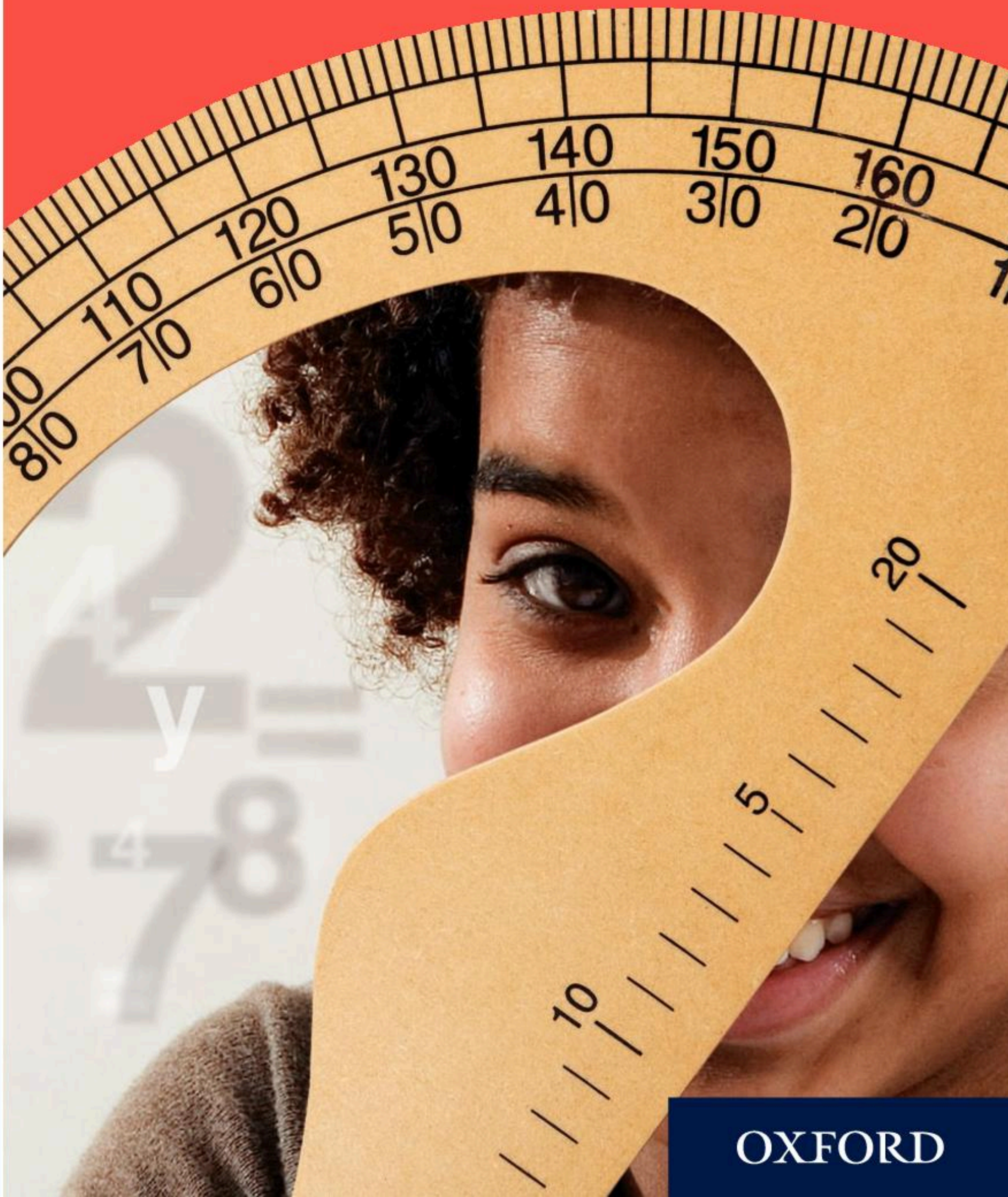
Oxford excellence for the Caribbean

Book 3

GRADE 9

STP Mathematics for Jamaica

SECOND EDITION



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Contents

Introduction

To the student

This new edition of *STP Mathematics for Jamaica Student Book 3* attempts to meet your needs as you begin your study of Mathematics at the secondary school level. Your learning experiences at this stage lay the foundation for future achievement in CSEC Mathematics and beyond. We are very conscious of your need for success and enjoyment in doing Mathematics, which comes from solving problems correctly. With this in mind, we have divided most of the exercises into three types of question:

Type 1 questions

These are identified by numbers written in bold print, e.g. **12**. They help you to see if you understand the topic being discussed and should be attempted in every chapter you study.

Type 2 questions

These are identified by a single underline under the bold print, e.g. $\underline{\mathbf{12}}$. They are extra questions for you to do and are not more difficult. They should be attempted if you need extra practice or want to do revision at a later time.

Type 3 questions

These are identified by a double underline under the bold print, e.g. $\underline{\underline{\mathbf{12}}}$. They are for those of you who completed Type 1 questions fairly easily and want to attempt questions that are more challenging.

Multiple choice questions

Multiple choice questions are included in the book to help you become more familiar with the format of your assessments at CSEC.

Mixed exercises

Most chapters end with Mixed exercises to help you advance your critical thinking, problem-solving and computational skills. These exercises will also help you revise what you have done, either when you have finished the chapter or as you prepare for examinations.

Use of calculator

You should be able to use a calculator accurately before you leave school. We suggest that you use a calculator mainly to check your answers. Whether you use a calculator or do the computations yourself, always estimate your answer first and always ask the question, 'Does my answer make sense?'

Suggestions for use of student book

- Break up the material in a chapter into manageable parts.
- Have paper and a pencil with you always when you are studying mathematics.
- Write down and look up the meaning of all new vocabulary you encounter.
- Read all questions carefully and rephrase them in your own words.
- Remember that each question contains all the information you need to solve the problem. Do not look only at the numbers that are given.
- Practise your mathematics. This will ensure your success!

You are therefore advised to try to solve as many problems as you can.
Above all, don't be afraid to make mistakes as you are learning. The greatest mathematicians all made many mistakes as they tried to solve problems.

You are now on your way to success in mathematics – GOOD LUCK!

To the teacher

In writing this series, the authors attempted to present the topics in such a way that students will understand the connections among topics in mathematics, and be encouraged to see and use mathematics as a means to make sense of the real world. The exercises have been carefully graded to make the content more accessible to students.

This new edition is designed to:

1 Assist you in helping students to

- attain important mathematical skills
- connect mathematics to their everyday lives and understand its role in the development of our contemporary society
- see the importance of critical thinking skills in everyday problems
- discover the fun of doing mathematics both individually and collaboratively
- develop a positive attitude towards doing mathematics.

2 Encourage you to include historical information about mathematics in your teaching.

Topics from the history of mathematics have been incorporated to ensure that mathematics is not dissociated from its past. This should lead to an increase in the level of enthusiasm, interest and fascination among students, thus enriching the teaching and learning experiences in the mathematics lessons.

Investigations

'Investigation' is included in this revised STP Mathematics for Jamaica series. This is in keeping with the requirements of the latest CSEC syllabus.

Investigations are used to provide students with the opportunity to explore hands-on and minds-on mathematics. At the same time, teachers are presented with open-ended explorations to enhance their mathematical instruction.

It is expected that the tasks will

- encourage problem solving and reasoning
- develop communication skills and the ability to work collaboratively
- connect various mathematical concepts and theories.

Suggestions

1 At the start of each lesson, give a brief outline of the topic to be covered in the lesson. As examples are given, refer back to the outline to show how the example fits into it.

2 List terms that you consider new to the students and solicit additional words from them. Encourage students to read from the text and make their own vocabulary list. Remember that mathematics is a foreign language. The ability to communicate mathematically must involve the careful use of the correct terminology.

3 Have students construct different ways to phrase questions. This helps students to see mathematics as a language. Students, especially in the junior classes, tend to concentrate on the numerical or 'maths' part of the question and pay little attention to the information that is required to solve the problem.

4 When solving problems, have students identify their own problem-solving strategies and listen to the strategies of others. This practice should create an atmosphere of discussion in the class centred on different approaches to solving the same problem.

As the students try to solve problems on their own they will make mistakes. This is expected, as this was the experience of the inventors of mathematics: they tried, guessed, made many mistakes and worked for hours, days and sometimes years before reaching a solution.

There are enough problems in the exercises to allow the students to try and try again. The excitement, disappointment and struggle with a problem until a solution is found will create rewarding mathematical experiences.

1 Algebraic fractions

At the end of this chapter you should be able to...

1 reduce a given algebraic fraction to its lowest form
2 multiply or divide two algebraic expressions
3 find the LCM of a set of algebraic expressions
4 add or subtract two algebraic fractions
5 solve simple equations containing algebraic fractions.

You need to know...

✔ how to cancel fractions
✔ index notation
✔ how to find the LCM of a set of numbers
✔ how to remove brackets
✔ how to multiply out expressions such as $3(2x-4)$.

Key words

common denominator, denominator, equivalent fraction, lowest common multiple (LCM), numerator, product, reciprocal, simplify

Simplifying fractions

We *simplify* a fraction such as $\frac{10}{50}$ by recognising that 10 is a common factor of the *numerator* and *denominator* and then cancelling that common factor,

i.e. $$\frac{10}{50} = \frac{\cancel{10}^{1}}{5 \times \cancel{10}_{1}} = \frac{1}{5}$$

To simplify an algebraic fraction, we do exactly the same: we find and then cancel the common factors of the numerator and denominator.

Note that we do not have to write the number 50 as 5×10 but that when the factors are letters it helps at this stage to put in the multiplication sign.

For example $\quad xy$ can be written as $x \times y$

and $\quad 2(a+b)$ can be written as $2 \times (a+b)$

Exercise 1a

Simplify: **a** $\dfrac{2xy}{6y}$ **b** $\dfrac{2a}{a^2b}$

a $\dfrac{2xy}{6y} = \dfrac{2 \times x \times y}{6 \times y} = \dfrac{x}{3}$

b $\dfrac{2a}{a^2b} = \dfrac{2 \times a}{a \times a \times b} = \dfrac{2}{ab}$

Simplify:

1	$\dfrac{2x}{8}$	**4**	$\dfrac{a^2}{ab}$	**7**	$\dfrac{2ab}{4bc}$	**10**	$\dfrac{a^2b}{abc}$	**13**	$\dfrac{b^2}{bd}$	**16**	$\dfrac{10x}{15xy}$
2	$\dfrac{ab}{2b}$	**5**	$\dfrac{xy}{y^2}$	**8**	$\dfrac{6p}{3pq}$	**11**	$\dfrac{7a}{14}$	**14**	$\dfrac{4}{12x}$	**17**	$\dfrac{m^2n}{kmn}$
3	$\dfrac{p^2}{pq}$	**6**	$\dfrac{3}{6a}$	**9**	$\dfrac{5p^2q}{10p}$	**12**	$\dfrac{yz}{2y}$	**15**	$\dfrac{3pq}{6p}$	**18**	$\dfrac{5s^2}{20st}$

Factors

We know that $3 \times 2 = 6$ but neither $3 + 2$ nor $3 - 2$ is equal to 6.

We can write a number as the *product* of its factors but, in general, we cannot write a number as the sum or difference of its factors.

Thus $\begin{cases} p \text{ and } q \text{ are factors of } pq \\ a \text{ and } (a-b) \text{ are factors of } a(a-b) \end{cases}$

but in general $\begin{cases} p \text{ is } \textit{not} \text{ a factor of } p+q \\ b \text{ is } \textit{not} \text{ a factor of } a-b \end{cases}$

This means that in the fraction $\dfrac{p+q}{pq}$ we *cannot* cancel q because q is not a factor of the numerator.

Sometimes the common factors in a fraction are not very obvious.

Consider $\dfrac{x-2}{y(x-2)}$

Placing the numerator in brackets and using the multiplication sign gives $\dfrac{(x-2)}{y \times (x-2)}$

Now we can see clearly that $(x-2)$ is a common factor, so

$$\frac{(x-2)}{y \times (x-2)} = \frac{1}{y}$$

Exercise 1b

Simplify where possible: **a** $\dfrac{2a(a-b)}{a-b}$ **b** $\dfrac{pq}{p-q}$

a $\dfrac{2a(a-b)}{a-b} = \dfrac{2\times a\times \cancel{(a-b)}^{1}}{\cancel{(a-b)}_{1}}$ (place the denominator in brackets)

$= 2a$

b $\dfrac{pq}{p-q} = \dfrac{p\times q}{(p-q)}$ which cannot be simplified.

Simplify where possible:

1 $\dfrac{x-y}{x(x-y)}$

2 $\dfrac{st}{s(s-t)}$

3 $\dfrac{2a}{a-b}$

4 $\dfrac{p+q}{2p}$

5 $\dfrac{4x}{8(x-y)}$

6 $\dfrac{3(a+b)}{6ab}$

7 $\dfrac{(p-q)(p+q)}{p+q}$

8 $\dfrac{(4+a)}{(4+a)(4-a)}$

9 $\dfrac{(a-b)}{3(a+b)}$

10 $\dfrac{u-v}{v(u-v)}$

11 $\dfrac{xy}{x(x+y)}$

12 $\dfrac{s-t}{2(s-t)}$

13 $\dfrac{10a}{15(a-b)}$

14 $\dfrac{8(x-y)}{12xy}$

15 $\dfrac{s-t}{3s}$

16 $\dfrac{(u+v)(u-v)}{u+v}$

17 $\dfrac{x+y}{2(x-y)}$

18 $\dfrac{s+6}{(s+6)(s-6)}$

Multiplying and dividing fractions

Reminder: The product of two fractions is found by multiplying the numerators and multiplying the denominators,

e.g. $\dfrac{2}{3}\times\dfrac{4}{5}=\dfrac{2\times 4}{3\times 5}=\dfrac{8}{15}$

and $\dfrac{p}{q}\times\dfrac{(a-b)}{(a+b)}=\dfrac{p(a-b)}{q(a+b)}$

To divide by a fraction, we multiply by its *reciprocal*,

e.g. $\dfrac{2}{3}\div\dfrac{3}{4}=\dfrac{2}{3}\times\dfrac{4}{3}=\dfrac{8}{9}$, $\dfrac{p}{q}\div r=\dfrac{p}{q}\times\dfrac{1}{r}=\dfrac{p}{qr}$

and $\dfrac{p}{q}\div\dfrac{a}{(a-b)}=\dfrac{p}{q}\times\dfrac{(a-b)}{a}=\dfrac{p(a-b)}{qa}$

Exercise 1c

Find:

1 $\frac{a}{b} \times \frac{c}{d}$

2 $\frac{a}{b} \div \frac{c}{d}$

3 $\frac{x-y}{2} \times \frac{5}{x}$

4 $\frac{x-y}{2} \div \frac{5}{x}$

5 $\frac{a}{b} \div c$

6 $\frac{a}{b} \times c$

7 $\frac{(a-b)}{4} \div \frac{(a+b)}{3}$

8 $\frac{(x-2)}{3} \times (x+3)$

9 $\frac{(x-2)}{3} \div (x+3)$

10 $\frac{p}{q} \div \frac{1}{r}$

Remember that to divide by a fraction you multiply by its reciprocal.

Simplify $\frac{ab}{4} \times \frac{8}{a^2}$

As is the case in number fractions, it is sometimes possible to simplify before multiplying.

$$\frac{ab}{4} \times \frac{8}{a^2} = \frac{\overset{1}{\cancel{a}}b}{\underset{1}{\cancel{4}}} \times \frac{\overset{2}{\cancel{8}}}{\underset{a}{\cancel{a^2}}}$$

$$= \frac{2b}{a}$$

Simplify:

11 $\frac{2a}{b} \div \frac{a^2}{3b^2}$

12 $\frac{pq}{6} \times \frac{3}{p^2}$

13 $\frac{4xy}{3} \times \frac{9}{x^2}$

14 $\frac{2ab}{5} \div \frac{a}{b}$

15 $\frac{2p^2}{3} \times \frac{q}{4p}$

16 $\frac{x^2}{4} \div \frac{xy}{2}$

17 $\frac{1}{b^2} \div \frac{2}{b}$

18 $\frac{7p}{5q} \times \frac{10q}{21p^2}$

19 $\frac{a^2}{2b} \div 2a$

20 $\frac{a}{b} \times \frac{2a}{3b} \div \frac{2b}{3a}$

Investigation

Multiply 32 547 891 by 6. Compare your answer with the original number.

What do you notice?

Investigate similar relationships. For example, is there an 8-digit number using every digit from 1 to 9 except 7 which, when multiplied by 7, gives a 9-digit answer which uses every digit from 1 to 9 once?

Lowest common multiple

Before we can simplify $\frac{2}{3}+\frac{1}{5}$ we must change both $\frac{2}{3}$ and $\frac{1}{5}$ into *equivalent fractions* with the same denominator. This *common denominator* must contain both 3 and 5 as factors; there are many numbers we could choose but 15 is the lowest such number, i.e. 15 is the *lowest common multiple* (LCM) of 3 and 5.

To simplify $\frac{3}{x}+\frac{2}{y}$ we follow the same pattern. We need a common denominator with both x and y as factors. Again there are many we could use, but the simplest is xy; this is the LCM of x and y.

Exercise 1d

Find the LCM of ab and c

The LCM is abc

Find the LCM of:

1	p, q	**3**	2, 3, 5	**5**	x, y, wz	**7**	v, uw
2	r, st	**4**	a, b, c	**6**	a, d	**8**	3, 7, 8

Find the LCM of: **a** 4, 10 **b** ab, a^2 **c** $2x, 6x$

a $4=2\times 2$ and $10=2\times 5$

(The LCM is the *lowest* number that 4 and 10 divide into exactly, so the factors it must include are

2×2 from 4 and 5 from 10

The factor of 2 from 10 is not needed as 2 is already included.)

$\therefore$ the LCM of 4 and 10 is $2\times 2\times 5=20$

b $ab=a\times b$ and $a^2=a\times a$

$\therefore$ the LCM is $a\times b\times a=a^2b$

c $2x=2\times x$ and $6x=2\times 3\times x$

$\therefore$ the LCM is $2\times 3\times x=6x$

Find the LCM of:

9	x, xy	**14**	s, st	**19**	$4x, 8x$	**24**	$4x, 6x$
10	$x^2, 2x$	**15**	$3p, p^2$	**20**	$6a, 9a$	**25**	$3y, 5y$
11	$pq, 3p$	**16**	$5a, ab$	**21**	6, 4, 10	**26**	$2x, 3x, 4x$
12	$x^2, 2xy$	**17**	$3pq, q^2$	**22**	a, ab, a^2		
13	ab, bc	**18**	$2x, 3x$	**23**	$10x, 15x$		

Addition and subtraction of fractions

To add or subtract fractions we first have to change them into equivalent fractions with a common denominator.

Thus to find $\frac{2}{3}+\frac{1}{5}$, we choose a common denominator of 15 which is the LCM of 3 and 5.

Now $\frac{2}{3}=\frac{2\times5}{3\times5}=\frac{10}{15}$ and $\frac{1}{5}=\frac{1\times3}{5\times3}=\frac{3}{15}$

Therefore $\frac{2}{3}+\frac{1}{5}=\frac{10+3}{15}=\frac{13}{15}$

To simplify $\frac{3}{x}+\frac{2}{y}$ we follow the same pattern:

xy is the LCM of x and y.

$$\frac{3}{x}=\frac{3\times y}{x\times y}=\frac{3y}{xy} \quad \text{and} \quad \frac{2}{y}=\frac{2\times x}{y\times x}=\frac{2x}{xy}$$

$$\therefore \quad \frac{3}{x}+\frac{2}{y}=\frac{3y+2x}{xy}$$

Exercise 1e

Simplify $\frac{1}{2a}+\frac{1}{b}$

($2ab$ is the LCM of $2a$ and b)

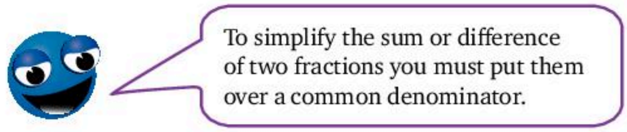
$$\frac{1}{2a}+\frac{1}{b}=\frac{(1)\times(b)+(1)\times(2a)}{2ab}=\frac{b+2a}{2ab}$$

Simplify $\frac{3}{4x}-\frac{1}{6x}$

($12x$ is the LCM of $4x$ and $6x$)

$$\frac{3}{4x}-\frac{1}{6x}=\frac{(3)\times(3)-(1)\times(2)}{12x}=\frac{7}{12x}$$

Simplify:



1 $\frac{1}{x}+\frac{1}{y}$

2 $\frac{3}{p}+\frac{2}{q}$

3 $\frac{2}{s}-\frac{1}{t}$

4 $\frac{3}{a}+\frac{1}{2b}$

5 $\frac{1}{3x}-\frac{2}{5y}$

6 $\frac{1}{a}+\frac{5}{2b}$

7 $\frac{2}{x}-\frac{3}{y}$

8 $\frac{4}{3p}+\frac{2}{q}$

9 $\frac{3}{x}-\frac{2}{y}$

10 $\frac{5}{7a}+\frac{3}{4b}$

11 $\frac{1}{2x}+\frac{1}{3x}$

12 $\frac{2}{5x}-\frac{3}{7x}$

13 $\frac{2}{y}-\frac{3}{4y}$

14 $\frac{3}{8p}-\frac{1}{4p}$

15 $\frac{1}{a}+\frac{5}{8a}$

16 $\frac{1}{3x}-\frac{1}{7x}$

17 $\frac{4}{7x}-\frac{2}{5x}$

18 $\frac{1}{y}-\frac{2}{3y}$

Simplify $\frac{4a}{3b}-\frac{b}{6a}$

($3b = 3 \times b$ and $6a = 2 \times 3 \times a$, $\therefore$ LCM $= 6ab$)

$$\frac{4a}{3b}-\frac{b}{6a}=\frac{(4a)\times(2a)-(b)\times(b)}{6ab}$$

$$=\frac{8a^2-b^2}{6ab}$$

Simplify:

19 $\frac{1}{2a}+\frac{3}{4b}$

20 $\frac{a}{2b}-\frac{a^2}{b^2}$

21 $\frac{3}{x}-\frac{4}{xy}$

22 $\frac{2}{p^2}-\frac{3}{2p}$

23 $\frac{3a}{4b}+\frac{b}{6a}$

24 $\frac{5}{2p}-\frac{3}{4q}$

25 $\frac{s}{t^2}+\frac{s^2}{2t}$

26 $\frac{5}{2a}+\frac{2}{3ab}$

27 $\frac{1}{x^2}+\frac{2}{3x}$

28 $\frac{2y}{3x}-\frac{3x}{2y}$

29 $\frac{5}{8x}+\frac{2}{4y}$

30 $\frac{p}{3q}+\frac{p^2}{q^2}$

31 $\frac{5}{7x}-\frac{3}{14xy}$

32 $\frac{9}{a^2}-\frac{3}{2ab}$

33 $\frac{3x}{2y}-\frac{3y}{2x}$

34 $\frac{7}{9p}-\frac{5}{6q}$

35 $\frac{a^2}{b^2}+\frac{4a}{5b}$

36 $\frac{7}{5pq}+\frac{8}{15q}$

Exercise 1f

Simplify $\frac{x-2}{3}-\frac{x-4}{2}$

$$\begin{aligned}\frac{(x-2)}{3}-\frac{(x-4)}{2} &= \frac{2(x-2)-3(x-4)}{6}\\ &= \frac{2x-4-3x+12}{6}\\ &= \frac{-x+8}{6}\\ &= \frac{8-x}{6}\end{aligned}$$

(Notice that we placed brackets round the numerators *before* putting the fractions over a common denominator. This ensured that each numerator was kept together and that the signs were not confused.)

Simplify:

1 $\frac{x+2}{5}+\frac{x-1}{4}$

2 $\frac{x+3}{4}-\frac{x+1}{3}$

3 $\frac{2x-1}{3}+\frac{x+2}{5}$

4 $\frac{2x+3}{4}-\frac{x-2}{6}$

5 $\frac{x+3}{7}-\frac{x+2}{5}$

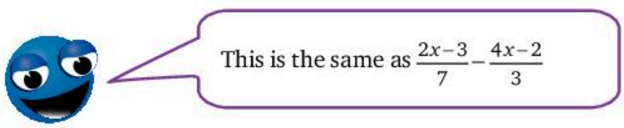
6 $\frac{x+4}{5}+\frac{x-1}{2}$

7 $\frac{2x-1}{7}-\frac{x-2}{5}$

8 $\frac{3x+1}{14}-\frac{2x+3}{21}$

Simplify:

9 $\frac{1}{7}(2x-3)-\frac{1}{3}(4x-2)$



10 $\frac{1}{4}(5x-1)-\frac{1}{3}(2x-3)$

11 $\frac{5-2x}{3}+\frac{4-3x}{2}$

12 $\frac{1}{4}(3-x)+\frac{1}{6}(1-2x)$

13 $\frac{1}{8}(5x+4)-\frac{1}{3}(4x-1)$

14 $\frac{2x+3}{5}-\frac{3x-2}{4}$

15 $\frac{3-x}{2}+\frac{1-2x}{6}$

16 $\frac{2+5x}{8}-\frac{3-4x}{6}$

17 $\frac{1}{5}(4-3x)+\frac{1}{10}(3-x)$

18 $\frac{1}{9}(4-x)-\frac{1}{6}(2+3x)$

Simplify $\frac{2(x+1)}{3} - \frac{3(x-2)}{5}$

$$\frac{2(x+1)}{3} - \frac{3(x-2)}{5} = \frac{5 \times 2(x+1) - 3 \times 3(x-2)}{15}$$

$$= \frac{10(x+1) - 9(x-2)}{15}$$

(Now multiply out the brackets.)

$$= \frac{10x + 10 - 9x + 18}{15}$$

$$= \frac{x+28}{15}$$

Simplify:

19 $\frac{4(x+2)}{3} + \frac{2(x-1)}{5}$

20 $\frac{3(x-1)}{4} + \frac{2(x+1)}{3}$

21 $\frac{2(x-2)}{3} - \frac{3(x-1)}{7}$

22 $\frac{5(2x-1)}{2} - \frac{4(x+3)}{5}$

23 $\frac{3(x-1)}{2} + \frac{3(x+4)}{7}$

24 $\frac{7(x-3)}{3} - \frac{2(x+5)}{9}$

25 $\frac{2(3x-1)}{5} + \frac{4(2x-3)}{15}$

26 $\frac{3(x-2)}{5} - \frac{7(x-4)}{6}$

Simplify $\frac{2}{x} - \frac{1}{x+2}$

$$\frac{2}{x} - \frac{1}{(x+2)} = \frac{(2)(x+2) - (1)(x)}{x(x+2)}$$

$$= \frac{2x+4-x}{x(x+2)}$$ (Multiplying out the brackets in the numerator.)

$$= \frac{x+4}{x(x+2)}$$

(Notice that we placed the two-term denominator in brackets. Notice also that we left the common denominator in factorised form.)

Simplify:

27 $\frac{2}{a} + \frac{1}{a+3}$

28 $\frac{4}{x+2} + \frac{2}{x}$

29 $\frac{3}{x-4} + \frac{1}{2x}$

30 $\frac{2}{2x+1} - \frac{3}{4x}$

31 $\frac{3}{a} + \frac{2}{a+4}$

32 $\frac{3}{x-1} + \frac{4}{x}$

33 $\frac{3}{2x+1} + \frac{1}{3x}$

34 $\frac{5}{2x+3} - \frac{2}{5x}$

Investigation

You now know how to express the sum of two fractions such as

$\frac{1}{x+1}+\frac{1}{x-1}$ as a single fraction,

i.e. $\frac{1}{x+1}+\frac{1}{x-1}=\frac{(x-1)+(x+1)}{(x-1)(x+1)}=\frac{2x}{(x-1)(x+1)}$

Therefore starting with a single fraction such as $\frac{2x}{(x-1)(x+1)}$ it must be possible to reverse the process and express it as the sum (or difference) of two fractions.

Investigate how $\frac{2}{(x+1)(x-1)}$ can be expressed as the sum or difference of two fractions.

If you think you have found a method that will work with any such fraction, try it by expressing $\frac{4}{(x+2)(2x-1)}$ as the sum or difference of two fractions.

Mixed questions

Exercise 1g

Simplify:

1 $\frac{2}{a}-\frac{b}{c}$

2 $\frac{pq}{r}\times\frac{r^3}{p^2}$

3 $\frac{x+2}{4}+\frac{x-5}{3}$

4 $\frac{8}{x+1}\div\frac{4}{3x}$

5 $\frac{3}{4x}-\frac{2}{3x}$

6 $\frac{x+1}{6}\times\frac{3x}{x+1}$

7 $\frac{4x^2}{3}\div\frac{5x}{12}$

8 $\frac{4}{x^2}-\frac{2}{3x}$

9 $\frac{1}{x}-\frac{3}{x+1}$

10 $\frac{a^2}{bc}\div\frac{a}{b^2}$

11 $\frac{2}{5x}\div\frac{3}{4x}$

12 $\frac{2}{5x}+\frac{3}{4x}$

13 $\frac{2}{5x}\times\frac{3}{4x}$

14 $\frac{x+4}{5}+\frac{2x-1}{10}$

15 $\frac{x+4}{5}\times\frac{2x-1}{10}$

16 $\frac{5}{4x}+\frac{5}{6x}$

17 $\frac{5}{4x}\times\frac{5}{6x}$

18 $\frac{5}{4x}\div\frac{5}{6x}$

19 $\frac{1}{3x}+\frac{6}{x-1}$

20 $\frac{1}{3x}\times\frac{6}{x-1}$

21 $\frac{3}{2a}-\frac{2}{a-1}$

22 $\frac{3}{2a}\times\frac{2}{a-1}$

23 $\frac{3}{4y-3}\div\frac{y}{4y-3}$

24 $\frac{3}{4y-3}-\frac{4y}{4y-3}$

Solving equations with fractions

Remember that when solving an equation we *must* keep the equality true. This means that if we alter the size of one side of the equation then we must alter the other side in the same way.

Consider the equation $\frac{1}{x}+\frac{1}{2x}=\frac{5}{6}$

If we choose to multiply each side by the LCM of the denominators, we can remove all fractions from the equation.

The LCM of x, $2x$ and 6 is $6x$.

Multiplying each side by $6x$ gives

$$6x\left(\frac{1}{x}+\frac{1}{2x}\right)=6x\times\frac{5}{6}$$

$$\therefore \qquad \frac{6x}{1}\times\frac{1}{x}+\frac{6x}{1}\times\frac{1}{2x}=\frac{6x}{1}\times\frac{5}{6}$$

$$\therefore \qquad 6+3=5x$$

$$9=5x$$

$$\frac{9}{5}=x \quad \text{i.e.} \quad x=1\frac{4}{5}$$

Exercise 1h

Solve the following equations:

1 $\frac{1}{2}+\frac{4}{x}=1$

2 $\frac{2}{3}-\frac{1}{x}=\frac{13}{15}$

3 $\frac{3}{4}-\frac{2}{x}=\frac{5}{12}$

4 $\frac{1}{x}-\frac{1}{3x}=\frac{1}{2}$

5 $\frac{1}{2x}+\frac{2}{x}=\frac{1}{4}$

6 $\frac{3}{x}+\frac{3}{10}=\frac{9}{10}$

7 $\frac{3}{8}-\frac{2}{x}=\frac{1}{6}$

8 $\frac{3}{2x}+\frac{1}{4x}=\frac{1}{3}$

9 $\frac{1}{x}-\frac{1}{2}=\frac{3}{2x}$

10 $\frac{3}{2x}+\frac{2}{5}=\frac{5}{x}$

Solve the equation $\frac{x-2}{4} - \frac{x-3}{6} = 2$

$$\frac{(x-2)}{4} - \frac{(x-3)}{6} = 2$$

Multiply each side by 12:

$$12\left[\frac{(x-2)}{4} - \frac{(x-3)}{6}\right] = 12\times 2$$

$$\therefore \quad \frac{\overset{3}{\cancel{12}}}{1}\times\frac{(x-2)}{\underset{1}{\cancel{4}}} - \overset{2}{\cancel{12}}\times\frac{(x-3)}{\underset{1}{\cancel{6}}} = 24$$

$$\therefore \quad 3(x-2) - 2(x-3) = 24$$

$$3x - 6 - 2x + 6 = 24$$

$$x = 24$$

Solve the following equations:

11 $\frac{x+2}{4} + \frac{x-3}{2} = \frac{1}{2}$

12 $\frac{x}{4} - \frac{x+3}{3} = \frac{1}{2}$

13 $\frac{x}{5} + \frac{x+1}{4} = \frac{8}{5}$

14 $\frac{2x}{5} - \frac{x-3}{8} = \frac{1}{10}$

15 $\frac{x-4}{3} - \frac{x+1}{4} = \frac{1}{6}$

16 $\frac{x+3}{5} + \frac{x-2}{4} = \frac{3}{10}$

17 $\frac{2}{3} - \frac{x+1}{9} = \frac{5}{6}$

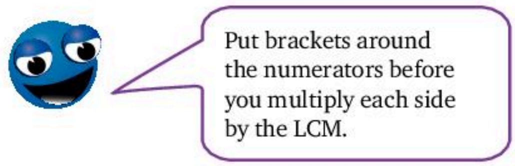
18 $\frac{x+3}{4} - \frac{x}{2} = 5$

19 $\frac{3x}{20} + \frac{x-2}{8} = \frac{3}{10}$

20 $\frac{x+3}{7} - \frac{x-4}{3} = 1$

21 $\frac{2x-1}{7} + \frac{3x-3}{4} = \frac{1}{7}$

22 $\frac{2x}{9} - \frac{3x+2}{4} = \frac{7}{12}$



Mixed exercises

Exercise 1i

1 Simplify:

a $\frac{ab^2}{2ab}$ **b** $\frac{a(a+b)}{a+b}$

2 Simplify:

a $\frac{1}{x} + \frac{1}{3x}$ **b** $\frac{1}{x} \times \frac{1}{3x}$

3 Solve the equation $\frac{x+1}{3} - \frac{x-1}{2} = 3$

4 **a** Simplify $\frac{1}{2}(x-1) + \frac{1}{3}(x-2)$

b Solve the equation $\frac{1}{2}(x-1) + \frac{1}{3}(x-2) = \frac{1}{4}$

Exercise 1j

1 Simplify:

a $\frac{uv^2}{wu^2v}$ **b** $\frac{a-b}{(b-a)(b-2a)}$

2 Simplify:

a $\frac{3s}{t} \div \frac{1}{6st}$ **b** $\frac{3}{4x-1} - \frac{2}{x}$

3 Solve the equation $\frac{x-2}{4} - \frac{x-3}{5} = \frac{3}{10}$

4 **a** Simplify $\frac{1}{2}(x-2) - \frac{1}{3}(x-3)$

b Solve the equation $\frac{1}{2}(x-2) - \frac{1}{3}(x-3) = 5$

5 Solve the equations:

a $\frac{x}{5} - \frac{x}{8} = 3$ **b** $\frac{x}{3} - \frac{x}{7} = 4$ **c** $\frac{x}{6} - \frac{x}{11} = 5$

What pattern do you notice in the above?

6 Solve for x:

$\frac{x}{a} - \frac{x}{b} = b - a$

Exercise 1k

Select the letter that gives the correct answer.

1 $\frac{9xy}{6x^2} =$

A $\frac{y}{x}$ **B** $\frac{3y}{2}$ **C** $\frac{3y}{2x}$ **D** $\frac{3y}{x^2}$

2 $\frac{2}{x} \div \frac{4}{x^2} =$

A $\frac{x}{4}$ **B** $\frac{x^2}{4}$ **C** $\frac{x}{2}$ **D** $\frac{8}{x}$

3 $\frac{1}{3a} - \frac{1}{4a} =$

A $\frac{1}{12a}$ **B** $\frac{1}{4a}$ **C** $\frac{1}{3a}$ **D** $\frac{1}{7a}$

4 $\frac{2}{3}(x+1)+\frac{1}{2}(x-2)=$

A $\frac{x}{2}$ **B** $\frac{x}{3}$ **C** $\frac{7x-2}{6}$ **D** $\frac{7x+2}{6}$

5 The solution of the equation $\frac{3}{2x}-\frac{1}{x}=\frac{3}{5}$ is

A $\frac{3}{5}$ **B** $\frac{5}{6}$ **C** $1\frac{1}{6}$ **D** $1\frac{1}{5}$

Did you know?

On what day of the week were you born? Can you answer this question? There is a formula which you may use to help you.

If d = day of the month, y = year and m = month, then

$$w = d + 2m + [3(m+1)/5] + y + [y/4] - [y/100] + [y/400] + 2$$

January is taken as the 13th month of the previous year, and February as the 14th month. All other months are given their regular number.

The numbers in the 'square' brackets denote 'the greatest whole number less than the number'

e.g. $[7.6] = 7$ and $[18.39] = 18$.

When you have a value for w, divide it by 7. Your remainder is the day of the week. Sunday is the first day and Saturday is day 0.

On what day of the week did New Year's Day fall in 1982?

$d = 1$, $m = 13$, $y = 1981$ (using the previous year for January) we have,

$$w = 1 + 2(13) + [3(14)/5] + 1981 + [1981/4] - [1981/100] + [1981/400] + 2$$
$$= 1 + 26 + [8.4] + 1981 + [495.25] - [19.81] + [4.9525] + 2$$
$$= 1 + 26 + 8 + 1981 + 495 - 19 + 4 + 2 = 2498$$

2498/7 = 356 remainder 6. The day of the week is therefore Friday.

Try the following exercises:

1 Find the day of the week on which you were born.

2 On what day of the week will your birthday fall in the year 2030?

In this chapter you have seen that...

- ✔ algebraic expressions can often be simplified by cancelling factors that are common to the numerator and denominator
- ✔ two fractions can be multiplied together by multiplying their numerators and multiplying their denominators
- ✔ to divide by a fraction, turn it upside down and multiply
- ✔ to add or subtract algebraic fractions, express each fraction as an equivalent fraction with the LCM of all the denominators as the denominator
- ✔ to solve equations with fractions, multiply every term by the LCM of the denominators. The resulting equation should be of a type you are familiar with.

2 Algebraic products

At the end of this chapter you should be able to...

1 calculate the product of expressions in two brackets, each of which contains two terms

2 square an expression of the form $ax + b$

3 factorise expressions of the form

$$ax + ab, \quad ax^2 + bx, \quad ax + ay + bx + by.$$

Did you know?

The branch of mathematics called algebra gets its name from the Arabic word *al-jebr*, which means 'the pulling together of broken parts'. In the seventeenth century it was used to describe the surgical treatment of fractures!

You need to know...

✔ how to work with directed numbers

✔ how to multiply out expressions such as $3(5x - 2)$ and $3a(3b + 2c)$.

Key words

common factor, expanding, expression, factorising, grouping, like terms, product

Brackets

Remember that $5(x+1) = 5x + 5$

and that $4x(y+z) = 4xy + 4xz$

Multiplying out the brackets to give a single *expression* is known as *expanding* the brackets.

Exercise 2a

Expand:

1 $2(x+1)$

2 $3(x-1)$

3 $4(x+3)$

4 $5(a+4)$

5 $3(b+7)$

6 $3(1-a)$

7 $5(1-b)$

8 $2(3a-1)$

9 $4(2+3b)$

10 $5a(b-c)$

11 $4a(b-2c)$

12 $3a(2a+b)$

13 $5x(3y+z)$

14 $4y(4x+3z)$

15 $2n(3p-5q)$

16 $8r(2t-s)$

17 $3a(b-5c)$

18 $4x(3y+2z)$

The product of two brackets

Frequently, we wish to find the *product* of two brackets, each of which contains two terms, e.g. $(a+b)(c+d)$. The meaning of this product is that each term in the first bracket has to be multiplied by each term in the second bracket.

Always multiply the brackets together in the following order:

1 the first terms in the brackets

2 the outside terms

3 the inside terms

4 the second terms in the brackets.

Thus

$$(a+b)(c+d) = ac + ad + bc + bd$$

Exercise 2b

Expand $(x+2y)(2y-z)$

$$(x+2y)(2y-z) = 2xy - xz + 4y^2 - 2yz$$

Multiply the first term in the first bracket by each term in the second, then the second term in the first bracket by each term in the second. Try to keep the same order when you multiply two brackets. You are less likely to leave a term out.

Expand:

1 $(a+b)(c+d)$

2 $(p+q)(s+t)$

3 $(2a+b)(c+2d)$

4 $(5x+2y)(z+3)$

5 $(x+y)(z-4)$

6 $(a-b)(c+d)$

7 $(x+y)(y+z)$

8 $(2a+b)(3c+d)$

9 $(5x+4y)(z+2)$

10 $(3x-2y)(5-z)$

11 $(p+q)(2s-3t)$

12 $(a-2b)(c-d)$

13 $(6u-5v)(w-5r)$

14 $(3a+4b)(2c-3d)$

15 $(3x+2y)(3z+2)$

16 $(3p-q)(4r-3s)$

17 $(3a-4b)(3c+4d)$

18 $(7x-2y)(3-2z)$

We get a slightly simpler form when we find the product of two brackets such as $(x+2)$ and $(x+3)$,

i.e. using the order we chose earlier

$$(x+2)(x+3) = x^2 + 3x + 2x + 6$$

$$= x^2 + 5x + 6 \text{ (since } 2x \text{ and } 3x \text{ are } \textit{like terms}\text{)}$$

i.e. $(x+2)(x+3) = x^2 + 5x + 6$

Exercise 2c

Expand:

1 $(x+3)(x+4)$

2 $(x+2)(x+4)$

3 $(x+1)(x+6)$

4 $(x+5)(x+2)$

5 $(x+8)(x+3)$

6 $(a+4)(a+5)$

7 $(b+2)(b+7)$

8 $(c+4)(c+6)$

9 $(p+3)(p+12)$

Expand $(x-4)(x-6)$

$$(x-4)(x-6) = x^2 - 6x - 4x + 24$$

Collect the like terms: $= x^2 - 10x + 24$

Expand:

10 $(x-2)(x-3)$

11 $(x-5)(x-7)$

12 $(a-2)(a-8)$

13 $(x-10)(x-3)$

14 $(b-5)(b-5)$

15 $(x-3)(x-4)$

16 $(x-4)(x-8)$

17 $(b-4)(b-2)$

18 $(a-4)(a-4)$

Remember to keep to the same order when you multiply out the brackets.

Expand $(x+3)(x-6)$

$$(x+3)(x-6) = x^2 - 6x + 3x - 18$$

Collect the like terms: $= x^2 - 3x - 18$

Expand:

19 $(x+3)(x-2)$

20 $(x-4)(x+5)$

21 $(x-7)(x+4)$

22 $(a+3)(a-10)$

23 $(p+5)(p-5)$

24 $(x+7)(x-2)$

25 $(x-5)(x+6)$

26 $(x+10)(x-1)$

27 $(b-8)(b-7)$

Investigation

Four 5s can be written as $5 \times 5 \div (5 \div 5)$. The answer is 25.

Investigate other ways of writing four 5s, together with any of the signs +, –, × or ÷, to give an answer of 25.

Finding the pattern

You may have noticed in the previous exercise, that when you expanded the brackets and simplified the answers, there was a definite pattern,

e.g. $(x+5)(x+9) = x^2 + 9x + 5x + 45$

$= x^2 + 14x + 45$

We could have written it

$(x+5)(x+9) = x^2 + (9+5)\,x + (5) \times (9)$

$= x^2 + 14x + 45$

Similarly $(x+4)(x-7) = x^2 + (-7+4)\,x + (4) \times (-7)$

$= x^2 - 3x - 28$

and $(x-3)(x-8) = x^2 + (-8-3)\,x + (-3) \times (-8)$

$= x^2 - 11x + 24$

In each case there is a pattern:

- the product of the two numbers in the brackets gives the number term in the expansion
- collecting the two numbers in the brackets gives the number of x's.

Exercise 2d

Use the pattern given above to expand the following products:

1 $(x+4)(x+5)$

2 $(a+2)(a+5)$

3 $(x-4)(x-5)$

4 $(a-2)(a-5)$

5 $(x+8)(x+6)$

6 $(a+10)(a+7)$

7 $(x-8)(x-6)$

8 $(a-10)(a-7)$

9 $(a+2)(a-5)$

10 $(y-6)(y+3)$

11 $(z+4)(z-10)$

12 $(p+5)(p-8)$

13 $(a-10)(a+7)$

14 $(y+10)(y-2)$

15 $(z-12)(z+1)$

16 $(p+2)(p-13)$

17 $(x-5)(x-1)$

18 $(b+9)(b+7)$

19 $(a+4)(a-4)$

20 $(r-14)(r+2)$

21 $(p+12)(p+2)$

22 $(t+5)(t-12)$

23 $(c-5)(c+8)$

24 $(x+5)(x-5)$

The pattern is similar when the brackets are slightly more complicated.

Exercise 2e

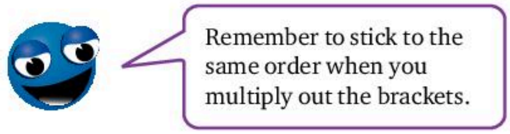
Expand the product $(2x+3)(x+2)$

$$(2x+3)(x+2) = 2x^2 + 4x + 3x + 6$$

Collect like terms: $\quad = 2x^2 + 7x + 6$

Expand the following products:

1 $(2x+1)(x+1)$
2 $(x+2)(5x+2)$
3 $(5x+2)(x+3)$
4 $(3x+4)(x+5)$
5 $(3x+2)(x+1)$
6 $(x+3)(3x+2)$
7 $(4x+3)(x+1)$
8 $(7x+2)(x+3)$



Expand the product $(3x-2)(2x+5)$

$$(3x-2)(2x+5) = 6x^2 + 15x - 4x - 10$$

$$= 6x^2 + 11x - 10$$

Expand:

9 $(3x+2)(2x+3)$
10 $(4x-3)(3x-4)$
11 $(5x+6)(2x-3)$
12 $(7a-3)(3a-7)$
13 $(5x+3)(2x+5)$
14 $(7x-2)(3x-2)$
15 $(3x-2)(4x+1)$
16 $(3b+5)(2b-5)$
17 $(2a+3)(2a-3)$
18 $(3b-7)(3b+7)$
19 $(7y-5)(7y+5)$
20 $(5a+4)(4a-3)$
21 $(4x+3)(4x-3)$
22 $(5y-2)(5y+2)$
23 $(3x-1)(3x+1)$
24 $(4x-7)(4x+5)$

Expand $(3x-2)(5-2x)$

$$(3x-2)(5-2x) = 15x - 6x^2 - 10 + 4x \qquad ((-2x)\times(-2) = +4x)$$

$$= 19x - 6x^2 - 10$$

$$= -6x^2 + 19x - 10$$

Expand:

25 $(2x+1)(1+3x)$

26 $(5x+2)(2-x)$

27 $(6x-1)(3-x)$

28 $(5a-2)(3-7a)$

29 $(3x+2)(4-x)$

30 $(4x-5)(3+x)$

31 $(5x+2)(4+3x)$

32 $(7x+4)(3-2x)$

33 $(4x-3)(3-5x)$

34 $(3-p)(4+p)$

35 $(x-5)(2+x)$

36 $(4x-3)(3+x)$

Important products

Three very important products are:

- $(x+a)^2 = (x+a)(x+a)$

  $= x^2 + xa + ax + a^2$

  $= x^2 + 2ax + a^2$ (since xa is the same as ax)

  i.e. $(x+a)^2 = x^2 + 2ax + a^2$

  so $(x+3)^2 = x^2 + 6x + 9$

- $(x-a)^2 = (x-a)(x-a)$

  $= x^2 - xa - ax + a^2$

  i.e. $(x-a)^2 = x^2 - 2ax + a^2$

  so $(x-4)^2 = x^2 - 8x + 16$

- $(x+a)(x-a) = x^2 - xa + ax - a^2$

  $= x^2 - a^2$

  i.e. $(x+a)(x-a) = x^2 - a^2$

  so $(x+5)(x-5) = x^2 - 25$

  and $(x-3)(x+3) = x^2 - 9$

You should learn these three results thoroughly, for they will appear time and time again. Given the left-hand side you should know the right-hand side, and vice versa.

Did you know?

Euclid of Alexandria – a mathematician more closely associated with geometry than algebra – was the first person to expand $(a+b)^2$ as $a^2 + 2ab + b^2$.

Exercise 2f

Expand $(x+5)^2$

Comparing with $(x+a)^2 = x^2 + 2ax + a^2$, $a = 5$

$$(x+5)^2 = x^2 + 10x + 25$$

Expand:

1	$(x+1)^2$	5	$(t+10)^2$	9	$(x+y)^2$	13	$(p+q)^2$
2	$(x+2)^2$	6	$(x+12)^2$	10	$(y+z)^2$	14	$(a+b)^2$
3	$(a+3)^2$	7	$(x+8)^2$	11	$(c+d)^2$	15	$(e+f)^2$
4	$(b+4)^2$	8	$(p+7)^2$	12	$(m+n)^2$	16	$(u+v)^2$

Expand $(2x+3)^2$

$$(2x+3)^2 = (2x)^2 + 2(2x)(3) + (3)^2$$

i.e. $$(2x+3)^2 = 4x^2 + 12x + 9$$

Expand:

17	$(2x+1)^2$	19	$(5x+2)^2$	21	$(3a+1)^2$	23	$(3a+4)^2$
18	$(4b+1)^2$	20	$(6c+1)^2$	22	$(2x+5)^2$	24	$(4y+3)^2$

Expand $(2x+3y)^2$

$$(2x+3y)^2 = (2x)^2 + 2(2x)(3y) + (3y)^2$$

i.e. $$(2x+3y)^2 = 4x^2 + 12xy + 9y^2$$

Expand:

25	$(x+2y)^2$	27	$(2x+5y)^2$	29	$(3a+b)^2$	31	$(7x+2y)^2$
26	$(3x+y)^2$	28	$(3a+2b)^2$	30	$(p+4q)^2$	32	$(3s+4t)^2$

Expand $(x-5)^2$

$$(x-5)^2 = x^2 - 10x + 25$$

Expand:

33	$(x-2)^2$	35	$(a-10)^2$	37	$(x-3)^2$	39	$(a-b)^2$
34	$(x-6)^2$	36	$(x-y)^2$	38	$(x-7)^2$	40	$(u-v)^2$

Expand $(2x-7)^2$

$$(2x-7)^2 = (2x)^2 + 2(2x)(-7) + (-7)^2$$

i.e. $$(2x-7)^2 = 4x^2 - 28x + 49$$

Expand:

41 $(3x-1)^2$
42 $(5z-1)^2$
43 $(10a-9)^2$
44 $(4x-3)^2$
<u>**45**</u> $(2a-1)^2$
<u>**46**</u> $(4y-1)^2$
<u>**47**</u> $(7b-2)^2$
<u>**48**</u> $(5x-3)^2$

Expand $(7a-4b)^2$

$$(7a-4b)^2 = (7a)^2 + 2(7a)(-4b) + (-4b)^2$$

i.e. $$(7a-4b)^2 = 49a^2 - 56ab + 16b^2$$

Expand:

49 $(2y-x)^2$
50 $(5x-y)^2$
51 $(3m-2n)^2$
52 $(7x-3y)^2$
<u>**53**</u> $(a-3b)^2$
<u>**54**</u> $(m-8n)^2$
<u>**55**</u> $(5a-2b)^2$
<u>**56**</u> $(3p-5q)^2$

Exercise 2g

Expand **a** $(a+2)(a-2)$ **b** $(2x+3)(2x-3)$

a $(a+2)(a-2) = a^2 - 4$

b $(2x+3)(2x-3) = 4x^2 - 9$

Expand:

1 $(x+4)(x-4)$
2 $(b+6)(b-6)$
3 $(c-3)(c+3)$
4 $(x+12)(x-12)$
<u>**5**</u> $(x+5)(x-5)$
<u>**6**</u> $(a-7)(a+7)$
<u>**7**</u> $(q+10)(q-10)$
<u>**8**</u> $(x-8)(x+8)$
9 $(2x-1)(2x+1)$
10 $(3x+1)(3x-1)$
11 $(7a+2)(7a-2)$
12 $(5a-4)(5a+4)$
<u>**13**</u> $(5x+1)(5x-1)$
<u>**14**</u> $(2a-3)(2a+3)$
<u>**15**</u> $(10m-1)(10m+1)$

Expand $(3x+2y)(3x-2y)$

$$(3x+2y)(3x-2y) = (3x)^2 - (2y)^2$$
$$= 9x^2 - 4y^2$$

Expand:

16 $(3x+4y)(3x-4y)$

17 $(2a-5b)(2a+5b)$

18 $(1-2a)(1+2a)$

19 $(7y+3z)(7y-3z)$

20 $(10a-9b)(10a+9b)$

21 $(5a-4b)(5a+4b)$

22 $(1+3x)(1-3x)$

23 $(3-5x)(3+5x)$

24 $(5m+8n)(5m-8n)$

The results from Exercise 2g are very important when written the other way around,

i.e. $a^2-b^2=(a+b)(a-b)$

We refer to this as 'factorising the difference between two squares'.

Harder expansions

Exercise 2h

Simplify $(x+2)(x+5)+2x(x+7)$

Work out the brackets first:

$$(x+2)(x+5)+2x(x+7)=x^2+5x+2x+10+2x^2+14x$$

Collect like terms: $=3x^2+21x+10$

Simplify:

1 $(x+3)(x+4)+x(x+2)$

2 $x(x+6)+(x+1)(x+2)$

3 $(x+4)(x+5)+6(x+2)$

4 $(a-6)(a-5)+2(a+3)$

5 $(a-5)(2a+3)-3(a-4)$

6 $(x+3)(x+5)+5(x+2)$

7 $(x-3)(x+4)-3(x+3)$

8 $(x+7)(x-5)-4(x-3)$

9 $(2x+1)(3x-4)+(2x+3)(5x-2)$

10 $(5x-2)(3x+5)-(3x+5)(x+2)$

Expand $(xy-z)^2$

$$(xy-z)^2=(xy)^2-2(xy)(z)+z^2$$

i.e. $$(xy-z)^2=x^2y^2-2xyz+z^2$$

Expand:

11 $(xy-3)^2$

12 $(5-yz)^2$

13 $(xy+4)^2$

14 $(3pq+8)^2$

15 $(a-bc)^2$

16 $(ab-2)^2$

17 $(6-pq)^2$

18 $(mn+3)^2$

19 $(uv-2w)^2$

Summary

So far we have removed brackets and expanded expressions. Here is a summary of the most important types of examples considered so far.

1 $2(3x+4)=6x+8$

2 $(x+2)(x+3)=x^2+5x+6$

3 $(x-2)(x-3)=x^2-5x+6$

4 $(x-2)(x+3)=x^2+x-6$

5 $(2x+1)(3x+2)=6x^2+7x+2$

6 $(2x-1)(3x-2)=6x^2-7x+2$

7 $(2x+1)(3x-2)=6x^2-x-2$

Note that **a** if the signs in the brackets are the same, i.e. both + or both –, then the number term is + (numbers **2, 3, 5** and **6**)

whereas **b** if the signs in the brackets are different, i.e. one + and one –, then the number term is – (numbers **4** and **7**)

c the middle term is given by collecting the product of the outside terms in the brackets and the product of the inside terms in the brackets, i.e.

in **2** the middle term is $3x+2x$ or $5x$

in **3** the middle term is $-3x-2x$ or $-5x$

in **4** the middle term is $3x-2x$ or x

in **5** the middle term is $4x+3x$ or $7x$

in **6** the middle term is $-4x-3x$ or $-7x$

in **7** the middle term is $-4x+3x$ or $-x$.

Most important of all we must remember the general expansions:

$$(x+a)^2=x^2+2ax+a^2$$

$$(x-a)^2=x^2-2ax+a^2$$

$$(x+a)(x-a)=x^2-a^2$$

Mixed questions

Exercise 2i

Expand:

1 $5(x+2)$
2 $8p\,(3q-2r)$
3 $(3a+b)(2a-5b)$
4 $(4x+1)(3x-5)$
5 $(x+6)(x+10)$
6 $(x-8)(x-12)$
7 $(4y+3)(4y-7)$
8 $(4y-9)(4y+9)$
9 $(5x+2)^2$
10 $(2a-7b)^2$

Exercise 2j

Expand:

1 $4(2-5x)$
2 $8a\,(2-3a)$
3 $(4a+3)(3a-11)$
4 $(x+11)(x-9)$
5 $(2x+5)(1-10x)$
6 $(y+2z)^2$
7 $(6y-z)(6y+5z)$
8 $(4a+1)^2$
9 $(5a-7)^2$
10 $(6z-13y)^2$

Did you know?

Did you know that the way of writing decimals is not the same throughout the world? In most English-speaking countries, one point five is written as 1.5, but in much of mainland Europe it is written 1,5.

Finding factors

Frequently we need to find the factors of an expression. This is called *factorising*.

In the expression $7a+14b$ we could write the first term as $7\times a$ and the second term as $7\times 2b$,

i.e. $7a+14b=7\times a+7\times 2b$

The 7 is a *common factor*.

However, we already know that $7(a+2b)=7\times a+7\times 2b$

$\therefore$ $7a+14b=7\times a+7\times 2b=7(a+2b)$

Exercise 2k

Factorise $3x - 12$

3 is a factor of $3x$ and of 12,

so $$3x - 12 = (3 \times x) - (3 \times 4)$$
$$= 3(x - 4)$$

Factorise:

1 $4x + 4$
2 $12x - 3$
3 $6a + 2$
4 $5a - 10b$
5 $3t - 9$
6 $10a - 5$
7 $12a + 4$
8 $2a + 4b$
9 $14x - 7$

Factorise $x^2 - 7x$

$$x^2 - 7x = x \times x - 7 \times x \quad \text{so } x \text{ is a common factor.}$$
$$= x(x - 7)$$

Factorise:

10 $x^2 + 2x$
11 $x^2 - 7x$
12 $a^2 + 6a$
13 $2x^2 + x$
14 $4t - 2t^2$
15 $x^2 + 5x$
16 $x^2 - 4x$
17 $b^2 + 4b$
18 $4a^2 - a$

Factorise $9ab + 12bc$

$$9ab + 12bc = 3b \times 3a + 3b \times 4c$$
$$= 3b(3a + 4c)$$

You may not at first see that both 3 and b are common factors. If you spot only 3 you would have $9ab + 12bc = 3(3ab + 4bc)$, then a check inside the bracket shows that b is also a common factor.

Factorise:

19 $2x^2 - 6x$
20 $2z^3 + 4z$
21 $25a^2 - 5a$
22 $12x^2 + 16x$
23 $5ab - 10bc$
24 $3y^2 + 27y$
25 $2a^2 - 12a$
26 $6p^2 + 2p$
27 $9y^2 - 6y$

Always check the terms inside the bracket to make sure that you have not missed any common factors.

Factorise $ab + 2bc + bd$

$$ab + 2bc + bd = b(a + 2c + d)$$

Factorise:

28 $2x^2 + 4x + 6$

29 $10a^2 - 5a + 20$

30 $ab + 4bc - 3bd$

31 $8x - 4y + 12z$

32 $9ab - 6ac - 3ad$

33 $3x^2 - 6x + 9$

34 $4a^2 + 8a - 4$

35 $5xy + 4xz + 3x$

36 $5ab + 10bc + 5bd$

37 $2xy - 4yz + 8yw$

Factorise $8x^3 - 4x^2$

$$8x^3 - 4x^2 = 4x^2(2x - 1)$$

You may do this in stages:

$$\begin{aligned} 8x^3 - 4x^2 &= 4(2x^3 - x^2) && \text{(take out 4)} \\ &= 4x(2x^2 - x) && \text{(take out } x\text{)} \\ &= 4x^2(2x - 1) && \text{(take out another } x\text{)} \end{aligned}$$

Factorise:

38 $x^3 + x^2$

39 $x^2 - x^3$

40 $20a^2 - 5a^3$

41 $12x^3 - 16x^2$

42 $4x^4 + 12x^2$

43 $a^2 + a^3$

44 $b^3 - b^2$

45 $4x^3 - 2x^2$

46 $27a^2 - 18a^3$

47 $10x^2 - 15x^4$

Factorise:

48 $12x + 8$

49 $8x^2 + 12x$

50 $9x^2 - 6x + 12$

51 $5x^3 - 10x$

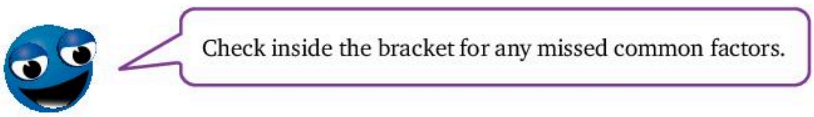
52 $8pq + 4qr$

53 $x^2 - 8x$

54 $12 + 9y^2$

55 $12xy + 16xz + 8x$

56 $4x^3 + 6x$



Factorise $\frac{1}{3}Mv - \frac{1}{3}mv$

$$\begin{aligned} \frac{1}{3}Mv - \frac{1}{3}mv &= \frac{1}{3}v \times M - \frac{1}{3}v \times m \\ &= \frac{1}{3}v(M - m) \quad (\tfrac{1}{3}v \text{ is a common factor, and note that } M \text{ and } m \text{ are not the same.}) \end{aligned}$$

57 $\frac{1}{2}ah + \frac{1}{2}bh$

58 $mg - ma$

59 $\frac{1}{2}mv^2 + \frac{1}{2}mu^2$

60 $P + \frac{PRT}{100}$

61 $2\pi r^2 + \pi rh$

62 $\pi R^2 + \pi r^2$

63 $2gh_1 - 2gh_2$

64 $\frac{1}{2}mv^2 - mgh$

65 $\frac{4}{3}\pi r^3 - \frac{1}{3}\pi r^2 h$

66 $3\pi r^2 + 2\pi rh$

67 $\frac{1}{2}mu^2 + \frac{1}{2}mu$

68 $\frac{1}{2}bc - \frac{1}{4}ca$

Factorising by grouping

The expression $ax + ay + bx + by$ can be factorised by *grouping* the terms in pairs. If we group the first two terms followed by the remaining two terms, i.e. $(ax + ay)$ followed by $(bx + by)$, and factorise each group, we have $a(x + y)$ and $b(x + y)$

Therefore $\qquad ax + ay + bx + by = a(x + y) + b(x + y)$

We can think of this as $aB + bB$, where B stands for $(x + y)$.

We have reduced the original four terms to two terms and these two terms now have the bracket B as a common factor.

Therefore

$$\begin{aligned} ax + ay + bx + by &= a(x + y) + b(x + y) \\ &= aB + bB \\ &= B(a + b) \\ &= (x + y)(a + b) \end{aligned}$$

If, on the other hand, we pair the first and third terms followed by the remaining terms, we have

$$\begin{aligned} ax + ay + bx + by &= x(a + b) + y(a + b) \\ &= (a + b)(x + y) \end{aligned}$$

This shows that, while it is often possible to pair the terms with a common factor in more than one way, the result is the same. Always check your factors by multiplying out.

Exercise 2I

Factorise $xy + 2x + 3y + 6$

$$\begin{aligned} xy + 2x + 3y + 6 &= xy + 2x && \text{(common factor } x\text{)} \\ &\quad + 3y + 6 && \text{(common factor 3)} \\ &= x(y + 2) + 3(y + 2) \\ & && \text{(common factor } y + 2\text{)} \\ &= (y + 2)(x + 3) \end{aligned}$$

Alternatively

$$\begin{aligned} xy + 2x + 3y + 6 &= y(x + 3) + 2(x + 3) \\ &= (x + 3)(y + 2) \end{aligned}$$

Check your answer by expanding the brackets.

Factorise the following expressions by grouping:

1 $xy + 3x + 3y + 9$

2 $a + ab + 2b + 2b^2$

3 $a^2 + ab + ac + bc$

4 $xy - 3y + 2x - 6$

5 $xz + z + xy + y$

6 $xy + 4x + 2y + 8$

7 $ac + 4a + bc + 4b$

8 $xy - 2x + 4y - 8$

9 $pr + ps + qr + qs$

10 $xy - 3y + 4x - 12$

11 $xy - 5x + 2y - 10$

12 $pr - ps + qr - qs$

13 $ab - 3a + 2b - 6$

14 $pr - qr + ps - qs$

15 $2p + pq + 4q + 8$

16 $6 + 2b + 3a + ab$

Factorise $2x - 2xy - y + y^2$

$$2x - 2xy - y + y^2 = 2x(1 - y) - y(1 - y)$$

Check: $2x(1 - y) = 2x - 2xy$ and $-y(1 - y) = -y + y^2$

$$\therefore \quad 2x - 2xy - y + y^2 = (1 - y)(2x - y)$$

Check: $(1 - y)(2x - y) = 2x - y - 2xy + y^2$

Factorise:

17 $pr - ps - qr + qs$

18 $9a - 3b - 3ab + b^2$

19 $2a - b - 2ab + b^2$

20 $a^2 + 2ab - 2a - 4b$

21 $6 - 3x - 2y + xy$

22 $4a^2 - ab - 8a + 2b$

23 $6a^2 - 9a - 2ab + 3b$

24 $2m - 3n - 2mn + 3n^2$

25 $t^2 + tr + st + sr$

26 $x^2 - x + xy - y$

27 $4a - 4a^2 + 2b - 2ab$

28 $x - xy + y - y^2$

29 $4a + 6b - 6a^2 - 9ab$

30 $2a^2 + 2ab + bc + ac$

31 $4x - 4xy + 2y - 2y^2$

32 $xy + xz + y^2 + yz$

Factorise $ac - ad + bd - bc$

$$ac - ad + bd - bc = a(c - d) + b(d - c)$$

$d - c = -1(c - d)$ so $b(d - c) = -b(c - d)$

$$\therefore \quad ac - ad + bd - bc = a(c - d) - b(c - d)$$

$$= (c - d)(a - b)$$

Factorise:

33 $5x - xy + 2y - 10$

34 $ab - 3a - 12 + 4b$

35 $xy - xz - 3z + 3y$

36 $2p - pq + 4q - 8$

37 $6 - 2b + ab - 3a$

38 $3a - ab + 4b - 12$

Factorise $a^2 - ab + a - b$

$$a^2 - ab + a - b = a(a-b) + 1(a-b)$$
$$= (a-b)(a+1)$$

Alternatively $a^2 - ab + a - b = a^2 + a - ab - b = a(a+1) - b(a+1) = (a-b)(a+1)$

Factorise:

39 $m^2 + mn + m + n$

40 $a^2 - ab + a - b$

41 $2p^2 - 4pq + p - 2q$

42 $x - xy + 1 - y$

43 $a^2 + ab + a + b$

44 $a^2 - ab + a - b$

45 $x^2 + xy - x - y$

46 $2a^2 + ab - 2a - b$

47 $5x^2 + 10xy - x - 2y$

48 $mn - m - n + 1$

49 $3x^2 + xy - 3x - y$

50 $2p^2 + 4pq - p - 2q$

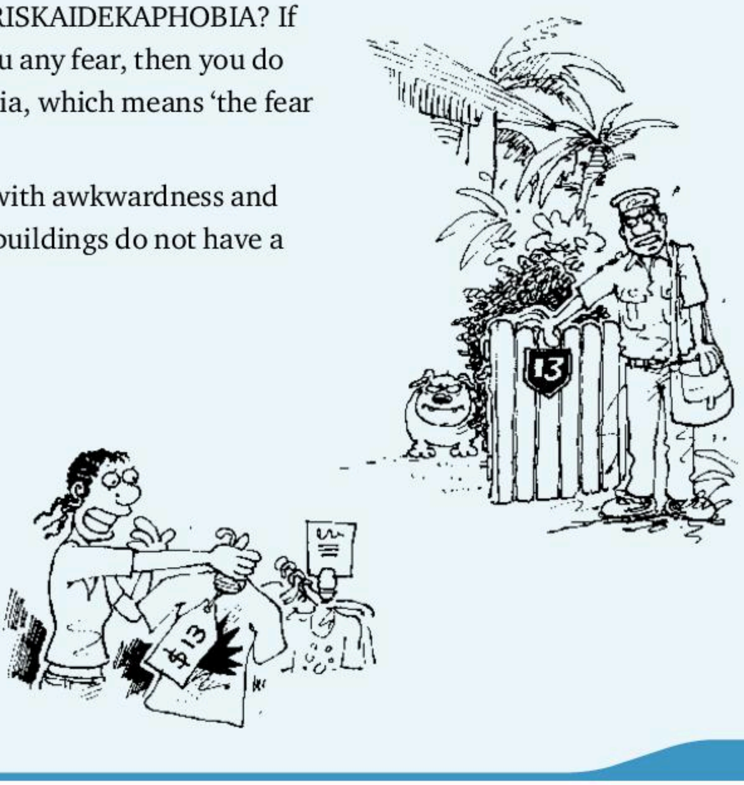
51 $3a + b - 3a^2 - ab$

52 $2x + y - 2xz - yz$

Did you know?

Do you know the meaning of TRISKAIDEKAPHOBIA? If the number 13 did not cause you any fear, then you do not suffer from triskaidekaphobia, which means 'the fear of the number 13'.

'Thirteen' is usually associated with awkwardness and bad luck. For this reason many buildings do not have a 13th floor.



Mixed exercise

Exercise 2m

Select the letter that gives the correct answer.

1 $7(x-3) =$

A $x-21$ **B** $7x-21$ **C** $7x-3$ **D** $7x+21$

2 $(x+4)(x+6) =$

A $x^2+2x+24$ **B** $x^2+4x+10$ **C** $x^2+6x+10$ **D** $x^2+10x+24$

3 $(x-5)(x+2) =$

A $x^2-7x-10$ **B** $x^2-3x-10$ **C** $x^2+3x-10$ **D** $x^2-3x+10$

4 $(2x+3)(1-4x) =$

A $3-10x-8x^2$ **B** $3-10x+8x^2$ **C** $3+10x-8x^2$ **D** $8x^2+10x-3$

5 $(x+2y)^2 =$

A $x^2+4xy+y^2$ **B** $x^2+2xy+4y^2$ **C** x^2+4y+y^2 **D** $x^2+4xy+4y^2$

6 $(4x-3y)(4x+3y) =$

A $16x^2-24xy-9y^2$ **B** $16x^2-9y^2$ **C** $16x^2+9y^2$ **D** $16x^2+24xy+9y^2$

7 $2x^2-4x =$

A $2(2x-4)$ **B** $2(x-4)$ **C** $2x(x-2)$ **D** $x(x-4)$

8 $\frac{1}{3}ab+\frac{1}{6}bc =$

A $\frac{1}{3}a(b+\frac{1}{3}c)$ **B** $\frac{1}{6}b(a+c)$ **C** $\frac{1}{3}b(a+2c)$ **D** $\frac{1}{6}b(2a+c)$

9 $ax+by+ay+bx =$

A $(a+b)(x+y)$ **B** $(a-b)(x-y)$ **C** $(a+x)(b+y)$ **D** $(a+y)(b+x)$

10 $2a^2+ab-2a-b =$

A $(a+b)(2a-1)$ **B** $(a+2)(a-b)$ **C** $(a+1)(2a-b)$ **D** $(a-1)(2a+b)$

In this chapter you have seen that...

- two brackets are multiplied together by multiplying each term in one bracket by every term in the other bracket, e.g.

  $(3a + 2b)(2c - 3d) = 6ac - 9ad + 4bc - 6bd$

  (Always try to do the multiplying in the same order.)

- three important products which you should commit to memory are:

  $(x + a)^2 = x^2 + 2ax + a^2$

  $(x - a)^2 = x^2 - 2ax + a^2$

  $(x + a)(x - a) = x^2 - a^2$ and $(x - a)(x + a) = x^2 - a^2$

- multiplying brackets of the form

  $(x + a)(x + b)$ gives $x^2 + (a + b)x + ab$.

3 Simultaneous equations

At the end of this chapter you should be able to...

1 solve a pair of simultaneous equations by

- **a** the method of elimination
- **b** the substitution method

2 solve problems using simultaneous equations

3 identify a pair of equations which have

- **a** no solution
- **b** an infinite number of solutions.

Did you know?

Equations containing several unknown quantities were known to the ancient Egyptians, Greeks and Indians. The Hindus used the names of colours to distinguish the unknown quantities.

You need to know...

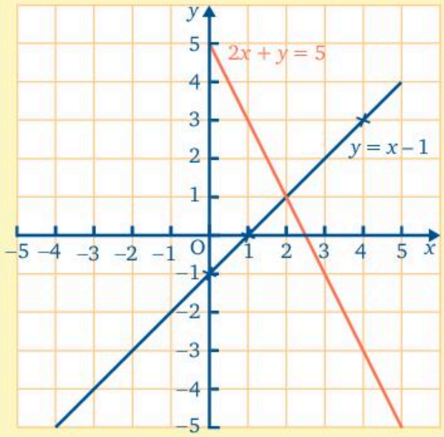
- ✔ how to solve a linear equation in one unknown
- ✔ the rules for working with negative numbers
- ✔ how to draw the graph of a straight line from its equation.

Key words

elimination, equation, infinite, simultaneous, substitution

Graphical solution of simultaneous equations

We saw in Grade 8 that when we are given an *equation* of the form $y = mx + c$ we can represent it by a straight-line graph.



Two such equations give us two straight lines which usually cross one another.

Consider the two equations $y = x - 1$, $2x + y = 5$. The graphs of these equations show that where the two lines cross, at the point with the coordinates (2, 1), the values of x and y satisfy the equations simultaneously.

Therefore $x = 2$ and $y = 1$ are the solutions of the pair of equations.

Algebraic solution of simultaneous equations

Looking at the equation $y = x - 1$ we can see that there are many possible values of x and y which will fit, for example $x = 1$ and $y = 0$, or $x = 4$ and $y = 3$. We could also have $x = 0$ and $y = -1$ or even $x = 2.632$ and $y = 1.632$. Indeed there is an *infinite* set of pairs of solutions.

If however we are also told that x and y must also satisfy $2x + y = 5$, we find that not every pair of numbers that satisfies the first equation also satisfies the second equation. While $x = 1, y = 0$ satisfies the first equation, it does not satisfy $2x + y = 5$.

These two equations together form a pair of *simultaneous* equations. 'Simultaneous' means that the two equations are both satisfied by the same values of x and y; that is, when x has the same value in both equations then y has the same value in both. We saw above that we could find this common pair of values by drawing a graph but now we are going to solve them algebraically without drawing a graph. Drawing a graph takes a long time and frequently does not give the most accurate value.

There are several different methods for solving simultaneous equations algebraically. We start with the simplest.

Elimination method

Whenever we meet a new type of equation, we try to reorganise it so that it is similar to equations we have already met.

Previous equations have had only one unknown quantity, so we try to *eliminate* one of the two unknowns.

Consider the pair of equations

$$2x + y = 8 \qquad [1]$$
$$x + y = 5 \qquad [2]$$

In this case, if we try subtracting the second equation from the first we find that the y term disappears but the x term does not:

i.e. [1] – [2] gives $x = 3$

Then, substituting 3 for x in equation [2], we see that $3 + y = 5$ so $y = 2$

We can check that $x = 3$ and $y = 2$ also satisfy equation [1].

Notice that it is essential to number the equations and to say that you are subtracting them.

Sometimes it is easier to subtract the first equation from the second rather than the second equation from the first. (In this case we would write equation [1] again, underneath equation [2].)

Sometimes we can eliminate x rather than y.

Exercise 3a

Solve the equations $\begin{aligned} x + y &= 5 \\ 3x + y &= 7 \end{aligned}$

$$x + y = 5 \qquad [1]$$
$$3x + y = 7 \qquad [2]$$
$$x + y = 5 \qquad [1]$$

[2] – [1] gives

$$2x = 2$$
$$x = 1$$

(To find y choose the simpler equation, i.e. the first.)
Substituting 1 for x in [1] gives

$$1 + y = 5$$
$$y = 4$$

(Check in the equation *not* used for finding y.)

Check in [2] left-hand side $= 3 + 4$
$= 7 =$ right-hand side

Therefore the solution is $x = 1, y = 4$

Solve the following pairs of equations:

1 $x + y = 5$
$4x + y = 14$

2 $5x + y = 14$
$3x + y = 10$

3 $2a + b = 11$
$4a + b = 17$

4 $2x + 3y = 23$
$x + 3y = 22$

5 $5x + 2y = 14$
$7x + 2y = 22$

6 $x + 2y = 12$
$x + y = 7$

7 $4p + 3q = -5$
$7p + 3q = -11$

8 $12x + 5y = 65$
$9x + 5y = 50$

9 $3x + 4y = 15$
$3x + 2y = 12$

10 $9c + 2d = 54$
$c + 2d = 6$

11 $2x + 3y = -8$
$2x + y = -4$

12 $9x + 5y = 45$
$4x + 5y = 45$

Not all pairs of simultaneous equations can be solved by subtracting one from the other.

Consider $4x + y = 6$ [1]
$2x - y = 0$ [2]

If we subtract we get $2x + 2y = 6$ which is no improvement.

On the other hand, if we add we get $6x = 6$ which eliminates y.

If the signs in front of the letter to be eliminated are the same we should *subtract*; if the signs are different we should *add*.

Exercise 3b

Solve the equations $x - 2y = 1$
$3x + 2y = 19$

$x - 2y = 1$ [1]
$3x + 2y = 19$ [2]

[1] + [2] gives $4x = 20$
$x = 5$

(It is easier to use the equation with the + sign to find y.)

Substitute 5 for x in [2] $15 + 2y = 19$

Subtract 15 from both sides $2y = 4$
$y = 2$

Check in [1] left-hand side $= 5 - 4 = 1 =$ right-hand side

Therefore the solution is $x = 5, y = 2$

Solve the following pairs of equations:

1 $x - y = 2$
$3x + y = 10$

2 $2x - y = 6$
$3x + y = 14$

3 $p + 2q = 11$
$3p - 2q = 1$

4 $3a - b = 10$
$3a + b = 2$

5 $6x + 2y = 19$
$x - 2y = 2$

6 $4x + y = 37$
$2x - y = 17$

<u>7</u> $x + y = 2$
$2x - y = 10$

<u>8</u> $5p + 3q = 5$
$4p - 3q = 4$

<u>9</u> $3x - 4y = -24$
$5x + 4y = 24$

To solve the following equations, first decide whether to add or subtract:

10 $3x + 2y = 12$
$x + 2y = 8$

11 $x - 2y = 6$
$4x + 2y = 14$

12 $x + 3y = 12$
$x + y = 8$

13 $9x + 2y = 48$
$x - 2y = 2$

14 $4x + y = 19$
$3x + y = 15$

<u>15</u> $2x + 3y = 13$
$2x + 5y = 21$

<u>16</u> $5x - 2y = 24$
$x + 2y = 0$

<u>17</u> $x + 3y = 0$
$x - y = -4$

<u>18</u> $5p - 3q = 9$
$4p + 3q = 9$

Solve the equations $4x - y = 10$
$x - y = 1$

$$4x - y = 10 \qquad [1]$$
$$x - y = 1 \qquad [2]$$

(The signs in front of the y terms are the same so we subtract:
remember that $-y - (-y) = -y + y = 0$)

[1] – [2] gives $3x = 9$

$x = 3$

Substitute 3 for x in [2] $3 - y = 1$

Add y to both sides $3 = 1 + y$

Subtract 1 from both sides $2 = y$

Check in [1] left-hand side = 12 – 2 = 10 = right-hand side

Therefore the solution is $x = 3, y = 2$

Solve the following pairs of equations:

19 $2x - y = 4$
$x - y = 1$

20 $2p - 3q = -7$
$4p - 3q = 1$

21 $x - y = 3$
$3x - y = 9$

<u>22</u> $6x - y = 7$
$2x - y = 1$

<u>23</u> $5x - 2y = -19$
$x - 2y = -7$

<u>24</u> $2x - 3y = 14$
$2x - y = 10$

25 $3x - 2y = 14$
$x + 2y = 10$

26 $3p - 5q = -3$
$4p - 5q = 1$

27 $3p + 5q = 17$
$4p + 5q = 16$

<u>28</u> $3p - 5q = 7$
$4p + 5q = -14$

<u>29</u> $3p + 5q = 35$
$4p - 5q = 0$

30 $3x - y = 10$
$x + y = -2$

Harder elimination

Equations are not always as simple as the ones we have had so far.

Consider $2x + 3y = 4$ [1]
$4x + y = -2$ [2]

Whether we add or subtract neither letter will disappear, so it is necessary to do something else first.

If we multiply the second equation by 3 to give $12x + 3y = -6$, we have the same number of y's in each equation. Then we can use the same method as before:

[2] × 3 $12x + 3y = -6$ [3]
$2x + 3y = 4$ [1]

[3] – [1] gives $10x = -10$
$x = -1$

Substitute –1 for x in [2] $-4 + y = -2$

Add 4 to both sides $y = 2$

Therefore the solution is $x = -1, y = 2$

Exercise 3c

Solve the equations $3x - 2y = 1$
$4x + y = 5$

	$3x - 2y = 1$	[1]
	$4x + y = 5$	[2]
[2] × 2 gives	$8x + 2y = 10$	[3]
	$3x - 2y = 1$	[1]
[1] + [3] gives	$11x = 11$	
	$x = 1$	
Substitute 1 for x in [2]	$4 + y = 5$	
Subtract 4 from both sides	$y = 1$	
Check in [1]	left-hand side $= 3 - 2$	
	$= 1 =$ right-hand side	

Therefore the solution is $x = 1, y = 1$

Solve the following pairs of equations:

1 $2x + y = 7$
$3x + 2y = 11$

2 $5x - 4y = -3$
$3x + y = 5$

3 $9x + 7y = 10$
$3x + y = 2$

4 $5x + 3y = 21$
$2x + y = 3$

5 $6x - 4y = -4$
$5x + 2y = 2$

6 $4x + 3y = 25$
$x + 5y = 19$

Solve the following pairs of equations:

7 $5x + 3y = 11$
$4x + 6y = 16$

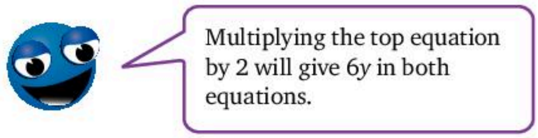
8 $2x - 3y = 1$
$5x + 9y = 19$

9 $2x + 5y = 1$
$4x + 3y = 9$

10 $9x + 5y = 15$
$3x - 2y = -6$

11 $4x + 3y = 1$
$16x - 5y = 21$

12 $7p + 2q = 22$
$3p + 4q = 11$



Puzzle

The Wilberforce hotel has 134 rooms and can accommodate up to 253 residents in single or double rooms. How many double rooms and how many single rooms are there?

Exercise 3d

Solve the equations $3x + 5y = 6$
$2x + 3y = 5$

$3x + 5y = 6$ [1]
$2x + 3y = 5$ [2]

We can choose to either get the same number of x's ([1] × 2 and [2] × 3 does this) or the same number of y's, which is what we will do.

[1] × 3 gives $9x + 15y = 18$ [3]
[2] × 5 gives $10x + 15y = 25$ [4]
$9x + 15y = 18$ [3]
[4] – [3] gives $x = 7$
Substitute 7 for x in [2] $14 + 3y = 5$
Subtract 14 from both sides $3y = -9$
Divide both sides by 3 $y = -3$
Check in [1] left-hand side $= 21 - 15$
$= 6 =$ right-hand side

Therefore the solution is $x = 7, y = -3$

Solve the following pairs of equations:

1 $2x + 3y = 12$
$5x + 4y = 23$

2 $3x - 2y = -7$
$4x + 3y = 19$

3 $2x - 5y = 1$
$5x + 3y = 18$

4 $6x + 5y = 9$
$4x + 3y = 6$

5 $14x - 3y = -18$
$6x + 2y = 12$

6 $6x - 7y = 25$
$7x + 6y = 15$

7 $5x + 4y = 21$
$3x + 6y = 27$

8 $9x + 8y = 17$
$2x - 6y = -4$

9 $9x - 2y = 14$
$7x + 3y = 20$

10 $5x + 4y = 11$
$2x + 3y = 3$

11 $4x + 5y = 26$
$5x + 4y = 28$

12 $2x - 6y = -6$
$5x + 4y = -15$

13 $5x - 6y = 6$
$2x + 9y = 10$

14 $3p + 4q = 5$
$2p + 10q = 18$

15 $6x + 5y = 8$
$3x + 4y = 1$

16 $7x - 3y = 20$
$2x + 4y = -4$

17 $10x + 3y = 12$
$3x + 5y = 20$

18 $6x - 5y = 4$
$4x + 2y = -8$

19 $5x + 3y = 8$
$3x + 5y = 8$

20 $7x + 2y = 23$
$3x - 5y = 4$

21 $6x - 5y = 17$
$5x + 4y = 6$

22 $3x + 8y = 56$
$5x - 6y = 16$

23 $7x + 3y = -9$
$2x + 5y = 14$

24 $7x + 6y = 0$
$5x - 8y = 43$

25 $2x + 6y = 30$
$3x + 10y = 49$

26 $4x - 3y = -7$
$3x + 2y = 16$

27 $17x - 2y = 47$
$5x - 3y = 9$

28 $8x + 3y = -17$
$7x - 4y = 5$

Mixed questions

Exercise 3e

Solve the following pairs of equations:

1 $x+2y=9$
$2x-y=-2$

2 $x+y=4$
$x+2y=9$

3 $2x+3y=0$
$3x+2y=5$

4 $3x-y=-10$
$4x-y=-4$

5 $5x+2y=16$
$2x-3y=-5$

6 $3x+2y=-5$
$3x-4y=1$

<u>7</u> $x+y=6$
$x-y=1$

<u>8</u> $3x-5y=13$
$2x+5y=-8$

<u>9</u> $7x+3y=35$
$2x-5y=10$

<u>10</u> $9x+2y=8$
$7x+3y=12$

<u>11</u> $2x-5y=1$
$3x+4y=13$

<u>12</u> $3x-2y=-2$
$5x-y=-15$

Sometimes the equations are arranged in an awkward fashion and need to be rearranged before solving them.

Exercise 3f

Solve the equations $x=4-3y$
$2y-x=1$

$$x=4-3y \qquad [1]$$
$$2y-x=1 \qquad [2]$$

(We must first arrange the equations so that the letters are in the same corresponding positions in both equations.

By adding $3y$ to both sides, equation [1] can be written $3y+x=4$)

$$3y+x=4 \qquad [3]$$
$$2y-x=1 \qquad [2]$$

[3] + [2] gives $5y=5$

$y=1$

Substitute 1 for y in [1] $x=4-3$

$x=1$

Check in [2] left-hand side $= 2-1=1=$ right-hand side

Therefore the solution is $x=1$, $y=1$

Solve the following pairs of equations:

1 $y = 6 - x$
$2x + y = 8$

2 $x - y = 2$
$2y = x + 1$

3 $3 = 2x + y$
$4x + 6 = 10y$

<u>4</u> $9 + x = y$
$x + 2y = 12$

<u>5</u> $2y = 16 - x$
$x - 2y = -8$

<u>6</u> $3x + 4y = 7$
$2x = 5 - 3y$

So far, we have rearranged the equations so the letter terms are in the same order on the left-hand side and the right-hand side is just the number term. However, as long as the *x* and *y* and number terms are in corresponding positions in the two equations, we can work with them without rearrangement.

Solve the equations $y = x + 5$
$y = 7 - x$

	$y = x + 5$	[1]
	$y = 7 - x$	[2]
Rewrite [1] as	$y = 5 + x$	[3]
[2] + [3] gives	$2y = 12$	
	$y = 6$	
Substitute 6 for *y* in [1]	$6 = x + 5$	
	$x = 1$	
Check in [2]	left-hand side = 6	
	right-hand side = 7 − 1 = 6	

Therefore the solution is $x = 1, y = 6$

Solve the following pairs of equations:

7 $y = 9 + x$
$y = 11 - x$

8 $x = 3 + y$
$2x = 4 - y$

9 $y = 4 - x$
$y = x + 6$

<u>10</u> $2y = 4 + x$
$y = x + 8$

<u>11</u> $x + 4 = y$
$y = 10 - 2x$

<u>12</u> $x + y = 12$
$y = 3 + x$

Special cases

Some pairs of equations have no solution and some have an infinite number of solutions.

Exercise 3g

Try solving the following pairs of equations. Comment on why the method breaks down:

1 $x + 2y = 6$
$x + 2y = 7$

2 $3x + 4y = 1$
$6x + 8y = 2$

3 $y = 4 + 2x$
$y - 2x = 6$

4 $9x = 3 - 6y$
$3x + 2y = 1$

5 Make up other pairs of equations which either have no solution or have an infinite set of solutions.

Substitution method

A second method for solving simultaneous equations avoids adding or subtracting equations. This is the *substitution* method. We start with one of the equations and rearrange it to make one unknown on its own on one side of the equals sign.

For example, for the equations $2x - y = 9$ [1]
and $3y + x = 1$ [2]

we can choose [2] to 'solve' for x: $x = 1 - 3y$ (subtracting $3y$ from both sides)

The next step is to substitute $1 - 3y$ for x in equation [1].

This gives $2(1 - 3y) - y = 9$

Solving this equation gives

$2 - 6y - y = 9$ (expanding the bracket)

$2 - 7y = 9$ (collecting like terms)

$2 = 9 + 7y$ (adding $7y$ to both sides)

$-7 = 7y$ (subtracting 9 from both sides)

$-1 = y$ or $y = -1$

Then using $x = 1 - 3y$ and substituting -1 for y gives $x = 1 - 3(-1)$
$= 1 + 3 = 4$

So the solution is $x = 4$ and $y = -1$

Exercise 3h

Use the substitution method to solve the following pairs of equations.

1 $2x + y = 4$
$5x - 2y = 1$

2 $3a - 2b = 1$
$2a = 5 - b$

3 $3x - y = 8$
$y = 2 - 2x$

4 $2x + 3y = 4$
$4x = 1 - y$

5 $3s - 2t = 1$
$3t = 4s$

6 $2a = b - 3$
$2a - 5b = 1$

Use the substitution method to solve questions **7** to **12** of Exercise 3f.

Puzzle

There are 12 identical coins. One of them is a forgery and its weight is different from the others. It is not known if the forged coin is heavier or lighter than the genuine coins. How can you find the forged coin by three weighings on a simple balance?

Problems

You can solve problems involving two unknowns by forming a pair of simultaneous equations. Then use either elimination or substitution to solve the equations.

Exercise 3i

I think of two numbers. If I add three times the smaller number to the bigger number I get 14. If I subtract the bigger number from twice the smaller number I get 1. Find the two numbers.

First allocate letters to the unknown numbers.

Let the smaller number be x and the bigger number be y.

Now interpret the information in terms of these letters.

Second sentence ⇒	$3x + y = 14$	[1]
Third sentence ⇒	$2x - y = 1$	[2]
[1] + [2] gives	$5x = 15$	
	$x = 3$	

Substitute 3 for x in [1]	$9 + y = 14$
Subtract 9 from both sides	$y = 5$

Therefore, the two numbers are 3 and 5.

(Check by reading the original statements to see if the numbers fit.)

Solve the following problems by forming a pair of simultaneous equations:

1 The sum of two numbers is 20 and their difference is 4. Find the numbers.

2 The sum of two numbers is 16 and they differ by 6. What are the numbers?

3 I think of two numbers. If I double the first and add the second I get 18. If I double the first and subtract the second I get 14. What are the numbers?

4 Three times a number added to a second number is 33. The first number added to three times the second number is 19. Find the two numbers.

5 Find two numbers such that twice the first added to the second is 26 and the first added to three times the second is 28.

6 Find the two numbers such that twice the first added to the second gives 27 and twice the second added to the first gives 21.

A shop sells bread rolls. If five brown rolls and six white rolls cost \$980 while three brown rolls and four white rolls cost \$620, find the cost of each type of roll.

First allocate letters to the unknown quantities.

Let one brown roll cost \$$x$ and one white roll cost \$$y$.

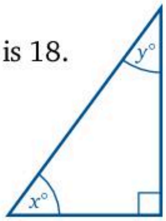
Now interpret the information in terms of x and y.

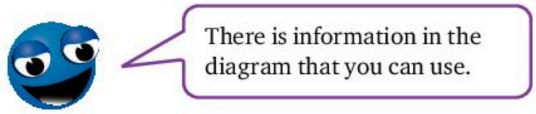
	$5x + 6y = 980$	[1]
	$3x + 4y = 620$	[2]
[1] × 2 gives	$10x + 12y = 1960$	[3]
[2] × 3 gives	$9x + 12y = 1860$	[4]
[3] – [4] gives	$x = 100$	
Substitute 100 for x in [1]	$500 + 6y = 980$	
Subtract 500 from both sides	$6y = 480$	
	$y = 80$	

Therefore one brown roll costs \$100 and one white roll costs \$80.

7 I buy x choc ices and y orange ices and spend \$230. I buy ten ices altogether. The choc ices cost \$30 each and the orange ices cost \$20 each. How many of each do I buy?

8 x is bigger than y.
The difference between x and y is 18.
Find x and y.



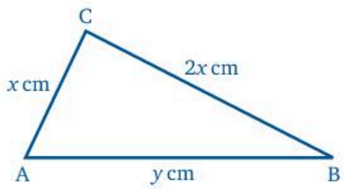


9 A cup and saucer cost \$315 together. A cup and two saucers cost \$450. Find the cost of a cup and of a saucer.

10 The cost of two roti is the same as the cost of three patties. One roti and one patty together cost \$350. What do they each cost?

11 In a test, the sum of Harry's marks and Adam's marks is 42. Sam has twice as many marks as Adam, and the sum of Harry's and Sam's marks is 52. What are the marks of each of the three boys?

12 The perimeter of triangle ABC is 14 cm. AB is 2 cm longer than AC. Find x and y.



13 The perimeter of the rectangle is 31 cm. The difference between the lengths of AB and BC is $3\frac{1}{2}$ cm.

Find the lengths of AB and BC.

14 The equation of a straight line is $y = mx + c$. When $x = 1, y = 6$ and when $x = 3, y = 10$. Form two equations for m and c and hence find the equation of the line.

Puzzle

Andy wants to invest some money now that will give him enough to pay a deposit on a house in 5 years' time. He sees an advertisement offering a bond that states that after 5 years it would be worth an amount equivalent to earning simple interest paid at 5% per annum.

How much does he need to invest so that the bond will be worth \$2 000 000 when he withdraws it?

Mixed exercise

Exercise 3j

Select the letter that gives the correct answer.

1 If $2x + y = 7$ and $x - y = 2$, then

A $x = 2, y = -3$ **B** $x = 2, y = -1$ **C** $x = 2, y = 3$ **D** $x = 3, y = 1$

2 The values of x and y that satisfy the equations $5x + y = 13$ and $3x - y = 3$ are

A $x = 2, y = 3$ **B** $x = 2, y = 5$ **C** $x = 3, y = 2$ **D** $x = 15, y = 2$

3 The solutions of the equations $4x + 5y = 11$ and $3x + 2y = 3$ are

A $x = -1, y = 2$ **B** $x = -1, y = 3$ **C** $x = 1, y = 2$ **D** $x = 1, y = 3$

4 In a right-angled triangle the difference between the other two angles is 18°.
The other two angles are

A 33° and 57° **B** 34° and 56° **C** 37° and 47° **D** 37° and 57°

5 Two numbers are such that twice the first added to the second gives 17, whereas twice the second added to the first gives 19.
The two numbers that satisfy these statements are

A 3 and 11 **B** 4 and 9 **C** 5 and 7 **D** 5 and 7

Puzzle

A teaset includes one milk jug and six teacups. The jug holds 130 ml more than a cup and two cups hold 140 ml more than the jug.

What is the capacity of a cup and of the jug?

In this chapter you have seen that...

