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Brakes and Dynamometers

19.1. Introduction

A *brake* is a device by means of which artificial frictional resistance is applied to a moving machine member, in order to retard or stop the motion of a machine. In the process of performing this function, the brake absorbs either kinetic energy of the moving member or potential energy given up by objects being lowered by hoists, elevators etc. The energy absorbed by brakes is dissipated in the form of heat. This heat is dissipated in the surrounding air (or water which is circulated through the passages in the brake drum) so that excessive heating of the brake lining does not take place. The capacity of a brake depends upon the following factors :

1. The unit pressure between the braking surfaces,
2. The coefficient of friction between the braking surfaces,
3. The peripheral velocity of the brake drum,
4. The projected area of the friction surfaces, and
5. The ability of the brake to dissipate heat equivalent to the energy being absorbed.

The major functional difference between a clutch and a brake is that a clutch is used to keep the driving and driven member moving together, whereas brakes are used to stop a moving member or to control its speed.

19.2. Materials for Brake Lining

The material used for the brake lining should have the following characteristics :

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1. It should have high coefficient of friction with minimum fading. In other words, the coefficient of friction should remain constant with change in temperature.
2. It should have low wear rate.
3. It should have high heat resistance.
4. It should have high heat dissipation capacity.
5. It should have adequate mechanical strength.
6. It should not be affected by moisture and oil.

The materials commonly used for facing or lining of brakes and their properties are shown in the following table.

Table 19.1. Properties of materials for brake lining.

Material for braking lining	*Coefficient of friction* (μ)			*Allowable pressure* (p)
	Dry	*Greasy*	*Lubricated*	*N/mm^2*
Cast iron on cast iron	0.15 – 0.2	0.06 – 0.10	0.05 – 0.10	1.0 – 1.75
Bronze on cast iron	–	0.05 – 0.10	0.05 – 0.10	0.56 – 0.84
Steel on cast iron	0.20 – 0.30	0.07 – 0.12	0.06 – 0.10	0.84 – 1.40
Wood on cast iron	0.20 – 0.35	0.08 – 0.12	–	0.40 – 0.62
Fibre on metal	–	0.10 – 0.20	–	0.07 – 0.28
Cork on metal	0.35	0.25 – 0.30	0.22 – 0.25	0.05 – 0.10
Leather on metal	0.30 – 0.5	0.15 – 0.20	0.12 – 0.15	0.07 – 0.28
Wire asbestos on metal	0.35 – 0.5	0.25 – 0.30	0.20 – 0.25	0.20 – 0.55
Asbestos blocks on metal	0.40 – 0.48	0.25 – 0.30	–	0.28 – 1.1
Asbestos on metal (Short action)	–	–	0.20 – 0.25	1.4 – 2.1
Metal on cast iron (Short action)	–	–	0.05 – 0.10	1.4 – 2.1

19.3. Types of Brakes

The brakes, according to the means used for transforming the energy by the braking elements, are classified as :

1. Hydraulic brakes *e.g.* pumps or hydrodynamic brake and fluid agitator,

2. Electric brakes *e.g.* generators and eddy current brakes, and

3. Mechanical brakes.

The hydraulic and electric brakes cannot bring the member to rest and are mostly used where large amounts of energy are to be transformed while the brake is retarding the load such as in laboratory dynamometers, high way trucks and electric locomotives. These brakes are also used for retarding or controlling the speed of a vehicle for down-hill travel.

The mechanical brakes, according to the direction of acting force, may be divided into the following two groups :

(*a*) *Radial brakes.* In these brakes, the force acting on the brake drum is in radial direction. The radial brakes may be

Simple bicycle brakes.

sub-divided into *external brakes* and *internal brakes.* According to the shape of the friction elements, these brakes may be *block* or *shoe brakes* and *band brakes.*

  (b) Axial brakes. In these brakes, the force acting on the brake drum is in axial direction. The axial brakes may be disc brakes and cone brakes. The analysis of these brakes is similar to clutches.

  Since we are concerned with only mechanical brakes, therefore, these are discussed, in detail, in the following pages.

19.4. Single Block or Shoe Brake

  A single block or shoe brake is shown in Fig. 19.1. It consists of a block or shoe which is pressed against the rim of a revolving brake wheel drum. The block is made of a softer material than the rim of the wheel. This type of a brake is commonly used on railway trains and tram cars. The friction between the block and the wheel causes a tangential braking force to act on the wheel, which retard the rotation of the wheel. The block is pressed against the wheel by a force applied to one end of a lever to which the block is rigidly fixed as shown in Fig. 19.1. The other end of the lever is pivoted on a fixed fulcrum O.
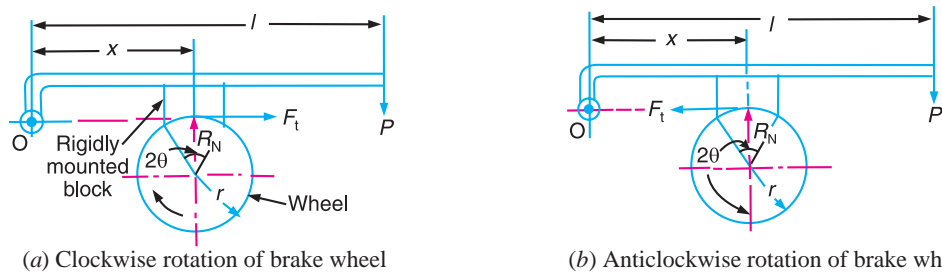

(a) Clockwise rotation of brake wheel   (b) Anticlockwise rotation of brake wheel.
Fig. 19.1. Single block brake. Line of action of tangential force passes through the fulcrum of the lever.

Let P = Force applied at the end of the lever,

       R_N = Normal force pressing the brake block on the wheel,

       r = Radius of the wheel,

       2θ = Angle of contact surface of the block,

       μ = Coefficient of friction, and

       F_t = Tangential braking force or the frictional force acting at the contact surface of the block and the wheel.

  If the angle of contact is less than 60°, then it may be assumed that the normal pressure between the block and the wheel is uniform. In such cases, tangential braking force on the wheel,

$$F_t = \mu.R_N \qquad ...(i)$$

and the braking torque, $T_B = F_t.r = \mu.R_N.r \qquad ...(ii)$

  Let us now consider the following three cases :

  Case 1. When the line of action of tangential braking force (F_t) passes through the fulcrum O of the lever, and the brake wheel rotates clockwise as shown in Fig. 19.1 (a), then for equilibrium, taking moments about the fulcrum O, we have

$$R_N \times x = P \times l \text{ or } R_N = \frac{P \times l}{x}$$

∴   Braking torque,

$$T_B = \mu.R_N.r = \mu \times \frac{P.l}{x} \times r = \frac{\mu.P.l.r}{x}$$

When brakes are on, the pads grip the wheel rim from either side, friction between the pads and the rim converts the cycle's kinetic energy into heat as they reduce its speed.
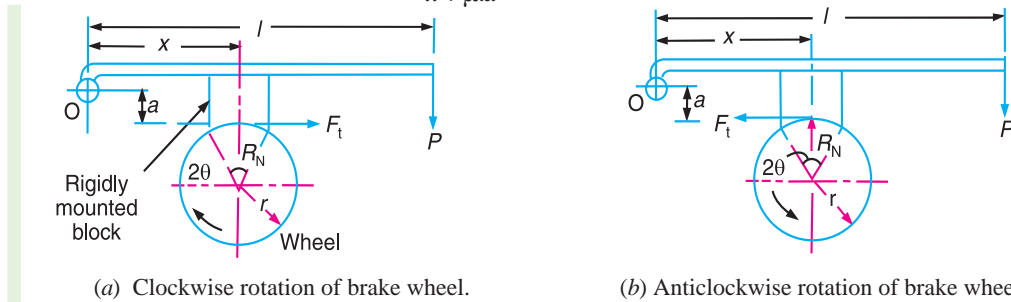
It may be noted that when the brake wheel rotates anticlockwise as shown in Fig. 19.1 (*b*), then the braking torque is same, *i.e.*

$$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x}$$

Case 2. When the line of action of the tangential braking force (F_t) passes through a distance '*a*' below the fulcrum *O*, and the brake wheel rotates clockwise as shown in Fig. 19.2 (*a*), then for equilibrium, taking moments about the fulcrum *O*,

$$R_N \times x + F_t \times a = P.l \quad \text{or} \quad R_N \times x + \mu\,R_N \times a = P.l \quad \text{or} \quad R_N = \frac{P.l}{x + \mu.a}$$

and braking torque, $\qquad T_B = \mu R_N.r = \dfrac{\mu.p.l.r}{x + \mu.a}$

| (*a*) Clockwise rotation of brake wheel. | (*b*) Anticlockwise rotation of brake wheel. |

Fig. 19.2. Single block brake. Line of action of F_t passes below the fulcrum.

When the brake wheel rotates anticlockwise, as shown in Fig. 19.2 (*b*), then for equilibrium,

$$R_N.x = P.l + F_t.a = P.l + \mu.R_N.a \qquad \qquad ...(i)$$
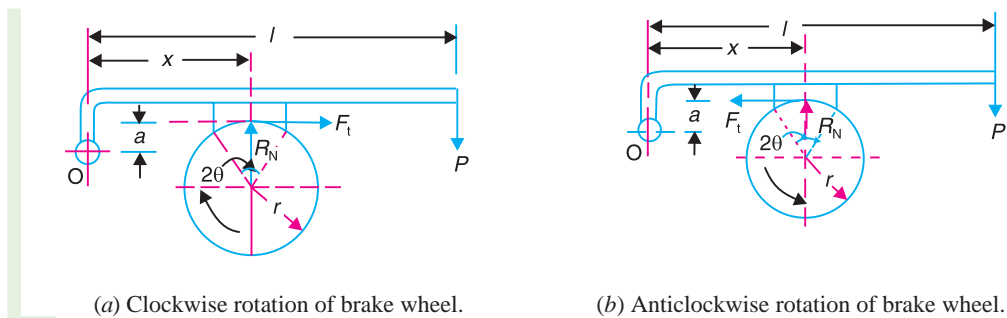
or $\qquad R_N (x - \mu.a) = P.l \quad \text{or} \quad R_N = \dfrac{P.l}{x - \mu.a}$

and braking torque, $\qquad T_B = \mu.R_N.r = \dfrac{\mu.P.l.r}{x - \mu.a}$

Case 3. When the line of action of the tangential braking force (F_t) passes through a distance '*a*' above the fulcrum *O*, and the brake wheel rotates clockwise as shown in Fig. 19.3 (*a*), then for equilibrium, taking moments about the fulcrum *O*, we have

$$R_N.x = P.l + F_t . a = P.l + \mu.R_N.a \qquad \qquad . . . (ii)$$

or $\qquad R_N (x - \mu.a) = P.l \qquad \text{or} \qquad R_N = \dfrac{P.l}{x - \mu.a}$

| (*a*) Clockwise rotation of brake wheel. | (*b*) Anticlockwise rotation of brake wheel. |

Fig. 19.3. Single block brake. Line of action of F_t passes above the fulcrum.

and braking torque, $\qquad T_B = \mu.R_N.r = \dfrac{\mu.P.l.r}{x - \mu.a}$

When the brake wheel rotates anticlockwise as shown in Fig. 19.3 (*b*), then for equilibrium, taking moments about the fulcrum *O*, we have

$$R_N \times x + F_t \times a = P.l \quad \text{or} \quad R_N \times x + \mu.R_N \times a = P.l \quad \text{or} \quad R_N = \frac{P.l}{x + \mu.a}$$

and braking torque, $\quad T_B = \mu.R_N.r = \dfrac{\mu.P.l.r}{x + \mu.a}$

Notes : 1. From above we see that when the brake wheel rotates anticlockwise in case 2 [Fig. 19.2 (*b*)] and when it rotates clockwise in case 3 [Fig. 19.3 (*a*)], the equations (*i*) and (*ii*) are same, *i.e.*

$$R_N \times x = P.l + \mu.R_N.a$$

From this we see that the moment of frictional force ($\mu.R_N.a$) adds to the moment of force (*P.l*). In other words, the frictional force helps to apply the brake. Such type of brakes are said to be *self energizing brakes*. When the frictional force is great enough to apply the brake with no external force, then the brake is said to be **self-locking brake**.

From the above expression, we see that if $x \leq \mu.a$, then *P* will be negative or equal to zero. This means no external force is needed to apply the brake and hence the brake is self locking. Therefore the condition for the brake to be self locking is

$$x \leq \mu.a$$

The self locking brake is used only in back-stop applications.

2. The brake should be self energizing and not the self locking.

3. In order to avoid self locking and to prevent the brake from grabbing, *x* is kept greater than μ . *a.*

4. If A_b is the projected bearing area of the block or shoe, then the bearing pressure on the shoe,

$$p_b = R_N / A_b$$

We know that $\quad A_b$ = Width of shoe × Projected length of shoe = $w(2r\sin\theta)$

5. When a single block or shoe brake is applied to a rolling wheel, an additional load is thrown on the shaft bearings due to heavy normal force (R_N) and produces bending of the shaft.

In order to overcome this drawback, a double block or shoe brake is used, as discussed in Art. 19.6.

19.5. Pivoted Block or Shoe Brake

We have discussed in the previous article that when the angle of contact is less than 60°, then it may be assumed that the normal pressure between the block and the wheel is uniform. But

when the angle of contact is greater than 60°, then the unit pressure normal to the surface of contact is less at the ends than at the centre. In such cases, the block or shoe is pivoted to the lever, as shown in Fig. 19.4, instead of being rigidly attached to the lever. This gives uniform wear of the brake lining in the direction of the applied force. The braking torque for a pivoted block or shoe brake (*i.e.* when $2\theta > 60°$) is given by

Fig. 19.4. Pivoted block or shoe brake.

$$T_B = F_t \times r = \mu'.R_N.r$$

where $\qquad \mu'$ = Equivalent coefficient of friction = $\dfrac{4\mu\sin\theta}{2\theta + \sin 2\theta}$, and

$\qquad \mu$ = Actual coefficient of friction.

These brakes have more life and may provide a higher braking torque.
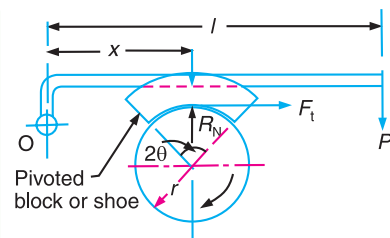
Example 19.1. *A single block brake is shown in Fig. 19.5. The diameter of the drum is 250 mm and the angle of contact is 90°. If the operating force of 700 N is applied at the end of a lever and the coefficient of friction between the drum and the lining is 0.35, determine the torque that may be transmitted by the block brake.*
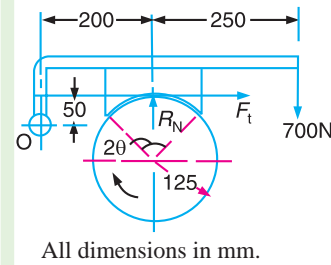

All dimensions in mm.

Fig. 19.5

Solution. Given : $d = 250$ mm or $r = 125$ mm ; $2\theta = 90°$ $= \pi/2$ rad ; $P = 700$ N ; $\mu = 0.35$

Since the angle of contact is greater than 60°, therefore equivalent coefficient of friction,

$$\mu' = \frac{4\mu \sin \theta}{2\theta + \sin 2\theta} = \frac{4 \times 0.35 \times \sin 45°}{\pi/2 + \sin 90°} = 0.385$$

Let R_N = Normal force pressing the block to the brake drum, and

F_t = Tangential braking force = $\mu'.R_N$

Taking moments about the fulcrum O, we have

$$700(250 + 200) + F_t \times 50 = R_N \times 200 = \frac{F_t}{\mu'} \times 200 = \frac{F_t}{0.385} \times 200 = 520 \, F_t$$

or $\quad\quad\quad 520 \, F_t - 50 F_t = 700 \times 450 \quad$ or $\quad F_t = 700 \times 450/470 = 670$ N

We know that torque transmitted by the block brake,

$$T_B = F_t \times r = 670 \times 125 = 8\,3750 \text{ N-mm} = 83.75\text{N-m } \textbf{Ans.}$$

Example 19.2. *Fig. 19.6 shows a brake shoe applied to a drum by a lever AB which is pivoted at a fixed point A and rigidly fixed to the shoe. The radius of the drum is 160 mm. The coefficient of friction at the brake lining is 0.3. If the drum rotates clockwise, find the braking torque due to the horizontal force of 600 N at B.*
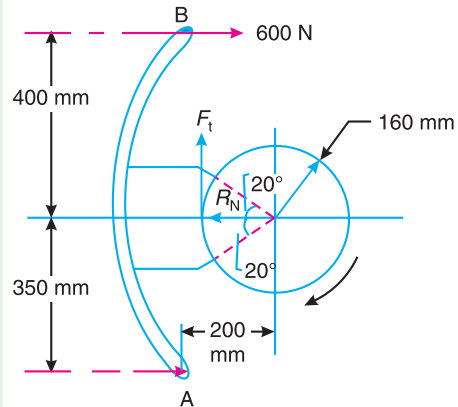

Fig. 19.6

Solution. Given : $r = 160$ mm $= 0.16$ m ; $\mu = 0.3$; $P = 600$ N

Since the angle subtended by the shoe at the centre of drum is 40°, therefore we need not to calculate the equivalent coefficient of friction μ'.

