CHAPTER 32
SHOCK AND VIBRATION ISOLATORS AND ISOLATION SYSTEMS

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INTRODUCTION

The first part of this chapter is devoted to various types of shock and vibration isolators, as well as their characteristics. The next topic considered is the properties of combinations of isolators in series and in parallel. A discussion is presented on the selection, installation, and specification of isolators. Then consideration is given to isolators that are combined with masses and damping, forming a vibration control system that can, for example, permit equipment to function as intended, often lengthening its operable life; protect sensitive equipment mounted on a structure from damage as a result of shock and vibration occurring in the structure; and reduce the level of noise and vibration near the equipment, or provide greater comfort to nearby occupants of a building.

The last section of this chapter considers the principles of *active vibration control systems* that differ from passive (conventional) control systems, described earlier, in that they supply additional power (controlled by one or more sensors) that is fed into the system so as to modify its behavior. In many special cases, this additional complication is worthwhile in that it can provide the system with benefits not otherwise obtainable.

TYPES AND CHARACTERISTICS OF ISOLATORS

Isolators are commercially available in many different resilient materials, in countless shapes and sizes, and with widely diverse characteristics. In the U.S.A. there are well over 100 elastomeric isolator manufacturers, each offering a range of models in a variety of synthetic elastomeric compounds and natural rubbers. The number is significantly higher if manufacturers of plastic, metal, pneumatic, and other-material isolators are included.

The properties of a given isolator are dependent not only on the material of which it is fabricated, but also on its configuration and overall construction with respect to the structural material used within the body of the isolator, as explained below. Data on these parameters can be found in the catalogs of the various isolator manufacturers.

ELASTOMERIC ISOLATORS

An *elastomer* is a natural rubber or any polymer having elastic properties similar to those of natural rubber, described in detail in Chap. 33. Such materials are widely used in isolators because they may be conveniently molded into many desired shapes and selected to provide a wide range of stiffnesses, they have more internal damping than metal springs, they usually require a minimum of space and weight, and they can be bonded to metallic inserts adapted for simplified attachment to the isolated structures.

The most commonly used type of isolator is fabricated of an elastomer. Figure 32.1 illustrates some typical elastomeric isolators. Such isolators are able to sustain large deformations and then return to their approximate original state with virtually no damage or change of shape. Elastomeric isolators are superior to other types of isolators in that, for a given amount of elasticity, deflection capacity, energy storage, and dissipation, they require less space and less weight; also, they may be molded into many different configurations of many different types—generally at a lower cost than other types of isolators.

Elastomers have exceptional extensibility and deformability: They can be utilized at elongations of up to about 300 percent, with ultimate elongations of some elastomers to about 1000 percent. They may be stressed as much as 1000 to 1500 psi (0.145 to 0.218 Pa) or more before their elastic limit is reached. Their great capacity for storing energy permits them to tolerate high stress. Upon release of the stress, there is virtually total recovery from the deformation. The inherent damping of elastomers is often useful in preventing excessive vibration amplitude at resonance; the amplitude is much lower than if coil metal springs were used.

Of the various elastomers, *natural rubber* probably embodies the most favorable combination of mechanical properties, such as minimum drift, maximum tensile strength, and maximum elongation at failure. Its usefulness is restricted by its limited resistance to deterioration under the influence of hydrocarbons, ozone, and high ambient temperatures. *Neoprene* and *Buna N* (*nitrile*) exhibit superior resistance to hydrocarbons and ozone, Buna N being particularly satisfactory for applications involving relatively high ambient temperatures. *Buna S* is a good general-purpose synthetic rubber for use in vibration isolators.

Silicone rubber is a costly elastomer. Its properties are remarkably stable, and it provides effective isolation over a very wide temperature range: –65 to +350°F (–54 to 177°C). By comparison, neoprene is limited in use to a range of about –40 to +200°F (–40 to 93°C). The upper temperature limit depends on the properties of the particular compound, the degree of deterioration which is permissible as a result of continued exposure at high temperatures, and the duration of exposure. For silicone, a temperature substantially greater than 300°F (149°C) is permissible for several hours. The outstanding ability of silicone elastomers to withstand extremes of temperature is offset somewhat by their inferior strength, tear resistance, and abrasion resistance.

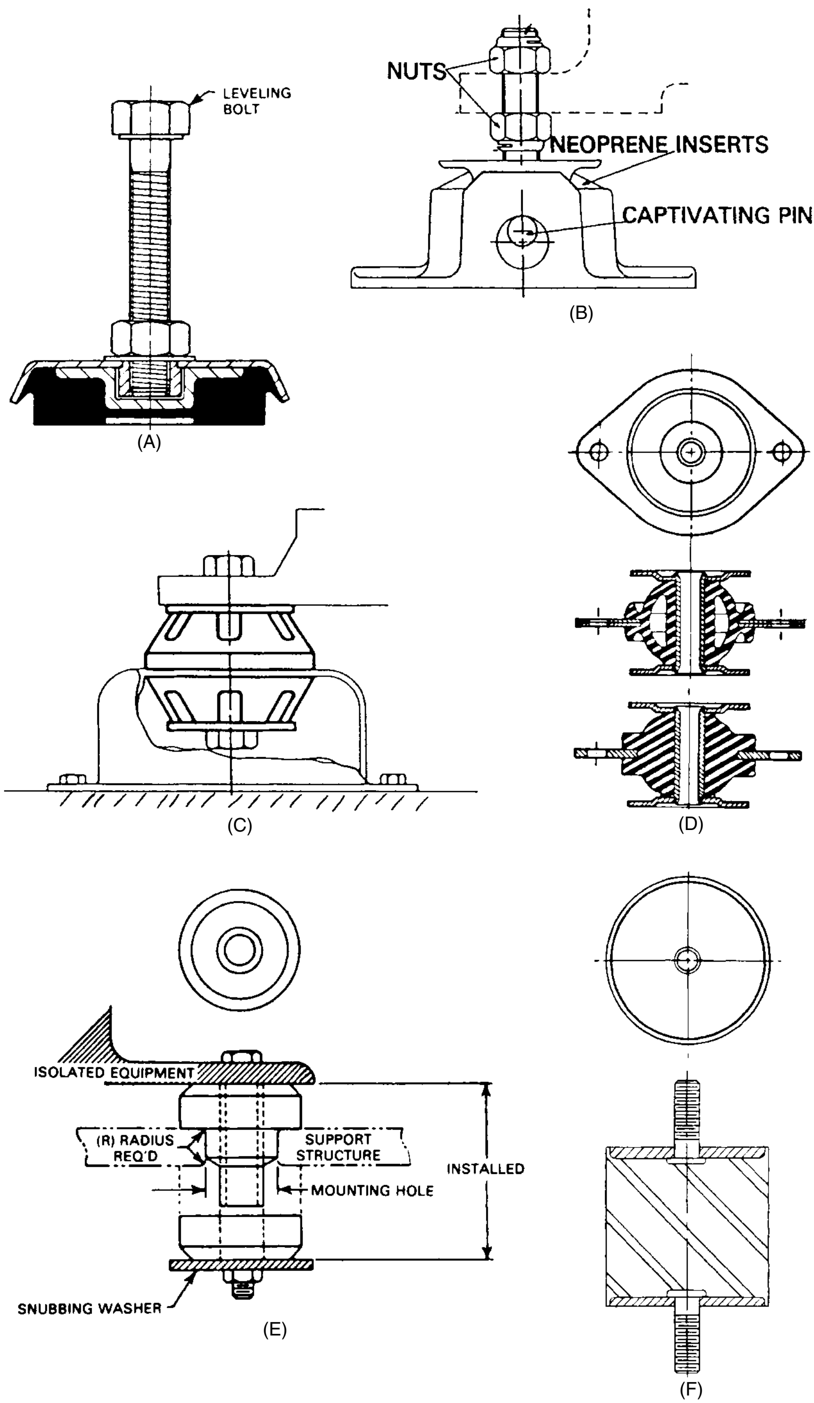

FIGURE 32.1 Typical elastomeric isolators. (*A*) Machinery mount. (*B*) Marine engine isolator. (*C*) Pedestal isolator. (*D*) Plate form instrument isolator. (*E*) General-purpose isolator. (*F*) Cylindrical stud isolator.

Isolators fabricated of elastomers are complex in behavior because of the viscoelastic nature (somewhere between that of a solid and that of a liquid) of elastomers in performance, because of their indefinite yield point, and because their physical properties vary with time, temperature, and environment. For example, rubber is a substantially incompressible material (it has a Poisson's ratio of approximately 0.5). Thus the stiffness of a rubber spring when it is strained in compression depends, to a considerable extent, on the area of the surface available for lateral expansion. In contrast, the stiffness of a rubber spring in shear is substantially independent of the shape of the rubber member. As a rough rule of thumb, it may be assumed that the minimum likely compression stiffness of a given rubber isolator is five times its shear stiffness. The maximum compression stiffness may be several times as great as the minimum value if lateral expansion of the rubber is constrained.

Fatigue Failure and Premature Failure. Regardless of geometry, both elastomers and metals exhibit fatigue failure as a result of repeated cyclic loadings. Unlike a metal, an elastomer does not experience catastrophic-type fatigue failure. Instead, the failure begins as a tear at the point of highest cyclic shear strain, which is generally on the outer extremity (and therefore visible in many cases), and gradually propagates through the body of the elastomer. The result is a gradual reduction in stiffness that usually becomes apparent before there is total failure.

Most elastomeric isolators should not be subject to large static strains over long periods of time. An isolator with a large static deflection may give satisfactory performance temporarily, but the deflection tends to creep (increase) excessively over a long period. In general, elastomers should not be statically strained continuously more than 10 to 15 percent in compression, or more than 25 to 50 percent in shear.

A factor contributing to the premature failure of an elastomeric isolator is the effect of the minimum strain on fatigue life. For elastomers which crystallize under high strains (such as neoprene and natural rubber), fatigue life is greatly increased if the minimum cyclic stress is always either plus or minus and never passes through zero. Proper static *precompression* of the isolator within the limits specified above is often an effective way to prevent the minimum cyclic stress from passing through zero under dynamic conditions. Local stress concentrations, which result in premature failure, often can be avoided by using fillets, radii, and generous overhangs of the elastomeric section. For example, sharp corners of metal inserts and support structures should be carefully rounded off wherever they contact the elastomer. Metal snubbing washers and/or support structures in contact with the elastomer should be large enough to prevent their edges from cutting into the elastomeric surfaces.

Bonded versus Unbonded Elastomeric Isolators. Elastomeric isolators may be designed in both bonded and unbonded configurations. In the bonded isolator, metal inserts are bonded to the elastomer on all load-carrying surfaces. In the unbonded or semibonded isolator, the elastomeric load-bearing surface rests directly on the support structure. Bonded parts usually cost more because of the special chemical preparation required to achieve a bond with strength in excess of that of the elastomer itself. Bonded parts are generally preferred since they may be more highly stressed for a given deflection. With higher stress they provide higher spring constants and higher elastic energy-storage capacity.

Bonded isolators can be designed to provide proper load distribution in shear, compression, tension, or combination loading. A more uniform stress distribution in the elastomer is obtained by bonding inserts on all the load-bearing elastomer surfaces. The bonded inserts reduce unit stress by distributing the stress more uniformly throughout the volume of the elastomer. In contrast, unbonded parts usually fail to distribute the load uniformly, resulting in local areas of stress concentration in the elastomer body which shorten the life of the isolator.

A significant difference between bonded and unbonded elastomeric isolators relates to how elastomers behave under load. When an elastomer pad is compressed under load, its volume remains constant—only its shape is changed. The rubber "bulges" under load. When this ability to bulge is controlled, the load-deflection characteristics of the isolator are controlled. In a bonded isolator, the load-carrying surfaces have a fixed degree of bulge because the elastomer cannot move along the bond line, and so it remains in a fixed position regardless of the load or environmental conditions.

In an unbonded isolator, this is not the case. The ability of the elastomer to bulge depends to a considerable degree on the maintenance of friction at the elastomer–support structure interface. When all surfaces are clean and dry, the difference between the ability of a bonded and an unbonded isolator to bulge is negligible. But if oil or sand works its way into the elastomer-to-metal interface of the unbonded isolator, the ability of the elastomer to bulge is greatly increased; consequently, its original load-deflection characteristics no longer exist. Then the isolator can exhibit load-deflection characteristics that are 50 percent less than when it was new; in many cases, this can cause the isolator to malfunction. Thus, where consistent load-deflection characteristics are required for the life of the equipment, bonded isolators should be used. Although the initial cost of unbonded isolators is lower, in many applications the cost of extra machining of the support structure and the reduced service life may well make unbonded isolators a poor selection.

Types of Isolator Loading. Elastomer isolators may be used with different types of loading: compression, shear, tension, or buckling, or any combination of these types.

Compression Loading. The word *compression* is used to indicate a reduction in the dimension (thickness) of an elastomeric element in the line of the externally applied force. The stiffness characteristic of elastomers stressed in compression exhibit a nonlinearity (hardening) which becomes especially pronounced for strains above 30 percent. Compression loading, illustrated in Fig. 32.2*A,* is most effective when used with simple unbonded isolators and is effective where gradual snubbing (motion limiting) is required. Compression loading is frequently employed to provide a low initial stiffness for vibration isolation and a relatively high final stiffness to limit the dynamic deflection under shock excitations. Because of the nonlinear hardening characteristics of compression loading, it is the least effective type of loading for energy storage and therefore is not recommended where the attenuation of force or acceleration transmission is the primary concern. (The energy stored by any spring is the area under the load-deflection curve.)

