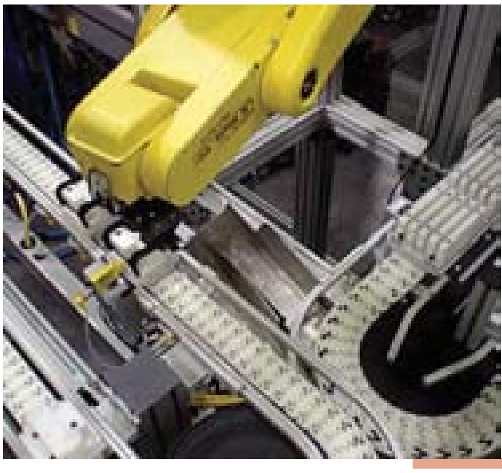
CONTENTS



# 26

# Automatic Control

**Features**



## 26.1. Introduction

The automatic control of system (or machine) is a very accurate and effective means to perform desired function by the system in which the human operator is replaced by a device thereby relieving the human operator from the job thus saving physical strength. The automatic control systems are also called as ***self-activated systems.*** *The* centrifugally actuated ball governor which controls the throttle valve to maintain the constant speed of an engine is an example of an automatically controlled system.

The automatic control systems are very fast, produces uniform and quality products. It reduces the requirement of human operators thus minimising wage bills.

## 26.2. Terms used in Automatic Control of Systems

The following terms are generally used in automatic control of systems :

**1. *Command.*** The result of the act of adjustment, *i.e.* closing a valve, moving a lever, pressing a button etc., is known as command.

**2. *Response.*** The subsequent result of the system to the command is known as response.

**3. *Process control.*** The automatic control of variables *i.e.* change in pressure, temperature or speed etc. in machine is termed as process control.



CONTENTS


A rail-track maintenance machine.
Note : This picture is given as additional information and is not a direct example of the current chapter.

**4. *Process controller.*** The device which controls a process is called a process controller.

**5. *Regulator.*** The device used to keep the variables at a constant desired value is called as regulator.

**6. *Kinetic control.*** The automatic control of the displacement or velocity or acceleration of a member of a machine is called as kinetic control.

**7. *Feed back.*** It is defined as measuring the output of the machine for comparison with the input to the machine.

**8. *Error detector.*** A differential device used to measure the actual controlled quantity and to compare it continuously with the desired value is called an error detector. It is also known as ***deviation sensor.***

**9. *Transducer.*** It is a device to change a signal which is in one physical form to a corresponding signal in another physical form. A Bourdon tube is an example of transducer because it converts a pressure signal into a displacement, thereby facilitating the indication of the pressure on a calibrated scale. The other examples of transducer are a loud speaker (because it converts electrical signal into a sound) and a photo-electric cell (because it converts a light signal into an electric signal). Similarly, the primary elements of all the many different forms of thermometers are transducers.

**10. *Amplification.*** It is defined as increasing the amplitude of the signal without affecting its waveform. For example, an error detector itself has insufficient power output to actuate the correcting mechanism and hence the error signal has to be amplified. This is generally done by employing mechanical or hydraulic or pneumatic amplifying elements like levers, gears and venturimeters etc.

## 26.3. Types of Automatic Control System

The automatic control systems are of the following two types :

**1. *Open-loop or unmonitored system.*** When the input to a system is independent of the output from the system, then the system is called an open-loop or unmonitored system. It is also called as a **calibrated system.** Most measuring instruments are open-loop control systems, as for the same input signal, the readings will depend upon things like ambient temperature and pressure. Following are the examples of open-loop system :

**(*a*)** A simple Bourdon tube pressure gauge commonly used for measuring pressure.

**(*b*)** A simple carburettor in which the air-fuel ratio adjusted through venturi remains same irrespective of load conditions.

**(*c*)** In traffic lights system, the timing of lights is preset irrespective of intensity of traffic.

**2. *Closed-loop or monitored system.*** When output of a system is measured and is continuously compared with the required value, then it is known as **closed-loop or monitored system.** In this system, the output is measured and through a feedback transducer, it is sent to an error detector which detects any error in the output from the required value thus adjusting the input in a way to get the required output. Following are the examples of a closed-loop system :

**(*a*)** In a traffic control system, if the flow of traffic is measured either by counting the number of vehicles by a person or by counting the impulses due to the vehicles passing over a pressure pad and then setting the time of signal lights.

