$\boxed{17.2\text{-}1}$

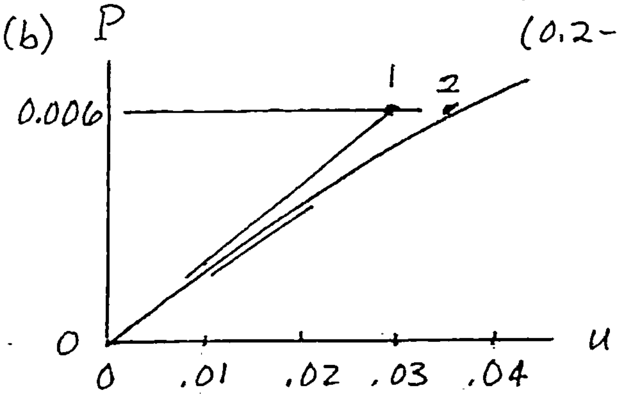
(a) $(0.2-u)u = 0.006$; $u_{exact} = 0.0367544$

Direct substitution: $(0.2-0)u_1 = 0.006$, $u_1 = 0.030$

$(0.2-0.030)u_2 = 0.006$, $u_2 = 0.03529$

$(0.2-0.03529)u_3 = 0.006$, $u_3 = 0.03643$

(b)



Step 0-1 is tangent to curve at $u = 0$

(c) $(0.2-u)u = P$

$P = 0.2u - u^2$

$k_t = \dfrac{dP}{du} = 0.2 - 2u$

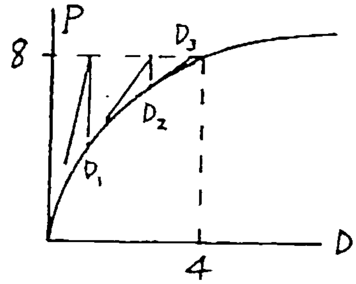
17.2-2

(a) $\dfrac{10}{(D+1)^2}\Delta D = P - R_c = 8 - R_c$ ← resisting force

$$\frac{10}{(D_i+1)^2}\Delta D_{i+1} = 8 - \frac{10 D_i}{D_i+1} \qquad D_{i+1} = D_i + \Delta D_{i+1}$$

Start with $D=0$ when $i=0$. Thus

$D_1 = 0.800$
$D_2 = 1.952$
$D_3 = 3.161$
$D_4 = 3.859$
$D_5 = 3.996$

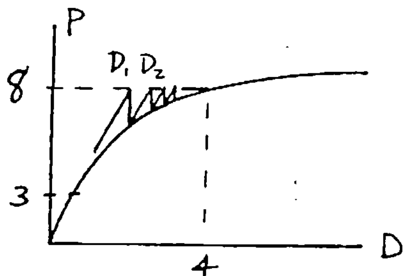


(b) At $D=3$, $k_t = \dfrac{dP}{dD} = \dfrac{10}{(3+1)^2} = 0.625$

$0.625\Delta D = P - R_c$, $\quad 0.625\Delta D_{i+1} = 8 - \dfrac{10D_i}{D_i+1}$

and $D_{i+1} = D_i + \Delta D_{i+1}$. Start with $D=3$ at $i=0$.

$D_1 = 3.800$
$D_2 = 3.933$
$D_3 = 3.977$
$D_4 = 3.992$
$D_5 = 3.997$

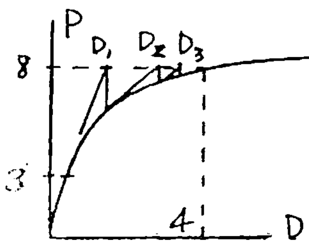


(c) Obtain $D_1 = 3.8$ as in part (b). Then $k_t = \dfrac{10}{(3.8+1)^2} = 0.43403$. Then

$0.43403\,\Delta D_{i+1} = 8 - \dfrac{10D_i}{D_i+1}$

We obtain

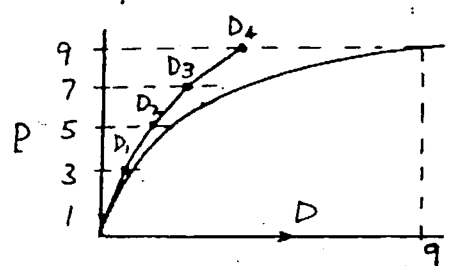
$D_2 = 3.992$
$D_3 = $ [illegible]
$D_4 = 3.99995$



(d) $\dfrac{10}{(D_i+1)^2}\Delta D_{i+1} = \Delta P = 2, \quad D_{i+1} = D_i + \Delta D_{i+1}$

Start with $P=1$ at $D=\frac{1}{9}$.

$P_0 = 1 \quad D_0 = 1/9$
$P_1 = 3 \quad D_1 = 0.358$
$P_2 = 5 \quad D_2 = 0.727$
$P_3 = 7 \quad D_3 = 1.323$
$P_4 = 9 \quad D_4 = 2.403$

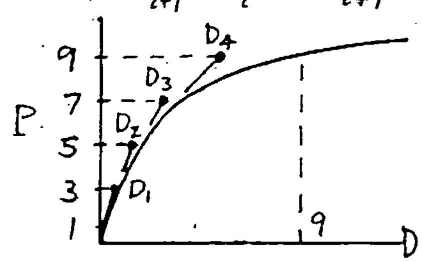


(e) After calc. of $D_1 = 0.358$ in part (d), write

$$\frac{10}{(D_i+1)^2}\Delta D_{i+1} = 2 + \left(P_i - \frac{10}{D_i+1}\right)$$

where P_i is the load applied to get D_i, not the load associated with $D_{i+1} = D_i + \Delta D_{i+1}$.

