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*Frege on Knowing the Third Realm*¹

TYLER BURGE

Anyone who reads Frege with moderate care is struck by a puzzle about the central objective of his work. His main project is to explain the foundations of arithmetic in such a way as to enable us to understand the nature of our knowledge of arithmetic. But he says very little about our knowledge of the foundations. A full treatment of this and associated puzzles would require more room than I have here.² I want to give a short solution to the puzzle, and then discuss one aspect of it that I find interesting.

The short solution is that Frege accepted the traditional rationalist account of knowledge of the relevant primitive truths, truths of logic. This account, which he associated with the Euclidean tradition, maintained that basic truths of geometry and logic are self-evident. Frege says on several occasions that such primitive truths—as well as basic rules of inference and certain relevant definitions—are self-evident. He did not develop these remarks because he thought they admitted little development. The interesting problems for him were finding and understanding the primitive truths, and showing how they, together with inference rules and definitions, could be used to derive the truths of arithmetic.

This short solution seems to me correct—as far as it goes. It does, however, leave out a lot. Frege thought that knowledge of the axioms of geometry required intuition—an imaginative or broadly perceptual capacity (1968, pp. 19-21). Knowledge of the basic truths of logic simply required reason. He regarded both types of basic truths as self-evident, but the differences between the two types of knowledge are significant. That is one complication. Another is that Frege uses a variety of terms that are translated “self-evident”. His sophisticated understanding of the notion is neither psychologistic nor purely proof-theoretic. He does not mean by it what most contemporary philosophers would mean by it. His uses of it relate in interesting ways to his basic philosophical views. A third complication is that there are complex relations between Frege’s appeals to self-evidence and an appeal he makes to pragmatic epistemological considerations. This appeal

¹ I am indebted to Tom Ricketts for clarifying his views, discussed in note 16. I have also benefited from remarks from various participants at a conference on early analytical philosophy held at the University of Chicago in honour of Leonard Linsky.

² An auxiliary puzzle attends this primary one. Most of Frege’s philosophical work is directed at correcting what he regards as the misunderstandings embedded in normal practice and language—misunderstandings that he thought had prevented a correct understanding of the fundamental notions present in his account of the foundations. But he has even less to say about the epistemology of his analysis and elucidation of the notions that interested him than he does about knowledge of the foundations.

makes his rationalism original and gives it, I think, special relevance to modern problems. Although these points are worth developing, I will not discuss them here. Instead, I shall discuss an intensification of the puzzle in the light of the short solution that I have just given.

Frege assumes that only truths are self-evident. He also assumes that it is rational to believe what is self-evident, given that it is well understood. Frege believes in other types of purely mathematical justification for arithmetical judgments besides self-evidence and derivation from self-evident truths.³ But these other types also involve only reason. The key idea in what follows is that Frege assumes that we can know arithmetic and its foundations purely through reason, and that individuals are reasonable and justified in believing basic foundational truths (e.g. 1979, p. 175; 1983, p. 190).

Frege held that both the thought contents that constitute the proof-structure of mathematics and the subject matter of these thought contents (extensions, functions) exist. He also thought that these entities are non-spatial, non-temporal, causally inert, and independent for their existence and natures from any person's thinking them or thinking about them. Frege proposed a picturesque metaphor of thought contents as existing in a "third realm". This "realm" counted as "third" because it was comparable to but different from the realm of physical objects and the realm of mental entities. I think that Frege held, in the main body of his career, that not only thought contents, but numbers and functions were members of this third realm.⁴ (Cf. 1968, p.viii; 1967a, pp. 15-16; 1962, p. xvii). Entities in the other realms depended for determinate identities on functions (concepts) in the third realm. Since logic was committed to this realm, and since all sciences contained logic, all sciences were committed to and were partly about elements of this realm. Broadly speaking, Frege was a Platonist about logical objects (like numbers and truth values), functions, and thought contents. I shall say more about Frege's Platonism later, but I think that I have said enough to enable me to introduce the problem that I want to discuss.

The problem is that of understanding how reason alone could justify one in believing that a thought is true, when the thought has a subject matter that is as independent of anyone's thinking as Frege indicates it is. How could mere reasoning give one any ground for believing that a realm of entities is one way rather than another, when that realm is so independent of that reasoning? How could reasoning and understanding have any tendency to tell one how things in such a realm really are?

³ I distinguish purely mathematical justifications from justifications of mathematics that derive from applications to the empirical world—which he also seems to have believed in, but which I lay aside.

⁴ Frege's logic is not committed to thought contents, only to extensions and functions. But this is an artifact not of his views about logic, but of his interests in deriving arithmetic from logic. For that, he did not need to refer to thought contents (*Gedanken*). But he clearly envisioned a logic which was committed to thought contents. In the correspondence with Russell, for example, he indicates the need for special names of senses to avoid the "ambiguity" of indirect discourse or propositional attitude attributions (cf. 1980, p. 153; 1976, p. 236).

This problem is clearly kin to a problem about the relation between the knowledge and truth of mathematics that is commonly discussed today.⁵ The contemporary problem is that of understanding how our beliefs about mathematics could have any tendency to be true, given that we do not appear to bear causal-perceptual relations to the subject matter of mathematics. This may be seen as a problem for Frege. But it is not one that he would have naturally formulated for himself. His attitude toward the point that numbers and thought contents are not causally effective (“wirklich”) seems to have been “so what?”.⁶ He showed no special interest in the causal theory of knowledge, or in cashing out his occasional physical-contact metaphors of “grasping” thoughts. The idea that mathematical or logical knowledge should be judged by reference to the standard of empirical knowledge would have seemed foreign to him.

Like Frege I see no reason to think that mathematical or logical knowledge is questionable because it apparently lacks causal-perceptual relations to its subject matter. But I formulated a problem that made no reference to causal-perceptual relations. This formulation seems not to import assumptions foreign to Frege. A theory of knowledge should not make it puzzling how being reasonable could be conducive to having true beliefs. Frege’s rationalist theory of knowledge combines with his Platonism to raise a question at just this point. Why did he not discuss the question?