***(b)*** In a thermostatically controlled water heater, whenever the temperature of water heater rises above the required point, the thermostate senses it and switches the water heater off so as to bring the temperature down to the required point. Similarly, when the temperature falls below the required point, the thermostate switches on the water heater to raise the temperature of water to the required point.

## 26.4. Block Diagrams

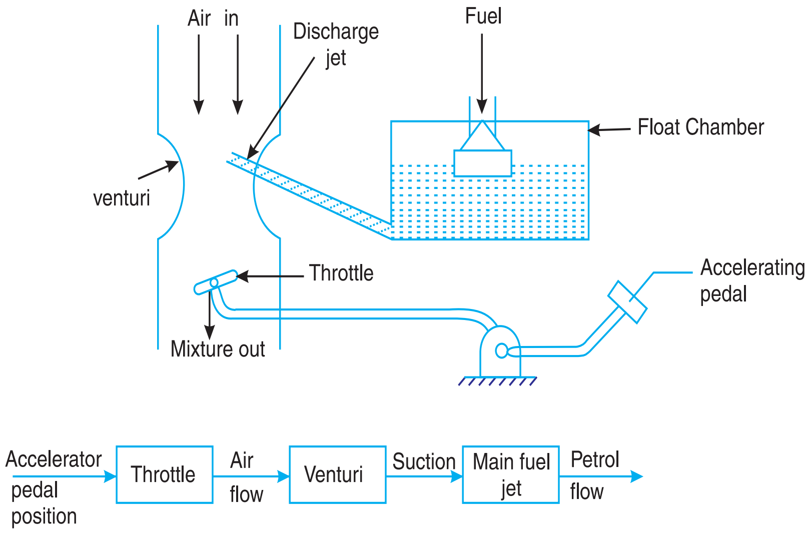



**Fig. 26.1.** Block diagram of a single carburettor.

The block diagrams are used to study the automatic control systems in a simplified way. In this, the functioning of a system is explained by the interconnected blocks where each block represents a labelled rectangle and is thought of as a block box with a definite function. These blocks are connected to other blocks by lines with arrow marks in order to indicate the sequence of events that are taking place. Fig. 26.1 shows the diagram of a simple carburettor. The reduction of a control system to a block diagram greatly facilitates the analysis of the system performance or response.

## 26.5. Lag in Response

We know that response is the subsequent result of the system to the command. In any control system, there is a delay in response (output) due to some inherent cause and it becomes difficult to measure the input and output simultaneously. This delay in response is termed as ***lag in response.*** For example, in steam turbines, with the sudden decrease in load, the hydraulic relay moves in the direction to close the valve. But unless the piston valve ports are made with literally zero overlap, there would be some lag in operation, since the first movement of the piston valve would not be sufficient to open the ports. This lag increases the probability of unstable operation.

## 26.6 Transfer Function

The transfer function is an expression showing the relation between output and the input to each unit or block of a control system. Mathematically,

$$\text{Transfer function} = \theta_o / \theta_i$$

where $\theta_o$ = Output signal of the block of a system, and

$\theta_i$ = Input signal to the block of a system.

Thus, the output from an element may be obtained by multiplying the input signal with the transfer function.

**Note :** From the transfer function of the individual blocks, the equation of motion of system can be formulated.

## 26.7 Overall Transfer Function

In the previous article, we have discussed the transfer function of a block. A control system actually consists of several such blocks which are connected in series. The overall transfer function of the series is the product of the individual transfer function. Consider a block diagram of any control system represented by the three blocks as shown in Fig. 26.2.

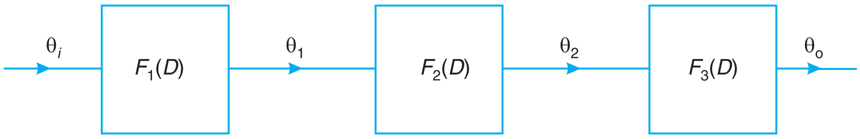



**Fig. 26.2.** Overall transfer function.

Thus, if $F_1$ (*D*), $F_2$ (*D*), $F_3$ (*D*) are individual transfer functions of three blocks in series, then the overall transfer function of the system is given as

$$\frac{\theta_o}{\theta_i} = \frac{\theta_1}{\theta_i} \times \frac{\theta_2}{\theta_1} \times \frac{\theta_o}{\theta_2} = F_1(D) \times F_2(D) \times F_3(D) = KG(D)$$

where $K$ = Constant representing the overall amplification or gain, and

$G(D)$ = Some function of the operator $D$.

