ARTICLE 23

The '120 Polyhedron' and the '144 Polyhedron' as the Exterior and Interior of the Inner Tree of Life

by

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Abstract

The 2-dimensional, polygonal form of the inner Tree of Life consists of two identical sets of seven enfolded, regular polygons. Their beautiful properties expressed in terms of the integers 1, 2, 3 & 4 symbolised by the Pythagorean tetractys are prescribed by the Godnames assigned to the ten Sephiroth of the outer Tree of Life. Constructed from 47 tetractyses, the seven enfolded polygons contain 264 yods. 120 yods lie on the boundaries of the polygons, inside of which are 144 yods. The 120 yods shaping the inner form of the Tree of Life symbolise the 120 faces of the **120 Polyhedron**. The 144 internal yods symbolise the 144 faces of the **144 Polyhedron** inside the **120 Polyhedron**. Of the 15 Archimedean solids, the rules of construction of polygons from tetractyses permit two possible candidates for the former — the cuboctahedron and the truncated cuboctahedron. Constructed from tetractyses, each is made up of 650 yods. This is the number of yods other than polygonal corners that are associated with the seven enfolded polygons when their sectors are constructed from three tetractyses. The 48 vertices of the truncated cuboctahedron makes it a natural candidate for the **144 Polyhedron** if the 'cones of light' have as axes lines joining these vertices to the centre of the solid. The **120 Polyhedron** and the **144 Polyhedron** are the polyhedral exterior and interior of the inner Tree of Life — the cosmic blueprint.

The outer form of the Tree of Life (Fig. 1) is a 3-dimensional object consisting of 16 triangles with 10 corners joined by 22 straight lines, where $10 = 1 + 2 + 3 + 4$, $16 = 4^2$

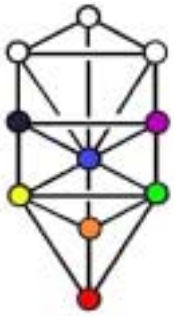


Figure 1. The outer form of the Tree of Life.

and $22 = 1^4 + 2^3 + 3^2 + 4^1$. The number of vertices, sides and triangles = 48. As we shall see later, this is also a fundamental geometrical parameter of the inner form of the Tree of Life (Fig. 2), which consists of two identical sets of seven regular polygons — the

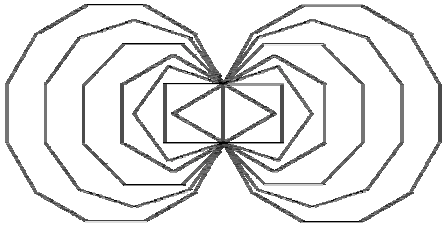


Figure 2. The inner Tree of Life.

triangle, square, pentagon, hexagon, octagon, decagon and dodecagon. They are enfolded in one another so that they share a common edge called the 'root edge.' When the 16 triangles of the outer Tree of Life are transformed into tetractyses, they contain

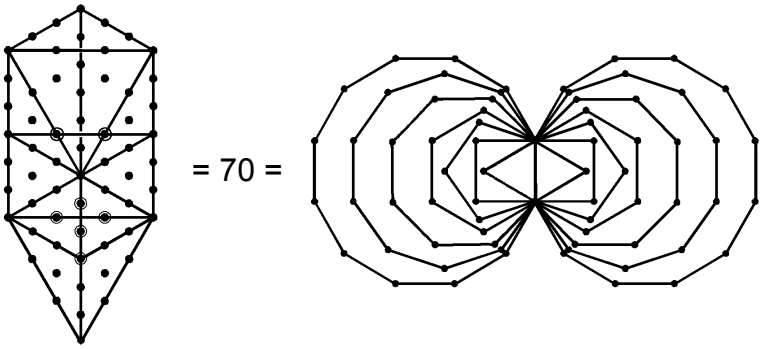


Figure 3. The 70 yods in the 16 tetractyses making up the outer Tree of Life have their counterpart in the inner Tree of Life as the 70 corners of the two sets of seven enfolded regular polygons.

70 yods. They denote degrees of freedom whose counterparts in the inner Tree of Life are the 70 corners of the polygons (Fig. 3).

