CHAPTER 40
MACHINE-TOOL VIBRATION

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INTRODUCTION

Machining and measuring operations are invariably accompanied by relative vibration between workpiece and tool. These vibrations are due to one or more of the following causes: (1) inhomogeneities in the workpiece material; (2) variation of chip cross section; (3) disturbances in the workpiece or tool drives; (4) dynamic loads generated by acceleration/deceleration of massive moving components; (5) vibration transmitted from the environment; (6) self-excited vibration generated by the cutting process or by friction (machine-tool chatter).

The tolerable level of relative vibration between tool and workpiece, i.e., the maximum amplitude and to some extent the frequency, is determined by the required surface finish and machining accuracy as well as by detrimental effects of the vibration on tool life (see *The Effect of Vibration on Tool Life*) and by the noise which is frequently generated.

This chapter discusses the sources of vibration excitation in machine tools, machine-tool chatter (i.e., self-excited vibration which is induced and maintained by forces generated by the cutting process), and methods of control of machine-tool vibration.

SOURCES OF VIBRATION EXCITATION

VIBRATION DUE TO INHOMOGENEITIES IN THE WORKPIECE

Hard spots or a crust in the material being machined impart small shocks to the tool and workpiece, as a result of which free vibrations are set up. If these transients are rapidly damped out, their effect is usually not serious; they simply form part of the general "background noise" encountered in making vibration measurements on machine tools. Cases in which transient disturbances do not decay but build up to vibrations of large amplitudes (as a result of dynamic instability) are of great practical importance, and are discussed later.

When machining is done under conditions resulting in discontinuous chip removal, the segmentation of chip elements results in a fluctuation of the cutting thrust. If the frequency of these fluctuations coincides with one of the natural frequencies of the structure, forced vibration of appreciable amplitude may be excited. However, in single-edge cutting operations (e.g., turning), it is not clear whether the segmentation of the chip is a primary effect or whether it is produced by other vibration, without which continuous chip flow would be encountered.

The breaking away of a built-up edge from the tool face also imparts impulses to the cutting tool which result in vibration. However, marks left by the built-up edge on the machined surface are far more pronounced than those caused by the ensuing vibration; it is probably for this reason that the built-up edge has not been studied from the vibration point of view. The built-up edge frequently accompanies certain types of vibration (chatter), and instances have been known when it disappeared as soon as the vibration was eliminated.

VIBRATION DUE TO CROSS-SECTIONAL VARIATION OF REMOVED MATERIAL

Variation in the cross-sectional area of the removed material may be due to the shape of the machined surface (e.g., in turning of a nonround or slotted part) or to the configuration of the tool (e.g., in milling and broaching when cutting tools have multiple cutting edges). In both cases, pulses of appreciable magnitude may be imparted to the tool and to the workpiece, which may lead to undesirable vibration. The pulses have relatively shallow fronts for turning of nonround or eccentric parts, and steep fronts for turning of slotted parts and for milling/broaching. These pulses excite transient vibrations of the frame and of the drive whose intensity depends on the pulse shape and the ratio between the pulse duration and the natural periods of the frame and the drive (Chap. 8). If the vibrations are decaying before the next pulse occurs, they can still have a detrimental effect on tool life and leave marks on the machined surface. In cylindrical grinding and turning, when a workpiece which contains a slot is machined, visible marks frequently are observed near the "leaving edge" of the slot or keyway. These are due to a "bouncing" of the grinding wheel or the cutting tool on the machined surface. They may be eliminated or minimized by closing the recess with a plug or with a filler.

When the transients do not significantly decay between the pulses, dangerous resonance vibrations of the frame and/or the drive can develop with the fundamental and higher harmonics of the pulse sequence. The danger of the resonance increases with higher cutting speeds.

Simultaneous engagement of several cutting edges with the workpiece results in an increasing dc component of the cutting force and effective reduction of the pulse intensity,[1] while runout of a multiedge cutter and inaccurate setup of the cutting edges enrich the spectral content of the cutting force and enhance the danger of resonance. Computational synthesis of the resulting cutting force is reasonably accurate.

DISTURBANCES IN THE WORKPIECE AND TOOL DRIVES

Forced vibrations result from rotating unbalanced masses; gear, belt, and chain drives; bearing irregularities; unbalanced electromagnetic forces in electric motors; pressure oscillations in hydraulic drives; etc.

Vibration Caused by Rotating Unbalanced Members. Forced vibration induced by rotation of some unbalanced member may affect both surface finish and tool life, especially when its rotational speed falls near one of the natural frequencies of the machine-tool structure. This vibration can be eliminated by careful balancing, the procedure being basically similar to that described in Chap. 39-I, or by self-centering due to resilient mounting of bearings.[2,3]

When a new machine is designed, a great deal of trouble can be forestalled by placing rotating components in a position in which the detrimental effect of their unbalance is likely to be relatively small. Motors should not be placed on the top of slender columns, and the plane of their unbalance should preferably be parallel to the plane of cutting. In some cases, vibration resulting from rotating unbalanced members can be eliminated by mounting them using vibration-isolation techniques (Chaps. 30 and 32).

Grinding and boring are most sensitive to vibration because of the high surface finish resulting from the operations. In cylindrical grinding, marks resulting from unbalance of the grinding wheel or of some other component are readily recognizable. They appear in the form of equally spaced, continuous spirals with a constant slope, as shown in Fig. 40.1*A*. From these marks, the machine component responsible for their existence is found by considering that its speed in rpm must be equal to $\pi Dn/a$, where D is the workpiece diameter in inches (millimeters), a is the pitch of the marks in inches (millimeters), and n is the workpiece speed in rpm. An analogous procedure also can be applied to peripheral surface grinding. Marks produced in one pass of the wheel are shown in Fig. 40.1*B*. The speed of the responsible component in rpm is equal to the number of marks (produced in one pass) which fall into a distance equal to that traveled by the workpiece (or wheel) in 1 min.

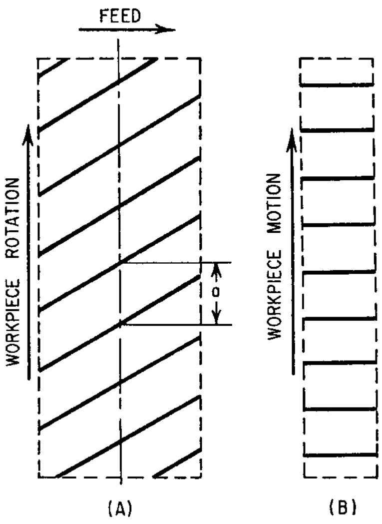

FIGURE 40.1 Grinding marks resulting from unbalance of grinding wheel or some other component. (*A*) Cylindrical grinding; (*B*) peripheral grinding. Marks which are unequally spaced or which have a varying slope are due to inhomogeneities in the wheel.

Since centrifugal force magnitudes are proportional to the square of rpm, high-speed machine tools are more sensitive to unbalance of toolholders and small asymmetrical tools (e.g., boring bars). Lathes may be sensitive to workpiece unbalance due to asymmetrical geometry or the nonuniform allowance (e.g., forged parts).

Marks Caused by Inhomogeneities in the Grinding Wheel. Although grinding marks usually indicate the presence of a vibration, this vibration may not necessarily be the primary cause of the marks. Hard spots on the cutting surface of the wheel result in similar, though generally less pronounced, marks. Grinding wheels usually are not of equal hardness throughout. A hard region on the wheel circumference rapidly becomes glazed in use and establishes itself as a high spot on the wheel (since it retains the grains for a longer period than the softer parts). These high spots eventually break down or shift to other parts of the wheel; in cylindrical grinding, this manifests itself as a sudden change in the slope of the spiral marks. Marks which appear to be due to an unbalanced member rotating at two or three times the speed of the wheel and which are nonuniformly spaced are always due to two or three hard spots.

The Effect of Vibration on the Wheel Properties. If vibration exists between wheel and workpiece, normal forces are produced which react on the wheel and tend to alter the wheel shape and/or the wheel's cutting properties. In soft wheels the dominating influence of vibration appears to be inhomogeneous wheel wear, and in hard wheels inhomogeneous loading (i.e., packing of metal chips on and in crevasses between the grits). These effects result in an increased fluctuation of the normal force, which produces further changes in the wheel properties. The overall effect is that a vibration once initiated tends to grow.[4] When successive cuts or passes overlap, the inhomogeneous wear and loading of the wheel may cause a regenerative chatter effect which makes the cutting process dynamically unstable (see *Dynamic Stability*).

