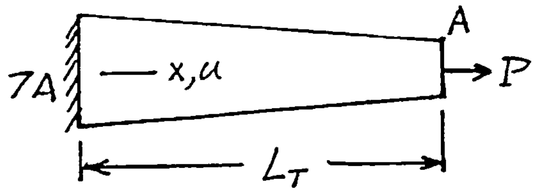
$\boxed{1.3\text{-}1}$



(a)

For $0 \le x \le L_T$,

$$A_x = A\left(7 - \frac{6x}{L_T}\right)$$

and $u = u(x) = \displaystyle\int_0^x \frac{P\,dx}{A_x E} = \frac{P}{EA}\int_0^x \frac{dx}{7 - \frac{6x}{L_T}}$

$$u = \frac{P}{EA}\left[-\frac{\ln(7 - 6x/L_T)}{6/L_T}\right]_0^x$$

$$u = \frac{PL_T}{6EA}\left[\ln 7 - \ln\left(7 - \frac{6x}{L_T}\right)\right]$$

	$x = L_T/3$	$x = 2L_T/3$	$x = L_T$
u	0.0561	0.1412	0.3243

$\underbrace{\qquad\qquad\qquad}_{*\ PL_T/EA}$

By FEA:

At $x = L_T/3$, $u = \dfrac{P(L_T/3)}{6EA} = 0.0556\dfrac{PL_T}{EA}$

At $x = 2L_T/3$, $u = 0.0556\dfrac{PL_T}{EA} + \dfrac{P(L_T/3)}{4EA}$

$$= 0.1389\frac{PL_T}{EA}$$

At $x = L_T$, $u = 0.1389\dfrac{PL_T}{EA} + \dfrac{P(L_T/3)}{2EA}$

$$= 0.3056\frac{PL_T}{EA}$$

(b) FEA stresses at element midpoints:

$$\sigma_{1-2} = \frac{E}{L_T/3}\,0.0556\frac{PL_T}{EA} = 0.167\frac{P}{A}$$

$$\sigma_{2-3} = \frac{E}{L_T/3}(0.1389 - 0.0556)\frac{PL_T}{EA} = 0.250\frac{P}{A}$$

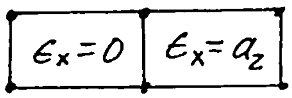
$$\sigma_{3-4} = \frac{E}{L_T/3}(0.3056 - 0.1389)\frac{PL_T}{EA} = 0.500\frac{P}{A}$$

These stresses are exact.

$\boxed{1.3\text{-}2}$ $u = a_1 + a_2 x + a_3 y + a_4 xy$

$$\epsilon_x = \frac{\partial u}{\partial x} = a_2 + a_4 y$$

Continuity of ϵ_x: might have $a_2 = a_4 = 0$ in one element and $a_2 \neq 0$, $a_4 = 0$ in its neighbor. Thus



and ϵ_x is discontinuous across the shared boundary.

$\boxed{1.3\text{-}3}$ In all parts, substitute coordinates of nodes into Eq. 1.3-5.

(a) $\phi_1 = a_1 \qquad \therefore\; a_1 = \phi_1$

$\phi_2 = a_1 + a a_2 \qquad \therefore\; a_2 = (\phi_2 - \phi_1)/a$

$\phi_3 = a_1 + b a_3 \qquad \therefore\; a_3 = (\phi_3 - \phi_1)/b$

$$\phi = \phi_1 + \frac{\phi_2 - \phi_1}{a} x + \frac{\phi_3 - \phi_1}{b} y$$

(b) $\left.\begin{array}{l} \phi_1 = a_1 + a_2 a \quad (A) \\ \phi_2 = a_1 + a_2 a + a_3 b \end{array}\right\} a_3 = \dfrac{\phi_2 - \phi_1}{b} \quad (B)$

$\phi_3 = a_1 + a_3 b \quad (C)$

(B) & (C) yield $a_1 = \phi_1 - \phi_2 + \phi_3 \quad (D)$

(A) & (D) yield $a_2 = \dfrac{\phi_2 - \phi_3}{a}$

$$\phi = (\phi_1 - \phi_2 + \phi_3) + \frac{\phi_2 - \phi_3}{a} x + \frac{\phi_2 - \phi_1}{b} y$$

$$\phi = \left(1 - \frac{y}{b}\right)\phi_1 + \left(-1 + \frac{x}{a} + \frac{y}{b}\right)\phi_2 + \left(1 - \frac{x}{a}\right)\phi_3$$

(c) $\left.\begin{array}{l} \phi_1 = a_1 - a_2 a \\ \phi_2 = a_1 + a_2 a \end{array}\right\} \begin{array}{l} a_1 = (\phi_1 + \phi_2)/2 \\ a_2 = (\phi_2 - \phi_1)/2a \end{array}$

