$$\sigma_B = \frac{20 \times 10^3 \times \cos(-25.49) \times 0.206}{1.592E-4}$$

$$+ \frac{20 \times 10^3 \times \sin(-25.49) \times 0.2158}{5.246E-4}$$

$$= 23.36 - 3.54$$

$$\sigma_B = 19.82 \ MN/m^2$$

6.

$$\delta_u = \frac{-W\sin\theta\ell^3}{3EI_v} = \frac{-4 \times 10^3 \times \sin(17.64)3^3}{3 \times 2 \times 10^{11} \times 1.741E-6}$$

$$\delta_u = \underline{-0.031m}$$

$$\delta_v = \frac{-W\cos\theta\ell^3}{3EI_u} = \frac{4 \times 10^3 \times \cos(17.64)3^3}{3 \times 2 \times 10^{11} \times 1.381E-5}$$

$$\delta_v = -0.0124m$$

7.

$$\delta_u = \frac{-20 \times 10^3 \times \sin(-25.49) \times 4^3}{48. \times 1 \times 10^{11} \times 5.246E-4} = \underline{2.19E-4m}$$

$$\delta_v = \frac{-20 \times 10^3\cos(-25.49) \times 4^3}{48 \times 1 \times 10^{11} \times 1.592E-4} = \underline{-1.51E-3m}$$

CHAPTER 14

1. Maximum shearing force occurs at the ends,

where $F_{max} = w\ell/2 = 200 \times 3/2$

$F_{max} = 300$ kN

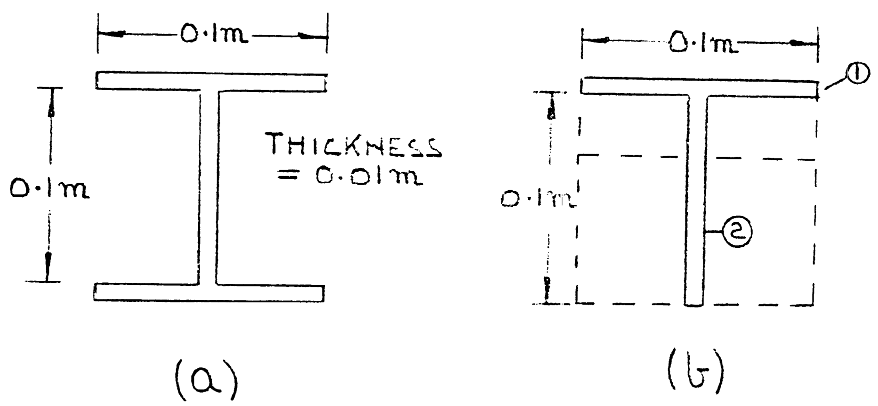
$$\text{Maximum shearing stress} = \frac{F_{max} \times 1.5}{bd}$$

or $$\hat{\tau} = \frac{300 \times 1.5}{0.1 \times 0.2} \times \frac{1E3}{1E6}$$

$\underline{\hat{\tau} = 22.5 \text{ MPa}}$

$\underline{\text{@ mid-span } \tau = F = 0}$

2.



$$I = \frac{0.1 \times 0.12^3}{12} - \frac{0.09 \times 0.1^3}{12} = 6.9E-6$$

Flange

$$\hat{\tau}_F = \frac{0.045 \times 0.01 \times 0.055}{0.01 \times 6.9E-6} \times 100E3 = \underline{35.87\ MN/m^2}$$

Web

$$\hat{\tau}_w = \frac{(0.1 \times 0.01 \times 0.055 + 0.05 \times 0.01 \times 0.025) \times 100E3}{0.01 \times 6.9E-6}$$

$\underline{\hat{\tau}_w = 97.83\ MN/m^2}$

b.

Section	a	y	ay	ay^2	i
1 2	1E-3 1E-3	0.105 0.05	1.05E-4 5E-5	1.103E-5 2.5E-6	8.33E-9 8.33E-7
Σ	2E-3	-	1.55E-4	1.353E-5	8.416E-7

