ARTICLE 48

The Holistic Nature of the First (4+4) Regular Polygons of the Inner Tree of Life

by

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Abstract

As a development of earlier work showing that certain subsets of the (7+7) polygons of the inner form of the Tree of Life have analogous properties to the latter, this article proves that the first (4+4) polygons also constitute such a subset. It analyses their geometrical composition and their yod populations generated from tetractyses of 10 points ("yods") by regarding them as either Type A or Type B polygons. Through their gematria number values, the Godnames of the 10 Sephiroth of the Tree of Life prescribe the first (4+4) polygons. The Decad determines the 120 geometrical elements outside the shared, root edge of the (4+4) enfolded Type A polygons because 120 is the sum of the first 10 odd integers after 1. Previous articles have established that the number 90 is a parameter of holistic systems, e.g., the five Platonic solids have 90 edges and the sum of the integers in the Platonic Lambda Tetractys is 90. It appears in the first four separate Type B polygons as the 90 sides of their 54 triangles and as their 90 corners & triangles. The first four enfolded Type A polygons also have 90 yods outside their root edge. The 108 sectors of the (4+4) separate Type B polygons have 260 corners & sides. Their counterparts in the inner Tree of Life are the 260 yods in the seven enfolded Type A polygons outside their root edge. The first four separate Type B polygons embody the 248 roots of the superstring gauge symmetry group E_8 as the 248 yods other than corners that are outside their root edges. Enfolded, these polygons have 247 yods. Their counterparts in the inner Tree of Life are the 247 yods lining sides of tetractys sectors of the seven separate Type A polygons. 192 yods outside the root edge line the sides of the tetractyses making up each set of the first four enfolded Type B polygons. This 192:192 division is characteristic of representations of holistic systems, being found, for example, in the 192 lines and 192 broken lines that make up the 64 hexagrams in the I Ching table. The (248+248) yods outside the root edge of two joined, Type C hexagons symbolise the (248+248) roots of the heterotic superstring gauge symmetry group $E_8{\times}E_8'$. With 2nd-order tetractyses as sectors, two joined hexagons have 840 yods outside the root edge. This is the number of circular turns in the inner or outer halves of each helical whorl of the basic unit of matter described by the Theosophists Annie Besant and C.W. Leadbeater. The 10 pairs of Type A hexagons enfolded in 10 overlapping Trees of Life have (840+840) yods that symbolise the (840+840) turns in a whorl. The 80 corners of the 108 triangles in the first (4+4) separate Type B polygons correspond to the 80 corners of the 94 sectors of the (7+7) enfolded Type A polygons of the inner Tree of Life. They also correspond to the 80 yods in the lowest Tree of Life. These and other correspondences demonstrate that the first (4+4) polygons constitute a holistic structure that is isomorphic to the greater whole of the inner Tree of Life.

Table 1. Gematria number values of the 10 Sephiroth in the four Worlds.

	SEPHIRAH	GODNAME	ARCHANGEL	ORDER OF ANGELS	MUNDANE CHAKRA
1	Kether (Crown) **620**	EHYEH (I am) **21**	Metatron (Angel of the Presence) **314**	Chaioth ha Qadesh (Holy Living Creatures) **833**	Rashith ha Gilgalim First Swirlings. (Primum Mobile) **636**
2	Chokmah (Wisdom) **73**	YAHVEH, YAH (The Lord) **26**, **15**	Raziel (Herald of the Deity) **248**	Auphanim (Wheels) **187**	Masloth (The Sphere of the Zodiac) **140**
3	Binah (Understanding) **67**	ELOHIM (God in multiplicity) **50**	Tzaphkiel (Contemplation of God) **311**	Aralim (Thrones) **282**	Shabathai Rest. (Saturn) **317**
	Daath (Knowledge) **474**				
4	Chesed (Mercy) **72**	EL (God) **31**	Tzadkiel (Benevolence of God) **62**	Chasmalim (Shining Ones) **428**	Tzadekh Righteousness. (Jupiter) **194**
5	Geburah (Severity) **216**	ELOHA (The Almighty) **36**	Samael (Severity of God) **131**	Seraphim (Fiery Serpents) **630**	Madim Vehement Strength. (Mars) **95**
6	Tiphareth (Beauty) **1081**	YAHVEH ELOHIM (God the Creator) **76**	Michael (Like unto God) **101**	Malachim (Kings) **140**	Shemesh The Solar Light. (Sun) **640**
7	Netzach (Victory) **148**	YAHVEH SABAOTH (Lord of Hosts) **129**	Haniel (Grace of God) **97**	Tarshishim or Elohim **1260**	Nogah Glittering Splendour. (Venus) **64**
8	Hod (Glory) **15**	ELOHIM SABAOTH (God of Hosts) **153**	Raphael (Divine Physician) **311**	Beni Elohim (Sons of God) **112**	Kokab The Stellar Light. (Mercury) **48**
9	Yesod (Foundation) **80**	SHADDAI EL CHAI (Almighty Living God) **49**, **363**	Gabriel (Strong Man of God) **246**	Cherubim (The Strong) **272**	Levanah The Lunar Flame. (Moon) **87**
10	Malkuth (Kingdom) **496**	ADONAI MELEKH (The Lord and King) **65**, **155**	Sandalphon (Manifest Messiah) **280**	Ashim (Souls of Fire) **351**	Cholem Yesodeth The Breaker of the Foundations. The Elements. (Earth) **168**

The Sephiroth exist in the four Worlds of Atziluth, Beriah, Yetzirah and Assiyah. Corresponding to them are the Godnames, Archangels, Order of Angels and Mundane Chakras (their physical manifestation, traditionally symbolised by celestial bodies). This table gives their number values obtained by the ancient practice of gematria, wherein a number is assigned to each letter of the alphabet, thereby giving to a word a number value that is the sum of the numbers of its letters.

*(All numbers in this table referred to in the article are written in **boldface**).*

1. Geometrical properties of the first four polygons

Many previous articles have established the role of the Tetrad Principle in determining the properties of holistic systems embodying the universal design or pattern found in the sacred geometries of some major religions, as well as in objects well-known to mathematicians. They have revealed numerous examples of how the Tetrad (4) and the integers 1, 2, 3 & 4 symbolized by the four rows of dots in Pythagoras' tetractys define or express the attributes of these systems. In view of this, the first four regular polygons of the inner Tree of Life and the hexagon — the fourth polygon — might therefore be expected to display defining parameters of these systems, such as the numbers 384, **248** & **168**. This expectation will be confirmed by examining their geometrical and yod composition. The appearance of so many of the 50 gematria number values (shown in boldface) of the ancient Hebrew names of the 10 Sephiroth of the
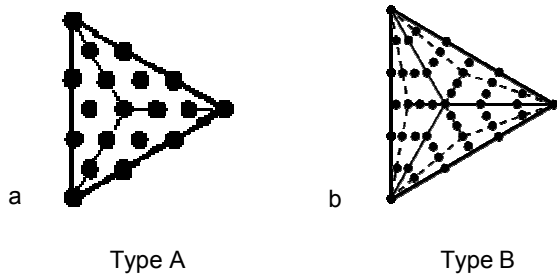
a   b

Type A   Type B

Figure 1. Type A & Type B triangles.

Tree of Life and their manifestation in the four Kabbalistic Worlds of Atziluth, Beriah, Yetzirah & Assiyah (Table 1) cannot, plausible, be explained away in terms of coincidence because their occurrence is too frequent to be due to chance. Instead, these numbers are signatures of the underlying sacred geometrical nature of these polygons. Previous articles have shown that various subsets of the (7+7) regular polygons making up the inner Tree of Life are counterparts of the whole set in the sense that their properties are analogous, being measured by the same set of parameters. The first (4+4) regular polygons are another example of this amazing fractal-like property of the inner Tree of Life, wherein smaller and smaller subsets of the 14 polygons display analogous properties quantified by the *same* set of numbers, namely, the number values of the 10 Sephiroth, their Godnames, Archangels, Orders of Angels & Mundane Chakras.

A polygon with n sides is known in mathematics as an 'n-gon.' It is transformed into a Type A polygon by dividing it up into its n sectors, which may then be further turned into tetractyses. Figure 1a shows the Type A triangle, or 3-gon with its three sectors turned into tetractyses. When each sector of the n-gon is divided into three sectors and each new sector turned into a tetractys, it becomes a Type B n-gon (Fig. 1b shows the Type B triangle). This transformation may be extended indefinitely, creating Type C n-gons, Type D n-gons, etc. This article will consider only Type A, Type B & Type C n-gons.

Instead of dividing sectors into ever smaller triangles/tetractyses, they may be regarded as members of an infinite sequence of nth-order tetractyses as ever greater differentiations of Unity (the Monad):

0th-order   1st-order   2nd-order
tetractys   tetractys   tetractys
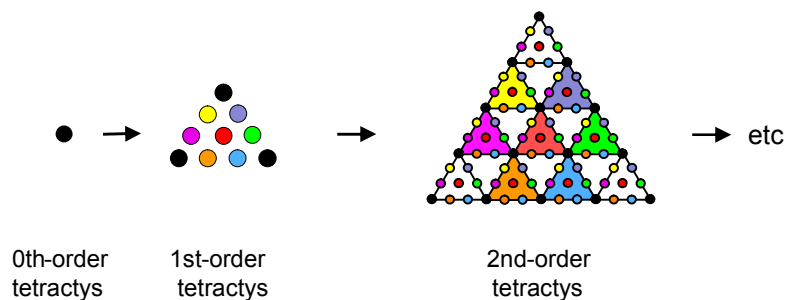
We shall consider only the 1st- and 2nd-order tetractyses. In this section, we shall examine the geometrical properties of the triangle, square, pentagon & hexagon, considering, firstly, the case when they are Type A polygons and then when they are Type B. The next section will analyse their construction from tetractyses in each case.