Some recent interpretations of Frege suggest that it is a question that is somehow precluded by his philosophy, or that it rests on fundamental misreadings of his views. One might question the notion of “subject matter” that the formulation of the problem uses. Or one might claim that Frege’s notion of truth or of logic blocks a “meta-standpoint” from which one could raise the question. Or one could doubt whether Frege’s Platonism should be understood in the way that the “third realm” metaphor suggests, and maintain that in talking about numbers or thought contents, Frege was really talking about our language or our cognitive practices in such a way that no gap between our beliefs and the numbers was even formulable. I will not criticize in detail all such lines for short-circuiting our question for Frege, though I will remark on some of them in a general way. I think none provides good grounds for ignoring the question. In fact, Frege himself gives an answer to it. The reason why he did not discuss it in detail is similar to the reason why he did not discuss knowledge of the foundations in detail. He believed that he had little to add to a traditional answer. I think that his answer is worth understanding.

Let us back up a bit. I want to explain in more detail what I mean by saying that Frege was a Platonist about logical objects, functions, and thought contents.

⁵ Benacerraf (1983).

⁶ Actually, he does provide an argument: objective sense-perception requires perceptual belief; but perceptual belief requires grasp of thoughts in the third realm—a non-causal relation; so one cannot cite the element of causal interaction in sense perception as providing grounds for thinking that knowledge cannot involve non-causal cognitive relations to abstract entities (1984, pp. 369-370; 1967b, p. 360).

First, some preliminary disclaimers. Although I think that Frege maintained a metaphysical view about numbers and other such entities, I do not believe that this view dominated his thinking. His is, for the most part, the relaxed Platonism of a mathematician who simply assumes that there are numbers, functions, and so on, and who regards these as an abstract subject matter which can be accepted without special philosophical explanation, which is clearly different from mental or physical subject matters, and which mathematics seeks to characterize correctly. One can see this attitude toward functions very prominently in "On Function and Concept". Frege highlighted the inter-subjective objectivity of scientific theorizing. He believed that standard mathematical practice told one most of what was true about mathematical entities, and he thought that one could know mathematical truths independently of any philosophy. Indeed, he assumes that ordinary mathematical practice yields "certain" knowledge even prior to the execution of his foundationalist program (1977, §13; 1968, §2).

Most of Frege's uses of his metaphysical view are defensive. His metaphysical remarks ward off idealist, physicalistic, psychologistic, reductive, or deflationary positions because he thinks that they prevent clear understanding of the fundamental notions of logic and arithmetic. As I shall later show, he does give his Platonism extra-mathematical work. But he does not think out this side of his philosophy as someone would who was concerned about certainty or who believed that logic and mathematics had no other cognitive underpinning than that provided by philosophy.

Another preliminary point about Frege's Platonism is that although he uses the Platonic metaphor of vision on occasion, when characterizing our knowledge, he shows no interest in developing the metaphor. He appeals to no faculty other than reason in his account of our mathematical knowledge. Moreover, as I have intimated earlier, his epistemological views are complex, and involve not only Platonic elements, but elements not at all associated with traditional Platonism. The discussion in what immediately follows will be concerned with the Platonic character of Frege's *ontology*. For now, I lay epistemology aside.

As is well-known, Frege thought that extensions—including numbers—functions—including concepts—and thought contents are imperceptible, non-spatial, atemporal, and causally inert.⁷ He emphasizes that numbers (1968, p. 108), concepts (1968, p. vii), and thought contents (1967a, p. 23; 1962, p. xxiv) are discovered—not created. He sharply distinguishes the act of thinking, which does occur

⁷ Numbers are counted imperceptible (1968, p. 85; 1979, p. 265; 1983, p. 284). Thoughts are termed imperceptible (1984, p. 369; 1967b, p. 360). Numbers are counted non-spatial (1968, pp. 58, 61, 85, 93). Thoughts are counted non-spatial (1984, pp. 369-370; 1967b, p. 360). Concepts or other functions are counted atemporal and by implication imperceptible, non-spatial, and causally inert (1968, p. vii, 1968, p. 37; 1984, p. 133; 1967b, p. 122). He also suggests these points about concepts indirectly (1967a, p. 23; 1962, p. xxiv; 1984, p. 198; 1967b, pp. 181-182). Numbers are counted atemporal (1984, p. 230; 1967b, p. 212). Thoughts are counted atemporal (1984, pp. 369-370; 1967b, p. 360). Numbers are counted causally inert (1968, p. 85; 1967a, pp. 15-16; 1962, p. xviii). Thoughts are said to be causally inert (1967a, p. 23; 1962, p. xxiv; 1979, pp. 137-138; 1983, pp. 149-150; 1984, pp. 230, 371; 1967b, pp. 212, 361-362).

in time, from the thought contents that we “grasp” or think, which are timeless. So in coming to know thought contents that denote numbers, concepts, and the like, one discovers objects, concepts, and relations that are what they are timelessly, independently of any causal influence. One comes to “stand in relation”, as Frege says, with non-spatial, atemporal entities. (1984, pp. 363, 369; 1967b, pp. 353-4, 360; 1967a, p. 23; 1962, p. xxiv.)

Frege calls numbers, concepts, and thought contents “objective”. By this he means, partly, that they are not intrinsically borne by a mind, as a pain or an after image is. He says that they are subject to laws. They are common property to different rational beings (1968, p. 26; 1984, pp. 363ff.; 1967b, pp. 355ff.). Much of Frege’s discussion of atemporal entities centres on their objectivity. For many of his purposes, the intersubjectivity and lawfulness of logic are its key properties.

