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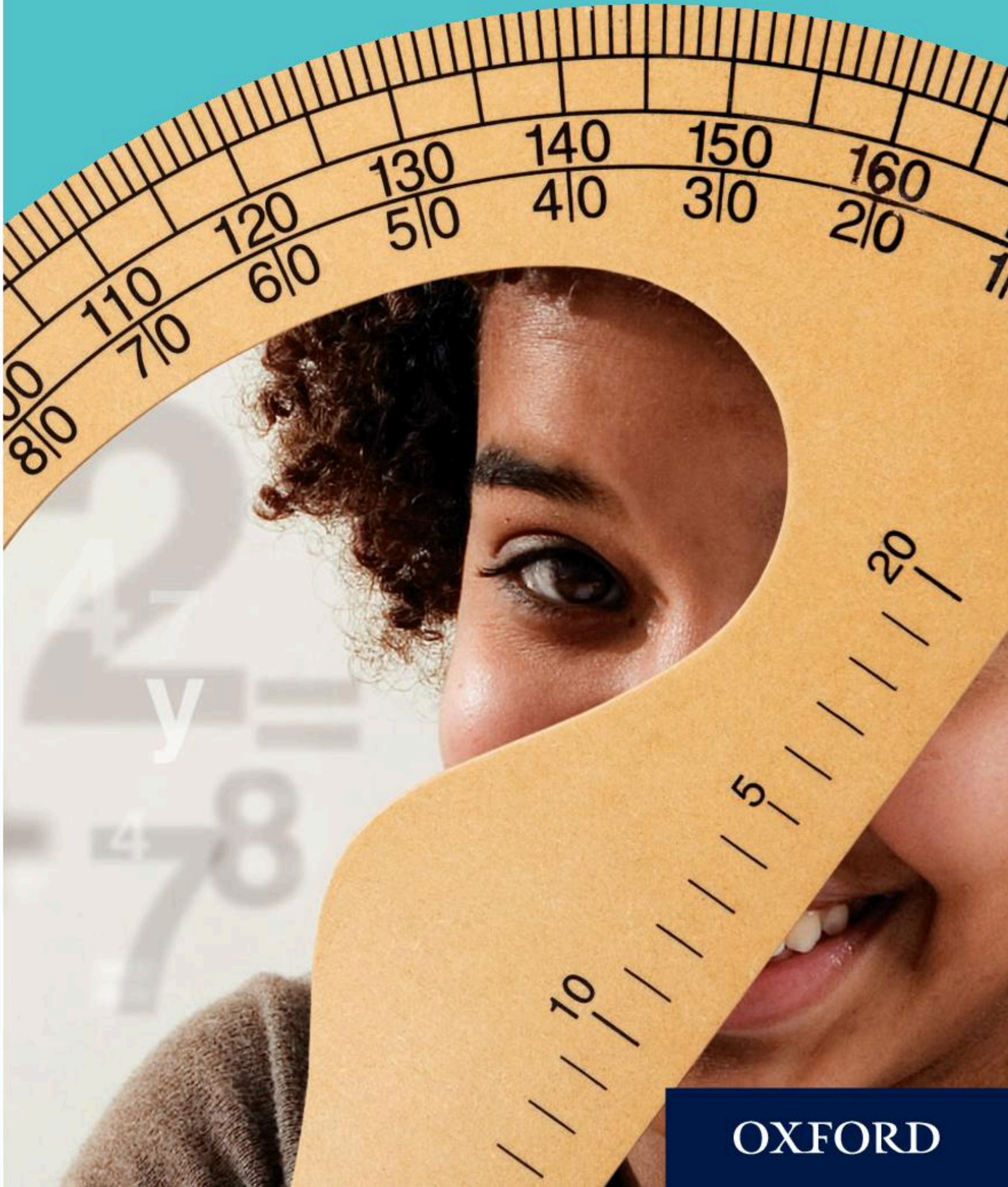
Book 2

STP Mathematics for Jamaica

GRADE 8

SECOND EDITION



S Chandler
E Smith
T Benjamin
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Introduction

To the student

This new edition of *STP Mathematics for Jamaica Student Book 2* attempts to meet your needs as you begin your study of Mathematics at the secondary school level. Your learning experiences at this stage lay the foundation for future achievement in CSEC Mathematics and beyond. We are very conscious of your need for success and enjoyment in doing Mathematics, which comes from solving problems correctly. With this in mind, we have divided most of the exercises into three types of question:

Type 1 questions

These are identified by numbers written in bold print, e.g. **12**. They help you to see if you understand the topic being discussed and should be attempted in every chapter you study.

Type 2 questions

These are identified by a single underline under the bold print, e.g. $\underline{\mathbf{12}}$. They are extra questions for you to do and are not more difficult. They should be attempted if you need extra practice or want to do revision at a later time.

Type 3 questions

These are identified by a double underline under the bold print, e.g. $\underline{\underline{\mathbf{12}}}$. They are for those of you who completed Type 1 questions fairly easily and want to attempt questions that are more challenging.

Multiple choice questions

Multiple choice questions are included in the book to help you become more familiar with the format of your assessments at CSEC.

Mixed exercises

Most chapters end with Mixed exercises to help you advance your critical thinking, problem-solving and computational skills. These exercises will also help you revise what you have done, either when you have finished the chapter or as you prepare for examinations.

Use of calculator

You should be able to use a calculator accurately before you leave school. We suggest that you use a calculator mainly to check your answers. Whether you use a calculator or do the computations yourself, always estimate your answer first and always ask the question, ‘Does my answer make sense?’

Suggestions for use of student book

- Break up the material in a chapter into manageable parts.
- Have paper and a pencil with you always when you are studying mathematics.
- Write down and look up the meaning of all new vocabulary you encounter.
- Read all questions carefully and rephrase them in your own words.
- Remember that each question contains all the information you need to solve the problem. Do not look only at the numbers that are given.
- Practise your mathematics. This will ensure your success!

You are therefore advised to try to solve as many problems as you can. Above all, don't be afraid to make mistakes as you are learning. The greatest mathematicians all made many mistakes as they tried to solve problems.

You are now on your way to success in mathematics – GOOD LUCK!

To the teacher

In writing this series, the authors attempted to present the topics in such a way that students will understand the connections among topics in mathematics, and be encouraged to see and use mathematics as a means to make sense of the real world. The exercises have been carefully graded to make the content more accessible to students.

This new edition is designed to:

1. Assist you in helping students to
    - attain important mathematical skills
    - connect mathematics to their everyday lives and understand its role in the development of our contemporary society
    - see the importance of critical thinking skills in everyday problems
    - discover the fun of doing mathematics both individually and collaboratively
    - develop a positive attitude towards doing mathematics.
2. Encourage you to include historical information about mathematics in your teaching.

    Topics from the history of mathematics have been incorporated to ensure that mathematics is not dissociated from its past. This should lead to an increase in the level of enthusiasm, interest and fascination among students, thus enriching the teaching and learning experiences in the mathematics lessons.

Investigations

'Investigation' is included in this revised STP Mathematics for Jamaica series. This is in keeping with the requirements of the latest CSEC syllabus.

Investigations are used to provide students with the opportunity to explore hands-on and minds-on mathematics. At the same time, teachers are presented with open-ended explorations to enhance their mathematical instruction.

It is expected that the tasks will

- encourage problem solving and reasoning
- develop communication skills and the ability to work collaboratively
- connect various mathematical concepts and theories.

Suggestions

1 At the start of each lesson, give a brief outline of the topic to be covered in the lesson. As examples are given, refer back to the outline to show how the example fits into it.

2 List terms that you consider new to the students and solicit additional words from them. Encourage students to read from the text and make their own vocabulary list. Remember that mathematics is a foreign language. The ability to communicate mathematically must involve the careful use of the correct terminology.

3 Have students construct different ways to phrase questions. This helps students to see mathematics as a language. Students, especially in the junior classes, tend to concentrate on the numerical or 'maths' part of the question and pay little attention to the information that is required to solve the problem.

4 When solving problems, have students identify their own problem-solving strategies and listen to the strategies of others. This practice should create an atmosphere of discussion in the class centred on different approaches to solving the same problem.

As the students try to solve problems on their own they will make mistakes. This is expected, as this was the experience of the inventors of mathematics: they tried, guessed, made many mistakes and worked for hours, days and sometimes years before reaching a solution.

There are enough problems in the exercises to allow the students to try and try again. The excitement, disappointment and struggle with a problem until a solution is found will create rewarding mathematical experiences.

1 Working with numbers

At the end of this chapter you should be able to...

1. understand the laws and properties of numbers
2. multiply numbers written in index form
3. divide numbers written in index form
4. find the value of numbers written in index form
5. write numbers in standard form
6. write numbers correct to a given number of significant figures
7. approximate numbers to a given degree of accuracy.

Did you know?

The googol (10^{100}) is said to have been so named by Milton Sirotta, the 9-year-old nephew of the American mathematician Edward Kasner.

You need to know...

- ✓ the meaning of place value in numbers
- ✓ how to work with fractions, decimals and percentages
- ✓ the meaning of integers, rational numbers and real numbers.

Key words

approximation, closure, decimal place, identity element, index (plural indices), integer, inverse element, reflexive, rough estimate, scientific notation, significant figures, standard form, symmetry, transitive, trichotomy

Properties of numbers

When we use real numbers, we rely on the following properties.

- The *reflexive* property states that any number is always equal to itself. For example, $2 = 2$, or to put it another way, 2 is always 2.
- The *symmetry* property states that for any two numbers, a and b, if $a = b$ then $b = a$. For example, if $2 = x$ then $x = 2$.
- The *transitive* property says that if $a = b$ and $b = c$, then $a = c$. For example if $a = 6$ and $6 = c$, then $a = c$.
- The *trichotomy* property of any two real numbers a and b states that either a is greater than b or $a = b$ or a is less than b.

We saw in Grade 7 that the *identity element* does not change the value of a number when it is added to or subtracted from that number. For example, $1.5 + 0 = 1.5$ and $3 - 0 = 0$. Zero is the identity element when added to or subtracted from any number.

One is the identity element for multiplication and division. For example, $2.6 \times 1 = 2.6$ and $\frac{1}{2} \div 1 = \frac{1}{2}$

We also saw that combining a number with its *inverse element* under addition or multiplication results in the identity element for that operation. For example, -8 is the inverse of 8 under addition as $8 - 8 = 0$ and $\frac{1}{8}$ is the inverse of 8 under multiplication as $8 \times \frac{1}{8} = 1$.

Closure

When we add, subtract, multiply or divide two numbers from a set of numbers, we may or may not get another number in the set.

For example, if you start with a number from the set $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$ and perform any of the operations +, – or × the result is still a member of the set. However, if you divide one member of the set by another you may get a number that is not in the set, e.g. $17 \div 8$.

We say that the numbers in the set $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$ are closed under +, – and × but not under ÷.

For any set, *closure* under an operation is when performing that operation on members of the set always gives a member of the same set.

Exercise 1a

1 Which property do the following statements illustrate?

a $-1 = x$ so $x = -1$ **b** $a = b$ and $b = c$, so $a = c$.

2 a, b and c are integers. Which of the following statements are always true?

A $a + b = b + a$

B $a - b = b - a$

C $a + b - c = a - c + b$

D $a \times b$ is an integer

E $a \div b$ is an integer

F $a + b \times c = a \times c + b$

G $a \div b \times c = a \times c \div b$

3 Which part of question **2** shows the symmetry property?

4 a and b are any two rational numbers, i.e. fractions.

Is the set of fractions closed under multiplication and division?

Give a reason for your answer.

5 a is an odd number, b is an even number, and $a + b = c$.

Which of the following statements are true?

c is

A the identity element under addition

B the inverse of b under addition

C an odd number

D an even number.

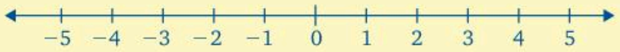
6 a and b are integers. The operation $a{*}b$ means $\frac{a+b}{a}$.

a Find 2*4.

b Is the set of integers closed under this operation?

Working with integers

Integers are all the positive and negative whole numbers including zero.
The number line is a useful tool in representing integers.



The list below summarises the rules for working with integers that you met in Grade 7.

The rules for adding and subtracting integers are:

- adding a positive number and subtracting a negative number is equal to adding the number (e.g. 2 + (+3) = 2 + 3 and 2 – (–3) = 2 + 3)
- adding a negative number and subtracting a positive number is equal to subtracting the number (e.g. 5 + (–2) = 5 – 2 and 5 – (+2) = 5 – 2)

The rules for multiplying integers are:

- when two positive numbers are multiplied the answer is positive (e.g. $3 \times 4 = 12$)
- when two negative numbers are multiplied the answer is positive (e.g. $-3 \times -4 = 12$)
- when a positive number and a negative number are multiplied the answer is negative (e.g. $-3 \times 4 = -12$ and $3 \times -4 = -12$).

The rules for dividing integers are:

- when a negative number is divided by a positive number and when a positive number is divided by a negative number the answer is negative (e.g. $-12 \div 3 = -4$ and $12 \div -3 = -4$)
- when a negative number is divided by a negative number the answer is positive (e.g. $-12 \div -3 = 4$).

The rules for a calculation that involves a mixture of brackets, multiplication, division, addition and subtraction are:

- work out calculations inside brackets first
- then do multiplication and division
- finally do addition and subtraction.

For example, $10 + 3 \times (6 - 8) = 10 + 3 \times -2 = 10 - 6 = 4$

The mnemonic **B**less **M**y **D**ear **A**unt **S**ally may help you remember this order.

Puzzle

Everton Giles stands on the middle rung of a ladder. He climbs 3 rungs higher but has forgotten something so descends 7 rungs to get it. He now goes up 16 rungs and reaches the top of the ladder. How many rungs are there to the ladder?

Exercise 1b

Calculate:

1 $2 \times (-3)$

2 $2 - (-4)$

3 $(-50) \div (-10)$

4 $(-6) \div (-12)$

5 $4(5+1) \times (-3)(3 - 2)$

6 $8 - 2(15 - 12)$

7 $12 \times 4 - 3(6 - 18)$

8 $2 - 4 \div 3(6 - 7)$

9 $\frac{2-8}{3-5}$

10 $\frac{2 \times (16-5)}{(12+10) \times (-3)}$

11 $(-8)(5 - 10) \div 2(4 - 16)$

12 $3 \times 3 + 16 \times 6 - 2 \div (-8)$

Changing between fractions, decimals and percentages

- **To change a fraction to a decimal, divide the numerator by the denominator.**

  The fraction $\frac{3}{8}$ means $3 \div 8$.

  You can calculate $3 \div 8$: $\begin{array}{r} 0.375 \\ 8\overline{)3.000} \end{array}$

  So $\frac{3}{8} = 0.375$

  Any fraction can be treated like this.

- **To change a decimal to a fraction, express it as a number of tenths or hundredths, etc. and, if possible, simplify.**

  The decimal 0.6 can be written $\frac{6}{10}$, which simplifies to $\frac{3}{5}$,

  and the decimal 1.85 can be written $1\frac{85}{100}$, which simplifies to $1\frac{17}{20}$.

- **To change a percentage to a decimal, divide the percentage by 100.**

  To express a percentage as a decimal, start by expressing it as a fraction, but *do not simplify,* because dividing by 100, or by a multiple of 100, is easy.

  For example $44\% = \frac{44}{100} = 44 \div 100 = 0.44$ and $12.5\% = \frac{12.5}{100} = 12.5 \div 100 = 0.125$

- **To change a decimal to a percentage simply multiply by 100.**

  For example $0.34 = 34\%$ and $1.55 = 155\%$.

- **To change a percentage to a fraction, divide by 100 and simplify.**

  We know that 20% of the cars in a car park means $\frac{20}{100}$ of the cars there.

  Now $\frac{20}{100}$ can be simplified to the equivalent fraction $\frac{1}{5}$, i.e. $20\% = \frac{1}{5}$.

  Similarly, 45% of the sweets in a bag means the same as $\frac{45}{100}$ of them

  and $\frac{45}{100} = \frac{9}{20}$, i.e. $45\% = \frac{9}{20}$.

- **To change a fraction to a percentage, change it to a decimal, then multiply by 100.**

  You can write a fraction as a percentage in two steps.

  First write the fraction as a decimal. For example, $\frac{4}{5} = 4 \div 5 = 0.8$

  Then change the decimal to a percentage: $0.8 \times 100\% = 80\%$.

There are different methods for converting between fractions, decimals and percentages. Use whatever method works for you.

Exercise 1c

1 Work out each fraction as a decimal.

a $\frac{3}{4}$ **b** $\frac{3}{5}$ **c** $\frac{3}{10}$ **d** $\frac{3}{20}$ **e** $\frac{7}{8}$ **f** $\frac{6}{25}$

2 Work out $\frac{9}{20}$ as a decimal.

Now decide which is larger, $\frac{9}{20}$ or 0.47?

3 Write each decimal as a fraction in its lowest terms, using mixed numbers where necessary.

a 0.06 **b** 0.004 **c** 15.5 **d** 2.01 **e** 3.25

In questions **4** and **5** write each decimal as a fraction in its lowest terms.

4 It is estimated that 0.86 of the families in Northgate Street own a car.

5 There were 360 seats on the aircraft and only 0.05 of them were vacant.

6 Write these decimals as percentages.

a 0.3 **b** 0.2 **c** 0.7 **d** 0.035 **e** 0.925

7 Write these decimals as percentages.

a 1.32 **c** 2.4 **e** 2.555
b 1.5 **d** 1.05

Remember that 1 is 100%, so
1.66 = 100% + 66% = 166%.

8 Write these percentages as decimals.

a 45% **b** 60% **c** 95% **d** 5.5% **e** 12.5%

9 Express each percentage as a fraction in its lowest terms.

a 40% **b** 65% **c** 54% **d** 25%

10 Express each fraction as a percentage.

a $\frac{2}{5}$ **b** $\frac{3}{20}$ **c** $\frac{21}{50}$ **d** $\frac{15}{40}$

In questions **11** to **14** express the given percentage as a fraction in its lowest terms.

11 Last summer 60% of the pupils in my class went on holiday.

12 At my youth club only 35% of the members are boys.

13 The postal service claims that 95% of the letters posted arrive the following day.

14 A survey showed that 32% of the pupils in a year group needed to wear glasses.

In each question from **15** to **18** express the fraction as a percentage.

15 At a youth club $\frac{17}{20}$ of those present took part in at least one sporting activity.

16 About $\frac{17}{50}$ of first-year pupils watch more than 20 hours of television a week.

17 Approximately $\frac{3}{5}$ of sixteen-year-olds have a Saturday job.

18 Recently, at the local garage, $\frac{1}{8}$ of the cars tested failed to get a test certificate.

19 Copy and complete the following table.

Fraction	Percentage	Decimal
$\frac{3}{5}$	60%	0.6
$\frac{4}{5}$		
	75%	
		0.7
$\frac{11}{20}$		
	44%	

20 The registers showed that only 0.05 of the pupils in the first year had 100% attendance last term.

a What fraction is this?

b What percentage of the first-year pupils had a 100% attendance last term?

21 Marion spends $\frac{21}{50}$ of her income on food and lodgings.

a What percentage is this?

b As a decimal, what part of her total income does she spend on food and lodging?

22 Marmalade consists of 28% fruit, $\frac{3}{5}$ sugar and the remainder water.

a What fraction of the marmalade is fruit?

b What percentage of the marmalade is sugar?

c What percentage is water?

23 An alloy is 60% copper, $\frac{7}{20}$ nickel and the remainder is tin.

a What fraction is copper?

b What percentage is **i** nickel **ii** either nickel or copper?

c Express the part that is tin as a decimal.

Positive indices

We have seen that 3^2 means 3×3

and that $2 \times 2 \times 2$ can be written as 2^3

The small number at the top is called the *index* or *power*. (The plural of index is indices.)

It follows that 2 can be written as 2^1, although we would not normally do so.

5^1 means 5

Exercise 1d

Find:

1 3^2

2 4^1

3 10^2

4 5^3

5 10^3

6 3^4

7 2^7

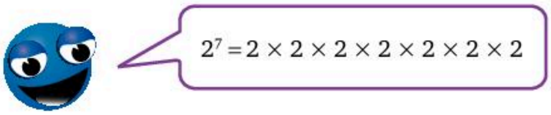
8 10^1

9 4^3

10 10^4

11 10^6

12 3^3



Find the value of:

13 7.2×10^3

14 8.93×10^2

15 6.5×10^4

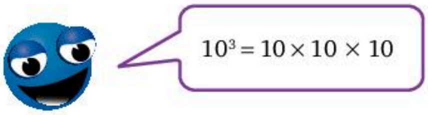
16 3.82×10^3

17 2.75×10^1

18 5.37×10^5

19 4.63×10^1

20 5.032×10^2



21 7.09×10^2

22 6.978×10^1

Multiplying numbers written in index form

We can write $2^2 \times 2^3$ as a single number in index form because

$$2^2 \times 2^3 = (2 \times 2) \times (2 \times 2 \times 2)$$

$$= 2 \times 2 \times 2 \times 2 \times 2$$

$$= 2^5$$

$$\therefore \quad 2^2 \times 2^3 = 2^{2+3} = 2^5$$

But we cannot do the same with $2^2 \times 5^3$ because the numbers multiplied together are not all 2s (nor are they all 5s).

We can multiply together different powers of the *same* number by adding the indices but we cannot multiply together powers of different numbers in this way.

Exercise 1e

Write as a single expression in index form:

1 $3^5 \times 3^2$

2 $7^5 \times 7^3$

3 $9^2 \times 9^8$

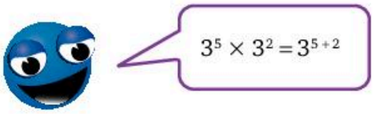
4 $2^4 \times 2^7$

5 $b^3 \times b^2$

6 $5^4 \times 5^4$

7 $12^4 \times 12^5$

8 $p^6 \times p^8$



9 $4^7 \times 4^9$

10 $r^5 \times r^3$

Dividing numbers written in index form

If we want to write $2^5 \div 2^2$ as a single number in index form then

$$2^5 \div 2^2 = \frac{2^5}{2^2} = \frac{\cancel{2}^{1} \times \cancel{2}^{1} \times 2 \times 2 \times 2}{\cancel{2}_{1} \times \cancel{2}_{1}} = \frac{2^3}{1}$$

i.e. $$\frac{2^5}{2^2} = 2^{5-2} = 2^3$$

We can divide different powers of the *same* number by subtracting the indices.

Exercise 1f

Write each as a single expression in index form:

1 $4^4 \div 4^2$

2 $7^9 \div 7^3$

3 $5^6 \div 5^5$

4 $10^8 \div 10^3$

5 $q^9 \div q^5$

6 $15^8 \div 15^4$

7 $6^{12} \div 6^7$

8 $b^7 \div b^5$

9 $9^{15} \div 9^{14}$

10 $p^4 \div p^3$

11 $6^4 \times 6^7$

12 $3^9 \div 3^6$

13 $2^8 \div 2^7$

14 $a^9 \times a^3$

15 $c^6 \div c^3$

16 $2^2 \times 2^4 \times 2^3$

17 $4^2 \times 4^3 \div 4^4$

18 $a^2 \times a^2 \div a^3$

19 $3^6 \div 3^2 \times 3^4$

20 $b^2 \times b^3 \times b^4$

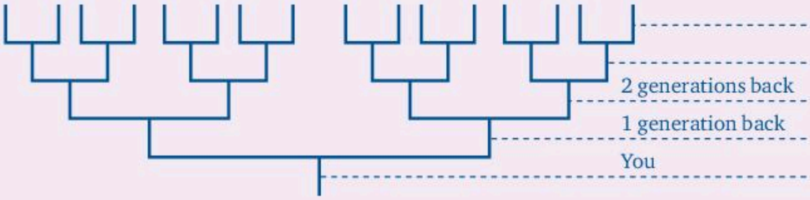
Investigation

1 Everyone has two biological parents.

Going back one generation, each of your parents has two biological parents.

Copy and complete the tree – fill in the number of ancestors for five generations back.

Do not fill in names!



2 Giving your answers as a power of 2, how many ancestors does this table suggest you have

- **a** five generations back
- **b** six generations back
- **c** ten generations back?

3 If we assume that each generation spans 25 years, how many generations are needed to go back 1000 years?

4 Find the number of ancestors the table suggests that you would expect to have 6000 years back. Give your answer as a power of 2. What assumptions are made to get this answer?

5 About 6000 years ago, according to the Bible, Adam and Eve were the only people on Earth. This contradicts the answer from part **4**. Suggest some reasons for this contradiction.

Mixed questions on indices

Exercise 1g

Find the value of:

1 2^2

2 4^3

3 5^3

4 3^4

5 4^1

6 2.41×10^3

7 4.971×10^2

8 5.92×10^4

9 7.834×10^2

10 3.05×10^4

Write each as a single number in index form:

11 $2^3 \times 2^4$

12 $4^6 \div 4^3$

13 $3^2 \times 3^4$

14 $a^4 \times a^3$

15 $a^7 \div a^3$

Standard form

The nearest star beyond our solar system (Alpha Centauri) is about 25 million million miles away, or to put it another way, 25 thousand billion miles away. Written in figures this very large number is 25 000 000 000 000.

The diameter of an atom is roughly 2 ten-thousand-millionths of a metre, or 0.000 000 000 2 metres, and this is very small.

These numbers are cumbersome to write down and, until we have counted the zeros, we cannot tell their size. We need a way of writing such numbers in a shorter form from which it is easier to judge their size: the form that we use is called *standard form*. It is also called *scientific notation*.

Written in standard form 25 000 000 000 000 is 2.5×10^{13}.

Standard form is a number between l and 10 multiplied by a power of 10.

So 1.3×10^2 and 2.86×10^4 are in standard form, but 13×10^3 is not in standard form because the first number is not between 1 and 10.

Exercise 1h

Each of the following numbers is written in standard form. Write them as ordinary numbers.

1 3.78×10^3

2 1.26×10^3

3 5.3×10^6

4 7.4×10^{14}

5 1.3×10^4

6 3.67×10^6

7 3.04×10^4

8 8.503×10^4

9 4.25×10^{12}

10 6.43×10^8

Changing numbers into standard form

To change 6800 into standard form, the decimal point has to be placed between the 6 and the 8 to give a number between l and 10.

Counting then tells us that, to change 6.8 to 6800, we need to move the decimal point three places to the right (i.e. to multiply by 10^3)

i.e. $6800 = 6.8 \times 1000 = 6.8 \times 10 \times 10 \times 10 = 6.8 \times 10^3$

Exercise 1i

Change the following numbers into standard form:

1 2500
2 630
3 15 300
4 260 000
5 9900
6 39 070
7 4500 000
8 530 000 000
9 40 000
10 80 000 000 000
11 26 030
12 547 000
13 30 600
14 4 060 000
15 704
16 79.3
17 80 600
18 60.5
19 7 080 000
20 560 800
21 5 300 000 000 000
22 708 000
23 40.5
24 503 000 000
25 99 000 000
26 84
27 351
28 36
29 5090
30 268 000
31 30.7

Investigation

If you read about computers, you will notice specifications such as ‘4 GB RAM’ or ‘1 TB hard disk’. GB stands for gigabytes and TB stand for terabytes.

‘Giga’ and ‘tera’ are prefixes used to describe very large numbers.

There are other prefixes used to describe very small numbers.

Find out what giga and tera mean.

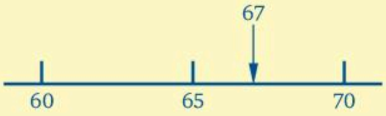
Find out what other prefixes are used to describe very large and very small numbers and what they mean.

Approximations: whole numbers

We saw in Grade 7 that it is sometimes necessary to approximate given numbers by rounding them off to the nearest 10, 100, … For example, if you measured your height in millimetres as 1678 mm, it would be reasonable to say that you were 1680 mm tall to the nearest 10 mm.

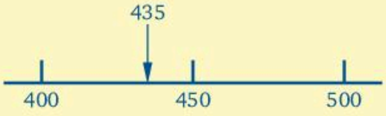
The rule is that if you are rounding off to the nearest 10 you look at the units. If there are 5 or more units you add one on to the tens. If there are fewer than 5 units you leave the tens alone.

For example, 67 is closer to 70 than to 60, as you can see on the number line. So 67 = 70 to the nearest ten.



Similar rules apply to rounding off to the nearest 100 (look at the tens); to the nearest 1000 (look at the hundreds); and so on.

For example, 435 is closer to 400 than to 500, so 453 = 400 to the nearest hundred.



Exercise 1j

Round off 1853 to

a the nearest ten

b the nearest hundred

c the nearest thousand.

a 185¦3 = 1850 to the nearest 10 (put a cut-off line (¦) after the 10s)

b 18¦53 = 1900 to the nearest 100 (put a cut-off line (¦) after the 100s)

c 1¦853 = 2000 to the nearest 1000 (put a cut-off line (¦) after the 1000s)

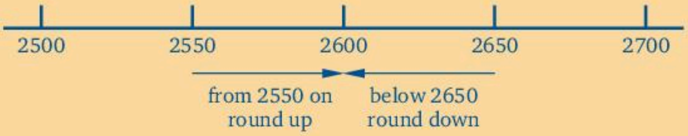
Round off each of the following numbers to

a the nearest ten **b** the nearest hundred **c** the nearest thousand.

1 1547
2 8739
3 2750
4 36 835
5 68 414
6 5729
7 4066
8 7507
9 53 804
10 6007
11 4981
12 8699

A building firm stated that, to the nearest 100, it built 2600 homes last year. What is the greatest number of homes that it could have built and what is the least number of homes that it could have built?

Look at this number line.



The smallest whole number that can be rounded up to 2600 is 2550.

The biggest whole number that can be rounded down to 2600 is 2649.

So the firm built at most 2649 homes and at least 2550 homes.

13 A bag of marbles is said to contain 50 marbles to the nearest 10. What is the greatest number of marbles that could be in the bag and what is the least number of marbles that could be in the bag?

14 To the nearest thousand, the attendance at a particular international football match was 45 000. What is the largest number that could have been there and what is the smallest number that could have attended?

15 1500 people came to the school bazaar. If this number is correct to the nearest hundred, give the maximum and the minimum number of people that could have come.

16 The annual accounts of Scrub plc (soap manufacturers) gave the company's profit as \$3 000 000 to the nearest million. What is the least amount of profit that the company could have made?

17 The chairman of A. Brick (Builders) plc said that they employ 2000 people. If this number is correct to the nearest 100, what is the least number of employees that the company can have?

Approximations: decimals

If you measure your height in centimetres as 167.8 cm, it would be reasonable to say that, to the nearest centimetre, you are 168 cm tall.

We write 167.8 = 168 correct to the nearest unit.

If you measure your height in metres as 1.678 m, it would be reasonable to say that, to the nearest $\frac{1}{100}$ m. you are 1.68 m tall.

Hundredths are represented in the second decimal place so we say that 1.678 = 1.68 correct to 2 *decimal places* (abbreviated to 2 d.p.).

Exercise 1k

Give each of the following numbers correct to

a 2 decimal places **b** 1 decimal place **c** the nearest unit:

1 2.758
2 7.371
3 16.987
4 23.758
5 9.858
6 3.896
7 8.936
8 73.649
9 6.896
10 55.575

You may find it helpful to draw a cut-off line.

Give the following numbers correct to the number of decimal places given in brackets:

11 5.07 (1)
12 0.0087 (3)
13 7.897 (2)
14 34.82 (1)
15 0.007831 (4)
16 0.9752 (3)
17 5.5508 (3)
18 285.59 (1)
19 6.749 (1)
20 9.999 (2)

Puzzle

What is the largest number you can make with two nines?

Significant figures

In the previous two sections we used a height of 1678 mm as an example. This height was measured in three different units and then rounded off:

in the first case to	1680 mm	correct to the nearest 10 mm,
in the second case to	168 cm	correct to the nearest centimetre,
in the third case to	1.68 m	correct to 2 d.p.

We could also give this measurement in kilometres, to the same degree of accuracy, as 0.001 68 km correct to 5 d.p.
Notice that the three figures 1, 6 and 8 occur in all four numbers and that it is the 8 that is the corrected figure.

The figures 1, 6 and 8 are called the *significant figures* and in all four cases the numbers are given correct to 3 significant figures. Note that significant figure is abbreviated to s.f.

Using significant figures rather than place values (i.e. tens, units, first d.p., second d.p., ...) has advantages. For example, if you are asked to measure your height and give the answer correct to 3 significant figures, then you can choose any convenient unit. You do not need to be told which unit to use and which place value in that unit to correct your answer to.

The first significant figure is the **non-zero** digit with the highest place value.

For example, for the number 170.6, the digit with the highest place value is 1. So l is the first significant figure in 170.6.

The second significant figure is the next digit to the right (7 in this case). The third significant figure is the next digit to the right again (0 in this case), and so on.

The first significant figure can never be zero, but zeros after the first significant figure are significant figures.

In 0.0305, the first non-zero digit is 3, so this is the first significant figure. The next figure to the right of 3 in 0.0305 is 0, so 0 is the second significant figure.

Exercise 1I

For the number 0.001 503, write

a the first significant figure **b** the third significant figure.

a The highest non-zero place value digit in 0.001 503 is 1 so the first s.f. is 1.

b The third s.f. is 0. Note that zeros after the first significant figure *are* included.

For each of the following numbers write the significant figure specified in the bracket:

1 36.2 (1st)
2 378.5 (3rd)
3 0.0867 (2nd)
4 3.786 (3rd)
5 47 632 (2nd)
6 5.083 (3rd)
7 34.807 (4th)
8 0.07603 (3rd)
9 54.06 (3rd)
10 5.7087 (4th)

Exercise 1m

Give 32 685 correct to 1 s.f.

The highest non-zero place value digit is 3, so 3 is the first significant figure.

(As before, to correct to 1 s.f. we look at the second s.f.: if it is 5 or more we add one to the first s.f.; if it is less than 5 we leave the first s.f. alone.)

So 3¦2685 = 30 000 to 1 s.f.

Give the following numbers correct to 1 s.f.:

Remember that the digits must not change their place value.

1 59 727

2 4164

3 4 396 185

4 586 359

5 80 755

6 476

7 51 488

8 4099

9 667 505

10 908

11 26

12 980

Give the following numbers correct to 2 s.f.:

13 4673

14 57 341

15 59 700

16 892 759

17 6992

18 9973

19 72 601

20 444

21 50 047

22 53 908

23 476

24 597

Give 0.021 94 correct to 3 s.f.

2 is the first s.f. so 9 is the third s.f.

(The fourth s.f. is 4 so we leave the third s.f. alone.)

So 0.0219¦4 = 0.0219 to 3 s.f.

Give the following numbers correct to 3 s.f.:

25 0.008 463

26 0.825 716

27 5.8374

28 78.49

29 46.8451

30 0.007 854 7

31 7.5078

32 369.649

33 0.989 624

34 53.978

Give each of the following numbers correct to the number of significant figures indicated in the brackets.

35 46.931 06 (2)

36 0.006 845 03 (4)

37 576 335 (1)

38 497 (2)

39 7.824 38 (3)

40 4537 (1)

41 37.856 72 (3)

42 6973 (2)

43 0.070 865 (3)

44 0.067 34 (1)

Find 50 ÷ 8 correct to 2 s.f.

(To give an answer correct to 2 s.f. we first work to 3 s.f.)

$$\begin{array}{r} 6.25 \\ 8\overline{)50.00} \end{array}$$

So $50 \div 8 = 6.3$ to 2 s.f.

Give, correct to 2 s.f.:

45 20 ÷ 6

46 10 ÷ 6

47 25 ÷ 2

48 53 ÷ 4

49 125 ÷ 9

50 143 ÷ 5

51 73 ÷ 3

52 0.7 ÷ 3

53 0.23 ÷ 9

54 0.0013 ÷ 3

Rough estimates

If you were asked to find 1.397 × 62.54 you could do it by long multiplication or you could use a calculator. Whichever method you choose, it is essential first to make a *rough estimate* of the answer. You will then know whether the actual answer you get is reasonable or not.

One way of estimating the answer to a calculation is to write each number correct to 1 significant figure.

So $1.397 \times 62.57 \approx 1 \times 60 = 60$

Exercise 1n

Correct each number to 1 s.f. and hence give a rough answer to:

a 9.524 × 0.0837 **b** 54.72 ÷ 0.761

a $9.524 \times 0.0837 \approx 10 \times 0.08 = 0.8$

b $\dfrac{54.72}{0.761} \approx \dfrac{50}{0.8} = \dfrac{500}{8} = 62.5$

$= 60$ (giving 500 ÷ 8 to 1 s.f.)

Correct each number to 1 s.f. and hence give a rough answer to each of the following calculations:

1 4.78×23.7
2 56.3×0.573
3 $0.0674 \div 5.24$
4 354.6×0.0475
5 576×256
6 $82.8 \div 146$
7 0.632×0.845
8 0.0062×574
9 $7.835 \div 6.493$
10 4736×729
11 34.7×21
12 8.63×0.523
13 $34.9 \div 15.8$
14 $0.47 \div 0.714$
15 $985 \div 57.2$
16 $0.0326 \div 12.4$
17 $0.007\,24 \times 0.783$
18 $3581 \div 45$
19 1097×94
20 45.07×0.0327

Correct each number to 1 s.f. and hence estimate $\frac{0.048 \times 3.275}{0.367}$ to 1 s.f.

$$\frac{0.048 \times 3.275}{0.367} \approx \frac{0.05 \times 3}{0.4} = \frac{0.15}{0.4} = \frac{1.5}{4}$$

$$= 0.4 \text{ (to 1 s.f.)}$$

21 $\frac{3.87 \times 5.24}{2.13}$

22 $\frac{0.636 \times 2.63}{5.47}$

23 $\frac{21.78 \times 4.278}{7.96}$

24 $\frac{6.38 \times 0.185}{0.628}$

25 $\frac{43.8 \times 3.62}{4.72}$

26 $\frac{89.03 \times 0.07937}{5.92}$

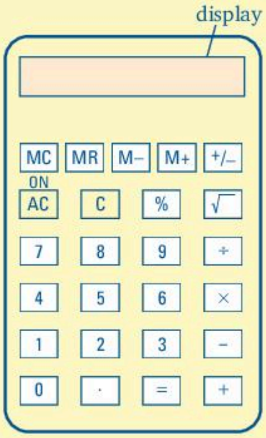
27 $\frac{975 \times 0.636}{40.78}$

28 $\frac{8.735}{5.72 \times 5.94}$

29 $\frac{0.527}{6.41 \times 0.738}$

30 $\frac{57.8}{0.057 \times 6.93}$

Calculations: multiplication and division



When you key in a number on your calculator it appears on the display. Check that the number on display is the number that you intended to enter.

Also check that you press the correct operator, i.e. press × to multiply and ÷ to divide.

To find 38.4×0.67, first estimate:

$$38.4 \times 0.67 \approx 40 \times 0.7 = 28$$

On your calculator, press $\boxed{3}\,\boxed{8}\,\boxed{.}\,\boxed{4}\,\boxed{\times}\,\boxed{0}\,\boxed{.}\,\boxed{6}\,\boxed{7}\,\boxed{=}$

The display shows 25.728, so $38.4 \times 0.67 = 25.7$ correct to 3 s.f.

Exercise 1p

First make a rough estimate of the answer. Then use your calculator to give the answer correct to 3 significant figures.

1 2.16×3.28

2 2.63×2.87

3 1.48×4.74

4 4.035×2.116

5 3.142×2.925

6 6.053×1.274

7 2.304×3.251

8 8.426×1.086

9 $5.839 \div 3.618$

10 $6.834 \div 4.382$

11 $9.571 \div 2.518$

12 $5.393 \div 3.593$

13 $7.384 \div 2.51$

14 $4.931\div 3.204$

15 $8.362 \div 5.823$

16 23.4×56.7

17 384×21.8

18 45.8×143.7

19 $537.8 \div 34.6$

20 $45.35 \div 6.82$

21 63.8×2.701

22 $40.3 \div 2.74$

23 $400 \div 35.7$

24 $(34.2)^2$

25 5007×2.51

26 $5703 \div 154.8$

27 39.03×49.94

28 $2000 \div 52.66$

29 $(36.8)^2$

30 $29\,006 \div 2.015$

31 0.366×7.37

32 0.0526×0.372

33 $6.924 \times 0.007\,93$

34 0.638×825

35 52×0.0895

36 0.0826×0.582

37 24.78×0.0724

38 0.00835×0.617

39 0.5824×6.813

40 $(0.74)^2$

41 $0\ .583 \div 4.82$

42 $0.628 \div 7.61$

43 $0.493 \div 1.253$

44 $0.518 \div 5.047$

45 $82.7 \div 593$

46 $89.5 \div 0.724$

47 $38.07 \div 0.682$

48 $5.71 \div 0.0623$

49 $7.045 \div 0.0378$

50 $6.888 \div 0.0072$

51 $45.37 \div 0.925$

52 $8.41 \div 0.000\,748$

53 $6.934 \div 0.0829$

54 $0.824 \div 0.362$

55 $0.572 \div 0.851$

56 $0.528 \div 0.0537$

57 $0.571 \div 0.824$

58 $0.0455 \div 0.0613$

59 $0.006 \div 0.04\,703$

60 $0.824 \div 0.000\,08$

61 $5000 \div 0.789$

62 $(0.078)^2$

63 0.0608×573

64 $(78.5)^3$

65 $\dfrac{3.782 \times 0.467}{4.89}$

66 $4.88 \times 0.004\,17$

67 $0.9467 \div 7683$

68 $0.0467 \div 0.000\,074$

69 $(0.00031)^2$

70 $\dfrac{54.9 \times 36.6}{0.406}$

71 $68.41 \div 392.9$

72 $0.0482 \div 0.002\,89$

73 $(0.0527)^3$

74 $\dfrac{0.857 \times 8.109}{0.5188}$

Mixed exercises

Exercise 1q

1. Find the value of 4^3.
2. Simplify $b^5 \div b^2$.
3. Find the value of $\frac{3^2 \times 3^3}{3^5}$.
4. Write 36 400 in standard form.
5. Give 57 934 correct to 1 s.f.
6. Give 0.061 374 correct to 3 s.f.
7. Find 0.582 × 6.382, giving your answer correct to 3 s.f.
8. Find 45.823 ÷ 15.89, giving your answer correct to 3 s.f.

Exercise 1r

1. Find the value of 6^3.
2. Write $\frac{2^4 \times 2^2}{2^3}$ as a single number in index form.
3. Find the value of $5^6 \div 5^3$.
4. Simplify $a^2 \times a^4 \times a$.
5. Write 650 000 000 in standard form.
6. Give 45 823 correct to 2 s.f.
7. The organisers of a calypso show hope that, to the nearest thousand, 8000 people will buy tickets. What is the minimum number of tickets that they hope to sell?
8. Find the value of 12.07 ÷ 0.008 97 giving your answer correct to 3 s.f.
9. Find the value of $(0.836)^2$ giving your answer correct to 3 s.f.
10. Change 35% into

    a a fraction in its lowest terms **b** a decimal.

Exercise 1s

1 Find the value of $5^2 \times 5^3$.

2 Simplify $\frac{a^8}{a^3 \times a^2}$.

3 Find the value of $3^2 \times 3^4 \div 3^6$.

4 Write 78 260 in standard form.

5 Give 9764 correct to 1 s.f.

6 Give 0.050 806 correct to 3 s.f.

7 Correct to 1 significant figure, there are 70 matches in a box. What is the difference between the maximum and the minimum number of matches that could be in the box?

8 Find 0.0468 ÷ 0.004 73 giving your answer correct to 3 s.f.

9 Find $\frac{56.82 \times 0.714}{8.625}$ giving your answer correct to 3 s.f.

10 Change $\frac{5}{8}$ into

a a percentage **b** a decimal.

Exercise 1t

Select the letter that gives the correct answer.

1 $\frac{7}{20}$ as a decimal is

A 0.33 **B** 0.35 **C** 0.36 **D** 0.37

2 30% as a fraction in its lowest terms is

A $\frac{3}{10}$ **B** $\frac{1}{3}$ **C** $\frac{3}{8}$ **D** $\frac{2}{5}$

3 $\frac{9}{40}$ as a percentage is

A 20% **B** 22.5% **C** 24.5% **D** 25%

4 0.006 as a fraction in its lowest terms is

A $\frac{1}{600}$ **B** $\frac{1}{300}$ **C** $\frac{3}{500}$ **D** $\frac{1}{125}$

5 The value of 4.36×10^5 is

A 4360 **B** 43 600 **C** 436 000 **D** 4360 000

6 As a single expression in index form $3^{10} \div 3^6$ is

A 3^4 **B** 3^5 **C** 3^6 **D** 3^{16}

7 8.743×10^2 is

A 8.743 **B** 87.43 **C** 874.3 **D** 8743

8 Expressed in standard form, 60 800 is

A 6.08×10^1 **B** 6.08×10^2 **C** 6.08×10^3 **D** 6.08×10^4

9 Correct to the nearest 100, the number 8507 is

A 8400 **B** 8500 **C** 8600 **D** 8700

10 Correct to two decimal places, 63.549 is

A 63.00 **B** 63.54 **C** 63.55 **D** 63.56

11 Correct to two significant figures, 4.5707 is

A 4.5 **B** 4.6 **C** 4.7 **D** 4.71

12 Correct to 3 s.f., 279.540 is

A 279 **B** 279.5 **C** 279.55 **D** 280

13 A rough estimate for the value of $0.005\,27 \times 0.717$ is

A 0.0035 **B** 0.035 **C** 0.055 **D** 0.35

14 47% as a decimal is

A 0.047 **B** 0.47 **C** 4.7 **D** 47

15 The property that $10 = x$ so $x = 10$ illustrates is

A the reflexive property **C** the transitive property

B the symmetry property **D** the trichotomy property

16 Which of the following statements shows the symmetry property?

A $100 \times 56 = 5600$ **C** $100 \times 56 = 5.6 \times 10^3$

B $100 \times 56 = 56 \times 100$ **D** $100 \times 56 = 6000$ (correct to 1 s.f.)

17 a and b are members of the set of positive rational numbers, i.e. they are fractions whose numerators and denominators are positive integers. Under which of the following operations is the set closed?

A $a - b$ **B** $\frac{a}{b-a}$ **C** $\frac{a}{b}$ **D** $b - a$

18 $2 - 4(8 - 10) =$

A 4 **B** 6 **C** 8 **D** 10

19 $\frac{3-1}{1-3} =$

A -1 **B** 1 **C** 2 **D** 3

20 $\frac{2\times3-1}{4+2\times(-3)} =$

A $-\frac{2}{9}$ **B** $-\frac{1}{3}$ **C** $-2\frac{1}{2}$ **D** $-5\frac{1}{2}$

Did you know?

Did you know these facts?

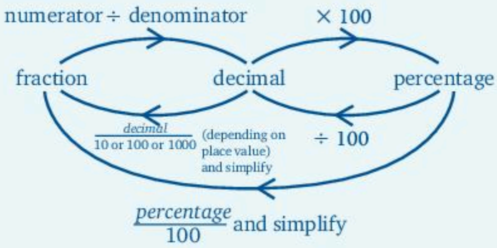
1. 1729 is the smallest positive integer that can be represented in two ways as the sum of two cubes:

$$9^3 + 10^3 \quad \text{or} \quad 1^3 + 12^3$$

2. The total number of gifts given in the song 'The Twelve Days of Christmas' is 364. That is one gift for each day of the year except Christmas Day. Check it.
3. $2592 = 2^5 \times 9^2$.

In this chapter you have seen that...

- ✔ the identity element is 0 for addition and 1 for multiplication
- ✔ the inverse element for any whole number under addition is minus it, and for multiplication it is the reciprocal, e.g. the inverse of 4 under multiplication is $1 \div 4 \left(\text{or} \times \frac{1}{4}\right)$
- ✔ you can interchange fractions, percentages and decimals using these rules



- ✔ you can multiply different powers of the same number by adding the indices, e.g. $3^4 \times 3^3 = 3^{4+3} = 3^7$
- ✔ you can divide different powers of the same number by subtracting the indices, e.g. $5^7 \div 5^2 = 5^{7-2} = 5^5$
- ✔ a number in standard form or scientific notation is a number between 1 and 10 multiplied by a power of 10, e.g. 1.2×10^5 is in standard form
- ✔ the first significant figure is the first non-zero digit in a number. The next digit (zero or otherwise) is the second significant figure, and so on
- ✔ to correct a number to a given degree of accuracy, place a cut-off line after the place value required and look at the next digit – if it is 5 or more, round up, otherwise round down
- ✔ you can make a rough estimate of a calculation by first writing each number correct to one significant figure
- ✔ you need to be careful when you use a calculator to work out accurate answers.

2 Number bases

At the end of this chapter you should be able to...

1. use markers to represent groups of fives or powers of fives for a given number
2. write in figures the numbers represented by markers under the headings of five and powers of five
3. write, in headed columns, numbers to a given base
4. write, in base ten, numbers given in other bases
5. write a number given in base ten as a number to another given base
6. perform operations of addition, subtraction and multiplication in bases other than ten
7. determine the base in which given calculations have been done.

Did you know?

Computers work using different number systems from the one based on powers of 10 that we use in daily life. One of these is the hexadecimal system.

Find out what you can about hexadecimal numbers and why they are important.

You need to know...

✓ your multiplication tables – and this means instant recall.

Key words

base, binary, denary system, number base

Denary system (base ten)

We have ten fingers. This is probably why we started to count in tens and developed a system based on ten for recording large numbers. For example

$$\begin{aligned} 3125 &= 3 \text{ thousands} + 1 \text{ hundred} + 2 \text{ tens} + 5 \text{ units} \\ &= 3\times10^3 + 1\times10^2 + 2\times10^1 + 5\times10^0 \end{aligned}$$

Each place value is ten times the value of its right-hand neighbour. The *base* of this number system is ten and it is called the *denary system*.

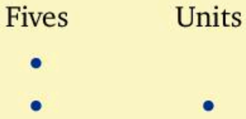
Base five

If humans had started to count using just one hand, we would probably have a system based on five.

Suppose we had eleven stones. Using one hand to count with, we could arrange them like this:

i.e. two handfuls + one }
or 2 fives + 1 }

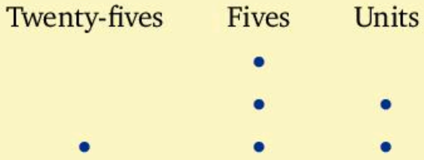
The next logical step is to use a single marker to represent each group of five. We also need to place these markers so that they are not confused with the marker representing the one unit. We do this by having separate columns for the fives and the units, with the column for the fives to the left of the units.



We can write this number as 21_5 and we call it 'two one to the base five'. We do *not* call it 'twenty-one to the base five', because the word 'twenty' means 'two tens'.

To cope with larger numbers, we can extend this system by adding further columns to the left such that each column is five times the value of its right-hand neighbour. Thus, in the column to the left of the fives column, each marker is worth twenty-five, or 5^2.

For example



The markers here represent 132_5 and this base five number means

$$(1 \times 5^2) + (3 \times 5) + 2$$

Exercise 2a

Write in figures the numbers represented by the markers:

5^3	5^2	5	Units
●	● ● ●		● ●

1	3	0	2

The number is 1302_5

(The '5' column is empty, so we write zero in this column.)

Write in figures the numbers represented by the markers in questions **1** to **4**:

	5^4	5^3	5^2	5	Units
1			● ●	●	● ● ●
2		● ●		●	● ● ● ●
3	● ● ● ●	●	● ●	● ● ● ●	
4	● ● ●		● ●		●

Write 120_5 in headed columns.

5^2	5	Units
1	2	0

Write the following numbers in headed columns:

5 31_5

6 42_5

7 410_5

8 231_5

9 34_5

10 10_5

11 204_5

12 400_5

Write 203_5 as a number to the base 10

5^2	5	Units
● ●		● ● ●
2	0	3

$$\begin{aligned} 203_5 &= (2 \times 5^2) + (0 \times 5) + 3 \\ &= \;\; 50_{10} \;\; + \;\; 0 \;\; + 3_{10} \\ &= 53_{10} \end{aligned}$$

(Although we do not normally write fifty-three as 53_{10}, it is sensible to do so when dealing with other bases as well.)

Write the following numbers as denary numbers, i.e. to base 10:

13 31_5

14 24_5

15 40_5

16 121_5

17 204_5

18 43_5

19 32_5

20 20_5

21 4_5

22 400_5

23 240_5

24 300_5

Write 38_{10} as a number to the base 5.

(To write a number to the base 5 we have to find how many … 125s, 25s, 5s and units the number contains.)

(Starting with the highest value column.)

38 contains no 125s.

$38 \div 25 = 1$ remainder 13 i.e. $38 = 1 \times 5^2 + 13$

$13 \div \;\; 5 = 2$ remainder 3 i.e. $13 = 2 \times 5 + 3$

$$\begin{aligned} \therefore \;\; 38 &= 1 \times 5^2 + 2 \times 5 + 3 \\ &= 123_5 \end{aligned}$$

Write the following numbers in base 5:

25 8_{10}

26 13_{10}

27 10_{10}

28 39_{10}

29 43_{10}

30 150_{10}

31 7_{10}

32 21_{10}

33 30_{10}

34 128_{10}

35 82_{10}

36 100_{10}

Other number bases

Any number can be used as a base for a number system.

If the *number base* is 6, we write the number in columns such that each column is 6 times the value of its right-hand neighbour. For example, writing 253_6 in headed columns gives:

6^2	6	Units
2	5	3

We see that 253_6 means $(2 \times 6^2) + (5 \times 6) + 3$

Similarly for 1011_2, we can write this in headed columns.

2^3	2^2	2	Units
●		●	●

Therefore 1011_2 means $(1 \times 2^3) + (0 \times 2^2) + (1 \times 2) + 1$.

Exercise 2b

Write 425_7 in headed columns and then write it as a denary number.

7^2	7	Units
4	2	5

$$\begin{aligned} \therefore \quad 425_7 &= (4 \times 7^2) + (2 \times 7) + 5 \\ &= 196_{10} + 14_{10} + 5_{10} \\ &= 215_{10} \end{aligned}$$

Write the following numbers

a in headed columns **b** as denary numbers.

1 23_4
2 15_7
3 131_4
4 101_2
5 57_8
6 204_5
7 210_3
8 574_9
9 21_3
10 18_9
11 24_6
12 175_8
13 303_4
14 1001_2
15 1211_3
16 1000_6

Write 29_{10} as a number

a to the base 9

b to the base 2.

a $29 \div 9 = 3$ remainder 2

so $29_{10} = 3 \times 9 + 2$

$\therefore$ $29_{10} = 32_9$

b

$2\overline{)29}$		
$2\overline{)14}$	remainder	1 (unit)
$2\overline{)\ 7}$	remainder	0 (twos)
$2\overline{)\ 3}$	remainder	1 (2^2)
$2\overline{)\ 1}$	remainder	1 (2^3)
0	remainder	1 (2^4)

$\therefore \quad 29_{10} = 11101_2$

Write the following denary numbers to the base indicated in brackets:

17 9 (4)

18 12 (5)

19 24 (7)

20 7 (2)

21 13 (5)

22 32 (6)

23 53 (8)

24 49 (7)

25 8 (3)

26 15 (6)

27 34 (9)

28 28 (3)

29 56 (7)

30 89 (9)

31 45 (2)

32 333 (3)

33 163 (8)

34 640 (4)

35 142 (2)

36 158 (6)

37 43 (6)

38 55 (5)

39 99 (2)

40 394 (7)

Express each of the following numbers as a number to the base indicated in brackets:

41 45_6 (4)

42 23_4 (6)

43 17_8 (2)

44 432_5 (3)

45 562_8 (4)

46 2120_3 (5)

47 11011_2 (8)

48 378_9 (3)

49 3020_4 (8)

Addition, subtraction and multiplication

Numbers with a base other than 10 do not need to be converted to base 10; provided that they have the same base they can be added, subtracted and multiplied in the usual way, as long as we remember which base we are working with.

For example, to find $132_5 + 44_5$ we work in fives, not tens. To aid memory, the numbers can be written in headed columns:

Twenty-fives	Fives	Units	
1	3	2	
①	4 ①	4	+
2	3	1	
	⑧	⑥	

Adding the units gives 6 units:

6 (units) = 1 (five) + 1 (unit)

We put 1 in the units column and carry the single five to the fives column.

Adding the fives gives 8 fives:

8 (fives) = 5 (fives) + 3 (fives)

= 1 (twenty-five) + 3 (fives)

We put 3 in the fives column and carry the 1 to the next column.

Adding the numbers in the last column gives 2,

i.e. $132_5 + 44_5 = 231_5$

If the numbers are not to the same base we cannot add them in this way.

For example, 432_7 and 621_8 cannot be added directly.

Exercise 2c

Find $174_8 + 654_8$

8^3	8^2	8	Units	
	1	7	4	
①	6 ①	5 ①	4	+
1	0	5	0	
	⑧	⑬	⑧	

$\therefore 174_8 + 654_8 = 1050_8$

Find:

1 $12_5 + 31_5$

2 $11_3 + 2_3$

3 $11_4 + 13_4$

4 $10_2 + 1_2$

5 $24_6 + 35_6$

6 $21_3 + 11_3$

7 $43_8 + 52_8$

8 $101_2 + 11_2$

9 $43_5 + 24_5$

10 $132_4 + 201_4$

11 $345_6 + 402_6$

12 $1101_2 + 111_2$

13 $122_3 + 101_3$

14 $231_4 + 103_4$

15 $635_7 + 62_7$

16 $10010_2 + 1111_2$

Make sure that you are clear about which base you are working in. Use headed columns if necessary.

Find $132_4 - 13_4$

(There are two methods of doing subtraction and we show both of them here.)

First method

4^2	4	Units	
1	$\cancel{3}$ ②	$\cancel{2}$ ⑥	
	1	3	–
1	1	3	

(We cannot take 3 from 2 so we take one 4 from the fours column, change it to 4 units and add it to the 2 units.)

Second method

If you use the 'pay back' method of subtraction, the calculation looks like this:

4^2	4	Units	
1	3	$\cancel{2}$ ⑥	
	1 ①	3	–
1	1	3	

In either case $132_4 - 13_4 = 113_4$

Find:

17 $153_6 - 24_6$
18 $110_3 - 2_3$
19 $32_4 - 23_4$
20 $52_7 - 14_7$
21 $210_3 - 1_3$
22 $30_5 - 14_5$
23 $253_8 - 25_8$
24 $10_2 - 1_2$
25 $231_4 - 32_4$
26 $153_7 - 64_7$
27 $205_6 - 132_6$
28 $100_2 - 10_2$
29 $144_6 - 53_6$
30 $1010_2 - 101_2$
31 $724_8 - 56_8$
32 $120_3 - 12_3$

Find $352_6 \times 4_6$

6^3	6^2	6	Units	
	3	5	2	
②	③	①	4	×
2	3	3	2	

$15 = 2 \times 6 + 3$ $21 = 3 \times 6 + 3$ $8 = 6 + 2$

$352_6 \times 4_6 = 2332_6$

Find:

33 $4_5 \times 3_5$

34 $2_4 \times 3_4$

35 $12_3 \times 2_3$

36 $20_4 \times 3_4$

37 $5_6 \times 4_6$

38 $13_5 \times 2_5$

39 $24_7 \times 3_7$

40 $56_9 \times 3_9$

41 $132_4 \times 2_4$

42 $501_6 \times 5_5$

43 $202_3 \times 2_3$

44 $241_5 \times 2_5$

Find:

45 $261_7 + 123_7$

46 $32_4 \times 2_4$

47 $434_5 - 142_5$

48 $36_7 \times 2_7$

49 $451_6 + 124_6$

50 $232_4 - 103_4$

51 $22_5 \times 4_5$

52 $365_8 + 173_8$

53 $121_3 - 112_3$

54 $34_8 \times 5_8$

Harder examples

Exercise 2d

Calculate $133_4 \times 32_4$

First place the number in headed columns, then use long multiplication, i.e. multiply by 2_4 and then by 30_4:

	4^4	4^3 (= 64)	4^2 (= 16)	4	Units
			1	3	3
				3	2
×2			3	3	2
			2 + 1 = 3	6 + 1 = 7 = 4 + 3	6 = 4 + 2
×30	1	1	3	1	0
		3 + 2 = 5 = 4 + 1	9 + 2 = 11 = 2 × 4 + 3	9 = 2 × 4 + 1	
+	1	2	3	0	2

$133_4 \times 32_4 = 12302_4$

Find:

1 $123_4 \times 23_4$

2 $413_5 \times 24_5$

3 $1001_2 \times 1101_2$

4 $73_8 \times 26_8$

5 $2120_3 \times 212_3$

6 $46_8 \times 35_8$

7 $234_5 \times 423_5$

8 $452_7 \times 324_7$

9 **a** Find $64_8 \times 27_8$ as a number to the base 8.

b Express 64_8 and 27_8 as denary numbers.

c Multiply together your two answers for **b**.

d Change your answer to **c** into a number to the base 8. Does this answer agree with your answer to **a**?

10 **a** Find $476_9 \times 57_9$ as a number to the base 9.

b Express 476_9 and 57_9 as denary numbers.

c Multiply together your two answers for **b**.

d Change your answer to **c** into a number to the base 9. Does this answer agree with your answer to **a**?

11 Find $55_8 \times 43_8$ and use the process described in questions **9** and **10** as a check on your working.

12 Choose a base and make up a long multiplication question of your own. Check your calculation using the process above.

Binary numbers

Numbers with a base of two are called *binary numbers*. We have singled binary numbers out for special attention because of the wide application that they have, especially in the world of computers.

Exercise 2e

1 If you have access to a computer and have done some programming using machine code, or have copied program listings from magazines, you will have seen instructions such as

BIN 1101, 1011, 11001, 100, 1100, 11101

'BIN' means 'binary'.

Convert the binary numbers given above to denary numbers.

2 Basically, computers are very simple; their fundamental computing parts can only be off (0) or on (1), i.e. computers count in binary numbers.

a Complete the adjacent addition table for binary numbers.

b Find $1011011_2 + 110101_2$.

+	0	1
0		
1		

3 **a** Subtract 23_{10} as many times as you can from 138_{10}. Hence find the value of $138_{10} \div 23_{10}$.

b Subtract 11_2 from 1111_2 as many times as you can.
Hence find $1111_2 \div 11_2$.

4 Complete the following multiplication table for binary numbers.

×	0	1
0		
1		

Investigation

1 Convert 7_{10}, 5_{10}, 10_{10}, 16_{10}, 19_{10}, 24_{10}, into binary numbers.

2 How many symbols are needed to represent all binary numbers?

3 Write each of the denary numbers 1 to 20 as binary numbers.

4 How can you see when a binary number is even?

5 How can you see when a binary number is odd?

6 Find some machine code and see if you can interpret it.

Mixed exercises

Exercise 2f

1 Convert the following denary numbers to base 3 numbers:

a 5 **b** 8 **c** 12 **d** 31

2 How many different symbols are needed to represent all numbers in base three?

3 Make up a multiplication table for base three numbers.

4 Use repeated subtraction to find the value of $1111_3 \div 101_3$.

5 Convert the following denary numbers to base 5 numbers:

a 27 **b** 18 **c** 153

6 What do you think 31.2_5 could mean?

7 If a number to the base 10 ends in 0, what does the same number to the base 5 end in?

8 Is it possible to write a number in base one? Give a reason for your answer.

9 How many digits are there in 5^3 written in base 5?

10 How many digits are there in 3^7 written in base 3?

11 **a** Convert 10_2, 10_3, 10_4, 10_5, 10_6, 10_7, 10_8 into denary numbers.
   b Find **i** $1101_2 \times 10_2$ **ii** $121_3 \times 10_3$ **iii** $175_8 \times 10_8$
   c What is the effect of multiplying a number by the base number?

Find the base in which the following calculation has been done:
$13 + 5 = 22$

$$\begin{array}{r} 13 \\ \textcircled{1}\,5+ \\ \hline 22 \\ \hline \textcircled{8} \end{array}$$

From the addition, $5 + 3 = 8$. To leave 2 in the units column, 6 units have been carried to the next column. The total in the next column is 2, so the 6 units have been carried as 1 to the next column.

∴ the base is 6

Find the bases in which the following calculations have been done.

12 $15 + 23 = 42$

13 $12 + 13 = 31$

14 $110 + 121 = 1001$

15 $134 + 213 = 350$

16 $13 - 4 = 4$

17 $21 - 2 = 17$

18 $13 \times 2 = 31$

19 $21 \times 3 = 103$

20 Is the following statement true or false? Give a reason for your answer.
A number written to any base is even if it ends in zero.

Exercise 2g

1 Write the following numbers as denary numbers:
a 12_4 **b** 101_2 **c** 403_6

2 Write the denary number 20 as a number to the given base:
a 5 **b** 3 **c** 8

3 Find:
a $204_5 + 132_5$ **b** $110_3 - 2_3$ **c** $212_8 \times 3_8$

4 There are four possible answers given below to the calculation $213_4 \times 2_4$.
Only one answer is correct. Which one is it?
A 426_4 **B** 2130_4 **C** 1032_4 **D** 221_4

Exercise 2h

Select the letter that gives the correct answer.

1 Expressed as a denary number 34_5 is

A 15 **B** 19 **C** 21 **D** 24

2 Written to the base 5, the number 31_{10} is

A 100_5 **B** 101_5 **C** 110_5 **D** 111_5

3 The denary number 54, written as a number to the base 8, is

A 52_8 **B** 57_8 **C** 66_8 **D** 67_8

4 Working to the base 2, the value of $1011_2 + 10111_2$ is

A 100010_2 **B** 100110_2 **C** 110110_2 **D** 111010_2

5 $254_8 - 24_8$ equals

A 222_8 **B** 224_8 **C** 230_8 **D** 234_8

6 $33_5 + 4_5$ equals

A 32_5 **B** 42_5 **C** 52_5 **D** 342_5

7 $31_5 \times 20_5$ equals

A 112_5 **B** 160_5 **C** 1120_5 **D** 1122_5

8 Expressed as a denary number $54_8 + 27_8$ equals

A 11 **B** 21 **C** 67 **D** 81

9 $165_8 - 20_8$ equals

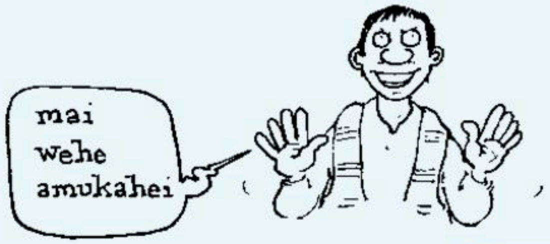
A 45_8 **B** 145_8 **C** 187_8 **D** 205_8

10 In what base has the calculation $23 - 4 = 13$ been made?

A 3 **B** 4 **C** 6 **D** 8

Did you know?

The Jivaro Indians of the Amazon rain forest express the number 'five' by the phrase 'wehe amukei', meaning 'I have finished one hand', and the number 'ten' by 'mai wehe amukahei', meaning 'I have finished both hands'.



In this chapter you have seen that...

- ✓ numbers can be expressed in any base
- ✓ if a number is in base *a*, the first column from the right is units, the second column gives the number of *a*'s, the next column gives the number of a^2's and so on. For example, 165_8 means
  $1 \times 8^2 + 6 \times 8^1 + 5$
- ✓ you can add, subtract and multiply numbers in a base other than ten in the usual way, but it is sensible to write them in headed columns so that you can keep track of your working.

3 Algebra

At the end of this chapter you should be able to...

1. construct formulae from given information
2. substitute numerical values in a formula
3. rearrange a formula to make a different letter the subject
4. solve an inequality.

Did you know?

The German mathematician Carl Friedrich Gauss (1777–1855) was the leading algebraist and theoretical astronomer of the day. He was born of poor parents but, because of his prodigious talent, his education was paid for by the Duke of Brunswick. Nearly all his fundamental mathematical discoveries were made before he was 22.

You need to know...

✓ how to add, subtract, multiply and divide numbers.

Key words

area, equation, expression, formula, inequality, perimeter, substitute

Expressions

In Grade 7 we introduced algebraic expressions.

An *expression* is defined as a collection of one or more algebraic terms.

Examples of expressions are $x+4$, $5x+2$, $5a^2+3b$ and $3(a+b+c)$.

We revise that work before we move on to consider formulae.

Exercise 3a

1 Form expressions from the following sentences:

- **a** Think of a number and subtract 10.
- **b** 4 is multiplied by an unknown number.
- **c** 5 times an unknown number is subtracted a different unknown number.

2 Simplify

- **a** $3x - x + 6x$
- **b** $6x - 3x + 2x - x$
- **c** $20x - 15x + 7x - 10\text{x}$

3 Simplify

- **a** $3(2x - 1)$
- **b** $2(1 - 4x)$
- **c** $7x + 9(2 + 3x)$

4 Simplify

- **a** $2x \times 8x$
- **b** $4x \times 5y$
- **c** $x \times 2y \times 3z$
- **d** $x \times 2x$
- **e** $x \times x \times 3 \times x$

5 Find the value of

a $5x + 3$ when	**i** $x = 3$	**ii** $x = 13$
b $12 - 2x$ when	**i** $x = 4$	**ii** $x = 5$
c $\frac{5x}{2}$ when	**i** $x = 6$	**ii** $x = 20$

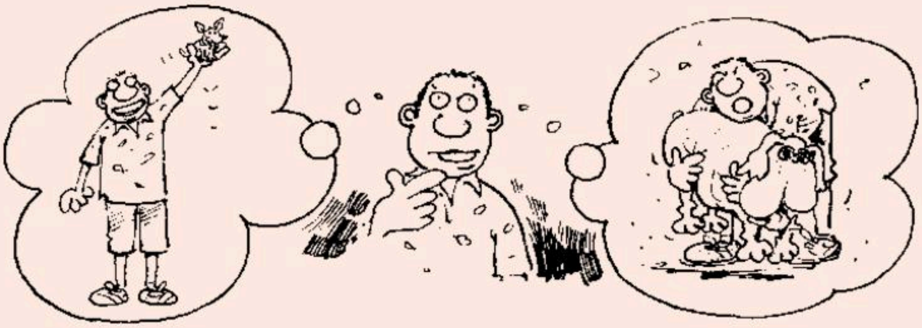
6 Simplify

- **a** $2x^2 \times 3x^2$
- **b** $x^2 \times 5x$

Puzzle

My dog weighs nine-tenths of its weight plus nine-tenths of a kilogram.

What does it weigh?



Formulae

For any triangle it is true that its *perimeter* is equal to the sum of the lengths of the three sides, provided that all measurements are made in the same units.

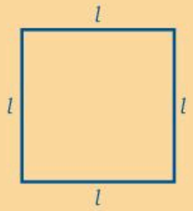
If we use letters for the variable quantities (P for the length of the perimeter and a, b and c for the lengths of the three sides), we can write the first sentence more briefly as a *formula*:

$$P = a + b + \mathrm{c}$$

Exercise 3b

The letters in the diagrams all stand for a number of centimetres.

The perimeter of the square below is P cm. Write a formula for P.



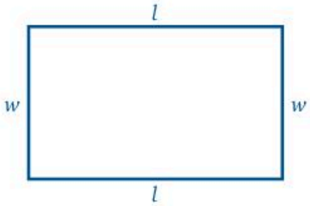
Start by writing the perimeter in terms of the letters in the diagram: this is $l + l + l + l$ (cm). As we are told that P cm is the perimeter we can write

$$P = l + l + l + l$$

Collect like terms $\quad P = 4l$

In each of the following figures the perimeter is P cm. Write a formula for P starting with $P =$

1



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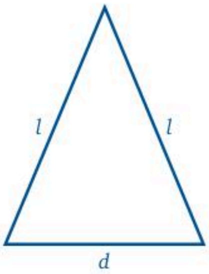

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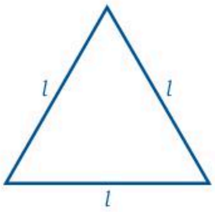

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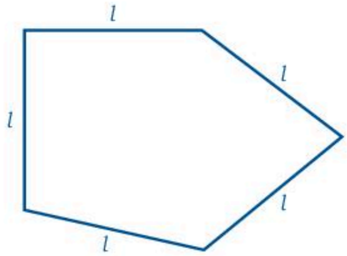

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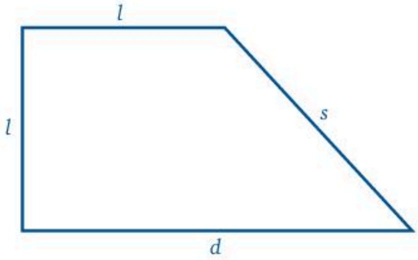


G is the number of girls and B is the number of boys in a class.

Given that T is the total number of students in the class, write a formula for T in terms of G and B.

Start by writing an expression for the total number of students.

The total number of students in the class is the sum of G and B, i.e. $G + B$.

As T is equal to the total number of students,

$$T = G + B$$

6 I buy x lb of apples and y lb of pears. Write a formula for W if W lb is the mass of fruit that I have bought.

Read each question carefully.

7 If l m is the length of a rectangle and b m is the breadth, write a formula for P if the perimeter of the rectangle is P m.

8 I start a game with N marbles and win another M marbles. Write a formula for the number, T, of marbles that I finish the game with.

9 I start a game with N marbles and lose L marbles. Write a formula for the number, T, of marbles that I finish with.

10 The length of the perimeter of a quadrilateral is P cm. The lengths of the four sides are p cm, q cm, r cm and s cm. Write down a formula for the length of the perimeter in terms of the lengths of the other sides.

11 I have a piece of string which is l cm long. I cut off a piece which is d cm long. Write a formula for L if the length of string which is left is L cm.

12 I had a bag of sweets with S sweets in it; I then ate T of them. Write a formula for the number, N, of sweets left in the bag.

A rectangle has an *area* of $A\text{ m}^2$. The rectangle is a m long and b m wide. Write down a formula for A in terms of a and b.

$$\text{Area of a rectangle} = \text{length} \times \text{breadth}$$

So $A = a \times b$ or $A = ab$ (we do not usually write the multiplication sign)

13 The side of a square is l m long. Write a formula for A if the area of the square is $A\text{ m}^2$.

14 Peaches cost n dollars each. Write a formula for N if the cost of 10 peaches is N dollars.

15 Oranges cost x dollars each and I buy n of these oranges. Write a formula for C where C dollars is the total cost of the oranges.

16 A rectangle is $2l$ m long and l m wide. Write a formula for P where P m is the perimeter of the rectangle.

17 Write down a formula for A where $A\text{ m}^2$ is the area of a rectangle $2l$ m long and l m wide.

18 A lorry has mass T tonnes when empty. Steel girders with a total mass of S tonnes are then loaded on to the lorry. Write a formula for W where W tonnes is the mass of the loaded lorry.

19 I started the term with a new packet of N felt-tipped pens. During the term I lost L of them and R of them were recycled. Write a formula for the number, S, that I had at the end of the term.

20 A truck travels p km in one direction and then it comes back q km in the opposite direction. If it is then r km from its starting point, write a formula for r.

21 One box of tinned fruit has mass K kg. The mass of n such boxes is W kg. Write a formula for W.

22 A fishing rod consists of three pieces. The first piece is x cm long, the second piece is y cm longer than the first piece and the third piece is y cm longer than the second piece. The total length of the fishing rod is L cm. Write down a formula for L in terms of x and y.

Substituting numerical values into a formula

The formula for the area of a rectangle is $A = lb$.

If a rectangle is 3 cm long and 2 cm wide, we can *substitute* the number 3 for l and the number 2 for b to give $A = 3 \times 2 = 6$.

So the area of that rectangle is 6 cm².

When you substitute numerical values into a formula you may have a mixture of operations, i.e. ( ), ×, ÷, +, −, to perform. Remember the order from the capital letters of 'Bless My Dear Aunt Sally'.

Exercise 3c

If $v = u + at$, find v when $u = 2$, $a = \frac{1}{2}$ and $t = 4$.

$$v = u + at$$

When $u = 2$, $a = \frac{1}{2}$, $t = 4$, $\quad v = 2 + \frac{1}{2} \times 4$ (Do multiplication first)

$$= 2 + 2$$

$$= 4$$

1 If $N = T + G$, find N when $T = 4$ and $G = 6$.

2 If $T = np$, find T when $n = 20$ and $p = 5$.

3 If $P = 2(l + b)$, find P when $l = 6$ and $b = 9$.

4 If $L = x - y$, find L when $x = 8$ and $y = 6$.

5 If $N = 4(l - s)$, find N when $l = 7$ and $s = 2$.

6 If $S = n(a + b)$, find S when $n = 20$, $a = 2$ and $b = 8$.

7 If $V = lbw$, find V when $l = 4$, $b = 3$ and $w = 2$.

8 If $A = \frac{PRT}{100}$, find A when $P = 100$, $R = 3$ and $T = 5$.

9 If $w = u(v - t)$, find w when $u = 5$, $v = 7$ and $t = 2$.

10 If $s = \frac{1}{2}(a + b + c)$, find s when $a = 5$, $b = 7$ and $c = 3$.

If $v = u - at$, find v when $u = 5$, $a = -2$ and $t = -3$.

$$v = u - at$$

When $u = 5$, $a = -2$, $t = -3$: $\quad v = 5 - (-2) \times (-3)$

$$\begin{aligned} &= 5 - (+6) \\ &= 5 - 6 \\ &= -1 \end{aligned}$$

(Notice that where negative numbers are substituted for letters they have been put in brackets. This makes sure that only one operation at a time is carried out.)

Put negative numbers in brackets.

11 If $N = p + q$, find N when $p = 4$ and $q = -5$.

12 If $C = RT$, find C when $R = 4$ and $T = -3$.

13 If $z = w + x - y$, find z when $w = 4$, $x = -3$ and $y = -4$.

14 If $r = u(v - w)$, find r when $u = -3$, $v = -6$ and $w = 5$.

15 Given that $X = 5(T - R)$, find X when $T = 4$ and $R = -6$.

16 Given that $P = d - rt$, find P when $d = 3$, $r = -8$ and $t = 2$.

17 Given that $v = l(a + n)$, find v when $l = -8$, $a = 4$ and $n = -6$.

18 If $D = \dfrac{a - b}{c}$, find D when $a = -4$, $b = -8$ and $c = 2$.

19 If $Q = abc$, find Q when $a = 3$, $b = -7$ and $c = -5$.

20 If $l = \dfrac{2}{3}(x + y - z)$, find l when $x = 4$, $y = -5$ and $z = -6$.

Given that $2S = d(a + l)$, find a when $S = 20$, $d = 2$ and $l = 16$.

$$2S = d(a + l)$$

Substituting $S = 20$, $d = 2$, $l = 16$ gives

$$40 = 2(a + 16)$$

We can now solve this *equation* for a.

Multiply out the brackets $\quad 40 = 2a + 32$

Take 32 from each side $\quad 8 = 2a$

Divide by 2 $\quad 4 = a \quad$ or $\quad a = 4$

21 Given that $N = G + B$, find B when $N = 40$ and $G = 25$.

22 If $R = t \div c$, find t when $R = 10$ and $c = 20$.

23 Given that $d = st$, find t when $d = 50$ and $s = 15$.

24 If $N = 2(p + q)$, find q when $N = 24$, and $p = 5$.

25 Given that $L = P(2 - a)$, find a when $L = 10$ and $P = 40$.

26 Given that $v = u + at$, find u when $v = 32$, $a = 8$ and $t = 4$.

27 If $v^2 = u^2 + 2as$, find a when $v = 3$, $u = 2$ and $s = 12$.

28 If $H = P(Q - R)$, find Q when $H = 12$, $P = 4$ and $R = -6$.

Problems

Exercise 3d

1 Given that $v = at$, find the value of

- **a** v when $a = 4$ and $t = 12$
- **b** v when $a = -3$ and $t = 6$
- **c** t when $v = 18$ and $a = 3$
- **d** a when $v = 25$ and $t = 5$.

2 Given that $N = 2(n - m)$, find the value of

- **a** N when $n = 6$ and $m = 4$
- **b** N when $n = 7$ and $m = -3$
- **c** n when $N = 12$ and $m = 2$
- **d** m when $N = 16$ and $n = -4$.

3 If $A = P + QT$, find the value of

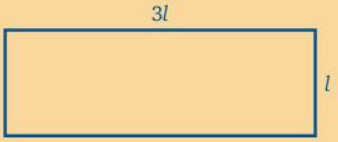
- **a** A when $P = 50$, $Q = \frac{1}{2}$ and $T = 4$
- **b** A when $P = 70$, $Q = 5$ and $T = -10$
- **c** P when $A = 100$, $Q = \frac{1}{4}$ and $T = 16$
- **d** T when $A = 25$, $P = -15$ and $Q = -10$.

4 If $P = 100r - t$, find the value of

- **a** P when $r = 0.25$ and $t = 10$
- **b** P when $r = 0.145$ and $t = 15.6$
- **c** t when $P = 18.5$ and $r = 0.026$
- **d** r when $P = 50$ and $t = -12$.

A rectangle is $3l$ cm long and l cm wide. If the area of the rectangle is A cm², write a formula for A.

Use your formula to find the area of this rectangle if it is 5 cm wide.



$$\text{Area} = \text{length} \times \text{width}$$

$$\therefore A = 3l \times l$$

$$\text{When } l = 5, \quad A = 3 \times 5 \times 5$$

$$= 75$$

$$\therefore \text{Area} = 75\,\text{cm}^2$$

5 Oranges cost n dollars each. If the cost of a box of 50 of these oranges is C dollars, write a formula for C. Use your formula to find the cost of a box of oranges if each orange costs \$24.

6 Lemons cost \$$n$ each. The cost of a box of 50 lemons is \$$L$. Write a formula for L. Use your formula to find the cost of a box of these lemons when they cost \$95 each.

7 A rectangle is a cm long and b cm wide. Write a formula for p if p cm is the perimeter of the rectangle. Use your formula to find the perimeter of a rectangle measuring 20 cm by 15 cm.

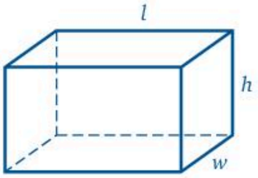
8 The length of a rectangle is twice its width. If the rectangle is x cm wide, write a formula for p if its perimeter is p cm. Use your formula to find the width of a rectangle that has a perimeter of 36 cm.

9 A roll of paper is L m long. N pieces each of length r m are cut off the roll. If the length of paper left is p m, write a formula for p. Use your formula to find the length of paper left from a roll that was 20 m long after 10 pieces, each of length 1.5 m, are cut off.

10 An equilateral triangle has sides each of length a cm. If the perimeter of the triangle is p cm, write a formula for p. Use your formula to find the lengths of the sides of an equilateral triangle whose perimeter is 72 cm.

11 Tins of baked beans weigh a g each. N of these tins are packed into a box. The empty box weighs p g. Write a formula for W where W g is the weight of the full box. Use your formula to find the number of tins that are in a full box if the full box weighs 10 kg, the empty box weighs 1 kg and each tin weighs 200 g.

Be careful with the units.



12 The rectangular box in the diagram is l cm long, w cm wide and h cm high. Write a formula for A if $A\,\text{cm}^2$ is the total surface area of the box (i.e. the area of all six faces). Use your formula to find the surface area of a rectangular box measuring 50 cm by 30 cm by 20 cm.

13 A person whose weight on Earth is W finds his weight on certain planets from these formulae:

a weight on Venus $0.85\,W$

b weight on Mars $0.38\,W$

c weight on Jupiter $2.64\,W$.

Calculate your weight on each of the above planets.

Changing the subject of a formula

Suppose that we have to use the formula $A = lb$ to find the value of l when $A = 20$ and $b = 5$. There are two ways of doing this.

Either we can substitute the numbers directly, giving $20 = l \times 5$, and solve this equation for l, which gives $l = 4$ or, by dividing both sides of the formula by b, we can rearrange the formula to $l = \frac{A}{b}$, then substitute in the numbers to give $l = \frac{20}{5} = 4$.

When the formula is in the form $A = lb$, A is called the subject of the formula.

When the formula is in the form $l = \frac{A}{b}$, l is called the subject of the formula.

Changing from $A = lb$ to $l = \frac{A}{b}$ is called changing the subject of the formula.

Exercise 3e

Make r the subject of the formula $p = q + r$

(To make r the subject of $p = q + r$ we have to 'solve' the formula for r. This means we have to isolate r on one side of the equals sign.)

$$p = q + r$$

Take q from both sides $\quad p - q = r$

or $\quad r = p - q$

Make the letter in brackets the subject of the following formulae.

1 $N = T + G \quad (T)$

2 $P = Q + R \quad (R)$

3 $L = X - Y \quad (X)$

4 $A = B - C \quad (C)$

5 $a = b + c + d \quad (d)$

6 $x - y = z \quad (x)$

7 $d+e=f$ (e)

8 $x+y=z$ (x)

9 $s=a+2b$ (a)

10 $v=u+t$ (u)

11 $S=d-t$ (d)

12 $P=2y+z$ (z)

13 $L=a+b+c$ (a)

14 $P=a+b$ (a)

15 $N=R+T$ (T)

16 $b=a+c+d$ (a)

17 $v=rt+u$ (u)

18 $x=y-z$ (y)

19 $P=ab+c$ (c)

20 $v=u+at$ (u)

Exercise 3f

Rearrange the formula $P=\frac{n}{3m}$ to make n the subject.

Multiplying both side by $3m$ gives $P\times 3m=n$

i.e. $n=3mP$

Make the letter in brackets the subject of the following formulae.

1 $z=xy$ (x)

2 $S=\frac{d}{t}$ (d)

3 $N=rn$ (n)

4 $C=RT$ (T)

5 $L=\frac{m}{n}$ (m)

6 $v=u+at$ (u)

7 $pq=r$ (p)

8 $X=y+2z$ (y)

9 $2a+b=3c$ (b)

10 $2s=a+b+c$ (a)

11 $nm=p$ (m)

12 $\frac{f}{g}=2j$ (f)

$\underline{\underline{13}}$ $2c-b=a$ (b)

$\underline{\underline{14}}$ $\frac{4p}{q}=r$ (q)

$\underline{\underline{15}}$ $2a+3b-c=L$ (c)

16 $a+2b+3c=4d$ (a)

17 $a^2+c=b^2$ (c)

$\underline{\underline{18}}$ $p^2-q^2=3-r$ (r)

19 Make r the subject of the formula $2s+r=t$

Inequality notation

Consider the statement

‘More than 1000 people came to see a firework display.’

This is an example of an *inequality*.

We do not know how many people were at the firework display, but if we use x as the variable number of people, we can write this statement as

$$x > 1000$$

where the symbol $>$ means 'is greater than'.

Now consider the statement

'At least 50 students belong to the chess club.'

This is also an inequality but it is slightly different from the first example because it means 50 or more students belong to the chess club. Using n for the number of students we can write this statement as

$$n \geqslant 50$$

where the symbol $\geqslant$ means 'is greater than or equal to'.

Other inequalities involve quantities that are less than, or less than or equal to, a given quantity. For example, 'The cost of oil is now less than \$60 a barrel' or 'Ten or fewer people came to the meeting'.

The symbols used in these cases are $<$ which means 'is less than' and $\leqslant$ which means 'is less than or equal to'.

Exercise 3g

Form an inequality from the statement 'more than 100 guests attended the wedding'.

$n > 100$ where n is the number of guests who attended the wedding.

Form an inequality from the following statements. For each question choose a letter to represent the variable and state what your letter stands for.

1 A school library can afford to buy fewer than 20 new books.

2 Fewer than 100 people attended a rally.

Read the questions **very** carefully to make sure you understand whether the inequality includes the number given or does not.

3 More than 20 albums were sold on the first day.

4 Fifty or more cars passed the school between 9 and 10 a.m.

5 The perimeter of a rectangle is not more than 50 cm.

6 The cost of making a widget is less than \$50.

7 Ceejay owns at least 3 goats.

8 There are more than 50 one dollar coins in a bag.

9 It takes at least 250 days to build a bungalow.

10 Victoria has more than 5 pens in her school bag.

11 Jesse has fewer than 5 rubbers in his school bag.

12 A box of fireworks contains at least 50 fireworks.

13 A bus journey takes 10 minutes or less to get to school.

14 There are at most twenty \$500 bank notes in a cash box.

15 Anna is at least two years older than her sister Charelle, who is 4 years old.

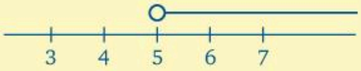
Using a number line to illustrate inequalities

Consider the statement

$$x > 5$$

This is an inequality (as opposed to $x = 5$ which is an equality or equation).

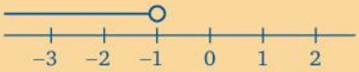
This inequality is true when x stands for any number that is greater than 5. Thus there is a range of numbers that x can stand for and we can illustrate this range on a number line.



The circle at the left-hand end of the range is 'open', because 5 is not included in the range.

Exercise 3h

Use a number line to illustrate the range of values of x for which $x < -1$



(The open circle means that –1 is not included. All values smaller than –1 are to the left of it on the number line.)

Use a number line to illustrate the range of values of x for which each of the following inequalities is true:

1 $x > 7$

2 $x < 4$

3 $x > -2$

4 $x > 0$

5 $x < -2$

6 $x > \frac{1}{2}$

7 $x < 5$

8 $x < 0$

9 $x < 1.5$

10 State which of the inequalities given in questions **1** to **9** are satisfied by a value of x equal to

a 2 **b** –3 **c** 0 **d** 1.5 **e** 0.0005

11 For each of the questions **1** to **9** give a number that satisfies the inequality and is

a a whole number **b** not a whole number.

12 Consider the true inequality $3 > 1$.

a Add 2 to each side.

b Add –2 to each side.

c Take 5 from each side.

d Take –4 from each side.

In each case state whether or not the inequality remains true.

13 Repeat question **12** with the inequality $-2 > -3$.

14 Repeat question **12** with the inequality $-1 < 4$.

15 Try adding and subtracting different numbers on both sides of a true inequality of your own choice.

Solving inequalities

From the last exercise we can see that:

> An inequality remains true when the *same* number is added to, or subtracted from, *both* sides.

Now consider the inequality $x - 2 < 3$.

Solving this inequality means finding the range of values of x for which it is true.

Adding 2 to each side gives $x < 5$.

We have now solved the inequality.

