

# Light Measurement Handbook

Alex Ryer

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# 1 What is Light?

## Electromagnetic Wave Theory

Light is just one portion of the various electromagnetic waves flying through space. The electromagnetic spectrum covers an extremely broad range, from radio waves with wavelengths of a meter or more, down to x-rays with wavelengths of less than a billionth of a meter. Optical radiation lies between radio waves and x-rays on the spectrum, exhibiting a unique mix of ray, wave, and quantum properties.

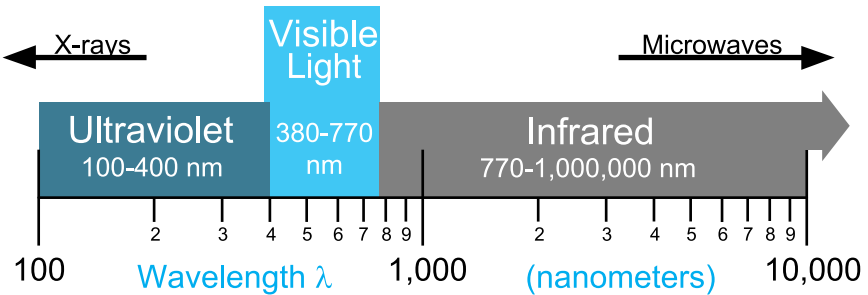
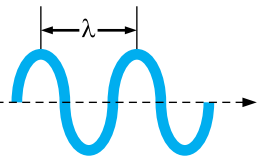


Fig. 1.1 The optical portion of the electromagnetic spectrum

At x-ray and shorter wavelengths, electromagnetic radiation tends to be quite particle like in its behavior, whereas toward the long wavelength end of the spectrum the behavior is mostly wavelike. The visible portion occupies an intermediate position, exhibiting both wave and particle properties in varying degrees.

Like all electromagnetic waves, light waves can interfere with each other, become directionally polarized, and bend slightly when passing an edge. These properties allow light to be filtered by wavelength or amplified coherently as in a laser.

In radiometry, light's propagating wavefront is modeled as a ray traveling in a straight line. Lenses and mirrors redirect these rays along predictable paths. Wave effects are insignificant in an incoherent, large scale optical system because the light waves are randomly distributed and there are plenty of photons.



## Ultraviolet Light

Short wavelength UV light exhibits more quantum properties than its visible and infrared counterparts. Ultraviolet light is arbitrarily broken down into three bands, according to its anecdotal effects.

UV-A is the least harmful and most commonly found type of UV light, because it has the least energy. UV-A light is often called black light, and is used for its relative harmlessness and its ability to cause fluorescent materials to emit visible light - thus appearing to glow in the dark. Most phototherapy and tanning booths use UV-A lamps.

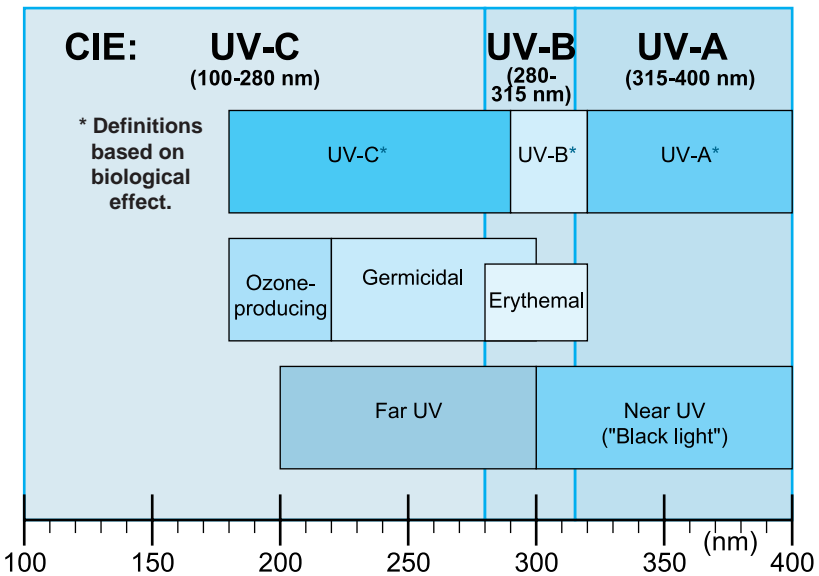


Fig. 1.2 Common ultraviolet band designations.

UV-B is typically the most destructive form of UV light, because it has enough energy to damage biological tissues, yet not quite enough to be completely absorbed by the atmosphere. UV-B is known to cause skin cancer. Since most of the extraterrestrial UV-B light is blocked by the atmosphere, a small change in the ozone layer could dramatically increase the danger of skin cancer.

Short wavelength UV-C is almost completely absorbed in air within a few hundred meters. When UV-C photons collide with oxygen atoms, the energy exchange causes the formation of ozone. UV-C is almost never observed in nature, since it is absorbed so quickly. Germicidal UV-C lamps are often used to purify air and water, because of their ability to kill bacteria.

## Visible Light

Photometry is concerned with the measurement of optical radiation as it is perceived by the human eye. The CIE 1931 Standard Observer established a standard based on the average human eye response under normal illumination with a 2° field of view. The tristimulus values graphed below represent an attempt to describe human color recognition using three sensitivity curves. The  $y(\lambda)$  curve is identical to the CIE  $V(\lambda)$  photopic vision function. Using three tristimulus measurements, any color can be fully described.

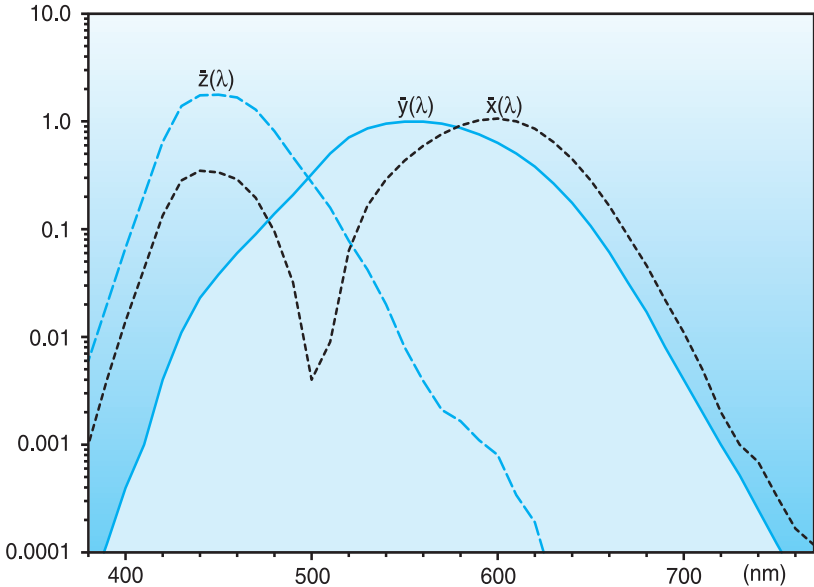


Fig. 1.3 CIE spectral tristimulus values

## Color Models

Most models of perceived color contain three components: hue, saturation, and lightness. In the CIE  $L^*a^*b^*$  model, color is modeled as a sphere, with lightness comprising the linear transform from white to black, and hues modeled as opposing pairs, with saturation being the distance from the lightness axis.

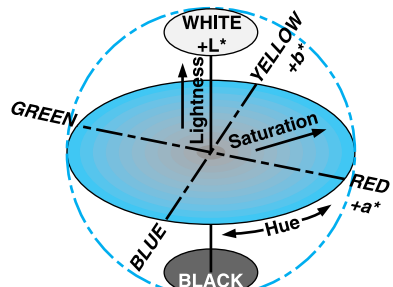


Fig. 1.4 CIE  $L^*a^*b^*$  color space.

## Infrared Light

Infrared light contains the least amount of energy per photon of any other band. Because of this, an infrared photon often lacks the energy required to pass the detection threshold of a quantum detector. Infrared is usually measured using a thermal detector such as a thermopile, which measures temperature change due to absorbed energy.

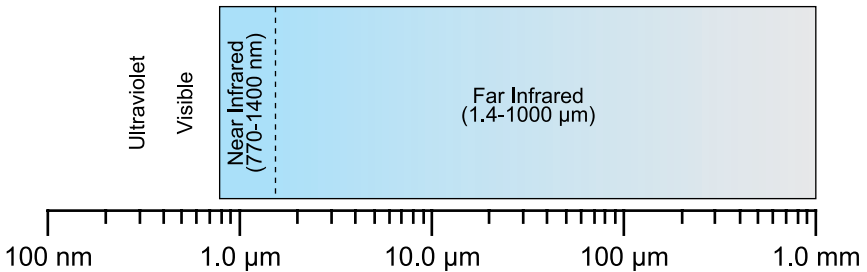


Fig. 1.5 The infrared spectrum.

While these thermal detectors have a very flat spectral responsivity, they suffer from temperature sensitivity, and usually must be artificially cooled. Another strategy employed by thermal detectors is to modulate incident light with a chopper. This allows the detector to measure differentially between the dark (zero) and light states.

Quantum type detectors are often used in the near infrared, especially below 1100 nm. Specialized detectors such as InGaAs offer excellent responsivity from 850 to 1700 nm. Typical silicon photodiodes are not sensitive above 1100 nm. These types of detectors are typically employed to measure a known artificial near-IR source without including long wavelength background ambient.

Since heat is a form of infrared light, far infrared detectors are sensitive to environmental changes - such as a person moving in the field of view. Night vision equipment takes advantage of this effect, amplifying infrared to distinguish people and machinery that are concealed in the darkness.

Infrared is unique in that it exhibits primarily wave properties. This can make it much more difficult to manipulate than ultraviolet and visible light. Infrared is more difficult to focus with lenses, refracts less, diffracts more, and is difficult to diffuse. Most radiometric IR measurements are made without lenses, filters, or diffusers, relying on just the bare detector to measure incident irradiance.



# 2 The Power of Light

## Quantum Theory

The watt (W), the fundamental unit of optical power, is defined as a rate of energy of one joule (J) per second. Optical power is a function of both the number of photons and the wavelength. Each photon carries an energy that is described by Planck's equation:

$$Q = hc / \lambda$$

where  $Q$  is the photon energy (joules),  $h$  is Planck's constant ( $6.623 \times 10^{-34}$  J s),  $c$  is the speed of light ( $2.998 \times 10^8$  m s<sup>-1</sup>), and  $\lambda$  is the wavelength of radiation (meters). All light measurement units are spectral, spatial, or temporal distributions of optical energy. As you can see in figure 2.1, short wavelength ultraviolet light has much more energy per photon than either visible or long wavelength infrared.

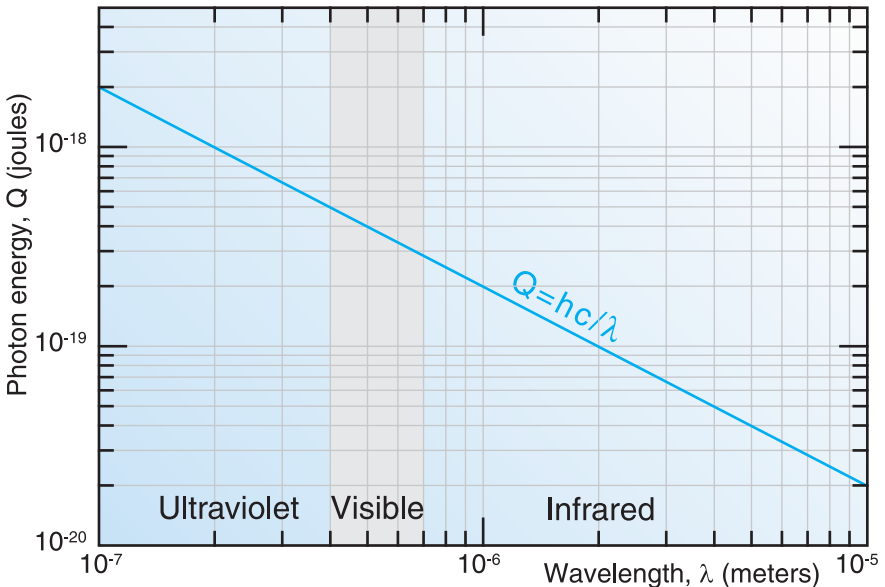


Fig. 2.1 Planck's equation showing photon energy vs. wavelength.

## Flat Response

Since silicon photodiodes are more sensitive to light at the red end of the spectrum than to light at the blue end, radiometric detectors filter the incoming light to even out the responsivity, producing a “flat response”. This is important for accurate radiometric measurements, because the spectrum of a light source may be unknown, or may be dependent on operating conditions such as input voltage.

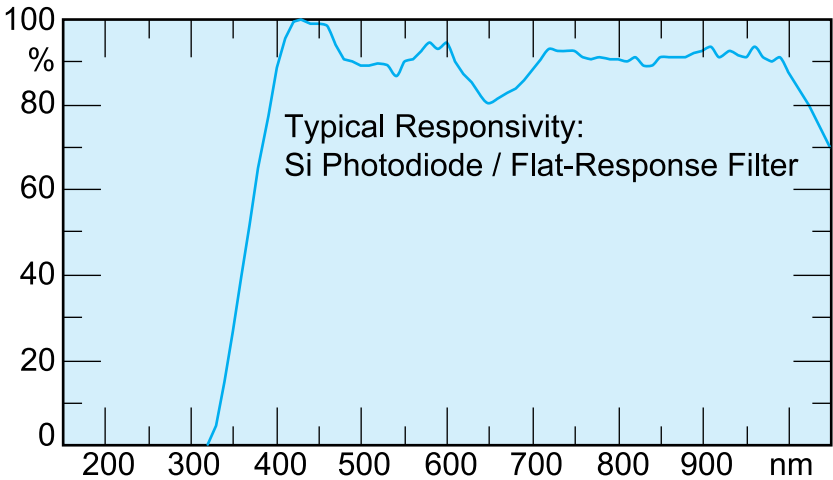


Fig. 2.2 Relative spectral responsivity of a flat response detector.

Most sources are continuums, emitting over a broad band of the spectrum. Incandescent lamps are a good example. The color temperature and output of these lamps vary significantly with input voltage. Flat response detectors measure only output power in watts, taking into consideration light at every wavelength.

Another approach is to use a narrow band filter to measure only within a small wavelength band. This is acceptable if the lamp has been fully characterized and the color temperature is carefully monitored. The difficulty with narrow band measurements, however, is that they only look at a single wavelength. If, for example, the color temperature of a lamp changes, it means that the energy distribution has shifted to a different peak wavelength. Single wavelength measurements do not reflect the total output power of the source, and may mislead you into adjusting the source.

Ratios between two narrow bands are quite useful, however, in monitoring color temperature. By measuring the red to blue ratio of a lamp, you can carefully monitor and adjust its spectral output.

## Visible Light

The lumen (lm) is the photometric equivalent of the watt, weighted to match the eye response of the “standard observer”. Yellowish-green light receives the greatest weight because it stimulates the eye more than blue or red light of equal radiometric power:

$$1 \text{ watt at } 555 \text{ nm} = 683.0 \text{ lumens}$$

To put this into perspective: the human eye can detect a flux of about 10 photons per second at a wavelength of 555 nm; this corresponds to a radiant power of  $3.58 \times 10^{-18} \text{ W}$  (or  $\text{J s}^{-1}$ ). Similarly, the eye can detect a minimum flux of 214 and 126 photons per second at 450 and 650 nm, respectively.

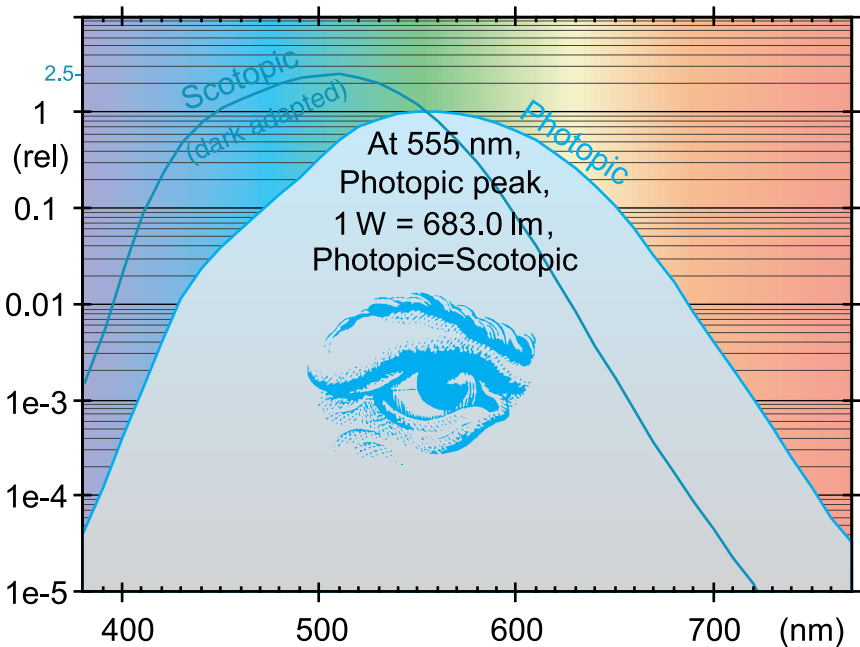


Fig. 2.3 CIE Photopic and Scotopic sensitivity curves.

Use of a photopic correction filter is important when measuring the perceived brightness of a source to a human. The filter weights incoming light in proportion to the effect it would produce in the human eye. Regardless of the color or spectral distribution of the source, the photopic detector can deliver accurate illuminance and luminance measurements in a single reading. Scotopic vision refers to the eye’s dark-adapted sensitivity (night vision).

## Effective Irradiance

Effective irradiance is weighted in proportion to the biological or chemical effect that light has on a substance. A detector and filter designed with a weighted responsivity will yield measurements that directly reflect the overall effect of an exposure, regardless of the light source.