Many of these things might be maintained by someone who was not a Platonist. One might make the remarks about imperceptibility, non-spatiality, atemporality, and causal inertness, if one glossed them as part of a practical recommendation or stipulation for a theoretical framework, having no cognitive import—or as otherwise not being theoretical claims or claims of reason. Carnap might have said at least some of those things, though only given certain background qualifications. Or one might have some other basis for qualifying these remarks, reading them as “non-metaphysical” or as lacking their apparent ontological import. Moreover, certain idealists might say these things. Kant might have said them, given certain background qualifications. He could have seen numbers as just as genuinely existent and discoverable as physical objects are. And he could see their objective status in terms of the possibility of inter-subjective agreement on laws governing them. Platonism has no monopoly on claims to lawlike or inter-subjective objectivity about non-spatial, atemporal entities. So we need to say more in order to distinguish Frege’s view from alternatives.

I would not take very seriously a reading of Frege as a Carnapian. Discussing my attitude would require going more into his methodology and epistemology than I plan to. I think it clear, however, that Frege was trying to provide a rational foundation for mathematics—in a way that Carnap would have regarded as misguided. Frege saw reason, not practical recommendation, as giving logical objects to us (e.g. 1968, p. 105). There is nothing remotely akin to Carnap’s Principle of Tolerance either in Frege’s philosophical pronouncements, or even more emphatically, in his temperament.

What interests me more is the distinction between Frege’s Platonism, on the one hand, and certain idealisms or certain vaguer “practice” oriented anti-Platonisms, on the other. Platonism, as I understand the doctrine, regards some entities (for Frege, some objects and all functions) as existing non-spatially and atemporally. Further, it avoids commenting on them as having special status, including being dependent for their existence or nature (as opposed to their discovery) on practice or mental activity. They are in *no way* derivative, instrumental, fictional, or otherwise second-class. The relevant entities are fundamental. It would be

incompatible with Platonism to regard them as essentially part of an appearance or perspective for a thinker—as Kant would have—though they may impose constitutive conditions on such appearances or perspectives. Platonism rejects any deeper philosophical commentary that would indicate that the nature or existence of these atemporal entities is to be regarded as in any way dependent on something mental, linguistic, communal, or on anything like a practice or activity that occurs in time. In Kant, we find a non-Platonic explanation of mathematical structures in terms of a mental activity, “synthesis”, that underlies the categories and the forms of spatial and temporal intuition. And in Hegel abstract structures are held to be abstractions from spirit in history. Recently, some philosophers have sought to avoid being “metaphysical”, contenting themselves with generalized remarks that mathematical objects are grounded in some unspecified way in linguistic or mathematical practice. Such views can admit non-spatio-temporal entities and can grant them objective status. But they are not Platonic in my sense. They regard atemporal entities as derivative from human practices—such as linguistic activity. I see such views as covertly idealist. Idealism regards actual activity or practice as implicated in the nature and existence of non-spatio-temporal structures. Platonism holds that structure is more fundamental than actual activity.

Frege’s Platonism shows itself in two ways. One is that he never enters the commentary that an idealist (or a deflationist) would enter on his claims about non-spatio-temporal entities, or about their objectivity or their discoverability. He takes them to be fundamental. The other is that he claims, more than once, that the assumption of the relevant entities explains the inter-subjective objectivity of science and communication. I will discuss these points briefly, in turn.

There is, as far as I can see, no evidence that Frege thought that the existence or nature of these non-spatio-temporal entities was to be explained in terms of human language, human inference, human practices (including the *activity* of judgment), or other patterns of human activity in time. Frege thought of extensions, functions, and thought contents as genuinely existing entities.⁸ He opposed thinking of such entities as having some derivative status. He inveighs against any suggestion that they are products of the mind, mere symbols, or otherwise dependent on events in time.⁹ Had he maintained that extensions, functions, or thought contents were dependent on human conceptualization or human language, judgment, or inference (actual or possible), he would have said so, and thereby qualified the numerous remarks that have traditionally invited the Platonic interpretation of his work. He never does say so. His claims that atemporal entities are independent of us are unqualified.

⁸ He quantified over them with quantifiers of different types. He used first-order quantifiers for the objects, second-order quantifiers for the functions. The quantifiers are appropriately read as involving existential commitments.

⁹ For extensions and numbers, cf. 1968, *passim*, 1967a, pp. 10, 12; 1962, pp. xiii, xiv; 1967a, pp. 15-16; 1962, p. xviii; 1984, p. 230; 1967b, p. 212. For concepts or functions, cf. 1968, p. viii; 1984, p. 133; 1967b, p. 122. For thoughts, cf. 1984, pp. 363, 370; 1967b, pp. 353-354, 360-361.

On several occasions, Frege compares the objectivity and existence of numbers, concepts, or thought contents with the existence and objectivity of physical objects. He compares numbers to the North Sea as regards objectivity (1968, p. 34). In doing so, he very explicitly indicates that the entity that we call "the North Sea" is what it is completely independently of our imposing the boundaries or making a map that we use to associate that entity with the name "the North Sea". He elaborates this comparison elsewhere:

Just as the geographer does not create a sea when he draws boundary lines and says: the part of the ocean's surface bounded by these lines I am going to call the Yellow Sea, so too the mathematician cannot really create anything by his defining. Nor can one by pure definition magically conjure into a thing a property that in fact it does not possess—save that of now being called by the name with which one has named it. (1967a, p. 11; 1962, p. xiii)

He compares a mathematician's relation to numbers with the astronomer's relation to the sun (1979, p. 7; 1983, p. 7) and to the planets (1968, p. 37). He says that like geographers, mathematicians cannot create, but can only discover "what is there and give it a name" (1968, p. 108; cf. also 1967a, pp. 23-24; 1962, p. xxiv; 1979, p. 137; 1983, p. 149). He compares our epistemic relation to numbers and concepts (and probably thought contents) to our grasping a pencil:

The picture of grasping is very well suited to elucidate the matter. If I grasp a pencil, many different events take place in my body... but the pencil exists independently of them. And it is essential for grasping that something be there which is grasped... In the same way, that which we grasp with the mind also exists independently of this activity... and it is neither identical with the totality of these events nor created by it as a part of our own mental life. (1967a, pp. 23-24; 1962, p. xxiv; cf. 1979, p. 137; 1983, p. 149)

Thought contents exist independently of thinking "*in the same way*", he says, that a pencil exists independently of grasping it. (The artifactual character of pencils plays no role in his understanding of the analogy, as other examples indicate.) He says that thought contents are true and bear their relations to one another (and presumably to what they are about) independently of anyone's thinking these thought contents—"just as a planet, even before anyone saw it, was in interaction with other planets" (1984, p. 363; 1967b, p. 354). And he compares a thought's independence of our grasping it to the star Algol's independence of anyone's being aware of it (1984, p. 369; 1967b, p. 359).