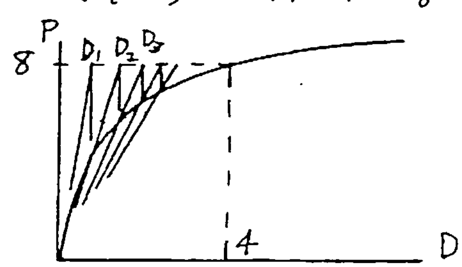
$P_0 = 1 \quad D_0 = 1/9$
$P_1 = 3 \quad D_1 = 0.358$
$P_2 = 5 \quad D_2 = 0.794$
$P_3 = 7 \quad D_3 = 1.622$
$P_4 = 9 \quad D_4 = 3.557$



(f) $\dfrac{10}{D_i+1} D_{i+1} = R$

$D_{i+1} = \dfrac{D_i+1}{10} R = 0.8(D_i+1)$. Start with $D_0 = 0$.

$D_1 = 0.800$
$D_2 = 1.440$
$D_3 = 1.952$
$D_4 = 2.362$
$D_5 = 2.689$



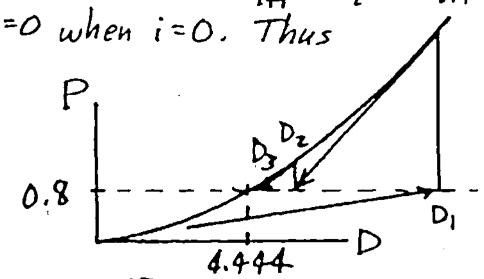
$\boxed{17.2\text{-}3}$

(a) $\dfrac{10}{(10-D)^2}\Delta D = P - R_c = 0.8 - R_c$ ← resisting force

$\dfrac{10}{(10-D_i)^2}\Delta D_{i+1} = 0.8 - \dfrac{D_i}{10-D_i}$ $\quad D_{i+1} = D_i + \Delta D_{i+1}$

Start with $D=0$ when $i=0$. Thus

$D_1 = 8.000$
$D_2 = 6.720$
$D_3 = 5.377$
$D_4 = 4.601$
$D_5 = 4.449$

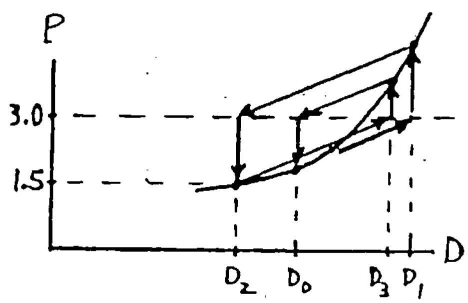


(b) At $D=6$, $k_t = \dfrac{dP}{dD} = \dfrac{10}{(10-6)^2} = 0.625$

$0.625\,\Delta D = P - R_c$, $\quad 0.625\,\Delta D_{i+1} = 3 - \dfrac{D_i}{10-D_i}$

and $D_{i+1} = D_i + \Delta D_{i+1}$. Start with $D=6$ at $i=0$.

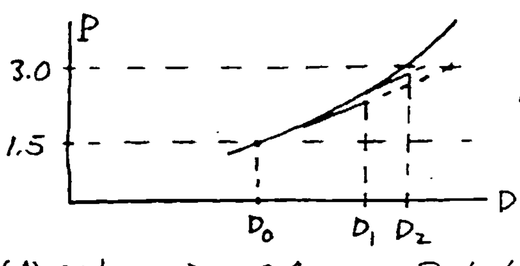
$D_1 = 8.400$
$D_2 = 4.800$
$D_3 = 8.123$
$D_4 = 5.998$
$D_5 = 8.400$



We are stuck in a loop.

(c)

i	D_i	ΔD_{i+1}	$0.6\Delta D_{i+1}$	D_{i+1}
0	6.000	2.400	1.440	7.440
1	7.440	0.150	0.090	7.530
2	7.530	−0.078	−0.047	7.483
3	7.483	0.042	0.025	7.509
4	7.509	−0.023	−0.014	7.495



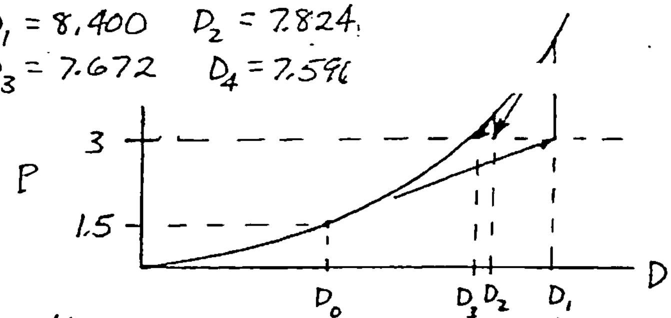
Exact result at $P=3$ is $D=7.50$.

(d) Obtain $D_1 = 8.4$ as in Part (b). Then

$k_t = \dfrac{10}{(10-8.4)^2} = 3.90625$, Then

$3.90625\,\Delta D_{i+1} = 3 - \dfrac{D_i}{10-D_i}$, $\quad D_{i+1} = D_i + \Delta D_{i+1}$

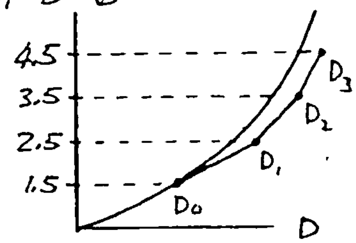
$D_1 = 8.400 \quad D_2 = 7.824$
$D_3 = 7.672 \quad D_4 = 7.596$



(e) $\dfrac{10}{(10-D_i)^2}\Delta D_{i+1} = \Delta P = 1$, $\quad D_{i+1} = D_i + \Delta D_{i+1}$

