# Chapter 4

**4-1** For a torsion bar, $k_T = T/\theta = Fl/\theta$, and so $\theta = Fl/k_T$. For a cantilever, $k_l = F/\delta, \delta = F/k_l$. For the assembly, $k = F/y$, or, $y = F/k = l\theta + \delta$

Thus

$$y = \frac{F}{k} = \frac{Fl^2}{k_T} + \frac{F}{k_l}$$

Solving for $k$

$$k = \frac{1}{\dfrac{l^2}{k_T} + \dfrac{1}{k_l}} = \frac{k_l k_T}{k_l l^2 + k_T} \qquad \textit{Ans.}$$

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**4-2** For a torsion bar, $k_T = T/\theta = Fl/\theta$, and so $\theta = Fl/k_T$. For each cantilever, $k_l = F/\delta_l$, $\delta_l = F/k_l$, and, $\delta_L = F/k_L$. For the assembly, $k = F/y$, or, $y = F/k = l\theta + \delta_l + \delta_L$.

Thus

$$y = \frac{F}{k} = \frac{Fl^2}{k_T} + \frac{F}{k_l} + \frac{F}{k_L}$$

Solving for $k$

$$k = \frac{1}{\dfrac{l^2}{k_T} + \dfrac{1}{k_l} + \dfrac{1}{k_L}} = \frac{k_L k_l k_T}{k_l k_L l^2 + k_T k_L + k_T k_l} \qquad \textit{Ans.}$$

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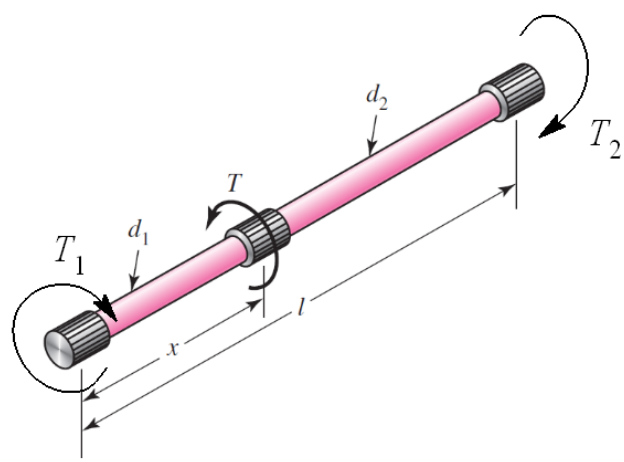
**4-3** (**a**) For a torsion bar, $k = T/\theta = GJ/l$.




Two springs in parallel, with $J = \pi d_i^{\,4}/32$, and $d_1 = d_1 = d$,

$$k = \frac{J_1 G}{x} + \frac{J_2 G}{l-x} = \frac{\pi}{32} G\left(\frac{d_1^4}{x} + \frac{d_2^4}{l-x}\right)$$

$$= \frac{\pi}{32} G d^4 \left(\frac{1}{x} + \frac{1}{l-x}\right) \qquad \textit{Ans.} \quad (1)$$

Deflection equation,

$$\theta = \frac{T_1 x}{JG} = \frac{T_2 (l-x)}{JG}$$

results in $\quad T_1 = \dfrac{T_2 (l-x)}{x} \qquad (2)$

From statics, $T_1 + T_2 = T = 1500$. Substitute Eq. (2)

$$T_2\left(\frac{l-x}{x}\right)+T_2=1500 \quad \Rightarrow \quad T_2=1500\frac{x}{l} \qquad Ans. \quad (3)$$

Substitute into Eq. (2) resulting in $\quad T_1=1500\dfrac{l-x}{l} \qquad Ans. \quad (4)$

(**b**) From Eq. (1), $k=\dfrac{\pi}{32}\left(0.5^4\right)11.5\left(10^6\right)\left(\dfrac{1}{5}+\dfrac{1}{10-5}\right)=28.2\left(10^3\right)$ lbf·in/rad *Ans.*

From Eq. (4), $T_1=1500\dfrac{10-5}{10}=750$ lbf·in *Ans.*

From Eq. (3), $T_2=1500\dfrac{5}{10}=750$ lbf·in *Ans.*

From either section, $\tau=\dfrac{16T_i}{\pi d_i^3}=\dfrac{16(1500)}{\pi\left(0.5^3\right)}=30.6\left(10^3\right)$ psi $=30.6$ kpsi *Ans.*

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**4-4** Deflection to be the same as Prob. 4-3 where $T_1$ = 750 lbf·in, $l_1 = l/2 = 5$ in, and $d_1 = 0.5$ in

$$\theta_1=\theta_2=\theta$$

$$\frac{T_1(4)}{\frac{\pi}{32}d_1^4G}=\frac{T_2(6)}{\frac{\pi}{32}d_2^4G}=\frac{750(5)}{\frac{\pi}{32}\left(0.5^4\right)G} \quad \Rightarrow \quad \frac{4T_1}{d_1^4}=\frac{6T_2}{d_2^4}=60\left(10^3\right) \qquad (1)$$

Or,

$$T_1=15\left(10^3\right)d_1^4 \qquad (2)$$

$$T_2=10\left(10^3\right)d_2^4 \qquad (3)$$

Equal stress, $\tau_1=\tau_2 \;\Rightarrow\; \dfrac{16T_1}{\pi d_1^3}=\dfrac{16T_2}{\pi d_2^3} \;\Rightarrow\; \dfrac{T_1}{d_1^3}=\dfrac{T_2}{d_2^3} \qquad (4)$

Divide Eq. (4) by the first two equations of Eq.(1) results in

$$\frac{\frac{T_1}{d_1^3}}{\frac{4T_1}{d_1^4}}=\frac{\frac{T_2}{d_2^3}}{\frac{4T_2}{d_2^4}} \quad \Rightarrow \quad d_2=1.5d_1 \qquad (5)$$

Statics, $T_1 + T_2 = 1500 \qquad (6)$

Substitute in Eqs. (2) and (3), with Eq. (5) gives

$$15\left(10^3\right)d_1^4+10\left(10^3\right)\left(1.5d_1\right)^4=1500$$

Solving for $d_1$ and substituting it back into Eq. (5) gives

$d_1 = 0.388\ 8$ in, $d_2 = 0.583\ 2$ in *Ans.*

From Eqs. (2) and (3),

$T_1 = 15(10^3)(0.388\ 8)^4 = 343$ lbf·in *Ans.*
$T_2 = 10(10^3)(0.583\ 2)^4 = 1\ 157$ lbf·in *Ans.*

Deflection of $T$ is $$\theta_1 = \frac{T_1 l_1}{J_1 G} = \frac{343(4)}{(\pi/32)(0.388\ 8^4)11.5(10^6)} = 0.053\ 18 \text{ rad}$$

Spring constant is $$k = \frac{T}{\theta_1} = \frac{1500}{0.053\ 18} = 28.2(10^3) \text{ lbf}\cdot\text{in} \quad \textit{Ans.}$$

The stress in $d_1$ is $$\tau_1 = \frac{16T_1}{\pi d_1^3} = \frac{16(343)}{\pi(0.388\ 8)^3} = 29.7(10^3) \text{ psi} = 29.7 \text{ kpsi} \quad \textit{Ans.}$$

The stress in $d_1$ is $$\tau_2 = \frac{16T_2}{\pi d_2^3} = \frac{16(1\ 157)}{\pi(0.583\ 2)^3} = 29.7(10^3) \text{ psi} = 29.7 \text{ kpsi} \quad \textit{Ans.}$$

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**4-5** (**a**) Let the radii of the straight sections be $r_1 = d_1/2$ and $r_2 = d_2/2$. Let the angle of the taper be $\alpha$ where $\tan\alpha = (r_2 - r_1)/2$. Thus, the radius in the taper as a function of $x$ is $r = r_1 + x\tan\alpha$, and the area is $A = \pi(r_1 + x\tan\alpha)^2$. The deflection of the tapered portion is

$$\delta = \int_0^l \frac{F}{AE}dx = \frac{F}{\pi E}\int_0^l \frac{dx}{(r_1 + x\tan\alpha)^2} = -\frac{F}{\pi E}\left.\frac{1}{(r_1 + x\tan\alpha)\tan\alpha}\right|_0^l$$
$$= \frac{F}{\pi E}\left[\frac{1}{r_1\tan\alpha} - \frac{1}{\tan\alpha(r_1 + l\tan\alpha)}\right] = \frac{F}{\pi E\tan\alpha}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$
$$= \frac{F}{\pi E\tan\alpha}\frac{r_2 - r_1}{r_1 r_2} = \frac{F}{\pi E\tan\alpha}\frac{l\tan\alpha}{r_1 r_2} = \frac{Fl}{\pi r_1 r_2 E}$$
$$= \frac{4Fl}{\pi d_1 d_2 E} \quad \textit{Ans.}$$

(**b**) For section 1,

$$\delta_1 = \frac{Fl}{AE} = \frac{4Fl}{\pi d_1^2 E} = \frac{4(1000)(2)}{\pi(0.5^2)(30)(10^6)} = 3.40(10^{-4}) \text{ in} \quad \textit{Ans.}$$

For the tapered section,
$$\delta = \frac{4}{\pi}\frac{Fl}{d_1 d_2 E} = \frac{4}{\pi}\frac{1000(2)}{(0.5)(0.75)(30)(10^6)} = 2.26(10^{-4}) \text{ in} \quad \textit{Ans.}$$
For section 2,

$$\delta_2 = \frac{Fl}{AE} = \frac{4Fl}{\pi d_1^2 E} = \frac{4(1000)(2)}{\pi(0.75^2)(30)(10^6)} = 1.51(10^{-4}) \text{ in} \qquad \textit{Ans.}$$

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**4-6** (**a**) Let the radii of the straight sections be $r_1 = d_1/2$ and $r_2 = d_2/2$. Let the angle of the taper be $\alpha$ where $\tan\alpha = (r_2 - r_1)/2$. Thus, the radius in the taper as a function of $x$ is $r = r_1 + x\tan\alpha$, and the polar second area moment is $J = (\pi/2)(r_1 + x\tan\alpha)^4$. The angular deflection of the tapered portion is

$$\theta = \int_0^l \frac{T}{GJ}dx = \frac{2T}{\pi G}\int_0^l \frac{dx}{(r_1 + x\tan\alpha)^4} = -\frac{1}{3}\frac{2T}{\pi G}\left.\frac{1}{(r_1 + x\tan\alpha)^3\tan\alpha}\right|_0^l$$

$$= \frac{2}{3\pi}\frac{T}{G}\left[\frac{1}{r_1^3\tan\alpha} - \frac{1}{\tan\alpha(r_1 + l\tan\alpha)^3}\right] = \frac{2}{3\pi}\frac{T}{G\tan\alpha}\left(\frac{1}{r_1^3} - \frac{1}{r_2^3}\right)$$

$$= \frac{2}{3\pi}\frac{T}{G\tan\alpha}\frac{r_2^3 - r_1^3}{r_1^3 r_2^3} = \frac{2}{3\pi}\frac{T}{G}\left(\frac{l}{r_2 - r_1}\right)\frac{r_2^3 - r_1^3}{r_1^3 r_2^3} = \frac{2}{3\pi}\frac{Tl}{G}\frac{(r_1^2 + r_1 r_2 + r_2^2)}{r_1^3 r_2^3}$$

$$= \frac{32}{3\pi}\frac{Tl}{G}\frac{(d_1^2 + d_1 d_2 + d_2^2)}{d_1^3 d_2^3} \qquad \textit{Ans.}$$

(**b**) The deflections, in degrees, are
For section 1,

$$\theta_1 = \frac{Tl}{GJ}\left(\frac{180}{\pi}\right) = \frac{32Tl}{\pi d_1^4 G}\left(\frac{180}{\pi}\right) = \frac{32(1500)(2)}{\pi(0.5^4)11.5(10^6)}\left(\frac{180}{\pi}\right) = 2.44 \text{ deg} \qquad \textit{Ans.}$$

For the tapered section,

$$\theta = \frac{32}{3\pi}\frac{Tl(d_1^2 + d_1 d_2 + d_2^2)}{G d_1^3 d_2^3}\left(\frac{180}{\pi}\right)$$

$$= \frac{32}{3\pi}\frac{(1500)(2)\left[0.5^2 + (0.5)(0.75) + 0.75^2\right]}{11.5(10^6)(0.5^3)(.75^3)}\left(\frac{180}{\pi}\right) = 1.14 \text{ deg} \qquad \textit{Ans.}$$

For section 2,

$$\theta_2 = \frac{Tl}{GJ}\left(\frac{180}{\pi}\right) = \frac{32Tl}{\pi d_2^4 G}\left(\frac{180}{\pi}\right) = \frac{32(1500)(2)}{\pi(0.75^4)11.5(10^6)}\left(\frac{180}{\pi}\right) = 0.481 \text{ deg} \qquad \textit{Ans.}$$

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**4-7** The area and the elastic modulus remain constant, however the force changes with respect to $x$. From Table A-5 the unit weight of steel is $\gamma = 0.282$ lbf/in$^3$, and the elastic modulus is $E = 30$ Mpsi. Starting from the top of the cable (i.e. $x = 0$, at the top).

$$F = \gamma(A)(l - x)$$

$$\delta_c = \int_o^l \frac{Fdx}{AE} = \frac{w}{E}\int_0^l (l-x)dx = \frac{\gamma}{E}\left(lx - \frac{1}{2}x^2\right)\Bigg|_0^l = \frac{\gamma l^2}{2E} = \frac{0.282\left[500(12)\right]^2}{2(30)10^6} = 0.169 \text{ in}$$

From the weight at the bottom of the cable,

$$\delta_W = \frac{Wl}{AE} = \frac{4Wl}{\pi d^2 E} = \frac{4(5000)\left[500(12)\right]}{\pi(0.5^2)30(10^6)} = 5.093 \text{ in}$$

$$\delta = \delta_c + \delta_W = 0.169 + 5.093 = 5.262 \text{ in} \qquad \textit{Ans.}$$

The percentage of total elongation due to the cable's own weight

$$\frac{0.169}{5.262}(100) = 3.21\% \qquad \textit{Ans.}$$

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**4-8** $\Sigma F_y = 0 = R_1 - F \;\Rightarrow\; R_1 = F$
$\Sigma M_A = 0 = M_1 - Fa \;\Rightarrow\; M_1 = Fa$
$V_{AB} = F,\ M_{AB} = F(x-a),\ V_{BC} = M_{BC} = 0$

Section *AB*:

$$\theta_{AB} = \frac{1}{EI}\int F(x-a)\,dx = \frac{F}{EI}\left(\frac{x^2}{2} - ax\right) + C_1 \qquad (1)$$

$\theta_{AB} = 0$ at $x = 0 \;\Rightarrow\; C_1 = 0$

$$y_{AB} = \frac{F}{EI}\int\left(\frac{x^2}{2} - ax\right)dx = \frac{F}{EI}\left(\frac{x^3}{6} - a\frac{x^2}{2}\right) + C_2 \qquad (2)$$

$y_{AB} = 0$ at $x = 0 \;\Rightarrow\; C_2 = 0$

$$\therefore \qquad y_{AB} = \frac{Fx^2}{6EI}(x - 3a) \qquad \textit{Ans.}$$

Section *BC*:

$$\theta_{BC} = \frac{1}{EI}\int (0)\,dx = 0 + C_3$$

From Eq. (1), at $x = a$ (with $C_1 = 0$), $\theta = \frac{F}{EI}\left(\frac{a^2}{2} - a(a)\right) = -\frac{Fa^2}{2EI} = C_3$. Thus,

$$\theta_{BC} = -\frac{Fa^2}{2EI}$$

$$y_{BC} = -\frac{Fa^2}{2EI}\int dx = -\frac{Fa^2}{2EI}x + C_4 \qquad (3)$$

From Eq. (2), at $x = a$ (with $C_2 = 0$), $y = \frac{F}{EI}\left(\frac{a^3}{6} - a\frac{a^2}{2}\right) = -\frac{Fa^3}{3EI}$. Thus, from Eq. (3)

$$-\frac{Fa^2}{2EI}a + C_4 = -\frac{Fa^3}{3EI} \quad \Rightarrow \quad C_4 = \frac{Fa^3}{6EI}$$ Substitute into Eq. (3)

$$\therefore \quad y_{BC} = -\frac{Fa^2}{2EI}x + \frac{Fa^3}{6EI} = \frac{Fa^2}{6EI}(a - 3x) \qquad \textit{Ans.}$$

The maximum deflection occurs at $x = l$,

$$y_{\max} = \frac{Fa^2}{6EI}(a - 3l) \qquad \textit{Ans.}$$

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**4-9** $\Sigma M_C = 0 = F(l/2) - R_1 l \;\Rightarrow\; R_1 = F/2$

$\Sigma F_y = 0 = F/2 + R_2 - F \;\Rightarrow\; R_2 = F/2$

Break at $0 \le x \le l/2$:

$V_{AB} = R_1 = F/2, \qquad M_{AB} = R_1 x = Fx/2$

Break at $l/2 \le x \le l$:

$V_{BC} = R_1 - F = -R_2 = -F/2, \qquad M_{BC} = R_1 x - F(x - l/2) = F(l - x)/2$

Section $AB$:

$$\theta_{AB} = \frac{1}{EI}\int \frac{Fx}{2}dx = \frac{F}{EI}\frac{x^2}{4} + C_1$$

From symmetry, $\theta_{AB} = 0$ at $x = l/2 \Rightarrow \frac{F\left(\frac{l}{2}\right)^2}{4EI} + C_1 = 0 \quad \Rightarrow \quad C_1 = -\frac{Fl^2}{16EI}$. Thus,

$$\theta_{AB} = \frac{F}{EI}\frac{x^2}{4} - \frac{Fl^2}{16EI} = \frac{F}{16EI}\left(4x^2 - l^2\right) \qquad (1)$$

$$y_{AB} = \frac{F}{16EI}\int\left(4x^2 - l^2\right)dx = \frac{F}{16EI}\left(\frac{4x^3}{3} - l^2 x\right) + C_2$$

$y_{AB} = 0$ at $x = 0 \quad \Rightarrow \quad C_2 = 0$, and,

$$\therefore \qquad y_{AB} = \frac{Fx}{48EI}\left(4x^2 - 3l^2\right) \qquad (2)$$

$y_{BC}$ is not given, because with symmetry, Eq. (2) can be used in this region. The maximum deflection occurs at $x = l/2$,

$$\therefore \qquad y_{\max} = \frac{F\left(\frac{l}{2}\right)}{48EI}\left[4\left(\frac{l}{2}\right)^2 - 3l^2\right] = -\frac{Fl^3}{48EI} \qquad \textit{Ans.}$$

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**4-10** From Table A-6, for each angle, $I_{1\text{-}1} = 207$ cm$^4$. Thus, $I = 2(207)(10^4) = 4.14(10^6)$ mm$^4$

From Table A-9, use beam 2 with $F = 2500$ N, $a = 2000$ mm, and $l = 3000$ mm; and beam 3 with $w = 1$ N/mm and $l = 3000$ mm.

$$\begin{aligned} y_{\max} &= \frac{Fa^2}{6EI}(a - 3l) - \frac{wl^4}{8EI} \\ &= \frac{2500(2000)^2}{6(207)10^3(4.14)10^6}\left[2000 - 3(3000)\right] - \frac{(1)(3000)^4}{8(207)(10^3)(4.14)(10^6)} \\ &= -25.4 \text{ mm} \qquad \textit{Ans.} \end{aligned}$$

$$\begin{aligned} M_O &= -Fa - (wl^2/2) \\ &= -2500(2000) - [1(3000^2)/2] = -9.5(10^6) \text{ N·mm} \end{aligned}$$

From Table A-6, from centroid to upper surface is $y = 29$ mm. From centroid to bottom surface is $y = 29.0 - 100 = -71$ mm. The maximum stress is compressive at the bottom of the beam at the wall. This stress is

$$\sigma_{\max} = -\frac{My}{I} = -\frac{-9.5(10^6)(-71)}{4.14(10^6)} = -163 \text{ MPa} \qquad \textit{Ans.}$$

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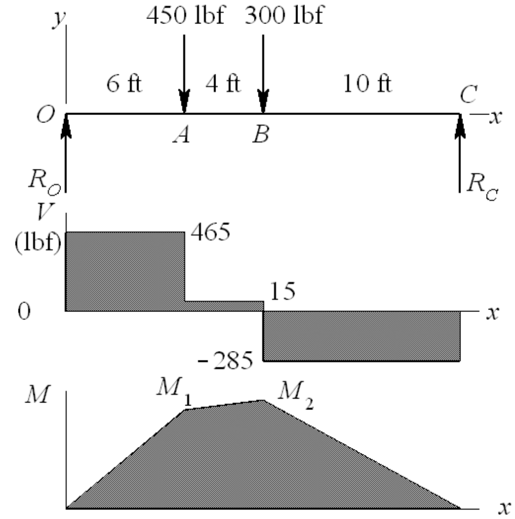
**4-11**

$$R_O = \frac{14}{20}(450) + \frac{10}{20}(300) = 465 \text{ lbf}$$

$$R_C = \frac{6}{20}(450) + \frac{10}{20}(300) = 285 \text{ lbf}$$




$M_1 = 465(6)12 = 33.48(10^3)$ lbf·in
$M_2 = 33.48(10^3) + 15(4)12$
$= 34.20(10^3)$ lbf·in

$$\sigma_{\max} = \frac{M_{\max}}{Z} \quad \Rightarrow \quad 15 = \frac{34.2}{Z} \qquad Z = 2.28 \text{ in}^3$$

For deflections, use beams 5 and 6 of Table A-9

$$y\Big|_{x=10\text{ft}} = \frac{F_1 a[l-(l/2)]}{6EIl}\left[\left(\frac{l}{2}\right)^2 + a^2 - 2l\frac{l}{2}\right] - \frac{F_2 l^3}{48EI}$$

$$-0.5 = \frac{450(72)(120)}{6(30)(10^6)I(240)}\left(120^2 + 72^2 - 240^2\right) - \frac{300(240^3)}{48(30)(10^6)I}$$

$I = 12.60 \text{ in}^4 \ \Rightarrow \ I/2 = 6.30 \text{ in}^4$

Select two 5 in-6.7 lbf/ft channels from Table A-7, $I = 2(7.49) = 14.98 \text{ in}^4$, $Z = 2(3.00) = 6.00 \text{ in}^3$

$$y_{\text{midspan}} = \frac{12.60}{14.98}\left(-\frac{1}{2}\right) = -0.421 \text{ in}$$

$$\sigma_{\max} = \frac{34.2}{6.00} = 5.70 \text{ kpsi}$$

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**4-12** $$I = \frac{\pi}{64}(1.5^4) = 0.2485 \text{ in}^4$$

From Table A-9 by superposition of beams 6 and 7, at $x = a = 15$ in, with $b = 24$ in and $l = 39$ in

$$y = \frac{Fba}{6EIl}[a^2 + b^2 - l^2] + \frac{wa}{24EI}(2la^2 - a^3 - l^3)$$

$$y_A = \frac{340(24)15}{6(30)10^6(0.2485)39}\left[15^2 + 24^2 - 39^2\right]$$

$$+ \frac{(150/12)(15)}{24(30)10^6(0.2485)}\left[2(39)(15^2) - 15^3 - 39^3\right] = -0.0978 \text{ in} \qquad \textit{Ans.}$$

At $x = l/2 = 19.5$ in

$$y = \frac{Fa[l-(l/2)]}{6EIl}\left[\left(\frac{l}{2}\right)^2 + a^2 - 2l\frac{l}{2}\right] + \frac{w(l/2)}{24EI}\left[2l\left(\frac{l}{2}\right)^2 - \left(\frac{l}{2}\right)^3 - l^3\right]$$

$$y = \frac{340(15)(19.5)}{6(30)(10^6)(0.2485)(39)}\left[19.5^2 + 15^2 - 39^2\right]$$

$$+ \frac{(150/12)(19.5)}{24(30)(10^6)(0.2485)}\left[2(39)(19.5^2) - 19.5^3 - 39^3\right] = -0.1027 \text{ in} \qquad Ans.$$

$$\% \text{ difference} = \frac{-0.1027 + 0.0978}{-0.0978}(100) = 5.01\% \qquad Ans.$$

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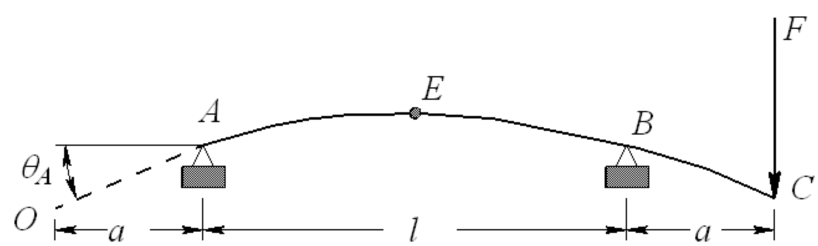
**4-13** $I = \frac{1}{12}(6)(32^3) = 16.384\left(10^3\right) \text{ mm}^4$

From Table A-9-10, beam 10

$$y_C = -\frac{Fa^2}{3EI}(l+a)$$

$$y_{AB} = \frac{Fax}{6EIl}\left(l^2 - x^2\right)$$

$$\frac{dy_{AB}}{dx} = \frac{Fa}{6EIl}(l^2 - 3x^2)$$




At $x = 0$, $\frac{dy_{AB}}{dx} = \theta_A$

$$\theta_A = \frac{Fal^2}{6EIl} = \frac{Fal}{6EI}$$

$$y_O = -\theta_A a = -\frac{Fa^2 l}{6EI}$$

With both loads,

$$y_O = -\frac{Fa^2 l}{6EI} - \frac{Fa^2}{3EI}(l+a)$$

$$= -\frac{Fa^2}{6EI}(3l + 2a) = -\frac{400(300^2)}{6(207)10^3(16.384)10^3}\left[3(500) + 2(300)\right] = -3.72 \text{ mm} \qquad Ans.$$

At midspan,

$$y_E = \frac{2Fa(l/2)}{6EIl}\left[l^2 - \left(\frac{l}{2}\right)^2\right] = \frac{3}{24}\frac{Fal^2}{EI} = \frac{3}{24}\frac{400(300)(500^2)}{207\left(10^3\right)16.384\left(10^3\right)} = 1.11 \text{ mm} \qquad Ans.$$

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**4-14** $I = \frac{\pi}{64}(2^4 - 1.5^4) = 0.5369 \text{ in}^4$

From Table A-5, $E$ = 10.4 Mpsi
From Table A-9, beams 1 and 2, by superposition

$$y_B = -\frac{F_B l^3}{3EI} + \frac{F_A a^2}{6EI}(a - 3l) = \frac{-200\left[4(12)\right]^3}{3(10.4)10^6(0.5369)} + \frac{300\left[2(12)\right]^2}{6(10.4)10^6(0.5369)}\left[2(12) - 3(4)(12)\right]$$

$$y_B = -1.94 \text{ in} \quad \textit{Ans.}$$

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**4-15** From Table A-7, $I$ = 2(1.85) = 3.70 in$^4$
From Table A-5, $E$ = 30.0 Mpsi
From Table A-9, beams 1 and 3, by superposition

$$y_A = -\frac{Fl^3}{3EI} - \frac{(w + w_c)l^4}{8EI} = -\frac{150(60^3)}{3(30)10^6(3.70)} - \frac{\left[5 + 2(5/12)\right](60^4)}{8(30)10^6(3.70)} = -0.182 \text{ in} \quad \textit{Ans.}$$

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**4-16** $I = \dfrac{\pi}{64}d^4$

From Table A-5, $E = 207(10^3)$ MPa
From Table A-9, beams 5 and 9, with $F_C = F_A = F$, by superposition

$$y_B = -\frac{F_B l^3}{48EI} + \frac{Fa}{24EI}(4a^2 - 3l^2) \quad \Rightarrow \quad I = \frac{1}{48Ey_B}\left[-F_B l^3 + 2Fa(4a^2 - 3l^2)\right]$$

$$I = \frac{1}{48(207)10^3\left(-2\right)}\left\{-550(1000^3) + 2\left(375\right)(250)\left[4(250^2) - 3(1000^2)\right]\right\}$$

$$= 53.624\left(10^3\right) \text{ mm}^4$$

$$d = \sqrt[4]{\frac{64}{\pi}I} = \sqrt[4]{\frac{64}{\pi}(53.624)10^3} = 32.3 \text{ mm} \quad \textit{Ans.}$$

---

**4-17** From Table A-9, beams 8 (region $BC$ for this beam with $a$ = 0) and 10 (with $a$ = $a$), by superposition

$$y_{AB} = \frac{M_A}{6EIl}\left(x^3 - 3lx^2 + 2l^2x\right) + \frac{Fax}{6EIl}\left(l^2 - x^2\right)$$

$$= \frac{1}{6EIl}\left[M_A\left(x^3 - 3lx^2 + 2l^2x\right) + Fax\left(l^2 - x^2\right)\right] \qquad \textit{Ans.}$$

$$y_{BC} = \left\{\frac{d}{dx}\left[\frac{M_A}{6EIl}\left(x^3 - 3lx^2 + 2l^2x\right)\right]\right\}_{x=l}(x - l) + \frac{F(x-l)}{6EI}[(x-l)^2 - a(3x - l)]$$

$$= -\frac{M_A l}{6EI}(x - l) + \frac{F(x-l)}{6EI}[(x-l)^2 - a(3x - l)]$$

$$= \frac{(x-l)}{6EI}\left\{-M_A l + F\left[(x-l)^2 - a(3x-l)\right]\right\} \quad Ans.$$

---

**4-18 Note to the instructor:** Beams with discontinuous loading are better solved using singularity functions. This eliminates matching the slopes and displacements at the discontinuity as is done in this solution.

