Understanding Physics: Student Guide

David Cassidy
Gerald Holton
James Rutherford

Springer

UNDERSTANDING PHYSICS

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"Exploring Heat Transfer and the Latent Heat of Fusion": This exploration is adapted from *Project Physics Handbook*, Experiment 3-11, pp. 128–131.

"Spacetime: A Computer Excursion into Relativity Theory": This exploration follows suggestions accompanying the program, *Spacetime* by Professor Edwin F. Taylor, Physics Academic Software, American Institute of Physics, College Park, MD.

"Exploring Electric Charges, Magnetic Poles, and Gravitation": The electrostatic portion of this exploration is adapted from *Project Physics Handbook*, Experiment 4-3, pp. 179–180. It was further inspired by the suggestions in A.B. Arons, *A Guide to Introductory Physics Teaching* (New York: Wiley, 1990), Chapter 6.

"Investigating Electric Currents I": Parts A and B of this exploration were inspired by L.C. McDermott et al., *Physics by Inquiry*, Vol. 2, pp. 383–389.

“Investigating Waves”: This exploration is adapted from *Project Physics Handbook*, Experiments 3-15 and 3-16, pp. 139–140.

“Avogadro’s Number and the Size and Mass of a Molecule”: This exploration is adapted from S.E. Kennedy et al., *Ideas, Investigation, and Thought: A General Chemistry Laboratory Manual*, 2nd ed., R.S. Wagner et al. (Garden City Park, NY: Avery, 1996), pp. 57–64.

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SUGGESTED MINI-LABORATORY EXPLORATIONS

SUGGESTED MAJOR LABORATORY EXPLORATIONS

In addition, one or more mini-laboratories might be joined together or expanded upon for use as a major laboratory on the later material.

Introduction

This book, a companion to the textbook, *Understanding Physics,* is your guide to observations and explorations in the world of physics. Prepare for challenging work, fun, and some surprises. One of the best ways to learn physics is by *doing* physics, in the laboratory and everywhere. One cannot rely on reading and class work alone. The explorations in this book are your opportunity to gain some actual, hands-on experience with physics. Many of these explorations will assist you to design your own experiments and to discover many of the important ideas of science yourself.

As you will see from the Contents, this *Student Guide* provides a variety of potentially helpful materials. Following the Introduction is a review of units, mathematics, and scientific notation, and a list of suggested further reading and Web Sites. However, a large portion of the *Student Guide* contains further materials relating to many of the textbook chapters, as well as to laboratory explorations. In the section containing "Further Chapter Materials" you will find elaborations on topics in many of the chapters, as well as derivations of important equations. A complete list of the suggested mini- and major-laboratory explorations is also given in the Contents. Each exploration is keyed to specific portions of the textbook, and lists are also provided of the explorations pertaining to each part of the text.

There are actually three types of laboratory explorations in this book: "mini-laboratories," "major laboratories," and some suggested "laboratory activities." The mini-laboratories are hands-on experiences and demonstrations that enable you to observe and study an event in nature, either in class or in a laboratory. The major laboratories are designed for more in-depth exploration. Finally, the laboratory activities provide ideas for ways in which you might design your own investigations. All three types of explorations are closely tied to the material in the book.

The textbook material and the laboratory explorations go hand-in-hand. You will get the most out of them by working on both together. All of the laboratories are deliberately designed to be as "low tech" as possible in order to provide you with as direct an experience as possible with the material and with the analysis of the data. As you become more familiar with the material, your instructor may introduce computer and other technological enhancements.

Scientific research is often performed in groups, and no research results are accepted in science until they have been reviewed and discussed by others. Your class may also work in pairs or in groups. This is a wonderful way to learn, as long as everyone does his or her best to contribute to the work. Group work is also a model used in many careers, and it is essential in nearly every career to be able to get along with your colleagues. Communicating your results to others in written and oral form is also important.

In studying the text and engaging in these laboratory explorations, we suggest that you keep a notebook or a journal of your work. This notebook should include your notes from reading the text, your answers to the questions at the end of each chapter, your questions on any of the material in this course, the results of your group discussions, and your work in the laboratory. You will notice that the laboratory explorations in this guide do not contain any tables for plugging in your data results. Part of the research experience will be to understand the data to such an extent that you can construct your own tables to organize and present the data in the way that you think it can be done most clearly—exactly as research scientists do.

A journal will also help you to keep all of your work together, enabling you to compare what you learn in the laboratory with what you find in the text, and it will help you in preparing for examinations. It will also provide

a record of your progress in the course. When you look back in the end, you will be amazed how far you have progressed.

Physics is not an easy subject, but it is no more difficult than most other academic subjects. Like other subjects, studying physics requires a certain amount of dedication, but the rewards are well worth the effort! Studying physics may give you an entirely new perspective on your world. It will help prepare you for the scientific age in which we live, and enhance your abilities in any career that you choose.

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A Word to Future and Current Teachers

Understanding Physics is an introductory course designed for non-science majors in general, and for future and current teachers, including those in K–12 classrooms, as well as in college. One aim of this course is to bring all undergraduate students at least to the level of science literacy in physics outlined in the recent national initiatives for the introductory physics course. These initiatives have been very influential in recent years at State level. Many States have been issuing more stringent education standards in science, and they are introducing new teacher certification examinations in line with these new standards. At present, the two most prominent national initiatives are:

- National Research Council. *National Science Education Standards* (Washington, DC: National Academy Press, 1996).
  Online text: http://www.nap.edu/readingroom/books/nses/html/
- Project 2061 (American Association for the Advancement of Science). *Benchmarks for Science Literacy* (New York: Oxford University Press, 1993).
  Online text: http://www.project2061.org/tools/benchol/bolframe.html/

In keeping with these developments, a second, related aim of this course is to equip future and in-service teachers with the knowledge and ability to teach the basic physical science recommended in these two national initiatives at different grade levels. In many instances you will find the same or very similar recommendations in your State standards for science education, and many of the concepts here will appear on teacher certification examinations.

Both of the above online sites may be accessed through the *Understanding Physics* Web site at: http://www.springer-ny.com/. At these sites you will find specific learning goals in science for different grade levels. You will also find links to cognitive research for each grade level, bibliographic references, and many other helpful materials.

Reviewing Units, Mathematics, and Scientific Notation

UNITS

Every measurement in science is made in the units of a standard measure appropriate for the property that is being measured. For example, length might be 3 m, or 2 in, or 8 cm. (It can never be just 3 or 2 or 8). Other measurements might be 8 s, 5 g, 16 l, 46°C, and so on.

Standard units in the sciences are those defined, accepted, and used by the scientific community. For instance, the standard unit of mass in the metric system is the kilogram (about 2.2 lb). The kilogram has been defined as the mass of a platinum–iridium cylinder kept by the International Bureau of Weights and Measures in Paris, with a duplicate in the United States in the National Institute of Standards and Technology (formerly the Bureau of Standards) in Washington, DC.

In the United States, two systems of units are often encountered: the *English* system and the *metric* system. The *English* system arose through common practice in the marketplace and most of it is ill-suited for used in the laboratory. It uses the following standard measures:

Distance:	inch (in), foot (ft), yard (yd), rod, furlong (fur), mile (mi) (statute and nautical).
Time:	second (s), minute (min), hour (hr), year (yr).
Mass:	ounce (oz), pound (lb).
Force:	ounce (oz), pound (lb), ton (t).
Volume:	ounce (oz), cup, pint (pt), quart (qt), gallon (gal).
Temperature:	degree Fahrenheit (°F).
Energy:	foot-pound (ft-lb), British thermal unit (Btu), calorie (cal).

As you can see, these units can be confusing—for instance, "ounce" may refer to volume, mass, or force. Notice also how difficult it is to convert smaller units to larger ones (feet to miles, ounces to quarts, rods to furlongs, etc.). In addition, the definitions of some units, such as gallons, differ from country to country. The British, Canadian, American, and Australian gallons are all different.

Because of these problems, the English system is avoided in scientific research. Instead, the metric system, based on units of ten, is used. In this system, the prefixes of the measures tell you the relationship to the standard measure. For instance, "milli" stands for 1/1000, "centi" stands for 1/100, and "kilo" stands for 1000.

The *metric* system uses the following standard measures:

Distance:	millimeter (mm), centimeter (cm), meter (m), kilometer (km).
Time:	second (s).
Mass:	milligram (mg), gram (g), kilogram (kg).
Force:	newton (1 kg-m/s^2) (N) or dyne (1 g-cm/s^2) (dyn).
Volume:	milliliter (ml), liter (l).
Temperature:	degree Celsius (centigrade) (°C) or absolute temperature (Kelvin) (K).
Energy:	erg (1 dyne-cm), joule (1 n-m) (J), calorie (cal)

When substituting actual measurements into an equation, always be careful to retain the units along with the numbers, since they provide the units of your final answer and serve as a check on your calculation. Generally you should convert all similar types of measurements to the same units. In multiplication and division, the units are treated like numbers, while in addition and subtraction the units are simply carried through.

▼ Examples

$$5 \text{ m} + 30 \text{ cm} = 5 \text{ m} + 0.3 \text{ m} = 5.3 \text{ m},$$
$$16 \text{ kg} \times 4 \text{ m/s}^2 = 64 \text{ kg-m/s}^2 = 64 \text{ N}.$$

During this course, when you need to convert from English to metric units or vice versa, you will be able to use the following (approximate) relations. (There is no need to memorize these.)

English	*Metric*	*Metric*	*English*
1 inch	2.54 centimeters	1 cm	0.39 in
1 foot	0.30 meters	1 m	3.28 ft
1 mile	1.61 kilometers	1 km	0.62 mi
1 gallon	3.79 liters	1 l	0.26 gal
1 pound weight	4.45 newtons	1 N	0.22 lb
°F	°C × $\frac{9}{5}$ + 32	°C	$\frac{5}{9}$ (°F − 32)

In the metric system, if measurements of mass, distance, and time are in grams, centimeters, and seconds, this is called the *cgs system*. If the measurements are in kilograms, meters, seconds, this is called the *mks system*. The units for force and energy in the cgs system are: dyne, calorie, erg. The units for force and energy in the mks system are: newton, kilocalorie (Calorie), joule.

The cgs system is usually used in chemistry or when the amounts of material studied in the laboratory are typically small. The mks system is usually used in physics, which often concerns itself with larger objects.

The erg (in cgs) and joule (in mks) are units of mechanical energy, while the calorie (cgs) and Calorie (mks) are units of heat energy. Since they are all units of energy, but in different forms, they are all related to each other according to the following:

$$1 \text{ Calorie} = 1000 \text{ calories} = 4190 \text{ joules} = 4190 \times 10^7 \text{ ergs.}$$

SIGNIFICANT FIGURES

The accuracy of every measurement is limited by the precision of the instrument being used. For example, if the length of a table is measured using a meter stick that is divided into centimeters and millimeters, you can measure the length to an accuracy of plus or minus 1 mm. Although it is possible to guess to a fraction of a millimeter, one cannot be more accurate than the nearest millimeter. Thus, you might measure the table to be 1.23 m long. Is it *exactly* 1.23 m long, or could it be 1.232 m or 1.229 m? One can't tell with this type of measuring instrument. A more precise instrument might yield a length of 1.23175 m. But then, could it be really 1.231749 m? It's possible, but this measurement can't tell us because, again, the instrument we are using is not that precise.

Every measuring instrument, no matter how precise, will have some imprecision. Because of the imprecision in every measurement, the last digit of a measurement is usually regarded only as an approximation. The last number is as "significant" as the other numbers in the measurement, but it is "uncertain." Thus, in the example above, for the table measured with a meter stick to be 1.23 m long, there are three significant figures, while the last figure, 3, is "uncertain." For the more precision measurement about, there are six significant figures, 1.23175, the 5 being approximate.

In calculations using measurements like the above, the answer you obtain can, of course, never be more precise than the measurements with which you started. If your answer has more digits than you started with, round off your answer to the same number of significant figures as the least number that you started with. This is especially important when you use a hand calculator, which can give eight to ten figures in an answer. For example,

$$\frac{1.23\ \text{m}}{3.69\ \text{s}} = 0.3333333\ \text{m/s} \quad \text{by using a hand calculator,}$$

but physically the answer is only 0.333 m/s.

Obviously, the original measurements were not made to the seventh decimal place. The extra decimals given by the calculator have no physical meaning, so the result has to be rounded to 0.333 m/s. Similarly,

$$\frac{18.25\ \text{m}}{6.8\ \text{s}} = 2.6838235\ \text{m/s} \quad \text{by calculator,}$$

but physically the answer is only 2.7 m/s.

If in a measurement you use all of the decimals available, you should still indicate the precision of your measurement by including zeros in your data. For instance, suppose you are using a meter stick that has centimeters and millimeters to measure the distance between two dots on a time tape. The measurement turns out to be exactly 4 cm. You may think to record 4 in your data table, but the number 4 alone does not convey the precision of your measurement. In fact, it conveys the impression that you measured only to the nearest centimeter. To indicate that you really measured to the nearest millimeter, this result should be recorded as 4.0 cm.

SCIENTIFIC NOTATION

Often when working with very large numbers or very small numbers, it is easier to express them in "scientific notation." This notation involves a decimal number, called the "argument," multiplied by 10 raised to an integer power (the exponent). The power of 10 is determined by the number of places that the decimal is moved to the left (positive) or to the right (negative).

▼ Examples

$$93{,}000{,}000 = 9.3 \times 10^7,$$
$$0.000000935 = 9.35 \times 10^{-7}.$$

When multiplying two numbers in scientific notation, first multiply the arguments, then add the exponents. Thus,

$$(6.3 \times 10^{7})(1.2 \times 10^{-5}) = 7.6 \times 10^{2}$$

$$(9.3 \times 10^{7})(8.6 \times 10^{-2}) = (9.3 \times 8.6) \times 10^{5} = 79.98 \times 10^{5},$$

$$= 7.998 \times 10^{6}.$$

When dividing two numbers, first divide the arguments, then subtract the exponent of the denominator from the exponent of the numerator. The result may be positive or negative. Thus,

$$\frac{6.3 \times 10^{7}}{1.2 \times 10^{-5}} = 5.3 \times 10^{12},$$

$$\frac{9.3 \times 10^{7}}{8.6 \times 10^{+2}} = \frac{9.3}{8.6} \times 10^{5} = 1.08 \times 10^{5}.$$

When adding or subtracting numbers in scientific notation, first express all of the numbers in scientific notation with the same exponent, then add or subtract the arguments, maintaining the same exponent in the result. Thus,

$$(9.30 \times 10^{7}) + (5.80 \times 10^{6}) = (9.30 \times 10^{7}) + (0.58 \times 10^{7}) = 9.88 \times 10^{7}.$$

To square or cube a number in scientific notation, you first square or cube the "argument" then multiply the power of 10 by 2 or 3.

▼ Example

$(9.0 \times 10^{7})^{3} = 729 \times 10^{21} = 7.29 \times 10^{23}.$

To take the cube root of a number in scientific notation, you must first express the power of 10 in a power that is divisible by 3. Then take the cube root of the argument and divide the power of 10 by 3.

▼ Example

$(0.8 \times 10^{7})^{1/3} = (8.0 \times 10^{6})^{1/3} = 2.0 \times 10^{2}.$

▼ Exercises

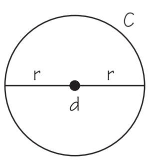
1. $(9.3 \times 10^7)(8.6 \times 10^{-2}) = ?$.
2. $(9.3 \times 10^7)/(8.6 \times 10^{-2}) = ?$.
3. $(9.3 \times 10^7) + (5.8 \times 10^6) + (1.23 \times 10^8) = ?$.
4. The speed of light is approximately 300,000 km/s. How many kilometers does light travel in 1 year? (This *distance* traveled by light in 1 year is confusingly called a "light year".)

GEOMETRY REVIEW

CIRCLES

Any line through the center of a circle and intersecting the circle is a diameter d. The center point divides the diameter into two equal halves, each of which is a radius r. The ratio of the circumference (C) of any circle to its diameter is a universal number known as pi (π):

$$\frac{C}{d} = \pi = 3.14159 \ldots.$$



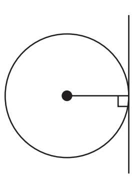
The circumference of a circle is

$$C = \pi \mathrm{d} = 2\pi \mathrm{r}.$$

The area of a circle is

$$A = \pi r^2.$$

The tangent to a circle at any point is perpendicular to the radius at that point.

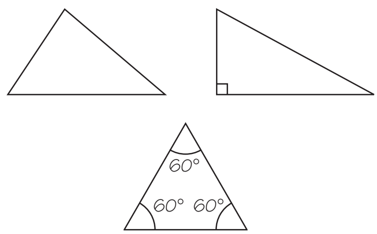


TRIANGLES

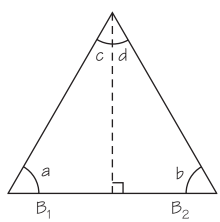
The sum of the angles of any triangle is 180°.

In a right triangle, one of the angles is 90°; while the sum of the other two angles is equal to 90°.

In an equilateral triangle, the sides are all of equal length and the three angles are each 60°.

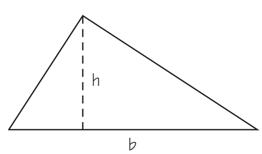


In an isosceles triangle, two of the sides are equal. The angles formed by those two sides and the third side are equal. In the triangle above, angle a = angle b. An altitude drawn from the third side to the opposite angle bisects the opposite angle, divides the third side in half, and forms a perpendicular with the third side. Thus, in the triangle above, angle c = angle d, and segment B_1 = segment B_2. The altitude thus divides an isosceles triangle into two congruent (identical) right triangles.



The area of a triangle is one-half of the base times the height of the triangle, or in symbols:

$$A = \frac{1}{2}bh.$$

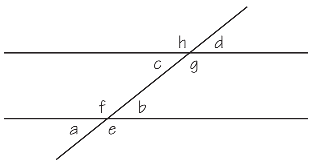


PARALLEL LINES

The following sets of angles are all equal to each other:

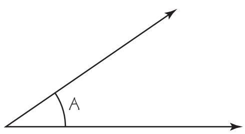
$$a = b = c = d,$$

$$e = f = g = h.$$

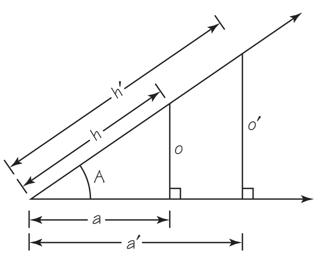


REVIEW OF BASIC TRIGONOMETRY

Two intersecting lines form an angle A, which stays constant no matter how far the two lines are extended. (This is a postulate of Euclidean geometry.)



If lines are dropped at various intervals from the upper line perpendicular to the lower line, right triangles are formed, all with the common angle A.



Since two of the angles in each of these right triangles are equal to each other (90° and angle A), the third angles are also equal. Since the sides are not equal to each other, the triangles are not congruent, but they are similar.

Because these right triangles are all similar, the ratios of their corresponding sides are all equal (although the sides themselves are all different). From this the following ratios of sides are equal to each other:

$$\frac{o}{h} = \frac{o'}{h'},$$

$$\frac{a}{h} = \frac{a'}{h'},$$

$$\frac{o}{a} = \frac{o'}{h'}.$$

The ratio of the corresponding sides of any right triangle with an angle *A* will be equated to one of the ratios above. These ratios are thus universal for all right triangles with the same size angles—from triangles on paper to the Earth–Sun–Moon system and beyond!

To simplify matters, these ratios are given special names. They are called the *sine* (abbreviation: sin), *cosine* (cos), and *tangent* (tan), and their values for a given angle may be found in standard tables or obtained on a pocket calculator. Referring to the first triangle formed, the definitions are

$$\sin A = \frac{o}{h},$$

$$\cos A = \frac{a}{h},$$

$$\tan A = \frac{o}{a}.$$

(The inverse ratios also have special names, but we will not consider them here.)

For example, if $A = 30°$, then o/h or o'/h', etc., will always have the ratio of 1:2 or 1/2; a/h or a'/h', etc., will be $\sqrt{3}/2$; o/a or o'/a, etc., will be $1/\sqrt{3}$.

The powerful advantage of these "trigonometric functions" is that, if you know the size of an angle in any right triangle and the length of one of the sides, you can find the other two sides simply by looking up the corresponding trigonometric functions and multiplying to obtain the unknown side. Conversely, if you know the ratio of any two sides of a right triangle, you can find the angles by finding the angle that yields the value of that trigonometric function.

An easy way to remember these functions is by remembering SOHCAHTOA. This word is made up of the first letters of: Sine is Opposite over Hypotenuse, Cosine is Adjacent over Hypotenuse, Tangent is Opposite over Adjacent.

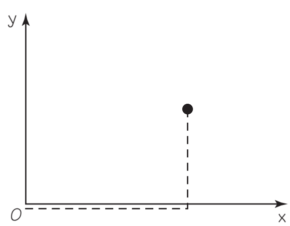
▼ Exercises

1. In a right triangle, one of the angles is 30° and the opposite side is 1 m long. Find the other angles and sides.

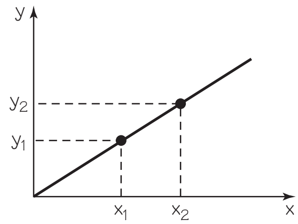
2. In an isosceles triangle, the base is 5 m long and the equal sides are each 6 m long. Find the angles and the length of the altitude from the base.

REVIEWING GRAPHS

A flat plane, such as a table top, has two dimensions, length and width, which can be labeled y and x. A point on the table or plane can be identified if we define a corner of the plane as the "origin," that is, the place from which we start measuring length and width, y and x. In this case, if the origin is defined as $x = 0$ and $y = 0$, any point on the plane can be designated simply by giving the values of x and y that one must move away from the origin to reach the designated point.



In graph 1 on the next page, a series of points were designated such that, as the value of the x coordinate increased by the same amount each time, the value of the y coordinate also increased each time by the same amount (not necessarily the amount that x increased). When this occurs, the points lie on a straight line, which is shown in graph 1.

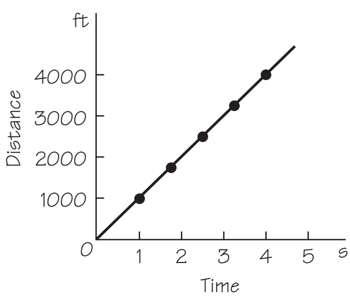


(1)

Here is an example: Measured from the origin, a plane has traveled a total distance of 1000 m in 1 s, 2000 m in 2 s, 3000 m in 3 s, and 4000 m in 4 s. These data can be placed in a table:

Time (s)	*Total distance (m)*
1	1000
2	2000
3	3000
4	4000

In graph 2 below, we plotted the position of the plane, which is the distance traveled (*d*), on the *y*-axis and the time (*t*) on the *x*-axis. Notice that each interval on the *t*-axis and on the *d*-axis of our graph has the exact same value: 1 s each for the time axis and 1000 m for each interval on the distance axis. Also, we set up the axes so that all of the data fit on the graph without going over the top or without being "scrunched" into the corner. We were also careful to label the units of each axis, seconds and feet.



(2)

The resulting data points produced a straight-line graph, as you might expect, since as time increased by 1 s at each point, the plane's position increased by 1000 m.

When the *y* and *x* variables increase or decrease together in this fashion, we say that *variable* y *is proportional to variable* x. This phrase may be written in symbols

$$y \propto x \quad \text{or in this case} \quad d \propto t.$$

Whenever two variables are proportional to each other, we can replace the proportional sign, $\propto$, by an equal sign, =, *if* we include a constant, which is called "the constant of proportionality." Usually this constant is given the symbol m.

$$y = mx \quad \text{where } m \text{ is a proportionality constant.}$$

It is important to note that two variables are proportional only if the points on a graph form a *straight line*. Otherwise, the variables are not proportional.

In the example of the airplane (graph 2), the straight line produced would indicate that $d \propto t$, or that $d = mt$.

How do we obtain the constant of proportionality, m? If the graph of y versus x does form a straight line, the line has a constant "slope." The slope is different for different lines, depending upon how fast or how slow the y variable changes compared to the change in the x variable. For instance, the line on a graph for a plane that travels 2000 m more every second, would have a steeper slope than for a plane that flew only 1000 m every second.

The ratio of the changes in the two variables gives the slope. More precisely, the ratio of the change in y between any two points on the line, $y_2 - y_1$, over the change in x between those same two points, $x_2 - x_1$, is the slope. For a straight line, this ratio is the same no matter which two points you choose. This constant value of the slope of the line is equal to the constant of proportionality, m in the equation $y = mx$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

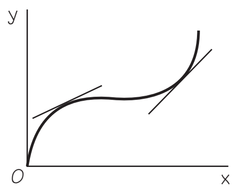
In the example of the first plane, the slope of the line measured between the first and last points is

$$m = \frac{4000 \text{ ft} - 1000 \text{ ft}}{4 \text{ s} - 1 \text{ s}} = \frac{3000 \text{ ft}}{3 \text{ s}} = 1000 \text{ ft/s}.$$

This is just the (constant) speed of the plane. So, in this case, the slope of the distance–time graph of an object moving at constant speed is the speed, v. So we can write

$$d = mt = vt.$$

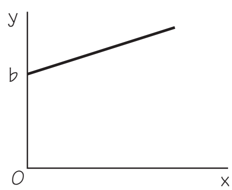
One can also find the slope of any portion of a graph that is linear over a small distance, as indicated in graph 3. In fact, a tangent can be drawn to a curve at any desired point on the curve, and the slope of the tangent can be found in the same way. This gives you a value for the slope at that point alone.



(3)

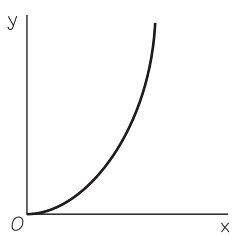
If the line does not go through the origin, as in graph 4, it will intersect the y-axis at some other point, $y = b$. In that case, the relationship between y and x for this line may be written

$$y = mx + b.$$



(4)

If a graph of a series of values for y and x does not yield a straight line, but an upward curve, as in graph 5, it may be a parabola. To see if it is, instead of graphing y versus x, try y versus x^2, that is, square each value of the x value (leaving the y values alone) and then graph those new numbers against y.



(5)

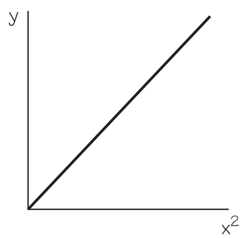
If the new graph turns out to be a straight line, then you have found a new proportionality between y and x^2, rather than between y and x. In this case

$$y \propto x^2 \qquad \text{or} \qquad y = kx^2.$$

This is the equation of a *parabola*. In this case, k is the slope of the new graph of y versus x^2.

Of course, one can substitute other variables for y and x, such as distance d and time t of a moving plane or any other object.

Although your graph does not have to start at the origin, most of the time it will. Always be sure to indicate the value of the origin variables; always indicate which variable is placed on each axis; and always clearly indicate the units for each axis.



(6)

Further Reading and Web Sites

SOME GENERAL READING

E.B. Bolles, ed., *Galileo's Commandment: 2,500 Years of Great Science Writing* (New York: Freeman, 1999).

J. Carey, ed., *Eyewitness to Science: Scientists and Writers Illuminate Natural Phenomena from Fossils to Fractals* (Cambridge, MA: Harvard University Press, 1995).

R.P. Feynman, *The Character of Physical Law* (New York: Modern Library, 1994).

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R. Pool, *Beyond Engineering: How Society Shapes Technology* (New York: Oxford University Press, 1997).

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M. Frayn, *Copenhagen* (London: Metheuen, 2000).

H. Kipphardt, *In the Matter of J. Robert Oppenheimer* (London: Hill and Wang, 1968).

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C. Djerassi, *Cantor's Dilemma* (New York: Penguin, 1991).

C. Minichino, *The Hydrogen Murder* (New York: Thomas Bouregy, 1997).

C. Minichino, *The Helium Murder* (New York: Avalon, 1998).

C. Minichino, *The Lithium Murder* (New York: William Morrow, 1999).
D. Sobel, *Galileo's Daughter: A Historical Memoir of Science, Faith, and Love* (New York: Walker, 1999).

Poetry and Art

D.H. Levy, *More Things in Heaven and Earth: Poets and Astronomers Read the Night Sky* (Wolfville, Nova Scotia: Wombat Press, 1997).
L. Shlain, *Art and Physics: Parallel Visions in Space, Time, and Light* (New York: Morrow, 1991).

(See also the end of each chapter.)

SLOAN TECHNOLOGY BOOK SERIES ON HISTORY OF TECHNOLOGY

R. Buderi, *The Invention That Changed the World: How a Small Group of Radar Pioneers Won the Second World War and Launched a Technological Revolution* (New York: Touchstone Books, 1998).
M. Campbell-Kelly and W. Aspray, *Computer: A History of the Information Machine* (New York: Basic Books, 1997).
C. Canine, *Dream Reaper: The Story of an Old-Fashioned Inventor in the High-Stakes World of Modern Agriculture* (Chicago: University of Chicago Press, 1997).
D.E. Fisher, and M.J. Fisher, *Tube: The Invention of Television* (New York: Harcourt Brace, 1997).
S.S. Hall, *A Commotion in the Blood: A Century of Using the Immune System to Battle Cancer and Other Diseases* (New York: Henry Holt, 1997).
J. Hect, *City of Light: The Story of Fiber Optics* (New York: Oxford University Press, 1999).
T.A. Heppenheimer, *Turbulent Skies: The History of Commercial Aviation* (New York: Wiley, 1998).
R. Kanigel, *The One Best Way: Frederick Winslow Taylor and the Enigma of Efficiency* (New York: Viking Press, 1997).
B.H. Kevles, *Naked to the Bone: Medical Imaging in the Twentieth Century*, (Piscataway, NJ: Rutgers University Press, 1997).
V. McElheny, *Insisting on the Impossible: The Life of Edwin Land, Inventor of Instant Photography* (New York: Perseus Press, 1999).
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R. Rhodes, *Dark Sun: The Making of the Hydrogen Bomb* (New York: Touchstone Books, 1996).
R. Rhodes, ed., *Visions of Technology: A Century of Vital Debate about Machines, Systems and the Human World* (New York: Simon and Schuster, 2000).

M. Riordan, and L. Hoddeson, *Crystal Fire: The Birth of the Information Age* (New York: Norton, 1997).
C.H. Townes, *How the Laser Happened: Adventures of a Scientist* (New York: Oxford University Press, 1999).

Some Web Sites

Web sites come and go. Visit the course Web site for an up-to-date list, at: http://www.springer-ny.com/.

A. Einstein: http://www.aip.org/history/einstein
A. Sakharov: http://www.aip.org/history/sakharov
M. Curie: http://www.aip.org/history/Curie/contents.html
Heisenberg and the Uncertainty Principle: http://www.aip.org/history/heisenberg
The Discovery of the Electron: http://www.aip.org/history/electron
Double Slit Experiment with Electrons or Photons: http://www.inkey.com/dslit
Virtual Physics Laboratories: http://explorescience.com
Discovery of the Transistor: http://www.pbs.org/transistor
Lasers: http://www.aip.org/success/industry/index.html
Superconducting Devices: http://superconductors.org and http://www.oml.gov/reports/m/ornlm3063r1/pt4.html
Todd's Intro to Quantum Mechanics: http://www-theory.chem.washington.edu/~trstedl/quantum/quantum.html
Nobel Prize Winners: http://nobelprizes.com/nobel/nobel.html
Women in Physics: http://www.physics.ucla.edu/~cwp
L. Kristick: "Physics: An Annotated List of Key Resources on the Internet," http://www.ala.org/acrl/resmar00.html
PhysLink (information resource on all aspects of physics): http://www.physlink.com
PhysicsEd: Physics Education Resources: http://www-hpcc.astro.washington.edu/scied/physics.html. A host of resource references on curricula, video, demonstration materials, software, and more.
Physics-2000: http://www.colorado.edu/physics/2000. Many interactive virtual experiments.
NASA: http://spacelink.nasa.gov
"How Stuff Works": http://www.howstuffworks.com
Physics Web: http://physicsweb.org/tiptop/lab
"Beyond Discovery Series," National Academy of Sciences: http://www.Beyond-Discovery.org
"Physics Success Stories": http://www.aip.org/success/
"Top 20 Engineering Achievements of the Twentieth Century," National Academy of Engineering: http://greatachievements.org
Flash-Card physics: http://hyperphysics.phys-astr.gsu.edu/hphys.html

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SOME FURTHER CHAPTER MATERIALS

PART ONE

CHAPTER 1. MOTION MATTERS

Instantaneous Speed

In Section 1.3 we discuss the average speed of an object, which is defined to be the ratio of the distance traveled, Δd, divided by the time interval, Δt:

$$v_{\text{av}} = \frac{\Delta d}{\Delta t}.$$

You may have noticed that we cannot measure the speed of an object in an instant of time. The average speed is the only kind of speed that we can actually measure, since we can only measure distance *intervals* and time *intervals*. We can use more sophisticated instruments to obtain the distance traveled in smaller and smaller time intervals. If the time interval Δt has approached zero, we are dealing with an *instant in time*, and the average speed becomes the actual speed at that instant. This is called the *instantaneous speed*. However, in any real experiment we can never actually achieve an instant in time, an infinitesimally small time interval, since every measurement, no matter how fast we can make it, still takes some amount of time.