$\underline{\bar{y} = 0.0775}$

$I_{XX} = 1.437E-5$

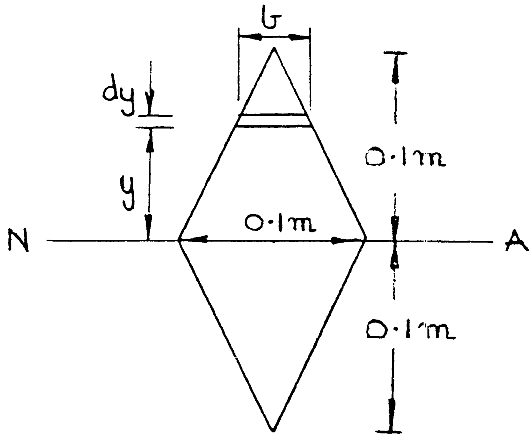
$I_{NA} = 2.358E-6m^4$

$$\hat{\tau}_F = \frac{100E3 \times 0.045 \times 0.01 \times 0.0275}{0.01 \times 2.358E-6} = \underline{52.48\ MPa}$$

$$\hat{\tau}_w = \frac{100E3(0.1 \times 0.01 \times 0.0275 + 0.0225^2/2 \times 0.01)}{0.01 \times 2.358E-6}$$

$= \underline{127.36\ MPa}$

3.



$$I_{NA} = \frac{0.1 \times 0.1^3}{12} \times 2 = \underline{1.667E{-5}m^4}$$

$b = 0.1\ (1 - 10y)$

$$\int ydA = \int bdy.y = \int_y^{0.1} 0.1\ (1 - 10y)\ .\ ydy$$

$$= 0.1 \left[\frac{y^2}{2} - \frac{10y^3}{3}\right]_y^{0.1} = \frac{0.1}{6}\left[3y^2 - 20y^3\right]_y^{0.1}$$

$$= \frac{0.1}{6}\left[0.01 - (3y^2 - 20y^3)\right]$$

$$\tau = \frac{0.5 \times 0.1}{6 \times 1.667E{-5}} \frac{[0.01 - (3y^2 - 20y^3)]}{0.1\ (1 - 10y)}$$

$$= 5000\ [0.01 - (3y^2 - 20y^3]/(1 - 10y)$$

For $\hat{\tau}$ $\frac{d\tau}{dy} = 0$

$(1 - 10y)[-(6y - 60y^2] - (-10)\ [0.01 - (3y^2 - 20y^3)] = 0$

$- 6y + 60y^2 + 60y^2 - 600y^3 + 0.1 - 30y^2 + 200y^3 = 0$

$- 400y^3 + 90y^2 - 6y + 0.1 = 0$

$\underline{+ 400y^3 - 90y^2 + 6y - 0.1 = 0 = \Psi}$

$1200y^2 - 180y + 6 = \frac{d\Psi}{dy} = 0$

$$y_1 = 0 + \frac{0.1}{6} = 0.017$$

$$y_2 = 0.017 + \frac{0.022}{3.29} = \underline{0.024}$$

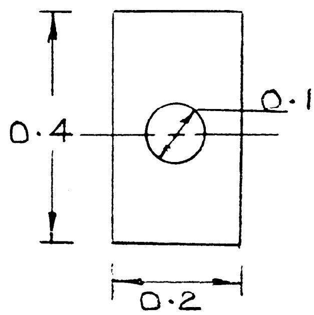
$$y_3 = 0.024 + \frac{2.31E-3}{2.37} = \underline{0.025}$$

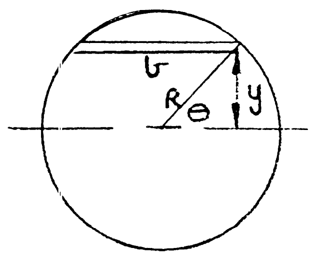
$\underline{y_4 = 0.025 - 0}$

$\underline{y = 0.025\text{ m}}$ perpendicular to NA

$$\hat{\tau} = \frac{5000 \times 8.4375E-3}{0.75} = \underline{56.25\ MN/m^2}$$

4.





$b = 2R\cos\theta$
$y = R\sin\theta$
$dy = R\cos\theta d\theta$

$\int ydA = \int bdy.y$

$$I_{NA} = \frac{0.2 \times 0.4^3}{12} - \frac{\pi \times 0.1^4}{64} = 1.062E-3$$

$$\int ydA = 0.2 \times 0.2 \times 0.1 - \int_0^{\pi/2} 2R\cos\theta.R\sin\theta.R\cos\theta d\theta$$

$$= 4E{-}3 + 2R^3 \int_0^{\pi/2} \cos^2\theta \, d(\cos\theta)$$

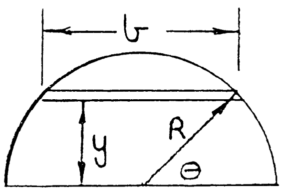
$$= 4E{-}3 + 2.5E{-}4 \left[\frac{\cos^3\theta}{3}\right]$$