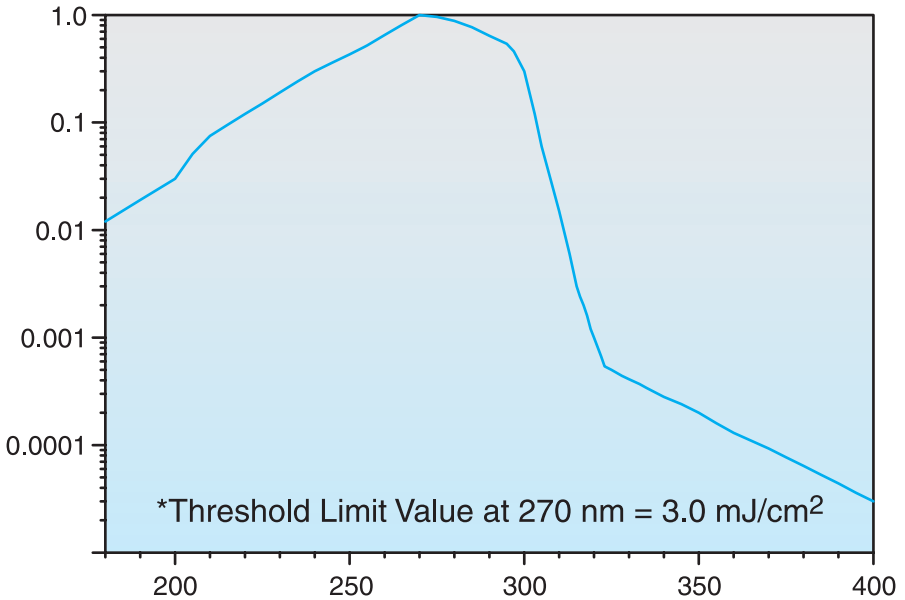


Fig. 2.4 ACGIH UV relative spectral effectiveness,  $S_\lambda$

Figure 2.4 shows the ACGIH spectral weighting function for actinic ultraviolet radiation on human skin, which is used to determine UV hazard. The threshold limit value peaks at 270 nm, representing the most dangerous segment of the UV spectrum. The harmful effect at 270 nm is two times greater than at the 254 and 297 nm mercury lines, and 9000 times greater than at the 365 nm mercury line.

The outlying extremes of the bandwidth are important to consider as well. If, for example, you are trying to assess the effective hazard of a UVA tanning lamp, which puts out most of its energy in the near UV and visible, you would want a fairly accurate match to the ACGIH curve up to the visible region of the spectrum..

Effective irradiance techniques are also used in many industries that employ UV cured inks, resins, and photoresists. A detector / filter combination is chosen that matches the chemical action spectrum of the substance that is being cured.

# 3 How Light Behaves

## Reflection

Light reflecting off of a polished or mirrored surface obeys the law of reflection: the angle between the incident ray and the normal to the surface is equal to the angle between the reflected ray and the normal.

Precision optical systems use first surface mirrors that are aluminized on the outer surface to avoid refraction, absorption, and scatter from light passing through the transparent substrate found in second surface mirrors.

When light obeys the law of reflection, it is termed a specular reflection. Most hard polished (shiny) surfaces are primarily specular in nature. Even transparent glass specularly reflects a portion of incoming light.

Diffuse reflection is typical of particulate substances like powders. If you shine a light on baking flour, for example, you will not see a directionally shiny component. The powder will appear uniformly bright from every direction.

Many reflections are a combination of both diffuse and specular components. One manifestation of this is a spread reflection, which has a dominant directional component that is partially diffused by surface irregularities.

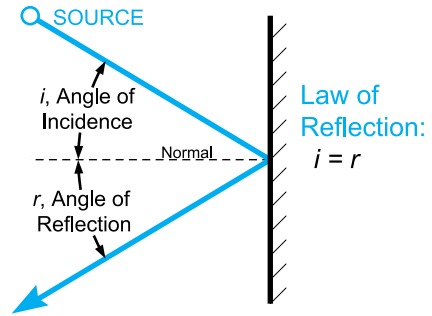


Fig. 3.1 Law of reflection.

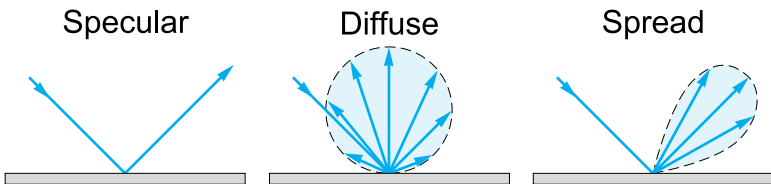


Fig. 3.2 Specular, diffuse, and spread reflection from a surface.

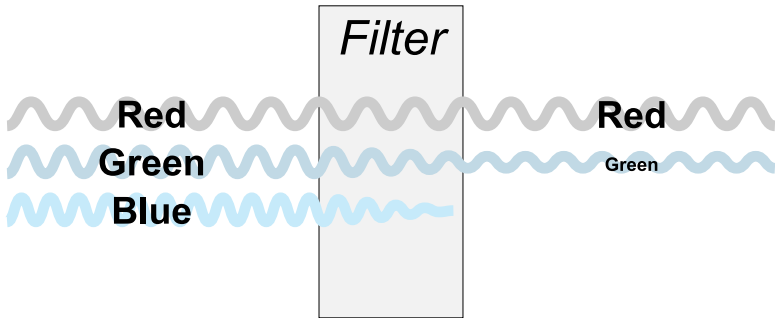


Fig. 3.3 Transmission through an optical filter.

### Transmission: Beer-Lambert or Bouguer's Law

Absorption by a filter glass varies with wavelength and filter thickness. Bouguer's law states the logarithmic relationship between internal transmission at a given wavelength and thickness.

$$\log_{10}(\tau_1) / d_1 = \log_{10}(\tau_2) / d_2$$

Internal transmittance,  $\tau_i$ , is defined as the transmission through a filter glass after the initial reflection losses are accounted for by dividing external transmission,  $T$ , by the reflection factor  $P_d$ .

$$\tau_i = T / P_d$$

*Example:* The external transmittance for a nominal 1.0 mm thick filter glass is given as  $T_{1.0} = 59.8\%$  at 330 nm. The reflection factor is given as  $P_d = 0.911$ . Find the external transmittance  $T_{2.2}$  for a filter that is 2.2 mm thick.

*Solution:*

$$\tau_{1.0} = T_{1.0} / P_d = 0.598 / 0.911 = 0.656$$

$$\tau_{2.2} = [\tau_{1.0}]^{2.2/1.0} = [0.656]^{2.2} = 0.396$$

$$T_{2.2} = \tau_{2.2} * P_d = (0.396)(0.911) = 0.361$$

So, for a 2.2 mm thick filter, the external transmittance at 330 nm would be 36.1%

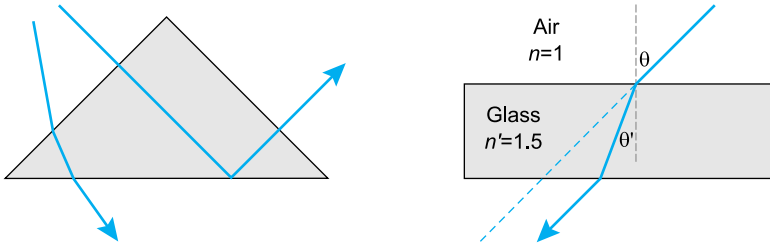


Fig. 3.4 Refraction.

## Refraction: Snell's Law

When light passes between dissimilar materials, the rays bend and change velocity slightly, an effect called refraction. Refraction is dependent on two factors: the incident angle,  $\theta$ , and the refractive index,  $n$  of the material, as given by Snell's law of refraction:

$$n \sin(\theta) = n' \sin(\theta')$$

For a typical air-glass boundary, (air  $n = 1$ , glass  $n' = 1.5$ ), a light ray entering the glass at  $30^\circ$  from normal travels through the glass at  $19.5^\circ$  and straightens out to  $30^\circ$  when it exits out the parallel side.

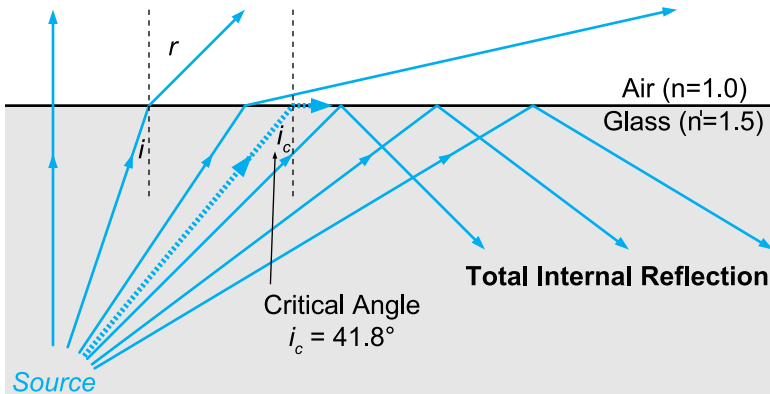


Fig. 3.5 Refraction and total internal reflection.

Note that since  $\sin(0^\circ) = 0$ , light entering or exiting normal to a boundary does not bend. Also, at the internal glass-air boundary, total internal reflection occurs when  $n' \sin(\theta') = 1$  (at  $\theta' = 41.8^\circ$  for  $n' = 1.5$  glass).

The index of refraction itself is also dependent on wavelength. This angular dispersion causes blue light to refract more than red, causing rainbows and allowing prisms to separate the spectrum.

## Diffraction

Diffraction is another wave phenomenon that is dependent on wavelength. Light waves bend as they pass by the edge of a narrow aperture or slit. This effect is approximated by:

$$\theta = \lambda / D$$

where  $\theta$  is the diffraction angle,  $\lambda$  the wavelength of radiant energy, and  $D$  the aperture diameter. This effect is negligible in most optical systems, but is exploited in monochromators. A diffraction grating uses the interference of waves caused by diffraction to separate light angularly by wavelength. Narrow slits then select the portion of the spectrum to be measured. The narrower the slit, the narrower the bandwidth that can be measured. However, diffraction in the slit itself limits the resolution that can ultimately be achieved.

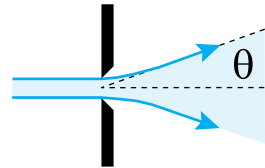


Fig. 3.6 Diffraction.

## Interference

When wave fronts overlap in phase with each other, the magnitude of the wave increases. When the wave fronts are out of phase, however, they cancel each other out. Interference filters use this effect to selectively filter light by wavelength. Thin metal or dielectric reflective layers separated by an optical distance of  $n'd = \lambda/2$ , or half the desired wavelength provide in phase transmission.

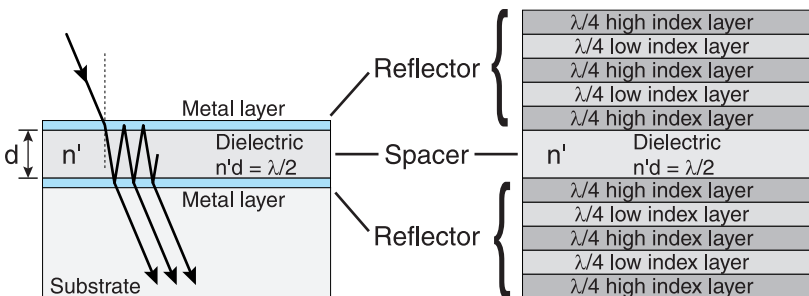


Fig. 3.7 A thin film metal interference filter and an all dielectric interference filter.

The center wavelength shifts with angle, since the optical path increases as the cosine of the angle. Special input optics are required to provide a cosine response while transmitting light through the filter at a near normal angle.



# 4 Manipulating Light

## Diffusion

It is often necessary to diffuse light, either through transmission or reflection. Diffuse transmission can be accomplished by transmitting light through roughened quartz, flashed opal, or polytetrafluoroethylene (PTFE, Teflon). Diffusion can vary with wavelength. Teflon is a poor IR diffuser, but makes an excellent visible / UV diffuser. Quartz is required for UV diffusion.

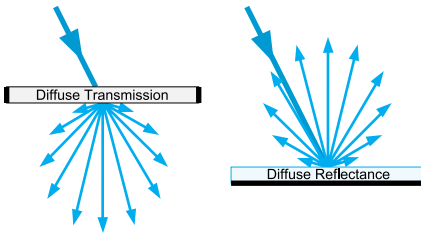


Fig. 4.1 Diffuse transmission and reflectance.

Integrating spheres are coated with  $\text{BaSO}_4$  or PTFE, which offer  $>97\%$  reflectance over a broad spectral range with near perfect diffusion. These coatings are, however, quite expensive and fragile.

## Collimation

Some lamps use collimating lenses or reflectors to redirect light into a beam of parallel rays. If the lamp filament is placed at the focal point of the lens, all rays entering the lens will become parallel. Similarly, a lamp placed in the focal point of a spherical or parabolic mirror will project a parallel beam. Lenses and reflectors can drastically distort inverse square law approximations, so should be avoided where precision distance calculations are required.

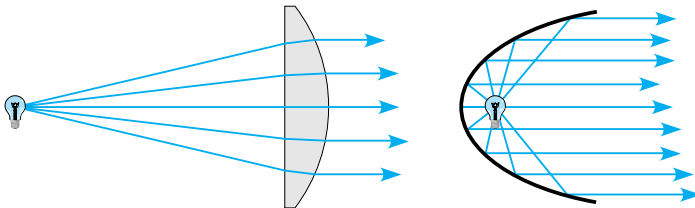
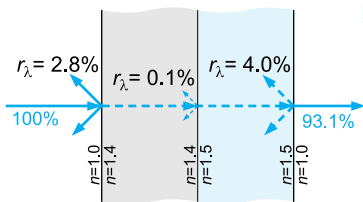


Fig. 4.2 Collimation using a lens and a parabolic reflector.

## Transmission Losses

When light passes between two materials of different refractive indices, a predictable amount of reflection losses can be expected. Fresnel's law quantifies this loss. If  $n_\lambda = 1.5$  between air and glass, then  $r_\lambda = 4\%$  for each surface. Two filters separated by air transmit 8% less than two connected by optical cement (or even water).



### Fresnel's Law:

The reflection loss,  $r_\lambda$ , at normal incidence, between two media with different refractive indexes. ( $n_\lambda = n/n_\lambda$ , the ratio of the indices)

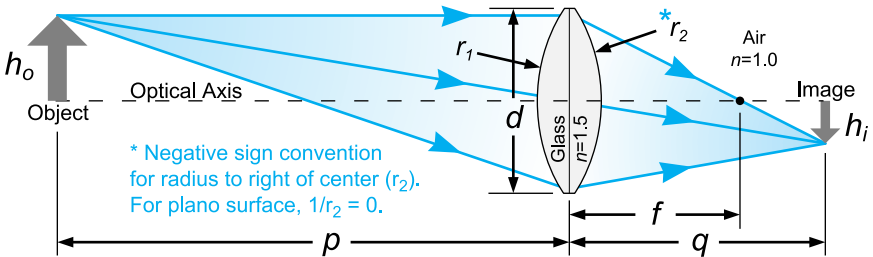
$$r_\lambda = \frac{(n_\lambda - 1)^2}{(n_\lambda + 1)^2}$$

Fig. 4.3 Fresnel's law of reflection, showing boundary reflection losses.

Precision optical systems use first surface mirrors to avoid reflection losses from entering and exiting a glass substrate layer.

## Focusing Lenses

Lenses are often employed to redirect light or concentrate optical power. The lens equation defines the image distance  $q$ , projected from a point that is a distance  $p$  from the lens, based on the focal distance,  $f$ , of the lens. The focal distance is dependent on the curvature and refractive index of the lens.



**Lens Equation:**  $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$     **Lens Maker's Equation:**  $\frac{1}{f} = (n-1) \left( \frac{1}{r_1} - \frac{1}{r_2} \right)$     **Magnification:**  $m = \frac{h_i}{h_o} = -\frac{q}{p}$     **F-number:**  $f/\# = \frac{f}{d}$

Fig. 4.4 Some useful lens equations.

Simply put, all rays parallel to the optical axis pass through the focal point. Since index of refraction is dependent on wavelength, chromatic aberrations can occur in simple lenses.

## Mirrors

When light reflects off of a rear surface mirror, the light first passes through the glass substrate, resulting in reflection losses, secondary reflections, and a change in apparent distance.

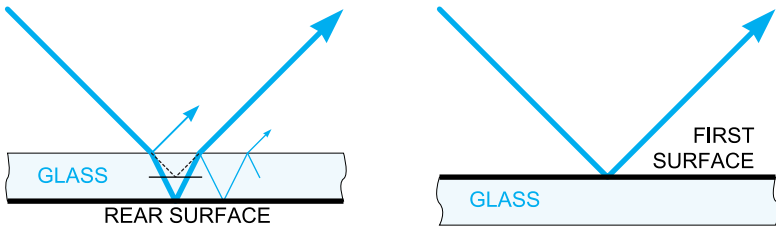
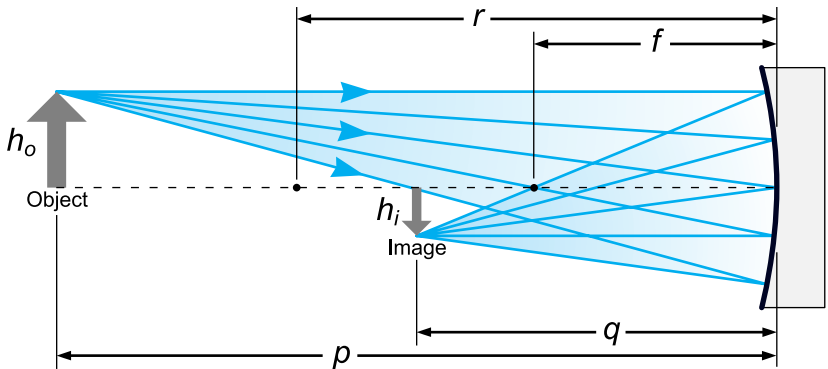


Fig. 4.5 Reflection from a second surface and first surface mirror.

First surface mirrors avoid this by aluminizing the front, and coating it with a thin protective SiO coating to prevent oxidation and scratching.

## Concave Mirrors

Concave mirrors are often used to focus light in place of a lens. Just as with a lens, a concave mirror has a principal focus,  $f$ , through which all rays parallel to the optical axis pass through. The focal length of a spherical concave mirror is one half the radius of the spherical surface. Reflective systems avoid the chromatic aberrations that can result from the use of lenses.



