

ARTICLE 27

How the Disdyakis Triacontahedron Embodies the Structural Parameter 1680 of the $E_8 \times E_8$ Heterotic Superstring

by

Stephen M. Phillips

Flat 3, 32 Surrey Road South. Bournemouth. Dorset BH4 9BP. England.

E-mail: stephen@smphillips.8m.com

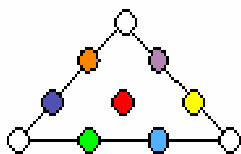
Website: <http://www.smphillips.8m.com>

Abstract

The Pythagorean tetractys of ten points ('yods') symbolises the geometrical sequence: point, line, triangle and tetrahedron. Its next higher-order differentiation has 85 yods that denote the 85 geometrical elements needed to construct the tetrahedron from tetractyses. The 15 Archimedean solids and their duals — the 15 Catalan solids — can likewise be constructed from this template. Starting from its centre, the construction of the truncated tetrahedron requires 248 geometrical elements. The simplest Archimedean solid therefore embodies the dimension 248 of the superstring gauge symmetry group E_8 . Its dual, the triakis tetrahedron, has 168 geometrical elements surrounding the axis passing through two opposite vertices. The simplest Catalan solid therefore embodies the superstring structural parameter 168 discussed by the author in many previous articles. Its 78 edges and 90 vertices and triangles correspond to the gematraic number values of the Hebrew words 'Cholem' and 'Yesodeth' in the Kabbalistic Mundane Chakra of Malkuth. This geometrical composition has its exact counterpart in the yod population of the first six regular polygons of the inner Tree of Life. When its faces are constructed from three tetractyses, the triakis tetrahedron has 240 elements surrounding its axis, i.e., 72 extra ones. E_8 has 240 non-zero roots, of which 72 are also non-zero roots of its exceptional subgroup E_6 . The disdyakis triacontahedron has 1680 geometrical elements surrounding its axis. This is the number of circularly polarised oscillations in each of the ten whorls of the 'ultimate physical atom', the basic unit of matter described paranormally 105 years ago by Annie Besant and C.W. Leadbeater and identified in earlier work by the author as the spin- $\frac{1}{2}$ subquark state of the $E_8 \times E_8$ heterotic superstring. The most complex Catalan solid therefore embodies in its geometry the number characterising the oscillatory form of the superstring. The geometrical composition of this solid is represented by the yod population of the first six polygons enfolded in ten Trees of Life. The disdyakis triacontahedron is their polyhedral counterpart. The numbers of geometrical elements in the triakis tetrahedron are the numbers of non-zero roots of E_8 , E_7 and E_6 . The disdyakis triacontahedron has corresponding numbers that are ten times as large, demonstrating its ten-fold Tree of Life nature.

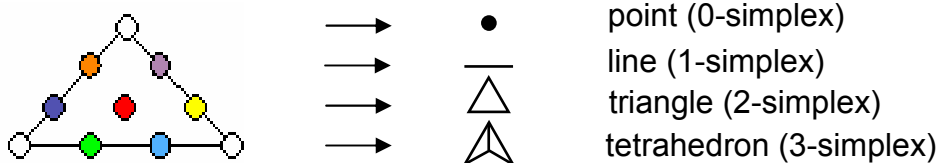
1. Introduction

The Pythagorean tetractys is a triangular array of ten dots:



These dots will be called 'yods.* The six yods at the corners of a hexagon and the yod at its centre will be called 'hexagonal yods.' They are shown above as coloured circles.

The four rows of points in the tetractys symbolise the sequence of what mathematicians call the first four "simplexes":



As the 3-simplex, the tetrahedron consists of 4 vertices (0-simplexes) and 6 edges (1-simplexes) (i.e., ten simplexes), 4 triangles and one tetrahedron (i.e., 5 simplexes), totalling 15 simplexes. The simplest regular polyhedron is prescribed by the Godname Yah (YH) assigned to Chokmah in the Tree of Life with number value 15, the value 10 of Y (yod) denoting the number of 0-simplexes and 1-simplexes and the value 5 of H (heh) denoting the number of 2-simplexes and 3-simplexes. The complete Godname Yahweh (YHVH) prescribes the sequence of the first four simplexes because it contains 26 vertices, edges, triangles and tetrahedra:

	0-simplexes	1-simplexes	2-simplexes	3-simplexes	
●	1	0	0	0	} 11
—	2	1	0	0	
△	3	3	1	0	} 15
△	4	6	4	1	
	6 = V		5 = H		} 26
	10 = Y		5 = H		

The letter values of Y and H are as above, the value 6 of V (vau) is the number of 0-simplexes in the first three simplexes (shown encircled above) and the value 5 of the second letter H is the number of 1- and 2-simplexes in the 1-simplex and the 2-simplex.

Any polyhedron can be constructed from tetractyses by dividing its polygonal faces into triangular sectors and then changing the latter into tetractyses — it does not matter whether these sectors are equilateral or isosceles triangles. The edges of the polyhedron are also edges of internal triangles formed by joining its centre to two adjacent vertices. These triangles can be constructed from three tetractyses. For consistency, a polyhedron that has both triangular faces and faces of other shapes must have the former divided into three tetractyses rather than be considered as single ones. However, if all its faces are triangular, there are two possible constructions: either each face is a tetractys (case A) or each face is divided into three tetractyses (case B).

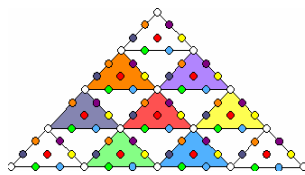
With its faces and internal triangles formed by its edges single tetractyses (case C), the tetrahedron contains also one internal vertex (its centre) and 4 internal edges of triangles (totalling 5 elements) and 6 triangles, a total of 11 geometrical elements. This is the same as the number of elements in the first three simplexes, as shown above. Starting from a point, 24 more vertices, edges and triangles have to be put in place before the interior and exterior of the tetrahedron can be constructed from triangles.

* The tenth letter of the Hebrew alphabet is yod (י), which looks like a dot.

With its faces and internal triangles divided into three tetractyses (case B), the geometrical composition of the tetrahedron is:

	vertices	+	edges	+	triangles	=	
faces:	8		18		12		38
interior:	7		22		18		47
Total =	<u>15</u>		<u>40</u>		<u>30</u>		<u>85</u>

It has 15 vertices prescribed by Yah with number value 15 and 26 vertices and edges in its faces defined by Yahweh with number value 26. It has 70 edges and triangles and 85 vertices, edges and triangles. This property proves its wholeness and perfection as the fundamental building block of solid geometry, for the next higher order tetractys:



has 85 yods. Each yod symbolises a geometrical element needed to construct the tetrahedron. The 15 (white) yods at the corners of the ten tetractyses denote the 15 vertices of the tetractyses forming the tetrahedron and the 70 hexagonal (coloured) yods denote the 70 edges and triangles. The Godname Eloha with number value 36 prescribes this symbol of the simplest regular polyhedron because 36 yods lie on its boundary. It is prescribed by the Godname El Chai assigned to Yesod with number value 49 because 49 yods are inside its boundary.

The Tetrad Principle¹ expresses this measure of a holistic system as

$$85 = \begin{array}{ccc} 4^0 & & 4^1 \\ & \square & \\ 4^3 & & 4^2 \end{array}$$

The number of geometrical elements surrounding the centre of the tetrahedron and symbolised by the 84 yods surrounding the central one is 84, where

$$84 = \begin{array}{ccc} 1^2 & & 3^2 \\ & \square & \\ 7^2 & & 5^2 \end{array}$$

i.e., 84 is the sum of the squares of the first four odd integers.

2. The Archimedean and Catalan Solids

There are 13 Archimedean solids (polyhedra with two or more types of regular polygons as faces). Two have different mirror images (chiral partners). Their duals (vertices replaced by faces and faces replaced by vertices) are the Catalan solids. The two sets have 26 members, showing how Yahweh prescribes this family of polyhedra, whilst Yah with number value 15 prescribes each set with the enantiomorphic members included. Their properties will now be calculated and then discussed.

Definitions: C = number of vertices of polyhedron.
E = number of polyhedral edges.

F = number of polyhedral faces.

C, E and F are related by Euler's equation:

$$C - E + F = 2.$$

m = number of triangular sectors of a face with m edges.

n(m) = number of faces with m sides in polyhedron.

$L \equiv \sum_m mn(m)$ = number of triangular sectors of faces.

The internal triangles formed by the centre of the polyhedron and two ends of an edge are divided into three tetractyses; any triangular faces are either considered as single tetractyses (case A) or divided into three tetractyses (case B).

Surface

Number of vertices in faces = C (case A)

$$= C + F = 2 + E \text{ (case B).}$$

Number of edges in faces = E (case A)

$$= E + \sum_m mn(m) = E + L \text{ (case B).}$$

Number of tetractyses in faces = F (case A)

$$= L \text{ (case B).}$$

Number of vertices, edges & tetractyses = C + E + F = 2E + 2 (case A)

$$= C + F + E + 2L = 2E + 2L + 2 \text{ (case B).}$$

Table 1. Population of geometrical elements in the Archimedean & Catalan solids

N'	F	E	C	Archimedean solid
244	8	18	12	truncated tetrahedron
322	14	24	12	cuboctahedron
490	14	36	24	truncated cube
490	14	36	24	truncated octahedron
646	26	48	24	rhombicuboctahedron
802	38	60	24	snub cube
802	38	60	24	snub cube (chiral partner)
808	32	60	30	icosidodecahedron
882	26	72	48	truncated cuboctahedron
1228	32	90	60	truncated icosahedron
1228	32	90	60	truncated dodecahedron
1618	62	120	60	rhombicosidodecahedron
2008	92	150	60	snub dodecahedron
2008	92	150	60	snub dodecahedron (chiral partner)
2458	62	180	120	truncated icosidodecahedron

Catalan solid	F	E	C	N	N'
triakis tetrahedron	12	18	8	168	240
rhombic dodecahedron	12	24	14	-	324
triakis octahedron	24	36	14	336	480
tetrakis hexahedron	24	36	14	336	480
deltoidal icositetrahedron	24	48	26	-	648
pentagonal icositetrahedron	24	60	38	-	816
pentagonal icositetrahedron (chiral partner)	24	60	38	-	816
rhombic triacontahedron	30	60	32	-	810
disdyakis dodecahedron	48	72	26	672	960
triakis icosahedron	60	90	32	840	1200
pentakis dodecahedron	60	90	32	840	1200
deltoidal hexacontahedron	60	120	62	-	1620
pentagonal hexacontahedron	60	150	92	-	2040
pentagonal hexacontahedron (chiral partner)	60	150	92	-	2040
disdyakis triacontahedron	120	180	62	1680	2400

Interior

Number of vertices = E + 1.

Number of edges = C + 3E.

Number of tetractyses = 3E.