All these comparisons suggest (and those of 1967a, pp. 23-24; 1962, p. xxiv; 1984, pp. 363, 369; 1967b, p. 354, 359 explicitly state) that numbers, functions, and thought contents are independent of thinkers "*in the same way*" that physical objects are. Frege nowhere asserts or clearly implies that he maintains any sort of idealism—Kantian or otherwise—about the physical objects studied by the physical sciences. He nowhere qualifies the ontological status of physical objects. It is dubious historical methodology to attribute to a philosopher with writings that stretch over decades, a large, controversial doctrine, if he nowhere clearly states it in his writings. If Frege had believed in

any such idealism about physical objects (or any doctrine qualifying their ontological status), he would have surely said he did.¹⁰ Doing so would have been necessary for a philosopher to balance the flat-out statements about mind-independence that Frege makes.¹¹

Frege thought that to know the physical world, one has to grasp thoughts (which bore for him eternal denotational relations to concepts and extensions) that are eternal and eternally true. Logic is embedded in the content of any knowledge. Since logic is about (denotes) concepts and other functions, relations, and logical objects, all knowledge is at least partly about non-spatio-temporal entities. Moreover, logic concerns the forms of correct judgment and inference; and logical structure is discovered by reflecting on patterns of correct judgment and inference. But Frege does not give the slightest indication that he thought that either the physical world or the non-spatio-temporal entities inevitably appealed to in knowing it depend in any way on any activities of judgment, inference, or linguistic practice.¹²

¹⁰ Such passages as 1979, p. 137; 1983, p. 149, or any of the various passages about independence of mind that I discuss below, would require strong qualification, which Frege nowhere makes, to be compatible with any sort of idealism or deflationary reading. For an interpretation of Frege as a Kantian idealist, see Sluga, 1980 e.g. pp. 59-60, 115-116. Sluga cites mainly considerations that are external to Frege's texts. He also writes, "the central role of the Fregean belief in the primacy of judgments over concepts would seem to be explicable only in the context of a Kantian point of view". Sluga does not explain this remark. I think it misleading. Judgments and inferences are a source of discovery. But logical theory is about the forms of correct judgment and inference—not about judgments and inferences. Frege regards judgment as a form. (Cf. 1984, pp. 383-385; 1967b, pp. 372-374.) I know of no evidence that he saw this form as dependent for its nature or existence on actual activities of judgment, or on anything like Kantian synthesis; there is substantial evidence that he did not.

¹¹ Some philosophers have suggested that Frege's use of the context principle somehow suggests a qualification on his Platonism. Issues surrounding Frege's context principle(s) are, of course, extremely subtle and complex. But it seems to me that the suggestion must involve some confusion. The context principles govern relations between linguistic expressions and their senses or referents. They do not bear directly on the nature of the senses or referents themselves at all. At most one of the principles might be coherently thought to rule out certain naive forms of epistemological Platonism (those that require that we have perception-like intuition of mathematical objects). There are many complex issues here, and some of them are not completely independent of ontology. But I think that any simple appeal to the context principles to motivate opposition to my interpretation will confuse language and epistemology with ontology.

¹² An interpretation of Frege similar to Sluga's is proposed in Weiner (1990). In interpreting the North Sea comparison (1968, p. 34), Weiner notes Frege's remark that if we should happen to draw the boundaries of what we call "the North Sea" differently, what we now call "the North Sea" would still exist, though perhaps unrecognized. But she continues, "It is important to realize, however, that the claim that such unrecognized objects exist need not be a substantive metaphysical claim. For... to claim that unrecognized objects exist is simply to claim that it is possible to formulate (heretofore unformulated) concepts under which exactly one object falls" (p. 171). Weiner cites no texts to support this reading. I see no reason to think that existence claims for Frege are "simply" claims about possibility or about formulations; he gives every indication that they are not about possibility, language, or activity at all. Later she correctly claims that Frege believed our knowledge requires language or drawing boundaries. But she moves without argument from this remark about knowledge to one about the world: "Frege's view is that the physical world is not articulated—that we impose structure on it" (p. 184). The language of imposition is not present or implied in Frege. That concepts mark boundaries of the ocean is nowhere

Frege not only compares non-spatio-temporal entities to physical objects in their independence of us; he makes unqualified statements about the independence of such entities from anything about us. He repeatedly claims that both the truth of thought contents and thought contents themselves are independent of individuals' and groups' thinking the thoughts or recognizing them to be true (1967a, pp. 15, 23; 1962, pp. xvii, xxvi; 1968, p. 60; 1984, p. 363; 1967b, p. 354). He writes: "What we want to assert in using that proposition [that the number three is prime] is something that always was and always will be objectively true, quite independently of our waking or sleeping, life or death, and irrespective of whether there were or will be other beings who recognize or fail to recognize this truth." (1984, p. 134; 1967b, p. 123)

The lack of qualification in his claims of independence is especially striking in two passages: one where he writes that someone's thinking a thought has "nothing to do" either with its truth or with the thought content itself (1984, p. 368; 1967b, p. 359); and another where he writes that thought contents are not only true independently of our recognizing them to be true, but they, the thought contents themselves, are "absolutely independent of our thinking" (1979, p. 133; 1983, p. 145). Independence is independence. Frege's repeated remarks about mind-independence of non-spatio-temporal entities would not have been literally true, if they had been backed by a set of unstated qualifications of the sort that an idealist (or deflationary) interpretation of them would require. Ultimately the idealist asserts dependence of the thought-contents and timeless objects on some underlying practice or activity that makes possible the framework in which attributions of objectivity are made. No idealist—and no deflationist who thought that non-spatio-temporal entities were dependent on our language, practices, or judgments, or who thought that general philosophical assertions about them were "non-factual"—would have made such statements without careful, explicit qualifications. Frege enters no such qualifications.