Shear Loading. Shear loading, illustrated in Fig. 32.2*B,* refers to the force applied to an elastomeric element so as to slide adjacent parts in opposite directions. An almost linear spring constant up to about 200 percent shear strain is characteristic of elastomer stress in shear. Because of this linear spring constant, shear loading is the preferred type of loading for vibration isolators because it provides a constant frequency response for both small and large dynamic shear strains in a simple spring-mass system. Shear loading is also useful for shock isolators where attenuating force or acceleration transmission is important, because of its more efficient energy-storage capacity when compared to compression loading. However, care

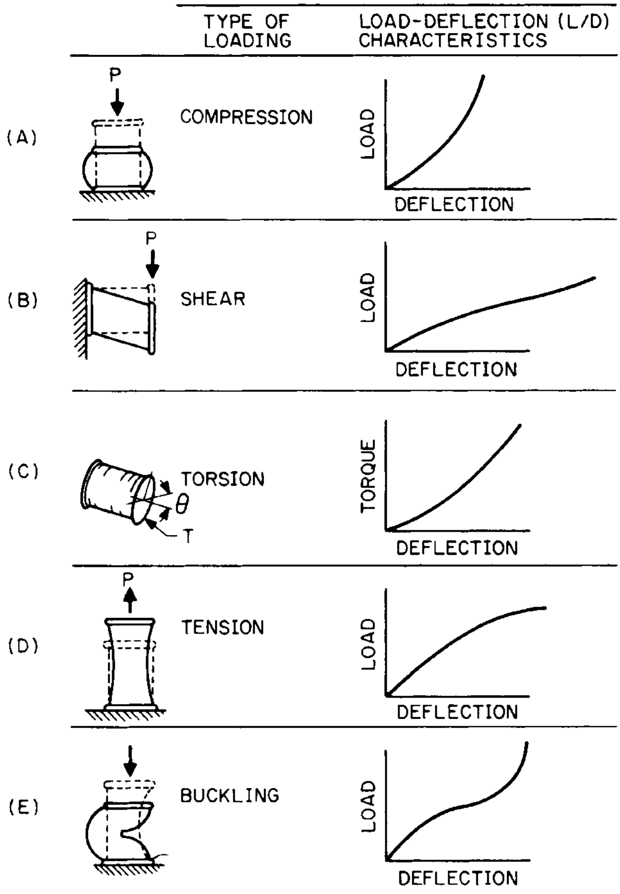

FIGURE 32.2 Load-deflection characteristics of typical elastomeric isolators.

must be taken to ensure that the expected dynamic loads do not result in shear strains that exceed the limits of the elastomer or that abrupt bottoming of the supported equipment does not occur.

Torsion Loading. A modification of shear loading that is sometimes listed as a separate type is torsion loading, shown in Fig. 32.2*C.* It consists of winding up a sandwich of laminated sections to strain the elastomer in torsion. When the strain in torsion exceeds about 150 percent, considerable axial thrust loads on connecting members are induced, if they are rigidly fixed parallel to each other, because of the reduction in the axial thickness of the elastomer.

Tension Loading. Tension loading, illustrated in Fig. 32.2*D,* refers to an increase in the dimension (thickness) of an elastomeric element in the line of the externally applied force. Elastomers stressed in tension exhibit a nonlinear (softening) spring constant. For a given deflection, tension loading stores energy more efficiently than either shear or compression loading. Because of this, tension loading has been occasionally used for shock isolation systems. However, in general, tension loading is not recommended because of the resulting loads on the elastomer-to-metal bond, which may cause premature failure of the material.

Buckling Loading. Buckling loading, illustrated in Fig. 32.2*E,* occurs when the externally applied load causes an elastomeric element to warp or bend in the direction of the applied load. Buckling stiffness characteristics may be used to derive the benefits of both softening stiffness characteristics (for the initial part of the load-deflection curve) and hardening characteristics (for the later part of the load-deflection curve). The buckling mode thus provides high energy-storage capacity and is useful for shock isolators where force or acceleration transmission is important and where snubbing (i.e., motion limiting) is required under excessively high transient dynamic loads. This type of stiffness characteristic is exhibited by certain elastomeric cushioning foam materials and by specially designed elastomeric isolators. However, it is important to note that even simple compressive elements will buckle when the slenderness ratio (the unloaded length/width ratio) exceeds 1.6.

Combinations of the types of loading described above are commonly used, which result in combined load-deflection characteristics. Consider, for example, a compression-type isolator which is installed at an angle instead of in the usual vertical position. Under these conditions, it acts as a compression-shear type of isolator when loaded in the vertical downward direction. When loaded in the vertical upward direction, it acts as a shear-tension combination type of isolator.

Static and Dynamic Stiffness. When the main load-carrying spring is made of rubber or a similar elastomeric material, the natural frequency calculated using the stiffness determined from a static load-deflection test of the spring almost invariably gives a value lower than that experienced during vibration. Thus the dynamic modulus appears greater than the static modulus. The ratio of moduli is approximately independent of the velocity of strain, and has a numerical value generally between 1 and 3. This ratio increases significantly as the durometer increases.

Damping Characteristics. Damping, to some extent, is inherent in all resilient materials. The damping characteristics of elastomers vary widely. A tightly cured elastomer may (within its proper operating range) store and return energy with more than 95 percent efficiency, while elastomers compounded for high damping have less than 30 percent efficiency. Damping increases with decreasing temperature because of the effects of crystallinity and viscosity in the elastomer. If the isolator remains at a low temperature for a prolonged period, the increase in damping may exceed 300 percent. Damping quickly decreases with low-temperature flexure, because of the crystalline structure deterioration and the heat generated by the high damping.

Where the nature of the excitation is difficult to predict (for example, random vibration), it is desirable that the damping in the isolator be relatively high. Damping in an isolator is of the greatest significance at the resonance frequency. Therefore, it is desirable that isolators embody substantial damping when they may operate at resonance, as is the case when the excitation is random over a broad frequency band or even momentary (as in the starting of a machine with an operating frequency greater than the natural frequency of the machine on its isolators). The relatively large amplitude commonly associated with resonance does not occur instantaneously, but rather requires a finite time to build up. If the forcing frequency is varied continuously as the machine starts or stops, the resonance condition may exist for such a short period of time that only a moderate amplitude builds. The rate of change of forcing frequency is of little importance for highly damped isolators, but it is of considerable importance for lightly damped isolators.

In general, damping in an elastomer increases as the frequency increases. The data of Figs. 33.5 and 33.6 can be used to predict transmissibility at resonance by estimating the frequency and the amplitude of dynamic shear strain; then the fraction of critical damping is obtained from the curves and used with Eq. (30.1) to calculate transmissibility at resonance.

Hydraulically Damped Vibration Isolators. Hydraulically damped vibration isolators combine a spring and a damper in a single compact unit that allows tuning of the spring and damper independently. This provides flexibility in matching the dynamic characteristics of the isolator to the requirements of the application. Hydraulic mounts have been used primarily as engine and operator cab isolators in vehicular applications. The hydraulically damped isolator, described in Ref. 2, has a flexible rubber element that encapsulates an incompressible fluid which is made to flow through a variety of ports and orifices to develop the dynamic characteristics required. The fluid cavity is divided into two chambers with an orifice between, so that motion of the elastomeric element causes fluid to flow from one chamber to the other, dissipating energy (and thus creating damping in the system).

Installations that require a soft isolator for good isolation may also require motion control under transient (shock) inputs or when operating close to the isolation system's resonant frequency. For good isolation, low damping is required. For motion control, high damping is required. Fluid-damped isolators accommodate these conflicting requirements. A hydraulically damped vibration isolator can also act as a tuned absorber by increasing the length of the orifice into an inertia track because the inertia of the fluid moving within the isolator acts as a tuned mass at a specific frequency (which is determined by the length of the orifice). This feature can be used where vibration isolation at a particular frequency is required.

PLASTIC ISOLATORS

Isolators fabricated of resilient plastics are available and have performance characteristics similar to those of the rubber-to-metal type of isolators of equivalent configuration. The structural elements are manufactured from a rigid thermoplastic and the resilient element from a thermoplastic elastomer. These elements are compatible in the sense that they are capable of being bonded one to another by fusion. The most commonly used materials are polystyrene for the structural elements and butadiene styrene for the resilient elastomer. The advantages of this type of spring are (1) low cost, (2) exceptional uniformity in dynamic performance and dimensional stability, and (3) ability to maintain close tolerances. The disadvantages are (1) limited temperature range, usually from a maximum of about 180°F (82°C) to a minimum of –40°F (–40°C), (2) creep of the elastomer element at high static strains, and (3) the structural strength of the plastic.

METAL SPRINGS

Metal springs are commonly used where large static deflections are required, where temperature or other environmental conditions make elastomers unsuitable, and (in some circumstances) where a low-cost isolator is required. Pneumatic (air) springs provide unusual advantages where low-frequency isolation is required; they can be used in many of the same applications as metal springs, but without certain disadvantages of the latter. Metal springs used in shock and vibration control are usually categorized as being of the following types: helical springs (coil springs), ring springs, Belleville (conical or conical-disc) springs, involute (volute) springs, leaf and cantilever springs, and wire-mesh springs.

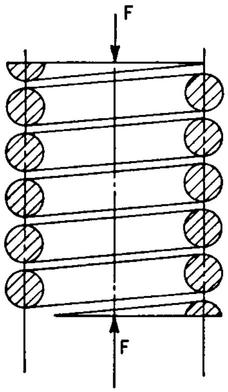

FIGURE 32.3 Cross section of a helical spring showing the direction of the applied force *F*.

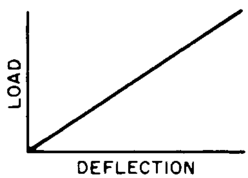

FIGURE 32.4 Load-deflection curve for a helical spring.

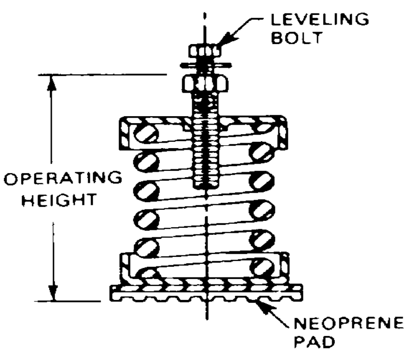

FIGURE 32.5 Helical spring isolator for mounting machinery.

Helical Springs (Coil Springs). Helical springs (also known as coil springs) are made of bar stock or wire coiled into a helical form, as illustrated in Fig. 32.3. The load is applied along the axis of the helix. In a *compression spring* the helix is compressed; in a *tension spring* it is extended. The helical spring has a straight load-deflection curve, as shown in Fig. 32.4. This is the simplest and most widely used energy-storage spring. Energy stored by the spring is represented by the area under the load-deflection curve.

Helical springs have the inherent advantages of low cost, compactness, and efficient use of material. Springs of this type which have a low natural frequency when fully loaded are available. For example, such springs having a natural frequency as low as 2 Hz are relatively common. However, the static deflection of such a spring is about 2.4 in. (61 mm). For such a large static deflection, the spring must have adequate lateral stability or the mounted equipment will tip to one side. Therefore, all forces on the spring must be along the axis of the spring. For a given natural frequency, the degree of lateral stability depends on the ratio of coil diameter to working height. Lateral stability also may be achieved by the use of a housing around the spring which restricts its lateral motion. Helical springs provide little damping, which results in transmissibility at resonance of 100 or higher. They effectively transmit high-frequency vibratory energy and therefore are poor isolators for structure-borne noise paths unless they are used in combination with an elastomer which provides the required high-frequency attenuation, as illustrated in Fig. 32.5.

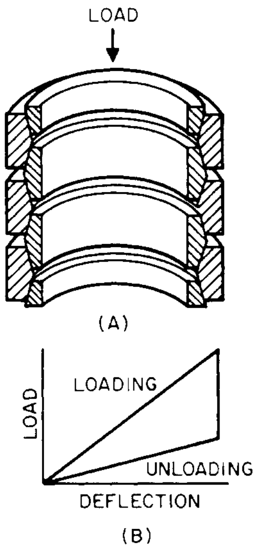

FIGURE 32.6 Ring spring. (*A*) Cross section. (*B*) Load-deflection characteristic when it is loaded and when it is unloaded.

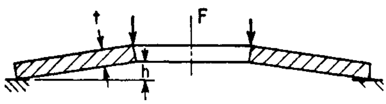

FIGURE 32.7 A Belleville spring made up of a coned disc of thickness *t* and height *h*, axially loaded by a force *F*.

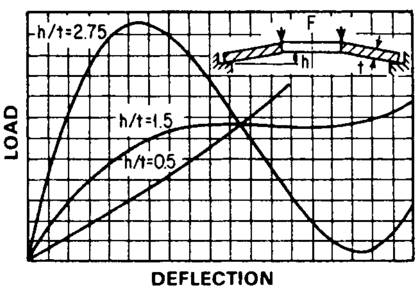

FIGURE 32.8 The load-deflection characteristic for a Belleville spring having various ratios of *h*/*t*.

Ring Springs. A ring spring, shown in Fig. 32.6*A*, absorbs the energy of motion in a few cycles, dissipating it as a result of friction between its sections. With a high load capacity for its size and weight, a ring spring absorbs linear energy with minimum recoil. It has a linear load-deflection characteristic, shown in Fig. 32.6*B*. Springs of this type often are used for loads of from 4000 to 200,000 lb (1814 to 90,720 kg), with deflections between 1 in. (25 mm) and 12 in. (305 mm).

Belleville Springs. Belleville springs (also called coned-disc springs), illustrated in Fig. 32.7, absorb more energy in a given space than helical springs. Springs of this type are excellent for large loads and small deflections. They are available as assemblies, arranged in stacks. Their inherent damping characteristics are like those of leaf springs: Oscillations quickly stop after impact. The coned discs of this type of spring have diametral cross sections and loading, as shown in Fig. 32.7. The shape of the load-deflection curve depends primarily on the ratio of the unloaded cone (or disc) height *h* to the thickness *t*. Some load-deflection curves are shown in Fig. 32.8 for different values of *h*/*t*, where the spring is supported so that it may deflect beyond the flattened position. For a ratio of *h*/*t* approximately equal to 0.5, the curve approximates a straight line up to a deflection equal to half the thickness; for *h*/*t* equal to 1.5, the load is constant within a few percent over a considerable range of deflection. Springs with ratios *h*/*t* approximating 1.5 are known as *constant-load* or *stiffness springs*. Advantages of Belleville springs include the small space requirement in the direction of the applied load, the ability to carry lateral loads, and load-deflection characteristics that may be changed by adding or removing discs. Disadvantages include nonuniformity of stress distribution, particularly for large ratios of outside to inside diameter.

