Week 1 solutions part 2

Groups 3.4 page 16

1.

$\odot$	[1]	[2]	[3]
[1]	[1]	[2]	[3]
[2]	[2]	[0]	[2]
[3]	[3]	[2]	[1]

This is not a group as it is not closed,
$[0] \notin \{ \mathbf{Z}_4 - [0] \}$

$\odot$	[1]	[2]	[3]	[4]	[5]	[6]
[1]	[1]	[2]	[3]	[4]	[5]	[6]
[2]	[2]	[4]	[6]	[1]	[3]	[5]
[3]	[3]	[6]	[2]	[5]	[1]	[4]
[4]	[4]	[1]	[5]	[2]	[6]	[3]
[5]	[5]	[3]	[1]	[6]	[4]	[2]
[6]	[6]	[5]	[4]	[3]	[2]	[1]

This is a group, it is closed, the identity is
[1], [2] is the inverse of [4], [3] is the
inverse of [5], [4] is the inverse of [2], [5] is
the inverse of [3], [6] is the inverse of [6].

2.

$\odot$	[2]	[4]	[8]
[2]	[4]	[8]	[2]
[4]	[8]	[2]	[4]
[8]	[2]	[4]	[8]

$\odot$	[8]	[2]	[4]
[8]	[8]	[2]	[4]
[2]	[2]	[4]	[8]
[4]	[4]	[8]	[2]

o $\{[2]_{14}, [4]_{14}, [8]_{14}\}(\odot)$ forms a group,
where $[8]_{14}$ is identity.

3.

$\odot$	[2]	[4]	[6]	[8]
[2]	[4]	[8]	[2]	[6]
[4]	[8]	[6]	[4]	[2]
[6]	[2]	[4]	[6]	[8]
[8]	[6]	[2]	[8]	[4]

(3 cont'd)
This is a group, it is closed, the identity is
[6], [2] is the inverse of [8], [4] is the
inverse of [4] and [8] is the inverse of [2].

4.

$\odot$	[1]	[2]	[4]	[6]	[8]	[10]	[12]
[1]	[1]	[2]	[4]	[6]	[8]	[10]	[12]
[2]	[2]	[4]	[8]	[12]	[3]*		

So $[2] \odot [8] = [3]$
    $\notin \{ [1], [2], [4], [6], [8], [10], [12] \}$
so the set does **not** form a group

ii)

$\odot$	[1]	[5]	[8]	[12]
[1]	[1]	[5]	[8]	[12]
[5]	[5]	[12]	[1]	[8]
[8]	[8]	[1]	[12]	[5]
[12]	[12]	[8]	[5]	[1]

So $\{[1]_{13}, [5]_{13}, [8]_{13}, [12]_{13}\}(\odot)$ forms a group.

5.

$\odot$	[2]	[4]	[8]	[10]	[14]	[16]
[2]	[4]	[2]	[16]	[2]	[10]	[14]
[4]	[8]	[16]	[14]	[4]	[2]	[10]
[8]	[16]	[14]	[10]	[8]	[4]	[2]
[10]	[2]	[4]	[8]	[10]	[14]	[16]
[14]	[10]	[2]	[4]	[14]	[16]	[8]
[16]	[14]	[10]	[2]	[16]	[8]	[4]

A group is formed by including the
elements [10], [14] and [16]