- ✔ two linear equations in two unknowns usually have one solution, that is one value for each unknown
- ✔ you can solve two simultaneous equations by adding or subtracting the equations when they both have the same number of one of the unknowns: this eliminates that unknown
- ✔ you may have to multiply one or both equations by numbers in order to get the same number of one unknown
- ✔ you can also solve two simultaneous equations by using one equation to express one letter in terms of the other letter, then substituting that expression into the other equation; this gives one equation in one unknown.

4 Indices

At the end of this chapter you should be able to...

1 multiply numbers written in index form
2 divide numbers written in index form
3 find the value of numbers written in index form
4 understand the meaning of zero and negative indices
5 write numbers in standard form or scientific notation.

Did you know?

Did you know that the ancient Greeks did not think of unity as a number? To them 3 was the first odd number.

You need to know...

✔ the meaning of place value in numbers

✔ how to work with fractions.

Key words

cube, cube root, exponent, index (plural indices), negative index, power, scientific notation, square, square root, standard form, zero index

Laws of indices

The laws of indices, as we know from Grade 8, are:

$$a^m \times a^n = a^{m+n}$$

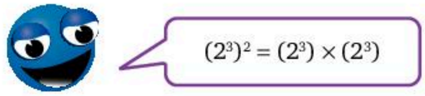
$$a^m \div a^n = a^{m-n}$$

and $$(a^m)^n = a^{mn}$$

We also know that a^n means n lots of a multiplied together. The superscript n is called the *power* or *index* or *exponent*.

Exercise 4a

1 Write each as a single number in index form.



a $(2^3)^2$ **c** $(5^2)^2$ **e** $4^8 \div 4^4$

b $(3^2)^2$ **d** $2^3 \times 2^6$ **f** $3^7 \div 3^3 \times 3^2$

Simplify $2(a^3b^2)^3$.

Notice that only the items in the bracket are cubed, so

$$2(a^3b^2)^3 = 2 \times (a^3)^3 \times (b^2)^3$$
$$= 2 \times a^9 \times b^6 = 2a^9b^6$$

2 Simplify:

a $3(t^3)^5$ **b** $(2d^5)^3$ **c** $2(a^2)^3$ **d** $(5p^3)^3$

3 Find the value of:

a $3(2^2)^3$ **c** $10(3^2)^2$

b $(5 \times 2^4)^2$ **d** $2(3^2 - 2^2)^2$

4 Simplify:

a $3(x^3y)^3$ **b** $5(ab^3)^4$ **c** $(4u^2v)^3$ **d** $2p^2(p^3q)^2$

Zero and negative indices

In Grade 8 you worked with positive indices. Now we look at the *zero index* and *negative indices*.

Consider $a^3 \div a^3$.

Subtracting indices gives $a^3 \div a^3 = a^0$

Dividing gives $a^3 \div a^3 = 1$

$$a^0 = 1 \quad \text{i.e. (any number)}^0 = 1$$

For example, 2^0, 10^0, 200^0 all equal 1.

Now consider $a^3 \div a^5$.

Subtracting indices gives $a^3 \div a^5 = a^{-2}$

Dividing gives $\frac{a^3}{a^5} = \frac{\cancel{a} \times \cancel{a} \times \cancel{a}}{\cancel{a} \times \cancel{a} \times \cancel{a} \times a \times a} = \frac{1}{a^2}$

Therefore a^{-2} means $\frac{1}{a^2}$.

A negative sign in front of the index means 'the reciprocal of'

i.e. $$a^{-b} = \frac{1}{a^b}$$

For example, $2^{-1} = \frac{1}{2}$, $10^{-1} = \frac{1}{10}$.

Exercise 4b

Find the value of 3^{-1}.

$$3^{-1} = \frac{1}{3^1} = \frac{1}{3}$$

Find the value of:

1 2^{-1} **3** 5^{-1} **5** 8^{-1} **7** a^{-1}

2 10^{-1} **4** 7^{-1} **6** 4^{-1} **8** x^{-1}

Find the value of:

a $\left(\frac{1}{2}\right)^{-1}$ **b** $\left(\frac{2}{5}\right)^{-1}$

a $\left(\frac{1}{2}\right)^{-1} = \left(\frac{2}{1}\right)^{1} = 2$ **b** $\left(\frac{2}{5}\right)^{-1} = \left(1 \div \frac{2}{5}\right)^{1} = \left(1 \times \frac{5}{2}\right)^{1} = \left(\frac{5}{2}\right)^{1} = 2\frac{1}{2}$

Find the value of:

9 $\left(\frac{1}{3}\right)^{-1}$ **11** $\left(\frac{1}{4}\right)^{-1}$ **13** $\left(\frac{1}{5}\right)^{-1}$ **15** $\left(\frac{1}{a}\right)^{-1}$

10 $\left(\frac{2}{3}\right)^{-1}$ **12** $\left(\frac{3}{4}\right)^{-1}$ **14** $\left(\frac{4}{5}\right)^{-1}$ **16** $\left(\frac{x}{y}\right)^{-1}$

Find the value of 3^{-2}.

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

Find the value of:

17 2^{-3} **19** 10^{-3} **21** 2^{-5} **23** 10^{-2}

18 5^{-2} **20** 6^{-2} **22** 10^{-4} **24** 4^{-3}

Find the value of $\left(\frac{1}{3}\right)^{-2}$.

$$\left(\frac{1}{3}\right)^{-2} = \left(1 \div \frac{1}{3}\right)^{2}$$

$$= \left(1 \times \frac{3}{1}\right)^{-2} = 3^2 = 9$$

Find the value of:

25 $\left(\frac{1}{5}\right)^{-3}$ **27** $\left(\frac{1}{2}\right)^{-5}$ **29** $\left(\frac{1}{8}\right)^{-3}$ **31** $\left(\frac{1}{2}\right)^{-3}$

26 $\left(\frac{1}{4}\right)^{-2}$ **28** $\left(\frac{1}{3}\right)^{-4}$ **30** $\left(\frac{1}{10}\right)^{-4}$ **32** $\left(\frac{1}{6}\right)^{-2}$

Find the value of $\left(\frac{2}{5}\right)^{-3}$.

$$\left(\frac{2}{5}\right)^{-3} = \left(\frac{5}{2}\right)^{3}$$

$$= \frac{5^3}{2^3} = \frac{125}{8} = 15\frac{5}{8}$$

Find the value of:

33 $\left(\frac{3}{4}\right)^{-2}$ **35** $\left(\frac{4}{9}\right)^{-2}$ **37** $\left(\frac{2}{3}\right)^{-4}$ **39** $\left(\frac{3}{10}\right)^{-4}$

34 $\left(\frac{2}{3}\right)^{-3}$ **36** $\left(\frac{2}{7}\right)^{-2}$ **38** $\left(\frac{3}{5}\right)^{-2}$ **40** $\left(\frac{5}{8}\right)^{-2}$

Find the value of:

41 $\left(\frac{1}{8}\right)^{-1}$ **43** 4^{-2} **45** $\left(\frac{1}{2}\right)^{0}$ **47** 5^3

42 $\left(\frac{2}{5}\right)^{-2}$ **44** 8^2 **46** $\left(\frac{2}{3}\right)^{0}$ **48** 9^{-1}

49 $\left(\frac{1}{2}\right)^{-4}$

50 6^0

<u>51</u> $\left(\frac{3}{4}\right)^{-3}$

<u>52</u> $\left(\frac{2}{7}\right)^{-1}$

<u>53</u> 5^0

<u>54</u> $\left(\frac{7}{10}\right)^{-3}$

<u>55</u> 2^{-2}

<u>56</u> $\left(\frac{4}{5}\right)^{3}$

<u>57</u> 12^{-1}

<u>58</u> 9^3

<u>59</u> $\left(\frac{1}{4}\right)^{-3}$

<u>60</u> $\left(\frac{3}{7}\right)^{0}$

Puzzle

Replace the stars with digits to complete this addition so that every digit from 1 to 9 is used once.

```
 * 6 *
 * 1 * +
 -----
 * 8 *
```

Powers and roots

The *square* of a number is the number multiplied by itself. For example, the square of 3 is called 3 squared and is equal to $3 \times 3 = 9$.

When a number can be expressed as two equal factors, that factor is the *square root* of the number. For example, $9 = 3 \times 3$ so 3 is the square root of 9.

A positive number has two square roots, for example, $9 = 3 \times 3$ and $9 = -3 \times -3$, so 3 and -3 are both square roots of 9.

A negative number has no square roots because there are no equal numbers whose product is negative.

By convention, the symbol $\sqrt{\ }$ is used to mean the positive square root only, so $\sqrt{4} = 2$

The *cube* of a number is the product of three of the number. For example, the cube of 2 is $2 \times 2 \times 2 = 8$.

So 2 cubed is equal to 8.

When a number can be expressed as the product of three equal factors, that factor is the *cube root* of the number.

As $8 = 2 \times 2 \times 2$, so 2 is the cube root of 8. The symbol used for the cube root of a number is $\sqrt[3]{\ }$.

A positive number has a positive cube root, and a negative number has a negative cube root.

For example, $\sqrt[3]{8} = 2$ and $\sqrt[3]{-8} = -2$

Fractional indices

We know that $a^n \times a^n = a^{2n}$ so it follows that $a^{\frac{1}{2}} \times a^{\frac{1}{2}} = a^1 = a$

Also we know that $\sqrt{a} \times \sqrt{a} = a$

Therefore $a^{\frac{1}{2}} = \sqrt{a}$

Similarly $a^n \times a^n \times a^n = a^{3n}$ so it follows that $a^{\frac{1}{3}} \times a^{\frac{1}{3}} \times a^{\frac{1}{3}} = a^1 = a$

Therefore $a^{\frac{1}{3}} = \sqrt[3]{a}$

Now we can see that $a^{\frac{3}{2}} = a^{\frac{1}{2}\times 3} = \sqrt{a^3}$

Exercise 4c

Write down the value of **a** $4^{\frac{1}{2}}$ **b** 5^0 **c** $9^{\frac{3}{2}}$

a $4^{\frac{1}{2}} = \sqrt{4} = 2$ **b** $5^0 = 1$ **c** $9^{\frac{3}{2}} = (9^{\frac{1}{2}})^3 = (\sqrt{9})^3 = 3^3 = 27$

Simplify:

1 **a** $9^{\frac{1}{2}}$ **b** $9^{-\frac{1}{2}}$ **c** 3^{-2} **d** $27^{-\frac{2}{3}}$

2 **a** $4^{\frac{1}{2}}$ **b** 4^{-1} **c** 4^0 **d** 4^{-2}

3 **a** $16^{\frac{1}{4}}$ **b** $16^{\frac{1}{2}}$ **c** $4^{\frac{3}{2}}$ **d** $32^{\frac{1}{5}}$

4 **a** $8^{\frac{1}{3}}$ **b** $8^{\frac{4}{3}}$ **c** $8^{-\frac{4}{3}}$ **d** $16^{-\frac{3}{4}}$

5 **a** $125^{\frac{1}{3}}$ **b** $125^{\frac{2}{3}}$ **c** 25^0 **d** 5^{-3}

6 **a** 2^{-2} **b** 20^0 **c** $64^{\frac{2}{3}}$ **d** $32^{\frac{2}{5}}$

7 **a** $49^{\frac{3}{2}}$ **b** $49^{-\frac{1}{2}}$ **c** $343^{\frac{1}{3}}$ **d** 7^{-2}

8 **a** $9^{\frac{3}{2}}$ **b** $9^{\frac{3}{2}} \times 9^{-1}$ **c** $(9^{\frac{3}{2}})^2$ **d** $9^{-\frac{3}{2}}$

9 **a** $4^{-\frac{3}{2}}$ **b** $\left(\frac{1}{4}\right)^2$ **c** $\left(\frac{1}{4}\right)^{-\frac{1}{2}}$ **d** $4^{-\frac{1}{2}}$

10 **a** $\frac{1}{10^2}$ **b** $100^{-\frac{1}{2}}$ **c** $10^0 \times 9^0$ **d** $\left(\frac{1}{100}\right)^{-\frac{1}{2}}$

11 **a** 2×2^{-4} **b** $2^{\frac{1}{2}} \times 2^{-\frac{1}{2}}$ **c** $2^{-3} \times 4^{\frac{1}{2}}$ **d** $2^2 \div 2^{-2}$

12 **a** $3^{\frac{2}{3}} \times 3^{\frac{1}{3}}$ **b** $\frac{1}{3^{-3}}$ **c** $12^{\frac{1}{2}} \times 3^{\frac{1}{2}}$

13 **a** $2^3 \div 2^4$ **b** $16^{-\frac{3}{4}} \times 2^2$

14 **a** $2^{\frac{1}{3}} \times 3^{\frac{1}{3}} \times 6^{\frac{2}{3}}$ **b** $24^{\frac{2}{3}} \times 16^{\frac{1}{3}} \div 18^{\frac{1}{3}}$

15 **a** $a^2 \times a^3$ **b** $a^5 \times a^3$ **c** $a^5 \div a^2$ **d** $a^9 \div a^5$

16 a $x^{12} \div x^5$ b $\frac{x^4}{x^3}$ c $x^3 \times x^0$ d $x^7 \times x^5$

17 a $x^9 \div x^5$ b $x^4 \div x^{-2}$ c $x^5 \times x^{-2}$ d $(x^3)^2$

18 a $(x^{-2})^3$ b $(x^{-\frac{3}{4}})^0$ c $(3x)^2$ d $x^3 \times x^{\frac{1}{2}}$

19 a $x^3 \div x^{\frac{1}{2}}$ b $x^6 \times x^{-4}$ c $x^6 \div x^{-4}$ d $(4x)^{-2}$

20 a $a^2 \times a^3 \times a^{-5}$ b $\sqrt{a^3 \times a^5}$ c $\sqrt{9a^4b^2}$

21 Which two of these have the same value?

2^6, 6^2, 3^4, 4^3

22 Which two of these have the same value?

$4^{\frac{1}{2}}$, $4^{-\frac{1}{2}}$, $16^{\frac{1}{4}}$, 2^0

Standard form (scientific notation)

Very large or very small numbers are more briefly written in *standard form* (also called *scientific notation*). It is easier to compare sizes of numbers written in standard form.

Standard form is a number between 1 and 10 multiplied by the appropriate power of 10.

Exercise 4d

The following numbers are given in standard form. Write them as ordinary numbers.

1 3.45×10^2

2 1.2×10^3

3 5.01×10^{-2}

4 4.7×10^{-3}

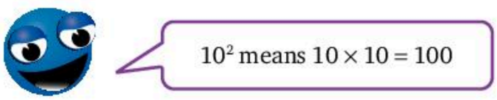
5 2.8×10^2

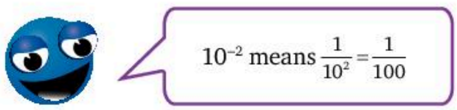
6 7.3×10^{-1}

7 9.02×10^5

8 6.37×10^{-4}

9 8.72×10^6





Write the following numbers in standard form:

a 3840 b 0.0025

(First, write the given digits as a number between 1 and 10 and then decide what power of 10 to multiply the number by to bring it back to the correct size.)

a $3840 = 3.84 \times 10^3$ (The decimal point has been moved 3 places to the left so the index of 10 is 3.)

b $0.0025 = 2.5 \times 10^{-3}$ (The decimal point has been moved 3 places to the right so the index of 10 is –3.)

Write the following numbers in standard form:

10 265
11 0.18
12 3020
13 0.019
14 76 700
15 390 000
16 0.000 85
17 7000
18 0.004
19 58 700
20 2600
21 450 000
22 0.000 007
23 0.8
24 0.000 56
25 24 000
26 39 000 000
27 0.000 000 000 08

Check your answers by writing them as ordinary numbers.

Mixed exercises

Exercise 4e

1 Simplify **a** $2a(b^2c)^3$ **b** $(3 \times 2^3)^2$

2 Find the value of **a** 4^{-1} **b** 9^{-1}

3 Find the value of **a** $\left(\frac{3}{5}\right)^{-1}$ **b** $\left(\frac{5}{7}\right)^{-1}$

4 Find the value of **a** $\left(\frac{3}{7}\right)^{-2}$ **b** $\left(\frac{3}{4}\right)^{0}$

5 Find the value of **a** $\left(\frac{1}{4}\right)^{0}$ **b** $\left(\frac{3}{4}\right)^{-3}$

6 Simplify **a** $x^3 \times x^4$ **b** $x^4 \times x^{-3}$

7 Simplify **a** $(3a)^3$ **b** $(2a^3)^2$

8 Which two of these have the same value?

6^2, 3^4, 9^2, 4^3

9 Write the following numbers, which are given in standard form, as ordinary numbers:

a 2.6×10^4 **b** 5.7×10^{-3}

10 Write the following numbers in scientific notation:

a 365 **b** 1200 **c** 0.000 007

Exercise 4f

In this exercise select the letter that gives the correct answer.

1 The value of $5 \times (10^2)^3$ is

A 5000 B 50 000 C 500 000 D 5 000 000

2 $(3a^4)^2$ simplifies to

A $3a^6$ B $3a^8$ C $9a^6$ D $9a^8$

3 The value of $\left(\frac{3}{2}\right)^{-1}$ is

A $\frac{2}{3}$ B 1 C $1\frac{1}{2}$ D 3

4 The value of $\left(\frac{1}{2}\right)^{-3}$ is

A $\frac{1}{8}$ B $\frac{1}{4}$ C 4 D 8

5 $4^{\frac{3}{2}}$ simplifies to

A 2 B 4 C 8 D 16

6 $x^6 \times x^{-3}$ simplifies to

A x B x^2 C x^3 D x^4

7 The value of $8^{\frac{2}{3}}$ is

A 2 B 4 C 8 D 16

8 The value of $3(3^2 - 2)^2$ is

A 49 B 147 C 231 D 441

9 3.7×10^{-2} written as an ordinary number is

A 0.000 37 B 0.0037 C 0.037 D 0.37

10 Written in standard form, 2 900 000 is

A 2.9×10^5 B 2.9×10^6 C 29×10^5 D 29×10^6

In this chapter you have seen that...

- ✔ you can multiply different powers of the same number by adding the indices, e.g. $2^3 \times 2^4 = 2^{3+4} = 2^7$ and $a^5 \times a^7 = a^{12}$
- ✔ you can divide different powers of the same number by subtracting the indices, e.g. $3^5 \div 3^2 = 3^{5-2} = 3^3$ and $p^8 \div p^3 = p^5$
- ✔ $(a^m)^n = a^{mn}$, e.g. $(5^2)^3 = 5^6$ and $(2x^2)^3 = 8x^6$
- ✔ $a^0 = 1$ for all non-zero values of a
- ✔ $\sqrt{a}$ means the square root of a, e.g. $\sqrt{16} = 4$ and $\sqrt{a^2 \times a^4} = \sqrt{a^6} = a^3$
- ✔ a number in standard form or scientific notation is a number between 1 and 10 multiplied by a power of 10, e.g. $130\,000 = 1.3 \times 10^5$ in standard form.

5 Pythagoras' theorem

At the end of this chapter you should be able to...

1 state the relationship between the hypotenuse and the other sides of a right-angled triangle
2 calculate the length of one side of a right-angled triangle, given the other two sides
3 identify Pythagorean triples
4 use Perigal's dissection
5 determine whether or not a triangle is right-angled given the lengths of the sides.

Did you know?

Did you know that the square root sign ($\sqrt{\ }$) comes from the first letter of the word *radix,* which was the Latin word for root?

You need to know...

✔ how to work with decimals
✔ the properties of the special quadrilaterals
✔ how to recognise right-angled triangles.

Key words

chord, converse, hypotenuse, irrational, Pythagoras' theorem, Pythagorean triple, significant figures

Squares and square roots

The following exercise covers the finding of squares and square roots. You should always find a rough estimate first.

Some numbers have exact square roots, for example $\sqrt{4}$, $\sqrt{9}$ and $\sqrt{36}$ are all exact. However most numbers do not have exact square roots. For example $\sqrt{2}$ is not exact. It is a never-repeating, never-ending decimal fraction. Numbers such as these are called *irrational numbers*.

Exercise 5a

Use a calculator to find the squares of:

a 2.3 **b** 23 **c** 2300 **d** 0.023

a $2.3^2 \approx 2 \times 2 = 4$

$2.3^2 = 5.29$

b $23^2 \approx 20 \times 20 = 400$

$23^2 = 529$

c $2300^2 \approx 2000 \times 2000 = 4\,000\,000$

$2300^2 = 5\,290\,000$

d $0.023^2 \approx 0.02 \times 0.02 = 0.0004$

$0.023^2 = 0.000\,529$

Always find a rough value of the square of a number because it is easy to press the wrong button on a calculator. Check that your calculator answer is sensible.

Find the squares of the following numbers, giving your answers correct to four *significant figures* where necessary:

1	6.2	**5**	0.71	**9**	3.12	**13**	5210
2	13.7	**6**	0.059	**10**	0.0312	**14**	52.1
3	242	**7**	0.0017	**11**	9.2	**15**	0.521
4	2780	**8**	312	**12**	92	**16**	0.0521

To estimate a square root, pair off the numbers each way from the decimal point. Then estimate the square root of the first pair of non-zero numbers.

Find the square root of 0.003 425, giving the answer correct to four significant figures.

$0.003\,425 = 0.\overset{\frown}{00}\overset{\frown}{34}\overset{\frown}{25}$

Therefore $\sqrt{0.003425} = \sqrt{0.\overset{\frown}{00}\overset{\frown}{34}\overset{\frown}{25}} \approx 0.05\ldots$ $\quad(\sqrt{00} = 0$ and $\sqrt{34} \approx 5)$

Using a calculator $\sqrt{0.003425} = 0.058\,5234\ldots$

$= 0.058\,52$ correct to 4 s.f.

Find the square roots of the following numbers, giving your answers correct to four significant figures:

17	9.87	**21**	0.0482	**25**	2.62	**29**	0.461
18	19.9	**22**	0.004 82	**26**	0.062	**30**	4.61
19	124	**23**	96	**27**	0.000 78	**31**	461
20	96 800	**24**	321	**28**	0.5	**32**	0.000 461

Pythagoras' theorem

Pythagoras was a native of Samos who travelled frequently to Egypt for the purpose of education. The Egyptians are believed to have known this theorem many years before Pythagoras was born. It is very likely that the Egyptian priests explained this theorem to Pythagoras. Indeed, it is claimed that Pythagoras offered a sacrifice to the Muses when the Egyptian priests explained to him the properties of the right-angled triangle.

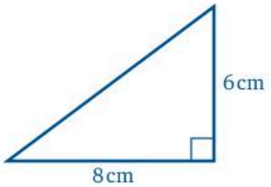
We can show that the properties involve a relationship between the lengths of the three sides.

Exercise 5b

First we will collect some evidence. Bear in mind that, however accurate your drawing, it is not perfect.

Construct the triangles in questions **1** to **6** and in each case measure the third side, the *hypotenuse*, which is the side opposite the right angle.

1



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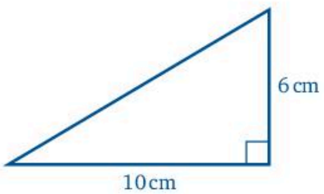

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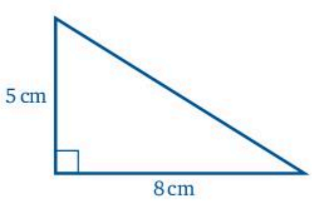

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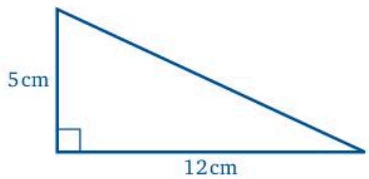

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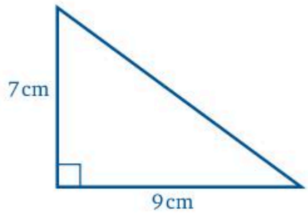
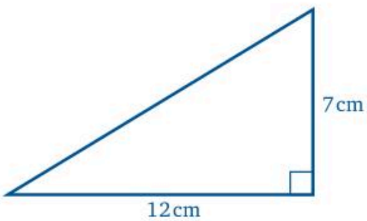


6



7 In each of the questions **1** to **6**, find the squares of the lengths of the three sides. Write the squares in ascending order (i.e. the smallest first). Can you see a relation between the first two squares and the third square?

If your drawings are reasonably accurate you will find that by adding the squares of the two shorter sides you get the square of the hypotenuse.

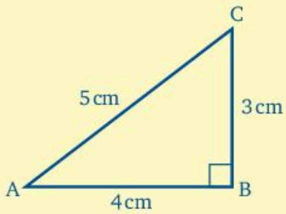
$AB^2 = 16$

$BC^2 = 9$

$AC^2 = 25$

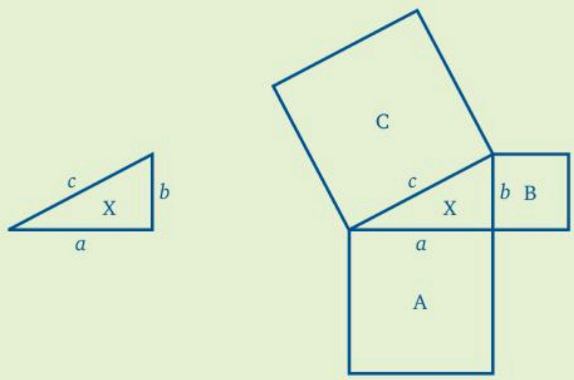
$25 = 16 + 9$

so $AC^2 = AB^2 + BC^2$

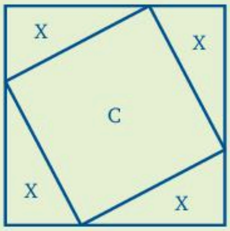


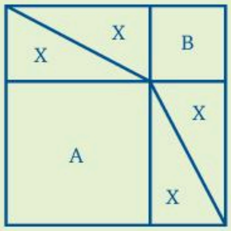
This result is called *Pythagoras' theorem*, which states that in a right-angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Activity



Draw any right-angled triangle and draw the square on each of the three sides. Mark the four areas A, B, C and X as shown in the diagram. Cut out one of each shape and make another three triangles identical to X. Arrange the shapes in two different ways as shown below. Sketch the two arrangements and mark in as many lengths as possible with a, b or c.





1 What can you say about the areas of these two diagrams? Justify your answer.

2 If the four triangles marked X are removed from each diagram, what can you say about the areas that remain? What relation does this give for

a areas A, B and C **b** lengths a, b and c?

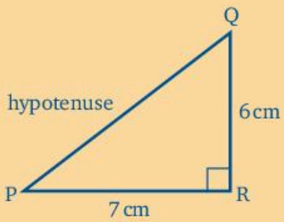
Finding the hypotenuse

Exercise 5c

Give your answers correct to 3 s.f.

In $\triangle PQR$, $\hat{R} = 90°$, PR = 7 cm and QR = 6 cm.

Find PQ.

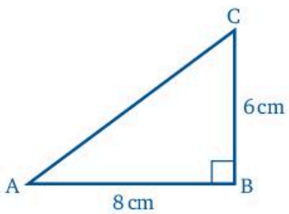


$$PQ^2 = PR^2 + QR^2 \qquad \text{(Pythagoras' theorem)}$$
$$= 7^2 + 6^2$$
$$= 49 + 36 = 85$$
$$PQ = \sqrt{85} \qquad (\approx 9)$$
$$PQ = 9.22\text{ cm} \qquad \text{correct to 3 s.f.}$$

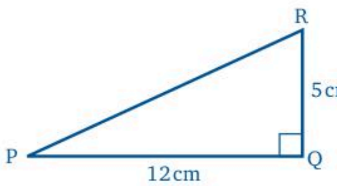
In the following right-angled triangles find the required lengths.

Start each question by labelling the hypotenuse.
Then write down Pythagoras' theorem in terms of the sides of this triangle.

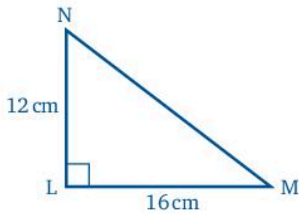
1 Find AC.



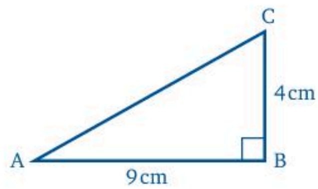
2 Find PR.



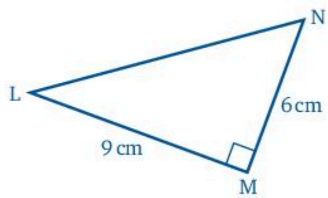
3 Find MN.



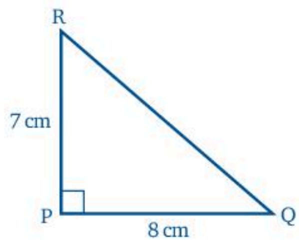
4 Find AC.



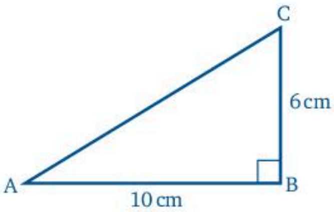
5 Find LN.



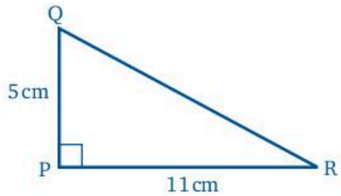
<u>6</u> Find QR.



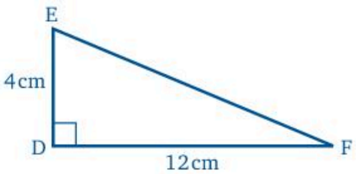
7 Find AC.



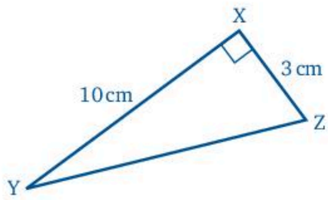
9 Find QR.



8 Find EF.



10 Find YZ.



11 In △ABC, $\hat{C} = 90°$, AC = 2 cm and BC = 3 cm. Find AB.

Start by drawing the triangle.

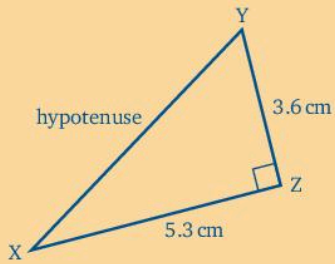
12 In △DEF, $\hat{E} = 90°$, DE = 7 cm and EF = 9 cm. Find DF.

13 In △ABC, $\hat{A} = 90°$, AB = 4 m and AC = 5 m. Find BC.

14 In △PQR, $\hat{Q} = 90°$, PQ = 11 m and QR = 3 m. Find PR.

15 In △XYZ, $\hat{X} = 90°$, YX = 12 cm and XZ = 2 cm. Find YZ.

In △XYZ, $\hat{Z} = 90°$, XZ = 5.3 cm and YZ = 3.6 cm.
Find XY.



$XY^2 = XZ^2 + ZY^2$ (Pythagoras' theorem)

$= 5.3^2 + 3.6^2$ $\quad 5.3^2 \approx 5 \times 5 = 25$

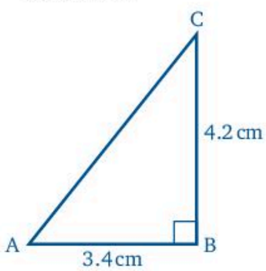
$= 28.09 + 12.96$ $\quad 3.6^2 \approx 4 \times 4 = 16$

$= 41.05$

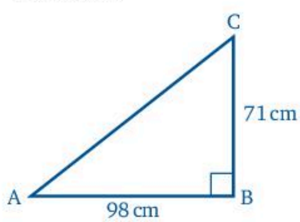
$XY = \sqrt{41.05} = 6.407\ldots$ $\quad (\sqrt{41.05} \approx 6)$

Length of XY = 6.41 cm correct to 3 s.f.

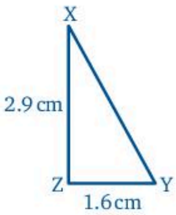
16 Find AC.



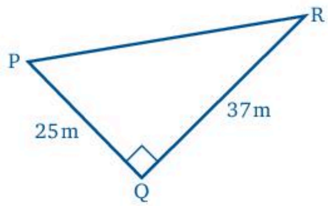
17 Find AC.



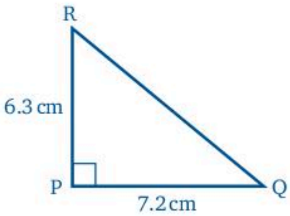
18 Find XY.



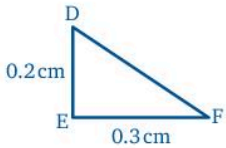
20 Find PR.



19 Find QR.



21 Find DF.



22 In $\triangle ABC$, $\hat{B} = 90°$, $AB = 7.9$ cm, $BC = 3.5$ cm. Find AC.

23 In $\triangle PQR$, $\hat{Q} = 90°$, $PQ = 11.4$ m, $QR = 13.2$ m. Find PR.

24 In $\triangle XYZ$, $\hat{Z} = 90°$, $XZ = 1.23$ cm, $ZY = 2.3$ cm. Find XY.

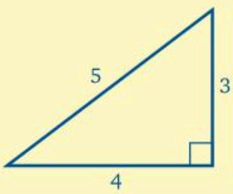
25 In $\triangle ABC$, $\hat{C} = 90°$, $AC = 32$ cm, $BC = 14.2$ cm. Find AB.

26 In $\triangle PQR$, $\hat{P} = 90°$, $PQ = 9.6$ m, $PR = 8.8$ m. Find QR.

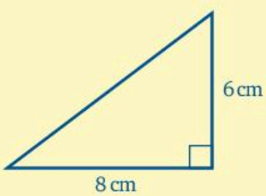
27 In $\triangle DEF$, $\hat{F} = 90°$, $DF = 10.1$ cm, $EF = 6.4$ cm. Find DE.

The 3, 4, 5 triangle

You will have noticed that, in most cases when two sides of a right-angled triangle are given and the third side is calculated using Pythagoras' theorem, the answer is not a rational number. There are a few special cases where all three sides are rational numbers.



The simplest one is the 3, 4, 5 triangle. Any triangle similar to this has sides in the ratio $3:4:5$, so whenever you spot this case you can find the missing side very easily.



For instance, in this triangle, $6 = 2 \times 3$ and $8 = 2 \times 4$. The triangle is similar to the 3, 4, 5 triangle, so the hypotenuse is 2×5 cm, that is, 10 cm.

The other triangle with exact sides which might be useful is the 5, 12, 13 triangle. Sets of numbers like {3, 4, 5} and {5, 12, 13} are called *Pythagorean triples*.

Exercise 5d

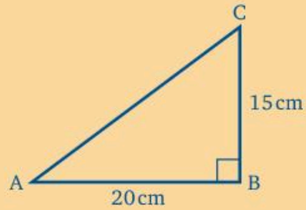
In $\triangle ABC$, $\hat{B} = 90°$, $AB = 20$ cm and $BC = 15$ cm. Find AC.

Notice that $BC = 3 \times 5$ cm and $AB = 4 \times 5$ cm so the sides about the right angle are in the ratio $3:4$.

ABC is therefore a '3, 4, 5 triangle'.

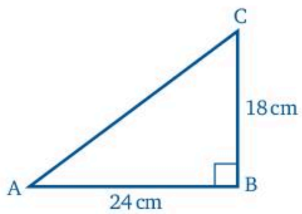
So $\quad AC = 5 \times 5$ cm $(3, 4, 5 \triangle)$

$= 25$ cm



In each of the following questions, decide whether the triangle is similar to the 3, 4, 5 triangle or to the 5, 12, 13 triangle or to neither. Find the hypotenuse, using the method you think is easiest.

1



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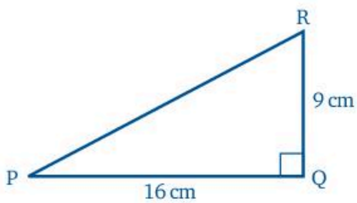

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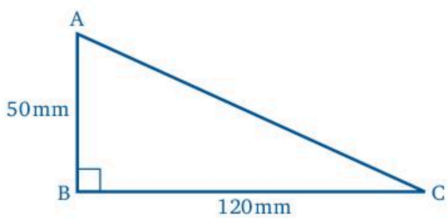

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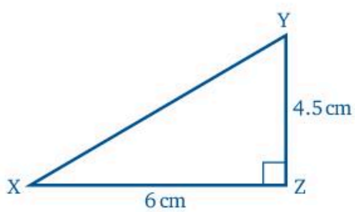

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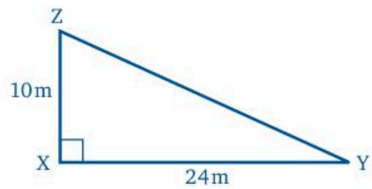

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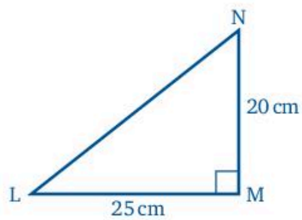

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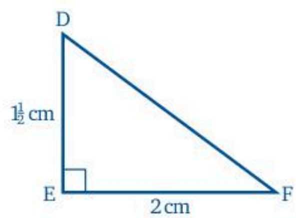

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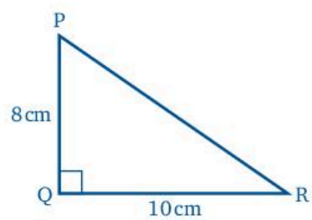


Investigation

Find more Pythagorean triples.

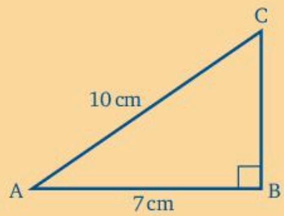
The two larger numbers are always consecutive whole numbers.

Finding one of the shorter sides

If we are given the hypotenuse and one other side we can find the third side.

Exercise 5e

In $\triangle ABC$, $\hat{B} = 90°$, $AB = 7$ cm and $AC = 10$ cm.
Find BC.



$$AC^2 = BC^2 + AB^2 \quad \text{(Pythagoras' theorem)}$$

$$10^2 = BC^2 + 7^2$$

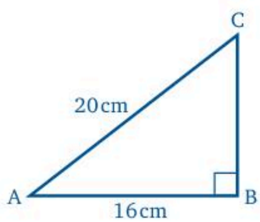
$$100 = BC^2 + 49$$

$$51 = BC^2 \quad \text{(subtracting 49 from both sides)}$$

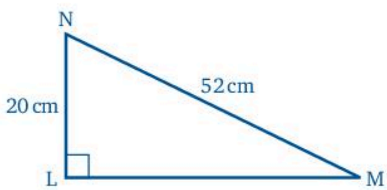
$$BC = \sqrt{51} = 7.141\ldots$$

Length of BC = 7.14 cm correct to 3 s.f.

1 Find BC.

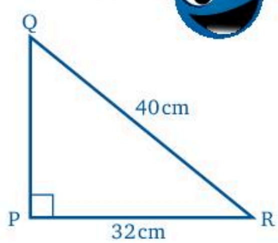


2 Find LM.

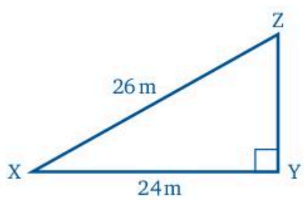


3 Find PQ.

Start each question by writing Pythagoras' theorem for the triangle.

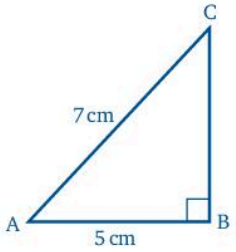


4 Find YZ.

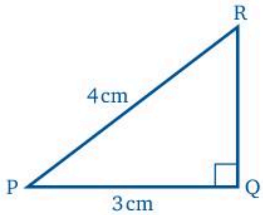


Give your answers to questions **5** to **14** correct to 3 s.f.

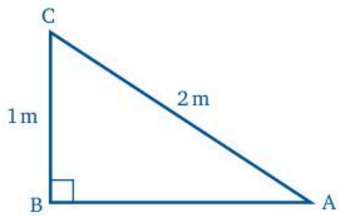
5 Find BC.



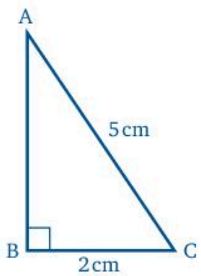
6 Find RQ.



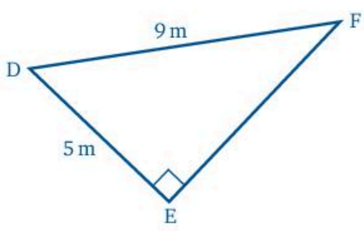
7 Find AB.



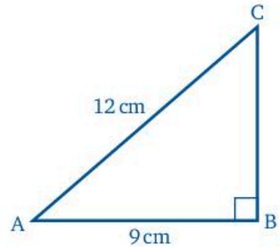
8 Find AB.



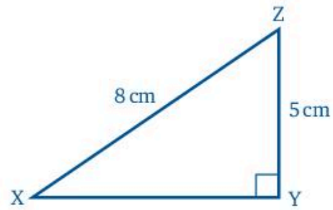
9 Find EF.



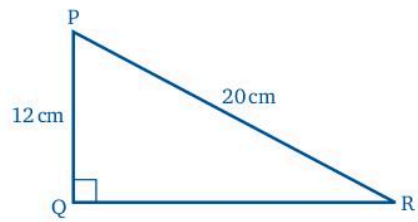
10 Find BC.



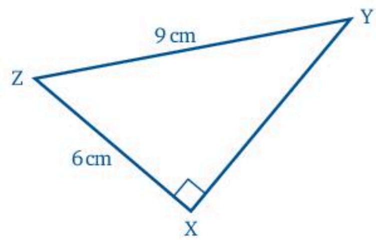
11 Find XY.



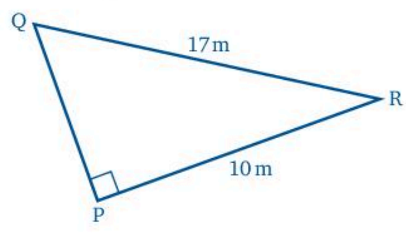
12 Find QR.



13 Find XY.



14 Find PQ.



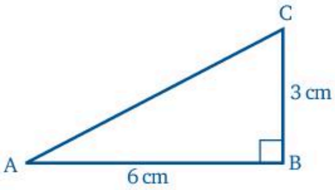
Mixed examples

Exercise 5f

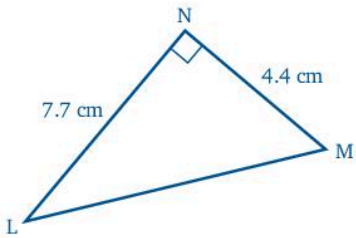
In each case find the length of the missing side. If any answers are not exact give them correct to 3 s.f.

If you notice a 3, 4, 5 triangle or a 5, 12, 13 triangle, you can use it to get the answer quickly.

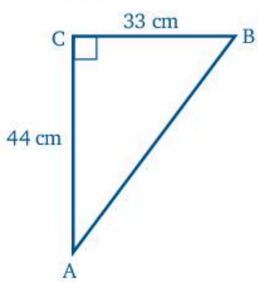
1 Find AC.



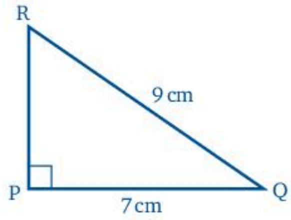
2 Find LM.



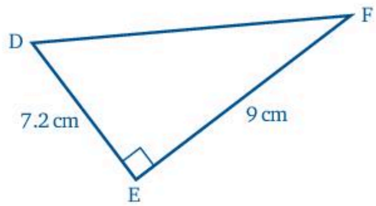
3 Find AB.



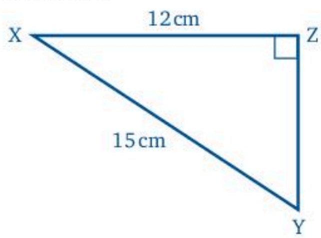
4 Find PR.



5 Find DF.



6 Find YZ.



7 In $\triangle ABC$, $\hat{B} = 90°$, AB = 2 cm, AC = 4 cm. Find BC.

8 In $\triangle ABC$, $\hat{B} = 90°$, AB = 1.25 m, CA = 8.25 m. Find BC.

9 In $\triangle PQR$, $\hat{Q} = 90°$, PQ = 65 cm, QR = 60 cm. Find PR.

10 One number in a Pythagorean triple is 25. Find the other two.

11 In $\triangle ABC$, $\hat{C} = 90°$, AB = 17.5 cm, AC = 16.8 cm. Use the Pythagorean triple you found in question **10** to find BC.

12 In $\triangle DEF$, $\hat{D} = 90°$, DE = 124 cm, DF = 234 cm. Find EF.

13 In $\triangle ABC$, $\hat{C} = 90°$, AC = 3.2 cm, AB = 9.81 cm. Find BC.

14 In $\triangle XYZ$, $\hat{Y} = 90°$, XY = 1.5 cm, YZ = 2 cm. Find XZ.

15 In △PQR, $\hat{P} = 90°$, PQ = 5.1 m, QR = 8.5 m. Find PR.

16 In △ABC, $\hat{C} = 90°$, AB = 92 cm, BC = 21 cm. Find AC.

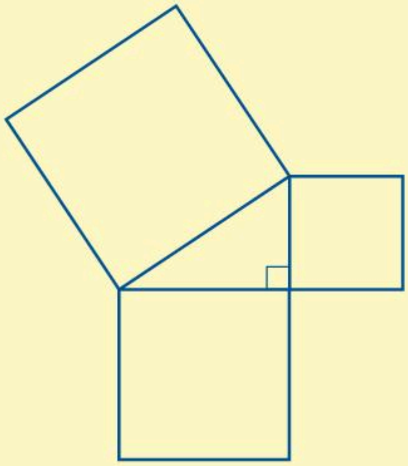
17 In △XYZ, $\hat{X} = 90°$, XY = 3.21 m, XZ = 1.43 m. Find YZ.

Pythagoras' theorem using areas

The area of a square is found by squaring the length of its side, so we can represent the squares of numbers by areas of squares.

This gives us a version of Pythagoras' theorem, using areas:

In a right-angled triangle, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides.

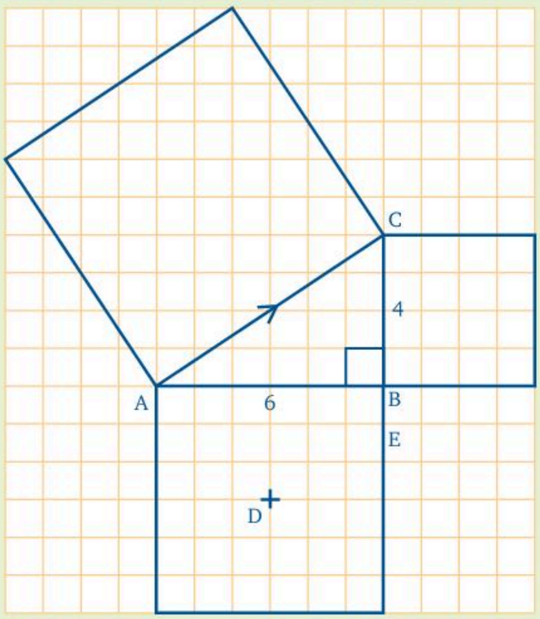


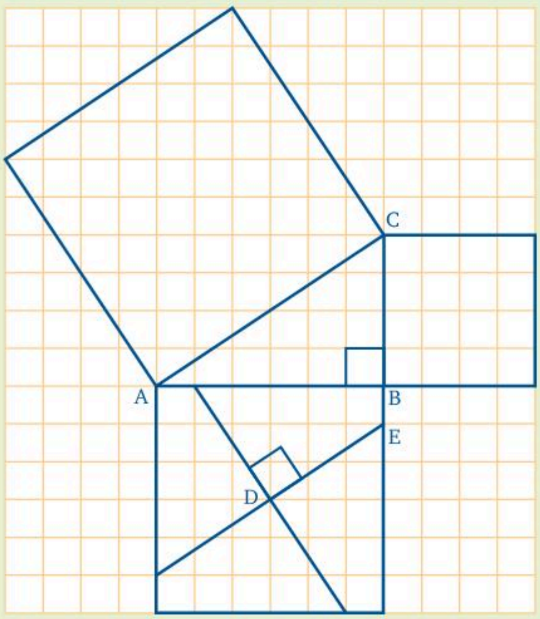
Activity

Perigal's dissection

On square grid paper, and using 1 cm to 1 unit, copy the left-hand diagram. Make sure that you draw an accurate square on the hypotenuse either by counting the squares or by using a protractor and a ruler. D is the centre of the square on AB.

Draw a vector $\overrightarrow{DE}$ so that $\overrightarrow{DE} = \frac{1}{2}\overrightarrow{AC}$, i.e. DE must be parallel to AC.





Now complete the drawing as in the right-hand diagram. Make sure that the angles at D are right angles.

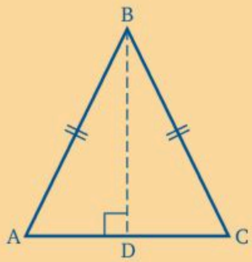
Cut out the smallest square and the four pieces from the middle-sized square. These five pieces can be fitted exactly, like a jigsaw, into the outline of the biggest square.

Finding lengths in an isosceles triangle

An isosceles triangle can be split into two right-angled triangles. This can sometimes help when finding missing lengths.

Exercise 5g

In $\triangle ABC$, $AB = BC = 12$ cm and $AC = 8$ cm.
Find the height of the triangle.



Join B to D, the midpoint of AC. Then we draw one of the right-angled triangles.

$AB^2 = AD^2 + BD^2$ (Pythagoras' theorem)

$12^2 = 4^2 + BD^2$

$144 = 16 + BD^2$

$128 = BD^2$ (subtracting 16 from both sides)

$BD = \sqrt{128}$

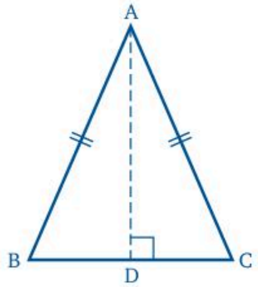
$BD = 11.31\ldots = 11.3$ (correct to 3 s.f.)

$\therefore$ length of BD is 11.3 cm

So the height of the triangle is 11.3 cm correct to 3 s.f.

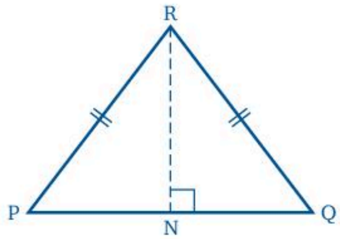
Give your answers correct to 3 s.f.

1



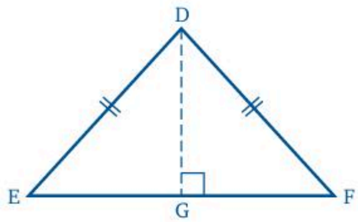
AB = AC = 16 cm. BC = 20 cm. Find the height of the triangle.

2



PQ = 12 cm, PR = RQ. The height of the triangle is 8 cm. Find PR.

3



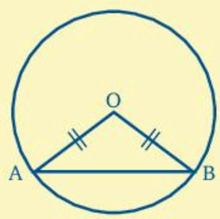
DE = DF = 20 cm. The height of the triangle is 13.2 cm. Find EG and hence EF.

4 In △ABC, AB = BC = 5.2 cm and AC = 6 cm. Find the height of the triangle.

5 In △PQR, PQ = QR = 9 cm and the height of the triangle is 7 cm. Find the length of PR.

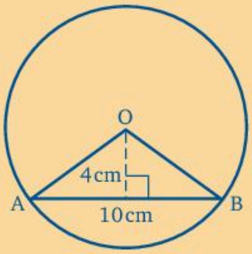
Finding the distance of a chord from the centre of a circle

AB is a *chord* of a circle with centre O. OA and OB are radii and so are equal. Hence triangle OAB is isosceles and we can divide it through the middle into two right-angled triangles.

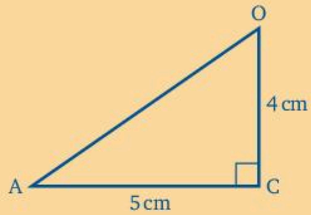


Exercise 5h

A chord AB of a circle with centre O is 10 cm long.
The chord is 4 cm from O. Find the radius of the circle.



Draw one of the triangles, then use Pythagoras' theorem on this triangle. Label the third vertex C.



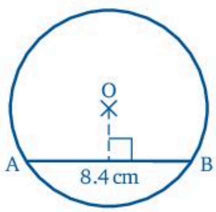
The distance from the centre is the perpendicular distance so OC = 4 cm. From symmetry AC = 5 cm.

$$OA^2 = AC^2 + OC^2 \quad \text{(Pythagoras' theorem)}$$
$$= 5^2 + 4^2$$
$$= 25 + 16$$
$$= 41$$
$$OA = \sqrt{41} = 6.403\ldots$$
$$OA = 6.40 \text{ correct to 3 s.f.}$$

The radius of the circle is 6.40 cm correct to 3 s.f.

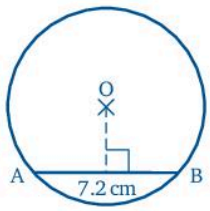
Give your answers correct to 3 s.f.

1



A circle with centre O has a radius of 5 cm. AB = 8.4 cm. Find the distance of the chord from the centre of the circle.

2



O is the centre of the circle and AB is a chord of length 7.2 cm. The distance of the chord from O is 3 cm. Find the radius of the circle.

3 In a circle with centre O, a chord AB is of length 7 cm. The radius of the circle is 11 cm. Find the distance of the chord from O.

4 In a circle with centre O and radius 17 cm, a chord AB is of length 10.4 cm. Find the distance of the chord from O.

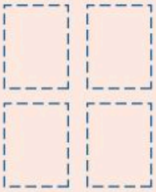
5 In a circle with centre P and radius 7.6 cm, a chord QR is 4.2 cm from P. Find the length of the chord.

Puzzle

Molly has two \$120 stamps and two \$60 stamps.

She wants to stick them on an envelope as a block of four as shown.

How many different arrangements are possible?

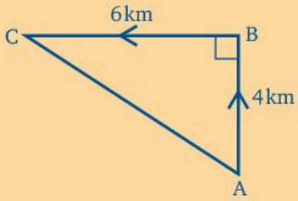


Problems using Pythagoras' theorem

Exercise 5i

A man starts from A and walks 4 km due north to B, then 6 km due west to C. Find how far C is from A.

Draw the triangle and then use Pythagoras' theorem.



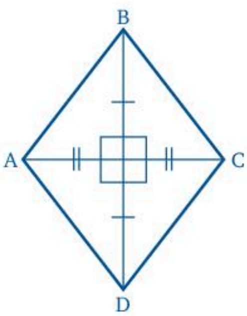
$$AC^2 = BC^2 + AB^2 \quad \text{(Pythagoras' theorem)}$$
$$= 6^2 + 4^2$$
$$= 36 + 16 = 52$$
$$AC = \sqrt{52} = 7.211\ldots$$
$$AC = 7.21 \quad \text{correct to 3 s.f.}$$

So the distance of C from A is 7.21 km, correct to 3 s.f.

Give your answers correct to 3 s.f.

1 A ladder 3 m long is leaning against a wall. Its foot is 1.5 m from the foot of the wall. How far up the wall does the ladder reach?

2 ABCD is a rhombus. AC = 10 cm and BD = 12 cm. Find the length of a side of the rhombus.

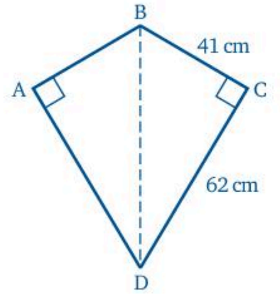


3 Find the length of a diagonal of a square of side 10 cm.

4 A hockey pitch measures 55 m by 90 m. Find the length of a diagonal of the pitch.

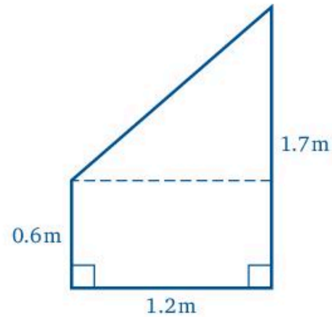
5 A wire stay 11 m long is attached to a telegraph pole at a point A, 8 m up from the ground. The other end of the stay is fixed to a point B, on the ground. How far is B from the foot of the telegraph pole?

6 In the kite ABCD, $\hat{A} = \hat{C} = 90°$. BC = 41 cm and DC = 62 cm. Find the length of the diagonal BD.

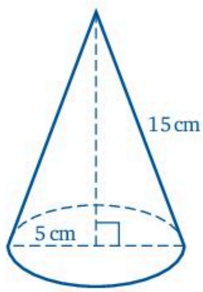


7 A diagonal of a football pitch is 130 m long and the long side measures 100 m. Find the length of the short side of the pitch.

8 The diagram shows the side view of a shed. Find the length of the slant edge.

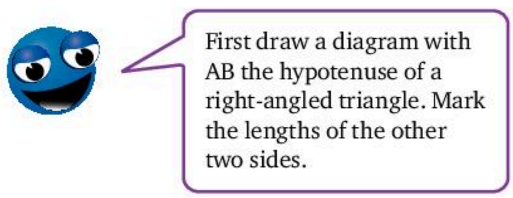


9 The slant height of a cone is 15 cm and the base radius is 5 cm. Find the height of the cone.



10 A man starts from A and walks 6.5 km due south to B; then he walks due east to C. He is then 9 km from A. How far is C from B?

11 A is the point (3, 1) and B is the point (7, 9). Find the length of AB.



12 A ship sails 32 nautical miles due north then 22 nautical miles due east. How far is it from its starting point?

13 A pole 4.5 m high stands on level ground. It is supported in a vertical position by two wires attached to its top and to points on opposite sides of the pole each 3.2 m from the foot of the pole. How long is each wire?

14 The diagonal AC of a rectangle ABCD is 0.67 m long and side AB is 0.32 m long. How long is side BC?

15 Find the length of the diagonal of a square of side 15 cm.

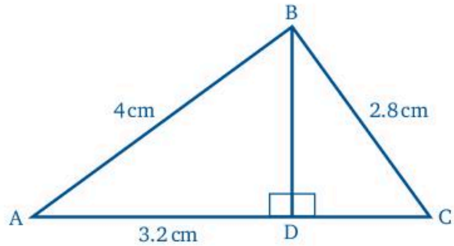
16 ABCD is a kite and AC is its line of symmetry. $\hat{B} = \hat{D} = 90°$, AB = 36 cm and BC = 16 cm. Find AC

17 In the diagram, $A\hat{D}B = 90°$
AB = 4 cm, AD = 3.2 cm and BC = 2.8 cm.

Find **a** BD **b** AC.

Is $A\hat{B}C$ a right angle?

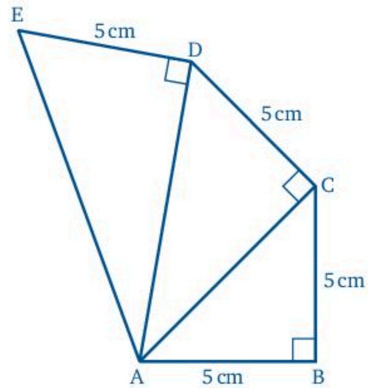
Give a reason for your answer.



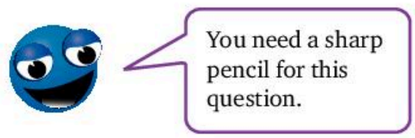
18 **a** *Construct* the figure in the diagram, starting with △ABC then adding △ADC and △ADE.

b Measure AC, AD and EA.

c *Calculate* AC, AD and AE and check the accuracy of your drawing.



19 Construct a right-angled triangle, choosing whole numbers of centimetres for the lengths of the two shorter sides, such that the hypotenuse will be $\sqrt{65}$ cm long. Check the accuracy of your drawing by measuring the hypotenuse and by calculating $\sqrt{65}$.



The converse of Pythagoras' theorem

If we are given three sides of a triangle, we can tell whether or not the triangle contains a right angle because if it does, the square on the longest side is equal to the sum of the squares on the other two sides. This is the *converse* of Pythagoras' theorem. Bear in mind that, *if* there is a right angle, then the longest side will be the hypotenuse.

Exercise 5j

Are the following triangles right-angled?

a $\triangle ABC$: AB = 17 cm, BC = 8 cm, CA = 15 cm

b $\triangle PQR$: PQ = 15 cm, PR = 7 cm, RQ = 12 cm

a $AB^2 = 17^2 = 289$

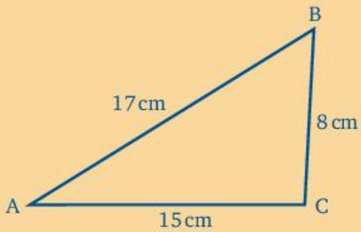
$AC^2 + BC^2 = 15^2 + 8^2$

$= 225 + 64$

$= 289$

$\therefore AC^2 + BC^2 = AB^2$

$\therefore$ by Pythagoras' theorem, $\hat{C} = 90°$.



b $PQ^2 = 15^2 = 225$

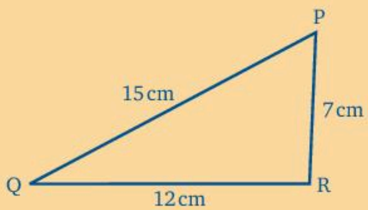
$PR^2 + RQ^2 = 12^2 + 7^2$

$= 144 + 49$

$= 193$

$\therefore PR^2 + RQ^2 \neq PQ^2$

$\therefore$ the triangle is not right-angled.



Are the following triangles right-angled?

Start by drawing a triangle and marking the sides. Then find the square of the longest side and compare this value with the sum of the squares of the other two sides.

1 Triangle ABC: AB = 48 cm, BC = 64 cm and CA = 80 cm.

2 Triangle PQR: PQ = 2.1 cm, QR = 2.8 cm and RP = 3.5 cm.

3 Triangle LMN: LM = 6 cm, MN = 7.2 cm and NL = 9 cm.

4 Triangle ABC: AB = 9.2 cm, BC = 6.3 cm and CA = 4.6 cm.

5 Triangle DEF: DE = 6.4 cm, EF = 12 cm and DF = 13.6 cm.

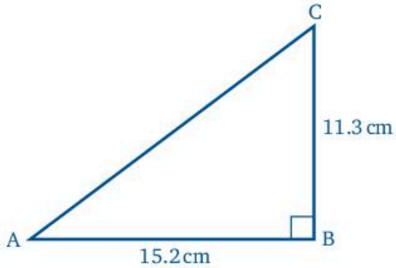
6 Triangle XYZ: XY = 32 cm, YZ = 40 cm and ZX = 48 cm.

Mixed exercises

Exercise 5k

Find the missing lengths in the following triangles:

1



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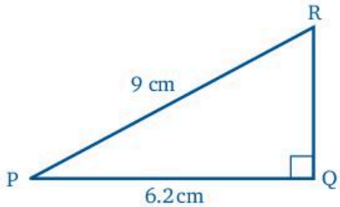

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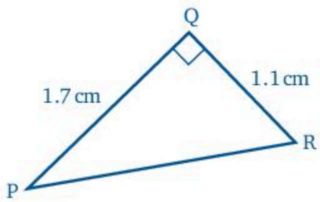

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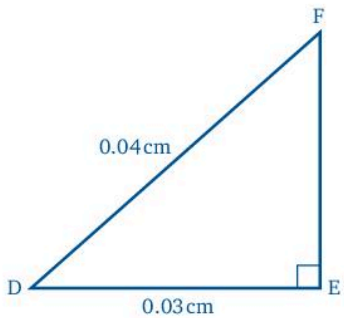

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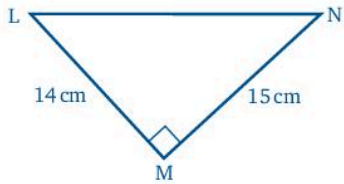

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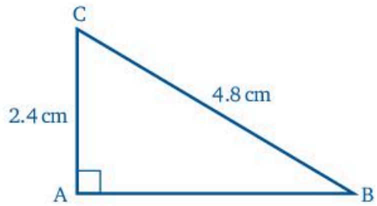

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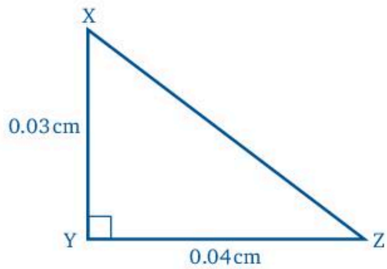

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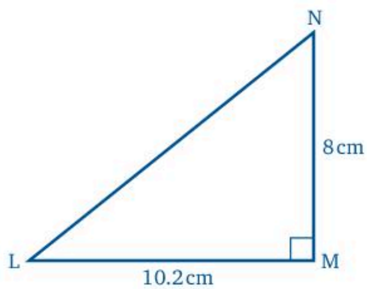


9 In triangle ABC, $\hat{A} = 90°$, AB = 3.2 cm and BC = 4.8 cm. Find AC.

Start by drawing a diagram and marking the sides.

10 In triangle PQR, $\hat{Q} = 90°$, PQ = 56 cm and QR = 32 cm. Find PR.

11 In triangle ABC, AB = 1 cm, BC = 2.4 cm and CA = 2.6 cm. Is $\hat{B}$ a right angle?

12 In triangle DEF, $\hat{F} = 90°$, DF = 2.8 cm and DE = 4.2 cm. Find EF.

13 In triangle XYZ, $\hat{Y} = 90°$, XY = 17 cm and YZ = 20 cm. Find XZ.

14 In triangle LMN, NL = 25 cm, LM = 24 cm and MN = 7 cm. Is the triangle right-angled? If it is, which angle is 90°?

Exercise 5I

Select the letter that gives the correct answer.

1 The square root of 200, correct to 3 s.f., is

A 12.1 **B** 13.1 **C** 14.1 **D** 15.1

2 AC is the hypotenuse of a right-angled triangle.
The lengths of the other two sides are 5 cm and 12 cm.
The length of AC is

A 13 cm **B** 14 cm **C** 15 cm **D** 16 cm

3 The two shorter sides of a right-angled triangle are of lengths 6 cm and 8 cm.

The length of the hypotenuse is

A 9 cm **B** 9.5 cm **C** 10 cm **D** 10.5 cm

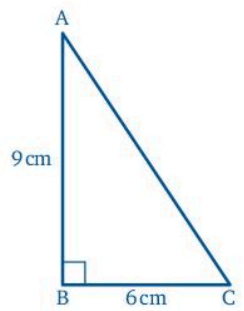
4 The length of AC, correct to 3 s.f., is

A 10.0 cm

B 10.8 cm

C 11.0 cm

D 11.5 cm



5 PQ is a chord of length 10 cm in a circle of radius 6 cm.
The distance of the chord from the centre of the circle, correct to 2 d.p., is

A 3.30 cm **B** 3.31 cm **C** 3.32 cm **D** 3.33 cm

6 The diagonals of a rhombus are of lengths 20 cm and 16 cm.

The length of a side of this rhombus, correct to 3 s.f., is

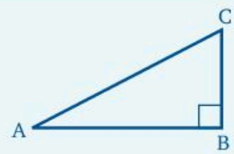
A 12.6 cm **B** 12.7 cm **C** 12.8 cm **D** 12.9 cm

Did you know?

Do you think that a person could be his own worst enemy?

A great mathematician named Evariste Galois (1811–1832) was considered such a person. He had a short unhappy life filled with hate and conceit. He hated school, and his teachers, whom he considered to be very stupid. The teachers thought that he was bad, stupid and strange. He studied mathematics and found that he was a genius in the subject. At the age of seventeen he wrote some ideas and sent them to the French Academy. While awaiting the reply on his work from the academy his father killed himself. This caused him to hate even more. He was expelled from university for inciting a riot for the French Revolution. He fell in love but his girlfriend left him. This caused him to hate even more. He was killed in a duel at the age of twenty years. The ideas he wrote down the night before his death were finally understood around 1900, many years after his death.

In this chapter you have seen that...



- ✔ Pythagoras' theorem states that, in a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides, i.e. in this triangle $AC^2 = AB^2 + BC^2$
- ✔ some special triplets of numbers like 3, 4, 5 and 5, 12, 13, or multiples of these, give a right-angled triangle whatever unit of measurement is used
- ✔ if, in a triangle ABC, $AC^2 = AB^2 + BC^2$, then the triangle contains a right angle and AC is the hypotenuse, but if AC is the longest side and $AC^2 \neq AB^2 + BC^2$, then the triangle is not right-angled.

6 Trigonometry

At the end of this chapter you should be able to...

1 define sine, cosine and tangent of an angle in a right-angled triangle
2 use a calculator to find the sine, cosine or tangent of a given angle
3 use a calculator to find an angle given its sine, cosine or tangent
4 calculate the size of an angle in a right-angled triangle, given the lengths of the sides of the triangle
5 calculate the length of a side of a right-angled triangle, given one side and another angle
6 draw diagrams to show angles of elevation or depression
7 use trigonometric ratios to solve problems on angles of elevation and depression
8 use Pythagoras' theorem and trigonometry to solve problems in three dimensions.

Did you know?

The word 'trigonometry' first appears in the English translation in 1614 of a book written by Bartholomeo Pitiscus (1561–1613) and published in 1595. The full title in the English translation is *Trigonometry: or The Doctrine of Triangles*.

You need to know...

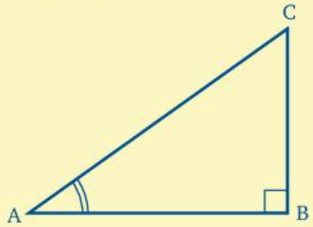
✔ how to work with decimals and fractions
✔ how to solve equations
✔ the properties of isosceles triangles
✔ the properties of similar triangles
✔ Pythagoras' theorem.

Key words

adjacent side, angle of depression, angle of elevation, clinometer, cosine of an angle, hypotenuse, opposite side, right-angled triangle, sine of an angle, tangent of an angle

The tangent of an angle

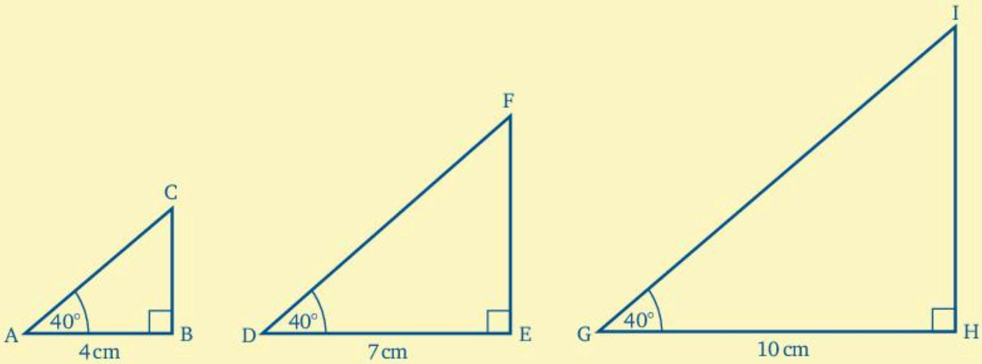
This chapter deals with finding angles and lengths in *right-angled triangles*.



In triangle ABC, AC is the *hypotenuse*, opposite to the right angle.

BC is the *opposite side* to angle A.

AB is the *adjacent side* (or neighbouring side) to angle A.



Draw these triangles as accurately as you can.

Then measure the side opposite the 40° angle in each triangle.

Now find the value of $\frac{BC}{AB}$, $\frac{EF}{DE}$ and $\frac{HI}{GH}$.

You should find that the values are all the same.

Now draw another right-angled triangle containing an angle of 40°.

In your triangle find the value of $\frac{\text{opposite side to angle } 40°}{\text{adjacent side to angle } 40°}$.

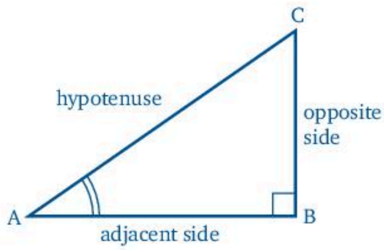
Again you should find that this value is equal to the values found earlier.

(Keep your diagrams, you will need them later in the chapter.)

This ratio is called the *tangent* of 40° or, in shortened form, tan 40°. Its size is stored, together with the tangents of other angles, in natural tangent tables and in scientific calculators.

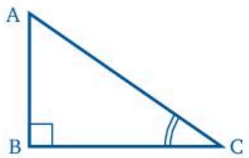
$$\tan \hat{A} = \frac{\text{opposite side}}{\text{adjacent side}}$$

Exercise 6a



In questions **1** to **6**, copy the diagram. Identify the hypotenuse and the sides opposite and adjacent to the marked angle:

1



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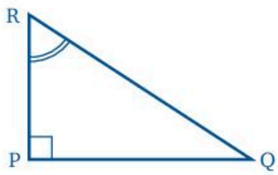

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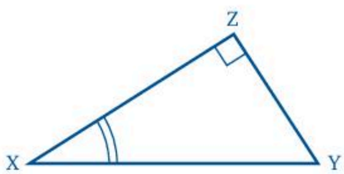

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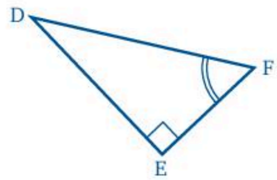

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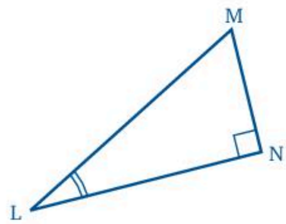

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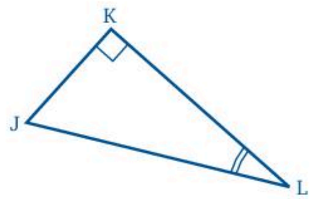
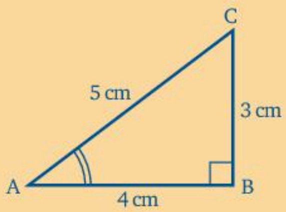
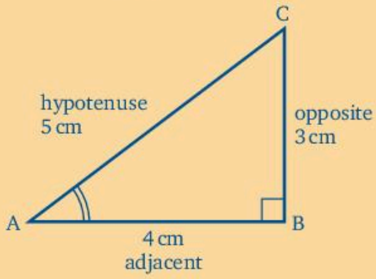




In $\triangle ABC$, $\hat{B} = 90°$, $AB = 4\,\text{cm}$, $BC = 3\,\text{cm}$ and $AC = 5\,\text{cm}$.

Write $\tan\hat{A}$ as a fraction and as a decimal.



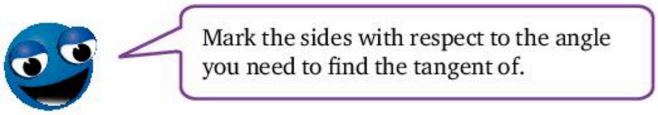
(First identify the sides and mark them on the diagram.)

$$\tan\hat{A} = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{3}{4}$$

$$= 0.75$$

In each of the following questions write the tangent of the marked angle as a fraction and as a decimal (correct to four decimal places where necessary):



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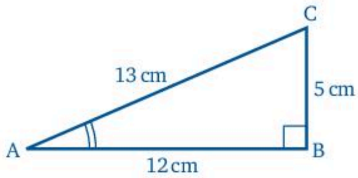

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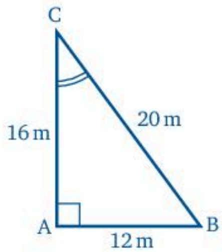

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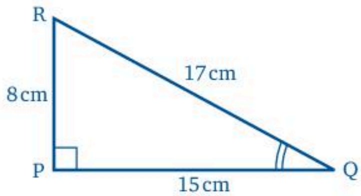

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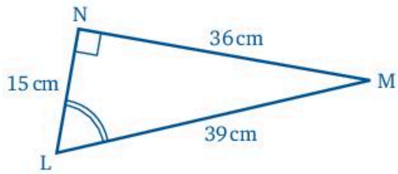

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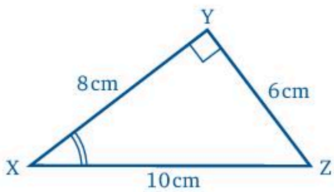

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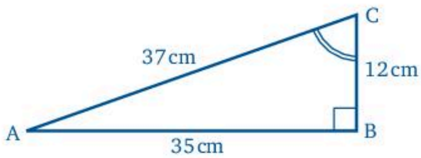


Using a calculator

To find the tangent of an angle, press the tan button on your calculator, enter the size of the angle and then press the equals button. Write the answer correct to four decimal places.

$$\tan 42.4° = 0.9131$$

To find an angle given its tangent, press the shift button followed by the tan button, enter the size of the angle then press the equals button. Write the size of the angle correct to one decimal place.

If these instructions do not work, consult the manual for your calculator.

Exercise 6b

Find the tangents of the following angles:

1 62°	**4** 16.8°	**7** 78.4°	**10** 48.2°
2 14°	**5** 4.6°	**8** 45°	**11** 3°
3 30.5°	**6** 72°	**9** 30°	**12** 29.4°

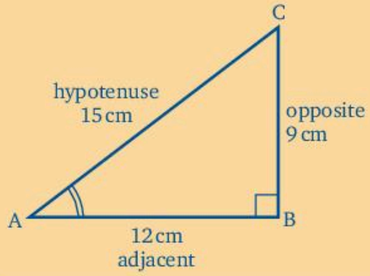
Find the angles whose tangents are given in questions **13** to **24**:

13 0.179	**16** 0.4326	**19** 0.9213	**22** 2.683
14 0.356	**17** 1.362	**20** 0.8	**23** 0.924
15 1.43	**18** 0.632	**21** 0.3214	**24** 0.0024

Finding an angle

Exercise 6c

In triangle ABC, $\hat{B} = 90°$, AB = 12 cm, BC = 9 cm and AC = 15 cm. Find $\hat{A}$.

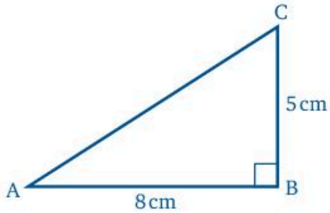


$$\tan \hat{A} = \frac{\text{opp}}{\text{adj}} = \frac{9}{12}$$
$$= 0.75$$
$$\hat{A} = 36.9° \text{ (to 1 d.p.)}$$

Use the information given on the diagrams to find $\hat{A}$ and give your answers correct to one decimal place.

Draw the triangle, mark the angle required, then label the sides with respect to this angle.

1



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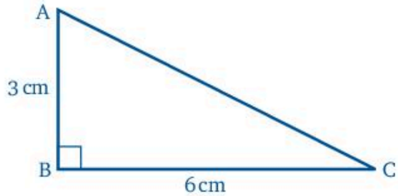

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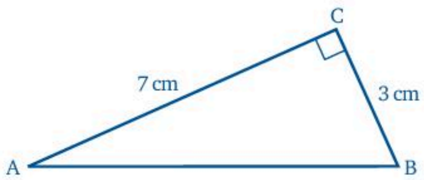

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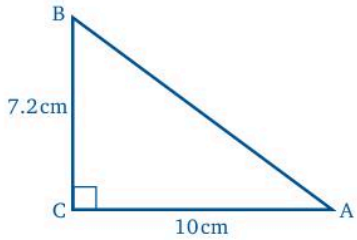


5 In triangle PQR, $\hat{P} = 90°$, QP = 6 cm and PR = 10 cm. Find $\hat{R}$.

6 In triangle XYZ, $\hat{Y} = 90°$, XY = 4 cm and YZ = 5 cm. Find $\hat{X}$.

7 In triangle LMN, $\hat{L} = 90°$, LM = 7.2 cm and LN = 6.4 cm. Find $\hat{N}$.

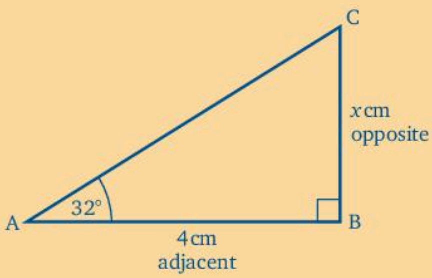
8 In triangle DEF, $\hat{D} = 90°$, DE = 210 cm and DF = 231 cm. Find $\hat{E}$.

9 In $\triangle$ABC, $\hat{C} = 90°$, AC = 3.2 m and BC = 4.7 m. Find $\hat{B}$.

Finding a side

Exercise 6d

In triangle ABC, $\hat{B} = 90°$, AB = 4 cm and $\hat{A} = 32°$. Find BC.



$$\tan A = \frac{\text{opp}}{\text{adj}}$$

$$\tan 32° = \frac{\text{opp}}{\text{adj}} = \frac{x}{4}$$

$$0.6248\ldots = \frac{x}{4}$$

$$4 \times 0.6248\ldots = {}^{1}\!\!\not{4} \times \frac{x}{\not{4}_{1}}$$

Multiply both sides of the equation by 4 to eliminate the denominator on the right-hand side.

Do not clear the display on your calculator.

Press [×] [4] [=]

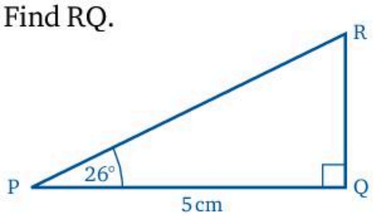
$$x = 2.4994\ldots$$

$\therefore$ BC = 2.50 cm (to 3 s.f.)

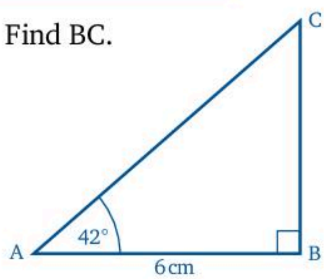
Use the information given in the diagram to find the required side.

When you use a calculator write down the first four figures in the display for intermediate steps. Do not clear the display; use the entry for the next step in the calculation.

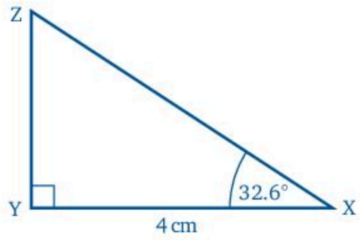
1 Find RQ.



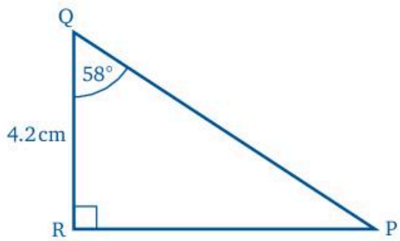
2 Find BC.



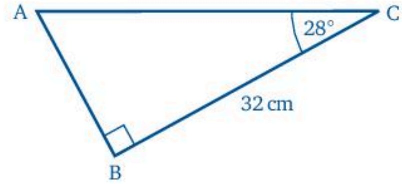
3 Find YZ.



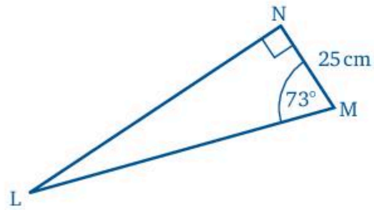
4 Find PR.



5 Find AB.



6 Find LN.



7 In triangle ABC, $\hat{B} = 90°$, $\hat{A} = 32°$ and AB = 9 cm. Find BC.

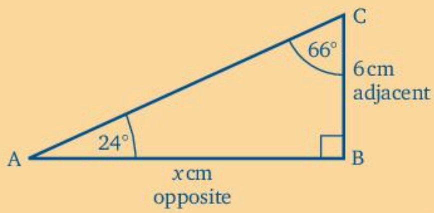
8 In triangle DEF, $\hat{D} = 90°$, $\hat{E} = 48°$ and DE = 20 cm. Find DF.

9 In triangle PQR, $\hat{R} = 90°$, $\hat{Q} = 10°$ and RQ = 16 cm. Find PR.

10 In triangle XYZ, $\hat{Z} = 90°$, $\hat{Y} = 67°$ and ZY = 3.2 cm. Find XZ.

In △ABC, $\hat{B} = 90°$, $\hat{A} = 24°$ and BC = 6 cm. Find AB.

(It is easier to find AB if it is on top of the tangent ratio, i.e. if AB is the opposite side. AB is opposite to $\hat{C}$, so find $\hat{C}$ first.)



$\hat{C} = 66°$ (∠s of a triangle)

$$\frac{x}{6} = \frac{\text{opp}}{\text{adj}} = \tan 66°$$

$$\frac{x}{6} = 2.246\ldots$$

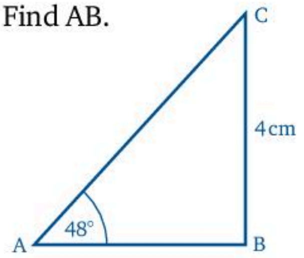
$$6 \times \frac{x}{6} = 6 \times 2.246\ldots$$

$$x = 13.476\ldots$$

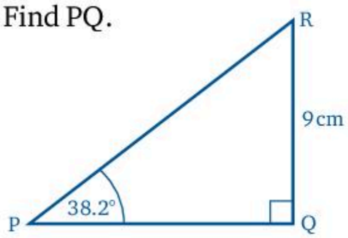
∴ AB is 13.5 cm (to 3 s.f.)

Use the information given in the diagram to find the required side. It may be necessary to find the third angle of the triangle first.

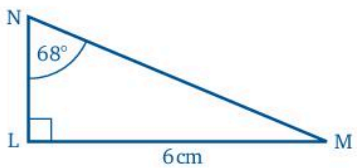
11 Find AB.



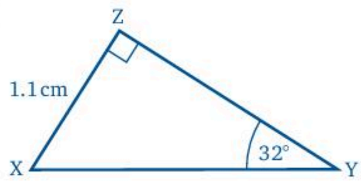
12 Find PQ.



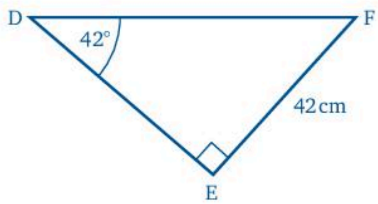
13 Find NL.



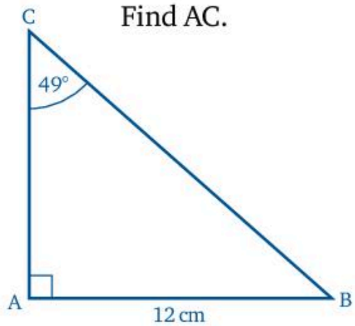
14 Find ZY.



15 Find DE.



16 Find AC.



17 In triangle PQR, $\hat{P} = 90°$, $\hat{Q} = 52°$ and PR = 6 cm. Find QP.

18 In triangle ABC, $\hat{A} = 90°$, $\hat{B} = 31°$ and AC = 220 cm. Find AB.

19 In triangle XYZ, $\hat{Z} = 90°$, $\hat{X} = 67°$ and YZ = 2.3 cm. Find XZ.

20 In triangle LMN, $\hat{L} = 90°$, $\hat{M} = 9°$ and LN = 11 m. Find LM.

Using the hypotenuse

So far, we have used only the opposite and adjacent sides. If we wish to use the hypotenuse we need different ratios.

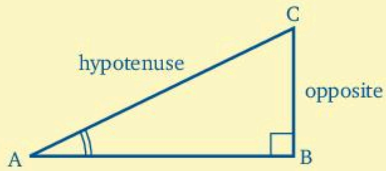
The sine of an angle

Use the triangles you drew earlier, but this time investigate the values of $\frac{\text{opposite side to angle } 40°}{\text{hypotenuse}}$ in each triangle. You should find that the values are all equal.

For an angle in a right-angled triangle, the name given to the ratio $\dfrac{\text{opposite side}}{\text{hypotenuse}}$ is the *sine* of the angle, where sine is abbreviated to sin.

In triangle ABC $\quad \dfrac{BC}{AC} = \sin\hat{A}$

The use of sines is similar to the use of tangents.



Exercise 6e

Find the sines of the following angles:

1 62.4°

2 70°

3 14.3°

4 9°

5 15.2°

6 37.5°

7 59.6°

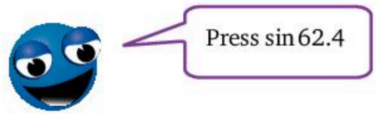
8 30°

9 82°

10 27.8°

11 15.8°

12 87.2°



Find the angles whose sines are given:

13 0.271

14 0.442

15 0.524

16 0.909

17 0.6664

18 0.3720

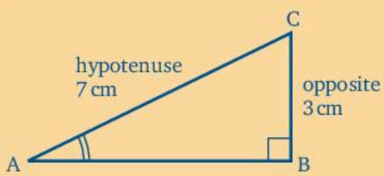
19 0.614

20 0.7283

21 0.1232

Exercise 6f

In triangle ABC, $\hat{B} = 90°$, BC = 3 cm and AC = 7 cm. Find $\hat{A}$.



$$\sin\hat{A} = \frac{\text{opp}}{\text{hyp}} = \frac{3}{7}$$

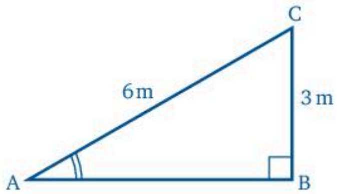
$$= 0.4285\ldots$$

$$\hat{A} = 25.4° \text{ (to 1 d.p.)}$$

Use the information given in the diagram to find the marked angle:

Draw the triangle and label the sides with respect to the required angle.

1



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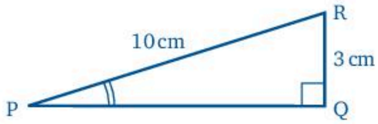

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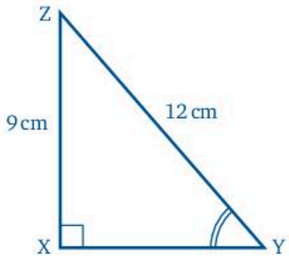

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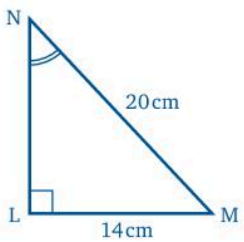
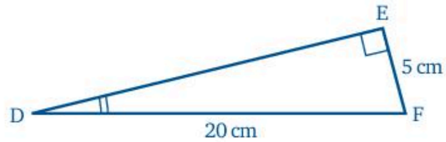
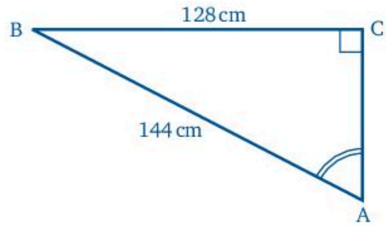


5



6



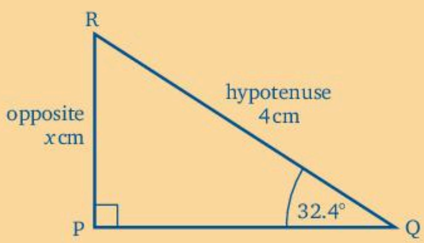
7 In triangle ABC, $\hat{C} = 90°$, BC = 7 cm and AB = 10 cm. Find $\hat{A}$.

8 In triangle PQR, $\hat{Q} = 90°$, PQ = 30 cm and PR = 45 cm. Find $\hat{R}$.

9 In triangle LMN, $\hat{M} = 90°$, MN = 3.2 cm and LN = 8 cm. Find $\hat{L}$.

10 In triangle DEF, $\hat{E} = 90°$, EF = 36 cm and DF = 108 cm. Find $\hat{D}$.

In triangle PQR, $\hat{P} = 90°$, $\hat{Q} = 32.4°$ and RQ = 4 cm. Find PR.



$$\frac{x}{4} = \frac{\text{opp}}{\text{hyp}} = \sin 32.4°$$

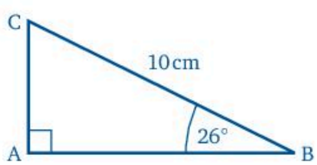
$$4 \times \frac{x}{4} = 4 \times 0.5358\ldots$$

$$x = 2.1433\ldots$$

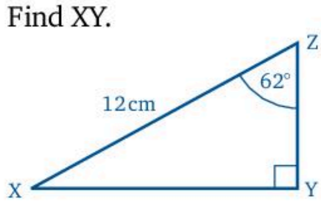
$\therefore$ PR = 2.14 cm (to 3 s.f.)

Use the information given in the diagram to find the required length:

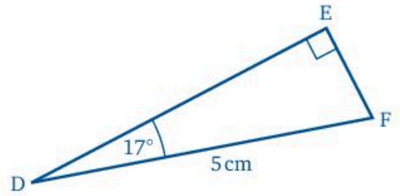
11 Find AC.



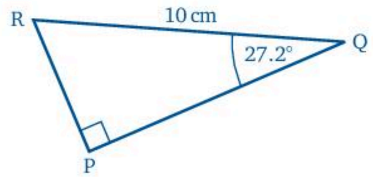
12 Find XY.



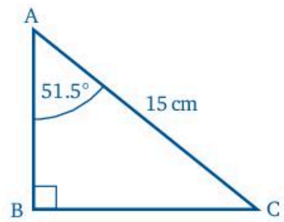
13 Find EF.



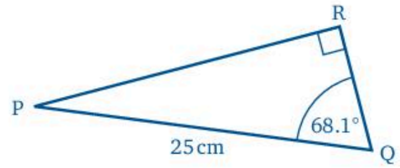
14 Find PR.



15 Find BC.



16 Find PR.



17 In triangle ABC, $\hat{A} = 90°$, BC = 11 cm and $\hat{C} = 35°$. Find AB.

18 In triangle PQR, $\hat{P} = 90°$, QR = 120 m and $\hat{Q} = 10.5°$. Find PR.

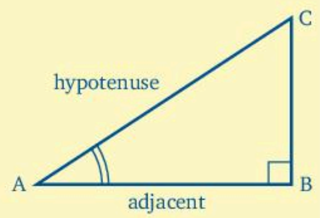
19 In triangle XYZ, $\hat{X} = 90°$, YZ = 3.6 cm and $\hat{Y} = 68°$. Find XZ.

20 In triangle DEF, $\hat{F} = 90°$, DE = 48 m and $\hat{D} = 72°$. Find EF.

The cosine of an angle

Investigate the values of $\dfrac{\text{adjacent side to angle } 40°}{\text{hypotenuse}}$ in each of your triangles. Again you should find that these values are all equal.

For an angle in a right-angled triangle, the name given to the ratio $\dfrac{\text{adjacent side}}{\text{hypotenuse}}$ is the *cosine* of the angle (cosine is abbreviated to cos).



