{As } 11^2 - 1 = 120 = \begin{array}{c} 3 \\ 5 \quad 7 \\ 9 \quad 11 \quad 13 \\ 15 \quad 17 \quad 19 \quad 21 \end{array},$$

i.e., as 120 is the sum of the first ten odd integers after 1, the number 240 is the sum of the ten even integers that start with 6 and are four units apart:

$$240 = \begin{array}{c} 6 \\ 10 \quad 14 \\ 18 \quad 22 \quad 26 \\ 30 \quad 34 \quad 38 \quad 42 \end{array}.$$

In terms of the numbers 1, 2, 3 & 4 symbolized by the four rows of dots in the tetractys, the factorization of 240 as 10×24 is expressed as $(1+2+3+4) \times 1 \times 2 \times 3 \times 4$.

As $5^2 - 1 = 24 = 3 + 5 + 7 + 9$, the number 240 is the sum of the tetractys arrays of the first four odd integers after 1:

$$240 = \begin{array}{c} 3 \\ 3 \quad 3 \\ 3 \quad 3 \quad 3 \\ 3 \quad 3 \quad 3 \quad 3 \end{array} + \begin{array}{c} 5 \\ 5 \quad 5 \\ 5 \quad 5 \quad 5 \\ 5 \quad 5 \quad 5 \quad 5 \end{array} + \begin{array}{c} 7 \\ 7 \quad 7 \\ 7 \quad 7 \quad 7 \\ 7 \quad 7 \quad 7 \quad 7 \end{array} + \begin{array}{c} 9 \\ 9 \quad 9 \\ 9 \quad 9 \quad 9 \\ 9 \quad 9 \quad 9 \quad 9 \end{array}.$$

The sum of the 12 odd integers at the corners of these tetractys is **72** and the sum of the remaining 28 odd integers is **168**. This is the arithmetic counterpart of the **72** corners and **168** hexagonal yods that line the two sets of the seven enfolded polygons. A beautiful harmony always exists between number and geometry when the latter is *sacred*. At the end of this article, it is proved that the number 16800 — the number of 1st-order spirillae in the UPA/superstring — is the sum of 240 odd integers.

The 10-fold UPA/superstring as the 10-fold division of the two Type A dodecagons

As **168** extra yods are needed to construct the Type A dodecagon from tetractyses, i.e., it embodies the very number that determines the structure of the UPA/heterotic superstring, it should come as no surprise that the geometry of the dodecagon also embodies the 10-fold pattern of this particle, as now proved.

The pair of Type A dodecagons (Fig. 4) has 24 corners, 24 red sides & 24 blue, radial sides, i.e., three sets of 24 geometrical elements. Each sector is divided into three triangles with three green sides and one internal corner (apart from the shared centre of the dodecagon). Hence, the 24 sectors are, additionally, composed of 24 internal corners, three sets of 24 green sides and three sets of 24 triangles, i.e., seven sets of 24 geometrical elements. The two Type A dodecagons, therefore, naturally comprise *ten* sets of 24 geometrical elements.

These sets are the counterpart of the ten whorls of the UPA/heterotic superstring, each of which carries 24 gauge charges of the superstring gauge symmetry group E_8 . The three sets of 24 corners & sides that create the 24 sectors of the two dodecagons correspond to the three major whorls of the UPA, each carrying 24 E_8 gauge charges. The seven extra sets of 24 geometrical elements needed to turn the dodecagon into a Type A dodecagon correspond to the seven minor whorls, each carrying 24 E_8 gauge charges. We saw previously that the 120 yods lining the 42 sides of the seven enfolded polygons divide into ten sets of 12, further divided into three sets (**36** corners) and seven sets (**168** hexagonal yods). Each yod corresponds to one of the geometrical elements making up the Type A dodecagon.

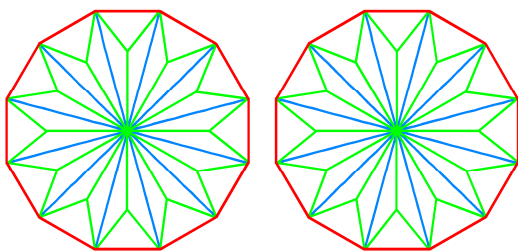
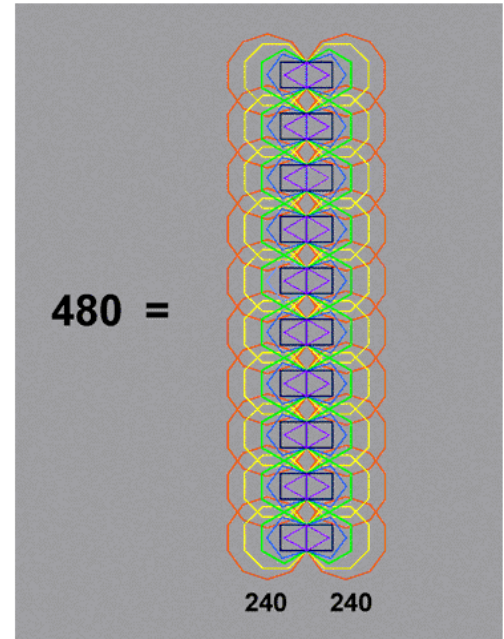
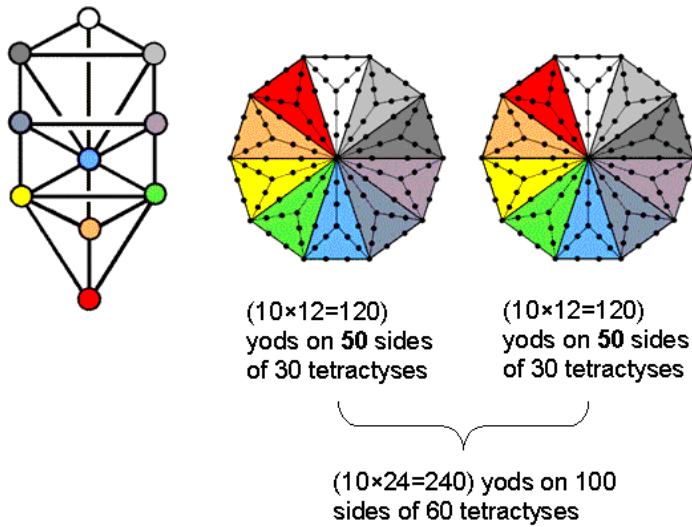


Figure 4. The two Type A dodecagons.