Let $\quad R_N$ = Normal force pressing the block to the brake drum, and

F_t = Tangential braking force = $\mu.R_N$

Taking moments about point A,

$$R_N \times 350 + F_t (200 - 160) = 600 (400 + 350)$$

$$\frac{F_t}{0.3} \times 350 + 40 F_t = 600 \times 750 \quad \text{or} \quad 1207 \, F_t = 450 \times 10^3$$

∴ $\quad\quad\quad F_t = 450 \times 10^3/1207 = 372.8$ N

We know that braking torque,

$$T_B = F_t \times r = 372.8 \times 0.16 = 59.6 \text{ N-m } \textbf{Ans.}$$

Example 19.3. *A bicycle and rider of mass 100 kg are travelling at the rate of 16 km/h on a level road. A brake is applied to the rear wheel which is 0.9 m in diameter and this is the only resistance acting. How far will the bicycle travel and how many turns will it make before it comes to rest ? The pressure applied on the brake is 100 N and μ = 0.05.*

Solution. Given : $m = 100$ kg, $v = 16$ km / h = 4.44 m / s ; $D = 0.9$ m ; $R_N = 100$ N ; $\mu = 0.05$

Distance travelled by the bicycle before it comes to rest

Let x = Distance travelled (in metres) by the bicycle before it comes to rest.

We know that tangential braking force acting at the point of contact of the brake and wheel,

$$F_t = \mu.R_N = 0.05 \times 100 = 5 \text{ N}$$

and work done $= F_t \times x = 5 \times x = 5x$ N-m   ... *(i)*

We know that kinetic energy of the bicycle

$$= \frac{m.v^2}{2} = \frac{100(4.44)^2}{2}$$

$$= 986 \text{ N-m} \qquad \ldots (ii)$$

In order to bring the bicycle to rest, the work done against friction must be equal to kinetic energy of the bicycle. Therefore equating equations *(i)* and *(ii)*,

$$5x = 986 \text{ or } x = 986/5 = 197.2 \text{ m } \textbf{Ans.}$$

Number of revolutions made by the bicycle before it comes to rest

Let N = Required number of revolutions.

We know that distance travelled by the bicycle (x),

$$197.2 = \pi DN = \pi \times 0.9 N = 2.83N$$

∴ $N = 197.2 / 2.83 = 70$ **Ans.**

Shoe brake.

Example 19.4. *A braking system has its braking lever inclined at an angle of 30° to the horizontal plane, as shown in Fig. 19.7. The mass and diameter of the brake drum are 218 kg and 0.54 m respectively.*
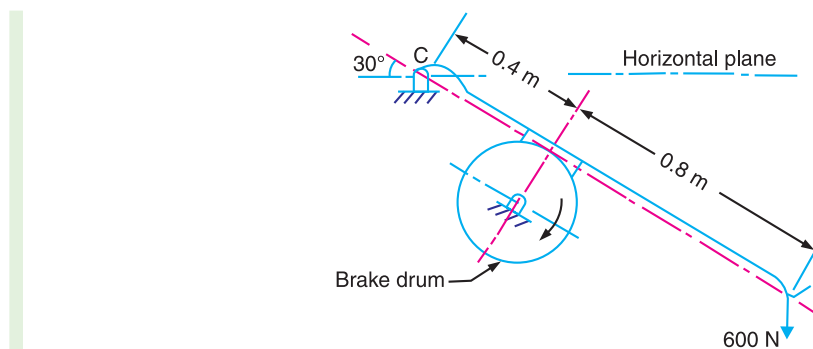

Fig. 19.7

At the instant the lever is pressed on the brake drum with a vertical force of 600 N, the drum is found to rotate at 2400 r.p.m. clockwise. The coefficient of friction between the brake shoe and the brake drum is 0.4. Assume that the lever and brake shoe are perfectly rigid and possess negligible weight. Find :
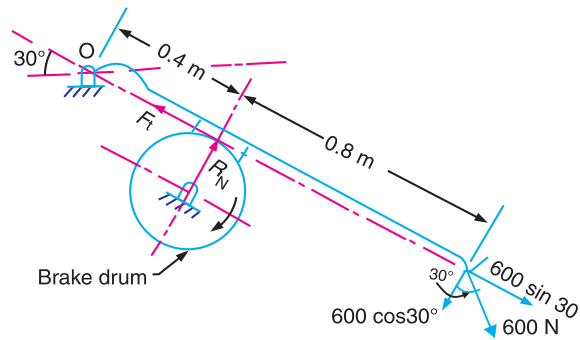
1. Braking torque, 2. Number of revolutions the drum will make before coming to rest from the instant of pressing the lever, and 3. Time taken for the drum to come to rest from the instant of pressing the lever.

Solution. Given : $m = 218$ kg ; $d = 0.54$ m or $r = 0.27$ m ; $P = 600$ N ; $N = 2400$ r.p.m.; $\mu = 0.4$

1. *Braking torque*

Let $\qquad\qquad R_N =$ Normal force pressing the block to the brake drum, and

$\qquad\qquad\qquad F_t =$ Tangential braking force.

The various forces acting on the braking system are shown in Fig. 19.8.

Fig. 19.8

Taking moments about the fulcrum *O*,

$$600 \cos 30° \times 1.2 = R_N \times 0.4 \qquad \text{or} \qquad 623.5 = 0.4\, R_N$$

$\therefore \qquad\qquad\qquad\qquad R_N = 623.5/0.4 = 1560$ N

and $\qquad\qquad\qquad F_t = \mu . R_N = 0.4 \times 1560 = 624$ N

We know that braking torque,

$$T_B = F_t \times r = 624 \times 0.27 = 168.5 \text{ N-m } \textbf{Ans.}$$

2. *Number of revolutions the drum will make before coming to rest*

Let $\qquad\qquad\qquad n =$ Required number of revolutions.

We know that kinetic energy of the brake drum

$$= \frac{m.v^2}{2} = \frac{218}{2}\left(\frac{\pi\, d.N}{60}\right)^2 = 109\left(\frac{\pi \times 0.54 \times 2400}{60}\right)^2 \text{ N-m}$$

$$= 502 \times 10^3 \text{ N-m} \qquad\qquad\qquad\qquad \ldots (i)$$

and work done by the brake drum due to braking torque

$$= T_B \times 2\pi\, n = 168.5 \times 2\pi\, n = 1060 n \text{ N-m} \qquad\qquad \ldots (ii)$$

Since the kinetic energy of the brake drum is used to overcome the work done due to braking torque, therefore equating equations *(i)* and *(ii)*,

$$n = 502 \times 10^3/1060 = 474 \textbf{ Ans.}$$

3. *Time taken for the drum to come to rest*

We know that time taken for the drum to come to rest *i.e.* time required for 474 revolutions,

$$t = \frac{n}{N} = \frac{474}{2400} = 0.2 \text{ min} = 12 \text{ s } \textbf{Ans.}$$

19.6. Double Block or Shoe Brake

When a single block brake is applied to a rolling wheel, an additional load is thrown on the shaft bearings due to the normal force (R_N). This produces bending of the shaft. In order to overcome this drawback, a double block or shoe brake, as shown in Fig. 19.9, is used. It consists of two brake blocks applied at the opposite ends of a diameter of the wheel which eliminate or reduces the unbalanced force on the shaft. The brake is set by a spring which pulls the upper ends of the brake arms together. When a force P is applied to the bell crank lever, the spring is compressed and the brake is released. This type of brake is often used on electric cranes and the force P is produced by an electromagnet or solenoid. When the current is switched off, there is no force on the bell crank lever and the brake is engaged automatically due to the spring force and thus there will be no downward movement of the load.
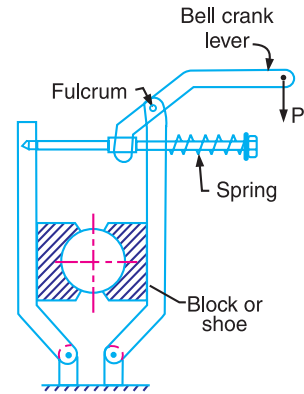
In a double block brake, the braking action is doubled by the use of two blocks and these blocks may be operated practically by the same force which will operate one. In case of double block or shoe brake, the braking torque is given by

$$T_B = (F_{t1} + F_{t2})\, r$$

where F_{t1} and F_{t2} are the braking forces on the two blocks.

Fig. 19.9. Double block or shoe brake.

Example 19.5. *A double shoe brake, as shown in Fig. 19.10, is capable of absorbing a torque of 1400 N-m. The diameter of the brake drum is 350 mm and the angle of contact for each shoe is 100°. If the coefficient of friction between the brake drum and lining is 0.4 ; find 1. the spring force necessary to set the brake ; and 2. the width of the brake shoes, if the bearing pressure on the lining material is not to exceed 0.3 N/mm².*
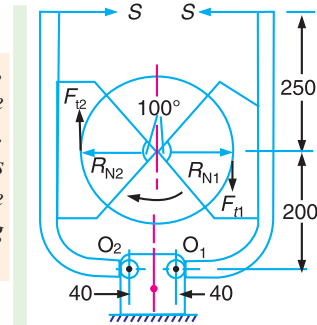
All dimensions in mm.
Fig. 19.10

Solution. Given : $T_B = 1400$ N-m $= 1400 \times 10^3$ N-mm ; $d = 350$ mm or $r = 175$ mm ; $2\theta = 100° = 100 \times \pi/180 = 1.75$ rad; $\mu = 0.4$; $p_b = 0.3$ N/mm²

1. *Spring force necessary to set the brake*

Let S = Spring force necessary to set the brake.

R_{N1} and F_{t1} = Normal reaction and the braking force on the right hand side shoe, and

R_{N2} and F_{t2} = Corresponding values on the left hand side shoe.

Since the angle of contact is greater than 60°, therefore equivalent coefficient of friction,

$$\mu' = \frac{4\mu \sin \theta}{2\theta + \sin 2\theta} = \frac{4 \times 0.4 \times \sin 50°}{1.75 + \sin 100°} = 0.45$$

Brakes on a railway coach.

Taking moments about the fulcrum O_1, we have

$$S \times 450 = R_{N1} \times 200 + F_{t1}(175 - 40) = \frac{F_{t1}}{0.45} \times 200 + F_{t1} \times 135 = 579.4\, F_{t1}$$

$$\ldots \left(\text{Substituting } R_{N1} = \frac{F_{t1}}{\mu'} \right)$$

∴ $F_{t1} = S \times 450 / 579.4 = 0.776\, S$

Again taking moments about O_2, we have

$$S \times 450 + F_{t2}(175 - 40) = R_{N2} \times 200 = \frac{F_{t2}}{0.45} \times 200 = 444.4\, F_{t2}$$

$$\ldots \left(\text{Substituting } R_{N2} = \frac{F_{t2}}{\mu'} \right)$$

$444.4\, F_{t2} - 135 F_{t2} = S \times 450$ or $309.4\, F_{t2} = S \times 450$

∴ $F_{t2} = S \times 450 / 309.4 = 1.454\, S$

We know that torque capacity of the brake (T_B),

$$1400 \times 10^3 = (F_{t1} + F_{t2})\, r = (0.776\, S + 1.454\, S)\, 175 = 390.25\, S$$

∴ $S = 1400 \times 10^3 / 390.25 = 3587$ N **Ans.**

2. *Width of the brake shoes*

Let b = Width of the brake shoes in mm.

We know that projected bearing area for one shoe,

$$A_b = b\,(2r \sin \theta) = b\,(2 \times 175 \sin 50°) = 268\, b \text{ mm}^2$$

Normal force on the right hand side of the shoe,

$$R_{N1} = \frac{F_{t1}}{\mu'} = \frac{0.776 \times S}{0.45} = \frac{0.776 \times 3587}{0.45} = 6186 \text{ N}$$

and normal force on the left hand side of the shoe,

$$R_{N2} = \frac{F_{t2}}{\mu'} = \frac{1.454 \times S}{0.45} = \frac{1.454 \times 3587}{0.45} = 11\,590 \text{ N}$$

We see that the maximum normal force is on the left hand side of the shoe. Therefore we shall find the width of the shoe for the maximum normal force *i.e.* R_{N2}.

We know that the bearing pressure on the lining material (p_b),

$$0.3 = \frac{R_{N2}}{A_b} = \frac{11\,590}{268\, b} = \frac{43.25}{b}$$

∴ $b = 43.25 / 0.3 = 144.2$ mm **Ans.**

19.7. Simple Band Brake

A band brake consists of a flexible band of leather, one or more ropes, or a steel lined with friction material, which embraces a part of the circumference of the drum. A band brake, as shown in Fig. 19.11, is called a ***simple band brake*** in which one end of the band is attached to a fixed pin or fulcrum of the lever while the other end is attached to the lever at a distance *b* from the fulcrum.

When a force *P* is applied to the lever at *C*, the lever turns about the fulcrum pin *O* and tightens the band on the drum and hence the brakes are applied. The friction between the band and the drum provides the braking force. The force *P* on the lever at *C* may be determined as discussed below :

Let T_1 = Tension in the tight side of the band,

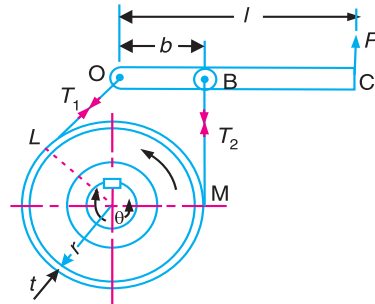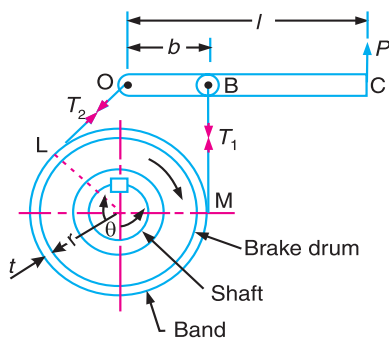 T_2 = Tension in the slack side of the band,

θ = Angle of lap (or embrace) of the band on the drum,

μ = Coefficient of friction between the band and the drum,

r = Radius of the drum,

t = Thickness of the band, and

r_e = Effective radius of the drum = $r + \dfrac{t}{2}$

Band brake

Bands of a brake shown separately

(*a*) Clockwise rotation of drum. (*b*) Anticlockwise rotation of drum.

Fig. 19.11. Simple band brake.

We know that limiting ratio of the tensions is given by the relation,

$$\frac{T_1}{T_2} = e^{\mu\theta} \qquad \text{or} \qquad 2.3\log\left(\frac{T_1}{T_2}\right) = \mu.\theta$$

and braking force on the drum = $T_1 - T_2$

∴ Braking torque on the drum,

$$T_B = (T_1 - T_2)\, r \qquad\qquad \text{... (Neglecting thickness of band)}$$

$$= (T_1 - T_2)\, r_e \qquad\qquad \text{... (Considering thickness of band)}$$

Now considering the equilibrium of the lever *OBC*. It may be noted that when the drum rotates in the clockwise direction, as shown in Fig. 19.11 (*a*), the end of the band attached to the fulcrum *O* will be slack with tension T_2 and end of the band attached to *B* will be tight with tension T_1. On the other hand, when the drum rotates in the anticlockwise direction, as shown in Fig. 19.11 (*b*), the tensions in the band will reverse, *i.e.* the end of the band attached to the fulcrum *O* will be tight with tension T_1 and the end of the band attached to *B* will be slack with tension T_2. Now taking moments about the fulcrum *O*, we have

$$P.l = T_1.b \qquad\qquad \text{... (For clockwise rotation of the drum)}$$

and $$P.l = T_2.b \qquad\qquad \text{... (For anticlockwise rotation of the drum)}$$

where l = Length of the lever from the fulcrum (OC), and

b = Perpendicular distance from O to the line of action of T_1 or T_2.

