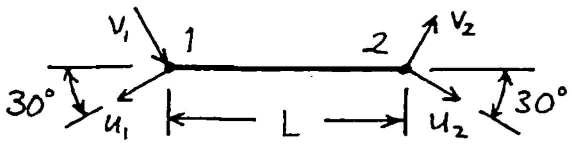
10.9-1



$v_1 = v_2 = 0$

Let F = axial force in bar. $F = k(0.866u_1)$ where $k = AE/L$. Component of F in u_1 direction is $0.866F = 0.75ku_1$.

Hence $[\underset{\sim}{K}]\{\underset{\sim}{D}\} = \{\underset{\sim}{R}\}$ is

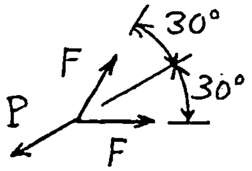
$$0.75k\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} P \\ 0 \end{Bmatrix}$$

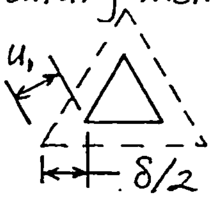
where P appears only once so as not to double the load.

Also $\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} u_1 = [\underset{\sim}{T}]u_1$

$$\left.\begin{aligned} [\underset{\sim}{T}]^T[\underset{\sim}{K}][\underset{\sim}{T}] &= 3k \\ [\underset{\sim}{T}]^T\{\underset{\sim}{R}\} &= P \end{aligned}\right\} 3ku_1 = P, \quad u_1 = \frac{P}{3k}$$

Check by elementary methods:





$u_1 = \dfrac{\delta/2}{\cos 30°}$

$u_1 = \dfrac{\delta}{\sqrt{3}}$

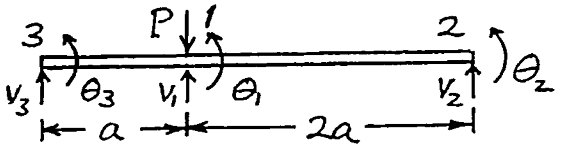
$2F\cos 30° = P$

$F = \dfrac{P}{2\cos 30°} = \dfrac{P}{\sqrt{3}}$

$\delta = \dfrac{FL}{AE} = \dfrac{PL}{\sqrt{3}AE}$

$u_1 = \dfrac{PL}{3AE} = \dfrac{P}{3k}$

$\boxed{10.9\text{-}2}$



For left portion, $[\underset{\sim}{k}]$ is (note signs)

$$\frac{EI}{a^3}\begin{array}{c}\begin{bmatrix} 12 & -6a & -12 & -6a \\ -6a & 4a^2 & 6a & 2a^2 \\ -12 & 6a & 12 & 6a \\ -6a & 2a^2 & 6a & 4a^2 \end{bmatrix}\end{array}\begin{array}{c} v_1 \\ \theta_1 \\ v_3 \\ \theta_3 \end{array}$$

For right portion, $[\underset{\sim}{k}]$ is

$$\frac{EI}{(2a)^3}\begin{bmatrix} 12 & 6(2a) & -12 & 6(2a) \\ 6(2a) & 4(2a)^2 & -6(2a) & 2(2a)^2 \\ -12 & -6(2a) & 12 & -6(2a) \\ 6(2a) & 2(2a)^2 & -6(2a) & 4(2a)^2 \end{bmatrix}\begin{array}{c} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{array}$$

Set $v_2 = v_3 = 0$ (supported); combine $[\underset{\sim}{k}]$'s

$$\frac{EI}{8a^3}\begin{array}{c}\begin{bmatrix} 96 & -48a & 0 & -48a \\ -48a & 32a^2 & 0 & 16a^2 \\ 0 & 0 & 0 & 0 \\ -48a & 16a^2 & 0 & 32a^2 \end{bmatrix} \\ \begin{array}{cccc} v_1 & \theta_1 & \theta_2 & \theta_3 \end{array}\end{array} + \frac{EI}{8a^3}\begin{array}{c}\begin{bmatrix} 12 & 12a & 12a & 0 \\ 12a & 16a^2 & 8a^2 & 0 \\ 12a & 8a^2 & 16a^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{array}{cccc} v_1 & \theta_1 & \theta_2 & \theta_3 \end{array}\end{array}$$