Exercise 3i

Solve the following inequalities and illustrate your solutions on a number line:

1 $x - 4 < 8$

2 $x + 2 < 4$

3 $x - 2 > 3$

4 $x - 3 > -1$

5 $x + 4 < 2$

6 $x - 5 < -2$

7 $x - 3 < -6$

8 $x + 7 < 0$

9 $x + 2 < -3$

Add 4 to each side first.

Solve the inequality $4 - x < 3$.

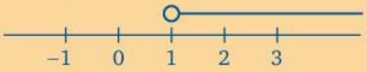
$4 - x < 3$

(Aim to get the x term on one side of the inequality and the number term in the other.)

Add x to each side $\quad 4 < 3 + x$

Take 3 from each side
If 1 is less than x then $\quad 1 < x$ or $x > 1$
x must be greater than 1.

An inequality remains true if the sides are reversed but you must remember to reverse the inequality sign.



Solve the following inequalities and illustrate your solutions on a number line:

10 $4 - x > 6$
11 $2 < 3 + x$
12 $7 - x > 4$
13 $5 < x + 5$
14 $5 - x < 8$
15 $2 > 5 + x$
16 $3 - x > 2$
17 $6 < x + 8$
18 $2 + x < -3$
19 $2 > x - 3$
20 $4 < 5 - x$
21 $1 < -x$
22 $3 - x < 3$
23 $5 < x - 2$
24 $7 > 2 - x$
25 $3 > -x$
26 $4 - x > -9$
27 $5 - x < -7$

28 Consider the true inequality $12 < 36$.

a Multiply each side by 2.
b Divide each side by 4.
c Multiply each side by 0.5.
d Divide each side by 6.
e Multiply each side by –2.
f Divide each side by –3.

In each case state whether or not the inequality remains true.
If the inequality does not remain true, what must be done so that it becomes true?

29 Repeat question **28** with the true inequality $36 > -12$.

30 Repeat question **28** with the true inequality $-18 < -6$.

31 Repeat question **28** with a true inequality of your own choice.

32 Can you multiply both sides of an inequality by any one number and be confident that the inequality remains true?

An inequality remains true when both sides are multiplied or divided by the same *positive* number.

Multiplication or division of an inequality by a negative number should be avoided, because it destroys the truth of the inequality.

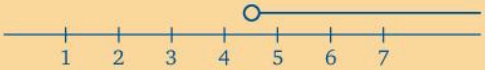
Exercise 3j

Solve the inequality $2x-4>5$ and illustrate the solution on a number line.

$$2x-4>5$$

Add 4 to both sides $\quad 2x>9$

Divide both sides by 2 $\quad x>4\frac{1}{2}$



Solve the inequalities and illustrate the solutions on a number line:

1 $3x-2<7$

2 $1+2x>3$

3 $4x-1>7$

4 $3+5x<8$

5 $5+2x<6$

6 $3x+1>5$

7 $4x-5<4$

8 $6x+2>11$

Solve the inequality $3-2x\leqslant 5$ and illustrate the solution on a number line.
($\leqslant$ means 'less than or equal to'.)

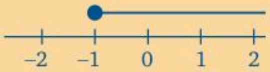
(As with equations, we collect the letter term on the side with the greater number to start with. In this case we collect on the right.)

$$3-2x\leqslant 5$$

Add $2x$ to each side $\quad 3\leqslant 5+2x$

Take 5 from each side $\quad -2\leqslant 2x$

Divide each side by 2 $\quad -1\leqslant x \quad$ i.e. which in reverse is $x\geqslant -1$



(A solid circle is used for the end of the range because –1 *is* included.)

Solve the inequalities and illustrate each solution on a number line:

9 $3 \leqslant 5 - 2x$

10 $5 \geqslant 2x - 3$

11 $4 - 3x \leqslant 10$

12 $4 \geqslant 9 - 5x$

13 $10 < 3 - 7x$

14 $8 - 3x \geqslant 2$

15 $x - 1 > 2 - 2x$

16 $2x + 1 \geqslant 5 - x$

17 $3x + 2 \leqslant 5x + 2$

18 $2x + 1 \leqslant 7 - 4x$

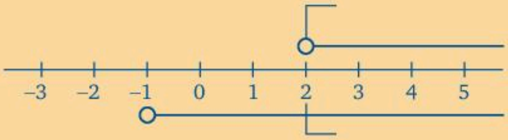
19 $1 - x > 2x - 2$

20 $2x - 5 > 3x - 2$

Find, where possible, the range of values of x which satisfy both of the inequalities:

a $x > 2$ and $x > -1$ **b** $x \leqslant 2$ and $x > -1$ **c** $x \geqslant 2$ and $x \leqslant -1$

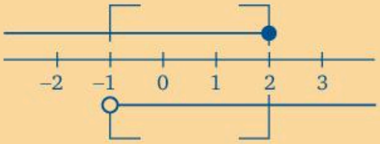
a



(Illustrating the ranges on a number line, we can see that both inequalities are satisfied for values on the number line where the ranges overlap.)

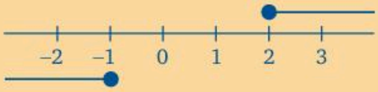
$\therefore$ $x > 2$ and $x > -1$ are both satisfied for $x > 2$.

b



$x \leqslant 2$ and $x > -1$ are both satisfied for $-1 < x \leqslant 2$.

c



There are no values of x for which $x \geqslant 2$ and $x \leqslant -1$ are both satisfied. (The lines do not overlap.)

Find, where possible, the range of values of x for which the two inequalities are both true:

21 **a** $x > 2$ and $x > 3$
  b $x \geqslant 2$ and $x \leqslant 3$
  c $x < 2$ and $x > 3$

22 **a** $x \geqslant 0$ and $x \leqslant 1$
  b $x \leqslant 0$ and $x \leqslant 1$
  c $x < 0$ and $x > 1$

23 **a** $x \leqslant 4$ and $x > -2$
  b $x \geqslant 4$ and $x < -2$
  c $x \leqslant 4$ and $x < -2$

24 **a** $x < -1$ and $x > -3$
  b $x < -1$ and $x < -3$
  c $x > -1$ and $x < -3$

Solve each of the following pairs of inequalities and then find the range of values of x which satisfy both of them:

25 $x - 4 < 8$ and $x + 3 > 2$

26 $3 + x \leqslant 2$ and $4 - x \leqslant 1$

27 $x - 3 \leqslant 4$ and $x + 5 \geqslant 3$

28 $2x + 1 > 3$ and $3x - 4 < 2$

29 $5x - 6 > 4$ and $3x - 2 < 7$

30 $3 - x > 1$ and $2 + x > 1$

31 $1 - 2x \leqslant 3$ and $3 + 4x < 11$

32 $0 > 1 - 2x$ and $2x - 5 \leqslant 1$

Find the values of x for which $x - 2 < 2x + 1 < 3$.

Some people call this a double-sided inequality.

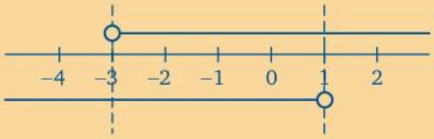
($x - 2 < 2x + 1 < 3$ represents two inequalities,

i.e. $x - 2 < 2x + 1$ and $2x + 1 < 3$, so solve each one separately.)

$x - 2 < 2x + 1$ $\qquad$ $2x + 1 < 3$

$-2 < x + 1$ $\qquad$ $2x < 2$

$-3 < x$ i.e. $x > -3$ $\qquad$ $x < 1$



So $-3 < x < 1$

Find the range of values of x for which the following inequalities are true:

33 $x + 4 > 2x - 1 > 3$

34 $x - 3 \leqslant 2x \leqslant 4$

35 $3x + 1 < x + 4 < 2$

36 $2 - x < 3x + 2 < 8$

37 $2 - 3x \leqslant 4 - x \leqslant 3$

38 $x - 3 < 2x + 1 < 5$

39 $2x < x - 3 < 4$

40 $4x - 1 < x - 4 < 2$

41 $4 - 3x < 2x - 5 < 1$

42 $x < 3x - 1 < x + 1$

Mixed exercises

Exercise 3k

1 There are three classes in the first year of Appletown School. There are a children in one class, b children in another class and c children in the third class. Write a formula for the number, N, of children in the first year.

2 Given that $n = N - ab$, find the value of N when $n = 6$, $a = -1$ and $b = 2$.

3 I think of a number and double it, then I add on 3 and double the result: this gives 14. If x stands for the number I first thought of, form an equation for x and then solve it.

4 Given that $r = s - vt$, find the value of r when $s = 4$, $v = 3$ and $t = -2$.

5 Solve the inequalities and illustrate each solution on a number line.

a $2x - 3 < 5$ $\qquad$ **b** $x + 3 \leqslant 4 - x < 5$

Exercise 3I

Select the letter that gives the correct answer.

1 $4x - 3x + 2x$ simplifies to

A x **B** $2x$ **C** $3x$ **D** $4x$

2 $4(2 - 3x)$ simplifies to

A $6 - 7x$ **C** $6 + 7x$

B $8 - 12x$ **D** $8 + 12xv$

3 $3x \times 2y \times 2z$ simplifies to

A $7xyz$ **B** $8xyz$ **C** $10xyz$ **D** $12xyz$

4 The value of $4x - 3$ when $x = 3$ is

A 6 **B** 9 **C** 12 **D** 15

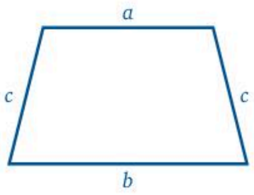
5 The value of $16 - 3x$ when $x = 4$ is

A 2 **B** 4 **C** 6 **D** 8

6 When $x = 4$, the value of $x^2 - 3x$ is

A 4 **B** 6 **C** 8 **D** 12

7

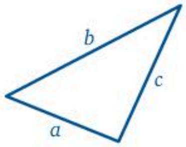


If P cm is the perimeter of this shape then $P =$

A $a + b - c$ **C** $a + b + 2c$

B $a - b + c$ **D** $2a + 2b - 2c$

8



If P cm is the perimeter of this shape then $P =$

A $a + b - c$ **C** $ab + 2c$

B $a + b + c$ **D** $2abc$

9 A rectangle is $4a$ cm long and $3a$ cm wide. The formula for P, where P cm is the perimeter of the rectangle is

A $P = 7a$ **C** $P = 11a$

B $P = 10a$ **D** $P = 14a$

10 If $P = 2(l + b)$, the value of P when $l = 9$ and $b = 4$ is

A 13 **B** 22 **C** 26 **D** 36

11 If $P = q - r - s$, the value of P when $q = 5$, $r = 3$ and $s = -1$ is

A 3 **B** 5 **C** 7 **D** 9

12 Given that $R = uvw$, the value of R when $u = 4$, $v = -5$ and $w = -3$ is

A -4 **B** -32 **C** -60 **D** 60

13 Given that $A = B + 2C$, the value of B when $A = 10$, and $C = -3$ is

A 4 **B** 7 **C** 16 **D** 60

14 Given that $v = u + at$, the value of u when $v = 21$, $a = 3$ and $t = 4$ is

A 9 **B** 10 **C** 14 **D** 15

15 If $P = \frac{q-r}{s}$, the value of P when $q = -3$, $r = -11$ and $s = 4$ is

A -2 **B** $-3\frac{1}{2}$ **C** 2 **D** $3\frac{1}{2}$

16 If the formula $P = a + 2b$ is rearranged then $a =$

A $p - 2b$ **C** $2b - p$

B $P + 2b$ **D** $2Pb$

17 If b is made the subject of the formula $4a + b = 3c$ then $b =$

A $4a - 3c$ **C** $7ac$

B $3c - 4a$ **D** $12ac$

18 Given that $p + 2q + 3r = 4s$ then $p =$

A $4s - 3r - 2q$ **C** $4s - 3r + 2q$

B $4s + 3r - 2q$ **D** $4s + 3r + 2q$

19 Which two of the numbers 3, 4, 6 and 7 satisfy the inequality $x > 5$?

A 3 and 7 **B** 4 and 6 **C** 4 and 7 **D** 6 and 7

20 The solution of the inequality $5 - x > 4$ is

A $x < 1$ **B** $x < 2$ **C** $x < 3$ **D** $x < 5$

21 If $4 \leqslant 7 - 3x$ then

A $x < -1$ **B** $x \leqslant -1$ **C** $x < 1$ **D** $x \leqslant 1$

22 If $x > 3$ and $x \leqslant 4$ then

A $2 < x \leqslant 4$ **B** $3 < x < 4$ **C** $3 < x \leqslant 4$ **D** $3 \leqslant x \leqslant 4$

23 Given that $x - 5 < 7$ and $x + 2 > 1$, then

A $-1 < x < 12$ **B** $-1 < x \leqslant 10$ **C** $1 < x < 10$ **D** $1 < x < 12$

Puzzle

Find two numbers, one of which is twice the other, such that the sum of their squares is equal to the cube of one of the numbers.

Did you know?

Optical illusions use patterns, shapes or colours to create images that are misleading.

Estimate which distance is the greater – from A to B or from B to C.

Now check your answer by measuring.

Investigate other optical illusions.

In this chapter you have seen that…

- ✔ you can construct formulae from given information
- ✔ you can substitute numerical values in a formula
- ✔ you can rearrange a formula to make a different letter the subject
- ✔ you can form an inequality from a given statement
- ✔ an inequality remains true when the same number is added to, or subtracted from, both sides
- ✔ an inequality remains true when both sides are multiplied or divided by the same **positive** number. Do not multiply or divide an inequality by a negative number. It changes the direction of the inequality.

4 Lines and angles

At the end of this chapter you should be able to...

1 identify acute, obtuse and reflex angles

2 define a degree as a fraction of a revolution

3 draw angles of given size using a protractor

4 state the properties of vertically opposite angles, angles on a straight line, supplementary angles, angles at a point and complementary angles

5 identify two parallel lines as lines that are always the same distance apart

6 identify corresponding angles

7 use the equality of corresponding angles to draw parallel lines

8 solve problems involving corresponding angles

9 identify alternate angles

10 state and use the equality property of alternate angles

11 use the interior angles property for a pair of parallel lines and a transversal.

Did you know?

The word 'geometry' means land measurement. Historians state that the term originated in Egypt where the River Nile, in its floods, overflowed its banks and changed the boundaries of neighbouring farms. 'Geo' is the Greek word for 'earth'. What other common words start with 'geo'?

You need to know...

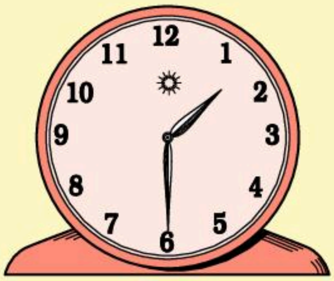
- ✔ how to find a fraction of a quantity
- ✔ how to use a protractor to measure angles
- ✔ how to read an analogue clock face
- ✔ what compass directions are
- ✔ the meaning of clockwise and anticlockwise.

Key words

acute angle, angles at a point, angles on a straight line, alternate angles, complementary angles, corresponding angles, degree, interior angles, obtuse angle, parallel lines, parallelogram, protractor, reflex angle, revolution, right angle, supplementary angles, transversal, vertex, vertically opposite angles

Angles

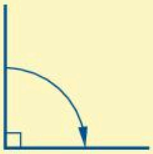
When the hand of a clock moves from one position to another it has turned through an angle.



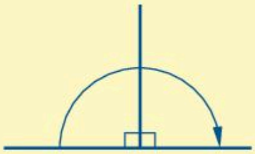
An angle measures an amount of turning.
One complete turn is called a *revolution*.

Right angles

A quarter of a revolution is called a *right angle*.

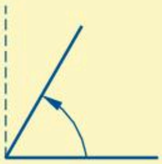


Half a revolution is two right angles.
Half a revolution gives a straight line.



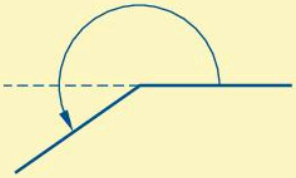
Acute, obtuse and reflex angles

Any angle that is smaller than a right angle is called an *acute angle*.



Any angle that is greater than one right angle and less than two right angles is called an *obtuse angle*.

Any angle that is greater than two right angles is called a *reflex angle*.



Exercise 4a

What type of angle is each of the following?

1

2

3

4

5

6

7

8

9

Degrees

One complete revolution is divided into 360 parts. Each part is called a *degree*. 360 degrees is written 360°.

360 seems a strange number of parts to have in a revolution but it is a good number because so many whole numbers divide into it exactly. This means that there are many fractions of a revolution that can be expressed as an exact number of degrees.

Exercise 4b

1 How many degrees are there in half a revolution?

2 How many degrees are there in one right angle?

3 How many degrees are there in three right angles?

How many degrees has the seconds hand of a clock moved through when it turns from 6 to 8?

Drawing the clock face shows that the hand moves through 2 out of 3 equal divisions of 90°, i.e. it moves through $\frac{2}{3}$ of 90° and $\frac{2}{3}$ of $90^\circ = \frac{2}{3} \times \frac{90^\circ}{1} = 60^\circ$.

Another way of looking at it is to say that the hand moves through 2 out of 12 equal divisions of a revolution, i.e. $\frac{2}{12}$ of 360°.

How many degrees has the seconds hand of a clock turned through when it moves from:

4 8 to 9

5 10 to halfway between 11 and 12

6 6 to 10

7 1 to 3

8 3 to halfway between 4 and 5

9 4 to 5

10 7 to 11

11 3 to 10

12 2 to 8

13 10 to 8

14 12 to 11

15 9 to 2

16 8 to 3

17 7 to 5

Drawing angles using a protractor

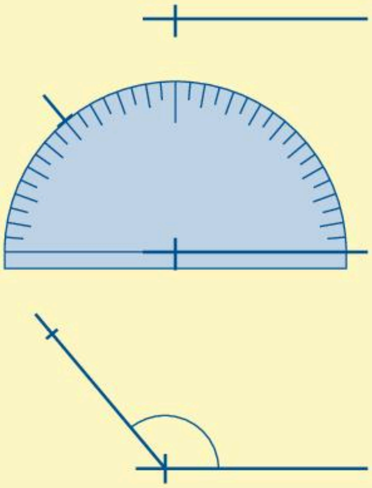
We saw how to use a *protractor* in Grade 7. This section revises that work.

To draw an angle of 130° start by drawing one arm and mark the *vertex*.

Place your protractor as shown in the diagram. Make sure that the vertex is at the centre of the base line.

Choose the scale that starts at 0° on your drawn line and mark the paper next to the 130° mark on the scale.

Remove the protractor and join your mark to the vertex.



Now look at your angle: does it look the right size?

Exercise 4c

Use your protractor to draw the following angles accurately:

1 35°

2 27°

3 65°

4 150°

5 93°

6 5°

7 120°

8 39°

9 75°

10 115°

11 165°

12 62°

13 145°

14 126°

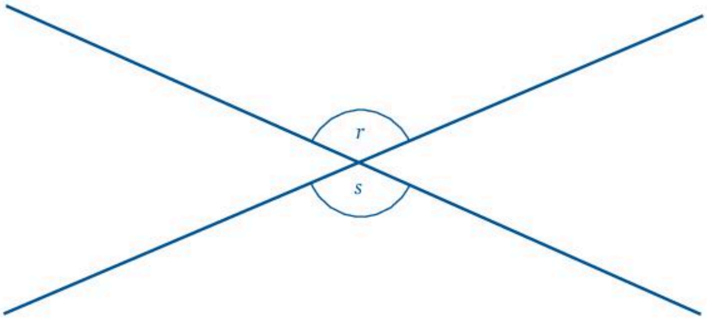
15 95°

Change books with your neighbour and measure each other's angles as a check.

Exercise 4d

In questions **1** and **2** first measure the angle marked *r*. Then estimate the size of the angle marked *s*. Check your estimate by measuring angle *s*.

1



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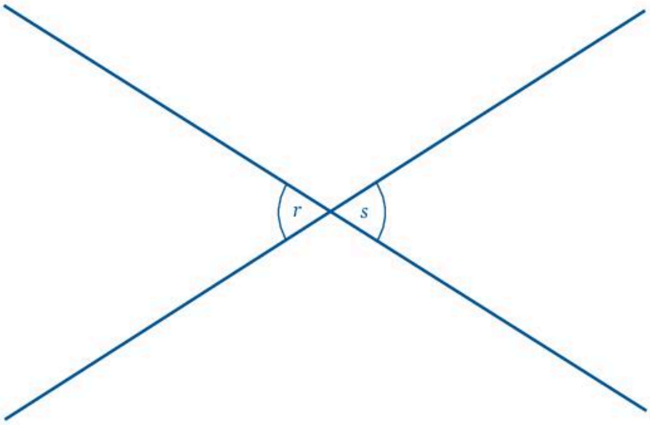


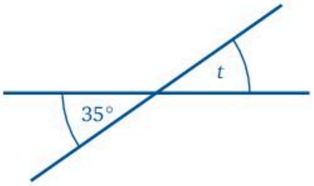
3 Draw some more similar diagrams and repeat questions **1** and **2**.

In questions **4** to **9**, write down the size of the angle marked *t*, without measuring it:

4

5

6



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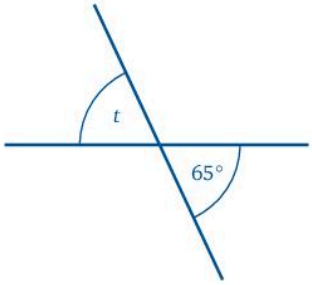

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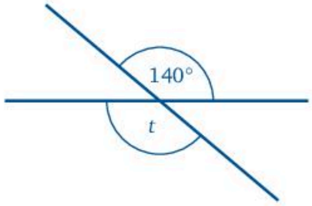

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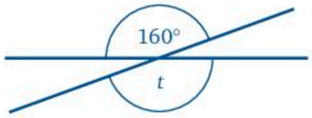
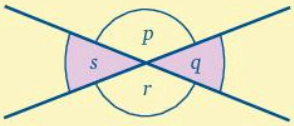


Vertically opposite angles

When two straight lines cross, four angles are formed.



The two angles that are opposite each other are called *vertically opposite angles*. After working through the last exercise you should now be convinced that

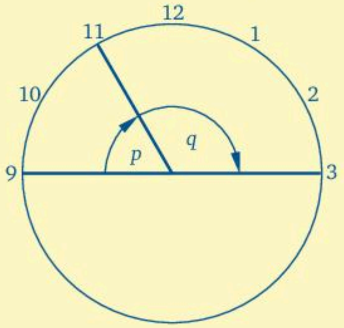
vertically opposite angles are equal

i.e. $p = r$ and $s = q$.

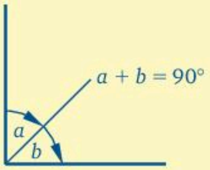
Angles on a straight line

The seconds hand of a clock starts at 9 and stops at 11 and then starts again and finally stops at 3.

Altogether the seconds hand has turned through half a revolution, so $p + q = 180°$.



Complementary angles



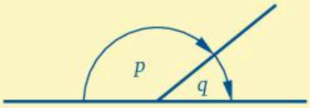
Angles in a right angle add up to 90°.

The angles a and b are adjacent. They share an arm.

Two angles that add up to 90° are called *complementary angles*.

Any two angles that add to 90° are complementary. They do not have to be adjacent angles.

Supplementary angles



Angles on a straight line add up to 180°.

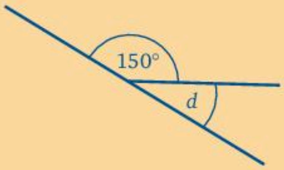
An angle of 180° is called a straight line.

Two angles that add up to 180° are called *supplementary angles*.

Any two angles that add to 180° are supplementary. They do not have to be adjacent angles.

Exercise 4e

Calculate the size of angle d.



Angles d and 150° together make a straight line.

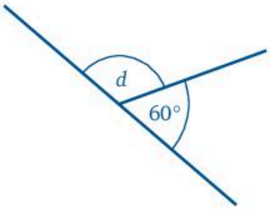
So $d + 150^\circ = 180^\circ$

$\therefore \qquad d = 30^\circ$

($\therefore$ means 'therefore' or 'it follows that')

In questions **1** to **12** calculate the size of the angle marked with a letter.
Give a reason for your answer.

1



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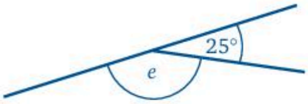

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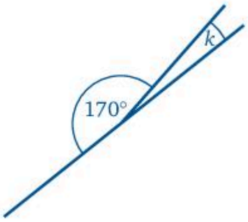

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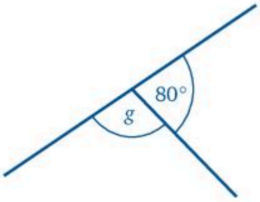

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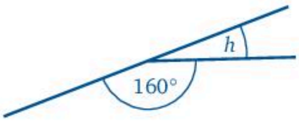

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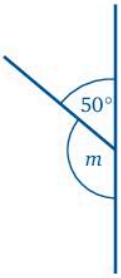

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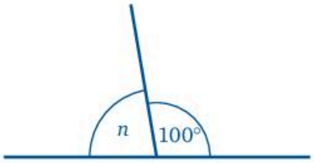

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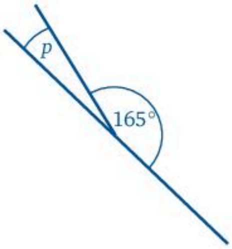

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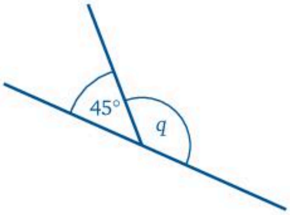

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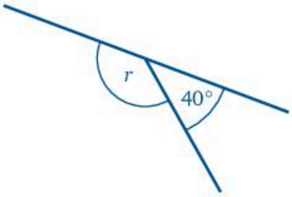

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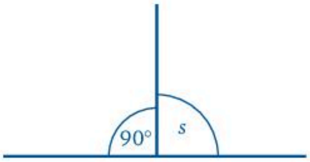

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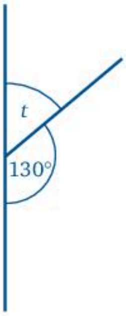
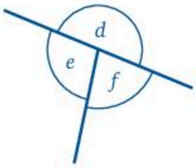


In questions **13** to **18** write down the pairs of angles that are supplementary:

13



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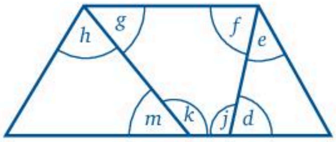

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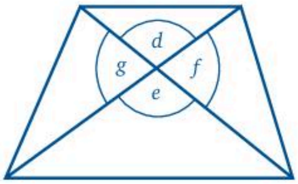

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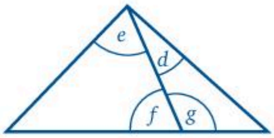

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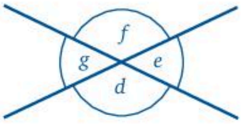

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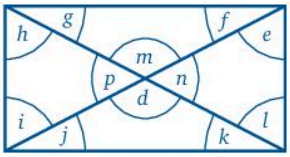
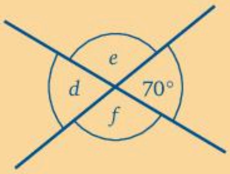


Calculate the sizes of angles *d*, *e* and *f*.



d and 70° are equal (they are vertically opposite)

$\therefore \quad d = 70°$

e and 70° add up 180° (they are angles on a straight line)

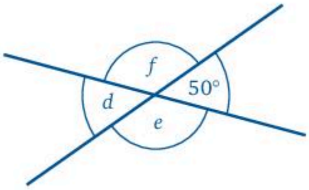
$\therefore \quad e = 110°$

f and *e* are equal (they are vertically opposite)

$\therefore \quad f = 110°$

In questions **19** to **26** calculate the size of the angles marked with a letter.
Give a reason for your answer.

19



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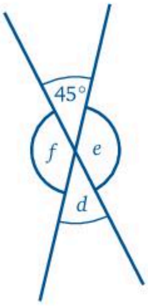

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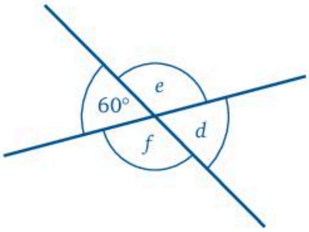

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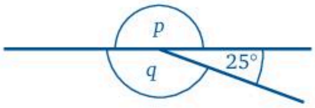

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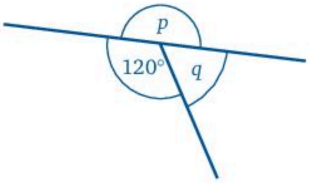

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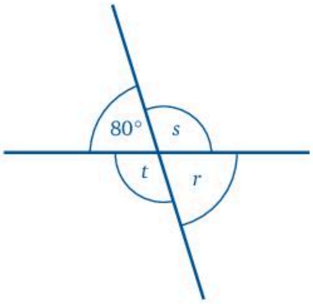

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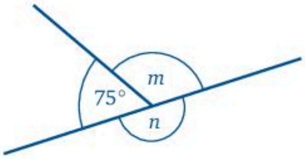

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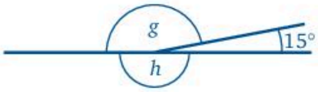
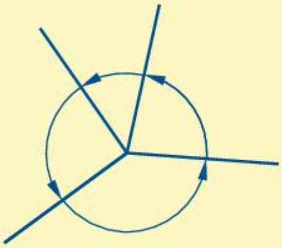


Angles at a point

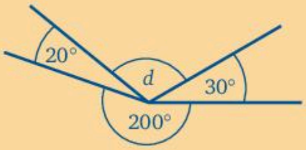
When several angles make a complete revolution they are called *angles at a point*.



Angles at a point add up to 360°.

Exercise 4f

Find the size of angle d. Give a reason for your answer.



The four angles at the point add up to 360°.

The three given angles add up to 250°.

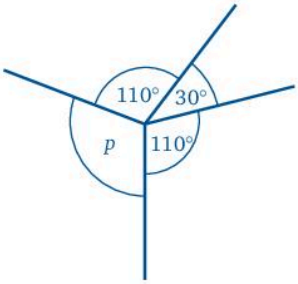
$$\therefore \quad d = 360° - 250°$$

$$d = 110°$$

$$\begin{array}{r} 30 \\ 200 \\ +20 \\ \hline 250 \\ \hline \end{array}$$

In questions **1** to **10** find the size of the angle marked with a letter. Give a reason for your answer.

1



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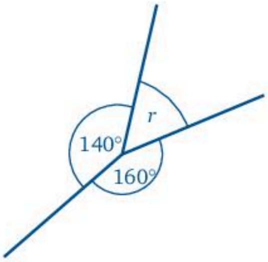
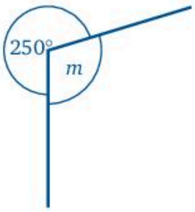


Remember that, however many angles there are at a point, their sum is 360°.

3



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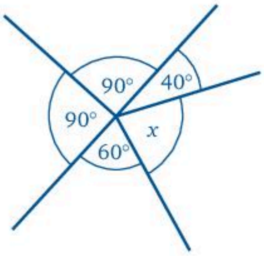

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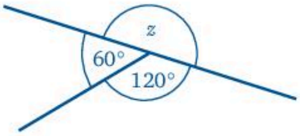

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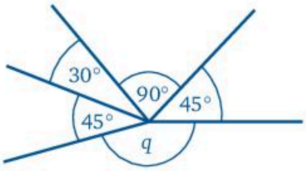

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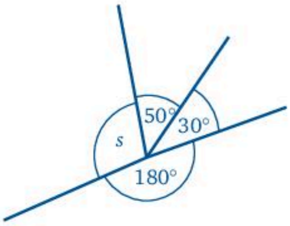

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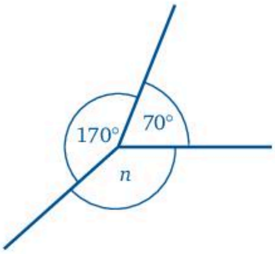

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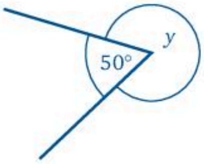

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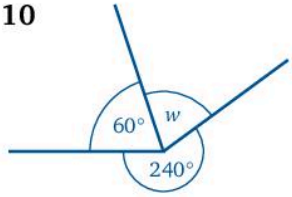
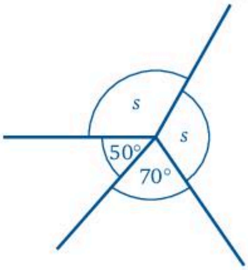


Problems

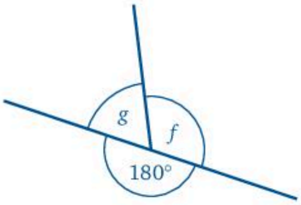
Exercise 4g

1 Find each of the equal angles marked *s*.

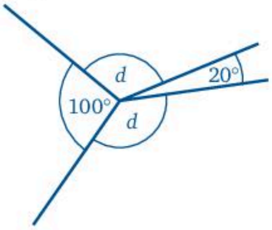


Do not always expect to see immediately how to find the angles asked for. Copy the diagram. On your copy, mark the sizes of any angles that you know or can work out. This may prompt you where to go next.

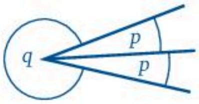
2 The angle marked *f* is twice the angle marked *g*. Find angles *f* and *g*.



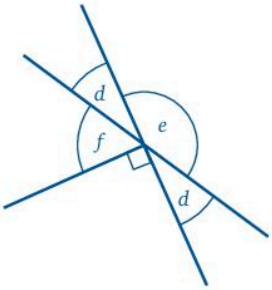
3 Find each of the equal angles marked *d*.



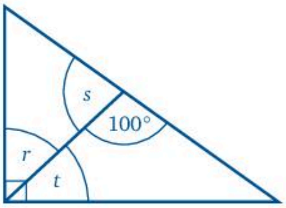
4 Each of the equal angles marked *p* is 25°. Find the reflex angle *q*.



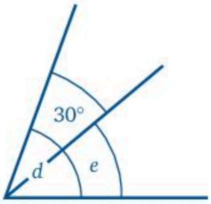
5 Each of the equal angles marked *d* is 30°. Angle *d* and angle *e* are supplementary. Find angles *e* and *f*. (An angle marked with a square is a right angle.)



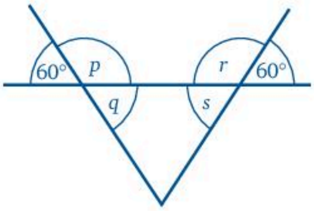
6 Angle *s* is twice angle *t*. Find angle *r*.



7 The angle marked *d* is 70°. Find angle *e*.



8 Find the angles marked *p*, *q*, *r* and *s*.



Investigation

1 How do you find the angle turned through by the hour hand of a clock in a given time?

Start by finding the angle turned through in 12 hours, then the angle for any other complete number of hours.

Next find the angle turned through for any fraction of an hour and, lastly, through a number of minutes.

2 Extend your investigation to the minute hand and the seconds hand.

3 How do you find the angle between the hands of a clock at any time? Start with times that give you the angles that are easiest to find. Remember that at 4.30 the minute hand will point to the 6 and the hour hand will be exactly halfway between 4 and 5.

4 Find out how many times there are in a day when the angle between the hour hand and the minute hand has a particular value, say 90°, 180°, or 120°.

5 What happens if the clock loses 10 minutes each hour? How many degrees would the minute hand turn through in 1 hour, or 15 minutes, or any other time?

6 What happens if the clock gains 5 minutes every hour?

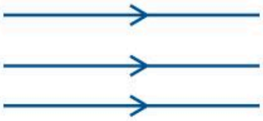
Parallel lines

Two straight lines that are always the same distance apart, however far they are drawn, are called *parallel lines*.

The lines in your exercise books are parallel. You can probably find many other examples of parallel lines.

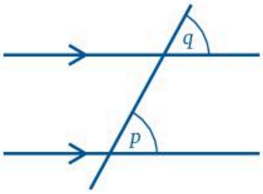
Exercise 4h

1 Using the lines in your exercise book, draw three lines that are parallel. Do not make them all the same distance apart. For example



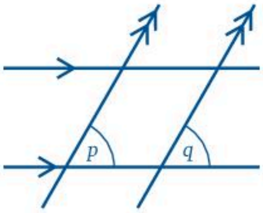
(We use arrows to mark lines that are parallel.)

2 Using the lines in your exercise book, draw two parallel lines. Make them fairly far apart. Now draw a slanting line across them. For example



Mark the angles in your drawing that are in the same position as those in the diagram. Are they acute or obtuse angles? Measure your angles marked p and q.

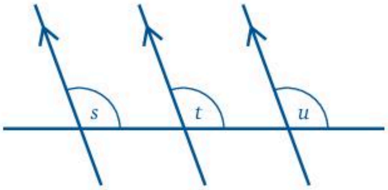
3 Draw a grid of parallel lines like the diagram below. Use the lines in your book for one set of parallels and use the two sides of your ruler to draw the slanting parallels.



Mark your drawing like the diagram. Are your angles p and q acute or obtuse? Measure your angles p and q.

4 Repeat question **3** but change the direction of your slanting lines.

5 Draw three slanting parallel lines like the diagram below, with a horizontal line cutting them. Use the two sides of your ruler and move it along to draw the third parallel line.



Mark your drawing like the diagram. Decide whether angles s, t and u are acute or obtuse and then measure them.

6 Repeat question **5** but change the slope of your slanting lines. What do you notice about the pairs of angles that you have measured?

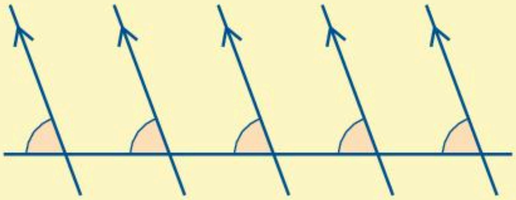
Corresponding angles

In the exercise above, lines were drawn that crossed a set of parallel lines.

A line that crosses a set of parallel lines is called a *transversal*.

When you have drawn several parallel lines you should notice that:

Two parallel lines on the same flat surface will never meet however far they are drawn.



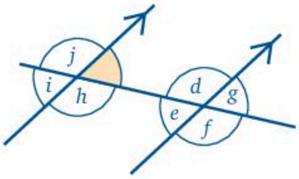
If you draw the diagram above by moving your ruler along you can see that all the shaded angles are equal. These angles are all in corresponding positions: in this diagram they are all above the transversal and to the left of the parallel lines. Angles like these are called *corresponding angles*.

When two or more parallel lines are cut by a transversal, the corresponding angles are equal.

Exercise 4i

In the diagrams below write down the letter that corresponds to the shaded angle:

1



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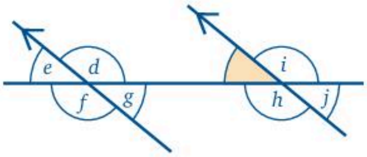

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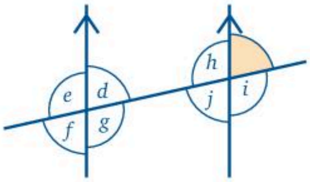

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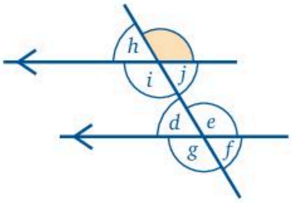

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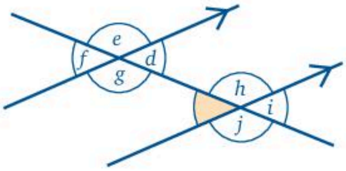

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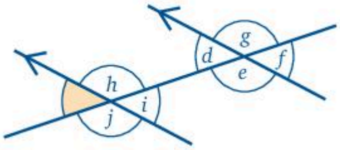

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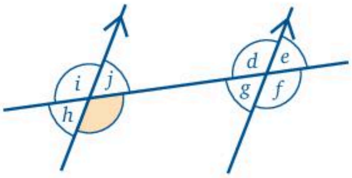

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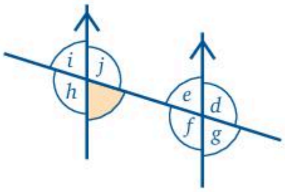


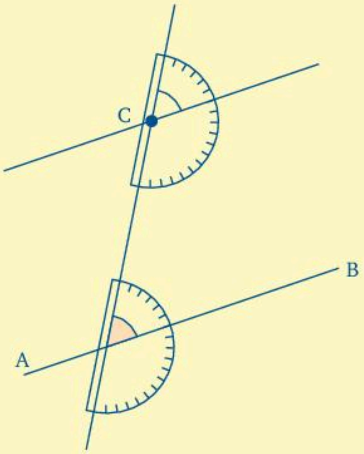
Drawing parallel lines (using a protractor)

The fact that the corresponding angles are equal gives us a method for drawing parallel lines.

If you need to draw a line through the point C that is parallel to the line AB, first draw a line through C to cut AB.

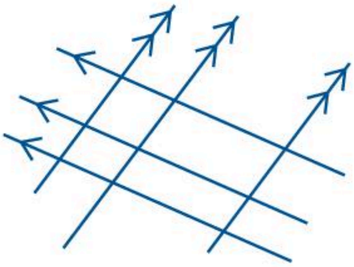
Use your protractor to measure the shaded angle. Place your protractor at C as shown in the diagram. Make an angle at C the same size as the shaded angle and in the corresponding position.

You can now extend the arm of your angle both ways, to give the parallel line.



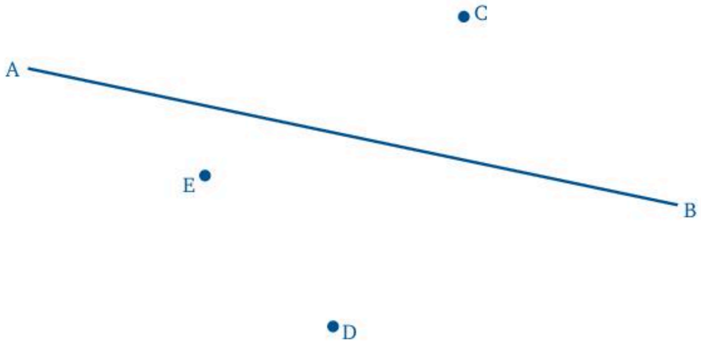
Exercise 4j

1 Using your protractor draw a grid of parallel lines like the one in the diagram. (It does not have to be an exact copy.)



Can you draw any conclusions about the relationship between the parallel lines and the angles formed by the lines which intersect them?

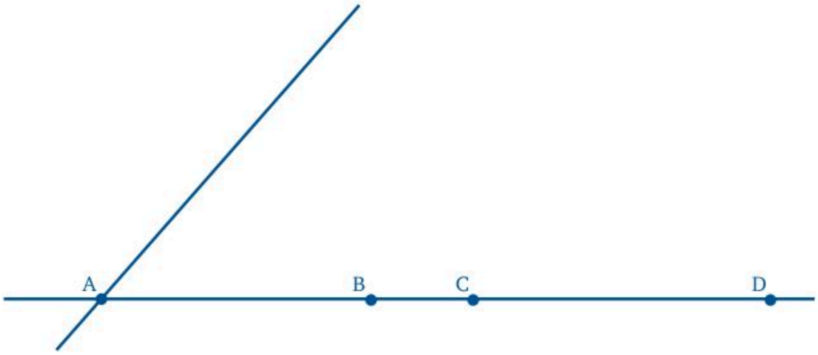
2 Trace the diagram below.



Now draw lines through the points C, D and E so that each line is parallel to AB.

3 Draw a sloping line on your exercise book. Mark a point C above the line. Use your protractor to draw a line through C parallel to your first line.

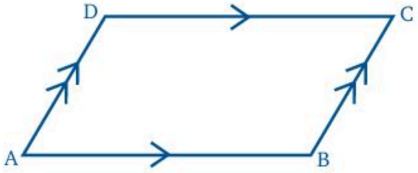
4 Trace the diagram below.



Measure the acute angle at A. Draw the corresponding angles at B, C and D. Extend the arms of your angles so that you have a set of four parallel lines.

5 Make an accurate drawing of the figure below where the side AB is 7 cm, the side AD is 4 cm and $\hat{A} = 60°$.

(A figure like this is called a *parallelogram*.)



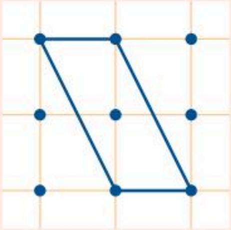
Puzzle

Copy this grid.

How many different-shaped parallelograms can you draw on this grid?

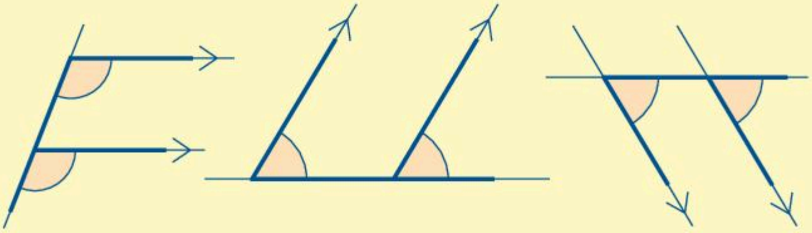
Each vertex must be on a dot. One has been drawn for you.

Do not include squares and rectangles.



Problems involving corresponding angles

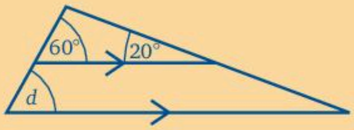
The simplest diagram for a pair of corresponding angles is an F shape.



Looking for an F shape may help you to recognise the corresponding angles.

Exercise 4k

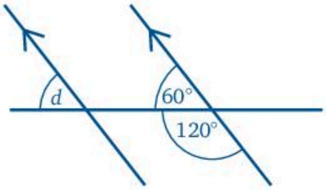
Write down the size of the angle marked *d*.



$d = 60°$ (*d* and the angle of 60° are corresponding angles.)

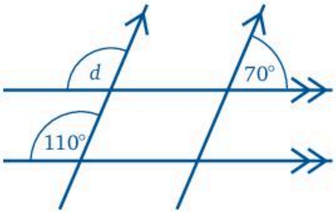
In questions **1** to **11**, write down the size of the angle marked *d* in each diagram.

1



Look for an F shape round the angle you need to find.

2



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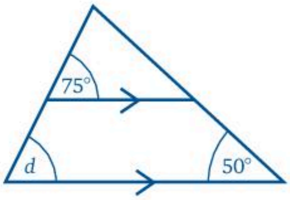

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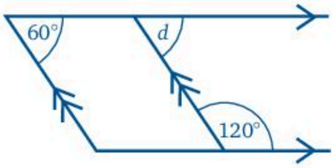

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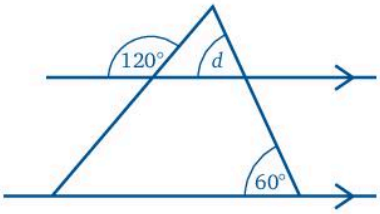

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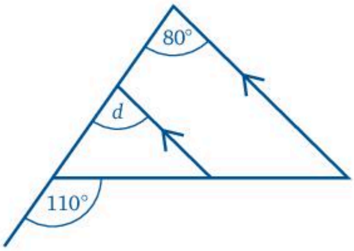

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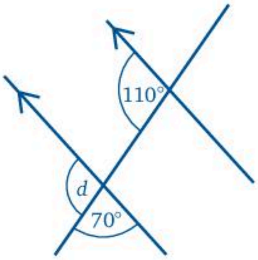

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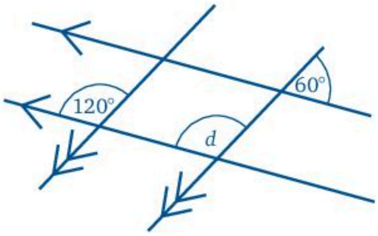

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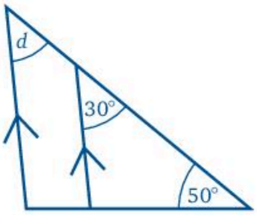

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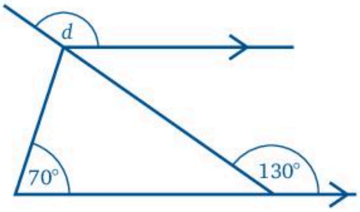

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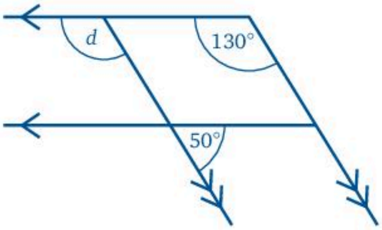
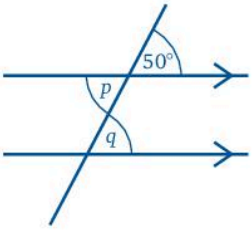


In questions **12** to **19**, find the size of each angle marked with a letter.

12



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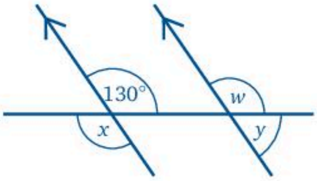

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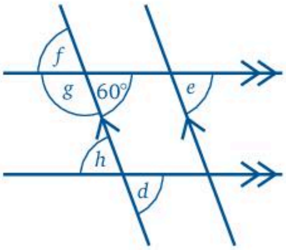

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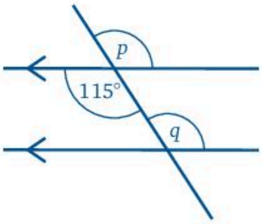

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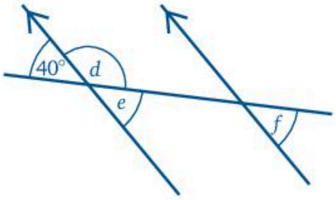

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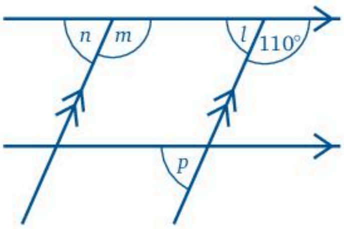

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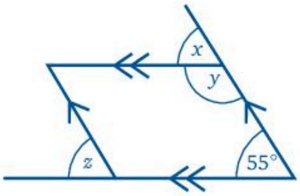

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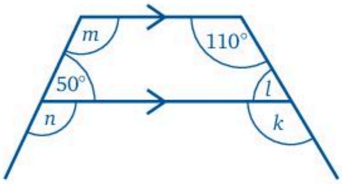


Alternate angles

Draw a large letter Z. Use the lines of your exercise book to make sure that the outer arms of the Z are parallel.

When this letter is turned through 180° about the point marked with a cross, the diagram looks exactly the same. This means that the two shaded angles are equal. Measure them to make sure.

Draw a large N, making sure that the outer arms are parallel.

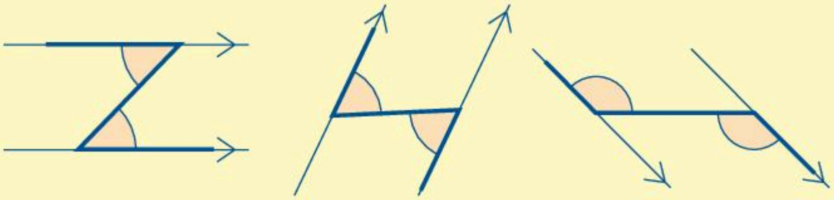
Also when this letter is turned through 180° about the point marked with a cross, the diagram looks exactly the same, so once again the shaded angles are equal. Measure them to make sure.

The pairs of shaded angles like those in the Z and N are between the parallel lines and on alternate sides of the transversal.

Angles like these are called *alternate angles*.

When two parallel lines are cut by a transversal, the alternate angles are equal.

The simplest diagram for a pair of alternate angles is a Z shape.



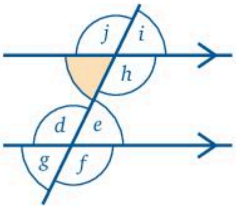
Looking for a Z shape may help you to recognise the alternate angles.

Exercise 4I

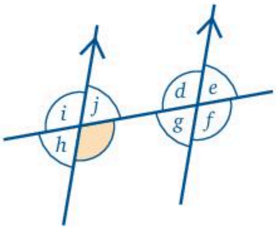
Look for a Z shape around the angle you want to find.

Write down the angle that is alternate to the shaded angle in the following diagrams:

1



4



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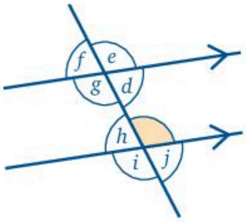

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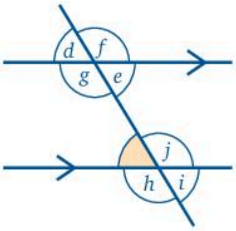

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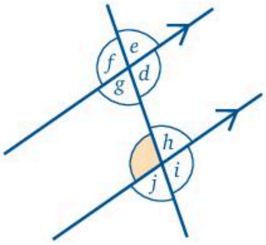

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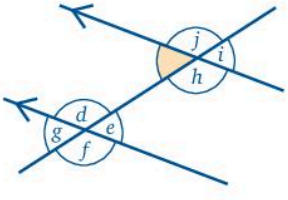

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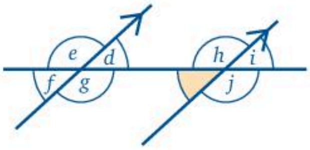

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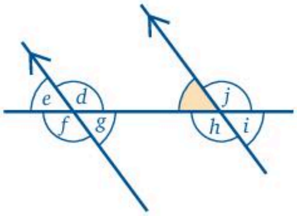

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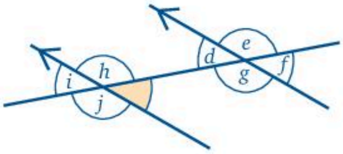
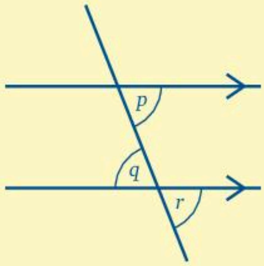


Problems involving alternate angles

Without doing any measuring we can show that alternate angles are equal by using the facts that we already know:



$p = r$ because they are corresponding angles

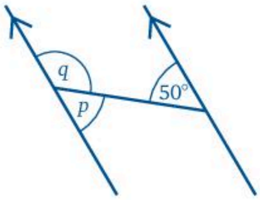
$q = r$ because they are vertically opposite angles

$\therefore\ p = q$ and these are alternate angles

Exercise 4m

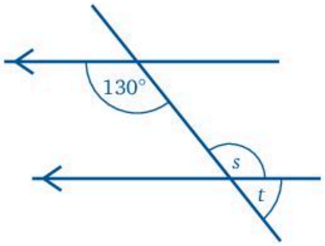
Find the size of each marked angle:

1



Remember that you can use any angle you know.

2



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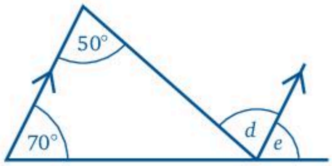

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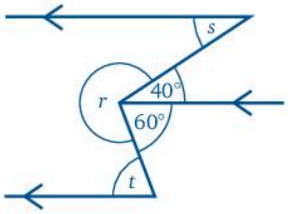

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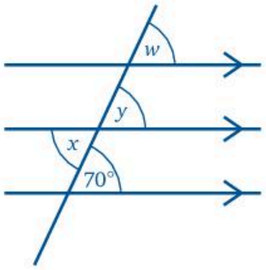

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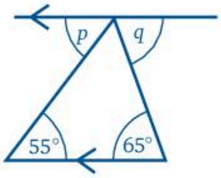

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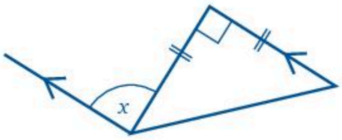

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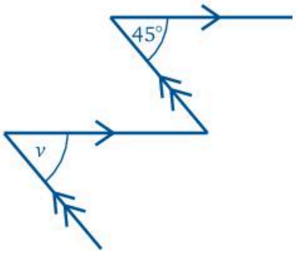
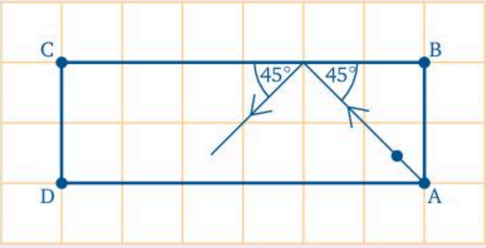


Investigation

This diagram represents a child's billiards table.



There is a pocket at each corner.

The ball is projected from the corner A at 45° to the sides of the table. It carries on bouncing off the sides at 45° until it goes down a pocket. (This is a very superior toy – the ball does not lose speed however many times it bounces!)

1 How many bounces are there before the ball goes down a pocket?

2 Which pocket does it go down?

3 What happens if the table is 2 squares by 8 squares?

4 Can you predict what happens for a 2 by 20 table?

5 Now try a 2 by 3 table.

6 Investigate for other sizes of tables. Start by keeping the width at 2 squares, then try other widths. Copy this table and fill in the results.

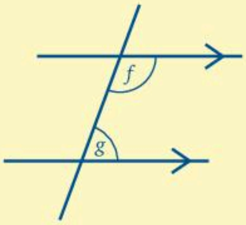
Size of table	Number of bounces	Pocket
2×6		
2×8		
2×3		
2×5		

7 Can you predict what happens with a 3×12 table?

Interior angles

In the diagram on the right, *f* and *g* are on the same side of the transversal and 'inside' the parallel lines.

Pairs of angles like *f* and *g* are called *interior angles*.

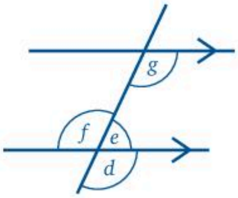


Exercise 4n

In the following diagrams, two of the marked angles are a pair of interior angles.
Name them:

You may find it helpful to look for a U shape.

1



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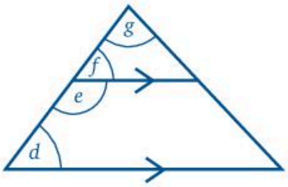

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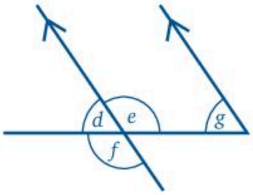

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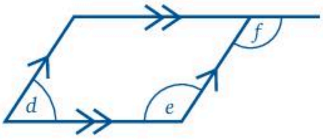

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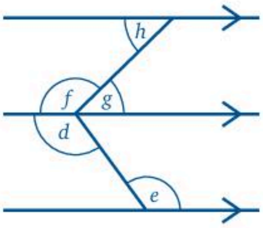

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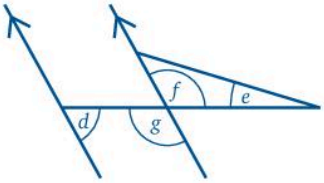
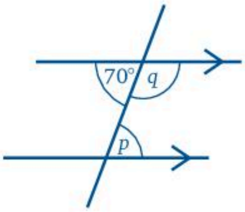


In the following diagrams, use the information given to find the size of p and of q. Then find the sum of p and q.

7



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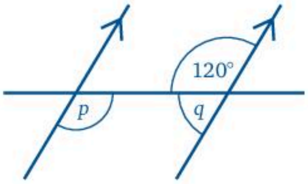

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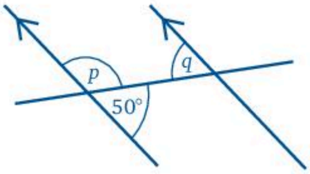

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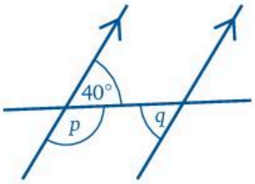
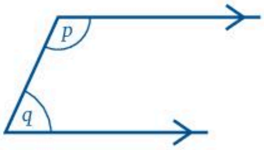


11 Make a large copy of the diagram below. Use the lines of your book to make sure that the outer arms of the 'U' are parallel.



Measure each of the interior angles p and q. Add them together.

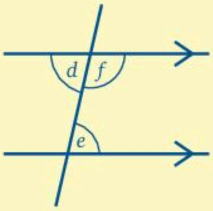
The sum of a pair of interior angles is 180°.

You will probably have realised this fact by now.
We can show that it is true from the following diagram.

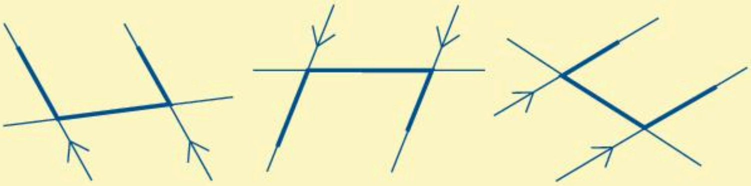
$d + f = 180°$ because they are angles on a straight line

$d = e$ because they are alternate angles

So $e + f = 180°$



The simplest diagram for a pair of interior angles is a U shape.

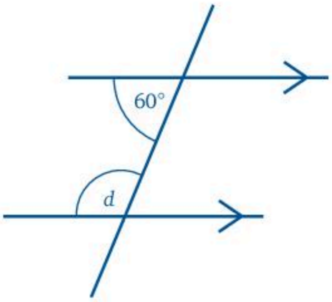


Looking for a U shape may help you to recognise a pair of interior angles.

Exercise 4p

Find the size of each marked angle:

1



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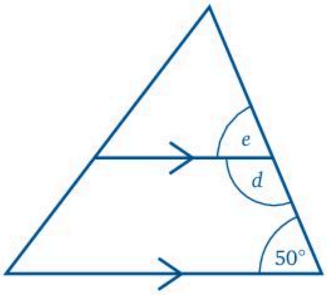

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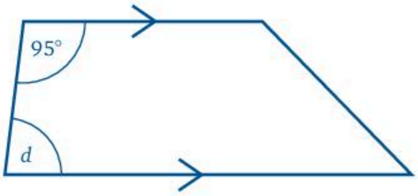

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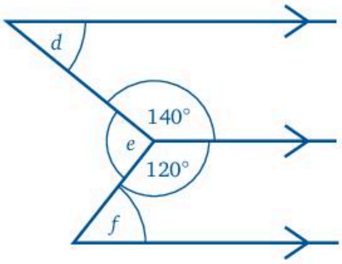

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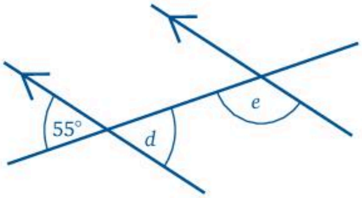

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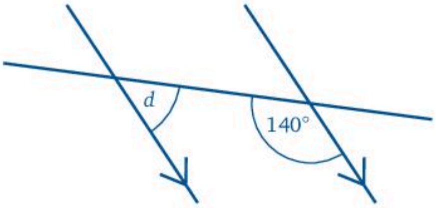

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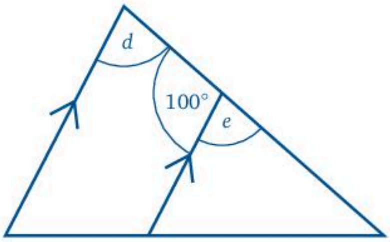

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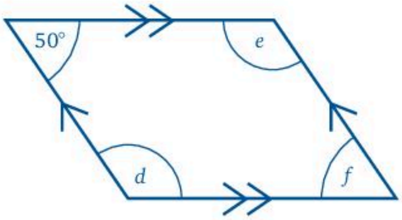

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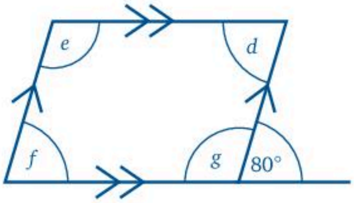

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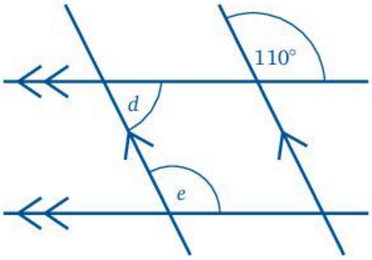


Mixed exercises

You now know that when a transversal cuts a pair of parallel lines:

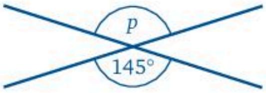
- the corresponding (F) angles are equal
- the alternate (Z) angles are equal
- the interior (U) angles add up to 180°.

You can use any of these facts, together with the other angle facts you know, to answer the questions in the following exercises.

Exercise 4q

Find the size of each marked angle.

1



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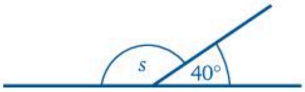

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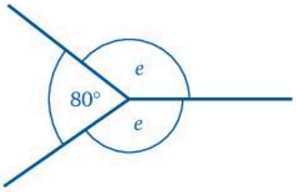

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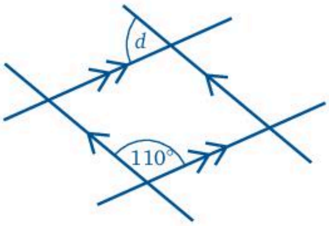

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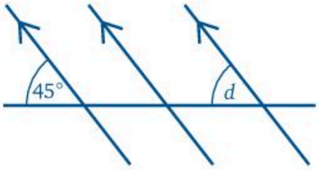

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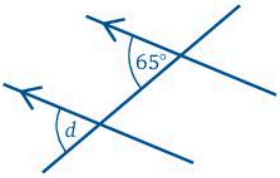

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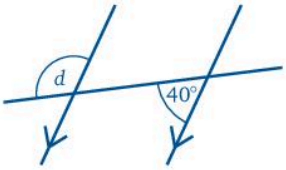

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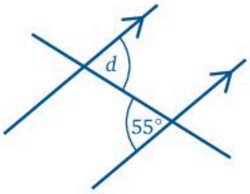

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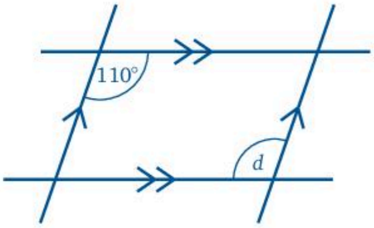
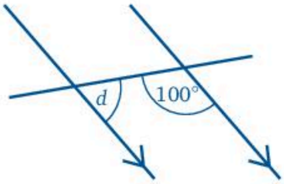


Exercise 4r

Find the size of each marked angle:

1



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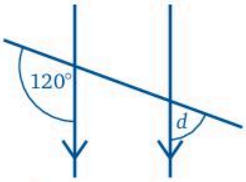

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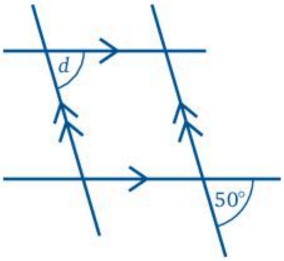

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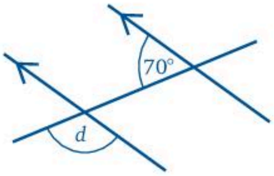

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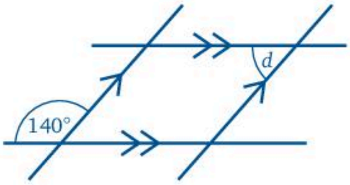

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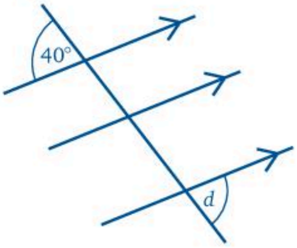
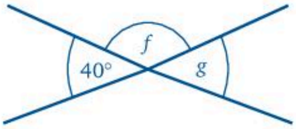
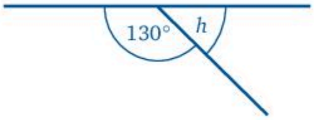


7 Write down the sizes of the angles marked *f* and *g*. Give a reason for your answer.



8 Write down the size of the angle marked *h*. Give a reason for your answer.



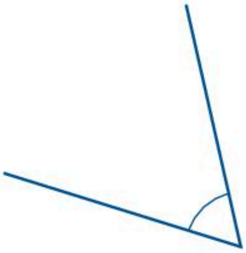
Exercise 4s

Select the letter that gives the correct answer.

1 The angle that the minute hand of a clock turns through as it moves from 1 to 9 is

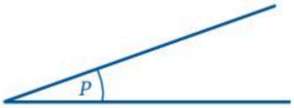
A 210° **B** 240° **C** 255° **D** 260°

2 Estimate the size of this angle.



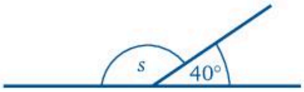
A 40°
B 45°
C 60°
D 75°

3 The size of the angle marked *p* is



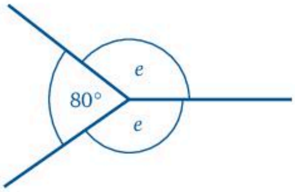
A 20°
B 30°
C 45°
D 60°

4 The size of the angle marked *s* is



A 110°
B 130°
C 140°
D 150°

5 The size of each the angles marked *e* is

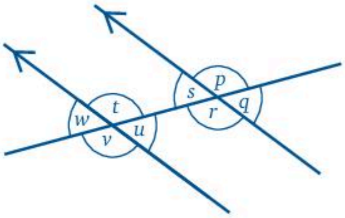


A 130°
B 135°
C 140°
D 145°

6 If you start facing east and turn clockwise through an angle of 270° the direction you are now facing is

A north **B** south **C** west **D** none of these

Use this diagram for questions **7** to **12**.



7 Which angle corresponds to *w*?

A *p* **B** *q* **C** *r* **D** *s*

8 Which angle corresponds to *r*?

A *t* **B** *u* **C** *v* **D** *w*

9 Which angle is alternate to s?

A t **B** u **C** v **D** w

10 Which angle is alternate to t?

A p **B** q **C** r **D** s

11 Which angle is vertically opposite q?

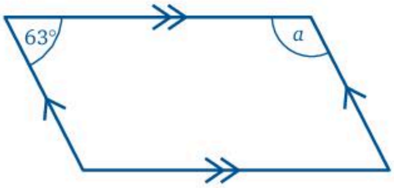
A p **B** r **C** s **D** u

12 The angle interior to angle u is

A p **B** q **C** r **D** s

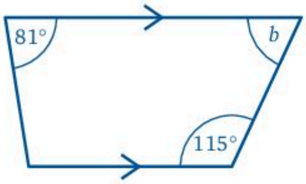
13 The angle marked a is

A 63°
B 107°
C 117°
D 127°



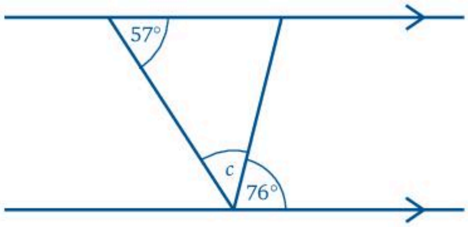
14 The angle marked b is

A 65°
B 81°
C 99°
D 115°



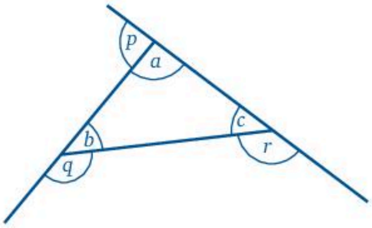
15 The angle marked c is

A 45°
B 47°
C 52°
D 58°



16 The sum of the six marked angles is

A 360°
B 450°
C 510°
D 540°



In this chapter you have seen that...

- ✓ a revolution can be divided into four right angles
- ✓ a revolution can be divided into 360°
- ✓ an acute angle is smaller than 90°
- ✓ an obtuse angle is larger than 90° but smaller than 180°
- ✓ a reflex angle is larger than 180°
- ✓ angles at a point add to give 360°
- ✓ vertically opposite angles are formed when two lines cross and they are equal
- ✓ angles on a straight line add up to 180°
- ✓ two angles that add up to 180° are called supplementary angles
- ✓ two angles that add up to 90° are called complementary angles
- ✓ parallel lines cut by a transversal give different types of angles – some are called corresponding angles, some alternate angles and others interior angles
- ✓ corresponding angles are equal; they can be recognised by an F shape
- ✓ alternate angles are equal; they can be recognised by a Z shape
- ✓ interior angles add up to 180°; they can be recognised by a U shape
- ✓ geometry problems can often be solved by starting with a copy of the diagram and filling in the sizes of the angles you know.

5 Transformations

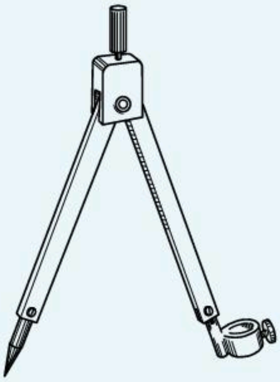
At the end of this chapter you should be able to...

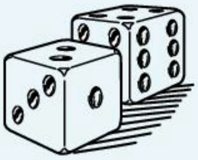
1. use a translation vector to describe and perform translations
2. perform reflections in the x-axis and in the y-axis
3. rotate an object about the origin by 90°, 180° or 270°.

Did you know?

Throughout the ages tools have been used in mathematics. Use an internet search to find out about the following tools, and match them to the possible uses given.

Tool	**Uses**
Vernier caliper	Drawing arcs
Dice	Drawing similar figures
Napier's bones	Scientific computation
Pantograph	Measuring small objects
Pair of compasses	Multiplication
Abacus	Early Peruvian computation
Quipu	Random-number generation
Slide rule	Manual four-function calculation





You need to know...

- ✔ how to use coordinates
- ✔ what a translation means
- ✔ the meaning of a reflection in a line
- ✔ the meaning of a rotation about a point.

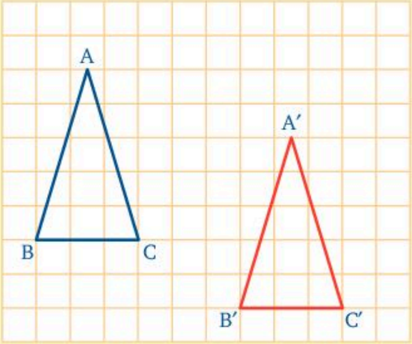
Key words

coordinates, displacement, mirror line, reflection, rotation, scalar, translation, vector

Translations

We saw in Grade 7 that a *translation* maps an object to an image by moving it without turning or reflecting it or changing its size.

This diagram shows a triangle ABC translated by 6 units horizontally to the right and 2 units vertically downwards.



A neater method of describing this movement is to use a translation *vector*.

Vectors

If you arranged to meet your friend 3 km from your home, this information would not be enough to ensure that you both went to the same place. You would also need to know which way to go.

Two pieces of information are required to describe where one place is in relation to another: the distance and the direction. Quantities that have both *size* (magnitude) and *direction* are called *vectors*.

A quantity that has magnitude but not direction is called a *scalar*. For example, the amount of money in your pocket or the number of pupils in your school are scalar quantities. On the other hand, the velocity of a hurricane, which states the speed and the direction in which it is moving, is a vector quantity.

Exercise 5a

State whether the following sentences refer to vector or scalar quantities:

1 There are 24 pupils in my class.

2 To get to school I walk $\frac{1}{2}$ km due north.

3 There are 11 players in a cricket team.

4 John walked at 6 km per hour.

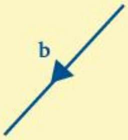
5 The vertical cliff face is 50 m high.

6 Give other examples of

  a vector quantities **b** scalar quantities.

Representing vectors

Because a vector has both size and direction we can represent a vector by a straight line and indicate its direction with an arrow. For example

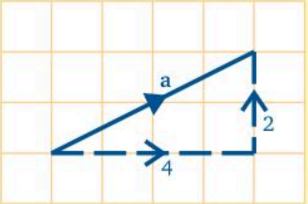


We use **a**, **b**, **c**,... to name the vectors.

When writing by hand it is difficult to write **a**, which is in heavy type, so we use $\underline{a}$.

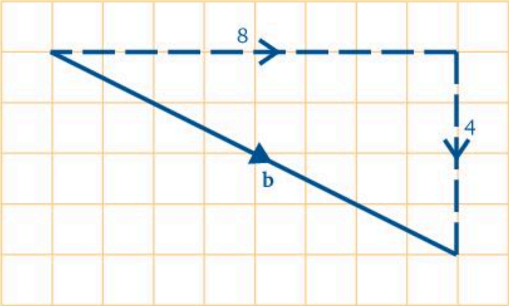
In the diagram on the right, the movement along **a** corresponds to 4 across and 2 up and we can write

$$\mathbf{a} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$



The vector **b** can be described as 8 across and 4 down. As with *coordinates*, which we looked at in Grade 7, we use negative numbers to indicate movement down or movement to the left.

Therefore $$\mathbf{b} = \begin{pmatrix} 8 \\ -4 \end{pmatrix}$$



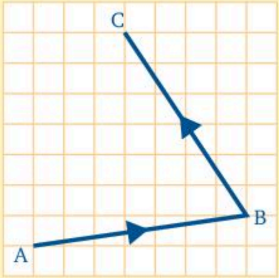
Notice that the top number represents movement across and that the bottom number represents movement up or down.

Capital letter notation

In the diagram A and B are two points.

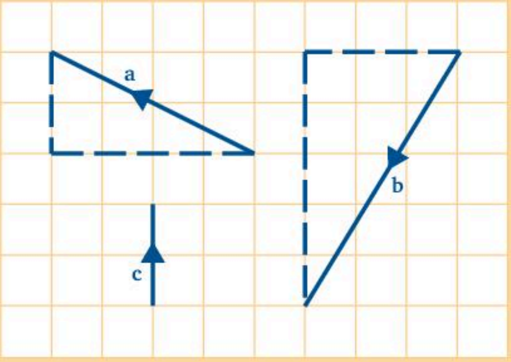
We can denote the vector from A to B as $\overrightarrow{AB}$ where $\overrightarrow{AB} = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$.

Similarly $\overrightarrow{BC} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$.



Exercise 5b

Write the following vectors in the form $\begin{pmatrix} p \\ q \end{pmatrix}$:



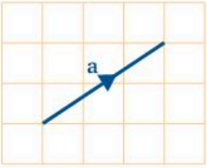
To move from the start to the end of **a**, you go 4 squares back (to the left) and 2 squares up: $\mathbf{a} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$.

For **b** you need to go 3 squares back and 5 squares down: $\mathbf{b} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$.

For **c** you do not need to go across, but you go 2 squares up: $\mathbf{c} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$.

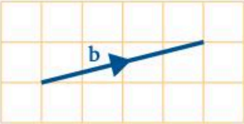
Write the following vectors in the form $\begin{pmatrix} p \\ q \end{pmatrix}$:

1



Move in the direction of the arrow and remember that the top number gives the distance across and the bottom number gives the distance up or down.

2



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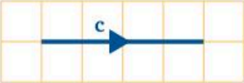

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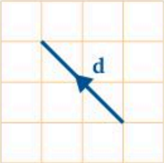

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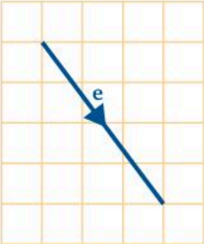

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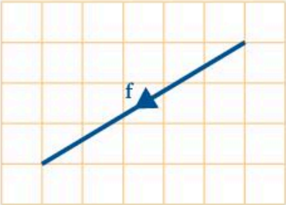

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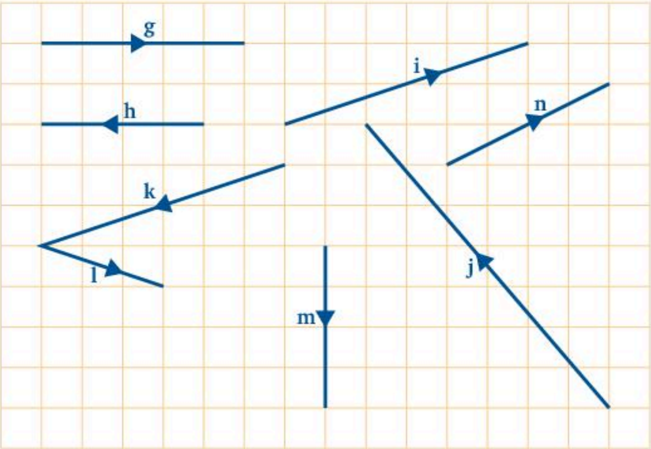


On square grid paper draw the following vectors. Label each vector with its letter and an arrow:

8 $\mathbf{a} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$

9 $\mathbf{b} = \begin{pmatrix} -4 \\ -3 \end{pmatrix}$

10 $\mathbf{c} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$

11 $\mathbf{d} = \begin{pmatrix} 6 \\ -12 \end{pmatrix}$

12 $\mathbf{e} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$

13 $\mathbf{f} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$

14 $\mathbf{g} = \begin{pmatrix} 6 \\ 10 \end{pmatrix}$

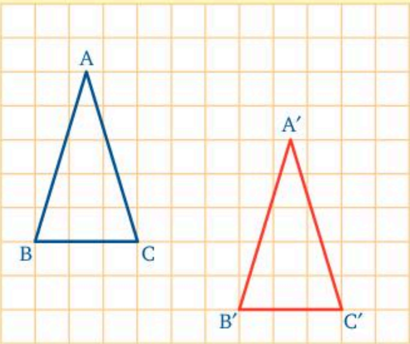
15 $\mathbf{h} = \begin{pmatrix} -1 \\ -5 \end{pmatrix}$

16 $\mathbf{i} = \begin{pmatrix} -6 \\ 2 \end{pmatrix}$

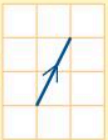
17 What do you notice about the vectors in questions **8** and **14**, and in questions **10** and **11**?

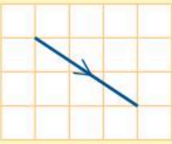
Translations using vectors

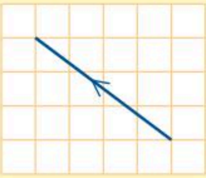
This translation (from page 93) can now be described by the vector $\begin{pmatrix} 6 \\ -2 \end{pmatrix}$.

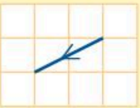


Remember that the top number gives the *displacement* parallel to the *x*-axis and the bottom number gives the displacement parallel to the *y*-axis.

$\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ 

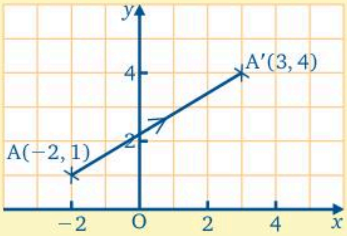
$\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ 

$\begin{pmatrix} -4 \\ 3 \end{pmatrix}$ 

$\begin{pmatrix} -2 \\ -1 \end{pmatrix}$ 

If the top number is negative, the displacement is to the left, and if the bottom number is negative, the displacement is downwards.

Consider the diagram:



$$\overrightarrow{AA'} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

A′ is the image of A under the translation described by the vector $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$.

A is mapped to A′ by the translation described by the vector $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$.

Exercise 5c

Find the images of the points given in questions **1** to **10** under the translations described by the given vectors.

1 (3, 1), $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$

2 (4, 5), $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$

3 (–2, 4), $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$

4 (3, 2), $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$

Draw the point (3, 1) then move 4 units to the right and 2 units up.

$\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ means 2 units to the left and 3 units up.

5 $(4, 5), \begin{pmatrix}-3\\-2\end{pmatrix}$

6 $(4, -4), \begin{pmatrix}2\\-3\end{pmatrix}$

7 $(-6, -3), \begin{pmatrix}4\\1\end{pmatrix}$

8 $(1, 1), \begin{pmatrix}-5\\-3\end{pmatrix}$

9 $(3, -2), \begin{pmatrix}6\\-4\end{pmatrix}$

10 $(7, 4), \begin{pmatrix}-5\\-4\end{pmatrix}$

In questions **11** to **16**, find the vectors describing the translations that map A to A′.

11 A(1, 2), A′(5, 3)

12 A(3, 8), A′(2, 9)

13 A(–4, –3), A′(0, 0)

14 A(–2, 6), A′(2, 6)

15 A(–3, –4), A′(–5, –6)

16 A(4, –2), A′(5, –1)

In questions **17** to **19**, the given point A′ is the image of an object point A under the translation described by the given vector. Find A.

17 $A'(7, 9), \begin{pmatrix}2\\3\end{pmatrix}$

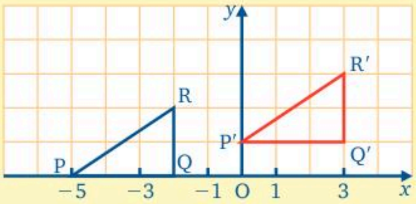
18 $A'(0, 6), \begin{pmatrix}2\\3\end{pmatrix}$

19 $A'(-3, -2), \begin{pmatrix}1\\3\end{pmatrix}$

A translation moves each point of an object the same distance in the same direction.

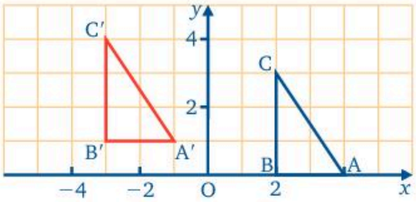
$\overrightarrow{PP'} = \begin{pmatrix}5\\1\end{pmatrix}$ $\quad \overrightarrow{RR'} = \begin{pmatrix}5\\1\end{pmatrix}$ $\quad \overrightarrow{QQ'} = \begin{pmatrix}5\\1\end{pmatrix}$

i.e. $\overrightarrow{PP'} = \overrightarrow{QQ'} = \overrightarrow{RR'}$

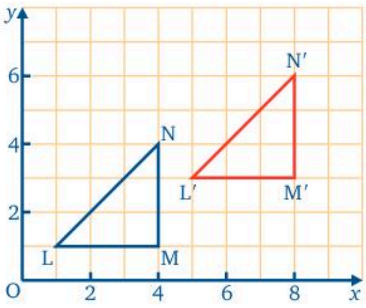


Exercise 5d

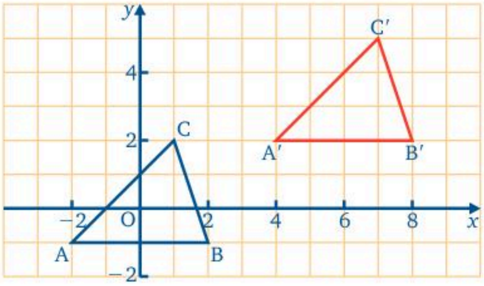
1 Given the following diagrams, find the vectors $\overrightarrow{AA'}$, $\overrightarrow{BB'}$ and $\overrightarrow{CC'}$. Are they all equal? Is the transformation a translation?



2 Given the following diagrams, find the vectors $\overrightarrow{LL'}$, $\overrightarrow{MM'}$ and $\overrightarrow{NN'}$. Are they all equal? Is the transformation a translation?

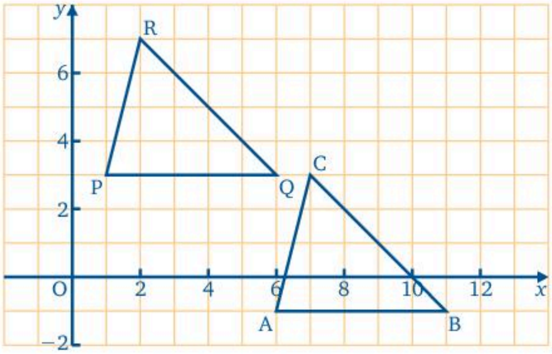


3 Find the vector that describes the translation mapping A to A′, B to B′ and C to C′.



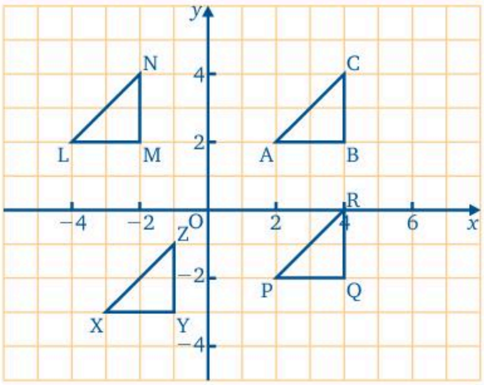
4 Give the vectors describing the translations that map

- **a** △ABC to △PQR
- **b** △PQR to △ABC.



5 Give the vectors describing the translations that map

- **a** △ABC to △PQR
- **b** △ABC to △LMN
- **c** △XYZ to △ABC
- **d** △ABC to △ABC.



6 Draw axes for x and y from -4 to 5. Draw the following triangles:
△ABC with A(2, 2), B(4, 2), C(2, 5)
△PQR with P(1, –2), Q(3, –2), R(1, 1)
△XYZ with X(–3, 1), Y(–1, 1), Z(–3, 4).

Give the vectors describing the translations that map

- **a** △ABC to △PQR
- **b** △PQR to △ABC
- **c** △PQR to △XYZ
- **d** △ABC to △ABC.

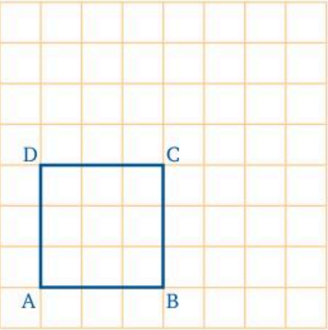
7 Draw axes for x and y from 0 to 9. Draw △ABC with A(3, 0), B(3, 3), C(0, 3) and △A′B′C′ with A′(8, 2), B′(8, 5), C′(5, 5).

Is △A′B′C′ the image of △ABC under a translation? If so, what is the vector describing the translation?

Join AA′, BB′ and CC′. What type of quadrilateral is AA′B′B? Give reasons. Name other quadrilaterals of the same type in the figure.

8 **a** Square ABCD is translated parallel to AB a distance equal to AB. Sketch the diagram and draw the image of ABCD.

b Square ABCD is translated parallel to AC a distance equal to AC. Sketch the diagram and draw the image of ABCD.



9 Draw axes for x and y from –2 to 7.
Draw △ABC with A(–2, 5), B(1, 3), C(1, 5).

Translate △ABC using the vector $\begin{pmatrix} 5 \\ 1 \end{pmatrix}$. Label this image $A_1B_1C_1$.

Then translate $\triangle A_1B_1C_1$ using the vector $\begin{pmatrix} -1 \\ -3 \end{pmatrix}$. Label this new image $A_2B_2C_2$.

