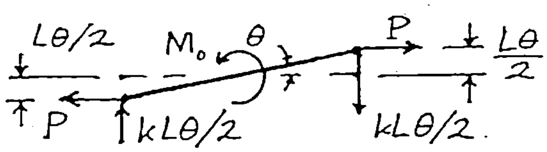
$\boxed{18.1\text{-}1}$

No. Force in rod (tension) = force in column (comp.). During a virtual lateral displacement, these forces do work of equal magnitude but opposite sign; i.e. the net work is zero. No membrane energy is lost, so no bending energy is to be gained. ∴ no buckling.

$\boxed{18.1\text{-}2}$



(a) $\Pi_p = 2\left[\frac{1}{2}k\left(\frac{\theta L}{2}\right)^2\right] + P\left(\frac{1}{2}L\theta^2\right) - M_o\theta$

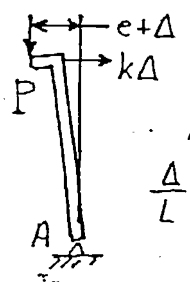
$$\frac{d\Pi_p}{d\theta} = 0 = \frac{kL^2}{2}\theta + PL\theta - M_o$$

$$\theta = \frac{M_o}{\frac{kL^2}{2} + PL}$$

(b) $\theta \to \infty$ if $\frac{kL^2}{2} + PL = 0$, so $P_{cr} = -\frac{kL}{2}$

or, discard M_o from Π_p, solve for P.

$\boxed{18.1\text{-}3}$



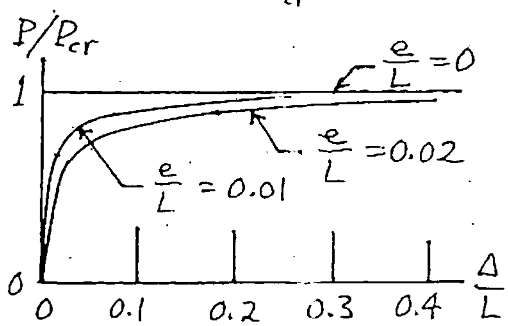
$$\Sigma M_A = 0 = P(e+\Delta) - k\Delta(L)$$

$$\Delta(kL-P) = 0, \quad \Delta = \frac{Pe}{kL-P}$$

$$\Delta \to \infty \text{ as } kL-P \to 0; \quad P_{cr} = kL$$

$$\frac{\Delta}{L} = \frac{\frac{P}{kL}\frac{e}{L}}{1-\frac{P}{kL}} = \frac{\frac{P}{P_{cr}}\frac{e}{L}}{1-\frac{P}{P_{cr}}} = f_P\frac{e}{L}$$

P/P_{cr}	f_P
0.4	0.667
0.7	2.33
0.9	9.00
0.95	19.0



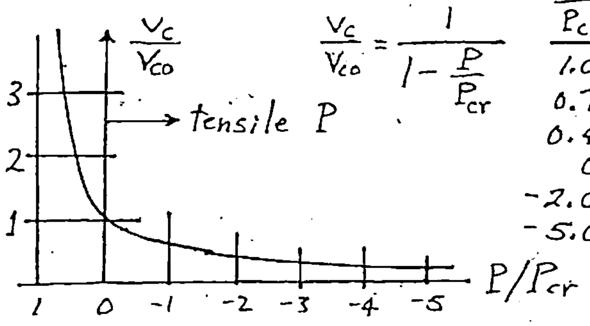
$\boxed{18.1-4}$

Eq. 18.1-6 with $P=0$: $\quad v_{co} = \dfrac{q_c L}{2k}$

$$\left(1+\frac{k_\sigma}{k}\right) v_c = \frac{q_c L}{2k} = v_{co}\,, \quad \frac{v_c}{v_{co}} = \frac{1}{1+\dfrac{k_\sigma}{k}}$$

$$\frac{k_\sigma}{k} = \frac{\pi^2 P}{2L}\,\frac{2L^3}{\pi^4 EI} = P\frac{L^2}{\pi^2 EI} = \frac{P}{-P_{cr}}.$$

$$\frac{v_c}{v_{co}} = \frac{1}{1-\dfrac{P}{P_{cr}}}$$



$\frac{P}{P_{cr}}$	$\frac{v_c}{v_{co}}$
1.0	∞
0.7	3.33
0.4	1.67
0	1.00
-2.0	0.33
-5.0	0.17

$\boxed{18.2-1}$

Given eqs. to start with are
$$\epsilon_x = u_{,x} + \frac{1}{2} v_{,x}^2 - y v_{,xx}$$
$$U = \int \frac{1}{2} E \epsilon_x^2 \, dV$$

Substitute 1st eq. into 2nd eq.; also $dV = dA\,dx$

$$U = \frac{E}{2}\iint \left(u_{,x}^2 + \frac{1}{4} v_{,x}^4 + y^2 v_{,xx}^2 + u_{,x} v_{,x}^2 - y\, v_{,x}^2 v_{,xx} - 2y\, u_{,x} v_{,xx}\right) dA\, dx$$

With $y=0$ at centroidal axis of A, terms linear in y integrate to zero over A. Also $\int E u_{,x} dA = P$ and $\int y^2 dA = I$. Thus

$$U = \frac{1}{2}\int u_{,x}^2 EA\,dx + \frac{1}{4}\int v_{,x}^4 EA\,dx + \frac{1}{2}\int v_{,xx}^2 EI\,dx + \frac{1}{2}\int v_{,x}^2 P\,dx$$

Discard the second integral as negligible in comparison with others. Also interpolate displacements in terms of nodal d.o.f. $\underset{\sim}{d}$.

$$u = \underset{\sim}{N}_u \underset{\sim}{d}_u \qquad v = \underset{\sim}{N}_v \underset{\sim}{d}_v$$

$$U = \frac{1}{2}\underset{\sim}{d}_u^T \int \underset{\sim}{N}_{u,x}^T \underset{\sim}{N}_{u,x} EA\,dx\, \underset{\sim}{d}_u + \frac{1}{2}\underset{\sim}{d}_v^T \int \underset{\sim}{N}_{v,xx}^T \underset{\sim}{N}_{v,xx} EI\,dx\, \underset{\sim}{d}_v$$
$$+ \frac{1}{2}\underset{\sim}{d}_v^T \int \underset{\sim}{N}_{v,x}^T \underset{\sim}{N}_{v,x} P\,dx\, \underset{\sim}{d}_v$$

or

$$U = \frac{1}{2}\underset{\sim}{d}_u^T \underset{\sim}{k}_{bar} \underset{\sim}{d}_u + \frac{1}{2}\underset{\sim}{d}_v^T \underset{\sim}{k}_{beam} \underset{\sim}{d}_v + \frac{1}{2}\underset{\sim}{d}_v^T \underset{\sim}{k}_\sigma \underset{\sim}{d}_v$$

$\boxed{18.2\text{-}2}$

In the formula for $[\underset{\sim}{k}]$, Eq. 3.3-14, EI becomes a function of x. The formula for $[\underset{\sim}{k}_\sigma]$, Eq. 18.2-3, says nothing about geometry of the cross section. Therefore $[\underset{\sim}{k}_\sigma]$ is not affected by taper if $\lfloor\underset{\sim}{N}\rfloor$ is not changed.

