Chapter 17

17-1 Given: F-1 Polyamide, $b = 6$ in, $d = 2$ in with $n = 1750$ rev/min, $H_{\text{nom}} = 2$ hp, $C = 9(12) = 108$ in, velocity ratio = 0.5, $K_s = 1.25$, $n_d = 1$

$$V = \pi d n / 12 = \pi(2)(1750)/12 = 916.3 \text{ ft/min}$$

$$D = d / \text{vel ratio} = 2/0.5 = 4 \text{ in}$$

Eq. (17-1): $$\theta_d = \pi - 2\sin^{-1}\frac{D-d}{2C} = \pi - 2\sin^{-1}\left[\frac{4-2}{2(108)}\right] = 3.123 \text{ rad}$$

Table 17-2: $t = 0.05$ in, $d_{\min} = 1.0$ in, $F_a = 35$ lbf/in, $\gamma = 0.035$ lbf/in^3, $f = 0.5$

$$w = 12\gamma bt = 12(0.035)6(0.05) = 0.126 \text{ lbf/ft}$$

(a) Eq. (*e*), p. 885: $$F_c = \frac{w}{g}\left(\frac{V}{60}\right)^2 = \frac{0.126}{32.17}\left(\frac{916.3}{60}\right)^2 = 0.913 \text{ lbf} \quad \textit{Ans.}$$

$$T = \frac{63\,025 H_{nom} K_s n_d}{n} = \frac{63\,025(2)(1.25)(1)}{1750} = 90.0 \text{ lbf} \cdot \text{in}$$

$$\Delta F = (F_1)_a - F_2 = \frac{2T}{d} = \frac{2(90.0)}{2} = 90.0 \text{ lbf}$$

Table 17-4: $C_p = 0.70$

Eq. (17-12): $(F_1)_a = bF_aC_pC_v = 6(35)(0.70)(1) = 147$ lbf *Ans.*

$F_2 = (F_1)_a - [(F_1)_a - F_2] = 147 - 90 = 57$ lbf *Ans.*

Do not use Eq. (17-9) because we do not yet know f'

Eq. (*i*), p. 886: $$F_i = \frac{(F_1)_a + F_2}{2} - F_c = \frac{147 + 57}{2} - 0.913 = 101.1 \text{ lbf} \quad \textit{Ans.}$$

Using Eq. (17-7) solved for f' (see step 8, p.888),

$$f' = \frac{1}{\theta_d}\ln\left[\frac{(F_1)_a - F_c}{F_2 - F_c}\right] = \frac{1}{3.123}\ln\left(\frac{147 - 0.913}{57 - 0.913}\right) = 0.307$$

The friction is thus underdeveloped.

(b) The transmitted horsepower is, with $\Delta F = (F_1)_a - F_2 = 90$ lbf, Eq. (j), p. 887: $H = \dfrac{(\Delta F)V}{33\,000} = \dfrac{90(916.3)}{33\,000} = 2.5 \text{ hp}$ *Ans.*

$$n_{fs} = \frac{H}{H_{\text{nom}}K_s} = \frac{2.5}{2(1.25)} = 1$$

Eq. (17-1): $\theta_D = \pi + 2\sin^{-1}\dfrac{D-d}{2C} = \pi + 2\sin^{-1}\left[\dfrac{4-2}{2(108)}\right] = 3.160 \text{ rad}$

Eq. (17-2): $L = [4C^2 - (D-d)^2]^{1/2} + (D\theta_D + d\theta_d)/2$

$$= [4(108)^2 - (4-2)^2]^{1/2} + [4(3.160) + 2(3.123)]/2 = 225.4 \text{ in}$$ *Ans.*

(c) Eq. (17-13): $\text{dip} = \dfrac{3C^2w}{2F_i} = \dfrac{3(108/12)^2(0.126)}{2(101.1)} = 0.151 \text{ in}$ *Ans.*

Comment: The solution of the problem is finished; however, a note concerning the design is presented here.

