APPENDIX 5. PROPERTIES OF SEA WATER

A5.1. The Equation of State

It is necessary to know the equation of state for the ocean very accurately to determine stability properties, particularly in the deep ocean. The equation of state defined by the Joint Panel on Oceanographic Tables and Standards (UNESCO, 1981) fits available measurements with a standard error of 3.5 ppm for pressure up to 1000 bar, for temperatures between freezing and 40°C, and for salinities between 0 and 42 (Millero *et al.*, 1980; Millero and Poisson, 1981). The density $\rho\,(\mathrm{kg\,m^{-3}})$ is expressed in terms of pressure p (bar), temperature t (°C), and practical salinity S. The last quantity is defined in such a way (Dauphinee, 1980) that its value (in practical salinity units or psu) is very close to the old value expressed in parts per thousand (‰ or ppt). Its relation to previously defined measures of salinity is given by Lewis and Perkin (1981).

The equation for ρ is obtained in a sequence of steps. First, the density ρ_w of pure water ($S = 0$) is given by

$$\begin{aligned}\rho_\mathrm{w} = {} & 999.842594 + 6.793952\times10^{-2}t - 9.095290\times10^{-3}t^2 + 1.001685\times10^{-4}t^3 \\ & - 1.120083\times10^{-6}t^4 + 6.536332\times10^{-9}t^5 \qquad [\mathrm{A5.1}]\end{aligned}$$

Second, the density at one standard atmosphere (effectively $p = 0$) is given by

$$\begin{aligned}\rho(S,t,0) = {} & \rho_\mathrm{w} + S(0.824493 - 4.0899\times10^{-3}t + 7.6438\times10^{-5}t^2 \\ & - 8.2467\times10^{-7}t^3 + 5.3875\times10^{-9}t^4) \\ & + S^{3/2}(-5.72466\times10^{-3} + 1.0227\times10^{-4}t \\ & - 1.6546\times10^{-6}t^2) + 4.8314\times10^{-4}S^2 \qquad [\mathrm{A5.2}]\end{aligned}$$

Finally, the density at pressure p is given by

$$\rho(S,t,p) = \frac{\rho(S,t,0)}{1 - p/K(S,t,p)} \qquad [\mathrm{A5.3}]$$

where K is the secant bulk modulus. The pure water value K_w is given by

$$K_\mathrm{w} = 19652.21 + 148.4206t - 2.327105t^2 + 1.360477\times10^{-2}t^3 - 5.155288\times10^{-5}t^4 \qquad [\mathrm{A5.4}]$$

The value at one standard atmosphere ($p = 0$) is given by

$$\begin{aligned}K(S,t,0) = {} & K_\mathrm{w} + S(54.6746 - 0.603459t + 1.09987\times10^{-2}t^2 - 6.1670\times10^{-5}t^3) \\ & + S^{3/2}(7.944\times10^{-2} + 1.6483\times10^{-2}t - 5.3009\times10^{-4}t^2) \qquad [\mathrm{A5.5}]\end{aligned}$$

and the value at pressure p by

$$\begin{aligned}K(S,t,p) = {} & K(S,t,0) + p(3.239908 + 1.43713\times10^{-3}t + 1.16092\times10^{-4}t^2 \\ & - 5.77905\times10^{-7}t^3) + pS(2.2838\times10^{-3} - 1.0981\times10^{-5}t - 1.6078\times10^{-6}t^2) \\ & + 1.91075\times10^{-4}pS^{3/2} + p^2(8.50935\times10^{-5} - 6.12293\times10^{-6}t + 5.2787\times10^{-8}t^2) \\ & + p^2S(-9.9348\times10^{-7} + 2.0816\times10^{-8}t + 9.1697\times10^{-10}t^2) \qquad [\mathrm{A5.6}]\end{aligned}$$