Involute Springs. An involute spring, shown in Fig. 32.9*A* and 32.9*B*, can be used to better advantage than a helical spring when the energy to be absorbed is high and space is rather limited. Isolators of this type have a nonlinear load-deflection characteristic, illustrated in Fig. 32.9*C*. They are usually much more complex in design than helical springs.

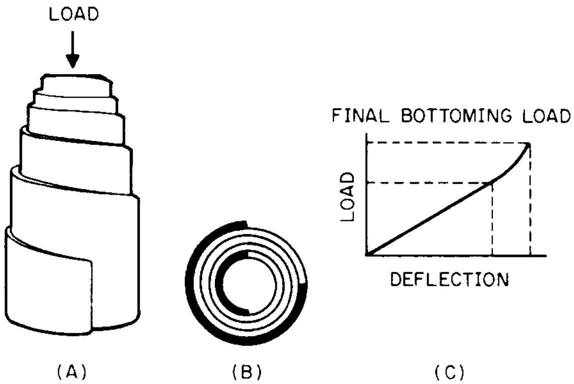

FIGURE 32.9 An involute spring. (*A*) Side view. (*B*) Cross section. (*C*) Load-deflection characteristic.

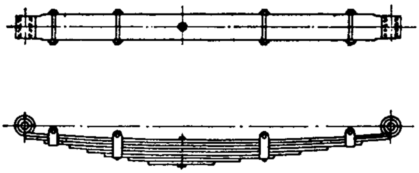

FIGURE 32.10 Semielliptic leaf spring.

Leaf Springs. Leaf springs are somewhat less efficient in terms of energy storage capacity per pound of metal than helical springs. However, leaf springs may be applied to function as structural members. A typical semielliptic leaf spring is shown in Fig. 32.10.

Wire-Mesh Springs. Knitted wire mesh acts as a cushion with high damping characteristics and nonlinear spring constants. A circular knitting process is used to produce a mesh of multiple, interlocking springlike loops. A wire-mesh spring, shown in Fig. 32.11, has a multidirectional orientation of the spring loops, i.e., each

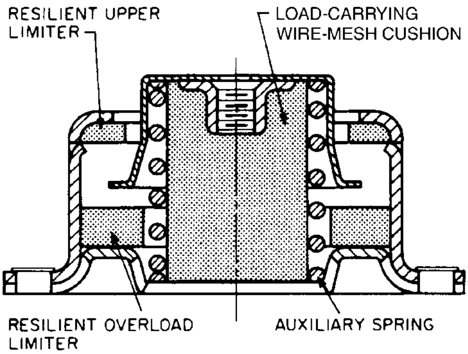

FIGURE 32.11 Wire-mesh spring, shown in section.

loop can move freely in three directions, providing a two-way stretch. Under tensile or compressive loads, each loop behaves like a small spring; when stress is removed, it immediately returns to its original shape. Shock loadings are limited only by the yield strength of the mesh material used. The mesh cushions, enclosed in springs, have characteristics similar to a spring and dashpot.

Commonly used wire mesh materials include such metals as stainless steel, galvanized steel, Monel, Inconel, copper, aluminum, and nickel. Wire meshes of stainless steel can be used outside the range to which elastomers are restricted, i.e., –65 to 350°F (–53 to 177°C); furthermore, stainless steel is not affected by various environmental conditions that are destructive to elastomers. Wire-mesh springs can be fabricated in numerous configurations, with a broad range of natural frequency, damping, and radial-to-axial stiffness properties. Wire-mesh isolators have a wide load tolerance coupled with overload capacity. The nonlinear load-deflection characteristics provide good performance, without excessive deflection, over a wide load range for loads as high as four times the static load rating.

Stiffness is nonlinear and increases with load, resulting in increased stability and gradual absorption of overloads. An isolation system has a natural frequency proportional to the ratio of stiffness to mass; therefore, if the stiffness increases in proportion to the increase in mass, the natural frequency remains constant. This condition is approached by the load-deflection characteristics of mesh springs. The advantages of such a nonlinear system are increased stability, resistance to bottoming out of the mounting system under transient overload conditions, increased shock protection, greater absorption of energy during the work cycle, and negligible drift rate. Critical damping of 15 to 20 percent at resonance is generally considered desirable for a wire-mesh spring. Environmental factors such as temperature, pressure, and humidity affect this value little, if at all. Damping varies with deflection: high damping at resonance and low damping at higher frequencies.

AIR (PNEUMATIC) SPRINGS

A pneumatic spring employs gas as its resilient element. Since the gas is usually air, such a spring is often called an air spring. It does not require a large static deflection; this is because the gas can be compressed to the pressure required to carry the load while maintaining the low stiffness necessary for vibration isolation. The energy-storage capacity of air is far greater per unit weight than that of mechanical spring materials, such as steel and rubber. The advantage of air is somewhat less than would be indicated by a comparison of energy-storage capacity per pound of material because the air must be contained. However, if the load and static deflection are large, the use of air springs usually results in a large weight reduction. Because of the efficient potential energy storage of springs of this type, their use in a vibration-isolation system can result in a natural frequency for the system which is almost 10 times lower than that for a system employing vibration isolators made from steel and rubber.

An air spring consists of a sealed pressure vessel, with provision for filling and releasing a gas, and a flexible member to allow for motion. The spring is pressurized with a gas which supports the load. Air springs generally have lower resonance frequencies and smaller overall length than mechanical springs having equivalent characteristics; therefore, they are employed where low-frequency vibration isolation is required. Air springs may require more maintenance than mechanical springs and are subject to damage by sharp and hot objects. The temperature limits are also restricted compared to those for mechanical springs.

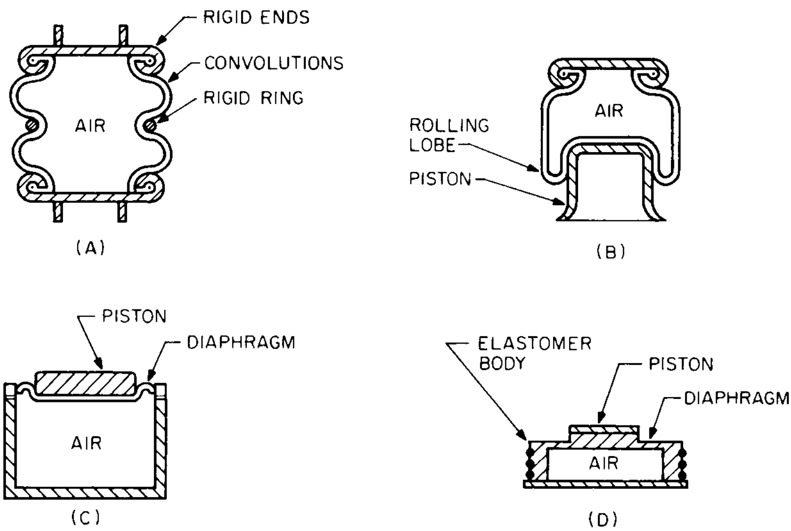

FIGURE 32.12 Four common types of air springs. (*A*) Air spring with convolutions. (*B*) A rolling lobe air spring. (*C*) Rolling diaphragm air spring. (*D*) Air spring having a diaphragm and an elastomeric sidewall.

Figure 32.12 shows four of the most common types of air springs. The air spring shown in Fig. 32.12*A* is available with one, two, and three convolutions. It has a very low minimum height and a stroke that is greater than its minimum height. The rolling lobe (reversible sleeve) spring shown in Fig. 32.12*B* has a large stroke capability and is used in applications which require large axial displacements, as, for example, in vehicle applications. The isolators shown in Fig. 32.12*A* and *B* may have insufficient lateral stiffness for use without additional lateral restraint. The rolling diaphragm spring shown in Fig. 32.12*C* has a small stroke and is employed to isolate low-amplitude vibration. The air spring shown in Fig. 32.12*D* has a low height and a small stroke capability. The thick elastomer sidewall can be used to cushion shock inputs.

The load F that can be supported by an air spring is the product of the gage pressure P and the effective area S (i.e., $F = PS$). For a given area, the pressure may be adjusted to carry any load within the strength limitation of the cylinder walls. Since the cross section of many types of air springs may vary, it is not always easy to determine. For example, the spring shown in Fig. 32.12*A* has a maximum effective area at the minimum height of the spring and a smaller effective area at the maximum height. The spring illustrated in Fig. 32.12*B* is acted on by a piston which is contoured to vary the effective area. In vehicle applications this is often done to provide a low spring stiffness near the center of the stroke and a higher stiffness at both ends of the stroke in order to limit the travel. The effective areas of the springs illustrated in Fig. 32.12*C* and *D* are usually constant throughout their stroke; the elastomeric diaphragm of the spring shown in Fig. 32.12*D* adds significantly to its stiffness. Air springs are commercially available in various sizes that can accommodate static loads that range from as low as 25 lb (11.3 kg) to as high as 100,000 lb (45,339 kg) with a usable temperature range of from –40 to 180°F (–40 to 83°C). System natural frequencies as low as 1 Hz can be achieved with air springs.

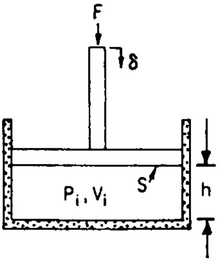

FIGURE 32.13 Illustration of a single-acting air spring consisting of a piston and a cylinder.

Stiffness. The stiffness of the air spring of Fig. 32.13 is derived from the gas laws governing the pressure and volume relationship. Assuming adiabatic compression, the equation defining the pressure-volume relationship is

$$PV^n = P_i V_i^n \tag{32.1}$$

where P_i = absolute gas pressure at reference displacement
V_i = corresponding volume of contained gas
n = ratio of specific heats of gas, 1.4 for air

If the area S is constant, and if the change in volume is small relative to the initial volume V_i [i.e., if $S\delta$ (where δ is the dynamic deflection) $\ll V_i$)], then the stiffness k is given by

$$k = \frac{nP_iS^2}{V_i} \tag{32.2}$$

Transverse Stiffness. The transverse stiffness (i.e., the stiffness to laterally applied forces) of the air springs illustrated in Fig. 32.12*A* and *B* varies from very small to moderate; the natural frequencies for such springs vary from 0 to 3 Hz. The spring illustrated in Fig. 32.12*C* has a higher transverse stiffness, with natural frequencies ranging from 2 to 8 Hz. The spring illustrated in Fig. 32.12*D* has a moderate transverse stiffness; the natural frequency varies in the range from 3 to 5 Hz. If an installation requires the selection of an air spring having insufficient transverse stiffness, additional springs in the transverse direction are often employed for stability, as shown in Fig. 32.14.

At frequencies above 3 Hz, the compression of gases used in air springs tends to be adiabatic and the ratio of specific heats n for both air and nitrogen has a value of 1.4. At frequencies below approximately 3 Hz, the compression tends to be isothermal and the ratio of specific heats n has a value of 1.0, unless the spring is thermally insulated. For thermally insulated springs, the transition from adiabatic to isothermal occurs at a frequency of less than 3 Hz. Gases other than air which are compatible with the air spring materials can also be used. For example, sulfur hexafluoride (SF_6) has a value of n equal to 1.09—a value that reduces the axial spring stiffness by 22 percent; it also has a considerably lower permeation (leakage through the air spring material) rate than air, which may reduce the frequency of recharging (repressurizing) for a closed (passive) air spring.

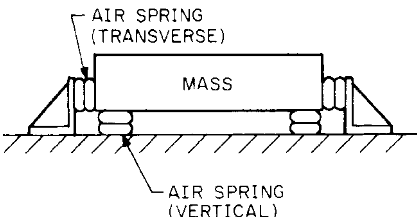

FIGURE 32.14 An air spring used to support a load and provide vibration isolation in the vertical direction. In addition, air springs are provided on the sides to increase the transverse stiffness.

Damping. Air springs have some inherent damping that is developed by damping in the flexible diaphragm or sidewall, friction, damping of the gas, and nonlinearity.

The damping varies with the vibration amplitude; however, it generally is between 1 and 5 percent of critical damping.

Natural Frequency. In U.S. Customary units, the natural frequency f_n of an undamped air spring is expressed by

$$f_n = 3.13\left(\frac{k_1}{W}\right)^{1/2} = \delta_1^{-1/2} \tag{32.3a}$$

where W = supported weight, lb
k_1 = stiffness of the air spring, lb/in.
δ_1 = static deflection, in.

In S.I. units, the natural frequency is given by

$$f_n = \delta_2^{-1/2} \tag{32.3b}$$

where δ_2 = static deflection, cm

ISOLATORS IN COMBINATION

When a number of isolators are used in a system, they are usually combined either in parallel or in series or in some combination thereof.

ISOLATORS IN PARALLEL

Most commonly, isolators are arranged in parallel. Figure 32.15 depicts three isolators schematically as springs in parallel. A number of vibration isolators are said to be in parallel if the static load supported is divided among them so that each isolator supports a portion of the load. If the stiffness of each of the n isolators in Fig. 32.15 is represented by k, the stiffness of the combination is given by

$$\text{Stiffness of } n \text{ isolators in parallel} = nk \tag{32.4}$$

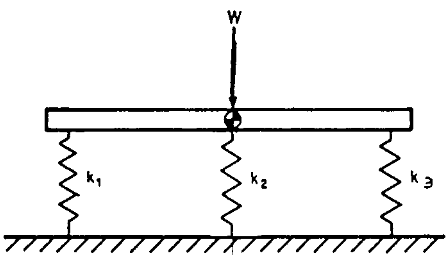

FIGURE 32.15 Schematic diagram of three springs in parallel. Individual spring loads are added to obtain the total weight. With the static load equal on all springs, the static deflection of each spring is the same.