Notes : 1. When the brake band is attached to the lever, as shown in Fig. 19.11 (*a*) and (*b*), then the force (*P*) must act in the upward direction in order to tighten the band on the drum.

2. If the permissible tensile stress (σ) for the material of the band is known, then maximum tension in the band is given by

$$T_1 = \sigma.w.t$$

where w = Width of the band, and

t = thickness of the band.

Example 19.6. *A band brake acts on the 3/4th of circumference of a drum of 450 mm diameter which is keyed to the shaft. The band brake provides a braking torque of 225 N-m. One end of the band is attached to a fulcrum pin of the lever and the other end to a pin 100 mm from the fulcrum. If the operating force is applied at 500 mm from the fulcrum and the coefficient of friction is 0.25, find the operating force when the drum rotates in the (**a**) anticlockwise direction, and (**b**) clockwise direction.*

Solution. Given : $d = 450$ mm or $r = 225$ mm $= 0.225$ m ; $T_B = 225$ N-m ; $b = OB = 100$ mm $= 0.1$ m ; $l = 500$ mm $= 0.5$ m ; $\mu = 0.25$

Let P = Operating force.

(a) Operating force when drum rotates in anticlockwise direction

The band brake is shown in Fig. 19.11. Since one end of the band is attached to the fulcrum at O, therefore the operating force P will act upward and when the drum rotates anticlockwise, as shown in Fig. 19.11 (*b*), the end of the band attached to O will be tight with tension T_1 and the end of the band attached to B will be slack with tension T_2. First of all, let us find the tensions T_1 and T_2.

We know that angle of wrap,

Drums for band brakes.

$$\theta = \frac{3}{4} \text{ th of circumference} = \frac{3}{4} \times 360° = 270°$$

$$= 270 \times \pi / 180 = 4.713 \text{ rad}$$

and

$$2.3 \log\left(\frac{T_1}{T_1}\right) = \mu.\theta = 0.25 \times 4.713 = 1.178$$

∴

$$\log\left(\frac{T_1}{T_2}\right) = \frac{1.178}{2.3} = 0.5123 \text{ or } \frac{T_1}{T_2} = 3.253 \qquad \ldots (i)$$

$$\ldots \text{(Taking antilog of 0.5123)}$$

We know that braking torque (T_B),

$$225 = (T_1 - T_2)\, r = (T_1 - T_2)\, 0.225$$

∴ $$T_1 - T_2 = 225 / 0.225 = 1000 \text{ N} \qquad \ldots (ii)$$

From equations (*i*) and (*ii*), we have

$$T_1 = 1444 \text{ N; and} \quad T_2 = 444 \text{ N}$$

Now taking moments about the fulcrum O, we have

$$P \times l = T_2.b \quad \text{or} \quad P \times 0.5 = 444 \times 0.1 = 44.4$$

∴ $$P = 44.4 / 0.5 = 88.8 \text{ N } \textbf{Ans.}$$

(*b*) *Operating force when drum rotates in clockwise direction*

When the drum rotates in clockwise direction, as shown in Fig.19.11 (*a*), then taking moments about the fulcrum *O*, we have

$$P \times l = T_1 . b \qquad \text{or} \qquad P \times 0.5 = 1444 \times 0.1 = 144.4$$

$$\therefore \qquad P = 144.4 / 0.5 = 288.8 \text{ N } \textbf{Ans.}$$

Example 19.7. *The simple band brake, as shown in Fig. 19.12, is applied to a shaft carrying a flywheel of mass 400 kg. The radius of gyration of the flywheel is 450 mm and runs at 300 r.p.m.*

If the coefficient of friction is 0.2 and the brake drum diameter is 240 mm, find :

 1. the torque applied due to a hand load of 100 N,

 2. the number of turns of the wheel before it is brought to rest, and

 3. the time required to bring it to rest, from the moment of the application of the brake.

All dimensions in mm.

Fig. 19.12

Solution. Given : $m = 400$ kg ; $k = 450$ mm $= 0.45$ m ; $N = 300$ r.p.m. or $\omega = 2\pi \times 300 / 60 = 31.42$ rad/s ; $\mu = 0.2$; $d = 240$ mm $= 0.24$ m or $r = 0.12$ m

1. *Torque applied due to hand load*

First of all, let us find the tensions in the tight and slack sides of the band *i.e.* T_1 and T_2 respectively.

From the geometry of the Fig. 19.12, angle of lap of the band on the drum,

$$\theta = 360° - 150° = 210° = 210 \times \frac{\pi}{180} = 3.666 \text{ rad}$$

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.2 \times 3.666 = 0.7332$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.7332}{2.3} = 0.3188 \qquad \text{or} \qquad \frac{T_1}{T_2} = 2.08 \qquad \qquad \ldots (i)$$

... (Taking antilog of 0.3188)

Taking moments about the fulcrum *O*,

$$T_2 \times 120 = 100 \times 300 = 30\,000 \qquad \text{or} \qquad T_2 = 30\,000/120 = 250 \text{ N}$$

$$\therefore \qquad T_1 = 2.08 T_2 = 2.08 \times 250 = 520 \text{ N} \qquad \qquad \ldots \text{[From equation } (i)]$$

We know that torque applied,

$$T_B = (T_1 - T_2)\, r = (520 - 250)\, 0.12 = 32.4 \text{ N-m } \textbf{Ans.}$$

2. *Number of turns of the wheel before it is brought to rest*

Let n = Number of turns of the wheel before it is brought to rest.

We know that kinetic energy of rotation of the drum

$$= \frac{1}{2} \times I.\omega^2 = \frac{1}{2} \times m.k^2.\omega^2 = \frac{1}{2} \times 400(0.45)^2 (31.42)^2 = 40\,000 \text{ N-m}$$

This energy is used to overcome the work done due to the braking torque (T_B).

$$\therefore \qquad 40\,000 = T_B \times 2\pi n = 32.4 \times 2\pi n = 203.6\, n$$

or

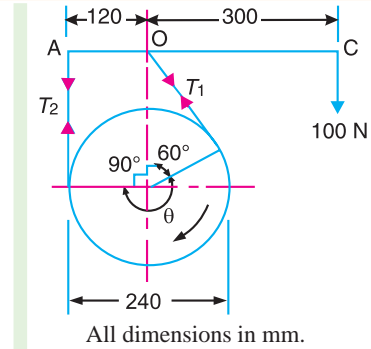$$n = 40\,000 / 203.6 = 196.5 \textbf{ Ans.}$$

3. Time required to bring the wheel to rest

We know that the time required to bring the wheel to rest

$$= n \,/\, N = 196.5 \,/\, 300 = 0.655 \text{ min} = 39.3 \text{ s } \textbf{Ans.}$$

Example 19.8. *A simple band brake operates on a drum of 600 mm in diameter that is running at 200 r.p.m. The coefficient of friction is 0.25. The brake band has a contact of 270°, one end is fastened to a fixed pin and the other end to the brake arm 125 mm from the fixed pin. The straight brake arm is 750 mm long and placed perpendicular to the diameter that bisects the angle of contact.*

1. *What is the pull necessary on the end of the brake arm to stop the wheel if 35 kW is being absorbed ? What is the direction for this minimum pull ?*

2. *What width of steel band of 2.5 mm thick is required for this brake if the maximum tensile stress is not to exceed 50 N/mm² ?*
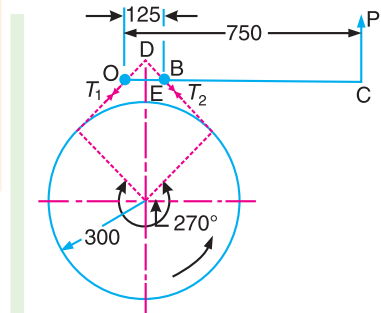
Solution. Given : $d = 600$ mm or $r = 300$ mm ; $N = 200$ r.p.m. ; $\mu = 0.25$; $\theta = 270° = 270 \times \pi/180 = 4.713$ rad ; Power = 35 kW = 35×10^3 W ; $t = 2.5$ mm ; $\sigma = 50$ N/mm²

1. Pull necessary on the end of the brake arm to stop the wheel

Let P = Pull necessary on the end of the brake arm to stop the wheel.

All dimensions in mm

Fig. 19.13

The simple band brake is shown in Fig. 19.13. Since one end of the band is attached to the fixed pin O, therefore the pull P on the end of the brake arm will act upward and when the wheel rotates anticlockwise, the end of the band attached to O will be tight with tension T_1 and the end of the band attached to B will be slack with tension T_2. First of all, let us find the tensions T_1 and T_2. We know that

$$2.3 \log\!\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.25 \times 4.713 = 1.178$$

∴ $$\log\!\left(\frac{T_1}{T_2}\right) = \frac{1.178}{2.3} = 0.5122 \text{ or } \frac{T_1}{T_2} = 3.25 \quad \text{... (Taking antilog of 0.5122) ... (i)}$$

Let $\qquad T_B$ = Braking torque.

We know that power absorbed,

$$35 \times 10^3 = \frac{2\pi \times N.T_B}{60} = \frac{2\pi \times 200 \times T_B}{60} = 21\, T_B$$

∴ $$T_B = 35 \times 10^3 \,/\, 21 = 1667 \text{ N-m} = 1667 \times 10^3 \text{ N-mm}$$

We also know that braking torque (T_B),

$$1667 \times 10^3 = (T_1 - T_2)\, r = (T_1 - T_2)\, 300$$

∴ $$T_1 - T_2 = 1167 \times 10^3 / 300 = 5556 \text{ N} \qquad\qquad\qquad ...(ii)$$

From equations (*i*) and (*ii*), we find that

$$T_1 = 8025 \text{ N}; \quad \text{and} \quad T_2 = 2469 \text{ N}$$

Now taking moments about O, we have

$$P \times 750 = T_2 \times {}^*OD = T_2 \times 62.5\sqrt{2} = 2469 \times 88.4 = 218\,260$$

$$\therefore \quad P = 218260 / 750 = 291 \text{ N Ans.}$$

2. Width of steel band

Let $\qquad w$ = Width of steel band in mm.

We know that maximum tension in the band (T_1),

$$8025 = \sigma.w.t = 50 \times w \times 2.5 = 125\,w$$

$$\therefore \qquad w = 8025 / 125 = 64.2 \text{ mm Ans.}$$

19.8. Differential Band Brake

In a differential band brake, as shown in Fig. 19.14, the ends of the band are joined at A and B to a lever AOC pivoted on a fixed pin or fulcrum O. It may be noted that for the band to tighten, the length OA must be greater than the length OB.

(a) Clockwise rotation of the drum.   (a) Anticlockwise rotation of the drum.
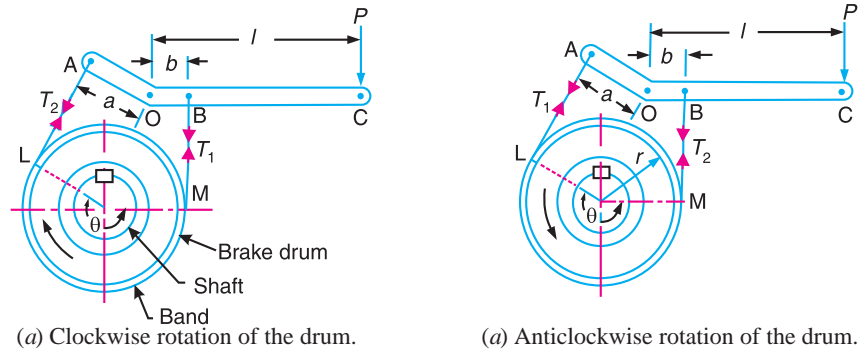
Fig. 19.14. Differential band brake.

The braking torque on the drum may be obtained in the similar way as discussed in simple band brake. Now considering the equilibrium of the lever AOC. It may be noted that when the drum rotates in the clockwise direction, as shown in Fig. 19.14 (a), the end of the band attached to A will be slack with tension T_2 and end of the band attached to B will be tight with tension T_1. On the other hand, when the drum rotates in the anticlockwise direction, as shown in Fig. 19.14 (b), the end of the band attached to A will be tight with tension T_1 and end of the band attached to B will be slack with tension T_2. Now taking moments about the fulcrum O, we have

$$P.l + T_1.b = T_2.a$$

... (For clockwise rotation of the drum)

or $\qquad P.l = T_2.a - T_1.b \qquad$ ... **(i)**

and $\qquad P.l + T_2.b = T_1.a$

... (For anticlockwise rotation of the drum)

or $\qquad P.l = T_1.a - T_2.b \qquad$ ... **(ii)**

Tractors are specially made to move on rough terrain and exert high power at low speeds.

Note : This picture is given as additional information and is not a direct example of the current chapter.

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* $\qquad OD$ = Perpendicular distance from O to the line of action of tension T_2.

$\qquad OE = EB = OB/2 = 125/2 = 62.5$ mm, and $\angle DOE = 45°$

$\therefore \qquad OD = OE \sec 45° = 62.5\sqrt{2}$ mm

We have discussed in block brakes (Art. 19.4), that when the frictional force helps to apply the brake, it is said to be self energizing brake. In case of differential band brake, we see from equations *(i)* and *(ii)* that the moment $T_1.b$ and $T_2.b$ helps in applying the brake (because it adds to the moment *P.l*) for the clockwise and anticlockwise rotation of the drum respectively.

We have also discussed that when the force *P* is negative or zero, then brake is self locking. Thus for differential band brake and for clockwise rotation of the drum, the condition for self locking is

$$T_2.a \le T_1.b \qquad \text{or} \qquad T_2 / T_1 \le b / a$$

and for anticlockwise rotation of the drum, the condition for self locking is

$$T_1.a \le T_2.b \qquad \text{or} \qquad T_1 / T_2 \le b / a$$

Notes : 1. The condition for self locking may also be written as follows :

For clockwise rotation of the drum,

$$T_1.b \ge T_2.a \qquad \text{or} \qquad T_1 / T_2 \ge a / b$$

and for anticlockwise rotation of the drum,

$$T_2.b \ge T_1.a \qquad \text{or} \qquad T_1 / T_2 \ge a / b$$

2. When in Fig. 19.14 (*a*) and (*b*), the length *OB* is greater than *OA*, then the force *P* must act in the upward direction in order to apply the brake. The tensions in the band, *i.e.* T_1 and T_2 will remain unchanged.

Example 19.9. *In a winch, the rope supports a load W and is wound round a barrel 450 mm diameter. A differential band brake acts on a drum 800 mm diameter which is keyed to the same shaft as the barrel. The two ends of the bands are attached to pins on opposite sides of the fulcrum of the brake lever and at distances of 25 mm and 100 mm from the fulcrum. The angle of lap of the brake band is 250° and the coefficient of friction is 0.25. What is the maximum load W which can be supported by the brake when a force of 750 N is applied to the lever at a distance of 3000 mm from the fulcrum ?*

Solution. Given : $D = 450$ mm or $R = 225$ mm ; $d = 800$ mm or $r = 400$ mm ; $OB = 25$ mm ; $OA = 100$ mm ; $\theta = 250° = 250 \times \pi/180 = 4.364$ rad ; $\mu = 0.25$; $P = 750$ N ; $l = OC = 3000$ mm

Since *OA* is greater than *OB*, therefore the operating force ($P = 750$ N) will act downwards.

First of all, let us consider that the drum rotates in clockwise direction.

