CHAPTER 27
APPLICATION OF DIGITAL COMPUTERS

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INTRODUCTION

This chapter introduces numerous applications and tools that are available on and with digital computers for the solution of shock and vibration problems. First, the types of computers that are used, the associated specialized processors, and their input and output peripherals, are considered. This is followed by a discussion of computer applications that fall into the following basic categories: (1) numerical analyses of dynamic systems, (2) experimental applications that require the synthesis of excitation (driving) signals for electrodynamic and electrohydraulic exciters (shakers), and (3) the acquisition of the associated responses and the digital processing of these responses to determine important structural characteristics.

The decision to employ a digital computer–based system for the solution of a shock or vibration problem should be made with considerable care. Before particular computer software or hardware is selected, the following matters should be carefully considered.

1. The existing hardware and/or software that is or is not available to perform the required task.
2. The extent to which the task or the existing software/hardware must be modified in order to perform the task.
3. If no applicable software/hardware exists, the extent of the development effort necessary to create the suitable software and/or hardware subsystems.
4. The detailed assumptions needed in the software/hardware in order to simplify its development (e.g., linearity, proportional damping, frequency content, sampling rates, etc.).
5. The ability of the software/hardware to measure and compute the output information required (e.g., absolute vs. relative motion, phase relationships, rotational information, etc.).

6. The detailed input and output limitations of the needed system software and/or hardware (types of excitation signals, voltage ranges, minimum detectable signal amplitudes, calculation rates, control speed, graphic outputs, setup parameters, etc.).
7. The processing power and time needed to perform the task.
8. The algorithms and hardware features that are needed to perform the task.

After these matters are resolved, the user must realize that the results obtained from the output of a computer system can be no better than its available inputs. For example, the quality of the natural frequencies and mode shapes obtained from a structural analysis software system depends heavily on the degree to which the mathematical model employed represents the actual mass, stiffness, and damping of the physical structure being analyzed (see Chap. 21). Likewise, a spectral analysis of a signal with poor signal-to-noise ratio will provide an accurate spectrum of the signal plus the measurement noise, but not of the signal amplitudes that fall below the noise floor (see Chap. 22).

DIGITAL COMPUTER TYPES

The digital computer types that are used to solve shock and vibration problems are varied. There are general-purpose or specialized digital computers. It is generally better to use general-purpose computers whenever possible, since these types of digital computers are supported with the best graphics, applications development, scientific and engineering tools, and the wider availability of preexisting applications software. However, even within these general categories, there are various processor or computer configurations available to help solve shock and vibration problems. The following sections provide definitions, descriptions, and discussions of the applicability of general-purpose computers and specialized processors that can help solve shock and vibration problems.

GENERAL PURPOSE

General-purpose computers are computers designed to solve a wide range of problems. They are optimized to allow many individual users to access the particular computer system's resources. They range from large central systems like mainframes, which can handle thousands of simultaneous users, to personal computers, which are designed to serve one interactive user at a time and provide direct and easy access to the computer system's computational capability through thousands of existing applications and its graphical user interface. These are personified by personal computers based on Wintel (i.e., Windows and Intel) or Power PC technologies. In the following, mainframes, workstations, personal computers, and palmtop digital computers are discussed from the viewpoint of their applicability to solve shock and vibration problems.

Mainframes. *Mainframe computers* are computer systems that are optimized to serve many users simultaneously. They typically have large memories, many parallel central processing units, large-capacity disk storage, and high-bandwidth local network and Internet connections. These systems, when available, can be used to solve the largest shock and vibration simulations, where very large finite element models or other discrete system models require large memories and the processing power that mainframes provide. They are also used for web or disk server functions to networked workstations and personal computers. Mainframe computers are increasingly being replaced by either powerful workstation or personal computer–based systems.

Workstations. *Workstations* are computer systems that provide dedicated computer processing for individual users that typically are involved in technically specialized and complex computing activities. These computer systems usually run a version of the UNIX operating system using a graphical user interface that is based on X-windows; *X-windows* is a set of libraries of graphical software routines, developed by an industry consortium that provide a standard access to the workstation's graphics hardware through a graphical user interface. Workstations often are based on reduced instruction set computer systems, to be discussed in a later section, with significant floating point processing power, sophisticated graphic hardware systems, and access to large disk and random access memory systems. This suits them for computer-assisted engineering activities like large-scale simulations, mechanical and electrical system design and drafting, significant applications in the experimental area that involve many channels of data acquisition and analysis, and the control of multiexciter vibration test systems. They are designed to efficiently serve one user, but are inherently multiuser, multitasking, and multiprocessor in nature, and can serve as a suitable replacement for mainframes in the server arena. These systems are now mature, with capability still expanding, but merging in the future with high-powered personal computers. However, due to their maturity, they have an inherent reliability advantage over personal computers, and thus have a higher suitability for mission-critical applications. Newer versions of UNIX, like LINUX, allow personal computer hardware to be used as a workstation, affording the power and reliability of workstations with the convenience of personal computer hardware.

Personal Computers. *Personal computers* (PCs) are computer systems that are intended to be used by casual users and are designed for simplicity of use. PCs originally were targeted to be used as home- and hobby-oriented computers. Over the years, PCs have evolved into systems that have central processing units that rival those of workstations and some older mainframes. PC operating systems have also evolved to provide access to large disk and random access memories, and a sophisticated graphical user interface. They have many applications in the shock and vibration arena that are available commercially. These applications include sophisticated word processors, spreadsheet processors, graphics processors, system modeling tools like Matlab™, design applications, and countless other computer-aided engineering applications.

There are also many experimental applications like modal analysis, signal analysis, and vibration control systems that are implemented using PCs. These types of systems are typically less expensive when they are built using PCs rather than workstations. At this time, however, workstations still provide greater performance and reliability than PCs. PC operating systems are not as robust as those that run on workstations, although this may change in the future. PCs, however, are ubiquitous and the hardware and software used to make them continues to expand in capability and reliability. It is likely that the PC and workstation categories will ultimately merge, hopefully preserving the best of both worlds. Currently, most PCs are based on Wintel technologies, with a smaller percentage based on Power PC technologies.

Palmtops. *Palmtop computers* (also called *hand-held computers*) are computer systems that are designed for extreme portability and moderate computing applications. This type of digital computer system is an outgrowth of electronic organizers. They are small enough to fit in a shirt pocket, are battery-powered, have small screens, and thus are useful for note-taking, simple calculations, simple word processing, and Internet access. They support simplified versions of popular personal computer applications with many also supporting handwriting and voice recognition. They can be employed in the shock and vibration field as remote data gatherers that can connect to a host computer to transfer the acquired data to it for further processing. The host computer is typically a personal computer or workstation.

SPECIALIZED PROCESSORS

Specialized processors are designed for a particular activity or type of calculation that is being performed. They consist of embedded, distributed, digital signal processors, and reduced instruction set computer processor architectures. These systems typically afford the most performance for shock and vibration applications, but at a higher level of complexity than that associated with the general purpose computers that were previously discussed. Included in this category are specialized peripherals such as analog-to-digital (A/D) converters and digital-to-analog (D/A) converters that provide the fundamental interfaces between computer systems and physical systems like transducers and exciters, which are used for many shock and vibration testing and analysis applications. Specialized processor architectures are used extensively in shock and vibration experimental applications, since they provide the necessary power and structure to be able to accomplish some of the more demanding applications like the control of single or multiple vibration test exciters, or applications that involve the measurement and analysis of many response channels from a shock and vibration test.

Embedded Processors. *Embedded processors* are computer systems that do not interact directly with the user and are used to accomplish a specialized application. This type of system is part of a larger system where the embedded portion serves as an intelligent peripheral for a general purpose computer host like a workstation or personal computer–based system. The embedded subsystem is used to perform time-critical functions that are not suitable for a general purpose system due to limitations in its operating systems. The operating system used for embedded processors is optimized for real-time response and dedicated, for example, to the signal synthesis, signal acquisition, and processing tasks. The embedded system typically communicates with the host processor through a high-speed interface like Ethernet, small computer system interconnect (SCSI), or a direct communication between the memory busses of the embedded and host computer systems. An embedded computer system does not interface directly with the computer system user, but uses the host computer system for this purpose. An example of an embedded system, which uses distributed processors, is shown in Fig. 27.1. Here the host computer is used to set the parameters for the particular activity, for example, shock and vibration control and analysis, and uses the embedded computer subsystem to accomplish the control and analysis task directly. This frees the host processor to simply receive the results of the shock and vibration task, and to create associated graphic displays for the system user.

Distributed Computer Systems. *Distributed computer systems* are digital computers that accomplish their task by using several computer processor systems in tandem to solve a problem that cannot be suitably solved by an individual computer

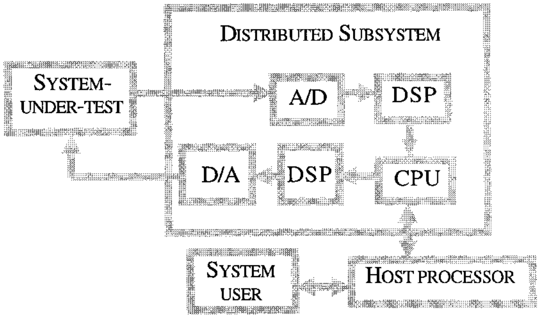

FIGURE 27.1 Diagram of distributed and embedded system.

or processor system. This type of computer system typically partitions its task in such a way that each part can be executed in parallel by its respective processor. This enables the use of several specialized processors to separately accomplish a demanding subtask, and thus the overall shock and vibration task, in a way that may not be possible with the use of a single general purpose computer system.

An example of this type of system, as shown in Fig. 27.1, is a distributed and embedded computer system that uses digital signal processors to process data being received from an A/D converter by filtering it and extracting the pertinent signal characteristics needed as part of a shock and vibration test. This filtered data, and its extracted characteristics, are subsequently sent to a more general processor to perform additional analysis on the data. The results of this more general analysis may yield a time-series data stream that is sent to another digital signal processor for filtering, and then sent to an output D/A converter to produce signals that are used to excite a system under test. Figure 27.1 also shows, in the form of a block diagram, a typical form and application of a distributed and embedded subsystem as it would be used in a shock and vibration test. A specialized embedded operating system is typically used by the distributed system's central processing unit (CPU) to coordinate the communications between and with the two digital signal processor subsystems. The host processing system is used to interface with the overall system's user.

Digital Signal Processors. *Digital signal processors* (DSPs) are specialized processors that are optimized for the multiply-accumulate operations that are used in digital filtering and linear algebra–related processing. They are used extensively in shock and vibration signal analysis and vibration control systems. These processors are ideal to implement digital filters, for sample-rate reduction and aliasing protection[1] (see Chap. 14), fast Fourier transform (FFT)–based algorithms (see Chap. 22), and digital control systems. Linear algebra problems, like those encountered in signal estimation, filtering, and prediction, are also performed efficiently by this architecture.[2,3] The previous example of an embedded and distributed system in Fig. 27.1 also shows a typical application of DSP technology. The development of this digital computer architecture has empowered much of the audio and video signal processing systems in current use. It has also enabled many of the shock and vibration experimental applications now in use.

Reduced Instruction Set Computer. *Reduced instruction set computer* (RISC) systems are computer systems based on specialized processors that are optimized to execute their computer instructions in a single CPU cycle. In order to execute instructions in a single cycle, these processors typically are designed to execute only simple instructions at a higher rate than is possible with *complex instruction set computers* (CISCs) like those used in many personal computer and mainframe computer systems. Current RISC systems also have multiple execution units that are part of the CPU and thus can execute several instructions in parallel. Such systems are called *super-scalar RISC* systems. RISC systems also have large internal memories, within the processor's integrated circuits, that are called *cache* memories, that keep the most recently executed instructions and data. This further speeds the computer's ability to execute instructions. Floating point instructions are also heavily optimized, which give this type of processor an advantage for shock and vibration applications. However, CISC processors are evolving. They are incorporating the best ideas from RISC designs and, as time passes, these two types of computer architectures will tend to merge.

RISC processors were originally developed for high-powered workstations that run the UNIX operating system. Now they are being used more in the embedded application arena for things like digital video, sophisticated game consoles, and increasingly in experimental applications for shock and vibration in systems like the example embedded system shown in Fig. 27.1. In these systems, the embedded and distributed system CPU is typically a RISC processor running an embedded *real-time operating system* (RTOS) to coordinate its activity and the activities of the other specialized processors that are used, as in Fig. 27.1.