Since in Fig. 32.15 isolator spacing is symmetrical in relation to the center-of-gravity and the same isolator is used at all support points, the stiffness of the combination is 3 times the stiffness of a single isolator, and the static deflection is the same at each isolator.

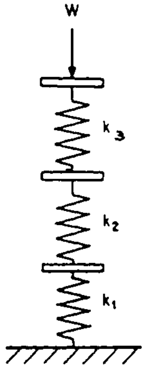

FIGURE 32.16 Schematic diagram of three springs in series. Individual spring deflections are added to obtain total deflection, but each spring carries the total load.

ISOLATORS IN SERIES

When three isolators are combined in series, as shown in Fig. 32.16, the static load is transmitted from one isolator to the next. If the static weight is supported by n isolators in series, each having the stiffness k, the stiffness of the combination is given by

$$\text{Stiffness of } n \text{ isolators in series} = \frac{k}{n} \tag{32.5}$$

Thus, if the mass is supported by three identical isolators (Fig. 32.16), the stiffness of this combination is one-third the stiffness of a single isolator, and the static deflection is the sum of the deflection of the individual isolators (or 3 times the static deflection of a single isolator).

ISOLATOR SELECTION

IMPORTANT FACTORS AFFECTING SELECTION

Stiffness and damping are the basic properties of an isolator which determine its use in a system designed to provide vibration isolation and/or shock isolation. These properties usually are found in isolator supplier literature. However, the following other important factors must be considered in the selection of an isolator:

Type and Direction of Disturbance. The source of a dynamic disturbance (shock or vibration) influences the selection of an isolator in several ways. For example, a decision can be made whether to isolate the source of the disturbance or to isolate the item being disturbed. This decision affects which isolator is to be used. Consider the operation of a heavy punch press which has an adverse effect on a nearby electronic instrument. Isolation of the punch press would reduce this effect but would require fairly large isolators which might have to be resistant to grease or oil. In contrast, isolation of the instrument would also provide the required protection, but the required isolators would be smaller and (since grease or oil would not be a consideration) could be fabricated of a preferred elastomer.

A knowledge of the source of the vibration can aid in defining the problem to be solved. Within a given industry there may be published material describing problems similar to the one under consideration. Such material may describe possible solutions plus equipment fragility, and/or dynamic characteristics of the equipment.

Type of Disturbance. The dynamic environment can be delineated into three categories: (1) periodic vibration—sinusoidal continuos motion or acceleration occurring at discrete frequencies, (2) random vibration—the simultaneous existence of any and all frequencies and amplitudes in any and all phase relationships as exemplified by noise, and (3) transient phenomenon (shock)—a nonperiodic sudden change of velocity, acceleration, or displacement. Usually some combination of these three categories occurs in most isolation systems. A knowledge of the dynamic disturbance is very important in the choice of an isolator. For example, in the case of an instrument supported by isolators, the resilient mounts permit the supported body to "stand still" by virtue of its own inertia while the support structure generates periodic or random vibration. In contrast, shock attenuation involves the storage by the isolators of the dynamic energy which impacts on the support structure and the subsequent release of the energy over a longer period of time at the natural frequency of the system. If only a vibration disturbance is present, a small isolator normally is suitable since vibration amplitudes usually are small relative to shock amplitudes. If a shock disturbance is the primary problem, then a larger isolator with more internal space for motion is required.

In selecting an isolator, ensure (1) that there is enough deflection capability in the isolator to accommodate the maximum expected motions from the dynamic environment, (2) that the load-carrying capacity of the isolator will not be exceeded; the maximum loads due to vibration and/or shock should be calculated and checked against the rated maximum dynamic load capacity of the isolator, and (3) that there will be no problem as a result of overheating of the isolator or fatigue deterioration due to long-term high-amplitude loading.

Direction of Disturbance. A factor that must be considered in the selection of an isolator is that of the directions (axes) of the disturbance. If the vibration or shock input occurs only in one direction, usually a simple isolator can be selected; its characteristics need be specified along only one axis. In contrast, if the vibration or shock is expected to occur along more than one axis, then the selected isolator must provide isolation (and its characteristics must be specified) along all the critical axes. For example, consider an industrial machine which produces troublesome vibration in the vertical direction and which must be isolated from its supporting structure. In this case, a standard plate-type isolator may be used. This type isolator is stiffer in the horizontal direction than in the vertical direction, which is the axis of the primary disturbance; the horizontal stiffness does not significantly affect the motion of the isolator in the vertical direction. Such horizontal stiffness adds to the lateral stability of the installation.

Allowable Response of a System to the Disturbance. The allowable response of a system is defined as the maximum allowable transmitted shock or vibration and the maximum displacements due to such disturbances. The allowable response of a system can be expressed in any of the following ways:

- Maximum acceleration loading due to a shock input
- Specific system natural frequency and maximum transmissibility at that frequency
- Maximum acceleration, velocity, or displacement allowable over a broad frequency range
- The allowable level of vibration at some critical frequency or frequencies
- Maximum displacement due to shock loading

The maximum acceleration which a piece of equipment can withstand without damage or malfunction is often called *fragility*. The definition of some allowable response is necessary for an appropriate isolator selection. If fragility data are not available for the specific equipment or installation at hand, then examples of similar situations should be used as a starting point. Suppose an isolator were chosen only for its load-carrying capability, with no regard for the fragility of a piece of equipment in a specific frequency range. Then, the natural frequency of the system might be incorrectly placed such that a resonance within the equipment might be excited by the isolation system.

Space and Locations Available for Isolators. Vibration and shock isolation should be considered as early as possible in the design of a system, and an estimate of isolator size should be made based on isolator literature. The size of the isolator depends on the nature and magnitude of the expected dynamic disturbances and the load to be carried. Typical literature describes the capabilities of isolators based on such factors.

The location of isolators is very important to the dynamics of the equipment mounted on them. For example, a center-of-gravity installation, as shown in Fig. 32.17, allows the mounted equipment to move only in straight translational modes (i.e., a force at the center-of-gravity does not cause rotation of the equipment). This

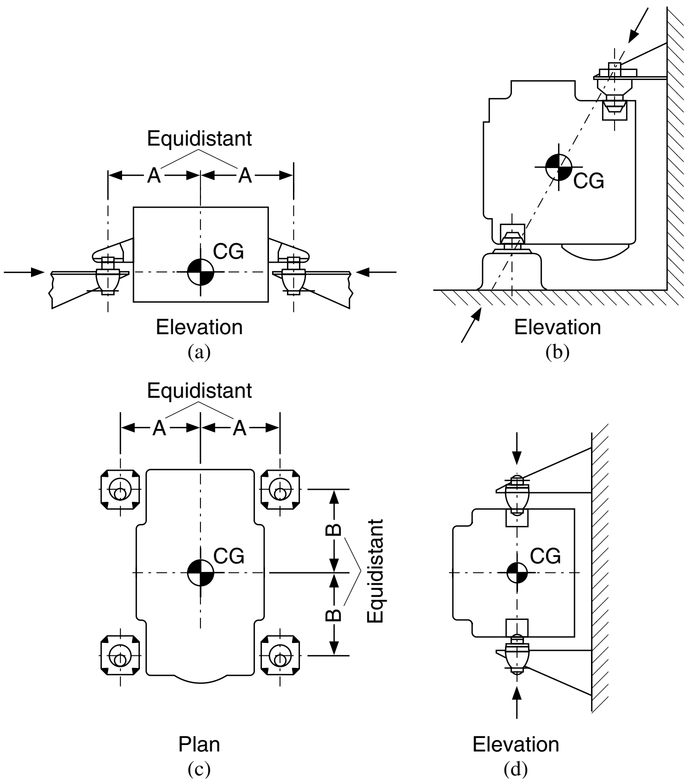

FIGURE 32.17 Center-of-gravity installations of vibration isolators: (*A*) Center-of-gravity horizontal support. (*B*) Center-of-gravity diagonal support. (*C*) Symmetrical spacing about the center-of-gravity. (*D*) Center-of-gravity vertical support. (*After Davey and Payne.*[2])

minimizes the motion of the corners of the equipment and allows the most efficient installation from the standpoint of space requirements and isolation efficiency.

If the isolators cannot be located so as to provide a center-of-gravity installation, then the system analysis is more difficult and more space must be allowed around the equipment to accommodate rocking motion (i.e., rotational modes) of the system. Finally, the isolators must be double-checked to ensure that they are capable of withstanding the additional loads and motions from the nontranslational movement of the equipment. This is particularly true when the center-of-gravity is a significant distance above or below the plane in which the isolators are located. Rule of thumb: The distance between the isolator plane and the center-of-gravity should be equal to or less than one-third of the minimum spacing between isolators. This helps to minimize rocking of the equipment and the resultant high stress in the isolators.

Weight and Center-of-Gravity of Supported Equipment. The weight and location of the center-of-gravity of the supported equipment should be determined. The location of the center-of-gravity is necessary for calculating the load supported on each mount. It is best to keep the equipment at least satatically balanced [essentially equal deflections on all isolators (see Fig. 32.17)]. The preferred approach is to use the same isolator at all points, choosing isolator locations such that static loads (and thus deflections) are equalized. If this is not practical, isolators of different load ratings may be required at different support points on the equipment for optimum isolation. The size of the equipment and the mass distribution are important in dynamic analyses of the isolated system.

Space Available for Equipment Motion. The choice of an isolator may depend on the space available (commonly called sway space) around a piece of equipment. The spring constant of the isolator should be chosen carefully so that motion is kept within defined space limits. The motion which must be considered is the sum of (1) the static deflection due to the weight supported by the isolator, (2) the deflection caused by the dynamic environment, and (3) the deflection due to any steady-state acceleration (such as in a maneuvering aircraft).

If there is a problem of excessive motion of the supported mass on the isolator, then a *snubber* (i.e., a device which limits the motion) can be used. A snubber may be an elastomeric compression element designed into an isolator. Captive-type isolators (see *Fail-Safe Installation*) have built-in motion-limiting stops. Also, elastomers stressed in compression have natural snubbing due to the nonlinear load-deflection characteristics. In some cases it may be necessary to limit motion by separately installed snubbers such as a compression pad at the point of excessive motion as shown in Fig. 32.18. The spring constant of such a snubber must be carefully selected to avoid transmission of high-impact loads into the supported equipment.

Ambient Environment. The environment in which an isolator is to be used affects its selection in two ways:

1. Some environmental conditions may degrade the physical integrity of the isolator and make it nonfunctional.
2. Some environmental conditions may change the operating characteristics of an isolator, without causing permanent damage.

This may alter the characteristics of the isolation system of the supported equipment; for example, frequency responses could change significantly with changes in

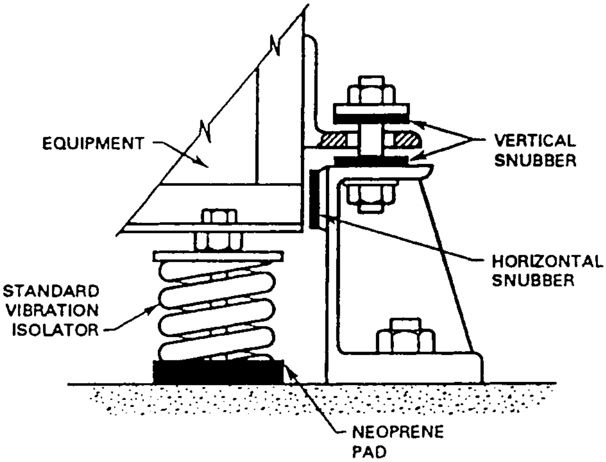

FIGURE 32.18 A vibration isolator provided with auxiliary elastomeric snubbers to limit the motion of the isolator in the horizontal and vertical directions; these snubbers provide a "cushion" stop to provide a lower shock force on the equipment than would be experienced with a metal-to-metal stop.

the ambient temperature. Thus, it is important to determine the operating environment of the isolation system and to select isolators that will function with desired characteristics in this environment.

Available Isolator Materials. Vibration and shock isolators are available in a wide variety of materials and configurations to fit many different situations. The type of isolator is chosen for the load and dynamic conditions under which it must operate. The material from which the isolator is made depends to a great extent on the ambient environment of an application and somewhat on the dynamic properties required. Guidance for the choice of isolator materials is given earlier in this chapter. Chapter 33 describes the engineering properties of rubber.

Metal-spring isolators are used primarily where operating temperatures are too high for elastomeric isolators. They can be used in a variety of applications.

By far, the majority of isolators in use today are elastomeric. The development of a vast array of elastomeric compounds has made it possible to use this type of isolator in almost any environment. Within a given type of elastomer, it is a simple matter to vary the stiffness (modulus, durometer) of the compound; this gives much flexibility in adapting an isolator to an application without changing the isolator's geometry.

Since the selection of material for an isolator depends so much on the environment in which the mount will be used, it is very important to learn as much as possible about the operating and storage environments.

Desired Service Life. The expected, or desired, length of service for an isolator can affect the type and size of the vibration isolator which is selected. For example, an isolator which must operate for 2000 hours under a given set of conditions typically is larger than one which must operate for only 500 hours under the same conditions.

In general, empirical data are used to estimate the operating life of an isolator. Accurate descriptions of the dynamic disturbances and ambient operating environment expected are needed to make an estimate of isolator life. A knowledge of the specific material and design factors in an isolator is necessary to make an estimate of fatigue life. Such information is best provided by the original manufacturer or designer of the isolator.

Requirement for Fail-Safe Operation. Many pieces of equipment must be mounted on isolators on which the equipment remains supported (in place) in the event of mechanical failure of the isolator, i.e., until it can be replaced. This feature may be provided by a metal-to-metal interlock, or it may be provided by snubbers, as illustrated in Fig. 32.18. A *snubber* is a component in a resilient isolator which limits the displacement of the isolator in the event of its failure.