We know that when the drum rotates in clockwise direction, the end of band attached to *A* will be slack with tension T_2 and the end of the band attached to *B* will be tight with tension T_1, as shown in Fig. 19.15. Now let us find out the values of tensions T_1 and T_2. We know that

750 N

All dimensions in mm.
Fig. 19.15

$$2.3 \log \left(\frac{T_1}{T_2} \right) = \mu.\theta = 0.25 \times 4.364 = 1.091$$

$$\therefore \quad \log \left(\frac{T_1}{T_2} \right) = \frac{1.091}{2.3} = 0.4743 \quad \text{or} \quad \frac{T_1}{T_2} = 2.98 \qquad \text{... (Taking antilog of 0.4743)}$$

and $\qquad T_1 = 2.98\, T_2 \qquad\qquad\qquad$ ... *(i)*

Now taking moments about the fulcrum *O*,

$$750 \times 3000 + T_1 \times 25 = T_2 \times 100$$
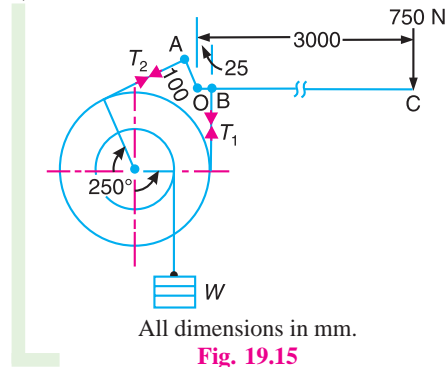
or $T_2 \times 100 - 2.98\, T_2 \times 25 = 2250 \times 10^3$ ... $(\because\ T_1 = 2.98\, T_2)$

$$25.5\, T_2 = 2250 \times 10^3 \quad \text{or} \quad T_2 = 2250 \times 10^3/25.5 = 88 \times 10^3 \text{ N}$$

and $T_1 = 2.98 T_2 = 2.98 \times 88 \times 10^3 = 262 \times 10^3 \text{ N}$

We know that braking torque,

$$T_B = (T_1 - T_2)\, r$$
$$= (262 \times 10^3 - 88 \times 10^3)\, 400 = 69.6 \times 10^6 \text{ N-mm} \qquad ...(i)$$

and the torque due to load W newtons,

$$T_W = W.R = W \times 225 = 225\, W \text{ N-mm} \qquad ...(ii)$$

Since the braking torque must be equal to the torque due to load W newtons, therefore from equations (i) and (ii),

$$W = 69.6 \times 10^6/225 = 309 \times 10^3 \text{ N} = 309 \text{ kN}$$

Now let us consider that the drum rotates in anticlockwise direction. We know that when the drum rotates in anticlockwise direction, the end of the band attached to A will be tight with tension T_1 and end of the band attached to B will be slack with tension T_2, as shown in Fig. 19.16. The ratio of tensions T_1 and T_2 will be same as calculated above, *i.e.*

All dimensions in mm.
Fig. 19.16

$$\frac{T_1}{T_2} = 2.98 \text{ or } T_1 = 2.98\, T_2$$

Now taking moments about the fulcrum O,

$$750 \times 3000 + T_2 \times 25 = T_1 \times 100$$

or $2.98\, T_2 \times 100 - T_2 \times 25 = 2250 \times 10^3$ ... $(\because\ T_1 = 2.98\, T_2)$

$$273\, T_2 = 2250 \times 10^3 \quad \text{or} \quad T_2 = 2250 \times 10^3/273 = 8242 \text{ N}$$
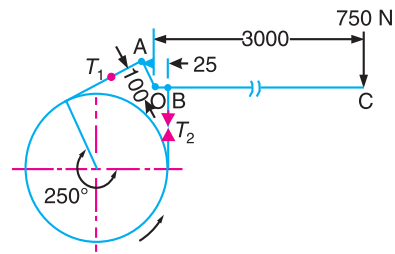
and $T_1 = 2.98\, T_2 = 2.98 \times 8242 = 24\,561 \text{ N}$

$\therefore$ Braking torque, $T_B = (T_1 \times T_2)\, r$

$$= (24\,561 - 8242)\,400 = 6.53 \times 10^6 \text{ N-mm} \qquad ...(iii)$$

From equations (ii) and (iii),

$$W = 6.53 \times 10^6/225 = 29 \times 10^3 \text{ N} = 29 \text{ kN}$$

From above, we see that the maximum load (W) that can be supported by the brake is 309 kN, when the drum rotates in clockwise direction. **Ans.**

Example 19.10. *A differential band brake, as shown in Fig. 19.17, has an angle of contact of 225°. The band has a compressed woven lining and bears against a cast iron drum of 350 mm diameter. The brake is to sustain a torque of 350 N-m and the coefficient of friction between the band and the drum is 0.3. Find : **1.** The necessary force (P) for the clockwise and anticlockwise rotation of the drum; and **2.** The value of 'OA' for the brake to be self locking, when the drum rotates clockwise.*

Solution. Given: $\theta = 225° = 225 \times \pi/180 = 3.93$ rad ; $d = 350$ mm   or $r = 175$ mm ; $T = 350$ N-m $= 350 \times 10^3$ N-mm

1. *Necessary force (P) for the clockwise and anticlockwise rotation of the drum*

When the drum rotates in the clockwise direction, the end of the band attached to A will be slack with tension T_2 and the end of the band attached to B will be tight with tension T_1, as shown in Fig. 19.18. First of all, let us find the values of tensions T_1 and T_2.
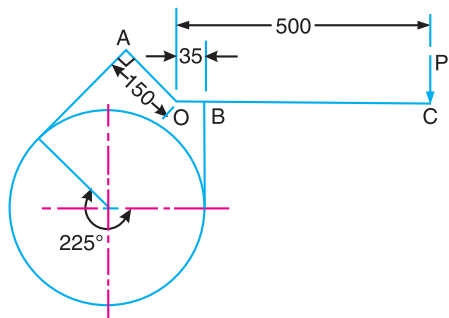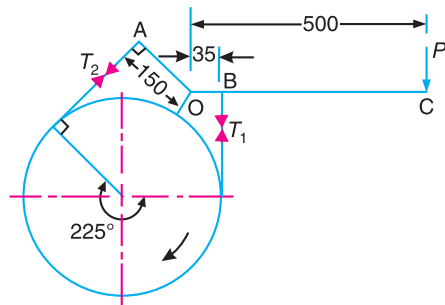
All dimensions in mm.
Fig. 19.17

Fig. 19.18

We know that

$$2.3\log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3 \times 3.93 = 1.179$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{1.179}{2.3} = 0.5126 \quad \text{or} \quad \frac{T_1}{T_2} = 3.255 \quad \text{... (Taking antilog of 0.5126) ... } (i)$$

and braking torque (T_B),

$$350 \times 10^3 = (T_1 - T_2)r = (T_1 - T_2) \, 175$$

$$\therefore \quad T_1 - T_2 = 350 \times 10^3/175 = 2000 \text{ N} \qquad \qquad \text{... } (ii)$$

From equations (*i*) and (*ii*), we find that

$$T_1 = 2887 \text{ N ; and } T_2 = 887 \text{ N}$$

Now taking moments about the fulcrum *O*, we have

$$P \times 500 = T_2 \times 150 - T_1 \times 35 = 887 \times 150 - 2887 \times 35 = 32 \times 10^3$$

$$\therefore \quad P = 32 \times 10^3/500 = 64 \text{ N } \textbf{Ans.}$$

When the drum rotates in the anticlockwise direction, the end of the band attached to A will be tight with tension T_1 and end of the band attached to *B* will be slack with tension T_2, as shown in Fig. 19.19. Taking moments about the fulcrum *O*, we have

$$P \times 500 = T_1 \times 150 - T_2 \times 35$$

$$= 2887 \times 150 - 887 \times 35$$

$$= 402 \times 10^3$$

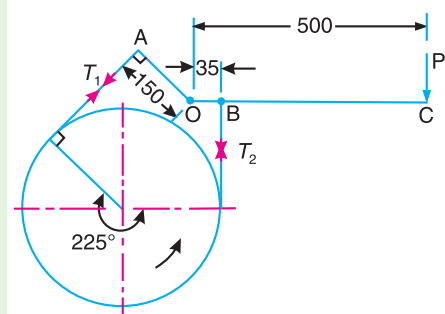$$P = 402 \times 10^3/500 = 804 \text{ N } \textbf{Ans.}$$

Fig. 19.19

2. *Value of 'OA' for the brake to be self locking, when the drum rotates clockwise*

The clockwise rotation of the drum is shown in Fig 19.18.

For clockwise rotation of the drum, we know that

$$P \times 500 = T_2 \times OA - T_1 \times OB$$

For the brake to be self locking, *P* must be equal to zero. Therefore

$$T_2 \times OA = T_1 \times OB$$

and

$$OA = \frac{T_1 \times OB}{T_2} = \frac{2887 \times 35}{887} = 114 \text{ mm } \textbf{Ans.}$$

19.9. Band and Block Brake

The band brake may be lined with blocks of wood or other material, as shown in Fig. 19.20 (*a*). The friction between the blocks and the drum provides braking action. Let there are '*n*' number of blocks, each subtending an angle 2θ at the centre and the drum rotates in anticlockwise direction.
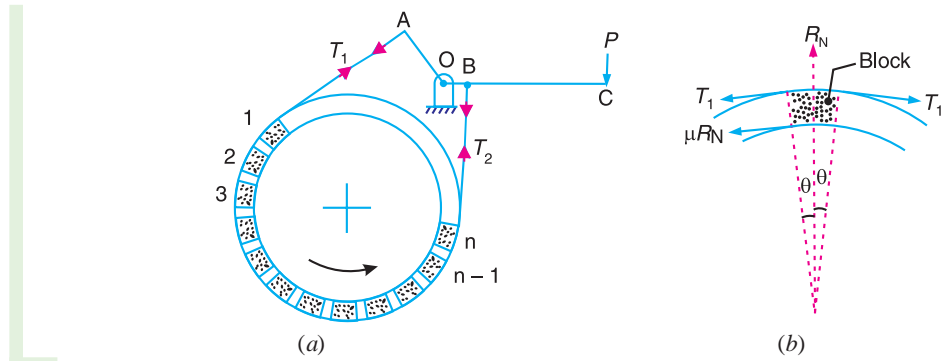

Fig. 19.20. Band and block brake.

Let
T_1 = Tension in the tight side,

T_2 = Tension in the slack side,

μ = Coefficient of friction between the blocks and drum,

T_1' = Tension in the band between the first and second block,

T_2', T_3' etc.= Tensions in the band between the second and third block,
between the third and fourth block etc.

Consider one of the blocks (say first block) as shown in Fig. 19.20 (*b*). This is in equilibrium under the action of the following forces :

1. Tension in the tight side (T_1),

2. Tension in the slack side (T_1') or tension in the band between the first and second block,

3. Normal reaction of the drum on the block (R_N), and

4. The force of friction ($\mu.R_N$).

Resolving the forces radially, we have

$$(T_1 + T_1') \sin \theta = R_N \qquad \qquad ... (i)$$

Resolving the forces tangentially, we have

$$(T_1 + T_1') \cos \theta = \mu.R_N \qquad \qquad ... (ii)$$

Dividing equation (*ii*) by (*i*), we have

$$\frac{(T_1 - T_1') \cos \theta}{(T_1 + T_1') \sin \theta} = \frac{\mu.R_N}{R_N}$$

or
$$(T_1 - T_1') = \mu \tan \theta (T_1 + T_1')$$

∴
$$\frac{T_1}{T_1'} = \frac{1 + \mu \tan \theta}{1 - \mu \tan \theta}$$

Similarly, it can be proved for each of the blocks that

$$\frac{T_1'}{T_2'} = \frac{T_2'}{T_3'} = \frac{T_3'}{T_4'} = ... \frac{T_{n-1}}{T_2} = \frac{1 + \mu \tan \theta}{1 - \mu \tan \theta}$$

$$\therefore \qquad \frac{T_1}{T_2} = \frac{T_1}{T_1'} \times \frac{T_1'}{T_2'} \times \frac{T_2'}{T_3'} \times \dots \times \frac{T_{n-1}}{T_2} = \left(\frac{1+\mu\tan\theta}{1-\mu\tan\theta}\right)^n \qquad \dots (iii)$$

Braking torque on the drum of effective radius r_e,

$$T_B = (T_1 - T_2)\, r_e$$
$$= (T_1 - T_2)\, r \qquad \dots \text{[Neglecting thickness of band]}$$

Note : For the first block, the tension in the tight side is T_1 and in the slack side is T_1' and for the second block, the tension in the tight side is T_1' and in the slack side is T_2'. Similarly for the third block, the tension in the tight side is T_2' and in the slack side is T_3' and so on. For the last block, the tension in the tight side is T_{n-1} and in the slack side is T_2.

Example 19.11. *In the band and block brake shown in Fig. 19.21, the band is lined with 12 blocks each of which subtends an angle of 15° at the centre of the rotating drum. The thickness of the blocks is 75 mm and the diameter of the drum is 850 mm. If, when the brake is in action, the greatest and least tensions in the brake strap are T_1 and T_2, show that*

$$\frac{T_1}{T_2} = \left(\frac{1+\mu\tan 7.5°}{1-\mu\tan 7.5°}\right)^{12}, \quad \text{where } \mu \text{ is the}$$
coefficient of friction for the blocks.

With the lever arrangement as shown in Fig.19.21, find the least force required at C for the blocks to absorb 225 kW at 240 r.p.m. The coefficient of friction between the band and blocks is 0.4.
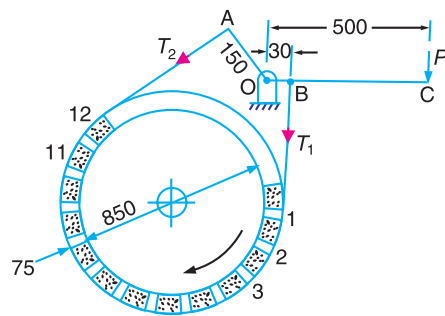
All dimensions in mm.

Fig. 19.21

Solution. Given : $n = 12$; $2\theta = 15°$ or $\theta = 7.5°$; $t = 75$ mm $= 0.075$ m; $d = 850$ mm $= 0.85$ m; Power $= 225$ kW $= 225 \times 10^3$ W; $N = 240$ r.p.m.; $\mu = 0.4$

Since $OA > OB$, therefore the force at C must act downward. Also, the drum rotates clockwise, therefore the end of the band attached to A will be slack with tension T_2 (least tension) and the end of the band attached to B will be tight with tension T_1 (greatest tension).

Consider one of the blocks (say first block) as shown in Fig. 19.22. This is in equilibrium under the action of the following four forces :

1. Tension in the tight side (T_1),

2. Tension in the slack side (T_1') or the tension in the band between the first and second block,

3. Normal reaction of the drum on the block (R_N), and

4. The force of friction ($\mu.R_N$).

Resolving the forces radially, we have

$$(T_1 + T_1')\sin 7.5° = R_N \qquad \dots (i)$$

Resolving the forces tangentially, we have

$$(T_1 - T_1')\cos 7.5° = \mu.R_N \qquad \dots (ii)$$

Dividing equation *(ii)* by *(i)*, we have

$$\frac{(T_1 - T_1')\cos 7.5°}{(T_1 + T_1')\sin 7.5°} = \mu \quad \text{or} \quad \frac{T_1 - T_1'}{T_1 + T_1'} = \mu\tan 7.5°$$

Fig. 19.22
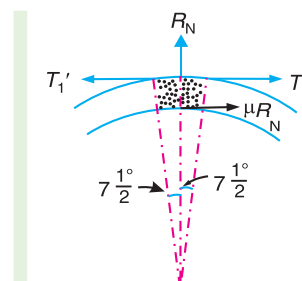
$$\therefore \qquad T_1 - T_1' = T_1.\mu \tan 7.5° + T_1'.\mu \tan 7.5°$$

or $\qquad T_1(1 - \mu \tan 7.5°) = T_1'(1 + \mu \tan 7.5°)$

$$\therefore \qquad \frac{T_1}{T_1'} = \left(\frac{1 + \mu \tan 7.5°}{1 - \mu \tan 7.5°}\right)$$

Similarly, for the other blocks, the ratio of tensions $\dfrac{T_1'}{T_2'} = \dfrac{T_2'}{T_3'}$ etc. remains constant.

Therefore for 12 blocks having greatest tension T_1 and least tension T_2 is

$$\frac{T_1}{T_2} = \left(\frac{1 + \mu \tan 7.5°}{1 - \mu \tan 7.5°}\right)^{12}$$

Least force required at C

Let $\qquad\qquad P$ = Least force required at C.

We know that diameter of band,

$$D = d + 2t = 0.85 + 2 \times 0.075 = 1 \text{ m}$$

$$\therefore \qquad \text{Power absorbed } = \frac{(T_1 - T_2)\pi D.N}{60}$$

or $\qquad T_1 - T_2 = \dfrac{\text{Power} \times 60}{\pi\, DN} = \dfrac{225 \times 10^3 \times 60}{\pi \times 1 \times 240} = 17\ 900 \text{ N} \qquad\qquad ...\ (iii)$

We have proved that

$$\frac{T_1}{T_2} = \left(\frac{1 + \mu \tan 7.5°}{1 - \mu \tan 7.5°}\right)^{12} = \left(\frac{1 + 0.4 \times 0.1317}{1 - 0.4 \times 0.1317}\right)^{12} = \left(\frac{1.0527}{0.9473}\right)^{12} = 3.55$$

$$...\ (iv)$$

From equations *(iii)* and *(iv)*, we find that

$$T_1 = 24\ 920 \text{ N, and } T_2 = 7020 \text{ N}$$

Now taking moments about O, we have

$$P \times 500 = T_2 \times 150 - T_1 \times 30 = 7020 \times 150 - 24\ 920 \times 30 = 305\ 400$$

$$\therefore \qquad\qquad P = 305\ 400 / 500 = 610.8 \text{ N } \textbf{Ans.}$$

Example 19.12. *A band and block brake, having 14 blocks each of which subtends an angle of 15° at the centre, is applied to a drum of 1 m effective diameter. The drum and flywheel mounted on the same shaft has a mass of 2000 kg and a combined radius of gyration of 500 mm. The two ends of the band are attached to pins on opposite sides of the brake lever at distances of 30 mm and 120 mm from the fulcrum. If a force of 200 N is applied at a distance of 750 mm from the fulcrum, find:*

1. maximum braking torque, 2. angular retardation of the drum, and 3. time taken by the system to come to rest from the rated speed of 360 r.p.m.