In triangle ABC $\dfrac{AB}{AC} = \cos\hat{A}$

Exercise 6g

Find the cosines of the following angles:

1 32° **3** 82° **5** 60° **7** 52.1°

2 41.8° **4** 47.8° **6** 15.6° **8** 49°

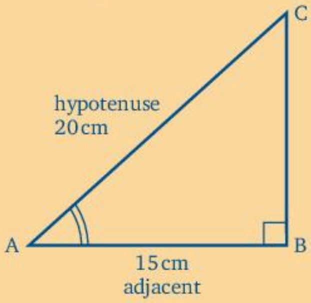
Find the angles whose cosines are given:

9 0.347 **11** 0.719 **13** 0.6281 **15** 0.865

10 0.936 **12** 0.349 **14** 0.3149 **16** 0.014

Exercise 6h

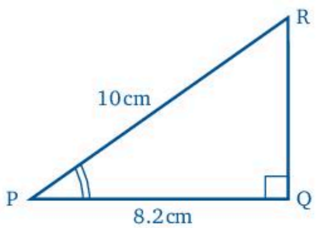
In triangle ABC, $\hat{B} = 90°$, AC = 20 cm and AB = 15 cm. Find $\hat{A}$.



$$\cos \hat{A} = \frac{\text{adj}}{\text{hyp}} = \frac{15}{20}$$
$$= 0.75$$
$$\hat{A} = 41.40\ldots°$$
$$= 41.4° \text{ (to 1 d.p.)}$$

Find the marked angles in the following triangles:

1



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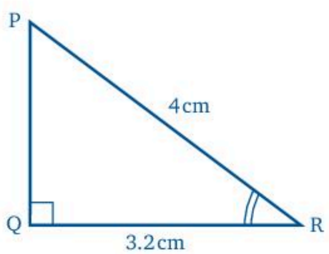

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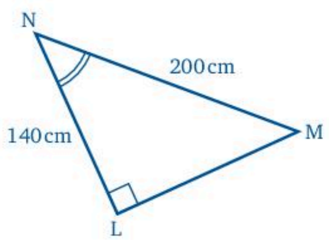

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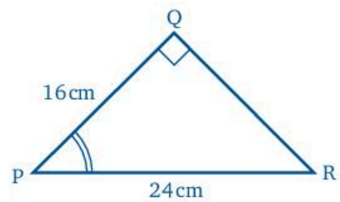

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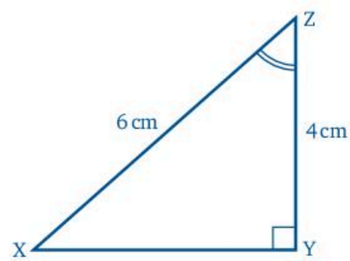
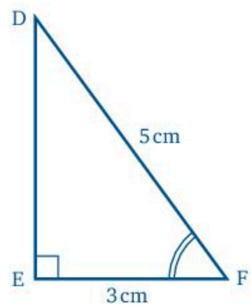


6



7 In triangle ABC, $\hat{B} = 90°$, AB = 3.2 cm and AC = 5 cm. Find $\hat{A}$.

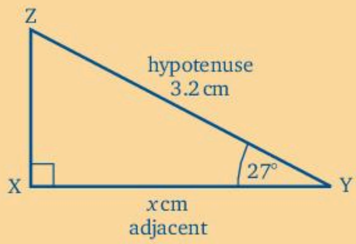
8 In triangle PQR, $\hat{P} = 90°$, QR = 12 cm and PQ = 4.8 cm. Find $\hat{Q}$.

9 In triangle LMN, $\hat{L} = 90°$, MN = 20 cm and ML = 3 cm. Find $\hat{M}$.

10 In triangle DEF, $\hat{F} = 90°$, DE = 18 cm and DF = 16.2 cm. Find $\hat{D}$.

11 In triangle XYZ, $\hat{Z} = 90°$, XY = 14 m and YZ = 11.6 m. Find $\hat{Y}$.

In triangle XYZ, $\hat{X} = 90°$, $\hat{Y} = 27°$ and ZY = 3.2 cm. Find XY.



$$\frac{x}{3.2} = \frac{\text{adj}}{\text{hyp}} = \cos 27°$$

$$\frac{x}{3.2} = 0.8910\ldots$$

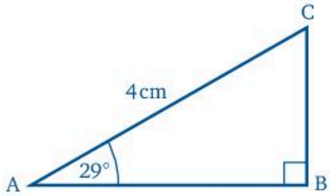
$$3.2 \times \frac{x}{3.2} = 0.8910\ldots \times 3.2$$

$$x = 2.8512\ldots$$

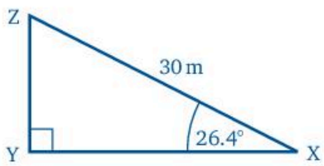
$$\therefore\ \text{XY} = 2.85\text{ cm (to 3 s.f.)}$$

Use the information given in the diagrams to find the required lengths.

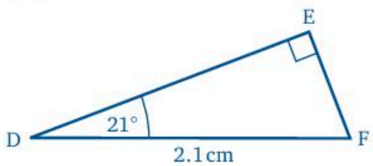
12 Find AB.



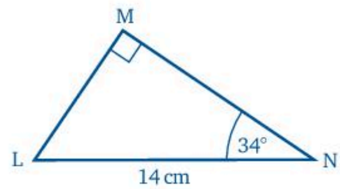
13 Find XY.



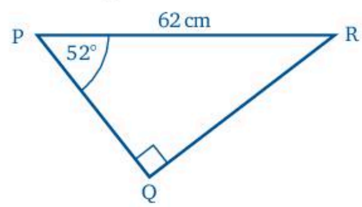
14 Find ED.



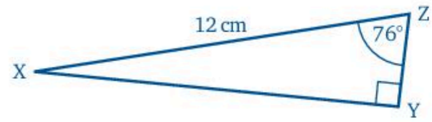
15 Find MN.



<u>**16**</u> Find PQ.



<u>**17**</u> Find YZ.



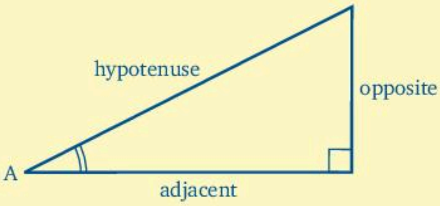
18 In triangle PQR, $\hat{Q} = 90°$, $\hat{P} = 31°$ and PR = 20 cm. Find PQ.

19 In triangle LMN, $\hat{N} = 90°$, $\hat{L} = 42°$ and LM = 3 cm. Find LN.

<u>**20**</u> In triangle DEF, $\hat{D} = 90°$, $\hat{E} = 68°$ and EF = 11 cm. Find DE.

<u>**21**</u> In triangle XYZ, $\hat{Z} = 90°$, $\hat{Y} = 15°$ and YX = 14 cm. Find ZY.

Summary



$$\text{Sin}\,\hat{A} = \frac{\textbf{O}\text{pposite}}{\textbf{H}\text{ypotenuse}} \quad \text{(SOH)}$$

$$\text{Cos}\,\hat{A} = \frac{\textbf{A}\text{djacent}}{\textbf{H}\text{ypotenuse}} \quad \text{(CAH)}$$

$$\text{Tan}\,\hat{A} = \frac{\textbf{O}\text{pposite}}{\textbf{A}\text{djacent}} \quad \text{(TOA)}$$

Some people remember these definitions by using the word 'SOHCAHTOA' or a sentence like 'Some Old Hangars Can Almost Hold Two Old Aeroplanes'.

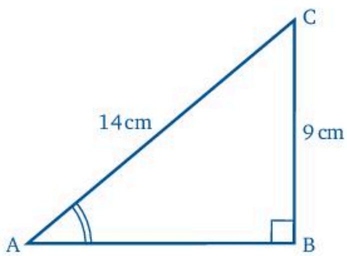
Sines, cosines and tangents

Exercise 6i

In questions **1** to **8**, find the marked angles.

Remember to first label the given sides with respect to the angle required and then decide which ratio you will have to use.

1



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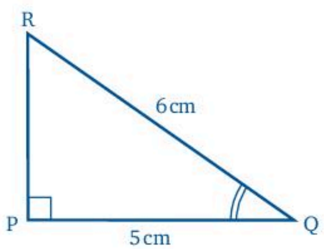
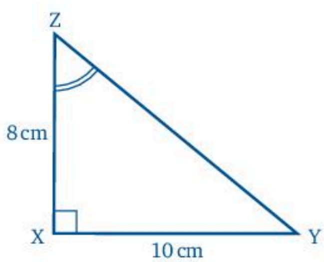


3



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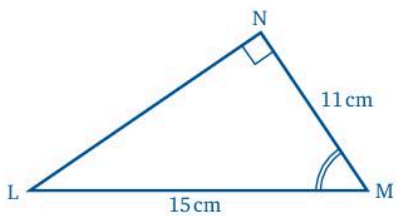

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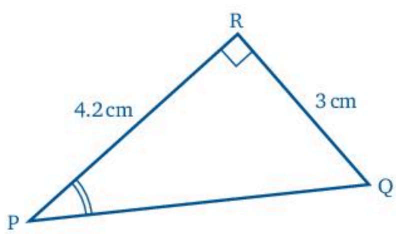
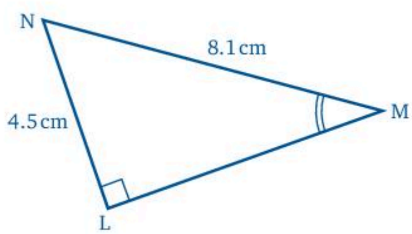


6



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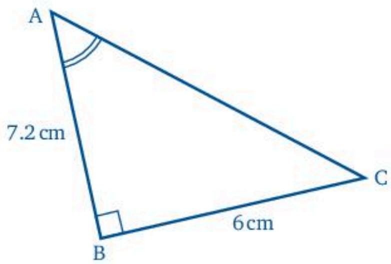

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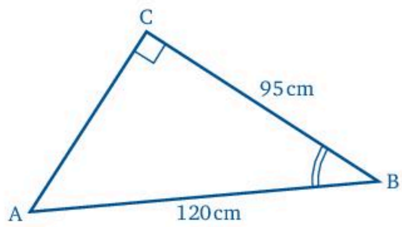

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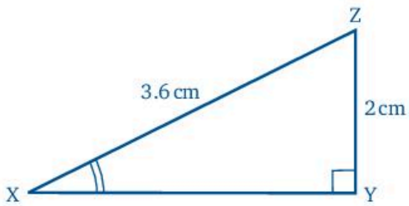

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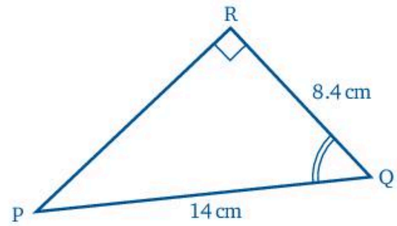


11 In triangle ABC, $\hat{B} = 90°$, AC = 60 cm and BC = 22 cm. Find $\hat{C}$.

12 In triangle PQR, $\hat{R} = 90°$, PQ = 24 cm and QR = 6 cm. Find $\hat{P}$.

13 In triangle ABC, $\hat{B} = 90°$, AC = 1.5 cm and BC = 0.82 cm. Find $\hat{C}$.

14 In triangle PQR, $\hat{R} = 90°$, RQ = 8 cm and RP = 6.2 cm. Find $\hat{Q}$.

15 In triangle DEF, $\hat{F} = 90°$, DF = 16.2 cm and EF = 19.8 cm. Find $\hat{E}$.

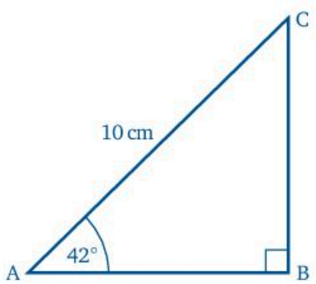
16 In triangle XYZ, $\hat{X} = 90°$, YZ = 1.6 m and XY = 1.32 m. Find $\hat{Z}$.

17 In triangle DEF, $\hat{E} = 90°$, DE = 1.9 m and EF = 2.1 m. Find $\hat{F}$.

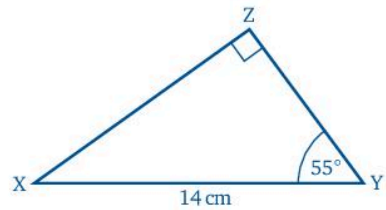
18 In triangle GHI, $\hat{H} = 90°$, GI = 52 cm and IH = 21 cm. Find $\hat{I}$.

Use the information given in the diagram to find the required length.

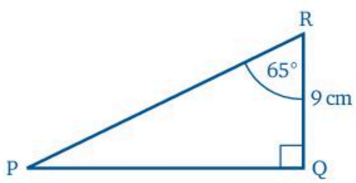
19 Find BC.



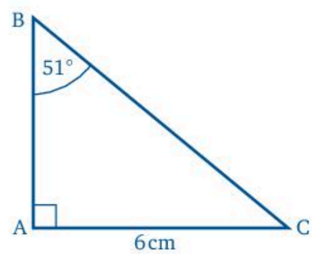
21 Find ZY.



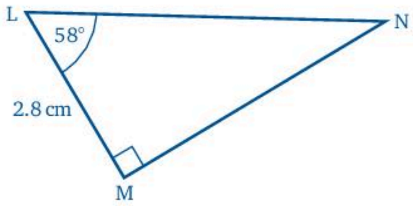
20 Find PQ.



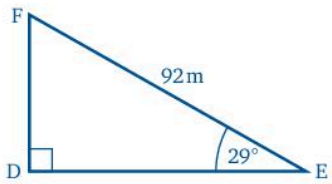
22 Find AB.



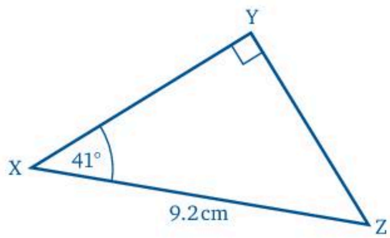
23 Find MN.



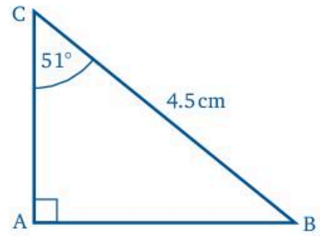
24 Find DE.



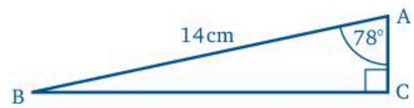
25 Find YZ.



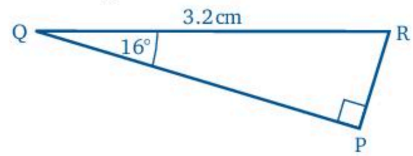
26 Find AB.



27 Find BC.



28 Find PQ.



29 In triangle ABC, $\hat{C} = 90°$, $\hat{A} = 78°$ and AC = 24 cm. Find BC.

30 In triangle PQR, $\hat{P} = 90°$, $\hat{Q} = 36°$ and QR = 3.2 cm. Find PQ.

31 In triangle XYZ, $\hat{X} = 90°$, $\hat{Y} = 36°$ and YZ = 17 cm. Find XZ.

32 In triangle DEF, $\hat{F} = 90°$, $\hat{E} = 51°$ and DF = 9.2 cm. Find EF.

33 In triangle LMN, $\hat{M} = 90°$, $\hat{N} = 25°$ and LN = 16 cm. Find MN.

34 In triangle LMN, $\hat{L} = 90°$, $\hat{M} = 56.2°$ and LN = 32 cm. Find ML.

35 In triangle ABC, $\hat{C} = 90°$, $\hat{B} = 72.8°$ and AB = 78 cm. Find AC.

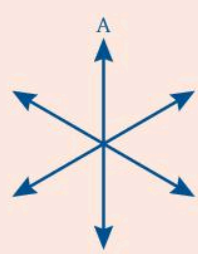
36 In triangle PQR, $\hat{R} = 90°$, $\hat{P} = 31.2°$ and PQ = 117 cm. Find QR.

Puzzle

As part of an aerobatics display, six aeroplanes fly at the same speed away from each other in a fan of 60° to each other.

This is what it looks like from the ground.

What does it look like to the pilot of aircraft A when he looks back?

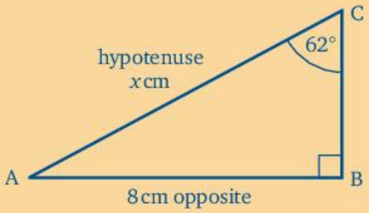


Finding the hypotenuse

Up to now, when finding the length of a side, we have been able to form an equation in which our unknown length is on the top of the fraction. If we wish to find the hypotenuse, this is not possible and the equation we form takes slightly longer to solve.

Exercise 6j

In triangle ABC, $\hat{B} = 90°$, AB = 8 cm and $\hat{C} = 62°$. Find AC.



$$\frac{8}{x} = \frac{\text{opp}}{\text{hyp}} = \sin 62°$$

$$\frac{8}{x} = \sin 62°$$

$$x \times \frac{8}{x} = x \times \sin 62° \quad \text{Multiplying both sides by } x$$

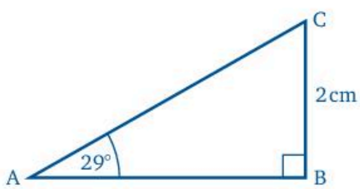
$$\frac{8}{\sin 62°} = x \quad \text{Dividing both sides by } \sin 62°$$

$$x = 9.060\ldots$$

$$\therefore \quad \text{AC} = 9.06 \text{ cm (to 3 s.f.)}$$

Use the information given in the diagram to find the hypotenuse:

1



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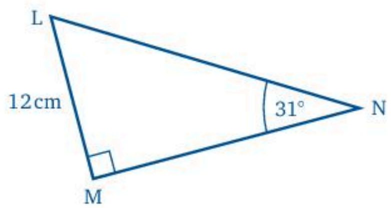

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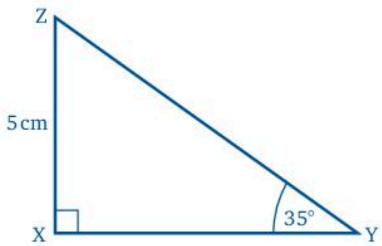

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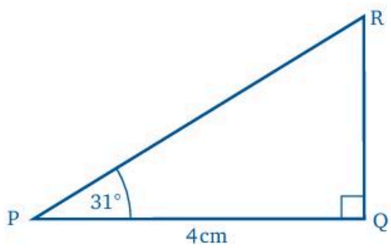

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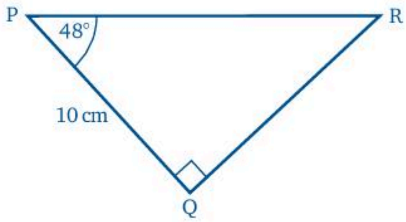

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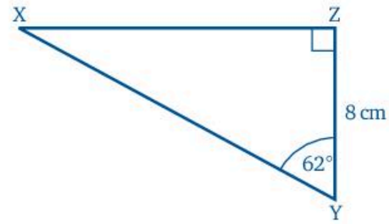


7 In triangle ABC, $\hat{B} = 90°$, $\hat{A} = 43°$ and BC = 3 cm. Find AC.

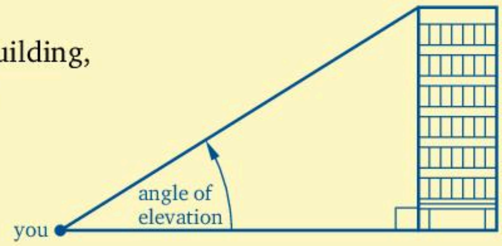
8 In triangle PQR, $\hat{P} = 90°$, $\hat{Q} = 28°$ and PR = 7 cm. Find QR.

9 In triangle LMN, $\hat{L} = 90°$, $\hat{M} = 14°$ and LN = 8 cm. Find MN.

10 In triangle XYZ, $\hat{Z} = 90°$, $\hat{Y} = 62°$ and ZY = 20 cm. Find XY.

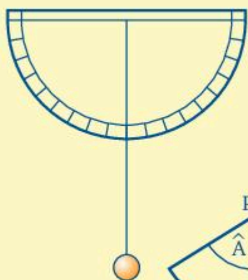
Angles of elevation

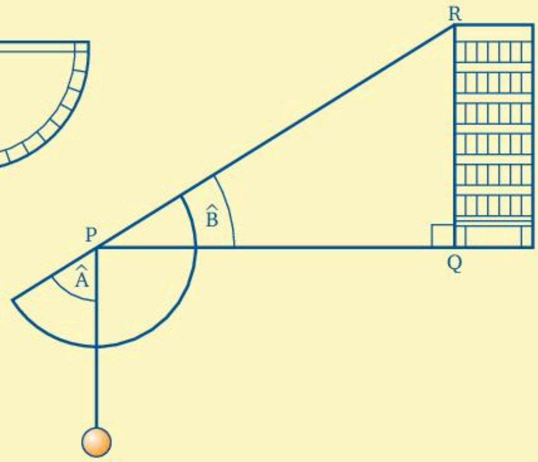
If you are standing on level ground and can see a tall building, you will have to look up to see the top of that building.



If you start by looking straight ahead and then look up to the top of the building, the angle through which you raise your eyes is called the *angle of elevation* of the top of the building.

There are instruments for measuring angles of elevation called *clinometers*. A simple one can be made from a large card protractor and a piece of string with a weight on the end.





You can read the size of $\hat{A}$.

Then the angle of elevation, $\hat{B}$, is given by $\hat{B} = 90° - \hat{A}$.

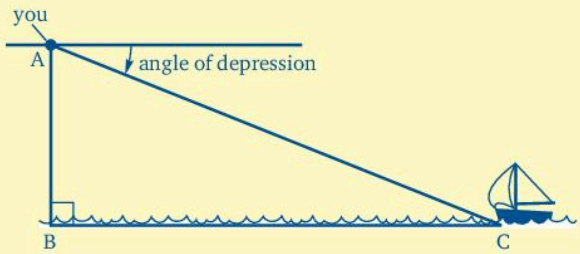
(Note that this method is not very accurate.)

If your distance from the foot of the building and the angle of elevation of the top are both known, you can use trigonometry to work out the height of the building.

Angles of depression

An *angle of depression* is the angle between the line looking straight ahead and the line looking down at an object below you.

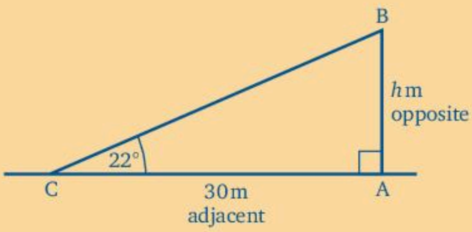
If, for example, you are standing on a cliff looking out to sea, the diagram shows the angle of depression of a boat.



If the angle of depression and the height of the cliff are both known, you can use trigonometry to work out the distance of the boat from the foot of the cliff.

Exercise 6k

A flagpole stands on level ground. From a point on the ground 30 m away from its foot, the angle of elevation of the top of the pole is 22°. Find the height of the pole.



$$\frac{h}{30} = \frac{\text{opp}}{\text{adj}} = \tan 22°$$

$$\frac{h}{30} = 0.4040\ldots$$

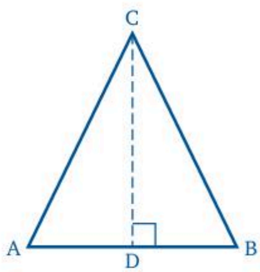
$$\cancel{30} \times \frac{h}{\cancel{30}} = 0.4040 \times 30$$

$$h = 12.12$$

The pole is 12.1 m high (to 3 s.f.)

Draw a diagram. Mark the sides and angles given and required. Label the sides with respect to the angles. Then you can see which ratio you need to use.

1 In triangle ABC, AC = CB = 10 m and $\hat{A} = 64°$.
Find the height of the triangle.



2 From a point on level ground 40 m from the base of a pine tree, the angle of elevation of the top of the tree is 50°. Find the height of the tree.

3 The angle of elevation of the top of a church tower, from a point on level ground 500 m away, is 16°. Find the height of the tower.

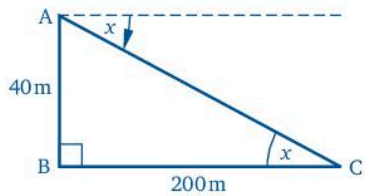
4 A , B and C are the points (2, 0), (8, 0) and (8, 5), respectively. Calculate the angle between AC and the x-axis.

5 ABCD is a rectangle with AB = 26 cm and BC = 48 cm. Find the angle between the diagonal AC and side AB.

6 A, B and C are the points (1, 2), (3, 2) and (1, 5), respectively. Find $A\hat{B}C$.

7 ABCD is a rhombus of side 15 cm. The diagonal AC is of length 20 cm. Find the angle between AC and the side CD.

8 A boat C is 200 m from the foot B of a vertical cliff, which is 40 m high. What is the angle of depression of the boat from the top of the cliff?



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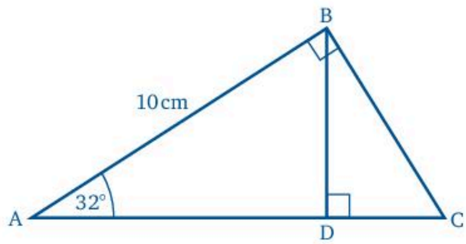


In the figure, AB = 10 cm, $\hat{A} = 32°$ and $A\hat{B}C = B\hat{D}C = 90°$. Copy the figure and then mark in the sizes of the remaining angles.

Find **a** BD **b** BC

10 Triangle ABC is an equilateral triangle of side 6 cm.

Find **a** its height **b** its area.

11 A lamp post stands on level ground. From a point which is 10 m from its foot, the angle of elevation of the top is 25°. How high is the lamp post?

12 From a point on the ground 60 m away, the angle of elevation of the top of a factory chimney is 42°. Find the height of the chimney.

13 From the top of a hill, which is 400 m above sea level, the angle of depression of a boathouse is 20°. The boathouse is at sea level. Find the distance of the boathouse from the top of the hill.

14 An aircraft flying at 5000 m measures the angle of depression of a point on the coast as 30°. At the moment that it measures the angle, how much further has the plane to fly before passing over the coastline?

15 A vertical radio mast is 250 m high. From a point A on the ground, the angle of elevation of the top of the mast is 30°. How far is the point A from the foot of the mast?

16 An automatic lighthouse is stationed 500 m from a point, A, on the coast. There are high cliffs at A and from the top of these cliffs, the angle of depression of the lighthouse is 15°. How high are the cliffs?

17 An airport controller measures the angle of elevation of an approaching aircraft as 20°. If the aircraft is then 1.6 km from the control building, at what height is it flying?

18 A surveyor standing 400 m from the foot of a church tower, on level ground, measures the angle of elevation of the top of the tower. If this angle is 35° how high is the tower?

Puzzle

There are three different routes from Ian's home to the nearest post box and two different routes from the post box to school. How many different ways are there for Ian to go to school if he must pass the post box on the way?

Mixed exercise

Exercise 6I

Select the letter that gives the correct answer.

1 The tangent of the angle 72.6°, correct to 4 s.f., is

A 3.078 **B** 3.190 **C** 3.191 **D** 3.271

2 The angle whose sine is 0.8829 is

A 28.0° **B** 41.4° **C** 48.6° **D** 62.0°

3 The angle whose cosine is 0.7071 is

A 41° **B** 43° **C** 45° **D** 47°

4 The angle whose tangent is 3.732 is

A 75.0° **B** 75.5° **C** 76.0° **D** 76.5°

5 In triangle PQR, $\hat{P} = 90°$, $\hat{Q} = 42°$ and PQ = 9.3 cm. The length of PR, correct to 3 s.f., is

A 8.36 cm **B** 8.37 cm **C** 8.38 cm **D** 8.39 cm

6 The angle of elevation of the top of a church spire from a point 600 m away on level ground is 15°. The height of the top of the spire, correct to the nearest metre, is

A 151 m **B** 156 m **C** 161 m **D** 166 m

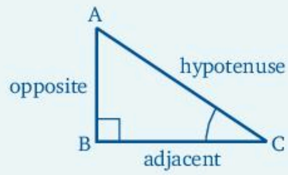
Puzzle

The integral triples {40, 42, 58}, {24, 70, 74} and {15, 112, 113} are Pythagorean triples. Calculate the areas of triangles having these triples as the lengths of their sides.

In this chapter you have seen that...

✔ in a right-angled triangle ABC,

$$\tan C = \frac{\text{opp}}{\text{adj}}, \quad \sin C = \frac{\text{opp}}{\text{hyp}}, \quad \cos C = \frac{\text{adj}}{\text{hyp}}$$



7 Circles

At the end of this chapter you should be able to...

1 state the relationship between the circumference and diameter of a circle
2 calculate the circumference of a circle given its diameter or radius
3 solve problems involving the calculation of circumferences of circles
4 calculate the radius of a circle of given its circumference
5 calculate the area of a circle of given radius
6 calculate the radius of a circle given its area
7 find the length of an arc of a circle, given the radius of the circle and the size of the angle at the centre of that circle
8 find the area of a sector of a circle given the radius and the angle at the centre
9 find the radius of a circle given the length of the arc and the angle at the centre
10 find the angle at the centre of a circle given the length of the arc and the radius of the circle
11 find the angle at the centre of a circle given the area of a sector and the radius of the circle
12 find the radius of a circle given the length of the arc and the area of the sector of the circle
13 find the area of a segment of a circle given the radius and the angle at the centre.

Did you know?

Circumference means 'the perimeter of a circle'. The word comes from the Latin *circumference* – 'to carry around'. The symbol π is the first letter of the Greek word for circumference – *perimetron*. In Germany, π is identified as the ludolphine number, because of the work of Ludolph van Ceulen, who tried to find a better estimate of the number.

You need to know...

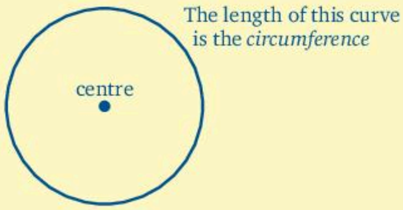
- ✔ the meaning of significant figures
- ✔ the units of area
- ✔ how to find one quantity as a fraction of another
- ✔ how to find the area of a rectangle
- ✔ how to find the area of a triangle.

Key words

annulus, arc, area, centre, chord, circumference, cone, cylinder, decimal place, diameter, perimeter, pi (π), quadrant, radius, revolution, sector, segment, semicircle, significant figure, subtend

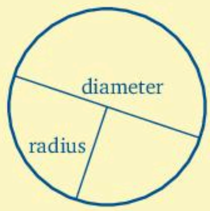
Diameter, radius and circumference

We first met circles in Grade 7. This is a reminder of the names for parts of a circle.



Any straight line joining the *centre* to a point on the circumference is a *radius*.

A *diameter* of a circle is a straight line through the centre from one point on the circumference to the opposite point on the circumference.



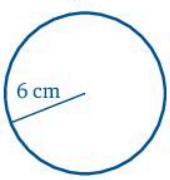
The diameter is twice as long as the radius. If d stands for the length of a diameter and r stands for the length of a radius, we can write this as a formula:

$$d = 2r$$

Exercise 7a

In questions **1** to **3**, write down the length of the diameter of the circle whose radius is given.

1

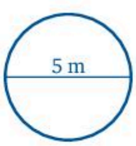


2 15 mm

3 3.5 cm

In questions **4** to **6**, write down the length of the radius of the circle whose diameter is given.

4



5 1 km

6 4.6 cm

Activity

For this activity you will need some thread and a *cylinder* (e.g. a tin of soup, a soft drink can, the cardboard tube from a roll of kitchen paper).

Measure across the top of the cylinder to get a value for the diameter. Wind the thread 10 times round the can. Measure the length of thread needed to do this and then divide your answer by 10 to get a value for the circumference. If C stands for the circumference and d for the length of the diameter, find, approximately, the value of $C \div d$.

Try this with a variety of different cylinders. You can improve the accuracy of your results by tracing the top of the cylinder, cutting out the circle and then folding it in half to measure the diameter.

(Note that you can also use the label from the cylindrical tin. If you are careful you can reshape it and measure the diameter and then unroll it to measure the circumference.)

Compare the results from the whole class for the value of $C \div d$.

Introducing π

From the activity above you will see that, for any circle,

$$\text{circumference} \approx 3 \times \text{diameter}$$

The number that you have to multiply the diameter by to get the circumference is slightly larger than 3.

This number cannot be written down exactly, either as a fraction or as a decimal: as a fraction it is approximately, but *not* exactly, $\frac{22}{7}$; as a decimal it is approximately 3.142, correct to 3 *decimal places*. This number is another example of an irrational number (like $\sqrt{2}$, which you met in Chapter 5).

Now with a computer to do the arithmetic we can find its value to as many decimal places as we choose: it is a never-ending, never-repeating decimal fraction. To sixty decimal places, the value of this number is

3.141592653589793238462643383279502884197169399375105820974944 ...

Because we cannot write it down exactly we use the Greek letter π (*pi*) to stand for this number. Then we can write a formula connecting the circumference and diameter of a circle in the form $C = \pi d$. But $d = 2r$ so we can rewrite this formula as

$$C = 2\pi r$$

where C = circumference and r = radius

The symbol π was first used by an English writer, William Jones, in 1706. It was later adopted in 1737 by Euler.

Calculating the circumference

Exercise 7b

Using 3.142 as an approximate value for π, find the circumference of a circle of radius 3.8 m.

To get your answer correct to 3 s.f. you must work to 4 s.f.

Using $C = 2\pi r$

with $\pi = 3.142$ and $r = 3.8$

gives $C = 2 \times 3.142 \times 3.8$

$= 23.9$ to 3 s.f.

Circumference = 23.9 m to 3 s.f.

Using 3.142 as an approximate value for π and giving your answers correct to 3 s.f., find the circumference of a circle of radius:

1	2.3 m	**4**	53 mm	**7**	36 cm	**10**	0.014 km	**13**	1.4 m
2	4.6 cm	**5**	8.7 m	**8**	4.8 m	**11**	7 cm	**14**	35 mm
3	2.9 cm	**6**	250 mm	**9**	1.8 m	**12**	28 mm	**15**	5.6 cm

For questions **16** to **23** you can use $C = 2\pi r$ or $C = \pi d$.
Read the question carefully before you decide which one to use.

Using $\pi \approx 3.14$ and giving your answer correct to 2 s.f., find the circumference of a circle of:

16	radius 154 mm	**20**	radius 34.6 cm
17	diameter 28 cm	**21**	diameter 511 mm
18	diameter 7.7 m	**22**	diameter 630 cm
19	radius 210 mm	**23**	diameter 9.1 m

In early times, $\sqrt{10}$ was used as an approximation for π.

Investigation

Count Buffon's experiment

Count Buffon was an 18th-century scientist who carried out many probability experiments. The most famous of these is his 'Needle Problem'. He dropped needles on to a surface ruled with parallel lines and considered the drop successful when a needle fell between two lines. His amazing discovery was that the number of successful drops divided by the number of unsuccessful drops was an expression involving π.

You can repeat his experiment and get a good approximation for the value of π from it:

- Take a matchstick or a similar small stick and measure its length.
- Take a sheet of paper measuring about $\frac{1}{2}$ m each way and fill the sheet with a set of parallel lines whose distance apart is equal to the length of the stick.
- With the sheet on the floor, drop the stick on to it from a height of about 1 m.
- Repeat this about a hundred times and keep a tally of the number of times the stick touches or crosses a line and of the number of times it is dropped.
- Find the value of

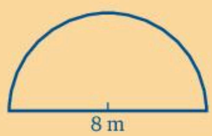
$$\frac{2 \times \text{number of times it is dropped}}{\text{number of times it crosses or touches a line}}$$

Problems

Exercise 7c

Use the value of π on your calculator and give your answers correct to 3 s.f.

Find the perimeter of the given *semicircle*.



(The prefix 'semi' means half.)

Remember that *perimeter* means distance all round, so you need to find half the circumference then add the length of the straight edge.

The complete circumference of the circle is $2\pi r$

The curved part of the semicircle is $\frac{1}{2} \times 2\pi r$

$$= \frac{1}{2} \times 2 \times \pi \times 4\,\text{m}$$

$= 12.57$ m (correct to 4 s.f.)

The perimeter = curved part + straight edge

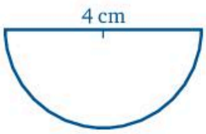
$= (12.57 + 8)\,\text{m}$

$= 20.57\,\text{m}$

$= 20.6\,\text{m}$ to 3 s.f.

Find the perimeter of each of the following shapes:

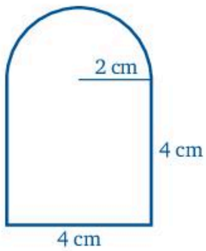
1



2

This is called a *quadrant*: it is one quarter of a circle.

3



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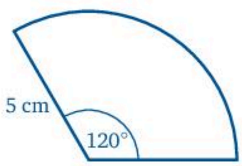
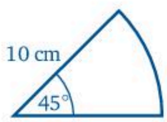


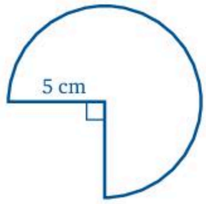
This is one-third of a circle because 120° is $\frac{1}{3}$ of 360°.

5



A 'slice' of a circle is called a *sector*. $\frac{45}{360} = \frac{1}{8}$, so this sector is $\frac{1}{8}$ of a circle.

6



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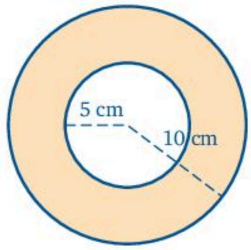
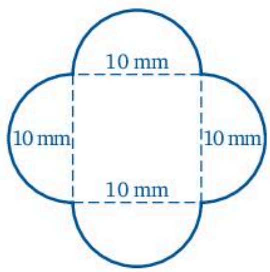


The perimeter is the distance round all the edges. The region between two concentric circles is called an *annulus*.

8



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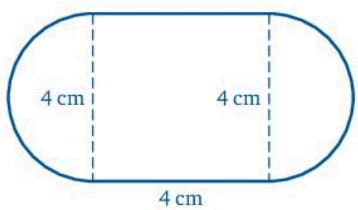

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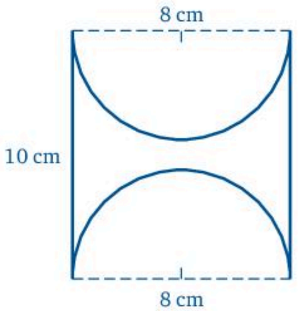

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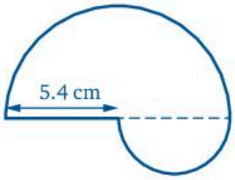

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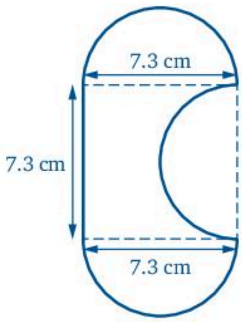

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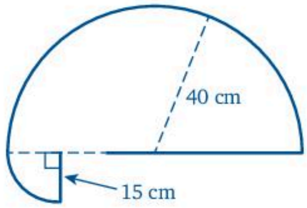

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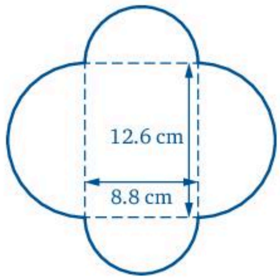
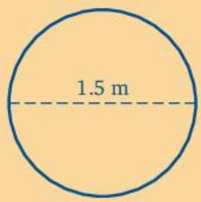


Exercise 7d

Use the value of π on your calculator and give your answers correct to 3 s.f.

A circular flower bed has a diameter of 1.5 m. A metal edging is to be placed round it. Find the length of edging needed and the cost of the edging if it is sold by the metre (i.e. you can only buy a whole number of metres) and costs \$60 a metre.

First find the circumference of the circle, then how many metres you need.



Using $C = \pi d$,

$$C = \pi \times 1.5$$
$$= 4.712\ldots$$

Length of edging needed = 4.71 m to 3 s.f.

(Note that if you use $C = 2\pi r$, you must remember to halve the diameter.)

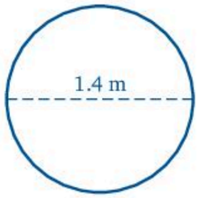
As the length is 4.71 m we have to buy 5 m of edging.

$$\text{Cost} = 5 \times \$60$$
$$= \$300$$

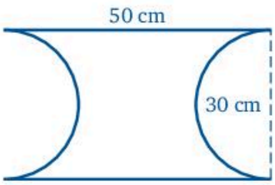
1 Measure the diameter, in millimetres, of a $20 coin. Use your measurement to find the circumference of a $20 coin.

2 Repeat question **1** with a $10 coin and a $5 coin.

3 A circular tablecloth has a diameter of 1.4 m. How long is the hem of the cloth?



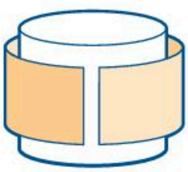
4 A rectangular sheet of metal measuring 50 cm by 30 cm has a semicircle of radius 15 cm cut from each short side as shown. Find the perimeter of the shape that is left.



5 A bicycle wheel has a radius of 28 cm. What is the circumference of the wheel?

6 How far does a bicycle wheel of radius 28 cm travel in one complete *revolution*? How many times will the wheel turn when the bicycle travels a distance of 352 m?

7 A cylindrical tin has a radius of 2 cm. What length of paper is needed to put a label on the tin if the edges of the paper just meet?



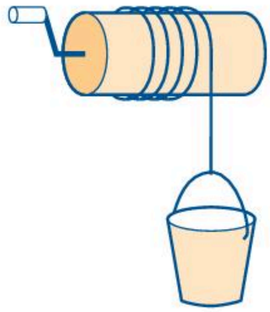
8 A square sheet of metal has sides of length 30 cm. A quadrant (one quarter of a circle) of radius 15 cm is cut from each of the four corners. Sketch the shape that is left and find its perimeter.

9 A boy flies a model aeroplane on the end of a wire 10 m long. If he keeps the wire horizontal, how far does his aeroplane fly in one revolution?

10 If the aeroplane described in question **9** takes 1 second to fly 10 m, how long does it take to make one complete revolution? If the aeroplane has enough power to fly for 1 minute, how many turns can it make?

11 A cotton reel has a diameter of 2 cm. There are 500 turns of thread on the reel. How long is the thread?

12 A bucket is lowered into a well by unwinding rope from a cylindrical drum. The drum has a radius of 20 cm and with the bucket out of the well there are 10 complete turns of the rope on the drum. When the rope is fully unwound the bucket is at the bottom of the well.
How deep is the well?



13 A garden hose is 100 m long. For storage it is wound on a circular hose reel of diameter 45 cm. How many turns of the reel are needed to wind up the hose?

14 The cage that takes miners up and down the shaft of a coal mine is raised and lowered by a rope wound round a circular drum of diameter 3 m. The rope does not wind on top of itself. It takes 10 revolutions of the drum to lower the cage from ground level to the bottom of the shaft. How deep is the shaft?

Investigation

Kevin entered a 50 km sponsored cycle ride. He wondered how many pedal strokes he made. The diameter of each wheel is 70 cm.

1. Investigate this problem if one pedal stroke gives one complete turn of the wheels.
2. What happens if Kevin uses a gear that gives two turns of the wheel for each pedal stroke?
3. Find out how the gears on a racing bike affect the ratio of the number of pedal strokes to the number of turns of the wheels.

Discuss the assumptions made in order to answer parts **1** and **2**.

Write a short report on how these assumptions affect the reasonableness of your answers.

Finding the radius of a circle given the circumference

If a circle has a circumference of 24 cm, we can find its radius from the formula $C = 2\pi r$

i.e. $24 = 2 \times 3.142 \times r$

and solving this equation for r.

Exercise 7e

Use the value of π on your calculator and give your answers correct to 3 s.f.

The circumference of a circle is 36 m. Find the radius of this circle.

Using $C = 2\pi r$ gives

$$36 = 2 \times \pi \times r$$

$$36 = 6.283 \times r$$

(writing down the first 4 digits in the calculator display)

$$\frac{36}{6.283} = r \quad \text{(dividing both sides by 6.283)}$$

$$= 5.729\ldots$$

$$r = 5.73\text{ m} \quad \text{to 3 s.f.}$$

Find the radius of the circle whose circumference is:

1	44 cm	**3**	550 m	**5**	462 mm	**7**	36.2 mm	**9**	582 cm
2	121 mm	**4**	275 cm	**6**	831 cm	**8**	391 m	**10**	87.4 m

11 Find the diameter of a circle whose circumference is 52 m.

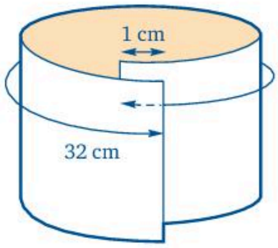
12 A roundabout at a major road junction is to be built. It has to have a minimum circumference of 188 m. What is the corresponding minimum diameter?

13 A bicycle wheel has a circumference of 200 cm. What is the radius of the wheel?

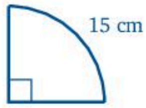
14 A car has a turning circle whose circumference is 63 m. What is the narrowest road that the car can turn round in without going on the sidewalk?

15 When the label is taken off a tin of soup it is found to be 32 cm long.

If there was an overlap of 1 cm when the label was on the tin, what is the radius of the tin?



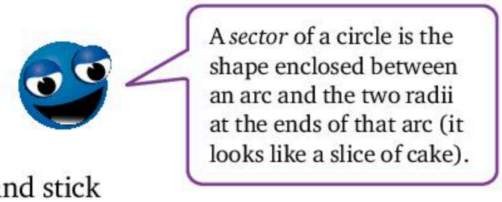
16 The diagram shows a quadrant of a circle. If the curved edge is 15 cm long, what is the length of a straight edge?



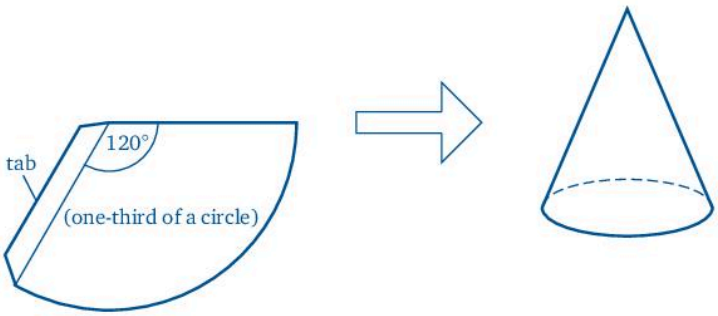
17 A tea cup has a circumference of 24 cm. What is the radius of the cup? Six of these cups are stored edge to edge in a straight line on a shelf. What length of shelf do they occupy?

18 Make a *cone* from a sector of a circle as follows:

On a sheet of paper draw a circle of radius 8 cm. Draw two radii at an angle of 90°. Make a tab on one radius as shown. Cut out the larger sector and stick the straight edges together. What is the circumference of the circle at the bottom of the cone?

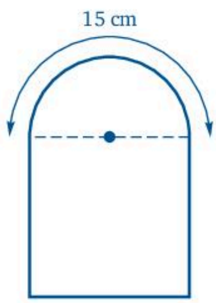


19 A cone is made by sticking together the straight edges of the sector of a circle, as shown in the diagram.



The circumference of the circle at the bottom of the finished cone is 10 cm. What is the radius of the circle from which the sector was cut?

20 The shape in the diagram on the right is made up of a semicircle and a square.
Find the length of a side of this square.



21 The curved edge of a sector which is $\frac{1}{6}$ of a circle is 10 cm.
Find the radius and the perimeter of the sector.

Puzzle

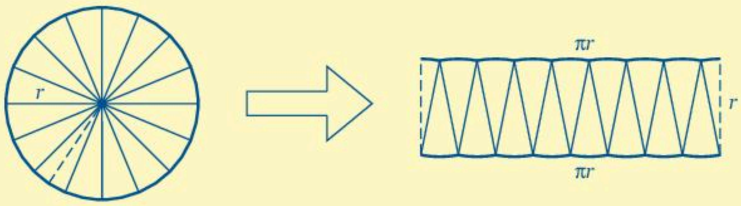
What is the exact time after 1 o'clock when the minute hand of a clock is immediately over the hour hand?

The area of a circle

The formula for finding the *area* of a circle is

$$A = \pi r^2$$

You can see this if you cut a circle up into sectors and place the pieces together as shown to get a shape which is roughly rectangular. Consider a circle of radius r whose circumference is $2\pi r$.



$$\begin{aligned}\text{Area of circle} &= \text{area of 'rectangle'}\\ &= \text{length} \times \text{width}\\ &= \pi r \times r = \pi r^2\end{aligned}$$

Exercise 7f

Use the value of π on your calculator and give your answers correct to 3 s.f.

Find the area of a circle of radius 2.5 cm.

Using $A = \pi r^2$

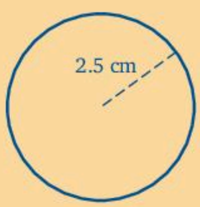
with $r = 2.5$

gives $A = \pi \times (2.5)^2$

$= 19.63\ldots$

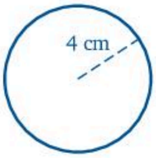
$= 19.6$ to 3 s.f.

Area is $19.6\,\text{cm}^2$ to 3 s.f.



Find the areas of the following circles:

1



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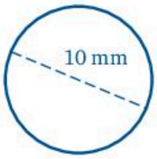

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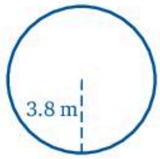

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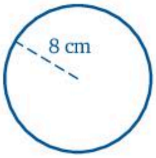

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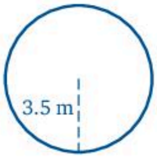

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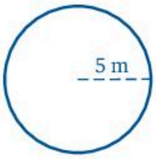

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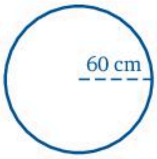

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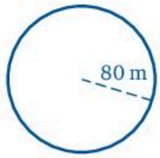
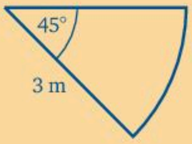




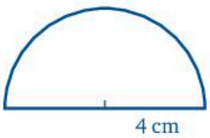
This sector is $\frac{1}{8}$ of a circle.

$\therefore$ area of sector $= \frac{1}{8}$ of area of circle of radius 3 m

$$\text{Area of sector} = \frac{1}{8} \text{ of } \pi r^2$$
$$= \frac{1}{8} \times \pi \times 9\,\text{m}^2$$
$$= 3.534\,\text{m}^2$$
$$= 3.53\,\text{m}^2 \text{ to 3 s.f.}$$

Find the areas of the following shapes:

10



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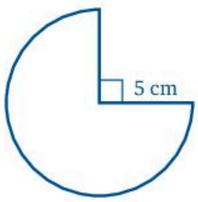

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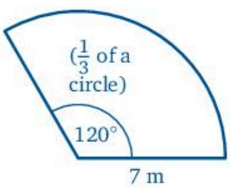

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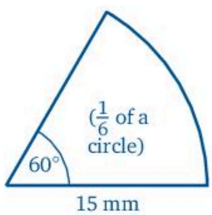

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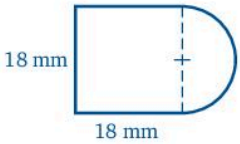

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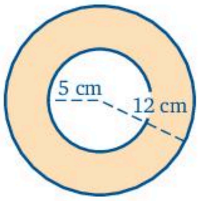

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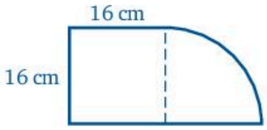

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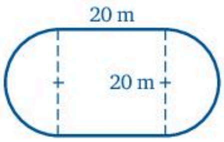

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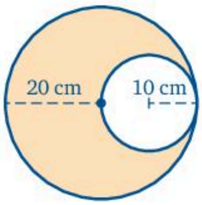

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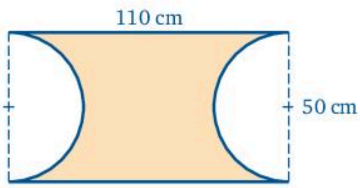

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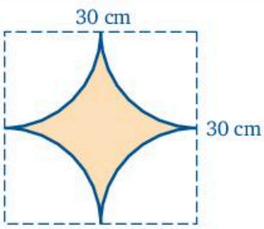

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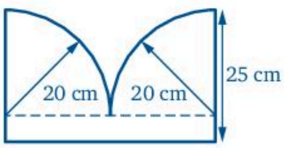

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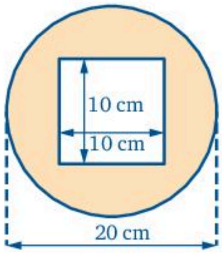

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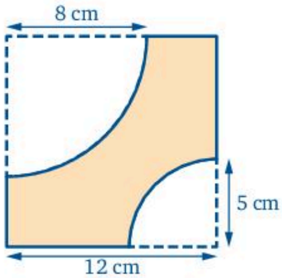

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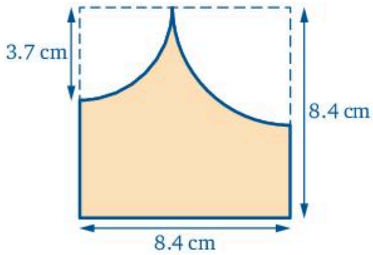

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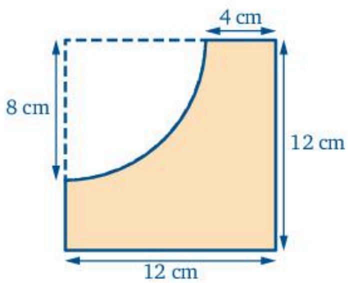

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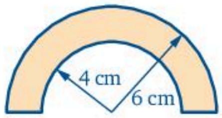

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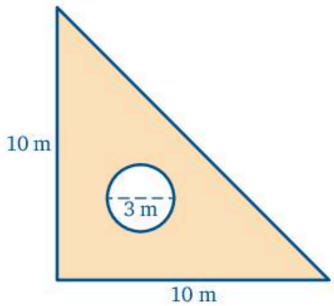


The area of a circle is 50 cm². Find the radius of the circle.

The area of a circle is πr^2.

Therefore $\pi r^2 = 50$

Giving $r^2 = \frac{50}{\pi}$ (dividing both sides by π)

$= 15.915...$ (taking the square root of each side)

therefore $r = 3.99$ correct to 3 s.f.

The radius is 3.99 cm² correct to 3 s.f.

28 A circle has an area of 76 cm². Find the radius of the circle.

29 A circle has an area of 110 cm². Find the diameter of the circle.

30 A circular flower bed has an area of 6 m². Find the diameter of the flower bed. Give your answer in centimetres.

Problems

Exercise 7g

Use the value of π on your calculator and make a rough sketch to illustrate each problem. Give your answers to 3 s.f.

A circular table has a radius of 75 cm. Find the area of the table top.
The top of the table is to be varnished. One tin of varnish covers 4 m².
Will one tin be enough to give the table top three coats of varnish?

The area to varnish is three times the area of the top of the table.

Area of table top is πr^2

$= \pi \times 75 \times 75 \text{ cm}^2$

$= 17671.4...$

$= 17670 \text{ cm}^2$ to 4 s.f.

$= (17670 \div 100^2) \text{ m}^2$

$= 1.767 \text{ m}^2$ to 4 s.f.

(To give an answer correct to 3 s.f. work to 4 s.f.)

For three coats, enough varnish is needed to cover

$$3 \times 1.767 \text{ m}^2 = 5.30 \text{ m}^2 \text{ to 3 s.f.}$$

So one tin of varnish is not enough.

1 The minute hand on a clock is 15 cm long. What area does it pass over in 1 hour?

2 What area does the minute hand described in question 1 cover in 20 minutes?

3 The diameter of a $20 coin is 23 mm. Find the area of one of its flat faces.

4 The hour hand of a clock is 10 cm long. What area does it pass over in 1 hour?

5 A circular lawn has a radius of 5 m. A bottle of lawn weedkiller says that the contents are sufficient to cover 50 m^2. Is one bottle enough to treat the whole lawn?

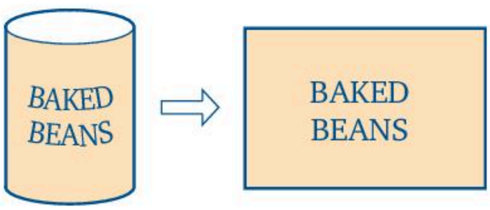
6 The largest possible circle is cut from a square of paper 10 cm by 10 cm. What area of paper is left?

7 Circular place mats of diameter 8 cm are made by stamping as many circles as possible from a rectangular strip of card measuring 8 cm by 64 cm. How many mats can be made from the strip of card and what area of card is wasted?

8 A wooden counter top is a rectangle measuring 280 cm by 45 cm. There are three circular holes in the counter, each of radius 10 cm. Find the area of the wooden top.

9 The surface of the counter top described in question **8** is to be given four coats of varnish. If one tin covers 3.5 m^2, how many tins will be needed?

10 Take a cylindrical tin of food with a paper label:



Measure the diameter of the tin and use it to find the length of the label. Find the area of the label. Now find the total surface area of the tin (two circular ends and the curved surface).

11 You need 5 mm squared paper for this question.

In Grade 7 we counted squares to find estimates for the areas of several different shapes. Let us now apply the method to finding the area of a circle.

a Draw a circle, of radius 3 cm, taking its centre at the intersection of two grid lines

b Count the number of squares within the circle. Include a square if at least half is inside the circle, otherwise do not.

c What is the area of one grid square? Give your answer
i in square millimetres **ii** in square centimetres.

d What area do the total number of squares you have counted cover?

e Use the formula for the area of a circle to calculate the area of a circle of radius 3 cm.
Give your answer correct to 1 decimal place.

f Now compare your answers for parts **d** and **e**. Does counting squares give a good estimate for the area of a circle?

Investigation

In addition to using the formula $A = \pi r^2$ to calculate the area of a circle, there are other methods that provide reasonably good results. One such method is by weighing.

This method requires floor tiles, linoleum, or other material of measurable weight that can be easily cut. Trace the circle whose area is required on the tile and cut out the resulting circular region.

Use the remaining tiles to cut out three 10 cm squares, five 10 cm by 1 cm rectangles, and ten 1 cm squares. You will need more cutouts if the circle is large. These rectangular cutouts will be used as weights.

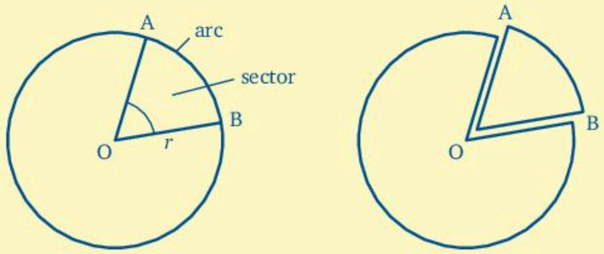
Place the circular cutout in the scale pan and use the rectangular pieces as weights. When the scale balances, remove the rectangular pieces and find their total area. This area will be the area of the circular cutout.

Why does this method of weighing to find area make sense?

The length of an arc

A shape that frequently occurs in everyday life is a slice of a circle. It may be a slice of cake or a piece of wood cut from the trunk of a tree.

Part of a circle is called an *arc*.
The shape formed by two radii and an arc is called a *sector*.



The length of an arc depends on the radius of the circle and the angle enclosed at the centre of the circle by the radii at the two ends of the arc. This angle is $A\hat{O}B$ in the diagram above; it is called the angle *subtended* by the arc AB at the centre, O, of the circle.

The length of the arc AB as a fraction of the circumference of the circle is $\frac{A\hat{O}B}{360°}$,
i.e. length of arc $AB = \frac{A\hat{O}B}{360°}$ of the circumference.

The circumference of a circle is $2\pi r$, so

$$\text{length of arc AB is } \frac{A\hat{O}B}{360°} \times 2\pi r = \pi r \times \frac{A\hat{O}B}{180°}$$

Exercise 7h

An arc subtends an angle of 60° at the centre of a circle of radius 6 cm.
Find the length of the arc:

a in terms of π

b correct to three *significant figures*.

a Arc length $= 6\pi \times \frac{60}{180}$ cm
$= 2\pi$ cm

b Arc length $= 2\pi$ cm
$= 2 \times 3.14...$ cm
$= 6.28$ cm

1 Find, in terms of π, the length of the arc that subtends an angle of 30° at the centre of a circle of radius 4 cm.

2 Find, in terms of π, the length of the arc that subtends an angle of $40°$ at the centre of a circle of radius 4 cm.

3 Find, in terms of π, the length of the arc that subtends an angle of $80°$ at the centre of a circle of radius 9 cm.

4 Find, in terms of π, the length of the arc that subtends an angle of $120°$ at the centre of a circle of radius 12 cm.

In questions **5** to **8** use the value of π on your calculator and give your answers correct to three significant figures.

5 Find the length of the arc that subtends an angle of $40°$ at the centre of a circle of radius 3.65 cm.

6 Find the length of the arc that subtends an angle of $65°$ at the centre of a circle of radius 7.43 cm.

7 Find the length of the arc that subtends an angle of $135°$ at the centre of a circle of radius 18.6 cm.

8 Find the length of the arc that subtends an angle of $100°$ at the centre of a circle of radius 48.5 cm.

Find the angle subtended at the centre of a circle of radius 8 cm by an arc of length 12 cm.

$$\text{Arc length} = \pi \times \text{radius} \times \frac{\text{angle subtended at centre of circle}}{180°}$$

$$\text{Therefore } 12 = \pi \times 8 \times \frac{\text{angle subtended at centre of circle}}{180°}$$

$$\text{i.e. (angle subtended at centre of circle)} \times \pi \times 8 = 12 \times 180°$$

$$\therefore \text{ angle subtended at centre of circle} = \frac{12 \times 180°}{8 \times \pi} = 85.9\ldots°$$

$$= 86° \text{ correct to the nearest degree.}$$

Give angles correct to the nearest degree and lengths correct to three significant figures.

9 Find the angle subtended at the centre of a circle of radius 5 cm by an arc of length 12 cm.

10 What is the size of the angle subtended at the centre of a circle of radius 65 mm by an arc of length 45 mm?

11 Find the radius of a circle in which an arc of length 15 cm subtends an angle of $66°$ at the centre.

12 The arc of a circle of length 34.5 cm subtends an angle of $110°$ at the

centre of the circle. Find the radius of the circle.

13 Find the radius of a circle in which an arc of length 24 cm subtends an angle of 116° at the centre.

14 The arc of a circle of length 18.2 cm subtends an angle of 77° at the centre of the circle. Find the radius of the circle.

15 Find the radius of a circle in which an arc of length 44 cm subtends an angle of 96° at the centre.

16 A circle has a radius of 16.5 cm. An arc of this circle subtends an angle of 19° at its centre. Calculate the length of the arc.

17 An arc of length 15 cm subtends an angle of 45° at the centre of the circle. Find the radius of the circle.

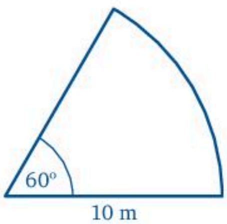
18 Calculate the angle subtended at the centre of a circle of radius 2.7 cm by an arc of length 6.9 cm.

19 Find the angle subtended at the centre of a circle of radius *a* cm by an arc of length 2*a* cm.

20 Find the angle subtended at the centre of a circle of radius *b* cm by an arc of length 1.5*b* cm.

In questions **21** to **26** find the perimeter of each shape.

21



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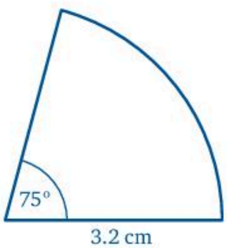

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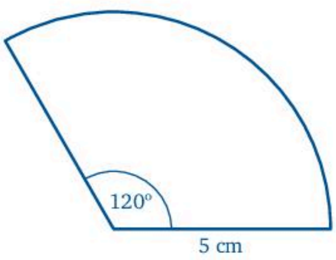

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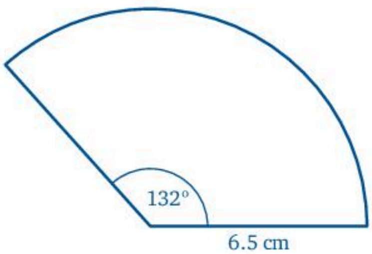

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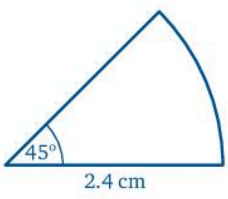

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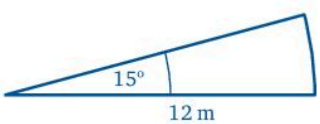
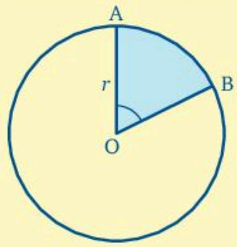


27 A company logo is made by removing a sector containing an angle of 67° from a circle of radius 25 mm. Find the perimeter of the logo.

28 A curve in the track of a railway line is a circular arc of length 400 m and radius 1200 m. Through what angle does the direction of the track turn?

The area of a sector of a circle

You have seen that the slice enclosed by the arc and the radii is called a sector.

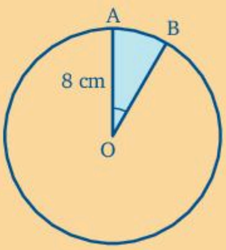


The area of the sector AOB as a fraction of the area of the circle is the same fraction as the arc is, i.e. $\frac{A\hat{O}B}{360°}$.

Therefore area of sector AOB $= \pi r^2 \times \frac{A\hat{O}B}{360°}$

Exercise 7i

Find the area of the shaded sector if angle AOB = 30°.



Area of sector $= \frac{30}{360} \times$ area of circle

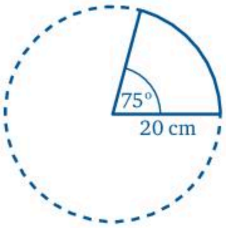
$= \frac{30}{360} \times \pi \times 8^2$ cm^2

$= 16.75\ldots$ cm^2

The area of the sector is 16.8 cm^2 correct to three significant figures.

In questions **1** to **8** find the area of the marked sector. Give each answer correct to three significant figures.

1



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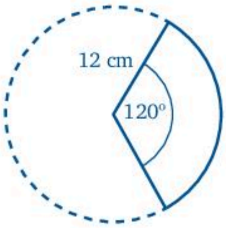

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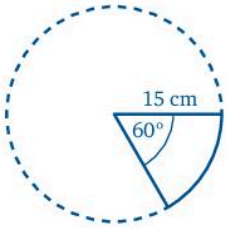

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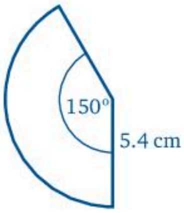

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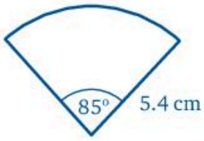

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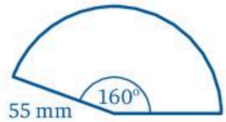

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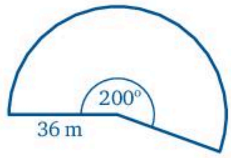

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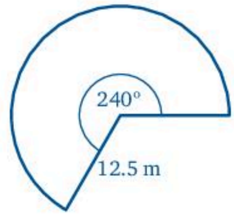


9 A sector of a circle of radius 4 cm contains an angle of 30°. Find the area of the sector.

10 A sector of a circle of radius 8 cm contains an angle of 135°. Find the area of the sector.

11 The area of a sector of a circle of radius 2 cm is π cm^2. Find the angle contained by the sector.

12 The area of a sector of a circle of radius 5 cm is 12 cm^2. Find the angle contained by the sector.

13 A sector of a circle of radius 10 cm contains an angle of 150°. Find the area of the sector.

14 A sector of a circle has an area of 3π cm^2 and contains an angle of 30°. Find the radius of the circle.

15 A sector of a circle has an area of 6π cm^2 and contains an angle of 45°. Find the radius of the circle.

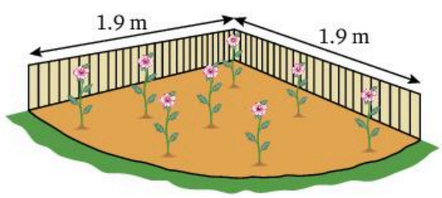
16 An arc of a circle of radius 12 cm is of length π cm. Find:

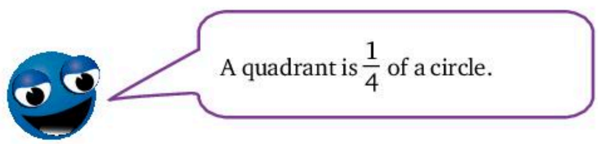
a the angle subtended by the arc at the centre of the circle

b the area of the sector in terms of π.

17 Calculate the angle at the centre of a circle of radius 83 mm contained in a sector of area 974 mm^2.

18



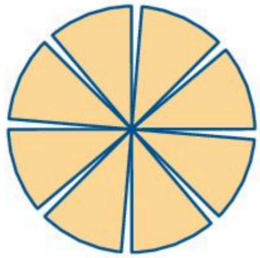


A flower bed is a quadrant of a circle.

a Find the length of the edging needed for the curved edge of the bed.

b Find the area of the flower bed.

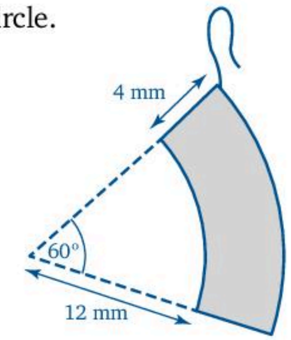
19 The diagram shows the plan of a herb garden. The beds are sectors of the circle, each one of which contains an angle of 40° at the centre. The radius of the circle is 0.9 m.



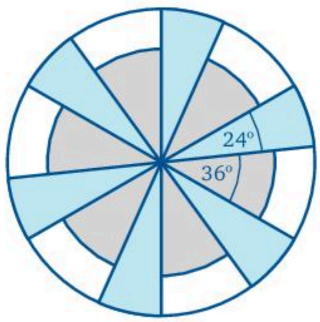
a What length of edging is required to surround all these beds?

b One handful of fertiliser covers one quarter of a square metre of soil.
How many handfuls are required to cover all the beds?

20 A silver earring pendant is part of a sector of a circle. Find the area of silver.



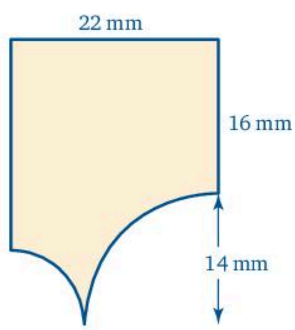
21 In this pattern, the blue sections are identical sectors of a circle of radius 9 cm and the grey sections are identical sectors of a circle of radius 7 cm. Which of the grey or blue sections of the pattern covers the greater area? Justify your answer.



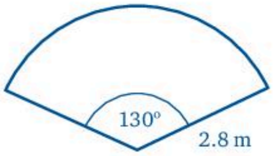
22 Harry is making a marquetry pattern. The diagram shows one of the pieces he has cut. The arcs are quadrants of circles.
Find:

a the perimeter of the piece

b its area.



23 Mr Gardener has designed a lawn in the shape of a sector of a circle with radius 2.8 m, as shown in the diagram. He plans to edge the whole lawn with a flexible edging which is sold by the metre at $325 a metre.



a Find the distance all round the edge of the lawn.

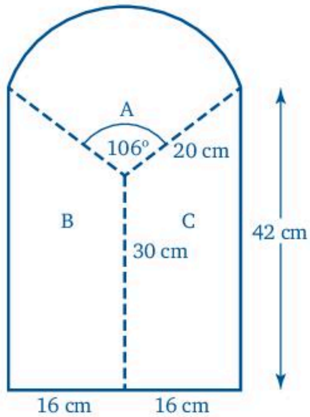
b What will the lawn edging cost?

c Find the area of the lawn.

d If grass seed is sown at 56 grams per square metre, find how much seed will be needed for the lawn.

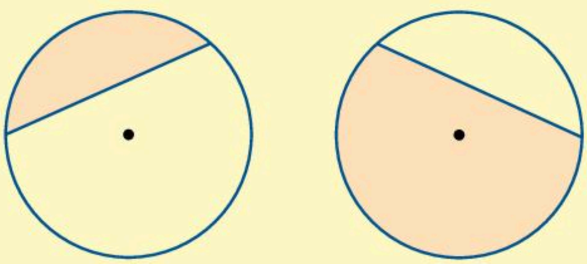
24 The diagram shows the cross-section of a low boundary wall. The cross-section can be divided into a sector of a circle of radius 20 cm and two equal trapeziums.

Find each of the areas marked A, B and C, and the total area.



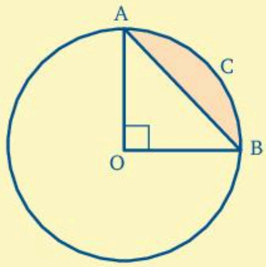
Segment of a circle

A *segment* of a circle is the shape enclosed between a *chord* and one of the arcs joining the ends of that chord. (A chord is a straight line segment joining two points on the circle.)



The diagrams show a minor segment, which is less than half the circle and a major segment, which is more than half the circle.

The area of the segment ACB is found by finding the area of the sector OACB and subtracting the area of triangle AOB from it.



For example, AB is a chord in a circle of radius 5 cm and angle AOB = 90°.

Area of sector OACB $= \frac{1}{4}\pi r^2 = \frac{1}{4}\pi \times 5^2\,\text{cm}^2$

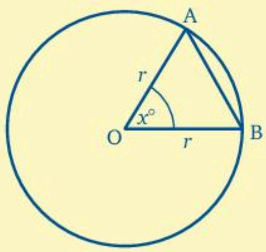
$= 19.6349\ldots\,\text{cm}^2$

$= 19.6\,\text{cm}^2$ (to 3 s.f.)

Area of triangle OAB $= \frac{1}{2}(5 \times 5) = 12.5\,\text{cm}^2$

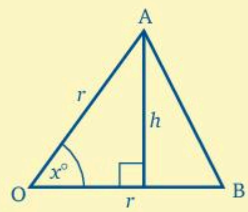
$\therefore$ area of segment $= (19.6 - 12.5)\,\text{cm}^2 = 7.1\,\text{cm}^2$

Not all segments contain an angle as easy to deal with as 90°. The areas of most segments will involve subtracting a triangle such as that in the diagram below.



The base and perpendicular height of triangle OAB are not known. So we need an alternative formula for the area of a triangle when the angle is not 90°.

Consider just triangle OAB.



The length of the perpendicular from A to OB is given by $r\sin x$.

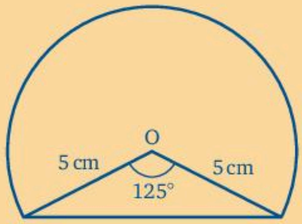
This is the perpendicular height of triangle OAB and the base is r.

Therefore the **area of triangle OAB $= \frac{1}{2}r^2 \sin x$**

Note that this formula only applies to an isosceles triangle where the lengths of the equal sides and the size of the angle between them is known.

Exercise 7j

The diagram shows a washer, which is a circular disc from which a segment has been removed.



Find the area of the washer.

The area of the complete disc is $\pi r^2 = \pi \times 25 = 78.539...$

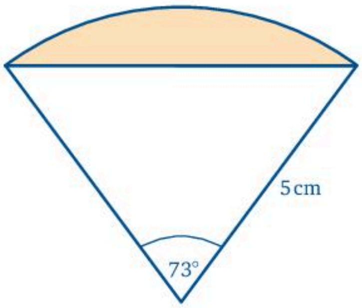
The area of the segment removed is given by

$$A = \frac{\pi r^2 \theta}{360} - \frac{1}{2}r^2 \sin\theta = \frac{25 \times 125 \times \pi}{360} - \frac{25}{2}\sin 125°$$
$$= 17.031$$

Therefore the area of the washer is $78.539 - 17.031 = 61.5\,\text{cm}^2$.

Find the area of each segment.

1



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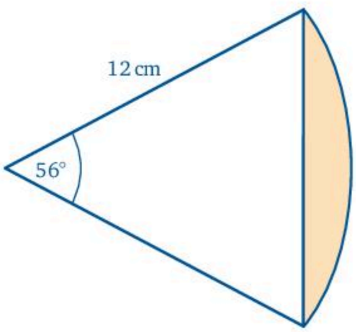

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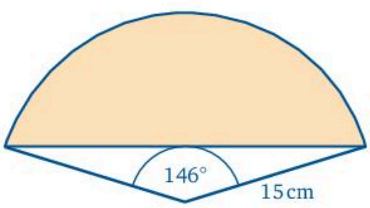

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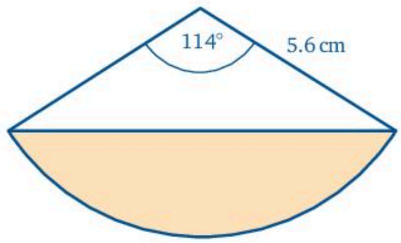

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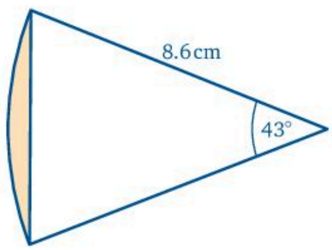
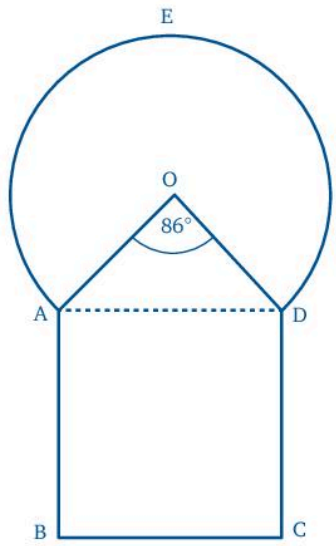


6 ABCD is a rectangle and AED a sector of a circle, centre O, radius 4 cm with $\hat{AOD} = 86°$.

Find

a the length of AD
b the area of triangle AOD
c the area of the major segment AED
d the area of the complete shape, given that AB = 7 cm.



Did you know?

Over the centuries mathematicians have spent a lot of time trying to find the true value of π. The ancient Chinese used 3. Three is also the value given in the Old Testament (1 Kings 7:23). The Egyptians (*c*. 1600 BCE) used $4 \times \left(\frac{8}{9}\right)^2$. Archimedes (*c*. 225 BCE) was the first person to use a sound method for finding its value and a mathematician called Van Ceulen (1540–1610) spent most of his life finding it to 35 decimal places!

Mixed exercises

Use the value of π on your calculator. Give your answers to 3 s.f.

Exercise 7k

1 Find the circumference of a circle of radius 2.8 mm.

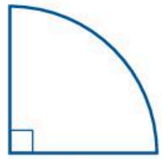
2 Find the radius of a circle of circumference 60 m.

3 Find the circumference of a circle of diameter 12 cm.

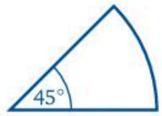
4 Find the area of a circle of radius 2.9 m.

5 Find the area of a circle of diameter 25 cm.

6 Find the perimeter of the quadrant in the diagram.



7 Find the area of the sector in the diagram, which is $\frac{1}{8}$ of a circle.



8 Find the radius of a circle whose area is $85\,\text{cm}^2$.

Exercise 7l

1 Find the circumference of a circle of diameter 20 m.

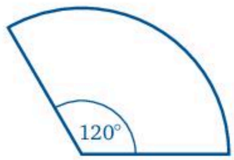
2 Find the area of a circle of radius 12 cm.

3 Find the radius of a circle of circumference 360 cm.

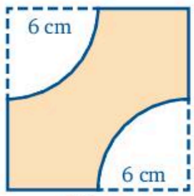
4 Find the area of a circle of diameter 8 m.

5 Find the diameter of a circle of circumference 280 mm.

6 Find the perimeter of the sector, which is $\frac{1}{3}$ of a circle.



7 Find the area of the shaded part of the diagram.



8 Find the diameter of a circle whose area is $25\,\text{cm}^2$.

Exercise 7m

1 Find the area of a circle of radius 2 km.

2 Find the circumference of a circle of radius 49 mm.

3 Find the radius of a circle of circumference 88 m.

4 Find the area of a circle of diameter 14 cm.

5 Find the area of a circle of radius 3.2 cm.

6

An ornamental pond in a garden is a rectangle with a semicircle on each short end. The rectangle measures 5 m by 3 m and the radius of each semicircle is 1 m. Find the area of the pond.

Exercise 7n

Select the letter that gives the correct answer.

1 The circumference of a circle of radius 8 cm is

A 25.1 cm **B** 50.2 cm **C** 50.3 cm **D** 50.4 cm

2 The radius of a circle of circumference 25 cm is

A 3.97 cm **B** 3.98 cm **C** 3.99 cm **D** 7.96 cm

3 The circumference of a circle of diameter 12 cm is

A 37.6 cm **B** 37.7 cm **C** 37.8 cm **D** 37.9 cm

4 The area of a circle of radius 15 cm is

A 707 cm² **B** 717 cm² **C** 827 cm² **D** 2830 cm²

5 The area of a circle of diameter 12 m is

A 113 m² **B** 114 m² **C** 453 m² **D** 454 m²

6 When the label is taken off a cylindrical tin it is 35 cm long. There was an overlap of 1 cm when the label was on the tin. The radius of the circular cross-section of the tin, correct to 3 s.f., is

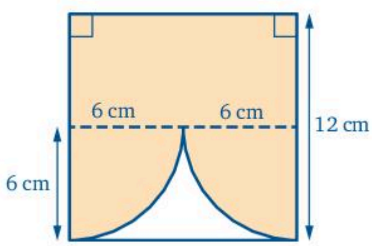
A 5.39 cm **B** 5.40 cm **C** 5.41 cm **D** 5.44 cm

7 The perimeter of this quadrant of a circle, correct to 3 s.f., is

A 20.6 cm **B** 25 cm **C** 28.5 cm **D** 28.6 cm

8 The shaded area of the shape shown below, correct to 3 s.f., is

A 100 cm² **B** 129 cm² **C** 135 cm² **D** 164 cm²



9 The circumference of a circle is 46 cm. Its area, correct to 3 s.f., is

A 168 cm² **B** 172 cm² **C** 176 cm² **D** 184 cm²

10 The area of a circle is 200 cm². The length of its circumference, correct to 2 s.f., is

A 50 cm **B** 51 cm **C** 58 cm **D** 60 cm

Exercise 7p

Select the letter that gives the correct answer.

1 The length of the arc, in terms of π, that subtends an angle of $50°$ at the centre of a circle of radius 6 cm is

A $\frac{5\pi}{6}$ cm **B** $\frac{7\pi}{6}$ cm **C** $\frac{4\pi}{3}$ cm **D** $\frac{5\pi}{3}$ cm

2 The size of the angle, correct to the nearest degree, subtended at the centre of a circle of radius 50 cm by an arc of length 80 cm is

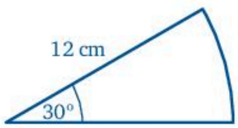
A $88°$ **B** $90°$ **C** $92°$ **D** $95°$

3 An arc of length 20 cm subtends an angle of $45°$ at the centre of the circle.

The radius of this circle, correct to the nearest whole number, is

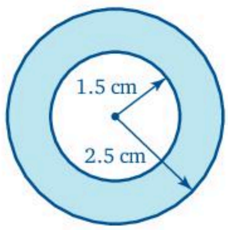
A 20 cm **B** 22 cm **C** 24 cm **D** 25 cm

4 The area of this sector of radius 12 cm, correct to 3 s.f., is



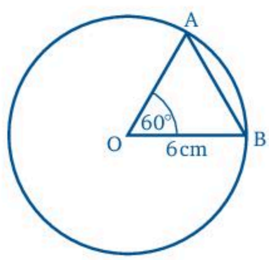
A 37 cm^2
B 37.6 cm^2
C 37.7 cm^2
D 38 cm^2

5 The area between these two circles is



A $2.25\pi\text{ cm}^2$
B $4\pi\text{ cm}^2$
C $5\pi\text{ cm}^2$
D $6.25\pi\text{ cm}^2$

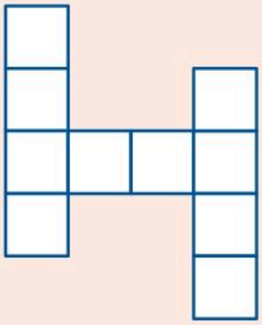
6 The area of the segment bounded by the chord AB is



A 1 cm^2 **C** 2 cm^2
B 1.63 cm^2 **D** 3.26 cm^2

Puzzle

Place the numbers from 1 to 10, one in each square, so that the total in each of the three lines of four squares is the same.

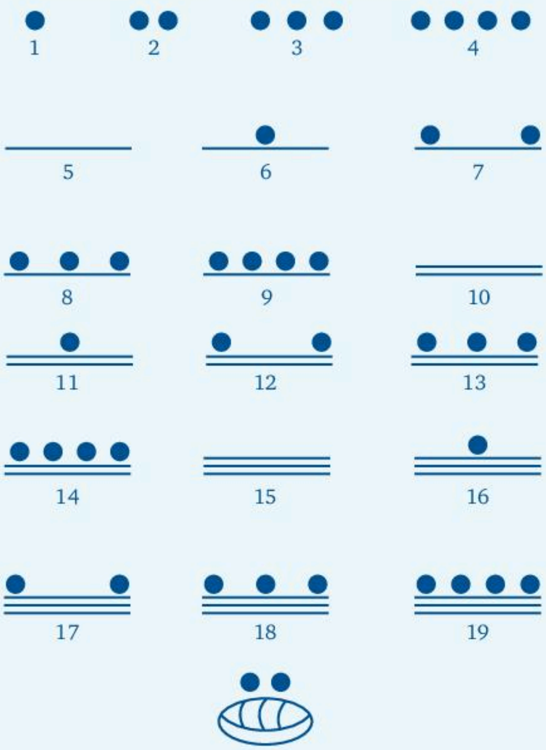


Did you know?

The number system used by the Mayans had only three symbols:

- a large dot ● represented one
- a bar ——— represented five
- a seashell  represented zero.

Numbers 1 to 20 were written as:



In this chapter you have seen that...

- ✔ for any circle the circumference divided by the diameter gives a fixed value; this value is denoted by π and its approximate value is 3.142
- ✔ you can find the circumference of a circle using either the formula $C = 2\pi r$ or the formula $C = \pi d$ when you know the radius or diameter of the circle
- ✔ you can use the formula $A = \pi r^2$ to find the area of a circle
- ✔ a sector of a circle is shaped like a slice of cake
- ✔ for a circle, centre O, radius r, the length of the arc AB is $\pi r \times \frac{A\hat{O}B}{180^\circ}$
- ✔ for a circle, centre O, radius r, the area of the sector AOB, formed by the arc AB and the radii OA and OB, is $\pi r^2 \times \frac{A\hat{O}B}{360^\circ}$
- ✔ the area of a segment of a circle is found by subtracting the area of the relevant triangle from the area of the sector.

REVIEW TEST 1: CHAPTERS 1–7

In questions **1** to **12**, choose the letter for the correct answer.

1 $2(a^3b^2)^3$ simplifies to

A $2a^6b^5$ B $2a^6b^6$ C $2a^9b^6$ D $2a^9b^5$

2 The value of $\left(\frac{2}{3}\right)^{-2}$ is

A $\frac{4}{9}$ B $\frac{2}{3}$ C $\frac{3}{2}$ D $\frac{9}{4}$

3 The value of $(4^2 - 2^3)^2$ is

A 16 B 32 C 48 D 64

4 The circumference of a circle of radius 12 cm is, correct to 3 s.f.,

A 37.7 cm B 75.3 cm C 75.4 cm D 75.5 cm

5 The area of a circle of radius 9 cm is, correct to 3 s.f.,

A 187 cm^2 B 254 cm^2 C 255 cm^2 D 1020 cm^2

6 The tangent of the angle 67°, correct to 4 s.f., is

A 1.355 B 1.356 C 2.355 D 2.356

7 The angle whose sine is 0.5592 is

A 30° B 31° C 32° D 34°

8 The angle whose cosine is 0.5592 is

A 55° B 56° C 57° D 58°

9 The angle of elevation of the top of a church tower from a point 500 m away on level ground is 14°. The height of the top of the spire, correct to the nearest metre, is

A 120 m B 124 m C 125 m D 126 m

10 $\frac{xy}{4} \div \frac{y}{2x^2} =$

A $\frac{y^2}{8x}$ B $\frac{8x}{y^2}$ C $\frac{x^3}{4}$ D $\frac{x^3}{2}$

11 The LCM of p, pq, and p^2 is

A pq B p^2q C pq^2 D p^3q

12 $\frac{3(2x-1)}{2}-\frac{4(x-5)}{5}$ simplifies to

A $\frac{38x+55}{10}$ **B** $\frac{4x-11}{2}$ **C** $\frac{22x+25}{10}$ **D** $\frac{22x+55}{10}$

13 Simplify: **a** 5^{-2} **b** $8^{\frac{2}{3}}$ **c** $\left(\frac{1}{9}\right)^{-\frac{1}{2}}$

14 Write **a** 0.000 003 in standard form
b 5.92×10^3 as an ordinary number
c 8.21×10^{-3} as an ordinary number.

15 Simplify: **a** $(2a-3b)^2$ **b** $(3x-2y)(3x+2y)$ **c** $(3x+4)(x-5)-(2x+1)(x-4)$

16 Factorise: **a** a^2-2ab **b** $4ab-8bc$ **c** $2x^2-2x-6$

17 Expand: **a** $(x+8)(x-1)$ **c** $(x+2)(x-4)+(2x+3)(x+2)$
b $(4x+5y)(4x-5y)$ **d** $(ab-4)^2$

18 Solve the equations:

a $\frac{4}{3x}+\frac{5}{12}=\frac{3}{x}$ **b** $\frac{x+3}{4}-\frac{x}{3}=\frac{1}{2}$

19 Simplify: **a** $\frac{x+3}{4}+\frac{2x-3}{5}$ **b** $\frac{3(x-2)}{4}+\frac{2(2x-5)}{3}$

20 Solve the following pairs of equations:

a $2x+y=8$
$5x-2y=11$

b $3x-5y=17$
$5x+2y=18$

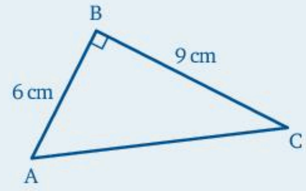
21 Four times a number added to second number is 33.
The first number added to twice the second number is 17.
Find the numbers.

22 Find, giving you answer correct to 3 s.f., the square of
a 5.21 **b** 0.817 **c** 0.042

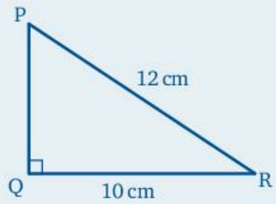
23 Find, giving you answer correct to 3 s.f., the square root of
a 12.64 **b** 0.0592 **c** 0.000 761

24 Does a triangle with sides of lengths 1.5 cm, 2 cm and 2.5 cm contain a right angle?
If you answer is 'yes' justify it and state the lengths of the two sides that form the arms of the right angle.

25 **a** Find, correct to 3 s.f., the length of AC.



b Find, correct to 3 s.f., the length of PQ.



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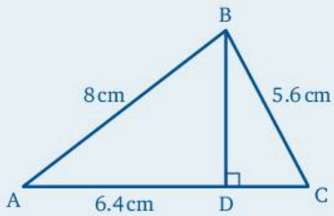
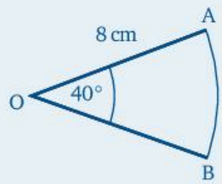
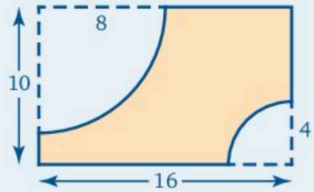


a Calculate the length of **i** BD **ii** DC.
b Hence determine whether or not triangle ABC is right-angled.
If it is, which angle is the right angle?
If it is not, justify your answer.

27 For this sector of a circle, find
a the length of the arc AB
b the area of the sector.

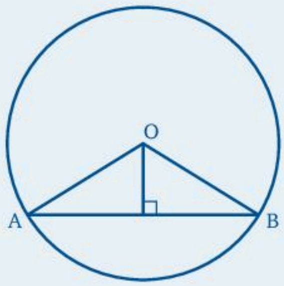


28 The diagram shows a rectangular piece of veneer from which quadrants of circles have been removed. Measurements are given in centimetres.



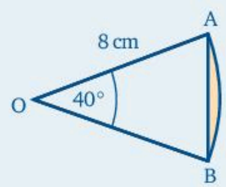
Find **a** the perimeter of the piece
b its area.

29 AB is a chord of length 10 cm which is 4 cm from the centre of the circle O.
Find the radius of the circle.



30 From the top of a vertical cliff, which is 56 m high, the angle of depression of a small boat directly out to sea is 15.5°.
How far is the boat from the foot of the cliff?

31 Find the area of the shaded segment.



8 Probability

At the end of this chapter you should be able to...

1 write the set of outcomes of an experiment

2 state the number of possible outcomes of an experiment

3 calculate the probability of an event A happening as

$$\frac{\text{the number of ways in which A can occur}}{\text{the total number of equally likely outcomes}}$$

4 calculate the probability that event A does not happen as

$1 - P(\text{A does happen})$

5 estimate the number of times an event might occur

6 perform experiments, collect data and hence calculate required probabilities.

Did you know?

Do you know how to play Tic-Tac-Toe in such a way that you never lose? If you do, you are using a topic in mathematics called Game Theory. We told you that maths was fun!

X	X	O
O	O	X
X	O	X

A game is won either by chance or by using a strategy.

Probability and statistics as mathematical sciences arose because game players wanted to use game odds to best advantage.

After studying mathematics carefully, mathematicians defined numbers called *probabilities* which helped them to explain how likely some 'outcome' or 'event' was to happen or fail to happen.

You need to know...

✔ how to simplify a fraction

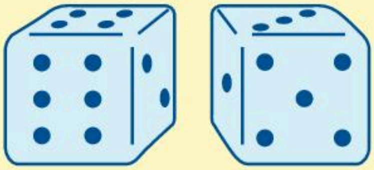
✔ how to add and subtract fractions

✔ the cards in an ordinary pack of playing cards.

Key words

approximation, biased, certain, chance, equally likely, estimate, event, experiment, fair, impossible, integer, odds, outcome, prime numbers, probability, random, unbiased

Outcomes of experiments



If you throw an ordinary die there are six possible scores that you can get. These are 1, 2, 3, 4, 5 or 6.
The act of throwing the die is called an *experiment*.
The score that you get is called an *outcome* or an *event*.
The set {1, 2, 3, 4, 5, 6} is called the set of all possible outcomes.

Exercise 8a

How many possible outcomes are there for the following experiments? Write down the set of all possible outcomes in each case.

1 Tossing a \$1 coin. (Assume that it lands flat.)

2 Taking one disc from a bag containing 1 red, 1 blue and 1 yellow disc.

3 Choosing one number from the first ten positive integers. (An *integer* is a whole number.)

4 Taking one crayon from a box containing 1 red, 1 yellow, 1 blue, 1 brown, 1 black and 1 green crayon.

5 Taking one item from a bag containing 1 packet of chewing gum, 1 packet of boiled sweets and 1 bar of chocolate.

6 Taking one coin from a bag containing one \$1 coin, one \$5 coin, one \$10 coin and one \$20 coin.

7 Choosing one card from part of a pack of ordinary playing cards containing just the suit of clubs.

8 Choosing one letter from the vowels of the alphabet.

9 Choosing one number from the first 5 *prime numbers*.

10 Choosing an even number from the first 20 whole numbers.

Probability

If you throw an ordinary die, what are the chances of getting a four? If you throw it fairly, it is reasonable to assume that you are as likely to throw any one score as any other, i.e. all outcomes are *equally likely*. As throwing a four is only 1 of the 6 equally likely outcomes you have a 1 in 6 *chance* of throwing a four.

Odds is another word in everyday language that is used to describe chances.

In mathematical language we use the word *probability* to describe chances.
We say that the probability of throwing a four is $\frac{1}{6}$.

This can be written more briefly as

$$P(\text{throwing a four}) = \frac{1}{6}$$

Probabilities can be given in various ways, either as a fraction or as a percentage, or even as a 1 in n chance as above.

We will now define exactly what we mean by 'the probability that something happens'.

If A stands for a particular event, the probability of A happening is written $P(\text{A})$ where

$$P(\text{A}) = \frac{\text{the number of ways in which A can occur}}{\text{the } \textit{total} \text{ number of equally likely outcomes}}$$

We can use this definition to work out, for example, the probability that if one card is drawn at *random* from a full pack of ordinary playing cards, it is the ace of spades.

(The phrase 'at random' means that any one card is as likely to be picked as any other.)

There are 52 cards in a full pack, so there are 52 equally likely outcomes.