The friction is under-developed. Narrowing the belt width to 5 in (if size is available) will increase f'. The limit of narrowing is $b_{\min} = 4.680$ in, whence

$w = 0.0983$ lbf/ft	$(F_1)_a = 114.7$ lbf
$F_c = 0.713$ lbf	$F_2 = 24.7$ lbf
$T = 90$ lbf · in (same)	$f' = f = 0.50$
$\Delta F = (F_1)_a - F_2 = 90$ lbf	dip $= 0.173$ in
$F_i = 68.9$ lbf	

Longer life can be obtained with a 6-inch wide belt by reducing F_i to attain $f' = 0.50$. Prob. 17-8 develops an equation we can use here

$$F_1 = \frac{(\Delta F + F_c)\exp(f\theta) - F_c}{\exp(f\theta) - 1}$$

$$F_2 = F_1 - \Delta F$$

$$F_i = \frac{F_1 + F_2}{2} - F_c$$

$$f' = \frac{1}{\theta_d}\ln\left(\frac{F_1 - F_c}{F_2 - F_c}\right)$$

$$\text{dip} = \frac{3C^2w}{2F_i}$$

which in this case, $\theta_d = 3.123$ rad, $\exp(f\theta) = \exp[0.5(3.123)] = 4.766$, $w = 0.126$ lbf/ft, $\Delta F = 90.0$ lbf, $F_c = 0.913$ lbf, and gives

$$F_1 = \frac{(0.913 + 90)4.766 - 0.913}{4.766 - 1} = 114.8 \text{ lbf}$$

$F_2 = 114.8 - 90 = 24.8$ lbf

$F_i = (114.8 + 24.8)/2 - 0.913 = 68.9$ lbf

$$f' = \frac{1}{3.123}\ln\left(\frac{114.8 - 0.913}{24.8 - 0.913}\right) = 0.50$$

$$\text{dip} = \frac{3(108/12)^2 0.126}{2(68.9)} = 0.222 \text{ in}$$

So, reducing F_i from 101.1 lbf to 68.9 lbf will bring the undeveloped friction up to 0.50, with a corresponding dip of 0.222 in. Having reduced F_1 and F_2, the endurance of the belt is improved. Power, service factor and design factor have remained intact.

17-2 Double the dimensions of Prob. 17-1.
In Prob. 17-1, F-1 Polyamide was used with a thickness of 0.05 in. With what is available in Table 17-2 we will select the Polyamide A-2 belt with a thickness of 0.11 in. Also, let $b = 12$ in, $d = 4$ in with $n = 1750$ rev/min, $H_{nom} = 2$ hp, $C = 18(12) = 216$ in, velocity ratio = 0.5, $K_s = 1.25$, $n_d = 1$.

$$V = \pi d n / 12 = \pi(4)(1750)/12 = 1833 \text{ ft/min}$$

$D = d$ / vel ratio $= 4/0.5 = 8$ in

Eq. (17-1): $$\theta_d = \pi - 2\sin^{-1}\frac{D-d}{2C} = \pi - 2\sin^{-1}\left[\frac{8-4}{2(216)}\right] = 3.123 \text{ rad}$$

Table 17-2: $t = 0.11$ in, $d_{min} = 2.4$ in, $F_a = 60$ lbf/in, $\gamma = 0.037$ lbf/in^3, $f = 0.8$

$w = 12\gamma bt = 12(0.037)12(0.11) = 0.586$ lbf/ft

(a) Eq. (*e*), p. 885: $$F_c = \frac{w}{g}\left(\frac{V}{60}\right)^2 = \frac{0.586}{32.17}\left(\frac{1833}{60}\right)^2 = 17.0 \text{ lbf} \quad \textit{Ans.}$$

$$T = \frac{63\,025 H_{nom} K_s n_d}{n} = \frac{63\,025(2)(1.25)(1)}{1750} = 90.0 \text{ lbf}\cdot\text{in}$$

$$\Delta F = (F_1)_a - F_2 = \frac{2T}{d} = \frac{2(90.0)}{4} = 45.0 \text{ lbf}$$