A/D and D/A Converters for Signal Sampling and Generation. A/D and D/A converters are fundamental to the applications of digital computers to the field of shock and vibration. They provide a fundamental interface between the analog nature of shock and vibration phenomena and the digital processing available from modern computing systems. These important subsystems are now realized by single *integrated circuits* (ICs), often incorporating most of the filtering needed for antialiasing (see Chaps. 13, 14, and 22) for A/D converters, and anti-imaging for D/A converters. This is particularly true of those A/D converters that use *sigma-delta* (ΣΔ) technology, which employs (1) simple analog signal preprocessing, (2) an internal sampling rate that is much higher than the signal's frequency bandwidth, (3) internal low accuracy A/D and D/A converters coupled with advanced feedback control processing, and (4) internal digital signal processing to reduce the output sampling rate and increase the output signal's resolution.[4] In practice, even when using ΣΔ technology, additional analog circuitry is needed to complete the antialiasing and anti-imaging function, and also to add needed signal amplification and conditioning to more fully utilize the resolution of modern A/D and D/A converters.

A/D Converters and Data Preparation. A/D converters furnish the analog-to-digital conversion function, which is the process by which an analog (continuous) signal is converted into a series of numerical values with a given binary digit (bit) resolution (see Chap. 22). This is the first step in any digital method. The A/D converter operation is generally built into self-contained digital analysis systems that use the A/D converter subsystem as a peripheral. The main CPU within the digital analysis system is typically a personal computer or a high-performance workstation. This CPU is used to set up the A/D converter's data-acquisition parameters such as the sampling rate, input-voltage range, frequency range, input data block size (duration of signal to be digitized), and the number of data blocks to be digitized. The acquired data may then be subsequently analyzed offline by the digital analysis system, or in real-time as the test progresses. Examples of A/D converter applications are shown in Figs. 27.1 and 27.2. If the digital processing is to be performed on a general-purpose scientific computer at another facility, then the data is captured to

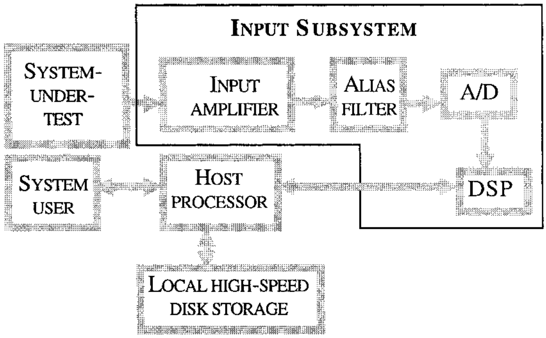

FIGURE 27.2 Diagram of typical A/D converter–based input subsystem.

local storage on the digital analysis system so that it can then be transported to the remote scientific computer, either by hard disk, by the Internet, or by other facility methods or networks.

A prime advantage of digital analysis methods is that the time-history needs to be digitized with an A/D converter and digitally recorded only once. Subsequently, the recorded data can be analyzed using various methods and at various times. Sometimes, the need to digitally record a time-history may be omitted if only real-time interactive signal analysis is needed. However, if the test data is digitized and stored during a test using real-time signal analysis, the problems associated with not anticipating the need for a particular signal analysis result during a test can be avoided by being able to reanalyze the test data that was digitally recorded.

In Fig. 27.2, the input signal from the system under test is amplified by the input amplifier to maximize the A/D converter's resolution. The amplified signal is then filtered to remove high-frequency energy in the input signal that could be aliased (see Chap. 22), and then is passed to the A/D converter for digitization. The digital time series that the A/D converter produces is then sent to a digital signal processor for additional filtering and perhaps sample-rate reduction, or other needed specialized processing before it is sent to the host processor. For each input channel, the combination of (1) the input amplifier, (2) the antialiasing filter, (3) the A/D converter, and (4) the DSP, is called the *input subsystem* and is used by digital vibration control systems to be discussed later.

The integrated circuits in many A/D converters, such as those shown in Figs. 27.1 and 27.2, employ ΣΔ technology.[4] The technology uses *oversampling* techniques to provide a higher *oversampling ratio* (the sampling frequency divided by the highest frequency of interest). This reduces the need for complexity in the antialias filter from that required for more conventional A/D converters, which use a lower oversampling ratio, like 2.56, and thus need complex antialias analog filters with very narrow *transition bands*[1,4] (the frequency region between the filter's cutoff frequency and the start of its stopband). ΣΔ A/D converters are typically implemented as shown in Fig. 27.3, which illustrates their usual structure in the form of a block diagram.

In Fig. 27.3, the ΣΔ *modulator* (the device that converts the analog input into its digital representation) and digital filter[1] operate at sampling rates K times higher than the A/D converter's output sampling rate f_s in samples per second (sps). In this example, the oversampling ratio of the modulator is K. The digital filter reduces the

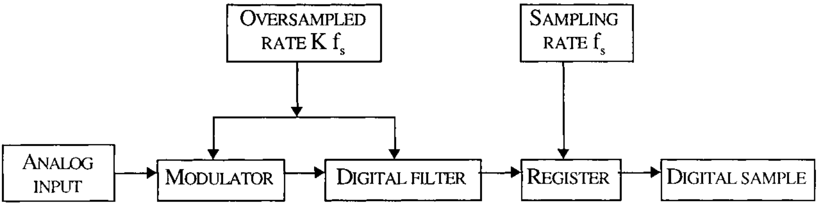

FIGURE 27.3 Oversampling sigma-delta (ΣΔ) A/D converter.

sampling rate from that used in the modulator section to f_s by successively filtering and decimating, usually in stages, to reduce the complexity of the digital filter. Most current A/D converter designs can provide alias-free output samples at an f_s that is 2.2 times the highest frequency of interest (the *acquisition bandwidth*). For example, if the A/D converter is operated with a 51.2 ksps sampling rate, then its output will be alias-free for an acquisition bandwidth (ABW) of 23.27 kHz. However, most digital systems used for shock and vibration testing applications, for example, typically process the data with an ABW of 20 kHz, thus using an effective oversampling ratio of 2.56. The modulator typically performs the initial sampling of the analog input signal with an internal oversampling ratio of 64, which results in an internal oversampled rate of 3.2768 Msps (64 times 51.2 ksps). The use of this internal sampling rate results in signal values that will alias if their frequency is above the Nyquist frequency f_A, defined as one-half the sample rate, that is, $f_A = f_s/2$ (see Chap. 22). However, only frequencies higher than 3.2568 MHz will alias into the 20 kHz ABW of this example.[13,31] The antialias filter thus only needs to attenuate signal frequencies larger than 3.2568 MHz to ensure alias-free data below 20 kHz, and thus can have a transition bandwidth from 20 kHz to 3.2568 MHz.[4,5] Since the complexity of the needed antialiasing filter is largely determined by the narrowness of its transition bandwidth, this large resultant transition bandwidth, which corresponds to the large oversampling ratio of 64, significantly simplifies the design of the needed antialias filter. Higher-signal ABWs can be obtained by operating the A/D converter at a higher sample rate. Output sample rates as high as 204.8 ksps, while maintaining good low-frequency performance, are becoming available, which provide an ABW of 80 kHz when using a 2.56 oversampling ratio.

The modulator[4] of the A/D converter shown in Fig. 27.3 is at the heart of the A/D converter design, and thus its structure is an important determinant of its resultant performance. An example of its internal structure is shown in Fig. 27.4, which presents an example of a first-order[4] modulator. Such first-order modulators show the basic ideas underlying ΣΔ technology. However, many current ΣΔ A/D converters employ higher-order modulators. These higher-order modulators use a number of integrators, as shown in Fig. 27.4, equal in number to the order of the ΣΔ modulator. These are either used in a cascade of first-order modulators, as in Fig. 27.4, or as a combination of integrators that are used in a multiple feedback loop, equal to the ΣΔ modulator order,[4] again as shown in Fig. 27.4.

At the input of the modulator shown in Fig. 27.4, there is a comparator that compares the value of the output voltage of the low-bit D/A converter and the analog input voltage, and passes this difference to an integrator. The integrated error voltage is passed to a low-bit A/D converter, typically with the same number of bits as the D/A converter, usually 1 or 2 bits, which then makes a digital output available from the ΣΔ modulator at its oversampled rate. The short-term averages of this low-resolution digital output sample can be made very close in value to the digitized value

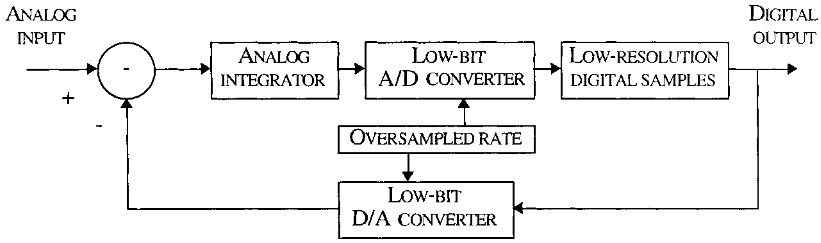

FIGURE 27.4 Typical first-order ΣΔ modulator.

of the analog input at a given bit resolution.[4] The digital filter that follows the ΣΔ modulator in Fig. 27.3 is designed to both average these samples and thereby increase their digital resolution, as well as reduce their sample rate while performing as a digital antialiasing filter.[4] The digital filter also causes delay effects in ΣΔ A/D converters that can cause problems when used with digital vibration control systems. This is due to the digital filter's *group delay,*[1,4] which is typically on the order of 34 samples, and which can cause closed-loop stability problems if not addressed properly.

D/A Converters and Signal Synthesis. As discussed previously, D/A converters convert a digital time series into an analog signal. This analog signal will have a "staircase" or zero-order hold nature.[5] This occurs because the D/A converter output signal is held constant for an output sample-rate period, and then is changed according to the next digital sample at the next sample-clock period. This staircase nature of the output D/A converter signal causes its analog output signal spectrum to have high-frequency terms, in addition to those present in its digital time series spectrum, with their frequency content centered about the D/A converter's sample-rate frequency, both below the sample rate and above the sample rate, and its integer multiples.[5] These somewhat symmetrical spectral lobes that appear in the D/A converter output signal spectrum, and that are centered at the sample-rate frequency and its harmonics, are called *signal images.*[5] These spectral lobes have a bandwidth double that of the bandwidth of the digital time series that is being sent to the D/A converter.[5] The spectrum of these signal images has a $\sin(x)/x$ envelope that is due to the zero-order hold nature of the D/A converter. They are the counterpart to aliasing that occurs with A/D converter sampling (see Chap. 22). These signal images should be removed before using the D/A converter output signal to excite a system under test. For this reason and others, the output subsystem should be organized as is shown in Fig. 27.5.

In Fig. 27.5, the signal flow is the reverse of that for the A/D converter–based input subsystem, as shown in Figs. 27.1 and 27.2. In Fig. 27.5, the output signal flows from a local high-speed disk storage subsystem into the host processor, which formats it for the digital signal processor in the output subsystem. The digital signal processor performs some filtering and perhaps increases the sample rate to minimize the impact of output signal images, moving them higher in frequency and lower in amplitude. This filtered and processed output time series is then sent to the D/A converter to produce an analog voltage. The D/A converter output voltage is filtered by an anti-imaging filter to remove any signal images that may still be present. This filtered signal is then passed to the output attenuator subsystem to set the final output signal amplitude. The attenuator is used to maximize the D/A converter output resolution. Typically, additional output filtering is provided by the analog circuitry that is part of the attenuator. Digital vibration control systems use the output subsystem shown in Fig. 27.5.

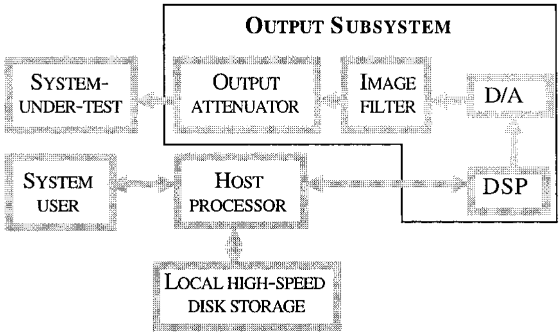

FIGURE 27.5 Typical D/A converter–based output subsystem.