Mirror Equation:

$$\frac{1}{p} - \frac{1}{q} = -\frac{2}{r}$$

Principal Focus:

$$f = \frac{r}{2}$$

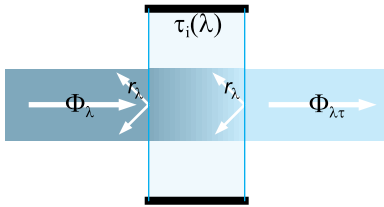
Magnification:

$$m = \frac{h_i}{h_o} = \frac{q}{p}$$

Fig. 4.6 Reflection from a spherical concave mirror.

## Internal Transmittance

Filter manufacturers usually provide data for a glass of nominal thickness. Using Bouguer's law, you can calculate the transmission at other thicknesses. Manufacturers usually specify  $P_d$ , so you can calculate the external transmittance from internal transmittance data.



**Transmittance:**

$$T(\lambda) = \Phi_{\lambda\tau} / \Phi_\lambda$$

**Internal Transmittance:**

$$\tau_i(\lambda) = T(\lambda) / P_d$$

**Reflection Factor:**

$$P_d = 1 - (2 * r_\lambda) = 0.918 \text{ typ}$$

Fig. 4.7 External vs. internal transmittance.

## Prisms

Prisms use glass with a high index of refraction to exploit the variation of refraction with wavelength. Blue light refracts more than red, providing a spectrum that can be isolated using a narrow slit.

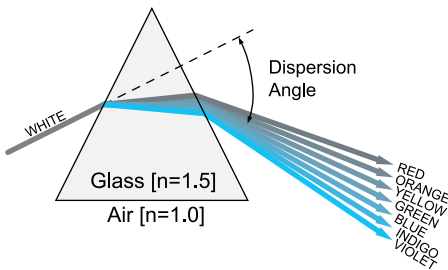
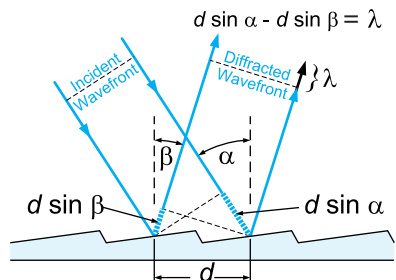


Fig. 4.8 Refraction through a prism.

Internal prisms can be used to simply reflect light. Since total internal reflection is dependent on a difference in refractive index between materials, any dirt on the outer surface will reduce the reflective properties, a property that is exploited in finger print readers.

## Diffraction Gratings

Most monochromators use gratings to disperse light into the spectrum. Gratings rely on interference between wavefronts caused by microscopically ruled diffraction lines on a mirrored surface. The wavelength of reflected light varies with angle, as defined by the grating equation, where  $m$  is the order of the spectrum (an integer).



**Grating Equation:**

$$m\lambda = d (\sin \alpha \pm \sin \beta)$$

Fig 4.9 Diffraction grating

# 5 Light Sources

## Blackbody Radiation

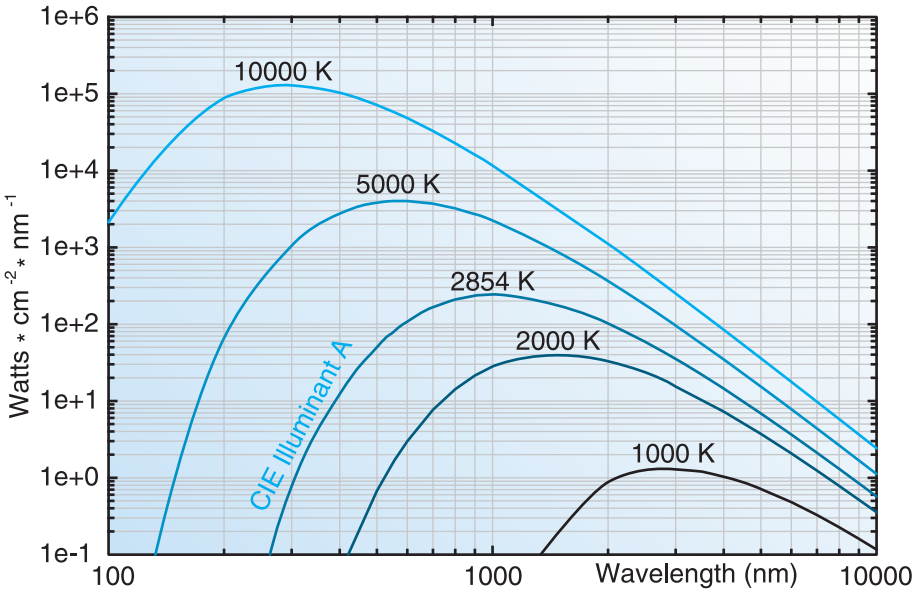


Fig. 5.1 Blackbody radiation at several color temperatures.

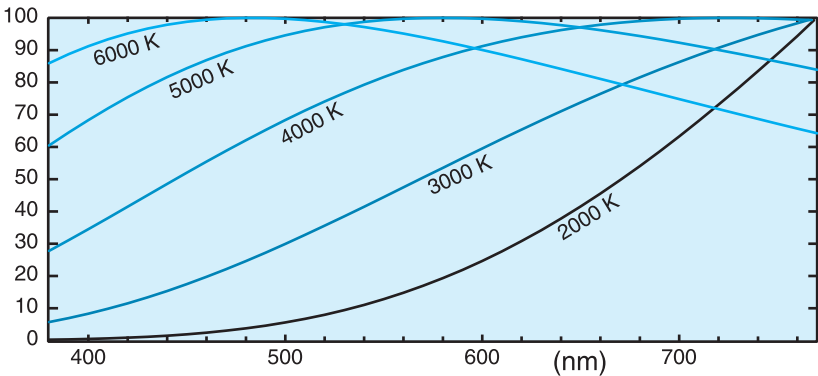


Fig. 5.2 Blackbody radiation plotted on a linear scale in the visible.

## Incandescent Sources

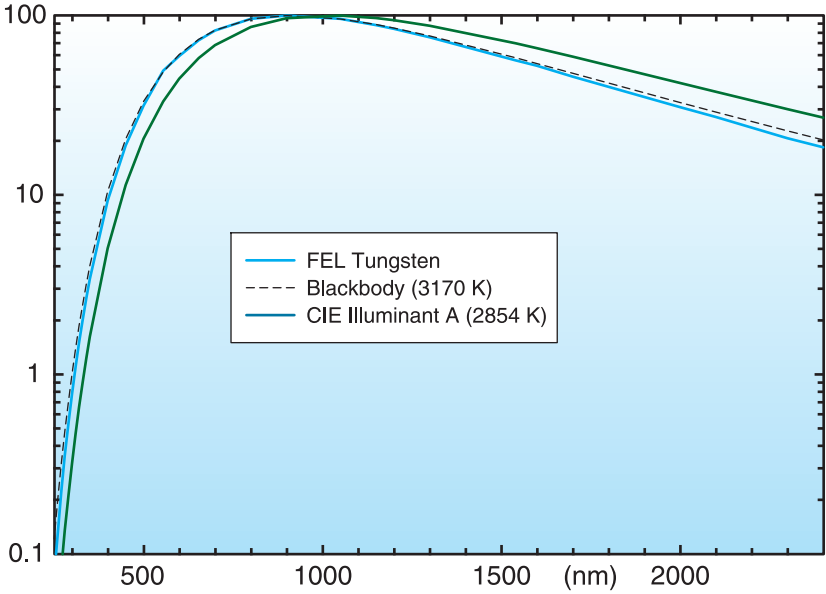


Fig. 5.3 Incandescent sources

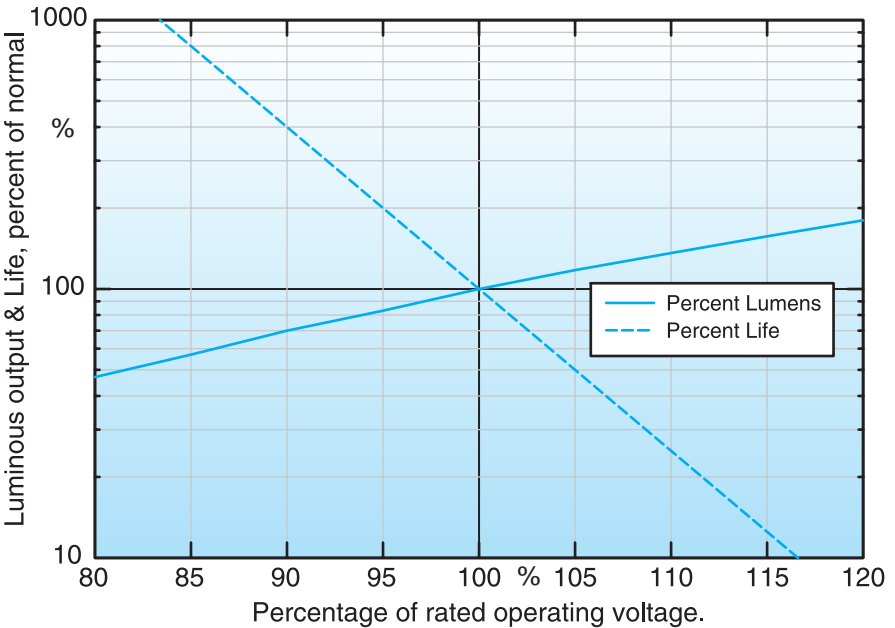


Fig. 5.4 Performance characteristics of tungsten lamps vary significantly with voltage.

## Luminescent Sources

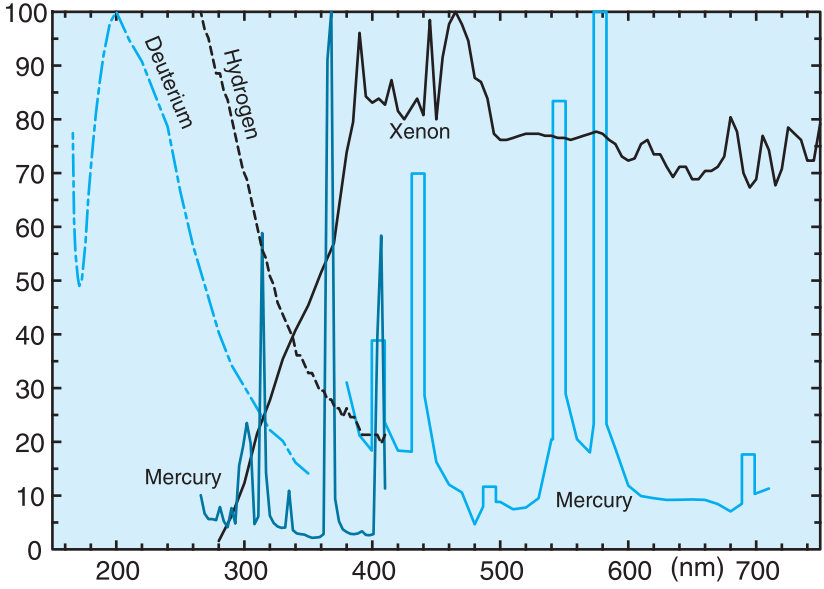


Fig. 5.5 Arc lamps.

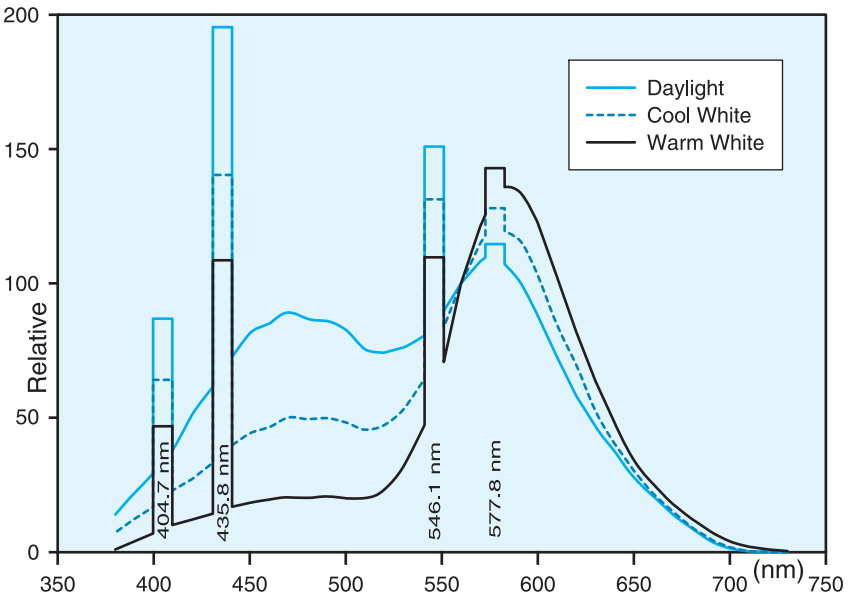


Fig. 5.6 Typical fluorescent sources.

## Sunlight

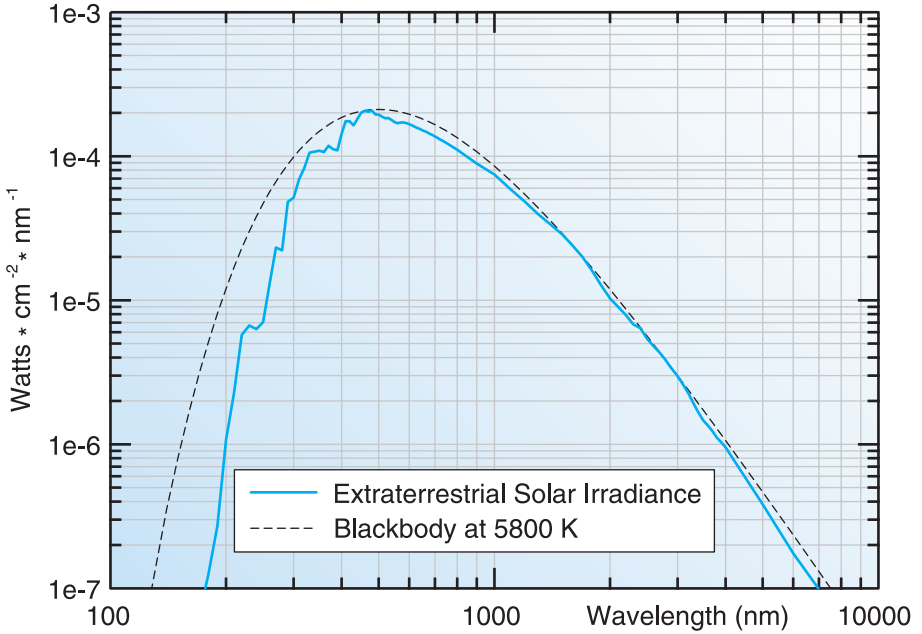


Fig. 5.7 Extraterrestrial solar irradiance compared to a blackbody.

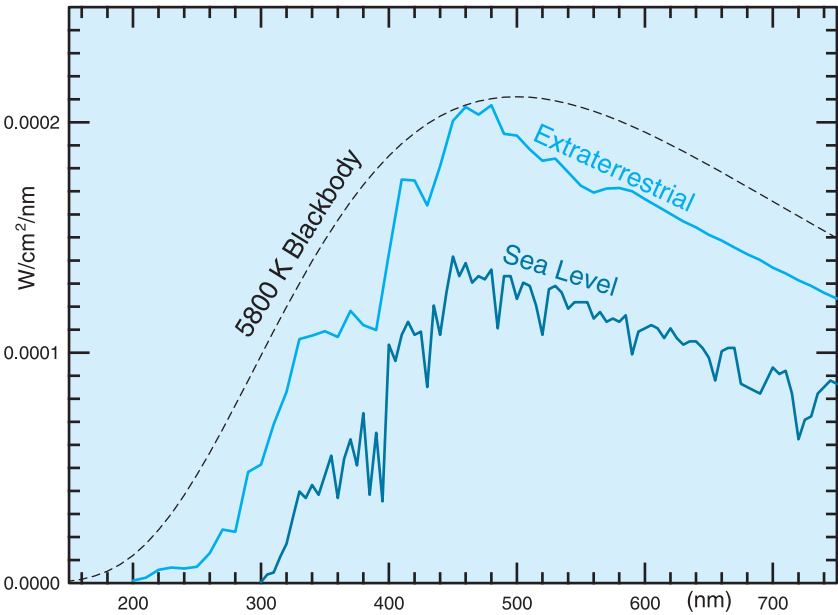


Fig. 5.8 Solar irradiance at sea level.



# 6 Basic Principles

## The Inverse Square Law

The inverse square law defines the relationship between the irradiance from a point source and distance. It states that the intensity per unit area varies in inverse proportion to the square of the distance.

$$E = I / d^2$$

In other words, if you measure 16 W/cm<sup>2</sup> at 1 meter, you will measure 4 W/cm<sup>2</sup> at 2 meters, and can calculate the irradiance at any other distance. An alternate form is often more convenient:

$$E_1 d_1^2 = E_2 d_2^2$$

Distance is measured to the first luminating surface - the filament of a clear bulb, or the glass envelope of a frosted bulb.

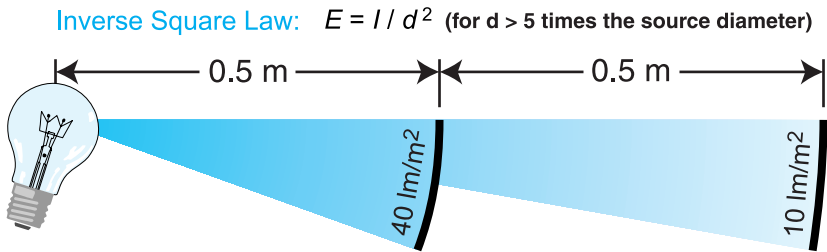


Fig. 6.1 Inverse square law.

*Example:* You measure 10.0 lm/m<sup>2</sup> from a light bulb at 1.0 meter. What will the flux density be at half the distance?

