CHAPTER 9
EFFECTS OF IMPACT ON STRUCTURES

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INTRODUCTION

This chapter discusses a particular phenomenon in the general field of shock and vibration usually referred to as *impact*.[1] An impact occurs when two or more bodies collide. An important characteristic of an impact is the generation of relatively large forces at points of contact for relatively short periods of time. Such forces sometimes are referred to as *impulse-type* forces.

Three general classes of impact are considered in this chapter: (1) impact between spheres or other rigid bodies, where a body is considered to be rigid if its dimensions are large relative to the wavelengths of the elastic stress waves in the body; (2) impact of a rigid body against a beam or plate that remains substantially elastic during the impact; and (3) impact involving yielding of structures.

DIRECT CENTRAL IMPACT OF TWO SPHERES

The elementary analysis of the central impact of two bodies is based upon an experimental observation of Newton.[2] According to that observation, the relative velocity of two bodies after impact is in constant ratio to their relative velocity before impact and is in the opposite direction. This constant ratio is the *coefficient of restitution;* usually it is designated by e.[3]

Let $\dot{u}$ and $\dot{x}$ be the components of velocity along a common line of motion of the two bodies before impact, and $\dot{u}'$ and $\dot{x}'$ the component velocities of the bodies in the same direction after impact. Then, by the observation of Newton,

$$\dot{u}' - \dot{x}' = -e(\dot{u} - \dot{x}) \tag{9.1}$$

Now suppose that a smooth sphere of mass m_u and velocity $\dot{u}$ collides with another smooth sphere having the mass m_x and velocity $\dot{x}$ moving in the same direction. Let the coefficient of restitution be e, and let $\dot{u}'$ and $\dot{x}'$ be the velocities of the two spheres, respectively, after impact. Figure 9.1 shows the condition of the two spheres just before collision. The only force acting on the spheres during impact is the force at the point of contact, acting along the line through the centers of the spheres.

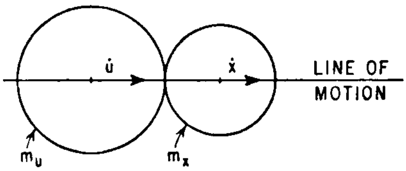

FIGURE 9.1 Positions of two solid spheres at instant of central impact.

According to the law of conservation of linear momentum:

$$m_u\dot{u}' + m_x\dot{x}' = m_u\dot{u} + m_x\dot{x} \tag{9.2}$$

Solving Eqs. (9.1) and (9.2) for the two unknowns, the velocities $\dot{u}'$ and $\dot{x}'$ after impact,

$$\dot{u}' = \frac{(m_u\dot{u} + m_x\dot{x}) - em_x(\dot{u} - \dot{x})}{m_u + m_x}$$

$$\dot{x}' = \frac{(m_u\dot{u} + m_x\dot{x}) + em_u(\dot{u} - \dot{x})}{m_u + m_x} \tag{9.3}$$

This analysis yields the resultant velocities for the two spheres on the basis of an experimental law and the principle of the conservation of momentum, without any specific reference to the force of contact F. A similar result is obtained for a ballistic pendulum used to measure the muzzle velocity of a bullet. A bullet of mass m_u and velocity $\dot{u}$ is fired into a block of wood of mass m_x which is at rest initially and finally assumes a velocity $\dot{x}'$ after the impact. Using only the principle of the conservation of momentum,

$$\dot{u} = \frac{(m_u + m_x)\dot{x}'}{m_u} \tag{9.4}$$

No knowledge of the complicated pattern of force acting on the bullet and the pendulum during the embedding process is required.

These simple facts are introductory to the more complicated problem involving the vibration of at least one of the colliding bodies, as discussed in a later section.

HERTZ THEORY OF IMPACT OF TWO SOLID SPHERES

The theory of two solid elastic spheres which collide with one another is based upon the results of an investigation of two elastic bodies pressed against one another under purely statical conditions.[4] For these static conditions, the relations between the sum of the displacements at the point of contact in the direction of the common line of motion and the resultant total pressure have been derived. The sum of these displacements is equal to the relative approach of the centers of the spheres, assuming that the spheres act as rigid bodies except for elastic compression at the point of contact. The relative approach varies as the two-thirds power of the total pressure; a formula is given for the time of duration of the contact.[4] The theory is valid only if the duration of contact is long in comparison with the period of the fundamental mode of vibration of either sphere.