Number of vertices, edges & tetractyses = C + 7E + 1.

$$\begin{aligned} \text{Number of vertices, edges \& tetractyses} &= 2C + 8E + F + 1 = C + 9E + 3 \text{ (case A)} \\ &= 2C + 8E + F + 2L + 1 \text{ (case B)} \\ &= C + 9E + 2L + 3 \text{ (case B)} \end{aligned}$$

The axis is made up of five geometrical elements (three vertices, two edges). In case A, the number of elements surrounding the axis $\equiv N = 2C + 8E + F + 1 - 5$
 $= 2C + 8E + F - 4 = C + 9E - 2$.

In case B, the number $\equiv N' = 2C + 8E + F + 2L + 1 - 5 = C + 9E + 2L - 2$.

Table 1 gives values of N and N' for the Archimedean and Catalan solids (as none of the former have just triangular faces, only values of N' can be listed for them). We find that the simplest Catalan solid — the triakis tetrahedron — has 168 geometrical elements surrounding an axis through two vertices, when its triangular faces are tetractyses (case A), and 240 elements if they are divided into three tetractyses (case B), that is, 72 more elements are needed to construct the faces from three tetractyses instead of from one. We also see that the most complex Catalan solid — the disdyakis triacontahedron — has 1680 (=168×10) geometrical elements in case A and 2400 (=240×10) elements in case B, i.e., 720 (=72×10) more elements are needed. *Numbers of elements for the last Catalan solid are exactly ten times their counterparts for the first one.* The first and last Catalan solids are equivalent in this sense. The division:

$$240 = 168 + 72$$

corresponds in the group mathematics of the superstring gauge symmetry group E_8 to the fact that, of its 240 non-zero roots, 72 are non-zero roots of its exceptional

Table 2. Number values of the Sephiroth in the four Kabbalistic worlds

Sephirah	Title	Godname	Archangel	Order of Angels	Mundane Chakra
Kether	620	21	314	833	636
Chokmah	73	15, 26	331	187	140
Binah	67	50	311	282	317
Chesed	72	31	62	428	194
Geburah	216	36	131	630	95
Tiphareth	1081	76	101	140	640
Netzach	148	129	97	1260	64
Hod	15	153	311	112	48
Yesod	80	49	246	272	87
Malkuth	496	65, 155	280	351	168

subgroup E_6 leaving 168 simple roots. Notice that 168, 240, 1680 and 2400 do not appear in the table of numbers for the Archimedean solids. Notice also that the tenth Catalan solid — the triakis icosahedron — has 840 geometrical elements surrounding its axis, whilst the 15th solid has 1680 such elements. This corresponds to the fact that the 1680 circularly polarized oscillations of each whorl of a subquark superstring comprise 840 oscillations making $2\frac{1}{2}$ outer revolutions around its axis and 840 oscillations making $2\frac{1}{2}$ revolutions in the core of the superstring (see Fig. 6). This division is prescribed by the Godname Yah (YH) of Chokmah. Its number 15 (see the

upper orange cell in Table 1) prescribes the disdyakis triacontahedron as the 15th Catalan solid, whilst the letter value Y = 10 in YH defines that Catalan solid which has 840 elements surrounding its axis. 168 is the number value of Cholem Yesodeth, the Mundane Chakra of Malkuth (see the last cell in Table 1).

The twelve faces of the triakis tetrahedron are isosceles triangles with one long side and two short sides. They are arranged as three-sided pyramids of height $\sqrt{6}/15$

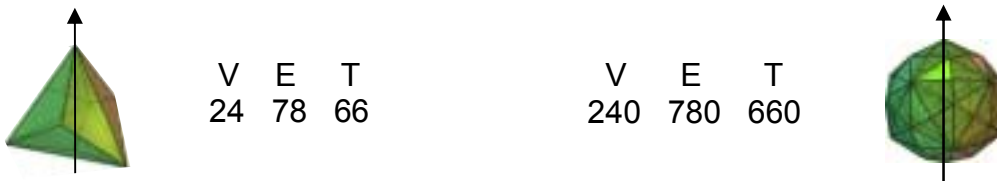


Figure 1. The first and last Catalan solids are Pythagorean counterparts of each other because their corresponding vertices (V), edges (E) and triangles (T) that surround an axis through two opposite vertices differ in number by a factor of 10.

constructed on the four faces of a regular tetrahedron with unit edge length (Fig. 1). The geometrical composition of the triakis tetrahedron in case A is given below:

	Vertices	Edges	Triangles
surface:	8	18	12
minus:	-2 (poles)		
	Subtotal = 6		
interior:	18	$8 + 3 \times 18 = 62$	$3 \times 18 = 54$
minus:		-2 (two edges)	
		Subtotal = 60	
	<u>24</u>	+	<u>78</u>
		+	<u>66</u>
			= 168

Compare these numbers with the geometrical composition of the disdyakis triacontahedron in case A:

	Vertices	Edges	Triangles
surface:	62	180	120
minus:	-2 (poles)		
	Subtotal = 60		
interior:	180	$62 + 3 \times 180 = 602$	$3 \times 180 = 540$
minus:		-2 (two edges)	
		Subtotal = 600	
	<u>240</u>	+	<u>780</u>
		+	<u>660</u>
			= 1680.

$$\begin{array}{rcc}
 \text{יְסוּדָאֵט} & \text{חֵלֶם} & \\
 \text{Th A D U S Y} & \text{M L Ch} & = 168 \\
 9 \ 1 \ 4 \ 6 \ 60 \ 10 & 40 \ 30 \ 8 & \\
 \underbrace{\hspace{10em}} & \underbrace{\hspace{10em}} & \\
 90 & 78 &
 \end{array}$$

Figure 2. The number value of Cholem Yesodeth, Mundane Chakra of Malkuth.

Every number of elements of a given type in the exterior and interior of the disdyakis triacontahedron is exactly *ten* times the corresponding number in the same colour in the triakis tetrahedron. The triakis tetrahedron is made up of 78 edges and 90 vertices and triangles, whilst the disdyakis triacontahedron is composed of 780 edges and 900 vertices and triangles. This division of the geometrical elements comprising the basic Catalan solid reflects the number value of Cholem Yesodeth, the Hebrew name of the

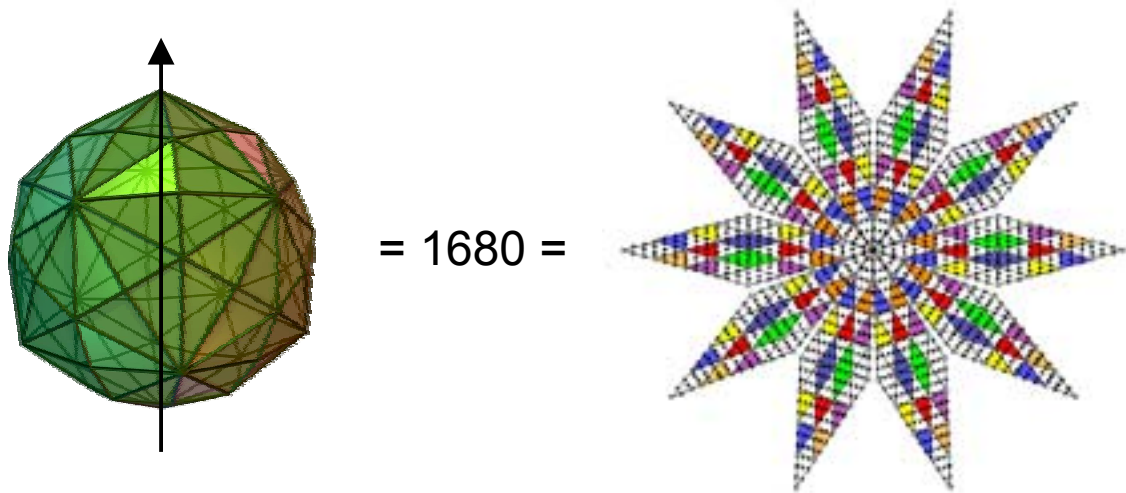


Figure 3. The faces and interior of the disdyakis triacontahedron have as many corners, sides and triangles surrounding its vertical axis as ten parallelograms constructed from tetractyses have yods surrounding their centre.

manifestation in Assiyah of the Sephirah Malkuth (Fig. 2).

The ten-fold difference between the composition of the first and last Catalan solids can be represented by a star with ten points, each one a parallelogram formed from 32 tetractyses with 168 yods below its point at the centre of the star (Fig. 3). The division:

$$168 = 90 + 78$$

reflects the 90 yods in one half of the parallelogram and the 78 yods in the other half. It

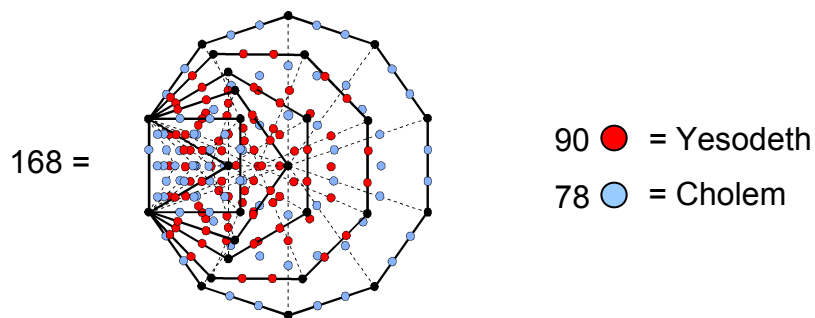


Figure 4. Associated with each set of the first six enfolded polygons are 168 yods other than corners. 78 of them are in the triangle, square and decagon and 90 yods are in the pentagon, hexagon and octagon.

is encoded in the inner form of the Tree of Life (Fig. 4). Associated with each set of its first six enfolded, regular polygons are 168 yods other than corners of the 35 tetractyses into which the six polygons can be divided. 90 of these representing the number of Yesodeth are in the square, hexagon and decagon, and 78 yods

representing the number value of Cholem are in the triangle, pentagon and octagon.