The outer and inner Tree of Life are analogous to the human cell and its nucleus containing the DNA that encodes its replication. They both embody 'bits' of information

of various kinds in the form of the different types of yods in the tetractyses from which they can be constructed. The polygons may be studied as separate objects. The

$$144 = \begin{matrix} 1^0 & 2^0 & 3^0 & 4^0 \\ 1^1 & 2^1 & 3^1 & 4^1 \\ 1^2 & 2^2 & 3^2 & 4^2 \\ 1^3 & 2^3 & 3^3 & 4^3 \end{matrix}$$

yods on the boundaries of the seven separate polygons become

$$120 = 4\times(1^2 + 2^2 + 3^2 + 4^2) = \begin{matrix} 2^2 & & 4^2 \\ & \square & \\ 8^2 & & 6^2 \end{matrix}$$

yods on the boundaries of the seven enfolded polygons, i.e., 24 (= 1×2×3×4) such yods disappear in the enfolding. The seven centres of the separate polygons and the 144 internal yods surrounding them become 144 yods inside the boundaries of the enfolded

Table 1

Sephirah	Godname	Number value
Kether	Ehyeh	21
Chokmah	Yah	15
	Yahweh	26
Binah	Elohim	50
Chesed	El	31
Geburah	Eloha	36
Tiphareth	Yahweh Elohim	76
Netzach	Yahweh Sabaoth	129
Hod	Elohim Sabaoth	153
Yesod	El Chai	49
Malkuth	Adonai	65

polygons, that is, seven yods disappear in the enfolding. A total of (24 + 7 = 31) yods disappear in the enfolding of the seven polygons. 31 is the number value of El ("God"), Godname of Chesed (Table 1), showing how it prescribes the properties of the inner Tree of Life. The 96 corners of both sets of separate polygons become 70 corners of the 14 enfolded polygons, that is, 26 corners disappear in the enfolding. According to Table 1, 26 is the number value of Yahweh, Godname of Chokmah. There are

$$288 = 1^1 + 2^2 + 3^3 + 4^4 = 1!\times 2!\times 3!\times 4!$$

$$= \begin{matrix} 3 & 5 & 7 & 9 \\ 11 & 13 & 15 & 17 \\ 19 & 21 & 23 & 25 \\ 27 & 29 & 31 & 33 \end{matrix} \qquad (33 = 1! + 2! + 3! + 4!)$$

yods inside both sets of polygons. This illustrates how the properties of the inner Tree of Life are expressed in terms of the Pythagorean integers 1, 2, 3 & 4.

The number of yods in the seven enfolded polygons = 144 + 120 = 264. There are 260 yods outside the root edge. This is the number of yods in 26 tetractyses. It is powerful

evidence of how the creative archetype embodied in Yahweh prescribes the yod population of the inner form of the Tree of Life (see the author's Article 4[1] for how the other Godnames of the ten Sephiroth prescribe the inner Tree of Life).

The 120 boundary yods comprise 36 corners, where 36 is the number value of Eloha, Godname of Geburah, and 84 hexagonal yods, where

$$84 = \begin{array}{cc} 1^2 & 3^2 \\ 7^2 & 5^2 \end{array}$$

36 is the sum of the first four even integers and the first four odd integers, again illustrating how the Tetrad Principle formulated in Article 1 defines properties of the Tree of Life.

The 120:144 differentiation between the outer boundary of the inner Tree of Life and its interior corresponds to the 120 faces of the **120 Polyhedron** and the 144 faces of the

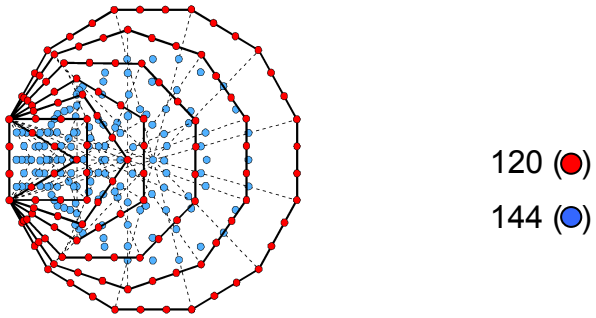


Figure 4. The 120 yods on the boundaries of the seven enfolded polygons and the 144 internal yods of the inner Tree of Life symbolise the 120 faces of the outer **120 Polyhedron** and the 144 faces of the inner **144 Polyhedron**.