Interaction with Support Structure. The support structure characteristics can also affect the selection of isolators. An isolator must deflect if it is to isolate vibration; generally the greater the deflection, the greater the isolation. The isolator functions by being soft enough to allow relative vibration amplitudes without transmitting excessive force to the support structure. It is often assumed, in the selection of vibration isolators, that the support structure is a rigid mass with infinite stiffness. This assumption is not true since if the foundation were infinitely stiff, it would not respond to a dynamic force and the isolator would not be needed. Since the foundation does respond to dynamic forces, its response must affect the components that are flexibly attached to it. In reality the support structure is a spring in series with the isolator (see *Isolators in Combination* above) and springs in series carry the same force and deflect proportionally to their respective spring constants. Thus if the stiffness of the isolator is high compared to the stiffness of the foundation, the foundation will deflect more than the isolator and actually nullify or limit the isolation provided from the isolator itself. To achieve maximum efficiency from the selected isolator, the spring constant of the support structure should be at least 10 times that of the spring constant of the isolator attached to it. This will assure that at least 90 percent of the total system spring constant is contributed by the isolators and only 10 percent by the support structure.

Because the structure supporting a piece of equipment has inherent flexibility, it has resonances which could cause amplification of vibration levels; these resonance frequencies must be avoided in relation to isolated system natural frequencies.

HOW TO SELECT ISOLATORS

The isolator selection process should proceed in the following steps:

Step 1. *Required isolation efficiency.* First, indicate the percentage of isolation efficiency that is desired. In general, an efficiency of 70 to 90 percent is desirable and is usually possible to attain.

Step 2. *Transmissibility.* From Table 32.1 determine the maximum transmissibility T of the system at which the required vibration isolation efficiency of Step 1 will be provided.

Step 3. *Forcing frequency.* Determine the value of the lowest forcing frequency f (i.e., the frequency of vibration excitation). For example, in the case of a motor, the

TABLE 32.1 Ratio of (f/f_n) Required to Achieve Various Values of Vibration Isolation Efficiency

Isolation efficiency, %	Maximum transmissibility	Required f/f_n
90	0.1	3.32
80	0.2	2.45
70	0.3	2.08
60	0.4	1.87
50	0.5	1.73
40	0.6	1.63
30	0.7	1.56
20	0.8	1.50
10	0.9	1.45
0	1.0	1.41

forcing frequency depends on the rotational speed, given in revolutions per minute (rpm); the rotational speed must be divided by 60 sec/minute to obtain the forcing frequency in cycles per second (Hz). The lowest forcing frequency is used because this is the worst condition, resulting in the lowest value of f/f_n (see Table 32.1). If a satisfactory value of isolation efficiency is attained at this frequency, the vibration reduction at higher frequencies will be even greater.

Step 4. *Natural frequency.* From Fig. 32.19, find the natural frequency f_n of the isolated system (i.e., the mass of the equipment supported on isolators) required to provide a transmissibility T, determined in Step 2 (which is equivalent to a corresponding percent vibration isolation efficiency) for a forcing frequency of f Hz (determined in Step 3).

Step 5. *Static deflection.* From Fig. 32.19, determine the static deflection required to provide a natural frequency of Step 4.

Step 6. *Stiffness of isolation system.* From Eq. (32.6), calculate the stiffness k required to provide a natural frequency f_n determined in Step 4:

$$f_n = \frac{[kg/W]^{1/2}}{2\pi} \tag{32.6}$$

where W = the weight in pounds of the supported mass
g = the acceleration due to gravity in inches per second per second

Step 7. *Stiffness of the individual vibration isolators.* Determine the stiffness of each of the n isolators from Eq. (32.4) or Eq. (32.5) depending on whether the vibration isolators are in parallel or in series. In general, they are in parallel so that the required stiffness of each vibration isolator is $1/n$ times the value obtained in Step 6—assuming that all isolators share the load equally.

Step 8. *Load on individual vibration isolators.* Now calculate the load on each individual isolator.

Step 9. *Isolator selection.* From a manufacturer's catalog, elect a vibration isolator which meets the stiffness requirement determined in Step 7 and which has a load-carrying capacity (i.e., load rating) equal to the value obtained in Step 8. The preferred approach is to use the same type and size isolator at all points of support;

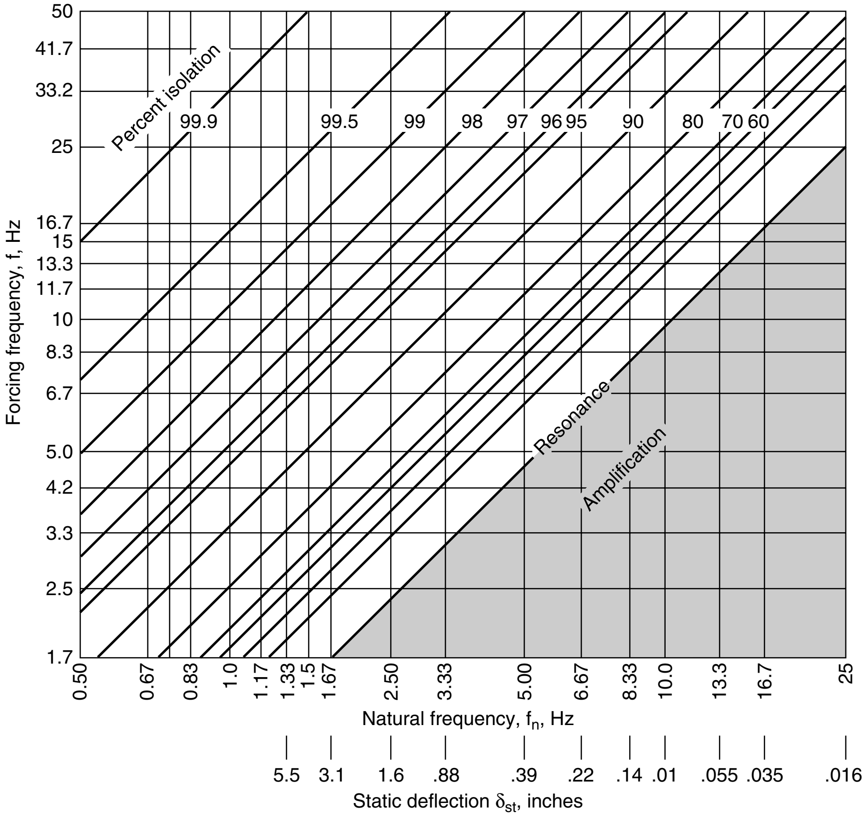

FIGURE 32.19 Isolation efficiency chart. The vibration efficiency, in percent, is given as a function of natural frequency of the isolated system (along the horizontal axis) and the forcing frequency, i.e., the frequency of excitation (along the vertical axis). The use of this chart is restricted to applications where the vibration isolators are supported by a floor structure having a vertical stiffness of at least 15 times the total stiffness of the isolation system. This may require that the isolated structure be placed along the length of a floor beam or that an additional floor beam be added to the structure.

choose isolator locations such that static loads (and thus deflections) are equalized. If this is not practical, isolators of different load ratings may be required at different support points on the equipment. If the vibration occurs only in one direction, usually a simple isolator can be selected; its characteristics need be specified along only one axis. In contrast, if the vibration is expected to occur along more than one direction, then the selected isolator must provide isolation along all the critical axes.

EXAMPLES

The following examples present specific applications. They show how isolators may be selected for some simple shock and vibration problems, but the steps used are basic and can be extended to many other situations. In the solution of these problems, the following simplifying assumptions are made:

1. The effect of damping is negligible, a valid assumption for many isolator applications.
2. All modes of vibration are uncoupled, i.e., the isolators are symmetrically located with respect to the mass center-of-gravity.
3. The static and dynamic spring constants of the isolators are equal, valid for low modulus elastomers with little damping.

Example 32.1: Vibration Isolation. When a shock or vibration disturbance originates in the supported equipment, isolators which support the equipment reduce the transmission of force to the supporting structure, thus protecting the structure or foundation, for example, in isolating a vehicle chassis from the vibration of an internal combustion engine or in reducing the transmission of machine vibration to adjacent structures.

Problem. An electric motor and pump assembly, rigidly mounted on a common base, rotates at a speed of 1800 rpm and transmits vibration to other components of a hydraulic system. The weight of the assembly and base is 140 lb (63 kg). Four isolators are to be located at the corners of the rectangular base. The center-of-gravity is centrally located in the horizontal plane near the base. The lowest vibratory forcing frequency is 1800 rpm and is a result of rotational unbalance. There also are higher frequencies due to magnetic and pump forces. The excitation is in both the horizontal and vertical directions.

Objective. To reduce the amount of vibration transmitted to the supporting structure and thus to other system components. A vibration isolation efficiency of 70 to 90 percent is usually possible to attain.

Solution:

1. Select a vibration isolation efficiency midway between 70 and 90 percent, i.e., 80 percent.

2. Find the transmissibility T which corresponds to an isolation efficiency of 80 percent. From Eq. (32.7) or Fig 32.19, this is a value of $T = 0.2$.

$$\text{Isolation efficiency} = 100(1 - T) \qquad \text{in percent} \tag{32.7}$$

where T = transmissibility.

3. Determine the lowest forcing frequency f by dividing the rotational speed in rpm by 60, yielding a value of 30 Hz.

4. Next calculate the natural frequency f_n required to provide the transmissibility $T = 0.2$ for a forcing frequency $f = 30$ Hz. According to Eq. (32.8), this is a value of 12.2 Hz.

$$T = \frac{1}{(f/f_n)^2 - 1} \tag{32.8}$$

where f = the forcing frequency (also called disturbing frequency) in Hz
f_n = system natural frequency in Hz

5. Then calculate the static deflection required to provide a natural frequency of 12.2 Hz. According to Eq. (32.9), this is a value of $\delta_{st} = 0.066$ in. (1.67 mm).

$$f_n = \frac{3.13}{(\delta_{st})^{1/2}} \tag{32.9}$$

where δ_{st} = the static deflection in inches
δ_{st} = 0.066 in. (1.67 mm)

[The same results may be obtained by using the isolation efficiency chart, Fig. 32.19, as follows. Find the point at which the horizontal line for a forcing frequency $f = 30$ Hz intersects the diagonal line for an isolation efficiency of 80 percent. From the point of intersection, project a vertical line to read the values of $\delta_{st} = 0.066$ in. (1.67 mm).]

6. Determine the stiffness of the required isolation system (i.e., combination of four isolators) required to provide a natural frequency of f_n. According to Eq. (32.6), the value of stiffness of the system, for a weight of 140 lb, is 2120 lb/in. (371 N/mm).

7. Calculate the stiffness of individual isolators if one is placed in each corner by dividing the value for the combination of isolators by 4, since all four support the load.

8. The load on the individual isolator is equal to the total weight of the load divided by the number of supporting isolators, i.e., 140/4 = 35 lb (15.8 kg) per isolator.

Example 32.2: Shock Isolation. Mechanical shock may be transmitted through a supporting structure to equipment, causing it to move. The transmitted motion and force are reduced by mounting the equipment on isolators, for example, to protect equipment from impacts during shipment.

Problem. A business machine is to be isolated so that it will not experience damage during normal shipping. The unit can withstand 25g of shock without damage. The suspended weight of 125 lb (56.2 kg) is to be equally distributed on four isolators. The disturbances expected are those from normal transportation handling, with no damage allowed after a 30-in. (762-mm) flat bottom drop. The peak vibration disturbances are normally in the range of 2 to 7 Hz.

Objective. To limit acceleration on the machine to 25g using the drop test as a simulation of the worst expected shock conditions. A natural frequency between 7 and 10 Hz is desired to avoid the peak vibration frequency range and still provide good shock protection.

Solution:

1. First, solve for the dynamic deflection δ_d (displacement) of the machine required to limit acceleration to X_F (expressed in g's) when the item is dropped from a height ($h = 30$ in.) using:

$$\delta_d = \frac{2h}{\ddot{x}_F} \tag{32.10}$$

Here $\ddot{x}_F$ = the fragility factor = 25g, so that $\delta_d = 2.4$ in. (61 mm).

2. Then determine the required *dynamic* natural frequency f_n to result in a dynamic deflection δ_d from Eq. (32.11), using a fragility $\ddot{x}_F = 25g$, $h = 30$ in., $W = 125$ lb (56.2 kg), and $\delta_d = 2.4$ in.:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{\ddot{x}_F g}{\delta_d}} \tag{32.11}$$

$f_n = 10$ Hz (a value also given by use of Fig. 32.19).

3. Calculate the system *dynamic* spring constant k required to provide a dynamic deflection δ_d from:

$$k = \frac{\text{force}}{\text{deflection}} = \frac{\ddot{x}_F W}{\delta_d} \tag{32.12}$$

$k = 1302$ lb/in. (228 N/mm) for the system.

4. From Eq. (32.4), calculate the system *static* spring constant of the n natural rubber isolators (for which the static and dynamic values are approximately equal). Here $n = 4$, yielding a stiffness value of k for each individual isolator of 325 lb/in. (56.9 N/mm).

5. Since the total weight is distributed equally on four identical isolators, the load per isolator is 125 lb divided by 4 or 31 lb (14 kg).

6. Sandwich-type isolators are often used to protect fragile items during shipment. The construction is typically two flat plates, bonded on either side of an elastomeric pad. Determine the minimum thickness of the elastomer (between the plates) needed to keep dynamic strain at an acceptable level. Use the following rule of thumb for rubber:

$$t_{\min} = \frac{\delta_d}{1.5} \tag{32.13}$$

For $\delta_d = 2.4$ in. (61 mm), the minimum elastomer thickness is 1.6 in. (40.6 mm).

7. Now choose a sandwich isolator for this application. Sandwich configuration permits sufficient deflection in two directions (shear) to absorb high shock loads. Sandwich isolators are readily available in a wide range of sizes, spring constants, and elastomers. From a catalog, select a part that has the capacity to support a static shear load of 31 lb (14 kg), has a minimum elastomer thickness of 1.6 in. (40.6 mm), and has a shear spring constant of 325 lb/in. (56.9 N/mm).

8. In designing or choosing the container, certain criteria must be considered. The four isolators should be installed equidistant from the center-of-gravity in the horizontal plane, oriented to act in shear in the vertical and fore-and-aft directions. The isolators should be attached on one end to a cradle which carries the machine and on the other end to the shipping container. There must be enough space allowed between the mounted unit and the container to prevent bottoming (contact) at impact, allowing a clearance space of at least $1.4\delta_d$.