The 10-fold UPA/superstring as the 10-fold division in the yods on edges of tetractyses in the pair of Type A decagons

The sixth regular polygon in the inner form of the Tree of Life is the decagon. Constructed from tetractyses, it is the polygonal representation of ten overlapping Trees of Life because its 61 yods correspond to the 61 SLs up to Chesed of the tenth Tree. Each sector is the counterpart of a Tree, the six yods per sector being the counterparts of the six SLs per Tree. The centre of the Type A decagon is surrounded by 120 yods that line the **50** sides of its 30 tetractyses. There are 12 boundary yods per sector. The two Type A decagons have $(120+120=240)$ yods surrounding their centres that line the 100



The 240 yods lying on the sides of the tetractyses in the two Type A decagons consist of 10 sets of 24

The 240 corners associated with the first 6 polygons enfolded in 10 Trees of Life consist of 10 sets of 24

Figure 5. The 240 yods on sides of tetractyses in the two Type A decagons correspond to the 240 corners associated with the first six polygons enfolded in ten Trees of Life.

sides of their 60 tetractyses (Fig. 5). They comprise $(12+12=24)$ boundary yods per pair of sectors. Therefore, the 240 boundary yods consist of ten sets of 24 yods. Three pairs of sectors (coloured white, grey & dark grey) have $(3 \times 24 = 72)$ boundary yods. The remaining seven pairs of sectors (coloured the seven colours of a rainbow) have $(7 \times 24 = 168)$ boundary yods. Starting from a point (the centre of a decagon), 240 yods are needed to delineate the tetractyses that make up two Type A decagons, 24 per pair of sectors. This **72:168** pattern is analogous to what was discussed earlier. Instead of ten geometrical elements, each repeated $(12+12)$ times, we now have $(12+12)$ yods that are repeated ten times.

The first six enfolded polygons up to the decagon have **26** corners. One of the corners is an endpoint of the root edge shared not only amongst the six polygons but also with the second set of six polygons — the mirror image of the first set. When sets of polygons enfolded in overlapping Trees of Life are considered, the topmost corner of the hexagon coincides with the lowest corner of the hexagon enfolded in the next higher Tree. This means that 25 corners of a set of six enfolded polygons are intrinsic to them. One corner (an endpoint of the root edge) can be associated with the set and one (the other endpoint) can be associated with the second set. Hence, 24 corners are associated with and intrinsic to the set of six polygons enfolded in each Tree. The 60 polygons of the first six types enfolded in ten overlapping Trees of Life have $(10 \times 24 = 240)$ corners, that is, 24 corners repeated *ten times*. This is the inner Tree of Life basis of the 24 E_8 gauge charges spread along each whorl of a UPA/subquark.

The Decad defines both the decagon with ten sides and the *tenth* polygon — the dodecagon. The pair of either polygon embodies ten sets of 24 geometrical elements or yods. Their subatomic counterparts are the 24 E_8 gauge charges carried by each of the ten whorls of a UPA, which is the subquark state of an $E_8 \times E_8$ heterotic superstring. Here is yet more evidence provided by sacred geometry of the ten-fold nature of the basic unit of matter paranormally described by Annie Besant & C.W. Leadbeater.

The 10-fold UPA/superstring as the 10-fold division in the 240 hexagonal yods in the seven separate polygons of the inner Tree of Life

The seven *separate* polygons of the inner Tree of Life have **48** sectors, where **48** is the number value of *Kokab*, the Mundane Chakra of Hod (see Table 1). As tetractyses, they contain 240 hexagonal yods that naturally group into ten sets of 24, as now explained.

2. The Platonic solids

Constructed from tetractyses, the Platonic solids have three types of hexagonal yods: 1. hexagonal yods at centres of tetractyses; 2. hexagonal yods on edges of solids, and 3. hexagonal yods on sides of tetractyses inside the boundaries of faces. Their numbers are calculated and tabulated below:

Table 2. Hexagonal yod populations in the faces of the five Platonic solids.

Platonic solid	Edges	Faces	Hexagonal yods at centres	Hexagonal yods on edges	Hexagonal yods on interior sides	Total
Tetrahedron	6	4	$4 \times 3 = 12 = 6 + 6$	$6 \times 2 = 12 = 6 + 6$	$4 \times 4 = 24 = 12 + 12$	$48 = 24 + 24$
Octahedron	12	8	$8 \times 3 = 24 = 12 + 12$	$12 \times 2 = 24 = 12 + 12$	$8 \times 6 = 48 = 24 + 24$	$96 = 48 + 48$
Cube	12	6	$6 \times 4 = 24 = 12 + 12$	$12 \times 2 = 24 = 12 + 12$	$6 \times 8 = 48 = 24 + 24$	$96 = 48 + 48$
Subtotal	30	18	$60 = 30 + 30$	$60 = 30 + 30$	$120 = 60 + 60$	$240 = 120 + 120$
Icosahedron	30	20	$20 \times 3 = 60 = 30 + 30$	$30 \times 2 = 60 = 30 + 30$	$20 \times 6 = 120 = 60 + 60$	$240 = 120 + 120$
Subtotal	60	38	$120 = 60 + 60$	$120 = 60 + 60$	$240 = 120 + 120$	$480 = 240 + 240$
Dodecahedron	30	12	$12 \times 5 = 60 = 30 + 30$	$30 \times 2 = 60 = 30 + 30$	$12 \times 10 = 120 = 60 + 60$	$240 = 120 + 120$
Total	90	50	$180 = 90 + 90$	$180 = 90 + 90$	$360 = 180 + 180$	$720 = 360 + 360$

(numbers of yods are further divided into their two halves in order to indicate the numbers of hexagonal yods in each half of the polyhedron). The first three Platonic solids have 240 hexagonal yods comprising 120 hexagonal yods either at centres of tetractyses or on edges and 120 hexagonal yods on interior sides (Fig. 7). This 120:120 division in the 240 hexagonal yods in the 18 faces of the first three Platonic solids re-occurs in both the icosahedron and the dodecahedron because each half-polyhedron has 120

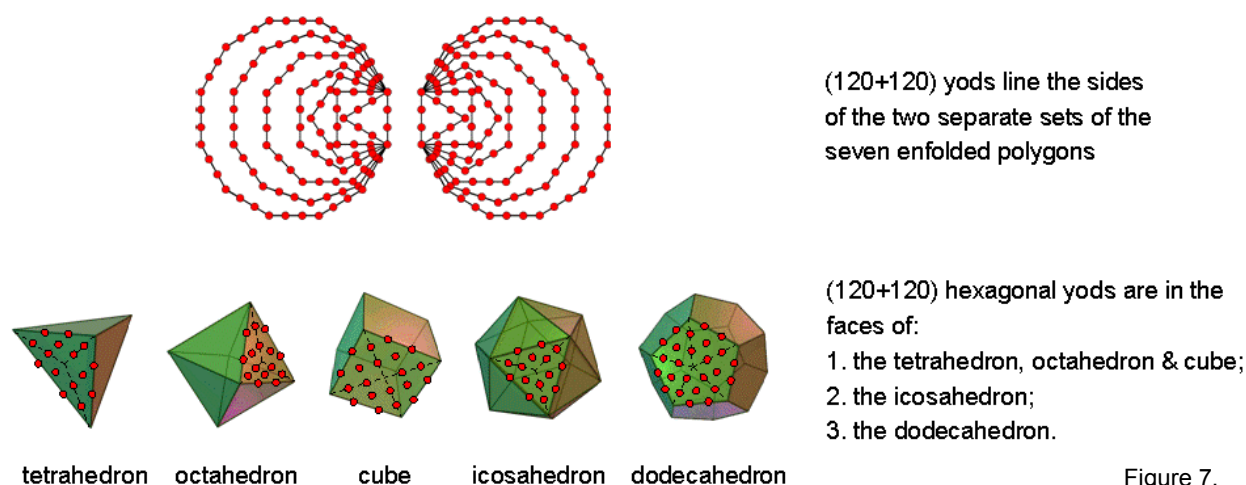


Figure 7.