$$= 4E{-}3 + 2.5E{-}4 \left[0 - \frac{1}{3}\right]$$

$$\int ydA = 3.917E{-}3m^3$$

$$\tau = \frac{0.5 \times 3.917E{-}3}{0.1 \times 1.062E{-}3}$$

$$= 18.44 \text{ MPa}$$



$b = 2R \cos \theta$
$y = R \sin \theta$
$dy = R \cos \theta d\theta$

Proof of $\int ydA$

$$\int y.dA = \int_0^{\pi/2} 2R\cos\theta.R\sin\theta.R\cos\theta d\theta$$

$$=2R^3 \int_0^{\pi/2} \cos^2\theta \sin\theta d\theta$$

$$= - 2R^3 \int \cos^2\theta d(\cos\theta)$$

$$= -2R^3 \left[\frac{\cos^3\theta}{3}\right]_0^{\pi/2}$$

$$= \frac{2R^3}{3}$$

$$dA = \frac{\pi R^2}{2}$$

$$\therefore \ \bar{y} = \frac{2R^3}{3} * \frac{2}{\pi R^2} = \frac{4R}{3\pi}$$

5. $$I = \frac{1 \times 1^3}{12} - \frac{\pi \times 0.6^4}{64} = 0.08333 - 6.362E{-}3$$

$\underline{I = 0.077 \text{ m}^4}$

$\underline{y = 0.5, \tau = 0}$

$\underline{y = 0.4 \text{ m}}$

$$\tau_{0.4} = \frac{50 \times 1 \times 0.1 \times 0.45}{1 \times 0.077} = 29.22$$

$\underline{y = 0.3 \text{ m}}$

$$\tau_{0.3} = \frac{50 \times 1 \times 0.2 \times 0.4}{1 \times 0.077} = 51.95$$

$b = 2R \cos \phi$; $y = R \sin \phi$; $dy = R \cos \phi . d \phi$

$$\int ydA = \int y.bdy = -\int 2R^3.\cos^2\phi \ d(\cos\phi)$$

$$= + \frac{2R^3}{3} \left[\cos^3\phi\right]_0^\phi$$

$\underline{y = 0.2 \text{ m}}$ $\quad \phi = \sin^{-1} (0.2/0.3) = \underline{41.81°}$

$\int ydA \quad = 1 \times 0.3 \times 0.35 - \frac{2}{3} \times 0.3^3 \cos^3 (41.81°)$

$= 0.105 - 7.454E{-}3 = \underline{0.0975 \text{ m}^3}$

$b \quad = 0.553$ m

$\tau \quad \underline{= 114.49}$

$\underline{y = 0.1 \text{ m}}$

$\phi \quad = \sin^{-1}(0.1/0.3) = 19.47°$

$\int ydA \quad = 1 \times 0.4 \times 0.3 - ⅔ \times 0.3^3 \cos^3 (19.47)$

$= 0.12 - 0.0151 = \underline{0.1049m^3}$

$\underline{b = 0.434 \text{ m}}$

$\underline{\tau = 156.95}$

$\underline{y = 0}\ \phi = 0$

$\int ydA = 1 \times 0.5 \times 0.25 - ⅔ \times 0.3^3 = \underline{0.107m^3}$

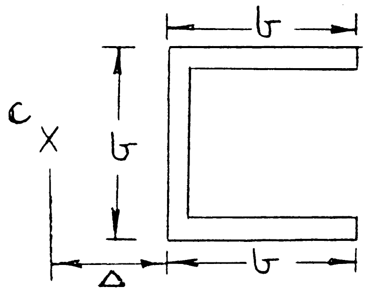
$b = 1 - 2R = \underline{0.4 \text{ m}}$

$\underline{\upsilon = 173.70}$

[@y = 0, $\tau_o = 173.7$; $\tau_{0.1} = 156.95$; $\tau_{0.2} = 114.49$;

$\tau_{0.3} = 51.95$; $\tau_{0.4} = 29.22$; @ y =0.5; $\tau_{0.5} = 0$]

6.