**Note:** The above equation is only true if there is no interaction between the blocks, that is the output from one block is not affected by its connection to the subsequent blocks.

## 26.8. Transfer Function for a System with viscous Damped Output

Consider a shaft, which is used to position a load (which may be pulley or gear) as shown in Fig. 26.3. The movement of the load is resisted by a viscous damping torque.

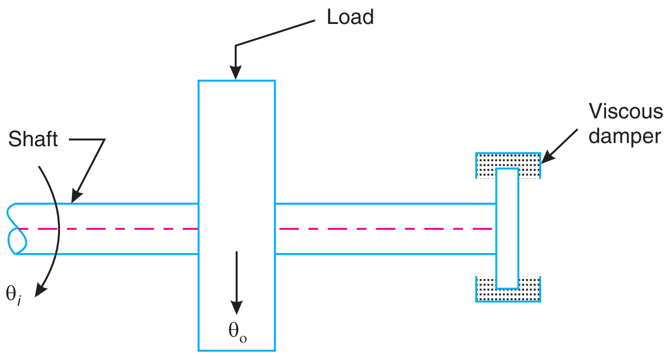



**Fig. 26.3.** Transfer function for a system with viscous damped output.

Let $\theta_i$ = Input signal to the shaft,

$\theta_o$ = Output signal of the shaft,

$q$ = Stiffness of the shaft,

$I$ = Moment of Inertia of the load, and

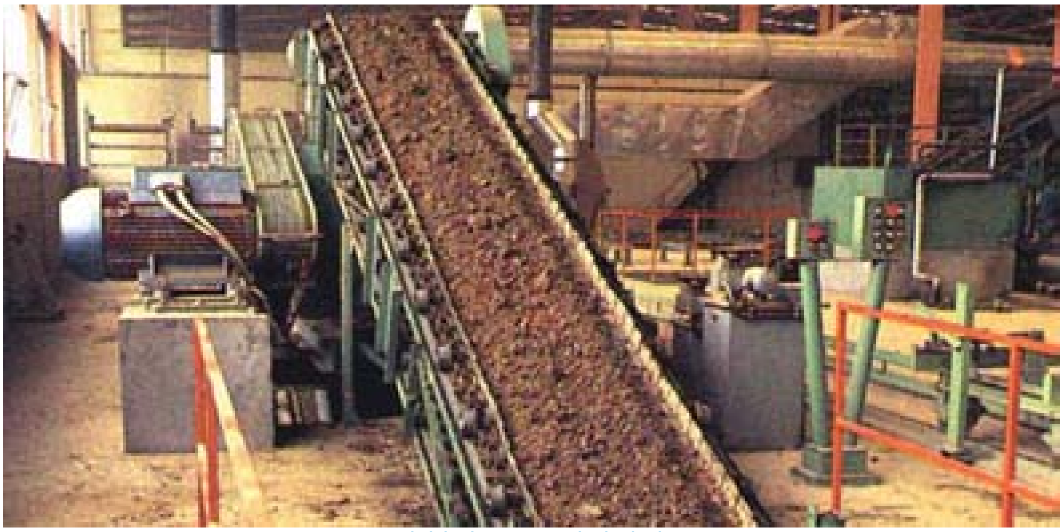
$T_d$ = Viscous damping torque per unit angular velocity.

After some time $t$,

Twist in the shaft $= \theta_i - \theta_o$

$\therefore$ Torque transmitted to the load $= q(\theta_i - \theta_o)$

We also know that damping torque $= T_d\,\omega_0 = T_d\left(\frac{d\theta_o}{dt}\right)$ ... $(\because \omega_0 = d\theta_o / dt)$



Material being moved via-belt conveyor.

Note : This picture is given as additional information and is not a direct example of the current chapter.