Table 17-4: $C_p = 0.73$

Eq. (17-12): $(F_1)_a = bF_aC_pC_v = 12(60)(0.73)(1) = 525.6$ lbf *Ans.*

$F_2 = (F_1)_a - [(F_1)_a - F_2] = 525.6 - 45 = 480.6$ lbf *Ans.*

Eq. (*i*), p. 886: $F_i = \dfrac{(F_1)_a + F_2}{2} - F_c = \dfrac{525.6 + 480.6}{2} - 17.0 = 486.1 \text{ lbf}$ *Ans.*

Eq. (17-9):

$$f' = \frac{1}{\theta_d}\ln\left[\frac{(F_1)_a - F_c}{F_2 - F_c}\right] = \frac{1}{3.123}\ln\left(\frac{525.6 - 17.0}{480.6 - 17.0}\right) = 0.0297$$

The friction is thus underdeveloped.

(b) The transmitted horsepower is, with $\Delta F = (F_1)_a - F_2 = 45$ lbf,

$$H = \frac{(\Delta F)V}{33\ 000} = \frac{45(1833)}{33\ 000} = 2.5 \text{ hp} \quad \textit{Ans.}$$

$$n_{fs} = \frac{H}{H_{\text{nom}}K_s} = \frac{2.5}{2(1.25)} = 1$$

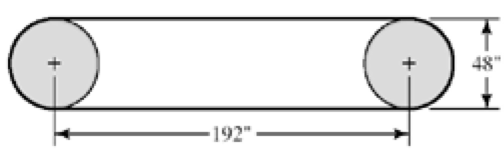
Eq. (17-1): $\theta_D = \pi + 2\sin^{-1}\dfrac{D-d}{2C} = \pi + 2\sin^{-1}\left[\dfrac{8-4}{2(216)}\right] = 3.160 \text{ rad}$

Eq. (17-2): $L = [4C^2 - (D-d)^2]^{1/2} + (D\theta_D + d\theta_d)/2$

$= [4(216)^2 - (8-4)^2]^{1/2} + [8(3.160) + 4(3.123)]/2 = 450.9$ in *Ans.*

(c) Eq. (17-13): $\text{dip} = \dfrac{3C^2w}{2F_i} = \dfrac{3(216/12)^2(0.586)}{2(486.1)} = 0.586 \text{ in}$ *Ans.*

17-3



As a design task, the decision set on p. 893 is useful.
A priori decisions:

- Function: $H_{\text{nom}} = 60$ hp, $n = 380$ rev/min, $C = 192$ in, $K_s = 1.1$
- Design factor: $n_d = 1$
- Initial tension: Catenary
- Belt material. Table 17-2: Polyamide A-3, $F_a = 100$ lbf/in, $\gamma = 0.042$ lbf/in^3, $f = 0.8$
- Drive geometry: $d = D = 48$ in
- Belt thickness: $t = 0.13$ in

Design variable: Belt width.
Use a method of trials. Initially, choose $b = 6$ in

$$V = \frac{\pi d n}{12} = \frac{\pi(48)(380)}{12} = 4775 \text{ ft/min}$$

$$w = 12\gamma b t = 12(0.042)(6)(0.13) = 0.393 \text{ lbf/ft}$$

$$F_c = \frac{wV^2}{g} = \frac{0.393(4775/60)^2}{32.17} = 77.4 \text{ lbf}$$

$$T = \frac{63\,025 H_{\text{nom}} K_s n_d}{n} = \frac{63\,025(60)(1.1)(1)}{380} = 10\,946 \text{ lbf}\cdot\text{in}$$