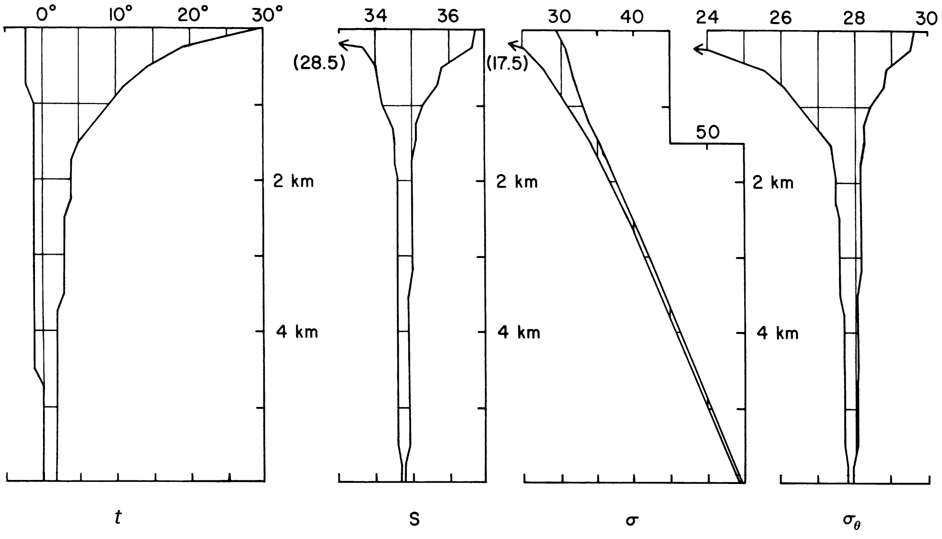

Figure A5.1 The ranges of temperature t (in °C) and salinity S for 98% of the ocean as a function of depth and the corresponding ranges of density σ and potential density σ_θ (From Bryan and Cox (1972).)

Values for checking the formula are $\rho(0,5,0) = 999.96675$, $\rho(35,5,0) = 1027.67547$, and $\rho(35,25,1000) = 1062.53817$.

Since ρ is always close to $1000\,\mathrm{kg\,m^{-3}}$, values quoted are usually those of the difference $(\rho - 1000)$ in $\mathrm{kg\,m^{-3}}$ as is done in **Table A5.1**. The table is constructed so that values can be calculated for 98% of the ocean (see **Figure A5.1**). The maximum errors in density on straight linear interpolation are $0.013\,\mathrm{kg\,m^{-3}}$ for both temperature and pressure interpolation and only 0.006 for salinity interpolation in the range of salinities between 30 and 40. The error when combining all types of interpolation for the 98% range of values is less than $0.03\,\mathrm{kg\,m^{-3}}$.

A5.2 Other Quantities Related to Density

Older versions of the equation of state usually gave formulas not for calculating the absolute density ρ, but for the *specific gravity* ρ/ρ_m, where ρ_m is the maximum density of pure water. Since this is always close to unity, a quantity called σ was defined by

$$1000\left(\frac{\rho}{\rho_\mathrm{m}} - 1\right) = \frac{1000}{\rho_\mathrm{m}}(\rho - \rho_\mathrm{m}) \qquad [\mathrm{A5.7}]$$

Since the value of ρ_m is

$$\rho_\mathrm{m} = 999.975 \quad \mathrm{kg\,m^{-3}} \qquad [\mathrm{A5.8}]$$

it follows that σ, as defined above, is related to the $(\rho - 1000)$ values by

$$\sigma = (\rho - 1000) + 0.025 \qquad [\mathrm{A5.9}]$$

i.e., 0.025 must be added to the values of $(\rho - 1000)$ on the table to obtain the old σ value. The notation σ_τ (sigma-tau) was used for the value of σ calculated at zero pressure, and σ_θ (sigma theta) for the quantity corresponding to potential density. Another quantity commonly used in oceanography is the specific volume (or steric) *anomaly* δ defined by

$$\delta = v_\mathrm{s}(S,t,p) - v_\mathrm{s}(35,0,p) \qquad [\mathrm{A5.10}]$$

and usually reported in units of $10^{-8}\,\mathrm{m^3\,kg^{-1}}$.

A5.3. Expansion Coefficients

The thermal expansion coefficient α is given in **Table A5.1** in units of $10^{-7}\,\mathrm{K}^{-1}$ along with its S derivative. The maximum error from pressure interpolation is two units, that from temperature interpolation is three units, and that for salinity interpolation ($30 < S < 40$) is two units plus a possible round-off error of two units. The salinity expansion coefficient β can be calculated by using the given values of $\partial\rho/\partial S$.