Nevertheless, we can use a graph of the motion to *calculate* a reasonable value for the instantaneous speed at an instant of time. We point out in Section 1.4 that the slope of the line on a distance–time graph is

$$\text{slope of line} = \frac{\Delta d}{\Delta t},$$

which is just the average speed during the time interval Δt. As the time interval becomes smaller and smaller, the line on the graph during the time interval becomes straighter and straighter. In such a situation, for very tiny time intervals, the average speed becomes, by definition, equal to the instantaneous speed at the center of the time interval. To put it differently, as the value of Δt approaches the limit of zero (which we cannot actually measure), the value for the average speed v_{av} approaches the instantaneous speed, which is given the symbol v. In this case, the slope of the line becomes a tangent to the curve at that instant. This means: *the instantaneous speed of an object at an instant of time* t *is defined as the tangent at time* t *to the line representing the object's motion on a distance–time graph.*

This can also be expressed in mathematical symbols as follows:

$$\lim_{\Delta t \to 0} \frac{\Delta d}{\Delta t} = v.$$

In words, this says that in the limit as the time interval approaches zero, the ratio of the distance traveled divided by the time interval approaches the instantaneous speed at the time t at the center of the original time interval. (Readers who have had some calculus may recognize this as a differential.)

Derivation of Galileo's Expression $d = \frac{1}{2}at^2$

Galileo's famous expression gives the distance (d) traveled by an object starting from rest and moving with uniform acceleration (a) during the time interval (t). Note that this expression does not contain the speed, only the distance and time, starting from zero, and the acceleration.

Galileo originally used a geometrical argument to derive this expression. Algebra was used more than 100 years later to derive the same expression.

Since it is more straightforward, we will use the algebraic derivation, along with some of Galileo's original assumptions.

We start with the definition of the average speed of a uniformly accelerating object during the time interval Δt. (This expression holds no matter how the object is moving.)

$$v_{av} = \frac{\Delta d}{\Delta t}.$$

We can rewrite this equation as

$$\Delta d = v_{av} \times \Delta t.$$

What would be the average velocity for a uniformly accelerating object? Galileo reasoned (as others had before him) that for any quantity that changes uniformly, the average value is just halfway between the beginning value and the final value. For uniformly accelerated motion starting from rest, the initial speed is zero, $v_{initial} = 0$. So, the average speed is halfway between 0 and v_{final}:

$$v_{av} = \tfrac{1}{2} v_{final}.$$

Substituting, we have

$$\Delta d = \tfrac{1}{2} v_{final} \times \Delta t.$$

Now we have to obtain a value for v_{final}. We can do this by starting with Galileo's *definition* of average acceleration

$$a_{av} = \frac{\Delta v}{\Delta t}.$$

In our case, a_{av} has a constant value, a, since the acceleration is uniform (constant). The value of Δv is $v_{final} - v_{initial}$, which is just v_{final}, since $v_{initial} = 0$. Substituting in the equation for a_{av}, we have

$$a = \frac{v_{final}}{\Delta t}.$$

Rearranging, we get

$$v_{final} = a \times \Delta t.$$

So now we can replace v_{final} in the expression for Δd, and we obtain

$$\Delta d = \tfrac{1}{2}v_{\text{final}} \times \Delta t,$$

$$\Delta d = \tfrac{1}{2}a(\Delta t)^2.$$

This is equivalent to Galileo's expression. If we measure the distance and the time interval from the position and the instant when the motion starts, then d_{initial} and t_{initial} are zero. We can then write this equation as

$$d_{\text{final}} = \tfrac{1}{2}a t_{\text{final}}{}^2.$$

Or, if we let $d_{\text{final}} = d$ and $t_{\text{final}} = t$, we have an even simpler expression

$$d = \tfrac{1}{2}at^2.$$

If we start with a nonzero initial speed, then we have

$$d = v_{\text{initial}}\, t + \tfrac{1}{2}at^2.$$

CHAPTER 3. UNDERSTANDING MOTION

Derivation of the Parabolic Trajectory of a Projectile

The motion of a projectile is composed of two independent motions: uniform velocity in the horizontal direction and uniform acceleration in the vertical direction. During the time interval t, the distance traveled by the projectile in the horizontal direction, x, with uniform speed v_x is

$$x = v_x t.$$

The distance the projectile moves in the vertical direction, y, during the same time interval t is

$$y = \tfrac{1}{2}gt^2.$$

Solving the equation $x = v_x\, t$ for t gives

$$t = \frac{x}{v_x}.$$

Because the time interval t is the same in both equations, we can substitute x/v_x for t in the equation for y. This gives

$$y = \tfrac{1}{2}g\left(\frac{x}{v_x}\right)^2,$$

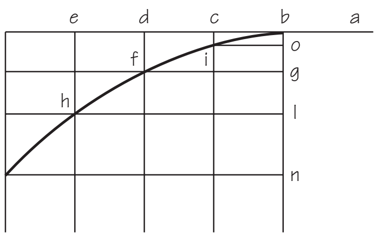
or

$$y = \left(\frac{g}{2{v_x}^2}\right)x^2.$$

This last equation contains two variables, x and y. It also contains three constant quantities: g, 2, and the horizontal speed v_x. The vertical distance y that the projectile falls is thus a constant times the square of the horizontal displacement x:

$$y = (\text{constant})x^2.$$

The mathematical curve represented by this relationship between x and y is called a parabola. Galileo deduced the parabolic shape of trajectories by an argument similar to the one used here. This discovery greatly simplified the study of projectile motion, because the geometry of the parabola had been established centuries earlier by Greek mathematicians.



Drawing of a parabolic trajectory from Galileo's Two New Sciences.

Derivation of the Equation for Centripetal Acceleration, $a_c = v^2/R$

Assume that a stone on the end of a string is moving uniformly in a circle of radius R. You can find the relationship between a_c, v, and R by treating a small part of the circular path as a combination of tangential motion and acceleration toward the center. To follow the circular path, the stone must accelerate toward the center through a distance h in the *same time* that it would move through a tangential distance d. The stone, with speed v, would travel a tangential distance d given by $d = v\Delta t$. In the same time Δt, the stone, with acceleration a_c, would travel toward the center through a distance h given by $h = ½a_c\ \Delta t^2$. (You can use this last equation because at $t = 0$, the stone's velocity toward the center is zero.)

You can apply the Pythagorean theorem to the triangle in the figure that follows:

$$R^2 - d^2 = (R+h)^2$$

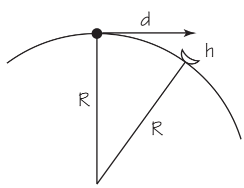
$$= R^2 + 2Rh + h^2.$$

When you subtract R^2 from each side of the equation, you are left with

$$d^2 = 2Rh + h^2.$$

You can simplify this expression by making an approximation. Since h is very small compared to R, h^2 will be very small compared to Rh. And since Δt must be vanishingly small to get the instantaneous acceleration, h^2 will become vanishingly small compared to Rh. So you can neglect h^2 and write

$$d^2 = 2Rh.$$



Also, $d = v\Delta t$ and $h = ½a_c\Delta t^2$; so you can substitute for d^2 and for h accordingly. Thus,

$$(v\ \Delta t)^2 = 2R \cdot ½a_c\ (\Delta t)^2,$$

$$v^2(\Delta t)^2 = Ra_c(\Delta t)^2,$$

$$v^2 = Ra_c,$$

or

$$a_c = \frac{v^2}{R}.$$

The approximation becomes better and better as Δt becomes smaller and smaller. In other words, v^2/R gives the magnitude of the *instantaneous* centripetal acceleration for a body moving on a circular arc of radius R. For uniform circular motion, v^2/R gives the magnitude of the centripetal acceleration at every point of the path. (Of course, it does not have to be a stone on a string. It can be a small particle on the rim of a rotating wheel, or a house on the rotating Earth, or a coin sitting on a rotating phonograph disk, or a car in a curve on the road, an electron in its path through a magnetic field, or the Moon going around the Earth in a nearly circular path.)

The relationship among a_c, v, and R was discovered by the Dutch scientists Christiaan Huygens and was published by him in 1673. Newton, however, must have known it in 1666, but he did not publish his proof until 1687, in the *Principia*.

We can substitute the relation $v = 2\pi Rf$ or $v = 2\pi R/T$ (see Section 4.11) into the equation for a_c:

$$\begin{aligned} a_c &= \frac{v^2}{R} \\ &= \frac{(2\pi Rf)^2}{R} \\ &= 4\pi^2 Rf^2 \end{aligned}$$

or

$$a_c = \frac{4\pi^2 R}{T^2}.$$

These two resulting expressions for a_c are entirely equivalent.

CHAPTER 4. NEWTON'S UNIFIED THEORY

"Weighing the Earth"

Now that we know how g arises in terms of Newton's law of universal gravitation, we can use the last equation above to find the mass of the Earth. This is possible because all of the terms in this equation are known, except for M_{Earth}. To find M_{Earth}, first solve for it in the equation, using simple algebra:

$$M_{\text{Earth}} = \frac{gR_{\text{Earth}}}{G}.$$

(Be sure that you understand each step in obtaining the answer below; look at the review of scientific notation in the *Student Guide*, if necessary.)

Now substitute the known values on the right side of the equation

$$M_{\text{Earth}} = \frac{(9.8 \text{ m/s}^2)(6.4 \times 10^6 \text{ m})^2}{(6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2)}.$$

To obtain a result from this expression, we perform all of the indicated arithmetic on the numbers and, separately, on the units. We'll first collect each of these together, which results in the following:

$$M_{\text{Earth}} = \frac{(9.8)(6.4 \times 10^6)^2}{6.67 \times 10^{-11}} \quad \frac{(\text{m/s}^2)(\text{m})^2}{\text{N m}^2/\text{kg}^2}.$$

For simplicity, let's first work on the numbers (but never forgetting the units, which we'll carry along). We start by squaring the term in the numerator

$$(6.4 \times 10^6)^2 = 40.96 \times 10^{12} \quad \frac{(\text{m/s}^2)(\text{m})^2}{\text{N m}^2/\text{kg}^2}.$$

So now we have

$$\frac{(9.8)(40.96 \times 10^{12})}{6.67 \times 10^{-11}} \quad \frac{(\text{m/s}^2)(\text{m})^2}{\text{N m}^2/\text{kg}^2}.$$

Multiply and divide the numbers, then subtract the exponent of the denominator from that of the numerator

$$M_{\text{Earth}} = \frac{(9.8)(40.96)}{6.67} \times 10^{12} \times 10^{11} \quad \frac{(\text{m/s}^2)(\text{m})^2}{\text{N m}^2/\text{kg}^2}$$

$$= 60.18 \times 10^{23} = 6.02 \times 10^{24} \quad \frac{(\text{m/s}^2)(\text{m})^2}{\text{N m}^2/\text{kg}^2}.$$

Now let's work on the units (carrying along the numerical value):

$$M_{\text{Earth}} = 6.02 \times 10^{24} \frac{(\text{m})(\text{m})^2\ (\text{kg})^2}{(\text{s}^2)(\text{Nm}^2)}$$

$$= 6.02 \times 10^{24} \ \frac{\text{m}^3\ \text{kg}^2}{\text{s}^2\text{Nm}^2}.$$

Cancel the m^2:

$$= 6.02 \times 10^{24} \ \frac{\text{m kg}^2}{\text{s}^2\text{N}}.$$

By definition 1 N = 1 kg m/s^2. Substituting for N we have

$$= 6.02 \times 10^{24} \ \frac{\text{m kg}^2\ \text{s}^2}{\text{s}^2\ \text{kg m}}.$$

Canceling as indicated, we are left simply with kg. So our final result is

$$M_{\text{Earth}} = 6.02 \times 10^{24} \text{ kg}.$$

This is a lot of mass, and the Earth is only one small blue planet just 4000 miles in radius! The value we have obtained agrees with the mass of the Earth obtained by other means, once again confirming Newton's theory.

Newton's Work: Impact and Reaction

Newton's work opened whole new lines of investigation, both theoretical and observational. In fact, much of our present science and also our technology had their effective beginnings with the work of Newton and those who followed in his spirit. New models, new mathematical tools, and a new confidence encouraged those followers to attack new problems, to open new vistas of research, and to answer long-standing questions. The modern view of science is that it is a continuing exploration of ever more interesting fields.

Newton's influence was not limited to science alone. The period following his death in 1727 was a period of further understanding and application of his discoveries and method. His influence was felt especially in philosophy and literature, but also in many other fields outside science. Let us round out our view of Newton by referring to some of these effects.

The eighteenth century is often called the Age of Reason, the apogee of the so-called Enlightenment. "Reason" was the motto of the eighteenth-century philosophers. Enlightened by reason, especially scientific reason, humanity would overcome the darkness of ignorance and usher in a new age of the flowering of human potential. Such ideals appeared, for instance, in the following excerpt from *Hymn to Science* by the poet Mark Akenside (1721–1770).*

Science! thou fair effusive ray
From the great source of mental day,
Free, generous, and refined!
Descend with all thy treasures fraught,
Illuminate each bewilder'd thought,
And bless my labouring mind. . . .

Oh! let thy powerful charms impart
The patient head, the candid heart,
Devoted to thy sway;
Which no weak passions e'er mislead,
Which still with dauntless steps proceed
Where reason points the way. . . .

Give me to learn each secret cause;
Let Number's, Figure's, Motion's laws
Reveal'd before me stand;
These to great Nature's scenes apply,
And round the globe, and through the sky,
Disclose her working hand.

Many thinkers of the Enlightenment believed they could extend the triumph of human reason in science to other areas of human endeavor. As a result, Newtonian physics, religious toleration, and republican government were all advanced by the same movement. However, their theories about improving religion and society were not convincingly connected. This does not mean there was really a logical link among these concepts. Nor were many eighteenth-century thinkers, in any field or nation, much bothered by other gaps in logic and feeling. For example, they believed that "all men

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* From *Poems of Science*, John Heath-Stubbs and Phillips Salman, eds. (New York: Penguin, 1984), pp. 150–152. We thank E.B. Sparberg for bringing this to our attention.

are created equal." Yet they did little to remove the chains of black slaves, the ghetto walls imprisoning Jews, or the laws that denied rights to women.

Still, compared with the previous century, the dominant theme of the eighteenth century was *moderation*, the "happy medium." The emphasis was on greater toleration of different opinions, restraint of excess, and balance of opposing forces. Even reason was not allowed to question religious faith too strongly. Atheism, which some philosophers thought would logically result from unlimited rationality, was still regarded with horror by most Europeans.

The Constitution of the United States of America is one of the most enduring achievements of this period. Its system of "checks and balances" was designed specifically to prevent any one group from getting too much power. It attempted to establish in politics a state of equilibrium of opposing trends. This equilibrium, some thought, resembled the balance between the Sun's gravitational pull and the tendency of a planet to fly off in a straight line. If the gravitational attraction upon the planet increased without a corresponding increase in planetary speed, the planet would fall into the Sun. If the planet's speed increased without a corresponding increase in gravitational attraction, it would escape from the solar system. When the opposing tendencies balanced, harmony resulted.

Political philosophers, some of whom used Newtonian physics as a model, hoped to create a similar balance in government. They tried to devise a system that would avoid the extremes of dictatorship and anarchy. According to James Wilson (1742–1798), who played a major role in writing the American Constitution:

> In government, the perfection of the whole depends on the balance of the parts, and the balance of the parts consists in the independent exercise of their separate powers, and, when their powers are separately exercised, then in their mutual influence and operation on one another. Each part acts and is acted upon, supports and is supported, regulates and is regulated by the rest.

Both Newton's life and his writings seemed to support the idea of political democracy. A former farm boy had attained the outermost reaches of the human imagination. What he had found there meant, first of all, that the same set of laws governed motion in the celestial and terrestrial spheres. This smashed the old beliefs about "natural place" and extended a new democracy throughout the Universe. Newton had shown that all matter, whether the Sun or an ordinary stone, was created equal; that is to say, all matter had the same standing before "the Laws of Nature and of Nature's God." (This phrase was used at the beginning of the Declaration of Independence to justify the desire of the people in the American colonies to throw off their oppressive political system and to become an independent people.) All political thought at this time was heavily influenced by Newtonian ideas. The *Principia* seemed to offer a parallel to theories about democracy. It seems logical that all people, like all natural objects, are created equal before nature's creator.

In literature, too, as already indicated, many welcomed the new scientific viewpoint. It supplied new ideas, convenient figures of speech, metaphors, parallels, and concepts which writers used in poems and essays. Many poems of the eighteenth century referred to Newton's discovery that white light is composed of colors (see Chapter 8). Samuel Johnson advocated that words drawn from the vocabulary of the natural sciences be used in literary works. He defined many such words in his *Dictionary* and illustrated their application in his *Rambler* essays.

However, not everyone welcomed the new rational, scientific viewpoint. That viewpoint was based on the idea that nature consists only of matter moving through empty space according to gravity and Newton's laws of motion. Many writers and artists of the Romantic movement were particularly disturbed by this so-called "mechanical world view" which, they argued, replaced the vibrancy and beauty of nature with an ugly, lifeless world of inert particles moving forever in empty space. Where in this system is there room for the beauty and warmth and feeling of a gorgeous rainbow, a melodious concerto, or the emotions of love and hate, ambition and pride, happiness and sorrow?

Romanticism started in Germany about 1780 among young writers inspired by the poet–philosopher Johann Wolfgang von Goethe. The most familiar examples of Romanticism in English literature are the poems and novels of Blake, Coleridge, Shelley, Byron, Scott, and Wordsworth. Most of the Romantics scorned the mathematical view of nature. They believed that any whole thing, whether a single human being or the entire Universe, is filled with a unique, nonmaterial spirit. This spirit cannot be explained by reason; it can only be *felt*. The Romantics insisted that phenomena cannot be meaningfully analyzed and reduced to their separate parts by mechanical explanations or pure reason alone. Contrast the following excerpt from William Wordsworth's (1770–1850) "The Tables Turned"* with Akenside's "Hymn to Science" quoted earlier:*

> Up! Up! my friend, and clear your looks,
> Why all this toil and trouble?

---

* ibid., p. 166.

Up! Up! my friend, and quit your books,
Or surely you'll grow double. . . .

Books! 'tis a dull and endless strife,
Come, hear the woodland linnet,
How sweet his music; on my life
There's more of wisdom in it.

And hark! How blithe the throstle sings!
And he is no mean preacher;
Come forth into the light of things,
Let Nature be your teacher.

The Romantic philosophers in Germany regarded Goethe as their greatest scientist as well as their greatest poet. They pointed in particular to his theory of color, which flatly contradicted Newton's theory of light. Goethe held that white light does not consist of a mixture of colors and that it is useless to "reduce" or "torture" a beam of white light by passing it through a prism to study its separate spectral colors. Rather, he charged, the colors of the spectrum are artificially produced in Newton's experiment using the prism, acting on and changing the light which is itself pure.

In the judgment of all modern scientists on this point, Newton was right and Goethe wrong. This does not mean that so-called *Nature Philosophy*, introduced by Friedrich Schelling in the early 1800s as the Romantic answer to Newtonian physics, was without any value. It encouraged speculation about ideas, even if they were so general that they could not be easily tested by experiment. At the time, it was condemned by most scientists for just this reason. Today, most historians of science agree that Nature Philosophy eventually played an important role in making possible certain scientific discoveries later on. Among these was the general principle of conservation of energy, which is described in the next two chapters. This principle asserted that all the "forces of nature," that is, the phenomena of heat, gravity, electricity, magnetism, and so forth, are forms of one underlying "force" (which we now call energy). This idea had agreed well with the viewpoint of Nature Philosophy. But it also could be put eventually in a scientifically acceptable form.

Movements hostile to conventional science have in fact occurred from time to time since Antiquity, in various forms, and are again visible today. Some modern artists, some intellectuals, and most members of the "alternative" or "new age" movements express deep-felt dislikes and mistrust of science. Their feelings are similar to, and historically related with, those of the Romantics. They are based in part on the mistaken notion that modern scientists dogmatically claim to be able to find (or have) a mechanical explanation for *everything*, whereas science is so powerful by being neither dogmatic, nor beholden only to "mechanics," nor ambitious to other fields in which it does not belong.

Even the Roman philosopher Lucretius (100–55 B.C.), who supported the atomic theory in his poem *On the Nature of Things*, wished to preserve some role for "free will" in the Universe, by suggesting that atoms might swerve randomly in their paths. This was not enough for Romantics, or even for some scientists. For example, Erasmus Darwin, a scientist and grandfather of evolutionist Charles Darwin, asked:

> Dull atheist, could a giddy dance
>   Of atoms lawless hurl'd
> Construct so wonderful, so wise,
>   So harmonised a world?

The Romantic Nature philosophers thought they could discredit the Newtonian scientists by forcing them to answer this question. To say "yes," they argued, would be absurd, and to say "no" would be disloyal to Newtonian beliefs. But the Newtonians succeeded quite well without committing themselves to any definite answer to Erasmus Darwin's question. They went on to discover immensely powerful and valuable laws of nature, which are discussed in the chapters ahead.

Questions

1. Describe some of the impacts of Newton's work outside the field of science.
2. What impact did Newtonian physics have on political thought?
3. Why did some people eventually reject the new physics?
4. Contrast the excerpts from the poems by Akenside and Wordsworth.
5. The poem by Erasmus Darwin asks a question. What is it in your own words? How did Nature Philosophers attempt to discredit Newtonian scientists?

CHAPTER 5. CONSERVING MATTER AND MOTION

An Example of Conservation of Momentum

(1) A space capsule at rest in space, far from the Sun or planets, has a mass of 1000 kg. A meteorite with a mass of 0.1 kg moves toward it with a speed

of 1000 m/s. How fast does the capsule (with the meteorite stuck in it) move after being hit?

m_A (mass of the meteorite) = 0.1 kg,

m_B (mass of the capsule) = 1000 kg,

v_A (initial speed of meteorite) = 1000 m/s,

v_B (initial speed of capsule) = 0,

v'_A (final speed of meteorite) = ?,

v'_B (final speed of capsule) = ?.

The law of conservation of momentum states

$$m_A\mathbf{v}_A + m_B\mathbf{v}_B = m_A\mathbf{v}'_A + m_B\mathbf{v}'_B.$$

Inserting the values given, we have

$$(0.1 \text{ kg})(1000 \text{ m/s}) + (1000 \text{ kg})\ (0)$$

$$= (0.1 \text{ kg})\ \mathbf{v}'_A + (1000 \text{ kg})\ \mathbf{v}'_B,$$

$$100 \text{ kg} \cdot \text{m/s} = (0.1 \text{ kg})\ \mathbf{v}'_A + (1000 \text{ kg})\ \mathbf{v}'_B.$$

Since the meteorite sticks to the capsule, $\mathbf{v}'_B = \mathbf{v}'_A$; so we can write

$$100 \text{ kg} \cdot \text{m/s} = (0.1 \text{ kg})\ v'_A + (1000 \text{ kg})\ v'_A,$$

$$100 \text{ kg} \cdot \text{m/s} = (1000 \cdot 1 \text{ kg})\ v'_A.$$

Therefore,

$$v_A{}' = \frac{100 \text{ kg} \cdot \text{m/s}}{1000.1 \text{ kg}}$$

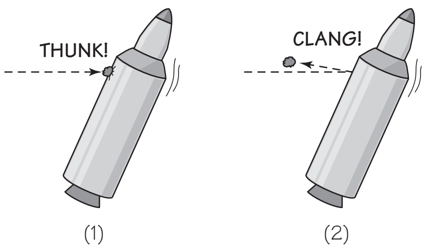
$$= 0.1 \text{ m/s}$$

(in the original direction of the motion of the meteorite). Thus, the capsule (with the stuck meteorite) moves on with a speed of 0.1 m/s.

Another approach to the solution is to handle the symbols first, and substitute the values as a final step. Substituting $\mathbf{v}'_A$ for $\mathbf{v}'_B$ and letting $\mathbf{v}'_B = 0$ would leave the equation $m_A\mathbf{v}_A = m_A\mathbf{v}'_A + m_B\mathbf{v}'_B = (m_A + m_B)\mathbf{v}$. Solving for v'_A we obtain

$$\mathbf{v}'_A = \frac{m_A\mathbf{v}_A}{(m_A + m_B)}.$$

This equation holds true for any projectile hitting (and staying with) a body initially at rest that moves on in a straight line after collision.



(2) An identical capsule at rest nearby is hit by a meteorite of the same mass as the other. However, this meteorite, hitting another part of the capsule, does not penetrate. Instead, it bounces straight back with almost no change of speed. How fast does the capsule move after being hit? Since all these motions are assumed to be along a straight line, we can drop the vector notation from the symbols and indicate the reversal in direction of the meteorite with a minus sign.

The same symbols are appropriate as in (1):

$m_A = 0.1$ kg, $\quad v_B = 0$,

$m_B = 1000$ kg, $\quad v'_A = 1000$ m/s,

$v_A = 1000$ m/s, $\quad v'_B = ?$.

The law of conservation of momentum states

$$m_A\mathbf{v}_A + m_B\mathbf{v}_B = m_A\mathbf{v}'_A + m_B\mathbf{v}'_B.$$

Here,

$$(0.1\ \text{kg})(1000\ \text{m/s}) + (1000\ \text{kg})(0),$$

$$= (0.1\ \text{kg})(-1000\ \text{m/s}) + (1000\ \text{kg})\ v'_B$$

$$100\ \text{kg} \cdot \text{m/s} = -100\ \text{kg} \cdot \text{m/s} + (1000\ \text{kg})\ v'_B,$$

$$v'_B = \frac{200\ \text{kg} \cdot \text{m/s}}{1000\ \text{kg}} = 0.2\ \text{m/s}.$$

Thus, the struck capsule moves on with about twice the speed of the capsule in (1). (A general symbolic approach to this solution can be taken, too. The result is valid only for the special case of a projectile rebounding perfectly elastically from a body of much greater mass.)

There is a general lesson here. It follows from the law of conservation of momentum that a struck object is given less momentum if it absorbs the projectile than if it reflects it. (A goalie who catches the soccer ball is pushed back less than one who lets the ball bounce off.) Some thought will help you to understand this idea: An interaction that merely stops the projectile is not as great as an interaction that first stops it and then propels it back again.

Doing Work on a Sled

Suppose a loaded sled of mass m is initially at rest on low-friction ice. You, wearing spiked shoes, exert a constant horizontal force F on the sled. The weight of the sled is balanced by the upward push exerted by the ice, so F is effectively the net force on the sled. You keep pushing, running faster and faster as the sled accelerates, until the sled has moved a total distance d.

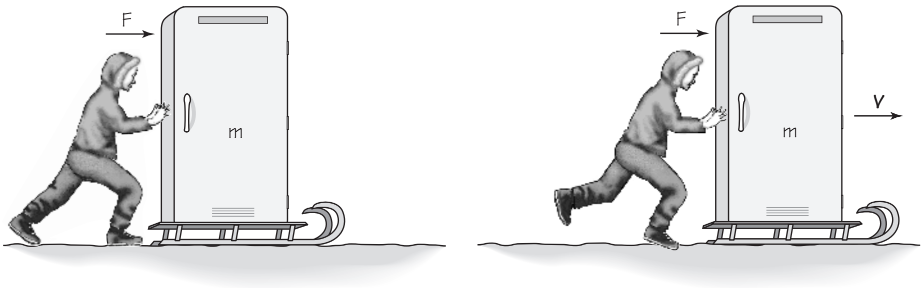
If the net force F is constant, the acceleration of the sled is constant. Two equations that apply to motion starting from rest with constant acceleration are

$$v = at$$

and

$$d = \tfrac{1}{2}at^2,$$

where a is the acceleration of the body, t is the time interval during which it accelerates (i.e., the time interval during which a net force acts on the body), v is the final speed of the body, and d is the distance it moves in the time interval t.



(a) (b)

According to the first equation, $t = v/a$. If we substitute this expression for t in the second equation, we obtain

$$d = \tfrac{1}{2}at^2 = \tfrac{1}{2}a\frac{v^2}{a^2} = \tfrac{1}{2}\,\frac{v^2}{a}.$$

The work done on the sled is $W = Fd$. From Newton's second law, $F = ma$, so

$$W = Fd$$

$$= ma \times \tfrac{1}{2}\frac{v^2}{a^2}.$$

The acceleration cancels out, giving

$$W = \tfrac{1}{2}mv^2.$$

Therefore, the work done in this case can be found from just the mass of the body and its final speed. With more advanced mathematics, it can be shown that the result is the same whether the force is constant or not.

More generally, we can show that the change in kinetic energy of a body already moving is equal to the work done on the body. By the definition of average speed

$$d = v_{av}t.$$

If we consider a uniformly accelerated body whose speed changes from v_0 to v, the average speed (v_{av}) during t is $\tfrac{1}{2}(v + v_0)$. Thus,

$$d = \frac{v + v_0}{2} \times t.$$

By the definition of acceleration, $a = \Delta v/t$; therefore, $t = \Delta v/a = (v - v_0)/a$. Substituting $(v - v_0)/a$ for t gives

$$d = \frac{v + v_0}{2} \times \frac{v - v_0}{a}$$

$$= \frac{(v + v_0)(v - v_0)}{2a}$$

$$= \frac{v^2 - {v_0}^2}{2a}.$$

The work W done is $W = Fd$, or, since $F = ma$:

$$W = ma \times d$$

$$= ma \times \frac{v^2 - v_0^2}{2a}$$

$$= \frac{m}{2}(v^2 - v_0^2)$$

$$= \tfrac{1}{2} mv^2 - \tfrac{1}{2} mv_0^2.$$

CHAPTER 6. THE DYNAMICS OF HEAT

You will often see energies expressed in terms of other units. A few of them are listed here.

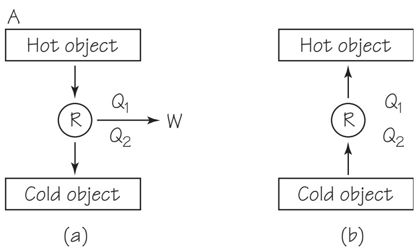
Unit name	*Symbol*	*Definition*	*Conversion*
kilowatt hour	kWhr	A watt (W) is 1 J/s, so 1 J = 1 W · s. A kWh is the amount of energy delivered in 1 hr if 1 kJ is delivered per second.	1 kWh = 3.60 MJ
Calorie (or kilocalorie)	Cal (or kcal)	The energy required to heat 1 kg of water by 1°C.	4.19 kJ
British thermal unit	Btu	The energy required to heat 0.454 kg by 0.556°C.	1.06 kJ

Carnot's Proof

Carnot's proof of maximum efficiency of ideal, reversible engines starts with the premise that when a cold object is in contact with a warmer one, the cold object does not spontaneously cool itself further and so give more heat to the warm object. However, an engine placed between the two bodies *can* move heat from a cold object to a hot one. Thus, a refrigerator can cool a cold bottle further, ejecting heat into the hot room. *You will see that this is not simple*. Carnot proposed that during any such experiment, the net result cannot be *only* the transfer of a given quantity of heat from a cold body to a hot one.

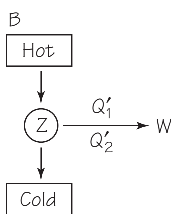
The engines considered in this case all work in cycles. At the end of each cycle, the engine itself is back to where it started. During each cycle, it has taken up and given off heat, and it has exerted forces and done work.

Consider an engine, labeled R in the figure, which suffers no internal friction, loses no heat because of poor insulation, and runs so perfectly that it can work backward in exactly the same way as forward (Figure A).

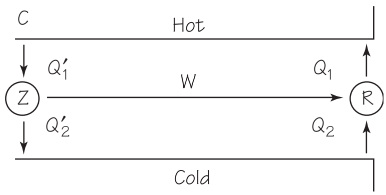


Now suppose someone claims to have invented an engine, labeled Z in the next figure, which is even more efficient than the ideal engine R. That is, in one cycle it makes available the same amount of work, W, as the R engine does, but takes less heat energy, Q', from the hot object to do it ($Q_1' < Q_1$). Since heat and energy are equivalent and since $Q_2 = Q_1 - W$ and $Q_2' = Q_1' - W$, it will also be true that $Q_2' < Q_2$ (Figure B).

$$Q_2' = Q_1' - W,$$



Suppose the two engines are connected so that the work from one can be used to drive the other. For example, the Z engine can be used to make the R engine work like a refrigerator (Figure C).