Start with $P = 1.5$ at $D = 6$

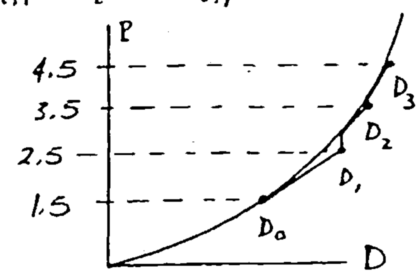
$P_0 = 1.5$, $D_0 = 6.000$
$P_1 = 2.5$, $D_1 = 7.600$
$P_2 = 3.5$, $D_2 = 8.176$
$P_3 = 4.5$, $D_3 = 8.509$



(f) After calc. of $D_1 = 7.600$ in Part (e), write

$\dfrac{10}{(10-D_i)^2}\Delta D_{i+1} = 1 + \left(P_i - \dfrac{D_i}{10-D_i}\right)$, where P_i is the load applied to obtain D_i, not the load associated with $D_{i+1} = D_i + \Delta D_{i+1}$.

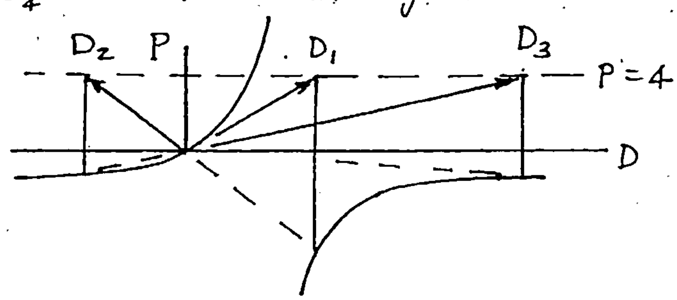
$P_0 = 1.5$, $D_0 = 6.000$
$P_1 = 2.5$, $D_1 = 7.600$
$P_2 = 3.5$, $D_2 = 7.792$
$P_3 = 4.5$, $D_3 = 8.265$



(g) $(10-D)P = D$

$D_{i+1} = P(10 - D_i) = 4(10 - D_i)$

$D_0 = 0$, $D_1 = 40$, $D_2 = -120$, $D_3 = 520$,
$D_4 = -2040$: diverges.



17.2-4

(a) Let L = length of bar.

$$L=\left[a^2+(c-D)^2\right]^{1/2}=a\left[1+\left(\frac{c-D}{a}\right)^2\right]^{1/2}$$

$$L\approx a\left[1+\frac{1}{2}\left(\frac{c-D}{a}\right)^2\right] \quad \text{for } c-D \ll a$$

$$L_o\approx a\left[1+\frac{1}{2}\left(\frac{c}{a}\right)^2\right] \quad \text{for } D=0.$$

$$\epsilon=\frac{L-L_o}{L_o}\approx\frac{L-L_o}{a}\approx\frac{1}{2}\left(\frac{c-D}{a}\right)^2-\frac{1}{2}\frac{c^2}{a^2}=\frac{-2cD+D^2}{2a^2}$$

$$\Pi_p=\int_0^L\frac{E}{2}\epsilon^2 A\,dx-PD\approx\frac{AE}{8a^3}\left(-2cD+D^2\right)^2-PD$$

(b) $$0=\frac{\partial\Pi_p}{\partial D}=\frac{AE}{4a^3}\left(-2cD+D^2\right)\left(-2c+2D\right)-P$$

or $$0=\frac{AE}{2a^3}\left(2cD-D^2\right)\left(c-D\right)-P \qquad (A)$$

(c) Eq. (A) above is

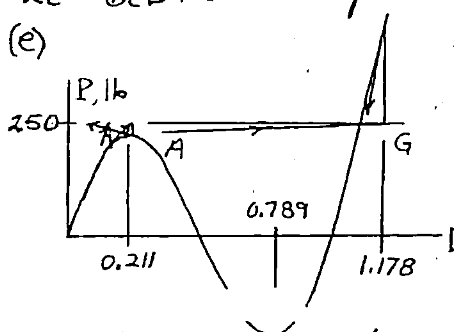
$$\left[\frac{AE}{2a^3}\left(2c^2-3cD+D^2\right)\right]D=P\text{, i.e. } K_{secant}D=P$$

$$\frac{dP}{dD}=\frac{AE}{2a^3}\left(2c^2-6cD+3D^2\right)=K_{tangent}$$

(d) Limit points, where $K_{tangent}=0$

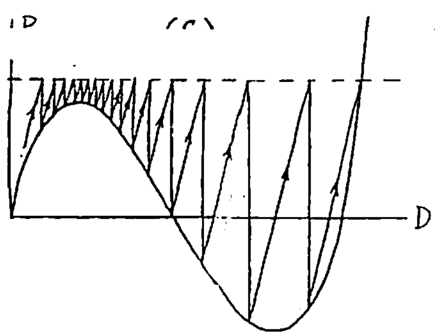
$2c^2-6cD+3D^2=0$ yields $D=c\left(1\pm\frac{\sqrt{3}}{3}\right)$

(e)



The solution dances about on top of the first hill, unable to find a D for which the resistance of the structure is equal to the load.

Only when we move along AG, which has a small enough slope that we hit $P=250$ at a D greater than 0.789, can we converge.