Example 32.3: Combined Shock and Vibration Isolation

Problem. A portable engine-driven air compressor, with a total weight of 2500 lb (1126 kg), is noisy in operation. An isolation system is required to isolate engine disturbances and to protect the unit from over-the-road shock excitation.

The engine and compressor are mounted on a common base which is to be supported by four isolators. The weight is not equally distributed. At the engine end the static load per isolator is 750 lb (338 kg); at the compressor end the static load per isolator is 500 lb (225 kg). The lowest frequency of the disturbance is at engine speed. The idling speed is 1400 rpm, and the operating speed is 1800 rpm. The unit is expected to be subjected to shock loads due to vehicle frame twisting when transported over rough roads.

Objective. To control force excitation vibration and provide secondary shock isolation. A compromise is required; the isolation system must have a stiffness that is low enough to isolate engine idling disturbance but high enough to limit shock motion. A system having a natural frequency of 12 to 20 Hz in the vertical direction is usually adequate. (Note: The tires and basic vehicle suspension will provide the primary shock protection.)

Solution:

1. First assume that the natural frequency of the system in the vertical direction is 12 Hz.

2. Next, convert the engine speeds to hertz (cycles per second) for use in the calculations. Divide the rpm values by 60 sec/min, yielding force frequencies f of 23.3 Hz at idling speed and 30 Hz at operating speed.

3. Then calculate the transmissibility T for $f_n = 12$ Hz from Eq. (32.8). At idling speed, using $f = 23.3$ Hz, yields $T = 0.36$ (36 percent). At operating speed, using $f = 30$ Hz, yields $T = 0.19$ (19 percent). Table 33.1 gives a vibration isolation T of 0.64 (64 percent) at idling speed and 0.81 (81 percent) under normal operation. For both conditions, performance with a natural frequency of 12 Hz is satisfactory.

4. Now determine the required static deflection δ_{st} to provide a natural frequency f_n from Eq. (32.9). For $f_n = 12$ Hz, this yields $\delta_{st} = 0.068$ in. (1.73 mm).

5. Select a general-purpose isolator (see Fig. 32.1*E*) for both ends of the unit. This type of isolator is simple and rugged and gives protection against shock loads expected here. It should be installed so that the axis of the bolt is vertical and the static weight rests on the disk portion. This isolator provides cushioning against upward (rebound) shock loads as well as against downward loads, and the isolation system is fail-safe. Each of the two isolators at the engine end should have a static load-carrying capacity of at least 750 lb (338 kg). Each of the two isolators at the compressor end should be able to support at least a 500-lb (225-kg) static load. For all isolators the static deflection should be close to 0.068 in. (1.73 mm) to give the desired natural frequency of 12 Hz.

AVOIDING ISOLATOR INSTALLATION PROBLEMS

There are usually two primary causes for unsatisfactory performance of an isolation system: (1) The isolator has been selected improperly or some important system parameter has been overlooked and (2) the isolator has been installed improperly. The following criteria can help obviate problems that can otherwise cause poor performance:

1. Do not overload the isolator, i.e., do not exceed the loading specified by the manufacturer. Overloading may shorten isolator life and affect performance.

2. In the case of coil-spring isolators, provide adequate space between coils at normal static load so that adjacent coils do not touch and there is no possibility of bottoming at the maximum load.

3. In the case of elastomeric compression-type isolators, do not overload the isolator so that it bulges excessively—the ratio of deflection at the static load to the original rubber thickness should not exceed 0.15. As indicated earlier, overloading an isolator may affect its performance. An elastomeric element loaded in compression has a nonlinear stiffness. Therefore, its effective dynamic stiffness (i.e., its effective stiffness when it is vibrating) will be higher than the published value. This raises the natural frequency and reduces its efficiency of isolation.

4. In the case of an elastomeric shear-type isolator, the ratio of the static deflection in shear (i.e., with metal plates moving parallel to one another) to the original thickness usually should not exceed 0.30.

5. To minimize rocking of the equipment and the resultant high stress in the isolators, the distance between the isolator plane and the center-of-gravity should be equal to or less than one-third of the minimum spacing between isolators.

6. The isolators and isolated equipment should be able to move freely under vibration and shock excitation. No part of the isolation system should be short-circuited by a direct connection rather than a resilient support.

7. The vibrating equipment should not contact adjacent equipment or a structural member. Space should be provided to avoid contact.

8. If an elastomeric pad has been installed beneath a machine, the resilient pad should not be short-circuited by hard-bolting the machine to its foundation.

9. The load on the isolator should be along the axis designed to carry the load. The isolator should not be distorted. Unless the isolator has built-in misalignment capability, installation misalignment can affect performance and shorten isolator life.

10. If an elastomeric mount is used, provide adequate clearance so that there is no solid object cutting the elastomer. There should be no evidence of bond separation between the elastomer and metal parts in the isolator. Cuts and tears in the elastomer surface can propagate during operation and destroy the spring element. If there are bonded surfaces in the isolator, a bond separation also can cause problems; growth in the separation can affect the performance of the isolator and ultimately cause failure.

11. The static deflection of all isolators should be approximately the same. There should be no evidence of improper weight distribution. Excessive tilt of the mounted equipment may affect its performance. For economic reasons and simplicity in installation, it is desirable to use the same isolator at all points in the system. In such a case, it is not usually a problem if the various isolators have slightly unequal static deflections. However, if one or more isolators exhibit excessive deflection, then corrective measures are required. If the spacing between isolators has been determined improperly, a correction of the spacing to equalize the load may be all that is required. If this is impractical, an isolator having a higher spring constant can be used at points supporting a higher static load. This will tend to equalize deflection.

SHOCK AND VIBRATION ISOLATOR SPECIFICATIONS

Often, shock and vibration isolators are overspecified; this can cause needless complication and increased cost. Overspecification is the practice of arbitrarily increasing shock or vibration load values to be safe (to make certain that the isolators have been chosen with a high margin of safety at the maximum load capability). The best isolator specification is one which defines the critical properties of the isolation system and the specific environment in which the system will operate. Extraneous requirements cause needless complications. For example, if the vibration level is an acceleration of $+1g$, it is not advisable to specify $+2g$ to be safe. Likewise, it is inadvisable to rigidly apply an entire specification to an isolator installation when only a small part of the specification is applicable.

Typically, specifications to which vibration and shock isolators are designed will include requirements regarding (1) vibration amplitudes, (2) shock amplitudes, (3) load to be carried, (4) required protection for equipment, (5) temperatures to be encountered (environmental factors, in general), and (6) steady acceleration loads superimposed on dynamic loading.

ACTIVE VIBRATION CONTROL SYSTEMS

The preceding sections of this chapter consider only *passive* vibration control systems and their components; active vibration control systems differ significantly from such conventional vibration control systems. An *active vibration control system* is a system in which one or more sensors is required to measure the absolute value or change in a physical quantity (such as position, motion, temperature, etc.); then such a change is converted to a signal used to modify the behavior of the system. Such modification requires the addition of external power, in contrast to a conventional (passive) vibration control system which does not require the addition of external power or the use of sensors. But in special cases, these additional complications, required in an active vibration control system, may be outweighed by benefits that can otherwise not be obtained with a conventional system, as illustrated in the following examples.

AN ACTIVE SYSTEM FOR RESILIENTLY SUPPORTING A BODY AT GIVEN POSITION DESPITE VARIATIONS IN THE APPLIED LOAD

Consider the active vibration control system shown in Fig. 32.20. A mass m is supported by a spring of stiffness k, with a damping coefficient c. Force F is slowly applied to the mass, as illustrated, causing the spring to stretch, resulting in a downward displacement δ of the mass. A sensor responds to the displacement, causing it to generate a signal proportional to the relative motion of the system. As a result, power is supplied by a servo-controlled motor that moves the supporting frame upward until the body returns to its original position with respect to the supporting plane. This active vibration control system thus maintains the supported body in its equilibrium position, despite the applied load, until another change in the force occurs. Thus there is zero displacement of the mass in the presence of a constant force F. This is a type of *negative feedback regulation,* so called because the servo-controlled motor applies a "feedback" force to the supported body which opposes its movement. A feedback control system is a system in which the value of some output quantity is controlled by feeding back the value of the controlled quantity and using it to manipulate an input quantity to bring the controlled quantity closer to the desired value. (In contrast, a *feedforward control system* is a system in which changes are detected at the process input, and anticipating correction is made before the process output is affected.)

The active vibration isolation system illustrated in Fig. 32.20 seeks as its equilibrium position a location at a distance h above the reference lane of the support, inde-

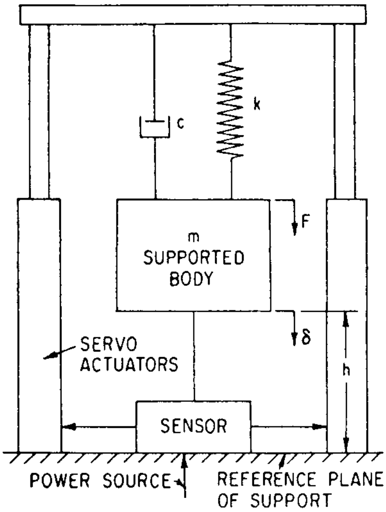

FIGURE 32.20 Schematic diagram of an active vibration-isolation system which maintains the supported body *m* a fixed distance *h* from the reference plane of the support, irrespective of the steady force *F* applied to the supported body.

pendent of the origin and magnitude of a steady force applied to the supported body. While there is no change in the position of the body in response to a very slowly applied load, if the applied load is suddenly removed, the servomechanism (providing the regulation) may be unable to respond fast enough to compensate for the tendency of the supported body to change position relative to the support; then the isolator can experience a significant deflection.

This example demonstrates that where the damped natural frequency of the isolation system must be relatively low, with the additional requirement that the supported body be maintained at a relatively constant distance from the base to which it is attached, the application of an active vibration control system may be of considerable benefit.

Controller Gain; Integral Control; Proportional Control. The computational element for the elimination of the isolator static deflection is that of an integrator and scaling term called a *controller gain.* This combination of sensing, computation, and actuation provides what is known as *integral control,* since the feedback force is proportional to the time integral of the sensor response. The computational elements for the control of the system resonance and low-frequency vibration isolation require only a scaling term. This combination of control elements is called *proportional control,* since the feedback force is proportional to the sensor response. The feedback elements added to a conventional isolation system must have an overall characteristic such that the output force is proportional to the sensed function times the control function of the computational element. The control function describes the operation of the computational element, which can be a simple constant as in proportional control, an integration function as in integral control, or an equation describing the action of one or more electric circuits. This corresponds to a spring which provides an output force proportional to the deflection of the spring, a viscous damper which provides a force proportional to the rate of deflection of the damper, or an electric circuit which produces a force signal proportional to the dynamics of a spring and viscous damper, in series, undergoing a motion proportional to the sensor response.

The sensing and actuation devices which provide integral control of the isolator relative displacement may take many forms. For example, the sensing element which measures the position of the supported body (relative to the reference plane of the support) may be a differential transformer which produces an electrical signal proportional to its extension relative to a neutral position. The sensing element is attached at one end to the supported body and at the other end to the isolator support structure in a manner such that the sensor is in its neutral position when the supported body is at its desired operating height. The electrical signal is integrated and amplified in the computational element, providing electric power to operate an electric motor actuation device. The differential transformer-integrator-motor system produces a force proportional to the integral of the signal from the differential transformer. The operation of this servomechanism can be visualized in the following manner:

1. A force F of constant magnitude F_0 is applied to the supported body, causing a relative deflection of the isolator spring element.
2. The sensing element (in this case a differential transformer) applies an electrical signal that is proportional to the isolator relative displacement to the integration and scaling functions in the computational element.
3. The response of the computational integration function generates an electrical signal that continues to increase in magnitude so long as the relative displacement δ is not zero.
4. The signal from the computational element is applied to the motor element, which generates a force in a direction that decreases the isolator deflection; the motor force follows the computational element signal and continues to increase in magnitude so long as the relative deflection δ is not zero.
5. At some point in time the force from the motor output will exactly equal the constant force F_0, requiring a relative displacement of zero.
6. The output from the differential transformer is zero; thus the output from the computational element integration function no longer increases but is maintained constant at the magnitude required for the motor element to generate a force exactly equal to the constant force F_0 applied to the supported body.

The isolation system remains in this equilibrium condition until the force applied to the supported body changes and causes a nonzero signal to be generated by the sensing element; then the process starts all over again. Alternatively, a proportionally scaled signal from the differential transformer may be used to operate an electromechanical servo valve, the flow response of the servo valve being proportional to its excitation signal. The servo-valve fluid-flow output is directed into the chamber of an air spring to produce the desired force applied to the supported body. The control function remains integral in nature since the actuator's internal pressure responds to the volume output from the servo valve, which is the integral of its flow output. Thus, in this case, no electrical integration of the sensor signal is needed. It is also possible to operate a mechanical servo valve through a direct mechanical coupling in such a way that the motion of the suspended body with respect to its support is used directly to provide the required servo-valve actuation. The possible combinations of elements and control devices are almost limitless. The choice of a suitable combination of sensor, computation element, and actuator is dictated by the type of power available, the supported body size, the weight, and the type of application, e.g., spacecraft, aircraft, automotive, or industrial.