said to depend in any way on anything about language or people. (Similarly, with concepts demarcating possible numerations in such cases as packs of cards.) Frege writes: "To bring an object under a concept is merely to recognize a relation that already existed beforehand" (1984, p. 198; 1967b, pp. 181-182; cf. 1979, p. 137; 1983, p. 149). Weiner glosses Frege's claims that mathematical truths are independent of us by *excepting* an alleged presupposed need to impose structure and formulate boundaries linguistically (p. 201ff.). She further writes, "discovering what is 'there' in the 'realm of the abstract' amounts to discovering what meets the descriptions that interest us" (p. 203). Weiner cites no texts to support either of these claims. Frege makes no exceptions or qualifications on his claims of independence; he notes no such presupposition. And it is at best deeply misleading to say that for Frege discovering mathematical structures "amounts to" discovering something associated with words, our interests, or ourselves. When our interests and descriptions happen to accord with mathematical truth, we do, of course, discover things that "meet" those interests and descriptions. But Frege explicitly says that our relation to logical truths and mathematical structures is "inessential" to their nature and existence (1984, p. 371; 1967b, p. 361).

Frege repeatedly inveighs against seeing logic (or mathematics) as embedded in language in the way that grammar is.¹³ He thought that thought contents, logical objects, and logical functions bore no such essential dependence relation to the actual practice of thinking or language use. For Frege, the subject matter of logic is not the nature of human thinking or practice (1967a, p. 13; 1962, p. xvi), *even when that practice accords with the laws of truth*: “But above all we should be wary of the view that it is the task of logic to investigate actual thinking and judging, insofar as it is in agreement with the laws of truth” (1979, p. 146; 1983, p. 158—the published translation is ambiguous in a way that does not match the German). This independence insures, for Frege, no scope for variation in the laws of logic between one group of thinkers and another (1979, pp. 7, 146; 1967b, pp. 7, 158; 1967a, p. 13; 1962, pp. xv-xvi).

Frege criticizes one Achelis who writes,

... the norms which hold in general for thinking and acting cannot be arrived at by the one-sided exercise of pure deductive abstraction alone; what is required is an empirico-critical determination of the objective principles of our psycho-physical organization which are valid at all times for the great consciousness of mankind.

Frege replies:

[It appears that according to Achelis] the laws in accordance with which judgments are made are set up as a norm for how judgments are to be made. But why do we need to do this?... Now what is our justification for isolating a part of the entire corpus of laws and setting it up as a norm?... Are [the laws of logic] like the grammar of a language which may, of course, change with the passage of time? This is a possibility we really have to face up to if we hold that the laws of logic derive their authority from a source similar to that of the laws of grammar... if it is normal to judge in accordance with our laws of logic as it is normal to walk upright (1979, p. 147; 1983, p. 159)

Frege thought one can discover logic by reflecting on linguistic and mathematical practice. But he makes it very clear that his logical theory is not about practices, and does not take its authority from such practices. They are not what ground the normative structures that logic articulates.¹⁴

¹³ Some interpreters of Frege have taken his views to be redescrptions of features of our practices of judgment or of linguistic use. Although Frege does describe logical structures that inform linguistic and cognitive practice, and does think that by reflecting on and reforming such practice we can discover these structures, I know of no evidence that Frege thought that the theory of judgment is really fundamentally about the activity or practice of judgment, much less linguistic practice. It is important to distinguish Frege's method of discovery (which does focus on language and activities of judgment) from Frege's views about the nature of thought contents and of judgment. It is also crucially important to realize that Frege was interested in judgment as a norm-yielding form, not in judgment as a human activity. Frege thought that thought contents and the form of judgment bore no essential relation to either language or activities (practices) of judgment, potential or actual, of human beings

¹⁴ Frege writes that there is no contradiction between something's being true and everyone's taking it to be false (1967a, p. 13; 1962, pp. xv-xvi), making it clear that he does not believe in some general connection between thought contents (or intentional contents, or what are expressed in language) and actual judgments and practice, that would close any

A second way Frege's Platonism shows itself lies in his attempts to explicate the success of science, the fact of intersubjectively objective cognitive practices, and indeed the authority of logic, in terms of the timelessness of the truths and structures of logic. In *The Basic Laws of Arithmetic* he states that the laws of logic (which he also calls the laws of truth) are authoritative *because* of their timelessness: "[the laws of truth] are boundary stones set in an eternal foundation, which our thought can overflow, but never displace. It is because of this that they have authority for our thought if it would attain to truth." (1967a, p. 13; 1962, p. xvi).

Frege frequently claims that only because individuals are not the bearers of thought contents is scientific communication possible (1967a, p. 17; 1962, p. xix; 1979, pp. 133, 137-138; 1983, pp. 144, 149-150; 1984, pp. 363, 368; 1967b, pp. 354, 359). But he sometimes goes further. In the Logic manuscript of 1897 he indicates that it is the timelessness of the subject matter of logic—the laws' not containing conditions that might be satisfied at some times and not at others—that enables logic to provide completely universal laws of truth (1979, p. 148; 1983, p. 160). The idea seems to be that all true thoughts are eternally true if they are true at all; but some have temporal subject matters. Some true laws even contain conditions that might be satisfied at certain times but not at others. But the laws of logic cannot be about temporal subject matters and cannot contain such conditions. For if the truth of some thought follows from the truth of others, then it must always follow. So to account for the universal aspect of entailment, one must assume that the subject matter of logic is eternal. (The conclusion of this argument, though not the argument itself is stated in 1967a, p. 13; 1962, p. xvi.)