$$\Delta F = \frac{2T}{d} = \frac{2(10\,946)}{48} = 456.1 \text{ lbf}$$

$$F_1 = (F_1)_a = bF_a C_p C_v = 6(100)(1)(1) = 600 \text{ lbf}$$

$$F_2 = F_1 - \Delta F = 600 - 456.1 = 143.9 \text{ lbf}$$

Transmitted power H

$$H = \frac{\Delta F(V)}{33\,000} = \frac{456.1(4775)}{33\,000} = 66 \text{ hp}$$

$$F_i = \frac{F_1 + F_2}{2} - F_c = \frac{600 + 143.9}{2} - 77.4 = 294.6 \text{ lbf}$$

$$f' = \frac{1}{\theta_d}\ln\frac{F_1 - F_c}{F_2 - F_c} = \frac{1}{\pi}\ln\left(\frac{600 - 77.4}{143.9 - 77.4}\right) = 0.656$$

Eq. (17-2): $L = [4(192)^2 - (48 - 48)^2]^{1/2} + [48(\pi) + 48(\pi)] / 2 = 534.8$ in

Friction is not fully developed, so $b_{\min}$ is just a little smaller than 6 in (5.7 in). Not having a figure of merit, we choose the most narrow belt available (6 in). We can improve the design by reducing the initial tension, which reduces F_1 and F_2, thereby increasing belt life (see the result of Prob. 17-8). This will bring f' to 0.80

$$F_1 = \frac{(\Delta F + F_c)\exp(f\theta) - F_c}{\exp(f\theta) - 1}$$

$$\exp(f\theta) = \exp(0.80\pi) = 12.345$$

Therefore

$$F_1 = \frac{(456.1 + 77.4)(12.345) - 77.4}{12.345 - 1} = 573.7 \text{ lbf}$$

$$F_2 = F_1 - \Delta F = 573.7 - 456.1 = 117.6 \text{ lbf}$$

$$F_i = \frac{F_1 + F_2}{2} - F_c = \frac{573.7 + 117.6}{2} - 77.4 = 268.3 \text{ lbf}$$

These are small reductions since f' is close to f, but improvements nevertheless.

$$f' = \frac{1}{\theta_d}\ln\frac{F_1 - F_c}{F_2 - F_c} = \frac{1}{\pi}\ln\left(\frac{573.7 - 77.4}{117.6 - 77.4}\right) = 0.80$$

$$\text{dip} = \frac{3C^2 w}{2F_i} = \frac{3(192 / 12)^2(0.393)}{2(268.3)} = 0.562 \text{ in}$$

17-4 From the last equation given in the problem statement,

$$\exp(f\phi) = \frac{1}{1 - \{2T / [d(a_0 - a_2)b]\}}$$

$$\left[1 - \frac{2T}{d(a_0 - a_2)b}\right]\exp(f\phi) = 1$$

$$\left[\frac{2T}{d(a_0 - a_2)b}\right]\exp(f\phi) = \exp(f\phi) - 1$$

$$b = \frac{1}{a_0 - a_2}\left(\frac{2T}{d}\right)\left[\frac{\exp(f\phi)}{\exp(f\phi) - 1}\right]$$

But $2T/d = 33\,000H_d/V$. Thus,

$$b = \frac{1}{a_0 - a_2}\left(\frac{33\,000H_d}{V}\right)\left[\frac{\exp(f\phi)}{\exp(f\phi) - 1}\right] \quad \textit{Q.E.D.}$$

17-5 Refer to Ex. 17-1 on p. 890 for the values used below.
(a) The maximum torque prior to slip is,

$$T = \frac{63\,025H_{\text{nom}}K_s n_d}{n} = \frac{63\,025(15)(1.25)(1.1)}{1750} = 742.8 \text{ lbf}\cdot\text{in} \quad \textit{Ans.}$$

The corresponding initial tension, from Eq. (17-9), is,

$$F_i = \frac{T}{d}\left(\frac{\exp(f\theta) + 1}{\exp(f\theta) - 1}\right) = \frac{742.8}{6}\left(\frac{11.17 + 1}{11.17 - 1}\right) = 148.1 \text{ lbf} \quad \textit{Ans.}$$