144 Polyhedron inside it (Fig. 4). Each yod symbolises a polyhedral face. Both polyhedra are the 3-dimensional representation of the boundary and interior of the inner Tree of Life.

There are 13 Archimedean solids[1]. Two of them — the snub cube and the snub dodecahedron exist in two chiral forms, so that, if their enantiomorphs are included, there are 15 Archimedean solids. This shows how the Divine Name Yah with number value 15 (see Table 1) prescribes the number of such solids. Table 2 lists the numbers of vertices, edges and faces in these solids. The cuboctahedron is unique amongst them in that it has the largest number of various combinations of vertices, edges and faces that are number values of the Godnames shown in Table 1. It has 26 vertices & faces, 36 vertices & edges and 50 vertices, edges & faces. Five other solids have either one or two combinations whose numbers are Godname numbers. It would not therefore be surprising if, being prescribed by the most Godnames, the cuboctahedron were the basis of **Polyhedron 144**. However, rather than make an assumption about its polyhedral identity that could turn out to be wrong, the list of Archimedean solids will be analysed in some detail with the hope of finding the correct candidate for **Polyhedron 144** by elimination. The criterion for selection is that the solid should have either 144 faces or 144 triangular sectors of its faces. Table 2 shows that none of the Archimedean solids has as many as 144 faces. In the case of the Platonic solids,

[1] An Archimedean solid is a convex polyhedron having at least two different regular polygons as faces.

	Archimedean solid	corners	edges	faces
	cuboctahedron	12	24	14
	icosidodecahedron	30	60	32
	truncated tetrahedron	12	18	8
	truncated cube	24	36	14
	truncated octahedron	24	36	14
	truncated dodecahedron	60	90	32
	truncated icosahedron	60	90	32
	rhombicuboctahedron	24	48	26
	truncated cuboctahedron	48	72	26
	rhombicosidodecahedron	60	120	62
	truncated icosidodecahedron	120	180	62
	snub cube (two chiral forms)	24	60	38
	snub cube	24	60	38
	snub dodecahedron (two chiral forms)	60	150	92
	snub dodecahedron	60	150	92

Table 2. The 15 Archimedean solids.

the icosahedron and dodecahedron display the largest number of sectors. However, this number is only 60. Of the Archimedean solids, Table 3 indicates that only the cuboctahedron with 48 triangular faces, each of which is further divided into three

sectors, and the truncated cuboctahedron with 144 triangular faces fit the bill. For the former, 144 triangles surround 48 centres of sectors of 14 faces. The latter has 48 corners and 144 sectors of its 26 faces (Fig. 5). The fact that 26 is the number value of

Table 3

Archimedean solid	types of faces	number of triangular faces
cuboctahedron	8 triangles + 6 squares	8×3 + 6×4 = 48
icosidodecahedron	20 triangles + 12 pentagons	20×3 + 12×5 = 120
truncated tetrahedron	4 triangles + 4 hexagons	4×3 + 4×6 = 36
truncated cube	8 triangles + 6 octagons	8×3 +6×8 = 72
truncated octahedron	6 squares + 8 hexagons	6×4 + 8×6 = 72
truncated dodecahedron	20 triangles +12 decagons	20×3 + 12×10 = 180
truncated icosahedron	12 pentagons + 20 hexagons	12×5 + 20×6 =180
rhombicuboctahedron	8 triangles + 18 squares	8×3 + 18×4 = 96
truncated cuboctahedron	12 squares + 8 hexagons + 6 octagons	12×4 + 8×6 + 6×8 = 144
rhombicosidodecahedron	20 triangles + 30 squares + 12 pentagons	20×3 + 30×4 + 12×5 = 240
truncated icosidodecahedron	30 squares + 20 hexagons + 12 decagons	30×4 + 20×6 + 12×10 = 360
snub cube	32 triangles + 6 squares	32×3 + 6×4 = 120
snub dodecahedron	80 triangles + 12 pentagons	80×3 + 12×5 = 300

Yahweh adds promise to this candidate. However, as we saw earlier, Godname numbers appear also in the geometrical properties of the cuboctahedron. Their properties therefore need to be compared further before an unambiguous choice between the two can be made.