According to Newton's Second law, the equation of motion of the system is given by

$$I\left(\frac{d^2\theta_o}{dt^2}\right) = q\,(\theta_i - \theta_o) - T_d\left(\frac{d\theta_o}{dt}\right) \quad \text{... }(i)$$

or

$$I\left(\frac{d^2\theta_o}{dt^2}\right) = q\,\theta_i - q\,\theta_o - T_d\left(\frac{d\theta_o}{dt}\right)$$

Replacing $d/dt$ by $D$ in above equation, we get

$$I(D^2\theta_o) = q\,\theta_i - q\,\theta_o - T_d\,(D\theta_o)$$

or

$$I(D^2\theta_o) + T_d\,(D\theta_o) + q\,\theta_o = q\,\theta_i$$

$$D^2\theta_o + \frac{T_d}{I}(D\,\theta_o) + \frac{q}{I}(\theta_o) = \frac{q}{I}(\theta_i)$$

$$D^2\theta_o + \frac{T_d}{I}(D\,\theta_o) + (\omega_n)^2\theta_o = (\omega_n)^2\theta \quad \text{... }(ii)$$

where $\omega_n$ = Natural frequency of the shaft $= \sqrt{\frac{q}{I}}$

Also we know that viscous damping torque per unit angular velocity,

$$T_d = 2I\xi\omega_n \quad \text{or} \quad T_d / I = 2\xi\omega_n$$

where $\xi$ = Damping factor or damping ratio.

The equation ***(ii)*** may now be written as

$$D^2\theta + 2\xi\omega_n(D\theta_o) + (\omega_n)^2\theta_o = (\omega_n)^2\theta_i$$

or $$[D^2 + 2\xi\omega_n D + (\omega_n)^2]\theta_o = (\omega_n)^2\theta_i$$

$\therefore$ $$\text{Transfer function } = \frac{\theta_o}{\theta_i} = \frac{(\omega_n)^2}{D^2 + 2\xi\omega_n D + (\omega_n)^2}$$

$$= \frac{1}{T^2D^2 + 2\xi T D + 1}$$

where $T$ = Time constant = $1/\omega_n$

**Note:** The time constant ($T$) may also be obtained by dividing the periodic time ($t_d$) of the undamped natural oscillations of the system by $2\pi$. Mathematically,

$$T = \frac{t_d}{2\pi} = \frac{2\pi}{\omega_n} \times \frac{1}{2\pi} = \frac{1}{\omega_n} \qquad \ldots \left(\because\ t_d = \frac{2\pi}{\omega_n}\right)$$

**Example 26.1.** *The motion of a pointer over a scale is resisted by a viscous damping torque of magnitude 0.6 N-m at an angular velocity of 1 rad / s. The pointer, of negligible inertia, is mounted on the end of a relatively flexible shaft of stiffness 1.2 N-m / rad, and this shaft is driven through a 4 to 1 reduction gear box. Determine its overall transfer function.*

*If the input shaft to the gear box is suddenly rotated through 1 completed revolution, determine the time taken by the pointer to reach a position within 1 percent of its final value.*

**Solution.** Given:

$T_d$ = 0.6/1 = 0.6 N-ms/rad;

$q$ = 1.2 N-m/rad

The control system along with its block diagram is shown in Fig 26.4 ($a$) and ($b$) respectively.

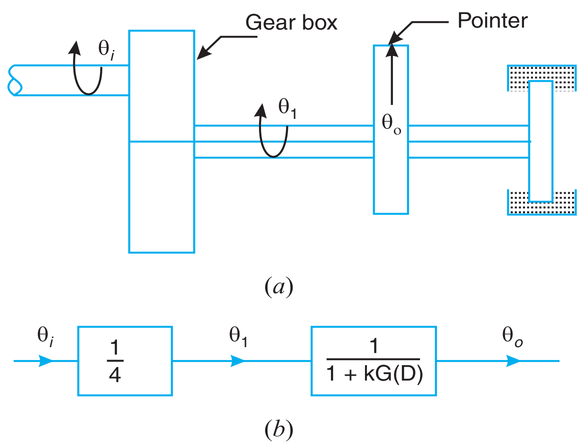



**Fig. 26.4**

### 1. *Overall transfer function*

Since the inertia of the pointer is negligible, therefore the torque generated by the twisting of the shaft has only to overcome the damping torque.

Therefore

$$q(\theta_1 - \theta_o) = T_d(d\theta_o/dt)$$

where $\theta_1$ = Output from the gear box.

