MA2004: Format of Seen Test 2 — One hour

1. (a) Convert

$$w = 2+i$$

into modulus-argument form.

(b) Find all the solutions to the equation

$$z^7 = 2 + i$$
.

Give your answers in x-y form correct to two decimal places.

2. Consider the surface defined by the equation

$$z = 2x^2 - 2xy + 2x + 7$$
.

- (a) Find the tangent plane which touches this surface at the point where x = 6 and y = -5.
- (b) Locate and describe all the critical points of the surface.
- 3. Consider the function of two variables given by

$$f(x,y) = x^2 + 2y^2$$
 where $\begin{cases} x = 4st + 6, \\ y = (t-3)e^s. \end{cases}$

(a) Calculate the Jacobian
$$\frac{\partial(x,y)}{\partial(s,t)} = \begin{vmatrix} \frac{\partial x}{\partial s} & \frac{\partial x}{\partial t} \\ \frac{\partial y}{\partial s} & \frac{\partial y}{\partial t} \end{vmatrix}$$

- (b) Using the chain rule for functions of two variables, find the second order partial derivatives f_{ss} , f_{st} and f_{tt} .
- 4. Evaluate

$$\iint_D 72\pi^2 x \cos(3\pi xy) \, dx \, dy$$

over the region D enclosed by the lines x = 5 and y = 7, and the curve xy = 140.