ΣΔ D/A converter IC designs are also used for shock and vibration applications. They use an internal signal flow that is the reverse of that for a ΣΔ A/D converter, as shown in Fig. 27.3, but are otherwise very similar.[4] It uses digital filters for output interpolation and to increase the sampling rate from the system sampling rate to an oversampling rate. This digital filter also causes group delay effects like those discussed for ΣΔ A/D converters. At this oversampling rate, a low-bit resolution D/A converter output is produced, but at this high output sample rate, the signal image filter shown in Fig. 27.5 is also simplified since the D/A converter signal images are now centered at the oversampling frequency, which is typically 3.2768 MHz, instead of the output sample rate frequency which is typically 51.2 kHz. As in the ΣΔ A/D converter case, this results in a large transition bandwidth image filter. The low-bit D/A converter output is filtered by the image filter to remove the signal images that are still present. The image filter also acts like a short-term averager, and thus a higher effective D/A converter resolution is obtained, again as in the associated discussion on ΣΔ A/D converters. For the ΣΔ D/A converter, the major design and research efforts are in the ΣΔ *de-modulator*[4] section (the device that converts the digital representation of the output signal into an equivalent analog output).

ANALYTICAL APPLICATIONS

The development of large-scale computers with a very short *cycle time* (i.e., the time required to perform a single operation, such as adding two numbers) and a very large memory permits detailed analyses of structural responses to shock and vibration excitations. In this chapter, programs developed to perform these analyses are categorized as general-purpose programs and special-purpose programs. References 3, 6, and 7 contain extensive discussions of both general-purpose and special-purpose analytical programs.

GENERAL-PURPOSE PROGRAMS

Programs may be classed as *general-purpose* if they are applicable to a wide range of structures and permit the user to select a number of options, such as damping (viscous or structural), and various types of excitations (sinusoidal vibration, random vibration, or transients).

Finite Element Methods. The most numerous programs are classed as *finite element* or *lumped-parameter* programs, as described in detail in Chap. 28, Part II. In a lumped-parameter program, the structure to be analyzed is represented in a model as a number of point masses (or inertias) connected by massless, spring-like elements. The points at which these elements are connected, and at which a mass may or may not be located, are the *nodes* of the system. Each node may have up to six degrees-of-freedom at the option of the analyst. The size of the model is determined by the sum of the degrees-of-freedom for which the mass or inertia is nonzero. The number of natural frequencies and normal modes that may be computed is equal to the number of dynamic degrees-of-freedom. However, the number of frequencies and modes that reliably represent the physical structure is generally only a fraction of the number that can be computed. Each program is limited in capacity to some combination of dynamic and zero mass degrees-of-freedom. The spring-like elements are chosen to represent the stiffness of the physical structure between the selected nodes and generally may be represented by springs, beams, or plates of specified shapes. The material properties, geometric properties, and boundary conditions for each element are selected by the analyst.

In the more general finite element programs, the spring-like elements are not necessarily massless, but may have distributed mass properties. In addition, lumped masses may be used at any of the nodes of the system. The equations of motion of the finite element model can be expressed in matrix form and solved by the methods described in Chap. 28, Part I. Regardless of the computational algorithms employed, the program computes the set of natural frequencies and orthogonal mode shapes of the finite-dimensional system. These modes and frequencies are sorted for future use in computing the response of the system to a specified excitation. For the latter computations, a damping factor must be specified. Depending on the programs, this damping factor may have to be equal for all modes, or it may have a selected value for each mode.

Component Mode Synthesis. The method of modeling described above leads to the creation of models with a very large number of degrees-of-freedom compared with the number of modes and frequencies actually of interest. Not only is this expensive, but it rapidly exceeds the capacity of many programs. To overcome these problems, *component mode synthesis*[8,9] techniques have been developed. Instead of developing a model of an entire physical system, several models are developed, each representing a distinct identifiable region of the total structure and within the capacity of the computer program. The modes and frequencies of interest in each of these models are computed independently. Where actual hardware exists for some or all components, modes and frequencies from an experimental modal analysis may be used (see Chap. 21). A model of the entire structure is then obtained by joining these several models, using the component model synthesis technique. This model retains the essential features of each substructure model, and thus the entire structure, with a greatly reduced number of degrees-of-freedom.

Reduction of Model Complexity. Companion methods developed to reduce the cost of analysis, permit the joining of several substructure models, and provide for correlation with experimental results are described under reduction techniques in Chap. 28, Part II. For cost reduction and joining of substructures, the objective is to reduce the mass and stiffness matrices to the minimum size consistent with retaining the modes and frequencies of interest, as well as other dynamic characteristics such as base impedance. For test/analysis correlation, the objective is to match the degrees-of-freedom of the test. It should be noted, however, that the Guyan reduction method (see Chap. 28, Part II) yields a mass matrix which is nondiagonal and which may be unacceptable for some computer programs. It is also of interest that the rigid-body mass properties (total masses and inertias of the structure) are not identifiable in the reduced mass matrix.

Boundary-Element Method. The *boundary-element method*[10–12] involves the transformation of a partial differential equation, which describes the behavior of an enclosed region, to an integral equation that describes the behavior of the region boundary. Once the numerical solution for the boundary is obtained, the behavior of the enclosed region is then calculated from the boundary solution. Using this method, three-dimensional problems can be reduced to two dimensions, and two-dimensional problems can be reduced to one dimension. It is then necessary to model in detail only the boundary of the enclosed region rather than the complete region. A volume can be described by its surface, and an area can be described by its edges. A discrete description of the boundary is much less detailed and less sensitive to mesh distortion than a finite element model of the same region. However, each boundary-element equation has a greater number of algebraic functions than the corresponding finite element equation, and more processing power is required.

Two types of boundary-element methods exist. The direct method solves directly for the physical variables on the surface. The system of equations is of a form where the matrices are full, complex, nonsymmetric, and a function of frequency. Boundary conditions for the direct method are the prescribed physical variables or impedance relationships at the nodes. The indirect method solves for single- and double-layer potentials on the surface, which can be postprocessed to obtain the physical variables. Matrices for the indirect method are complex-valued and symmetric, which enables coupling with finite element models.

The boundary-element method is particularly powerful for solving field or semi-infinite problems. It can be readily applied to coupled structural/acoustical analysis or to solve for the boundary conditions of a finite element model. The method assumes isotropic material properties and works well for structures that have a high volume-to-surface ratio, but is not suitable for plate and thin-shell problems.

Distributed (Continuous) System Methods. A number of specialized programs treating the analysis of distributed or continuous structural systems such as beams, plates, shells, rings, etc., have been developed.[6,7] Each program can be applied for a broad, selectable range of physical properties and dimensions of the particular structural shape. Not all programs employ the same theory of elasticity. Thus, the user must examine the theoretical basis on which the program was developed. For example, the user must determine if the program includes such effects as rotary inertia or shear deformation.

Preprocessing and Postprocessing of Shock and Vibration Data. Experience with the general-purpose analysis programs previously described indicates two major shortcomings: (1) a large amount of development time is required to debug the structural models, and (2) the large amount of tabulations and/or much of the results of the analysis are very difficult to evaluate. To alleviate these problems, programs have been written, called *preprocessors* and *postprocessors,* which use sophisticated interactive graphics in combination with algorithms. Such programs greatly simplify the construction and verification of the models, and presentation of the results of the analysis. These highly efficient programs often can be run on personal computers, independent of the larger computer required to exercise the model. Many organizations have developed their own preprocessors tailored to their product lines. Commercial software packages also are available for this purpose. Interfaces have been developed so computer-aided design (CAD) and the CAD database can exchange the obtained structural model data.

Statistical Energy Analysis. *Statistical energy analysis* (SEA)[13] is used to predict the structural response to broadband random excitation in frequency regions of high modal density (see Chap. 11). In these frequency regions, response predictions for individual normal modes are impractical. Structural response is treated in a statistical manner, that is, an estimate of the average response is computed in frequency bands wide enough to include many normal modes. The structural system is divided into components, with each component described by the parameters of modal density and loss factor. A third modeling parameter is the energy transmission characteristics of the structural coupling between components. SEA is valuable in predicting environments and responses for structures in the conceptual design phase, where detailed structural information is not available. Chapter 11 describes SEA in detail.

Personal Computer–Based Applications. Almost all analytical and experimental applications that are available on mainframe computers and workstations can also be found for personal computer systems.[14,15] Mainframes and workstations are often used for applications requiring large amounts of memory and disk space; fast processing speeds, such as large finite element models; and vibration control and data analysis for tests with a great number of control and response channels. However, for most other computation efforts, both analytical and experimental, personal computers can be employed. The following are examples of general-purpose applications that are widely used on the personal computer.

Technical computation packages are available that allow the user to obtain solutions to dynamics equations without resorting to programming. Equations can be entered using symbolic mathematical formulas that involve integrals, differentials, matrices, and vectors. Solutions can be plotted in two and three dimensions. Such equations may be solved using either symbolic or numerical methods. Additional capabilities include curve fitting, fast Fourier transform (FFT) calculation, symbolic manipulation, numerical integration, and the treatment of vectors and matrices as variables.

Spreadsheet software developed for accounting can also be used to manipulate vectors and matrices. Their graphical capabilities can be used to generate report-quality plots. Commercial data acquisition systems can store time- or frequency-domain information in files compatible with spreadsheets. Even ensemble averaging can be accomplished for the computation of statistical functions (see Chap. 22).

Graphical programming software exists for data acquisition and control, data analysis, and data presentation and visualization. Instruments such as oscilloscopes, spectrum analyzers, vibration controllers, etc., can be emulated in graphical form. These instruments can acquire, analyze, and graphically present data from plug-in data acquisition boards or from connected instruments.[15]

SPECIAL-PURPOSE APPLICATIONS

The need for a special-purpose program[6,7] may arise in several ways. First, for an engineering activity engaged in the design, on a repetitive basis, of what amounts analytically to the same structure, it may be economical to develop an analysis program that efficiently analyzes that particular structure. The analysis of vibration isolator systems, automobile suspension systems, piping systems, or rotating machinery, are examples. Similarly, parametric studies of a particular structure, either to gain an understanding or to optimize the design, may require a sufficient number of computer runs to justify the development of specialized software. A second type of special-purpose program includes programs that in some way perform an unusual type of analysis, for example, the analysis of nonlinear systems. Access to existing special-purpose programs is generally more restricted than is access to general-purpose programs, because they are often proprietary and their development requires a substantial investment.

EXPERIMENTAL APPLICATIONS

The classification *experimental applications* covers uses of computers which involve, in some way, the processing of shock and vibration information originally obtained during the test or field operation of equipment. Two development streams led to the applications described in later sections, namely, (1) the recognition of the computational efficiency of the fast Fourier transform (FFT) algorithm (see Chap. 14) and other advanced digital signal processing algorithms, and (2) the development of hardware FFT processors, using digital signal processor technology. These developments permit the use of digital computers for such tasks as vibration data analysis; shock data analysis; and shock, vibration, and modal testing. The information resulting from such applications is in digital form, which permits more sophisticated engineering evaluation of the information through further efficient digital processing, e.g., regression analysis, averaging, etc.

Digital computers are used extensively in experimental applications such as (1) the acquisition and processing of shock and vibration data associated with a test or field operation of equipment, (2) controlling the vibration testing machine used to accomplish many of these tests, and (3) modal testing. In each of these cases, a digital computer–based system, along with specialized signal acquisition, signal processing, and signal generation hardware and software, is used to accomplish these complex applications, as discussed in the following sections.

DIGITAL SHOCK AND VIBRATION DATA ANALYSIS[16]

The basic principles of digital shock and vibration data analysis are thoroughly covered in other chapters and their references, as summarized in Table 27.1. Only methods that are fundamental to the discussed applications of digital computers that are not presented elsewhere are discussed here. Specifically, this section discusses (1) the definition of the estimates of the spectral density and cross–spectral density matrices used with multiexciter random vibration control systems; (2) tracking filters for the measurement of the amplitude and phase, as a function of frequency, of response and control data taken during a swept-sine vibration test; (3) the synthesis of transient signals that achieve a predetermined shock response spectrum (see Chap. 26); and (4) frequency response estimation.

Spectral Density Matrix. The *spectral density matrix* (SDM) is a matrix that consists of both power spectral density values as its diagonal elements and cross–spectral density values as its off-diagonal elements. It is the natural extension to matrices of the concepts of power spectral density and cross–spectral density that are discussed in Chap. 22. A SDM is both a Hermitian and a nonnegative definite matrix.[17–22] It can be estimated as follows.