The range of validity of the Hertz theory is related to the possibility of exciting vibration in the spheres.[5] The dimensionless ratio of the maximum kinetic energy of vibration to the sum of the kinetic energies of the two spheres just before collision is approximately

$$R = \frac{1}{50} \frac{\dot{u} - \dot{x}}{\sqrt{E/\rho}} \tag{9.5}$$

where $\dot{u} - \dot{x}$ = relative velocity of approach, in./sec
E = Young's modulus of elasticity, assumed to be the same for each sphere, lb/in.^{2}
ρ = density of each sphere, lb-sec^2/in.^{4}
$\sqrt{E/\rho}$ = approximate velocity of propagation of dilatational waves, in./sec

The ratio R usually is a very small quantity; thus, the theory of impact set forth by Eq. (9.5) has wide application because vibration is not generated in the spheres to an appreciable degree under ordinary conditions. The energy of the colliding spheres remains translational, and the velocities after impact are deducible from the principles of energy and of momentum. The important point of plastic deformation at the point of contact is discussed in a later section.

Formulas for force between the spheres, the radius of the circular area of contact, and the relative approach of the centers of the spheres, all as functions of time, can be determined for any two given spheres.[6]

IMPACT OF A SOLID SPHERE ON AN ELASTIC PLATE

An extension of the Hertz theory of impact to include the effect of vibration of one of the colliding bodies involves a study of the transverse impact of a solid sphere upon an infinitely extended plate.[7] The plate has the role of the vibrating body. The coefficient of restitution is an important element in any analysis of the motion ensuing after the collision of two bodies.

The analysis is based on the assumption that the principal elastic waves of importance are flexural waves of half-period equal to the duration of impact. Let $2h$ and $2D$ be the thickness of plate and diameter of sphere, respectively; ρ_1, ρ_2 their densities; E_1, E_2 their Young's moduli; ν_1, ν_2 their values of Poisson's ratio; and τ_H the duration of impact. The velocity c of long flexural waves of wavelength λ in the plate is given by

$$c^2 = \frac{4\pi^2}{3} \frac{h^2}{\lambda^2} \frac{E_1}{\rho_1(1 - \nu_1^2)} \tag{9.6}$$

The radius a of the circle on the plate over which the disturbance has spread at the termination of impact is given by

$$a = c\tau_H = \frac{\lambda}{2} \tag{9.7}$$

Combining Eqs. (9.6) and (9.7),

$$a^2 = \pi\tau_H h \sqrt{\frac{E_1}{3\rho_1(1 - \nu_1^2)}} \tag{9.8}$$

The next step is to find the kinetic and potential energies of the wave motion of the plate. The kinetic energy may be determined from the transverse velocity of the plate at each point over the circle of radius a covered by the wave. Figure 9.2 shows an approximate distribution of velocity over the circle of radius a at the end of impact.[8] The direction of the impact also is shown. The kinetic energy in the wave at the end of impact is

$$T = \int_0^a ½ \cdot 2h \cdot \rho_1 \cdot 2\pi R \cdot \dot{w}^2 dR \tag{9.9}$$

where $\dot{w}$ is the transverse velocity at distance R from the origin. As an approximation it is assumed that the sum of the potential energy and the kinetic energy in the wave is $2T$. With considerable effort these energies can be calculated in terms of the motion of the plate, although the calculation may be laborious.

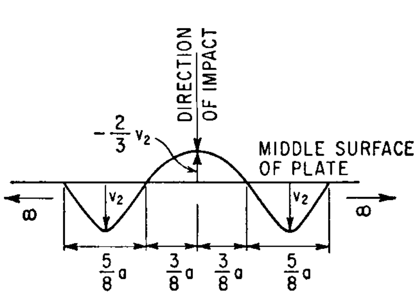

FIGURE 9.2 Distribution of transverse velocities in plate as a result of impact by a moving body. (*After Lamb*[8].)