The coefficient of friction between blocks and drum may be taken as 0.25.

Solution. Given : $n = 14$; $2\theta = 15°$ or $\theta = 7.5°$; $d = 1$ m or $r = 0.5$ m ; $m = 2000$ kg ; $k = 500$ mm $= 0.5$ m ; $P = 200$ N ; $N = 360$ r.p.m. ; $l = 750$ mm ; $\mu = 0.25$

1. *Maximum braking torque*

The braking torque will be maximum when $OB > OA$ and the drum rotates anticlockwise as shown in Fig. 19.23. The force P must act upwards and the end of the band attached to A is tight under tension T_1 and the end of the band attached to B is slack under tension T_2.

Taking moments about O,

$$200 \times 750 + T_1 \times 30 = T_2 \times 120$$

$$12\,T_2 - 3T_1 = 15\,000 \qquad \ldots (i)$$

We know that

$$\frac{T_1}{T_2} = \left(\frac{1+\mu\tan\theta}{1-\mu\tan\theta}\right)^n$$

$$= \left(\frac{1+0.25\tan 7.5°}{1-0.25\tan 7.5°}\right)^{14}$$

$$= \left(\frac{1+0.25\times0.1317}{1-.025\times0.1317}\right)^{14}$$

$$= (1.068)^{14} = 2.512 \ldots (ii)$$

From equations (i) and (ii),

$$T_1 = 8440 \text{ N, and } T_2 = 3360 \text{ N}$$

We know that maximum braking torque,

$$T_B = (T_1 - T_2)\,r = (8440 - 3360)0.5 = 2540 \text{ N-m} \quad \textbf{Ans.}$$

All dimensions in mm
Fig. 19.23

2. *Angular retardation of the drum*

Let $\quad\alpha$ = Angular retardation of the drum.

We know that braking torque (T_B),

$$2540 = I.\alpha = m.k^2.\alpha = 2000(0.5)^2\alpha = 500\alpha$$

$\therefore \qquad \alpha = 2540/500 = 5.08 \text{ rad/s}^2 \textbf{ Ans.}$

3. *Time taken by the system to come to rest*

Let $\quad t$ = Required time.

Since the system is to come to rest from the rated speed of 360 r.p.m., therefore

Initial angular speed, $\omega_1 = 2\pi \times 360/60 = 37.7 \text{ rad/s}$

and final angular speed, $\quad \omega_2 = 0$

We know that $\qquad \omega_2 = \omega_1 - \alpha.t \qquad \ldots (\text{– ve sign due to retardation })$

$\therefore \qquad t = \omega_1/\alpha = 37.7/5.08 = 7.42 \text{ s } \textbf{Ans.}$
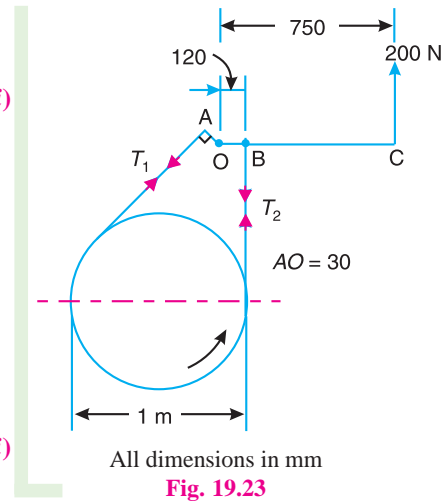
19.10. Internal Expanding Brake

An internal expanding brake consists of two shoes S_1 and S_2 as shown in Fig. 19.24. The outer surface of the shoes are lined with some friction material (usually with Ferodo) to increase the coefficient of friction and to prevent wearing away of the metal. Each shoe is pivoted at one end about a fixed fulcrum O_1 and O_2 and made to contact a cam at the other end. When the cam rotates, the shoes are pushed outwards against the rim of the drum. The friction between the shoes and the drum produces the braking torque and hence reduces the speed of the drum. The shoes are normally held in off position by a spring as shown in Fig. 19.24. The drum encloses the entire mechanism to keep out dust and moisture. This type of brake is commonly used in motor cars and light trucks.
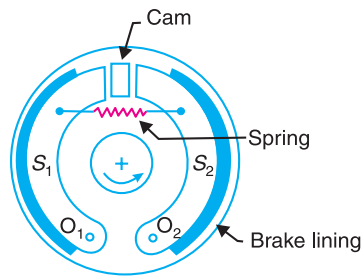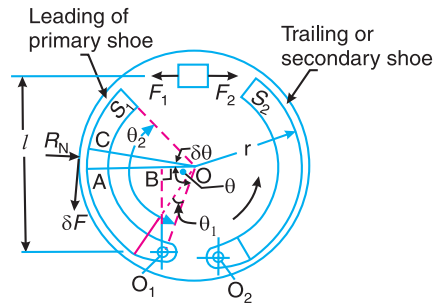
and total braking torque about O for whole of one shoe,

$$T_B = \mu\, p_1 b\, r^2 \int_{\theta_1}^{\theta_2} \sin\theta\, d\theta = \mu\, p_1 b\, r^2 \left[-\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$= \mu\, p_1 b r^2 (\cos\theta_1 - \cos\theta_2)$$

Moment of normal force δR_N of the element about the fulcrum O_1,

$$\delta M_N = \delta R_N \times O_1 B = \delta R_N (OO_1 \sin\theta)$$

$$= p_1 \sin\theta (b.r.\delta\theta)(OO_1 \sin\theta) = p_1 \sin^2\theta (b.r.\delta\theta)OO_1$$

∴ Total moment of normal forces about the fulcrum O_1,

$$M_N = \int_{\theta_1}^{\theta_2} p_1 \sin^2\theta (b.r.\delta\theta)OO_1 = p_1.b.r.OO_1 \int_{\theta_1}^{\theta_2} \sin^2\theta\, d\theta$$

Internal exparding brake.

$$= p_1.b.r.OO_1 \int_{\theta_1}^{\theta_2} \frac{1}{2}(1-\cos 2\theta)d\theta \qquad \ldots\left[\because\ \sin^2\theta = \frac{1}{2}(1-\cos 2\theta)\right]$$

$$= \frac{1}{2} p_1.b.r.OO_1 \left[\theta - \frac{\sin 2\theta}{2}\right]_{\theta_1}^{\theta_2}$$

$$= \frac{1}{2} p_1.b.r.OO_1 \left[\theta_2 - \frac{\sin 2\theta_2}{2} - \theta_1 + \frac{\sin 2\theta_1}{2}\right]$$

$$= \frac{1}{2} p_1.b.r.OO_1 \left[(\theta_2 - \theta_1) + \frac{1}{2}(\sin 2\theta_1 - \sin 2\theta_2)\right]$$

Moment of frictional force δF about the fulcrum O_1,

$$\delta M_F = \delta F \times AB = \delta F(r - OO_1 \cos\theta) \qquad \ldots(\because\ AB = r - OO_1 \cos\theta)$$

$$= \mu\, p_1 \sin\theta (b.r.\delta\theta)(r - OO_1 \cos\theta)$$

$$= \mu.p_1.b.r(r\sin\theta - OO_1 \sin\theta\cos\theta)\delta\theta$$

$$= \mu.p_1.b.r\left(r\sin\theta - \frac{OO_1}{2}\sin 2\theta\right)\delta\theta \qquad \ldots(\because\ 2\sin\theta\cos\theta = \sin 2\theta)$$

∴ Total moment of frictional force about the fulcrum O_1,

$$M_F = \mu\, p_1 b\, r \int_{\theta_1}^{\theta_2}\left(r\sin\theta - \frac{OO_1}{2}\sin 2\theta\right)d\theta$$

$$= \mu\, p_1 b\, r\left[-r\cos\theta + \frac{OO_1}{4}\cos 2\theta\right]_{\theta_1}^{\theta_2}$$

$$= \mu\, p_1 b\, r\left[-r\cos\theta_2 + \frac{OO_1}{4}\cos 2\theta_2 + r\cos\theta_1 - \frac{OO_1}{4}\cos 2\theta_1\right]$$

$$= \mu\, p_1 b\, r\left[r(\cos\theta_1 - \cos\theta_2) + \frac{OO_1}{4}(\cos 2\theta_2 - \cos 2\theta_1)\right]$$

Now for leading shoe, taking moments about the fulcrum O_1,

$$F_1 \times l = M_N - M_F$$

and for trailing shoe, taking moments about the fulcrum O_2,

$$F_2 \times l = M_N + M_F$$

Note : If $M_F > M_N$, then the brake becomes self locking.

Example 19.13. *The arrangement of an internal expanding friction brake, in which the brake shoe is pivoted at 'C' is shown in Fig. 19.26. The distance 'CO' is 75 mm, O being the centre of the drum. The internal radius of the brake drum is 100 mm. The friction lining extends over an arc AB, such that the angle AOC is 135° and angle BOC is 45°. The brake is applied by means of a force at Q, perpendicular to the line CQ, the distance CQ being 150 mm.*

The local rate of wear on the lining may be taken as proportional to the normal pressure on an element at an angle of 'θ' with OC and may be taken as equal to $p_1 \sin \theta$, where p_1 is the maximum intensity of normal pressure.

*The coefficient of friction may be taken as 0.4 and the braking torque required is 21 N-m. Calculate the force Q required to operate the brake when **1.** The drum rotates clockwise, and **2.** The drum rotates anticlockwise.*
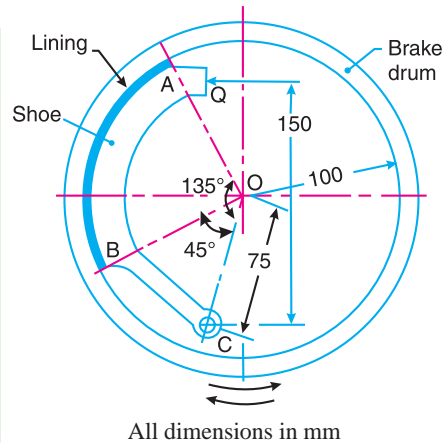

All dimensions in mm
Fig. 19.26

Solution. Given : $OC = 75$ mm ; $r = 100$ mm ;

$\theta_2 = 135° = 135 \times \pi/180 = 2.356$ rad ; $\theta_1 = 45° = 45 \times \pi/180 = 0.786$ rad ; $l = 150$ mm ;

$\mu = 0.4$; $T_B = 21$ N-m $= 21 \times 10^3$ N-mm

1. *Force 'Q' required to operate the brake when drum rotates clockwise*

We know that total braking torque due to shoe (T_B),

$$21 \times 10^3 = \mu.p_1.b.r^2 (\cos\theta_1 - \cos\theta_2)$$

$$= 0.4 \times p_1 \times b(100)^2 (\cos 45° - \cos 135°) = 5656 \, p_1.b$$

$\therefore \qquad p_1.b = 21 \times 10^3 / 5656 = 3.7$

Total moment of normal forces about the fulcrum C,

$$M_N = \frac{1}{2} p_1.b.r.OC \left[(\theta_2 - \theta_1) + \frac{1}{2}(\sin 2\theta_1 - \sin 2\theta_2) \right]$$

$$= \frac{1}{2} \times 3.7 \times 100 \times 75 \left[(2.356 - 0.786) + \frac{1}{2}(\sin 90° - \sin 270°) \right]$$

$$= 13\ 875\ (1.57 + 1) = 35\ 660 \text{ N-mm}$$

and total moment of friction force about the fulcrum C,

$$M_F = \mu.p_1.b.r \left[r(\cos\theta_1 - \cos\theta_2) + \frac{OC}{4}(\cos 2\theta_2 - \cos 2\theta_1) \right]$$

$$= 0.4 \times 3.7 \times 100 \left[100(\cos 45° - \cos 135°) + \frac{75}{4}(\cos 270° - \cos 90°) \right]$$

$$= 148 \times 141.4 = 20\ 930 \text{ N-mm}$$

Taking moments about the fulcrum C, we have

$$Q \times 150 = M_N + M_F = 35\ 660 + 20\ 930 = 56\ 590$$

∴ $Q = 56\ 590 / 150 = 377$ N **Ans.**

2. *Force 'Q' required to operate the brake when drum rotates anticlockwise*

Taking moments about the fulcrum C, we have

$$Q \times 150 = M_N - M_F = 35\ 660 - 20\ 930 = 14\ 730$$

∴ $Q = 14\ 730/150 = 98.2$ N **Ans.**

19.11. Braking of a Vehicle

In a four wheeled moving vehicle, the brakes may be applied to

1. the rear wheels only,

2. the front wheels only, and

3. all the four wheels.

In all the above mentioned three types of braking, it is required to determine the retardation of the vehicle when brakes are applied. Since the vehicle retards, therefore it is a problem of dynamics. But it may be reduced to an equivalent problem of statics by including the inertia force in the system of forces actually applied to the vehicle. The inertia force is equal and opposite to the braking force causing retardation.

Now, consider a vehicle moving up an inclined plane, as shown in Fig. 19.27.
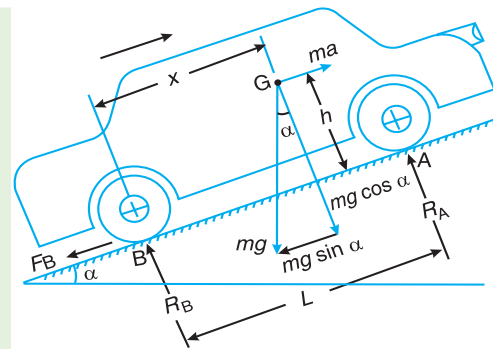
Fig. 19.27. Motion of vehicle up the inclined plane and brakes are applied to rear wheels only.

Let α = Angle of inclination of the plane to the horizontal,

         m = Mass of the vehicle in kg (such that its weight is $m.g$ newtons),

         h = Height of the C.G. of the vehicle above the road surface in metres,

         x = Perpendicular distance of C.G. from the rear axle in metres,

         L = Distance between the centres of the rear and front wheels (also called wheel base) of the vehicle in metres,

         R_A = Total normal reaction between the ground and the front wheels in newtons,

         R_B = Total normal reaction between the ground and the rear wheels in newtons,

         μ = Coefficient of friction between the tyres and road surface, and

         a = Retardation of the vehicle in m/s^2.

We shall now consider the above mentioned three cases of braking, one by one. In all these cases, the braking force acts in the opposite direction to the direction of motion of the vehicle.

1. *When the brakes are applied to the rear wheels only*

It is a common way of braking the vehicle in which the braking force acts at the rear wheels only.

Let F_B = Total braking force (in newtons) acting at the rear wheels due to the application of the brakes. Its maximum value is $\mu.R_B$.

The various forces acting on the vehicle are shown in Fig. 19.27. For the equilibrium of the vehicle, the forces acting on the vehicle must be in equilibrium.