AN ACTIVE SYSTEM FOR CONTROLLING ITS SYSTEM RESONANCE AND LOW-FREQUENCY VIBRATION ISOLATION

The mechanical system shown in Fig. 32.21 provides active control of its system resonance and the vibration isolation it provides at low frequencies. This system consists of a velocity sensor (for example, see Chap. 12), a proportional computational element, and a motor actuation device that also may take on many forms. The veloc-

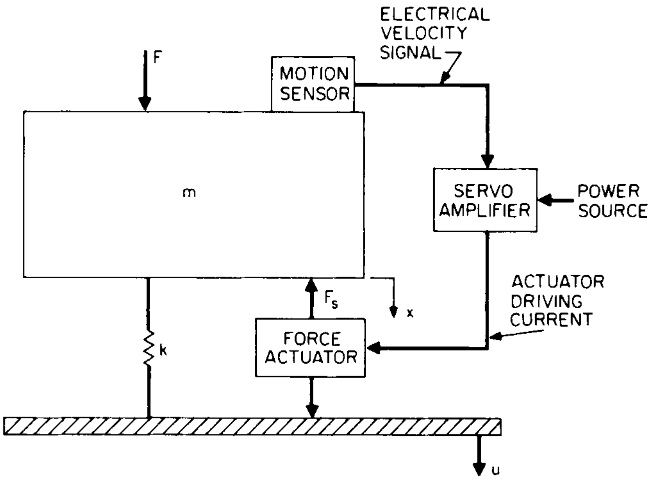

FIGURE 32.21 Schematic diagram of an active vibration control system which acts like a passive vibration-isolation mass and spring element with a viscous damping element connected between the supported body and motionless fixed space. The active damping servomechanism can eliminate the isolation system resonance, thereby providing vibration isolation starting at zero frequency.

ity of the supported body may be sensed by an electromagnetic sensor which measures velocity directly, or it may be obtained by integrating the response of an accelerometer. Figure 32.21 illustrates the elements of this servomechanism; the servo amplifier contains the system electronic devices which form the computational elements and the power elements required to operate the force actuator. The motor element is contained partly in the servo amplifier and partly in the force actuator. This shows that the three basic elements of a servomechanism are not always self-contained devices, but may be made up of the combined operation of system hardware components. The force actuator usually consists of an electrodynamic vibration exciter similar to those described in Chap. 25. Electronic amplifiers which drive the force motor must have a frequency response extending down to zero frequency, so as not to introduce timing errors into the control signal that can significantly alter the response of the servomechanism. The velocity sensor-amplifier-motor system making up this servomechanism applies a force to the supported body that is proportional to the body's velocity and thus acts in the same manner as a viscous damper connected to the supported body at one end and to motionless fixed space at the other end. This produces a form of damping within the active vibration control system which cannot be synthesized using passive damping elements alone. The action of this velocity-controlled servomechanism is referred to as *active damping,* and the active damping scaling term G_2, relating the supported body velocity to the force applied to the mass *m,* when divided by the critical damping term for the passive spring and mass elements $2\sqrt{km}$, is commonly referred to as the *active fraction of critical damping* G_2/c_c.

An active vibration-isolation system usually is described by a cubic or higher-order differential equation; because of the complexity of these equations, it is difficult to visualize the effect of changes in the system constants on the performance of the isolation system. This is particularly true when the actual nonideal response characteristics of the system sensing, computational, and motor elements are included in the system differential equation of motion and when additional computational elements, called compensation circuits, are added. The compensation circuits are used to alter the system frequency response, i.e., resonance frequency and peak transmissibility. In working with active vibration control systems of the type presented here, it is not uncommon to have differential equations as high as the twelfth order or more. The field of automatic control system synthesis has devised methods to deal with differential equations of such high orders from both a theoretical analysis and an actual system hardware point of view.

Because integral feedback of displacement requires that energy be fed into the control system, it is possible to make the active system dynamically unstable by improper proportioning of its constants. An active vibration control system that is dynamically unstable will undergo continuously increasing mechanical oscillations which, when not limited by available power, will increase until the system is destroyed. Therefore, one of the factors in achieving a satisfactory active vibration control system is the determination of the margin of dynamic stability of the entire system. Here too, the field of automatic control systems has devised methods to establish the system margin of dynamic stability. The margin of dynamic stability is a measure of the degree of change in system constants that is required for the active vibration control system to become unstable.

In the case of a conventional passive vibration control system, it is possible to determine many of the performance characteristics from the constants appearing in the differential equation. For example, the transmissibility T of a conventional system at the condition of resonance is approximately

$$T_r \simeq \frac{\sqrt{km}}{c} = \frac{1}{2(c/c_c)} \qquad \text{where } c/c_c < 0.2 \tag{32.14}$$

Similarly, the resonance frequency ω_r is approximately equal to the undamped natural frequency:

$$\omega_r \simeq \sqrt{\frac{k}{m}} \qquad \text{where } c/c_c < 0.2 \tag{32.15}$$

At high frequencies ($\omega >> \omega_n$), the transmissibility of a conventional system approaches the asymptotic value

$$T_i = \frac{c/c_c}{\omega/\omega_n} \qquad \text{where } \omega >> \omega_n \tag{32.16}$$

The transmissibility curve of a conventional isolator may be estimated from Eqs. (32.14) to (32.16) without plotting the transmissibility equation point by point. Somewhat similar relationships can be obtained for an active system if its equation of motion is not higher than the second order. A convenient way to obtain rules of thumb for the design of an active vibration control system is to compare the characteristic properties of a conventional vibration control system with those of the same isolation system but with active elements which provide integral relative displacement force feedback and proportional velocity force feedback added in parallel with a spring isolation element. The velocity feedback gain G_2 generally has a larger effect on the system response than the relative displacement gain term G_1. The feedback gain terms relate the sensed system motion term to the force applied to the supported body; therefore, the units of the velocity feedback gain term G_2 are the same as those for a viscous damper, or force per unit velocity; the gain term G_1 for the integral relative displacement feedback has no passive counterpart and has units of force per unit displacement multiplied by time. The active damping term dominates the system differential equation, affecting the system response both above and below the undamped natural frequency, while the effect of the relative displacement feedback on system performance is confined mainly to the frequency region below the undamped natural frequency. Setting the integral relative displacement gain term G_1 to zero gives an approximation for the transmissibility of the active vibration control system:

$$T = \sqrt{\frac{1}{[1-(\omega/\omega_n)^2]^2 + [2(G_2/c_c)(\omega/\omega_n)]^2}} \tag{32.17}$$

Using the above equation, the following response estimations can be formulated. The system transmissibility T at a frequency equal to the undamped natural frequency ω_n, formed by the passive spring and mass elements k and m, is

$$T_n = \frac{1}{2G_2/c_c} \qquad \text{where } \omega = \omega_n \tag{32.18}$$

The resonance frequency is less than the system undamped natural frequency, and with an active fraction of critical damping term of 1 or larger, there is no system resonance; i.e., at all frequencies the system transmissibility is less than 1. In the case where the relative displacement feedback gain is not zero, the mechanics of the system must always form a resonance condition. At excitation frequencies well above the system undamped natural frequency, the transmissibility of the active isolation system approaches the asymptotic value

$$T_i = (\omega/\omega_n)^2 \qquad \text{where } \omega >> \omega_n \tag{32.19}$$

In the above response estimation relationship function, the system transmissibility at the undamped natural frequency is less than unity when the velocity feedback gain exceeds a value giving an active fraction of critical damping of 0.5; i.e., $G_2/c_c = 0.5$. With an active fraction of critical damping of unity, the system transmissibility at the undamped natural frequency is 0.5. Active vibration control systems of this type typically exhibit velocity feedback gain magnitudes yielding an active fraction of critical damping ranging from a low of about 0.5 to a high of about 5. The incorporation of the integral relative displacement feedback servomechanism in conjunction with the velocity feedback servomechanism and the passive system elements forms a system described by a third-order differential equation. A resonance condition occurs well below the undamped natural frequency when the active fraction of critical damping is 0.5 or more. The simplified response estimations of transmissibility are valid for frequencies at and above the system undamped natural frequency in instances where the active fraction of critical damping is 0.5 or greater. As the active fraction of critical damping is decreased, the resonance frequency approaches the undamped natural frequency with an increasing peak transmissibility and an eventual dynamically unstable system.

In an ideal active vibration control system, the resonance frequency and peak transmissibility are a function of the passive system constants and the two feedback gain terms. In a nonideal active vibration control system, there are many other factors that influence the system resonance characteristics, such as the low-frequency response of the velocity sensor or a more complex passive system formed from many mass and spring elements. The resonance characteristics of the active vibration control system are manipulated through compensation functions formed using electric networks in the computation element of the velocity servomechanism. The function of these compensation networks is to alter the nature of the velocity feedback signal applied to the motor element, in a manner that provides for a dynamically stable system, and to raise or lower the resonance frequency, peak transmissibility, and transmissibility frequency response above the resonance frequency. The use of system compensation circuitry is extensive in the field of automatic control system synthesis as well as with active vibration control systems, which are a type of automatic control system. The result of system compensation is active vibration control systems with response characteristics similar but not limited to the response of the ideal system. The analysis of the transient and frequency-response characteristics of an active vibration control system having ideal elements shows many of the advantages of actual active vibration control systems when compared to the response of passive system elements alone.

In an active vibration control system, the element that provides integral control of relative displacement strives to maintain the supported body at a constant distance from the support base to which it is attached. When a step function of force is applied to the supported body, the response of the system gives a measure of the element's effectiveness in performing the desired function. A comparison of the transient response of the active vibration control system, i.e., one having integral relative displacement and absolute velocity force feedback, with that of the conventional passive vibration control system illustrates the advantage obtained from integral relative displacement feedback.

Transient Response. The equation of motion for the mass m of the passive control system is

$$m\ddot{x} + c\dot{x} + kx = F(t) \tag{32.20}$$

where the force $F(t)$ is a step function of force having a magnitude $F = F_0$ when $t > 0$ and $F = 0$ when $t < 0$. Writing the Laplace transform of Eq. (32.20),

$$\mathcal{L}[x(t)] = X(s) = \frac{F_0}{ms} \frac{1}{s^2 + (c/m)s + k/m} \tag{32.21}$$

where $X(s)$ designates the Laplace transform of x, a function of time. Letting $c/m = 2(c/c_c)\omega_n$ and $k/m = \omega_n^2$, Eq. (32.21) may be written as

$$X(s) = \frac{F_0}{ms} \frac{1}{s^2 + 2(c/c_c)\omega_n s + \omega_n^2} \tag{32.22}$$

The time solution of Eq. (32.22) is a damped sinusoid offset by the deflection of the spring caused by the constant force F_0. A typical time solution is shown by curve A of Fig. 32.22. The deflection of the isolator can be calculated by applying the final value theorem of Laplace transformations. This theorem states that if the Laplace transform of $x(t)$ is $X(s)$ and if the limit $x(t)$ as $t \to \infty$ exists, then

$$\lim_{s\to 0} sX(s) = \lim_{t\to\infty} x(t) \tag{32.23}$$

Applying the final value theorem using the Laplace transform of the passive isolator responding to the step function of force, Eq. (32.22), shows that the final deflection of the isolator is

$$\lim_{s\to 0} sX(s) = \lim_{t\to\infty} x(t) = \frac{F_0}{m\omega_n^2} \tag{32.24}$$

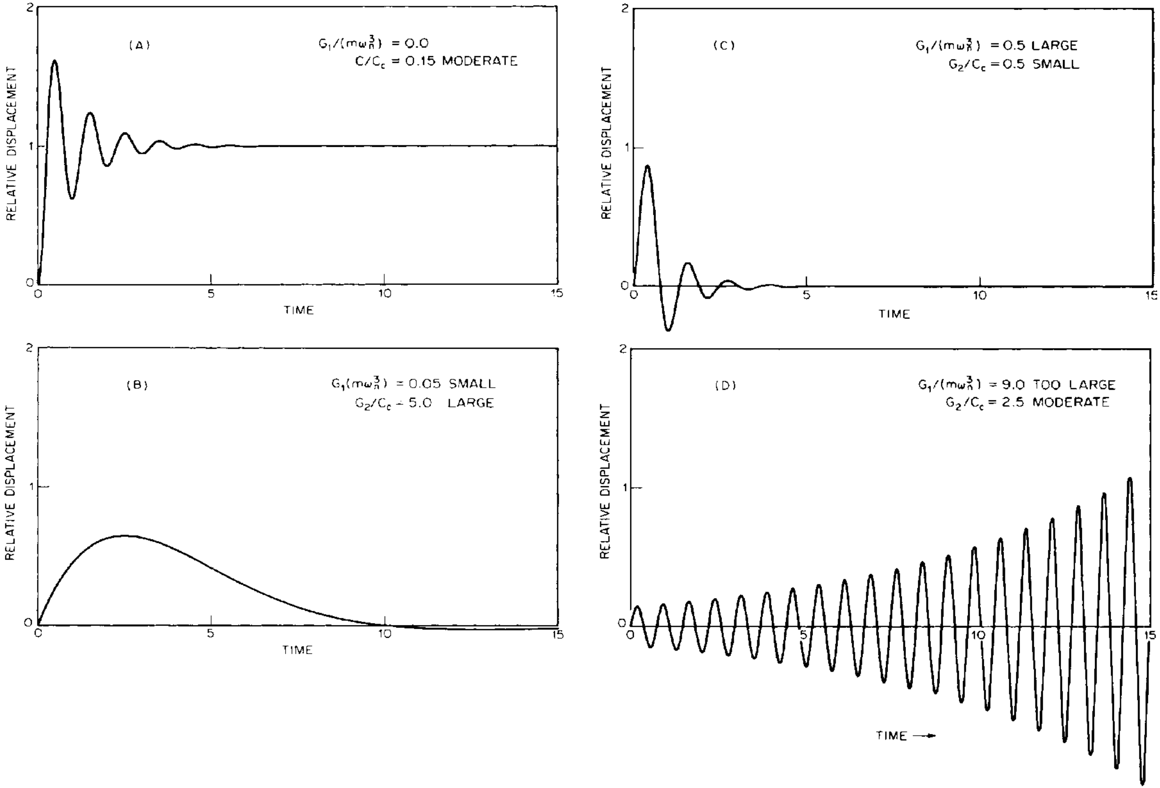

FIGURE 32.22 (*A*) Transient response of a passive vibration-isolation system to a step in force. (*B*), (*C*), and (*D*) show the transient response of an active vibration-isolation system to the same force step for different values of integral relative displacement and proportional velocity gains. The response is changed by changes in the feedback gain magnitude. In (*D*) the system is unstable as a result of the improper selectfion of the servomechanism constants; as a result, oscillations become increasingly large.