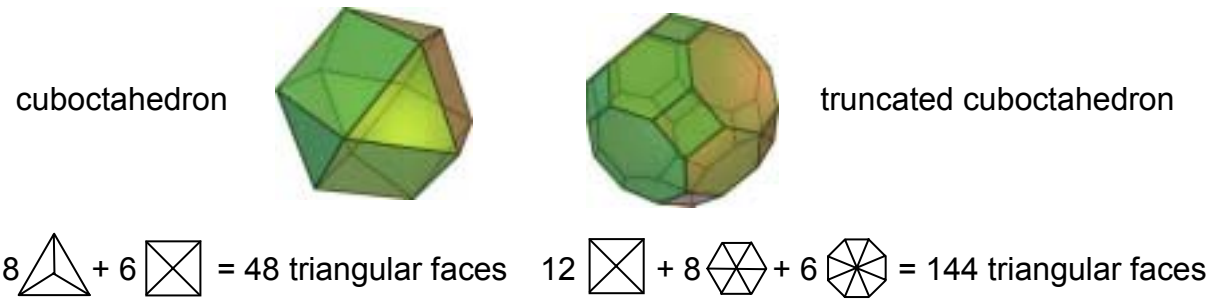


Figure 5.The cuboctahedron with 48 tetractyses in its 14 faces and the truncated cuboctahedron with 144 tetractyses in its 26 faces are the only Archimedean solids allowing 144 triangular faces.

En passim, reference needs to be made to Robert Gray's identification of the **144 Polyhedron.**[2] He starts with the spherical cuboctahedron and divides the square faces into eight sectors and the triangular faces into six sectors, creating 6×8 + 8×6 = 96 triangles. He then combines adjoining triangles from adjacent triangular and square faces, creating 48 new triangles that cover the sphere. Gray then places a tetrahedron over each of these triangles, creating a polyhedron with 3×48 = 144 faces. He wrongly gives their number of corners as 62. As each corner is either one of the 12 corners of

the cuboctahedron or a centre of one of its 14 faces, the 48 new triangles have (12 + 14 = 26) corners and the 144 faces of the tetrahedra placed over them have (48 + 26 = 74) corners (Gray added 14 to 48, making 62, but this missed the corners coinciding with the 12 vertices of the cuboctahedron). His calculation of the number of sides of the 144 faces is also wrong. It should be 216, not the 204 that he states. Quite apart from not explaining why he chose the cuboctahedron other than citing his work on R. Buckminster Fuller's geometry, his divisions of a square face into eight sectors and a

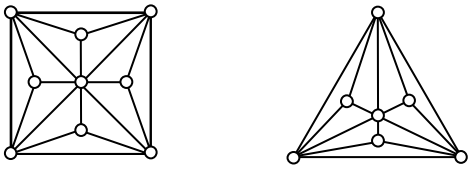
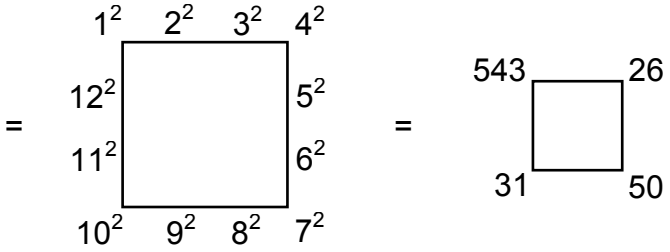


Figure 6. 12 tetractyses form a square and 9 tetractyses form a triangle.

triangular face into six sectors violate the fundamental rule of sacred geometry that all straight-line bodies possessing the latter are constructed from tetractyses, so that, for example, triangles must be regarded as three tetractyses and squares as four tetractyses. The reason behind this rule is that the n-sided polygon divided into n tetractyses is a representation of n overlapping Trees of Life up to Chesed of the highest tree. Any other division of polygons that ignores the template nature of the tetractys in building objects with true *sacred* geometry is, of course, permissible in pure mathematics. However, it has no relevance to the mathematics of the Divine Mind that thinks in terms of this holistic symbol.