$\therefore$ $$q\theta_1 - q\theta_o = T_d(D\theta_o) \qquad \ldots (\because\ d/dt = D)$$

or $$(q + T_d D)\theta_o = q\theta_1$$

$\therefore$ $$\frac{\theta_o}{\theta_1} = \frac{q}{q + T_d D} = \frac{1}{1 + (T_d/q)D} = \frac{1}{1 + TD} \qquad \ldots(i)$$

where $T$ = Time constant = $T_d/q$ = 0.6/1.2 = 0.5s

Substituting this value in equation *(i)*, we get

$$\frac{\theta_0}{\theta_1} = \frac{1}{1 + 0.5D}$$

We know that overall transfer function for the control system is

$$\frac{\theta_o}{\theta_i} = \frac{\theta_1}{\theta_i} \times \frac{\theta_2}{\theta_1} = \frac{1}{4} \times \frac{1}{(1 + 0.5D)} \quad \textbf{Ans.} \qquad \ldots \left[\because \theta_1 / \theta_i = \frac{1}{4} \text{(Given)}\right]$$

Aircraft engine is being assembled.

Note : This picture is given as additional information and is not a direct example of the current chapter.

**2. *Time taken by the pointer***

Let $t$ = Time taken by the pointer.

Since the input shaft to the gear box is rotated through 1 complete revolution, therefore $\theta_i = 2\pi$, a constant.

We know that transfer function for the control system is

$$\frac{\theta_o}{\theta_i} = \frac{1}{4} \times \frac{1}{(1 + 0.5D)} \quad \text{or} \quad (1 + 0.5D)\theta_o = \frac{\theta_i}{4}$$

$$\therefore \qquad 0.5\left(\frac{d\theta_o}{dt}\right) + \theta_o = \frac{\theta_i}{4} \qquad \ldots (\because D \equiv d/dt)$$

Substituting $\theta_i = 2\pi$ in the above equation, we get

$$0.5\left(\frac{d\theta_o}{dt}\right) + \theta_o = \frac{2\pi}{4} = \frac{\pi}{2}$$

or

$$0.5\left(\frac{d\theta_o}{dt}\right) = \frac{\pi}{2} - \theta_o$$

Separating the variables, we get

$$\frac{d\theta}{\pi - \theta_o} = 2\,dt$$

Integrating the above equation, we get

$$-\log_e\left(\frac{\pi}{2}-\theta_o\right)=2t+\text{constant} \quad \text{... (ii)}$$

Applying initial conditions to the above equation *i.e.* when $t = 0,\ \theta_o = 0$, we get

$$\text{constant} = -\log_e\left(\frac{\pi}{2}\right)$$

Substituting the value of constant in equation ***(i),***

$$-\log_e\left(\frac{\pi}{2}-\theta_o\right)=2t-\log_e\left(\frac{\pi}{2}\right)$$

or
$$\log_e\left(\frac{\pi}{2}-\theta_o\right)=-2t+\log_e\left(\frac{\pi}{2}\right)$$

∴
$$\frac{\pi}{2}-\theta_o=e^{-2t}\times\frac{\pi}{2}$$

or
$$\frac{\pi/2-\theta_o}{\pi/2}=e^{-2t}$$

*i.e*
$$\theta_o=\frac{\pi}{2}(1-e^{-2t}) \quad \text{... (iii)}$$

The curve depicted by above equation is shown in Fig. 26.5 and is known as ***simple exponential time delay curve.***

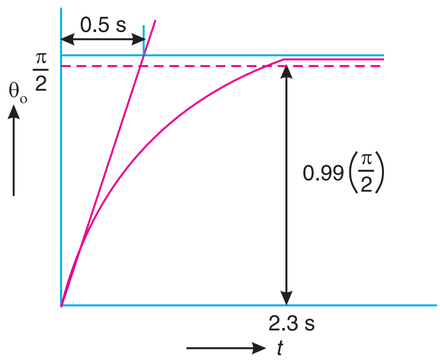



**Fig. 26.5**

The output $\theta_o$ will be within 1 percent of its final value when $\theta_0 = 0.99(\pi/2)$. Substituting this value in equation ***(iii),*** we get

$$0.99\left(\frac{\pi}{2}\right)=\frac{\pi}{2}\left(1-e^{-2t}\right)$$

$$0.99=1-e^{-2t} \quad \text{or} \quad e^{-2t}=0.01$$

∴
$$2t=\log_e 100=4.6 \quad \text{or} \quad t=2.3\text{s}$$ **Ans.**

## 26.9. Transfer Function of a Hartnell Governor

Consider a Hartnell governor* as shown in Fig. 26.6 (*a*). The various forces acting on the governor are shown in Fig. 26.6 (*b*).