The correspondence between the two sets of seven enfolded polygons and the Platonic solids

hexagonal yods. As we saw in Section 1, each of the two separate sets of seven enfolded polygons has 120 yods on their boundaries. Both sets have 240 yods on their boundaries that comprise **72** corners and **168** hexagonal yods. We saw there that these 240 boundary yods symbolize the 240 roots of the superstring gauge symmetry group E_8 , the **72** corners denoting the **72** roots of its exceptional subgroup E_6 , leaving **168** roots represented by the **168** hexagonal yods. The table indicates that their counterparts in the first three Platonic solids are the **72** hexagonal yods in one half of the tetrahedron and the octahedron and the **168** hexagonal yods in their other half and in the cube. The **72:168** division in the holistic parameter 240 manifests in the first three Platonic solids. Moreover, just as the **72** corners were found to be composed of three sets of 24 corners of a unique combination of polygons, so the **72** hexagonal yods consist of 24 hexagonal yods in one half of the tetrahedron, 24 hexagonal yods that either line edges of the octahedron or are centres of tetractyses and 24 hexagonal yods on sides of tetractyses inside the faces of the octahedron. Finally, just as the **168** hexagonal yods on the sides of the

two sets of polygons were found earlier to be distributed amongst the polygons in a way that naturally generates seven sets of 24, so, too, Table 2 shows that the **168** hexagonal yods in the other half of the tetrahedron & octahedron and in the cube group naturally into seven sets of 24:

1. 24 (6+6+12) hexagonal yods that are in that half of the tetrahedron;
2. 24 (12+12) hexagonal yods that either are centres of tetractyses or line edges of the other half of the octahedron;
3. 24 hexagonal yods that are sides of tetractyses in that half;
4. 24 (12+12) hexagonal yods at centres of tetractyses in the cube;
5. 24 (12+12) hexagonal yods that line its edges;
6. 24 hexagonal yods lining sides of tetractyses in one half of the cube, and
7. 24 hexagonal that line sides in the other half of the cube.

(the red numbers are those that appear as such in the table). The mathematical analogy between the two systems of sacred geometries exists because one encodes in the yods shaping the two-dimensional, regular polygons the same pattern as that what exists in the faces of the three-dimensional, regular polyhedra. As with the two sets of seven enfolded polygons, the 240 hexagonal yods in the 18 faces of the tetrahedron, octahedron & cube can be grouped into *ten* sets of 24. Here, therefore, are two sacred geometries that encode in analogous ways the 10-fold structure of the UPA as ten whorls, each carrying 24 gauge charges of E_8 . **The importance of this discovery for the future development of M-theory cannot be emphasized too strongly.** This as yet undiscovered 10-fold composition of the superstring is the manifestation in the subatomic world of the 10-fold nature of God, as represented by both the Tree of Life and Pythagoras' tetractys. Predicting this 10-foldness will be an authentic sign of the validity of any future candidate for the much-sought M-theory that unifies the five superstring theories with General Relativity.

The 240 hexagonal yods repeated in the fourth Platonic solid symbolize the 240 roots of the second, similar Lie group E_8' that is part of the gauge symmetry group $E_8 \times E_8'$ governing the unified force between one of the two types of heterotic superstrings. The first four Platonic solids embody not only the dimension **248** of E_8 as the **248** corners & sides of the sectors of its faces (see p. 12, Article 3) but also the numbers of roots of E_8 and $E_8 \times E_8'$. It has been proved on page 5 at <http://smphillips.8m.com/platonic-solid.html> that 1680 geometrical elements surround their axes when their faces are divided into their sectors and the face centres are joined to the centres of the solids. Here is irrefutable evidence that the UPA paranormally described over a century ago by Besant & Leadbeater is an $E_8 \times E_8'$ heterotic superstring, for both the basic structural parameter of the UPA and the root composition of this symmetry group are embodied in the first four Platonic solids that is so uncontrived that it is highly implausible that the presence of both numbers is mere coincidence.

It was pointed out in Section 1 that the **48** corners outside the root edge of the two separate sets of the first (6+6) enfolded polygons denote the **48** roots of F_4 , the rank-4 exceptional subgroup of E_6 whose **72** roots are denoted by the **72** corners of the two separate sets of the (7+7) enfolded polygons. What is their counterpart in the first three Platonic solids? Table 2 above indicates that the **72** hexagonal yods in the six faces making up half the tetrahedron and half the octahedron are:

	Hexagonal yods at centres	Hexagonal yods on edges	Hexagonal yods on interior sides	Total
Tetrahedron:	6	6	12	24
Octahedron:	12	12	24	48

The **48** hexagonal yods in the four faces in a half of the octahedron denote the **48** roots of F_4 .

In Section 4, we shall discuss the properties of the disdyakis triacontahedron, which is the 120-face Cartan solid that was shown in Article 23 to be the outer form of the polyhedral version of the Tree of Life. We shall find that it embodies both these properties in the same way that the first four Platonic solids do, namely, 1680 geometrical elements surround an axis of the disdyakis triacontahedron, whilst 240 hexagonal yods are in the 60 faces making up each half of it when they are turned into tetractyses. Like these Platonic solids, the disdyakis triacontahedron also embodies the number **248** (see Article 44). The re-appearance of the paranormally-obtained structural parameter of the superstring in a polyhedron that Article 23 identified as the outer form of the "Polyhedral Tree of Life" cannot be due to chance. Instead, it confirms the archetypal nature of this number, whose scientific significance awaits discovery by theoretical physicists.

3. The 2-d Sri Yantra

The geometrical make-up of the 2-dimensional Sri Yantra (Fig. 8) is tabulated below:

Table 3. Geometrical composition of the 2-dimensional Sri Yantra.

