$$\frac{P_R}{A} = \frac{350}{1 + 0.3125} = \underline{266.7}$$

For $\left(\frac{\ell}{k}\right) = 80$

$$\frac{P_R}{A} = \frac{350}{1 + 0.8} = \underline{194.4}$$

$A = 0.06 * 0.019 = 1.14 \text{ x } 10^{-3} \text{ m}^2$

$I = 3.4295E\text{-}8$

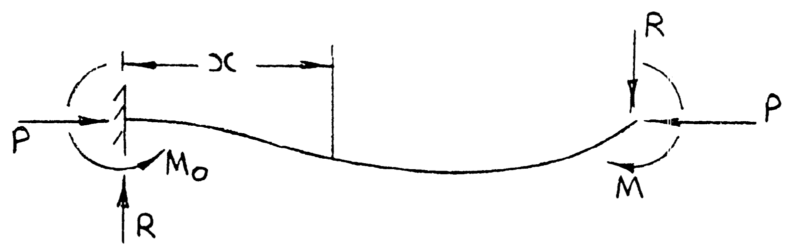
$k = 5.485E\text{-}3$

$$\frac{\ell}{k} = 72.93$$

$$P_R = \frac{350 * 1.14 \text{ x } 10^{-3}}{1 + \frac{1}{4} * \frac{1}{8000} * 5318.6}$$

$\underline{P_R = 342 \text{ kN}}$

8.



@ $x = 0$, $y = 0$ $\therefore$ $A = - M_o/(\alpha^2 EI)$

$$EI \frac{d^2y}{dx^2} = -Py + M_o - Rx$$

$$y = A \cos \alpha x + B \sin \alpha x + M_o/(\alpha^2 EI) - Rx/(\alpha^2 EI)$$

$$\frac{dy}{dx} = -\alpha A \sin \alpha x + \alpha B \cos \alpha x - R/(\alpha^2 EI)$$

$$\frac{d^2y}{dx^2} = -\alpha^2 A \cos \alpha x - \alpha^2 B \sin \alpha x$$

@ $x = 0$, $\frac{dy}{dx} = 0 \quad \therefore \alpha B = R/(\alpha^2 EI)$

or $B = R/(\alpha^3 EI)$

@ $x = \ell$, $y = 0$

$\therefore 0 = A \cos \alpha \ell + B \sin \alpha \ell + M_o/(\alpha^2 EI) - R\,\ell/(\alpha^2 EI)$

or $0 = -M_o/(\alpha^2 EI) \cos \alpha \ell + R/(\alpha^3 EI) \sin \alpha \ell$

$+ M_o/(\alpha^2 EI) - R\ell(\alpha^2 EI)$

or $0 = (M_o/\alpha^2 EI)(1 - \cos \alpha\ell) + R/(\alpha^2 EI)\left(\frac{\sin\alpha\ell}{\alpha} - \ell\right)$

$$\text{or } R = \frac{-M_o (1 - \cos\alpha\ell)}{\left(\frac{\sin\alpha\ell}{\alpha} - \ell\right)} \qquad 1$$

Moments about the left end

$M + R\ell = M_o$

$$\text{or } R = \frac{M_o - M}{\ell} \qquad 2$$

Equating 1 and 2

$$\frac{M_o - M}{\ell} = \frac{-M_o (1 - \cos\alpha\ell)}{\left[\frac{\sin\alpha\ell}{\alpha} - \ell\right]}$$

$$M_o - M = \frac{-M_o\ell(1 - \cos\alpha\ell)}{\left[\frac{\sin\alpha\ell}{\alpha} - \ell\right]}$$

$$M = \frac{M_o\ell(1 - \cos\alpha\ell)}{\left[\frac{\sin\alpha\ell}{\alpha} - \ell\right]} + M_o$$

$$= \frac{M_o \left[\ell(1 - \cos\alpha\ell) + \left[\frac{\sin\alpha\ell}{\alpha} - \ell\right]\right]}{\frac{\sin\alpha\ell}{\alpha} - \ell}$$

$$= \frac{M_o (-\ell\cos\alpha\ell + \frac{\sin\alpha\ell}{\alpha})}{\left[\frac{\sin\alpha\ell}{\alpha} - \ell\right]} \times \frac{\alpha}{\alpha}$$

$$M = \frac{M_o (- \alpha\ell\cos\alpha\ell + \sin\alpha\ell)}{(\sin\alpha\ell - \alpha\ell)}$$