Resolving the forces parallel to the plane,

$$F_B + m.g.\sin \alpha = m.a \qquad \qquad \dots (i)$$

Resolving the forces perpendicular to the plane,

$$R_A + R_B = m.g \cos \alpha \qquad \qquad \ldots (ii)$$

Taking moments about G, the centre of gravity of the vehicle,

$$F_B \times h + R_B \times x = R_A (L - x) \qquad \qquad \ldots (iii)$$

Substituting the value of $F_B = \mu.R_B$, and $R_A = m.g \cos \alpha - R_B$ [from equation (ii)] in the above expression, we have

$$\mu.R_B \times h + R_B \times x = (m.g \cos \alpha - R_B)(L - x)$$
$$R_B (L + \mu.h) = m.g \cos \alpha (L - x)$$

$$\therefore \qquad \qquad R_B = \frac{m.g \cos \alpha (L - x)}{L + \mu.h}$$

and

$$R_A = m.g \cos \alpha - R_B = m.g \cos \alpha - \frac{m.g \cos \alpha (L - x)}{L + \mu.h}$$

$$= \frac{m.g \cos \alpha (x + \mu.h)}{L + \mu.h}$$

We know from equation (i),

$$a = \frac{F_B + m.g \sin \alpha}{m} = \frac{F_B}{m} + g \sin \alpha = \frac{\mu.R_B}{m} + g \sin \alpha$$

$$= \frac{\mu.g \cos \alpha (L - x)}{L + \mu.h} + g \sin \alpha \qquad \qquad \ldots \text{(Substituting the value of } R_B)$$

Notes : 1. When the vehicle moves on a level track, then $\alpha = 0$.

$$\therefore \qquad R_B = \frac{m.g(L - x)}{L + \mu.h} ; R_A = \frac{m.g(x + \mu.h)}{L + \mu.h} \quad \text{and} \quad a = \frac{\mu.g(L - x)}{L + \mu.h}$$

2. If the vehicle moves down the plane, then equation (i) becomes

$$F_B - m.g \sin \alpha = m.a$$

$$\therefore \qquad a = \frac{F_B}{m} - g.\sin \alpha = \frac{\mu.R_B}{m} - g.\sin \alpha = \frac{\mu.g \cos \alpha (L - x)}{L + \mu.h} - g \sin \alpha$$

2. When the brakes are applied to front wheels only

It is a very rare way of braking the vehicle, in which the braking force acts at the front wheels only.

Let F_A = Total braking force (in newtons) acting at the front wheels due to the application of brakes. Its maximum value is $\mu.R_A$.
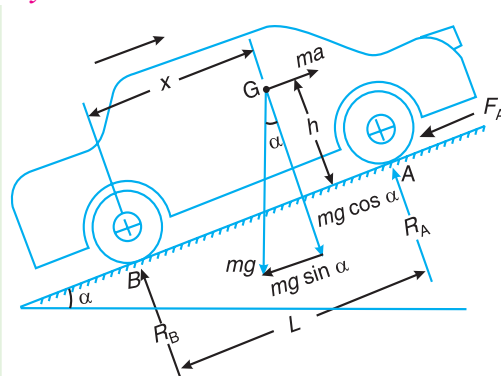
The various forces acting on the vehicle are shown in Fig. 19.28.

Resolving the forces parallel to the plane,

$$F_A + m.g \sin \alpha = m.a \qquad \ldots (i)$$

Resolving the forces perpendicular to the plane,

$$R_A + R_B = m.g \cos \alpha \qquad \ldots (ii)$$

Fig. 19.28. Motion of the vehicle up the inclined plane and brakes are applied to front wheels only.

Taking moments about G, the centre of gravity of the vehicle,

$$F_A \times h + R_B \times x = R_A (L - x)$$

Substituting the value of $F_A = \mu.R_A$ and $R_B = m.g \cos \alpha - R_A$ [from equation (*ii*)] in the above expression, we have

$$\mu.R_A \times h + (m.g \cos \alpha - R_A) x = R_A (L - x)$$

$$\mu.R_A \times h + m.g \cos \alpha \times x = R_A \times L$$

$$\therefore \qquad R_A = \frac{m.g \cos \alpha \times x}{L - \mu.h}$$

and

$$R_B = m.g \cos \alpha - R_A = m.g \cos \alpha - \frac{m.g \cos \alpha \times x}{L - \mu.h}$$

$$= m.g \cos \alpha \left(1 - \frac{x}{L - \mu.h} \right) = m.g \cos \alpha \left(\frac{L - \mu.h - x}{L - \mu.h} \right)$$

We know from equation (*i*),

$$a = \frac{F_A + m.g \sin \alpha}{m} = \frac{\mu.R_A + m.g \sin \alpha}{m}$$

$$= \frac{\mu.m.g \cos \alpha \times x}{(L - \mu.h)m} + \frac{m.g \sin \alpha}{m} \qquad \dots \text{(Substituting the value of } R_A)$$

$$= \frac{\mu.g \cos \alpha \times x}{L - \mu.h} + g \sin \alpha$$

Notes : 1. When the vehicle moves on a level track, then $\alpha = 0$.

$$\therefore \qquad R_A = \frac{m.g \times x}{L - \mu.h} \; ; \; R_B = \frac{m.g (L - \mu.h - x)}{L - \mu.h} ; \quad \text{and} \quad a = \frac{\mu.g \cdot x}{L - \mu.h}$$

2. When the vehicle moves down the plane, then equation (*i*) becomes

$$F_A - m.g \sin \alpha = m.a$$

$$\therefore \qquad a = \frac{F_A}{m} - g.\sin \alpha = \frac{\mu.R_A}{m} - g.\sin \alpha = \frac{\mu.g \cos \alpha \times x}{L - \mu.h} - g \sin \alpha$$

3. When the brakes are applied to all the four wheels

This is the most common way of braking the vehicle, in which the braking force acts on both the rear and front wheels.

Let F_A = Braking force provided by the front wheels = $\mu.R_A$, and

F_B = Braking force provided by the rear wheels = $\mu.R_B$.

A little consideration will show that when the brakes are applied to all the four wheels, the braking distance (*i.e.* the distance in which the vehicle is brought to rest after applying the brakes) will be the least. It is due to this reason that the brakes are applied to all the four wheels.

The various forces acting on the vehicle are shown in Fig. 19.29.
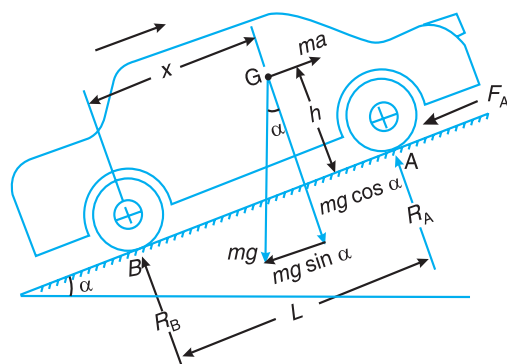
Fig. 19.29. Motion of the vehicle up the inclined plane and the brakes are applied to all the four wheels.

Resolving the forces parallel to the plane,

$$F_A + F_B + m.g \sin \alpha = m.a \qquad \qquad \dots (i)$$

Resolving the forces perpendicular to the plane,

$$R_A + R_B = m.g \cos \alpha \qquad \qquad \dots (ii)$$

Taking moments about G, the centre of gravity of the vehicle,

$$(F_A + F_B)h + R_B \times x = R_A(L-x) \qquad \qquad \dots (iii)$$

Substituting the value of $F_A = \mu.R_A$, $F_B = \mu.R_B$ and $R_B = m.g \cos \alpha - R_A$ [From equation (ii)] in the above expression,

$$\mu(R_A + R_B)h + (m.g \cos \alpha - R_A)x = R_A(L-x)$$

$$\mu(R_A + m.g \cos \alpha - R_A)h + (m.g \cos \alpha - R_A)x = R_A(L-x)$$

$$\mu.m.g \cos \alpha \times h + m.g \cos \alpha \times x = R_A \times L$$

$$\therefore \qquad R_A = \frac{m.g \cos \alpha(\mu.h + x)}{L}$$

and

$$R_B = m.g \cos \alpha - R_A = m.g \cos \alpha - \frac{mg \cos \alpha(\mu.h + x)}{L}$$

$$= m.g \cos \alpha \left[1 - \frac{\mu.h + x}{L}\right] = m.g \cos \alpha \left(\frac{L - \mu.h - x}{L}\right)$$

Now from equation (i),

$$\mu.R_A + \mu R_B + m.g \sin \alpha = m.a$$

$$\mu(R_A + R_B) + m.g \sin \alpha = m.a$$

$$\mu.m.g.\cos \alpha + m.g \sin \alpha = m.a \qquad \qquad \dots \text{[From equation (ii)]}$$

$$\therefore \qquad a = g(\mu.\cos \alpha + \sin \alpha)$$

Notes : 1. When the vehicle moves on a level track, then $\alpha = 0$.

$$\therefore \qquad R_A = \frac{m.g(\mu.h + x)}{L} ; R_B = m.g\left(\frac{L - \mu.h - x}{L}\right); \text{ and } a = g.\mu$$

2. If the vehicle moves down the plane, then equation (i) may be written as

$$F_A + F_B - m.g \sin \alpha = m.a$$

or

$$\mu(R_A + R_B) - m.g \sin \alpha = m.a$$

$$\mu.m.g \cos \alpha - m.g \sin \alpha = m.a$$

and

$$a = g(\mu.\cos \alpha - \sin \alpha)$$

Example 19.14. *A car moving on a level road at a speed 50 km/h has a wheel base 2.8 metres, distance of C.G. from ground level 600 mm, and the distance of C.G. from rear wheels 1.2 metres. Find the distance travelled by the car before coming to rest when brakes are applied,*
1. to the rear wheels, 2. to the front wheels, and 3. to all the four wheels.
The coefficient of friction between the tyres and the road may be taken as 0.6.

Solution. Given : $u = 50$ km/h $= 13.89$ m/s ; $L = 2.8$ m ; $h = 600$ mm $= 0.6$ m ; $x = 1.2$ m ; $\mu = 0.6$

Let $\qquad\qquad s = $ Distance travelled by the car before coming to rest.

1. When brakes are applied to the rear wheels

Since the vehicle moves on a level road, therefore retardation of the car,

$$a = \frac{\mu.g(L-x)}{L + \mu.h} = \frac{0.6 \times 9.81(2.8 - 1.2)}{2.8 + 0.6 \times 0.6} = 2.98 \text{ m/s}^2$$

We know that for uniform retardation,

$$s = \frac{u^2}{2a} = \frac{(13.89)^2}{2 \times 2.98} = 32.4 \text{ m } \textbf{Ans.}$$

2. *When brakes are applied to the front wheels*

Since the vehicle moves on a level road, therefore retardation of the car,

$$a = \frac{\mu.g.x}{L - \mu.h} = \frac{0.6 \times 9.18 \times 1.2}{2.8 - 0.6 \times 0.6} = 2.9 \text{ m/s}^2$$

We know that for uniform retardation,

$$s = \frac{u^2}{2a} = \frac{(13.89)^2}{2 \times 2.9} = 33.26 \text{ m } \textbf{Ans.}$$

3. *When the brakes are applied to all the four wheels*

Since the vehicle moves on a level road, therefore retardation of the car,

$$a = g.\mu = 9.81 \times 0.6 = 5.886 \text{ m/s}^2$$

We know that for uniform retardation,

$$s = \frac{u^2}{2a} = \frac{(13.89)^2}{2 \times 5.886} = 16.4 \text{ m } \textbf{Ans.}$$

Example 19.15. *A vehicle moving on a rough plane inclined at 10° with the horizontal at a speed of 36 km/h has a wheel base 1.8 metres. The centre of gravity of the vehicle is 0.8 metre from the rear wheels and 0.9 metre above the inclined plane. Find the distance travelled by the vehicle before coming to rest and the time taken to do so when **1.** The vehicle moves up the plane, and **2.** The vehicle moves down the plane.*

The brakes are applied to all the four wheels and the coefficient of friction is 0.5.

Solution. Given : $\alpha = 10°; u = 36 \text{ km / h} = 10 \text{ m / s }; L = 1.8 \text{ m }; x = 0.8 \text{ m }; h = 0.9 \text{ m }; \mu = 0.5$

Let s = Distance travelled by the vehicle before coming to rest, and

t = Time taken by the vehicle in coming to rest.

1. *When the vehicle moves up the plane and brakes are applied to all the four wheels*

Since the vehicle moves up the inclined plane, therefore retardation of the vehicle,

$$a = g \left(\mu \cos \alpha + \sin \alpha \right)$$

$$= 9.81 \left(0.5 \cos 10° + \sin 10° \right) = 9.81(0.5 \times 0.9848 + 0.1736) = 6.53 \text{ m/s}^2$$

We know that for uniform retardation,

$$s = \frac{u^2}{2a} = \frac{(10)^2}{2 \times 6.53} = 7.657 \text{ m } \textbf{Ans.}$$

and final velocity of the vehicle (v),

$$0 = u + a.t = 10 - 6.53 \ t \qquad \qquad \text{. . .(Minus sign due to retardation)}$$

∴ $t = 10 / 6.53 = 1.53 \text{ s } \textbf{Ans.}$

2. *When the vehicle moves down the plane and brakes are applied to all the four wheels*

Since the vehicle moves down the inclined plane, therefore retardation of the vehicle,

$$a = g \left(\mu \cos \alpha - \sin \alpha \right)$$

$$= 9.81(0.5 \cos 10° - \sin 10°) = 9.81(0.5 \times 0.9848 - 0.1736) = 3.13 \text{ m/s}^2$$

We know that for uniform retardation,

$$s = \frac{u^2}{2a} = \frac{(10)^2}{2 \times 3.13} = 16 \text{ m } \textbf{Ans.}$$

and final velocity of the vehicle (v),

$$0 = u + a.t = 10 - 3.13\,t \qquad \qquad \ldots \text{(Minus sign due to retardation)}$$

$$\therefore \qquad \qquad t = 10/3.13 = 3.2 \text{ s } \textbf{Ans.}$$

Example 19.16. *The wheel base of a car is 3 metres and its centre of gravity is 1.2 metres ahead the rear axle and 0.75 m above the ground level. The coefficient of friction between the wheels and the road is 0.5. Determine the maximum deceleration of the car when it moves on a level road, if the braking force on all the wheels is the same and no wheel slip occurs.*

Solution. Given : $L = 3$ m ; $x = 1.2$ m ; $h = 0.75$ m ; $\mu = 0.5$

Let a = Maximum deceleration of the car,

m = Mass of the car,

F_A and F_B = Braking forces at the front and rear wheels respectively, and

R_A and R_B = Normal reactions at the front and rear wheels respectively.

The various forces acting on the car are shown in Fig. 19.30.

Fig. 19.30

We shall consider the following two cases:

(*a*) *When the slipping is imminent at the rear wheels*

We know that when the brakes are applied to all the four wheels and the vehicle moves on a level road, then

$$R_B = m.g \left(\frac{L - \mu.h - x}{L} \right) = m \times 9.81 \left(\frac{3 - 0.5 \times 0.75 - 1.2}{3} \right) = 4.66 \, m \text{ N}$$

and $\qquad F_A + F_B = m.a \qquad$ or $\qquad 2\mu.\ R_B = m.a \qquad \ldots (\because F_B = F_A \text{ and } F_B = \mu.R_B)$

$\therefore \qquad 2 \times 0.5 \times 4.66\ m = m.a \qquad$ or $\qquad a = 4.66 \text{ m/s}^2$

(*b*) *When the slipping is imminent at the front wheels*

We know that when the brakes are applied to all the four wheels and the vehicle moves on a level road, then

$$R_A = \frac{m.g\,(\mu.h + x)}{L} = \frac{m \times 9.81 (0.5 \times 0.75 + 1.2)}{3} = 5.15 \, m \text{ N}$$

and $\qquad F_A + F_B = m.a \qquad$ or $\qquad 2\mu.\ R_A = m.a \qquad \ldots (\because F_A = F_B \text{ and } F_A = \mu.R_A)$

$\therefore \qquad 2 \times 0.5 \times 5.15\ m = m.a \qquad$ or $\qquad a = 5.15 \text{ m/s}^2$

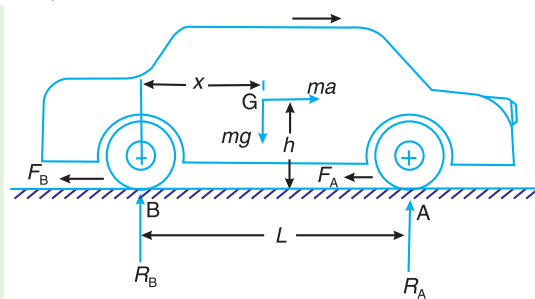Hence the maximum possible deceleration is 4.66 m/s^2 and slipping would occur first at the rear wheels. **Ans.**

19.12. Dynamometer

A dynamometer is a brake but in addition it has a device to measure the frictional resistance. Knowing the frictional resistance, we may obtain the torque transmitted and hence the power of the engine.

19.13. Types of Dynamometers

Following are the two types of dynamometers, used for measuring the brake power of an engine.

1. Absorption dynamometers, and **2.** Transmission dynamometers.

In the *absorption dynamometers,* the entire energy or power produced by the engine is absorbed by the friction resistances of the brake and is transformed into heat, during the process of measurement. But in the *transmission dynamometers,* the energy is not wasted in friction but is used for doing work. The energy or power produced by the engine is transmitted through the dynamometer to some other machines where the power developed is suitably measured.