Drives. Spindle and feed drives can be important sources of vibration caused by motors, power transmission elements (gears, traction drives, belts, screws, etc.), bearings, and guideways.

Electric motors can be sources of both rectilinear and torsional vibrations. Rectilinear vibrations are due to a nonuniform air gap between the stator and rotor, asymmetry of windings, unbalance, bearing irregularities, misalignment with the driven shaft, etc. Torsional vibrations (torque ripple) are due to various electrical irregularities.[5] Misalignment- and bearing-induced vibrations of spindle motors are reduced by integrating the spindle with the motor shaft.

Gear-induced vibrations can also be both rectilinear and torsional. They are due to production irregularities (pitch and profile errors, eccentricities, etc), assembly errors (eccentric fit on the shaft, key/spline errors, and backlash), or distortion of mesh caused by deformations of shafts, bearings, and housings under transmitted loads. Tight tolerances of the gears and design measures reducing their sensitivity to misalignment (crowning, flanking) should be accompanied by rigid shafts and housings and accurate fits. All gear faults, eccentricities, pitch errors, profile errors, etc., produce nonuniform rotation, which in some cases adversely affects surface finish, geometry, and possibly tool life. In precision machines, where a high degree of surface finish is required, the workpiece or tool spindle usually is driven by belts or by directly coupled motors.

In some high-precision systems, inertia drives are used, in which the energy is supplied to the flywheel between the cutting operations, but the cutting process is energized by the flywheel disconnected from the motor/transmission system. Such a system practically eliminates transmission of drive vibrations into the work zone.

Belt drives, used in some applications as filters to suppress high-frequency vibrations (especially torsional), can induce their own forced vibrations, both torsional and rectilinear. Any variation of the effective belt radius, i.e., the radius of the neutral axis of the belt around the pulley axis, produces a variation of the belt tension and the belt velocity. This causes a variation of the bearing load and of the rotational velocity of the pulley. The effective pulley radius can vary as a result of (1) eccentricity of the pulley or (2) variation of belt profile or inhomogeneity of belt material. Another source of belt-induced vibrations is variation of the elastic modulus along the belt length, which may excite parametric vibration (Chap. 5). Flat belts generate less vibration than V belts because of their better homogeneity and because the disturbing force is less dependent on the belt tension.

Grinding is particularly sensitive to disturbances caused by belts. Seamless belts or a direct motor drive to the main spindle is recommended for high-precision machines.[4] Vibration is minimized when the belt tension and the normal grinding force point in the same direction, as shown in Fig. 40.2*A*. The clearance between bearing and spindle is thus eliminated. With the arrangement shown in Fig. 40.2*B,* large amplitudes of vibration may arise when the normal grinding force is substan-

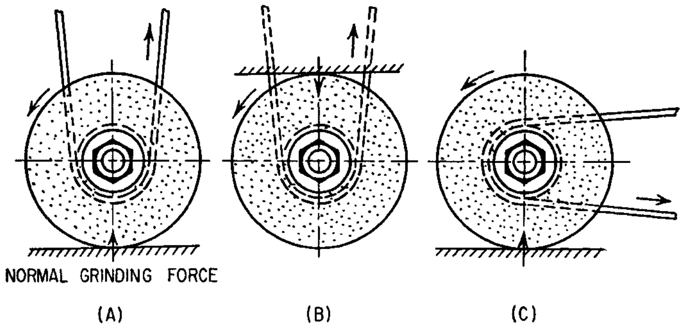

FIGURE 40.2 Direction of driving belt and its influence on performance. (*A*) Vibration is minimized when belt tension and normal grinding force point in the same direction. (*B*) Large amplitudes may arise when the normal grinding force is substantially equal to the belt tension. (*C*) Vibration due to centrifugal force is likely to be caused by an unbalance of the wheel. (*S. Doi.*[4])

tially equal to the belt tension and/or the peripheral surface of the wheel is nonuniform. Tests indicate that with the arrangement shown in Fig. 40.2*C,* vibration due to the centrifugal force is likely to be caused by an unbalance of the wheel.[4] The spindle pulley should preferably be placed between the spindle bearings (Fig. 40.3*A*) and not at the end of the spindle (Fig 40.3*B*), unless the pulley is "unloaded" (supported by its own bearings), as in Fig. 40.3*C.*[3]

Chain drives have inherent nonuniformity of transmission ratio and are a significant source of vibration, even when used for auxiliary drives.

Bearings. Dimensional inaccuracies of the components of ball or roller bearings and/or surface irregularities on the running surfaces (or the bearing housing) may give rise to vibration trouble in machines when high-quality surface finish is demanded. From the frequency of the vibration produced, it is sometimes possible to identify the component of the bearing responsible (Chap. 16). For conventional bearings frequently used in machine tools, the outer race is stationary and the inner race rotates at n_i rpm; the cage velocity is of the order of $n_c \cong 0.4\, n_i$, and the velocity of the balls or rollers is about $n_b \cong 2.4 n_i$. In some cases, a disturbing frequency of the order of $n_z = z n_c$ also can be detected, where z is the number of rolling elements. This is the frequency with which successive rolling elements pass through the "loaded zone" of the bearing, which is determined by the direction of the load. These disturbing frequencies are less pronounced with bearings having two rows of rolling elements, each unit of which lies halfway between units of the neighboring row. Because of the importance of spindle bearings' influence on accuracy of machining and on vibrations in the work zone, especially for precision and high-speed machine tools, both races and rolling bodies of spindle bearings must have high dimensional accuracy.

From the point of view of vibration control, both stiffness and damping of bearings should be maximized. Stiffness can be maximized by using roller bearings (with tapered or cylindrical rollers), by using rollers with two rows of rolling elements, by preloading the bearings in the radial direction, and by improving fits between bearings and shafts/housings.[3] Preloading eliminates clearances (play) in bearings, besides increasing their stiffness. However, increased preload is accompanied by decreased damping,[3] as well as by an increase in heat generation and a likely decrease in bearing life. Optimal preload values are recommended by bearing manufacturers. Roller bearings usually have higher damping than ball bearings. Sliding, and especially

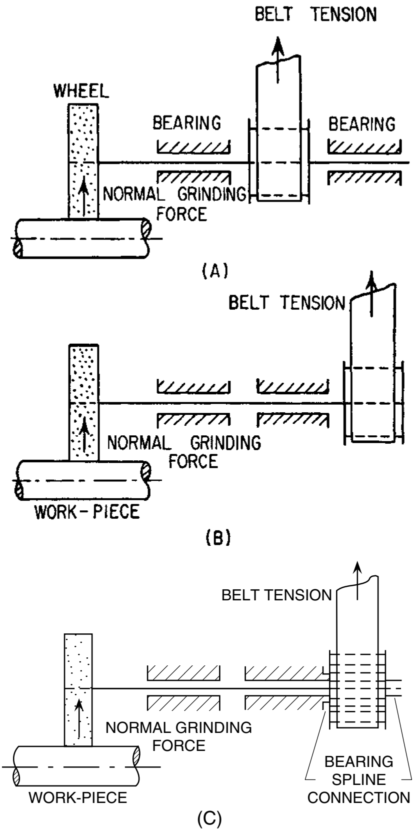

FIGURE 40.3 Effect of relative position of grinding wheel, bearings, and driving pulley on grinding performance. (*A*) Driving pulley should be placed between bearings, as shown in (*A*). Arrangement shown in (*B*) is constructionally simpler but is more liable to cause trouble. (*After S. Doi.*[4]) (*C*) Supporting of pulley by independent bearings eliminates bending and rectilinear vibrations of spindle by belt-induced forces.

hydrostatic, bearings have a greater damping capacity than antifriction bearings and are therefore superior with respect to vibration. Machine tools with hydrostatic bearings have extremely high chatter resistance.