$$\text{or } M_o = \frac{M(\alpha\ell - \sin\alpha\ell)}{(\alpha\ell \cos\alpha\ell - \sin\alpha\ell)}$$

$$P_{cr} = \frac{20.25 \text{ EI}}{\ell^2}$$

$$\therefore P = \frac{5.063 \text{ EI}}{\ell^2}$$

$$\alpha^2 = \frac{P}{EI} = \frac{5.063}{\ell^2}$$

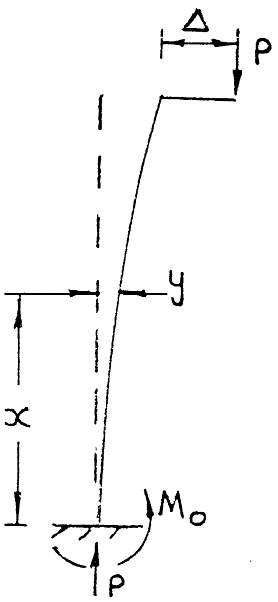
& $\alpha = 2.25/\ell$

$$\therefore M_o = \frac{M(2.25 - 0.778)}{2.25 \times (-0.628) - 0.778}$$

$$= \frac{-1.472M}{2.191}$$

$$M_o = -0.672\ M$$

9.



$$EI\frac{d^2y}{dx^2} = -Py + M_o$$

$$\frac{d^2y}{dx^2} + \alpha^2 y = \alpha^2\frac{M}{P}$$

$$y = A\cos\alpha x + B\sin\alpha x + \frac{M_o}{P}$$

$$\frac{dy}{dx} = -\alpha A\sin\alpha x + \alpha B\cos\alpha x$$

$$\frac{d^2y}{dx^2} = -\alpha^2 A\cos\alpha x - \mathcal{L}^2 B\sin\mathcal{L}x$$

@ $x = 0$, $y = 0$

$$0 = A + \frac{M_o}{P}$$

$$A = \frac{-M_o}{P}$$

@ $x = 0$, $\frac{dy}{dx} = 0$

$0 = \alpha B \therefore \underline{B = 0}$

$\underline{@\ x = \ell,\ M = P\Delta} = EI \left[\frac{d^2y}{dx^2}\right]_{x=\ell}$

$$\frac{P\Delta}{EI} = -\alpha^2 A\cos\mathscr{L}\ell$$

$$A = -\frac{P\Delta}{EI} \cdot \frac{EI}{P} \sec \alpha\ell = -\Delta \sec\alpha\ell$$

$$\therefore M = -P * -\Delta \sec \alpha\ell;\ \underline{\&\ M_0 = P\Delta\sec\alpha\ell}$$

hence

$\underline{y = -\Delta \sec\alpha\ \ell.\ \cos \alpha x + \Delta \sec \alpha\ell}$

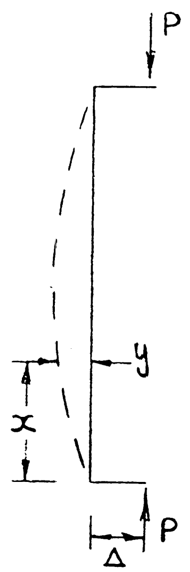
Deflection at the free end

$= -\Delta \sec \alpha\ \ell.\ \cos \alpha\ \ell + \Delta \sec \alpha\ell$

$\underline{= \Delta\ (\sec \alpha\ell - 1)}$

10.

Let Δ = eccentricity



$$EI \frac{d^2y}{dx^2} = -P(y + \Delta)$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI} y = -\frac{P}{EI} \Delta$$

$$\frac{d^2y}{dx^2} + \alpha^2 y = -\alpha^2\Delta$$

Complete solution is

$y = A \cos \alpha x + B \sin \alpha x - \Delta$
$: x = 0, y = 0$

$\therefore A = \Delta$

$\therefore x = \ell, y = 0$

$\therefore 0 = \Delta \cos \alpha \ell + B \sin \alpha \ell - \Delta$

$$B = \frac{\Delta(1 - \cos\alpha\ell)}{\sin\alpha\ell} = \frac{\Delta.2\sin^2\frac{\alpha\ell}{2}}{2\sin\frac{\alpha\ell}{2}\cos\frac{\alpha\ell}{2}} = \Delta \tan\frac{\alpha\ell}{2}$$

$$\underline{y = \Delta \left[\cos \alpha x + \tan \frac{\alpha\ell}{2} \sin (\alpha x) - 1\right]}$$