	Number of vertices	Number of sides	Number of triangles	Total
Central 'triangle'	1	3	0	4
1st group of triangles	$4 + 8 = 12$	$8 \times 3 = 24$	8	44
2nd group of triangles	$6 + 10 = 16$	$10 \times 3 = 30$	10	56
3rd group of triangles	$2 + 10 = 12$	$10 \times 3 = 30$	10	52
4th group of triangles	28	$14 \times 3 = 42$	14	84
Total	69	129	42	240

Mathematically speaking, the central, downward-pointing triangle does not count as a true, triangular area (hence the two inverted commas enclosing the word 'triangle' in the first row of the table) because the bindu at the centre of the 2-dimensional Sri Yantra is a point that lies *in* its plane, so that this triangle is

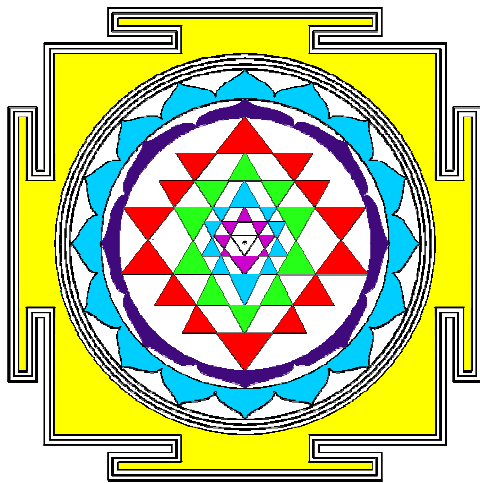


Figure 8. The Sri Yantra.

really only three lines and three points, the upper two of which coincide with corners of triangles belonging to the first group. The 42 triangles surrounding the centre have 126 sides, i.e., they comprise **168** lines & triangles. Including the lowest corner of the central 'triangle,' 69 points surround the centre of the 2-dimensional Sri Yantra. Three straight lines create the shape of the innermost 'triangle.' Hence, the 240 points, lines & triangles that surround its centre comprise ($69+3=72$) points & lines and **168** lines & triangles. This is the counterpart of the similar **72:168** division manifested by the two separate sets of seven enfolded polygons making up the inner Tree of Life and by the first three Platonic solids. We shall see in the next section that it also appears in the disdyakis triacontahedron — the outer form of the polyhedral Tree of Life. It is the counterpart of the **72** roots of E_6 and the remaining **168** roots of E_8 . Notice that the **168** lines & triangles comprise the 42 lines in the fourth group of triangles and 42 triangles, i.e., 84 lines & triangles, as well as 84 lines in the first three groups. Alternatively, the four groups of triangles in each half of the Sri Yantra comprise 63

sides and **21** triangles, i.e., 84 sides & triangles. This 84:84 division is characteristic of sacred geometries, e.g., in the case of the two sets of seven enfolded polygons discussed in Section 1, there are 84 hexagonal yods on the sides of the polygons in each set.

The first & third groups of triangles have ($12+12=24$) vertices. The central 'triangle' comprises one unshared vertex and three straight lines. They group into two sets: one that is made up of this vertex and the straight line opposite it and one that comprises the two lines that meet at this point. The second & fourth groups of triangles have ($16+28=44$) vertices, i.e., 22 vertices in each half of the Sri Yantra, making two sets of 22 vertices. Combining them with the set of point & line and with the set of two lines creates two sets of 24 vertices & sides. Hence, the **72** points & lines discussed in the previous paragraph divide into three sets of 24:

1. 1st & 3rd groups have 24 vertices;
2. Central 'triangle,' 2nd & 4th groups have 23 vertices & one side;
3. Central 'triangle,' 2nd & 4th groups have 22 vertices & two sides.

The third group of triangles has ten sets of three sides and the fourth group has 14 sets of three sides. Together, they form three sets of ($10+14=24$) sides and one set of ($10+14=24$) triangles. The first and second groups in the Sri Yantra have ($8+10=18$) triangles, i.e., nine in each half. The second group has **15** sides in each half. There are therefore ($9+15=24$) sides & triangles in each half of the first and second groups, as well as a set of 24 sides in the former group. The **168** sides & triangles in the four groups of triangles divide into seven sets of 24:

1. 3rd & 4th groups have set of 24 sides and one set of 24 triangles;
2. 3rd & 4th groups have set of 24 sides;
3. 3rd & 4th groups have set of 24 sides;
4. 3rd & 4th groups have set of 24 triangles;
5. 1st & 2nd groups have 24 sides & triangles in one half;
6. 1st & 2nd groups have 24 sides & triangles in other half;

7. 1st group has 24 sides.

In conclusion, the 240 geometrical elements surrounding the central bindu in the 2-dimensional Sri Yantra consist of ten sets of 24 elements that divide into three sets of vertices & lines and into seven sets of sides & triangles. They are the counterpart of the three major whorls and seven minor whorls of the UPA/subquark.

4. The disdyakis triacontahedron

As the outer form of the Polyhedral Tree of Life (see Article 23), the disdyakis triacontahedron is a Catalan solid[†] with **62** vertices, 180 edges & 120 triangular faces. The vertices comprise 30 A vertices, 12 B vertices & 20 C vertices (Fig. 9). The 180 edges consist of 60 AB, 60 BC & 60 AC edges. When its axis is a straight line passing through two diametrically opposite A vertices, its central 12-gon is formed by four A, four B & four C vertices. Four B vertices are above this plane and four B vertices are below it. Hence, there are three sets of four B vertices. Figure 8 shows that the 12 B vertices are those of an icosahedron

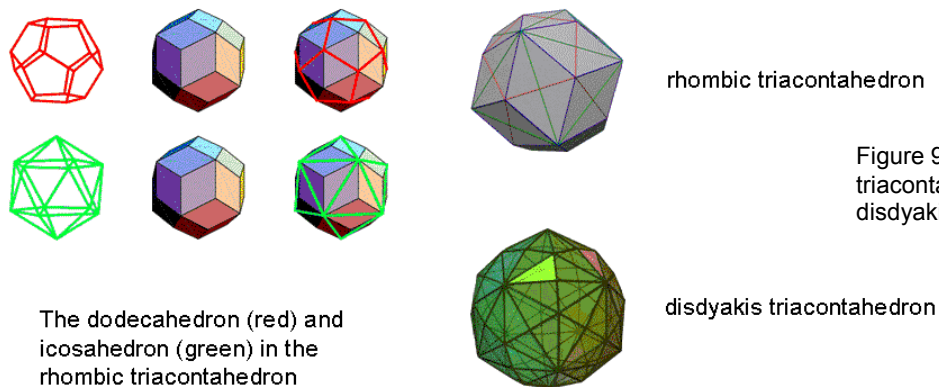


Figure 9. The rhombic triacontahedron & the disdyakis triacontahedron

with 30 green edges, whilst the 20 C vertices are those of a dodecahedron with 30 red edges. The 32 B & C vertices forming a rhombic triacontahedron with 30 Golden Rhombic faces. Three AC edges (raised, red half-edges of the dodecahedron) are associated with each of the 20 icosahedral triangular faces with three green sides. Hence, the 60 AC edges are composed of three sets of 20 AC edges. Adding them to the three sets of B vertices generates three sets of 24 geometrical elements (four B vertices & 20 AC edges):

1. Four B vertices in upper half & AC edge associated with 20 icosahedral faces;
2. Four B vertices in central 12-gon & AC edge associated with 20 icosahedral faces;
3. Four B vertices in lower half & AC edge associated with 20 icosahedral faces.