Guideways (Slides). The uniformity of feed motions is often disturbed by a phenomenon known as *stick-slip,* which is described in Chap. 5. When motion of a tool support is initiated, elastic deformations of the feed drive elements increase until the forces transmitted exceed the static frictional resistance of the tool support. Subsequently, the support commences to move, and the friction drops to its dynamic value. As a result of the drop of the friction force, the support receives a high acceleration and overshoots because of its inertia. At the end of the "jump," the transmission is wound up in the opposite sense; before any further motion can take place, this deformation must be unwound. This occurs during a period of standstill of the support. Subsequently, the phenomenon repeats itself. The physical sequence described falls into the category generally known as "relaxation oscillations" (Chap. 5).

The occurrence of stick-slip depends on the interaction of the following factors: (1) the mass of the sliding body, (2) the drive stiffness, (3) the damping present in the drive, (4) the sliding speed, (5) the surface roughness of the sliding surfaces, and (6) the lubricant used. It is encountered only at low sliding speeds; slide drives designed for stick-slip-free motion have small moving masses and a high drive stiffness. Excellent results also may be achieved by using cast iron and a suitable plastic material as mating surfaces. By keeping the oil film between the mating surfaces under a certain pressure (hydrostatic lubrication), the possibility of mixed dry and viscous friction is eliminated, and stick-slip cannot arise. High damping is another advantage of hydrostatic slides.

Rolling friction slides[6] do not exhibit stick-slip but may generate high-frequency vibrations because of the shape and dimensional imperfections of the rolling bodies. These can be reduced by increasing their dimensional accuracy and by introducing damping. Rolling friction slides have very low damping and as a result can amplify vibrations from other sources if their frequencies are close to resonance frequencies of the slide. High-precision systems require extremely low friction as well as the absence of vibration.[7]

IMPACTS FROM MASSIVE PART REVERSALS

Some machine tools have reciprocating massive parts whose reversals produce sharp impacts which excite both low-frequency solid-body vibrations of the machine (the system "machine on its mounts") and high-frequency structural modes. Such effects occur both in machine tools, such as surface grinders, and in high-speed computer numerically controlled (CNC) machining centers and coordinate measuring machines (CMM). In the CMMs the working process is associated with start-stop operations; in machining centers it is associated with changing magnitude and/or directions of feed motions of heavy tables, slides, spindle heads, etc., with accelerations as high as 2*g*. The driving forces causing such changes in magnitudes and directions of momentum of the massive units have impulsive character and cause free decaying vibrations in both solid-body and structural modes (Chap. 8). These vibrations excite relative displacements in the work zone between the workpiece and the cutting or measuring tool. Figure 40.4 shows oscillograms of the acceleration of the table of a surface grinder during its reversal (*A*) and the resulting relative displacements between the grinding wheel and the table (workpiece) for two cases of installation: the machine installed on rigid steel wedges (*B*) and on vibration isolators (*C*). In the latter case the relative displacements during the reversal process are much higher, although they are decaying at a faster rate due to higher damping in the isolators. The peak magnitude of acceleration, 7.9 m/s$^2 \cong 0.8g$, is typical for surface grinders, CMMs, etc. If these displacements exceed allowable limits, the working process cannot start before the vibrations decay. This adversely affects the machine productivity.

Reduction in the adverse effects of the impulsive forces can be achieved by

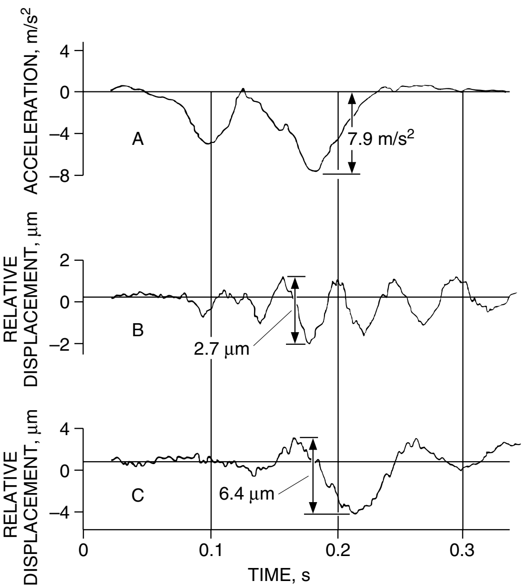

FIGURE 40.4 Effect of mounts on relative displacements between grinding wheel and table during reversal of table of surface grinder. (*A*) Acceleration of table; (*B*)(*C*) relative displacements [(*B*), machine installed on steel wedge mounts; (*C*), machine installed on vibration isolators]; table velocity 20 m/min. (*After Kaminskaya from Ref. 6.*)

enhancing the structural stiffness and natural frequencies, thus reducing the sensitivity of the machine to impulsive forces and accelerating the decay. A similar effect results from an increase in "solid-body frequencies" (the natural frequencies of the machine on its mounts) in the direction of the impulsive forces and from decoupling of vibratory modes in the vibration-isolation system, e.g., by increasing the distance between the mounts in the direction of acceleration. Increase of structural damping as well as damping of mounting elements (vibration isolators) also results in a reduction in the decay time.

VIBRATION TRANSMITTED FROM THE ENVIRONMENT

Shock and vibration generated in presses, machine tools, internal-combustion engines, compressors, cranes, carts, rail and road vehicles, etc., are transmitted through the foundation to other machines, which they may set into forced vibration. Vibration of the shop floor contains a wide frequency spectrum. It is almost inevitable that one of these frequencies should fall near a natural frequency of a particular machine tool. Although the amplitudes of the floor vibration usually are small, they may adversely affect precision machine tools and measuring instruments. The undesirable effects include irreversible shifts in structural joints of machine tools and their mounts, shape and surface finish distortions of machined parts, erroneous readings of measuring instruments, and chipping of cutting inserts.[19]

Vibration transmitted through the floor may be reduced by vibration isolation (Chaps. 30 and 32), i.e., the stationary machines which generate the vibration are placed upon vibration isolators. However, precision machine tools and measuring instruments are isolated to provide further reduction.

When applying vibration isolators to machine tools, some care must be exercised. The foundation constitutes the "end condition" of the machine-tool structure. Any alteration of the end condition affects equivalent stiffness and damping, and thus the natural frequencies and vibratory modes of the structure.[3]

If vibration isolators are not properly selected and located, the machine tool may become more susceptible to internal exciting forces, and its chatter behavior also may be affected in an undesirable way,[8] usually at the lower modes of vibration. Many undesirable effects can be eliminated or significantly reduced by using vibration isolators having a natural frequency that is independent of weight loads on isolators ("constant natural frequency" isolators); by using isolators with high damping; by assigning the mounting points locations that enhance the effective stiffness of the machine-tool frame; by increasing the stiffness of isolators and the distance between them in the directions of movements of heavy reciprocating masses; and by reducing modal coupling in the isolation system.[3] In general, machine-tool structures which are very stiff by themselves (i.e., without being bolted down) can be placed on vibration isolators safely (milling machines, grinding machines, and some lathes).

MACHINE-TOOL CHATTER

The cutting of metals is frequently accompanied by violent vibration of workpiece and cutting tool which is known as machine-tool chatter. *Chatter* is a self-excited vibration which is induced and maintained by forces generated by the cutting process. It is highly detrimental to tool life and surface finish, and is usually accompanied by considerable noise. Chatter adversely affects the rate of production since, in many cases its elimination can be achieved only by reducing the rate of metal removal. Cutting regimes for nonattended operations (such as computer numerically controlled machine tools and flexible manufacturing systems) are frequently assigned conservatively in order to avoid the possibility of chatter.

Machine-tool chatter is characteristically erratic since it depends on the design and configuration of both the machine and the tooling structures, on workpiece and cutting tool materials, and on machining regimes. Chatter resistance of a machine tool is usually characterized by a maximum stable (i.e., not causing chatter vibration) depth of cut $b_{\lim}$. Forced vibration effects in machine tools are more frequently detected in the development stage or during final inspection, and can be reduced or eliminated. The tendency for a certain machine to chatter may remain unobserved in the plant of the machine-tool manufacturer unless the machine is thoroughly tested.[9,10] If this tendency is encountered at the user facility, its elimination from a particular machining process may be highly time-consuming and laborious.

A distinction can be drawn[8] between regenerative and nonregenerative chatter. The former occurs when there is an overlap in the process of performing successive cuts such that part of a previously cut surface is removed by a succeeding pass of the cutter. Under regenerative cutting, a displacement of the tool can result in a vibration of the tool relative to the workpiece, resulting in a variation of the chip thickness. This in turn results in a variation in the cutting force during following revolutions. The regenerative chatter theory explains a wide variety of practical chatter situations in such operations as normal turning and milling.

