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1. Adding x to both sides yields $3 \le 3x + 9$ or $3x \ge -6$, and $x \ge -2$. We conclude that the solution set is $[-2, \infty)$.

2. $-2 \le 3x + 1 \le 7 \Rightarrow -3 \le 3x \le 6 \Rightarrow -1 \le x \le 2$, and the solution set is [-1,2].

3. The inequalities imply $x > 5$ or $x < -4$. So the solution set is $(-\infty, -4) \cup (5, \infty)$.

4. $2x^2 > 50 \Rightarrow x^2 > 25 \Rightarrow x > 5$ and $x < -5$ and the solution set is $(-\infty, -5) \cup (5, \infty)$.

5. $|-5+7| + |-2| = |2| + |-2| = 2 + 2 = 4.$

6. $\left|\frac{5-12}{-4-3}\right| = \frac{|5-12|}{|-7|} = \frac{|-7|}{7} = \frac{7}{7} = 1.$

7. $|2\pi - 6| - \pi = 2\pi - 6 - \pi = \pi - 6.$

8. $|\sqrt{3} - 4| + |4 - 2\sqrt{3}| = (4 - \sqrt{3}) + (4 - 2\sqrt{3}) = 8 - 3\sqrt{3}.$

9. $\left(\frac{9}{4}\right)^{3/2} = \frac{9^{3/2}}{4^{3/2}} = \frac{27}{8}.$

10. $\frac{5^6}{5^4} = 5^{6-4} = 5^2 = 25.$

11. $(3 \cdot 4)^{-2} = 12^{-2} = \frac{1}{12^2} = \frac{1}{144}.$

12. $(-8)^{5/3} = [(-8^{1/3})]^5 = (-2)^5 = -32.$

13. $\frac{(3 \cdot 2^{-3})(4 \cdot 3^5)}{2 \cdot 9^3} = \frac{3 \cdot 2^{-3} \cdot 2^2 \cdot 3^5}{2 \cdot (3^2)^3} = \frac{2^{-1} \cdot 3^6}{2 \cdot 3^6} = \frac{1}{4}.$

14. $\frac{3\sqrt[3]{54}}{\sqrt[3]{18}} = \frac{3 \cdot (2 \cdot 3^3)^{1/3}}{(2 \cdot 3^2)^{1/3}} = \frac{3^2 \cdot 2^{1/3}}{2^{1/3} \cdot 3^{2/3}} = 3^{4/3} = 3\sqrt[3]{3}.$

15. $\frac{4(x^2+y)^3}{x^2+y} = 4(x^2+y)^2.$

16. $\frac{a^6 b^{-5}}{(a^3 b^{-2})^{-3}} = \frac{a^6 b^{-5}}{a^{-9} b^6} = \frac{a^{15}}{b^{11}}.$

17. $\dfrac{\sqrt[4]{16x^5yz}}{\sqrt[4]{81xyz^5}} = \dfrac{(2^4x^5yz)^{1/4}}{(3^4xyz^5)^{1/4}} = \dfrac{2x^{5/4}y^{1/4}z^{1/4}}{3x^{1/4}y^{1/4}z^{5/4}} = \dfrac{2x}{3z}.$

18. $(2x^3)(-3x^{-2})(\frac{1}{6}x^{-1/2}) = -x^{1/2}.$

19. $\left(\dfrac{3xy^2}{4x^3y}\right)^{-2}\left(\dfrac{3xy^3}{2x^2}\right)^3 = \left(\dfrac{3y}{4x^2}\right)^{-2}\left(\dfrac{3y^3}{2x}\right)^3 = \left(\dfrac{4x^2}{3y}\right)^2\left(\dfrac{3y^3}{2x}\right)^3 = \dfrac{(16x^4)(27y^9)}{(9y^2)(8x^3)} = 6xy^7.$

20. $\sqrt[3]{81x^5y^{10}}\,\sqrt[3]{9xy^2} = \sqrt[3]{(3^4x^5y^{10})(3^2xy^2)}$
$= (3^6x^6y^{12})^{1/3} = 3^2x^2y^4 = 9x^2y^4.$

21. $-2\pi^2r^3 + 100\pi r^2 = -2\pi r^2(\pi r - 50).$

22. $2v^3w + 2vw^3 + 2u^2vw = 2vw(v^2 + w^2 + u^2).$

23. $16 - x^2 = 4^2 - x^2 = (4 - x)(4 + x).$

24. $12t^3 - 6t^2 - 18t = 6t(2t^2 - t - 3) = 6t(2t - 3)(t + 1).$

25. $8x^2 + 2x - 3 = (4x + 3)(2x - 1) = 0$ and $x = -3/4$ and $x = 1/2$ are the roots of the equation.