$\boxed{18.2\text{-}3}$

$$\frac{d}{dx} = \frac{1}{L}\frac{d}{d\xi}, \quad \frac{d}{dx}\left[\frac{L}{2}(\xi-\xi^2)\right] = \frac{1}{2}-\xi = \frac{1}{2}-\frac{x}{L}$$

$$v_{,x} = \underbrace{\left\lfloor -\frac{1}{L},\ \frac{1}{2}-\frac{x}{L},\ \frac{1}{L},\ -\frac{1}{2}+\frac{x}{L}\right\rfloor}_{[G]} \lfloor v_1\ \theta_1\ v_2\ \theta_2 \rfloor^T$$

$$\int_0^L \frac{1}{L^2}dx = \frac{1}{L}, \quad \int_0^L \frac{1}{L}\left(\frac{1}{2}-\frac{x}{L}\right)dx = 0, \quad \int_0^L \left(\frac{1}{2}-\frac{x}{L}\right)^2 dx = \frac{L}{12}$$

Hence

$$[\underset{\sim}{k}_\sigma] = \int_0^L \lfloor \underset{\sim}{G}\rfloor^T P \lfloor \underset{\sim}{G}\rfloor dx = \frac{P}{12L}\begin{bmatrix} 12 & 0 & -12 & 0 \\ 0 & L^2 & 0 & -L^2 \\ -12 & 0 & 12 & 0 \\ 0 & -L^2 & 0 & L^2 \end{bmatrix}$$

$\boxed{18.2\text{-}4}$

Impose rigid body rotation (no curvature). That is, impose rotation d.o.f. $\theta_1 = \theta_2 = \frac{v_2 - v_1}{L}$.

Thus, for $[\underset{\sim}{k}_\sigma]_{4\times4}$, use $\begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{Bmatrix} = [\underset{\sim}{T}]_{4\times4}\begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix}$, where

$$[\underset{\sim}{T}] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -1/L & 0 & 1/L \\ 0 & 0 & 0 & 1 \\ 0 & -1/L & 0 & 1/L \end{bmatrix}, \quad [\underset{\sim}{T}]^T([\underset{\sim}{k}_\sigma][\underset{\sim}{T}]) =$$

$$[\underset{\sim}{T}]^T \begin{bmatrix} 0 & 30 & 0 & -30 \\ 0 & 0 & 0 & 0 \\ 0 & -30 & 0 & 30 \\ 0 & 0 & 0 & 0 \end{bmatrix}\frac{P}{30L} = \frac{P}{L}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

18.2-5

As rigid bar rotates to angle β, where $\beta = v_2/c$, it exerts vertial force $\beta P = v_2 P/c$ on node 2.

$$([\underset{\sim}{k}]+[\underset{\sim}{k}_\sigma])\{\underset{\sim}{D}\}=\{\underset{\sim}{R}\}$$

$$\left(\frac{EI}{L^3}\begin{bmatrix}12 & -6L\\ -6L & 4L^2\end{bmatrix} - \frac{P}{30L}\begin{bmatrix}36 & -3L\\ -3L & 4L^2\end{bmatrix}\right)\begin{Bmatrix}v_2\\ \theta_2\end{Bmatrix} = \begin{Bmatrix}Q+P\frac{v_2}{c}\\ 0\end{Bmatrix}$$

$$\left(\frac{EI}{L^3}\begin{bmatrix}12 & -6L\\ -6L & 4L^2\end{bmatrix} - \frac{P}{30L}\begin{bmatrix}36+30\frac{L}{c} & -3L\\ -3L & 4L^2\end{bmatrix}\right)\begin{Bmatrix}v_2\\ \theta_2\end{Bmatrix} = \begin{Bmatrix}Q\\ 0\end{Bmatrix}$$

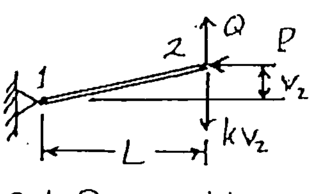
18.2-6

For small rotation v_2/L of bar 1-2, bar 1-2 carries force P and exerts downward force $P v_2/L$ on node 2. Sum of vertical forces on node 2 yields

$$Q - kv_2 - \frac{P}{L}v_2 = 0, \qquad v_2 = \frac{Q}{k+\frac{P}{L}}$$

(a) $P=0$, $v_2 = Q/k$
(b) $P = 0.96kL$, $v_2 = 0.510\,Q/k$
(c) $P = -0.96kL$, $v_2 = 25.0\,Q/k$

18.2-7



$$\Sigma M_1 = 0 = (Q-kv_2)L + Pv_2$$

$$v_2 = \frac{Q}{k} + \frac{P}{kL}v_2 \qquad (A)$$

Set $P = 0.96kL$ & write (A) in iterative form

$$(v_2)_i = \frac{Q}{k} + 0.96(v_2)_{i-1}$$

i	1	2	3	---	∞
$(v_2)_i$	Q/k	$1.96Q/k$	$2.8816Q/k$	---	$25.0Q/k$

18.2-8

(a) θ_2 is the only d.o.f.; $M_2 = Pe$ the only load.

$$\left(\frac{EI}{L^3}4L^2 - \frac{P}{30L}4L^2\right)\theta_2 = Pe$$

$$\theta_2 = \left(\frac{4EI}{L} - \frac{PL}{7.5}\right)^{-1} Pe$$

(b) Set $\left(\frac{4EI}{L} - \frac{PL}{7.5}\right) = 0$, get $P_{cr} = \frac{30EI}{L^2}$

This is $\approx 50\%$ high (exact $P_{cr} = \frac{20.2EI}{L^2}$)

18.2-9

Note: $P_{cr} = -\frac{\pi^2 EI}{4L^2} = -30.157\ N$

(a) The one element gives the exact result,

$$w_2 = \frac{0.1\,L^3}{3EI} = 0.008182\ m \qquad (\text{for } P=0)$$

(b) Standard beam $[k]$, $[k_\sigma]$ from Eq. 18.2-6:

$$\left(\frac{110}{27}\begin{bmatrix}12 & -6(3)\\ -6(3) & 4(9)\end{bmatrix} + \frac{-30}{30(3)}\begin{bmatrix}36 & -3(3)\\ -3(3) & 4(9)\end{bmatrix}\right)\begin{Bmatrix}v_2\\ \theta_2\end{Bmatrix} = \begin{Bmatrix}0.1\\ 0\end{Bmatrix}$$

$$\begin{bmatrix}36.889 & -70.333\\ -70.333 & 134.667\end{bmatrix}\begin{Bmatrix}v_2\\ \theta_2\end{Bmatrix} = \begin{Bmatrix}0.1\\ 0\end{Bmatrix} \quad \text{Solving,}$$

$$v_2 = \frac{0.1}{36.889 - 36.734} = 0.645\ m$$

(c) In the second matrix of part (b), change -30 to $+30$. Thus

$$\begin{bmatrix}60.889 & -76.333\\ -76.333 & 158.667\end{bmatrix}\begin{Bmatrix}v_2\\ \theta_2\end{Bmatrix} = \begin{Bmatrix}0.1\\ 0\end{Bmatrix}$$

$$v_2 = \frac{0.1}{60.889 - 36.723} = 0.00414\ m$$

$\boxed{8.2\text{-}10}$

(a) —— As before; $v_2 = 0.008182\ m$

(b) $\left(\frac{110}{27}\begin{bmatrix}12 & -18\\ -18 & 36\end{bmatrix} + \frac{-30}{36}\begin{bmatrix}12 & 0\\ 0 & 9\end{bmatrix}\right)\begin{Bmatrix}v_2\\ \theta_2\end{Bmatrix} = \begin{Bmatrix}0.1\\ 0\end{Bmatrix}$