*Solution:*

$$E_1 = (d_2 / d_1)^2 * E_2$$

$$E_{0.5m} = (1.0 / 0.5)^2 * 10.0 = 40 \text{ lm/m}^2$$

## Point Source Approximation

The inverse square law can only be used in cases where the light source approximates a point source. A general rule of thumb to use for irradiance measurements is the “five times rule”: the distance to a light source should be greater than five times the largest dimension of the source. For a clear enveloped lamp, this may be the length of the filament. For a frosted light bulb, the diameter is the largest dimension. Figure 6.2 below shows the relationship between irradiance and the ratio of distance to source radius. Note that for a distance 10 times the source radius (5 times the diameter), the error from using the inverse square is exactly 1 %, hence the “five times” approximation.

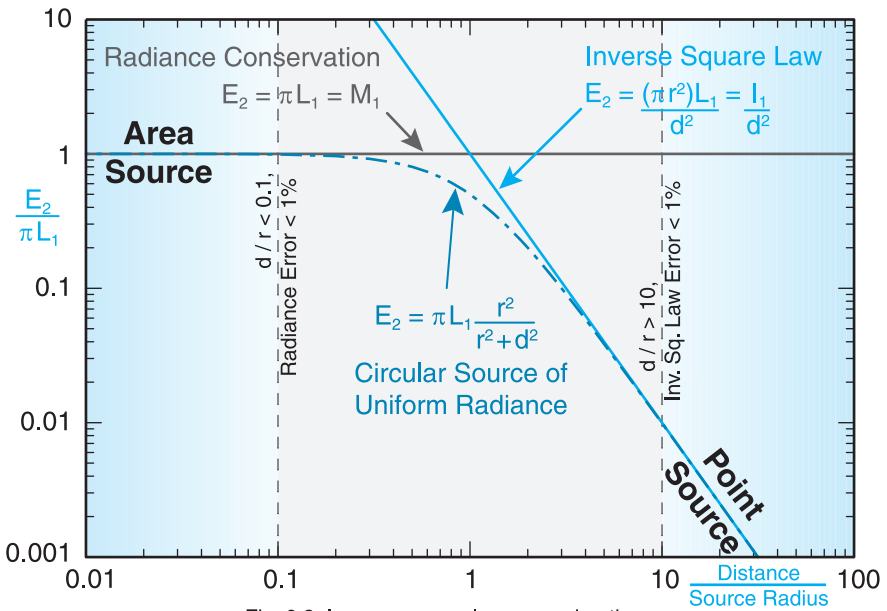


Fig. 6.2 Inverse square law approximation error.

Note also, that when the ratio of distance to source radius decreases to below 0.1 (1/20 the diameter of the source), changes in distance hardly affect the irradiance (< 1 % error). This is due to the fact that as the distance from the source decreases, the detector sees less area, counteracting the inverse square law. The graph above assumes a cosine response. Radiance detectors restrict the field of view so that the  $d/r$  ratio is always low, providing measurements independent of distance.



## Lambert's Cosine Law

The irradiance or illuminance falling on any surface varies as the cosine of the incident angle,  $\theta$ . The perceived measurement area orthogonal to the incident flux is reduced at oblique angles, causing light to spread out over a wider area than it would if perpendicular to the measurement plane.

To measure the amount of light falling on human skin, you need to mimic the skin's cosine response. Since filter rings restrict off-angle light, a cosine diffuser must be used to correct the spatial responsivity. In full immersion applications like the phototherapy booth shown above, off angle light is significant, requiring accurate cosine correction optics.

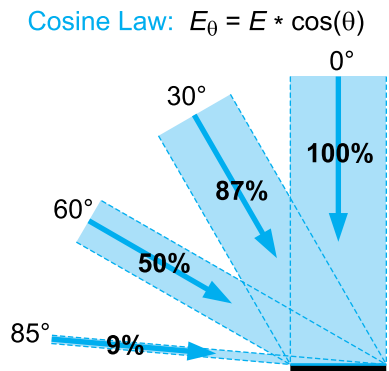


Fig. 6.3 Lambert's cosine law.

## Lambertian Surface

A Lambertian surface provides uniform diffusion of the incident radiation

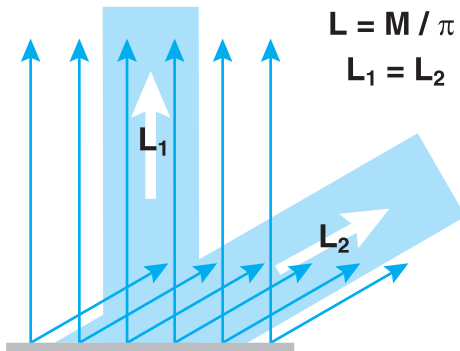


Fig. 6.4 Lambertian surface.

such that its radiance or luminance is the same in all directions from which it can be measured. Many diffuse surfaces are, in fact, Lambertian. If you view this Light Measurement Handbook from an oblique angle, it should look as bright as it did when held perpendicular to your line of vision. The human eye, with its restricted solid viewing angle, is an ideal luminance, or brightness, detector.

Figure 6.4 shows a surface radiating equally at 0° and at 60°.

Since, by the cosine law, a radiance detector sees twice as much surface area in the same solid angle for the 60° case, the average incremental reflection must be half the magnitude of the reflection in the 0° case.

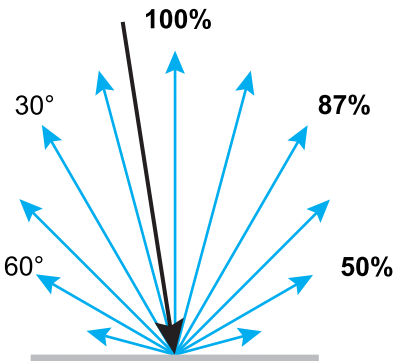


Fig. 6.5 Lambertian surface.

Figure 6.5 shows that a reflection from a diffuse Lambertian surface obeys the cosine law by distributing reflected energy in proportion to the cosine of the reflected angle.

A Lambertian surface that has a radiance of 1.0 W/cm<sup>2</sup>/sr will radiate a total of  $\pi \cdot A$  watts, where A is the area of the surface, into a hemisphere of 2π steradians. Since the radiant exitance of the surface is equal to the total power divided by the total area, the radiant exitance is  $\pi$  W/cm<sup>2</sup>. In other words, if you were to illuminate a surface with an irradiance of 3.1416 W/cm<sup>2</sup>, then you

will measure a radiance on that surface of 1.00 W/cm<sup>2</sup>/sr (if it is 100% reflective).

The next section goes into converting between measurement geometries in much greater depth.

# 7 Measurement Geometries

## Solid Angles

One of the key concepts to understanding the relationships between measurement geometries is that of the solid angle, or steradian. A sphere contains  $4\pi$  steradians. A solid angle which, having its vertex at the center of the sphere, cuts off a spherical surface area equal to the square of the radius of the sphere. For example, a one steradian section of a one meter radius spherical surface subtends a surface area of one square meter.

The sphere shown in cross section in figure 7.1 illustrates the concept. A cone with a solid angle of one steradian has been removed from the sphere. This removed cone is shown in figure 7.2. The solid angle,  $\Omega$ , in steradians, is equal to the spherical surface area,  $A$ , divided by the square of the radius,  $r$ .

Most radiometric measurements do not require an accurate calculation of the spherical surface area to convert between units. Flat area estimates can be substituted for spherical area when the solid angle is less than 0.03 steradians, resulting in an error of less than one percent. This roughly translates to a distance at least 5 times greater than the largest dimension of the detector. In general, if you follow the “five times rule” for approximating a point source (see Chapter 6), you can safely estimate using planar surface area.

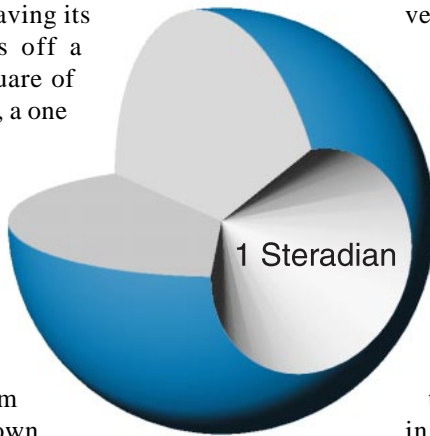


Fig. 7.1

A steradian is defined as the solid angle which, having its vertex at the center of a spherical surface the radius of the sphere is one meter, subtends a spherical surface area equal to the square of the radius of the sphere. This is shown in cross section in figure 7.1. A cone with a solid angle of one steradian has been removed from the sphere. This is shown in figure 7.2. The solid angle is equal to the spherical

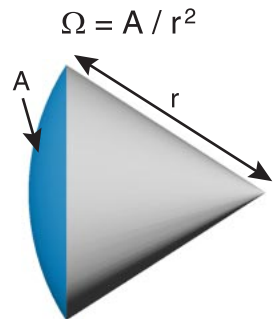


Fig. 7.2 Solid angle

## Radiant and Luminous Flux

Radiant flux is a measure of radiometric power. Flux, expressed in watts, is a measure of the rate of energy flow, in joules per second. Since photon energy is inversely proportional to wavelength, ultraviolet photons are more powerful than visible or infrared.

Luminous flux is a measure of the power of visible light. Photopic flux, expressed in lumens, is weighted to match the responsivity of the human eye, which is most sensitive to yellow-green.

Scotopic flux is weighted to the sensitivity of the human eye in the dark adapted state.

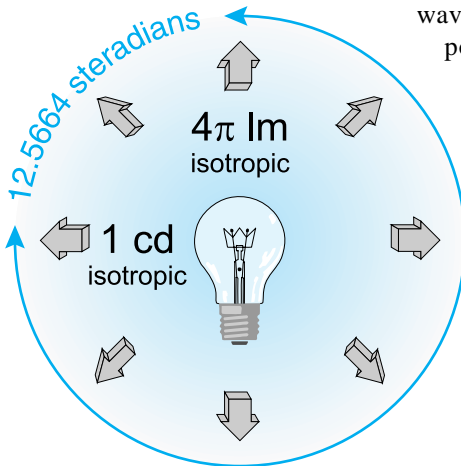


Fig. 7.3 total flux output.

### Units Conversion: Power

#### RADIANT FLUX:

- 1 W (watt)
  - = 683.0 lm at 555 nm
  - = 1700.0 scotopic lm at 507 nm
- 1 J (joule)
  - = 1 W\*s (watt \* second)
  - =  $10^7$  erg
  - = 0.2388 gram \* calories

#### LUMINOUS FLUX:

- 1 lm (lumen)
  - =  $1.464 \times 10^{-3}$  W at 555 nm
  - =  $1/(4\pi)$  candela (only if isotropic)
- 1 lm\*s (lumen \* seconds)
  - = 1 talbot (T)
  - =  $1.464 \times 10^{-3}$  joules at 555 nm

| $\lambda$<br>nm | Photopic               | Photopic             | Scotopic               | Scotopic             |
|-----------------|------------------------|----------------------|------------------------|----------------------|
|                 | Luminous<br>Efficiency | lm / W<br>Conversion | Luminous<br>Efficiency | lm / W<br>Conversion |
| 380             | 0.000039               | 0.027                | 0.000589               | 1.001                |
| 390             | .000120                | 0.082                | .002209                | 3.755                |
| 400             | .000396                | 0.270                | .009290                | 15.793               |
| 410             | .001210                | 0.826                | .034840                | 59.228               |
| 420             | .004000                | 2.732                | .096600                | 164.220              |
| 430             | .011600                | 7.923                | .199800                | 339.660              |
| 440             | .023000                | 15.709               | .328100                | 557.770              |
| 450             | .038000                | 25.954               | .455000                | 773.500              |
| 460             | .060000                | 40.980               | .567000                | 963.900              |
| 470             | .090980                | 62.139               | .676000                | 1149.200             |
| 480             | .139020                | 94.951               | .793000                | 1348.100             |
| 490             | .208020                | 142.078              | .904000                | 1536.800             |
| 500             | .323000                | 220.609              | .982000                | 1669.400             |
| 507             | <b>.444310</b>         | <b>303.464</b>       | <b>1.000000</b>        | <b>1700.000</b>      |
| 510             | .503000                | 343.549              | .997000                | 1694.900             |
| 520             | .710000                | 484.930              | .935000                | 1589.500             |
| 530             | .862000                | 588.746              | .811000                | 1378.700             |
| 540             | .954000                | 651.582              | .650000                | 1105.000             |
| 550             | .994950                | 679.551              | .481000                | 817.700              |
| 555             | <b>1.000000</b>        | <b>683.000</b>       | <b>.402000</b>         | <b>683.000</b>       |
| 560             | .995000                | 679.585              | .328800                | 558.960              |
| 570             | .952000                | 650.216              | .207600                | 352.920              |
| 580             | .870000                | 594.210              | .121200                | 206.040              |
| 590             | .757000                | 517.031              | .065500                | 111.350              |
| 600             | .631000                | 430.973              | .033150                | 56.355               |
| 610             | .503000                | 343.549              | .015930                | 27.081               |
| 620             | .381000                | 260.223              | .007370                | 12.529               |
| 630             | .265000                | 180.995              | .003335                | 5.670                |
| 640             | .175000                | 119.525              | .001497                | 2.545                |
| 650             | .107000                | 73.081               | .000677                | 1.151                |
| 660             | .061000                | 41.663               | .000313                | 0.532                |
| 670             | .032000                | 21.856               | .000148                | 0.252                |
| 680             | .017000                | 11.611               | .000072                | 0.122                |
| 690             | .008210                | 5.607                | .000035                | .060                 |
| 700             | .004102                | 2.802                | .000018                | .030                 |
| 710             | .002091                | 1.428                | .000009                | .016                 |
| 720             | .001047                | 0.715                | .000005                | .008                 |
| 730             | .000520                | 0.355                | .000003                | .004                 |
| 740             | .000249                | 0.170                | .000001                | .002                 |
| 750             | .000120                | 0.082                | .000001                | .001                 |
| 760             | .000060                | 0.041                |                        |                      |
| 770             | .000030                | 0.020                |                        |                      |

## Irradiance and Illuminance:

Irradiance is a measure of radiometric flux per unit area, or flux density. Irradiance is typically expressed in  $\text{W}/\text{cm}^2$  (watts per square centimeter) or  $\text{W}/\text{m}^2$  (watts per square meter).

Illuminance is a measure of photometric flux per unit area, or visible flux density. Illuminance is typically expressed in lux (lumens per square meter) or foot-candles (lumens per square foot).

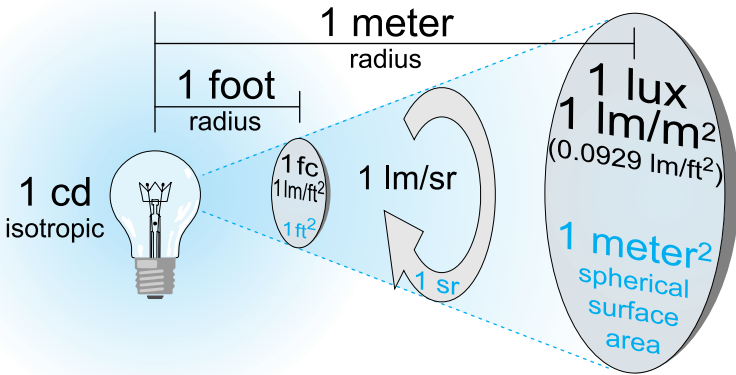


Fig. 7.4 Irradiance.

In figure 7.4, above, the lightbulb is producing 1 candela. The candela is the base unit in light measurement, and is defined as follows: a 1 candela light source emits 1 lumen per steradian in all directions (isotropically). A steradian is defined as the solid angle which, having its vertex at the center of the sphere, cuts off an area equal to the square of its radius. The number of steradians in a beam is equal to the projected area divided by the square of the distance.

So, 1 steradian has a projected area of 1 square meter at a distance of 1 meter. Therefore, a 1 candela (1 lm/sr) light source will similarly produce 1 lumen per square foot at a distance of 1 foot, and 1 lumen per square meter at 1 meter. Note that as the beam of light projects farther from the source, it expands, becoming less dense. In fig. 7.4, for example, the light expanded from 1 lm/ft<sup>2</sup> at 1 foot to 0.0929 lm/ft<sup>2</sup> (1 lux) at 3.28 feet (1 m).

## Cosine Law

Irradiance measurements should be made facing the source, if possible. The irradiance will vary with respect to the cosine of the angle between the optical axis and the normal to the detector.

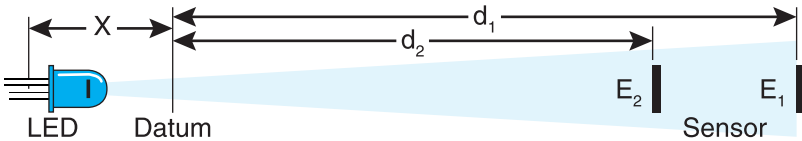


## Calculating Source Distance

Lenses will distort the position of a point source. You can solve for the virtual origin of a source by measuring irradiance at two points and solving for the offset distance,  $X$ , using the Inverse Square Law:

$$E_1(d_1 + X)^2 = E_2(d_2 + X)^2$$

Figure 7.5 illustrates a typical setup to determine the location of an LED's virtual point source (which is behind the LED due to the built-in lens). Two irradiance measurements at known distances from a reference point are all that is needed to calculate the offset to the virtual point source.



$$X = [d_1(E_1 / E_2)^{1/2} - d_2] \div [1 - (E_1 / E_2)^{1/2}]$$

Fig. 7.5 Sometimes you have to solve for the true distance from source to sensor.

### Units Conversion: Flux Density

#### IRRADIANCE:

- 1 W/cm<sup>2</sup> (watts per square centimeter)
- = 10<sup>4</sup> W/m<sup>2</sup> (watts per square meter)
- = 6.83 x 10<sup>6</sup> lux at 555 nm
- = 14.33 gram\*calories/cm<sup>2</sup>/minute

#### ILLUMINANCE:

- 1 lm/m<sup>2</sup> (lumens per square meter)
- = 1 lux (lx)
- = 10<sup>-4</sup> lm/cm<sup>2</sup>
- = 10<sup>-4</sup> phot (ph)
- = 9.290 x 10<sup>-2</sup> lm/ft<sup>2</sup>
- = 9.290 x 10<sup>-2</sup> foot-candles (fc)

## Radiance and Luminance:

Radiance is a measure of the flux density per unit solid viewing angle, expressed in  $\text{W}/\text{cm}^2/\text{sr}$ . Radiance is independent of distance for an extended area source, because the sampled area increases with distance, cancelling inverse square losses.