26. $-6x^2 - 10x + 4 = 0$, $3x^2 + 5x - 2 = (3x - 1)(x + 2) = 0$ and so $x = -2$ or $1/3$.

27. $-x^3 - 2x^2 + 3x = -x(x^2 + 2x - 3) = -x(x + 3)(x - 1) = 0$ and the roots of the equation are $x = 0$, $x = -3$, and $x = 1$.

28. $2x^4 + x^2 = 1$. Let $y = x^2$ and we can write the equation as
$$2y^2 + y - 1 = (2y - 1)(y + 1) = 0$$
giving $y = 1/2$ or $y = -1$. We reject the second root since $y = x^2$ must be nonnegative. Therefore, $x^2 = 1/2$ or $x = \pm 1/\sqrt{2} = \pm\sqrt{2}/2$.

29. Here we use the quadratic formula to solve the equation $x^2 - 2x - 5$. Then $a = 1$, $b = -2$, and $c = -5$. Thus,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-5)}}{2(1)}=\frac{2\pm\sqrt{24}}{2}=1\pm\sqrt{6}.$$

30. Here we use the quadratic formula to solve the equation $2x^2+8x+7=0$. Then $a=2$, $b=8$, and $c=7$. So

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(8)\pm\sqrt{(8)^2-4(2)(7)}}{4}=\frac{-8\pm2\sqrt{2}}{4}=-2\pm\tfrac{1}{2}\sqrt{2}.$$

31. $$\frac{(t+6)(60)-(60t+180)}{(t+6)^2}=\frac{60t+360-60t-180}{(t+6)^2}=\frac{180}{(t+6)^2}.$$

32. $$\frac{6x}{2(3x^2+2)}+\frac{1}{4(x+2)}=\frac{(6x)(2)(x+2)+(3x^2+2)}{4(3x^2+2)(x+2)}=\frac{12x^2+24x+3x^2+2}{4(3x^2+2)(x+2)}$$
$$=\frac{15x^2+24x+2}{4(3x^2+2)(x+2)}.$$

33. $$\frac{2}{3}\left(\frac{4x}{2x^2-1}\right)+3\left(\frac{3}{3x-1}\right)=\frac{8x}{3(2x^2-1)}+\frac{9}{3x-1}=\frac{8x(3x-1)+27(2x^2-1)}{3(2x^2-1)(3x-1)}$$
$$=\frac{78x^2-8x-27}{3(2x^2-1)(3x-1)}.$$

34. $$\frac{-2x}{\sqrt{x+1}}+4\sqrt{x+1}=\frac{-2x+4(x+1)}{\sqrt{x+1}}=\frac{2(x+2)}{\sqrt{x+1}}.$$

35. $$\frac{\sqrt{x}-1}{x-1}=\frac{\sqrt{x}-1}{x-1}\cdot\frac{\sqrt{x}+1}{\sqrt{x}+1}=\frac{(\sqrt{x})^2-1}{(x-1)(\sqrt{x}+1)}=\frac{x-1}{(x-1)(\sqrt{x}+1)}=\frac{1}{\sqrt{x}+1}.$$

36. $$\frac{\sqrt{x}-1}{2\sqrt{x}}=\frac{\sqrt{x}-1}{2\sqrt{x}}\cdot\frac{\sqrt{x}}{\sqrt{x}}=\frac{x-\sqrt{x}}{2x}.$$

37. The distance is

$$d=\sqrt{[1-(-2)]^2+[-7-(-3)]^2}=\sqrt{3^2+(-4)^2}=\sqrt{9+16}=\sqrt{25}=5.$$

38. The distance is

$$d=\sqrt{(-1/2-1/2)^2+(2\sqrt{3}-\sqrt{3})^2}=\sqrt{1+3}=\sqrt{4}=2$$

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39. An equation is $x = -2$.

40. An equation is $y = 4$.

41. The slope of L is $m=\dfrac{\frac{7}{2}-4}{3-(-2)}=-\dfrac{1}{10}$ and an equation of L is

$$y-4=-\tfrac{1}{10}\left[x-(-2)\right]=-\tfrac{1}{10}x-\tfrac{1}{5},$$

or $$y=-\tfrac{1}{10}x+\tfrac{19}{5}$$

The general form of this equation is $x + 10y - 38 = 0$.