An important characteristic feature of regenerative chatter is a "lobing" dependence of the maximum stable depth of cut $b_{\lim}$ on cutting speed (rpm of tool or workpiece).[8,11] This dependence is shown as the solid line in Fig. 40.5.[8,11] There is an area of absolute stability below the lobes' envelope, which is shown as a broken line in Fig. 40.5. The position of this envelope depends on the material and geometry of the cutting tool as well as the workpiece material. The lobing shape indicates that some speeds are characterized by much higher stability (larger $b_{\lim}$).

Nonregenerative chatter is found in such operations as shaping, slotting, and screw-thread cutting. In this type of cutting, chatter has been explained through the principle of mode coupling.[8] If a machining system can be modeled by a two degree-of-freedom mass-spring system, with orthogonal axes of major flexibilities and with a common mass, the dynamic motion of the tool end can take an elliptical path. If the major axis of motion (axis with the greater compliance) lies within the angle formed by the total cutting force and the normal to the workpiece surface, energy can be transferred into the machine-tool structure, thus producing an effective negative damping. The depth of cut for the threshold of stable operation is directly dependent upon the difference between the two principal stiffness values, and chatter tends to occur when the two principal stiffnesses are close in magnitude.

DYNAMIC STABILITY

Machine-tool chatter is essentially a problem of dynamic stability. A machine tool under vibration-free cutting conditions may be regarded as a dynamical system in steady-state motion. Systems of this kind may become dynamically unstable and break into oscillation around the steady motion. Instability is caused by an alteration of the cutting conditions produced by a disturbance of the cutting process (e.g., a hard spot in the material). As a result, a time-dependent thrust element dP is superimposed on the steady cutting thrust P. If this thrust element is such as to amplify the original disturbance, oscillations will build up and the system is said to be unstable.

This chain of events is most easily investigated theoretically by considering that the incremental thrust element dP is a function not only of the original disturbance but

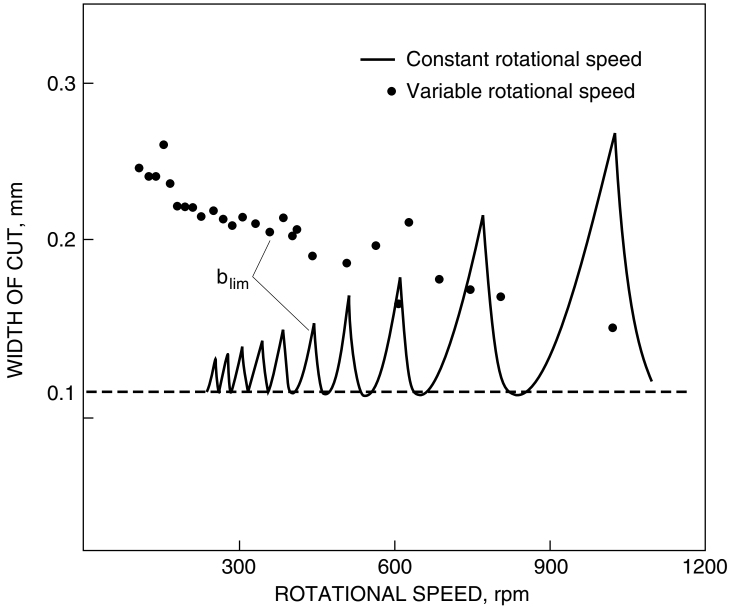

FIGURE 40.5 Dependence of maximum stable width of cut b_{lim} on cutting speed (stability chart) for turning; stable area is below the line, unstable, above the line. Dots indicate cutting with variable (modulated) cutting speed. (*After J. Sexton and B. Stone.*[11])

also of the velocity of this disturbance. Forces which are dependent on the velocity of a displacement are damping forces; they are additive to or subtractive from the damping present in the system (e.g., structural damping or damping introduced by special antivibration devices). When the damping due to dP is positive, the total damping (structural damping plus damping due to altered cutting conditions) also is positive and the system is stable. Any disturbance will then be damped out rapidly. However, the damping due to dP may be negative, in which case it will decrease the structural damping, which is always positive. If the negative damping due to dP predominates, the total damping is negative. Positive damping forces are energy-absorbing. Negative damping forces feed energy into the system; when the total damping is negative, this energy is used for the maintenance of oscillations (chatter).

From the practical point of view, the fully developed chatter vibration (self-induced vibration) is of little interest. Production engineers are almost entirely concerned with conditions leading to chatter (dynamic instability). The build-up of chatter is very difficult to observe, and experimental work has to be carried out mainly under conditions which are only indirectly relevant to the problem being investigated. Experimental results obtained from fully developed chatter vibration may, in some instances, be not really relevant to the problem of dynamic stability.

The influence of the machine-tool structure on the dynamic stability of the cutting process is of great importance. This becomes clear by considering that with a structure (including tool and workpiece) of infinite stiffness, the cutting process could not be disturbed in the first place because hard spots, for example, would not be able to produce the deflections necessary to cause such a disturbance. Further-more, it is clear that were the structural damping infinite, the total damping could not become negative and the cutting process would always be stable. This discussion indicates that an increase in structural stiffness and/or damping always has beneficial effects from the point of view of chatter.

In practically feasible machines, the interrelation between structural stiffness, damping, and dynamic stability is of considerable complexity. This is because machine-tool structures are systems with distributed mass, elasticity, and damping; their vibration is described by a large set of partial differential equations which can be analyzed using simplified models or more precise large finite-element models. Stiffness and damping play similar roles in determining the stability of a machine tool. The maximum stable depth of cut b_{lim} is proportional to a product of effective stiffness and effective damping coefficients. The cutting angles and the number and shape of the cutting edges of the cutting tool are important.

THE EFFECT OF VIBRATION ON TOOL LIFE

Inasmuch as the cutting speed and the chip cross section vary during vibration, it is to be expected that vibration affects tool life. The magnitude of this effect is unexpectedly large, even when impact loading of the tool is excluded. Elimination of vibration may significantly enhance tool life. Ceramic and diamond tools are especially sensitive to impact loading.

The life of face-mill blades may suffer considerably owing to torsional vibration executed by the cutter. The torsional vibration need not necessarily be caused by dynamic instability of the cutting process but may be forced vibration, because of resonance caused by one of the harmonics of impact excitation from interrupted chip removal, by tool runout, etc. Judiciously applied forced vibration of the tool and/or the workpiece may also significantly enhance tool life by reducing cutting forces, leading to enhanced dynamic stability.[12]

VIBRATION CONTROL IN MACHINE TOOLS

The vibration behavior of a machine tool can be improved by a reduction of the intensity of the sources of vibration, by enhancement of the effective static stiffness and damping for the modes of vibration which result in relative displacements between tool and workpiece, and by appropriate choice of cutting regimes, tool design, and workpiece design. Abatement of the sources is important mainly for forced vibrations. Stiffness and damping are important for both forced and self-excited (chatter) vibrations. Both parameters, especially stiffness, are critical for accuracy of machine tools, stiffness by reducing structural deformations from the cutting forces, and damping by accelerating the decay of transient vibrations. In addition, the application of vibration dampers and absorbers is an effective technique for the solution of machine-vibration problems. Such devices should be considered as a functional part of a machine, not as an add-on to solve specific problems.

STIFFNESS

Static stiffness k_s is defined as the ratio of the static force P_o, applied between tool and workpiece, to the resulting static deflection A_s between the points of force application. A force applied in one coordinate direction is causing displacements in three coordinate directions; thus the stiffness of a machine tool can be characterized by a stiffness matrix (three proper stiffnesses defined as ratios of forces along the coordinate axes to displacements in the same directions, and three reciprocal stiffnesses between each pair of the coordinate axes). Frequently only one or two stiffnesses are measured to characterize the machine tool.[3,6]

Machine tools are characterized by high precision, even at heavy-duty regimes (high magnitudes of cutting forces). This requires very high structural stiffness. While the frame parts are designed for high stiffness, the main contribution to deformations in the work zone (between tool and workpiece) comes from contact deformations in movable and stationary joints between components (contact stiffness[3,14]). Damping is determined mainly by joints (log decrement $\Delta \cong 0.15$), especially for steel welded frames (structural damping $\Delta \cong 0.001$). Cast iron parts contribute more to the overall damping ($\Delta \cong 0.004$), while material damping in polymer-concrete ($\Delta \cong 0.02$) and granite ($\Delta \cong 0.015$) is much higher. While the structure has many degrees-of-freedom, dangerous forced and self-excited vibrations occur at a few natural modes which are characterized by high intensity of relative vibrations in the work zone. Since machine tools operate in different configurations (positions of heavy parts, weights, dimensions, and positions of workpieces) and at different regimes (spindle rpm, number of cutting edges, cutting angles, etc.), different vibratory modes can be prominent depending on the circumstances.