$\boxed{17.2\text{-}5}$

$$P_1 = f_1(D_1, D_2) \qquad P_2 = f_2(D_1, D_2)$$

$$P_A = f_1(D_A, D_B) = f_1(D_A^* + \Delta D_A,\ D_B^* + \Delta D_B)$$

$$P_B = f_2(D_A, D_B) = f_2(D_A^* + \Delta D_A,\ D_B^* + \Delta D_B)$$

Apply truncated Taylor series.

$$P_A = f_1(D_A^*, D_B^*) + \Delta D_A \left.\frac{\partial f_1}{\partial D_1}\right|_{D_A^*, D_B^*} + \Delta D_B \left.\frac{\partial f_1}{\partial D_2}\right|_{D_A^*, D_B^*}$$

$$P_B = f_2(D_A^*, D_B^*) + \Delta D_A \left.\frac{\partial f_2}{\partial D_1}\right|_{D_A^*, D_B^*} + \Delta D_B \left.\frac{\partial f_2}{\partial D_2}\right|_{D_A^*, D_B^*}$$

Group terms

$$\underbrace{\begin{bmatrix} \partial f_1/\partial D_1 & \partial f_1/\partial D_2 \\ \partial f_2/\partial D_1 & \partial f_2/\partial D_2 \end{bmatrix}_{D_A^*, D_B^*}}_{\text{tangent stiffness at } D_A^*, D_B^*} \begin{Bmatrix} \Delta D_A \\ \Delta D_B \end{Bmatrix} = \underbrace{\begin{Bmatrix} P_A - f_1(D_A^*, D_B^*) \\ P_B - f_2(D_A^*, D_B^*) \end{Bmatrix}}_{\text{load imbalance}}$$

disp. increments

$\boxed{17.2-6}$

$$g'(x^*) = 1 - \frac{f'(x^*)^2 - f''(x^*)f(x^*)}{f'(x^*)^2}$$

$$g'(x^*) = 1 - 1 + \frac{f''(x^*)f(x^*)}{f'(x^*)^2}$$

But $f(x^*)=0$, so $g'(x^*)=0$. Also $g(x^*)=x^*$
Hence

$$x_{i+1} = g(x_i) = \underbrace{g(x^*)}_{=x^*} + \underbrace{g'(x^*)}_{=0}(x_i - x^*) + \frac{1}{2} g''(\bar{x}_i)(x_i - x^*)^2$$

Thus

$$|x_{i+1} - x^*| = \frac{1}{2}\left|g''(\bar{x}_i)\right|(x_i - x^*)^2$$

or $\quad e_{i+1} = Ce_i^2$

$\boxed{17.3\text{-}1}$

At start, $K_t = 2\frac{AE}{L} = 2\frac{(1)10{,}000}{10} = 2000$

$2000\,\Delta D = 20$, $\Delta D = 0.01$, $D = 0 + \Delta D = 0.01$

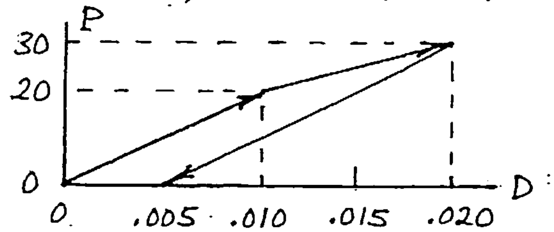
$\Delta\epsilon = 0.001$, $\epsilon = \pm 0.001$. Now $E_t = 0$ in left bar.

$K_t = \frac{AE}{L} = 1000$, $1000\,\Delta D = 1000$, $\Delta D \doteq 0.01$,

$D = 0.01 + \Delta D = 0.02$. Now unload (elastically)

$K_t = 2000$ again; $2000\,\Delta D = -30$,

$\Delta D = -0.015$, $D = 0.02 + (-0.015) = +0.005$



17.3-2

To yield point: $K = 2(AE/L) = 2000$, $KD = P$, where $P = 20$ & $D = 0.01$. Subsequent strain increments are entirely plastic in left el., $\Delta\epsilon^p = \Delta\epsilon$. For $\Delta P = 10$, $2000\,\Delta D = 10$, $\Delta D = 0.005$, $D = 0.01 + \Delta D = 0.015$. Load ΔR_s is $AE\Delta\epsilon^p = (1)10{,}000\,\Delta\epsilon^p$. Hence $2000\,\Delta D_i = 10{,}000\,\Delta\epsilon^p = 10{,}000\,(\Delta D_{i-1}/10)$

i.e. $\Delta D_i = 0.5\,\Delta D_{i-1}$

& $D_i = D_{i-1} + \Delta D_{i-1}$

Successive iterates are as shown at right.

ΔD	D
0.005	0.015
0.0025	0.0175
0.00125	0.01875
0.000625	0.019375
0.0003125	0.0196875
⋮	⋮
0.0	0.020

17.3-3

For each bar, $k = AE/L = 0.5$ until yielding; $k = 0$ after yielding. Before yielding, force in each bar is $F = 0.5v$.

<u>1st Step</u> $K = 3k = 1.5$. Let $\Delta P = 1$.

$1.5\,\Delta v = 1$, $\Delta v = \frac{2}{3}$, $v = 0 + \Delta v = \frac{2}{3}$

Scale solution by c to make $F_1 = 2$:

$F_1 = 2 = 0.5\left(\frac{2}{3}\right)c$, $c = 6$, so $v = 6\left(\frac{2}{3}\right) = 4$

<u>2nd Step</u> $K = 2k = 1$. Let $\Delta P = 1$.

$(1)\,\Delta v = 1$, $\Delta v = 1$. Scale to make $F_2 = 4$:

$0.5(4 + c\,\Delta v) = F_2 = 4$, $4 + c = 8$, $c = 4$

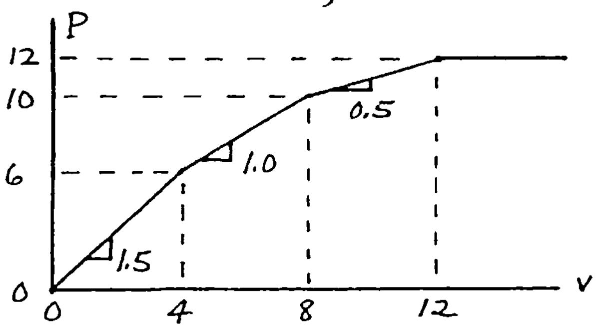
Now $v = 4 + 4(1) = 8$, $P = 6 + 4(1) = 10$

<u>3rd Step</u> $K = (1)k = 0.5$. Let $\Delta P = 1$.