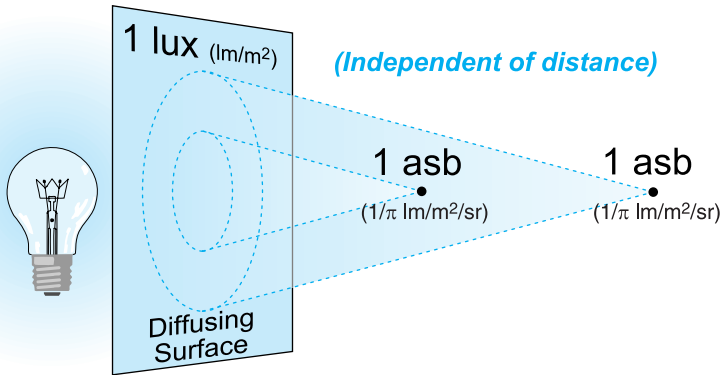


Fig. 7.6 Radiance

The radiance,  $L$ , of a diffuse (Lambertian) surface is related to the radiant exitance (flux density),  $M$ , of a surface by the relationship:

$$L = M / \pi$$

Some luminance units (asb,  $L$ , fL) already contain  $\pi$  in the denominator, allowing simpler conversion to illuminance units.

*Example:* Suppose a diffuse surface with a reflectivity,  $\rho$ , of 85% is exposed to an illuminance,  $E$ , of 100.0 lux ( $\text{lm}/\text{m}^2$ ) at the plane of the surface. What would be the luminance,  $L$ , of that surface, in  $\text{cd}/\text{m}^2$ ?

*Solution:*

1.) Calculate the luminous exitance of the surface:

$$M = E * \rho$$
$$M = 100.0 * 0.85 = 85.0 \text{ lm}/\text{m}^2$$

2.) Calculate the luminance of the surface:

$$L = M / \pi$$
$$L = 85.0 / \pi = 27.1 \text{ lm}/\text{m}^2/\text{sr} = 27.1 \text{ cd}/\text{m}^2$$

## Irradiance From An Extended Source:

The irradiance,  $E$ , at any distance from a uniform extended area source, is related to the radiance,  $L$ , of the source by the following relationship, which depends only on the subtended central viewing angle,  $\theta$ , of the radiance detector:

$$E = \pi L \sin^2(\theta/2)$$

So, for an extended source with a radiance of  $1 \text{ W/cm}^2/\text{sr}$ , and a detector with a viewing angle of  $3^\circ$ , the irradiance at any distance would be  $2.15 \times 10^{-3} \text{ W/cm}^2$ . This assumes, of course, that the source extends beyond the viewing angle of the detector input optics.

## Units Conversion: Radiance & Luminance

### RADIANCE:

$$\begin{aligned} 1 \text{ W/cm}^2/\text{sr} & \text{ (watts per sq. cm per steradian)} \\ & = 6.83 \times 10^6 \text{ lm/m}^2/\text{sr} \text{ at } 555 \text{ nm} \\ & = 683 \text{ cd/cm}^2 \text{ at } 555 \text{ nm} \end{aligned}$$

### LUMINANCE:

$$\begin{aligned} 1 \text{ lm/m}^2/\text{sr} & \text{ (lumens per sq. meter per steradian)} \\ & = 1 \text{ candela/m}^2 \text{ (cd/m}^2\text{)} \\ & = 1 \text{ nit} \\ & = 10^{-4} \text{ lm/cm}^2/\text{sr} \\ & = 10^{-4} \text{ cd/cm}^2 \\ & = 10^{-4} \text{ stilb (sb)} \\ & = 9.290 \times 10^{-2} \text{ cd/ft}^2 \\ & = 9.290 \times 10^{-2} \text{ lm/ft}^2/\text{sr} \\ & = \pi \text{ apostilbs (asb)} \\ & = \pi \text{ cd}/\pi\text{m}^2 \\ & = \pi \times 10^{-4} \text{ lamberts (L)} \\ & = \pi \times 10^{-4} \text{ cd}/\pi\text{cm}^2 \\ & = 2.919 \times 10^{-1} \text{ foot-lamberts (fL)} \\ & = 2.919 \times 10^{-1} \text{ lm}/\pi\text{ft}^2/\text{sr} \end{aligned}$$

## Radiant and Luminous Intensity:

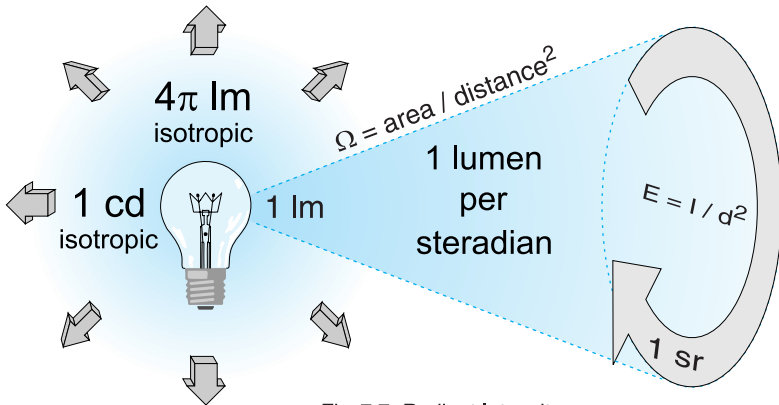


Fig. 7.7 Radiant Intensity.

Radiant Intensity is a measure of radiometric power per unit solid angle, expressed in watts per steradian. Similarly, luminous intensity is a measure of visible power per solid angle, expressed in candela (lumens per steradian). Intensity is related to irradiance by the inverse square law, shown below in an alternate form:

$$I = E * d^2$$

If you are wondering how the units cancel to get flux/sr from flux/area times distance squared, remember that steradians are a dimensionless quantity. Since the solid angle equals the area divided by the square of the radius,  $d^2=A/W$ , and substitution yields:

$$I = E * A / W$$

The biggest source of confusion regarding intensity measurements involves the difference between Mean Spherical Candela and Beam Candela, both of which use the candela unit (lumens per steradian). Mean spherical measurements are made in an integrating sphere, and represent the total output in lumens divided by  $4\pi$  sr in a sphere. Thus, a one candela isotropic lamp produces one lumen per steradian.

Beam candela, on the other hand, samples a very narrow angle and is only representative of the lumens per steradian at the peak intensity of the beam. This measurement is frequently misleading, since the sampling angle need not be defined.

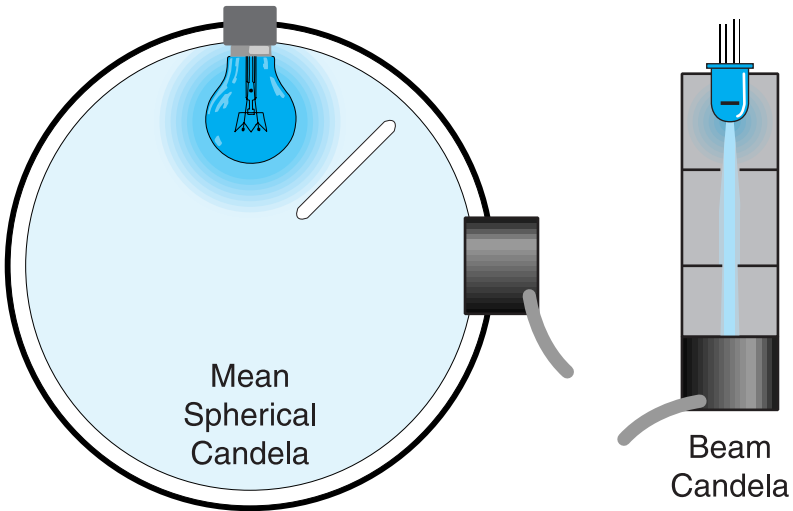


Fig. 7.8 Both mean spherical candela and beam candela are expressed in cd.

Suppose that two LED's each emit 0.1 lm total in a narrow beam: One has a  $10^\circ$  solid angle and the other a  $5^\circ$  angle. The  $10^\circ$  LED has an intensity of 4.2 cd, and the  $5^\circ$  LED an intensity of 16.7 cd. They both output the same total amount of light, however - 0.1 lm.

A flashlight with a million candela beam sounds very bright, but if its beam is only as wide as a laser beam, then it won't be of much use. Be wary of specifications given in beam candela, because they often misrepresent the total output power of a lamp.

### Units Conversion: Intensity

#### RADIANT INTENSITY:

$$\begin{aligned} 1 \text{ W/sr (watts per steradian)} \\ &= 12.566 \text{ watts (isotropic)} \\ &= 4 \cdot \pi \text{ W} \\ &= 683 \text{ candela at 555 nm} \end{aligned}$$

#### LUMINOUS INTENSITY:

$$\begin{aligned} 1 \text{ lm/sr (lumens per steradian)} \\ &= 1 \text{ candela (cd)} \\ &= 4 \cdot \pi \text{ lumens (isotropic)} \\ &= 1.464 \times 10^{-3} \text{ watts/sr at 555 nm} \end{aligned}$$

## Converting Between Geometries

Converting between geometry-based measurement units is difficult, and should only be attempted when it is impossible to measure in the actual desired units. You must be aware of what each of the measurement geometries implicitly assumes before you can convert. The example below shows the conversion between lux (lumens per square meter) and lumens.

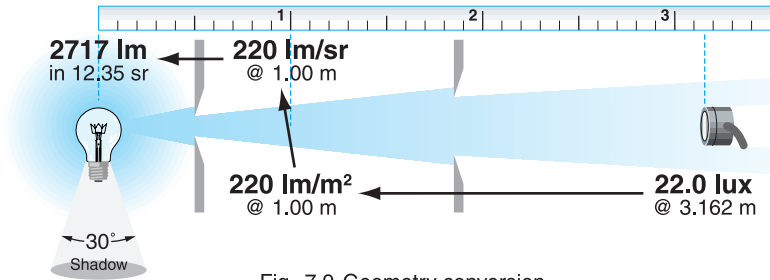


Fig. 7.9 Geometry conversion.

*Example:* You measure 22.0 lux from a light bulb at a distance of 3.162 meters. How much light, in lumens, is the bulb producing? Assume that the clear enveloped lamp is an isotropic point source, with the exception that the base blocks a 30° solid angle.

*Solution:*

1.) Calculate the irradiance at 1.0 meter:

$$E_1 = (d_2 / d_1)^2 * E_2$$

$$E_{1.0\text{ m}} = (3.162 / 1.0)^2 * 22.0 = 220 \text{ lm/m}^2$$

2.) Convert from lm/m² to lm/sr at 1.0 m:

$$220 \text{ lm/m}^2 * 1 \text{ m}^2/\text{sr} = 220 \text{ lm/sr}$$

3.) Calculate the solid angle of the lamp:

$$W = A / r^2 = 2\pi h / r = 2\pi[1 - \cos(\alpha / 2)]$$

$$W = 2\pi[1 - \cos(30 / 2)] = 12.35 \text{ sr}$$

4.) Calculate the total lumen output:

$$220 \text{ lm/sr} * 12.35 \text{ sr} = 2717 \text{ lm}$$

# 8 Setting Up An Optical Bench

## A Baffled Light Track

The best light measurement setup controls as many variables as possible. The idea is to prevent the measurement environment from influencing the measurement. Otherwise, the measurement will not be repeatable at a different time and place.

Baffles, for example, greatly reduce the influence of stray light reflections. A baffle is simply a sharp edged hole in a piece of thin sheet metal that has been painted black. Light outside of the optical beam is blocked and absorbed without affecting the optical path.

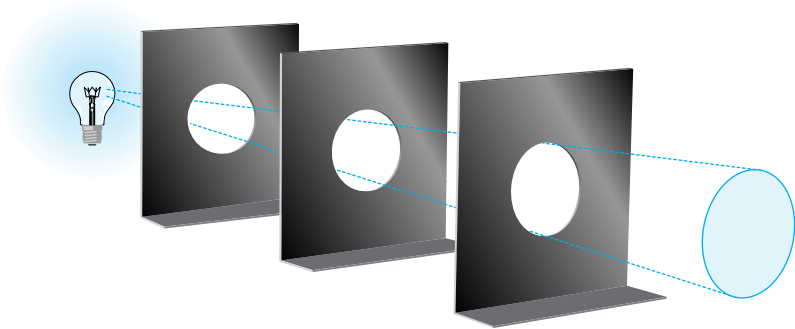


Fig. 8.1 Thin baffles help reduce stray reflections.

Multiple baffles are usually required in order to guarantee that light is trapped once it strikes a baffle. The best light trap of all, however, is empty space. It is a good idea to leave as much space between the optical path and walls or ceilings as is practical. Far away objects make weak reflective sources because of the Inverse Square Law. Objects that are near to the detector, however, have a significant effect, and should be painted with “black velvet” paint or moved out of view.

Closing a shutter, door, or light trap in one of the baffles allows you to measure the background scatter component and subtract it from future readings. The “zero” reading should be made with the source on, to maintain the operating temperature of the lamp as well as measure light that has defeated your baffling scheme.

## Kinematic Mounts

Accurate distance measurements and repeatable positioning in the optical path are the most important considerations when setting up an optical bench. The goal of an optical bench is to provide repeatability. It is not enough to merely control the distance to the source, since many sources have non-uniform beams. A proper detector mounting system provides for adjustment of position and angle in 3-D space, as well as interchangeability into a calibrated position in the optical path.

To make a kinematic fixture, cut a cone and a conical slot into a piece of metal using a 45° conical end mill (see fig. 8.2). A kinematic mount is a three point fixture, with the third point being any planar face. The three mounting points can be large bolts that have been machined into a ball on one end, or commercially available 1/4-80 screws with ball bearing tips (from Thorlabs, Inc.) for small fixtures.

The first leg rests in the cone hole, fixing the position of that leg as an X-Y point. The ball tip ensures that it makes reliable, repeatable contact with the cone surface. The second leg sits in the conical slot, fixed only in Yaw, or angle in the horizontal plane.

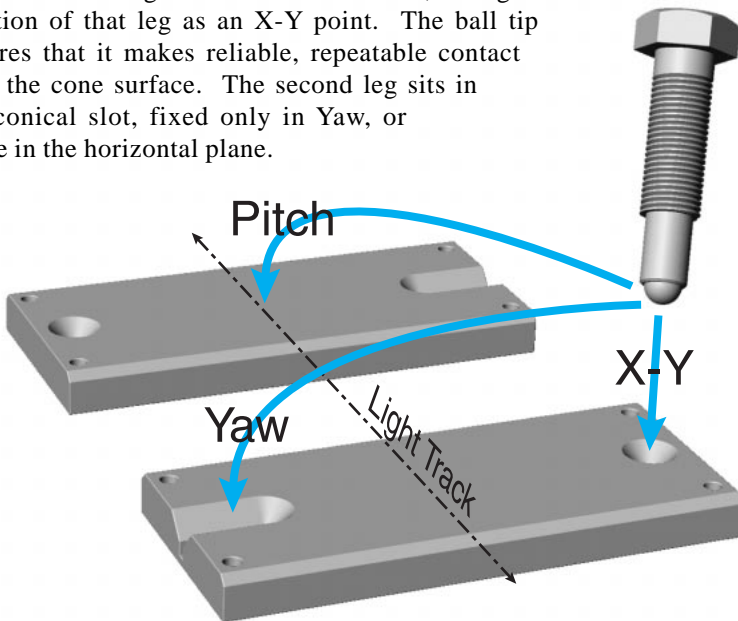


Fig. 8.2 Reversible kinematic mount.

The use of a slot prevents the Yaw leg from competing with the X-Y leg for control. The third leg rests on any flat horizontal surface, fixing the Pitch, or forward tilt, of the assembly.

A three legged detector carrier sitting on a kinematic mounting plate is the most accurate way to interchange detectors into the optical path, allowing intercomparisons between two or more detectors.



# 9 Graphing Data

## Line Sources

Discharge sources emit large amounts of irradiance at particular atomic spectral lines, in addition to a constant, thermal based continuum. The most accurate way to portray both of these aspects on the same graph is with a dual axis plot, shown in figure 9.1. The spectral lines are graphed on an irradiance axis ( $W/cm^2$ ) and the continuum is graphed on a band irradiance ( $W/cm^2/nm$ ) axis. The spectral lines ride on top of the continuum.

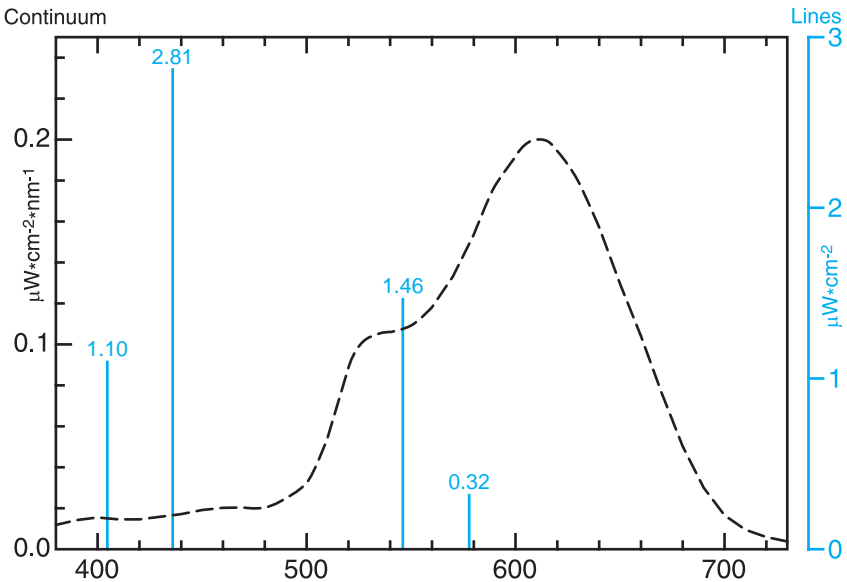


Fig. 9.1 Warm white fluorescent lamp continuum with spectral lines.