$$\sum M_C = 0 = R_1 l - wa\left(l - a + \frac{a}{2}\right) \quad \Rightarrow \quad R_1 = \frac{wa}{2l}(2l-a) \quad Ans.$$

$$\sum F_y = 0 = \frac{wa}{2l}(2l-a) + R_2 - wa \quad \Rightarrow \quad R_2 = \frac{wa^2}{2l} \quad Ans.$$

$$V_{AB} = R_1 - wx = \frac{wa}{2l}(2l-a) - wx = \frac{w}{2l}\left[2l(a-x) - a^2\right] \quad Ans.$$

$$V_{BC} = -R_2 = -\frac{wa^2}{2l} \quad Ans.$$

$$M_{AB} = \int V_{AB}\,dx = \frac{w}{2l}\left[2l\left(ax - \frac{x^2}{2}\right) - a^2 x\right] + C_1$$

$$M_{AB} = 0 \text{ at } x = 0 \;\therefore\; C_1 = 0 \quad \Rightarrow \quad M_{AB} = \frac{wx}{2l}\left[2al - a^2 - lx\right] \quad Ans.$$

$$M_{BC} = \int V_{BC}\,dx = \int -\frac{wa^2}{2l}dx = -\frac{wa^2}{2l}x + C_2$$

$$M_{BC} = 0 \text{ at } x = l \;\therefore\; C_2 = \frac{wa^2}{2} \quad \Rightarrow \quad M_{BC} = \frac{wa^2}{2l}(l-x) \quad Ans.$$

$$\theta_{AB} = \int \frac{M_{AB}}{EI}dx = \frac{1}{EI}\int \frac{wx}{2l}\left(2al - a^2 - lx\right)dx = \frac{1}{EI}\left[\frac{w}{2l}\left(alx^2 - \frac{1}{2}a^2x^2 - \frac{1}{3}lx^3\right) + C_3\right]$$

$$y_{AB} = \int \theta_{AB}\,dx = \frac{1}{EI}\int\left[\frac{w}{2l}\left(alx^2 - \frac{1}{2}a^2x^2 - \frac{1}{3}lx^3\right) + C_3\right]dx$$

$$= \frac{1}{EI}\left[\frac{w}{2l}\left(\frac{1}{3}alx^3 - \frac{1}{6}a^2x^3 - \frac{1}{12}lx^4\right) + C_3 x + C_4\right]$$

$$y_{AB} = 0 \text{ at } x = 0 \;\therefore\; C_4 = 0$$

$$\theta_{BC} = \int \frac{M_{BC}}{EI}dx = \frac{1}{EI}\int \frac{wa^2}{2l}(l-x)\,dx = \frac{1}{EI}\left[\frac{wa^2}{2l}\left(lx - \frac{1}{2}x^2\right) + C_5\right]$$

$$\theta_{AB} = \theta_{BC} \text{ at } x = a \;\therefore$$

$$\frac{1}{EI}\left[\frac{w}{2l}\left(ala^2 - \frac{1}{2}a^4 - \frac{1}{3}la^3\right) + C_3\right] = \frac{1}{EI}\left[\frac{wa^2}{2l}\left(la - \frac{1}{2}a^2\right) + C_5\right] \Rightarrow C_3 = \frac{wa^3}{6} + C_5 \qquad (1)$$

$$y_{BC}=\int\theta_{BC}dx=\frac{1}{EI}\int\left[\frac{wa^2}{2l}\left(lx-\frac{1}{2}x^2\right)+C_5\right]dx=\frac{1}{EI}\left[\frac{wa^2}{2l}\left(\frac{1}{2}lx^2-\frac{1}{6}x^3\right)+C_5x+C_6\right]$$

$$y_{BC}=0 \text{ at } x=l \ \therefore\ C_6=-\frac{wa^2l^2}{6}-C_5l$$

$$y_{BC}=\frac{1}{EI}\left[\frac{wa^2}{2l}\left(\frac{1}{2}lx^2-\frac{1}{6}x^3-\frac{1}{3}l^3\right)+C_5(x-l)\right]$$

$y_{AB}=y_{BC}$ at $x=a$ $\therefore$

$$\frac{w}{2l}\left(\frac{1}{3}ala^3-\frac{1}{6}a^5-\frac{1}{12}la^4\right)+C_3a=\frac{wa^2}{2l}\left(\frac{1}{2}la^2-\frac{1}{6}a^3-\frac{1}{3}l^3\right)+C_5(a-l)$$

$$C_3a=\frac{wa^2}{24l}\left(3la^2-4l^3\right)+C_5(a-l) \qquad (2)$$

Substituting (1) into (2) yields $C_5=\frac{wa^2}{24l}\left(-a^2-4l^2\right)$. Substituting this back into (2) gives

$C_3=\frac{wa^2}{24l}\left(4al-a^2-4l^2\right)$. Thus,

$$y_{AB}=\frac{w}{24EIl}\left(4alx^3-2a^2x^3-lx^4+4a^3lx-a^4x-4a^2l^2x\right)$$

$$\Rightarrow \quad y_{AB}=\frac{wx}{24EIl}\left[2ax^2(2l-a)-lx^3-a^2\left(2l-a\right)^2\right] \qquad \textit{Ans.}$$

$$y_{BC}=\frac{w}{24EIl}\left(6a^2lx^2-2a^2x^3-a^4x-4a^2l^2x+a^4l\right) \qquad \textit{Ans.}$$

This result is sufficient for $y_{BC}$. However, this can be shown to be equivalent to

$$y_{BC}=\frac{w}{24EIl}\left(4alx^3-2a^2x^3-lx^4-4a^2l^2x+4a^3lx-a^4x\right)+\frac{w}{24EI}(x-a)^4$$

$$y_{BC}=y_{AB}+\frac{w}{24EI}(x-a)^4 \qquad \textit{Ans.}$$

by expanding this or by solving the problem using singularity functions.

---

**4-19** The beam can be broken up into a uniform load $w$ downward from points $A$ to $C$ and a uniform load upward from points $A$ to $B$.

$$y_{AB}=\frac{wx}{24EIl}\left[2bx^2(2l-b)-lx^3-b^2\left(2l-b\right)^2\right]-\frac{wx}{24EIl}\left[2ax^2(2l-a)-lx^3-a^2\left(2l-a\right)^2\right]$$

$$=\frac{wx}{24EIl}\left[2bx^2(2l-b)-b^2\left(2l-b\right)^2-2ax^2(2l-a)+a^2\left(2l-a\right)^2\right] \qquad \textit{Ans.}$$

$$y_{BC}=\frac{w}{24EIl}\Big[2bx^3(2l-b)-lx^4-b^2x\left(2l-b\right)^2$$

$$-\left(4alx^3-2a^2x^3-lx^4-4a^2l^2x+4a^3lx-a^4x\right)-l(x-a)^4\Big] \qquad \textit{Ans.}$$

$$y_{CD} = \frac{w}{24EIl}\left[4blx^3 - 2b^2x^3 - lx^4 - 4b^2l^2x + 4b^3lx - b^4x + l(x-b)^4\right]$$

$$-\frac{w}{24EIl}\left[4alx^3 - 2a^2x^3 - lx^4 - 4a^2l^2x + 4a^3lx - a^4x + l(x-a)^4\right]$$

$$= \frac{w}{24EI}\left[(x-b)^4 - (x-a)^4\right] + y_{AB} \qquad Ans.$$

---

**4-20** **Note to the instructor:** See the note in the solution for Problem 4-18.

$$\sum F_y = 0 = R_B - \frac{wa^2}{2l} - wa \quad \Rightarrow \quad R_B = \frac{wa}{2l}(2l + a) \qquad Ans.$$

For region $BC$, isolate right-hand element of length $(l + a - x)$

$$V_{AB} = -R_A = -\frac{wa^2}{2l}, \qquad V_{BC} = w(l + a - x) \qquad Ans.$$

$$M_{AB} = -R_A x = -\frac{wa^2}{2l}x, \qquad M_{BC} = -\frac{w}{2}(l + a - x)^2 \qquad Ans.$$

$$EI\theta_{AB} = \int M_{AB}\,dx = -\frac{wa^2}{4l}x^2 + C_1$$

$$EIy_{AB} = -\frac{wa^2}{12l}x^3 + C_1x + C_2$$

$y_{AB} = 0$ at $x = 0 \quad \Rightarrow \quad C_2 = 0 \quad \therefore \quad EIy_{AB} = -\frac{wa^2}{12l}x^3 + C_1x$

$y_{AB} = 0$ at $x = l \quad \Rightarrow \quad C_1 = \frac{wa^2l}{12} \quad \therefore$

$$EIy_{AB} = -\frac{wa^2}{12l}x^3 + \frac{wa^2l}{12}x = \frac{wa^2x}{12l}\left(l^2 - x^2\right) \quad \Rightarrow \quad y_{AB} = \frac{wa^2x}{12EIl}\left(l^2 - x^2\right) \quad Ans.$$

$$EI\theta_{BC} = \int M_{BC}\,dx = -\frac{w}{6}(l + a - x)^3 + C_3$$

$$EIy_{BC} = -\frac{w}{24}(l + a - x)^4 + C_3x + C_4$$

$y_{BC} = 0$ at $x = l \quad \Rightarrow \quad -\frac{wa^4}{24} + C_3l + C_4 = 0 \quad \Rightarrow \quad C_4 = \frac{wa^4}{24} - C_3l \qquad (1)$

$\theta_{AB} = \theta_{BC}$ at $x = l \quad \Rightarrow \quad -\frac{wa^2l}{4} + \frac{wa^2l}{12} = \frac{wa^3}{6} + C_3 \quad \Rightarrow \quad C_3 = -\frac{wa^2}{6}(l + a)$

Substitute $C_3$ into Eq. (1) gives $\quad C_4 = \frac{wa^2}{24}\left[a^2 + 4l(l + a)\right]$. Substitute back into $y_{BC}$

$$y_{BC} = \frac{1}{EI}\left[-\frac{w}{24}(l + a - x)^4 - \frac{wa^2}{6}x(l + a) + \frac{wa^4}{24} + \frac{wa^2l}{6}(l + a)\right]$$

$$= -\frac{w}{24EI}\left[(l + a - x)^4 - 4a^2(l - x)(l + a) - a^4\right] \qquad Ans.$$

**4-21** Table A-9, beam 7,

$$R_1 = R_2 = \frac{wl}{2} = \frac{100(10)}{2} = 500 \text{ lbf } \uparrow$$

$$y_{AB} = \frac{wx}{24EI}\left(2lx^2 - x^3 - l^3\right) = \frac{100x}{24(30)10^6(0.05)}\left[2(10)x^2 - x^3 - 10^3\right]$$

$$= 2.7778\left(10^{-6}\right)x\left(20x^2 - x^3 - 1000\right)$$

Slope: $\theta_{AB} = \frac{d\,y_{AB}}{d\,x} = \frac{w}{24EI}\left(6lx^2 - 4x^3 - l^3\right)$

At $x = l$, $\theta_{AB}\big|_{x=l} = \frac{w}{24EI}\left(6l\,l^2 - 4l^3 - l^3\right) = \frac{wl^3}{24EI}$

$$y_{BC} = \theta_{AB}\big|_{x=l}(x-l) = \frac{wl^3}{24EI}(x-l) = \frac{100\left(10^3\right)}{24(30)10^6(0.05)}(x-10) = 2.7778\left(10^{-3}\right)(x-10)$$

From Prob. 4-20,

$$R_A = \frac{wa^2}{2l} = \frac{100\left(4^2\right)}{2(10)} = 80 \text{ lbf } \downarrow \qquad R_B = \frac{wa}{2l}(2l+a) = \frac{100(4)}{2(10)}\left[2(10)+4\right] = 480 \text{ lbf } \uparrow$$

$$y_{AB} = \frac{wa^2x}{12EIl}\left(l^2 - x^2\right) = \frac{100\left(4^2\right)x}{12(30)10^6(0.05)}\left(10^2 - x^2\right) = 8.8889\left(10^{-6}\right)x\left(100 - x^2\right)$$

$$y_{BC} = -\frac{w}{24EI}\left[(l+a-x)^4 - 4a^2(l-x)(l+a) - a^4\right]$$

$$= -\frac{100}{24(30)10^6(0.05)}\left[(10+4-x)^4 - 4\left(4^2\right)(10-x)(10+4) - 4^4\right]$$

$$= -2.7778\left(10^{-6}\right)\left[(14-x)^4 + 896x - 9216\right]$$

Superposition,

$R_A = 500 - 80 = 420 \text{ lbf } \uparrow \qquad R_B = 500 + 480 = 980 \text{ lbf } \uparrow$ *Ans.*

$y_{AB} = 2.7778\left(10^{-6}\right)x\left(20x^2 - x^3 - 1000\right) + 8.8889\left(10^{-6}\right)x\left(100 - x^2\right)$ *Ans.*

$y_{BC} = 2.7778\left(10^{-3}\right)(x-10) - 2.7778\left(10^{-6}\right)\left[(14-x)^4 + 896x - 9216\right]$ *Ans.*

The deflection equations can be simplified further. However, they are sufficient for plotting.

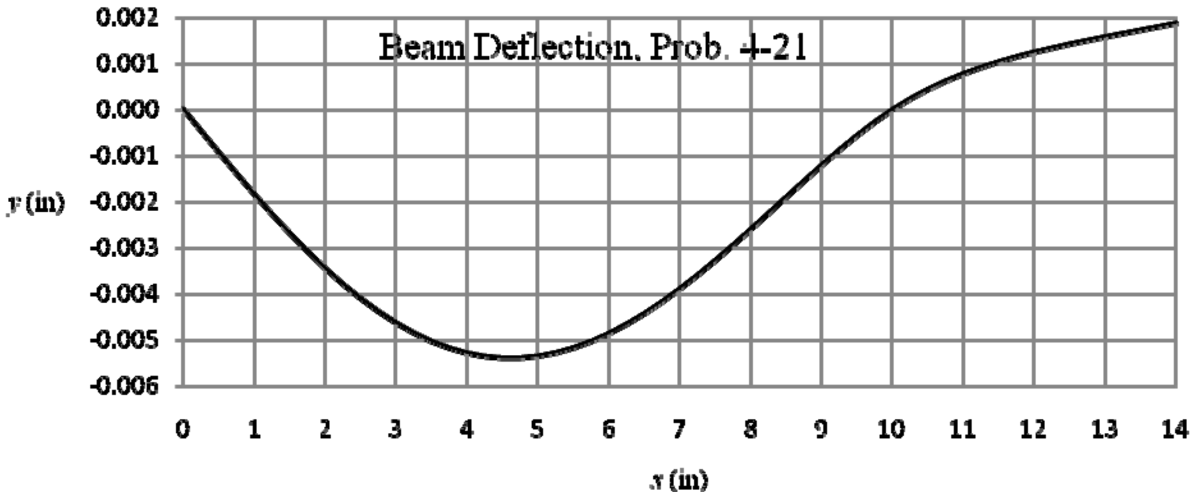
Using a spreadsheet,

| *x* | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 |
|---|---|---|---|---|---|---|---|---|
| *y* | 0.000000 | -0.000939 | -0.001845 | -0.002690 | -0.003449 | -0.004102 | -0.004632 | -0.005027 |

| *x* | 4 | 4.5 | 5 | 5.5 | 6 | 6.5 | 7 | 7.5 |
|---|---|---|---|---|---|---|---|---|
| *y* | -0.005280 | -0.005387 | -0.005347 | -0.005167 | -0.004853 | -0.004421 | -0.003885 | -0.003268 |

| $x$ | 8 | 8.5 | 9 | 9.5 | 10 | 10.5 | 11 | 11.5 |
|---|---|---|---|---|---|---|---|---|
| $y$ | -0.002596 | -0.001897 | -0.001205 | -0.000559 | 0.000000 | 0.000439 | 0.000775 | 0.001036 |

| $x$ | 12 | 12.5 | 13 | 13.5 | 14 |
|---|---|---|---|---|---|
| $y$ | 0.001244 | 0.001419 | 0.001575 | 0.001722 | 0.001867 |

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**4-22** (**a**) Useful relations

$$k = \frac{F}{y} = \frac{48EI}{l^3}$$

$$I = \frac{kl^3}{48E} = \frac{1800\left(36^3\right)}{48(30)10^6} = 0.05832 \text{ in}^4$$

From $I = bh^3/12$, and $b = 10\,h$, then $I = 5\,h^4/6$, or,

$$h = \sqrt[4]{\frac{6I}{5}} = \sqrt[4]{\frac{6(0.05832)}{5}} = 0.514 \text{ in}$$

$h$ is close to 1/2 in and 9/16 in, while $b$ is close to 5.14 in. Changing the height drastically changes the spring rate, so changing the base will make finding a close solution easier. Trial and error was applied to find the combination of values from Table A-17 that yielded the closet desired spring rate.

| $h$ (in) | $b$ (in) | $b/h$ | $k$ (lbf/in) |
|---|---|---|---|
| 1/2 | 5 | 10 | 1608 |
| 1/2 | 5½ | 11 | 1768 |
| 1/2 | 5¾ | 11.5 | 1849 |
| 9/16 | 5 | 8.89 | 2289 |
| 9/16 | 4 | 7.11 | 1831 |

$h$ = ½ in, $b$ = 5 ½ in should be selected because it results in a close spring rate and $b/h$ is still reasonably close to 10.

**(b)** $I = 5.5(0.5)^3/12 = 0.05729\ \text{in}^4$

$$\sigma = \frac{Mc}{I} = \frac{(Fl/4)c}{I} \quad \Rightarrow \quad F = \frac{4\sigma I}{lc} = \frac{4(60)10^3(0.05729)}{(36)(0.25)} = 1528\ \text{lbf}$$

$$y = \frac{Fl^3}{48EI} = \frac{(1528)(36^3)}{48(30)10^6(0.05729)} = 0.864\ \text{in} \qquad Ans.$$

---

**4-23** From the solutions to Prob. 3-68, $T_1 = 60$ lbf and $T_2 = 400$ lbf

$$I = \frac{\pi d^4}{64} = \frac{\pi(1.25)^4}{64} = 0.1198\ \text{in}^4$$

From Table A-9, beam 6,

$$z_A = \left[\frac{F_1 b_1 x}{6EIl}(x^2 + b_1^2 - l^2) + \frac{F_2 b_2 x}{6EIl}(x^2 + b_2^2 - l^2)\right]_{x=10\text{in}}$$

$$= \frac{(-575)(30)(10)}{6(30)10^6(0.1198)(40)}\left(10^2 + 30^2 - 40^2\right)$$

$$+ \frac{460(12)(10)}{6(30)10^6(0.1198)(40)}\left(10^2 + 12^2 - 40^2\right) = 0.0332\ \text{in} \qquad Ans.$$

$$(\theta_A)_y = -\left(\frac{dz}{dx}\right)_{x=10\text{in}} = -\left\{\frac{d}{dx}\left[\frac{F_1 b_1 x}{6EIl}(x^2 + b_1^2 - l^2) + \frac{F_2 b_2 x}{6EIl}(x^2 + b_2^2 - l^2)\right]\right\}_{x=10\text{in}}$$

$$= -\left\{\frac{F_1 b_1}{6EIl}(3x^2 + b_1^2 - l^2) + \frac{F_2 b_2}{6EIl}(3x^2 + b_2^2 - l^2)\right\}_{x=10\text{in}}$$

$$= -\frac{(575)(30)}{6(30)10^6(0.1198)(40)}\left[3\left(10^2\right) + 30^2 - 40^2\right]$$

$$- \frac{-460(12)}{6(30)10^6(0.1198)(40)}\left[3\left(10^2\right) + 12^2 - 40^2\right]$$

$$= 6.02(10^{-4})\ \text{rad} \qquad Ans.$$

---

**4-24** From the solutions to Prob. 3-69, $T_1 = 2880$ N and $T_2 = 432$ N

$$I = \frac{\pi d^4}{64} = \frac{\pi(30)^4}{64} = 39.76\left(10^3\right)\ \text{mm}^4$$

The load in between the supports supplies an angle to the overhanging end of the beam. That angle is found by taking the derivative of the deflection from that load. From Table A-9, beams 6 (subscript 1) and 10 (subscript 2),

$$y_A = \left[\theta_{BC}\big|_C (a_2)\right]_{\text{beam}\,6} + \left(y_A\right)_{\text{beam}\,10} \qquad (1)$$

$$\theta_{BC}\big|_C = \left\{\frac{d}{dx}\left[\frac{F_1 a_1 (l-x)}{6EIl}\left(x^2 + a_1^{\ 2} - 2lx\right)\right]\right\}_{x=l} = \left[\frac{F_1 a_1}{6EIl}\left(6lx - 3x^2 - a_1^{\ 2} - 2l^2\right)\right]_{x=l}$$

$$= \frac{F_1 a_1}{6EIl}\left(l^2 - a_1^{\ 2}\right)$$

Equation (1) is thus

$$y_A = \frac{F_1 a_1}{6EIl}\left(l^2 - a_1^{\ 2}\right)a_2 - \frac{F_2 a_2^{\ 2}}{3EI}(l + a_2)$$

$$= \frac{-3312(230)}{6(207)10^3(39.76)10^3(510)}\left(510^2 - 230^2\right)(300) - \frac{2070(300^2)}{3(207)10^3(39.76)10^3}(510 + 300)$$

$$= -7.99 \text{ mm} \qquad \textit{Ans.}$$

The slope at *A*, relative to the *z* axis is

$$\left(\theta_A\right)_z = \frac{F_1 a_1}{6EIl}(l^2 - a_1^{\ 2}) + \left\{\frac{d}{dx}\left[\frac{F_2 (x-l)}{6EI}\left[(x-l)^2 - a_2(3x - l)\right]\right]\right\}_{x=l+a_2}$$

$$= \frac{F_1 a_1}{6EIl}\left(l^2 - a_1^{\ 2}\right) + \frac{F_2}{6EI}\left[3(x-l)^2 - 3a_2(x-l) - a_2(3x-l)\right]_{x=l+a_2}$$

$$= \frac{F_1 a_1}{6EIl}(l^2 - a_1^{\ 2}) - \frac{F_2}{6EI}\left(3a_2^{\ 2} + 2la_2\right)$$

$$= \frac{-3312(230)}{6(207)10^3(39.76)10^3(510)}\left(510^2 - 230^2\right)$$

$$- \frac{2070}{6(207)10^3(39.76)10^3}\left[3(300^2) + 2(510)(300)\right]$$

$$= -0.0304 \text{ rad} \qquad \textit{Ans.}$$

---

**4-25** From the solutions to Prob. 3-70, $T_1 = 392.16$ lbf and $T_2 = 58.82$ lbf

$$I = \frac{\pi d^4}{64} = \frac{\pi (1)^4}{64} = 0.049\ 09 \text{ in}^4$$

From Table A-9, beam 6,

$$y_A = \left[\frac{F_1 b_1 x}{6EIl}\left(x^2 + b_1^{\,2} - l^2\right)\right]_{x=8\text{in}} = \frac{(-350)(14)(8)}{6(30)10^6(0.049\ 09)(22)}\left(8^2 + 14^2 - 22^2\right) = 0.0452 \text{ in} \quad \textit{Ans.}$$

$$z_A = \left[\frac{F_2 b_2 x}{6EIl}(x^2 + b_2^{\,2} - l^2)\right]_{x=8\text{in}} = \frac{(-450.98)(6)(8)}{6(30)10^6(0.049\ 09)(22)}\left(8^2 + 6^2 - 22^2\right) = 0.0428 \text{ in} \quad \textit{Ans.}$$

The displacement magnitude is $\delta = \sqrt{y_A^2 + z_A^2} = \sqrt{0.0452^2 + 0.0428^2} = 0.0622$ in  *Ans.*

$$(\theta_A)_z = \left(\frac{d\,y}{d\,x}\right)_{x=a_1} = \left\{\frac{d}{dx}\left[\frac{F_1 b_1 x}{6EIl}\left(x^2 + b_1^2 - l^2\right)\right]\right\}_{x=a_1} = \frac{F_1 b_1}{6EIl}(3a_1^2 + b_1^2 - l^2)$$

$$= \frac{(-350)(14)}{6(30)10^6(0.04909)(22)}\left[3\left(8^2\right) + 14^2 - 22^2\right] = 0.00242 \text{ rad} \quad \textit{Ans.}$$

$$(\theta_A)_y = \left(-\frac{d\,z}{d\,x}\right)_{x=a_1} = -\left\{\frac{d}{dx}\left[\frac{-F_2 b_2 x}{6EIl}(x^2 + b_2^2 - l^2)\right]\right\}_{x=a_1} = \frac{F_2 b_2}{6EIl}\left(3a_1^2 + b_2^2 - l^2\right)$$

$$= \frac{(450.98)(6)}{6(30)10^6(0.04909)(22)}\left[3\left(8^2\right) + 6^2 - 22^2\right] = -0.00356 \text{ rad} \quad \textit{Ans.}$$

The slope magnitude is $\Theta_A = \sqrt{0.00242^2 + (-0.00356)^2} = 0.00430$ rad  *Ans.*

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**4-26** From the solutions to Prob. 3-71, $T_1 = 250$ N and $T_2 = 37.5$ N

$$I = \frac{\pi d^4}{64} = \frac{\pi (20)^4}{64} = 7\ 854 \text{ mm}^4$$

$$y_A = \left[\frac{F_{1y} b_1 x}{6EIl}(x^2 + b_1^{\,2} - l^2)\right]_{x=300\text{mm}} = \frac{\left(-345 \sin 45^\circ\right)(550)(300)}{6(207)10^3(7\ 854)(850)}\left(300^2 + 550^2 - 850^2\right)$$

$$= 1.60 \text{ mm} \quad \textit{Ans.}$$

$$z_A = \left[\frac{F_{1z} b_1 x}{6EIl}(x^2 + b_1^{\,2} - l^2) + \frac{F_2 b_2 x}{6EIl}(x^2 + b_2^{\,2} - l^2)\right]_{x=300\text{mm}}$$

$$= \frac{\left(345 \cos 45^\circ\right)(550)(300)}{6(207)10^3(7\ 854)(850)}\left(300^2 + 550^2 - 850^2\right)$$

$$+ \frac{-287.5(150)(300)}{6(207)10^3(7\ 854)(850)}\left(300^2 + 150^2 - 850^2\right) = -0.650 \text{ mm} \quad \textit{Ans.}$$

The displacement magnitude is $\delta = \sqrt{y_A^2 + z_A^2} = \sqrt{1.60^2 + (-0.650)^2} = 1.73$ mm  *Ans.*

$$\left(\theta_A\right)_z = \left(\frac{d\,y}{d\,x}\right)_{x=a_1} = \left\{\frac{d}{dx}\left[\frac{F_{1y}b_1x}{6EIl}\left(x^2+b_1^2-l^2\right)\right]\right\}_{x=a_1} = \frac{F_{1y}b_1}{6EIl}(3a_1^2+b_1^2-l^2)$$

$$= \frac{-\left(345\sin 45^{\circ}\right)(550)}{6(207)10^3(7\ 854)(850)}\left[3\left(300^2\right)+550^2-850^2\right] = 0.00243\text{ rad} \qquad \textit{Ans.}$$

$$\left(\theta_A\right)_y = -\left(\frac{d\,z}{d\,x}\right)_{x=a_1} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}b_1x}{6EIl}\left(x^2+b_1{}^2-l^2\right)+\frac{F_2b_2x}{6EIl}\left(x^2+b_2{}^2-l^2\right)\right]\right\}_{x=a_1}$$

$$= -\frac{F_{1z}b_1}{6EIl}\left(3a_1^2+b_1^2-l^2\right)-\frac{F_2b_2}{6EIl}\left(3a_1^2+b_2^2-l^2\right)$$

$$= -\frac{\left(345\cos 45^{\circ}\right)(550)}{6(207)10^3(7\ 854)(850)}\left[3\left(300^2\right)+550^2-850^2\right]$$

$$-\frac{-287.5(150)}{6(207)10^3(7\ 854)(850)}\left[3\left(300^2\right)+150^2-850^2\right] = 1.91\cdot 10^{-4}\,\text{rad} \qquad \textit{Ans.}$$

The slope magnitude is $\Theta_A = \sqrt{0.00243^2+0.000191^2} = 0.00244\text{ rad}$ *Ans.*

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**4-27** From the solutions to Prob. 3-72, $F_B = 750$ lbf

$$I = \frac{\pi d^4}{64} = \frac{\pi(1.25)^4}{64} = 0.1198\text{ in}^4$$

From Table A-9, beams 6 (subscript 1) and 10 (subscript 2)

$$y_A = \left[\frac{F_{1y}b_1x}{6EIl}\left(x^2+b_1^2-l^2\right)+\frac{F_{2y}a_2x}{6EIl}\left(l^2-x^2\right)\right]_{x=16\text{in}}$$

$$= \frac{\left(-300\cos 20^{\circ}\right)(14)(16)}{6(30)10^6(0.119\ 8)(30)}\left(16^2+14^2-30^2\right)+\frac{\left(750\sin 20^{\circ}\right)(9)(16)}{6(30)10^6(0.119\ 8)(30)}\left(30^2-16^2\right)$$

$$= 0.0805\text{ in} \qquad \textit{Ans.}$$

$$z_A = \left[\frac{F_{1z}b_1x}{6EIl}\left(x^2+b_1{}^2-l^2\right)+\frac{F_{2z}a_2x}{6EIl}\left(l^2-x^2\right)\right]_{x=16\text{in}}$$

$$= \frac{\left(300\sin 20^{\circ}\right)(14)(16)}{6(30)10^6(0.119\ 8)(30)}\left(16^2+14^2-30^2\right)+\frac{\left(-750\cos 20^{\circ}\right)(9)(16)}{6(30)10^6(0.119\ 8)(30)}\left(30^2-16^2\right)$$

$$= -0.1169\text{ in} \qquad \textit{Ans.}$$

The displacement magnitude is $\delta = \sqrt{y_A^2+z_A^2} = \sqrt{0.0805^2+\left(-0.1169\right)^2} = 0.142\text{ in}$ *Ans.*

$$(\theta_A)_z = \left(\frac{dy}{dx}\right)_{x=a_1} = \left\{\frac{d}{dx}\left[\frac{F_{1y}b_1x}{6EIl}\left(x^2+b_1{}^2-l^2\right)+\frac{F_{2y}a_2x}{6EIl}\left(l^2-x^2\right)\right]\right\}_{x=a_1}$$

$$= \frac{F_{1y}b_1}{6EIl}\left(3a_1^2+b_1^2-l^2\right)+\frac{F_{2y}a_2}{6EIl}\left(l^2-3a_1^2\right)$$

$$= \frac{\left(-300\cos 20^\circ\right)(14)}{6(30)10^6(0.119\,8)(30)}\left[3\left(16^2\right)+14^2-30^2\right]$$

$$+\frac{\left(750\sin 20^\circ\right)(9)}{6(30)10^6(0.119\,8)(30)}\left[30^2-3\left(16^2\right)\right] = 8.06\left(10^{-5}\right)\text{rad} \qquad \textit{Ans.}$$

$$(\theta_A)_y = -\left(\frac{dz}{dx}\right)_{x=a_1} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}b_1x}{6EIl}\left(x^2+b_1^2-l^2\right)+\frac{F_{2z}a_2x}{6EIl}\left(l^2-x^2\right)\right]\right\}_{x=a_1}$$

$$= -\frac{F_{1z}b_1}{6EIl}\left(3a_1^2+b_1^2-l^2\right)-\frac{F_{2z}a_2}{6EIl}\left(l^2-3a_1^2\right)$$

$$= -\frac{\left(300\sin 20^\circ\right)(14)}{6(30)10^6(0.119\,8)(30)}\left[3\left(16^2\right)+14^2-30^2\right]-\frac{\left(-750\cos 20^\circ\right)(9)}{6(30)10^6(0.119\,8)(30)}\left[30^2-3\left(16^2\right)\right]$$

$$= 0.00115 \text{ rad} \qquad \textit{Ans.}$$

The slope magnitude is $\Theta_A = \sqrt{\left[8.06\left(10^{-5}\right)\right]^2+0.00115^2} = 0.00115 \text{ rad}$ *Ans.*