The following formulae are useful for examining geometrical properties of n-gons:

	Type A n-gon	Type B n-gon
Number of corners =	n+1	2n+1
Number of sides =	2n	5n
Number of triangles =	n	3n
Number of corners & sides =	3n+1	7n+1
Number of corners & triangles =	2n+1	5n+1
Number of sides & triangles =	3n	8n
Number of geometrical elements =	4n+1	10n+1

First four Type A polygons
The separate polygons will be considered first.

First four separate polygons
Table 2 shows the geometrical composition of each Type A polygon:

Table 2. The geometrical composition of the first four separate Type A polygons.

	triangle	square	pentagon	hexagon	Total
Number of corners of sectors =	3+1	4+1	5+1	6+1	18+4
Number of sides =	6	8	10	12	**36**
Number of triangles =	3	4	5	6	18
Number of corners & sides =	9+1	12+1	**15**+1	18+1	54+4
Number of corners & triangles =	6+1	8+1	10+1	12+1	**36**+4
Number of sides & triangles =	9	12	**15**	18	54
Number of corners, sides & triangles =	12+1	16+1	20+1	24+1	**72**+4

First (4+4) separate polygons
1. Number of corners of polygons = 2×18 = **36**.
2. Number of sides of polygons = 2×18 = **36**.
3. Number of corners & sides of polygons **= 2×36 = 72**.
4. Number of triangles = 2×18 = **36**.
5. Number of corners of triangles = 2×22 = 44.
6. Number of sides of triangles = 2×**36** = **72**.
7. Number of corners & sides of triangles = 2×58 = 116.
8. Numbers of corners & triangles = 2×40 = **80**.
9. Number of sides & triangles = 2×54 = 108.
10. Number of corners, sides & triangles = 2×**76** = 152. There are (152– 4×3 = **140**) geometrical elements outside the sides that become the root edge when the polygons are enfolded.

Comments
1) **72** corners, sides & triangles surround the centres of the four polygons. They have **76** geometrical elements. The two sets of polygons separated by the root edge have 2×**76** + 3 = **155** geometrical elements. ADONAI MELEKH, the Godname of Malkuth with number value **155** and YAHWEH ELOHIM, the Godname of Tiphareth with number value **76**, prescribe the geometrical composition of the polygons. **140** elements are outside the root edges of the (4+4) polygons, where **140** is the number value of *Malachim*, the Order of Angels assigned to Tiphareth. ELOHA, the Godname of Geburah with number value **36**, prescribes the number of sides of triangles in each set of four polygons, as well as the number of triangles in both sets. It also prescribes the total number of geometrical elements surrounding the centres of each set because **72** is the **36**th even integer.

2) (2×**72**=144) geometrical elements surround the centres of the (4+4) polygons, where

$$144 = \begin{matrix} 1^0 & 2^0 & 3^0 & 4^0 \\ 1^1 & 2^1 & 3^1 & 4^1 \\ 1^2 & 2^2 & 3^2 & 4^2 \\ 1^3 & 2^3 & 3^3 & 4^3 \end{matrix}$$

It demonstrates how the integers 1, 2, 3 & 4 express properties of holistic systems. The (4+4) polygons have **36** corners, where

$$36 = (1+3+5+7) + (2+4+6+8),$$

i.e., **36** is the sum of the first *four* odd integers and the first *four* even integers, showing how the Tetrad determines the number of corners of the (4+4) polygons. The 18 sectors of the first four polygons have 22 corners, where

$$22 = 1^4 + 2^3 + 3^2 + 4^1.$$

3) The (4+4) polygons have **80** corners & triangles, where **80** is the number value of Yesod.

First four enfolded polygons
1. Number of corners of polygons= 3 + (4–2=2) + (5–2=3) + (6–2=4) = 12; (12—2=10) corners are outside their root edges ("external").
2. Number of sides of polygons = 3 + (4–1=3) + (5–1=4) + (6–1=5) = **15**; (**15**–1=14) sides are external.
3. Number of corners & sides of polygons = 12 + **15** = 27; (27–3=24) corners & sides are external.
4. Number of triangles = 18 – 1 = 17 (the triangle fills one sector of the hexagon).

4

5. Number of corners of triangles = (3+1=4) + (2+1=3) + (3+1=4) + 4 = **15**; (**15**–2=13) are external.
6. Number of sides of triangles = (3+3=6) + (3+4=7) + (4+5=9) + (5+4=9) = **31**; (**31**–1=30) are external.
7. Number of corners & sides of triangles = **15** + **31** = 46; (46–3=43) are external.
8. Number of triangles & their sides = 17 + **31** = **48**.
9. Number of corners, sides & triangles = 46 + 17 = 63; (63–3=60) are external.

First (4+4) enfolded polygons
1. Number of corners of polygons = 2×10 + 2 = 22; (22–2=20) are external.
2. Number of sides of polygons = 2×14 + 1 = 29; (29–1=28) are external.
3. Number of corners & sides of polygons = 22 + 29 = 51; (51–3=**48**) are external.
4. Number of corners of triangles = 2×13 + 2 = 28; (28–2=**26)** are external.
5. Number of sides of triangles = 2×30 + 1 = 61; (61–1=60) are external.
6. Number of triangles = 2×17 = 34.
7. Number of corners & sides of triangles = 28 + 61 = 89; (89–3=86) are external.
8. Number of corners & triangles = 28 + 34 = **62;** (**62**–2=60) are external.
9. Number of triangles & their sides = 34 + 61 = **95**.
10. Number of corners, sides & triangles = 89 + 34 = 123; (123–3=120) are external.

Comments
1) The first four enfolded polygons have **15** sides and **15** corners of 17 sectors with **31** sides, showing how YAH, the Godname of Chokmah with number value **15**, and EL, the Godname of Chesed with number value **31**, prescribes the shape of the four polygons. They have 63 geometrical elements. The top corner of the hexagon coincides with the bottom corner of the hexagon enfolded in the next higher, overlapping Tree of Life. This means that **62** elements are intrinsic to each set of the first four enfolded polygons. The number **62** is the **31**st even integer and the number value of *Tzadkiel*, the Archangel of Chesed. The first (4+4) enfolded polygons also have **62** corners & sectors. YAHWEH with number value **26** prescribes the **26** corners of sectors outside the root edge.
2) 120 geometrical elements are outside the root edge of the first (4+4) enfolded polygons, where

$$120 = 2^2 + 4^2 + 6^2 + 8^2.$$

Two of the 123 geometrical elements (top corners of the two hexagons) in the first (4+4) enfolded polygons are shared with the corresponding polygons enfolded in the next higher Tree of Life. The number of intrinsic geometrical elements = $123 - 2 = 121 = 11^2$. i.e., it is the square of the *tenth* integer after 1, 120 is the sum of the first 10 odd integers after 1, showing how the Decad determines the geometrical composition of the first (4+4) polygons. 30 sides of sectors and 30 corners & sectors lie on either side of the root edge, where

$$30 = 1^2 + 2^2 + 3^2 + 4^2.$$

3) The 34 sectors of the first (4+4) enfolded polygons have 89 corners & sides. As the top corners of both hexagons coincide with the bottom corners of the hexagons enfolded in the next higher Tree of Life, there are **87** corners & sides that are intrinsic to the first (4+4) enfolded polygons. **87** is the number value of *Levanah*, the Mundane Chakra of Yesod.
4) The first (4+4) enfolded polygons have **95** sectors & sides, where **95** is the number value of *Madim*, the Mundane Chakra of Geburah.
5) Outside the root edge are **48** corners & sides of the first (4+4) enfolded polygons. **48** is the number value of *Kokab*, the Mundane Chakra of Hod, and the number of triangles & sides in the four enfolded polygons. The first (4+4) enfolded polygons have 51 corners & sides. 51 is the **50**th integer after 1. They have **49** intrinsic corners & sides. **49** is the number value of EL ChAI, the Godname of Yesod.

First four Type B polygons
We will next consider the first four Type B polygons, starting with the separate polygons.

First four separate polygons
Table 3 displays the properties of each separate Type B polygon:

Table 3. Geometrical composition of the first four separate Type B polygons.

	triangle	square	pentagon	hexagon	Total
Number of corners of sectors =	6+1	8+1	10+1	12+1	**36**+4
Number of sides of sectors =	**15**	20	25	30	90
Number of sectors =	9	12	**15**	18	54
Number of corners & sides of sectors =	**21**+1	28+1	35+1	42+1	126+4

Number of corners & sectors =	**15**+1	20+1	25+1	30+1	90+4
Number of sides & sectors =	24	32	40	**48**	144
Number of corners, sides & sectors =	30+1	40+1	**50**+1	60+1	180+4

Comments

1) The first four polygons have 40 corners, where 40 = 4 + 8 + 12 + 16. Both sets of polygons have **80** corners, where **80** is the number value of Yesod. **36** corners surround the centres of each set of polygons. **72** corners surround the centres of both sets.