At the end of one cycle, both Z and R are back where they started. No work has been done; the Z engine has transferred some heat to the cold object; and the R engine has transferred some heat to the hot object. The *net* heat transferred is $Q_1 - Q'_1$, and the net heat taken from the cold object is $Q_2 - Q_2$. These are, in fact, the same

$$Q_2 - Q'_2 = (Q_1 - W) - (Q'_1 - W)$$

$$= Q_1 - Q'_1.$$

Because Z is supposed to be more efficient than R, this quantity should be positive; that is, heat has been transferred from the cold object to the hot object. Nothing else has happened. But, according to the fundamental premise, this is impossible, and does not happen.

The only conclusion is that the Z engine was improperly "advertised" and that it is either impossible to build or that in actual operation it will turn out to be *less* efficient than R.

As for two different reversible engines, they must have the same efficiency. Suppose the efficiencies were different; then one would have to be more efficient than the other. What happens when the more efficient engine is used to drive the other reversible engine as a refrigerator? The same argument just used shows that heat would be transferred from a cold body to a hot one. This is impossible. Therefore, the two reversible engines must have the same efficiency.

To actually compute that efficiency, you must know the properties of one reversible engine; all reversible engines working between the same temperatures must have that same efficiency. (Carnot computed the efficiency of an engine that used an ideal gas instead of steam.)

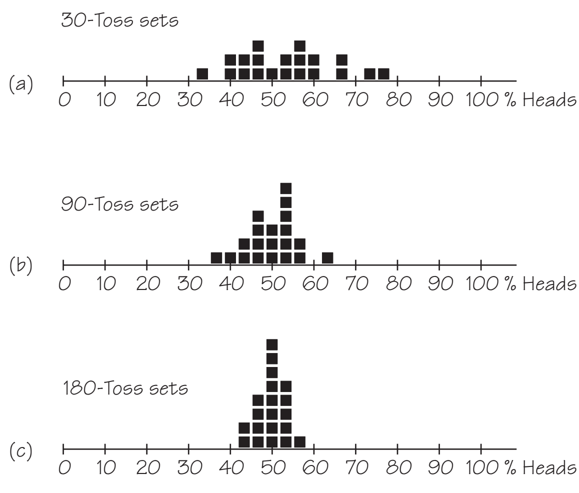
CHAPTER 7. HEAT—A MATTER OF MOTION

Averages and Fluctuations

Molecules are too small, too numerous, and too fast for us to measure the speed of any one molecule, its kinetic energy, or how far it moves before colliding with another molecule. For this reason, the kinetic theory of gases concerns itself with making predictions about *average* values. The theory enables us to predict quite precisely the *average* speed of the molecules in a sample of gas, the *average* kinetic energy, or the *average* distance the molecules move between collisions.

Any measurement made on a sample of gas reflects the combined effect of billions of molecules, averaged over some interval of time. Such average values measured at different times, or in different parts of the sample, will be slightly different. We assume that the molecules are moving randomly. Thus, we can use the mathematical rules of statistics to estimate just how different the averages are likely to be. We will call on two basic rules of statistics for random samples:

1. Large variations away from the average are less likely to occur than are small variations. (For example, if you toss 10 coins, you are less likely to get 9 heads and 1 tail than to get 6 heads and 4 tails.)
2. Percentage variations are likely to be smaller for large samples. (For example, you are likely to get nearer to 50% heads by flipping 1000 coins than by flipping just 10 coins.)



A simple statistical prediction is the statement that if a coin is tossed many times, it will land "heads" 50% of the time and "tails" 50% of the time. For small sets of tosses there will be many "fluctuations" (variations) to either side of the predicted average of 50% heads. Both statistical rules are evident in the charts. The top chart shows the percentage of heads in sets of 30 tosses each. Each of the 10 black squares represents a set of 30 tosses. Its position along the horizontal scale indicates the percentage of heads. As we would expect from Rule 1, there are more values near the theoretical 50% than far from it. The second chart is similar to the first, but here each square represents a set of 90 tosses. As before, there are more values near 50% than far from it. And, as we would expect from Rule 2, there are fewer values far from 50% than in the first chart.

The third chart is similar to the first two, but now each square represents a set of 180 tosses. Large fluctuations from 50% are less common still than for the smaller sets.

Statistical theory shows that the *average* fluctuation from 50% shrinks in proportion to the square root of the number of tosses. We can use this rule to compare the average fluctuation for sets of, say, 30,000,000 tosses with the average fluctuation for sets of 30 tosses. The 30,000,000-toss sets have 1,000,000 times as many tosses as the 30-toss sets. Thus, their average fluctuation in percent of "heads" should be 1,000 times smaller!

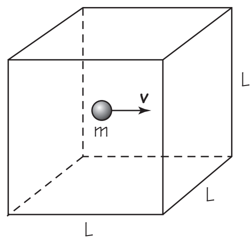
These same principles hold for fluctuations from average values of any randomly distributed quantities, such as molecular speed or distance between collisions. Since even a small bubble of air contains about a quintillion (10^{18}) molecules, fluctuations in the average value for any isolated sample of gas are not likely to be large enough to be measurable. A measurably large fluctuation is not *impossible*, but extremely unlikely.

Deriving an Expression for Pressure from the Kinetic Theory

We begin with the model of a gas described in Section 7.2: "a large number of very small particles in rapid, disordered motion." We can assume here that the particles are points with vanishingly small size, so that collisions between them can be ignored. If the particles did have finite size, the results of the calculation would be slightly different. But the approximation used here is accurate enough for most purposes.

The motions of particles moving in all directions with many different velocities are too complex as a starting point for a model. So we fix our attention first on one particle that is simply bouncing back and forth between two opposite walls of a box. Hardly any molecules in a real gas would actually move like this. But we will begin here in this simple way and later in this chapter extend the argument to include other motions. This later part of the argument will require that one of the walls be movable. Therefore, we will arrange for that wall to be movable, but to fit snugly into the box.

In Chapter 5, you saw how the laws of conservation of momentum and energy apply to cases like this. When a very light particle hits a more massive object, like the wall, very little kinetic energy is transferred. If the collision is elastic, the particle will reverse its direction with very little change in speed. In fact, if a force on the outside of the wall keeps it stationary against the impact from inside, the wall will not move during the collisions. Thus *no work* is done on it, and the particles rebound without any change in speed.



How large a force will these particles exert on the wall when they hit it? By Newton's third law the average force acting on the wall is equal and opposite to the average force with which the wall acts on the particles. The force on each particle is equal to the product of its mass times its acceleration ($\mathbf{F} = m\mathbf{a}$), by Newton's second law. The force can also be written as

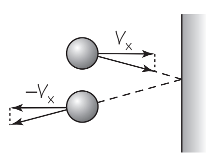
$$\mathbf{F} = \frac{\Delta(m\mathbf{v})}{\Delta t},$$

where $\Delta m\mathbf{v}$ is the change in momentum. Thus, to find the average force acting on the wall we need to find the change in momentum per second due to molecule–wall collisions.

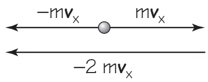
Imagine that a particle, moving with speed v_x (the component of $\mathbf{v}$ in the x direction) is about to collide with the wall at the right. The component of the particle's momentum in the x direction is mv_x. Since the particle collides elastically with the wall, it rebounds with the same speed. Therefore, the momentum in the x direction after the collision is $m(-v_x)$. The change in the momentum of the particle as a result of this collision is

final momentum − initial momentum = change in momentum,

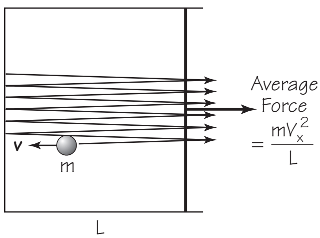
$$(-mv_x) - (mv_x) = (-2mv_x).$$



Note that all the vector quantities considered in this derivation have only two possible directions: to the right or to the left. We can therefore indicate direction by using a + or a − sign, respectively.



Now think of a single particle of mass m moving in a cubical container of volume L^3 as shown in the figure.



The time between collisions of one particle with the right-hand wall is the time required to cover a distance $2L$ at a speed of v_x; that is, $2L/v_x$. If $2L/v_x$ equals the time between collisions, then $v_x/2L$ equals the number of collisions per second. Thus, the change in momentum per second is given by

$$\begin{pmatrix}\text{change in momentum}\\ \text{in one collision}\end{pmatrix} \times \begin{pmatrix}\text{number of collisions}\\ \text{per second}\end{pmatrix} = \begin{pmatrix}\text{change in momentum}\\ \text{per second}\end{pmatrix},$$

$$(-2mv_x) \times \frac{v_x}{2L} = \frac{-mv_x^2}{L}.$$

The net force equals the rate of change of momentum. Thus, the average force acting on the molecule (due to the wall) is equal to $-mv_x^2/L$, and by Newton's third law, the average force acting on the wall (due to the molecule) is equal to $+mv_x^2/L$. So the average pressure on the wall due to the collisions made by one molecule moving with speed v_x is

$$P = \frac{F}{A} = \frac{F}{L^2} = \frac{mv_x^2}{L^3} = \frac{mv_x^2}{V},$$

where V (here L^3) is the volume of the cubical container.

Actually, there are not one but N molecules in the container. They do not all have the same speed, but we need only the average speed in order to find the pressure they exert. More precisely, we need the average of the square of their speeds in the x direction. We call this quantity $(v_x^2)_{av}$. The pressure on the wall due to N molecules will be N times the pressure due to one molecule, or

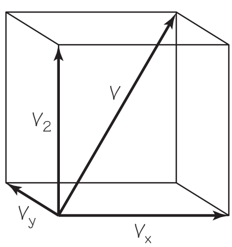
$$P = \frac{Nm(v_x^2)_{av}}{V}.$$

In a real gas, the molecules will be moving in all directions, not just in the x direction; that is, a molecule moving with speed v will have three components: v_x, v_y, and v_z. If the motion is random, then there is no preferred direction of motion for a large collection of molecules, and $(v_x^2)_{av} = (v_y^2)_{av} = (v_z^2)_{av}$. It can be shown from Pythagoras' theorem that $v^2 = v_x^2 + v_y^2 + v_z^2$. These last two expressions can be combined to give

$$(v^2)_{av} = 3(v_x^2)_{av}$$

or

$$(v_x^2) = 1/3(v^2)_{av}.$$



By substituting this expression for $(v_x^2)_{av}$ in the pressure formula, we get

$$P = \frac{Nm \times 1/3(v^2)_{av}}{V}$$
$$= 1/3\, \frac{Nm}{V}\, (v^2)_{av}.$$

Notice now that Nm is the total mass of the gas, and therefore Nm/V is just the density D. So

$$P = 1/3D(v^2)_{av}.$$

This is our theoretical expression for the pressure P exerted on a wall by a gas in terms of its density D and the molecular speed v.

CHAPTER 8. WAVE MOTION

Calculating the Wavelength from an Interference Pattern

$V = (S_1S_2)$ = separation between S_1 and S_2. (S_1 and S_2 may be actual sources that are in phase, or two slits through which a previously prepared wave front passes.)

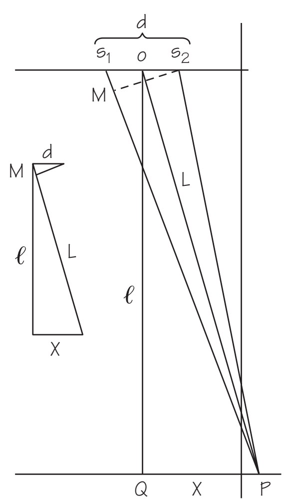
$l = OQ$ = distance from sources to a far-off line or screen placed parallel to the two sources,

x = distance from center axis to point P along the detection line,

$L = OP$ = distance to point P on detection line measured from sources.

Waves reaching P from S_1 have traveled farther than waves reaching P from S_2. If the extra distance is λ (or 2λ, 3λ, etc.), the waves will arrive at P in phase. Then P will be a point of strong wave disturbance. If the extra distance is $\frac{1}{2}\lambda$ (or $\frac{3}{2}\lambda$, $\frac{5}{2}\lambda$, etc.), the waves will arrive out of phase. Then P will be a point of weak or no wave disturbance.

With P as center, draw an arc of a circle of radius PS_2; it is indicated on the figure by the dotted line S_2M. Then line segment PS_2 equals line segment PM. Therefore, the extra distance that the wave from S travels to reach P is the length of the segment SM.



Now if d is very small compared to l, as you can easily arrange in practice, the circular arc S_2M will be a very small piece of a large-diameter circle, or nearly a straight line. Also, the angle S_1MS_2 is very nearly 90°. Thus, the triangle S_1S_2/M can be regarded as a right triangle. Furthermore, angle S_1S_2/M is equal to angle POQ. Then the right triangle S_1S_2M is similar to triangle POQ:

$$\frac{S_1M}{S_1S_2} = \frac{X}{OP} \qquad \text{or} \qquad \frac{S_1M}{d} = \frac{X}{L}.$$

If the distance l is large compared to x, the distances l and L are nearly equal. Therefore,

$$\frac{S_1M}{d} = \frac{x}{l}$$

But S_1/M is the extra distance traveled by the wave from source S_1. For P to be a point of maximum wave disturbance, S_1/M must be equal to $n\lambda$ (where $n = 0$ if P is at Q, and $n = 1$ if P is at the first maximum of wave disturbance found to one side of Q, etc.). So the equation becomes

$$\frac{n\lambda}{d} = \frac{x}{l}$$

and

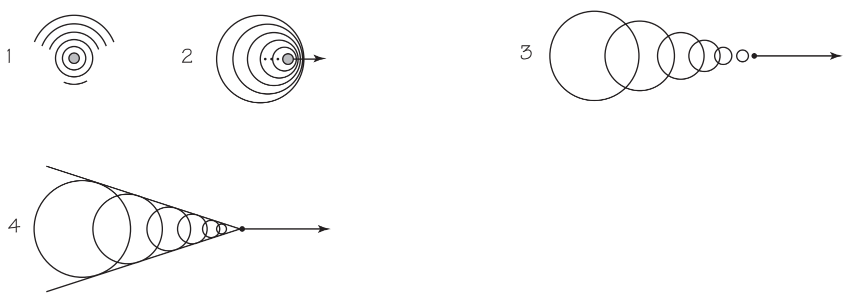
$$\lambda = \frac{dx}{nl}.$$

This important result says that if you measure the source separation d, the distance l, and the distance x from the central line to a wave disturbance maximum, you can calculate the wavelength λ.

The Sonic Boom

In the last half century a new kind of noise has appeared: the sonic boom. An explosion-like sonic boom is produced whenever an object travels through air at a speed greater than the speed of sound (supersonic speed). Sound travels in air at about 340 m/s. Many types of military airplanes can travel at two or three times this speed. Flying at such speeds, the planes unavoidably and continually produce sonic booms, which can cause physical damage, and anxiety in people and animals. SST (Supersonic Transport) planes such as the *Concorde* are now in civilian use in some countries. The unavoidable boom raises important questions. What are the consequences of this technological "progress"? Who gains, and what fraction of the population do they represent? Who and how many pay the price? *Must* we pay it; must SST's be used? How much say has the citizen in decisions that affect the environment so violently?

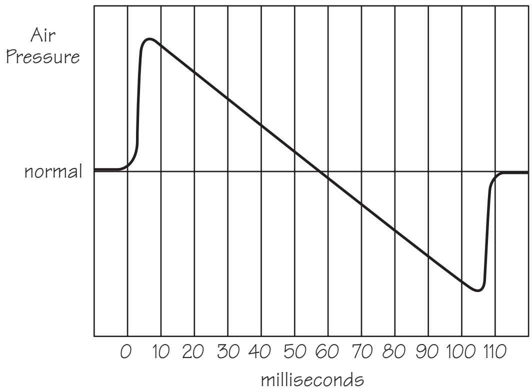
The formation of a sonic boom is similar to the formation of a wake by a boat. Consider a simple point source of waves. If the source remains in the same position in a medium, the wave it produces spreads out symmetrically around it, as in Diagram 1. If the source of the disturbance is *moving* through the medium, each new crest starts from a different point, as in Diagram 2.



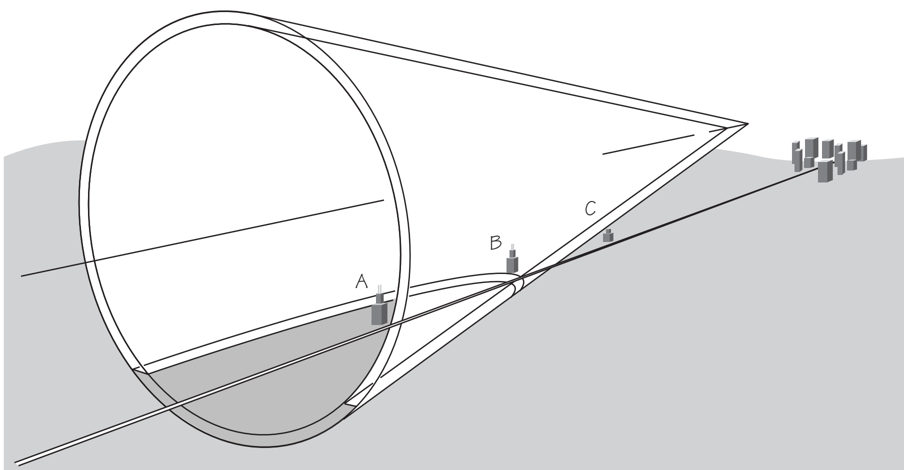
Notice that the wavelength has become shorter in front of the object and longer behind it. This is called the *Doppler effect*. The Doppler effect is the reason that the sound an object makes seems to have a higher pitch when it is moving toward you and a lower pitch when it is moving away from you. In Diagram 3, the source is moving through the medium *faster than the wave speed*. Thus, the crests and the corresponding troughs overlap and interfere with one another. The interference is mostly destructive everywhere except on the line tangent to the wave fronts, indicated in Diagram 4. The result is a wake that spreads like a wedge away from the moving source, as in the diagram.

All these concepts apply not only to water waves but also to sound waves, including those disturbances set up in air by a moving plane as it pushes the air out of the way. If the source of sound is moving faster than the speed of sound wave, then there is a cone-shaped wake (in three dimensions) that spreads away from the source.

Actually, two cones of sharp pressure change are formed. One cone originates at the front of the airplane and one at the rear, as indicated in the graph at the right.



Because the double shock wave follows along behind the airplane, the region on the ground where people and houses may be struck by the boom (the "sonic-boom carpet") is as long as the supersonic flight path itself. In such an area, typically thousands of kilometers long and 80 km wide, there may be millions of people. Tests made with airplanes flying at supersonic speed have shown that a single such cross-country flight by a 315-ton supersonic transport plane would break many thousands of dollars worth of windows, plaster walls, etc., and cause fright and annoyance to millions of people. Thus, the supersonic flight of such planes has been confined to over-ocean use. It may even turn out that the annoyance to people on shipboard, on islands, and on coastal areas near the flight paths is so great that over-ocean flights, too, will have to be restricted.



Model, Analogy, Hypothesis, Theory

Model, analogy, hypothesis, and *theory* have similar but distinct meanings when applied to physics. An *analogy* is a corresponding situation which, though perhaps totally unrelated to the situation at hand, helps you understand it. Many electronic circuits have analogs in mechanical systems. A *model* is a corresponding situation that may offer a picture of what "is really going on" and therefore can be taken more seriously as an explanation. An electron rotating around a nucleus is one model for an atom. A *hypothesis* is a statement that can usually be directly or indirectly tested. To Franklin, the statement "lightning is caused by electricity" was at first a hypothesis. A *theory* is a more general construction, perhaps putting together several models and hypotheses to explain a collection of effects that previously seemed unrelated. Newton's explanation of Kepler's laws, Galileo's experiments in mechanics and, finally, the Cavendish experiment were all part of the theory of universal gravitation. This is a good example of a theory.

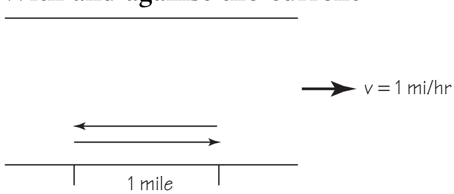
A well-tested theory, such as Newton's theory of gravitation or Einstein's theory of relativity, is a robust part of science, explaining a myriad of individual events or facts, and not to be confused with the vernacular use of "just a theory."

CHAPTER 9. EINSTEIN AND RELATIVITY THEORY

Differences in Speed for Light Waves Traveling Parallel and Perpendicular to the Ether Wind

Instead of light waves moving parallel and perpendicular to the ether wind, we examine an equivalent situation: a swimmer swimming at constant speed, first parallel and perpendicular to a current, in a river 1 mi wide. Assume the swimmer can swim at 2 mi/hr and the stream runs 1 mi/hr from left to right in the diagram below. We will calculate the time required for the swimmer to travel 1 mi each way with and against the current and 1 mi back and forth across the current.

With and against the current



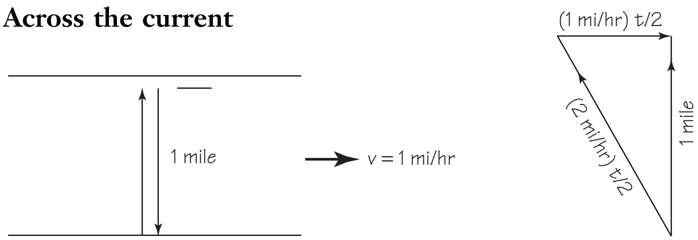
Traveling 1 mi with the current, the swimmer's speed is enhanced by the speed of the current, while traveling 1 mi back against the current, his speed is hindered by it. Thus the total time for the round trip is

$$t = \frac{1 \text{ mi}}{2 \text{ mi/hr} + 1 \text{ mi/hr}} + \frac{1 \text{ mi}}{2 \text{ mi/hr} - 1 \text{ mi/hr}}$$

$$= \frac{1 \text{ mi}}{3 \text{ mi/hr}} + \frac{1 \text{ mi}}{1 \text{ mi/hr}}$$

$$= 1.33 \text{ hr}$$

Across the current



In order to swim directly across the river from the starting point and back, the swimmer, in each direction, must head toward a point upstream from the destination point. The path taken *relative to the fixed shore* will be directly across and back, 1 mi in each direction. But the path taken in each direction by the swimmer *relative to the flowing water* will be along the hypothenuse of a right triangle formed by the 1-mi width of the river and the speed of the river current times one-half the total time for the round trip (1 mi/hr)t/2.

Using the Pythagorean theorem, the total distance traveled by the swimmer at the speed of 2 mi/hr is

$$(2 \text{ mi/hr})\ t = 2\sqrt{(1 \text{ mi})^2 + [(1 \text{ mi/hr})\ t/2]^2}.$$

In order to solve for t, cancel 2 on both sides and square both sides to get

$$\text{mi}^2/\text{hr}^2\ t^2 = \text{mi}^2 + \tfrac{1}{4}\ \text{mi}^2/\text{hr}^2\ t^2.$$

Cancel mi^2, multiply through by hr^2 and solve for t^2:

$$\tfrac{3}{4}t^2 = 1 \text{ hr}^2,$$

$$t = \sqrt{4/3} \text{ hr} = 1.15 \text{ hr}.$$

Note that the time to cross the river back and forth at constant swimming is less than the time it took in the earlier example to swim the same distance parallel to the current and back.

Michelson and Morley reasoned that exactly the same kind of result would occur for a light beam split in half—one-half sent "swimming" perpendicular to the supposed ether wind and back, the other half "swimming" parallel to the wind and back. Although the two halves of the beam started out together, the one sent parallel to the wind should return slightly behind the one sent parallel to the wind. The difference in time was expected to be small but detectable. Yet, when comparing the two light waves experimentally, they could find *no difference in the time of travel* of the two beams. We now know from Einstein's second postulate that the times had to be the same, and that the ether model, while usually appealing, is misleading.

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Some Further Chapter Materials

PART TWO

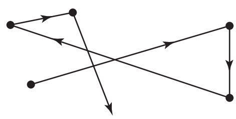
CHAPTER 10. ELECTRICITY AND MAGNETISM

Electrical Conduction in Metals

While charges cannot move freely through an insulator, they can move freely through a conductor. Yet when a conductor (say, a piece of copper wire or a steel knife blade) is connected between the two terminals of a battery, a steady current starts immediately and persists until the battery is discharged. This is puzzling. The battery sets up a potential difference between the two ends of the conductor and so there is an electric field along the conductor. This means that there is an electrical force on the charges. If this were the net force on the charges, they would be moving faster and faster. In that case, the current should increase with time, a situation not at all like what actually happens.

An acceptable model for a conductor must be a little more complex, then, than a substance "through which charge can move freely." One of the first useful models for a conductor (and one which is still used today) was constructed around 1900 by Drude and Lorentz. They pictured the atoms of a perfect crystal of metal locked into position in a regular array (called a lattice). Each atom has one or more electrons (depending on the metal) that are shared with all the other atoms in the metal. These mobile electrons are always in random motion at very high speeds (roughly 10^6 m/s for copper), very much like the molecules of a gas studied in Chapter 7. The electrons' motion is *much* faster, though, than that of the gas molecules at the same temperature (the reason for this was not discovered until about 1930 when quantum mechanics was applied to the problem).

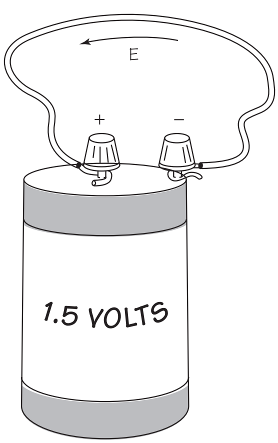
An electric current exists where there is *net* flow of charge along the wire. As long as the electrons are moving at random, the net flow is zero, on average. The electrons are constantly experiencing collisions with any metal atom which gets "out of line," for example, impurities in the metal or imperfections in the lattice, and with vibrations of the atoms caused by their own random thermal motion. On average, an electron travels freely for a time t between consecutive collisions (for copper, this time t is about 10^{-14} s).

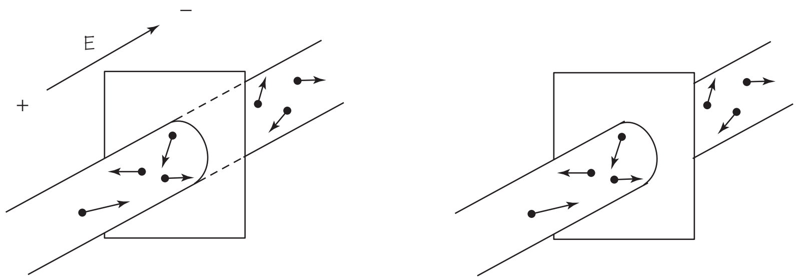


Path of an electron.

When a battery is connected to the metal, there is an electric field E created along the length of the conductor. This field does indeed accelerate the electrons, but since they move freely only for a time t, the change in their velocity caused by the field is just

$$\Delta v = at$$
$$= E\frac{q_e}{m}t.$$





No net flow of electrons past the surface. A net flow of electrons past the surface.

This *additional* velocity imparted to the electrons is called the "drift velocity" and is responsible for the conduction of electricity. Since E is proportional to the battery's voltage, it is easy to see that the current will be proportional to the voltage (Ohm's law) so long as the average time between collisions, t, does not change. For example, when a metal is cooled, the thermal motion of the atoms is reduced and collisions with these thermal vibrations become less frequent. Therefore, cooling a metal makes it a better conductor. Similarly, a very pure sample of copper is a better conductor than a sample with many impurities from which electrons are scattered as they move. A more quantitative model can also be described (though that is not necessary in understanding the basic model). Picture a piece of wire of length L, cross-sectional area A, with an average of n electrons in each cubic centimeter.

Ignore the *random* motion of the electrons, since this makes no contribution to the conduction, and picture all the electrons moving with the drift velocity

$$v_d = \Delta v = E\frac{q_e}{m}t.$$

The current is just the amount of charge crossing the surface each second:

$$I = \left(\begin{matrix}\text{number of electrons}\\ \text{crossing surface in 1 s}\end{matrix}\right) \times q_e.$$

The number of electrons crossing the surface each second is nAv_d (just as you calculated in Chapter 7 for gas molecules). Thus,

$$I = \left(\frac{nq_e^2 t_A}{m}\right)E.$$

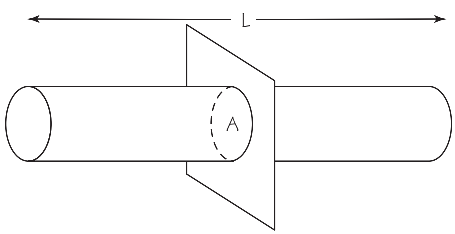
But $E = V/L$ if the wire is uniform so that the field is a constant along its length, and

$$I = \left(\frac{nq_e^2 t_A}{m}\,\frac{A}{L}\right)V.$$

That is, $I \propto V$. But this is Ohm's law! Thus, this model determines the resistance of a wire as

$$R = \left(\frac{mL}{nq_e^2 t_A}\right),$$

where $R = V/I$. It follows that, for a given material, doubling the length should double the resistance; doubling the cross-sectional area should halve the resistance. This is just what is found experimentally.



CHAPTER 14. A QUANTUM MODEL OF THE ATOM

Bohr's Quantization Rule and the Size of Orbits

The magnitude of the charge on the electron is q_e; the charge on a nucleus is Zq_e, and for hydrogen ($Z = 1$) it is just q_e. The electric force with which the hydrogen nucleus attracts its electron is therefore

$$F_{\rm el} = k\,\frac{q_e q_e}{r^2},$$

where k is the coulomb constant, and r is the center-to-center distance. If the electron is in a stable circular orbit of radius r around the nucleus, moving at a constant speed v, then the centripetal force is equal to mv^2/r. Since the centripetal force is provided by the electric attraction,

$$\frac{mv^2}{r} = k\,\frac{q_e^2}{r^2}.$$

In the last equation, m, q_e, and k are constants; r and v are variables, whose values are related by the equation. What are the possible values of v and r for stationary states of the atom?

You can begin to get an answer if you write the last equation in slightly different form. Multiplying both sides by r^2 and dividing both sides by v, you get

$$mvr = \frac{kq_e^2}{v}.$$

The quantity on the left side of this equation is the product of the momentum of the electron and the radius of the orbit. You can use this quantity to characterize the stable orbits. According to classical mechanics, the radius of the orbit could have any value, so the quantity mvr could also have any value. Of course, classical physics also seemed to deny that there could be *any* stable orbits in the hydrogen atom. But Bohr's first postulate implies that certain stable orbits (and only those) are permitted. So Bohr needed to find the rule that decides *which* stable orbits are possible. Here Bohr appears to have been largely guided by his intuition. He found that what was needed was the recognition that the quantity mvr does not take on just any value, but only certain *allowed values*. These values are defined by the relation

$$mvr = n\,\frac{h}{2\pi},$$

where h is Planck's constant, and n is a positive integer; that is, $n = 1, 2, 3, 4, \ldots$ (but not zero). When the possible values of mvr are restricted in this way, the quantity mvr is said to be *quantized*. The integer n that appears in the formula is called the *quantum number*. The main point is that each quantum number ($n = 1, 2, 3 \ldots$) corresponds to one allowed, stable orbit of the electron.

If you accept this rule, you can at once describe the "allowed" states of the atom, for example, in terms of the radii r of the possible orbits. You can combine the last expression above with the classical centripetal force relation as follows. The quantization rule is

$$mvr = n\,\frac{h}{2\pi}$$

so

$$r = \frac{nh}{2\pi mv}$$

and

$$r^2 = \frac{n^2h^2}{4\pi^2m^2v^2}.$$

From classical mechanics

$$\frac{mv^2}{r} = k\,\frac{q_e^2}{r}$$

so

$$v^2 = \frac{kq_e^2}{mr}.$$

Substituting this "classical" value for v^2 into the quantization expression for r^2 gives

$$r^2 = \frac{n^2h^2}{4\pi^2m^2\left(\dfrac{kq_e^2}{mr}\right)}$$

Simplifying, you get the expression for the allowed radii, r_n:

$$r_n = \frac{n^2h^2}{4\pi^2kmq_e^2} = \left(\frac{h^2}{4\pi^2kmq_e^2}\right)n^2.$$

CHAPTER 15. QUANTUM MECHANICS

The de Broglie Wavelength: Examples

A body of mass 1 kg moves with a speed of 1 m/s. What is the de Broglie wavelength?

$$\lambda = \frac{h}{mv},$$

$$h = 6.6 \times 10^{-34}\ \mathrm{J \cdot s},$$

$$mv = 1\ \mathrm{kg \cdot m/s},$$

$$\lambda = \frac{6.6 \times 10^{-34}\ \mathrm{J \cdot s}}{1\ \mathrm{kg \cdot m/s}},$$

so

$$\lambda = 6.6 \times 10^{-34}\ \mathrm{m}.$$

The de Broglie wavelength is many orders of magnitude smaller than an atom. Thus, it is much too small to be detected. There are, for example, no slits or obstacles small enough to show diffraction effects. You would expect to detect no wave aspects in the motion of this body.