The fact that the numbers of geometrical elements in the faces and interior of the last Catalan solid are ten times the numbers of corresponding elements in the faces and interior of the first Catalan solid is further evidence of the unique status of the disdyakis triacontahedron as the completion of the Catalan solids. For the Pythagorean tetractys symbolises holistic (that is, *whole*) systems — systems that are complete in themselves rather than mere intermediate members of some developing sequence. The triakis tetrahedron is the unit, or yod, of a tetractys representing the disdyakis triacontahedron:



Viewed as part of the complete sequence of 35 solids:

5 Platonic solids → 15 Archimedean solids → 15 Catalan solids

the triakis tetrahedron is the 21st member, whilst the disdyakis triacontahedron is the 31st member when the enantiomorphic versions of solids are excluded. This shows how the Godname Ehyeh of Kether with number value 21 prescribes the first Catalan solid with 168 geometrical elements surrounding the axis joining two opposite vertices and how the Godname EL of Chesed with number value 31 prescribes the last Catalan solid with 1680 geometrical elements surround its axis. The Godname Yahweh with number 26 prescribes the number of Archimedean solids and their duals, the Catalan

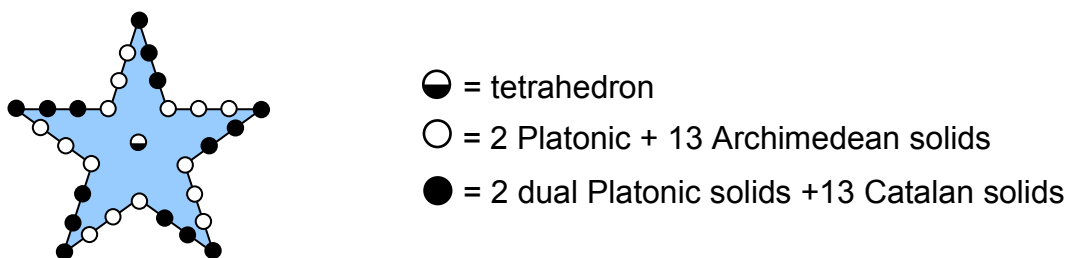


Figure 5. The pentagram represents the Godname EL with number value 31 because 31 yods define its boundary and centre. The 30 yods on the former divide into two alternating sets of 15 yods, one set symbolising 15 Platonic/Archimedean solids and the other set symbolising their duals. The central yod denotes the self-dual tetrahedron.

solids, when enantiomorphs are excluded, whilst its older version, Yah, with number value 15, is the number of each type when their enantiomorphs are included. The 31 solids comprise the tetrahedron, 15 Platonic/Archimedean solids and their duals. This 1:30 pattern is reflected in the name EL, where E = 1 and L = 30. It is also represented by the pentagram because 30 yods lie on its edge when each side of the point is made up of four yods (Fig. 5). The central yod denotes the self-dual tetrahedron and the three yods on each side of a point symbolise Platonic or Archimedean solids and their duals.

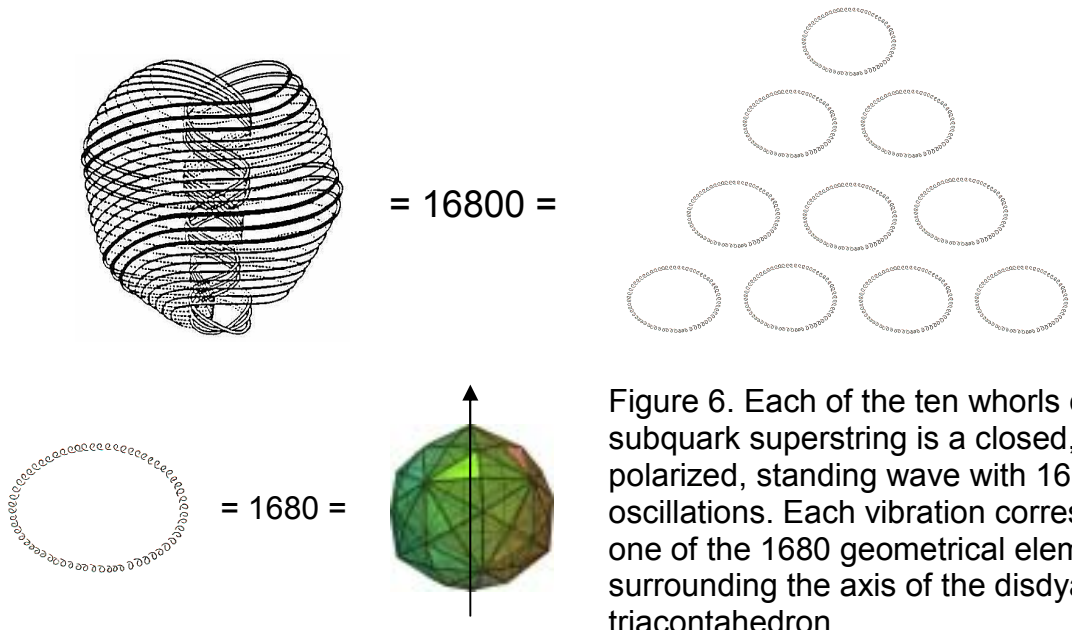


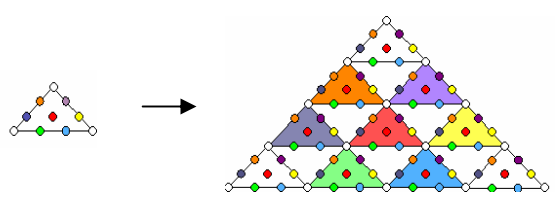
Figure 6. Each of the ten whorls of the subquark superstring is a closed, circularly polarized, standing wave with 1680 oscillations. Each vibration corresponds to one of the 1680 geometrical elements surrounding the axis of the disdyakis triacontahedron.

The tetractys pattern of ten triakis tetrahedra that is equivalent to the disdyakis triacontahedron has its counterpart in the subquark superstring, which consists of ten closed standing waves, each with 1680 circularly polarised oscillations (Fig. 6). These vibrations are the subatomic counterpart of the 1680 geometrical elements surrounding the axis of the disdyakis triacontahedron. The correspondence exists because the former is the manifestation of the Tree of Life in the 10-dimensional space-time of superstrings and the latter is its polyhedral manifestation in the 4-dimensional space-time of human beings.

The fact that the triakis tetrahedron has 168 geometrical elements surrounding its axis in case A and 240 elements in case B, i.e., 72 extra elements, itself reflects the tetractys arrangement of the number 24, where $24 = 1 \times 2 \times 3 \times 4$:

$$\begin{aligned}
 240 &= \begin{array}{cccc} & & 24 & \\ & & 24 & 24 \\ 24 & = & 24 & 24 & 24 \\ & & 24 & 24 & 24 & 24 \end{array} = \begin{array}{c} 24 \\ \triangle \\ 24 \quad 24 \end{array} + \begin{array}{c} 24-24 \\ \triangle \\ 24-24 \end{array} \\
 &= 72 + 168.
 \end{aligned}$$

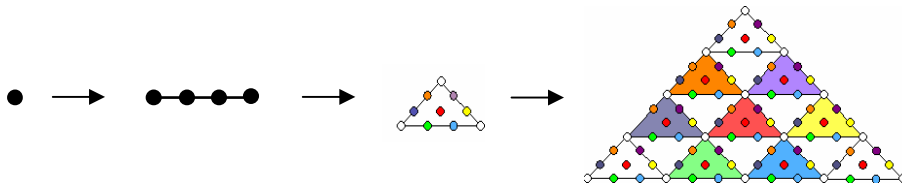
The first Catalan solid is constructed by what mathematicians call “cumulation” of a tetrahedron by a pyramid. Constructed from tetractyses, the tetrahedron in case B has 15 vertices, 40 edges and 30 triangles, totalling 85 geometrical elements. It conforms to the next higher differentiation of the tetractys:



because this consists of 85 yods, of which 15 are corners of the ten tetractyses and 70 are hexagonal yods (shown coloured). These yods signify degrees of freedom. For the tetrahedron, the 15 corners symbolise the vertices of its 30 triangles and the 70 hexagonal yods denote its 70 edges and triangles. The geometrical sequence:

point → line → triangle → tetrahedron

has its Pythagorean counterpart:



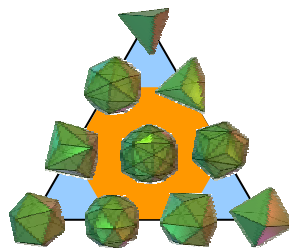
So the higher-order tetractys symbolises the geometrical composition of the tetrahedron — the simplest regular polyhedron.

Ignoring enantiomorphic Archimedean solids, the 21st solid in the sequence:

5 Platonic solids → 15 Archimedean solids → 15 Catalan solids

is the triakis octahedron. It has $F = 24$, $E = 36$, $C = 14$ and $N = 336$. Comparing these values with the values $F = 120$, $E = 180$, $C = 62$ and $N = 1680$ for the disdyakis triacontahedron, we see that the latter has five times as many faces and five times as many edges, whilst the number of polyhedral vertices surrounding an axis passing through two opposite ones = $62 - 2 = 60$, comparing with $(14 - 2 = 12)$ such vertices for the former, that is, five times as many vertices of this type. The disdyakis triacontahedron has therefore five times as many elements of each variety surrounding its axis as the triakis octahedron has surrounding its axis. This difference of a factor of 5 appears in the subquark superstring described by Annie Besant and C.W. Leadbeater (Fig. 6). Each of its ten whorls make five revolutions about its spin axis, the number of oscillations made in one revolution being $1680/5 = 336$, which is the number of geometrical elements surrounding the axis of the triakis octahedron.

Three Platonic solids (tetrahedron, octahedron & icosahedron) have all triangular faces. None of the Archimedean solids has this property, whilst seven of the 15 Catalan solids have all triangular faces: the triakis tetrahedron, the triakis octahedron, the tetrakis hexahedron, the disdyakis dodecahedron, the triakis icosahedron, the pentakis dodecahedron and the disdyakis triacontahedron. Ten of the 31 different solids have faces that are all triangular, and 21 have either non-triangular faces or a combination of both, showing that the Godname Ehyeh with number value 21 prescribes the latter. The ten polyhedra with triangular faces form a tetractys array:



The Platonic solids are at the corners and the seven Catalan solids are at the corners and centre of a hexagon (shown in orange above), the disdyakis triacontahedron occupying the latter position because this corresponds to the Sephirah Malkuth, whose superstring manifestation the polyhedron embodies. The ten solids have 210 polyhedral vertices, where

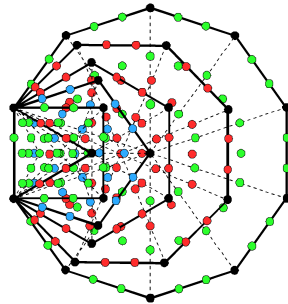
$$210 = \begin{array}{cccc} & & 21 & \\ & & 21 & 21 \\ & 21 & 21 & 21 \\ 210 = & 21 & 21 & 21 & 21, \end{array}$$

triakis tetrahedron



= 168 =

Yesodeth = 90
 24 vertices
 78 edges → Cholem
 66 triangles



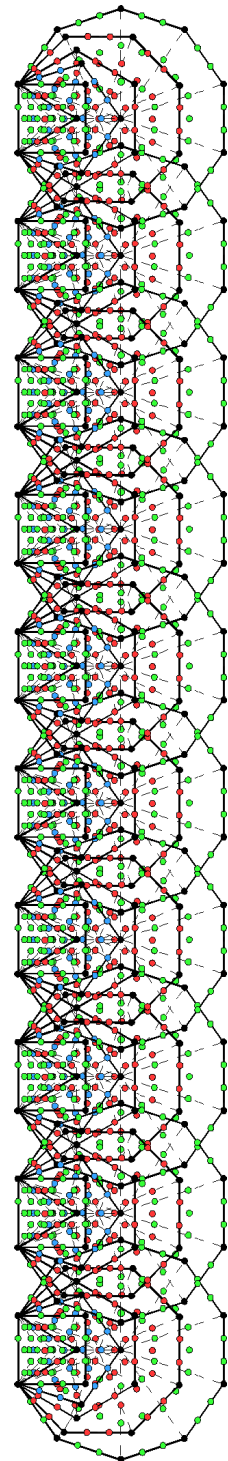
24(●) + 78(●) + 66(●)
 pentagon triangle hexagon
 square octagon
 decagon

disdyakis triacontahedron



= 1680 =

240 vertices
 780 edges
 660 triangles



240(●) + 780(●) + 660(●)

Figure 7. The 168 yods associated with the first 6 polygons of the inner Tree of Life other than their 26 corners symbolise the 168 geometrical elements surrounding an axis of the triakis tetrahedron. The 1680 yods associated with the 60 polygons in ten trees other than their 251 corners symbolise the 1680 geometrical elements surrounding the axis of the disdyakis triacontahedron. Blue yods denote vertices, green yods denote edges and red yods denote triangles.

showing how Ehyeh prescribes this set of polyhedra.