Give the vectors describing the translations that map

a △ABC to $\triangle A_2B_2C_2$

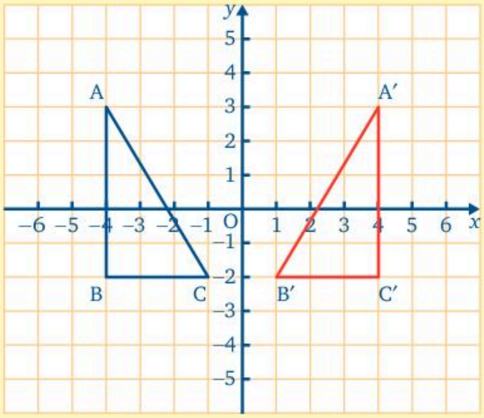
b $\triangle A_2B_2C_2$ to △ABC

c $\triangle A_2B_2C_2$ to $\triangle A_1B_1C_1$.

Reflections

We saw in Grade 7 that when we reflect an object in a line (called the *mirror line*), the object and its image together form a symmetrical shape.

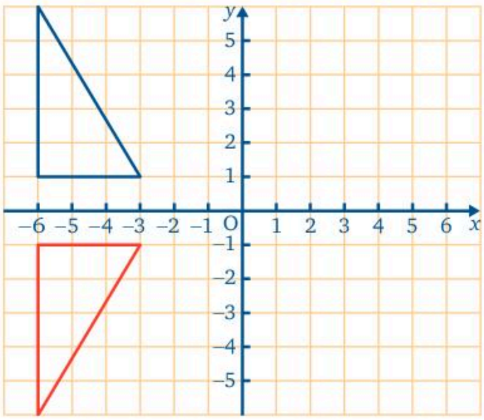
This diagram shows the image of triangle ABC when it is reflected in the y-axis. Triangle A′B′C′ is the *reflection* of triangle ABC in the y-axis.



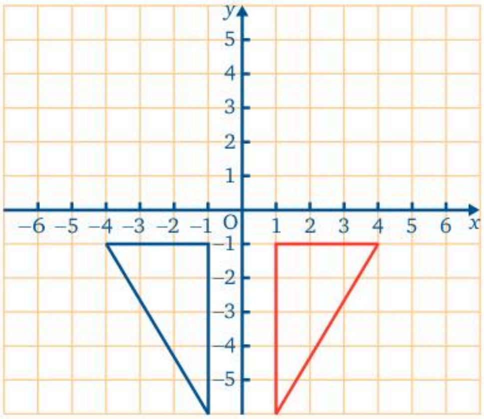
Exercise 5e

In questions **1** to **6**, describe the transformation that maps the object (blue) to its image (red).

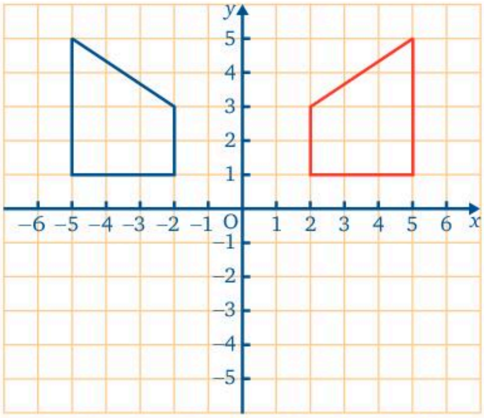
1



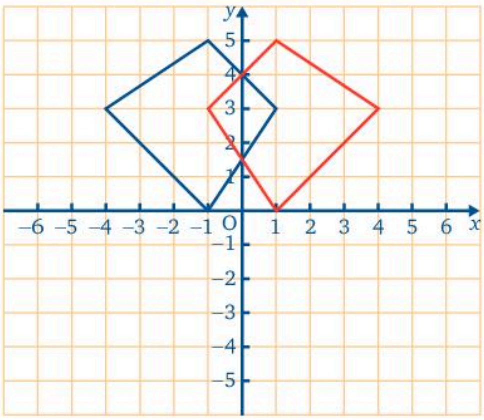
2



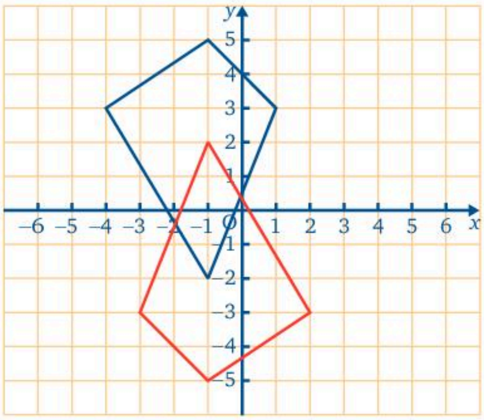
3



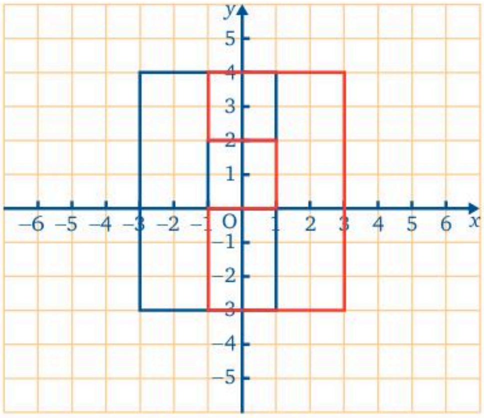
4



5



6



In questions **7** to **9** use graph paper and a scale of 1 cm to 1 unit.

7 Draw axes, for x from –5 to 5 and for y from 0 to 5. Draw triangle ABC by plotting A(1, 2), B(3, 2) and C(3, 5). Draw the image A′B′C′ when ABC is reflected in the y-axis.

8 Draw axes, for x from 0 to 5 and for y from –2 to 2. Draw triangle PQR where P is (1, –1), Q is (5, –1) and R is (4, 0). Draw the image P′Q′R′ when ΔPQR is reflected in the x-axis.

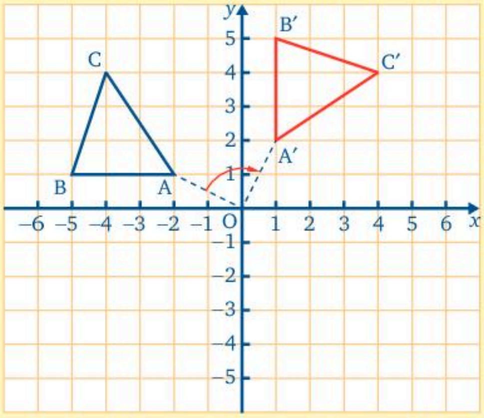
9 Draw axes for x and y from –6 to 6. Draw triangle ABC when A is (–6, –2), B is (–3, –4) and C is (–2, –1). Draw the following images of triangle ABC:

a triangle $A_1B_1C_1$ by reflection in the y-axis

b triangle $A_2B_2C_2$ by reflecting $A_1B_1C_1$ in the x-axis

c write down the coordinates of the vertices of triangle $A_2B_2C_2$.

Rotations



In Grade 7, we saw that we can rotate an object to give an image.

In this diagram, the image is obtained by rotating the object by 90° about the origin.

When we draw the lines joining corresponding vertices to the origin, the angle between them is 90°.

If OA is rotated about O through 90° clockwise the point A, which has coordinates (–2, 1), moves to the point A′, which has coordinates (1, 2).

Similarly OB rotates through 90° so that B, which has coordinates (−5, 1), moves to the point B′, which has coordinates (1, 5). OC rotates through 90° so that C, which has coordinates (–4, 4), moves to the point C′, which has coordinates (4, 4).

Remember that a clockwise *rotation* is a negative angle while an anticlockwise rotation is a positive angle.

Exercise 5f

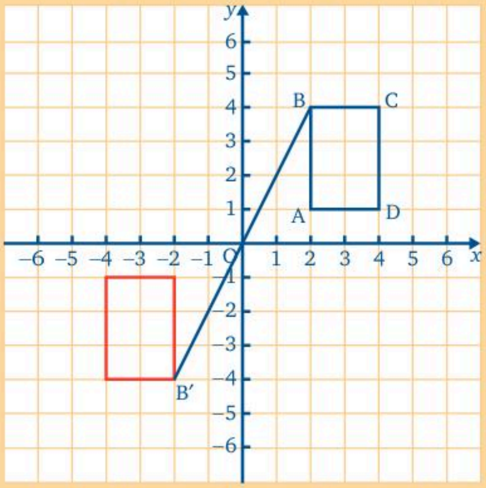
Rectangle ABCD is rotated about the origin to give rectangle A′B′C′D′.

Give the angle of rotation.

Join B to O and B′ to O where B′ is the image of B under a rotation about the origin.

Since BOB′ is a straight line the object has rotated through 180° to give the image.

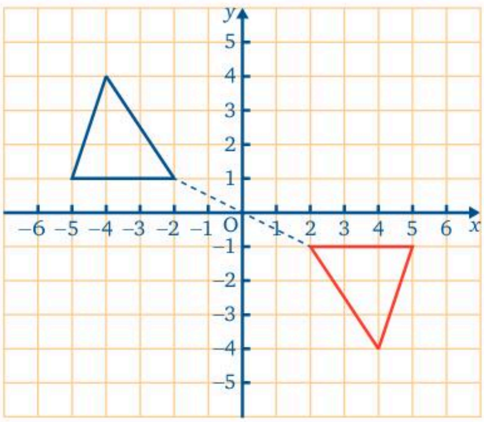
We do not need to say that the rotation is clockwise or anticlockwise since the result is the same.



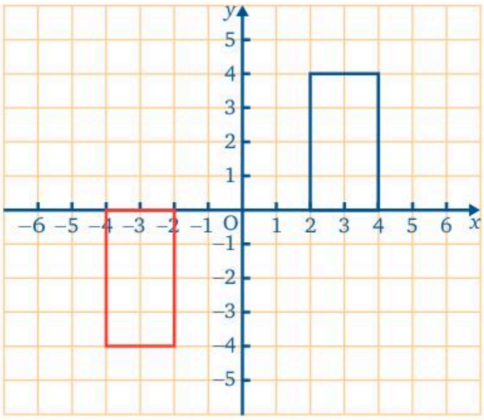
In questions **1** to **4** the object is rotated about the origin. Give the angle of rotation.

Join corresponding vertices to the origin. The first one has been done for you.

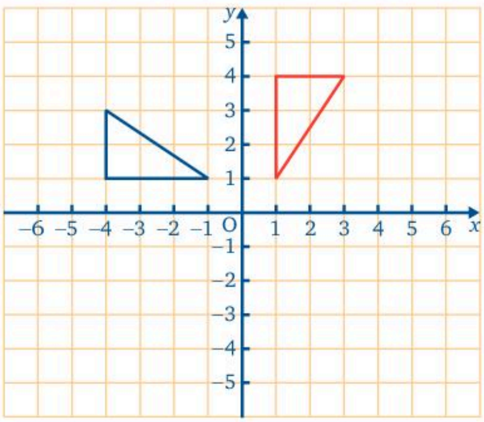
1



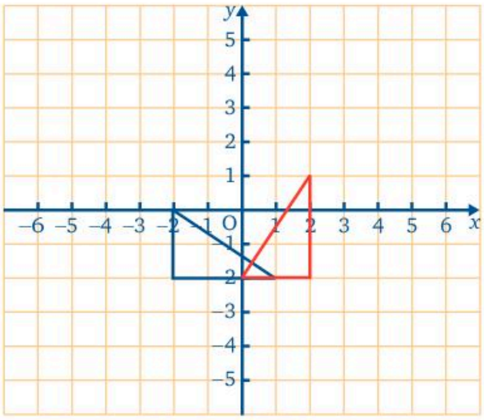
2



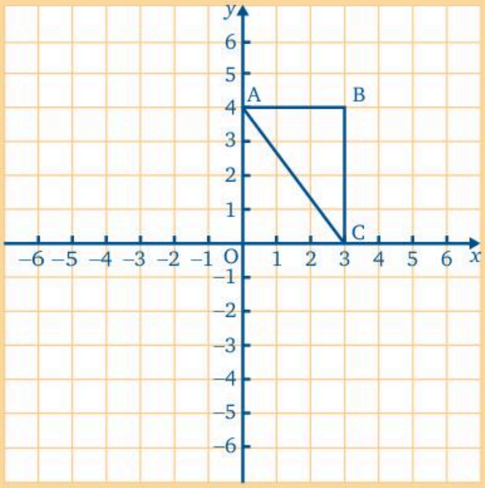
3



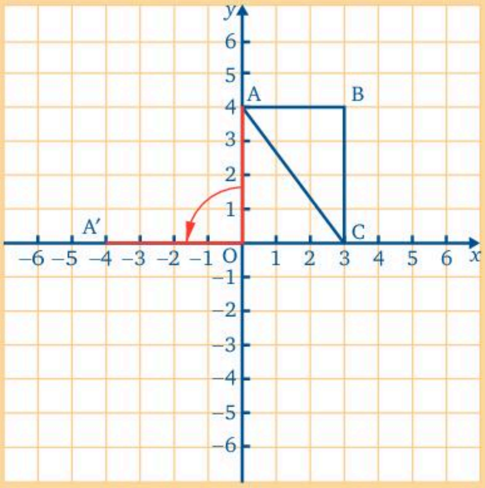
4



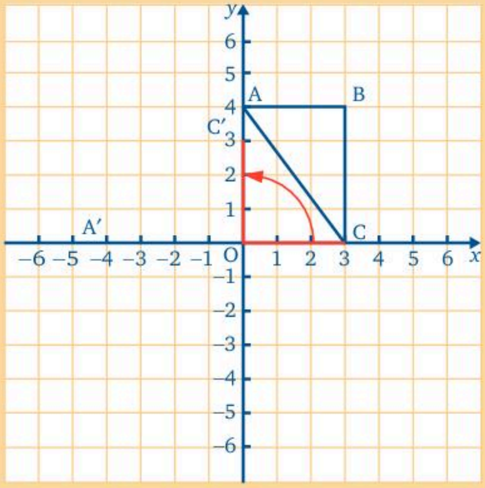
Copy the diagram. Find the image of triangle ABC when it is rotated 90° anticlockwise about the origin. Label the image A′B′C′ and give the coordinates of A′, B′ and C′.



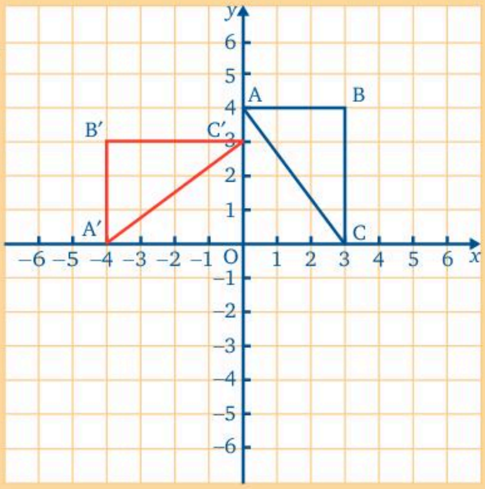
Draw the line OA, and now rotate this line 90° anticlockwise. Mark A′, which is the same distance from O as A, which is 4 squares. The coordinates of A are (0, 4) and the coordinates of A′ are (−4, 0).



OC will rotate so the OC′ lies on the *y*-axis. So C′ has coordinates (0, 3).



To find B′, first join OB. Now rotate OB through 90° anticlockwise. This moves B to B′, where B′ is the point with coordinates (–4, 3).

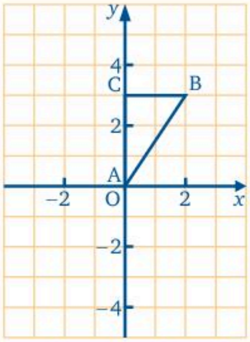


Copy the diagrams in questions **5** to **8** using a scale of 1 cm to one unit.

Find the image of the given object when rotated about the origin by the angle given.

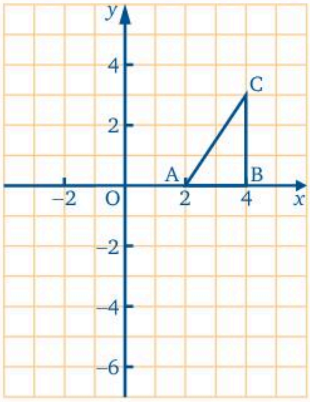
In each case label the image A′B′C′ and give the coordinates of A′, B′ and C′.

5



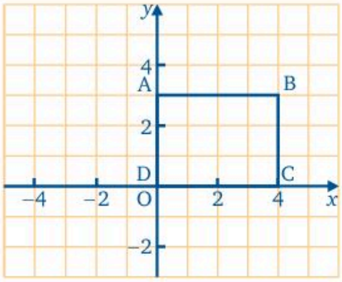
180°

6



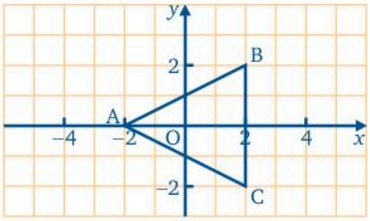
90° clockwise

7



90° anticlockwise

8



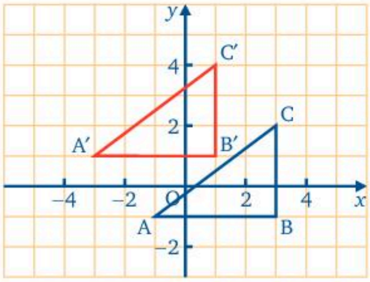
270° clockwise

Mixed exercises

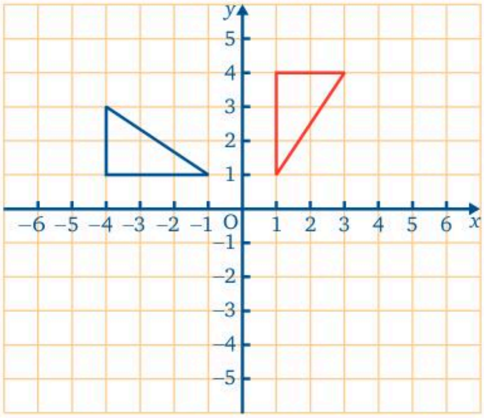
Exercise 5g

In questions **1** to **4** name the transformations, describing them fully. The blue shape is the object and the red shape is the image. Give the transformation vector for a translation, the mirror line for a reflection and the angle turned through for rotation.

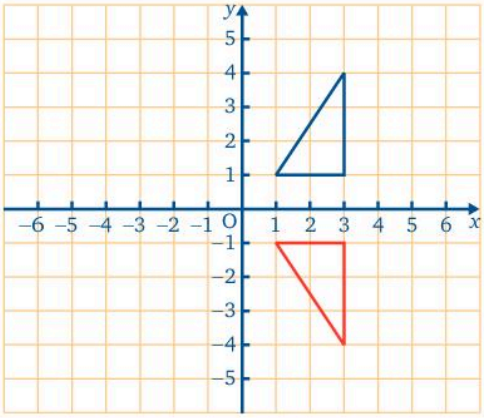
1



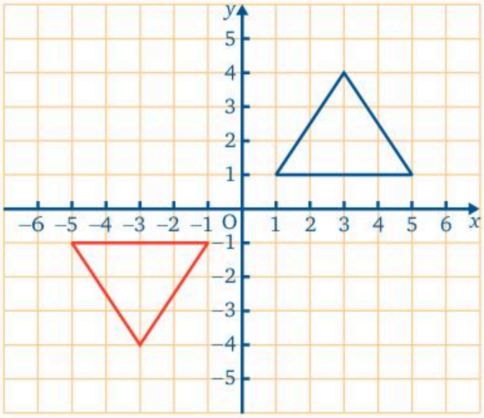
2



3

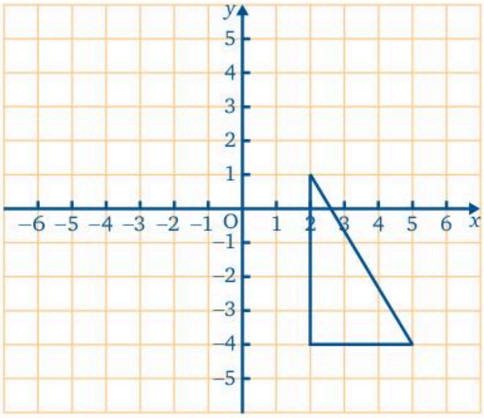


4



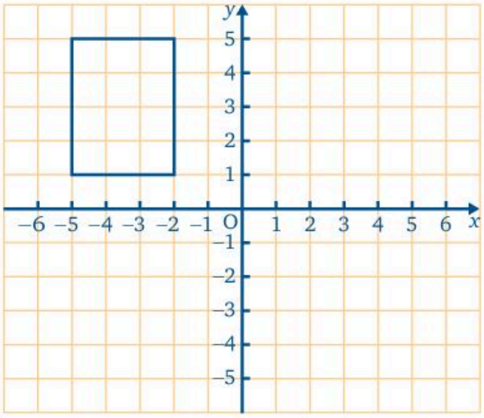
In questions **5** to **8** copy the diagram on square grid paper using one square to one unit. Draw the image under the transformation described.

5



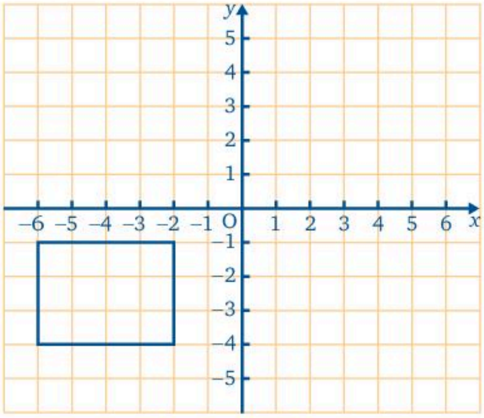
Translation under the vector $\begin{pmatrix} -4 \\ 5 \end{pmatrix}$.

6



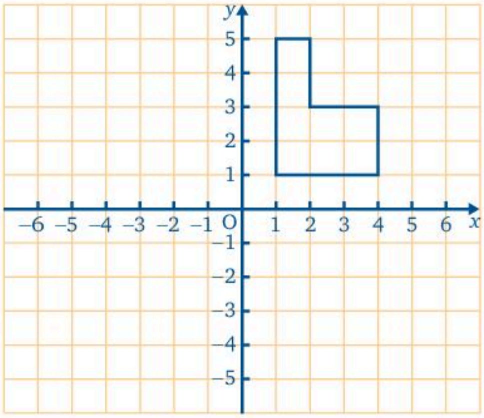
Reflection in the *x*-axis.

7



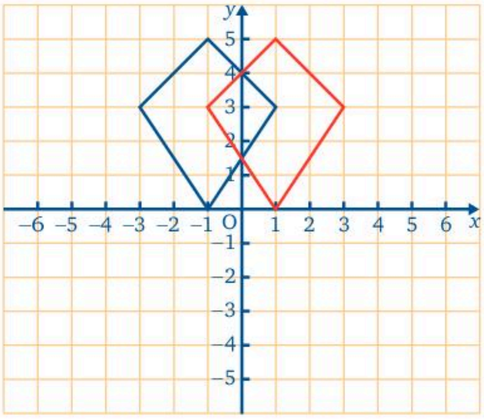
Rotation about O by 90° clockwise.

8



Translation under the vector $\begin{pmatrix} -1 \\ -4 \end{pmatrix}$.

9 Name two different transformations that will map the object to the image, describing them fully.



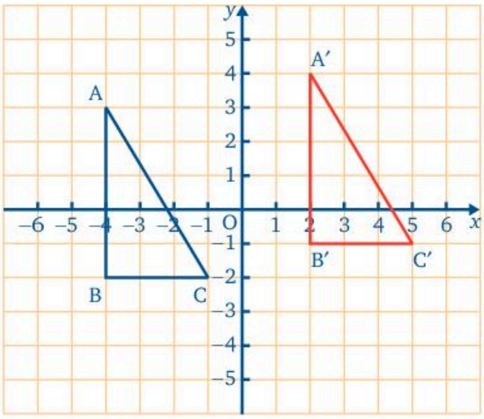
10 This diagram shows the object triangle ABC and the image triangle A′B′C′ under a transformation.

Copy the diagram using a scale of 2 cm to one unit.

a Name and describe the transformation.

b Reflect triangle A′B′C′ in the y-axis and label the vertices A″B″C″.

c Rotate triangle A″B″C″ about the origin by 90°.

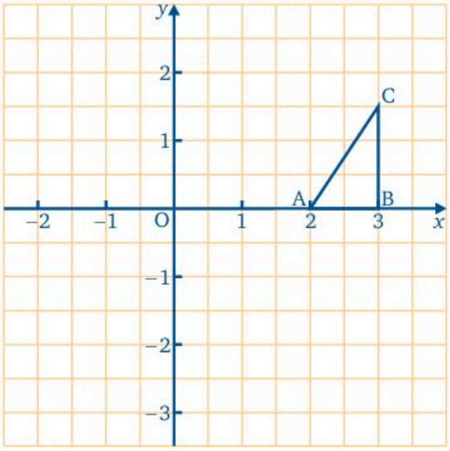


11 Copy the diagram using a scale of 1 cm to 1 unit.

Reflect triangle ABC in the x-axis to give triangle $A_1B_1C_1$.

Then reflect triangle $A_1B_1C_1$ in the y-axis to give triangle $A_2B_2C_2$.

What single transformation will map triangle ABC to triangle $A_2B_2C_2$?

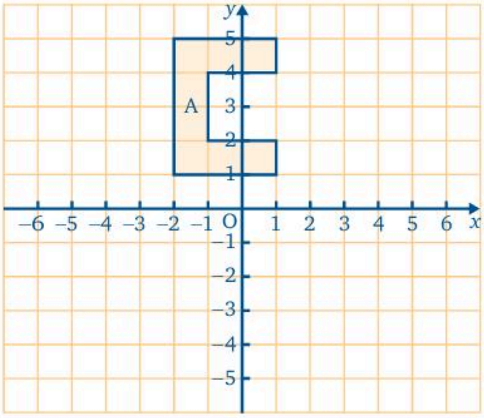


12 Copy the diagram using a scale of 1 cm to 1 unit.

a Reflect the shape A in the y-axis and label the image B.

b Rotate B about O by 180° and label the image C.

c Describe a single transformation that maps A to C.



Exercise 5h

1. Which of these sentences refer to a vector quantity?
   - **i** Clive walked to school at 6 km/h.
   - **ii** The number of pupils in my class is 27.
   - **iii** The nearest post office from my home is 1 km away in the direction northwest.
   - **iv** To get to the leisure centre my journey takes me due east.

   A **i** and **ii** **B** **i** and **iii** **C** **iii** **D** **iii** and **iv**

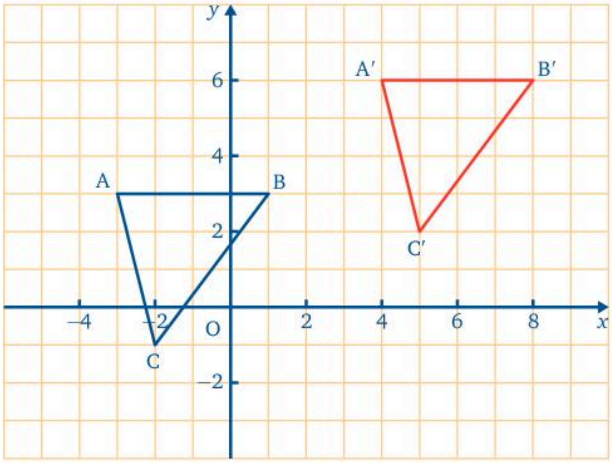
2. Which of the sentences in question **1** refer to a scalar quantity?

   A **i** and **ii** **B** **i** and **iii** **C** **iii** **D** **iii** and **iv**

3. The coordinates of the image of the point (2, 5) under a translation described by the vector $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$ are

   A (2, −3) **B** (2, 2) **C** (6, 2) **D** (6, 3)

Use this diagram for questions **4** and **5**.



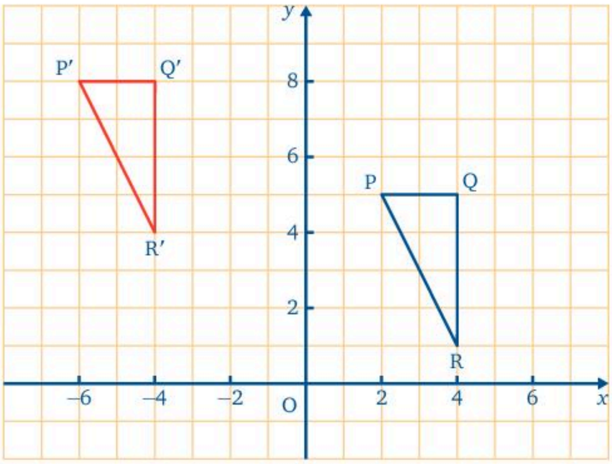
4. The vector that describes the translation mapping △ABC to △A′B′C′ is

   A $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$ **B** $\begin{pmatrix} 6 \\ 3 \end{pmatrix}$ **C** $\begin{pmatrix} 7 \\ 3 \end{pmatrix}$ **D** $\begin{pmatrix} 8 \\ 3 \end{pmatrix}$

5. The vector that describes the translation mapping △A′B′C′ to △ABC is

   A $\begin{pmatrix} -8 \\ -3 \end{pmatrix}$ **B** $\begin{pmatrix} -7 \\ -3 \end{pmatrix}$ **C** $\begin{pmatrix} -7 \\ -2 \end{pmatrix}$ **D** $\begin{pmatrix} -6 \\ 3 \end{pmatrix}$

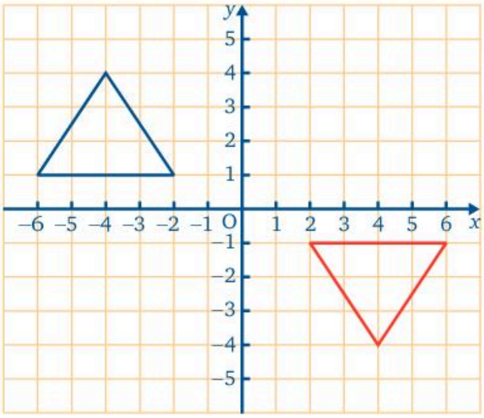
6



The vector that describes the translation mapping △PQR to △P′Q′R′ is

A $\begin{pmatrix} -9 \\ -3 \end{pmatrix}$ **B** $\begin{pmatrix} -8 \\ -3 \end{pmatrix}$ **C** $\begin{pmatrix} -8 \\ 3 \end{pmatrix}$ **D** $\begin{pmatrix} 9 \\ 3 \end{pmatrix}$

7

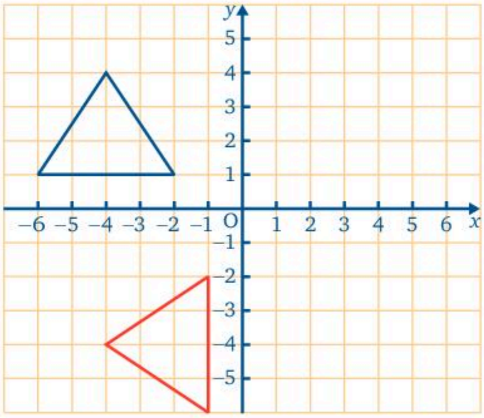


The blue triangle is rotated about the origin to give the red image.

The angle of rotation is

A 90° **B** 180° **C** 270° **D** 360°

8

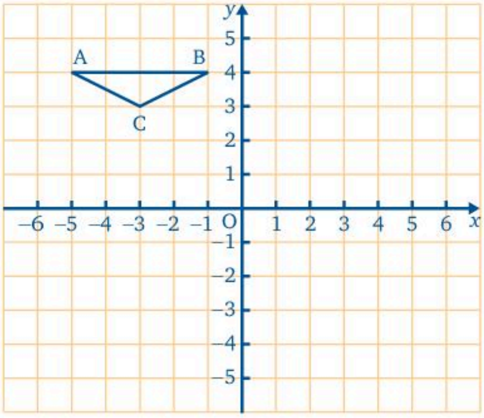


The blue triangle is rotated about the origin to give the red image.

The angle of rotation is

A 90° **B** 180° **C** 270° **D** 360°

Use this diagram for questions **9** to **11**.



9 When triangle ABC is rotated by 180° about the origin, the coordinates of A′, the image of A, are

A (−1, −4) **B** (−5, −4) **C** (5, −4) **D** (4, 1)

10 When triangle ABC is rotated by 90° about the origin, the coordinates of C′, the image of C, are

A (1, 4) **B** (−3, −3) **C** (3, 3) **D** (4, 1)

11 When triangle ABC is rotated by 270° about the origin, the coordinates of B′, the image of B, are

A (−1, −4) **B** (−4, −1) **C** (1, 4) **D** (4, 1)

Puzzle

Kim, David, Jenny and Clare are Emma's family. They are Emma's mother, father, younger brother and younger sister.

1 David is older than Emma.

2 Clare is not Emma's younger brother.

3 Jenny is not Emma's father. She is also not Emma's younger sister.

Who is the mother, father, younger brother and younger sister?

In this chapter you have seen that...

- ✔ a vector has size (magnitude or length) and direction
- ✔ a scalar has size only
- ✔ a vector can be represented by an ordered pair of numbers written vertically. The top number gives movement across and the bottom number gives movement up or down
- ✔ a translation moves an object without turning it or reflecting it
- ✔ a translation can be described by a vector
- ✔ when an object is reflected in a mirror line, the object and the image are symmetrical about the mirror line
- ✔ when an object is rotated about a point, the angle between lines joining corresponding points on the object and image to the centre of rotation are equal to the angle of rotation.

REVIEW TEST 1: CHAPTERS 1–5

1 **a** Find the value of 5^4.

b Simplify **i** $3^3 \times 3^2$ **ii** $4^6 \div 4^3$

c Write 29 430 in standard form.

2 **a** Give 7294 correct to 2 significant figures.

b Give 0.060 705 correct to 3 s.f.

3 **a** Find 0.642×4.725, giving your answer correct to 3 s.f.

b Find $54.258 \div 16.78$, giving your answer correct to 3 s.f.

c Change $\frac{7}{8}$ into **i** a percentage **ii** a decimal.

4 Write **a** 131_5 as a number to base 10.

b 14_{10} as a number to base 5

c 30_{10} as a number to base 9.

5 Find

a $34_5 + 14_5$ **b** $123_4 + 321_4$ **c** $10101_2 + 1111_2$

6 Find

a $21_4 \times 3_4$ **b** $211_3 - 112_3$ **c** $1010_2 \times 1011_2$

7 Simplify

a $5x - 2x + 6x$

b $4(1 - 3x)$

c $2x \times 3x$

d $4x^2 \times 5x$

8 Find the value of $7x - 2$ when **a** $x = 3$ **b** $x = -3$

9 Find the value of $2(10 - 3x)$ when **a** $x = 2$ **b** $x = -3$

10 I go shopping with $\$a$ in my purse. At the first shop I spend $\$b$ and at the second shop I spend half of what I have remaining in my purse.
How much do I have left?

11 If $Q = 2r - s$, find Q when $r = 5$ and $s = 4$.

12 Given that $D = E + F$, find E when $D = 30$ and $F = 18$.

13 If $R = \frac{s-t}{u}$, find R when $s = -3$, $t = -11$ and $u = 4$.

14 The length of a rectangle is three times its width.
If the rectangle is x cm long, write down a formula for P in terms of x.
Use your formula to find the length of the rectangle if it has a perimeter of 64 cm.

15 **a** Make f the subject of the formula $E = f - g$
b Make p the subject of the formula $Q = \frac{p}{r}$
c Make c the subject of the formula $3a + 2b + c = R$

16 Solve the inequalities
a $x + 3 < 5$ **b** $x - 4 < -5$ **c** $x + 4 < -2$

17 Solve the inequalities
a $4x - 1 < 11$ **b** $6x + 3 > 15$ **c** $3x + 2 < 4$

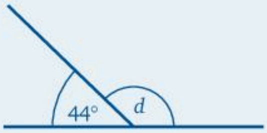
18 Solve the inequalities
a $5 \leqslant 7 - 2x$ **b** $6 - x > 2x - 3$ **c** $x + 2 \leqslant 8$

19 Solve the inequalities **a** $x - 2 \leqslant 5$ and $x + 6 \geqslant 4$
b $3 - 2x \geqslant -9$ and $1 + 4x > 9$

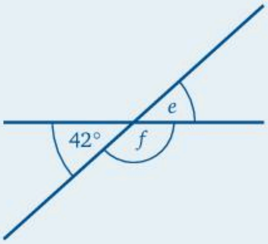
20 How many degrees are there
a in one-third of a revolution
b between the hands of a clock at 4 a.m.?

In questions **21** to **23** find the size of each angle marked with a letter.

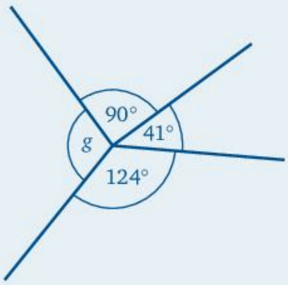
21 **a**



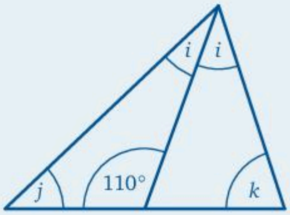
b



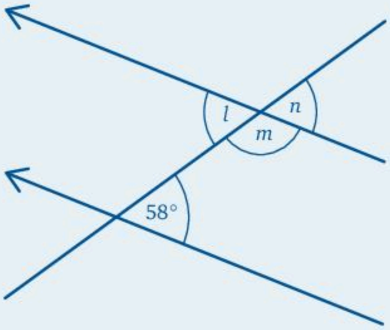
22 **a**



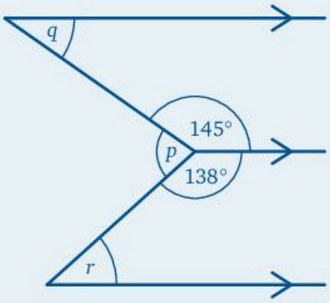
b



23 **a**



b

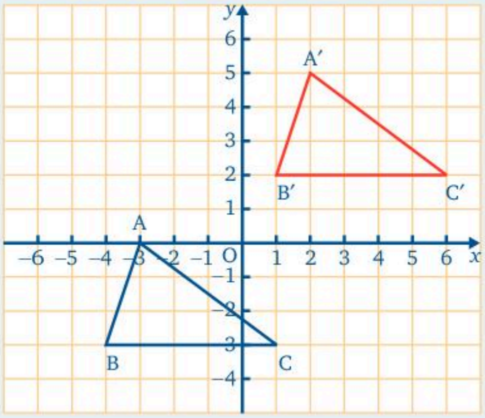


24 Find the image of the point (4, 5) under a translation described by the vector $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$.

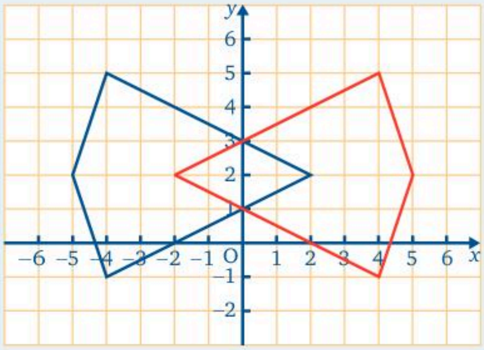
25 Find the image of the point (3, –2) under a translation described by the vector $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$.

26 Find the vector describing the translation that maps the point (2, –5) to the point (–1, 4).

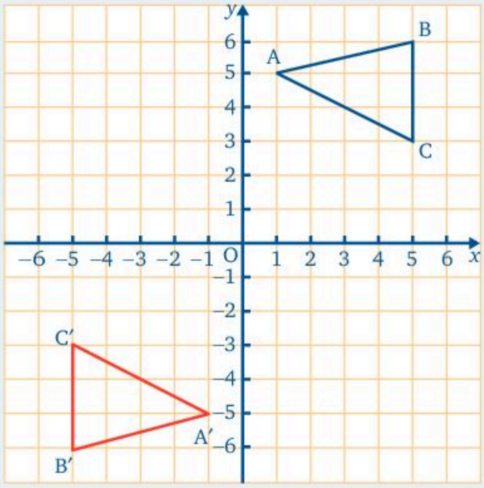
27 Find the vector that maps A to A′, B to B′ and C to C′.
Are they all equal?
Is the transformation a translation?



28 Describe the transformation that maps the object (blue) to the image (red).

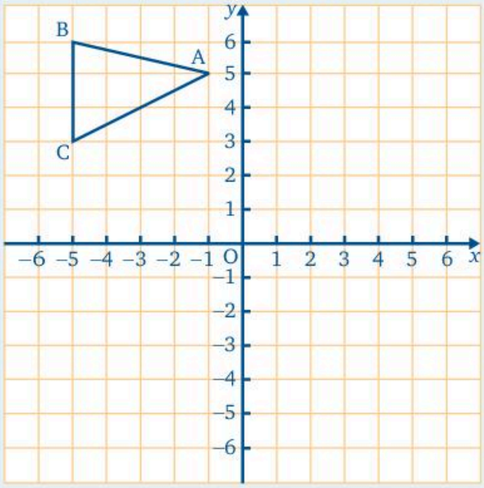


29 Describe the transformation that maps the object (blue) to the image (red).



30 Triangle ABC is rotated about O through 180°. Find the coordinates of the image of

a A **b** B **c** C



6 Angles and polygons

At the end of this chapter you should be able to...

1 state the sum of the angles of a triangle
2 calculate the third angle of a triangle given the other two angles
3 classify triangles as isosceles or equilateral
4 identify equal sides (angles) of an isosceles triangle, given the angles (sides)
5 state the sum of the interior angles of a quadrilateral
6 classify polygons in terms of their number of sides
7 identify regular polygons
8 state the sum of the exterior angles of a given polygon with *n* sides as 360°
9 calculate the size of an exterior or interior angle of a regular polygon
10 use the formula for the sum of the exterior angles of a polygon and that for interior angles to solve problems.

Did you know?

The area between two concentric circles (circles having the same centre) is called an annulus.

You need to know...

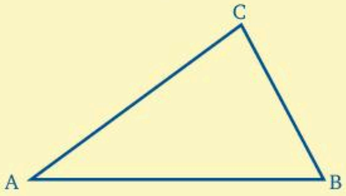
- ✔ that vertically opposite angles are equal
- ✔ that angles on a straight line add up to 180°
- ✔ the properties of the angles formed when a transversal cuts a pair of parallel lines
- ✔ how to solve simple linear equations.

Key words

equilateral triangle, exterior angle, hexagon, interior angle, isosceles triangle, kite, octagon, parallelogram, pentagon, polygon, quadrilateral, rectangle, regular, rhombus, square, trapezium, triangle, vertex (plural vertices)

Triangles

A *triangle* has three sides and three angles.



The corners of the triangle are called *vertices*. (One corner is called a *vertex*.) So that we can refer to one particular side, or to one particular angle, we label the vertices using capital letters. In the diagram above we use the letters A, B and C so we can now talk about 'the triangle ABC' or '△ABC'.

The side between A and B is called 'the side AB' or AB.

The side between A and C is called 'the side AC' or AC.

The side between B and C is called 'the side BC' or BC.

The angle at the vertex A is called 'angle A' or $\widehat{A}$ for short.

Angles of a triangle

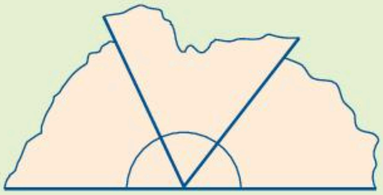
Activity

Draw a large triangle of any shape. Use a straight edge to draw the sides. Measure each angle in this triangle, turning your page to a convenient position when necessary. Add up the sizes of the three angles.

Draw another triangle of a different shape. Again measure each angle and then add up their sizes.

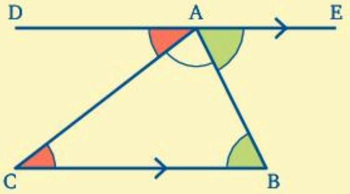
Now try this: on a piece of paper draw a triangle of any shape and cut it out. Next tear off each corner and place the three corners together.

They should look like this:



What conclusion can you make about the sum of the angles of a triangle?

The activity showed that the three angles of the triangle make a straight angle. We can prove this is true for any triangle using angles and parallel lines.



In the diagram, the line DE is drawn parallel to the side CB of triangles ABC.

Then the two red angles are equal because they are alternate angles and the two green angles are equal because they are also alternate angles.

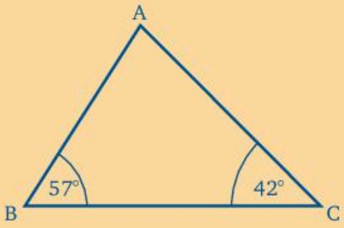
The three angles at A are angles on a straight line, so they add up to 180°.

Therefore the angles of the triangle at A, B and C add up to 180°, that is, $A + B + C = 180°$.

The three angles of a triangle add up to 180°.

Exercise 6a

In each triangle, find the size of angle A (an angle marked with a square is a right angle). Give reasons for each step in your argument, as shown in the worked example



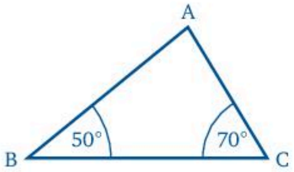
$\hat{A} + 57° + 42° = 180°$ (angles of △ add up to 180°)

$\therefore \quad \hat{A} = 180° - 99°$

$= 81°$

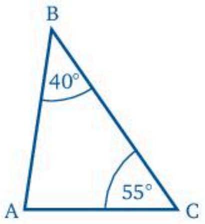
$$\begin{array}{r} 57 \\ +42 \\ \hline 99 \\ \hline \end{array}$$

1



Remember to give reasons for each step of your working.

2



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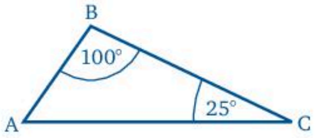

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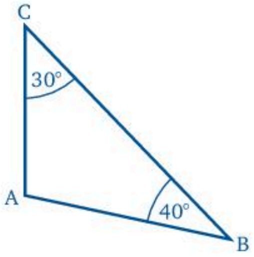

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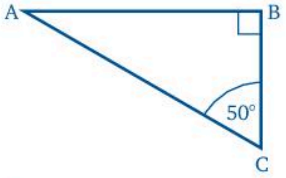

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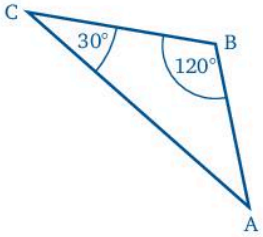

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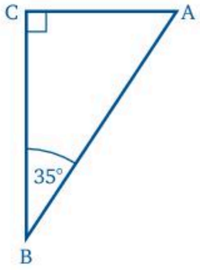

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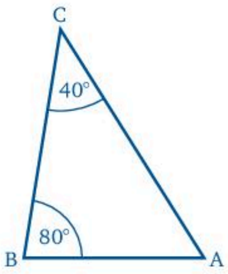

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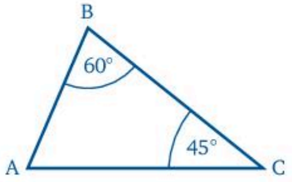

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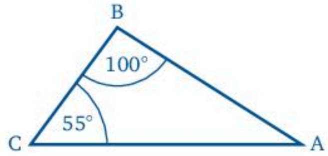

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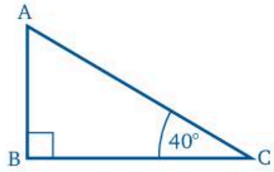

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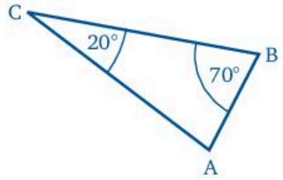

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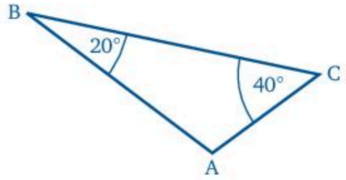

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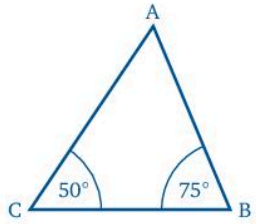

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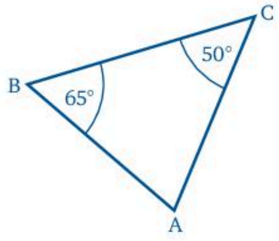


Problems

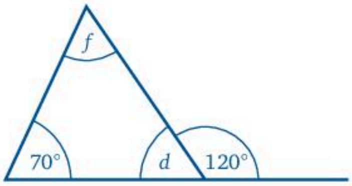
Reminder: Vertically opposite angles are equal.
Angles on a straight line add up to 180°.

You will need these facts in the next exercise.

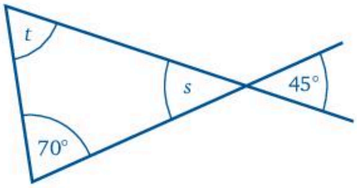
Exercise 6b

In each question make a rough copy of the diagram and mark the sizes of the angles that you are asked to find. You do not need to find them in alphabetical order. You can also mark in any other angles that you know. This may help you find the angles asked for.

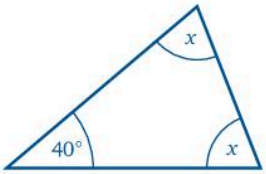
1 Find angles d and f.



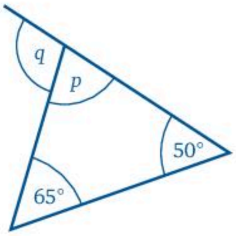
2 Find angles s and t.



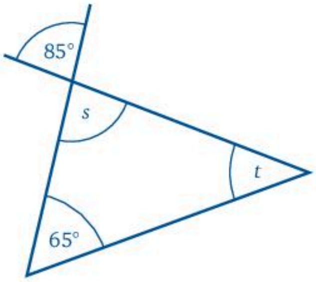
3 Find each of the equal angles x.



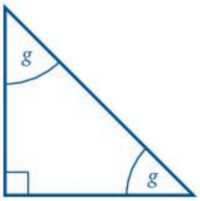
4 Find angles p and q



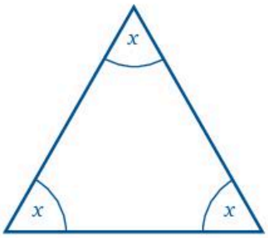
5 Find angles s and t.



6 Find each of the equal angles g.

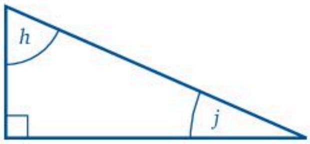


7 Find each of the equal angles x.

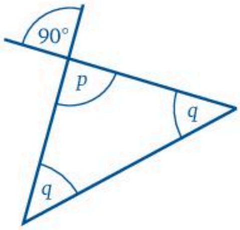


8 Angle h is twice angle j.

Find angles h and j

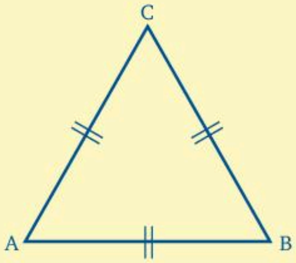


9 Find each of the equal angles q, and angle p.

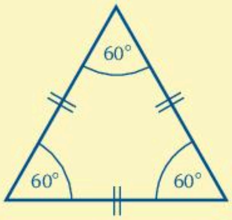


Some special triangles: equilateral and isosceles

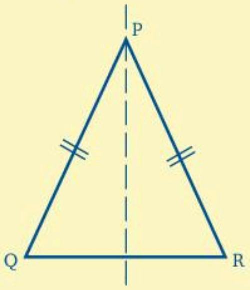
A triangle in which all three sides are the same length is called an *equilateral triangle*.



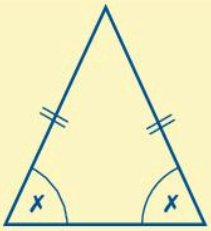
In an equilateral triangle all three sides are the same length and each of the three angles is 60°.



A triangle in which two sides are equal is called an *isosceles triangle*.



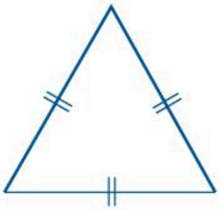
In an isosceles triangle two sides are equal and the two angles opposite the equal sides are equal.



Exercise 6c

In questions **1** to **10** make a rough sketch of the triangle and mark angles that are equal:

1



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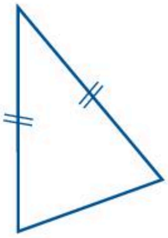

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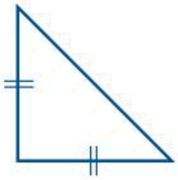

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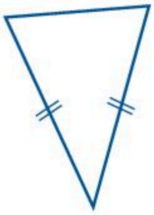

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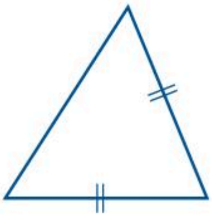

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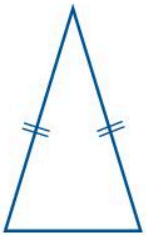

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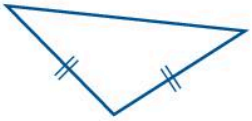

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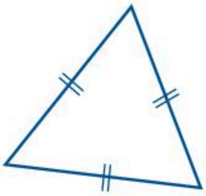

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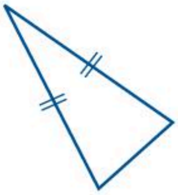

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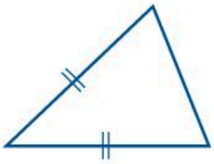
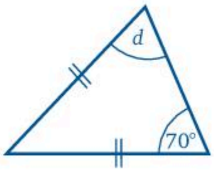


In questions **11** to **22** find angle d:

11



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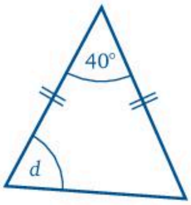

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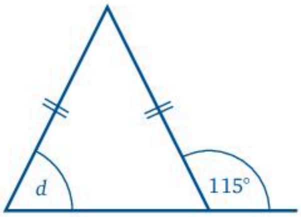

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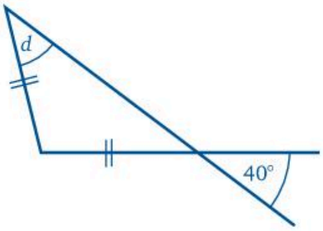

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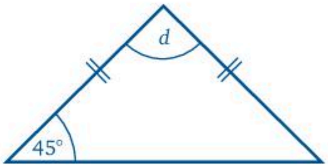

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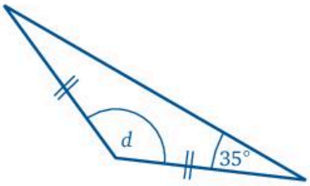

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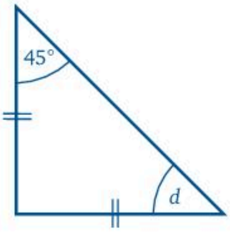

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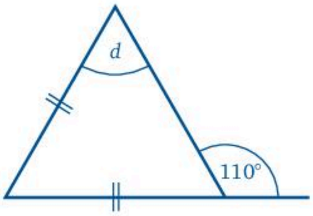

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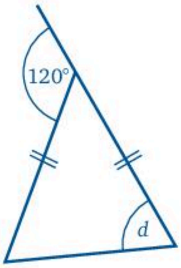

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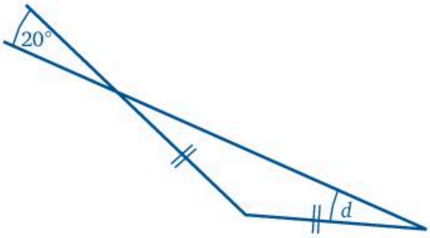

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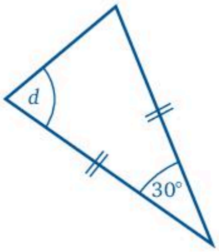

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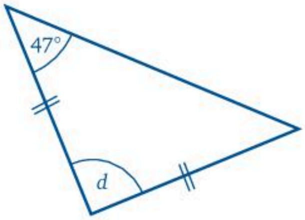
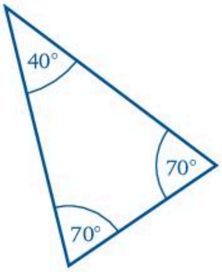


In questions **23** to **26** make a rough sketch of the triangles and mark the equal sides:

23



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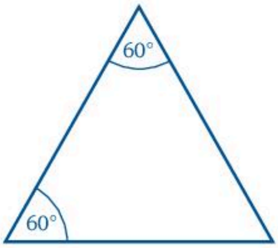

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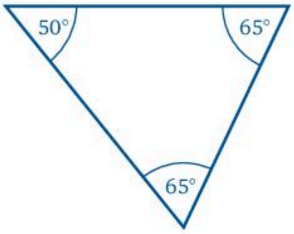

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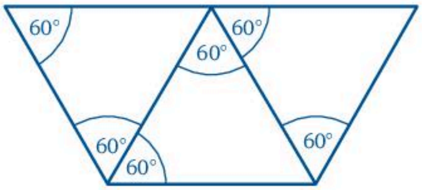
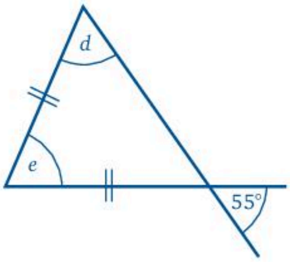


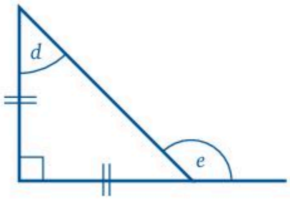
In questions **27** to **32** find angles *d* and *e*:

27



There are two equal sides in an isosceles triangle. The third side is the base.

28



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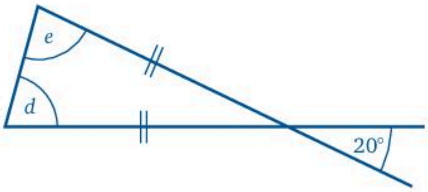

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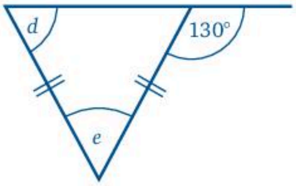

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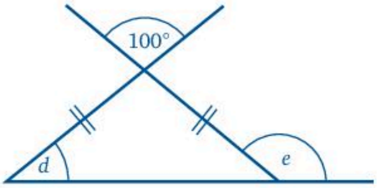

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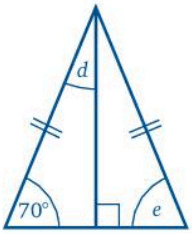
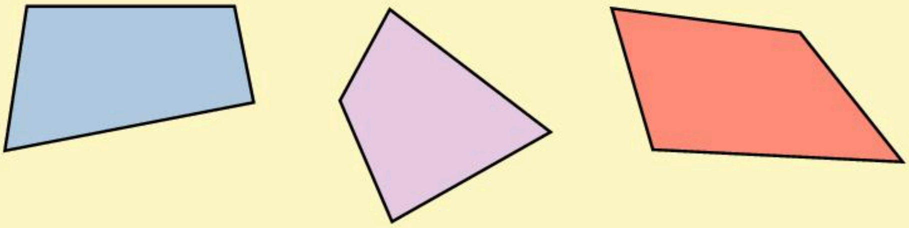


Quadrilaterals

A *quadrilateral* is bounded by four straight sides. These shapes are examples of quadrilaterals:



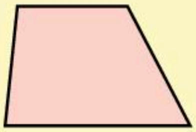
The following diagrams are also quadrilaterals, but each one is a 'special' quadrilateral with its own name:


square


rectangle


parallelogram


rhombus


trapezium

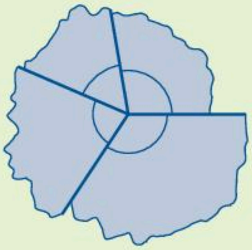

kite

Activity

Draw yourself a large quadrilateral, but do not make it one of the special cases. Measure each angle and then add up the sizes of the four angles.

Do this again with another three quadrilaterals.

Now try this: on a piece of paper draw a quadrilateral. Tear off each corner and place the vertices together. It should look like this:



What conclusion can make about the sum of the four angles of a quadrilateral?

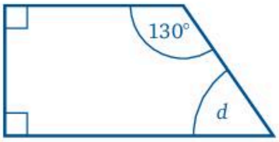
Can you tell why? Hint: draw one of its diagonals.

Exercise 6d

In the Activity you found that the sum of the four angles of a quadrilateral is 360°.
Use this fact in questions **1** to **10** find the size of the angle marked *d*.

Make a rough copy of the following diagrams and mark on your diagram the sizes of the required angles. You can also write in the sizes of any other angles that you can: this may help you find the angles you need.

1



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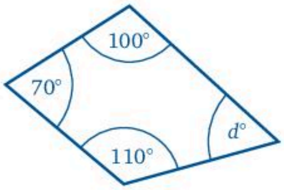

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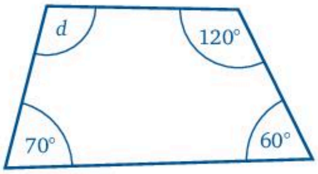

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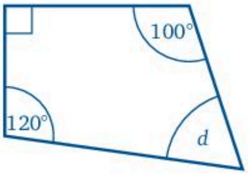

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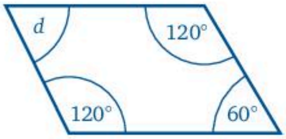

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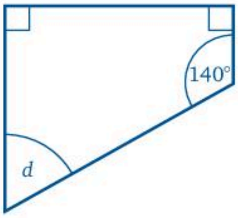

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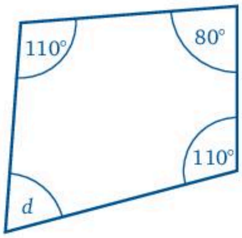

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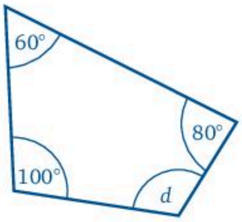

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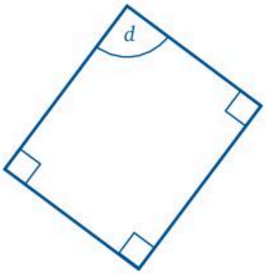
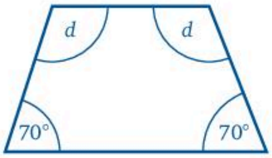
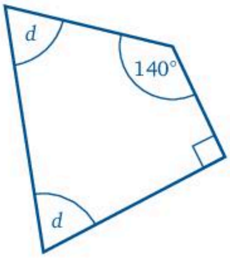


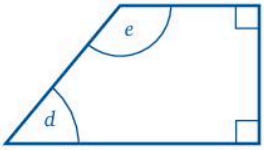
11 Find each of the equal angles *d*.



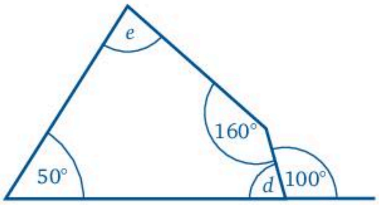
12 Find each of the equal angles *d*.



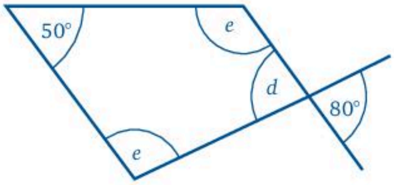
13 Angle *e* is twice angle *d*.
Find angles *d* and *e*.



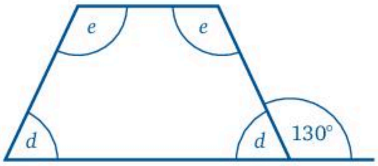
14 Find angles *d* and *e*.



15 Find d and each of the equal angles e.

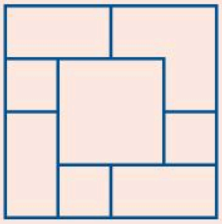


16 Angles d and e are supplementary.
Find d and each of the equal angles e.



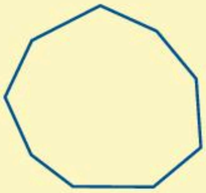
Puzzle

Eight square serviettes are placed flat but overlapping on a table and give the outlines shown in the diagram. In which order must they be removed if the top one is always next?



Polygons

A *polygon* is a plane (flat) figure formed by three or more points joined by line segments. The points are called vertices (singular 'vertex'). The line segments are called sides.

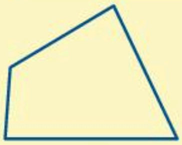


This is a nine-sided polygon.

Some polygons have names which you already know:

a three-sided polygon is a *triangle*

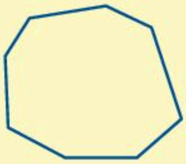
a four-sided polygon is a *quadrilateral*



a five-sided polygon is a *pentagon*

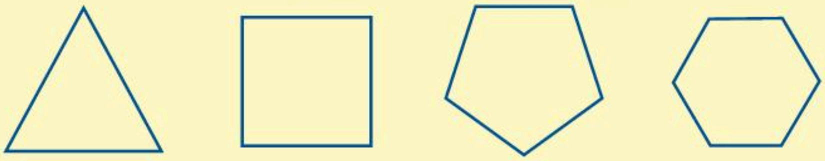
a six-sided polygon is a *hexagon*

an eight-sided polygon is an *octagon*.



Regular polygons

A polygon is called *regular* when all its sides are the same length *and* all its angles are the same size. The polygons below are all regular:



Exercise 6e

State which of the following figures are regular polygons. Give a brief reason for your answer.

1 Rhombus
2 Square
3 Rectangle
4 Parallelogram
5 Isosceles triangle
6 Right-angled triangle
7 Equilateral triangle
8 Circle

Make a rough sketch of each of the following polygons. (Unless you are told that a polygon is regular, you must assume that it is *not* regular.)

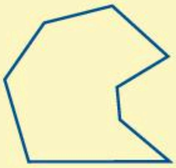
9 A regular quadrilateral
10 A hexagon
11 A triangle
12 A regular triangle
13 A regular hexagon
14 A pentagon
15 A quadrilateral
16 A ten-sided polygon

Convex and concave polygons

When the vertices of a polygon all point outwards, the polygon is convex.

Sometimes one or more of the vertices point inwards, in which case the polygon is concave.

convex polygon

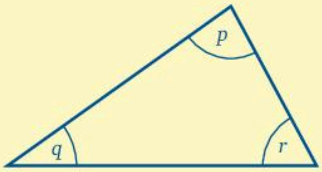


concave polygon

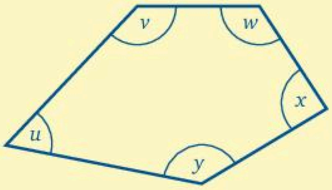
In this chapter we consider only convex polygons.

Interior angles

The angles enclosed by the sides of a polygon are the *interior angles*.
For example:



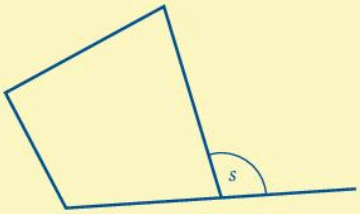
p, q and r are the interior angles of the triangle



u, v, w, x and y are the interior angles of the pentagon.

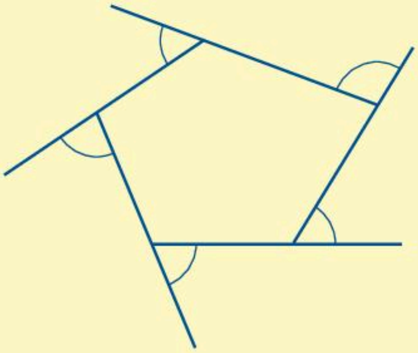
Exterior angles

If we produce (extend) one side of a polygon, an angle is formed outside the polygon. It is called an *exterior angle.*



For example, s is an exterior angle of the quadrilateral.

If we produce all the sides in order we have all the exterior angles.



Exercise 6f

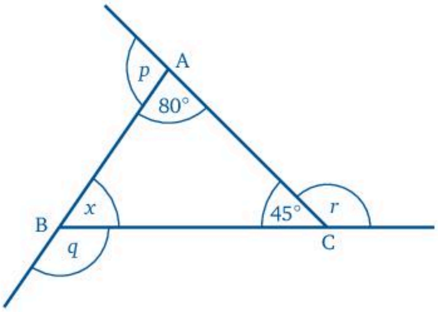
1 What is the sum of the interior angles of any triangle?

2 What is the sum of the interior angles of any quadrilateral?

3 In triangle ABC, find

a the size of each marked angle

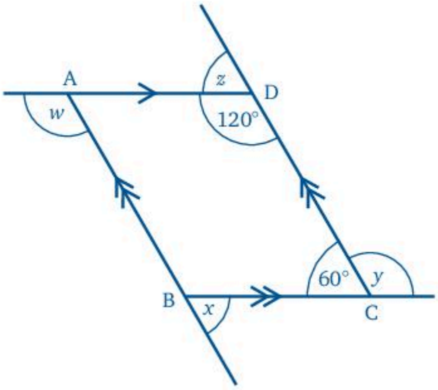
b the sum of the exterior angles.



4 ABCD is a parallelogram. Find

a the size of each marked angle

b the sum of the exterior angles.



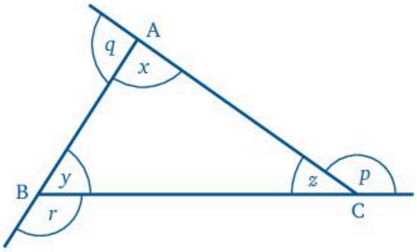
5 In triangle ABC, write down the value of

a $x + q$

b the sum of all six marked angles

c the sum of the interior angles

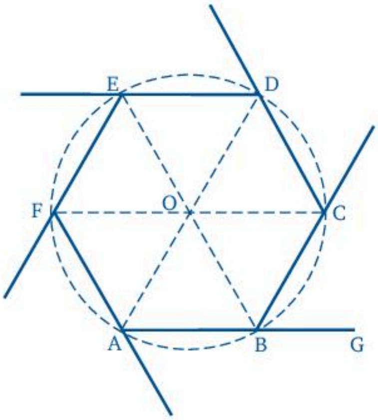
d the sum of the exterior angles.



6 Draw a pentagon. Produce the sides in order to form the five exterior angles. Measure each exterior angle and then find their sum.

7 Construct a regular hexagon of side 5 cm. (Start with a circle of radius 5 cm and then with your compasses still open to a radius of 5 cm, mark off points on the circumference in turn.) Produce each side of the hexagon in turn to form the six exterior angles.

If O is the centre of the circle, joining O to each vertex forms six triangles.



a What kind of triangle is each of these triangles?

b What is the size of each interior angle in these triangles?

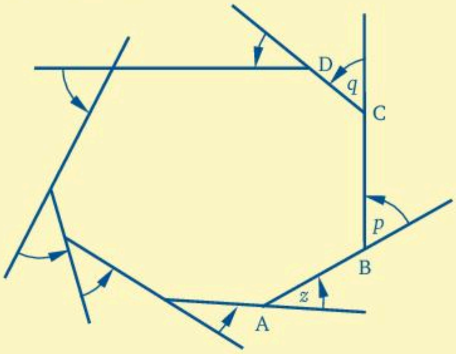
c Write down the value of $A\hat{B}C$.

d Write down the value of $C\hat{B}G$.

e Write down the value of the sum of the six exterior angles of the hexagon.

The sum of the exterior angles of a polygon

In the last exercise, we found that the sum of the exterior angles is 360° in each case. This is true of any polygon, whatever its shape or size.

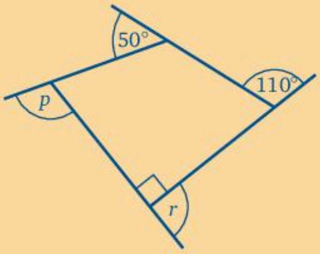


Consider walking round this polygon. Start at A and walk along AB. When you get to B you have to turn through angle p to walk along BC. When you get to C you have to turn through angle q to walk along CD, ... and so on until you return to A. If you then turn through angle z you are facing in the direction AB again. You have now turned through each exterior angle and have made just one complete turn, i.e.

The sum of the exterior angles of a polygon is 360°.

Exercise 6g

Find the size of the angle marked p.



$p + r + 110° + 50° = 360°$ (sum of exterior angles of a polygon)

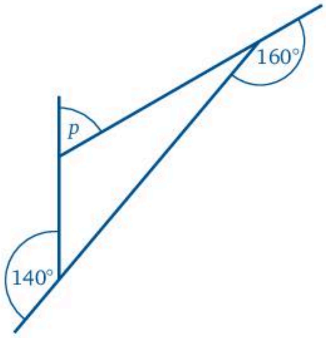
but $r = 90°$ (angles on a straight line)

∴ $p = 360° - 90° - 110° - 50°$

$p = 110°$

In each case find the size of the angle marked *p*:

1



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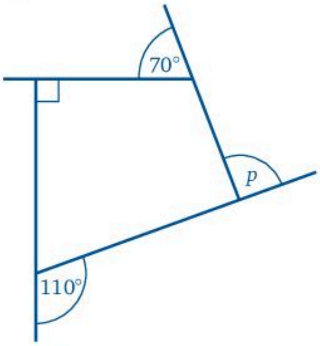

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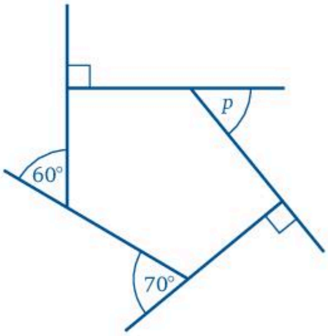

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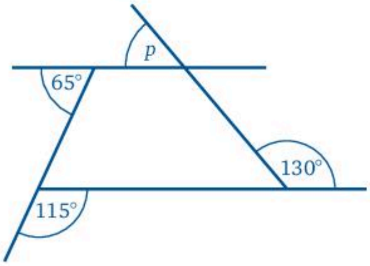

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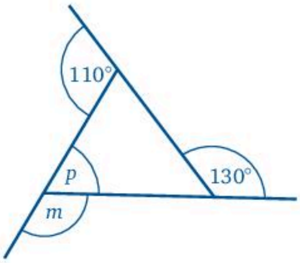

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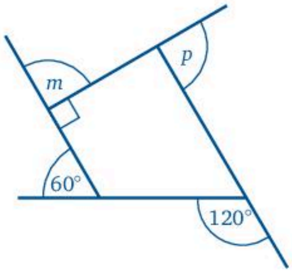

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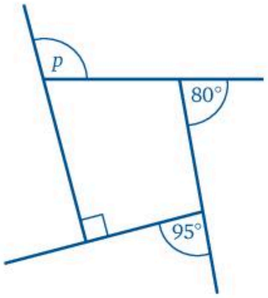

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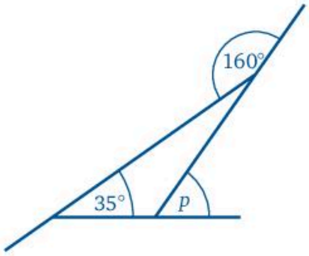

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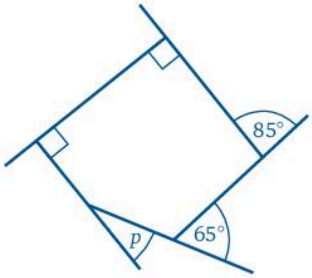

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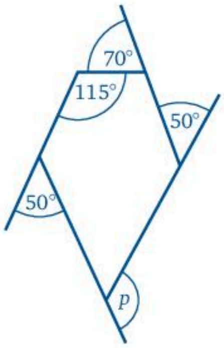
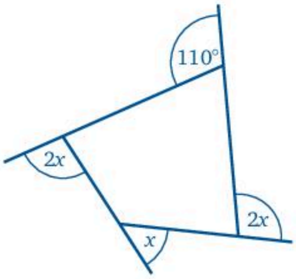
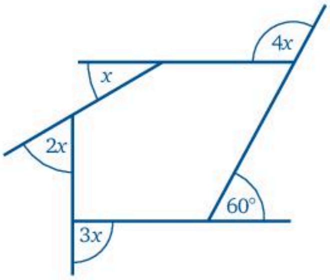


In questions **11** and **12** find the value of x.

11



12



13 The exterior angles of a hexagon are x, $2x$, $3x$, $4x$, $3x$ and $2x$. Find the value of x.

14 Find the number of sides of a polygon if each exterior angle is

a 72° **b** 45°.

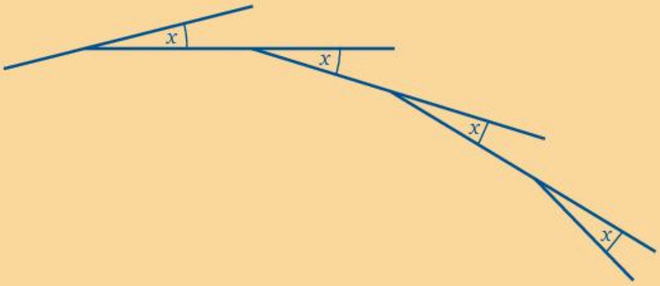
The exterior angle of a regular polygon

You know that the sum of the exterior angles of any polygon is 360°. If a polygon is regular, all its exterior angles are the same size. Since we know that the sum of the exterior angles is 360°, the size of one exterior angle is easily found; we just divide 360° by the number of sides of the polygon, i.e.

in a *regular* polygon with n sides, the size of an exterior angle is $\frac{360°}{n}$.

Exercise 6h

Find the size of each exterior angle of a 24-sided regular polygon.



(There are 24 exterior angles, each of the same size. Their sum is 360°)

Each exterior angle $= \frac{360°}{24} = 15°$.

Find the size of each exterior angle of a regular polygon with:

1 10 sides

2 8 sides

3 12 sides

4 6 sides

5 15 sides

6 18 sides

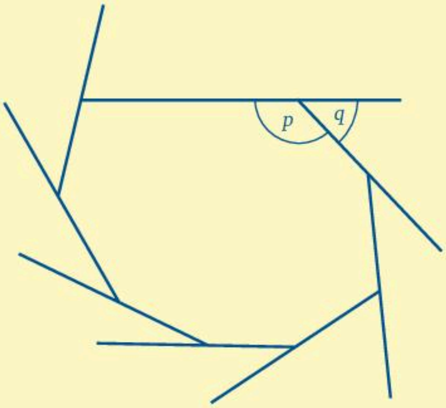
7 9 sides

8 16 sides

9 20 sides

The sum of the interior angles of a polygon

Consider an octagon:



At each vertex there is an interior angle and an exterior angle and the sum of these two angles is 180° (angles on a straight line), i.e. $p + q = 180°$ at each one of the eight vertices.

Therefore, the sum of the interior angles and exterior angles together is

$$8 \times 180° = 1440°$$

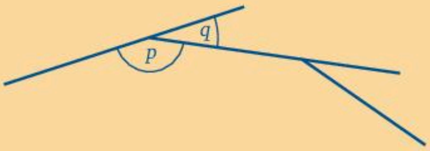
The sum of the eight exterior angles is 360°.

Therefore, the sum of the interior angles is

$$1440° - 360° = 1080°$$

Exercise 6i

Find the sum of the interior angles of a 14-sided polygon.



At each vertex $p + q = 180°$

If there are 14 sides there are 14 pairs of exterior and interior angles.

$\therefore$ sum of interior angles and exterior angles is

$$14 \times 180° = 2520°$$

$\therefore$ sum of interior angles $= 2520° - 360°$

$= 2160°$

Find the sum of the interior angles of a polygon with:

1 6 sides

2 5 sides

3 10 sides

4 4 sides

5 7 sides

6 12 sides

<u>7</u> 18 sides

<u>8</u> 9 sides

<u>9</u> 15 sides

Formula for the sum of the interior angles

If a polygon has n sides, the sum of the interior and exterior angles together is $n \times 180° = 180n°$. Therefore the sum of the interior angles only is $180n° - 360°$. As $360° = 180° \times 2$, this can be written as $(n-2)180°$,

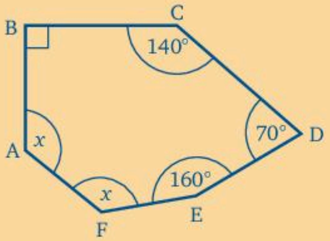
i.e. in a polygon with n sides, the sum of the interior angles is

$$(180n - 360)° \qquad \text{or} \qquad (n-2)180°$$

Exercise 6j

1 Find the sum of the interior angles of a polygon with

a 20 sides **b** 16 sides **c** 11 sides.



In the hexagon ABCDEF, the angles marked x are equal. Find the value of x.

The sum of the interior angles is $180° \times 6 - 360° = 1080° - 360° = 720°$

$$\therefore 90° + 140° + 70° + 160° + 2x = 720°$$

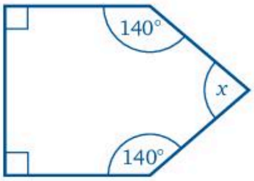
$$460° + 2x = 720°$$

$$2x = 260°$$

$$x = 130°$$

In each of the following questions find the size of the angle(s) marked x:

2



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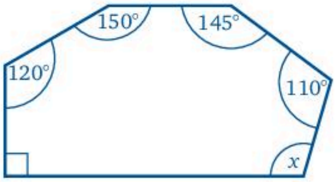

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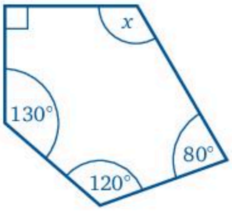

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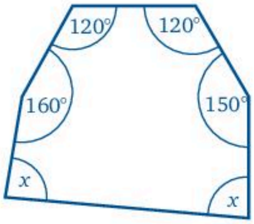

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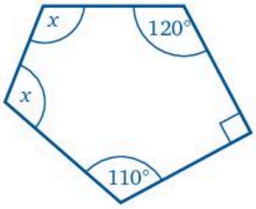

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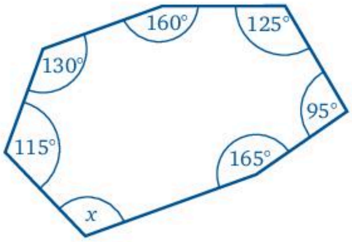


Find the size of each interior angle of a regular nine-sided polygon.

(As the polygon is regular, all the exterior angles are equal and all the interior angles are equal.)

Method 1 Sum of exterior angles = 360°

∴ each exterior angle $= 360° \div 9 = 40°$

∴ each interior angle $= 180° - 40° = 140°$

Method 2 Sum of interior angles $= 180° \times 9 - 360° = 1260°$

∴ each interior angle $= 1260° \div 9 = 140°$

Find the size of each interior angle of each shape:

8 a regular pentagon

9 a regular hexagon

10 a regular octagon

11 a regular ten-sided polygon

12 a regular 12-sided polygon

13 a regular 20-sided polygon.

14 How many sides has a regular polygon if each exterior angle is

a 20° **b** 15°?

15 How many sides has a regular polygon if each interior angle is

a 150° **b** 162°?

Find the exterior angle first.

16 Is it possible for each exterior angle of a regular polygon to be

a 30° **c** 50° **e** 70°

b 40° **d** 60° **f** 90°?

In those cases where it is possible, give the number of sides.

17 Is it possible for each interior angle of a regular polygon to be

a 90° **c** 180° **e** 170°

b 120° **d** 175° **f** 135°?

In those cases where it is possible, give the number of sides.

18 Construct a regular pentagon with sides 5 cm long.

19 Construct a regular octagon of side 5 cm.

Find the size of each interior angle, then use your protractor.

Puzzle

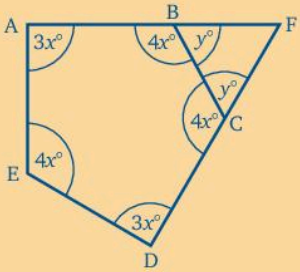
Arrange ten counters in such a way as to form five rows with four counters only in each row.

Mixed problems

Exercise 6k

ABCDE is a pentagon, in which the interior angles at A and D are each $3x°$ and the interior angles at B, C and E are each $4x°$. AB and DC are produced until they meet at F.

Find $B\hat{F}C$.



Sum of the interior angles of a pentagon $= 180^\circ \times 5 - 360^\circ$

$$= 540^\circ$$

$$\therefore \quad 3x + 4x + 3x + 4x + 4x = 540$$

$$18x = 540$$

$$x = 30$$

$$\therefore \quad A\hat{B}C = 120^\circ \quad \text{and} \quad B\hat{C}D = 120^\circ$$

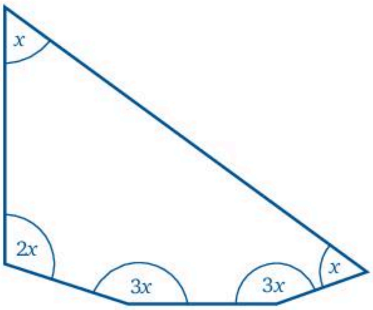
so $\quad y = 60 \quad$ (angles on a straight line)

$\therefore \quad B\hat{F}C = 180^\circ - 2 \times 60^\circ \quad$ (angle sum of $\triangle BFC$)

$$= 60^\circ$$

In questions **1** to **10** find the value of x:

1



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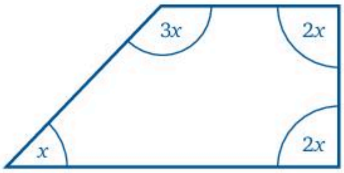

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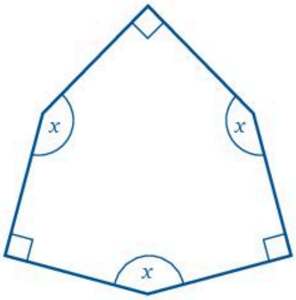

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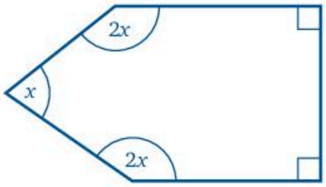

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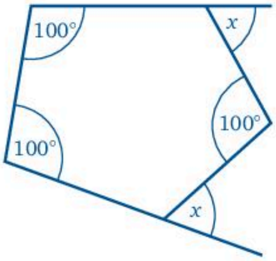

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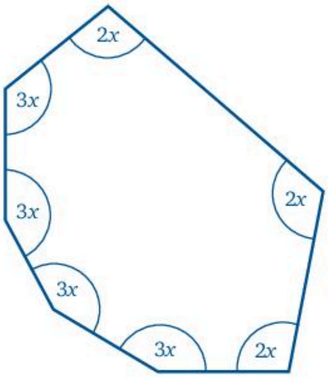

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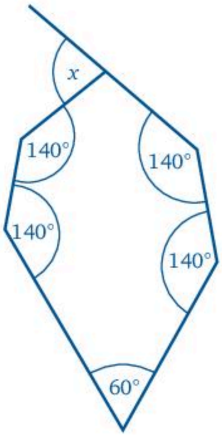

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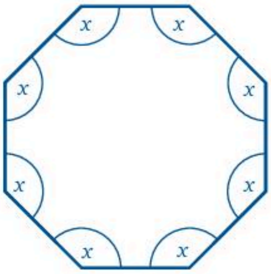

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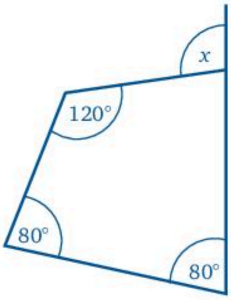

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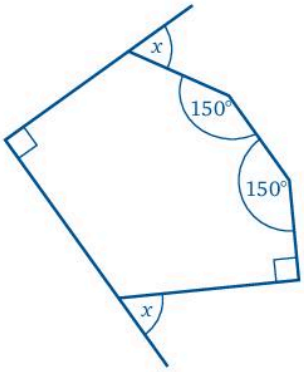
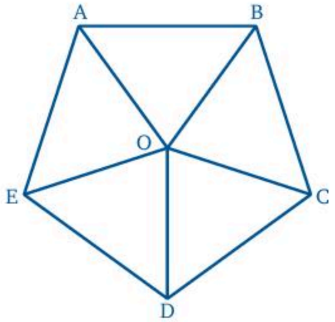


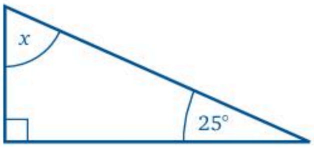
11 ABCDE is a regular pentagon.
OA = OB = OC = OD = OE.
Find the size of each angle at O.



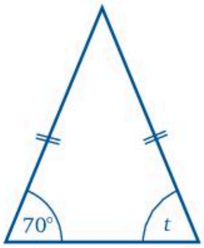
Mixed exercises

Exercise 6I

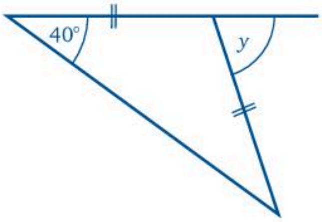
1 Find the size of the angle marked x.



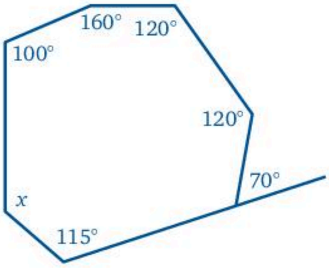
2 Find the size of the angle marked t.



3 Find the size of the angle marked y.

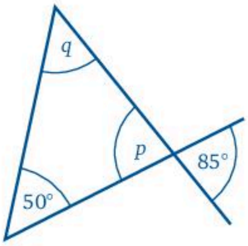


4 Find the size of x.

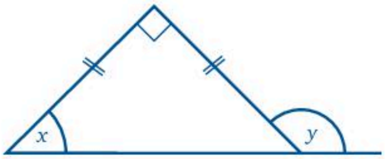


Exercise 6m

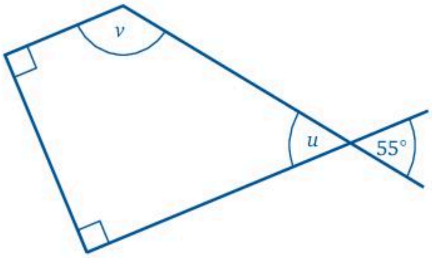
1 Find the size of the angles marked p and q.