$$I = \frac{tb^3}{12} + b \times t \times \left(\frac{b}{2}\right)^2 \times 2$$

$\underline{I = 0.5833\ tb^3}$

$$\hat{\tau} = \frac{F}{tI}\, bt \times \frac{b}{2} = \frac{0.5tb^2F}{t.I} = \frac{0.5b^2F}{0.5833tb^3}$$

$$\hat{\tau} = \frac{0.8572F}{bt}$$

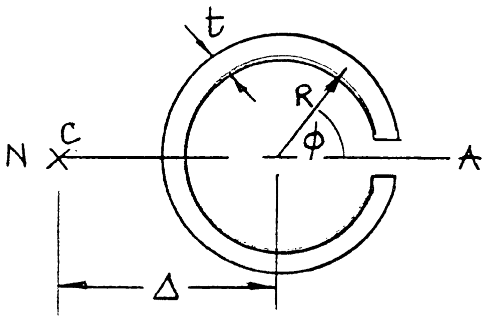
$F_F = (0.429F/bt) * bt = 0.429F$

Moments about centre of web

$$F\Delta \quad = \quad 0.429\ F \times \frac{b}{2} \times 2$$

$\Delta \quad = \underline{0.429b}$

6b



$$I_{NA} = \int_0^{2\pi} (t.R.d\phi)\ (R\sin\phi)^2$$

$$= tR^3 \int_0^{2\pi} \frac{[1 - \cos 2\phi]}{2}$$

$\underline{I_{NA} = \pi t R^3}$

$$\int_o^{\phi} ydA = \int tRd\phi/R\sin\phi = tR^2\ [-\cos\phi]_o^{\phi}$$

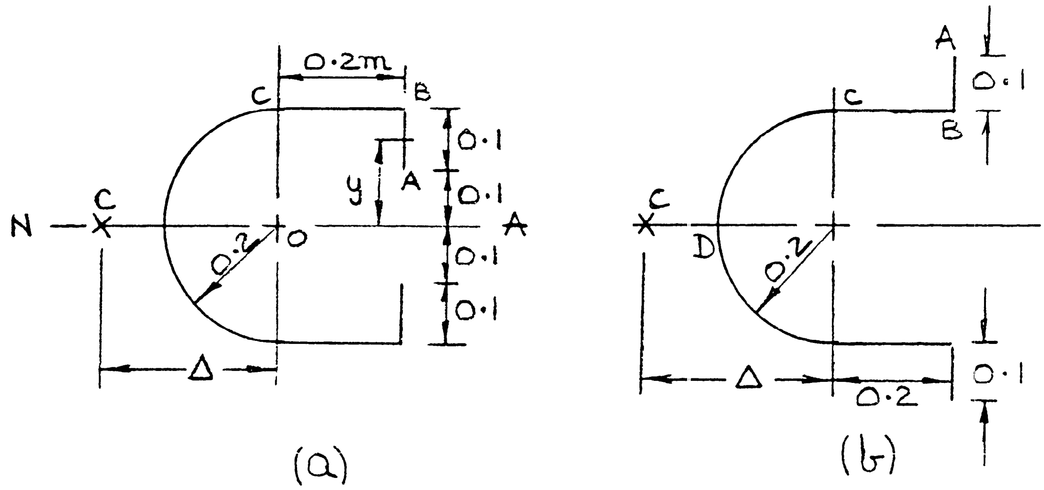
$$= tR^2\ [-\cos\phi + 1] = tR^2\ (1 - \cos\phi)$$

$$\tau_\phi = \frac{F}{\pi t^2 R^3}\ .\ tR^2\ (1 - \cos\phi) = \frac{F(1 - \cos\phi)}{\pi t R}$$

$$F\Delta = \int \tau_\phi\ .\ t.R^2 d\phi = \frac{Ft}{\pi t R} R^2\ [\phi - \sin\phi]_o^{2\pi}$$

$\underline{\Delta = 2R}$

7a.



$$I_{NA} = 0.2t \times 0.2^2 \times 2 + \frac{t \times 0.1^3}{12} \times 2 + 0.1 \times t \times 0.15^2 \times 2$$

$$= \int_0^{\pi} (0.2 \times \cos \phi)^2 (t \times 0.2 \, d\phi)$$

$$= 0.016\,t + 1.667E{-}4t + 4.5E{-}3t$$

$$+8E{-}3t \int_0^{\pi} \frac{(1 + \cos 2\phi)}{2} \, d\phi$$

$$= 0.0207t + 0.01257\,t$$

$$I = 0.0333t$$

<u>AB</u>

$$F_{AB} = \int_{0.1}^{0.2} \tau \, t dy = \frac{Ft}{I} \left[\frac{y^3}{6} - 5E{-}3y \right]_{0.1}^{0.2}$$

$$= \frac{Ft}{I} \{3.333E{-}4) \; ({-}3.333E{-}4)\}$$

$$= \frac{6.6667E{-}4Ft}{0.0333t} = \underline{0.02F}$$