The maximum deflection occurs : $x = \ell/2$

$$\text{ie } \delta = \Delta \left(\cos \frac{\alpha\ell}{2} + \tan \frac{\alpha\ell}{2}.\sin\frac{\alpha\ell}{2} - 1 \right)$$

$$= \Delta \cos \frac{\alpha\ell}{2}\left(1 + \tan^2\frac{\alpha\ell}{2} - \frac{1}{\cos\frac{\alpha\ell}{2}}\right) = \Delta\cos\frac{\alpha\ell}{2}\left(\sec^2 \frac{\alpha\ell}{2} - \frac{1}{\cos\frac{\alpha\ell}{2}}\right)$$

$$\delta = \Delta \left[\sec \left(\frac{\alpha\ell}{2}\right) - 1\right]$$

The maximum bending moment = M_{max}

where $M_{max} = P (\delta + \Delta)$

$$\text{or } M_{max} = P \Delta \sec \left(\sqrt{\frac{P}{EI}} \frac{\ell}{2}\right)$$

$$I = \frac{\pi (70^4 - 50^4)}{64} = 871790 \text{ mm}^4$$

$EI = 1.744E11$

$\alpha = \sqrt{(114.7E3/1.744E11)}$

$\alpha = 8.111E\text{-}4$

$$\frac{\alpha \ell}{2} = 1.318$$

$$\cos \frac{\alpha \ell}{2} = 0.25$$

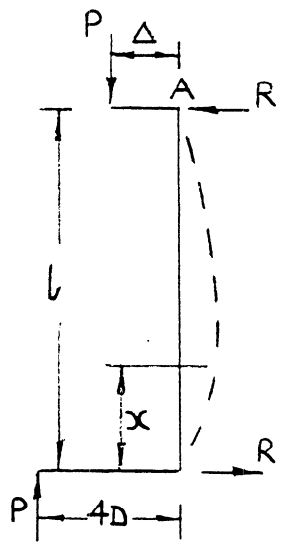
$$\therefore \sec \frac{\alpha \ell}{2} = 4$$

$$\delta = \Delta\left[\sec\left(\frac{\alpha \ell}{2}\right) - 1\right]$$

$15 = \Delta\ (4\text{-}1)$

<u>$\Delta = 5$ mm</u>

11. <u>Moments about A</u>



$P\Delta + R\ell = P \times 4\Delta$

$R = 3\ P\Delta/\ell$

$$EI\frac{d^2y}{dx^2} = -P(4\Delta + y) + Rx$$

$$= -P\Delta(4 - 3x/\ell) - Py$$

$$\text{or } \frac{d^2y}{dx^2} + \alpha^2 y = -\alpha^2\ \Delta\ (4 - 3x/\ell)$$

$$\text{ie } y = A\cos\alpha x + B\sin\alpha x - \Delta(4 - 3x/\ell)$$

$$\frac{dy}{dx} = -\alpha\ A\sin\alpha x + \alpha B\cos\alpha x + 3\Delta/\ell$$

@ $x = 0, y = 0$

$\therefore$ <u>$A = 4\Delta$</u>

$\therefore x = \ell, y = 0$

$\therefore 0 = A \cos \alpha\ell + B \sin \alpha - \Delta.1$

$$B = \frac{\Delta}{\sin \alpha\ell} (-4 \cos\alpha\ell + 1)$$

$$\therefore B = \frac{\Delta - A \cos\alpha\ell}{\sin\alpha\ell}$$

$$= \Delta (1 - 4\cos \alpha\ell)/\sin \alpha\ell$$

$\therefore y = 4\Delta \cos \alpha x + \Delta (1 - 4 \cos \alpha\ell).\sin \alpha x/\sin \alpha\ell - \Delta (4 - 3x/\ell)$

& $\frac{dy}{dx}$ $- 4\alpha\Delta \sin \alpha x + \alpha\Delta (1 - 4 \cos \alpha\ell) \cos \alpha x/\sin \alpha\ell + 3 \Delta/\ell$

For maximum y, $\frac{dy}{dx} = 0$

$\therefore 0 = - 4\alpha\Delta \sin \alpha x + \alpha\Delta (1 - 4 \cos \alpha\ell) \cos \alpha x/\sin \alpha\ell + 3\Delta \ell$