An electron mass 9.1×10^{-31} kg moves with a speed of 2×10^6 m/s. What is its de Broglie wavelength?

$$\lambda = \frac{h}{mv},$$

$$h = 6.6 \times 10^{-34}\ \mathrm{J \cdot s},$$

$$mv = 1.82 \times 10^{-24}\ \mathrm{kg \cdot m/s},$$

$$\lambda = \frac{6.6 \times 10^{-34}\ \mathrm{J \cdot s}}{1.82 \times 10^{-24}\ \mathrm{kg \cdot m/s}},$$

so

$$\lambda = 3.6 \times 10^{-10}\ \mathrm{m}.$$

The de Broglie wavelength is of atomic dimensions. For example, it is of the same order of magnitude as the distances between atoms in a crystal. So you can expect to see wave aspects in the interaction of electrons with crystals.

The Uncertainty Principle: Examples

Applied to a large mass

Consider a car, with a mass of 1000 kg, moving with a speed of about 1 m/s. Suppose that in this experiment the inherent uncertainty Δv in the measured speed is 0.1 m/s (10% of the speed). What is the minimum uncertainty in the position of the car?

$$\Delta x \, \Delta p \geq \frac{h}{2\pi},$$

$$\Delta p = m \, \Delta v = 100 \text{ kg} \cdot \text{m/s},$$

$$h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s},$$

$$\Delta x \geq \frac{6.63}{6.28} \times \frac{10^{-34} \text{ J} \cdot \text{s}}{10^{2} \text{ kg} \cdot \text{m/s}},$$

$$\Delta x \geq 1 \times 10^{-36} \text{ m}.$$

This uncertainty in position, which is many orders smaller than the size of an atom, is much too small to be observable. In this case, you can determine the position of the body with as high an accuracy as you would ever need.

Applied to a small mass

Consider an electron, with a mass of 9.1×10^{-31} kg, moving with a speed of about 2×10^{6} m/s. Suppose that the uncertainty Δv in the speed is 0.2×10^{6} m/s (10% of the speed). What is the minimum uncertainty in the position of the electron?

$$\Delta x \, \Delta p \geq \frac{h}{2\pi}$$

$$\Delta p = m\Delta v = 1.82 \times 10^{-25} \text{ kg} \cdot \text{m/s},$$

$h = 6.63 \times 10^{-34}\ \mathrm{J \cdot s}$,

$$\Delta x \geq \frac{6.63}{6.28} \times \frac{10^{-34}\ \mathrm{J \cdot s}}{1.82 \times 10^{-25}\ \mathrm{kg \cdot m/s}},$$

$$\Delta x \geq 5 \times 10^{-10}\ \mathrm{m}.$$

The uncertainty in position is of the order of atomic dimensions and is significant in atomic problems. It is impossible to specify exactly where an electron is in an atom.

The reason for the difference between these two results is that Planck's constant h is very small, so small that the uncertainty principle becomes important only on the atomic scale. For ordinary-sized objects, the equations give the same result as if h had the value zero.

CHAPTER 17. PROBING THE NUCLEUS

The Mathematics of Decay

The *activity* of a sample, the number of disintegrations per second, the decay rate are alternative expressions for the same quantity. Using the letter N to represent generally the number of atoms of a given kind present in a radioactive sample, the activity is $\Delta N/\Delta t$, where ΔN is the number of atoms disintegrating in the same interval Δt. But $\Delta N/\Delta t$ depends both on the type of atom involved, and how many happen to be in the sample. Therefore, a more useful quantity is needed. If, in a time interval Δt, ΔN atoms disintegrate out of a total number N, the *fraction* of atoms disintegrating is $\Delta N/\Delta N$. The *fraction* of atoms *disintegrating per unit time is* $\Delta N/N/\Delta t$. (This quantity can be thought of as the ratio of the activity $\Delta N/\Delta t$ to the total number, N.) This quantity, usually called λ or the decay constant, will be important, as you will see at once below. It is analogous to the death rate in a human population. In the United States, for example, about 5,000 persons die each day out of a population of about 200,000,000. The death rate is therefore one person per 40,000 per day (or one person per day per 40,000).

The beautifully simple mathematical aspect of radioactive decay is that the fraction of atoms decaying per second does not change with time. If initially there are N_0 atoms, and a certain fraction λ decay in 1 s, the actual number of atoms decaying in 1 s is λN_0. Then, at any later time t, when there are only N_t, atoms remaining, the *fraction* that decay in 1 s will still be λ, but the *number* of atoms decaying in 1 s is now λN_t, a smaller number than before.

The constant fraction λ of atoms decaying per unit time is called the *decay constant*. The value of this constant λ can be found for each radioactive species. For example, λ for radium is 1.36×10^{-11} per second, which means that on average 0.0000000000136th of the total number of atoms in any sample of radium will decay in 1 s.

The fact that λ is a constant can be represented by the expression

$$\lambda = \frac{\Delta N/\Delta t}{N} = \text{constant}$$

which can be rewritten as

$$\frac{\Delta N}{\Delta t} = \text{constant} \times N \qquad \text{or} \qquad \frac{\Delta N}{\Delta t} \propto N.$$

This form of the relation expresses clearly the fact that the decay rate depends directly on the number of atoms left.

By using calculus, a relation of this type can be turned into an expression for N as a function of elapsed time t:

$$\frac{N_t}{N_0} = e^{-\lambda t} \qquad \text{or} \qquad N_t = N_0 e^{-\lambda t},$$

where N_0 is the number of atoms at $t = 0$, N_t is the number remaining unchanged at time t, and e is a mathematical constant that is approximately equal to 2.718. The factor $e^{-\lambda t}$ has the value 1 when $t = 0$, and decreases toward 0 as t increases. Since the decay constant appears as an exponent, the decay is called "exponential" and takes the form shown by the graph in Section 17.9.

The relationship between the half-life $T_{1/2}$ and the decay constant λ can be derived as follows. Write the exponential decay equation in logarithmic form by taking the logarithm of both sides of the equation

$$\log \frac{N_t}{N_0} = \log e^{-\lambda t} = -\lambda t \log e.$$

After a time equal to the half-life $T_{1/2}$, the ratio $N_t/N_0 = ½$. So you can substitute ½ for N_t/N_0 if you substitute $T_{1/2}$ for t in the above equation, and get

$$\log(½) = -\lambda T_{1/2} \log e.$$

The value of log (½) is −0.301 and the value of log $e = 0.4343$; therefore,

$$-0.301 = -\lambda T_{1/2}(0.4343),$$

and

$$\lambda T_{1/2} = 0.693.$$

So the product of the decay constant and the half-life is always equal to 0.693. Knowing either one allows you to compute the other.

For example, radium-226 has a decay constant $\lambda = 1.36 \times 10^{-11}$ per second; so

$$(1.36 \times 10^{-11}\ \mathrm{s}^{-1})T_{1/2} = 0.693,$$

$$T_{1/2} = \frac{0.693}{1.36 \times 10^{-11}\ \mathrm{s}^{-1}},$$

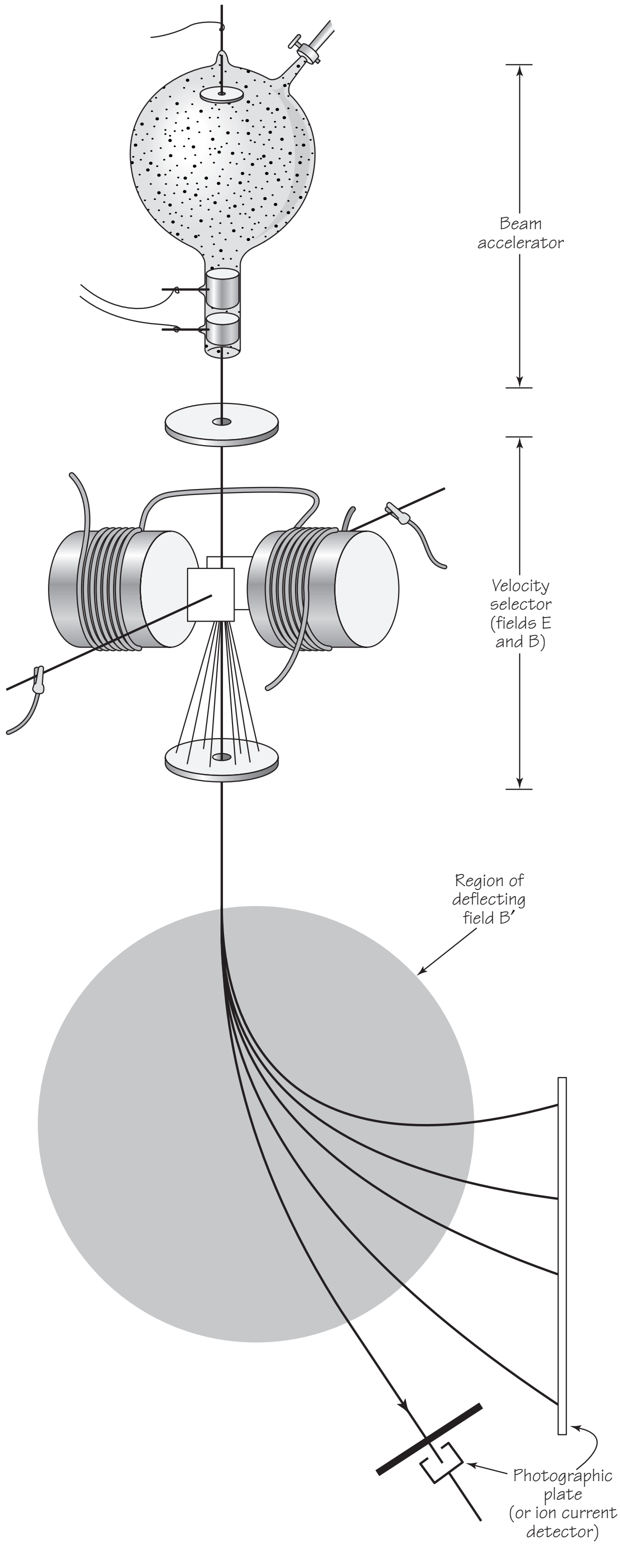
$$T_{1/2} = 5.10 \times 10^{10}\ \mathrm{s}.$$

Thus, the half-life of radium-226 is 5.10×10^{10} s (about 1620 yr).

The Mass Spectrograph

The magnetic separation of isotopes begins by electrically charging the atoms of a sample of material, for example, by means of an electric discharge through a sample of gas. The resulting ions are then further accelerated by means of the electric potential difference between the lower pair of electrodes, and a beam emerges.

Before the different isotopes in the beam are separated, there is usually a preliminary stage that allows only those ions with a certain velocity to pass through. In one type, the ion beam initially enters a region of crossed magnetic fields B and E, produced by current in coils and charged plates as shown. There, each ion experiences a magnetic force of magnitude qvB and an electric force of magnitude qE. The magnetic and electric forces act on an ion in opposite directions, and only for ions of a certain speed will


Beam
accelerator
Velocity
selector
(fields E
and B)
Region of
deflecting
field B′
Photographic
plate
(or ion current
detector)

the forces be balanced, allowing them to pass straight through the crossed fields and the hole in the diaphragm below them. For each of these ions, $qvB = qE$; so their speed $v = E/B$. Because only ions with this speed in the original direction remain in the beam, this portion of the first part of the apparatus is called a *velocity selector*.

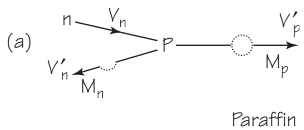
The separation of isotopes in the beam is now accomplished in another magnetic field of strength B'. As the beam enters this field, the magnetic field causes a centripetal force to act on each ion, deflecting it into a circular arc whose radius R depends upon the ion's charge-to-mass ratio. That is, $qvB' = mv^2/r$, and so $q/m = vB'R$.

The divided beams of ions fall on either a photographic plate (in a mass spectro*graph*) or a sensitive ion current detector (in a mass spectro*meter*), allowing the radii R of their deflections to be calculated from the geometry of the apparatus. Since v, B', and R can be determined from measurements, the charge-to-mass ratio of each beam of ions can be calculated directly.

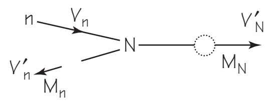
Because this method uses electric and magnetic fields, it is called the *electromagnetic method of separation of isotopes*.

CHAPTER 18. THE NUCLEUS AND ITS APPLICATIONS

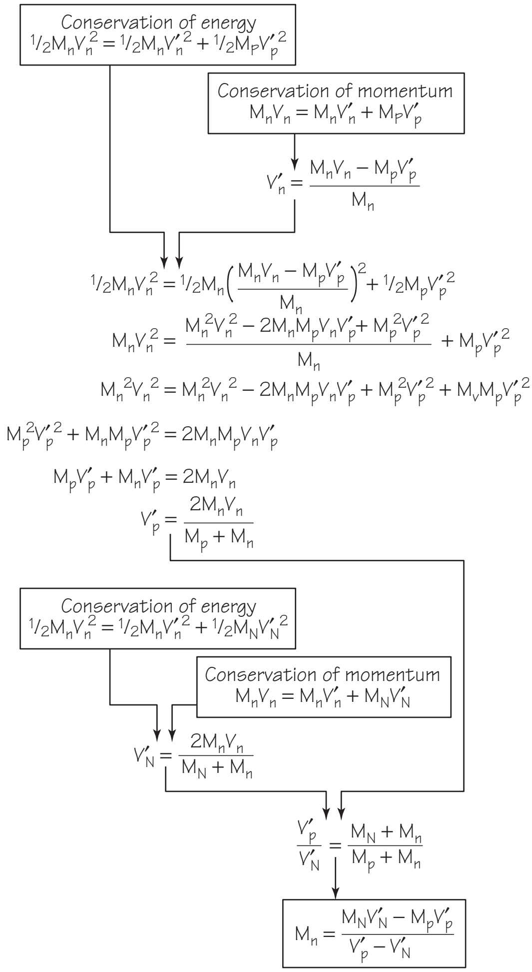
Determining the Neutron's Mass



(a) The sketch in (a) represents an elastic collision of a neutron (n) and a proton (p). If it were a head-on collision, the neutron would rebound straight back and the proton would be seen to emerge along the *same* line. To determine the mass of the neutron, m_n, you may use the principles of conservation of kinetic energy and conservation of momentum, which provide two algebraic equations that must both hold. The case is particularly simple if you consider a perfectly elastic head-on collision. As shown in (c), an expression for the proton's recoil speed v'_p can be derived by combining the equations algebraically (solving the momentum equation for v_n, substituting the resulting expression for v'_n in the energy equation, expanding, collecting terms, and solving for v'_p). However, this expression includes the term v_n, the neutron's initial speed, which cannot be measured directly. You can elim-



(b)



(c)

inate v_n from the equation by analyzing another collision and combining the results with what you already have.

(b) The sketch in (b) represents a perfectly elastic collision between a neutron (n) and a nitrogen nucleus (N). When the collision is head-on, you can write energy and momentum equations similar to what you wrote before, but this time *leading to an expression for the recoil speed of the nitrogen nucleus*, v'_N. This expression also includes the unmeasurable quantity v_n.

(c) The v_p equation and v'_N equation are then combined algebraically (eliminating v_n), and solved for m_n. The expression for m_n now contains only terms that can be measured, so the mass of the neutron, m_n, can be calculated. Note that only the ideas developed for ordinary elastic collisions are used here.

LABORATORY EXPLORATIONS

Physics is an experimental science. With few exceptions, the great advances in physics have arisen in close association with experimental evidence. Direct, hands-on experience with the phenomena is essential to understanding concepts in physics.

GOALS OF THE LABORATORY EXPLORATIONS

Most of you will be pursuing careers in fields other than physics, perhaps in other sciences: medicine, the liberal arts, business, or teaching. Whatever your personal goal, these explorations will provide a useful introduction to the fundamental principles of physics and to the principles underlying experimentation of any kind. Here are some of the goals of the laboratory work:

- *Conceptual Learning.* The explorations are meant to provide hands-on experience and to help reinforce some of the fundamental concepts you are learning in the other parts of the course.
- *Collaborative Learning.* Collaboration in small groups is a very beneficial way of learning. Working in groups should also help you develop collaborative skills that are vital to success in many lifelong endeavors.
- *Experimental and Analytical Skills.* During the course of this semester you will be making observations, recording measurements, analyzing experimental results, and drawing conclusions at various levels of sophistication, ranging from purely qualitative to highly quantitative.

Suggested Mini-Laboratory Explorations

1. OUR PLACE IN SPACE (SECTIONS P.2, 14.4)

Most drawings of the solar system are badly out of scale, because it is impossible to show both the sizes of the Sun and planets and their relative distances on an ordinary-sized piece of paper. Constructing a simple scale model of the distances and sizes of objects in the solar system will help you develop a better picture of the real dimensions of the solar system—in a sense, your greater home.

A. Scale of Distances

1. To begin, find a straight stretch of sidewalk, street, empty ball court, stadium, etc., that is at least 102 yd, or 60 m, in length. This can also be done on a football field or another open area having an equivalent length.
2. The table on the next page lists the radii of the orbits of all the planets, in miles and kilometers. Convert these distances to “scale” inches or centimeters, where 1 in = 1,000,000 mi, or 1 cm = 1,000,000 km.
3. Calculate the distance between planets, scaled to inches or centimeters. Using the new scale, measure and label strings to represent the distances between each pair of planets.
4. Beginning with the Sun, lay out the scale distances in the entire solar system on the sidewalk, street, field, etc., you have chosen, and mark the location of each planet. Include the Earth’s moon, which is about 384,000 km, or 240,000 mi, from the Earth.
5. Survey your result and record your observations.
6. The nearest star to our Sun is Alpha Centauri. How far away from the Sun would this star be on your scale? (See Section 2 of the Prologue.)
7. Using the scale on a local map, or driving in a car or bus, find a landmark or building that is approximately at the position of Alpha Centauri on your scale.
8. What fraction of a light year is represented by the distance between the Earth and the Sun?
9. The distance to the farthest part of the visible universe, as observed by the Hubble Telescope in space, is about thirteen billion light years. How many inches or centimeters would this be on your scale? How many miles or kilometers would this be?

B. Scale of Sizes

Let a tennis ball about 7 cm in diameter represent the Sun. Since the diameter of the Sun is about 1,400,000 km, in this model 1 cm will represent about 200,000 km. The Earth has an approximate diameter of 13,000 km, so on this scale model it would have a diameter of only 0.065 cm. This is about the size of a pinhead.

The table below lists the approximate diameters of the planets and our Moon. Fill in the table, giving the approximate size on this scale. Try to find a sample object of this size, and use it in your scale model of distances.

C. The Size of An Atom (Section 14.4)

1. Using the same method as above, create a scale model of the distances involved in a hydrogen atom, the smallest atom. Use a handbook or ref-

A Scale Model of the Solar System

Object	Solar Average Distance km (×10⁶)	Solar Average Distance mi (×10⁶) (cm/in)	(approx. km)	Diameter Model (cm)	Sample Object
Sun	—	—	1,400,000	7	Tennis ball
Mercury	58	36	4,600		
Venus	107	67	12,000		
Earth	150	93	13,000	0.065	Pinhead
Mars	228	141	6,600		
Jupiter	780	484	140,000		
Saturn	1408	879	120,000		
Uranus	2870	1780	48,000		
Neptune	4470	2790	45,000		
Pluto	5886	3674	1,300		

erences in the text to find the size of the nucleus (a single proton) and of an electron, and the radius of the first Bohr orbit of the electron.

2. Assign a reasonable scale to these measurements, then lay out the distance scales on the long sidewalk or street or field you have chosen.
3. Record your impressions of the result.

2. REVIEWING GRAPHS (CHAPTER 1 AND MAJOR LABORATORIES)

You may want to read first the section on graphs in the essay "Reviewing Units, Mathematics, and Scientific Notation."

The following table records the growth of a tomato plant from a seedling of zero height over a period of 7 weeks.

Week	*Approx. height (cm)*
1	7
2	14
3	22
4	29
5	35
6	42
7	50

1. Examine the data and draw some conclusions about the trend over the period of observation.

2. By inspection of these data, what would you expect the graph to look like?
3. Now make such a graph, or "picture," of the data by placing the week on the horizontal axis and the height on the vertical axis. The horizontal axis should be divided evenly into weeks, starting from 0. The vertical axis should also be divided evenly. Start the vertical axis at 0 cm. Make sure that the numbers (data) fill as much of each axis as possible without going beyond the end. Label the axes and their units.
4. Graph each pair of points and connect the points.
5. Describe in your own words what the plant did during this period. Was its growth exactly the same each week? What was the overall trend?
6. Do your observations of the graph in Question 5 agree with your expectations in Question 2?

You know from the study of graphs that any time you obtain a straight line, the two variables are considered to be "proportional," or in symbols: $y \propto x$. We can replace the proportional sign, $\propto$, by an equals sign, $=$, if we multiply the x variable by a constant. Call the constant m. So, instead of

$$y \propto x,$$

we have

$$y = mx.$$

If the line intersects the y-axis at the value $y = b$, then we have the equation

$$y = mx + b.$$

You may recognize this as the general formula for a straight line. How do we obtain the value of m? As you may recall, it is just the numerical value of the "slope" of the line.

7. The data points on your graph probably do not form an exact straight line, since the slope tends to vary slightly from week to week. However, we can find the "average slope" by choosing the slope between the first and last data points. Find this average slope.
8. Using your result for the average slope, and assuming an approximate straight line for the graph, write an equation for the approximate straight line. Extrapolate your data back to height at week 0. Including the y intercept, b, in your equation.

9. When a graph involves time on the x-axis, the slope has a special meaning. It tells us the *rate* that the y variable is changing, for example, in units of centimeters per second for speed, or centimeters per week week in our case. What was the overall rate of growth of the plant during this period?
10. You now have an exact equation for the height of the plant for the period of its recorded growth. Using your equation, what would be the height of the plant 4 weeks from the last data point, assuming this trend continued? One year? (This absurd result shows the weakness of "linear extrapolation" in many situations.)

Now you try it

1. Obtain your own data on a variable that changes over a period of time. Examples might include the daily temperature, the growth of a baby, the maximum height of a local tide, the ups and downs of the stock market, etc. Examine the numbers and attempt by inspection to predict what a graph of these data will look like. Then graph the result and compare with your prediction.
2. You may know that an object moving from rest with constant acceleration (a) covers the distance d in the time interval t given by Galileo's famous equation

$$d = \tfrac{1}{2}at^2.$$

Here is a table of distances covered during different time intervals for an object moving with constant acceleration from rest. Find the acceleration by the graph method.

Distance covered (cm)	*Time interval (s)*
3.60	0.1
14.5	0.2
32.0	0.3
57.5	0.4
90.2	0.5
129.5	0.6
176.4	0.7
230.0	0.8
291.5	0.9

Using a spread sheet

1. If your class has access to computers and to a so-called spreadsheet program, enter the table of data for the tomato plant in the spreadsheet.

2. Use the graphing function of the spreadsheet to create a graph of data similar to your earlier graph. Make sure the program labels the axes.
3. In what ways, if any, does the spreadsheet graph differ from your own graph? Examine, for instance, the spacing of data on the x-axis.
4. Define a cell on the spreadsheet in such a way that it gives the slope of the line between any two of your data points, and provide a label in a neighboring cell.
5. Try this again with your own data, obtained above.

3. FALLING OBJECTS (SECTION 1.9)

The study of falling objects is an important part of Chapter 2. It is the gateway (and was historically) to understanding the new mechanics.

1. Try dropping different types of objects at the same time from the same height and compare when they hit the ground. Is there any difference? If there is, what do you think are the reasons for the difference?
2. Predict what will happen if you drop a book and a piece of unfolded paper simultaneously to the floor from the same height.
3. Try this. Is the result what you predicted? Explain what happened.
4. Now crumple up the paper tightly into a ball and try the experiment again. Explain what you observe.
5. It has been reported that, in order to slow down the fierce speeds of serve during world championship tennis, the size of the official tennis ball is to be increased by a small amount. Explain how this would accomplish the purpose.
6. Using Galileo's formula $d = \frac{1}{2}at^2$, explain why two objects dropped from the same height should hit the ground at the same time. What assumption is necessary?

4. KEPLER'S THIRD LAW (SECTION 2.10)

Review Kepler's third law of planetary motion in the text.

1. A table of the periods and radii of the orbits of the planets, and of the distances of the Sun and fixed stars, is given in Section 2.6, as first obtained by Copernicus. Examine the data in this table, from the Sun to

the planets and fixed stars. What is the harmony that Copernicus saw in these numbers?

2. What is an "astronomical unit" (AU)? How large is it?
3. If you know the radii of the orbits and the periods of all the planets, how could you test the accuracy of Kepler's third law? Using the method you devised, test the law using the data in the table in Section 2.6.

   Notice, however, that the periods of planets are given in days for some planets and years for others, while all of the radii are in AU. The periods must all be in the same unit for this comparison. Chose a convenient unit and then convert the periods to that unit before testing the data.
4. What do you conclude about the validity of Kepler's third law? Give the reasons for your conclusion.

5. RELATIVE MOTION (CHAPTER 2, AND SECTIONS 3.9, 9.3)

In this investigation two different observers will observe the same event, but report seeing two different phenomena. The difference between these two observers is that the first observer is at rest relative to the event, while to the second observer the event is in motion relative to that second observer.

The event will be a ball dropping to the ground. The first observer will be the person who drops the ball as he or she walks forward at constant speed and direction. The second observer will be a person standing still in the room.

1. One person walks forward on a straight line at constant speed while holding the ball over his head and to one side. While steadily walking forward, he lets the ball drop, and he carefully watches its motion.
2. At the same time, a student standing in the room near the walking student is also carefully observing the motion of the ball. Each observer should then draw the path of the ball as he or she observed it, from the position of the hand on release to the place where the ball landed.
3. Record your observations, repeating the experiment several times if necessary.
4. Compare the observations made by the observers, one moving, the other stationary relative to the horizontal motion of the ball. What do you conclude about the effect that relative motion has on the observations of two different observers?

5. If, instead of walking, the first observer was inside a ship moving smoothly forward relative to the shore, would his observation on the trajectory of the ball give any clue that he is actually moving with respect to the shore?

Now you try it
Repeat the above observations and analyses, only this time the walking student will toss the ball straight up and catch it as it returns to his hand.

Thought Experiments

1. One argument against the moving Earth was that a ball dropped from a high tower would land behind the tower, since the tower is moving forward during the time the ball is dropping. Since, in fact, the ball always lands at the base of the tower, people concluded that the Earth cannot be moving. Does the first part of this experiment support or refute that argument?
2. Two observers are observing the setting of the Sun as seen by a person on Earth. One observer is on Earth, but the other is on the Moon. How does each one account for the observation of the first observer, that the Sun is "setting"?
3. If you are the observer on Earth, is there any way that you could determine whether it is the Sun or the Earth that is moving as the Sun "sets"?
4. The opening sentence of this mini-laboratory states: "In this investigation two different observers will observe one and the same event, but report seeing two different phenomena." How can this be? How would you explain it to someone?

6. GALILEO AND INERTIA (SECTIONS 3.1, 3.8, 3.9, 5.9, 5.10)

A. The Pendulum

The text describes an experiment with a pendulum in which the string hits a peg at the center of the line.

Observe two or three swings of a pendulum without the peg, and compare the height to which the bob rises on each side. Allowing for friction and air resistance, are they nearly equal?

Now let the string hit a peg or other obstruction, and compare the height to which the pendulum bob rises on each side.

Again taking friction and air resistance into account, what do you conclude from this experiment?

B. Two Inclined Planes

Galileo reasoned that he should obtain the same result as above if, instead of a pendulum, he used a ball rolling on two inclined planes facing each other.

1. Test this result by letting a ball roll down one incline and up the other. The inclines must be arranged so that there is a smooth transition at the bottom from one to the other. Carefully observe the starting and stopping points.
2. Taking all factors into account, do your observations confirm Galileo's prediction?
3. Galileo then predicted that this result should be the same, even if the angle of the second incline with respect to the horizontal is much less than the first one. Test this prediction and write your conclusion.
4. Finally, Galileo predicted that the same result should hold, even with a zero incline of the second inclined plane (a flat table). In a laboratory where the curvature of the Earth can be neglected, he predicted that the ball will keep on rolling in a straight line at uniform speed until it is stopped by a wall or falls off the table). Try it. Does Galileo's statement seem reasonable?

C. Kinetic and Potential Energy (Sections 5.9, 5.10)

1. Examine the results of this experiment by using the concepts of kinetic energy and potential energy.
2. Explain why, neglecting friction and air resistance, the pendulum and the ball always rise to the same height on both sides, even with a peg in the way or with a different incline.

7. FINDING THE CENTRIPETAL ACCELERATION VECTOR (SECTIONS 3.3, 3.12)

Take a ball and string and whirl the ball in a vertical circle in a counterclockwise direction. Make sure that the direction of the ball, moving at any point on the circle, is not pointing at anyone else or any fragile objects in the room.

When you are certain it is safe to do so, release the ball just when it is at the top of its circular motion. Carefully note the direction in which the ball moves. Do this several times, carefully observing each case.

1. How does the ball move immediately after you release it?
2. Draw a circle and an arrow representing the velocity vector, which will be tangent to the circle at the point where you released it. The length of the arrow represents the speed. Let this be 5 cm. Its direction will represent the direction in which the ball moved. Label this vector $\mathbf{v}_1$.
3. Where would the ball be if you had released it a fraction of a second later? Draw an arrow to represent the velocity vector at that point. Since the speed is constant, the length of the arrow should be the same; but the direction should be different. Label this vector $\mathbf{v}_2$.

Acceleration is a vector, and it is defined as the change in the velocity vector per unit of time.

$$\mathbf{a} = \frac{\Delta \mathbf{v}}{t} = \frac{\mathbf{v}_2 - \mathbf{v}_1}{t}.$$

In the case of centripetal acceleration, the velocity is changing in direction but not in speed. Assume the time interval is 1 s, and solve for $\mathbf{v}_2$:

$$\mathbf{v}_2 = \mathbf{v}_1 + \mathbf{a}t.$$

This equation states that the second velocity vector is the vector sum of the first velocity vector and the acceleration vector multiplied by the time elapsed. In other words, the acceleration (times time) transforms the initial velocity $\mathbf{v}_1$ into the final velocity $\mathbf{v}_2$.

Let's find out what vector quantity we need to add to.

As discussed in Section 3.3, the representations of vectors by arrows can be moved around on a piece of paper, as long as the same length is maintained and as long as they remain parallel to the original vector. The vector arrows are added by placing them together head to tail to form a chain. The sum or resultant is then represented by the vector arrow that goes from the starting tail to the ending head.

In this case we know the starting vector ($\mathbf{v}_1$) and the resultant vector ($\mathbf{v}_2$). But we don't know the second vector ($\mathbf{a}t$) that adds to $\mathbf{v}_1$ to obtain $\mathbf{v}_2$. We'll find out what it is by drawing.

1. Move the arrow for vector $\mathbf{v}_1$ to another place on the paper, keeping the same length and direction. The arrow $\mathbf{v}_2$ will represent the resultant. It will connect the tail of $\mathbf{v}_1$ to the head of $\mathbf{a}t$.

2. Draw arrow $\mathbf{v}_2$, placing its tail at the tail of $\mathbf{v}_1$, keeping the same direction and length as the original.
3. Arrow $\mathbf{a}t$ will make up the difference. Draw $\mathbf{a}t$ from the head of $\mathbf{v}_1$ to the head of $\mathbf{v}_2$. Label it $\mathbf{a}t$. This arrow represents the direction and magnitude of the acceleration vector (times the elapsed time).
4. Place a dot on the circle between the positions of $\mathbf{v}_1$ and $\mathbf{v}_2$. Move the arrow representing $\mathbf{a}t$ to the circle at that point, placing its tail on the dot. Be sure to keep the same direction and length as in your drawing.
5. Draw a tangent to the circle at the position of arrow $\mathbf{a}t$. What angle does it form with $\mathbf{a}t$?
6. You will recall from geometry that a radius is always perpendicular to the tangent of a circle at that point. What do you conclude about the direction of the acceleration vector $\mathbf{a}$ for uniform circular motion?