42. The line passes through the points $(-2, 4)$ and $(3, 0)$. So its slope is $m = (4 - 0)/(-2 - 3)$ or $m = -4/5$. An equation is

$$y-0=-\tfrac{4}{5}(x-3) \quad \text{or} \quad y=-\tfrac{4}{5}x+\tfrac{12}{5}.$$

43. Writing the given equation in the form $y=\frac{5}{2}x-3$, we see that the slope of the given line is 5/2. So a required equation is

$$y-4=\tfrac{5}{2}(x+2) \quad \text{or} \quad y=\tfrac{5}{2}x+9$$

The general form of this equation is $5x - 2y + 18 = 0$.

44. Writing the given equation in the form $y=-\frac{4}{3}x+2$, we see that the slope of the given line is $-4/3$. Therefore, the slope of the required line is 3/4 and an equation of the line is $y-4=\frac{3}{4}(x+2)$ or $y=\frac{3}{4}x+\frac{11}{2}$.

45. Rewriting the given equation in the slope-intercept form, we have $4y = -3x + 8$

or $$y=-\tfrac{3}{4}x+2$$

and conclude that the slope of the required line is $-3/4$. Using the point-slope form of the equation of a line with the point (2,3) and slope $-3/4$, we obtain

$$\begin{aligned} y-3&=-\tfrac{3}{4}(x-2)\\ y&=-\tfrac{3}{4}x+\tfrac{6}{4}+3\\ &=-\tfrac{3}{4}x+\tfrac{9}{2}. \end{aligned}$$

The general form of this equation is $3x + 4y - 18 = 0$.

46. The slope of the line joining the points (– 3,4) and (2,1) is $m = \dfrac{1-4}{2-(-3)} = -\dfrac{3}{5}$.

Using the point-slope form of the equation of a line with the point (– 1,3) and slope – 3/5, we have

$$y-3 = -\tfrac{3}{5}[x-(-1)], \quad y = -\tfrac{3}{5}(x+1)+3 = -\tfrac{3}{5}x + \tfrac{12}{5}.$$

47. The slope of the line passing through (-2,-4) and (1,5) is

$$m = \frac{5-(-4)}{1-(-2)} = \frac{9}{3} = 3.$$

So the required line is

$$y-(-2) = 3[x-(-3)]$$

$$y+2 = 3x+9$$

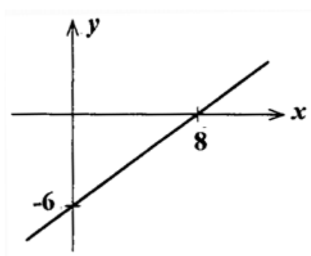
or $$y = 3x+7$$

48. Rewriting the given equation in the slope-intercept form $y = \tfrac{2}{3}x - 8$, we see that the slope of the line with this equation is 2/3. The slope of the required line is – 3/2. Using the point-slope form of the equation of a line with the point (– 2,– 4) and slope – 3/2, we have

$$y-(-4) = -\tfrac{3}{2}[x-(-2)] \quad \text{or} \quad y = -\tfrac{3}{2}x - 7 .$$

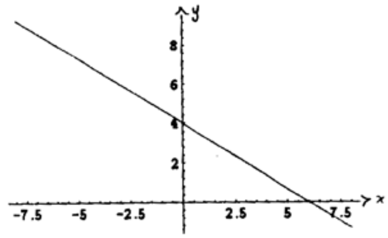
The general form of this equation is $3x + 2y + 14 = 0$.

49. Setting $x = 0$ gives $y = -6$ as the y-intercept. Setting $y = 0$ gives $x = 8$ as the x-intercept. The graph of the equation $3x - 4y = 24$ is follows.



50. Using the point-slope form of an equation of a line, we have

$y-2 = -\dfrac{2}{3}(x-3)$ or $y = -\dfrac{2}{3}x+4$. If $y = 0$, then $x = 6$, and if $x = 0$, then $y = 4$. A sketch of the line follows.



51. $2(1.5C + 80) \leq 2(2.5C - 20) \Rightarrow 1.5C + 80 \leq 2.5C - 20$, so $C \geq 100$ and the minimum cost is \$100.

52.
$$\begin{aligned} 3(2R - 320) &\leq 3R + 240 \\ 6R - 960 &\leq 3R + 240 \\ 3R &\leq 1200 \\ R &\leq 400. \end{aligned}$$

We conclude that the maximum revenue is \$400.