There is only one ace of spades, so there is only one way of drawing that card,

i.e. $$P(\text{ace of spades}) = \frac{1}{52}$$

Exercise 8b

In the following questions, assume that all possible outcomes are equally likely.

1 One letter is chosen at random from the letters in the word SALE. What is the probability that it is A?

2 What is the probability that a red pencil is chosen from a box containing 10 different coloured pencils, one of which is red?

3 What is the probability of choosing a prime number from the numbers 6, 7, 8, 9, 10?

4 What is the probability of choosing the most expensive car from a range of six different new cars in a showroom?

5 What is the probability of choosing an integer that is exactly divisible by 5 from the set {6, 7, 8, 9, 10, 11, 12}?

6 In a raffle 200 tickets are sold. If you have bought one ticket, what is the probability that you will win first prize? Give your answer as a percentage.

7 One card is chosen at random from a pack of 52 ordinary playing cards. What is the probability that it is the ace of hearts?

8 What is the probability of choosing the colour blue from the colours of the rainbow?

9 A whole number is chosen from the first 15 whole numbers. What is the probability that it is exactly divisible both by 3 and by 4?

Puzzle

Charlie keeps his socks in a drawer. They are all either brown or grey. One night, in the dark, due to an electricity cut, he pulls out some socks to put on. What is the smallest number of socks he must pull out to be certain of having a pair of the same colour?

Experiments where an event can happen more than once

If a card is picked at random from an ordinary pack of 52 playing cards, what is the probability that it is a five?

There are 4 fives in the pack: the five of spades, the five of hearts, the five of diamonds and the five of clubs.

That is, there are 4 ways in which a five can be chosen.

Altogether there are 52 cards that are equally likely to be chosen,

therefore $P(\text{choosing a five}) = \frac{4}{52} = \frac{1}{13}$

Now consider a bag containing 3 white discs and 2 black discs.

If one disc is taken from the bag it can be black or white. But these are not equally likely events: there are three ways of choosing a white disc and two ways of choosing a black disc, so

$P(\text{choosing a white disc}) = \frac{3}{5}$

and $P(\text{choosing a black disc}) = \frac{2}{5}$

Exercise 8c

A letter is chosen at random from the letters of the word DIFFICULT. How many ways are there of choosing the letter I? What is the probability that the letter I will be chosen?

There are 2 ways of choosing the letter I and there are 9 letters in DIFFICULT.

$$P(\text{choosing I}) = \frac{2}{9}$$

1 How many ways are there of choosing an even number from the first 10 whole numbers?

2 A prime number is picked at random from the set {4, 5, 6, 7, 8, 9, 10, 11}. How many ways are there of doing this?

3 A card is taken at random from an ordinary pack of 52 playing cards. How many ways are there of taking a black card?

4 An ordinary six-sided die is thrown. How many ways are there of getting a score that is greater than 4?

5 A lucky dip contains 50 boxes, only 10 of which contain a prize, the rest being empty. How many ways are there of choosing a box that contains a prize?

6 A number is chosen at random from the first 10 integers. What is the probability that it is

a an even number
b an odd number
c a prime number
d exactly divisible by 3?

7 One card is drawn at random from an ordinary pack of 52 playing cards. What is the probability that it is

a an ace
b a red card
c a heart
d a picture card (include the aces)?

8 One letter is chosen at random from the word DIFFICULT. What is the probability that it is

a the letter F
b the letter D
c a vowel
d one of the first five letters of the alphabet?

9 An ordinary unbiased six-sided die is thrown. What is the probability that the score is

Unbiased means that any possible score is equally likely.

a greater than 3
b at least 5
c less than 3?

10 A book of 150 pages has a picture on each of 20 pages. If one page is chosen at random, what is the probability that it has a picture on it?

11 One counter is picked at random from a bag containing 15 red counters, 5 white counters and 5 yellow counters. What is the probability that the counter removed is

a red **b** yellow **c** not red?

12 If you bought 10 raffle tickets and a total of 400 were sold, what is the probability that you win first prize?

13 A number wheel is spun. What is the probability that when it stops it will be pointing to

a an even number

b an odd number

c a number less than 10 excluding zero?

(The numbers on the number wheel go from 0 to 36, and zero is counted as neither an even number nor an odd number.)

14 One letter is chosen at random from the letters of the alphabet. What is the probability that it is a consonant?

15 A number is chosen at random from the set of two-digit numbers (i.e. the numbers from 10 to 99). What is the probability that it is exactly divisible both by 3 and by 4?

16 A bag of sweets contains 4 caramels, 3 fruit centres and 5 mints. If one sweet is taken out, what is the probability that it is

a a mint **b** a caramel **c** not a fruit centre?

Certainty and impossibility

Consider a bag that contains 5 red discs only. If one disc is removed it is absolutely *certain* that it will be red. It is *impossible* to take a blue disc from that bag.

$$P(\text{disc is red}) = \frac{5}{5} = 1$$

$$P(\text{disc is blue}) = \frac{0}{5} = 0$$

In all cases

$$P(\text{an event that is certain}) = 1$$

$$P(\text{an event that is impossible}) = 0$$

Most events fall somewhere between the two, so

$$0 \leqslant P(\text{that an event happens}) \leqslant 1$$

Exercise 8d

Discuss the probability that the following events will happen. Try to class them as certain, impossible or somewhere in between.

1. You will swim the Atlantic Ocean.
2. You will weigh 80 kg.
3. You will be late home from school at least once this term.
4. You will grow to a height of 2 m.
5. The sun will not rise tomorrow.
6. You will run a mile in $3\frac{1}{2}$ minutes.
7. You will have a drink sometime today.
8. Guyana will win next year's Digicel Four Day Championship.
9. A card chosen from an ordinary pack of playing cards is either red or black.
10. A coin that is tossed lands on its edge.
11. Give some examples of events that are likely or unlikely to happen. For example: you will own a car; your home will burn down.

Probability that an event does not happen

If one card is drawn at random from an ordinary pack of playing cards, the probability that it is a club is given by

$$P(\text{a club}) = \frac{13}{52} = \frac{1}{4}$$

Now there are 39 cards that are not clubs so the probability that the card is not a club is given by

$$P(\text{not a club}) = \frac{39}{52} = \frac{3}{4}$$

i.e. $$P(\text{not a club}) + P(\text{a club}) = \frac{3}{4} + \frac{1}{4} = 1$$

Hence $$P(\text{not a club}) = 1 - P(\text{a club})$$

This relationship is true in any situation because

$$\begin{pmatrix}\text{the number of ways}\\ \text{in which an event, A,}\\ \text{can } \textit{not} \text{ happen}\end{pmatrix} = \begin{pmatrix}\text{the total number of}\\ \text{possible outcomes}\end{pmatrix} - \begin{pmatrix}\text{the number of ways}\\ \text{in which A can}\\ \text{happen}\end{pmatrix}$$

i.e. $$P(\text{A does not happen}) = 1 - P(\text{A does happen})$$

'A does not happen' is shortened to $\bar{A}$, where $\bar{A}$ is read as 'not A'.

$\bar{A}$ is sometimes written as A′.

Therefore $$P(\bar{A}) = 1 - P(A)$$

Exercise 8e

A letter is chosen at random from the letters of the word PROBABILITY. What is the probability that it is not B?

Method 1: There are 11 letters and 2 of them are Bs

$$\therefore\ P(\text{letter is B}) = \frac{2}{11}$$

Hence $$P(\text{letter is not B}) = 1 - \frac{2}{11} = \frac{9}{11}$$

Method 2: There are 11 letters and 9 of them are not Bs

$$\therefore\ P(\text{letter is not B}) = \frac{9}{11}$$

1 A number is chosen at random from the first 20 whole numbers. What is the probability that it is not a prime number?

2 A card is drawn at random from an ordinary pack of playing cards. What is the probability that it is not a two?

3 One letter is chosen at random from the letters of the alphabet. What is the probability that it is not a vowel?

4 A box of 60 coloured crayons contains a mixture of colours, 10 of which are red. If one crayon is removed at random, what is the probability that it is not red?

5 A number is chosen at random from the first 10 whole numbers. What is the probability that it is not exactly divisible by 3?

6 One letter is chosen at random from the letters of the word ALPHABET. What is the probability that it is not a vowel?

7 In a raffle, 500 tickets are sold. If you buy 20 tickets, what is the probability that you will not win first prize?

8 If you throw an ordinary fair six-sided die, what is the probability that you will not get a score of 5 or more?

A dice is *fair* when all scores are equally likely.

9 There are 200 packets hidden in a lucky dip. Five packets contain \$5 and the rest contain \$1. What is the probability that you will not draw out a packet containing \$5?

10 When an ordinary pack of playing cards is cut, what is the probability that the card showing is not a picture card? (The picture cards are the jacks, queens and kings.)

11 A letter is chosen at random from the letters of the word SUCCESSION. What is the probability that the letter is

a N
b S
c a vowel
d not S?

12 A card is drawn at random from an ordinary pack of playing cards. What is the probability that it is

a an ace
b a spade
c not a club
d not a seven or an eight?

13 A bag contains a set of snooker balls (i.e. 15 red and 1 each of the following colours: white, yellow, green, brown, blue, pink and black). What is the probability that one ball removed at random is

a red
b not red
c black
d not red or white?

14 There are 60 cars in the parking lot. Of the cars, 22 are British made, 24 are Japanese made and the rest are European but not British. What is the probability that the first car to leave is

a Japanese
b not British
c European but not British
d American?

The number of times an event is likely to happen

Sometimes it is useful to be able to estimate how often an event *might* happen.

For example, Sue is organising a game at a bazaar. The game involves rolling a die. When a six shows, you win a prize.
Sue needs to estimate the number of prizes that will be won.

On one turn, the probability of winning is $\frac{1}{6}$. So there will be about 1 win in every 6 turns.

Sue estimates that there will be about 300 turns. So there will be about $\frac{1}{6}$ of 300 wins.

Now $\frac{1}{6}$ of 300 = 300 ÷ 6 = 50, so Sue will need about 50 prizes.

$$\left(\begin{array}{c}\text{The number of times that an event}\\ \text{is likely to happen}\end{array}\right) = \left(\begin{array}{c}\text{probability that it}\\ \text{will happen once}\end{array}\right) \times \left(\begin{array}{c}\text{the number of times}\\ \text{it is tried}\end{array}\right)$$

When we flip a fair coin, the probability that it will show a head is $\frac{1}{2}$.

If we flip this coin 20 times, we expect to get about $\frac{1}{2} \times 20$ heads, i.e. 10 heads.

What we expect to get is an *estimate*. It is **not** the same as what we will get.

Suppose we flip a coin 20 times and get 15 heads. This is more than the 10 heads we expect. This can happen by chance, or it could be that the coin is *biased*. Biased means that some outcomes are more likely than others.

Exercise 8f

Fair means the same as unbiased, i.e. any outcome is as likely to happen as any other possible outcome.

1 A fair coin is flipped 100 times.

Write the number of heads expected.

2 A fair ordinary die is rolled 60 times.

Find the number of times you expect it to show 1.

3 This spinner is fair. It is spun 100 times.

Work out the number of times you expect it to show 4.

4 This spinner is fair. It is spun 40 times.

a Find the number of times you expect it will show 1.

b Work out the number of times you expect that it will show 3.

5 This fair spinner is spun 50 times.

How often do you expect it will show an even number?

6 A ordinary fair die is rolled 90 times.

a Work out the number of times you expect it to show 6.

b Work out the number of times you expect it to show an odd number.

c Work out the number of times you expect it to show a number less than 3.

7 Jamestown Airport has 500 flights leaving each day.

The probability that a plane is delayed is $\frac{1}{25}$.

How many delayed flights are expected on one day?

8 Derek rolls two ordinary fair six-sided dice 360 times.

The probability that they show a double six is $\frac{1}{36}$.

Work out the number of double sixes Derek is likely to get.

9 The probability of winning a prize on a game of chance is $\frac{1}{50}$.
500 people have a go at this game. Find an estimate for the number of prizes that were won.

10 The probability that a lift will break down each time it is used is $\frac{1}{1000}$.
The lift is used about 5000 times each year. Estimate the number of times that it is likely to break down.

11 Sam goes to work by train. The probability that his train is late is $\frac{1}{8}$.

Sam works 200 days each year. Estimate the number of these days when Sam's train is late.

12 **a** One card is picked at random from an ordinary pack of playing cards.
Write the probability that it is an ace.

b Joe has 40 packs of ordinary playing cards.
One card is picked at random from each pack.
Estimate the number of aces that are picked.

13 **a** Write the probability of getting a head on one flip of a fair coin.

b Work out the number of heads you expect to get if you flipped the coin 60 times.

c Angela said, 'If you flip a fair coin 10 times, you will get 5 heads.'
Is Angela correct? Give a reason for your answer.

d Sam tossed one coin 100 times. He got 10 heads.
Sam said, 'This coin is biased.'
Is Sam correct? Give a reason for your answer.

14 Harry plays a game of chance. The probability of winning is $\frac{1}{10}$.

Each go at the game costs $1 and the prize is $5.

Harry plays this game 50 times.

a How much does Harry spend on the game?

b How many times is Harry likely to win?

c How much money is Harry likely to win?

d Is Harry likely to lose money or make money on this game?

Give a reason for your answer.

15 The probability of winning a prize of $25 from a scratch card is $\frac{1}{50}$.

Scratch cards cost $1 each.

Tom buys 100 cards.

a Work out how many times Tom is likely to win.

b Work out how much Tom is likely to win.

c Calculate the difference between the money Tom spends on scratch cards and his likely winnings.

16 The probability of getting a broken egg from a batch of 800 eggs is 8%.

How many broken eggs are expected in this batch?

Investigation

This investigation should be done as a class exercise.

A company runs the following promotional offer with tubes of sweets.

Each lid has a number 1, 2, 3, 4 or 6 printed inside.

Collect 4 lids with the same number and get a free T-shirt.

What are the chances of getting four numbers the same if you just buy four tubes?

What is the most likely number of tubes that you need to buy to collect enough lids?

1 We could answer the questions by collecting evidence, i.e. buying tubes of these sweets until we have four lids with the same number on them, and repeating this until we think we have reliable results.

What are the disadvantages of this?

2 We can avoid the disadvantages of having to buy tubes of these sweets by simulating the situation as follows:

First assume that any one of the numbers 1 to 6 is equally likely to be inside a lid.

Now throw a die to simulate the number we would get if we bought a tube, and carry on throwing it until you get four numbers the same. We will need several tally charts to keep a record of the number of throws needed on each occasion.

This shows the start of the simulation.

Score	Tally
1	/
2	///
3	/
4	//
5	////
6	/
Total	12

Number of throws needed to get 4 numbers the same	4	5	6	7	8	9	10	11	12	13	14	15	16	…
Tally									/					

Using probability to help decision making

There are many occasions when we have to choose between two or more options and probability can sometimes help us decide which option to take.

For example, suppose you want to have a barbeque on either Saturday or Sunday evening. The probability it will rain on Saturday evening is 20%. The probability it will rain on Sunday evening is 60%. Unless there are other reasons to choose Sunday, Saturday would be the better choice.

Exercise 8g

1 Jason visits a school bazaar. He cannot decide whether to spend money on the tombola or the lucky dip. The probability that he will win a prize in the tombola is 1 in 5. The probability that he will win a prize on the lucky dip is 1 in 8. Which one should he choose?

It is easier to compare probabilities when probabilities are written as percentages.

2 The chance of a hurricane occurring in August is 30%. The chance of a hurricane occurring in September is 44%.

Mr Khan wants to build a shed in his garden. Would he be wise to build it in September?

3 There are two medicines available to treat a particular virus. Medicine A has a 40% chance of curing an infection quickly.

Medicine B has a 70% chance of curing the infection quickly.

a Using just the information above, which medicine is the better choice?

b If the chance of serious side-effects with Medicine B is 1 in 10 and the chance of serious side-effects with Medicine A is 1 in 100, would this be likely to change the choice?

4 Angela needs to buy a new printer. Printer A costs $5000 and the probability that it will last for 10 years is 40%. Printer B costs $2000 and the probability it will last for 10 years is 15%.

Based on this information, which printer is it sensible to buy? Give reasons for your answer.

What other considerations could affect the decision?

5 Grace drives from home to her place of work every day. She can use two routes. If the roads are clear of traffic, using Route 1 takes 10 minutes less time than using Route 2. However, the probability of traffic congestion on Route 1 is $\frac{1}{3}$. The probability of traffic congestion on Route 2 is $\frac{2}{11}$. Which of these two routes is the better choice?

6 An engaged couple are choosing a date for their wedding. They would like a sunny day. Research shows the probability of sun on 10 June is $\frac{3}{7}$, the probability of sun on 17 June is $\frac{2}{5}$ and the probability of sun on 24 June is $\frac{5}{9}$. Based on just this information, which date should they choose? Give reasons for your answer.

Finding probability by experiment

We have assumed that if you toss a coin it is equally likely to land head up or tail up so that $P(\text{a head}) = \frac{1}{2}$. This is called a theoretical probability. Coins like this are called 'fair' or 'unbiased'.

Most coins are likely to be unbiased but it is not necessarily true of all coins. A particular coin may be slightly bent or even deliberately biased so that there is not an equal chance of getting a head or a tail.

The only way to find out if a particular coin is unbiased is to toss it several times and count the number of times that it lands head up.

Then for that coin

$$P(\text{a head}) \approx \frac{\text{number of heads}}{\text{total number of tosses}}$$

This is called an experimental probability.

The *approximation* gets nearer to the truth as the number of tosses gets larger.

Exercise 8h

Work with a partner or collect information from the whole class.

1 Toss a \$5 coin 100 times and count the number of times it lands head up and the number of times it lands tails up.

Use tally marks, in groups of 5, to count as you toss.
Illustrate your results in a bar chart.
Find, approximately, the probability of getting a head with this coin.

2 Repeat question 1 with a \$10 coin.

3 Repeat question 1 with the \$5 coin that you used first but this time stick a small piece of modelling clay on one side.

4 Choose two \$5 coins and toss them both once. What do you think is the probability of getting two heads? Now toss the two coins 100 times and count the number of times that both coins land head up together. Use tally marks to count as you go: you will need to keep two tallies, one to count the total number of tosses and one to count the number of times you get two heads. Use your results to find approximately the probability of getting two heads.

5 Take an ordinary pack of playing cards and keep them well shuffled. If the pack is cut, what do you think is the probability of getting a red card? Cut the pack 100 times and keep count, using tally marks as before, of the number of times that you get a red card. Illustrate your results with an appropriate graph. Now find an approximate value for the probability of getting a red card.

6 Using the pack of cards again, what do you think is the probability of getting a spade? Now find this probability by experiment.

7 Use an ordinary six-sided die. Toss it 25 times and keep count of the number of times that you get a six. Use your results to find an approximate value for the probability of getting a six. Now toss the die another 25 times and add the results to the last set. Use these to find again the probability of getting a six. Now do another 25 tosses and add the results to the last two sets to find another value for the probability. Carry on doing this in groups of 25 tosses until you have done 200 tosses altogether.

You know that the probability of getting a six is $\frac{1}{6}$. Now look at the sequence of results obtained from your experiment. What do you notice? (1t is easier to compare your results if you use your calculator to change the fractions into decimals correct to 2 d.p.)

8 Remove all the diamonds from an ordinary pack of playing cards. Shuffle the remaining cards well and then cut the pack. What do you think is the probability of getting a black card? Shuffle and cut the pack 100 times and use the results to find approximately the probability of cutting a black card.

9 Roll two dice together 100 times. Find the total for each toss and put a mark beside a number each time you toss that number. Use a copy of the chart on the right.

Total	Number of times
2	
3	
4	
5	
.	
.	
12	

a What is the probability of getting a total of 7?

b What is the probability of getting

i a total 2 **ii** a total 12?

Mixed exercise

Exercise 8i

Select the letter that gives the correct answer.

1 I roll an ordinary die. The probability that it shows an even number is

A $\frac{1}{6}$ **B** $\frac{1}{4}$ **C** $\frac{1}{3}$ **D** $\frac{1}{2}$

2 I choose a number from the first 10 whole numbers. The probability that it is a prime number is

A $\frac{1}{3}$ **B** $\frac{2}{5}$ **C** $\frac{1}{2}$ **D** $\frac{3}{5}$

3 A whole number is chosen from the first 10 whole numbers.

The probability that it is exactly divisible by 3 and 4 is

A 0 **B** $\frac{1}{10}$ **C** $\frac{1}{7}$ **D** $\frac{1}{5}$

4 An ordinary fair die is rolled 120 times. The number of times I expect it to show a 6 is

A 15 **B** 18 **C** 20 **D** 24

5 An ordinary fair die is rolled 60 times. The number of times I expect it to show a 2 or a 4 is

A 10 **B** 12 **C** 16 **D** 20

6 A letter is chosen at random from the letters in the word EXCELLENCE. The probability that it is an E is

A $\frac{1}{5}$ **B** $\frac{3}{10}$ **C** $\frac{2}{5}$ **D** $\frac{1}{2}$

In this chapter you have seen that...

- ✔ the probability that an event will happen = $\dfrac{\text{the number of ways that the event can happen}}{\text{the total number of equally likely outcomes}}$
- ✔ the probability that an event will happen lies between 0 and 1; it is 0 when the event is impossible and it is 1 when the event is certain
- ✔ the probability that an event will not happen is
  1 – the probability that it will happen
- ✔ an estimate for the number of times an event will happen
  = (probability that it will happen once) × (the number of times it is tried).

9 Relations and functions

At the end of this chapter you should be able to...

1 describe a relation
2 define a function and use the notation associated with functions
3 distinguish between the graph of a function and the graph of a relation.

Did you know?

Do you know how dependent we all are on notation and convention?
Ask any mathematician to solve the equation $ax = b$ and you will get $x = \frac{b}{a}$.
If you get $a = \frac{b}{x}$, you would probably say that it is wrong. But is it? Without realising it, we use the convention that letters near the end of the alphabet represent unknowns and letters near the beginning represent known quantities.

You need to know...

- ✔ set notation
- ✔ how to draw a mapping diagram
- ✔ how to draw a graph of a straight line
- ✔ how to divide by a fraction.

Key words

domain, function, range, relation

Relations

We met relations in Grade 8. This section revises that work.

Consider the set $A = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$.

Each element is a pair of numbers in which the second number is three times the first number.

In the set, $B = \{$(UK, Jamaica), (UK, Trinidad), (Jamaica, Trinidad)$\}$, each element is a pair of countries where the second country is south of the first country.

Both of these sets are examples of a relation.

A *relation* is a set of ordered pairs with a rule that connects the two objects in each pair.

We say that the first object maps to the second object.

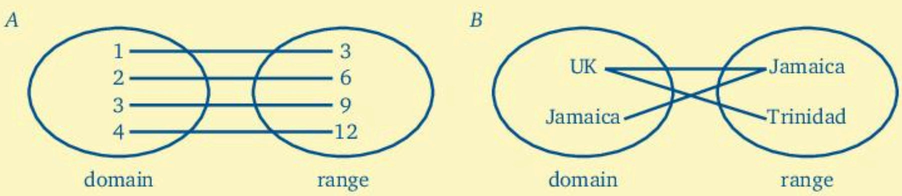
The set of the first objects in the ordered pairs is called the *domain* of the relation.

The domain of A is $\{1, 2, 3, 4\}$ and the domain of B is {UK, Jamaica}.

The set of the second objects in the ordered pairs is called the *range* of the relation.

The range of A is $\{3, 6, 9, 12\}$ and the range of B is {Jamaica, Trinidad}.

We can draw mapping diagrams to illustrate these sets:



In the set A, each member of the domain is different. Also each member of the domain maps to a different member of the range. This relation is called a one-to-one $(1:1)$ relation.

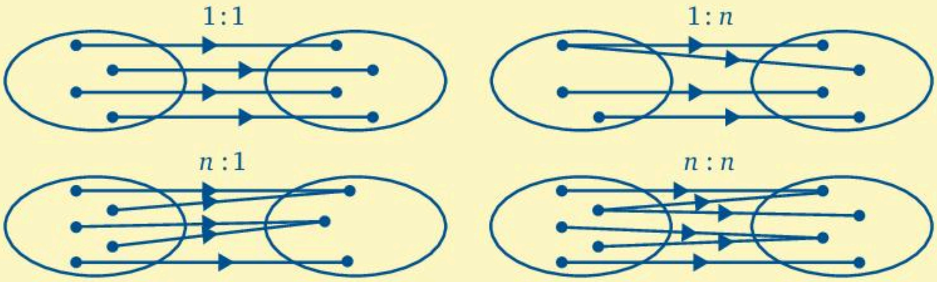
In the set B, the members of the domain are not all different, nor are the members of the range. This type of relation is called a many-to-many $(n:n)$ relation.

There are two other types of relation, one-to-many $(1:n)$ and many-to-one $(n:1)$.

In a $(1:n)$ relation, the members of the domain are not all different, but the members of the range are.

In a $(n:1)$ relation, the members of the domain are all different, but the members of the range are not.

The mapping diagrams summarise this:



Exercise 9a

1 The second number in each pair in a relation is the cube of the first number. The domain of the relation is {1, 2, 3, 4}.

Write down the relation as a set of ordered pairs.

2 A relation is defined by $\{(a, b): a > b, \quad a, b \in \{1, 2, 3\}\}$

a List the set of ordered pairs in the relation.

b Write down the domain and the range of this relation.

3 Draw a mapping diagram to represent these relations and state the type of relation.

a {(1, 4), (4, 1), (5, 4), (3, 1)}

b {(1, 4), (–1, 4), (2, 1), (–2, 1)}

c {(1, 4), (2, 1), (5, 2), (3, 3)}

d {(1, 4), (1, 1), (5, 2), (5, 3)}

e $\{(a, b), (b, c), (c, 2a), (d, 3b)\}$

Functions

In the relation {(1, 3), (2, 6), (3, 9), (4, 12)}, the first numbers in the ordered pair are all different.

This means that each number in the domain is paired with just one number in the range.

This relation is an example of a *function*.

A function is a relation where each member of the domain is paired with just one member of the range.

So a function is a $1:1$ or $n:1$ relation.

In the relation $B = \{$(UK, Jamaica), (UK, Trinidad), (Jamaica, Trinidad)$\}$, the first objects in the ordered pairs are all not all different because the UK is paired with two different countries. The relation B is not a function.

We can write the relation A above as $\{(x, y): y = 3x \text{ for } x \in \{1, 2, 3, 4\}\}$

As we know that this relation is a function, we can describe it as $\text{f}: x \rightarrow 3x, x \in \{1, 2, 3, 4\}$ which we read as 'the function such that x maps to $3x$ for $x = 1, 2, 3$ and 4.'

This means that x is the first number in the ordered pair and $3x$ is the second number in the ordered pair.

$3x$ is sometimes called the image of x under the function f.

We can describe this function even more briefly as $\text{f}(x) = 3x, x \in \{1, 2, 3, 4\}$.

We read $\text{f}(x) = 3x$ as 'f of x equals $3x$' and it means that the second number in each ordered pair is three times the first number.

For example, if $\text{f}(x) = x^2 - 1, x \in \{1, 2\}$
the ordered pairs are $(1, 1^2 - 1)$ and $(2, 2^2 - 1)$, i.e. $(1, 0)$ and $(2, 3)$.

Exercise 9b

1 A relation is such that apples are cheaper than mangoes, bananas are cheaper than apples.

- **a** List the ordered pairs where the first item is cheaper than the second item.
- **b** Is this relation a function? Give a reason for your answer.

2 In this list of ordered pairs, the first person is older than the second person.

(Anne, James), (James, Cheryl), (Anne, Cheryl), (Damien, Usain)

Is this relation a function? Give a reason for your answer.

3 The second number in each pair in a relation is 5 times the first number. The domain of the relation is $\{-1, 0, 1, 2\}$.

List the ordered pairs in the relation and determine whether this relation is a function.

4 A relation is defined by $\{(x, y): x < y, x, y \in \{1, 4, 6\}\}$.

List the ordered pairs in this relation and determine whether or not it is a function.

5 A relation in defined by $\{(x, y): y = x^2 + 1, x \in \{0, 2, 4\}\}$.

List the ordered pairs in the relation and determine whether it is a function.

6 A function is given by $f: x \to 5x$ for $x \in \{3, 7, 9\}$.

List the ordered pairs for this function.

7 A function is given by $f: x \to 2x^2 - 1$ for $x \in \{0, 1, 2\}$.

List the ordered pairs for this function.

8 A function is defined as $f(x) = x^2 - 2x$ for $x \in \{-1, 0, 2\}$.

List the ordered pairs for this function.

9 A function is defined as $f(x) = 5x - 4$ for $x \in \{2, 4, 6\}$.

List the ordered pairs for this function.

In questions **10** to **14** the domain of the function is $\mathbb{R}$. The symbol $\mathbb{R}$ stands for the set of real numbers, which is the set of all possible numbers that can be shown on a number line.

10 Which of these ordered pairs are members of the function $f: x \to 3x - 1, x \in \mathbb{R}$?

a (3, 1) **b** (2, 5) **c** (–1, 2)

The ordered pairs that are members of f are $(x, 3x - 1)$ for all values of x. When $x = 3$, $3x - 1 = 3 \times 3 - 1 = 8$, so (3, 1) is not a member of f.

11 Which of these ordered pairs are members of the function $f(x) = 2x^2, x \in \mathbb{R}$?

a (2, 8) **b** (2, 16) **c** (–1, 2)

12 A function f is given by $f(x) = 3x - 5, x \in \mathbb{R}$. Find

a f(2) **b** f(5) **c** f(–1)

f(2) means the second number in the ordered pair when the first number is 2, i.e. $f(2) = 3(2) - 5$.

13 A function f is given by $f(x) = x^3 + x, x \in \mathbb{R}$. Find

a f(1) **b** f(0) **c** f(–1)

14 A function g is given by $g(x) = 2x^2 - 3x + 1, x \in \mathbb{R}$. Find

a g(1) **b** g(0) **c** g(–1)

15 Determine which of the relations given in Exercise 9a, question 3, are functions.

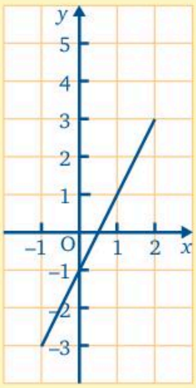
Representing functions graphically

Consider the function $f(x) = 2x - 1, x \in \mathbb{R}$

If we let $y = f(x)$, i.e. $y = 2x - 1$, we can represent this function by drawing the graph of $y = 2x - 1$.

We cannot draw the graph for all possible values of x, but we can draw part of the graph by choosing a range of values for x.

This graph represents $f(x) = 2x - 1$ for values of x from -1 to 2.

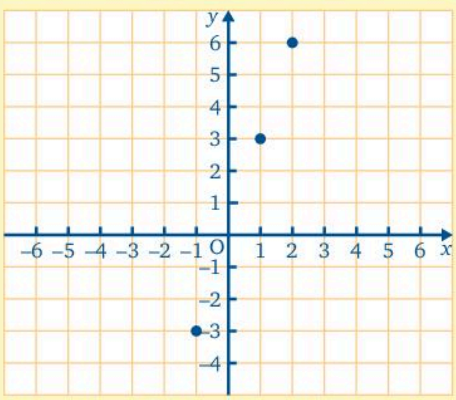


When the domain is a set of separate numbers, we can represent a function as points.

For example, to represent the function $f(x) = 3x$ for $x \in \{-1, 1, 2\}$, first list the ordered pairs in a table:

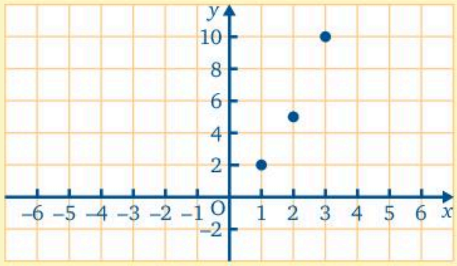
x	-1	1	2
$y = f(x)$	-3	3	6

The graph looks like this.

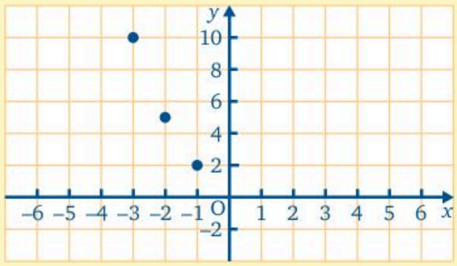


Now consider the function $f(x) = x^2 + 1$.

When the domain is $\{1, 2, 3\}$, the range is $\{2, 5, 10\}$ and the graph looks like this:



But when the domain is $\{-1, -2, -3\}$ the range is again $\{2, 5, 10\}$, and the graph looks like this:



This shows that the graph of a given function depends on the domain.

It also shows that the function $f(x) = x^2 + 1$ for $x \in \{1, 2, 3\}$

and the function $f(x) = x^2 + 1$ for $x \in \{-1, -2, -3\}$

are not the same function.

Exercise 9c

Use square grid paper and 1 square for 1 unit on both axes.

1 Sketch the graph that represents the function.

a $f(x) = x + 2$ for $-3 \leqslant x \leqslant 1$

b $f(x) = 3 - x$ for $-1 \leqslant x \leqslant 4$

Let $y = f(x)$ and use what you learnt in Grade 8 about drawing graphs of straight lines.

2 Draw the graph that represents $f(x) = x^2 - 1$ for $-2 \leqslant x \leqslant 2$

Make a table using half unit values of x. Join the points with a smooth curve.

3 **a** Draw the graph that represents the function $f(x) = 3x - 1$ for the domain $-2 \leqslant x \leqslant 0$.

b Draw the graph that represents the function $f(x) = 3x - 1$ for the domain $0 \leqslant x \leqslant 3$.

c Explain why the functions in parts **a** and **b** are not the same.

Make a table of values using half unit values of x. Join the points with a smooth curve.

4 **a** Draw the graph of $f(x) = x^3$ for the domain $-2 \leqslant x \leqslant 2$.

b Draw the graph of $f(x) = x^3$ for the domain $-2 \leqslant x \leqslant 0$.

c Explain why the functions in parts **a** and **b** are not the same.

5 **a** Draw the graph of $f(x) = \frac{1}{x}$ for the domain $x \in \left\{\frac{1}{4}, \frac{1}{3}, \frac{1}{2}\right\}$.

b Draw the graph of $f(x) = \frac{1}{x}$ for the domain $x \in \left\{-\frac{1}{2}, -\frac{1}{3}, -\frac{1}{4}\right\}$.

Graphs of relations and functions

We can graph a relation $\{(x, y)\}$ when we know the relationship between x and y and the domain.

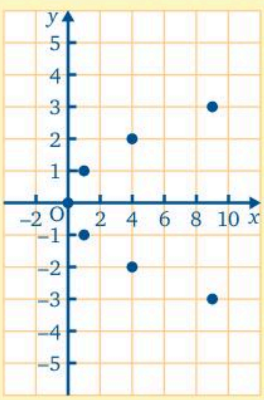
For example, consider the relation $\{(x, y): y^2 = x \text{ for } x \in \{0, 1, 4, 9\}\}$.

Making a table of values gives:

x	9	4	1	0
y	3 or −3	2 or −2	1 or −1	0

(Remember that a positive number has two square roots, one positive and one negative, so when $y^2 = 9$, $y = \pm\sqrt{9} = \pm 3$)

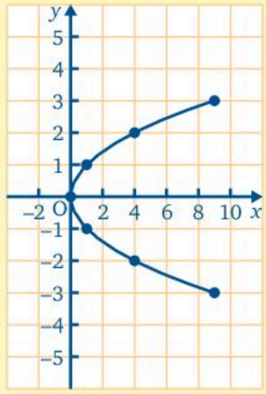
The graph looks like this:



It is clear from both the table and the graph that, except for $x = 0$, one value of x gives two values of y.

Therefore this is a $1:n$ relation, and so it is not a function.

It is even clearer if we draw the graph of $\{(x, y): y^2 = x$ for $0 \leqslant x \leqslant 9\}$.
We can do this by using the same table of values and drawing a smooth curve through the points.



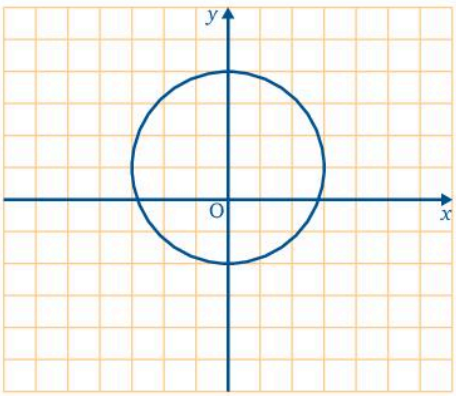
If we draw a vertical line through any value of x, it cuts the curve in two places, giving two values of y.

You can use the vertical line test through any graph.
If there are values where the line cuts the curve in more than one place, the relationship is not a function.

Exercise 9d

Determine whether each of the following graphs represents a function.

1



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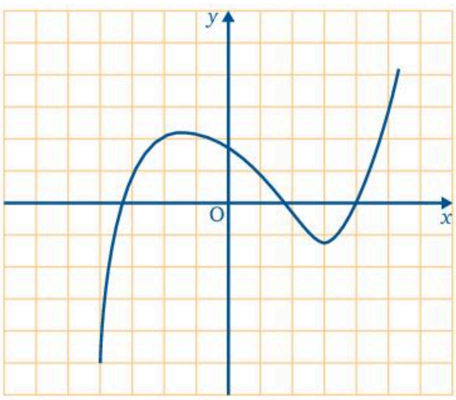

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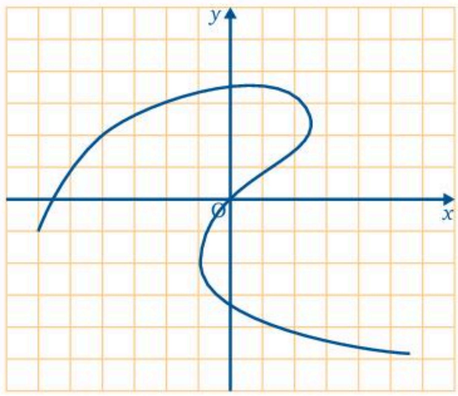

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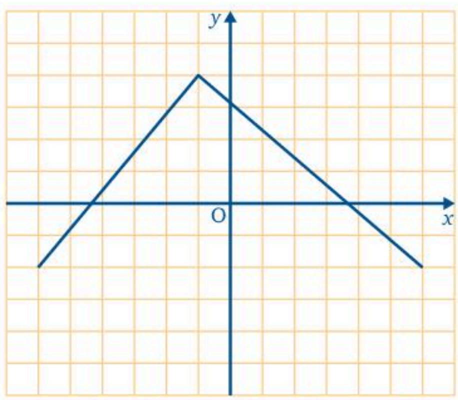

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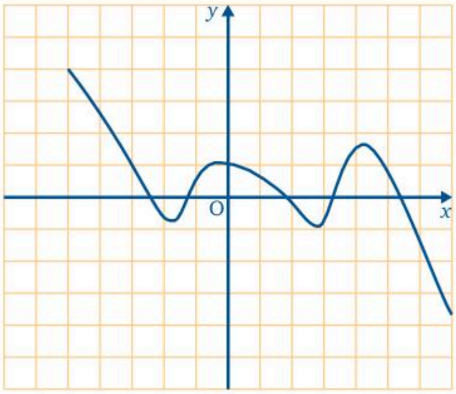

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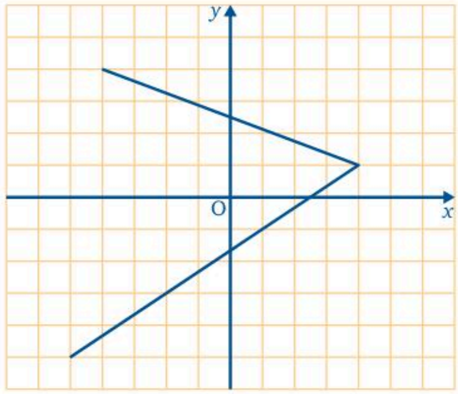


Mixed exercise

Exercise 9e

1 A relation is defined by $\{(a, b) : a < b, a, b \in \{2, 3, 5\}\}$.

a List the set of ordered pairs in the relation.

b Give the domain and range of the relation.

c State, with a reason, whether this relation is a function.

2 Draw a mapping diagram to represent each relation and state the type of relation.

a $(2, -1), (3, 5), (4, -1)$

b $(a, b), (a, c), (b, c)$

3 A function is given by $f(x) = x^2 + 2$.

Find

a $f(-1)$ b $f(4)$ c $f\left(\frac{1}{2}\right)$

4 A relation is given by $y = 2 - x^2$ for $x \in \{-2, -1, 0, 1, 2\}$.

a List the ordered pairs in this relation.

b Determine, with a reason, whether the relation is a function.

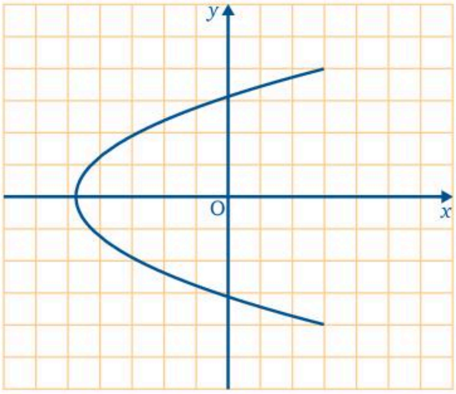
5 Draw the graph of $f(x) = \frac{2}{x}$ for the domain

a $(1, 2, 3)$

b $\left(\frac{1}{4}, \frac{1}{3}, \frac{1}{2}\right)$

6 Determine whether the graphs represent functions

a



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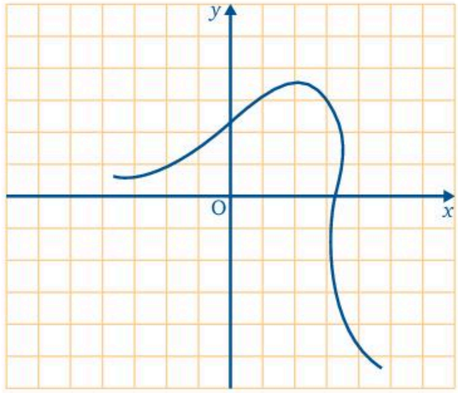

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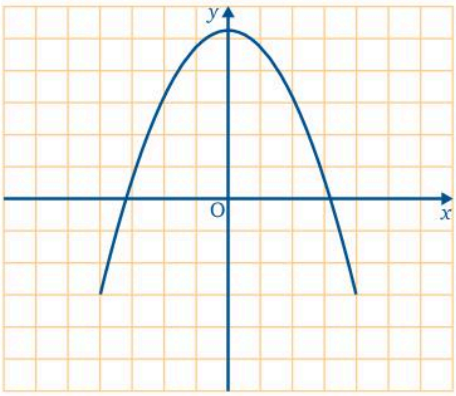

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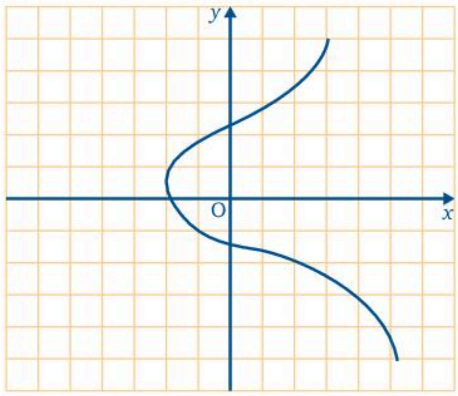


Investigation

1 Choose any odd number n, greater than 1, and calculate:

$\frac{n^2-1}{2}, \frac{n^2+1}{2}$

What do you notice about the values of n, $\frac{n^2-1}{2}$ and $\frac{n^2+1}{2}$?

2 Choose any number n, greater than 1, and calculate $2n$, $n^2 - 1$ and $n^2 + 1$. What do you notice about these numbers?

Did you know?

Towards the end of the 19th century, mathematicians tried to formalise mathematics by attempting to define every mathematical object as a set. As part of this quest, Dirichlet and Lobachevsky independently, and almost at the same time, gave the modern definition of a function.

In this chapter you have seen that...

- ✔ a relation is a set of ordered pairs with a rule that connects the objects in each pair
- ✔ the domain of a relation is the set of the first objects in each pair and the range is the set of the second objects in each pair
- ✔ a function is a relation where the first objects in each pair are all different
- ✔ the ordered pairs in a function are $(x, \mathrm{f}(x))$
- ✔ a function can be represented graphically by graphing the equation $y = \mathrm{f}(x)$
- ✔ a graph represents a function if there is nowhere that a vertical line through the x-axis intersects the graph in more than one place.

10 Ratio and proportion

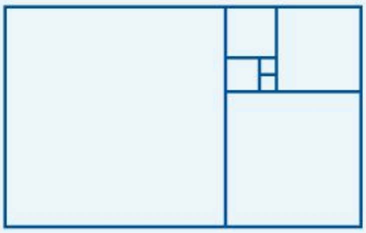
At the end of this chapter you should be able to...

1 reduce a given ratio to its simplest form
2 compare two ratios in terms of size
3 express a given ratio in the form $n : 1$, where n is written correct to a stated number of significant figures
4 find the ratio of two quantities measured in different units
5 calculate the missing quantity, given two equivalent ratios
6 use ratios to solve problems in sharing
7 express quantities which are in direct or inverse proportion in terms of ratio
8 solve problems involving direct or inverse proportion
9 use the unitary method to solve problems.

Did you know?

The Golden Ratio

Early Greek mathematicians were fascinated by a rectangle, the ratio of whose sides is such that if a square is cut off one end, you are left with a rectangle of the same shape as the original, i.e. the ratios of their corresponding sides are equal. This process can be repeated as often as you like.



Because such a rectangle is thought to be pleasing to the eye, the ratio of the sides is also called the divine proportion.

You need to know...

- ✔ how to work with fractions and decimals
- ✔ the units of length and the relationships between them
- ✔ the units of mass and the relationships between them
- ✔ how to find the areas of squares and triangles
- ✔ how to solve equations involving fractions.

Key words

denominator, direct proportion, inverse proportion, lowest common multiple, proportional, ratio, reciprocal, unitary method

The first few exercises are to remind you of what you learnt about ratios in Grades 7 and 8. Remember that ratios compare the sizes of related quantities.

Simplifying ratios

A *ratio* can be divided or multiplied throughout by the same number.

Exercise 10a

Give the ratios **a** $225:105$ **b** $\frac{4}{5}:\frac{7}{8}:\frac{1}{2}$ in their simplest forms

a $225:105 = 45:21$ (dividing by 5)

$= 15:7$ (dividing by 3)

b 40 is the *lowest common multiple* of the *denominators* so multiply by 40 to get rid of the fractions

$$\frac{4}{5}:\frac{7}{8}:\frac{1}{2} = 40\times\frac{4}{5} : 40\times\frac{7}{8} : 40\times\frac{1}{2}$$

$$= 32:35:20$$

Give the following ratios in their simplest form:

1 $12:18$

2 $3:6:9$

3 $3.5:2.5$

4 $320:480$

5 $288:128:144$

6 $\frac{1}{2}:\frac{3}{4}:\frac{1}{4}$

7 $3.2:7.2$

8 $\frac{1}{2}:\frac{5}{6}:\frac{2}{3}$

9 $36:54:18$

Which ratio is larger, $3:2$ or $14:9$?

(To compare the sizes of the ratios, we write them as fractions and then convert them to equivalent fractions with a common denominator.)

$$\frac{3}{2} = \frac{27}{18} \quad \text{and} \quad \frac{14}{9} = \frac{28}{18}$$

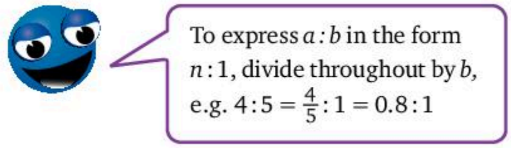
The second ratio is the larger.

Which ratio is the larger?

10 6 : 11 or 2 : 5 **11** 15 : 4 or 11 : 3 **12** 20 : 3 or 31 : 4 **13** 2 : 7 or 5 : 16

Express the following ratios in the form n : 1, giving n correct to three significant figures where necessary:

14	3 : 2	**17**	30 : 11	**20**	4 : 3
15	12 : 5	**18**	3 : 5	**21**	3 : 4
16	6 : 7	**19**	21 : 8	**22**	10 : 7



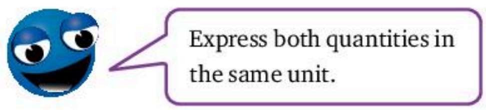
Mixed units and problems

If we are asked to compare two quantities expressed in different units, we need to change one or both so that the two quantities are in the same unit. It is easier to change to smaller units (where multiplication is required) rather than to larger units (where division is required).

Exercise 10b

Simplify the following ratios:

1	45 cm : 0.1 m	**4**	32 g : 2 kg
2	42 cm : 1.05 m	**5**	450 mg : 1 g
3	340 m : 1.2 km	**6**	2.2 t : 132 kg



Find the ratio of 14 c per gram to $120 per kilogram.

(In order to compare we will use both prices in $ per kg.)

$$\begin{aligned} 14\text{c per g} &= 14\,000\text{c per kg} \\ &= \$140 \text{ per kg} \\ 14\text{c per g} : \$120 \text{ per kg} &= \$140 \text{ per kg} : \$120 \text{ per kg} \\ &= 140 : 120 \\ &= 7 : 6 \end{aligned}$$

Find the ratios of the following prices :

7 40 c per kilogram to $380 per tonne.

8 $6 each to $70 per dozen.

9 $1620 per metre to $15 per centimetre.

10 $72 for twenty to $4 each.

Give the ratio of the cost of 6 m of material at \$240 per metre to the cost of 8 m at \$220 per metre.

We need to find the cost of each length of material first.

$$\text{First cost} = \$6 \times 240 = \$1440$$
$$\text{Second cost} = \$8 \times 220 = \$1760$$
$$\text{Ratio of costs} = \$1440 : \$1760$$
$$= 1440 : 1760$$
$$= 18 : 22 = 9 : 11$$

11 In a school of 1029 pupils, 504 are girls. What is the ratio of the number of boys to the number of girls?

12 I spend \$360 on groceries and \$240 on vegetables. What is the ratio of the cost of

a groceries to vegetables

b vegetables to groceries

c groceries to the total?

13 One rectangle has a length of 6 cm and a width of $4\frac{1}{2}$ cm. A second rectangle has a length of 9 cm and a width of $2\frac{1}{2}$ cm. Find the ratios of

a their lengths

b their widths

c their perimeters

d their areas.

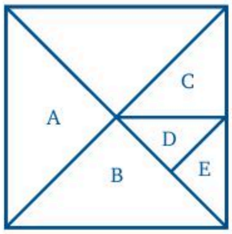
14 Find the ratio of the cost of 12 m^2 of carpet at \$720 per m^2 to the cost of 50 carpet tiles at \$240 per tile.

15 Find the ratios of the following areas

a B : A **b** C : B

c E : A + B **d** E : D

e E : C + D **f** C : whole square

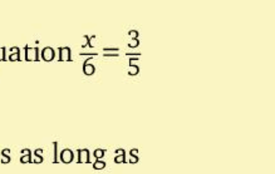


16 The areas of the small triangles are equal. Find the ratios of the following areas:

a A : whole figure

b A : A + B + C + D

c B + E + F + G : whole figure



Finding missing quantities

If we are given the ratio $x : 6 = 3 : 5$ we may write this as the equation $\frac{x}{6} = \frac{3}{5}$ and solve the equation for x.

It does not matter in which order we compare the two quantities as long as we are consistent. If the ratio is $4 : x = 5 : 3$ we may rewrite it as $x : 4 = 3 : 5$.

Exercise 10c

Find x if **a** $x:6=13:15$ **b** $4:x=3:2$

a $\frac{x}{6}=\frac{13}{15}$

$6\times\frac{x}{6}=6\times\frac{13}{15}$ (multiply both sides by 6)

$x=\frac{26}{5}=5.2$

b $x:4=2:3$ (rearrange the ratio so that x comes first)

$\frac{x}{4}=\frac{2}{3}\Rightarrow 4\times\frac{x}{4}=4\times\frac{2}{3}$ (multiply both sides by 4)

$x=\frac{8}{3}=2\frac{2}{3}$

Find x in questions **1** to **6**:

1 $x:5=2:9$

2 $x:3=1:7$

3 $x:6=5:4$

4 $5:x=7:2$

5 $3:8=9:x$

6 $15:2=x:3$

Complete the following ratios:

7 $4:\quad=3:7$

8 $\quad:5=6:11$

9 $3:\quad=5:2$

10 $9:5=\quad:4$

11 $6:5=4:\quad$

12 $\quad:12=5:7$

In a town, the ratio of the number of males to the number of females is 80 : 81. There are 9680 males. How many females are there? What is the total population?

Suppose there are x females,

then $x:9680=81:80$ (Notice the change of order so that x comes first)

$\frac{x}{9680}=\frac{81}{80}$

$9680\times\frac{x}{9680}=9680\times\frac{81}{80}$

$x=9801$

There are 9801 females and the total population is 19 481.

13 The numbers of Mr and Mrs James' grandsons and granddaughters are in the ratio 4 : 3. There are nine granddaughters. How many grandsons are there? What is the ratio of the number of granddaughters to the number of grandchildren?

14 The ratio of the number of cats to the number of dogs owned by the children in one year group in a school is 5 : 3. There are 95 cats. How many cats and dogs are there altogether?

15 The ratio of the lengths of two rectangles is 6 : 5. The length of the second is 8.4 cm. What is the length of the first?

16 The ratio of the numbers of orange flowers to white flowers in a garden is 6 : 11. There are 144 orange flowers. How many white flowers are there?

Division in a given ratio

Exercise 10d

Share \$72 amongst three people so that their shares are in the ratio 3 : 4 : 5.

There are 12 portions (i.e. $3+4+5$).

First share is 3 out of 12 portions, i.e. $\frac{3}{12}$ of \$72 $= \frac{3}{12} \times \$72 = \$18$

Second share is 4 out of 12 portions, i.e. $\frac{4}{12}$ of \$72 $= \frac{4}{12} \times \$72 = \$24$

Third share $= \frac{5}{12} \times \$72 = \$30$

Check: $\$18 + \$24 + \$30 = \$72$

1 Divide \$45 into two parts in the ratio 4 : 5

2 Divide 96 m into two parts in the ratio 9 : 7

3 Divide 5 kg into three parts in the ratio 1 : 2 : 5

4 Divide seven hours into three parts in the ratio 1 : 5 : 8

5 There are 32 children in a class. The ratio of the number of boys to the number of girls is 9 : 7. How many boys and how many girls are there?

6 The angles of a triangle are in the ratio 6 : 5 : 7. Find the sizes of the three angles.

7 Share the contents of a box containing 30 chocolates amongst Anne, Mary and Sue in the ratio 3 : 4 : 3. How many chocolates will each get?

8 A marksman fires at a target and the ratio of hits to misses is 11 : 4. He fires 90 times. How many hits does he score and how many times does he miss?

Mixed questions

Exercise 10e

1 Simplify the ratio $324:252$

2 Divide 72 m into two parts in the ratio $5:7$

3 Complete the ratio $4:7 = 3:$

4 Find x if $4:x = 9:5$

5 The ratio of two lengths is $4:5$. If the first length is 22 cm what is the second?

6 If $p:q = 2:3$, what is $5p:2q$?

7 Find the ratio of 20 c per gram to \$21.20 per kilogram.

8 Simplify $4\frac{2}{3}:3\frac{1}{2}$

9 In a block of apartments, 24 have two bedrooms and 32 have one bedroom. Give the ratio of the number of two-bedroomed apartments to the number of one-bedroomed apartments.

10 Simplify the ratio $3.2:4.8$

Simple direct proportion

If we know the cost of one article, we can easily find the cost of ten similar articles, or if we know that someone is paid for one hour's work, we can find what the pay is for five hours.

Exercise 10f

If 1 cm^3 of lead has a mass of 11.3 g, what is the mass of

a 6 cm^3 **b** 0.8 cm^3?

1 cm^3 has a mass of 11.3 g

a 6 cm^3 has a mass of $11.3 \times 6\,g = 67.8\,g$

b 0.8 cm^3 has a mass of $11.3 \times 0.8\,g = 9.04\,g$

1 The cost of 1 kg of sugar is \$160. What is the cost of

a 3 kg **b** 12 kg?

2 In one hour an electric fire uses $1\frac{1}{2}$ units. Find how much it uses in

a four hours **b** $\frac{1}{2}$ hour.

3 One litre of petrol takes a car 18 km. At the same rate, how far does it travel on

a four litres **b** 6.6 litres?

4 A knitting pattern states that, at the correct tension, five rows measure 1 cm. How many rows must be knitted to measure

a 7 cm **b** 8.4 cm?

5 The cost of 1 kg of tomatoes is \$425. Find the cost of

a $\frac{1}{2}$ kg **b** 2.4 kg.

We can reverse the process and, for instance, find the cost of one article if we know the cost of three similar articles. In general, if we know about one unit of a quantity, we can find what we need to know about any number of units. This process can also be reversed.

Exercise 10g

18 cm³ of copper has a mass of 162 g. What is the mass of 1 cm³?

$$18\text{ cm}^3 \text{ has a mass of } 162\text{ g}$$

$$1\text{ cm}^3 \text{ has a mass of } \frac{162}{18}\text{ g} = 9\text{ g}$$

1 Six pens cost \$720. What is the cost of one pen?

2 A car uses eight litres of petrol to travel 124 km. At the same rate, how far can it travel on one litre?

3 A man walks steadily for three hours and covers 13 km. How far does he walk in one hour?

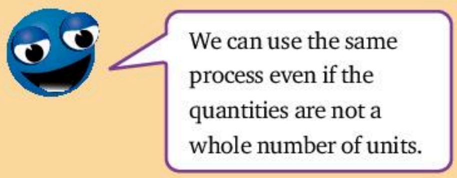
4 Dress material costs \$580 for 4 m. What is the cost of 1 m?

5 A carpet costs \$92 400. Its area is 12 m². What is the cost of 1 m²?

The mass of 0.6 cm³ of a metal is 3 g. What is the mass of 1 cm³?

$$\text{The mass of } 0.6\text{ cm}^3 \text{ is } 3\text{ g}$$

$$\text{The mass of } 1\text{ cm}^3 \text{ is } \frac{3}{0.6}\text{ g} = 5\text{ g}$$



6 The cost of 2.8 m of material is \$1176. What is the cost of 1 m?

7 8.6 m² of carpet cost \$71 380. What is the cost of 1 m²?

8 The cost of running a refrigerator for 3.2 hours is \$2.40. What is the cost of running the refrigerator for one hour?

9 A bricklayer takes 0.8 hour to build a wall 1.2 m high. How high a wall (of the same length) could he build in one hour?

10 A piece of webbing is 12.4 cm long and its area is 68.2 cm². What is the area of a piece of this webbing that is 1 cm long?

Direct proportion

We saw in Grades 7 and 8 that if two varying quantities are always in the same ratio, they are in *direct proportion*. The quantities are said to be *directly proportional* to one another (or sometimes simply *proportional*).

For example, when buying pens which each cost the same amount, the total cost is proportional to the number of pens. The ratio of the cost of 11 pens to the cost of 14 pens is 11 : 14, and if we know the cost of 11 pens, we can find the cost of 14 pens.

One method for solving problems involving direct proportion uses ratio, another uses the ideas in the last two exercises. This method is called the *unitary method* because it makes use of the cost of *one* article or the time taken by *one* man to complete a piece of work.

Either method will work, whether the numbers are complicated or simple. Even if the question is about something unfamiliar, it is sufficient to know that the quantities are proportional.

Exercise 10h

If the mass of 16 cm³ of a metal is 24 g, what is the mass of 20 cm³?

Method 1 (using ratios)
Let the mass of 20 cm³ be x grams

Then $x : 24 = 20 : 16$ (The ratio of the masses = the ratio of the volumes)

$$\frac{x}{24} = \frac{20}{16}$$

$$\overset{1}{\cancel{24}} \times \frac{x}{\underset{1}{\cancel{24}}} = \overset{3}{\cancel{24}} \times \frac{\overset{10}{\cancel{20}}}{\underset{\underset{1}{\cancel{2}}}{\cancel{16}}}$$

$$x = 30$$

The mass of 20 cm³ is 30 g.

Method 2 (unitary method)
(Write the first sentence so that it ends with the quantity you want, i.e. the mass.)

16 cm³ has a mass of 24 g.

1 cm³ has a mass of $\frac{24}{16}$ g. (There is no need to work out the value $\frac{24}{16}$ yet.)

20 cm³ has a mass of $\overset{10}{\cancel{20}} \times \frac{\overset{3}{\cancel{24}}}{\underset{\underset{1}{\cancel{2}}}{\cancel{16}}}$ g = 30 g

1 At a steady speed a car uses four litres of petrol to travel 75 km. At the same speed how much petrol is needed to travel 60 km?

2 A hiker walked steadily for four hours, covering 16 km. How long did he take to cover 12 km?

3 An electric fire uses $7\frac{1}{2}$ units in three hours. How many units does it use in five hours?

4 How long does the same electric fire take to use 9 units?

5 A taxi journey of 30 km costs $3600. At the same rate per kilometre

a what would be the cost of travelling 25 km

b how far could you travel for $4200?

6 It costs $10 800 to turf a lawn of area 63 m^2. How much would it cost to turf a lawn of area 56 m^2?

7 A machine in a soft drinks factory fills 840 bottles in six hours. How many could it fill in five hours?

8 A 6 kg bag of beans costs $1980. At the same rate, what would an 8 kg bag cost?

9 A knitting pattern states that the correct tension is such that 55 rows measure 10 cm. How many rows should be knitted to give 12 cm?

10 A scale model of a ship is such that the mast is 9 cm high and the mast of the original ship is 12 m high. The length of the original ship is 27 m. How long is the model ship?

In a spring balance, the extension in the spring is proportional to the load. If the extension is 2.5 cm when the load is 8 newtons, what is the extension when the load is 3.6 newtons?

Ratio method

Let the extension be x cm.

$$x : 2.5 = 3.6 : 8$$
$$\frac{x}{2.5} = \frac{3.6}{8}$$
$$2.5 \times \frac{x}{2.5} = 2.5 \times \frac{3.6}{8}$$
$$x = 1.125$$

The extension is 1.125 cm.

Unitary method

If a load of 8 newtons gives an extension of 2.5 cm, then a load of 1 newton gives an extension of $\frac{2.5}{8}$ cm.

$\therefore$ a load of 3.6 newtons gives an extension of

$$3.6 \times \frac{2.5}{8} \text{ cm} = 1.125 \text{ cm}$$

11 It costs $19600 to hire scaffolding for 42 days. How much would it cost to hire the same scaffolding for 36 days at the same rate per day?

12 The rates of currency exchange published in the newspapers on a certain day showed that £14 could be exchanged for J$4200. How many dollars could be obtained for £112 ?

13 At a steady speed, a car uses 15 litres of petrol to travel 164 km. At the same speed, what distance could be travelled if six litres were used?

14 If a 2 kg bag of sugar contains 9×10^6 crystals, how many crystals are there in

a 5 kg **b** 1.8 kg **c** 0.03 kg?

15 The current flowing through a lamp is proportional to the voltage across the lamp. If the voltage across the lamp is ten volts the current is 0.6 amp. What voltage is required to make a current of 0.9 amp flow?

16 The amount of energy carried by an electric current is proportional to the number of coulombs. If five coulombs carry 19 joules of energy, how many joules are carried by 6.5 coulombs?

17 A recipe for date squares uses the following quantities:

Ingredients	*Costs*
125 g of brown sugar	1 kg cost $152.50
75 g of oats	750 g cost $204
75 g of flour	$1\frac{1}{2}$ kg cost $176
100 g of margarine	250 g cost $72
100 g of dates	250 g cost $168
Pinch of bicarbonate of soda	–
Squeeze of lemon juice	50 c

Find the cost of making these date squares as accurately as possible, then give your answer correct to the nearest dollar.

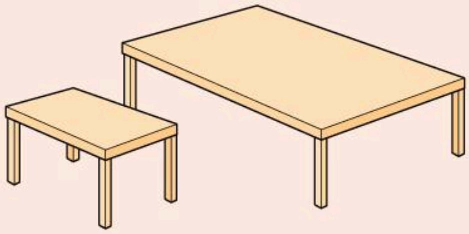
18 A-do-it-yourself enthusiast makes a base for a table.

Materials	*Costs*
4 legs each 30 cm long	2 m cost $960
4 stretchers each 70 cm long	3 m cost $630
4 stretchers each 35 cm long	2 m cost $450
3 pieces each 80 cm long	3 m cost $1890
$\frac{3}{4}$ litre of varnish	1 litre costs $1440
12 screws	20 screws cost $240

What is the total cost of the materials that are actually used?

Puzzle

Roger makes two tables. They are the same height. The top of the first table is 0.5 m by 1 m. The top of the second table is twice the area of the first.



The wood for the table tops costs $450 a square metre and the legs cost $250 each.

What is the ratio of the costs of the tables?

Inverse proportion

Some quantities are not directly proportional to one another, although there is a connection between them. As one increases in size, the other may decrease at the same rate, so that the *reciprocal*, or inverse, of the second is proportional to the first. This relationship is called *inverse proportion*.

Suppose, for example, that a certain amount of food is available for several days. If each person eats the same amount each day, the more people there are, the shorter is the time that the food will last. The number of days the food will last is *inversely proportional* to the number of people eating it.

Exercise 10i

In this exercise, assume that the rates are constant.

Four bricklayers can build a certain wall in ten days. How long would it take five bricklayers to build it?

Ratio method

Suppose it takes five bricklayers x days to build it.

The ratio of the number of bricklayers is $\frac{4}{5}$.

The ratio of the time taken in $\frac{10}{x}$.

(Five bricklayers will take a shorter time so we use the inverse ratio.)

$$\frac{x}{10}=\frac{4}{5}$$

$$10\times\frac{x}{10}=10\times\frac{4}{5}$$

$$x=8$$

It would take them 8 days.

Unitary method

Four bricklayers take 10 days.

One bricklayer would take 40 days.

Five bricklayers would take $\frac{40}{5}$ days $=8$ days

1 Eleven taps fill a tank in three hours. How long would it take to fill the tank if only six taps are working?

2 Nine children share out equally the chocolates in a large tin and get eight each. If there were only six children, how many would each get?

3 The length of an essay is 174 lines with an average of 14 words per line. If it is rewritten with an average of 12 words per line, how many lines will be needed?

4 A field of grass feeds 24 cows for six days. How long would the same field feed 18 cows?

5 The dimensions of a block of stamps are 30 cm wide by 20 cm high. The same number of stamps could also have been arranged in a block 24 cm wide. How high would this second block be?

6 A batch of bottles was packed in 25 boxes taking 12 bottles each. If the same batch had been packed in boxes taking 15 each, how many boxes would be filled?

7 When knitting a scarf 48 stitches wide, one ball of wool will give a length of 18 cm. If there had been 54 stitches instead, how long a piece would the same ball give?

8 In a school, 33 classrooms are required if each class has 32 pupils. How many classrooms would be required if the class size was reduced to 22?

9 A factory requires 42 machines to produce a given number of articles in 63 days. How many machines would be required to produce the same number of articles in 54 days?

Mixed questions

Exercise 10j

Some of the following questions cannot be answered because the quantities are neither in direct nor in inverse proportion. In these cases write 'There is no answer'. For those questions that can be solved, give answers correct to three significant figures where necessary.

1 The list of exchange rates states that J$100 = 102 yen and J$100 = 590 Costa Rican Colón

  a How many Costa Rican Colón can 315 yen be exchanged for?

  b How many yen are 100 Costa Rican Colón worth?

2 A man earned $4120 for an eight-hour day. How much would he earn at the same rate for a 38-hour week?

3 A typist typed 3690 words in $4\frac{1}{2}$ hours. How long would it take to type 2870 words at the same rate?

4 At the age of twelve, a boy is 1.6 m tall. How tall will he be at the age of eighteen?

5 A ream of paper (500 sheets) is 6.2 cm thick. How thick is a pile of 360 sheets of the same paper?

6 If I buy balloons at $14 each, I can buy 63 of them. If the price of a balloon increases to $18, how many can I buy for the same amount of money?

7 A boy's mark for a test is 18 out of a total of 30 marks. If the test had been marked out of 40 what would the boy's mark have been?

8 Twenty-four identical mathematics text books occupy 60 cm of shelf space. How many books will fit into 85 cm?

9 A lamp post 4 m high has a shadow 3.2 m long cast by the sun. A man 1.8 m high is standing by the lamp post. At the same moment, what is the length of his shadow?

10 A contractor decides that he can build a barn in nine weeks using four men. If he employs two more men, how long will the job take? Assume that all the men work at the same rate.

11 A 12-year-old girl gained 27 marks in a competition. How many marks did her six-year-old sister gain?

12 For a given voltage, the current flowing is inversely proportional to the resistance. When the current flowing is 2.5 amps the resistance is 0.9 ohm. What is the current when the resistance is 1.5 ohms?

Mixed exercise

Exercise 10k

1 Simplify the ratio 7.35 : 2.45

2 Complete the ratio : 9 = 2 : 5

3 Divide 56 m into three parts in the ratio 1 : 2 : 4

4 A car uses seven litres of petrol for a 100 km journey. At the same rate, how far could it go on eight litres?

5 Eight typists together could complete a task in five hours. If all the typists work at the same rate, how long would six typists take?

6 Simplify the ratio $7\frac{1}{2} : 2\frac{1}{2} : 1\frac{1}{4}$

7 The ratio of the numbers of eleven-year-olds to twelve-year-olds in a class is 8 : 3. There are 24 eleven-year-olds. How many twelve-year-olds are there?

8 Give the ratio 6 : 5 in the form $n : 1$

Investigation

Sweets at a 'Pick and Mix' counter are sold by mass at $56 per 100 grams.

x	20	50	100	200	500	1000
y						

1 If x grams cost $$y$, copy and complete this table giving values of y corresponding to values of x.

2 Use a scale of 1 cm for 50 units on the x-axis and a scale of 2 cm for 50 units on the y-axis to plot these points on a graph.

3 What do you notice about these points? Can you use the graph to find the cost of 162 grams?

4 The cost and mass of these sweets are directly proportional. Investigate the graphical relationship between other quantities that are directly proportional. What do you notice? Is this always true?

5 Extend your work to investigate the graphical relationship between two quantities that are inversely proportional.

Did you know?

If you are given the ratio of two numbers and you know either their sum, or their difference or their product, you can find the two numbers. Try it.

In this chapter you have seen that...

- ✔ to compare the size of two quantities as a ratio, both quantities must be in the same units
- ✔ ratios can be simplified by multiplying or dividing all parts of the ratio by the same number
- ✔ when two varying quantities are directly proportional, they are always in the same ratio
- ✔ when two varying quantities are inversely proportional, the reciprocal of one of them is proportional to the other.

11 Scale drawings

At the end of this chapter you should be able to...

1 draw a given shape to scale
2 calculate actual measurements from scale drawings
3 understand and use map scales.

You need to know...

✔ how to use a ruler, a protractor and a pair of compasses
✔ the angle sum of a triangle
✔ the sum of angles round a point
✔ how to construct triangles, perpendicular lines and angles of 30°, 45°, 60° and 90°
✔ how to simplify ratios
✔ the meaning of angles of elevation and depression.

Key words

map ratio, scale, scale drawing

Scale drawing

An architect uses a computer to make an accurate drawing of a building before it is built. Everything is shown much smaller than it will be in the completed building, but it is all carefully drawn to scale so that all the proportions are correct.

Likewise a motor engineer makes accurate drawings when he or she designs small parts for a new car. However, these drawings usually show small components much larger than they will be when they are made.

In these, as in many other occupations, *scale drawings* are essential to produce high-quality products.

Activity

This is a class discussion.

1. Discuss other occupations in which accurate drawings are used. Say whether each occupation tends to make scale drawings that show each object larger or smaller than the finished product.
2. If you had a map of your area within 20 miles of where you live, and the scale was not given, would you be able to decide how far one place was from another?

   Would you give the same answer if the map was of a foreign country where you wanted to take a holiday?
3. What information do you think is needed on any scale drawing?

Accurate drawing with scaled down measurements

If you are asked to draw a parking lot that is a rectangle measuring 50 m by 25 m, you obviously cannot draw it full size. To fit it on to your page you will have to scale down the measurements. In this case you could use 1 cm to represent 5 m on the parking lot. This is called the *scale*; it is usually written as $1\text{ cm} \equiv 5\text{ m}$, and must *always* be stated on any scale drawing.

It may also be written as a ratio, i.e. $1\text{ cm} : 5\text{ m} = 1 : 500$.

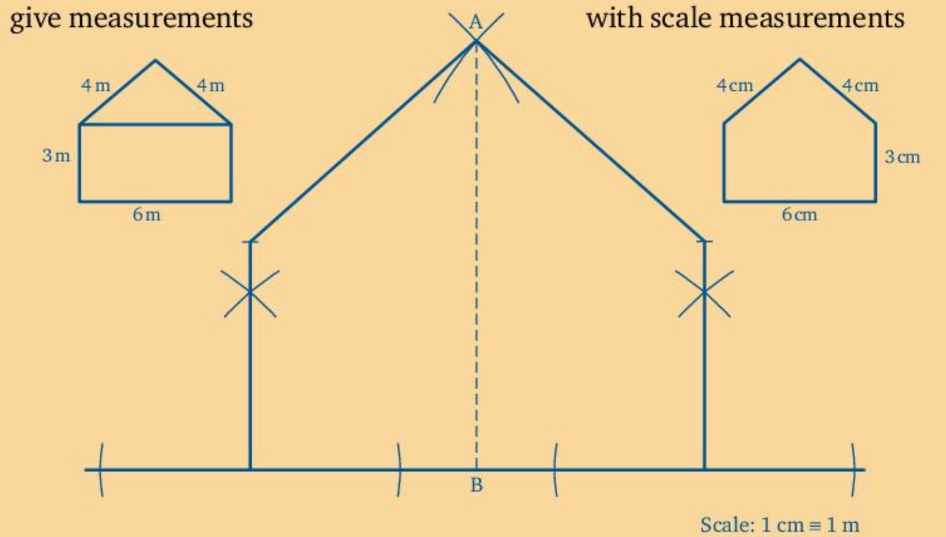
Exercise 11a

Start by making a rough drawing of the object you are asked to draw to scale. Mark all the full-size measurements on your sketch. Next draw another sketch and put the scaled measurements on this one. Then do the accurate scale drawing. Always give the scale on your drawing.

The end wall of a bungalow is a rectangle with a triangular top. The rectangle measures 6 m wide by 3 m high. The base of the triangle is 6 m and the sloping sides are 4 m long. Using a scale of 1 cm to 1 m, make a scale drawing of this wall. Use your drawing to find, to the nearest tenth of a metre, the distance from the ground to the ridge of the roof.

Rough sketch of wall to give measurements

Rough sketch of scale drawing with scale measurements

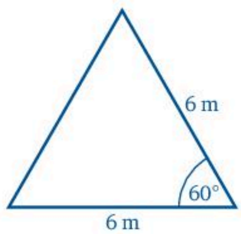


From the drawing, AB measures 5.6 cm.

So the height of the wall is $5.6 \times 1\,\text{m} = 5.6\,\text{m}$.

In questions **1** to **5**, use a scale of 1 cm to 1 m to make a scale drawing.

1



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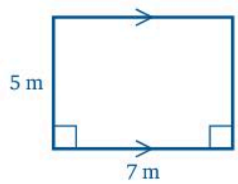

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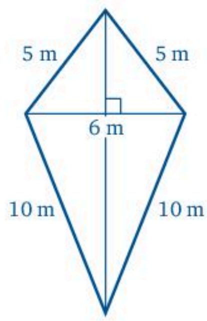

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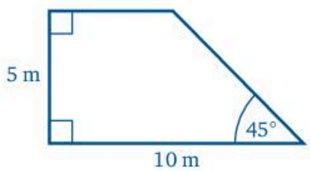

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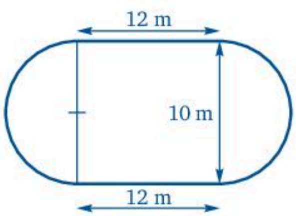
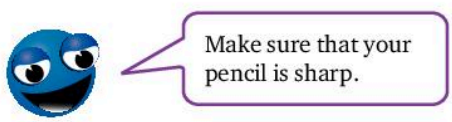
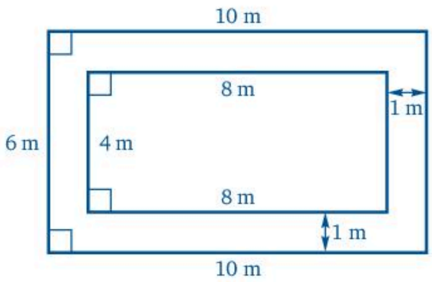




In questions **6** to **10**, choose your own scale.

Choose a scale that gives lines that are long enough to draw easily; in general, the lines on your drawing should be at least 5 cm long. Avoid scales that give lengths involving awkward fractions of a centimetre, such as thirds; $\frac{1}{3}$ cm cannot easily be read from your ruler.

6



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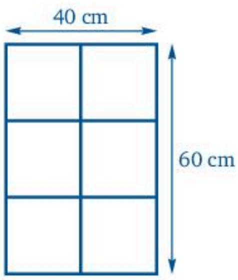
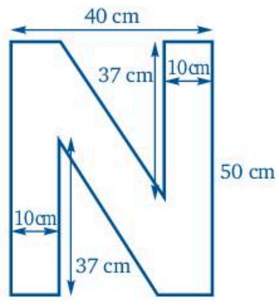


A casement window with equally spaced glazing bars.

8



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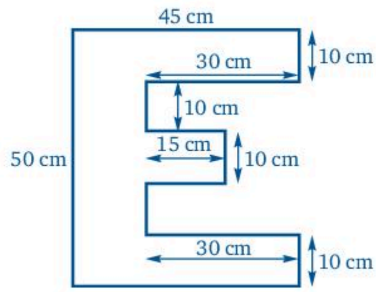
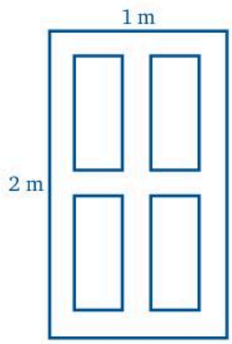
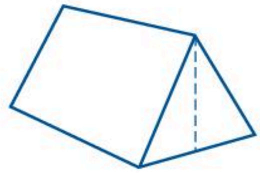


10 A rectangular door with four rectangular panels, each 35 cm by 70 cm, and 10 cm from the edges of the door.



11 A field is rectangular in shape. It measures 300 m by 400 m. A land drain goes in a straight line from one corner of the field to the opposite corner. Using a scale of 1 cm to 50 m, make a scale drawing of the field and use it to find the length of the land drain.



12 The end wall of a ridge tent is a triangle. The base is 2 m and the sloping edges are each 2.5 m. Using a scale of 1 cm to 0.5 m, make a scale drawing of the triangular end of the tent and use it to find the height of the tent.

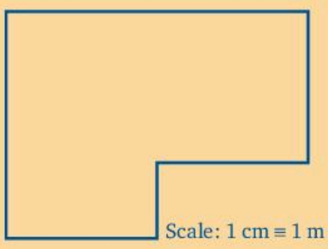
13 The surface of a swimming pool is a rectangle measuring 25 m by 10 m. Choose your own scale and make a scale drawing of the pool. Now compare and discuss your drawing with other pupils.

14 The whole class working together can collect the information for this question. Measure your classroom and make a rough sketch of the floor plan. Mark the position and width of doors and windows. Choosing a suitable scale, make an accurate scale drawing of the floor plan of your classroom.

Scale drawings without measurements

Exercise 11b

This is a scale drawing of Sally's kitchen.



a How long, in centimetres, on the drawing, is Sally's kitchen?

b How long is the actual kitchen?

c How wide, in centimetres, on the drawing is the kitchen at its widest point?

d How wide is the actual kitchen?

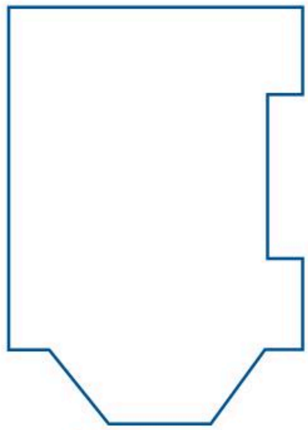
a The length of the drawing of the kitchen is 4 cm.

b Since 1 cm represents 1 m, the length of the actual kitchen is 4×1 m, i.e. 4 m.

c The width of the kitchen in the drawing at its widest point is 3 cm.

d The width of the actual kitchen is $3 \times 1\text{ m} = 3\text{ m}$.

1 This is a scale drawing of Joe's lounge.



Scale: 1 cm ≡ 1 m

a On the drawing, how long is the room, in centimetres, from the back wall to the front of the bay window?

b How far is it in the actual room from the back wall to the front of the bay window?

c How wide is the room on the drawing at the widest point?

d What is the actual width of the room?

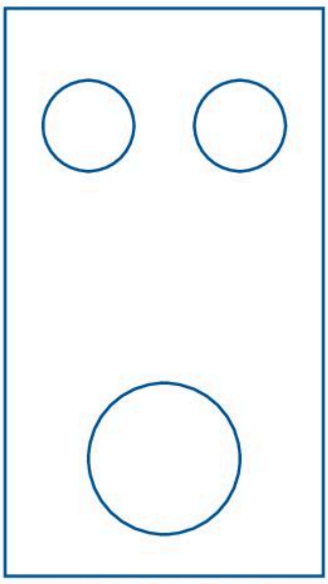
2 This is a scale drawing of one of the set of metal plates needed to manufacture an HK180 earth-moving machine.

What is the actual measurement of

a the length of the plate

b the diameter of one of the small holes

c the radius of the large hole?



Scale: 1 cm ≡ 5 cm

3 These are scale drawings for the goalposts on the soccer and rugby pitches at Windford Recreation Ground.

a On the drawing, how wide, in centimetres, are the goalposts for

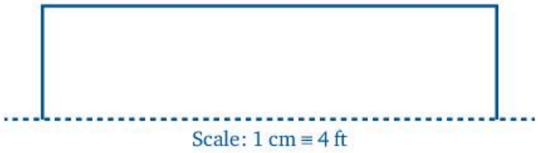
i soccer **ii** rugby?

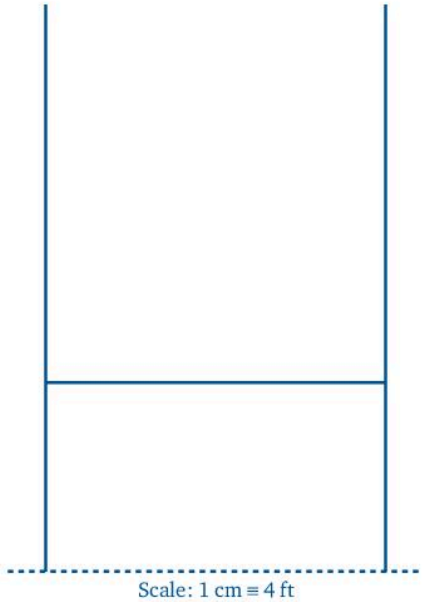
b How far apart are the actual posts for

i soccer **ii** rugby?

c How high is each crossbar actually above the ground?

d How high is one of the rugby posts at the Recreation Ground?





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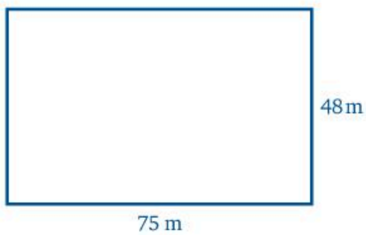


This sketch is of an area of ground that is to be laid with tarmac. It is required to make a scale drawing of the area using a scale of 1 cm to represent 5 m.

a How long should the rectangle be in the drawing?

b How wide should it be?

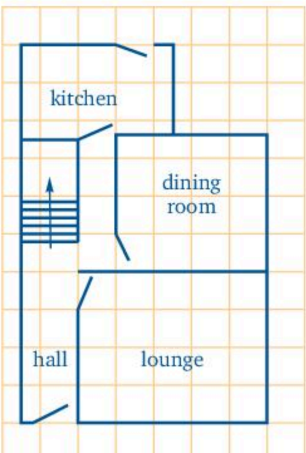
5 Given here is a scale drawing of the ground floor of a house. Each square of the grid has a side of 1 cm and represents 1 m.

a Find the actual length and breadth of

i the lounge **ii** the dining room.

b How wide is the actual hall at its widest part?

c What is the actual area of the lounge?
How much would it cost to carpet at $130 a square metre?



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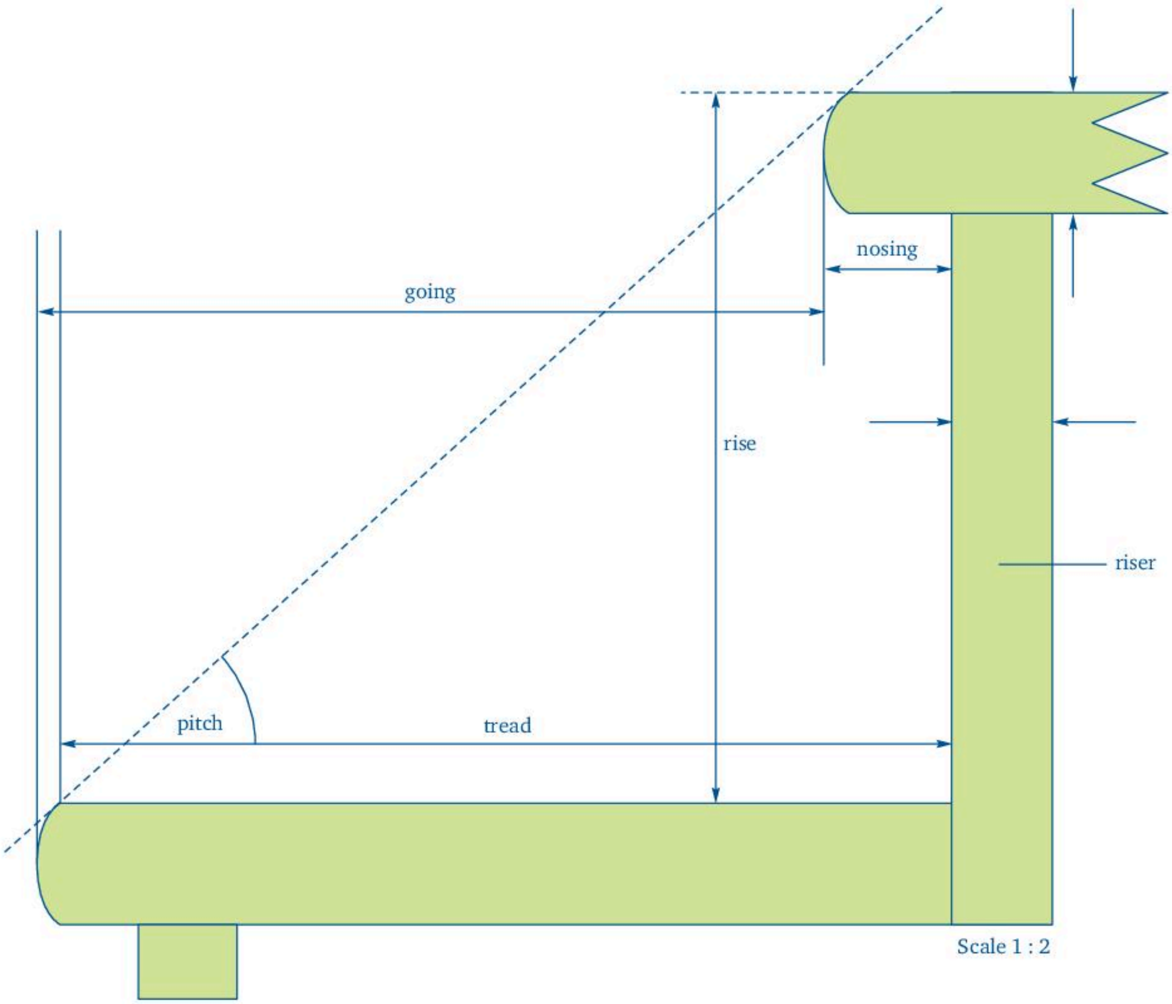


This is an accurate drawing, drawn half-size, of the cross-section of part of a domestic staircase.

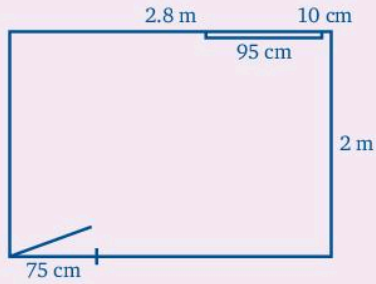
a Use the drawing to find the actual length of
i the tread **ii** the rise **iii** the going **iv** the nosing.

b What thickness of timber has been used for
i the tread **ii** the riser?

c A safety regulation states that the sum of the going plus twice the rise must be more than 500 mm but less than 700 mm. Do these stairs satisfy this regulation?

d The maximum permitted pitch of a staircase is 42°. Use your protractor to check whether or not this staircase complies with the regulation.

Investigation



The sketch shows the measurements of Ken's bathroom. There is only one outside wall and the bottom of the window is 120 cm above the level of the floor. Draw an accurate diagram of the floor, using a scale of 1 cm to 10 cm.

Ken wants a new bathroom suite. The units he would like to install, together with their measurements, are:

- bath: 170 cm × 75 cm
- handbasin: 60 cm × 42 cm, the longest edge against a wall
- shower tray: 80 cm square
- toilet: 70 cm × 50 cm, the shorter measurement against a wall
- bidet: 55 cm × 35 cm, the shorter measurement against a wall.

Using the same scale make accurate drawings of the plans of these units, then cut them out and see if you can place them on your plan in acceptable positions.

If they will not all fit into the room, which unit(s) would you be prepared to do without? Give reasons for your answer.

Is it possible to arrange your chosen units so that all the plumbing is

1 against the outside wall

2 not on more than two walls at right angles, one of which is the outside wall?

Illustrate your answer with a diagram.

Map ratio (or representative fraction)

The *map ratio* of a map is the ratio of a length on the map to the length it represents on the ground. This ratio or fraction is given on most maps in addition to the scale.

Two towns are 40 km apart.

On a map this distance is 20 cm so the map ratio is

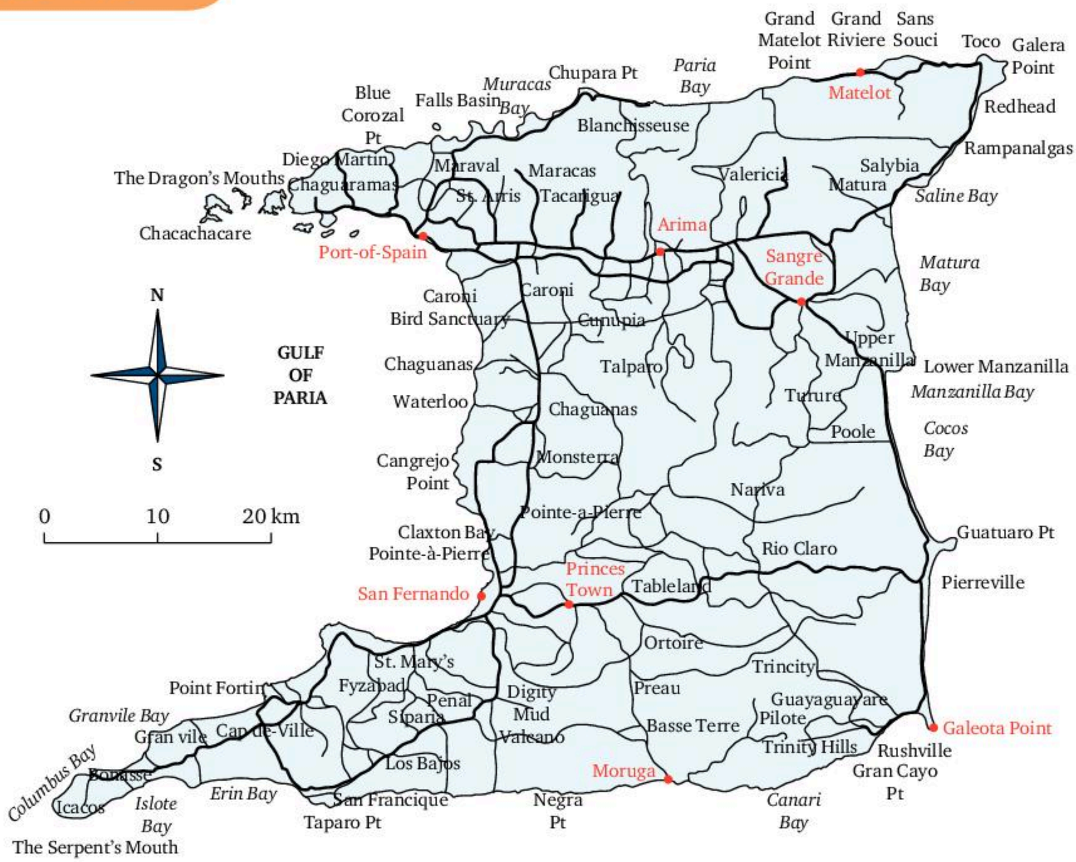
$$\begin{aligned} 20\text{ cm} : 40\text{ km} &= 20\text{ cm} : 4\,000\,000\text{ cm} \\ &= 1 : 200\,000 \end{aligned}$$

So the map ratio is 1 : 200 000.

Any length on the ground is 200 000 times the corresponding length on the map.

Exercise 11c

1



Use this map of Trinidad and the given scale to find

a the representative fraction for this map

b the distance from Port-of-Spain to Arima

c the distance from Princes Town to Port-of-Spain

d the distance from Galeota Point to Princes Town

e the distance from San Fernando to Sangre Grande.

On a map the area of a park is $2.4\,\text{cm}^2$. The scale of the map is $1:20\,000$.

Find the actual area of the park in hectares.

1 cm on the map represents 20 000 cm

so $1\,\text{cm}^2$ on the map represents $20\,000 \times 20\,000\,\text{cm}^2$ on the ground

$$1\,\text{cm}^2 = \frac{20\,000}{100} \times \frac{20\,000}{100}\,\text{m}^2$$
$$= 200 \times 200\,\text{m}^2$$
$$= 40\,000\,\text{m}^2$$
$$2.4\,\text{cm}^2 = 2.4 \times 40\,000\,\text{m}^2$$
$$= \frac{96\,000}{10\,000}\ \text{hectares} \qquad (1\ \text{hectare} = 10\,000\,\text{m}^2)$$
$$= 9.6\ \text{hectares}$$

2 On a map the area of a piece of land is $7.2\,\text{cm}^2$. The scale of the map is 1 : 10 000. What is the actual area of the land in hectares?

3 The scale of the map is 1 : 50 000.

a Find the actual distance between two places that are 15 cm apart on the map.

b The area of a farm is 100 hectares. What is its area on the map?

4 The scale of the map is 1 : 50 000.

a Find the actual distance between two places that are 4.8 cm apart on the map.

b On the map the area of an airfield is $11.2\,\text{cm}^2$.

Find, in square kilometres, the actual area of the airfield.

5 The scale of the model of a building is 5 cm to 150 metres.

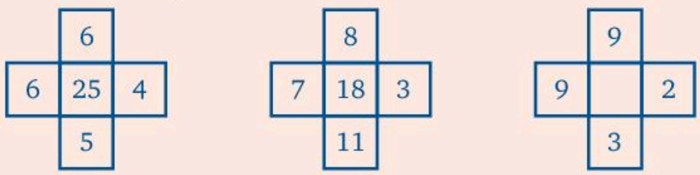
a Find this scale in the form $1 : n$

b How high is the building if the height of the model is 6.4 cm?

c The area of one wall of the building is $1600\,\text{m}^2$.
What is the area of this wall on the model?