TABLE 27.1 Summary of Data Analysis Applications

Application	Chapter
Spectral analysis for stationary vibration data	11, 14, 22
Spectral analysis for nonstationary vibration data	22
Correlation analysis for stationary vibration data	11
Probability analysis for stationary vibration data	11, 22
Fourier and shock response spectral analysis of shock data	23
Modal analysis of structural systems from shock and vibration data	21
Multiple input/output analysis of shock and vibration data	21
Average values and tolerance limits for shock and vibration data	20
Other statistical analysis of shock and vibration data	22
Matrix methods of analysis for shock and vibration data	28, Part I

Let $\{x(t)\}$ be an N-dimensional column-vector of time-histories, whose components are the waveforms $x_1(t), \ldots, x_N(t)$. These waveforms could, for example, be the acceleration responses of a system under test, at N measurement points, that is being excited by N vibration exciters with the use of N stationary Gaussian drive signals that are partially correlated (see Chap. 22). If their complex finite Fourier transform is defined as in Eq. (22.3), with $x(t)$ successively replaced by the $x_i(t)$ waveforms, the complex vector $\{X(f,T)\}$ is obtained, with the finite Fourier transforms, $X_1(f,T), \ldots, X_N(f,T)$, as its components. If the time-history vector $\{x(t)\}$ has a duration much longer than T, then as in Chap. 22 it can be partitioned into a series of nonoverlapping segments of data (often called *frames*), each of duration T, such that the average can be defined as

$$[W_{XX}(f,T)] = \frac{2}{n_d T} \sum_{i=1}^{n_d} \begin{Bmatrix} X_1(f,T) \\ X_2(f,T) \\ \vdots \\ X_N(f,T) \end{Bmatrix}_i \{X_1^*(f,T)\ X_2^*(f,T) \cdots X_N^*(f,T)\}_i \tag{27.1}$$

or using a more compact matrix notation as

$$[W_{XX}(f,T)] = \frac{2}{n_d T} \sum_{i=1}^{n_d} \{X(f,T)\}_i \{X(f,T)\}_i^H \tag{27.2}$$

In Eqs. (27.1) and (27.2), (1) the average is taken as in Table 22.3, where the estimates for the power and cross-spectra are defined using a finite Fourier transform, (2) $X_1^*(f,T)$ is the complex conjugate of $X_1(f,T)$, (3) $\{X(f,T)\}_i^H$ is the complex conjugate transpose of the vector $\{X(f,T)\}_i$, and (4) the subscript i refers to the ith nonoverlapping frame. As is shown in Refs. 17 to 19, the above average is an unbiased estimator for the spectral density matrix of the N-dimensional Gaussian stationary process $\{X(t)\}$, which converges to the true spectral density matrix of the process, $\{x(t)\}$, as T and n_d approach infinity. The use of windowing[17–19] in the definition of the $X_i(f,T)$ that are used in Eqs. (27.1), (27.2), and (27.3) reduces the errors associated with spectral side-lobe leakage (see Chap. 14).

Cross–Spectral Density Matrix. The *cross–spectral density matrix* (CSDM) is a matrix that consists of cross–spectral densities between the components of two multidimensional Gaussian stationary random processes. It is defined similarly as the previously discussed spectral density matrix. It is the natural extension of the cross–spectral density concepts that are discussed in Chap. 22. The CSDM is further discussed in Refs. 17 to 22. For simplicity and without loss of generality, the CSDM estimate is defined in the following discussion for the case where the two random process vectors have the same dimension.

Let $\{x(t)\}$ and $\{y(t)\}$ be two N-dimensional column vectors of time-histories, which respectively consist of the waveforms $x_1(t), \ldots, x_N(t)$ and $y_1(t), \ldots, y_N(t)$. The $\{x(t)\}$ waveform vector can, for example, be the vector of random drive signals that are used to excite the system under test, as in Fig. 27.8. The $\{y(t)\}$ waveform vector in this case will be the vector of responses, at the N instrumented points located on a system under test, that is being excited by N-exciters with the use of the drive vector $\{x(t)\}$. If the finite Fourier transform vectors $\{X(f,T)\}$ and $\{Y(f,T)\}$ are similarly defined, with components $X_1(f,T), \ldots, X_N(f,T)$ and $Y_1(f,T), \ldots, Y_N(f,T)$, it is found that the average cross-spectrum can be defined as

$$[W_{YX}(f,T)] = \frac{2}{n_d T} \sum_{i=1}^{n_d} \{Y(f,T)\}_i \{X(f,T)\}_i^H \tag{27.3}$$

where the above average is taken as in Eqs. (27.1) and (27.2) but with the use of the vector $\{Y(f,T)\}_i$ instead of the vector $\{X(f,T)\}_i$ for the ith nonoverlapping frame. As in the spectral density matrix estimator in Eqs. (27.1) and (27.2), and as is shown in Refs. 17 to 19, the above average is an unbiased estimator for the cross-spectral density matrix between the N-dimensional Gaussian stationary processes $\{x(t)\}$ and $\{y(t)\}$, which converges to the true CSDM as T and n_d approach infinity. There are also convergence results for fixed T when $\{x(t)\}$ and $\{y(t)\}$ are ergodic (see Chap. 1) and with the use of a window function as n_d approaches infinity for Eqs. (27.1) through (27.3).[17]

Tracking Filters. *Tracking filters* are specialized filters that implement a narrow bandpass filter, of selectable bandwidth, centered about the instantaneous frequency of a sine wave with a frequency that is changing with time (commonly called a *sweeping* sine wave).[23] These filters are used to extract the amplitude of the sweeping response sine wave, as well as its phase with respect to the modulating signal used in the tracking filter implementation. This algorithm, based on proprietary technologies, provides essentially a time-varying estimate of the Fourier spectral amplitude, in essentially a continuous manner, of a sweeping sine wave,[23] as illustrated in Fig. 22.7.

A simplified implementation of a tracking filter is shown in Fig. 27.6. It accepts a sweeping sine wave response from a system under test that is being excited by a sweeping sine wave. This response signal is shown as $A\sin(\omega t + \theta) + n(t)$, with a frequency of ω radians/sec, an amplitude A, a phase of θ with respect to the modulating signals $\sin(\omega t)$ and $\cos(\omega t)$, and an additive distortion and noise term $n(t)$. By modulating the input signal with the sine and cosine terms shown in Fig. 27.6, the energy at the sweep frequency ω is translated to 0 Hz, hence the name 0-Hz intermediate frequency (IF) detector, where the data detection[23] is accomplished by the two low-pass filters that produce the imaginary and real-term estimates of the complex amplitude of the sweeping sine wave response of the system under test. From these filter outputs, the amplitude A and phase θ, with respect to the modulating signal, are estimated. By analyzing several response signals in this manner with separate tracking filters that use the same modulating signals, the relative phase between several sweeping sine wave responses can be measured since their individual phase measurements have a common phase reference. In this way, tracking filters can be used for such diverse applications as frequency response function and matrix estimation, and multiexciter and single-exciter swept sine wave control.

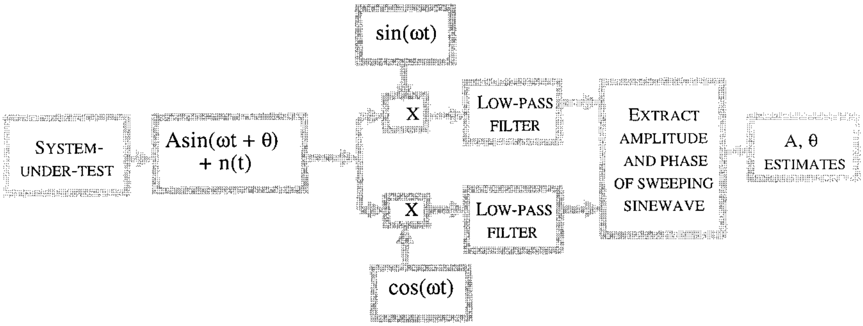

FIGURE 27.6 Tracking filter using 0-Hz intermediate frequency (IF) detector.

The tracking filter operation shown in Fig. 27.6 provides an estimate of the complex amplitude at the modulating signal's frequency, which is typically the same as the swept sine wave's frequency. It is important that the modulating signals and the drive signals used to excite the system under test be in frequency and phase synchronization for the best results. Because it can track a sweeping sine wave, it provides a way of measuring the nonstationary spectral amplitudes associated with swept sine wave tests and rotating machinery vibration analysis. By its nature, it discards other terms not centered at the sweep frequency, like unwanted harmonic and nonharmonic distortion terms. Tracking filters can also be used to track frequencies other than the fundamental response frequency, like the frequencies of harmonics. Some modern digital vibration control systems provide the function of Fig. 27.6 by using dedicated digital signal processors to implement a digital tracking filter subsystem. These can provide an estimate of a sweeping sine wave's amplitude and phase at their sampling rate. Some provide estimates of as many as four to eight times per cycle of the drive signal.[23]

Shock Response Spectrum Transient/Shock Synthesis. *Signal synthesis* techniques are used in transient testing where the test's reference response is specified as a shock response spectrum, as discussed later in this chapter. This type of application is often referred to as *shock response spectrum synthesis.* The primary goal is to create or synthesize a transient signal with a predetermined shock response spectrum. Since the same shock response spectrum is possible for a large range of signals (see Chaps. 23 and 26), many such synthesis techniques are possible. Some are based on wavelet expansions[24,25] for pyroshock testing, and others on a transient created by windowing a stationary random signal (see Chap. 26, Part II).

The methods employed for pyroshock testing are based on the use of a weighted sum of *wavelets,* which are defined as a set of orthogonal functions with finite durations. The wavelets used for shock synthesis are either windowed sine waves with an odd number of half cycles or damped sinusoids.[24,25] These are used in an inverse wavelet transform process[24–26] to represent the transients. The transients are chosen as sums of these wavelets. The amplitude of the wavelets is modified so that the sum of such wavelets is a transient that achieves the prescribed shock response spectrum.[24,25] Since the shock response spectrum definition (see Chap. 23) allows for many waveforms to have the same shock response spectrum, this many-to-one relationship allows for the further optimization of the resulting shock-synthesized transients.[25,27,28] They can be optimized, for example, to produce the least peak acceleration for a given peak shock response spectrum. This type of optimization can increase the peak amplitude of the shock response spectra that are possible with a particular system under test (see Fig. 27.8), thus extending the performance range of vibration test machines used for transient/shock testing.

The method used for seismic simulation involves windowed sections of broadband Gaussian stationary noise, also known as *burst-random transients.* These random transients are generated using a prescribed magnitude Fourier spectrum, assigning random phase to it, and using the inverse FFT to create a random transient with the specified magnitude spectrum. This transient is windowed (see Chap. 14) and its shock response spectrum is calculated. The calculated shock response spectrum is compared with the prescribed shock response spectrum, and the discrepancy is used to modify the magnitude of its Fourier spectrum. The synthesis iteration is repeated until the shock response spectrum of the synthesized windowed transient agrees with the prescribed shock response spectrum within some acceptable error. Again, the many-to-one characteristic of the shock response spectrum allows for further optimization of the synthesized random transient.

Frequency Response Function and Frequency Response Matrix Measurements. The computation of frequency response functions and frequency response matrices make use of the digital signal processor, A/D converter, D/A converter, and embedded distributed computer systems discussed in a previous section. The objective of these applications is to excite the system under test in such a way that its frequency response characteristics can be measured. This type of measurement is done as part of modal-testing, single-exciter, and multiexciter control systems applications to be discussed later in this chapter.

Single Input, Multiple Output (SIMO) Methods. In this method, a single drive signal is used to excite the system under test at any one time. A digital system, like those shown in Figs. 27.1, 27.2 and 27.5, can be used to drive a system under test and acquire multiple response signals from instrumentation on the system under test. The excitation signals can be impulsive, continuous broadband noise, transient noise, or swept sine waves. In all these cases, the complex-amplitude spectra are measured for both the drive and response signals by the digital system. The cross–spectral densities between the various response signals and the drive signal, as measured at the input to the system under test, are divided by the drive signal's power spectral density to obtain a frequency response function estimate between the single drive signal and the response signals (see Table 22.3). Typically broadband noise and swept sine wave excitations produce the best estimates for the needed frequency response functions, but at the expense of longer test times that may stress the test article or system under test. Frequency response functions can be measured, while using swept sine wave excitation, by using the tracking filters discussed previously.