In "Thoughts" Frege gives two more arguments that scientific objectivity (of communication and of knowledge of the physical world, respectively) is explicable only on the view that thought contents belong to a "third realm" that is neither mental nor physical. In the first argument (1984, p. 363; 1967b, p. 354) he holds that scientific communication cannot be understood on the assumption that thought contents are ideas in particular people's minds. He had previously maintained that thought contents are clearly not perceptible or knowable on the basis of perceptions (1984, pp. 354, 360; 1967b, pp. 345, 351). He concludes (1984, p. 363; 1967b, p. 354) that in order to understand the objectivity of the communal scientific enterprise, one "must recognize" the third realm. The timelessness of the truths of this realm and the fact that their truth is independent of whether anyone takes them to be true are clearly seen as part of an

possible gap between mind and subject matter. There is more evidence for this fact in his discussion of scepticism in "The Thought". The example he gives in 1967a, p. 13 and 1962, pp. xv-xvi concerns an empirical truth. As I discuss below, he indicates that some truths—simple truths of arithmetic and basic logical truths—can be denied only through madness, and that any attempt to deny them in a thoroughgoing way will undermine judgment itself.

account of how a “science common to many on which many could work” is possible.¹⁵

In the second argument (1984, p. 368; 1967b, p. 359), Frege indicates that a “firm foundation of science” must be facts that are independent of men’s varying states of consciousness. Facts are, he maintains, true thoughts. True thoughts have the requisite independence: not only are they not part of anyone’s “inner” mental world; their truth “has nothing to do with” someone’s thinking them. The work of science consists in the discovery of true thoughts (which provide a “firm foundation” for the science). Moreover, Frege argues that the applicability of mathematical truths to investigations at any time (he cites application of mathematics in an astronomical investigation into events in the distant past) is possible *because* a mathematical thought’s truth, and the thought content itself, are timeless. So, he concludes, explicating the objectivity of science and the temporally neutral applicability of mathematics requires that both the thoughts and their truth be timeless.¹⁶

These arguments take for granted the existence of the objectivity manifested in intersubjectively accepted norms for communication and the checking and

¹⁵ Although this argument is not explicit in his 1968, the attitude behind it is not hard to discern in the introduction (1968, pp. vii–viii). I think that the argument is the least interesting of the three arguments I am discussing.

¹⁶ Thomas G. Ricketts (1986) opposes reading Frege as “the archetypical metaphysical platonist” (p. 65), according to which “the mind-independent existence of things is for Frege a presupposition of the representational operation of language: it explains how our statements are determinately true or false apart from our ability to make or understand them”. This description of Platonism does not fit the Platonism I attribute. Frege was clearly not trying to give a general explanation of linguistic representation or even of intentionality in judgment. But—in contrast to Ricketts—I think that Frege thought, as the previously cited passages indicate, that assuming the mind-independence of all thought contents, concepts, and logical objects, was necessary to understanding the objectivity of scientific practice and the universal applicability of logic and mathematics. I do not think that he thought that such objectivity would somehow be in jeopardy in the absence of such an explanation. Logic was for him epistemically prior to philosophy of logic. It is rather that such an explanation accounts for what is involved in judgment, logical inference, and logical truth. Ricketts elaborates: “The crucial feature of this [Platonist] line of interpretation is its taking ontological notions, especially that of an independently existing thing, as prior to and available apart from logical ones, from notions of judgment, assertion, inference, and truth” (p. 66). Ricketts also thinks that Frege’s claims about the objectivity associated with judgment are not meant to be factual claims, and that there is therefore no possible explanation for Frege of objectivity. As I indicate in the text, I see no evidence for a relevantly applicable distinction in Frege between factual and non-factual claims. Moreover, Frege’s Platonism does not involve any claim about the priority of ontological notions over logical notions. (I do not see even initial plausibility to attributing this assumption to Frege.) Logic and ontology are mutually entangled in Frege. Logic is about what is, as Frege says (1984, p. 351; 1967b, p. 342); it has an ontology. But logic is the most general science. So no thought about being could be independent of its notions. Moreover, Frege’s most fundamental ontological categories (function and object) are logical categories. Nor does Frege’s appeal to ontology in his account of the objectivity of science and the universality of logic imply that he thought that ontological notions were prior to logical ones, much less available apart from them. The explanation is not a definition, derivation, or reduction. All the key ontological notions he uses both presuppose and include logical notions. Rather he thought that a full understanding of logic involved appeal to notions like logical object, function, thought content, mind-independence, timelessness,

confirming of scientific results. Frege thinks that we need no reassurance about its solidity. He is not concerned with scepticism. He regards ordinary certainties as certain (1977 §13; 1968, p. 2). He does not seek foundations, nor does he appeal to his Platonism, to bolster confidence in an otherwise doubtful scientific enterprise. He does not view philosophy in the grand manner, as protecting science against otherwise dangerous philosophical worries. He articulates his Platonism because he finds a refusal to qualify the timelessness of mathematical structures, or to explain them in terms of something more familiar and temporal—such as our minds or practices—provides the best understanding of scientific inter-subjective objectivity. He thinks his view shows why practices that have been found to be firm are in fact firm (1968, p. 2).

Let us turn to Frege's views about how we know this third realm of entities. As I indicated earlier, I am prescinding from complexities in Frege's epistemology. What is important for our purposes is that Frege thought that our knowledge of the primitive logical truths and inference rules depended on a logical faculty—reason (1968, p. 21; 1980, pp. 37, 57; 1976, 37, 89; 1962 II, §74; 1984, p. 405; 1967b, p. 393; 1979, pp. 267-273; 1983, pp. 286-292.) The question is: how could Frege believe that reason alone could give one knowledge of an atemporal realm of entities that are completely independent for their existence, nature, and relations to one another, of anyone's reasoning?