Another useful way to graph mixed sources is to plot spectral lines as a rectangle the width of the monochromator bandwidth. (see fig. 5.5) This provides a good visual indication of the relative amount of power contributed by the spectral lines in relation to the continuum, with the power being bandwidth times magnitude.

## Polar Spatial Plots

The best way to represent the responsivity of a detector with respect to incident angle of light is by graphing it in Polar Coordinates. The polar plot in figure 9.2 shows three curves: A power response (such as a laser beam underfilling a detector), a cosine response (irradiance overfilling a detector), and a high gain response (the effect of using a telescopic lens). This method of graphing is desirable, because it is easy to understand visually. Angles are portrayed as angles, and responsivity is portrayed radially in linear graduations.

The power response curve clearly shows that the response between  $-60$  and  $+60$  degrees is uniform at 100 percent. This would be desirable if you were measuring a laser or focused beam of light, and underfilling a detector. The uniform response means that the detector will ignore angular misalignment.

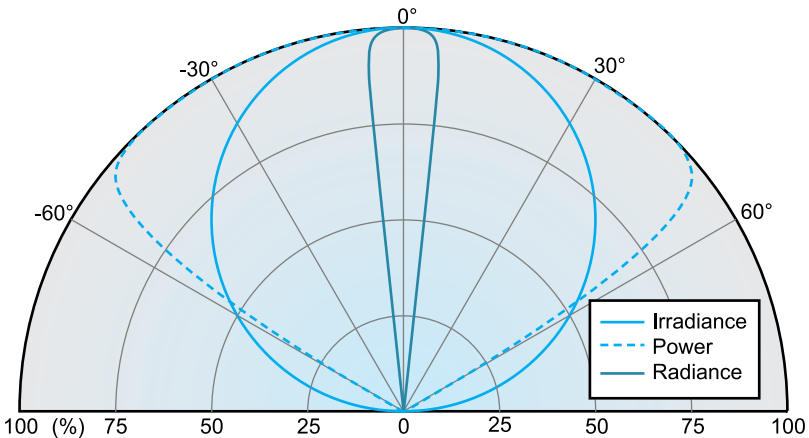


Fig. 9.2 Polar plot of ideal detector spatial responses.

The cosine response is shown as a circle on the graph. An irradiance detector with a cosine spatial response will read 100 percent at 0 degrees (straight on), 70.7 percent at 45 degrees, and 50 percent at 60 degrees incident angle. (Note that the cosines of  $0^\circ$ ,  $45^\circ$  and  $60^\circ$ , are 1.0, 0.707, and 0.5, respectively).

The radiance response curve has a restricted field of view of  $\pm 5^\circ$ . Many radiance barrels restrict the field of view even further ( $\pm 1^\circ$  is common). High gain lenses restrict the field of view in a similar fashion, providing additional gain at the expense of lost off angle measurement capability.

## Cartesian Spatial Plots

The cartesian graph in figure 9.3 contains the same data as the polar plot in figure 9.2 on the previous page. The power and high gain curves are fairly easy to interpret, but the cosine curve is more difficult to visually recognize. Many companies give their detector spatial responses in this format, because it masks errors in the cosine correction of the diffuser optics. In a polar plot the error is easier to recognize, since the ideal cosine response is a perfect circle.

In full immersion applications such as phototherapy, where light is coming from all directions, a cosine spatial response is very important. The skin (as well as most diffuse, planar surfaces) has a cosine response. If a cosine response is important to your application, request spatial response data in polar format.

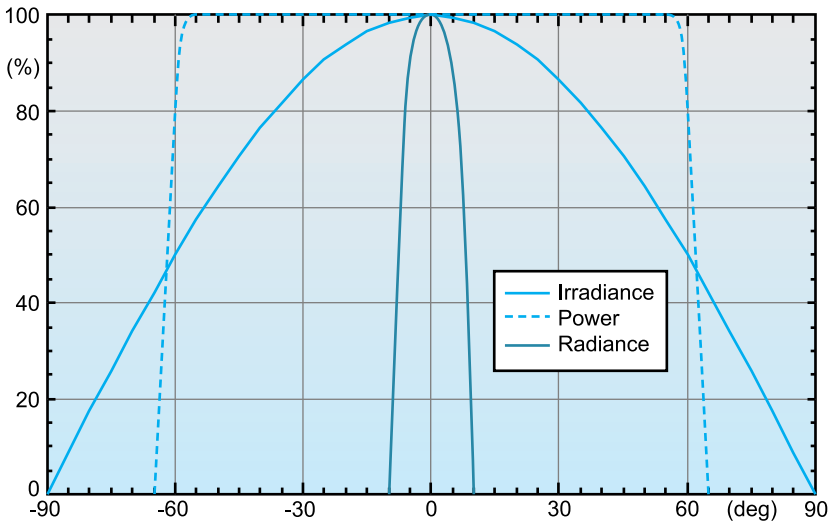


Fig. 9.3 Cartesian plot of ideal detector spatial responses.

At the very least, the true cosine response should be superimposed over the Cartesian plot of spatial response to provide some measure of comparison.

Note: Most graphing software packages do not provide for the creation of polar axes. Microsoft Excel™, for example, does have “radar” category charts, but does not support polar scatter plots yet. SigmaPlot™, an excellent scientific graphing package, supports polar plots, as well as custom axes such as log-log etc. Their web site is: <http://www.spss.com/>

## Logarithmically Scaled Plots

A log plot portrays each 10 to 1 change as a fixed linear displacement. Logarithmically scaled plots are extremely useful at showing two important aspects of a data set. First, the log plot expands the resolution of the data at the lower end of the scale to portray data that would be difficult to see on a linear plot. The log scale never reaches zero, so data points that are 1 millionth of the peak still receive equal treatment. On a linear plot, points near zero simply disappear.

The second advantage of the log plot is that percentage difference is represented by the same linear displacement everywhere on the graph. On a linear plot, 0.09 is much closer to 0.10 than 9 is to 10, although both sets of numbers differ by exactly 10 percent. On a log plot, 0.09 and 0.10 are the same distance apart as 9 and 10, 900 and 1000, and even 90 billion and 100 billion. This makes it much easier to determine a spectral match on a log plot than a linear plot.

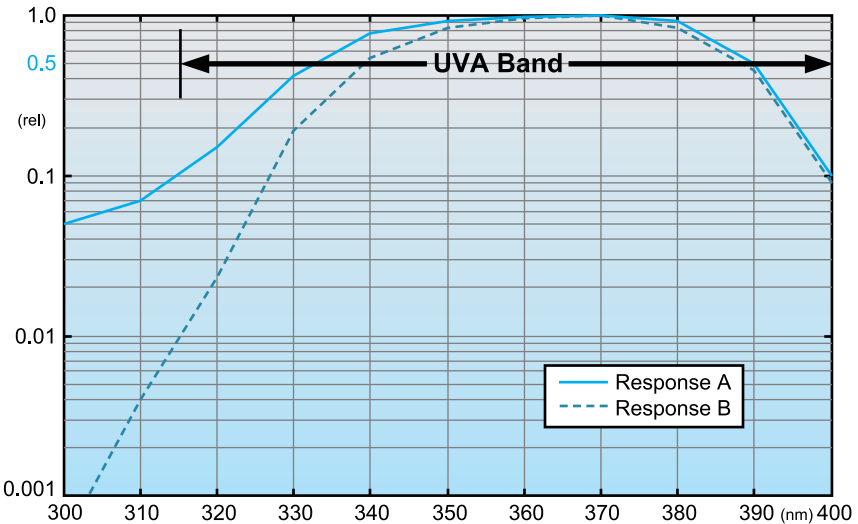


Fig. 9.4 Log plot of two detector responsivities.

As you can clearly see in figure 9.4, response B is within 10 percent of response A between 350 and 400 nm. Below 350 nm, however, they clearly mismatch. In fact, at 315 nm, response B is 10 times higher than response A. This mismatch is not evident in the linear plot, figure 9.5, which is plotted with the same data.

One drawback of the log plot is that it compresses the data at the top end, giving the appearance that the bandwidth is wider than it actually is. Note that Figure 9.4 appears to approximate the UVA band.

## Linearly Scaled Plots

Most people are familiar with graphs that utilize linearly scaled axes. This type of graph is excellent at showing bandwidth, which is usually judged at the 50 percent power points. In figure 9.5, it is easy to see that response A has a bandwidth of about 58 nm (332 to 390 nm). It is readily apparent from this graph that neither response A nor response B would adequately cover the entire UVA band (315 to 400 nm), based on the location of the 50 percent power points. In the log plot of the same data (fig. 9.4), both curves appear to fit nicely within the UVA band.

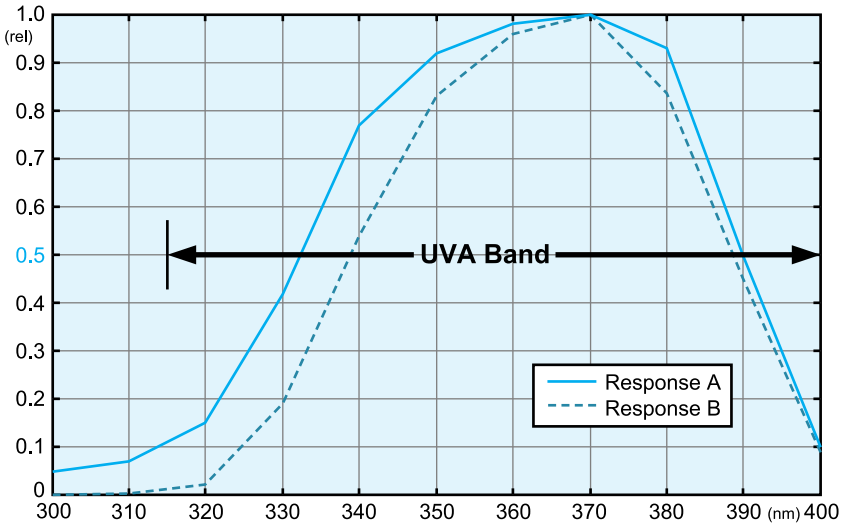


Fig. 9.5 Linear plot of two detector responsivities.

This type of graph is poor at showing the effectiveness of a spectral match across an entire function. The two responses in the linear plot appear to match fairly well. Many companies, in an attempt to portray their products favorably, graph detector responses on a linear plot in order to make it seem as if their detector matches a particular photo-biological action spectrum, such as the Erythral or Actinic functions. As you can clearly see in the logarithmic curve (fig. 9.4), response A matches response B fairly well above 350 nm, but is a gross mismatch below that. Both graphs were created from the same set of data, but convey a much different impression.

As a rule of thumb - half power bandwidth comparisons and peak spectral response should be presented on a linear plot. Spectral matching should be evaluated on a log plot.

## Linear vs. Diabatic Spectral Transmission Curves

The Diabatic scale (see fig. 9.7) is a log-log scale used by filter glass manufacturers to show internal transmission for any thickness. The Diabatic value,  $\theta(\lambda)$ , is defined as follows according to DIN 1349:

$$\theta(\lambda) = 1 - \log(\log(1/\tau))$$

Linear transmission curves are only useful for a single thickness (fig. 9.6). Diabatic curves retain the same shape for every filter glass thickness, permitting the use of a transparent sliding scale axis overlay, usually provided by the glass manufacturer. You merely line up the key on the desired thickness and the transmission curve is valid.

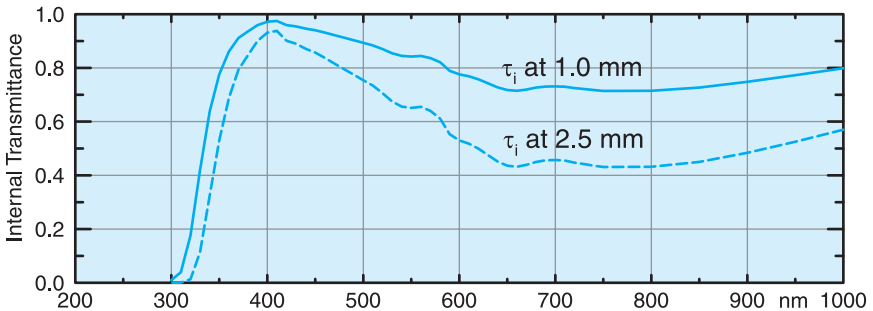


Fig. 9.6 Hoya LB-60 internal transmittance at 1.0 & 2.5 mm.

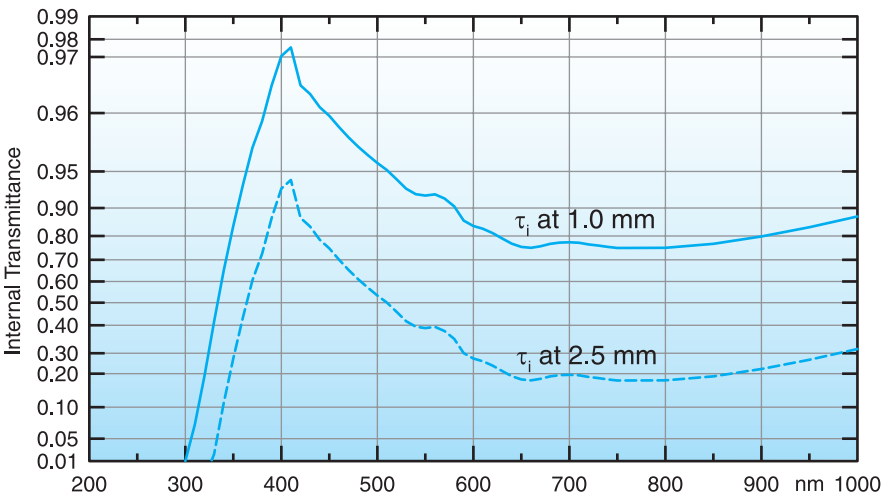


Fig. 9.7 Diabatic plot of Hoya LB-60 at 1.0 & 2.5 mm.

# 10 Choosing a Detector

## Sensitivity

Sensitivity to the band of interest is a primary consideration when choosing a detector. You can control the peak responsivity and bandwidth through the use of filters, but you must have adequate signal to start with. Filters can suppress out of band light but cannot boost signal.

Another consideration is blindness to out of band radiation. If you are measuring solar ultraviolet in the presence of massive amounts of visible and infrared light, for example, you would select a detector that is insensitive to the long wavelength light that you intend to filter out.

Lastly, linearity, stability and durability are considerations. Some detector types must be cooled or modulated to remain stable. High voltages are required for other types. In addition, some can be burned out by excessive light, or have their windows permanently ruined by a fingerprint.

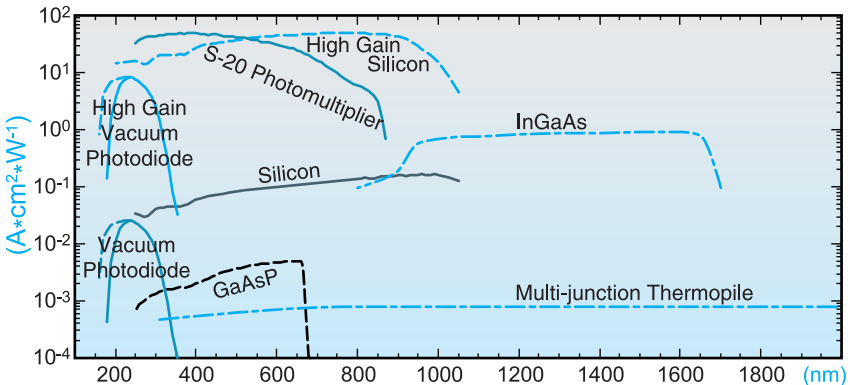
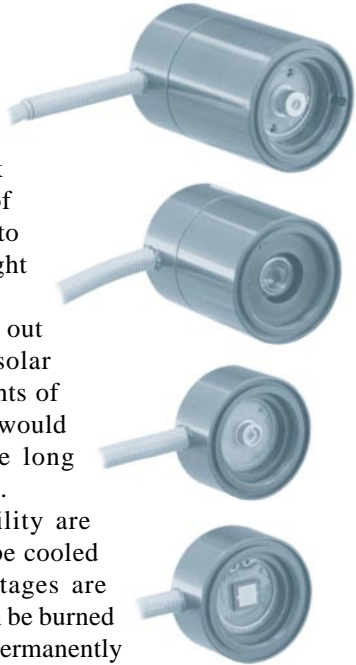
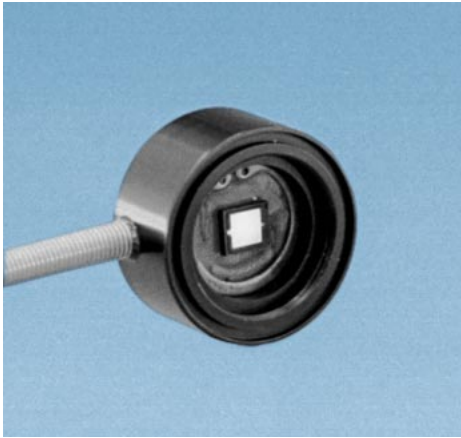


Fig. 10.1 Common detector types - absolute responsivity, unfiltered.

## Silicon Photodiodes

Planar diffusion type silicon photodiodes are perhaps the most versatile and reliable sensors available. The P-layer material at the light sensitive surface and the N material at the substrate form a P-N junction which operates as a photoelectric converter, generating a current that is proportional to the incident light. Silicon cells operate linearly over a ten decade dynamic range, and remain true to their original calibration longer than any other type of sensor. For this reason, they are used as transfer standards at NIST.



Silicon photodiodes are best used in the short-circuit mode, with zero input impedance into an op-amp. The sensitivity of a light-sensitive circuit is limited by dark current, shot noise, and Johnson

(thermal) noise. The practical limit of sensitivity occurs for an irradiance that produces a photocurrent equal to the dark current (Noise Equivalent Power, NEP = 1).