The impulse in the plate produced by the colliding body is

$$J = \int_0^a ½ \cdot 2h \cdot \rho_1 \cdot 2\pi R \cdot \dot{w}\, dr \tag{9.10}$$

The integration should be carried out with due regard to the sign of velocity. If m_u is the mass of colliding body, $\dot{u}$ its velocity before impact, and e the coefficient of restitution, the following relations are obtained on the assumption that the energy is conserved:

$$½ m_u \dot{u}^2(1 - e^2) = 2T \tag{9.11}$$

$$m_u \dot{u}(1 + e) = J \tag{9.12}$$

Equation (9.11) represents the energy lost to the moving sphere as a result of impact and Eq. (9.12) represents the change in momentum of the sphere.

The coefficient of restitution e is determined by evaluating the integrals for T and J and substituting their values in Eq. (9.12). The necessary integrations can be performed by taking the function for transverse velocity in Fig. 9.2 as arcs of sine curves. The resultant expression for e is

$$e = \frac{h\rho_1 a^2 - 0.56 m_u}{h\rho_1 a^2 + 0.56 m_u} \tag{9.13}$$

where a, the radius of the deformed region, is given by Eq. (9.8) and τ_H, the time of contact between sphere and plate, is given by Hertz's theory of impact to a first approximation.[4] The mass of the sphere is m_u; the mass of the plate is assumed to be infinite. Large discrepancies between theory and experiment occur when the diameter of the sphere is large compared with the thickness of the plate. The duration of impact τ_H is

$$\tau_H = 2.94\,\frac{\alpha}{\dot{u}}$$

where
$$\alpha = \left[\frac{15}{16}\,\nu_1^2\left(\frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2}\right)m_u\right]^{2/5} R_s^{-1/5} \tag{9.14}$$

The radius of the striking sphere is R_s and its velocity before impact is $\dot{u}$. Subscripts 1 and 2 represent the properties of the sphere and plate, respectively. The value of τ_H may be substituted in Eq. (9.8) above.

Experimental results verify the theory when the limitations of the theory are not violated. The velocity of impact must be sufficiently small to avoid plastic deformation. When the collision involves steel on steel, the velocity usually must be less than 1 ft/sec. However, useful engineering results can be obtained with this approach even though plastic deformation does occur locally.[9,10]

TRANSVERSE IMPACT OF A MASS ON A BEAM

If $F(t)$ is the force acting between the sphere and the beam during contact, the distance traveled by the sphere in time t after collision is[11]

$$\dot{u}t - \frac{1}{m_u}\int_0^t F(t_v)\,(t-t_v)\,dt_v \tag{9.15}$$

where $\dot{u}$ = velocity of sphere before collision (beam assumed to be at rest initially)
m_u = mass of solid sphere

The beam is assumed to be at rest initially.

For example, the deflection of a simply supported beam under force $F(t_v)$ at its center is

$$\sum_{1,3,5\ldots}^{\infty} \frac{1}{m_b}\int_0^t F(t_v)\,\frac{\sin\omega_n(t-t_v)}{\omega_n}\,dt_v \tag{9.16}$$

where m_b = one-half of mass of beam
ω_n = angular frequency of the nth mode of vibration

Equation (9.16) represents the transverse vibration of a beam. While the present case is only for direct central impact, the cases for noncentral impact depend only on the corresponding solution for transverse vibration. Oblique impact also is treated readily.

The expression for the relative approach of the sphere and beam, i.e., penetration of beam by sphere, is[11]

$$\alpha = \kappa_1 F(t)^{2/3} \tag{9.17}$$

where κ_1 is a constant depending on the elastic and geometrical properties of the sphere and the beam at the point of contact, and α is given by Eq. (9.14). Consequently, the equation that defines the problem is

$$\alpha = K_1F^{2/3} = \dot{u}t - \frac{1}{m_u}\int_0^t F(t_v)(t-t_v)\,dt_v - \sum_{1,3,5}^{\infty} \frac{1}{m_b}\int_0^t F(t_v)\,\frac{\sin\omega_n(t-t_v)}{\omega_n}\,dt_v \tag{9.18}$$

Equation (9.18) has been solved numerically for two specific problems by subdividing the time interval 0 to t into small elements and calculating, step by step, the displacements of the sphere.[11] The results are not general but rather apply only to the cases of beam and sphere.