$\begin{bmatrix}4200 & -7920\\ -7920 & 15{,}030\end{bmatrix}\begin{Bmatrix}v_2\\ \theta_2\end{Bmatrix} = \begin{Bmatrix}10.8\\ 0\end{Bmatrix}$. Solving,

$$v_2 = \frac{1}{(63.1260 - 62.7264)(10)^6}\begin{bmatrix}15{,}030 & 7920\\ 7920 & 4200\end{bmatrix}\begin{Bmatrix}10.8\\ 0\end{Bmatrix}$$

$v_2 = 0.406\ m$

(c) In the second matrix of part (b), change -30 to $+30$. Thus $\begin{bmatrix}6360 & -7920\\ -7920 & 16{,}650\end{bmatrix}\begin{Bmatrix}v_2\\ \theta_2\end{Bmatrix} = \begin{Bmatrix}10.8\\ 0\end{Bmatrix}$

$$v_2 = \frac{1}{(105{,}894 - 62.7264)10^6}\begin{bmatrix}16{,}650 & 7920\\ 7920 & 6360\end{bmatrix}\begin{Bmatrix}10.8\\ 0\end{Bmatrix}$$

$v_2 = 0.00417\ m$

$\boxed{18.2\text{-}11}$

From $\lfloor \underset{\sim}{B} \rfloor$ in Eq. 3.3-13 at $x=0$,

$$M_0 = EI(v_{,xx})_{x=0} = EI\left\lfloor -\frac{6}{L^2} \quad -\frac{4}{L} \quad \frac{6}{L^2} \quad -\frac{2}{L}\right\rfloor\begin{Bmatrix}v_1\\ \theta_1\\ v_2\\ \theta_2\end{Bmatrix}$$

$v_1 = \theta_1 = 0$

$v_2 = 0.645\ m$, $\theta_2 = \frac{70.333}{134.667} v_2 = 0.337$

$M_0 = 110\left\lfloor \frac{6}{9} \quad -\frac{2}{3}\right\rfloor\begin{Bmatrix}0.645\\ 0.337\end{Bmatrix} = 22.6\ N\cdot m$

By force $\times$ displacement,

$M_0 = 0.1(3) + 30(0.645) = 19.7\ N\cdot m$

$\boxed{18.2\text{-}12}$

We must obtain $[\underset{\sim}{k}_\sigma]\{\underset{\sim}{d}\} = \{\underset{\sim}{0}\}$ if $\{\underset{\sim}{d}\}$ represents rigid-body translation. Hence, if $[\underset{\sim}{k}_\sigma]$ is diagonal & its nonzero terms operate only on translational d.o.f., $[\underset{\sim}{k}_\sigma]$ must be null.

$\boxed{18.4\text{-}1}$

$\{\underset{\sim}{\delta}\} = [\underset{\sim}{G}]\{\underset{\sim}{d}\}$, where $\{\underset{\sim}{\delta}\}$ is given by Eq. 18.4-3, and $\{\underset{\sim}{d}\} = \lfloor u_1\ u_2 \cdots u_8\ v_1 \cdots w_7\ w_8 \rfloor^T$.

Also $\quad u = \sum_1^8 N_i u_i \qquad v = \sum_1^8 N_i v_i \qquad w = \sum_1^8 N_i w_i$

$$\begin{Bmatrix} u_{,x} \\ u_{,y} \\ u_{,z} \end{Bmatrix} = \underset{3\times 8}{[\underset{\sim}{G}_r]} \begin{Bmatrix} u_1 \\ u_2 \\ \vdots \\ u_8 \end{Bmatrix}$$ and similarly for derivatives of v and w, where

$$\underset{3\times 8}{[\underset{\sim}{G}_r]} = \begin{bmatrix} N_{1,x} & N_{2,x} & \cdots & N_{8,x} \\ N_{1,y} & N_{2,y} & \cdots & N_{8,y} \\ N_{1,z} & N_{2,z} & \cdots & N_{8,z} \end{bmatrix}, \quad \underset{9\times 24}{[\underset{\sim}{G}]} = \begin{bmatrix} \underset{\sim}{G}_r & 0 & 0 \\ 0 & \underset{\sim}{G}_r & 0 \\ 0 & 0 & \underset{\sim}{G}_r \end{bmatrix}$$

$$N_{i,x} = \Gamma_{11} N_{i,\xi} + \Gamma_{12} N_{i,\eta} + \Gamma_{13} N_{i,\zeta}$$
$$N_{i,y} = \Gamma_{21} N_{i,\xi} + \Gamma_{22} N_{i,\eta} + \Gamma_{23} N_{i,\zeta}$$
$$N_{i,z} = \Gamma_{13} N_{i,\xi} + \Gamma_{32} N_{i,\eta} + \Gamma_{33} N_{i,\zeta}$$

$\boxed{18.4\text{-}2}$

(a) Discard terms in $[\underset{\sim}{s}]$ with z subscripts. Also, $w=0$; only u & v have derivatives w.r.t. x & y.

$$\{\underset{\sim}{\delta}\} = \begin{Bmatrix} u_{,x} \\ u_{,y} \\ v_{,x} \\ v_{,y} \end{Bmatrix} = \underset{4\times 6}{[\underset{\sim}{G}]}\{\underset{\sim}{d}\} = \begin{bmatrix} \underset{\sim}{B} & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{B} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ v_1 \\ v_2 \\ v_3 \end{Bmatrix};$$ where,

from Eq. 7.2-6,

$$[\underset{\sim}{B}] = \frac{1}{2A} \begin{bmatrix} y_{23} & y_{31} & y_{12} \\ x_{32} & x_{13} & x_{21} \end{bmatrix}$$

$$[\underset{\sim}{s}] = \begin{bmatrix} \sigma_{xo} & \tau_{xyo} \\ \tau_{xyo} & \sigma_{yo} \end{bmatrix}$$

$$\underset{6\times 6}{[\underset{\sim}{k}_\sigma]} = \iint \begin{bmatrix} \underset{\sim}{B}^T & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{B}^T \end{bmatrix} \underset{4\times 4}{\begin{bmatrix} \underset{\sim}{s} & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{s} \end{bmatrix}} \begin{bmatrix} \underset{\sim}{B} & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{B} \end{bmatrix} t\, dx\, dy$$

(b) $$[\underset{\sim}{k}_\sigma] = \iint \underset{3\times 2}{[\underset{\sim}{B}]^T} \underset{2\times 2}{[\underset{\sim}{s}]} \underset{2\times 3}{[\underset{\sim}{B}]}\, t\, dx\, dy, \quad \{\underset{\sim}{d}\} = \begin{Bmatrix} w_1 \\ w_2 \\ w_3 \end{Bmatrix}$$

$\boxed{18.5\text{-}1}$

We will solve for both roots. Expansion of the determinant by cofactors shows at once that only the southeast 2 by 2 submatrices need be used. Let $s = \frac{\lambda_{cr}}{30L}\frac{L^3}{2EI} = \frac{\lambda_{cr}L^2}{60EI}$.