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**4-28** From the solutions to Prob. 3-73, $F_B = 22.8\ (10^3)$ N

$$I = \frac{\pi d^4}{64} = \frac{\pi\left(50^4\right)}{64} = 306.8\left(10^3\right)\ \text{mm}^4$$

From Table A-9, beam 6,

$$y_A = \left[\frac{F_{1y}b_1x}{6EIl}(x^2+b_1{}^2-l^2)+\frac{F_{2y}b_2x}{6EIl}(x^2+b_2{}^2-l^2)\right]_{x=400\text{mm}}$$

$$= \frac{\left[11\left(10^3\right)\sin 20^\circ\right](650)(400)}{6(207)10^3(306.8)10^3(1050)}\left(400^2+650^2-1050^2\right)$$

$$+\frac{\left[22.8\left(10^3\right)\sin 25^\circ\right](300)(400)}{6(207)10^3(306.8)10^3(1050)}\left(400^2+300^2-1050^2\right)$$

$$= -3.735 \text{ mm} \qquad \textit{Ans.}$$

$$z_A = \left[ \frac{F_{1z} b_1 x}{6EIl}(x^2 + b_1^2 - l^2) + \frac{F_{2z} b_2 x}{6EIl}(x^2 + b_2^2 - l^2) \right]_{x=400\text{mm}}$$

$$= \frac{\left[11\left(10^3\right)\cos 20^\circ\right](650)(400)}{6(207)10^3(306.8)10^3(1050)}\left(400^2 + 650^2 - 1050^2\right)$$

$$+ \frac{\left[-22.8\left(10^3\right)\cos 25^\circ\right](300)(400)}{6(207)10^3(306.8)10^3(1050)}\left(400^2 + 300^2 - 1050^2\right) = 1.791 \text{ mm} \qquad \textit{Ans.}$$

The displacement magnitude is $\delta = \sqrt{y_A^2 + z_A^2} = \sqrt{\left(-3.735\right)^2 + 1.791^2} = 4.14 \text{ mm}$ *Ans.*

$$\left(\theta_A\right)_z = \left(\frac{d\,y}{d\,x}\right)_{x=a_1} = \left\{ \frac{d}{dx}\left[ \frac{F_{1z} b_1 x}{6EIl}\left(x^2 + b_1^2 - l^2\right) + \frac{F_{2z} b_2 x}{6EIl}\left(x^2 + b_2^2 - l^2\right) \right] \right\}_{x=a_1}$$

$$= \frac{F_{1y} b_1}{6EIl}\left(3a_1^2 + b_1^2 - l^2\right) + \frac{F_{2y} b_2}{6EIl}\left(3a_1^2 + b_2^2 - l^2\right)$$

$$= \frac{\left[11\left(10^3\right)\sin 20^\circ\right](650)}{6(207)10^3(306.8)10^3(1050)}\left[3\left(400^2\right) + 650^2 - 1050^2\right]$$

$$+ \frac{\left[22.8\left(10^3\right)\sin 25^\circ\right](300)}{6(207)10^3(306.8)10^3(1050)}\left[3\left(400^2\right) + 300^2 - 1050^2\right]$$

$$= -0.00507 \text{ rad} \qquad \textit{Ans.}$$

$$\left(\theta_A\right)_y = -\left(\frac{d\,z}{d\,x}\right)_{x=a_1} = -\left\{ \frac{d}{dx}\left[ \frac{F_{1z} b_1 x}{6EIl}\left(x^2 + b_1^2 - l^2\right) + \frac{F_{2z} b_2 x}{6EIl}\left(x^2 + b_2^2 - l^2\right) \right] \right\}_{x=a_1}$$

$$= -\frac{F_{1z} b_1}{6EIl}\left(3a_1^2 + b_1^2 - l^2\right) - \frac{F_{2z} b_2}{6EIl}\left(3a_1^2 + b_2^2 - l^2\right)$$

$$= -\frac{\left[11\left(10^3\right)\cos 20^\circ\right](650)}{6(207)10^3(306.8)10^3(1050)}\left[3\left(400^2\right) + 650^2 - 1050^2\right]$$

$$- \frac{\left[-22.8\left(10^3\right)\cos 25^\circ\right](300)}{6(207)10^3(306.8)10^3(1050)}\left[3\left(400^2\right) + 300^2 - 1050^2\right]$$

$$= -0.00489 \text{ rad} \qquad \textit{Ans.}$$

The slope magnitude is $\Theta_A = \sqrt{\left(-0.00507\right)^2 + \left(-0.00489\right)^2} = 0.00704 \text{ rad}$ *Ans.*

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**4-29** From the solutions to Prob. 3-68, $T_1$ = 60 lbf and $T_2$ = 400 lbf , and Prob. 4-23, $I$ = 0.119 8 in$^4$. From Table A-9, beam 6,

$$\left(\theta_O\right)_y = -\left(\frac{d\,z}{d\,x}\right)_{x=0} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}b_1x}{6EIl}\left(x^2+b_1^2-l^2\right)+\frac{F_{2z}b_2x}{6EIl}\left(x^2+b_2^2-l^2\right)\right]\right\}_{x=0}$$

$$= -\frac{F_{1z}b_1}{6EIl}\left(b_1^{\,2}-l^2\right)-\frac{F_{2z}b_2}{6EIl}\left(b_2^{\,2}-l^2\right) = -\frac{-575(30)}{6(30)10^6(0.119\ 8)(40)}\left(30^2-40^2\right)$$

$$-\frac{460(12)}{6(30)10^6(0.119\ 8)(40)}\left(12^2-40^2\right) = -0.00468\text{ rad} \qquad \textit{Ans.}$$

$$\left(\theta_C\right)_y = -\left(\frac{d\,z}{d\,x}\right)_{x=l} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}a_1\left(l-x\right)}{6EIl}\left(x^2+a_1^2-2lx\right)+\frac{F_{2z}a_2\left(l-x\right)}{6EIl}\left(x^2+a_2^2-2lx\right)\right]\right\}_{x=l}$$

$$= -\left[\frac{F_{1z}a_1}{6EIl}\left(6lx-2l^2-3x^2-a_1^2\right)+\frac{F_{2z}a_2}{6EIl}\left(6lx-2l^2-3x^2-a_2^2\right)\right]_{x=l}$$

$$= -\frac{F_{1z}a_1}{6EIl}\left(l^2-a_1^2\right)-\frac{F_{2z}a_2}{6EIl}\left(l^2-a_2^2\right)$$

$$= -\frac{-575(10)\left(40^2-10^2\right)}{6(30)10^6(0.119\ 8)(40)}-\frac{460(28)\left(40^2-28^2\right)}{6(30)10^6(0.119\ 8)(40)} = -0.00219\text{ rad} \qquad \textit{Ans.}$$

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**4-30** From the solutions to Prob. 3-69, $T_1$ = 2 880 N and $T_2$ = 432 N, and Prob. 4-24, $I$ = 39.76 ($10^3$) mm$^4$. From Table A-9, beams 6 and 10

$$\left(\theta_O\right)_z = \left(\frac{d\,y}{d\,x}\right)_{x=0} = \left\{\frac{d}{dx}\left[\frac{F_1b_1x}{6EIl}(x^2+b_1^2-l^2)+\frac{F_2a_2x}{6EIl}(l^2-x^2)\right]\right\}_{x=0}$$

$$= \left[\frac{F_1b_1}{6EIl}(3x^2+b_1^2-l^2)+\frac{F_2a_2}{6EIl}(l^2-3x^2)\right]_{x=0} = \frac{F_1b_1}{6EIl}(b_1^2-l^2)+\frac{F_2a_2l}{6EI}$$

$$= \frac{-3\ 312(280)}{6(207)10^3(39.76)10^3(510)}\left(280^2-510^2\right)+\frac{2\ 070(300)(510)}{6(207)10^3(39.76)10^3}$$

$$= 0.0131\text{ rad} \qquad \textit{Ans.}$$

$$\left(\theta_C\right)_z = \left(\frac{d\,y}{d\,x}\right)_{x=l} = \left\{\frac{d}{dx}\left[\frac{F_1a_1(l-x)}{6EIl}(x^2+a_1^2-2lx)+\frac{F_2a_2x}{6EIl}(l^2-x^2)\right]\right\}_{x=l}$$

$$= \left[\frac{F_1a_1}{6EIl}(6lx-2l^2-3x^2-a_1^2)+\frac{F_2a_2}{6EIl}(l^2-3x^2)\right]_{x=l} = \frac{F_1a_1}{6EIl}(l^2-a_1^{\,2})-\frac{F_2a_2l}{3EI}$$

$$= \frac{-3\ 312(230)}{6(207)10^3(39.76)10^3(510)}(510^2-230^2)-\frac{2\ 070(300)(510)}{3(207)10^3(39.76)10^3}$$

$$= -0.0191\text{ rad} \qquad \textit{Ans.}$$

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**4-31** From the solutions to Prob. 3-70, $T_1$ = 392.19 lbf and $T_2$ = 58.82 lbf , and Prob. 4-25, $I$ = 0.0490 9 in$^4$. From Table A-9, beam 6

$$(\theta_O)_z = \left(\frac{dy}{dx}\right)_{x=0} = \left\{\frac{d}{dx}\left[\frac{F_{1y}b_1x}{6EIl}\left(x^2+b_1^2-l^2\right)\right]\right\}_{x=0} = \frac{F_{1y}b_1}{6EIl}(b_1^{\,2}-l^2)$$

$$= \frac{-350(14)}{6(30)10^6(0.04909)(22)}\left(14^2-22^2\right) = 0.00726 \text{ rad} \qquad \textit{Ans.}$$

$$(\theta_O)_y = -\left(\frac{dz}{dx}\right)_{x=0} = -\left\{\frac{d}{dx}\left[\frac{F_{2z}b_2x}{6EIl}\left(x^2+b_2^2-l^2\right)\right]\right\}_{x=0} = -\frac{F_{2z}b_2}{6EIl}\left(b_2^2-l^2\right)$$

$$= -\frac{-450.98(6)}{6(30)10^6(0.04909)(22)}\left(6^2-22^2\right)$$

$$= -0.00624 \text{ rad} \qquad \textit{Ans.}$$

The slope magnitude is $\Theta_O = \sqrt{0.00726^2 + (-0.00624)^2} = 0.00957$ rad *Ans.*

$$(\theta_C)_z = \left(\frac{dy}{dx}\right)_{x=l} = \left\{\frac{d}{dx}\left[\frac{F_{1y}a_1(l-x)}{6EIl}\left(x^2+a_1^2-2lx\right)\right]\right\}_{x=l}$$

$$= \left[\frac{F_{1y}a_1}{6EIl}\left(6lx-2l^2-3x^2-a_1^2\right)\right]_{x=l} = \frac{F_{1y}a_1}{6EIl}(l^2-a_1^{\,2})$$

$$= \frac{-350(8)}{6(30)10^6(0.0491)(22)}\left(22^2-8^2\right) = -0.00605 \text{ rad} \qquad \textit{Ans.}$$

$$(\theta_C)_y = -\left(\frac{dz}{dx}\right)_{x=l} = -\left\{\frac{d}{dx}\left[\frac{F_{2z}a_2(l-x)}{6EIl}\left(x^2+a_2^2-2lx\right)\right]\right\}_{x=l}$$

$$= -\left[\frac{F_{2z}a_2}{6EIl}\left(6lx-2l^2-3x^2-a_2^2\right)\right]_{x=l} = -\frac{F_{2z}a_2}{6EIl}\left(l^2-a_2^2\right)$$

$$= -\frac{-450.98(16)}{6(30)10^6(0.04909)(22)}\left(22^2-16^2\right) = 0.00846 \text{ rad} \qquad \textit{Ans.}$$

The slope magnitude is $\Theta_C = \sqrt{(-0.00605)^2 + 0.00846^2} = 0.0104$ rad *Ans.*

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**4-32** From the solutions to Prob. 3-71, $T_1$ =250 N and $T_1$ =37.5 N, and Prob. 4-26, $I$ = 7 854 mm$^4$. From Table A-9, beam 6

$$(\theta_O)_z = \left(\frac{dy}{dx}\right)_{x=0} = \left\{\frac{d}{dx}\left[\frac{F_{1y}b_1x}{6EIl}\left(x^2+b_1^2-l^2\right)\right]\right\}_{x=0} = \frac{F_{1y}b_1}{6EIl}(b_1^{\,2}-l^2)$$

$$= \frac{\left[-345\sin 45^\circ\right](550)}{6(207)10^3(7\,854)(850)}\left(550^2-850^2\right) = 0.00680 \text{ rad} \qquad \textit{Ans.}$$

$$(\theta_O)_y = -\left(\frac{dz}{dx}\right)_{x=0} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}b_1x}{6EIl}\left(x^2+b_1^2-l^2\right)+\frac{F_{2z}b_2x}{6EIl}\left(x^2+b_2^2-l^2\right)\right]\right\}_{x=0}$$

$$= -\frac{F_{1z}b_1}{6EIl}\left(b_1^2-l^2\right)-\frac{F_{2z}b_2}{6EIl}\left(b_2^2-l^2\right) = -\frac{\left[345\cos 45^{\circ}\right](550)}{6(207)10^3(7\ 854)(850)}\left(550^2-850^2\right)$$

$$-\frac{-287.5(150)}{6(207)10^3(7\ 854)(850)}\left(150^2-850^2\right) = 0.00316 \text{ rad} \qquad \textit{Ans.}$$

The slope magnitude is $\Theta_O = \sqrt{0.00680^2+0.00316^2} = 0.00750$ rad *Ans.*

$$(\theta_C)_z = \left(\frac{dy}{dx}\right)_{x=l} = \left\{\frac{d}{dx}\left[\frac{F_{1y}a_1(l-x)}{6EIl}\left(x^2+a_1^2-2lx\right)\right]\right\}_{x=l} = \left[\frac{F_{1y}a_1}{6EIl}\left(6lx-2l^2-3x^2-a_1^2\right)\right]_{x=l}$$

$$= \frac{F_{1y}a_1}{6EIl}(l^2-a_1^2) = \frac{\left[-345\sin 45^{\circ}\right](300)}{6(207)10^3(7\ 854)(850)}\left(850^2-300^2\right) = -0.00558 \text{ rad} \qquad \textit{Ans.}$$

$$(\theta_C)_y = -\left(\frac{dz}{dx}\right)_{x=l} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}a_1(l-x)}{6EIl}\left(x^2+a_1^2-2lx\right)+\frac{F_{2z}a_2(l-x)}{6EIl}\left(x^2+a_2^2-2lx\right)\right]\right\}_{x=l}$$

$$= -\frac{F_{1z}a_1}{6EIl}\left(l^2-a_1^2\right)-\frac{F_{2z}a_2}{6EIl}\left(l^2-a_2^2\right) = -\frac{\left[345\cos 45^{\circ}\right](300)}{6(207)10^3(7\ 854)(850)}\left(850^2-300^2\right)$$

$$-\frac{-287.5(700)}{6(207)10^3(7\ 854)(850)}\left(850^2-700^2\right) = 6.04\left(10^{-5}\right) \text{ rad} \qquad \textit{Ans.}$$

The slope magnitude is $\Theta_C = \sqrt{(-0.00558)^2+\left[6.04\left(10^{-5}\right)\right]^2} = 0.00558$ rad *Ans.*

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**4-33** From the solutions to Prob. 3-72, $F_B$ = 750 lbf, and Prob. 4-27, $I$ = 0.119 8 in$^4$. From Table A-9, beams 6 and 10

$$(\theta_O)_z = \left(\frac{dy}{dx}\right)_{x=0} = \left\{\frac{d}{dx}\left[\frac{F_{1y}b_1x}{6EIl}\left(x^2+b_1^2-l^2\right)+\frac{F_{2y}a_2x}{6EIl}\left(l^2-x^2\right)\right]\right\}_{x=0}$$

$$= \left[\frac{F_{1y}b_1}{6EIl}\left(3x^2+b_1^2-l^2\right)+\frac{F_{2y}a_2}{6EIl}\left(l^2-3x^2\right)\right]_{x=0} = \frac{F_{1y}b_1}{6EIl}\left(b_1^2-l^2\right)+\frac{F_{2y}a_2l}{6EI}$$

$$= \frac{\left[-300\cos 20^{\circ}\right](14)}{6(30)10^6(0.119\ 8)(30)}\left(14^2-30^2\right)+\frac{\left[750\sin 20^{\circ}\right](9)(30)}{6(30)10^6(0.119\ 8)} = 0.00751 \text{ rad} \qquad \textit{Ans.}$$

$$(\theta_O)_y = -\left(\frac{dz}{dx}\right)_{x=0} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}b_1x}{6EIl}\left(x^2+b_1^2-l^2\right)+\frac{F_{2z}a_2x}{6EIl}\left(l^2-x^2\right)\right]\right\}_{x=0}$$

$$= -\left[\frac{F_{1z}b_1}{6EIl}\left(3x^2+b_1^2-l^2\right)+\frac{F_{2z}a_2}{6EIl}\left(l^2-3x^2\right)\right]_{x=0} = -\frac{F_{1z}b_1}{6EIl}\left(b_1^2-l^2\right)-\frac{F_{2z}a_2l}{6EI}$$

$$= -\frac{\left[300\sin 20^{\circ}\right](14)}{6(30)10^6(0.119\ 8)(30)}\left(14^2-30^2\right)-\frac{\left[-750\cos 20^{\circ}\right](9)(30)}{6(30)10^6(0.119\ 8)} = 0.0104 \text{ rad} \qquad Ans.$$

The slope magnitude is $\Theta_O = \sqrt{0.00751^2+0.0104^2} = 0.0128$ rad *Ans.*

$$(\theta_C)_z = \left(\frac{dy}{dx}\right)_{x=l} = \left\{\frac{d}{dx}\left[\frac{F_{1y}a_1(l-x)}{6EIl}\left(x^2+a_1^2-2lx\right)+\frac{F_{2y}a_2x}{6EIl}\left(l^2-x^2\right)\right]\right\}_{x=l}$$

$$= \left[\frac{F_{1y}a_1}{6EIl}\left(6lx-2l^2-3x^2-a_1^2\right)+\frac{F_{2y}a_2}{6EIl}\left(l^2-3x^2\right)\right]_{x=l} = \frac{F_{1y}a_1}{6EIl}(l^2-a_1^{\,2})-\frac{F_{2y}a_2l}{3EI}$$

$$= \frac{\left[-300\cos 20^{\circ}\right](16)}{6(30)10^6(0.119\ 8)(30)}\left(30^2-16^2\right)-\frac{\left[750\sin 20^{\circ}\right](9)(30)}{3(30)10^6(0.119\ 8)} = -0.0109 \text{ rad} \qquad Ans.$$

$$(\theta_C)_y = -\left(\frac{dz}{dx}\right)_{x=l} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}a_1(l-x)}{6EIl}\left(x^2+a_1^2-2lx\right)+\frac{F_{2z}a_2x}{6EIl}\left(l^2-x^2\right)\right]\right\}_{x=l}$$

$$= -\left[\frac{F_{1z}a_1}{6EIl}\left(6lx-2l^2-3x^2-a_1^2\right)+\frac{F_{2z}a_2}{6EIl}\left(l^2-3x^2\right)\right]_{x=l} = -\frac{F_{1z}a_1}{6EIl}\left(l^2-a_1^2\right)+\frac{F_{2z}a_2l}{3EI}$$

$$= -\frac{\left[300\sin 20^{\circ}\right](16)}{6(30)10^6(0.119\ 8)(30)}\left(30^2-16^2\right)+\frac{\left[-750\cos 20^{\circ}\right](9)(30)}{3(30)10^6(0.119\ 8)} = -0.0193 \text{ rad} \qquad Ans.$$

The slope magnitude is $\Theta_C = \sqrt{\left(-0.0109\right)^2+\left(-0.0193\right)^2} = 0.0222$ rad *Ans.*

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**4-34** From the solutions to Prob. 3-73, $F_B$ = 22.8 kN, and Prob. 4-28, $I$ = 306.8 ($10^3$) mm$^4$. From Table A-9, beam 6

$$(\theta_O)_z = \left(\frac{dy}{dx}\right)_{x=0} = \left\{\frac{d}{dx}\left[\frac{F_{1y}b_1x}{6EIl}\left(x^2+b_1^2-l^2\right)+\frac{F_{2y}b_2x}{6EIl}\left(x^2+b_2^2-l^2\right)\right]\right\}_{x=0}$$

$$= \frac{F_{1y}b_1}{6EIl}\left(b_1^2-l^2\right)+\frac{F_{2y}b_2}{6EIl}\left(b_2^2-l^2\right) = \frac{\left[11\left(10^3\right)\sin 20^{\circ}\right](650)}{6(207)10^3(306.8)10^3(1050)}\left(650^2-1050^2\right)$$

$$+\frac{\left[22.8\left(10^3\right)\sin 25^{\circ}\right](300)}{6(207)10^3(306.8)10^3(1050)}\left(300^2-1050^2\right) = -0.0115 \text{ rad} \qquad Ans.$$

$$(\theta_O)_y = -\left(\frac{dz}{dx}\right)_{x=0} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}b_1x}{6EIl}\left(x^2+b_1^2-l^2\right)+\frac{F_{2z}b_2x}{6EIl}\left(x^2+b_2^2-l^2\right)\right]\right\}_{x=0}$$

$$= -\frac{F_{1z}b_1}{6EIl}\left(b_1^2-l^2\right)-\frac{F_{2z}b_2}{6EIl}\left(b_2^2-l^2\right)$$

$$= -\frac{\left[11\left(10^3\right)\cos 20^{\circ}\right](650)}{6(207)10^3(306.8)10^3(1050)}\left(650^2-1050^2\right)$$

$$-\frac{\left[-22.8\left(10^3\right)\cos 25^{\circ}\right](300)}{6(207)10^3(306.8)10^3(1050)}\left(300^2-1050^2\right) = -0.00427 \text{ rad} \qquad \textit{Ans.}$$

The slope magnitude is $\Theta_O = \sqrt{\left(-0.0115\right)^2+\left(-0.00427\right)^2} = 0.0123$ rad *Ans.*

$$(\theta_C)_z = \left(\frac{dy}{dx}\right)_{x=l} = \left\{\frac{d}{dx}\left[\frac{F_{1y}a_1(l-x)}{6EIl}\left(x^2+a_1^2-2lx\right)+\frac{F_{2y}a_2(l-x)}{6EIl}\left(x^2+a_2^2-2lx\right)\right]\right\}_{x=l}$$

$$= \left[\frac{F_{1y}a_1}{6EIl}(6lx-2l^2-3x^2-a_1^2)+\frac{F_{2y}a_2}{6EIl}\left(6lx-2l^2-3x^2-a_2^2\right)\right]_{x=l}$$

$$= \frac{F_{1y}a_1}{6EIl}\left(l^2-a_1^2\right)+\frac{F_{2y}a_2}{6EIl}\left(l^2-a_2^2\right) = \frac{\left[11\left(10^3\right)\sin 20^{\circ}\right](400)}{6(207)10^3(306.8)10^3(1050)}\left(1050^2-400^2\right)$$

$$+\frac{\left[22.8\left(10^3\right)\sin 25^{\circ}\right](750)}{6(207)10^3(306.8)10^3(1050)}\left(1050^2-750^2\right) = 0.0133 \text{ rad} \qquad \textit{Ans.}$$

$$(\theta_C)_y = -\left(\frac{dz}{dx}\right)_{x=l} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}a_1(l-x)}{6EIl}\left(x^2+a_1^2-2lx\right)+\frac{F_{2z}a_2(l-x)}{6EIl}\left(x^2+a_2^2-2lx\right)\right]\right\}_{x=l}$$

$$= -\left[\frac{F_{1z}a_1}{6EIl}\left(6lx-2l^2-3x^2-a_1^2\right)+\frac{F_{2z}a_2}{6EIl}\left(6lx-2l^2-3x^2-a_2^2\right)\right]_{x=l}$$

$$= -\frac{F_{1z}a_1}{6EIl}\left(l^2-a_1^2\right)-\frac{F_{2z}a_2}{6EIl}\left(l^2-a_2^2\right) = -\frac{\left[11\left(10^3\right)\cos 20^{\circ}\right](400)}{6(207)10^3(306.8)10^3(1050)}\left(1050^2-400^2\right)$$

$$-\frac{\left[-22.8\left(10^3\right)\cos 25^{\circ}\right](750)}{6(207)10^3(306.8)10^3(1050)}\left(1050^2-750^2\right) = 0.0112 \text{ rad} \qquad \textit{Ans.}$$

The slope magnitude is $\Theta_C = \sqrt{0.0133^2+0.0112^2} = 0.0174$ rad *Ans.*

---

**4-35** The required new slope in radians is $\theta_{\text{new}} = 0.06(\pi/180) = 0.00105$ rad.
In Prob. 4-29, $I = 0.119\ 8$ in$^4$, and it was found that the greater angle occurs at the bearing at $O$ where $(\theta_O)_y = -\ 0.00468$ rad.

Since $\theta$ is inversely proportional to $I$,

$$\theta_{new}\, I_{new} = \theta_{old}\, I_{old} \quad \Rightarrow \quad I_{new} = \pi\, d_{new}^4/64 = \theta_{old}\, I_{old}/\theta_{new}$$

or,

$$d_{new} = \left( \frac{64}{\pi} \left| \frac{\theta_{old}}{\theta_{new}} \right| I_{old} \right)^{1/4}$$

The absolute sign is used as the old slope may be negative.

$$d_{new} = \left( \frac{64}{\pi} \left| \frac{-0.00468}{0.00105} \right| 0.119\,8 \right)^{1/4} = 1.82 \text{ in} \qquad \textit{Ans.}$$

---

**4-36** The required new slope in radians is $\theta_{new} = 0.06(\pi/180) = 0.00105$ rad.
In Prob. 4-30, $I = 39.76\ (10^3)\ \text{mm}^4$, and it was found that the greater angle occurs at the bearing at $C$ where $(\theta_C)_y = -\,0.0191$ rad.

See the solution to Prob. 4-35 for the development of the equation

$$d_{new} = \left( \frac{64}{\pi} \left| \frac{\theta_{old}}{\theta_{new}} \right| I_{old} \right)^{1/4}$$

$$d_{new} = \left( \frac{64}{\pi} \left| \frac{-0.0191}{0.00105} \right| 39.76\left(10^3\right) \right)^{1/4} = 62.0 \text{ mm} \qquad \textit{Ans.}$$

---

**4-37** The required new slope in radians is $\theta_{new} = 0.06(\pi/180) = 0.00105$ rad.
In Prob. 4-31, $I = 0.0491\ \text{in}^4$, and the maximum slope is $\Theta_C = 0.0104$ rad.

See the solution to Prob. 4-35 for the development of the equation

$$d_{new} = \left( \frac{64}{\pi} \left| \frac{\theta_{old}}{\theta_{new}} \right| I_{old} \right)^{1/4}$$

$$d_{new} = \left( \frac{64}{\pi} \left| \frac{0.0104}{0.00105} \right| 0.0491 \right)^{1/4} = 1.77 \text{ in} \qquad \textit{Ans.}$$

---

**4-38** The required new slope in radians is $\theta_{new} = 0.06(\pi/180) = 0.00105$ rad.
In Prob. 4-32, $I = 7\,854\ \text{mm}^4$, and the maximum slope is $\Theta_O = 0.00750$ rad.

See the solution to Prob. 4-35 for the development of the equation

$$d_{\text{new}} = \left( \frac{64}{\pi} \left| \frac{\theta_{\text{old}}}{\theta_{\text{new}}} \right| I_{\text{old}} \right)^{1/4}$$

$$d_{\text{new}} = \left( \frac{64}{\pi} \left| \frac{0.00750}{0.00105} \right| 7\,854 \right)^{1/4} = 32.7 \text{ mm} \qquad \textit{Ans.}$$

---

**4-39** The required new slope in radians is $\theta_{\text{new}} = 0.06(\pi/180) = 0.00105$ rad.
In Prob. 4-33, $I = 0.119\,8 \text{ in}^4$, and the maximum slope $\Theta = 0.0222$ rad.

See the solution to Prob. 4-35 for the development of the equation

$$d_{\text{new}} = \left( \frac{64}{\pi} \left| \frac{\theta_{\text{old}}}{\theta_{\text{new}}} \right| I_{\text{old}} \right)^{1/4}$$

$$d_{\text{new}} = \left( \frac{64}{\pi} \left| \frac{0.0222}{0.00105} \right| 0.119\,8 \right)^{1/4} = 2.68 \text{ in} \qquad \textit{Ans.}$$

---

**4-40** The required new slope in radians is $\theta_{\text{new}} = 0.06(\pi/180) = 0.00105$ rad.
In Prob. 4-34, $I = 306.8\ (10^3)\ \text{mm}^4$, and the maximum slope is $\Theta_C = 0.0174$ rad.

See the solution to Prob. 4-35 for the development of the equation

$$d_{\text{new}} = \left( \frac{64}{\pi} \left| \frac{\theta_{\text{old}}}{\theta_{\text{new}}} \right| I_{\text{old}} \right)^{1/4}$$

$$d_{\text{new}} = \left( \frac{64}{\pi} \left| \frac{0.0174}{0.00105} \right| 306.8\left(10^3\right) \right)^{1/4} = 100.9 \text{ mm} \qquad \textit{Ans.}$$

---

**4-41** $I_{AB} = \pi\, 1^4/64 = 0.04909 \text{ in}^4$, $J_{AB} = 2\, I_{AB} = 0.09818 \text{ in}^4$, $I_{BC} = (0.25)(1.5)^3/12 = 0.07031 \text{ in}^4$, $I_{CD} = \pi\,(3/4)^4/64 = 0.01553 \text{ in}^4$. For Eq. (3-41), p. 102, $b/c = 1.5/0.25 = 6 \Rightarrow \beta = 0.299$.