2 Find the size of the angles marked x and y.

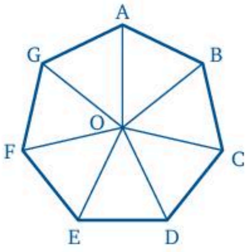


3 Find the size of the angles marked u and v.



4 ABCDEFG is a regular seven-sided polygon.

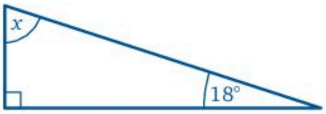
OA = OB = OC = OD = OE = OF = OG. Find the size of each angle at O.



Exercise 6n

Select the capital letter that gives the correct answer.

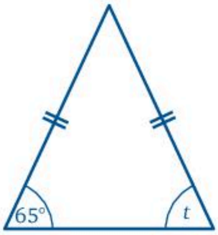
1



The size of the angle marked *x* is

A 48° **B** 62° **C** 68° **D** 72°

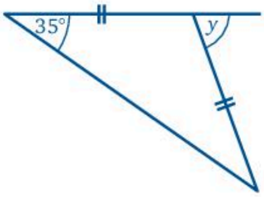
2



The size of the angle marked *t* is

A 25° **B** 45° **C** 65° **D** 75°

3

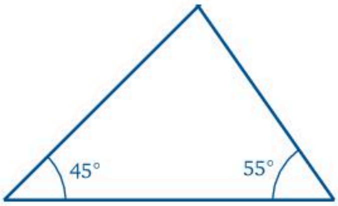


The size of the angle marked *y* is

A 45° **B** 70° **C** 75° **D** 100°

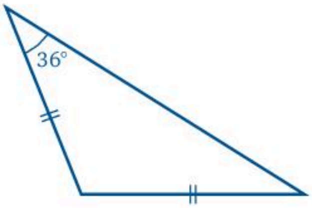
4 The size of the third angle in this triangle is

A 40° **B** 77° **C** 80° **D** 90°



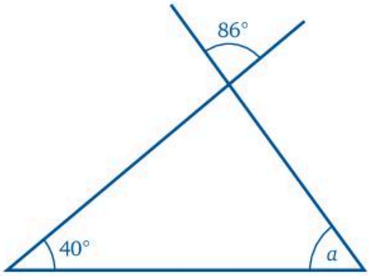
5 The size of the obtuse angle in this triangle is

A 98° **B** 106° **C** 108° **D** 118°



6 The value of the angle marked a in this diagram is

A 40° **B** 44° **C** 46° **D** 54°



7 A regular polygon has 18 sides. The size of one of its exterior angles is

A 18° **B** 20° **C** 22° **D** 24°

8 The exterior angle of a regular polygon is $22\frac{1}{2}°$. The number of sides this polygon has is

A 16 **B** 18 **C** 20 **D** 22

9 The sum of the interior angles of a polygon with 10 sides is

A 1080° **B** 1440° **C** 1800° **D** 2340°

10 Which two of these angles are possible exterior angles for a regular polygon?

A 40° and 45° **B** 40° and 50° **C** 40° and 55° **D** 50° and 55°

11 The value of x in this regular polygon is

A 120°

B 140°

C 150°

D 160°

12 How many sides has a regular polygon if each interior angle is 160°?

A 12 **B** 15 **C** 18 **D** 20

Puzzle

Find a way of cutting up an equilateral triangle into four pieces so that the pieces fit together to form a square.

In this chapter you have seen that...

- ✓ the three angles of a triangle add up to 180°
- ✓ the four angles of a quadrilateral add up to 360°
- ✓ an equilateral triangle has three equal sides and each of its angles is 60°
- ✓ an isosceles triangle has two equal sides and the two angles at the base of these sides are equal
- ✓ the sum of the exterior angles of any polygon is 360°
- ✓ if a polygon is regular (i.e. equal sides and angles) the exterior angles are equal and the size of each one is 360° ÷ the number of sides
- ✓ for a polygon with n sides the sum of the interior angles is $(180n - 360)°$ or, in a slightly more useful form, $(n - 2)180°$.

7 Statistics

At the end of this chapter you should be able to...

1 find the mode, mean and median from data given in a frequency table

2 draw pie charts and interpret information from them

3 draw line graphs

4 draw pictographs

5 draw bar charts and interpret information from them.

Did you know?

William Playfair (1759–1823) is believed to have been the inventor of the pie chart, bar chart and line graph. Born in Scotland, he was at one time personal assistant to James Watt, the Scotsman who did such important early work on the development of the steam engine.

You need to know...

✓ how to work with whole numbers and with decimals

✓ the meaning of mean, median and mode.

Key words

arithmetic average, bar chart, circle, data, distribution, fraction, frequency, frequency table, line graph, mean, median, mode, pictograph, pie chart, protractor

Mean, mode and median

In Grade 7 we saw that, when we have a set of numbers, there are three different measures we can use that attempt to give a 'typical member' that is representative of the set.

Mean

The *mean* (*arithmetic average*) of a set of n numbers is the sum of the numbers divided by n. The mean of the set 2, 6, 8, 8, 10, 10, 12, is

$$\frac{2+6+8+8+10+10+12}{7}=\frac{56}{7}=8$$

The mean value of a set of numbers is the most frequently used form of average, so much so that the word 'average' is often used for 'mean value'.

Mode

In a set of numbers, the *mode* is the number that occurs most often. For example, for the set 2, 2, 4, 4, 4, 5, 6, 6, the mode is 4 as the number 4 occurs more often than any of the other numbers.

The mode is easier to find if the numbers are arranged in order of size.
If the numbers in a set are all different, there is no mode.
For example, the set 1, 2, 3, 5, 8, 10, has no mode.

If there are two (or more) numbers which equally occur most often, there are two (or more) modes.

For example, in the set 1, 2, 2, 3, 5, 5, 8, both 2 and 5 are modes.

Median

If we arrange a set of numbers in order of size, the *median* is the number in the middle. For example, for the seven numbers in the set 2, 4, 5, 7, 7, 8, 9, the median is 7.

When there is an even number of numbers in the set, the median is the mean of the two middle numbers. For example, for the eight numbers in the set 2, 3, 4, 4, 5, 6, 7, 7, the median is the mean of 4 and 5, i.e. 4.5.

For a small set of numbers, say 15, it is easy to find the median and we can see that it is the $\left(\frac{15+1}{2}\right)$th value, i.e. the 8th value. From examples such as these we deduce that, for n numbers arranged in order of size, the median is the $\left(\frac{n+1}{2}\right)$th number.

For example, for 59 numbers, the median is the $\left(\frac{59+1}{2}\right)$th number, i.e. the 30th number. For 60 numbers, the median is the $\left(\frac{60+1}{2}\right)$th number, i.e. the $\left(30\frac{1}{2}\right)$th number. This means the average of the 30th and 31st numbers.

Exercise 7a

A page from a novel by George Lamming was chosen at random and the number of letters in each of the first twenty words on that page was recorded:

3, 4, 5, 3, 7, 8, 3, 3, 6, 2, 4, 6, 4, 6, 3, 13, 4, 3, 3, 2

Find the mean, mode and median of the number of letters per word.

Arranging the numbers in size order:

$$2,\ 2,\ 3,\ 3,\ 3,\ 3,\ 3,\ 3,\ 3,\ \underset{\text{10th}}{\underset{/}{4}},\ \underset{\text{11th}}{\underset{\backslash}{4}},\ 4,\ 4,\ 5,\ 6,\ 6,\ 6,\ 7,\ 8,\ 13$$

The mean is $\frac{92}{20} = 4.6$

The mode is 3

The median is the value of the $\left(\frac{20+1}{2}\right)$th number,

i.e. the $\left(10\frac{1}{2}\right)$th number, which is the average of the 10th and 11th numbers

$\therefore$ the median is 4

Find the mean, mode and median of the sets of numbers in questions **1** to **4**. Remember to arrange the numbers in order of size first. Give answers correct to three significant figures where necessary:

1 3, 6, 2, 5, 9, 2, 4

2 13, 16, 12, 14, 19, 12, 14, 13

3 1.6, 2.4, 3.9, 1.7, 1.6, 0.2, 1.3, 2.0

4 1.3, 1.8, 1.7, 1.9, 1.4, 1.5, 1.3, 1.8, 1.2

5 Ten music students took a Grade 3 piano examination. They obtained the following marks:

106, 125, 132, 140, 108, 102, 75, 135, 146, 123

Find the mean and median marks. Which of these two representative measures would be most useful to the teacher who entered the students? (Give *brief* reasons – do not write an essay on the subject.)

6 A small firm employs ten people. The monthly salaries of the employees are as follows:

$3000 000, $800 000, $500 000, $500 000, $500 000, $500 000, $500 000, $400 000, $300 000, $150 000

Find the mean, mode and median salary.

Which of these three figures is a trade union official unlikely to be interested in, and why?

Finding the mode from a frequency table

The *frequency table* shows the number of houses in a village that are occupied by different numbers of people:

Number of people living in one house	0	1	2	3	4	5	6
Frequency	2	10	8	15	25	12	4

The highest *frequency* is 25 so there are more houses with four people living in them than any other number, i.e. the modal number of people living in one house is 4.

Finding the mean from a frequency table

The pupils in class 3G were asked to state the number of children in their own family and the following frequency table was made:

Number of children per family	1	2	3	4	5
Frequency	7	15	5	2	1

Adding the frequencies gives the total number of families as 30.

We have seven families with one child giving seven children, 15 with two children giving 30 children and so on, giving the total number of children as

$$(7 \times 1) + (15 \times 2) + (5 \times 3) + (2 \times 4) + (1 \times 5) = 65$$

There are 30 numbers in the set, so the mean is

$$\frac{65}{30} = 2.2 \text{ (to 1 d.p.)}$$

i.e. there are, on average, 2.2 children per family.

In this set, the most common number of children is 2, but because some families have more children, the average is higher. No family has 2.2 children!

To avoid unnecessary errors, this kind of calculation needs to be done systematically and it helps if the frequency table is written vertically.

We can then add a column for the number of children in each group and sum the numbers in this column for the total number of children.

Number of children per family x	**Frequency f**	fx
1	7	7
2	15	30
3	5	15
4	2	8
5	1	5
	No. of families = 30	No. of children = 65

mean $= \frac{65}{30} = 2.2$ (to 1 d.p.)

Exercise 7b

1 A survey of the number of people living in the properties of a street gave the following data.

You saw how to make frequency tables in Grade 7.

5 2 1 3 4 5 2 4

2 4 4 2 3 4 5 2

3 5 0 1 1 4 4 1

3 2 4 5 6 5 3 0

1 4 2 3 2 4 0 4

2 3 4 4 2 1 6 3

Make a frequency table for this data. Find

a the mode **b** the mean.

2

Number of tickets bought per person for a football match	1	2	3	4	5	6	7
Frequency	250	200	100	50	10	3	1

Find the mean number of tickets bought per person.

Take this data and draw a vertical frequency table, like the one at the top of the page, with a third column.

3 This table shows the results of counting the number of prickles per leaf on 50 holly bushes.

Number of prickles	1	2	3	4	5	6
Frequency	4	2	8	7	20	9

Find

a the mean number of prickles per leaf **b** the mode.

4 A six-sided die was thrown 50 times. The table gives the number of times each score was obtained.

Score	1	2	3	4	5	6
Frequency	7	8	10	8	5	12

Find

a the mean score per throw **b** the mode.

5 Three coins were tossed together 30 times and the number of heads per throw was recorded.

Number of heads	0	1	2	3
Frequency	3	12	10	5

Find

a the mean number of heads per throw **b** the mode.

Finding the median from a frequency table

Exercise 7c

A group of students gathered this information about themselves.

Number of children in each family	**Frequency**
1	8
2	12
3	4
4	2

Find the median number of children per family.

You want to find the middle value.

First find the number of families (add up the frequencies).

$$8 + 12 + 4 + 2 = 26$$

You can find where in the order the middle value is by adding 1 to the total, then dividing this by 2. This tells you which family or families you want.

$$(26 + 1) \div 2 = 13\tfrac{1}{2}$$

So the family you want is not in the first 8 families, but is in the next 12 families, i.e. 9 to 20. So the median number of children per family is 2.

1 The number of words in each line of a page taken at random from a book is shown in the table.

Number of words in a line	Frequency
2	1
3	4
4	9
5	14
6	20
7	24
8	31
9	1

a How many lines are there on the page?

b Find

i the median **ii** the mean **iii** the mode,

for the number of words per line on the page.

2 Once every five minutes, Debbie counted the number of people queuing at a checkout. Her results are shown in this table.

Number of people queuing at a supermarket checkout	Frequency
0	4
1	6
2	5
3	2
4	2

Write down the median number of people queuing.

3 This frequency table shows the *distribution* of scores when a die is rolled 20 times.

Score	1	2	3	4	5	6
Frequency	3	2	5	3	3	4

Find the median score.

4 In a shooting competition a competitor fired 50 shots at a target and got the following scores

Score	1	2	3	4	5
Frequency	3	4	18	16	9

Find

a the median score **b** the mode **c** the mean.

Explain what each of the mean, mode and median means for this information.

5 The table shows the distribution of goals scored by the home teams one Saturday.

Score	0	1	2	3	4	5
Frequency	3	8	4	3	5	2

Find

a the median score **b** the mode **c** the mean.

Explain what each of the mean, mode and median means for this information.

Puzzle

A certain crystal doubles in size every minute. At 12 noon it is placed in a container and by 2 p.m. fills it completely. At what time would the container be

a half full b three-quarters empty?

Pie charts

A *pie chart* is used to represent information when some quantity is shared out and divided into different categories.

This pie chart shows the proportions, within a group, of people with eyes of certain colours. The *circle* representing the whole group is divided into slices.

The size of the ‘pie slice’ represents the size of the group. We can see without looking at the numbers that there are about the same number of people with brown eyes as with grey eyes and that there are about twice as many with grey eyes as with blue. The size of the pie slice is given by the size of the angle at the centre, so to draw a pie chart we need to calculate the sizes of the angles.

The number of people is 60.

As there are 12 blue-eyed people, they form $\frac{12}{60}$ of the whole group and are therefore represented by that *fraction* of the circle.

Blue: $\frac{12}{\cancel{60}_{1}} \times \frac{\cancel{360°}^{6}}{1} \times 1 = 72°$

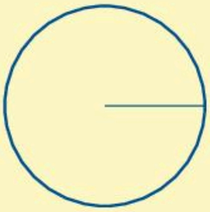
Grey: $\frac{20}{\cancel{60}_{1}} \times \frac{\cancel{360°}^{6}}{1} = 120°$

Hazel: $\frac{6}{\cancel{60}_{1}} \times \frac{\cancel{360°}^{6}}{1} = 36°$

Brown: $\frac{22}{\cancel{60}_{1}} \times \frac{\cancel{360°}^{6}}{1} = 132°$

Total 360°

Now draw a circle of radius about 5 cm (or whatever is suitable). Draw one radius as shown and complete the diagram using a *protractor*, turning your page into the easiest position for drawing each new angle.



Label each 'slice'.

Exercise 7d

Draw pie charts to represent the following information, first working out the angles.

1 A box of 60 coloured balloons contains the following numbers of balloons of each colour:

Colour	Red	Yellow	Green	Blue	White
Number of balloons	16	22	10	7	5

Find the number of balloons in each category as a fraction of the total number of balloons. Then find this fraction of 360°.

2 Ninety people were asked how they travelled to work and the following information was recorded:

Transport	Car	Bus	Walk	Motorcycle	Bicycle
Number of people	32	38	12	6	2

3 On a cornflakes packet the composition of 120 g of cornflakes is given in grams as follows:

Protein	**Fat**	**Carbohydrate**	**Other ingredients**
101	1	10	8

4 Of 90 cars passing a survey point it was recorded that 21 had two doors, 51 had four doors, 12 had three (two side doors and a hatchback) and 6 had five doors.

5 A large flower arrangement contained 18 dark red roses, 6 pale pink roses, 10 white roses and 11 deep pink roses.

6 The children in a class were asked what pets they owned and the following information was recorded:

Animal	Dog	Cat	Bird	Rabbit	Fish
Frequency	8	10	3	6	3

The eye colours of 54 people were recorded:

Eye colour	Blue	Grey	Hazel	Brown
Frequency	10	19	5	20

The total number here is 54, which is not as convenient as in the previous problems. In problems like these we may have to find an angle correct to the nearest degree.

Blue:

$$\frac{10}{\cancel{54}_{\cancel{6}\,3}} \times \frac{\cancel{360°}^{\cancel{40}\,20}}{1} = \frac{200°}{3}$$

$$= 66\frac{2}{3}° = 67° \qquad \text{(to the nearest degree)}$$

Grey:

$$\frac{19}{\cancel{54}_{3}} \times \frac{\cancel{360°}^{20}}{1} = \frac{380°}{3}$$

$$= 126\frac{2}{3}° = 127° \qquad \text{(to the nearest degree)}$$

Hazel:

$$\frac{5}{\cancel{54}_{3}} \times \frac{\cancel{360°}^{20}}{1} = \frac{100°}{3}$$

$$= 33\frac{1}{3}° = 33° \qquad \text{(to the nearest degree)}$$

Brown:

$$\frac{20}{\cancel{54}_{3}} \times \frac{\cancel{360°}^{20}}{1} = \frac{400°}{3}$$

$$= 133\frac{1}{3}° = 133° \qquad \text{(to the nearest degree)}$$

Total $= 67 + 127 + 33 + 133 = 360°$

Draw pie charts to represent the following information, working out the angles first and, where necessary, giving the angles correct to the nearest degree.

7 300 people were asked whether they lived in a condo, a house, a studio, an apartment or in some other type of accommodation and the following information was recorded:

Type of accommodation	Condo	House	Studio	Apartment	Other
Frequency	90	150	33	15	12

8 In a street in which 80 people live the numbers in various age groups are as follows:

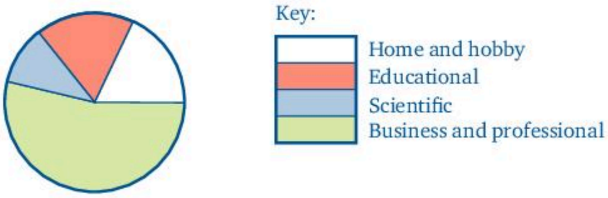
Age group (years)	0–15	16–21	22–34	35–49	50–64	65 and over
Number of people	16	3	19	21	12	9

9 Use the information in Exercise 7b, question 5 on page 150.

Interpreting pie charts

Exercise 7e

1 This pie chart shows the uses of personal computers in 2019:

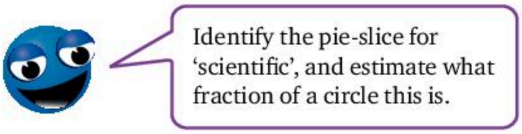


a For which purpose were computers used most?

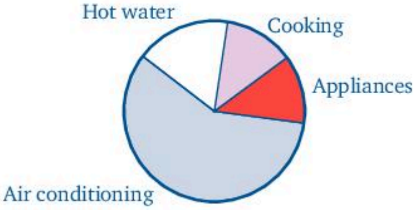
b Estimate the fraction of the total used for

  i scientific purposes

  ii home and hobbies.



2 The pie chart below shows how fuel is used for different purposes in the average house in the Caribbean:

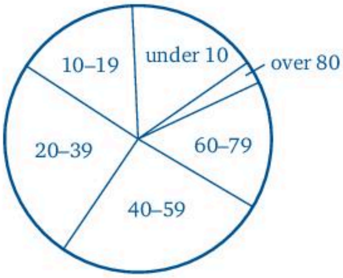


a For which purpose is most fuel used?

b How does the amount used for cooking compare with the amount used for hot water?

Is the angle of the slice for 'cooking' bigger or smaller than the angle of the slice for 'hot water' and by roughly how much?

3 The pie chart shows the age distribution of a population in years in 2018:



a Estimate the size of the fraction of the population in the age groups

- **i** under 10 years
- **ii** 20–39 years.

b State which groups are of roughly the same size.

Remember that a whole turn is 360°. What fraction of a whole turn represents each age group?

Line graphs

A *line graph* is used to show how a quantity changes over time.
For example, temperature, share prices, cost of living and profits are all quantities that change over time.

This table shows the profits of AB Manufacturing Company as declared at the end of each year from 2010 to 2017.

2010	2011	2012	2013	2014	2015	2016	2017
$25 000 000	$15 000 000	$10 000 000	$12 000 000	$13 000 000	$18 000 000	$26 000 000	$29 000 000

We can draw a line graph to illustrate these figures by plotting the profits against the years and joining the points with straight lines.

The lines help to show how the profits are changing but values between the lines mean nothing. For example, it is not possible to give the profits half way between the end of 2016 and the end of 2017 for two reasons: there is no point on the graph for this time and half-year profits may or may not be half the yearly profits.

Exercise 7f

1 This table shows the average daily temperature at noon, in degrees Celsius, for each month of a year on an island.

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
14	16	20	23	24	27	30	32	25	19	17	16

Draw a line graph by plotting the temperature against the month and joining the points in order with straight lines.

a Which month gave the highest average daily temperature?

b Which month had the higher average daily temperature, May or September?

c Which month gave the lower average daily temperature, March or November?

2 This table shows the annual rainfall on an island.

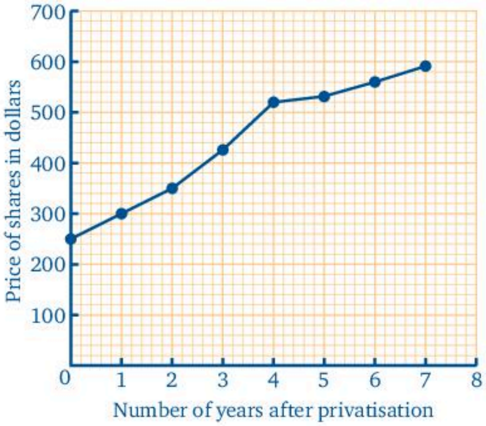
Year	2012	2013	2014	2015	2016	2017	2018	2019
Rainfall, in mm	500	440	530	560	530	490	570	540

a Plot these values on a graph.

b Join the points with straight lines.

c Is it true that the annual rainfall on the island is tending to increase? Give a reason for your answer.

3 The line graph shows the price of a share in a privatised company at yearly intervals after privatisation.



a What was the price of the share when issued?

b Copy and complete the following table.

Time after privatisation (years)	Value of share (dollars)
1	
2	
3	
4	
5	
6	
7	

c In which year after privatisation did the price of the share rise most?

d What trend do you notice in the price of the share?

4 **a** Use the graph given in question **3** to estimate the price of the share, 8 years after privatisation.

b Do you think that, 8 years after privatisation, the share price will be the same as your estimate?
Give a reason for your answer.

5 Use the graph for question **2** for these questions.

   a Can the graph be used to find the half-monthly sales figures for the year? Explain your answer.

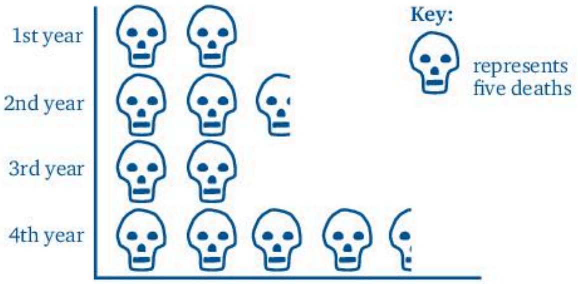
   b Looking at this graph, the managing director asked why the sales had fallen in the first half of July. The sales director replied that they had not; they had in fact increased for the first two weeks of July. How could the sales director justify this statement and, assuming it is correct, describe what happened to the sales in the second half of July.

Pictographs

To attract attention, *pictographs* are often used on posters and in newspapers and magazines. The best pictographs give the numerical information as well; the worst give the wrong impression.

Exercise 7g

1 Road deaths in the past 4 years at an accident black spot:

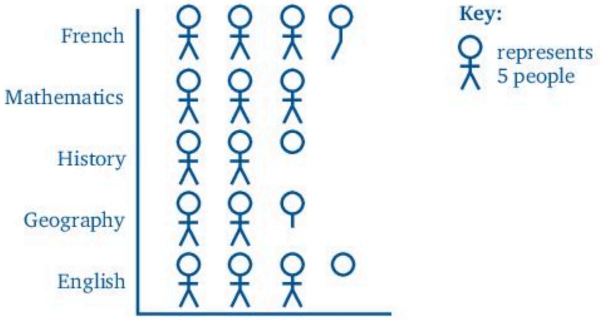


   a Give an estimate of the number of deaths in each year.

   b What message is the poster trying to convey?

   c How effective do you think it is?

2 The most popular subject among Grade 8 pupils:

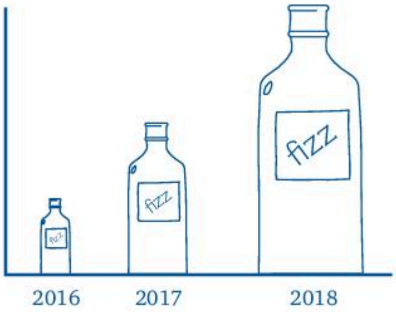


   a Which is the most popular subject?

   b How many pupils chose each subject and how many were asked altogether?

   c Is this a good way of presenting the information?

3 This pictograph is an advertisement showing the consumption of Fizz lemonade.



This looks impressive but it could be misleading. Why?

Drawing pictographs

Make sure when using drawings that each drawing takes up the same amount of space and is simple and clear.

Exercise 7h

1 Eighty-five people were asked how they travelled to work and the following information was recorded:

Transport	Car	Bus	Walk	Bicycle
Number of people	30	40	10	5

Draw a pictograph using one drawing to 5 people.

2 Thirty pupils in a class were asked what they were writing with. The following information was recorded:

Writing implement	Black pen	Blue pen	Pencil
Frequency	12	9	9

Draw a pictograph using one drawing to 3 pens.

3 Some children were asked what pets they owned:

Pet	Dog	Cat	Bird	Small animal	Fish
Frequency	9	7	6	10	2

Use one drawing to one pet. Make the symbols simple.

The symbol for fish could be 

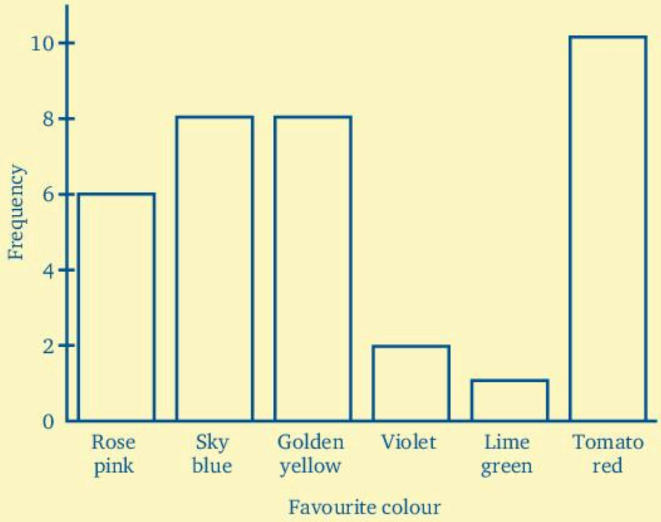
Bar charts

When information (or *data*) is collected, it can be illustrated in various ways and one of the most common is the *bar chart*.

This data is from a group of people who were asked to select their favourite colour from a card showing six colours. The following results were recorded.

Colour	Rose pink	Sky blue	Golden yellow	Violet	Lime green	Tomato red
Number of people (frequency)	6	8	8	2	1	10

We can illustrate this data using bars where the height of each bar represents the frequency of each category.



The bars must all be the same width but they do not have to touch. The spaces between the bars must all be the same.

Notice that the groups are arranged along the base line and the frequencies are marked on the vertical axis.

Exercise 7i

In questions **1** to **3** draw bar charts to show the information given in the frequency tables. Mark the frequency on the vertical axis and label the bars below the horizontal axis.

1 Types of vehicles moving along a busy road during one hour:

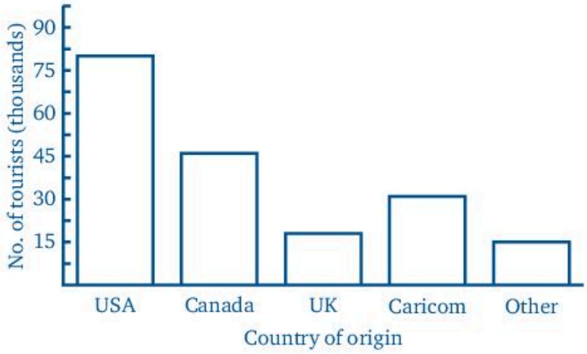
Vehicle	Cars	Vans	Lorries	Motorcycles	Bicycles
Frequency	62	11	15	10	2

2 Thirty pupils were asked to state their favourite subject chosen from their school timetable:

Subject	English	Mathematics	French	PE	History	Geography
Frequency	5	7	4	3	7	4

3 The number of tourist arrivals by country, to a certain Caribbean island, in the first six months of 2018 (to the nearest thousand) were:

Country	USA	Canada	UK	Caricom	Other
No. of tourists	80 000	46 000	18 000	31 000	15 000



In an attempt to save space, in the bar chart shown above the scale for the number of tourists was started at 15 000 instead of at 0.

a Redraw the bar chart with the scale for the number of tourists going from 0 to 90 000 (suggested scale 1 cm to 15 000).

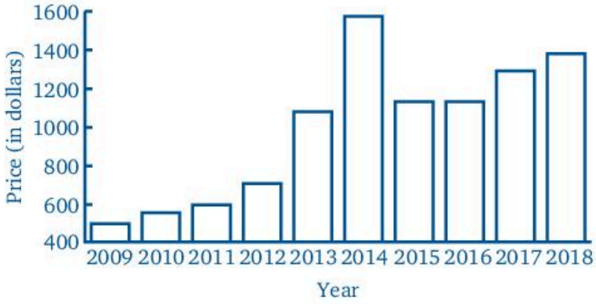
b Compare the two bar charts. The impression given by one of them is misleading. Why?

Interpreting bar charts

Bar charts can be used to represent information other than frequencies and can appear in different forms. The bars are usually vertical but occasionally they are horizontal.

Exercise 7j

1 The average price per tonne of sugar earned by a Caribbean country in the ten-year period 2009–2018:



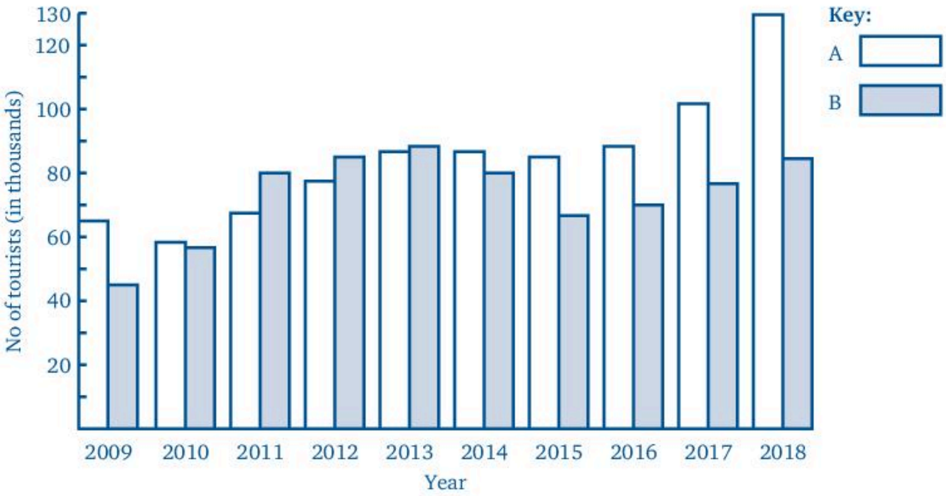
a In which year was the price lowest? Highest?

b In which year was the price increase from the previous year the greatest?

c In which year was the price of sugar above $1400 per tonne?

d In which two years did the price remain the same?

2 The tourist arrivals (in thousands) to destinations A and B for the period 2009–2018:

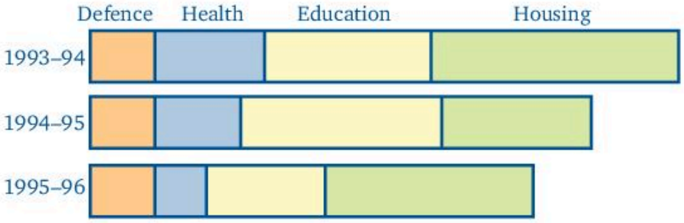


a Did more tourists visit B or A during the period 2009–2018?

b In which year did destination B have fewest tourists?

c In which years did B have better tourist seasons than A?

3 Cost of defence, health, education and housing over a three-year period:



No numbers are given, but we can get a good idea about the relative costs.

a In which year was the most money spent?

b On what was the least money spent?

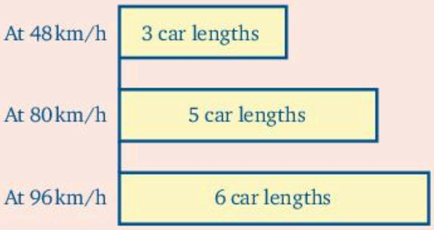
c In which year did education cost most?

d In which year did health cost least?

e In which year was the least money spent?

Puzzle

A rough guide to the distance to keep behind another car on the road:



Can you puzzle out what rule has been used to decide on the distances?

Why is the guide only 'rough'? What other factors should be taken into account?

Exercise 7k

1 **a** Draw a bar chart to show the number of full lorries leaving a quarry each day for one seven-day period.

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Number of lorries	30	50	25	27	40	10	0

b During which day were the most full lorries leaving the quarry?

c Give a reason why no lorries left on Sunday.

d The quarry produced material by blasting the rock. When do you think blasting took place? Give a reason.

2 **a** Draw a bar chart to show the birth rates per 1000 population in ten Caribbean countries in 1996.

Country	Antigua	Barbados	Dominica	Grenada	Guadeloupe	Jamaica	St Kitts	Trinidad	St Vincent
Birth rate per 1000	24	21	39	30	28	34	25	27	36

b Which country has the highest birth rate?

c Which country has the lowest birth rate?

Investigation

Collect the information; where it is necessary, you may need to divide the data into groups, for example heights correct to the nearest centimetre in groups 131–135 cm, 136–140 cm, 141–145 cm, and so on. Record the information in a frequency table. Write a short report on any problems you had collecting the information.

Suggestions for class projects

1. Heights of children in the class.
2. Masses of children in the class.
3. Handspan. Stretch your hand out as wide as it will go on a piece of paper and mark the positions of the end of the thumb and of the little finger. Measure the distance between these points to the nearest centimetre.
4. Times of journeys to school in minutes.
5. Times of arrival at school.

For projects **6** to **12**, illustrate your information. Decide whether a bar chart or a pictograph would be more suitable for representing the data.

6. Find out the size of the family of each student in your class. Find out the numbers of boys and girls in each family. Compare the fraction of boys over girls to the National data for this fraction.
7. Pets owned.
8. Pets you would *like* to own, but decide on categories first before collecting the information.
9. Birthday months.
10. Number of houses in the street where a pupil lives. Decide what to record if houses are isolated or the pupil lives in an apartment block.
11. Colours of cars seen passing during, say, 20 minutes.
12. Number of people in cars travelling at a given time of day, say on the way to school.

Suggestions for individual projects

For projects **13** to **18**, also find, where possible, the mode, mean and median.

Add to your report which one of these is best at representing the data. Also add anything else you notice about the data.

13 Throw one die 120 times and record the scores.

14 Throw two dice 120 times and record the combined score each time.

15 Choose a page of a book of plain text and record the occurrence of the different letters of the alphabet.

16 Choose a page of text in a different language and repeat number 15. Compare the two sets of results.

17 Choose pages of text from a book and record the lengths of, say, 60 sentences.

18 Choose a page of text and record the number of letters used in each word. Decide beforehand what to do about words with hyphens.

In this chapter you have seen that...

- ✔ the arithmetic average or mean of a set of numbers is the sum divided by the number of them
- ✔ the mode is the number that occurs most often
- ✔ the median is the middle number when they have been placed in order. When there are two middle values the median is half-way between them
- ✔ large quantities of information can be made sense of by grouping the information and putting it into a frequency table. The frequency of a group is the number of items in that group
- ✔ frequency tables can be represented by pie charts or bar charts
- ✔ the heights of the bars correspond to the frequencies of the groups
- ✔ the slices on a pie chart represent the fractions that the groups are of the total
- ✔ a line graph shows how a quantity changes with time
- ✔ you can draw a line graph by plotting the values of the quantity at given times and joining the points with straight lines.

8 Ratio and proportion

At the end of this chapter you should be able to...

1 increase a quantity in a given ratio
2 decrease a quantity in a given ratio
3 divide a quantity in a given ratio
4 solve problems on direct proportion
5 calculate the distance travelled in a given time by an object moving at a constant speed
6 calculate the time taken to travel a given distance at a constant speed
7 calculate the average speed of a body covering different distances at different speeds.

Did you know?

When the illustrious English mathematician G. H. Hardy went to visit the brilliant Indian mathematician Srinivasa Ramanujan in hospital, he began by telling him that he had driven there in a taxi with the uninteresting number of 1729. Ramanujan replied that 1729 was a very interesting number. It is the smallest number that could be expressed as the sum of two cubes in two different ways.

Can you find them?

You need to know...

- ✓ the meaning of equivalent fractions
- ✓ how to simplify fractions
- ✓ how to multiply a fraction by a whole number
- ✓ the meaning of LCM
- ✓ how to increase a quantity by a given fraction.

Key words

average speed, constant speed, denominator, directly proportional, equivalent fractions, knot, numerator, ratio, speed

Ratio

We introduced *ratio* in Grade 7. The next few sections revise that work.

Reminder: a ratio is a comparison between two related quantities.

For example if one piece of wood is 30 cm long and another is 50 cm long, we say that their lengths are in the ratio 30 cm to 50 cm or 30 cm : 50 cm.

Since they are in the same units we write this as 30 : 50.

The ratio 30 : 50 simplifies to 15 : 25 (dividing each number in the given ratio by 2) and to 3 : 5 (dividing each number in the given ratio by 10).

Exercise 8a

Express the following ratios in their simplest form:

1 12 : 10

2 30 : 15

3 4 cm : 12 cm

4 64 c : 96 c

5 500 g : 1 kg

6 $5\,\text{cm}^2 : 50\,\text{mm}^2$

7 72 c : \$2.16

8 1.25 kg : 600 g

9 $1\,\text{m}^2 : 2000\,\text{cm}^2$

Remember that the quantities have to be measured in the same unit before they can be compared.

Simplify the following ratios:

10 3 : 9 : 18

11 14 : 70 : 56

12 2 : 14 : 22

13 60 : 24 : 72

14 45 : 72 : 63

15 24 : 32 : 40

16 24 : 40 : 56

17 36 : 52 : 28

18 171 : 153 : 117

Express the ratio $\frac{3}{4} : \frac{2}{5}$ in its simplest form.

$$\frac{3}{4} : \frac{2}{5} = 20 \times \frac{3}{4} : 20 \times \frac{2}{5}$$

(multiplying both numbers by 20, which is the lowest common multiple of 4 and 5)

$$= 15 : 8$$

Express the following ratios in their simplest forms:

19 $4 : \frac{1}{3}$

20 $\frac{2}{3} : \frac{1}{2}$

21 $\frac{1}{4} : \frac{3}{4}$

22 $\frac{3}{8} : \frac{1}{2}$

23 $\frac{5}{6} : \frac{7}{24}$

24 $\frac{2}{3} : \frac{11}{15}$

25 $4\frac{1}{2} : 1\frac{2}{5}$

26 $2\frac{1}{4} : 1\frac{3}{4}$

27 $4\frac{1}{2}:3$

28 $9\frac{1}{2}:11\frac{1}{2}$

29 $8:12:16$

30 $\frac{1}{8}:\frac{1}{12}:\frac{1}{16}$

31 $\frac{3}{5}:\frac{2}{3}$

32 $\frac{1}{4}:\frac{5}{8}:\frac{7}{12}$

33 $\frac{3}{4}:\frac{1}{8}:\frac{5}{15}$

34 $\frac{3}{4}:\frac{4}{5}:\frac{5}{6}$

Equivalent ratios

For a given ratio we can find the equivalent ratio when we know either the *numerator* or the *denominator*.

Exercise 8b

Find the missing numbers in the following ratios:

a $9:5 = \quad :25$ **b** $\frac{5}{3} = \frac{15}{} = \frac{}{48}$

a $9:5 = \quad :25$

25 is 5×5, so the missing number is $9 \times 5 = 45$

b $\frac{5}{3} = \frac{15}{} = \frac{}{48}$

15 is 5×3 so the denominator is also multiplied by 3

so the denominator is 3×3, i.e. 9

Similarly with the third fraction the denominator is 48, i.e. 3×16

so the numerator is $5 \times 16 = 80$

Hence $\frac{5}{3} = \frac{15}{9} = \frac{80}{48}$

Find the missing numbers in the following ratios:

1 $4:3 = \quad :15$

2 $7:2 = \quad :12$

3 $12: \quad = 60:5$

4 $3:5 = 21:$

5 $\quad :7 = 56:49$

6 $9:5 = \quad :40$

7 $\frac{2}{5} = \frac{}{20} = \frac{14}{}$

8 $\frac{3}{7} = \frac{12}{} = \frac{}{56}$

9 $\frac{14}{13} = \frac{}{39} = \frac{70}{}$

10 $\frac{7}{11} = \frac{56}{} = \frac{}{121}$

11 $\frac{}{9} = \frac{12}{27} = \frac{60}{}$

12 $\frac{7}{} = \frac{49}{56} = \frac{}{168}$

Comparing ratios

The ratio 30 : 50 can be expressed as a fraction,

i.e. $30:50 = \frac{30}{50}$ which simplifies to $\frac{3}{5}$.

Sometimes we are given two ratios and need to know which one is the larger or the smaller.

For example, to find which ratio is the larger, 7 : 4 or 15 : 7, we can write 7 : 4 as the fraction $\frac{7}{4}$ and 15 : 7 as $\frac{15}{7}$. To compare the sizes of $\frac{7}{4}$ and $\frac{15}{7}$, we express each fraction as an *equivalent fraction* with the same denominator.

$$\frac{7}{4} = \frac{49}{28} \quad \text{and} \quad \frac{15}{7} = \frac{60}{28}$$

So 15 : 7 is larger than 7 : 4.

In a similar way we can determine which of two ratios is the smaller.

Exercise 8c

1 Which ratio is the larger, 5 : 9 or 3 : 4?

2 Which ratio is the smaller, 9 : 4 or 7 : 3?

3 Which ratio is the larger, $\frac{11}{7}$ or $\frac{19}{11}$?

4 Which ratio is the smaller, $\frac{7}{13}$ or $\frac{5}{12}$?

In the following sets of ratios some are equal to one another. In each question identify the equal ratios.

5 $12:18$, $\frac{2}{3}:1$, $\frac{30}{45}:1$, $18:12$

6 $\frac{7}{9}:1$, $35:50$, $63:81$

7 $5:8$, $30:48$, $\frac{25}{16}:2$, $\frac{10}{16}:1$

8 $1:\frac{7}{12}$, $14:24$, $36:21$, $2:\frac{7}{6}$

Increasing and decreasing quantities in a given ratio

Increasing or decreasing a quantity in a given ratio means that

the changed quantity : the original quantity = given ratio.

For example, to change \$100 by the ratio 3 : 2 mean

changed quantity : \$100 = 3 : 2

Exercise 8d

Increase \$500 in the ratio 12 : 5

required quantity : \$500 = 12 : 5

$$\text{i.e. } \frac{\text{required quantity}}{\$500} = \frac{12}{5}$$

Comparing denominators, multiplying both numerator and denominator of $\frac{12}{5}$ by 100 will give equal fractions.

$$\text{i.e. } \frac{\text{required quantity}}{\$500} = \frac{1200}{500}$$

so the required quantity is \$1200.

Decrease 36 cm in the ratio 4 : 9

$$\frac{\text{decreased length}}{36} = \frac{4}{9} = \frac{16}{36}$$

The decreased length is 16 cm.

1. Increase \$12 000 in the ratio 5 : 2
2. Increase \$5600 in the ratio 13 : 8
3. Increase 588 cm in the ratio 12 : 7
4. Increase 15.3 m in the ratio 4 : 3
5. Increase 165 g in the ratio 9 : 5
6. Decrease \$16 360 in the ratio 3 : 4
7. Decrease \$84 in the ratio 7 : 12
8. Decrease 323 mm in the ratio 6 : 19
9. Decrease 28 km in the ratio 5 : 8
10. Decrease 39.1 kg in the ratio 12 : 17

Division in a given ratio

If a mother divides $1000 between two children giving $700 to one and $300 to the other, we can think of the $1000 as being made up of 10 equal parts. She gives 7 parts to one child and 3 parts to the other child. We say she has divided the money between the two children in the ratio 7 : 3.

Exercise 8e

Divide $5000 between Daniel and Mia in the ratio 3 : 4.

Give each amount correct to the nearest dollar.

Who has slightly more than their share?

We need to divide $5000 into 3 + 4, i.e. 7 equal shares. Then Daniel has 3 shares and Mia has 4 shares.

$\text{One share} = \$5000 \div 7 = \$714.28\ldots$

$\therefore$ Daniel's share is $\$714.28\ldots \times 3 = \$2142.8\ldots = \$2143$ correct to the nearest $

and Mia's share is $\$714.28\ldots \times 4 = 2857.14\ldots = \$2857$ correct to the nearest $.

This shows that Daniel gets slightly more than his fair share (and Mia gets slightly less than her fair share).

1 Divide $42 in the ratio 4 : 3

2 Divide $550 in the ratio 2 : 3

3 Divide 54 cm in the ratio 7 : 11

4 Divide 3.6 m in the ratio 4 : 5

5 Divide 20 km in the ratio 3 : 7

In questions **6** to **10** give each answer correct to the nearest whole number.

6 Divide $660 in the ratio 10 : 7

7 Divide 32 m in the ratio 4 : 5

8 Divide 54 cm in the ratio 8 : 5

9 Divide 265 km in the ratio 15 : 25

10 Divide $64\,\text{cm}^2$ in the ratio 14 : 10

Direct proportion

Reminder: If two quantities are *directly proportional*, they are always in the same ratio.

If 1 jar of jam costs \$1400 then the ratio of the number of jars to the cost in dollars is 1 : 1400.

So 3 jars will cost $3 \times \$1400 = \$4200$

and 7 jars will cost $7 \times \$1400 = \$9800$

Since the total cost increases as the number of jars increases, the cost is said to increase in direct proportion to the number of jars bought.

Exercise 8f

If a car uses 3 gallons of petrol for a journey of 162 miles, how far will it travel on 8 gallons?

On 3 gallons the car will travel 162 miles

$\therefore$ on 1 gallon it will travel $\frac{162}{3}$ miles = 54 miles

$\therefore$ on 8 gallons it will travel 54×8 miles = 432 miles

If 5 coaches are required to transport 260 supporters to a football match, how many similar coaches would be required to transport 1092 supporters?

260 supporters require 5 coaches

$\therefore$ 1 supporter requires $\frac{5}{260}$ coaches

so 1092 supporters require $\frac{5}{260} \times 1092$ coaches = 21 coaches

i.e. 1092 supporters require 21 coaches

1 If 12 bars of soap cost \$9000, how much will 20 bars cost?

2 If 5 boxes of tea cost \$4690, how much will 12 boxes cost?

3 Twenty-seven articles cost \$32 130. Find the cost of 44 at the same rate.

4 Thirty-five packets of sweets cost \$8820. Find the cost of twenty-five similar packets.

5 If 12 oranges cost \$1176, find the cost of 25 oranges.

6 A man earns \$201 600 by working for 36 hours.

How much would he earn by working for 44 hours at the same rate?

7 A car travels $192\frac{1}{2}$ miles on 5 gallons of petrol.

How far would it travel on 8 gallons?

8 A small car will run 266 km on 19 litres of petrol.

How far will it run on 32 litres?

9 A motorcycle requires 14 litres of petrol to cover 308 km.

How many litres will be required for a journey of 341 km?

10 A hotel charges \$362 600 per person per week.

What would be the charge for 16 days at the same rate?

11 The airfare for a flight of 1350 miles is \$283 500.

What would be the fare for a flight of 3440 miles at the same rate?

12 The cost of publishing a book with 105 pages is \$2352 per copy.

How many pages could be expected in a book costing \$5264 per copy if the cost per page is considered constant?

Puzzle

32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47
48	49	50	51	52	53	54	55
56	57	58	59	60	61	62	63

A

8	9	10	11	12	13	14	15
24	25	26	27	28	29	30	31
40	41	42	43	44	45	46	47
56	57	58	59	60	61	62	63

B

4	5	6	7	12	13	14	15
20	21	22	23	28	29	30	31
36	37	38	39	44	45	46	47
52	53	54	55	60	61	62	63

C

1	3	5	7	9	11	13	15
17	19	21	23	25	27	29	31
33	35	37	39	41	43	45	47
49	51	53	55	57	59	61	63

D

2	3	6	7	10	11	14	15
18	19	22	23	26	27	30	31
34	35	38	39	42	43	46	47
50	51	54	55	58	59	62	63

E

16	17	18	19	20	21	22	23
24	25	26	27	28	29	30	31
48	49	50	51	52	53	54	55
56	57	58	59	60	61	62	63

F

Rohan places these six cards in front of Justin. Rohan then asks Justin to choose one number from any card but not tell him what it is or which card he chose it from.

The following conversation then takes place.

Rohan: Now select all the cards that have your number on it.

Justin: Right. I have cards B, D and F.

Rohan: The number you chose was 25.

Justin: Correct. How did you do that?

Can you solve this puzzle?

As soon as you can, try it on family and friends. You will impress them with your number skills!

Speed, time and distance

When Wesley drives on the freeway he aims to travel at a steady speed (or *constant speed*) of 80 kilometres per hour (km/h).

This means that in 1 hour he travels 80 km, i.e. at 80 km/h
in 2 hours he travels $80 \times 2 = 160$ km
and in 3 hours he travels $80 \times 3 = 240$ km.

More generally **Distance travelled = Speed × Time**

The icon 

may help you to remember how these quantities are related.

Put your finger over the quantity you want to find. What is left is the formula you need.

It follows that $D = S \times T$, $S = \frac{D}{T}$ and $T = \frac{D}{S}$

If the speed is measured in kilometres per hour and the time is measured in hours, the distance travelled will be measured in kilometres.

Exercise 8g

A coach travels at 60 km/h. How far will it travel in

a 3 hours **b** $5\frac{1}{2}$ hours?

a Distance = speed × time
Therefore distance travelled in 3 hours $= 60 \times 3$ km
$= 180$ km

b Distance travelled in $5\frac{1}{2}$ hours $= 60 \times 5\frac{1}{2}$ km
$= 60 \times \frac{11}{2}$ km
$= 330$ km

1 An express train travels at 200 km/h. How far will it travel in

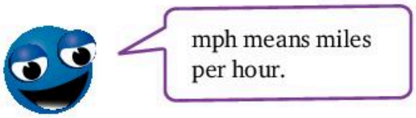
a 4 hours **b** $5\frac{1}{2}$ hours?

2 Ken cycles at 24 km/h. How far will he travel in

a 2 hours **b** $3\frac{1}{2}$ hours **c** $2\frac{1}{4}$ hours?

3 An aeroplane flies at 300 mph. How far will it travel in

a 4 hours **b** $5\frac{1}{2}$ hours?



4 A bus travels at 60 km/h. How far will it travel in

a $1\frac{1}{2}$ hours **b** $2\frac{1}{4}$ hours?

5 Susan can cycle at 12 mph. How far will she ride in

a $\frac{3}{4}$ hour **b** $1\frac{1}{4}$ hours?

6 An athlete can run at 10.5 m/s. How far will he travel in

a 5 seconds **b** 8.5 seconds?

m/s means metres per second.

7 A boy cycles at 12 mph. How far will he travel in

a 2 hours 40 minutes **b** 3 hours 10 minutes?

8 Majid can walk at 8 km/h. How far will he walk in

a 30 minutes **b** 20 minutes **c** 1 h 15 minutes?

9 A racing car travels at 111 mph. How far will it travel in

a 20 minutes **b** 1 hour 40 minutes?

10 A bullet travels at 100 m/s. How far will it travel in

a 5 seconds **b** $8\frac{1}{2}$ seconds?

11 A Boeing 747 travels at 540 mph. How far does it travel in

a 3 hours 15 minutes **b** 7 hours 45 minutes?

12 A racing car travels around a 2 km circuit at 120 km/h. How many laps will it complete in

a 30 minutes **b** 1 hour 12 minutes?

Calculating the time taken

If you know the speed something is travelling at, then you can find out how long it takes to travel a given distance. For example, suppose Georgina walks at 6 km/h. How long will it take her to walk 24 km?

If she takes 1 hour to walk 6 km, she will take $\frac{24}{6}$ hours, i.e. 4 hours, to walk 24 km.

Similarly, if the distance is 15 km, then she will take $\frac{15}{6}$ hours, i.e. $2\frac{1}{2}$ hours, to walk 15 km.

So

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Exercise 8h

1 How long will Zena, walking at 5 km/h, take to walk

a 10 km   **b** 15 km?

2 How long will a car travelling at 80 km/h take to travel

a 400 km   **b** 260 km?

3 How long will it take David, running at 10 mph, to run

a 5 miles   **b** $12\frac{1}{2}$ miles?

4 How long will it take an aeroplane flying at 450 mph to fly

a 1125 miles   **b** 2400 miles?

5 A cowboy rides at 14 km/h. How long will it take him to ride

a 21 km   **b** 70 km?

6 A rally driver drives at 50 mph. How long does it take him to cover

a 75 miles   **b** 225 miles?

7 An athlete runs at 8 m/s. How long does it take her to cover

a 200 m   **b** 1600 m?

8 A dog runs at 20 km/h. How long will it take him to travel

a 8 km   **b** 18 km?

9 A liner cruises at 28 nautical miles per hour. How long will it take to travel

a 6048 nautical miles   **b** 3528 nautical miles?

10 A car travels at 56 mph. How long does it take to travel

a 70 miles   **b** 154 miles?

11 A cyclist cycles at 12 mph. How long will it take her to cycle

a 30 miles   **b** 64 miles?

12 How long will it take a car travelling at 64 km/h to travel

a 48 km   **b** 208 km?

Average speed

Russell Compton left home at 8 a.m. to travel the 50 km to his place of work. He arrived at 9 a.m. Although he had travelled at many different speeds during his journey he covered the 50 km in exactly 1 hour. We say that his *average speed* for the journey was 50 kilometres per hour, or 50 km/h. If he had travelled at the same speed all the time, he would have travelled at 50 km/h.

Judy Smith travelled the 135 miles from her home to Georgetown in 3 hours. If she had travelled at the same speed all the time, she would have travelled at $\frac{135}{3}$ mph., i.e. 45 mph. We say that her average speed for the journey was 45 mph.

In each case: $$\text{average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

This formula can also be written:

$$\text{distance travelled} = \text{average speed} \times \text{time taken}$$

and $$\text{time taken} = \frac{\text{distance travelled}}{\text{average speed}}$$

Suppose that a car travels 35 km in 30 min, and we wish to find its speed in kilometres per hour. To do this we must express the time taken in hours instead of minutes,

i.e. $$\text{time taken} = 30\text{ min} = \frac{1}{2}\text{ hour}$$

Then $$\text{average speed} = \frac{35}{\frac{1}{2}}\text{ km/h} = 35 \times \frac{2}{1}\text{ km/h}$$

$$= 70\text{ km/h}$$

Great care must be taken with units. If we want a speed in kilometres per hour, we need the distance in kilometres and the time in hours. If we want a speed in metres per second, we need the distance in metres and the time in seconds.

Exercise 8i

Find the average speed for each of the following journeys:

1 80 km in 1 hour

2 120 km in 2 hours

3 60 miles in 1 hour

4 480 miles in 4 hours

5 80 m in 4 seconds

6 135 m in 3 seconds

7 150 km in 3 hours

8 520 km in 8 hours

9 245 miles in 7 hours

10 104 miles in 13 hours

11 252 m in 7 seconds

12 255 m in 15 seconds

Find the average speed in km/h for a journey of 39 km which takes 45 minutes.

To find a speed in km/h you need the distance in kilometres and the time in hours.

First, convert the time taken to hours:

$$45 \text{ min} = \frac{45}{60} \text{ hour} = \frac{3}{4} \text{ hour}$$

Then

$$\begin{aligned} \text{average speed} &= \frac{\text{distance travelled}}{\text{time taken}} \\ &= \frac{39 \text{ km}}{\frac{3}{4} \text{ hour}} \\ &= 39 \times \frac{4}{3} \text{ km/h} \\ &= 52 \text{ km/h} \end{aligned}$$

Find the average speed in km/h for a journey of:

13 40 km in 30 min

14 60 km in 40 min

15 48 km in 45 min

16 66 km in 33 min

Find the average speed in km/h for a journey of:

17 4000 m in 20 min

18 6000 m in 45 min

19 40 m in 8 s

20 175 m in 35 s

Make sure that the time is in hours and the distance is in kilometres.

Find the average speed in mph for a journey of:

21 27 miles in 30 min

22 18 miles in 20 min

23 25 miles in 25 min

24 28 miles in 16 min

The following table shows the distances in kilometres between various towns in the West Indies.

	St John's	Roseau	Castries	Basseterre	Kingstown	St Georges	Port of Spain
Roseau	174						
Castries	382	211					
Basseterre	100	478	621				
Kingstown	446	272	74	557			
St Georges	549	570	554	1040	118		
Port of Spain	723	659	534	1218	528	176	
Georgetown	1234	1224	1099	1694	1093	741	565

Use this table to find the average speeds for journeys between:

25 St John's, leaving at 1025 h, and Kingstown, arriving at 1625 h

26 St Georges, leaving at 0330 h, and Castries, arriving at 0730 h

27 Basseterre, leaving at 1914 h, and St Georges, arriving at 2044 h

28 Port of Spain, leaving at 0620 h, and St Johns, arriving at 0750 h

29 Roseau, leaving at 1537 h, and St Georges, arriving at 1907 h

30 Castries, leaving at 1204 h, and Georgetown, arriving at 1624 h

31 Roseau, leaving at 1014 h, and Port of Spain, arriving at 1638 h

Problems frequently occur where different parts of a journey are travelled at different speeds in different times but we wish to find the average speed for the whole journey.

Consider for example a motorist who travels the first 50 miles of a journey at an average speed of 25 mph and the next 90 miles at an average speed of 30 mph.

One way to find his average speed for the whole journey is to complete the following table by using the relationship:

$$\text{time in hours} = \frac{\text{distance in miles}}{\text{speed in mph}}$$

	Speed in mph	**Distance in miles**	**Time in hours**
First part of journey	25	50	2
Second part of journey	30	90	3
Whole journey		**140**	**5**

We can add the distances to give the total length of the journey, and add the times to give the total time taken for the journey.

$$\text{average speed for whole journey} = \frac{\text{total distance}}{\text{total time}}$$

$$= \frac{140\text{ miles}}{5\text{ hours}} = 28\text{ mph}$$

Note: Never add or subtract average speeds.

We could also solve this problem, without using a table, as follows:

$$\text{time to travel 50 miles at 25 mph} = \frac{\text{distance}}{\text{speed}}$$

$$= \frac{50\text{ miles}}{25\text{ mph}} = 2\text{ hours}$$

$$\text{time to travel 90 miles at 30 mph} = \frac{\text{distance}}{\text{speed}}$$

$$= \frac{90\text{ miles}}{30\text{ mph}} = 3\text{ hours}$$

∴ total distance of 140 miles is travelled in 5 hours

i.e. $$\text{average speed for whole journey} = \frac{\text{total distance}}{\text{total time}}$$

$$= \frac{140\text{ miles}}{5\text{ hours}} = 28\text{ mph}$$

Exercise 8j

To find the average speed you need the *total distance* travelled and the *total time* taken.

1. I walk for 24 km at 8 km/h, and then jog for 12 km at 12 km/h. Find my average speed for the whole journey.
2. A cyclist rides for 23 miles at an average speed of $11\frac{1}{2}$ mph before his cycle breaks down, forcing him to push his cycle the remaining distance of 2 miles at an average speed of 4 mph. Find his average speed for the whole journey.
3. An athlete runs 6 miles at 8 mph, then walks 1 mile at 4 mph. Find her average speed for the total distance.
4. A woman walks 3 miles at an average speed of $4\frac{1}{2}$ mph and then runs 4 miles at 12 mph. Find her average speed for the whole journey.

5 A motorist travels the first 30 km of a journey at an average speed of 120 km/h, the next 60 km at 60 km/h, and the final 60 km at 80 km/h. Find the average speed for the whole journey.

6 Phil Sharp walks the 1 km from his home to the bus stop in 15 min, and catches a bus immediately which takes him the 9 km to the airport at an average speed of 36 km/h. He arrives at the airport in time to catch the plane which takes him the 240 km to Antigua at an average speed of 320 km/h. Calculate his average speed for the whole journey from home to Antigua.

7 A liner steaming at 24 knots takes 18 days to travel between two ports. By how much must it increase its speed to reduce the length of the voyage by 2 days? (A *knot* is a speed of 1 nautical mile per hour.)

Puzzle

Two ships, the Antes and the Postes, leave port A together on a journey of 1080 nautical miles to port B. The Antes sails out at an average speed of 24 knots, spends 12 hours unloading, and then sails back to A at an average speed of 30 knots. The Postes sails both ways at an average speed of 27 knots and also spends 12 hours at B unloading. Which ship arrives back at port A first? How much later does the second ship dock at A?

Mixed exercises

Exercise 8k

1 Express the ratio 28 : 35 in its lowest terms.

2 Express the ratio $\frac{4}{5}:\frac{2}{3}$ in its lowest terms.

3 Which is the larger, 8 : 5 or 10 : 7?

4 Find the missing number in the ratio 12 : 5 =   : 25

5 Decrease 396 m in the ratio 6 : 11.

6 Divide $1020 in the ratio 7 : 10.

7 The difference between two numbers is 24. The two numbers are in the ratio 8 : 5.

Find the two numbers.

8 A car can run 192 km on 12 litres of petrol. At the same rate of consumption, how many litres are required to drive 136 km?

9 If 5 boxes of screws cost \$1550, find the cost of 12 similar boxes.

10 A train journey of 60 miles costs \$4920. Find the cost of a similar train journey of 48 miles if the cost per mile is exactly the same.

11 Brianna takes 3 hours to pedal 42 kilometres. Find her average speed.

12 An animal runs at 20 km/h.

How long will it take to run **a** 16 km **b** 9 km?

13 An athlete takes 4 minutes to run 1500 metres. Find his average speed.

14 Alex walks at 5 km/h.

How long will it take him to walk **a** 1.5 km **b** 8 km?

15 An aircraft travels for 4 hours at an average speed of 400 mph, but then, because of a headwind, reduces its average speed to 350 mph for the remaining hour of the journey. Find

a the total distance travelled

b the total time taken

c the average speed for the whole journey.

16 Neal walks the $\frac{1}{2}$ mile from his home to the bus stop at an average speed of 4 mph and immediately catches the bus that takes him the 10 miles to the city centre at an average speed of 20 mph. Find his average speed for the whole journey.

Exercise 8I

1 Which is the smaller, $7:5$ or $9:7$?

2 Which is the larger, $4:7$ or $9:16$?

3 Express the ratio $\frac{5}{12}:\frac{10}{11}$ in its lowest terms.

4 Find the missing number in the ratio $9:4 = 36:$

5 Which of the following ratios is the odd one out? Why?

$1:\frac{7}{5}$ $25:49$ $35:49$ $5:7$

6 Increase 144 g in the ratio $9:4$

7 Decrease \$384 in the ratio $7:8$

8 A packet of 8 bulbs for a particular flower costs \$320.

Find the cost of a similar packet containing 20 bulbs.

9 A car uses 6 gallons of petrol to travel 255 miles.

How far should it travel on 10 gallons if fuel is consumed at the same rate?

10 Divide \$10 080 between Jess and Chad in the ratio 7 : 11.
How much more than Jess does Chad receive?

11 An aeroplane travels at an average speed of 650 mph for 3 hours.
How far does it travel?

12 A train takes 25 minutes to travel 40 km. Find its average speed in km/h.

13 Tim's journey to his holiday destination took $3\frac{1}{4}$ hours.

If he drove there at an average speed of 68 km/h, how far away was his holiday destination?

14 In training, a long-distance runner takes 2 hours 20 minutes to run 14 miles.

Find his average speed in miles per hour.

15 An international sprinter ran 200 metres in 24 seconds.

Give his average speed in

a metres per second　　**c** metres per hour
b metres per minute　　**d** kilometres per hour.

16 Benny wants to make the 110 km trip to Singleton in 2 hours. He travelled the first 60 kilometres at an average speed of 45 km/h, and the next 30 kilometres at an average speed of 90 km/h. What must his average speed for the final 20 kilometres be if he is to arrive on time?

Exercise 8m

In this exercise choose the capital letter that gives the correct answer.

1 Expressed in its simplest terms the ratio 72 : 108 is

A 2:3　**B** 4:6　**C** 6:9　**D** 18:27

2 Expressed in its simplest terms the ratio 1 m^2 : 4000 cm^2 is

A 2:5　**B** 4:5　**C** 5:2　**D** 5:4

3 The ratio 12 : 24 : 30 simplifies to

A 1:2: 3　**B** 2:3:4　**C** 2:4:5　**D** 1:3:4

4 The largest ratio of the ratios 3 : 2, 3 : 4, 6 : 4 and 4 : 2 is

A 3:2　**B** 3:4　**C** 6:4　**D** 4:2

5 The missing number in the ratio $\frac{5}{3} : \frac{\quad}{36}$ is

A 25　**B** 48　**C** 60　**D** 120

6 If \$660 is increased in the ratio 4 : 3, the increased value is

A \$770　**B** \$880　**C** \$990　**D** \$1000

7 If \$8100 is decreased in the ratio 5 : 9, the decreased value is

A \$4500 **B** \$4800 **C** \$5100 **D** \$5400

8 When \$108 is divided in the ratio 4 : 5, the larger part is

A \$48 **B** \$60 **C** \$72 **D** \$84

9 When \$114 is divided in the ratio 3 : 16, the size of the smaller part is

A \$18 **B** \$24 **C** \$30 **D** \$96

10 If 6 jars of coffee cost \$4830, 8 jars will cost

A \$6440 **B** \$7245 **C** \$7445 **D** \$7640

11 A car will run 162 km on 9 litres of petrol. How many litres are required for a journey of 432 km?

A 20 **B** 24 **C** 28 **D** 30

12 Jenny earns \$205 200 by working for 38 hours. How much would she earn if she worked for 42 hours and was paid at the same rate?

A \$226 800 **B** \$228 400 **C** \$230 500 **D** \$246 800

13 Twelve boxes of a particular chocolate cost \$3216. What would 20 boxes of the same chocolate cost?

A \$5300 **B** \$5360 **C** \$5560 **D** \$5720

14 Three-fifths of a sum of money is \$3960. What is one-sixth of the same sum?

A \$960 **B** \$1000 **C** \$1100 **D** \$1110

15 Which ratio is the odd one out? 3 : 5, 12 : 15, 12 : 20, 15 : 25

A 3 : 5 **B** 12 : 15 **C** 12 : 20 **D** 15 : 25

16 A coach travels at an average speed of 56 mph. The distance it travels in $2\frac{1}{2}$ hours is

A 120 miles **B** 130 miles **C** 140 miles **D** 150 miles

17 A journey of 33 miles takes three-quarters of an hour. This gives an average speed of

A 40 mph **B** 44 mph **C** 46 mph **D** 48 mph

18 Morgan travels from his home to a concert, a distance of 63 miles, at an average speed of 45 mph. The time this journey takes is

A 1 h 12 min **B** 1 h 15 min **C** 1 h 20 min **D** 1 h 24 min

19 Carrie cycles at 12 mph. The distance she will travel in 2 hours 10 minutes is

A 24 miles **B** 25 miles **C** 26 miles **D** 28 miles

20 A ship cruises at 26 knots for a voyage of 754 nautical miles.
The time this voyage takes is

A 26 hours **B** 27 hours **C** 28 hours **D** 29 hours

21 A journey of 77 km takes 1 hour 10 minutes. At the same average speed a similar journey of 121 km would take

A 1 h 30 min **B** 1 h 40 min **C** 1 h 50 min **D** 1 h 55 min

22 A car travels for 40 miles at 80 mph. and a further 20 miles at 20 mph.
Its average speed for the whole journey is

A 35 mph **B** 40 mph **C** 45 mph **D** 50 mph

Puzzle

A train, 400 m long and travelling at 120 km/h, enters a tunnel that is 5.6 km long. For what time is any part of the train in the tunnel?

Did you know?

Most countries measure speed in kilometres per hour.

However, a group of countries measure speed in miles per hour. The largest are the USA and the UK.

There are also some Caribbean Islands in this group such as Antigua, Grenada, St Kitts and Nevis, and St Lucia.

Many analogue speedometers in cars have scales showing both kilometres per hour and miles per hour.

In this chapter you have seen that…

- ✓ a ratio can be simplified by multiplying (or dividing) all parts in the same ratio by the same number
- ✓ you can compare ratios by expressing them with the same denominator
- ✓ you can divide a quantity in the ratio $a : b$ by first dividing it into $(a + b)$ equal parts
- ✓ two quantities that are directly proportional are always in the same ratio
- ✓ the formula 'distance = speed × time' can be used to find one quantity when the other two are known
- ✓ when you are working out speeds, you must make sure that the units are consistent, e.g. to find a speed in kilometres per hour, the distance must be in kilometres and the time must be in hours
- ✓ the average speed for a journey is equal to the total distance travelled divided by the total time taken.

9 Consumer arithmetic

At the end of this chapter you should be able to...

1 find the sales tax on an article given the percentage tax

2 work out discounts given the percentage reduction

3 find the simple interest on a sum of money given the interest rate and the time

4 work out problems involving compound interest

5 write a cheque and know how to look after a bank account

6 understand hire-purchase agreements.

Did you know?

54 is the lowest number that can be written as the sum of three squares in three different ways.

One way is 5 squared + 5 squared + 2 squared.

Can you find the other two?

You need to know...

- ✔ how to work with decimals and fractions
- ✔ how to work with percentages
- ✔ how to substitute numbers into expressions
- ✔ how to correct a number to a given place value.

Key words

amount, balance, bank statement, cheque, compound interest, debt, deposit, deposit slip, discount, General Consumption Tax (GCT), hire-purchase, interest, investing, payment, per annum, percentage, principal, purchase price, rate per cent, sales tax, selling price, simple interest, withdrawal, withdrawal slip

Sales tax

The government is forever looking for ways of extracting money from us to pay for its spending. One such way is to put a tax on almost everything that is sold. This *sales tax* is usually a fixed percentage of the *selling price* (also called the *purchase price*). In Jamaica this tax is also called *General Consumption Tax* (GCT).

Exercise 9a

A cell phone is priced at \$1450 plus sales tax at 16%.

Find: **a** the sales tax **b** the price to be paid for the cell phone.

a The sales tax is 16% of \$1450

$$= \$1450 \times \frac{16}{100}$$

$$= \$1450 \times 0.16$$

$$= \$232$$

b The price to be paid = \$1450 + \$232

$$= \$1682$$

In questions **1** to **4** find the total purchase price of the item. The rate of sales tax is $16\frac{1}{2}\%$

1 An electric stove marked \$80 000 + sales tax.

2 A calculator costing \$1900 + sales tax.

3 A car marked \$3 350 000 + sales tax.

4 The price tag on a camera gives \$12 500 plus sales tax.
What price does the customer have to pay?

5 Calculate the selling price of a tea set marked \$7200 + sales tax at 18%.

6 A table is marked \$12 960 + sales tax. How much must I pay for it if the rate of sales tax is 20%?

7 A chest of drawers is priced at \$13 500 + sales tax at 16%.

- **a** Calculate the sales tax.
- **b** What is the purchase price of the chest?

8 In March, Niki looked at a camera costing \$65 000 plus sales tax. The sales tax at the time was $16\frac{1}{2}$%. How much would the camera have cost in March?

Niki decided to wait until June to buy the camera, but by then the sales tax had been raised to 20%.

a How much did the camera cost Niki?

b How much more was this than if Niki had bought the camera in March?

9 A gas stove was priced in a showroom at \$120 000 plus sales tax at 16%.

a What was the price to the customer?

Later in the year sales tax was increased to $17\frac{1}{2}$%. The showroom manager placed a notice that read:

Due to the increase in sales tax this stove will now cost you \$141 288.

b Was the manager correct?

c If your answer 'Yes', state how the manager calculated the new price.
If your answer is 'No', give a reason and find the correct price.

Discount

Stores often have promotions or sales when they offer a reduction of some prices. This is called a *discount*. Sometimes the discount is an amount of money. For example, '\$50 off when you spend \$7500'. On other occasions the discount is a percentage of the price. For example, 'Sale: 10% off all sale items'.

Exercise 9b

The online store goodbuys.com is offering a discount of 10% on all items.

Keron orders a table priced \$90 000. What is the price of the table after the discount is applied?

$$10\% \text{ of } \$90\,000 = \$90\,000 \times \frac{10}{100}$$
$$= \$9000$$

Therefore the discounted price is \$90 000 − \$9000 = \$81 000

In a sale, a shop offers a discount of 20%. What is the cash price for each of the following articles?

1 A dress marked \$4500

2 A lawnmower marked \$25 000

3 A pair of shoes marked \$6000

4 A set of gardening tools marked \$850

5 Light fittings marked \$27 500 each

6 A pair of jeans marked \$3500

In a sale, a department store offers a discount of 50%. What would be the cash price for each of the following?

7 A pair of curtains marked \$5200

8 A leather jacket marked \$21 000

9 A girl's dress marked \$2890

10 A leather football marked \$8500

11 In order to clear a large quantity of goods a shopkeeper put them on sale at a discount of $33\frac{1}{3}\%$.

Find the cash price of

a a shirt marked \$2700 **b** a skirt marked \$4500.

Simple interest

Everybody wishes to borrow something at one time or another. Perhaps you want to borrow a video camera to record a wedding, a dress to wear to an important event or even a bicycle for a few minutes. In the same way, the time will come when you will wish to borrow money to buy a motorcycle, a car, furniture or even a house.

The cost of hiring or borrowing money is called the *interest*. The sum of money borrowed (or lent) is called the *principal* and the interest is usually an agreed *percentage* of the sum borrowed.

For example, if \$10 000 is borrowed for a year at an interest rate of 12% per year, then the interest due is $\frac{12}{100}$ of \$10 000, i.e. \$1200.

The interest due on \$20 000 for one year at 12% would be $\$20\,000 \times \frac{12}{100} = \$2400$, and on $\$P$ for one year at 12% would be $\$P \times \frac{12}{100}$.

If we double the period of the loan, we double the interest due, and so on. The interest on $\$P$ borrowed for T years at 12% would therefore be $\$P \times \frac{12}{100} \times T$.

If the interest rate was $R\%$ instead of the given 12%, the interest, I, would be

$$\$P \times \frac{R}{100} \times T.$$

When interest is calculated this way it is called *simple interest*.

Therefore $$\boxed{I = \frac{PRT}{100}}$$

where I is the simple interest in \$s
P is the principal in \$s
R is the *rate per cent* per year
T is the time in years.

Unless stated otherwise, $R\%$ will always be taken to mean $R\%$ each year or *per annum*.

When we put money in a savings account, it is called *investing*. That money is being borrowed by the bank and they pay us interest.

Exercise 9c

Find the simple interest on:

\$10 000 is the principal, 2 years is the time and 10% is the rate, so $I = \frac{10\,000 \times 10 \times 2}{100}$.

1 \$10 000 for 2 years at 10%
2 \$10 000 for 2 years at 12%
3 \$10 000 for 3 years at 4%
4 \$100 000 for 4 years at 2%
5 \$100 000 for 7 years at 3%
6 \$200 000 for 2 years at 5%
7 \$200 000 for 5 years at 6%
8 \$300 000 for 4 years at 4%
9 \$4 670 000 for 7 years at 5%
10 \$650 000 for 3 years at 6%
11 \$1 250 000 for 8 years at 3%
12 \$640 000 for 7 years at 5%

Find the simple interest on \$134 660 for 5 years at 3% giving your answer correct to the nearest \$100.

$I = \frac{PRT}{100}$ where $P = 134\,660$, $R = 3$ and $T = 5$

$$\begin{aligned}\therefore \text{ simple interest} &= \frac{\$134\,660 \times 3 \times 5}{100}\\ &= \$20\,199\\ &= \$20\,200 \text{ correct to the nearest } \$100\end{aligned}$$

Find, giving your answers correct to the nearest $1000, the simple interest on

13 $5 265 200 for 2 years at 3%

14 $94 560 000 for 5 years at 2%

15 $14 216 000 for 10 years at 4%

16 $55 545 000 for 7 years at 6%

17 $12 372 000 for 4 years at 4%

18 $54 389 000 for 7 years at 5%

19 $1 544 000 for $8\frac{1}{2}$ years at 3%

20 $3 738 000 for $4\frac{1}{2}$ years at 2%

21 $52 749 000 for $12\frac{3}{4}$ years at 4%

22 $43 615 000 for $7\frac{1}{2}$ years at 5%

23 $84 720 000 for $4\frac{1}{4}$ years at 2%

24 $203 446 000 for $1\frac{1}{2}$ years at 5%

25 $61 327 000 for $3\frac{1}{4}$ years at 2%

Write $8\frac{1}{2}$ as 8.5.

Find, giving your answer correct to the nearest dollar, the simple interest on

26 $320 000 for 100 days at 3%

27 $41 300 for 150 days at 4%

28 $1 000 000 for 300 days at 2%

29 $2 825 000 for 214 days at 5%

30 $6 139 400 for 98 days at $3\frac{1}{2}$%

31 $7 293 200 for 22 days at $2\frac{1}{2}$%

T must be in years so change the number of days to a fraction of a year. Use 365 as the number of days in a year.

100 days = $\frac{100}{365}$ yrs

Puzzle

Four married couples met at a party.

Everyone shook hands with everyone else except each husband with his own wife.

How many handshakes were there?

Amount

If I borrow $250 000 for 3 years at 4% simple interest, the sum of the interest and the principal is the total I must repay to clear the *debt*. This sum is called the *amount* and is denoted by A\$.

i.e.

$$A = P + I$$

In this case

$$I = \frac{\$250000 \times 4 \times 3}{100}$$

$$= \$30\,000$$

So

$$\text{amount} = \$250\,000 + \$30\,000$$

$$= \$280\,000$$

Exercise 9d

Give answers that are not exact correct to the nearest $1.

Find the amount of:

1 $35 000 for 5 years at 4%

2 $420 000 for 2 years at 2%

3 $650 000 for 4 years at 3%

4 $513 000 for 4 years at 5%

5 $820 000 for 8 years at 2%

6 $970 000 for 7 years at 3%

7 $492 000 for 5 years at $5\frac{1}{2}$%

8 $654 200 for 4 years at 4%

9 $738 000 for $3\frac{1}{2}$ years at 8%

10 $186 000 for $4\frac{1}{4}$ years at 10%

11 $285 000 for 9 years at $2\frac{1}{2}$%

12 $826 500 for 6 years at $3\frac{1}{2}$%

13 $192 630 for 5 years at $3\frac{1}{4}$%

14 $564 270 for $6\frac{1}{2}$ years at 5%

15 $718 550 for $4\frac{1}{4}$ years at 7%

16 $318 000 for $5\frac{3}{4}$ years at $5\frac{1}{2}$%

Compound percentage problems

There are many occasions when a percentage increase or decrease happens more than once. Suppose that a plot of land is bought for $2 000 000 and increases in value (appreciates) by 10% of its value each year.

After one year, its value will be 110% of its initial value, i.e.

$$\frac{110}{100} \times \$2\,000\,000 = \$2\,200\,000$$

The next year it will increase by 10% of the $2 200 000 it was worth at the beginning of the year, i.e. its value after two years will be

$$\frac{110}{100} \times \$2\,200\,000 = \$2\,420\,000$$

While some things increase in value year after year, many things decrease in value (depreciate) each year. If you buy a car or a motorcycle it will probably depreciate in value more quickly than anything else you buy.

If you invest money in a savings account at a fixed rate of interest and do not spend the interest, your money will increase by larger amounts each year.

This kind of interest is called *compound interest*.

Exercise 9e

Find the compound interest on $260 000 invested for 2 years at 8% p.a.

We need to find the interest for the first year, then add this to $260 000 to find the principal for the second year.

$$\begin{aligned}\text{Simple interest for first year at } 8\% &= \frac{8}{100} \times \$260\,000\\ &= \$\,\frac{2\,080\,000}{100}\\ &= \$20\,800\end{aligned}$$

$$\begin{aligned}\text{New principal} &= \text{original principal} + \text{interest}\\ &= \$260\,000 + \$20\,800 = \$280\,800\end{aligned}$$

Now we can find the interest for the second year.

Simple interest for second year at 8%

$$= \frac{8}{100} \times \$280\,800 = \$22\,464$$

The compound interest is the sum of the interests for the two years.

∴ total of interest for the two years is

$$\$20\,800 + \$22\,464 = \$43\,264$$

i.e. compound interest on $260 000 for 2 years at 8% is $43 264

Find the compound interest on:

Read these questions carefully to make sure that you know what you are being asked to find.

1. $20 000 for 2 years at 10% p.a.
2. $30 000 for 2 years at 12% p.a.
3. $40 000 for 3 years at 8% p.a.
4. $65 000 for 3 years at 9% p.a.
5. $52 000 for 2 years at 13% p.a.
6. $69 000 for 2 years at 14% p.a.
7. $62 400 for 3 years at 12% p.a.
8. A house is bought for $80 000 000 and appreciates at 10% each year. What will it be worth in 2 years' time?
9. A postage stamp increases in value by 15% each year. If it is bought for $5000, what will it be worth in 3 years' time?
10. A motorcycle bought for $60 000 depreciates in value by 10% each year. Find its value after 3 years.
11. A motor car bought for $2 000 000 depreciates in any one year by 20% of its value at the beginning of that year. Find its value after 2 years.

Banking

Banks, such as the National Commercial Bank Jamaica Limited, are clearing banks. 'Clearing' denotes all the activities from the time a commitment is made for a transaction to take place until it is settled. The process starts with a promise to pay, using a *cheque* or an electronic payment, to the movement of the amount agreed from one account to another.

Credit unions and building societies are different. They are owned and controlled by their members, and operate on the principle of people helping people. They usually offer higher interest rates on deposits and lower interest rates on loans.

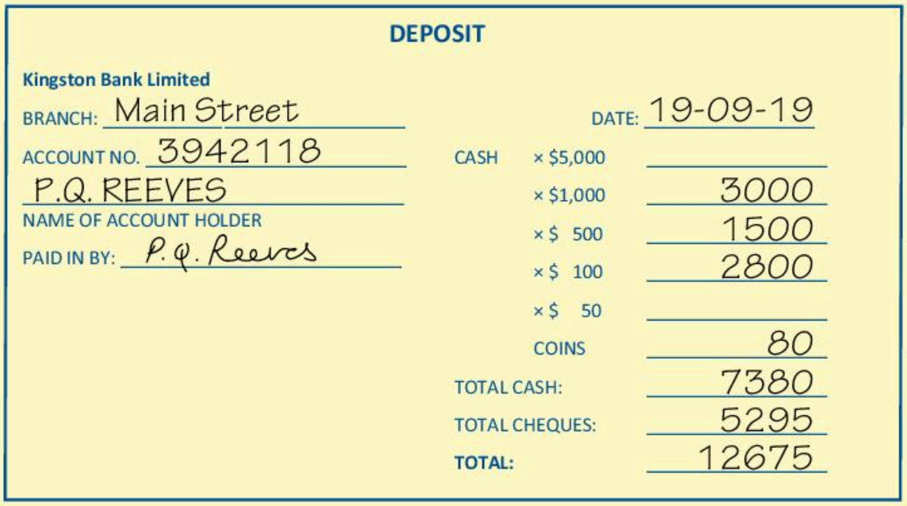
Investigation

Find out the services offered by a clearing bank near you. Also find out about the services offered by a credit union.

Write a short essay comparing the two. Include the advantages and disadvantages of each.

Managing a bank account

Most people find it convenient to have a bank account. When you pay cash or cheques into a bank account you need to fill in a form. This form, called a *deposit slip*, gives details of how the amount paid in is made up. It shows the number $100, $500 and $1000 notes, etc., as well as the total value of the cheques. The value of each individual cheque is written on the back of the slip. An example of a deposit slip is given below.



DEPOSIT

Kingston Bank Limited

BRANCH: Main Street

DATE: 19-09-19

ACCOUNT NO. 3942118

P.Q. REEVES

NAME OF ACCOUNT HOLDER

PAID IN BY: P.Q. Reeves

CASH	× $5,000	
	× $1,000	3000
	× $ 500	1500
	× $ 100	2800
	× $ 50	
	COINS	80
TOTAL CASH:		7380
TOTAL CHEQUES:		5295
TOTAL:		12675

Exercise 9f

Use the deposit slip given above to answer the following questions.

1 **a** How much was paid in, in $1000 bank notes?

b How many $100 bank notes were paid in?

2 **a** How much was paid in?

b How much was paid in, in coins?

3 How much was paid in, in cash?

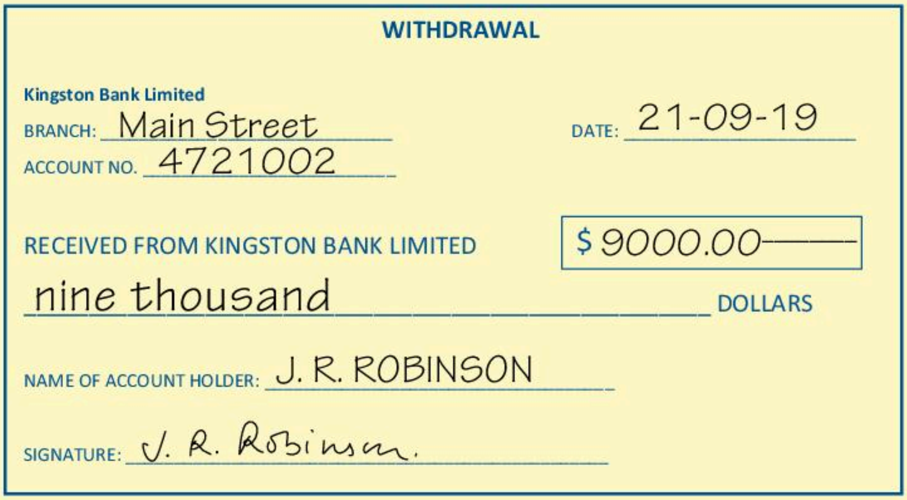
4 How much more was paid in, in cash, than was paid in through cheques?

5 Three cheques were paid in. The value of one was $2632 and the value of another was $1999.

What was the value of the third cheque?

6 Make a copy of the deposit slip shown above before it was filled in. Now enter the following details on it: bank notes – three $1000, eight $500 and twelve $100.

You can take cash out of a bank account at an ATM machine using a debit card, but for some accounts you may need to fill in a *withdrawal slip* when you want to take out money. An example is shown below. You sign your name to show that you have received the money, and when stamped by the bank this forms a record of the transaction.



WITHDRAWAL

Kingston Bank Limited

BRANCH: Main Street

DATE: 21-09-19

ACCOUNT NO. 4721002

RECEIVED FROM KINGSTON BANK LIMITED $ 9000.00———

nine thousand DOLLARS

NAME OF ACCOUNT HOLDER: J. R. ROBINSON

SIGNATURE: J. R. Robinson.

It is important to keep track of all the payments that have been made into a bank account (called *deposits*) and all the payments that have been made from the account (called *payments* or *withdrawals*). At monthly intervals you will receive a *bank statement* listing all the transactions from the previous month. The statement is sometimes printed on paper, but is often provided online.

An example of a paper statement is given below. The figures in the right-hand column show the amount in the account (called the *balance*) after each transaction. A minus sign in front of the balance figure means the account has been overdrawn, and you owe the bank money. The last figure in this column shows the amount you have in your account at the close of business on the day that the statement was prepared.

P.R. JAMES
Account no. 5798234

Midway Bank plc
29 Penford Street
Kingston

Statement of account
Statement date: 31 August 2019
Statement no. 67

2019	Description	Payment	Deposit	Balance
1 Aug	Balance from statement 66 dated 31 July 2019			65 949
8 Aug	101184	12 343		53 606
10 Aug	101182	6 721		46 885
13 Aug	101185	1 289		45 596
17 Aug	101186	396		45 200
23 Aug	Victoria Mutual Build. Soc.	35 267		9 933
26 Aug	MOTOR FINANCE	4 476		5 457
26 Aug	101187	2 543		2 914
29 Aug	JPS	5 692		–2 778
30 Aug	Hanover Parish Council		86 466	83 688
31 Aug	CHARGES	686		83 002

Exercise 9g

Use the bank statement above to answer the following questions:

1. What was the balance in the account on

   a 5 Aug **b** 23 Aug **c** 28 Aug?

2. Did the account go overdrawn at any time? If so, by how much and for how long?

3. On which day(s) was there

   a most in the account **b** least in the account?

4. How much did the bank deduct for bank charges?

5. Percy James pays Motor Finance the same amount every month by direct debit (a way of paying automatically each month).

   How much does he pay Motor Finance in a year?

6 Why do you think the cheque numbers do not appear on the statement in numerical order, even though they were probably written in numerical order?