Puzzle

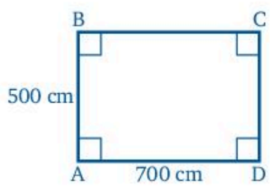
What number should go in the centre of the last cross?



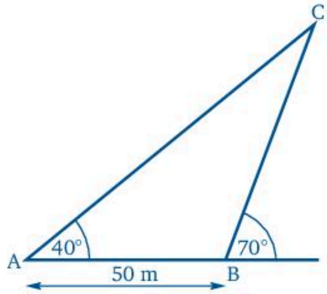
Mixed exercises

Exercise 11d

1 Using a scale of 1 cm to 100 cm, make a scale drawing of the figure below. Use your drawing to find the length of the diagonal AC.



2 Use a scale of 1 cm to 10 m to make a scale drawing of the figure below. Use your scale drawing to find the length of AC.



3 The scale of a map is 1 : 500. A road on the map is 6 cm long. Find the length of the actual road.

4 An aircraft is flying at a height of 1000 m. From a point A on the ground, the angle of elevation of the aircraft is 60°. Use a scale of 1 cm to 100 m, make a scale drawing. Hence find the horizontal distance of the aircraft form the point A.

5 From the top of a cliff, the angle of depression of a boat is 30°. The cliff is 50 m high. Choose your own scale to make a scale drawing and use it to find the distance of the boat from the foot of the cliff.

Exercise 11e

Select the letter that gives the correct answer.

Questions **1** to **3** refer to the scale drawing of the plan of a house where the scale is 1 cm ≡ 1 metre.

1 On the plan the length of the lounge is 4.5 cm. The actual length of the lounge is

A 2.25 m **B** 4 m **C** 4.5 m **D** 5 m

2 On the plan the width of the lounge is 3 cm. The actual width of the lounge is

A 2.5 m **B** 3 m **C** 3.5 m **D** 6 m

3 On the plan the length of the kitchen is 3.5 cm. The actual length of the kitchen is

A 1.5 m **B** 2.5 m **C** 3 m **D** 3.5 m

4 The map ratio of a map is 1 : 25 000. The distance between two villages that are 3.4 cm apart on the map is

A 2.5 km **B** 0.85 km **C** 3.8 km **D** 6.8 km

5 The map ratio of a map is 1 : 1000. A straight road on the ground is 3 km long. The length of the road on the map is

A 3 cm **B** 30 cm **C** 300 cm **D** 3000 cm

Did you know?

Up to the middle of the 18th century, because of the difficulty of measuring longitude, it was impossible to fix your exact position at sea. Because of this, thousands of sailors had perished. The problem was solved by John Harrison (1693–1776), a self-taught Yorkshire clockmaker, who spent 40 years designing and building a clock that would keep perfect time at sea. John Harrison's clock, tested on a journey from the UK to Jamaica, gave an error of 5 seconds or less than 1 nautical mile.

In this chapter you have seen that...

- ✔ the map ratio of a map is the ratio of a length on the map to the length it represents on the ground
- ✔ you can use scale diagrams to find heights and distances.

12 Consumer arithmetic

At the end of this chapter you should be able to...

1 calculate gross and net wages
2 calculate commission and bonus incentives
3 calculate income tax and other deductions from a person's pay
4 calculate the amount due on telephone, water and electricity bills, given the necessary information
5 calculate insurance and assurance premiums.

You need to know...

✔ how to work with decimals and fractions
✔ how to use a calculator
✔ the meaning of percentages
✔ how to find the percentage of a quantity
✔ how to find one quantity as a percentage of another.

Key words

allowances, assurance, bonus, commission, deduction, Education Tax, gross wage, income tax, inflation, insurance, kilowatt-hour, National Housing Trust (NHT), National Insurance Scheme (NIS), net wage, overtime, premium, salary, statutory income

Wages

Everybody who goes to work expects to get paid. Some people are paid an annual amount or *salary*, but most people are paid a wage at a fixed amount per hour.

There is usually an agreed length to the working week and any hours worked over and above this (*overtime*) may be paid for at a higher rate.

If John Duffy works for 37 hours for an agreed hourly rate of \$1400, he receives payment of $\$1400 \times 37$, i.e. \$51 800. This figure is called his *gross* *wage* for the week. From this, statutory deductions are made for such things as *National Insurance Scheme* contributions (NIS), pension contributions and income tax. Other deductions include *Education Tax* and *National Housing Trust* (NHT) payments.

Additional voluntary deductions, which are allowable for tax, may also be made for extra payments into a pension scheme. After all the deductions have been made he receives his *net wage* or 'take-home' pay.

All this information is gathered together by the employer on a pay slip, an example of which is given below.

Company: ABC Ltd		Date: 25 August 2019	
Employee name: J. Morgan		Employee no. 01035932	
Income	Amount	Deductions	Amount
Basic salary	$213000.00	Pension	$12 300.00
Overtime (7h time and half)	$60500.00	NIS	$3 437.50
		NHT	$5 460.00
		Ed Tax	$5 799.66
		Income tax	$33 188.63
		Total Deductions	$60 185.79
Gross Pay	$273500.00	Net Pay	$213 314.21

Exercise 12a

Calculate the gross weekly wage for each of the following factory workers.

	Name	Number of hours worked	Hourly rate of pay
1	E. D. Nisbett	40	$800
2	A. Dexter	35	$850
3	T. Wilson	$38\frac{1}{2}$	$1000
4	A. Smith	44	$1120
5	D. Thomas	$39\frac{1}{2}$	$1250

In the questions that follow, it is assumed that the meal breaks are unpaid.

Sally Green works a five-day week Monday to Friday. She starts work every day at 8 a.m. and finishes at 4.30 p.m. She has 1 hour off for lunch.

How many hours does she work in a week?

Find her gross pay if her rate is \$984 for each hour worked.

Number of hours from 8 a.m. to 4.30 p.m. is $8\frac{1}{2}$.

She has 1 hour off for lunch, so number of hours worked each day is $7\frac{1}{2}$.

Number of hours worked each week is $7\frac{1}{2} \times 5 = 37\frac{1}{2}$

Gross pay for the week $= \$984 \times 37\frac{1}{2} = \$36\,900$

6 Edna Fozzard works a five-day week. She starts work each day at 7.30 a.m. and finishes at 4.15 p.m. She has 45 minutes for lunch and a 10-minute break each morning and afternoon. How long does she actually work

a in a day **b** in a week?

c If her hourly rate is \$932, calculate her gross wage for the week.

7 Martin Jones starts work each day at 7 a.m. and finishes at 4.30 p.m. He has a 45-minute lunch break. How many hours does he work in a normal five-day week? Find his gross weekly wage if his rate of pay is \$1048 per hour.

8 Elaine Mock works 'afternoons'. She starts every day at 2 p.m. and finishes at 10.30 p.m., and is entitled to a meal break from 6 p.m. to 6.45 p.m. How many hours does she work

a in a day **b** in a five-day week?

c Calculate her gross weekly wage if she is paid \$904 per hour.

Mary Killick gets paid \$824 per hour for her normal working week of $37\frac{1}{2}$ hours. Any overtime is paid at time-and-a-half. Find her gross pay in a week when she works $45\frac{1}{2}$ hours.

$$\text{Basic weekly pay} = \$824 \times 37.5 = \$30\,900$$

$$\text{Number of hours overtime} = (45\tfrac{1}{2} - 37\tfrac{1}{2}) \text{ hours} = 8 \text{ hours}$$

Because overtime is paid at time-and-a-half, which is one-and-a-half times the hourly rate, the rate of overtime pay is $\$824 \times 1.5 = \$1286$ per hour

$$\text{Payment for overtime} = \$1286 \times 8 = \$9888$$

$$\begin{aligned}\text{Total gross pay} &= \text{basic pay} + \text{overtime pay}\\ &= \$30\,900 + \$9888\\ &= \$40\,788\end{aligned}$$

9 Tom Shepherd works for a builder who pays $620 per hour for a basic week of 38 hours. If overtime worked is paid at time-and-a-half, how much will he earn in a week when he works for

a 38 hours

b 48 hours

c 50 hours?

10 Maxine Brown works in a factory where the basic hourly rate is $672 for a 35-hour week. Any overtime is paid at time-and-a-half. How much will she earn in a week when she works for 46 hours?

11 Walter Markland works a basic week of $37\frac{1}{2}$ hours. Overtime is paid at time-and-a-quarter. How much does he earn in a week when he works $44\frac{1}{2}$ hours if the hourly rate is $680?

12 Peter Ambler's time sheet showed that he worked 7 hours overtime in addition to his basic 38-hour week. If his basic hourly rate is $632 and overtime is paid at time-and-a-half, find his gross pay for the week.

13 During a certain week Adelle Dookham worked $8\frac{1}{2}$ hours Monday to Friday together with 4 hours on Saturday. The normal working day was 7 hours and any time worked in excess of this was paid at time-and-a-half, with Saturday working being paid at double time. Calculate her gross wage for the week if her basic pay is $864 per hour.

14 Diana Read works a basic week of 39 hours. Overtime is paid at time-and-a-half. How much does she earn in a week when she works $47\frac{1}{2}$ hours if the basic hourly rate is $728?

15 Joan Danby's pay slip showed that she worked $5\frac{1}{2}$ hours overtime in addition to her basic 37-hour week. If her basic rate of pay is $592 and overtime is paid at time-and-a-half, find her gross pay for the week.

16 The timesheet for Anne Stent showed that during the last week in November she worked as follows:

Day	Morning		Afternoon	
	In	Out	In	Out
Monday	7.45 a.m.	12 noon	1.00 p.m.	5.45 p.m.
Tuesday	7.45 a.m.	12 noon	1.00 p.m.	4.15 p.m.
Wednesday	7.45 a.m.	12 noon	1.00 p.m.	4.15 p.m.
Thursday	7.45 a.m.	12 noon	1.00 p.m.	4.15 p.m.
Friday	7.45 a.m.	12 noon	1.00 p.m.	4.15 p.m.

a What is the length of her normal working day?

b How many hours make up her basic working week?

c Calculate her basic weekly wage if the hourly rate is $568.

d How much overtime was worked?

e Calculate her gross wage if overtime is paid at time-and-a-half. Explain how you got your answer.

Commission and bonus incentive

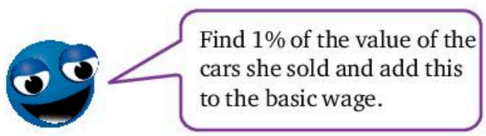
Some workers, such as salesmen and representatives, are paid in a different way. They are given a fairly low basic wage but they also get *commission* on every order they secure. The commission is usually a percentage of the value of the order.

Other workers get paid a fixed wage plus an amount that depends on the amount of work they do, called a *bonus*.

For example, Pete gets paid $12 000 a week plus $40 for every article he produces after the first 30.

Exercise 12b

1 In addition to a basic weekly wage of $14 000, Miss Black receives a commission of 1% for selling second-hand cars. Calculate her gross wage for a week when she sells cars to the value of $5 000 000.



2 A salesman receives a basic wage of \$5000 per week plus commission at 6% on the value of the goods he sells. Find his income in a week when sales amount to \$530 000.

3 Tom Hannah receives a basic wage of \$8500 per week and receives a commission of 2% on all sales over \$100 000. Find his income for a week when he sells goods to the value of \$1 880 000.

4 Sue Renner receives a basic wage of \$12 000 per week plus a commission of 2% on her sales. Find her income for a week when she sells goods to the value of \$2 120 000.

5 Penny George is paid a basic wage of \$10 500 per week plus a commission of $1\frac{1}{2}$% on her sales over \$150 000. Find her income for a week when she sells goods to the value of \$2 130 000.

6 Alan McKay is paid a basic wage of \$15 000 per week plus a commission of 3% on all sales over \$240 000. Find his income for a week when he sells goods to the value of \$1 740 000.

7 In addition to a weekly wage of \$17 000, Olive MacCarthy receives commission of $1\frac{1}{2}$% on the sales of antique furniture. Calculate her gross wage in a week when she sells furniture to the value of \$1 550 000.

8 Don Smith receives a guaranteed weekly wage of \$ 26 000 plus a bonus of \$40 for every circuit board he completes each day after the first 20. During a particular week the number of boards he produced are as follows:

Monday 53, Tuesday 48, Wednesday 55, Thursday 51, Friday 47.
Calculate his gross wage for the week.

9 Audley Davis gets paid \$40 for each article he completes up to 100 per day. For every article above this figure he receives \$45. In a particular week his production figures are

Mon	Tues	Wed	Thurs	Fri
216	192	234	264	219

a How many articles does he produce in the week?
b For how many of these is he paid \$40 each?
c For how many of these is he paid \$45 each?
d Find his earnings for the week.

10 The table shows the number of electric light fittings produced by five factory workers each day for a week.

	Mon	Tues	Wed	Thurs	Fri
Ms Arnold	34	38	34	39	41
Mr Beynon	37	40	37	44	–
Miss Capstick	35	40	43	37	39
Mr Davis	42	45	40	52	46
Mrs Edmunds	39	38	37	35	42

The rate of payment is: \$95 for each fitting up to 20 per day and \$145 for each fitting above 20 per day.

a How many fittings does each person produce in the week?

b For each person find

i how many fittings are paid at \$95 each

ii how many fittings are paid at \$145 each.

c Find each person's income for the week.

d On which day of the week does this group of workers produce the greatest number of fittings? Explain how you got your answer.

Income tax

Income tax is deducted from every person's taxable income, that is, on their earnings over and above their *allowances*.

The basic rate of income tax varies depending on how much the government needs to raise, a typical rate being 25%. Sometimes there is a lower rate for part of your income. There is usually a higher rate once your income exceeds a certain amount. This can change from one year to the next.

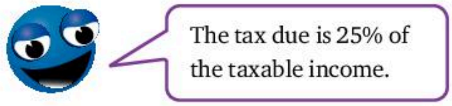
Exercise 12c

1 Jane Axe has a taxable income of \$20 000 a week. She pays income tax on this at 20%.
How much tax must she pay?

2 Freddy Davis has a taxable income of \$520 000 a year. He pays income tax at 25%.
How much tax does he pay?

Assuming that the basic rate of income tax is 25%, find the yearly income tax on the following taxable incomes:

3	\$1 200 000	5	\$650 000	7	\$645 000
4	\$800 000	6	\$1 250 000	8	\$826 000



Find the yearly income tax due on a taxable income of:

9 \$1 000 000 if the basic tax rate is 33%

10 \$800 000 if the basic tax rate is 28%

11 \$1 600 000 if the basic tax rate is 25%

12 \$2 400 000 if the basic tax rate is 32%

Sally's gross pay is \$250 000 a month.

The allowances against income tax total \$130 000. The income tax rate is 23%.

a Calculate Sally's taxable income.

b Work out how much income tax she pays each month.

a The taxable income is the gross pay minus the allowances.

\$250 000 – \$130 000 = \$120 000

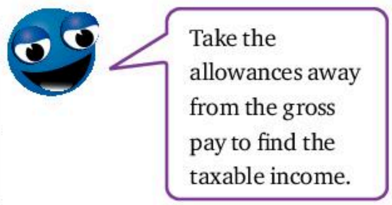
Sally's taxable income is \$120 000.

b Income tax due = 0.23 × \$120 000 = \$27 600

Sally pays \$27 600 income tax each month.

Use the following details to find the income tax due in each case:

	Name	Gross pay	Allowances	Basic tax rate
13	Miss Deats	\$2 400 000	\$600 000	30%
14	Mr Evans	\$3 000 000	\$900 000	30%
15	Mrs Khan	\$4 500 000	\$960 000	30%
16	Mr Amos	\$2 700 000	\$780 000	33%
17	Miss Eyles	\$6 000 000	\$1 425 000	28%



Taxable income and total deductions

You have seen how to work out the tax due on taxable income. We now look at how to work out taxable income from gross pay, and how to work out the total deductions.

Taxable income

When calculating taxable income, the government deducts pension contributions and NIS payments from gross pay. The NIS payment is 2.75% of gross pay, but there is a maximum amount payable of $3437.50 per month.

The income left after these two deductions is called *statutory income*.

statutory income = gross pay – pension contribution – NIS

The tax-free allowance is subtracted from statutory income to give the taxable income.

taxable income = statutory income – tax-free allowance

Other deductions from pay

Some statutory deductions are made from the amount left after income tax has been deducted. The rates change from year to year.

- National Housing Trust is charged at 2% of gross pay.
- Education Tax is charged at 2.25% of statutory income.

Many people choose to have voluntary or non-statutory deductions made from their pay. These are not allowances. One person may choose to have a fixed amount deducted to go into a holiday fund, another to give money to the local sports club or to the particular religious organisation to which they belong. The employer is usually quite happy to help with these deductions.

Exercise 12d

What is the NIS due on gross monthly earnings of **a** $90 000 **b** $150 000?

(Use an NIS rate of 2.75%, with a maximum monthly NIS deduction of $3437.50.)

a $\$90\,000 \times \frac{2.75}{100} = \$2475$

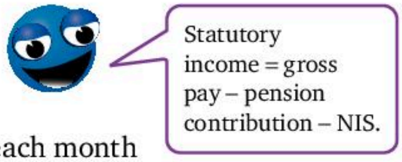
This is less than the maximum of $3437.50, so the NIS due is $2475

b $\$150\,000 \times \frac{2.75}{100} = \$4125$

This is more than the maximum of $3437.50, so the NIS due is $3437.50

In this exercise, use an NIS rate of 2.5%, with a maximum monthly NIS deduction of $3000.

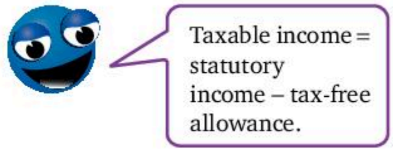
1 Sam's gross pay is $100 000 per month. He pays $8000 per month into his pension fund.



a What is the NIS due?

b What is Sam's statutory income?

2 Alice's gross pay is $256 000 per month. She pays $25 000 each month into her pension fund.



a What is Alice's statutory income?

b The monthly tax-free allowance is $125 000. What is Alice's taxable income?

c The tax rate is 25%. How much income tax does Alice pay each month?

Mrs Carter has gross monthly pay of $200 000. Her monthly deductions for pension and NIS amount to $25 000.

a What is Mrs Carter's statutory income?

b Education tax is calculated at 2.5% of statutory income and NHT at 2% of gross pay. Calculate the deductions for these two taxes.

a Statutory income = $200 000 – $25 000 = $175 000

b Education tax = 0.025 × $175 000 = $4375

NHT = 0.02 × $200 000 = $4000

(Convert the percentages to decimals.)

3 Copy and complete this table. Use the tax rates given in the table.

	Gross monthly pay	Pension and NIS	Statutory income	NHT 2% of gross pay	Ed tax 2.5% of statutory income
Mrs P	$240 000	$27 000			
Mr W	$325 000	$33 000			
Ms M	$100 000	$12 500			
Mr D	$85 000	$8000			
Mrs E	$125 000	$14 750			
Ms B	$140 000	$18 000			

4 Mr Bolt is paid $290 000 a month gross. The table below shows how his net income is calculated. Copy and complete this table. The letters at the right identify the rows.

Monthly gross pay	$290 000	A
Pension contribution	$25 000	B
NIS (lesser of $3000 or 2.5% of A)		C
Statutory income (A – B – C)		D
Tax-free allowance	$125 000	E
Taxable income (D – E)		F
Income tax at 25% of taxable income (25% of F)		G
NHT (2% of A)		H
Ed tax (2.25% of D)		I
Total deductions (B + C + G + H + I)		J
Net income = gross pay – total deductions (A – J)		K

5 Sid's gross weekly wage is $65 000. He decides to pay an extra 2% into his pension fund, an amount which is allowable against his income tax. If the rate of income tax is 25%, how much

a extra a week is paid into Sid's pension fund

b does it actually cost him, taking into account the income tax benefit?

6 Nicki earns $75 000 a week gross. She decides to have 3% of her wage deducted and placed in a holiday fund. There are no tax allowances for this deduction.

a How much a week goes into her holiday fund?

b Calculate the total saved in a year of 52 weeks.

7 Shirley Stewart earns $280 000 a calendar month gross. Her total statutory deductions come to $96 000. Other deductions are 2% of her net pay after tax for her holiday fund and 1% to her church.

Calculate

a her net pay after the statutory deductions

b the amount that goes into her holiday fund

c the amount she gives to her church

d her monthly take-home pay after all the deductions.

8 Tim Corry is paid $368 000 gross a month and pays $48 000 into a pension fund. The table shows how his net income is calculated.

Copy this table and complete it.

Monthly gross pay	$368 000
Pension contribution	
NIS (lesser of 2.5% of gross pay or $3000)	
Statutory income	
Tax-free allowance	$125 000
Taxable income	
Income tax at 25% of taxable income	
NHT at 2% of gross pay	
Ed tax at 2.5% of statutory income	
Total of these deductions (pension, NIS, income tax, NHT, Ed tax)	
Non-statutory deductions, 5% of gross pay	
Net income = gross pay – total of all deductions	

Telephone bills

There are fixed line telephones and mobile phones.

The cost of a telephone call from a fixed line normally depends on three factors:

1 the distance between the caller and the person being called, for example if the call is local or international

2 the time of day and/or the day of the week on which the call is being made

3 the length of the call.

These three factors are put together in various ways to give metered units of time, each unit being charged at a fixed rate.

There is also a fixed monthly charge, called the line rental.

The cost of a telephone call on a mobile phone depends on the contract bought. You can buy a prepaid sim card or a variety of monthly contracts. The monthly cost of a contract depends on what is included.

Exercise 12e

Chris Reynolds' telephone account for the last month showed that his telephone had been used for 546 minutes of local calls. If the line rental charge was \$799 and each minute cost \$1.99, work out how much he must pay for the month on line rental and calls.

$$\text{Cost of 546 minutes at \$1.99 per unit} = 546 \times \$1.99 = \$1086.54$$

$$\text{Line rental charge} = \$799.00$$

$$\text{Total cost} = \text{Cost of minutes} + \text{Line rental}$$

$$= \$1086.54 + \$799.00 = \$1885.54$$

$\therefore$ the line rental and call charge for the month was \$1885.54.

For questions **1** to **5**, the table show the number of minutes used in a month for different types of calls made from a landline. The monthly line rental charge is \$999.

Calls to local landline numbers cost \$2.50 per minute and calls to local mobile numbers cost \$3.10 per minute. International calls cost \$12.00 per minute.

Calculate the total cost of line rental and calls.

	Minutes to landlines	**Minutes to mobiles**	**International minutes**
1	120	305	52
2	15	150	100
3	26	830	14
4	96	185	12
5	42	1700	75

6 Rick's Basic home phone package costs \$1500 per month before tax. This includes line rental and 100 free minutes of calls to local landline numbers. Calls outside the package cost \$2.55 per minute to local landline numbers and \$2.85 per minute to local mobile numbers.

The table lists the calls made over a period of four months.

Month	April	May	June	July
Minutes to local landlines	95	110	95	120
Minutes to local mobiles	102	300	48	120

a Work out the total cost for each month.

b A Super package costing $2100 per month before tax includes 1000 free minutes of calls to local mobile and landline numbers. Would Rick be better off with the Super package? Give a reason for your answer.

Investigation

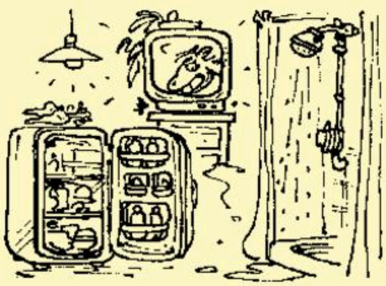
1 Find a bill for a landline telephone. Use the bill to work out how the company charges for the calls made.

The questions you can try to answer are:

- is there a fixed charge irrespective of the number of calls made
- what is the cost for 1 minutes and is there a minimum cost for a call
- is there a cheaper rate for off-peak usage
- what period of time does the bill cover
- are there any taxes applied to the bill?

2 Investigate the different mobile phone contracts available.

Electricity: kilowatt-hours



A *kilowatt-hour* is a unit of energy equivalent to one kilowatt of electrical power used for one hour. The electric company charges one rate for the first number of kilowatt-hours and a higher rate for additional usage.

We all use electricity in some form and we know that some appliances cost more to run than others. For example, an electric water heater costs much more to run than a light bulb. Electricity is sold in units called kilowatt-hours (kW h) and each appliance has a rating that tells us how many kilowatt-hours it uses each hour.

A typical rating for an electric water heater is 2 kW. This tells us that it will use 2 kWh each hour, i.e. 2 units per hour. On the other hand, a low-energy light bulb can have a rating of 10 W. Because 1 kilowatt = 1000 watts (kilo means 'thousand', as we have already seen in kilometre and kilogram), the light bulb uses $\frac{1}{100}$ kWh each hour, or $\frac{1}{100}$ of a unit.

Exercise 12f

How many units (i.e. kilowatt-hours) will each of the given appliances use in 1 hour?

1 a 3 kW water heater

2 a 100 W bulb

3 a $1\frac{1}{2}$ kW sanding machine

4 a 1200 W hair dryer

<u>5</u> a 60 W video recorder

<u>6</u> a 20 W radio

<u>7</u> an 8 kW stove

<u>8</u> a 2 kW dishwasher

With the help of an adult, find the rating of any of the following appliances that you might have at home. The easiest place to find this information is probably from the instructions.

9 an electric kettle

10 the refrigerator

11 the washing machine

12 the television set

13 a bedside lamp

14 the electric stove

How many units of electricity would

15 a 2 kW water heater use in 8 hours

16 a 100 W bulb use in 10 hours

17 an 8 kW stove use in $1\frac{1}{2}$ hours

<u>18</u> a 12 W bulb use in 50 hours

19 a 150 W refrigerator use in 12 hours

20 a 12 W radio use in 12 hours

<u>21</u> an 8 W night bulb use in a week at 10 hours per night

<u>22</u> a 5 W clock use in 1 week?

For how long could the following appliances be run on one unit of electricity?

23 a 25 W bulb

24 a 2 kW water heater

25 a 100 W television set

26 a 360 W electric drill

In the following questions assume that 1 unit of electricity costs \$6.

How much does it cost to run

27 a 100 W bulb for 5 hours

28 a 250 W television set for 8 hours

29 a 3 W clock for 1 week

30 a 3 kW kettle for 5 minutes?

Electricity bills

It is clear from the questions in the previous exercise that lighting from electricity is cheap but heating is expensive.

Domestic electricity bills are calculated by charging every household a fixed amount, together with a charge for each unit used. The amount used is recorded on a meter, the difference between the readings at the beginning and end of a month showing how much has been used.

The following table shows a simplified electricity bill.

Current reading	**Previous reading**	**Description**	**Usage**	**Rate (\$/kWh)**	**Charge (\$)**
14261	13978	Energy first 100 kWh	100	9	900
		Energy next kWh	183	22	4026
		Customer charge			445
		Subtotal			5371
		Fuel & IPP charge	283	18	5094
				Total	**10465**

This bill shows the number of kWh registered on the meter at the beginning and end of the charging period (usually one month). The number of units used is the difference between these two values.

The cost of the first 100 kWh is \$9 per unit and the cost of the remaining units used is \$22 per unit. In addition there is a fixed or Customer charge. Working these amounts out and adding them gives the subtotal.

However, there is also a Fuel and IPP charge for every unit. This covers the cost, which varies from month to month, of the fuel to generate the electricity and the cost of the electricity supplied by Independent Power Producers (IPP).

In the table,

Cost of energy first 100 units $= 100 \times \$9 = \$900$

Cost of energy next units $= 183 \times \$22 = \$4026$

Fuel & IPP charge $= 283 \times \$18 = \$5094$

It is also possible that General Consumption Tax (GCT) may be added to the total to give the amount the householder must pay. You can find the current charges at:

www.myjpsco.com/residential/understanding_your_bill.php

Exercise 12g

Mrs Comerford used 196 units of electricity last month. Apart from a Customer charge of $110 she is charged $5.70 per unit for the first 100 units and $11.00 per unit for the remainder.
How much does her electricity cost for the month?

Cost of 100 units at $5.70 per unit = 100 × $5.70 = $570

Cost of remaining 96 units at $11 per unit = 96 × $11.00 = $1056

Customer charge = $110

Total cost = $570 + $1056 + $110 = $1736

1 For each householder find the total cost of their electricity for the month. For these questions we have neglected the Fuel and IPP charge.

Name	Number of units used	Cost per unit for first 100 units ($)	Cost per unit for remainder ($)	Customer charge ($)	Total
Mr George	250	5.50	11.20	120	
Miss Newton	320	5.60	12.00	130	
Mr Khan	225	6.20	12.50	105	
Mrs Wilton	174	7.20	14.20	135	
Mr Barnes	385	6.45	12.50	128	

2 The government introduced a General Consumption Tax of 10% for all electricity bills. For each person listed in question **1** how much extra will they have to pay?

3 For each householder find the total cost of their electricity for the month. Note that for these householders the Fuel and IPP charge is included.

Name	Number of units used	Cost per unit for first 100 units ($)	Cost per unit for remainder ($)	Customer charge ($)	Fuel & IPP charge per unit ($)	Total
Mrs Wan	240	5.10	10.00	100	16	
Mr Davis	270	6.20	12.60	130	18	
Mr Deats	166	5.75	13.00	100	15	
Mrs Beale	342	5.35	11.70	125	17	

4 The government introduced a General Consumption Tax of 10% for all electricity bills. For each person listed in question **3** work out the total they will now have to pay.

In questions **5** to **7** copy the table and fill in the blanks.

5

Current	Previous	Description	Usage	Rate ($)	Charge ($)
9421	9175	Energy first	100	6.00	600
		Energy next	146	11.50	
		Customer charge			110
		Subtotal			
		Fuel & IPP charge	246	18.00	
				Total	

6

Current	Previous	Description	Usage	Rate ($)	Charge ($)
8432	8156	Energy first	100	6.50	650
		Energy next		12.50	
		Customer charge			130
		Subtotal			
		Fuel & IPP charge	276	17.00	4692
				Total	

7

Current	Previous	Description	Usage	Rate ($)	Charge ($)
	10762	Energy first	120	5.90	
		Energy next	159	13.00	
		Customer charge			140
		Subtotal			
		Fuel & IPP charge	279	19.00	
				Total	

Water bills

In most countries water is supplied by the state as a public service. Businesses and householders pay for this service depending on how much water they consume.

Water usage is measured with a meter. The newest meters show consumption in cubic metres, but the bills show it in litres. One cubic metre is equal to 1000 litres.

The charges for water and sewerage depend on the amount used; this charge increases the more water is used. There is a service charge that depends on the diameter of the pipe that supplies the water. There is also an adjustment to the total bill to reflect costs incurred by the water supplier. This adjustment is a percentage of the total and changes from month to month.

You can find out about these different charges from **www.nwcjamaica.com**

Payment is usually monthly.

The table shows the rates for a typical domestic customer.

Description	Usage	Rate per 1000 litres ($)
Water charge	First 14 000 litres	103.67
	Next 13 000 litres	182.80
	Next 14 000 litres	197.38
Sewerage charge	First 14 000 litres	94.09
	Next 13 000 litres	165.90
	Next 14 000 litres	179.13

Service charge (monthly)	$830.00
Adjustment	14%

In January Mr Smith used 18 cubic metres (18 000 litres) of water. We can use the table to work out his bill. We start with the charge for water.

Charge for the first 14 000 litres is $\$103.67 \times 14 = \$1451.38$

Charge for remaining 4000 litres is $\$182.80 \times 4 = \$731.20$

Water charge is $\$1451.38 + \$731.20 = \$2182.58$

In the same way, we can work out the charge for sewerage.

Charge for the first 14 000 litres is $\$94.09 \times 14 = \$1317.26$

Charge for remaining 4000 litres is $\$165.90 \times 4 = \$663.60$

Sewerage charge is $\$1317.26 + \$663.60 = \$1980.86$

Therefore the charges before the adjustment are

$$\text{water use} + \text{sewerage} + \text{service charge} = \$2182.58 + \$1980.86 + \$830.00$$
$$= \$4993.44$$

$$\text{Adjustment} = 14\% \text{ of } \$4993.44 = \$4993.44 \times 0.14$$
$$= \$699.08 \text{ (to the nearest cent)}$$

$$\text{Mr Smith's bill} = \$4993.44 + \$699.08 = \$5692.52$$

Exercise 12h

Use the table on page 216 to calculate the charges. Give answers that are not exact to the nearest cent.

The water charge is just the cost of supplying the water. It does not include the sewerage charge, the service charge or adjustments.

1 Mr Barnes used 10 000 litres of water in June.
Calculate Mr Barnes's water charge for June.

2 Mrs Khan used 5000 litres of water in April.
Calculate Mrs Khan's water charge for April.

3 Mrs Weeks used 20 000 litres of water in February.
Calculate Mrs Weeks's water charge for February.

4 Mr Anthony used 30 000 litres of water in August.
Calculate Mr Anthony's water charge for August.

5 Mr Layne used 10 000 litres of water in December.
- **a** Calculate Mr Layne's charges for water, sewerage and service charge before the adjustment.
- **b** Calculate Mr Layne's bill.

6 Mrs Arnold used 16 000 litres of water in September.
- **a** Calculate Mrs Arnold's charges for water, sewerage and service charge before the adjustment.
- **b** Calculate Mrs Arnold's bill.

7 Mr Amish used 28 000 litres of water in May.
- **a** Calculate Mr Amish's charges for water, sewerage and service charge before the adjustment.
- **b** Calculate Mr Amish's bill.

8 At the end of June the meter reading on Mrs Wright's water meter was 38 cubic metres. At the end of July the reading was 51 cubic metres.
Calculate Mrs Wright's bill for July. Explain how you got your answer.

Puzzle

I have two old coins. One is marked Elizabeth I and the other George VI. A knowledgeable friend told me that one was probably genuine but that the other was definitely a fake. How could she be so certain?

Insurance and assurance

Insurance is a financial product we buy from an insurance company to cover an event we hope will never happen, such as a fire, burglary or accident with the car. We pay the company an amount of money called a *premium*. The company will then pay us to restore the damage or replace the object if we lose something. Insurance is available for almost any risk but the higher the risk the greater the premium. At the most basic level most people would insure their property against fire or burglary. They may also decide to use it to cover the possible loss of such things as jewellery, paintings, china or furniture.

Often an insurance company will offer a lower premium if they do not have to pay the whole of the claim. They could ask the policy-holder to pay the first $10 000. This is called the *excess*.

Assurance is for something that is certain to happen, for example a person's death.

Exercise 12i

Mrs Owen wants to insure the contents of her home, valued at $3000 000 against loss or damage.

One insurance company quotes a premium of $16 p.a. per $1000 worth of property.

Calculate the annual premium payable.

Premium payable = $\$16 \times 3000 = \$48\,000$

If the company had offered a premium of \$12.50 p.a. per \$1000, with an excess of \$10 000, would Mrs Owen have been better off?

Premium payable = \$12.50 × 3000 = \$37 500

The cost of the damage to Mrs Owen is therefore \$10 000 + \$37 500 = \$47 500

She is therefore better off the taking the policy that has an excess that she pays.

1 Calculate the annual premium on the contents of a property valued at \$2.5 m, if the rate is \$25 per annum per \$1000 insured.

2 Calculate the annual insurance premium on a house valued at \$7 million if the rate is \$36 per annum per \$1000.

What would the premium be next year if the premium is linked to *inflation* and the rate of inflation is 5%?

3 Phil's house is valued at \$6.5 million and the contents at \$2.5 million. He wishes to cover \$1 million of the contents under the 'all risks' section.

The rates for insurance are \$10 per \$1000 of the property, \$30 per \$1000 for the contents and \$70 per \$1000 for 'all risks' cover.

Calculate the total premium due **a** per annum **b** per week.

4 Find the cost of insuring my bicycle, valued at \$60 000, if the rate is \$180 per \$1000.

The table gives the premiums payable per person for holiday insurance.

Use the table to answer questions **5** to **7**.

Number of days	Adult	Child under 16
up to 8	\$4800	\$2800
9 to 14	\$6000	\$3300
15 to 20	\$8000	\$3600

5 Calculate the premium for a family of four – father, mother and two children under 16 – to go on a 10-day holiday.

6 Calculate the premium for a family of three – father, mother and one child under 16 – to go on a 15-day holiday.

7 Mr and Mrs Peacock take their three children, aged 7, 12 and 17 for a 12-day holiday.

Calculate the cost of their holiday insurance.

8 A company sells motor insurance and offers the following No Claims discounts:

1st renewal 20%

2nd renewal 30%

3rd renewal 40%

4th renewal 50%

Tim Chesham is quoted an annual premium of $80 000 when he takes out insurance on his car.

Calculate has annual premium on

a the first renewal
b the second renewal
c the fourth renewal.

Explain how you got your answers.

Mixed exercises

Exercise 12j

1 Tom Watkins worked $38\frac{1}{2}$ hours at $720 per hour.

Find his gross wage.

2 One week Elaine Eastman worked her basic week of 37 plus 5 hours overtime.

If her basic hourly rate is $950 and she is paid time-and-a-half for overtime, calculate her gross pay for the week.

3 Shirley Green receives a basic weekly wage of $20 000 plus commission of 3% on her sales.

Find her gross income for a week when she sells goods to the value of $3100 000.

4 When the rate of income tax is 26%, find the tax due on Sarah's annual taxable income of $1900 000.

5 John Swift earned $154 000 gross last month. His pension contribution was $26 000 and he paid $3000 in NIS.

a Calculate John's statutory income.
b The tax-free allowance is $105 000 and the income tax rate is 26%. How much income tax did John pay?
c Other deductions amounted to $6870. Calculate the total of all deductions from gross pay.
d Find John's net income.

6 Rosmah's gross monthly income is $112 000. Calculate how much is deducted for

a National Insurance Scheme at 2.75%

b National Housing Trust at 2%.

7 Ed Taylor's gross weekly wage is $75 000. He agrees to pay 0.5% of his net pay to his favourite charity. If deductions from his gross pay amount to $23 500, how much does his favourite charity receive?

8 Norma Khan's monthly electricity bill shows that she has used 105 units at $9.40 a unit.

If there was a customer charge of $420, calculate Norma's bill for the month.

9 a How many units (kWh) will a 40 W bulb use in 12 hours?

b How many units will a 3 kW water heater use in 6 hours?

10 Mrs Franka's electricity bill showed that she had used 420 units. The first 100 units were priced at $11 a unit and the remainder at $35 per unit. In addition there was a customer charge of $542. Calculate the total cost of Mrs Franka's electricity.

Exercise 12k

In this exercise write the letter that gives the correct answer.

1 Rohan works $7\frac{3}{4}$ hours each day from Monday to Friday. For this he is paid $1100 an hour.

His gross weekly pay is

A $40 625 B $42 625 C $43 625 D $48 650

2 Peg works from 7.30 a.m. to 4.30 p.m. each day from, Monday to Friday. She gets $\frac{3}{4}$ hour for lunch and a 15-minute break both morning and afternoon.

The number of hours she works in a week is

A $38\frac{1}{4}$ B $38\frac{3}{4}$ C 39 D $39\frac{1}{4}$

3 Miss Brown is paid a basic weekly wage of $74 000 plus commission at 5% on the value of the goods she sells. Find her gross pay in a week when she sells goods to the value of $1 300 000.

A $108 800 B $116 000 C $127 000 D $139 000

4 How much income tax is due on a taxable income of $950 000 when the rate of income tax is 32%?

A $285 000 B $304 000 C $313 500 D $340 000

5 How much income tax is due on a gross income of $1000 000 if 35% of this amount is not taxable? Assume that the rate of income tax is 30%.

A $195 000 B $205 000 C $215 000 D $225 000

6 My telephone bill shows that I used 970 minutes at $7 a minute and that the monthly line rental is $3750. The total I must pay is

A $9460 B $10 450 C $10 540 D $11 450

7 The number of units of electricity used by a $1\frac{1}{2}$ kW electric water heater when it is used for 18 hours is

A 2.7 B 13.5 C 20.5 D 27

8 Last month I used 315 units of electricity. These units were charged at $804 per 100 units.

In addition there is a customer charge of $280. The cost of my electricity for the month is

A $2532.60 B $2812.60 C $3012.60 D $3112.60

9 Calculate the annual insurance premium on a property valued at $3.7m if the rate of insurance tax is $24 p.a. per $1000.

A $46 800 B $48 600 C $88 800 D $88 400

10 Mr Johnston used 18 000 litres of water last month. If water costs $60 per 1000 litres for the first 14 000 litres, $92 per 1000 litres for the next 13 000 litres, and there is a service charge of $420, the total cost of water for the month is

A $1628 B $1720 C $1835 D $1916

In this chapter you have seen how to...

- ✔ calculate gross and net wages
- ✔ determine commission and bonus incentives
- ✔ calculate income tax and other deductions from a person's pay
- ✔ check utility bills
- ✔ calculate the number of units of electricity used by an appliance in a given time
- ✔ calculate insurance and assurance premiums.

13 Enlargement and reflection

At the end of this chapter you should be able to...

1 find, by drawing, the centre of enlargement, given an object and its image
2 calculate the scale factor of an enlargement, given an object and its image
3 draw the image of a given figure, knowing the scale factor and the centre of enlargement
4 classify an image in terms of relative size and position, depending on whether the scale factor is greater than 1 or less than 1
5 find the image of an object given the object and the mirror line
6 find the mirror line given an object and its image.

You need to know...

- ✔ the sum of the three angles in a triangle
- ✔ how to find one quantity as a fraction of another
- ✔ how to draw x and y axes and plot points
- ✔ how to find the equation of a straight line given two points on the line
- ✔ how to construct the perpendicular bisector of a line.

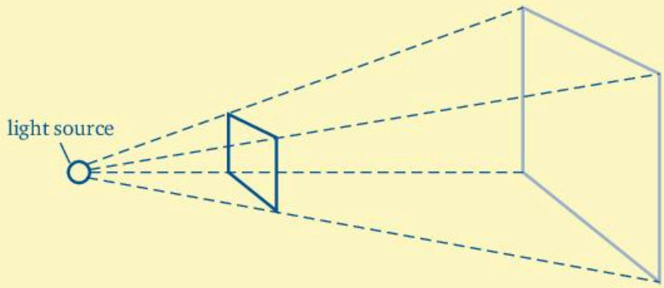
Key words

centre of enlargement, enlargement, fractional scale factor, guidelines, image, invariant point, mirror line, object, reflection, scale factor

Enlargements

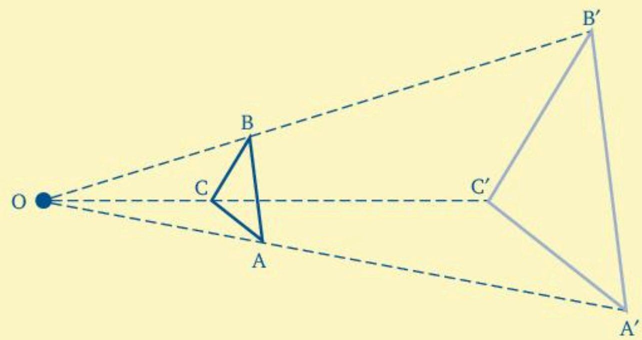
All the transformations we have used so far (i.e. reflections, translations and rotations) have moved the *object* and perhaps turned it over to produce the *image*, but its shape and size have not changed. Next we come to a transformation that keeps the shape but alters the size.

Think of the picture thrown on the screen when a projector is used.



The picture on the screen is the same as the original picture but it is very much bigger.

We can use the same idea to enlarge any shape.



△A′B′C′ is the image of △ABC under an *enlargement*, centre O.

O is the *centre of enlargement*.

We call the dotted lines *guidelines*.

Centre of enlargement

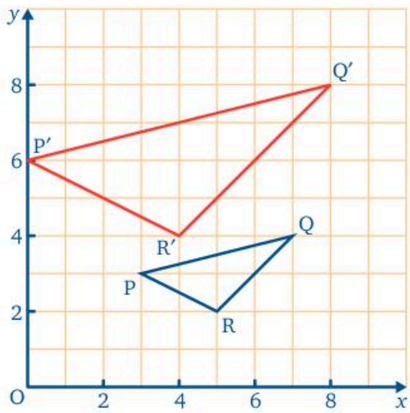
In all the questions in Exercise 13a, one triangle is an enlargement of the other.

Exercise 13a

1 Copy the diagram using 1 cm to 1 unit. Draw P′P, Q′Q and R′R and continue all three lines until they meet.

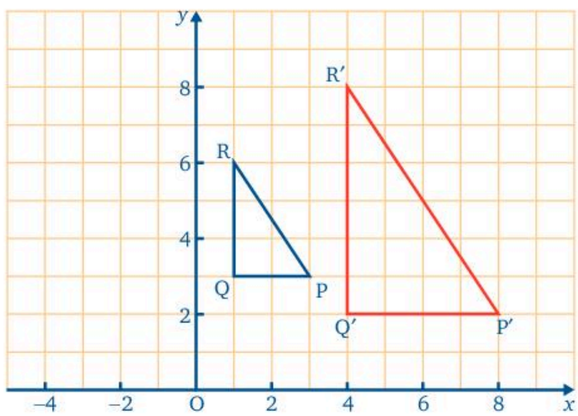
The point where the lines meet is called the centre of enlargement.

Give the coordinates of the centre of enlargement.



Repeat question **1** using the diagrams in questions **2** and **3**.

2



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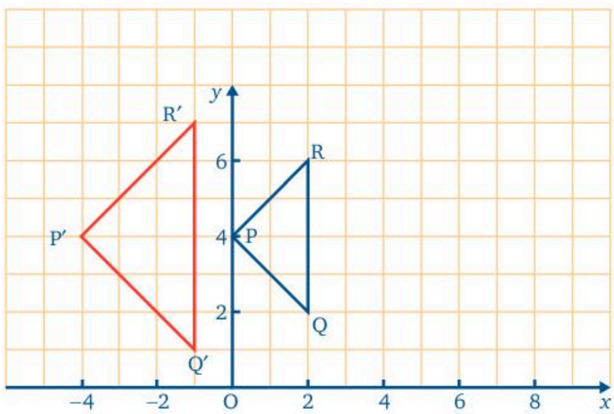


4 In questions **1** to **3**, name pairs of lines that are parallel.

5 Draw axes for x and y from 0 to 9 using 1 cm as 1 unit.

Draw △ABC: A(2, 3), B(4, 1), C(5, 4).

Draw △A′B′C′: A′(2, 5), B′(6, 1), C′(8, 7).

Draw A′A, B′B and C′C and extend these lines until they meet.

a Give the coordinates of the centre of enlargement.

b Measure the sides and angles of the two triangles. What do you notice?

6 Repeat question **5** with △ABC: A(8, 4), B(6, 6), C(6, 4) and △A′B′C′: A′(6, 2), B′(0, 8), C′(0, 2).

7 Draw axes for x and y from 0 to 10 using 1 cm as 1 unit.

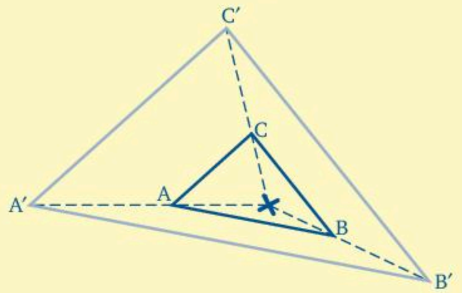
Draw △XYZ with X(8, 2), Y(6, 6) and Z(5, 3) and △X′Y′Z′ with X′(6, 2), Y′(2, 10) and Z′(0, 4).

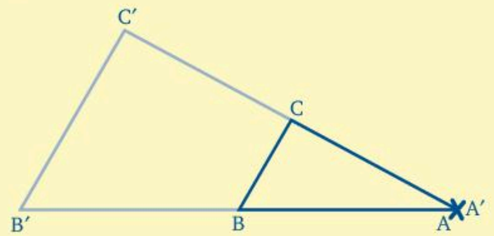
Find the centre of enlargement and label it P.

Measure PX, PX′, PY, PY′, PZ, PZ′. What do you notice?

The centre of enlargement can be anywhere, including a point inside the object or a point on the object.

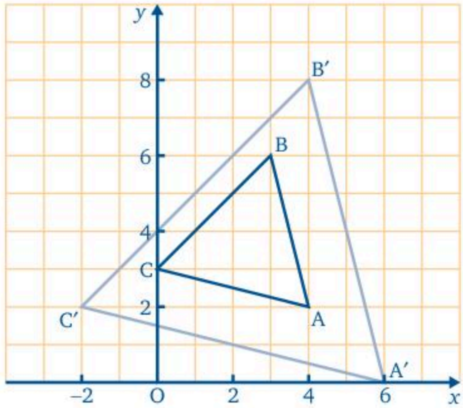
The centres of enlargement in the diagrams below are marked with a cross.



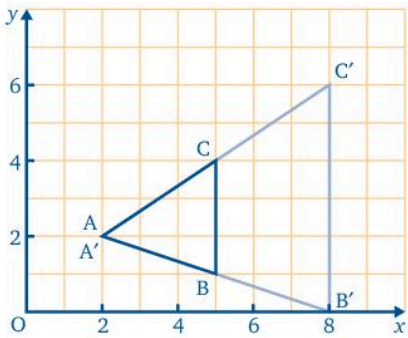


Exercise 13b

1 Copy the diagram using 1 cm as 1 unit. Draw A′A, B′B and C′C and extend the lines until they meet. Give the coordinates of the centre of enlargement.



2 In the diagram below, which point is the centre of enlargement?



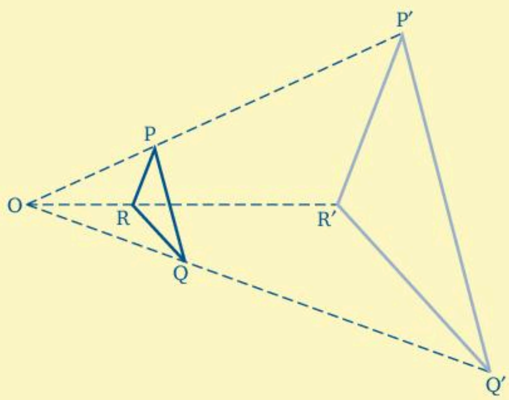
3 Draw axes for x and y from –3 to 10 using 1 cm as 1 unit. Draw △ABC with A(4, 0), B(4, 4) and C(0, 2). Draw △A′B′C′ with A′(5, –2), B′(5, 6) and C′(–3, 2). Find the coordinates of the centre of enlargement.

4 Repeat question **3** with A(1, 4), B(5, 2), C(5, 5) and A′(–3, 6), B′(9, 0), C′(9, 9).

Scale factors

If we measure the lengths of the sides of the two triangles PQR and P′Q′R′ and compare them, we find that the lengths of the sides of △P′Q′R′ are three times those of △PQR.

We say that △P′Q′R′ is the image of △PQR under an enlargement, centre O, with *scale factor* 3.



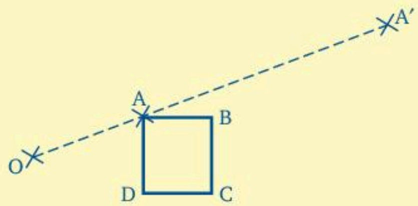
Finding an image under enlargement

If we measure OR and OR′ in the diagram on page 224, we find R′ is three times as far from O as R is. This enables us to work out a method for enlarging an object with a given centre of enlargement (say O) and a given scale factor (say 3).

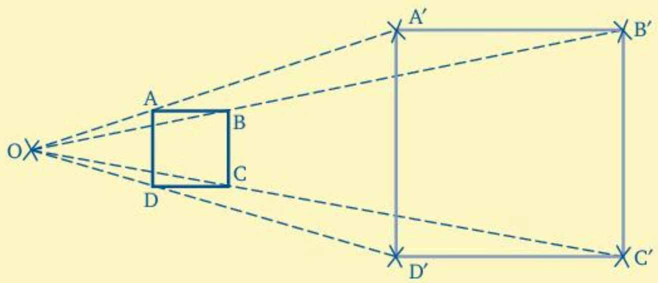
Measure OA. Multiply it by 3. Mark A′ on the guideline three times as far from O as A is.

$OA' = 3 \times OA$

Repeat for B and the other vertices of ABCD.

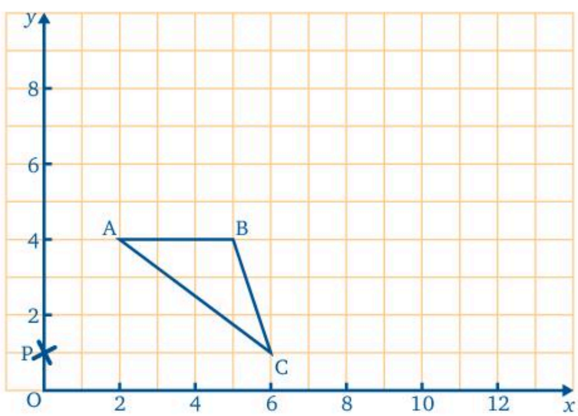


Then A′B′C′D′ is the image of ABCD. To check, measure A′B′ and AB. A′B′ should be three times as large as AB.

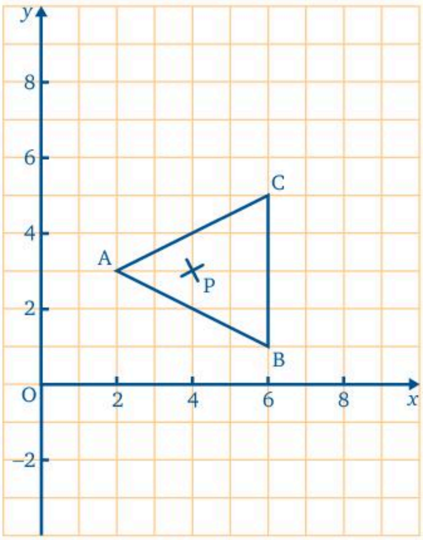


Exercise 13c

1 Copy the diagram using 1 cm as 1 unit. P is the centre of enlargement. Draw the image of △ABC under an enlargement scale factor 2.



2 Repeat question **1** using this diagram.



In questions **3** to **6**, draw axes for *x* and *y* from 0 to 10, using 1 cm as 1 unit.
In each case, find the image A′B′C′ of △ABC using the given enlargement.
Check by measuring the lengths of the sides of the two triangles.

3 △ABC: A(3, 3), B(6, 2), C(5, 6).

Enlargement with centre (5, 4) and scale factor 2.

4 △ABC: A(1, 2), B(3, 2), C(1, 5).

Enlargement with centre (0, 0) and scale factor 2.

What do you notice about the coordinates of A′ compared with those of A?

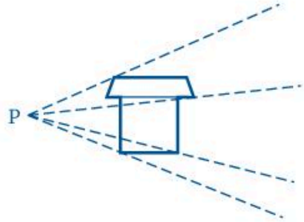
5 △ABC: A(2, 1), B(4, 1), C(3, 4).

Enlargement with centre (1, 1) and scale factor 3.

6 △ABC: A(1, 2), B(7, 2), C(1, 6).

Enlargement with centre (1, 2) and scale factor $1\frac{1}{2}$.

7 On plain paper, mark a point P near the left-hand edge. Draw a small object (a pin man perhaps, or a square house) between P and the middle of the page. Using the method of enlargement, draw the image of the object with centre P and scale factor 2.

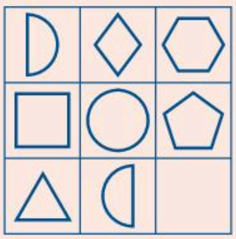


8 Repeat question **7** with other objects and other scale factors. Think carefully about the space you will need for the image.

9 Draw axes for x and y from 0 to 10 using 1 cm as 1 unit. Draw △ABC with A(2, 2), B(5, 1) and C(3, 4). Taking the origin as the centre of enlargement and a scale factor of 2, draw the image of △ABC by counting squares and without drawing the guidelines.

10 Draw axes for x and y from 0 to 8 using 1 cm as 1 unit. Draw △ABC with A(1, 2), B(5, 2) and C(2, 5). Taking (3, 2) as the centre of enlargement and a scale factor of 2, draw the image △ABC by counting squares and without drawing the guidelines.

? Puzzle



Which of these shapes goes into the blank box?

Fractional scale factors

We can reverse the process of enlargement and shrink or reduce the object, producing a smaller image. If the lengths of the image are one-third of the lengths of the object, then the scale factor is $\frac{1}{3}$.

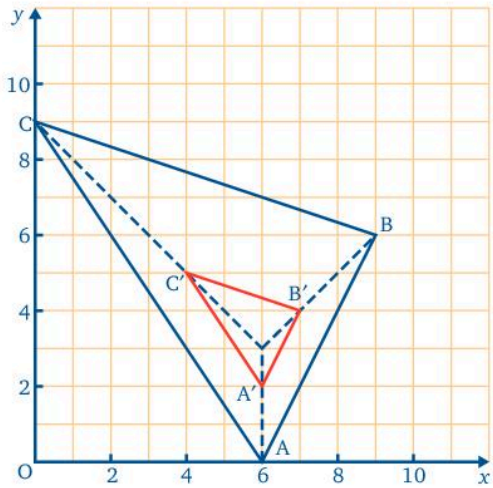
There is no satisfactory word to cover both enlargement and shrinking (some people use 'dilation' and some 'scaling') so 'enlargement' tends to be used for both. An enlargement may therefore be defined as a transformation which maps an object on to an image of similar shape. If the scale factor is less than 1, the image is smaller than the object. If the scale factor is greater than 1, the image is larger than the object.

A scale factor between 0 and 1 is known as a *fractional scale factor*.

Exercise 13d

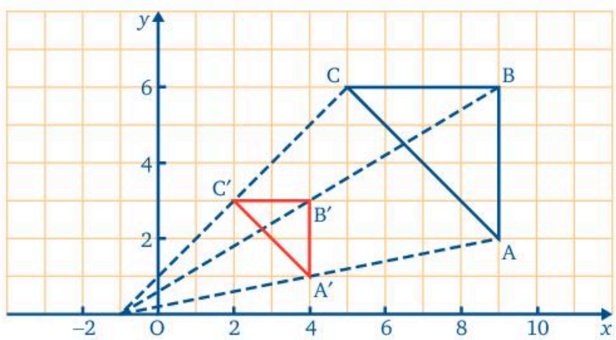
In questions **1** to **4**, △A′B′C′ is the image of △ABC. Give the centre of enlargement and the scale factor.

1



To find the scale factor, find the length of a side on the image as a fraction of the length of the corresponding side on the object.

2



3 Draw axes for x and y from –2 to 8 using 1 cm as 1 unit. Draw △ABC with A(–1, 4), B(5, 1) and C(5, 7), and △A′B′C′ with A′(2, 4), B′(4, 3) and C′(4, 5).

4 Draw axes for x and y from 0 to 9 using 1 cm as 1 unit. Draw △ABC with A(1, 2), B(9, 2) and C(9, 6), and △A′B′C′ with A′(1, 2), B′(5, 2) and C′(5, 4).

In questions **5** and **6**, draw axes for x and y from –1 to 11 using 1 cm as 1 unit. Find the image of △ABC under the given enlargement.

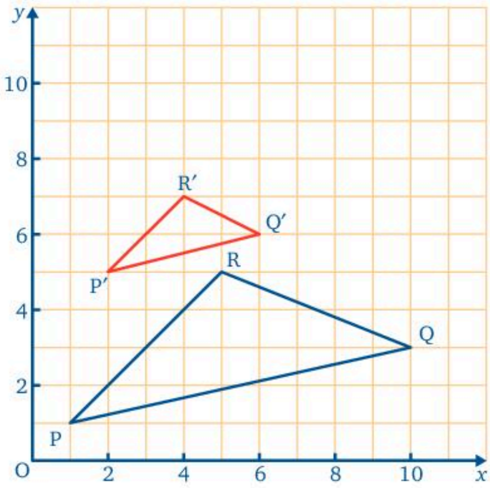
5 △ABC: A(9, 1), B(11, 5), C(7, 7). Centre (–1, 1), scale factor $\frac{1}{2}$.

6 △ABC: A(4, 0), B(10, 9), C(1, 6). Centre (4, 3), scale factor $\frac{1}{3}$.

Exercise 13e

Select the letter that gives the correct answer.

Use this diagram for questions **1** and **2**.



1 Triangle P′Q′R′ is the image of triangle PQR under an enlargement.

Join PP′, QQ′ and RR′. These lines show that the coordinates of the centre of enlargement are

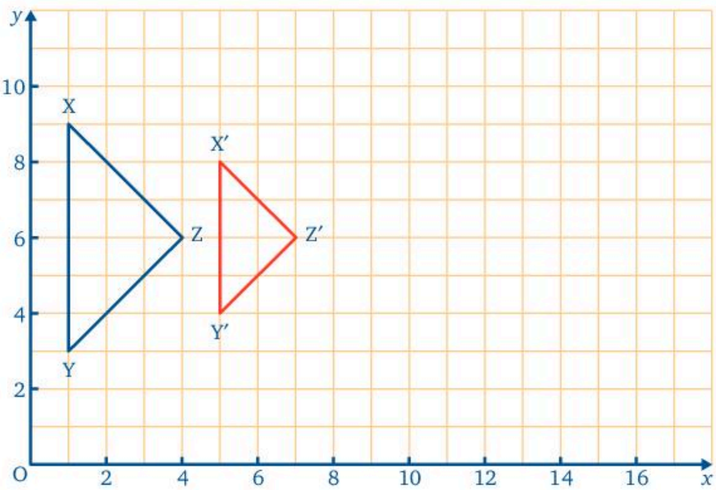
A (3, 8) **C** (4, 8)

B (3, 9) **D** (4, 9)

2 The scale factor that enlarges triangle PQR to triangle P′Q′R′ is

A $\frac{1}{4}$ **B** $\frac{1}{3}$ **C** $\frac{1}{2}$ **D** $\frac{2}{3}$

Use this diagram for questions **3** and **4**.



3 Triangle X′Y′Z′ is the image of triangle XYZ under an enlargement. The coordinates of the centre of enlargement are

A (12, 6) **B** (13, 6) **C** (14, 6) **D** (15, 6)

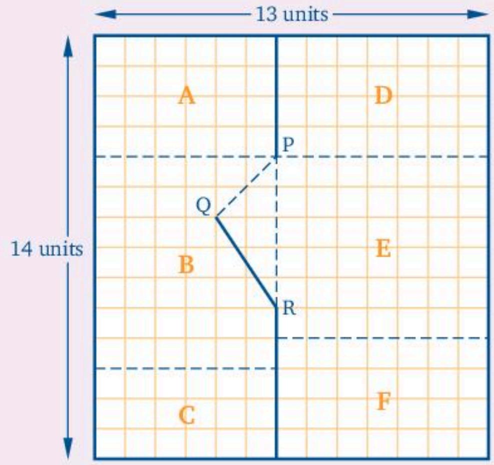
4 The scale factor that enlarges triangle XYZ to triangle X′Y′Z′ is

A $\frac{1}{3}$ **B** $\frac{1}{2}$ **C** $\frac{2}{3}$ **D** $\frac{3}{2}$

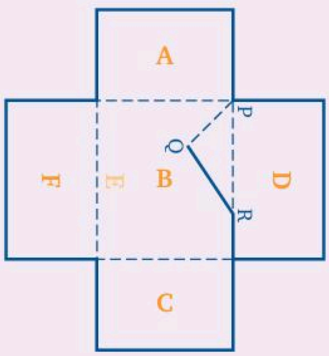
Investigation

Daniel designs packaging. The sketch shows a design that will fold to make a package suitable for sending books of various sizes by post. The design can be produced in several different sizes if it is enlarged by a given scale factor.

The basic rectangle is divided into six smaller rectangles, marked A, B, C, D, E and F on the sketch. It is cut along the heavy lines and fold marks are pressed into it along the broken lines.



By folding along PQ and PR, you will find that rectangle B now fits over rectangle E and this acts as the base of the package. The remaining four rectangles fold up and over, allowing books of different thicknesses to be packed.



1 On 5 mm squared paper, draw an enlargement of the diagram with a scale factor of 2.

2 Cut it out and fold it to show how it is used.

3 If you have a much larger sheet of paper or card, repeat parts **1** and **2** using a larger scale factor, for example, 4.

4 **a** Can the fold PQ be drawn at any angle?

b Can the cut QR be made at any angle?

Give reasons for your answers.

5 What is the ratio of the length of the original rectangle to its breadth?
If you alter this ratio, can you still make a satisfactory package?
Investigate.

6 Can you improve on the design?

Reflections

In Grades 7 and 8 we saw how to draw the *reflection* of an object in a *mirror line* and how to find the mirror line given an object and its image. We revise that work here.

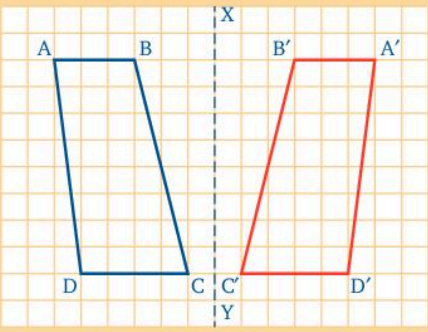
When we reflect an object in a line (called the mirror line), the object and its image together form a symmetrical shape.

Exercise 13f

Reflect quadrilateral ABCD in the mirror line XY.

Mark the image A′B′C′D′.

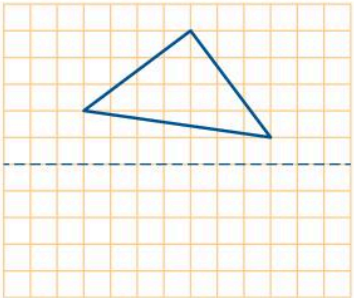
Remember that a point and its image are equidistant from the mirror line. Also a line joining a point and its image is perpendicular to the mirror line.



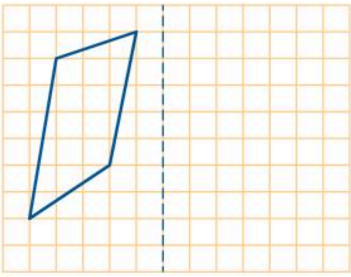
A′B′C′D′ is the reflection of ABCD in the mirror line XY.

In questions **1** to **6** copy the object and the mirror line on to square grid paper and draw the image of each object.

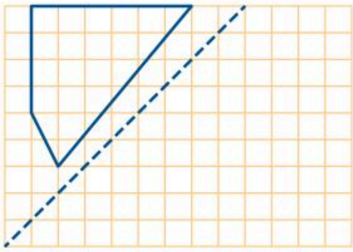
1



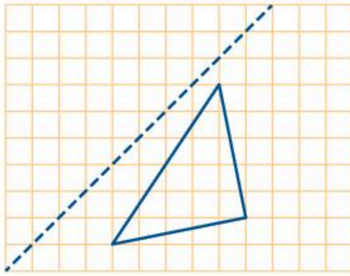
2



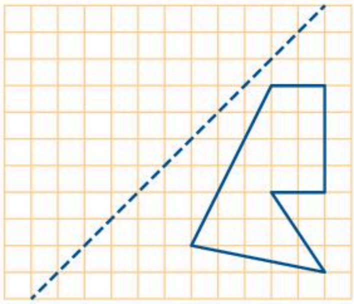
3



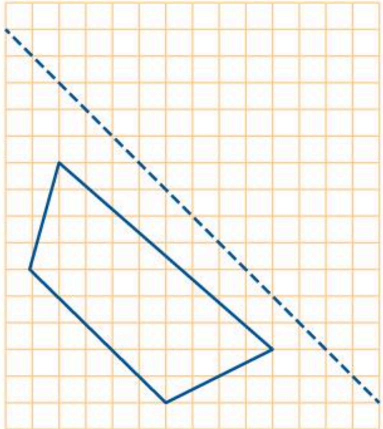
5



4



6

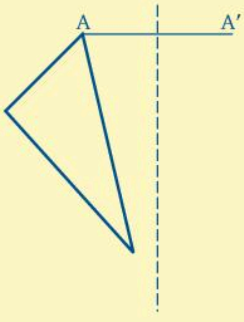


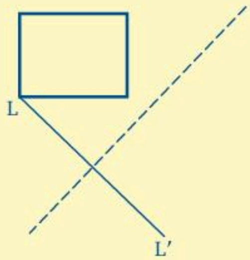
In questions **7** and **8** use graph paper and a scale of 1 cm to 1 unit.

7 Draw axes for x and y from –5 to 1. Draw rectangle WXYZ: W is (–3, –1), X is (–3, –2), Y is (–5, –2) and Z is (–5, –1). Draw the mirror line $y = x$. Draw the image W′X′Y′Z′ when WXYZ is reflected in the mirror line.

8 Draw axes for x and y from –1 to 9. Plot the points A(2, 1), B(5, 1), C(7, 3) and D(4, 3). Draw the parallelogram ABCD and its image by reflection in the line $y = x$.

Finding the mirror line





We first met this diagram in Grade 7. We know from these diagrams and from the previous work that the object and the image points are equal distances from the mirror line, and the lines joining them (AA′ and LL′) are perpendicular, i.e. at right angles, to the mirror line.

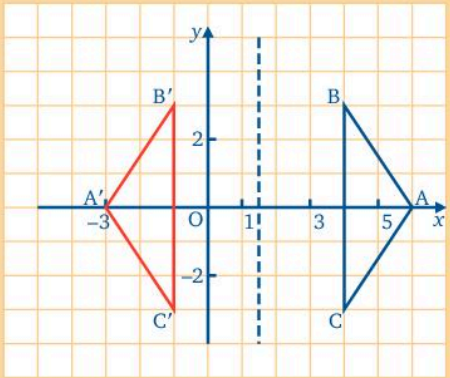
Exercise 13g

Find the mirror line if △A′B′C′ is the image of △ABC.

The mirror line is halfway between an object point and its image and perpendicular to the line through them.

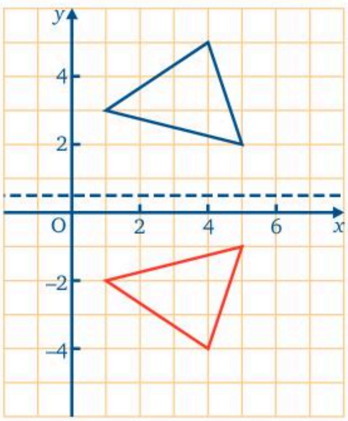
So the mirror line is halfway between B and B′ and perpendicular to the line BB′. Check that it also goes through the midpoint of CC′.

The mirror line is the line $x = 1.5$.

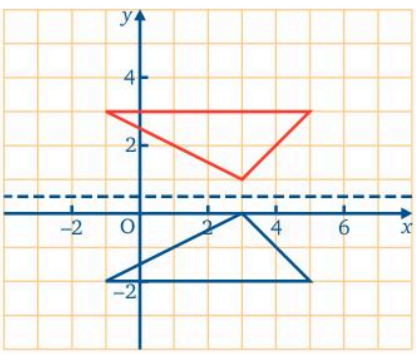


Copy the diagrams in questions **1** to **4** and draw in the mirror lines.

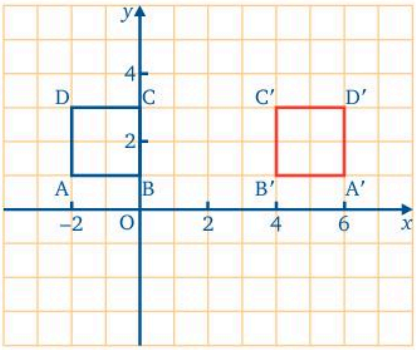
1



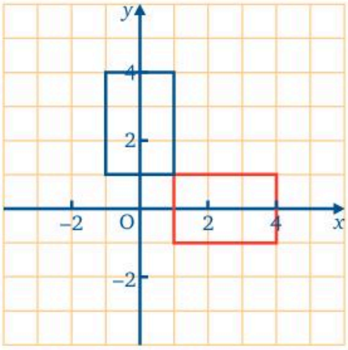
2



3



4



Draw axes for x and y from –5 to 5 for each of questions **5** to **8**.

5 Draw square PQRS: P(1, 1), Q(4, 1), R(4, 4), S(1, 4). Draw square P′Q′R′S′: P′(–2, 1), Q′(–5, 1), R′(–5, 4), S′(–2, 4). Draw the mirror line so that P′Q′R′S′ is the reflection of PQRS and write its equation.

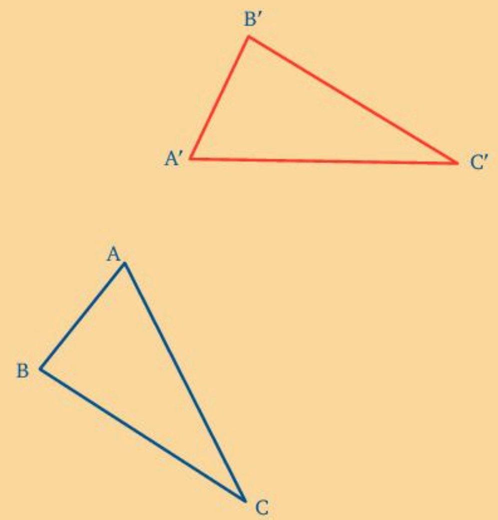
6 Draw △XYZ: X(2, 1), Y(4, 4), Z(–2, 4), and △X′Y′Z′: X′(2, 1), Y′(4, –2), Z′(–2, –2). Draw the mirror line so that △X′Y′Z′ is the reflection of △XYZ and write its equation. Are there any invariant points? If there are, name them.

An *invariant point* is where a point on the object and the corresponding point on the image are in the same place.

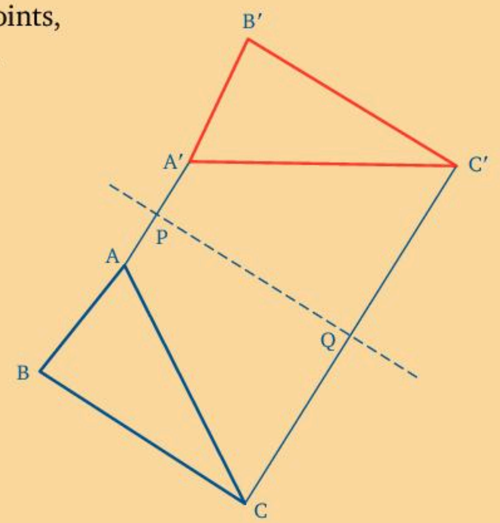
7 Draw △ABC: A(–2, 0), B(0, 2), C(–3, 3), and △PQR: P(3, –1), Q(4, –4), R(1, –3). Draw the mirror line so that △PQR is the reflection of △ABC. Which point is the image of A? Are there any invariant points? If there are, name them.

<u>8</u> Draw lines AB and PQ: A(2, –1), B(4, 4), P(–2, –1), Q(–5, 4). Is PQ a reflection of AB? If it is, draw the mirror line. If not, give a reason.

If A′B′C′ is the reflection of ABC, draw the mirror line.



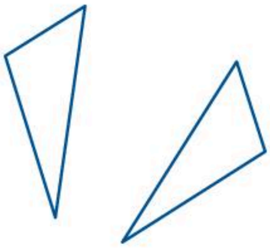
(Join AA′ and CC′ and find their midpoints, marking them P and Q. Then PQ is the mirror line.)



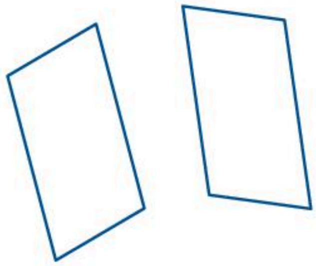
Whenever you attempt to draw a mirror line in this way, always check that the mirror line is at right angles to AA′ and BB′. If it is not, then A′B′C′ cannot be a reflection of ABC.

Trace the diagrams in questions **9** to **14** and draw the mirror lines.

9



12



10

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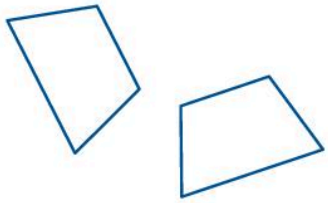
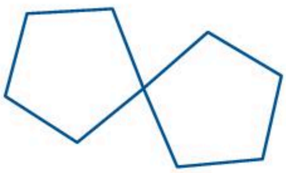
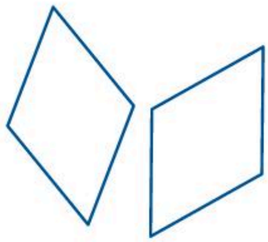


11



14



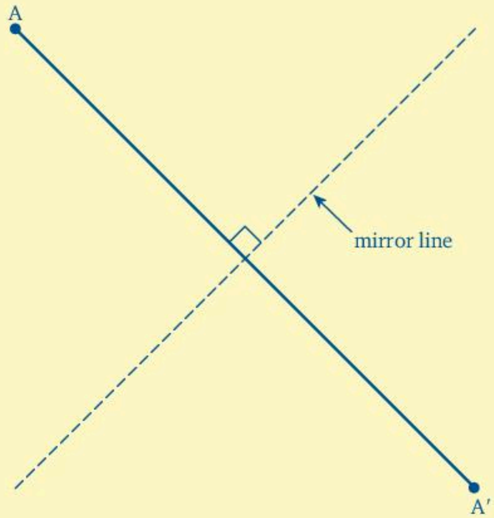
15 Draw axes for x and y from –4 to 5. Draw △ABC: A(3, 1), B(4, 5), C(1, 4), and △A′B′C′: A′(0, –2), B′(–4, –3), C′(–3, 0). Draw the mirror line so that △A′B′C′ is the image of △ABC.

16 Draw axes for x and y from –4 to 4. Draw lines AB and PQ: A(–4, 3), B(0, 4), P(1, –2), Q(2, 2). Draw the mirror line so that AB is the image of PQ.

17 Draw axes for x and y from –3 to 5. Draw △XYZ: X(3, 2), Y(5, 2), Z(3, 5), and △LMN: L(0, –3), M(0, –1), N(–3, –1). Draw the mirror line so that △LMN is the image of △XYZ.

Construction of the mirror line

You saw how to construct a mirror line in Grade 7. If we have only one point and its image, and we cannot use squares to guide us, we can use the fact that the mirror line goes through the midpoint of AA′ and is perpendicular to AA′. The mirror line is therefore the perpendicular bisector of AA′ and can be drawn.



Exercise 13h

1 On plain paper mark two points P and P′ about 10 cm apart in the middle of the page and the perpendicular bisector of PP′. Join PP′ and check that it is cut in half by the line you have drawn and that the two lines cut at right angles. Are we correct in saying that P′ is the reflection of P in the drawn line?

2 On square grid paper draw axes for x and y from –5 to 5, using 1 cm to 1 unit. A is the point (5, 2) and A′ is the point (–3, –3). Draw the mirror line so that A′ is the reflection of A.

3 Draw axes for x and y from –1 to 8, using 1 cm to 1 unit. B is the point (–1, 0) and B′ is the point (6, 3). Draw the mirror line so that B′ is the reflection of B.

Investigation

Palindromes

A palindrome is a word or sentence or number that reads the same from right to left as it does from left to right.

Examples of single word palindromes are DID and CIVIC.

An example of a sentence is

ABLE WAS I ERE I SAW ELBA.

Examples with numbers are much easier to find.

For example: 242, 1551 and 3672763.

These numbers are called palindromic numbers.

Consider the times that appear on a digital watch or clock. For a clock set for a 12-hour cycle midday is shown as 1200. The time shown on the face continues 1201, 1202, 1203, … until it gets around to midnight and starts again at 1200.

1 For a clock set for a 12-hour cycle investigate:

a all the possible palindromic times

b the shortest number of minutes between two consecutive palindromic times

c the longest time between two consecutive palindromic times.

2 Repeat these questions for a digital clock set for a 24-hour cycle. For a watch or clock set in 24-hour mode the cycle ends at midnight with the face showing 0000.

There are therefore many more possible palindromic times when the timepiece is set in this way.

3 The dates when important buildings were opened are often shown on them, for example 1754 or 1991. The second of these dates is a palindrome. What do you notice if you turn this date upside down?

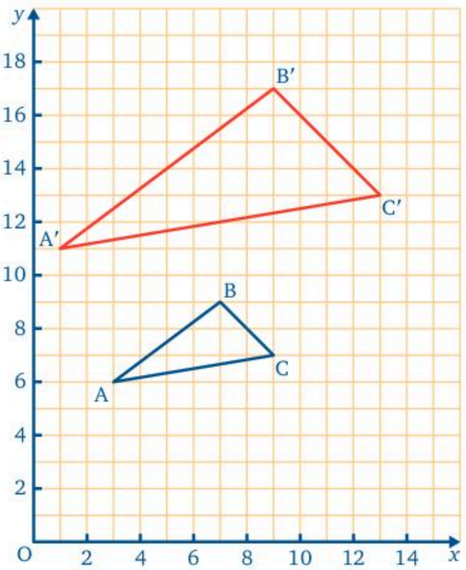
Is it possible to find a date in the last 200 years that is not only a palindrome but reads the same when turned upside down? Investigate.

If you find this investigation interesting try it with words; it is more difficult. The more letters in the word the more difficult it becomes. Should you want a really difficult challenge, try to create a palindromic sentence.

Mixed exercise

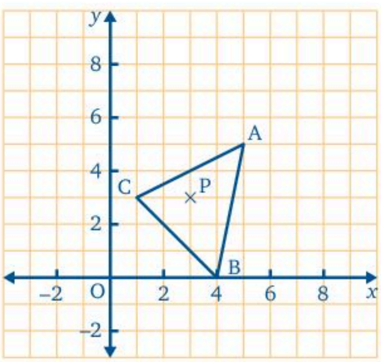
Exercise 13i

1 Triangle A′B′C′ is the image of triangle ABC under an enlargement.

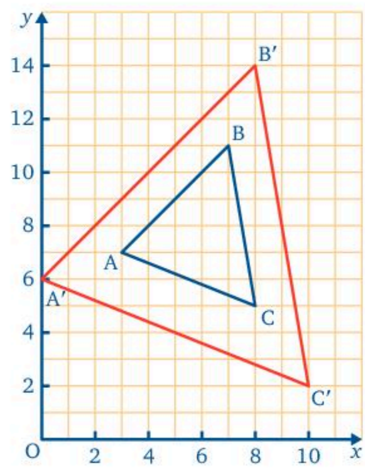


Find the centre of enlargement and the scale factor.

2 Copy the diagram using 1 cm as 1 unit.
P is the centre of enlargement.
Draw the image, △A′B′C′, of △ABC under an enlargement of 2.
Write down the coordinates of A′, B′ and C′.



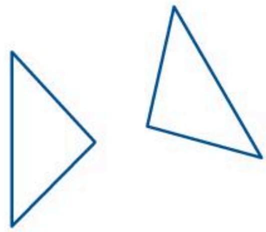
3 △A′B′C′ is an enlargement of △ABC.
Find the coordinates of the centre of enlargement.



4 On square grid paper draw axes for x and y from –4 to 8. Draw triangle ABC by plotting A(8, 3), B(4, 2) and C(2, –4).
Draw the image A′B′C′ when ABC is reflected in the line $y = x$.

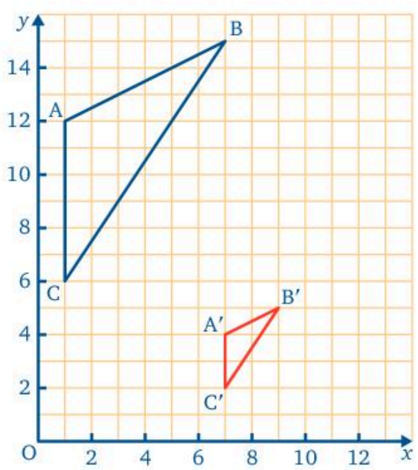
5 On square grid paper draw axes for x and y from –6 to 6 using 1 cm to 1 unit.
P is the point (2, –3) and P′ is the point (–4, 1).
Draw the mirror line so that P′ is the reflection of P.
Which of the points (1, 2), (–1, –1) and (–3, –4) lie on the mirror line?

6 Trace the diagram and draw the mirror line.



7 On square grid paper draw axes for x and y from –6 to 6 using 1 cm to 1 unit.
Draw triangle ABC where A is the point (0, 4), B the point (5, 0) and C the point (0, 0).
Draw the image of △ABC using the line $y = -x$ as the mirror line.
Mark the image A′B′C′. Is there an invariant point? If so, name it.

8 △A′B′C′ is the image of △ABC under an enlargement.
Give the centre of enlargement and the scale factor.



In this chapter you have seen that...

- ✔ to find the centre of enlargement, you can draw guidelines between corresponding points on the object and the image
- ✔ you can find the scale factor by comparing lengths of corresponding sides on the object and the image
- ✔ to find an image of a given shape under an enlargement you need the scale factor and centre of enlargement
- ✔ a scale factor less than 1 gives an image smaller than the object whereas a scale factor larger than 1 gives an image larger than the object
- ✔ when an object is reflected in a mirror line, the object and the image are symmetrical about the mirror line
- ✔ the mirror line is the perpendicular bisector of the line joining a point on the object to the corresponding point on the image.

14 Constructing quadrilaterals

At the end of this chapter you should be able to...

1 construct a line parallel to a given line through a point
2 construct various quadrilaterals including a square, a rectangle, a trapezium, a parallelogram, a rhombus and a kite.

Did you know?

The ratio of the circumference to the diameter of a circle which we know as π cannot be expressed as the ratio of two whole numbers. Mathematicians tried for centuries to do this but in modern times it was found impossible. Numbers such as π belong to a subset of the set of irrational numbers. They are called transcendental numbers.

You need to know...

- ✔ the angle properties of a triangle
- ✔ the angle sum of a quadrilateral
- ✔ the properties of angles formed by a pair of parallel lines and a transversal
- ✔ the properties of isosceles and equilateral triangles
- ✔ the properties of special quadrilaterals
- ✔ how to construct triangles and quadrilaterals using a ruler and a pair of compasses
- ✔ how to bisect an angle
- ✔ how to construct an angle equal to a given angle
- ✔ how to bisect a straight line
- ✔ how to drop the perpendicular from a point to a line.

Key words

alternate angles, arc, bisect, diagonal, in-circle, parallelogram, perpendicular, perpendicular bisector, quadrilateral, radius, rectangle, rhombus, square, symmetry, trapezium

Basic constructions

In Grades 7 and 8 you saw how to do a variety of constructions. Exercise 14a revises these skills.

Exercise 14a

You may use a protractor to draw and measure the angles in questions **1** to **3**.

1 Construct △ABC in which AB = 6 cm, ∠BAC = 70° and ∠ABC = 40°.

a Measure and record the size of ∠ACB.

b What special name do we give to this triangle?

2 Construct △DEF in which DE = 8 cm, DF = 10 cm and ∠EDF = 35°. Measure and record

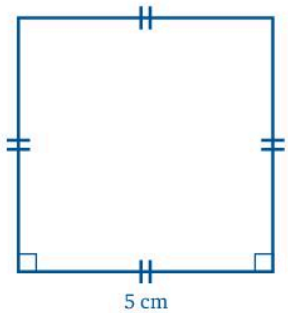
a the length of EF **b** the size of ∠DFE.

3 Construct △GHI in which GH = 10 cm, GI = 8 cm and HI = 6 cm. Measure its angles. What special name do we give to this triangle?

In questions **4** to **12** construct the figures using only a ruler and a pair of compasses.

For an angle of 30° construct an angle of 60° and *bisect* it, and for an angle of 45° construct an angle of 90° and bisect it.

4



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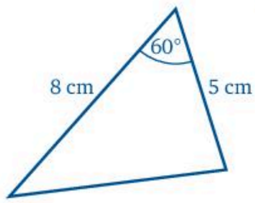

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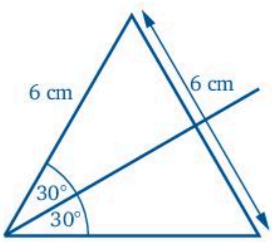


For questions **7** to **12**, draw a rough sketch before starting the construction.

7 Construct a *quadrilateral*, PQRS, in which PQ = 10 cm, $\hat{P}$ = 60°, PS = 5 cm, $\hat{Q}$ = 60° and QR = 5 cm. What can you say about the lines PQ and SR?

8 Construct a triangle, ABC, in which AB = 12 cm, ∠ABC = 30° and ∠BAC = 45°. What size should ∠ACB be? How accurate is your construction?

9 Construct a triangle, ABC, in which AB = 6 cm, ∠ABC = 90° and ∠BAC = 45°.

What length do you expect BC to be?

What size do you expect ∠BCA to be?

How accurate is your construction?

10 Construct a triangle, DEF, in which DE = 9.5 cm, ∠EDF = 45° and ∠DEF = 60°. What size do you expect ∠DFE to be? How accurate is your construction?

11 Construct a triangle ABC in which AB = 12 cm, ∠ABC = ∠BAC = 30°.

Construct the *perpendicular* from C to AB to cut AB at D.

What special point is D on the line segment AB?

12 Construct a triangle DEF in which DE = 9 cm, EF = 6 cm and ∠DEF = 90°.

Construct the *perpendicular bisectors* of DE and EF.

Mark O the point where they intersect.

Measure the distance of O from to each of the vertices D, E and F.

How do they compare?

The square and the rectangle

Exercise 14b

In this exercise use only a ruler and a pair of compasses.

1 **a** Construct a *square* ABCD of side 9 cm.

b Now measure the angles in the corner of your square with a protractor.

How accurate is your construction?

c Draw the *diagonals* of the square and mark their point of intersection E.

With the point of your compasses on E and *radius* AE draw a circle.

What other points should this circle pass through?

d Construct the perpendicular from E to the side AB. Mark the point F where this perpendicular intersects AB.

With centre E and radius EF draw a circle. Does this circle touch the other three sides? Justify why this should be so.

e What name do we give to the space between the two circles?

2 **a** Construct a *rectangle* ABCD in which AB = 10 cm and BC = 8 cm.

b Draw the diagonals AC and BD. Mark their point of intersection E.

With the point of your compasses on E and radius ED draw a circle.

What other points does this circle pass through?

c Measure AC. Hence find AC^2 and $AB^2 + BC^2$.

How do these two answers compare?

What theorem does this justify? State it.

d Use your protractor to measure angle BAC and use your calculator to find tan BAC.

Now find the ratio $\frac{BC}{AB}$. How do the two values compare?

3 Construct a square ABCD, of side 8 cm.
Construct the *in-circle* (i.e. the circle that touches all four sides of the square).

4 Construct a square ABCD whose diagonal, AC, is 8 cm long. Measure the side AC.

5 Construct a square in which each of the diagonals is 6.6 cm long.

What is the length of a side of this square?

6 In a rectangle ABCD, AB = 6 cm and the diagonal BD is 10 cm. Make a rough sketch of the rectangle and then construct ABCD. Measure AD.

7 Construct a rectangle with diagonals of length 12 cm containing an angle of 30°.

Measure and record the lengths of the sides of the rectangle.

8 Construct a rectangle with diagonals of length 12.6 cm containing an angle of 45°.

Measure and record the lengths of the sides of the rectangle.

9 Construct a rectangle PQRS is which PQ = SR = 9 cm and PS = QR = 6 cm.

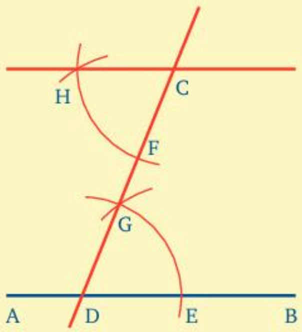
Draw the diagonals and measure their length.

10 Construct a rectangle ABCD in which the diagonals AC and BD are each 8 cm in length and which intersect at O such that ∠BOC = 60°.

Measure and record the length of AB and BC.

To construct a line parallel to a given line through a given point

You first saw this construction in Grade 7. This is a reminder of how it is done.



C is a point above the line AB.

To draw a line through C parallel to AB proceed as follows:

- Draw a line through C to cut AB at D.
- With centre D and any suitable radius (it must not be too small) draw an *arc* to cut AB at E and CD at G.
- With centre C and the same radius draw an arc to cut CD at F.
- With centre E open out your compasses to a radius that passes through G.
- With this radius place the point of you compasses at F and draw the first arc at H.
- Join CH.

CH is parallel to AB as you have constructed angles EDG and FCH as equal angles. You have constructed *alternate angles*.

Exercise 14c

1 Construct triangle ABC such that AB = 9.5 cm, angle A = 65° and AC = 6.4 cm. Construct the perpendicular bisector of AC to cut AC at E. Finally construct the line through E parallel to AB to intersect BC at F. Measure EF. How does the length of EF compare with the length of AB?

2 R is a point on a straight line PQ.

a Construct an angle QRS such that $\angle QRS = 45°$ and RS = 4.4 cm.

b Construct the line through S parallel to PQ. Mark a point on it T such that ST = RQ.

c Construct the perpendicular from S to PQ to meet PQ at U.

d Measure RU and SU. How do they compare? Is this what you expected? Justify your answer.

e Measure QT. How does its length compare with RS?

f What name do we give to the quadrilateral RSTQ?

3 Draw AB = 8.5 cm. Construct ∠ABC = 60° and mark BC = 10 cm. Construct the perpendicular bisector of BC to intersect BC at D. Construct the line through D parallel to AB and mark the point E on it such that DE = 5 cm and E is on the same side of BC as A. Join AE. What name do we give to the quadrilateral ABDE?

4 Construct a quadrilateral LMNO with LM = 8.4 cm, angle MLO = 60°, LO = 5 cm, ON parallel to LM and MN = 5.2 cm.

What name do we give to this quadrilateral?

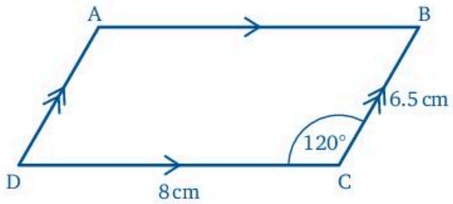
The parallelogram and the trapezium

Exercise 14d

In this exercise use only a ruler and a pair of compasses to construct the figure asked for.

Always draw a rough sketch first.

1



Join AC and BD. Hence write down the length of each diagonal.

2 Construct a *parallelogram* ABCD with diagonals intersecting at X given that AC = 8 cm, BD = 10 cm and ∠AXD = 60°. Measure and record the lengths of the sides of the parallelogram.

3 Construct a parallelogram ABCD in which AB = 7 cm, AD = 5 cm and ∠BAD = 45°.

4 Construct a parallelogram ABCD whose diagonals intersect at X given that AC = 10.6 cm, BD = 8.2 cm and ∠AXD = 60°. Measure the lengths of the sides of the parallelogram.

5 Construct a quadrilateral ABCD in which AB = 12 cm, ∠A = ∠B = 60°, AD = 6 cm and BC = 6 cm.

What can you say about the lines AB and DC?

What name do you give to this shape?

6 Construct a *trapezium* PQRS in which PQ = 9 cm, ∠QPS = 75°, ∠PQR = 65° and PS = 5 cm.

  a Which side is parallel to PQ?

  b Measure the four angles of the trapezium. Is the total what you expected?

  c Find ∠PQR + ∠SRQ. Is the total what you expected? Justify your answer.

7 Construct a trapezium PQRS in which PQ = 9.3 cm, angle P = 45°, angle Q = 60° and QR = 4.4 cm. Measure and record the lengths of PS and SR.

8 Construct a trapezium ABCD given AB = 12.2 cm, BC = 7.3 cm, DC = 8.5 cm and ∠ABC = 60°. Measure and record the lengths of AD, AC and BD.

9 Construct triangle ABC such that AB = 4 cm, ∠ABC = ∠BAC = 60°.

Through C construct the line segment parallel to AB. Mark D and E on opposite sides of C such that CD = CE = 4 cm. Join AE and BD.