(b) See Prob. 17-4 statement. The final relation can be written

$$b_{\min} = \frac{1}{F_a C_p C_v - (12\gamma t / 32.174)(V / 60)^2} \left\{ \frac{33\,000 H_a \exp(f\theta)}{V[\exp(f\theta) - 1]} \right\}$$

$$= \frac{1}{100(0.7)(1) - \{[12(0.042)(0.13)] / 32.174\}(2749 / 60)^2} \left[\frac{33\,000(20.6)(11.17)}{2749(11.17 - 1)} \right]$$

$$= 4.13 \text{ in} \quad \textit{Ans.}$$

This is the minimum belt width since the belt is at the point of slip. The design must round up to an available width.

Eq. (17-1):

$$\theta_d = \pi - 2\sin^{-1}\left(\frac{D - d}{2C}\right) = \pi - 2\sin^{-1}\left[\frac{18 - 6}{2(96)}\right]$$
$$= 3.016\,511 \text{ rad}$$
$$\theta_D = \pi + 2\sin^{-1}\left(\frac{D - d}{2C}\right) = \pi + 2\sin^{-1}\left[\frac{18 - 6}{2(96)}\right]$$
$$= 3.266\,674 \text{ rad}$$

Eq. (17-2):

$$L = [4(96)^2 - (18 - 6)^2]^{1/2} + \frac{1}{2}[18(3.266\,674) + 6(3.016\,511)]$$
$$= 230.074 \text{ in} \quad \textit{Ans.}$$

(c)

$$\Delta F = \frac{2T}{d} = \frac{2(742.8)}{6} = 247.6 \text{ lbf}$$
$$(F_1)_a = bF_a C_p C_v = F_1 = 4.13(100)(0.70)(1) = 289.1 \text{ lbf}$$
$$F_2 = F_1 - \Delta F = 289.1 - 247.6 = 41.5 \text{ lbf}$$
$$w = 12\gamma bt = 12(0.042)4.13(0.130) = 0.271 \text{ lbf/ft}$$
$$F_c = \frac{w}{g}\left(\frac{V}{60}\right)^2 = \frac{0.271}{32.17}\left(\frac{2749}{60}\right)^2 = 17.7 \text{ lbf}$$
$$F_i = \frac{F_1 + F_2}{2} - F_c = \frac{289.1 + 41.5}{2} - 17.7 = 147.6 \text{ lbf}$$

Transmitted belt power H

$$H = \frac{\Delta F(V)}{33\,000} = \frac{247.6(2749)}{33\,000} = 20.6 \text{ hp}$$
$$n_{fs} = \frac{H}{H_{\text{nom}} K_s} = \frac{20.6}{15(1.25)} = 1.1$$

Dip: $dip = \frac{3C^2 w}{2F_i} = \frac{3(96/12)^2(0.271)}{2(147.6)} = 0.176$ in

(**d**) If you only change the belt width, the parameters in the following table change as shown.

	Ex. 17-1	This Problem
b	6.00	4.13
w	0.393	0.271
F_c	25.6	17.7
$(F_1)_a$	420	289
F_2	172.4	41.5
F_i	270.6	147.6
f'	0.33*	0.80**
dip	0.139	0.176

*Friction underdeveloped
**Friction fully developed

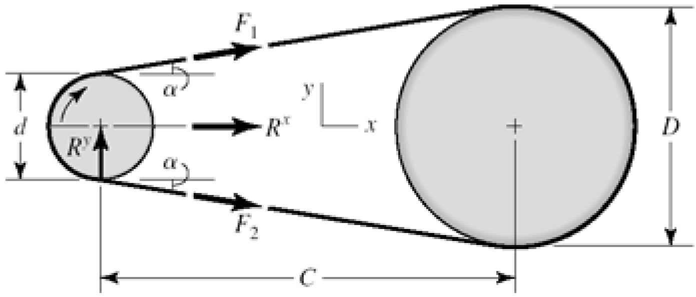
17-6 The transmitted power is the same.