We found earlier that, in case A, where its faces are single tetractyses, the triakis tetrahedron has 66 triangles with 24 vertices and 78 edges surrounding its axis, these numbers generating the number values of the Hebrew words ‘Cholem’ and ‘Yesodeth’ in the name of the Mundane Chakra of Malkuth. They are symbolised in Fig. 7 by the 168 yods other than corners associated with the first six regular polygons enfolded in

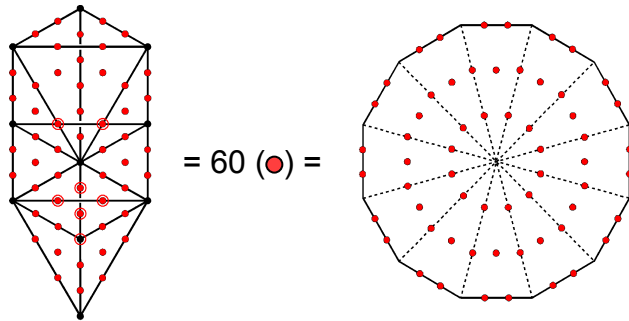


Figure 8. Constructed from tetractyses, the dodecagon has as many hexagonal yods as the Tree of Life.

the inner Tree of Life. The 24 vertices are denoted by the 24 yods of the pentagon outside the shared edge. The 78 edges are symbolised by the 78 yods of the triangle, square and decagon and the 66 triangles correspond to the 66 yods of the hexagon and octagon. We also found that, in case A, the disdyakis triacontahedron has 660 triangles with 240 vertices and 780 edges surrounding its axis. As the first Catalan solid, the triakis tetrahedron is the polyhedral manifestation of the first six polygons enfolded in the inner Tree of Life. As the last Catalan solid, the disdyakis triacontahedron is the

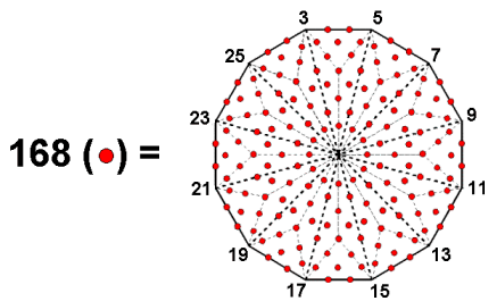


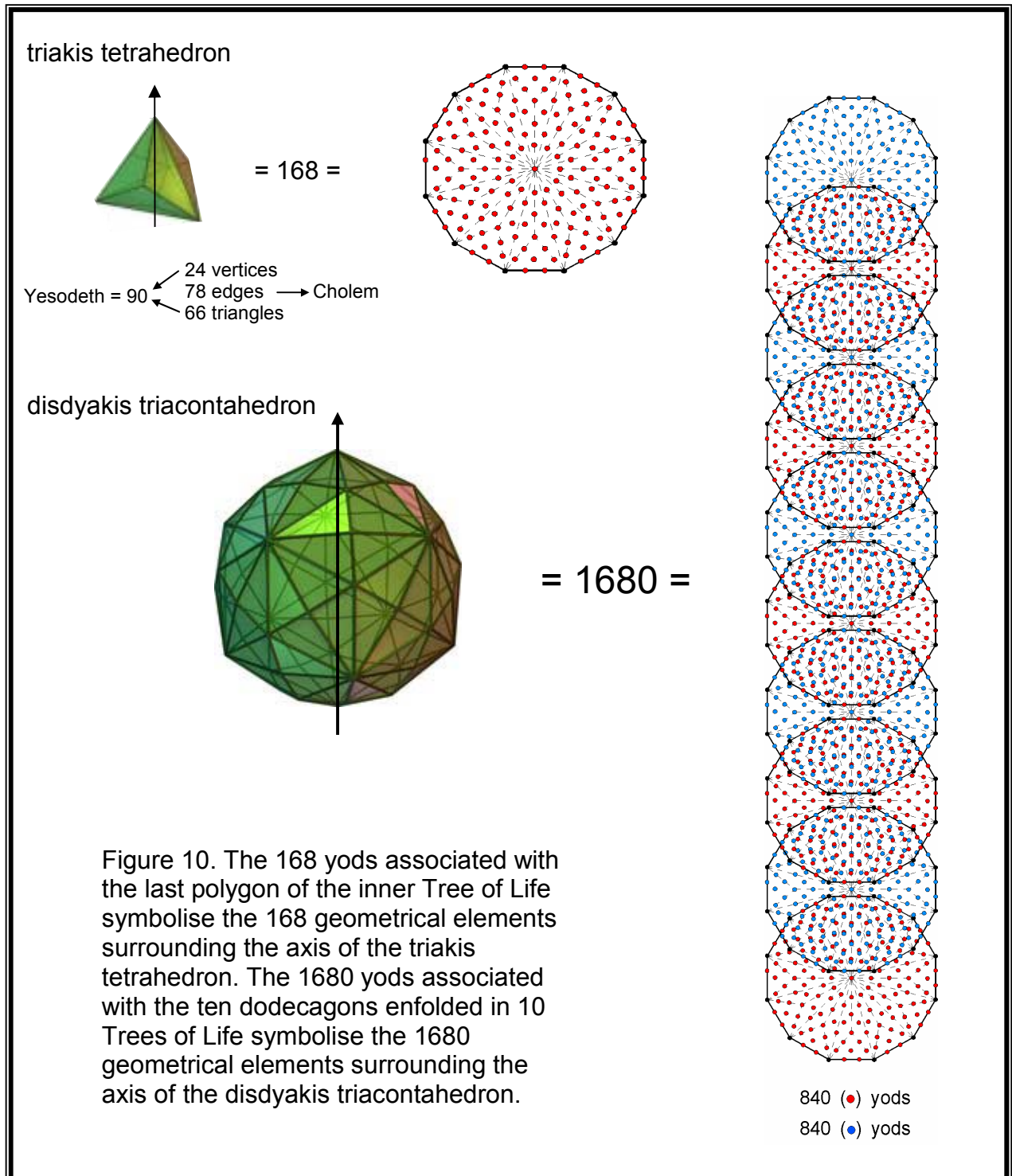
Figure 9. Divided into 12 sectors, a dodecagon needs 168 more yods to transform each sector into three tetractyses. This is the sum of the first 12 odd integers after 1 that can be assigned to its corners.

polyhedral version of the 60 polygons enfolded in ten overlapping Trees of Life, each yod symbolising a vertex, edge or triangle. Because each Sephirah of the Tree of Life is itself ten-fold, the complete, expanded version of the latter is the set of ten overlapping Trees of Life. This illustrates the holistic nature of the disdyakis triacontahedron: its remarkable geometry makes it the counterpart of the holistic system of ten Trees of Life. Table 1 indicates that none of the Archimedean solids has the property that the number of geometrical elements surrounding an axis passing through two vertices is ten times that of another polyhedron of this type. The disdyakis triacontahedron is unique in this regard for the Catalan solids as well.

When constructed from tetractyses, the dodecagon has 73 yods. Twelve of these are at its corners and one is at its centre, so that 60 extra yods are needed to turn its sectors into tetractyses (Fig. 8). This is the number of hexagonal yods in the 16 triangles of the Tree of Life — a sign (which will be confirmed shortly) that the last of the seven regular polygons making up the inner Tree of Life embodies properties of its outer form. With its sectors divided into three tetractyses, the dodecagon has 181 yods, i.e., 168 yods other than its corners and centre (Fig. 9). As

$$13^2 - 1 = 168 = 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25,$$

this is the sum of the twelve odd integers after 1 that can be assigned to the corners of the dodecagon. Just as there are 168 yods other than corners associated with the first six enfolded polygons, so there are 168 yods other than corners associated with the last polygon enfolded on each side of the central Pillar of Equilibrium. The dodecagons



enfolded on one side of ten overlapping Trees of Life have 1680 such yods — exactly the same number of yods as in the first six polygons enfolded in ten Trees of Life. What is embodied in the first six polygons as potential degrees of freedom manifest in the seventh polygon as actual degrees of freedom determining its form. Fig. 10 indicates

the correspondence between the triakis tetrahedron and the dodecagon and the correspondence between the disdyakis triacontahedron and the dodecagons enfolded in ten Trees of Life.

We found earlier that, in case B, when its faces are constructed from three tetractyses, 72 more geometrical elements are added to the 168 elements surrounding the axis of the triakis tetrahedron. The counterpart of this in the inner Tree of Life is that there are 168 yods other than corners associated with the first six polygons and 72 yods associated with the dodecagon. Likewise, 720 more geometrical elements are added to the 1680 elements surrounding the axis of the disdyakis triacontahedron when its faces are constructed from three tetractyses. Its counterpart in the inner Tree of Life is that

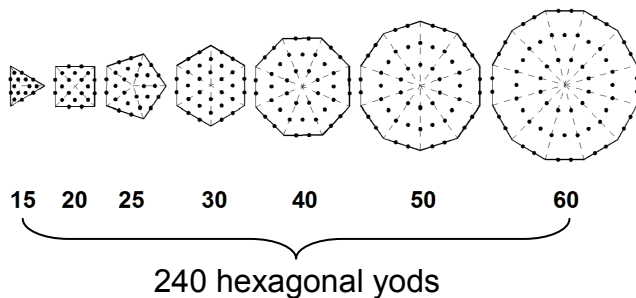


Figure 11. The seven separate polygons have 240 hexagonal yods.

there are 720 yods associated with the ten dodecagons enfolded in ten Trees of Life. Construction of both Catalan solids according to Case A generates the counterpart of the first six regular polygons and construction according to case B creates the polyhedral counterpart of all seven polygons. The 240 geometrical elements of the triakis tetrahedron are symbolised by the 240 hexagonal yods belonging to the seven *separate* polygons (Fig. 11). The extra 720 geometrical elements in the 120 faces of the disdyakis triacontahedron for case B are symbolised by the 720 yods in the seven

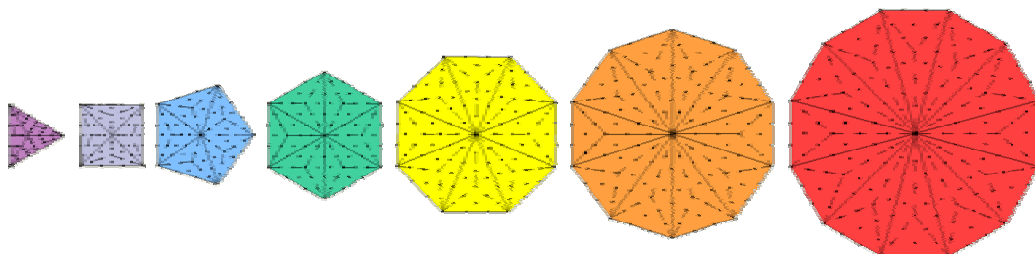


Figure 12. 720 yods surround the centres of the seven separate polygons when their sectors are divided into three tetractyses.

separate polygons that surround their centres when their 48 triangular sectors are each turned into three tetractyses (Fig. 12). It cannot be coincidence that the same number should appear in contexts where the polygonal and polyhedral forms of the inner Tree of Life are transformed in the same way. When constructed from tetractyses, the five Platonic solids have 820 yods in their 50 faces, of which 720 are hexagonal yods.² The very surfaces of the five regular polyhedra embody this number as well.