The deflection can be broken down into several parts

1. The vertical deflection of $B$ due to force and moment acting on $B$ ($y_1$).
2. The vertical deflection due to the slope at $B$, $\theta_{B1}$, due to the force and moment acting on $B$ ($y_2 = \overline{CD}\,\theta_{B1} = 2\theta_{B1}$).

3. The vertical deflection due to the rotation at $B$, $\theta_{B2}$, due to the torsion acting at $B$ ($y_3 = \overline{BC}\,\theta_{B1} = 5\theta_{B1}$).
4. The vertical deflection of $C$ due to the force acting on $C$ ($y_4$).
5. The rotation at $C$, $\theta_C$, due to the torsion acting at $C$ ($y_3 = \overline{CD}\,\theta_C = 2\theta_C$).
6. The vertical deflection of $D$ due to the force acting on $D$ ($y_5$).

1. From Table A-9, beams 1 and 4 with $F = -200$ lbf and $M_B = 2(200) = 400$ lbf·in

$$y_1 = -\frac{-200(6^3)}{3(30)10^6(0.04909)} + \frac{400(6^2)}{2(30)10^6(0.04909)} = 0.01467 \text{ in}$$

2. From Table A-9, beams 1 and 4

$$\theta_{B1} = \left\{\frac{d}{dx}\left[\frac{Fx^2}{6EI}(x-3l) + \frac{M_B x^2}{2EI}\right]\right\}_{x=l} = \left[\frac{Fx}{6EI}(3x-6l) + \frac{M_B x}{EI}\right]_{x=l}$$

$$= \left\{\frac{l}{2EI}\left[-Fl + 2M_B\right]\right\} = \frac{6}{2(30)10^6(0.04909)}\left[-(-200)(6) + 2(400)\right] = 0.004074 \text{ rad}$$

$y_2 = 2(0.004072) = 0.00815$ in

3. The torsion at $B$ is $T_B = 5(200) = 1000$ lbf·in. From Eq. (4-5)

$$\theta_{B2} = \left(\frac{TL}{JG}\right)_{AB} = \frac{1000(6)}{0.09818(11.5)10^6} = 0.005314 \text{ rad}$$

$y_3 = 5(0.005314) = 0.02657$ in

4. For bending of $BC$, from Table A-9, beam 1

$$y_4 = -\frac{-200(5^3)}{3(30)10^6(0.07031)} = 0.00395 \text{ in}$$

5. For twist of $BC$, from Eq. (3-41), p. 102, with $T = 2(200) = 400$ lbf·in

$$\theta_C = \frac{400(5)}{0.299(1.5)0.25^3(11.5)10^6} = 0.02482 \text{ rad}$$

$y_5 = 2(0.02482) = 0.04964$ in

6. For bending of $CD$, from Table A-9, beam 1

$$y_6 = -\frac{-200(2^3)}{3(30)10^6(0.01553)} = 0.00114 \text{ in}$$

Summing the deflections results in

$$y_D = \sum_{i=1}^{6} y_i = 0.01467 + 0.00815 + 0.02657 + 0.00395 + 0.04964 + 0.00114 = 0.1041 \text{ in} \quad \textit{Ans.}$$

This problem is solved more easily using Castigliano's theorem. See Prob. 4-71.

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**4-42** The deflection of $D$ in the $x$ direction due to $F_z$ is from:

1. The deflection due to the slope at $B$, $\theta_{B1}$, due to the force and moment acting on $B$ ($x_1 = \overline{BC}\,\theta_{B1} = 5\theta_{B1}$).
2. The deflection due to the moment acting on $C$ ($x_2$).

1. For $AB$, $I_{AB} = \pi\, 1^4/64 = 0.04909$ in$^4$. From Table A-9, beams 1 and 4

$$\theta_{B1} = \left\{ \frac{d}{dx}\left[ \frac{Fx^2}{6EI}(x-3l) + \frac{M_B x^2}{2EI} \right] \right\}_{x=l} = \left[ \frac{Fx}{6EI}(3x-6l) + \frac{M_B x}{EI} \right]_{x=l}$$

$$= \left\{ \frac{l}{2EI}\left[ -Fl + 2M_B \right] \right\} = \frac{6}{2(30)10^6(0.04909)}\left[ -(100)(6) + 2(-200) \right] = -0.002037 \text{ rad}$$

$$x_1 = 5(-0.002037) = -0.01019 \text{ in}$$

2. For $BC$, $I_{BC} = (1.5)(0.25)^3/12 = 0.001953$ in$^4$. From Table A-9, beam 4

$$x_2 = \frac{M_C l^2}{2EI} = \frac{2(-100)5}{2(30)10^6(0.001953)} = -0.04267 \text{ in}$$

The deflection of $D$ in the $x$ direction due to $F_x$ is from:

3. The elongation of $AB$ due to the tension. For $AB$, the area is $A = \pi\, 1^2/4 = 0.7854$ in$^2$

$$x_3 = \left( \frac{Fl}{AE} \right)_{AB} = \frac{-150(6)}{0.7854(30)10^6} = -3.82\left(10^{-5}\right) \text{ in}$$

4. The deflection due to the slope at $B$, $\theta_{B2}$, due to the moment acting on $B$ ($x_1 = \overline{BC}\,\theta_{B2} = 5\theta_{B2}$). With $I_{AB} = 0.04907$ in$^4$,

$$\theta_{B2} = \frac{M_B l}{EI} = \frac{5(-150)6}{30\left(10^6\right)0.04909} = -0.003056 \text{ rad}$$

$x_4 = 5(-\,0.003056) = -\,0.01528$ in

5. The deflection at $C$ due to the bending force acting on $C$. With $I_{BC} = 0.001953$ in$^4$

$$x_5 = \left(-\frac{Fl^3}{3EI}\right)_{BC} = -\frac{150\left(5^3\right)}{3(30)10^6\left(0.001953\right)} = -0.10667 \text{ in}$$

6. The elongation of $CD$ due to the tension. For $CD$, the area is $A = \pi(0.75^2)/4 = 0.4418$ in$^2$

$$x_6 = \left(\frac{Fl}{AE}\right)_{CD} = \frac{-150(2)}{0.4418(30)10^6} = -2.26\left(10^{-5}\right) \text{ in}$$

Summing the deflections results in

$$x_D = \sum_{i=1}^{6} x_i = -0.01019 - 0.04267 - 3.82\left(10^{-5}\right)$$
$$-\,0.01528 - 0.10667 - 2.26\left(10^{-5}\right) = -0.1749 \text{ in} \quad \textit{Ans.}$$

---

**4-43** $J_{OA} = J_{BC} = \pi(1.5^4)/32 = 0.4970$ in$^4$, $J_{AB} = \pi(1^4)/32 = 0.09817$ in$^4$, $I_{AB} = \pi(1^4)/64 = 0.04909$ in$^4$, and $I_{CD} = \pi(0.75^4)/64 = 0.01553$ in$^4$.

$$\theta = \left(\frac{Tl}{GJ}\right)_{OA} + \left(\frac{Tl}{GJ}\right)_{AB} + \left(\frac{Tl}{GJ}\right)_{BC} = \frac{T}{G}\left(\frac{l_{OA}}{J_{OA}} + \frac{l_{AB}}{J_{AB}} + \frac{l_{BC}}{J_{BC}}\right)$$
$$= \frac{250(12)}{11.5(10^6)}\left(\frac{2}{0.4970} + \frac{9}{0.09817} + \frac{2}{0.4970}\right) = 0.0260 \text{ rad} \qquad \textit{Ans.}$$

Simplified

$$\theta_s = \frac{Tl}{GJ} = \frac{250(12)(13)}{11.5\left(10^6\right)\left(0.09817\right)}$$

$\theta_s = 0.0345$ rad *Ans.*

Simplified is 0.0345/0.0260 = 1.33 times greater *Ans.*

$$y_D = \frac{F_y l_{OC}{}^3}{3EI_{AB}} + \theta_s\left(l_{CD}\right) + \frac{F_y l_{CD}{}^3}{3EI_{CD}} = \frac{250\left(13^3\right)}{3(30)10^6\left(0.04909\right)} + 0.0345(12) + \frac{250\left(12^3\right)}{3(30)10^6\left(0.01553\right)}$$

$y_D = 0.847$ in *Ans.*

---

**4-44** Reverse the deflection equation of beam 7 of Table A-9. Using units in lbf, inches

$$y = -\frac{wx}{24EI}\left(2lx^2 - x^3 - l^3\right) = -\frac{(3000/12)x}{24(30)10^6(485)}\left\{2(25)x^2 - x^3 - \left[25(12)\right]^3\right\}$$
$$= 7.159\left(10^{-10}\right)x\left[27\left(10^6\right) - 600x^2 + x^3\right] \qquad Ans.$$

The maximum height occurs at $x = 25(12)/2 = 150$ in

$$y_{\max} = 7.159\left(10^{-10}\right)150\left[27\left(10^6\right) - 600\left(150^2\right) + 150^3\right] = 1.812 \text{ in} \qquad Ans.$$

---

**4-45** From Table A-9-6,

$$y_L = \frac{Fbx}{6EIl}\left(x^2 + b^2 - l^2\right)$$
$$y_L = \frac{Fb}{6EIl}\left(x^3 + b^2x - l^2x\right)$$
$$\frac{dy_L}{dx} = \frac{Fb}{6EIl}\left(3x^2 + b^2 - l^2\right)$$
$$\left.\frac{dy_L}{dx}\right|_{x=0} = \frac{Fb\left(b^2 - l^2\right)}{6EIl}$$

Let $\xi = \left.\frac{dy_L}{dx}\right|_{x=0}$ and set $I = \frac{\pi d_L^4}{64}$. Thus,

$$d_L = \left|\frac{32Fb\left(b^2 - l^2\right)}{3\pi El\xi}\right|^{1/4} \qquad Ans.$$

For the other end view, observe beam 6 of Table A-9 from the back of the page, noting that *a* and *b* interchange as do *x* and –*x*

$$d_R = \left|\frac{32Fa\left(l^2 - a^2\right)}{3\pi El\xi}\right|^{1/4} \qquad Ans.$$

For a uniform diameter shaft the necessary diameter is the larger of $d_L$ and $d_R$.

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**4-46** The maximum slope will occur at the left bearing. Incorporating a design factor into the solution for $d_L$ of Prob. 4-45,

$$d=\left[\frac{32nFb\left(l^2-b^2\right)}{3\pi El\xi}\right]^{1/4}$$

$$d=\sqrt[4]{\frac{32(1.28)(3000)(200)\left(300^2-200^2\right)}{3\pi(207)10^3(300)(0.001)}}$$

$$d=38.1\text{ mm} \quad \textit{Ans.}$$

$$I=\frac{\pi\left(38.1^4\right)}{64}=103.4\left(10^3\right)\text{ mm}^4$$

From Table A-9, beam 6, the maximum deflection will occur in *BC* where $dy_{BC}/dx = 0$

$$\frac{d}{dx}\left[\frac{Fa(l-x)}{6EIl}\left(x^2+a^2-2lx\right)\right]=0 \quad\Rightarrow\quad 3x^2-6lx+\left(a^2+2l^2\right)=0$$

$$3x^2-6(300)x+\left[100^2+2\left(300^2\right)\right]=0 \quad\Rightarrow\quad x^2-600x+63333=0$$

$$x=\frac{1}{2}\left[600\pm\sqrt{600^2-4(1)63\,333}\right]=463.3,\ 136.7\text{ mm}$$

$x$ = 136.7 mm is acceptable.

$$y_{\max}=\left[\frac{Fa(l-x)}{6EIl}\left(x^2+a^2-2lx\right)\right]_{x=136.7\,\text{mm}}$$

$$=\frac{3\left(10^3\right)100(300-136.7)}{6(207)10^3(103.4)10^3(300)}\left[136.7^2+100^2-2(300)136.7\right]=-0.0678\text{ mm} \quad \textit{Ans.}$$

---

**4-47** $I = \pi(1.25^4)/64 = 0.1198$ in$^4$. From Table A-9, beam 6

$$\delta=\sqrt{\left[\frac{F_1a_1(l-x)}{6EIl}(x^2+a_1{}^2-2lx)\right]^2+\left[\frac{F_2b_2x}{6EIl}(x^2+b_2{}^2-l^2)\right]^2}$$

$$=\left\{\left[\frac{150(5)(20-8)}{6(30)10^6(0.1198)(20)}\left(8^2+5^2-2(20)(8)\right)\right]^2\right.$$

$$\left.+\left[\frac{250(10)(8)}{6(30)10^6(0.1198)(20)}\left(8^2+10^2-20^2\right)\right]^2\right\}^{1/2}$$

$$=0.0120\text{ in} \quad \textit{Ans.}$$

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**4-48** $I = \pi(1.25^4)/64 = 0.1198$ in$^4$. For both forces use beam 6 of Table A-9.

For $F_1 = 150$ lbf:

$\underline{0 \le x \le 5}$

$$y = \frac{F_1 b_1 x}{6EIl}\left(x^2 + b_1^2 - l^2\right) = \frac{150(15)x}{6(30)10^6(0.1198)(20)}\left(x^2 + 15^2 - 20^2\right)$$

$$= 5.217\left(10^{-6}\right)x\left(x^2 - 175\right) \qquad (1)$$

$\underline{5 \le x \le 20}$

$$y = \frac{F_1 a_1 (l-x)}{6EIl}\left(x^2 + a_1^2 - 2lx\right) = \frac{150(5)(20-x)}{6(30)10^6(0.1198)(20)}\left[x^2 + 5^2 - 2(20)x\right]$$

$$= 1.739\left(10^{-6}\right)(20-x)\left(x^2 - 40x + 25\right) \qquad (2)$$

For $F_2 = 250$ lbf:

$\underline{0 \le x \le 10}$

$$z = \frac{F_2 b_2 x}{6EIl}\left(x^2 + b_2^2 - l^2\right) = \frac{250(10)x}{6(30)10^6(0.1198)(20)}\left(x^2 + 10^2 - 20^2\right)$$

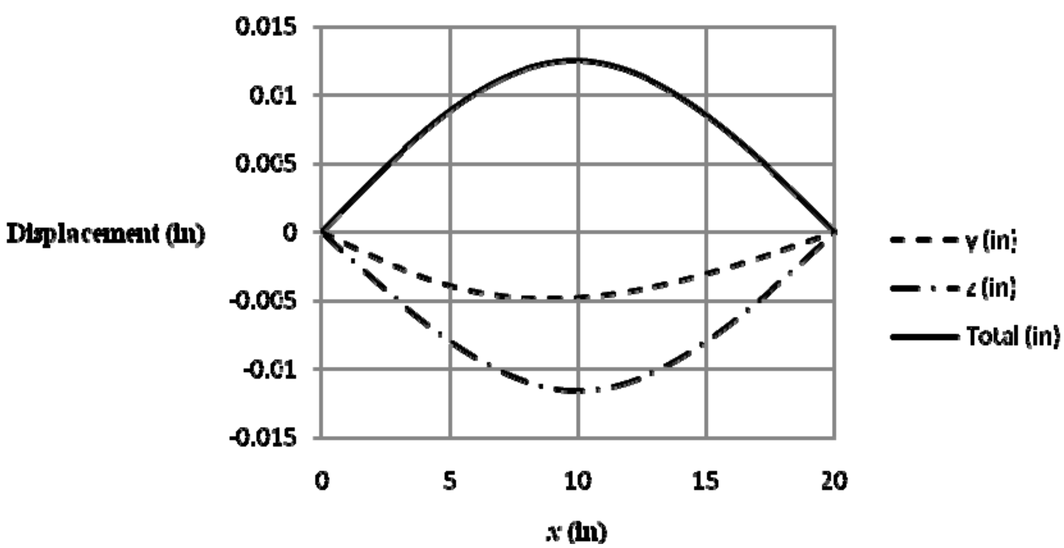
$$= 5.797\left(10^{-6}\right)x\left(x^2 - 300\right) \qquad (3)$$

1$\underline{0 \le x \le 20}$

$$z = \frac{F_2 a_2 (l-x)}{6EIl}\left(x^2 + a_2^2 - 2lx\right) = \frac{250(10)(20-x)}{6(30)10^6(0.1198)(20)}\left[x^2 + 10^2 - 2(20)x\right]$$

$$= 5.797\left(10^{-6}\right)(20-x)\left(x^2 - 40x + 100\right) \qquad (4)$$

Plot Eqs. (1) to (4) for each 0.1 in using a spreadsheet. There are 201 data points, too numerous to tabulate here but the plot is shown below, where the maximum deflection of $\delta = 0.01255$ in occurs at $x = 9.9$ in. *Ans.*



**4-49** The larger slope will occur at the left end.
From Table A-9, beam 8

$$y_{AB} = \frac{M_B x}{6EIl}(x^2 + 3a^2 - 6al + 2l^2)$$

$$\frac{dy_{AB}}{dx} = \frac{M_B}{6EIl}(3x^2 + 3a^2 - 6al + 2l^2)$$

With $I = \pi d^4/64$, the slope at the left bearing is

$$\left.\frac{dy_{AB}}{dx}\right|_{x=0} = \theta_A = \frac{M_B}{6E\left(\pi d^4/64\right)l}(3a^2 - 6al + 2l^2)$$

Solving for $d$

$$d = \sqrt[4]{\frac{32M_B}{3\pi E\theta_A l}\left(3a^2 - 6al + 2l^2\right)} = \sqrt[4]{\frac{32(1000)}{3\pi(30)10^6(0.002)(10)}\left[3(4^2) - 6(4)(10) + 2\left(10^2\right)\right]}$$

$$= 0.461 \text{ in} \qquad \textit{Ans.}$$

---

**4-50** From Table A-5, $E = 10.4$ Mpsi
$\Sigma M_O = 0 = 18\,F_{BC} - 6(100) \quad \Rightarrow \quad F_{BC} = 33.33$ lbf
The cross sectional area of rod $BC$ is $A = \pi(0.5^2)/4 = 0.1963$ in$^2$.

The deflection at point $B$ will be equal to the elongation of the rod $BC$.

$$y_B = \left(\frac{FL}{AE}\right)_{BC} = \frac{33.33(12)}{(0.1963)30\left(10^6\right)} = 6.79\left(10^{-5}\right) \text{ in} \qquad \textit{Ans.}$$

---

**4-51** $\Sigma M_O = 0 = 6\,F_{AC} - 11(100) \quad \Rightarrow \quad F_{AC} = 183.3$ lbf

The deflection at point $A$ in the negative $y$ direction is equal to the elongation of the rod $AC$. From Table A-5, $E_s = 30$ Mpsi.

$$y_A = -\left(\frac{FL}{AE}\right)_{AC} = -\frac{183.3(12)}{\left[\pi\left(0.5^2\right)/4\right]30\left(10^6\right)} = -3.735\left(10^{-4}\right) \text{ in}$$

By similar triangles the deflection at $B$ due to the elongation of the rod $AC$ is

$$\frac{y_A}{6} = \frac{y_{B1}}{18} \quad \Rightarrow \quad y_{B1} = 3y_A = 3(-3.735)10^{-4} = -0.00112 \text{ in}$$

From Table A-5, $E_a = 10.4$ Mpsi

The bar can then be treated as a simply supported beam with an overhang $AB$. From Table A-9, beam 10

$$y_{B2} = \left(\overline{BD}\right)\left(\frac{dy_{BC}}{dx}\bigg|_{x=l+a}\right) - \frac{Fa^2}{3EI}(l+a) = 7\left\{\frac{d}{dx}\left(\frac{F(x-l)}{6EI}\left[(x-l)^2 - a(3x-l)\right]\right)\right\}_{x=l+a} - \frac{Fa^2}{3EI}(l+a)$$

$$= 7\frac{F}{6EI}\left[3(x-l)^2 - 3a(x-l) - a(3x-l)\right]\big|_{x=l+a} - \frac{Fa^2}{3EI}(l+a) = -\frac{7Fa}{6EI}(2l+3a) - \frac{Fa^2}{3EI}(l+a)$$

$$= -\frac{7(100)5}{6(10.4)10^6\left(0.25(2^3)/12\right)}\left[2(6)+3(5)\right] - \frac{100\left(5^2\right)}{3(10.4)10^6\left(0.25(2^3)/12\right)}(6+5)$$

$$= -0.01438 \text{ in}$$

$y_B = y_{B1} + y_{B2} = -\ 0.00112 - 0.01438 = -\ 0.0155$ in *Ans.*

---

**4-52** From Table A-5, $E$ = 207 GPa, and $G$ = 79.3 GPa.

$$|y_B| = \left(\frac{Tl}{GJ}\right)_{OC} l_{AB} + \left(\frac{Tl}{GJ}\right)_{AC} l_{AB} + \frac{Fl_{AB}^{\ 3}}{3EI_{AB}} = \frac{Fl_{OC}l_{AB}^{\ 2}}{G\left(\pi d_{OC}^{\ 4}/32\right)} + \frac{Fl_{AC}l_{AB}^{\ 2}}{G\left(\pi d_{AC}^{\ 4}/32\right)} + \frac{Fl_{AB}^{\ 3}}{3E\left(\pi d_3^{\ 4}/64\right)}$$

$$= \frac{32Fl_{AB}^{\ 2}}{\pi}\left[\frac{l_{OC}}{Gd_{OC}^{\ 4}} + \frac{l_{AC}}{Gd_{AC}^{\ 4}} + \frac{2l_{AB}}{3Ed_{AB}^{\ 4}}\right]$$

The spring rate is $k = F/\,|y_B|$. Thus

$$k = \left\{\frac{32l_{AB}^{\ 2}}{\pi}\left[\frac{l_{OC}}{Gd_{OC}^{\ 4}} + \frac{l_{AC}}{Gd_{AC}^{\ 4}} + \frac{2l_{AB}}{3Ed_{AB}^{\ 4}}\right]\right\}^{-1}$$

$$= \left\{\frac{32\left(200^2\right)}{\pi}\left[\frac{200}{79.3\left(10^3\right)18^4} + \frac{200}{79.3\left(10^3\right)12^4} + \frac{2(200)}{3(207)10^3\left(8^4\right)}\right]\right\}^{-1}$$

$$= 8.10 \text{ N/mm}$$ *Ans.*

---

**4-53** For the beam deflection, use beam 5 of Table A-9.

$$R_1 = R_2 = \frac{F}{2}$$

$$\delta_1 = \frac{F}{2k_1}, \text{ and } \delta_2 = \frac{F}{2k_2}$$

$$y_{AB} = -\delta_1 + \frac{\delta_1 - \delta_2}{l}x + \frac{Fx}{48EI}(4x^2 - 3l^3)$$

$$y_{AB} = F\left[-\frac{1}{2k_1} + \frac{k_2 - k_1}{2k_1k_2l}x + \frac{x}{48EI}(4x^2 - 3l^3)\right]$$ *Ans.*

For *BC*, since Table A-9 does not have an equation (because of symmetry) an equation will need to be developed as the problem is no longer symmetric. This can be done easily using beam 6 of Table A-9 with $a = l/2$

$$y_{BC} = \frac{-F}{2k_1} + \frac{Fk_2 - Fk_1}{2k_1k_2l}x + \frac{F(l/2)(l-x)}{EIl}\left(x^2 + \frac{l^2}{4} - 2lx\right)$$

$$= F\left[-\frac{1}{2k_1} + \frac{k_2 - k_1}{2k_1k_2l}x + \frac{(l-x)}{48EI}\left(4x^2 + l^2 - 8lx\right)\right] \qquad \textit{Ans.}$$

---

**4-54**

$$R_1 = \frac{Fa}{l}, \text{ and } R_2 = \frac{F}{l}(l+a)$$

$$\delta_1 = \frac{Fa}{lk_1}, \text{ and } \delta_2 = \frac{F}{lk_2}(l+a)$$

$$y_{AB} = -\delta_1 + \frac{\delta_1 - \delta_2}{l}x + \frac{Fax}{6EIl}(l^2 - x^2)$$

$$y_{AB} = F\left\{-\frac{a}{k_1l} + \frac{x}{k_1k_2l^2}\left[k_2a - k_1(l+a)\right] + \frac{ax}{6EIl}(l^2 - x^2)\right\} \qquad \textit{Ans.}$$

$$y_{BC} = -\delta_1 + \frac{\delta_1 - \delta_2}{l}x + \frac{F(x-l)}{6EI}\left[(x-l)^2 - a(3x-l)\right]$$

$$y_{BC} = F\left\{-\frac{a}{k_1l} + \frac{x}{k_1k_2l^2}\left[k_2a - k_1(l+a)\right] + \frac{(x-l)}{6EI}\left[(x-l)^2 - a(3x-l)\right]\right\} \qquad \textit{Ans.}$$

---

**4-55** Let the load be at $x \geq l/2$. The maximum deflection will be in Section *AB* (Table A-9, beam 6)

$$y_{AB} = \frac{Fbx}{6EIl}\left(x^2 + b^2 - l^2\right)$$

$$\frac{dy_{AB}}{dx} = \frac{Fb}{6EIl}\left(3x^2 + b^2 - l^2\right) = 0 \qquad \Rightarrow \qquad 3x^2 + b^2 - l^2 = 0$$

$$x = \sqrt{\frac{l^2 - b^2}{3}}, \quad x_{\max} = \sqrt{\frac{l^2}{3}} = 0.577l \qquad \textit{Ans.}$$

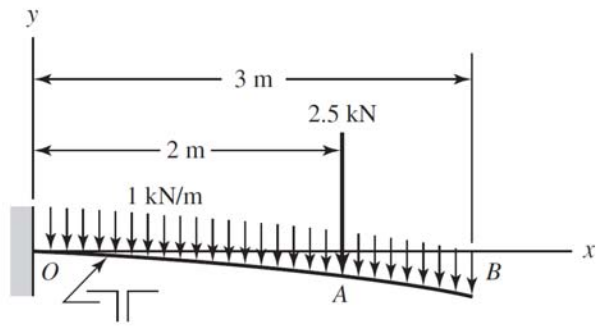
For $x \leq l/2$, $x_{\min} = l - 0.577l = 0.423l$ *Ans.*

---

**4-56**

$$M_O = 1(3000)(1500) + 2500(2000)$$
$$= 9.5\left(10^6\right) \text{ N·mm}$$
$$R_O = 1(3000) + 2500 = 5\ 500 \text{ N}$$

From Prob. 4-10, $I = 4.14(10^6)\text{ mm}^4$




$$M = -9.5\left(10^6\right) + 5500x - \frac{x^2}{2} - 2500\langle x\text{-}2000\rangle^1$$

$$EI\frac{dy}{dx} = -9.5\left(10^6\right)x + 2750x^2 - \frac{x^3}{6} - 1250\langle x - 2000\rangle^2 + C_1$$

$$\frac{dy}{dx} = 0 \text{ at } x = 0 \quad \therefore C_1 = 0$$

$$EI\frac{dy}{dx} = -9.5\left(10^6\right)x + 2750x^2 - \frac{x^3}{6} - 1250\langle x - 2000\rangle^2$$

$$EIy = -4.75\left(10^6\right)x^2 + 916.67x^3 - \frac{x^4}{24} - 416.67\langle x - 2000\rangle^3 + C_2$$

$y = 0$ at $x = 0$ $\quad \therefore C_2 = 0$, and therefore

$$y = -\frac{1}{24EI}\left[114\left(10^6\right)x^2 - 22\left(10^3\right)x^3 + x^4 + 10\left(10^3\right)\langle x - 2000\rangle^3\right]$$

$$y_B = -\frac{1}{24(207)10^3(4.14)10^6}\left[114\left(10^6\right)3000^2 - 22\left(10^3\right)3000^3 + 3000^4 + 10\left(10^3\right)(3000 - 2000)^3\right]$$

$$= -25.4 \text{ mm} \qquad \textit{Ans.}$$

$M_O = 9.5\ (10^6)$ N·m. The maximum stress is compressive at the bottom of the beam where $y = 29.0 - 100 = -71$ mm

$$\sigma_{\max} = -\frac{My}{I} = -\frac{-9.5\left(10^6\right)(-71)}{4.14(10^6)} = -163\left(10^6\right)\text{Pa} = -163\text{MPa} \qquad \textit{Ans.}$$

The solutions are the same as Prob. 4-10.

---

**4-57** See Prob. 4-11 for reactions: $R_O = 465$ lbf and $R_C = 285$ lbf. Using lbf and inch units

$M = 465\,x - 450\,\langle x - 72\rangle^1 - 300\,\langle x - 120\rangle^1$

$$EI\frac{dy}{dx} = 232.5x^2 - 225\langle x-72\rangle^2 - 150\langle x-120\rangle^2 + C_1$$

$EIy = 77.5\,x^3 - 75\,\langle x - 72\rangle^3 - 50\,\langle x - 120\rangle^3 - C_1 x$

$y = 0$ at $x = 0 \quad \Rightarrow \quad C_2 = 0$

$y = 0$ at $x = 240$ in

$0 = 77.5(240^3) - 75(240 - 72)^3 - 50(240 - 120)^3 + C_1\,x \quad \Rightarrow \quad C_1 = -\,2.622(10^6)$ lbf·in$^2$

and,

$EIy = 77.5\,x^3 - 75\,\langle x - 72\rangle^3 - 50\,\langle x - 120\rangle^3 - 2.622(10^6)\,x$

Substituting $y = -\,0.5$ in at $x = 120$ in gives

$$30(10^6)\,I\,(-\,0.5) = 77.5\,(120^3) - 75(120 - 72)^3 - 50(120 - 120)^3 - 2.622(10^6)(120)$$

$I = 12.60$ in$^4$

Select two 5 in × 6.7 lbf/ft channels; from Table A-7, $I = 2(7.49) = 14.98$ in$^4$

$$y_{\text{midspan}} = \frac{12.60}{14.98}\left(-\frac{1}{2}\right) = -0.421 \text{ in} \qquad \textit{Ans.}$$

The maximum moment occurs at $x = 120$ in where $M_{\max} = 34.2(10^3)$ lbf·in

$$\sigma_{\max} = \frac{Mc}{I} = \frac{34.2(10^3)(2.5)}{14.98} = 5\,710 \text{ psi} \qquad \text{O.K.}$$

The solutions are the same as Prob. 4-17.