A multiple-reference frequency response matrix estimate can be obtained by exciting the system with a hammer or a vibration exciter, one excitation at a time but at different locations, to successively obtain one column of the frequency response matrix estimate using this SIMO methodology. These methods may have problems with repeatability since the structure's characteristics may change between excitations (see Chap. 21).

Multiple Input, Multiple Output (MIMO) Methods. These methods excite the system under test with a digital system as in the previous section, but drive it with multiple simultaneous excitation signals, acquire the associated response signals, and process the thus-acquired response and drive signals to obtain the needed system frequency response matrix estimates. Most estimators used are based on the response equations[17–19]

$$[W_{cd}(f)] = [H(f)][W_{dd}(f)] \qquad \text{or} \qquad [W_{cc}(f)] = [H(f)][W_{dc}(f)] \tag{27.4}$$

where $[W_{cd}(f)]$ is an estimate of the cross–spectral density matrix between the response vector $\{c(t)\}$ and drive-signal vector $\{d(t)\}$, as defined in Eq. (27.3). $[W_{cc}(f)]$ and $[W_{dd}(f)]$ are estimates of the spectral density matrices of the response vector $\{c(t)\}$ and the drive-signal vector $\{d(t)\}$, as defined in Eqs. (27.1) and (27.2), and $[W_{dc}(f)]$ is the complex-conjugate and matrix transpose of $[W_{cd}(f)]$.[17–19] The above two equations that are part of Eq. (27.4) can be solved separately for $[H(f)]$. The left equation is relatively insensitive to measurement noise but sensitive to drive-signal noise, and the right equation exhibits the reverse condition. These types of frequency response matrix estimates are very similar to the type 1 and type 2 frequency response estimators discussed in Chap. 21. Here the emphasis is on the use of Eq. (27.4) with the spectral density matrix and cross–spectral density matrix estimates, defined in Eqs. (27.1) through (27.3), to estimate $[H(f)]$. The use of Eq. (27.4) for system identification will also be discussed as part of the sections on multiexciter digital vibration control and modal testing.

Note that to use Eq. (27.4), either the matrix $[W_{dd}(f)]$ or $[W_{dc}(f)]$ needs to be inverted. For this reason, the left side of Eq. (27.4) is typically used because it is easier to guarantee that $[W_{dd}(f)]$ is not singular rather than $[W_{dc}(f)]$. In many cases, $[W_{dc}(f)]$ is not a square matrix because the dimensions of $\{c(t)\}$ and $\{d(t)\}$ are not equal and clearly $[W_{dc}(f)]$ is singular in that case. Some digital systems make an additional simplification by exciting the system with mutually uncorrelated random drive signals and thus "ensure" that $[W_{dd}(f)]$ is a diagonal matrix. This simplification can cause additional problems since the measured $[W_{dd}(f)]$ will typically not be diagonal even if the drive signals are uncorrelated due to unavoidable measurement and exciter noise. Hence, in practice, it is better to measure $[W_{dd}(f)]$ and invert it as a matrix rather than just inverting its diagonal elements and assuming that its matrix inverse is diagonal. This is the preferred way to characterize the system under test for multiexciter control applications to be discussed later. In many of these cases, the drive signals are measured as inputs to the test article by load cells (see Chap. 12). The use of MIMO methods can separate modes that correspond to the same repeated root or eigenvalue (see Chap. 28, Part I), whereas SIMO methods may not (see Chap. 21).

DIGITAL CONTROL SYSTEMS FOR SHOCK AND VIBRATION TESTING

The vibratory motions specified for the majority of vibration tests are either sinusoidal[23,29] or random[29] (see Chap. 20). A smaller percentage of the vibration tests are prescribed to be either a classical-shock transient[27] (see Chap. 26, Part I), a shock response spectrum synthesized transient (see Chap. 26, Part II), a long-term response waveform,[30] or mixed-mode[31] (sine-on-random or narrow bandwidth random-on-random) vibratory motions. These specified environments are typically represented by a reference response signal, in either the time or frequency domain, that the digital control system servo uses as a control reference to achieve the specified control response at the chosen control point or points that are associated with the test (see Chap. 20).

The reference response is either a frequency-domain or time-domain signal that represents the specified vibration environment associated with a shock or vibration test. It is typically specified as a reference spectrum, which describes the vibration environment in the frequency domain to which the control response spectrum is compared as part of the digital vibration control process. It could be a power spectral density for a random vibration test, an amplitude vs. frequency profile for a swept-sine test, a shock response spectrum for a shock test, or a finite Fourier spectrum (see Chaps. 20 and 22) for a generalized transient or a long-term reference response waveform test. Time-domain vibration environments, like transient and long-term response waveforms, are represented by a reference pulse or reference waveform, whereas frequency-domain-specified environments like random, swept-sine, and shock response spectrum synthesis shock tests, are specified with an appropriate reference spectrum. Typically, the time-domain reference signals are converted to the frequency domain as part of the feedback control and drive-signal synthesis process, using an appropriate time-to-frequency and frequency-to-time transformation process.

Vibration tests are accomplished with the use of vibration test machines, as discussed in Chap. 25, and a *digital vibration control system* (DVCS). The DVCS employed to control the vibration level(s) during the test typically utilizes the output signal from a control transducer (usually an accelerometer) mounted at an appropriate location on the vibration exciter's test fixture (part of the vibration test machine) or the unit under test (UUT) to provide a feedback signal to its servo system. The servo system in turn drives the power supply of the vibration testing machine used for the shock or vibration test. The servo system is largely implemented digitally using analog-to-digital (A/D) converters, digital-to-analog (D/A) converters, digital signal processors (DSPs), embedded processors, and general-purpose processors, to adjust the drive-signal amplitude and spectrum for the system under test so as to maintain the control transducer's response level and waveform characteristics as close to the test's specified reference response as possible.

The overall block diagram of the vibration test system, when using electrodynamics exciters and accelerometers for control transducers, is shown in Fig. 27.7. In this case, the DVCS drives the system under test with an analog drive signal, $d(t)$, such that the control response at the chosen control-point location on the system under test agrees with the specified reference response with an acceptable error. The DVCS consists of (1) an input subsystem, which acquires the response waveform of the system under test, $c(t)$; (2) the digital servo subsystem, which creates the digital drive signal through a closed-loop process that causes $c(t)$ to agree with a suitable description of the specified test reference signal; and (3) the output subsystem,

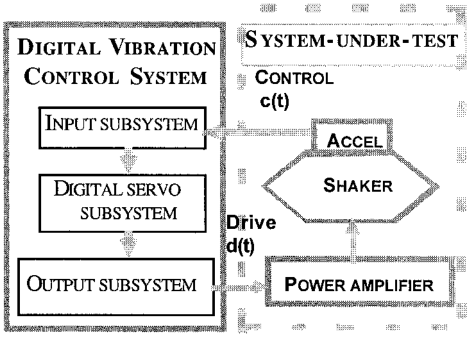

FIGURE 27.7 General setup for vibration test system.

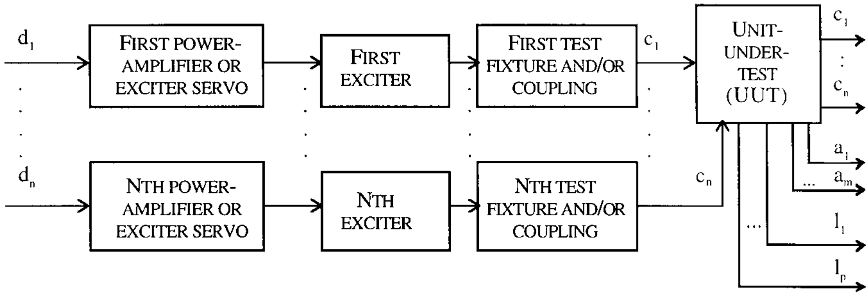

FIGURE 27.8 General setup for multiple-exciter vibration test system.

which converts the digital description of the generated drive signal into an equivalent analog drive signal, $d(t)$, used to drive the system under test.

A typical system-under-test configuration for both single and multiple exciters is shown in Fig. 27.8. If there is only one exciter involved, then only the top leg of the block diagram in Fig. 27.8 is used. Here, d_i means the drive signal generated by the DVCS that is used to drive the ith exciter. This drive signal is sent to the exciter's power amplifier (when using electrodynamic exciters), which in turn drives the exciter. For electrohydraulic exciters, this drive signal is sent to the exciter's servo amplifier, which in turn drives the hydraulic servo-valve subsystem, as discussed in Chap. 25. The exciter, either electrohydraulic or electrodynamic, then drives a test fixture (see Chap. 20), which in turn drives the unit under test. The test is either instrumented by mounting control transducers, which are typically accelerometers (see Chap. 12), on the test fixture, here shown by the signal c_1 through c_n, or on the UUT as shown by the signals c_1 through c_n in Fig. 27.8. These chosen control signals are then sent to the input subsystem of the DVCS where they are either averaged or their maximum or minimum, as a function of frequency, is extracted to create a composite response spectrum.

The signals a_1 through a_m in Fig. 27.8 are additional or auxiliary responses of the UUT that are monitored during the test as additional signal channels to be analyzed as part of the test. The signals l_1 through l_p are input channels that are to be used for *limiting* during the test. This limiting may involve either limits on the response or limits on the applied force to the UUT, as discussed in Chap. 20. For multiexciter applications, there are n exciter systems with n drive signals, d_1 through d_n. These drive signals are processed as in the single exciter case discussed before. The basic difference is that the n exciters will drive the UUT jointly through the fixture that connects the UUT to the multiple exciters. The response to this vector of drive signals is also a vector comprised of the control responses c_1 through c_n. This test configuration and its associated control methods are further discussed in a subsequent section. In either the single- or multiexciter control configuration, the control feedback signals, auxiliary response signals, and the limit signals are routed to the input subsystem of the DVCS.

A block diagram of the input subsystem is shown in Fig. 27.9. Here only the control-feedback signals are shown as inputs to the DVCS's input subsystem. These feedback signals, also called control-response channels, or simply control signals, are each sensed through an input signal conditioning system and analog-to-digital (A/D) converter subsystem. The input signal conditioning typically consists of an instrumentation amplifier, followed by a ranging amplifier to optimize the signal's amplitude as presented to the A/D converter, and an antialiasing filter (see the input

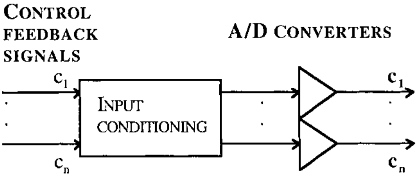

FIGURE 27.9 Input subsystem for digital vibration control system.

subsystem in Fig. 27.2). This conditioned analog signal representing the chosen response signal is finally presented to the A/D converter subsystem for conversion into a digital time-history.

Typically, other points on the UUT or on the vibration test machine are also monitored by the digital control system for subsequent vibration analysis or limiting. The input subsystem then sends digitized versions of the control signals, here represented by the c_1 through c_n, to the DVCS's servo subsystem, as shown in Fig. 27.10. The digital control-response time-series, c_1 through c_n, are then sent to a time-to-frequency block shown in Fig. 27.10. The function of this block varies with the type of vibration control. For random vibration testing, this block estimates the control-response power spectral density. For swept-sine vibration testing, this block typically produces either the fundamental amplitude or the overall response root-mean-square (rms) estimate using tracking filters or variable time-constant rms detectors.[29] For other types of vibration testing, this block is typically an FFT estimator (see Chap. 23). These estimates are further processed to produce either a single control-response spectrum, C_1, for single shaker control, or a control-response vector, with components C_1 through C_n, for multishaker control. The type of processing is again application-specific. These control-response amplitude estimates are then sent to a block that updates the drive-signal amplitude and spectrum to minimize the difference between these control-response amplitudes and the specified test reference for single-shaker control, or the test's reference-response vector for multishaker control applications. The updated drive amplitude(s) and their respective spectra are then sent to a frequency-to-time transformation block, which converts the spectral representation of the drive signal(s) into a digital time series of the time-domain drive that will be used to excite the system under test as previously described. This digital time-series signal or vector, comprised of d_1 through d_n for multishaker control, is then sent to the output subsystem (see Figs. 27.5 and 27.11) for conversion into an analog signal or signals to be used to drive the previously discussed system under test in Fig. 27.8. The output subsystem is shown in Fig. 27.11. The digital version of the drive signal or signals are synthesized to analog-driving voltages by the system's output subsystem. These digital drive signals are then converted into analog signals by the subsystem's D/A converters. The D/A converter output signals are filtered to eliminate the images generated by the D/A converters, and the final output is attenuated from the D/A converter's full-scale voltage to produce the proper amplitude exciter drive signal d_1 for single-shaker control or drive-

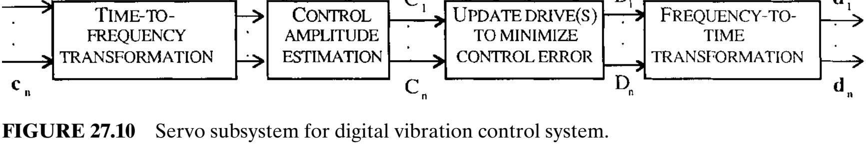

FIGURE 27.10 Servo subsystem for digital vibration control system.