It cannot also be coincidence that the disdyakis triacontahedron has 62 vertices (31 vertices and their inversions), in view of the facts that the number value of Chesed, the first Sephirah of Construction, is 72, its Godname El has number value 31 and its Archangel Tzadkiel (“Benevolence of God”) has number value 62 (Table 2).

As $49^2 - 1 = 2400 = 3 + 5 + 7 + 9 + \dots + 97$, the number (2400) of geometrical elements

in the disdyakis triacontahedron in case B is the sum of the first 48 odd integers after 1. As Table 2 indicates, Kokab, the Mundane Chakra of Hod, has the number value 48 and Haniel ("Grace of God"), the Archangel of Netzach, has the number value 97.

The integers 1, 2 3 & 4 symbolised by the tetractys express the number 2400 as:

$$2400 = 1 \times 2 \times 3 \times 4 (1^3 + 2^3 + 3^3 + 4^3) = 1 \times 2 \times 3 \times 4 (1 + 2 + 3 + 4)^2.$$

As $24 = 5^2 - 1 = 3 + 5 + 7 + 9$,

$$2400 = 24 \times 100 = \begin{array}{|c|c|} \hline 3 & 5 \\ \hline \square & \\ \hline 9 & 7 \\ \hline \end{array} \times \begin{array}{|c|c|} \hline 1^3 & 2^3 \\ \hline \square & \\ \hline 4^3 & 3^3 \\ \hline \end{array}$$

illustrating how the Tetrad Principle³ determines this number.

The disdyakis triacontahedron has 30 polyhedral vertices and their 30 inversions surrounding its axis, where

$$30 = \begin{array}{|c|c|} \hline 2^1 & 2^2 \\ \hline \square & \\ \hline 2^4 & 2^3 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 1^2 & 2^2 \\ \hline \square & \\ \hline 4^2 & 3^2 \\ \hline \end{array}$$

This again shows how the Tetrad Principle determines properties of this solid.

We found earlier that, as the first Catalan solid, the triakis tetrahedron has 168 geometrical elements surrounding its axis, where

$$168 = \begin{array}{|c|c|c|c|} \hline 3 & 5 & 7 & 9 \\ \hline 25 & \square & 11 & \\ \hline 23 & & 13 & \\ \hline 21 & 19 & 17 & 15 \\ \hline \end{array}$$

This further illustrates the Tetrad Principle, for the odd integers 3, 5, 7, etc are arranged along the sides of a *four*-sided square, *four* to a side.

Since $168 = 13^2 - 1$,

$$16800 = 100 \times 168 = 10^2 \times (13^2 - 1) = 130^2 - 100.$$

But

$$1300 = 1^5 + 2^5 + 3^5 + 4^5$$

and

$$100 = 1^3 + 2^3 + 3^3 + 4^3 = (1 + 2 + 3 + 4)^2.$$

Therefore,

$$16800 = \frac{(1^5 + 2^5 + 3^5 + 4^5)^2 - (1^3 + 2^3 + 3^3 + 4^3)^2}{(1 + 2 + 3 + 4)^2}$$

and

$$1680 = \frac{(1^5 + 2^5 + 3^5 + 4^5)^2 - (1^3 + 2^3 + 3^3 + 4^3)^2}{(1 + 2 + 3 + 4)^3}.$$

This shows how the integers 1, 2, 3 & 4 express the superstring parameters 16800 and 1680. The former number has also the remarkable representation:

$$16800 = \left(\begin{array}{cccccc} 4^1 & & & & & \\ 3^1 & 3^1 & & & & \\ 2^1 & 2^1 & 2^1 & & & \\ 1^1 & 1^1 & 1^1 & 1^1 & & \\ & 4^2 & & & & \\ & 3^2 & 3^2 & & & \\ & 2^2 & 2^2 & 2^2 & & \\ & 1^2 & 1^2 & 1^2 & 1^2 & \\ & & 4^3 & & & \\ & & 3^3 & 3^3 & & \\ & & 2^3 & 2^3 & 2^3 & \\ & & 1^3 & 1^3 & 1^3 & 1^3 \\ & & & 4^1 & 4^1 & 4^1 & 4^1 \\ & & & 2^2 & 2^2 & & \\ & & & 3^3 & 3^3 & 3^3 & \\ & & & 4^2 & 4^2 & 4^2 & 4^2 \\ & & & & 4^3 & 4^3 & 4^3 & 4^3 \end{array} \right)_{1 \times 2 \times 3 \times 4}$$

The number 2400 has the representation:

$$2400 = \begin{array}{cccccc} & 1^1 & & 1^2 & & 1^3 & & 1^4 & & 4^3 & & 4^4 \\ & 2^1 & 2^1 & & 2^2 & 2^2 & & 2^4 & 2^4 & & 3^3 & 3^3 & & 3^4 & 3^4 \\ 3^1 & 3^1 & 3^1 & + & 3^2 & 3^2 & 3^2 & + & 3^3 & 3^3 & 3^3 & + & 3^4 & 3^4 & 3^4 \\ & 4^1 & 4^1 & 4^1 & 4^1 & 4^2 & 4^2 & 4^2 & 4^2 & 4^3 & 4^3 & 4^3 & 4^3 & 4^4 & 4^4 & 4^4 \end{array}$$

There are 240 gauge fields corresponding to the 240 non-zero roots of the superstring gauge symmetry group E_8 . Each of these fields has ten components because superstring space-time is 10-dimensional. This means that the 240 fields have 2400 components. Their counterparts in the disdyakis triacontahedron are the 2400 geometrical elements surrounding its axis that are needed to define its shape when it is constructed from the building block of the tetractys.

Table 1 shows that the truncated tetrahedron — the dual of the triakis tetrahedron — has 244 geometrical elements surrounding its axis, which consists of five elements (three vertex points and two edges). The total number of elements is therefore 249. Surrounding the centre of the solid are 248 elements. Starting from the mathematical point, 248 more geometrical elements are needed to build the simplest Archimedean solid. Remarkably, this is the dimension of the superstring gauge symmetry group E_8 ! The tetractys therefore reveals the following beautiful properties of four solids that speak eloquently of their archetypal nature:



The simplest Platonic solid is the polyhedral manifestation of the holistic tetractys symbolising the integers 1, 2, 3 and 4.



The simplest Archimedean solid embodies the 248 spin-1 gauge bosons of E_8 that transmit the unified superstring force.



The simplest Catalan solid embodies the 168 circularly polarised oscillations made in a half-revolution of each whorl of a superstring.



The most complex Catalan solid embodies the 1680 circularly polarised oscillations in a whorl.

It was because of its power to describe the nature of the universe that the followers of Pythagoras swore the following oath:

“I swear by the discoverer of the tetractys
Which is the spring of all our wisdom
The perennial fount and root of Nature.”

They would have been delighted by the connections between superstring physics and solid geometry that the tetractys uncovers. This universal template has application both on a metaphysical level as the symbol of the ten-fold nature of God and on a physical level as the representation of holistic systems such as the superstring and — in its higher-order version — the tetrahedron, the basic unit of solid geometry.

Let us now calculate how many yods make up the Archimedean and Catalan solids

when they are constructed from tetractyses. Formulae are simplified by making use of Euler's equation connecting the number of vertices (C), edges (E) and faces (F) in a polyhedron:

$$C - E + F = 2.$$

Surface

Number of corners surrounding axis $\equiv V_s = C + F - 2 = E$;

Number of hexagonal yods surrounding axis $\equiv H_s = 2E + 3L$;

Number of yods surrounding axis $\equiv N_s = V_s + H_s = 3E + 3L$;

Interior

Number of corners surrounding axis $\equiv V_i = E$;

Number of hexagonal yods surrounding axis $\equiv H_i = 2C + 9E - 4 = 11E - 2F$;

Number of yods surrounding axis $\equiv N_i = V_i + H_i = 12E - 2F$;

Total number of corners surrounding axis $\equiv V = V_s + V_i = 2E$;

Total number of hexagonal yods surrounding axis $\equiv H = H_s + H_i = 13E - 2F + 3L$;

Total number of yods surrounding axis $\equiv N = N_s + N_i = 15E - 2F + 3L$.

Table 3. Populations of yods in the Archimedean and Catalan solids.

Archimedean solid	L	V _s	H _s	N _s	V _i	H _i	N _i	V	H	N
truncated tetrahedron	36	18	144	162	18	182	200	36	326	362
cuboctahedron	48	24	192	216	24	236	260	48	428	476
truncated cube	72	36	288	324	36	368	404	72	656	728
truncated octahedron	72	36	288	324	36	368	404	72	656	728
rhombicuboctahedron	96	48	384	432	48	476	524	96	860	956
snub cube	120	60	480	540	60	584	644	120	1064	1284
icosidodecahedron	120	60	480	540	60	596	656	120	1076	1196
truncated cuboctahedron	144	72	576	648	72	740	812	144	1316	1460
truncated icosahedron	180	90	720	810	90	926	1016	180	1646	1826
truncated dodecahedron	180	90	720	810	90	926	1016	180	1646	1826
rhombicosidodecahedron	240	120	960	1080	120	1196	1316	240	2156	2396
snub dodecahedron	300	150	1200	1350	150	1466	1616	300	2666	2966
truncated icosidodecahedron	360	180	1440	1620	180	1856	2036	360	3296	3656

Catalan solid	L	V _s	H _s	N _s	V _i	H _i	N _i	V	H	N
triakis tetrahedron	36	18	144	162	18	174	192	36	318	354
rhombic dodecahedron	48	24	192	216	24	240	264	48	432	480
triakis octahedron	72	36	288	324	36	348	384	72	636	708
tetrakis hexahedron	72	36	288	324	36	348	384	72	636	708
deltoidal icositetrahedron	96	48	384	432	48	480	528	96	864	960
pentagonal icositetrahedron	120	60	480	540	60	612	672	120	1092	1212
rhombic triacontahedron	120	60	480	540	60	600	660	120	1080	1200
disdyakis dodecahedron	144	72	576	648	72	696	768	144	1272	1416
triakis icosahedron	180	90	720	810	90	870	960	180	1590	1770
pentakis dodecahedron	180	90	720	810	90	870	960	180	1590	1770
deltoidal hexacontahedron	240	120	960	1080	120	1220	1340	240	2180	2420
pentagonal hexacontahedron	300	150	1200	1350	150	1530	1680	300	2730	3030
disdyakis triacontahedron	360	180	1440	1620	180	1740	1920	360	3180	3540

Table 3 lists the various yod populations of the Archimedean and Catalan solids (enantiomorphic partners are excluded because their numbers are the same). The disdyakis triacontahedron has $V = 360 (= 36 \times 10)$ vertices surrounding its axis, showing how the Godname Eloha of Geburah with number value 36 prescribes this solid. It also prescribes the triakis tetrahedron because it has 36 vertices surrounding its axis and

354 yods surrounding its axis, i.e., it has 361 yods in total, 360 of them surrounding its centre. As there are three vertices on the axis, the total number of vertices in the disdyakis triacontahedron is 363. This is the number value of Shaddai El Chai (“Almighty Living God”), the Godname of Yesod, the penultimate Sephirah (see Table 2). If its faces are single tetractyses, it has 2460 (= 246×10) yods surrounding its axis. 246 is the number value of Gabriel (“Strong Man of God”), the Archangel of Yesod.