For the impact of a mass on a beam, the sum of the kinetic and the potential energies may be expressed in terms of the unknown contact force.[12] Also, the impulse integral J in terms of the contact force may be expressed as

$$J = \int_0^t F(t)\, dt = m_u \dot{u}\,(1 + e) \tag{9.19}$$

A satisfactory approximation to $F(t)$ is defined in terms of a normalized force $\bar{F}$:

$$F(t) = m_u \dot{u}\,(1 + e)\,\bar{F}(t) \tag{9.20}$$

Thus, from Eqs. (9.19) and (9.20),

$$\int_0^t \bar{F}\, dt = 1 \tag{9.21}$$

The value of this integral is independent of the shape of $F(t)$. The normalized force is defined such that its maximum value equals the maximum value of the corresponding normalized Hertz force.[12] To perform the necessary integrations, a suitable function for defining $F(t)$ is chosen as follows:

$$\begin{aligned} \bar{F}(t) &= \frac{\pi}{2\tau_L} \sin \frac{\pi}{\tau_L} t \qquad && [0 < t < \tau_L] \\ \bar{F}(t) &= 0 && [|t| > \tau_L] \end{aligned} \tag{9.22}$$

Results for particular problems solved in this manner agree well with those obtained for the same problems by the numerical solution of the exact integral equation.[12]

To apply these results to a specific beam impact problem, it is necessary to express the deflection equation for the beam in terms of known quantities. One of these quantities is the coefficient of restitution; a formula must be provided for its determination in terms of known functions. This is given by Eq. (9.31).

IMPACT OF A RIGID BODY ON A DAMPED ELASTICALLY SUPPORTED BEAM

For the more general case of impact of a rigid body on a damped, elastically supported beam, it is assumed that there is external damping, damping determined by the Stokes' law of stress-strain, and an elastic support attached to the beam along its length in such a manner that resistance is proportional to deflection.[13] The differential equation for the deflection of the beam is

$$EI \frac{\partial^4 w}{\partial x^4} + c_1 I \frac{\partial^5 w}{\partial x^4\, \partial t} + c_2 \frac{\partial w}{\partial t} + kw + \rho S \frac{\partial^2 w}{\partial t^2} = F(x,t) \tag{9.23}$$

where w = deflection, in.
E = Young's modulus, lb/in.^{2}
I = moment of inertia for cross section (constant), in.^{4}

c_1 = internal damping coefficient, lb/in.^{2}-sec (Stokes' law)
c_2 = external damping coefficient, lb/in.^{2}-sec
k = foundation modulus, lb/in.^{2}
ρ = density, lb-sec^2/in.^{4}
S = area of cross section (constant), in.^{2}
$\frac{\partial^2 w}{\partial t^2}$ = acceleration, in./sec^2
t = time, sec
$F(x,t)$ = driving force per unit length of beam, lb/in.

For example, to illustrate the application of specific boundary conditions, consider a simply supported beam of length l. The moments and deflections must vanish at the ends. The beam is assumed undeflected and at rest just before impact, and central impact is assumed although with some additional computation this restriction may be dropped. The solution may be written as follows:

$$w(x,t) = \sum^{\infty} \sin \frac{n\pi x}{l} \sin \frac{n\pi}{2} \frac{1}{m} \frac{1}{\sqrt{\omega_n^2 - \delta_n^2}}$$

$$\times \int_0^t e^{-\delta_n (t-\tau)} \sin \left[\sqrt{\omega_n^2 - \delta_n^2} \cdot (t - \tau) \right] F_1(\tau)\, d\tau \tag{9.24}$$

where e = base of natural logarithms

δ_n = damping numbers $= \frac{1}{2}\left(r_i \frac{n^4\pi^4}{l^4} + r_e \right)$

$r_i = \frac{c_1 I}{\rho S}$

$r_e = \frac{c_2}{\rho S}$

ω_n = angular frequencies

$m = \frac{1}{2}\rho A l$

A satisfactory analytical expression for the contact force $F_1(t)$, a particularization of $F(x,t)$ in Eq. (9.23), must be developed. Although $F_1(t)$ is assumed to act at the center of the beam, the methods apply with only minor alterations if the impact occurs at any other point of the beam.