	b = 6 in	b = 12 in	n-Fold Change
F_c	25.65	51.3	2
F_i	270.35	664.9	2.46
$(F_1)_a$	420	840	2
F_2	172.4	592.4	3.44
H_a	20.62	20.62	1
n_{fs}	1.1	1.1	1
f'	0.139	0.125	0.90
dip	0.328	0.114	0.34

If we relax F_i to develop full friction ($f = 0.80$) and obtain longer life, then

	b = 6 in	b = 12 in	n-Fold Change
F_c	25.6	51.3	2
F_i	148.1	148.1	1
F_1	297.6	323.2	1.09
F_2	50	75.6	1.51
f'	0.80	0.80	1
dip	0.255	0.503	2

17-7



Find the resultant of F_1 and F_2:

$$\alpha = \sin^{-1}\frac{D - d}{2C}$$

$$\sin\alpha = \frac{D - d}{2C}$$

$$\cos\alpha \doteq 1 - \frac{1}{2}\left(\frac{D - d}{2C}\right)^2$$

$$R^x = F_1\cos\alpha + F_2\cos\alpha = (F_1 + F_2)\left[1 - \frac{1}{2}\left(\frac{D - d}{2C}\right)^2\right] \quad \textit{Ans.}$$

$$R^y = F_1\sin\alpha - F_2\sin\alpha = (F_1 - F_2)\frac{D - d}{2C} \quad \textit{Ans.}$$

From Ex. 17-2, $d = 16$ in, $D = 36$ in, $C = 16(12) = 192$ in, $F_1 = 940$ lbf, $F_2 = 276$ lbf

$$\alpha = \sin^{-1}\left[\frac{36 - 16}{2(192)}\right] = 2.9855^{\circ}$$

$$R^x = (940 + 276)\left[1 - \frac{1}{2}\left(\frac{36 - 16}{2(192)}\right)^2\right] = 1214.4 \text{ lbf}$$

$$R^y = (940 - 276)\left[\frac{36 - 16}{2(192)}\right] = 34.6 \text{ lbf}$$

$$T = (F_1 - F_2)\left(\frac{d}{2}\right) = (940 - 276)\left(\frac{16}{2}\right) = 5312 \text{ lbf}\cdot\text{in}$$

17-8 Begin with Eq. (17-10),

$$F_1 = F_c + F_i\frac{2\exp(f\theta)}{\exp(f\theta) - 1}$$

Introduce Eq. (17-9):

$$F_1 = F_c + d\left[\frac{\exp(f\theta)+1}{\exp(f\theta)-1}\right]\left[\frac{2\exp(f\theta)}{\exp(f\theta)+1}\right] = F_c + \frac{2T}{d}\left[\frac{\exp(f\theta)}{\exp(f\theta)-1}\right]$$

$$F_1 = F_c + \Delta F\,\frac{\exp(f\theta)}{\exp(f\theta)-1}$$

Now add and subtract $F_c\left[\frac{\exp(f\theta)}{\exp(f\theta)-1}\right]$

$$\begin{aligned}
F_1 &= F_c + F_c\left[\frac{\exp(f\theta)}{\exp(f\theta)-1}\right] + \Delta F\left[\frac{\exp(f\theta)}{\exp(f\theta)-1}\right] - F_c\left[\frac{\exp(f\theta)}{\exp(f\theta)-1}\right]\\
&= (F_c+\Delta F)\left[\frac{\exp(f\theta)}{\exp(f\theta)-1}\right] + F_c - F_c\left[\frac{\exp(f\theta)}{\exp(f\theta)-1}\right]\\
&= (F_c+\Delta F)\left[\frac{\exp(f\theta)}{\exp(f\theta)-1}\right] - \frac{F_c}{\exp(f\theta)-1}\\
&= \frac{(F_c+\Delta F)\exp(f\theta) - F_c}{\exp(f\theta)-1} \quad \textit{Q.E.D.}
\end{aligned}$$