We found earlier that the numbers of vertices, edges and triangles surrounding the axis of the disdyakis triacontahedron are ten times the corresponding numbers for the triakis tetrahedron. Table 3 indicates that the numbers of corners or hexagonal yods in its faces and interior that surround the axis are ten times the corresponding numbers for the triakis tetrahedron. This is true only for *surrounding* numbers, not for the total numbers. It is also true only for the first and last members of the Catalan solids, not for the first and last members of the Archimedean solids, although some of their corresponding numbers do have this property.

The number of boundary yods in the faces of a polyhedron surrounding its axis = $C + 2E + 2L + F - 2 = 3E + 2L$. The triakis tetrahedron with 12 triangular faces has 126 such yods on the boundaries of its (12×3=36) tetractyses in its faces, and the disdyakis triacontahedron has 1260 boundary yods in the (120×3=360) tetractyses of its 120 faces, i.e., ten times as many yods. The number of boundary yods inside a polyhedron

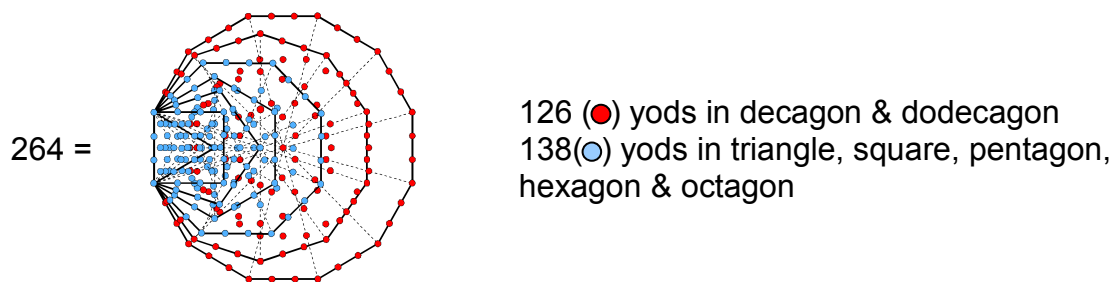


Figure 13. The first five polygons have 138 yods corresponding to the 138 yods on the boundaries of tetractyses inside the triakis tetrahedron. The last two polygons have 126 yods corresponding to the 126 boundary yods in its faces.

surrounding its axis = $2C + 7E - 4 = 9E - 2F$. The triakis tetrahedron has 138 such yods and the disdyakis triacontahedron has 1380 such yods. The former has (126+138=264) boundary yods and the latter has (1260+1380=2640) boundary yods. We see that the numbers of yods on the boundaries of tetractyses inside the disdyakis triacontahedron and on its faces are ten times the corresponding numbers for the triakis tetrahedron. The division $264 = 126 + 138$ created by the interior and surface of the triakis tetrahedron appears in the inner form of the Tree of Life as the 126 yods of the first five polygons and the 138 yods of the last two polygons. The fact that the number of boundary yods in the disdyakis triacontahedron is ten times that for the triakis tetrahedron and ten times the yod population of the inner Tree of Life is further evidence that this solid is the polyhedral counterpart of the inner Tree of Life.

1260 is the number of yods in 126 tetractyses. It is remarkable that the number 126 is the sum of the number values of the four types of combinations of the letters A, H and I in Ehyeh (AHIH), the Godname of Kether:

$$\begin{array}{rcl}
 & & A = 1, H = 5, I = 10 \\
 1. & A + H + I & = 16 \\
 2. & AH + HI + AI + HH & = 42
 \end{array}$$

$$\begin{array}{rcl}
3. & \text{AHI} + \text{HIH} + \text{AHH} & = 47 \\
4. & \text{AHIH} & = 21 \\
& & \hline
& \text{Total} & = 126
\end{array}$$

This shows how Ehyeh prescribes the population of yods that shape the 360 tetractyses in the faces of the disdyakis triacontahedron. It is further evidence of the way the Godnames of the ten Sephiroth prescribe the disdyakis triacontahedron as the polyhedral form of the inner Tree of Life.

The number 126 has the remarkable property that it is the arithmetic mean of the first 26 triangular numbers, where 26 is the number value of Yahweh:

$$126 = \frac{\sum_{n=1}^{26} T_n}{26}$$

This shows how Yahweh prescribes the shape of the disdyakis triacontahedron by mathematically determining the number of yods on boundaries of tetractyses in its surface.

According to Table 3, the number of yods in its faces that surround its axis is 1620. Of these, 60 are vertices of the polyhedron, leaving 1560 (=156×10) extra yods needed to create its faces. 156 is the sum of the number values of the four types of combinations of the letters Y, H and V in YHVH, the Godname of Chokmah:

$$Y = 10, H = 5, V = 6$$

$$\begin{array}{rcl}
1. & Y + H + V & = 21 \\
2. & YH + HV + YV + HH & = 52 \\
3. & YHV + HVH + HYH & = 57 \\
4. & YHVV & = 26 \\
& & \hline
& \text{Total} & = 156
\end{array}$$

It illustrates the creative quality embodied in this well-known Godname, for the yods in 156 tetractyses are needed to form the faces of the disdyakis triacontahedron marked out by its 62 vertices. 156 is the 155th integer after 1, showing how Adonai Melekh, the Godname of Malkuth with number value 155 (see Table 2), prescribes this polyhedron.

156 is the sum of the first 12 even integers that can be arranged in a square:

$$\begin{array}{cccc}
& & 2 & 4 & 6 & 8 \\
& & 24 & & & 10 \\
156 = & & 22 & & & 12 \\
& & 20 & 18 & 16 & 14
\end{array}$$

This illustrates how the Tetrad Principle determines this number as a square array of even integers, starting with 2, the first even integer.

Let us now consider a polyhedron whose faces and internal triangles are single tetractyses (case C). The number of internal yods = 2C + E + 1. The number of yods in its faces = C + 2E + F = 3E + 2. The total number of yods = 2C + 4E + 3. The triakis tetrahedron has 91 yods, that is, 84 yods surround its axis made up of seven yods. The disdyakis triacontahedron has 847 yods, that is, 840 yods surround its axis. Once again, we see that the yod population of the last Catalan solid is ten times that of the first one. The number 840 was encountered in Article 26 as the number of geometrical

elements inside the faces of the disdyakis triacontahedron in case B.⁴ It is also the number of circularly polarised oscillations in an outer or an inner half of each whorl of the heterotic superstring (Fig. 6). The triakis tetrahedron with single tetractyses as its faces has as many yods surrounding its axis as the tetrahedron has geometrical elements surrounding its centre (case B).

For case C:

Number of vertices $\equiv V = C + 1$. Number of vertices surrounding axis $\equiv V' = C - 2$.

Number of edges $\equiv e = C + E$. Number of edges surrounding axis $\equiv e' = C + E - 2$.

Number of triangles $\equiv T = E + F$. Number of triangles surrounding axis = $E + F$.

Total number of geometrical elements $\equiv N = 2C + 2E + F + 1 = C + 3E + 3$.

Number of geometrical elements surrounding axis $\equiv N' = C + 3E - 2$.

Table 4. Numbers of geometrical elements in the Catalan solids.

Catalan solid	V	V'	e	e'	T	N	N'
triakis tetrahedron	9	6	26	24	30	65	60
rhombic dodecahedron	-	-	-	-	-	-	-
triakis octahedron	15	12	50	48	60	125	120
tetrakis hexahedron	15	12	50	48	60	125	120
deltoidal icositetrahedron	-	-	-	-	-	-	-
pentagonal icositetrahedron	-	-	-	-	-	-	-
rhombic triacontahedron	-	-	-	-	-	-	-
disdyakis dodecahedron	27	24	98	96	120	245	240
triakis icosahedron	33	30	122	120	150	305	300
pentakis dodecahedron	33	30	122	120	150	305	300
deltoidal hexacontahedron	-	-	-	-	-	-	-
pentagonal hexacontahedron	-	-	-	-	-	-	-
disdyakis triacontahedron	63	60	242	240	300	605	600

Table 4 lists the various numbers of elements in the Catalan solids with triangular faces. The triakis tetrahedron has 26 edges prescribed by Yahweh and 65 geometrical elements prescribed by Adonai, Godname of Malkuth, which has number value 65 (see Table 2). The latter property is remarkable because, together with the 168 elements surrounding its axis in case A, the solid embodies number values of the *same* Sephirah (Malkuth) in *two* Kabbalistic worlds — Atziluth and Assiyah. This is yet another indication that the simplest Catalan solid, which is prescribed by the Godname Ehyeh of Kether with number value 21 because it is the 21st member of the set of Platonic, Archimedean and Catalan solids, is one that has a Tree of Life pattern. This is

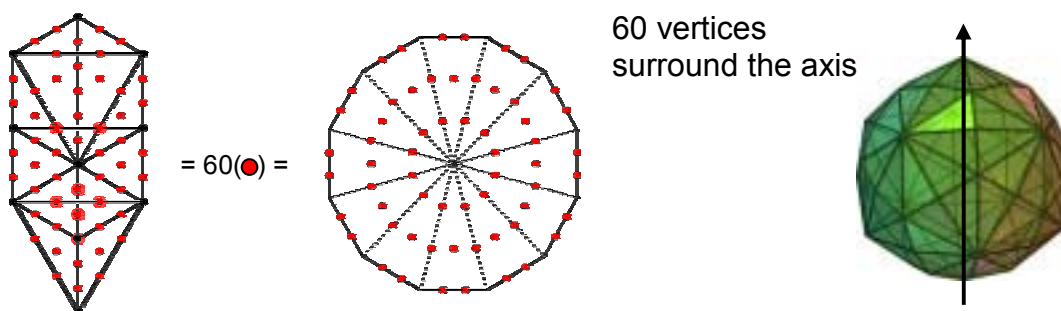


Figure 14. The disdyakis triacontahedron has as many vertices surrounding its axis as the Tree of Life needs more yods to be constructed from tetractyses. It is the number of extra (●) yods needed to construct the dodecagon from tetractyses.

confirmed by the fact that the disdyakis triacontahedron has numbers of geometrical elements that — once again — are ten times their counterparts in the triakis tetrahedron. As was discussed in Article 26,⁵ Adonai prescribes the lowest ten Trees of Life of an extended sequence of trees because they have 65 Sephirothic emanations or levels (what in previous articles were called ‘SLs’). The 65 bits of geometrical information needed to define the triakis tetrahedron are signalling these ten trees, the properties of their enfolded polygons being embodied in the disdyakis triacontahedron.⁶

The 60 vertices surrounding the axis of the disdyakis triacontahedron are degrees of freedom symbolised by the 60 extra yods needed to construct the Tree of Life from tetractyses (Fig. 14). As the last of the seven regular polygons constituting the inner Tree of Life, the dodecagon — the tenth regular polygon — has 60 hexagonal yods symbolising these degrees of freedom.