Properly constructed from tetractyses, the cuboctahedron has 12 vertices, five corners of triangles within each of its six square faces and four centres within each of its eight triangular faces (Fig. 6), making a total of 74 corners of 144 triangles. As a triangular face has 12 internal sides of triangles and a square face has 16 internal sides, the number of sides of the 144 triangles in the cuboctahedron = 24 + 6×16 + 8×12 = 216 = 6^3. Transformed into 9 tetractyses, a triangle has 9 yods at centres of triangles and a square has 12 such yods. Each side has two yods between its endpoints, which are corners of triangles. The number of yods in the 144 tetractyses = 12 + 216×2 + 6×12 + 8×9 + 6×5 + 8×4 = 650



where 543, 26, 50 and 31 are the number values of the Godnames of the first *four* Sephirah of the Tree of Life:

Kether: Ehyeh Asher Ehyeh = 543,
Chokmah: Yahweh = 26,
Binah: Elohim = 50,
Chesed: El = 31.

Ehyeh Asher Ehyeh, the Name of God that God is said in the Old Testament to have imparted to Moses when he received the Ten Commandments on Mount Sinai, is

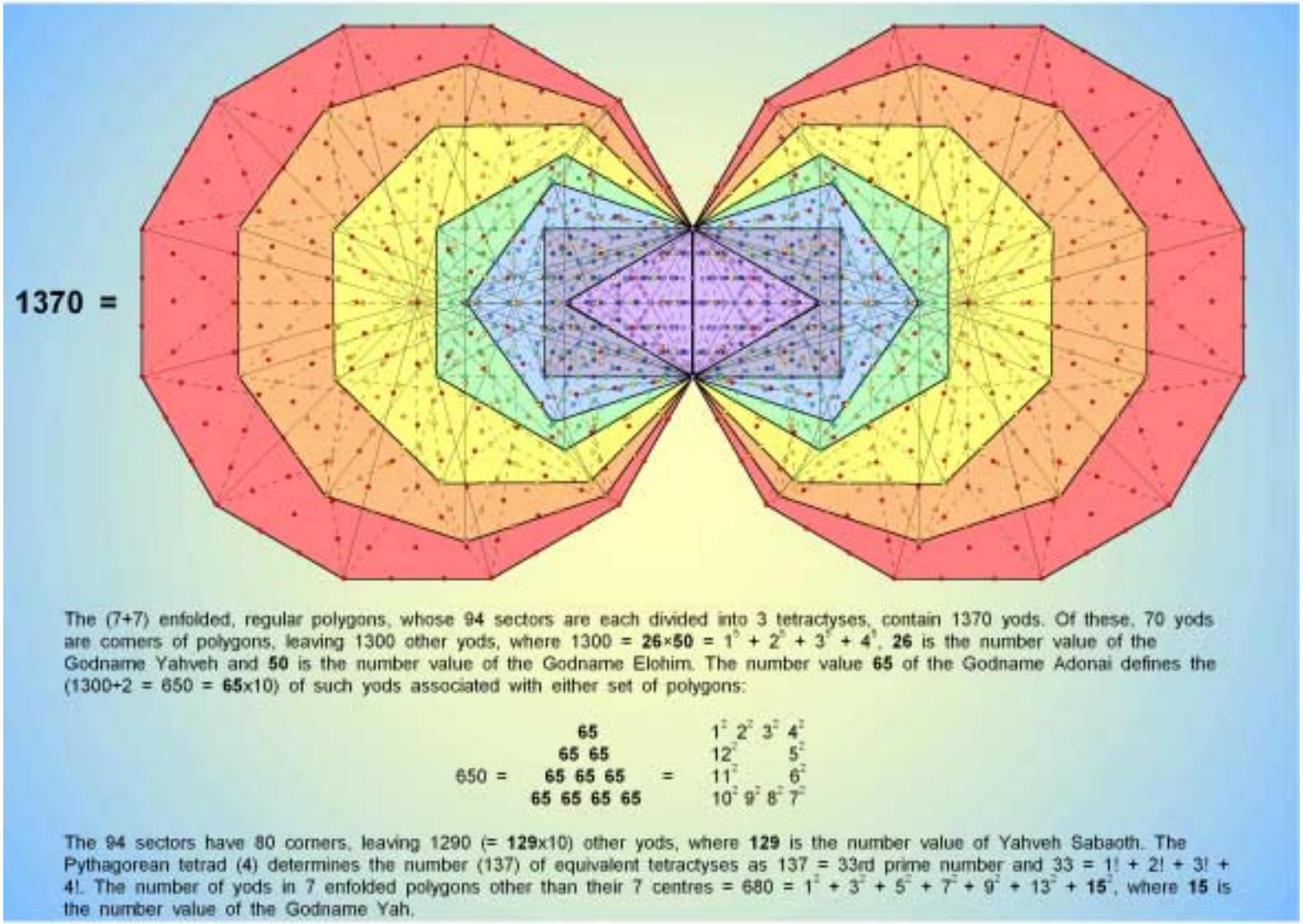


Figure 7. The inner Tree of Life has the yods in 137 tetractyses. 137 is one of the ‘magic numbers of physics because its reciprocal is known as the fine structure constant determining the strength of electromagnetic interactions.

translated as “I am that I am.” In view of the fact that it is the sum of the first four Godname numbers, the number 650 might be expected to have particular significance vis-à-vis the Tree of Life. This is, indeed, the case. When the 94 sectors of the two sets of seven enfolded polygons constituting the inner Tree of Life are divided into three triangles and each triangle then turned into a tetractys, the resulting yod population of the (94×3 = 282) tetractyses is 1370. 282 is the number value of Aralim (Thrones), the Order of Angels assigned to Binah. The yods in 137 tetractyses are therefore needed to form the inner Tree of Life. The number 137 is known to physicists as one of the most important numbers in Nature because its reciprocal is approximately equal to the fine structure constant that sets the energy scale of atomic physics because it measures the strength of the coupling of the electron to the electromagnetic field. The 14 enfolded polygons have 70 corners, leaving 1300 yods other than corners. In other words, associated with each set of seven polygons are (1300/2 = 650) yods other than their defining corners. This is the number of yods in 65 tetractyses, where 65 is the number value of Adonai (“The Lord”), Godname of Malkuth. This Godname is read by Jews in substitute of Yahweh, the sacred Divine Name that they are forbidden to pronounce and whose number value is 26. 650 is related to the latter by