One of the conditions which the contact force must satisfy is that its time integral for the duration of impact equal the change in momentum of the striking body. The change of momentum is

$$m\dot{z} - m\dot{z}' = m\dot{z}\left(1 - \frac{\dot{z}'}{\dot{z}}\right) \tag{9.25}$$

where m = mass of rigid body, lb-sec^2/in.
$\dot{z}$ = velocity of rigid body just before collision, in./sec
$\dot{z}'$ = velocity of rigid body just after collision, in./sec

When the velocity of the beam is zero, Eq. (9.1) may be written

$$e = -\frac{\dot{z}'}{\dot{z}} \tag{9.26}$$

Equation (9.26) may be written

$$m\dot{z}\left(1 - \frac{\dot{z}'}{\dot{z}}\right) = m\dot{z}(1 + e) \tag{9.27}$$

From the equivalence of impulse and momentum:

$$\int_0^{\tau_0} F_1(t)\, dt = m\dot{z}(1 + e) \tag{9.28}$$

where τ_0 is the time of contact.

It can then be shown[13] that the impact force may be written

$$\begin{aligned} F_1(t) &= m\dot{z}(1 + e)\,\frac{\pi}{2\tau_L}\,\sin\frac{n\pi t}{\tau_L} && [0 < t < \tau_L] \\ F_1 &= 0 && [t > \tau_L] \end{aligned} \tag{9.29}$$

It can be shown further[13] that

$$\tau_L = 3.28\left[\frac{m^2}{\dot{z}R}\cdot\frac{(1 - \nu^2)}{E^2}\right]^{1/5} \tag{9.30}$$

where $R =$ radius of sphere, in.
$\nu =$ Poisson's ratio

The time interval τ_L is a special value of the time of contact T_0. It agrees well with experimental results.

The coefficient of restitution e is[13]

$$e = \frac{1 - \dfrac{m}{m_b}\displaystyle\sum_1^\infty \Phi_n - \dfrac{m}{m_b}\displaystyle\sum_1^\infty \Psi_n}{1 + \dfrac{m}{m_b}\displaystyle\sum_1^\infty \Phi_n + \dfrac{m}{m_b}\displaystyle\sum_1^\infty \Psi_n} \tag{9.31}$$

where $m =$ mass of sphere
$m_b =$ half mass of beam

The functions Φ_n and Ψ_n are given in the form of curves in Figs. 9.3 and 9.4; the symbol $\beta_n = \delta_n/\omega_n$ represents fractional damping and $Q_n = \omega_n\tau_L/2\pi$ is a dimensionless frequency where ω_n = angular frequency of nth mode of vibration of undamped vibration of beam, rad/sec, and τ_L = length of time the sinusoidal pulse is assumed to act on beam [see Eq. (9.30)]. If damping is neglected, the functions Ψ_n vanish from Eq. (9.31).

The above theory may be generalized to apply to the response of plates to impact. The deflection equation of a plate subjected to a force applied at a point is required. The various energy distributions at the end of impact are arrived at in a manner analogous to that for the beam.

The theory has been applied to columns and continuous beams[14,15] and also could be applied to transverse impact on a ring. Measurement of the force of impact illustrates the large number of modes of vibration that can be excited by an impact.[16,17,22]

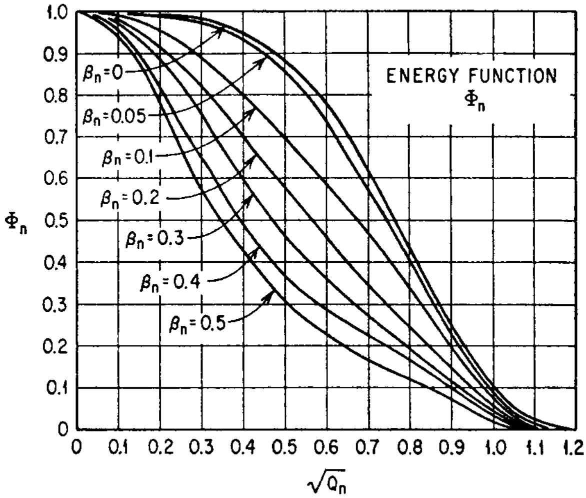

FIGURE 9.3 Energy functions Φ_n used with Eq. (9.31) to determine the coefficient of restitution from the impact of a rigid body on a damped elastically supported beam.