3. The Triakis Tetrahedron as the Cosmic Tree of Life

Because the superphysical and physical cosmos constitute a whole, they are together represented by the tetractys. Or, rather, following the ancient hermetic axiom “as above, so below,” a tetractys that is modified to express this statement. This is accomplished by replacing the tetractys at the centre of the next higher-order tetractys by another

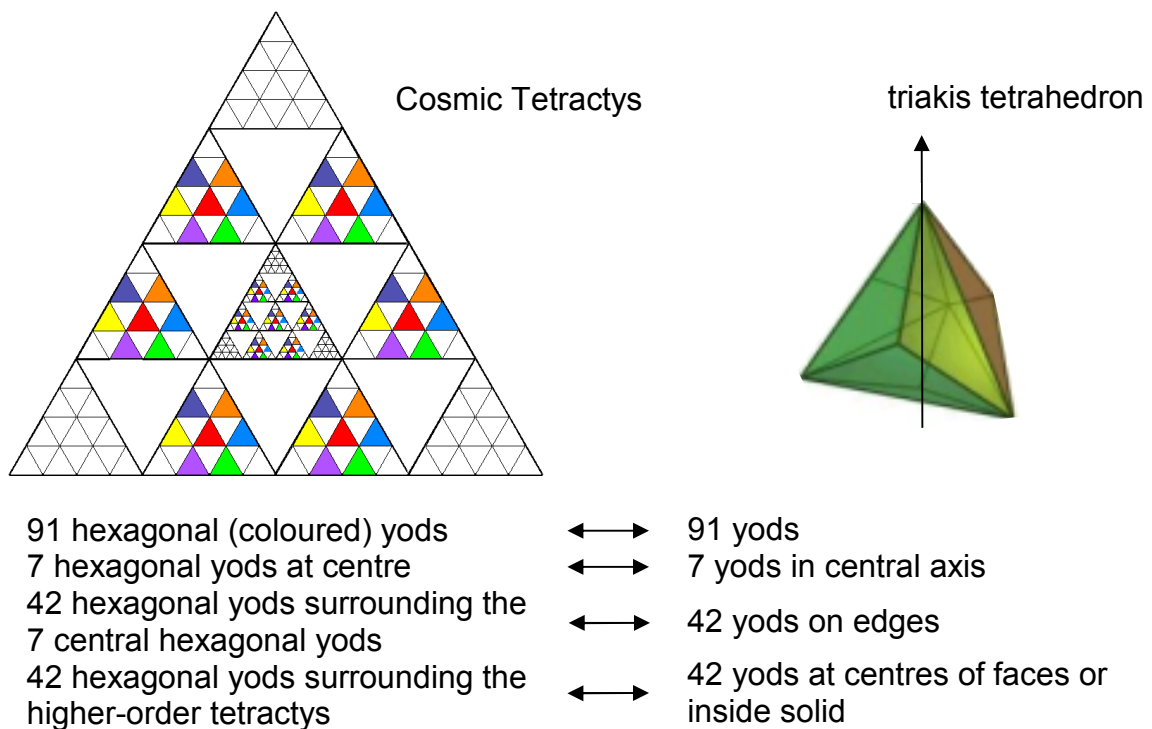


Figure 15. The Cosmic Tetractys compared with the triakis tetrahedron.

such tetractys (Fig. 15). This is the ‘Cosmic Tetractys.’ It maps what the author has called the ‘Cosmic Tree of Life,’ namely, 91 overlapping Trees of Life, each tree representing a possible jump in evolutionary consciousness.⁷ This map of all levels of consciousness is encoded in the inner Tree of Life. The seven hexagonal yods at the centre of its central tetractys (shown coloured in Fig. 15) symbolise the seven physical subplanes of consciousness discussed in Theosophical literature. The (6×7=42) hexagonal yods in the central higher-order tetractys denote the 42 subplanes of the six planes of consciousness above the physical plane: astral, mental, buddhic, atmic, anupadaka and adi planes. The 42 hexagonal yods surrounding these denote the 42

triangles are single tetractyses. This, indeed, is true. 90 yods surround its centre (Fig. 16). This is the sum of the ten integers in the 'Lambda Tetractys' formed from the Lambda by extrapolating the three numbers shown in red. The six yods on the axis on either side of its centre correspond to the number 6 at the centre of the tetractys. The 18 edges of the solid have 36 hexagonal yods, and there are 12 hexagonal yods at centres of its faces. The 48 hexagonal yods in its faces correspond to the sum of the six integers in the Lambda Tetractys at the corners of a hexagon: $2 + 3 + 9 + 18 + 12 + 4 = 48$. The remaining 36 yods (6 vertices and 30 internal yods) correspond to the sum of the integers at the corners of the tetractys: $1 + 8 + 27 = 36$.

An alternative correspondence would have the central integer corresponding to the six vertices surrounding the axis and the sum 36 of the integers at the three corners corresponding to the 36 internal yods. As the number 6 is the musical proportion of the tonic, the lowest note of the musical scale, and as its position in the Lambda tetractys corresponds to Malkuth, the physical universe of seven subplanes symbolised (as we

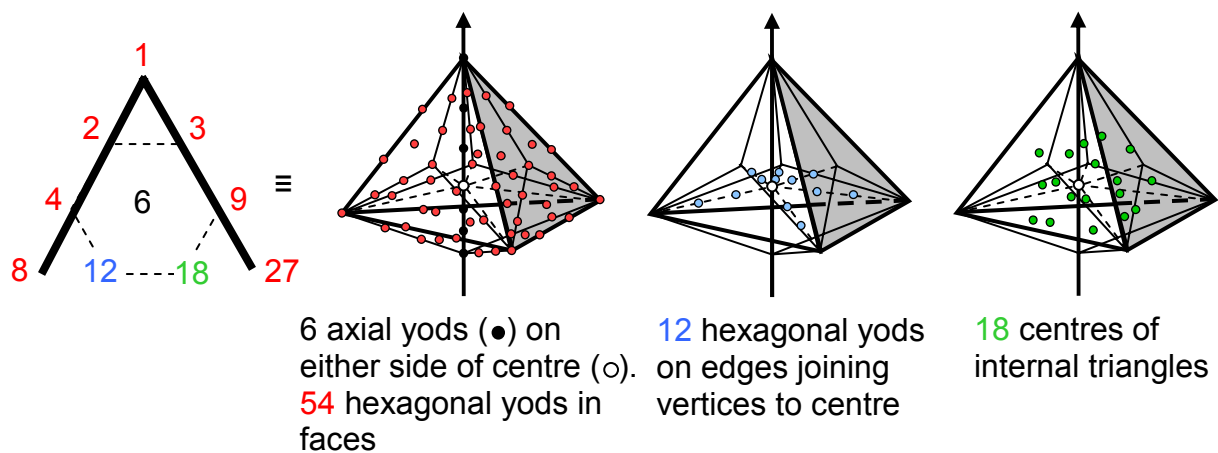


Figure 17. The triakis tetrahedron is the polyhedral form of Plato's Lambda Tetractys.

found earlier) by the seven yods on the axis, the correspondence that is more consistent with the analogy between the Cosmic Tetractys and the triakis tetrahedron is the one discussed first in which the central integer 6 denotes the six yods on the axis on either side of its centre.

Another alternative correspondence scheme would have, firstly, the sum (54) of the seven integers making up the Lambda denoting the number of yods in the faces of the solid that surround its axis (6 vertices, 48 hexagonal yods) and, secondly, the sum (36) of the three extrapolated integers (shown in red in Fig. 17) denoting the number of yods in its interior. As before, the central integer 6 would signify the number of yods on the axis on either side of its centre, but now the extrapolated number 12 would denote the 12 internal hexagonal yods on the lines joining the centre to vertices surrounding the axis and the number 18 would denote the 18 hexagonal yods at the centres of the triangles formed by the centre and 18 edges of the triakis tetrahedron. This is intuitively appealing because the three extrapolated (i.e., *hidden*) numbers would then determine the number of interior yods, although the correspondence is not perfect because the number 6 would also determine the two vertices at the ends of the axis and these are *external*, not internal, yods.

Whatever scheme is favoured, this correspondence exists because the Lambda Tetractys and the triakis tetrahedron are, respectively, the arithmetic and geometric expressions of mathematical archetypes working through a law of correspondence to link the psychological qualities of the notes of the musical scale to the pattern of

varieties of consciousness in the spiritual cosmos.

It was pointed out in Article 26⁹ that the sums of the integers in the four diagonal rows of the Lambda Tetractys are the numbers of yods in the first four polygons of the inner

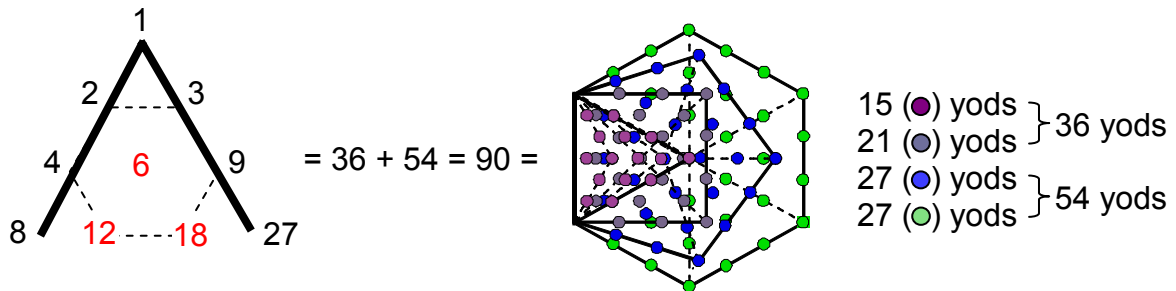
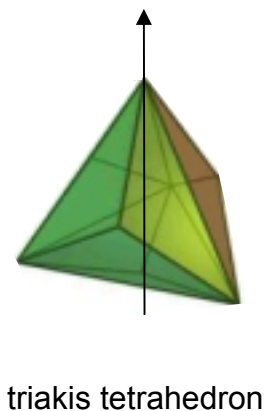


Figure 18. The sum (90) of the ten integers of the Lambda Tetractys is the number of yods in the first four polygons of the inner Tree of Life that are outside their shared edge (or, alternatively, the number of yods surrounding the centres of these polygons).