$$650 = 26^2 - 26.$$

This means that 650 is the sum of the first 25 even integers.

Article 22 provided evidence that the **120 Polyhedron** is the polyhedral counterpart of the 2-dimensional, polygonal form of the inner Tree of Life at the first level of differentiation of the latter, namely, with each sector of a polygon turned into tetractyses. Having the same name of yods as that associated with the seven enfolded polygons with each sector now divided into three tetractyses means that the cuboctahedron can be regarded as the polyhedral version of the 2-dimensional, inner Tree at the next level of differentiation of its seven polygons. In fact, it is easily proved[3] that, apart from their corners, there are 650 yods associated with the seven regular polygons enfolded in each overlapping Tree of Life, that the number 650 represents the number of new degrees of freedom intrinsic to the tetractys-transformed geometry of the inner form of successive, overlapping Trees of Life. This is why this number has a special significance and is prescribed by Adonai, for its number 65 actually quantifies the yods other than corners making up the inner form of each overlapping Tree of Life.

The complete Godname associated with Malkuth is Adonai Malekh (“Lord and King”). Its number value is 155. Constructed from tetractyses, the **120 Polyhedron** has 120 faces with 422 yods on their 180 edges. If each face is further divided into three

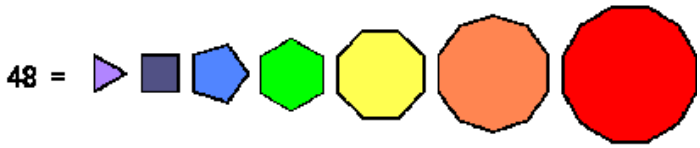


Figure 8. The 7 separate, regular polygons making up the inner Tree of Life have 48 corners.

tetractyses, (120×10 = 1200) yods are added, making a total of (422 + 1200 = 1622) yods. As the **120 Polyhedron** has 62 corners, its construction from tetractyses generates (1622 – 62 = 1560) yods. This is the number of yods in 156 tetractyses, where 156 is the 155th integer after 1. This shows how the number value of Adonai Malekh determines how many extra yods are needed to construct the **120 Polyhedron** from 360 tetractyses. Adonai prescribes the **144 Polyhedron** and Adonai Melekh

prescribes the **120 Polyhedron**.