---

**4-58** $I = \pi(1.5^4)/64 = 0.2485$ in$^4$, and $w = 150/12 = 12.5$ lbf/in.

$$R_O = \frac{1}{2}(12.5)39 + \frac{24}{39}(340) = 453.0 \text{ lbf}$$

$$M = 453.0x - \frac{12.5}{2}x^2 - 340\langle x-15\rangle^1$$

$$EI\frac{dy}{dx} = 226.5x^2 - \frac{12.5}{6}x^3 - 170\langle x-15\rangle^2 + C_1$$

$$EIy = 75.5x^3 - 0.5208x^4 - 56.67\langle x-15\rangle^3 + C_1 x + C_2$$

$y = 0$ at $x = 0 \;\Rightarrow\; C_2 = 0$

$y = 0$ at $x = 39$ in $\quad\Rightarrow\quad C_1 = -6.385(10^4)$ lbf·in$^2$ Thus,

$$y = \frac{1}{EI}\left[75.5x^3 - 0.5208x^4 - 56.67\langle x-15\rangle^3 - 6.385\left(10^4\right)x\right]$$

Evaluating at $x = 15$ in,

$$y_A = \frac{1}{30(10^6)(0.2485)}\left[75.5\left(15^3\right) - 0.5208\left(15^4\right) - 56.67(15-15)^3 - 6.385\left(10^4\right)(15)\right]$$
$$= -0.0978 \text{ in} \quad Ans.$$

$$y_{\text{midspan}} = \frac{1}{30(10^6)(0.2485)}\left[75.5\left(19.5^3\right) - 0.5208\left(19.5^4\right) - 56.67(19.5-15)^3 - 6.385\left(10^4\right)(19.5)\right]$$
$$= -0.1027 \text{ in} \quad Ans.$$

5 % difference *Ans.*

The solutions are the same as Prob. 4-12.

---

**4-59** $I = 0.05 \text{ in}^4$, $R_A = \frac{3(14)100}{10} = 420 \text{ lbf}\uparrow$ and $R_B = \frac{7(14)100}{10} = 980 \text{ lbf}\uparrow$

$M = 420x - 50x^2 + 980\langle x - 10\rangle^1$

$$EI\frac{dy}{dx} = 210x^2 - 16.667x^3 + 490\langle x-10\rangle^2 + C_1$$

$$EIy = 70x^3 - 4.167x^4 + 163.3\langle x-10\rangle^3 + C_1x + C_2$$

$y = 0$ at $x = 0 \Rightarrow C_2 = 0$
$y = 0$ at $x = 10$ in $\Rightarrow C_1 = -2\,833 \text{ lbf·in}^2$. Thus,

$$y = \frac{1}{30\left(10^6\right)0.05}\left[70x^3 - 4.167x^4 + 163.3\langle x-10\rangle^3 - 2833x\right]$$
$$= 6.667\left(10^{-7}\right)\left[70x^3 - 4.167x^4 + 163.3\langle x-10\rangle^3 - 2833x\right] \quad Ans.$$

The tabular results and plot are exactly the same as Prob. 4-21.

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**4-60** $R_A = R_B = 400$ N, and $I = 6(32^3)/12 = 16\,384 \text{ mm}^4$.
First half of beam,
$M = -400x + 400\langle x - 300\rangle^1$

$$EI\frac{dy}{dx} = -200x^2 + 200\langle x-300\rangle^2 + C_1$$

From symmetry, $dy/dx = 0$ at $x = 550$ mm $\Rightarrow$ $0 = -200(550^2) + 200(550-300)^2 + C_1$

$\Rightarrow$ $C_1 = 48(10^6) \text{ N·mm}^2$

$EIy = -66.67x^3 + 66.67\langle x - 300\rangle^3 + 48(10^6)x + C_2$

$y = 0$ at $x = 300$ mm $\Rightarrow$ $C_2 = -12.60(10^9)$ N·mm$^3$.

The term $(EI)^{-1} = [207(10^3)16\ 384]^{-1} = 2.949\ (10^{-10})$ Thus

$$y = 2.949\ (10^{-10})\ [-66.67\ x^3 + 66.67\ \langle x - 300 \rangle^3 + 48(10^6)\ x - 12.60(10^9)]$$

$y_O = -3.72$ mm *Ans.*

$$y|_{x = 550\text{ mm}} = 2.949\ (10^{-10})\ [-66.67\ (550^3) + 66.67\ (550 - 300)^3 + 48(10^6)\ 550 - 12.60(10^9)] = 1.11 \text{ mm} \quad \textit{Ans.}$$

The solutions are the same as Prob. 4-13.

---

**4-61**

$$\sum M_B = 0 = R_1 l + Fa - M_A \quad \Rightarrow \quad R_1 = \frac{1}{l}\left(M_A - Fa\right)$$

$$\sum M_A = 0 = M_A + R_2 l - F(l + a) \quad \Rightarrow \quad R_2 = \frac{1}{l}\left(Fl + Fa - M_A\right)$$

$$M = R_1 x - M_A + R_2 \left\langle x - l \right\rangle^1$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_1 x^2 - M_A x + \frac{1}{2}R_2 \left\langle x - l \right\rangle^2 + C_1$$

$$EIy = \frac{1}{6}R_1 x^3 - \frac{1}{2}M_A x^2 + \frac{1}{6}R_2 \left\langle x - l \right\rangle^3 + C_1 x + C_2$$

$y = 0$ at $x = 0$ $\Rightarrow$ $C_2 = 0$

$y = 0$ at $x = l$ $\Rightarrow$ $C_1 = -\frac{1}{6}R_1 l^2 + \frac{1}{2}M_A l$. Thus,

$$EIy = \frac{1}{6}R_1 x^3 - \frac{1}{2}M_A x^2 + \frac{1}{6}R_2 \left\langle x - l \right\rangle^3 + \left(-\frac{1}{6}R_1 l^2 + \frac{1}{2}M_A l\right)x$$

$$y = \frac{1}{6EIl}\left[\left(M_A - Fa\right)x^3 - 3M_A x^2 l + \left(Fl + Fa - M_A\right)\left\langle x - l\right\rangle^3 + \left(Fal^2 + 2M_A l^2\right)x\right] \qquad \textit{Ans.}$$

In regions,

$$y_{AB} = \frac{1}{6EIl}\left[\left(M_A - Fa\right)x^3 - 3M_A x^2 l + \left(Fal^2 + 2M_A l^2\right)x\right]$$

$$= \frac{x}{6EIl}\left[M_A\left(x^2 - 3lx + 2l^2\right) + Fa\left(l^2 - x^2\right)\right] \qquad \textit{Ans.}$$

$$y_{BC} = \frac{1}{6EIl}\left[(M_A - Fa)x^3 - 3M_A x^2 l + (Fl + Fa - M_A)(x-l)^3 + (Fal^2 + 2M_A l^2)x\right]$$

$$= \frac{1}{6EIl}\left\{M_A\left[x^3 - 3x^2 l - (x-l)^3 + 2xl^2\right] + F\left[-ax^3 + (l+a)(x-l)^3 + axl^2\right]\right\}$$

$$= \frac{1}{6EIl}\left\{-M_A(x-l)l^2 + Fl(x-l)\left[(x-l)^2 - a(3x-l)\right]\right\}$$

$$= \frac{(x-l)}{6EI}\left\{-M_A l + F\left[(x-l)^2 - a(3x-l)\right]\right\} \qquad Ans.$$

The solutions reduce to the same as Prob. 4-17.

---

**4-62** $\sum M_D = 0 = R_1 l - w(b-a)\left[l - b + \frac{1}{2}(b-a)\right] \quad \Rightarrow \quad R_1 = \frac{w(b-a)}{2l}(2l - b - a)$

$$M = R_1 x - \frac{w}{2}\langle x-a\rangle^2 + \frac{w}{2}\langle x-b\rangle^2$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_1 x^2 - \frac{w}{6}\langle x-a\rangle^3 + \frac{w}{6}\langle x-b\rangle^3 + C_1$$

$$EIy = \frac{1}{6}R_1 x^3 - \frac{w}{24}\langle x-a\rangle^4 + \frac{w}{24}\langle x-b\rangle^4 + C_1 x + C_2$$

$y = 0$ at $x = 0 \quad \Rightarrow \quad C_2 = 0$
$y = 0$ at $x = l$

$$C_1 = -\frac{1}{l}\left[\frac{1}{6}R_1 l^3 - \frac{w}{24}(l-a)^4 + \frac{w}{24}(l-b)^4\right]$$

$$y = \frac{1}{EI}\left\{\frac{1}{6}\frac{w(b-a)}{2l}(2l-b-a)x^3 - \frac{w}{24}\langle x-a\rangle^4 + \frac{w}{24}\langle x-b\rangle^4 \right.$$
$$\left. - x\frac{1}{l}\left[\frac{1}{6}\frac{w(b-a)}{2l}(2l-b-a)l^3 - \frac{w}{24}(l-a)^4 + \frac{w}{24}(l-b)^4\right]\right\}$$

$$= \frac{w}{24EIl}\left\{2(b-a)(2l-b-a)x^3 - l\langle x-a\rangle^4 + l\langle x-b\rangle^4 \right.$$
$$\left. - x\left[2(b-a)(2l-b-a)l^2 - (l-a)^4 + (l-b)^4\right]\right\} \qquad Ans.$$

The above answer is sufficient. In regions,

$$y_{AB}=\frac{w}{24EIl}\left\{2(b-a)(2l-b-a)x^3-x\left[2(b-a)(2l-b-a)l^2-(l-a)^4+(l-b)^4\right]\right\}$$
$$=\frac{wx}{24EIl}\left[2(b-a)(2l-b-a)x^2-2(b-a)(2l-b-a)l^2+(l-a)^4-(l-b)^4\right]$$

$$y_{BC}=\frac{w}{24EIl}\left\{2(b-a)(2l-b-a)x^3-l(x-a)^4\right.$$
$$\left.-x\left[2(b-a)(2l-b-a)l^2-(l-a)^4+(l-b)^4\right]\right\}$$

$$y_{CD}=\frac{w}{24EIl}\left\{2(b-a)(2l-b-a)x^3-l(x-a)^4+l(x-b)^4\right.$$
$$\left.-x\left[2(b-a)(2l-b-a)l^2-(l-a)^4+(l-b)^4\right]\right\}$$

These equations can be shown to be equivalent to the results found in Prob. 4-19.

---

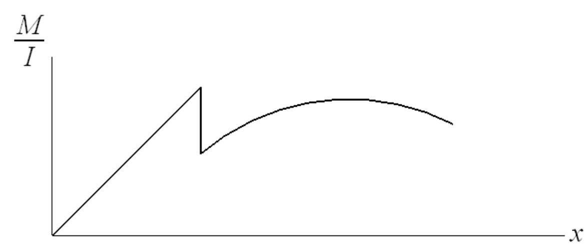
**4-63** $I_1=\pi(1.375^4)/64=0.1755\text{ in}^4$, $I_2=\pi(1.75^4)/64=0.4604\text{ in}^4$,

$R_1=0.5(180)(10)=900$ lbf

Since the loading and geometry are symmetric, we will only write the equations for half the beam

For $\underline{0\le x\le 8\text{ in}}$ $\quad M=900x-90\langle x-3\rangle^2$

At $x=3$, $M=2700$ lbf·in




Writing an equation for $M/I$, as seen in the figure, the magnitude and slope reduce since $I_2>I_1$. To reduce the magnitude at $x=3$ in, we add the term, $-2700(1/I_1-1/I_2)\langle x-3\rangle^0$. The slope of 900 at $x=3$ in is also reduced. We account for this with a ramp function, $\langle x-3\rangle^1$. Thus,

$$\frac{M}{I}=\frac{900x}{I_1}-2700\left(\frac{1}{I_1}-\frac{1}{I_2}\right)\langle x-3\rangle^0-900\left(\frac{1}{I_1}-\frac{1}{I_2}\right)\langle x-3\rangle^1-\frac{90}{I_2}\langle x-3\rangle^2$$
$$=5128x-9520\langle x-3\rangle^0-3173\langle x-3\rangle^1-195.5\langle x-3\rangle^2$$

$$E\frac{dy}{dx}=2564x^2-9520\langle x-3\rangle^1-1587\langle x-3\rangle^2-65.17\langle x-3\rangle^3+C_1$$

Boundary Condition: $\dfrac{dy}{dx}=0$ at $x=8$ in

$$0 = 2564(8)^2 - 9520(8-3) - 1587(8-3)^2 - 65.17(8-3)^3 + C_1 \Rightarrow$$

$C_1 = -68.67\ (10^3)$ lbf/in$^2$

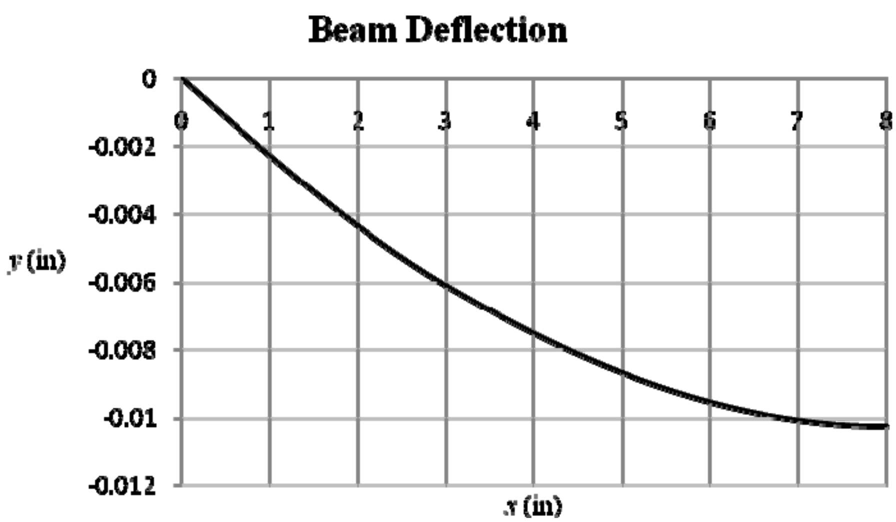
$$Ey = 854.7x^3 - 4760\langle x-3\rangle^2 - 529\langle x-3\rangle^3 - 16.29\langle x-3\rangle^4 - 68.67(10^3)x + C_2$$

$y = 0$ at $x = 0 \quad \Rightarrow \quad C_2 = 0$

Thus, for $\underline{0 \le x \le 8 \text{ in}}$

$$y = \frac{1}{30(10^6)}\left[854.7x^3 - 4760\langle x-3\rangle^2 - 529\langle x-3\rangle^3 - 16.29\langle x-3\rangle^4 - 68.7(10^3)x\right] \quad \textit{Ans.}$$

Using a spreadsheet, the following graph represents the deflection equation found above




The maximum is $y_{max} = -0.0102$ in at $x = 8$ in *Ans.*

---

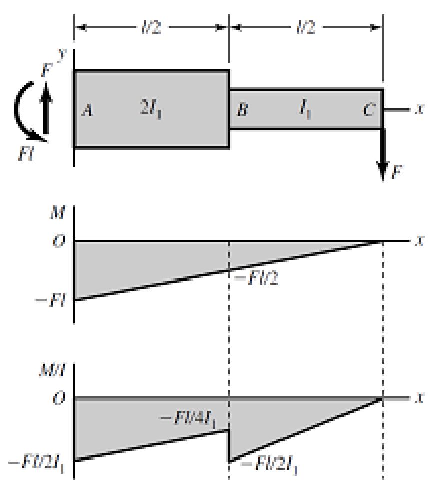
**4-64** The force and moment reactions at the left support are $F$ and $Fl$ respectively. The bending moment equation is

$M = Fx - Fl$

Plots for $M$ and $M/I$ are shown.

$M/I$ can be expressed using singularity functions

$$\frac{M}{I} = \frac{F}{2I_1}x - \frac{Fl}{2I_1} - \frac{Fl}{4I_1}\left\langle x - \frac{l}{2}\right\rangle^0 + \frac{F}{2I_1}\left\langle x - \frac{l}{2}\right\rangle^1$$



where the step down and increase in slope at $x = l/2$ are given by the last two terms. Integrate

$$E\frac{dy}{dx} = \frac{F}{4I_1}x^2 - \frac{Fl}{2I_1}x - \frac{Fl}{4I_1}\left\langle x-\frac{l}{2}\right\rangle^1 + \frac{F}{4I_1}\left\langle x-\frac{l}{2}\right\rangle^2 + C_1$$

$dy/dx = 0$ at $x = 0 \quad \Rightarrow \quad C_1 = 0$

$$Ey = \frac{F}{12I_1}x^3 - \frac{Fl}{4I_1}x^2 - \frac{Fl}{8I_1}\left\langle x-\frac{l}{2}\right\rangle^2 + \frac{F}{12I_1}\left\langle x-\frac{l}{2}\right\rangle^3 + C_2$$

$y = 0$ at $x = 0 \quad \Rightarrow \quad C_2 = 0$

$$y = \frac{F}{24EI_1}\left(2x^3 - 6lx^2 - 3l\left\langle x-\frac{l}{2}\right\rangle^2 + 2\left\langle x-\frac{l}{2}\right\rangle^3\right)$$

$$y\Big|_{x=l/2} = \frac{F}{24EI_1}\left[2\left(\frac{l}{2}\right)^3 - 6l\left(\frac{l}{2}\right)^2 - 3l(0) + 2(0)\right] = -\frac{5Fl^3}{96EI_1} \qquad \textit{Ans.}$$

$$y\Big|_{x=l} = \frac{F}{24EI_1}\left[2(l)^3 - 6l\left(l^2\right) - 3l\left(l-\frac{l}{2}\right)^2 + 2\left(x-\frac{l}{2}\right)^3\right] = -\frac{3Fl^3}{16EI_1} \qquad \textit{Ans.}$$

The answers are identical to Ex. 4-10.

---

**4-65** Place a dummy force, $Q$, at the center. The reaction, $R_1 = wl/2 + Q/2$

$$M = \left(\frac{wl}{2} + \frac{Q}{2}\right)x - \frac{wx^2}{2} \qquad \frac{\partial M}{\partial Q} = \frac{x}{2}$$

Integrating for half the beam and doubling the results

$$y_{\max} = \left(2\frac{1}{EI}\int_0^{l/2} M\left(\frac{\partial M}{\partial Q}\right)dx\right)_{Q=0} = \frac{2}{EI}\int_0^{l/2}\left[\left(\frac{wl}{2}\right)x - \frac{wx^2}{2}\right]\left(\frac{x}{2}\right)dx$$

Note, after differentiating with respect to $Q$, it can be set to zero

$$y_{\max} = \frac{w}{2EI}\int_0^{l/2} x^2(l-x)dx = \frac{w}{2EI}\left(\frac{x^3 l}{3} - \frac{x^4}{4}\right)\Bigg|_0^{l/2} = \frac{5w}{384EI} \qquad \textit{Ans.}$$

---

**4-66** Place a fictitious force $Q$ pointing downwards at the end. Use the variable $\bar{x}$ originating at the free end at positive to the left

$$M = -Qx - \frac{wx^2}{2} \qquad \frac{\partial M}{\partial Q} = -x$$

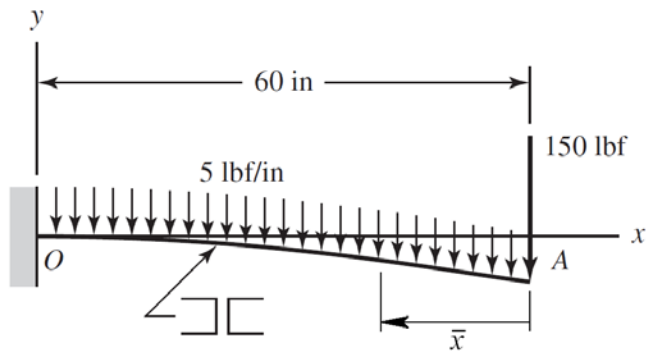
$$y_{max} = \left[\frac{1}{EI}\int_0^l M\left(\frac{\partial M}{\delta Q}\right)dx\right]_{Q=0} = \frac{1}{EI}\int_0^l\left(-\frac{wx^2}{2}\right)(-x)\,dx = \frac{w}{2EI}\int_0^l x^3 dx$$

$$= \frac{wl^4}{8EI} \qquad \textit{Ans.}$$

---

**4-67** From Table A-7, $I_{1\text{-}1}$ = 1.85 in$^4$. Thus, $I$ = 2(1.85) = 3.70 in$^4$

First treat the end force as a variable, $F$. Adding weight of channels of 2(5)/12 = 0.833 lbf/in. Using the variable $\bar{x}$ as shown in the figure




$$M = -F\bar{x} - \frac{5.833}{2}\bar{x}^2 = -F\bar{x} - 2.917\bar{x}^2$$

$$\frac{\partial M}{\partial F} = -\bar{x}$$

$$\delta_A = \frac{1}{EI}\int_0^{60} M\frac{\partial M}{\partial F}d\bar{x} = \frac{1}{EI}\int_0^{60}(F\bar{x} + 2.917\bar{x}^2)(\bar{x})\,d\bar{x}$$

$$= \frac{(150/3)(60^3) + (2.917/4)(60^4)}{30(10^6)(3.70)} = 0.182 \text{ in} \quad \text{in the direction of the 150 lbf force}$$

$$\therefore y_A = -0.182 \text{ in} \qquad \textit{Ans.}$$

---

**4-68** The energy includes torsion in $AC$, torsion in $CO$, and bending in $AB$.

Neglecting transverse shear in $AB$

$$M = Fx, \quad \frac{\partial M}{\partial F} = x$$

In $AC$ and $CO$,

$$T = Fl_{AB}, \quad \frac{\partial T}{\partial F} = l_{AB}$$

The total energy is

$$U = \left(\frac{T^2 l}{2GJ}\right)_{AC} + \left(\frac{T^2 l}{2GJ}\right)_{CO} + \int_0^{l_{AB}}\frac{M^2}{2EI_{AB}}dx$$

The deflection at the tip is

$$\delta = \frac{\partial U}{\partial F} = \frac{Tl_{AC}}{GJ_{AC}}\frac{\partial T}{\partial F} + \frac{Tl_{CO}}{GJ_{CO}}\frac{\partial T}{\partial F} + \int_0^{l_{AB}} \frac{M}{EI_3}\frac{\partial M}{\partial F}dx = \frac{Tl_{AC}l_{AB}}{GJ_{AC}} + \frac{Tl_{CO}l_{AB}}{GJ_{CO}} + \frac{1}{EI_{AB}}\int_0^{l_{AB}} Fx^2 dx$$

$$\delta = \frac{Tl_{AC}l_{AB}}{GJ_{AC}} + \frac{Tl_{CO}l_{AB}}{GJ_{CO}} + \frac{Fl_{AB}^3}{3EI_{AB}} = \frac{Fl_{AC}l_{AB}^2}{G\left(\pi d_{AC}^4/32\right)} + \frac{Fl_{CO}l_{AB}^2}{G\left(\pi d_{CO}^4/32\right)} + \frac{Fl_{AB}^3}{3E\left(\pi d_{AB}^4/64\right)}$$

$$= \frac{32Fl_{AB}^2}{\pi}\left(\frac{l_{AC}}{Gd_{AC}^4} + \frac{l_{CO}}{Gd_{CO}^4} + \frac{2l_{AB}}{3Ed_{AB}^4}\right)$$

$$k = \frac{F}{\delta} = \frac{\pi}{32l_{AB}^2}\left(\frac{l_{AC}}{Gd_{AC}^4} + \frac{l_{CO}}{Gd_{CO}^4} + \frac{2l_{AB}}{3Ed_{AB}^4}\right)^{-1}$$

$$= \frac{\pi}{32\left(200^2\right)}\left(\frac{200}{79.3\left(10^3\right)18^4} + \frac{200}{79.3\left(10^3\right)12^4} + \frac{2(200)}{3(207)10^3\left(8^4\right)}\right)^{-1} = 8.10 \text{ N/mm} \quad \textit{Ans.}$$

---

**4-69** $I_1 = \pi(1.375^4)/64 = 0.1755 \text{ in}^4$, $I_2 = \pi(1.75^4)/64 = 0.4604 \text{ in}^4$

Place a fictitious force $Q$ pointing downwards at the midspan of the beam, $x = 8$ in

$$R_1 = \frac{1}{2}(10)180 + \frac{1}{2}Q = 900 + 0.5Q$$

For $\underline{0 \le x \le 3 \text{ in}}$ $\quad M = \left(900 + 0.5Q\right)x \qquad \frac{\partial M}{\partial Q} = 0.5x$

For $\underline{3 \le x \le 13 \text{ in}}$ $\quad M = \left(900 + 0.5Q\right)x - 90(x-3)^2 \qquad \frac{\partial M}{\partial Q} = 0.5x$

By symmetry it is equivalent to use twice the integral from 0 to 8

$$\delta = \left(2\int_0^8 \frac{M}{EI}\frac{\partial M}{\partial Q}dx\right)_{Q=0} = \frac{1}{EI_1}\int_0^3 900x^2 dx + \frac{1}{EI_2}\int_3^8 \left[900x - 90\left(x-3\right)^2\right]x\,dx$$

$$= \frac{300x^3}{EI_1}\Bigg|_0^3 + \frac{1}{EI_2}\left[300x^3 - 90\left(\frac{1}{4}x^4 - 2x^3 + \frac{9}{2}x^2\right)\right]\Bigg|_3^8$$

$$= \frac{8100}{EI_1} + \frac{1}{EI_2}\left[145.5\left(10^3\right) - 25.31\left(10^3\right)\right] = \frac{8100}{30\left(10^6\right)0.1755} + \frac{120.2\left(10^3\right)}{30\left(10^6\right)0.4604}$$

$$= 0.0102 \text{ in} \qquad \textit{Ans.}$$

---

**4-70** $I = \pi(0.5^4)/64 = 3.068\ (10^{-3})\ \text{in}^4$, $J = 2I = 6.136\ (10^{-3})\ \text{in}^4$, $A = \pi(0.5^2)/4 = 0.1963\ \text{in}^2$.

Consider *x* to be in the direction of *OA*, *y* vertically upward, and *z* in the direction of *AB*. Resolve the force *F* into components in the *x* and *y* directions obtaining 0.6 *F* in the horizontal direction and 0.8 *F* in the negative vertical direction. The 0.6 *F* force creates strain energy in the form of bending in *AB* and *OA*, and tension in *OA*. The 0.8 *F* force creates strain energy in the form of bending in *AB* and *OA*, and torsion in *OA*. Use the dummy variable $\bar{x}$ to originate at the end where the loads are applied on each segment,

<u>0.6 *F*</u>: *AB* $\quad M = 0.6F\,\bar{x} \qquad \dfrac{\partial M}{\partial F} = 0.6\,\bar{x}$

*OA* $\quad M = 4.2F \qquad \dfrac{\partial M}{\partial F} = 4.2$

$F_a = 0.6F \qquad \dfrac{\partial F_a}{\partial F} = 0.6$

<u>0.8 *F*</u>: *AB* $\quad M = 0.8F\,\bar{x} \qquad \dfrac{\partial M}{\partial F} = 0.8\,\bar{x}$

*OA* $\quad M = 0.8F\,\bar{x} \qquad \dfrac{\partial M}{\partial F} = 0.8\,\bar{x}$

$T = 5.6F \qquad \dfrac{\partial T}{\partial F} = 5.6$

Once the derivatives are taken the value of *F* = 15 lbf can be substituted in. The deflection of *B* in the direction of *F* is*

$$\begin{aligned}
(\delta_B)_F &= \frac{\partial U}{\partial F} = \left(\frac{F_a L}{AE}\right)_{OA}\frac{\partial F_a}{\partial F} + \left(\frac{TL}{JG}\right)_{OA}\frac{\partial T}{\partial F} + \frac{1}{EI}\sum\int M\frac{\partial M}{\partial F}\,d\bar{x} \\
&= \frac{0.6(15)15}{0.1963(30)10^6}(0.6) + \frac{5.6(15)15}{6.136\left(10^{-3}\right)11.5\left(10^6\right)}(5.6) \\
&\quad + \frac{15}{30\left(10^6\right)3.068\left(10^{-3}\right)}\int_0^7 (0.6\bar{x})^2\,d\bar{x} + \frac{15\left(4.2^2\right)}{30\left(10^6\right)3.068\left(10^{-3}\right)}\int_0^{15} d\bar{x} + \\
&\quad + \frac{15}{30\left(10^6\right)3.068\left(10^{-3}\right)}\int_0^7 (0.8\bar{x})^2\,d\bar{x} + \frac{15}{30\left(10^6\right)3.068\left(10^{-3}\right)}\int_0^{15} (0.8\bar{x})^2\,d\bar{x} \\
&= 1.38\left(10^{-5}\right) + 0.1000 + 6.71\left(10^{-3}\right) + 0.0431 + 0.0119 + 0.1173 \\
&= 0.279\ \text{in} \qquad \textit{Ans.}
\end{aligned}$$

*Note. This is not the actual deflection of point *B*. For this, dummy forces must be placed on *B* in the *x*, *y*, and *z* directions. Determine the energy due to each, take derivatives, and then substitute the values of $F_x = 9$ lbf, $F_y = -12$ lbf, and $F_z = 0$. This can be done separately and then use superposition. The actual deflections of *B* are

$$\boldsymbol{\delta_B} = 0.0831\ \mathbf{i} - 0.2862\ \mathbf{j} - 0.00770\ \mathbf{k} \text{ in}$$

From this, the deflection of *B* in the direction of *F* is

$$(\delta_B)_F = 0.6(0.0831) + 0.8(0.2862) = 0.279 \text{ in}$$

which agrees with our result.

---

**4-71** Strain energy. *AB*: Bending and torsion, *BC*: Bending and torsion, *CD*: Bending.
$I_{AB} = \pi(1^4)/64 = 0.04909$ in$^4$, $J_{AB} = 2\,I_{AB} = 0.09818$ in$^4$, $I_{BC} = 0.25(1.5^3)/12 = 0.07031$ in$^4$, $I_{CD} = \pi(0.75^4)/64 = 0.01553$ in$^4$.
For the torsion of bar *BC*, Eq. (3-41) is in the form of $\theta = TL/(JG)$, where the equivalent of $J$ is $J_{eq} = \beta bc^3$. With $b/c = 1.5/0.25 = 6$, $J_{BC} = \beta bc^3 = 0.299(1.5)0.25^3 = 7.008\,(10^{-3})$ in$^4$.