2) The first four polygons have 90 sides. 25 sides belong to the pentagon and **65** belong to the other three polygons. This 25:**65** division has its counterpart in the Lambda Tetractys because the sum of its uppermost six integers is 25 and the sum of the four integers in its base is **65**:

$$
\begin{array}{c}
1 \\
2 \quad 3 \\
4 \quad 6 \quad 9
\end{array} \rightarrow 25
$$
$$
8 \quad 12 \quad 18 \quad 27 \rightarrow \mathbf{65}
$$
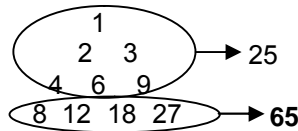
The 90 corners & triangles surround their centres comprise **36** corners and 54 triangles. Many previous articles have illustrated how this **36**:54 division is characteristic of holistic systems, the sum of the integers at the corners of the Lambda Tetractys being **36** and the sum of the seven remaining

36 corners surround centres
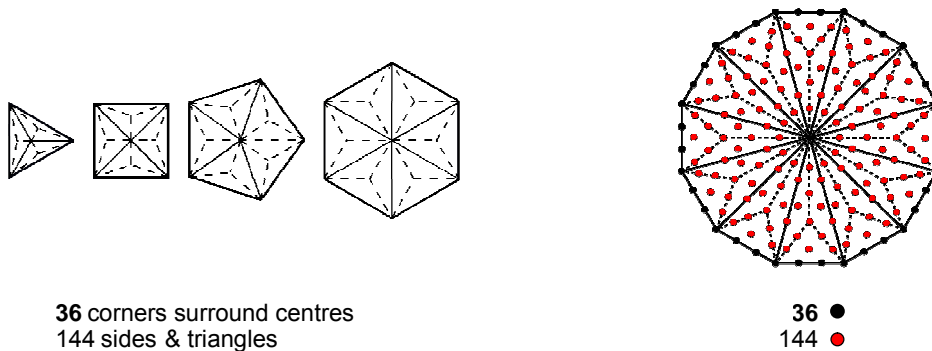144 sides & triangles

36 ●
144 ●

Figure 2. The 180 yods surrounding the centre of the Type B dodecagon symbolise the 180 geometrical elements that surround the centres of the first four polygons.

integers being 54. This conformity to the archetypal Lambda Tetractys is a clear indication of the holistic nature of the first four polygons. The counterparts of this division in the five Platonic solids are the **36** sides of the tetrahedron and dodecahedron and the 54 sides of the octahedron, cube & icosahedron.

3) The first four polygons have 144 sides & triangles, where

$$
144 = \begin{array}{cccc}
1^0 & 2^0 & 3^0 & 4^0 \\
1^1 & 2^1 & 3^1 & 4^1 \\
1^2 & 2^2 & 3^2 & 4^2 \\
1^3 & 2^3 & 3^3 & 4^3
\end{array}
$$

The first (4+4) polygons have 288 sides & triangles, where

$$288 = 1^1 + 2^2 + 3^3 + 4^4.$$

36 corners surround the centres of each set of four polygons, where **36** = (1+3+5+7) + (2+4+6+8), i.e., this number is the sum of the first *four* odd integers and the first *four* even integers. 360 geometrical elements surround the centres of both sets. This is the sum of the Decad (10) assigned to each of the **36** yods that surround the centre of the Type A hexagon (*fourth* polygon). These properties demonstrate how the Tetrad and the integers 1, 2, 3 & 4 symbolised by the tetractys express properties of this set of polygons.

4) 180 corners, sides & triangles surround the centres of the first four polygons. They comprise **36** corners and 144 sides & triangles. The counterpart of this in the Type B dodecagon is the 180 yods that surround its centre. They comprise **36** yods that line its boundary and 144 yods in its interior (Fig. 2). Alternatively, there are **36** corners & sides shaping the polygons and 144 interior, geometrical elements. They are symbolised by the corresponding **36** yods on the boundary of the dodecagon and by the 144 yods inside it.

5) The 54 sectors of the first four polygons have 130 corners & sides, where 130 is the **65**th even integer and the **129**th integer after 1. The (4+4) polygons have 260 (=**26**×10) corners & sides. The seven enfolded polygons with their 47 sectors turned into tetractyses have 260 yods outside their root edge (Fig. 3a), whilst 260 yods other than the corners and centres of two joined, Type B dodecagons line
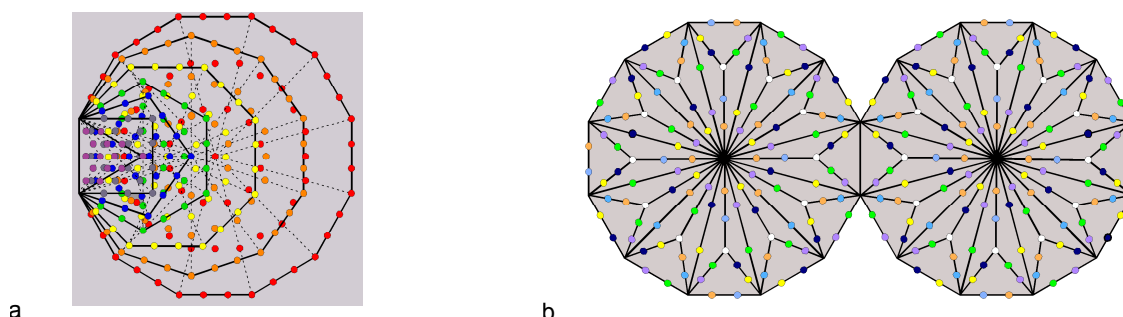

a   b

Figure 3. The Godname YAHWEH with number value **26** prescribes the 260 yods outside the root edge of the seven enfolded Type A polygons of the inner Tree of Life and the 260 yods other than corners & centres of the Type B dodecagons that are outside the root edge on sides of tetractyses.

the sides of their **72** tetractyses outside their shared edge (Fig. 3b). It is evidence that the first (4+4) polygons constitute a holistic system having properties analogous to those of the inner Tree of Life.

First four enfolded polygons
One of the sectors of the hexagon is replaced by the three sectors of the triangle when they become enfolded in one another. Table 4 displays the properties of the first four enfolded polygons:

Table 4. Geometrical composition of the first four, enfolded Type B polygons.

	4 enfolded polygons	(4+4) enfolded polygons
Number of corners of sectors =	40 – 3×2 – 2 = 32	2×30 + 2 = **62**
Number of sides of sectors =	90 – 3 – 5 = 82	2×81 + 1 = 163
Number of sectors =	54 – 3 = 51	2×51 = 102
Number of corners & sides of sectors =	32 + 82 = 114	**62** + 163 = 225
Number of corners & sectors =	32 + 51 = 83	**62** + 102 = 164
Number of sides & sectors =	82 + 51 = 133	163 + 102 = 265
Number of corners, sides & triangles =	32 + 82 + 51 = 165	**62** + 163 + 102 = 327

Comments
1) The four polygons have 32 corners, where 32 is the **31**st integer after 1. The (4+4) polygons have **62** corners, where **62** is the number value of *Tzadkiel*, the Archangel of Chesed, which is the *fourth* Sephirah from the top of the Tree of Life. **31** corners are associated with each set, where **31** is the number value of EL, the Godname of Chesed. One is an endpoint of the root edge and there are 30 corners outside it. This reflects the number values E = 1 and L = 30.
2) The four polygons have 51 sectors, where 51 is the **50**th integer after 1. The 114 corners & sides of their sectors comprise 12 corners and **15** sides of polygons, leaving **87** corners and sides generated by their transformation, where **87** is the number value of *Levanah*, the Mundane Chakra of Yesod.
3) The (4+4) polygons have 265 sides & triangles, that is, 264 sides & triangles are outside the root edge. This is the number of yods in the seven enfolded, Type A polygons (see Fig. 3).
4) There are 165 corners, sides & triangles in the four polygons, where

$$165 = 1^2 + 3^2 + 5^2 + 7^2 + 9^2.$$

Such a beautiful property is no accident but, instead, the manifestation of perfect mathematical design. They include the 12 polygonal corners, leaving **153** geometrical elements, where **153** is the number value of ELOHIM SABAOTH, the Godname of Hod. The (4+4) polygons have 327 geometrical elements. They include the three corners and two sides on each side pillar of the outer Tree of Life. This means that (327–5–5=**317**) geometrical elements are unshared with it. **317** is the number value of *Shabathai*, the Mundane Chakra of Binah.

2. Yod composition of the first four polygons
The following formulae are useful for studying the yod composition of n-gons:

	Type A	Type B
Number of yods at corners of tetractyses =	n + 1	2n + 1
Number of hexagonal yods =	5n	13n
Number of yods =	6n + 1	**15**n + 1

First four Type A polygons

The first four separate polygons are considered first.