On what line does it always lie?
Why is it called "centripetal acceleration"?

8. THREE STATES OF MATTER (CHAPTER 7, SECTION 16.2, MAJOR LABORATORY "HEAT TRANSFER AND LATENT HEAT OF FUSION")

1. In what ways do solids, liquids, and gases differ from one another?
   Adding heat to a substance usually causes the temperature of a substance to rise. It might also cause the state or "phase" of the substance to change. In the following you will constantly add heat to ice until it melts in water, then continue adding heat until the water boils and finally evaporates completely.
2. Predict what a graph of the temperature, plotted over the entire time of the experiment, will look like.
3. Now place some crushed ice in a pyrex glass container, put a thermometer in the ice, and gently apply heat until all of the ice melts, then boils, then evaporates. Carefully record the temperature and the time every 10 s until all of the water has boiled away. Note the time when each of the phase transitions occurs. (Take care with the source of heat and the boiling water.)
4. Construct a graph of the temperature versus the time and indicate what is happening during each block of time.
5. Compare the results with your predictions in Question 2.
6. Attempt to account for what you observe by using the kinetic–molecular theory of matter.

9. HOW DO WE KNOW THAT ATOMS REALLY EXIST? THE BROWNIANSCOPE* (SECTION 7.8, CHAPTER 13)

The Brownianscope is a 200-power microscope that is able to focus on a chamber at one end containing microscopic smoke particles. The smoke particles are about 50,000 to 100,000 times larger than the air molecules. If the air molecules really do exist, then, in analogy with Einstein's results for small objects such as pollen grains suspended in a liquid, the smoke particles suspended in air should exhibit random motions caused by the random bombarding they receive from fast-moving air molecules. This scope is designed to test this predicted observation.

1. To create the smoke for the chamber, burn two matches about half way down, then blow out the flame. Holding the chamber over the smoke, capture the smoke, then (keeping it vertical) place the chamber over the end of the microscope.
2. Point the objective lens of the microscope at the bare light bulb for a light source. Wrap your fingers around the other end to block out the light from the source. If you wear glasses, try to observe this without them.
3. Try to focus the microscope on the smoke particles in the chamber. Note that, because of the arrangement of the optics, they will appear light against a dark background.
4. What do you observe?
5. How would you account for your observations?

10. LIGHT AND COLOR (CHAPTER 8, PART 2; SECTION 14.1)

The amount of refraction that a light wave experiences when it moves from air into glass, then back into air, depends upon the frequency of the light wave. We can use this dependence to separate visible white light into its constituent frequencies, which we observe as colors. This is the principle behind the operation of a *glass prism*. In a diffraction grating, the light waves diffract into different angles depending upon the frequency of the light. The result is the separation of the light into its constituent frequencies,

* Available inexpensively from Frey Scientific, Beckley Cardy Group, Mansfield, OH, 1-888-222-1332. A bare light bulb is also needed.

which we again observe as colors. This is the principle behind the *diffraction grating*. Both of these devices enable us to observe the *visible spectrum*.

1. Use the prism and the diffraction grating to observe the visible spectrum. For the prism you will need direct sunlight for the best result. For the diffraction grating, use any artificial light source. DO NOT LOOK AT THE SUN THROUGH THE DIFFRACTION GRATING!
2. What do you think would happen if you filtered the incoming light through a color filter?
3. Use one of the color filters and record your result.

Adding and subtracting colors

1. Your instructor may have different color filters which can be placed over the light of an overhead projector. What happens as the filters are added?
2. Shine white light through a color filter onto objects of the same and different colors in a darkened space. Record your observations. How would you explain what you observe?

11. SPECTROSCOPY (CHAPTER 14)

Equipment: A diffraction-grating spectroscope (Cenco Scientific), a light bulb source, a discharge lamp, and single-element source, flourescent lights in room (optional).

A spectroscope is a device for viewing the spectrum and measuring the frequencies or wavelengths of the light observed. Our spectroscope uses a diffraction grating. The numbers on the scale read from 4 to 7, indicating wavelengths from 400 nm to 700 nm (nm is the abbreviation for nanometer, which is 10^{-9} m.)

1. Use the spectroscope to observe the visible spectrum emitted by the incandescent light bulb. Record the wavelength at the center of each of the colors you observe. (Do not use a flourescent light or the Sun.)
2. Observe the spectrum emitted by a flourescent light (if one is available). How does it differ from the spectrum emitted by the light bulb?
3. Your instructor will set out a discharge lamp emitting rays from a single element. Observe the spectrum of the element and record the wavelengths of the observed lines. What you see is the visible portion of the bright-line or emission spectrum of that element.
4. Again observe the spectrum from the flourescent light. How would you account for what you see?

5. How does Bohr's quantum model of the atom account for the emission spectrum you observed? Why are there lines only at certain frequencies?

12. RADIOACTIVITY AND NUCLEAR HALF-LIFE (CHAPTER 17)

This investigation uses plastic simulated atoms in a kit provided by Frey Scientific, S16402. An age determination using the decay of carbon-14 is simulated through instructions provided with the kit.

Suggested Major Laboratory Explorations

1. INVESTIGATING MEASUREMENTS AND UNCERTAINTY

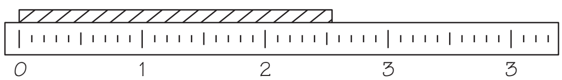
INTRODUCTION

Precision and uncertainty

Physics can claim to be one of the most precise sciences. For example, by laser pulse reflection the distance to the Moon is known to about 1 cm, and some physical constants are known to one part per billion. But there is a paradox.

On the one hand, obviously most experimental work in physics involves measurement. Reliable measurements are essential for the accumulation of accurate data that can lead to major laws and theories, and for the testing of new theories. Many of the measurements in physics involve such basic properties as distance, time, mass, voltage, and temperature. From these, more complicated properties, such as speed, force, and energy, can be constructed.

On the other hand, measuring instruments are never able to help us obtain absolutely exact measurements of any quantity, no matter how carefully made or how sophisticated the instruments may be. Every measuring instrument has a limit to its *precision*. For instance, look carefully at the divisions or marks on a meter stick. The numbered divisions are centimeters (1 cm = 0.01 m). The smallest divisions are millimeters (1 mm = 0.1 cm). Can you read your meter stick more accurately than to the nearest millimeter? If you are like most people, you read it to the nearest mark of 0.1 cm (the nearest millimeter) and *estimate* the next digit between the marks for the nearest tenth of a millimeter (0.01 cm), as illustrated in the diagram below.



In the same way, whenever you read the divisions of any measuring device, you should read accurately to the nearest division or mark and then estimate the next digit in the measurement. Then probably your measurement, including your estimate of a digit between divisions, is not more than half a division in error. It is not likely, for example, that in the above diagram you would read more than half a millimeter away from where the edge being measured comes between the divisions. In this case, in which the divisions on the ruler are millimeters, you are at most no more than 0.5 mm (0.05 cm) in error. So, in recording this measurement, you would record the best estimate of the distance and indicate the likely error as plus or minus 0.05 cm. This is written

$$2.58 \pm 0.05 \text{ cm.}$$

The ± 0.05 is called the *uncertainty* of your measurement. The uncertainty for a single measurement is commonly taken to be half a scale division. With many measurements, this uncertainty may be even less.

Error and uncertainty for repeated measurements of a single quantity using a single instrument

Many experiments involve a series of repeated measurements of a quantity, such as the distance traveled by an object in uniform motion in fixed time intervals. However, because of the precision of the instruments, or simplifications such as the neglect of air resistance, or simple carelessness, the recorded measurements are often not identical.

For instance, suppose you measure the length of a book page four times and obtain the following values: 27.61 cm, 27.59 cm, 27.70 cm, and 27.64 cm.

Is there any way to decide from the data which is the "true" value for the length of the page? Unfortunately, the answer is no. But we can pick the average value as most likely the closest to the true value, on the assumption that half the time the measurement will be too high, and half the time too low. (This assumption becomes more likely, the more measurements we include.) In our example, the average is 27.635 cm. However, since our data are given only to the second decimal place, we are allowed only four significant figures. We must round off 27.635 cm to 27.64 cm. The average value is selected as the accepted value of our measurement.

How can we indicate the possible error in our accepted value as represented by the variation in our individual measurements? The difference between each measurement and the accepted value is −0.03 cm, −0.05 cm, 0.06 cm, 0.00 cm. The average of these differences, *without regard to sign* is 0.035 cm. This average deviation is taken as the experimental error for these measurements of a single quantity. The result of this measurement would then be given as the average value, plus or minus the average deviation, rounded off to the hundredth decimal place

$$27.64 \pm 0.04 \text{ cm}.$$

Relative error: Comparing an experimental result with an "accepted value"

Finally, there is an experimental error that is associated with the relative deviation of a measured quantity from the standard value for that quantity. For instance, the generally accepted value for the acceleration of gravity (as a result of many measurements) is 9.8 m/s^2; but in an experiment you might obtain a value for the acceleration of gravity of 9.7 m/s^2 (both at sea level). The difference between your result and the accepted result is 0.1 m/s^2. Is your result off by a lot or by a little? It depends upon how much difference there is in comparison with the number. The ratio of the difference to the size of the accepted value, expressed as a percent, is known as the *relative error*. It may be defined in symbols as follows:

$$\text{relative error} = \frac{|\text{experimental value} - \text{accepted value}|}{\text{accepted value}} \times 100\%.$$

In our example

$$\text{relative error} = \frac{|9.7\ \text{m/s}^2 - 9.8\ \text{m/s}^2|}{9.8\ \text{m/s}^2} \times 100\%$$

$$= \frac{|-0.1\ \text{m/s}^2|}{9.8\ \text{m/s}^2} \times 100\%$$

$$= \frac{0.1}{9.8} = \times 100\% = 0.01 \times 100\% = 1\%.$$

In the following you will explore these concepts with some concrete examples.

Exploration

1. Your instructor will have set up various stations around the room. At each one, you are to make a measurement. Everyone will use the same instrument located at each station. The stations might include measurements of a voltage across a resistor in a circuit with a constant current, the temperature of ice water, and the length of a strip of paper.
2. You may work together on this, but each student should make each measurement and record the result in a table in his or her notebook. The table should include the object measured, the precision of the instrument you used, and the result of your measurement with the uncertainty and units indicated. Make your measurements as carefully as possible.

Together in class

1. After you have completed your measurements, your instructor will collect the results and write them in a large table on the board. He/she will select one of the measurements that has an accepted or a predicted value, such as the voltage, which may be compared with the result obtained from Ohm's law.
2. Working with your instructor, you will obtain the best value for the voltage and the average of the class's measurements.
3. The class will also obtain the experimental error in the result from the average deviations of the measurements from the average value for the voltage.
4. Together with your instructor plot the class's results for the voltage, indicating the value and the frequency that each value appeared. This

will result in a "curve." Indicate on the graph the position of the average value and the experimental error on each side of the average.

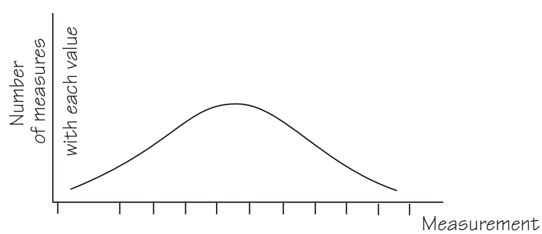
5. Finally, since in this case there is a theoretically predicted value for the voltage, given the current and resistance in the circuit, obtain the relative error of the class's result for the voltage.

Now you try it

1. Find the class's experimental result for the other two quantities measured and the experimental error in both cases.
2. Plot the results for both measurements, indicating the value and the frequency that each value appeared. Indicate the average values and the errors on each "curve."
3. Examine the class's results for each of the objects measured. What do you observe? Did everyone obtain exactly the same results? If not, why do you think this is so?
4. Is there an accepted value for either of these measurements? If so, find the relative error.

Think about it

1. When we obtain and plot a lot of data as in the above, an "error curve" or "bell-shaped curve" is formed. This is an important concept in the understanding of all types of measurement.



2. Sometimes the scores of all members of a class of substantial size on a test are also plotted on a curve of this type, and grades are computed on the basis of where the scores fall on the curve in relation to other students in the class. This is sometimes called "grading on the curve." What are some of the advantages and disadvantages of this type of grading system?
3. Your friend who has not had this course measures the width of a standard 8½ × 11 in sheet of paper one time with a metric ruler and obtains 21.5 cm. How would you explain to your friend that the "true" width of the sheet of paper might not be 21.5 cm?
4. How would your friend obtain the most likely value for the width, along with the error?

2. EXPLORING MOTION (CHAPTER 1)

A. Observing Motion

How can we recognize and distinguish acceleration, deceleration, and uniform motion? In order to answer this question, at first in *qualitative* terms, assemble the apparatus and place the cart on the track with the timing tape attached, but with the timer off.

1. Give the cart a push and describe what happens to the speed of the cart.
2. Now try it again with the timer on. The timer tells us where the tape (hence the cart) was at each tenth of a second. Describe what happens to the distance the cart travels in each ⅒ s at the start, middle, and end of the run.
3. Compare your answers to Questions 1 and 2. How would you recognize that an object is *decelerating*?
4. Now lift up the timer end of the track to its highest position. Give the cart a push to describe what happens to the speed of the cart as it moves down the track.
5. Now try it again with the timer on. Describe what happens to the distance the cart travels in each ⅒ s at the start, middle, and end of the run.
6. Compare you answers to Questions 1 and 2. How would you recognize that an object is *accelerating*?
7. From the above, how would you recognize *uniform speed*, in which there is no acceleration or deceleration?
8. What would you have to do to give a cart uniform speed?
9. Try to obtain uniform speed before turning on the timer. Then, when you think you have obtained it, turn on the timer and compare the results with your answer to Question 8. You may have to try this several times to obtain an as close to uniform motion as possible.

B. Uniform Speed

Instead of analyzing a photograph, as in Section 2.3, you will use the timer to analyze the motion of the cart you observed in Part A. A timer places a dot on the tape every ⅒ s. Each dot tells us where the tape (hence, the cart) is at each ⅒ s. So the distance from the start of the tape to a dot tells us how far the cart has moved since the timer was turned on.

1. Notice that the timer tape introduces a lot of friction, which acts to decelerate the motion. Attempt to compensate for this friction and

obtain a motion for the cart that is as uniform as possible (see Part A). Turn on the timer and let it record the motion. Remove the tape and mark the starting dot.

2. Place a meter stick alongside the tape, starting with zero at the starting dot. The distance of each dot from the start provides a position reading for the cart at each $\frac{1}{10}$ s to the end of the run. Let's see if there is a relationship between these two variables.
   Use the symbol d for the position reading of each dot, and the symbol t for the time reading that goes with it.

Draw a table, like the table in Section 1.3, and record the values of d and t for the entire run. (Leave room on the right for three more columns.) Be sure to include the units in your table. Use only the metric scale for distances.

3. Add two more columns to your table. One is for Δd, the distance traveled in the each time interval (the distance covered between times when the dots are produced). Another column is for Δt, the corresponding time interval. Indicate the units.
4. Why is the top line of the table in the text left blank?
5. Examine your data so far and carefully describe the motion of the cart overall and during the early, middle, and later parts of the motion.
6. Now add a fifth column to your table for the rate or the average velocity, v_{av}, in each time interval, and complete the table. Indicate the units.

Drawing conclusions

7. Once again examine your results. What can you say about the average speed of the cart during the run? Take into account the variations due to the uncertainty in the measurements of distance.
8. What is the overall average of the separately obtained average speeds? What is the average of the values in the fifth column? Write this at the bottom of that column.
9. Now obtain the average speed for the entire run from the total distance covered and the total elapsed time.
10. How does the overall average of the average speeds (Question 8) compare with the average speed for the total run (Question 9)?
11. Is the answer to Question 10 what you expected?
12. Can you give an experimentally testable definition of uniform speed?
13. Is the velocity vector in this case also uniform? How do you know?

Picturing the motion

1. If you graphed the distance and time measurements for the data in your table, what do you expect the resulting graph to look like?
2. Now plot such a graph using the first two columns in your table. Place the distance on the vertical axis and the corresponding time on the horizontal axis. Be careful to label the axes and the units and to fit the data onto the axes so that the data points do not go off the end, nor are they "scrunched" into one corner.
3. If the data points are in a line, use a ruler to draw a straight line through them. If they are not, draw a smooth curve.
4. Using your graph, describe the motion of the cart throughout the run.
5. For a straight section of the line on your graph, obtain the speed of the cart from the slope of the line.
6. Compare the speed with the average speeds in the table for that segment of the motion. Finally, compare it with the overall average of the average speeds. Take into account the uncertainties in measurement.
7. Is this what you expected to obtain? Try to account for any differences that you see.
8. Finally, obtain the *instantaneous speed* of the cart from the graph at a chosen instant of time. Indicate the instant of time and show how you obtained the speed.

Using a spreadsheet

1. Using the spreadsheet functions and features, recreate your data table above. Have the program automatically compute the values for Δd, Δt, and v_{av}, as well as the overall average velocity.
2. Use the program to create automatically a distance–time graph and to compute the slope of the line. If you are adept at using macros, create a pop-up data-input table that is activated by a "button" and then automatically enters the data in the proper cells on the table.
3. Save your work for Part C.

C. Now You Try It

Design an experiment to examine the motion of an object, other than a cart. The object might be a ball rolling down the hallway, a car traveling on a road, or a person riding a bike or roller blading or swimming or walking.

You will need to measure the time at which the person or object reaches various distances from the starting point. Your instructor can help you find any equipment you might need.

In a brief report, describe the purpose of your experiment, your procedure, the data you obtained, your analysis of the data using both a table and a graph, your conclusions based on the data and graph, and the difficulties encountered, and sources of error.

If you used the spreadsheet in Part B, replace the old data with your data in this part of the experiment, and allow the spreadsheet to analyze your data automatically.

D. Changing the Speed

This investigation will parallel the study of uniform speed in Part B, but it will involve uniform acceleration instead of uniform speed.

Since the force of friction plays a large part in the motion of a cart dragging a piece of tape, we can balance the friction force with the force of gravity. This can be done by raising one end of the track or table so that the force of gravity balances the force of friction retarding the motion, as in Part A of this investigation. Alternatively, a hanging mass may be attached by a string to the cart over a pulley.

1. With the timer tape attached but the timer off, give the cart a push and attempt to obtain acceleration, either by increasing the incline or by increasing the mass of the hanging weight. When you have achieved accelerated motion, start over: turn on the timer and let it record the motion. Remove the tape and mark on it the point where the measurements start.
2. Place a meter stick alongside the tape, starting with zero at the start of the timer marks. As before, the distance of each dot from the start corresponds to the position of the cart at that instant of time.

   As before, create a table displaying the position (d) corresponding to each dot, the time reading (t) that goes with it, the time interval Δt between successive dots, the distance Δd traveled in each time interval, and the average speed in each time interval, which is $v_{av} = \Delta d/\Delta t$. Be sure to indicate the units for each column.

3. Examine your data so far and describe the motion of the cart overall and during the early, middle, and later parts of the motion.
4. Now add a sixth column to your table for the rate of change of the average speed, that is, the average acceleration in each time interval which is, by definition, $a_{av} = \Delta v_{av}/\Delta t$. Complete the table.

Drawing conclusions

5. Examine your results. What can you say about the overall average acceleration of the cart during the entire run? Take into account the variations due to the uncertainty in the measurements of distance.
6. If any of your values for Δv_{av} came out negative, what does this mean?
7. If any of your values for a_{av} came out negative, what does this mean?
8. What is the overall average of the individual average accelerations? Write this at the bottom of the last column of your table.
9. Now obtain the average acceleration for the entire run from the total change in speed and the total elapsed time from start to finish.
10. How does the average in Question 9 compare with the overall average acceleration for the total run (Question 8)?
11. Is your answer to Question 9 what you expected?
12. Can you give an experimentally testable definition of uniform acceleration?
13. On a sketch of your apparatus draw and label an arrow representing the acceleration vector. Is the acceleration vector in this case uniform? How do you know?

Picturing the motion

1. If you graphed the average speed against the corresponding time measurements in your table, what do you expect the resulting graph to look like?
2. Now plot a graph of the average speed, v_{av}, and the total elapsed time, t, for each velocity from your table. Place the speed on the vertical axis and the time on the horizontal axis. Be careful to label the axes and the units and to fit the data onto the axes so that the numbers do not go off the end nor are they "scrunched" into one corner.
3. If the data points are in a line, use a ruler to draw a straight line through them. If not, draw a smooth curve.
4. Using your graph, describe the motion of the cart throughout the run.
5. For a straight section of the line on your graph, obtain the acceleration of the cart from the slope of the line.
6. Compare the acceleration for that segment of the motion with the average acceleration in the table. Finally, compare the acceleration with the overall average of the average accelerations. Take into account the uncertainties in measurement.
7. Is this what you expected to obtain? Try to account for any differences that you see.
8. Finally, obtain the *instantaneous acceleration* of the cart from the graph at a chosen instant of time. Indicate the instant of time and show how you obtained the acceleration.

Using a spreadsheet
As in Part C, use a spreadsheet to recreate your table, once the data are provided, and to render a velocity–time graph and the slope of the resulting line.

E. Now You Try It

Design an experiment to examine the acceleration (or deceleration) of an object. The object might be a ball rolling down an inclined plane, or a speeding cart that rolls onto a rough surface, or a person speeding up and slowing down deliberately as they walk, ride a bike, jog, or roller blade a brief distance.

You will need to measure the time and distance. Your instructor can help you find any equipment you might need.

In a report describe the purpose of your experiment, your procedure, the data gathered, your analysis of the data using both a table and a graph, your conclusions based on the data and graph, and the difficulties encountered and sources of error.

If you created the spreadsheet in Part D, replace the data with your new data and allow the spreadsheet to perform an automatic analysis.

F. Checking Galileo's Result

You saw in Section 1.9 of the textbook that Galileo could not test his hypothesis directly that free fall is an example of uniformly accelerated motion. Instead, he had to test it indirectly by studying the motion of a rolling ball on an inclined plane.

But there was another difficulty: He could not measure short time intervals during the motion or the corresponding distances traveled in order to obtain the changes in speed. He found a way around this problem, too: he derived a formula that did not include speed at all. It included only the total time (t) and the total distance (d) covered in that time, in addition to the acceleration (a). If the acceleration is uniform for the entire time and distance, and the object starts from rest at time 0 and distance 0, Galileo obtained the formula

$$d = \tfrac{1}{2}at^2.$$

Then, in either a thought experiment or a real experiment (historians still debate this), Galileo studied a ball rolling down a long inclined plane at various angles of incline. He hypothesized that, if the acceleration is constant for a fixed inclination, the distance covered and the square of the time needed should be directly proportional to each other, the constant of proportionality being $\frac{1}{2}a$:

$$d \propto t^2.$$

Indeed, this is what can be obtained experimentally. Galileo claimed that this relation also applied to freely falling objects if air resistance and friction are neglected. Devise an experiment to check Galileo's result using an inclined plane. Your instructor will have some equipment available for you to use. Since the distances are short and the falling objects are heavy, you will not need to worry about air resistance or friction.

Cautions

When testing different inclines, do not go over about 20° inclination, because beyond that the ball will begin to slide as well as roll, which changes the experiment.

Because of inherent uncertainties in the measurements, take at least four readings for each distance and time, then average the results.

Extrapolating to free fall

Although he could not test higher inclinations of the incline, Galileo noticed that the acceleration increased roughly as the angle of the incline increased. (This is true only for small angles.)

From your results, is it reasonable to conclude that an "incline" of 90°, when free fall occurs, will also be an example of uniform acceleration? Why or why not?

3. EXPLORING THE HEAVENS (CHAPTER 2)

INTRODUCTION

Each of the following inquiries may be performed as individual units. In addition, the tasks outlined in each unit may be divided among various groups.

A. The Seasons in the Heliocentric System

Celestial observations were made for thousands of years by Egyptian, Mayan, and Chinese people (and others). In this exploration we shall repeat some of their efforts with apparatus different but not more sophisticated than theirs (but during a relatively brief time).

The "planetarium" is a very helpful, hands-on working model of the relative positions of the Sun, Earth, and Moon in the heliocentric system. The distances, however, are not to scale.

1. Notice that N–S axis of the Earth is tilted at 23½° to the plane of its orbit around the Sun. Move the model representing the Earth around the Sun. What do you observe about the axis of the Earth as it moves around the Sun?
2. Now place the Earth in the position on its orbit where the North Pole is pointing most directly at the Sun. Draw a sketch of this arrangement in your notebook. Then, using a straight edge and a different pen or pencil, draw a series of parallel lines from the Sun across the entire width of the Earth. These will represent rays of light from the Sun. Since the Sun is actually much farther away from the Earth than in this model, the rays from the Sun are effectively parallel.
3. Examine your drawing. Where on the Earth do the Sun's rays hit most directly? Where do they hit least directly?
4. Where will the temperature on the Earth's surface be the warmest, where will it be the coldest?
5. To the people in the northern hemisphere, what season is it on this day? What about in the southern hemisphere? What is the special name for this day?
6. Will the time of illumination by the Sun's rays during a day be any longer in the northern hemisphere at any other position on the Earth's orbit than at this position? Try some other positions to test your answer.
7. From your drawing can you tell where on the Earth the Sun would be directly overhead at noon?
8. Is there any place on the Earth where the Sun never sets?
9. Is there any place where it never rises?

A second situation

This time, move the "Earth" to the position on its orbit where its north pole is pointing at the greatest angle *away* from the Sun. Draw a sketch of this arrangement in your notebook. Then, using a straight edge and a different pen or pencil, draw a series of parallel lines from the Sun across the entire width of the Earth. These will represent rays of light from the Sun.

Work through Questions 3–8 above for this arrangement.

Two other positions

Let the "Earth" now move forward slowly in its orbit. Notice the behavior of the N–S axis. Stop the motion when the Earth has moved about one-quarter of the way around its annual orbit and three-quarters of the way around. At these point the Sun's rays should be hitting a level surface at the equator at a 90° angle (the Sun is directly overhead).

1. Draw a sketch of the Earth–Sun relationship at one of these two positions and include the direction of the Earth's axis.

2. Draw another sketch, as before, of the Earth and Sun, along with a series of parallel lines representing the rays of light from the Sun.
3. Examine your drawing. Where on the Earth do the Sun's rays hit most directly? Where do they hit least directly?
4. Compare the amount of daylight in the northern hemisphere with the amount in the southern hemisphere?
5. To the people in the northern hemisphere, what season do you think it is on this day? What about in the southern hemisphere? What special names do these 2 days have?

Inquiry

1. Find your location on "Earth." As the Earth revolves around the Sun in 1 year, predict what will happen to the length of the day at your location on Earth during that year.
2. Now move the Earth around its orbit and identify its position at each of the four seasons.
3. As seen from your location on Earth, how does your prediction compare with your observations about the time of daily sunlight during the course of 1 year?

Solar noon

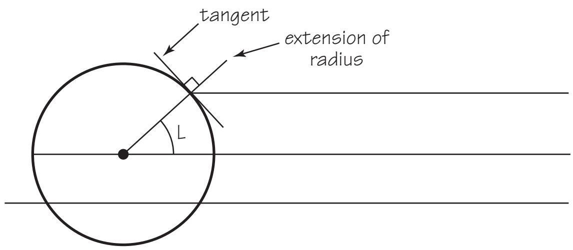
Solar noon is the time during the day when the Sun is at its maximum "altitude," the angle of the Sun with respect to a plane to the horizon. Solar noon may not occur exactly at 12:00 noon, or at 1:00 p.m. if Daylight Savings Time (DST) is in effect.

1. Why wouldn't solar noon occur everywhere in your time zone at exactly at 12:00 noon (or 1:00 p.m.)? At a given location on Earth, could it occur at 12:08 p.m. (1:08 p.m. DST)?
2. In your local newspaper find the time when the Sun will next rise and set. Since the Sun appears to move in a circle across the sky, solar noon will be the time exactly half-way between rising and setting. Figure out when this will be.
3. Use the length of the shadow cast by a tree or a stick you have placed in the ground to gain a qualitative measure of the altitude of the Sun ½ hr before the predicted solar noon, at solar noon, and ½ hr after solar noon. Does this confirm your prediction of solar noon in Step 2? *Caution:* Never look directly at the Sun, even with sunglasses. It can cause permanent eye damage. This is only a qualitative measurement.

A geometric inquiry

1. Draw the Earth represented by a perfect circle, and draw the equator through the center. Locate your latitude on the Earth and draw and label the latitude angle with respect to the equator. Now draw a line

that is an extension of the equator all the way to the edge of the paper. This line will also represent a ray of light coming from the Sun at solar noon on the Vernal Equinox. Draw another line from the edge of the paper, parallel to this one, hitting the Earth at your location. This represents a ray from the Sun at your location at solar noon on the Vernal Equinox. Using geometry, find the altitude of the Sun with respect to the horizon at your location at solar noon on the Vernal Equinox.



An Example
L = Latitude angle at your location

2. During the course of a year will the Sun ever be directly overhead at solar noon at your location on the Earth?
3. To check your answer to Question 2, find by construction the altitude of the Sun at its highest point in the sky in your hemisphere—that is, on either the Summer or Winter Solstice. On this day the Sun's rays are exactly perpendicular to a plane surface at noon at 23½° north (or south) latitude. Using this information and your latitude, find the altitude of the Sun with respect to the horizon at your location at solar noon on the Summer or Winter Solstice.
4. Do the same for the opposite solstice.
5. Describe the changes in the Sun's altitude at solar noon at your location during an entire year. Be precise about the maximum and minimum values.

B. Observing the Sun's Apparent Motion

1. On a clear day, find in a newspaper the time when the Sun is scheduled to set. Go outside at that time, and just after the Sun has set (so you are not looking at it directly), record the date, time, and position

Caution: Never look directly at the Sun; it can cause permanent eye damage. Do not depend on sunglasses or fogged photographic film; they do *not* provide enough protection. In this experiment you will never need to look directly at the Sun.

where it has set. In most places today, the horizon is obstructed by buildings or trees. In these cases, the "setting" of the Sun will involve its disappearance behind the object obstructing the horizon. In this case, record the position by noting exactly where the Sun disappears behind a building or tree or fence. Even better, if you have a compass, record the angle at which the Sun sets on a scale from 0° to 360° (due north is 0° or 360°, due east is 90°, due south is 180°, and due west is 270°). Draw a simple sketch of the horizon and indicate the location where the Sun set. Note also your exact observation point. (You could also do this experiment just before sunrise, if that is more convenient.)

2. Predict how you would expect these observations to change, if at all, 1 week later.
3. Actually repeat these observations once a week on the same day for 1 or, preferably, 2 months. Some scheduled days you may not be able to make the observations if the sky is overcast. In that case, observe on the next available day (or the day before if the forecast is for cloudy conditions).
4. Record how the position and time of the Sun's setting has changed from the week before. How do your observations compare with what you expected?
5. At the end of the observation period, draw some general conclusions about the changing position of the Sun's setting and the length of the day with reference to the season in which you made these observations.
6. The title of this investigation is "Observing the Sun's Apparent Motion." Your laboratory partner might argue that it is really the Earth that is moving and the Sun that is stationary. Is there any way to determine from your observations whether it is really the Sun or the Earth that is moving? Which is the more plausible from your observations?
7. How do your observations fit with the heliocentric theory, in which the Earth is indeed rotating on its axis and the Sun is stationary?

C. Observing the Sun Pass through Solar Noon

You will need a large sheet of cardboard (sometimes known as "oaktag"), a pointed stick about 25 cm in length, a meter stick, a penny, a piece of string, and tape.

Caution: Once again, never look directly at the Sun; it can cause permanent eye damage. Do not depend on sunglasses or fogged photographic film; they do *not* provide enough protection.

In this experiment you will observe the motion of the Sun as it crosses solar noon. But you will do this only indirectly by observing the motion of the shadow cast by the Sun onto the cardboard sheet.