Let $m$ = Mass of the ball

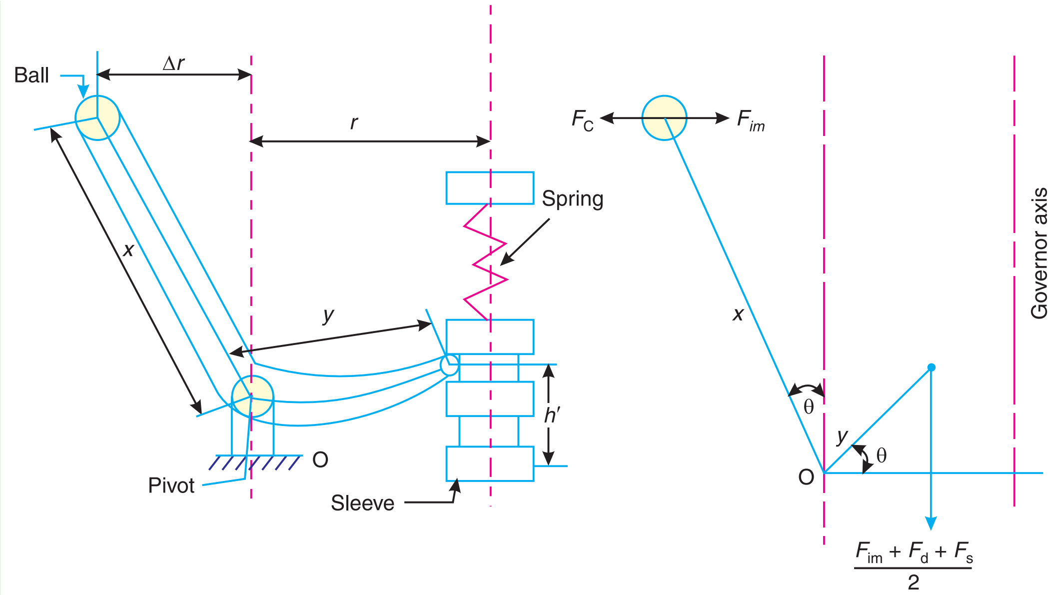
$M$ = mass of the sleeve,

$r$ = Radius of rotation of the governor in mid position,

$\Delta r$ = Change in radius of rotation,

$\omega$ = Angular speed of rotation in mid position,

$\Delta\omega$ = Change in angular speed of rotation,




(*a*) Hartnell governor. (*b*) Forces acting on a Hartnell governor.

**Fig. 26.6**

$x$ = Length of the vertical or ball arm of the lever,

$y$ = Length of the horizontal or sleeve arm of the lever,

$h$ = compression of spring with balls in vertical position,

$h'$ = Displacement of the sleeve,

$s$ = Stiffness of the spring,

$c$ = Damping coefficient *i.e.* damping force per unit velocity, and

$\xi$ = Damping factor.

Bucket conveyor

Note : This picture is given as additional information and is not a direct example of the current chapter.

---

* For details on Hartnell governor, refer chapter 18, Art. 18.8.

The various forces acting on the governor at the given position are as follows :

**1.** Centrifugal force due to ball mass,

$$F_c = m(r + \Delta r)\,(\omega + \Delta\omega)^2$$

$$= m\left(r + \frac{x}{y}(h')\right)(\omega + \Delta\omega)^2$$

**2.** Inertia force of the balls, $\quad F_{im} = m\left(\frac{x}{y}\right)\left(\frac{d^2h'}{dt^2}\right)$

**3.** Inertia force of the sleeve mass, $\quad F_{iM} = M\left(\frac{d^2h'}{dt^2}\right)$

**4.** Damping force, $\quad F_d = c\left(\frac{dx}{dt}\right)$

**5.** Spring force, $\quad F_s = s(h + h')$

It is assumed that the load on the sleeve, weight of the balls and the friction force are negligible as compared to the inertia forces. Now, taking moments about the fulcrum *O*, considering only one half of the governor,

$$m\left(r + \frac{x}{y}h'\right)(\omega + \Delta\omega)^2\,x = m\times\frac{x}{y}\left(\frac{d^2h'}{dt^2}\right)x + \frac{1}{2}\times M\left(\frac{d^2h'}{dt^2}\right)y$$

$$+\frac{1}{2}\times c\left(\frac{dh'}{dt}\right)y + \frac{1}{2}\times s(h + h')y$$