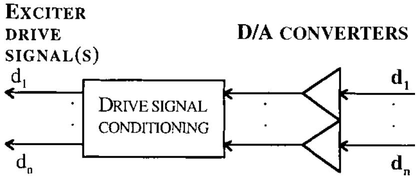

FIGURE 27.11 Output subsystem for digital vibration control system.

signal vector for multiexciter control (see Fig. 27.5). These conditioned analog drive signals are output by the DVCS to drive the system under test.

Initially, with the advent of dedicated FFT processors and minicomputers, it became possible to perform spectral analysis of random processes rapidly enough to permit the use of digital control systems for random vibration testing. Further developments in digital signal processors, embedded and distributed processors, personal computers, and workstation technologies extended the range of vibration testing to include swept-sine, transient waveform, long-term waveform, and multishaker testing. Most shock and vibration testing remains based on single-shaker methods, but multishaker testing is becoming more important when the size and weight of the UUT dictates its need, or when the prescribed vibratory motions are inherently multiaxis or otherwise consist of multiple degree-of-freedom vibratory motions.[30,32,33] Enough differences exist between single- and multishaker digital control systems for these to be discussed separately in the following sections. The previous discussion, however, illustrates the areas where they are similar.

Single-Exciter Testing Applications. The great majority of shock and vibration testing is specified and accomplished with the use of single exciters or shakers. These are typically single-axis tests. Multiaxis test specifications are accomplished one axis at a time when using single exciters. Random, swept-sine, mixed-mode, transient waveform, and long-term response waveform vibration applications can be accomplished as long as the vibration test machine capabilities and the weight and size of the unit under test allow it (see Chap. 25).

In many single-exciter vibration tests, especially random and swept-sine tests, even though only a single drive signal is employed, multiple control accelerometer input channels are used. In these cases, the multiple control signals are combined by averaging them or by selecting the largest or smallest response, as a function of frequency, to create a composite control-response spectrum, with the control-estimation block in Fig. 27.10. Often multiple input channels are additionally used for limit control, as discussed earlier. The single-shaker control applications that use a single drive signal to excite the system under test, and use multiple input control signals and/or limit signals, are called *multiple input, single output* (MISO) control systems.

Random. These systems excite a test item with an approximation of a stationary Gaussian random vibration (see Chap. 2). Digital random vibration control systems use signal processing that mimics analog methods in their fundamental control and measurement methods [see Eq. (22.7)] and offer significant user-interface and graphics subsystems that provide greater system tailoring and varied displays and graphs of ongoing test conditions. Digital systems also afford greater stability, more freedom in the control methods, and superior accuracy than those control systems that directly use analog methods.[29]

The control-response waveforms from the system under test are low-pass filtered to prevent aliasing (see Chaps. 13 and 22) and converted to a sequence of control samples by the input subsystem of the digital system as previously discussed. The averaging control, the spectrum analyzer, and the display are implemented by the time-to-frequency and control-amplitude estimation blocks. These blocks use a discrete Fourier transform (DFT), as discussed in Chap. 14, to estimate the power-spectral density (see Table 22.3) of the control responses $c_1(t)$ through $c_n(t)$. The random noise generator and the analog equalizer, used in previous analog random vibration systems, are replaced by an analogous digital process using a DFT and a time-domain randomization algorithm.[29] This is accomplished in the frequency-to-time processing block within the DVCS in Fig. 27.10. The lines of the DFT (see Chap. 14) in the digital system play the role of the contiguous narrowband filters in the equalizer of the analog system.[29] Equalization is the adjustment of the amplitude of the output of a bank of narrowband DFT filters, which is an FFT equivalent (see Chap. 22), whose amplitude is given by the drive signal's spectrum amplitude, $D_1(f)$, that correspond to the center frequency of each DFT filter, such that the power spectral density of the control response matches that of the test-prescribed reference.

The equalization of the drive waveform can be accomplished directly, by generating an error correction from the difference between the control power-spectral density and the reference spectral density. The equalization can also be accomplished indirectly through a knowledge of the system frequency response function magnitude. The required system frequency response function (see Chap. 21) is the ratio of the Fourier transform of the control response (usually an acceleration) and the Fourier transform of the drive-voltage signal, as is discussed in an earlier section. Only the magnitude of the frequency response function is required for random control, since the relative phase between frequencies is random and not controlled.

The drive spectrum D_1, that results from the "update drive to minimize error" block in Fig. 27.10, is multiplied by a random phase sequence and its inverse FFT is calculated to create the corrected drive time series $d_1(t)$. Samples of the corrected digital drive time series, $d_1(t)$, are fed through the output subsystem in Fig. 27.11 within the DVCS, converted to an analog signal, low-pass filtered to remove the images caused by the D/A converter, further amplified, and then sent as the analog signal d_1 to the power amplifier input of the system under test, which completes the loop. Corrections to the drive are not made continuously in the digital random-vibration control system. Many samples of the drive (often thousands) are output between corrections. Many digital systems use a *time-domain randomization process*[29] that converts the finite duration $d_1(t)$ drive block into an indefinite duration signal with a continuous power spectral density that has the same values as $d_1(t)$'s at the discrete frequencies at which the FFT was evaluated. The time between drive corrections is called the *loop time.* The loop time for digital random vibration control systems can be from a fraction of a second to a few seconds depending on the type of averaging used for control-response power spectral density estimation.

The speed at which the system can correct the control spectrum is determined by two factors. The first is the loop time, and the second is the number of spectral averages required to generate a statistically sound estimate of the control power spectral density (see Chap. 22). The loop time is usually the shorter of the two. Typically, a compromise is required; an estimate of the power spectral density with a significant error is used, but only a fraction of the correction is made in each loop. The type of spectrum average, linear or exponential, also has a large effect on the averaging time where the exponential average affords a shorter averaging period, but only a fraction of a correction is made in each control loop to ensure system closed-loop stability.[29] In such cases, multiple corrections occur within the averaging period. The equivalent bandwidth of the DFT filters is dependent on the number of lines in the DFT, the type of spectral window that is used (see Chap. 14), and the sampling rate of the D/A and A/D converters. These parameters are usually options chosen by the operator either directly or indirectly. The averaging parameters are also typically operator-specified.

Swept-Sine. The objective of a digital sine wave vibration test control system is to drive a system under test, as shown in Fig. 27.8, with a sweeping sine wave excitation such that the control-response signals, when processed by the control-response estimation block shown in Fig. 27.10, agree with the specified test reference within some acceptable error. The control-response outputs, c_1 through c_n, of the system under test are filtered and digitized with the input subsystem of the DVCS. The needed tracking filters,[23] variable time-constant rms detectors,[29] averaging control, and control signal selection are implemented within the appropriate blocks in Fig. 27.10 by the use of an embedded DSP subsystem for the required specialized signal-processing functions. It is however nontrivial to implement tracking filters digitally,[23] as previously discussed. Many systems, in the interest of simplicity, do not use true tracking filters, but approximate this function by using FFT methods. In any case, these are implemented in the time-to-frequency transformation and control-amplitude estimation blocks within the servo subsystem in Fig. 27.10 within the DVCS.

The sine-wave generator is implemented by using samples of a digitally generated sine wave, usually by a digital signal processor subsystem within the frequency-to-time transformation block in Fig. 27.10, which are sent to the output subsystem in Figs. 27.5 and 27.11, to be used to drive the system under test in Fig. 27.8. The swept-sine test parameters are entered by the test operator through the DVCS's graphical user interface to be stored in a test parameter file for use in a subsequent test. The control-response servo subsystem shown in Fig. 27.10 is implemented by an algorithm that compares the computed amplitude of the control waveform with the required control amplitude, as defined by the test setup, and generates a corrected sampled drive waveform. This function is accomplished by the "update drive to minimize control error" block shown in the DVCS's servo subsystem block diagram in Fig. 27.10. The sampled drive waveform is converted to an analog drive waveform by the D/A converter and sent to the low-pass filter and output attenuator shown in Fig. 27.5, which illustrates the DVCS's output subsystem block diagram shown in Fig. 27.11. This resultant analog drive signal, d_1, is used as the input to the power amplifier within the system-under-test block diagram in Fig. 27.8 to complete the closed loop.

Swept-sine vibration tests can require that the frequency be stepped in a sequence of fixed frequencies, or swept in time over a range of frequencies. However, the stepped approach can generate vibration transients every time the frequency of the sine-wave drive signal is changed. A swept sine is the changing of the frequency from one frequency to another in a smooth continuous manner. This is the preferred drive-signal generation method since it creates no significant transients as the frequency is changed. Again, many commercial control systems use the stepped-frequency method because of its simpler implementation. The rate of change of frequency with respect to time is called *sweep rate.* Both logarithmic and linear swept sines are required. For a *logarithmic sweep,* the change in the logarithm of the frequency per unit of time is a constant. For a *linear sweep,* the change in frequency per unit of time is a constant. Because the drive waveform is usually generated in blocks of samples, care must be taken in swept-sine vibration tests to ensure that the frequency and amplitude change is continuous. The correction of the drive amplitude in a digital system is not continuous, but discrete. The time between amplitude corrections is also called the loop time, and is controlled by the number of samples that must be taken to define the control-waveform amplitude and the required computations to compute the corrected drive waveform. Here as in the other DVCS applications, a control loop iteration is the completion of one complete cycle from the correction of one drive waveform to the next.

The control-response amplitude can vary rapidly as the frequency changes due to system resonance, and the required loop time is measured in small fractions of a second. For stability, the complete correction of the drive waveform is not usually made in each loop. The maximum rate of drive waveform correction is called the *compression speed*[29] and is usually expressed as decibels per second (dB/sec). If the compression speed is too fast, system instabilities can occur. If the compression speed is too slow, the correct amplitude will not be maintained. The required compression speed is a function of (1) frequency, (2) sweep rate, (3) the system dynamics, (4) the amount of noise present in the response measurement, and (5) the degree to which the response of the system under test is nonlinear. Limited operator control of the compression speed is usually provided. The bandwidth of the digital tracking filter[23,29] will affect the stability of the system. Specifically, as the bandwidth of the tracking filter decreases, the delay in the output of the tracking filter increases.[23] As the filter delay increases, the compression speed must be decreased to maintain stability.[29] Some of the more advanced DVCSs used for this purpose accommodate the change in correction rate automatically to ensure a good compromise between control speed and accuracy. However, the user needs to make the required compromise by selecting the bandwidth of the tracking filter or the time constant of the rms measurement to be used during the swept-sine test, which trades off the ability to reject components in the control waveform at frequencies other than the drive frequency, and the ability of the control system to respond quickly to changes in the control waveform amplitude.

Transient/Shock. Sometimes it is desirable to perform shock or transient testing using electrodynamic or electrohydraulic vibration test machines.[24] The ability to employ this method is dependent on such parameters as the stroke (the maximum allowable motion of the vibration exciter); the peak amplitude, spectral characteristics of the specified transient waveform; the amount of moving mass during the test; and the test time. If the required test is within the performance capability of an available vibration test machine, the ability to obtain and control the desired motion has been greatly expanded by the use of digital control equipment.[24,27] In general, the servo control of a shock test parallels that used for the other vibration-control methods but, in this case, the controller compares the control accelerometer time-history response to a reference waveform as part of the control process. The primary difference here is that the time-to-frequency and frequency-to-time transformations in Fig. 27.10 are accomplished using an FFT of the transient with the forward or inverse transformations, respectively. If necessary, the controller drive signal is altered to minimize the deviation of the control accelerometer response from the reference based on the comparison between the control-response and reference-response FFT spectrum. This discrepancy is used to update the drive spectrum in the "update drive to minimize control error" processing block within the DVCS's servo subsystem in Fig. 27.10.