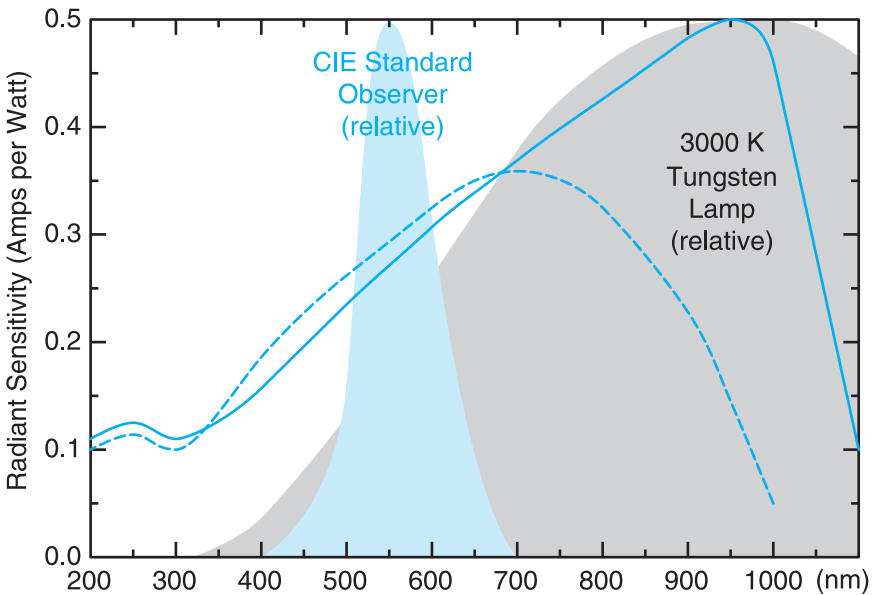


Fig 10.2 Spectral responsivities of two silicon photodiodes.



## Solar-Blind Vacuum Photodiodes

The phototube is a light sensor that is based on the photoemissive effect. The phototube is a bipolar tube which consists of a photoemissive cathode surface that emits electrons in proportion to incident light, and an anode which collects the emitted electrons. The anode must be biased at a high voltage (50 to 90 V) in order to attract electrons to jump through the vacuum of the tube. Some phototubes use a forward bias of less than 15 volts, however.



The cathode material determines the spectral sensitivity of the tube. Solar-blind vacuum photodiodes use Cs-Te cathodes to provide sensitivity only to ultraviolet light, providing as much as a million to one long wavelength rejection. A UV glass window is required for sensitivity in

the UV down to 185 nm, with fused silica windows offering transmission down to 160 nm.

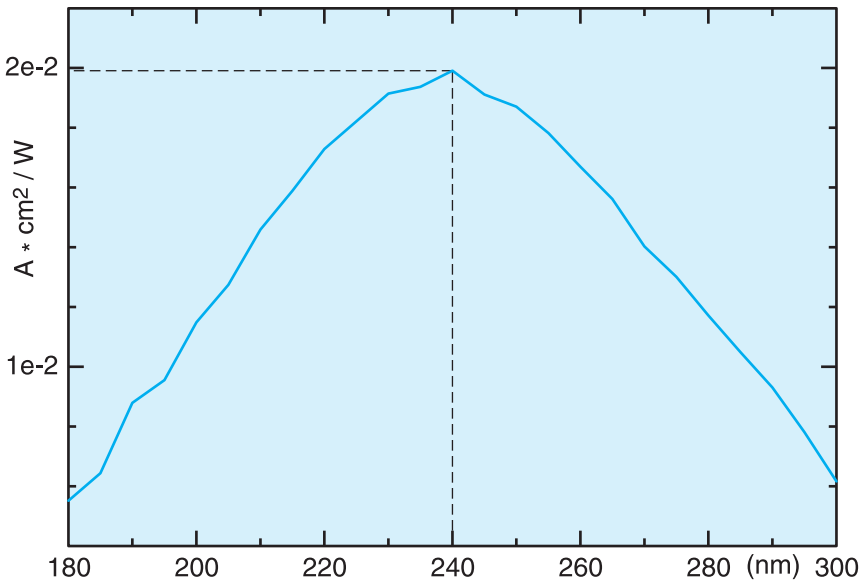


Fig. 10.3 Responsivity of a Solar-Blind Vacuum Photodiode.

## Multi-Junction Thermopiles

The thermopile is a heat sensitive device that measures radiated heat. The sensor is usually sealed in a vacuum to prevent heat transfer except by radiation. A thermopile consists of a number of thermocouple junctions in

series which convert energy into a voltage using the Peltier effect. Thermopiles are convenient sensor for measuring the infrared, because they offer adequate sensitivity and a flat spectral response in a small package. More sophisticated bolometers and pyroelectric detectors need to be chopped and are generally used only in calibration labs.

Thermopiles suffer from temperature drift, since the reference portion of the detector is constantly absorbing heat. The best method of operating a thermal detector is by chopping incident radiation, so that

drift is zeroed out by the modulated reading.

The quartz window in most thermopiles is adequate for transmitting from 200 to 4200 nm, but for long wavelength sensitivity out to 40 microns, Potassium Bromide windows are used.

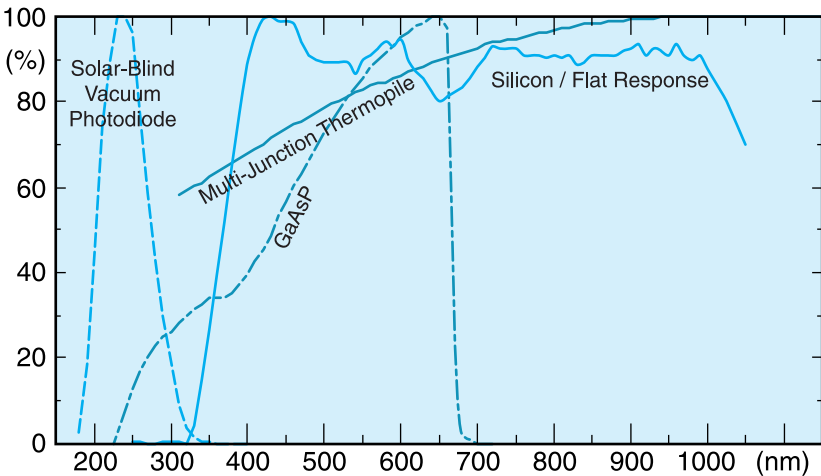


Fig. 10.4 Responsivities: Vac. photodiode, Si / flat response, GaAsP, & Thermopile.

# 11 Choosing a Filter

## Spectral Matching

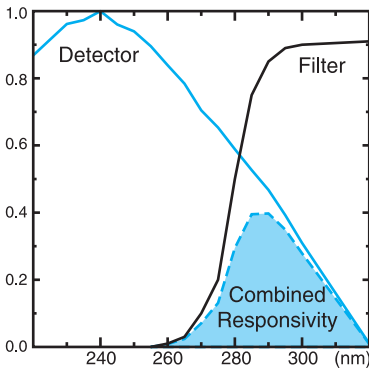
A detector's overall spectral sensitivity is equal to the product of the responsivity of the sensor and the transmission of the filter. Given a desired overall sensitivity and a known detector responsivity, you can then solve for the ideal filter transmission curve.

A filter's bandwidth decreases with thickness, in accordance with Bouguer's law (see Chapter 3). So by varying filter thickness, you can selectively modify the spectral responsivity of a sensor to match a particular function. Multiple filters cemented in layers

give a net transmission equal to the product of the individual transmissions. At International Light, we've written simple algorithms to iteratively adjust layer thicknesses of known glass melts and minimize the error to a desired curve.

Filters operate by absorption or interference. Colored glass filters are doped with materials that selectively absorb light by

wavelength, and obey Bouguer's law. The peak transmission is inherent to the additives, while bandwidth is dependent on thickness. Sharp-cut filters act as long pass filters, and are often used to subtract out long wavelength radiation in a secondary measurement. Interference filters rely on thin layers of dielectric to cause interference between wavefronts, providing very narrow bandwidths. Any of these filter types can be combined to form a composite filter that matches a particular photochemical or photobiological process.



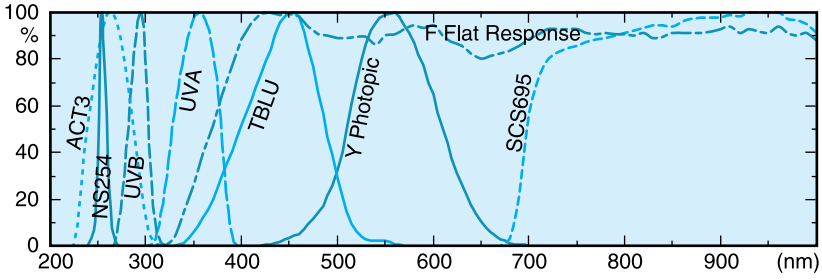


Fig. 11.1 Common detector-filter combined responsivities.

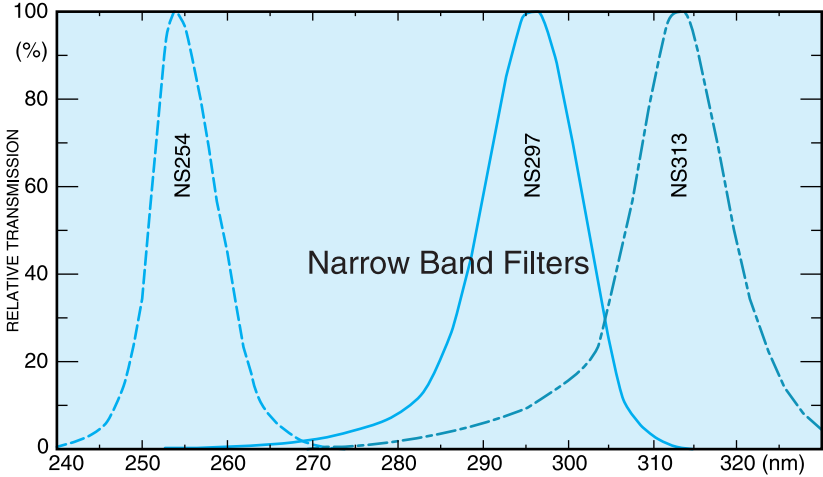


Fig. 11.2 All dielectric, multicavity interference, narrow band filters.

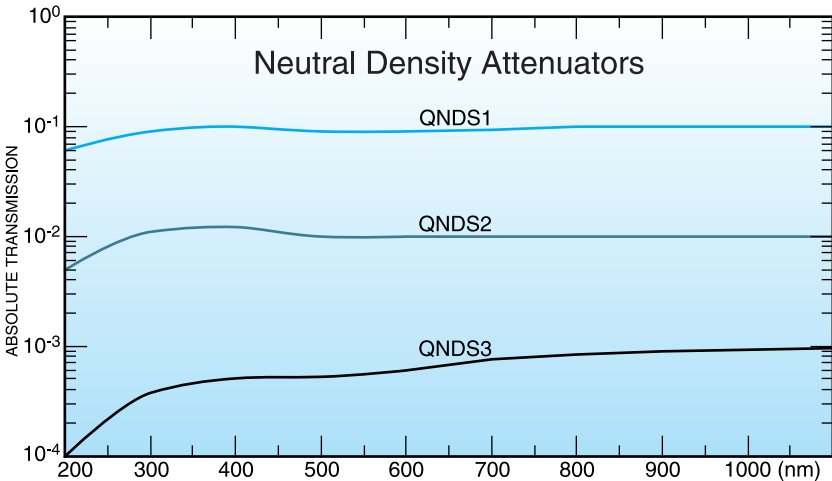


Fig. 11.3 Multilayer thin film neutral density filters (flat attenuators).

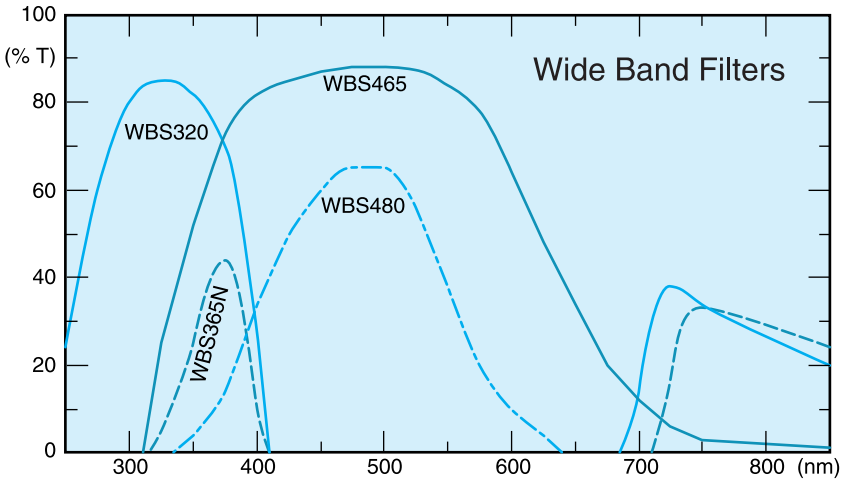


Fig. 11.4 Wide Band Optical Glass Filters.

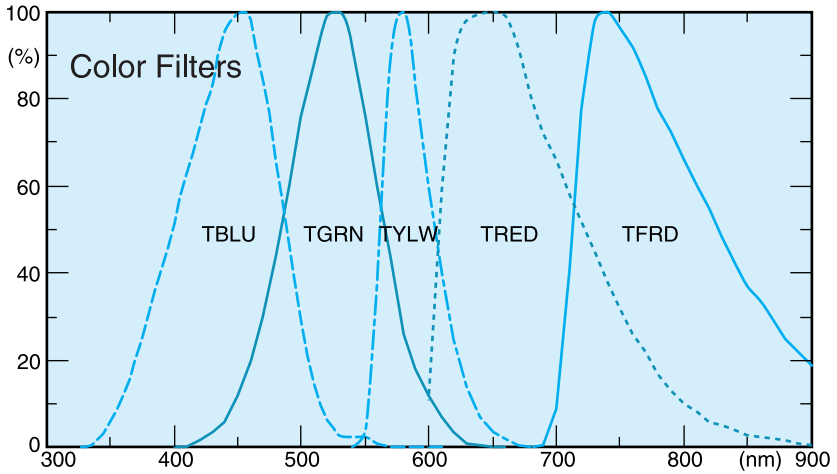


Fig. 11.5 Visible Color Filters.

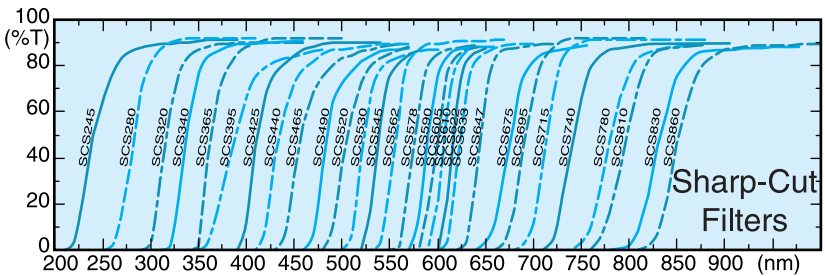


Fig. 11.6 Sharp-Cut Filters (long-pass).

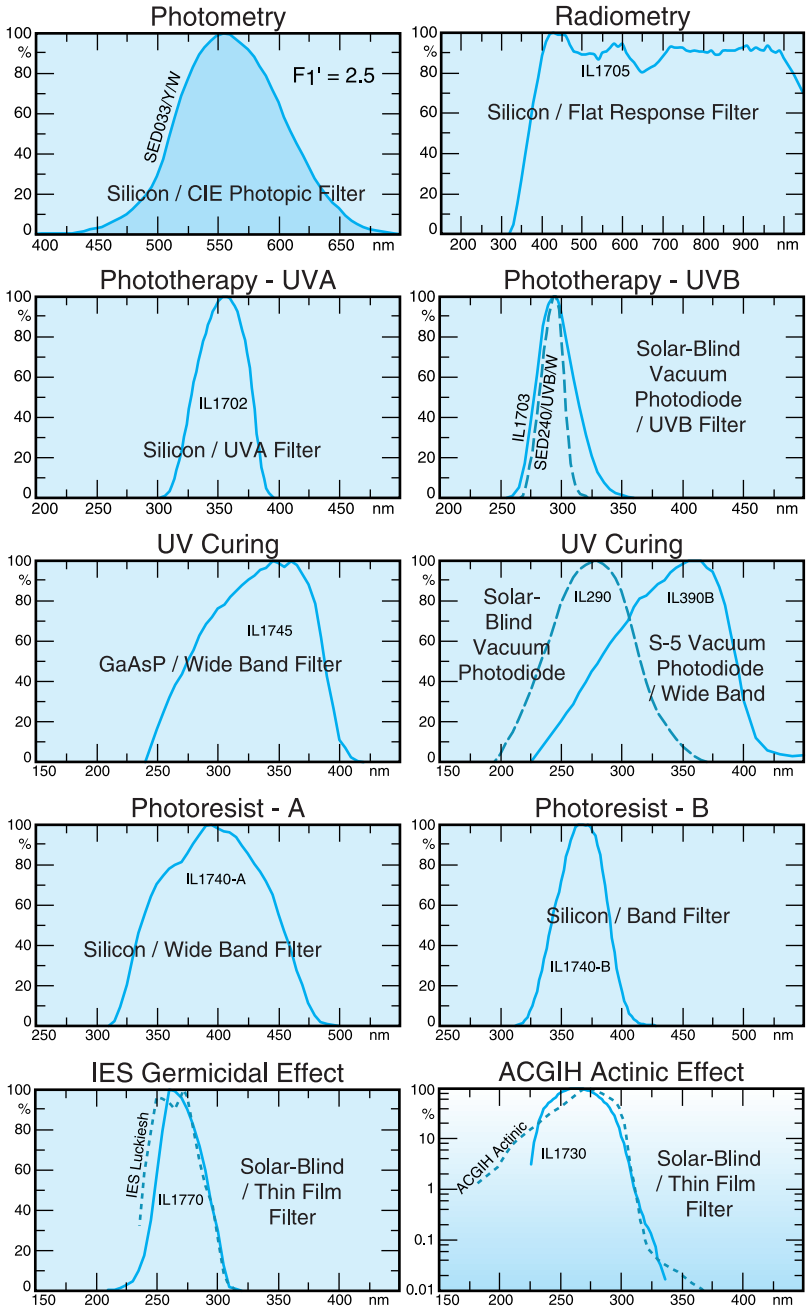


Fig. 11.7 Common light measurement applications

# 12 Choosing Input Optics

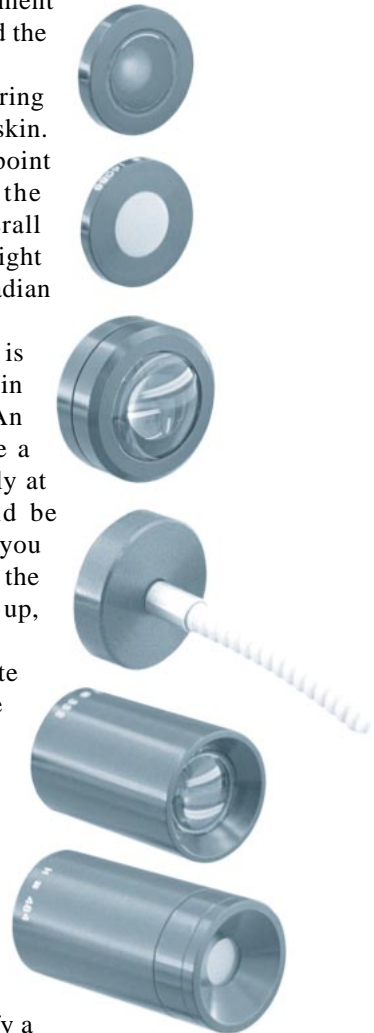
When selecting input optics for a measurement application, consider both the size of the source and the viewing angle of the intended real-world receiver.