Neglecting the product of small terms, we get

$$mr\omega^2 x + m\times\frac{x}{y}\times h'\omega^2 x + 2mr\omega(\Delta\omega)x$$

$$= \frac{mx^2}{y}\left(\frac{d^2h'}{dt^2}\right) + \frac{1}{2}\times M\;y\left(\frac{d^2h'}{dt^2}\right) + \frac{1}{2}\times c\,y\left(\frac{dh'}{dt}\right) + \frac{1}{2}\times s\;y(h + h') \qquad \ldots(i)$$

Also, we know that at equilibrium position,

$$mr\omega^2 x = \frac{1}{2}\times s\;h\;y$$

Now the equation ***(i)*** may be written as

$$\frac{1}{2}\times s\,h\;y + m\times\frac{x}{y}\times h'\omega^2 x + 2mr\;\omega(\Delta\omega)x = \frac{mx^2}{y}(D^2h') + \frac{1}{2}My(D^2h') + \frac{1}{2}\,c\;y(Dh')$$

$$+\frac{1}{2}sy(h + h') \qquad \ldots(\because d/dt = D)$$

or $$\left(\frac{mx^2}{y}+\frac{1}{2}My\right)D^2h'+\left(\frac{1}{2}\times cy\right)Dh'+\left(\frac{1}{2}sy-\frac{mx^2}{y}\times D^2\right)h'=2mr\,\omega(\Delta\omega)x$$

Multiplying the above equation throughout by $2y$, we get

$$(2mx^2+My^2)D^2h'+(c\,y^2)dh'+(sy^2-2mx^2\omega^2)h'=4mr\,\omega(\Delta\omega)x\,y$$

$$(2mx^2+My^2)\left(D^2+\frac{cy^2}{2mx^2+My^2}+\frac{sy^2-2mx^2\omega^2}{2mx^2+My^2}\right)h'=4mr\,\omega(\Delta\omega)xy$$

or $$\left(D^2+\frac{cy^2}{2mx^2+My^2}\times D+\frac{sy^2-2mx^2\omega^2}{2mx^2+My^2}\right)h'=\frac{4mr\omega(\Delta\omega)xy}{2mx^2+My^2}$$

or $$D^2+2\xi\omega_n\,D+(\omega_n^2)h'=\frac{4mr\,\omega(\Delta\omega)xy}{2mx^2+My^2}$$

∴ $$h'=\frac{4mr\omega(\Delta\omega)xy}{2mx^2+My^2}\times\frac{1}{D^2+2\xi\omega_n D+(\omega_n)^2}$$

where $$2\xi\,\omega_n=\frac{cy^2}{2mx^2+My^2}$$

$\xi$ = Damping factor, and

$$\omega_n = \text{Natural frequency} = \sqrt{\frac{sy^2-2mx^2\omega^2}{2mx^2+My^2}}$$

Thus, transfer function for the Hartnell governor,

$$=\frac{\text{Output signal}}{\text{Input signal}}=\frac{\text{Displacement of sleeve }(h')}{\text{Change in speed }(\Delta\omega)}$$

$$=\frac{4mr\,\omega\,xy}{2mx^2+My^2}\times\frac{1}{D^2+2\,\xi\omega_n D+(\omega_n)^2}$$

## 26.10. Open-Loop Transfer Function

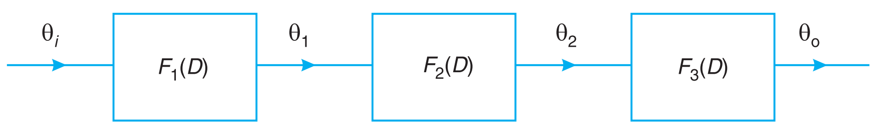



**Fig. 26.7.** Open loop control system.


**Fig. 26.8** Simplified open loop control system.

The open loop transfer function is defined as the overall transfer function of the forward path elements. Consider an open loop control system consisting of several elements having individual transfer function such $F_1(D)$, $F_2(D)$, $F_3\,(D)$ as shown in Fig. 26. 7. Thus

Open loop transfer function $= \dfrac{\theta_o}{\theta_i} = \dfrac{\theta_1}{\theta_i} \times \dfrac{\theta_2}{\theta_1} \times \dfrac{\theta_o}{\theta_2}$

$$= F_1(D) \times F_2(D) \times F_3(D) = KG(D)$$

The simplified block diagram of open loop transfer function is shown in Fig. 26.8.