1. As you saw in the earlier section on solar noon, the time of solar noon is around our clock-time of 12:00 noon, or 1:00 p.m. if DST is in effect. But, because of the width of your time zone on the Earth's surface, solar noon at your location is very probably not exactly on the hour by the clock. To anticipate this, you will begin the investigation 30 min before the hour and continue until 30 min after the hour.
2. Find a level, preferably a paved area, with an unobstructed view of the Sun when it is overhead on a clear day. Before starting, note the location of the Sun in the sky in relation to the "cardinal points" (north, south, east, west). How do you expect the Sun to move from its present position during the course of this experiment?
3. Hold the pointed stick upright and notice the shadow the Sun casts. From your answer to Question 2, how do you expect the shadow to change during the course of this experiment?
4. Place the cardboard behind the stick so that the shadow falls on the cardboard and there is room on the cardboard for the expected movement of the shadow. Hold the cardboard in place with weights or books, so that it does not move during this experiment. Also mark the exact position of the end of the stick at the edge of the cardboard.
5. It is important that the stick is always in the same location and perpendicular to the ground. To help ensure that the stick is perpendicular, tape a penny to the string and tape the other end of the string near the upper end of the pointed stick. The penny and string should now lie flat against the stick as long as it remains perpendicular. Now you are ready to begin.
6. Beginning exactly 30 min before the predicted solar noon, set up the stick and mark the approximate point of the end of the shadow. (The end of the shadow is actually a bit fuzzy, because the Sun is a bright ball, not a point source of light, so the several shadows cast by different parts of the Sun overlap to form the shadow on the cardboard.) Use the meter stick to draw on the cardboard a line connecting the end of the shadow with the position of the low end of the stick. Record the time on the line, and in your notebook. Record also the length of the line.
7. Repeat this procedure exactly every 5 min for 1 hr. (If the motion of the shadow takes it off the cardboard, reposition the cardboard and start again.)
8. From the type of observations you have made so far, how would you

know approximately when solar noon occurred? Write down the approximate time.

9. Why is this only the approximate time of solar noon?
10. How would you determine from your data the direction of due north and due south? Indicate the cardinal points on your cardboard.
11. Your instructor may review with you the definition of the tangent of an angle in trigonometry. How could you use this definition to determine the altitude of the Sun (angle with respect to the horizon) at solar noon?
12. Find the altitude of the Sun at solar noon.
13. How did the actual motion of the shadow compare with your expectations?
14. Is there any way to determine from your observations of the Sun's moving shadow whether it is really the sun or the Earth that is moving? Which is the more plausible from your observations?
15. How do your observations fit with the heliocentric theory, in which the Earth is moving and the Sun is stationary?

D. Phases of the Moon

1. The phases of the Moon, as seen from the Earth, occur because of the different positions of the Moon in relation to the Sun and Earth during the course of its monthly orbit around the Earth. Use the planetarium to observe how the Moon's phases change as seen from the Earth during the course of one orbit of the Moon around the Earth.
2. Sketch the relative positions of the Earth, Moon, and Sun at full moon, half moon, and new moon.
3. What would the positions of the three objects be for the appearance of the Moon in which one-quarter, and three-quarters, of the Moon's face is visible? Is there more than one position for each?

Now you try it

1. During the course of 1 month, either in the evening or in the morning, observe the Moon and sketch its phases; record the corresponding date and time. Note or guess the approximate location of the Sun at each observation.
2. From the position of the Sun and the brightened portion of the Moon, sketch the positions of the Sun, Moon, and Earth for each observation.
3. How does the position of the Moon in the sky in relation to the Sun change from day to day?
4. Assume that the Sun is stationary. Is there any way to determine from your observations whether it is really the Moon or the Earth that is moving? Which is the more plausible from your observations?

5. Ancient observers (and some people to this day!) believed the phases of the Moon are caused by the interposition of the Earth in the path of light beams from the Sun to the Moon. Give some arguments against this view.
6. Why are there only occasional eclipses of the Sun at new moon, and occasional eclipses of the Moon at full moon? Answer this in terms of
   (a) the geocentric system, and
   (b) the heliocentric system.
7. Draw the relative positions of the Sun, Moon, and Earth at the occurrence of solar and lunar eclipses. Include rays of sunlight representing the limits of the shadows in each case.

E. The Motion of an Outer Planet

You saw in Section 2.4 that, seen from the Earth, the planets appear to "wander," that is, they appear to fall behind the stars each day, as do the Sun and Moon. This is called their eastward drift. But every once in a while, they also tend to move forward (to the west) faster than the stars. This is called their retrograde motion. The result is a looping or S-shaped motion as seen against the background of the stars. (See the text Section 2.4).

You also saw how Ptolemy explained this motion in the geocentric view by placing the planet on a circle that rode on another circle. (See figure 2.7 in the text.) Copernicus presented a quite different explanation in the heliocentric model, one that is also more plausible (and the one we accept today.)

Below is a top-view diagram of the Earth, Mars, and the Sun in the heliocentric model (not to scale). The diagram shows several locations of the Earth and the planet Mars at intervals of 1 month apart. The background of stars is also represented.

1. Which path represents the orbit of the Earth, and which represents the orbit of Mars? In which direction is each planet traveling? Explain your reasoning.
2. Copy the diagram onto a separate sheet of paper. Number the positions at each month from 1 to 7 for the Earth and Mars.

3. For each month shown on the diagram, draw a straight line from the Earth to the corresponding position of Mars, continuing the line to the background of stars. Each line represents the line of sight for an astronomer on the Earth who is observing Mars against the background of the stars. Number the ends of the lines from 1 to 7.
4. From the lines of sight you have drawn, determine how Mars appears to change location with respect to the background stars. Explain your reasoning.
5. During which of the months shown, if any, does Mars appear to move *eastward* with respect to the background of stars (eastward drift)? Explain how you can tell from your diagram.
6. During which of the months shown, if any, does Mars appear to move *westward* with respect to the background of stars (retrograde motion)? Explain your reasoning.
7. Do any of these answers change if the experiment is done on the Earth's southern hemisphere?

Evaluating your results

1. What was the purpose of this laboratory exploration?
2. Summarize the overall steps as well as the procedure in your own investigation in relation to the purpose of this exploration.
3. What conclusions did you make? What is the supporting evidence for them? What are the sources of error?
4. What difficulties did you encounter? How did you overcome them? How could this exploration be improved?
5. Connections: How does this laboratory relate to the material in the textbook and to your own daily experiences?

4. SKYGLOBE: A COMPUTER PLANETARIUM (CHAPTER 2)

Skyglobe is a computer planetarium that simulates many of the astronomical observations discussed in the text. It is by no means a substitute for actual observations, which physically connect you to the real universe which is our home. But the program does allow you to obtain a sense of what you would see if you were observing the actual events. Study each observation for a while and try to imagine yourself being outside and looking up at the sky on a beautiful clear evening.

Because of the complexity of the calculations involved in displaying the observations, *Skyglobe* is a DOS-based program (written in Assembly language). So when you start it up, the computer switches to the DOS operating system. It will switch back when you exit.

STARTING AND SETTING UP THE PROGRAM

After you have started the computer and it has completed booting to Windows, click the Start button, then click Run. Type the location of the program, then click OK or press Enter.

The program starts in a full-screen DOS window. Press the space bar to make the logo disappear.

Except where requested, do not use the mouse, as it changes the angle of observation too quickly.

You can exit the program at any time by pressing Esc twice.

The upper left corner indicates some of the settings. We will change most of them.

Set up the program by pressing the following

NumLock	Turn it off (so the green light goes off).
F1	Remove the help list of key commands. (You can access it anytime by pressing F1 again.)
+ (at right of keyboard)	Press several times to increase number of stars to maximum.
L	To obtain a list of locations from which to observe. Go to a city closest to your location using the arrow keys or the mouse and click or hit Enter.
F6	Remove the Sun's ecliptic path, the dotted red line. (We'll leave the planets and their names.)
N	This will place you facing due north.

The circular green line is your horizon line. You cannot see anything below the horizon line (even though the program shows some stars).

Up and down arrows	These raise and lower your view of the sky. Place N at the bottom of the screen.

The red lines are drawn to indicate the various constellations.

C	To remove or replace most of the constellations. Note the locations of the Big Dipper and Little Dipper.
F10	Removes all constellations and returns them. Leave them turned off for now.
F9	Remove names of constellations.
F8	Remove star names.

The grid lines are some of the latitude and longitude lines of the celestial sphere.

F7	Remove the grid lines on the celestial sphere for now.
F4	Remove the deep-space objects, which we can't see anyway without a telescope.
K	Turns the Milky Way on and off. Be sure it is on.
M, D, H, T	Sets date and time forward. Shift-M, D, T, or H moves each one backward. Be sure it's set to the current date and time.

The little green + sign is the Zenith (90° altitude), and the green line is the horizon (0°).

You are now ready to begin.

OBSERVING THE CELESTIAL SPHERE

The settings you just entered enable you to see the celestial sphere as it would look right now at your location, facing due north, if the Sun were not visible and no buildings were in the way.

1. Look in the other directions on your horizon plane by pressing S, E, W. You can also slowly "turn" in different directions by pressing the left and right arrow keys, ← and →. Try both of these.
2. If it is still daylight outside, observe the position of the Sun in the southern sky. Place the mouse over the Sun, and its altitude and azimuth will be given in the lower left corner. Record the result. (The azimuth runs from 0 at north to 180, due south, to 359.)
   Altitude:
   Azimuth:
3. Are the Moon and any planets above the horizon at this time? Which ones?
4. Return to looking due north, by hitting N. Place N on the bottom of the screen, if it is not there already, by using the arrow keys. Press C several times until just the Big and Little Dippers and a few other constellations appear.

   You can now set the celestial sphere in motion by pressing A. To stop it, press A again. However, the motion will probably be too fast to be meaningful. Slow it down to "slow motion" by pressing shift ←. You can speed it back up, if necessary, by pressing shift →.

   Observe for a while what happens to the stars and constellations as time advances. While the sphere is rotating, turn the grid lines on and off by pressing F7.

In what general direction are the stars moving?

What do you observe about the motion of the sphere?

Do all of the visible stars and constellations rise and set each day?

5. Stop the motion by pressing A. Place the mouse over the central star and look in the lower left corner of the screen.

   What star is this?

   What is its altitude?

   How does its altitude compare with the latitude of the location nearest you?

   How could you use the Big and Little Dippers to find this star?

   Try to find this star tonight, if it is a clear night and if your view is not blocked.

6. Set the celestial sphere in slow motion again (A). Now look to the E, S, W. Turn the grid lines on and off (F7). What is the apparent motion of the stars as observed in each of these directions? (Stop the motion when you finish.)
7. Now let's look at the celestial sphere after traveling to two other important locations.

   Press L to obtain the location list. Select "More Locations," the North Pole at bottom right.

   Be sure the Little and Big Dippers and a few other constellations are on by pressing C.

   Where is the "central star" from Question 5 in relation to the zenith?

   Set the sphere in slow motion once again, turning the grid lines on and off. Look in all four cardinal directions and use the left or right arrow key to turn completely around in a circle.

   Is there any difference in your observations in any direction?

   How could you describe the overall motion of the celestial sphere as seen from the North Pole?

8. Press L and go to an observation point on the Equator. Where is the "central star" this time?

   Set the sphere in motion. Look at each of the four cardinal directions. Notice that the central grid line (celestial equator) is directly on the E and W points. Describe the apparent motions of the stars at each of the cardinal points.

   How could you describe the overall motion of the celestial sphere as seen from the equator?

9. Finally, return to the location nearest you (press L). Perform the same observations as in Questions 7 and 8. How could you now describe the overall motion of the celestial sphere as seen from this location?
10. How could these observations be used to argue that we are observing the celestial sphere from a position on a sphere?

OBSERVING THE APPARENT MOTION OF THE SUN

Set the date to today's date and the time to 12:00 noon, if Standard Time, or to 1:00 p.m., if DST. Use the M, D, H, and T keys to do this. If you overshoot a setting, use the shift key with M, D, H, or T to go backward.

Look to the south (press S) and turn on the grid lines (F7). Be sure the ecliptic path (red line) is still off (F6). Turn off all constellations (F10).

1. Is the Sun due south in the sky at noon today? If not, why not?
2. Is the Sun directly overhead (at the zenith) at noon today?
3. Use the mouse to place the cursor on the Sun. What are the altitude and azimuth of the sun at noon today?
   Altitude:
   Azimuth:
4. Notice where the Sun is in relation to a few nearby stars. You can move the celestial sphere ahead exactly 24 hr by pressing the D key. Press the D key several times, pausing briefly each time to observe the position of the Sun in relation to the nearby stars. Try this again a few more times. (Ignore the planets for now.)
   Carefully describe what you observe about the position of the Sun in relation to the fixed stars.
5. Turn on the ecliptic path (red line) by pressing F6. Hold down the D key and observe the motion of the Sun. (Again ignore the planets and the Moon, which flies across the screen every month.) Remember that each jump forward of the sphere represents the motion of an entire day.
   Is the Sun staying up with the stars, or is it falling behind? What is this motion called?
6. Observe the motion of the Sun for an entire year against the background of the celestial sphere. Notice how the altitude of the Sun changes as it moves eastward along the ecliptic path. (There are two jumps by an hour each year as the time shifts into and out of DST, indicated by D in front of the time.)
7. Return to today's date and time and continue to look south. Find the date and altitude of the Sun at solar noon on the two solstices during the next year.
   Summer Soltice:
   Winter Solstice:

Caution: NEVER LOOK DIRECTLY AT THE SUN. It can damage your eyes permanently. Therefore, briefly glancing past it, obtain only a *very approximate* confirmation of (a).

8. At your location is the Sun ever directly overhead at any time during the year? (Remember that the green + mark is the zenith point.) What is the highest it ever gets in the sky?
9. Return to today's date and time.
   (a) What is the location of the Sun right now? (Change your view with the arrow keys, if necessary.)
   (b) What time will (or did) the Sun set today?

   If feasible, go outside right now and *approximately* confirm (a). If feasible, confirm prediction (b) either by direct observation or, if necessary, in a newspaper.

OBSERVING THE APPARENT MOTION OF THE MOON

Find the location of the Moon by pressing F, then select Moon. Use the arrow keys to place the moon in a comfortable viewing position for when the sphere begins to turn.

Notice where the Moon is in relation to a grid line. Then press A and move forward in time exactly 24 hr. You can use shift → to speed up the motion.

1. What two motions does the Moon exhibit relative to the stars?
2. Leave the Sun's ecliptic path on (F6). Carefully notice the location of the Moon in relation to a longitude line. Press D, pausing briefly to count each time, as you advance a day at a time. Carefully keeping your eye on the grid line as the Moon disappears.

   How many days does it take for the Moon to be completely "lapped" by the stars? This is known as a lunar month.
3. The Moon moves near the sun's ecliptic. Why don't we observe lunar or solar eclipses every month.
4. Finally, set the date to today and the time to sometime this evening when you will have a chance to observe the Moon (if it's not cloudy and if the Moon is not in new phase).

   What will be the location of the Moon at that time?

   Confirm your prediction this evening.

OBSERVING THE PLANETS

1. Return to today's date and set the time for around noon. Find the Sun. Turn the grid lines off and turn off all constellations (F10), but leave the ecliptic path on (F6).
2. Note the location of the planet Mercury. Advance by days at a time by holding down the D key and carefully observe the apparent motion of Mercury. (Again ignore the Moon's rapid motion across the screen.)

3. Using the A and D keys, carefully attempt to observe all three motions of Mercury: the daily westward motion, eastward drift, and retrograde motion.
4. Sketch the path of motion that you observe for Mercury over the course of time. Include the Sun and the ecliptic in your sketch.
5. Try to observe the retrograde motion of another planet.

Now you try it
Go to a date of your choosing (such as your birth date) at your location and record when the Sun rises and sets, the time and altitude of the Sun at solar noon, and the Moon and any planets that are visible in the sky at sunset.

5. EXPLORING FORCES (SECTION 3.4)

In this investigation, as is often done in scientific research, you will first confirm previously obtained results (here, those discussed in Section 3.4 of the textbook). Then you will go on to explore new territory, using the results you obtained from the first part of this investigation.

Question: An unbalanced force causes an acceleration. How are force and acceleration related to each other? To investigate this, you will apply different forces to a given mass, then the same force to different masses, observing the accelerations that result. Let's start with an unbalanced force on one object.

A. Acceleration With a Constant Force Acting On a Constant Mass (1.0 kg)

We do not want any part of the force that we apply to the cart to be balanced out by friction, nor do we want gravity to add to the force. So, before starting, we'll arrange the set-up so that gravity on the cart balances the force of friction on the cart.

1. How would you arrange the apparatus so that the effects of gravity and of friction balance?
2. How should the cart move when the effects of gravity and of friction just balance?
3. Use the available masses to obtain a total mass for the cart, with its load, of just 1.0 kg.
4. On the basis of your above answers, carefully arrange the cart and track

so that the balance of the forces described above is achieved as closely as possible. Note that, if you are using a tape to record the time intervals, it should be included as a source of friction.

5. Now obtain by experiment the average acceleration of the cart (mass of 1.0 kg) when you apply a constant force of, say, 1.0 N parallel to the track by means of the spring scale. Remember that to obtain the average acceleration, you must start with the observed distance and the time, and then obtain from your data the average velocity, the change in average velocity, the average acceleration for each time interval, and the overall average acceleration. Construct a table for your data and obtain the overall average acceleration for this force.
6. From your results so far, what do you conclude about the validity of Newton's second law of motion regarding the relationship between a constant force and the acceleration of a given mass?

B. Acceleration with Different Forces Acting On the Same Mass (1.0 kg)

So far, you have investigated the effect of only one force on the acceleration of the 1.0 kg cart.

1. If you applied larger and smaller forces to the cart, what do you predict will happen to the acceleration of the cart on the basis of Newton's second law?
2. How would you test these predictions? (You may want to refer to Part A.)
3. Discuss the testing procedure with your group and with your instructor. Once you have decided upon a procedure, proceed with the test.
4. Construct appropriate tables to display your data. Construct a summary table, showing the different forces applied and the resulting average accelerations for the 1.0 kg mass. You might use a spreadsheet program to present your data, if this is available and your instructor recommends it.
5. Examine your data and compare your results with your predictions in Questions 1 and 2 above. Discuss the reasons for any disagreement with what you expected to find. What are the sources of error?

Finding a pattern

1. You now have the results for the average accelerations on the 1.0-kg mass caused by different forces. Is there a pattern in the relationship between force and acceleration for a single mass?

2. To see any pattern more clearly, create a graph with force on the *y*-axis and average acceleration on the *x*-axis. Take care in constructing the scales and labeling the axes. If the data points appear to fall on a straight line, use a ruler to draw a straight line through the data points.
3. What does this pattern tell you about the relationship between the two variables, force and acceleration?
4. Now obtain the slope of the line and be careful to include the units of the slope.
5. Express the relationship between the force and the acceleration as an equation, and including the actual mass of the cart in this equation.
6. How does your result compare with Newton's second law of motion? Discuss whether it confirms the law or conflicts with it, and why.

C. Acceleration with Different Masses and the Same Force

So far you have found the relationship between forces and accelerations for one mass.

In this investigation you will test the relationship to see if it holds for other masses, as it should if it is a law of nature. This procedure is common in actual research: first testing a result to see if it holds in one case, then testing to see if it holds in other cases under various conditions.

You will test only two other masses. (You may perform other tests if you wish.)

1. Chose other total masses for the cart that are simple multiples of each other and of the 1-kg mass used in Part A; for example, 2.0 kg, 0.5 kg.
2. As before, arrange the apparatus so that the effects of friction are balanced out by the effects of gravity.
3. Using the same amount of force as in Part B (1.0 N) on each total mass of the cart, obtain the overall average acceleration of the cart from the distance and time data.
4. Create a summary table of the different masses, including the mass in Part B, and the corresponding overall average accelerations. Can you discern a pattern in the relationship between mass and acceleration?
5. In order to obtain a clearer picture of the relationship, graph your results. From the type of line formed by this graph, what do you conclude about the relationship between mass and acceleration?
6. How does your conclusion compare with what you might expect from Newton's second law?
7. Theoretically, what should be the value of the slope of the line? Obtain the slope of the line. Does it equal this expected quantity?

D. Conclusions

1. Write your conclusions regarding:
   (a) the relationship between force and acceleration, from Part B;
   (b) the relationship between mass and acceleration, from Part C;
   (c) the overall relationship between force, mass, and acceleration.
2. Does this experiment so far confirm or deny Newton's second law of motion? Explain.

E. Exploring the Unknown

Once scientists are confident of a general result, they can rely on it as a tool for exploring the unknown. Newton's second law of motion has been tested and confirmed so many times in many different situations that it now accepted as a law of nature (at speeds much less than the speed of light).

1. Your instructor will give you an unknown mass to place on the cart. Using your results above, design an experiment to obtain the value of the mass (without using a scale) and then carry it out.
2. Confirm your result by using a scale. What are your conclusions about using Newton's law?
3. Once you know the mass, you can predict the acceleration it will have when a known force is applied to it.
4. Design an experiment to do just that and carry it out. (Again, use a spreadsheet if available and recommended by your instructor.)
5. What do you conclude from this result?

F. A Practical Application

Place a volunteer from the class on a smoothly running cart or on roller blades or on a scooter. On a stretch of sidewalk or other open area, measure out a distance of 10 m.

1. When your volunteer is on wheels and some distance from the measured 10 m, push him or her from rest until you reach the 10-m measured stretch, then release the volunteer to cover the 10-m stretch at constant speed. Using two stop watches, record:
   (a) the time during which the acceleration occurred;
   (b) the time it takes your volunteer to cover the 10-m stretch;
   (c) the time it takes your volunteer to stop after completing the 10-m stretch.

   Record also the total mass of your volunteer and the device on which they are riding.

2. From these data obtain:
   (a) the acceleration of the volunteer;
   (b) the force required to produce this acceleration;
   (c) the amount of force required to stop the volunteer. What provides this force?
3. Why is the mass in Newton's second law sometimes called "inertial mass"?
4. Describe with the aid of a sketch the action and reaction forces in the process of accelerating your volunteer.

6. EXPLORING FORCE, WORK, ENERGY, AND POWER (CHAPTERS 3, 5, SECTION 6.3)

How are all of the basic physical concepts of force, work, energy, and power related to each other? The textbook states that work done by a force on an object is defined as the force exerted times the distance the object moves under the action of the force. The textbook also referred to two types of mechanical energy: potential energy and kinetic energy.

Write down in your own words the definitions of these other concepts below. Refer to the textbook as needed, but put these in your own words. If it is still not clear to you what these concepts actually mean, ask for help.

Force:
Work:
Kinetic energy:
Potential energy:
Power:
Law of conservation of energy:

A. The Force of a Spring

In this exploration you will investigate how the above concepts apply to a simple mechanical device—a spring. Just as scientists do in investigating a new phenomenon, you will make observations, develop and test hypotheses, change your hypotheses as needed, draw conclusions, and apply your conclusions to new situations.

Since the concept of force is essential to the concept of work, we first need to obtain an expression for the amount of force that a spring exerts when it is pulled off equilibrium (or squeezed together).

1. Pull on the two different springs to obtain a sense of how the force changes as the spring is stretched more and more. Try this several times.

(Do not pull the spring so much that it is bent and does not go back to its original position.) What do you conclude about the force as you increase the elongation of the spring?

2. Your answer to the above question can be regarded as a rough hypothesis about the force exerted by a stretched spring. Now you will test this hypothesis and obtain an exact relationship between the force and the elongation of the spring.
3. It is difficult to measure the force exerted by your hand. Instead you will use several masses, each of which, by its weight, owing to the gravitational pull of the Earth, will exert a precise, measurable force on the spring. The weight of an object on the Earth's surface is equal to the mass m of the object times the acceleration of gravity, g. Or

$$W = m \times g.$$

In this equation g has a constant value at a given location. In general, it is taken to be $g = 980 \text{ cm/s}^2$.

4. If the mass is not given on each of the available masses, measure the masses in grams using a scale.
5. Place the shorter spring on a cross bar and hook the 50-g hanger to the end of the spring. In order to give the spring a stretch before starting the investigation, place the 500-g mass on the hanger. This will be the starting position. We will define this as the position when there is *zero added mass* on the spring, even though we know that there are already 550 g attached to the spring.

   Measure the length of the spring from its top to the end of the spring (not to the end of the hanger).
6. Now place increasing amounts of added mass on the hanger from 100 g to 500 g in 100-g intervals. In each case the elongation of the spring increases. Measure the length of the spring, again from the top of the spring to the end of the spring.
7. Create a table with four columns in which to present your data, giving the units in each case:
   - In the first column, list the mass added to the hanger and 500-g mass, starting with zero mass.
   - Leave the second column blank for now.
   - In the third column give the total length of the spring corresponding to each mass. Call it l.
   - In the fourth column give the *increase* in length l with each mass, call it x. For zero added mass, $x = 0$.
8. The second column of your data table will list the weight of each mass. Call it the force F. Using the definition of the weight, find the

weight for each mass and present the results in the table, including the units.

In this experiment we are measuring mass in grams, distance in centimeters, and time in seconds. Using these measures, in the metric system the unit of weight (and any other force) is

1 g cm/s^2 which has a special name; it is called a "dyne."

Revising your hypothesis

Carefully examine the results in your table and compare with your earlier tentative hypothesis (Question 1). What do you conclude from your table about the stretch of the spring as the force increases?

Examining the data

1. Your table gives the increase in the length of the spring for added weight (force) applied to it. In order to see more clearly how the two variables of force and length increase (x) are related, we can use graphical representation of the data, in other words, a graph.
2. Using a sheet of graph paper, place the added force (weight), F, on the vertical axis and the increase in length, x, on the horizontal axis. Then plot your data. Use a ruler to connect your data points.
3. Examine the result of your graph. How would you now revise your earlier hypothesis about the relationship between the F on a spring and the increased length, x?

Obtaining a precise equation

1. If your graph turned out to be close to a straight line, you can obtain an exact equation relating the two variables by obtaining the slope of the line. Call the slope of the line k. Here $k = \Delta F/\Delta x$. Obtain the value of k, including its units, and write an equation relating the force, F, and the increase in the length of the spring, x. Show all your work.
2. If the length of the spring with zero added force (neglecting the initial weight added) is *defined* as zero length, then all increases are measured from zero length. So Δl, which we have called x, is simply l. Rewrite the above equation for the situation when l is measured from the position $l = 0$. This equation was first obtained by Newton's colleague Robert Hooke in the 1600s. It is known as *Hooke's law*.

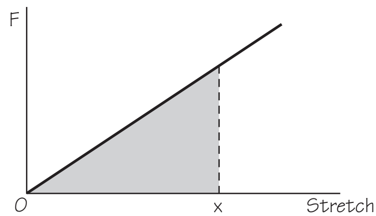
Using Hooke's law

1. How could Hooke's law be used to measure the unknown weight of an object?
2. Find the unknown mass of the object.

B. Work and Energy (Chapter 5)

Attach the spring to the crossbar and attach the hanger and the 700 g of mass to the spring. Observe that the spring extends and comes to rest.

1. The weight of the mass that you attached to the spring does work on the spring, while stretching it. What happened to the energy that was created by this amount of work? Did it disappear?
2. We want to find a value for the work performed by each added mass as it stretches the spring. Unfortunately, to obtain the amount of work done, we cannot simply multiply the amount of stretch, x, times the force, F, because the force is constantly increasing as the spring stretches. Instead, the amount of work can be represented by the area in the triangle under the graphed line:



Work is numerally equal to the area $= \frac{1}{2}$(base)(height)
$= \frac{1}{2}x\,(k\,x) = \frac{1}{2}k(x)^2$.

3. Using this equation and your data from Part A, obtain the amount of additional energy stored in the spring for each mass that you added (beyond the initial mass). Present your results in a table and include the units. The table should have four columns: the mass applied to the spring, the force, the increase in length (x), and the work done, $W = \frac{1}{2}k(x)^2$.
4. When a weight is at rest on the spring, we say it is in equilibrium. What does Newton's first law of motion say about the forces on a weight at equilibrium?

Observing energy transformations

1. With an added mass of 200 g on the hanger (in addition to the initial mass), gently pull the spring slightly off equilibrium and let it go. What do you observe?
2. Using either energy or work concepts, what is happening during the motion of the spring right after you release it as it moves back to the point of equilibrium?

3. What is happening in terms of either energy or work during the part of the motion from equilibrium to the top of the oscillation?
4. Starting from your initial pull off equilibrium, carefully trace all of the energy transformations that are occurring and where they occur. Assume that the mechanical energy is conserved.
5. Where does each of the following have its largest and smallest values?
   (a) the elastic potential energy of the spring;
   (b) the kinetic energy of the mass;
   (c) the gravitational potential energy of the mass.

Obtaining quantitative results

1. With the 200-g additional mass still on the spring, pull the spring off equilibrium by exactly 3.0 cm and let it go. See if you can find a way to give a quantitative value to the maximum amount of the following. (These numbers will be very large, because the units used, ergs, are very small.)
   (a) the elastic potential energy of the spring;
   (b) the gravitational potential energy of the spring.
2. What is the speed of the mass as it passes through equilibrium?

C. Now You Try It (Section 6.3)

Power is defined as the rate of doing work. Two examples might be a person walking up a flight of stairs and another person running up the same flight of stairs. In both cases they perform the same amount of work, but they apply different amounts of power. In order to see this, examine the difference.

1. Find a staircase in your building. After measuring the height of one step, obtain the height from one floor to the next.
2. Let one student casually walk up the stairs. Then let the same or another student run up the same flight of stairs. Time each student with a stopwatch.
3. You will need to convert the weight of the student(s) to newtons.
4. From these data, obtain the work performed and the power output of each student for their ascent. Show your calculations.
5. Compare each with the power output of a 100-W light bulb.

D. Devise Your Own Experiment

Devise an experiment to measure the power output of a person riding a bicycle. Describe exactly what you would do, and how you would obtain your results.

Try the experiment if time and equipment permit.

7. FINDING THE MECHANICAL EQUIVALENT OF HEAT (SECTION 6.1)

INTRODUCTION

Scientific research may involve a variety of aims. Some research is directed at finding whether known laws of nature hold under extreme conditions; other research may seek to test a new theory or prediction, or to understand a puzzling or new phenomenon; and some research is undertaken to obtain a precise measurement of a given quantity or variable. This investigation is similar to the last of these. It involves a measurement of the quantity known as the mechanical equivalent of heat—one of the fundamental constants of nature.

Before starting: review the section on scientific notation in "Reviewing Units, Mathematics, and Scientific Notation."

Mechanical work and heat are different manifestations of one overall concept—energy. Although each is a manifestation of energy, they are measured in different units. In the study of mechanical work, we speak of foot-pounds, ergs, or joules. In the study of heat, we speak of calories, kilocalories (equal to 1000 calories), or Btus (British thermal units). Briefly defined, 1 cal is the amount of heat required to raise the temperature of 1 g of liquid water 1°C.

Since heat and work are both manifestations of energy, we should be able

to convert from one system of units to the other. That is, there should be a conversion factor that tells us how much heat energy is equivalent to a certain amount of work, and vice versa. Unfortunately, there is no theoretical procedure that gives us this factor. We have to resort to experiment. By directly measuring the amount of heat produced by a given amount of work, we can find a numerical factor with which we can multiply calories of heat to obtain the equivalent number of ergs or joules of work. This factor has been named "the mechanical equivalent of heat." In this experiment we will make a direct measurement of this factor.

The mechanical equivalent of heat was first measured by Joule in England in 1840, and many times thereafter. The method by which he achieved his most accurate result was one in which a mass of water was churned by a set of paddle wheels set in rotation by a series of falling weights. The heat developed in the water came as a result of the work done on the water by the paddle wheels, kept churning by their connection to the weights as they fell.

The mechanical equivalent of heat (J) is today usually defined in joules per calorie and the accepted value is $\mathcal{J}$ = 4.19 joules/calorie—which is very close to Joules' original result. However, this mixes units of the mks (joules) and the cgs (calories) systems of metric units. The measurements in our experiment will be done using centimeters and grams, so we will want $\mathcal{J}$ in cgs units. Since calories are already in cgs units, we only need to transform joules into ergs to obtain $\mathcal{J}$ in the proper units. Since 1 J = 10^7 ergs, what is the value of $\mathcal{J}$ in cgs units?

$\mathcal{J}$ = ______________________ erg/cal.

The mechanical equivalent of heat may be defined in symbols by the relationship

$$W = \mathcal{J}Q,$$

where W is the amount of mechanical work in ergs, Q is the equivalent number of calories of heat, and $\mathcal{J}$ is the conversion factor, the mechanical equivalent of heat.

Or, one may write

$$\mathcal{J} = \frac{W}{Q}.$$

The above indicates that $\mathcal{J}$ can also be thought of as the number of ergs of work necessary to produce the same effect on a system as that produced by the absorption of 1 cal of heat.

INVESTIGATION

Materials

Bits of copper and tin, PVC tubes of 1-m length with corks or stoppers to insert in each end, thermometer, meter stick, beakers, plastic cup.

Procedure

In this experiment, the mechanical equivalent of heat is to be obtained by measuring the heat produced by the expenditure of a definite amount of work. Quantities of first lead shot and then copper bits are allowed to fall many times from end to end in a cardboard tube. Knowing the mass of each metal and the length of fall, one can calculate the mechanical work done on each metal by gravity. By measuring the temperature change of each metal resulting from this work, the heat energy gained can be determined from the known specific heat of the metal.