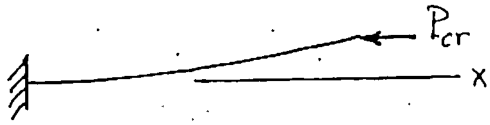
$$\begin{vmatrix} 6-36s & -3L+3Ls \\ -3L+3Ls & 2L^2-4sL^2 \end{vmatrix} = 45s^2-26s+1=0$$

$$s = \frac{26 \pm \sqrt{496}}{90} \Rightarrow \begin{cases} s_1 = 0.04143, \ \lambda_1 = 2.486\frac{EI}{L^2} \\ s_2 = 0.53635, \ \lambda_2 = 32.18\frac{EI}{L^2} \end{cases}$$

Mode, with $\theta_2 = 1$ and $s = s_1$:

$$(6-36s_1)v_2 + (-3L+3Ls_1)(1) = 0$$

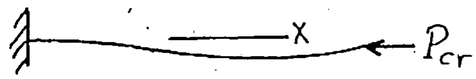
$$v_2 = \frac{1-s_1}{2-12s_1}L = 0.6379L$$



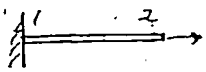
Mode, with $\theta_2 = 1$ and $s = s_2$:

$$(6-36s_2)v_2 + (-3L+3Ls_2)(1) = 0$$

$$v_2 = \frac{1-s_2}{2-12s_2}L = -0.1045L$$



$\boxed{18.5\text{-}2}$



P (assumed tensile here)

$$\left(\frac{EI}{L^3}\begin{bmatrix} 12 & -6L \\ -6L & 4L^2 \end{bmatrix} + \frac{P_{cr}}{12L}\begin{bmatrix} 12 & 0 \\ 0 & L^2 \end{bmatrix}\right)\begin{Bmatrix} v_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Let $s = \frac{P_{cr}L^2}{24EI}$, then $\begin{vmatrix} 6+12s & -3L \\ -3L & 2L^2+sL^2 \end{vmatrix} = 0$

$12s^2+30s+3=0$, $s = -0.10436$, $P_{cr} = -2.505\frac{EI}{L^2}$

$\boxed{18.5\text{-}3}$

(a) Use $[k_\sigma]$ from Eq. 18.2-4.

$$\left(\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} - \frac{P_{cr}}{a}\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}\right)\begin{Bmatrix} v_2 \\ v_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}. \quad \text{Let } s = \frac{P_{cr}}{a}$$

$$\begin{vmatrix} k-2s & s \\ s & k-s \end{vmatrix} = s^2-3ks+k^2=0, \quad s_{cr} = \frac{3-\sqrt{5}}{2}k$$

$$P_{cr} = 0.382ka$$

(b) $k[v_2 - 0.382(2v_2 - v_3)] = 0$; set $v_2 = 1$, then $v_3 = -0.618$

$$[k_\sigma]\begin{Bmatrix} v_2 \\ v_3 \end{Bmatrix} = 0.382\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} 1 \\ -0.618 \end{Bmatrix} = k\begin{Bmatrix} 1 \\ -0.618 \end{Bmatrix}$$

$0.618k$ These forces resisted by spring forces.

$\boxed{18.5\text{-}4}$

(a)

$$\left(\frac{EI}{a^3}\begin{bmatrix}24 & 0 & 6a\\ 0 & 8a^2 & 2a^2\\ 6a & 2a^2 & 4a^2\end{bmatrix} - \frac{P}{30a}\begin{bmatrix}72 & 0 & 3a\\ 0 & 8a^2 & -a^2\\ 3a & -a^2 & 4a^2\end{bmatrix}\right)\begin{Bmatrix}v_2\\ \theta_2\\ \theta_3\end{Bmatrix} = \begin{Bmatrix}F\\ 0\\ 0\end{Bmatrix}$$

$$[\underset{\sim}{T}] = \begin{bmatrix} 1 \\ -\dfrac{1}{28a^4}\begin{bmatrix}4a^2 & -2a^2\\ -2a^2 & 8a^2\end{bmatrix}\begin{Bmatrix}0\\ 6a\end{Bmatrix}\end{bmatrix} = \begin{Bmatrix}1\\ 3/7a\\ -12/7a\end{Bmatrix}$$

Condensed arrays $[\underset{\sim}{T}]^T[\cdot\,][\underset{\sim}{T}]$ are

$$\frac{EI}{a^3}\left\lfloor 1 \quad \frac{3}{7a} \quad -\frac{12}{7a}\right\rfloor\begin{Bmatrix}13.7143\\ 0\\ 0\end{Bmatrix} = \frac{13.7143\,EI}{a^3}$$

$$\frac{P}{30a}\left\lfloor 1 \quad \frac{3}{7a} \quad -\frac{12}{7a}\right\rfloor\begin{Bmatrix}66.8571\\ 5.1429a\\ -4.2857a\end{Bmatrix} = \frac{2.5469P}{a}$$

$$\left(\frac{13.7143EI}{a^3} - \frac{2.5469P}{a}\right)v_2 = F \qquad (A)$$

(b) $F=0$, $$P_{cr} = \frac{13.7143}{2.5469}\frac{EI}{a^2} = 5.385\frac{EI}{a^2} \qquad (B)$$

exact $P_{cr} = 20.2\dfrac{EI}{L^2} = 5.05\dfrac{EI}{a^2}$ ↖ 6.6% high

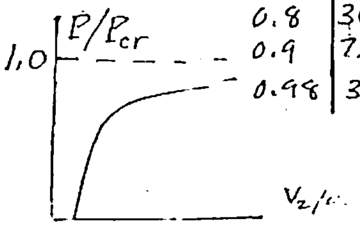
(c) Divide (A) by P_{cr} from (B).

$$\left(\frac{13.7143EI/a^2}{5.385EI/a^2} - \frac{2.5469P}{P_{cr}}\right)\frac{v_2}{a} = \frac{0.1EI/a^2}{5.385EI/a^2}$$

$$\left(2.547 - 2.547\frac{P}{P_{cr}}\right)\frac{v_2}{a} = 0.01857$$

$$\frac{v_2}{a} = \frac{[\text{illegible}])^{-3}}{1 - \frac{P}{P_{cr}}}$$

$\frac{P}{P_{cr}}$	$\frac{v_2}{a}\times 10^3$
0	7.292
0.4	12.15
0.8	36.46
0.9	72.92
0.98	364.6