Associated with each of the 12 faces of the underlying red dodecahedron are five green half-edges, which become raised into five AB edges, and five blue edges, which are BC edges (see the rhombic triacontahedron). Hence, there are 12 sets of five AB edges and 12 sets of five BC edges. This may be seen as five sets of 12 AB edges and five sets of 12 BC edges, i.e., as five sets of 24 edges (12AB & 12BC).

The 20 C vertices consist of 10 C vertices in one half of the polyhedron and 10 C vertices in its other half. The 28 A vertices surrounding the axis passing through two opposite A vertices comprise 14 A vertices in each half. Hence, (10+14=24) A & C vertices surround the axis in each half, i.e., there are two sets of 24 A & C vertices. There are, therefore, seven more sets of 24 geometrical elements: two sets of 24 A & C vertices and five sets of 12 AB & 12 BC edges.

Surrounding the A-A axis of the disdyakis triacontahedron are (60+180=240) vertices & edges. They consist of **72** B vertices & AC edges grouped into three sets of 24 and **168** vertices & edges grouped into seven sets of 24 (two sets of 24 vertices and five sets of edges). In other words, the 240 vertices & edges form ten sets of 24 that divide into groups of **72** and **168**. This is the same pattern as found earlier for the inner Tree of Life, the two Type A dodecagons, the two Type A decagons, the first three Platonic solids and the 2-dimensional Sri Yantra. It is intrinsic to *all* sacred geometries because they embody the root composition of the superstring gauge symmetry group E_8 and its subgroups, in particular E_6 . This

[†] Catalan solids are the duals of the 13 different Archimedean solids, created by replacing their vertices with faces and vice versa.

fundamental 10-fold division manifests in the subquark state of the $E_8 \times E_8$ heterotic superstring as the ten whorls of the UPA described by Besant & Leadbeater. The symmetry breakdown from E_8 to E_6 appears in the difference that Leadbeater reported between the major and minor whorls, along which are spread, respectively, the **72** E_8 gauge charges of E_6 and the **168** remaining gauge charges of E_8 . The major whorls are made thicker by the inclusion of an extra $(n+1)$ th-order spirilla in every 25th spirillae, so that, instead of the latter being composed of 175 $(n+1)$ th-order spirillae (as in the minor whorls), in a major whorl they contain 176 $(n+1)$ th-order spirillae, where $n = 1-6$.

10-fold geometrical composition of the disdyakis triacontahedron

Article 27 showed that, when both the internal triangles and the faces of the disdyakis triacontahedron are Type A triangles, 2400 corners, sides & triangles surround any axis passing through two diametrically opposite vertices (see Table 1, p. 4). It will now be proved that the 10-fold division of vertices & edges found above for this polyhedron extends to its *total geometrical composition*. In other words, the 2400 geometrical elements divide *naturally* into ten sets of 240 elements that divide further into three sets of

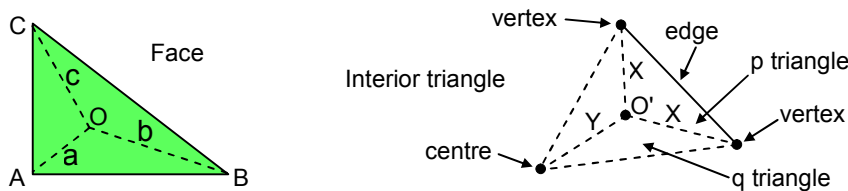


Figure 10. The sectors of a face and an internal triangle.

240 and seven sets of 240. The meaning of this 10-fold division is that, associated with the 240 roots of E_8 are 240 gauge fields, each with ten components, so that 240 components of these fields are defined by each of the 10 dimensions of superstring space-time. Each geometrical element making up the disdyakis triacontahedron that surrounds its axis therefore bears a formal correspondence to a component of the 240 gauge fields of E_8 .

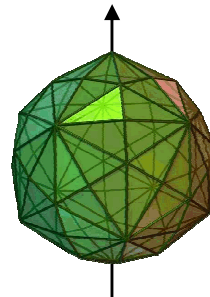
Figure 10 shows one of the 120 faces with centre O. a, b & c are the internal sides of its three sectors that terminate, respectively, on A, B & C vertices. It also depicts an internal triangle with polyhedral vertices at two corners and the centre of the polyhedron at its third corner. The three sides meeting at O' are of two types: one (X) that terminates on a vertex and one (Y) that ends at the centre. The three sectors of an internal triangle are of two types: one (p) that has a polyhedral edge as one side and two (q) that do not have an edge as a side. The edges are of three types: AB, BC & AC. As the disdyakis triacontahedron has 180 edges, there are 60 edges of each type, each one generating in an internal triangle two X sides & one Y side, one triangle of class p & two triangles of class q, as well as a centre O'. Table 4 shows the numbers of geometrical elements in the faces and interior of the polyhedron that surround its axis:

Table 4. Geometrical composition of the disdyakis triacontahedron.

	Corners	Sides	Triangles
Faces	120	$120a + 120b + 120c = 3 \times 120$	3x120
	60	AB: 60 BC: 60 AC: 60	
Interior		60	
	60	60X + 120Y	60p + 120q
	60	60X + 120Y	60p + 120q
	60	60X + 120Y	60p + 120q

In the faces are three sets of 120 sides and three sets of 120 triangles, i.e., three sets of 240 sides & triangles, or 720 (=72x10) geometrical elements. There are also 120 corners that are centres of faces and 60 vertices, as well as 60 sides joining the latter to the centre of the polyhedron. These form a set of 240 corners & sides. There are three sets of 120 Y sides & 120 q triangles (indicated in orange, yellow & green), i.e., three sets of 240 geometrical elements, and there are three sets of 60 edges, 60 internal corners, 60 internal sides & 60 internal triangles (indicated in blue, indigo & violet), i.e., three sets of 240 elements. Hence, there are seven more sets of 240 geometrical elements.