A5.4. Specific Heat

The specific heat at surface pressure is given by Millero *et al.* (1973) and can be calculated in two stages. First, the value in $\mathrm{J\,kg^{-1}\,K^{-1}}$ for fresh water is given by

$$c_p(0,t,0) = 4217.4 - 3.720283t + 0.1412855t^2 - 2.654387\times10^{-3}t^3 + 2.093236\times10^{-5}t^4 \qquad [\mathrm{A5.11}]$$

Second,

$$\begin{aligned} c_p(S,t,0) &= c_p(0,t,0) + S(-7.6444 + 0.107276t - 1.3839\times10^{-3}t^2) \\ &\quad + S^{3/2}(0.17709 - 4.0772\times10^{-3}t + 5.3539\times10^{-5}t^2) \end{aligned} \qquad [\mathrm{A5.12}]$$

The formula can be checked against the result $c_\mathrm{p}(40,40,0) = 3981.050$. The standard deviation of the algorithm fit is 0.074. Values at nonzero pressures can be calculated by using eqn [A5.13] and the equation of state.

$$\left(\frac{\partial c_p}{\partial p}\right)_T = -T\left(\frac{\partial^2 v_\mathrm{s}}{\partial T^2}\right)_p \qquad [\mathrm{A5.13}]$$

The values in **Table A5.1** are based on the above formula and a polynomial fit for higher pressures derived from the equation of state by N. P. Fofonoff. The intrinsic interpolation errors in the table are 0.4, 0.1, and $0.3\,\mathrm{J\,kg^{-1}\,K^{-1}}$ for pressure, temperature, and salinity interpolation, respectively, and there are additional obvious round-off errors.

A5.5. Potential Temperature

The *adiabatic lapse rate* Γ is given by

$$\Gamma = \frac{g\alpha T}{c_p} \qquad [\mathrm{A5.14}]$$

and therefore can be calculated from the above formulas. The definition of potential temperature

$$\frac{\theta}{T} = \left(\frac{p_r}{p}\right)^\kappa \qquad [\mathrm{A5.15}]$$

where p_r is a reference pressure level (usually 1 bar) and $\kappa = (\gamma - 1)/\gamma$ where γ is the ratio of specific heats at constant pressure and at constant volume, can then be used to obtain θ. The following algorithm, however, was derived by Bryden (1973), using experimental compressibility data, to give θ(°C) as a function of salinity S, temperature t(°C), and pressure p (bar) for $30 < S < 40$, $2 < t < 30$, and $0 < p < 1000$:

$$\begin{aligned} \theta(S,t,p) &= t - p(3.6504\times10^{-4} + 8.3198\times10^{-5}t - 5.4065\times10^{-7}t^2 \\ &\quad + 4.0274\times10^{-9}t^3) - p(S-35)(1.7439\times10^{-5} \\ &\quad - 2.9778\times10^{-7}t) - p^2(8.9309\times10^{-7} - 3.1628\times10^{-8}t \\ &\quad + 2.1987\times10^{-10}t^2) + 4.1057\times10^{-9}(S-35)p^2 \\ &\quad - p^3(-1.6056\times10^{-10} + 5.0484\times10^{-12}t) \end{aligned} \qquad [\mathrm{A5.16}]$$

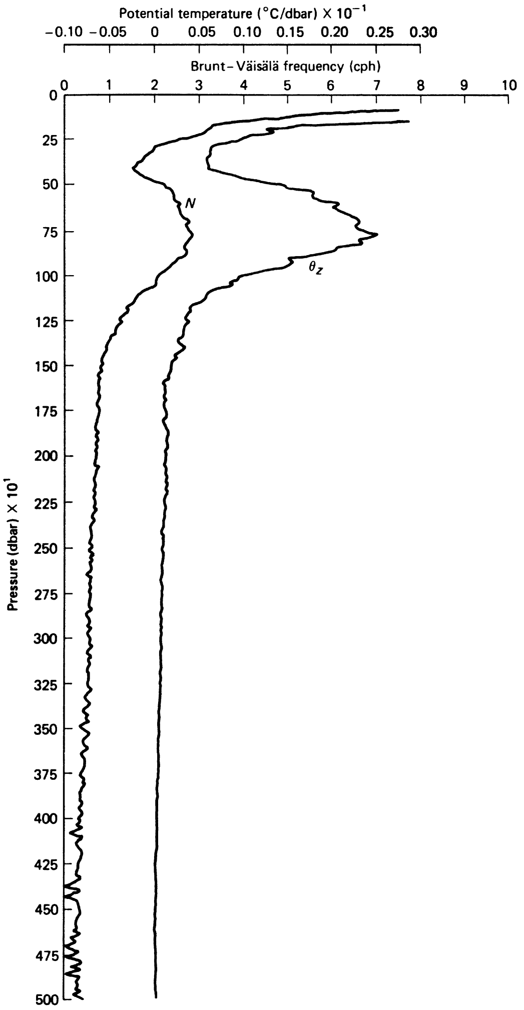

Figure A5.2 A profile of buoyancy frequency N in the ocean. [From the North Atlantic near 28°N, 70°W, courtesy of Dr. R. C. Millard.]