First four separate polygons

Table 5 shows the numbers of yods in the first four Type A polygons:

Table 5. Yod composition of the first four separate Type A polygons.

	triangle	square	pentagon	hexagon	Total
Number of corners of tetractyses =	3+1	4+1	5+1	6+1	18+4
Number of hexagonal yods =	**15**	20	25	30	90
Number of yods =	18+1	24+1	30+1	**36**+1	108+4
Number of yods outside root edge =	14+1	20+1	**26**+1	32+1	92+4

1. Number of yods at corners of 18 tetractyses = 22.
2. Number of hexagonal yods = **15** + 20 + 25 + 30 = 90.
3. Number of yods = 19 + 25 + **31** + 37 = **112;** 108 yods surround their centres.
4. Number of yods on boundaries of polygons = 18 + 2×18 = 54; (2×18=**36**) are hexagonal.
5. Number of yods on boundaries of tetractyses = 54 + 2×18 + 4 = 94; (94–22=**72**) are hexagonal.

First (4+4) separate polygons
1. Number of yods at corners of tetractyses = 2×22 = 44.
2. Number of hexagonal yods = 2×90 = 180.
3. Number of yods = 2×**112** = 224. (2×108=**216**) yods surround their centres.
4. Number of yods on boundaries of polygons = 2×54 = 108; (2×**36**=**72**) are hexagonal.
5. Number of yods on boundaries of tetractyses = 2×94 = 188; (2×**72**=144) are hexagonal.

First four enfolded polygons
1. Number of yods at corners of 17 tetractyses = 22 – 3×2 – 1 = **15** (13 outside root edge).
2. Number of hexagonal yods = 90 – 3×2 – 2 – 2 – 1 = 79 (77 outside root edge).
3. Number of yods = **15** + 79 = 94 (90 outside root edge).

First (4+4) enfolded polygons
1. Number of yods at corners of 34 tetractyses = 2 + 2×13 = 28.
2. Number of hexagonal yods = 2 + 2×77 = 156; (156–2=154) are external.
3. Number of yods = 28 + 156 = 184; (184–4=180) are external.
4. Number of yods on 29 sides of polygons with 22 corners = 2×29 + 22 = **80**; (**80**–4=**76**) are external.
5. Number of yods on sides of 34 tetractyses = 184 – 34 = 150 = **15**×10; (150–4=146) are external, **73** on each side. Number of yods on sides of tetractyses other than 20 corners of polygons outside root edge = 150 – 20 = 130.

First four Type B polygons

The separate polygons are considered first.

First four separate polygons

Table 6 shows the numbers of yods in the four separate polygons:

Table 6. Yod composition of the first four separate Type B polygons.

	triangle	square	pentagon	hexagon	Total
Number of corners of tetractyses =	6+1	8+1	10+1	12+1	**36**+4
Number of hexagonal yods =	39	52	**65**	78	234
Number of yods =	45+1	60+1	75+1	90+1	270+4
Number of yods outside root edge =	41+1	56+1	71+1	86+1	254+4

Comments
1) The first four polygons have 274 yods (18 corners and 256 other yods, where $256 = 4^4$). This is a remarkable example of how the Tetrad expresses properties of holistic systems like the four polygons.
2) There are 258 yods outside their sides that become the root edge when they are enfolded in one

8

another. The hexagon (fourth polygon) has **87** such yods, where **87** is the number value of *Levanah*, the Mundane Chakra of Yesod. Number of yods in the polygons outside their 'root edges' other than their 10 corners = 258 − 10 = **248**. This is the number value of *Raziel*, the Archangel of Binah. It is the dimension of the superstring gauge symmetry group E_8, the rank-8 exceptional Lie group. The polygons contribute to this number as follows:

	triangle	square	pentagon	hexagon
248 =	41	55	69	83

= 22 non-polygonal corners + 226 hexagonal yods.

There are 54 hexagonal yods at centres of tetractyses, so that there are (54+22=**76**) yods that are either centres of tetractyses or their non-polygonal corners outside their root edges. This is how the Godname YAHWEH ELOHIM with number value **76** prescribes the embodiment of the **248** roots of E_8 in the four

7 ●
72 ○
168 ● } 240
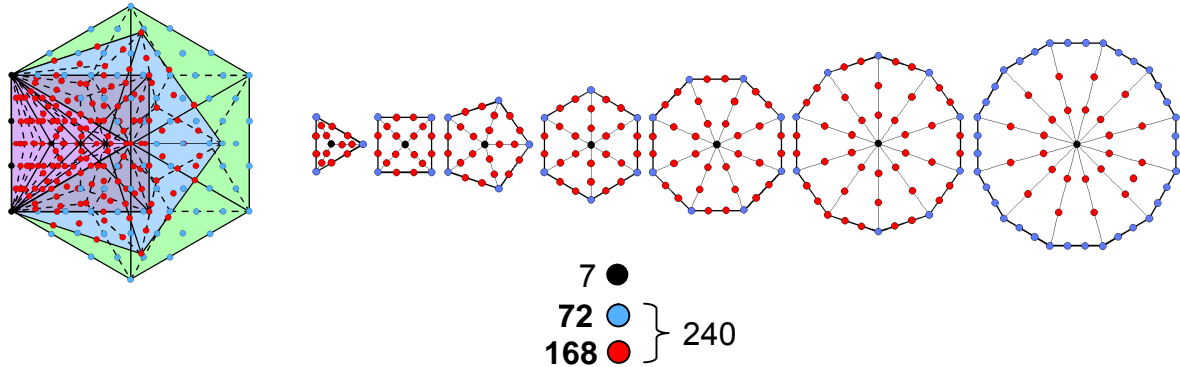
Figure 4. The first four enfolded Type B polygons have 240 yods outside the root edge surrounding their four centres. This is the number of yods on sides of tetractyses that surround the centres of the seven polygons of the inner Tree of Life. The **72** yods in the hexagon correspond to the **72** yods that either lie on the sides of the dodecagon or are corners of the first six polygons. The **168** yods outside the root edge surrounding the centres of the first three enfolded polygons correspond to the **168** yods surrounding the centres of the seven polygons that are either inside the dodecagon or are not corners of the first six polygons.

separate Type B polygons. As the hexagon has 78 hexagonal yods, it has **76** hexagonal yods outside its root edge.

First four enfolded polygons
There are 19 yods (**15** outside root edge) in the sector of the hexagon which coincides with the triangle. Four corners of tetractyses in the hexagon disappear when the polygons become enfolded. Table 7 shows the numbers of yods in the four enfolded polygons.

Table 7. Yod composition of first four enfolded Type B polygons.

	triangle	square	pentagon	hexagon	Total
Number of corners of tetractyses =	7	9–2=7	11–2=9	13–4=9	32
Number of hexagonal yods =	39	52–2=**50**	**65**–2=63	78–2–13=63	215
Number of yods =	46	57	**72**	**72**	247
Number of external yods surrounding centre =	46–4–1=41	57–1=56	**72**–1=71	**72**	240

Comments
1) The first four enfolded polygons have 247 yods. This is the number of yods on the sides of the **48** sectors of the 7 separate polygons of the inner Tree of Life when the sectors are tetractyses (Fig. 4) 240 yods outside the root edge surround the centres of the polygons. **168** such yods are in the first
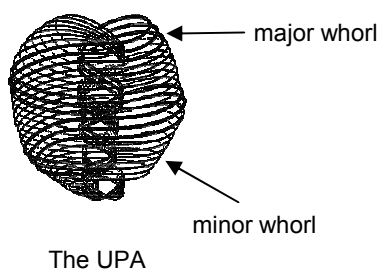

major whorl

minor whorl

The UPA

Figure 5. The three major whorls and seven minor whorls of the UPA ("ultimate physical atom"), the unit of matter described with micro-psi (a form of remote-viewing) by the Theosophists Annie Besant and C.W. Leadbeater.

three and **72** yods are in the fourth. As illustrated in many previous articles, this **72**:**168** division is characteristic of holistic systems. It signifies the distinction between the **72** roots of E_6, an exceptional subgroup of E_8, and the remaining **168** roots of E_8. It manifests in the $E_8{\times}E_8$ heterotic superstring itself (Fig. 5) as the distinction between its three major whorls and its seven minor whorls, each of which carries 24 gauge charges of E_8, according to the author's work. There are also 240 yods in the first 4

490 yods outside the root edge   490 yods

550 =
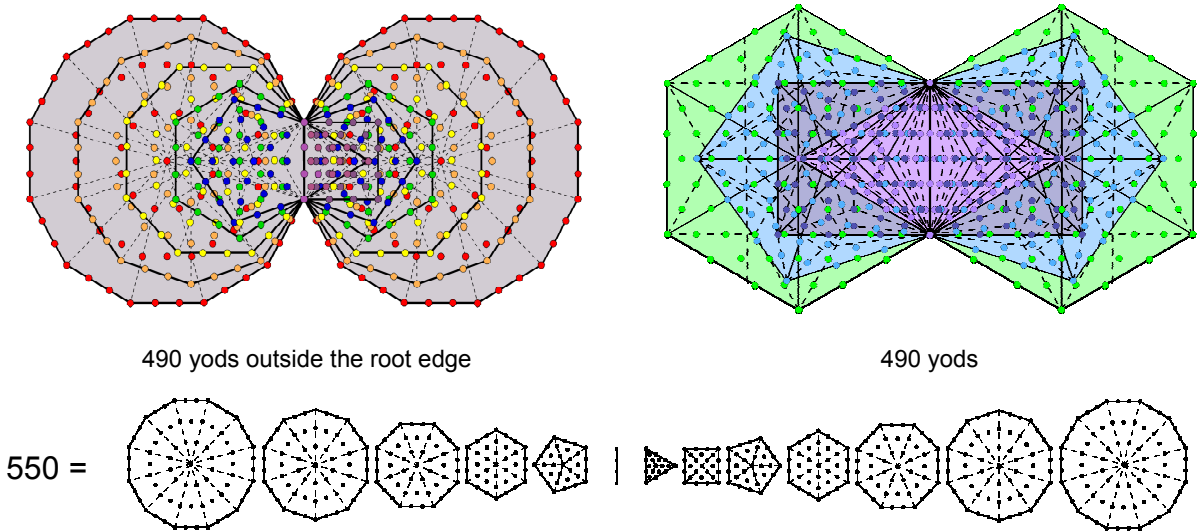
Figure 6. The first (4+4) enfolded Type B polygons have the same number (490) of yods as there are outside the root edge of the (7+5) enfolded Type A polygons which, together with the root edge, separately have 550 yods.

enfolded polygons outside the root edge that do not coincide with the 3 Sephiroth on the side pillars of the Tree of Life.