We see that the 2400 geometrical elements comprise ten sets of 240 that are made up of three sets in the faces and seven sets that comprise two triplets of sets and a single set. This 3:3:3:1 pattern reproduces the pattern of Sephiroth of the Tree of Life:

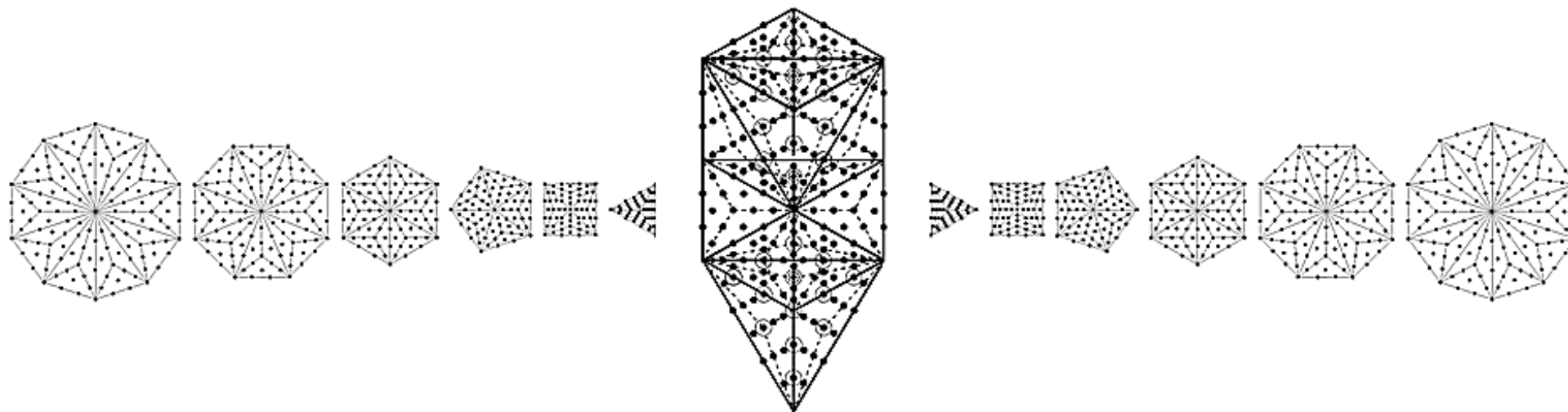


Disdyakis triacontahedron

720 geometrical elements in the upper half of the disdyakis triacontahedron surround its axis

240 vertices & edges surround the axis

720 geometrical elements in the lower half of the disdyakis triacontahedron surround its axis



720 yods surround centres

240 yods other than Sephiroth

720 yods surround centres

Figure 11. The 720:240:720 pattern of yods in the two sets of seven polygons and the 1-tree corresponds to the pattern of distribution of the 1680 geometrical elements that surround an axis of the disdyakis triacontahedron.

Kether-Chokmah-Binah
 Chesed-Geburah-Tiphareth
 Netzach-Hod-Yesod
 Malkuth

There are 720 elements in the faces that are unshared with the remaining 1680 elements. This pattern is the counterpart of the **72** roots of E_6 and the **168** other roots of E_8 . The 1680 elements comprise 60 vertices & 180 edges, i.e., 240 elements in the faces, leaving 1440 elements in the interior, 720 elements being in each half of the disdyakis triacontahedron. The division:

$$1680 = 720 + 240 + 720$$

Is embodied in the 240 yods other than SLs in the 1-tree and the 720 yods that surround the centres of the two sets of seven Type B polygons of the inner Tree of Life (Fig. 11). Every yod or geometrical element corresponds to a circular turn of each helical whorl of the UPA/subquark superstring.

5. The 1-tree

The 10-fold UPA/superstring as the 10-fold division in the 1-tree

When its 19 triangles are turned into Type A triangles, i.e., when each triangle is divided into its three sectors which are then changed into tetractyses, the 1-tree contains 251 yods (Fig. 12). Their 11 corners are “Sephirothic levels” (SLs), as Daath is Yesod of the next higher Tree. Hence, the transformation *generates* (251-11=240) yods that belong to (19×3=57) tetractyses. The 19 triangles have 25 sides. Inside them are (19×3 = 57) sides of tetractyses. (251-57=194) yods are on the (25+57=82) sides of these tetractyses, where **194** is the number value of *Tzadekh*, the Mundane Chakra of Chesed (see Table 1). The **194** yods comprise (2×82=164) hexagonal yods and (11+19=30) corners. The 1-tree contains (251-30=221) hexagonal yods.

The 1-tree is the lowest Tree of a set of overlapping Trees of Life. Below its apex (Kether) and outside the 1-tree on each side are four red hexagonal yods (Fig. 13). One is at the centre of a tetractys, two are on a side of this tetractys and another is on the vertical side of another tetractys. These eight hexagonal yods belonging to the next higher Tree of Life consist of two that are at centres of two tetractyses and six that

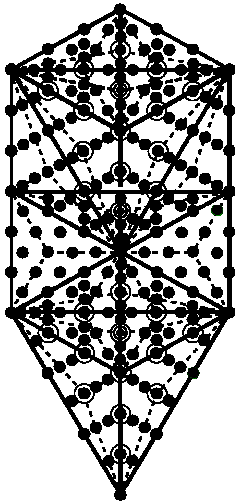
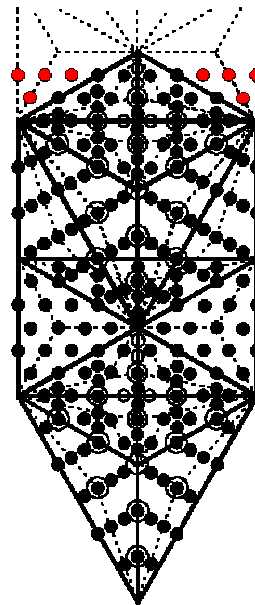


Figure 12. The 251 yods in the 1-Tree.



= 248

240(●) yods → 240 non-zero roots of E_8

8 (●) yods → 8 zero roots of E_8

Figure 13. Below the apex of the 1-tree are **248** yods other than SLs. They correspond to the **248** roots of the superstring gauge symmetry group E_8 and to the **248** gauge bosons that transmit the unified superstring force.

are on four sides of four tetractyses. Below the apex of the 1-tree are therefore eight red, hexagonal yods that belong to the second Tree and 240 black yods other than Sephiroth that belong to the lowest Tree,

that is, **248** yods other than Sephiroth, where **248** is the number value of *Raziel*, the Archangel of Chokmah. The eight red hexagonal yods outside the 1-tree symbolize the eight simple roots of E_8 , the rank-8, exceptional Lie group with dimension **248**; the 240 black yods denote its 240 roots. As the latter include 221 hexagonal yods in the 1-tree, there are $(8+221=229)$ hexagonal yods in the **248** yods that are below the top of the 1-tree. 229 is the **50th** prime number, showing how ELOHIM, the Godname of Binah

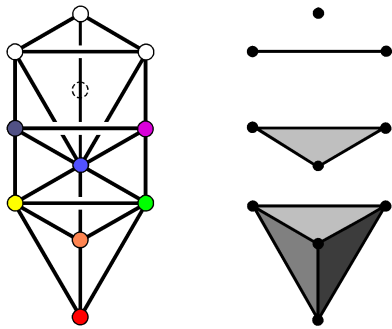


Figure 14. The Tree of Life & its trunk.

with number value **50**, prescribes the number of hexagonal yods below the top of the lowest Tree of Life. As the eight hexagonal yods outside it belong to four tetractyses, the **248** yods belong to $(57+4=61)$ tetractyses, where 61 is the **31st** odd integer and **31** is the number value of EL, the Godname of Chesed. They comprise 19 corners, $(57+2=59)$ hexagonal yods at centres of 61 tetractyses & $(164+6=170)$ hexagonal yods on their $(82+4=86)$ sides.