The specific heat has the symbol C, which is not to be confused with the kilocalorie of the Celsius unit. The specific heat of any substance is defined as the amount of heat in calories required to raise the temperature of 1 g of the substance by 1°C. Thus, the amount of heat Q absorbed by each metal with a temperature change of ΔT is

$$Q = mC\Delta T,$$

where m is the mass of the metal, C is the specific heat (given in the table below), and ΔT is the increase in temperature.

To compensate for heat gained from and lost to the room, the shot will first be cooled below room temperature.

Part I

Since there are many pieces of metal falling down a narrow tube, they cannot all fall the entire length of the tube. To determine the average distance that a piece falls with the corks inserted in each end, perform the following measurements, recording them in the data table on the following page.

1. Place one of the corks tightly into the end of the tube and, inserting the meter stick, measure the length to the opposite end. Call this l_1.
2. Remove the meter stick and, holding the cork in place, carefully pour the tin bits into the tube. Holding the tube vertically, measure the distance again to the open end of the tube. Call this l_2.
3. Half the difference between the two distances in Steps 1 and 2 is the position of the average piece of lead from the bottom of the tube. Let $(l_1 - l_2)/2 = h$.
4. Place the other cork securely into the other end. Measure the amount of cork (c) that enters the tube. Subtract this amount from l_1.

5. When flipping the tube, the average piece of metal will fall from a position h above the bottom of the tube to a position h above the cork at the other end of the tube. The average distance that it falls is thus the net distance in Step 4, less $2h$. Call this distance d.

	TIN	*COPPER*
1. Finding average distance of fall, d.		
(a) l_1		
(b) l_2		
(c) $(l_1 - l_2)/2 = h$		
(d) $d = l_1 - c - 2h$		

(Write your result for (d) in row 1 of the table on the next page.)

Part II
You will now measure the mass and temperature of the tin pieces.

6. Determine the mass of the tin by weighing it in the beaker. Since the scale reads only to 300 g, you must weigh the tin in two or more batches. Then subtract the number of weighings times the mass of the beaker. Record your data in the data table.
7. Before beginning, cool the tin in the ice bath, being careful to keep the tin dry. Pour the tin from the beaker into the plastic cup. Insert the thermometer probe into the tin pieces in the plastic cup, covering it as much as possible with the tin. When the tin has reached a temperature around 4° below room temperature, record the temperature in the table. You are now ready to begin.
8. Remove the thermometer and pour the tin into the long tube, holding the cap tightly on the tube at the opposite end. Be careful not to lose any of the bits. Place the other cap tightly on the tube.
9. Holding the caps tightly in place to prevent any bits from escaping, sharply flip the tube over to a vertical position *100* times, keeping careful count. Try not to allow the tin to slide down the tube. Perform each inversion quickly in such a manner that all the pieces fall vertically from one end to the other. Do not raise or lower the tube during the inversions.
10. Carefully but quickly pour the tin back into the beaker. Measure and record the final equilibrium temperature.
11. Using your measurements, calculate the work done on the lead shot (Step **10** in the table); the heat Q produced by this work (Step **11** in the table); and the mechanical equivalent of heat (Step **12** in the table).

Part III

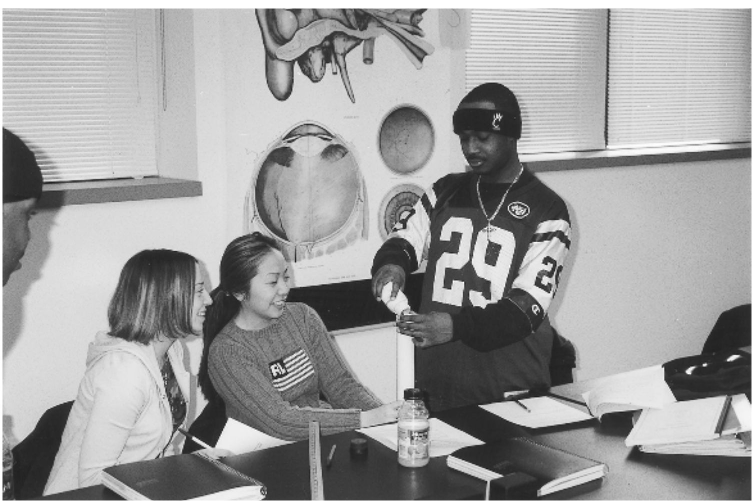
Repeat the above procedure using the copper bits.

Data and Analysis

	TIN	*COPPER*
1. Average distance of fall, *d*.		
2. Mass of metal and beaker		
(a) First batch		g
(b) Second batch		g
(c) Third batch		g
(d) Total		g
3. Mass of beaker		g
4. Mass of metal		g
5. Initial temperature of metal		°C
6. Number of times the metal falls		N
7. Final temperature of the metal		°C
8. Specific heat of metal (cal/g°C)	**0.054**	**0.092**
9. Acceleration of gravity (g)	**980**	**980** cm/s^2
10. Work done on the metal $W = Nmgd$		erg
11. Heat produced by this work $Q = mC\Delta T$		cal
12. Mechanical equivalent of heat $\mathcal{J} = \frac{W}{Q}$		erg/cal

13. Mechanical equivalent of heat, average of your results for two metals in the above table: ________________erg/cal.

14. Mechanical equivalent of heat, accepted value: ________________erg/cal.



THOUGHT QUESTIONS

1. How do the measured and accepted values for J compare? Express this as a relative error (percentage difference).
2. List some of the sources of experimental error in this experiment.
3. Why did we use a PVC tube and not an aluminum tube?
4. An unknown metal has a specific heat only half that for tin. If you performed this experiment on the metal, would you expect the temperature increase to be less than, equal to, or greater than that for lead? Explain.
5. Joule's biographer reports that Joule took a thermometer with him on his honeymoon to the Swiss Alps in order to measure the temperature increase of water landing at the base of a high water fall. Niagara Falls is 59-m high (5900 cm). If Joule were to honeymoon at Niagara Falls, what temperature increase of the water would he find? What assumptions should be made in this measurement? Use the accepted value of J.

   (*Hint:* Note that $mgd = JQ = JmC\Delta T$, and C for water is 1 cal/(g°C). Thus, $\Delta T = gd/JC$. Note that you do not need to know the mass of the water!)

8. EXPLORING HEAT TRANSFER AND THE LATENT HEAT OF FUSION (CHAPTER 7, SECTION 16.2)

INTRODUCTION

Most substances can appear in three states, or phases: solid (frozen), liquid, and vapor (gas). The amount of heat energy—or, in the old caloric theory, the amount of caloric fluid—that each atom possesses determines the state of the substance: the least energy in the solid state, the most energy in the gaseous state.

Heat flowing from one substance to another often simply warms up the substance that gains the heat, increasing its temperature. However, if sufficient heat is absorbed, the substance can undergo a change of state, e.g., from solid to liquid, or from liquid to vapor. In this case, the added heat does not increase the temperature of the substance while the change of state takes place. Instead it serves to break up the structure of the substance as it transfers to the next state. For instance, a certain amount of heat energy is required simply to melt a substance in a frozen state, turning it into liquid at the same temperature. Since this heat seems to "disappear" (i.e., the temperature does not change during the phase transition), it is called "latent" heat.

There are three parts to this investigation.

PART I. HEAT TRANSFER IN AIR

As everyone knows, hot water in contact with air will cool down. This is said colloquially because heat gradually "flows" from the hot water to the cooler air. The reverse occurs when cold water is exposed to air.

But does the heat "flow" as quickly in each direction?

Does the presence of frozen ice in cold water have any effect on the rate of temperature change?

1. Before you start, how would you answer each of these questions?
2. Now let's see what actually happens in each of these three cases. The three cases are:
   (a) plain hot water;
   (b) a mixture of ice and cold water;
   (c) plain cold water.
3. Use the insulated cups, hot water, and crushed ice to prepare each of these situations. Why should you keep the lids on the cups until you are ready to begin?
4. Since you are investigating the rate of heat transfer in these three situations, the time will be an important factor, as well as the temperature. Before starting, create a table in which to record the temperature and the time for each cup.
5. Record the starting temperature in each cup, the room temperature, and the time.
6. Now *remove the tops from the cups*, so that heat can flow from and to the room.
7. Continue to measure the temperature of the water in each cup every minute for 5 min, and record.
8. Put these cups in a safe place to the side (*without tops*). During the rest of the experiment, continue to measure the temperature of each cup about *every 10 min* until after you have finished Parts II and III of this experiment. Be sure to record both the temperatures and the times of observation in the data tables. (This will require some organization of time and equipment by your group.)

Analysis

1. After you have completed all measurements. You can observe and compare any trends more easily if, *on a single sheet* of graph paper, you plot the temperature on the vertical axis and the elapsed time in minutes on the horizontal axis (starting time is 0 min) for each of the three sets of data. Draw a smooth curve through each set of points. Label your axes and curves. Also indicate room temperature on your graph and draw a horizontal line at that temperature.

2. Examine your results. Describe any differences you see between the three curves.
3. How do results compare with what you expected to find (Question 1)?
4. How would you account for the differences between these three curves, using the kinetic–molecular theory and the idea of latent heat?

PART II. CONSERVATION OF HEAT IN MIXTURES

(Remember to continue recording every 10 min the temperature of the water and the time of observation for the three cups in Part I.)

In this part of the experiment, you will test whether or not heat is conserved (not lost) when it flows from a liquid at high temperature to another liquid at a lower temperature.

In this case, the two samples of liquid are both water. They will be mixed together until they reach a uniform temperature (this is when heat ceases to flow). To determine if heat is conserved, we can compare the observed final temperature with the value predicted on the assumption that heat is conserved. Naturally, there is some systematic error, since some heat is lost to the air and surroundings. So the predicted and observed values may not exactly match. But if they are close, then we know that heat was conserved within the limits of experimental uncertainty.

As discussed in the text, when heat flows into or out of any substance without a change of phase, its temperature changes. The amount of heat, Q, is proportional to the temperature change, $\Delta T = T_f - T_i$. If ΔT is positive, heat flows into the body; if ΔT is negative, heat flows out. The amount of heat is also proportional to the mass m, and there is a proportionality constant C, which is different for each substance and is called the specific heat. Thus

$$Q = mC\Delta T.$$

Since we don't yet know the starting temperatures of the liquids, we can't make any predictions at this time. We will have to start with observation, then compare with our predicted result for this particular case.

1. Using a graduated cylinder and insulated cups with tops, prepare 50 ml of hot water and 50 ml of cold water without ice. (Be careful not to burn yourself in obtaining the hot water.)
2. Measure and record the temperature of the water in each cup.
3. Now pour one cup into the other, keeping the top on as much as possible.
4. Record the equilibrium temperature.

5. Now repeat this procedure, but use *different* quantities of cold and hot water, for example, 75 ml cold water and 25 ml hot water, and record your measurements. Create a table in which to present your data in both cases.

Analysis

1. Now let's see if *experimentally* the heat gained by the cold water is equal to the heat lost by the hot water. To do this, use the equation for the heat gained or lost, $Q = mC\Delta T$, to find the heat gained by the cold water and the heat lost by the hot water.

   Here m is the mass of each amount of water, C for water is just 1 cal/(g°C), and $\Delta T = T_f - T_i$. Negative values for ΔT and Q simply indicate a loss of heat.

   (It is fortunate that the metric units are defined in such a way that for water 1 ml contains 1 g of water. The specific heat C for water is defined by convention to be 1 cal/g°C.)

   Trial 1
   $Q_c = m_1 C\Delta T_c =$,
   $Q_h = m_2 C\Delta T_h =$.

   Trial 2
   $Q_c = m_1 C\Delta T_c =$,
   $Q_h = m_2 C\Delta T_h =$.

2. If heat is conserved in each mixture, how should you be able to determine this from the above calculations?
3. Taking into account uncontrolled losses and gains of heat, from your results so far would you say that heat was conserved or not in each case?
4. If the two numbers in each case are not exactly equal, how much net heat was lost or gained? Indicate whether heat is lost or gained.

   Trial 1:
   Trial 2:

Making a theoretical prediction

You can calculate *theoretically* what the value of the equilibrium temperature T_e in Trial 1 *should have been* by assuming that heat is conserved and that absolutely no heat is lost or gained from the surroundings.

If heat is conserved, then

$$\text{heat gained} = \text{heat lost}$$
$$m_1 C\Delta T_c = -m_2 C\Delta T_c,$$
$$(T_e - T_1) = (T_2 - T_e).$$

C cancels out since only liquid water is used; no change of state occurs. The masses also cancel out, since they are both the same (50 ml = 50 g). This is now an equation for one unknown, T_e, since all of the other factors are known from the above table.

Substitute in your values for T_1 and T_2 from Trial 1 above and solve the equation for the unknown T_e.

Comparing the theoretical and experimental results

1. How does the observed final temperature compare with the predicted value in Trial 1?
2. *Taking experimental error into account*, what does this say about the conservation of heat in mixtures?

Extra credit

Make the same prediction of the equilibrium temperature in the case of Trial 2 above. Note that C cancels out again, but that the masses are now different.

PART III. LATENT HEAT OF FUSION OF ICE

(Remember to continue recording every 10 min the temperature of the water and the time of observation for the three cups in Part I.)

A. Observing Latent Heat

1. If you placed several ice cubes in a glass or metal container and started heating the container, the ice would melt. That's obvious, but what will happen to the temperature of the mixture of ice and liquid water as the ice melts?
2. Now try the above experiment. Record the temperature every few minutes, until several minutes after all of the ice is melted.
3. Compare your observations with your prediction in Question 1.
4. How would you explain what you observed?

Extra Credit

Make a graph of your results as a function of time.

B. Measuring Latent Heat

In this part of the experiment, you will actually measure the latent heat of fusion of ice. We will make several assumptions about the melting process, based on Parts I and II. The main assumption is the conservation of heat, with only negligible loss of heat to the surroundings. By observing the cooling of hot water, when an ice cube at melting temperature (0°C) is placed into it, we also assume that all of the heat extracted from the hot water is used for the melting of the ice cube, followed by the heating of the water from the melted ice (which starts at 0°C) to the equilibrium temperature of the mixture. This may be expressed as follows in symbols and words:

$$Q(\text{lost by hot water}) = Q(\text{latent of ice}) + Q(\text{gained by cold water obtained from melted ice})$$

or

$$Q(\text{latent of ice}) = Q(\text{lost by hot water}) - Q(\text{gained by cold water obtained from melted ice}).$$

Since there is no temperature change of the ice while the ice melts, the heat Q required to melt the ice is the number of calories per gram. Instead of Q, the symbol for latent heat is usually given as L. The value of L, the latent heat of fusion of per gram of ice, has been found experimentally to be

$$L = 79.4 \text{ cal/g}.$$

If there is more or less than 1 g of ice, the heat required to melt the ice is simply

$$Q(\text{to melt ice of mass } m) = mL.$$

Thus, to turn 10 g of ice at 0°C into 10 g of water at the same temperature requires an amount of heat equal to $Q = mL = 794$ cal. Note that water obtained from melting ice is always at an initial temperature of 0°C.

We can now rewrite the earlier equations for heat transfer in the melting of an ice cube in hot water as follows:

$$Q(\text{lost by hot water}) = mL + Q(\text{gained by cold water obtained from melted ice})$$

or

$$mL = Q(\text{lost by hot water}) - Q(\text{gained by cold water obtained from melted ice}).$$

The two expressions for heat on the right side of the last equation, Q(lost by hot water) and Q(gained by cold water obtained from melted ice), involve the familiar relationship $Q = mC\Delta T$.

In order to obtain the value of L from the second equation, we need to know the masses of the ice cube and the hot water, and the change of temperature of the hot water and the cold water from the melted ice cube.

Now you try it

Find the latent heat of fusion of ice, L. Start by measuring the mass and temperature of 100 ml of hot water (be sure not to include the container in the mass). Obtain an ice cube and allow it to begin melting, which indicates that it is at the melting temperature of 0°C. Drop the ice cube into the water and measure the equilibrium temperature after all of the ice has melted. Find the mass of the ice and resulting water by measuring the new mass of the water and subtracting the initial mass of the hot water from your measurement. Create a table in which to present all of your measurements. Finally, using the data in your table, calculate each of the two heat expressions on the right side of the above equation, then solve for L.

This will require some thought and discussion, and perhaps some help, but you should be able to carry this out and obtain a fairly good result.

Analysis

1. How does your result for the latent heat L compare with the accepted value? Express your answer as a percentage difference.
2. How do you account for the variation from the accepted value?
3. Calculate your contribution to the net entropy gain of the universe in the melting of the ice cube. Note that $\Delta S = \Delta Q/T$, where T is kelvins (not celsius). At the freezing point of water $T(\text{K}) = 0°\text{C} + 273°\text{C}$.

(Complete Part I of this investigation.)

9. INVESTIGATING WAVES (CHAPTER 8)

PART I. WAVES ON SPRINGS

Many waves are too fast or too small to observe easily. Using a long metal spring and a Slinky you can make large waves that move slowly enough to study.

A. Longitudinal Waves

1. Together with a partner, pull the Slinky out across the laboratory table or on the floor to a length of about 14 ft (6 m). (Do not pull it so far that the spring is bent.) Create a longitudinal pulse from one end by grasping the stretched spring about 20 cm from the end with your free hand. Pull the spring together toward your end, then release it, being careful not to let go of the fixed end with your other hand.
2. Try this from either end and then from both ends simultaneously. Write down everything you observe.

3. What happens when the two waves meet? This is called *interference*.
4. In order to see the longitudinal wave more easily, tie pieces of string to a loop of the spring at several places. What do you observe about their motion as the pulse passes?
5. Does a pulse carry matter all along its path, or does it carry something else? Explain your reasoning.

B. Transverse Waves

1. Leave the strings attached to the Slinky. To create a single transverse pulse, move your hand quickly back and forth once at right angles to the stretched spring. Try this from each end of the Slinky, while the other end is held steady.

   Perform experiments to answer the following questions about transverse pulses.

   (a) What is the motion of the attached strings as a transverse pulse passes by?

   (b) Does the size of the pulse change as it travels along the spring? If so, in what way?

(c) Does the pulse *reflected* from the fixed far end return to you on the same side of the spring as the original pulse, or on the opposite side?
(d) What happens when two pulses on opposite sides of the spring interfere? Try to draw what happens before, during, and after.
(e) What happens when two pulses on the same side of the spring interfere? Again, try to draw this.

C. Standing waves

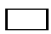
1. Use the thin helical spring. Note that it is much more taut than the Slinky. Again with a partner, stretch it to about 14 ft (6 m). Repeat your observations and conclusions about transverse pulses. Is the speed of the pulses any different compared with the Slinky?
2. By vibrating your hand steadily with the same amplitude, back and forth perpendicular to the spring, you can create a train of pulses, a *periodic wave*. When this wave reflects off the opposite, fixed end, it interferes with itself and forms a *standing wave*. Try this.

3. A standing wave is also created if periodic transverse waves are sent from both ends of the spring. The waves must be of the same size and the frequency. Try this.
4. In either case, how does the frequency of the oscillation affect the wavelength of the standing wave? Can you express this as a proportionality?

PART II. WAVES ON WATER

So far you have observed reflection, refraction, and interference of waves moving along one dimension. In order that you can make more realistic comparisons with other forms of traveling energy, especially light waves, you can first observe the same wave properties spread out over a two-dimensional surface, the surface of water.

Pulses, waves, and interference

1. Put a large yellow or other bright color cafeteria tray on a horizontal table and fill it with water almost to the top, but don't fill it to the top. Orient the tray sideways in order to minimize the effects of reflection off the sides of the tray. 
2. To see what a single pulse looks like on water, gently touch the surface with the eraser of a pencil or the cap end of a ball-point pen. Then with the pipette dropper held only about a centimeter above the surface, let a single drop of water fall onto the surface. Sketch and describe the pulses on the water surfaces which you observe.
3. Using two pipette droppers at opposite sides of the tray, let a single drop of water fall onto the surface simultaneously from each dropper. Carefully observe what happens when the two pulses meet. Describe and sketch your observations before, during, and after the interaction. This is an example of *interference*.
4. You can generate a single straight pulse by moving the small plastic ruler back and forth sharply once in the water. Use the large ruler to act as a barrier in the water, about 8 in from the source. Then observe what happens when a continuous series of straight pulses forming a periodic wave strikes the barrier.
5. By changing the frequency of the motion of the small ruler, you can set up a standing wave. Sketch and describe your observations.

Diffraction patterns

Orienting the tray sideways as before, use a continuous series of straight pulses forming a periodic wave to observe what happens in the following situations. Describe and sketch what you see. These are examples of *diffraction*.

1. Use the larger ruler to generate a periodic wave that hits the smaller ruler straight on. Observe what happens on the sides of the barrier.
2. Place the 100 g mass in the water to act as a small barrier. Create straight waves of wavelength about the size of the mass, striking the mass. Observe and sketch what you see.
3. Now allow the straight waves to strike a barrier with a gap in the middle that is about the size of a wavelength. Use two small rulers to form the barrier and gap.

Interfering waves from point sources

1. Set up a standing wave in the tray by using a pencil eraser or a ballpoint pen as a single point source. Strike the surface gently at a constant frequency near one of the long sides of the tray. Change the frequency and observe what happens to the wavelength. Describe and sketch your observations.
2. Observe what happens when the waves emitted simultaneously by two point sources near each other interfere. Use two pipettes or two pencils held together by their points so as to form a double source. Vibrate the source rapidly near one side of the tray to set up the standing wave interference pattern on the surface of the water.

   If you look carefully, you can observe the patterns of constructive and destructive interference (nodal lines) that spread across the tray to the opposite side. Notice how they form a pattern that is much wider than the distance between the two point sources. Carefully sketch what you observe and label the lines representing constructive and destructive interference. What type of line is at the center of the pattern?

Now you try it

Investigate any other properties of one- or two-dimensional waves that you would like to know more about.

1. Write down a question in advance.
2. Describe what you did to answer it.
3. Describe the answer you found. Draw the result, if appropriate.

Thought question

If the tray had been filled with small particles instead of liquid, in what ways would particles behave differently from water waves in some of the above observations?

10. SPACETIME: A COMPUTER EXCURSION INTO RELATIVITY THEORY (CHAPTER 9)

Spacetime is a DOS program created by Professor Edwin F. Taylor and his students at MIT. It is used under license from Physics Academic Software, American Institute of Physics, College Park, MD.

STARTING AND EXITING THE PROGRAM

After you have started the computer and it has completed booting to Windows, click the Start button, then click Run. Type the location of the program, then click OK or press Enter. The program starts in a full-screen DOS window.

Make sure "1 Run SPACETIME" is highlighted, then press Enter twice. Then press any key.

You can exit the program at any time, by typing Q. Then hold down the Ctrl key and type C.

THE DISPLAY

The display you see is called the *Highway Display*. The large blank area in the middle of the window represents a "superhighway" running from left to right across the screen. Different lanes on the *Highway* are for objects traveling at different speeds. Objects lying on the horizontal line through the middle of the screen are on the center strip of the *Highway* and do not move. They are at zero velocity relative to the computer screen (and to you). Objects above the center move to the right; the farther above the center, the faster they move. Objects in the very top lane move to the right with the speed of light; only light flashes (and neutrinos) can occupy this lane.

Objects below the center of the screen move to the left; the farther below the center, the faster they move. Objects in the very bottom lane move to the left with the speed of light; only light flashes (and neutrinos) can occupy this lane.

The *Highway* convention is British, but modified; vehicles drive on the left side, but slow lanes are near the center strip, fast lanes are near the edge of the road.

To understand what appears on the *Highway*, think of a movie of all the objects traveling along the *Highway*. At any given instant, you are looking at a single "still picture" of the movie. As you change the time, you change the movie as you progress from one still to the next. You are going to learn how to make such movies and how to step time forward and backward through the stills of the movie.

A vertical ruler, at the left of the screen, shows β (Greek beta), the velocity as a fraction of the speed of light: $\beta = v/c$. β is also indicated at the bottom of the screen. The range of β extends from $+1$ at the top ($v = c$, light moving to the right), through 0 at the center, to -1 at the bottom ($v = -c$, light moving to the left). Notice that this is not a linear scale; equal vertical lane separations do not correspond to equal changes in β. This is done so that more of the interesting velocities near $\beta = v/c = 1.0$ can fit on the screen. The other parameter is called γ (gamma). It is equal to $1/\sqrt{1-\beta^2}$.

The square object at the center of the screen represents the Earth. The vertical red line indicates the position $x = 0$. The number inside the Earth's square represents the time registered on a clock attached to the Earth. The Earth starts the position $x = 0$ at time $t = 0.0$. We'll let the units on the clock represent minutes.

You're now ready to start.

RELATIVITY OF LENGTH

Press R (for rod). A cross bar appears.

Press $\rightarrow$ several times to move the rod to the position $x = +4.0000$. You can see it on the x-axis, or read the position at the bottom of the screen.

Press Enter to set the rod at this position.

A rectangle appears representing a rod at $x = 4.0$ and $v = 0$ relative to the Earth.

We'll place a second rod at a different place:

Press R. A crossbar appears.

Press the up arrow to set $\beta = +0.9000$ (or $v = 0.9000c$), as indicated at the bottom of the screen.

Press $\leftarrow$ several times to move the rectangle to $x = -4.0000$, also indicated at the bottom of the screen.

Hit Enter. Another rectangle appears.

It is a rod of the same length as the first rod when at rest on the Earth, but now we have a "snapshot" of the rod flying toward the Earth at $v = 0.9c$ as seen from the Earth.

Now let time move forward by holding down the up arrow. What has happened to this identical moving rod as seen from the stationary Earth?

Notice that there is an effect only on the length of the rod, not on its width.

Release the up arrow and press zero (0) to return to time 0.0 on the Earth.

Changing reference frames

Suppose you could jump from the Earth to the moving rod, so that you are riding at rest on it (at rest in its reference frame) and the Earth and the first rod are now moving toward you.

Will there be any change in the length of the first rod as measured by you? If so, what change?

Will there be any change in the distance between the Earth and the first rod? If so, what change?

Now let's make the jump:

Press F6 to select an object.

Press C to select the moving rod. A box appears around it to indicate selection.

Type J to jump to the frame of the moving rod. Now it's at rest on the center line and the Earth and first rod are in the speed lanes moving to the left.

What has happened to the length of the previously moving rod? Why?

What has happened to the length of the previously stationary rod? Why?

What has happened to the distance between the Earth and the rod moving with it? Why?

What has happened to the time registered on the Earth's clock?

(The reason for this is the relativity of simultaneity. A clock on the rod at rest and a clock on the moving Earth are not synchronized.)

Press the up arrow to let time move forward.

Return to time 0.0 by pressing 0.

Now jump (press J) to rod B, which is at rest relative to the Earth, and note your observations about the Earth and the other rod.

Trying different relative speeds

Press N twice to start over.

Now place rods at different distances from Earth and different speeds up to the highest + and − values for β you can obtain.

What happens to the lengths of the rods as β increases? Why?

What happens to the widths of the rods as β increases? Why?

What is the highest value for β that you can obtain? Why can't you go any higher?

RELATIVITY OF TIME

Press N twice to start over, with the Earth at rest in the center at $x = 0$ at time $t = 0.0$.
Press C (for clock). A crossbar appears.
Use the arrows to move it to $x = -4.0000$ and $\beta = +0.900$.
Press Enter to place the clock in this position.
Notice that the time indicated on the moving clock is not 0.0 but 8.3 min. Again, this is because of the relativity of simultaneity. We cannot synchronize a moving clock with a stationary clock.

Write the starting times of the Earth clock and the moving clock in the table below:

Table of clock readings. Clock is moving relative to stationary Earth.

	Start	*Stop*	*Elapsed time*
Earth's clock			
Moving clock			

Now press the up arrow to let time move forward on the Earth to 9.0 min as the moving clock flies to the right past the Earth at speed $v = 0.9c$.
If you pass 9.0, move time backward by pressing the down arrow (unfortunately not possible in real life).
Record the new clock readings in the above table and obtain the elapsed time recorded by each clock.
What do you conclude about the rate of the moving clock compared with the rate of the Earth's clock at rest?

Changing the reference frame

Suppose you jumped from the Earth to the moving clock, so that you are riding at rest on it (at rest in its reference frame) and the Earth is now moving toward you.

From this perspective, will there be any change in the rate of the two clocks from what you just observed? If so, what change do you predict?

Let's make the jump, as before.
Press 0 to go back to time 0.0 on the Earth's clock.
Press F6 then B to select the moving clock (B).
Jump to the moving clock by pressing J.

You now have a still picture of the previously moving clock, which is now at rest in your reference frame on the center line, with the Earth and its clock moving toward you from the right.

Again the two clocks are not synchronized, and you can see a contraction in the distance between your clock and the Earth.

Record the starting time on the two clocks in the table below:

Table of clock readings. Earth is moving relative to stationary clock.

	Start	*Stop*	*Elapsed time*
Earth's clock (moving)			
Stationary clock			

Now let time move forward for a while by pressing the up arrow. *Stop the motion before the Earth goes off the screen.*

Record the new clock readings in the above table and obtain the elapsed time recorded by each clock.

What do you conclude about the rate of the clock on the moving Earth compared with the rate of the clock at rest relative to you?

Compare your two observations of the rate of time as measured from the two reference frames of the Earth and the clock. Is there a contradiction, or are they consistent with each other?

When you are finished, press N twice to return to the opening screen.

OPTIONAL

A TRIP TO ALPHA CENTAURI: THE TWIN PARADOX

Now let's take a longer trip. The visible star nearest to our Sun is Alpha Centauri, about 4 light years distant. You will remain on Earth while your identical twin will travel on the Space Shuttle at a speed of $v = 0.9c$ to Alpha Centauri and back. Assume that the distance units on the x-axis display are light years and the time units on the clocks are years.

As before, the reference clock in the center of the start-up screen represents Earth.

Press C, then use the up arrow, and Enter, to place a second clock, representing Alpha Centauri, on the center strip ($\beta = 0$) at a distance of 4 light years to the right of Earth ($x = +4.0$).

(Notice that this time the two clocks are synchronized, since they are at rest relative to each other.)

Now prepare the Space Shuttle for the trip to Alpha Centauri.

Press S (for Space Shuttle) and use the up arrow to move the Space Shuttle straight up to the speed lane $\beta = +0.900$ at the Earth's position, at $x = 0.0$.

Press Enter to create the Space Shuttle.

Press the up arrow four or five times to step time forward. Watch the Space Shuttle move toward Alpha Centauri from you position on Earth. Note that its clock runs slower than the clocks at rest, as we expect.

We need details of the Space Shuttle's position in order to line it up with Alpha Centauri. Get the Space Shuttle details this way:

- Press F6, then press S to select the Space Shuttle.
- When the box appears around the Space Shuttle, press I for information.
- Details of the Space Shuttle's position, speed, and time will appear across the bottom of the screen. This information is updated as you move the Space Shuttle toward Alpha Centauri.

Keep changing time until the Space Shuttle is approximately lined up with Alpha Centauri (when its position is approximately $x = 4$ as shown at the bottom of the screen). The lineup with Alpha Centauri will not be perfect, actually $x = 4.0500$.

Now we need to turn the Space Shuttle around by placing it in a lower lane so that it can head for home.

Press P. A crossbar appears at the position of the Space Shuttle.

Move the cursor down with down arrow key until it is in a lane below the center strip with $\beta = -0.900$. (Note the minus sign.)

Press Enter, when you get to the correct lane.

Now the Space Shuttle is turned around and ready to head for home.

Press the up arrow to move time forward again.

Bring the Space Shuttle back to Earth ($x = 0$).

Welcome your twin home after all this time.

Read the number of years on the Earth's clock and the number of years on the Space Shuttle's clock. These are the number of years that have passed for each clock.

Earth's clock: ______________________________

Space Shuttle's clock: ______________________

Who has aged less: you or your twin, the Space Shuttle pilot?

But wait a minute. Your twin could say that he or she had actually gone nowhere, but that it was the Earth that had flown 4 light years away and returned. So, when you return, riding with the Earth, it is you who should be younger, not your twin.

Obviously you both can't be younger. So who's right? This is a way of phrasing the so-called *Twin Paradox.*

Let's try to see this from your twin's perspective.

Press 0 to reset the time to zero.

Press F6, then S to select the Space Shuttle, if it's not already selected.

Press J to jump to the Space Shuttle.

Now the Space Shuttle is at rest with respect to you on the center line at $x = 0$. Earth is lined up just below you, at $x = 0$ but in a leftward-moving lane. Alpha Centauri is in the same lane as Earth, but farther to the right.

Earth and Alpha Centauri are moving in your rest frame; therefore, the distance between them is contracted, just as the length of a moving rod was contracted.