Tree of Life that are outside their shared edge (Fig. 18). As if this 1:1 correlation between the numbers and the geometry were not remarkable enough, it is further extraordinary that:

1. the sum (36) of the integers at the corners of the tetractys is the number of yods outside the root edge in the triangle and square. This is also the number of vertices and internal yods of the triakis tetrahedron;
2. the sum (54) of the integers arranged at the corners and centre of a hexagon is the number of external yods in the pentagon and hexagon. This is the number of hexagonal yods on the faces of the triakis tetrahedron and yods on its axis on either side of its centre.



triakis tetrahedron

	Faces	Interior
basic polyhedron:	6 vertices	6 edges
	18 edges	18 triangles
	12 triangles	
	Subtotal = 36 elements	24 elements
	Total = 60	

one vertex, 3 edges & 2 triangles are added in 12 faces: $(6 \times 12 = 72)$ new elements $\rightarrow E_6$.

3 edges are added in 18 internal triangles: $(3 \times 18 = 54)$ new elements.

$(72 + 54 = 126)$ new elements $\rightarrow E_7$.

one vertex + 2 triangles are added in 18 internal triangles: $(18 \times 3 = 54)$ new elements

Total number of elements = $60 + 72 + 54 + 54 = 240 \rightarrow E_8$.

Figure 17. The numbers of geometrical elements surrounding the axis of the triakis tetrahedron are the numbers of the non-zero roots of E_6 , E_7 & E_8 .

5. Root Structures of E_8 , E_7 & E_6 and Their Polyhedral Representation

The superstring gauge symmetry group E_8 has 248 simple roots made up of eight zero roots and 240 non-zero roots. Its largest exceptional subgroup E_7 has 133 simple roots made up of 7 zero roots and 126 non-zero roots. E_7 has the rank-6, exceptional subgroup E_6 with 78 simple roots that comprise six zero roots and 72 non-zero roots. Let us compare this root structure with the geometrical composition of the triakis tetrahedron. Fig. 17 shows that, when we start with case C, in which the faces and internal triangles are single tetractyses, the solid has 60 geometrical elements surrounding its axis. Construction of the triangular faces from three tetractyses adds 72

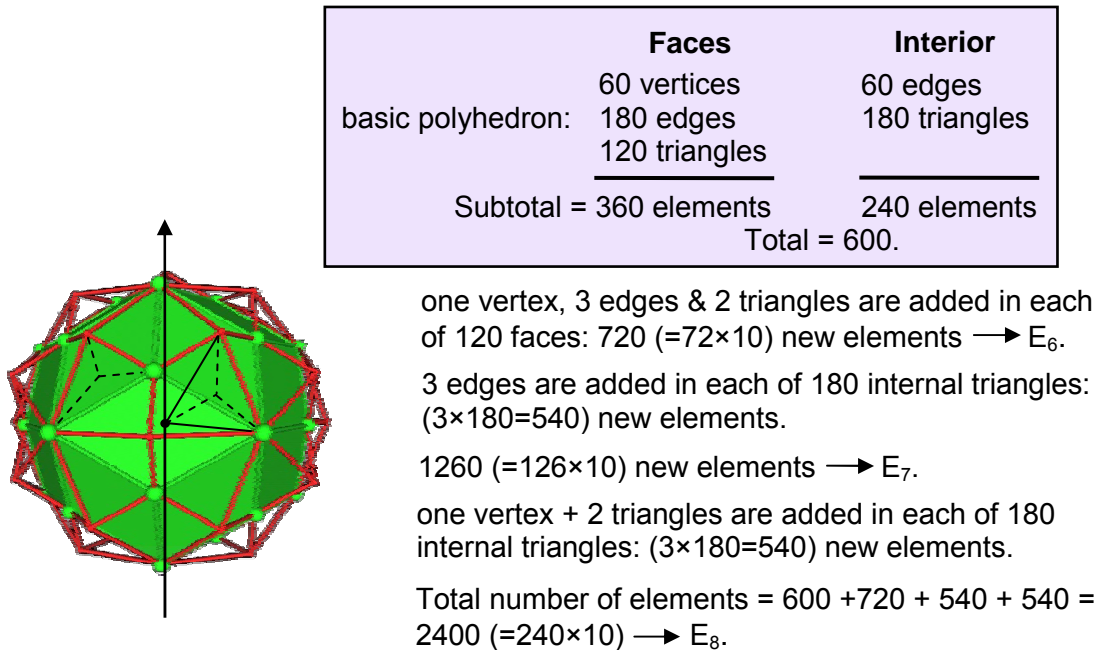


Figure 18. The numbers of geometrical elements surrounding the axis of the disdyakis triacontahedron are ten times the numbers of non-zero roots of E_6 , E_7 & E_8 . (part of image from: <http://www.rwgrayprojects.com/Lynn/P120BUP/rest/rest.html>)

new elements. Construction of the internal triangles from three tetractyses requires adding three edges within each triangle, i.e., 54 for the interior of the solid, therefore adding $(72+54=126)$ elements for its interior and faces. As well as adding 54 more edges, division of each internal triangle into three tetractyses adds one vertex and two triangles, creating 54 more elements. The total number of elements = $60 + 72 + 54 + 54 = 240$. Constructed from tetractyses, the triakis tetrahedron has as many geometrical elements surrounding its axis as E_8 has non-zero roots. The 72 new elements needed to construct its faces from three tetractyses correspond to the 72 non-zero roots of E_6 , whilst the minimal number of 126 new elements needed to construct both faces and internal triangles from tetractyses correspond to the 126 non-zero roots of E_7 . The remaining $(60+54=114)$ elements correspond to the $(240-126=114)$ non-zero roots of E_8 that do not belong to E_7 . The simplest Catalan solid embodies the non-zero root structure. As we saw earlier, the simplest Archimedean solid — the truncated tetrahedron — has, when constructed from tetractyses, 248 geometrical elements surrounding its centre (eight vertices at the centres of its faces, 240 other elements). This solid represents the eight zero roots and 240 non-zero roots of E_8 ; its dual expresses the non-zero root structure of E_8 .

Fig. 18 displays the same type of calculation for the disdyakis triacontahedron as that

given in Fig. 17 for the triakis tetrahedron. It shows a representative face and an internal triangle divided into three triangles. Amazingly, all corresponding numbers differ by a factor of 10. The non-zero root structures of the superstring gauge symmetry group E_8 and its exceptional subgroups E_7 and E_6 are embodied in the geometrical composition of the disdyakis triacontahedron. E_6 is determined by its faces and E_7 encompasses both faces and its interior.

The superstring parameter 168 is the number of geometrical elements in the triakis tetrahedron in case A. In terms of the corresponding non-zero roots:

$$168 = 54 + 114,$$

where the first number is the number of non-roots of E_7 that do not belong to E_6 and the second term is the number of non-zero roots of E_8 that do not belong to E_7 . In terms of vertices, edges and triangles, there are in the triakis tetrahedron 24 vertices (six external, 18 internal), 78 edges (18 external, 60 internal) and 66 triangles (12 external, 54 internal). The total number of 168 elements is the number value 168 of Cholem Yesodeth. The number value 78 of the word 'Cholem' measures the 78 edges and the number value 90 of the word 'Yesodeth' measures the number of vertices and triangles. The triakis tetrahedron is composed of 36 geometrical elements in its faces (six vertices, 18 edges & 12 triangles) and 132 internal elements (18 vertices, 60 edges & 54 triangles). This shows how Eloha, the Godname assigned to Geburah with number value 36 (see Table 2), prescribes this solid and thus the superstring parameter 168 as the number of geometrical elements surrounding its axis. Surrounding the axis of the disdyakis triacontahedron are 360 elements in its faces (60 vertices, 180 edges & 120 triangles) and 1320 elements in its interior (180 vertices, 600 edges & 540 triangles). Both the oscillatory structure and the E_8 -symmetric forces of the $E_8 \times E_8$ heterotic superstring are represented in the geometry of this remarkable polyhedron.

The 1680 circularly polarised oscillations of each whorl of the $E_8 \times E_8$ heterotic superstring are the string manifestation of the 240 gauge charges of E_8 associated with the 240 generators defined by its non-zero roots. These gauge charges are spread along the ten whorls, 24 to a whorl. The seven minor whorls of the subquark superstring carry 168 gauge charges of E_8 and the three major whorls carry ($3 \times 24 = 72$) gauge charges that are also the gauge charges of its subgroup E_6 . The distinction between the two types of whorl (see amounts to a difference in how each turn in their helices is compounded from helices formed by the winding of each stringy whorl around six successively smaller circular dimensions of a 6-torus and does not need to be discussed here. Their difference indicates symmetry breaking of E_8 into E_6 . Its counterpart in the disdyakis triacontahedron is the topological distinction between the interior of the polyhedron and its exterior, which requires 720 extra geometrical elements to construct it in order to make its outer fashioned in the same way from tetractyses as its inner. Analogous to a hypothetical universe where only superstring physics governed by E_6 operates (this symmetry is further reduced for the real world) would be one allowing only the faces of polyhedra to be visible, so that their interiors never played any *noticeable* role, despite always being present. It would be one in which, like an ant crawling over the surfaces of things, one could be conscious only of surfaces, never of depth. The form of superstrings is determined, however, not by E_6 or any of its subgroups (this determines only the difference between the major and minor whorls) but by that part of E_8 that is *not* also its subgroup E_6 . In this sense, the analogy between the disdyakis triacontahedron and the $E_8 \times E_8$ heterotic superstring indicates that what is now visible as matter is determined by what is now unobservable because the physics operated in a former time when the universe was, momentarily, one of

perfect E_8 symmetry and is locked out by the lesser symmetry of E_6 and its subgroups that survived. It is not that the universe today is less perfect in a mathematical sense than before. It is just that the perfection of the complete symmetry is hidden from view — as the interior of a disdyakis triacontahedron would be to an ant crawling over it. Only when consciousness has the attribute of recognising spatial depth can the mathematical beauty of its solid structure be appreciated and its archetypal meaning discerned.

References

- ¹ Phillips, Stephen M. Article 1: “The Pythagorean Nature of Superstring and Bosonic String Theories,” <http://www.smpillips.8m.com>, p. 5.
- ² Phillips, Stephen M. Article 3: “The Sacred Geometry of the Platonic Solids,” <http://www.smpillips.8m.com>, Table 1, p. 20.
- ³ Ref. 1.
- ⁴ Phillips, Stephen M. Article 26: “How the Seven Musical Scales Determine the Disdyakis Triacontahedron As the ‘120 Polyhedron,’” <http://www.smpillips.8m.com>, p. 29.
- ⁵ Ibid., p. 21.
- ⁶ Ibid., pp. 21, 34–36.
- ⁷ Phillips, Stephen M. Article 5: “The Superstring as Microcosm of the Spiritual Macrocosm,” <http://www.smpillips.8m.com>.
- ⁸ Ref. 4.
- ⁹ Ibid., p. 28.