Dynamometers measure the power of the engines.

19.14. Classification of Absorption Dynamometers

The following two types of absorption dynamometers are important from the subject point of view :

1. Prony brake dynamometer, and **2.** Rope brake dynamometer.

These dynamometers are discussed, in detail, in the following pages.

19.15. Prony Brake Dynamometer

A simplest form of an absorption type dynamometer is a prony brake dynamometer, as shown in Fig. 19.31. It consists of two wooden blocks placed around a pulley fixed to the shaft of an engine whose power is required to be measured. The blocks are clamped by means of two bolts and nuts, as shown in Fig. 19.31. A helical spring is provided between the nut and the upper block to adjust the pressure on the pulley to control its speed. The upper block has a long lever attached to it and carries a weight W at its outer end. A counter weight is placed at the other end of the lever which balances the brake when unloaded. Two stops S, S are provided to limit the motion of the lever.
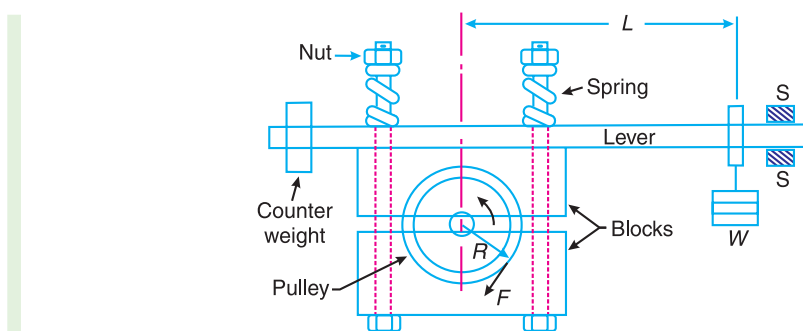

Fig. 19.31. Prony brake dynamometer.

When the brake is to be put in operation, the long end of the lever is loaded with suitable weights W and the nuts are tightened until the engine shaft runs at a constant speed and the lever is in horizontal position. Under these conditions, the moment due to the weight W must balance the moment of the frictional resistance between the blocks and the pulley.

Let W = Weight at the outer end of the lever in newtons,

   L = Horizontal distance of the weight W from the centre of the pulley in metres,

   F = Frictional resistance between the blocks and the pulley in newtons,

   R = Radius of the pulley in metres, and

   N = Speed of the shaft in r.p.m.

We know that the moment of the frictional resistance or torque on the shaft,

$$T = W.L = F.R \text{ N-m}$$

Work done in one revolution

= Torque × Angle turned in radians

$$= T \times 2\pi \text{ N-m}$$

∴   Work done per minute

$$= T \times 2\pi N \text{ N-m}$$

We know that brake power of the engine,

Another dynamo

$$B.P. = \frac{\text{Work done per min.}}{60} = \frac{T \times 2\pi N}{60} = \frac{W.L \times 2\pi N}{60} \text{ watts}$$

Notes : 1. From the above expression, we see that while determining the brake power of engine with the help of a prony brake dynamometer, it is not necessary to know the radius of the pulley, the coefficient of friction between the wooden blocks and the pulley and the pressure exerted by tightening of the nuts.

   2. When the driving torque on the shaft is not uniform, this dynamometer is subjected to severe oscillations.

19.16. Rope Brake Dynamometer

It is another form of absorption type dynamometer which is most commonly used for measuring the brake power of the engine. It consists of one, two or more ropes wound around the flywheel or rim of a pulley fixed rigidly to the shaft of an engine. The upper end of the ropes is attached to a spring balance while the lower end of the ropes is kept in position by applying a dead weight as shown in Fig. 19.32. In order to prevent the slipping of the rope over the flywheel, wooden blocks are placed at intervals around the circumference of the flywheel.

In the operation of the brake, the engine is made to run at a constant speed. The frictional torque, due to the rope, must be equal to the torque being transmitted by the engine.

Let W = Dead load in newtons,

   S = Spring balance reading in newtons,

   D = Diameter of the wheel in metres,

   d = diameter of rope in metres, and

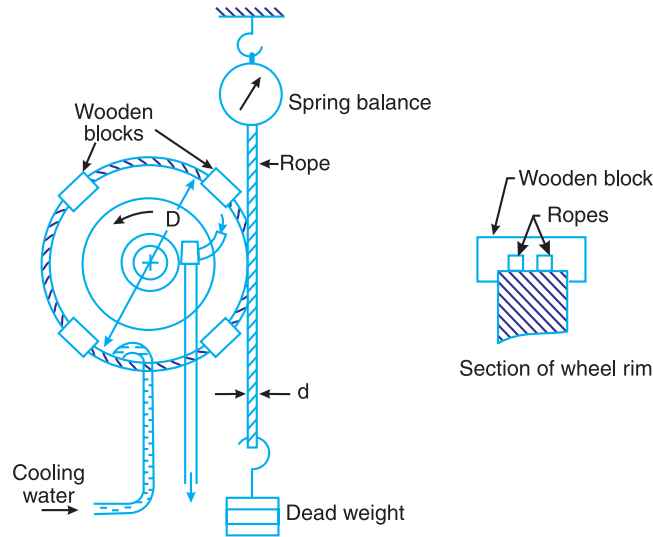 N = Speed of the engine shaft in r.p.m.

∴ Net load on the brake

$$= (W - S) \text{ N}$$

We know that distance moved in one revolution

$$= \pi(D + d) \text{ m}$$

∴   Work done per revolution

$$= (W - S) \pi (D + d) \text{ N-m}$$

and work done per minute

$$= (W - S) \pi (D + d) \, N \text{ N-m}$$

Fig. 19.32. Rope brake dynamometer.

∴   Brake power of the engine,

$$\text{B.P} = \frac{\text{Work done per min}}{60} = \frac{(W - S) \, \pi \, (D + d) N}{60} \text{ watts}$$

If the diameter of the rope (*d*) is neglected, then brake power of the engine,

$$\text{B.P.} = \frac{(W - S) \pi D \, N}{60} \text{ watts}$$

Note: Since the energy produced by the engine is absorbed by the frictional resistances of the brake and is transformed into heat, therefore it is necessary to keep the flywheel of the engine cool with soapy water. The flywheels have their rims made of a channel section so as to receive a stream of water which is being whirled round by the wheel. The water is kept continually flowing into the rim and is drained away by a sharp edged scoop on the other side, as shown in Fig. 19.32.

Example 19.17. *In a laboratory experiment, the following data were recorded with rope brake:*

Diameter of the flywheel 1.2 m; diameter of the rope 12.5 mm; speed of the engine 200 r.p.m.; dead load on the brake 600 N; spring balance reading 150 N. Calculate the brake power of the engine.

Solution. Given : $D = 1.2$ m ; $d = 12.5$ mm $= 0.0125$ m ; $N = 200$ r.p.m ; $W = 600$ N ; $S = 150$ N

An engine is being readied for testing on a dynamometer

We know that brake power of the engine,

$$\text{B.P.} = \frac{(W - S)\,\pi\,(D + d)N}{60} = \frac{(600 - 150)\,\pi\,(1.2 + 0.0125)200}{60} = 5715 \text{ W}$$

$$= 5.715 \text{ kW } \textbf{Ans.}$$

19.17. Classification of Transmission Dynamometers

The following types of transmission dynamometers are important from the subject point of view :

1. Epicyclic-train dynamometer, **2.** Belt transmission dynamometer, and **3.** Torsion dynamometer.

We shall now discuss these dynamometers, in detail, in the following pages.
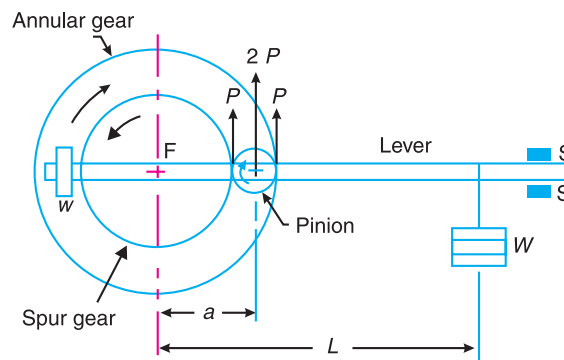
19.18. Epicyclic-train Dynamometer

Fig. 19.33. Epicyclic train dynamometer.

An epicyclic-train dynamometer, as shown in Fig. 19.33, consists of a simple epicyclic train of gears, *i.e.* a spur gear, an annular gear (a gear having internal teeth) and a pinion. The spur gear is keyed to the engine shaft (*i.e.* driving shaft) and rotates in anticlockwise direction. The annular gear is also keyed to the driving shaft and rotates in clockwise direction. The pinion or the intermediate gear meshes with both the spur and annular gears. The pinion revolves freely on a lever which is pivoted to the common axis of the driving and driven shafts. A weight *w* is placed at the smaller end of the lever in order to keep it in position. A little consideration will show that if the friction of the pin on which the pinion rotates is neglected, then the tangential effort *P* exerted by the spur gear on the pinion and the tangential reaction of the annular gear on the pinion are equal.

Since these efforts act in the upward direction as shown, therefore total upward force on the lever acting through the axis of the pinion is 2*P*. This force tends to rotate the lever about its fulcrum and it is balanced by a dead weight *W* at the end of the lever. The stops *S, S* are provided to control the movement of the lever.

For equilibrium of the lever, taking moments about the fulcrum *F*,

$$2P \times a = W.L \quad \text{or} \quad P = W.L / 2a$$

Let $\quad R = $ Pitch circle radius of the spur gear in metres, and

$\quad N = $ Speed of the engine shaft in r.p.m.

∴ Torque transmitted, $\quad T = P.R$

and power transmitted $\qquad = \dfrac{T \times 2\pi N}{60} = \dfrac{P.R \times 2\pi N}{60} \text{ watts}$

19.19. Belt Transmission Dynamometer-Froude or Throneycroft Transmission Dynamometer

When the belt is transmitting power from one pulley to another, the tangential effort on the driven pulley is equal to the difference between the tensions in the tight and slack sides of the belt. A belt dynamometer is introduced to measure directly the difference between the tensions of the belt, while it is running.
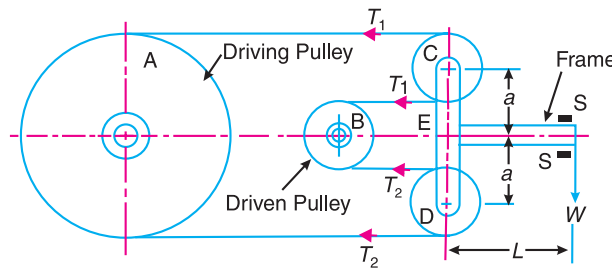
Fig. 19.34. Froude or Throneycroft transmission dynamometer.

A belt transmission dynamometer, as shown in Fig. 19.34, is called a Froude or Throneycroft transmission dynamometer. It consists of a pulley *A* (called driving pulley) which is rigidly fixed to the shaft of an engine whose power is required to be measured. There is another pulley *B* (called driven pulley) mounted on another shaft to which the power from pulley *A* is transmitted. The pulleys *A* and *B* are connected by means of a continuous belt passing round the two loose pulleys *C* and *D* which are mounted on a *T*-shaped frame. The frame is pivoted at *E* and its movement is controlled by two stops *S,S*. Since the tension in the tight side of the belt (T_1) is greater than the tension in the slack side of the belt (T_2), therefore the total force acting on the pulley *C* (*i.e.* $2T_1$) is greater than the total force acting on the pulley *D* (*i.e.* $2T_2$). It is thus obvious that the frame causes movement about *E* in the anticlockwise direction. In order to balance it, a weight *W* is applied at a distance *L* from *E* on the frame as shown in Fig. 19.34.

Now taking moments about the pivot *E*, neglecting friction,

$$2T_1 \times a = 2T_2 \times a + W.L \qquad \text{or} \qquad T_1 - T_2 = \frac{W.L}{2a}$$

Let   *D* = diameter of the pulley *A* in metres, and

                       N = Speed of the engine shaft in r.p.m.

∴   Work done in one revolution = $(T_1 - T_2) \pi D$ N-m

and workdone per minute   = $(T_1 - T_2) \pi DN$ N-m

∴   Brake power of the engine, $\text{B.P.} = \dfrac{(T_1 - T_2) \pi DN}{60}$ watts

Example 19.18. *The essential features of a transmission dynamometer are shown in Fig. 19.35. A is the driving pulley which runs at 600 r.p.m. B and C are jockey pulleys mounted on a horizontal beam pivoted at D, about which point the complete beam is balanced when at rest. E is the driven pulley and all portions of the belt between the pulleys are vertical. A, B and C are each 300 mm diameter and the thickness and weight of the belt are neglected. The length DF is 750 mm.*

Find : **1.** *the value of the weight W to maintain the beam in a horizontal position when 4.5 kW is being transmitted, and* **2.** *the value of W, when the belt just begins to slip on pulley A. The coefficient of friction being 0.2 and maximum tension in the belt 1.5 kN.*
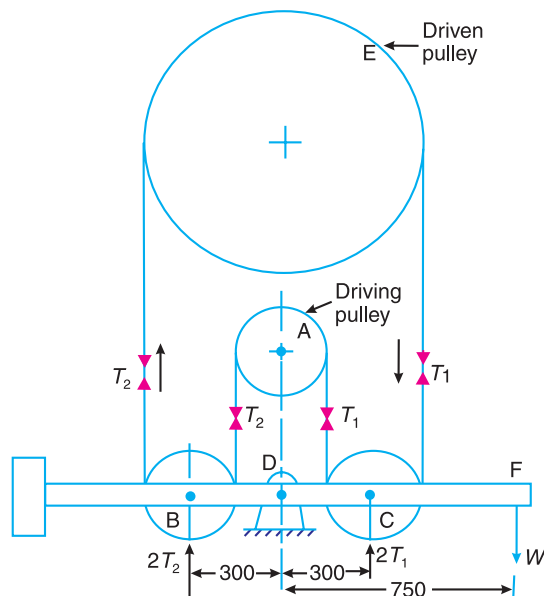
Fig. 19.35. All dimensions in mm.

Solution. Given : $N_A = 600$ r.p.m. : $D_A = D_B = D_C = 300$ mm $= 0.3$ m

1. *Value of the weight W to maintain the beam in a horizontal position*

Given : Power transmitted $(P) = 4.5$ kW $= 4500$ W

Let T_1 = Tension in the tight side of the belt on pulley A, and

T_2 = Tension in the slack side of the belt on pulley A.

∴ Force acting upwards on the pulley $C = 2T_1$

and force acting upwards on the pulley $B = 2T_2$

Now taking moments about the pivot D,

$$W \times 750 = 2T_1 \times 300 - 2T_2 \times 300 = 600 (T_1 - T_2)$$

∴ $T_1 - T_2 = W \times 750 / 600 = 1.25\ W$ N

We know that the power transmitted (P),

$$4500 = \frac{(T_1 - T_2)\,\pi\, D_A\, N_A}{60} = \frac{1.25\, W \times \pi \times 0.3 \times 600}{60} = 11.78\ W$$

∴ $W = 4500 / 11.78 = 382$ N **Ans.**

2. *Value of W, when the belt just begins to slip on A*

Given : $\mu = 0.2$; $T_1 = 1.5$ kN $= 1500$ N

We know that

$$2.3\log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.2 \times \pi = 0.6284 \qquad\qquad \ldots (\because \theta = 180° = \pi \text{ rad})$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.6284}{2.3} = 0.2732 \quad \text{or} \quad \frac{T_1}{T_2} = 1.876 \qquad\qquad \ldots \text{(Taking antilog of 0.2732)}$$

∴ $T_2 = T_1 / 1.876 = 1500 / 1.876 = 800$ N

Now taking moments about the pivot D,

$$W \times 750 = 2T_1 \times 300 - 2T_2 \times 300 = 2 \times 1500 \times 300 - 2 \times 800 \times 300$$

$$= 420 \times 10^3$$

∴ $W = 420 \times 10^3 / 750 = 560$ N **Ans.**

19.20. Torsion Dynamometer

A torsion dynamometer is used for measuring large powers particularly the power transmitted along the propeller shaft of a turbine or motor vessel. A little consideration will show that when the power is being transmitted, then the driving end of the shaft twists through a small angle relative to the driven end of the shaft. The amount of twist depends upon many factors such as torque acting on the shaft (T), length of the shaft (l), diameter of the shaft (D) and modulus of rigidity (C) of the material of the shaft. We know that the torsion equation is

$$\frac{T}{J} = \frac{C.\theta}{l}$$

where θ = Angle of twist in radians, and

J = Polar moment of inertia of the shaft.