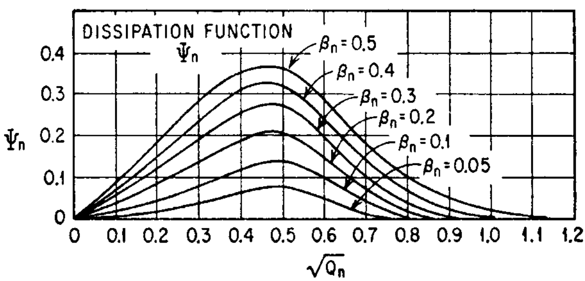

FIGURE 9.4 Dissipative (damping) functions Ψ_n used with Eq. (9.31) to determine the coefficient of restitution from the impact of a rigid body on a damped elastically supported beam.

Principal qualitative results of the foregoing analysis are:

1. Impacts by bodies of relatively small mass moving with low velocities develop significant bending strains in beams.
2. External damping of the type assumed above has a rapidly decreasing effect on reducing deflection and strain as the number of the mode increases.
3. Internal damping of the viscous type here assumed reduces deflection and strain appreciably in the higher modes. For a sufficiently high mode number, the vibration becomes aperiodic.
4. Increasing the modulus for an elastic foundation reduces the energy absorbed by the structure from the colliding body.

5. Impacts from collision produce sharp initial rises in strain which are little influenced by damping.
6. Because of result 5, the fatigue problem for machines and structures, in which the impact conditions are repeated many times, can be serious. Ordinary damping affords little protection.
7. The structure seldom can be treated as a single degree-of-freedom system with any degree of reliability in predicting strain.[13,19]

LONGITUDINAL AND TORSIONAL IMPACT ON BARS

If a mass strikes the end of a long bar, the response may be investigated by means of the Hertz contact theory.[11] The normal modes of vibration must be known so the displacement at each part of the bar can be calculated in terms of a contact force. In a similar manner, the torsional vibration of a long bar can be studied, using the normal modes of torsional vibration.

PLASTIC DEFORMATION RESULTING FROM IMPACT

Many problems of interest involve plastic deformation rather than elastic deformation as considered in the preceding analyses. Using the concept of the plastic hinge, the large plastic deformation of beams under transverse impact[23] and the plastic deformation of free rings under concentrated dynamic loads[24] have been studied. In such analyses, the elastic portion of the vibration usually is neglected. To make further progress in analyses of large deformations as a result of impact, a realistic theory of material behavior in the plastic phase is required.

An attempt to solve the problem for the longitudinal impact on bars has been made using the static engineering-type stress-strain curve as a part of the analysis.[25] An extension of the work to transverse impact also was attempted.[26]

Figure 9.5 illustrates the impact of a large body m colliding axially with a long rod. The body m has an initial velocity $\dot{u}$ and is sufficiently large that the end of the rod may be assumed to move with constant velocity $\dot{u}$. At any time t a stress wave will have moved into the bar a definite distance; by the condition of continuity (no break in the material), the struck end of the bar will have moved a distance equal to the total elongation of the end portion of the bar:

$$\dot{u}t = \epsilon \cdot l \tag{9.32}$$

The velocity c of a stress wave is $c = l/t$, and Eq. (9.32) becomes

$$\epsilon = \frac{\dot{u}}{c} \tag{9.33}$$

The stress and strain in an elastic material are related by Young's modulus. Substituting for strain from Eq. (9.33),

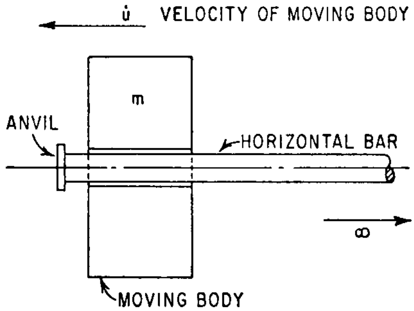

FIGURE 9.5 Longitudinal impact of moving body on end of rod.