$0.5\,\Delta v = 1$, $\Delta v = 2$. Scale to make $F_3 = 6$:

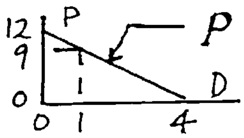
$0.5(8 + c\,\Delta v) = F_3 = 6$, $4 + c = 6$, $c = 2$

Now $v = 8 + 2(2) = 12$, $P = 10 + 2(1) = 12$



17.3-4

(a) when $D<1$, $P<19$, $K=19$



$P = 12 - \frac{12}{4}D$ for bar #2

When $D>1$, stiffness eq. is

$$10D + \left(12 - \frac{12}{4}D\right) = P \quad (A)$$

For $P = 24$, $D = 12/7 = 1.714286$

(b) When $D=1$, $P = 10+9 = 19$

Load left to apply is $24-19=5$

From Eq. (A), for $D>1$, $K = 10 - \frac{12}{4} = 7$

Hence $K\Delta D = \Delta P$ is $7\Delta D = 5$ and

$\Delta D = \frac{5}{7} = 0.714286$, $D = 1 + \Delta D = 1.714286$

17.3-5

To start, assume standard linearly elastic structure. Form $[\underset{\sim}{K}]$; apply a reference load; calculate $\{\underset{\sim}{D}\}$ and bar forces. For each bar, calculate ratio of actual load to yield point load (or buckling load for compression members). Select the largest of these ratios and scale the entire solution by dividing by it. We are now at the end of the linear regime.

Now remove the bar about to yield or buckle. Load the structure nodes to which it was attached by the yield or buckling force in the bar removed. Form $[\underset{\sim}{K}]$ for the remaining structure; apply load increment; calculate $\{\Delta\underset{\sim}{D}\}$ and increments in bar forces. Determine r_i, the load increment ratio needed to cause each remaining bar to fail, i.e.

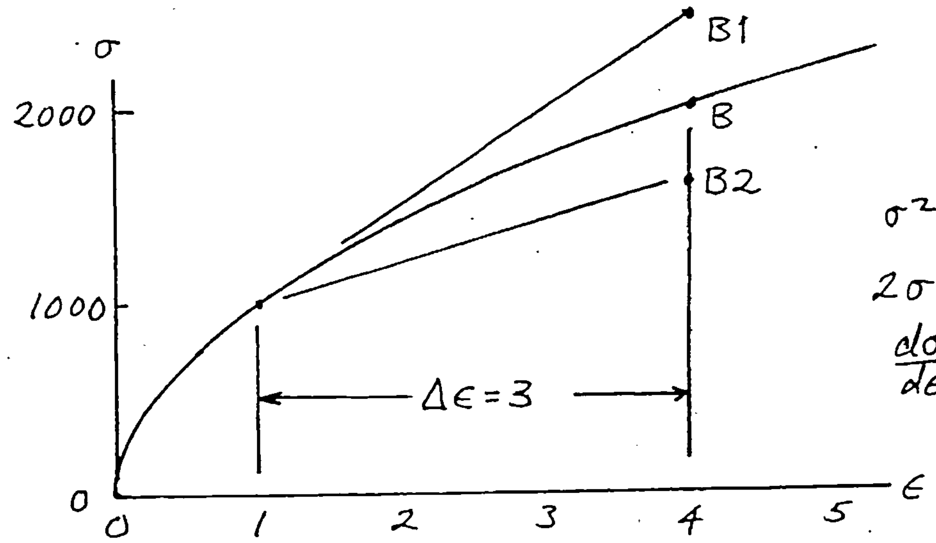
$$(F_i)_{yield} = (F_i)_{previous} + r_i(\Delta F_i)$$

Scale the solution update by multiplying it by the smallest r_i, then update.

Repeat to fail one more bar. Structure collapses when $[\underset{\sim}{K}]$ becomes singular.

(If the truss is statically determinate initially, it will collapse when the first bar yields or buckles.)

$\boxed{17.3\text{-}6}$



$$\sigma^2 = 10^6 \epsilon$$

$$2\sigma\, d\sigma = 10^6 d\epsilon$$

$$\frac{d\sigma}{d\epsilon} = \frac{10^6}{2\sigma}$$

$$\Delta\sigma_1 = \frac{10^6}{2\sigma}\Delta\epsilon = \frac{10^6}{2(1000)} 3 = 1500$$

$$\sigma_{B1} = 1000 + 1500 = 2500$$

$$\Delta\sigma_2 = \frac{10^6}{2(2500)} 3 = 600$$

Now Eq. 17.3-7: $\sigma_B \simeq 1000 + \frac{1}{2}(1500 + 600) = 2050$ (2.5% high)

Exact σ_B is 2000

$\boxed{17.5\text{-}1}$

(a) Gauss rule does not detect start of yielding: no sampling points at ends of interval (surfaces), where yield begins

(b) Let $d=2$, width $=1$. Exact M is

$$M = 2\left[\left(\sigma_a \frac{1}{2}\right)\frac{3}{4} + \left(\frac{\sigma_a}{2}\frac{1}{2}\right)\left(\frac{2}{3}\frac{1}{2}\right)\right] = 0.91667\,\sigma_a$$