From Ex. 17-2: $\theta_d = 3.037$ rad, $\Delta F = 664$ lbf, $\exp(f\theta) = \exp[0.80(3.037)] = 11.35$, and $F_c = 73.4$ lbf.

$$\begin{aligned}
F_1 &= \frac{(73.4+664)11.35-73.4}{(11.35-1)} = 802\text{ lbf}\\
F_2 &= F_1 - \Delta F = 802 - 664 = 138\text{ lbf}\\
F_i &= \frac{802+138}{2} - 73.4 = 396.6\text{ lbf}\\
f' &= \frac{1}{\theta_d}\ln\left(\frac{F_1-F_c}{F_2-F_c}\right) = \frac{1}{3.037}\ln\left(\frac{802-73.4}{138-73.4}\right) = 0.80 \quad \textit{Ans.}
\end{aligned}$$

17-9 This is a good class project. Form four groups, each with a belt to design. Once each group agrees internally, all four should report their designs including the forces and torques on the line shaft. If you give them the pulley locations, they could design the line shaft.

17-10 If you have the students implement a computer program, the design problem selections may differ, and the students will be able to explore them. For $K_s = 1.25$, $n_d = 1.1$, $d = 14$ in and $D = 28$ in, a polyamide A-5 belt, 8 inches wide, will do ($b_{\min} = 6.58$ in)

17-11 An efficiency of less than unity lowers the output for a given input. Since the object of the drive is the output, the efficiency must be incorporated such that the belt's capacity is increased. The design power would thus be expressed as

$$H_d = \frac{H_{\text{nom}} K_s n_d}{\text{eff}} \qquad \textit{Ans.}$$

17-12 Some perspective on the size of F_c can be obtained from

$$F_c = \frac{w}{g}\left(\frac{V}{60}\right)^2 = \frac{12\gamma bt}{g}\left(\frac{V}{60}\right)^2$$

An approximate comparison of non-metal and metal belts is presented in the table below.

	Non-metal	Metal
γ, lbf/in^3	0.04	0.280
b, in	5.00	1.000
t, in	0.20	0.005

The ratio w / w_m is

$$\frac{w}{w_m} = \frac{12(0.04)(5)(0.2)}{12(0.28)(1)(0.005)} \doteq 29$$

The second contribution to F_c is the belt peripheral velocity which tends to be low in metal belts used in instrument, printer, plotter and similar drives. The velocity ratio squared influences any $F_c / (F_c)_m$ ratio.

It is common for engineers to treat F_c as negligible compared to other tensions in the belting problem. However, when developing a computer code, one should include F_c.

17-13 Eq. (17-8):

$$\Delta F = F_1 - F_2 = (F_1 - F_c)\frac{\exp(f\theta) - 1}{\exp(f\theta)} \doteq F_1 \frac{\exp(f\theta) - 1}{\exp(f\theta)}$$

Assuming negligible centrifugal force and setting $F_1 = ab$ from step 3, p. 897,

$$b_{\min} = \frac{\Delta F}{a}\frac{\exp(f\theta)}{\exp(f\theta) - 1} \qquad (1)$$

Also,

$$H_d = H_{\text{nom}} K_s n_d = \frac{(\Delta F)V}{33\,000}$$

$$\Delta F = \frac{33\,000 H_{\text{nom}} K_s n_d}{V}$$

Substituting into Eq. (1), $$b_{\min} = \frac{1}{a}\left(\frac{33\,000H_d}{V}\right)\frac{\exp(f\theta)}{\exp(f\theta)-1} \quad \textit{Ans.}$$

17-14 The decision set for the friction metal flat-belt drive is:

A priori decisions

- Function: $H_{\text{nom}} = 1$ hp, $n = 1750$ rev/min, $VR = 2$, $C \doteq 15$ in, $K_s = 1.2$, $N_p = 10^6$ belt passes.
- Design factor: $n_d = 1.05$
- Belt material and properties: 301/302 stainless steel
  Table 17-8: $S_y = 175$ kpsi, $E = 28$ Mpsi, $\nu = 0.285$
- Drive geometry: $d = 2$ in, $D = 4$ in
- Belt thickness: $t = 0.003$ in