Suppose, for example, that you were measuring the erythema (sunburn) effect of the sun on human skin. While the sun may be considered very much a point source, skylight, refracted and reflected by the atmosphere, contributes significantly to the overall amount of light reaching the earth's surface. Sunlight is a combination of a point source and a  $2\pi$  steradian area source.

The skin, since it is relatively flat and diffuse, is an effective cosine receiver. It absorbs radiation in proportion to the incident angle of the light. An appropriate measurement system should also have a cosine response. If you aimed the detector directly at the sun and tracked the sun's path, you would be measuring the maximum irradiance. If, however, you wanted to measure the effect on a person laying on the beach, you might want the detector to face straight up, regardless of the sun's position.

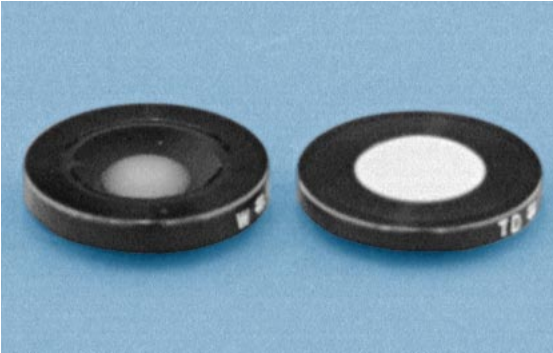
Different measurement geometries necessitate specialized input optics. Radiance and luminance measurements require a narrow viewing angle ( $< 4^\circ$ ) in order to satisfy the conditions underlying the measurement units. Power measurements, on the other hand, require a uniform response to radiation regardless of input angle to capture all light.

There may also be occasions when the need for additional signal or the desire to exclude off-angle light affects the choice of input optics. A high gain lens, for example, is often used to amplify a distant point source. A detector can be calibrated to use any input optics as long as they reflect the overall goal of the measurement.



## Cosine Diffusers

A bare silicon cell has a near perfect cosine response, as do all diffuse planar surfaces. As soon as you place a filter in front of the detector, however, you change the spatial responsivity of the cell by restricting off-angle light.



Fused silica or optical quartz with a ground (rough) internal hemisphere makes an excellent diffuser with adequate transmission in the ultraviolet. Teflon is an excellent alternative for UV and visible applications, but is not an

effective diffuser for infrared light. Lastly, an integrating sphere coated with  $\text{BaSO}_4$  or PTFE powder is the ideal cosine receiver, since the planar sphere aperture defines the cosine relationship.

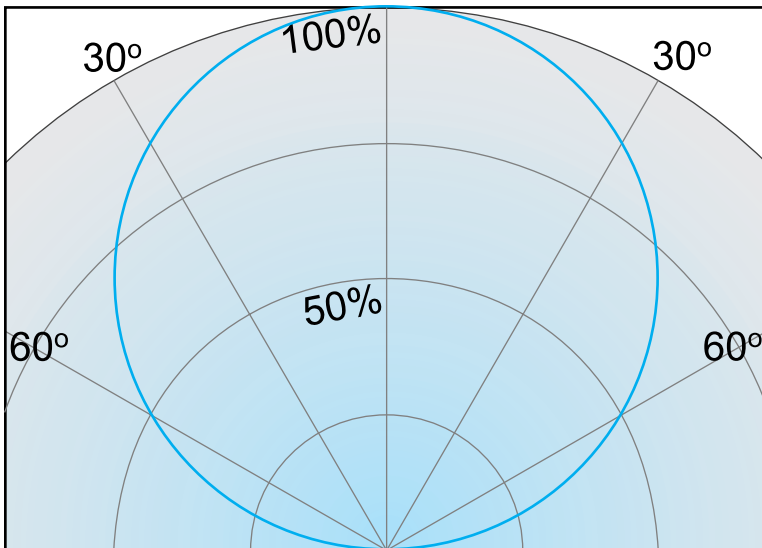


Fig. 12.1 Relative spatial response of an ideal cosine diffuser.



## Radiance Lens Barrels

Radiance and luminance optics frequently employ a dual lens system that provides an effective viewing angle of less than  $4^\circ$ . The tradeoff of a restricted viewing angle is a reduction in signal. Radiance optics merely limit the viewing angle to less than the extent of a uniform area source. For very small sources, such as a single element of an LED display, microscopic optics are required to “underfill” the source.

The Radiance barrel shown at right has a viewing angle of  $3^\circ$ , but due to the dual lenses, the extent of the beam is the full diameter of the first lens; 25 mm. This provides increased signal at close distances, where a restricted viewing angle would limit the sampled area.

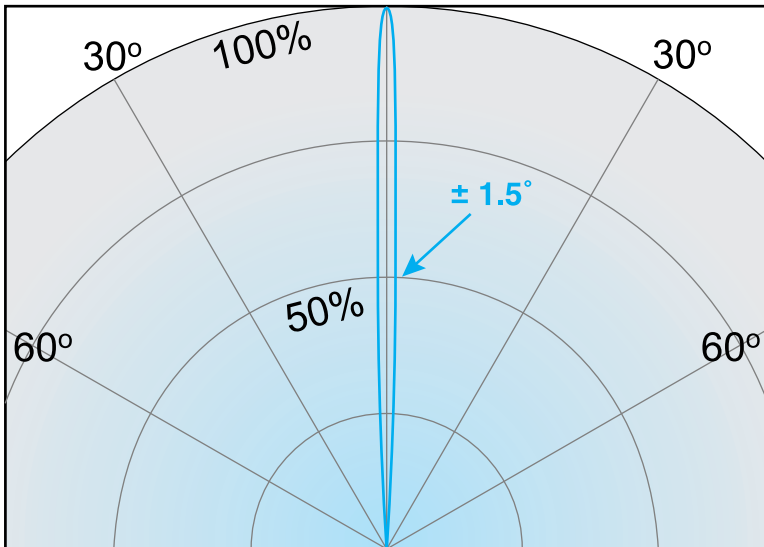


Fig. 12.2 Relative spatial response of an ideal radiance lens assembly.

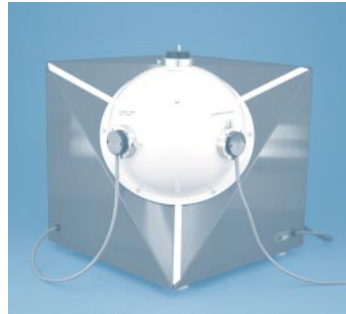
## Fiber Optics

Fiber optics allow measurements in tight places or where irradiance levels and heat are very high. Fiber optics consist of a core fiber and a jacket with an index of refraction chosen to maximize total internal reflection. Glass fibers are suitable for use in the visible, but quartz or fused silica is required for transmission in the ultraviolet. Fibers are often used to continuously monitor UV curing ovens, due to the attenuation and heat protection they provide. Typical fiber optics restrict the field of view to about  $\pm 20^\circ$  in the visible and  $\pm 10^\circ$  in the ultraviolet.



## Integrating Spheres

An integrating sphere is a hollow sphere coated inside with Barium Sulfate, a diffuse white reflectance coating that offers greater than 97% reflectance between 450 and 900 nm. The sphere is baffled internally to block direct and first-bounce light. Integrating spheres are used as sources of uniform radiance and as input optics for measuring total power. Often, a lamp is placed inside the sphere to capture light that is emitted in any direction.



## High Gain Lenses

In situations with low irradiance from a point source, high gain input optics can be used to amplify the light by as much as 50 times while ignoring off angle ambient light. Flash sources such as tower beacons often employ fresnel lenses, making near field measurements difficult. With a high gain lens, you can measure a flash source from a distance without compromising signal strength. High gain lenses restrict the field of view to  $\pm 8^\circ$ , so cannot be used in full immersion applications where a cosine response is required.



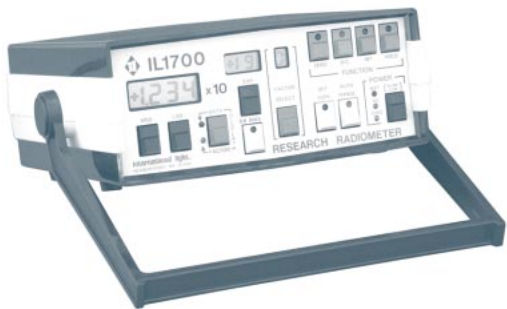
# 13 Choosing a Radiometer

Detectors translate light energy into an electrical current. Light striking a silicon photodiode causes a charge to build up between the internal "P" and "N" layers. When an external circuit is connected to the cell, an electrical current is produced. This current is linear with respect to incident light over a 10 decade dynamic range.

A wide dynamic range is a prerequisite for most applications. The radiometer should be able to cover the entire dynamic range of any detector that will be plugged into it. This usually means that the instrument should be able to cover at least 7 decades of dynamic range with minimal linearity errors. The current or voltage measurement device should be the least significant source of error in the system.

| Billion-to-One Dynamic Range |             |
|------------------------------|-------------|
| Sunny Day                    | 100000. lux |
| Office Lights                | 100. lux    |
| Full Moon                    | 0.1 lux     |
| Overcast Night               | 0.0001 lux  |

The second thing to consider when choosing a radiometer is the type of features offered. Ambient zeroing, integration ability, and a "hold" button should be standard. The ability to multiplex several detectors to a single radiometer or control the instrument remotely may also be desired for certain applications. Synchronous detection capability may be required for low level signals. Lastly, portability and battery life may be an issue for measurements made in the field.

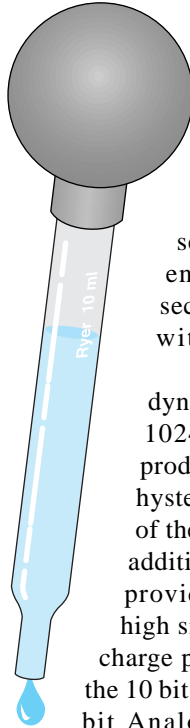


## Floating Current to Current Amplification

International Light radiometers amplify current using a floating current-to-current amplifier (FCCA), which mirrors and boosts the input current directly while “floating” completely isolated. The FCCA current amplifier covers an extremely large dynamic range without changing gain. This proprietary amplification technique is the key to our unique analog to digital conversion, which would be impossible without linear current preamplification.

We use continuous wave integration to integrate (or sum) the incoming amplified current as a charge, in a capacitor. When the charge in the capacitor reaches a threshold, a charge packet is released. This is analogous to releasing a drop from an eye dropper. Since each drop is an identical known volume, we can determine the total volume by counting the number of drops. The microprocessor simply counts the number of charge packets that are released every 500 milliseconds. Since the clock speed of the computer is much faster than the release of charge packets, it can measure as many as 5 million, or as few as 1 charge packet, each 1/2 second. On the very low end, we use a rolling average to enhance the resolution by a factor of 4, averaging over a second period. The instrument can cover 6 full decades without any physical gain!

In order to boost the dynamic range even further, we use a single gain change of 1024 to overlap two 6 decade ranges by three decades, producing a 10 decade dynamic range. This “range hysteresis” ensures that the user of the working ranges without the addition, the two ranges are locked together at a single point, providing a step free transition high signal level, the instrument is still sensitive to the smallest charge packet, for a resolution of 21 bits within each range! With the 10 bit gain change, we overlap two 21 bit ranges to achieve a 32 bit Analog to Digital conversion, yielding valid current measurements from a resolution of 100 femtoamps ( $10^{-13}$  A) to 2.0 milliamps ( $10^{-3}$  A). The linearity of the instrument over its entire dynamic range is guaranteed, since it is dependent only on the microprocessor's ability to keep track of time and count, both of which it does very well.



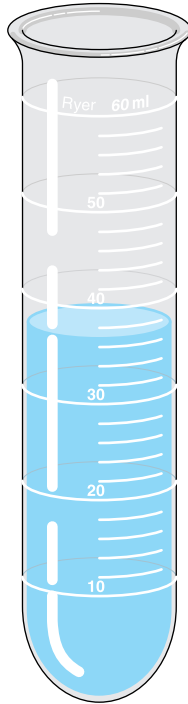
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## Transimpedance Amplification

Transimpedance amplification is the most common type of signal amplification, where an op-amp and feedback resistor are employed to amplify an instantaneous current. Transimpedance amplifiers are excellent for measuring within a fixed decade range, but must change gain by switching feedback resistors in order to handle higher or lower signal levels. This gain change introduces significant errors between ranges, and precludes the instrument from measuring continuous exposures.

A graduated cylinder is a good analogy for describing some of the limitations of transimpedance amplification. The graduations on the side of the cylinder are the equivalent



converter. The more greater the resolution in the cannot measure volumes lacks the resolution for must switch to a different measurement range - the in an amplifier.

In a simple light meter, voltage, which is amplified using an analog-to-digital converter provides a total of and 1 volt, allowing you to 1000 to an accuracy of 3 measure between 10 and 10 a factor of 100 to get three

In transimpedance for each range have to be absolute standard. It is occur between the 10% point point of the range below it. offset error is magnified at this

of bit depth in an A-D graduating lines, the measurement. A beaker greater than itself, and smaller measurements. You size container to expand the equivalent of changing gain

incoming light induces a and converted to digital converter. A 10 bit A-D 1024 graduations between 0 measure between 100 and significant digits. To 100, however, you must 10, because the resolution digits. Similarly, to you must boost the gain by digit resolution again.

systems, the 100% points adjusted and set to an expected for a mismatch to of one range and the 100% Any nonlinearity or zero 10% point.

is sampled instantaneously, it suffers from a lower S/N ratio than an integrating amplifier. Transimpedance amplifiers simulate integration by taking multiple samples and calculating the average reading. This technique is sufficient if the sampling rate is at least double the frequency of the measured signal.

## Integration

The ability to sum all of the incident light over a period of time is a very desirable feature. Photographic film is a good example of a simple integration. The image on the emulsion becomes more intense the longer the exposure time. An integrating radiometer sums the irradiance it measures continuously, providing an accurate measure of the total exposure despite possible changes in the actual irradiance level.

The primary difficulty most radiometers have with integration is range changes. Any gain changes in the amplification circuitry mean a potential loss of data. For applications with relatively constant irradiance, this is not a concern. In flash integration, however, the change in irradiance is dramatic and requires specialized amplification circuitry for an accurate reading.

Flash integration is preferable to measuring the peak irradiance, because the duration of a flash is as important as its peak. In addition, since the total power from a flash is low, an integration of 10 flashes or more will significantly improve the signal to noise ratio and give an accurate average flash. Since International Light radiometers can cover a large dynamic range (6 decades or more) without changing gain, the instruments can accurately subtract a continuous low level ambient signal while catching an instantaneous flash without saturating the detector.

The greatest benefit of integration is that it cancels out noise. Both the signal and the noise vary at any instant in time, although they are presumably constant in the average. International Light radiometers integrate even in signal mode, averaging over a 0.5 second sampling period to provide a significant improvement in signal to noise ratio.

## Zero

The ability to subtract ambient light and noise from readings is a necessary feature for any radiometer. Even in the darkest room, electrical “dark current” in the photodiode must be subtracted. Most radiometers offer a “Zero” button that samples the ambient scatter and electrical noise, subtracting it from subsequent readings.

Integrated readings require ambient subtraction as well. In flash measurements especially, the total power of the DC ambient could be higher than the power from an actual flash. An integrated zero helps to overcome this signal to noise dilemma.

# 14 Calibration

“NIST-traceable” metrology labs purchase calibrated transfer standard detectors directly from the [National Institute of Standards and Technology](#) in Gaithersburg, MD. From 400 to 1100 nm, this transfer standard is a Hamamatsu S1337-1010BQ photodiode, a 10 x 10 mm planar silicon cell coated with synthetic quartz. The photodiode is mounted behind a precisely measured 7.98 mm diameter circular aperture, yielding an active area of 0.5 cm<sup>2</sup>. The responsivity is usually given every 5 nanometers.

The calibration labs then use this transfer standard to calibrate their intercomparison working standards using a monochromatic light source. These working standards are typically identical to the equipment that will be calibrated. The standards are rotated in the lab, tracked over time to monitor stability, and periodically recalibrated.

Detectors are most often calibrated at the peak wavelength of the detector / filter / diffuser combination using identical optics for the intended application. The key to this calibration transfer is a reliable kinematic mount that allows exchangeability of detectors in the optical path, and a stable, power regulated light source. Complete spectroradiometric responsivity scans or calibration at an alternate wavelength may be preferred in certain circumstances.

Although the working standard and the unknown detector are fixed in precise kinematic mounts in front of carefully regulated light sources, slight errors are expected due to transfer error and manufacturing tolerances. An overall uncertainty to absolute of 10% or less is considered very good for radiometry equipment, and is usually only achievable by certified metrology labs. An uncertainty of 1% is considered state of the art, and can only be achieved by NIST itself.

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