2) There are (247−4=243) yods outside the root edge. Number of yods in the (4+4) enfolded polygons = 4 + 2×243 = 490 = **49**×10, showing how EL CHAI, the Godname of Yesod with number value **49**, prescribes the (4+4) enfolded polygons. This is also the number of yods outside the root edge of the (7+5) enfolded polygons which, separately and together with the four yods of the root edge, have 550 yods that symbolize the 550 SLs of CTOL (Fig. 6) [1]. The 490 yods in the (4+4) enfolded polygons comprise **62** corners (**31** corners associated with each set), 326 hexagonal yods on 163 sides of 102 tetractyses & 102 hexagonal yods at their centres, i.e., (326+102=**428**) hexagonal yods. (490−102=388) yods line their sides, **194** yods per set. **31** is the number value of EL, Godname of Chesed, **62** is the number value of its Archangel *Tzadkiel*, **428** is the number value of *Chasmalim*, its Order of Angels and **194** is the number value of *Tzadekh*, the Mundane Chakra of Chesed. Here is a remarkable example of how the number values of the *same* Sephirah in the four Kabbalistic Worlds of Atziluth, Beriah, Yetzirah and Assiyah measure properties of an object possessing sacred geometry.

The (7+7) enfolded Type B polygons of the inner Tree of Life have 1370 yods [2]. The last (3+3) enfolded polygons have (1370−490=880) yods outside the root edge. 880 = 88×10, where 88 is the **87**th integer after 1 and **87** is the number value of *Levanah*, the Mundane Chakra of Yesod. They have **50** corners (**48** outside the root edge), where **50** is the number of ELOHIM and **48** is the number of *Kokab*, the Mundane Chakra of Hod. The last 3 enfolded polygons have **26** corners, where **26** is the number value of YAHWEH.
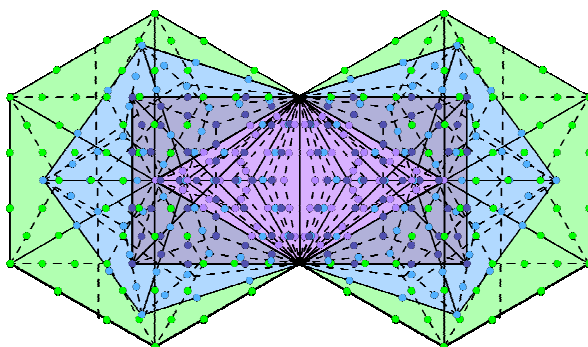

Figure 7. 384 yods outside the root edge line the 162 sides of the 102 tetractyses in the first (4+4) enfolded Type B polygons.

3) As the (4+4) enfolded Type B polygons have 102 tetractyses, each with a hexagonal yod at its centre, there are (490–102=388) yods lining their 163 sides. (388–4=384) yods outside the root edge line 162 sides, 192 in each set (Fig. 7). This 192:192 division is another characteristic of holistic systems, being found, for example, in the 192 lines and 192 broken lines making up the 64 hexagrams used in the ancient Chinese system of divination known as I Ching and in the 192 yods associated with each set of the first six enfolded polygons and unshared with polygons enfolded in the next higher Tree of Life (Fig. 8). 24 yods line the nine sides of the triangle, square & pentagon. They correspond to the 24 lines & broken lines in the upper or lower trigrams in the eight hexagrams forming the diagonal of the 8×8 square array of hexagrams and to the 24 corners associated with each set of the first six enfolded
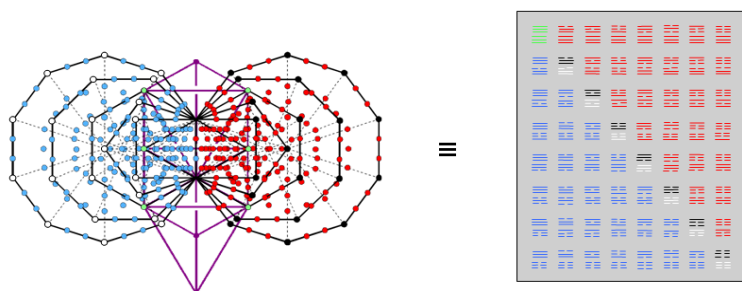
Figure 8. The equivalence of the 384 lines & broken lines of the **64** hexagrams of the I Ching table and the 384 yods intrinsic to the first (6+6) enfolded Type A polygons.

polygons unshared with the polygons enfolded in the next higher Tree of Life. The remaining **168** yods correspond to the **168** lines & broken lines in the 28 off-diagonal hexagrams in either diagonal half of the array and to the **168** yods in the first six enfolded polygons that are not corners of polygons.

The number 384 is the sum of the first nine squares after 1:

$$384 = 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2.$$

It illustrates the fundamental connection between the Pythagorean Decad and this measure of holistic systems represented by sacred geometries.

3. How the Godnames prescribe the first four polygons

Set out below is a summary of the way in which the Godnames of the 10 Sephiroth prescribe the first four polygons and the (4+4) polygons:

Sephirah	Godname	
Kether	EHYEH = **21**	22 corners of first (4+4) enfolded polygons, where 22 = **21**st integer after 1.
Chokmah	YAH = **15**	**15** sides of first 4 enfolded polygons; **15** corners of 17 sectors of first 4 enfolded Type A polygons.
	YAHWEH = **26**	**26** corners outside root edge of 34 sectors of first (4+4) enfolded Type A polygons.
Binah	ELOHIM = **50**	51 corners & sides of 34 sectors of first (4+4) enfolded Type A polygons, where 51 = **50**th integer after 1. 51 sectors of first 4 enfolded Type B polygons.
Chesed	EL = **31**	**31** sides of 17 sectors of first 4 enfolded Type A polygons. **62** corners & triangles in first (4+4) enfolded Type A polygons, where **62** = **31**st even integer. **62** intrinsic elements in first 4 enfolded Type A polygons.
Geburah	ELOHA = **36**	**36** corners/sides of first 4 separate Type A polygons. **36** corners surround centres of sectors in first 4 separate Type B polygons. **72** geometrical elements surround centres of first 4 separate Type A polygons, where **72** = **36**th even integer. **72** corners & sides of first (4+4) separate Type A polygons.

		360 (=**36**×10) geometrical elements surround centres of first (4+4) separate Type B polygons.
Tiphareth	YAHWEH ELOHIM = **76**	**76** geometrical elements in first 4 separate Type A polygons. **76** non-polygonal corners & hexagonal yods at centres of 54 tetractyses outside root edges of first 4 separate Type B polygons.
Netzach	YAHWEH SABAOTH = **129**	130 corners & sides of 54 sectors of first 4 separate Type B polygons, where 130 = **129**th integer after 1. 130 yods on sides of 34 tetractyses in first (4+4) enfolded Type A polygons other than external corners of polygons.
Hod	ELOHIM SBAOTH = **153**	**153** geometrical elements in first 4 enfolded Type B polygons other than their corners. 154 hexagonal yods outside root edge of first (4+4) enfolded Type A polygons, where 154 = **153**rd integer after 1.
Yesod	EL ChAI = **49**	**49** intrinsic corners & sides of first (4+4) enfolded Type A polygons.
Malkuth	ADONAI = **65**	130 corners & sides of 54 sectors of first 4 separate Type B polygons, where 130 = **65**th even integer. 130 yods on sides of 34 tetractyses in first (4+4) enfolded Type A polygons other than external corners of polygons.
	ADONAI MELEKH = **155**	**155** geometrical elements in root edge and in first (4+4) separate Type A polygons. 156 hexagonal yods in first (4+4) enfolded Type A polygons, where 156 = **155**th integer after 1.