...

$$[\underset{\sim}{K}] = \frac{EI}{8a^3}\begin{bmatrix} 108 & -36a & 12a^2 & -48a \\ -36a & 48a^2 & 8a^2 & 16a^2 \\ 12a & 8a^2 & 16a^2 & 0 \\ -48a & 16a^2 & 0 & 32a^2 \end{bmatrix}\begin{array}{c} v_1 \\ \theta_1 \\ \theta_2 \\ \theta_3 \end{array}$$

$$\begin{Bmatrix} v_1 \\ \theta_1 \\ \theta_2 \\ \theta_3 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{Bmatrix} v_1 \\ \theta_1 \\ \theta_2 \end{Bmatrix} = [\underset{\sim}{T}]\begin{Bmatrix} v_1 \\ \theta_1 \\ \theta_2 \end{Bmatrix}$$

$[\underset{\sim}{T}]^T[\underset{\sim}{K}][\underset{\sim}{T}]$ yields

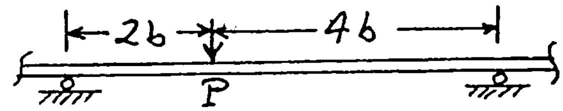
$$\frac{EI}{8a^3}\begin{bmatrix} 108 & -36a & -36a \\ -36a & 48a^2 & 24a^2 \\ -36a & 24a^2 & 48a^2 \end{bmatrix}\begin{Bmatrix} v_1 \\ \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} -P \\ 0 \\ 0 \end{Bmatrix}$$

Last 2 eqs. yield $\theta_1 = \theta_2$. Hence the first 2 eqs. yield

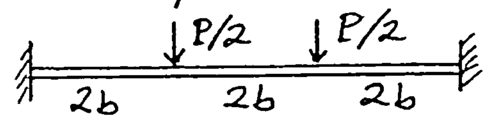
$$\begin{aligned} 108\,v_1 - 72a\theta_1 &= \frac{8Pa^3}{EI} \\ -36\,v_1 + 72a\,\theta_1 &= 0 \\ \hline 72\,v_1 &= -\frac{8Pa^3}{EI} \end{aligned}$$

$$v_1 = -\frac{Pa^3}{9EI}\ \downarrow, \quad \theta_1 = \theta_2 = \frac{v_1}{2a} = -\frac{Pa^2}{18EI}$$

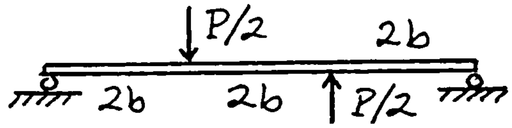
Check by elementary methods: let $a = 2b$



is equal to sum of —

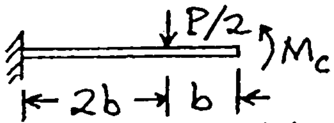


(a)



(b)

(a) Is same as



Now apply beam formulas:

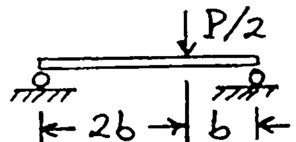
$$\theta_c = 0 = \frac{(P/2)(2b)^2}{2EI} - \frac{M_c(3b)}{EI}$$

$\therefore M_c = Pb/3$ At load $P/2$,

$$v_a = \frac{(P/2)(2b^3)}{3EI} - \frac{M_c(2b)^2}{2EI}$$

$$v_a = \frac{4Pb^3}{3EI} - \frac{2Pb^3}{3EI} = \frac{2Pb^3}{3EI}$$

(b) Is same as



At load $P/2$,

$$v_b = \frac{(P/2)(2b)^2 b^2}{3EI(3b)} = \frac{2Pb^3}{9EI}$$

Net result:

$$v_a + v_b = \frac{Pb^3}{EI}\left(\frac{2}{3} + \frac{2}{9}\right) = \frac{8Pb^3}{9EI} = \frac{Pa^3}{9EI}$$

(downward)