Frege is aware that foundational questions about our knowledge of mathematical structures ultimately come down to questions about knowledge of the primitive truths and inference rules. He is admirably clear that logic does not answer these questions: "The question why and with what right we acknowledge a law of logic to be true, logic can answer only by reducing it to another law of logic. Where that is not possible, logic can give no answer" (1967a, p. 15; 1962, p. xvii); "We justify a judgment either by going back to truths that have been recognized already or without having recourse to other judgments. Only the first case, inference, is the concern of Logic" (1979, p. 175; 1983, p. 190). Frege lays fundamental epistemological questions aside in much of his work, especially in *Basic Laws of Arithmetic*. But it would be a serious misunderstanding to think that he thought that the questions were off limits.¹⁷ For he expresses a consistent interest in them.

Of course, he thought that one could not and need not argue for the basic logical truths. But he did see them as a source for the justification of the belief in them by a person who understood them. He thought that they were self-evident.

causal inertness, imperceptibility, non-spatiality. Frege thought of himself as describing the ontological features that logic must have. Logical and ontological notions are interrelated for Frege; and all the relevant logical objects and functions are timelessly related to the relevant notions (thought components). Frege sees the whole logical structure, not just objects, in a Platonic fashion.

¹⁷ Cf. also 1979, pp. 3, 175; 1983, pp. 3, 190. Contrast Ricketts (1986, p. 81) "There is, as far as Frege is concerned, nothing to be said about the justification for our recognition of those basic laws of logic to be truths"; and Weiner (1990, pp. 71-72). Frege says a good bit about the epistemology of belief in the basic laws, scattered through his writings. I shall not discuss these passages in this paper, however.

We justify our judgment of the basic truths, as he said, without having recourse to other judgments (1979, p. 175; 1983, p. 190).

One needs to bear in mind here a three-fold distinction that Frege often carries along in his writings (it is very explicit in "The Thought" 1984, p. 352; 1967b, pp. 342-343): a) psychological explanation of belief or judgment, including an account of its acquisition, (b) justification of our belief or judgment, and (c) grounding for logical truth. Frege always lays aside psychological explanation. But he repeatedly discusses the justification of "our" belief or judgment in logical truths as well as the grounding of logical truth. Understanding grounding of truth is a matter of understanding the natural order of truths, which is independent of thinking or practice, and the "same for all men" (1968, p. ii, 17; 1967a, pp. 13, 15; 1962, pp. xvi, xvii). One understands the grounding of truth when one understands the natural order of logical and mathematical proofs, and the primitive truths on which such proofs rest. What grounds logical truths are the primitive logical truths.

One of Frege's primary motivations for understanding logical truth and the proof structure of logic was to understand the nature of justification for our mathematical judgments. In *Foundations of Arithmetic* Frege begins in §§1 and 2 by announcing an interest in the proof structure of mathematics. But he immediately associates this structure with the question of the justification of belief. In §2 he says that the aim of proof is, partly, to place a proposition beyond doubt. In §3 he says that "philosophical motives" underly his inquiry into the foundations of mathematics: The motives turn out to centre on answering the question "Whence do we derive the justification of our assertion [of mathematical truths]?" The question of whether arithmetic is analytic turns out to concern the justification for making a judgment. He refines this to read, it concerns "the ultimate ground upon which rests the justification for holding [a proposition] to be true".

What is important about this passage is not only Frege's concentration on justification for judgment, but also his belief that the justification of an arithmetical judgment derives from the mathematical foundation (*Grund*)—from the primitive truths. The problem [of finding the justification for assertion or judgment], he says, is to be solved by "finding the proof of the proposition and following it back to the primitive truths." (1968, p. 4). One might ask, how can a problem of the justification of our beliefs or judgments be solved by citing primitive truths? How can such truths be primary in an account of justification?

Frege's line is made clearer in "The Thought" where he characterizes laws of truth as general laws which concern not "what happens" but "what is". Speaking of these laws about "what is" in the third realm, Frege says that "from the laws of truth there *follow* ["*ergibt sich*"]—a non-technical term] prescriptions about asserting, thinking, judging, inferring" (1984, p. 351; 1967b, p. 342). Frege then calls these prescriptive epistemic laws "laws of thought" (1984, p. 351; 1967b, p. 342). This is a paradigmatic Platonic direction of explanation: from what *is* in an abstract realm to what is reasonable. What could be the nature of this derivation from general laws of truth—which concern logical objects and functions—to pre-

scriptive laws about judgment? Frege writes: “[the laws of truth] are boundary stones set in an eternal foundation, which our thought can overflow, but never displace. It is because of this that they have authority for our thought *if it would attain to truth*” (1967a, p. 13; 1962, p. xvi).

Is it contingent that a judging subject “would attain to truth”? Frege is certainly insistent that the laws of truth are independent of their being taken to be true by anyone (1967a, pp. 13, 15; 1962, pp. xvi, xvii). Moreover, he thinks it not contradictory to suppose something’s being true which everyone takes to be false (1967a, p. 13; 1962, pp. xv–xvi). On the other hand, Frege sees judgment as an advance from thought content to truth value. The function or aim of judgment is to reach truth. So to be a judging subject, one must have this aim or function insofar as one makes judgments. In this sense, the prescriptions of the laws of truth must apply to the judgments of judging subjects.

There is a second way in which Frege thinks that there is a deep, non-contingent relation between the laws of truth and prescriptive laws about judgment. To be rational, he thinks, one must be disposed to acknowledge the simplest logical truths. Judgments in contradiction with the laws of logic would constitute a kind of madness (1967a, p. 14; 1962, p. xvi). In fact, Frege appears to believe that failure to acknowledge primitive logical laws, like the principle that every object is identical with itself, and even certain truths of arithmetic, would throw thought into confusion and undermine the possibility of genuine judgment and thought (1968, p. 21). This suggests that he was inclined to believe that a disposition to acknowledge basic logical truths and inferences—and a disposition not to deny non-basic but relatively simple truths of arithmetic—was a condition not only for being rational but for being a judge or thinker at all.