For a solid shaft of diameter D, the polar moment of inertia

$$J = \frac{\pi}{32} \times D^4$$

and for a hollow shaft of external diameter D and internal diameter d, the polar moment of inertia,

$$J = \frac{\pi}{32}(D^4 - d^4)$$

From the above torsion equation,

$$T = \frac{C.J}{l} \times \theta = k.\theta$$

where $k = C.J/l$ is a constant for a particular shaft. Thus, the torque acting on the shaft is proportional to the angle of twist. This means that if the angle of twist is measured by some means, then the torque and hence the power transmitted may be determined.

We know that the power transmitted

$$P = \frac{T \times 2\pi N}{60} \text{ watts, where } N \text{ is the speed in r.p.m.}$$

A number of dynamometers are used to measure the angle of twist, one of which is discussed in Art. 19.21. Since the angle of twist is measured for a small length of the shaft, therefore some magnifying device must be introduced in the dynamometer for accurate measurement.

Example 19.19. *A torsion dynamometer is fitted to a propeller shaft of a marine engine. It is found that the shaft twists 2° in a length of 20 metres at 120 r.p.m. If the shaft is hollow with 400 mm external diameter and 300 mm internal diameter, find the power of the engine. Take modulus of rigidity for the shaft material as 80 GPa.*

Solution. Given : θ = 2° = 2 × π /180 = 0.035 rad ; l = 20 m ; N = 120 r.p.m. ; D = 400 mm = 0.4 m ; d = 300 mm = 0.3 m ; C = 80 GPa = 80 × 10⁹ N/m²

We know that polar moment of inertia of the shaft,

$$J = \frac{\pi}{32}(D^4 - d^4) = \frac{\pi}{32}\left[(0.4)^4 - (0.3)^4\right] = 0.0017 \text{m}^4$$

and torque applied to the shaft,

$$T = \frac{C.J}{l} \times \theta = \frac{80 \times 10^9 \times 0.0017}{20} \times 0.035 = 238 \times 10^3 \text{ N-m}$$

We know that power of the engine,

$$P = \frac{T \times 2\pi N}{60} = \frac{238 \times 10^3 \times 2\pi \times 120}{60} = 2990 \times 10^3 \text{ W} = 2990 \text{ kW } \textbf{Ans.}$$

19.21. Bevis-Gibson Flash Light Torsion Dynamometer
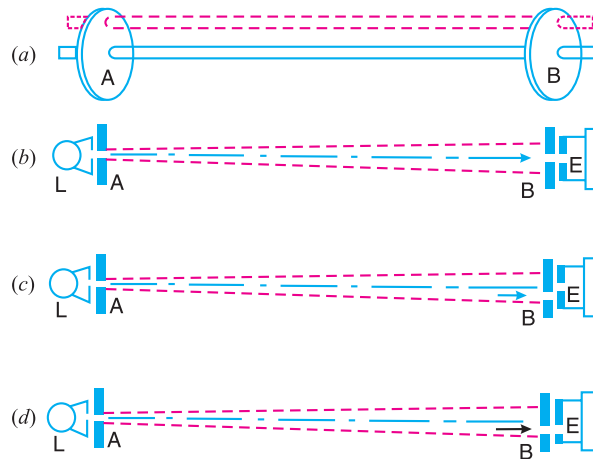

Fig. 19.36. Bevis-Gibson flash light torsion dynamometer.

It depends upon the fact that the light travels in a straight line through air of uniform density and the velocity of light is infinite. It consists of two discs A and B fixed on a shaft at a convenient distance apart, as shown in Fig. 19.36 (*a*). Each disc has a small radial slot and these two slots are in the same line when no power is transmitted and there is no torque on the shaft. A bright electric lamp L, behind the disc A, is fixed on the bearing of the shaft. This lamp is masked having a slot directly opposite to the slot of disc A. At every revolution of the shaft, a flash of light is projected through the slot in the disc A towards the disc B in a direction parallel to the shaft. An eye piece E is fitted behind the disc B on the shaft bearing and is capable of slight circumferential adjustment.

When the shaft does not transmit any torque (*i.e.* at rest), a flash of light may be seen after every revolution of the shaft, as the positions of the slit do not change relative to one another as shown in Fig. 19.36 (*b*). Now when the torque is transmitted, the shaft twists and the slot in the disc B changes its position, though the slots in L, A and E are still in line. Due to this, the light does not reach to the eye piece as shown in Fig. 19.36 (*c*). If the eye piece is now moved round by an amount equal to the lag of disc B, then the slot in the eye piece will be opposite to the slot in disc B as shown in Fig. 19.36 (*d*) and hence the eye piece receives flash of light. The eye piece is moved by operating a micrometer spindle and by means of scale and vernier, the angle of twist may be measured upto 1/100th of a degree.

The torsion meter discussed above gives the angle of twist of the shaft, when the uniform torque is transmitted during each revolution as in case of turbine shaft. But when the torque varies during each revolution as in reciprocating engines, it is necessary to measure the angle of twist at several different angular positions. For this, the discs A and B are perforated with slots arranged in the form of spiral as shown in Fig. 19.37. The lamp and the eye piece must be moved radially so as to bring them into line with each corresponding pair of slots in the discs.
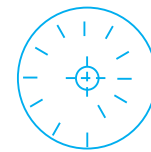

Fig. 19.37. Perforated disc.

EXERCISES

1. A single block brake, as shown in Fig. 19.38, has the drum diameter 250 mm. The angle of contact is 90° and the coefficient of friction between the drum and the lining is 0.35. If the operating force of 650 N is applied at the end of the lever, determine the torque that may be transmitted by the block brake. **[Ans. 65.6 N-m]**
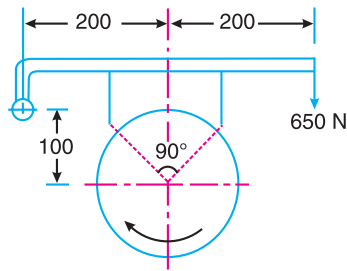
All dimensions in mm.
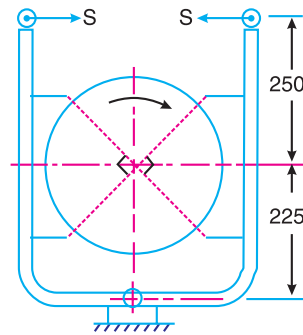
Fig. 19.38   **Fig. 19.39**

2. The layout and dimensions of a double shoe brake is shown in Fig. 19.39. The diameter of the brake drum is 300 mm and the contact angle for each shoe is 90°. If the coefficient of friction for the brake lining and the drum is 0.4, find the spring force necessary to transmit a torque of 30 N-m. Also determine the width of the brake shoes, if the bearing pressure on the lining material is not to exceed 0.28 N/mm^2.   **[Ans. 98.4 N ; 5 mm]**

3. The arrangements of a transmission brake is shown in Fig. 19.40. The arms are pivoted at O_1 and O_2 and when force is applied at the end of a hand lever, the screw AB rotates. The left and right hand threads working in nuts on the ends of the arms move the arms together and thus apply the brake. The force on the hand lever is applied 400 mm from the axis of the screw.

    The drum is 240 mm in diameter and the angle subtended by each is 90°. The screw has six square threads with a mean diameter of 20 mm and a lead of 55 mm. Assuming a coefficient of friction for the braking surface as 0.3 and for the threads 0.15, determine the force on the hand lever required to set the brake when the torque on the drum is 245 N-m.   **[Ans. 86.5 N]**
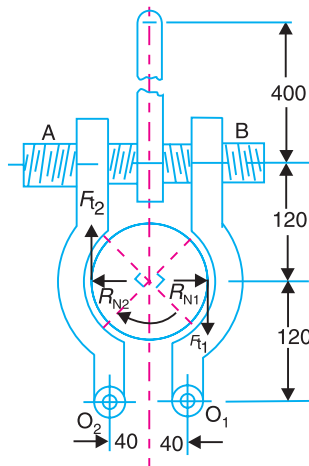
All dimensions in mm.   All dimensions in mm.
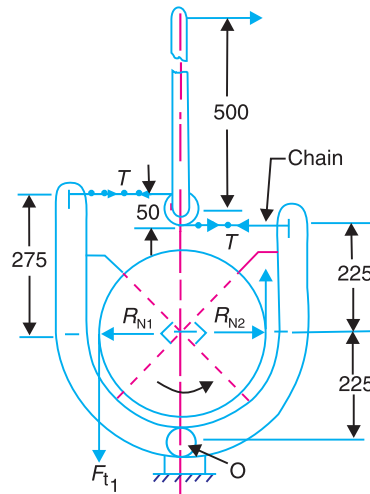
Fig. 19.40   **Fig. 19.41**

4. The layout and dimensions of the block brake are shown in Fig. 19.41. The diameter of the wheel is 300 mm and the contact angle for each block is 90°. If the coefficient of friction for the brake lining and wheel is 0.4 and the torque on the wheel is 30 N-m, find the force P on the operating arm required to set the brake for anticlockwise rotation of the wheel.   **[Ans. 10 N]**

5. A simple band brake is operated by a lever of length 500 mm. The brake drum has a diameter of 500 mm and the brake band embraces 5/8 of the circumference. One end of the band is attached to the fulcrum of the lever while the other end is attached to a pin on the lever 100 mm from the fulcrum. If the effort applied to the end of the lever is 2 kN and the coefficient of friction is 0.25, find the maximum braking torque on the drum.   **[Ans. 4.2 kN-m]**

6. A differential band brake acting on the 3/4 th of the circumference of a drum of 450 mm diameter, is to provide a braking torque of 225 N-m. One end of the band is attached to a pin 100 mm from the fulcrum of the lever and the other end to another pin 25 mm from the fulcrum on the other side of it where the operating force is also acting. If the operating force is applied at 500 mm from the fulcrum and the coefficient of friction is 0.25, find the two values of the operating force corresponding to two directions of rotation of the drum. **[Ans. 16.6 N for clockwise ; 266.6 N for anticlockwise]**

7. A differential band brake is shown in Fig. 19.42. The diameter of the drum is 800 mm. The coefficient of friction between the band and the drum is 0.3 and the angle of embrace is 240°.
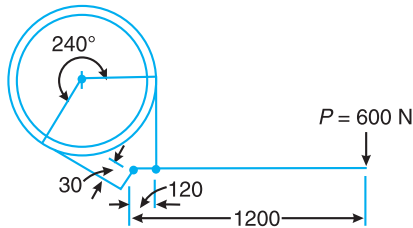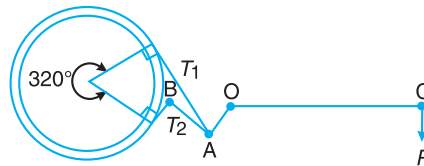

Fig. 19.42   **Fig. 19.43**

When a force of 600 N is applied at the free end of the lever, find for clockwise and anticlockwise rotation of the drum: 1. the maximum and minimum forces in the band ; and 2. the torque which can be applied by the brake. **[Ans. 176 kN, 50 kN, 50.4 kN-m ; 6.46 kN, 1.835 kN, 1.85 kN-m]**

8. A differential band brake is shown in Fig. 19.43. The diameter of the drum is 1 metre and rotates at 1200 r.p.m. in the anticlockwise direction. The angle of contact is 320°. The various lengths are : $OA = 30$ mm; $AB = 150$ mm and $OC = 700$ mm. Find the pull required at the end C of a lever to absorb 40 kW. Also find the length of AB for self locking. The coefficient of friction may be taken as 0.2. **[Ans. 25.7 N ; 91.8 mm]**

9. In a band and block brake, the band is lined with 14 blocks, each of which subtends an angle of 20° at the drum centre. One end of the band is attached to the fulcrum of the brake lever and the other to a pin 150 mm from the fulcrum. Find the force required at the end of the lever 1 metre long from the fulcrum to give a torque of 4 kN-m. The diameter of the brake drum is 1 metre and the coefficient of friction between the blocks and the drum is 0.25. **[Ans. 1712 N]**
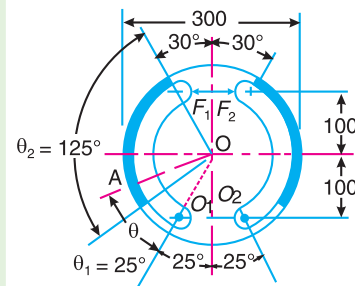
All dimensions in mm.

Fig. 19.44

10. Fig. 19.44 shows the particulars of two brake shoes which act on the internal surface of a cylindrical brake drum. The braking forces F_1 and F_2 are applied as shown, and each shoe pivots on its fixed fulcrum O_1 and O_2.

The width of the brake lining is 35 mm. The intensity of pressure at any point A is 0.4 sin θ N/mm², where θ is measured as shown from either pivot. The coefficient of friction is 0.4. Determine the braking torque and the magnitude of the forces F_1 and F_2. **[Ans. 373 N-m ; 685 N, 2323 N]**

11. A lorry is moving on a level road at a speed of 36 km/h. Its centre of gravity lies at a distance of 0.6 m from the ground level. The wheel base is 2.4 metres and the distance of C.G. from the rear wheels is 0.9 m. Find the distance travelled by the car before coming to rest when brakes are applied,
(*a*)   to the rear wheels, (*b*) to the front wheels, and (*c*) to all the four wheels.
The coefficient of friction between the tyres and the road surface is 0.45.

[Ans. 21.55 m; 26.82 m; 11.36 m]

12. A torsion dynamometer is fitted on a turbine shaft to measure the angle of twist. It is observed that the shaft twists 1.5° in a length of 5 metres at 500 r.p.m. The shaft is solid and has a diameter of 200 mm. If the modulus of rigidity for the shaft material is 85 GPa, find the power transmitted by the turbine.

[Ans. 3662 kW]

DO YOU KNOW ?

1.   Distinguish between brakes and dynamometers.
2.   Discuss the various types of the brakes.
3.   Show that, in a band and block brake, the ratio of the maximum and minimum tensions in the brake straps is

$$\frac{T_0}{T_n} = \left(\frac{1 + \mu \tan\theta}{1 - \mu \tan\theta}\right)^n$$

where T_0 = Maximum tension,

T_n = Minimum tension

μ = Coefficient of friction between the blocks and drum, and

2θ = Angle subtended by each block at the centre of the drum.

4.   Describe with the help of a neat sketch the principles of operation of an internal expanding shoe. Derive the expression for the braking torque.
5.   What are the leading and trailing shoes of an internal expanding shoe brake ?
6.   What is the difference between absorption and transmission dynamometers ? What are torsion dynamometers ?
7.   Describe the construction and operation of a prony brake or rope brake absorption dynamometer.
8.   Describe with sketches one form of torsion dynamometer and explain with detail the calculations involved in finding the power transmitted.
9.   Explain with neat sketches the Bevis-Gibson flash light dynamometer.

OBJECTIVE TYPE QUESTIONS

1.   The brakes commonly used in railway trains is
     (*a*)   shoe brake   (*b*) band brake
     (*c*)   band and block brake   (*d*) internal expanding brake
2.   The brake commonly used in motor cars is
     (*a*)   shoe brake   (*b*) band brake
     (*c*)   band and block brake   (*d*) internal expanding brake
3.   In a differential band brake, as shown in Fig. 19.45, the length *OA* is greater than *OB*. In order to apply the brake, the force *P* at *C* should

Fig. 19.45

     (*a*)   be zero   (*b*) act in upward direction
     (*c*)   act in downward direction
4.   For the brake to be self locking, the force *P* at *C* as shown in Fig. 19.45, should
     (*a*)   be zero
     (*b*)   act in upward direction
     (*c*)   act in downward direction
5.   When brakes are applied to all the four wheels of a moving car, the distance travelled by the car before it is brought to rest, will be
     (*a*) maximum   (*b*) minimum
6.   Which of the following is an absorption type dynamometer ?
     (*a*) prony brake dynamometer   (*b*) rope brake dynamometer
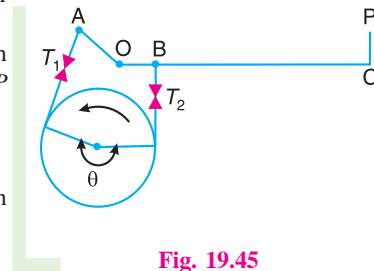   (*c*) epicyclic-train dynamometer   (*d*) torsion dynamometer

ANSWERS

| **1.** (*a*) | **2.** (*d*) | **3.** (*c*) | **4.** (*a*) | **5.** (*b*) | **6.** (*a*), (*b*) |