7 Percy James is employed by Hanover Parish Council. How often do you think they pay him?

8 How much was paid out of the account during the month of August?

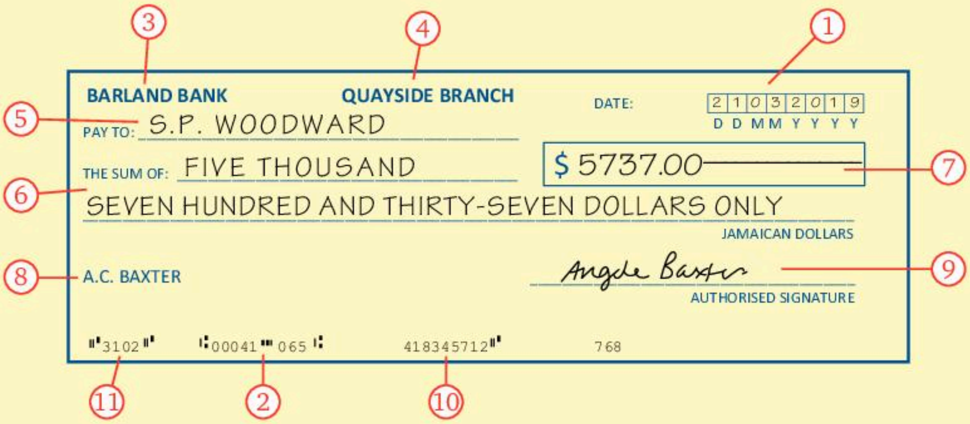
9 What was the value of

- **a** the largest cheque
- **b** the smallest cheque that Percy wrote?

10 When the next statement arrives

- **a** what sheet number will it show
- **b** how much will the starting balance be?

In order to operate a bank account fully you need to be able to write a cheque correctly.



BARLAND BANK QUAYSIDE BRANCH

DATE: 2 1 0 3 2 0 1 9

D D M M Y Y Y Y

PAY TO: S.P. WOODWARD

THE SUM OF: FIVE THOUSAND

SEVEN HUNDRED AND THIRTY-SEVEN DOLLARS ONLY

$ 5737.00

JAMAICAN DOLLARS

A.C. BAXTER

Angela Baxter

AUTHORISED SIGNATURE

3102 00041 065 418345712 768

An example of a correctly written cheque from A.C. Baxter to pay S.P. Woodward $5737 is given above. The numbers refer to different parts of the cheque and have the following meanings:

1 Today's date. A cheque should not be dated in the future.

2 The bank code. All cheques drawn on accounts in this branch of the bank have this number on them.

3 The name of the bank.

4 The name of the branch of the bank where the account is held.

5 The name of the person or organisation to whom the cheque is paid.

6 The amount to be paid written in words followed by the word 'only'. This discourages other words being added.

7 The amount to be paid written in figures followed by a line so that other figures cannot be added.

8 The name of the account holder is printed on the blank cheque.

9 The signature of the account holder.

10 The number of the account, which is unique to the account holder.

11 The number of the cheque. This also appears on the slip or stub which you keep. This allows you to trace one of your cheques easily.

Hire-purchase

Hire-purchase (HP) is a popular and convenient way of buying things when you cannot afford the full price out of your income and don't wish to spend your savings. It is frequently used to buy such things as a washing machine, camera, motorcycle or car. As the term implies, you don't really own the article until you have made the last payment. An article bought on HP always costs more, sometimes much more, than if you can pay cash.

Exercise 9h

A motorcycle is priced at \$22 500. If bought on hire-purchase the terms are: $\frac{1}{3}$ deposit plus 36 monthly payments of \$640. Find the HP price.

Deposit of $\frac{1}{3}$ of \$22 500 = \$7500

$$\begin{aligned}\text{Total of 36 monthly payments of } \$640 &= \$640 \times 36\\ &= \$23\,040\end{aligned}$$

$$\begin{aligned}\text{Total HP price} &= \$7500 + \$23\,040\\ &= \$30\,540\end{aligned}$$

Find the total hire-purchase cost in each of the following cases:

1 No deposit, 12 monthly payments of \$124

2 No deposit, 12 monthly payments of \$252

3 No deposit, 12 monthly payments of \$744

4 Deposit \$5120 plus 12 monthly payments of \$461

5 Deposit \$3120 plus 24 monthly payments of \$258

6 Deposit \$17 370 plus 24 monthly payments of \$940

7 Deposit \$624 plus 52 weekly payments of \$103

8 Deposit \$8670 plus weekly payments of \$521 for 3 years

9 Deposit \$24 130 plus monthly payments of \$2123 for 2 years

10 Deposit \$19 370 plus monthly payments of \$1650 for 3 years.

The complete furnishings for a lounge display in a department store amount to \$128 160. If cash is paid a 5% discount is given, but if sold on hire-purchase the terms are: a deposit of 25% plus a monthly payment of \$3472 for three years.

Find **a** the cash price

b the deposit

c the total HP price

d the amount saved by paying cash.

a Cash price = \$128 160 – discount of 5%

$$\text{Discount} = \$128\,160 \times \frac{5}{100} = \$6408$$

$$\begin{aligned}\text{Cash price} &= \$128\,160 - \$6408\\ &= \$121\,752\end{aligned}$$

b $\text{Deposit} = \$128\,160 \times \frac{25}{100} = \$32\,040$

c Total HP price = deposit + 36 monthly payments of \$3472

$$\begin{aligned}&= \$32\,040 + 36 \times \$3472\\ &= \$32\,040 + \$124\,992\\ &= \$157\,032\end{aligned}$$

d Saving by paying cash = HP cost – cash price

$$\begin{aligned}&= \$157\,032 - \$121\,752\\ &= \$35\,280\end{aligned}$$

11 The cash price of a dining suite is \$7840. Hire-purchase terms require 25% deposit together with 24 monthly repayments of \$282. Calculate the amount saved by paying cash.

12 An electric lawn mower is offered for sale at \$28 500. If bought on hire-purchase a deposit of $\frac{1}{5}$ is required, followed by 24 equal monthly payments of \$1824. How much is saved by paying cash?

13 A camera is advertised at \$2240. If bought on HP, the terms are: 25% deposit plus 12 monthly payments of \$179. How much is saved by paying cash?

14 A grand piano is advertised at \$220 000. If bought on hire-purchase, the terms are: 20% deposit plus 18 monthly payments of \$12 600. How much is saved by paying cash?

15 The cash price of a cut-glass water set is \$1710. The hire-purchase terms are $\frac{1}{5}$ deposit plus 52 weekly payments of \$34.50. How much is saved by paying cash?

16 A man's suit can be bought for \$2520 cash or for a deposit of \$840 plus 12 monthly instalments of \$194.

a How much more does the suit cost if bought on the instalment plan compared with the cash price?

b Express the additional cost as a percentage of the cash price.

17 A motorcycle is offered for sale at \$11 200. If bought on hire-purchase a deposit of $\frac{1}{4}$ is required, together with 24 monthly payments of \$435. Calculate the difference between the cash price and the hire-purchase price.

18 The marked price of a three-piece suite is \$25 800. A 5% discount is offered for a cash sale, but if bought on HP, the deposit is $\frac{1}{3}$, followed by 18 monthly payments of \$1166. Find the cash difference in the two ways of paying for the suite and express this difference as a percentage of the cash price, giving your answer correct to three significant figures.

19 The marked price of an electric stove is \$3660. If bought for cash, a discount of $2\frac{1}{2}\%$ is given, but if bought on hire-purchase, the terms are: $\frac{1}{3}$ deposit plus 24 monthly payments of \$132. How much more does the stove cost if bought on hire-purchase?

20 A bus company is offered a second-hand coach for \$200 700. Since it cannot afford to pay cash it has two options:

Option 1: 6 half-yearly payments of \$38 880

Option 2: a deposit of $\frac{1}{3}$ plus 12 three-monthly payments of \$14 580.

Which option is the cheaper, and by how much?

21 Retiling a house will cost \$114 000. If paid for on hire-purchase, a deposit of $\frac{1}{5}$ is required together with 60 monthly payments of \$2128. Find the additional cost when bought on HP and express this as a percentage of the cash price.

22 A motorist decides to buy a new car, the list price of which is \$223 200. If he sells his old car privately for \$69 000 and then pays cash for the new car, he is given a discount of $12\frac{1}{2}\%$. However, if he offers his car in part-exchange, it is valued at \$75 000 and in addition he must make 36 monthly payments of \$5760. How much will he save if he sells his car privately and pays cash?

23 A carpet, which is suitable for use in a lounge measuring 5 m by 4 m, is offered for sale at $428 per square metre. Hire-purchase terms are as follows: $33\frac{1}{3}$% deposit, the balance to be increased by 12% and divided by 12 to give the monthly repayments for 1 year.

Find:

a the monthly repayments

b the increased cost if bought on hire-purchase

c the increased cost expressed as a percentage of the cash price.

24 A food mixer may be bought by paying a deposit of $292 together with 26 equal payments of $43.40. If this is $170.80 more than the cash price, find the cash price.

25 A professional standard football may be bought by paying a deposit of $114.80 together with 52 equal instalments of $9.20. If this is $78.80 more than the cash price, find the cash price.

26 The cash price of an outfit is $5292. Alternatively it may be paid for with a cash deposit of $1323 followed by 23 monthly payments of $241. How much cheaper is it to pay cash?

27 The cash price of a television set is $11 550. On the instalment plan a deposit of 20% is followed by monthly payments of $839 for one year. For the second and subsequent years the set may be insured against failure for $960 p.a. If the same set had been rented, the rental fee would have been $284 per month for the first year and $278 for every additional month. Compare the hire-purchase costs with the rental costs over a 6-year period. Which is the cheaper and by how much?

28 An electrical discount store calculates its HP prices as follows:

i a deposit of 25% of the cash price

ii the balance is charged interest at $12\frac{1}{2}$%

iii the balance plus interest is divided by the number of monthly instalments paid.

Using this information calculate

a the monthly repayments over one year on a tablet marked $35 300

b the total cost of a webcam, the list price of which is $3400, if it is paid for over an 8-month period.

Mixed exercises

Exercise 9i

1 A shop gives a discount of 20% on all items in a sale.

Find the sale price of a soccer ball whose full price is \$3150.

2 The price of a desk is \$48 000 plus sales tax at 16.5%.

Find the full purchase price.

3 A watch is priced at \$25 000 plus sales tax of 17.5%.

Find the full purchase price.

4 Find the simple interest on \$68 200 for 3 years at 4%.

5 Find the simple interest on \$8500 for $2\frac{1}{2}$ years at 5%.

6 Find the amount of \$950 000 invested for 5 years at 4%.

7 Find the compound interest on \$8 500 000 for 2 years at 5%.

8 The hire-purchase terms for an article are: a deposit of \$5750 plus \$2980 a month for 24 months. How much does the article cost the buyer? Give your answer correct to the nearest \$1000.

9 A bathroom suite is marked \$65 000, to which must be added sales tax at $17\frac{1}{2}$% of the marked price to give the selling price. How much does the suite actually cost the customer?

Exercise 9j

1 A grocery store is offering \$1000 off all bills above \$10 000. Zane's shopping came to \$13 000 before the discount. Find the percentage discount on Zane's bill.

2 Find the simple interest on \$500 000 borrowed for 10 years at 2%.

3 Find the simple interest on \$53 700 for 5 years at $5\frac{1}{2}$%.

4 Find the amount of \$7370 for 8 years at 6%.

5 Find the compound interest on \$540 000 if it is invested for 3 years at 7%.

6 The marked price of an electric stove is \$42 000. If bought for cash, a discount of $2\frac{1}{2}$% is given, but if bought on hire-purchase, the terms are: $\frac{1}{3}$ deposit plus 24 monthly payments of \$1460.

How much more does the stove cost if bought on hire-purchase rather than for cash?

Exercise 9k

In this exercise choose the letter that gives the correct answer.

1 An electrical appliance is marked \$4840 + sales tax at $17\frac{1}{2}\%$.

The cost of this appliance to a buyer is

A \$5663 **B** \$5687 **C** \$5768 **D** \$5808

2 A jacket, marked \$18 000, is offered in a sale at a discount of 15%.

The reduced price is

A \$15 300 **B** \$16 200 **C** \$16 900 **D** \$17 300

3 In a sale a football cost Ainsley \$2992. This was the sale price after a discount of 20%.

The pre-sale price was

A \$2394 **B** \$3590 **C** \$3740 **D** \$3840

4 Correct to the nearest \$1000, the simple interest on \$765 000 invested for 3 years at 3% is

A \$6000 **B** \$68 000 **C** \$69 000 **D** \$70 000

5 I borrow \$900 000 for 2 years at 6% simple interest.

The amount I must pay back at the end of the 2 years to clear the debt is

A \$108 000 **B** \$100 800 **C** \$1 000 800 **D** \$1 008 000

In this chapter you have seen that ...

- ✔ sales tax is a percentage of the selling price
- ✔ a discount is a reduction of the price of an item. It can be a sum of money or a percentage of the price
- ✔ you can find the simple interest of a sum of money by using the formula

$$I = \frac{PRT}{100}$$

- ✔ the ‘amount’ is the sum of the principal and the interest
- ✔ you can calculate the compound interest on a sum of money by finding the interest due each year on the increased amount
- ✔ you need to know how to write a cheque correctly and how to operate a bank account
- ✔ you can calculate hire-purchase deposits and total costs.

10 Areas

At the end of this chapter you should be able to...

1 find the area of a parallelogram
2 find the area of a trapezium
3 find the area of a compound shape involving rectangles
4 find the area of a triangle
5 calculate the surface area of a prism
6 calculate the surface area of a cuboid
7 find missing measurements for various shapes.

You need to know...

✓ how to multiply fractions and decimals
✓ the units of length and area.

Key words

base, centimetre, cross-section, cube, cuboid, face, kilometre, metre, millimetre, net, parallel, parallelogram, perpendicular height, polygon, prism, right prism, slant height, square units, surface area, trapezium, triangular prism

Changing units of area

Remember that in the metric system the standard units of measurement are the *kilometre* (km), *metre* (m), *centimetre* (cm) and *millimetre* (mm) where

$$1\,\text{km} = 1000\,\text{m}$$
$$1\,\text{m} = 100\,\text{cm}$$

and $$1\,\text{cm} = 10\,\text{mm}$$

From these we have the units of area where

$$\begin{aligned} 1\,\text{km}^2 &= 1000 \times 1000\,\text{m}^2 \\ &= 1\,000\,000\,\text{m}^2 \\ 1\,\text{m}^2 &= 100 \times 100\,\text{cm}^2 \\ &= 10\,000\,\text{cm}^2 \end{aligned}$$

and $$\begin{aligned} 1\,\text{cm}^2 &= 10 \times 10\,\text{mm}^2 \\ &= 100\,\text{mm}^2 \end{aligned}$$

Exercise 10a

Express $4\,\text{m}^2$ in **a** cm^2 **b** mm^2

a Since $1\,\text{m}^2 = 100 \times 100\,\text{cm}^2$

$$4\,\text{m}^2 = 4 \times 100 \times 100\text{cm}^2$$
$$= 40\,000\,\text{cm}^2$$

b Since $1\,\text{cm}^2 = 100\,\text{mm}^2$

$$40\,000\,\text{cm}^2 = 40\,000 \times 100\,\text{mm}^2$$
$$= 4000\,000\,\text{mm}^2$$

1 Express in cm^2

a 2m **b** $8\,\text{m}^2$ **c** $3.5\,\text{m}^2$ **d** $5\frac{1}{2}\text{m}^2$

2 Express in mm^2

a $20\,\text{cm}^2$ **b** $4\,\text{cm}^2$ **c** $5.5\,\text{cm}^2$ **d** $12\frac{1}{2}\text{cm}^2$

3 Express in $0.043\,\text{m}^2$ in **a** cm^2 **b** mm^2

Express $554\,000\,000\text{mm}^2$ in **a** cm^2 **b** m^2

a Since $100\,\text{mm}^2 = 1\,\text{cm}^2$

$$554\,000\,000\,\text{mm}^2 = \frac{554\ 000\ 000}{100}\,\text{cm}^2$$
$$= 5\,540\,000\,\text{cm}^2$$

b Since $100 \times 100\,\text{cm}^2 = 1\,\text{m}^2$

$$5\,540\,000\,\text{cm}^2 = \frac{554\,000\,000}{100 \times 100}\,\text{m}^2$$
$$= 554\,\text{m}^2$$

4 Express in cm^2

a $600\,\text{mm}^2$ **b** $3900\,\text{mm}^2$

5 Express in m^2

a 7500cm^2 **b** $304\,000\,000\,\text{cm}^2$

6 Express in km^2

a $25\,000\,000\,\text{m}^2$ **b** $620\,000\,\text{m}^2$ **c** $70\,000\,\text{m}^2$

Area of a rectangle and a square

Reminder: The area of a rectangle is found by multiplying its length by its breadth (or width).

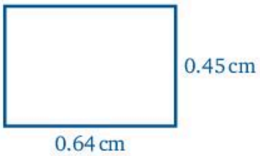
Area = length × breadth

or $A = l \times b$ (the length and the breadth must be measured in the same units)

A square is a rectangle whose length and width are equal. So if one side of a square is a units long, the area of the square is given by $A = a^2$.

Exercise 10b

1



Find the area of this rectangle.

2 A rectangle measures 500 mm by 84 mm. Find its area in cm^2.

3 A rectangle measures 240 cm by 85 cm.

Find its area in **a** cm^2 **b** m^2

4 Find the area, in cm^2, of a rectangle measuring $3\frac{3}{4}$ cm by $1\frac{1}{3}$ cm.

5 A rectangle, which is 2.5 cm long, has an area of 2 cm^2. Find its width.

6 The width of a rectangle is 3.4 m and its area is 15.3 m^2.

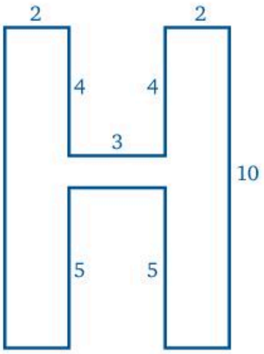
Find its length in **a** metres **b** cm

7 Find the area of a square piece of land whose side is 20 m.

In questions **8** to **10**, find the area of each shape in square centimetres.

All measurements are given in centimetres.

8



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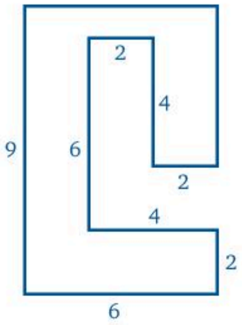

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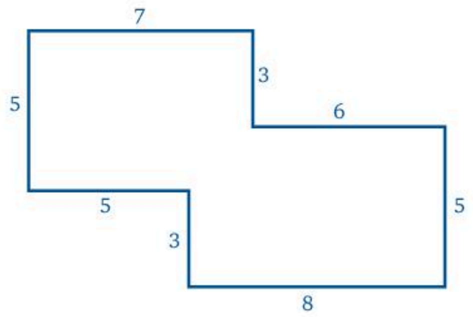
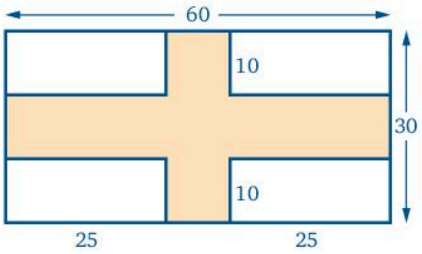


11 Find the area, in square centimetres,

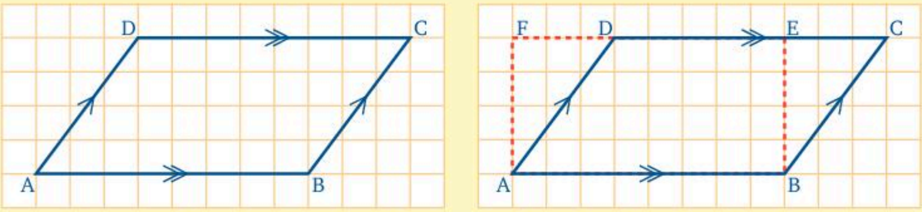
a that is white

b that is coloured.



Area of a parallelogram

Knowing how to find the area of a rectangle helps us to work out the area of a *parallelogram*.



Copy the first diagram on to squared paper. Draw the line BE and remove triangle BEC from the right-hand side. Draw an equal triangle, AFD, at the left-hand side to replace triangle BEC. Then you can see that the area of the parallelogram ABCD is equal to the area of rectangle ABEF.

You can also do this by drawing a parallelogram on paper. Then cut off triangle BEC and tape it to the other side of the parallelogram.

Area of parallelogram = AB × BE

= *base* × perpendicular height

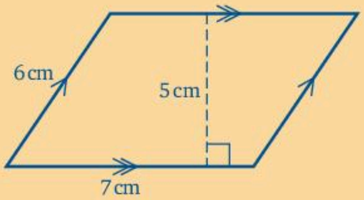
When we use the word *height* we mean the *perpendicular height* BE, not the *slant height* BC, so we can say

Area of parallelogram = base × height

Exercise 10c

Find the area of a parallelogram of base 7 cm, height 5 cm and slant height 6 cm.

The height of a parallelogram means the perpendicular height, which is the perpendicular distance between a pair of parallel sides.



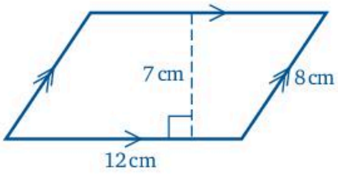
$$\begin{aligned}\text{Area} &= \text{base} \times \text{height}\\ &= 7 \times 5\ \text{cm}^2\\ &= 35\ \text{cm}^2\end{aligned}$$

(Notice that we do not use the length of the 6 cm side to find the area of the parallelogram.)

Find the areas of the following parallelograms:

Make sure that you use the perpendicular height.

1



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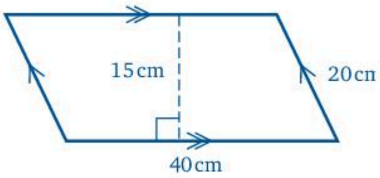
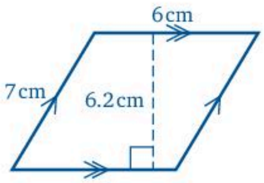


3



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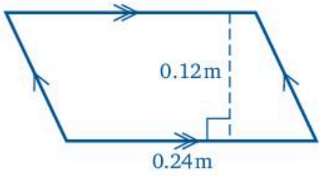

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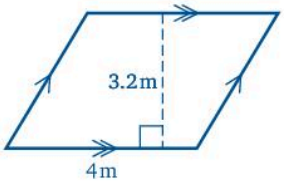

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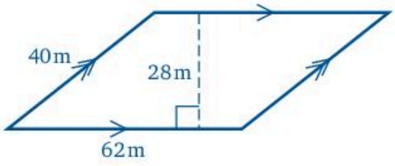

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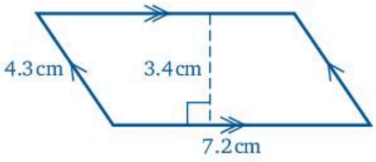

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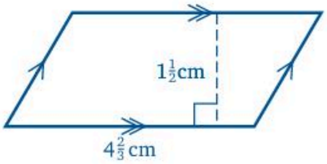
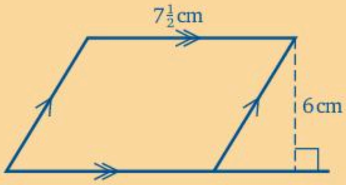
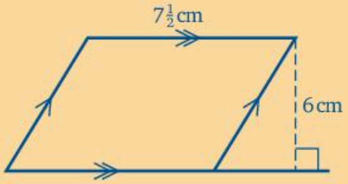


Find the area of the parallelogram.



(Notice that it does not matter if the height is measured inside or outside of the parallelogram.)



$$\text{Area} = \text{base} \times \text{height}$$

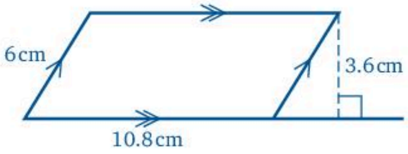
$$= 7\frac{1}{2} \times 6\,\text{cm}^2$$

$$= \frac{15}{\cancel{2}_1} \times \cancel{6}^{3}\ \text{cm}^2$$

$$= 45\,\text{cm}^2$$

Find the areas of these parallelograms.

9



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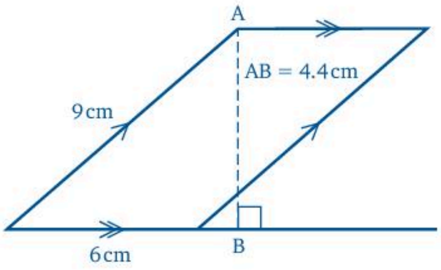

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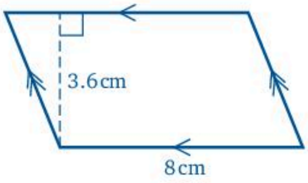

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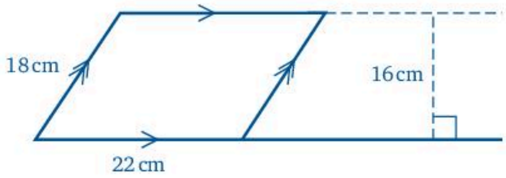
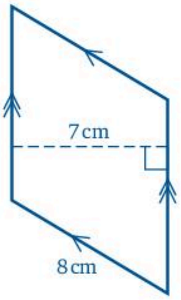


In questions **13** to **18**, turn the page round if necessary so that you can see which is the base and which the height.

13



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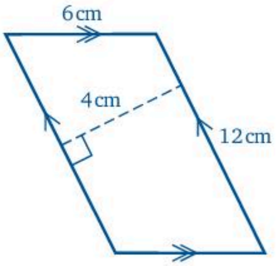

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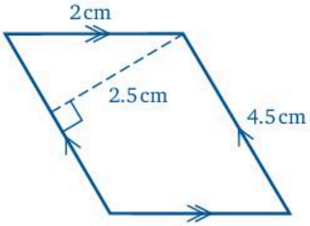

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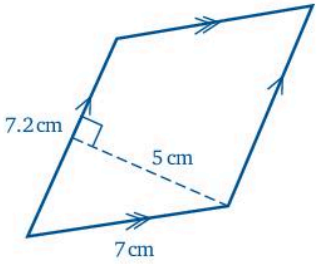

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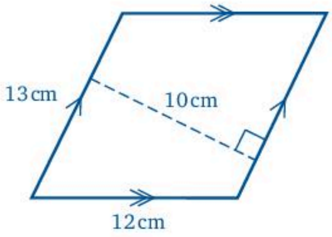

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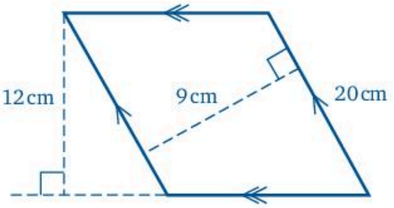


19 In questions **15** to **18** which of the given measurements did you not use?

Puzzle

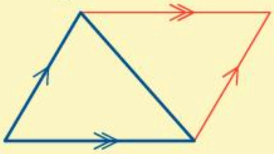
Divide this trapezium into four pieces, all of which are exactly the same shape and size.

Area of a triangle

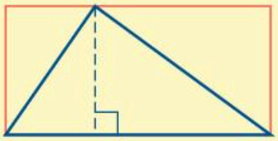
There are two ways of finding how to calculate the area of a triangle.

First, if we think of a triangle as half a parallelogram we get

$$\text{Area of triangle} = \frac{1}{2} \times \text{area of parallelogram}$$
$$= \frac{1}{2}\,(\text{base} \times \text{height})$$



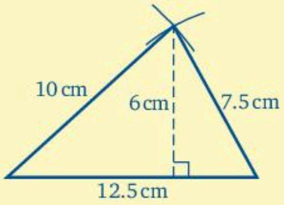
Second, if we enclose the triangle in a rectangle we see again that the area of the triangle is half the area of the rectangle.



These diagrams can be drawn on squared paper and then cut out to show how the pieces fit.

Height of a triangle

As with the parallelogram, when we talk about the height of a triangle we mean its perpendicular height and not its slant height.

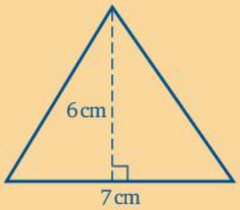


If we draw the given triangle accurately on squared paper, we can see that the height of the triangle is not 10 cm or 7.5 cm but 6 cm. (We can also see that the foot of the perpendicular is *not* the midpoint of the base.)

Finding areas of triangles

Exercise 10d

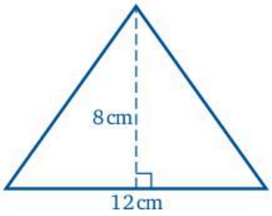
Find the area of a triangle with base 7 cm and height 6 cm.



$$\text{Area} = \frac{1}{2}(\text{base} \times \text{height})$$
$$= \frac{1}{\cancel{2}_1} \times 7 \times \cancel{6}^3 \text{ cm}^2$$
$$= 21 \text{ cm}^2$$

Find the areas of the following triangles.

1



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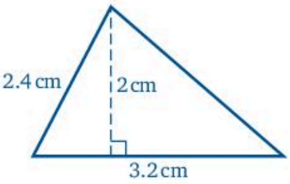

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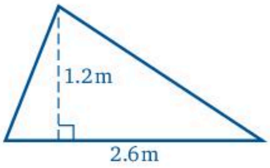

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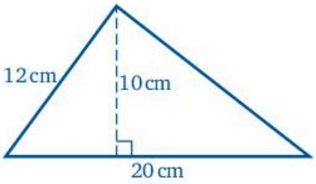

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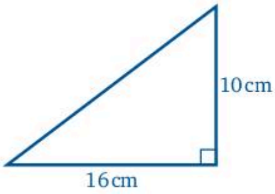

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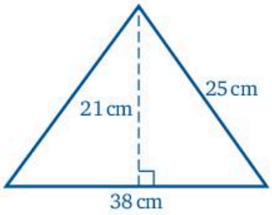
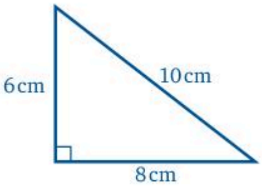


7 In questions **4**, **5** and **6**, one of the given measurements is redundant. Which one is it?

Remember that you want the perpendicular height, which is the perpendicular distance from the base to the opposite vertex.

8



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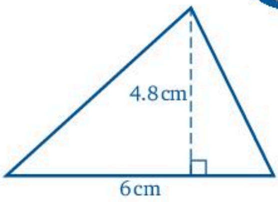

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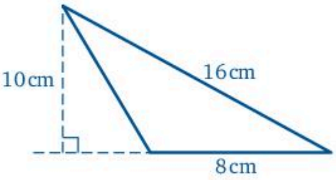

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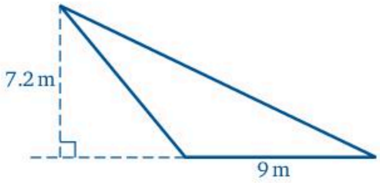

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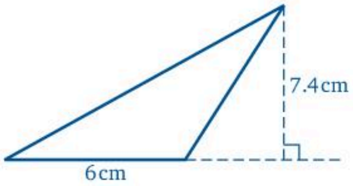

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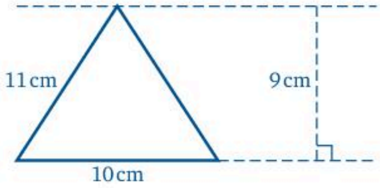
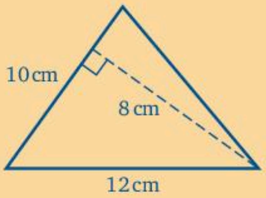
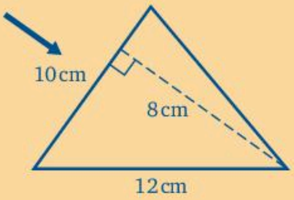


Find the area of the triangle.



Look at this diagram from the direction of the arrow.



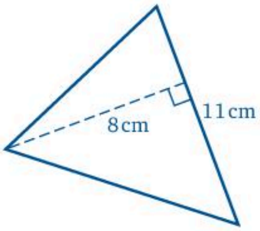
$$\text{Area} = \frac{1}{2}\,(\text{base} \times \text{height})$$

$$= \frac{1}{\cancel{2}_{1}} \times 10 \times \cancel{8}^{4}\ \text{cm}^2$$

$$= 40\ \text{cm}^2$$

If necessary turn the page round and look at the triangle from a different direction.

14



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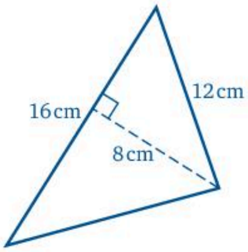

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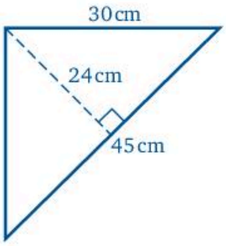

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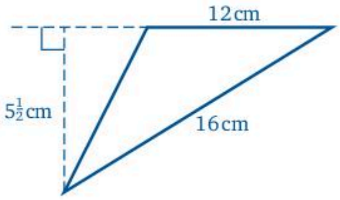

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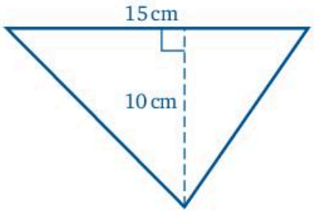

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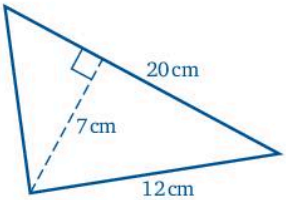

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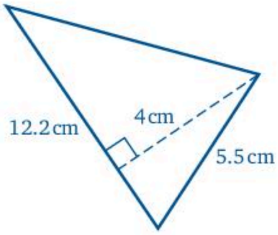

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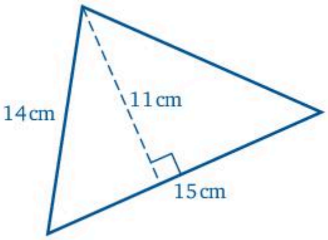

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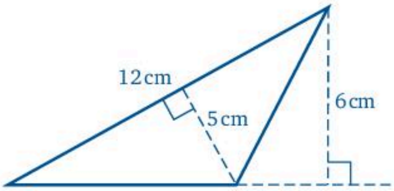

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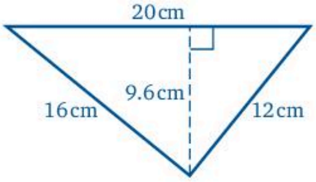

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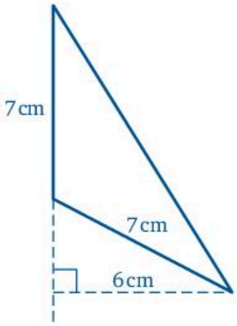

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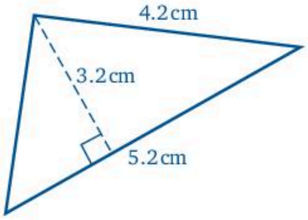
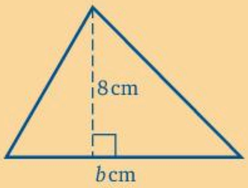


Finding missing measurements

Exercise 10e

The area of a triangle is 20 cm². The height is 8 cm. Find the length of the base.

Let the base be *b* cm long.



Using the formula for the area of a triangle, we substitute the values we know and b for the base.

$$\text{Area} = \frac{1}{2}\,(\text{base} \times \text{height})$$

$$20 = \frac{1}{\cancel{2}_{1}} \times b \times \cancel{8}^{4}$$

Now we can see that

$$20 = 4b$$

$$b = 5$$

The base is 5 cm long.

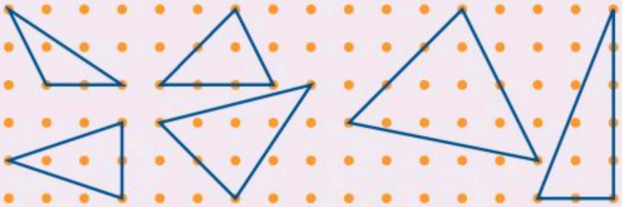
Find the missing measurements of the following triangles.

	Area	Base	Height
1	24 cm²	6 cm	
2	30 cm²		10 cm
3	48 cm²		16 cm
4	10 cm²	10 mm	
5	36 cm²	24 cm	
6	108 cm²		6 cm

	Area	Base	Height
7	96 cm²		64 cm
8	4 cm²		3 cm
9	2 cm²	10 cm	
10	1.2 cm²	0.4 cm	
11	72 cm²		18 cm
12	1.28 cm²	0.64 cm	

Investigation

These triangles are drawn on a grid of dots 1 cm apart.



1 Copy and complete this table for each triangle.

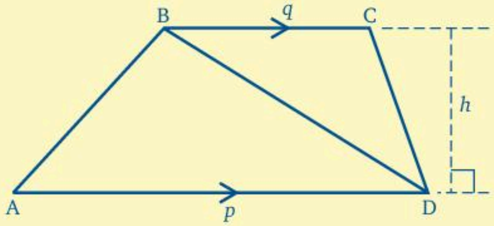
Number of dots on edge	Number of dots inside	Area (cm²)

2 Find a relationship between the number of dots on the edge, the number of dots inside and the area of each shape. Does this relationship hold for any triangle drawn on the grid?

3 Investigate the relationship between the number of dots on the edge, the number of dots inside and the areas of rectangles and parallelograms.

Area of a trapezium

A *trapezium* is a shape that occurs often enough to justify finding a formula for its area.



$$\text{Area of } \triangle\text{ABD} = \frac{1}{2}\text{ base} \times \text{height} = \frac{1}{2}p \times h$$

$$\text{Area of } \triangle\text{BCD} = \frac{1}{2}\text{ base} \times \text{height} = \frac{1}{2}q \times h$$

The perpendicular heights of both triangles are the same, as each is the distance between the *parallel* sides of the trapezium.

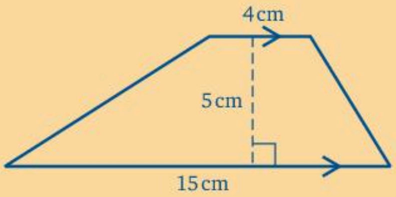
$\therefore$ total area of ABCD $= \frac{1}{2}ph + \frac{1}{2}qh = \frac{1}{2}(p+q) \times h$

i.e. the area of a trapezium is equal to

$\frac{1}{2}$ (sum of parallel sides) × (distance between them)

Exercise 10f

Find the area of the trapezium in the diagram.



$$\text{Area} = \frac{1}{2}\,(\text{sum of parallel sides}) \times (\text{distance between them})$$

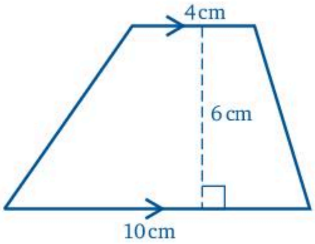
$$= \frac{1}{2}\,(4 + 15) \times 5\,\text{cm}^2$$

$$= \frac{1}{2} \times 19 \times 5\,\text{cm}^2$$

$$= 47.5\,\text{cm}^2$$

Find the area of each of the following trapeziums:

1



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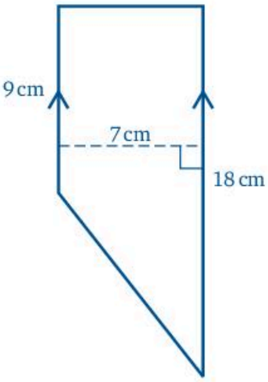

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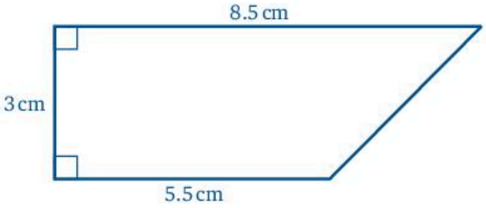

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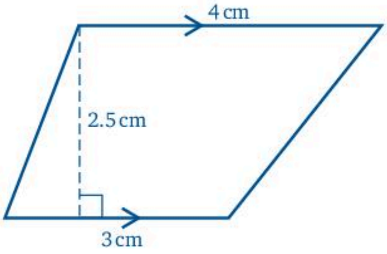


For questions **5** to **10** use square grid paper and draw axes for x and y using ranges $-6 \leqslant x \leqslant 6$ and $-6 \leqslant y \leqslant 6$ and a scale of one square to 1 unit. Plot the points and join them up in alphabetical order. Find, in *square units*, the area of the resulting shape.

5 A(6, 1), B(4, –3), C(–2, –3), D(–3, 1)

6 A(4, 4), B(–2, 2), C(–2, –2), D(4, –3)

7 A(3, 5), B(−4, 4), C(−4, −2), D(3, −5)

8 A(1, 0), B(5, 0), C(5, 3), D(3, 5), E(1, 3)

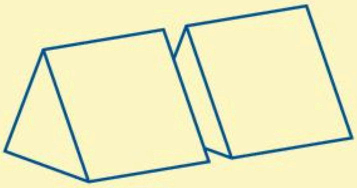
9 A(6, −4), B(6, 1), C(2, 5), D(−5, 3), E(−5, −4)

10 A(2, 0), B(6, 4), C(−4, 4), D(−4, −2), E(5, −2)

Prisms

A *prism* is solid with two identical ends and flat faces between them.

A *right prism* has two faces that are identical *polygons*. The other faces are rectangles.



This prism has triangular ends.

When a prism is cut parallel to one of its ends, we get the same shape as an end.

This shape is called the *cross-section* and is constant, i.e. it is the same shape throughout the prism.

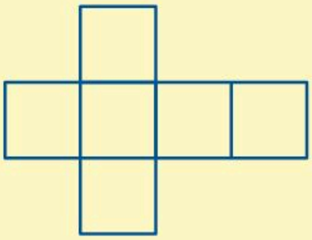
A *cube* is a right prism with square ends. Its six faces are identical squares.

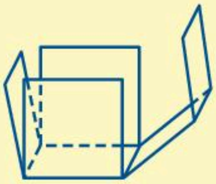
A *cuboid* is a right prism in which all the faces are rectangles.

A *triangular prism* has a triangular cross-section.

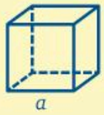
Nets

A *net* is a flat drawing that can be cut out and folded to make a solid shape. You can make a cube from a net.



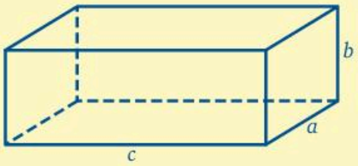


Surface area of a prism



A cube of side a has 6 square faces. The area of each *face* is a^2 so the total *surface area* of a cube with side a is $6a^2$.

Similarly a cuboid has 6 faces.



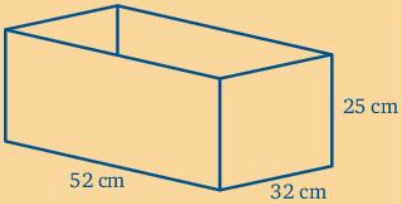
This cuboid has two rectangular faces measuring $a \times b$
two measuring $b \times c$
and two measuring $a \times c$

The total surface area of a cuboid is therefore

$$2(ab + bc + ac).$$

The surface area of a prism is the sum of the areas of all the faces.

Exercise 10g



This cuboid, which is not drawn to scale, represents an open box.

The internal dimensions are 52 cm by 32 cm by 25 cm.

Find the total surface area of the inside of the box.

There are two rectangular faces measuring 52 cm by 25 cm, two measuring 32 cm by 25 cm, and the base measuring 52 cm by 32 cm.

The surface area of the inside of the box $= 2(52 \times 25) + 2(32 \times 25) + 52 \times 32\text{ cm}^2$

$$= 5864\text{ cm}^2$$

1 Find the surface area of a cube of side 3 cm.

2 Find the surface area of a cube of side 6 cm.

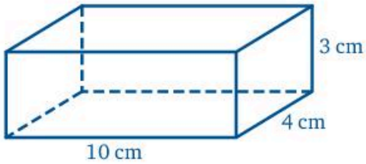
3 The total surface area of a cube is 150 cm^2. Find:

- **a** the area of one face
- **b** the length of an edge of the cube.

4 The edge of a cube measures 4 cm. Find:

- **a** the area of one face
- **b** the total surface area of the cube.

5 Find the total surface area of this cuboid.



6 A cuboid measures 10 cm by 5 cm by 4 cm. Find:

a the area of one of the largest faces

b the area of one of the smallest faces

c its total surface area.

7 A prism with a rectangular cross-section measuring 5 cm by 6 cm, is 2.5 m long. Find:

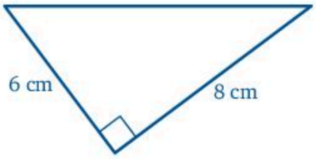
a the area of cross-section

b the total surface area of the prism.

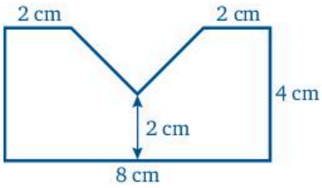
8 The diagram shows the cross-section of a water channel which is 15 m long and open at the top. Find, in square metres:

a the area of the channel in contact with the water

b the area of water open to the elements.



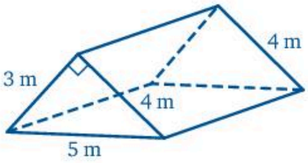
9 The diagram shows the cross-section of a block of wood which is 12 cm long. Find the area of cross-section.



10 The diagram shows the roof-space of a workshop which is 8 m long. The sloping sides of the roof are at right angles. Find:

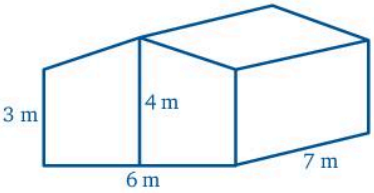
a the area of the cross-section

b the total area of the sloping faces.



11 The cross-section of a building is a rectangle surmounted by a triangle. It is 6 m wide, 3 m high at the eaves and it is 4 m from the floor to the ridge.

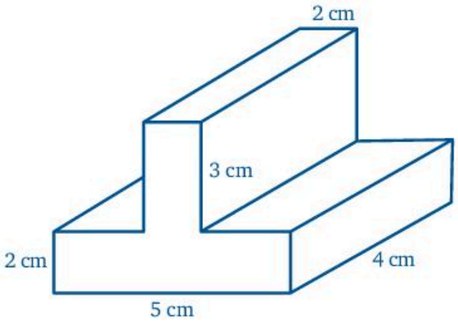
Find the surface area of the building excluding the floor and the roof.



12 Use the measurements on the diagram to find

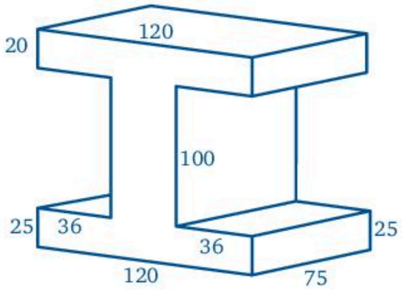
a the surface area of the solid

b the area of cross-section of the solid.



13 The measurements on the diagram are given in centimetres.

The solid is to be painted. One tin of spray covers 4 m^2. How many tins must be purchased to give enough paint for three coats?



Mixed exercise

Exercise 10h

In this exercise choose the capital letter that gives the correct answer.

1 5 m^2 expressed in cm^2 is

A 50 cm^2 **B** 500 cm^2 **C** 5000 cm^2 **D** 50 000 cm^2

2 40 cm^2 expressed in mm^2 is

A 400 mm^2 **B** 4000 mm^2 **C** 40 000 mm^2 **D** 400 000 mm^2

3 6000 mm^2 expressed in cm^2 is

A 0.6 cm^2 **B** 6 cm^2 **C** 60 cm^2 **D** 600 cm^2

4 80 000 000 m^2 expressed in km^2 is

A 0.08 km^2 **B** 0.8 km^2 **C** 8 km^2 **D** 80 km^2

5 The area, in cm^2, of a rectangle measuring 90 cm by 120 cm is

A 10.8 cm^2 **B** 108 cm^2 **C** 1080 cm^2 **D** 10 800 cm^2

6 The area, in cm^2, of a rectangle measuring 440 cm by 150 mm is

A 66 cm^2 **B** 660 cm^2 **C** 6600 cm^2 **D** 66 000 cm^2

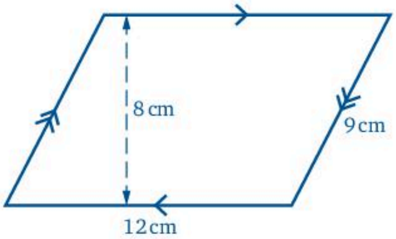
7 The area, in cm^2, of a rectangle measuring $4\frac{1}{2}$ cm by $3\frac{1}{3}$ cm is

A $7\frac{5}{6}$ cm^2 **B** 12 cm^2 **C** $12\frac{5}{6}$ cm^2 **D** 15 cm^2

8 The area of a rectangle is 14.3 cm^2. If it is 2.6 cm wide its length is

A 4.4 cm **B** 4.5 cm **C** 5.5 cm **D** 5.6 cm

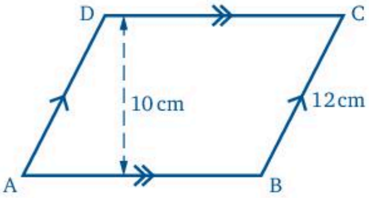
9



The area of this shape is

A 80 cm^2 **B** 84 cm^2 **C** 96 cm^2 **D** 108 cm^2

10



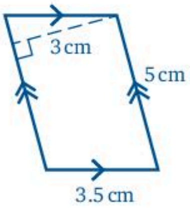
The area of this parallelogram is 120 cm^2. The length of AB is

A 8 cm **B** 10 cm **C** 12 cm **D** 14 cm

11 The area of this parallelogram is

A 10.5 cm^2 **C** 15 cm^2

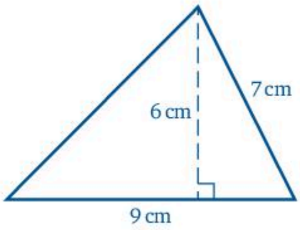
B 12.5 cm^2 **D** 16.5 cm^2



12 The area of this triangle is

A 21 cm^2 **C** 31.5 cm^2

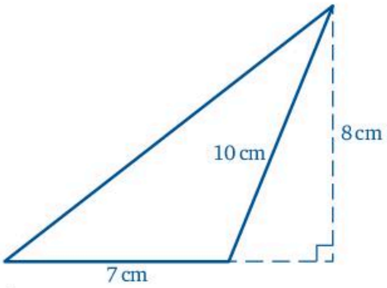
B 27 cm^2 **D** 54 cm^2



13 The area of this triangle is

A 28 cm^2 **C** 56 cm^2

B 35 cm^2 **D** 70 cm^2



14 The area of a triangle is 30 cm^2. If its height is 6 cm its width is

A 4 cm **B** 5 cm **C** 6 cm **D** 10 cm

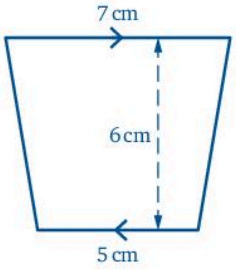
15 The area of a triangle is 20.5 cm^2. It is 5 cm high. The length of its base is

A 4.1 cm **B** 6.3 cm **C** 8.2 cm **D** 9.5 cm

16 The area of this trapezium is

A 36 cm^2 C 60 cm^2

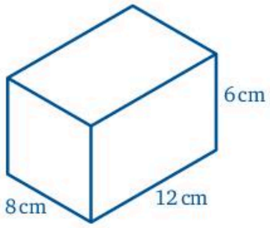
B 42 cm^2 **D** 72 cm^2



17 The total surface area of this cuboid is

A 192 cm^2 C 384 cm^2

B 240 cm^2 **D** 432 cm^2



18 The total surface area of a cube is 96 cm^2.

The length of an edge of this cube is

A 1 cm **B** 2 cm **C** 3 cm **D** 4 cm

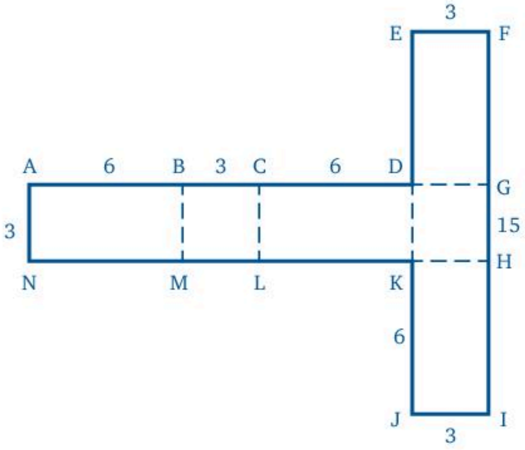
19 If a prism has triangular ends, how many rectangular surfaces does it have?

A 1 **B** 2 **C** 3 **D** 4

20 If the cross-section of a prism is a hexagon, how many rectangular surfaces does it have?

A 5 **B** 6 **C** 7 **D** 8

Use the net shown, which will make a cuboid, to answer questions **21** to **25**.



21 Which corner meets with A?

A D **C** F

B E **D** G

22 Which corner meets with L?

A K **C** I

B J **D** H

23 Which corner meets with M?

A J **C** H

B I **D** G

24 The total surface area of the resulting solid is

A 64 cm^2 **B** 72 cm^2 **C** 90 cm^2 **D** 108 cm^2

25 The number of corners this cuboid has is

A 4 **B** 5 **C** 6 **D** 8

In this chapter you have seen that...

- ✔ the area of a rectangle is given by 'length × breadth'
- ✔ the area of a parallelogram is given by 'length × perpendicular height'
- ✔ the area of a trapezium is equal to '½ sum of the parallel sides × the distance between them'
- ✔ the area of a triangle is equal to '½ base × perpendicular height'
- ✔ the cross-section of a prism is constant, i.e. the same throughout its length
- ✔ the surface area of a cube is the sum of the areas of six identical squares
- ✔ the surface area of a cuboid is the sum of the areas of six rectangles
- ✔ a right prism has two faces that are identical polygons, the remaining faces are rectangles.

11 Volume and capacity

At the end of this chapter you should be able to...

1 calculate the volume of a prism

2 calculate the volume of certain composite solids.

Did you know?

You can multiply a two-digit number by 11 by adding the digits together and putting the result in the middle:

$54 \times 11 = 594$ ($5 + 4 = 9$ and 9 placed between the digits 5 and 4 gives 594)

You need to know...

- ✔ how to change from one metric unit to another
- ✔ how to find the area of a triangle, a parallelogram and a trapezium
- ✔ how to find the volume of a cube and a cuboid
- ✔ the meaning of capacity.

Key words

capacity, cross-section, cube, cuboid, dimension, litre, prism, volume

Volume and capacity

In Grade 7 we introduced *volume* and *capacity*.

There we saw that the volume of a solid is a measure of the amount of space it occupies.

The volume of a *cube* of side a cm is $a \times a \times a\,\text{cm}^3$, i.e. $a^3\,\text{cm}^3$ and the volume of a *cuboid* measuring $l\,\text{cm} \times b\,\text{cm} \times h\,\text{cm}$ is $lbh\,\text{cm}^3$

The capacity of a container is the amount of space within it. For example, the capacity of a cup is a measure of the amount it will hold. In the metric system the most common unit of capacity is the *litre*. We buy bottled water and petrol by the litre.

To calculate volume or capacity the measurements multiplied together must be in the same unit.

For example, if a rectangular block measures 2 m × 6 cm × 8 cm its volume can

either be calculated as $200\,\text{cm} \times 6\,\text{cm} \times 8\,\text{cm} = 9600\,\text{cm}^3$

or $2\,\text{m} \times 0.06\,\text{m} \times 0.08\,\text{m} = 0.0096\,\text{m}^3$

Exercise 11a

1 Find the volume of a cuboid measuring

a 5 cm by 6 cm by 10 cm **b** 4 m by 5.2 m by 0.5 m

2 Find the volume of a cube, every edge of which has length

a 7 cm **b** 0.3 cm **c** $\frac{2}{3}$ cm

3 a Find the volume occupied by a cube of side 6 cm.

b How many cubes of side 2 cm are needed to fill exactly the same space?

4 Express $3.2\,\text{m}^3$ in **a** cm^3 **b** mm^3

5 Express in mm^3 **a** $6\,\text{cm}^3$ **b** $0.004\,\text{m}^3$

6 Express in cm^3 **a** 3 litres **b** 0.75 litres

7 Express in litres

a $8000\,\text{cm}^3$ **b** $759\,\text{cm}^3$ **c** $2\,\text{m}^3$ **d** $0.046\,\text{m}^3$

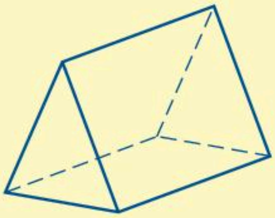
8 Find, in cubic centimetres, the volume of a cuboid measuring 5 cm by 50 mm by 12 cm.

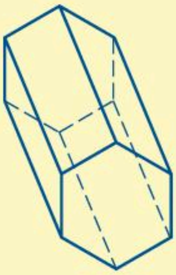
9 Find, in cubic centimetres, the volume of a cuboid measuring 8 cm by 12 mm by 10 mm.

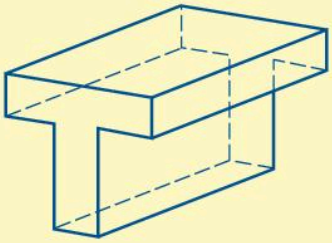
10 How many rectangular packets measuring 6 cm × 3 cm × 2 cm, may be packed in a rectangular box measuring 24 cm × 6 cm × 6 cm?

Volume of a prism

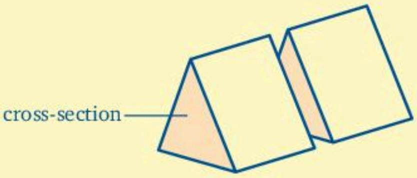
The solids below are all *prisms*.







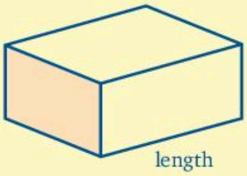
When we cut through any one of these solids, parallel to the ends, we always get the same shape as the end.



This shape is called the *cross-section*.
It is said to be uniform or constant.

We can use this property to find a formula for the volume of a prism.

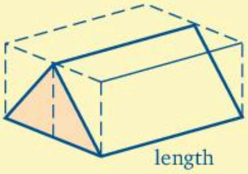
First consider a cuboid (which can also be thought of as a rectangular prism).



$$\begin{aligned}\text{Volume} &= \text{length} \times \text{width} \times \text{height}\\ &= (\text{width} \times \text{height}) \times \text{length}\\ &= \text{area of shaded end} \times \text{length}\\ &= \text{area of cross-section} \times \text{length}\end{aligned}$$

Now consider a triangular prism. If we enclose it in a cuboid we can see that its volume is half the volume of the cuboid.

$$\begin{aligned}\text{Volume} &= \left(\frac{1}{2} \times \text{width} \times \text{height}\right) \times \text{length}\\ &= \text{area of shaded triangle} \times \text{length}\\ &= \text{area of cross-section} \times \text{length}\end{aligned}$$



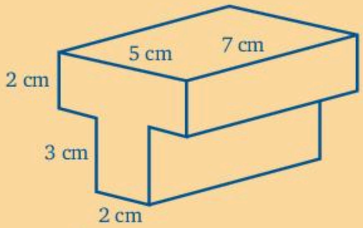
This is true of any prism so that

Volume of a prism = area of cross-section × length

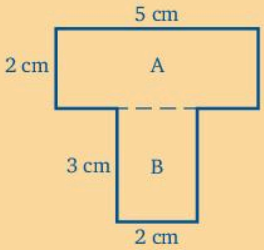
Exercise 11b

Find the volume of the solid below.

To find the volume you need first to find the area of the cross-section.



Draw the cross-section, then divide it into two rectangles.



$$\text{Area of A} = 2 \times 5\,\text{cm}^2 = 10\,\text{cm}^2$$

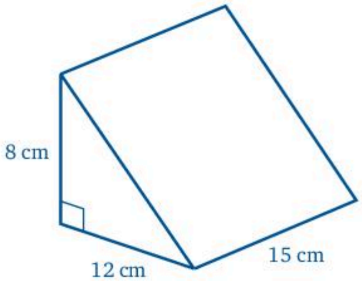
$$\text{Area of B} = 2 \times 3\,\text{cm}^2 = 6\,\text{cm}^2$$

$$\text{Area of cross-section} = 16\,\text{cm}^2$$

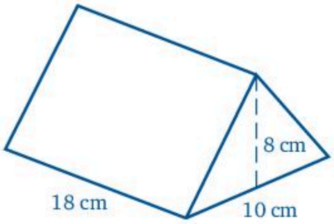
$$\begin{aligned}\text{Volume} &= \text{area} \times \text{length}\\ &= (16 \times 7)\,\text{cm}^3\\ &= 112\,\text{cm}^3\end{aligned}$$

Find the volumes of the following prisms. Draw a diagram of the cross-section but do *not* draw a picture of the solid.

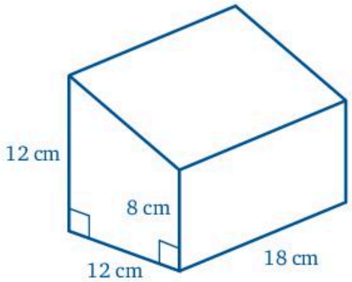
1



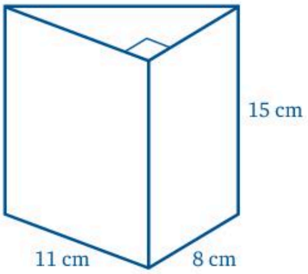
3



2

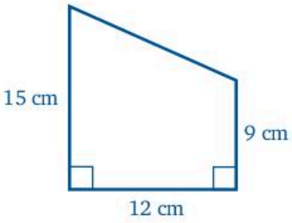


4



This solid is standing on its end so the vertical measurement is its length.

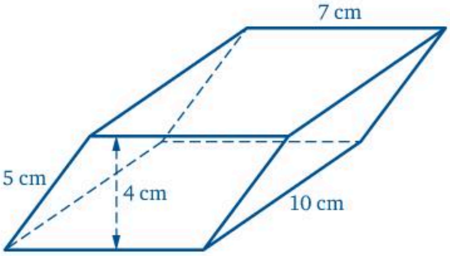
5 The diagram shows the cross-section of the prism.
The length is given.



length 20 cm

6 A tent is in the shape of a triangular prism. Its length is 2.4 m, its height 1.8 m and the width of the triangular end is 2.4 m. Find the volume enclosed by the tent.

7

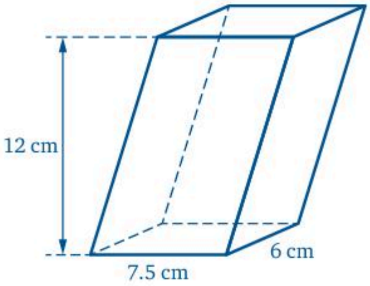


This solid has a constant cross-section and is 10 cm long.

- **a** What name do we give to the shape of the cross-section?
- **b** Find the area of the cross-section.
- **c** Find the volume of the solid.

8 This solid has a constant cross-section.

- **a** How high is it?
- **b** What name do we give to the shape of the cross-section?
- **c** Find the area of the cross-section.
- **d** Find the volume of the solid.



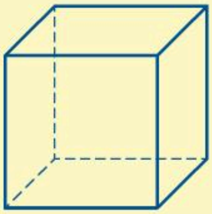
Drawing cubes and prisms

A solid object such as a cuboid has three *dimensions*: length, width and height.

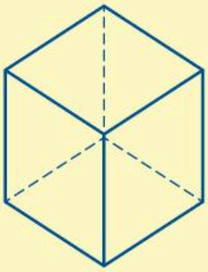
When we draw a solid on paper, we are making a two-dimensional representation of a three-dimensional object.

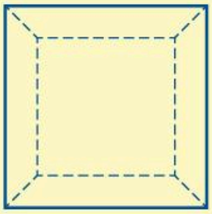
We do this by using solid lines for edges that can be seen and broken lines for edges that are hidden from view.

For example, this cube is drawn as though we were looking at it with one face directly in front of us.

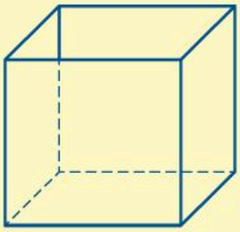


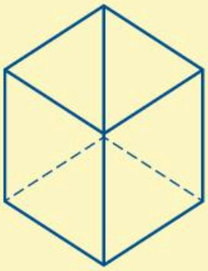
We can also draw the same cube with an edge directly in front of us or looking down from the top of the cube.

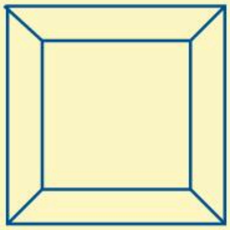




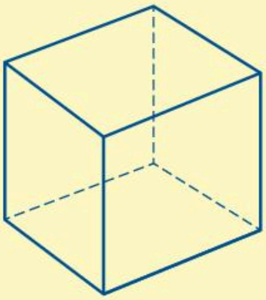
If the cube represents a box without a lid, we can see some of the inside edges. The diagrams then look like this:







The clearest representation is usually when looking at a face directly in front or at a slight angle, like this cube:



Edges do not then appear hidden behind other edges.

Exercise 11c

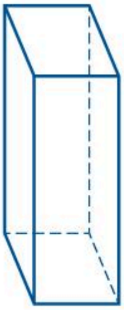
1 Sketch this cuboid with

- **a** a smallest face in front
- **b** standing on one of the smallest faces.

2 Sketch this cuboid with

a a smallest face in front

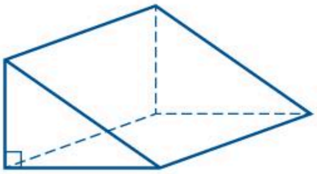
b looking down on one of the larger faces.



3 Sketch this wedge when it is standing on

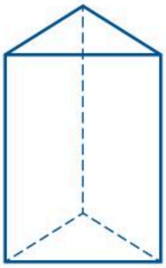
a a triangular face

b the smallest rectangular face.



4 This wedge is standing on a triangular face.

Sketch this wedge when it is standing on the face that is directly facing you in the diagram.

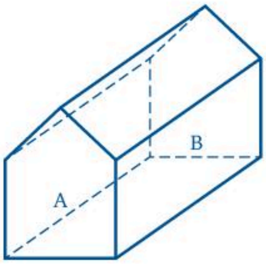


5 This diagram represents a shed.

Sketch this shed when

a it is standing on the end marked A

b it is standing on the side marked B.

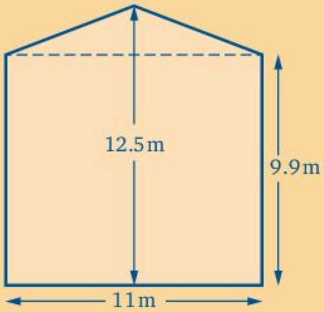


Composite solids

Some solids are the result of putting two prisms together.

Exercise 11d

The diagram shows the cross-section of a barn 11 m wide and 25 m long.



The barn is 12.5 m high at the ridge and 9.9 m high at its sides.

Calculate the volume of hay that can be stored in it.

The height of the triangle is 12.5 m – 9.9 m = 2.6 m

Area of cross-section = area of rectangle + area of triangle

$$= (11 \times 9.9)\,\text{m}^2 + \tfrac{1}{2}(11 \times 2.6)\,\text{m}^2$$

$$= (108.9 + 14.3)\,\text{m}^2$$

$$= 123.2\,\text{m}^2$$

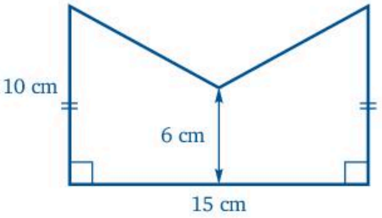
Capacity of barn = area of cross-section × length

$$= 123.2\,\text{m}^2 \times 25\,\text{m}$$

$$= 3080\,\text{m}^3$$

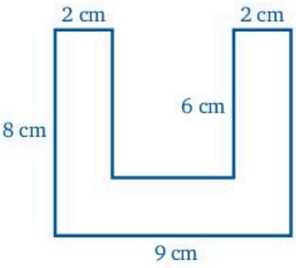
In questions **1** to **7** the cross-sections of the prisms and their lengths are given. Find their volumes.

1



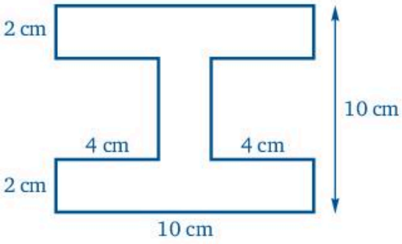
length 20 cm

2



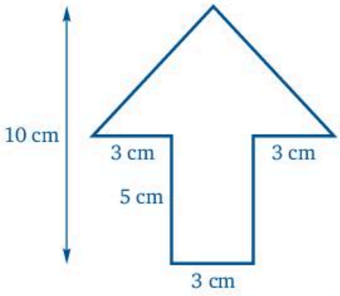
length $7\frac{1}{2}$ cm

3



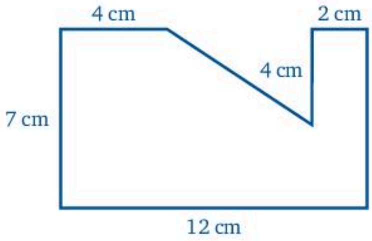
length 12 cm

4



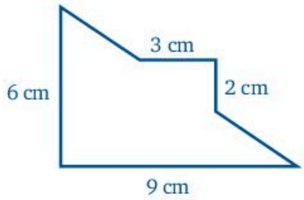
length 12 cm

5



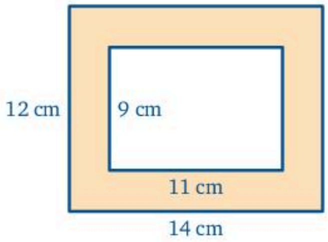
length 12 cm

7

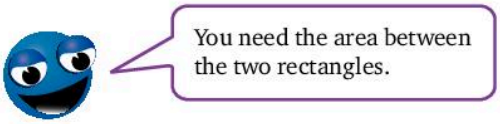


length 24 cm

6



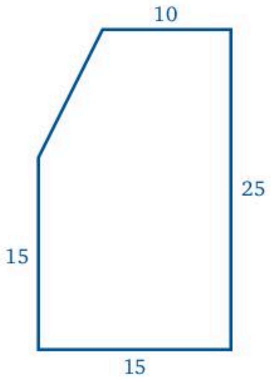
length 10 cm



In questions **8** to **11** all dimensions are given in centimetres.

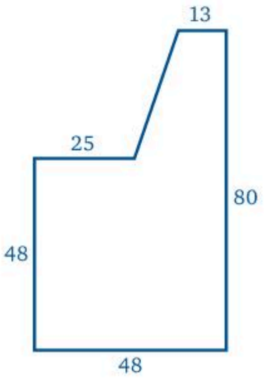
8 The diagram shows the cross-section of a kerbstone which is 1 metre long.

Calculate its volume in
a cubic centimetres
b cubic metres.

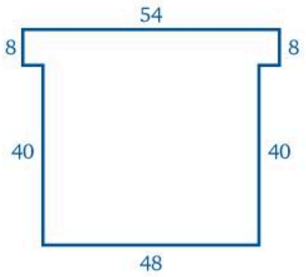


9 The diagram shows the cross-section of the concrete ends for a park seat.

If the structures are 6 cm thick, calculate the volume of concrete used in the manufacture of one seat.



10 A foam cushion for a chair is in the form of a letter T with dimensions as shown in the diagram. If the cushion is 8 cm thick, find the volume of foam used.



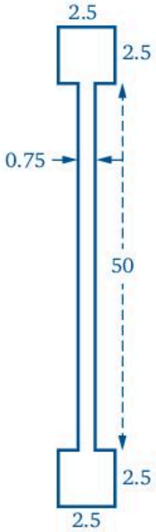
11 The diagram shows the cross-section through a domestic central heating radiator.

The centre section is 50 cm high and 0.75 cm thick.

At the top and bottom of the radiator is a square of side 2.5 cm.

a Calculate the area of cross-section of the radiator.

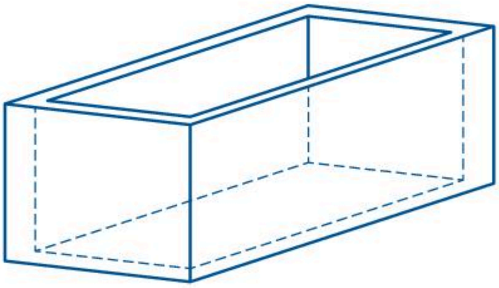
b If the radiator is 2 m long, how much water will it contain when full?
Give your answer **i** in cubic centimetres **ii** in litres.



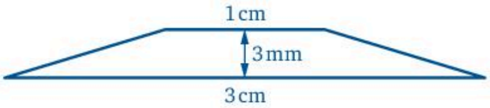
12 An open metal mould is used for casting concrete blocks measuring 30 cm by 20 cm by 10 cm. The walls of the mould are 0.5 cm thick.

a Calculate the volume of metal used to make the mould.

b The mould is melted down and recast, without any change in volume, as a cube. Calculate the length of an edge of this cube.



13

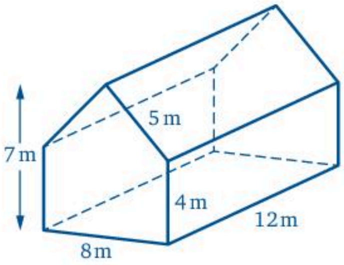


The diagram shows the cross-section through a plastic ruler which is 31.5 cm long.

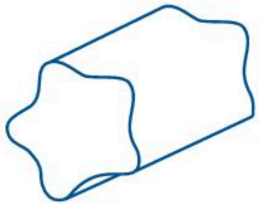
Find the volume of material used.

14 The diagram shows a warehouse.

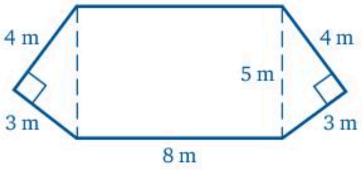
Calculate its capacity.



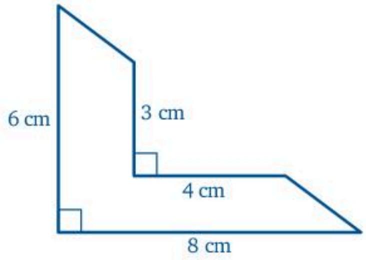
15 The area of the cross-section of the given solid is 42 cm² and the length is 32 cm. Find its volume.



16 A solid of uniform cross-section is 12 m long. Its cross-section is shown in the diagram. Find its volume.



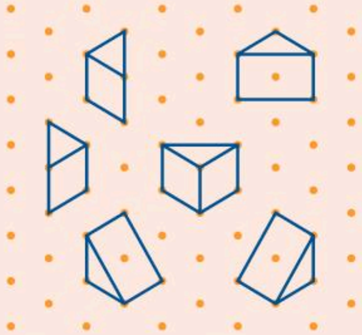
17

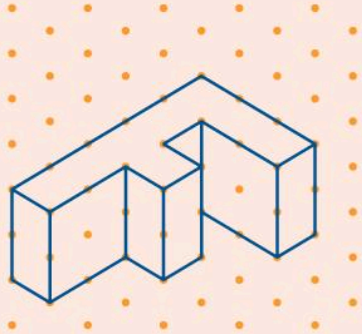


The diagram shows the uniform cross-section of a prism. The prism is 30 cm long. Find its volume.

Puzzle

Leela has a bag of identical triangular wooden blocks.





She uses some of them to make the letter F.

How many blocks does she need?

Mixed exercises

Exercise 11e

1 Express 3.2 m³ in

a cm³ **b** mm³.

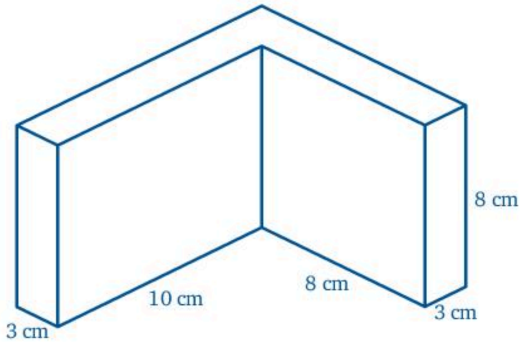
2 Express 1.6 litres in cm³.

3 Find the volume of a cube of side 4 cm.

4 Find the volume, in cm³, of a cuboid measuring 2 m by 25 cm by 10 cm.

5 Find the volume, in mm³, of a cuboid measuring 5 cm by 3 cm by 9 mm.

6 Find the volume of this prism.



Exercise 11f

1 Express 8 cm³ in

a mm³ **b** m³.

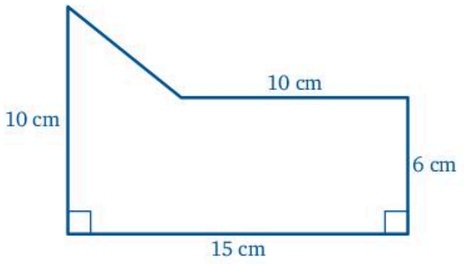
2 Express 3500 cm³ in litres.

3 Find the volume of a cuboid measuring 10 cm by 5 cm by 6 cm.

4 Find, in cm³, the volume of a cube of side 8 mm.

5 Find the volume, in cm³, of a cuboid measuring 50 cm by 1.2 m by 20 cm.

6 The diagram shows the uniform cross-section of a prism. The prism is 12 cm long. Find its volume.



Exercise 11g

1 Express 0.009 m³ in

   a cm³ **b** mm³.

2 Express 0.44 litres in cm³.

3 Find the volume of a cube of side 6 cm.

4 Find the volume of a cuboid measuring 12 cm by 6 cm by 4 cm.

5 Find the capacity, in litres, of a rectangular tank measuring 2 m by 1.5 m by 80 cm.

6 How many rectangular packets, measuring 12 cm by 8 cm by 4 cm, may be packed into a rectangular cardboard box measuring 24 cm by 16 cm by 12 cm?

Exercise 11h

1 Express 900 cm³ in m³.

2 Express 10 800 cm³ in litres.

3 Express 0.075 m³ in litres.

4 Find, in cm³, the volume of a cube of side 20 mm.

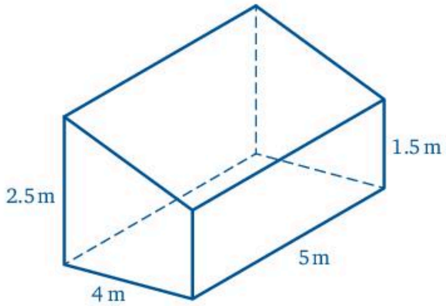
5 Find, in m³, the volume of a cuboid measuring 150 cm by 100 cm by 80 cm.

6 The diagram shows a glasshouse at a garden centre.

   Find

   a the area of the cross-section of the glasshouse

   b the volume of the glasshouse in cubic metres.



Exercise 11i

In this exercise choose the capital letter that gives the correct answer.

1 Expressed in cubic centimetres, 0.024 cubic metres is

A 24 cm^3 **B** 240 cm^3 **C** 2400 cm^3 **D** 24 000 cm^3

2 Expressed in cubic centimetres, 3.6 litres is

A 360 cm^3 **B** 3600 cm^3 **C** 36 000 cm^3 **D** 360 000 cm^3

3 Expressed in cubic metres, 120 000 cubic centimetres is

A 0.0012 m^3 **B** 0.012 m^3 **C** 0.12 m^3 **D** 1.2 m^3

4 Expressed in cubic centimetres, 0.065 litres is

A 6.5 cm^3 **B** 65 cm^3 **C** 650 cm^3 **D** 6500 cm^3

5 The volume, in cubic centimetres, of a cuboid measuring 12 cm by 9 cm by 6 cm is

A 64.8 cm^3 **B** 648 cm^3 **C** 1296 cm^3 **D** 6480 cm^3

6 The volume of a cuboid measuring 5 m by 2.5 m by 75 cm is

A 9.375 m^3 **B** 93.75 m^3 **C** 937.5 m^3 **D** 9375 m^3

7 The capacity of a rectangular tank measuring 1 metre by 80 cm by 40 cm is

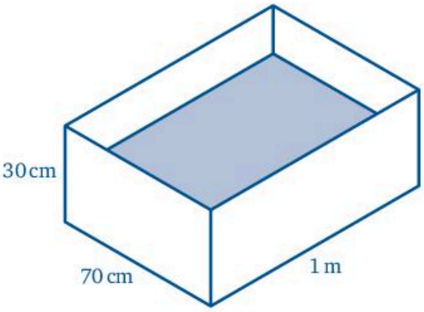
A 32 litres **B** 320 litres **C** 3200 litres **D** 6400 litres

8 The cross-section of a prism is a pentagon.

The number of rectangular faces this prism has is

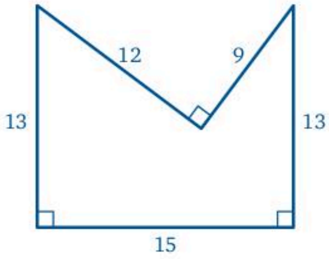
A 0 **B** 3 **C** 4 **D** 5

9 This open rectangular tank is half-full of water.



The number of litres of water required to fill it is

A 70 **B** 105 **C** 175 **D** 210



10 The diagram shows the cross-section of a prism.

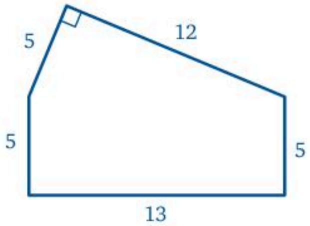
All the dimensions are given in centimetres.

If the prism is 60 cm long, its volume is

A 5220 cm³ **C** 8460 cm³

B 6430 cm³ **D** 11 700 cm³

11



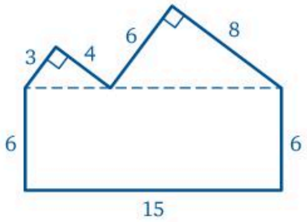
The diagram shows the cross-section through a warehouse.

All the measurements are in metres.

If the warehouse is 40 metres long, its capacity, in cubic metres, is

A 3800 **B** 3750 **C** 4200 **D** 5000

12



The diagram shows the cross-section through a bus depot.

All the measurements are in metres

If the depot is 30 metres long, its total capacity is

A 3600 m³ **B** 3780 m³ **C** 4320 m³ **D** 4500 m³

Did you know?

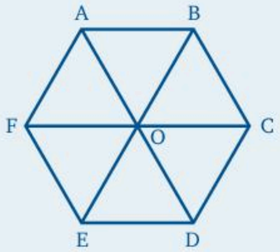
A Moebius strip is named after the German mathematician August Ferdinand Moebius. You can make one: take a strip of paper, twist it once and join the ends together. It has only one surface and only one edge (try it – draw a line along its surface – what happens?).

In this chapter you have seen that...

✔ any solid with a uniform cross-section is called a prism and its volume is equal to the area of the cross-section multiplied by its length.

REVIEW TEST 2: CHAPTERS 6–11

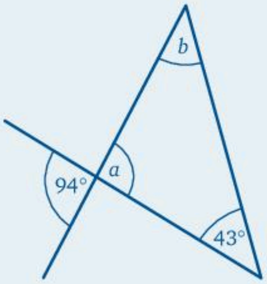
1



ABCDEF is a regular hexagon. OA = OB = OC = OD = OE = OF.
Find the size of the following angles:

a angle OAB **b** angle AOB **c** angle BCD

2 Find the size of the angles marked a and b.

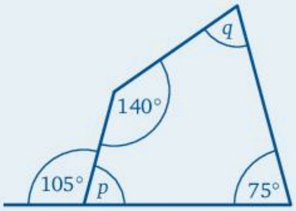


3 Find the size of an interior angle of a regular polygon with 15 sides.

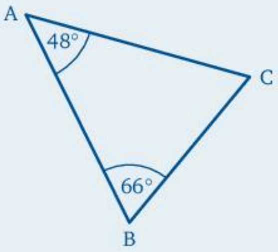
4 Is it possible for each exterior angle of a regular polygon to be

a 36° **b** 42° **c** 20° ?

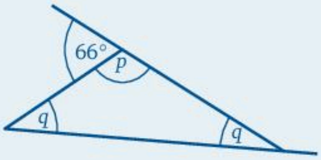
5 Find the size of the angles marked p and q.



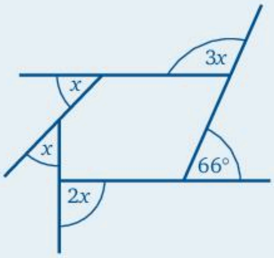
6 Which sides, if any, are equal sides in this diagram?



7 Find the value of the angles marked p and q.
What special name describes this triangle?



8 What name do you give to this shape?
Find the value of x.



9 A regular polygon has 15 sides.
Find the size of **a** an exterior angle **b** an interior angle.

10 Find the sum of the interior angles of a polygon with 13 sides.

11 Find the mode, mean and median of the following set of numbers:

2.7, 2.9, 2.4, 2.6, 2.8, 2.5, 2.2, 2.3, 2.8

Remember to arrange the numbers in order of size first.

Give answers correct to 3 s.f. where necessary.

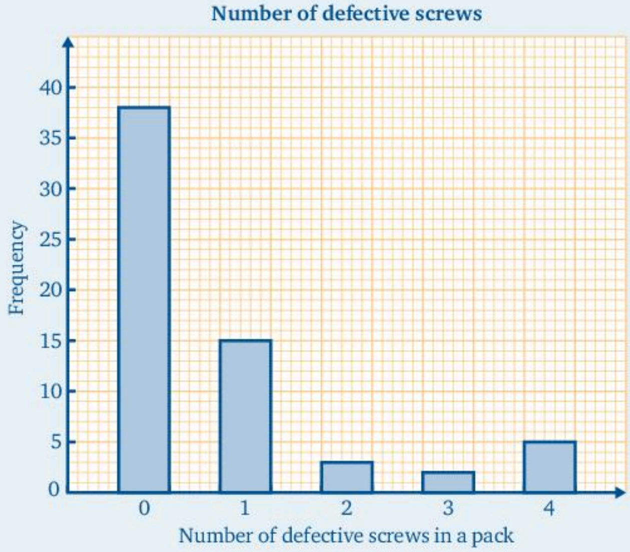
12 Sally stood outside her home and counted the number of occupants in the cars that passed. Her results are shown in the table.

Number of occupants per car	1	2	3	4	5	6
Frequency	13	19	8	3	1	1

a How many cars passed while Sally was watching?
b What was the total number of occupants in these cars?
c Calculate the mean number of occupants per car.
d Find the mode for this data.
e What is the median?

13 Draw a pie chart for the data given in question **12**.

14 A machine makes 6-inch screws and puts them in packs of five. A small number of packs are selected at random and the number of defective screws in each pack noted. The results for one afternoon shift are shown in the bar chart.



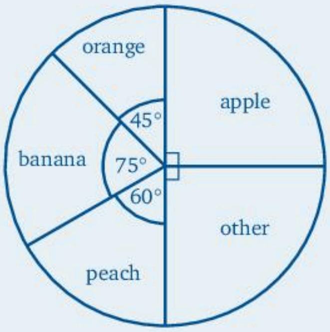
a How many packs contained
  i no defective screws **ii** 2 defective screws?

b How many packs were checked?

c How many defective screws were there altogether?

d What percentage of the screws checked were defective? Give your answer correct to 3 s.f.

15 120 people were asked to name their favourite fruit. Their answers are shown in the pie chart.



a What fraction chose
  i an apple **ii** a banana?

b What fraction did not choose an orange or a banana?

c How many chose
  i an apple
  ii a peach
  iii an apple or a banana?

16 Ninety-five people were asked how they had travelled to the shops and the following information was recorded.

Transport	walk	bus	car	bicycle
Number of people	15	40	35	5

Draw a pictograph using one drawing to 5 people. 

17 **a** Which is the larger ratio, 4 : 7 or 5 : 8?
   b Express $3 : \frac{2}{3}$ as a ratio in its simplest form.

18 **a** Increase \$7000 in the ratio 12 : 7.
   b Decrease 40 cm in the ratio 5 : 3.

19 **a** Divide 312 km in the ratio 5 : 8.
   b Find the missing number in the ratio $7 : 4 = 28 : \square$.

20 If 5 boxes of sweets cost \$900, find the cost of 14 similar boxes.

21 A bed costs \$54 000 + sales tax at 16%. How much will it cost me?

22 A retailer offers a discount of 15% on an electric appliance marked at \$36 000.
Calculate the discount.

23 Calculate the simple interest on \$35 000 invested for 5 years at 3%.

24 Calculate, correct to the nearest \$100, the compound interest on \$81 000 invested for 2 years at 8%.

25 The marked price of lounge suite is \$450 000. If bought for cash there is a discount of 5%.
If bought on hire-purchase the terms are:

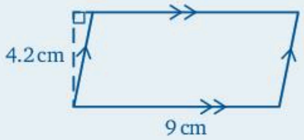
$\frac{1}{3}$ deposit + 36 monthly payments of \$11 260.

Calculate
- **a** the cash price
- **b** the deposit if bought on HP
- **c** the total of the monthly payments
- **d** the difference between the discounted price and the hire-purchase price.

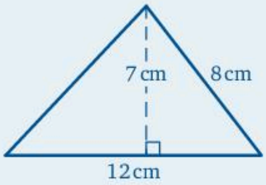
26 Express **a** 9200 cm² in m²
b 470 000 m² in km².

27 Find the area, in cm², of a rectangle measuring $2\frac{1}{4}$ cm by $1\frac{1}{3}$ cm.

28 Find the area of this parallelogram.

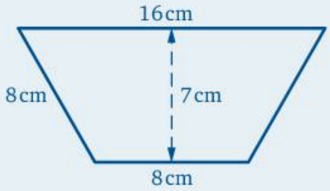


29 **a** Find the area of this triangle.



b The area of a triangle, which has a base of length 18 cm, is 36 cm².
Find the height of the triangle.

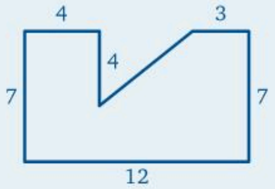
30 Find the area of this trapezium.



31 A cuboid measures 9 cm by 6 cm by 4 cm.
Find **a** the area of one of the largest sides
b the area of one of the smallest sides
c the total surface area of the cuboid.