What special name do we give to

  a triangles ACE and BDC

  b quadrilateral ABDC

  c quadrilateral ABDE?

10 Construct a trapezium ABCD such that AB = 7.5 cm, ∠A = 75°, ∠B = 60° and BC = 4 cm.

  a Measure and record the lengths of AD and DC.

  b Join AC and BD. Measure and record their lengths.

The rhombus

This exercise investigates some properties of the *rhombus*.

Exercise 14e

1 Draw a line 12 cm long across your page. Label the ends A and C. Open your compasses to a radius of 9 cm. With the point on A, draw an arc above AC and another arc below AC. Keeping the same radius, move the point of your compasses to C. Draw arcs above and below AC to cut the first pair of arcs. Where the arcs intersect (i.e. cross), label the points B and D.

Join A to B, B to C, C to D and D to A. ABCD is called a rhombus.

Questions **2** to **9** refer to the figure that you have constructed in question **1**.

2 Without measuring them, what can you say about the lengths of AB, BC, CD and DA?

3 ABCD has two lines of *symmetry*. Name them.

4 If ABCD is folded along BD, where is A in relation to C?

5 If ABCD is folded along AC, where is D in relation to B?

6 Where AC and BD cut, label the point E. With ABCD unfolded, where is E in relation to A and C?

7 Where is E in relation to B and D?

8 If ABCD is folded first along BD and then folded again along AE, what is the size of the angle at E?

9 With ABCD unfolded, what are the sizes of the four angles at E?

10 In a rhombus ABCD, AB = 7 cm and the diagonal AC = 12 cm. Make a rough sketch of the rhombus and then construct ABCD. Measure BD.

11 In a rhombus ABCD, the diagonal AC is 8 cm and the diagonal BD is 6 cm. Construct the rhombus and measure AB. (Remember first to make a rough sketch.)

12 Construct a rhombus ABCD in which the sides are 5 cm long and the diagonal AC is 8 cm long. Measure the diagonal BD.

13 Construct a rhombus PQRS with sides of length 6 cm and angle PQR = 120°.

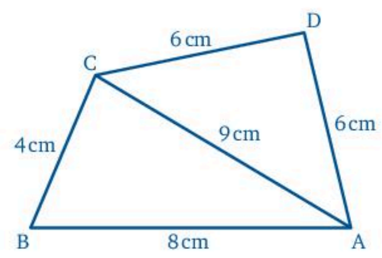
14 Construct a rhombus ABCD in which diagonal AC = 6 cm and diagonal BD = 6 cm. Measure and record the length of a side of the rhombus.

15 Construct a rhombus with diagonals of length 5 cm and 7 cm. Measure and record the length of a side of this rhombus.

Other quadrilaterals

Exercise 14f

In this exercise use only a ruler and a pair of compasses to construct the figure asked for.

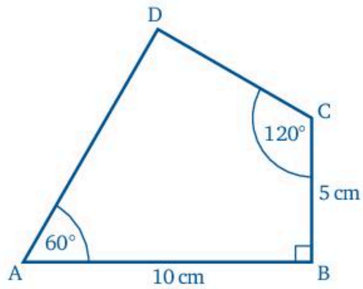


1 **a** Join BD, measure it and write down its length.

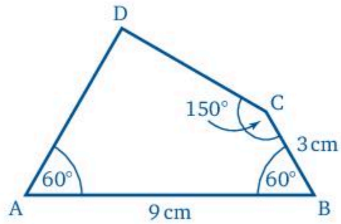
b Use a protractor to measure the sizes of ∠ACD and ∠CAD.

c What special name is given to triangle ACD?

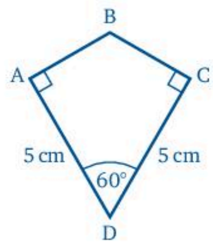
2 a Measure the length of AD and DC
b Measure ∠ADC.
How does it compare with ∠ABC?
Justify your answer.



3 a Measure the length of AD and DC.
b Measure ∠ADC. Is it what you expect?
Justify your answer.
c Draw the diagonals AC and DB.
Measure them and write down their length.



4 a Measure and record the lengths of AB and BC.
b Measure ∠ABC. Is it what you expect?
Give a reason for your answer.
c What special name do we give to this shape?



5 Construct a quadrilateral ABCD in which AB = BD = 8.5 cm, ∠ABD = 30°, BC = 3.5 cm and DC = 7 cm.
Join AC and write down its length.

6 Draw a circle of radius 4 cm. Mark its centre O and A any point on the circumference.

Construct the perpendicular bisector of AO to cut the circle at B and C, and to cut AO at X. (Mark the points so that you move clockwise round the circle from A to B.)

Now construct the perpendicular bisector of CX to cut CX at D and the circumference of the circle at E, where E is on the same side of BC as O.

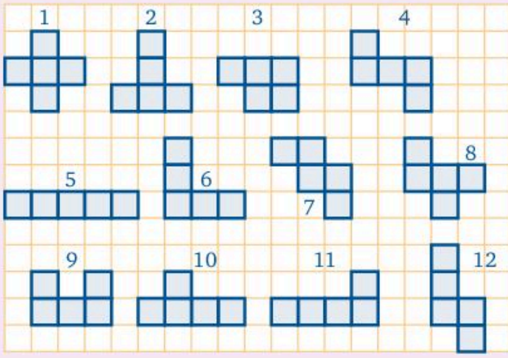
Next construct the perpendicular bisector of DE to cut DE at F and the circumference of the circle at G, where G is on the opposite side of DE from O.

a What name do we give to quadrilateral CDFG?
b Use a protractor to measure the angles DCG and FGC. What is their sum?
Is this what you expected? Justify your answer.

Investigation

Pentominoes

In 1954, Solomon Golomb, a Californian mathematician, introduced the recreational game of pentominoes to the world. The set, as shown in the diagram, consists of 12 pieces. Each shows a different way of putting 5 squares together along their edges.



1 How many squares are there in any five pieces?

2 Can you arrange the four pieces numbered 2, 3, 6 and 7 to form a 5×4 rectangle?

3 Now form a square using pieces 1, 3, 6, 9 and 11.

If you want a bigger challenge:

4 arrange six pieces of your choice to form a 6×5 rectangle

5 form rectangles measuring 7×5 and 10×4.

In this chapter you have seen that...

✔ using a ruler and a pair of compasses you can construct
- a square and a rectangle
- a line through a given point parallel to a given line
- a parallelogram and a trapezium
- a rhombus and a kite
- a general quadrilateral.

REVIEW TEST 2: CHAPTERS 8–14

In questions **1** to **12**, choose the letter for the correct answer.

1 The probability that a particular car will break down when travelling one kilometre is $\frac{1}{10\,000}$.
The number of times the car is likely to break down in 200 000 kilometres is
A 2 B 20 C 200 D 2000

2 A number is chosen at random from the first 10 whole numbers.
The probability that it is not a prime number is
A $\frac{2}{5}$ B $\frac{1}{2}$ C $\frac{3}{5}$ D $\frac{4}{5}$

3 A letter is chosen at random from the word CANTANKEROUS.
The probability that it is a vowel is
A $\frac{1}{4}$ B $\frac{1}{3}$ C $\frac{5}{12}$ D $\frac{1}{2}$

4 The domain of the relation {(2, 4), (4, 10), (6, 16), (8, 22)} is
A {2, 4, 6, 8} B {2, 4, 6} C {4, 10, 16} D {4, 10, 16, 22 }

5 The range of the relation {(3, 6), (4, 9), (6, 15), (7, 18)} is
A {3, 4, 6, 7} B {4, 6, 7} C {6, 9, 15} D {6, 9, 15, 18}

6 Draw a mapping diagram to represent the relation
{(2, 3), (0, 3), (3, 0), (–1, 0)}
This relation is said to be
A 1 : 1 B 1 : n C n : 1 D n : n

7 A car uses 8 litres of fuel for a journey of 224 km. At the same rate the distance it can travel on 9 litres is
A 248 km B 252 km C 262 km D 264 km

8 The ratio $4\frac{2}{3}:5\frac{5}{6}$ simplifies to
A 4 : 5 B 5 : 4 C 2 : 5 D none of these

9 The tax due on $1050 000 when the rate of income tax is 30% is
A $31 500 B $315 000 C $301 000 D $3150 000

10 The monthly charge for my home phone package is $1699. Last month I made international calls totalling 140 minutes outside the package, charged at $9.99 per minute. The total charge before taxes is

A $1398.60 **B** $1838.86 **C** $3096.70 **D** $3097.60

11 In May I used 240 units of electricity. These units were charged at $9.00 per unit for the first 100 units and $21.50 for the additional units. The customer charge and additional charges came to $5158. The total cost of my electricity is

A $7318 **B** $8568 **C** $9068 **D** $10 318

12 The number of units of electricity a 60 W light bulb will use in a week if it burns for 8 hours every day is

A 0.48 **B** 2.88 **C** 3.36 **D** 4.48

13 A letter is chosen at random from the letters in the word CANTEEN. What is the probability that it is

a A **b** B **c** a vowel **d** not a vowel?

14 A letter is chosen at random from the letters in the word PARALLEL. What is the probability that it is

a A **b** L **c** S?

15 A relation is given by $y = 3 - x^2$ for $x \in \{-3, -1, 0, 1, 2\}$.

a List the ordered pairs in this relation.

b Determine, with a reason, whether or not the relation is a function.

16 A function f is defined by $f(x) = 2x^2 - 3x$ for $x \in \{-2, 0, 1, 2\}$.
List the ordered pairs for this function

17 Rectangle A measures 10 cm by 8 cm and rectangle B measures 8 cm by 5 cm.
Find the ratio of

a the length of A to the length of B

b the width of A to the width of B

c the perimeter of A to the perimeter of B

d the size of an angle of A to the size of an angle of B.

18 The map ratio of a map is 1 : 25 000

a A straight road is 1.2 km long. What is the length of the road on the map?

b The distance between two farms on the map is 2.6 cm. How far apart are the farms?

19 Divide a length of rope, which is 21 metres long, into two parts in the ratio 2 : 5.

20 Enid George earns $96 000 a week gross. Her total statutory deductions come to $31 200. Other non-statutory deductions are 2% of her net pay into a holiday fund and 0.5% to the local leisure centre.

Calculate

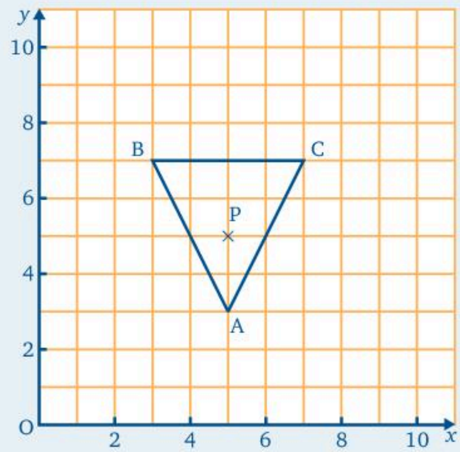
a her net pay after the statutory deductions

b the amount the local leisure receives

c her take-home pay for the week.

21 Copy the diagram using 1 cm as 1 unit. P is the centre of enlargement.

Draw the image of triangle ABC under an enlargement of scale factor 2.

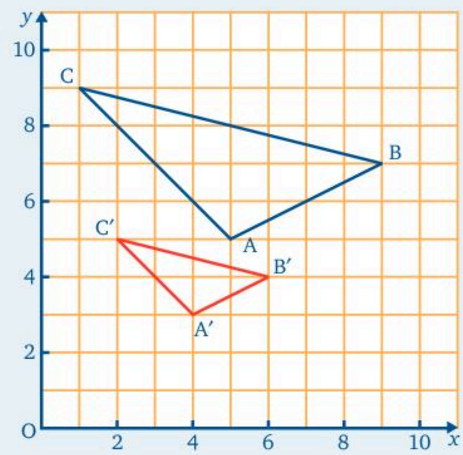


22 Triangle A′B′C′ is the image of triangle ABC under an enlargement

Find

a the coordinates of the centre of enlargement.

b the scale factor.



23 Draw axes for x and for y from 0 to 10. Draw triangle ABC by plotting the points A(3, 5), B(4, 8) and C(8, 9).

Draw the image A′B′C′ when it is reflected in the line $y = x$.

Write down the coordinates of A′, B′ and C′.

24 Construct a triangle ABC in which AB = BC = 7 cm and ∠ABC = 60°.

Measure, and write down, the length of AC and the size of angle ACB.

Are your values what you expected? Justify your answers.

25 Construct a rectangle ABCD in which AB = 9.6 cm and BC = 7.2 cm.

Measure and record

a the length of the diagonal BD **b** the size of angle ACD.

26 Construct a parallelogram ABCD in which the diagonals intersect at X, AC = 10 cm, ∠AXB = 120° and BD = 11 cm.
Measure and record the lengths of the sides of the parallelogram.

15 Sets

At the end of this chapter you should be able to...

1 remember all the facts relating to sets
2 solve harder problems using Venn diagrams.

Did you know?

One of the best selling mathematics books ever published was *Mathematics for the Million*.

It was written by Lancelot Hogben, an English zoologist and geneticist who held academic posts in the UK, Canada and South Africa. He applied mathematical principles to genetics and was concerned with the way statistical methods could be used in the biological and behavioural sciences. This book was published in 1933.

You need to know...

- ✔ the meaning of: equal sets, empty or null set, finite and infinite sets, intersection and union of two sets, subset, universal set
- ✔ the meaning of the complement of a set
- ✔ the meaning of the symbols $\in$, $\notin$, $\subset$, $\subseteq$, $\cup$, $\cap$, $\varnothing$ and U
- ✔ how to draw a Venn diagram.

Key words

element, complement, disjoint sets, empty set, equal set, equivalent set, finite set, infinite set, intersection of sets, member, null set, proper subset, set, subset, union of sets, universal set, Venn diagram, the symbols $\in$, $\notin$, $\subset$, $\subseteq$, $\cup$, $\cap$, $\varnothing$ and U

Reminders

Sets were first introduced in Grade 7 and extended in Grade 8. We begin this chapter by revising that work.

- a *set* is a collection of things that have something in common
- things that belong to a set are called *elements* or *members* and are written between curly brackets
- the symbol $\in$ means 'belongs to' whereas the symbol $\notin$ means 'does not belong to'.

  Hence Maisie $\in$ {girls' names} means 'Maisie belongs to the set of girls' names'

  whereas cruise ship $\notin$ (aeroplanes} means that 'a cruise ship is not a type of aeroplane'
- in a *finite set* all the elements can, in theory, be counted. For example you can count the number of elements in the set {integers from 1 to 30} and, in theory but not in practice, you can count the elements in the set {grains of sand in the Sahara Desert}
- in an *infinite set* there is no limit to the number of elements it has
- two sets are *equal* if they contain exactly the same elements or members, though not necessarily in the same order
- two sets are *equivalent* if they have the same number of elements. The elements do not have to be the same
- a set with no members is called the *empty set* or *null set*. It is denoted by { } or $\varnothing$
- a set that contains all the elements of the sets under consideration (and possibly some more) is called the *universal set*. It is denoted by U or $\mathcal{E}$
- the *complement* of a set A (denoted by A') is the set containing all the members of U that are not members of set A
- if all the members of a set B are also members of a set A, then B is a *subset* of A. This is written $B \subseteq A$. If B does not contain all the elements of A, B is called a *proper subset* of A. This is denoted by $B \subset A$
- A and B are *disjoint sets* when they have no members in common
- the number of elements in a set A is denoted by $n(A)$.

Notation

We use $\mathbb{N}$ for the set of natural numbers 1, 2, 3, 4, …

$\mathbb{W}$ for the set of whole numbers 0, 1, 2, 3, …

$\mathbb{Z}$ for the set of integers …, −2, −1, 0, 1, 2, …

$\mathbb{Q}$ for the set of rational numbers, that is numbers of the form $\frac{a}{b}$ where a and b are integers

$\mathbb{R}$ for the set of real numbers.

The real numbers include $\mathbb{W}$, $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ and the irrational numbers.

We can describe a set in words, e.g. the set of natural numbers that are less than 20.

We can also describe this set by listing the elements enclosed in curly brackets, e.g.

$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$.

A shorter way of describing this set is to write $\{x:1 \leqslant x < 20, x \in \mathbb{N}\}$, where the colon (:) means 'such that'.

$\{x : 1 \leqslant x < 20, x \in \mathbb{N}\}$ reads 'the values of x such that x is greater than or equal to 1 and less than 20 and x is a member of the set of natural numbers.'

Exercise 15a

1 Describe in words the given sets.

a {3, 5, 7, 11, 13} **b** {Jamaica, St. Kitts, Dominica, Grenada}

2 Write each of the following statements in set notation:

a Geography is a member of the set of school subjects.

b August is not a day of the week.

c Ohio is a member of the set of American states.

d A golf club is not a kitchen utensil.

3 State whether or not the following statements are true:

a 147 $\notin$ {even numbers}

b Peru $\in$ {South American cities}

c Hollywood $\notin$ {American states}

d Ethiopia $\in$ {African countries}

4 Are the following sets finite or infinite sets?

a {the number of people on the Earth}

b {multiples of 5}

c {fractions between 0 and 1}

d {the number of litres of water in the Atlantic Ocean}

5 For each set A find $n(A)$.

a A = {players in a hockey team}

b A = {whole numbers between 10 and 30 that are multiples of both 2 and 3}

c A = {P, Q, R, S, T}

d A = {prime numbers between 6 and 20}

6 Suggest a universal set for:

a {4, 6, 8, 12} and {2, 6, 10, 14, 16, 18}

b {Volkswagen, Rolls Royce} and {Jaguar, Citroen, BMW}

c {Kenya, Zimbabwe, Cameroon} and {Nigeria, Angola, Libya}

d {relativity, arithmetic, trigonometry} and {algebra, geometry, dynamics}

7 $U = \{x$, a whole number, such that $3 < x \leqslant 20\}$
$A = \{$multiples of 4$\}$, $B = \{$multiples of 7$\}$, $C = \{$multiples of 10$\}$.
Find $n(A)$, $n(B)$ and $n(C)$.

8 $U = \{$integers from 10 to 30 inclusive$\}$
$A = \{$prime numbers$\}$, $B = \{$integers exactly divisible by 2 and by 3$\}$,
$C = \{$factors of 18$\}$.

List the sets A, B and C.

9 List the members of the set $\{x : x \leqslant 10, x \in \mathbb{N}\}$

10 a List any ten members of the set $\{x : x = 2n, n \in \mathbb{N}\}$

b Is this set finite or infinite?

11 List the members of the set $\{x: -2 < x < 3, x \in \mathbb{Z}\}$

12 List the members of the set $A = \{(x, y): y = 3x, -1 \leqslant x \leqslant 1, x \in \mathbb{Z}\}$

13 Find $n(A)$ where $A = \{x : x = \sqrt{m}, 0 < m < 20, x \in \mathbb{Z}\}$

14 $U = \{x : 1 \leqslant x < 15, x \in \mathbb{N}\}$
$P = \{$multiples of 3$\}$, $Q = \{$even numbers$\}$, $R = \{$multiples of 5$\}$.
List the sets P, Q and R.

15 $U = \{x : -6 \leqslant x < 6, x \in \mathbb{Z}\}$
$A = \{$even numbers$\}$, $B = \{$negative numbers$\}$, $C = \{$positive prime numbers$\}$.
List the sets A, B and C.

? Puzzle

Signposts are placed at 1-mile intervals along the road joining Aston and Barryton, which are 23 miles apart. Each signpost shows the distance to each town. The signpost that is 10 miles from Aston shows Aston 11 miles, Barryton 12 miles. This signpost uses the digit 1 three times and the digit 2 once, i.e. it uses 2 digits only.

1 How many signposts are there altogether?

(Don't forget the one in Aston and the one in Barryton.)

2 How many signposts between the two towns use exactly two digits? (0 counts as a digit.)

3 How many signposts use exactly three digits?

4 Which digit is used most? How many times is this?

Number of members in a set

This exercise revises and extends work using *Venn diagrams* to find the number of members in the *union* of sets A and B, $n(A \cup B)$, and the *intersection* of sets A and B, $n(A \cap B)$.

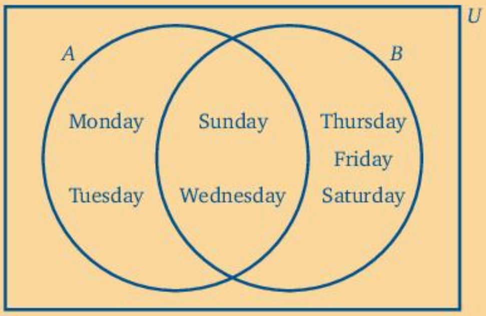
Exercise 15b

Illustrate on a Venn diagram the sets A and B if

$A = \{$Sunday, Monday, Tuesday, Wednesday$\}$
$B = \{$Wednesday, Thursday, Friday, Saturday, Sunday$\}$

Use your diagram to find

a $n(A)$ **b** $n(B)$ **c** $n(A \cup B)$ **d** $n(A \cap B)$



Counting the number of elements in the various regions gives

a $n(A) = 4$

b $n(B) = 5$

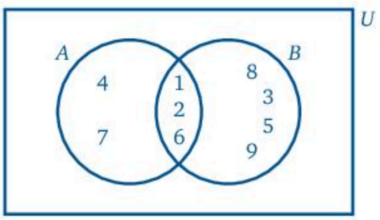
c $n(A \cup B) = 7$ (the number of members in either set A or set B)

d $n(A \cap B) = 2$ (the number of members in both set A and set B)

In questions **1** to **4** count the number of elements in the various regions to find

a $n(A)$ **b** $n(B)$ **c** $n(A \cup B)$ **d** $n(A \cap B)$

1



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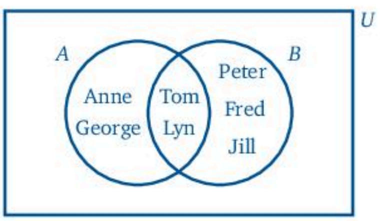

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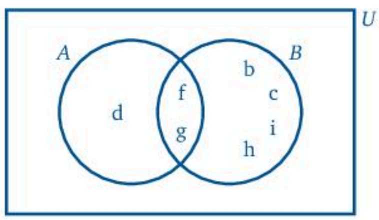

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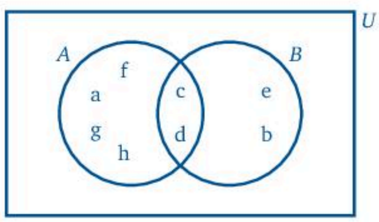
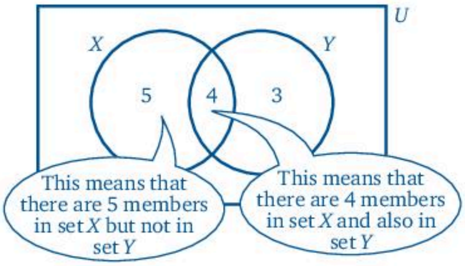


In the remaining questions in this exercise, the numbers in the various regions of the Venn diagrams show the **number** of elements in, or members of, the set in that region.

In questions **5** to **8** use the information given in the Venn diagrams to find

a $n(X)$ **b** $n(Y)$ **c** $n(X \cup Y)$ **d** $n(X \cap Y)$

5



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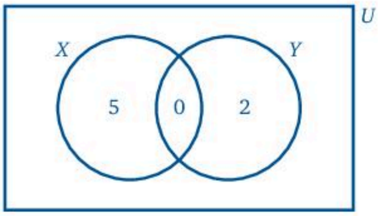

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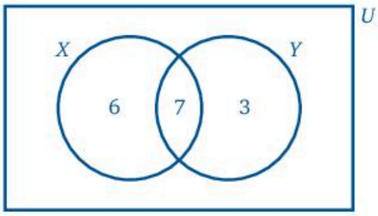

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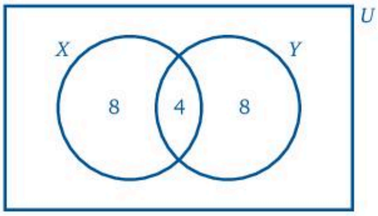
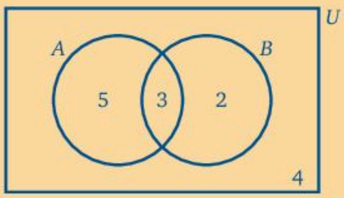


Use the information given in the Venn diagram to find

$n(A)$, $n(B)$, $n(A')$, $n(B')$, $n(A \cup B)$, $n(A \cap B)$, $n(A' \cup B')$, $n[(A \cap B)']$ for the given sets.



The numbers in the regions show the number of elements in that region.

$n(A) = 8$ (the number of elements in set A, i.e. $5 + 3$)

$n(B) = 5$ (the number of elements within the B circle, i.e. $3 + 2$)

$n(A') = 6$ (the number of elements not in set A)

$n(B') = 9$ (the number of elements not in set B)

$n(A \cup B) = 10$ (the sum of the numbers in either A or B, i.e. $5 + 3 + 2$)

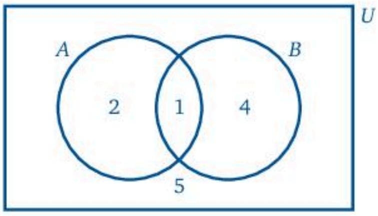
$n(A \cap B) = 3$ (the number in both A and B)

$n(A' \cup B') = 11$ (the number not in set A $(2 + 4)$ plus the number not in set B and not already accounted for (5))

$n[(A \cap B)'] = 11$, (the number not in both A and B)

In questions **9** to **16**, use the information given in the Venn diagrams to find $n(A)$, $n(B)$, $n(A')$, $n(B')$, $n(A \cup B)$, $n(A \cap B)$, $n(A' \cup B')$ and $n[(A \cap B)']$ for each of the given pairs of sets.

9



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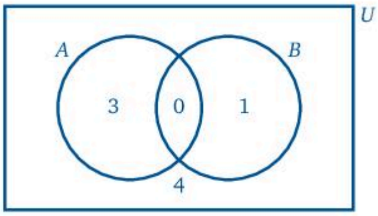

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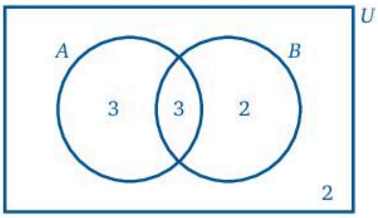

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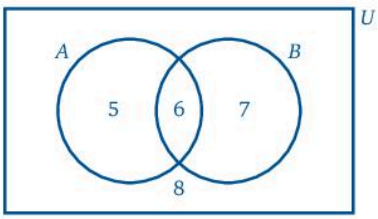

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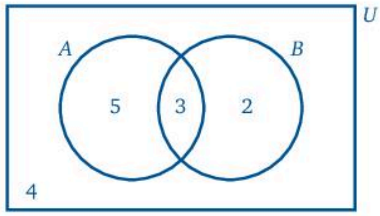

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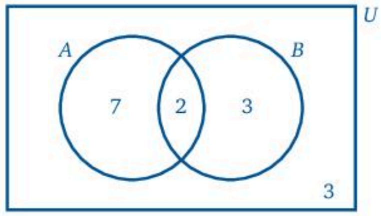

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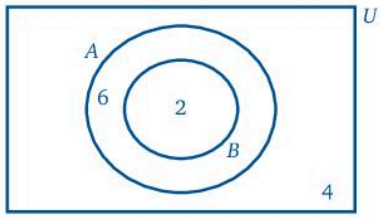

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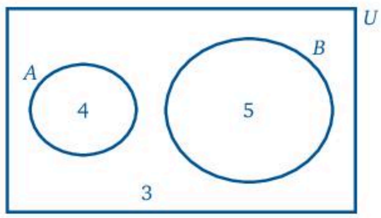
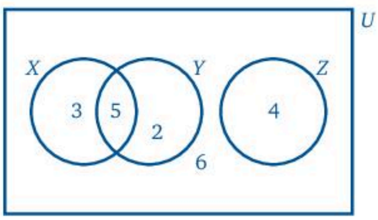


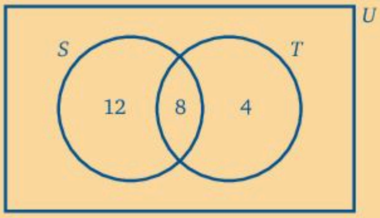
17 Use the information given in the Venn diagram below to find $n(X)$, $n(Y)$, $n(Z)$, $n(X \cap Y)$, $n(X \cup Z)$, $n(Y \cup Z)$ and $n(X \cup Y \cup Z)$.



Problems

Exercise 15c

The Venn diagram shows how many pupils in a class have smartphones (S) and tablets (T).

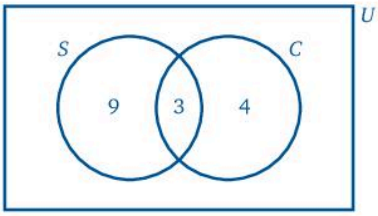


How many pupils have:

a both a smartphone and a tablet
b a smartphone
c a smartphone and/or a tablet
d a tablet but not a smartphone?

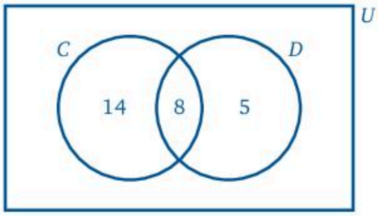
a The number of pupils with both is 8. (The number in both circles.)
b The number of pupils with a smartphone is $12 + 8$, i.e. 20. (The sum of the numbers in circle S.)
c The number of pupils with at least one of the two is $12 + 8 + 4$, i.e. 24. (The sum of the numbers in either circle.)
d 4 pupils have a tablet but not a smartphone. (The number in T but not in S.)

1 The Venn diagram shows the number of boys in a class who play soccer (S) and who play cricket (C).



How many boys play
a both games
b only cricket
c soccer
d exactly one of these games?

2 The students in a form were asked if they did any cooking (C) or dressmaking (D) at home. Their replies are shown in the Venn diagram.



If all the students in the form took part in at least one of these activities, how many students
a are there in the form
b did only cooking
c did both
d did exactly one of these activities?

3 In a group of 24 children, each had a dog or a cat or both. If 18 kept a dog and 5 of these also kept a cat, show this information on a Venn diagram and hence find the number of children who kept
a a cat
b only a dog
c just one of these as a pet.

4 A group of 50 television addicts were asked if they watched sport programmes and nature programmes. The replies revealed that 21 watched both sport and nature programmes but 9 watched nature programmes only. All 50 people watched either sport or nature programmes or both. Show this information on a Venn diagram and use it to find the numbers of viewers who

a watched sport

b did not watch nature programmes

c watched either sport or nature programmes but not both.

5 In a youth club 35 teenagers said that they went to football matches, discos or both. Of the 22 who said they went to football matches, 12 said they also went to discos. Show this information on a Venn diagram. How many went to football matches or discos, but not to both?

6 There are 28 pupils in a form, all of whom take history or geography or both. If 14 take history, 5 of whom also take geography, show this information on a Venn diagram and hence find the number of pupils who take

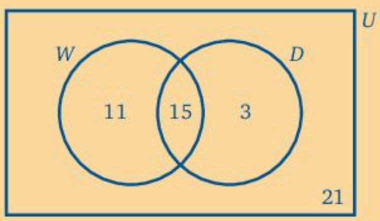
a geography

b history but not geography

c just one of these subjects.

7 In a squad of 35 cricketers 20 said that they could bat and 8 said that they could bat and bowl. Show this information on a Venn diagram. How many more were willing to bowl than to bat?

The Venn diagram shows how many houses in a street have new windows (W) and how many have new front doors (D).



How many houses

a are there in the street

b do not have a new front door

c have either a new front door or new windows but not both?

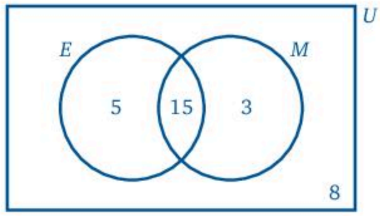
a Number of houses in the street is $11 + 15 + 3 + 21$, i.e. 50 (The sum of all the numbers in U.)

b Number of houses without a new front door is $11 + 21$, i.e. 32 (The sum of the numbers outside D.)

c The numbers with either a new front door or new windows but not both is $11 + 3$, i.e. 14.

(The number in either W or D but not in both.)

8 The Venn diagram shows how many pupils in a class passed the English examination (E) and how many passed the mathematics examination (M).



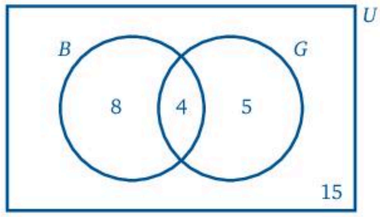
How many pupils

a passed in only one examination

b did not pass in English

c passed in at least one examination?

9 The Venn diagram shows how many pupils in a class kept goldfish (G), budgerigars (B) or both.



How many pupils

a were there in the class

b did not have a budgerigar

c had at least one of these pets?

10 The passengers on a coach were questioned about the newspapers and weekly magazines they bought. Their replies showed that 3 bought both a daily newspaper and a weekly magazine, 15 bought a daily newspaper, 8 bought a weekly magazine and 8 did not buy either a daily paper or a weekly magazine. Show this information on a Venn diagram.

a How many passengers were there on the coach?

b How many passengers bought a daily newspaper, a weekly magazine or both?

11 One evening all 78 members of a Youth Club were asked whether they liked swimming (S) and/or dancing (D). It was found that 34 liked swimming, 41 liked dancing and 19 liked both. Show this information on a Venn diagram. How many members were

a swimmers but not dancers

b dancers or swimmers but not both

c neither dancers nor swimmers?

12 During April, 36 cars were taken to an Island Traffic Authority Examination Depot for their annual inspection. The results showed that 8 had defective brakes and lights, 10 had defective brakes, and 13 had defective lights. How many cars

a failed the inspection

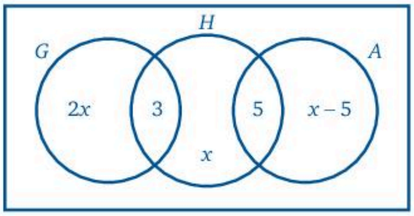
b passed the inspection

c had exactly one defect?

13 The 32 pupils in a class were asked whether they studied French or art or both. It was found that 8 studied both, 13 studied French and 6 did not study either subject. How many pupils studied

a art but not French

b French or art but not both?

14 

The Venn diagram shows the number of students taking geography (G), history (H) and accounts (A) in a class of 43. Every student takes at least one of these subjects

a Write down an expression, in terms of x, for the number of students who take history.

b Write down an expression, in terms of x, which shows the information given in the Venn diagram.

c Work out the number of students who take geography only.

d Work out the number of students who take accounts.

15 The universal set U contains the sets A, B and C such that

$$B \subset A,\ C \subset A,\ B \cap C = \varnothing$$

Draw a Venn diagram to show the relation between the sets A, B and C.

16 The universal set U contains the sets P, Q and R such that $P \cap Q = \varnothing$, $P \cap R \neq \varnothing$, $Q \cap R \neq \varnothing$.

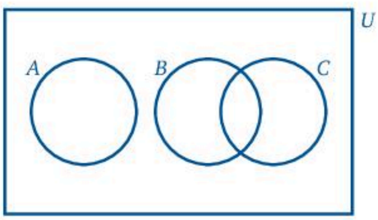
Draw a Venn diagram to show the relation between the sets P, Q and R.

17 Given $U = \{x\text{: } 3 \leqslant x < 15, x \in \mathbb{N}\}$

$A = \{\text{prime numbers}\}$

$B = \{\text{even numbers}\}$

$C = \{\text{odd numbers that are not prime}\}$



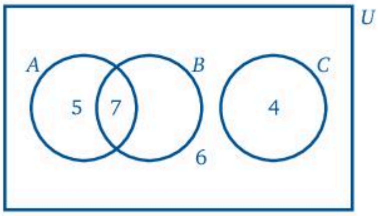
a Copy this diagram and use it to show sets A, B and C. Hence find $n(A)$, $n(B)$ and $n(C)$.

b List the elements (if any) in the set $B \cap C$.

c Find $n(A \cup B)$ and $n(B \cup C)$.

d How would you describe the set $B \cap C$?

18



The Venn diagram shows the reading habits of the 30 students in a class. Set A enjoy female authors, set B enjoy male authors, while set C prefer anonymous authors.

How many students

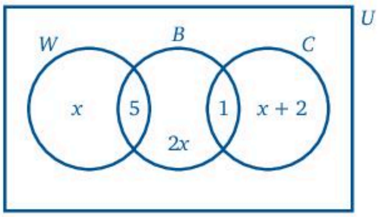
a enjoy both male and female authors

b do no reading

c enjoy male authors

d do not read any books by authors who are known to be either male or female?

19



The Venn diagram shows three sets W, B and C contained within a universal group which illustrate how the students in a class of 24 come to school. W = {students who walk to school}, B = {students who come to school by bus}, C = {students who come to school by car}.

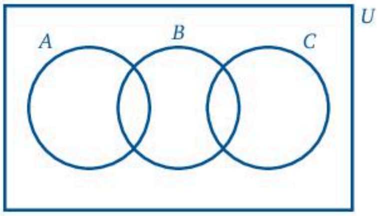
Every student uses at least one mode of transport.

a Write an expression in terms of x, for the number of students who use a bus.

b Write an expression, in terms of x, for the total number of students in the class.

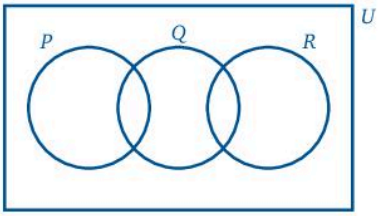
Hence find the value of x.

c How many students come by car?

20 

The Venn diagram shows three sets A, B and C, contained within a universal set U.

Copy the diagram and enter numbers on the correct region of your diagram using the following information:
$n(A \cap B) = 3$, $n(B \cap C) = 1$, $n(A) = 9$, $n(B) = 14$, and $n(C) = 7$
Use your diagram to find $n(A \cup B \cup C)$.

21 

The Venn diagram shows three sets P, Q and R, contained within a universal set U.

Copy the diagram and enter numbers on the correct region of your diagram using the following information:
$n(P \cap Q) = 2$, $n(Q \cap R) = 3$, $n(P) = 9$, $n(Q) = 13$, and $n(R) = 8$
Use your diagram to find $n(P \cup Q \cup R)$.

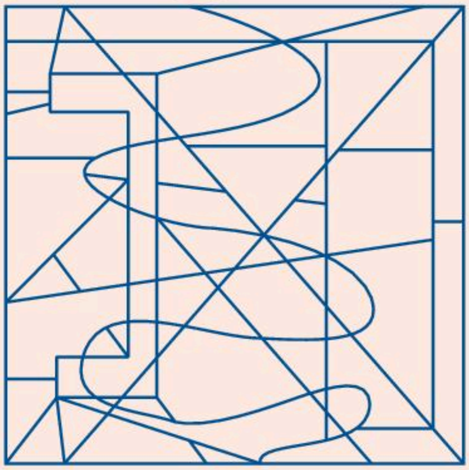
22 $U = \{p, q, r, s, t, u, v, w, x, y, z\}$
$A = \{p, s, t, y\}$
$B = \{q, s, u, v, z\}$
$C = \{r, u, x, z\}$

a Draw a Venn diagram showing the sets A, B and C.

b List the members of the set represented by $(A \cap B) \cup C$.

Puzzle

Every map or shape divided into regions can be coloured so that no two touching edges of any regions are the same colour using not more than four different colours. Make a copy of the diagram given below and colour it so that no two touching edges of any regions are the same colour.



If you are successful, try using three different colours. Is it possible?

Mixed exercises

Exercise 15d

1 Write each statement in set notation:
 a Nebraska is an American state.
 b Pound sterling is not a currency used in the West Indies.

2 State whether or not the following statements are true:
 a Dominica $\in$ {islands in the Pacific Ocean}
 b 47 $\in$ {prime numbers}

3 Are the following sets finite or infinite?
 a the number of positive fractions where the numerator is larger than the denominator
 b the number of three-digit numbers that can be written using the digits 3, 4, 5 and 6, if each digit is used no more than once

4 If A = {prime numbers between 15 and 25}

a find $n(A)$

b write down the subset of A whose elements are odd numbers. Is this a proper subset of A?

c write down the subset of A whose elements are even numbers. What special name do we give to this set?

5 $U = \{4, 5, 6, \ldots, 16\}$, C = {multiples of 3} and D = {even numbers}. Show these sets in a Venn diagram. Use this diagram to list the elements in $C \cap D$.

6 U = {different letters in the word COMPLEMENT}

A = {different letters in the word TEMPT}

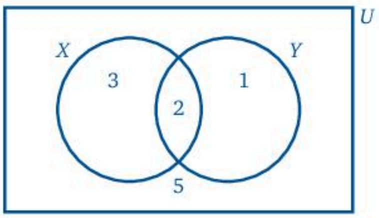
B = {different letters in the word CEMENT}

Show U, A and B on a Venn diagram, entering all the elements.

Hence list the following sets:

a $A \cup B$ **b** $A \cap B$

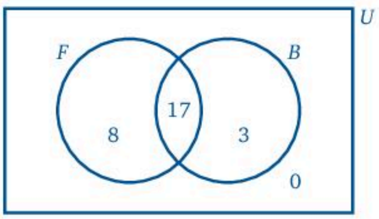
7



Use the information given in the Venn diagram to find:

a $n(X)$ **b** $n(Y)$ **c** $n(X \cup Y)$ **d** $n(X \cap Y)$

8



The Venn diagram shows how many houses in a street have windows in the back wall (B) and how many have windows in the front wall (F).

How many houses

a have windows at the back and front of the house

b have windows at the back only

c do not have any windows at the back or the front

d are there in the street?

9 The 28 students in a class were asked whether or not they played draughts or chess, or both. Their replies showed that 4 played both, 7 played chess and 15 played only draughts. The remainder did not play either game.

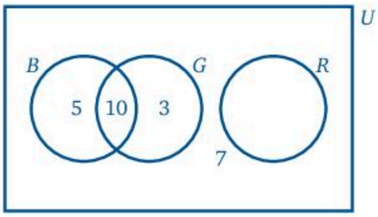
Show this information on a Venn diagram and hence find how many students

a played chess only

b played draughts

c did not play either game.

10



A group of 30 students were asked to choose the colours they liked from the colours red (R), green (G) and blue (B). Their answers are shown in the Venn diagram.

How many

a chose red

b chose both blue and green

c did not choose a colour

d did not choose blue or green

e chose a single colour

f chose blue?

Exercise 15e

In this exercise choose the letter that gives the correct answer.

1 For the statements

i $56 \in \{\text{multiples of } 7\}$ and **ii** $40 \notin \{\text{prime numbers}\}$

which statements are true?

A **i** **B** **ii** **C** **i** and **ii** **D** neither

2 If $U = \{\text{a whole number such that } 4 < x \leqslant 28\}$ and $P = \{\text{multiples of 4 or 6}\}$, then $P =$

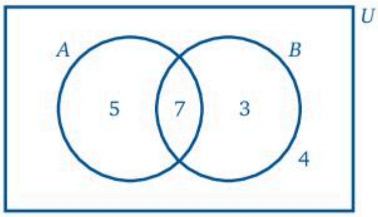
A {4, 6, 12, 18, 24}

B {6, 8, 12, 16}

C {8, 12, 16, 18, 20}

D {6, 8, 12, 16, 18, 20, 24, 28}

Use this diagram to answer questions **3** and **4**.



3 The value of $n(A \cup B)$ is

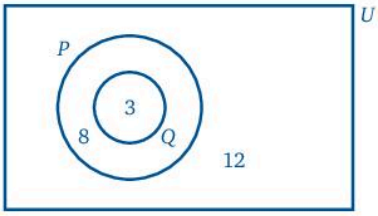
A 8 **B** 10 **C** 15 **D** 19

4 The value of $n(A \cap B)$ is

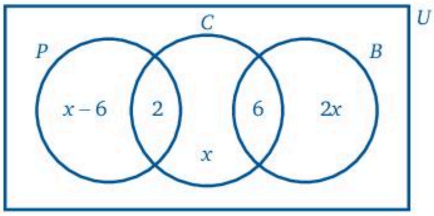
A 3 **B** 4 **C** 5 **D** 7

5 The value of $n(P \cup Q)$ is

A 3 **B** 8 **C** 11 **D** 12



Use this diagram for questions **6** to **8**.
The Venn diagram shows the number of students taking physics, chemistry and biology in a class of 34. Every student takes at least one of these subjects.



6 The value of x is

A 4 **B** 6 **C** 8 **D** 10

7 The number of students who take chemistry is

A 8 **B** 10 **C** 12 **D** 16

8 The number of students who do not take biology is

A 8 **B** 10 **C** 12 **D** 16

Did you know?

The famous French mathematician René Descartes (1596–1650) believed that the whole of human knowledge should be founded on the belief *cogito ergo sum* – I think, therefore I am. He also believed that the entire material universe could be explained in terms of mathematical physics. You will come across his name again in the branch called Cartesian or coordinate geometry.

In this chapter you have seen that...

✔ many problems involving details about two or three sets can be solved using Venn diagrams.

16 Logic

At the end of this chapter you should be able to...

1. draw a truth table
2. establish the truth value of compound propositions using truth tables
3. determine if two propositions are logically equivalent
4. explain what an argument is
5. describe the difference between a valid argument and a valid conclusion
6. determine the validity of an argument
7. describe the difference between a valid argument and a sound argument.

You need to know...

- ✔ the meaning of a proposition
- ✔ the language of logic
- ✔ the difference between a simple proposition and a compound proposition
- ✔ what an implication means.

Key words

argument, compound proposition, conclusion, conjunction, disjunction, equivalent, implication, negation, premise, proposition, sound argument, truth table, valid argument

Did you know?

Aristotle (*c*. 384–322 BCE) was a Greek philosopher who, among many achievement, was the first to give a formal structure to logic. He demonstrated the principles of reasoning by employing variables to show the underlying logical form of an argument.

Symbolic logic

Propositions, negation, conjunction and disjunction are introduced in Grade 8. We revise them here.

A *proposition* is a statement that is either true or false. There is no in-between or maybe.

'$2 + 2 = 4$' is a proposition (it is true) and '$2 + 3 = 4$' is also a proposition (it is false).

'$x + 2 = 4$' is not a proposition because we do not know the value of x.

For p: $2 + 2 = 4$ (true), $\sim p$ is the *negation* of p, i.e. $\sim p : 2 + 2 \neq 4$ (false).

For q: $2 + 3 = 4$ (false), $\sim q$ is the negation of q, i.e. $\sim q$: $2 + 3 \neq 4$ (true).

These two examples illustrate that for any statement p,

if p is true, $\sim p$ is false

if p is false, $\sim p$ is true.

The *conjunction* of two propositions, p and q, is written as $p \wedge q$ and means p and q.
For example for p: $2 + 2 = 4$, q: $2 + 3 = 4$, $p \wedge q$ means '$2 + 2 = 4$ and $2 + 3 = 4$'.

The *disjunction* of two propositions, p and q, is written as $p \vee q$ and means p or q.
For example for p: $2 + 2 = 4$, q: $2 + 3 = 4$, $p \vee q$ means '$2 + 2 = 4$ or $2 + 3 = 4$'.

An *implication* is an *if… then…* connection between two statements.
For example **if** $2x = 4$ **then** $x = 2$.

The statements may or may not be propositions.
In the example above, $2x = 4$ is not a proposition because we cannot know whether it is true or false, but we can say that $2x = 4 \Rightarrow x = 2$. So $2x = 4 \Rightarrow x = 2$ is true but $2x = 7 \Rightarrow x = 2$ is false.

Exercise 16a

For each of the following propositions p, write in words the meaning of $\sim p$.

1 p: It is snowing.

2 p: 2 is a positive integer.

3 p: 5 is a prime number.

4 p: All prime numbers are even numbers.

For each of the following propositions p and q, write in words the meaning of

a $p \wedge q$ **b** $p \vee q$ **c** $p \wedge \sim q$ **d** $\sim p \vee q$

5 p: I am eating, q: I am hungry.

6 p: All cars have four wheels, q: All bicycles have two wheels.

7 p: I have a headache, q: I like chocolate.

8 p: 2 is an even number, q: 2 is a prime number.

9 p: a is a vowel, q: b is a vowel.

Determine whether the following implications are true or false.

10 4 is an even number $\Rightarrow$ 4 is divisible by 2.

11 $x + 1 = 2 \Rightarrow x = 1$

12 $x + 1 \neq 2 \Rightarrow x \neq 1$

13 It is raining $\Rightarrow$ the grass is wet.

14 It is not raining $\Rightarrow$ the grass is not wet.

Truth tables

Truth tables are used to determine if a *compound proposition* is true or false.

For the statements p: $2 + 2 = 4$, q: $2 + 3 = 4$, the conjunction $p \wedge q$ means '$2 + 2 = 4$ and $2 + 3 = 4$'.
p is true and q is false, so it is clear that '$2 + 2 = 4$ and $2 + 3 = 4$' is false.

For any two statements, p and q,

$p \wedge q$ is true only when both p and q are true.
If either p or q or both are false then $p \wedge q$ is false.

We can show all combinations of true and false for p, q and $p \wedge q$ in a table, called a *truth table*.

To make a truth table for p and q, we tabulate all the possible combinations for true (T) and false (F) in the first two columns. Then, as $p \wedge q$ is true only when both p and q are true, we can fill in the last column.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

We can make a similar table for the disjunction of two propositions.
Consider again the statements p: $2+2=4$, q: $2+3=4$. The disjunction $p \vee q$ means '$2+2=4$ or $2+3=4$' and this is clearly true.

For any two statements, p and q,

> $p \vee q$ is true when p or q or both are true.
> If p and q are both false then $p \vee q$ is false.

The truth table for $p \vee q$ is

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Exercise 16b

1 Given p: '24 is divisible by 12' and q: '12 is divisible by 5'

- **a** use the appropriate truth table to determine whether $p \wedge q$ is true
- **b** use the appropriate truth table to determine whether $p \vee q$ is true.

2 Repeat question **1** for p: 'a is a vowel' and q: 'b is a consonant'.

3 **a** Copy and complete this truth table.

p	$\sim p$	q	$\sim p \vee q$
T	F	T	
T	F	F	
F		T	
F		F	

Remember that if p is true, $\sim p$ is false and vice versa.

b Given p: 'a is a vowel' and q: 'b is a consonant', use the truth table to determine whether $\sim p \vee q$ is true or false.

c Repeat part **b** for p: '$2 + 2 = 4$' and q: '$2 + 3 = 4$'.

4 **a** Copy and complete this truth table.

p	$\sim p$	q	$\sim p \wedge q$
T	F	T	
T	F	F	
F		T	
F		F	

b Given p: 'a is a vowel' and q: 'b is a consonant', use the truth table to determine whether $\sim p \vee q$ is true or false.

c Repeat part **b** for p: '$2 + 2 = 4$' and q: '$2 + 3 = 4$'.

5 **a** Copy and complete this truth table.

p	q	$\sim q$	$p \wedge \sim q$
T	T	F	
T	F	T	
F	T		
F			

b Given p: 'there are 7 days in a week' and q: 'there are 12 months in a year', use the truth table to determine whether $\sim p \vee q$ is true or false.

c Repeat part **b** for p: '16 is a multiple of 4' and q: '16 is divisible by 3'.

<u>6</u> **a** Copy and complete this truth table for $\sim p \wedge \sim q$.

p	$\sim p$	q	$\sim q$	$\sim p \wedge \sim q$
T		T	F	
T		F	T	
F				

b Make a truth table for $\sim p \vee \sim q$.

Logical equivalence

Two propositions are *equivalent* if they have the same truth value.
For example, $2+2=4$ and $1+3=4$ are both true, so they are equivalent.
Also, $2+2=5$ and $1+3=5$ are both false, so they are also equivalent.

Two compound propositions are equivalent if they have the same truth tables.
Here are the truth tables for $p \wedge q$ and $\sim p \wedge \sim q$.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	T
T	F	F
F	T	F
F	F	F

The values of T and F are the same, so $p \wedge q$ and $\sim p \wedge \sim q$ are equivalent.

Two implications are also equivalent if they are both true. For example,
$2x=4 \Rightarrow x=2$ and $2x \neq 4 \Rightarrow x \neq 2$ are both true so they are equivalent.

This is the truth table for an implication.

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

These examples illustrate the reasoning behind the truth values for $p \Rightarrow q$.

If p: 4 is an even number **then** q: 4 is a multiple of 2
then p is true and q is true, so $p \Rightarrow q$ is true.

If p: 4 is an even number **then** q: 4 is a multiple of 3
then p is true but q is false, so $p \Rightarrow q$ is false.

If p: 4 is an odd number **then** q: 4 is a multiple of 2
so p is false and q is true, then, because the conclusion is correct, $p \Rightarrow q$ is taken to be true.

(This is a difficult concept to accept, but think of it this way: 4 **is** a multiple of 2, so it doesn't matter what p is.)

If p: 4 is an odd number **then** q: 4 is a multiple of 3
then p and q are both false. This is equivalent to p: 4 is not an odd number $\Rightarrow q$: 4 is not a multiple of 3, so $p \Rightarrow q$ is true.

Exercise 16c

1 Given p: '5 is a prime number' and q: '5 is an odd number', determine which of the following are equivalent.

A $p \Rightarrow q$ **B** $q \Rightarrow p$ **C** $\sim p \Rightarrow \sim q$ **D** $\sim p \Rightarrow q$

2 Given p: 'It is raining' and q: 'The grass is wet', determine which of the following are equivalent.

A $p \Rightarrow q$ **B** $q \Rightarrow p$ **C** $\sim p \Rightarrow \sim q$ **D** $\sim p \Rightarrow q$

3 Given p: 'It is sunny' and q: 'I will go for a walk', determine which of the following are equivalent.

A $p \Rightarrow q$ **B** $q \Rightarrow p$ **C** $\sim p \Rightarrow \sim q$ **D** $\sim p \Rightarrow q$

4 Given p: '4 is a multiple of 2' and q: '$4 \div 2$ is an integer', determine which of the following are equivalent.

A $p \Rightarrow q$ **B** $q \Rightarrow p$ **C** $\sim p \Rightarrow \sim q$ **D** $\sim p \Rightarrow q$

5 Given p: 'All politicians are liars' and q: 'The Prime Minister is a liar', determine which of the following are equivalent.

A $p \Rightarrow q$ **B** $q \Rightarrow p$ **C** $\sim p \Rightarrow \sim q$ **D** $\sim p \Rightarrow q$

6 Given p: '$x + 2 = 5$' and q: '$x = 3$', determine which of the following are equivalent.

A $p \Rightarrow q$ **B** $q \Rightarrow p$ **C** $\sim p \Rightarrow \sim q$ **D** $\sim p \Rightarrow q$

Arguments

We can think of an implication as an *argument*. For example **if** 6 is an even number **then** 6 is divisible by 2.

'6 is an even number' is called the *premise* and '6 is divisible by 2' is called the *conclusion*.

This form of argument is deductive, because we rely on known facts, i.e. all even numbers are divisible by 2, so 6 is divisible by 2.

A *valid argument* is one in which the premise (true or false) has a correct conclusion.

For example,

'If 6 is an even number then 6 is divisible by 2'

is a valid argument.

'If $x+3=5$ and $x+2=5$ then $3=2$'

is also a valid argument because it uses the fact that if $a+b=c$ and $a+d=c$, then $b=d$, so the deduction is correct.
(This corresponds to the row in the truth table for an implication where p is false and q is true, so $p \Rightarrow q$ is true.)

Another example of a valid argument is

'If it is raining gold coins then gold coins can be picked up from the ground.'

However,

'If x is a positive integer then $x=-1$'

is not a valid argument because the deduction is wrong.

A *sound argument* is valid and *both* the premise and the conclusion are correct.

For example,

'If 6 is an even number then 6 is divisible by 2'

is a sound argument because 6 *is* an even number and 6 *is* divisible by 2.

However,

'If $x + 3 = 5$ and $x + 2 = 5$ then $3 = 2$'

is not a sound argument because 3 is not equal to 2.

Also, 'If it is raining gold coins then gold coins can be picked up from the ground.' is not a sound argument because it is not true that gold coins rain down.

Arguments do not always use 'if… then…'. There are other ways of expressing an implication.

For example, '6 is an even number so 6 is divisible by 2'

or '6 is an even number therefore 6 is divisible by 2'

or 'because 6 is an even number, it is divisible by 2'

or 'given that 6 is an even number, 6 is divisible by 2'

Exercise 16d

Determine if the following arguments are not valid, valid, or valid and sound. Give a reason for your answer.

1 2 is a prime number so all prime numbers are even numbers.

2 The Moon is made of green cheese so the man in the Moon can eat green cheese.

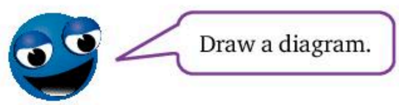
3 If the Earth is flat then you will fall off the Earth if you go over the edge.

4 If a pack of ten pencils costs $1000 then the cost of one pencil is $100.

5 If a person cannot swim then they will drown if they fall into the sea.

6 The year 2019 was the hottest year on record so this proves that the climate is warming.

7 The triangle ABC has a right angle at A so $AB^2 + AC^2 = BC^2$.



8 Ethan is a basketball player so Ethan is tall.

9 All fish are mammals so my goldfish is a mammal.

10 The Earth is not flat so you cannot fall off the edge of the Earth.

11 If a quadrilateral has four sides of equal length then the quadrilateral is a square.

12 All tennis players are right-handed so Cheryl is a tennis player and is right-handed.

Did you know?

Boolean algebra is based on the work on logic by George Boole (1815–1864), who was an English mathematician. He laid the foundations for the binary algebra used in computer chips.

Digital technology is based on ones and zeros called bits. Using 1 for true and 0 for false, a truth table for $A \vee B$ looks like his:

A	**B**	**$A \vee B$**
1	1	1
1	0	1
0	1	1
0	0	0

Calculations similar to this are performed inside computer chips using devices called logic gates.

In this chapter you have seen that...

- ✔ a proposition is a statement that is either true or false
- ✔ for any statement p, if p is true, $\sim p$ is false and if p is false, $\sim p$ is true
- ✔ the conjunction of two propositions, p and q, is written as $p \wedge q$ and means p and q
- ✔ the disjunction of two propositions, p and q, is written as $p \vee q$ and means p or q
- ✔ an implication, $p \Rightarrow q$ is an if… then… connection between two statements
- ✔ a truth table shows all combinations of true and false for p, q and one of $p \wedge q$, $p \vee q$ or $p \Rightarrow q$ in a table
- ✔ two propositions are equivalent if they have the same truth value
- ✔ two compound propositions are equivalent if they have the same truth tables
- ✔ an argument is an implication if p then q. An argument may be worded differently, e.g. p so q
- ✔ a valid argument is one in which the premise p (true or false) has a correct conclusion q
- ✔ an argument is sound when *both* the premise, p, and the conclusion, q, are correct.

17 Matrices

At the end of this chapter you should be able to...

1 arrange a set of data in a rectangular array called a matrix
2 state the size of a given matrix
3 write down the entry in a matrix, given its row and column
4 add and subtract two or more matrices
5 multiply a matrix by a constant.

Did you know?

The symbol '=', which stands for 'is equal to', was first proposed by a physician named Robert Recorde (1512–1558) in a book published in 1557 entitled *The Whetstone of Witte*. However, it was a long time before the symbol was generally adopted.

You need to know...

✔ how to work with directed numbers
✔ the meaning of commutative and distributive.

Key words

column, element, entry, matrix (plural matrices), order, rectangular array, row, scalar, vector

Shopping lists

Mrs Smith and Mrs Jones go shopping for oranges, lemons and grapefruit. Mrs Smith buys six oranges, two lemons and three grapefruit while Mrs Jones buys five oranges, one lemon and two grapefruit. We can arrange this information in a table:

	Oranges	Lemons	Grapefruit
Mrs Smith	6	2	3
Mrs Jones	5	1	2

We can write this information more briefly in the form $\begin{pmatrix} 6 & 2 & 3 \\ 5 & 1 & 2 \end{pmatrix}$

All the numerical information is given, though we have missed out the descriptions of what the various rows and columns mean.

A *rectangular array* of numbers is called a *matrix* (plural *matrices*). It is held together by curved brackets. Each item is called an *entry* or *element*. A matrix can have any number of *rows* and *columns;* rows go across and columns go down. If we had the shopping lists of three people buying four different sorts of fruit we would have a matrix with three rows and four columns.

e.g. $\begin{pmatrix} 4 & 3 & 1 & 2 \\ 1 & 2 & 0 & 4 \\ 9 & 0 & 1 & 1 \end{pmatrix}$

We use the number of rows and the number of columns to describe the size or *order* of a matrix. This matrix is a 3×4 matrix (number of rows first, then the number of columns). A matrix with the same number of rows and columns is called a square matrix. For example, $\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ is a square matrix.

Exercise 17a

a Give the order of the matrix $\begin{pmatrix} 1 & 4 & 3 & 2 \\ 7 & 0 & 6 & 5 \end{pmatrix}$.

b What is the entry in the second row, third column?

a The matrix has 2 rows and 4 columns so it is a 2×4 matrix.

b The entry in the second row, third column is 6.

Give the order of the matrices in questions **1** to **6**.

1 $\begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}$

2 $\begin{pmatrix} 2 & 3 & 4 \\ 1 & 1 & 4 \end{pmatrix}$

3 $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

4 $\begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$

5 (2)

6 $\begin{pmatrix} 2 & 3 \\ 4 & 5 \\ 6 & 7 \end{pmatrix}$

7 In the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ give the entry in

a the second row, third column
b the third row, second column
c the first row, second column
d the third row, first column

8 **a** Write down the second row and the third column of the matrix
$\begin{pmatrix} 5 & 2 & 4 \\ 3 & 1 & 7 \\ 9 & 6 & 2 \end{pmatrix}$.

b Give the entry in the second row of the third column.
c Give the entry in the third row of the second column.
d Give the entry in the first row of the third column.

9 Write down the matrix with three rows and three columns in which the first row is a row of zeros, the second row is a row of ones and the third row is a row of twos.

10 Write down the 3×2 matrix in which the first column is a column of threes and the second column is a column of ones.

Addition of matrices

Mrs Smith's and Mrs Jones' first shopping lists were represented by the matrix $\begin{pmatrix} 6 & 2 & 3 \\ 5 & 1 & 2 \end{pmatrix}$

The next week, Mrs Smith buys three oranges, no lemons and one grapefruit and Mrs Jones buys four oranges, two lemons and one grapefruit. We can arrange this in a table:

	Oranges	Lemons	Grapefruit
Mrs Smith	3	0	1
Mrs Jones	4	2	1

and represent it by the matrix $\begin{pmatrix} 3 & 0 & 1 \\ 4 & 2 & 1 \end{pmatrix}$

We see that over the two weeks Mrs Smith buys nine oranges, two lemons and four grapefruit and Mrs Jones buys nine oranges, three lemons and three grapefruit. This information can be written briefly in the form

$$\begin{pmatrix} 6 & 2 & 3 \\ 5 & 1 & 2 \end{pmatrix} + \begin{pmatrix} 3 & 0 & 1 \\ 4 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 9 & 2 & 4 \\ 9 & 3 & 3 \end{pmatrix}$$ Add the corresponding elements e.g. $6 + 3 = 9$

Each of these matrices is a 2×3 matrix.

We have added the entries in corresponding positions, e.g. in the first row, third column, $3 + 1 = 4$

In the third week, Mrs Smith buys two oranges and one lemon and Mrs Jones buys four oranges and one lemon.

The matrix showing this information is $\begin{pmatrix} 2 & 1 \\ 4 & 1 \end{pmatrix}$

This is a 2×2 matrix.

We do not know how many grapefruit were bought in the third week. This makes it impossible to add the matrices for the second and third weeks

i.e. $\begin{pmatrix} 3 & 0 & 1 \\ 4 & 2 & 1 \end{pmatrix} + \begin{pmatrix} 2 & 1 \\ 4 & 1 \end{pmatrix}$ cannot be worked out.

We can add matrices if they are the same order, but not if they are different orders.

Exercise 17b

Find, where possible

a $\begin{pmatrix} 6 & 3 \\ 1 & 2 \end{pmatrix} + \begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$ **b** $\begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 4 & 1 \\ 0 & 3 \end{pmatrix}$

a $\begin{pmatrix} 6 & 3 \\ 1 & 2 \end{pmatrix} + \begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 10 & 4 \\ 4 & 4 \end{pmatrix}$

b $\begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 4 & 1 \\ 0 & 3 \end{pmatrix}$ not possible since the matrices are not the same order.

Find, where possible:

1 $\begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 10 \\ 12 \end{pmatrix}$

2 $\begin{pmatrix} 9 & 2 \\ 4 & 1 \end{pmatrix} + \begin{pmatrix} 6 & 2 \\ 3 & 0 \end{pmatrix}$

3 $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 & 6 \\ 4 & 7 \\ 5 & 8 \end{pmatrix}$

4 $(6 \quad 1) + (3 \quad 4)$

5 $\begin{pmatrix} 4 & 1 & 2 \\ 3 & 5 & 6 \end{pmatrix} + \begin{pmatrix} 7 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$

6 $\begin{pmatrix} 7 & 8 \\ 9 & 1 \end{pmatrix} + \begin{pmatrix} 4 & 3 \\ 2 & 4 \end{pmatrix}$

7 $(2 \quad 1 \quad 4) + (3 \quad 2 \quad 1)$

8 $\begin{pmatrix} 1 \\ 2 \end{pmatrix} + (3 \quad 4)$

9 $\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} + \begin{pmatrix} 5 & 6 \\ 4 & 3 \\ 2 & 1 \end{pmatrix}$

10 $(6 \quad 3) + (4 \quad 5)$

Negative numbers can be used.

Find $\begin{pmatrix} 4 & -3 \\ 4 & -1 \end{pmatrix} + \begin{pmatrix} -3 & 2 \\ 1 & 4 \end{pmatrix}$

$$\begin{pmatrix} 4 & -3 \\ 4 & -1 \end{pmatrix} + \begin{pmatrix} -3 & 2 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 4+(-3) & -3+2 \\ 4+1 & -1+4 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 5 & 3 \end{pmatrix}$$

Find, where possible:

11 $\begin{pmatrix} 2 & 4 \\ -1 & 4 \end{pmatrix} + \begin{pmatrix} -1 & 4 \\ -3 & 3 \end{pmatrix}$

12 $\begin{pmatrix} 1 \\ -4 \end{pmatrix} + \begin{pmatrix} -3 \\ 6 \end{pmatrix}$

13 $\begin{pmatrix} 4 & 3 \\ -2 & 1 \end{pmatrix} + \begin{pmatrix} -6 & 4 \\ -3 & 2 \end{pmatrix}$

14 $(1 \quad 2) + (3 \quad 4)$

15 $\begin{pmatrix} -2 \\ -3 \\ 4 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$

16 $(1 \quad -6 \quad 1) + (-4 \quad 1)$

17 $\begin{pmatrix} 3 & 6 \\ 2 & -5 \end{pmatrix} + \begin{pmatrix} -1 & 4 \\ 3 & 2 \end{pmatrix}$

18 $\begin{pmatrix} 1 & -2 \\ -2 & 1 \end{pmatrix} + \begin{pmatrix} 4 & -3 \\ 5 & -1 \end{pmatrix}$

<u>**19**</u> $\begin{pmatrix} 3 & 2 \\ -1 & -4 \end{pmatrix} + \begin{pmatrix} -3 & 6 \\ 9 & 2 \end{pmatrix}$

<u>**20**</u> $\begin{pmatrix} 4 \\ -2 \end{pmatrix} + \begin{pmatrix} 4 & -2 \end{pmatrix}$

<u>**21**</u> $\begin{pmatrix} 3 & 2 & 0 \\ 1 & 4 & 3 \end{pmatrix} + \begin{pmatrix} 2 & 4 \\ -3 & 4 \\ 1 & 2 \end{pmatrix}$

<u>**22**</u> $\begin{pmatrix} 1 & 4 & -3 \end{pmatrix} + \begin{pmatrix} 0 & 2 & 0 \end{pmatrix}$

23 $\begin{pmatrix} 3 & -1 & 1 \\ 5 & 6 & -7 \end{pmatrix} + \begin{pmatrix} -1 & 4 & 3 \\ 0 & -6 & -5 \end{pmatrix}$

Multiples of matrices

If Mrs Smith and Mrs Jones each have the same shopping list for three weeks running we can see that

$$\begin{pmatrix} 3 & 1 & 4 \\ 1 & 2 & 1 \end{pmatrix} + \begin{pmatrix} 3 & 1 & 4 \\ 1 & 2 & 1 \end{pmatrix} + \begin{pmatrix} 3 & 1 & 4 \\ 1 & 2 & 1 \end{pmatrix} = 3\begin{pmatrix} 3 & 1 & 4 \\ 1 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 9 & 3 & 12 \\ 3 & 6 & 3 \end{pmatrix}$$

In the same way $5\begin{pmatrix} 1 & 4 \\ 3 & -2 \end{pmatrix} = \begin{pmatrix} 5 & 20 \\ 15 & -10 \end{pmatrix}$

When we multiply a matrix by five, we multiply *every* entry by five.

Subtraction of matrices

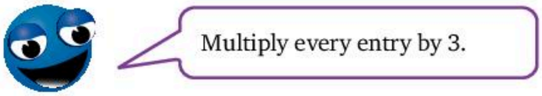
We can subtract matrices if they are the same order.

$$\underset{\text{Two weeks' shopping}}{\begin{pmatrix} 7 & 9 & 2 \\ 6 & 4 & 1 \end{pmatrix}} - \underset{\text{First week's shopping}}{\begin{pmatrix} 3 & 2 & 1 \\ 3 & 3 & 1 \end{pmatrix}} = \underset{\text{Second week's shopping}}{\begin{pmatrix} 4 & 7 & 1 \\ 3 & 1 & 0 \end{pmatrix}}$$

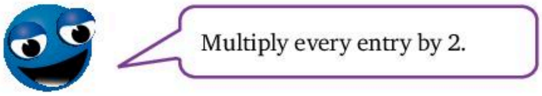
Exercise 17c

Find, where possible:

1 $3\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$



2 $2\begin{pmatrix} 1 & 4 & 0 \\ 2 & -1 & 3 \end{pmatrix}$



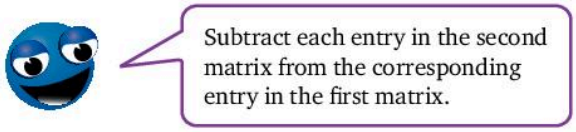
3 $\frac{1}{2}\begin{pmatrix} 2 & 4 \\ 1 & 6 \\ 3 & 8 \end{pmatrix}$

4 $6\begin{pmatrix} 1 & 4 \\ 3 & -2 \end{pmatrix}$

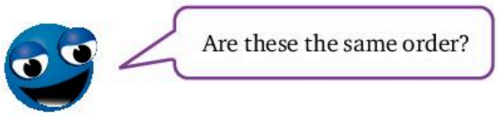
5 $6\begin{pmatrix} -1 & -5 \\ 1 & 2 \end{pmatrix}$

6 $\frac{2}{3}\begin{pmatrix} 6 & 0 \\ 1 & 2 \\ 3 & 5 \end{pmatrix}$

7 $\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} - \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$



8 $\begin{pmatrix} 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix}$



9 $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$

10 $\begin{pmatrix} 4 & 5 & 3 \\ 1 & -2 & 1 \end{pmatrix} - \begin{pmatrix} 2 & 1 & 1 \\ 4 & 1 & 2 \end{pmatrix}$

11 $(1 \quad 6 \quad 2) - (4 \quad 3)$

12 $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} - \begin{pmatrix} 4 & 3 & 1 \\ -5 & 0 & 2 \\ 6 & -3 & 4 \end{pmatrix}$

Mixed questions

Exercise 17d

Find, where possible:

1 $\begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix} + \begin{pmatrix} -2 & 4 \\ 3 & -1 \end{pmatrix}$

2 $\begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix} - \begin{pmatrix} -2 & 4 \\ 3 & -1 \end{pmatrix}$

3 $(1 \quad 2 \quad 4) - (3 \quad 2)$

4 $\begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix} + \begin{pmatrix} 4 & -3 \\ 1 & 0 \end{pmatrix}$

5 $\frac{1}{3}\begin{pmatrix} 4 \\ 5 \\ -1 \end{pmatrix}$

6 $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$

7 $2\begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$

8 $\frac{1}{2}\begin{pmatrix} -2 & 4 \\ 3 & -1 \end{pmatrix}$

9 $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ 5 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$

<u>10</u> $4\begin{pmatrix} 6 & 2 & -1 \\ 4 & 3 & 4 \end{pmatrix}$

<u>11</u> $\begin{pmatrix} 6 & 2 & 1 \\ 4 & 3 & 4 \end{pmatrix} + \begin{pmatrix} -2 & 4 \\ 3 & -1 \end{pmatrix}$

<u>12</u> $\begin{pmatrix} 6 & 2 & -1 \\ 4 & 3 & 4 \end{pmatrix} - \begin{pmatrix} -2 & 4 & 1 \\ 3 & -1 & 0 \end{pmatrix}$

Puzzle

The large square is divided into sixteen smaller squares.

Each small square is coloured red (R), white (W), yellow (Y) or blue (B).

Divide the large square into four pieces so that each piece is made up of four small squares with four different colours.

R	R	W	Y
W	Y	R	B
B	R	W	Y
Y	W	B	B

Use of letters

In Grade 8 we saw that we could use a single small letter in heavy type to denote a *vector*,

e.g. $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

In the same way we can represent a matrix by giving it a capital letter in heavy type:

e.g. $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 4 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 4 & 1 \\ 3 & 0 \end{pmatrix}$

then $\mathbf{A} + \mathbf{B} = \begin{pmatrix} 1 & 2 \\ 4 & 1 \end{pmatrix} + \begin{pmatrix} 4 & 1 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} 5 & 3 \\ 7 & 1 \end{pmatrix}$

and $2\mathbf{A} = 2\begin{pmatrix} 1 & 2 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 8 & 2 \end{pmatrix}$

We cannot *write* **A**, so we write <u>A</u>.

Exercise 17e

The questions in this exercise refer to the following matrices:

$\mathbf{A} = \begin{pmatrix} 4 & 3 & 1 \\ 1 & 2 & 3 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ $\mathbf{C} = \begin{pmatrix} 2 & 3 \\ 1 & -2 \end{pmatrix}$ $\mathbf{D} = \begin{pmatrix} 6 & 2 \\ 1 & 4 \end{pmatrix}$

$\mathbf{E} = (6 \quad -1 \quad 2)$ $\mathbf{F} = (3 \quad 2)$ $\mathbf{G} = \begin{pmatrix} 5 & 1 & 3 \\ 6 & -1 & 4 \end{pmatrix}$

Give the order of **A**.

A is a 2×3 matrix (It has 2 rows and 3 columns.)

Find, where possible **a** $\mathbf{C} - \mathbf{D}$ **b** $\mathbf{B} + \mathbf{F}$

a $\mathrm{C} - \mathrm{D} = \begin{pmatrix} 2 & 3 \\ 1 & -2 \end{pmatrix} - \begin{pmatrix} 6 & 2 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} -4 & 1 \\ 0 & -6 \end{pmatrix}$

b $\mathbf{B} + \mathbf{F} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} + (3 \quad 2)$ not possible since B and F are not the same order.

1 Give the orders of the matrices **B** to **G**.

Find, where possible:

2 $\mathbf{A} + \mathbf{G}$	**5** $3\mathbf{A}$	**8** $\mathbf{G} + \mathbf{B}$	**11** $\frac{3}{4}\mathbf{D}$
3 $\mathbf{B} + \mathbf{C}$	**6** $\mathbf{E} + \mathbf{F}$	**9** $\mathbf{G} - \mathbf{A}$	**12** $\mathbf{C} + \mathbf{D} + \mathbf{G}$
4 $\mathbf{D} - \mathbf{C}$	**7** $\frac{1}{2}\mathbf{F}$	**10** $6\mathbf{B}$	**13** $\mathbf{F} - \mathbf{E}$

Commutative and distributive laws

Consider the sum $\begin{pmatrix} 1 & 2 & 1 \\ 3 & 0 & 2 \\ -1 & 4 & -3 \end{pmatrix} + \begin{pmatrix} 3 & 3 & 1 \\ 4 & 5 & 6 \\ 6 & -4 & 2 \end{pmatrix}$. We know that to find this sum, we add the corresponding elements.

We also know that the addition of real numbers is commutative, so it follows that

$$\begin{pmatrix} 1 & 2 & 1 \\ 3 & 0 & 2 \\ -1 & 4 & -3 \end{pmatrix} + \begin{pmatrix} 3 & 3 & 1 \\ 4 & 5 & 6 \\ 6 & -4 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 3 & 1 \\ 4 & 5 & 6 \\ 6 & -4 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 2 & 1 \\ 3 & 0 & 2 \\ -1 & 4 & -3 \end{pmatrix} \quad \left(= \begin{pmatrix} 4 & 5 & 2 \\ 7 & 5 & 8 \\ 5 & 0 & 4 \end{pmatrix} \right)$$

This is true of the sum of any two matrices of the same order, i.e. $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$

Therefore addition of matrices is commutative.

Now consider $2\left(\begin{pmatrix} 1 & 2 & 1 \\ 3 & 0 & 2 \\ -1 & 4 & -3 \end{pmatrix} + \begin{pmatrix} 3 & 3 & 1 \\ 4 & 5 & 6 \\ 6 & -4 & 2 \end{pmatrix}\right)$.

If we add the matrices first this gives $2\begin{pmatrix} 4 & 5 & 2 \\ 7 & 5 & 8 \\ 5 & 0 & 4 \end{pmatrix} = \begin{pmatrix} 8 & 10 & 4 \\ 14 & 10 & 16 \\ 10 & 0 & 8 \end{pmatrix}$

If we multiply each matrix by 2 and then add them this gives

$$\begin{pmatrix} 2 & 4 & 2 \\ 6 & 0 & 4 \\ -2 & 8 & -6 \end{pmatrix} + \begin{pmatrix} 6 & 6 & 2 \\ 8 & 10 & 12 \\ 12 & -8 & 4 \end{pmatrix} = \begin{pmatrix} 8 & 10 & 4 \\ 14 & 10 & 16 \\ 10 & 0 & 8 \end{pmatrix}$$

This shows that $2(\mathbf{A} + \mathbf{B}) = 2\mathbf{A} + 2\mathbf{B}$

Therefore multiplication of a sum or difference of matrices by a number (*scalar*) is distributive.

Exercise 17f

The questions in this exercise refer to the following matrices.

$\mathbf{A} = (3 \quad -2 \quad 4)$ $\quad \mathbf{B} = \begin{pmatrix} 4 & -2 & 6 \\ 2 & 3 & -1 \end{pmatrix}$ $\quad \mathbf{C} = \begin{pmatrix} 3 & -3 \\ 5 & 6 \end{pmatrix}$ $\quad \mathbf{D} = (-4 \quad 0 \quad 3)$

$\mathbf{E} = \begin{pmatrix} -1 & 2 & -2 \\ 2 & 0 & -4 \end{pmatrix}$ $\quad \mathbf{F} = \begin{pmatrix} -4 & -3 \\ 2 & -8 \end{pmatrix}$ $\quad \mathbf{G} = (10 \quad 6 \quad -5)$ $\quad \mathbf{H} = \begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix}$ $\quad \mathbf{I} = \begin{pmatrix} -6 \\ -2 \\ 9 \end{pmatrix}$

Find, where possible:

1 $\mathbf{A} + 2\mathbf{D}$ **2** $3(\mathbf{D} + \mathbf{G})$ **3** $4\mathbf{I} - \mathbf{H}$ **4** $\mathbf{C} - \mathbf{F}$ **5** $2(\mathbf{G} - \mathbf{D})$

6 $\mathbf{E} - 3\mathbf{B}$ **7** $\mathbf{D} + \mathbf{H}$ **8** $3(\mathbf{C} + \mathbf{F})$ **9** $4(\mathbf{B} + \mathbf{E})$ **10** $5(\mathbf{F} - 4\mathbf{C})$

Mixed exercise

Exercise 17g

Questions **1** to **5** refer to the matrices

$$\mathbf{A}=\begin{pmatrix}5 & 2\\ 1 & 4\end{pmatrix}\qquad \mathbf{B}=\begin{pmatrix}3 & -1\\ 4 & -3\end{pmatrix}\qquad \mathbf{C}=\begin{pmatrix}1\\ 4\end{pmatrix}$$

1 What are the orders of **A** and **C**?

2 Find **A** – **C** if it is possible.

3 Find 3**B**.

4 Find 2**A** + **B** if it is possible.

5 What is the entry in the second row of the first column of **C**?

Questions **6** to **9** refer to the matrices

$$\mathbf{P}=\begin{pmatrix}2 & 1 & -1\\ 4 & 3 & 1\end{pmatrix}\qquad \mathbf{Q}=\begin{pmatrix}3 & 2\\ -1 & 4\end{pmatrix}\qquad \mathbf{R}=\begin{pmatrix}3\\ 4\end{pmatrix}$$

6 Find 2**P**.

7 Find 2**P** + **Q** if it is possible.

8 What is the entry in the first row of the first column of **R**?

9 What is the entry in the second row of the third column of **P**?

Investigation

Consider the matrix $\begin{pmatrix}5 & -2 & 3\\ 0 & 2 & 4\\ 1 & 6 & -1\end{pmatrix}$

Find the sum of the terms in each row, column and diagonal.

For example, sum in column 1 is $5+0+1=6$.

What do you notice about the sum in each case?

Now consider another matrix whose terms are 2^x, where x is the entry in the matrix above.

For example, the element corresponding to 5 above is $2^5 = 32$.

Now complete this matrix.

Now find the product of the terms in each row, in each column, in each diagonal.

What do you notice about the products?

If we call our first matrix an 'additive magic square', we may refer to this matrix as a 'multiplicative magic square'.

Do you notice any relationship between the sum of entries in the 'additive square' and the product of entries in the 'multiplicative square'?

Try to form other multiplicative magic squares using bases other than 2.

In this chapter you have seen that...

- ✔ matrices that have the same number of rows and the same number of columns can be added together by adding the corresponding entries, e.g.

$$\begin{pmatrix} 2 & 3 & 5 \\ 3 & -1 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 3 & 2 \\ 5 & 4 & 3 \end{pmatrix} = \begin{pmatrix} 3 & 6 & 7 \\ 8 & 3 & 5 \end{pmatrix}$$

- ✔ matrices that have the same number of rows and the same number of columns can be subtracted by subtracting the corresponding entries, e.g.

$$\begin{pmatrix} 2 & 3 & 5 \\ 3 & -1 & 2 \end{pmatrix} - \begin{pmatrix} 1 & 3 & 2 \\ 5 & 4 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 3 \\ -2 & -5 & -1 \end{pmatrix}$$

- ✔ matrices can be added in any order, i.e. $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$
- ✔ multiplication of a sum or difference of matrices is distributive, i.e. $2(\mathbf{A} + \mathbf{B}) = 2\mathbf{A} + 2\mathbf{B}$.

18 Vectors

At the end of this chapter you should be able to...

1 recall the difference between position vectors and relative position vectors
2 calculate the length of a vector
3 recall the meaning of a reverse vector
4 find the resultant of two vectors
5 recall the meaning of a position vector.

You need to know...

- ✔ the difference between a vector and a scalar
- ✔ that a vector can be represented by a straight line with an arrow to show its direction
- ✔ that a translation vector can be represented in the *xy* plane by a column, e.g. $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$
- ✔ how to use Pythagoras' theorem
- ✔ the properties of triangles, special quadrilaterals and regular polygons
- ✔ how to divide a length in a given ratio.

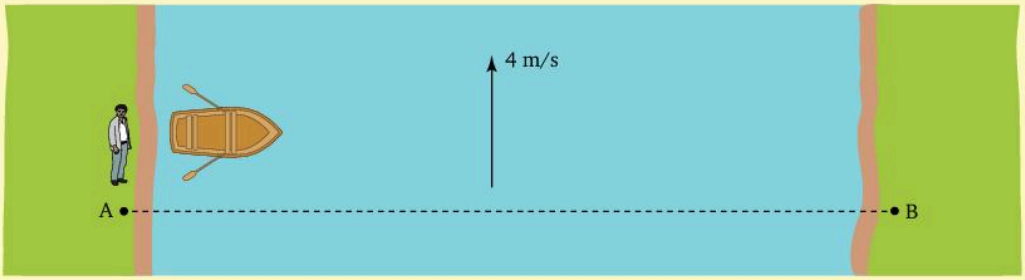
Key words

collinear, coordinates, direction, displacement vector, magnitude, non-collinear, parallel vectors, position vector, relative position vector, resultant vector, scalar, translation, vector

Did you know?

Vectors were developed in the second half of the nineteenth century primarily to work with the new discoveries about electricity. They have since been used to describe any quantity that needs two separate pieces of information to define it, such as velocity or force.

The magnitude of a vector



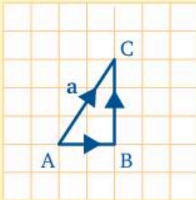
The traveller tries to row his boat by pointing it straight across the river, hoping to land at B on the opposite bank.

That will not happen because the water is flowing at 4 m/s, so it will push him in the *direction* of the flow. His landing point will depend on the result of two *vectors* – the flow of the water in the river and how fast he can row straight across still water.

This chapter will help you to solve problems similar to this.

In Grade 8 we used *vectors* to describe *translations*.

For example, the vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ means move two units right and 3 units up.



This is equivalent to the single movement from A to C.

This is the vector $\overrightarrow{AC}$ or **a** and is the *resultant* of $\overrightarrow{AB}$ followed by $\overrightarrow{BC}$.

Because each vector shows how to get from one point to another, they are called *displacement vectors*.

The *magnitude* of **a** is represented by the length of AC.

We can find this using Pythagoras' theorem: $AC^2 = AB^2 + BC^2$

$= 4 + 9 = 13$

So $AC = \sqrt{13} = 3.61$ to 3 s.f.

Exercise 18a

1 On square grid paper, draw a single vector to show the result of

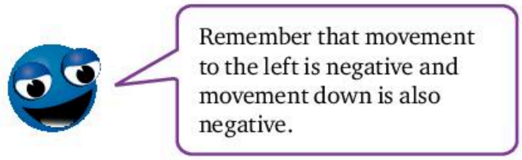
a moving two squares right followed by 2 squares up

b moving two squares left followed by 3 squares up

c moving 4 squares right followed by 1 square down.

2 Find the magnitude of each of the following vectors.

a $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ **b** $\begin{pmatrix} 5 \\ -12 \end{pmatrix}$ **c** $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ **d** $\begin{pmatrix} -2 \\ -5 \end{pmatrix}$



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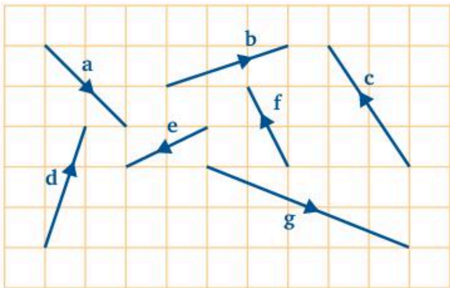
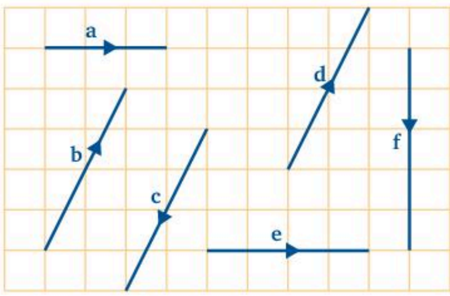


For each vector in the diagram

a express the vector in the form $\begin{pmatrix} a \\ b \end{pmatrix}$

b find the magnitude of the vector.

4



a Find the magnitude of each vector.

b Give the names of two vectors that are the same magnitude and in the same direction.

c Give the names of two vectors that are the same magnitude but in opposite directions.

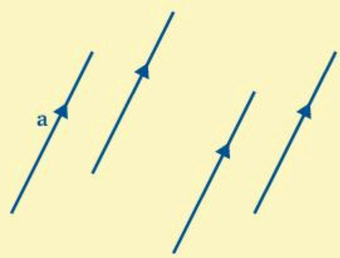
Equal vectors

Two vectors are equal when they are the same length and in the same direction.

So, if **a** = **b** then the lengths and directions of **a** and **b** are the same

This means that any vector can be represented by any line of the right length and direction.

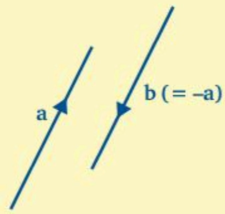
Therefore all the lines in the diagram represent the vector **a**.



Reverse vectors

When vector **b** is the same length as vector **a** but **b** is in the opposite direction to that of **a**, then **b** is called the reverse of **a**.

We say that **a** and **b** are equal and opposite and we can say that **b** = –**a**.



The vector –**a** is also called a negative vector.

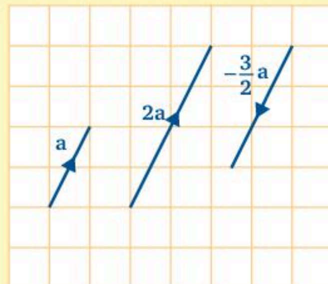
Any negative vector, –**x**, is the same length but in the opposite direction to the vector **x**.

Multiplication of a vector by a scalar

Any vector can be multiplied by a *scalar*, i.e. a number.

For example, 2**a** means a vector in the same direction as **a** but twice as long and $-\frac{3}{2}$ **a** means a vector $\frac{3}{2}$ times the length of **a** but in the opposite direction.

Expressed in the form $\begin{pmatrix} a \\ b \end{pmatrix}$, $2\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$.



Parallel vectors

Two vectors are parallel if they are in the same direction and one is a positive scalar multiple of the other.

In the diagram above, **a** and 2**a** are *parallel vectors*.

The vectors $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ -2 \end{pmatrix}$ are parallel because $\begin{pmatrix} 4 \\ -2 \end{pmatrix} = 2\begin{pmatrix} 2 \\ -1 \end{pmatrix}$.

However, $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} -4 \\ 2 \end{pmatrix}$ are not parallel because $\begin{pmatrix} -4 \\ 2 \end{pmatrix} = -2\begin{pmatrix} 2 \\ -1 \end{pmatrix}$, so they are in opposite directions.

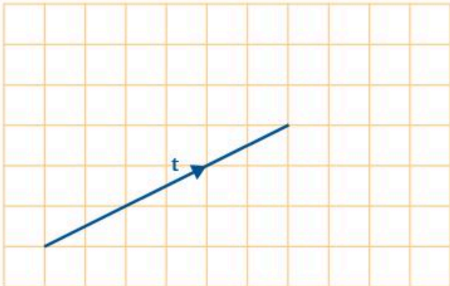
Exercise 18b

1 Copy the vector **t** on to square grid paper.

On the same diagram draw vectors to represent the vectors

a $-\mathbf{t}$ **c** $\frac{1}{2}\mathbf{t}$ **e** $-2\mathbf{t}$

b $2\mathbf{t}$ **d** $3\mathbf{t}$ **f** $-\frac{1}{2}\mathbf{t}$

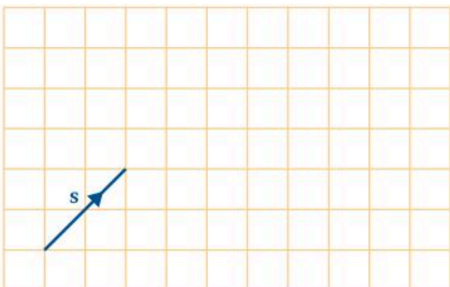


2 Copy the vector **s** on to square grid paper.

On the same diagram draw vectors to represent the vectors

a $-\mathbf{s}$ **c** $\frac{1}{3}\mathbf{s}$ **e** $-3\mathbf{s}$

b $3\mathbf{s}$ **d** $-\frac{1}{3}\mathbf{s}$ **f** $4\mathbf{s}$

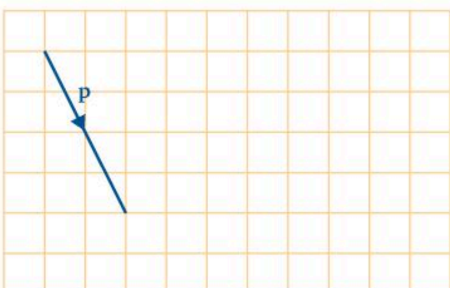


3 Copy the vector **p** on to square grid paper.

On the same diagram draw vectors to represent the vectors

a $-\mathbf{p}$ **c** $\frac{1}{2}\mathbf{p}$ **e** $-3\mathbf{p}$

b $2\mathbf{p}$ **d** $-\frac{1}{2}\mathbf{p}$ **f** $3\mathbf{p}$

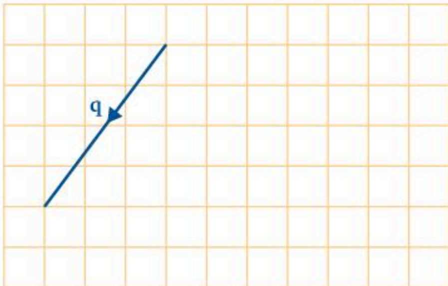


4 Copy the vector **q** on to square grid paper.

On the same diagram draw vectors to represent the vectors

a $-\mathbf{q}$ **c** $\frac{1}{2}\mathbf{q}$ **e** $-2\mathbf{q}$

b $2\mathbf{q}$ **d** $-\frac{1}{3}\mathbf{q}$ **f** $-3\mathbf{q}$



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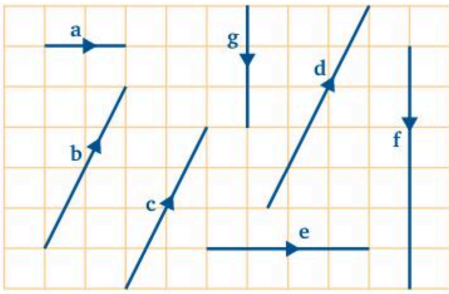


Use the vectors in this diagram.

a Name two vectors that are parallel to **b**.

b Which vector is parallel to **a**? Write this vector as a multiple of **a**.

c Which vector is parallel to **f**? Write this vector as a multiple of **f**.

6 The vector $\mathbf{a} = \begin{pmatrix} 6 \\ 12 \end{pmatrix}$. Find in the form $\begin{pmatrix} a \\ b \end{pmatrix}$

a the vector **b** where $\mathbf{b} = \frac{1}{2}\mathbf{a}$

b the vector **c** where $\mathbf{c} = 3\mathbf{a}$.

Position vectors

We use *coordinates* to give the position of a point relative to the origin on a set of *xy* axes.

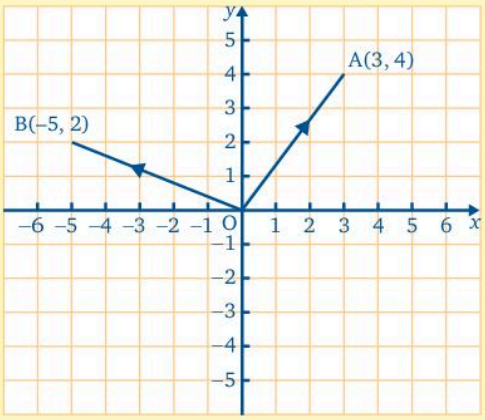
For example, in the diagram on page 301, A is the point (3, 4).

We can also use the vector from the origin to A, $\overrightarrow{OA}$, to give the position vector of A.

In the same way, we can use the vector $\overrightarrow{OB}$ to give the position of the point B(−5, 2).

$\overrightarrow{OA}$ and $\overrightarrow{OB}$ are called the *position vectors* of A and B respectively, where

$\overrightarrow{OA} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$.