$\alpha = \sqrt{} (5000/20000)$

<u>or $\alpha = 0.5$</u>

<u>To calculate x</u>

Try <u>$x = 1.5$ m</u> $\therefore \alpha x = 42.97° \ \& \ \alpha\ell = 85.94°$

Substituting

$\therefore 0 = -4 \times 0.5 \times 0.682 + 0.5 (1 - 0.2829) \times 0.732/0.997 + 1$

or $0 = - 1.364 + 0.263 + 1 = - 0.101$ <u>incorrect</u>

Try x = 1.4 m $\therefore$ $\alpha x = 40.11°$

$\therefore$ $0 = -1.288 + 0.3586 \times 0.764/0.997 + 1 = \underline{-0.0129}$

Try x = 1.35 m $\therefore$ $\alpha x = 38.67°$

$\therefore$ $0 = -1.2498 + 0.3586 \times 0.78/0.997 + 1 = +0.003$

Try $\underline{x = 1.38 \text{ m}}$ $\therefore$ $\alpha x = 39.53°$

$\therefore$ $0 = - 1.273 + 0.3596 \times 0.771 + 1 = 4.356E\text{-}3$

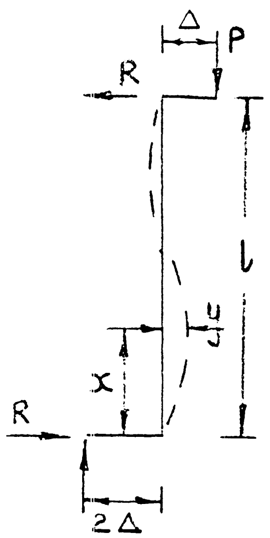
ie $\underline{x = 1.38 \text{ m}}$

& $\delta = 4 \times 0.01 \times 0.771 + 0.01 \times 0.719 \times 0.636 - 0.01 (4 - 1.38)$

$= 0.03084 + 4.573E\text{-}3 - 0.0262$

$\underline{\delta = 9.21E\text{-}3\text{m}}$

12.



$R\ell = 3P\Delta$

$$R = \frac{3P\Delta}{\ell}$$

@ x

$$EI\frac{d^2y}{dx^2} = -P(y + 2\Delta) + Rx$$

CS is

$$y = A \cos x + B \sin \alpha x + \frac{Rx}{P} - 2\Delta$$

@ x = 0   y = 0 $\therefore$ $\underline{A = 2\Delta}$

@ $x = \ell$ y = 0

$$\therefore 0 = 2\,\Delta \cos \alpha\ell + B \sin \alpha\ell + \frac{3P\Delta\ell}{P\ell} - 2\Delta$$

$$0 = 2\,\Delta \cos \alpha\ell + B \sin \ell + \Delta$$

$$B = \frac{-\Delta(1 + 2\cos\alpha\ell)}{\sin\alpha\ell}$$

$$\therefore y = 2\Delta \cos\alpha x - \frac{\Delta(1 + 2\cos\alpha\ell)}{\sin\alpha\ell}\sin\alpha x + 3\Delta x/\ell - 2\Delta$$

$$\frac{dy}{dx} = \Delta\left[-2\alpha\sin\alpha x - \alpha\frac{(1 + 2\cos\alpha\ell)}{\sin\alpha\ell}\cos\alpha x + 3/\ell\right]$$

$$\frac{d^2y}{dx^2} = \Delta\alpha^2\left[-2\cos\alpha x + \frac{(1 + 2\cos\alpha\ell)}{\sin\alpha\ell}\sin\alpha x\right]$$

$$M = EI\frac{d^2y}{dx^2} = P\Delta\left[-2\cos\alpha x + \frac{(1 + 2\cos\alpha\ell)}{\sin\alpha\ell}\,.\,\sin\alpha x\right]$$

13.

$$\text{Let } y_0 = \frac{4\Delta x(1 - x)}{\ell^2} = \frac{4\Delta x}{\ell^2} - \frac{4x^2\Delta}{\ell^2}$$

$$\frac{dy_o}{dx} = \frac{4\Delta}{\ell^2} - \frac{8x\Delta}{\ell^2}$$

$$\frac{d^2y_o}{dx^2} = -\frac{8\Delta}{\ell^2}$$