Now let us consider the truncated cuboctahedron — the other candidate for the **144 Polyhedron**. It has 12 square faces, 8 hexagonal faces and 6 octagonal faces. A square divided into four tetractyses contains 25 yods, of which 13 are inside it. A hexagon similarly divided has 37 yods (19 internal) and an octagon has 49 yods (25 internal). The truncated cuboctahedron has 48 polyhedral corners and 72 edges with

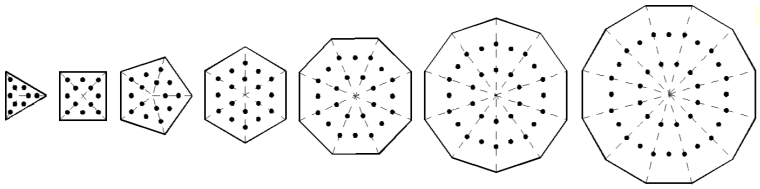


Figure 9. The **144 Polyhedron** is a Tree of Life pattern because its '48 cones of light' correspond to the 48 corners of the 7 separate polygons — primary geometrical degrees of freedom — and its 144 faces correspond to the 144 yods inside the polygons.

two yods between the ends of each one. The total number of yods in the 144 tetractyses in its 26 faces = 48 + 72×2 + 12×13 + 8×19 + 6×25 = 650. Amazingly, both candidates for the **144 Polyhedron** contain the same number of yods in their faces! Whereas, however, the cuboctahedron has 48 corners of triangles within each of its 14 faces (24 in its square faces, 24 in its triangular faces), the truncated octahedron has 48 vertices, that is, corners of triangles that define the *shape* of the Archimedean solid instead of being just corners of triangles that become apices of tetrahedra when raised above the planes of the faces. Moreover, the number 48 is the number of corners of the seven separate polygons making up the inner Tree of Life (Fig. 8). It is the number of Kokab ("The Stellar Light"), the Mundane Chakra of Hod, associated in Kabbalah with the planet Mercury. Its fundamental significance vis-à-vis the Tree of Life is that it quantifies the polygonal nature of the inner form of the Tree of Life in a *minimal* way — no smaller number can define its geometry. The number 48 *defines* the shape of this inner form in the same sense that it defines the shape of the truncated cuboctahedron.

Table 4

	C	**2E**	$\sum_i n_i F(n_i)$	**S**
cuboctahedron	12	24×2 = 48	8×10 + 6×13 = 158	218
icosidodecahedron	30	60×2 = 120	20×10 + 12×16 = 392	542
truncated tetrahedron	12	18×2 = 36	4×10 + 4×19 = 116	164
truncated cube	24	36×2 = 72	8×10 + 6×25 = 230	326
truncated octahedron	24	36×2 = 72	6×13 + 8×19 = 230	326
truncated dodecahedron	60	90×2 = 180	20×10 + 12×31 = 572	812
truncated icosahedron	60	90×2 = 180	12×16 + 20×19 = 572	812
rhombicuboctahedron	24	48×2 = 96	8×10 + 18×13 = 314	434
truncated cuboctahedron	48	72×2 = 144	12×13 +8×19 + 6×25 = 458	650
rhombicosidodecahedron	60	120×2 = 240	20×10 + 30×13 + 12×16 = 782	1082
truncated icosidodecahedron	120	180×2 = 360	30×13 + 20×19 + 12×31 = 1142	1622
snub cube	24	60×2 = 120	32×10 + 6×13 = 398	542
snub dodecahedron	60	150×2 = 300	80×10 + 12×16 = 992	1352

The 144 faces of the **144 Polyhedron** correspond to the 144 hexagonal yods inside the seven separate polygons (Fig. 9), showing how it is the polyhedral counterpart of the *interior* of the inner form of the Tree of Life, thus explaining why it is *inside* the **120 Polyhedron**. The **144 Polyhedron** must therefore be the truncated cuboctahedron, not the cuboctahedron, with the 48 "cones of light" that symbolise the 48 corners of the inner Tree of Life emanating from its vertices, *not* from faces.