Two Gauss pts.: $M_c = \int_{-1}^{1} \sigma(1)\,y\,dy = 2\sigma_{GP}\,y_{GP}$

$$M_c = 2\sigma_a \frac{\sqrt{3}}{3} = 1.1547\sigma_a \quad (26.0\%\text{ high})$$

Three Gauss pts.:

$$M_c = 2\left(\frac{5}{9}\sigma_a\sqrt{0.6}\right) = 0.8607\sigma_a \quad (6.1\%\text{ low})$$

(c) $$\int_a^b f(y)\,dy = \frac{b-a}{n}\left(\frac{1}{2}f_0 + f_1 + f_2 + \cdots + f_{n-1} + \frac{1}{2}f_n\right)$$

$$M_{c3} = \frac{2}{2}\left(\frac{1}{2}\sigma_a + 0 + \frac{1}{2}\sigma_a\right) = \sigma_a \quad (9.1\%\text{ high})$$

$$M_{c5} = \frac{2}{4}\left(\frac{1}{2}\sigma_a + \sigma_a\frac{1}{2} + 0 + \sigma_a\frac{1}{2} + \frac{1}{2}\sigma_a\right) = \sigma_a$$

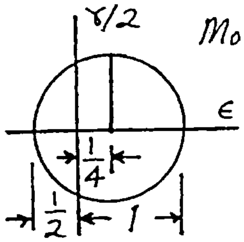
$$M_{c7} = \frac{2}{6}\left(\frac{1}{2}\sigma_a + \sigma_a\frac{2}{3} + \frac{2\sigma_a}{3}\frac{1}{3} + 0 + \frac{2\sigma_a}{3}\frac{1}{3} + \sigma_a\frac{2}{3} + \frac{1}{2}\sigma_a\right) = 0.9259\sigma_a \quad (1.0\%\text{ hi})$$

$$M_{c9} = \frac{2}{8}\left(\frac{1}{2}\sigma_a + \sigma_a\frac{3}{4} + \sigma_a\frac{1}{2} + \frac{\sigma_a}{2}\frac{1}{4} + 0 + \frac{\sigma_a}{2}\frac{1}{4} + \sigma_a\frac{1}{2} + \sigma_a\frac{3}{4} + \frac{1}{2}\sigma_a\right) = 0.9375\sigma_a$$

$(2.3\%\text{ hi})$

17.5-2

In what follows, ϵ and γ represent strain increments.



Mohr's strain circle:

$$\text{radius} = \frac{1}{2}\left(\frac{3}{2}\right) = \frac{3}{4}$$

$$\epsilon_x^P = \epsilon_y^P = \frac{1}{4}, \quad \epsilon_z^P = -\frac{1}{2}$$

$$\gamma_{xy}^P = 2\left(\frac{3}{4}\right) = \frac{3}{2}, \quad \gamma_{yz}^P = \gamma_{zx}^P = 0$$

$$\epsilon_{ef}^P = \frac{\sqrt{2}}{3}\left[0^2 + \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^2 + \frac{3}{2}\left(\frac{3}{2}\right)^2\right]^{1/2}$$

$$\epsilon_{ef}^P = \frac{\sqrt{2}}{3}\left[\frac{18}{16} + \frac{27}{8}\right]^{1/2} = \frac{\sqrt{2}}{3}\left[\frac{36}{8}\right]^{1/2} = \frac{\sqrt{2}}{3}\,\frac{6}{2\sqrt{2}} = 1$$

$\boxed{17.5\text{-}3}$

- 1. Apply a load; calculate elastic solution & yield function F_i at each sampling point i (all $F_i < 0$ if no yielding). Calculate factor that will place most highly stressed sampling point at yield. Multiply elastic solution by this factor.
- 2. Determine $[\underset{\sim}{E}_{ep}]$ for sampling points that have yielded or will yield in next step. Form $[\underset{\sim}{K}_t] = \Sigma[k]$, using $[\underset{\sim}{E}]$ or $[\underset{\sim}{E}_{ep}]$ as appropriate for the various sampling points.
- 3. Apply a load increment; solve for increments of displacement and stress. Write Eq. 17.4-13 in form $F = S - \sigma_Y$. For each sampling point not yet yielded, calculate factor r that will initiate yield:

$$\sigma_Y = S_{previous} + r\,\Delta S_{current}$$

Scale solution increment by smallest r and update the solution using the scaled increment. Go to step 2.

17.5-4

Similar to Problem 17.5-3. Calculate σ_{max} at each sampling point. In Step 1, find sampling point with largest σ_{max}; compute $r = \sigma_t / \sigma_{max}$; scale solution by r. In Step 2, calculate $[\underset{\sim}{E}_{ep}]$ as follows. Let

$$[\underset{\sim}{E}'] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & \beta G \end{bmatrix} \begin{matrix} \sigma_{max} \\ \sigma_{min} \\ \tau \end{matrix}$$

(referred to prin. stress directions)

where $0 < \beta < 1$ to retain partial shear stiffness. Also, set $E = 0$ in σ_{max} and σ_{min} exceed σ_t. Transform to global directions, $[\underset{\sim}{E}_{ep}] = [\underset{\sim}{T}]^T [\underset{\sim}{E}'] [\underset{\sim}{T}]$.

In Step 3, use the equation

$$\sigma_t = (\sigma_{max})_{previous} + r(\Delta\sigma_{max})_{current}$$

Collapse indicated by very large $\{\Delta \underset{\sim}{D}\}$ or by unsolvable equations.

$\boxed{17.9-1}$

Green strain: $\epsilon_x = \frac{u_2}{L} + \frac{1}{2}\left(\frac{u_2^2}{L^2} + \frac{v_2^2}{L^2}\right)$

Let $r = \frac{u_2}{L}$, which is the engineering definition of strain.