From Eq. (32.24), the mass takes a new position of static equilibrium at a distance $F_0/(m\omega_n^2)$ from the original position as $t \to \infty$. The final deflection term may be eliminated from Eq. (32.24) by adding an integral relative displacement control servomechanism. This added element produces a force proportional to the integral of displacement x with respect to time. The system damping element is replaced by an active damping control servomechanism. Active damping in this case acts in the same manner as the passive damping element used for Eq. (32.20) since x is the only system motion. The differential equation of motion for the supported body of the active vibration control system is

$$m\ddot{x} + G_2\dot{x} + kx + G_1 \int x\,dt = F(t) \tag{32.25}$$

The Laplace transform of the active vibration control system differential equation is

$$\mathfrak{L}[x(t)] = X(s) = \frac{F_0}{ms} \frac{1}{s^2 + (G_2/m)s + k/m + G_1/ms} \tag{32.26}$$

Placing the above equation in a form similar to Eq. (32.22) gives

$$X(s) = \frac{F_0}{m} \frac{1}{s^3 + 2(G_2/c_c)s^2 + \omega_n^2 s + (G_1/m\omega_n^3)\omega_n^3} \tag{32.27}$$

The term G_2/c_c represents the active fraction of critical damping. The term containing the active relative displacement feedback gain $G_1/m\omega_n^3$ is called the *dimensionless relative displacement feedback gain.* The use of the dimensionless gain terms, active fraction of critical damping and dimensionless relative displacement feedback gain, allows the response characteristics of the active vibration control system to be represented in a generalized manner where the numerical values of the passive system elements are not required.

Applying the final value theorem to the transient response of the active vibration control system represented by Eq. (32.27) gives the deflection of the supported body in its final equilibrium position:

$$\lim_{s \to 0} sX(s) = \lim_{t \to \infty} x(t) = 0 \tag{32.28}$$

The final equilibrium position for the supported body of the active vibration control system is zero so long as the dimensionless relative displacement feedback gain is not zero. The final position of the supported body is zero even with a very small dimensionless relative displacement feedback gain because of the integration operation provided by the relative displacement servomechanism. The magnitudes of the two servomechanism gain terms affect the motion of the supported body during the transient. Figure 32.22*A* shows the transient response of a passive vibration control system to a step in force which is applied to the supported body. In Fig. 32.22*B, C,* and *D* the transient response of an active system subjected to the same step force is shown for various values of the dimensionless feedback gain. The two servomechanisms in the active vibration control system interact, but their effect can be generalized:

1. Increasing the magnitude of the dimensionless relative displacement gain increases the rate at which the system relative displacement approaches the final equilibrium position.
2. Increasing the active fraction of critical damping decreases the peak magnitude of the system relative displacement during the transient event and lowers the damped natural frequency.

The degree of oscillation exhibited by the active vibration control system is a function of the magnitude and relative magnitude of the dimensionless gains of the two servomechanisms. In general, small magnitudes of the dimensionless relative displacement gain and large magnitudes of the active fraction of critical damping lead to little system oscillation, as depicted by the curve of Fig. 32.22*B*. Likewise, large magnitudes of the dimensionless relative displacement gain and small magnitudes of the active fraction of critical damping tend to increase the amount of oscillation. The dimensionless relative displacement gain can be increased too much in relation to the active fraction of critical damping and will then produce a condition of instability, as shown by the curve of Fig. 32.22*D*. The conditions resulting in system instability are presented in the last part of this section.

The relative displacement response of this ideal active vibration control system to constant acceleration of the isolator support, such as that produced by gravity or by the sustained acceleration of a missile, cannot be represented by applying a constant force to the supported body, as is frequently done with passive vibration control systems. The reason for this is that active vibration control systems which utilize absolute motion feedback, as in active damping of the type presented in this chapter, respond differently to forces applied to the supported body than to a constant acceleration of the support. In the case of a constant force applied to the supported body, presented above, the velocity servomechanism output force approaches zero as the transient motions of the system die out. In the case of a constant acceleration of the support, the velocity of the supported body continually increases in a manner similar to the increase in velocity of the support. The output of the velocity servomechanism increases constantly with time since the output force is proportional to the velocity of the supported body. This leads to a system which cannot work because the velocity servomechanism will rapidly reach its maximum force output, at which time all active damping is lost. In this situation, active vibration control is reobtained by placing an electric filter in the active damping servomechanism computational element. The filter forms a control function which produces a zero output for a ramp input. The use of such a filter is part of the compensation process often required with automatic control systems; this process is presented in more detail in the next section.

Many active vibration control systems of the ideal type presented in this chapter are used to isolate angular vibration, on which gravity has no effect. The active isolation of angular vibration uses the same system equations presented above except that the motions are angular, the mass is a moment of inertia, and the passive spring element applies a torque to the supported body that is proportional to the relative rotational displacement between the supported body and the support. The integral relative displacement servomechanism operates by measuring the rotation of the supported body relative to the support and applying a torque to the supported body that is proportional to the time integral of the sensed rotation. The relative angular displacement may be sensed using a rotational differential transformer or a linear potentiometer.

The active damping servomechanism operates by sensing the absolute rotational velocity of the supported body using a rate gyroscope which has an output response proportional to its rotational velocity. The active damping torque applied to the supported body is proportional to the output of the rate gyroscope. Many times the passive spring element is replaced by a servomechanism where the integral relative displacement control function in the computational element of the servomechanism is modified to produce an output proportional to the sum of the relative displacement and its first integral. Such a servomechanism has proportional plus integral control.

Steady-State Response. A comparison of the steady-state response of the active and passive vibration control systems illustrates some of the advantages and disadvantages associated with a servo-controlled vibration control system. In Fig. 32.21, assume that $F(t) = 0$ and that the vibration excitation is caused by the motion $u(t)$ of the support base. Then the equation of motion for the supported body of the active vibration control system having both the active damping servomechanism shown by Fig. 32.21 and the integral relative displacement control servomechanism shown by Fig. 32.20 is

$$m\ddot{x} + G_2\dot{x} + kx + G_1 \int x\,dt = ku + G_1 \int u\,dt \tag{32.29}$$

The response of this isolation system, when the vibration excitation $u(t)$ is sinusoidal in nature and steady with respect to time, may be expressed in terms of transmissibility:

$$T = \sqrt{\frac{(G_1/m\omega_n^3)^2 + (\omega/\omega_n)^2}{(\omega/\omega_n - \omega^3/\omega_n^3)^2 + [G_1/m\omega_n^3 - 2(G_2/c_c)(\omega^2/\omega_n^2)]^2}} \tag{32.30}$$

Figure 32.23 is a plot of Eq. (32.30) for four values of the relative displacement dimensionless gain term and six values of the velocity dimensionless gain term, $G_1/(m\omega_n^3)$ and G_2/c_c, respectively. The corresponding expression for the transmissibility for the conventional passive vibration control system differs from that for an active system, i.e., Eq. (32.30), because of the nature of the force feedback terms acting upon the supported body. At frequencies well above the vibration control system undamped natural frequency ω_n, the active and passive system transmissibility equations differ because of the presence of a damping term in the numerator of the passive system equation. At these higher frequencies, the passive system transmissibility has the characteristic that as $\omega \to \infty$, $T \to 2(c/c_c)\,(\omega_n/\omega)$. The active system, however, tends to act as an undamped vibration control system wherein the transmissibility at high frequencies has the characteristic that as $\omega \to \infty$, $T \to \omega_n^2/\omega^2$. Thus the active vibration control system provides a lower transmissibility at frequencies above the system natural frequency, especially for large values of the active and passive damping terms.

At excitation frequencies close to the system natural frequency, both the active and passive vibration control systems exhibit a resonance condition when the system damping terms are small. The peak value of the system transmissibility at the system resonance frequency is controllable by the addition of damping. In the passive vibration control system, as the fraction of critical damping is increased, the peak transmissibility is lowered, reaching a value of unity for an infinite value of the fraction of critical damping. Although the passive system damping controls the peak transmissibility, high values of damping greatly degrade the system's main function of isolating vibration; in fact, very large magnitudes of the system damping term yield little to no vibration isolation, since the damper tends to become a rigid link between the control system vibrating base and the supported body. The effect of damping on the active vibration control system is similar to that on the passive vibration-isolation system when the active fraction of critical damping is small. However, as the active system damping is increased, an increasingly more rigid link is placed between the supported body and motionless space; thus, increasing the active fraction of critical damping always decreases the system transmissibility at frequencies above the natural frequency. With a relative displacement gain G_1 of zero, the active system resonance will disappear when the active fraction of critical damping exceeds unity, as is shown by the curve of Fig. 32.23*A*. With an active fraction of critical damping of unity, the peak transmissibility is also unity and occurs at zero frequency, and for all

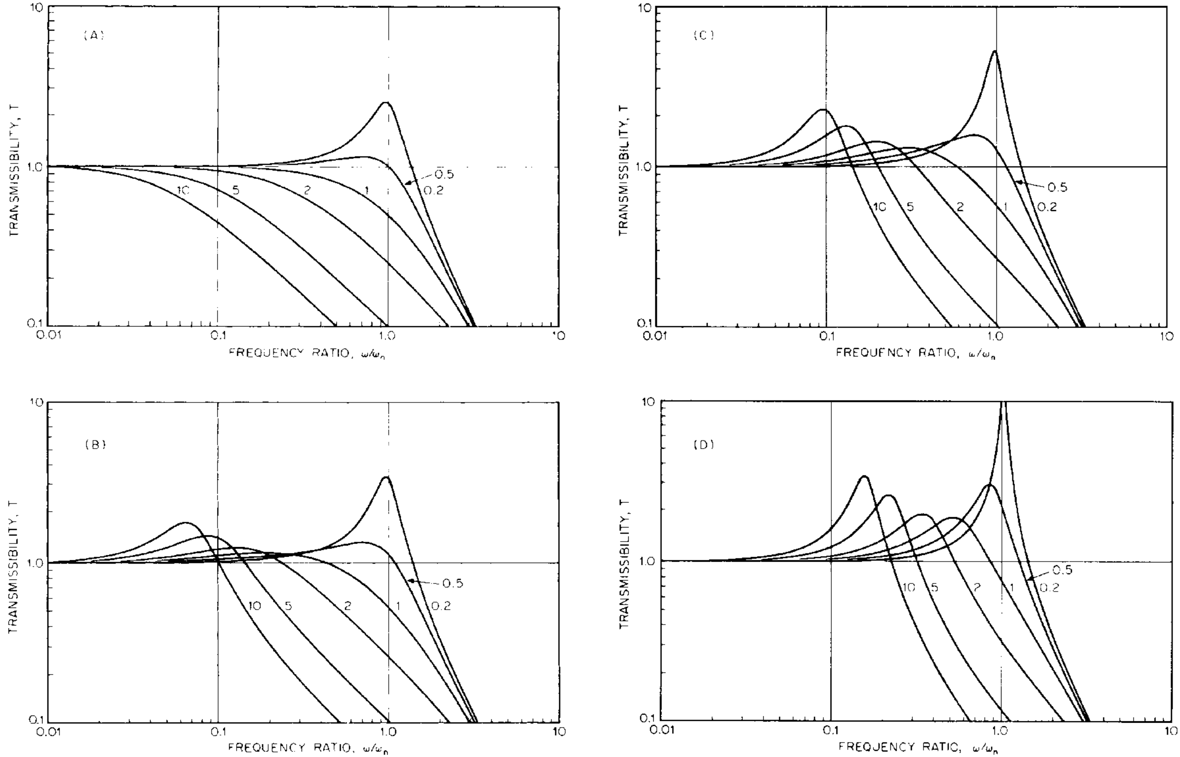

FIGURE 32.23 Steady-state frequency response for an active vibration control system having an ideal active damping servomechanism. The transmissibility is plotted against the frequency ratio ω/ω_n. In (A) there is no integral relative displacement control servomechanism, i.e., $G_1/m\omega_n^3 = 0$; in (B), (C), and (D) such a control mechanism has been added and this ratio has values of 0.1, 0.2, and 0.5, respectively. For each of these illustrations a set of curves is shown for the following values of the ratio G_2/C_c: 0.2, 0.5, 1, 2, 5, and 10. Changes in the servomechanism feedback constants affect the response characteristics through their dynamic interactions, which alter the frequency response at low excitation frequencies.

other frequencies the system transmissibility is less than 1, having the approximate magnitude of $1/[2(G_2/c_c)\ (\omega/\omega_n)]$ at frequencies from zero to about twice the system natural frequency and ω_n^2/ω^2 at higher frequencies.

The addition of the relative displacement integral control has little influence on transmissibility at high frequencies and thus has no important effect on the ability of the complete system to isolate vibration. However, the effect at lower frequencies is significant, as is shown in Fig. 32.23*B, C,* and *D.* As the dimensionless gain $G_1/m\omega_n^3$ of the displacement control loop is increased, the transmissibility of the system in the region of resonance increases. If the dimensionless displacement gain term equals twice the active fraction of critical damping, the active vibration control system becomes dynamically unstable. Under these conditions, if the supported body receives the slightest disturbance, a system oscillation will develop and continue indefinitely, as would be the case with a passive system without damping. Increasing the relative displacement gain term above this critical value results in a condition where the system's automatic control functions continually add energy to the supported body and passive spring element in the form of ever-increasing oscillations, which continue to increase in amplitude until motor saturation or destruction of the system occurs.

Stability of Active Vibration Control Systems. Operation of a dynamically unstable active vibration control system exhibits one or more of the following characteristics:

1. The active vibration control system acts like an undamped passive vibration control system.
2. The system exhibits oscillations that increase with time and can become very large in magnitude.
3. The system moves to one of its excursion stroke limits and stays there.

The ensurance of a dynamically stable active vibration control system is important at both the design and hardware stages of development and can become a complex design task. Much of the field of automatic control system analysis and synthesis deals with establishing the limits of feedback gains beyond which the system becomes unstable.

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