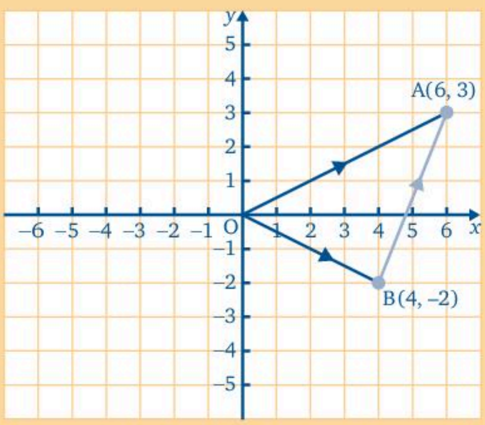


Relative position vectors

In the diagram above, moving from B to A gives vector $\overrightarrow{BA}$. The vector $\overrightarrow{BA}$ is called a *relative position vector*. It is the the position vector of A relative to B.

Exercise 18c

A is the point (6, 3) and B is the point (4, –2).



a Give the position vectors of A and B in the form $\begin{pmatrix} a \\ b \end{pmatrix}$.

b Find the vector $\overrightarrow{BA}$ in the form $\begin{pmatrix} a \\ b \end{pmatrix}$.

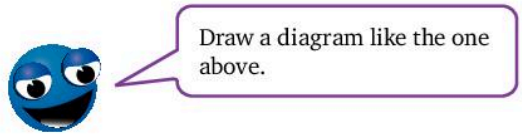
a $\overrightarrow{OA} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$.

b To get from B to A, we need to go 2 units to the right and 5 units up.

Therefore $\overrightarrow{BA} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$.

Note that $\overrightarrow{BA}$ is the position vector of A relative to B.

1 A is the point (2, 3) and B is the point (4, 5).



a Give the position vectors of A and B in the form $\begin{pmatrix} a \\ b \end{pmatrix}$.

b Find the vector $\overrightarrow{BA}$ in the form $\begin{pmatrix} a \\ b \end{pmatrix}$.

2 A is the point (–1, 5) and B is the point (3, –6).

a Give the position vectors of A and B in the form $\begin{pmatrix} a \\ b \end{pmatrix}$.

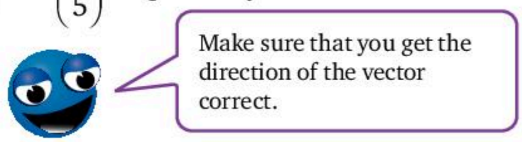
b Find the vector $\overrightarrow{BA}$ in the form $\begin{pmatrix} a \\ b \end{pmatrix}$.

3 A, B and C are the points (1, 4), (3, –1) and (–4, 6) respectively.

a Give the position vectors of A, B and C in the form $\begin{pmatrix} a \\ b \end{pmatrix}$.

b Find the vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}$ in the form $\begin{pmatrix} a \\ b \end{pmatrix}$.

4 The position vectors of the points A and B are $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ 5 \end{pmatrix}$ respectively.



a Give the coordinates of the points A and B.

b Find the vector $\overrightarrow{AB}$ in the form $\begin{pmatrix} a \\ b \end{pmatrix}$.

5 The position vectors of the points A and B are $\begin{pmatrix} -3 \\ -6 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ -1 \end{pmatrix}$ respectively.

a Give the coordinates of the points A and B.

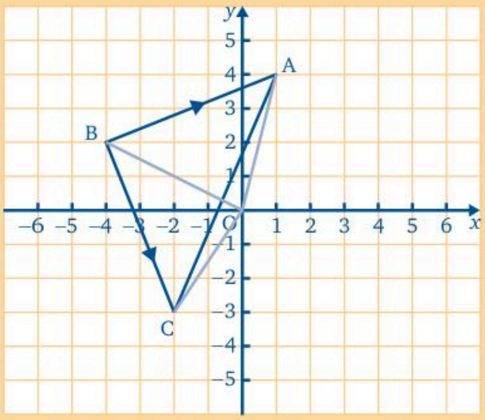
b Find the vector $\overrightarrow{AB}$ in the form $\begin{pmatrix} a \\ b \end{pmatrix}$.

The vertices of a triangle ABC are A(1, 4), B(–4, 2) and C(–2, –3).

a Find the position vectors of the points A, B and C in the form $\begin{pmatrix} a \\ b \end{pmatrix}$.

b Find the position vector of A relative to B and the position vector of C relative to B.

c Hence show that triangle ABC is isosceles.



First draw a diagram.

a The position vector of A is $\overrightarrow{OA} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$,

the position vector of B is $\overrightarrow{OB} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$,

the position vector of C is $\overrightarrow{OC} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$.

b The position vector of A relative to B is $\overrightarrow{BA}$

and from the diagram, $\overrightarrow{BA} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$.

The position vector of C relative to B is $\overrightarrow{BC}$

and from the diagram, $\overrightarrow{BC} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$.

c The length of BA is $\sqrt{5^2 + 2^2} = \sqrt{29}$.

The length of BC is $\sqrt{2^2 + (-5)^2} = \sqrt{29}$.

BA = BC, therefore triangle ABC is isosceles.

6 The vertices of a quadrilateral ABCD are (1, 0), (2, 4), (7, 6) and (6, 2) respectively.

a Find in the form $\begin{pmatrix} a \\ b \end{pmatrix}$ the vectors $\overrightarrow{AB}$ and $\overrightarrow{DC}$.

b Hence show that AB and DC are equal and parallel.

c Which of the special quadrilaterals is ABCD?

7 The vertices of a triangle ABC are (–1, –3), (1, 5) and (4, 0) respectively.

a Find in the form $\begin{pmatrix} a \\ b \end{pmatrix}$ the vectors $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{CA}$.

b Show that triangle ABC is a right-angled triangle.

8 The vertices of a quadrilateral ABCD are (0, 4), (3, 3), (5, –1) and (–1, 1) respectively.

a Find in the form $\begin{pmatrix} a \\ b \end{pmatrix}$ the vectors $\overrightarrow{AB}$ and $\overrightarrow{DC}$.

b Are AB and DC equal? Justify your answer.

c Are AB and DC parallel? Justify your answer.

d What type of quadrilateral is ABCD?

9 ABCD is a quadrilateral where A is the point (–1, 1), B is the point (2, 5), C is the point (5, 1) and D is the point (2, –3).

The four sides of a rhombus are the same length and each pair of opposite sides are parallel.

a Find in the form $\begin{pmatrix} a \\ b \end{pmatrix}$ the vectors $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$ and $\overrightarrow{DA}$.

b Prove that ABCD is a rhombus.

10 ABC is a triangle where A is point (1, 3), B is the point (5, 1) and C is the point (4, –2).

a Find in the form $\begin{pmatrix} a \\ b \end{pmatrix}$ the vector $\overrightarrow{AB}$.

b Find in the form $\begin{pmatrix} a \\ b \end{pmatrix}$ the vector $\overrightarrow{CD}$ such that $\overrightarrow{CD} = 2\overrightarrow{AB}$.

c Write down the coordinates of the point D.

11 A is the point (3, 1) and B is the point (6, 4).

D is the point (2, 5). Find in the form $\begin{pmatrix} a \\ b \end{pmatrix}$ the vector $\overrightarrow{CD}$ such that $\overrightarrow{CD}$ is parallel to $\overrightarrow{AB}$.

12 ABCD is a square. The position vector of the point A is $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and the position vector of the point B is $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$.

a Find the vector $\overrightarrow{AB}$ in the form $\begin{pmatrix} a \\ b \end{pmatrix}$.

b C is the point (5, 0). Find the coordinates of D.

13 ABCD is a parallelogram where $\overrightarrow{OA} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and $\overrightarrow{OD} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$.

Find the position vector of C.

14 A(2, 2), B(5, 6), C(8, 2) and D(5, –2) form a rhombus. E is the midpoint of AC.

a Find the vector $\overrightarrow{AC}$ and hence the vector $\overrightarrow{AE}$.

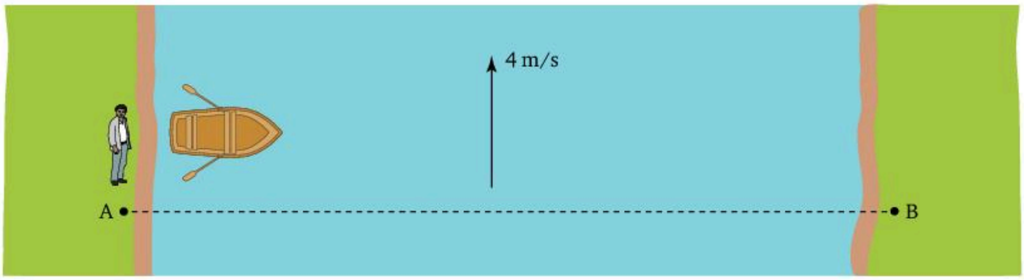
b Find the vector $\overrightarrow{BE}$ and hence show that E is also the midpoint of BD.

15 A boy starts from A and walks for 50 m in the direction of the vector $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$.

He then walks 52 m in the direction of the vector $\begin{pmatrix} 5 \\ 12 \end{pmatrix}$, ending up at B.

Find the vector $\overrightarrow{AB}$ and hence find the distance AB to the nearest metre.

16



Here is the traveller again. He rows his boat pointing straight across the river, hoping to land at B, directly opposite where he starts at A. The river is flowing at 4 m/s and he can row at 2.5 m/s across still water. The river is 20 m wide.

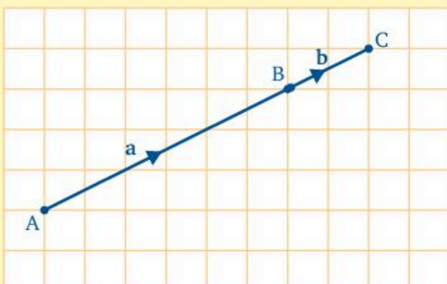
The traveller rows in the direction AB. How long does it take him to row across the river? Use your answer to find how far the current moves the boat downstream in the same time.

Find how far downstream he lands from B.

Collinear vectors

Most vectors do not lie on the same straight line. They are *non-collinear* vectors.

In the diagram, the vectors **a** and **b** are *collinear*, i.e. they are in the same straight line.



We can show this using the fact that $\overrightarrow{AB} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

So $\overrightarrow{AB} = 3\overrightarrow{BC}$, showing that $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel.

B is a point on both vectors, so $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are collinear.

From the diagram we can also see that B divides the line AC into two parts in the ratio 3 : 1.

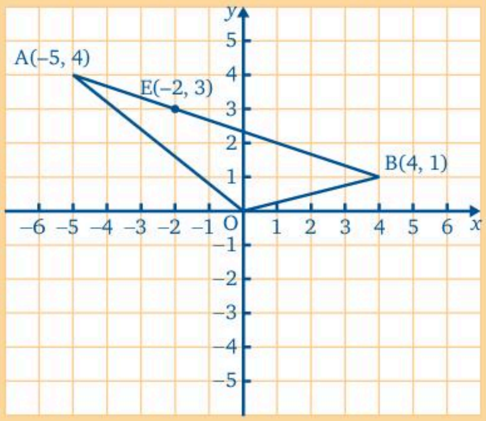
Exercise 18d

A(–5, 4), B(4, 1) and E(–2, 3) are three points.

a Show that A, E and B are collinear.

b Find the ratio in which E divides the line AB.

First draw a diagram.



a $\overrightarrow{AE} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ and $\overrightarrow{EB} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$ so $\overrightarrow{AE} = \frac{1}{3}\overrightarrow{EB}$

Therefore $\overrightarrow{AE}$ and $\overrightarrow{EB}$ are parallel.
E is common to both $\overrightarrow{AE}$ and $\overrightarrow{EB}$ so
A, E and B are collinear.

b As $\overrightarrow{AE} = \frac{1}{3}\overrightarrow{EB}$, AE : EB = 1 : 3

Therefore E divides AB in the ratio 1 : 3.

1 A is the point (–4, –3) and B is the point (5, 3).
E is the point (2, 1).

a Find $\overrightarrow{AE}$ and $\overrightarrow{EB}$ and hence show that A, E and B are collinear.

b Find the ratio AE : EB.

2 A, B and C are three points such that $\overrightarrow{OA} = \begin{pmatrix} -6 \\ -2 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$
and $\overrightarrow{OC} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$.

a Show that A, B and C are collinear.

b Find the ratio AC : BC.

3 P, Q and R are three points such that $\overrightarrow{OP} = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$, $\overrightarrow{OQ} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$
and $\overrightarrow{OR} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$.

a Show that P, Q and R are collinear.

b Find the ratio PQ : QR.

A is the point (–6, –2) and B is the point (4, 3).

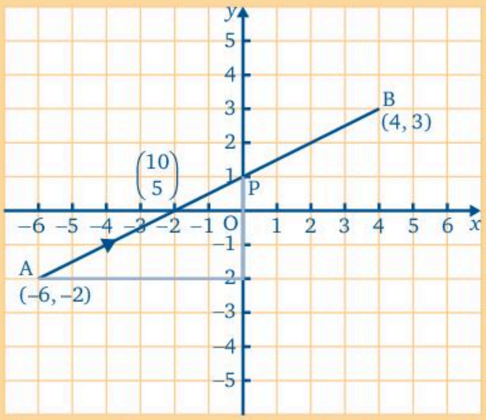
Find the vector $\overrightarrow{OP}$ such that P divides the line AB in the ratio 3 : 2.

$$\overrightarrow{AB} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$$

For P to divide AB in the ratio 3 : 2,

$$\overrightarrow{AP} = \frac{3}{5}\overrightarrow{AB} = \frac{3}{5}\begin{pmatrix} 10 \\ 5 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$$

Therefore P is the point (0, 1), so $\overrightarrow{OP} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.



4 A is the point (–6, 0) and B is the point (2, 2).
Find the vector $\overrightarrow{OP}$ such that P is the midpoint of AB.

5 A is the point (–6, –3) and B is the point (6, 0).
Find the vector $\overrightarrow{OP}$ such that P divides AB in the ratio 2 : 1.

6 A is the point (–6, 5) and B is the point (4, 0).
Find the vector $\overrightarrow{OP}$ such that P divides AB in the ratio 1 : 4.

7 A is the point (8, 4) and B is the point (–6, –3).
Find the vector $\overrightarrow{OP}$ such that P divides AB in the ratio 5 : 2.

8 A is the point (5, 12) and B is the point (–3, –4).
Use vectors to find the coordinates of D such that D divides AB in the ratio 5 : 3.

Mixed exercises

Exercise 18e

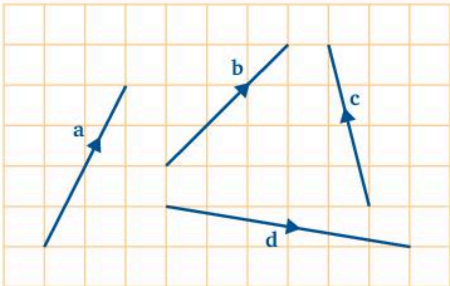
1 Find the magnitude of each of the following vectors:

a $\begin{pmatrix} 5 \\ -12 \end{pmatrix}$ **b** $\begin{pmatrix} -4 \\ -3 \end{pmatrix}$ **c** $\begin{pmatrix} 1 \\ 6 \end{pmatrix}$

2 For each vector in the diagram

a find its magnitude

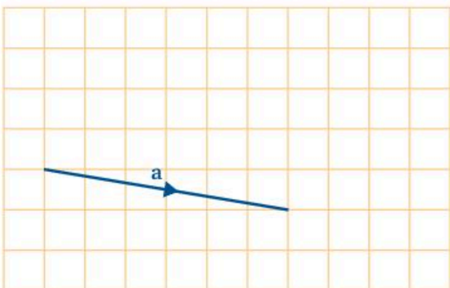
b express it in the form $\begin{pmatrix} a \\ b \end{pmatrix}$.



3 Copy the vector **a** on to square grid paper.

On the same diagram draw vectors to represent these vectors:

a $-\mathbf{a}$ **b** $3\mathbf{a}$ **c** $\frac{1}{2}\mathbf{a}$ **d** $-\frac{1}{2}\mathbf{a}$



4 A is the point (2, –5) and B is the point (4, –8).

a Give the position vectors of A and B in the form $\begin{pmatrix} a \\ b \end{pmatrix}$.

b Find the vector $\overrightarrow{BA}$ in the form $\begin{pmatrix} a \\ b \end{pmatrix}$.

5 The position vectors of the points A and B are $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 6 \end{pmatrix}$ respectively.

a Give the coordinates of the points A and B.

Make sure that you get the direction of the vector correct.

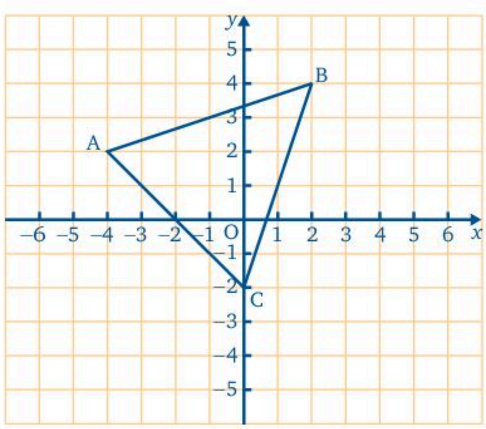
b Find the vector $\overrightarrow{AB}$ in the form $\begin{pmatrix} a \\ b \end{pmatrix}$.

6 ABCD is a square. $\overrightarrow{OA} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} 3 \\ -3 \end{pmatrix}$.

Find $\overrightarrow{OD}$.

7 a Use the diagram to find $\overrightarrow{AB}$ and $\overrightarrow{CB}$.

b Prove that triangle ABC is isosceles.



8 A is the point (–1, –5), B is the point (2, 4) and C is the point (1, 1).

a Find $\overrightarrow{AC}$ and $\overrightarrow{CB}$ and hence explain why A, B and C are collinear.

b Find the ratio in which C divides AB.

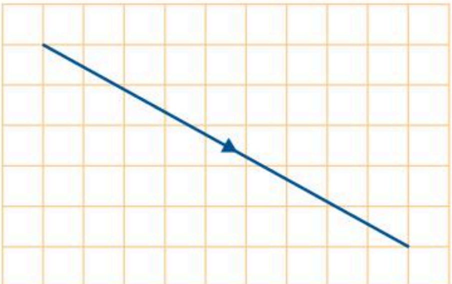
Exercise 18f

Select the letter that gives the correct answer.

1 The magnitude of the vector $\begin{pmatrix} 8 \\ -6 \end{pmatrix}$ is

A 2 B 10 C 14 D 100

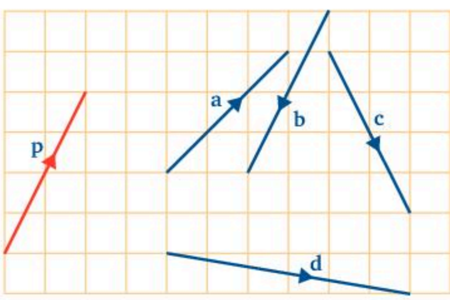
2



The magnitude of the vector in the diagram is

A $\sqrt{14}$ B 4 C $\sqrt{106}$ D 14

3



The vector equal to $-\mathbf{p}$ is

A **a** B **b** C **c** D **d**

4 A is the point (–4, 5) and B is the point (7, –8). $\overrightarrow{AB} =$

A $\begin{pmatrix} -4 \\ 5 \end{pmatrix}$ **B** $\begin{pmatrix} 3 \\ -3 \end{pmatrix}$ **C** $\begin{pmatrix} -11 \\ 13 \end{pmatrix}$ **D** $\begin{pmatrix} 11 \\ -13 \end{pmatrix}$

5 ABCD is a square. A is the point (0, 4), B is the point (4, 2) and C is the point (2, –2). $\overrightarrow{OD} =$

A $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$ **B** $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$ **C** $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$ **D** $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$

6 The vectors $\mathbf{a} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 2 \\ -6 \end{pmatrix}$ are such that

A $2\mathbf{a} = -\mathbf{b}$ **B** $\mathbf{a} = -2\mathbf{b}$ **C** $-\mathbf{a} = 2\mathbf{b}$ **D** $\mathbf{a} = \mathbf{b}$

7 A is the point (–5, –1), B is the point (3, 5) and E is the midpoint of AB. $\overrightarrow{OE} =$

A $\begin{pmatrix} -5 \\ -1 \end{pmatrix}$ **B** $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$ **C** $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ **D** $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$

8 A is the point (–6, –2), B is the point (3, 1) and C is the point (6, 2).

A A, B and C are collinear

B A, B and C are three vertices of a triangle

C $\overrightarrow{AB} = \overrightarrow{CB}$

D A, B and C are three vertices of a square.

In this chapter you have seen that…

- ✔ a position vector gives the position of a point relative to the origin and a relative position vector gives the position of a point relative to another point
- ✔ the magnitude of a vector in the form $\begin{pmatrix} a \\ b \end{pmatrix}$ is $\sqrt{a^2 + b^2}$
- ✔ two vectors are equal when they are the same length and in the same direction
- ✔ the reverse of a vector **a** is the same magnitude as **a** but in the opposite direction and is denoted by –**a**
- ✔ a vector can be multiplied by a number, e.g. 3**a**, where 3**a** is in the same direction as **a** but has three times the magnitude of **a**
- ✔ vectors are parallel when they are in the same direction
- ✔ collinear vectors are in the same straight line.

19 Quadratic mapping

At the end of this chapter you should be able to...

1 construct a table of values when an equation of the form $y = ax^2 + bx + c$ is given
2 draw a smooth curve through plotted points
3 use a graph to find the value of one quantity, given the corresponding value of the other
4 use a graph to find the least or greatest value of a quadratic function
5 interpret the meaning of the points where a quadratic graph crosses the axes.

Did you know?

Maria Agnesi (1718–1799) was born into a rich Italian family. By the age of nine she was able to speak five languages. As a teenager she argued about mathematics with teachers who visited her home. It is said that the queen sent her a diamond ring, the Pope sent her a letter of praise and a school in Italy was named after her. She was a kind and smart woman. In one of her books however she gave a Latin name to a curve. In a translation the word 'versoria' meaning 'free to move in every direction' was read as 'versiera' meaning 'a witch'. She became known as 'The witch of Agnesi'.

You need to know...

- ✔ the meaning of the words relation, mapping, function
- ✔ how to substitute values into a formula
- ✔ how to find squares and square roots
- ✔ how to plot points on a set of xy axes.

Key words

intercept, quadratic equation, quadratic function, quadratic graph

Quadratic functions

In Chapter 2 we saw that an expression such as $(2x-3)(x+4)$ is equal to $2x^2+5x-12$ when expanded.

Any expression of the form ax^2+bx+c (as long as a is not equal to zero) is called a *quadratic* expression.

For example, x^2, $3x^2-2$, $2x^2+3x$ are all quadratic expressions.

and $y=x^2$, $y=3x^2-2$, $y=2x^2+3x$ are all quadratic relations.

When the domain is $x \in \mathbb{R}$, then, in each of these relations, one value of x gives just one value for y, but for each value of y there can be two values of x.

Therefore these mappings are all $n:1$, so $y=x^2$, $y=3x^2-2$, $y=2x^2+3x$ are all *quadratic functions*.

By drawing up a table of values for a quadratic function, we can plot the points and so draw a graph to represent the function.

Quadratic graphs

A very important family of curves give what are called *quadratic graphs*. The simplest of these is the graph of $y=x^2$.

We will draw the graph of $y=x^2$ for values of x ranging from –3 to +3 at half-unit intervals. The corresponding values for x and y are given in the table below:

x	–3	–2.5	–2	–1.5	–1	–0.5	0	0.5	1	1.5	2	2.5	3
$y(=x^2)$	9	6.25	4	2.25	1	0.25	0	0.25	1	2.25	4	6.25	9

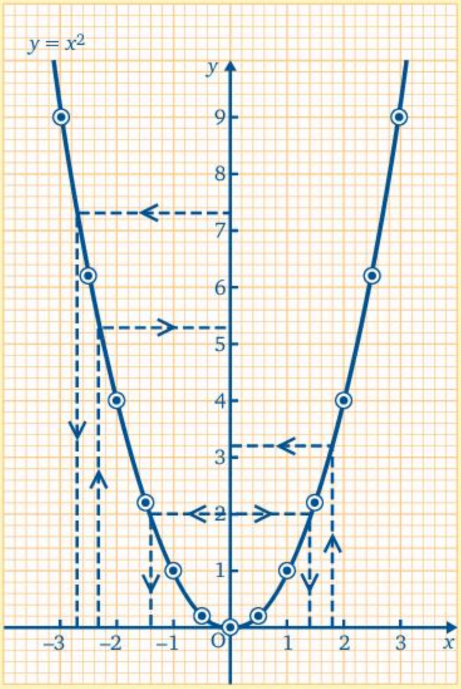
A suitable scale for you to take is 2 cm to represent 1 unit on both axes but we have taken 1 cm to represent 1 unit on both axes.

We draw the y-axis vertically in the centre of the page, since the x-values range from –3 to +3, i.e. they are symmetrical about O.

The x-axis is drawn along the bottom of the page since all the y-values are positive.

The table gives us thirteen points. In drawing any quadratic graph, aim for at least ten points. It is especially important to have plenty of points where the graph is changing direction most quickly. For quadratic graphs this is about the lowest (or highest) point.

From the graph we can find the value of y that corresponds to any value of x within the range –3 to +3. For any value of y between 0 and 9 we can find the two corresponding values of x.



For example:

a if $x = 1.8$, $y = 3.2$

b if $x = -2.3$, $y = 5.3$

c if $y = 2$, $x = 1.4$ or -1.4
(There are two values of x for which $y = 2$)

Points to remember when drawing graphs of curves

1 Do not take too few points. About ten are usually necessary.

2 To decide where you need to draw the y-axis, look at the range of x-values.

3 To decide where to draw the x-axis, look at the range of y-values.

4 In some questions you will be given most of the y-values but you may have to calculate a few more for yourself. In this case always plot first those points that you were given and, from these, get an idea of the shape of the curve. Then you can plot the points you calculated and see if they fit on to the curve you have in mind. If they do not, go back and check your calculations.

5 When you draw a smooth curve to pass through the points, always turn the page into a position where your wrist is on the inside of the curve.

Exercise 19a

1 Draw on the same axes the graphs of $y = x^2$, $y = 2x^2$ and $y = 3x^2$, taking half-unit intervals for x in the range -2 to $+2$. Take 4 cm as the unit on the x-axis and 2 cm as the unit on the y-axis for values of y from 0 to 12. What can you deduce about the graph of $y = ax^2$ for any positive value of a?

2 Draw the graph of $y = x^2 + 3$ for values of x in the range -3 to 3. Take 4 cm as the unit for x and 1 cm as the unit for y.

a Use your graph to find the values of x when $y = 6$.

b Are there any values of x for which y has the value 0?

3 Draw the graph of $y = -x^2 + 4$ for values of x in the range -3 to 3. Take 2 cm as the unit for both x and y. Use your graph to find the value of x when

a $y = \mathbf{0}$ **b** $y = 3$.

Is the graph upside down compared with those you drew in questions 1 and 2?

4 Draw on the same axes the graphs of $y = x^2$, $y = x^2 + 4$ and $y = x^2 - 4$. Use values of x from -3 to $+3$ at unit intervals, taking 2 cm as 1 unit on the x-axis and 1 cm as 1 unit on the y-axis. Let the scale on the y-axis range from -5 to $+14$.

What can you say about the shapes of the three graphs?

What can you deduce about the graph of $y = x^2 + c$ for different positive or negative values of c?

5 Complete the following table which gives values of $x(x - 3)$ for values of x in the range -1 to 4 at half-unit intervals.

x	-1	-0.5	0	0.5	1	1.5	2	2.5	3	3.5	4
$x - 3$	-4			-2.5		-1.5					1
$x(x - 3)$	4			-1.25		-2.25					4

Hence draw the graph of $y = x(x - 3)$ within the given range taking 2 cm as the unit on both axes. Use your graph to write down

a the values of x where the graph crosses the x-axis

b the values of x when $x(x - 3) = 3$ (i.e. when $y = 3$).

6 Draw the graph of $y = x(2x - 3)$ for values of x in the range -2 to 3 taking values of x at half-unit intervals. Use a scale of 2 cm for 1 unit on the x-axis and 1 cm for 1 unit on the y-axis. Use your graph to find

a the values of x where the graph crosses the x-axis

b the lowest value of $x(2x - 3)$, i.e. the lowest value of y and the corresponding value of x.

7 Draw the graph of $y = 2x(2 + x)$ for values of x in the range -5 to 3. Take values of x at unit intervals, with extra values where you think they are needed. Let the scale on your y-axis range from -4 to $+32$. Let 1 cm represent 2 units. Use your graph to find

a the smallest value of $2x(2 + x)$ and the value of x for which it occurs

b the value of $2x(2 + x)$ when $x = -3.5$

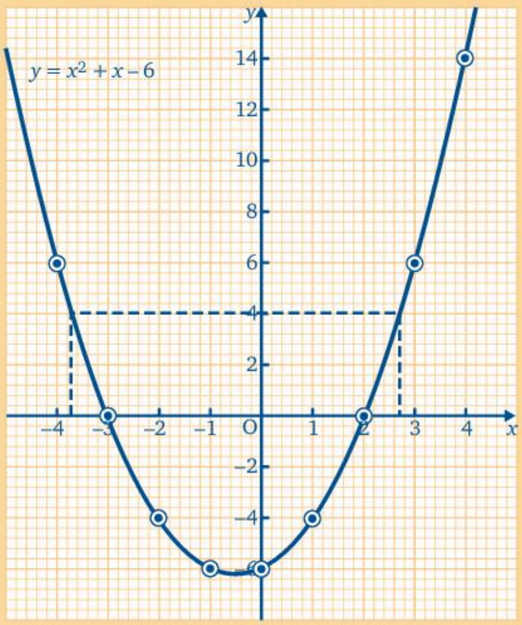
c the values of x when $2x(2 + x) = 0$

Draw the graph of $y = x^2 + x - 6$ for whole number values of x from -4 to $+4$. Take 1 cm as 1 unit on the x-axis and 1 cm as 2 units on the y-axis. Use your graph to find

a the lowest value of $x^2 + x - 6$ and the corresponding value of x

b the values of x when $x^2 + x - 6$ is 4.

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
x	-4	-3	-2	-1	0	1	2	3	4
-6	-6	-6	-6	-6	-6	-6	-6	-6	-6
x^2+x-6	6	0	-4	-6	-6	-4	0	6	14



a From the graph, the lowest value of $x^2 + x - 6$ is $-6\frac{1}{4}$. This occurs when $x = -\frac{1}{2}$.

b The values of x when $x^2 + x - 6$ is 4 are -3.70 and 2.70

8 Draw the graph of $y = x^2 - 2x - 3$ for whole number values of x in the range -3 to 5. Take 2 cm as 1 unit for x and 1 cm as 1 unit for y. Use your graph to find

- **a** the lowest value of $x^2 - 2x - 3$ and the corresponding value of x
- **b** the values of x when $x^2 - 2x - 3$ has a value of **i** 1 **ii** 8

9 Draw the graph of $y = 6 + x - x^2$ for whole number values of x from -3 to 4. Take 2 cm as 1 unit on both axes. Use your graph to find

- **a** the highest value of $6 + x - x^2$ and the corresponding value of x
- **b** the values of x when $6 + x - x^2$ has a value of **i** -2 **ii** 4

The intercepts of a quadratic graph on the axes

When we draw the graph of $y = ax^2 + bx + c$, we can read the values of x where the curve crosses the axes. These are called the *intercepts* on the x-axis.

The y-intercept is the value of y where the graph crosses the y-axis. This is where $x = 0$, i.e. where $y = c$. This is the point $(0, c)$.

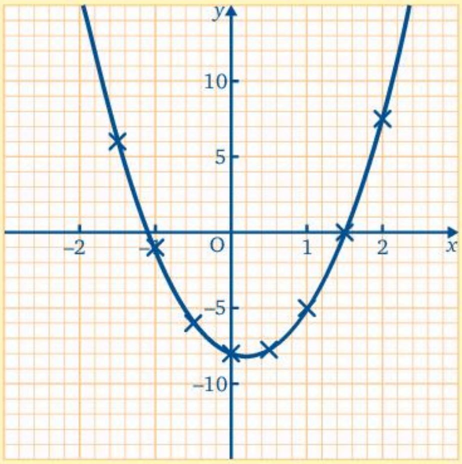
So for the function $y = 4x^2 - x + 1$, the value of c is 1. Therefore the y-intercept is 1 and the graph crosses the y-axis at the point $(0, 1)$. We can use this fact when making a table of values, by including the coordinates $x = 0$ and $y = 1$.

The curve crosses the x-axis where $y = 0$, i.e. where $ax^2 + bx + c = 0$.

Now $ax^2 + bx + c = 0$ is an equation, called a *quadratic equation*.

Therefore the values of x where the graph of $y = ax^2 + bx + c$ crosses the x-axis are the solutions, called roots, of the quadratic equation $ax^2 + bx + c = 0$.

Therefore we can solve a quadratic equation, such as $5x^2 - 2x - 8 = 0$, by first drawing the graph of $y = 5x^2 - 2x - 8$.



As accurately as we can read from the graph, the curve cuts the x-axis where $x = -1.1$ and $x = 1.5$.

Therefore the solution of the equation $5x^2 - 2x - 8 = 0$ are $x = -1.1$ and $x = 1.5$.

Note that the accuracy of graphical solutions depends on how accurately the graph is drawn and on the scale used. The larger the scale, the more accurate the solutions.

Exercise 19b

Draw the graphs the following quadratic functions.

Use scales of 1 cm ≡ 1 unit on both axes for values of x between –4 and 4.

1 $y = x^2 - x - 1$ **2** $y = 2x^2 + 3x - 3$ **3** $y = 4 - 2x - x^2$ **4** $y = x(2x - 3)$

Use your graphs to write down the solutions of the following equations.

5 $x^2 - x - 1 = 0$ **6** $2x^2 + 3x - 3 = 0$ **7** $4 - 2x - x^2 = 0$ **8** $x(2x - 3) = 4$

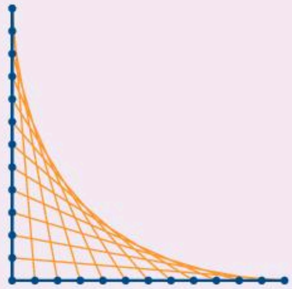
9 Draw the graph of $y = x^2 + 1$ using scales of 1 cm ≡ 1 unit on both axes for values of x between –4 and 4.

Where does this graph cross the x-axis? What does that tell you about the solution of the equation $x^2 + 1 = 0$?

10 Repeat question **9** for $y = x^2 - 4x + 8$.

Investigation

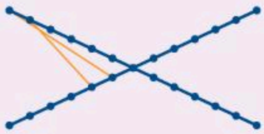
Curve stitching



Interesting curves can be produced by joining points that are equally spaced on straight lines.

This example uses two straight lines drawn at right angles.

The coloured lines form what is called the envelope of the curve.



1 Draw two lines, approximately 10 cm long that bisect each other, as shown in the diagram. Mark equally spaced points on the lines about 5 mm apart. Use a ruler or compasses (which should give a more accurate result if used carefully) to mark the points. Use a ruler and a coloured line to join the points as shown in the diagram, then continue the pattern to complete the curve. Repeat the pattern on the other half of the diagram to give two curves.

2 Investigate with two lines drawn at different angles.

3 Now investigate with more than two lines.

Did you know?

Apollonius of Perga (250–175 BCE) was an astronomer. He gained immortality from his work with double cones which he sliced to get different shaped curves – called conic sections.

You have drawn examples of one of these in this chapter – the quadratic curve (also called a parabola). The diagram shows this together with two of the other curves Apollonius investigated.

In this chapter you have seen that…

- ✔ the simplest relationship that gives a quadratic graph is $y = x^2$
- ✔ a quadratic graph comes from any relationship of the form $y = ax^2 + bx + c$, where a, b and c are numbers, as long as a is not equal to 0
- ✔ the value of c in the equation $y = ax^2 + bx + c$ is the y-intercept
- ✔ the values of x where the graph of $y = ax^2 + bx + c$ cuts the x-axis give the solutions of the equation $ax^2 + bx + c = 0$.

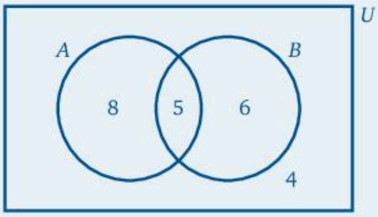
REVIEW TEST 3: CHAPTERS 15–19

In questions **1** to **12**, choose the letter for the correct answer.

1 $A = \{x: 10 \leqslant x \leqslant 40, x \text{ is a multiple of 5 or 11}\}, n(A) =$

A 7 **B** 8 **C** 9 **D** 10

This diagram shows the number of members in $\{A\}$, $\{B\}$ and $\{U\}$. Use this diagram for questions **2** to **4**.



2 $n(U) =$

A 4 **B** 18 **C** 23 **D** 28

3 $n(A \cap B) =$

A 5 **B** 11 **C** 19 **D** 23

4 $n(A') =$

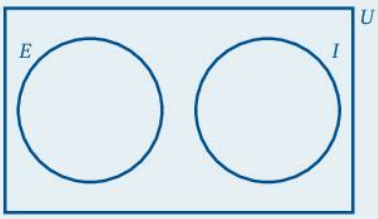
A 4 **B** 10 **C** 15 **D** 23

5 $E = \{\text{equilateral triangles}\}$

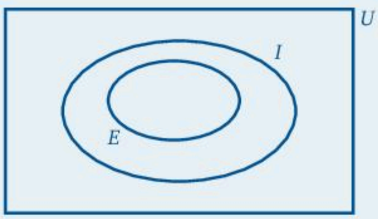
$I = \{\text{isosceles triangles}\}$

The diagram illustrating E and I is:

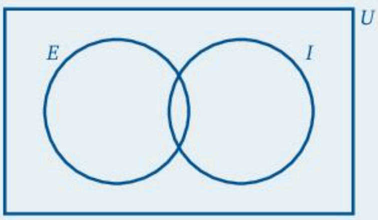
A



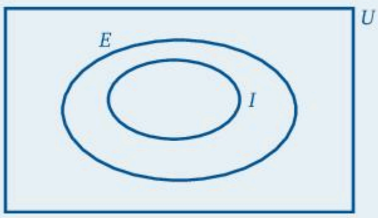
C



B



D



6 The intercept on the y-axis of the graph of $x^2 - 3x + 4 = 0$ is

A -3 **B** 1 **C** 4 **D** 16

7 The graph of $x^2 - 25 = 0$ cuts the x-axis where $x =$

A 0 and 5 **B** -1 and 1 **C** -5 and 5 **D** 25

8 p: 'The water is cold' and q: 'The water heater is broken'.
The statement 'The water is cold but the water heater is not broken' can be denoted as

A $p \wedge q$ **B** $p \vee q$ **C** $p \vee \sim q$ **D** $p \wedge \sim q$

9 The truth table for $p \vee q$ is

p	q	$p \vee q$
T	T	T
T	F	T
F	T	
F	F	F

The missing entry is

A F **B** T **C** FT **D** TT

10 $\begin{pmatrix} 1 & 0 & -1 \\ -2 & 5 & 0 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 2 \\ 0 & -3 & 0 \end{pmatrix} =$

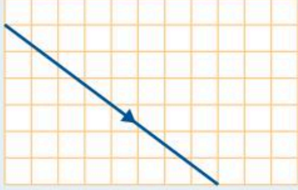
A $\begin{pmatrix} 2 & -1 & 1 \\ -2 & 8 & 0 \end{pmatrix}$ **B** $\begin{pmatrix} 3 & -1 & 1 \\ -2 & 2 & 0 \end{pmatrix}$ **C** $\begin{pmatrix} 3 & -1 & 1 \\ 0 & 2 & 0 \end{pmatrix}$ **D** $\begin{pmatrix} 0 & -1 & -1 \\ -2 & 2 & 0 \end{pmatrix}$

11 Given $\mathbf{A} = \begin{pmatrix} 2 & 4 \\ 1 & -1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 0 & 2 \\ 2 & -1 \end{pmatrix}$, $2\mathbf{A} - \mathbf{B} =$

A $\begin{pmatrix} 4 & 6 \\ 0 & -1 \end{pmatrix}$ **B** $\begin{pmatrix} 2 & 2 \\ -1 & 0 \end{pmatrix}$ **C** $\begin{pmatrix} 4 & 10 \\ 4 & -3 \end{pmatrix}$ **D** $\begin{pmatrix} 2 & 6 \\ 3 & -2 \end{pmatrix}$

12 The magnitude of the vector in the diagram is

A 8 **B** 10 **C** 12 **D** 14



13 The position vector of the point A(1, 3) relative to the point B(3, 9) is

A $\begin{pmatrix} -2 \\ -6 \end{pmatrix}$ **B** $\begin{pmatrix} 2 \\ 6 \end{pmatrix}$ **C** $\begin{pmatrix} 2 \\ -6 \end{pmatrix}$ **D** $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$

14 $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{3, 6, 9\}$

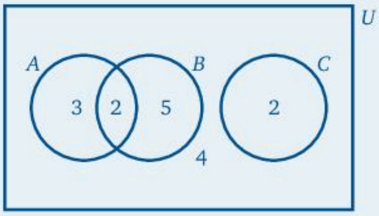
$B = \{4, 8\}$

$C = \{4, 6, 8\}$

a Draw a Venn diagram showing the sets A, B and C.

b List the members of the set represented by $(A \cap C) \cup B$.

15



This Venn diagram shows the number of members in each set. Use the information in the Venn diagram to find

a $n(A)$ **b** $n(U)$ **c** $n(A \cup B)$ **d** $n(A \cap C)$

16 Determine if the following argument is **a** valid **b** sound.

$x + 2 = 3$ and $x + 1 = 3$ so $2 = 1$.

17 Draw a truth table for $p \vee q$.

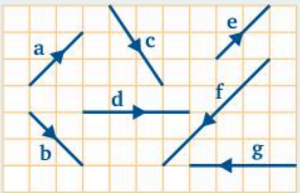
Questions **18** to **20** refer to these matrices:

$\mathbf{A} = \begin{pmatrix} 2 & -2 & 0 \\ 1 & 0 & 3 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} -4 & 5 & 1 \\ 0 & -6 & 1 \end{pmatrix}$

18 Find $\mathbf{A} - \mathbf{B}$.

19 Find $4\mathbf{A}$.

20 Find $2\mathbf{A} + 3\mathbf{B}$.

21 

For the vectors in the diagram, name the vector(s)

a equal to **a**

b equal to –**d**

c equal to – 2**a**

d equal in magnitude to **b**.

22 A, B and C are three points such that $\overrightarrow{OA} = \begin{pmatrix} -6 \\ -5 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$.

a Show that A, B and C are collinear.

b Find the ratio AB : BC.

23 Draw the graph of $y = x^2 - 4$ for values of x in the range –3 to 3, taking values of x at half-unit values.

Scale the y-axis from –5 to 2. Use your graph to find

a the values of x when $y = 1$ to 2 significant figures

b the solutions of the equation $x^2 - 4 = 0$.

24 Draw the graph of $y = x^2 - x - 6$ for whole number values of x in the range –4 to 3.

Take 2 cm as 1 unit for x and 1 cm as 1 unit for y. Use your graph to find

a the lowest value of $x^2 - x - 6$ and the corresponding value of x

b the value of x when $x^2 - x - 6$ has a value of 1.

REVIEW TEST 4: CHAPTERS 1–19

In questions **1** to **11**, choose the letter for the correct answer.

1 The value of $\left(\frac{5}{3}\right)^{-1}$ is

A $\frac{2}{5}$ **B** $\frac{3}{5}$ **C** $\frac{5}{3}$ **D** $\frac{5}{2}$

2 2.8×10^{-3} written as an ordinary number is

A 0.000 28 **B** 0.0028 **C** 0.028 **D** 0.28

3 The area of a circle of radius 18 cm, correct to 3 s.f., is

A 323 cm^2 **B** 324 cm^2 **C** 1018 cm^2 **D** 1020 cm^2

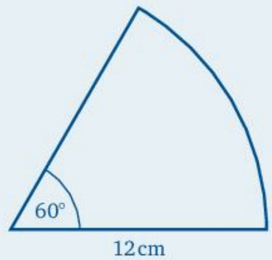
4 The radius of a circle with a circumference of 700 cm, correct to 3 s.f., is

A 110 cm **B** 111 cm **C** 220 cm **D** 222 cm

5 The area of this sector of a circle of radius 12 cm, correct to 3 s.f., is

A 57.4 cm^2 **C** 75.4 cm^2

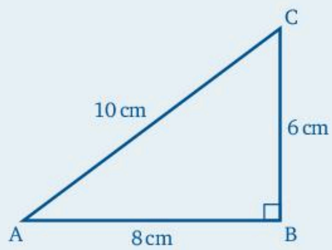
B 62.3 cm^2 **D** 84.3 cm^2



6 For this triangle cos A =

A $\frac{3}{5}$ **C** $\frac{4}{5}$

B $\frac{3}{4}$ **D** none of these



7 The solution of the equation $\frac{5}{2x} - \frac{2}{x} = \frac{1}{5}$ is $x =$

A $\frac{2}{5}$ **B** $\frac{3}{5}$ **C** $1\frac{1}{2}$ **D** $2\frac{1}{2}$

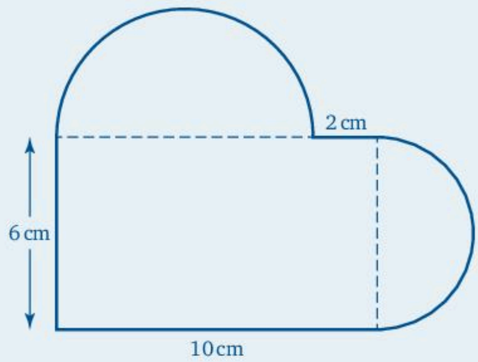
8 $\frac{3(x-2)}{4} - \frac{5(x-3)}{8}$ simplifies to

A $\frac{x-3}{8}$ **B** $\frac{x+3}{8}$ **C** $\frac{x-27}{8}$ **D** $\frac{11x+3}{8}$

9 The diagram shows a rectangle to which two semicircles are attached. Its perimeter is

A 32 cm C 40 cm

B 35.5 cm D 44 cm



10 If **M** and **N** are both 2×2 matrices and $\mathbf{M} + \mathbf{N} = \mathbf{M}$, then **N** is

A $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ B $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ C $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ D $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

11 Which of the vectors represented by $\mathbf{a}\begin{pmatrix} 1 \\ -2 \end{pmatrix}$, $\mathbf{b}\begin{pmatrix} 2 \\ -3 \end{pmatrix}$, $\mathbf{c}\begin{pmatrix} 4 \\ -6 \end{pmatrix}$ and $\mathbf{d}\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ are parallel?

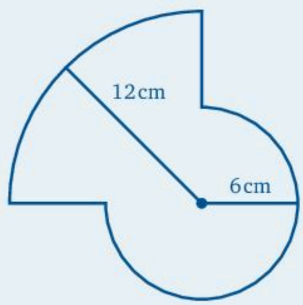
A **a** and **b** B **a** and **c** C **a** and **d** D **b** and **c**

12 Simplify a $x^4 \times x^2$ b $x^3 \times x^{-4}$ c $(2a^2)^3$

13 Find, for this shape

a the perimeter

b its area.



14 The area of the sector of a circle of radius 8 cm is 25 cm^2. Find the angle contained by the sector.

15 Find the length of the hypotenuse of a right-angled triangle if the sides containing the right angle are of length 10 cm and 12 cm.

16 In $\triangle ABC$, $\angle B = 90°$, AB = 12.4 cm and AC = 16.5 cm. Find BC.

17 The distance of a chord, of length 8.6 cm, from the centre of a circle centre O, is 3.4 cm. Find the radius of the circle.

18 Find **a** the angle whose tangent is 2.0503
b the angle whose cosine is 0.9063
c the angle whose sine is 0.8290.

19 In △DEF, ∠F = 90°, ∠E = 38° and ED = 24 cm. Find EF.

20 Simplify **a** $\frac{4xy-2x}{6x^2}$ **b** $\frac{1}{c^2}\div\frac{3}{c}$ **c** $\frac{5ab}{a+ab}$

21 Find the LCM of **a** x^2y, xy^2 **b** a, a^2b, b^2

22 Simplify **a** $\frac{5}{6x}-\frac{7}{12xy}$ **b** $\frac{x+3}{5}-\frac{x+2}{6}$

23 Solve the equation $\frac{x}{6}+\frac{x+1}{4}=\frac{3}{2}$

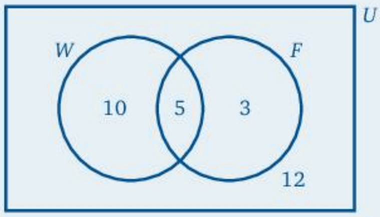
24 Expand **a** $(x-2y)(x-y)$ **b** $(2x-1)(1-3x)$ **c** $(5a-b)^2$

25 Factorise **a** a^3-2a^2 **b** $a^2-ab+2a-2b$

26 Solve each pair of simultaneous equations
a $2x+3y=11$
$x+y=4$
b $3x+5y=9$
$2x+3y=5$

27 Three times a number added to twice a second number is 19.
Three times the second number minus twice the first number is 9.
What are the numbers?

28 U = {different letters in the word SEASONAL}
A = {different letters in the word LANE}
B = {different letters in the word EASE}
Show U, A and B on a Venn diagram, entering all the elements.
Hence list the subsets
a A' **d** $(A\cup B)'$
b B' **e** $A'\cup B'$
c $A\cup B$ **f** $A'\cap B'$

29 

The Venn diagram shows the number of houses in a street that are painted white (W) and the number that have a front gate (F).
How many houses

a are painted white

b do not have a front gate

c are neither painted white nor have a front gate?

30 A relation is defined by $\{(p, q): p > q, p, q \in \{2, 3, 4\}\}$

a List the set of ordered pairs in the relation.

b Write down the domain and range of this relation.

31 A relation is defined by $\{(x, y): y = x^2 - 2, x \in \{0, 2, 3\}\}$
List the ordered pairs in the relation and determine whether or not it is a function.

32 A letter is chosen at random from the letters in the word CANADA.
What is the probability that it is

a the letter A **b** the letter B **c** a vowel **d** not a vowel?

33 a What is the probability of choosing an integer that is exactly divisible by 6 from the set
$\{8, 9, 10, 11, 12, 13, 14\}$?

b A whole number is chosen from the set of whole numbers from 10 to 30 inclusive.
What is the probability that it is exactly divisible by 3 and 4?

34 A card is chosen at random from a pack of 52 playing cards.
What is the probability that it is

a a queen **b** a black card **c** a club?

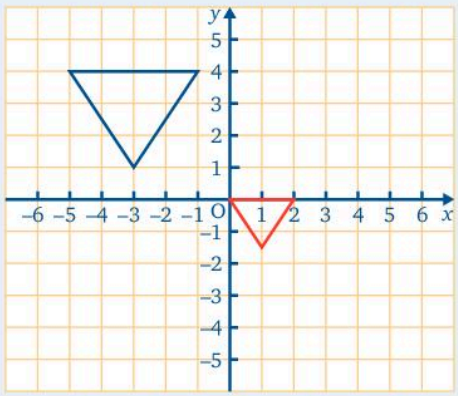
35 Find **a** $\begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix} + \begin{pmatrix} -2 & 3 \\ 5 & 2 \end{pmatrix}$ **b** $\begin{pmatrix} 3 & 4 \\ 2 & 5 \end{pmatrix} - \begin{pmatrix} 2 & -3 \\ 4 & 2 \end{pmatrix}$

36 Find **a** $\begin{pmatrix} 5 & 6 & 7 \\ 8 & 9 & 1 \\ 2 & 3 & 4 \end{pmatrix} - \begin{pmatrix} 9 & -6 & 5 \\ 3 & 8 & -4 \\ -2 & 1 & 7 \end{pmatrix}$ **b** $5\begin{pmatrix} -3 & 4 \\ 2 & 1 \end{pmatrix}$

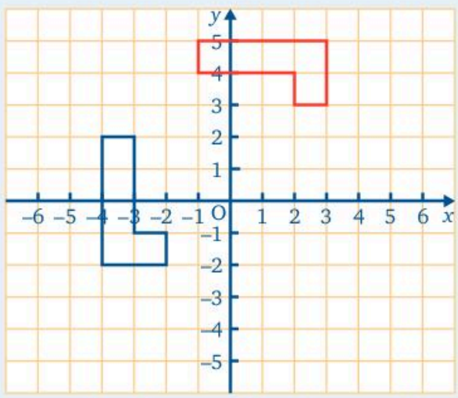
37 Draw the graph of $y = x^2 - 6x + 4$ for values of x between 0 and 6.
Use 1 cm as unit on both axes and scale the y-axis from –6 to 4.
Use your graph to solve the equation $x^2 - 6x + 4 = 0$.

38 Anne and Dylan share a bag of sweets in the ratio of their ages. Anne is 12 years old and Dylan is 8 years old. Anne's share is 15 sweets. How many sweets were in the bag?

39 The red triangle is the image of the blue triangle under an enlargement.
Find **a** the scale factor
b the centre of enlargement.



40 The red shape is the reflection of the blue shape in a mirror line. Find the equation of the mirror line.



Glossary

adjacent side	next to an angle
allowances	amounts of money deducted from gross earnings before income tax is calculated
alternate angles	equal angles on opposite sides of a transversal, e.g. 
angle of depression	the angle that is turned through down from the horizontal to view an object below the viewpoint
angle of elevation	the angle that is turned through up from the horizontal to view an object above the viewpoint
annulus	area between two concentric circles
approximation	an estimate of the value of a calculation or quantity
arc	a part of a curve
area	amount of a surface covered
argument	the logical process that leads from a premise to a conclusion
assurance	a financial product that provides cover for something that is certain to happen, for example a person's death, which then pays out an agreed sum of money
biased	where each outcome of an experiment is not equally likely
bisect	divide into two equal parts
bonus	extra wages that employees are paid
centre	(of a circle) the point that is the same distance from any point on the circumference (edge)
centre of enlargement	the point at which construction lines connecting corresponding vertices of an object and an image meet
certain	an outcome or event that will definitely happen; the probability of a certain event is 1
chance	another name for probability
chord	a straight line joining two points on a curve
circumference	the total distance round a circle
clinometer	an instrument for measuring angles of elevation or depression
collinear	vectors that lie on the same straight line
column (of a matrix)	the numbers listed vertically in the matrix array
commission	pay earned by successful selling
common denominator	a denominator that is divisible by all of the original denominators in a calculation involving fractions
common factor	a number that divides exactly into two or more other numbers

complement (of a set)	the members of the universal set not included in the given set
compound proposition	two connected propositions, e.g. a conjunction of two propositions or a disjunction of two propositions
conclusion	a statement arrived at by applying a set of logical rules to a set of premises
cone	a pyramid with a circular base
conjunction	two propositions connected with the word 'and'
converse	reverses the two objects in a relationship, e.g. the converse of 'a rectangle is a quadrilateral' is 'a quadrilateral is a rectangle'
coordinates	an ordered pair of numbers giving the position of a point on a grid
cosine of an angle	in a right-angled triangle, the cosine of an angle is equal to the length of the side adjacent to the angle divided by the length of the hypotenuse
cube	the cube of a number is the product of three of the number, e.g. the cube of 2 is $2 \times 2 \times 2 = 8$
cube root	when a number can be expressed as the product of three equal factors, that factor is the cube root of the number. As $8 = 2 \times 2 \times 2$, so 2 is the cube root of 8
cylinder	a prism with a uniform circular cross-section
decimal place	the position of a figure after the decimal point
deduction	an amount of money subtracted from earnings
denominator	the bottom of a fraction
diagonal	a line from one corner to another in a figure
diameter	a straight line through the centre of a circle from one point on the circumference to the opposite point on the circumference
direct proportion	the relationship between two quantities when the ratio between them is constant
direction	denotes the path along which a vector is moving or pointing (e.g. from west to east)
disjoint sets	sets that have no common elements
disjunction	two propositions connected with the word 'or'
displacement vector	a vector showing the distance and direction from one point to another
domain	the set of the first objects in the set of the ordered pairs of a relation
Education Tax	a statutory deduction from earnings to provide support to the state education system
element (of a set)	a member of a set
element (of a matrix)	a figure in a matrix
elimination	a method of solving simultaneous equations that uses addition or subtraction to remove one variable, giving an equation in a single variable that can be solved

empty set	a set with no members, shown by the symbol ∅ or { }
enlargement	a transformation where an object is increased or reduced in size but retains its shape
entry	a figure in a matrix
equal set	sets are equal when they contain identical members
equally likely	in probability, the assumption that you are as likely to get any particular result as any other
equation	two expressions connected by an equals sign
equivalent (logic)	two propositions are equivalent when they have the same truth table
equivalent fraction	measures the same part of a quantity
equivalent set	sets are equivalent when they contain the same number of elements; the elements in equivalent sets are not usually the same
estimate	the number of times an outcome is expected to occur based on the known probability of it happening
event	the result of an experiment, e.g. tossing a coin
expanding	multiplying out brackets
experiment	a procedure that can be repeated an infinite number of times and has a set of possible outcomes, e.g. the act of throwing a die is an experiment
exponent	another name for index
expression	a collection of algebraic terms connected with plus and minus signs, without an equals sign
factorising	the reverse of expanding brackets; finding what factors multiply together to form a given expression
fair	where each outcome of an experiment is equally likely
finite set	a set whose members are limited in number
fractional scale factor	a scale factor between zero and one, which results in the image being smaller than the object
function	a relation where each member of the domain is paired with just one member of the range
gross wage	wage amount before any deductions
grouping	a factorising method where terms are grouped together
guidelines	construction lines passing through the corresponding vertices of the object and image that meet at the centre of enlargement
hypotenuse	the longest side in a right-angled triangle
image	the resulting shape after a transformation of an object
implication	when one sentence implies another sentence
impossible	an outcome or event that will definitely not happen; the probability of an impossible event is 0

in-circle	the circle drawn inside a shape that just touches all its sides
income tax	a tax due on income levied by the government
index (plural indices)	a superscript to a number that tells you how many of those numbers are multiplied together
infinite	without end
infinite set	a set with an unlimited number of members
inflation	a percentage increase in the cost of living, calculated for the year up to the date it is given
insurance	a financial product that provides cover for an event we hope will never happen, such as a fire, burglary or a car accident. The insurance company will pay to restore damage done or replace objects that have been lost
integer	a positive or negative whole number
intercept	the distance from the origin to where a line crosses an axis
intersection of sets	the set of elements common to two or more sets, e.g. $A \cap B$ is the set of members in both A and B
invariant point	a point that does not change under a transformation
inverse proportion	relationship between two quantities where one increases in size while the other decreases at the same rate
irrational	a type of number that cannot be written as a ratio of two integers, e.g. $\sqrt{2}$ or π
kilowatt-hour	a unit of energy
like terms	terms that contain the same combination of letters, e.g. $3xy^2$ and $7xy^2$ are like terms but xy^2 and xy are not
lowest common multiple (LCM)	the lowest number that two or more other numbers divide into exactly
magnitude	the size of a quantity
map ratio	the ratio between the distance of two points on a map and the corresponding two points on the ground
matrix (plural matrices)	a rectangular array of numbers enclosed by curved brackets
member	an item that belongs to a set of items; the symbol $\in$ means 'is a member of' and the symbol $\notin$ means 'is not a member of' a particular set
mirror line	the line in which an object is reflected to give its image
National Housing Trust (NHT)	an organisation set up by the Government of Jamaica to lend money at low interest rates to people who wish to build, buy or repair their homes
National Insurance Scheme (NIS)	a statutory deduction from gross earning to provide a range of social security benefits, such as sickness benefit, maternity allowance and retirement pension

negation	the opposite, e.g. the negation of the proposition 'It is raining' is 'It is not raining'
negative index	an index or power with a negative sign means 'the reciprocal of', e.g. $5^{-2} = \frac{1}{5^2}$
net wage	wage amount after deductions
non-collinear	vectors that don't lie on the same straight line
null set	a set with no members, also called an empty set
numerator	the top of a fraction
object	the original shape before a transformation is performed
odds	a word that describes chances
opposite side	the side opposite a given angle in a right-angled triangle
order	a description of a matrix in terms of the numbers of rows and columns in the array, e.g. a matrix of order 3×4 has 3 rows and 4 columns
outcome	the result of an experiment, e.g. tossing a coin
overtime	time worked over and above the agreed length to the working week that entitles the worker to a higher rate of pay
parallel vectors	two vectors that are in the same direction and one is a positive scalar multiple of the other
parallelogram	a four-sided figure whose opposite sides are parallel
perimeter	the total distance round the edge of a plane figure
perpendicular	at right angles to a line or surface
perpendicular bisector	a line that bisects a given line segment at right angles
pi (π)	the ratio of the circumference of a circle to its diameter
position vector	a vector that describes the position of a point relative to the origin
power	another name for index
premise	an assumption that something is true
premium	the payment we make to a company to provide insurance or assurance
prime number	a number whose only factors are 1 and itself (1 is not a prime number)
probability	the likelihood of an event happening expressed as a fraction or as a decimal
product	the result of multiplying two or more numbers together
proper subset	a set of some, but not all, of the elements in another set; in symbols $A \subset B$ means A is a proper subset of B
proportional	when two quantities are in the same ratio
proposition	a statement that can be true or false
Pythagoras' theorem	a theory that states that, in a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides

Pythagorean triple	any set of three positive integers a, b and c such that $a^2 + b^2 = c^2$, i.e. these lengths form the sides and hypotenuse of a right-angled triangle
quadrant	a quarter of a circle
quadratic equation	an equation that can be written in the form $ax^2 + bx + c = 0$, where $a \neq 0$
quadratic function	a mapping of the form $f(x) = ax^2 + bx + c$, where $a \neq 0$
quadratic graph	a graph in the form of a parabola that represents a quadratic function
quadrilateral	a plane figure bounded by four straight lines
radius	the distance from the centre of a circle to the edge
random	a selection where each member of a set has an equal chance of being chosen
range	the set of the second objects in the ordered pairs in a relation
ratio	a comparison of the size of two quantities
reciprocal	the reciprocal of a number n is $\frac{1}{n}$
rectangle	a quadrilateral whose angles are each 90°
rectangular array	an arrangement of objects into rows and columns that form a rectangle, i.e. each row contains the same number of items
reflection	a transformation in which any two corresponding points in the object and the image are both the same distance from a fixed straight line
relation	a set of ordered pairs with a rule that connects the objects in each pair
relative position vector	a vector that describes the position of a point relative to another point
resultant vector	the combination of two or more single vectors that has the same effect as applying the individual vectors one after the other
revolution	a complete turn
rhombus	a four-sided figure whose sides are all the same length, e.g. ◊
right-angled triangle	a triangle that contains a right angle
row (of a matrix)	the numbers listed horizontally in the matrix array
salary	a yearly sum paid to employees, usually in monthly amounts
scalar	a quantity that has size but not direction, e.g. speed
scale	the ratio between the size of an object in a drawing and its actual size, e.g. 1 cm : 5 m
scale drawing	an accurate drawing of a real object where each length on the object is enlarged or reduced by the same ratio
scale factor	the ratio of the lengths of corresponding sides of an image and the original object
scientific notation	another name for standard form
sector	part of a circle enclosed by two radii and an arc
segment	part of a circle cut off by a chord

semicircle	half a circle
set	a collection of items having something in common
significant figure	position of a figure in a number, e.g. in 2731 the third significant figure is 3
simplify (a fraction)	reduce the size of the numerator and denominator by dividing them by common factors
simultaneous	when two equations are both satisfied by the same values of x and y
sine of an angle	in a right-angled triangle, the sine of an angle is the length of the side opposite the angle divided by the length of the hypotenuse
sound argument	an argument which is valid and also both premise and conclusion are correct
square	the square of a number is the number multiplied by itself, e.g. the square of 3 is called 3 squared and is equal to $3 \times 3 = 9$
square	a four-sided figure whose sides are all the same length and each of whose angles is a right angle
square root	when a number can be expressed as two equal factors, that factor is the square root of the number, e.g. $9 = 3 \times 3$ so 3 is the square root of 9
standard form	a number between 1 and 10 multiplied by a power of 10
statutory	required by law
statutory income	income remaining after pension contributions and NIS payments have been subtracted from gross pay
subset	a set whose members are also members of another set; we use the symbol $A \subseteq B$ rather than $A \subset B$ if we don't know whether B could be equal to A
substitution	a method for solving simultaneous equations where one variable is expressed in terms of the other, giving an equation in a single variable that can be solved
subtend (an angle)	form an angle at a point opposite an arc by connecting the ends of the arc to the point
symmetry	having congruent parts each side of a line or around a point
tangent of an angle	in a right-angled triangle, the tangent of an angle is the length of the side opposite the angle divided by the length of the side adjacent to the angle
translation	movement of an object in a straight line from one position to another without change in size or shape
trapezium	a four-sided figure with one pair of sides parallel
truth table	a diagram in table format showing all possible combinations of true and false for a proposition in relation to its component statements
unbiased	where each outcome of an experiment is equally likely

union of sets	the set containing all the different elements of two or more sets, e.g. $A \cup B$ is the set of all members of A and B
unitary method	a method for solving direct proportion problems that finds the value for a single unit and then multiplies to find the required value
universal set	the set containing all elements, shown by the symbol U
valid argument	an argument where the premise (which may be true or false) has a correct conclusion
vector	a quantity that has size and direction, e.g. velocity
Venn diagram	a diagram used to show the elements in two or more sets
zero index	the number zero used as a power; any non-zero number raised to the power 0 is equal to 1

Answers

CHAPTER 1

Exercise 1a page 2

1 $\frac{x}{4}$
2 $\frac{a}{2}$
3 $\frac{p}{q}$
4 $\frac{a}{b}$
5 $\frac{x}{y}$
6 $\frac{1}{2a}$
7 $\frac{a}{2c}$
8 $\frac{2}{q}$
9 $\frac{pq}{2}$
10 $\frac{a}{c}$
11 $\frac{a}{2}$
12 $\frac{z}{2}$
13 $\frac{b}{d}$
14 $\frac{1}{3x}$
15 $\frac{q}{2}$
16 $\frac{2}{3y}$
17 $\frac{m}{k}$
18 $\frac{s}{4t}$

Exercise 1b page 3

1 $\frac{1}{x}$
2 $\frac{t}{s-t}$
3 not possible
4 not possible
5 $\frac{x}{2(x-y)}$
6 $\frac{(a+b)}{2ab}$
7 $p-q$
8 $\frac{1}{(4-a)}$
9 not possible
10 $\frac{1}{v}$
11 $\frac{y}{x+y}$
12 $\frac{1}{2}$
13 $\frac{2a}{3(a-b)}$
14 $\frac{2(x-y)}{3xy}$
15 not possible
16 $u-v$
17 not possible
18 $\frac{1}{(s-6)}$

Exercise 1c page 4

1 $\frac{ac}{bd}$
2 $\frac{ad}{bc}$
3 $\frac{5(x-y)}{2x}$
4 $\frac{x(x-y)}{10}$
5 $\frac{a}{bc}$
6 $\frac{ac}{b}$
7 $\frac{3(a-b)}{4(a+b)}$
8 $\frac{(x-2)(x+3)}{3}$
9 $\frac{x-2}{3(x+3)}$
10 $\frac{pr}{q}$
11 $\frac{6b}{a}$
12 $\frac{q}{2p}$
13 $\frac{12y}{x}$
14 $\frac{2b^2}{5}$
15 $\frac{pq}{6}$
16 $\frac{x}{2y}$
17 $\frac{1}{2b}$
18 $\frac{2}{3p}$
19 $\frac{a}{4b}$
20 $\frac{a^3}{b^3}$

Exercise 1d page 5

1 pq
2 rst
3 30
4 abc
5 $wxyz$
6 ad
7 uvw
8 168
9 xy
10 $2x^2$
11 $3pq$
12 $2x^2y$
13 abc
14 st
15 $3p^2$
16 $5ab$
17 $3pq^2$
18 $6x$
19 $8x$
20 $18a$
21 60
22 a^2b
23 $30x$
24 $12x$
25 $15y$
26 $12x$

Exercise 1e page 6

1 $\frac{x+y}{xy}$
2 $\frac{3q+2p}{pq}$
3 $\frac{2t-s}{st}$
4 $\frac{6b+a}{2ab}$
5 $\frac{5y-6x}{15xy}$
6 $\frac{2b+5a}{2ab}$
7 $\frac{2y-3x}{xy}$
8 $\frac{4q+6p}{3pq}$
9 $\frac{3y-2x}{xy}$
10 $\frac{20b+21a}{28ab}$
11 $\frac{5}{6x}$
12 $-\frac{1}{35x}$
13 $\frac{5}{4y}$
14 $\frac{1}{8p}$
15 $\frac{13}{8a}$
16 $\frac{4}{21x}$
17 $\frac{6}{35x}$
18 $\frac{1}{3y}$
19 $\frac{3a+2b}{4ab}$
20 $\frac{ab-2a^2}{2b^2}$
21 $\frac{3y-4}{xy}$
22 $\frac{4-3p}{2p^2}$
23 $\frac{9a^2+2b^2}{12ab}$
24 $\frac{10q-3p}{4pq}$
25 $\frac{2s+ts^2}{2t^2}$
26 $\frac{15b+4}{6ab}$
27 $\frac{3+2x}{3x^2}$
28 $\frac{4y^2-9x^2}{6xy}$
29 $\frac{5y+4x}{8xy}$
30 $\frac{pq+3p^2}{3q^2}$
31 $\frac{10y-3}{14xy}$
32 $\frac{18b-3a}{2a^2b}$
33 $\frac{3x^2-3y^2}{2xy}$
34 $\frac{14q-15p}{18pq}$
35 $\frac{5a^2+4ab}{5b^2}$
36 $\frac{21+8p}{15pq}$

Exercise 1f page 8

1 $\frac{9x+3}{20}$
2 $\frac{5-x}{12}$
3 $\frac{13x+1}{15}$
4 $\frac{4x+13}{12}$
5 $\frac{1-2x}{35}$
6 $\frac{7x+3}{10}$
7 $\frac{3x+9}{35}$
8 $\frac{5x-3}{42}$
9 $\frac{5-22x}{21}$
10 $\frac{7x+9}{12}$
11 $\frac{22-13x}{6}$
12 $\frac{11-7x}{12}$
13 $\frac{20-17x}{24}$
14 $\frac{22-7x}{20}$
15 $\frac{10-5x}{6}$
16 $\frac{31x-6}{24}$
17 $\frac{11-7x}{10}$
18 $\frac{2-11x}{18}$
19 $\frac{26x+34}{15}$
20 $\frac{17x-1}{12}$
21 $\frac{5x-19}{21}$
22 $\frac{42x-49}{10}$
23 $\frac{27x+3}{14}$
24 $\frac{19x-73}{9}$
25 $\frac{26x-18}{15}$
26 $\frac{-17x+104}{30}$
27 $\frac{3a+6}{a(a+3)}$
28 $\frac{6x+4}{x(x+2)}$
29 $\frac{7x-4}{2x(x-4)}$
30 $\frac{2x-3}{4x(2x+1)}$
31 $\frac{5a+12}{a(a+4)}$
32 $\frac{7x-4}{x(x-1)}$
33 $\frac{11x+1}{3x(2x+1)}$
34 $\frac{21x-6}{5x(2x+3)}$

Exercise 1g page 10

1 $\frac{2c-ab}{ac}$
2 $\frac{qr^2}{p}$
3 $\frac{7x-14}{12}$
4 $\frac{6x}{x+1}$
5 $\frac{1}{12x}$
6 $\frac{x}{2}$
7 $\frac{16x}{5}$
8 $\frac{12-2x}{3x^2}$
9 $\frac{1-2x}{x(x+1)}$
10 $\frac{ab}{c}$
11 $\frac{8}{15}$
12 $\frac{23}{20x}$
13 $\frac{3}{10x^2}$
14 $\frac{4x+7}{10}$
15 $\frac{(x+4)(2x-1)}{50}$
16 $\frac{25}{12x}$
17 $\frac{25}{24x^2}$
18 $\frac{3}{2}$
19 $\frac{19x-1}{3x(x-1)}$
20 $\frac{2}{x(x-1)}$
21 $\frac{-a-3}{2a(a-1)}$
22 $\frac{3}{a(a-1)}$
23 $\frac{3}{y}$
24 -1

Exercise 1h page 11

1 8
2 -5
3 6
4 $1\frac{1}{3}$
5 10
6 5
7 $9\frac{3}{5}$
8 $5\frac{1}{4}$
9 -1
10 $8\frac{3}{4}$
11 2
12 -18
13 3
14 -1
15 21
16 $\frac{4}{9}$
17 $-2\frac{1}{2}$
18 -17
19 2
20 4
21 1
22 $-2\frac{1}{19}$

Exercise 1i page 12

1 a $\frac{b}{2}$ b a
2 a $\frac{4}{3x}$ b $\frac{1}{3x^2}$
3 -13
4 a $\frac{5x-7}{6}$ b $1\frac{7}{10}$

Exercise 1j page 13

1 a $\frac{v}{uw}$ b $\frac{1}{2a-b}$
2 a $18s^2$ b $\frac{2-5x}{x(4x-1)}$
3 4
4 a $\frac{x}{6}$ b 30
5 a $x = 40$ b $x = 21$ c $x = 66$
x equals the LCM
6 $x = ab$

Exercise 1k page 13

1 C 2 C 3 A 4 C 5 B

CHAPTER 2

Exercise 2a page 17

1 $2x+2$
2 $3x-3$
3 $4x+12$
4 $5a+20$
5 $3b+21$
6 $3-3a$
7 $5-5b$
8 $6a-2$
9 $8+12b$
10 $5ab-5ac$
11 $4ab-8ac$
12 $6a^2+3ab$
13 $15xy+5xz$
14 $16xy+12yz$
15 $6np-10nq$
16 $16rt-8rs$
17 $3ab-15ac$
18 $12xy+8xz$

Exercise 2b page 18

1 $ac+ad+bc+bd$
2 $ps+pt+qs+qt$
3 $2ac+4ad+bc+2bd$
4 $5xz+15x+2yz+6y$
5 $xz-4x+yz-4y$
6 $ac+ad-bc-bd$
7 $xy+xz+y^2+yz$
8 $6ac+2ad+3bc+bd$
9 $5xz+10x+4yz+8y$
10 $15x-3xz-10y+2yz$
11 $2ps-3pt+2qs-3qt$
12 $ac-ad-2bc+2bd$
13 $6uw-30ur-5vw+25vr$
14 $6ac-9ad+8bc-12bd$
15 $9xz+6x+6yz+4y$
16 $12pr-9ps-4qr+3qs$
17 $9ac+12ad-12bc-16bd$
18 $21x-14xz-6y+4yz$

Exercise 2c page 18

1 $x^2+7x+12$
2 x^2+6x+8
3 x^2+7x+6
4 $x^2+7x+10$
5 $x^2+11x+24$
6 $a^2+9a+20$
7 $b^2+9b+14$
8 $c^2+10c+24$
9 $p^2+15p+36$
10 x^2-5x+6
11 $x^2-12x+35$
12 $a^2-10a+16$
13 $x^2-13x+30$
14 $b^2-10b+25$
15 $x^2-7x+12$
16 $x^2-12x+32$
17 b^2-6b+8
18 $a^2-8a+16$
19 x^2+x-6
20 x^2+x-20
21 $x^2-3x-28$
22 $a^2-7a-30$
23 p^2-25
24 $x^2+5x-14$
25 x^2+x-30
26 $x^2+9x-10$
27 $b^2-15b+56$

Exercise 2d page 20

1 $x^2+9x+20$
2 $a^2+7a+10$
3 $x^2-9x+20$
4 $a^2-7a+10$
5 $x^2+14x+48$
6 $a^2+17a+70$
7 $x^2-14x+48$
8 $a^2-17a+70$
9 $a^2-3a-10$
10 $y^2-3y-18$
11 $z^2-6z-40$
12 $p^2-3p-40$
13 $a^2-3a-70$
14 $y^2+8y-20$
15 $z^2-11z-12$
16 $p^2-11p-26$
17 x^2-6x+5
18 $b^2+16b+63$
19 a^2-16
20 $r^2-12r-28$
21 $p^2+14p+24$
22 $t^2-7t-60$
23 $c^2+3c-40$
24 x^2-25

Exercise 2e page 21

1 $2x^2+3x+1$
2 $5x^2+12x+4$
3 $5x^2+17x+6$
4 $3x^2+19x+20$
5 $3x^2+5x+2$
6 $3x^2+11x+6$
7 $4x^2+7x+3$
8 $7x^2+23x+6$
9 $6x^2+13x+6$
10 $12x^2-25x+12$
11 $10x^2-3x-18$
12 $21a^2-58a+21$
13 $10x^2+31x+15$
14 $21x^2-20x+4$
15 $12x^2-5x-2$
16 $6b^2-5b-25$
17 $4a^2-9$
18 $9b^2-49$

19 $49y^2 - 25$
20 $20a^2 + a - 12$
21 $16x^2 - 9$
22 $25y^2 - 4$
23 $9x^2 - 1$
24 $16x^2 - 8x - 35$
25 $6x^2 + 5x + 1$
26 $-5x^2 + 8x + 4$
27 $-6x^2 + 19x - 3$
28 $-35a^2 + 29a - 6$
29 $8 + 10x - 3x^2$
30 $4x^2 + 7x - 15$
31 $15x^2 + 26x + 8$
32 $-14x^2 + 13x + 12$
33 $-20x^2 + 27x - 9$
34 $12 - p - p^2$
35 $x^2 - 3x - 10$
36 $4x^2 + 9x - 9$

Exercise 2f page 23

1 $x^2 + 2x + 1$
2 $x^2 + 4x + 4$
3 $a^2 + 6a + 9$
4 $b^2 + 8b + 16$
5 $t^2 + 20t + 100$
6 $x^2 + 24x + 144$
7 $x^2 + 16x + 64$
8 $p^2 + 14p + 49$
9 $x^2 + 2xy + y^2$
10 $y^2 + 2yz + z^2$
11 $c^2 + 2cd + d^2$
12 $m^2 + 2mn + n^2$
13 $p^2 + 2pq + q^2$
14 $a^2 + 2ab + b^2$
15 $e^2 + 2ef + f^2$
16 $u^2 + 2uv + v^2$
17 $4x^2 + 4x + 1$
18 $16b^2 + 8b + 1$
19 $25x^2 + 20x + 4$
20 $36c^2 + 12c + 1$
21 $9a^2 + 6a + 1$
22 $4x^2 + 20x + 25$
23 $9a^2 + 24a + 16$
24 $16y^2 + 24y + 9$
25 $x^2 + 4xy + 4y^2$
26 $9x^2 + 6xy + y^2$
27 $4x^2 + 20xy + 25y^2$
28 $9a^2 + 12ab + 4b^2$
29 $9a^2 + 6ab + b^2$
30 $p^2 + 8pq + 16q^2$
31 $49x^2 + 28xy + 4y^2$
32 $9s^2 + 24st + 16t^2$
33 $x^2 - 4x + 4$
34 $x^2 - 12x + 36$
35 $a^2 - 20a + 100$
36 $x^2 - 2xy + y^2$
37 $x^2 - 6x + 9$
38 $x^2 - 14x + 49$
39 $a^2 - 2ab + b^2$
40 $u^2 - 2uv + v^2$
41 $9x^2 - 6x + 1$
42 $25z^2 - 10z + 1$
43 $100a^2 - 180a + 81$
44 $16x^2 - 24x + 9$
45 $4a^2 - 4a + 1$
46 $16y^2 - 8y + 1$
47 $49b^2 - 28b + 4$
48 $25x^2 - 30x + 9$
49 $4y^2 - 4yx + x^2$
50 $25x^2 - 10xy + y^2$
51 $9m^2 - 12mn + 4n^2$
52 $49x^2 - 42xy + 9y^2$
53 $a^2 - 6ab + 9b^2$
54 $m^2 - 16mn + 64n^2$
55 $25a^2 - 20ab + 4b^2$
56 $9p^2 - 30pq + 25q^2$

Exercise 2g page 24

1 $x^2 - 16$
2 $b^2 - 36$
3 $c^2 - 9$
4 $x^2 - 144$
5 $x^2 - 25$
6 $a^2 - 49$
7 $q^2 - 100$
8 $x^2 - 64$
9 $4x^2 - 1$
10 $9x^2 - 1$
11 $49a^2 - 4$
12 $25a^2 - 16$
13 $25x^2 - 1$
14 $4a^2 - 9$
15 $100m^2 - 1$
16 $9x^2 - 16y^2$
17 $4a^2 - 25b^2$
18 $1 - 4a^2$
19 $49y^2 - 9z^2$
20 $100a^2 - 81b^2$
21 $25a^2 - 16b^2$
22 $1 - 9x^2$
23 $9 - 25x^2$
24 $25m^2 - 64n^2$

Exercise 2h page 25

1 $2x^2 + 9x + 12$
2 $2x^2 + 9x + 2$
3 $x^2 + 15x + 32$
4 $a^2 - 9a + 36$
5 $2a^2 - 10a - 3$
6 $x^2 + 13x + 25$
7 $x^2 - 2x - 21$
8 $x^2 - 2x - 23$
9 $16x^2 + 6x - 10$
10 $12x^2 + 8x - 20$
11 $x^2y^2 - 6xy + 9$
12 $25 - 10yz + y^2z^2$
13 $x^2y^2 + 8xy + 16$
14 $9p^2q^2 + 48pq + 64$
15 $a^2 - 2abc + b^2c^2$
16 $a^2b^2 - 4ab + 4$
17 $36 - 12pq + p^2q^2$
18 $m^2n^2 + 6mn + 9$
19 $u^2v^2 - 4uvw + 4w^2$

Exercise 2i page 27

1 $5x + 10$
2 $24pq - 16pr$
3 $6a^2 - 13ab - 5b^2$
4 $12x^2 - 17x - 5$
5 $x^2 + 16x + 60$
6 $x^2 - 20x + 96$
7 $16y^2 - 16y - 21$
8 $16y^2 - 81$
9 $25x^2 + 20x + 4$
10 $4a^2 - 28ab + 49b^2$

Exercise 2j page 27

1 $8 - 20x$
2 $16a - 24a^2$
3 $12a^2 - 35a - 33$
4 $x^2 + 2x - 99$
5 $-20x^2 - 48x + 5$
6 $y^2 + 4yz + 4z^2$
7 $36y^2 + 24yz - 5z^2$
8 $16a^2 + 8a + 1$
9 $25a^2 - 70a + 49$
10 $36z^2 - 156zy + 169y^2$

Exercise 2k page 28

1 $4(x + 1)$
2 $3(4x - 1)$
3 $2(3a + 1)$
4 $5(a - 2b)$
5 $3(t - 3)$
6 $5(2a - 1)$
7 $4(3a + 1)$
8 $2(a + 2b)$
9 $7(2x - 1)$
10 $x(x + 2)$
11 $x(x - 7)$
12 $a(a + 6)$
13 $x(2x + 1)$
14 $2t(2 - t)$
15 $x(x + 5)$
16 $x(x - 4)$
17 $b(b + 4)$
18 $a(4a - 1)$
19 $2x(x - 3)$
20 $2z(z^2 + 2)$
21 $5a(5a - 1)$
22 $4x(3x + 4)$
23 $5b(a - 2c)$
24 $3y(y + 9)$
25 $2a(a - 6)$
26 $2p(3p + 1)$
27 $3y(3y - 2)$
28 $2(x^2 + 2x + 3)$
29 $5(2a^2 - a + 4)$
30 $b(a + 4c - 3d)$
31 $4(2x - y + 3z)$
32 $3a(3b - 2c - d)$
33 $3(x^2 - 2x + 3)$
34 $4(a^2 + 2a - 1)$
35 $x(5y + 4z + 3)$
36 $5b(a + 2c + d)$
37 $2y(x - 2z + 4w)$
38 $x^2(x + 1)$
39 $x^2(1 - x)$
40 $5a^2(4 - a)$
41 $4x^2(3x - 4)$
42 $4x^2(x^2 + 3)$
43 $a^2(1 + a)$
44 $b^2(b - 1)$
45 $2x^2(2x - 1)$
46 $9a^2(3 - 2a)$
47 $5x^2(2 - 3x^2)$
48 $4(3x + 2)$
49 $4x(2x + 3)$
50 $3(3x^2 - 2x + 4)$
51 $5x(x^2 - 2)$
52 $4q(2p + r)$
53 $x(x - 8)$
54 $3(4 + 3y^2)$
55 $4x(3y + 4z + 2)$
56 $2x(3x^2 + 3)$
57 $\frac{1}{2}h(a+b)$
58 $m(g - a)$
59 $\frac{1}{2}m(v^2+u^2)$
60 $P\left(1+\frac{RT}{100}\right)$
61 $\pi r(2r + h)$
62 $\pi(R^2 + r^2)$
63 $2g(h_1 - h_2)$
64 $m\left(\frac{1}{2}v^2 - gh\right)$
65 $\frac{\pi r^2}{3}(4r - h)$
66 $\pi r(3r + 2h)$
67 $\frac{1}{2}mu\,(u+1)$
68 $\frac{1}{4}c\,(2b - a)$

Exercise 2l page 30

1 $(x + 3)\,(y + 3)$
2 $(a + 2b)\,(1 + b)$
3 $(a + b)\,(a + c)$
4 $(x - 3)\,(y + 2)$
5 $(x + 1)\,(y + z)$
6 $(x + 2)\,(y + 4)$
7 $(a + b)\,(c + 4)$
8 $(x + 4)\,(y - 2)$
9 $(p + q)\,(r + s)$
10 $(y + 4)\,(x - 3)$
11 $(x + 2)\,(y - 5)$
12 $(p + q)\,(r - s)$
13 $(a + 2)\,(b - 3)$
14 $(p - q)\,(r + s)$
15 $(p + 4)\,(q + 2)$
16 $(2 + a)\,(3 + b)$
17 $(p - q)\,(r - s)$
18 $(3a - b)\,(3 - b)$
19 $(2a - b)\,(1 - b)$
20 $(a - 2)\,(a + 2b)$
21 $(2 - x)\,(3 - y)$
22 $(a - 2)\,(4a - b)$
23 $(3a - b)\,(2a - 3)$
24 $(2m - 3n)\,(1 - n)$
25 $(t + r)\,(t + s)$
26 $(x - 1)\,(x + y)$
27 $2\,(1 - a)\,(2a + b)$
28 $(x + y)\,(1 - y)$

29 $(2a+3b)(2-3a)$
30 $(a+b)(2a+c)$
31 $2(2x+y)(1-y)$
32 $(x+y)(y+z)$
33 $(x-2)(5-y)$
34 $(a+4)(b-3)$
35 $(x+3)(y-z)$
36 $(p-4)(2-q)$
37 $(a-2)(b-3)$
38 $(a-4)(3-b)$
39 $(m+n)(m+1)$
40 $(a+1)(a-b)$
41 $(2p+1)(p-2q)$
42 $(x+1)(1-y)$
43 $(a+b)(a+1)$
44 $(a-b)(a+1)$
45 $(x+y)(x-1)$
46 $(a-1)(2a+b)$
47 $(x+2y)(5x-1)$
48 $(n-1)(m-1)$
49 $(x-1)(3x+y)$
50 $(2p-1)(p+2q)$
51 $(1-a)(3a+b)$
52 $(1-z)(2x+y)$

Exercise 2m page 33

1 B **2** D **3** B **4** A **5** D **6** B **7** C **8** D **9** A **10** D

CHAPTER 3

Exercise 3a page 37

1 3, 2 **2** 2, 4 **3** 3, 5 **4** 1, 7 **5** 4, −3 **6** 2, 5 **7** −2, 1 **8** 5, 1 **9** $3, 1\frac{1}{2}$ **10** 6, 0 **11** −1, −2 **12** 0, 9

Exercise 3b page 38

1 3, 1 **2** 4, 2 **3** 3, 4 **4** 2, −4 **5** $3, \frac{1}{2}$ **6** 9, 1 **7** 4, −2 **8** 1, 0 **9** 0, 6 **10** 2, 3
11 4, −1 **12** 6, 2 **13** $5, 1\frac{1}{2}$ **14** 4, 3 **15** $\frac{1}{2}, 4$ **16** 4, −2 **17** −3, 1 **18** $2, \frac{1}{3}$ **19** 3, 2 **20** 4, 5
21 3, 0 **22** $1\frac{1}{2}, 2$ **23** −3, 2 **24** 4, −2 **25** 6, 2 **26** 4, 3 **27** −1, 4 **28** −1, −2 **29** 5, 4 **30** 2, −4

Exercise 3c page 41

1 3, 1 **2** 1, 2 **3** $\frac{1}{3}, 1$ **4** −12, 27 **5** 0, 1 **6** 4, 3
7 1, 2 **8** 2, 1 **9** 3, −1 **10** 0, 3 **11** 1, −1 **12** $3, \frac{1}{2}$

Exercise 3d page 42

1 3, 2 **2** 1, 5 **3** 3, 1 **4** $1\frac{1}{2}, 0$ **5** 0, 6 **6** 3, −1 **7** 1, 4 **8** 1, 1 **9** 2, 2
10 3, −1 **11** 4, 2 **12** −3, 0 **13** $2, \frac{2}{3}$ **14** −1, 2 **15** 3, −2 **16** 2, −2 **17** 0, 4 **18** −1, −2 **19** 1, 1
20 3, 1 **21** 2, −1 **22** 8, 4 **23** −3, 4 **24** $3, -3\frac{1}{2}$ **25** 3, 4 **26** 2, 5 **27** 3, 2 **28** −1, −3

Exercise 3e page 43

1 1, 4 **2** −1, 5 **3** 3, −2 **4** 6, 28 **5** 2, 3 **6** −1, −1
7 $3\frac{1}{2}, 2\frac{1}{2}$ **8** 1, −2 **9** 5, 0 **10** 0, 4 **11** 3, 1 **12** −4, −5

Exercise 3f page 43

1 2, 4 **2** 5, 3 **3** 1, 1 **4** −2, 7
5 4, 6 **6** 1, 1 **7** 1, 10 **8** $2\frac{1}{3}, -\frac{2}{3}$
9 −1, 5 **10** −12, −4 **11** 2, 6 **12** $4\frac{1}{2}, 7\frac{1}{2}$

Exercise 3h page 46

1 1, 2 **2** $1\frac{4}{7}, 1\frac{6}{7}$ **3** 2, −2
4 $\frac{1}{2}, 1$ **5** 3, 4 **6** −2, −1
7 1, 10 **8** $2\frac{1}{3}, -\frac{2}{3}$ **9** −1, 5
10 −12, −4 **11** 2, 6 **12** $4\frac{1}{2}, 7\frac{1}{2}$

Exercise 3i page 46

1 12, 8 **2** 11, 5 **3** 8, 2 **4** 10, 3 **5** 10, 6 **6** 11, 5
7 3 choc ices, 7 orange ices
8 54, 36
9 cup \$180, saucer \$135
10 patties \$140 each, rotis \$210 each
11 Harry 32, Adam 10, Sam 20
12 3, 5
13 $AB = 9\frac{1}{2}$ cm, BC = 6 cm
14 $m = 2, c = 4, y = 2x + 4$

Exercise 3j page 49

1 A **2** A **3** D **4** D **5** C

CHAPTER 4

Exercise 4a page 51

1 **a** 2^6 **b** 3^4 **c** 5^4 **d** 2^9 **e** 4^4 **f** 3^6
2 **a** $3t^{15}$ **b** $8d^{15}$ **c** $2a^6$ **d** $125p^9$
3 **a** 192 **b** 6400 **c** 810 **d** 50
4 **a** $3x^9y^3$ **b** $5a^4b^{12}$ **c** $64u^6v^3$ **d** $2p^8q^2$

Exercise 4b page 52

1 $\frac{1}{2}$ **2** $\frac{1}{10}$ **3** $\frac{1}{5}$ **4** $\frac{1}{7}$ **5** $\frac{1}{8}$ **6** $\frac{1}{4}$ **7** $\frac{1}{a}$ **8** $\frac{1}{x}$
9 3 **10** $1\frac{1}{2}$ **11** 4 **12** $1\frac{1}{3}$ **13** 5 **14** $1\frac{1}{4}$ **15** a **16** $\frac{y}{x}$
17 $\frac{1}{8}$ **18** $\frac{1}{25}$ **19** $\frac{1}{1000}$ **20** $\frac{1}{36}$ **21** $\frac{1}{32}$ **22** $\frac{1}{10000}$ **23** $\frac{1}{100}$ **24** $\frac{1}{64}$
25 125 **26** 16 **27** 32 **28** 81 **29** 512 **30** 10 000 **31** 8 **32** 36

33 $1\frac{7}{9}$
34 $3\frac{3}{8}$
35 $5\frac{1}{16}$
36 $12\frac{1}{4}$
37 $5\frac{1}{16}$
38 $2\frac{7}{9}$
39 $123\frac{37}{81}$
40 $2\frac{14}{25}$
41 8
42 $6\frac{1}{4}$
43 $\frac{1}{16}$
44 64
45 1
46 1
47 125
48 $\frac{1}{9}$
49 16
50 1
51 $2\frac{10}{27}$
52 $3\frac{1}{2}$
53 1
54 $2\frac{314}{343}$
55 $\frac{1}{4}$
56 $\frac{64}{125}$
57 $\frac{1}{12}$
58 729
59 64
60 1

Exercise 4c page 55

1 a 3 **b** $\frac{1}{3}$ **c** $\frac{1}{9}$ **d** $\frac{1}{9}$
2 a 2 **b** $\frac{1}{4}$ **c** 1 **d** $\frac{1}{16}$
3 a 2 **b** 4 **c** 8 **d** 2
4 a 2 **b** 16 **c** $\frac{1}{16}$ **d** $\frac{1}{8}$
5 a 5 **b** 25 **c** 1 **d** $\frac{1}{125}$
6 a $\frac{1}{4}$ **b** 1 **c** 16 **d** 4
7 a 343 **b** $\frac{1}{7}$ **c** 7 **d** $\frac{1}{49}$
8 a 27 **b** 3 **c** 729 **d** $\frac{1}{27}$
9 a $\frac{1}{8}$ **b** $\frac{1}{16}$ **c** 2 **d** $\frac{1}{2}$
10 a $\frac{1}{100}$ **b** $\frac{1}{10}$ **c** 1 **d** 10
11 a 2^{-3} **b** 1 **c** 2^{-2} **d** 2^4
12 a 3 **b** 27 **c** 6
13 a $\frac{1}{2}$ or 2^{-1} **b** $\frac{1}{2}$
14 a 6 **b** 8
15 a a^5 **b** a^8 **c** a^3 **d** a^4
16 a x^7 **b** x^1 **c** x^3 **d** x^{12}
17 a x^4 **b** x^6 **c** x^3 **d** x^6
18 a x^{-6} **b** 1 **c** $9x^4$ **d** $x^{\frac{7}{2}}$
19 a $x^{\left(\frac{5}{2}\right)}$ **b** x^2 **c** x^{10} **d** $\frac{1}{16x^2}$
20 a $a^0 = 1$ **b** a^4 **c** $3a^2b$
21 $2^6, 4^3$
22 $4^{1/2}, 2^0$

Exercise 4d page 56

1 345
2 1200
3 0.0501
4 0.0047
5 280
6 0.73
7 902 000
8 0.000 637
9 8 720 000
10 2.65×10^2
11 1.8×10^{-1}
12 3.02×10^3
13 1.9×10^{-2}
14 7.67×10^4
15 3.9×10^5
16 8.5×10^{-4}
17 7×10^3
18 4×10^{-3}
19 5.87×10^4
20 2.6×10^3
21 4.5×10^5
22 7×10^{-6}
23 8×10^{-1}
24 5.6×10^{-4}
25 2.4×10^4
26 3.9×10^7
27 8×10^{-11}

Exercise 4e page 57

1 a $2ab^6c^3$ **b** 576
2 a $\frac{1}{4}$ **b** $\frac{1}{9}$
3 a $\frac{5}{3}$ **b** $\frac{7}{5}$
4 a 3 **b** 1
5 a 1 **b** $\frac{64}{27}$
6 a x^7 **b** x
7 a $27a^3$ **b** $4a^6$
8 $3^4, 9^2$
9 a 26 000 **b** 0.0057
10 a 3.65×10^2 **b** 1.2×10^3 **c** 7×10^{-6}

Exercise 4f page 58

1 C
2 D
3 A
4 D
5 C
6 C
7 B
8 B
9 B
10 A

CHAPTER 5

Exercise 5a page 61

1 38.44
2 187.69
3 58 564
4 7 728 400
5 0.5041
6 0.003 481
7 0.000 002 89
8 97 344
9 9.7344
10 0.000 973 44
11 84.64
12 8464
13 27 140 000
14 2714
15 0.2714
16 0.002 714
17 3.142
18 4.461
19 11.14
20 311.1
21 0.2195
22 0.069 43
23 9.798
24 17.92
25 1.619
26 0.2490
27 0.027 93
28 0.7071
29 0.6790
30 2.147
31 21.47
32 0.021 47

Exercise 5b page 62

1 10 cm
2 11.7 cm
3 9.43 cm
4 13 cm
5 11.4 cm
6 13.9 cm
7 The square of the third side is equal to the sum of the squares of the other two.