32 Express **a** 5 litres in cm³
b 0.04 m³ in cm³
c 1.6 cm³ in mm³.

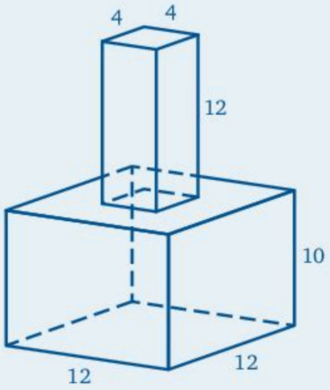
33 The diagram shows the cross-section of a prism that is 20 cm long.
All dimensions are in centimetres.



Find **a** the area of cross-section of the prism
b its volume.

34 A rectangular water tank is 1.5 m long, 80 cm wide and 50 cm deep.
Find its capacity in **a** cm^3 **b** litres.

35 The diagram shows an unusual design for a vase.
All dimensions are in centimetres.



Water is poured into the vase until it is half-way up the funnel.
Calculate the amount of water in the vase

a in cubic centimetres **b** in litres.

12 Straight-line graphs

At the end of this chapter you should be able to...

1. find the gradient of a straight line
2. calculate the gradient of a line, given the coordinates of two points on the line
3. state the gradient of a line whose equation is given
4. relate the gradient of a graph to the rate of change of quantities
5. draw a line with equation $y = mx + c$, by finding points on the line
6. use the equation of a line to calculate the value of y for a given x
7. write the equation of a line given its gradient and intercept on the y-axis
8. write the equation of a line through two given points
9. find the equation of a straight line parallel to the x- or y-axis
10. draw lines of the form $y = k$ and $x = h$ where h and k are constants
11. determine if two lines whose equations are given are parallel or perpendicular
12. find the point of intersection of two straight lines graphically.

Did you know?

Although we always tend to think that the shortest distance between two points is a straight line, this is not always the case. The shortest distance between two points on a globe or curved surface is a geodesic.

You need to know...

- ✓ how to draw and scale a set of axes and plot points on the resulting grid
- ✓ how to read the coordinates of a point on a line
- ✓ how to solve a linear equation
- ✓ how to work with fractions and decimals
- ✓ how to work with directed numbers.

Key words

axes, coefficient, coordinates, gradient, parallel, perpendicular, y-intercept

Plotting points and equations of straight lines

Remember that in Grade 7 we showed how points can be plotted on x- and y-*axes* given their *coordinates*. We also showed how to draw a straight line given its equation. We revise that work here.

Exercise 12a

1 Draw axes with both x and y values from -6 to 6 for this question.

a Plot the points A(2, 4), B(0, 0) and C(-1, -2).
Draw the straight line through A, B and C.

b Does the point D(-2, -4) lie on the line?

c Does the point E(3, 5) lie on this line?

2 Find the y-coordinates of the points on the line $y = 3x$ that have x-coordinates of

a 1 **b** 0 **c** -2 **d** 4

3 Find the x-coordinates of the points on the line $y = -3x$ that have y-coordinates of

a 0 **b** 2 **c** -1 **d** 9

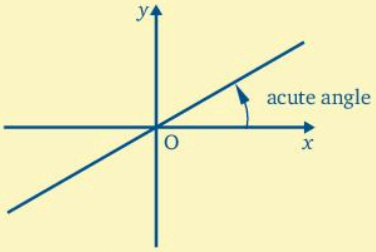
Draw your own set of axes with both x and y values from -6 to 6. In questions **4** to **7**, make a table of values for the coordinates of three points on the line and draw the graph of the line on the set of axes you have drawn. Write the equation of the line on it. Use the same set of axes for all four lines.

4 $y = 4x$ **5** $y = 2x$ **6** $y = -4x$ **7** $y = -\frac{3}{4}x$

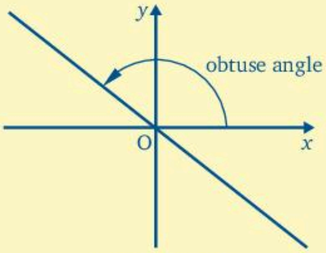
Gradient of a straight line

The graphs of the lines in questions **4** to **7** show that

- an equation of the form $y = mx$ is a straight line that passes through the origin
- when m, the *coefficient* of x, is positive, the line makes an acute angle with the positive x-axis and when m is negative, the line makes an obtuse angle with the positive x-axis

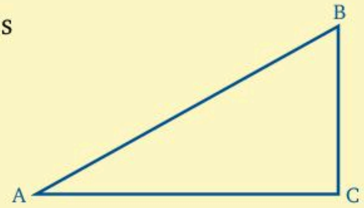


positive m and
positive gradient
(upward diagonal)



negative m and
negative gradient
(downward diagonal)

- the larger the value of m, the steeper is the line.



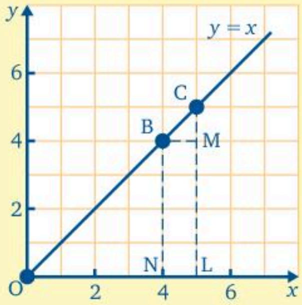
The *gradient* or slope of a line is defined as the amount the line rises vertically divided by the distance moved horizontally,

i.e. gradient or slope of AB $= \dfrac{\text{CB}}{\text{AC}}$

The gradient of any line is defined in a similar way.

Considering any two points on a line, the gradient of the line is given by

$$\frac{\text{the increase in } y \text{ value}}{\text{the increase in } x \text{ value}}$$



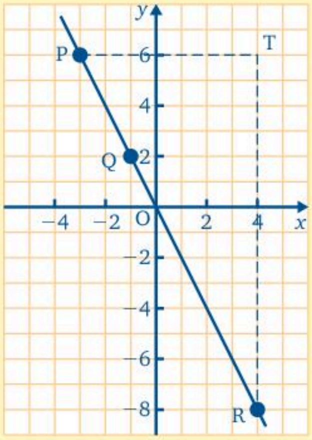
If we plot the points O(0, 0), B(4, 4) and C(5, 5), all of which lie on the line with equation $y = x$, then:

$$\text{gradient of OC} = \frac{\text{LC}}{\text{OL}} = \frac{5}{5} = 1$$

$$\text{gradient of OB} = \frac{\text{NB}}{\text{ON}} = \frac{4}{4} = 1$$

$$\text{gradient of BC} = \frac{\text{MC}}{\text{BM}} = \frac{5-4}{5-4} = \frac{1}{1} = 1$$

These show that, whichever two points are taken, the gradient of the line is 1.



Similarly, if we plot the points P(–3, 6), Q(–1, 2) and R(4, –8), all of which lie on the line with equation $y = –2x$, then:

$$\text{gradient of PR} = \frac{\text{increase in } y \text{ value from P to R}}{\text{increase in } x \text{ value from P to R}}$$

$$= \frac{y\text{-coordinate of R} - y\text{-coordinate of P}}{x\text{-coordinate of R} - x\text{-coordinate of P}}$$

$$= \frac{(-8)-(6)}{(4)-(-3)}$$

$$= \frac{-8-6}{4+3} = \frac{-14}{7} = -2$$

We can choose any two points on a line to calculate the gradient.

If these two points are (x_1, y_1) and (x_2, y_2) then

the gradient of the line is given by $\dfrac{y_1 - y_2}{x_1 - x_2}$

Exercise 12b

Draw axes for x and y, for values between –6 and +6, taking 1 cm as 1 unit on each axis.

Plot the points A(–4, 4), B(2, –2) and C(5, –5), all of which lie on the line $y = -x$. Find the gradient of

a AB **b** BC **c** AC

a Gradient of AB

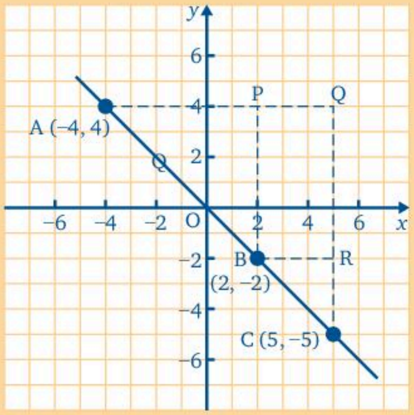
$$= \frac{(-2) - (4)}{(2) - (-4)} = \frac{-6}{6} = -1$$

b Gradient of BC

$$= \frac{(-5) - (-2)}{(5) - (2)} = \frac{-3}{3} = -1$$

c Gradient of AC

$$= \frac{(-5) - (4)}{(5) - (-4)} = \frac{-9}{9} = -1$$



1 Using 2 cm to 1 unit on each axis, draw axes that range from 0 to 6 for x and from 0 to 10 for y. Plot the points A(2, 4), B(3, 6) and C(5, 10), all of which lie on the line $y = 2x$. Find the gradient of

a AB **b** BC **c** AC

What do you notice about the gradients?

2 Draw the x-axis from –4 to 4 taking 2 cm as 1 unit, and the y-axis from –16 to 12 taking 0.5 cm as 1 unit. Plot the points X(–3, 12), Y(–1, 4) and Z(4, –16), all of which lie on the line $y = -4x$. Find the gradient of

a XY **b** YZ **c** XZ

What do you notice about the gradients?

3 Choosing your own scale and range of values for both x and y, plot the points D(–2, –6), E(0, 0) and F(4, 12), all of which lie on the line $y = 3x$. Find the gradient of

a DE **b** EF **c** DF

What do you notice about the gradients?

4 Taking 2 cm as 1 unit for x and 1 cm as 1 unit for y, draw the x-axis from –1.5 to 2.5 and the y-axis from –10 to 6. Plot the points A(–1.5, 4.5), B(0.5, –1.5) and C(2.5, –7.5), all of which lie on the line $y = -3x$. Find the gradient of

a AB **b** BC **c** AC

What do you notice about the gradients?

5 Copy and complete the following table and use it to draw the graph of $y = 2.5x$.

x	–3	–1	0	2	4
y					

Choose your own pairs of points to find the gradient of this line at least twice.

6 Copy and complete the following table and use it to draw the graph of $y = -0.5x$.

x	–6	–2	3	4
y				

Choose your own pairs of points to find the gradient of this line at least twice.

7 Look back at your answers to questions **1** to **6**.

a Write the equations that give graphs with a negative gradient in order from shallowest slope to steepest slope.

b Write the equations that give graphs with a positive gradient in order from shallowest slope to steepest slope.

8 Determine whether the straight lines with the following equations have positive or negative gradients:

a $y = 5x$ **c** $y = 12x$ **e** $3y = -x$

b $y = -7x$ **d** $y = -\frac{1}{4}x$ **f** $5y = -12x$

These exercises confirm that

- the larger the value of m the steeper is the slope
- lines with positive values for m make an acute angle with the positive x-axis
- lines with negative values for m make an obtuse angle with the positive x-axis.

Exercise 12c

For each of the following pairs of lines, state which line is the steeper. Show both lines on the same sketch.

1 $y = 5x, \quad y = \frac{1}{5}x$

2 $y = 2x, \quad y = 5x$

3 $y = \frac{1}{2}x, \quad y = \frac{1}{3}x$

4 $y = -2x, \quad y = -3x$

5 $y = 10x, \quad y = 7x$

6 $y = -\frac{1}{2}x, \quad y = \frac{1}{-4}x$

7 $y = -6x, \quad y = -3x$

8 $y = 0.5x, \quad y = 0.75x$

Determine whether each of the following straight lines makes an acute angle or an obtuse angle with the positive x-axis.

9 $y = 4x$

10 $y = -3x$

11 $y = -\frac{1}{2}x$

12 $y = 3.6x$

13 $y = \frac{1}{3}x$

14 $y = 0.7x$

15 $y = 10x$

16 $y = 0.5x$

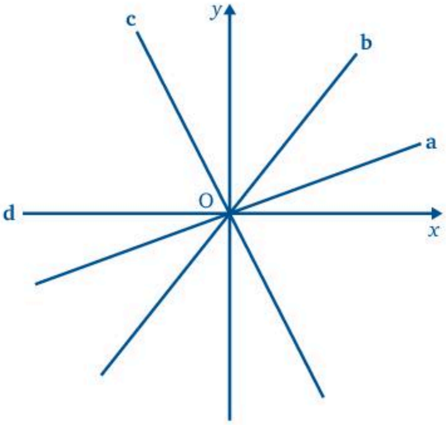
17 $y = -6x$

18 $y = -\frac{2}{3}x$

19 $y = -\frac{3}{4}x$

20 $y = -0.4x$

21 Estimate the gradient of each of the lines shown in the sketch.



Puzzle

Here is a very ingenious method of guessing the values of three dice thrown by a friend, without seeing them.

- Tell your friend to think of the first die.
- Multiply by 2. Add 5. Multiply by 5.
- Add the value of the second die.
- Multiply by 10. Add the value of the third die.
- Now ask for the total. From this total subtract 250.
- The three digits of your answer will be the values of the three dice.

As an example, if the total was 706, then 706 – 250 = 456. The three dice were therefore 4, 5 and 6. Try it and see. Why does it work?

Exercise 12d

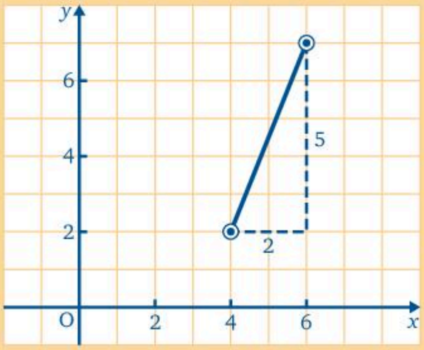
Find the gradients of the lines joining the points

a (4, 2) and (6, 7) **b** (2, 3) and (4, –3)

a *Either* from the diagram

the gradient is $\frac{5}{2}$

(In moving from left to right you go up 5 units (i.e. +5) and go across 2 units to the right, i.e. +2.)



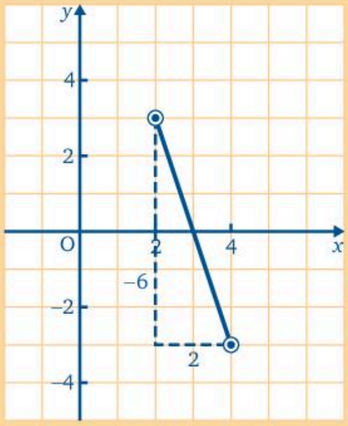
Or using $\frac{y_1 - y_2}{x_1 - x_2}$ with $(x_1, y_1) = (4, 2)$ and $(x_2, y_2) = (6, 7)$ gives

$$\text{gradient} = \frac{2-7}{4-6} = \frac{-5}{-2} = \frac{5}{2}$$

(We could also use $\frac{y_1 - y_2}{x_1 - x_2}$ in the form $\frac{y_2 - y_1}{x_2 - x_1}$ to avoid minus signs.)

b *Either* from the diagram the gradient is $\frac{-6}{2} = -3$

(In moving from left to right you go down 6 units (–6) and across to the right 2 units, i.e. +2.)



Or using $\frac{y_1 - y_2}{x_1 - x_2}$ with $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (4, -3)$ gives

$$\text{gradient} = \frac{3-(-3)}{2-4} = \frac{6}{-2} = -3$$

Find the gradients of the lines joining the following pairs of points:

1 (5, 1) and (7, 9)

2 (3, 6) and (5, 2)

3 (3, 4) and (6, 7)

4 (–2, 4) and (2, 1)

5 (1, 2) and (6, –7)

6 (–3, 4) and (–6, 2)

If you understand what is going on and can find the gradient without plotting the points on a grid, do so.

7 Find the gradient of the line joining the points (4, 3) and (7, 3).

8 Which axis is parallel to the line joining the points (4, 3) and (4, 6)? What happens when you try to work out the gradient?

9 If lines are drawn joining the following pairs of points, state which lines have zero gradient and which are parallel to the y-axis:

a (6, 4) and (6, –2)

b (3, 2) and (–10, 2)

c (–6, –3) and (–2, –3)

d (–4, 6) and (–4, 12)

10 If (2, 1) is a point on a line and its gradient is 3, draw the line and find the coordinates of two other points on it.

11 State the gradient of the line $y = 4x + 1$, without calculation if possible.

12 Give the gradients of the lines with equations

a $y = 4x + 4$

b $y = 2 - 3x$

c $y = x - 3$

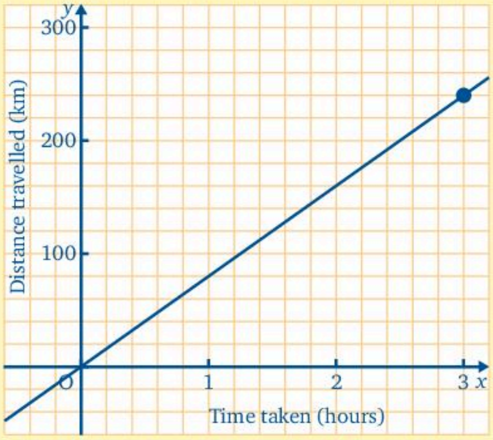
d $y = \frac{1}{2}x + 1$

Gradient of a graph and rate of change

In real-life graphs, the gradient of the line represents a rate of change.

In Chapter 8 you saw that speed is the distance travelled in a given time. You can draw a graph to show how distance travelled changes with time.

Wesley travels for 3 hours at a steady speed. In that time he has travelled 240 km. A graph of his journey looks like this.



The gradient represents the rate of change of distance. Choose two convenient points on the line to find the gradient.

A useful point is the origin, (0, 0). After 3 hours Wesley has travelled 240 km, which is the point (3, 240) on the graph.

Gradient $= \frac{240-0}{3-0} = \frac{240\text{km}}{3\text{h}} = 80\,\text{km/h}$

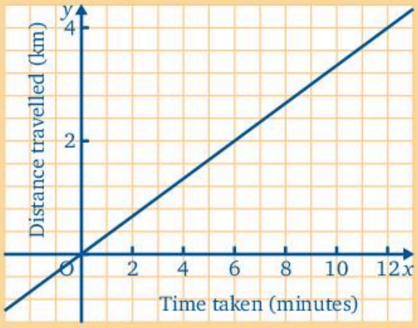
The gradient of the graph gives the speed.

Exercise 12e

The graph shows the time, in minutes, that a cyclist takes to ride 4 km.

a Calculate the gradient of the line.

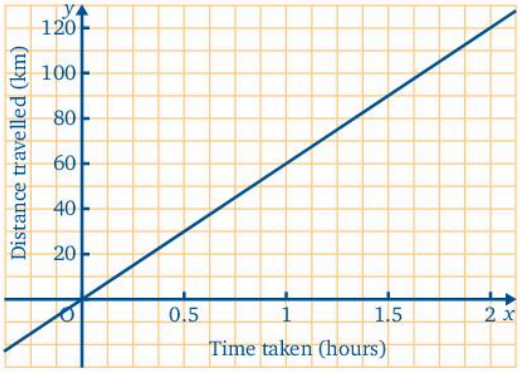
b Hence determine the speed of the cyclist in km per minute.



a Gradient $= \frac{4-0}{12-0} = \frac{1\text{km}}{3\text{min}}$

b Since the gradient is the rate of change of distance with respect to time (i.e. speed), the speed of the cyclist is $\frac{1}{3}$km per minute.

1 The graph shows the distance, in km, travelled by a car in two hours.

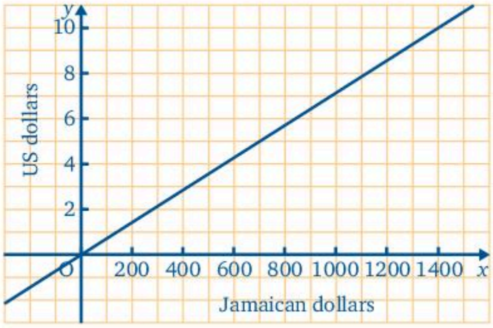


a Calculate the gradient of the line.

b Hence determine the speed of the car in km per hour.

Be careful with the units of time.

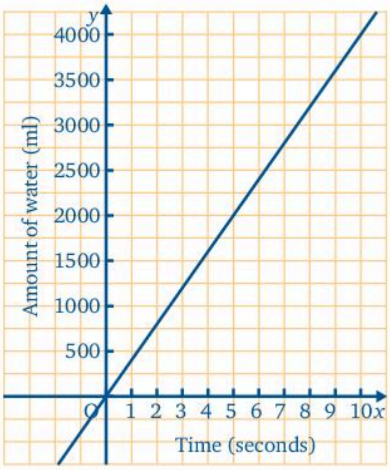
2 The graph shows the number of Jamaican dollars that can be exchanged for different numbers of US dollars.



a Calculate the gradient of the line.

b Hence determine the number of Jamaican dollars that can be exchanged for 1 US dollar.

3 The graph shows the amount of water, in millilitres, that a vase contains as it is filled from a tap.



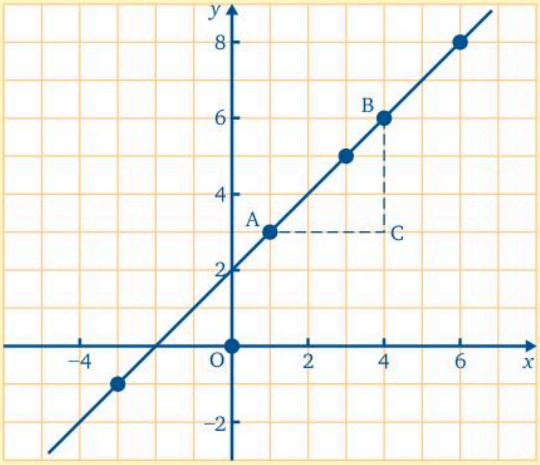
a Calculate the gradient of the line.

b Hence determine the rate, in ml per second, that the vase is filled.

The *y*-intercept

If we plot the points (–3, –1), (1, 3), (3, 5), (4, 6) and (6, 8), and draw the straight line that passes through these points, we find that the line does not pass through the origin. We can describe the line by finding

- its gradient
- the distance from the origin to the point where the line crosses the y-axis.



Using the points A and B, the gradient of the line is given by

$$\frac{CB}{AC}, \quad \text{i.e.} \quad \frac{3}{3}=1$$

The line crosses the y-axis at the point (0, 2), which is 2 units above the origin. This value of y where x equals zero is called the *y-intercept*.

This line has the equation $y = x + 2$. In the next exercise you will find out how the y-intercept is related to the equation of the line.

Exercise 12f

Draw the graph of $y = -2x + 3$ for values of x between -4 and $+4$.
Hence find

a the gradient of the line

b its y-intercept.

Compare the values for the gradient and the y-intercept with the number of x's and the number term on the right-hand side of the equation.

x	-4	0	4
y	11	3	-5

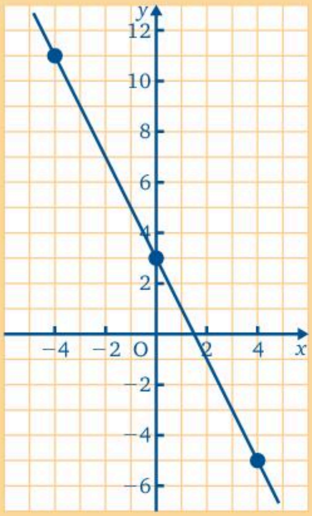
a Gradient of line $= \dfrac{3-11}{0-(-4)} = -\dfrac{8}{4}$

$= -2$

b The y-intercept is 3.

The number of x's on the right-hand side is -2, which is the same as the gradient of the line.

The number term on the right-hand side is 3, which is the same as the y-intercept.



In the following questions, draw a graph for each of the given equations. In each case find the gradient and the y-intercept for the resulting straight line. Take 1 cm as 1 unit on each axis, together with suitable values of x within the range -4 to $+4$. Choose your own range for y when you have completed the table.

Compare the values you get for the gradient and the y-intercept with

a the number of x's

b the number term on the right-hand side of the equation.

1 $y = 2x - 2$

2 $y = -2x + 4$

3 $y = 3x - 4$

4 $y = \frac{1}{2}x + 3$

5 $y = -\frac{3}{2}x + 3$

6 $y = 2x + 5$

7 $y = -2x - 7$

8 $y = -3x + 2$

As long as the equation of the line is of the form $y = mx + c$, for example $y = 2x + 3$ or $y = -5x + 1$, then

the number term c tells us where the line cuts the y-axis and m, the coefficient of x, tells us the gradient.

The equation $y = mx + c$ is the standard form of the equation of a straight line.

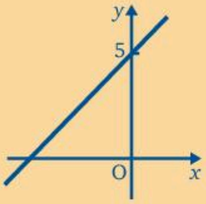
If the equation is $y = 3x$ then the number term is 0, i.e. the y-intercept is 0. Therefore, the line goes through the origin.

Exercise 12g

Give the gradient and the intercept on the y-axis of the line with equation $y = x + 5$. Sketch the line.

The gradient is the coefficient of x when the equation is written in the form $y = mx + c$, i.e. 1.

The intercept on the y-axis is 5.



Give the gradients and the intercepts on the y-axis of the lines with the following equations. *Sketch* each line:

1 $y = 2x + 4$

2 $y = 5x + 3$

3 $y = 3x - 4$

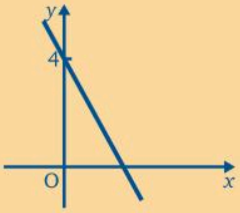
4 $y = x - 6$

Give the gradient and the intercept on the y-axis of the line with equation $y = 4 - 2x$. Sketch the line.

Rewrite the equation in the form $y = mx + c$, i.e. as $y = -2x + 4$

The gradient is –2 so the line slopes back to the left.

The intercept on the y-axis is 4.



Give the gradients and the intercepts on the y-axis of the lines with the following equations. Sketch each line:

5 $y = 3 - 2x$

6 $y = -4x + 2$

7 $y = 2 + 5x$

8 $y = \frac{1}{2}x - 1$

9 $y = -\frac{1}{3}x + 4$

10 $y = 3x - 7$

11 $y = 7 - 3x$

12 $y = \frac{1}{3}x + 7$

13 $y = 9 - 0.4x$

14 $y = 4 + 5x$

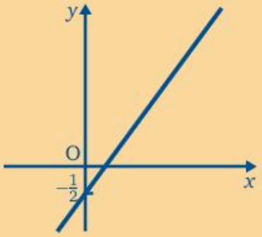
Give the gradient and the intercept on the y-axis of the line with equation $2y = 3x - 1$. Sketch the line.

$2y = 3x - 1$ (Write this equation in the form $y = mx + c$)

Divide both sides by 2 $y = \frac{3}{2}x - \frac{1}{2}$

The gradient is $\frac{3}{2}$ so the line slopes up to the right.

The intercept on the y-axis is $-\frac{1}{2}$.



Give the gradients and the intercepts on the y-axis of the lines with the following equations. Sketch each line:

15 $2y = 4x + 5$ **16** $3y = x - 6$ **17** $5y = 5 + 2x$ **18** $4y = 8 - 3x$

A line has a gradient of –2 and the intercept on the y-axis is 4. Find the equation of the line.

Comparing with the equation $y = mx + c$, $m = -2$ and $c = 4$.

The equation is $y = -2x + 4$

Write down the equations of the lines with the given gradients and intercepts on the y-axis (y-intercepts):

	Gradient	y-intercept
19	2	7
20	3	1
21	1	3
22	2	–5

	Gradient	y-intercept
23	$\frac{1}{2}$	6
24	–2	1
25	1	–2
26	$-\frac{1}{2}$	4

Puzzle

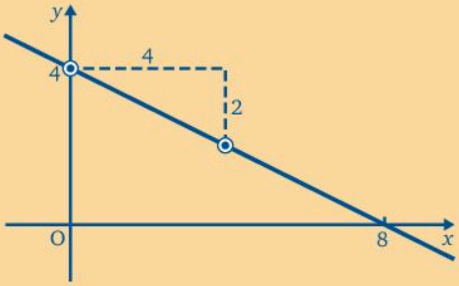
A farmer grows cabbages on a square plot. He says he has 151 more cabbages this year than last year, when he also had a square plot. How many cabbages did he raise last year?

The equation of a line through two given points

Exercise 12h

Find the gradient and the intercept on the y-axis of the line which passes through the points (4, 2) and (0, 4). Hence give the equation of the line.

(A sketch only is needed.)



The equation of any line is $y = mx + c$, so to find the equation of this line we need to find the value m, i.e. the gradient, and c, i.e. where it cuts the y-axis.

$$\text{Gradient} = \frac{2-4}{4-0} = \frac{-2}{4} = -\frac{1}{2}$$

When $x = 0, y = 4$ so the y-intercept $= 4$

$\therefore$ the equation is $y = -\frac{1}{2}x + 4$

(Multiplying by 2 and rearranging, allows this equation to be written in the form $2y = 8 - x$ or $2y + x = 8$.)

Find the gradient and the intercept on the y-axis of the line through the given points. Hence give the equation of the line:

1 (0, 4) and (3, 0)

2 (0, 7) and (2, 3)

3 (5, 4) and (0, 1)

4 (0, 2) and (3, –2)

5 (0, –4) and (2, 3)

6 (–6, –3) and (0, –1)

7 (6, 2) and (0, 1)

8 (5, 1) and (0, –3)

9 (0, –4) and (3, 1)

10 (–5, 0) and (0, –5)

11 (0, 12) and (–6, 0)

12 (–6, 1) and (0, 6)

13 A, B and C are the points (0, 4), (5, 6) and (3, –1). Find the equations of the lines AB and AC.

Find the gradient and the equation of the line through (6, 2) and (4, 8).

The gradient is $\frac{2-8}{6-4} = \frac{-6}{2} = -3$

(We do not know the intercept on the y-axis.)

Let the equation be $y = -3x + c$,

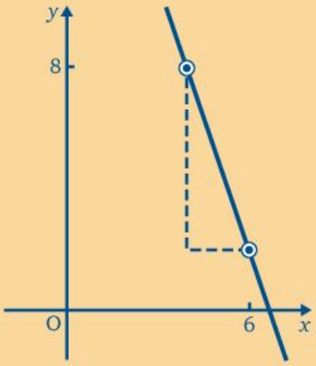
i.e. $m = -3$ in the equation $y = mx + c$.

When $x = 6, y = 2$ so $2 = -18 + c$

$\therefore$ $c = 20$

$\therefore$ the equation is $y = -3x + 20$

(*Check*: when $x = 4$, the equation gives $y = -12 + 20$, i.e. $y = 8$, which is correct.)



In questions **14** to **34**, find the gradient and the equation of the line through the given pair of points:

14 (4, 1) and (7, 10)

15 (1, 4) and (2, 1)

16 (3, 7) and (5, 12)

17 (–2, 3) and (–1, 5)

18 (4, –1) and (3, –6)

19 (5, –2) and (–4, 7)

20 (–6, 7) and (1, 0)

21 (4, –3) and (2, –7)

22 (–9, –3) and (6, 0)

23 (4, 0) and (0, 5)

24 (3, 0) and (0, 2)

25 (3, 0) and (0, –2)

26 (2, 0) and (0, 6)

27 (4, 2) and (6, 8)

28 (0, 4) and (3, 1)

29 (2, 1) and (0, –6)

30 (–1, 4) and (5, –2)

31 (0, 5) and (–2, 0)

32 (–6, 3) and (5, 5)

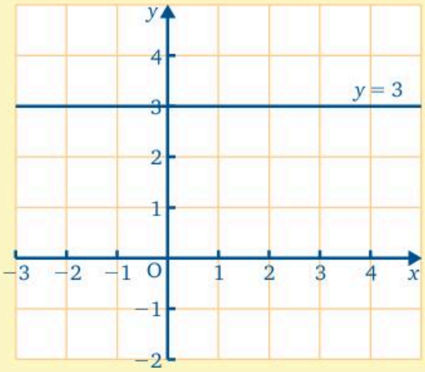
33 (–1, –2) and (–5, –6)

34 (3, 2) and (7, 1)

Lines parallel to the axes

Now we will see what happens when the gradient of a line is zero.

Think, for example, of the equation $y = 3$.



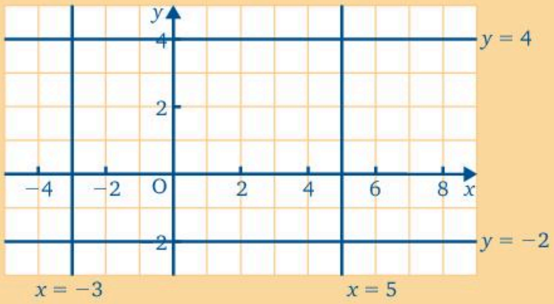
For every value of x the y-coordinate is 3. This means that the graph of $y = 3$ is a straight line parallel to the x-axis at a distance 3 units above it.

$y = c$ is therefore the equation of a straight line parallel to the x-axis at a distance c units away from it. If c is positive, the line is above the x-axis, and if c is negative, the line is below the x-axis.

Similarly $x = b$ is the equation of a straight line parallel to the y-axis at a distance b units from it.

Exercise 12i

Draw, on the same diagram, the straight-line graphs of $x = -3$, $x = 5$, $y = -2$ and $y = 4$.



In the following questions, take both x and y in the range –8 to +10, Let 1 cm be 1 unit on each axis.

1 Draw the straight-line graphs of the following equations in a single diagram:

$x = 2, x = -5, y = \frac{1}{2}, y = -3\frac{1}{2}$

2 Draw the straight-line graphs of the following equations in a single diagram:

$y = -5, x = -3, x = 6, y = 5.5$

3 On one diagram, draw graphs to show the following equations:

$x = 5, y = -5, y = 2x$

Write down the coordinates of the three points where these lines intersect. What kind of triangle do they form?

4 On one diagram, draw the graphs of the straight lines with equations

$x = 4, y = -\frac{1}{2}x, y = 3$

Write down the coordinates of the three points where these lines intersect. What kind of triangle is it?

5 On one diagram, draw the graphs of the straight lines with equations

$y = 2x + 4, y = -5, y = 4 - 2x$

Write down the coordinates of the three points where these lines intersect. What kind of triangle is it?

Parallel lines

Two lines are parallel when their gradients are equal.

Exercise 12j

1 Draw axes with both x and y values from -6 to 6. Draw these lines on your grid.

a $y = 2x + 2$

b $y = -x - 3$

c $y = 2x - 3$

d $y = 5 - x$

e $y = \frac{1}{2}x + \frac{1}{2}$

State the gradient of each line. Which lines are parallel?

Which of the lines with the following equations are parallel:

$y = 2x + 3, \quad y = 4 - 2x, \quad y = 4 + 2x, \quad 2y = x + 1, \quad y = x + 3?$

Written in the form $y = mx + c$ the equations of these lines are

$y = 2x + 3, \quad y = -2x + 4, \quad y = 2x + 4, \quad y = \frac{1}{2}x + \frac{1}{2}$ and $y = x + 3$.

The gradients of the lines are 2, -2, 2, $\frac{1}{2}$ and 1, so the first and third lines are parallel.

In questions **2** to **5**, state which of the lines with the given equations are parallel.

2 $y = 3x + 1$, $y = \frac{1}{3}x - 4$, $y = x + 1$,
$y = 4 - 3x$, $y = 5 + 3x$, $y = 3x - 4$

Write each equation in the form $y = mx + c$ first.

3 $y = 2 - x$, $y = x + 2$, $y = 4 - x$,
$2y = 3 - 2x$, $y = -x + 1$, $y = -x$

4 $3y = x$, $y = \frac{1}{3}x + 2$, $y = \frac{1}{3} + x$, $y = \frac{1}{3} + \frac{1}{3}x$, $y = \frac{1}{3}x - 4$

5 $y = \frac{1}{2}x + 2$, $y = 2 - \frac{1}{2}x$, $y = -x - 4$, $y = \frac{1}{2}x - 1$, $2y = 3 - x$

6 What is the gradient of the line with equation $y = 2x + 1$? Give the equation of the line that is parallel to the first line and which cuts the y-axis at the point (0, 3).

7 What is the gradient of the line with equation $y = 6 - 3x$?
If a parallel line goes through the point (0, 1), what is its equation?

8 Give the equation of the line through the origin that is parallel to the line with equation $y = 4x + 2$.

9 Give the equations of any three lines that are parallel to the line with equation $y = 4 - x$.

10 Give the equations of the lines through the point (0, 4) that are parallel to the lines with equations

a $y = 4x + 1$ **b** $y = 6 - 3x$ **c** $y = \frac{1}{2}x + 1$

11 Give the equations of the lines, parallel to the line with equation $y = \frac{1}{3}x + 1$, that pass through the points

a (0, 6) **b** (0, 0) **c** (0, –3)

12 Give the equations of the lines with gradient 2 which pass through the points

a (0, 2) **b** (0, 10) **c** (0, –4)

13 Which two of the lines with the following equations are parallel?

$y = 3 + 2x$, $y = 3 - 2x$, $y = 2x - 3$

14 Find the gradients and the intercepts on the y-axis of the lines with equations $y = 4 - 3x$ and $y = 4x - 3$. Give the equation of the line that is parallel to the first line and cuts the y-axis at the same point as the second line.

15 A line of gradient –4 passes through the origin.

a Give its equation.

b Give the equation of the line that is parallel to the first line and that passes through the point (0, –7).

Perpendicular lines

Two lines are *perpendicular* when the product of their gradients is –1.

For example, the lines $y = 2x - 1$ and $y = -\frac{1}{2}x + 4$ have gradients 2 and $-\frac{1}{2}$ respectively.

The product of 2 and $-\frac{1}{2}$ is $2 \times -\frac{1}{2} = \frac{\cancel{2}^{1}}{1} \times -\frac{1}{\cancel{2}_{1}} = -1$,

therefore the lines $y = 2x - 1$ and $y = -\frac{1}{2}x + 4$ are perpendicular.

Exercise 12k

1 Draw axes with both x and y values from –6 to 6. Draw these lines on your grid.

- **a** $y = -x + 4$
- **b** $y = -4x - 5$
- **c** $y = \frac{1}{4}x + 5$
- **d** $y = \frac{1}{2}x + 2$
- **e** $y = x + \frac{1}{2}$

State the gradient of each line. Which lines are perpendicular? Write the product of the gradients of the perpendicular lines.

In questions **2** to **7** determine whether the lines are parallel or perpendicular or neither.

2 $y = 5x - 2$ and $y = 3 - \frac{1}{5}x$

3 $y = \frac{2}{3}x - 3$ and $y = \frac{3}{2}x + 4$

4 $2y = 5 - x$ and $y = 2x + 4$

Divide both sides of the first equation by 2.

5 $3y = 2x - 4$ and $3y = 2x + 5$

6 $y = 5 - x$ and $y = x - 5$

7 $2y - 5 = x$ and $y + 2x = 1$

Find the equation of the line through the point (1, 2) that is perpendicular to the line $y = 4 - 3x$.

The gradient of the line $y = 4 - 3x$ is -3, so if the gradient of the perpendicular line is m, then $-3m = 1$.

$$-3m = -1$$

Dividing both sides by -3 gives $m = \frac{1}{3}$

Therefore the equation of the required line is $y = \frac{1}{3}x + c$

When $x = 1, y = 2$ so $2 = \frac{1}{3}(1) + c$

giving $2 - \frac{1}{3} = c$

so $c = 1\frac{2}{3}$

The equation of the required line is $y = \frac{1}{3}x + 1\frac{2}{3}$

Multiplying both sides by 3 gives $3y = x + 5$

8 What is the gradient of the line with equation $y = x - 4$?

Give the equation of the line that is perpendicular to the first line and which cuts the y-axis at the point (0, 2).

9 What is the gradient of the line with equation $y = 2x + 3$?

Give the equation of the line that is perpendicular to the first line and which cuts the x-axis at the point (1, 0).

10 Give the equation of the line through the origin that is perpendicular to the line with equation $2y = 5 - x$.

11 Which two of the lines with the following equations are perpendicular and which two are parallel?

$y = x - 5, \quad y = 2 - x, \quad y = x + 1$

12 **a** Find the gradients and the intercepts on the y-axis of the lines with equations $y = 3 - 3x$ and $y = 3x + 4$.

b Give the equation of the line that is perpendicular to the first line and cuts the y-axis at the same point as the second line.

13 A line with gradient $-\frac{2}{3}$ cuts the y-axis at the point (0, 1).

a Give the equation of this line.

b Find the equation of the line that is perpendicular to this line and goes through the origin.

Different forms of the equation of a straight line

The terms in the equation of a straight line can only be x terms, y terms or number terms.

An equation containing terms like $x^2, y^2, \frac{1}{x}, \frac{1}{x^2}$ is not the equation of a straight line.

Sometimes the equation of a straight line is not given exactly in the form $y = mx + c$.

It could be $2x + y = 6$ or $\frac{x}{4} + \frac{y}{2} = 1$.

An easy way to draw a line when its equation is in one of these forms is to start by finding the points where it cuts each axis.

Exercise 12I

Draw on graph paper the line with equation

$$3x - 4y = 12 \text{ (use } 0 \leqslant x \leqslant 5)$$

Find the gradient of the line.

$$3x - 4y = 12$$

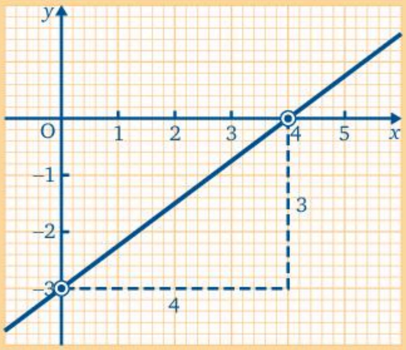
When $x = 0$, $-4y = 12$, i.e. $y = -3$

When $y = 0$, $3x = 12$, i.e. $x = 4$

So the points $(0, -3)$ and $(4, 0)$ lie on the line.

(Draw the line using these two points only.

Choose a point on the line, e.g. $(2, -1\frac{1}{2})$, to check.)



Check: when $x = 2$, $y = -1\frac{1}{2}$

$3x - 4y = 6 - 4 \times (-1\frac{1}{2}) = 6 + 6 = 12$, which is correct.

From the graph, the gradient $= \frac{3}{4}$ (lines that slope up to the right have a positive gradient).

Draw on graph paper the lines with the following equations. Use 1 cm to 1 unit.

Find the gradient of each line:

1 $3x + 5y = 15$ $\quad 0 \leqslant x \leqslant 6$

2 $2x + 6y = 12$ $\quad 0 \leqslant x \leqslant 6$

3 $x - 4y = 8$ $\quad 0 \leqslant x \leqslant 8$

4 $x + y = 6$ $\quad 0 \leqslant x \leqslant 7$

5 $2x + y = 5 \quad -2 \leqslant x \leqslant 4$

6 $x + 3y = 5 \quad 0 \leqslant x \leqslant 5$

7 $x - 3y = 6 \quad 0 \leqslant x \leqslant 6$

8 $2x - y = 3 \quad -2 \leqslant x \leqslant 2$

9 On the same axes ($-6 \leqslant x \leqslant 6$ and $-6 \leqslant y \leqslant 6$) draw the lines with equations $x + y = 1$, $x + y = 4$, $x + y = 6$, $x + y = -4$. Find their gradients.

Exercise 12m

Draw on graph paper the line with equation

$$\frac{x}{3} + \frac{y}{2} = 1 \quad (-1 \leqslant x \leqslant 4)$$

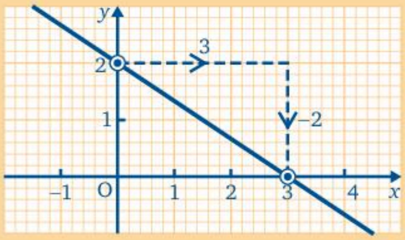
Find its gradient.

When $x = 0$, $\frac{y}{2} = 1$, so $y = 2$

When $y = 0$, $\frac{x}{3} = 1$, so $x = 3$

Check: from the graph, when $x = 1\frac{1}{2}$, $y = 1$

Therefore $\frac{x}{3} + \frac{y}{2} = \frac{1\frac{1}{2}}{3} + \frac{1}{2} = \frac{1}{2} + \frac{1}{2} = 1$ which is correct.



The gradient is $-\frac{2}{3}$ (lines that slope up to the left have a negative gradient).

Draw on graph paper the lines whose equations are given below. Find the gradient of each line:

1 $\frac{x}{4} + \frac{y}{3} = 1$

2 $\frac{x}{5} + \frac{y}{3} = 1$

3 $\frac{x}{4} - \frac{y}{2} = 1$

4 $\frac{x}{3} + \frac{y}{6} = 1$

5 $\frac{x}{1} - \frac{y}{2} = 1$

6 $\frac{y}{3} - \frac{x}{4} = 1$

7 Without drawing a diagram, state where the lines with the following equations cut the axes:

a $\frac{x}{2} + \frac{y}{4} = 1$ **b** $\frac{x}{12} - \frac{y}{9} = 1$

8 Form the equations of the lines which cut the axes at

a (0, 5) and (6, 0) **b** (0, −3) and (4, 0).

9 Sketch the line with equation $\frac{x}{6} + \frac{y}{2} = 1$ and find its gradient.

Getting information from the equation of a line

From the last exercise, we can see that if the equation of a line is in the form

$$\frac{x}{a}+\frac{y}{b}=1 \quad \text{(i.e. like questions } \mathbf{1} \text{ to } \mathbf{6}\text{)},$$

then the line cuts the x-axis at $x=a$

and the y-axis at $y=b$.

Then if we sketch the line we can work out the gradient.

If the equation is in the form $ax+by=c$, like those in Exercise 14k, we need to rearrange the equation so that it is in the form $y=mx+c$. Then the gradient and the intercept on the y-axis can be seen.

Exercise 12n

Find the gradient and the intercept on the y-axis of the line with equation $2x+3y=6$.

Change the equation to the form $y=mx+c$.

Subtract $2x$ from both sides $\quad 3y=6-2x$

Divide both sides by 3 $\quad y=2-\frac{2}{3}x$

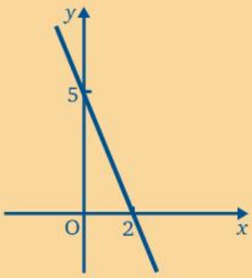
i.e. $\quad y=-\frac{2}{3}x+2$

The gradient is $-\frac{2}{3}$ and the intercept on the y-axis is 2.

Find the gradient and the intercept on the y-axis of each of the following lines:

1	$3x+5y=15$	**3**	$x-4y=8$	**5**	$y-3x=6$
2	$2x+6y=12$	**4**	$x-3y=6$	**6**	$x+3y=6$

Find the gradient and the intercept on the y-axis of the line with equation $\frac{x}{2}+\frac{y}{5}=1$.



Either

If $y = 0$ then $x = 2$ and if $x = 0$ then $y = 5$ so this line cuts the axes at (2, 0) and (0, 5).

Hence the gradient is $-\frac{5}{2}$ and the intercept on the y-axis is 5.

Or $\qquad \frac{x}{2} + \frac{y}{5} = 1$

Multiply both sides by 5 $\qquad \frac{5x}{2} + y = 5$

Subtract $\frac{5x}{2}$ from both sides $\qquad y = 5 - \frac{5x}{2}$

i.e. $\qquad y = -\frac{5}{2}x + 5$

Then the gradient is $-\frac{5}{2}$ and the intercept is 5.

Find the gradient and the intercept on the y-axis of each of the following lines:

7 $\frac{x}{4} + \frac{y}{3} = 1$

8 $\frac{x}{5} + \frac{y}{3} = 1$

9 $\frac{x}{4} - \frac{y}{2} = 1$

10 $\frac{x}{2} + \frac{y}{6} = 1$

11 $\frac{x}{3} + \frac{y}{4} = 1$

12 $\frac{x}{3} - \frac{y}{4} = 1$

13 $y = 4x + 2$

14 $x + y = 4$

15 $\frac{x}{2} + \frac{y}{4} = 1$

16 $2x + 5y = 15$

17 $y = 5 - \frac{1}{2}x$

18 $2y = 4x + 5$

19 $\frac{x}{2} - \frac{y}{4} = 1$

20 $x + y = -3$

21 $3x + 4y = 12$

Problems

Exercise 12p

1 Find the equation of the line through the point (6, 2) which is parallel to the line $y = 3x - 1$.

Remember that parallel lines have the same gradient.

2 On the same pair of axes, for $-8 \leqslant x \leqslant 8$ and $-12 \leqslant y \leqslant 4$, using 1 cm to 1 unit, draw the four lines $y = 2x - 4$, $y = 2x + 6$, $x + 2y + 10 = 0$ and $2y + x = 0$. What type of quadrilateral is formed by the four lines?

3 On the same pair of axes, for $-4 \leqslant x \leqslant 10$ and $-6 \leqslant y \leqslant 10$ using 1 cm to 1 unit, draw the four lines $y = 3x + 4$, $y = 3x - 6$, $y = -3x$ and $y = 10 - 3x$. Name the type of quadrilateral formed by the lines.

4 Find the point where the lines $y = 2x + 2$ and $y = 4 - 2x$ meet.

5 Find the equation of the line with gradient -2 that passes through the midpoint of the line joining the points (3, 2) and (7, 4). (The midpoint can be found either from a drawing on square grid paper or from a rough sketch.)

6 Find the equation of the line that is parallel to the line joining (–1, 4) and (3, 2) and which passes through the point (4, 0).

7 On the same pair of axes, for $-2 \leqslant x \leqslant 5$ and $-5 \leqslant y \leqslant 2$, using 1 cm to 1 unit, draw the four lines $4y = x + 1$, $4y = x - 16$, $4x + y - 13 = 0$ and $4x + y + 4 = 0$. Name the type of quadrilateral formed by the lines.

Intersecting lines

When we are given the equation of a straight line we can draw a graph. Two equations give us two straight lines which usually cross (intersect).

Consider the two equations $x + y = 4$, $y = 1 + x$

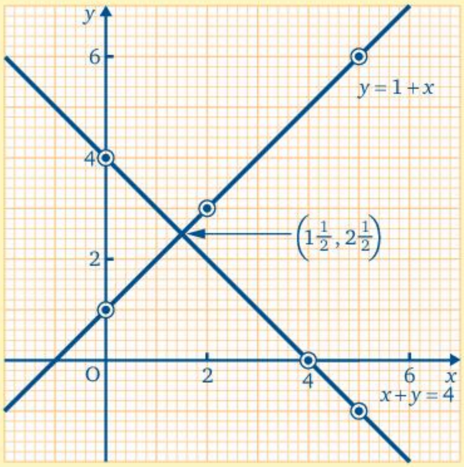
Suppose we know that the x-coordinate of the point of intersection is in the range $0 \leqslant x \leqslant 5$: this means that we can plot these lines in that range.

$x + y = 4$

x	0	4	5
y	4	0	–1

$y = 1 + x$

x	0	2	5
y	1	3	6



The point where the lines cross is called the point of intersection.

For these two lines, the coordinates of the point of intersection is $\left(1\frac{1}{2}, 2\frac{1}{2}\right)$.

This means that $x = 1\frac{1}{2}$ and $y = 2\frac{1}{2}$ satisfy both $x + y = 4$ and $y = 1 + x$.

We say that these values of x and y satisfy the equations simultaneously.

Exercise 12q

Find the coordinates of the point of intersection for each of the following pairs of equations.
Draw axes for x and y using the range shown, taking 2 cm to 1 unit.

1 $x+y=6$ $\quad 0 \leqslant x \leqslant 6, 0 \leqslant y \leqslant 6$
$y=3+x$

2 $x+y=5$ $\quad 0 \leqslant x \leqslant 6, 0 \leqslant y \leqslant 6$
$y=2x+1$

3 $y=4+x$ $\quad 0 \leqslant x \leqslant 6, 0 \leqslant y \leqslant 6$
$y=1+3x$

4 $x+y=1$ $\quad -3 \leqslant x \leqslant 2, -2 \leqslant y \leqslant 4$
$y=x+2$

5 $2x+y=3$ $\quad 0 \leqslant x \leqslant 3, -3 \leqslant y \leqslant 3$
$x+y=2\frac{1}{2}$

6 $y=5-x$ $\quad 0 \leqslant x \leqslant 5, 0 \leqslant y \leqslant 7$
$y=2+x$

7 $3x+2y=9$ $\quad 0 \leqslant x \leqslant 4, -2 \leqslant y \leqslant 5$
$2x-2y=3$

8 $2x+3y=4$ $\quad -2 \leqslant x \leqslant 2, 0 \leqslant y \leqslant 4$
$y=x+2$

9 $x+3y=6$ $\quad 0 \leqslant x \leqslant 5, 0 \leqslant y \leqslant 5$
$3x-y=6$

10 $x=2y-3$ $\quad -2 \leqslant x \leqslant 3, 0 \leqslant y \leqslant 4$
$y=2x+1$

11 Draw the graphs of $y+x=2$ and $y+x=5$ on the same set of axes. Explain why there is no point of intersection.

12 The point (2, 1) is a point on the line with equation $y=-2x+5$. The point (2, 1) is also a point on the line with equation $2y=3x-4$. Explain the significance of the point (2, 1).

Mixed exercises

Exercise 12r

1 What is the gradient of the line with equation $y=4-3x$?

2 Does the point (6, –1) lie on the line $3x+11y=8$?

3 What is the equation of the line that passes through the origin and has a gradient of –4?

4 At what point does the line with equation $y=4-x$ cut the y-axis?

5 At what points does the line with equation $x+y=6$ cut the two axes?

6 Find the gradient of the line through the points (3, 1) and (5, –2).

7 Give the equation of the line that is parallel to the line with equation $y=4+\frac{1}{2}x$ and which passes through the origin.

8 At what points does the line $\frac{x}{2}+\frac{y}{3}=1$ cut the two axes?

Exercise 12s

Select the letter that gives the correct answer.

The equation of a line is $\frac{x}{3}-\frac{y}{5}=1$. Use this information for questions **1** to **3**.

1 This line cuts the x-axis at the point where $x=$

A $\frac{5}{3}$ **B** 1 **C** 3 **D** 5

2 The gradient of this line is

A $\frac{3}{5}$ **B** $\frac{5}{3}$ **C** 3 **D** 5

3 The y-intercept of this line is

A $\frac{3}{5}$ **B** $\frac{5}{3}$ **C** 3 **D** -5

4 The equation of a line is $2x-4y=6$. Which point lies on this line?

A (5, 1) **B** (2, 4) **C** (2, 6) **D** (4, 1)

5 The gradient of the line through the points (5, –1) and (–2, 4) is

A $-\frac{5}{7}$ **B** $-\frac{7}{5}$ **C** $\frac{1}{3}$ **D** 1

Puzzle

A hunter met two shepherds. One shepherd had three small loaves while the other shepherd had five similar loaves. All the loaves were the same size. They decided to divide the eight loaves equally between the three of them. The hunter thanked the shepherds and paid them eight dinars.

How should the shepherds divide the money?

Did you know?

There is something special about the integers 1, 2 and 3. Their sum is equal to their product. There is only one other set of three integers that have this property. Can you find them?

In this chapter you have seen that...

- ✔ the equation $x = h$ gives a straight line parallel to the y-axis
- ✔ the equation $y = k$ gives a straight line parallel to the x-axis
- ✔ if you know one coordinate of a point on a line you can use the equation of the line to find the other coordinate
- ✔ the gradient of the straight line joining two points is
$$\frac{\text{the difference in the } y\text{-coordinates}}{\text{the difference in the } x\text{-coordinates}} = \frac{y_1 - y_2}{x_1 - x_2}$$
- ✔ a line that slopes up to the right has a positive gradient while a line that slopes the other way has a negative gradient; the bigger the gradient the steeper the slope
- ✔ any equation that can be arranged in the form $y = mx + c$ represents a straight line, with gradient m, that crosses the y-axis at the point $(0, c)$; c is called the y-intercept. For example, the equation $2x + 3y = 8$ can be rewritten as $y = -\frac{2}{3}x + \frac{8}{3}$ and so represents a straight line with gradient $\frac{-2}{3}$ and y-intercept $\frac{8}{3}$
- ✔ lines that have the same gradient are parallel
- ✔ two lines are perpendicular when the product of their gradients is -1
- ✔ the coordinates of the point of intersection of two lines satisfy both equations.

13 Illustrating inequalities

At the end of this chapter you should be able to...

1 represent inequalities on a diagram

2 identify the region satisfying a given inequality.

Did you know?

You know that there are an infinite number of counting numbers: 1 is the 1st, 2 is the 2nd, 3 is the third, 4 is the 4th, and so on.

Did you know that there are exactly the same number of positive even numbers? 2 is the 1st, 4 is the 2nd, 6 is the 3rd, 8 is the 4th, and so on.

You need to know...

✓ how to solve linear inequalities

✓ how to plot a graph given its equation

✓ how to find the equation of a straight line given its graph.

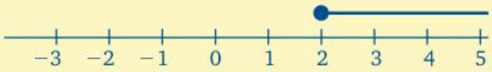
Key words

boundary line, Cartesian plane, inequality, region, *xy* plane

Illustrating inequalities in the Cartesian plane

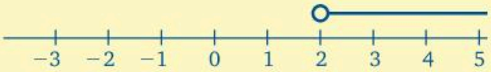
We now look at illustrating inequalities in a more visual way.

If we have the *inequality* $x \geqslant 2$, x can take any value greater than or equal to 2. This can be represented by the following diagram.



On this number line, x can take any value on the heavy part of the line including 2 itself, as indicated by the solid circle at 2.

If $x > 2$ then the diagram is as shown below.

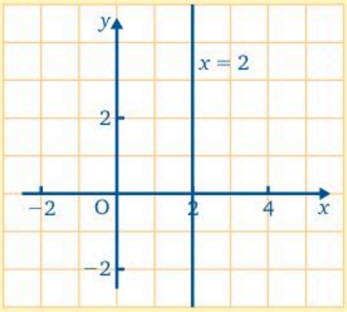


In this case, x cannot take the value 2 and this is shown by the open circle at 2.

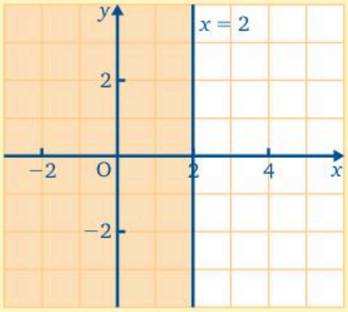
It is sometimes more useful to use two-dimensional space with x and y axes, rather than a one-dimensional line. This space is called the *Cartesian plane* or *xy plane*.

We represent $x \geqslant 2$ by the set of points whose x-coordinates are greater than or equal to 2. (y is not mentioned in the inequality so y can take any value.)

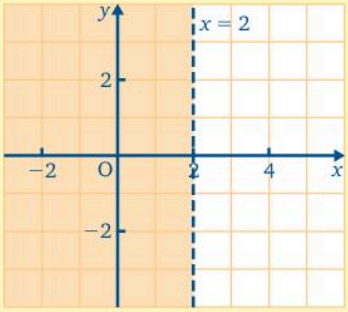
The *boundary line* represents all the points for which $x = 2$ and the region to the right contains all points with x-coordinates greater than 2.



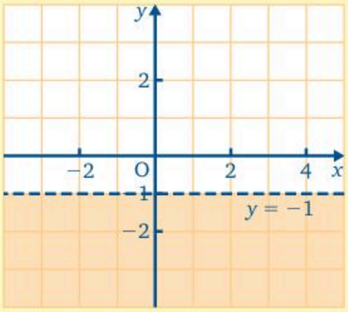
To indicate this, and to make future work easier, we use a continuous line for the boundary when it is included and we shade the *region* we do *not* want.



The inequality $x > 2$ tells us that x may not take the value 2. In this case we use a broken line for the boundary.



We can draw a similar diagram for $y > -1$



Exercise 13a

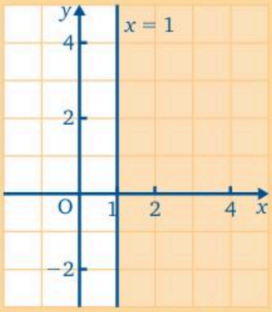
Draw diagrams to represent these inequalities:

a $x \leqslant 1$ **b** $2 < y$

a $x \leqslant 1$

The boundary line is $x = 1$ (included).

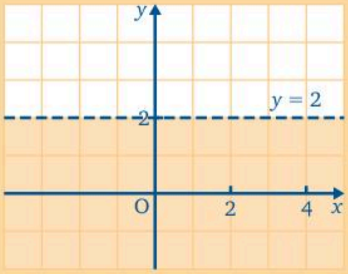
The unshaded region represents $x \leqslant 1$



b $2 < y$

The boundary line is $y = 2$ (not included).

The unshaded region represents $2 < y$

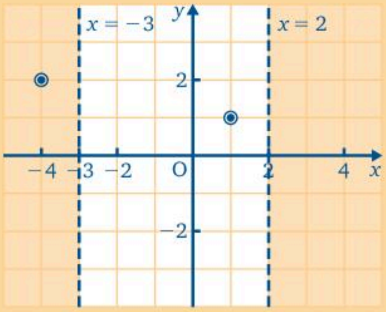


Draw diagrams to represent the following inequalities:

1	$x \geqslant 2$	**3**	$x > -1$	**5**	$x \geqslant 0$	**7**	$x \leqslant -4$
2	$y \leqslant 3$	**4**	$y < 4$	**6**	$0 > y$	**8**	$2 < x$

Draw a diagram to represent $-3 < x < 2$ and state whether or not the points (1, 1) and (–4, 2) lie in the given region.

$-3 < x < 2$ gives two inequalities, $-3 < x$ and $x < 2$, so the boundary lines are $x = -3$ and $x = 2$ (neither included). Shade the regions not wanted.



The unshaded region represents $-3 < x < 2$

Plot the points. Then you see that (–4, 2) does not lie in the given region.

(1, 1) lies in the given region.

Draw diagrams to represent the following pairs of inequalities:

9 $2 \leqslant x \leqslant 4$

10 $-3 < x < 1$

11 $-1 \leqslant y \leqslant 2$

12 $4 < y < 5$

13 $0 \leqslant x < 4$

14 $-2 < y \leqslant 3$

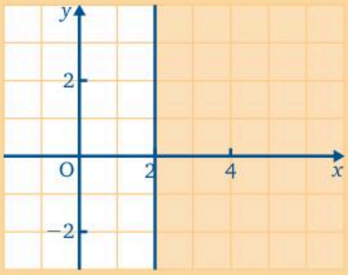
15 $-\frac{1}{2} \leqslant x \leqslant 1\frac{1}{2}$

16 $-2 \leqslant y < -1$

17 $3 \leqslant x < 5$

18 In each of the questions **9** to **11**, state whether or not the point (1, 4) lies in the unshaded region.

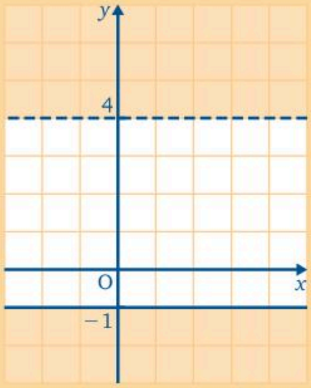
Give the inequality that defines the unshaded region.



Boundary line $x = 2$ (included)

Inequality is $x \leqslant 2$

Give the inequalities that define the unshaded region.



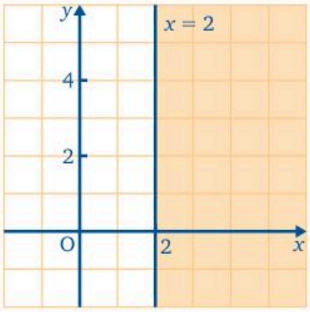
Boundary lines $y = 4$ (not included)

and $y = -1$ (included)

The inequalities are $y < 4$ and $y \geqslant -1$ or $-1 \leqslant y < 4$.

Give the inequalities that define the unshaded regions:

19



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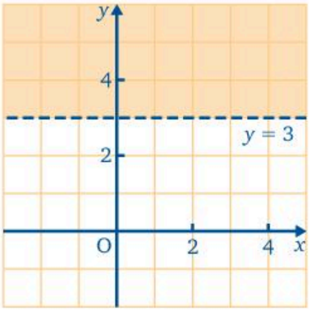

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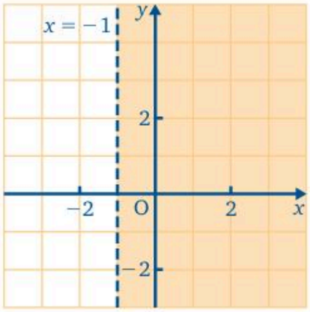

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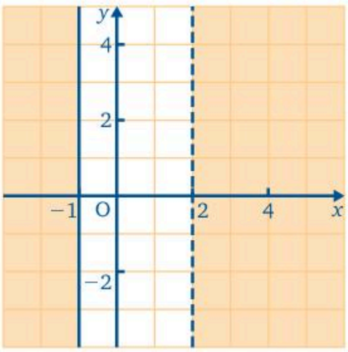

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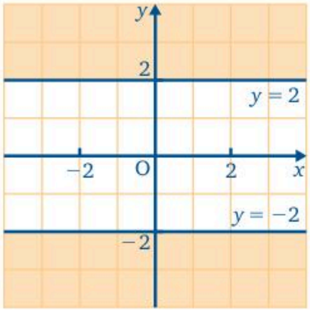

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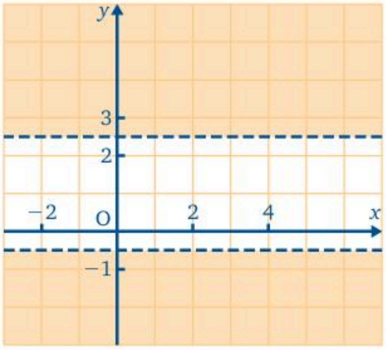
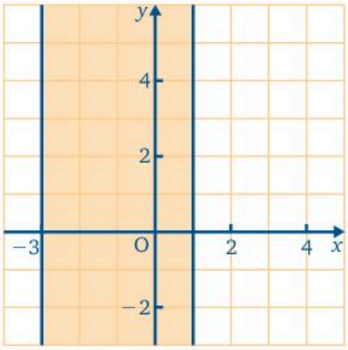


25 In each of the questions **19** to **24** state whether or not the point (2, –1) is in the unshaded region.

In questions **26** to **29** give the inequalities that define the *shaded* regions:

26



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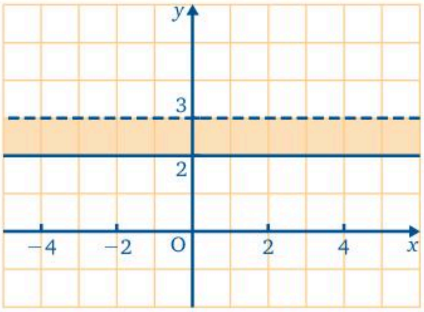

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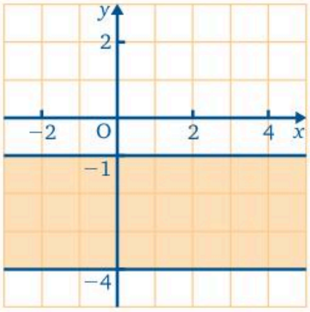

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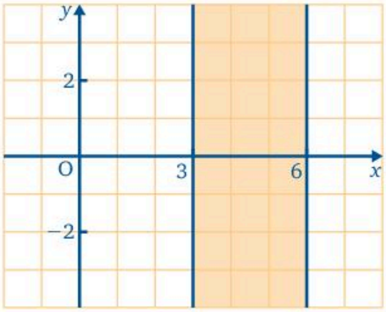


30 In each of the questions **26** to **29** state whether or not the point (0, 2) is in the shaded region.

Exercise 13b

Draw a diagram to represent the region defined by the set of inequalities $-1 \leqslant x \leqslant 2$ and $-5 \leqslant y \leqslant 0$

There are four inequalities here: $-1 \leqslant x$, $x \leqslant 2$, $-5 \leqslant y$ and $y \leqslant 0$.

The boundary lines are

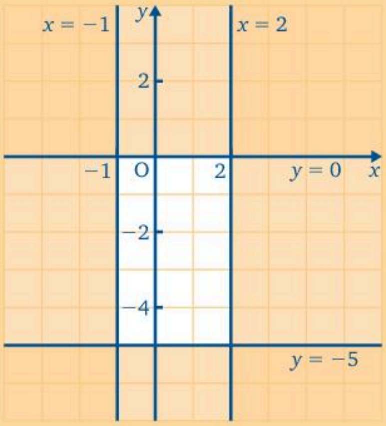
$x = -1$: for $-1 \leqslant x$, shade the region on the left of the line

$x = 2$: for $x \leqslant 2$, shade the region on the right of the line

$y = -5$: for $-5 \leqslant y$, shade the region below this line

$y = 0$: for $y \leqslant 0$, shade the region above the line

The unshaded region represents the inequalities.



Draw diagrams to represent the regions described by the following sets of inequalities. In each case, draw axes for values of x and y from -5 to 5.

1 $2 \leqslant x \leqslant 4, -1 \leqslant y \leqslant 3$

2 $-2 < x < 2, -2 < y < 2$

3 $-3 < x \leqslant 2, -1 \leqslant y$

4 $0 \leqslant x \leqslant 4, 0 \leqslant y \leqslant 3$

5 $-4 < x < 0, -2 < y < 2$

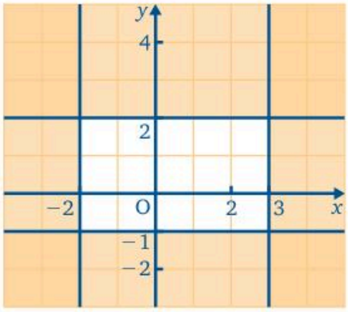
6 $-1 < x < 1, -3 < y < 1$

7 $x \geqslant 0; y \geqslant 0$

8 $x \geqslant 1, -1 \leqslant y \leqslant 2$

Give the sets of inequalities that describe the unshaded regions:

9



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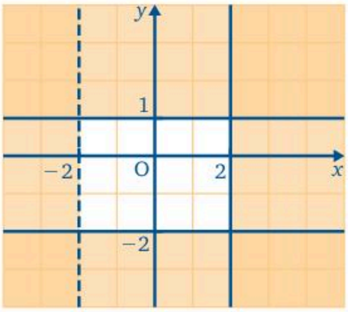
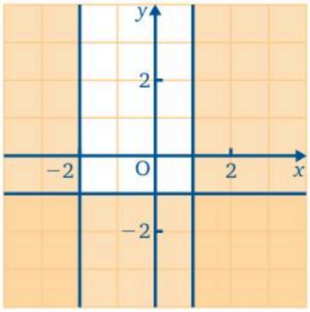


11 Is the point $(2\frac{1}{2}, 0)$ in either of the unshaded regions in questions **9** and **10**?

Give the sets of inequalities that describe the unshaded regions:

12



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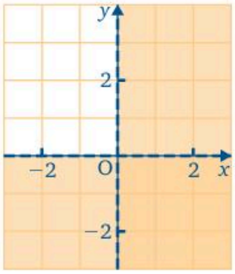

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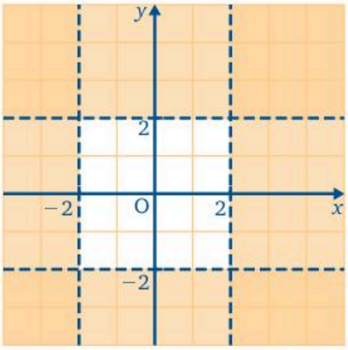

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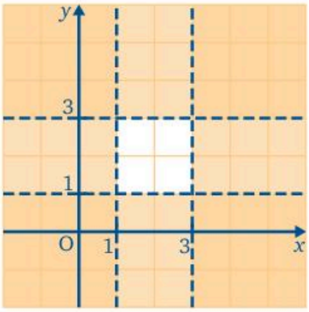


Solve each of the following inequalities and find the range of values that satisfies them both. Illustrate your solution as a region of the *xy* plane.

16 $x+3>1$ and $x+2<3$

17 $y+4 \geqslant 5$ and $y+3 \leqslant 5$

18 $2x>-4$ and $3-x>0$

19 $x>0$ and $y \leqslant 3$

20 $x+1 \geqslant 1, 2x \leqslant 6, y \geqslant 0$ and $2y \leqslant 8$

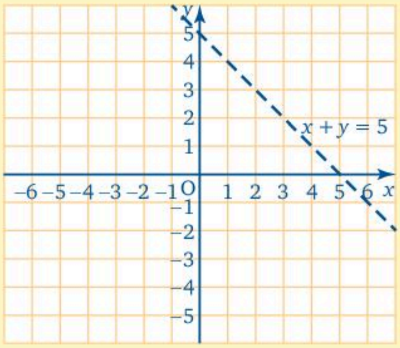
Inequalities involving two variables

The boundary line for an inequality is parallel to the x or y axis when the inequality contains either x or y but not both.

Now consider an inequality containing both x and y, for example $x + y < 5$.

The boundary line is $x + y = 5$ and as it is not included in the region it is shown as a broken line.

The boundary line divides the xy plane into two regions, one on each side of the line.



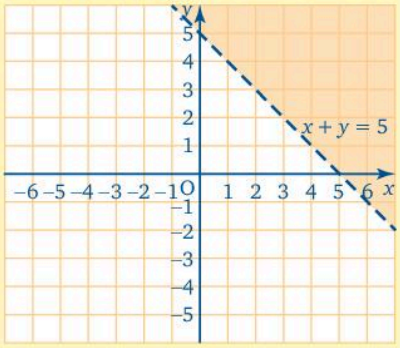
We need to decide which of the two regions we require, so we test a point in one of the regions.

The origin (0, 0) is in the region below the line.

Substituting $x = 0$ and $y = 0$ in the inequality gives

$$0 + 0 < 5 \qquad \text{or} \qquad 0 < 5$$

This is a true statement so the region we require is below the line.



To check, test a point in the region above the line, for example (4, 4). Substituting $x = 4$ and $y = 4$ gives

$$8 < 5$$

which is false. Thus the region above the line is not the region we need.

As before we shade the unwanted region.

Exercise 13c

Find the inequality defining the unshaded region.

The boundary line is $y - x = 2$.

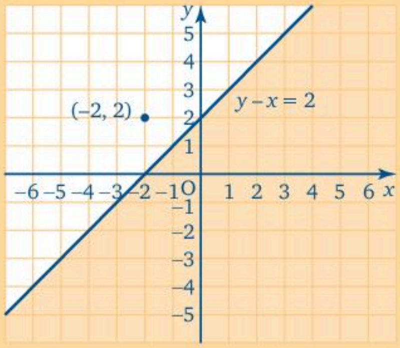
Because the boundary line is solid, it is included.

Test the point (–2, 2), which is in the unshaded region.

When $x = -2$ and $y = 2$, $y - x = 2 - (-2) = 4$

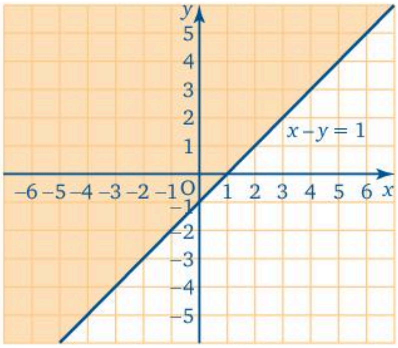
$4 > 2$, so $y - x > 2$ is the unshaded region.

As the boundary line is solid, the inequality that defines the unshaded region is $y - x \geqslant 2$.



Find the inequalities that define the unshaded regions.

1



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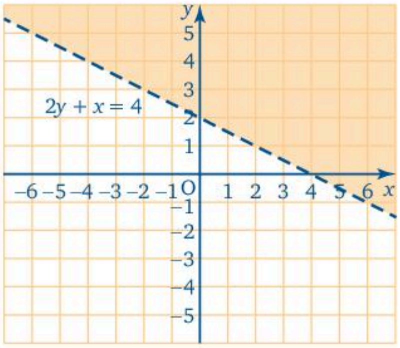

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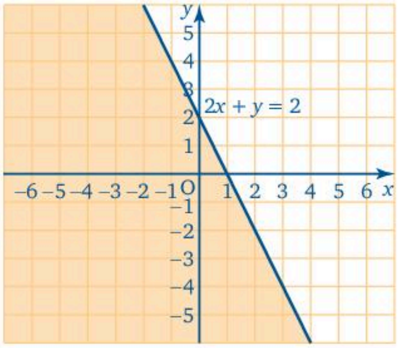

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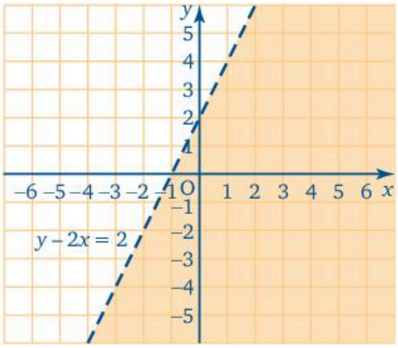

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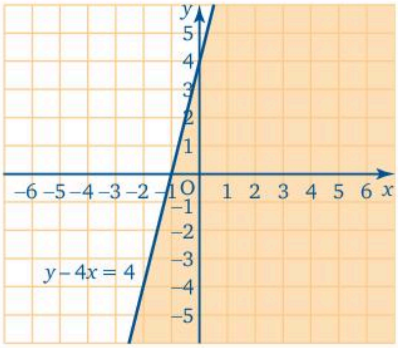

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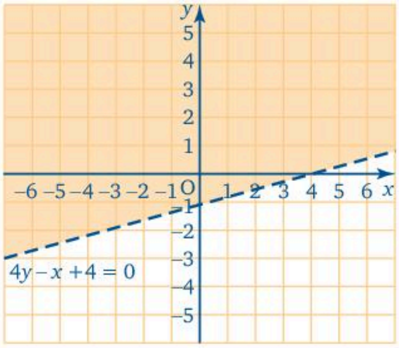
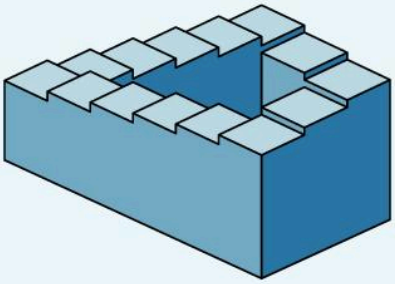


Did you know?

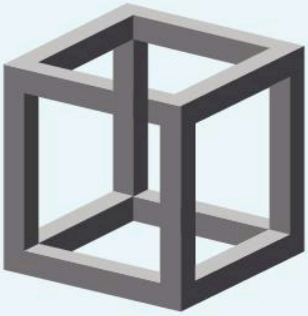


This 'impossible staircase' was created in 1958 by the English psychologist Lionel Penrose and his son Roger Penrose, a mathematician. Moving around the staircase one way, the stairs seem to go up endlessly, while moving the other way they seem to go down forever.

The Penrose drawing inspired the Dutch graphic artist M.C. Escher to draw his own optical illusion of an impossible staircase, called **Ascending and Descending**, which he produced in 1960.

Escher drew many impossible versions of everyday objects. The impossible cube is one of his well-known drawings.

You can find out more about the Penrose drawing and Escher's work online.



In this chapter you have seen that…

- ✔ inequalities such as $x > 2$ or $y \leqslant 3$ can be shown as a region of the xy plane
- ✔ when the inequality is 'greater than' or 'less than', the boundary line is broken
- ✔ when the inequality is 'greater than or equal to' or 'less than or equal to', the boundary line is solid
- ✔ when the boundary line is a sloping line, use a point not on the line to determine if it lies in the required region.

14 Constructions

At the end of this chapter you should be able to...

1 construct a triangle given

either two sides and the angle between the two sides

or one side and two angles

or three sides

2 construct angles of 30°, 45°, 60° and 90°

3 bisect an angle

4 construct a triangle using only a ruler and a pair of compasses

5 construct various quadrilaterals.

Did you know?

Isaac Newton (1642–1727) wrote:

The description of right lines and circles, upon which geometry is founded, belongs to mechanics. Geometry does not teach us to draw these lines, but requires them to be drawn.

You need to know...

✔ how to use a protractor

✔ how to use a pair of compasses

✔ the properties of the special quadrilaterals.

Key words

bisect, construct, equilateral triangle, isosceles triangle, line of symmetry, protractor, radius

Puzzle

There are 5 Saturdays, 5 Sundays and 5 Mondays in January.

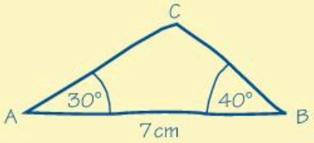
On what day of the week will 1 February fall?

Constructing triangles given one side and two angles

If we are given enough information about a triangle we can make an accurate drawing of that triangle. The mathematical word for 'make an accurate drawing of' is *construct*.

For example: construct $\triangle ABC$ in which $AB = 7\,\text{cm}$, $\hat{A} = 30^\circ$ and $\hat{B} = 40^\circ$.

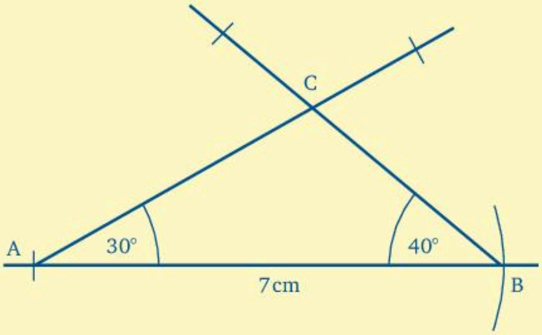
First make a rough sketch of $\triangle ABC$ and put all the given measurements in your sketch.



Next draw the line AB making it 7 cm long. Label the ends.

Then use your *protractor* to make an angle of 30° at A.

Next make an angle of 40° at B. If necessary extend the arms of the angles until they cross; this is the point C.



We can calculate $\hat{C}$ because $\hat{A} + \hat{B} + \hat{C} = 180^\circ$ so $\hat{C} = 110^\circ$. Now as a check we can measure $\hat{C}$ in our construction.

You can also use the following fact to check your construction.

In any triangle, the longest side is opposite the largest angle.

Exercise 14a

Construct the following triangles; calculate the third angle in each triangle and then measure this angle to check the accuracy of your construction.

1 $\triangle ABC$ in which $AB = 8\text{ cm}$, $\hat{A} = 50°$, $\hat{B} = 20°$

2 $\triangle PQR$ in which $QR = 5\text{ cm}$, $\hat{Q} = 30°$, $\hat{R} = 90°$

3 $\triangle DEF$ in which $EF = 6\text{ cm}$, $\hat{E} = 50°$, $\hat{F} = 60°$

4 $\triangle XYZ$ in which $YZ = 5.5\text{ cm}$, $\hat{Y} = 100°$, $\hat{Z} = 40°$

5 $\triangle UVW$ in which $\hat{V} = 35°$, $VW = 5.5\text{ cm}$, $\hat{W} = 75°$

6 $\triangle FGH$ in which $\hat{F} = 55°$, $\hat{G} = 70°$, $FG = 4.5\text{ cm}$

7 $\triangle KLM$ in which $KM = 10\text{ cm}$, $\hat{K} = 45°$, $\hat{M} = 45°$

8 $\triangle BCD$ in which $\hat{B} = 100°$, $BC = 8.5\text{ cm}$, $\hat{C} = 45°$

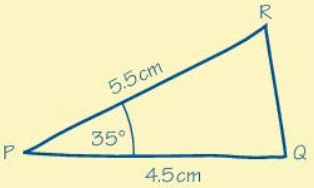
9 $\triangle GHI$ in which $GI = 7\text{ cm}$, $\hat{G} = 25°$, $\hat{I} = 45°$

10 $\triangle JKL$ in which $\hat{J} = 50°$, $JL = 6.5\text{ cm}$, $\hat{L} = 35°$

Start by drawing a rough sketch of the triangle and then put all the given measurements on your sketch. Make sure your pencil is *sharp*.

Constructing triangles given two sides and the angle between the two sides

To construct $\triangle PQR$ in which $PQ = 4.5\text{ cm}$, $PR = 5.5\text{ cm}$ and $\hat{P} = 35°$, first draw a rough sketch of $\triangle PQR$ and put in all the measurements that you are given.



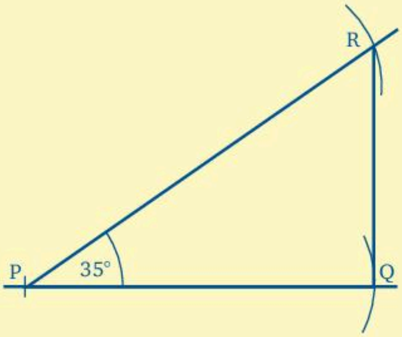
Draw one of the sides whose length you know; we will draw PQ.

Now using your protractor make an angle of 35° at P. Make the arm of the angle quite long.

Next use your compasses to measure the length of PR on your ruler.

Then with the point of your compasses at P, draw an arc to cut the arm of the angle. This is the point R.

Now join R and Q.



Exercise 14b

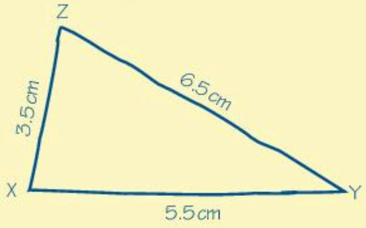
Construct each of the following triangles and measure the third side and the other two angles:

1 $\triangle ABC$ in which AB = 5.5 cm, BC = 6.5 cm, $\hat{B} = 40°$
2 $\triangle PQR$ in which PQ = 6 cm, QR = 8 cm, $\hat{Q} = 35°$
3 $\triangle XYZ$ in which XZ = 4.5 cm, YZ = 6.5 cm, $\hat{Z} = 70°$
4 $\triangle DEF$ in which DE = 6 cm, $\hat{E} = 50°$, EF = 11 cm
5 $\triangle HJK$ in which HK = 4.2 cm, $\hat{H} = 45°$, HJ = 5.3 cm
6 $\triangle ABC$ in which AC = 6.3 cm, $\hat{C} = 48°$, CB = 5.1 cm
7 $\triangle XYZ$ in which $\hat{Y} = 65°$, XY = 3.8 cm, YZ = 4.2 cm
8 $\triangle PQR$ in which $\hat{R} = 52°$, RQ = 5.8 cm, PR = 7 cm
9 $\triangle LMN$ in which $\hat{N} = 73°$, LN = 4.1 cm, MN = 6.3 cm
10 $\triangle ABC$ in which AC = 5.2 cm, BA = 7.3 cm, $\hat{A} = 56°$

Remember to start with a rough sketch with all the given measurements on it. Make sure that your pencil is *sharp*.

Constructing triangles given the lengths of three sides

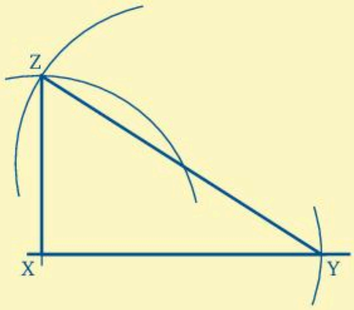
To construct, $\triangle XYZ$, in which, XY = 5.5 cm, XZ = 3.5 cm, YZ = 6.5 cm, first draw a rough sketch of the triangle and put in all the given measurements.



Next draw one side; we will draw XY.

Then with your compasses measure the length of XZ from your ruler. With the point of your compasses at X draw a wide arc.

Next use your compasses to measure the length of YZ from your ruler. Then with your compasses point at Y draw another large arc to cut the first arc. Where the two arcs cross is the point Z. Join ZX and ZY.



Exercise 14c

Construct the following triangles:

1 △ABC in which AB = 7 cm, BC = 8 cm, AC = 12 cm

2 △PQR in which PQ = 4.5 cm, PR = 6 cm, QR = 8 cm

3 △XYZ in which XZ = 10.4 cm, XY = 6 cm, YZ = 9.6 cm

4 △DEF in which DE = 8 cm, DF = 10 cm, EF = 6 cm

5 △ABC in which AB = 7.3 cm, BC = 6.1 cm, AC = 4.7 cm

6 △DEF in which DE = 10.4 cm, EF = 7.4 cm, DF = 8.2 cm

7 △PQR in which PQ = 8.8 cm, QR = 6.6 cm, PR = 11 cm

8 △LMN in which LN = 7 cm, NM = 5.3 cm, LM = 6.1 cm

9 △XYZ in which XY = 12 cm, YZ = 5 cm, XZ = 13 cm

10 △ABC in which AB = 5.5 cm, BC = 6 cm, AC = 6.5 cm

Constructing angles without using a protractor

Some angles can be made without using a protractor: one such angle is 60°.

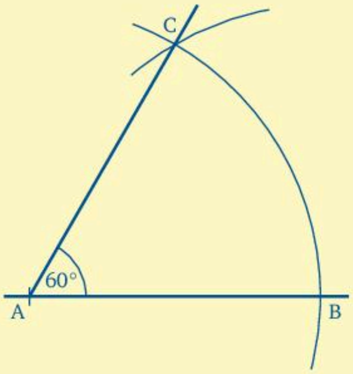
Every *equilateral triangle*, whatever its size, has three angles of 60°. To make an angle of 60° we construct an equilateral triangle but do not draw the third side.

To construct an angle of 60°

Start by drawing a straight line and marking a point, A, near one end of the line.

Next open your compasses to a *radius* of about 4 cm (this will be the length of the sides of your equilateral triangle).

With the point of your compasses on A, draw an arc to cut the line at B, continuing the arc above the line.



Move the point to B and draw an arc above the line to cut the first arc at C.

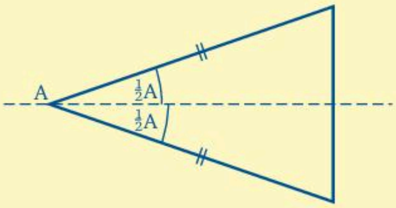
Draw a line through A and C. Then $\hat{A}$ is 60°.

△ABC is the equilateral triangle so *be careful not to alter the radius on your compasses during this construction.* Why is △ABC always equilateral?

Bisecting angles

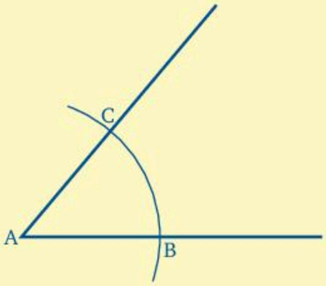
Bisect means 'cut exactly in half'.

The construction for bisecting an angle makes use of the fact that, in an *isosceles triangle*, the *line of symmetry* cuts $\hat{A}$ in half.



To bisect $\hat{A}$, open your compasses to a radius of about 6 cm.