The stiffness of a structure is determined primarily by the stiffness of the most flexible component in the path of the force. To enhance the stiffness, this flexible component must be reinforced. To assess the influence of various structural components on the overall stiffness, a breakdown of deformation (or compliance) at the cutting edge must be constructed analytically or experimentally on the machine.[3] Breakdown of deformation (compliance) in torsional systems (transmissions) can be critically influenced by transmission ratios between the components.[3] In many cases the most flexible components of the breakdown are local deformations in joints, i.e., bolted connections between relatively rigid elements such as column and bed, column and table, etc. Some points to be considered in the design of connections are illustrated in Fig. 40.6.[13] To avoid bending of the flange in Fig. 40.6*A*, the bolts should be placed in pockets or between ribs, as shown in Fig. 40.6*B*. Increasing the flange thickness does not necessarily increase the stiffness of the connection, since this requires longer bolts, which are more flexible. There is an optimum flange thickness (bolt length), the value of which depends on the elastic deformation in the vicinity of the connection. Deformation of the bed is minimized by placing ribs under connecting bolts.[13]

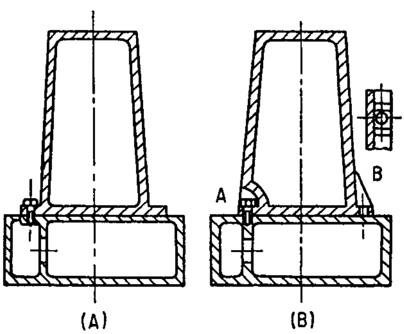

FIGURE 40.6 Load transmission between column and bed. (*A*) Old design, relatively flexible owing to deformation of flange. (*B*) New design, bolt placed in a pocket (*A*) or flange stiffened with ribs on both sides of bolt (*B*). (*After H. Opitz.*[13])

The efficiency of bolted connections, and other static and dynamic structural problems, is conveniently investigated by scaled model analysis[13] and finite-element analysis techniques described in Chap. 28, Part II. Figure 40.7 shows the results of successive stages of a model experiment in which the effect of the design of bolt connections on the bending rigidity (*X* and *Y* directions) and the torsional rigidity of a column were investigated. The relative

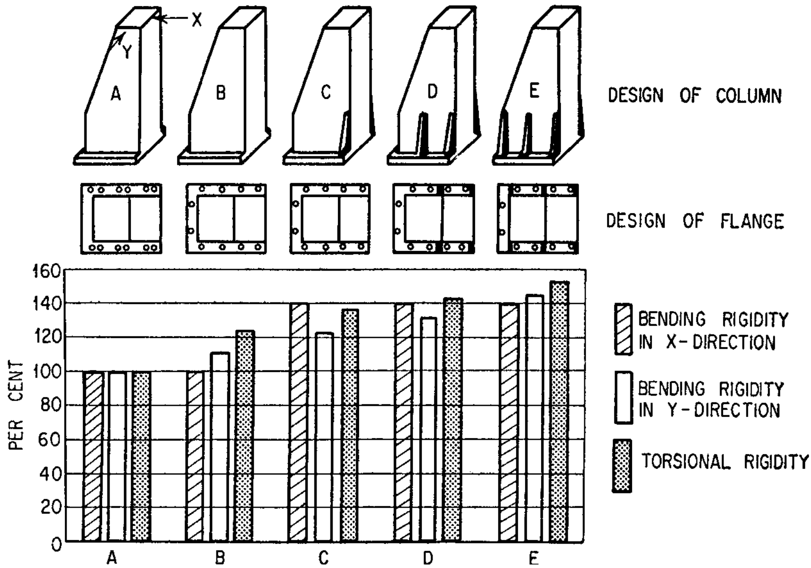

FIGURE 40.7 Successive stages in the improvement of a flange connection. (*H. Opitz.*[13])

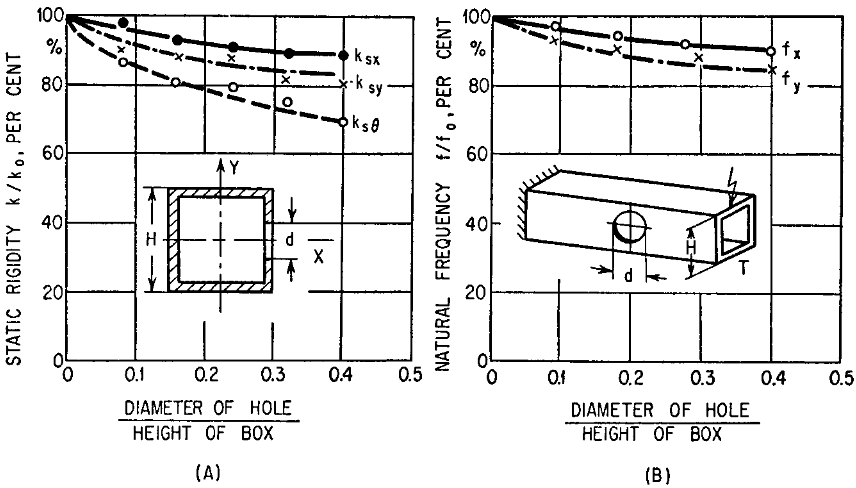

FIGURE 40.8 Influence of a hole in the wall of a box column on the static stiffness and natural frequency. (*A*) Static stiffness; (*B*) natural frequency. (*H. Opitz.*[13])

rigidities are shown by the length of bars. In the design of Fig. 40.7*A*, the connection consists of 12 bolts (diameter of ⅝ in.) arranged in pairs along both sides of the column. In the design of Fig. 40.7*B*, the number of bolts is reduced to 10, arranged as shown. With the addition of ribs, shown in succeeding figures, the bending stiffness in the direction *X* was raised by 40 percent, that in the direction *Y* by 45 percent, and the torsional stiffness by 53 percent, compared to the original design.[13]

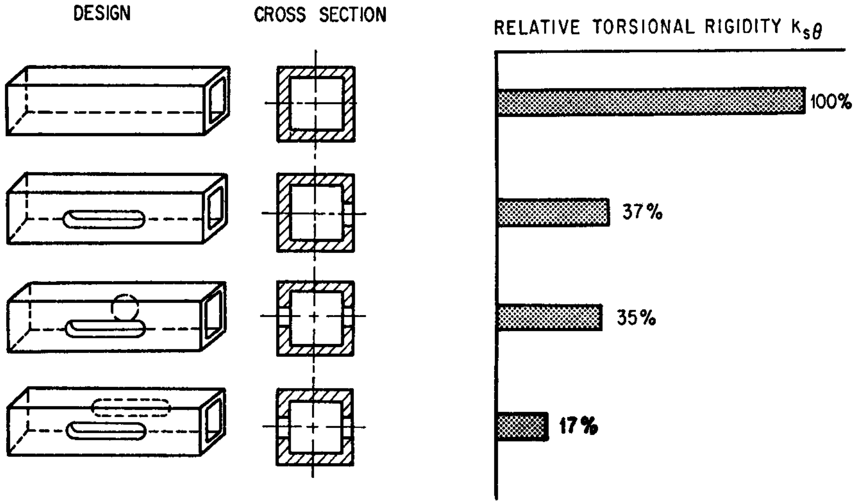

FIGURE 40.9 Torsional stiffness of box columns with different holes in walls. (*H. Opitz.*[13])