Use the dummy variable $\bar{x}$ to originate at the end where the loads are applied on each segment,

*AB*: Bending $\quad M = F\bar{x} + 2F \qquad \dfrac{\partial M}{\partial F} = \bar{x} + 2$

Torsion $\quad T = 5F \qquad \dfrac{\partial T}{\partial F} = 5$

*BC*: Bending $\quad M = F\bar{x} \qquad \dfrac{\partial M}{\partial F} = \bar{x}$

Torsion $\quad T = 2F \qquad \dfrac{\partial T}{\partial F} = 2$

*CD*: Bending $\quad M = F\bar{x} \qquad \dfrac{\partial M}{\partial F} = \bar{x}$

$$\delta_D = \frac{\partial U}{\partial F} = \sum \frac{Tl}{JG}\frac{\partial T}{\partial F} + \sum \frac{1}{EI}\int M \frac{\partial M}{\partial F}\,d\bar{x}$$

$$= \frac{5F(6)}{0.09818(11.5)10^6}(5) + \frac{2F(5)}{7.008(10^{-3})11.5(10^6)}2 + \frac{1}{30(10^6)0.04909}\int_0^6 F(\bar{x}+2)^2 d\bar{x}$$

$$+ \frac{1}{30(10^6)0.07031}\int_0^5 F\bar{x}^2 d\bar{x} + \frac{1}{30(10^6)0.01553}\int_0^2 F\bar{x}^2 d\bar{x}$$

$$= 1.329(10^{-4})F + 2.482(10^{-4})F + 1.141(10^{-4})F + 1.98(10^{-5})F + 5.72(10^{-6})F$$

$$= 5.207(10^{-4})F = 5.207(10^{-4})200 = 0.104 \text{ in} \qquad \textit{Ans.}$$

---

**4-72** $A_{AB} = \pi(1^2)/4 = 0.7854$ in$^2$, $I_{AB} = \pi(1^4)/64 = 0.04909$ in$^4$, $I_{BC} = 1.5\,(0.25^3)/12 = 1.953$ $(10^{-3})$ in$^4$, $A_{CD} = \pi(0.75^2)/4 = 0.4418$ in$^2$, $I_{AB} = \pi(0.75^4)/64 = 0.01553$ in$^4$. For $(\delta_D)_x$ let $F = F_x = -150$ lbf and $F_z = -100$ lbf . Use the dummy variable $\bar{x}$ to originate at the end where the loads are applied on each segment,

$CD$: $\quad M_y = F_z\bar{x} \qquad \dfrac{\partial M_y}{\partial F} = 0$

$\quad F_a = F \qquad \dfrac{\partial F_a}{\partial F} = 1$

$BC$: $\quad M_y = F\,\bar{x} + 2F_z \qquad \dfrac{\partial M_y}{\partial F} = \bar{x}$

$\quad F_a = F_z \qquad \dfrac{\partial F_a}{\partial F} = 0$

$AB$: $\quad M_y = 5F + 2F_z + F_z\bar{x} \qquad \dfrac{\partial M_y}{\partial F} = 5$

$\quad F_a = F \qquad \dfrac{\partial F_a}{\partial F} = 1$

$$(\delta_D)_x = \frac{\partial U}{\partial F} = \left(\frac{FL}{AE}\right)_{CD}\frac{\partial F_a}{\partial F} + \frac{1}{EI_{BC}}\int_0^5 \left(F\,\bar{x} + 2F_z\right)\bar{x}\,d\,\bar{x}$$

$$+\frac{1}{EI_{AB}}\int_0^6 \left(5F + 2F_z + F_z\bar{x}\right)(5)\,d\,\bar{x} + \left(\frac{FL}{AE}\right)_{AB}\frac{\partial F_a}{\partial F}$$

$$= \frac{F(2)}{0.4418(30)10^6}(1) + \frac{1}{30\left(10^6\right)1.953\left(10^{-3}\right)}\left[\frac{F}{3}(5)^3 + F_z\left(5^2\right)\right]$$

$$+\frac{1}{30\left(10^6\right)0.04909}\left[25F(6) + 10F_z(6) + \frac{F_z}{2}\left(6^2\right)5\right] + \frac{F(6)}{0.7854(30)10^6}(1)$$

$$= 1.509\left(10^{-7}\right)F + 7.112\left(10^{-4}\right)F + 4.267\left(10^{-4}\right)F_z + 1.019\left(10^{-4}\right)F$$

$$+1.019\left(10^{-4}\right)F_z + 2.546\left(10^{-7}\right)F = 8.135\left(10^{-4}\right)F + 5.286\left(10^{-4}\right)F_z$$

Substituting $F = F_x = -150$ lbf and $F_z = -100$ lbf gives

$$(\delta_D)_x = 8.135\left(10^{-4}\right)(-150) + 5.286\left(10^{-4}\right)(-100) = -0.1749 \text{ in} \qquad \textit{Ans.}$$

---

**4-73** $I_{OA} = I_{BC} = \pi(1.5^4)/64 = 0.2485$ in$^4$, $J_{OA} = J_{BC} = 2\,I_{OA} = 0.4970$ in$^4$, $I_{AB} = \pi(1^4)/64 =$ $0.04909$ in$^4$, $J_{AB} = 2\,I_{AB} = 0.09818$ in$^4$, $I_{CD} = \pi(0.75^4)/64 = 0.01553$ in$^4$

Let $F_y = F$, and use the dummy variable $\bar{x}$ to originate at the end where the loads are applied on each segment,

$OC$: $\quad M = F\,\bar{x} \quad \dfrac{\partial M}{\partial F} = \bar{x}, \qquad T = 12F \quad \dfrac{\partial T}{\partial F} = 12$

$DC$: $\quad M = F\,\bar{x} \quad \dfrac{\partial M}{\partial F} = \bar{x}$

$$(\delta_D)_y = \frac{\partial U}{\partial F} = \sum \left(\frac{TL}{JG}\right)_{OC} \frac{\partial T}{\partial F} + \sum \frac{1}{EI} \int M \frac{\partial M}{\partial F}\, d\,\bar{x}$$

The terms involving the torsion and bending moments in *OC* must be split up because of the changing second-area moments.

$$(\delta_D)_y = \frac{12F(4)}{0.4970(11.5)10^6}(12) + \frac{12F(9)}{0.09818(11.5)10^6}(12) + \frac{1}{30(10^6)0.2485}\int_0^2 F\,\bar{x}^2 d\,\bar{x}$$
$$+ \frac{1}{30(10^6)0.04909}\int_2^{11} F\,\bar{x}^2 d\,\bar{x} + \frac{1}{30(10^6)0.2485}\int_{11}^{13} F\,\bar{x}^2 d\,\bar{x} + \frac{1}{30(10^6)0.01553}\int_0^{12} F\,\bar{x}^2 d\,\bar{x}$$
$$= 1.008(10^{-4})F + 1.148(10^{-3})F + 3.58(10^{-7})F$$
$$+ 2.994(10^{-4})F + 3.872(10^{-5})F + 1.2363(10^{-3})F$$
$$= 2.824(10^{-3})F = 2.824(10^{-3})250 = 0.706 \text{ in} \qquad \textit{Ans.}$$

For the simplified shaft *OC*,

$$(\delta_B)_y = \frac{12F(13)}{0.09818(11.5)10^6}(12) + \frac{1}{30(10^6)0.04909}\int_0^{13} F\,\bar{x}^2 d\,\bar{x} + \frac{1}{30(10^6)0.01553}\int_0^{12} F\,\bar{x}^2 d\,\bar{x}$$
$$= 1.6580(10^{-3})F + 4.973(10^{-4})F + 1.2363(10^{-3})F = 3.392(10^{-3})F = 3.392(10^{-3})250$$
$$= 0.848 \text{ in} \qquad \textit{Ans.}$$

Simplified is 0.848/0.706 = 1.20 times greater *Ans.*

---

**4-74** Place a dummy force *Q* pointing downwards at point *B*. The reaction at *C* is

$R_C = Q + (6/18)100 = Q + 33.33$

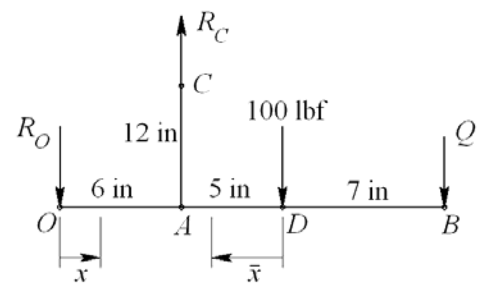
This is the axial force in member *BC*. Isolating the beam, we find that the moment is not a function of *Q*, and thus does not contribute to the strain energy. Thus, only energy in the member *BC* needs to be considered. Let the axial force in *BC* be *F*, where

$$F = Q + 33.33 \qquad \frac{\partial F}{\partial Q} = 1$$

$$\delta_B = \left.\frac{\partial U}{\partial Q}\right|_{Q=0} = \left[\left(\frac{FL}{AE}\right)_{BC}\frac{\partial F}{\partial Q}\right]_{Q=0} = \frac{(0+33.33)12}{\left[\pi\left(0.5^2\right)/4\right]30\left(10^6\right)}(1) = 6.79\left(10^{-5}\right)\text{ in} \qquad \textit{Ans.}$$

---

**4-75** $I_{OB} = 0.25(2^3)/12 = 0.1667 \text{ in}^4$




$A_{AC} = \pi(0.5^2)/4 = 0.1963 \text{ in}^2$

$\Sigma M_O = 0 = 6\,R_C - 11(100) - 18\,Q$

$R_C = 3Q + 183.3$

$\Sigma M_A = 0 = 6\,R_O - 5(100) - 12\,Q \quad \Rightarrow \quad R_O = 2Q + 83.33$

Bending in *OB*.

*BD*: Bending in *BD* is only due to *Q* which when set to zero after differentiation gives no contribution.

*AD*: Using the variable $\bar{x}$ as shown in the figure above

$$M = -100\bar{x} - Q\left(7+\bar{x}\right) \qquad \frac{\partial M}{\partial Q} = -\left(7+\bar{x}\right)$$

*OA*: Using the variable *x* as shown in the figure above

$$M = -\left(2Q + 83.33\right)x \qquad \frac{\partial M}{\partial Q} = -2x$$

Axial in *AC*:

$$F = 3Q + 183.3 \qquad \frac{\partial F}{\partial Q} = 3$$

$$\delta_B = \left(\frac{\partial U}{\partial Q}\right)_{Q=0} = \left[\left(\frac{FL}{AE}\right)\frac{\partial F}{\partial Q}\right]_{Q=0} + \left(\frac{1}{EI}\sum M\frac{\partial M}{\partial Q}dx\right)_{Q=0}$$

$$= \frac{183.3(12)}{0.1963(30)10^6}(3) + \frac{1}{EI}\int_0^5 (100\,\bar{x})(7+\bar{x})\,d\,\bar{x} + \int_0^6 2(83.33)x^2 dx$$

$$= 1.121(10^{-3}) + \frac{1}{10.4(10^6)0.1667}\left[100\int_0^5 \bar{x}(7+\bar{x})\,d\,\bar{x} + 166.7\int_0^6 x^2 dx\right]$$

$$= 1.121(10^{-3}) + 5.768(10^{-7})\left[100(129.2) + 166.7(72)\right] = 0.0155 \text{ in} \qquad Ans.$$

---

**4-76** There is no bending in *AB*. Using the variable $\theta$, rotating counterclockwise from *B*

$$M = PR\sin\theta \qquad \frac{\partial M}{\partial P} = R\sin\theta$$

$$F_r = P\cos\theta \qquad \frac{\partial F_r}{\partial P} = \cos\theta$$

$$F_\theta = P\sin\theta \qquad \frac{\partial F_\theta}{\partial P} = \sin\theta$$

$$\frac{\partial MF_\theta}{\partial P} = 2PR\sin^2\theta$$

$A = 6(4) = 24\text{ mm}^2,\ \ r_o = 40 + \tfrac{1}{2}(6) = 43\text{ mm},\ \ r_i = 40 - \tfrac{1}{2}(6) = 37\text{ mm},$

From Table 3-4, p.121, for a rectangular cross section

$$r_n = \frac{6}{\ln(43/37)} = 39.92489\text{ mm}$$

From Eq. (4-33), the eccentricity is $e = R - r_n$ =40 – 39.92489 = 0.07511 mm

From Table A-5, $E = 207(10^3)$ MPa, $G = 79.3(10^3)$ MPa

From Table 4-1, $C = 1.2$

From Eq. (4-38)

$$\delta = \int_0^{\frac{\pi}{2}} \frac{M}{AeE}\left(\frac{\partial M}{\partial P}\right)d\theta + \int_0^{\frac{\pi}{2}} \frac{F_\theta R}{AE}\left(\frac{\partial F_\theta}{\partial P}\right)d\theta - \int_0^{\frac{\pi}{2}} \frac{1}{AE}\frac{\partial(MF_\theta)}{\partial P}d\theta + \int_0^{\frac{\pi}{2}} \frac{CF_r R}{AG}\left(\frac{\partial F_r}{\partial P}\right)d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{P(R\sin\theta)^2}{AeE}d\theta + \int_0^{\frac{\pi}{2}} \frac{PR(\sin\theta)^2}{AE}d\theta - \int_0^{\frac{\pi}{2}} \frac{2PR\sin^2\theta}{AE}d\theta + \int_0^{\frac{\pi}{2}} \frac{CPR(\cos\theta)^2}{AG}d\theta$$

$$= \frac{\pi PR}{4AE}\left(\frac{R}{e} + 1 - 2 + \frac{EC}{G}\right) = \frac{\pi(10)(40)}{4(24)(207\cdot 10^3)}\left(\frac{40}{0.07511} + 1 - 2 + \frac{(207\cdot 10^3)(1.2)}{79.3\cdot 10^3}\right)$$

$\delta = 0.0338$ mm $\qquad$ *Ans.*

---

**4-77** Place a dummy force $Q$ pointing downwards at point $A$. Bending in $AB$ is only due to $Q$ which when set to zero after differentiation gives no contribution. For section $BC$ use the variable $\theta$, rotating counterclockwise from $B$

$$M = PR\sin\theta + Q\left(R + R\sin\theta\right) \qquad \frac{\partial M}{\partial Q} = R\left(1+\sin\theta\right)$$

$$F_r = \left(P+Q\right)\cos\theta \qquad \frac{\partial F_r}{\partial Q} = \cos\theta$$

$$F_\theta = \left(P+Q\right)\sin\theta \qquad \frac{\partial F_\theta}{\partial Q} = \sin\theta$$

$$MF_\theta = \left[PR\sin\theta + QR\left(1+\sin\theta\right)\right]\left(P+Q\right)\sin\theta$$

$$\frac{\partial MF_\theta}{\partial Q} = PR\sin^2\theta + +PR\sin\theta\left(1+\sin\theta\right) + 2QR\sin\theta\left(1+\sin\theta\right)$$

But after differentiation, we can set $Q = 0$. Thus,

$$\frac{\partial MF_\theta}{\partial Q} = PR\sin\theta\left(1+2\sin\theta\right)$$

$$A = 6(4) = 24\text{ mm}^2,\quad r_o = 40 + \tfrac{1}{2}(6) = 43\text{ mm},\quad r_i = 40 - \tfrac{1}{2}(6) = 37\text{ mm},$$

From Table 3-4, p.121, for a rectangular cross section

$$r_n = \frac{6}{\ln(43/37)} = 39.92489\text{ mm}$$

From Eq. (4-33), the eccentricity is $e = R - r_n = 40 - 39.92489 = 0.07511$ mm
From Table A-5, $E = 207(10^3)$ MPa, $G = 79.3(10^3)$ MPa
From Table 4-1, $C = 1.2$
From Eq. (4-38)

$$\delta = \int_0^{\frac{\pi}{2}} \frac{M}{AeE}\left(\frac{\partial M}{\partial Q}\right)d\theta + \int_0^{\frac{\pi}{2}} \frac{F_\theta R}{AE}\left(\frac{\partial F_\theta}{\partial Q}\right)d\theta - \int_0^{\frac{\pi}{2}} \frac{1}{AE}\frac{\partial\left(MF_\theta\right)}{\partial Q}d\theta + \int_0^{\frac{\pi}{2}} \frac{CF_r R}{AG}\left(\frac{\partial F_r}{\partial Q}\right)d\theta$$

$$= \frac{PR^2}{AeE}\int_0^{\frac{\pi}{2}}\sin\theta\left(1+\sin\theta\right)d\theta + \frac{PR}{AE}\int_0^{\frac{\pi}{2}}\sin^2\theta d\theta - \frac{PR}{AE}\int_0^{\frac{\pi}{2}}\sin\theta\left(1+2\sin\theta\right)d\theta$$

$$+\frac{CPR}{AG}\int_0^{\frac{\pi}{2}}\cos^2\theta d\theta$$

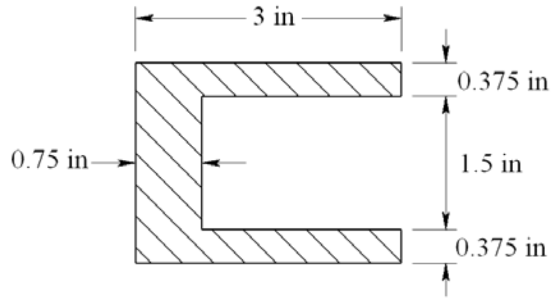
$$= \left(\frac{\pi}{4}+1\right)\frac{PR^2}{AeE} + \frac{\pi}{4}\frac{PR}{AE} - \left(\frac{\pi}{4}+2\right)\frac{PR}{AE} + \frac{\pi}{4}\frac{CPR}{AG} = \frac{PR}{AE}\left[\left(\frac{\pi}{4}+1\right)\frac{R}{e} - 2 + \frac{\pi}{4}\frac{CE}{G}\right]$$

$$= \frac{10\left(40\right)}{24\left(207\right)10^3}\left[\left(\frac{\pi}{4}+1\right)\frac{40}{0.07511} - 2 + \frac{\pi}{4}\frac{1.2\left(207\right)10^3}{79.3\left(10^3\right)}\right]$$

$$= 0.0766\text{ mm} \qquad \textit{Ans.}$$

---

**4-78** **Note to the Instructor**. The cross section shown in the first printing is incorrect and the solution presented here reflects the correction which will be made in subsequent printings. The corrected cross section should appear as shown in this figure. We apologize for any inconvenience.




Section *A-A*

$A = 3(2.25) - 2.25(1.5) = 3.375\ \text{in}^2$

$$R = \frac{(1+1.5)(3)(2.25)-(1+0.75+1.125)(1.5)(2.25)}{3.375} = 2.125\ \text{in}$$

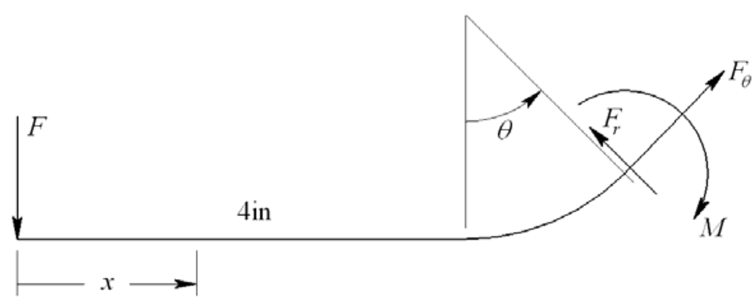
Section is equivalent to the "T" section of Table 3-4, p. 121,

$$r_n = \frac{2.25(0.75)+0.75(2.25)}{2.25\ln[(1+0.75)/1]+0.75\ln[(1+3)/(1+0.75)]} = 1.7960\ \text{in}$$

$$e = R - r_n = 2.125 - 1.7960 = 0.329\ \text{in}$$

For the straight section

$$I_z = \frac{1}{12}(2.25)\left(3^3\right) + 2.25(3)(1.5-1.125)^2$$
$$-\left[\frac{1}{12}(1.5)\left(2.25^3\right) + 1.5(2.25)\left(0.75+\frac{2.25}{2}-1.125\right)^2\right]$$
$$= 2.689\ \text{in}^4$$




For $\underline{0 \le x \le 4\ \text{in}}$

$$M = -Fx \quad \frac{\partial M}{\partial F} = -x, \qquad V = F \quad \frac{\partial V}{\partial F} = 1$$

For $\underline{\theta \le \pi/2}$

$$F_r = F\cos\theta \quad \frac{\partial F_r}{\partial F} = \cos\theta, \qquad F_\theta = F\sin\theta \quad \frac{\partial F_\theta}{\partial F} = \sin\theta$$

$$M = F(4+2.125\sin\theta) \quad \frac{\partial M}{\partial F} = (4+2.125\sin\theta)$$

$$MF_\theta = F(4+2.125\sin\theta)F\sin\theta \qquad \frac{\partial MF_\theta}{\partial F} = 2F(4+2.365\sin\theta)\sin\theta$$

Use Eqs. (4-31) and (4-24) (with $C = 1$) for the straight part, and Eq. (4-38) for the curved part, integrating from 0 to $\pi/2$, and double the results

$$\delta = \frac{2}{E}\left\{\frac{1}{I}\int_0^4 Fx^2 dx + \frac{F(4)(1)}{3.375(G/E)} + \int_0^{\pi/2} F\frac{(4+2.125\sin\theta)^2}{3.375(0.329)}d\theta\right.$$
$$+\int_0^{\pi/2}\frac{F\sin^2\theta(2.125)}{3.375}d\theta - \int_0^{\pi/2}\frac{2F(4+2.125\sin\theta)\sin\theta}{3.375}d\theta$$
$$\left.+\int_0^{\pi/2}\frac{(1)F\cos^2\theta(2.125)}{3.375(G/E)}d\theta\right\}$$

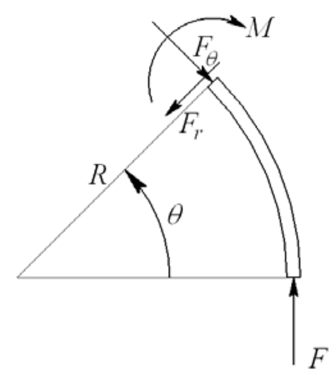
Substitute $I = 2.689\ \text{in}^4$, $F = 6700$ lbf, $E = 30\ (10^6)$ psi, $G = 11.5\ (10^6)$ psi

$$\delta = \frac{2(6700)}{30(10^6)}\left\{\frac{4^3}{3(2.689)} + \frac{4}{3.375(11.5/30)} + \frac{1}{3.375(0.329)}\left[16\left(\frac{\pi}{2}\right) + 17(1) + 4.516\left(\frac{\pi}{4}\right)\right]\right.$$
$$\left.+\frac{2.125}{3.375}\left(\frac{\pi}{4}\right) - \frac{2}{3.375}\left[4(1) + 2.125\left(\frac{\pi}{4}\right)\right] + \frac{2.125}{3.375(11.5/30)}\left(\frac{\pi}{4}\right)\right\}$$
$$= 0.0226 \text{ in} \qquad \textit{Ans.}$$

---

**4-79** Since $R/h = 35/4.5 = 7.78$ use Eq. (4-38), integrate from 0 to $\pi$, and double the results

$$M = FR(1 - \cos\theta) \qquad \frac{\partial M}{\partial F} = R(1 - \cos\theta)$$
$$F_r = F\sin\theta \qquad \frac{\partial F_r}{\partial F} = \sin\theta$$
$$F_\theta = F\cos\theta \qquad \frac{\partial F_\theta}{\partial F} = \cos\theta$$
$$MF_\theta = F^2 R\cos\theta(1 - \cos\theta)$$
$$\frac{\partial(MF_\theta)}{\partial F} = 2FR\cos\theta(1 - \cos\theta)$$




From Eq. (4-38),

$$\delta = 2\left[\frac{FR^2}{AeE}\int_0^\pi (1-\cos\theta)^2 d\theta + \frac{FR}{AE}\int_0^\pi \cos^2\theta d\theta\right.$$
$$\left.-\frac{2FR}{AE}\int_0^\pi \cos\theta(1-\cos\theta)d\theta + \frac{1.2FR}{AG}\int_0^\pi \sin^2\theta d\theta\right]$$
$$= \frac{2FR}{AE}\left(\frac{3\pi}{2}\frac{R}{e} + \frac{3\pi}{2} + 0.6\pi\frac{E}{G}\right)$$

$A = 4.5(3) = 13.5\ \text{mm}^2$, $E = 207\ (10^3)\ \text{N/mm}^2$, $G = 79.3\ (10^3)\ \text{N/mm}^2$, and from Table 3-4, p. 121,

$$r_n = \frac{h}{\ln \frac{r_o}{r_i}} = \frac{4.5}{\ln \frac{37.25}{32.75}} = 34.95173 \text{ mm}$$

and $e = R - r_n = 35 - 34.95173 = 0.04827$ mm. Thus,

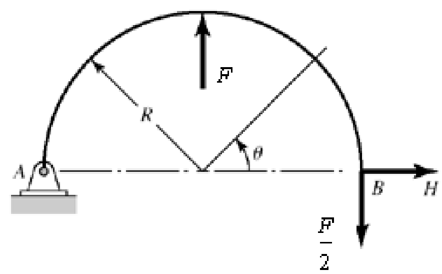
$$\delta = \frac{2F(35)}{13.5(207)10^3}\left(\frac{3\pi}{2}\frac{35}{0.04827} + \frac{3\pi}{2} + 0.6\pi\frac{207}{79.3}\right) = 0.08583F$$

where $F$ is in N. For $\delta$ = 1 mm, $F = \frac{1}{0.08583} = 11.65$ N *Ans.*

Note: The first term in the equation for $\delta$ dominates and this is from the bending moment. Try Eq. (4-41), and compare the results.

---

**4-80** $R/h = 20 > 10$ so Eq. (4-41) can be used to determine deflections. Consider the horizontal reaction, to applied at $B$, subject to the constraint $(\delta_B)_H = 0$.




$$M = \frac{FR}{2}(1-\cos\theta) - HR\sin\theta \qquad \frac{\partial M}{\partial H} = -R\sin\theta \qquad 0 < \theta < \frac{\pi}{2}$$

By symmetry, we may consider only half of the wire form and use twice the strain energy Eq. (4-41) then becomes,

$$(\delta_B)_H = \frac{\partial U}{\partial H} \doteq \frac{2}{EI}\int_0^{\pi/2}\left(M\frac{\partial M}{\partial H}\right)Rd\theta = 0$$

$$\int_0^{\pi/2}\left[\frac{FR}{2}(1-\cos\theta) - HR\sin\theta\right](-R\sin\theta)R\, d\theta = 0$$

$$-\frac{F}{2} + \frac{F}{4} + H\frac{\pi}{4} = 0 \Rightarrow H = \frac{F}{\pi} = \frac{30}{\pi} = 9.55 \text{ N} \quad \textit{Ans.}$$

Reaction at $A$ is the same where $H$ goes to the left. Substituting $H$ into the moment equation we get,

$$M = \frac{FR}{2\pi}\left[\pi(1-\cos\theta) - 2\sin\theta\right] \qquad \frac{\partial M}{\partial F} = \frac{R}{2\pi}[\pi(1-\cos\theta) - 2\sin\theta] \qquad 0 < \theta < \frac{\pi}{2}$$

$$\delta_P = \frac{\partial U}{\partial P} \doteq \int \frac{2}{EI}\left(M\,\frac{\partial M}{\partial F}\right)Rd\theta = \frac{2}{EI}\int_0^{\pi/2} \frac{FR^2}{4\pi^2}[\pi(1-\cos\theta) - 2\sin\theta]^2 R\; d\theta$$

$$= \frac{FR^3}{2\pi^2 EI}\int_0^{\pi/2} (\pi^2 + \pi^2\cos^2\theta + 4\sin^2\theta - 2\pi^2\cos\theta - 4\pi\sin\theta + 4\pi\sin\theta\cos\theta)\; d\theta$$

$$= \frac{FR^3}{2\pi^2 EI}\left[\pi^2\left(\frac{\pi}{2}\right) + \pi^2\left(\frac{\pi}{4}\right) + 4\left(\frac{\pi}{4}\right) - 2\pi^2 - 4\pi + 2\pi\right]$$

$$= \frac{(3\pi^2 - 8\pi - 4)}{8\pi}\frac{FR^3}{EI} = \frac{(3\pi^2 - 8\pi - 4)}{8\pi}\frac{(30)(40^3)}{207\left(10^3\right)\left[\pi\left(2^4\right)/64\right]} = 0.224 \text{ mm} \qquad \textit{Ans.}$$

---

**4-81** The radius is sufficiently large compared to the wire diameter to use Eq. (4-41) for the curved beam portion. The shear and axial components will be negligible compared to bending.

Place a fictitious force *Q* pointing to the left at point *A*.

$$M = PR\sin\theta + Q(R\sin\theta + l) \qquad \frac{\partial M}{\partial Q} = R\sin\theta + l$$

Note that the strain energy in the straight portion is zero since there is no real force in that section.

From Eq. (4-41),

$$\delta = \left[\int_0^{\pi/2} \frac{1}{EI}\left(M\,\frac{\partial M}{\partial Q}\right)Rd\theta\right]_{Q=0} = \frac{1}{EI}\int_0^{\pi/2} PR\sin\theta\left(R\sin\theta + l\right)Rd\theta$$

$$= \frac{PR^2}{EI}\int_0^{\pi/2}\left(R\sin^2\theta + l\sin\theta\right)d\theta = \frac{PR^2}{EI}\left(\frac{\pi}{4}R + l\right) = \frac{1(5^2)}{30\left(10^6\right)\left[\pi\left(0.125^4\right)/64\right]}\left(\frac{\pi}{4}(5) + 4\right)$$

$$= 0.551 \text{ in} \qquad \textit{Ans.}$$

---

**4-82** Both the radius and the length are sufficiently large to use Eq. (4-41) for the curved beam portion and to neglect transverse shear stress for the straight portion.