Shock-test requirements may be specified in one of two ways. The first and more direct method specifies a certain acceleration waveform, such as a half-sine pulse of specified duration and maximum acceleration. These are called classical-shock transients (see Chap. 26, Part I). The DVCS in this case needs to modify such classical pulses by adding a pre- and postpulse to the overall test pulse waveform[27] to ensure that the response of the system under test returns to a zero acceleration, velocity, and displacement conditions at the end of the shock test. Typical pulses used as the reference-response waveform, $r(t)$, in addition to the previously discussed half-sine pulse, include final and initial-peak sawtooth, rectangular, and trapezoidal pulses of varying duration and amplitudes (see Chap. 26, Part I). The control method that is used is a subset of what is used for long-term response-waveform control, discussed in a later section, usually without a need for the overlap and add indirect convolution method.[1]

The second method employs the shock response spectrum (see Chaps. 23 and 26, Part II) as the means of characterizing the response of the control points.[25,28] In this case, the control-response spectrum, $C(f)$, and the reference-response spectrum, $R(f)$, are specified as a shock response spectrum. The requirements for the reference shock response spectrum must specify the frequency range, frequency spacing, damping factor, type of spectrum, and either maximum or nominal values with an allowable tolerance on spectrum values.[24,28] Reference pulses are generated using one of the shock response spectrum synthesis techniques[24,25] discussed previously. The control method that is used is called the *wavelet amplitude equalization* (WAE) method. If the test requirements are specified as a shock response spectrum reference, $R(f)$, then during the test the shock response spectrum of the control-response waveform is computed and compared with the prescribed $R(f)$. The difference is then used to update the drive signal, which is expressed as a weighted sum of wavelets. The weights in the sum represent the amplitude of the various wavelets. These amplitudes are varied as a function of the discrepancy of the control-response shock response spectrum and the reference shock response spectrum. Care is required when this difference is large since the control problem is highly nonlinear due to the nonlinear dependence of the control-response shock response spectrum to the wavelet amplitudes of the drive signal. Because of this, the control corrections are iterative and yield an approximate shock response spectrum for the control response.

Mixed-Mode. Digital vibration test control systems are available which can control several sine waves superimposed on a stationary random vibration test.[31] This is called *sine-on-random vibration testing* or *swept-sine-on-random vibration testing.* Systems are also available that can control swept narrow bandwidths of nonstationary random superimposed on a stationary random vibration test. This is called *swept-narrow-bandwidth-random-on-random testing.* It uses a variation of the random vibration control methods, previously discussed, by modifying the reference-response spectrum during the test to create sweeping narrow bandwidths of random that are superimposed on a broad-bandwidth random background.[31] The control or servo-process for the case of sine-on-random works as a parallel connection of a random vibration and a swept-sine control system. A simplified block diagram of this process is shown in Fig. 27.12.

The two critical differences between mixed-mode controllers and individual random and swept-sine controllers are the presence of the bandpass/reject and synthesize composite subblocks in Fig. 27.12. The bandpass/reject subblock in Fig. 27.12 separates the swept-sine and random backgrounds into two separate signal streams. The swept-sine component is fed into the sine control section and the random background section is fed into the random control section. These separate controllers, with needed synchronization between each other, then create separate drive-amplitude updates for control of their respective component. These separate drive-amplitude updates are combined into a composite drive signal, containing the random and swept-sine components in a single drive signal, by the synthesize composite section in Fig. 27.12. This composite drive is then sent to the system under test to complete the control loop. The bandpass/reject section should employ advanced signal-estimation techniques to determine the phase and amplitude of the control-response sinusoids that are masked by the background random noise contained in the composite control-response signal, $c(t)$.

Long-Term Response Waveform Control. The objective of a *long-term response waveform control,* or simply *waveform control,* test is to drive the system

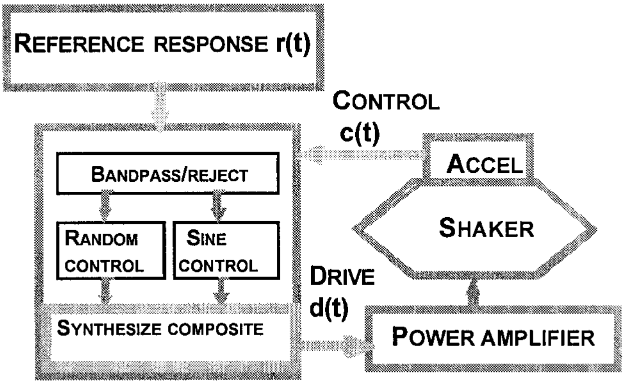

FIGURE 27.12 Swept-sine on random vibration control system.

under test in Fig. 27.8 with a drive signal, $d(t)$, such that the time-domain response of the chosen control transducer [$c_1(t)$ in Fig. 27.8] matches the test-specified reference waveform $r(t)$ within an operator-specified error margin. The same type of DVCS shown in Figs. 27.7 through 27.11 can be used for this application. The DVCS is tasked with finding the drive signal, $d(t)$, which achieves the objective of the waveform control test.

This type of testing is sometimes called *waveform replication testing* and uses an estimate of the system-under-test's frequency response function to control the response of the system under test. The frequency response function estimate relates the control-response waveform, $c_i(t)$, to the electrical drive waveform, $d(t)$, that the DVCS uses to control the system under test. It is the principal quantity that is used in the waveform control process. The frequency response function needs to be estimated prior to the vibration test. It is measured by exciting the system under test with a drive-voltage waveform having a bandwidth of at least that of $r(t)$, which is output through the DVCS's output subsystem to the system under test. During this test phase, which is often called system identification or characterization, the response of the chosen control point, $c_i(t)$, is measured and the drive signal, $d(t)$, which is used to achieve this response, is also stored. These two signals, $c_i(t)$ and $d(t)$, are then used to calculate the system-under-test frequency response function $H(f)$ (see Table 22.3). The functions $H^{-1}(f)$ and $r(t)$ are then used in conjunction with an overlap-and-add fast indirect-convolution method[1] to generate a drive signal that should cause the system-under-test's control response, $c(t)$, to agree with the specified reference-response, $r(t)$, within an acceptable error margin.[30,32] Often multiple control iterations that use $H^{-1}(f)$, $r(t)$, and $c(t)$, within the DVCS's servo subsystem, as part of an overlap-and-add fast indirect-convolution method, are needed to achieve the test's goal.[30,32]

The unit under test needs to be part of the system under test, as shown in Fig. 27.8, during the system identification test phase, since feedback from the test article or unit under test will change the system's frequency response function $H(f)$. Numerous waveforms can be used for the excitation including an impulsive transient, the predetermined reference-response waveform, a continuous random waveform, or repeated short bursts of random vibration with the transient noise having frequency-domain characteristics like those of the continuous noise. The last two methods are most commonly used. Continuous random noise produces better results in practice, but at the expense of longer vibration times for the unit under test during this phase. In all cases, it is important for the excitation drive signal to have energy at all frequencies of interest, but of sufficiently small amplitude so the test item is not damaged from this excitation, but a large enough amplitude such that a linear extrapolation to full-test level will not cause significant control errors. Averaging, as part of the frequency response function estimation, can mitigate the effects of nonlinear response and measurement noise (see Chap. 22) on the quality of the estimate.

Multiexciter Testing Applications. The simplest example of *multiple-exciter testing* is when multiple exciters are connected to independent systems under test and are controlled simultaneously. This configuration corresponds to several single-exciter control systems operating in parallel and will not be further discussed. The more complex and more interesting case is when the multiple exciters act on the same test fixture and unit under test simultaneously, as shown in Fig. 27.8 and discussed in more detail in Chap. 25. The attachments of the multiple exciters to the test fixture can be at several points in a single direction, or at one point in several directions, or combinations of both.[33] This is the type of configuration that is represented in the block diagram of the multiexciter system under test in Fig. 27.8. If any of the drives, $d_1(t)$ through $d_n(t)$, is capable of causing a response on more than one of the control responses, $c_1(t)$ through $c_n(t)$, then the multiexciter control system has cross-coupling between control responses. In this situation, the measured frequency response matrix, $[H(f)]$, between the drive-signal vector, $\{d(t)\}$, and the control-response vector, $\{c(t)\}$, will have offdiagonal elements that compare in order to the diagonal elements.

Systems that have cross-coupling between the control-response signals, $c_1(t)$ through $c_n(t)$, and which are elements of the vector of control-response waveforms, $\{c(t)\}$, require the DVCS to have provisions for control of these cross-coupling effects. These are typically controlled using the measured frequency response matrix in a manner similar to how the system frequency response function, $H(f)$, is used for long-term response waveform control. The needed frequency response matrix is measured using the multiple input, multiple output (MIMO) system identification techniques discussed in association with Eq. (27.4). The specifics of how this is done vary with each application dictated by the type of MIMO shock and vibration testing that needs to be accomplished. These are typically multiexciter tests that use a MIMO methodology within the DVCS employed to control such multiexciter tests. These shock and vibration control applications are called MIMO random, MIMO swept-sine, MIMO shock, and MIMO long-term response waveform control tests. Good mechanical design (the design of the excitation, fixturing subsystems, how the test article is attached, and where the control points are located on the system under test) is very important and can reduce the severity of system identification and control problems that can arise during multiexciter testing. Poor mechanical design can make the MIMO system under test and the corresponding DVCS unusable, no matter how advanced the control technology may be.

The complexity of building these systems (i.e., designing the control system) and specifying the test parameters increases much faster than the rate of increase in the number of exciters. To a first order, the control and test specification complexity increases by at least the square of the number of exciters that are used due to the use of n-dimensional signal-processing methods and their use of n-by-n complex matrices. The design complexity of the system under test in Fig. 27.8 also increases, but for other reasons (see Chap. 25). The resultant physical constraints of achievable system-under-test designs typically limits many MIMO control and excitation systems to frequencies less than 2 kHz. The significant displacements encountered in low-frequency MIMO testing also increase the complexity of the design of the vibration fixture that interconnects the exciters and the unit under test, and lets the exciters move independently from each other. However, at lower frequencies, large MIMO test systems are possible. For example, long-term response waveform control systems that have as many as 18 exciters are used to simulate road conditions in the automobile industry. An example of this is shown in Fig. 25.10.

MIMO Random. For MIMO random, the test's vibratory motions are specified in terms of a reference response spectral density matrix $[R(f)]$. This is a matrix that consists of both power spectral densities along the diagonal and cross-spectral densities along the offdiagonal elements of the matrix. The elements at the ith diagonal of the reference spectral density matrix, $R_{ii}(f)$, represents the reference power spectral density to be used for the ith reference response for the control response $c_i(t)$. The ijth offdiagonal matrix elements of the reference spectral density matrix, $R_{ij}(f)$, represent the reference response cross-spectral density to control the control-response cross-spectral density between the ith and jth control response, $c_i(t)$ and $c_j(t)$, as in Eq. (27.1). This cross-spectral density can also be described by the ordinary coherence and phase between $c_i(t)$ and $c_j(t)$ (see Chap. 22), as well as their respective power-spectral densities.[18,30,32,33] The objective of the MIMO random vibration test control system is to create a drive signal vector, $\{d(t)\}$, that consists of the exciter drive signals, $d_1(t)$ through $d_n(t)$, which causes the spectral density matrix of the control-response vector, $[W_{cc}(f)]$, to agree, within some acceptable error, with the MIMO random test reference spectral density matrix, $[R(f)]$. The issues associated with spectrum averaging and input-signal windowing that were discussed for single-exciter random vibration control also need to be considered.