For Green strain to be 105% of $\frac{u_2}{L}$, $\quad 0.05\,r = \frac{1}{2}\left(r^2 + \frac{v_2^2}{L^2}\right) \quad (A)$

(a) $v_2 = 0$; (A) gives $r = 0.10$

(b) $v_2 = u_2$; (A) becomes $0.05r = r^2$, so $r = 0.05$

(c) $v_2 = 100\,u_2$; (A) becomes $0.05r = \frac{1}{2}(101r^2)$, so $r = \frac{0.1}{101} \approx 0.001$

If instead we set Green strain to be 95% of $\frac{u_2}{L}$, we obtain the same magnitudes of r but with a negative sign on each.

$\boxed{17.9-2}$

$$\epsilon_x = u_{,x} + \frac{1}{2}(u_{,x}^2 + v_{,x}^2)$$

$$\epsilon_y = v_{,y} + \frac{1}{2}(u_{,y}^2 + v_{,y}^2)$$

$$\gamma_{xy} = u_{,y} + v_{,x} + (u_{,x}u_{,y} + v_{,x}v_{,y})$$

$$u = a_1 + (\cos\theta - 1)x - (\sin\theta)y$$

$$v = a_4 + (\sin\theta)x + (\cos\theta - 1)y$$

$$\epsilon_x = \cos\theta - 1 + \frac{1}{2}(\cos^2\theta - 2\cos\theta + 1 + \sin^2\theta)$$

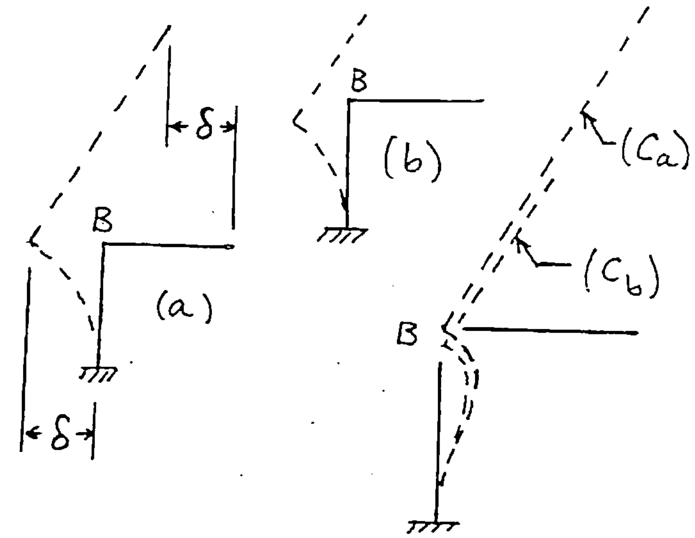
$$= \cos\theta - 1 + (1 - \cos\theta) = 0$$

$$\epsilon_y = \cos\theta - 1 + \frac{1}{2}(\sin^2 + \cos^2\theta - 2\cos\theta + 1)$$

$$= \cos\theta - 1 + (1 - \cos\theta) = 0$$

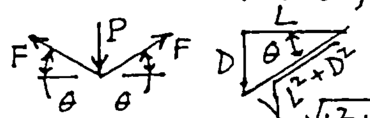
$$\gamma_{xy} = -\sin\theta + \sin\theta + (\cos\theta - 1)(-\sin\theta) + (\sin\theta)(\cos\theta - 1) = 0$$

17.9-3



$\boxed{17.9\text{-}4}$

$$P = 2F\sin\theta\,,\quad F = k_o\left(\sqrt{L^2+D^2} - L\right)$$



$$\sin\theta = \frac{D}{\sqrt{L^2+D^2}}$$

Hence $P = 2k_o D \dfrac{\sqrt{L^2+D^2} - L}{\sqrt{L^2+D^2}}$ or

$$P = 2k_o D \frac{\sqrt{1+D^2/L^2} - 1}{\sqrt{1+D^2/L^2}} \qquad (A)$$

For small D,

$$P = 2k_o D \frac{1+\frac{1}{2}\frac{D^2}{L^2} - 1}{1+\frac{D^2}{2L^2}} = 2k_o D \frac{D^2}{2L^2} = k_o \frac{D^3}{L^2} \quad (B)$$

$$k_{secant} = P/D = \frac{k_o D^2}{L^2}$$

(b) From Eq. (B), $\dfrac{dP}{dD} = 3k_o \dfrac{D^2}{L^2} = k_t$

(c) From Eq. (A), $\dfrac{P}{D} = 2k_o \dfrac{\sqrt{1+D^2/L^2} - 1}{\sqrt{1+D^2/L^2}}$

(d) At $D = \frac{1}{2}$, $k_t = \dfrac{3(800)\left(\frac{1}{2}\right)^2}{10^2} = 6$

Apply $\Delta P = 7$ in $k_t \Delta D = \Delta P$

$6\,\Delta D = 7$, $\Delta D = 1.167$, $D = \frac{1}{2} + \Delta D$

$D = 1.667$

Equilibrium iterations:

$$k_t = \frac{3(800)(1.667)^2}{10^2} = 66.67$$

$$\ldots = 8 - 800\,\frac{1.667^3}{10^2} = -29.04$$

$\Delta D = -0.436$, $D = 1.231$

$$k_t = \frac{3(800)(1.231)^2}{10^2} = 36.37$$

$$36.37\,\Delta D = 8 - 800\,\frac{1.231^3}{10^2} = -6.92$$

$\Delta D = -0.190$, $D = 1.041$

$$k_t = \frac{3(800)(1.041)^2}{10^2} = 25.99$$

$$25.99\,\Delta D = 8 - 800\,\frac{1.041^3}{10^2} = -1.016$$

$\Delta D = -0.039$, $D = 1.002$

From eq. (B),

$$D^3 = \frac{PL^2}{k_o} = \frac{8(10)^2}{800} = 1,\quad D = 1$$

$\boxed{17.9\text{-}5}$

Let α be the rigid-body rotation angle.