Straight portion: $\quad M_{AB} = Px \qquad \dfrac{\partial M_{AB}}{\partial P} = x$

Curved portion: $\quad M_{BC} = P\left[R(1-\cos\theta) + l\right] \qquad \dfrac{\partial M_{BC}}{\partial P} = \left[R(1-\cos\theta) + l\right]$

From Eq. (4-41) with the addition of the bending strain energy in the straight portion of the wire,

$$
\begin{aligned}
\delta &= \int_0^l \frac{1}{EI}\left(M_{AB}\frac{\partial M_{AB}}{\partial P}\right)dx + \int_0^{\pi/2}\frac{1}{EI}\left(M_{BC}\frac{\partial M_{BC}}{\partial P}\right)R\,d\theta \\
&= \frac{P}{EI}\int_0^l x^2 dx + \frac{PR}{EI}\int_0^{\pi/2}\left[R(1-\cos\theta)+l\right]^2 d\theta \\
&= \frac{Pl^3}{3EI} + \frac{PR}{EI}\int_0^{\pi/2}\left[R^2(1-2\cos\theta+\cos^2\theta)+2Rl(1-\cos\theta)+l^2\right]d\theta \\
&= \frac{Pl^3}{3EI} + \frac{PR}{EI}\int_0^{\pi/2}\left[R^2\cos^2\theta-\left(2R^2+2Rl\right)\cos\theta+(R+l)^2\right]d\theta \\
&= \frac{Pl^3}{3EI} + \frac{PR}{EI}\left[\frac{\pi}{4}R^2-\left(2R^2+2Rl\right)+\frac{\pi}{2}(R+l)^2\right] \\
&= \frac{P}{EI}\left[\frac{l^3}{3}+\frac{\pi}{4}R^3-R\left(2R^2+2Rl\right)+\frac{\pi}{2}R(R+l)^2\right] \\
&= \frac{1}{30\left(10^6\right)\pi\left(0.125^4\right)/64}\left[\frac{4^3}{3}+\frac{\pi}{4}(5^3)-5\left[2(5^2)+2(5)(4)\right]+\frac{\pi}{2}\left(5\right)\left(5+4\right)^2\right] \\
&= 0.850 \text{ in} \qquad \textit{Ans.}
\end{aligned}
$$

---

**4-83** Both the radius and the length are sufficiently large to use Eq. (4-41) for the curved beam portion and to neglect transverse shear stress for the straight portion.

Place a dummy force, $Q$, at $A$ vertically downward. The only load in the straight section is the axial force, $Q$. Since this will be zero, there is no contribution.

In the curved section

$$M = PR\sin\theta + QR\left(1-\cos\theta\right) \qquad \frac{\partial M}{\partial Q} = R\left(1-\cos\theta\right)$$

From Eq. (4-41)

$$
\begin{aligned}
\delta &= \left[\int_0^{\pi/2}\frac{1}{EI}\left(M\frac{\partial M}{\partial Q}\right)R\,d\theta\right]_{Q=0} = \frac{1}{EI}\int_0^{\pi/2} PR\sin\theta\left[R\left(1-\cos\theta\right)\right]R\,d\theta \\
&= \frac{PR^3}{EI}\int_0^{\pi/2}\left(\sin\theta-\sin\theta\cos\theta\right)d\theta = \frac{PR^3}{EI}\left(1-\frac{1}{2}\right) = \frac{PR^3}{2EI} \\
&= \frac{1\left(5^3\right)}{2\left(30\right)10^6\left[\pi\left(0.125^4\right)/64\right]} = 0.174 \text{ in} \qquad \textit{Ans.}
\end{aligned}
$$

---

**4-84** Both the radius and the length are sufficiently large to use Eq. (4-41) for the curved beam portion and to neglect transverse shear stress for the straight portion.

Place a dummy force, $Q$, at $A$ vertically downward. The load in the straight section is the axial force, $Q$, whereas the bending moment is only a function of $P$ and is not a function of $Q$. When setting $Q = 0$, there is no axial or bending contribution.

In the curved section

$$M = P\left[R(1-\cos\theta)+l\right]-QR\sin\theta \qquad \frac{\partial M}{\partial Q} = -R\sin\theta$$

From Eq. (4-41)

$$\delta = \left[\int_0^{\pi/2}\frac{1}{EI}\left(M\frac{\partial M}{\partial Q}\right)Rd\theta\right]_{Q=0} = \frac{1}{EI}\int_0^{\pi/2}P\left[R(1-\cos\theta)+l\right](-R\sin\theta)Rd\theta$$

$$= -\frac{PR^2}{EI}\int_0^{\pi/2}(R\sin\theta - R\sin\theta\cos\theta + l\sin\theta)\,d\theta = -\frac{PR^2}{EI}\left(R+l-\frac{1}{2}R\right) = -\frac{PR^2}{2EI}(R+2l)$$

$$= -\frac{1(5^2)}{2(30)10^6\left[\pi(0.125^4)/64\right]}\left[5+2(4)\right] = -0.452 \text{ in}$$

Since the deflection is negative, $\delta$ is in the opposite direction of $Q$. Thus the deflection is

$$\delta = 0.452 \text{ in } \uparrow \qquad Ans.$$

---

**4-85** Consider the force of the mass to be $F$, where $F = 9.81(1) = 9.81$ N. The load in $AB$ is tension

$$F_{AB} = F \qquad \frac{\partial F_{AB}}{\partial F} = 1$$

For the curved section, the radius is sufficiently large to use Eq. (4-41). There is no bending in section $DE$. For section $BCD$, let $\theta$ be counterclockwise originating at $D$

$$M = FR\sin\theta \qquad \frac{\partial M}{\partial F} = R\sin\theta \qquad 0 \le \theta \le \pi$$

Using Eqs. (4-29) and (4-41)

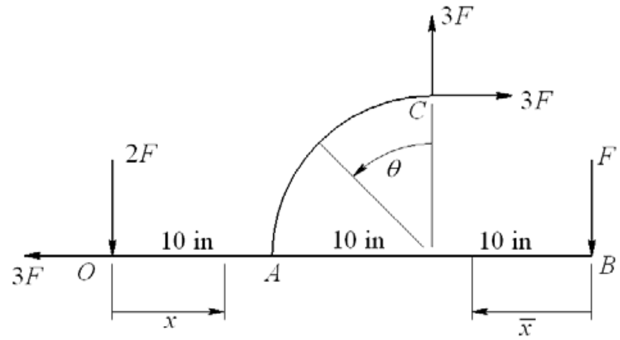
$$\delta = \left(\frac{Fl}{AE}\right)_{AB}\frac{\partial F_{AB}}{\partial F} + \int_0^{\pi}\frac{1}{EI}\left(M\frac{\partial M}{\partial F}\right)Rd\theta = \frac{Fl}{AE}(1) + \int_0^{\pi}\frac{FR^3}{EI}\sin^2\theta\,d\theta$$

$$= \frac{Fl}{AE} + \frac{\pi FR^3}{2EI} = \frac{F}{E}\left(\frac{l}{A}+\frac{\pi R^3}{2I}\right) = \frac{9.81}{207(10^3)}\left[\frac{80}{\left[\pi(2^2)/4\right]} + \frac{\pi(40^3)}{2\left[\pi(2^4)/64\right]}\right]$$

$$= 6.067 \text{ mm} \qquad Ans.$$

---

**4-86** $A_{OA} = 2(0.25) = 0.5$ in$^2$,
$I_{OAB} = 0.25(2^3)/12 = 0.1667$ in$^4$,
$I_{AC} = \pi(0.5^4)/64 = 3.068\ (10^{-3})$ in$^4$




Applying a force $F$ at point $B$, using statics, the reaction forces at $O$ and $C$ are as shown.

$OA$: Axial $F_{OA} = 3F \qquad \dfrac{\partial F_{OA}}{\partial F} = 3$

Bending $M_{OA} = -2Fx \qquad \dfrac{\partial M_{OA}}{\partial F} = -2x$

$AB$: Bending $M_{AB} = -F\,\bar{x} \qquad \dfrac{\partial M_{AB}}{\partial F} = -\bar{x}$

$AC$: Isolating the upper curved section

$$M_{AC} = 3FR(\sin\theta + \cos\theta - 1) \qquad \frac{\partial M_{AC}}{\partial F} = 3R(\sin\theta + \cos\theta - 1)$$

$$\delta = \left(\frac{Fl}{AE}\right)_{OA}\frac{\partial F_{OA}}{\partial F} + \frac{1}{(EI)_{OAB}}\int_0^{10} 4Fx^2 dx + \frac{1}{(EI)_{OAB}}\int_0^{20} F\,\bar{x}^2 d\,\bar{x}$$

$$+\frac{9FR^3}{(EI)_{AC}}\int_0^{\pi/2}(\sin\theta + \cos\theta - 1)^2\, d\theta$$

$$= \frac{3F(10)}{0.5(10.4)10^6}(3) + \frac{4F(10^3)}{3(10.4)10^6(0.1667)} + \frac{F(20^3)}{3(10.4)10^6(0.1667)}$$

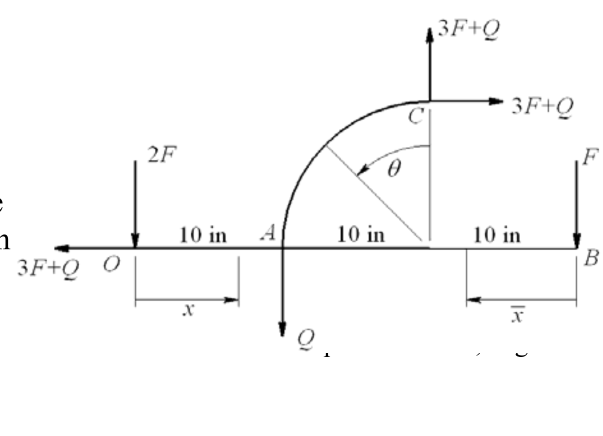
$$+\frac{9F(10^3)}{30(10^6)3.068(10^{-3})}\int_0^{\pi/2}(\sin^2\theta + 2\sin\theta\cos\theta - 2\sin\theta + \cos^2\theta - 2\cos\theta + 1)\, d\theta$$

$$= 1.731(10^{-5})F + 7.691(10^{-4})F + 1.538(10^{-3})F + 0.09778F\left(\frac{\pi}{4} + 1 - 2 + \frac{\pi}{4} - 2 + \frac{\pi}{2}\right)$$

$$= 0.0162F = 0.0162(100) = 1.62 \text{ in} \qquad \textit{Ans.}$$

---

**4-87** $A_{OA} = 2(0.25) = 0.5$ in$^2$,
$I_{OAB} = 0.25(2^3)/12 = 0.1667$ in$^4$,
$I_{AC} = \pi(0.5^4)/64 = 3.068\ (10^{-3})$ in$^4$
Applying a vertical dummy force, $Q$, at $A$, from statics the reactions are as shown. The dummy force is transmitted through section



*OA* and member *AC*.

$$OA: \quad F_{OA} = 3F + Q \qquad \frac{\partial F_{OA}}{\partial Q} = 1$$

$$AC: \quad M_{AC} = (3F+Q)R\sin\theta - (3F+Q)R(1-\cos\theta) \qquad \frac{\partial M_{AC}}{\partial Q} = R(\sin\theta + \cos\theta - 1)$$

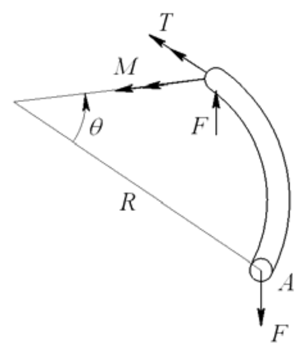
$$\delta = \left[\left(\frac{Fl}{AE}\right)_{OA}\left(\frac{\partial F_{OA}}{\partial Q}\right) + \left(\frac{1}{EI}\right)_{AC}\int_0^{\pi/2} M_{AC}\frac{\partial M_{AC}}{\partial Q}R d\theta\right]_{Q=0}$$

$$= \frac{3Fl_{OA}}{(AE)_{OA}} + \frac{3FR^3}{(EI)_{AC}}\int_0^{\pi/2}(\sin\theta + \cos\theta - 1)^2 d\theta$$

$$= \frac{3(100)10}{10.4(10^6)0.5} + \frac{3(100)10^3}{30(10^6)3.068(10^{-3})}\left(\frac{\pi}{4} + 1 - 2 + \frac{\pi}{4} - 2 + \frac{\pi}{2}\right) = 0.462 \text{ in} \qquad Ans.$$

---

**4-88** $I = \pi(6^4)/64 = 63.62 \text{ mm}^4$




$\underline{0 \le \theta \le \pi/2}$

$$M = FR\sin\theta \qquad \frac{\partial M}{\partial F} = R\sin\theta$$

$$T = FR(1-\cos\theta) \qquad \frac{\partial T}{\partial F} = R(1-\cos\theta)$$

According to Castigliano's theorem, a positive $\partial U/\partial F$ will yield a deflection of *A* in the negative *y* direction. Thus the deflection in the positive *y* direction is

$$(\delta_A)_y = -\frac{\partial U}{\partial F} = -\left\{\frac{1}{EI}\int_0^{\pi/2} F(R\sin\theta)^2 R\, d\theta + \frac{1}{GJ}\int_0^{\pi/2} F[R(1-\cos\theta)]^2 R\, d\theta\right\}$$

Integrating and substituting $J = 2I$ and $G = E/[2(1+\nu)]$

$$(\delta_A)_y = -\frac{FR^3}{EI}\left[\frac{\pi}{4} + (1+\nu)\left(\frac{3\pi}{4} - 2\right)\right] = -[4\pi - 8 + (3\pi - 8)\nu]\frac{FR^3}{4EI}$$

$$= -[4\pi - 8 + (3\pi - 8)(0.29)]\frac{(250)(80)^3}{4(200)10^3(63.62)} = -12.5 \text{ mm} \qquad Ans.$$

---

**4-89** The force applied to the copper and steel wire assembly is

$$F_c + F_s = 400 \text{ lbf} \qquad (1)$$

Since the deflections are equal, $\delta_c = \delta_s$

$$\left(\frac{Fl}{AE}\right)_c = \left(\frac{Fl}{AE}\right)_s$$

$$\frac{F_c l}{3(\pi/4)(0.1019)^2(17.2)10^6} = \frac{F_s l}{(\pi/4)(0.1055)^2(30)10^6}$$

Yields, $F_c = 1.6046F_s$. Substituting this into Eq. (1) gives

$$1.604F_s + F_s = 2.6046F_s = 400 \quad \Rightarrow \quad F_s = 153.6 \text{ lbf}$$

$$F_c = 1.6046F_s = 246.5 \text{ lbf}$$

$$\sigma_c = \frac{F_c}{A_c} = \frac{246.5}{3(\pi/4)(0.1019)^2} = 10\ 075 \text{ psi} = 10.1 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_s = \frac{F_s}{A_s} = \frac{153.6}{(\pi/4)(0.1055^2)} = 17\ 571 \text{ psi} = 17.6 \text{ kpsi} \quad \textit{Ans.}$$

$$\delta = \left(\frac{Fl}{AE}\right)_s = \frac{153.6(100)(12)}{(\pi/4)(0.1055)^2(30)10^6} = 0.703 \text{ in} \quad \textit{Ans.}$$

---

**4-90** **(a)** Bolt stress $\sigma_b = 0.75(65) = 48.8$ kpsi *Ans.*

Total bolt force $F_b = 6\sigma_b A_b = 6(48.8)\left(\frac{\pi}{4}\right)(0.5^2) = 57.5$ kips

Cylinder stress $\sigma_c = -\frac{F_b}{A_c} = \frac{57.43}{(\pi/4)(5.5^2 - 5^2)} = -13.9$ kpsi *Ans.*

**(b)** Force from pressure

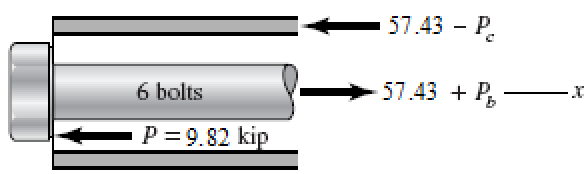
$$P = \frac{\pi D^2}{4} p = \frac{\pi(5^2)}{4}(500) = 9817 \text{ lbf} = 9.82 \text{ kip}$$

$\Sigma F_x = 0$

$P_b + P_c = 9.82$ (1)

Since $\delta_c = \delta_b$,

$$\frac{P_c l}{(\pi/4)(5.5^2 - 5^2)E} = \frac{P_b l}{6(\pi/4)(0.5^2)E}$$




$P_c = 3.5\ P_b$ (2)

Substituting this into Eq. (1)

$P_b + 3.5\ P_b = 4.5\ P_b = 9.82 \ \Rightarrow P_b = 2.182$ kip. From Eq. (2), $P_c = 7.638$ kip

Using the results of (**a**) above, the total bolt and cylinder stresses are

$$\sigma_b = 48.8 + \frac{2.182}{6(\pi/4)(0.5^2)} = 50.7 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_c = -13.9 + \frac{7.638}{(\pi/4)(5.5^2 - 5^2)} = -12.0 \text{ kpsi} \quad \textit{Ans.}$$

---

**4-91** $T_c + T_s = T$ (1)

$$\theta_c = \theta_s \quad \Rightarrow \quad \frac{T_c l}{(JG)_c} = \frac{T_s l}{(JG)_s} \quad \Rightarrow \quad T_c = \frac{(JG)_c}{(JG)_s} T_s \qquad (2)$$

Substitute this into Eq. (1)

$$\frac{(JG)_c}{(JG)_s} T_s + T_s = T \quad \Rightarrow \quad T_s = \frac{(JG)_s}{(JG)_s + (JG)_c} T$$

The percentage of the total torque carried by the shell is

$$\% \text{ Torque} = \frac{100(JG)_s}{(JG)_s + (JG)_c} \qquad \textit{Ans.}$$

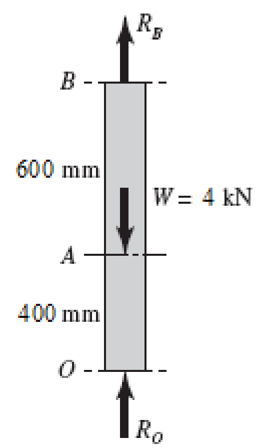
---

**4-92** $R_O + R_B = W$ (1)

$\delta_{OA} = \delta_{AB}$

$$\left(\frac{Fl}{AE}\right)_{OA} = \left(\frac{Fl}{AE}\right)_{AB}$$

$$\frac{400R_O}{AE} = \frac{600R_B}{AE} \quad \Rightarrow \quad R_O = \frac{3}{2} R_B \qquad (2)$$




Substitute this unto Eq. (1)

$$\frac{3}{2} R_B + R_B = 4 \quad \Rightarrow \quad R_B = 1.6 \text{ kN} \quad \textit{Ans.}$$

From Eq. (2) $\quad R_O = \frac{3}{2} 1.6 = 2.4 \text{ kN} \quad$ *Ans.*

$$\delta_A = \left(\frac{Fl}{AE}\right)_{OA} = \frac{2\,400(400)}{10(60)(71.7)(10^3)} = 0.0223 \text{ mm} \quad \textit{Ans.}$$

---

**4-93** See figure in Prob. 4-92 solution.

Procedure 1:

1. Let $R_B$ be the redundant reaction.

2. Statics. $R_O + R_B = 4\,000$ N $\Rightarrow$ $R_O = 4\,000 - R_B$ (1)

3. Deflection of point *B*. $\delta_B = \dfrac{R_B(600)}{AE} + \dfrac{(R_B - 4000)(400)}{AE} = 0$ (2)

4. From Eq. (2), *AE* cancels and $R_B = 1\,600$ N *Ans.*
and from Eq. (1), $R_O = 4\,000 - 1\,600 = 2\,400$ N *Ans.*

$$\delta_A = \left(\frac{Fl}{AE}\right)_{OA} = \frac{2\,400(400)}{10(60)(71.7)(10^3)} = 0.0223 \text{ mm} \quad \textit{Ans.}$$

---

**4-94** (**a**) Without the right-hand wall the deflection of point *C* would be

$$\delta_C = \sum \frac{Fl}{AE} = \frac{5(10^3)8}{(\pi/4)0.75^2(10.4)10^6} + \frac{2(10^3)5}{(\pi/4)0.5^2(10.4)10^6}$$
$$= 0.01360 \text{ in} > 0.005 \text{ in} \quad \therefore \text{ Hits wall} \quad \textit{Ans.}$$

(**b**) Let $R_C$ be the reaction of the wall at *C* acting to the left (←). Thus, the deflection of point *C* is now

$$\delta_C = \frac{\left[5(10^3) - R_C\right]8}{(\pi/4)0.75^2(10.4)10^6} + \frac{\left[2(10^3) - R_C\right]5}{(\pi/4)0.5^2(10.4)10^6}$$
$$= 0.01360 - \frac{4R_C}{\pi(10.4)10^6}\left(\frac{8}{0.75^2} + \frac{5}{0.5^2}\right) = 0.005$$

or,

$$0.01360 - 4.190(10^{-6})R_C = 0.005 \quad \Rightarrow \quad R_C = 2\,053 \text{ lbf} = 2.05 \text{ kip} \leftarrow \quad \textit{Ans.}$$

Statics. Considering $\rightarrow +$, $5\,000 - R_A - 2\,053 = 0$ $\Rightarrow$ $R_A = 2\,947$ lbf = 2.95 kip ← *Ans.*

Deflection. *AB* is 2 947 lbf in tension. Thus

$$\delta_B = \delta_{AB} = \frac{R_A(8)}{A_{AB}E} = \frac{2947(8)}{(\pi/4)0.75^2(10.4)10^6} = 5.13(10^{-3}) \text{ in} \rightarrow \quad \textit{Ans.}$$

---

**4-95** Since $\theta_{OA} = \theta_{AB}$,

$$\frac{T_{OA}(4)}{JG} = \frac{T_{AB}(6)}{JG} \quad \Rightarrow \quad T_{OA} = \frac{3}{2}T_{AB} \quad (1)$$

Statics. $T_{OA} + T_{AB} = 200$ (2)

Substitute Eq. (1) into Eq. (2),

$$\frac{3}{2}T_{AB} + T_{AB} = \frac{5}{2}T_{AB} = 200 \qquad \Rightarrow \qquad T_{AB} = 80 \text{ lbf}\cdot\text{in} \qquad \textit{Ans.}$$

From Eq. (1) $T_{OA} = \frac{3}{2}T_{AB} = \frac{3}{2}80 = 120 \text{ lbf}\cdot\text{in}$ *Ans.*

$$\theta_A = \frac{80(6)}{(\pi/32)0.5^4(11.5)10^6}\frac{180}{\pi} = 0.390^0 \quad \textit{Ans.}$$

$$\tau_{\max} = \frac{16T}{\pi d^3} \qquad \Rightarrow \qquad \tau_{OA} = \frac{16(120)}{\pi(0.5^3)} = 4890 \text{ psi} = 4.89 \text{ kpsi} \qquad \textit{Ans.}$$

$$\tau_{AB} = \frac{16(80)}{\pi(0.5^3)} = 3260 \text{ psi} = 3.26 \text{ kpsi} \qquad \textit{Ans.}$$

---

**4-96** Since $\theta_{OA} = \theta_{AB}$,

$$\frac{T_{OA}(4)}{(\pi/32)0.5^4 G} = \frac{T_{AB}(6)}{(\pi/32)0.75^4 G} \qquad \Rightarrow \qquad T_{OA} = 0.2963 T_{AB} \qquad (1)$$

Statics. $T_{OA} + T_{AB} = 200$ (2)

Substitute Eq. (1) into Eq. (2),

$$0.2963T_{AB} + T_{AB} = 1.2963T_{AB} = 200 \qquad \Rightarrow \qquad T_{AB} = 154.3 \text{ lbf}\cdot\text{in} \qquad \textit{Ans.}$$

From Eq. (1) $T_{OA} = 0.2963T_{AB} = 0.2963(154.3) = 45.7 \text{ lbf}\cdot\text{in}$ *Ans.*

$$\theta_A = \frac{154.3(6)}{(\pi/32)0.75^4(11.5)10^6}\frac{180}{\pi} = 0.148^0 \quad \textit{Ans.}$$

$$\tau_{\max} = \frac{16T}{\pi d^3} \qquad \Rightarrow \qquad \tau_{OA} = \frac{16(45.7)}{\pi(0.5^3)} = 1862 \text{ psi} = 1.86 \text{ kpsi} \qquad \textit{Ans.}$$

$$\tau_{AB} = \frac{16(154.3)}{\pi(0.75^3)} = 1862 \text{ psi} = 1.86 \text{ kpsi} \qquad \textit{Ans.}$$

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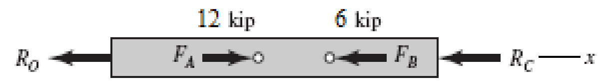
**4-97** Procedure 1.

1. Arbitrarily, choose $R_C$ as a redundant reaction.
2. Statics. $\Sigma F_x = 0$,




$$12(10^3) - 6(10^3) - R_O - R_C = 0$$

$$R_O = 6(10^3) - R_C \qquad (1)$$

3. The deflection of point $C$.

$$\delta_C = \frac{\left[12(10^3) - 6(10^3) - R_C\right](20)}{AE} - \frac{\left[6(10^3) + R_C\right](10)}{AE} - \frac{R_C(15)}{AE} = 0$$

4. The deflection equation simplifies to

$$-45\,R_C + 60(10^3) = 0 \quad \Rightarrow \quad R_C = 1\,333 \text{ lbf} \doteq 1.33 \text{ kip} \quad \textit{Ans.}$$

From Eq. (1), $R_O = 6(10^3) - 1\,333 = 4\,667 \text{ lbf} \doteq 4.67 \text{ kip}$ *Ans.*

$$F_{AB} = F_B + R_C = 6 + 1.333 = 7.333 \text{ kips compression}$$

$$\sigma_{AB} = \frac{F_{AB}}{A} = \frac{-7.333}{(0.5)(1)} = -14.7 \text{ kpsi} \qquad \textit{Ans.}$$

Deflection of $A$. Since $OA$ is in tension,

$$\delta_A = \delta_{OA} = \frac{R_O l_{OA}}{AE} = \frac{4\,667(20)}{(0.5)(1)(30)10^6} = 0.00622 \text{ in} \qquad \textit{Ans.}$$

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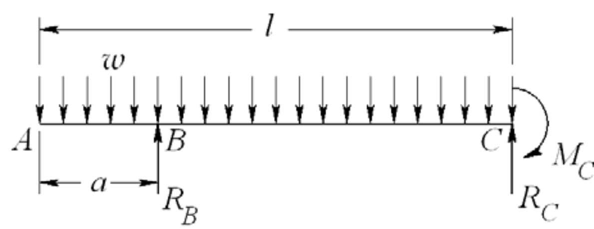
**4-98** Procedure 1.




1. Choose $R_B$ as redundant reaction.
2. Statics. $R_C = wl - R_B$ (1)

$$M_C = \frac{1}{2}wl^2 - R_B(l-a) \qquad (2)$$

3. Deflection equation for point $B$. Superposition of beams 2 and 3 of Table A-9,

$$y_B = \frac{R_B(l-a)^3}{3EI} + \frac{w(l-a)^2}{24EI}\left[4l(l-a) - (l-a)^2 - 6l^2\right] = 0$$

4. Solving for $R_B$.

$$R_B = \frac{w}{8(l-a)}\left[6l^2 - 4l(l-a) + (l-a)^2\right]$$

$$= \frac{w}{8(l-a)}\left(3l^2 + 2al + a^2\right) \qquad \textit{Ans.}$$

Substituting this into Eqs. (1) and (2) gives

$$R_C = wl - R_B = \frac{w}{8(l-a)}\left(5l^2 - 10al - a^2\right) \qquad Ans.$$

$$M_C = \frac{1}{2}wl^2 - R_B(l-a) = \frac{w}{8}\left(l^2 - 2al - a^2\right) \qquad Ans.$$

---

**4-99** See figure in Prob. 4-98 solution.

Procedure 1.

1. Choose $R_B$ as redundant reaction.

2. Statics. $R_C = wl - R_B$ (1)

$$M_C = \frac{1}{2}wl^2 - R_B(l-a) \qquad (2)$$

3. Deflection equation for point $B$. Let the variable $x$ start at point $A$ and to the right. Using singularity functions, the bending moment as a function of $x$ is

$$M = -\frac{1}{2}wx^2 + R_B\langle x-a\rangle^1 \qquad \frac{\partial M}{\partial R_B} = \langle x-a\rangle^1$$

$$y_B = \frac{\partial U}{\partial R_B} = \frac{1}{EI}\int_0^l M\frac{\partial M}{\partial R_B}dx$$

$$= \frac{1}{EI}\int_0^l -\frac{1}{2}wx^2(0)\,dx + \frac{1}{EI}\int_a^l \left[-\frac{1}{2}wx^2 + R_B(x-a)\right](x-a)\,dx = 0$$

or,

$$-\frac{1}{2}w\left[\frac{1}{4}\left(l^4 - a^4\right) - \frac{a}{3}\left(l^3 - a^3\right)\right] + \frac{R_B}{3}\left[(l-a)^3 - (a-a)^3\right] = 0$$

Solving for $R_B$ gives

$$R_B = \frac{w}{8(l-a)^3}\left[3\left(l^4 - a^4\right) - 4a\left(l^3 - a^3\right)\right] = \frac{w}{8(l-a)}\left(3l^2 + 2al + a^2\right) \qquad Ans.$$

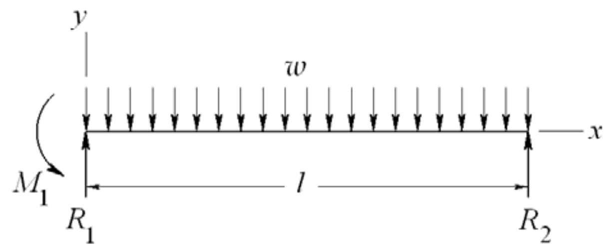
From Eqs. (1) and (2)

$$R_C = wl - R_B = \frac{w}{8(l-a)}\left(5l^2 - 10al - a^2\right) \qquad Ans.$$

$$M_C = \frac{1}{2}wl^2 - R_B(l-a) = \frac{w}{8}\left(l^2 - 2al - a^2\right) \qquad Ans.$$

**4-100** **Note:** When setting up the equations for this problem, no rounding of numbers was made. It turns out that the deflection equation is very sensitive to rounding.

Procedure 2.