$\boxed{18.5\text{-}5}$

(a)

$$\left(\frac{EI}{L^3}\begin{bmatrix}4L^2 & 2L^2\\ 2L^2 & 4L^2\end{bmatrix}+\frac{P}{30L}\begin{bmatrix}4L^2 & -L^2\\ -L^2 & 4L^2\end{bmatrix}\right)\begin{Bmatrix}\theta_1\\ \theta_2\end{Bmatrix}=\begin{Bmatrix}0\\ 0\end{Bmatrix}$$

Let $s = PL^2/60EI$

$$\begin{vmatrix}2+4s & 1-s\\ 1-s & 2+4s\end{vmatrix}=15s^2+18s+3=0,\ s=-0.2$$

$$P_{cr}=-\frac{12EI}{L^2}$$

(b) Let $a=L/2$

$$\left(\frac{EI}{a^3}\begin{bmatrix}4a^2 & -6a\\ -6a & 12\end{bmatrix}+\frac{P}{30a}\begin{bmatrix}4a^2 & -3a\\ -3a & 36\end{bmatrix}\right)\begin{Bmatrix}\theta_1\\ v_2\end{Bmatrix}=\begin{Bmatrix}0\\ 0\end{Bmatrix}$$

Let $s = Pa^2/60EI$

$$\begin{vmatrix}2a^2+4sa^2 & -3a-3as\\ -3a-3as & 6+36s\end{vmatrix}=(135s^2+78s+3)a^2=0$$

$$s=-0.0414327,\ P_{cr}=-2.4859\frac{EI}{a^2}=-9.9436\frac{EI}{L^2}$$

(c)

$$\left(\frac{EI}{L^3}\begin{bmatrix}12 & -6L\\ -6L & 4L^2\end{bmatrix}+\begin{bmatrix}\frac{2EI}{L^3} & 0\\ 0 & 0\end{bmatrix}+\frac{P}{30L}\begin{bmatrix}36 & -3L\\ -3L & 4L^2\end{bmatrix}\right)\begin{Bmatrix}v_2\\ \theta_2\end{Bmatrix}=\begin{Bmatrix}0\\ 0\end{Bmatrix}$$

Let $s = PL^2/60EI$

$$\begin{vmatrix}7+36s & -3L-3Ls\\ -3L-3Ls & 2L^2+4L^2s\end{vmatrix}=(135s^2+82s+5)L^2=0$$

$$s=-0.068759,\ P_{cr}=-4.126\frac{EI}{L^2}$$

(d)

$$\left(\frac{EI}{L^3}\begin{bmatrix}12 & -6L\\ -6L & 4L^2\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & \frac{EI}{L}\end{bmatrix}+\frac{P}{30L}\begin{bmatrix}36 & -3L\\ -3L & 4L^2\end{bmatrix}\right)\begin{Bmatrix}v_2\\ \theta_2\end{Bmatrix}=\begin{Bmatrix}0\\ 0\end{Bmatrix}$$

Let $s = PL^2/30EI$

$$\begin{vmatrix}12+36s & -6L-3Ls\\ -6L-3Ls & 5L^2+4L^2s\end{vmatrix}=(135s^2+192s+24)L^2=0$$

$$s=-0.13848,\ P_{cr}=-4.155\frac{EI}{L^2}$$

(e) Set $v_2=0$ in equations of part (c).

$$\left[\frac{EI}{L^3}(4L^2)+0+\frac{P}{30L}(4L^2)\right]\theta_2=0,\ P_{cr}=-30\frac{EI}{L^2}$$

(f) Set $\theta_2=0$ in equations of part (d).

$$\left[\frac{EI}{L^3}(12)+0+\frac{P}{30L}(36)\right]v_2=0,\ P_{cr}=-\frac{10EI}{L^2}$$

$\boxed{18.5-6}$

(a)

$$\left(\frac{EI}{L^3}\begin{bmatrix}4L^2 & 2L^2\\ 2L^2 & 4L^2\end{bmatrix}+\frac{P}{12L}\begin{bmatrix}L^2 & -L^2\\ -L^2 & L^2\end{bmatrix}\right)\begin{Bmatrix}\theta_1\\ \theta_2\end{Bmatrix}=\begin{Bmatrix}0\\0\end{Bmatrix}$$

Let $s = PL^2/24EI$

$$\begin{vmatrix}2+s & 1-s\\ 1-s & 2+s\end{vmatrix} = 6s+3=0,\ s=\frac{-1}{2},\ P_{cr}=-\frac{12EI}{L^2}$$

(Same as Prob. 18.5-5a because both activate the constant-curvature state, for which $[\underset{\sim}{k}]$ $[\underset{\sim}{k}_\sigma]$ are the same in these two problems.)

(b) Let $a = L/2$

$$\left(\frac{EI}{a^3}\begin{bmatrix}4a^2 & -6a\\ -6a & 12\end{bmatrix}+\frac{P}{12a}\begin{bmatrix}a^2 & 0\\ 0 & 12\end{bmatrix}\right)\begin{Bmatrix}\theta_1\\ v_2\end{Bmatrix}=\begin{Bmatrix}0\\0\end{Bmatrix}$$

Let $s = Pa^2/24EI$

$$\begin{vmatrix}2a^2+sa^2 & -3a\\ -3a & 6+12s\end{vmatrix}=(12s^2+30s+3)a^2=0$$

$$s=-0.10436,\ P_{cr}=-2.505\frac{EI}{a^2}=-10.02\frac{EI}{L^2}$$

(c)

$$\left(\frac{EI}{L^3}\begin{bmatrix}12 & -6L\\ -6L & 4L^2\end{bmatrix}+\begin{bmatrix}\frac{2EI}{L^3} & 0\\ 0 & 0\end{bmatrix}+\frac{P}{12L}\begin{bmatrix}12 & 0\\ 0 & L^2\end{bmatrix}\right)\begin{Bmatrix}v_2\\ \theta_2\end{Bmatrix}=\begin{Bmatrix}0\\0\end{Bmatrix}$$

Let $s = PL^2/24EI$

$$\begin{vmatrix}7+12s & -3L\\ -3L & 2L^2+L^2s\end{vmatrix}=12s^2+31s+5=0$$

$$s=-0.17286,\ P_{cr}=-4.149\frac{EI}{L^2}$$

(d)

$$\left(\frac{EI}{L^3}\begin{bmatrix}12 & -6L\\ -6L & 4L^2\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & \frac{EI}{L}\end{bmatrix}+\frac{P}{12L}\begin{bmatrix}12 & 0\\ 0 & L^2\end{bmatrix}\right)\begin{Bmatrix}v_2\\ \theta_2\end{Bmatrix}=\begin{Bmatrix}0\\0\end{Bmatrix}$$

Let $s = PL^2/12EI$

$$\begin{vmatrix}12+12s & -6L\\ -6L & 5L^2+L^2s\end{vmatrix}=(12s^2+72s+24)L^2=0$$

$$s=-0.3542,\ P_{cr}=-4.251\frac{EI}{L^2}$$

(e) Set $v_2=0$ in equations of part (c).

$$\left[\frac{EI}{L^3}(4L^2)+0+\frac{P}{12L}(L^2)\right]\theta_2=0,\ P_{cr}=-48\frac{EI}{L^2}$$

(f) Set $\theta_2=0$ in equations of part (d).

$$\left[\frac{EI}{L^3}(12)+0+\frac{P}{12L}(12)\right]v_2=0,\ P_{cr}=-12\frac{EI}{L^2}$$

$\boxed{18.5\text{-}7}$

(a) A buckling load is not obtained: $[\underset{\sim}{k}_\sigma]$ that operates on θ d.o.f. is null.