A check value is $\theta(25, 10, 1000) = 8.4678516$, and the standard deviation of Bryden's polynomial fit was 0.001 K. Values in **Table A5.1** are given millidegrees, the intrinsic interpolation errors being 2, 0.3, and 0 millidegrees for pressure, temperature, and salinity interpolation, respectively (**Figure A5.2**).

A5.6. Speed of Sound

The speed of sound c_s can be calculated from the equation of state, using eqn [A5.17]

$$c_s^2 = \left(\frac{\partial p}{\partial \rho}\right)_{\theta,s} \qquad [A5.17]$$

Values given in **Table A5.1** use algorithms derived by Chen and Millero (1977) on the basis of direct measurements. The formula applies for $0 < S < 40$, $0 < t < 40$, $0 < p < 1000$ with a standard deviation of $0.19\,\mathrm{m\,s^{-1}}$. Values in the table are given in meters per second, the intrinsic interpolation errors being 0.05, 0.10, and $0.04\,\mathrm{m\,s^{-1}}$ for pressure, temperature, and salinity interpolation, respectively.

A5.7. Freezing Point of Sea Water

The freezing point t_f of sea water (°C) is given (Millero, 1978) by

$$t_f(S,p) = -0.0575S + 1.710523 \times 10^{-3} S^{3/2} - 2.154996 \times 10^{-4} S^2 - 7.53 \times 10^{-3} p \qquad [A5.18]$$

The formula fits measurements to an accuracy of $\pm 0.004\,\mathrm{K}$.

Table A5.1

p (bar)	S	t ($^\circ C$)	$\rho - 1000$ ($kg\,m^{-3}$)	$\partial\rho/\partial S$	α ($10^{-7} K^{-1}$)	$\partial\alpha/\partial S$	c_p ($J kg^{-1} K^{-1}$)	$\partial c_p/\partial S$	θ ($10^{-3}\,^\circ C$)	$\partial\theta/\partial S$	c_s ($m\,s^{-1}$)	$\partial c_s/\partial S$
0	35	− 2	28.187	0.814	254	33	3989	− 6.2	− 2000	0	1439.7	1.37
0	35	− 0	28.106	0.808	526	31	3987	− 6.1	0	0	1449.1	1.34
0	35	− 2	27.972	0.801	781	28	3985	− 5.9	2000	0	1458.1	1.31
0	35	− 4	27.786	0.796	1021	26	3985	− 5.8	4000	0	1466.6	1.29
0	35	− 7	27.419	0.788	1357	23	3985	− 5.6	7000	0	1478.7	1.25
0	35	10	26.952	0.781	1668	20	3986	− 5.5	10000	0	1489.8	1.22
0	35	13	26.394	0.775	1958	17	3988	− 5.3	13000	0	1500.2	1.19
0	35	16	25.748	0.769	2230	15	3991	− 5.2	16000	0	1509.8	1.16
0	35	19	25.022	0.764	2489	14	3993	− 5.1	19000	0	1518.7	1.13
0	35	22	24.219	0.760	2734	12	3996	− 4.9	22000	0	1526.8	1.10
0	35	25	23.343	0.756	2970	11	3998	− 4.9	25000	0	1534.4	1.08
0	35	28	22.397	0.752	3196	9	4000	− 4.8	28000	0	1541.3	1.06
0	35	31	21.384	0.749	3413	8	4002	− 4.7	31000	0	1547.6	1.03
100	35	− 2	32.958	0.805	552	31	3953	− 5.8	− 2029	− 2	1456.1	1.38
100	35	− 0	32.818	0.799	799	28	3953	− 5.7	− 45	− 2	1465.5	1.35
100	35	− 2	32.629	0.793	1031	26	3954	− 5.6	1939	− 2	1474.5	1.33
100	35	− 4	32.393	0.788	1251	24	3955	− 5.5	3923	− 2	1483.1	1.30
100	35	− 7	31.958	0.781	1559	21	3957	− 5.3	6901	− 1	1495.1	1.26
100	35	10	31.431	0.774	1844	18	3960	− 5.2	9879	− 1	1506.3	1.22
100	35	13	30.818	0.769	2111	16	3963	− 5.1	12858	− 1	1516.7	1.19
100	35	16	30.126	0.763	2363	14	3967	− 5.0	15838	− 1	1526.4	1.16
100	35	19	29.359	0.759	2603	13	3970	− 4.9	18819	− 1	1535.3	1.13
200	35	− 2	37.626	0.797	834	28	3922	− 5.5	− 2076	− 3	1472.8	1.39
200	35	0	37.429	0.791	1058	26	3923	− 5.4	− 107	− 3	1482.3	1.36
200	35	− 2	37.187	0.786	1269	24	3925	− 5.3	1862	− 3	1491.2	1.33
200	35	− 4	36.903	0.781	1469	22	3927	− 5.2	3832	− 3	1499.8	1.30
200	35	− 7	36.402	0.774	1750	19	3931	− 5.1	6789	− 3	1511.8	1.26
300	35	− 2	42.191	0.789	1101	26	3893	− 5.2	− 2140	− 5	1489.9	1.39
300	35	− 0	41.941	0.783	1303	24	3896	− 5.1	− 186	− 5	1499.3	1.36
300	35	− 2	41.649	0.778	1494	22	3899	− 5.0	1771	− 5	1508.2	1.33
300	35	− 4	41.319	0.774	1676	20	3903	− 5.0	3728	− 5	1516.6	1.30
400	35	− 2	46.658	0.781	1351	24	3867	− 4.9	− 2221	− 7	1507.2	1.39
400	35	− 0	46.356	0.776	1534	22	3871	− 4.8	− 279	− 6	1516.5	1.36
400	35	− 2	46.017	0.771	1707	20	3876	− 4.8	1665	− 6	1525.3	1.33
400	35	− 4	45.643	0.767	1872	19	3880	− 4.7	3610	− 6	1533.7	1.30
500	35	− 2	51.029	0.773	1587	22	3844	− 4.7	− 2316	− 8	1524.8	1.38
500	35	− 0	50.678	0.769	1751	20	3849	− 4.6	− 386	− 8	1534.0	1.35
500	35	− 2	50.293	0.764	1907	19	3854	− 4.6	1546	− 7	1542.7	1.32
600	35	− 2	55.305	0.766	1807	20	3824	− 4.4	− 2426	− 9	1542.6	1.37
600	35	− 0	54.908	0.762	1954	18	3829	− 4.4	− 506	− 9	1551.6	1.34
600	35	− 2	54.481	0.758	2094	17	3835	− 4.4	1416	− 9	1560.2	1.31