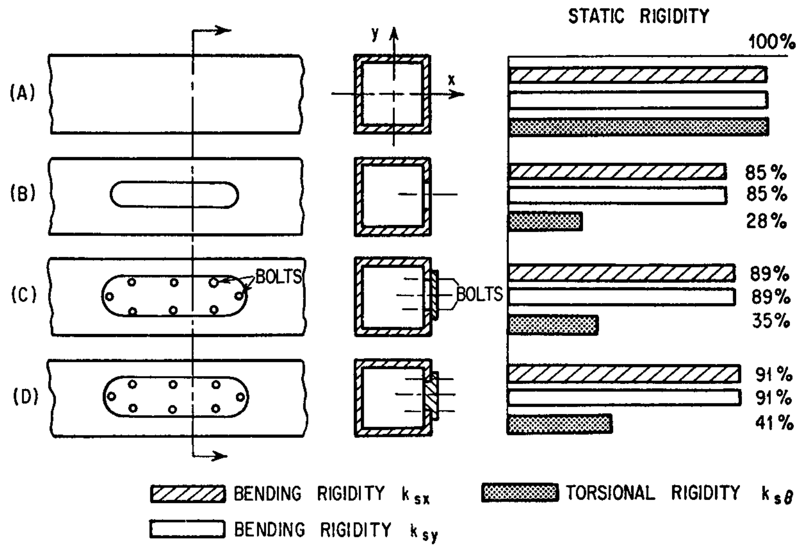

FIGURE 40.10 Influence of cover plate and lid on static stiffness of box column. (*A*) Column without holes, (*B*) one hole uncovered, (*C*) hole covered with cover plate, and (*D*) hole covered with substantial lid, firmly attached. (*After H. Opitz.*[13])

Openings in columns should be as small as possible. Figure 40.8 shows the loss of static flexural stiffness k_{sx}, k_{sy}, and torsional stiffness $k_{s\theta}$, and the decrease of the flexural natural frequencies f_x, f_y, resulting from the introduction of a hole in a box-type column. Smaller holes result in relatively smaller decreases of stiffness and natural frequency than larger ones. The torsional rigidity $k_{s\theta}$ of a box-type column is particularly sensitive to openings, as shown in Fig. 40.9.[13] Lids or doors used for covering these openings do not restore the stiffness. The influence of covers depends on their thickness, mode of attachment, and design, as shown in Fig. 40.10.[13] However, covers may increase damping and thereby partly compensate for the detrimental effect of loss of stiffness.

Welded structural components are usually stiffer than cast iron components but have a lower damping capacity. Some damping is generated because welds are never perfect; consequently, rubbing takes place between joined members. A considerable increase in damping can be achieved by using interrupted welds, but at a price of reduced stiffness. Welded ribs may be necessary not so much to increase rigidity as to prevent "drumming" (membrane vibration) of large unsupported areas.

Not all deformations in machine tools are objectionable, but only those which influence relative displacements in the work zone between the tool and the workpiece. The magnitude of the relative displacement in the work zone under external or internal forces (weight, cutting force, inertia force) determines *effective* stiffness.

Effective stiffness of machine-tool frames is significantly influenced by their interaction with the supporting structures (foundations). For large, low-aspect-ratio machine-tool frames, a rigid attachment to a properly dimensioned[6] foundation substantially improves dynamic stability. Medium- and small-size machine tools are usually attached to the reinforced floor plate by discrete mounts (rigid wedge or screw mounts or vibration isolators). A rational assignment of number and location of mounts noticeably enhances the effective stiffness of machine tools and in some cases may allow direct mounting of rather large machine tools on vibration isolators. Examples of influence of number and location of mounts on the effective stiffness are given in Fig. 40.11, which shows three schematics of a mounting for a jig borer on rigid wedge mounts. The table of the jig borer is in the lower end of the illustration. Relative displacements in the work zone when the table travels from right to left for the scheme in Fig. 40.11*C* are three times smaller than for Fig. 40.11*A* and 1.5 times smaller than for Fig. 40.11*B,* notwithstanding the fact that in the latter case there are seven mounts vs. three mounts in Fig. 40.11*C.* In the case shown in Fig. 40.11*A,* the large weight of the moving table creates a twisting of the supporting frame about the single front mount, while the column is rigidly positioned by two mounts. In case of Fig. 40.11*C,* the front end is well supported, but the column can tilt on its single mount and follow small deformations of the front part, thus resulting in smaller relative deformations and higher effective stiffness. For example, in the case of a precision grinder having a bed 3.8 m long, it was found that mounting the grinder on seven carefully located (offset from the ends) vibration isolators resulted in higher effective stiffness than installation on 15 rigid mounts.[3]

The effective static stiffness of a machine tool may vary within wide limits. High stiffness values are ensured by the use of steady rests, by placing tool and workpiece in a position where the relative dynamic displacement between them is small (i.e., by

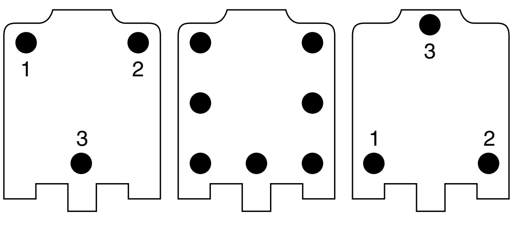

FIGURE 40.11 Mounting schemes of a jig borer. (*After V. Kaminskaya from Ref. 6.*)

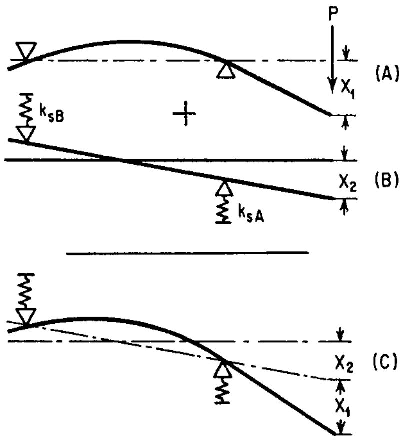

FIGURE 40.12 Deflection of machine-tool spindle and bearings. A machine-tool spindle can be regarded as a beam on flexible supports. The total deflection under the force P consists of the sum of (A) the deflection X_1 of a flexible beam on rigid supports and (B) the deflection X_2 of a rigid beam on flexible supports. (*H. Opitz.*[13])

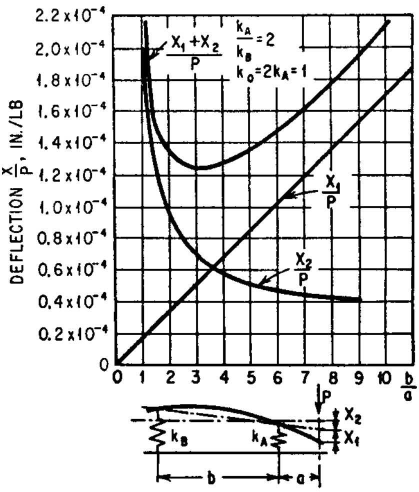

FIGURE 40.13 Deflection of a beam on elastic supports as a function of the bearing distance. Bearing stiffness k_A and k_B, spindle stiffness k_o. (*After H. Opitz.*[13])

placing them near the main column, etc.), by using rigid tools and clamps, by using jigs which rigidly clamp (and if necessary support) the workpiece, by clamping securely all parts of the machine which do not move with respect to each other, etc., and by the optimization of mounting conditions mentioned above.

The static and dynamic behavior of machine tools is influenced significantly by the design of the spindle and its bearings. The static deflection of the spindle consists of two parts, X_1 and X_2, as shown in Fig. 40.12. The deflection X_1 corresponds to the deflection of a flexible beam on rigid supports, and X_2 corresponds to the deflection of a rigid beam on flexible supports which represent the flexibility of the bearings. The deflection of the spindle amounts to 50 to 70 percent of the total deflection, and the bearings 30 to 50 percent of the total, depending on the relation of spindle cross section to bearing stiffness and span. The stiffness of antifriction bearings depends on their design, accuracy, preload, and the fit between the outer race and the housing (responsible for 10 to 40 percent of the bearing deformation[3]).

The distance between the bearings has considerable influence on the effective stiffness of the spindle, as shown in Fig. 40.13. The ordinate of the figure corresponds to the deflection in inches per pound, and the abscissa represents the ratio of bearing distance b to cantilever length a. The straight line refers to the deflection of the spindle, and the hyperbola refers to the deflection of the bearings. The total deflection is obtained by the addition of the two curves; the minimum of the curve of total deflection corresponds to the optimum bearing distance. For a short cantilever length a, the optimum value of b/a lies between 3 and 5; for a long cantilever length a, the optimum $b/a = {\sim}2$.