The trunk of the Tree of Life (Fig. 14) consists of the mathematical sequence of the point (Kether), straight line (Chokmah-Binah Path), triangle (corners at Chesed, Geburah & Tiphareth) & tetrahedron (vertices at Netzach, Hod, Yesod & Malkuth). It expresses the emergence of 3-dimensional form from the dimensionless point. The ten points in space needed to achieve this represent the integers 1, 2, 3 & 4, which are also symbolized by the four rows of the Pythagorean tetractys. The trunk is composed of **26** geometrical elements (ten points, ten lines, five triangles & one tetrahedron), where **26** is the number value of YAHWEH, the Godname of Chokmah (see Table 1). When these triangles are Type A triangles, the trunk has **15** corners of **15** triangular sectors with 25 sides, i.e., 55 geometrical elements, where 55 is the *tenth* triangular number:

$$55 = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$$

and the *tenth* Fibonacci number. This demonstrates how the Decad (10) arithmetically determines the geometrical composition of the trunk of the Tree of Life that represents this number. Table 5 lists the numbers of yods in the trunk other than those coinciding with Sephiroth:

Table 5. Non-Sephirothic yod population of trunk of Tree of Life.

	Corners	Hexagonal yods at centres	Hexagonal yods on sides	Total
Point	0	0	0	0
Line	0	0	2	2
Triangle	1	3	12	4 + 12
Tetrahedron	4	12	12 + 24 = 36	4 + (12+12=24) + 24
Total	5	15	50	$(2+4+4=10) + 12 + 24 + 24 = 70$

ADONAI, the Godname of Malkuth with number value **65**, prescribes the trunk because its **15** tetractyses have **65** hexagonal yods. ELOHIM, the Godname of Binah with number value **50**, shapes the trunk because **50** hexagonal yods line its 25 sides. The branches of the 1-tree are all those parts of it other than its trunk. They have $(240-70=170)$ yods. As the 240 yods comprise 19 corners of 57 tetractyses & 221 hexagonal yods, the branches contain $(19-5=14)$ corners & $(221-65=156)$ hexagonal yods in $(57-15=42)$ tetractyses, i.e., $(14+156=170)$ yods. The distinction between the trunk & branches of the 1-tree therefore generates the division: $240 = 70 + 170$, where $70 = 5$ corners + **65** hexagonal yods and $170 = 14$ corners + 156 hexagonal yods. The 240 yods include $(19+57=76)$ corners & centres of tetractyses, where **76** is the number value of YAHWEH ELOHIM, the Godname of Tiphareth.

According to Table 5, the numbers of yods in the trunk are:

$$70 = 10 + 12 + 24 + 24.$$

Suppose that we add the two hexagonal yods on the Path joining Daath and Tiphareth. The projection of this line onto the plane of the $(7+7)$ enfolded polygons making up the inner Tree of Life is the side that they all share — what we have called the "root edge." This Path is its counterpart in the outer Tree of Life. It is what may be called the "root" of the Tree of Life. Its trunk and root constitute a set of **72** yods, where

$$72 = (10+2) + 12 + 24 + 24 = (12+12) + 24 + 24 = 24 + 24 + 24.$$

The **72** yods consist of *three sets of 24 yods* (Fig. 15). This leaves $(170-2=168)$ yods in the branches made up of 14 corners and 154 hexagonal yods in 14 Type A triangles with two hexagonal yods on each of their 14 sides (there are actually **15** sides but one is the line between Daath & Tiphareth, the two

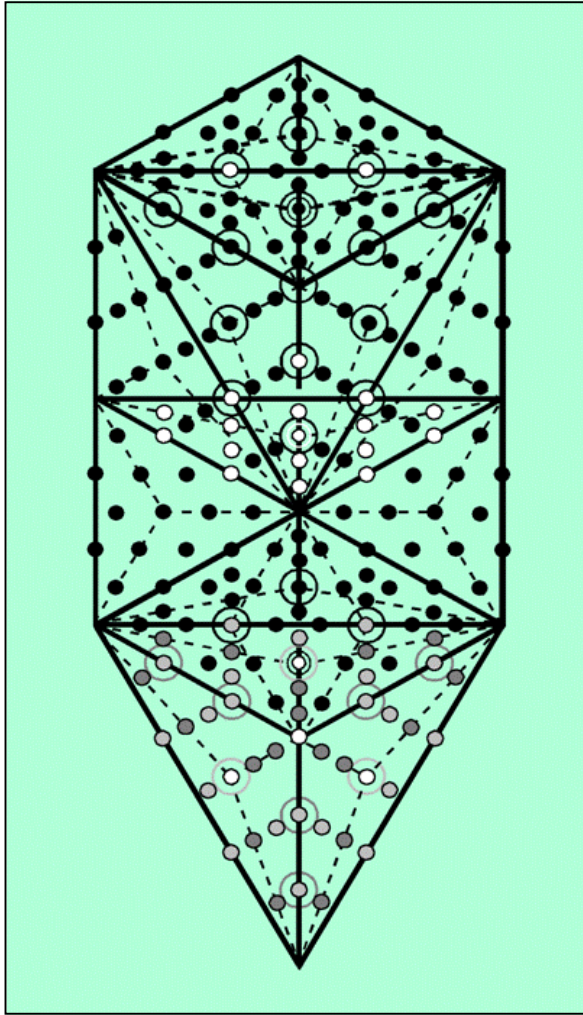
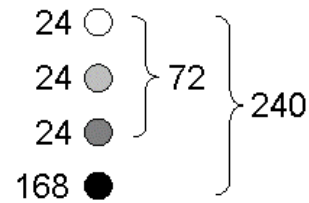


Figure 15. The 1-tree embodies the 240 roots of the superstring gauge symmetry E_8 . Circles represent yods that are behind other yods.



**The trunk and root edge of the 1-tree has 72 yods;
its branches have 168 yods. They symbolize the
240 roots of the rank-8 Lie group E_8 .**

hexagonal yods on which have been added to the 70 yods in the trunk). There are nine hexagonal yods inside each Type A triangle, so that the **168** yods comprise 14 corners, 14 sets of nine hexagonal yods and 14 pairs of hexagonal yods, i.e., 14 sets of 12 yods, or *seven sets of 24 yods*. Each set of 24 yods comprises two centres of triangles of the 1-tree, four hexagonal yods on two Paths & 18 hexagonal yods inside two triangles. The latter consist of six hexagonal yods at centres of six tetractyses & 12 hexagonal yods on six sides of them, so that the 24 yods comprise two corners of tetractyses, six hexagonal yods at their centres and $(4+12=16)$ hexagonal yods on their sides. This 2:6:16 pattern is identical to what is found for the 24 transverse dimensions of bosonic strings, for they comprise two large-scale dimensions, six dimensions of the compactified space of superstrings and 16 higher dimensions. The following correspondences exist:

1. 2 corners of tetractyses \leftrightarrow 2 large-scale, transverse dimensions;
2. 6 hexagonal yods at centres of tetractyses \leftrightarrow 6 compactified, superstring dimensions;
3. 16 hexagonal yods on sides of tetractyses \leftrightarrow 16 compactified, bosonic string dimensions.