(b) Let $a = L/2$.

$$\left(\frac{EI}{a^3}\begin{bmatrix} 4a^2 & -6a \\ -6a & 12 \end{bmatrix} + \frac{P}{a}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right)\begin{Bmatrix} \theta_1 \\ v_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}. \text{ Let } s = \frac{Pa^2}{2EI}$$

$$\begin{vmatrix} 2a^2 & -3a \\ -3a & 6+s \end{vmatrix} = (2s+3)a^2 = 0$$

$$s = \frac{3}{2}, \quad P_{cr} = -3\frac{EI}{a^2} = -12\frac{EI}{L^2}$$

(c)

$$\left(\frac{EI}{L^3}\begin{bmatrix} 12 & -6L \\ -6L & 4L^2 \end{bmatrix} + \begin{bmatrix} \frac{2EI}{L^3} & 0 \\ 0 & 0 \end{bmatrix} + \frac{P}{L}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right)\begin{Bmatrix} v_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Let $s = PL^2/2EI$

$$\begin{vmatrix} 7+s & -3L \\ -3L & 2L^2 \end{vmatrix} = (2s+5)L^2 = 0 \qquad s = -2.5 \qquad P_{cr} = -5\frac{EI}{L^2}$$

(d)

$$\left(\frac{EI}{L^3}\begin{bmatrix} 12 & -6L \\ -6L & 4L^2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{EI}{L} \end{bmatrix} + \frac{P}{L}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right)\begin{Bmatrix} v_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Let $s = PL^3/EI$

$$\begin{vmatrix} 12+s & -6L \\ -6L & 5L^2 \end{vmatrix} = (5s+24)L^2 = 0 \qquad s = -4.8 \qquad P_{cr} = -4.8\frac{EI}{L^2}$$

(e) Set $v_2 = 0$ in equations of part (c).

$$\left[\frac{EI}{L^3}(4L^2) + 0 + \frac{P}{L}(0)\right]\theta_2 = 0$$

No buckling: $\theta_2 = 0$ is the only solution.

(f) Set $\theta_2 = 0$ in equations of part (d).

$$\left[\frac{EI}{L^3}(12) + 0 + \frac{P}{L}(1)\right]v_2 = 0, \quad P_{cr} = -12\frac{EI}{L^2}$$

$\boxed{18.5\text{-}8}$

(a) In dynamics, we choose as masters the D_i for which M_{ii}/K_{ii} is large. By analogy, for buckling & stress-stiffing problem we choose as masters the D_i for which $K_{\sigma ii}/K_{ii}$ is large. Thus, slaves are apt to be rotational d.o.f. (see e.g. Eqs. 3.3-14 and 18.2-6).

(b)
$$[\mathbf{T}] = \begin{bmatrix} 1 \\ 70.333/134.667 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.52228 \end{bmatrix}$$

$$[\mathbf{T}]^T([\mathbf{K}][\mathbf{T}]) = [\mathbf{T}]^T \begin{Bmatrix} 0.1554 \\ 0 \end{Bmatrix} = 0.1554$$

$$[\mathbf{T}]^T\{R\} = [\mathbf{T}]^T \begin{Bmatrix} 0.1 \\ 0 \end{Bmatrix} = 0.1, \quad v_2 = \frac{0.1}{0.1554} = 0.644\,m$$

(c)
$$[\mathbf{T}] = \begin{bmatrix} 1 \\ 76.333/158.667 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.48109 \end{bmatrix}$$

$$[\mathbf{T}]^T([\mathbf{K}][\mathbf{T}]) = [\mathbf{T}]^T \begin{Bmatrix} 24.165 \\ 0 \end{Bmatrix} = 24.165$$

$$[\mathbf{T}]^T\{\mathbf{R}\} = [\mathbf{T}]^T \begin{Bmatrix} 0.1 \\ 0 \end{Bmatrix} = 0.1, \quad v_2 = \frac{0.1}{24.165} = 0.00414\ m$$

(d) Keep v_2 as master.

$$[\mathbf{T}] = \begin{bmatrix} -(-6a)/4a^2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.5/a \\ 1 \end{bmatrix}$$ Apply to $[\mathbf{K}]$ and to $[\mathbf{K}_\sigma]$.

$$[\mathbf{T}]^T[\mathbf{K}][\mathbf{T}] = \frac{3EI}{a^3}, \quad [\mathbf{T}]^T[\mathbf{K}_\sigma][\mathbf{T}] = 36\frac{P}{30a} = \frac{1.2P}{a}$$

$$\left(\frac{3EI}{a^3} + \frac{1.2P}{a}\right)v_2 = 0, \quad P_{cr} = -2.5\frac{EI}{a^2} = -10\frac{EI}{L^2}$$

(e) Same $[\mathbf{T}]$ as in part (d).

[illegible] $\frac{EI}{a^3}$, $[\mathbf{T}]^T[\mathbf{K}_\sigma][\mathbf{T}] = \frac{14.25P}{12a}$

$$\left(\frac{3EI}{a^3} + \frac{14.25P}{12a}\right)v_2 = 0, \quad P_{cr} = -2.526\frac{EI}{a^2} = -10.11\frac{EI}{L^2}$$

$\boxed{18.5-9}$

$$\left(\frac{AE}{L}+\frac{AE}{\alpha L}\right)u_B = P, \quad u_B = \frac{\alpha}{1+\alpha}\frac{PL}{AE}$$

$$P_{AB} = \frac{AE}{L}u_B = \frac{\alpha P}{1+\alpha}, \quad P_{BC} = -\frac{AE}{\alpha L}u_B = -\frac{P}{1+\alpha}$$

$$(k_{AB}+k_{BC}+k_{\sigma AB}+k_{\sigma BC})\theta_B = 0$$

$$\left[\left(\frac{4EI}{L}+\frac{4EI}{(1+\alpha)L}\right)+\underbrace{\left(\frac{2(\alpha P)L}{15}+\frac{-2P(\alpha L)}{15}\right)}_{=0}\right]\theta_B = 0$$

The only solution is $\theta_B = 0$: no buckling.