It is often important to consider the dynamic behavior of a spindle before establishing an optimum bearing span. Maximizing the stiffness of a spindle at one point does not establish its dynamic properties. Care must be taken to investigate both bending and rocking modes of the spindle before accepting a final optimum span.

For example, a large overhang on the rear of a spindle could produce an undesirable low-frequency rocking mode of the spindle even if the "optimum span" as defined previously were satisfied. The optimum bearing span for minimum deflection as well as the dynamic characteristics of spindles may be computed with the help of available computer programs.

The influence of the ratio of bore diameter to outside diameter on the stiffness of a hollow spindle is shown in Fig. 40.14.[13] A 25 percent decrease in stiffness occurs only at a diameter ratio of $d/D = 0.7$, where D is the outside diameter and d the bore diameter. This is important for the dynamic behavior of the spindle. A solid spindle has nearly the same stiffness, but a substantially greater mass. Consequently, the natural frequency of the solid spindle is considerably lower, which is undesirable. A stiff spindle does not always assure the required high stiffness at the cutting edge of the tool because of potentially large contact deformations in the toolholder/spindle interface. Measurements have shown that in a tapered connection, these deformations may constitute up to 50 percent of the total deflection at the tool edge.[3] These deformations can be significantly reduced by replacing tapered connections by face contact between the toolholder and the spindle. The face connection must be loaded by a high axial force.[12]

A significant role (frequently up to 50 percent) in the breakdown of deformations between various parts of machine tool structures is played by contact deformations between conforming (usually flat, cylindrical, or tapered) contacting surfaces in structural joints and slides.[3,14] Contact deformations are due to surface imperfections on contacting surfaces. These deformations are highly nonlinear and are influenced by lubrication conditions. Figure 40.15 shows contact deformation between flat steel parts as a function of contact pressure for different lubrication conditions in the joint. Joints are also responsible for at least 90 percent of structural damping in machine-tool frames due to micromotions in the joints during vibratory processes. Contact deformations for the same contact pressure can be significantly reduced by increasing accuracy (fit) and improving the surface finish of the mating surfaces. The nonlinear load-deflection characteristic of joints, Fig. 40.15, allows enhancement of their stiffness by preloading. However, preloading reduces micromotions in the joints and thus results in a lower damping.

This explains why in some cases old machines are less likely to chatter than new machines of identical design. The situation may result from wear and tear of the slides, which increases the damping and effects an improvement in performance. Also, in some cases chatter is eliminated by loosening the locks of slides. However, it would be wrong to conclude that lack of proper attention and maintenance is desirable. Proper attention to slides, bearings (minimum play), belts, etc., is necessary for satisfac-

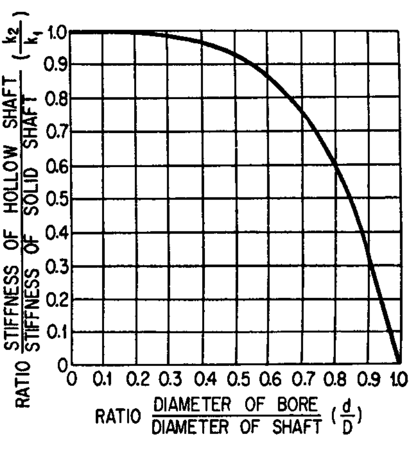

FIGURE 40.14 Effect of bore diameter on stiffness of hollow spindle where k_1 = stiffness of solid spindle, k_2 = stiffness of hollow spindle, D = outer spindle diameter, d = bore diameter, J_2 = second moment of area of hollow spindle, and J_1 = second moment of area of solid spindle. The curve is defined by $k_2/k_1 = J_2/J_1 = 1 - (d/D)^4$. (*H. Opitz.*[13])

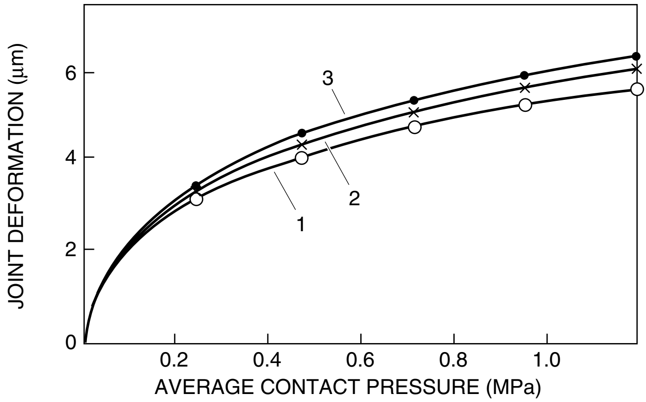

FIGURE 40.15 Load-deflection characteristics for flat, deeply scraped surfaces (overall contact area 80 cm^2). 1, no lubrication; 2, lightly lubricated (oil content 0.8×10^{-3} gram/cm^2); 3, richly lubricated (oil content 1.8×10^{-3} gram/cm^2). (*After Z. Levina and D. Reshetov.*[14])

tory performance. It would be wrong also to conclude that a highly polluted workshop atmosphere is desirable because some new machines exposed to workshop dirt for a sufficiently long time, even when not used, appear to improve in their chatter behavior. The explanation is that dirty slides increase the damping.

When the rigidity of some machine element is intentionally *reduced,* but this reduction is accompanied by a greater damping at the cutter, the increase in damping may outweigh the reduction in rigidity.[3] Although a loss of rigidity in machine tools is generally undesirable, it may be tolerated when it leads to a desirable shift in natural frequencies or is accompanied by a large increase in damping or by a beneficial change in the ratio of stiffnesses along two orthogonal axes, which can result in improved nonregenerative chatter stability.[8]

A very significant improvement in chatter resistance can be achieved by an intentional measured reduction of stiffness in the direction along the cutting speed (orthogonal to the direction of the principal component of cutting force). The benefits of this approach have been demonstrated for turning and boring operations.[12,15]

DAMPING

The overall damping capacity of a structure with cast iron or welded steel frame components is determined only to a small extent by the damping capacity of its individual components. The major part of the damping results from the interaction of joined components at slides or bolted joints.[3,14] The interaction of the structure with the foundation or highly damped vibration isolators also may produce a noticeable damping.[3,8] A qualitative picture of the influence of the various components of a lathe on the total damping is given in Fig. 40.16. The damping of the various modes of vibration differs appreciably; the values of the logarithmic decrement shown in the figure correspond to an average value for all the modes which play a significant part.

The overall damping of various types of machine tool differs, but the log decrement is usually in the range of from 0.15 to 0.3. While structural damping is significantly higher for frame components made of polymer-concrete compositions or

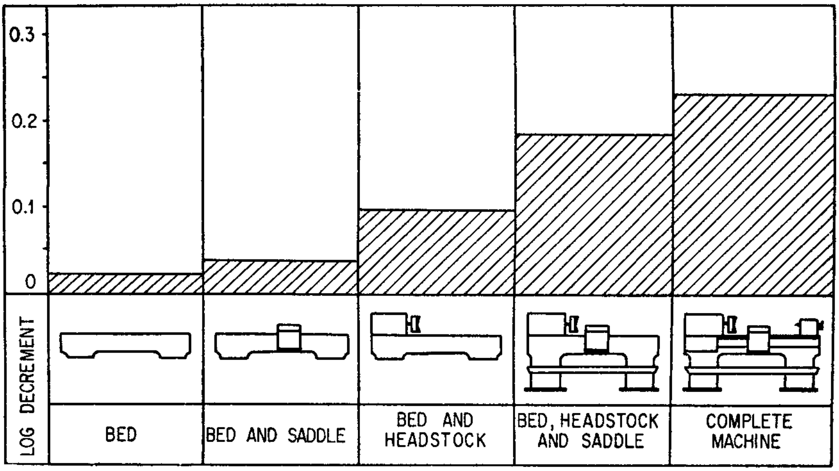

FIGURE 40.16 Influence of various components on total damping of lathes. The major part of the damping is generated at the mating surfaces of the various components. (*K. Loewenfeld.*[16])

granite (see above), the overall damping does not change very significantly since the damping of even these materials is small compared with damping from joints.