Here it is worth looking very carefully at a famous passage in *Basic Laws*. Frege considers a supposed possibility in which beings had laws of thought (prescriptions for judgment) that contradicted ours. He claims that such beings would exhibit a “hitherto unknown type of madness”, and indicates that such beings’ procedures for taking things to be true would be in radical disaccord with the laws of truth (1967a, p. 14; 1962, p. xvii). Shortly afterwards, he writes:

If we step away from logic, we may say: we are compelled to make judgments by our own nature and by external circumstances; and if we do so, we cannot reject this law—of Identity, for example; we must acknowledge it unless we wish to reduce our thought to confusion and finally do without all judgment whatever. I shall neither dispute nor support this view; I shall merely remark that what we have here is not a logical consequence. What is given is not a ground (*Grund*) for something’s being true, but for our taking it to be true. Not only that: this impossibility of our rejecting the law in question hinders us not at all in supposing beings who do reject it; where it hinders us in supposing that these beings are right in so doing... Anyone who has once acknowledged a law of truth has by the same token acknowledged a law that prescribes the way in which one ought to judge, no matter where, or when, or by whom the judgment is made. (1967a, p. 15; 1962, p. xvii)

Frege is taking a hands-off attitude toward the epistemological issues for the purpose of his mathematical treatise. But, given his own beliefs, what does he neither dispute nor support in the view he states? Some have thought that in citing the limits of logic, he is prescinding from any judgment about grounds for our taking something to be true, as opposed to the ground for its being true. Some have even held that grounds for our taking something to be true are thought by Frege to be psychologistic, and of no interest to him. These are serious misreadings of the passage.

Understanding grounds for our *taking* something to be true had long been what motivated his inquiry into the foundations of arithmetic. (Cf. especially 1968, p. 3, where he uses exactly the same German terms as he does in the above-cited passage: grounds for taking [holding] something to be true.) One page earlier in *Basic Laws* Frege characterized the laws of logic in the double way I have described: not only as laws of truth but as laws that “prescribe the way in which one ought to think” (1967a, p. 14; 1962, p. xvi)—as laws of thought. What Frege takes no position on is whether we are compelled to acknowledge the laws by our own nature and by external circumstances.

This is indeed a psychological matter. He thinks that any such psychological law would admit of conceivable exceptions—mad beings that do reject the law. But where he writes, “we must acknowledge it unless we wish to reduce our thought to confusion and finally do without all judgment whatever”, he is speaking in his own voice. For he had already indicated that he believes that renouncing the laws of arithmetic (which are less basic for him than the basic laws of logic) would be to reduce thought to confusion and make thinking impossible (1968, p. 21). Frege thinks that acknowledging these laws, at least implicitly in one’s actual thinking, is necessary for having reason and for being a non-degenerate thinking and judging subject. (He apparently believed that although a mad person could reject a law, abiding by such rejection would reduce thought to confusion, and by degrees undermine judgment altogether.) These are normative not psychological judgments. Although they are not logical consequences, they are part of Frege’s epistemic view.

So let us summarize the view that Frege maintains. He holds that justification for holding logical laws to be true rests on and follows from primitive laws of truth. He spells out this dependence of epistemic justification on the laws of truth in two ways. He thinks that laws of truth indicate how one ought to think “if one would attain to truth”. But a judging subject necessarily would attain to truth, insofar as it engages in judgment. So any judgment by a particular person necessarily is subject to the prescriptive laws set out by the primitive laws of logic. One is justified in acknowledging them because doing so is necessary to fulfilling one’s aim and function as a judging subject.

Frege’s second way is: acknowledgement of certain laws of truth is necessary for having reason and for engaging in non-degenerate thinking and judging. One is rationally entitled to judge the primitive laws of logic to be true because the nature of reason—and even non-degenerate judgment—is partly constituted by the prescription that one acknowledge at least the simple and basic laws of truth.

To put it crudely, reason and judgment—indeed mind—are partly defined in terms of acknowledging the basic laws of truth.¹⁸

Our problem was to explain how, for Frege, mere reason could give grounds to believe that a subject matter is any particular way, given that the subject matter is atemporal, causally inert, and independent of thinking about it. Most current approaches to the substantive problem look for some analog of causal interrelation in our mathematical knowledge. More traditional views—both Platonic and idealist—see the relation as individuating or constitutive.

An idealist line is to make the subject matter constitutively dependent on thinking, synthesis, or practice. Frege's line is to hold that, although the laws of truth are independent of judging subjects, judging subjects are in two ways not independent of the laws of truth. First, to be a judging subject is to be subject to the prescriptions of reason, which in turn are provided by the laws of truth (logic). For judgment has the function of attaining truth; and the laws of logic—which are constituted by atemporal thoughts and atemporal subject matter—provide universal prescriptions of how one ought to think, given that one's thinking has the function of attaining truth. Second, being a judging subject is to have or have had some degree of reason. Having or having had some degree of reason requires acknowledging, at least implicitly in one's thinking, the simplest, most basic logical truths and inferences; and doing so commits one to an atemporal subject matter. Questions of "access" to the third realm are on reflection seen to be misconceived. For, to reverse somewhat Gertrude Stein's dictum about Oakland, there is no there there.

Why was this line not more prominent in Frege's philosophy? He thought that his primary contribution lay in identifying primitive truths and inference rules, and in deriving arithmetic from them. He accepted the traditional rationalist-Platonist line about the relation between reason and primitive truths. He did not think it needed substantial elaboration. Like Frege, I think that this neglected line is not to be dismissed. Unlike Frege, I think it may be worth developing.

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¹⁸ One can see this view alluded to in the passage where Frege claims that logic can, with anti-idealist and anti-psychologistic qualifications, be seen as the study of not minds but Mind (*Geist*) (1984, p. 369; 1967b, p. 359). One can also see it in his claim: "We might with alteration of a well-known proposition say: the proper object of reason is reason. In arithmetic we are not concerned with objects which we come to know as something alien from without through the medium of the senses, but with objects given directly to reason, which as her most proper objects are completely transparent to her" (1968, p. 115). Cf. also 1977 § 23; and 1968, §26. These quotes are not idealist, as they have sometimes been taken. They are expressions of the view that the basic forms and objects of logic constitutively inform minds, and help define what it is to be mind or reason.

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