The control-response spectral density matrix, $[W_{cc}(f)]$, of the control-response vector can be modeled by the following result from linear system dynamics and multidimensional stationary stochastic process theory,[17–19] which states that the control-response spectral density matrix is given by

$$[W_{cc}(f)] = [H(f)][W_{dd}(f)][H(f)]^H \tag{27.5}$$

Equation (27.5) can be solved for the initial drive signals using the measured frequency response matrix, $[H(f)]$, and the test-prescribed reference-response spectral density matrix, $[R(f)]$. This result gives the spectral density matrix, $[W_{dd}(f)]$, of the drive signals as

$$[W_{dd}(f)] = [H(f)]^{-1}[W_{cc}(f)][H(f)]^{-H} \tag{27.6}$$

The resultant drive spectral density matrix, $[W_{dd}(f)]$, can be further factored using a Cholesky decomposition[2,18,32,34] as

$$[W_{dd}(f)] = [\Gamma_d(f)][\Gamma_d(f)]^H \tag{27.7}$$

where $[\Gamma_d(f)]$ is the *Cholesky factor* of $[W_{dd}(f)]$, which is a lower-triangular complex matrix, with real and nonnegative diagonal elements, that plays the same role as the drive spectrum plays in single-shaker control (see Refs. 24 and 34 for details). This Cholesky factor is also associated with the general study of partial coherence,[17,20,21] and the partial coherence that will exist between drive signals that are synthesized using it. It is used, with the frequency-to-time processing block of Fig. 27.10, to create a vector of drive signals, $\{d(t)\}$, that has $[W_{dd}(f)]$ as its spectral density matrix.[32,34] These are further randomized by a MIMO time-domain randomization process, similar to what is done in single-exciter random, but with the use of a lower-triangular matrix of waveforms obtained from $[\Gamma_d(f)]$.[22,32] By this means, the coherence and phase between the control-response signals is controlled as well as each individual control response's power spectral density.[30,32] The drive vector, $\{d(t)\}$, then has the matrix $[W_{dd}(f)]$ as its spectral density matrix, and should cause the MIMO system under test to respond with a control-response vector, $\{c(t)\}$, that has as its spectral density matrix, $[W_{cc}(f)]$, which agrees with the test-specified reference-response spectral density matrix, $[R(f)]$, within some acceptable error margin.

MIMO random, similar to waveform control, uses the matrix-inverse of the measured frequency response matrix, $[H(f)]$, to create the initial drive. The *impedance matrix,* $[Z(f)]$, of the system under test, is given by

$$[Z(f)] = [H(f)]^{-1} \tag{27.8}$$

This matrix needs to be measured prior to the test in the system identification testing phase, as discussed in previous sections on frequency response matrix estimation. The accuracy of this measured matrix, which is computed before the vibration test, is critical to the success of the control task. The method used to estimate $[H(f)]$[17–19,30,35] typically uses the left expression in Eq. (27.4) to solve for $[H(f)]$ from the computed spectral density matrix $[W_{dd}(f)]$ and the measured cross–spectral density matrix $[W_{cd}(f)]$ as

$$[H(f)] = [W_{cd}(f)][W_{dd}(f)]^{-1} \tag{27.9}$$

The MIMO control system uses the frequency response matrix, measured before the MIMO test with the use of Eq. (27.9), to construct the initial drive signals as in Eq. (27.6). A further MIMO control iteration is used to refine the drive and approximately account for the possible nonlinearities in the control responses.[30,32,33,35] The control iteration uses $[Z(f)]$ to compute the contribution that the control errors at each of the control points make to each of the drive signals. It effectively decouples the control errors so they can be used to adjust the drive signal's relative phase and coherence to achieve control[22,30,32,34,35] according to their respective contribution. In MIMO random, unlike in MISO random testing, phase cannot be ignored since the relative phase between the control responses and the drive signals, and also between the drive vector and the control response vector, is critical to the success of the MIMO test. Also, since the impedance matrix, $[Z(f)]$, which is the inverse of $[H(f)]$, is being used for control, special care is needed in its calculation at those frequencies where $[H(f)]$ is singular or nearly singular.[30,32,35]

For MIMO random testing, the system characterization is done by operating all exciters in the system under test simultaneously with band-limited Gaussian noise. These system identification drive signals typically have a uniform, bandwidth-limited spectrum covering the maximum frequency of interest. They are also uncorrelated among themselves. The response levels for the system characterization should be chosen as high above the noise floor as possible to maximize the accuracy of the $[Z(f)]$ estimate, but below a level that might cause undue stress or damage to the test article during the system identification operation. With the system excited in this way, the spectral density matrix $[W_{dd}(f)]$ and the cross–spectral density matrix $[W_{cd}(f)]$ are estimated using the methods associated with Eqs. (27.1) through (27.3).

Equation (27.9) is used to compute the estimate of $[H(f)]$, and Eq. (27.6) is used to generate the initial drive signals based on the Cholesky factor $[\Gamma_d(f)]$ discussed as part of Eq. (27.7).

MIMO Swept-Sine. MIMO swept-sine control systems operate much like the MIMO random control systems discussed previously with differences in the control objective. The objective of a MIMO swept-sine test is to apply a controlled excitation to a structure at specified points with a series of exciters connected to the structure so that the response motion at a chosen number of control points on the system under test (see Fig. 27.8), as described by the control-response vector, $\{C(f)\}$, match a specified reference-response vector, $\{R(f)\}$, within some acceptable error margin.[30,35] In this case, if there are n exciters and n control transducers, the complex vectors of spectra, $\{C(f)\}$, with components $C_1(f)$ through $C_n(f)$, and $\{R(f)\}$, with components $R_1(f)$ through $R_n(f)$, are of dimension n for each frequency within the test range. To accomplish this goal, the linear system model of system response is solved for the initial drive by

$$\{D(f)\} = [H(f)]^{-1}\{R(f)\} \tag{27.10}$$

As in other MIMO control applications, Eq. (27.10) is solved for the initial drive vector $\{D(f)\}$, using the system-under-test's frequency response matrix that is obtained prior to the test. In MIMO sine, the additional problem is that random noise excitation, as used in other MIMO applications, is many times not suitable for the system identification. This is because the system's frequency response characteristics can be quite different for swept-sine excitation, as opposed to a random excitation. For this reason, the system identification should be done with a swept-sine excitation, one exciter at a time. This can be time-consuming and may cause undue fatigue to the structure under test in Fig. 27.13. Other approaches that are used involve stepped-sweep methods with a single exciter at time or with multiple exciters using multiple phases at each step frequency. There is at least one commercial system, which uses patented adaptive control technology, that can estimate the $[H(f)]$ matrix during the swept-sine test, and thus minimize the initial system identification phase.[35]

The overall block diagrams of the MIMO swept-sine control system and the MIMO sweep-sine controller are shown in Figs. 27.13 and 27.14, respectively. As can be seen in the block diagram of the overall system in Fig. 27.13, a vector-tracking filter subsystem plays the role of the time-to-frequency conversion in the DVCS. As

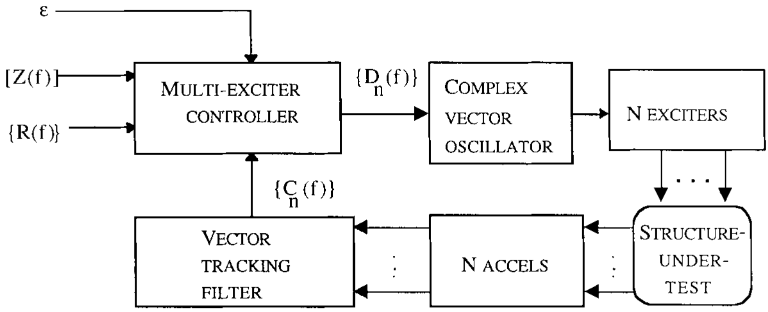

FIGURE 27.13 Overall multiexciter vibration control system.

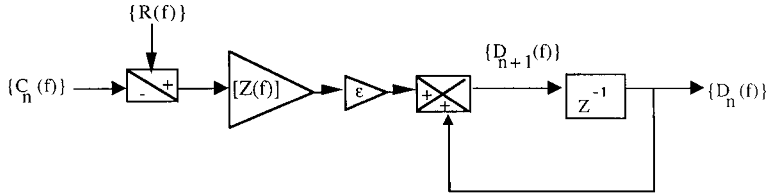

FIGURE 27.14 Multiexciter swept-sine vibration control system.

discussed in a previous section, tracking filters estimate the complex amplitude of the sweeping sine-wave control-response signals, $c_1(t)$ through $c_n(t)$. The resulting complex control-response vector, $\{C(f)\}$, is then compared by the DVCS with the specified test reference-response vector, $\{R(f)\}$. The control-error vector is then multiplied by the impedance matrix, $[Z(f)]$, to get the contribution of the control errors at each control location to each drive signal sent to each exciter. A percentage of this error, given by ε, is added to the previous complex-drive signal's amplitude spectrum to obtain the next drive signal's vector spectrum amplitude, as shown in the multiexciter swept-sine controller block diagram in Fig. 27.14. This corrected drive signal, with updated amplitude and relative phase, is then sent to the vector oscillator, which plays the role of the frequency-to-time transformation subsystem within the DVCS. It provides control of the amplitude of the output drive signals and the relative phase with respect to the modulating signal used by the vector-tracking filter shown in Fig. 27.13. Each component of $\{C(f)\}$ is an output of an individual tracking filter, within the vector-tracking filter in Fig. 27.13 given by Fig. 27.6, which all use the same modulating signal. There is also a common phase and frequency reference for the drive signals generated by the complex vector oscillator in Fig. 27.13. The system is driven as the frequency of the drive-signal vector is swept continuously through the sweep range of the MIMO swept-sine wave test.

MIMO Transient/Shock. MIMO transient waveform control methods are an extension of single-shaker transient/shock and MIMO swept-sine control methods previously discussed. This type of control is used principally for seismic simulations. The application uses shock response spectrum synthesis techniques to create the waveforms that are to be used as the specified reference-response vector, $\{r(t)\}$. In this case, the control process matches the specified shock response spectrum indirectly by using waveform control to make the control response, $\{c(t)\}$, match $\{r(t)\}$, thereby indirectly matching the specified shock response spectrum. This vector of waveforms, $\{r(t)\}$, typically consist of random transients that have been synthesized such that each such transient matches a specified shock response spectrum to be used as the spectral reference response for each control point, as discussed in the section on shock response spectrum synthesis. In other applications, these transient waveforms sometimes represent data that have been measured in the field. Many times, they are actual earthquake time-domain response data, from remote sensors that are located to measure an earthquake's ground motion when and where it occurs. The block diagram of this type of control system is similar to that of MIMO sine. The predominant difference is that the time-to-frequency transformation is accomplished by an FFT, with a frame size large enough to accommodate the transient, but still avoid circular convolution errors.[1] Spectral leakage errors (see Chap. 14) are mitigated by using windowing.

MIMO Long-Term Response Waveform Control. This application is an extension of MIMO transient waveform control discussed in the previous section. The primary difference is in the fact that the test-specified reference-response vector, $\{r(t)\}$, consists of waveforms that cannot be processed within a single FFT frame. For this reason, like in the discussion about single-exciter waveform control methods, an overlap-and-add technique[1] has to be used in both the time-to-frequency and frequency-to-time transformations within the DVCS used for MIMO long-term response waveform control. The issues that are associated with the use of the overlap-and-add indirect convolution technique need to be considered and addressed.[1,30,32]

Again, as in MIMO random, MIMO sine, and MIMO transient/shock applications, the MIMO system under test is driven with a vector of time-histories, $\{d(t)\}$, such that the control-response vector, $\{c(t)\}$, in this case a vector of time-histories, agrees within an acceptable error margin with the test-specified reference-response vector $\{r(t)\}$, which is also a vector of time-histories.

Modal Testing. *Modal testing* is conducted to excite a system under test, acquire its drive and response signals, and estimate its frequency response characteristics to determine experimentally the natural frequencies, mode shapes, and associated damping factors of a structure via modal analysis. Modal analysis is discussed thoroughly in Chap. 21. Typically, much of the DVCS hardware and its shock and vibration data acquisition and analysis software is usable for this application.

Currently, digital computers are applied in modal testing in two distinct ways. First, for sinusoidal excitation, computers are employed as an aid in obtaining the desired purity of the modal excitation as well as in acquiring and processing data, usually with operator adjustments of the frequency, the relative phase, and the amplitude of several sine-wave outputs. These are used to drive a system under test so as to achieve a particular relative phase and amplitude between chosen response points on the system under test that is characteristic of a particular normal mode response. The use of MIMO sine control methods can simplify this process. Second, and more commonly, the DVCS is used to excite the system under test with either a broad bandwidth random or a transient excitation, usually with several such outputs. The response and drive signals are acquired and processed using FFT computations with the methods discussed on frequency response function and frequency response matrix estimation, using Eq. (27.4). The use of MIMO random control methods can simplify this process. The frequency response functions are typically measured between chosen response points on the system under test, while exciting the system under test with the chosen excitation at prespecified excitation points, as discussed previously and in Chap. 21. The frequency response functions and/or frequency response matrices thus estimated are subsequently passed to modal analysis software for further processing and extraction of the pertinent modal data using the methods of Chap. 21.

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