4. The hexagon embodies the dimension 248 of E$_8$

The pair of joined Type A hexagons has 70 yods when their 12 sectors are transformed into tetractyses. This is the number of yods in the Tree of Life when its 16 triangles are tetractyses (Fig. 9). The 10 corners of the two hexagons correspond to the 10 Sephirothic corners of the triangles making up the Tree of Life.
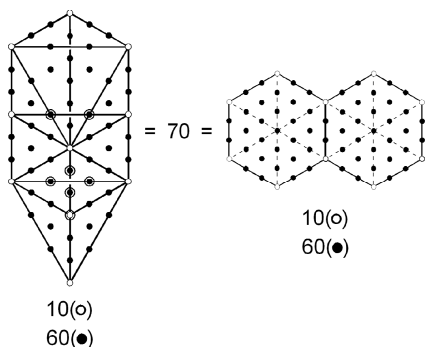
Figure 9. The 70 yods of the two joined Type A hexagons with their 12 sectors turned into tetractyses are the counterpart of the Tree of Life with its 16 triangles turned into tetractyses. The 10 corners of the two hexagons correspond to the 10 Sephirothic corners of the Tree of Life. Each system has 60 hexagonal yods.

Their 60 hexagonal yods are the counterpart of the 60 hexagonal yods in the latter. This correspondence suggests that the two joined hexagons are the single polygonal counterpart of the Tree of Life. It should therefore not be surprising that they embody parameters of holistic systems, as we now show. The next level of transformation of the Type B polygon is the Type C polygon in which each triangle of the former is
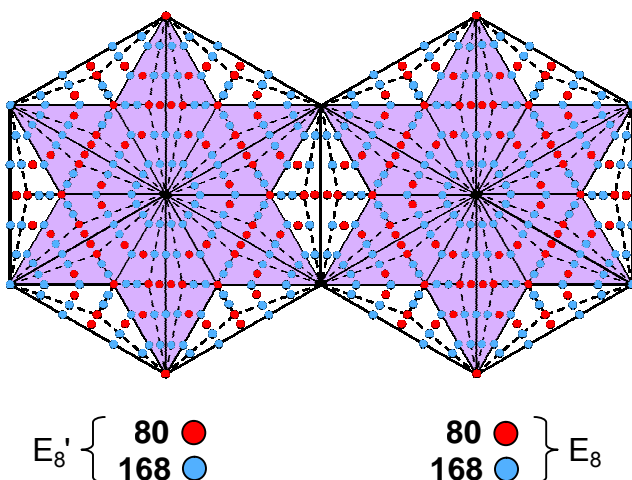
Figure 10. The **496** yods outside the root edge of two joined Type C hexagons symbolise the **496** gauge bosons of the heterotic superstring symmetry group E$_8$×E$_8$'. The **248** yods in each hexagon comprise **168** yods on the 83 sides of 54 tetractyses and **80** yods either at centres of tetractyses or at the top & bottom corners shared with hexagons enfolded in adjacent Trees of Life.

E$_8$' { **80** ● **168** ○   **80** ● **168** ○ } E$_8$

replaced by a Type A triangle. The Type C n-gon has (42n+1) yods. A Type C hexagon (n=6) has 252 yods surrounding its centre. (252–4=**248**) yods in each hexagon outside their root edge surround their centres (Fig. 10). 500 (=**50**×10) yods surround the two centres, **496** yods being outside the root edge, where **496** is the number value of Malkuth. This is how ELOHIM, the Godname of Binah with number value **50**, prescribes the polygonal representation of the dimension **496** of $E_8 \times E_8$, one of the two gauge symmetry groups governing the unified interaction of heterotic superstrings.

E_6 subgroup

Inside each sector of a Type C n-gon are 13 yods that are either corners of tetractyses or hexagonal yods at their centres. A Type C hexagon has (6×13=78) such yods. 78 is the dimension of E_6, the rank-6 exceptional subgroup of E_8. The six yods at the centres of each sector denote the six simple root of E_6

248 (◉)
168 (●)
80 (●)

← 31st SL

a

168 ●
80 ●

b

c

24 corners
60 sides
36 triangles

24 corners
60 sides
36 triangles

168 corners & sides
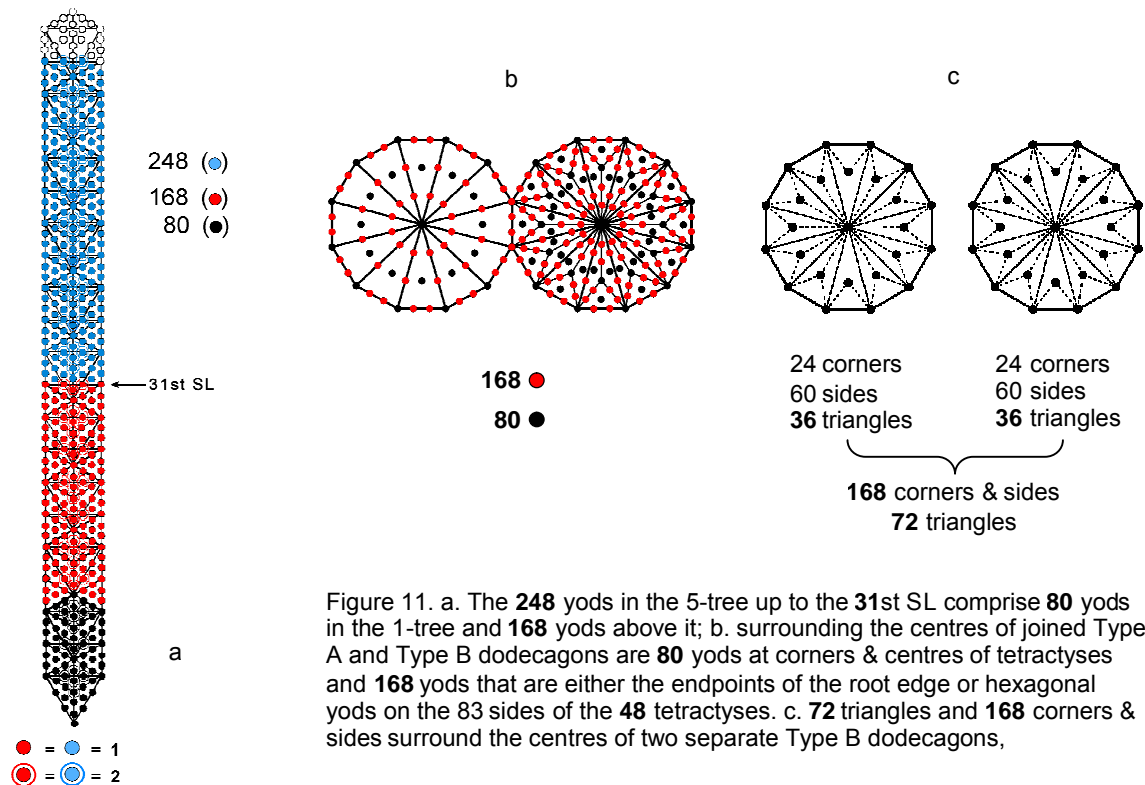**72** triangles

● = ◉ = 1
◉ = ◉ = 2

Figure 11. a. The **248** yods in the 5-tree up to the **31**st SL comprise **80** yods in the 1-tree and **168** yods above it; b. surrounding the centres of joined Type A and Type B dodecagons are **80** yods at corners & centres of tetractyses and **168** yods that are either the endpoints of the root edge or hexagonal yods on the 83 sides of the **48** tetractyses. c. **72** triangles and **168** corners & sides surround the centres of two separate Type B dodecagons,

and the **72** other yods denote its **72** roots. The top and bottom corners of the hexagon (those which are shared with the hexagons enfolded in the adjacent Trees of Life) denote the remaining simple roots of E_8. The number **248** divides into **80** yods that denote the eight simple roots of E_8 and the **72** roots of E_6 and **168** yods denoting the **168** roots of E_8 that are not roots of E_6. This **80**:**168** division of the number **248**, which is essentially the **72**:**168** division of the holistic parameter 240 discussed earlier, manifests in the context of the outer Tree of Life as follows: there are **80** yods in the 1-tree and **168** yods above it up to Chesed of the 5-tree, which is the **31**st SL prescribed by the Godname EL of Chesed (Fig. 11a). It also appears in the dodecagons because joined Type A and Type B dodecagons have **80** yods at corners and centres of their **48** tetractyses and **168** yods that are either the endpoints of the root edge or hexagonal
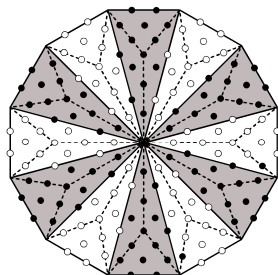

Figure 12. **168** extra yods are needed to transform the 12 sectors of a dodecagon into a Type B dodecagon.

yods on the 83 sides of tetractyses (Fig 11b). The **72**:**168** division appears in a pair of separate Type B dodecagons as their **72** triangles and as the **168** corners & sides that surround their centres (Fig. 11c). It exists because, as the tenth type of polygon determined by the Decad, the dodecagon is a holistic object

that displays the characteristics of holistic systems. For example, 24 corners of triangles surround the centre of each separate Type B dodecagon, comparing with the 24 corners of the first six enfolded polygons on either side of the root edge with its two endpoints, the properties of these two sets of polygons being shown in Article 4 [3] to be prescribed by the Godnames of the 10 Sephiroth and to display properties characteristic of such systems. Another example is the fact that **168** more yods are
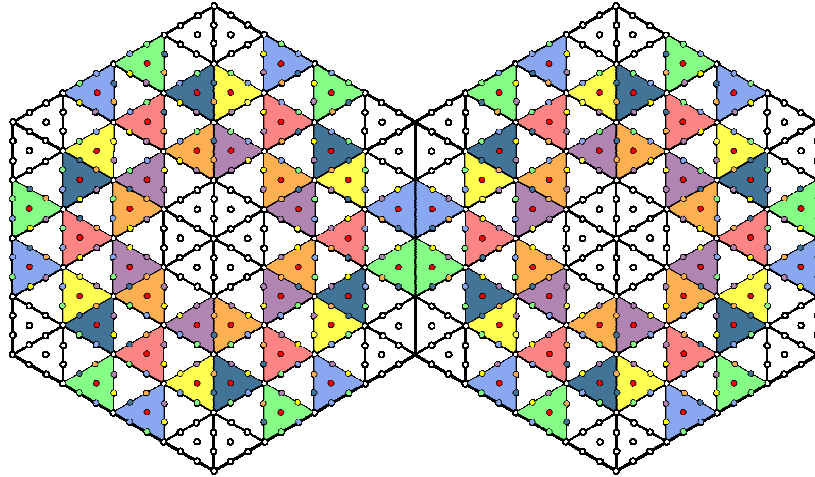

Figure 13. Outside the root edge of two joined hexagons with 2nd-order tetractyses as sectors are 840 yods.

needed to transform the 12 sectors of the dodecagon with 13 corners into a Type B dodecagon with 181 yods (Fig. 12).