A significant damping increase can be achieved by filling internal cavities of the frame parts with a granular material, e.g., sand. For cast parts it can also be achieved by leaving cores in blind holes inside the casting. A similar, sometimes even more pronounced, damping enhancement can be achieved by placing auxiliary longitudinal structural members inside longitudinal cavities within a frame part, with offset from the bending neutral axis of the latter. The auxiliary structural member interacts with the frame part via a high viscous layer, thus imparting energy dissipation during vibrations.

Damping can be increased without impairing the static stiffness and machining accuracy of the machine by the use of dampers and dynamic vibration absorbers. These are basically similar to those employed in other fields of vibration control (Chaps. 6, 32, and 41). Dampers are effective only when placed in a position where vibration amplitudes are significant.

The tuned dynamic vibration absorber (Chap. 6) has been employed with considerable success on milling machines, machining centers, radial drilling machines, gear hobbing machines, grinding machines, and boring bars.[15,17] A design variant of this type of absorber is shown in Fig. 40.17. In this design a plastic ring element combines both the elastic and the damping elements of the absorber. The auxiliary mass may be attached to the top of a column (Fig. 40.17*C*), as shown in Fig. 40.17*A*. Alternatively, the auxiliary mass may be suspended on the underside of a table (Fig. 40.17*C*), using the design shown in Fig. 40.17*B*. In either case, several plastic ring elements may support one large auxiliary mass, as shown in Fig. 40.17*C*. In a boring bar, shown in Fig. 40.18*A*, elastic and damping properties are combined in O-rings made of a high-damping rubber. Tuning of the absorber can be changed by varying the radial preload force on the O-ring. The natural frequency of this absorber can be varied over a range of more than 3:1.

A variation of the *Lanchester damper* (Chap. 6) is frequently used in boring bars to good advantage.[16] This consists of an inertia weight fitted into a hole bored in the end of a quill. To ensure effective operation, a relatively small radial clearance of

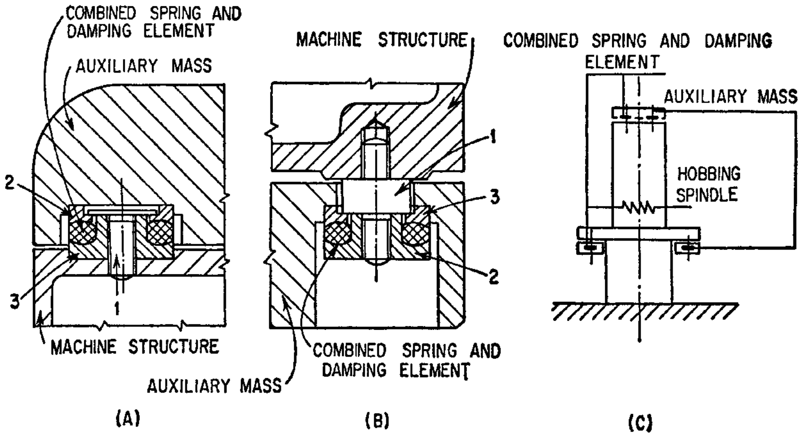

FIGURE 40.17 Auxiliary mass damper with combined elastic and damping element. The combined element lies between two retainer rings, of which one (3) is attached with bolt 1 to the machine structure. The other ring (2) takes the weight of the auxiliary mass. (*A*) Arrangement when auxiliary mass is being supported. (*B*) Arrangement when auxiliary mass is being suspended. (*C*) Application of both types of arrangements to a hobbing machine. (*After F. Eisele and H. W. Lysen.*[17])

about 1 to $5 \times 10^{-3}d$ must be provided, where d is the diameter of the inertia weight. An axial clearance of about 0.006 to 0.010 in. (0.15 to 0.25 mm) is sufficient. A smooth surface finish of both plug and hole is desirable. The clearance values given refer to dry operation, using air as the damping medium. Oil also can be used as a damping medium, but it does not necessarily result in improved performance. When applying oil, clearance gaps larger than those stated above have to be ensured, depending on the viscosity of the oil. In general, Lanchester dampers are less effective than tuned vibration absorbers.

Since the effectiveness of both Lanchester dampers and tuned vibration absorbers depends on the mass ratio between the inertia mass and the effective mass of the structure (Chap. 6), heavy materials such as lead and, especially, machinable sintered tungsten alloys are used for inertia masses in cases where the dimensions of the inertia mass are limited (as in the case of boring bars in Fig. 40.18). The mass ratio and the effectiveness of the absorber can be significantly enhanced by using a combination structure. In such a structure the overhang segment of the boring bar or other cantilever structure, which does not significantly influence its stiffness but determines its effective mass, is made of a light material, while the root segment, which determines the stiffness but does not significantly influence the effective mass, is made from a high Young's modulus material.[15]

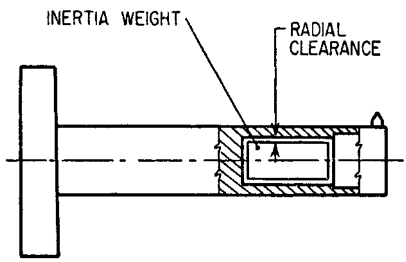

FIGURE 40.18 Lanchester damper for the suppression of boring bar vibration. (*After R. S. Hahn.*[18])

Dynamic absorbers can be active (servo-controlled). Such devices can be designed to be self-optimizing (capable of self-adjustment of the spring rate to minimize vibration amplitude under changing excitation conditions) or to use a vibration cancellation approach. The self-optimizing feature is achieved by placing vibration transducers on both the absorber mass and the main system. A control circuit measures the phase angle between the motions and activates a spring-modifying mechanism to maintain a 90° phase difference between the two measured motions. It has been demonstrated that the 90° phase relationship guarantees minimum motion of the main vibrating mass. In the vibration-cancellation devices, the actuator applies force to the structure which is opposite in phase to structural vibrations.

Dynamic analysis of a machine tool structure can identify potentially unstable natural modes of vibration and check the effectiveness of the applied treatments. In another approach, transfer functions between the selected points on the machine tool are measured and processed through a computational technique which indicates at which location stiffness and/or damping should be modified or a dynamic vibration absorber installed in order to achieve specified dynamic characteristics of the machine tools.[3]

Tool Design. Sharp tools are more likely to chatter than slightly blunted tools. In the workshop, the cutting edge is often deliberately dulled by a slight honing. Consequently, a beveling of the leading face of a lathe tool has been suggested. This bevel has a leading edge of –80° and a width of about 0.080 in. (0.2 mm). Tests show that the negative bevel does not in all cases eliminate vibration and that the life of the bevel is short. Appreciably worn cutting edges cause violent chatter.

Since narrow chips are less likely to lead to instability, a reduction of the approach angle of the cutting tool results in improved chatter behavior. With lathe tools, an increase in the rake angle may result in improvement, but the influence of changes in the relief angle is relatively small.

Reduction of both forced and chatter vibrations in cutting with tools having multiple cutting edges (e.g., milling cutters, reamers) can be achieved by making the distance between the adjacent cutting edges nonequal and/or making the helix angle of the cutting edges different for each cutting edge. However, such treatment results in nonuniform loading of the cutting edges and may lead to a shortened life of the more heavily loaded edges as well as deteriorating surface finish as a result of different deformations of the tool when lighter or heavier loaded edges are engaged.

Reduction of cutting forces by low-friction (e.g., diamond) coating of the tool or by application of ultrasonic vibrations to the tool usually improves chatter resistance.

Variation of Cutting Conditions. In the elimination of chatter, cutting conditions are first altered. In some cases of regenerative chatter, a small increase or decrease in speed may stabilize the cutting process. In high-speed or unattended computer numerically controlled machine tools, this can be achieved by continuous computer monitoring of vibratory conditions and, as chatter begins to develop, a shifting of the spindle rpm toward the stable area.

Cutting with a variable cutting speed (constant speed modulated by a sinusoidal or other oscillatory component) acts similarly with regard to undulations in the positioning of the cutting edges (see above) and results in increased chatter resistance. The dots in Fig. 40.5 show the stabilizing effect of the sinusoidal modulation of the cutting speed.[11]

An increase in the feed rate is also beneficial in some types of machining (drilling, face milling, and the like). For the same cross-sectional area, narrow chips (high feed rate) are less likely to lead to chatter than wide chips (low feed rate), since the chip thickness variation effect results in a relatively smaller variation of the cross-sectional area in the former (smaller dynamic cutting force).

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