$\phi_3 = a_1 + a_3 b$

$$a_3 = \frac{\phi_3 - a_1}{b} = \frac{2\phi_3 - \phi_1 - \phi_2}{2b}$$

$$\phi = \frac{\phi_1 + \phi_2}{2} + \frac{\phi_2 - \phi_1}{2a} x + \frac{2\phi_3 - \phi_1 - \phi_2}{2b} y$$

[illegible] $\frac{x}{2a} - \frac{y}{2b}\Big)\phi_1 + \left(\frac{1}{2} + \frac{x}{2a} - \frac{y}{2b}\right)\phi_2 + \frac{y}{b}\phi_3$

(d) $\phi_1 = a_1 \qquad \therefore\; a_1 = \phi_1$

$\left.\begin{array}{l} \phi_2 = a_1 + a_2 a + a_3 b \\ \text{[illegible]}\; a + a_3 b \end{array}\right\} \therefore\; a_2 = \dfrac{\phi_2 - \phi_3}{2a}$

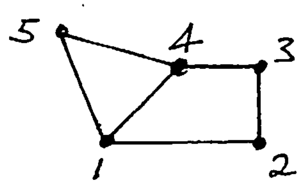
Add: $\phi_2 + \phi_3 = 2\phi_1 + 2a_3 b$; $a_3 = \dfrac{\phi_2 + \phi_3 - 2\phi_1}{2b}$

$$\phi = \phi_1 + \frac{\phi_2 - \phi_3}{2a} x + \frac{\phi_2 + \phi_3 - 2\phi_1}{2b} y$$

$$\phi = \left(1 - \frac{y}{b}\right)\phi_1 + \left(\frac{x}{2a} + \frac{y}{2b}\right)\phi_2 + \left(-\frac{x}{2a} + \frac{y}{2b}\right)\phi_3$$

$\boxed{1.3-4}$

Along sides 1-2 and 3-4, y is constant, so ϕ varies linearly with x.
Along side 2-3, x is constant, so ϕ varies linearly with y.
Along side 4-1, $x=y$, so ϕ varies quadratically with x (or with y).
Imagine that adjacent element 1-4-5 is a three-node triangle, for which $\phi = a_1 + a_2 x + a_3 y$, hence ϕ is linear in x (or y) along each edge. Hence, a gap or overlap is expected along 1-4.



$\boxed{1.3-4}$

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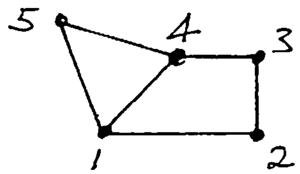
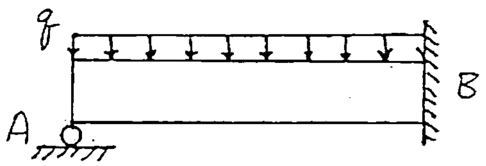


1.4-1



- Point support at A not really possible.
- Support at A may actually apply a horizontal component of force.
- Load q may not be quite uniform.
- Rigid base at A and rigid wall at B not really possible.
- Plane conditions implied; it is actually 3D to some extent.

1.4-2

- In actual pipe, T_1 and T_2 may vary along the length.
- In a horizontal pipe, temperatures may vary somewhat from top to bottom (e.g. from convection on outside, partial fill of liquid on inside).
- Pipe supports change geometry locally and may add or subtract heat.
- Transients may be important (as from sudden fill with hot water).

$\boxed{\begin{array}{l}1.4\text{-}3\\1.4\text{-}4\end{array}}$

- Omitted from sketch: inspection tunnels (often running normal to the paper), penstocks (running parallell to paper), possible cracks in the concrete, variation normal to paper of the geometry of the rock-concrete interface.
- Considerations: Are loads static or dynamic? If dynamic (e.g. earthquake), what characteristics of the reservoir are important? —level of fill, extent & shape, sloshing of contents?
- Needed: data on geometry (lengthwise variation of geometry, including abutments), material properties. Properties of rock include layering, orentation of layers and any fracturing.
- For seepage analysis, need the foregoing data on geometry and material properties. Pertinent properties are permeabilities. Rock [illegible]ties (excluding cracks) are likely to be orthotropic. Regarding the problem as plane may be an over-simplification.

1.4-5

- In the edge view, loads shown are not collinear. Is this correct? — what realignment or additional moments will appear, perhaps after loading begins?
- Does the rivet fill the hole? Will gaps open or close as load increases? Should friction between plates be credited with carrying appreciable load?
- Is yielding involved?
- Cooling of a hot rivet sets up residual stresses, whose magnitudes influence the load at which yielding begins. What are the residual stresses?