$\beta_0 = y_0 = D_1 = D_2 = 0$

$x_6 = L = L_0 \qquad D_3 = D_6 = \alpha$

$D_5 = L_0 \sin\alpha$

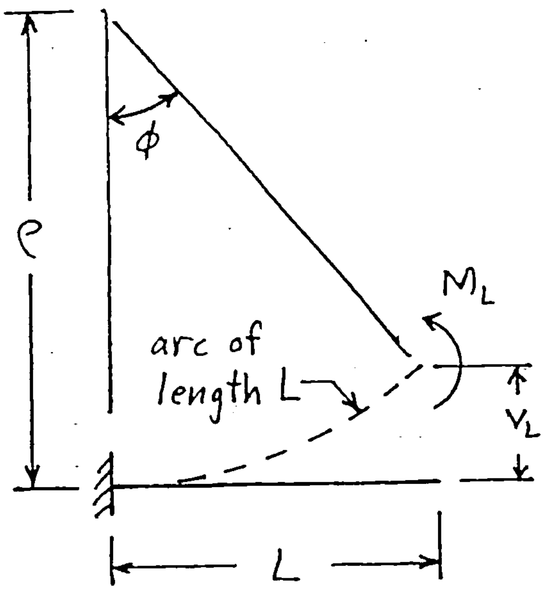
$D_4 = -L_0(1-\cos\alpha)$

$$u_2 = \frac{1}{2L_0}\left[2L_0 - L_0(1-\cos\alpha)\right]\left[-L_0(1-\cos\alpha)\right] + \frac{1}{2L_0}\left[L_0\sin\alpha\right]\left[L_0\sin\alpha\right]$$

$$u_2 = \frac{L_0}{2}\left[-2(1-\cos\alpha) + (1-2\cos\alpha+\cos^2\alpha) + \sin^2\alpha\right]$$

$$u_2 = \frac{L_0}{2}\left[-1 + \cos^2\alpha + \sin^2\alpha\right] = 0$$

17.9-6



$$\left.\begin{aligned} \frac{1}{\rho} &= \frac{M}{EI} \\ \phi &= \frac{L}{\rho} = \frac{ML}{EI} \end{aligned}\right\} \quad v_L = \rho(1-\cos\phi) = \frac{EI}{M}\left(1-\cos\frac{ML}{EI}\right)$$

$$\cos\phi = 1 - \frac{\phi^2}{2} + \frac{\phi^4}{24} - \cdots$$

For small ϕ, $1-\cos\phi \approx \frac{\phi^2}{2}$

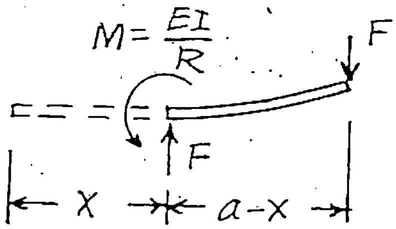
Thus $1-\cos\frac{ML}{EI}$ becomes $\frac{1}{2}\left(\frac{ML}{EI}\right)^2$, and

$$v_L \approx \frac{--}{M}\,\frac{1}{2}\left(\frac{ML}{EI}\right)^2 = \frac{ML^2}{2EI}$$

17.9-7

Becomes straight when change of curvature is $\frac{1}{R}$. But $\frac{1}{R} = \frac{M}{EI}$, so at left end, where $M = Fa$, $\frac{1}{R} = \frac{Fa}{EI}$ and $F = \frac{EI}{Ra}$

For $F > \frac{EI}{Ra}$:

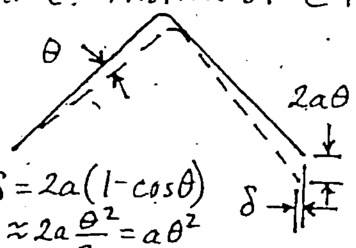


$$M = F(a-x)$$

$$x = a - \frac{M}{F} = a - \frac{EI}{FR}$$

17.9-8

Let θ = rigid body rotation about A, with no force at C. Motion of C is

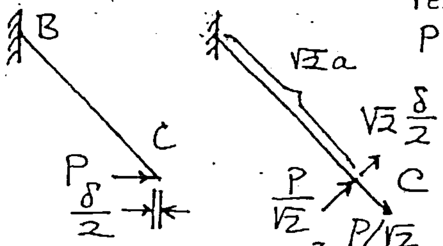


$\delta = 2a(1-\cos\theta)$

$\delta \approx 2a\frac{\theta^2}{2} = a\theta^2$

This is the motion we are to calculate.

Displacement δ must now be restored by bending of fram in response to reaction P of wall at C.



Consider right half of frame, as shown:

$$\frac{\sqrt{2}\,\delta}{2} = \frac{(P/\sqrt{2})(\sqrt{2}a)^3}{3EI}, \quad \delta = a\theta^2 = \frac{2\sqrt{2}\,Pa^3}{3EI},$$

$P = \frac{3EI\theta^2}{2\sqrt{2}\,a^2}$. Moments about A: $Fa = (2a\theta)P$,

hence $F = 2\theta P = \frac{3EI\theta^3}{\sqrt{2}\,a^2}$, $\theta = \left(\frac{\sqrt{2}\,Fa^2}{3EI}\right)^{1/3}$

$$2a\theta = \left(8a^3\frac{\sqrt{2}\,Fa^2}{3EI}\right)^{1/3} = \left(\frac{8\sqrt{2}\,Fa^5}{3EI}\right)^{1/3}$$