## 26.11. Closed - Loop Transfer Function

The closed loop transfer function is defined as the overall transfer function of the entire control system. Consider a closed loop transfer function consisting of several elements as shown in Fig. 26.9.

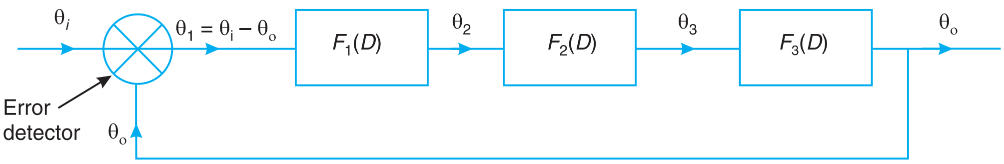



**Fig 26.9** Closed-loop transfer function.

Now, for the forward path element, we know that

$$\frac{\theta_o}{\theta_1} = \frac{\theta_o}{\theta_i - \theta_o} = K\,G(D)$$

where $$K\,G(D) = F_1(D) \times F_2(D) \times F_3(D)$$

On rearranging, we get

$$\theta_o = K\,G\,(D)\theta_i - K\,G\,(D)\theta_o$$

or $$[1 + K\,G\,(D)]\,\theta_o = K\,G\,(D)\theta_i$$

$\therefore$ $$\frac{\theta_o}{\theta_i} = \frac{K\,G(D)}{1 + K\,G(D)} = \frac{\text{Open loop } TF}{1 + \text{Open loop } TF}$$

The above expression shows the transfer function for the closed-loop control system.

Thus the block diagram may be further simplified as shown in Fig. 26.10, where the entire system is represented by a single block.


**Fig. 26.10.** Simplified closed-loop system.

## EXERCISES

**1.** Define the following terms:
(*a*) Response (*b*) Process control
(*c*) Regulator (*d*) Transducer

**2.** What do you understand by open-loop and closed loop control system? Explain with an example.

**3.** Discuss the importance of block diagrams in control systems.

4. Draw the block diagrams for the following control systems:
   (*a*) A simple carburettor,
   (*b*) A thermostatically controlled electric furnace.
5. What is a transfer function ?

## OBJECTIVE TYPE QUESTIONS

1. The device used to keep the variables at a constant desired value is called a
   (*a*) process controlled (*b*) regulator
   (*c*) deviation sensor (*d*) amplifier
2. The transfer function of a 4 to 1 reduction gear box is
   (*a*) 4 (*b*) 2
   (*c*) 1/4 (*d*) 1/2
3. A simple Bourdon tube pressure gauge is a
   (*a*) closed-loop control system
   (*b*) open-loop control system
   (*c*) manually operated system
   (*d*) none of the above
4. The overall transfer function of three blocks connected in series is
   (*a*) $\dfrac{F_1(D)\times F_2(D)}{F_3(D)}$ (*b*) $\dfrac{F_1(D)\times F_3(D)}{F_2(D)}$
   (*c*) $F_1(D)\times F_2(D)\times F_3(D)$ (*d*) $\dfrac{1}{F_1(D)\times F_2(D)\times F_3(D)}$
   where $F_1$ (*D*), $F_2$ (*D*) and $F_3$ (*D*) are the individual transfer functions of the three blocks.
5. The transfer function for a closed-loop control system is
   (*a*) $\dfrac{K\,G\,(D)}{1+K\,G\,(D)}$ (*b*) $K\,G(D)[1+KG(D)]$
   (*c*) $\dfrac{1+K\,G(D)}{KG(D)}$ (*d*) $\dfrac{K\,G(D)}{1-K\,G(D)}$

## ANSWERS

1. (*b*) 2. (*c*) 3. (*b*) 4. (*c*) 5. (*a*)

GO To FIRST