1. Statics. $R_1 + R_2 = wl$ (1)

$$R_2 l + M_1 = \frac{1}{2} w l^2 \qquad (2)$$

2. Bending moment equation.

$$M = R_1 x - \frac{1}{2} w x^2 - M_1$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_1 x^2 - \frac{1}{6} w x^3 - M_1 x + C_1 \qquad (3)$$

$$EIy = \frac{1}{6} R_1 x^3 - \frac{1}{24} w x^4 - \frac{1}{2} M_1 x^2 + C_1 x + C_2 \qquad (4)$$

$EI = 30(10^6)(0.85) = 25.5(10^6)$ lbf·in$^2$.

3. Boundary condition 1. At $x = 0$, $y = -R_1/k_1 = -R_1/[1.5(10^6)]$. Substitute into Eq. (4) with value of $EI$ yields $C_2 = -17 R_1$.

Boundary condition 2. At $x = 0$, $dy/dx = -M_1/k_2 = -M_1/[2.5(10^6)]$. Substitute into Eq. (3) with value of $EI$ yields $C_1 = -10.2 M_1$.

Boundary condition 3. At $x = l$, $y = -R_2/k_3 = -R_1/[2.0(10^6)]$. Substitute into Eq. (4) with value of $EI$ yields

$$-12.75 R_2 = \frac{1}{6} R_1 l^3 - \frac{1}{24} w l^4 - \frac{1}{2} M_1 l^2 - 10.2 M_1 l - 17 R_1 \qquad (5)$$

Equations (1), (2), and (5), written in matrix form with $w = 500/12$ lbf/in and $l = 24$ in, are

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 24 & 1 \\ 2287 & 12.75 & -532.8 \end{pmatrix} \begin{Bmatrix} R_1 \\ R_2 \\ M_1 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 12 \\ 576 \end{Bmatrix} \left(10^3\right)$$

Solving, the simultaneous equations yields

$R_1 = 554.59$ lbf, $R_2 = 445.41.59$ lbf, $M_1 = 1310.1$ lbf·in *Ans.*

For the deflection at $x = l/2 = 12$ in, Eq. (4) gives

$$y\Big|_{x=12\text{ in}} = \frac{1}{25.5\left(10^6\right)}\left[\frac{1}{6}\left(554.59\right)12^3 - \frac{1}{24}\frac{500}{12}12^4 - \frac{1}{2}\left(1310.1\right)12^2 - 10.2\left(1310.1\right)12 - 17\left(554.59\right)\right]$$

$$= -5.51\left(10^{-3}\right) \text{ in} \qquad \textit{Ans.}$$

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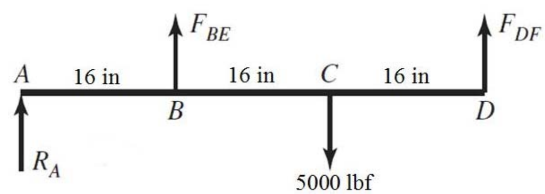
**4-101** Cable area, $A = \frac{\pi}{4}(0.5^2) = 0.1963 \text{ in}^2$




Procedure 2.

1. Statics. $R_A + F_{BE} + F_{DF} = 5(10^3)$ (1)

$3\,F_{DF} + F_{BE} = 10(10^3)$ (2)

2. Bending moment equation.

$$M = R_A x + F_{BE}\left\langle x-16\right\rangle^1 - 5000\left\langle x-32\right\rangle^1$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 + \frac{1}{2}F_{BE}\left\langle x-16\right\rangle^2 - 2500\left\langle x-32\right\rangle^2 + C_1 \qquad (3)$$

$$EIy = \frac{1}{6}R_A x^3 + \frac{1}{6}F_{BE}\left\langle x-16\right\rangle^3 - \frac{2500}{3}\left\langle x-32\right\rangle^3 + C_1 x + C_2 \qquad (4)$$

3. B.C. 1: At $x = 0$, $y = 0$ $\Rightarrow$ $C_2 = 0$

B.C. 2: At $x = 16$ in,

$$y_B = -\left(\frac{Fl}{AE}\right)_{BE} = -\frac{F_{BE}(38)}{0.1963(30)10^6} = -6.453(10^{-6})F_{BE}$$

Substituting into Eq. (4) and evaluating at $x = 16$ in

$$EIy_B = 30(10^6)(1.2)(-6.453)(10^{-6})F_{BE} = \frac{1}{6}R_A\left(16^3\right) + C_1(16)$$

Simplifying gives $682.7\,R_A + 232.3\,F_{BE} + 16\,C_1 = 0$ (5)

B.C. 2: At $x = 48$ in,

$$y_D = -\left(\frac{Fl}{AE}\right)_{DF} = -\frac{F_{DF}(38)}{0.1963(30)10^6} = -6.453(10^{-6})F_{DF}$$

Substituting into Eq. (4) and evaluating at $x = 48$ in,

$$EIy_D = -232.3F_{DF} = \frac{1}{6}R_A\left(48^3\right) + \frac{1}{6}F_{BE}(48-16)^3 - \frac{2500}{3}(48-32)^3 + 48C_1$$

Simplifying gives $18\,432\,R_A + 5\,461\,F_{BE} + 232.3\,F_{DF} + 48\,C_1 = 3.413(10^6)$ (6)

Equations (1), (2), (5) and (6) in matrix form are

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & 0 \\ 682.7 & 232.3 & 0 & 16 \\ 18\,432 & 5\,461 & 232.3 & 48 \end{pmatrix} \begin{Bmatrix} R_A \\ F_{BE} \\ F_{DF} \\ C_1 \end{Bmatrix} = \begin{Bmatrix} 5\,000 \\ 10\,000 \\ 0 \\ 3.413\left(10^6\right) \end{Bmatrix}$$

Solve simultaneously or use software. The results are

$R_A = -\,970.5$ lbf, $F_{BE} = 3956$ lbf, $F_{DF} = 2015$ lbf, and $C_1 = -\,16\,020$ lbf·in$^2$.

$$\sigma_{BE} = \frac{3956}{0.1963} = 20.2 \text{ kpsi}, \quad \sigma_{DF} = \frac{2015}{0.1963} = 10.3 \text{ kpsi} \quad \textit{Ans.}$$

$EI = 30(10^6)(1.2) = 36(10^6)$ lbf·in$^2$

$$y = \frac{1}{36\left(10^6\right)}\left(-\frac{970.5}{6}x^3 + \frac{3956}{6}\langle x-16\rangle^3 - \frac{2500}{3}\langle x-32\rangle^3 - 16\,020x\right)$$

$$= \frac{1}{36\left(10^6\right)}\left(-161.8x^3 + 659.3\langle x-16\rangle^3 - 833.3\langle x-32\rangle^3 - 16\,020x\right)$$

$B$: $x = 16$ in, $\quad y_B = \dfrac{1}{36\left(10^6\right)}\left[-161.8\left(16^3\right) - 16\,020(16)\right] = -0.0255$ in $\quad$ *Ans.*

$C$: $x = 32$ in,

$$y_C = \frac{1}{36\left(10^6\right)}\left[-161.8\left(32^3\right) + 659.3(32-16)^3 - 16\,020(32)\right]$$

$$= -0.0865 \text{ in} \quad \textit{Ans.}$$

$D$: $x = 48$ in,

$$y_D = \frac{1}{36\left(10^6\right)}\left[-161.8\left(48^3\right) + 659.3(48-16)^3 - 833.3(48-32)^3 - 16\,020(48)\right]$$

$$= -0.0131 \text{ in} \quad \textit{Ans.}$$

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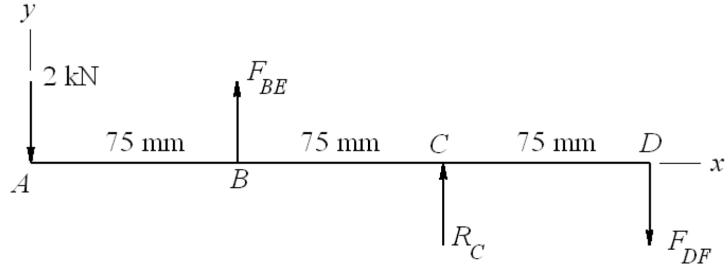
**4-102** Beam: $EI = 207(10^3)21(10^3)$
$= 4.347(10^9)$ N·mm$^2$.
Rods: $A = (\pi/4)8^2 = 50.27$ mm$^2$.




Procedure 2.

1. Statics.

$R_C + F_{BE} - F_{DF} = 2\,000$ (1)

$R_C + 2F_{BE} = 6\,000$ (2)

2. Bending moment equation.

$M = -2\,000\,x + F_{BE}\,\langle x - 75\rangle^1 + R_C\,\langle x - 150\rangle^1$

$$EI\frac{dy}{dx} = -1000x^2 + \frac{1}{2}F_{BE}\left\langle x-75\right\rangle^2 + \frac{1}{2}R_C\left\langle x-150\right\rangle^2 + C_1 \qquad (3)$$

$$EIy = -\frac{1000}{3}x^3 + \frac{1}{6}F_{BE}\left\langle x-75\right\rangle^3 + \frac{1}{6}R_C\left\langle x-150\right\rangle^3 + C_1x + C_2 \qquad (4)$$

3. <u>B.C 1</u>. At $x = 75$ mm,

$$y_B = -\left(\frac{Fl}{AE}\right)_{BE} = -\frac{F_{BE}(50)}{50.27(207)10^3} = -4.805\left(10^{-6}\right)F_{BE}$$

Substituting into Eq. (4) at $x = 75$ mm,

$$4.347\left(10^9\right)\left[-4.805\left(10^{-6}\right)F_{BE}\right] = -\frac{1000}{3}\left(75^3\right) + C_1(75) + C_2$$

Simplifying gives

$$20.89\left(10^3\right)F_{BE} + 75C_1 + C_2 = 140.6\left(10^6\right) \qquad (5)$$

<u>B.C 2</u>. At $x = 150$ mm, $y = 0$. From Eq. (4),

$$-\frac{1000}{3}\left(150^3\right) + \frac{1}{6}F_{BE}(150-75)^3 + C_1(150) + C_2 = 0$$

or,

$$70.31\left(10^3\right)F_{BE} + 150C_1 + C_2 = 1.125\left(10^9\right) \qquad (6)$$

<u>B.C 3</u>. At $x = 225$ mm,

$$y_D = \left(\frac{Fl}{AE}\right)_{DF} = \frac{F_{DF}(65)}{50.27(207)10^3} = 6.246\left(10^{-6}\right)F_{DF}$$

Substituting into Eq. (4) at $x = 225$ mm,

$$4.347\left(10^{9}\right)\left[6.246\left(10^{-6}\right)F_{DF}\right]=-\frac{1000}{3}\left(225^{3}\right)+\frac{1}{6}F_{BE}\left(225-75\right)^{3}$$

$$+\frac{1}{6}R_{C}\left(225-150\right)^{3}+C_{1}\left(225\right)+C_{2}$$

Simplifying gives

$$70.31\left(10^{3}\right)R_{C}+562.5\left(10^{3}\right)F_{BE}-27.15\left(10^{3}\right)F_{DF}+225C_{1}+C_{2}=3.797\left(10^{9}\right) \qquad (7)$$

Equations (1), (2), (5), (6), and (7) in matrix form are

$$\begin{pmatrix} 1 & 1 & -1 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 \\ 0 & 20.89\left(10^{3}\right) & 0 & 75 & 1 \\ 0 & 70.31\left(10^{3}\right) & 0 & 150 & 1 \\ 70.31\left(10^{3}\right) & 562.5\left(10^{3}\right) & -27.15\left(10^{3}\right) & 225 & 1 \end{pmatrix}\begin{Bmatrix} R_{C} \\ F_{BE} \\ F_{DF} \\ C_{1} \\ C_{2} \end{Bmatrix}=\begin{Bmatrix} 2\left(10^{3}\right) \\ 6\left(10^{3}\right) \\ 140.6\left(10^{6}\right) \\ 1.125\left(10^{9}\right) \\ 3.797\left(10^{9}\right) \end{Bmatrix}$$

Solve simultaneously or use software. The results are

$R_C$ = – 2378 N, $F_{BE}$ = 4189 N, $F_{DF}$ = – 189.2 N *Ans*.
and $C_1$ = 1.036 ($10^7$) N·mm$^2$, $C_2$ = – 7.243 ($10^8$) N·mm$^3$.

The bolt stresses are $\sigma_{BE}$ = 4189/50.27 = 83.3 MPa, $\sigma_{DF}$ = – 189/50.27= – 3.8 MPa *Ans*.

The deflections are

From Eq. (4) $y_A=\dfrac{1}{4.347\left(10^{9}\right)}\left[-7.243\left(10^{8}\right)\right]=-0.167\text{ mm}$ *Ans*.

For points *B* and *D* use the axial deflection equations*.

$$y_B=-\left(\frac{Fl}{AE}\right)_{BE}=-\frac{4189(50)}{50.27(207)10^{3}}=-0.0201\text{ mm} \qquad Ans.$$

$$y_D=\left(\frac{Fl}{AE}\right)_{DF}=\frac{-189(65)}{50.27(207)10^{3}}=-1.18\left(10^{-3}\right)\text{ mm} \qquad Ans.$$

*Note. The terms in Eq. (4) are quite large, and due to rounding are not very accurate for calculating the very small deflections, especially for point *D*.

---

**4-103** (**a**) The cross section at *A* does not rotate. Thus, for a single quadrant we have

$$\frac{\partial U}{\partial M_A} = 0$$

The bending moment at an angle $\theta$ to the $x$ axis is

$$M = M_A - \frac{FR}{2}(1 - \cos\theta) \qquad \frac{\partial M}{\partial M_A} = 1$$

The rotation at $A$ is

$$\theta_A = \frac{\partial U}{\partial M_A} = \frac{1}{EI}\int_0^{\pi/2} M \frac{\partial M}{\partial M_A} R d\theta = 0$$

Thus, $$\frac{1}{EI}\int_0^{\pi/2}\left[M_A - \frac{FR}{2}(1-\cos\theta)\right](1) R d\theta = 0 \quad \Rightarrow \quad \left(M_A - \frac{FR}{2}\right)\frac{\pi}{2} + \frac{FR}{2} = 0$$

or,

$$M_A = \frac{FR}{2}\left(1 - \frac{2}{\pi}\right)$$

Substituting this into the equation for $M$ gives

$$M = \frac{FR}{2}\left(\cos\theta - \frac{2}{\pi}\right) \qquad (1)$$

The maximum occurs at $B$ where $\theta = \pi/2$

$$M_{\max} = M_B = -\frac{FR}{\pi} \qquad \textit{Ans.}$$

(**b**) Assume $B$ is supported on a knife edge. The deflection of point $D$ is $\partial U/\partial F$. We will deal with the quarter-ring segment and multiply the results by 4. From Eq. (1)

$$\frac{\partial M}{\partial F} = \frac{R}{2}\left(\cos\theta - \frac{2}{\pi}\right)$$

Thus,

$$\delta_D = \frac{\partial U}{\partial F} = \frac{4}{EI}\int_0^{\pi/2} M \frac{\partial M}{\partial F} R d\theta = \frac{FR^3}{EI}\int_0^{\pi/2}\left(\cos\theta - \frac{2}{\pi}\right)^2 d\theta = \frac{FR^3}{EI}\left(\frac{\pi}{4} - \frac{2}{\pi}\right)$$

$$= \frac{FR^3}{4\pi EI}\left(\pi^2 - 8\right) \quad \textit{Ans.}$$

---

**4-104**

$$P_{\text{cr}} = \frac{C\pi^2 EI}{l^2}$$

$$I = \frac{\pi}{64}\left(D^4 - d^4\right) = \frac{\pi D^4}{64}\left(1 - K^4\right) \qquad \text{where } K = \frac{d}{D}$$

$$P_{\text{cr}} = \frac{C\pi^2 E}{l^2}\left[\frac{\pi D^4}{64}\left(1 - K^4\right)\right]$$

$$D = \left[ \frac{64 P_{\text{cr}} l^2}{\pi^3 C E \left(1 - K^4\right)} \right]^{1/4} \qquad Ans.$$

---

**4-105** $A = \dfrac{\pi}{4} D^2 \left(1 - K^2\right), \quad I = \dfrac{\pi}{64} D^4 \left(1 - K^4\right) = \dfrac{\pi}{64} D^4 \left(1 - K^2\right)\left(1 + K^2\right)$, where $K = d / D$.

The radius of gyration, $k$, is given by

$$k^2 = \frac{I}{A} = \frac{D^2}{16}\left(1 + K^2\right)$$

From Eq. (4-46)

$$\frac{P_{\text{cr}}}{(\pi / 4) D^2 \left(1 - K^2\right)} = S_y - \frac{S_y^2 l^2}{4\pi^2 k^2 C E} = S_y - \frac{S_y^2 l^2}{4\pi^2 \left(D^2 / 16\right)\left(1 + K^2\right) C E}$$

$$4 P_{\text{cr}} = \pi D^2 \left(1 - K^2\right) S_y - \frac{4 S_y^2 l^2 \pi D^2 \left(1 - K^2\right)}{\pi^2 D^2 \left(1 + K^2\right) C E}$$

$$\pi D^2 \left(1 - K^2\right) S_y = 4 P_{\text{cr}} + \frac{4 S_y^2 l^2 \left(1 - K^2\right)}{\pi \left(1 + K^2\right) C E}$$

$$D = \left[ \frac{4 P_{\text{cr}}}{\pi S_y \left(1 - K^2\right)} + \frac{4 S_y^2 l^2 \left(1 - K^2\right)}{\pi \left(1 + K^2\right) C E \pi \left(1 - K^2\right) S_y} \right]^{1/2}$$

$$= 2 \left[ \frac{P_{\text{cr}}}{\pi S_y \left(1 - K^2\right)} + \frac{S_y l^2}{\pi^2 C E \left(1 + K^2\right)} \right]^{1/2} \qquad Ans.$$

---

**4-106** **(a)** $\Sigma M_A = 0, \quad (0.75)(800) - \dfrac{0.9}{\sqrt{0.9^2 + 0.5^2}} F_{BO}(0.5) = 0 \quad \Rightarrow \quad F_{BO} = 1373 \text{ N}$

Using $n_d = 4$, design for $F_{\text{cr}} = n_d F_{BO} = 4(1373) = 5492$ N

$$l = \sqrt{0.9^2 + 0.5^2} = 1.03 \text{ m}, \quad S_y = 165 \text{ MPa}$$

In-plane:

$$k = \left(\frac{I}{A}\right)^{1/2} = \left(\frac{bh^3 / 12}{bh}\right)^{1/2} = 0.2887h = 0.2887(0.025) = 0.007\,218 \text{ m}, \qquad C = 1.0$$

$$\frac{l}{k} = \frac{1.03}{0.007218} = 142.7$$

$$\left(\frac{l}{k}\right)_1 = \left(\frac{2\pi^2 (207)(10^9)}{165(10^6)}\right)^{1/2} = 157.4$$

Since $(l/k)_1 > (l/k)$ use Johnson formula.

Try 25 mm x 12 mm,

$$P_{cr} = 0.025(0.012)\left\{165\left(10^6\right) - \left[\frac{165\left(10^6\right)}{2\pi}(142.7)\right]^2 \frac{1}{1(207)10^9}\right\} = 29.1 \text{ kN}$$

This is significantly greater than the design load of 5492 N found earlier. Check out-of-plane.

Out-of-plane: $k = 0.2887(0.012) = 0.003\ 464$ in, $C = 1.2$

$$\frac{l}{k} = \frac{1.03}{0.003\ 464} = 297.3$$

Since $(l/k)_1 < (l/k)$ use Euler equation.

$$P_{cr} = 0.025(0.012)\frac{1.2\pi^2\left(207\right)10^9}{297.3^2} = 8321 \text{ N}$$

This is greater than the design load of 5492 N found earlier. It is also significantly less than the in-plane $P_{cr}$ found earlier, so the out-of-plane condition will dominate. Iterate the process to find the minimum $h$ that gives $P_{cr}$ greater than the design load.

With $h = 0.010$, $P_{cr} = 4815$ N (too small)
$h = 0.011$, $P_{cr} = 6409$ N (acceptable)

Use 25 mm x 11 mm. If standard size is preferred, use 25 mm x 12 mm. *Ans.*

(**b**) $\sigma_b = -\dfrac{P}{dh} = -\dfrac{1373}{0.012(0.011)} = -10.4\left(10^6\right) \text{ Pa} = -10.4 \text{ MPa}$

*No,* bearing stress is not significant. *Ans.*

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**4-107** This is an open-ended design problem with no one distinct solution.

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**4-108** $F = 1500(\pi/4)2^2 = 4712$ lbf. From Table A-20, $S_y = 37.5$ kpsi
$P_{cr} = n_d F = 2.5(4712) = 11\ 780$ lbf

(**a**) Assume Euler with $C = 1$

$$I = \frac{\pi}{64}d^4 = \frac{P_{cr}l^2}{C\pi^2 E} \quad \Rightarrow \quad d = \left(\frac{64P_{cr}l^2}{\pi^3 CE}\right)^{1/4} = \left[\frac{64\left(11790\right)50^2}{\pi^3\left(1\right)30\left(10^6\right)}\right]^{1/4} = 1.193 \text{ in}$$

Use $d = 1.25$ in. The radius of gyration, $k = (I/A)^{1/2} = d/4 = 0.3125$ in

$$\frac{l}{k} = \frac{50}{0.3125} = 160$$

$$\left(\frac{l}{k}\right)_1 = \left(\frac{2\pi^2 CE}{S_y}\right)^{1/2} = \left(\frac{2\pi^2(1)30\left(10^6\right)}{37.5\left(10^3\right)}\right)^{1/2} = 126 \qquad \therefore \text{ use Euler}$$

$$P_{cr} = \frac{\pi^2(30)10^6(\pi/64)1.25^4}{50^2} = 14194 \text{ lbf}$$

Since 14 194 lbf > 11 780 lbf, $d$ = 1.25 in is satisfactory. *Ans.*

(**b**)

$$d = \left[\frac{64(11780)16^2}{\pi^3(1)30\left(10^6\right)}\right]^{1/4} = 0.675 \text{ in, so use } d = 0.750 \text{ in}$$

$k$ = 0.750/4 = 0.1875 in

$$\frac{l}{k} = \frac{16}{0.1875} = 85.33 \qquad \text{use Johnson}$$

$$P_{cr} = \frac{\pi}{4}\left(0.750^2\right)\left\{37.5\left(10^3\right) - \left[\frac{37.5\left(10^3\right)}{2\pi}85.33\right]^2 \frac{1}{1(30)10^6}\right\} = 12\,748 \text{ lbf}$$

Use $d$ = 0.75 in.

(**c**)

$$n_{(a)} = \frac{14194}{4712} = 3.01 \qquad \textit{Ans.}$$

$$n_{(b)} = \frac{12\,748}{4712} = 2.71 \qquad \textit{Ans.}$$

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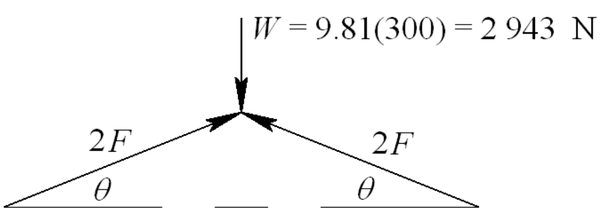
**4-109** From Table A-20, $S_y$ = 180 MPa




$4F \sin\theta$ = 2 943

$$F = \frac{735.8}{\sin\theta}$$

In range of operation, $F$ is maximum when $\theta$ = 15°

$$F_{max} = \frac{735.8}{\sin 15^\circ} = 2\,843 \text{ N per bar}$$

$P_{cr} = n_d F_{max}$ = 3.50 (2 843) = 9 951 N

$l$ = 350 mm, $h$ = 30 mm

Try $b$ = 5 mm. Out of plane, $k = b/\sqrt{12} = 5/\sqrt{12} = 1.443$ mm

$$\frac{l}{k} = \frac{350}{1.443} = 242.6$$

$$\left(\frac{l}{k}\right)_1 = \left[\frac{2\pi^2(1.4)207(10^9)}{180(10^6)}\right]^{1/2} = 178.3 \qquad \therefore \text{ use Euler}$$

$$P_{\text{cr}} = A\frac{C\pi^2 E}{(l/k)^2} = 5(30)\frac{1.4\pi^2(207)10^3}{(242.6)^2} = 7\,290 \text{ N}$$

Too low. Try $b$ = 6 mm. $k = 6/\sqrt{12} = 1.732$ mm

$$\frac{l}{k} = \frac{350}{1.732} = 202.1$$

$$P_{\text{cr}} = A\frac{C\pi^2 E}{(l/k)^2} = 6(30)\frac{1.4\pi^2(207)10^3}{(202.1)^2} = 12\,605 \text{ N}$$

O.K. Use 25 × 6 mm bars *Ans.* The factor of safety is

$$n = \frac{12\,605}{2\,843} = 4.43 \qquad \textit{Ans.}$$

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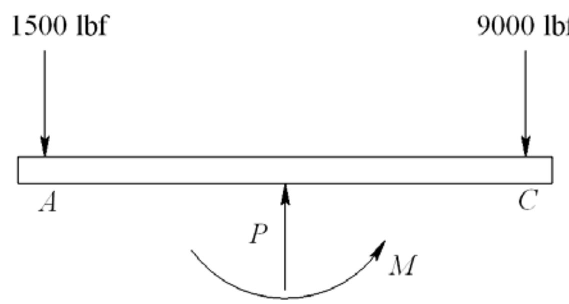
**4-110** $P$ = 1 500 + 9 000 = 10 500 lbf *Ans.*




$\Sigma M_A$ = 10 500 (4.5/2) − 9 000 (4.5) +$M$ = 0

$M$ = 16 874 lbf·in

$e = M / P$ = 16 874/10 500 = 1.607 in *Ans.*

From Table A-8, $A$ = 2.160 in$^2$, and $I$ = 2.059 in$^4$. The stresses are determined using Eq. (4-55)

$$k^2 = \frac{I}{A} = \frac{2.059}{2.160} = 0.953 \text{ in}^2$$

$$\sigma_c = -\frac{P}{A}\left(1+\frac{ec}{k^2}\right) = -\frac{10\,500}{2.160}\left[1+\frac{1.607(3/2)}{0.953}\right] = -17157 \text{ psi} = -17.16 \text{ kpsi} \qquad \textit{Ans.}$$

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**4-111** This is a design problem which has no single distinct solution.

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**4-112** Loss of potential energy of weight = $W(h+\delta)$

Increase in potential energy of spring = $\frac{1}{2}k\delta^2$

$$W(h+\delta) = \frac{1}{2}k\delta^2$$

or, $\delta^2 - \frac{2W}{k}\delta - \frac{2W}{k}h = 0$. $W$ = 30 lbf, $k$ = 100 lbf/in, $h$ = 2 in yields

$$\delta^2 - 0.6\,\delta - 1.2 = 0$$

Taking the positive root (see discussion on p. 192)

$$\delta_{\max} = \frac{1}{2}\left[0.6 + \sqrt{(-0.6)^2 + 4(1.2)}\right] = 1.436 \text{ in} \qquad \textit{Ans.}$$

$$F_{\max} = k\,\delta_{\max} = 100\,(1.436) = 143.6 \text{ lbf} \qquad \textit{Ans.}$$

---

**4-113** The drop of weight $W_1$ converts potential energy, $W_1 h$, to kinetic energy $\frac{1}{2}\frac{W_1}{g}v_1^2$.

Equating these provides the velocity of $W_1$ at impact with $W_2$.

$$W_1 h = \frac{1}{2}\frac{W_1}{g}v_1^2 \qquad \Rightarrow \qquad v_1 = \sqrt{2gh} \qquad (1)$$

Since the collision is inelastic, momentum is conserved. That is, $(m_1 + m_2)\,v_2 = m_1\,v_1$, where $v_2$ is the velocity of $W_1 + W_2$ after impact. Thus

$$\frac{W_1 + W_2}{g}v_2 = \frac{W_1}{g}v_1 \qquad \Rightarrow \qquad v_2 = \frac{W_1}{W_1 + W_2}v_1 = \frac{W_1}{W_1 + W_2}\sqrt{2gh} \qquad (2)$$

The kinetic and potential energies of $W_1 + W_2$ are then converted to potential energy of the spring. Thus,

$$\frac{1}{2}\frac{W_1 + W_2}{g}v_2^2 + \left(W_1 + W_2\right)\delta = \frac{1}{2}k\delta^2$$

Substituting in Eq. (1) and rearranging results in

$$\delta^2 - 2\frac{W_1 + W_2}{k}\delta - 2\frac{W_1^2}{W_1 + W_2}\frac{h}{k} = 0 \qquad (3)$$

Solving for the positive root (see discussion on p. 192)

$$\delta = \frac{1}{2}\left[2\frac{W_1 + W_2}{k} + \sqrt{4\left(\frac{W_1 + W_2}{k}\right)^2 + 8\frac{W_1^2}{W_1 + W_2}\frac{h}{k}}\right] \qquad (4)$$

$W_1$ = 40 N, $W_2$ = 400 N, $h$ = 200 mm, $k$ = 32 kN/m = 32 N/mm.

$$\delta = \frac{1}{2}\left[2\left(\frac{40+400}{32}\right)+\sqrt{4\left(\frac{40+400}{32}\right)^2 + 8\frac{40^2}{40+400}\frac{200}{32}}\right] = 29.06 \text{ mm} \qquad \textit{Ans.}$$

$F_{max} = k\delta$ = 32(29.06) = 930 N *Ans.*

---

**4-114** The initial potential energy of the $k_1$ spring is $V_i = \frac{1}{2}k_1a^2$. The movement of the weight $W$ the distance $y$ gives a final potential of $V_f = \frac{1}{2}k_1(a-y)^2 + \frac{1}{2}k_2y^2$. Equating the two energies give

$$\frac{1}{2}k_1a^2 = \frac{1}{2}k_1(a-y)^2 + \frac{1}{2}k_2y^2$$

Simplifying gives

$$(k_1+k_2)y^2 - 2ak_1y = 0$$

This has two roots, $y$ = 0, $\frac{2k_1a}{k_1+k_2}$. Without damping the weight will vibrate between these two limits. The maximum displacement is thus $y_{max} = \frac{2k_1a}{k_1+k_2}$ *Ans.*

With $W$ = 5 lbf, $k_1$ = 10 lbf/in, $k_2$ = 20 lbf/in, and $a$ = 0.25 in

$$y_{max} = \frac{2(0.25)10}{10+20} = 0.1667 \text{ in} \qquad \textit{Ans.}$$

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