Now hold down the up arrow to replay the movie of the earlier trip to Alpha Centauri and back to Earth, this time while riding on the Space Shuttle.

The Earth will turn around automatically and return to the Space Shuttle.

The Earth is always the object to the left of Alpha Centauri. Try to line it up just above the Space Shuttle in the center of the screen.

The motion is very rapid because the distances are contracted. If you want to repeat the motion, press 0 (zero), then the up arrow.

Now what are the times registered on each of the clocks?

Which twin would be younger?

How can we resolve the twin paradox?

The answer is that the two situations are not quite identical. There is a crucial difference between the motion of the twin who left on the Space Shuttle and the motion of the twin who stayed at home. The twin who stayed at home remained at zero or constant velocity for the entire time. But the twin who left on the Space Shuttle had to turn around, and when the Space Shuttle turned around, he or she experienced an *acceleration*. Because of the acceleration both twins can then determine that it was really the one on the Space Shuttle who traveled while the other one stayed at home. Therefore the traveling twin will be younger.

11. EXPLORING ELECTRIC CHARGES, MAGNETIC POLES, AND GRAVITATION (CHAPTER 10)

A. Comparing the Three Forces

- In what ways are the gravitational, electric, and magnetic forces similar to one another?
- In what ways are they different?

1. Your instructor will provide you with a variety of small objects to subject to electric, magnetic, and gravitational forces. By observing what happens in each case, you will be able to draw some conclusions in answering the above questions.
2. In order to organize your observations, construct a table in your notebook with four columns. Label the first column "Objects" and list all of the objects you have. Label the other columns "Gravitational Force," "Electric Force," and "Magnetic Force."

Gravitational force

1. How could you determine whether or not an object is subject to the gravitational force of the Earth? In the second column of your table, indicate which of the objects is subject to the gravitational force downward toward the Earth.
2. Was there anything you had to do to initiate the gravitational force to act on the objects?

Electric force

1. Separate the objects from each other. Rub the clear plastic (acrylic) rod with the silk, or the dark plastic (delvin) rod with the fur. Pass the rod over each object and observe what happens. If there is an attraction upward, this is the result of an *electrical interaction* between the rod and the object. The upward motion is due to an *electric force*. Indicate in your table which objects respond to an electric force.
2. Compare the electric force upward to the rod with the gravitational force downward to the Earth. Which force is stronger?
3. Is it possible that the electric force is actually due to a gravitational attraction of the objects to the rod? Explain.
4. Was there anything you had to do to initiate the electric force?

Note: The term *charge* is used to describe the property of an object that enables it to engage in electrical interactions with other objects. Notice that the other object does not have to be charged, although it can be. You'll see examples later in this investigation.

The rubbing of the rod produces a build up of charge on the rod. This charge is often called an *electrostatic charge*, since the charge is static (not moving). You encounter an electrostatic charge when you experience electrostatic cling on a cold day with low humidity, or after running a comb through your hair. Humidity enables the charge to escape from the rod by clinging onto water molecules in the air. The charge can be produced again by rubbing (friction).

Magnetic force

1. Perform the same test on the objects as you performed with the electric force, only this time use the bar magnet. Write your results in the table.
2. What do you conclude from your results about the types of objects that are subject to the magnetic force? What types of objects are not subject to the magnetic force?
3. Compare the magnetic force upward to the bar magnet with the gravitational force downward to the Earth. Which force is stronger? Explain your reasoning.
4. Is it possible that the magnetic force is actually due to a gravitational attraction of the objects to the magnet? Or an electrical attraction of the objects to the magnet? Explain in each case.
5. Was there anything *you* had to do to initiate the action of the magnetic force?

Conclusions

On the basis of your observations so far, as recorded in your tables and in your answers to the questions, answer the two opening questions:

- In what ways are the gravitational, electric, and magnetic forces similar to each other?
- In what ways are they different?

B. Like and Unlike Charges

- How many types of electric charges are there?
- How do they alter the direction of the electric force?

1. You can easily generate an electrostatic charge by pulling a piece of Scotch tape off a clean dry surface. Take a piece of Scotch tape, about

5 to 8 cm in length. Bend the end of it over to form a little "handle." Tape it onto a clean table top or other surface, then peel it off briskly and stick it to the crossbar, just below the "handle." Be careful not to let the tape curl back to touch your hand or any other object. If it does touch another object, or if you need to recharge it, simply stick it again to the surface and briskly peel it off.

2. After you have prepared one tape, prepare a second tape pulled from the same surface. Bring it near the first tape (sticky sides facing away). What do you observe?
3. Construct a table listing at least three different surfaces—such as wood, glass, plastic—in the first column, and the same surfaces across the top. Now compare strips pulled from the same and different surfaces, and record your results in the table. Do not let the strips touch each other.
4. Strips that are pulled from the same surface always have like charges, since they are prepared in exactly the same way. What do you conclude about the electric force between like charges?
5. Strips pulled from different surfaces usually may have unlike charges. What do you surmise from your observations about the force between unlike charges?

How many?

Let's see if we can determine how many different types of charges there are.

1. You can create unlike charges by again using Scotch tape. Stick one piece of tape (with a handle) to a surface. Then stick another piece directly over it. Keeping both pieces stuck together, briskly pull both pieces of tape together off the table. Now carefully separate the two pieces. Do not allow them to curl back to your hand or to touch each other after separated.

   After they are separated, bring the two strips near each other, back to back. What do you observe?
2. What does your observation tell you about the nature of the charges on the two strips? (Repeat this again, until you are convinced of your conclusion.)
3. If you found these charges to be unlike and mutually attracting, you can use them to test other charges. To do so, prepare another two strips of unlike charges and place them carefully on the crossbar. Replace or recharge them as necessary in the following.
4. Construct a table with different charged objects in the first column and Strip 1 and Strip 2 in the next two columns. Bring the charged clear and plastic rods (using silk and fur, respectively) near each strip and

observe what happens. Do not touch the strips with the rods. Try charging other objects, such as a balloon or a comb, by friction and bring each one near the charged strips of Scotch tape. Observe whether the interaction with the strip is attractive or repulsive with each strip. Be sure that each object is indeed charged. Record the result in your table.

Conclusions

1. Carefully examine your table. What conclusions can you draw from this?
2. Is there any charged object that repels or attracts *both* strips?
3. If a charged object attracts one of the two strips, what does it do to the other strip. Is there any exception to this?
4. You saw previously that like charges always repel. What do your observations say about unlike charges?
5. What do you conclude from your observations about the number of different types of charges? Support your conclusion.

   Note: By agreement, the two different charges have been called "positive" and "negative." But they could have been called "red" and "green," or "up" and "down," or any other names. Ben Franklin chose "positive" and "negative" for various historical reasons. The "positive" charge has been defined as the charge produced on the clear rod when rubbed with silk. The negative charge is defined as the one produced on the dark plastic rod when it is rubbed with fur.

   The electrical interaction between these types of charges is either attractive (between unlike charges) or repulsive (between like charges).
6. How would you now answer the opening questions to this section, and what evidence would you use to support your conclusions?
   - How many types of charges are there?
   - How do they affect the attractive or repulsive nature of the electric force?

C. Neutral Objects

You have learned that all matter is made up of atoms, which contain charges inside them. However, the atoms themselves are usually neutral, because the positive charge of the nucleus is exactly balanced by the negative charge of the electrons orbiting the nucleus. Some materials can have some of their electrons removed when they are rubbed with another material. This is how objects are made to carry net electric charge.

A neutral object has no charges added or removed, so it has a net charge of zero. This does not mean that it has no charge in it. It simply means that the numbers of like and unlike, positive and negative, charges are equal.

What happens when you bring a charged object near a neutral object?

1. To find out, attach the aluminum ball on the thread to a crossbar and bring a charged rod near it but without touching the ball. What do you observe?
2. Now bring the other type of rod (with an opposite charge) near the ball. What do you observe?
3. Does this violate our recent conclusion that "we have never found a charged object that either attracts or repels both of the two strips that attract each other"?

   Note: The answer is no. The key word in the conclusion is *charged* object. The object tested is a neutral object, one without net charge. What is happening here is that the charged glass rod (positive) is attracting the negative charges in atoms of the ball, and repelling the positive charges. The negative charges move toward the front of the ball, and the positive charges toward the back. Because the electric force is proportional to $1/r^2$ (see Section 3.4), the attractive force is stronger than the repulsive force, because the negative charges are closer (r is smaller) to the positive glass rod than the positive charges.
4. Make sure the ball is electrically neutral by holding it in your hand for a moment. Charges from your hand will cancel out any net charge on the ball. Recharge the dark plastic rod with the fur and this time touch the ball, after bringing it close. Carefully observe what happens before and after you touch the ball.
5. How would you explain this?

   Note: Remember, the clear rod charged with silk is (by definition) positive, so negative charges are drawn to the front of the ball. When the rod touches the ball, some negative charges move to the rod, leaving behind a ball that now has a net positive charge; the ball is then repelled by the positive rod.
6. To test this explanation, try the experiment again. This time, bring the negatively charged plastic rod (the dark plastic rod rubbed with fur) near the ball. What do you observe?
7. Does this agree or disagree with the explanation?

D. Magnets

Now let's look at some of the similar properties of magnets.

Properties

1. Using two bar magnets, examine the attractions and repulsions between the ends as well as the middle of each magnet. Be careful to experience

yourself the actual push and pull. If the magnets are strong enough, attempt to experience why scientists (such as Faraday) believed that there is a "field" that exerts the repulsion. Lift up one magnet by the other. What does this say about the strength of gravity on the magnet compared to magnetism?

Like and unlike poles

How many poles does a magnet have?

2. To find out, tie one magnet in the middle and hang it from the crossbar. Place a sticker near one end of the magnet to distinguish the two sides. Now bring one end of the other magnet near the marked end. What do you observe?
3. Now bring the other end of the magnet in your hand toward that end. What do you observe?
4. Repeat this for the other end of the dangling magnet, and record your observations.
5. Is there any side to a magnet that attracts both ends of another magnet or repels both ends?
6. Is there any end of a magnet that attracts one end and does not repel the other?
7. What do you conclude from this?

   Note: The end sides of a magnet are called the *poles* of the magnet. As with charges, like poles repel and unlike poles attract. One pole is called the "north" pole and the other pole is called the "south" pole. There was a good reason for this. As discussed in Section 10.1 of the text, Gilbert discovered that the Earth itself is a magnet. The end of the magnet that seeks the geographic North Pole of the Earth is called the "north-seeking pole." It is actually the "south pole" of the magnet. The pole that seeks the Earth's South Pole is the magnet's "north pole."
8. Using the little compass, determine the directions of north, south, east, and west in your room. Be sure that you are far away from any nearby magnets. Now compare the approximate alignment of the dangling bar magnet with the geographic directions. (Be sure the string is not twisted.) Use a piece of tape to indicate which end of the bar magnet is its north pole and which is its south pole.
9. To continue the investigation in Part A, do magnets have an effect on the electrically charged Scotch tapes? Prepare Scotch tapes with unlike charges and see if there is any effect. What do you conclude from this about the electric and magnetic forces?

E. Magnetic Fields

You may already have experienced the repulsion generated by the magnetic field between the opposite poles of two bar magnets. Can a thin piece of material block the magnetic field?

1. To find out, place a sheet of paper vertically near the dangling magnet, then bring the other bar magnet close to the first magnet but behind the sheet of paper. Does the paper block the field?
2. Try some other objects, such as aluminum foil, glass, a piece of copper, or steel, a hand, etc. Which ones block the field and which do not?
3. You can trace out the field by using iron filings spread over a transparency or a piece of paper lying on top of the magnet. *Do not put the filings directly on the magnet(s)*. Sprinkle the filings over the transparency lying on top of a single bar magnet and sketch the result.
4. Place two like poles near each other. Place a sheet of paper or a transparency over the region between them and use the iron filings to sketch the result.
5. Do the same with unlike poles near each other, but not touching.
6. What are the characteristics of the field for attraction and repulsion?

Mapping the field

1. The magnetic field is a vector, and you can "map" the field near a bar magnet by using a small compass. The direction of the magnetic field at any point is defined as the direction in which the north pole of a compass at that position is pointing. The compass needle is tangent to the magnetic field line at that position. Note that the end of the compass that points to the magnetic north of the Earth is actually the south pole of the compass needle.
2. Using the small compass, plot the magnetic field at various positions around a bar magnet and draw your result. Indicate the direction of the field at each position, and the north and south poles of the bar magnet. Draw your result here.

12. INVESTIGATING ELECTRIC CURRENTS I (CHAPTERS 10, 16)

A. Let There Be Light!

You are given a battery, a light bulb, and some wires. The battery has a voltage of only 1.5 V and will not cause a shock or any harm to you.

Working together, think of the right arrangements ("circuits") to get the light bulb to light; then try them.

Sketch each arrangement that you try, including those that do not work.

When you find an arrangement that works, try to find another, similar arrangement that will also work.

Conclusions

1. What is the common feature of the arrangements that work?
2. What is the common feature of the arrangements that do not work?

B. The Bulb and Battery Holders

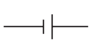
For convenience in making electrical connections, bulbs are usually screwed into sockets and batteries placed into holders.

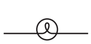
1. Carefully examine the bulb socket and the battery holder. Then place the bulb and battery into their holders and hook up the wires to obtain the lighted bulb. Include a switch in order to open or close the "circuit."
2. Why is this arrangement called a "circuit"?
3. The protrusion on one end of the cylindrical battery is the positive end of the battery. The flat rear side is the negative end. Sketch the circuit again and trace the current flow through the circuit from the positive end of the battery through the switch and bulb and back to the negative side of the battery.

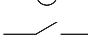
Note: The current enters the bulb through the pointed metal protrusion at the base, and it leaves through the metal threads of the base that are screwed into the holder.

C. Circuit Diagrams

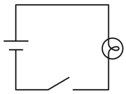
Instead of drawing realistic sketches, engineers have invented a way of diagraming circuits that includes special symbols for each component in the circuit. Here are some of the symbols and the components they represent:


a DC battery or power source; the long line represents the positive side.

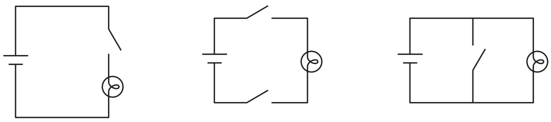

a light bulb.


a switch.

This is what a light bulb circuit would look like with these symbols:



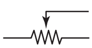
Here are some circuit diagrams:

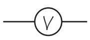


1. In which of the above diagrams would the light bulb light after closing the switches? In which one would it not light?
2. How can you tell from a circuit diagram whether or not the bulb will light?
3. What do the terms "closed circuit" and "open circuit" mean?

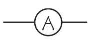
   Here are several more electrical symbols and the objects they represent:

 a resistor.

 a variable resistor (allows changing the resistance).



a voltmeter (measures the potential difference in volts).



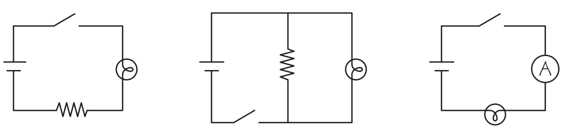
an ammeter (measures the current in amperes).

4. Use the circuit board, the bulb holder, and other components to make the following circuits.

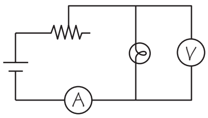
   In each case the bulb should light. If it does not light, check to make sure you have created a closed circuit.

   Always leave the switch open until you have completed the set up.

   To save the battery, use the converter from AC to DC current, setting it at 3 V.



5. Create the following circuit with the variable resistor. Use the variable resistor to vary the amount of current in the circuit. What happens to the light from the light bulb?



6. Using Ohm's law, $V = IR$ (V is potential difference in volts, I is the current in amperes, and R is the resistance in ohms), obtain the resistance of the light bulb from your circuit.

Please note:

(a) If you are using analog meters, the ammeter and voltmeter have positive (red post) and negative (black post) sides. In any circuit, the positive side should always be closest to the positive side of the power source. If this is reversed, the needle on the scale will go in the negative direction, and may be damaged.

Therefore, when you close the switch, watch the needle. If it goes negative, instantly open the switch and reverse the leads to the meter.

(b) Each of the meters has scales for different amounts of current and voltage. If you close the switch and the needle is pinned to the right, off the scale, instantly open the switch and transfer the meter to the highest scale.

D. The Light Bulb's Power

After you have found the resistance of the light bulb, you decide to apply your result to the useful task of finding out how much power the bulb consumes. You remember from the text that the power output is the square of the current times the resistance

$$P = I^2R.$$

1. For the current you are using, what is the power output of the bulb? Show your work.
2. You leave the bulb on for 10 s. How much energy is released by the bulb?
3. Is this energy all in the form of light energy, or is it converted into other forms of energy? How do you know?

E. Thought Questions

1. An insulator does not allow any significant current to pass through it. What is its resistance?

2. A superconductor allows all of the current to pass through unhindered. What is its resistance?
3. Voltage is the amount of work required to move a charge from one point to another. Why does a larger resistance require a larger voltage to yield the same current?

13. INVESTIGATING ELECTRIC CURRENTS II (CHAPTER 10)

IDEAS

In electricity there are two concepts that are basic to all other studies. These are voltage (potential difference) and current. The first refers to the work necessary to move a unit of positive charge from one point to the other; the second refers to the amount of electric charge that is transported per second between the two points in question. One is measured in volts, the other in amperes. One ampere is 1 coulomb per second.

Is there a relationship between the voltage and the current between two points? In 1851 Georg Ohm discovered that there is. If one measures the voltage on and the current through several objects, such as a copper wire, a salt solution, and a bar of silver, no relationship seems to exist between the measured volts and amps. However, by keeping the copper wire as a constant factor and varying the amount of voltage, while noting the amount of current that flows through the wire, Ohm found a simple relationship, known as "Ohm's law," between the volts and amps for the copper wire. According to this law, the voltage is directly proportional to the current, where the constant of proportionality is the resistance.

This may be expressed in symbols as follows:

$$V \propto I \qquad \text{or} \qquad V = IR,$$

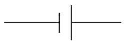
where V is the voltage, I is the current, and the constant R is the resistance of the wire. Resistance is measured in units of "ohms," 1 Ω, whereby 1 Ω = 1 V/A. We will test this relationship and utilize it in today's experiment.

We are going to follow the way actual research is done when a new law is proposed. First, in Part I, you will test the law to see if it is valid. Then, in Part II, you will assume it is valid and make predictions to see if they are accurate. Once Ohm's law has passed those two tests, we can be so confident it is valid that we can use it to explore the unknown (Part III)—in this case the value of unknown resistances.

INVESTIGATION

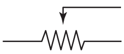
Materials: 12-V power source, switch, variable resistor (rheostat), ammeter, voltmeter, resistors, connectors, circuit board.

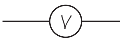
The following symbols are used:

 a DC power source; the long line represents the positive side.

 a resistor.

 a switch.

 a variable resistor (rheostat).



a voltmeter.

an ammeter (mA refers to milliamps or 10^{-3} A).

Before you begin, please note the following concerning the circuit components (assuming you are using analog meters):

- Always leave the switch open while wiring the circuits. Do not close it until the instructor has approved the wiring connections.
- The voltmeter and ammeter have a positive and a negative side. In any circuit, the negative side should always be closest to the negative side of the power source. This is also indicated in the circuits later in these instructions. If you should close the switch and the needle moves to the left, instead of to the right, instantly open the circuit and reverse the leads to the meter.
- Each of the meters has scales for different amounts of current and voltage. In most cases here, the scale to be used is indicated. If it is not, or you are uncertain which scale to use, always start with the scale for the largest amount of voltage or amperage and decrease in sequence as necessary. Also, if you close the switch and the needle is pinned to the right of the scale, quickly open the switch and transfer to a higher scale.
- To help keep the positive and negative sides of the circuit apparent, the circuit board has black and red binding posts. As is standard in electrical equipment, the *black signifies negative, and the red signifies positive.*

The variable resistance, or rheostat, is used not only to vary current in the circuit, but also to protect the meters and other components from an overload. The principle of the rheostat is that the longer the length of wire that the current from the battery must transverse the more the resistance. The length of the wire, hence the resistance, is controlled by the slide wire at the top. When the rheostat is connected at the lower left corner and the top right, all the way to the right is maximum resistance; all the way to the left is minimum resistance. To begin with, slide the wire all the way to the right for maximum resistance (hence minimum current in the circuit).

PART I. OHM'S LAW

You are a researcher in your school's laboratory and Dr. Ohm has just reported in the latest journal that he has concluded from his research that the current and voltage are related to each other for these types of resistors according to the simple relationship

$$V = IR$$

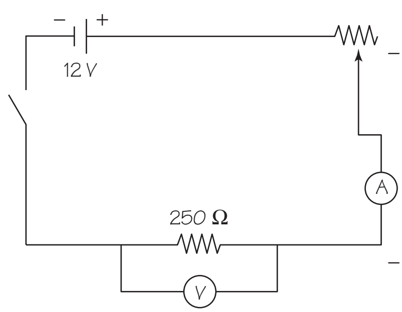
which he is calling "Ohm's law."

You are very excited to read of his discovery, because it relates the three basic electrical properties so simply. However, being a good scientist, you want to check it out for yourself before you accept it.

Here is one way to test to see if Ohm's law is indeed valid.

1. Connect the circuit shown below, placing the power source, open switch, rheostat, 250-Ω resistance, and ammeter in series (i.e., on one continuous line). Note that the voltmeter is connected in parallel with (or across) the resistance being studied. Place the transformer on 12 V, positive (+) polarity. Be sure that the little red light is lit. Do not close the switch until the instructor has approved the wiring.

   Use the *50-mA scale* on the ammeter and the *15-V scale* on the voltmeter.



2. Slowly decrease the variable resistance until the ammeter reads almost full scale. Take six readings of the potential difference across

the 250-Ω resistance and the current through it as the variable resistance is increased.

Reading #	*Voltage* (V)	*Current* (I)
1		
2		
3		
4		
5		
6		

3. To see any regularity in the relation of V and I, plot your values of V and I on a sheet of graph paper, with V on the y-axis and I on the x-axis.
4. Note that, as printed on the resistors, the resistors are accurate only to $\pm 10\%$. Within this limit of precision, do you see a smooth pattern?
5. If your graph is a straight line, what does this tell you about the relationship between the variables V and I?
6. If your graph is a straight line, find the slope of the straight line and compare with what you would expect the slope to be from Ohm's law.

   Expected result:

   Slope:

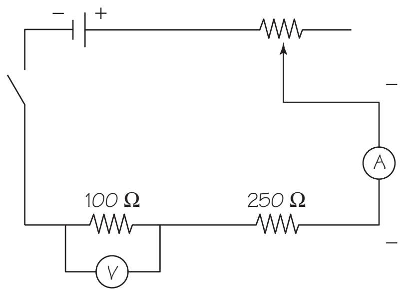
7. Do your data confirm or refute Ohm's law? Explain.
   Turn in your graph with your laboratory report.

PART II. SERIES CIRCUITS

Now that you have tested the validity of Ohm's law, you will want use it to make predictions about series circuits and see if these predictions agree with the observed phenomena.

Two circuit elements are in series if they are connected end to end in a continuous line.

1. Connect the circuit below, this time with the 100-Ω resistor in series with the 250-Ω resistor. Put the variable resistance at maximum before closing the switch. Leave the voltmeter disconnected for the time being. Use the **50-mA scale on the ammeter** and the **15-V scale on the voltmeter.**



2. By changing the variable resistor, *set the current at 25 mA*.
3. Knowing the resistance of each resistor and the current through each one (25 mA), use Ohm's law to predict the voltage drop across each resistor and across both resistors together. Note that they should *not* be the same (why not?), even though the battery voltage stays constant. Show how you got these results in the table below.
4. Now measure the voltage drop across each resistor and across both of them together.

Voltage	*Predicted*	*Observed*
V_1		
V_2		
$V_1 + V_2$		

THOUGHT QUESTIONS

1. So far we have tried one resistance and two resistances in series. Can you make an inductive generalization about Ohm's law—that is, about the relationship between V and I—for any number of resistors in series? Write this as an equation.

PART III. STUDYING UNKNOWN RESISTANCES

Now that you have tested Dr. Ohm's conclusions and used his new law to make predictions that are confirmed by actual measurement, you are confident enough of the validity of Ohm's law to use it as a tool to explore the unknown.

1. In this case the unknown consists of two conductors of electricity with resistances unknown to you. They are a unknown resistor and a con-

ducting solution. Using the equipment available to you, try to determine the resistance of each of these conductors without asking the instructor. Show any calculations you make. Ask the instructor if you are stuck.

Resistance of unknown resistor:

Resistance of conducting solution:

2. Leave the circuit connected for a while to the salt solution. Carefully observe and note everything that you see occurring in the solution.

THOUGHT QUESTIONS

1. An insulator does not allow any significant current to pass through it. What is its resistance? Explain using Ohm's law.
2. A superconductor allows all of the current to pass through unhindered. What is its resistance? Explain using Ohm's law.
3. If voltage is the amount of work required to move a charge from one point to another, why does a larger resistance require a higher voltage to yield the same current?

14. AVOGADRO'S NUMBER AND THE SIZE AND MASS OF A MOLECULE (CHAPTERS 7, 13)

INTRODUCTION

The acceptance of Avogadro's hypothesis enabled the determination of the relative masses of many atoms and molecules. Atomic weights and molecular weights (really "masses") were defined in terms of an accepted standard. The isotope ^{12}C was chosen as the standard and an atom of this isotope was defined as 12.000 u, where u is the standard symbol for atomic mass units (amu).

If the weight of an element is known in amu, then the same number of grams of that element or compound is called the gram atomic weight or the gram molecular weight. Each of these contains a standard "package", or mole, of atoms or molecules. One gram mole of ^{12}C contains a mass of 12.000 g.

It is a fact of nature that 1 g-mol of every substance contains the same number of atoms or molecules. The name *Avogadro's number* was given to the number of molecules or atoms in 1 g-mol. ("Loschmidt's number" refers to the number of atoms in 1 kg-mol). This number has been determined to be 6.02×10^{23}. Thus, for example, 1 g-mol of water, H_2O, would have a gram molecular mass of 2 (H) + 1 (O), or 2 (1.0080) + 1 (15.999) = 18.015 g. Thus, this small amount of water would contain 6.02×10^{23} water (15.999) = 18.015 g. Thus, this small amount of water would contain 6.02×10^{23} water molecules. As you see, Avogadro's number is extremely large, because atoms and molecules are extremely small.

Avogadro's number has been determined by various methods, all of which yield the same results, within the limits of experimental error. The method we shall use, although relatively primitive, yields surprisingly good results which are of the right order of magnitude (power of ten) if the experiment is carefully performed. It utilizes an interesting property of certain large molecules, such as fatty acids. If a drop of fatty acid is placed on the surface of water, it will spread out to form an extremely thin film on the surface of the water. Observations of this sort were recorded as long ago as 1773 by Benjamin Franklin, who noted that one teaspoon of oil spread out to form a film of about 22,000 ft^2 on a pond near London.

That this extremely thin film is probably the thickness of one long-chain molecule may be demonstrated by placing a wire across the surface of a shallow container filled to the brim with water, and allowing a drop of oil to fall on the water to one side of the wire. The oil will spread out over the water surface and attach itself to the wire and to the edges of the container because of intermolecular forces. If the wire is moved to stretch the film, the film breaks in places, and islands of water are visible.

Stearic acid and oleic acid, because of their large intermolecular forces, and their uncomplicated straight chain structure, are often used to study single-molecule films. In this experiment, the fatty acids used must be quite dilute. One drop of *pure* oleic acid will cover a water surface of about $200/m^2$ (about 2000 ft^2)!

In this experiment the concentration of oleic acid used is only 0.25% (by volume). The thickness of the film, which is the thickness of one molecule, can be calculated from a measurement of the size of the film made by one drop and a knowledge of the volume and concentration of the drop. If the simplifying assumption is made that the molecules are cubes, than the volume of one molecule can be calculated from the size and thickness of the film. Avogadro's number can be obtained from the known density of oleic acid and its gram molecular weight. Finally, the mass of one molecule can be obtained from Avogadro's number and the molecular weight.

Note: Since we will be multiplying and dividing numbers expressed in scientific notation, do not perform this investigation until you have reviewed the section on scientific notation in the Mathematics Review.

Equipment

Cafeteria tray, medicine dropper bulb, micropipet, 25 ml Erlenmeyer flask and stopper, 10 ml graduated cylinder, powder, 0.25% (volume) solution of oleic acid in methyl or ethyl alcohol.

INVESTIGATION

1. Withdraw about 5 ml of the oleic acid solution from the stock bottle and place in the clean, dry Erlenmeyer flask. Keep this closed with the stopper, when not in use. Otherwise the alcohol will evaporate, changing the concentration of the oleic acid.
2. In the following, use only the micropipet, *not* the medicine dropper. Determine the volume of one drop of oleic acid solution delivered by the micropipet. This can be done by first placing exactly 2 ml of this solution in the 10 ml graduated cylinder. Then count the number of drops necessary to increase this volume to exactly 3 ml. 1 ml is equal to 1 cm^3. In reading the volume, hold the cylinder at eye level and measure to the bottom of the miniscus.
3. Add tap water to the tray until it is completely covered with water up to the rim.
4. Evenly dust the surface with a very thin layer of the powder. The powder makes the boundaries of the oleic acid film easily visible. However, if there is too much powder, it prevents the oleic acid from spreading out completely. Try not to breathe in this powder.
5. Discard the first drop. Then put one drop of oleic acid solution on the surface of the water and wait about 30 s. The alcohol in the solution will evaporate upward and dissolve downward into the water, leaving a layer of pure oleic acid.
6. Measure the diameters of the film in two directions at right angles, record, and average.

DATA AND ANALYSIS (IMPORTANT: YOU MUST SHOW YOUR WORK.)

1. Number of drops in 1 cm^3 of 0.25% oleic acid solution.
2. Volume of one drop of oleic acid solution (in units of cm^3).
3. Volume of pure oleic acid in one drop. This value takes into account the fact that only 0.25% of the volume of the drop is actually oleic acid.
4. Diameters of the film (in cm) in two perpendicular directions.
5. Average diameter and radius of film (in cm).

6. Area of film, assuming a circle

$$A = \pi r^2$$

$$= \quad ,$$

7. Thickness of the film $= \dfrac{\text{volume of the acid}}{\text{area}}$

$$= \quad .$$

8. *Volume of one molecule*, assuming the molecules are cubes and that they are in contact with each other. The thickness of the film tells you the length of the edge of the cube.
9. Gram molecular weight of oleic acid as determined from its formula, which is $C_{18}H_{34}O_2$. Consult the periodic table.
10. Volume occupied by 1 mol of oleic acid. This can be determined from the density (0.098 g/cm^3) and the gram molecular weight.
11. *Avogadro's number:* The number of molecules in 1 mol, assuming the molecules are cubes. This is determined by knowing the volume of one molecule and the volume occupied by a mole of molecules.
12. Write down the accepted value of Avogadro's number.
13. *Mass of one molecule*, determined from your value of Avogadro's number and the molecular weight.

THOUGHT QUESTIONS

1. How do the measured and accepted values of Avogadro's number compare? Note that since we are dealing with such large numbers and making such great assumptions, good agreement is attained if the numbers are within a "ball park" of each other (i.e., within a power of 10).
2. Define Avogadro's number in words.
3. In this experiment, we made a number of simplifying assumptions. What were some of these assumptions? Which were the most important? How would each of these assumptions influence our calculation of Avogadro's constant?
4. To gain an idea how tiny a molecule of oleic acid really is, how many molecules would you have to line up end to end to make 1 mm of length, the smallest interval on a meter stick? Assume the molecules are cubes and use the length of one side determined in Exercise 7.
5. To gain an idea how enormous Avogadro's number is, assume that each molecule of a mole of oleic acid is the size of a cube 1 ft on a side. If Avogadro's number of such cubes were placed into a cubic box,

how long would one side of the box be in feet and in miles (1 mi = 5000 ft)? *Hint*: First find the volume of the box, then find the length of one side by taking the cube root. Compare your result to the size of the Earth (diameter about 8000 mi).

ADDITIONAL INVESTIGATIONS

The following "mini-laboratories" may be utilized or extended to serve as major explorations pertaining to the latter chapters of the textbook.

- ***How Do We Know That Atoms Really Exist? The Brownianscope*** (Chapter 13).
- ***Light and Color*** (Section 14.1).
- ***Spectroscopy*** (Chapter 14).
- ***Radioactivity and Nuclear Half-Life*** (Chapter 17).
- "The Photoelectric Effect," an investigation using light and an electroscope, described by P. Hewitt, in *Conceptual Physics Laboratory Manual* (Boston, MA: Addison-Wesley), pp. 305–307.