This is not coincidental, because the 240 yods symbolize the 240 roots of E_8 , whose associated gauge charges are spread along the ten whorls of the $E_8 \times E_8$ heterotic superstring, ten to a whorl, and each gauge charge is the result of an oscillation along each of the 24 transverse dimensions of space. Nevertheless, this analogy between the hidden yod composition of the 1-tree and the pattern of dimensions of space predicted by the theories of bosonic strings and superstrings is remarkable.

Notice that the **72** yods, which symbolize the **72** roots of E_6 , the rank-6, exceptional subgroup of E_8 , have

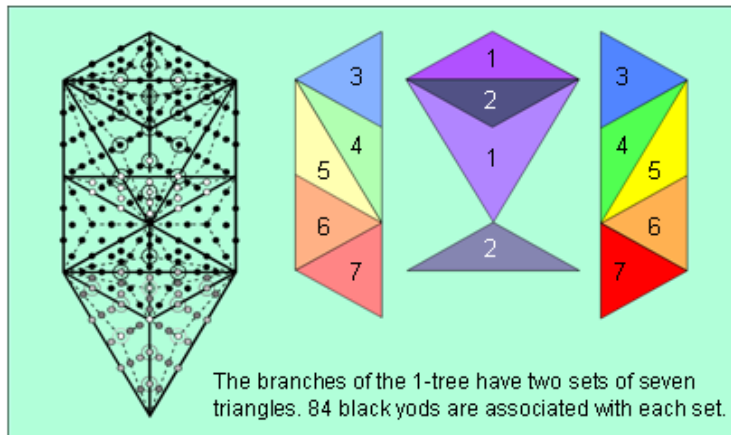


Figure 16. The (7+7) triangles making up the branches of the 1-tree.

24. This 3:7 division is the Tree of Life counterpart of the three major whorls and seven minor whorls of the UPA/subquark state of the $E_8 \times E_8$ heterotic superstring (see Fig. 2). The three major whorls are the manifestation of the root and trunk of the 1-tree and the seven minor whorls are the manifestation of its branches. The factorization: $168 = 7 \times 24$ is made possible by the facts that: 1. the branches of the 1-tree contain 14 triangles, and 2. each side of the 1-tree is the mirror image of the other, so that they consist of two sets of seven triangles. Five triangles (coloured dark or light shades of red, orange, yellow, green & blue) are on each side of the central Pillar of Equilibrium, which is straddled by four triangles coloured light & dark shades of indigo and violet (Fig. 16). The dark shades of these two colours can be associated with one side of the 1-tree and the light shades can be associated with the other side. Therefore, the factor of seven in the factorization arises simply from the fact that seven triangles can be associated with the left-hand side of the 1-tree and seven triangles can be associated with its right-hand side. There are two hexagonal yods on a side shared by two triangles. One hexagonal yod is associated with one triangle and the second one should be regarded as associated with the other triangle. This allows 12 yods to be associated with each triangle. Each set of 24 yods consists of 12 yods associated with a triangle in each side of the 1-tree, so that the **168** yods divide into two sets of 84 yods associated with each half of it. This 84:84 division was encountered in earlier analyses of sacred geometries in this article and has been discussed in previous articles in many different contexts. It manifests in the subquark as the outer and inner halves of each helical whorl of the UPA, each having 840 circular turns that make $2\frac{1}{2}$ revolutions around its axis of spin, that is, **168** turns make a half-revolution and 84 turns make a quarter-revolution. Figure 1 indicates that the **168** roots of E_8 that are not roots of E_6 consist of three sets of 28 (i.e., 84 roots), a set of 56 & another set of 28 (i.e., 84 roots). So these **168** roots divide *naturally* into two sets of 84 roots. Their counterparts in the branches of the 1-tree are the 84 yods that belong to the seven triangles associated with its left-hand or right-hand half.

Four corners of tetractyses, four hexagonal yods at their centres & eight hexagonal yods on their sides (i.e., 16 yods) are aligned with the central Pillar of Equilibrium straddled by the four triangles. Hence, $(168 - 16 = 152)$ yods belong to the branches in both halves of the 1-tree. **76** yods in the branches belong to each half, where **76** is the number value of YAHWEH ELOHIM. The eight hexagonal yods symbolizing the eight simple roots of E_8 consist of four yods *outside* the 1-tree on either side of the Pillar of Equilibrium (see Fig. 13). Outside the trunk are $(76 + 4 = 80)$ yods on either side, where **80** is the number value of Yesod. This is also the number of yods in the trunk when the (10+1) Sephirothic corners of the 19 triangles are included.

The 1-tree consists of 30 corners of 57 triangles with 82 sides, i.e., **112** corners & sides, where **112** is the number value of *Beni Elohim*, the Order of Angels assigned to Hod, and **87** corners & triangles, where **87** is the number value of *Levanah*, the Mundane Chakra of Yesod. The 1-tree consists of 169 points, lines & triangles. In other words, below its apex are **168** geometrical elements. The number value of *Cholem Yesodeth*, the Mundane Chakra of Malkuth, is the number of geometrical elements needed to construct the 1-tree, starting from the mathematical point represented by its apex. Here is yet more, truly amazing evidence that the gematria number values of the Hebrew names of the Sephiroth, their Godnames, etc have a geometrical basis.

Endnotes

Proof that 16800 is the sum of 240 odd integers

Using the identity

been compounded from a set of 70 yods making up the trunk of the 1-tree and the two hexagonal yods of the Daath-Tiphareth Path. Figure 1 indicates that the **72** roots *also* consist of a set of 70 roots and two roots represented by the 8-tuples $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ and $(-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2})$. So the group-theoretical composition of the **72** roots is paralleled by the yod composition of the trunk and the root of the 1-tree.

The 240 yods needed to construct the 1-tree from Type A triangles divide into ten sets of 24 yods, which divide further into three sets of 24 and seven sets of

$$168 = 13^2 - 1,$$

$$16800 = 168 \times 10^2 = (13^2 - 1)10^2 = 130^2 - 10^2 = (2 \times 65)^2 - (2 \times 5)^2 = 4(65^2 - 5^2).$$

$$65^2 = 1 + 3 + 5 + \dots + 129$$

and

$$5^2 = 1 + 3 + 5 + 7 + 9.$$

Therefore,

$$16800 = 4(11 + 13 + 15 + \dots + 129),$$

i.e., 16800 is the sum of $4(65-5) = 4 \times 60 = 240$ odd integers, the largest of which is **129**, the number of YAHWEH SABAOTH, the Godname of Netzach, which is the *fourth* Sephirah from Malkuth (see Table 1).