Further Reading

Bryan K and Cox MD (1972) An approximate equation of state for numerical models of ocean circulation. *Journal of Physical Oceanography* **2**: 510–514.

Bryden HL (1973) New polynomials for thermal expansion, adiabatic temperature gradient and potential temperature gradient of sea water. *Deep Sea Research* **20**: 401–408.

Chen C-T and Millero FJ (1977) Speed of sound in sea-water at high pressures. *Journal of the Acoustical Society of America* **62**: 1129–1135.

Dauphinee TM (1980) Introduction to the special issue on the Practical Salinity Scale 1978. *IEEE, Journal of Oceanic Engineering* **OE-5**: 1–2.

Gill AE (1982) *Atmosphere–Ocean Dynamics*, International Geophysics Series Volume 30. San Diego: Academic Press.

Kraus EB (1972) *Atmosphere–Ocean Interaction*. London: Oxford University Press.

Lewis EL and Perkin RG (1981) The Practical Salinity Scale 1978: conversion of existing data. *Deep Sea Research* **28A**: 307–328.

Millero FJ (1978) Freezing point of seawater. In: *Eigth Report of the Joint Panel on Oceanographic Tables and Standards*, UNESCO Technical Papers in Marine Science No. 28, Annex 6. Paris: UNESCO

Millero FJ and Poisson A (1981) International one-atmosphere equation of state for seawater. *Deep Sea Research* **28A**: 625–629.

Millero FJ, Chen C-T, Bradshaw A and Schleicher K (1980) A new high pressure equation of state for seawater. *Deep Sea Research* **27A**: 255–264.

Millero FJ, Perron G and Desnoyers JE (1973) Heat capacity of seawater solutions from 5 to 25°C and 0.5 to 22% chlorinity. *Journal of Geophysical Research* **78**: 4499–4507.

UNESCO (1981) *Tenth Report of the Joint Panel on Oceanographic Tables and Standards*, UNESCO Technical papers in Marine Science No. 36. Paris: UNESCO.