Exercise 5c page 64

1 10 cm
2 13 cm
3 20 cm
4 9.85 cm
5 10.8 cm
6 10.6 cm
7 11.7 cm
8 12.6 cm
9 12.1 cm
10 10.4 cm
11 3.61 cm
12 11.4 cm
13 6.40 m
14 11.4 m
15 12.2 cm
16 5.40 cm
17 121 cm
18 3.31 cm
19 9.57 cm
20 44.7 m
21 0.361 cm
22 8.64 cm
23 17.4 m
24 2.61 cm
25 35.0 cm
26 13.0 cm
27 12.0 cm

Exercise 5d page 67

1 30 cm
2 18.4 cm
3 130 mm
4 7.5 cm
5 26 m
6 32.0 cm
7 $2\frac{1}{2}$ cm
8 12.8 cm

Exercise 5e page 68

1 12 cm
2 48 cm
3 24 cm
4 10 m
5 4.90 cm
6 2.65 cm
7 1.73 m
8 4.58 cm
9 7.48 m
10 7.94 cm
11 6.24 cm
12 16.0 cm
13 6.71 cm
14 13.7 m

Exercise 5f **page 70**

1 6.71 cm
2 8.87 cm
3 55 cm
4 5.66 cm
5 11.5 cm
6 9 cm
7 3.46 cm
8 8.15 m
9 88.5 cm
10 7, 24
11 4.9 cm
12 265 cm
13 9.27 cm
14 2.5 cm
15 6.8 m
16 89.6 cm
17 3.51 cm

Exercise 5g **page 72**

1 12.5 cm
2 10 cm
3 15.0 cm, 30 cm
4 4.25 m
5 11.3 cm

Exercise 5h **page 74**

1 2.71 cm
2 4.69 cm
3 10.4 cm
4 16.2 m
5 12.7 cm

Exercise 5i **page 75**

1 2.60 m
2 7.81 cm
3 14.1 cm
4 105 m
5 7.55 m
6 74.3 cm
7 83.1 m
8 1.63 m
9 14.1 cm
10 6.22 km
11 8.94 units
12 38.8 n.m.
13 5.52 m
14 0.589 m
15 21.2 cm
16 a 39.4 cm
17 a 2.4 cm b 4.64 cm
no; $AC^2 \neq AB^2 + BC^2$
18 c AC = 7.07 cm
AD = 8.66 cm
AE = 10 cm
19 use 7 cm and 4 cm
or 8 cm and 1 cm
$\sqrt{65} = 8.06$

Exercise 5j **page 78**

1 yes
2 yes
3 no
4 no
5 yes
6 no

Exercise 5k **page 79**

1 18.9 cm
2 6.52 cm
3 2.02 cm
4 0.0265 cm
5 20.5 cm
6 4.16 cm
7 0.05 cm
8 130 cm
9 3.58 cm
10 64.5 cm
11 yes
12 3.13 cm
13 26.2 cm
14 yes, M = 90°

Exercise 5l **page 80**

1 C 2 A 3 C 4 B 5 C 6 C

CHAPTER 6

Exercise 6a **page 84**

7 $\frac{5}{12}$, 0.4167
8 $\frac{8}{15}$, 0.5333
9 $\frac{3}{4}$, 0.75
10 $\frac{3}{4}$, 0.75
11 $\frac{12}{5}$, 2.4
12 $\frac{35}{12}$, 2.917

Exercise 6b **page 86**

1 1.8807
2 0.2493
3 0.5890
4 0.3019
5 0.0805
6 3.0777
7 4.8716
8 1
9 0.5774
10 1.1184
11 0.0524
12 0.5635
13 10.1°
14 19.6°
15 55.0°
16 23.4°
17 53.7°
18 32.3°
19 42.7°
20 38.7°
21 17.8°
22 69.6°
23 42.7°
24 0.1°

Exercise 6c **page 86**

1 32.0°
2 63.4°
3 23.2°
4 35.8°
5 31.0°
6 51.3°
7 48.4°
8 47.7°
9 34.2°

Exercise 6d **page 87**

1 2.44 cm
2 5.40 cm
3 2.56 cm
4 6.72 cm
5 17.0 cm
6 81.8 cm
7 5.62 cm
8 22.2 cm
9 2.82 cm
10 7.54 cm
11 3.60 cm
12 11.4 cm
13 2.42 cm
14 1.76 cm
15 46.6 cm
16 10.4 cm
17 4.69 cm
18 366 cm
19 0.976 cm
20 69.5 cm

Exercise 6e **page 90**

1 0.8862
2 0.9397
3 0.2470
4 0.1564
5 0.2622
6 0.6088
7 0.8625
8 0.5
9 0.9903
10 0.4664
11 0.2723
12 0.9988
13 15.7°
14 26.2°
15 31.6°
16 65.4°
17 41.8°
18 21.8°
19 37.9°
20 46.7°
21 7.1°

Exercise 6f **page 90**

1 30°
2 17.5°
3 48.6°
4 44.4°
5 14.5°
6 62.7°
7 44.4°
8 41.8°
9 23.6°
10 19.5°
11 4.38 cm
12 10.6 cm
13 1.46 cm
14 4.57 cm
15 11.7 cm
16 23.2 cm
17 6.31 cm
18 21.9 m
19 3.34 cm
20 45.7 cm

Exercise 6g **page 92**

1 0.8480
2 0.7455
3 0.1392
4 0.6717
5 0.5
6 0.9632
7 0.6143
8 0.6561
9 69.7°
10 20.6°
11 44.0°
12 69.6°
13 51.1°
14 71.6°
15 30.1°
16 89.2°

Exercise 6h **page 93**

1 34.9°
2 36.9°
3 45.6°
4 48.2°
5 48.2°
6 53.1°
7 50.2°
8 66.4°
9 81.4°
10 25.8°
11 34.0°
12 3.50 cm
13 26.9 cm
14 1.96 cm
15 11.6 cm
16 38.2 cm
17 2.90 cm
18 17.1 cm
19 2.23 cm
20 4.12 cm
21 13.5 cm

Exercise 6i **page 95**

1 40.0°
2 33.6°
3 51.3°
4 42.8°
5 35.5°
6 33.7°
7 39.8°
8 33.7°
9 37.7°
10 53.1°
11 68.5°
12 14.5°
13 56.9°
14 37.8°
15 39.3°
16 55.6°
17 42.1°
18 66.2°
19 6.69 cm
20 19.3 cm
21 8.03 cm
22 4.86 cm
23 4.48 cm
24 80.5 cm
25 6.04 cm
26 3.50 cm
27 13.7 cm
28 3.08 cm
29 113 cm
30 2.59 cm
31 9.99 cm
32 7.45 cm
33 14.5 cm
34 21.4 cm
35 74.5 cm
36 60.6 cm

Exercise 6j **page 98**

1 4.13 cm
2 8.72 cm
3 23.3 cm
4 4.67 cm
5 14.9 cm
6 17.0 cm
7 4.40 cm
8 14.9 cm
9 33.1 cm
10 42.6 cm

Exercise 6k page 100

1 8.99 cm **2** 47.7 m **3** 143 m **4** 39.8° **5** 61.6° **6** 56.3° **7** 48.2° **8** 11.3°
9 **a** 5.30 cm **b** 6.25 cm
10 **a** 5.20 cm **b** 15.6 cm^2
11 4.66 m **12** 54 m **13** 1099 m **14** 8660 m **15** 433 m **16** 134 m **17** 582 m **18** 280 m

Exercise 6l page 102

1 C **2** D **3** C **4** A **5** B **6** C

CHAPTER 7

Exercise 7a page 105

1 12 cm **2** 30 mm **3** 7 cm **4** 2.5 m **5** 500 m **6** 2.3 cm

Exercise 7b page 107

1 14.5 m **2** 28.9 cm **3** 18.2 cm **4** 333 mm **5** 54.7 m **6** 1570 mm **7** 226 cm **8** 30.2 m **9** 11.3 m **10** 0.0880 km **11** 44.0 cm **12** 176 mm **13** 8.80 m **14** 220 mm **15** 35.2 cm **16** 970 mm **17** 88 cm **18** 24 m **19** 1300 mm **20** 220 cm **21** 1600 mm **22** 2000 cm **23** 29 m

Exercise 7c page 108

1 10.3 cm **2** 10.7 cm **3** 18.3 cm **4** 20.5 cm **5** 27.9 cm **6** 33.6 cm **7** 94.2 cm **8** 62.8 mm **9** 20.6 cm **10** 45.1 cm **11** 30.9 cm **12** 41.7 cm **13** 229.2 cm **14** 67.2 cm

Exercise 7d page 110

1 78.5 mm
2 62.8 mm, 88.0 mm
3 4.4 m
4 194 cm
5 176 cm
6 176 cm, 200
7 12.6 cm
8 94.2 cm

9 62.8 m
10 6.28 s, 9.55 revolutions
11 3140 cm
12 12.6 m
13 70.7
14 94.3 m

Exercise 7e page 113

1 7.00 cm **2** 19.3 mm **3** 87.5 m **4** 43.8 cm **5** 73.5 mm **6** 132 cm **7** 5.76 mm **8** 62.2 m **9** 92.6 cm **10** 13.9 m **11** 16.5 m **12** 59.8 m **13** 31.8 cm **14** 20.1 m **15** 4.93 cm **16** 9.55 cm each **17** 3.82 cm, 45.8 cm **18** 37.7 cm **19** 4.77 cm **20** 9.55 cm **21** 9.55 cm, 29.1 cm

Exercise 7f page 115

1 50.3 cm^2 **2** 201 m^2 **3** 78.6 m^2 **4** 78.6 mm^2 **5** 38.5 m^2 **6** 11 300 cm^2 **7** 45.4 m^2 **8** 9.62 km^2 **9** 20 100 m^2 **10** 25.1 cm^2 **11** 51.3 m^2 **12** 58.9 cm^2 **13** 118 mm^2 **14** 451 mm^2 **15** 374 cm^2 **16** 457 cm^2 **17** 714 m^2 **18** 942 cm^2 **19** 3540 cm^2 **20** 193 cm^2 **21** 828 cm^2 (3 s.f.) **22** 214 cm^2 **23** 74.1 cm^2 **24** 42.5 cm^2 **25** 93.7 cm^2 **26** 31.4 cm^2 **27** 42.9 m^2 **28** 4.92 cm **29** 11.8 cm **30** 276 cm

Exercise 7g page 118

1

707 cm^2

2

236 cm^2

3 415 mm^2

4

26.2 cm^2

5 no
6 21.5 cm^2
7 8, 110 cm^2
8 11 700 cm^2
9 2

Exercise 7h page 121

1 $\frac{2\pi}{3}$ cm **2** $\frac{8\pi}{9}$ cm **3** 4π cm **4** 8π cm **5** 2.55 cm **6** 8.43 cm **7** 43.8 cm **8** 84.7 cm **9** 137° **10** 40° **11** 13.0 cm **12** 18 cm **13** 11.9 cm **14** 13.5 cm **15** 26.3 cm **16** 5.47 cm **17** 19.1 cm **18** 146° **19** 115° **20** 85.9° **21** 30.5 m **22** 10.6 cm **23** 20.5 cm **24** 28 cm **25** 6.69 cm **26** 27.1 m **27** 77.9 mm **28** 19.1°

Exercise 7i page 124

1 262 cm^2 **2** 151 cm^2 **3** 118 cm^2 **4** 38.2 cm^2 **5** 21.6 cm^2 **6** 4220 mm^2 **7** 2260 m^2 **8** 327 m^2 **9** 4.19 cm^2 **10** 75.4 cm^2 **11** 90° **12** 55° **13** 131 cm^2 **14** 6 cm **15** 6.93 cm
16 **a** 15° **b** 6π cm^2
17 16°
18 **a** 2.98 m **b** 2.84 m^2
19 **a** 19.4 m **b** 9.05
20 58.7 mm^2
21 blue by 9.42 cm^2
22 **a** 94.6 mm **b** 456 mm^2
23 **a** 12.0 m **b** \$3900 **c** 8.90 m^2 **d** 498 g
24 A = 370 cm^2, B = C = 576 cm^2; total 1522 cm^2

Exercise 7j page 129

1 74.6 cm^2
2 442 cm^2
3 483 cm^2
4 81.7 cm^2
5 230 cm^2
6 **a** 2.73 cm **b** 5.46 cm^2 **c** 38.3 cm^2 **d** 92.7 cm^2

Exercise 7k **page 130**

1 17.6 mm
2 9.55 m
3 37.7 cm
4 $26.4\,m^2$
5 $491\,cm^2$
6 28.6 mm
7 $7.95\,cm^2$
8 5.20 cm

Exercise 7l **page 131**

1 62.8 m
2 $452\,cm^2$
3 57.3 cm
4 $50.3\,m^2$
5 89.1 mm
6 40.9 cm
7 $87.5\,cm^2$
8 5.64 cm

Exercise 7m **page 131**

1 $12.6\,km^2$
2 308 mm
3 14 m
4 $154\,cm^2$
5 $32.2\,cm^2$
6 $18.1\,m^2$

Exercise 7n **page 132**

1 C 2 B 3 B 4 A 5 A 6 C 7 D 8 B 9 A 10 A

Exercise 7p **page 133**

1 D 2 C 3 D 4 C 5 B 6 D

REVIEW TEST 1 **page 136**

1 C 2 D 3 D 4 C 5 C 6 D 7 D 8 B 9 C 10 D 11 B 12 C

13 a $\frac{1}{25}$ b 4 c 3
14 a 3×10^{-6} b 5920 c 8.21×10^{-3}
15 a $4a^2-12ab+3b^2$ b $9x^2-4y^2$ c $x^2-6x-16$
16 a $a(a-2b)$ b $4b(a-2c)$ c $2(x^2-x-3)$
17 a x^2+7x-8 b $16x^2-25y^2$ c $3x+5x-2$ d $ab^2-8ab+16$
18 a $x=4$ b $x=3$
19 a $\frac{13x+3}{20}$ b $\frac{25x-58}{12}$
20 a $x=3,\ y=2$ b $x=4,\ y=-1$
21 $x=7,\ y=5$
22 a 27.1 b 0.667 c 0.00176
23 a 3.56 b 0.243 c 0.0276
24 Yes because $a^2+b^2=c^2$
25 a $x=10.8$ cm b $x=6.63$ cm
26 a i $x=4.8$ cm ii $x=2.88$ cm
b No
27 a 5.59 cm b $22.3\,cm^2$
28 a 46.9 cm b $97.2\,cm^2$
29 6.40
30 202 m
31 $199\,cm^2$

CHAPTER 8

Exercise 8a **page 141**

1 2, {H, T}
2 3, {R, B, Y}
3 10, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
4 6, {R, Y, B, Brown, Black, G}
5 3, {chewing gum, boiled sweets, bar of chocolate}
6 4, {1 c, 10 c, 20 c, 50 c}
7 13, {A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K}
8 5, {a, e, i, o, u}
9 5, {2, 3, 5, 7, 11}
10 10, {2, 4, 6, 8, 10, 12, 14, 16, 18, 20}

Exercise 8b **page 142**

1 $\frac{1}{4}$
2 $\frac{1}{10}$
3 $\frac{1}{5}$
4 $\frac{1}{6}$
5 $\frac{1}{7}$
6 $\frac{1}{200}$
7 $\frac{1}{52}$
8 $\frac{1}{7}$
9 $\frac{1}{15}$

Exercise 8c **page 144**

1 5 2 3 3 26 4 2 5 10
6 a $\frac{1}{2}$ b $\frac{1}{2}$ c $\frac{2}{5}$ d $\frac{3}{10}$
7 a $\frac{1}{13}$ b $\frac{1}{2}$ c $\frac{1}{4}$ d $\frac{4}{13}$
8 a $\frac{2}{9}$ b $\frac{1}{9}$ c $\frac{1}{3}$ d $\frac{2}{9}$
9 a $\frac{1}{2}$ b $\frac{1}{3}$ c $\frac{1}{3}$
10 $\frac{2}{15}$
11 a $\frac{3}{5}$ b $\frac{1}{5}$ c $\frac{2}{5}$
12 $\frac{1}{40}$
13 a $\frac{18}{37}$ b $\frac{18}{37}$ c $\frac{9}{37}$
14 $\frac{21}{26}$
15 $\frac{4}{45}$
16 a $\frac{5}{12}$ b $\frac{1}{3}$ c $\frac{3}{4}$

Exercise 8d **page 146**

1 0, impossible
2 0.3, unlikely to be this heavy
3 1, almost certain
4 0.001, possible but unlikely
5 0, most unlikely!
6 0, impossible
7 1, certain
8 0, virtually impossible
9 1, it must be
10 0, almost impossible
11 Likely: you will watch TV this week, you will get maths homework this week.
Unlikely: you will be a US dollar millionaire, it will snow in Jamaica on mid-summer's day.

Exercise 8e **page 147**

1 $\frac{3}{5}$ 2 $\frac{12}{13}$ 3 $\frac{21}{26}$ 4 $\frac{5}{6}$ 5 $\frac{7}{10}$ 6 $\frac{5}{8}$ 7 $\frac{24}{25}$ 8 $\frac{2}{3}$ 9 $\frac{39}{40}$ 10 $\frac{10}{13}$
11 a $\frac{1}{10}$ b $\frac{3}{10}$ c $\frac{2}{5}$ d $\frac{7}{10}$
12 a $\frac{1}{13}$ b $\frac{1}{4}$ c $\frac{3}{4}$ d $\frac{11}{13}$
13 a $\frac{15}{22}$ b $\frac{7}{22}$ c $\frac{1}{22}$ d $\frac{3}{11}$
14 a $\frac{2}{5}$ b $\frac{19}{30}$ c $\frac{7}{30}$ d 0

Exercise 8f **page 149**

1 about 50
2 10
3 20
4 a 10 b 20
5 20

6 a 15 **b** 45 **c** 30
7 20
8 10
9 10
10 5
11 25
12 a $\frac{1}{13}$ **b** 3
13 a $\frac{1}{2}$ **b** 30
c no, possible but not certain
d probably, you'd expect about 50
14 a \$10 **b** 5 **c** \$5
d lose, spending greater than likely winnings
15 a 2 **b** \$50
c spends \$50 more than winnings.
16 64

Exercise 8g page 152

1 the tombola (1 in 5 = 20% and is greater than 1 in 8 = 13%)
2 no, August is the better choice
3 a Medicine B
b probably, because the chance of serious side effects is lot lower with Medicine A
4 a Printer A as it is likely to last longer
b the size of the printers; the advance in technology over the life span of the printers; any other sensible answers
5 $\frac{2}{11}$ is less than $\frac{1}{3}$ so route 2 is the better choice
6 $\frac{5}{9} > \frac{3}{7}$ and $\frac{2}{5}$ so 24 June is the best choice

Exercise 8i page 155

1 D **2** B **3** A **4** C **5** D **6** C

CHAPTER 9

Exercise 9a page 159

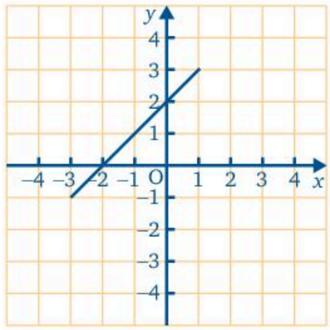
1 {(1, 1), (2, 8), (3, 27), (4, 64)}
2 a {(2, 1), (3, 1), (3, 2)}
b domain = {2, 3}, range = {1, 2}
3 a one-to-many; 1 : n
b one-to-many; 1 : n
c one-to-one: 1 : 1
d many-to-one; n : 1
e one-to-one; 1 : 1

Exercise 9b page 160

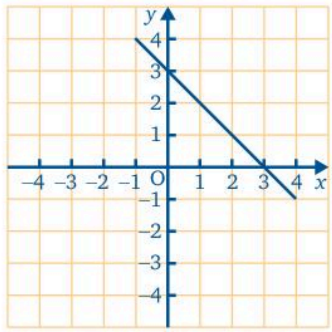
1 a {(apple, mango), (banana, apple), (banana, mango)}
b no; banana in the domain is paired with more than one member of the range
2 no; Anne in the domain is paired with more than one member of the range
3 (−1, −5), (0, 0), (1, 5), (2, 10); yes
4 (1, 4), (1, 6), (4, 6); no, I occurs twice as the first number
5 (0, 1), (2, 5), (4, 17); yes
6 (3, 15), (7, 35), (9, 45)
7 (0, −1), (1, 0), (2, 7)
8 (−1, 3), (0, 0), (2, 0)
9 (2, 6), (4, 16), (6, 26)
10 a no **b** yes **c** no
11 a yes **b** no **c** yes
12 a 1 **b** 10 **c** −8
13 a 2 **b** 0 **c** −2
14 a 0 **b** 1 **c** 6
15 a, b, c, e are functions

Exercise 9c page 163

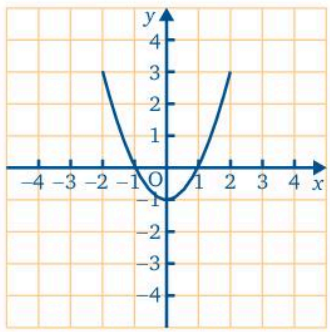
1 a



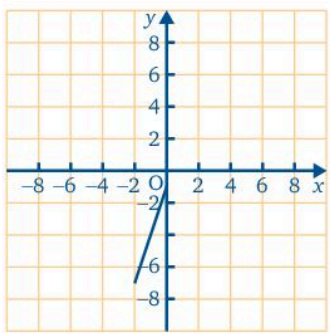
b



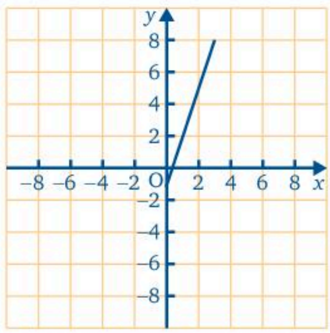
2



3 a

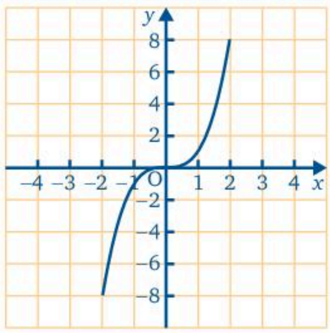


b

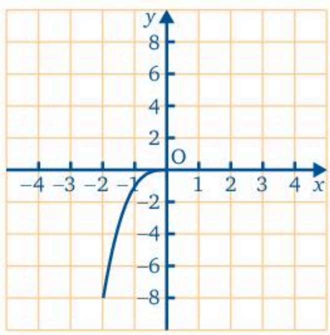


c The two functions have different domains and ranges.

4 a

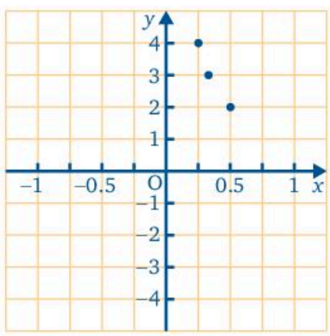


b

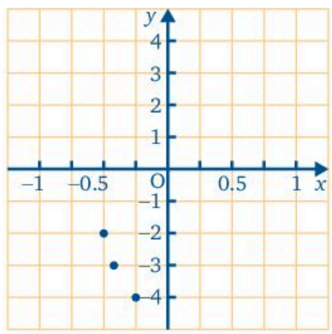


c The two functions have different domains and ranges.

5 a



b



Exercise 9d page 165

1 no **2** yes **3** no **4** yes **5** yes **6** no

Exercise 9e page 166

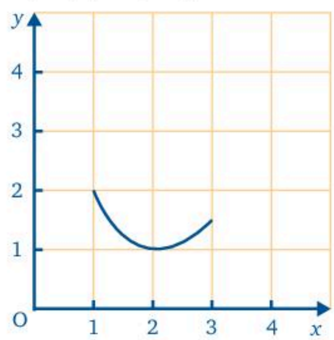
1 a {(2, 3), (2, 5), (3, 5)}
b {2, 3}, {3, 5}
c no; the number 2 from the domain is paired with two members of the range

2 a one-to-many; $1:n$ **b** many-to-many; $n:n$

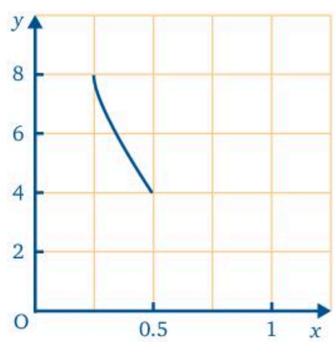
3 a 3 **b** 18 **c** $2\frac{1}{4}$

4 a {(−2, 0), (−1, 1), (0, 2), (1, 1), (2, −2)}
b yes; each member of the domain is paired with only one member of the range

5 a



b



6 a no **b** yes **c** no **d** no

CHAPTER 10

Exercise 10a page 170

1 2 : 3
2 1 : 2 : 3
3 7 : 5
4 2 : 3
5 18 : 8 : 9
6 2 : 3 : 1
7 4 : 9
8 3 : 5 : 4
9 2 : 3 : 1
10 6 : 11
11 15 : 4
12 31 : 4
13 5 : 16
14 1.5 : 1
15 2.4 : 1
16 0.857 : 1
17 2.73 : 1
18 0.6 : 1
19 2.63 : 1
20 1.33 : 1
21 0.75 : 1
22 1.43 : 1

Exercise 10b page 171

1 9 : 2
2 2 : 5
3 17 : 60
4 2 : 125
5 9 : 20
6 50 : 3
7 20 : 19
8 36 : 35
9 27 : 25
10 9 : 10
11 25 : 24
12 a 3 : 2 b 2 : 3 c 3 : 5
13 a 2 : 3 b 9 : 5 c 21 : 23 d 6 : 5
14 18 : 25
15 a 1 : 1 b 1 : 2 c 1 : 8 d 1 : 1 e 1 : 3 f 1 : 8
16 a 1 : 9 b 1 : 4 c 4 : 9

Exercise 10c page 173

1 $1\frac{1}{9}$, or 1.11
2 $\frac{3}{7}$ or 0.429
3 $7\frac{1}{2}$ or 7.5
4 $1\frac{3}{7}$ or 1 : 43
5 24
6 $22\frac{1}{2}$
7 $9\frac{1}{3}$ or 9.33
8 $2\frac{8}{11}$ or 2.73
9 $1\frac{1}{5}$ or 1.2
10 $7\frac{1}{5}$ or 7.2
11 $3\frac{1}{3}$ or 3.33
12 $8\frac{4}{7}$ or 8.57
13 12 grandsons; 3 : 7
14 152
15 10.08 cm
16 264

Exercise 10d page 174

1 \$20, \$25
2 54 m, 42 m
3 0.625 kg, 1.25 kg, 3.125 kg
4 $\frac{1}{2}$ h, $2\frac{1}{2}$ h, 4 h
5 18 boys, 14 girls
6 60°, 50°, 70°
7 9, 12, 9
8 66 hits, 24 misses

Exercise 10e page 175

1 9 : 7
2 30 m, 42 m
3 $5\frac{1}{4}$
4 $2\frac{2}{9}$ or 2.22
5 27.5 cm
6 5 : 3
7 500 : 53
8 4 : 3
9 3 : 4
10 2 : 3

Exercise 10f page 175

1 a \$480 b \$1920
2 a 6 units b $\frac{3}{4}$ unit
3 a 72 km b 118.8 km
4 a 35 rows b 42 rows
5 a \$212.50 b \$1020

Exercise 10g page 176

1 \$120
2 15.5 km
3 $4\frac{1}{3}$ or 4.33 km
4 \$145
5 \$7700
6 \$420
7 \$8300
8 75 c
9 1.5 m
10 5.5 cm^2

Exercise 10h page 177

1 3.2 litres
2 3 hours
3 $12\frac{1}{2}$ units
4 3.6 hours
5 a \$3000 b 35 km
6 \$9600
7 700
8 \$2640
9 66 rows
10 20.25 cm
11 \$16 800
12 \$33 600
13 65.6 km
14 a 2.25×10^7 b 8.1×10^6 c 1.35×10^5
15 15 V
16 24.7 joules
17 \$148
18 \$4216

Exercise 10i page 180

1 $5\frac{1}{2}$ hours
2 12
3 203
4 8 days
5 25 cm
6 20
7 16 cm
8 48
9 49

Exercise 10j page 182

1 a 1822.06 b 17.29
2 \$19 570
3 $3\frac{1}{2}$ hours
4 no answer
5 4.46 cm
6 49
7 24
8 34
9 1.44 m
10 6 weeks
11 no answer
12 1.5 amps

Exercise 10k page 183

1 3 : 1
2 $3\frac{3}{5}$ or 3.6
3 8 m, 16 m, 32 m
4 114 km (3 s.f)
5 6 hours 40 mins
6 6 : 2 : 1
7 9
8 $\frac{6}{5}$: 1 or 1.2 : 1

CHAPTER 11

Exercise 11a page 186

1 to **10** students' own drawings
11 500 m
12 2.29 m
13 and **14** students' own drawings

Exercise 11b page 189

1 a 5.5 cm b 5.5 m c 3.5 cm d 3.5 m
2 a 33 cm b 6 cm c 5 cm
3 a i 6 cm ii 4.5 cm b i 24 ft ii 18 ft c soccer 6 ft, rugby 10 ft d 30 ft
4 a 15 cm b 9.6 cm
5 a i 5 m by 4 m ii 4 m by 3.5 m b 2.5 m c 20 m^2 d \$2600
6 a i 236 mm ii 188 mm iii 208 mm iv 34 mm b i 32 mm ii 26 mm c yes, going + twice the rise = 584 mm d yes

Exercise 11c page 194

1 a $\frac{3}{2\,000\,000}$ b $21\frac{1}{3}$ km c $35\frac{1}{3}$ km d 34 km e 38 km
2 72 000 m^2 = 7.2 hectares
3 a 7.5 km b 100 cm^2
4 a 2.4 km b 2.8 km^2
5 a 1 : 3000 b 192 m c $\frac{16}{9}$ cm^2 or 1.78 cm^2 (3 s.f.)

Exercise 11d page 195

1 860 cm
2 94 m
3 30 m
4 577 m
5 86.6 m

Exercise 11e page 196

1 C 2 B 3 D 4 B 5 C

CHAPTER 12

Exercise 12a page 199

1 $32 000
2 $29 750
3 $38 500
4 $49 280
5 $49 375
6 a 7 h 40 min b 38 h 20 min c $35 726.67
7 $43\frac{3}{4}$ h, $45 850
8 a $7\frac{3}{4}$ b $38\frac{3}{4}$ c $35 030
9 a $23 560 b $32 860 c $34 720
10 $34 608
11 $31 450
12 $30 652
13 $46 872
14 $37 674
15 $26 788
16 a $7\frac{1}{2}$ h b $37\frac{1}{2}$ c $21 300
d $1\frac{1}{2}$ h e $22 578

Exercise 12b page 202

1 $64 000 3 $44 100 5 $40 200 7 $40 250
2 $36 800 4 $54 400 6 $60 000 8 $32 160
9 a 1125 b 500 c 625 d $48 125

10

	a	bi	bii	c
Ms Arnold	186	100	86	$21 970
Mr Beynon	158	80	78	$18 910
Miss Capstick	194	100	94	$23 130
Mr Davis	225	100	125	$27 625
Mr Edmunds	191	100	91	$22 695

d Thursday; the total for Thursday is the greatest

Exercise 12c page 204

1 $4000
2 $130 000
3 $300 000
4 $200 000
5 $162 500
6 $312 500
7 $161 250
8 $206 500
9 $330 000
10 $224 000
11 $400 000
12 $768 000
13 $540 000
14 $630 000
15 $1 062 000
16 $633 600
17 $1 281 000

Exercise 12d page 206

1 a $2500 b $89 500
2 a $228 000 b $103 000 c $25 750
3 The values in order are:
Mrs P $213 000, $4800, $5325
Mr W $292 000, $6500, $7300
Ms M $87 500, $2000, $2187.50
Mr D $77 000, $1700, $1925
Mrs E $110 250, $2500, $2756.25
Ms B $122 000, $2800, $3050
4 NIS: $3000, Statutory income: $262 000, Taxable income: $137 000, Income tax: $34 250, NHT: $5800, Ed tax: $5895, Total deductions: $73 945, Net income: $216 055
5 a $1300 b $975
6 a $2250 b $117 000
7 a $184 000 b $3680 c $1840 d $178 480
8 Pension fund: $48 000, NIS: $3000, Statutory income: $317 000, Taxable income: $192 000, Income tax: $48 000, NHT: $7360, Ed tax: $7925, Total of these deductions: $114 285, Non-statutory deductions: $18 400, Net income: $235 315

Exercise 12e page 210

1 $2868.50
2 $2701.50
3 $3805.00
4 $1956.50
5 $7274.00
6 a April: $1790.70, May: $2380.50, June: $1636.80, July: $1893.00
b No, as he only exceeded the $2100 cost in one month out of the four; Rick's average cost is $1925.25

Exercise 12f page 212

1 3
2 $\frac{1}{10}$
3 $1\frac{1}{2}$
4 1.2
5 0.06
6 0.02
7 8
8 2
15 16
16 1
17 12
18 0.6
19 1.8
20 0.144
21 0.056
22 0.84
23 40 hours
24 $\frac{1}{2}$ hour
25 10 hours
26 $2\frac{7}{9}$ hours
27 $3
28 $12
29 $3024
30 $$1\frac{1}{2}$

Exercise 12g page 214

1 Mr George $2350, Miss Newton $3330, Mr Khan $2287.50, Mrs Wilton $1905.80, Mr Barnes $4335.50
2 Mr George $235, Miss Newton $333, Mr Khan $228.75, Mrs Wilton $190.58, Mr Barnes $433.55
3 Mrs Wan $5850, Mr Davis $7752, Mr Deats $4023, Miss Beale $9305.40
4 Mrs Wan $6435, Mr Davis $8527.20, Mr Deats $4425.30, Miss Beale $10 235.94.

5

Current	Previous	Description	Usage	Rate($)	Charge($)
9421	9175	Energy first	100	6.00	600
		Energy next	146	11.50	1679
		Customer charge			110
		Subtotal			2389
		Fuel & IPP charge	246	18.00	4428
				Total	6817

6

Current	Previous	Description	Usage	Rate($)	Charge($)
8432	8156	Energy first	100	6.50	650
		Energy next	176	12.50	2200
		Customer charge			130
		Subtotal			2980
		Fuel & IPP charge	276	17.00	4692
				Total	7672

7

Current	Previous	Description	Usage	Rate($)	Charge($)
11 041	10762	Energy first	120	5.90	708
		Energy next	159	13.00	2067
		Customer charge			140
		Subtotal			2915
		Fuel & IPP charge	279	19.00	5301
				Total	8216

Exercise 12h page 217

1 $1036.70 **2** $518.35 **3** $2548.18 **4** $4419.92
5 **a** $2807.60 **b** $3200.66
6 **a** $4296.04 **b** $4897.49
7 **a** $8508.25 **b** $9699.41
8 $3877.00; Mrs Wright used 51 – 38 = 13 cubic metres, which is 13 000 litres

Exercise 12i page 218

1 $62 500.00
2 $252 000.00
$264 600.00
3 **a** $180 000.00 **b** $3461.54
4 $10 800.00
5 $18 600.00
6 $19 000.00
7 $24 600.00
8 **a** $64 000 **b** $56 000 **c** $48 000

Exercise 12j page 220

1 $27 720
2 $42 275.00
3 $113 000.00
4 $494 000
5 **a** $125 000 **b** $5200 **c** $41 070 **d** $112 930
6 **a** $3080 **b** $2240
7 $257.50
8 $1407
9 **a** 0.48 **b** 18
10 $12 842

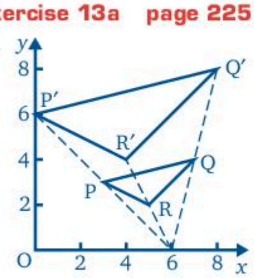
Exercise 12k page 221

1 B
2 B
3 D
4 B
5 A
6 C
7 D
8 B
9 C
10 A

CHAPTER 13

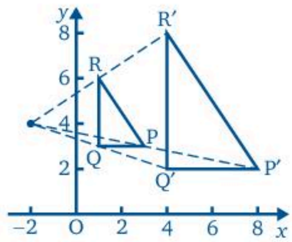
Exercise 13a page 225

1



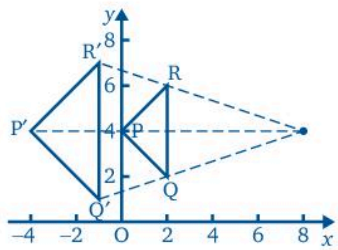
Centre of enlargement is (6, 0).

2



Centre of enlargement is (–2, 4).

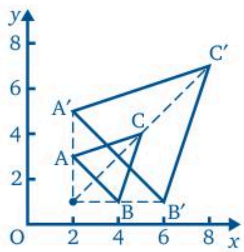
3



Centre of enlargement is (8, 4).

4 In 1 PQ||P′Q′, PR||P′R′, RQ||R′Q′
In 2 PQ||P′Q′, PR||P′R′, RQ||R′Q′
In 3 PQ||P′Q′, PR||P′R′, RQ||R′Q′

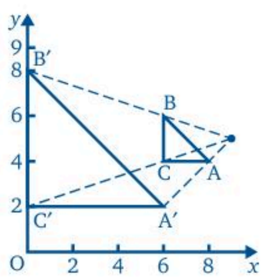
5



a Centre of enlargement is (2, 1).

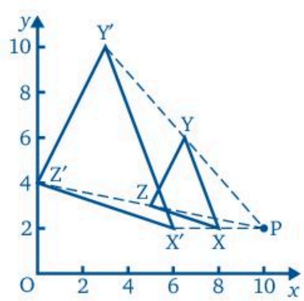
b Sides are different lengths but angles are the same.

6



Centre of enlargement is (9, 5).

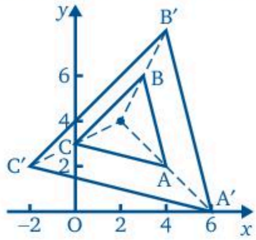
7



Centre of enlargement is (10, 2).

Exercise 13b page 227

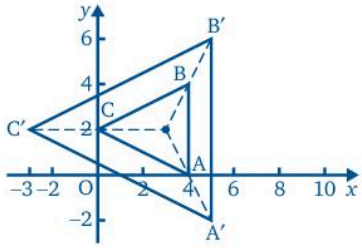
1



Centre of enlargement is (2, 4).

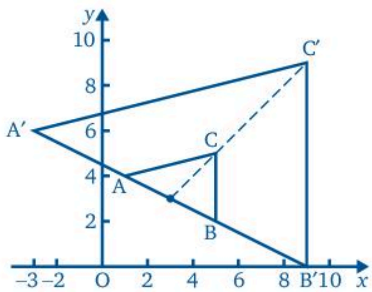
2 Centre of enlargement is (2, 2).

3



Centre of enlargement is (3, 2).

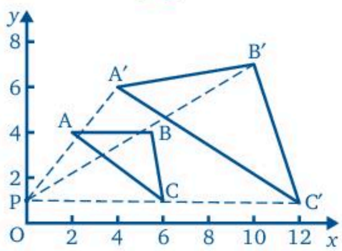
4



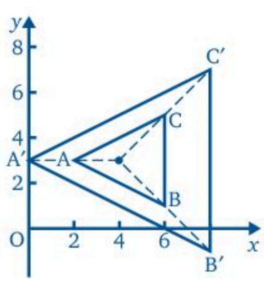
Centre of enlargement is (3, 3).

Exercise 13c page 228

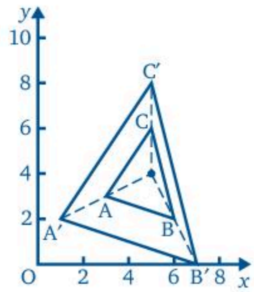
1



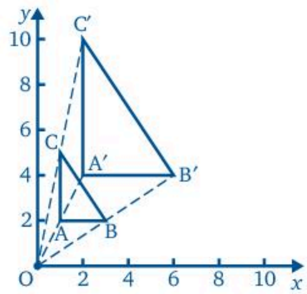
2



3

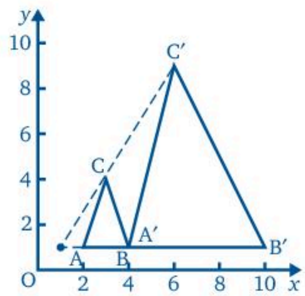


4

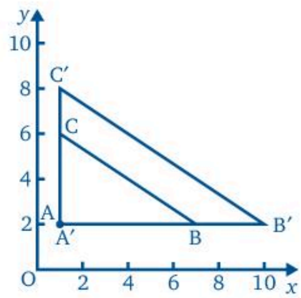


Coordinates of A′ are double the coordinates of A.

5



6



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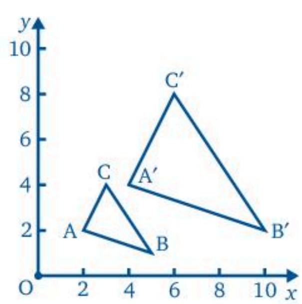
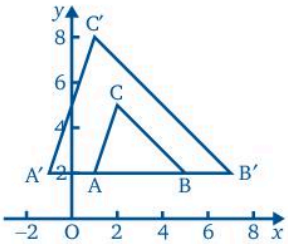


10



Exercise 13d page 231

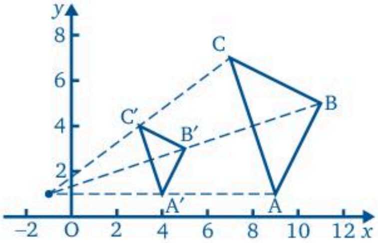
1 $(6, 3), \frac{1}{3}$

2 $(-1, 0), \frac{1}{2}$

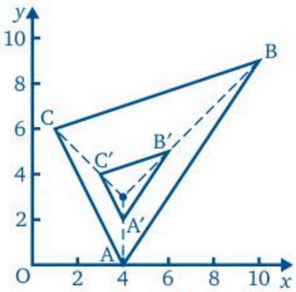
3 $\left(3\frac{1}{2}, 4\right), \frac{1}{3}$

4 $(1, 2), \frac{1}{2}$

5



6

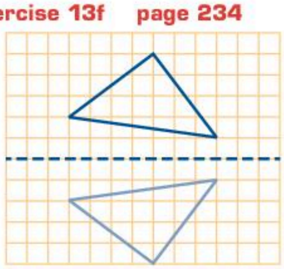


Exercise 13e page 232

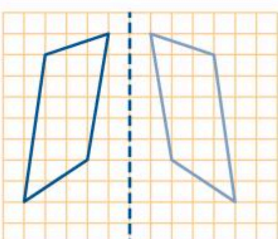
1 D **2** C **3** B **4** C

Exercise 13f page 234

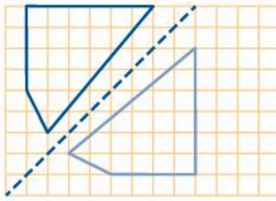
1



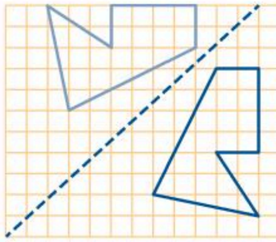
2



3



4



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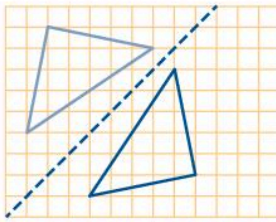

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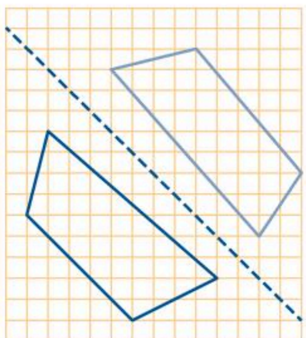
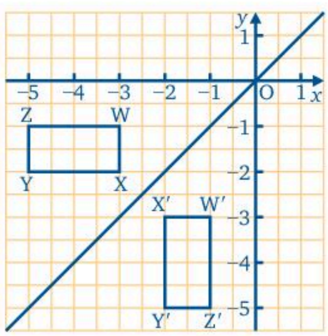
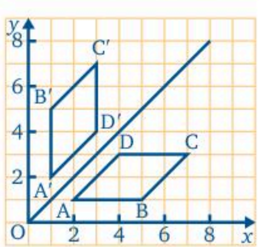


7

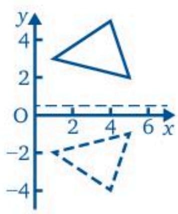


8

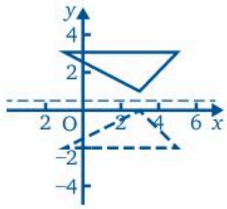


Exercise 13g page 236

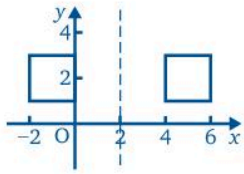
1



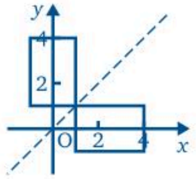
2



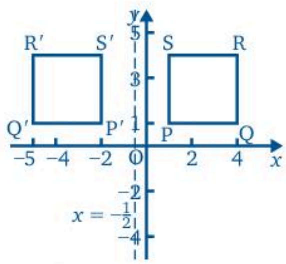
3



4

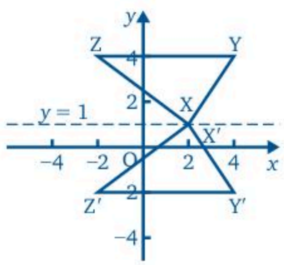


5



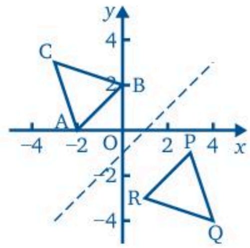
$x = -\frac{1}{2}$

6



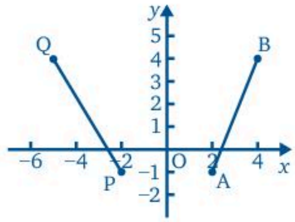
X, X′ are invariant points

7 (0,0) is an invariant point.



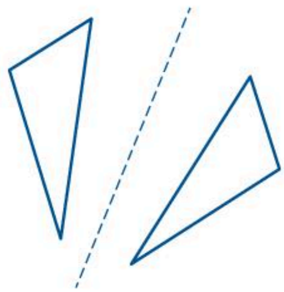
R is the image of A. There are no invariant points.

8

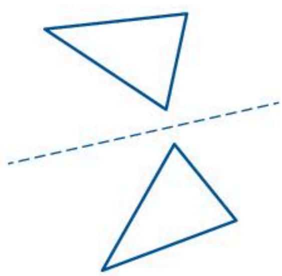


If there is a mirror line it has to be the perpendicular bisector of AP. But this line does not pass through the midpoint of QB, so PQ is not the reflection of AB.

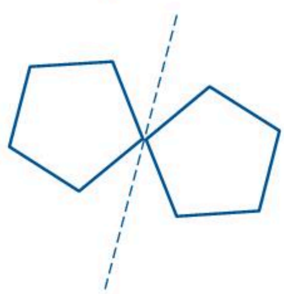
9



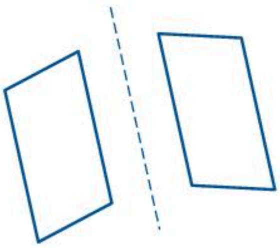
10



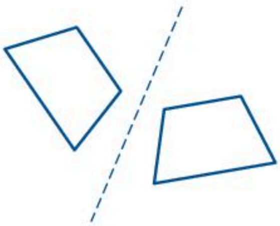
11



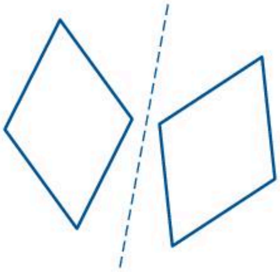
12



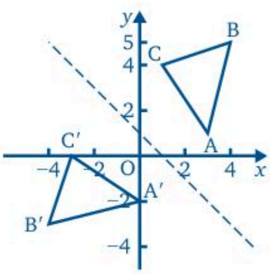
13



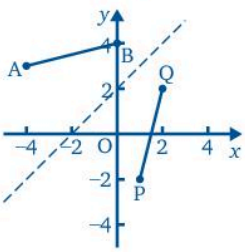
14



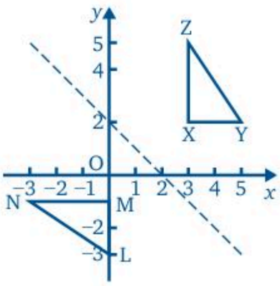
15



16



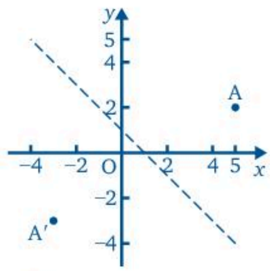
17



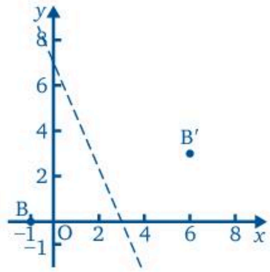
Exercise 13h page 239

1 yes

2



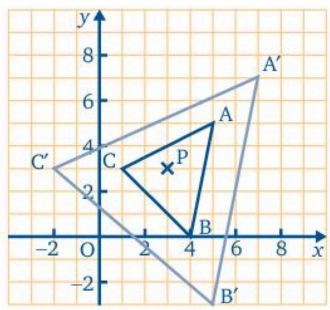
3



Exercise 13i page 241

1 (5, 0), scale factor 2

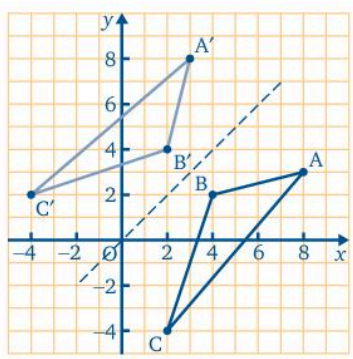
2



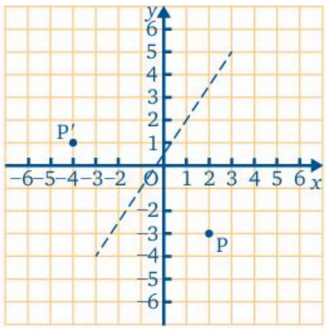
A′(7, 7), B′((5, −3), C′(−2, 3)

3 (6, 8)

4

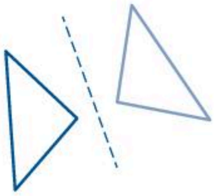


5

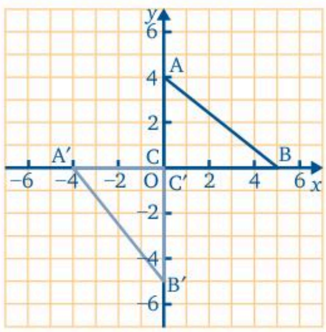


(−1, −1)

6



7 (0, 0) is an invariant point



8 (10, 0), scale factor 3

CHAPTER 14

Exercise 14a page 245

1 **a** 70° **b** isosceles

2 **a** DF = 11.1 cm, EF = 6.5 cm **b** 45°

3 ∠G = 36.9°, ∠H = 53.1°, ∠I = 90°, a right angle

4–6 constructions

7 PQ and SR are parallel

8 105°

9 BC = 6 cm, angle BCA = 45°

10 75°

11 midpoint

12 they are equal

Exercise 14b page 246

1 **c** B, C and D **d** yes; E is the centre of the square
e annulus

2 **b** A, B and C
c they are equal; Pythagoras' theorem states that in a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides
d they are the same

3 construction

4 5.7 cm

5 4.7 cm

6 8 cm

7 6 cm, 10.4 cm

8 8.9 cm

9 10.8 cm

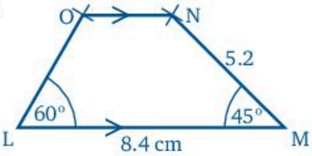
10 AB = 6.9 cm, BC = 4 cm

Exercise 14c page 248

1 EF = $\frac{1}{2}$AB

2 **d** RU = SU **e** QT = RS **f** parallelogram

3 trapezium



4 trapezium

Exercise 14d page 249

1 AC = 7.4 cm, BD = 12.6 cm

2 AB = 7.8 cm, BC = 4.6 cm

3 construction

4 AB = 8.2 cm, BC = 4.8 cm

5 AB is twice as long as DC; this is a trapezium

6 **a** SR
b ∠PSR = 105°, ∠QRS = 115°; total of all four angles = 360°. This is as expected as the interior angles of a quadrilateral sum to 360°.
c 180°; as expected, since PQ and SR are parallel, ∠PQR and ∠SRQ are interior angles which sum to 180°.

7 PS = 5.4 cm, SR = 3.3 cm

8 AD = 6.0 cm, AC = 10.6 cm, BD = 13.7 cm

9 **a** isosceles
b rhombus
c trapezium

10 **a** AD = 3.6 cm, DC = 4.6 cm
b AC = 6.5 cm, BD = 7.4 cm

Exercise 14e page 250

2 they are equal

3 AC and BD

4 coincident

5 coincident

6 at the midpoint of AC

7 at the midpoint of BD

8 90°

9 each is 90°

10 3.6 cm

11 5 cm

12 6 cm

13 construction

14 4.2 cm

15 4.3 cm

Exercise 14f page 251

1 **a** BD = 8 cm
b ∠ACD = ∠CAD = 41°
c isosceles

2 **a** AD = 9.3 cm, DC = 6.2 cm
b 90°; same as ∠ABC. The interior angles of a quadrilateral sum to 360°.

3 **a** AD = 6 cm, DC = 5.2 cm
b 90°; this is expected, as the interior angles of a quadrilateral sum to 360°.
c AC = 7.9 cm, DB = 7.0 cm

4 **a** AB = BC = 2.9 m
b 120°; yes, since the interior angles of a quadrilateral sum to 360°.
c kite

5 AC = 8.8 cm

6 **a** trapezium
b ∠DCG = 100°, ∠CGF = 80°; sum is 180° as expected. CD and GF are parallel with GC as the transversal. Interior angles sum to 180° (and the interior angles of a quadrilateral sum to 360°).

REVIEW TEST 2 page 254

1 B	**4** A	**7** B	**10** D
2 C	**5** D	**8** A	**11** C
3 C	**6** B	**9** B	**12** C

13 **a** $\frac{1}{7}$ **b** 0 **c** $\frac{3}{7}$ **d** $\frac{4}{7}$

14 **a** $\frac{1}{4}$ **b** $\frac{3}{8}$ **c** 0

15 **a** (−3, −6) (−1, 2) (0, 3) (1, 2) (2, −1)
b No it is not a function as it's one to many.

16 (−2, 14) (0, 0) (1, − 1) (2, 2)

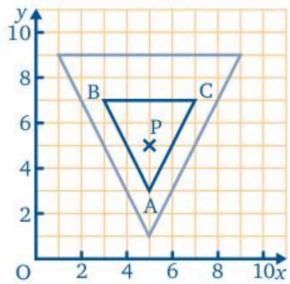
17 **a** 5 : 4 **b** 8 : 5 **c** 18 : 13 **d** 1 : 1

18 **a** 4.8 cm **b** 650 m

19 6 m and 15 m

20 **a** $64 800 **b** $324 **c** $63 180

21



22 **a** (3, 1) **b** 2

23 A′(5, 3) B′(8, 4) C′(9, 8)

24 7 cm. It's an equilateral triangle. All angles are 60° and all sides equal.

25 **a** 12 cm **b** 45°

26 AB = CD = 9.1 cm, BC = DA = 5.3 cm

CHAPTER 15

Exercise 15a page 260

1 **a** odd numbers from 3 to 13 excluding 9
b islands in the West Indies

2 **a** Geography ∈ {school subjects}
b August ∉ {days of the week}
c Ohio ∈ {American states}
d golf club ∉ {kitchen utensils}

3 **a** true **b** false **c** false **d** true

4 **a** finite **b** infinite **c** infinite **d** finite

5 **a** 11 **b** 3 **c** 5 **d** 5

6 **a** {even numbers from 2 to 18 inclusive}
b {makes of car}
c {countries in Africa}
d {branches of mathematics}

7 $n(A) = 5$, $n(B) = 2$, $n(C) = 2$

8 $A = \{11, 13, 17, 19, 23, 29\}$, $B = \{12, 18, 24, 30\}$, $C = \{18\}$

9 {1, 2, 3, 4, 5 ,6, 7, 8, 9, 10}

10 **a** {2, 4, 6, 8, 10, 12, 14, 16, 18, 20} **b** infinite

11 {−1, 0, 1, 2}

12 $A = \{(-1, -3), (0, 0), (1, 3)\}$

13 $n(A) = 8$

14 $P = \{3, 6, 9, 12\}$, $Q = \{2, 4, 6, 8, 10, 12, 14\}$, $R = \{5, 10\}$

15 $A = \{-6, -4, -2, 0, 2, 4\}$, $B = \{-6, -5, -4, -3, -2, -1\}$, $C = \{2, 3, 5\}$

Exercise 15b page 262

	$n(A)$	$n(B)$	$n(A \cup B)$	$n(A \cap B)$
1	5	7	9	3
2	4	5	7	2
3	3	6	7	2
4	6	4	8	2
	$n(X)$	$n(Y)$	$n(X \cup Y)$	$n(X \cap Y)$
5	9	7	12	4
6	5	2	7	0
7	13	10	16	7
8	12	12	20	4

	9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
$n(A)$	3	3	6	11	8	9	8	4
$n(B)$	5	1	5	13	5	5	2	5
$n(A')$	9	5	4	15	6	6	4	8
$n(B')$	7	7	5	13	9	10	10	7
$n(A \cup B)$	7	4	8	18	10	12	8	9
$n(A \cap B)$	1	0	3	6	3	2	2	0
$n(A' \cup B')$	11	8	7	20	11	13	10	12
$n[(A \cap B)']$	11	8	7	20	11	13	10	12

17 $n(X) = 2$, $n(Y) = 2$, $n(Z) = 1$, $n(X \cap Y) = 1$, $n(X \cup Z) = 3$, $n(Y \cup Z) = 3$, $n(X \cup Y \cup Z) = 4$

Exercise 15c page 265

1 **a** 3 **b** 4 **c** 12 **d** 13

2 **a** 27 **b** 14 **c** 8 **d** 19

3 **a** 11 **b** 13 **c** 19

4 **a** 41 **b** 20 **c** 29

5 23

6 **a** 19 **b** 9 **c** 23

7 3

8 **a** 8 **b** 11 **c** 23

9 **a** 32 **b** 20 **c** 17

10 **a** 28 **b** 20

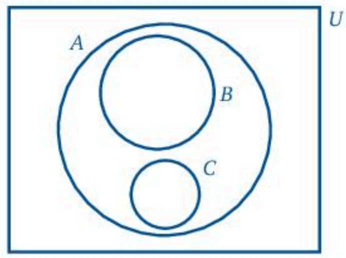
11 **a** 15 **b** 37 **c** 22

12 **a** 15 **b** 21 **c** 7

13 **a** 13 **b** 18

14 **a** $x + 8$
b $2x + 3 + x + 5 + x - 5 = 43$, i.e. $4x = 40$
c 20 **d** 10

15



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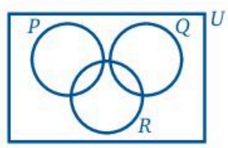
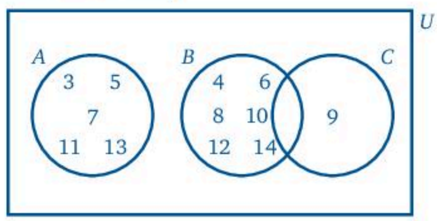


17 **a**

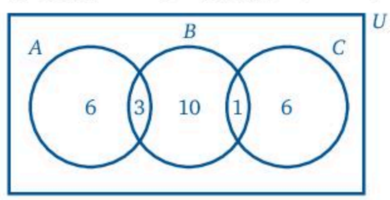


$n(A) = 5$, $n(B) = 6$, $n(C) = 1$
b none **c** $n(A \cup B) = 11$, $n(B \cup C) = 7$ **d** null set

18 **a** 7 **b** 6 **c** 0 **d** 4

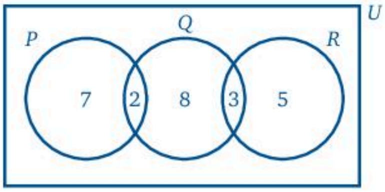
19 **a** $2x + 6$ **b** $4x + 8$; $x = 4$ **c** 7

20



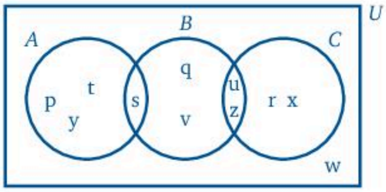
$n(A \cup B \cup C) = 26$

21



$n(P \cup Q \cup R) = 25$

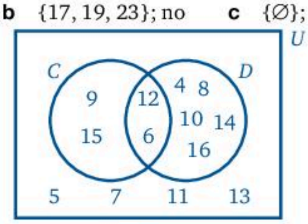
22 a



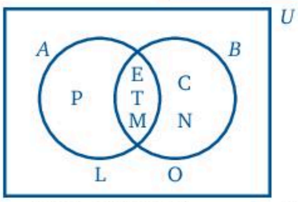
b $(A \cap B) \cup C = \{r, s, u, x, z\}$

Exercise 15d page 271

1 a Nebraska ∈ {American states}
b £ sterling ∉ {currencies used in the West Indies}
2 a false **b** true
3 a infinite **b** finite
4 a 3 **b** {17, 19, 23}; no **c** {∅}; the null set
5 $C \cap D = \{6, 12\}$



6



a {P, E, C, T, M, N} **b** {E, T, M}
7 a 5 **b** 3 **c** 6 **d** 2
8 a 17 **b** 3 **c** 0 **d** 28
9 a 3 **b** 19 **c** 6
10 a 0 **c** 7 **e** 8
b 10 **d** 7 **f** 15

Exercise 15e page 273

1 B **3** C **5** C **7** D
2 D **4** D **6** C **8** C

CHAPTER 16

Exercise 16a page 277

1 It isn't snowing
2 2 is not a positive integer
3 5 is not a prime number
4 All prime numbers are not even.
5 a I am eating and I am hungry
b I am eating or I am hungry
c I am eating and I am not hungry
d I am not eating or I am hungry
6 a All cars have four wheels and all bicycles have two wheels
b All cars have four wheels or all bicycles have two wheels
c All cars have four wheels and all bicycles do not have two wheels
d All cars do not have four wheels or all bicycles have two wheels
7 a I have a headache and I like chocolate
b I have a headache or I like chocolate
c I have a headache and I do not like chocolate
d I don't have a headache or I do like chocolate
8 a 2 is an even number and 2 is a prime number
b 2 is an even number or 2 is a prime number
c 2 is an even number and 2 is not a prime number
d 2 is not an even number or 2 is a prime number
9 a A is a vowel and B is a vowel
b A is a vowel or B is a vowel
c A is a vowel and B is not a vowel
d A is not a vowel or B is a vowel
10 True
11 True
12 True
13 False
14 False

Exercise 16b page 278

1 a False **b** True
2 a True **b** True
3 a

p	$\sim p$	q	$\sim p \vee q$
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	T

b True
c False
4 a

p	$\sim p$	q	$\sim p \wedge q$
T	F	T	F
T	F	F	F
F	T	T	T
F	T	F	F

b False
c False
5 a

p	q	$\sim q$	$p \wedge \sim q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

b False
c True
6 a

p	$\sim p$	q	$\sim q$	$\sim p \wedge \sim q$
T	F	T	F	F
T	F	F	T	F
F	T	T	F	F
F	T	F	T	T

b

p	$\sim p$	q	$\sim q$	$\sim p \wedge \sim q$
T	F	T	F	F
T	F	F	T	T
F	T	T	F	T
F	T	F	T	T

Exercise 16c page 281

1 A, B, C are equivalent
2 A, C
3 A, B, C, D
4 A, B, C
5 A, C
6 A, B, C

Exercise 16d page 283

1 Valid
2 Not valid
3 Valid
4 Valid and sound
5 Valid and sound
6 Valid and sound
7 Valid and sound
8 Valid
9 Valid and sound
10 Valid and sound
11 Valid and sound
12 Valid

CHAPTER 17

Exercise 17a page 287

1 2×2
2 2×3
3 2×1
4 1×3
5 1×1
6 3×2
7 **a** 6 **b** 8 **c** 2 **d** 7
8 **a** 3 1 7; 4 7 2 **b** 7 **c** 6 **d** 4

9 $\begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{pmatrix}$

10 $\begin{pmatrix} 3 & 1 \\ 3 & 1 \\ 3 & 1 \end{pmatrix}$

Exercise 17b page 289

1 $\begin{pmatrix} 12 \\ 15 \end{pmatrix}$

2 $\begin{pmatrix} 15 & 4 \\ 7 & 1 \end{pmatrix}$

3 not possible

4 $(9, 5)$

5 $\begin{pmatrix} 11 & 2 & 2 \\ 6 & 7 & 7 \end{pmatrix}$

6 $\begin{pmatrix} 11 & 11 \\ 11 & 5 \end{pmatrix}$

7 $(5 \;\; 3 \;\; 5)$

8 not possible

9 $\begin{pmatrix} 6 & 8 \\ 7 & 7 \\ 7 & 7 \end{pmatrix}$

10 $(10 \;\; 8)$

11 $\begin{pmatrix} 1 & 8 \\ -4 & 7 \end{pmatrix}$

12 $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$

13 $\begin{pmatrix} -2 & 7 \\ -5 & 3 \end{pmatrix}$

14 $(4 \;\; 6)$

15 $\begin{pmatrix} -3 \\ -3 \\ 6 \end{pmatrix}$

16 not possible

17 $\begin{pmatrix} 2 & 10 \\ 5 & -3 \end{pmatrix}$

18 $\begin{pmatrix} 5 & -5 \\ 3 & 0 \end{pmatrix}$

19 $\begin{pmatrix} 0 & 8 \\ 8 & -2 \end{pmatrix}$

20 not possible

21 not possible

22 $(1 \;\; 6 \;\; -3)$

23 $\begin{pmatrix} 2 & 3 & 4 \\ 5 & 0 & -12 \end{pmatrix}$

Exercise 17c page 291

1 $\begin{pmatrix} 3 \\ 6 \\ 12 \end{pmatrix}$

2 $\begin{pmatrix} 2 & 8 & 0 \\ 4 & -2 & 6 \end{pmatrix}$

3 $\begin{pmatrix} 1 & 2 \\ \frac{1}{2} & 3 \\ 1\frac{1}{2} & 4 \end{pmatrix}$

4 $\begin{pmatrix} 6 & 24 \\ 18 & -12 \end{pmatrix}$

5 $\begin{pmatrix} -6 & -30 \\ 6 & 12 \end{pmatrix}$

6 $\begin{pmatrix} 4 & 0 \\ \frac{2}{3} & 1\frac{1}{3} \\ 2 & 3\frac{1}{3} \end{pmatrix}$

7 $\begin{pmatrix} 2 & -2 \\ 1 & 3 \end{pmatrix}$

8 not possible

9 $\begin{pmatrix} -3 \\ 3 \\ 0 \end{pmatrix}$

10 $\begin{pmatrix} 2 & 4 & 2 \\ -3 & -3 & -1 \end{pmatrix}$

11 not possible

12 $\begin{pmatrix} -3 & -1 & 2 \\ 9 & 5 & 4 \\ 1 & 11 & 5 \end{pmatrix}$

Exercise 17d page 292

1 $\begin{pmatrix} -1 & 8 \\ 6 & 1 \end{pmatrix}$

2 $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

3 not possible

4 $\begin{pmatrix} 7 & -1 \\ 5 & -1 \end{pmatrix}$

5 $\begin{pmatrix} 1\frac{1}{3} \\ 1\frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}$

6 $\begin{pmatrix} -3 \\ -3 \\ -3 \end{pmatrix}$

7 $\begin{pmatrix} 2 & 8 \\ 6 & 4 \end{pmatrix}$

8 $\begin{pmatrix} -1 & 2 \\ 1\frac{1}{2} & -\frac{1}{2} \end{pmatrix}$

9 $\begin{pmatrix} 8 \\ 9 \\ 3 \end{pmatrix}$

10 $\begin{pmatrix} 24 & 8 & -4 \\ 16 & 12 & 16 \end{pmatrix}$

11 not possible

12 $\begin{pmatrix} 8 & -2 & -2 \\ 1 & 4 & 4 \end{pmatrix}$

Exercise 17e page 294

1 B 2×1, C 2×2, D 2×2, E 1×3, F 1×2, G 2×3

2 $\begin{pmatrix} 9 & 4 & 4 \\ 7 & 1 & 7 \end{pmatrix}$

3 not possible

4 $\begin{pmatrix} 4 & -1 \\ 0 & 6 \end{pmatrix}$

5 $\begin{pmatrix} 12 & 9 & 3 \\ 3 & 6 & 9 \end{pmatrix}$

6 not possible

7 $\begin{pmatrix} 1\frac{1}{2} & 1 \end{pmatrix}$

8 not possible

9 $\begin{pmatrix} 1 & -2 & 2 \\ 5 & -3 & 1 \end{pmatrix}$

10 $\begin{pmatrix} 24 \\ 6 \end{pmatrix}$

11 $\begin{pmatrix} 4\frac{1}{2} & 1\frac{1}{2} \\ \frac{3}{4} & 3 \end{pmatrix}$

12 not possible

13 not possible

Exercise 17f page 295

1 $\begin{pmatrix} -5 & -2 & 10 \end{pmatrix}$

2 $\begin{pmatrix} 18 & 18 & -6 \end{pmatrix}$

3 $\begin{pmatrix} -29 \\ -8 \\ 34 \end{pmatrix}$

4 $\begin{pmatrix} 7 & 0 \\ 3 & 14 \end{pmatrix}$

5 $\begin{pmatrix} 28 & 12 & -16 \end{pmatrix}$

6 $\begin{pmatrix} 11 & 8 & -20 \\ -4 & -9 & -1 \end{pmatrix}$

7 not possible

8 $\begin{pmatrix} -3 & 0 \\ 9 & 42 \end{pmatrix}$

9 $\begin{pmatrix} 12 & 0 & 16 \\ 16 & 12 & -20 \end{pmatrix}$

10 $\begin{pmatrix} -80 & 45 \\ -90 & -160 \end{pmatrix}$

Exercise 17g page 296

1 2×2, 2×1

2 not possible

3 $\begin{pmatrix} 9 & -3 \\ 12 & -9 \end{pmatrix}$

4 $\begin{pmatrix} 13 & 3 \\ 6 & 5 \end{pmatrix}$

5 4

6 $\begin{pmatrix} 4 & 2 & -2 \\ 8 & 6 & 2 \end{pmatrix}$

7 not possible

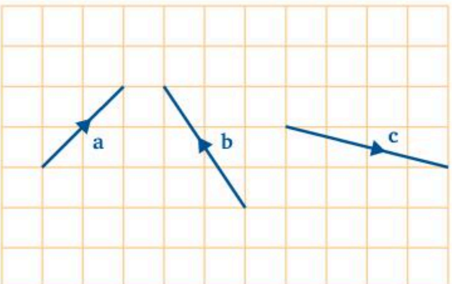
8 3

9 1

CHAPTER 18

Exercise 18a page 299

1



2 a 5 b 13 c 2.24 d 5.39

3 a $\mathbf{a} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$, $\mathbf{c} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$, $\mathbf{d} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$,

$\mathbf{e} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$, $\mathbf{f} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$, $\mathbf{g} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$

b 2.83, 3.16, 3.61, 3.16, 2.24, 2.24, 5.39

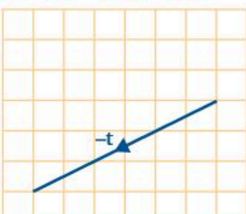
4 a 3, 4.47, 4.47, 4.47, 4, 5

b **b** and **d**

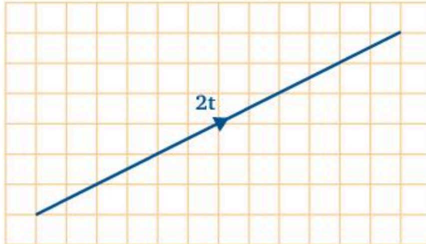
c **b** and **c** (or **d** and **c**)

Exercise 18b page 302

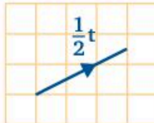
1 a



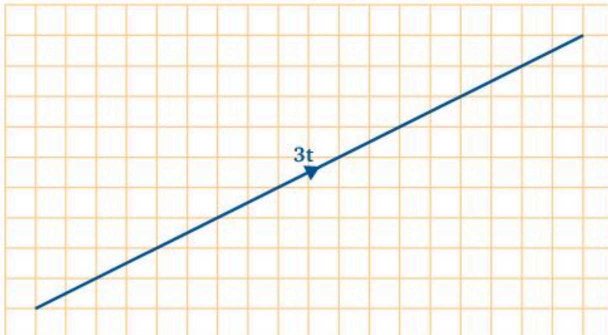
b



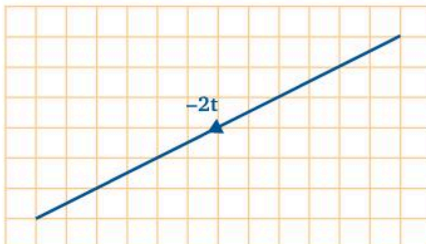
c



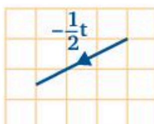
d



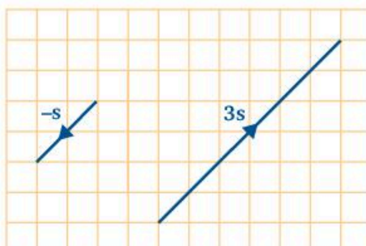
e



f

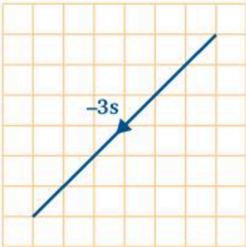


2 a–b

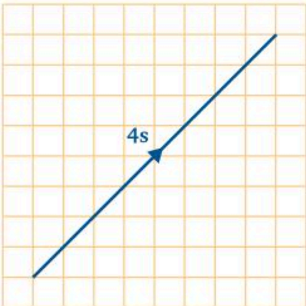


c–d

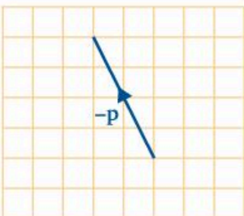
e



f



3 a



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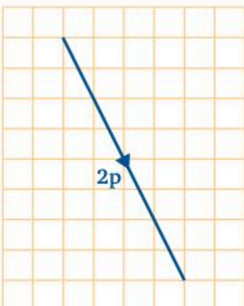
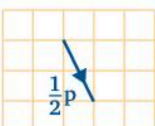
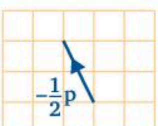


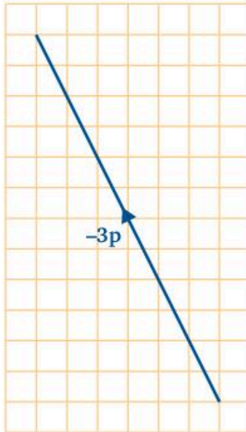
c



d



e



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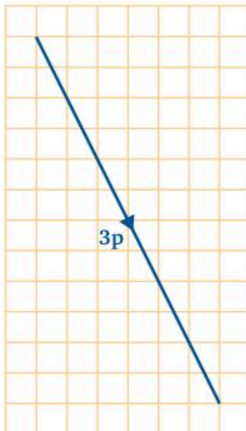
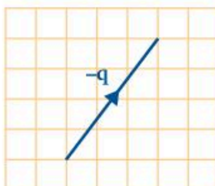
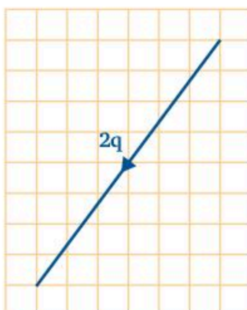


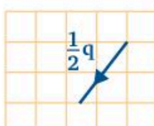
4 a



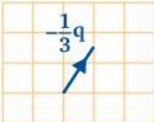
b



c



d



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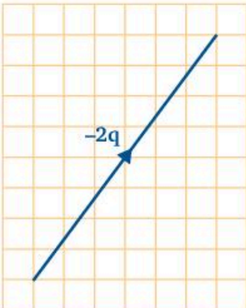

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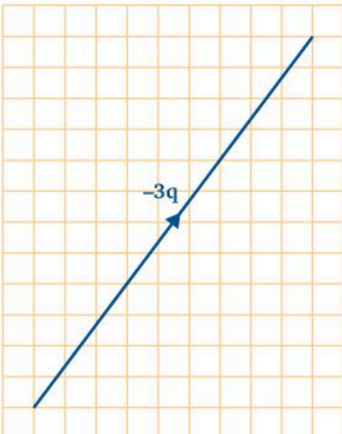


5 **a** **c** and **d**
b **e** = 2**a**
c $\mathbf{g} = \frac{1}{2}\mathbf{f}$

6 **a** $\mathbf{b} = \begin{pmatrix}3\\6\end{pmatrix}$ **b** $\mathbf{c} = \begin{pmatrix}18\\36\end{pmatrix}$

Exercise 18c page 304

1 **a** $\overrightarrow{OA} = \begin{pmatrix}2\\3\end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix}4\\5\end{pmatrix}$ **b** $\overrightarrow{BA} = \begin{pmatrix}-2\\-2\end{pmatrix}$

2 **a** $\overrightarrow{OA} = \begin{pmatrix}-1\\5\end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix}3\\-6\end{pmatrix}$ **b** $\overrightarrow{BA} = \begin{pmatrix}-4\\11\end{pmatrix}$

3 **a** $\overrightarrow{OA} = \begin{pmatrix}1\\4\end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix}3\\-1\end{pmatrix}, \overrightarrow{OC} = \begin{pmatrix}-4\\6\end{pmatrix}$

b $\overrightarrow{BA} = \begin{pmatrix}-2\\5\end{pmatrix}, \overrightarrow{BC} = \begin{pmatrix}-7\\7\end{pmatrix}$

4 **a** A(2, 2), B(6, 5) **b** $\overrightarrow{AB} = \begin{pmatrix}4\\3\end{pmatrix}$

5 **a** A(−3, −6), B(6, −1) **b** $\overrightarrow{AB} = \begin{pmatrix}9\\5\end{pmatrix}$

6 **a** $\overrightarrow{AB} = \begin{pmatrix}1\\4\end{pmatrix}$

$\overrightarrow{DC} = \begin{pmatrix}1\\4\end{pmatrix}$

b The two vectors are parallel as they are in the same direction and one is a positive scalar multiple of the other. The multiple is 1 so they are also equal.
c ABCD is a parallelogram.

7 **a** $\overrightarrow{AB} = \begin{pmatrix}2\\8\end{pmatrix}$

$\overrightarrow{BC} = \begin{pmatrix}3\\-5\end{pmatrix}$

$\overrightarrow{CA} = \begin{pmatrix}-5\\-3\end{pmatrix}$

b $AB^2 = 68$, $BC^2 = 34$, $CA^2 = 34$, so $AB^2 = BC^2 + CA^2$.
By Pythagoras' theorem, ABC is a right-angled triangle

8 **a** $\overrightarrow{AB} = \begin{pmatrix}3\\-1\end{pmatrix}$

$\overrightarrow{DC} = \begin{pmatrix}6\\-2\end{pmatrix}$

b No, the lengths are not the same
c Yes; they are parallel – same direction and one is a positive scalar multiple of the other
d a trapezium

9 **a** $\overrightarrow{AB} = \begin{pmatrix}3\\4\end{pmatrix}$

$\overrightarrow{BC} = \begin{pmatrix}3\\-4\end{pmatrix}$

$\overrightarrow{CD} = \begin{pmatrix}-3\\-4\end{pmatrix}$

$\overrightarrow{DA} = \begin{pmatrix}-3\\4\end{pmatrix}$

b Yes; the four sides are the same length and each pair of opposite sides are parallel.

10 **a** $\overrightarrow{AB} = \begin{pmatrix}4\\-2\end{pmatrix}$

b $\overrightarrow{CD} = 2\,\overrightarrow{AB} = \begin{pmatrix}8\\-4\end{pmatrix}$

c (12, –6)

11 $\overrightarrow{CD} = \begin{pmatrix}5\\8\end{pmatrix}$

12 **a** $\overrightarrow{AB} = \begin{pmatrix}-1\\-3\end{pmatrix}$

b (4, –3)

13 (2, –2)

14 **a** $\overrightarrow{AC} = \begin{pmatrix}6\\0\end{pmatrix}$

$\overrightarrow{AE} = \begin{pmatrix}3\\0\end{pmatrix}$

b $\overrightarrow{BE} = \begin{pmatrix}0\\-4\end{pmatrix}$

15 $\overrightarrow{AB} = \begin{pmatrix}50\\88\end{pmatrix}$, distance AB = 101 m

16 32 m

Exercise 18d page 309

1 a $\overrightarrow{AE}=\begin{pmatrix}6\\4\end{pmatrix}$ and $\overrightarrow{EB}=\begin{pmatrix}3\\2\end{pmatrix}$ so $\overrightarrow{AE}=2\overrightarrow{EB}$. Therefore $\overrightarrow{AE}$ and $\overrightarrow{EB}$ are parallel. E is common to both $\overrightarrow{AE}$ and $\overrightarrow{EB}$ so A, E and B are collinear.

b 2 : 1

2 a $\overrightarrow{AB}=\begin{pmatrix}9\\3\end{pmatrix}$ and $\overrightarrow{BC}=\begin{pmatrix}3\\1\end{pmatrix}$ so $\overrightarrow{AB}=3\overrightarrow{BC}$

Therefore $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel. B is common to both $\overrightarrow{AB}$ and $\overrightarrow{BC}$ so A, B and C are collinear.

b $\overrightarrow{AC}=\begin{pmatrix}12\\4\end{pmatrix}$ and $\overrightarrow{BC}=\begin{pmatrix}3\\1\end{pmatrix}$ so $\overrightarrow{AC}=4\overrightarrow{BC}$.

Ratio AC : BC = 4 : 1.

3 a $\overrightarrow{PQ}=\begin{pmatrix}4\\-6\end{pmatrix}$ and $\overrightarrow{QR}=\begin{pmatrix}2\\-3\end{pmatrix}$ so $\overrightarrow{PQ}=2\overrightarrow{QR}$

Therefore $\overrightarrow{PQ}$ and $\overrightarrow{QR}$ are parallel. Q is common to both $\overrightarrow{PQ}$ and $\overrightarrow{QR}$ so P, Q and R are collinear.

b 2 : 1

4 $\overrightarrow{OP}=\begin{pmatrix}-2\\1\end{pmatrix}$ **6** $\overrightarrow{OP}=\begin{pmatrix}-4\\4\end{pmatrix}$ **8** $\overrightarrow{OP}=\begin{pmatrix}0\\2\end{pmatrix}$

5 $\overrightarrow{OP}=\begin{pmatrix}-2\\1\end{pmatrix}$ **7** $\overrightarrow{OP}=\begin{pmatrix}-2\\-1\end{pmatrix}$

Exercise 18e page 310

1 a 13 **b** 5 **c** 6.08

2 a The magnitudes are:
a 4.47
b 4.24
c 4.12
d 6.08

b Expressed in the form $\begin{pmatrix}a\\b\end{pmatrix}$

$\mathbf{a}=\begin{pmatrix}2\\4\end{pmatrix}$

$\mathbf{b}=\begin{pmatrix}3\\3\end{pmatrix}$

$\mathbf{c}=\begin{pmatrix}-1\\4\end{pmatrix}$

$\mathbf{d}=\begin{pmatrix}6\\-1\end{pmatrix}$

3 a $-\mathbf{a}=\begin{pmatrix}-6\\1\end{pmatrix}$ **c** $\frac{1}{2}\mathbf{a}=\begin{pmatrix}3\\-0.5\end{pmatrix}$

b $3\mathbf{a}=\begin{pmatrix}18\\-3\end{pmatrix}$ **d** $-\frac{1}{2}\mathbf{a}=\begin{pmatrix}-3\\0.5\end{pmatrix}$

4 a $\overrightarrow{OA}=\begin{pmatrix}2\\-5\end{pmatrix}$ and $\overrightarrow{OB}=\begin{pmatrix}4\\-8\end{pmatrix}$

b $\overrightarrow{BA}=\begin{pmatrix}-2\\3\end{pmatrix}$

5 a A(–1, –2), B (3, 6)

b $\overrightarrow{AB}=\begin{pmatrix}4\\8\end{pmatrix}$

6 $\overrightarrow{OD}=\begin{pmatrix}-2\\-3\end{pmatrix}$

7 a $\overrightarrow{AB}=\begin{pmatrix}6\\2\end{pmatrix}$ and $\overrightarrow{CB}=\begin{pmatrix}2\\6\end{pmatrix}$

b The length of AB is $\sqrt{6^2+2^2}=\sqrt{40}$
The length of CB is $\sqrt{2^2+6^2}=\sqrt{40}$
AB = CB, therefore triangle ABC is isosceles

8 a $\overrightarrow{AC}=\begin{pmatrix}2\\6\end{pmatrix}$ and $\overrightarrow{CB}=\begin{pmatrix}1\\3\end{pmatrix}$ so $\overrightarrow{AC}=2\overrightarrow{CB}$

Therefore $\overrightarrow{AC}$ and $\overrightarrow{CB}$ are parallel. C is common to both $\overrightarrow{AC}$ and $\overrightarrow{CB}$ so A, B and C are collinear

b AC : CB = 2 : 1

Exercise 18f page 312

1 B	**3** B	**5** D	**7** C
2 C	**4** D	**6** A	**8** A

CHAPTER 19

Exercise 19a page 317

1 The graph passes through the origin O, which is also the lowest value for y.

2 a 1.73 or –1.73 **b** No

3 a 2 and –2 **b** 1 and –1, Yes

4 They all have the same shape.
They all have the same shape but cross the y-axis at different points.

5 a When $x=0$ and $x=3$
b –0.79 and 3.79

6 a When $x=0$ and 1.5
b $-1\frac{1}{8}$ when $x=\frac{3}{4}$

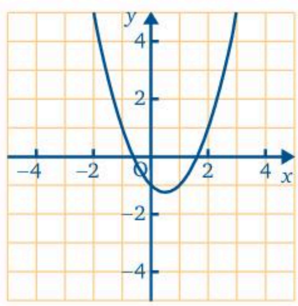
7 a –2 when $x=-1$ **c** 0 and –2
b 10.5

8 a –4 when $x=1$
b i –1.24 and 3.24 **ii** –2.46 and 4.46

9 a 6.25 when $x=0.5$
b i –2.37 and 3.37 **ii** –1 and 2

Exercise 19b page 320

1



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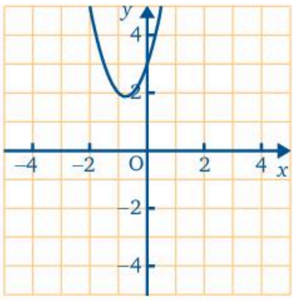

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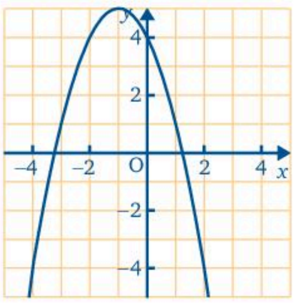

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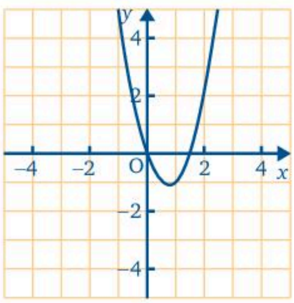
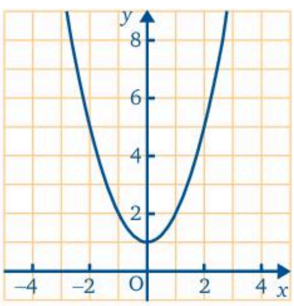
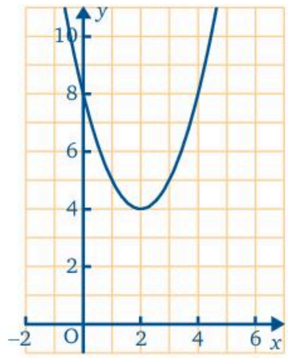


5 −0.6, 1.6
6 −2.2, 0.7
7 −3.2, 1.2
8 −0.9, 2.4
9


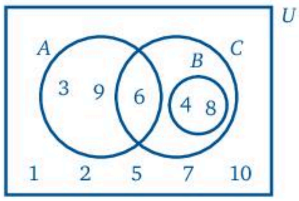
It does not cross the axis; there is no solution

10


It does not cross the axis; there is no solution

REVIEW TEST 3 page 322

1 D **4** B **7** C **10** B **13** A
2 C **5** C **8** D **11** A
3 A **6** C **9** B **12** B

14 a


b {4, 6, 8}

15 a 5 **b** 16 **c** 10 **d** 0

16 the argument is sound

17

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

18 $\begin{pmatrix} 6 & -7 & -1 \\ 1 & 6 & 2 \end{pmatrix}$

19 $\begin{pmatrix} 8 & -8 & 0 \\ 4 & 0 & 12 \end{pmatrix}$

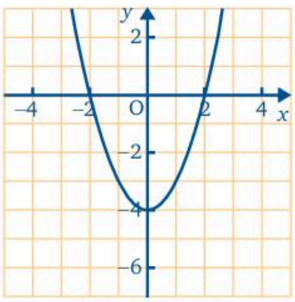
20 $\begin{pmatrix} -8 & 11 & 3 \\ 2 & -18 & 9 \end{pmatrix}$

21 a e **b** g **c** f **d** a, e

22 a $\overrightarrow{AB} = \begin{pmatrix} 6 \\ 6 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$ so $\overrightarrow{AB} = 2\overrightarrow{BC}$. Therefore $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel. B is common to both $\overrightarrow{AB}$ and $\overrightarrow{BC}$ so A, B and C are collinear.

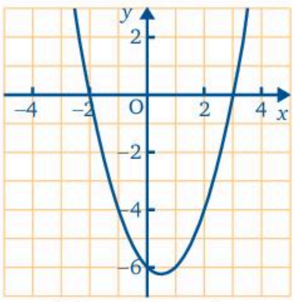
b 2 : 1

23



a $x = \pm 2.2$ **b** $x = \pm 2$

24



a −6.25, $x = 0.5$ **b** $x = -2.2, 3.2$

REVIEW TEST 4 page 326

1 B **2** B **3** C **4** B **5** C **6** C **7** D **8** B **9** C **10** B **11** D

12 a x^6 **b** x^{-1} **c** $8a^6$

13 a 54.4 **b** 198 cm²

14 44.8° **15** 15.6 cm **16** 10.9 cm **17** 5.48 cm

18 a 64° **b** 25° **c** 56°

19 18.9 cm

20 a $\frac{2y-1}{3x}$ **b** $\frac{1}{3x}$ **c** $\frac{5b}{1+b}$

21 a x^2y^2 **b** a^2b^2

22 a $\frac{10y-7}{12xy}$ **b** $\frac{x+8}{30}$

23 $x = 3$

24 a $x^2 - 3xy = 2y^2$
b $-6x^2 + 5x - 1$
c $25a^2 - b^2 - 10ab$

25 a $a^2(a-2)$ **b** $(a-b)(a+2)$

26 a $y = 3,\ x = 1$ **b** $y = 3,\ x = -2$

27 3 and 5

28 a {O, S} **b** {O, L, N} **c** {S, E, A, L, N} **d** {O} **e** {O} **f** {O}

29 a 15 **b** 22 **c** 12

30 a (3, 2) (4, 2) (4, 3)
b Domain (3, 4)
Range (2, 3)

31 (0, −2) (2, 2) (3, 7)
Yes; it is a function as it is 1 : 1

32 a $\frac{1}{2}$ **b** 0 **c** $\frac{1}{2}$ **d** $\frac{1}{2}$

33 a $\frac{1}{7}$ **b** $\frac{2}{21}$

34 a $\frac{1}{13}$ **b** $\frac{1}{2}$ **c** $\frac{1}{4}$

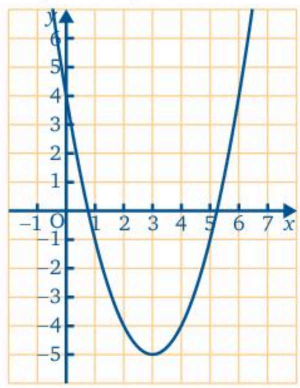
35 a $\begin{pmatrix} 0 & 4 \\ 9 & 5 \end{pmatrix}$

b $\begin{pmatrix} 1 & 7 \\ -2 & 3 \end{pmatrix}$

36 a $\begin{pmatrix} -4 & 12 & 2 \\ 5 & 1 & 5 \\ 0 & 2 & -3 \end{pmatrix}$

b $\begin{pmatrix} -15 & 20 \\ 10 & 5 \end{pmatrix}$

37



$x = 5.2$ or $x = 0.8$

38 25

39 a $\frac{1}{2}$ **b** (6, −5)

40 $y = 1 - x$

Index

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3

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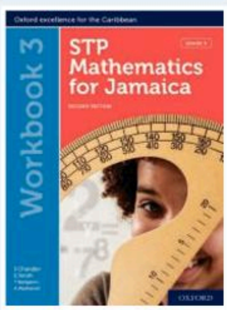

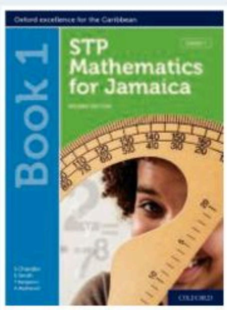

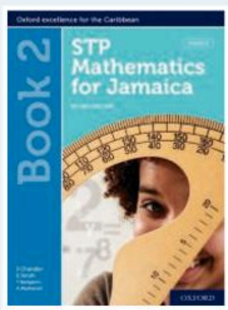

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