5. The hexagon embodies superstring structural parameters

The number of yods in an n-gon with 2nd-order tetractyses as sectors is **72**n + 1 [4]. A hexagon has 433 yods. Thirteen yods lie on an side, leaving 420 yods outside it. Two joined hexagons have 840 yods
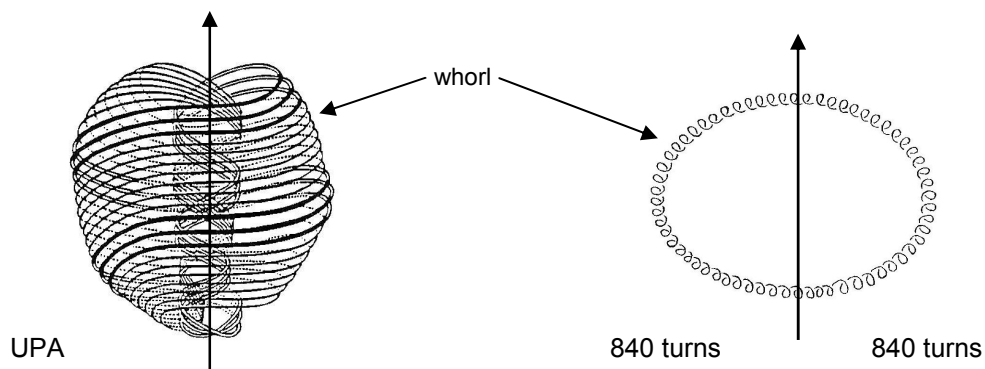

Figure 14. The UPA, the unit of matter described by Besant & Leadbeater, is the $E_8 \times E_8'$ heterotic superstring. It consists of 10 closed curves, or "whorls," each a helix with 1680 circular turns.
Every whorl winds 2½ times around the spin axis of the UPA in an outer helical spiral of 840 turns and returns to its top by twisting 2½ more times in a tighter, inner spiral of 840 turns.

outside their root edge (Fig. 13). The two hexagons embody not only the number of gauge bosons mediating the unified superstring force but also the number of coils in an outer or inner half of a helical whorl of the UPA/heterotic superstring (Fig. 14) described by Annie Besant & C.W. Leadbeater [5].

This information about the structure of each string-like whorl of the UPA is embodied in the pair of joined hexagons with Type A triangles as sectors. Each such hexagon has 91 yods, that is, 85 yods other than their six corners, where

$$85 = 4^0 + 4^1 + 4^2 + 4^3.$$

This demonstrates how the Tetrad expresses the *extra* number of yods needed to construct a Type B
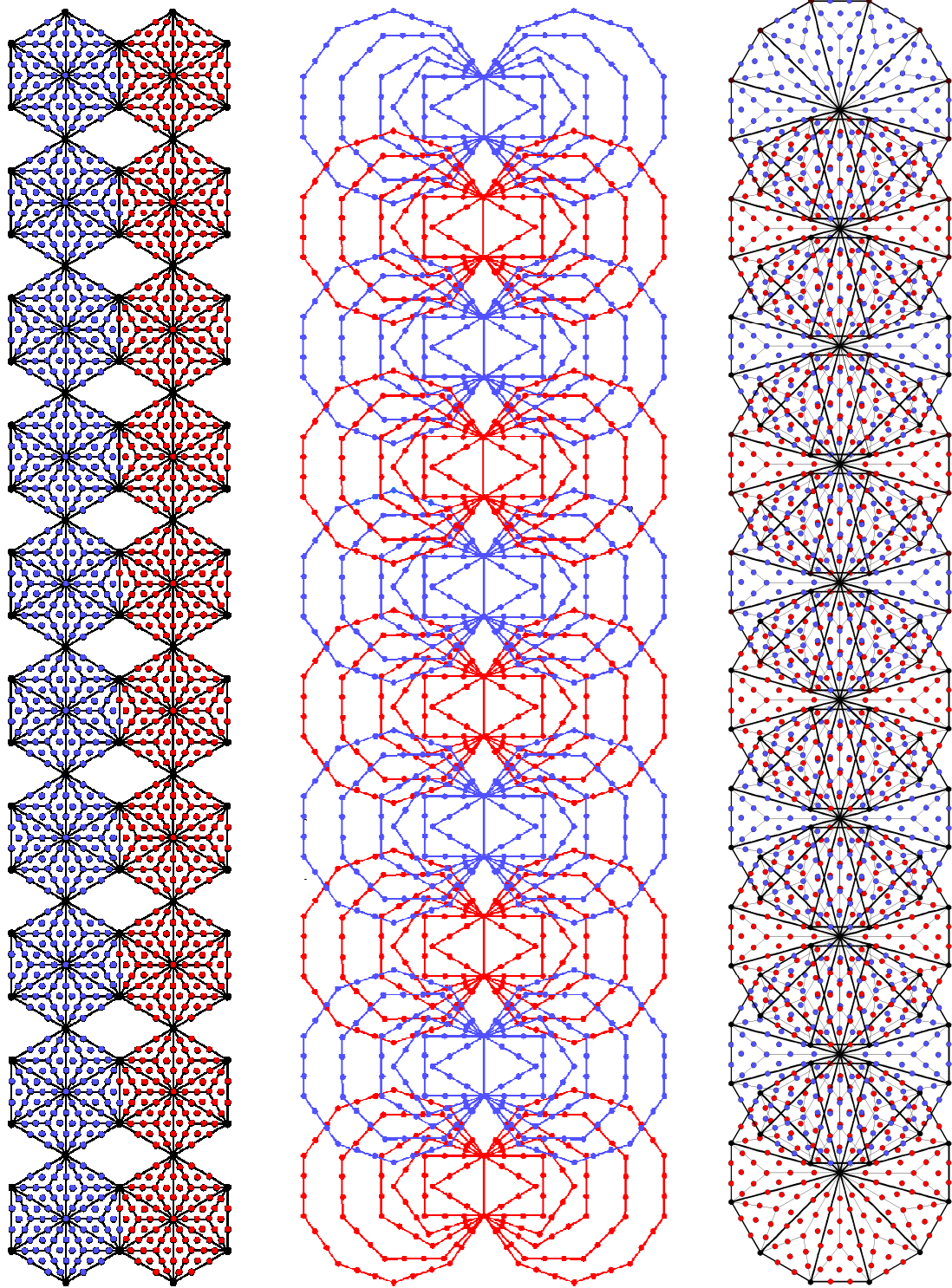
a

b

c

840 🔴

840 🔵

Figure 15. The 840 helical turns in each half of a whorl in the $E_8 \times E_8'$ heterotic superstring are symbolised by the yods: a. in the 20 hexagons; b. on the boundaries of the (60+60) polygons of the first 6 types, and c. in the 10 Type B dodecagons enfolded in 10 overlapping Trees of Life.

hexagon from 18 tetractyses. The two joined Type B hexagons therefore have 85 + 85 − 2 = **168** yods other than corners. 84 such yods are associated with each hexagon. Enfolded in 10 overlapping Trees of Life are 20 hexagons with (840+840=1680) yods other than corners (Fig. 15a). The same information is embodied in the first (6+6) polygons enfolded in 10 overlapping Trees of Life (Fig. 15b) because 1680 yods line their sides outside their root edges. It is also embodied in the 10 Type B dodecagons enfolded in 10 Trees of Life as the1680 yods other than corners that surround their centres (Fig. 15c). Here are
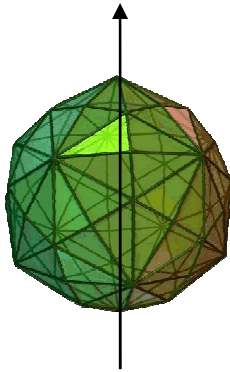

Figure 16. The disdyakis triacontahedron embodies the superstring structural parameter 1680 as the 840 vertices, sides & triangles in each half of the polyhedron that surround an axis passing through two opposite vertices.

equivalent holistic objects embodying the same information, namely, the number of circular polarized oscillations in each vibrating whorl. This structural parameter of the heterotic superstring is prescribed by ADONAI, the Godname of Malkuth, because the lowest 10 overlapping Trees of Life have **65** SLs. This Sephirah is appropriate, as Malkuth signifies the outer, physical form of any holistic object.