$$\sigma = \epsilon \cdot E = E\frac{\dot{u}}{c} \tag{9.34}$$

where $\dot{u}$ = velocity of end of rod, in./sec
l = distance stress wave travels in time t, in.
t = time, sec
σ = stress, lb/in.^{2}
ϵ = strain (uniform), in./in.
E = Young's modulus, lb/in.^{2}
c = velocity of stress wave (dilatational), in./sec

When the yield point of the material is exceeded, Eq. (9.34) is inapplicable. Extensions of the analysis, however, lead to some results in the case of plastic deformation.[25] The differential equation for the elastic case is

$$E\frac{\partial^2 u}{\partial x^2} = \rho\frac{\partial^2 u}{\partial t^2} \tag{9.35}$$

where u = displacement, in.
x = coordinate along rod, in.
t = time, sec
E = Young's modulus, lb/in.^{2}
ρ = mass density, lb-sec^2/in.^{4}

The velocity of the elastic dilatational wave obtained from Eq. (9.35) is

$$c = \sqrt{\frac{E}{\rho}}$$

The modulus E is the slope of the stress-strain curve in the initial linear elastic region. Replacing E by $\partial\sigma/\partial\epsilon$ for the case in which plastic deformation occurs, the slope of the static stress-stress curve can be determined at any value of the strain ϵ.[25] Equation (9.35) then becomes

$$\frac{\partial\sigma}{\partial\epsilon}\,\frac{\partial^2 u}{\partial x^2} = \rho\frac{\partial^2 u}{\partial t^2} \tag{9.36}$$

Equation (9.36) is nonlinear; its general solution never has been obtained. For the simple type of loading discussed above and an infinitely long bar, the theory predicts a so-called critical velocity of impact because the velocities of the plastic waves are much smaller than those for the elastic waves and approach zero as the strain is indefinitely increased.[25] Since the impact velocity $\dot{u}$ is an independent quantity, it can be made larger and larger while the wave velocities are less than the velocity for elastic waves. Hence a point must be reached at which the continuity of the material is violated. Experimental data illustrate this point.[27]

ENERGY METHOD

Many problems in the design of machines and structures require knowledge of the deformation of material in the plastic condition. In statical problems the method of limit design[28] may be used. In dynamics, the most useful corresponding concept is less theoretical and may be termed the *energy method;* it is based upon the impact test used for the investigation of brittleness in metals. Originally, the only purpose of this test was to break a standard specimen as an index of brittleness or ductility. The general method, using a tension specimen, may be used in studying the dynamic resistance of materials.[27] An axial force is applied along the length of the specimen and causes the material to rupture ultimately. The energy of absorption is the total amount of energy taken out of the loading system and transferred to the specimen to cause the plastic deformation. The elastic energy and the specific mode of build-up of stress to the final plastic state are ignored. Such an approach has value only to the extent that the material has ductility. For example, in a long tension-type specimen of medium steel, the energy absorbed before neck-down and rupture is of the order of 500 ft-lb per cubic inch of material. Thus, if the moving body in Fig. 9.5 weighs 200 lb and has an initial velocity of 80 ft/sec, it represents 20,000 ft-lb of kinetic energy. If the tension bar subjected to the impact is 10 in. long and 0.5 in. in diameter, it will absorb approximately 1,000 ft-lb of energy. Under these circumstances it will rupture. On the other hand, if the moving body m weighs only 50 lb and has an initial velocity of 30 ft/sec, its kinetic energy is approximately 700 ft-lb and the bar will not rupture.

If the tension specimen were severely notched at some point along its length, it would no longer absorb 500 ft-lb per cubic inch to rupture. The material in the immediate neighborhood of the notch would deform plastically; a break would occur at the notch with the bulk of the material in the specimen stressed below the yield stress for the material. A practical structural situation related to this problem occurs when a butt weld is located at some point along an unnotched specimen. If the weld is of good quality, the full energy absorption of the entire bar develops before rupture; with a poor weld, the rupture occurs at the weld and practically no energy is absorbed by the remainder of the material. This is an important consideration in applying the energy method to design problems.

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