$\boxed{18.5-10}$

$$\left[\left(\frac{4EI}{1.3a}+\frac{4EI}{a}\right)+\frac{2}{15}\left(-1.3aP\cos\beta - aP\sin\beta\right)\right]\theta_B = 0$$

$$(1.3\cos\beta+\sin\beta)P = \frac{30EI}{a^2}\left(\frac{1}{1.3}+1\right) = 53.077\frac{EI}{a^2}$$

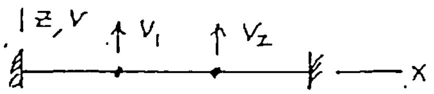
$$P = 53.077\frac{EI}{a^2}\,\frac{1}{1.3\cos\beta+\sin\beta} \qquad (A)$$

Want to minimize $1.3\cos\beta+\sin\beta$.

$-1.3\sin\beta+\cos\beta = 0, \quad \beta = \arctan\frac{1}{1.3} = 37.6°$

By using adjacent angles in Eq. (A), we see that $\beta = 37.6°$ provides a min. of P, not a max.

$\boxed{18.6\text{-}1}$



(a) After discarding fixed d.o.f. at the ends, $[\underset{\sim}{K}_\sigma]\{\underset{\sim}{D}\}=\{\underset{\sim}{R}\}$ becomes (g = accel. of gravity)

$$\frac{T}{L}\begin{bmatrix}2 & -1\\ -1 & 2\end{bmatrix}\begin{Bmatrix}v_1\\ v_2\end{Bmatrix}=\begin{Bmatrix}-mg\\ -mg\end{Bmatrix},\quad \begin{Bmatrix}v_1\\ v_2\end{Bmatrix}=-\frac{mgL}{T}\begin{Bmatrix}1\\ 1\end{Bmatrix}$$

(b) With $[\underset{\sim}{K}]$ and $\{\underset{\sim}{R}\}$ both zero, Eq. 14.6-1 becomes

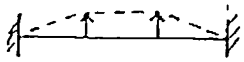
$$\left(\frac{T}{L}\begin{bmatrix}2 & -1\\ -1 & 2\end{bmatrix}-m\omega^2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right)\begin{Bmatrix}v_1\\ v_2\end{Bmatrix}=\begin{Bmatrix}0\\ 0\end{Bmatrix}$$

Let $s=m\omega^2L/T$

$$\begin{vmatrix}2-s & -1\\ -1 & 2-s\end{vmatrix}=s^2-4s+3=0,\quad \begin{cases}s=1\\ s=3\end{cases}$$

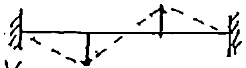
For $s=1$, $\omega^2=\dfrac{T}{mL}$

$$\begin{bmatrix}2-1 & -1\\ -1 & 2-1\end{bmatrix}\begin{Bmatrix}v_1\\ v_2\end{Bmatrix}=\begin{Bmatrix}0\\ 0\end{Bmatrix},\quad v_1=v_2$$



For $s=3$, $\omega^2=\dfrac{3T}{mL}$

$$\begin{bmatrix}2-3 & -1\\ -1 & 2-3\end{bmatrix}\begin{Bmatrix}v_1\\ v_2\end{Bmatrix}=\begin{Bmatrix}0\\ 0\end{Bmatrix},\quad v_1=-v_2$$



$\boxed{18.6\text{-}2}$

(a)

$$\frac{T}{L}\begin{bmatrix}2 & -1\\ -1 & 2\end{bmatrix}\begin{Bmatrix}v_1\\ v_2\end{Bmatrix}=\begin{Bmatrix}-2mg\\ [illegible]\end{Bmatrix},\quad \begin{Bmatrix}v_1\\ v_2\end{Bmatrix}=-\frac{mgL}{3T}\begin{Bmatrix}5\\ 4\end{Bmatrix}$$

(b)

$$\left(\frac{T}{L}\begin{bmatrix}2 & -1\\ -1 & 2\end{bmatrix}-\omega^2\begin{bmatrix}2m & 0\\ 0 & m\end{bmatrix}\right)\begin{Bmatrix}v_1\\ v_2\end{Bmatrix}=\begin{Bmatrix}0\\ 0\end{Bmatrix}$$

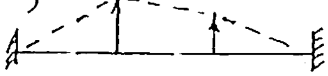
Let $s=m\omega^2L/T$

$$\begin{vmatrix}2-2s & -1\\ -1 & 2-s\end{vmatrix}=2s^2-6s+3=0,\quad \begin{cases}s=0.634\\ s=2.366\end{cases}$$

For $s=0.634$, $\omega^2=0.634\,T/mL$

$[2-2(0.634)]v_1-v_2=0;$

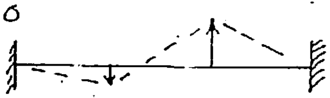
$v_1=1.366\,v_2$



For $s=2.366$, $\omega^2=2.366\,T/mL$

$[2-2(2.366)]v_1-v_2=0$

$v_1=-0.366\,v_2$



$\boxed{18.6-3}$

(a) Let $L = 2a$, $[\underset{\sim}{K}] = [\underset{\sim}{0}]$ in Eq. 18.6-2.

$$\left(\frac{TL}{30}\begin{bmatrix}4 & -1\\ -1 & 4\end{bmatrix} - \omega^2\frac{\rho L^3}{420}\begin{bmatrix}4 & -3\\ -3 & 4\end{bmatrix}\right)\begin{Bmatrix}\theta_1\\ \theta_2\end{Bmatrix} = \begin{Bmatrix}0\\ 0\end{Bmatrix}$$

Let $s = \omega^2 \rho L^2 / 14T$

$$\begin{vmatrix}4-4s & -1+3s\\ -1+3s & 4-4s\end{vmatrix} = 0 \quad \begin{cases} s = \frac{5}{7}, \ \omega_1^2 = 10\frac{T}{\rho L^2} = 2.5\frac{T}{\rho a^2} \\ s = 3, \ \omega_3^2 = 42\frac{T}{\rho L^2} = 10.5\frac{T}{\rho a^2}\end{cases}$$

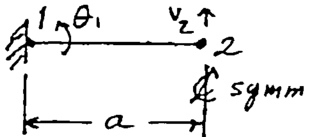
Mode 1: $\left(-1+3\frac{5}{7}\right)\theta_1 + \left(4-4\frac{5}{7}\right)\theta_2 = 0$

$$\theta_1 = -\theta_2$$

Mode 2: $(-1+9)\theta_1 + (4-12)\theta_2 = 0$

$$\theta_1 = \theta_2$$

(b)



Thus we exclude mode 2; we obtain only mode 1 and mode 3.

$$\left(\frac{T}{30a}\begin{bmatrix}4a^2 & -3a\\ -3a & 36\end{bmatrix} - \frac{\rho\omega^2}{420}\begin{bmatrix}4a^3 & 13a^2\\ 13a^2 & 156a\end{bmatrix}\right)\begin{Bmatrix}\theta_1\\ v_2\end{Bmatrix} = \begin{Bmatrix}0\\ 0\end{Bmatrix}$$

Let $s = 30\rho\omega^2 a^2 / 420T$

$$\begin{vmatrix}4a^2-4sa^2 & -3a-13sa\\ -3a-13sa & 36-156s\end{vmatrix} = (455s^2 - 846s + 135)a^2 = 0$$

Mode 1: $s = 0.17629, \ \omega_1^2 = 2.468\frac{T}{\rho a^2}$

$$4a^2(1-0.17629)\theta_1 - a[3+13(0.17629)]v_2 = 0$$

$$\theta_1 = 1.606\frac{v_2}{a}$$

Mode 3: $s = 1.68305, \ \omega_3^2 = 23.56\frac{T}{\rho a^2}$

$$\ldots 15)\theta_1 - a[3+13(1.68305)]v_2 = 0$$

$$\theta_1 = -9.106\frac{v_2}{a}$$

$\boxed{18.6\text{-}4}$

(a) Cubic $[\underset{\sim}{M}]$

18.6-3(a): $[\underset{\sim}{K}_\sigma]$ is null if d.o.f. are θ_1 and θ_2 only; no solution.