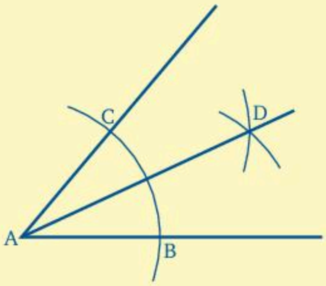
With the point on A, draw an arc to cut both arms of $\hat{A}$ at B and C. (If we joined BC, △ABC would be isosceles.)



With the point on B, draw an arc between the arms of $\hat{A}$. Move the point to C (being careful not to change the radius) and draw an arc to cut the other arc at D.

Join AD.

The line AD then bisects $\hat{A}$.



Exercise 14d

1 Construct an angle of 60°.

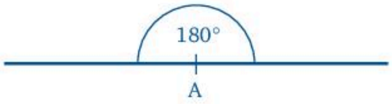
2 Draw an angle of about 50°. Bisect this angle. Measure both halves of your angle.

Make sure that your pencil is sharp.

3 Construct an angle of 60°. Now bisect this angle. What size should each new angle be? Measure both of them.

4 Use what you learnt from the last question to construct an angle of 30°.

5 Draw a straight line and mark a point A near the middle.

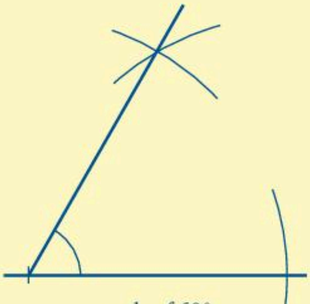


You now have an angle of 180° at A.

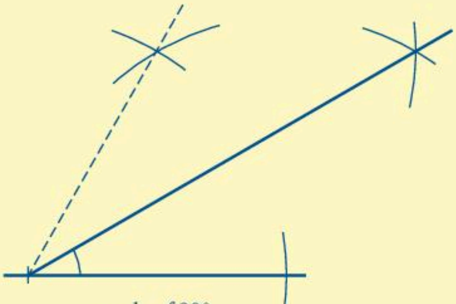
Now bisect the angle of 180° you have drawn. What is the size of each new angle? Measure each of them.

Construction of angles of 60°, 30°, 90°, 45°

You constructed these angles in the last exercise. Here is a summary of these constructions.

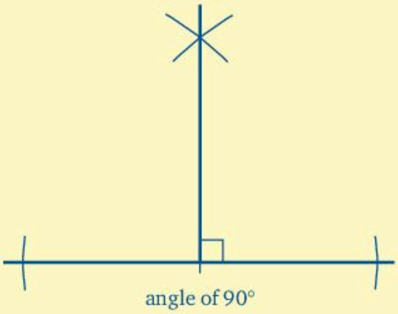


angle of 60°

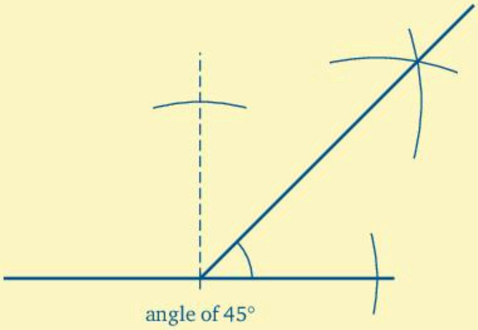


angle of 30°

(This is bisecting a 60° angle.)



angle of 90°



angle of 45°

(This is bisecting a 90° angle.)

Exercise 14e

Construct the following triangles:

1 $\triangle ABC$ in which $AB = 10\,cm$, $\hat{A} = 60°$ and $AC = 8\,cm$

2 $\triangle DEF$ in which $DE = 9\,cm$, $\hat{D} = 45°$ and $\hat{E} = 60°$

3 $\triangle PQR$ in which $PQ = 8\,cm$, $\hat{Q} = 90°$ and $QR = 6\,cm$

4 $\triangle ABC$ in which $AB = 8.5\,cm$, $\hat{B} = 30°$ and $BC = 6.5\,cm$

5 $\triangle DEF$ in which $DF = 9.4\,cm$, $\hat{D} = 90°$ and $\hat{F} = 30°$

6 $\triangle XYZ$ in which $XY = 11\,cm$, $\hat{X} = 45°$ and $\hat{Y} = 30°$

7 $\triangle ABC$ in which $AC = 9.2\,cm$, $\hat{A} = \hat{C} = 60°$

8 $\triangle DEF$ in which $DE = 4.8\,cm$, $\hat{E} = 90°$ and $EF = 5.4\,cm$

9 $\triangle PQR$ in which $PQ = 6\,cm$, $\hat{Q} = 120°$ and $QR = 5.6\,cm$

10 $\triangle ABC$ in which $AB = 7.5\,cm$, $\hat{B} = 90°$ and $\hat{A} = 30°$

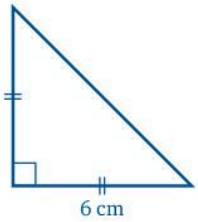
We can use the work we have done so far to include constructing special quadrilaterals.

Exercise 14f

Construct the following figures using only a ruler and a pair of compasses:

Make sure that your pencil is sharp.

1



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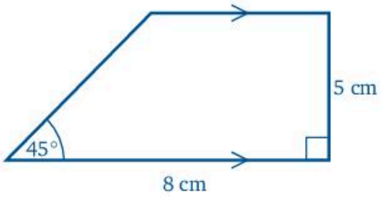

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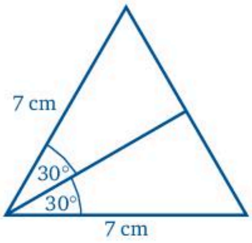

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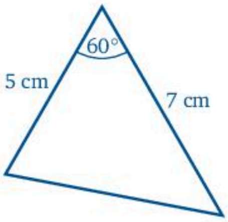

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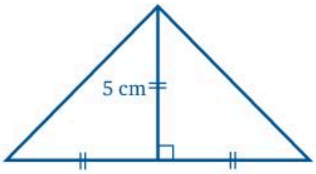

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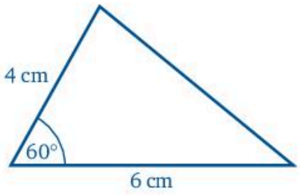

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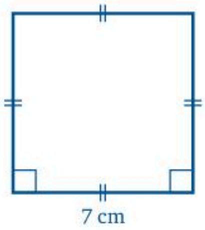

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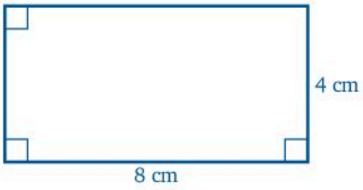

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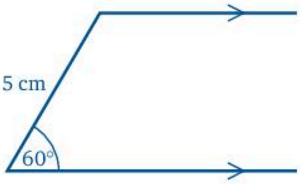

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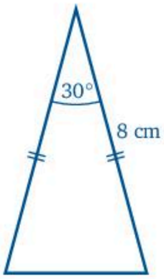


For questions **11** to **15**, draw a rough sketch before starting the construction.

11 Draw a line, AB, 12 cm long. Construct an angle of 60° at A. Construct an angle of 30° at B. Label with C the point where the arms of $\hat{A}$ and $\hat{B}$ cross. What size should $\hat{C}$ be? Measure $\hat{C}$ as a check on your construction.

12 Construct a triangle, ABC, in which AB is 10 cm long, $\hat{A}$ is 90° and AC is 10 cm long. What size should $\hat{C}$ and $\hat{B}$ be? Measure $\hat{C}$ and $\hat{B}$ as a check.

13 Construct a square, ABCD, with a side of 6 cm.

14 Construct a quadrilateral, ABCD, in which AB is 12 cm, $\hat{A}$ is 60°, AD is 6 cm, $\hat{B}$ is 60° and BC is 6 cm. What can you say about the lines AB and DC?

15 Construct an angle of 120°. Label it BAC (so that A is the vertex and B and C are at the ends of the arms). At C, construct an angle of 60° so that $\hat{C}$ and $\hat{A}$ are on the same side of AC. You have constructed a pair of parallel lines; mark them and devise your own check.

Mixed exercise

Exercise 14g

1 Construct triangle ABC in which AB = 7.5 cm, BC = 8.3 cm and CA = 5.6 cm.

2 Construct ABC in which AB = 9.2 cm, $\hat{A} = 30°$ and $\hat{B} = 60°$.

Measure and record

a the length of AC **b** the length of BC **c** the size of $\hat{C}$.

3 Construct triangle DEF in which DF = 8.7 cm, $\hat{D} = 45°$ and $\hat{F} = 30°$.

4 Construct triangle ABC in which AB = 10.4 cm, $\hat{A} = 45°$ and AC = 8.5 cm.

You may use a protractor to measure the angles in questions **5** and **6**.

5 Construct triangle XYZ such that XY = 9.2 cm, $\hat{X} = 56°$ and $\hat{Y} = 34°$.

Measure and record the size of angle Z. Is it what you expect? Justify your answer.

6 Construct triangle LMN in which LM = 6.8 cm, LN = 8.2 cm and $\hat{L} = 76°$.

Measure and record the size of $\hat{M}$ and $\hat{N}$. Which angle is the greater?

Is this what you expected? Justify your answer.

7 Construct a quadrilateral ABCD in which $AD = 12\text{ cm}$, $\hat{A} = 60°$, $AB = 9\text{ cm}$, $\hat{D} = 90°$ and $\hat{B} = 120°$.

Measure and record the length of BC.

How are the lines AD and BC related?

8 **a** Construct triangle ABC in which $A\hat{B}C = 90°$ and $AB = BC = 6.5\text{ cm}$.

b Now construct BD, the bisector of angle ABC.

c Measure AD and CD. How do their lengths compare? Is this what you expected? Give a reason for your answer.

9 Construct a quadrilateral ABCD in which $AB = 8\text{ cm}$, $A\hat{B}C = 45°$, $B\hat{A}D = 90°$, $AD = 3.8\text{ cm}$ and $B\hat{A}C = 60°$.

Measure and record the length of **a** BC **b** CD.

Puzzle

When a car travels 30 000 km it wears out 6 tyres. Each tyre is used for exactly the same distance. For how many kilometres does each tyre last?

In this chapter you have seen that...

- ✔ you can construct a triangle using a ruler and protractor given

  one side and two angles

  or two sides and the angle between the two sides

  or three sides
- ✔ you can bisect an angle
- ✔ you can construct angles of 60°, 30°, 90° and 45°
- ✔ you can construct various triangles using only a ruler and a pair of compasses
- ✔ you can construct various quadrilaterals using only a ruler and a pair of compasses.

15 Sets

At the end of this chapter you should be able to...

1. use correctly the symbols $\in$, $\notin$, $\subset$, $\subseteq$, $\cup$, $\cap$, $\varnothing$, { } and U
2. classify sets as finite or infinite
3. determine when two sets are equal
4. identify empty sets and use the correct symbol for a set
5. find all the possible subsets of a set with up to four elements
6. give a suitable universal set for a given set
7. find the union or intersection of sets
8. draw Venn diagrams to show the union or intersection of sets
9. solve simple problems using Venn diagrams.

Did you know?

Venn diagrams are named after John Venn (1834–1923), an Englishman born in Yorkshire who studied logic at Cambridge University.

You need to know...

- ✓ the meaning of: equal sets, equivalent sets, empty or null set, finite and infinite sets, intersection and union of two sets, subset, proper subset, universal set
- ✓ the meaning of the symbols $\in$, $\notin$, $\subset$, $\subseteq$, $\cup$, $\cap$, $\varnothing$, { } and U
- ✓ how to draw a Venn diagram to show the union or intersection of sets.

Key words

disjoint sets, element, empty set, equal set, equivalent set, finite set, infinite set, intersection of sets, member, null set, proper subset, set, subset, union of sets, universal set, Venn diagram, the symbols $\in$, $\notin$, $\subset$, $\subseteq$, $\cup$, $\cap$, $\varnothing$, { } and U

Set notation

A *set* is a collection of things having something in common.

Things that belong to a set are called *members* or *elements*. When written down, these members or elements are usually separated by commas and enclosed by curly brackets or braces.

Instead of writing 'the set of Jamaican reggae artists', we write {Jamaican reggae artists}.

The symbol $\in$ means 'is a member of' so that 'History is a member of the set of school subjects' may be written History $\in$ {school subjects}.

Similarly the symbol $\notin$ means 'is not a member of'.

'Elm is not a breed of dog' may be written Elm $\notin$ {breeds of dogs}.

Exercise 15a

1 Use the correct set notation to write the following sets:

a the set of teachers in my school **b** the set of books I have read.

2 Write two members from each of the sets given in question **1**.

Describe in words the set {2, 4, 6, 8, 10, 12}.

{2, 4, 6, 8, 10, 12} = {even numbers from 2 to 12 inclusive}

3 Describe in words the given sets:

a {1, 3, 5, 7, 9}

b {Monday, Tuesday, Wednesday, Thursday, Friday}

Note that these descriptions must be very precise,
e.g. it is correct to say
{1, 2, 3, 4, 5} = {first five natural numbers}
but it is incorrect to say
{alsation, boxer} = {breeds of dogs}
because there are many more breeds than the two that are given.

4 Describe a set that includes the given members of the following sets and state another member of each.

a Hungary, Poland, Slovakia, Bulgaria **b** 10, 20, 30, 40, 50

Write each of the following statements in set notation.

5 John is a member of the set of boys' names.

6 English is a member of the set of school subjects.

7 June is not a day of the week.

8 Monday is not a member of the set of domestic furniture.

State whether the following statements are true or false.

9 $32 \in$ {odd numbers}

10 Washington $\in$ {American states}

11 Washington $\in$ {capital cities}

12 $1 \notin$ {prime numbers}

Finite, infinite, empty, equal and equivalent sets

When we can write down all the members of a set, the set is called a *finite set*, e.g. $A =$ {days of the week} is a finite set because there are seven days in a week. If we denote the number of members in the set A by $n(A)$, then $n(A) = 7$.

Similarly if $B = \{5, 10, 15, 20, 25, 30\}$, $n(B) = 6$
and if $C =$ {letters in the alphabet}, $n(C) = 26$.

If there is no limit to the number of members in a set, the set is called an *infinite set*, e.g. {even numbers} is an infinite set because we can go on adding 2 time and time again.

A set that has no members is called an *empty set* or *null* set. It is denoted by $\varnothing$ or { }.

Two sets are *equal* if they contain exactly the same elements, not necessarily in the same order,

e.g. if $A =$ {prime numbers greater than 2 but less than 9}

and $B =$ {odd numbers between 2 and 8}

then $A = B$, i.e. they are equal sets.

Sets are *equivalent* when they contain the same number of elements,

e.g. $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c, d\}$ are equivalent sets.

Exercise 15b

Are the following sets finite or infinite sets?

1 {odd numbers}

2 {the number of leaves on a particular tree}

3 {trees more than 60 m tall}

4 {the decimal numbers between 0 and 1}

Find the number of elements in each of the following sets.

5 $A = \{\text{vowels}\}$

6 $C = \{\text{prime numbers less than } 20\}$

If $n(A)$ is the number of elements in set A, find $n(A)$ for each of the following sets.

7 $A = \{5, 10, 15, 20, 25, 30\}$

8 $A = \{\text{the consonants}\}$

9 $A = \{\text{players in a soccer team}\}$

State whether or not the following sets are equal.

10 $A = \{8, 4, 2, 12\}$, $B = \{2, 4, 6, 8\}$

11 $C = \{\text{letters of the alphabet except consonants}\}$, $D = \{\text{i, o, u, a, e}\}$

12 $X = \{\text{integers between 2 and 14 that are exactly divisible by 3 or 4}\}$,

$Y = \{3, 4, 6, 8, 9, 12\}$

Determine whether or not the following sets are null sets.

13 {animals that have travelled in space}

14 {multiples of 11 between 12 and 20}

15 {prime numbers less than 2}

16 {consonants}

Universal sets

Think of the set {pupils in my class}.

With this group of pupils in mind we might well think of several other sets,

i.e. $A = \{\text{pupils wearing spectacles}\}$

$B = \{\text{pupils wearing brown shoes}\}$

$C = \{\text{pupils with long hair}\}$

$D = \{\text{pupils more than 150 cm tall}\}$

We call the set {pupils in my class} a *universal set* for the sets A, B, C and D.

All the members of A, B, C and D must be found in a universal set, but a universal set may contain other members as well.

We denote a universal set by U or $\mathscr{E}$.

{pupils in my year at school} or {pupils in my school} would also be suitable universal sets for the sets A, B, C and D given above.

Exercise 15c

Suggest a universal set for $\{5, 10, 15, 20\}$ and $\{6, 18, 24\}$.

$U = \{\text{integers}\}$

In questions **1** to **3** suggest a universal set for:

1 {knife, teaspoon}, {fork, spoon}

2 $\{10, 20, 30, 40\}, \{15, 25, 35\}$

3 $\{8, 12, 16, 20, 24\}, \{9, 12, 15, 18, 21, 24\}$

4 $U = \{\text{integers from 1 to 20 inclusive}\}$

$A = \{\text{prime numbers}\}$ $B = \{\text{multiples of 3}\}$

Find $n(A)$ and $n(B)$.

5 $U = \{\text{positive integers less than 16}\}$

$A = \{\text{factors of 12}\}$ $B = \{\text{prime numbers}\}$

$C = \{\text{integers that are exactly divisible by 2 and by 3}\}$

List the sets A, B and C.

6 $U = \{x, \text{ a whole number, such that } 4 \leqslant x \leqslant 20\}$

$A = \{\text{multiples of 5}\}$ $B = \{\text{multiples of 7}\}$ $C = \{\text{multiples of 4}\}$

Find $n(A)$, $n(B)$ and $n(C)$.

Subsets

If all the members of a set B are also members of a set A, then the set B is called a *subset* of the set A. This is written $B \subseteq A$. We use the symbol $\subseteq$ rather than $\subset$ if we don't know whether B could be equal to A.

Subsets that do not contain all the members of A are called *proper subsets*. If B is such a subset we write $B \subset A$.

Consider the set $A = \{q\}$. $n(A) = 1$. Set A has 2 possible subsets, $\{\ \}$ and $\{q\}$.

Consider the set $B = \{a, b\}$. $n(B) = 2$. We can list all the possible subsets as:
$\{\ \}, \{a\}, \{b\}, \{a, b\}$

Now consider the set $C = \{x, y, z\}$. $n(C) = 3$. The possible subsets of C are:
$\{\ \}, \{x\}, \{y\}, \{z\}, \{x, y\}, \{x, z\}, \{y, z\}, \{x, y, z\}$

So for a set with 1 element there are 2 possible subsets, i.e. 2^1 subsets

for a set with 2 elements there are 4 possible subsets, i.e. 2^2 subsets

and for a set with 3 elements there are 8 possible subsets, i.e. 2^3 subsets.

Exercise 15d

If A = {David, Edward, Fritz, Harry}, write down all the subsets of A with exactly three members.

The subsets of A with exactly three members are

{David, Edward, Fritz}

{David, Edward, Harry}

{David, Fritz, Harry}

{Edward, Fritz, Harry}

1 If A = {John, Joy, Peter, Anora, Tissha}, write down all the subsets of A with exactly two female members.

2 If N = {positive integers from 1 to 15 inclusive}, list the following subsets of N:

A = {odd numbers from 1 to 15 inclusive}

B = {prime numbers less than 15}

C = {multiples of 3 that are less than or equal to 15}

Do sets A and B have any element in common?

3 If A = {even numbers from 2 to 20 inclusive}, list the following subsets of A:

B = {multiples of 3}

C = {prime numbers}

D = {numbers greater than 12}

4 If X = {Ava, Betty}, how many possible subsets are there? List them.

How many of these are proper subsets?

5 If A = {2, 4, 6}, how many proper subsets are there? List them.

6 If C = {p, q, r, s}, determine the number of possible subsets. List them.

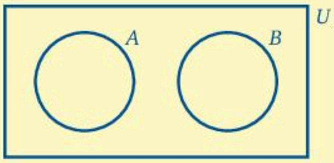
Make sure you list the subsets systematically, so you do not miss any. One way to do this is to start with the empty set, then the subsets with one element, and so on.

Puzzle

During the day, because of the heat, the pendulum of a clock lengthens, causing it to gain half a minute during daylight hours. During the night the pendulum cools, causing it to lose one-third of a minute. The clock shows the correct time at dawn on the first of August. When will it be five minutes fast?

Venn diagrams

In the *Venn diagram* the universal set (U) is usually represented by a rectangle and the subsets of the universal set by circles within the rectangle.



If U = {families}, A = {families with one car} and B = {families with more than one car} the Venn diagram would be as shown.

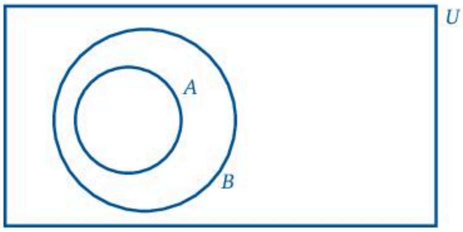
No family can have just one car and, at the same time, more than one car,

i.e. A and B have no members in common.

Two such sets are called *disjoint sets*.

Exercise 15e

1



You are given the following information:
U = {pupils in my year}

A = {pupils in my class who are my friends}

B = {pupils in my class}

a Copy the Venn diagram and shade the region that shows the pupils in my class that are not my friends.

b Are all my friends in my class?

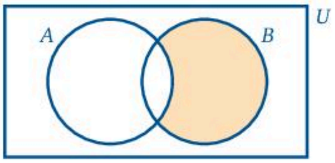
In questions **2** to **6**:
U = {pupils who attend my school}

A = {pupils who like coming to my school}

B = {pupils who are my friends}

In each case describe, in words, the shaded area.

2



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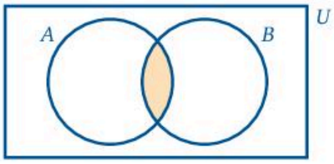

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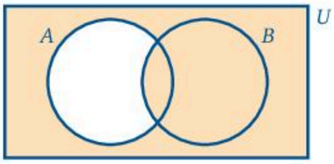

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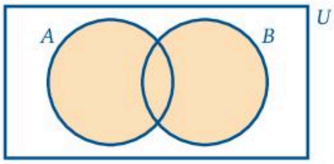

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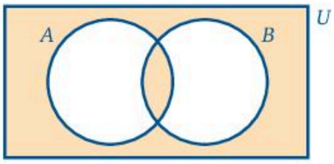


Union and intersection of two sets

If we write down the set of all the members that are in either set A or set B we have what we call the *union* of the sets A and B.

The union of A and B is written $A \cup B$.

The set of all the members that are members both of set A and of set B is called the *intersection* of A and B, and is written $A \cap B$.

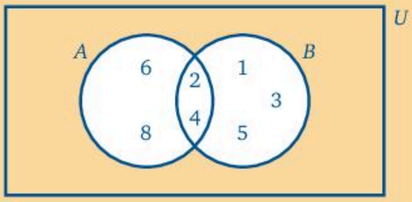
Exercise 15f

$U = \{1, 2, 3, 4, 5, 6, 7, 8\}$

If $A = \{2, 4, 6, 8\}$ and $B = \{1, 2, 3, 4, 5\}$ find $A \cup B$ illustrating these sets on a Venn diagram.

$A \cup B = \{1, 2, 3, 4, 5, 6, 8\}$

We could show this on a Venn diagram as follows.



The white area represents the set $A \cup B$.

In questions **1** to **3** find the union of the two given sets, illustrating your answer on a Venn diagram.

1 $U = \{$girls' names beginning with the letter J$\}$

$A = \{$Janet, Jill, Jamila$\}$ $B = \{$Judith, Janet, Jacky$\}$

2 $U = \{$positive integers from 1 to 16 inclusive$\}$

$X = \{4, 8, 12, 16\}$ $Y = \{2, 6, 10, 14, 16\}$

3 $U = \{$letters of the alphabet$\}$

$P = \{$letters in the word GEOMETRY$\}$

$Q = \{$letters in the word TRIGONOMETRY$\}$

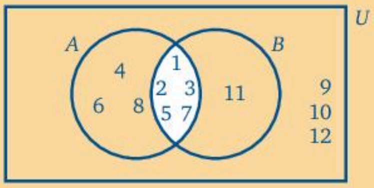
4 Draw suitable Venn diagrams to show the unions of the following sets, and describe these unions in words as simply as possible.

- **a** $U = \{$quadrilaterals$\}$ $A = \{$parallelograms$\}$ $B = \{$trapeziums$\}$
- **b** $U = \{$angles$\}$ $P = \{$obtuse angles$\}$ $Q = \{$reflex angles$\}$

$U = \{$integers from 1 to 12 inclusive$\}$

If $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $B = \{1, 2, 3, 5, 7, 11\}$ find $A \cap B$ and show it on a Venn diagram.

$A \cap B = \{1, 2, 3, 5, 7\}$



The white area represents the set $A \cap B$.

Draw suitable Venn diagrams to show the intersections of the following sets. In each case write the intersection in set notation.

5 $U = \{$integers from 4 to 12 inclusive$\}$

$X = \{4, 5, 6, 7, 10\}$ $Y = \{5, 7, 11\}$

6 $U = \{$colours of the rainbow$\}$

$A = \{$red, orange, yellow$\}$ $B = \{$blue, red, violet$\}$

7 $U = \{$positive whole numbers$\}$

$C = \{$positive whole numbers that divide exactly into 24$\}$

$D = \{$positive whole numbers that divide exactly into 28$\}$

8 $U = \{$integers less than 25$\}$

$A = \{$multiples of 3 between 7 and 23$\}$

$B = \{$multiples of 4 between 7 and 23$\}$

Simple problems involving Venn diagrams

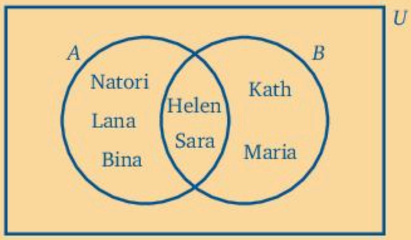
Exercise 15g

If U = {girls in my class}

A = {girls who play netball} = {Helen, Bina, Natori, Sara, Lana} and
B = {girls who play tennis} = {Kath, Sara, Helen, Maria}

Illustrate A and B on a Venn diagram. Use this diagram to write the following sets:

a {girls who play both netball and tennis}
b {girls who play netball but not tennis}
c If $n(U) = 30$ find the number of girls who play neither netball nor tennis.



From the Venn diagram

a {girls who play both netball and tennis} = {Helen, Sara}
b {girls who play netball but not tennis} = {Natori, Lana, Bina}
c n(girls who play neither netball nor tennis) = 30 – 7 = 23

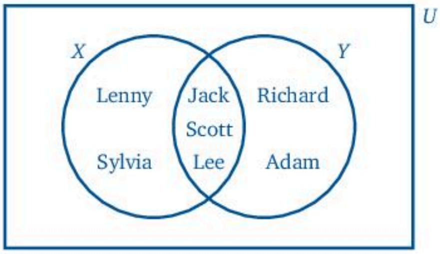
1 U = {the pupils in a class}

X = {pupils who like history}

Y = {pupils who like geography}

List the set of pupils who

a like history but not geography
b like geography but not history
c like both subjects.



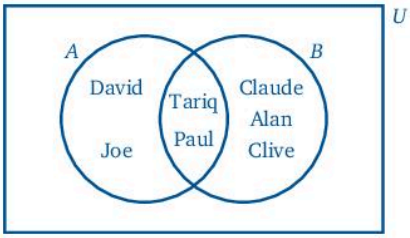
2 U = {boys in my class}

A = {boys who play soccer}

B = {boys who play rugby}

Write the sets of boys who

a play soccer
b play both games
c play rugby but not soccer.



3 $U = \{\text{my friends}\}$

$P = \{\text{friends who wear glasses}\}$

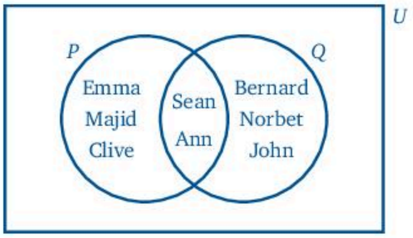
$Q = \{\text{friends who wear brown shoes}\}$

List all my friends who

a wear glasses

b wear glasses but not brown shoes

c wear both glasses and brown shoes.



4 $U = \{\text{whole numbers from 1 to 14 inclusive}\}$

$A = \{\text{even numbers between 3 and 13}\}$

$B = \{\text{multiples of 3 between 1 and 14}\}$

Illustrate this information on a Venn diagram and hence find

a the even numbers between 3 and 13 that are multiples of 3

b $n(A)$ and $n(B)$.

5 $U = \{\text{letters of the alphabet}\}$

$P = \{\text{different letters in the word SCHOOL}\}$

$Q = \{\text{different letters in the word SQUASH}\}$

Show these on a Venn diagram and hence find

a $n(P)$

b $n(P \cup Q)$

c $n(P \cap Q)$

$U = \{\text{months of the year}\}$

$A = \{\text{months of the year beginning with the letter J}\}$

$B = \{\text{months of the year ending with the letter Y}\}$

a Find $n(U)$, $n(A)$ and $n(B)$

Hence find

b $n(A \cap B)$

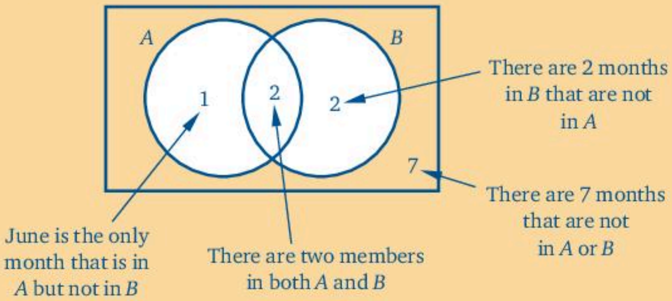
c $n(A \cup B)$

a $n(U) = 12$ (there are 12 months in a year)

$A = \{\text{January, June, July}\}$ so $n(A) = 3$

$B = \{\text{January, February, May, July}\}$ so $n(B) = 4$

b We can illustrate these sets with a Venn diagram using the numbers in each region, rather than the members.



This shows that $n(A \cap B) = 2$.

c $n(A \cup B) = 5$

Alternatively, we know that $A \cup B$ is the set of months in both A and B. However two months, January and July, are in both A and B. This means that we cannot find $n(A \cup B)$ just by adding $n(A)$ and $n(B)$, because that includes the two months in $(A \cap B)$ twice.

Hence

$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ &= 3 + 4 - 2 = 5 \end{aligned}$$

For any two sets, A and B, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

6 $U = \{\text{letters of the alphabet}\}$

$P = \{\text{letters used in the word LIBERAL}\}$

$Q = \{\text{letters used in the word LABOUR}\}$

a Find $n(U)$, $n(P)$ and $n(Q)$.

b Show these on a Venn diagram.

c Hence find **i** $n(P \cap Q)$ **ii** $n(P \cup Q)$ describing each of these sets.

7 $U = \{\text{counting numbers less than 12}\}$

$C = \{\text{prime numbers}\}$

$D = \{\text{odd numbers}\}$

a Find $n(U)$, $n(C)$ and $n(D)$.

b Show these on a Venn diagram.

Hence find **i** $n(C \cap D)$ **ii** $n(C \cup D)$.

8 $U = \{\text{whole numbers from 1 to 35 inclusive}\}$

$R = \{\text{multiples of 4}\}$

$S = \{\text{multiples of 6}\}$

a Find $n(U)$, $n(R)$ and $n(S)$.

b Find **i** $n(R \cap S)$ **ii** $n(R \cup S)$.

9 A and B are two sets such that $n(A) = 8$, $n(B) = 5$ and $n(A \cap B) = 3$. Find $n(A \cup B)$.

Exercise 15h

Select the letter that gives the correct answer.

1 Which of the following are finite sets?

i vowels in the alphabet
ii trees less than 30 m tall
iii decimal numbers between 1 and 2
iv the number of leaves on a particular tree

A **i** and **ii** **B** **i**, **ii** and **iii** **C** **i**, **ii** and **iv** **D** **i**, **iii** and **iv**

2 If $P = \{2, 4, 6, 8, 10, 12, 14\}$, $n(P) =$

A 5 **B** 6 **C** 7 **D** 8

3 Which of these three sets are null sets?

$P = \{\text{multiples of 7 between 8 and 16}\}$

$Q = \{\text{prime numbers less than 2}\}$

$R = \{\text{multiples of 9 between 10 and 17}\}$

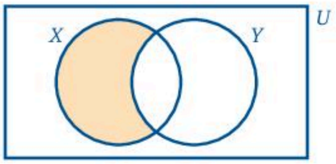
A P and Q **B** P and R **C** P, Q and R **D** Q and R

4 $U = \{x, \text{ a whole number such that } 5 \leqslant x \leqslant 20\}$.
If $A = \{\text{multiples of 5}\}$ then $n(A)$ is

A 1 **B** 2 **C** 3 **D** 4

5 If $U = \{\text{pupils who attend my school}\}$
$X = \{\text{pupils who walk to school}\}$
$Y = \{\text{pupils who are my friends}\}$

the shaded area in this Venn diagram represents

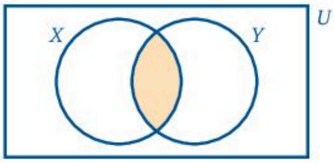


A pupils who are my friends who walk to school
B pupils who are my friends who do not walk to school
C pupils who are not my friends who walk to school
D pupils who are not my friends who do not walk to school.

6 If $U = \{\text{pupils who attend my school}\}$

$X = \{\text{pupils who walk to school}\}$

$Y = \{\text{pupils who are my friends}\}$



the shaded area in this Venn diagram represents

A pupils who are my friends who walk to school

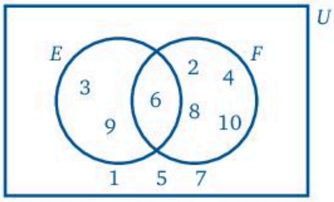
B pupils who are my friends who do not walk to school

C pupils who are not my friends who walk to school

D pupils who are not my friends who do not walk to school.

7 $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

The set $\{2, 3, 4, 6, 8, 9, 10\}$ are the members in the set



A E

B $E \cup F$

C $E \cap F$

D F

8 Using the same data as in question **7**, the members 3, 6, 9 are the only elements in the set

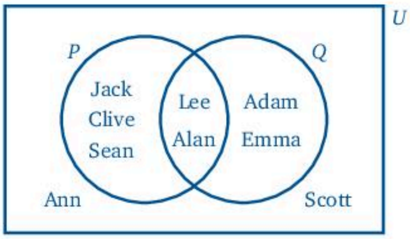
A E **B** $E \cup F$ **C** $E \cap F$ **D** F

Use this Venn diagram for questions **9** and **10**.

$U = \{\text{pupils in my class}\}$

$P = \{\text{pupils in my class who like maths}\}$

$Q = \{\text{pupils in my class who like science}\}$



9 The pupils who like both maths and science are

A Adam and Emma

B Alan and Lee

C Ann and Scott

D Clive, Jack and Sean

10 The pupils who like maths but not science are

A Adam and Emma

B Alan and Lee

C Ann and Scott

D Clive, Jack and Sean

Investigation

Ask any 12 members of your class these questions:

Do you swim? Do you play cricket? Do you play football?

Now write the following sets:

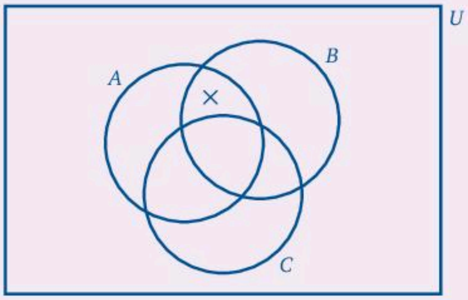
A = {pupils in my class who swim}

B = {pupils in my class who play cricket}

C = {pupils in my class who play football}

Now write each name in the correct place in this Venn diagram.

For example a classmate who swims and plays cricket but does not play football goes in the region that is inside circle *A*, inside circle *B* but outside circle *C*. This is marked with a ×.



Write down a possible universal set.

Are there any empty sets? If there are, write a sentence to explain what each one means.

In this chapter you have seen that…

- ✔ an infinite set has no limit on the number of members in it
- ✔ in a finite set, all the members can be counted or listed
- ✔ the intersection of two sets contains the elements that are in both sets
- ✔ a proper subset of a set *A* contains some, but not all, of the members of *A*
- ✔ the union of two sets contains all the members of the first set together with the members of the second set that have not already been included
- ✔ when two sets have exactly the same members, they are said to be equal
- ✔ a set that has no members is called an empty or null set and is written { } or $\varnothing$
- ✔ $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

16 Logic

At the end of this chapter you should be able to...

1. understand the meaning of a proposition
2. know and use the language of logic
3. know the difference between a simple proposition and a compound proposition
4. use algebra to express propositions.

You need to know...

✓ how to draw Venn diagrams.

Key words

proposition, negation, conjunction, disjunction, equivalence, implication

Did you know?

Studies of logic go back to ancient times in India, China and Greece. The first known writings on logical theory were written by Aristotle (384–322 BCE). He identified the most important forms of reasoning and is considered the father of formal logic. The use of symbols to represent logical were not introduced until the middle of the 1800s.

Propositions

A *proposition* is a statement that can be true or false. You do not need to know whether the statement is true or not, just whether it is possible that it is true or false.

For example,
 'I own a smart phone.'
 'It is snowing.'
are both propositions. They may or may not be true.

If you cannot say that a sentence could be either true or false, it is not a proposition.

For example, 'Is it hot today?'
'Put your phone away.'
are not propositions.

In algebra '$x < 10$' is not a proposition because we do not know what x is so cannot say whether this is true or not.

But '$12 < 10$' is a proposition because it is false.

You may find it helpful to decide whether a sentence is a proposition by putting 'Is it true that …?' in front of the sentence and see if it makes sense. For example, 'Is it true that $x < 10$?' is impossible to answer without knowing the value of x.

Exercise 16a

State whether the following sentences are propositions.

1 I do not like cheese.

2 Do you like cheese?

3 The sun will rise tomorrow.

4 The sun will not rise tomorrow.

5 Take off your earrings.

6 The sum of two odd numbers is even.

7 The sum of two even numbers is odd.

8 There is a man in the moon.

9 I own a tablet.

10 In a class of 30 students, 15 own a smart phone.

11 $4 + 5 = 10$

12 What is the value of 356×274?

Remember, you are not asked whether the sentences are true or false.

Negation

The *negation* of a proposition means the opposite. For example, the negation of the proposition 'It is raining' is 'It is not raining'.

We use a lower case letter to denote a proposition, usually p.
The negation of p means the opposite of p and is denoted by $\sim p$.

So if p is 'I own a smart phone', then $\sim p$ is 'I do not own a smart phone'.

Exercise 16b

For each of the following propositions p, write the meaning of $\sim p$.

1 It is cold today.

2 $2 + 3 = 7$

3 Jayden said, 'Close the door.'

4 Anna plays basketball.

5 Harry does not play football.

6 I eat sweet potatoes.

7 I do not like lettuce.

8 $15 > 10$

9 The Earth revolves round the Sun.

10 When $a = b$ then $b = a$.

Conjunction

The *conjunction* of two propositions connects them with the word 'and'.

For example, the conjunction of
p: I eat sweet potatoes.
q: I do not like lettuce.
is 'I eat sweet potatoes and I do not like lettuce.'
(This can also be expressed as 'I eat sweet potatoes but I do not like lettuce'.)

The conjunction of p and q is denoted by $p \wedge q$.

Exercise 16c

Write the meaning in words of $p \wedge q$ for each of the following propositions.

Remember that a proposition does not need to be true.

1 p: I play football; q: I play cricket.

2 p: Kingston is the capital of Jamaica;
q: Apples grow on trees.

3 p: The Red Sox is a basketball team; q: Manchester United is a soccer team.

4 p: The sky is blue; q: The sea is pink.

5 p: Macaroni cheese contains pasta; q: Spaghetti grows on trees.

Disjunction

The *disjunction* of two propositions connects them with the word 'or' (this does not include 'and or').

For example the disjunction of
p: I wear trainers
q: I wear boots
is 'I wear trainers or I wear boots.'

The disjunction of p and q is denoted by $p \vee q$.

Exercise 16d

Write the meaning in words of $p \vee q$ for each of the propositions in Exercise 16c.

Symbolic logic

Symbolic logic is the use of symbols to denote the connections between propositions. Some of these symbols are introduced above such as $\sim p$, $p \wedge q$, $p \vee q$.

Simple and compound propositions

The propositions p and q are simple propositions whereas $p \wedge q$ and $p \vee q$ are compound propositions.

For example, 'I do not like cheese' is a simple proposition, whereas 'I do not like cheese but I like bread' is a compound proposition.

Exercise 16e

1 Given p: 'I wear trainers' and q: 'I wear boots', write sentences for the meaning of

a $\sim p$ **b** $p \vee q$ **c** $\sim p \wedge q$ **d** $\sim p \wedge \sim q$ **e** $\sim p \vee \sim q$

2 Given p: 'I drink tea' and q: 'I drink cola', write the following sentences in symbolic language.

a I do not drink tea.

b I drink tea and I drink cola.

c I drink tea and I do not drink cola.

d I drink tea or I drink cola.

e I do not drink tea or I do not drink cola.

3 Write the following sentences in symbolic language given p: 'I go to bed at 9 p.m.' and q: 'I get up at 7 a.m.'.

a I go to bed at 9 p.m. and I get up at 7 a.m.

b I do not go to bed at 9 p.m. and I get up at 7 a.m.

c I do not get up at 7 a.m. but I go to bed at 9 p.m.

d I go to bed at 9 p.m. or I get up at 7 a.m.

4 If p: $4 = 2$ and q: $2 = 2$ write the following in symbolic language.

a $4 = 2$ and $2 = 2$ **c** $2 = 2$ or $4 = 2$

b $4 \neq 2$ and $2 \neq 2$ **d** $2 = 2$ and $4 \neq 2$

The symbol ≠ means 'is not equal to'.

5 State whether the following propositions are simple or compound.

a I go swimming every day.

b I go swimming every day and my sister goes swimming every day.

c My friends play cricket but I do not play cricket.

d My friends do not play tennis.

Activity

This is a class activity.

First make a copy of this table:

	p	$\sim p$
q		
$\sim q$		

p is the proposition 'I have a cell phone' and q is the proposition 'I read at least one book last week'.

Each student in the class needs to mark ticks in the boxes for what is true for them.

Before you start, discuss what $\sim p \wedge q$ means.

If this applies to you, which box should you tick? Now complete your table.

Next count up the results for the whole class for the numbers of ticks in each box.

If A is the set {students who have a cell phone} and B is the set {students who read at least one book last week}, discuss how to work out the value of $n(A)$ and $n(B)$.

Now draw a Venn diagram to illustrate $n(A)$ and $n(B)$.

Discuss which region of the Venn diagram represents the number of ticks for

a $p \wedge q$ **b** $\sim p \wedge \sim q$ **c** $p \wedge \sim q$ **d** $p \vee q$

If …then …

You will already be familiar with sentences such as '**If** $2x - 1 = 3$ **then** $x = 2$'.

We can also read this as $2x - 1 = 3$ **implies** $x = 2$.

This is an example of what is called an *implication*. We can use the symbol $\Rightarrow$ to mean implies so we can also write $2x - 1 = 3 \Rightarrow x = 2$.

An implication may not be correct. For example, it is not true to say that $2x - 1 = 3 \Rightarrow x = 4$.

If p is the proposition 'The swimming pool has water in it' and q is 'I will get wet if I go in the swimming pool', then we can say that '**If** the swimming pool has water in it **then** I will get wet if I go in the swimming pool'.

Therefore $p \Rightarrow q$.

Equivalence

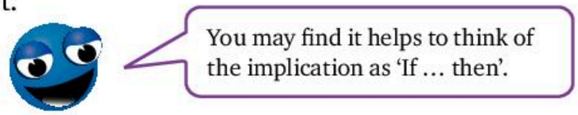
Using the first example from page 319, we can see that since '**If** $2x - 1 = 3$ **then** $x = 2$' is true, it is also true that '**If** $2x - 1 \neq 3$ **then** $x \neq 2$'.

For propositions p and q on page 319, $\sim p$ is 'The swimming pool has no water in it' and $\sim q$ is
'I will not get wet if I go in the swimming pool'. So $\sim p \Rightarrow \sim q$.

The relationship between the propositions p and q such that $p \Rightarrow q$ and $\sim p \Rightarrow \sim q$ is an example of *equivalence*.

Exercise 16f

Determine if the following implications are correct.



1 $x+1=2 \Rightarrow x=1$

2 $x+1=2 \Rightarrow x=3$

3 Two of the angles in a triangle are each 60° ⇒ The third angle in the triangle is 60°.

4 The equation of a straight line is $y = 2x - 4 \Rightarrow$ The gradient of the line is 2.

5 It is night ⇒ The sun is not visible.

6 The sun is not visible ⇒ It is night.

7 When p: 'I am cold' $\Rightarrow q$: 'I need to warm up', write down the meaning of $\sim p \Rightarrow \sim q$

Did you know?

Lewis Carroll is best known among the general public as the author of *Alice in Wonderland*.

Lewis Carroll, whose real name was Charles Dodgson, was also a mathematician who did a lot of work on mathematical logic and reasoning, including writing several books on symbolic logic.

In this chapter you have seen that…

- ✓ a proposition is a sentence that is either true or false
- ✓ the conjunction of two propositions connects them with the word ‘and’
- ✓ the disjunction of two propositions combines them with the word ‘or’
- ✓ in symbolic logic, the conjunction of propositions p and q is denoted by $p \wedge q$ and the disjunction is denoted by $p \vee q$
- ✓ an implication is when one sentence implies another sentence
- ✓ when $p \Rightarrow q$ and $\sim p \Rightarrow \sim q$ the implications are equivalent.

REVIEW TEST 3: CHAPTERS 12–16

1 Find the y-coordinate of the points on the line $y = 5x - 2$ that have x-coordinates of

a 1 **b** 4 **c** –2

2 Find the x-coordinate of the points on the line $y = 3 - 2x$ that have y-coordinates of

a 3 **b** 0 **c** –3

3 Determine whether the straight lines whose equations are given have positive or negative gradients.

a $y = -4x$ **b** $y = 3x + 7$ **c** $y = 5 - 2x$

4 Determine whether the straight lines whose equations are given make acute or obtuse angles with the positive x-axis.

a $y = 0.6x$ **b** $y = \frac{3}{4}x$ **c** $y = 4 - 3x$

5 Find the gradient of the straight line joining the following pairs of points:

a (2, 5) and (0, 2) **b** (4, 2) and (6, 4) **c** (3, –4) and (–2, 6)

6 Give the gradient and intercept on the y-axis for the lines with equations

a $y = 4 - 3x$ **b** $y = 5x + 3$ **c** $4y = 7 - 3x$

7 Draw a suitable graph to find the coordinates of the point of intersection of the two straight lines with equations

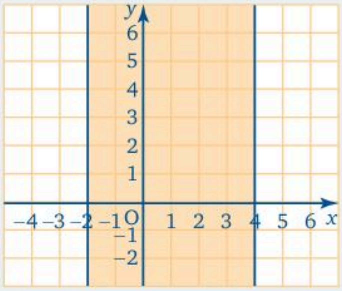
$y = x - 2$ and $x + 3y = 6$

Draw x and y axes for $-3 \leqslant x \leqslant 6$ and $-4 \leqslant y \leqslant 5$. Take 1 cm for 1 unit on each axis.

8 Draw diagrams to represent the following inequalities:

a $x > -2$ **b** $y \leqslant 4$ **c** $x \geqslant 3$

9 Give the inequalities that define the shaded region.

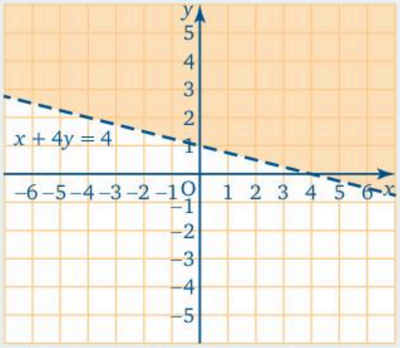


10 Draw a diagram to represent the region described by the pair of inequalities

$-2 \leqslant x < 4$ and $-3 \leqslant y < 5$.

Draw axes for values of x and y from −6 to 6.

11 Find the inequality that defines the unshaded region.



12 Using only a ruler and a pair of compasses, construct a triangle ABC in which AB = 8 cm, angle ABC = 90° and angle BAC = 30°.

Construct the bisector of angle ABC to meet the line AC at D.

Measure, and write down, the value of AD, DC and BD.

13 Using only a ruler and a pair of compasses, construct a triangle PQR in which PQ = 9 cm, angle QPR = 60° and angle PQR = 45°.

Measure, and record, the length of PR, RQ and the size of angle PRQ.

14 Construct a quadrilateral ABCD in which AB = 10 cm, angle ABC = 120°, BC = 8 cm, angle BCD = 60° and angle BAD = 90°.

Measure and record the length of CD and AD.

How are the lines AB and CD related?

15 If A = {even numbers from 5 to 30 inclusive}, list the following subsets of A:

- **a** multiples of 6
- **b** prime numbers
- **c** even numbers greater than 20.

16 U = {integers from 5 to 20 inclusive}

$A = \{6, 8, 10, 14, 15, 16\}$

$B = \{6, 7, 8, 15\}$

- **a** Draw a Venn diagram to show the intersection of these two sets.
- **b** Write the intersection in set notation.
- **c** Write down the set $A \cup B$.

17 U = {pupils in my class}

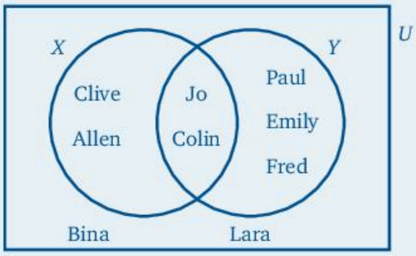
X = {pupils who like music}

Y = {pupils who like maths}

List the pupils who

- **a** like music but not maths
- **b** like maths but not music
- **c** do not like either subject.

What can you say about the likes and/or dislikes of Jo and Colin?



18 Are the following sets finite or infinite?

- **a** {multiples of 5}
- **b** {the number of shops in Kingston}
- **c** {the fractions between 1 and 2}

19 Write each of the following statements in set notation:

- **a** Castor and Pollux belong to the set of stars.
- **b** French is a member of the set of modern languages.
- **c** 39 is not a member of the set of prime numbers.

20 State whether the following statements are true or false:

- **a** $10 \notin$ {prime numbers}
- **b** $42 \in$ {odd numbers}
- **c** Berlin $\in$ {European cities}

21 State whether or not the following sentences are propositions:

- **a** Do you like cola?
- **b** I like to play cricket.
- **c** The sum of an odd number and an even number is always odd.

22 For each proposition p write the meaning of $\sim p$.

a $3+4=9$

b I do not like bananas.

c $8>5$

23 Given p: 'I wear a tie' and q: 'I wear shorts', write sentences in symbolic logic for

a I do not wear a tie.

b I wear a tie and I wear shorts.

c I do not wear a tie and I do not wear shorts.

24 Determine whether or not the following implications are correct.

a $x+3=2 \Rightarrow x=1$

b Two of the angles of a triangle are 30° and 70° $\Rightarrow$ the third angle is 80°.

c The equation of a straight line is $y=3x-7 \Rightarrow$ the gradient is 3.

REVIEW TEST 4: CHAPTERS 1–16

1 What is the property of numbers that illustrates the fact that if $2 = 1 + x$ then $1 + x = 2$?

2 **a** Write $\frac{3}{7}$ as a decimal correct to 3 significant figures.

b Write 48% as a decimal.

c Write 365.4 in standard form.

3 **a** Find the value of 1.2^2.

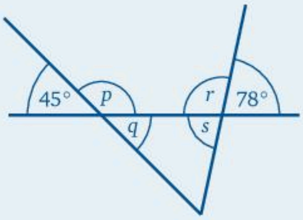
b Write $\frac{a^3 \times a^4}{a^2}$ as a single letter in index form.

4 **a** Write 243_5 as a denary number.

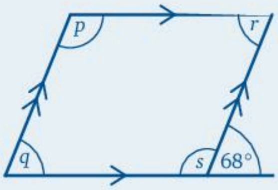
b Write 27_{10} as a binary number.

c Calculate $201_3 + 1102_3$.

5 Find the size of each angle marked with a letter.



6 Find the size of each angle marked with a letter.

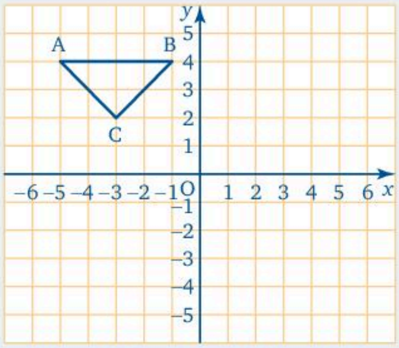


7 On square grid paper draw the following vectors. Label each one with its letter.

a $\mathbf{a} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ **b** $\mathbf{b} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ **c** $\mathbf{c} = \begin{pmatrix} -4 \\ -2 \end{pmatrix}$

8 Find the images of the points given under the transformation defined by the given vector.

a $(1, 2), \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ **b** $(-1, 3), \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ **c** $(4, -2), \begin{pmatrix} 0 \\ -4 \end{pmatrix}$

9 

Copy the diagram above. Use your copy to draw the image of triangle ABC when

a it is translated by the vector $\begin{pmatrix} 6 \\ -1 \end{pmatrix}$. Label the image A′B′C′

b it is reflected in the x-axis. Label the image A″B″C″

c it is rotated about the origin by 180°. Label the image A‴B‴C‴.

10 One banana costs $\$c$. The cost of five bananas is $\$P$. Write down a formula for P.

11 Given $p = 3x - 2y$, find the value of p when $x = 1.2$ and $y = -0.5$.

12 a Rearrange the formula $x = 2y + z$ to make z the subject of the formula.

b Rearrange the formula $P = \frac{a}{3}$ to make a the subject of the formula.

13 Solve the following inequalities and illustrate each solution on a number line.

a $3x - 2 > 4$

b $2 - 3x \leqslant x + 6$

c $x + 4 < 2x + 3 \leqslant 11$

14 a Simplify the ratio $3 : 2\frac{1}{2}$.

b Which ratio is smaller, $4 : 7$ or $5\frac{1}{2} : 9$?

c Find the missing number in the ratio $4 : 7 = \square : 56$

15 a Increase \$36 in the ratio $12 : 5$.

b Decrease 44 m in the ratio $3 : 11$.

16 Give your answers to this question correct to the nearest dollar.

a Divide \$2000 in the ratio $3 : 7$.

b Divide \$56 000 in the ratio $9 : 17$.

17 Fifty books cost \$25 000. Assuming each book costs the same, find the cost of 140 books.

18 A bicycle travels at a constant speed of 50 km/h and takes 36 minutes to complete one lap of a course.
How far will the bike travel when it completes 20 laps?

19 A car takes 20 minutes to cover the first 16 km of a journey and 1 hour to cover the remaining 60 km of the journey. Find the average speed for the whole journey.

20 A car is priced \$1150 000 plus sales tax of $16\frac{1}{2}\%$.

a Find the sales tax payable.

b Calculate the total purchase price of the car.

21 Find the simple interest on \$150 000 for 5 years at 3% per annum.

22 Find the compound interest on \$1000 000 for 2 years at $4\frac{1}{3}\%$ per annum.
Give your answer correct to the nearest dollar.

23 A motor bike is for sale at a cash price of \$280 000. Hire-purchase terms are a deposit of 10% plus 36 monthly payments of \$9500.

a Find the hire-purchase price.

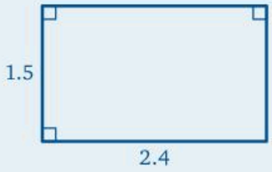
b How much is saved by paying cash?

24 Express 0.0025 m^2 in

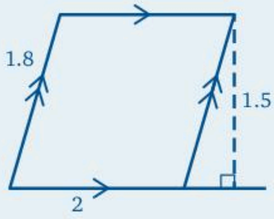
a cm^2 **b** mm^2

25 Find the area of each of the following figures. All measurements are given in centimetres.

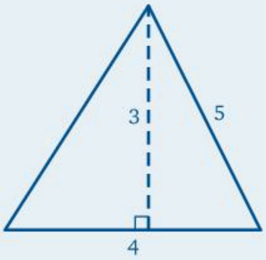
a



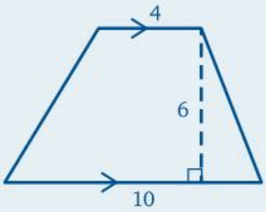
b



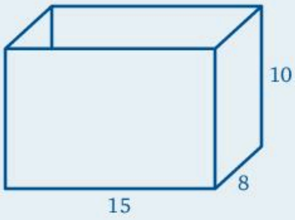
c



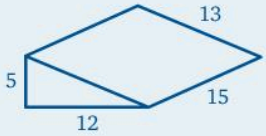
d



26 Find the surface area of this open cuboid. All measurements are given in centimetres.



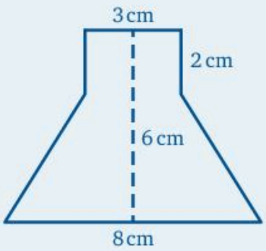
27 Find the surface area of this closed prism. All measurements are given in centimetres.



28 **a** Find the capacity of the cuboid in question **26**.

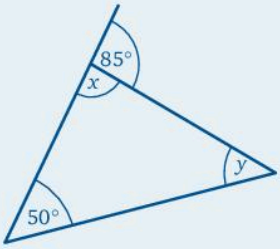
b Find the volume of the prism in question **27**.

29 The diagram shows the cross-section through the top rail of a fence. The rail is 12 m long. Find the volume of this rail.

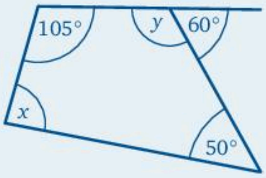


30 In the following diagrams, find the size of each angle marked with a letter.

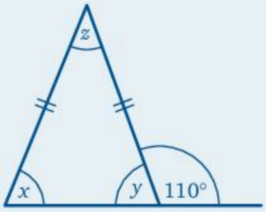
a



b



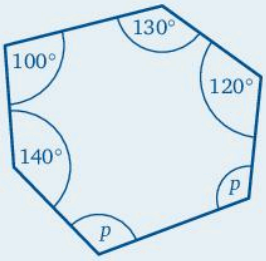
c



31 Find the size of each exterior angle of a regular 12-sided polygon.

32 Four of the exterior angles of a pentagon are 54°, 78°, 40° and 120°. Find the size of the fifth exterior angle.

33 Find the size of the equal angles marked p.



34 Find the number of sides of a regular polygon where each interior angle is 120°.

35 The frequency table shows the number of heads that appeared when four coins were repeatedly tossed together.

Number of heads	0	1	2	3	4
Frequency	2	4	8	5	3

a How many times were the coins tossed?
b Find the mean number of heads per toss.
c What is the modal number of heads?
d Find the median number of heads.

36 The frequency table shows the number of people in different age groups living in a village.

Age group (years)	0–19	20–39	40–59	60 and over
Number of people	17	8	20	45

Draw a pie chart to show this information.

37 The table shows the average price of a second-hand small Ford car for a period of ten years.

Year	2005	2006	2007	2008	2009	2010	2011	2012	2013
Price ($)	140 k	145 k	130 k	150 k	170 k	150 k	160 k	180 k	175 k

(140 k means 140 thousand.)
Draw a line graph to show this information.

38 Draw a bar chart to illustrate the information in question **37**.

39 State whether the following statements are true or false.

a $2 \in$ {prime numbers}

b Kingston $\notin$ {capitals of Caribbean countries}

40 **a** Is the set {players in a basketball team} finite or infinite?

b Find $n(A)$ where $A =$ {even numbers between 11 and 41}.

41 List all the subsets of $A = \{1, 2, 3\}$.

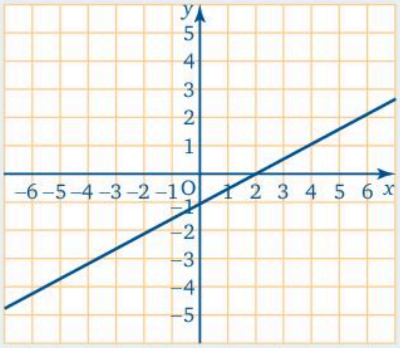
42 Draw a Venn diagram to show $A \cup B$ where
$U =$ {letters of the alphabet}
$A =$ {letters in the word ALGEBRA}
$B =$ {letters in the word ARITHMETIC}

43 Use the information in question **42** to draw a Venn diagram to show $A \cap B$.

44 $U =$ {the integers from -3 to 10 inclusive}
$A =$ {positive numbers}
$B =$ {numbers divisible by 3}
Find **a** $n(U), n(A), n(B)$
b $n(A \cup B)$
c $n(A \cap B)$

45 Use only a ruler and pair of compasses to construct triangle ABC where AB = 6 cm, angle CAB = 60° and angle BCA = 45°.

46 Find the gradient and y-intercept of the line in the diagram. Hence write down the equation of the line.



47 Find the equation of the line that is perpendicular to the line $y = 3 - 5x$ and which passes through the point (2, 2).

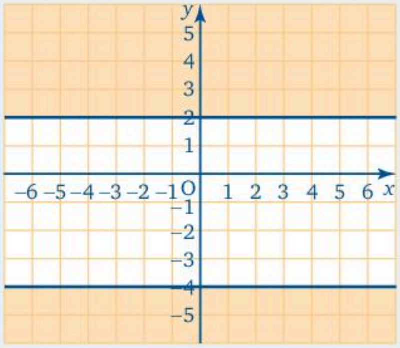
48 **a** Draw axes for x and y using the ranges $-1 \leqslant x \leqslant 4$ and $-2 \leqslant y \leqslant 4$. Plot the graphs of the following pair of lines on these axes:

$y = x - 2$

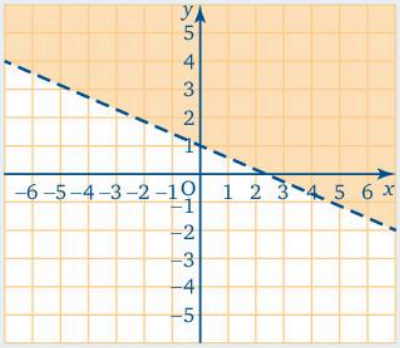
$y = 4 - x$

b Write the coordinates of the point of intersection of these lines.

49 Give the inequalities that define the unshaded region.



50 Find the inequality that defines the unshaded region.



51 State whether the following sentences are propositions.

- **a** The Moon is made of green cheese.
- **b** Eat your vegetables.
- **c** The bus leaves at 1600 hours.

52 Given the proposition p: 'School starts at 0830 hours', write the meaning of $\sim p$.

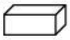
53 Given p: 'I have to wear school uniform' and q: 'I wear trainers after school', write sentences for the meaning of

a $p \wedge q$ **b** $\sim p \vee q$ **c** $\sim p \wedge \sim q$

54 Determine if the following implications are correct.

- **a** $x + 5 = 10 \Rightarrow x = 15$
- **b** The light is on ⇒It is dark.

Glossary

acute angle	an angle less than 90°
alternate angles	equal angles on opposite sides of a transversal, e.g. 
amount	the sum of the interest and the principal (the original money invested or borrowed)
angles at a point	a group of angles round a point that make a complete revolution, e.g. 
angles on a straight line	a group of angles that together make a straight line, e.g. 
approximation	finding an estimate of the value of a calculation or quantity
area	the amount of surface covered
arithmetic average	the sum of a set of values divided by the number of values
average speed	the total distance travelled divided by the total time taken
axis (plural axes)	a fixed line against which the positions of points are measured, for example points on a graph
balance	the amount of money in a bank account
bank statement	this shows details of all the payments that have been made into or out of a bank account
bar chart	a diagram of bars; each represents a quantity. The height of the bar represents the number (frequency) of that quantity
base	the line from which the perpendicular height of a plane figure is measured
binary (number)	any number with a base of two
bisect	divide into two equal parts
boundary line	a line showing the edge of an area or a region
capacity	a measure of three-dimensional space, i.e. volume
Cartesian plane	the plane containing the x and y axes
centimetre	a measure of length
cheque	a request to a bank to pay a stated sum from one person's to another person's account, written on a special slip of paper
circle	a curve made by moving one point at a fixed distance from another
closure	a set is closed under an operation such as addition where the result is always a member of the set
coefficient	the number multiplied by a variable
complementary angles	two angles that add up to 90°
compound interest	the interest accumulated over a given time when each successive interest is added to the principal before calculating the next interest payable
conjunction	the conjunction of two propositions connects them with the word 'and'
constant speed	steady speed over a fixed period of time
construct	make an accurate drawing of
coordinates	an ordered pair of numbers giving the position of a point on a grid
corresponding angles	equal angles formed when two or more parallel lines are cut by a transversal
cross-section	the shape formed by a plane cutting a solid
cube	a solid with six faces, each of which is a square, e.g. 
cuboid	a solid with six faces, each of which is a rectangle, e.g. 
data	a collection of facts or figures
debt	a sum of money that is owed or due
decimal place	a fraction expressed by numbers on the right of a point, e.g. $0.2 = \frac{2}{10}$
degree	the unit of measure for an angle (1 turn = 360°) or for temperature, e.g. °F or °C
denary system	a counting system based on 10
denominator	the bottom of a fraction
deposit	an amount of money that is paid into an account
deposit slip	a form that you fill in when you pay money into an account that gives details of how the amount paid in is made up

directly proportional	if two varying quantities are directly proportional, they are always in the same ratio
discount	the amount an item is reduced by
disjoint sets	sets that have no common elements
disjunction	the disjunction of two propositions connects them with the word 'or'
displacement	the distance and direction of an object from some fixed point
distribution	the possible values a set of data can take and how often each value appears
element	an item that belongs to a set of items; the symbol $\in$ means 'is a member of' and the symbol $\notin$ means 'is not a member of' a particular set
empty set	a set with no members, shown by the symbol $\varnothing$ or $\{\ \}$
equal set	sets are equal when they contain identical members
equation	two expressions connected by an equals sign
equilateral triangle	a triangle whose sides are all the same length
equivalence	an example of equivalence is when one statement implies another, and the negation of the first statement implies the negation of the second, i.e. $p \Rightarrow q$ and $\sim p \Rightarrow \sim q$
equivalent set	sets are equivalent when they contain the same number of elements; the elements in equivalent sets are not usually the same
equivalent fractions	measures the same part of a quantity
expression	a collection of algebraic terms connected with plus and minus signs, without an equals or inequality sign
exterior angle	the angle between the extension of one side of a polygon and another side, e.g. 
face	a surface on a solid
finite set	a set whose members are limited in number
formula	a general rule for expressing one quantity in terms of other quantities
fraction	part of a quantity
frequency	the number of times that a value or group of values occurs
frequency table	a table listing the number of each quantity or group of quantities
General Consumption Tax (GCT)	a sales tax on many goods and services, charged at 16.5% of the selling price
gradient	a measure of the slope of a line
hexagon	a six-sided polygon
hire-purchase	paying a deposit to hire an article and the balance in instalments after which it becomes yours
identity element	when the identity element for an operation is combined with a number using that operation, the result is the original number; for addition the identity element is 0, since $4 + 0 = 4$
implication	when one sentence implies another sentence
index (plural indices)	a superscript to a number that tells you how many of those numbers are multiplied together
inequality	the relationship between two quantities that are not equal
infinite set	a set with an unlimited number of members
integer	a positive or negative whole number
interest	the money paid for the use of money lent or borrowed
interior angle	the angle between a transversal and two parallel lines that add to 180°, e.g. 
intersection of sets	the set of elements common to two or more sets, e.g. $A \cap B$ is the set of members in both A and B
inverse element	combining an element with its inverse element under an operation 'undoes' the operation, leaving the result unchanged, e.g. adding 6 and −6 to any number leaves the number unchanged, so −6 is the inverse of 6 under addition
investing	putting money into an account that pays interest
isosceles triangle	a triangle with two equal sides
kilometre	a metric measure of length

kite	a quadrilateral with two pairs of adjacent sides that are equal, e.g. 
knot	a measure of speed on water; a knot is a speed of 1 nautical mile per hour
line graph	a type of graph that is used to show how a quantity (such as temperature, share prices, cost of living) changes over time
line of symmetry	a line that divides a figure into two identical shapes
litre	a measure of capacity
mean	the sum of a set of values divided by the number of values
median	the middle item of a set of items arranged in order of size
member	an item that belongs to a set of items; the symbol $\in$ means 'is a member of' and the symbol $\notin$ means 'is not a member of' a particular set
metre	a metric measure of length
millimetre	a metric measure of length
mirror line	the line in which an object is reflected to give its image
mode	the most frequent item in a set
negation	the negation of a proposition means the opposite; for example, the negation of the proposition 'It is raining' is 'It is not raining
net	a flat shape that can be folded to make a solid
null set	a set with no members, also called an empty set
number base	the number of distinct digits, including zero, used to represent numbers, e.g. numbers to base five use the digits 0, 1, 2, 3 and 4
numerator	the top of a fraction
obtuse angle	an angle whose size is between 90° and 180°
octagon	a plane figure bounded by eight straight line segments
parallel	two lines that are always the same distance apart
parallelogram	a four-sided figure whose opposite sides are parallel
payment	money paid out of an account to another account, for example to pay a bill
pentagon	a five-sided polygon
per annum	each year
percentage	out of a hundred, i.e. a fraction whose denominator is 100
perimeter	the total distance round the edge of a figure
perpendicular	at right angles to a line or surface
perpendicular height	the height of an object measured at right angles to an edge or surface
pictograph	a diagram showing the frequencies of values in the form of small pictures
pie chart	a circle divided into slices where each slice shows the fraction that one category is of the whole information
polygon	a plane figure bounded by straight line segments
principal	the amount of money lent or borrowed
prism	a solid with two identical ends and flat faces between them
proper subset	a set of some, but not all, of the elements in another set; in symbols $A \subset B$ means A is a proper subset of B
proposition	a statement that can be true or false
protractor	an instrument for measuring angles
purchase price	the price someone pays for an item
quadrilateral	a plane figure bounded by four straight lines
radius	the distance from the centre of a circle to the edge
rate per cent	an amount of money charged for something (like interest), displayed as a percentage
ratio	the comparison between the sizes of two quantities
rectangle	a quadrilateral whose angles are each 90°
reflection	a transformation in which any two corresponding points in the object and the image are both the same distance from a fixed straight line
reflex angle	an angle whose size is between 180° and 360°

reflexive (property)	a mathematical property that states that any number is always equal to itself
region	an area on a graph that contains the points that satisfy a particular condition
regular (polygon)	a polygon is described as *regular* when all its sides are the same length and all its angles are the same size
revolution	a complete turn
rhombus	a four-sided figure whose sides are all the same length, e.g. ◊
right angle	one quarter of a revolution (90°)
right prism	a prism that has two faces that are identical polygons and the other faces are rectangles
rotation	a transformation in which a figure is turned about a given point
rough estimate	an approximate value
sales tax	a fixed percentage of the selling price of something, payable to the government
scalar	a quantity that has size but not direction, e.g. speed
scientific notation	another term for standard form
selling price	the price that a business sells something for, usually in order to make a profit
set	a collection of items
significant figures	position of a figure in a number, e.g. in 2731 the third significant figure is 3
simple interest	interest that is always calculated on the original principal
slant height	the length of a sloping edge or curved surface
speed	the rate at which an object covers distance
square	a four-sided figure whose sides are all the same length and each of whose angles is a right angle
square units	a measure of area
standard form	a number between 1 and 10 multiplied by a power of 10
subset	a set whose members are also members of another set; we use the symbol $A \subseteq B$ rather than $A \subset B$ if we don't know whether B could be equal to A
substitute	replacing one thing with another
supplementary angles	angles whose sum is 180°
surface area	the sum of the areas of all the faces of a three-dimensional object
symmetry (property)	a property that states that, for any two numbers a and b, if $a = b$ then $b = a$
transitive (property)	a property that states that if $a = b$ and $b = c$, then $a = c$
translation	a movement in one direction
transversal	a line that crosses two or more parallel lines
trapezium	a four-sided figure with one pair of unequal sides parallel
trapezium	a four-sided figure with one pair of unequal sides parallel
triangle	a three-sided figure
triangular prism	a prism with a triangular cross-section
trichotomy property	a property applying to any two real numbers, for example a and b, in which either a is greater than b, or $a = b$, or a is less than b
union of sets	the set containing all the different elements of two or more sets, e.g. $A \cup B$ is the set of all members of A and B
universal set	the set containing all elements
vector	a quantity that has size and direction, e.g. velocity
Venn diagram	a diagram used to show the elements in two or more sets
vertex (plural vertices)	corner
vertically opposite angles	the pair of angles opposite each other where two lines cross
volume	a measure of space
withdrawal	money taken out of an account as a cheque or cash
withdrawal slip	record of withdrawal of money from a savings or chequing account
***xy* plane**	the plane containing the x- and y-axes
***y*-intercept**	the value of y where a graph cross the y-axis, which is the point at which x equals zero

Answers

CHAPTER 1

Exercise 1a page 3

1 **a** symmetry **b** transitive
2 A, C, D, G
3 A
4 yes, any fraction when multiplied or divided by another fraction gives a fraction as an answer
5 C
6 **a** $\frac{3}{2}$ **b** no

Exercise 1b page 5

1 −6
2 6
3 5
4 $\frac{1}{2}$
5 −72
6 2
7 84
8 $3\frac{1}{3}$
9 3
10 $-\frac{1}{3}$
11 $-\frac{5}{3}$
12 105.25

Exercise 1c page 6

1 **a** 0.75 **b** 0.6 **c** 0.3 **d** 0.15 **e** 0.875 **f** 0.24
2 0.47
3 **a** $\frac{3}{50}$ **b** $\frac{1}{250}$ **c** $15\frac{1}{2}$ **d** $2\frac{1}{100}$ **e** $3\frac{1}{4}$
4 $\frac{43}{50}$ **5** $\frac{1}{20}$
6 **a** 30% **b** 20% **c** 70% **d** 3.5% **e** 92.5%
7 **a** 132% **b** 150% **c** 240% **d** 105% **e** 255.5%
8 **a** 0.45 **b** 0.6 **c** 0.95 **d** 0.055 **e** 0.125
9 **a** $\frac{2}{5}$ **b** $\frac{13}{20}$ **c** $\frac{27}{50}$ **d** $\frac{1}{4}$
10 **a** 40% **b** 15% **c** 42% **d** 37.5%
11 $\frac{3}{5}$
12 $\frac{7}{20}$
13 $\frac{19}{20}$
14 $\frac{8}{25}$
15 85%
16 34%
17 60%
18 12.5%
19

Fraction	Percentage	Decimal
$\frac{3}{5}$	60%	0.6
$\frac{4}{5}$	80%	0.8
$\frac{3}{4}$	75%	0.75
$\frac{7}{10}$	70%	0.7
$\frac{11}{20}$	55%	0.55
$\frac{11}{25}$	44%	0.44

20 **a** $\frac{1}{20}$ **b** 5%
21 **a** 42% **b** 0.42
22 **a** $\frac{7}{25}$ **b** 60% **c** 12%
23 **a** $\frac{3}{5}$ **b** **i** 35% **ii** 95% **c** 0.05

Exercise 1d page 8

1 9
2 4
3 100
4 125
5 1000
6 81
7 128
8 10
9 64
10 10000
11 1 000 000
12 27
13 7200
14 893
15 65 000
16 3820
17 27.5
18 537 000
19 46.3
20 503.2
21 709
22 69.78

Exercise 1e page 9

1 3^7
2 7^8
3 9^{10}
4 2^{11}
5 b^5
6 5^8
7 12^9
8 p^{14}
9 4^{16}
10 r^8

Exercise 1f page 9

1 4^2
2 7^6
3 5^1
4 10^5
5 q^4
6 15^4
7 6^5
8 b^2
9 9^1
10 p^1
11 6^{11}
12 3^3
13 2^1
14 a^{12}
15 c^3
16 2^9
17 4^1
18 a^1
19 3^8
20 b^9

Exercise 1g page 10

1 4
2 64
3 125
4 81
5 4
6 2410
7 497.1
8 59 200
9 783.4
10 30 500
11 2^7
12 4^3
13 3^6
14 a^7
15 a^4

Exercise 1h page 11

1 3780
2 1260
3 5 300 000
4 740 000 000 000 000
5 13 000
6 3 670 000
7 30 400
8 85 030
9 4 250 000 000 000
10 643 000 000

Exercise 1i page 12

1 2.5×10^3
2 6.3×10^2
3 1.53×10^4
4 2.6×10^5
5 9.9×10^3
6 3.907×10^4
7 4.5×10^6
8 5.3×10^8
9 4×10^4
10 8×10^{10}
11 2.603×10^4
12 5.47×10^5
13 3.06×10^4
14 4.06×10^6
15 7.04×10^2
16 7.93×10^1
17 8.06×10^4
18 6.05×10^1
19 7.08×10^6
20 5.608×10^5
21 5.3×10^{12}
22 7.08×10^5
23 4.05×10^1
24 5.03×10^8
25 9.9×10^7
26 8.4×10^1
27 3.51×10^2
28 3.6×10^1
29 5.09×10^3
30 2.68×10^5
31 3.07×10^1

Exercise 1j page 13

1 1550, 1500, 2000
2 8740, 8700, 9000
3 2750, 2800, 3000
4 36 840, 36 800, 37 000
5 68 410, 68 400, 68 000
6 5730, 5700, 6000
7 4070, 4100, 4000
8 7510, 7500, 8000
9 53 800, 53 800, 54 000
10 6010, 6000, 6000
11 4980, 5000, 5000
12 8700, 8700, 9000

13 54, 45
14 45 499, 44 500
15 1549, 1450
16 $2 500 000
17 1950

Exercise 1k page 15

1 2.76, 2.8, 3
2 7.37, 7.4, 7
3 16.99, 17.0, 17
4 23.76, 23.8, 24
5 9.86, 9.9, 10
6 3.90, 3.9, 4
7 8.94, 8.9, 9
8 73.65, 73.6, 74
9 6.90, 6.9, 7
10 55.58, 55.6, 56
11 5.1
12 0.009
13 7.90
14 34.8
15 0.0078
16 0.975
17 5.551
18 285.6
19 6.7
20 10.00

Exercise 1l page 16

1 3
2 8
3 6
4 8
5 7
6 8
7 0
8 0
9 0
10 8

Exercise 1m page 17

1 60000
2 4000
3 4 000 000
4 600 000
5 80000
6 500
7 50000
8 4000
9 700 000
10 900
11 30
12 1000
13 4700
14 57000
15 60000
16 890 000
17 7000
18 10000
19 73 000
20 440
21 50 000
22 54 000
23 480
24 600
25 0.00846
26 0.826
27 5.84
28 78.5
29 46.8
30 0.00785
31 7.51
32 370
33 0.990
34 54.0
35 47
36 0.006 845
37 600000
38 500
39 7.82
40 5000
41 37.9
42 7000
43 0.0709
44 0.07
45 3.3
46 1.7
47 13
48 13
49 14
50 29
51 24
52 0.23
53 0.026
54 0.00 043

Exercise 1n page 18

1 100
2 36
3 0.014
4 20
6 180 000
6 0.8
7 0.48
8 3.6
9 1.3
10 3 500 000
11 600
12 4.5
13 1.5
14 0.7
15 17
16 0.003
17 0.0056
18 80
19 90 000
20 1.5
21 10
22 0.36
23 10
24 2
25 32
26 1.2
27 15
28 0.25
29 0.12
30 140

Exercise 1p page 20

1 7.08
2 7.55
3 7.02
4 8.54
6 9.19
6 7.71
7 7.49
8 9.15
9 1.61
10 1.56
11 3.80
12 1.50
13 2.94
14 1.54
16 1.44
16 1330
17 8370
18 6580
19 15.5
20 6.65
21 172
22 14.7
23 11.2
24 1170
25 12 600
26 36.8
27 1950
28 38.0
29 1350
30 14 400
31 2.70
32 0.0196
33 0.0549
34 526
35 4.65
36 0.0481
37 1.79
38 0.005 15
39 3.97
40 0.548
41 0.121
42 0.0825
43 0.393
44 0.103
45 0.139
46 124
47 55.8
48 91.7
49 186
50 957
51 49.0
52 11 200
53 83.6
54 2.28
55 0.672
56 9.83
57 0.693
58 0.742
59 0.128
60 10300
61 6340
62 0.00608
63 34.8
64 484 000
65 0.361
66 0.0203
67 0.000 123
68 631
69 0.000 000 0961
70 4950
71 0.174
72. 16.7
73 0.000 146
74 13.4

Exercise 1q page 21

1 64
2 b^3
3 1
4 3.64×10^4
5 60 000
6 0.0614
7 3.71
8 2.88

Exercise 1r page 21

1 216
2 2^3
3 5^3
4 a^7
5 6.5×10^8
6 46 000
7 7500
8 1350
9 0.699
10 **a** $\frac{7}{20}$ **b** 0.35

Exercise 1s page 22

1 $5^5 = 3125$
2 a^3
3 1
4 7.826×10^4
5 10 000
6 0.0508
7 9
8 9.89
9 4.70
10 **a** 62.5% **b** 0.625

Exercise 1t page 22

1 B
2 A
3 B
4 C
5 C
6 A
7 C
8 D
9 B
10 C
11 B
12 D
13 A
14 B
15 B
16 B
17 D
18 D
19 A
20 C

CHAPTER 2

Exercise 2a page 27

1 213_5
2 2014_5
3 41240_5
4 30201_5

	5^3	5^2	5	Units
5			3	1
6			4	2
7		4	1	0
8		2	3	1
9			3	4
10			1	0
11		2	0	4
12		4	0	0

13 16_{10}
14 14_{10}
15 20_{10}
16 36_{10}
17 54_{10}
18 23_{10}
19 17_{10}
20 10_{10}
21 4_{10}
22 100_{10}
23 70_{10}
24 75_{10}
25 13_5
26 23_5
27 20_5
28 124_5
29 133_5
30 1100_5
31 12_5
32 41_5
33 110_5
34 1003_5
35 312_5
36 400_5

Exercise 2b page 29

1 a

4	Units
2	3

b 11_{10}

2 a

7	Units
1	5

b 12_{10}

3 a

4^2	4	Units
1	3	1

b 29_{10}

4 a

2^2	2	Units
1	0	1

b 5_{10}

5 a

8	Units
5	7

b 47_{10}

6 a

5^2	5	Units
2	0	4

b 54_{10}

7 a

3^2	3	Units
2	1	0

b 21_{10}

8 a

9^2	9	Units
5	7	4

b 472_{10}

9 a

	3	Units
	2	1

b 7_{10}

10 a

	9	Units
	1	8

b 17_{10}

11 a

	6	Units
	2	4

b 16_{10}

12 a

8^2	8	Units
1	7	5

b 125_{10}

13 a

4^2	4	Units
3	0	3

b 51_{10}

14 a

2^3	2^2	2	Units
1	0	0	1

b 9_{10}

15 a

3^3	3^2	**3**	Units
1	2	1	1

b 49_{10}

16 a

6^3	6^2	6	Units
1	0	0	0

b 216_{10}

17 21_4
18 22_5
19 33_7
20 111_2
21 23_5
22 52_6
23 65_8
24 100_7
25 22_3
26 23_6
27 37_9
28 1001_3
29 110_7
30 108_9
31 101101_2
32 110100_3
33 243_8
34 $22\,000_4$
35 10001110_2
36 422_6
37 111_6
38 210_5
39 1100011_2
40 1102_7
41 131_4
42 15_6
43 1111_2
44 11100_3
45 11302_4
46 234_5
47 33_8
48 $102\,122_3$
49 310_8

Exercise 2c page 31

1 43_5
2 20_3
3 30_4
4 11_2
5 103_6
6 102_3
7 115_8
8 1000_2
9 122_5
10 333_4
11 1151_6
12 $10\,100_2$
13 1000_3
14 1000_4
15 1030_7
16 100001_2
17 125_6
18 101_3
19 3_4
20 35_7
21 202_3
22 11_5
23 226_8
24 1_2
25 133_4
26 56_7
27 33_6
28 10_2
29 51_6
30 101_2
31 646_8
32 101_3
33 22_5
34 12_4
35 101_3
36 120_4
37 32_6
38 31_5
39 105_7
40 180_9
41 330_4
42 4105_6
43 1111_3
44 1032_5
45 414_7
46 130_4
47 242_5
48 105_7
49 1015_6
50 123_4
51 143_5
52 560_8
53 2_3
54 214_8

Exercise 2d page 33

1 $10\,221_4$
2 $22\,022_5$
3 $1\,110\,101_2$
4 2422_8
5 $2\,011\,210_3$
6 2116_8
7 $222\,142_5$
8 $220\,041_7$
9 a 2254_8
b 52, 23
c 1196_{10}
d 2254_8
10 a $31\,026_9$
b 393, 52
c 20436
d $31\,026_9$
11 3047_8

Exercise 2e page 34

1 13, 11, 25, 4, 12, 29

2 a

+	0	1
0	0	1
1	1	10

b $10010\,000_2$

3 a 6 **b** 101_2

4

×	0	1
0	0	0
1	0	1

Exercise 2f page 35

1 a 12_3 **b** 22_3 **c** 110_3 **d** 1011_3
2 three

3

×	0	1	2
0	0	0	0
1	0	1	2
2	0	2	11

4 11_3
5 **a** 102_5 **b** 33_5 **c** 1103_5
6 $31_5 + \frac{2}{5}$ or 16.4_{10}
7 0
8 no
9 four, $5^3 = 1000_5$
10 eight, $3^7 = 10000000_3$
11 **a** 2, 3, 4, 5, 6, 7, 8
b **i** 11010_2 **ii** 1210_3 **iii** 1750_8
c The digits move one column to the left; i.e. the effect is the same as multiplying a denary number by ten.
12 6
13 4
14 3
15 7
16 5
17 9
18 5
19 6
20 false

Exercise 2g page 36

1 **a** 6 **b** 5 **c** 147
2 **a** 40_5 **b** 202_3 **c** 24_8
3 **a** 341_5 **b** 101_3 **c** 636_8
4 C

Exercise 2h page 37

1 B **3** C **5** C **7** C **9** B
2 D **4** A **6** B **8** C **10** B

CHAPTER 3

Exercise 3a page 40

1 **a** $x - 10$ **b** $4x$ **c** $y - 5x$
2 **a** $8x$ **b** $4x$ **c** $2x$
3 **a** $6x - 3$ **b** $2 - 8x$ **c** $34x + 18$
4 **a** $16x^2$ **b** $20xy$ **c** $6xyz$
d $2x^2$ **e** $3x^3$
5 **a** **i** 18 **ii** 68 **b** 4, 2 **c** **i** 15 **ii** 50
6 **a** $6x^4$ **b** $5x^3$

Exercise 3b page 41

1 $P = 2(l + w)$
2 $P = 2l + d$
3 $P = 3l$
4 $P = 5l$
5 $P = 2l + s + d$
6 $W = x + y$
7 $P = 2(l + b)$
8 $T = N + W$
9 $T = N - L$
10 $P = p + q + r + s$
11 $L = l - d$
12 $N = S - T$
13 $A = l^2$
14 $N = 10n$
15 $c = nx$
16 $P = 6l$
17 $A = 2l^2$
18 $W = T + S$
19 $S = N - L - R$
20 $r = p - q$
21 $W = nk$
22 $L = 3(x + y)$

Exercise 3c page 44

1 10
2 100
3 30
4 2
5 20
6 200
7 24
8 15
9 25
10 $7\frac{1}{2}$
11 −1
12 −12
13 5
14 33
15 50
16 19
17 16
18 2
19 105
20 $3\frac{1}{3}$
21 15
22 200
23 $3\frac{1}{3}$
24 7
25 $1\frac{3}{4}$
26 0
27 $\frac{5}{24}$
28 −3

Exercise 3d page 46

1 **a** 48 **b** −18 **c** 6 **d** 5
2 **a** 4 **b** 20 **c** 8 **d** −12
3 **a** 52 **b** 20 **c** 96 **d** −4
4 **a** 15 **b** −1.1 **c** −15.9 **d** 0.38

5 $C = 50n$, \$1200
6 $L = 50n$, \$4750
7 $P = 2a + 2b$, 70 cm
8 $P = 6x$, 6 cm
9 $P = L - Nr$, 5 m
10 $P = 3a$, 24 cm
11 $W = Na + p$, 45
12 $A = 2lw + 2lh + 2hw$, 6200 cm²

Exercise 3e page 48

1 $T = N - G$
2 $R = P - Q$
3 $X = L + Y$
4 $c = B - A$
5 $d = a - b - c$
6 $x = y + z$
7 $e = f - d$
8 $x = z - y$
9 $a = s - 2b$
10 $u = v - t$
11 $d = S + t$
12 $z = P - 2y$
13 $a = L - b - c$
14 $a = P - b$
15 $T = N - R$
16 $a = b - c - d$
17 $u = v - rt$
18 $y = x + z$
19 $c = P - ab$
20 $u = v - at$

Exercise 3f page 49

1 $x = \frac{z}{y}$
2 $d = St$
3 $n = \frac{N}{r}$
4 $T = \frac{C}{R}$
5 $m = Ln$
6 $u = v - at$
7 $p = \frac{r}{q}$
8 $y = X - 2z$
9 $b = 3c - 2a$
10 $a = 2s - b - c$
11 $m = \frac{p}{n}$
12 $f = 2gj$
13 $b = 2c - a$
14 $q = \frac{4p}{r}$
15 $c = 2a + 3b - L$
16 $a = 4d - 2b - 3c$
17 $c = b^2 - a^2$
18 $r = 3 - p^2 + q^2$
19 $r = t - 2s$

Exercise 3g page 50

1 $a < 20$ where a is the number of library books a school can afford to buy.
2 $b < 100$ where b is the number of people who attended a rally.
3 $c > 20$ where c is the number of albums sold on the first day.
4 $d \geqslant 50$ where d is the number of cars that passed the school between 9 a.m. and 10 a.m.
5 $p \leqslant 50$ where p is the perimeter of a rectangle in cm.
6 $c < 50$ where c is the cost in dollars of making a widget.
7 $g \geqslant 3$ where g is the number of goats Ceejay owns.
8 $d > 50$ where d is the number of \$1 coins in a bag.
9 $d \geqslant 250$ where d is the number of days it takes to build a bungalow.
10 $p > 5$ where p is the number of pens Victoria has in her school bag.
11 $r < 5$, where r is the number of rubbers Jesse has in his school bag.
12 $f \geqslant 50$, where f is the number of fireworks there are in a box.
13 $t \leqslant 10$, where t is the number of minutes a bus takes to get to school.
14 $n \leqslant 20$ where n is the number of \$500 bank notes in a cash box.
15 $a \geqslant 6$ where a is Anna's age in years.