The 840 yods other than corners associated with each set of hexagons enfolded in 10 overlapping Trees of Life have their counterpart in the polyhedral Tree of Life [6]. Its outer form is the disdyakis triacontahedron (Fig. 16), which has **62** vertices, 180 edges & 120 triangular faces. Surrounding any axis passing through two diametrically opposite vertices are 840 vertices, edges & triangles in each half of the polyhedron when the 180 interior triangles formed by its edges and centre are Type A triangles:

I	vertices	edges	triangles	
Exterior:	60	180	120	
Interior	180	60 + 3×180 = 600	180×3 = 540	
Total =	240 +	780 +	760	= 1680

Moreover, when its exterior and interior triangles are turned into tetractyses, there are 840 yods surrounding the axis of the disdyakis triacontahedron:

	vertices	hexagonal yods on edges	hexagonal yods at centres of tetractyses	
Exterior:	60	180×2 = 360	120	
Interior:	0	60×2 = 120	180	
Total =	60 +	480 +	300	= 840

As the hexagon is the fourth regular polygon, this is confirmation of the Tetrad Principle, which states that the fourth member of a class of mathematical object embodies numbers that are characteristic of holistic systems. It is the fourth member of a set of polygons that constitute a holistic system displaying properties
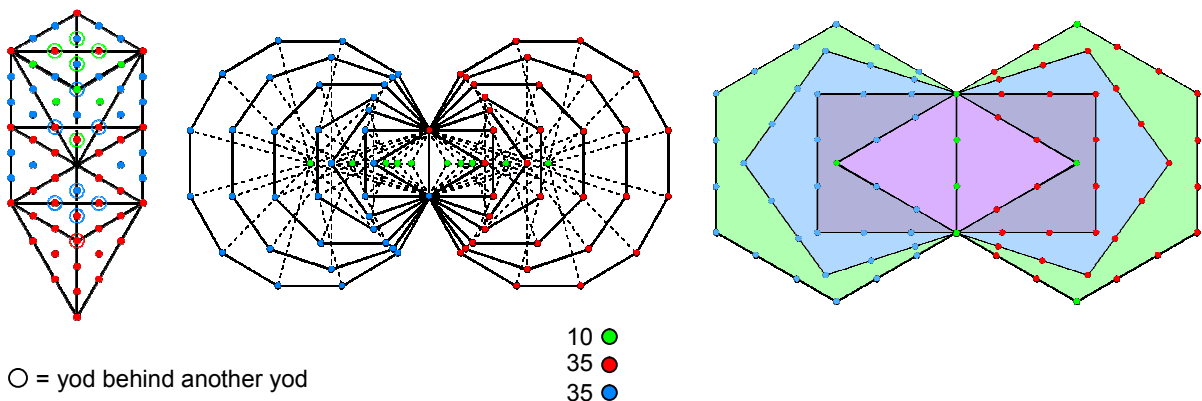

○ = yod behind another yod

10 ●
35 ●
35 ●

Figure 17. The equivalence of the 1-tree, the inner Tree of Life and the first (4+4) enfolded polygons.

that are analogous to the Tree of Life. The most obvious example of this is that, as was found in the analysis of the yod composition of the first (4+4) enfolded Type A polygons, **80** yods line the sides of these polygons. This is the number value of Yesod and the number of yods in the 1-tree (Fig. 17) when its 19 triangles are turned into tetractyses, as well as the number of corners of the 94 sectors of the (7+7) enfolded polygons of the inner Tree of Life. The 35 red yods making up its 'trunk':

> Point (Kether)
> Line (Path connecting Chokmah & Binah)
> Triangle ((triangle with Chesed, Geburah & Tiphareth at its corners)
> Tetrahedron (tetrahedron with Netzach, Hod, Yesod & Malkuth at its vertices)

have their counterpart in the 35 red corners associated with one set of seven enfolded polygons and in the 35 red yods lining the outer sides of one set of the first four enfolded polygons, apart from the corner of the triangle and the top and bottom corners of the hexagon, which coincide with Sephiroth on the Pillar of Mercy of the Tree of Life, i.e., the 35 red yods are *intrinsic* to the four polygons themselves). The 35 blue yods making up the 'branches' of the 1-tree (the region outside its trunk) correspond to the 35 blue corners associated with the other set of seven enfolded polygons and to the 35 blue yods lining the outer sides of the other set of the first four enfolded polygons, apart from the corner of the triangle and the top and bottom corners of the hexagon, which coincide with Sephiroth on the Pillar of Judgement. The 10 green yods added by conversion of the Tree of Life with 70 yods into the 1-tree with **80** yods correspond to the 10 green centres of the (7+7) enfolded polygons that do not coincide with Sephiroth and to the 10

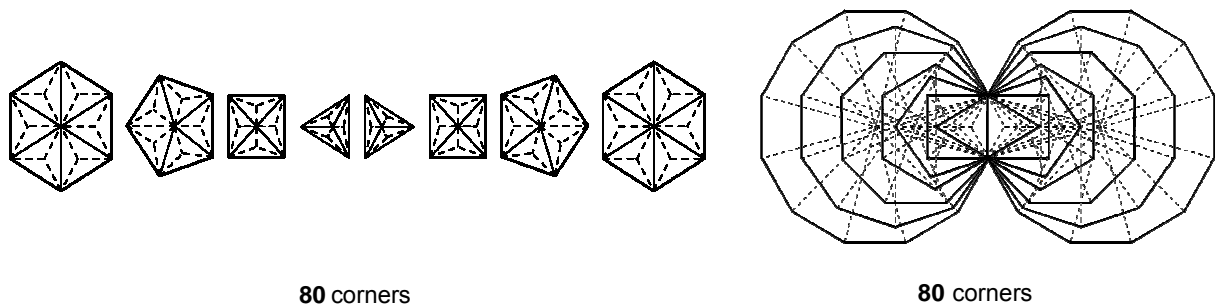**80** corners   **80** corners

Figure 18. The 102 triangles in the (4+4) separate Type B polygons have **80** corners. They are the counterpart of the **80** corners of the 94 sectors of the (7+7) enfolded Type A polygons of the inner Tree of Life.

green yods in the first (4+4) enfolded polygons that either coincide with Sephiroth or are in the root edge. Such natural, one-to-one correspondence with both the outer and inner forms of the Tree of Life is clear evidence that the first (4+4) enfolded polygons constitute what in previous articles has been called a "Tree of Life pattern" that embodies properties of holistic systems, such as the **248** gauge bosons that transmit the $E_8 \times E_8$'-invariant interaction between heterotic superstrings and the structural parameters 840 and 1680 of the $E_8 \times E_8$' heterotic superstring.

This conclusion about the holistic character of the first (4+4) polygons is given further support by the fact that the 108 triangles making up the first (4+4) separate Type B polygons have **80** corners (see comment 1 on page 6). They correspond to the **80** corners of the 94 sectors of the (7+7) enfolded Type A polygons (Fig. 18). The eight centres of the two sets of the first four polygons correspond to the eight corners of the latter that coincide with the positions of Daath, Tiphareth and the Sephiroth on the side pillars of the Tree of Life. The first (4+4) separate Type A polygons have **80** corners & sectors (see comment 8 on page 4). Truly, the Tetrad reveals here its power in defining members of classes of mathematical objects that have a holistic character because they are equivalent to, or embody properties of, the universal blueprint of the Tree of Life and other sacred geometries.

References

1) For a discussion of CTOL, see Article 5 at http://smphillips.8m.com/html/articles.html.

2) Phillips, Stephen M. Article 36: "The Sri Yantra-like pattern of the 15 layers of vertices in the disdyakis triacontahedron and its scientific meaning," http://www.smphillips.8m.com/article36.pdf, p. 10. Here is the proof that the (7+7) enfolded Type B polygons have 1370 yods:

	triangle	square	pentagon	hexagon	octagon	decagon	dodecagon
Number of yods =	46	61	76	91	121	151	181

Of the 727 yods in the 7 separate polygons, the centre of the decagon coincides with a corner of the pentagon when they become enfolded. Fifteen yods in a sector of the hexagon disappear when it is replaced by the triangle in the enfolded set of polygons. (6×4=24) yods in the root edges of 6 polygons disappear when they become enfolded. Number of yods in the 7 enfolded polygons = 727 – 1 – **15** – 24 = 687. Number of yods in the (7+7) enfolded polygons = 687 + 687 – 4 = 1370.

3) Phillips, Stephen M. Article 4: "The Godnames prescribe the inner Tree of Life," http://www.smphillips.8m.com/article04.pdf, p. 4.

4) Proof: a 2nd-order tetractys contains 85 yods. 13 yods lie along each side. An n-gon with its n sectors turned into 2nd-order tetractyses has (85–13=**72**)n + 1 yods.

5) Besant, A, and Leadbeater, C.W. "Occult Chemistry," 3rd ed., Theosophical Publishing House, Adyar, Madras, India, 1951.

6) Phillips, Stephen M. Article 23: "The polyhedral Tree of Life," http://www.smphillips.8m.com/article23.pdf.