18.6-3(b): $$\left(\frac{T}{a}\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix} - \frac{\rho\omega^2}{420}\begin{bmatrix}4a^3 & 13a^2\\13a^2 & 156a\end{bmatrix}\right)\begin{Bmatrix}\theta_1\\v_2\end{Bmatrix} = \begin{Bmatrix}0\\0\end{Bmatrix}$$

Let $s = \rho\omega^2 a^2/420T$

$$\begin{vmatrix}-4sa^2 & -13sa\\-13sa & 1-156s\end{vmatrix} = (455s-4)sa^2 = 0$$

$s = 0.0087912$, $\omega^2 = 3.692\dfrac{T}{\rho a^2}$

$0 = -4sa^2\theta_1 - 13sa\,v_2$, $\theta_1 = -\dfrac{13}{4}\dfrac{v_2}{a}$

This result is physically meaningless.

(b) Lumped $[\underset{\sim}{M}]$

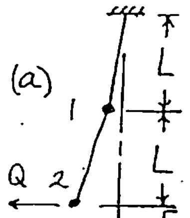
18.6-3(a): As before, no solution.

18.6-3(b): $$\left(\frac{T}{a}\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix} - \omega^2\frac{\rho a}{2}\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}\right)\begin{Bmatrix}\theta_1\\v_2\end{Bmatrix} = \begin{Bmatrix}0\\0\end{Bmatrix}$$

$\left(\dfrac{T}{a} - \omega^2\dfrac{\rho a}{2}\right)v_2 = 0$, $\omega^2 = 2\dfrac{T}{\rho a^2}$

18.6-5

(a)



Let g = accel. of gravity. Tension is mg in lower part, $2mg$ in upper part. Write $[\underset{\sim}{K}_\sigma]\{\underset{\sim}{D}\}=\{\underset{\sim}{R}\}$ where $\{\underset{\sim}{D}\}$ = lateral deflections.

$$\frac{mg}{L}\begin{bmatrix}2+1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}v_1\\ v_2\end{Bmatrix}=\begin{Bmatrix}0\\ Q\end{Bmatrix}, \quad \begin{Bmatrix}v_1\\ v_2\end{Bmatrix}=\frac{QL}{2mg}\begin{Bmatrix}1\\ 3\end{Bmatrix}$$

(b) With $[\underset{\sim}{K}]=\{\underset{\sim}{R}\}=\{\underset{\sim}{0}\}$, Eq. 18.6-1 becomes

$$\left(\frac{mg}{L}\begin{bmatrix}3 & -1\\ -1 & 1\end{bmatrix}-\omega^2 m\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right)\begin{Bmatrix}v_1\\ v_2\end{Bmatrix}=\begin{Bmatrix}0\\ 0\end{Bmatrix}. \quad \text{Let } s=\frac{\omega^2 L}{g}$$

$$\begin{vmatrix}3-s & -1\\ -1 & 1-s\end{vmatrix}=s^2-4s+2=0 \quad \begin{cases} s=0.586\\ s=3.414\end{cases}$$

For $s=0.586$, $\omega^2=0.586\,g/L$

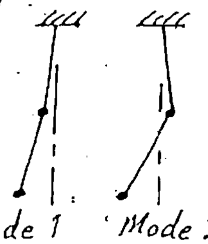
$-v_1+(1-0.586)v_2=0$

$v_1=0.414\,v_2$

For $s=3.414$, $\omega^2=3.414\,g/L$

$-v_1+(1-3.414)v_2=0$

$v_1=-2.414\,v_2$



18.6-6

Use Eq. 18.6-2 for this problem.

$$\left(\frac{300}{1^3}\begin{bmatrix}4 & 2\\ 2 & 4\end{bmatrix}-\omega^2\frac{2100(0.0002)}{420}\begin{bmatrix}4 & -3\\ -3 & 4\end{bmatrix}+\frac{T}{30}\begin{bmatrix}4 & -1\\ -1 & 4\end{bmatrix}\right)\begin{Bmatrix}\theta_1\\ \theta_2\end{Bmatrix}=\begin{Bmatrix}0\\ 0\end{Bmatrix}$$

Set $\theta_2=-\theta_1$; then

$$\left(600-\frac{7\omega^2}{1000}+\frac{T}{6}\right)\theta_1=0$$

(a) $T=0$; $\omega=\left[\dfrac{6(10)^5}{7}\right]^{1/2}=293$ rad/s

(b) $\omega^2=347^2$; $T=6\left[\dfrac{7(347)^2}{1000}-600\right]\doteq 1457\,N$

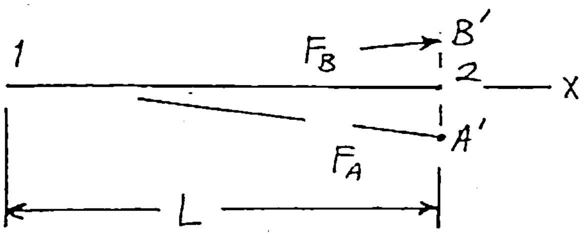
(c) $T=-1200$;

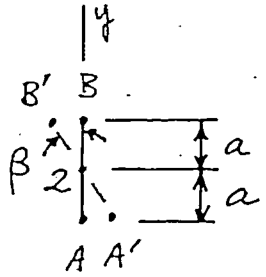
$$\omega=\left[\frac{1000}{7}\left(600+\frac{-1200}{6}\right)\right]^{1/2}=239\ \frac{\text{rad}}{\text{s}}$$

$\boxed{18.6\text{-}7}$

Imagine a small rotation of the bars of length a about the x axis.

View along the y axis:





$$\alpha = \frac{\beta a}{L} \quad \text{(a small angle)}$$

$$F_A = F_B = mL\Omega^2$$

Torque about x axis due to F_A and F_B is

$$T = 2(F_A \alpha)a = 2mL\Omega^2 \alpha a = 2mL\Omega^2 \frac{\beta a^2}{L} = 2m\Omega^2 \beta a^2$$

Torque generated due to twist of bar 1-2 is

$$\text{rotation} = \frac{TL}{GK} \quad \text{so} \quad T = \frac{GK\beta}{L}$$

Equate torques

$$\frac{GK\beta}{L} = 2m\Omega^2 \beta a^2 \quad \text{hence} \quad \Omega^2 = \frac{GK}{2mLa^2}$$