Exercise 3h page 51

1


2


3


4


5


6


7


8


9


10 **a** 2, 3, 4, 6, 7 **d** 2, 3, 4, 6, 7
b 2, 5, 7, 8, 9 **e** 2, 3, 4, 7, 9
c 2, 3, 7, 9
12 **a** $5 > 3$; yes **c** $-2 > -4$; yes
b $1 > -1$; yes **d** $7 > 5$; yes
13 **a** $0 > -1$; yes **c** $-7 > -8$; yes
b $-4 > -5$; yes **d** $2 > 1$; yes
14 **a** $1 < 6$; yes **c** $-6 < -1$; yes
b $-3 < 2$; yes **d** $3 < 8$; yes

Exercise 3i page 52

1 $x < 12$


2 $x < 2$


3 $x > 5$


4 $x > 2$


5 $x < -2$


6 $x < 3$


7 $x < -3$


8 $x < -7$


9 $x < -5$


10 $x < -2$


11 $x > -1$


12 $x < 3$


13 $x > 0$


14 $x > -3$


15 $x < -3$


16 $x < 1$


17 $x > -2$


18 $x < -5$


19 $x < 5$


20 $x < 1$


21 $x < -1$


22 $x > 0$


23 $x > 7$


24 $x > -5$


25 $x > -3$


26 $x < 13$


27 $x > 12$


28 **a** $24 < 72$ **c** $6 < 18$ **e** $-24 < -72$
b $3 < 9$ **d** $2 < 6$ **f** $-4 < -12$
a yes **c** yes **e** no
b yes **d** yes **f** no
29 **a** $72 > -24$ **c** $18 > -6$ **e** $-72 > 24$
b $9 > -3$ **d** $6 > -2$ **f** $-12 > 4$
c $18 > -6$
a yes **c** yes **e** no
b yes **d** yes **f** no
30 **a** $-36 < -12$ **c** $-9 < -3$ **e** $36 < 12$
b $-4\frac{1}{2} < -1\frac{1}{2}$ **d** $-3 < -1$ **f** $6 < 2$
a yes **c** yes **e** no
b yes **d** yes **f** no

32 Only when you are multiplying by a positive number.

Exercise 3j page 54

1


$x < 3$

2


$x > 1$

3


$x > 2$

4


$x < 1$

5


$x < \frac{1}{2}$

6


$x > 1\frac{1}{3}$

7


$x < 2\frac{1}{4}$

8


$x > 1\frac{1}{2}$

9


$x \leqslant 1$

10


$x \leqslant 4$

11


$x \geqslant -2$

12


$x \geqslant 1$

13


$x < -1$

14


$x \leqslant 2$

15


$x > 1$

16


$x \geqslant 1\frac{1}{3}$

17


$x \geqslant 0$

18 

$x \leq 1$

19 

$x < 1$

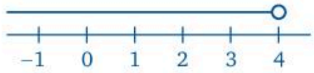
20 

$x < -3$

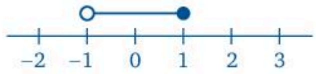
21 **a** $x > 3$ **b** $2 \leq x \leq 3$ **c** no values of x
22 **a** $0 \leq x \leq 1$ **b** $x \leq 0$ **c** no values of x
23 **a** $-2 < x \leq 4$ **b** no values of x **c** $x < -2$
24 **a** $-3 < x < -1$ **b** $x < -3$ **c** no values of x
25 $x < 12; x > -1; -1 < x < 12$
26 $x \leq -1; x \geq 3;$ no values of x
27 $x \leq 7; x \geq -2; -2 \leq x \leq 7$
28 $x > 1; x < 2; 1 < x < 2$
29 $x > 2; x < 3; 2 < x < 3$
30 $x < 2; x > -1; -1 < x < 2$
31 $x \geq -1; x < 2; -1 \leq x < 2$
32 $x > \frac{1}{2}; x \leq 3; \frac{1}{2} < x \leq 3$
33 $2 < x < 5$
34 $-3 \leq x \leq 2$
35 $x < -2$
36 $0 < x < 2$
37 $x \geq 1$
38 $-4 < x < 2$
39 $x < -3$
40 $x < -1$
41 $1\frac{4}{5} < x < 3$
42 $\frac{1}{2} < x < 1$

Exercise 3k page 56

1 $N = a + b + c$
2 4
3 2
4 10
5 **a** $x < 4$



b $-1 < x \leq \frac{1}{2}$



Exercise 3l page 57

1 C
2 B
3 D
4 B
5 B
6 A
7 C
8 B
9 D
10 C
11 A
12 D
13 C
14 A
15 C
16 A
17 B
18 A
19 D
20 A
21 D
22 C
23 A

CHAPTER 4

Exercise 4a page 62

1 obtuse
2 acute
3 reflex
4 reflex
5 acute
6 acute
7 reflex
8 obtuse
9 obtuse

Exercise 4b page 62

1 180°
2 90°
3 270°
4 30°
5 45°
6 120°
7 60°
8 45°
9 30°
10 120°
11 210°
12 180°
13 300°
14 330°
15 150°
16 210°
17 300°

Exercise 4d page 64

4 150°
5 20°
6 35°
7 65°
8 140°
9 160°

Exercise 4e page 67

1 120°
2 155°
3 10°
4 100°
5 20°
6 130°
7 80°
8 15°
9 135°
10 140°
11 90°
12 50°
13 *e* and *f*
14 *m* and *k*, *j* and *d*
15 *d* and *f*, *f* and *e*, *e* and *g*, *g* and *d*
16 *f* and *g*
17 *f* and *g*, *g* and *d*, *d* and *e*, *e* and *f*
18 *n* and *d*, *d* and *p*, *p* and *m*, *m* and *n*
19 50°, 130°, 130°
20 60°, 120°, 120°
21 180°, 60°
24 180°, 155°
22 105°, 180°
25 80°, 100°, 100°
23 45°, 135°, 135°
26 165°, 180°

Exercise 4f page 70

1 110°
2 60°
3 110°
4 80°
5 180°
6 150°
7 100°
8 120°
9 310°
10 60°

Exercise 4g page 71

1 120°
2 120°, 60°
3 120°
4 310°
5 150°, 60°
6 50°
7 40°
8 120°, 60°, 120°, 60°

Exercise 4i page 75

1 *g*
2 *e*
3 *d*
4 *e*
5 *f*
6 *f*
7 *d*
8 *g*

Exercise 4k page 78

1 60°
2 110°
3 75°
4 60°
5 60°
6 80°
7 110°
8 120°
9 30°
10 130°
11 130°
12 50°, 50°
13 130°, 130°, 50°
14 60°, 60° 60°, 120°, 60°
15 115°, 115°
16 140°, 40°, 40°
17 70°, 110°, 70°, 70°
18 55°, 125°, 55°
19 110°, 70°, 130°, 130°

Exercise 4l page 81

1 *e*
2 *e*
3 *d*
4 *d*
5 *d*
6 *g*
7 *g*
8 *e*
9 *d*

Exercise 4m page 82

1 50°, 130°
2 130°, 50°
3 50°, 70°
4 260°, 40°, 60°
5 70°, 70°, 70°
6 55°, 65°
7 45°
8 90°

Exercise 4n page 84

1 *e*, *g*
2 *e*, *d*
3 *e*, *g*
4 *e*, *d*
5 *h*, *f*
6 *d*, *g*
7 70°, 110°, 180°
8 130°, 50°, 180°
9 140°, 40°, 80°
10 120°, 60°, 180°

Exercise 4p page 86

1 120°
2 130°, 50°
3 85°
4 40°, 100°, 60°
5 55°, 125°
6 40°
7 80°, 80°
8 130°, 130°, 50°
9 80°, 100°, 80°, 100°
10 70°, 110°

Exercise 4q page 87

1 145°
2 140°
3 140°
4 70°
5 45°
6 65°
7 140°
8 55°
9 110°

Exercise 4r page 88

1 80°
2 60°
3 50°
4 110°
5 40°
6 40°
7 $f = 140°$, $g = 40°$
8 50°

Exercise 4s page 88

1 B
2 A
3 B
4 C
5 C
6 A
7 D
8 C
9 B
10 C
11 D
12 C
13 C
14 A
15 B
16 D

CHAPTER 5

Exercise 5a page 94

1 scalar
2 vector
3 scalar
4 scalar
5 vector

Exercise 5b page 95

1 $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

2 $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$

3 $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$

4 $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$

5 $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$

6 $\begin{pmatrix} -5 \\ -3 \end{pmatrix}$

7 $\mathbf{g} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$

$\mathbf{h} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}$

$\mathbf{i} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$

$\mathbf{j} = \begin{pmatrix} -6 \\ 7 \end{pmatrix}$

$\mathbf{k} = \begin{pmatrix} -6 \\ -2 \end{pmatrix}$

$\mathbf{l} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

$\mathbf{m} = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$

$\mathbf{n} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$

17 Both pairs are parallel.

Exercise 5c page 97

1 (7, 3)
2 (6, 9)
3 (2, 7)
4 (1, 5)
5 (1, 3)
6 (6, –7)
7 (–2, –2)
8 (–4, –2)
9 (9, –6)
10 (2, 0)
11 $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$
12 $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$
13 $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$
14 $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$
15 $\begin{pmatrix} -2 \\ -2 \end{pmatrix}$
16 $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
17 (5, 6)
18 (–2, 3)
19 (–4, –5)

Exercise 5d page 98

1 $\overrightarrow{AA'} = \begin{pmatrix} -5 \\ 1 \end{pmatrix}$, $\overrightarrow{BB'} = \begin{pmatrix} -5 \\ 1 \end{pmatrix}$, $\overrightarrow{CC'} = \begin{pmatrix} -5 \\ 1 \end{pmatrix}$

yes, yes

2 $\overrightarrow{LL'} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$, $\overrightarrow{MM'} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$, $\overrightarrow{NN'} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$,

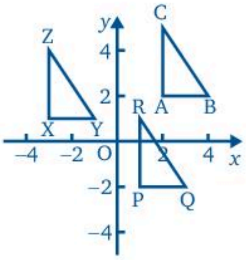
yes, yes

3 $\begin{pmatrix} 6 \\ 3 \end{pmatrix}$

4 **a** $\begin{pmatrix} -5 \\ 4 \end{pmatrix}$ **b** $\begin{pmatrix} 5 \\ -4 \end{pmatrix}$

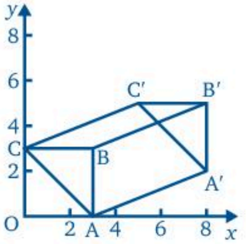
5 **a** $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$ **b** $\begin{pmatrix} -6 \\ 0 \end{pmatrix}$ **c** $\begin{pmatrix} 5 \\ 5 \end{pmatrix}$ **d** $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

6



a $\begin{pmatrix} -1 \\ -4 \end{pmatrix}$ **b** $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$ **c** $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$ **d** $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

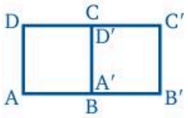
7



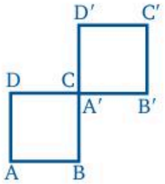
yes, $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$, parallelogram – the opposite sides are parallel.

AA′C′C, BB′C′C

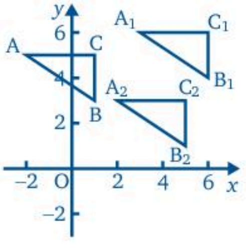
8 **a**



b



9

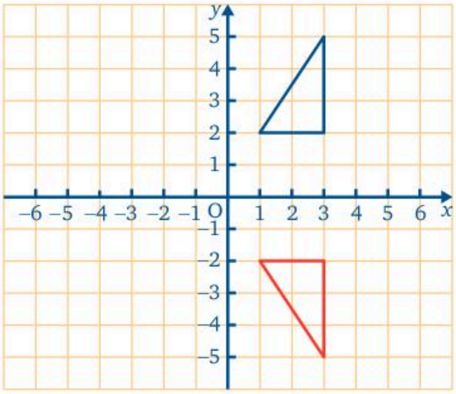


a $\begin{pmatrix} 4 \\ -2 \end{pmatrix}$ **b** $\begin{pmatrix} -4 \\ 2 \end{pmatrix}$ **c** $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$

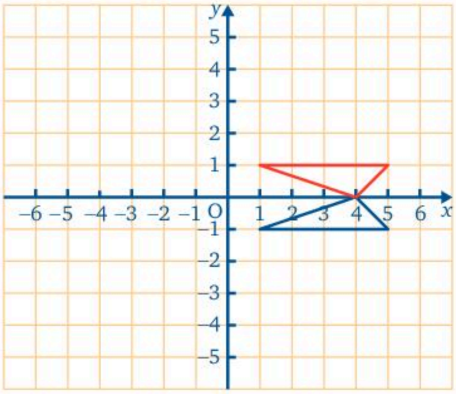
Exercise 5e page 101

1 reflection in the *x*-axis
2 reflection in the *y*-axis
3 reflection in the *y*-axis
4 reflection in the *y*-axis
5 reflection in the *x*-axis
6 reflection in the *y*-axis

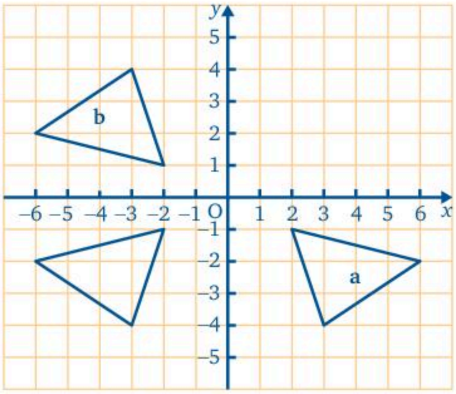
7



8



9



c $A_2(-6, 2)$, $B_2(-3, 4)$, $C_2(-2, 1)$

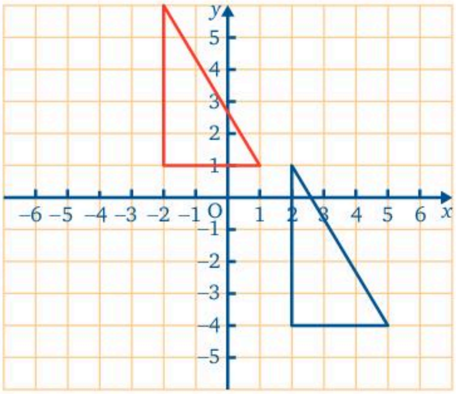
Exercise 5f page 103

1 180°
2 180°
3 90°
4 270°
5 A′(0, 0), B′(2, –3), C′(0, –3)
6 A′(0, –2), B′(0, –4), C′(3, –3)
7 A′(0, –3) B′(–3, 4) C′(0, 4) D′(0, 0)
8 A′(0, –2), B′(–2, 2), C′(2, 2)

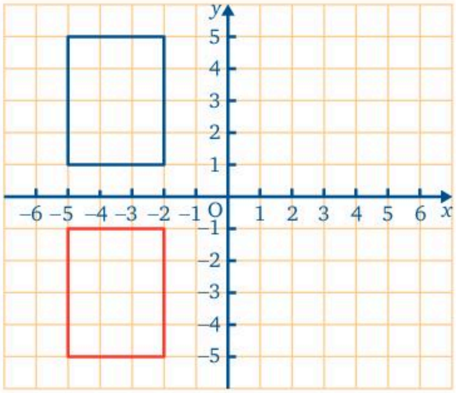
Exercise 5g page 106

1 translation by $\begin{pmatrix}-2\\2\end{pmatrix}$
2 rotation about O by 90°
3 reflection in the *x*-axis
4 rotation about O by 180°

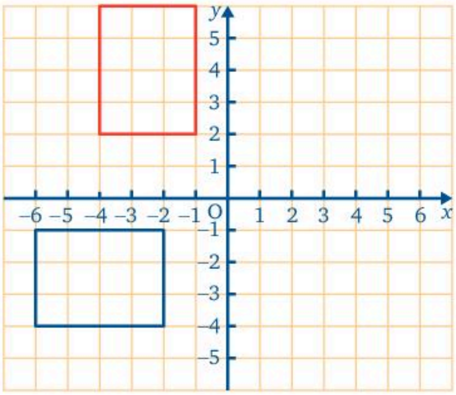
5



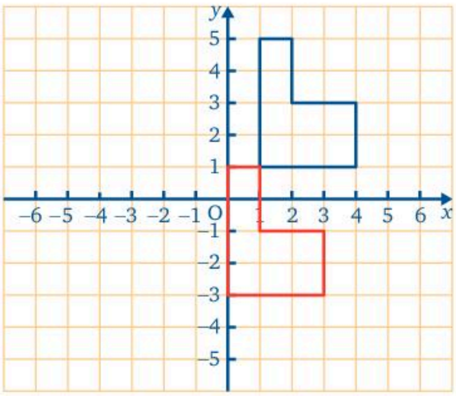
6



7

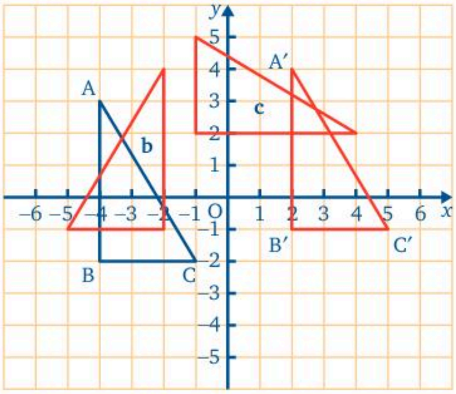


8

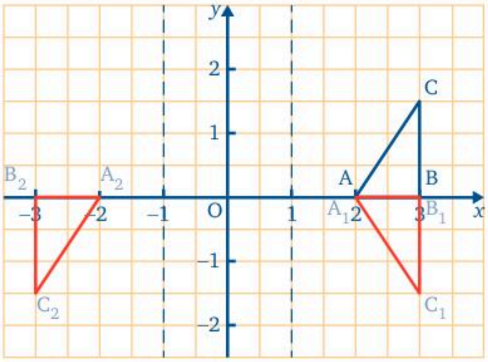


9 translation by $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$, reflection in the *y*-axis

10 a translation by $\begin{pmatrix} 6 \\ 1 \end{pmatrix}$

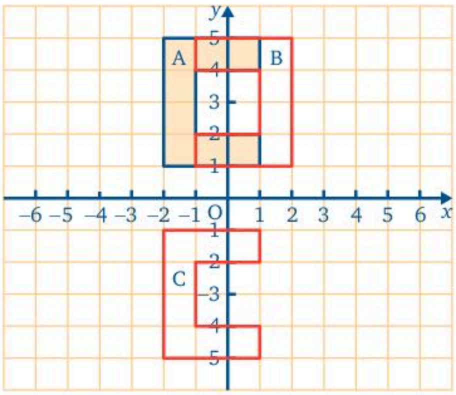


11



rotation about O by 180°

12



c reflection in the *x*-axis or a translation by $\begin{pmatrix} 0 \\ -7 \end{pmatrix}$

Exercise 5h page 109

1 D **2** A **3** C **4** C **5** B **6** C **7** B
8 C **9** C **10** C **11** B

REVIEW TEST 1 page 113

1 a 625 **b i** 3^5 **ii** 4^3 **c** 2.943×10^4
2 a 7300 **b** 0.0607
3 a 3.03 **b** 3.23 **c i** $87\frac{1}{2}\%$ **ii** 0,875
4 a 41_{10} **b** 24_5 **c** 33_9
5 a 103_5 **b** 1110_4 **c** 100100_2
6 a 123_4 **b** 22_3 **c** 1101110_2
7 a $9x$ **b** $4-12x$ **c** $6x^2$ **d** $20x^3$
8 a 19 **b** −23
9 a 8 **b** 38
10 $\frac{a-b}{2}$ **11** 6 **12** 12 **13** 2 **14** 8 cm
15 a $f=E+g$ **b** $p=Qr$ **c** $c=R-3a-2b$
16 a $x<2$ **b** $x<-1$ **c** $x<-6$
17 a $x<3$ **b** $x>2$ **c** $x<\frac{2}{3}$
18 a $x<1$ **b** $x<3$ **c** $x\leqslant 6$
19 a $-2\leqslant x\leqslant 7$ **b** $2<x\leqslant 6$
20 a 120 **b** 120
21 a $d=136°$ **b** $e=42^2, f=138°$
22 a $g=105°$ **b** $i=45°, j=25°, k=65°$
23 a $l=58°, m=122°, n=58°$
b $p=77°, q=35°, r=42°$
24 (6, 9)
25 (1, 3)
26 $\begin{pmatrix} -3 \\ 9 \end{pmatrix}$
27 $\begin{pmatrix} 5 \\ 5 \end{pmatrix}$, yes, yes
28 reflection in the *y*-axis
29 Triangle ABC is rotated about O through 180°.
30 a (−1, −5) **b** (−5, −6) **c** (−5, −3)

CHAPTER 6

Exercise 6a page 119

1	60°	**4**	110°	**7**	55°	**10**	25°	**13**	120°
2	85°	**5**	40°	**8**	60°	**11**	50°	**14**	55°
3	55°	**6**	30°	**9**	75°	**12**	90°	**15**	65°

Exercise 6b page 121

1 60°, 50°
2 65°, 45°
3 70°
4 65°, 115°
5 85°, 30°
6 45°
7 60°
8 60°, 30°
9 90°, 45°

Exercise 6c page 123

11 70°
12 70°
13 65°
14 40°
15 90°
16 110°
17 45°
18 70°
19 60°
20 20°
21 75°
22 86°
27 55°, 70°
28 45°, 135°
29 80°, 80°
30 50°, 80°
31 40°, 140°
32 20°, 70°

Exercise 6d page 127

1 50°
2 80°
3 110°
4 50°
5 60°
6 40°
7 90°
8 60°
9 120°
10 90°
11 110°
12 65°
13 60°, 120°
14 80°, 70°
15 80°, 115°
16 50°, 130°

Exercise 6e page 129

1 no, angles not equal
2 yes
3 no, sides not equal
4 no, $\begin{cases}\text{sides not equal}\\ \text{angles not equal}\end{cases}$
5 no, $\begin{cases}\text{sides not equal}\\ \text{angles not equal}\end{cases}$
6 no, $\begin{cases}\text{sides not equal}\\ \text{angles not equal}\end{cases}$
7 yes
8 no, not bounded by straight lines

Exercise 6f page 131

1 180°
2 360°
3 **a** $p = 100°, r = 135°, x = 55°, q = 125°$
  b 360°
4 **a** $w = 120°, x = 60°, y = 120°, z = 60°$
  b 360°
5 **a** 180° **b** 540° **c** 180° **d** 360°
6 360°
7 **a** equilateral
  b 60°
  c 120°
  d 60°
  e 360°

Exercise 6g page 132

1 60°
2 90°
3 50°
4 50°
5 60°
6 90°
7 95°
8 55°
9 30°
10 125°
11 $x = 50°$
12 $x = 30°$
13 $x = 24°$
14 **a** 5 **b** 8

Exercise 6h page 134

1 36°
2 45°
3 30°
4 60°
5 24°
6 20°
7 40°
8 22.5°
9 18°

Exercise 6i page 135

1 720°
2 540°
3 1440°
4 360°
5 900°
6 1800°
7 2880°
8 1260°
9 2340°

Exercise 6j page 136

1 **a** 3240° **b** 2520° **c** 1620°
2 80°
3 120°
4 110°
5 105°
6 85°
7 110°
8 108°
9 120°
10 135°
11 144°
12 150°
13 162°
14 **a** 18 **b** 24
15 **a** 12 **b** 20
16 **a** yes, 12
  b yes, 9
  c no
  d yes, 6
  e no
  f yes, 4
17 **a** yes, 4
  b yes, 6
  c no
  d yes, 72
  e yes, 36
  f yes, 8

Exercise 6k page 138

1 54°
2 45°
3 150°
4 72°
5 60°
6 50°
7 80°
8 135°
9 100°
10 60°
11 72°

Exercise 6l page 140

1 65°
2 70°
3 80°
4 175°

Exercise 6m page 141

1 $p = 85°, q = 45°$
2 $x = 45°, y = 135°$
3 $u = 55°, v = 125°$
4 51.4°

Exercise 6n page 142

1 D
2 C
3 B
4 C
5 C
6 D
7 B
8 A
9 B
10 A
11 B
12 C

CHAPTER 7

Exercise 7a page 147

	Mean	Mode	Median
1	4.43	2	4
2	14.1	12, 13 and 14	13.5
3	1.84	1.6	1.65
4	1.54	1.3 and 1.8	1.5

5 mean 119.2 median 124
The median; one very low mark brings down the mean.
6 mean \$715 000, mode \$500 000, median \$500 000

Exercise 7b page 149

1

Number of people	0	1	2	3	4	5	6
Frequency	3	6	10	8	13	6	2

a 4 **b** 3
2 2.00 (to 3 s.f)
3 **a** 4.28 **b** 5
4 **a** 3.64 **b** 6
5 **a** 1.57 **b** 1

Exercise 7c page 150

1 **a** 104 **b** **i** 7 **ii** 6.4 **iii** 8
2 1
3 3.5
4 **a** 3.5 **b** 3 **c** 3.48
5 **a** 2 **b** 1 **c** 2.2

Exercise 7d page 153

1 96°, 132°, 60°, 42°, 30°
2 128°, 152°, 48°, 24°, 8°
3 303°, 3°, 30°, 24°
4 84°, 204°, 48°, 24°
5 144°, 48°, 80°, 88°
6 96°, 120°, 36°, 72°, 36°
7 108°, 180°, 40°, 18°, 14°
8 72°, 13.5°, 85.5°, 94.5°, 54°, 40.5°
9 36°, 144°, 120°, 60°

Exercise 7e page 155

1 a business and professional
b i $\frac{1}{12}$
ii $\frac{1}{6}$
2 a air conditioning
b a little less
3 a i $\frac{1}{6}$
ii $\frac{1}{4}$
b 20–39 and 40–59; under 10, 10–19 and 60–79

Exercise 7f page 157

1 a August **b** September **c** November
2 c Although rainfall does fluctuate it is increasing over time.
3 a $250
b

Time after privatisation (years)	Value of share (dollars)
1	300
2	350
3	430
4	520
5	530
6	560
7	590

c the fourth year
d an upward trend
4 a about $620
b The share price is unlikely to maintain a steady $30 rise for two years in a row.
5 a No; half-monthly sales figures would be about half the monthly sales, but sales can fluctuate a lot from week to week.
b The overall sales for July were down on June; if sales increased in the first half of July, they must have slumped badly during the second half to give this downward trend for the month as a whole.

Exercise 7g page 159

1 a 10, 14, 10, 22
b danger
c very effective
2 a French
b 18, 15, 11, 12, 16: total 72
3 a Consumption is rising each year
b Impression is given by the volume in the bottle which goes up more quickly than the height of the bottle.

Exercise 7j page 163

1 a lowest = 2009; highest: 2014
b 2014
c 2014
d 2015, 2016
2 a A
b 2009
c 2011, 2012, 2013
3 a 1993–94
b defence
c 1994–95
d 1995–96
e 1995–96

Exercise 7k page 164

1 b Tuesday
c all of the material blasted during the week had already been taken away
d Monday to Friday, when most lorries left the quarry
2 b Dominica **c** Barbados

CHAPTER 8

Exercise 8a page 168

1 6:5
2 2:1
3 1:3
4 2:3
5 1:2
6 10:1
7 1:3
8 25:12
9 5:1
10 1:3:6
11 1:5:4
12 1:7:11
13 5:2:6
14 5:8:7
15 3:4:5
16 3:5:7
17 9:13:7
18 19:17:13
19 12:1
20 4:3
21 1:3
22 3:4
23 20:7
24 10:11
25 45:14
26 9:7
27 3:2
28 19:23
29 2:3:4
30 6:4:3
31 9:10
32 6:15:14
33 90:15:40
34 45:48:50

Exercise 8b page 169

1 20
2 42
3 1
4 35
5 8
6 72
7 $\frac{2}{5}=\frac{8}{20}=\frac{14}{35}$
8 $\frac{3}{7}=\frac{12}{28}=\frac{24}{56}$
9 $\frac{14}{13}=\frac{42}{39}=\frac{70}{65}$
10 $\frac{7}{11}=\frac{56}{88}=\frac{77}{121}$
11 $\frac{4}{9}=\frac{12}{27}=\frac{60}{135}$
12 $\frac{7}{8}=\frac{49}{56}=\frac{147}{168}$

Exercise 8c page 170

1 3:4
2 7:3
3 $\frac{19}{11}$
4 $\frac{7}{13}$
5 $12:18, \frac{2}{3}:1, \frac{30}{45}:1$
6 $\frac{7}{9}:1, \frac{63}{81}$
7 $5:8, 30:48, \frac{10}{16}:1$
8 $1:\frac{7}{12}, \frac{36}{21}, 2:\frac{7}{6}$

Exercise 8d page 171

1 $30 000
2 $9100
3 1008 cm
4 20.4 m
5 297 g
6 $12 270
7 $49
8 102 mm
9 17.5 km
10 27.6 kg

Exercise 8e page 172

1 $24, $18
2 $220, $330
3 21 cm, 33 cm
4 1.6 m, 2.0 m
5 6 km, 14 km
6 $388, $272
7 14 m, 18 m
8 33 cm, 21 cm
9 99 km, 166 km
10 37 cm, 27 cm

Exercise 8f page 173

1 $15 000
2 $11 256
3 $52 360
4 $6300
5 $2450
6 $246 400
7 308 miles
8 448 km
9 $15\frac{1}{2}l$
10 $828 800
11 $722 400
12 235 pages

Exercise 8g page 176

1 **a** 800 km **b** 1100 km
2 **a** 48 km **b** 84 km **c** 54 km
3 **a** 1200 miles **b** 1650 miles
4 **a** 90 km **b** 135 km
5 **a** 9 miles **b** 15 miles
6 **a** 52.5 m **b** 89.25 m
7 **a** 32 miles **b** 38 miles
8 **a** 4km **b** $2\frac{2}{3}$ km **c** 10 km
9 **a** 37 miles **b** 185 miles
10 **a** 500 m **b** 850 m
11 **a** 1755 miles **b** 4185 miles
12 **a** 30 **b** 72

Exercise 8h page 178

1 **a** 2 hours **b** 3 hours
2 **a** 5 hours **b** $3\frac{1}{4}$ hours
3 **a** $\frac{1}{2}$ hour **b** $1\frac{1}{4}$ hours
4 **a** $2\frac{1}{2}$ hours **b** $5\frac{1}{3}$ hours
5 **a** $1\frac{1}{2}$ hours **b** 5 hours
6 **a** $1\frac{1}{2}$ hours **b** $4\frac{1}{2}$ hours
7 **a** 25 seconds **b** 200 seconds
8 **a** 24 minutes **b** 54 minutes
9 **a** 216 hours = 9 days **b** $5\frac{1}{4}$ days = 126 hours
10 **a** $1\frac{1}{4}$ hours **b** $2\frac{3}{4}$ hours
11 **a** $2\frac{1}{2}$ hours **b** 5 hours 20 minutes
12 **a** $\frac{3}{4}$ hour **b** $3\frac{1}{4}$ hours

Exercise 8i page 180

1 80 km/h
2 60 km/h
3 60 mph
4 120 mph
5 20 m/s
6 45 m/s
7 50 km/h
8 65 km/h
9 35 mph
10 8 mph
11 36 m/s
12 17 m/s
13 80 km/h
14 90 km/h
15 64 km/h
16 120 km/h
17 12 km/h
18 8 km/h
19 18 km/h
20 18 km/h
21 54 mph
22 54 mph
23 60 mph
24 105 mph
25 74.33 km/h
26 138.5 km/h
27 693.3 km/h
28 482 km/h
29 162.86 km/h
30 253.62 km/h
31 102.97 km/h

Exercise 8j page 182

1 9 km/h
2 10 mph
3 7 mph
4 7 mph
5 75 km/h
6 200 km/h
7 3 knots

Exercise 8k page 183

1 4 : 5
2 6 : 5
3 8 : 5
4 60
5 216 m
6 $420 : $600
7 64 and 40
8 $8\frac{1}{2}l$
9 $3720
10 $3936
11 14km/h
12 **i** 48min **ii** 27min
13 375 m/min
14 **i** 18min **ii** 1 h 36min
15 **a** 1950 miles **b** 5 h **c** 390 mph
16 16.8mph

Exercise 8l page 184

1 9 : 7
2 4 : 7
3 11 : 24
4 16
5 25 : 49
6 324 g
7 $336
8 $800
9 425 miles
10 Jess $3920, Chad $6160
11 1950 miles
12 96km/h
13 221 km
14 6 mph
15 **a** $8\frac{1}{3}$m/s **b** 500 m/min
c 30000m/h **d** 30km/h
16 60km/h

Exercise 8m page 185

1 A
2 C
3 C
4 D
5 C
6 B
7 A
8 B
9 A
10 A
11 B
12 A
13 B
14 C
15 B
16 C
17 B
18 D
19 C
20 D
21 C
22 B

CHAPTER 9

Exercise 9a page 190

1 $93 200
2 $2213.50
3 $3902 750
4 $14 562.50
5 $8496
6 $15 552
7 **a** $2160 **b** $15 660
8 **a** $75 725 **b** $78 000 **c** $2275
9 **a** $139 200
b no
c it should be $120 000 × 1.175 = $141 000. The manager found $139 200 × 1.5% and added it to $139 200.

Exercise 9b page 191

1 $3600
2 $20 000
3 $4800
4 $680
5 $22 000
6 $2800
7 $2600
8 $10 500
9 $1445
10 $4250
11 **a** $1800 **b** $3000

Exercise 9c page 193

1 $2000
2 $2400
3 $1200
4 $8000
5 $21000
6 $20000
7 $60000
8 $48000
9 $1634 500
10 $117000
11 $300000
12 $2240 00

13 $316000
14 $9456000
15 $5686000
16 $23329000
17 $1980000
18 $19036000
19 $394000
20 $336000
21 $26902000
22 $16356000
23 $7201000
24 $15258000
25 $3986000
26 $2630
27 $679
28 $16438
29 $82815
30 $57694
31 $10990

Exercise 9d page 195

1 $42000
2 $436800
3 $728000
4 $615600
5 $951200
6 $1173700
7 $627300
8 $758872
9 $944640
10 $265050
11 $349125
12 $1000065
13 $223932
14 $747658
15 $932319
16 $418568

Exercise 9e page 196

1 $4200
2 $7632
3 $10388
4 $19177
5 $14399
6 $20672
7 $25268
8 $96800000
9 $7604
10 $43740
11 $1280000

Exercise 9f page 198

1 **a** $3000 **b** 28
2 **a** $12675 **b** $80
3 $7380
4 $2085
5 $664

Exercise 9g page 200

1 **a** $65949 **b** $9933 **c** $2914
2 yes, on 29–30 August
3 **a** August 30 **b** 29 August
4 $686
5 $53712
6 paid in at different times
7 monthly
8 $69413.00
9 **a** $12343.00 **b** $396
10 **a** 68 **b** $83002.00

Exercise 9h page 202

1 $1488
2 $3024
3 $8928
4 $10652
5 $9312
6 $39930
7 $5980
8 $89946
9 $75082
10 $78770
11 $888
12 $20976
13 $468
14 $50800
15 $426
16 **a** $648 **b** 25.7%
17 $2040
18 $5078, 20.7%
19 $819.50
20 Option 1 by $8580
21 $36480, 32%
22 $62940
23 **a** $532.62 **b** $684.80 **c** 8%
24 $1250 3 s.f.
25 $514 3 s.f.
26 $1574
27 Option 1 by $2910
28 **a** $2482.03 **b** $3718.75

Exercise 9i page 206

1 $2520
2 $55920
3 $29375
4 $8184
5 $1062.50
6 $1140000
7 $871250
8 $77000
9 $76375

Exercise 9j page 206

1 7.69% 3 s.f.
2 $100000
3 $14800 3 s.f.
4 $10900 3 s.f.
5 $122000 3 s.f.
6 $8090

Exercise 9k page 207

1 B **2** A **3** C **4** C **5** A

CHAPTER 10

Exercise 10a page 209

1 **a** 20000 cm^2 **b** 80000 cm^2 **c** 35000 cm^2 **d** 55000 cm^2
2 **a** 2000 mm^2 **b** 400 mm^2 **c** 550 mm^2 **d** 1250 mm^2
3 **a** 430 cm^2 **b** 43000 mm^2
4 **a** 6 cm^2 **b** 39 cm^2
5 **a** 0.75 m^2 **b** 30400 m^2
6 **a** 25 km^2 **b** 0.62 km^2 **c** 0.07 km^2

Exercise 10b page 210

1 0.288 cm^2
2 420 cm^2
3 **a** 20400 cm^2 **b** 2.04 m^2
4 5 cm^2
5 0.8 cm
6 **a** 4.5 m **b** 450 cm
7 400 m^2
8 43 cm^2
9 38 cm^2
10 71 cm^2
11 800 cm^2 shaded, 1000 cm^2 unshaded

Exercise 10c page 212

1 84 cm^2
2 600 cm^2
3 37.2 cm^2
4 0.0288 m^2
5 12.8 m^2
6 1736 m^2
7 24.48 cm^2
8 7 cm^2
9 38.88 cm^2
10 28.8 cm^2
11 26.4 cm^2
12 352 cm^2
13 63 cm^2
14 11.25 cm^2
15 130 cm^2
16 48 cm^2
17 36 cm^2
18 180 cm^2
19 12 cm, 6 cm, 7 cm, 12 cm

Exercise 10d page 216

1 48 cm^2
2 1.56 m^2
3 80 cm^2
4 3.2 cm^2
5 100 cm^2
6 399 cm^2
7 2.4 cm, 12 cm, 25 cm
8 24 cm^2
9 14.4 cm^2
10 40 cm^2
11 32.4 m^2
12 22.2 cm^2

13 45 cm^2
14 44 cm^2
15 64 cm^2
16 540 cm^2
17 33 cm^2
18 75 cm^2
19 70 cm^2
20 24.4 cm^2
21 82.5 cm^2
22 30 cm^2
23 96 cm^2
24 21 cm^2
25 8.32 cm^2

Exercise 10e page 218

1 8 cm
2 6 cm
3 6 cm
4 20 cm
5 3 cm
6 36 cm
7 3 cm
8 $2\frac{2}{3}$ cm
9 0.4 cm
10 6 cm
11 8 cm
12 4 cm

Exercise 10f page 221

1 42 cm^2
2 94.5 cm^2
3 21 cm^2
4 8.75 cm^2
5 30 sq. units
6 33 sq. units
7 56 sq. units
8 16 sq. units
9 84 sq. units
10 47 sq. units

Exercise 10g page 223

1 54 cm^2
2 216 cm^2
3 **a** 25 cm^2 **b** 5 cm
4 **a** 16 cm^2 **b** 96 cm^2
5 164 cm^2
6 **a** 50 cm^2 **b** 20 cm^2 **c** 220 cm^2
7 **a** 30 cm^2 **b** 5560 cm^2
8 **a** 21 000 cm^2 **b** 15 000 cm^2
9 28 cm^2
10 **a** 6 m^2 **b** 64 m^2
11 84 m^2
12 **a** 112 cm^2 **b** 16 cm^2
13 6 tins (SA is 7.095 m^2, 3 coats 21.285 m^2 needs 5.32 tins)

Exercise 10h page 225

1 D
2 B
3 C
4 D
5 D
6 C
7 D
8 C
9 C
10 C
11 C
12 B
13 A
14 D
15 C
16 A
17 D
18 D
19 C
20 B
21 D
22 B
23 B
24 C
25 D

CHAPTER 11

Exercise 11a page 230

1 **a** 300 cm^3 **b** 10.4 m^3
2 **a** 343 cm^3 **b** 0.27 cm^3 **c** $\frac{8}{27}$ cm^3
3 **a** 216 cm^3 **b** 27
4 **a** 3200 000 cm^3 **b** 3200 000 000 mm^3
5 **a** 6000 mm^3 **b** 4000 000 mm^3
6 **a** 3000 cm^3 **b** 750 cm^3
7 **a** 8l **b** 0.759l **c** 2000l **d** 46l
8 300 cm^3
9 9.6 cm^3
10 24

Exercise 11b page 232

1 720 cm^3
2 2160 cm^3
3 720 cm^3
4 660 cm^3
5 2880 cm^3
6 5.184 m^3
7 **a** parallelogram **b** 28 cm^2 **c** 280 cm^3
8 **a** 12 cm **c** 90 cm^2
b parallelogram **d** 540 cm^3

Exercise 11d page 236

1 2400 cm^3
2 315 cm^3
3 624 cm^3
4 450 cm^3
5 864 cm^3
6 690 cm^3
7 720 cm^3
8 **a** 35 000 cm^3 **b** 0.035 m^3
9 17 280 cm^3
10 18 816 cm^3
11 **a** 50 cm^2 **b** **i** 10000 cm^3 **ii** 10l
12 **a** 565.125 cm^3 **b** 8.27 cm (3 s.f.)
13 18.9 cm^3
14 528 m^3
15 1344 cm^3
16 624 m^3
17 540 cm^3

Exercise 11e page 240

1 **a** 3200000 cm^3 **b** 3200 000 000 mm^3
2 1600 cm^3
3 64 cm^3
4 50 000 cm^3
5 13 500 mm^3
6 504 cm^3

Exercise 11f page 240

1 **a** 8000 mm^3 **b** 0.000 008 m^3
2 3.5l
3 300 cm^3
4 0.512 cm^3
5 120000 cm^3
6 1200 cm^3

Exercise 11g page 241

1 **a** 9000 cm^3 **b** 9000 000 mm^3
2 440 cm^3
3 216 cm^3
4 288 cm^3
5 2400l
6 12

Exercise 11h page 241

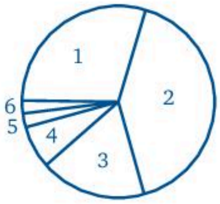
1 0.009 m^3
2 10.8l
3 75l
4 8 cm^3
5 1.2 m^3
6 **a** 8 m^2 **b** 40 m^3

Exercise 11i page 242

1 D
2 B
3 C
4 B
5 B
6 A
7 B
8 D
9 B
10 C
11 A
12 A

REVIEW TEST 2 page 244

1 **a** 60° **b** 60° **c** 120°
2 **a** 94° **b** 43°
3 156°
4 **a** yes **b** no **c** yes
5 $p = 75°$, $q = 70°$
6 AB and AC
7 $p = 114°$, $q = 33°$, isosceles triangle
8 pentagon, 42°
9 **a** 24° **b** 156°
10 1980°
11 mode 2.8, mean 2.58, median 2.6
12 **a** 45 **b** 98 **c** 2.18 **d** 2 **e** 2
13 Number of occupants in the cars in a survey



14 **a** **i** 38 **ii** 3
b 62.5 **c** 47 **d** 20%

15 **a** **i** $\frac{1}{4}$ **ii** $\frac{5}{24}$
b $\frac{2}{3}$
c **i** 30 **ii** 20 **iii** 55

16

	Number of people using various means of transport
Walk	🯅 🯅 🯅
Bus	🯅 🯅 🯅 🯅 🯅 🯅 🯅 🯅
Car	🯅 🯅 🯅 🯅 🯅 🯅 🯅
Bicycle	🯅

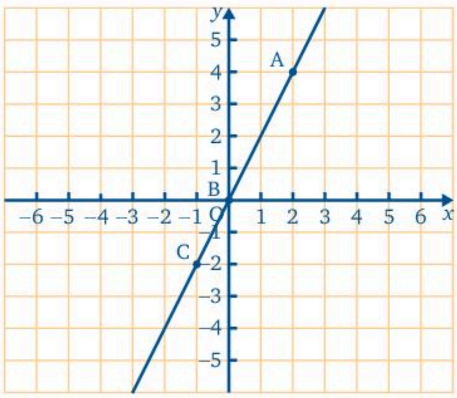
🯅 Represents 5 persons

17 **a** 5 : 8 **b** 9 : 2
18 **a** \$12 000 **b** 24 cm
19 **a** 120 km, 192 km **b** 16
20 \$2520 **23** \$5250
21 \$62 640 **24** \$5400
22 \$5400
25 **a** \$427 500 **c** \$405 360
b \$150 000 **d** \$127 860
26 **a** 0.92 m² **b** 0.47 km²
27 3 cm² **28** 37.8 cm²
29 **a** 42 cm² **b** 4 cm
30 84 cm²
31 **a** 54 cm² **b** 24 cm² **c** 228 cm²
32 **a** 5000 cm³ **b** 40 000 cm³ **c** 1600 mm³
33 **a** 74 cm² **b** 1480 cm³
34 **a** 600 000 cm³ **b** 600 *l*
35 **a** 1536 cm³ **b** 1.536 *l*

CHAPTER 12

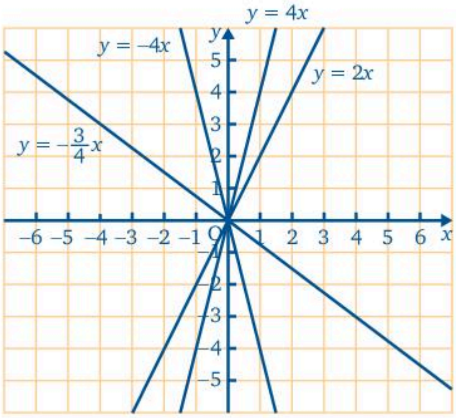
Exercise 12a page 251

1 **a**



b yes
c no
2 a 3 **b** 0 **c** −6 **d** 12
3 a 0 **b** $-\frac{2}{3}$ **c** $\frac{1}{3}$ **d** −3

4–7

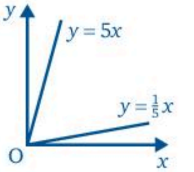


Exercise 12b page 253

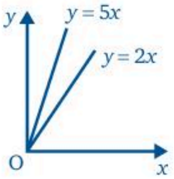
1 **a** 2 **b** 2 **c** 2
2 **a** −4 **b** −4 **c** −4
3 **a** 3 **b** 3 **c** 3
4 **a** −4 **b** −4 **c** −4
5 2.5
6 −0.5
7 **a** $y = 0.5x$ **b** $y = 2x$
$y = -3x$ $y = 2.5x$
$y = -4x$ $y = 3x$
8 **a** + **c** + **e** −
b − **d** − **f** −

Exercise 12c page 255

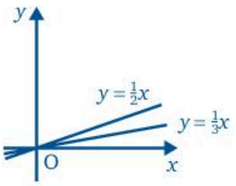
1 $y = 5x$



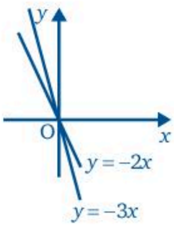
2 $y = 5x$



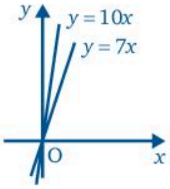
3 $y = \frac{1}{2}x$



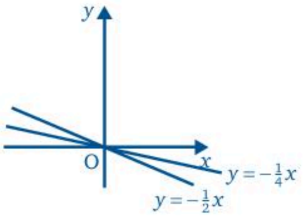
4 $y = -3x$



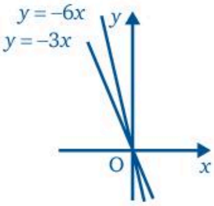
5 $y = 10x$



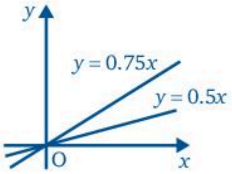
6 $y = -\frac{1}{2}x$



7 $y = -6x$



8 $y = 0.75x$



9 acute
10 obtuse
11 obtuse
12 acute
13 acute
14 acute
15 acute
16 acute
17 obtuse
18 obtuse
19 obtuse
20 obtuse

Exercise 12d page 256

1 4
2 −2
3 1
4 $-\frac{3}{4}$
5 $-\frac{9}{5}$
6 $\frac{2}{3}$
7 0
8 y-axis; you find yourself dividing by zero
9 **a** parallel to the y-axis
  b zero gradient
  c zero gradient
  d parallel to the y-axis
10 e.g. (1, −2), (3, 4)
11 4
12 **a** 4 **b** −3 **c** 1 **d** $\frac{1}{2}$

Exercise 12e page 258

1 **a** 60 **b** 60 km per hour
2 **a** $\frac{1}{140}$ **b** 140 Jamaican dollars per US dollar
3 **a** 400 **b** 400 ml per second

Exercise 12f page 261

1 gradient 2, y-intercept −2
2 gradient −2, y-intercept 4
3 gradient 3, y-intercept −4
4 gradient $\frac{1}{2}$, y-intercept 3
5 gradient $-\frac{3}{2}$, y-intercept 3
6 gradient 2, y-intercept 5
7 gradient −2, y-intercept −7
8 gradient −3, y-intercept +2

Exercise 12g page 262

1 2, 4
2 5, 3
3 3, −4
4 1, −6
5 −2, 3
6 −4, 2
7 5, 2
8 $\frac{1}{2}, -1$
9 $-\frac{1}{3}, 4$
10 3, −7
11 −3, 7
12 $\frac{1}{3}, 7$
13 −0.4, 9
14 5, 4
15 $2, 2\frac{1}{2}$
16 $\frac{1}{3}, -2$
17 $\frac{2}{5}, 1$
18 $-\frac{3}{4}, 2$
19 $y = 2x + 7$
20 $y = 3x + 1$
21 $y = x + 3$
22 $y = 2x - 5$
23 $y = \frac{1}{2}x + 6$
24 $y = -2x + 1$
25 $y = x - 2$
26 $y = -\frac{1}{2}x + 4$

Exercise 12h page 264

1 $-\frac{4}{3}, 4; y = -\frac{4}{3}x + 4$
2 $-2, 7; y = -2x + 7$
3 $\frac{3}{5}, 1; y = \frac{3}{5}x + 1$
4 $-\frac{4}{3}, 2; y = -\frac{4}{3}x + 2$
5 $\frac{7}{2}, -4; y = \frac{7}{2}x - 4$
6 $\frac{1}{3}, -1; y = \frac{1}{3}x - 1$
7 $\frac{1}{6}, 1; y = \frac{1}{6}x + 1$
8 $\frac{4}{5}, -3; y = \frac{4}{5}x - 3$
9 $\frac{5}{3}, -4; y = \frac{5}{3}x - 4$
10 $-1, -5; y = -x - 5$
11 $2, 12; y = 2x + 12$
12 $\frac{5}{6}, 6; y = \frac{5}{6}x + 6$
13 AB, $5y = 2x + 20$;
AC, $3y = -5x + 12$
14 $3, y = 3x - 11$
15 $-3, y = -3x + 7$
16 $\frac{5}{2}, y = \frac{5}{2}x - \frac{1}{2}$
17 $2, y = 2x + 7$
18 $5, y = 5x - 21$
19 $-1, y = -x + 3$
20 $-1, y = -x + 1$
21 $2, y = 2x - 11$
22 $\frac{1}{5}, y = \frac{1}{5}x - \frac{6}{5}$
23 $-\frac{5}{4}, \frac{x}{4} + \frac{y}{5} = 1$ or $y = -\frac{5}{4}x + 5$
24 $-\frac{2}{3}, \frac{x}{3} + \frac{y}{2} = 1$ or $y = -\frac{2}{3}x + 2$
25 $\frac{2}{3}, \frac{x}{3} - \frac{y}{2} = 1$ or $y = \frac{2}{3}x - 2$
26 $-3, \frac{x}{2} + \frac{y}{6} = 1$ or $y = -3x + 6$
27 $3, y = 3x - 10$
28 $-1, y = -x + 4$
29 $\frac{7}{2}, y = \frac{7}{2}x - 6$
30 $-1, y = -x + 3$
31 $\frac{5}{2}, -\frac{x}{2} + \frac{y}{5} = 1$ or $y = \frac{5}{2}x + 5$

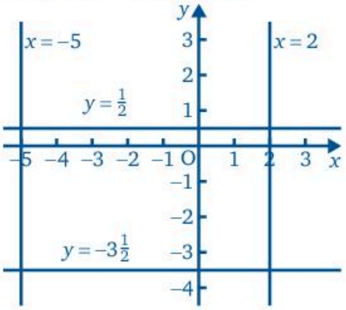
32 $\frac{2}{11}, y=\frac{2}{11}x+\frac{45}{11}$

33 $1, y=x-1$

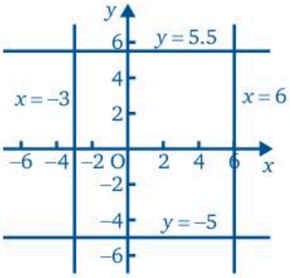
34 $-\frac{1}{4}, y=-\frac{1}{4}x+\frac{11}{4}$

Exercise 12i page 266

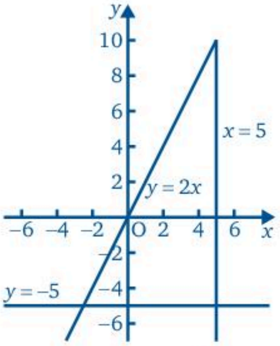
1



2

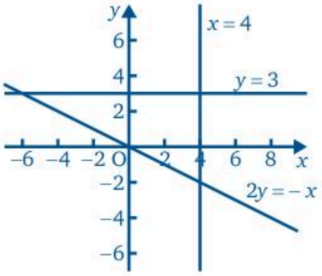


3



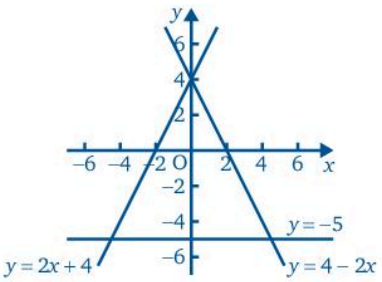
(5, 10), (5,−5), (−2.5, −5)
a right-angled triangle

4



(4, 3), (4, −2), (−6, 3)
a right-angled triangle

5



(0, 4), (4.5, −5), (−4.5, −5)
an isosceles triangle

Exercise 12j page 267

1 **a** 2 **c** 2 **e** $\frac{1}{2}$
b −1 **d** −1

2 $y = 3x + 1, y = 5 + 3x, y = 3x - 4$

3 $y = 2 - x, y = 4 - x, 2y = 3 - 2x, y = -x + 1, y = -x$

4 $3y = x, y = \frac{1}{3}x + 2, y = \frac{1}{3} + \frac{1}{3}x, y = \frac{1}{3}x - 4$

5 $y = \frac{1}{2}x + 2$ and $y = \frac{1}{2}x - 1$; $y = 2 - \frac{1}{2}x$ and $2y = 3 - x$

6 2; $y = 2x + 3$

7 −3; $y = -3x + 1$

8 $y = 4x$

9 e.g. $y = 6 - x, y = -x, y = -2 - x$

10 **a** $y = 4x + 4$ **c** $y = \frac{1}{2}x + 4$
b $y = -3x + 4$

11 **a** $y = \frac{1}{3}x + 6$ **c** $y = \frac{1}{3}x - 3$
b $y = \frac{1}{3}x$

12 **a** $y = 2x + 2$ **c** $y = 2x - 4$
b $y = 2x + 10$

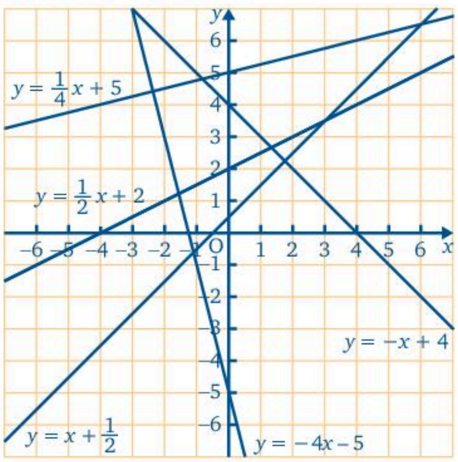
13 $y = 3 + 2x$ and $y = 2x - 3$

14 −3, 4; 4, −3; $y = -3x - 3$

15 **a** $y = -4x$ **b** $y = -4x - 7$

Exercise 12k page 269

1



2 yes, perpendicular
3 neither
4 yes, perpendicular
5 yes, parallel
6 yes, perpendicular
7 yes, perpendicular
8 1, $y = 2 - x$
9 2, $y = \frac{1}{2} - \frac{1}{2}x$
10 $y = 2x$
11 $y = x - 5$, $y = x + 1$ are *parallel* and $y = 2 - x$ is perpendicular to both $y = x - 5$, and $y = x + 1$
12 **a** −3, 3 and 3, 4 **b** $y = \frac{1}{3}x + 4$
13 **a** $y = 1 - \frac{2}{3}x$ **b** $y = \frac{3}{2}x$

Exercise 12l page 271

1 $-\frac{3}{5}$
2 $-\frac{1}{3}$
3 $\frac{1}{4}$
4 −1
5 −2
6 $-\frac{1}{3}$
7 $\frac{1}{3}$
8 2
9 −1 in each case

Exercise 12m page 272

1 $-\frac{3}{4}$
2 $-\frac{3}{5}$
3 $\frac{1}{2}$
4 −2
5 2
6 $\frac{3}{4}$
7 **a** (2, 0), (0, 4) **b** (12, 0), (0, −9)
8 **a** $\frac{x}{6} + \frac{y}{5} = 1$ **b** $\frac{x}{4} - \frac{y}{3} = 1$
9 $-\frac{1}{3}$

Exercise 12n page 273

1 $-\frac{3}{5}, 3$
2 $-\frac{1}{3}, 2$
3 $\frac{1}{4}, -2$
4 $\frac{1}{3}, -2$
5 3, 6
6 $-\frac{1}{3}, 2$
7 $-\frac{3}{4}, 3$
8 $-\frac{3}{5}, 3$
9 $\frac{1}{2}, -2$
10 −3, 6
11 $-\frac{4}{3}, 4$
12 $\frac{4}{3}, -4$
13 4, 2
14 −1, 4
15 −2, 4
16 $-\frac{2}{5}, 3$
17 $-\frac{1}{2}, 5$
18 $2, \frac{5}{2}$
19 2, −4
20 −1, −3
21 $-\frac{3}{4}, 3$

Exercise 12p page 274

1 $y = 3x - 16$
2 square
3 rhombus
4 $\left(\frac{1}{2}, 3\right)$
5 midpoint is (5, 3); $y = -2x + 13$
6 $2y = -x + 4$
7 square

Exercise 12q page 276

1 $1\frac{1}{2}, 4\frac{1}{2}$
2 $1\frac{1}{3}, 3\frac{2}{3}$
3 $1\frac{1}{2}, 5\frac{1}{2}$
4 $-\frac{1}{2}, 1\frac{1}{2}$
5 $\frac{1}{2}, 2$
6 $1\frac{1}{2}, 3\frac{1}{2}$
7 $2\frac{2}{5}, \frac{9}{10}$
8 $-\frac{2}{5}, 1\frac{3}{5}$
9 $2\frac{2}{5}, 1\frac{1}{5}$
10 $\frac{1}{3}, 1\frac{2}{3}$
11 the lines are parallel
12 it is the point of intersection of the two lines

Exercise 12r page 276

1 −3
2 no
3 $y = -4x$
4 (0, 4)
5 (6, 0) and (0, 6)
6 $-\frac{3}{2}$
7 $y = \frac{1}{2}x$
8 (2, 0), (0, 3)

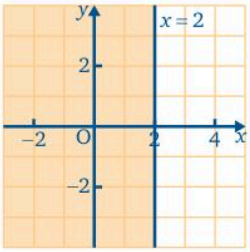
Exercise 12s page 277

1 C **2** B **3** D **4** A **5** A

CHAPTER 13

Exercise 13a page 281

1



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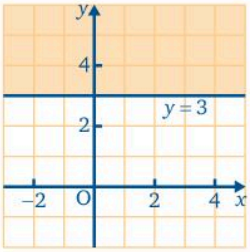

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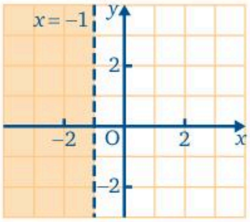

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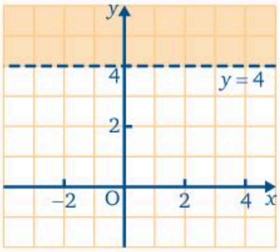

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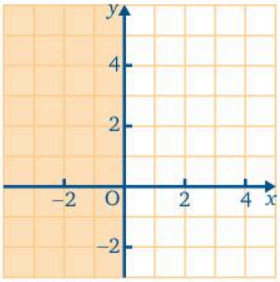

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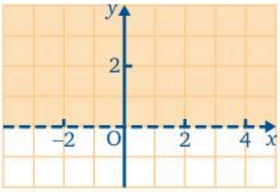

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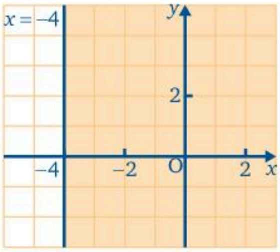

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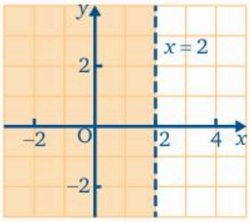

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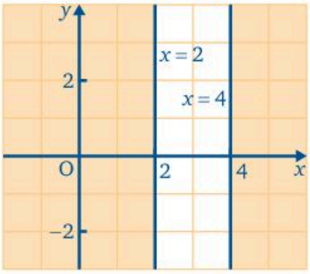

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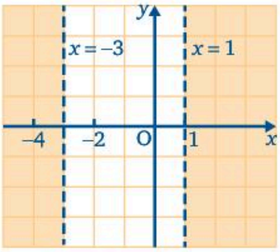

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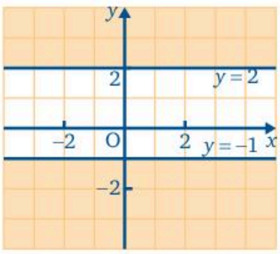

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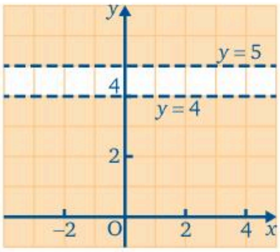

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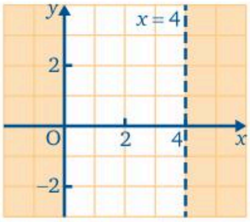

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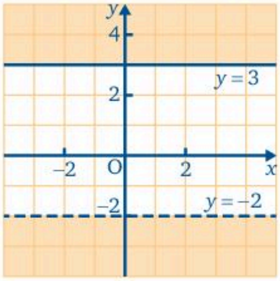

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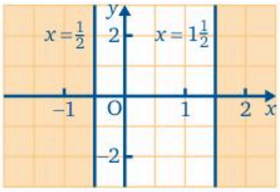

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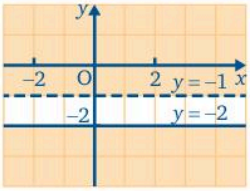

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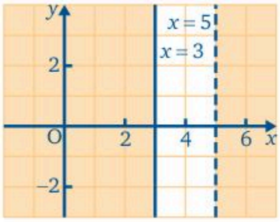


18 9: no 10: no 11: no

19 $x \leqslant 2$

20 $y < 3$

21 $x < -1$

22 $-2 \leqslant y \leqslant 2$

23 $-1 \leqslant x < 2$

24 $-\frac{1}{2} < y < 2\frac{1}{2}$

25 19: yes 20: yes 21: no 22: yes 23: no 24: no

26 $-3 \leqslant x \leqslant 1$

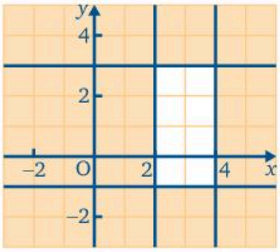
27 $-4 \leqslant y \leqslant -1$

28 $2 \leqslant y < 3$ **29** $3 \leqslant x \leqslant 6$

30 26: yes 27: no 28: yes 29: no

Exercise 13b page 284

1



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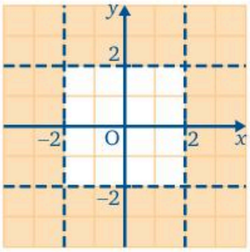

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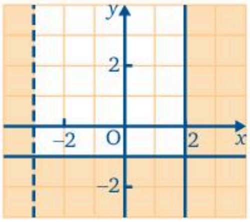

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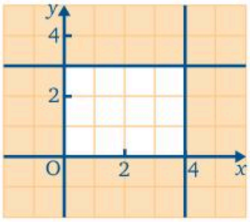

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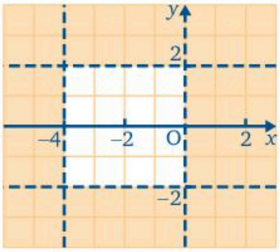

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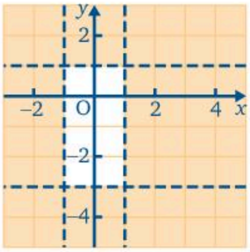

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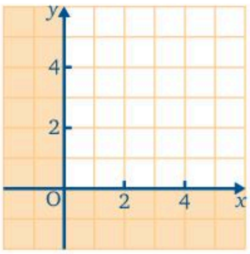

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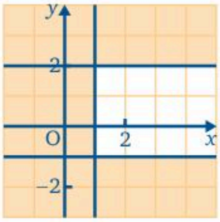


9 $-2 \leqslant x \leqslant 3, -1 \leqslant y \leqslant 2$

10 $-2 < x \leqslant 2, -2 \leqslant y \leqslant 1$

11 9: yes 10: no

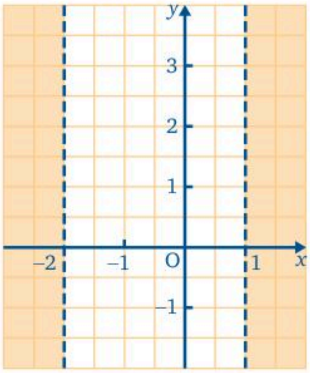
12 $-2 \leqslant x \leqslant 1, y \geqslant -1$

13 $x < 0, y > 0$

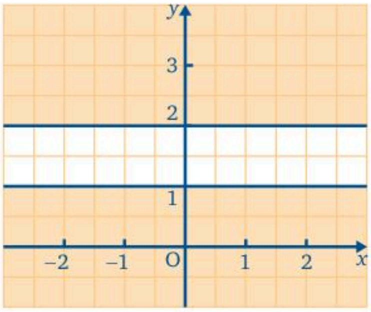
14 $-2 < x < 2, -2 < y < 2$

15 $1 < x < 3, \quad 1 < y < 3$

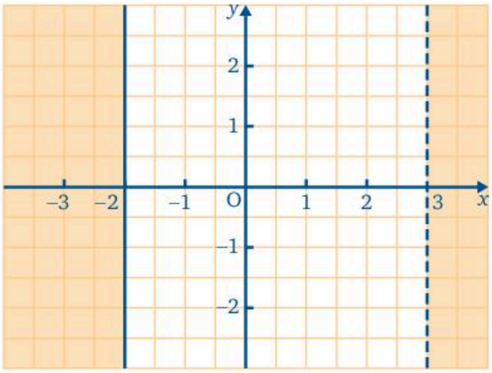
16 $-2 < x < 1$



17 $1 \leqslant y \leqslant 2$



18 $-2 \leqslant x < 3$



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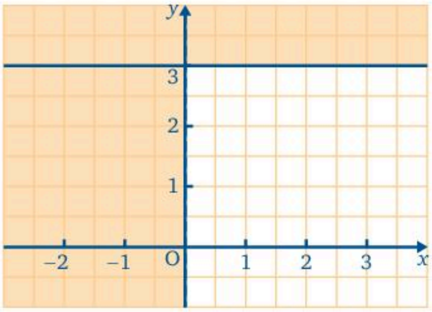

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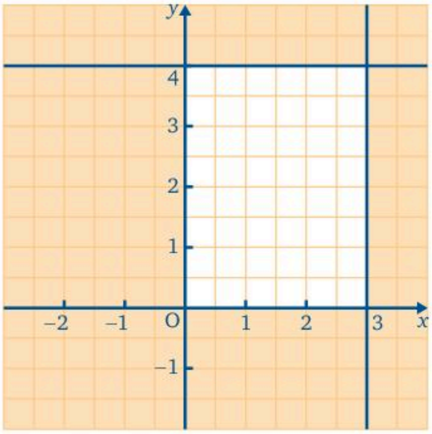


Exercise 13c page 286

1 $x - y \geqslant 1$
2 $2x + y < 4$
3 $2x + y \geqslant 2$
4 $y - 2x > 2$
5 $y - 4x \geqslant 4$
6 $4y + x + 1 < 0$

CHAPTER 14

Exercise 14a page 291

1 110°
2 60°
3 70°
4 40°
5 70°
6 55°
7 90°
8 35°
9 110°
10 95°

Exercise 14b page 292

1 4.2 cm, 57°, 83°
2 4.6 cm, 97°, 48°
3 6.5 cm, 70°, 40°
4 8.5 cm, 97°, 33°
5 3.8 cm, 52°, 83°
6 4.8 cm, 79°, 53°
7 4.3 cm, 53°, 62°
8 5.7 cm, 53°, 75°
9 6.4 cm, 38°, 69°
10 6.2 cm, 44°, 80°

Exercise 14c page 293

Teacher to check

Exercise 14d page 294

Teacher to check

Exercise 14e page 295

Teacher to check

Exercise 14f page 296

11 90°
12 45°
14 They are parallel.

Exercise 14g page 297

2 **a** AC = 7.97 cm (2 d.p.) **b** BC = 4.6 cm **c** 90°
5 90°, yes (180° − 56° − 34° or angles of a triangle)
6 ∠M = 59°, ∠N = 45°, ∠ is greatest (greatest angle opposite longest side)
7 BC = 7.5 cm, they are parallel
8 **c** AD = CD = 4.6 cm, △ABC is isosceles
9 **a** 7.1 cm **b** 3.2 cm

CHAPTER 15

Exercise 15a page 300

1 **a** {teachers in my school}
 b {books I have read}
3 **a** odd numbers up to 9
 b the days of the week from Monday to Friday
4 **a** {European countries}, France
 b {multiples of 10}, 60
5 John ∈ {boys' names}
6 English ∈ {school subjects}
7 June ∉ {days of the week}
8 Monday ∉ {domestic furniture}
9 false **10** true **11** true **12** true

Exercise 15b page 301

1 infinite
2 infinite
3 finite
4 infinite
5 5
6 8
7 6
8 21
9 11
10 no
11 yes
12 yes
13 no
14 yes
15 yes
16 no

Exercise 15c page 303

1 cutlery
2 whole numbers less than 50
3 whole numbers less than 25
4 $n(A) = 8, n(B) = 6$
 $B = \{3, 6, 9, 12, 15, 18\}$
5 $A = \{1, 2, 3, 4, 6, 12\}$ $B = \{2, 3, 5, 7, 11, 13\}$
 $C = \{6, 12\}$
6 $n(A) = 4, n(B) = 2, n(C) = 5$

Exercise 15d page 304

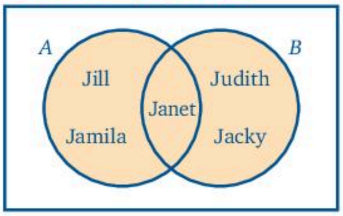
1 {Joy, Anora}, {Joy, Tissha}, {Tissha, Anora}
2 $A = \{1, 3, 5, 7, 9, 11, 13, 15\}$,
 $B = \{2, 3, 5{,}7, 11, 13\}$, $C = \{3, 6, 9, 12, 15\}$
 yes, 3, 5, 7, 11, 13
3 $B = \{6, 12, 18\}$, $C = \{2\}$,
 $D = \{13, 14, 15, 16, 17, 18, 19, 20\}$
4 4: { }, {Ava}, {Betty}, {Ava, Betty}; 3
5 7: { }, {2}, {4}, {6}, {2, 4}, {2, 6}, {4, 6}
6 16: { }, {p}, {q}, {r}, {s}, {p, q}, {p, r}, {p, s}, {q, r}, {q, s}, {r, s}, { p, q, r}, {p, q, s}, {p, r, s}, {q, r, s}, {p, q, r, s}

Exercise 15e page 305

1 your own answers
2 my friends who do not like coming to my school
3 my friends who like coming to my school
4 all pupils at my school except those who are not my friends and like coming to school
5 pupils who like coming to school or who are not my friends
6 my friends who like coming to school, and the pupils who are not my friends and do not like coming to school

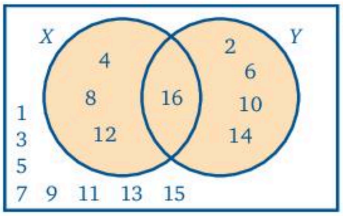
Exercise 15f page 306

1



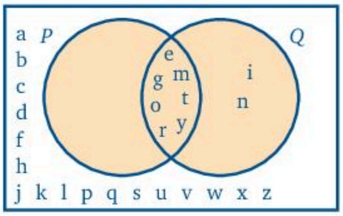
$A \cup B = \{\text{Janet, Jill, Jamila, Judith, Jacky}\}$

2



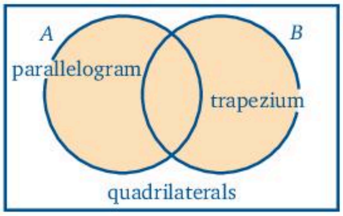
$X \cup Y = \{2, 4, 6, 8, 10, 12, 14, 16\}$

3



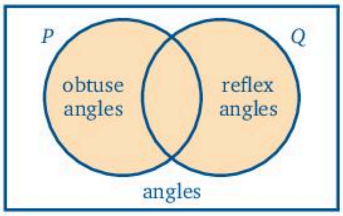
$P \cup Q = \{e, g, i, m, n, o, t, r, y\}$

4 a



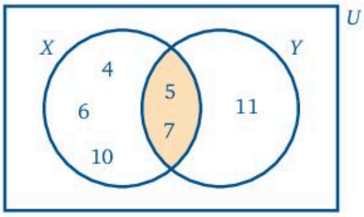
$A \cup B = \{\text{all parallelograms and trapeziums}\}$

b



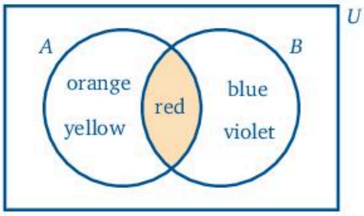
$P \cup Q = \{\text{angles that are either obtuse or reflex}\}$

5



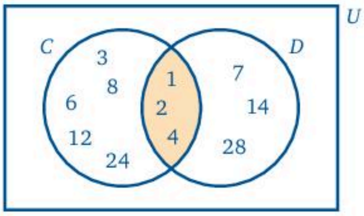
$X \cap Y = \{5, 7\}$

6



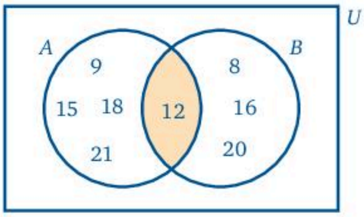
$A \cap B = \{\text{red}\}$

7



$C \cap D = \{1, 2, 4\}$

8

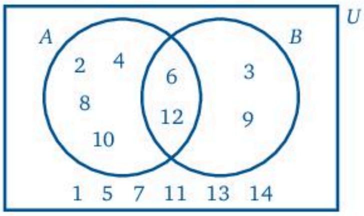


$A \cap B = \{12\}$

Exercise 15g page 308

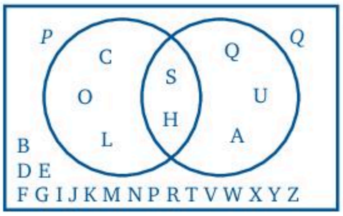
1 a Lenny, Sylvia
  b Adam, Richard
  c Jack, Scott, Lee
2 a David, Joe, Tariq, Paul
  b Tariq, Paul
  c Claude, Alan, Clive
3 a Emma, Majid, Clive, Sean, Ann
  b Emma, Majid, Clive
  c Sean, Ann

4



a 6, 12
b $n(A) = 6, n(B) = 4$

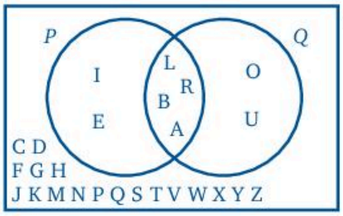
5



a 5 **b** 8 **c** 2

6 a 26, 6, 6

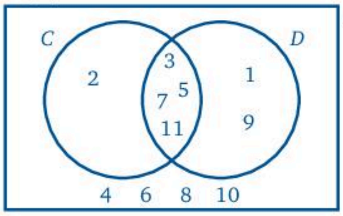
b



c i 4 **ii** 8

7 a 11, 5, 6

b



i 4 **ii** 7

8 a 35, 8, 5

b i 2 **ii** 11

9 10

Exercise 15h page 311

1 C
2 C
3 D
4 D
5 C
6 A
7 B
8 A
9 B
10 D

CHAPTER 16

Exercise 16a page 315

1 yes
2 no
3 yes
4 yes
5 no
6 yes
7 yes
8 yes
9 yes
10 yes
11 yes
12 no

Exercise 16b page 316

1 It is not cold today
2 $2+3 \neq 7$
3 Jayden did not say ‘Close the door’.
4 Anna does not play basketball.
5 Harry does play football.
6 I do not eat sweet potatoes.
7 I do like lettuce.
8 $15 \ngtr 10$
9 The Earth does not revolve round the Sun.
10 When $a = b$ then $b \neq a$.

Exercise 16c page 317

1 I play football and I play cricket.
2 Kingston is the capital of Jamaica and apples grow on trees.
3 The Red Sox is a basketball team and Manchester United is a soccer team.
4 The sky is blue and the sea is pink.
5 Macaroni cheese contains pasta and spaghetti grows on trees.

Exercise 16d page 317

1 I play football or I play cricket.
2 Kingston is the capital of Jamaica or apples grow on trees.
3 The Red Sox is a basketball team or Manchester United is a soccer team.
4 The sky is blue or the sea is pink.
5 Macaroni cheese contains pasta or spaghetti grows on trees.

Exercise 16e page 318

1 a I do not wear trainers
  b I wear trainers or I wear boots
  c I do not wear trainers and I wear boots
  d I do not wear trainers and I do not wear boots
  e I do not wear trainers or I do not wear boots
2 a $\sim p$ **b** $p \wedge q$ **c** $p \wedge \sim q$ **d** $p \vee q$ **e** $\sim p \vee \sim q$
3 a $p \wedge q$ **b** $\sim p \wedge q$ **c** $\sim q \wedge p$ **d** $p \vee q$
4 a $p \wedge q$ **b** $\sim p \wedge \sim q$ **c** $p \vee q$ **d** $p \wedge \sim q$
5 a simple **b** compound **c** compound **d** simple

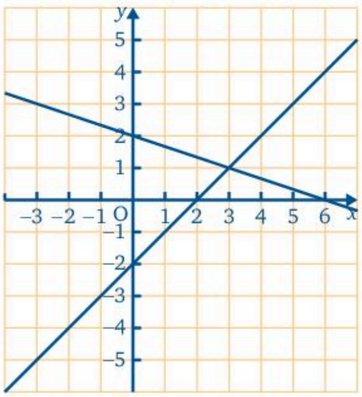
Exercise 16f page 320

1 yes
2 no
3 yes
4 yes
5 yes
6 no (it might be cloudy)
7 If I am not cold then I do not need to warm up.

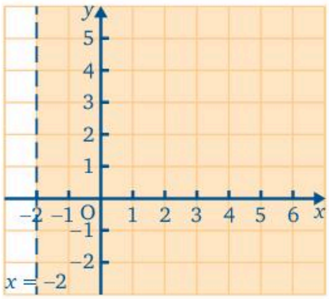
REVIEW TEST 3 page 322

1 a 3 **b** 18 **c** 12
2 a 0 **b** $\frac{3}{2}$ **c** 3
3 a negative **b** positive **c** negative
4 a acute **b** acute **c** obtuse
5 a $\frac{3}{2}$ **b** 1 **c** -2
6 a $-3, 4$ **b** 5, 3 **c** $-\frac{3}{4}, \frac{7}{4}$

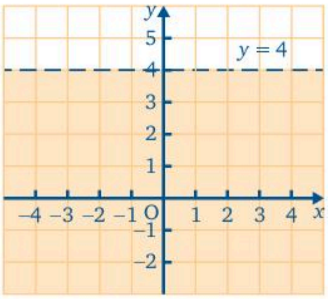
7



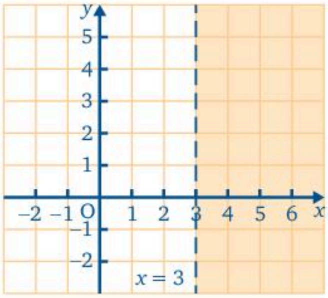
8 a



b

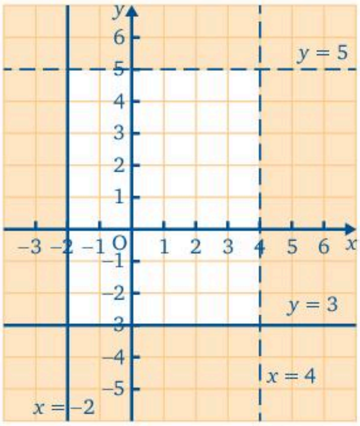


c



9 $-2 \leqslant x \leqslant 4$

10



11 $x + 4y > 4$

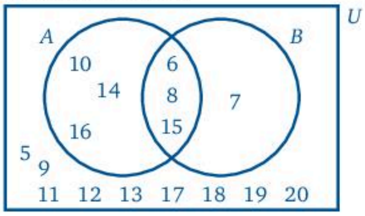
12 AD = 5.9 cm, DC = 3.4 cm, BD = 4.1 cm

13 PR = 6.5 cm, RQ = 8 cm, angle PRQ = 75°

14 CD = 14 cm, AD = 7 cm ; the lines AB and CD are parallel

15 **a** {6, 12, 18, 24, 30}
b {7, 11, 13, 17, 19, 23, 29}
c {22, 24, 26, 28, 30}

16 **a**



b $A \cap B = \{6, 8, 15\}$
c $A \cup B = \{6, 7, 8, 10, 14, 15, 16\}$

17 **a** Clive, Allen
b Paul, Emily, Fred
c Bina, Lara They like both

18 **a** infinite **b** finite **c** infinite

19 **a** Castor, Pollux $\in$ {stars}
b French $\in$ {modern languages}
c $39 \notin$ {prime numbers}

20 **a** true **b** false **c** true

21 **a** no **b** yes **c** yes

22 a $3 + 4$ is not equal to 9
b I like bananas
c 8 is not larger than 5

23 **a** $\sim p$ **b** $p \wedge q$ **c** $\sim p \wedge \sim q$

24 **a** no **b** yes **c** yes

REVIEW TEST 4 page 326

1 symmetry

2 **a** 0.429 **b** 0.48 **c** 3.654×10^2

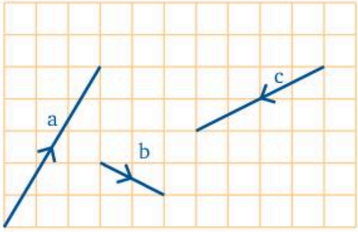
3 **a** 1.44 **b** a^5

4 **a** 73_{10} **b** 11011_2 **c** 2010_3

5 $p = 135°$, $q = 45°$, $r = 92°$, $s = 78°$

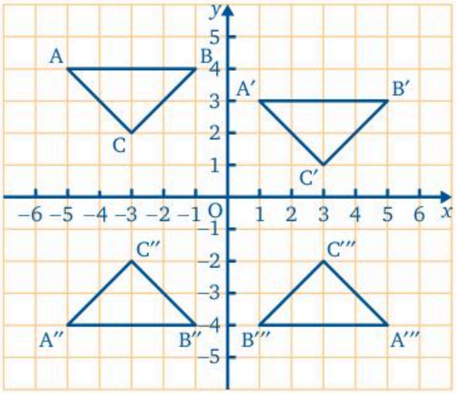
6 $p = 112°$, $q = 68°$, $r = 68°$, $s = 112°$

7



8 **a** (3, 5) **b** (−2, 6) **c** (4, −6)

9

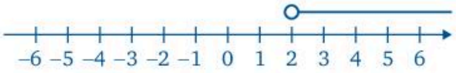


10 $P = 5c$

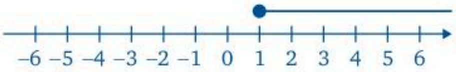
11 4.6

12 **a** $z = x - 2y$
b $a = 3p$

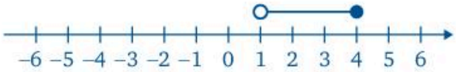
13 **a** $x > 2$



b $x \geqslant 1$



c $x > 1$ and $x \leqslant 4$



14 **a** 6 : 5 **b** 4 : 7 **c** 32

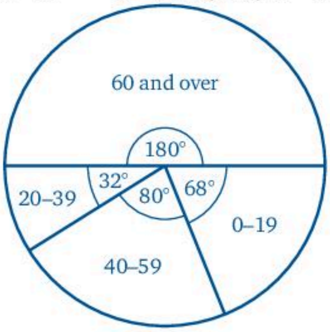
15 **a** \$86.40 **b** 12 m

16 **a** \$600 and £1400 **b** \$19385 and \$36615

17 \$70000

18 600 km

19 57 km/h
20 **a** \$189 750 **b** \$1339 750
21 \$22 500
22 \$4521
23 **a** \$370 000 **b** \$90 000
24 **a** 25 cm^2 **b** 2500 mm^2
25 **a** 3.6 cm^2 **b** 3 cm^2 **c** 12 cm^2 **d** 42 cm^2
26 580 cm^2
27 570 cm^2
28 **a** 1200 cm^3 **b** 450 cm^3
29 33 600 cm^3
30 **a** $x = 95°, y = 35°$
b $x = 90°, y = 120°$
c $x\ y = 70°, z = 40°$
31 30°
32 68°
33 110°
34 6
35 **a** 22 **b** 2.14 (3 s.f.) **c** 2 **d** 2
36



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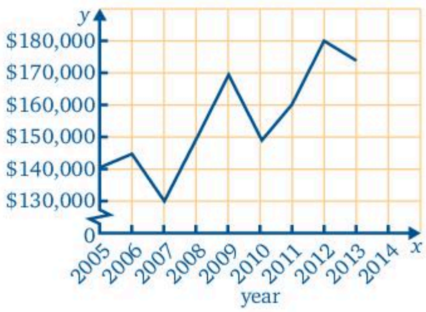

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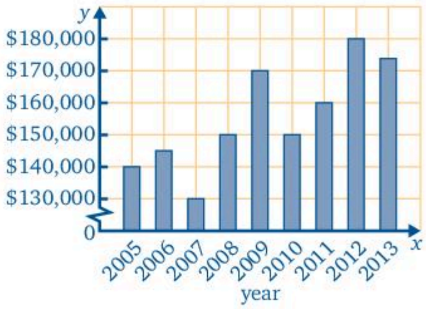
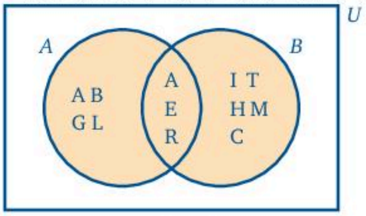


39 **a** true **b** false
40 **a** finite **b** 15
41 {1}, {2},{3},{1, 2},{2, 3},{1, 3}, {1, 2, 3}
42



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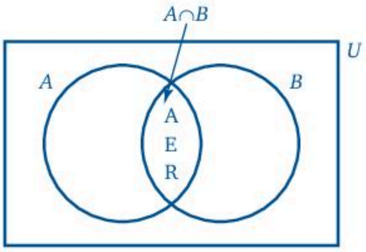
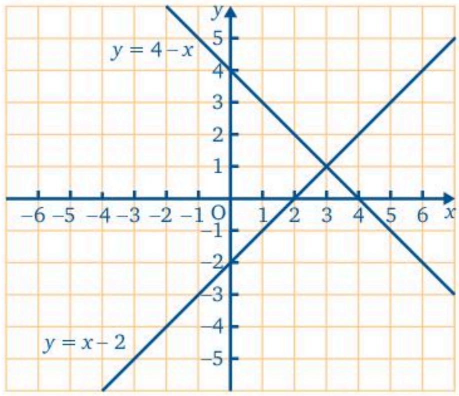


44 **a** 14, 10, 5 **b** 11 **c** 4
45 teacher to check
46 gradient $\frac{1}{2}$, y-intercept -1, $y = \frac{1}{2}x - 1$
47 $5y = x + 6$
48 **a**



b (3, 1)
49 $-4 \leqslant y \leqslant 2$
50 $y < 1 - \frac{1}{2}x$
51 **a** yes **b** no **c** yes
52 School does not start at 0830 hours
53 **a** I have to wear school uniform and I wear trainers after school.
b I do not have to wear school uniform or I wear trainers after school.
c I do not have to wear school uniform and I do not wear trainers after school.
54 **a** no
b no (It doesn't have to be dark for a light to be put on.)

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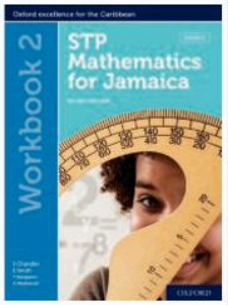

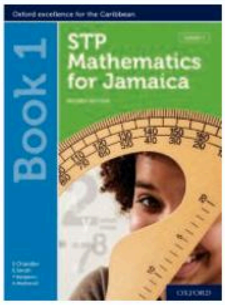

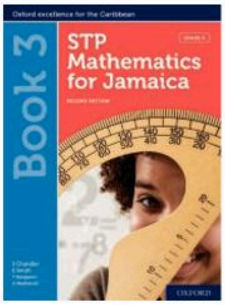

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