The number of yods in an Archimedean solid constructed from tetractyses with n_i faces shaped like an N_i-sided regular polygon, n_j faces shaped like an N_j-sided regular polygons, etc is given by

$$N = C + 2E + \sum_i n_i F(N_i),$$

where C = number of vertices, E = number of solid edges, $F(N_i)$ is the number of yods inside the N_i-sided polygon and

$$\sum_i n_i = F,$$

where C, E and F are given by Euler's equation:

$$C - E + F = 2$$

Table 4 lists the number of yods in the Archimedean solids. Only the truncated cuboctahedron has a yod population equal to 650. Moreover, none of the others is a Godname number or ten times a Godname number. This proves the uniqueness of the yod population of the truncated cuboctahedron in being prescribed by a Godname.

The number of yods on the 72 sides of the truncated cuboctahedron is 192. The significance of this number is that it is the Bode number of the planet Uranus, which was shown in Article 17[4] to be its distance (in units of 1/10 AU) from the asymptotic centre of the logarithmic spiral of the primordial planetary nebula that spawned the Solar System. The I Ching table, which was analysed in Article 18[5] and shown to encode the distances of all the known planets, has 192 yang lines and 192 yin lines. The presence of such an astronomical parameter in characterising the shape of the truncated cuboctahedron is further evidence that it is the **144 Polyhedron**.

The number of yods inside the 26 faces of the truncated cuboctahedron = 650 – 192 = 458. This is the 229th even integer, where 229 is the 50th prime number. This shows how Elohim, Godname of Binah with number value 50, prescribes the population of internal yods of the truncated cuboctahedron. The seven enfolded polygons with their sectors divided into three tetractyses have 141 tetractyses with 229 sides. (229×2 = 458) hexagonal yods lie along them. This demonstrates the Tree of Life pattern embodied in this Archimedean solid.

Conclusion

The truncated cuboctahedron is unique amongst the Archimedean solids in having features that correspond to the **144 Polyhedron**. It is also unique in embodying the number of yods associated with the seven enfolded polygons constituting the inner form of the Tree of Life — a number prescribed by the Godname of the Sephirah signifying the outer, physical form of the Tree of Life. The 2-dimensional, inner Tree of Life has an exterior delineated by 120 yods and an interior filled with 144 yods. The **120 Polyhedron** is the 3-dimensional version of the former and the **144 Polyhedron** is the 3-dimensional version of the latter, each yod denoting a face. The distinction between the exterior and interior of the inner Tree of Life is expressed by the **144 Polyhedron** being inside the **120 Polyhedron**.

References

[1] Phillips, Stephen M. Article 4: “Godnames Prescribe Inner Tree of Life,” http://www.smphillips.8m.com.

[2] http://www.rwgrayprojects.com – Lynnclaire Dennis’ Geometry.

[3] Proof: of the 1370 yods making up the (7+7) enfolded polygons, four belong to the shared root edge, leaving 1366 yods outside it, that is, 683 in each set of 7. There are therefore (683 + 4 = 687) yods in the 7 enfolded polygons. The topmost corner of the hexagon coincides with the lowest corner of the hexagon enfolded in the next higher tree. The number of yods in the 7n polygons enfolded in n Trees of Life = 686n + 1. As each set of 7 polygons has 36 corners, the number of corners of the 7n polygons enfolded in n Trees of Life = 35n + 1. The number of yods other than corners in the 7n polygons = 686n + 1 – (35n + 1) = 651n. Each of the two yods in each root edge other than corners is associated with a set of polygons. The number of yods other than corners associated with the 7n polygons = 651n – n = 650n. The number of yods associated with the seven polygons enfolded in successive trees = 650(n+1) – 650n = 650.

[4] Phillips, Stephen M. Article 17: “The Logarithmic Spiral Basis of the Titius-Bode Law,” http://www.smphillips.8m.com.

[5] Phillips, Stephen M. Article 18: “Encoding of Planetary Distance and Superstring Structural Parameters in the I Ching Table,” http://www.smphillips.8m.com.