Design variables:

- Belt width, b
- Belt loop periphery

Preliminaries

$$H_d = H_{\text{nom}}K_s n_d = 1(1.2)(1.05) = 1.26 \text{ hp}$$
$$T = \frac{63\,025(1.26)}{1750} = 45.38 \text{ lbf}\cdot\text{in}$$

A 15 in center-to-center distance corresponds to a belt loop periphery of 39.5 in. The 40 in loop available corresponds to a 15.254 in center distance.

$$\theta_d = \pi - 2\sin^{-1}\left[\frac{4-2}{2(15.254)}\right] = 3.010 \text{ rad}$$
$$\theta_D = \pi + 2\sin^{-1}\left[\frac{4-2}{2(15.274)}\right] = 3.273 \text{ rad}$$

For full friction development

$$\exp(f\theta_d) = \exp[0.35(3.010)] = 2.868$$
$$V = \frac{\pi dn}{12} = \frac{\pi(2)(1750)}{12} = 916.3 \text{ ft/s}$$
$$S_y = 175 \text{ kpsi}$$

Eq. (17-15):
$$S_y = 14.17\left(10^6\right)N_p^{-0.407} = 14.17\left(10^6\right)\left(10^6\right)^{-0.407} = 51.212\left(10^3\right) \text{ psi}$$

From selection step 3, p. 897,

$$a = \left[S_f - \frac{Et}{(1-\nu^2)d} \right] t = \left[51.212(10^3) - \frac{28(10^6)(0.003)}{(1-0.285^2)(2)} \right](0.003)$$

$= 16.50$ lbf/in of belt width

$$(F_1)_a = ab = 16.50b$$

For full friction development, from Prob. 17-13,

$$b_{\min} = \frac{\Delta F}{a} \frac{\exp(f\theta_d)}{\exp(f\theta_d) - 1}$$

$$\Delta F = \frac{2T}{d} = \frac{2(45.38)}{2} = 45.38 \text{ lbf}$$

So

$$b_{\min} = \frac{45.38}{16.50}\left(\frac{2.868}{2.868-1}\right) = 4.23 \text{ in}$$

Decision #1: $b = 4.5$ in

$$F_1 = (F_1)_a = ab = 16.5(4.5) = 74.25 \text{ lbf}$$

$$F_2 = F_1 - \Delta F = 74.25 - 45.38 = 28.87 \text{ lbf}$$

$$F_i = \frac{F_1 + F_2}{2} = \frac{74.25 + 28.87}{2} = 51.56 \text{ lbf}$$

Existing friction

$$f' = \frac{1}{\theta_d} \ln\left(\frac{F_1}{F_2}\right) = \frac{1}{3.010} \ln\left(\frac{74.25}{28.87}\right) = 0.314$$

$$H_t = \frac{(\Delta F)V}{33\,000} = \frac{45.38(916.3)}{33\,000} = 1.26 \text{ hp}$$

$$n_{fs} = \frac{H_t}{H_{\text{nom}} K_s} = \frac{1.26}{1(1.2)} = 1.05$$

This is a non-trivial point. The methodology preserved the factor of safety corresponding to $n_d = 1.1$ *even* as we rounded $b_{\min}$ up to b.

Decision #2 was taken care of with the adjustment of the center-to-center distance to accommodate the belt loop. Use Eq. (17-2) as is and solve for C to assist in this. Remember to subsequently recalculate θ_d and θ_D .

17-15 Decision set:

A priori decisions

- Function: $H_{\text{nom}} = 5$ hp, $N = 1125$ rev/min, $VR = 3$, $C \doteq 20$ in, $K_s = 1.25$, $N_p = 10^6$ belt passes
- Design factor: $n_d = 1.1$
- Belt material: BeCu, $S_y = 170$ kpsi, $E = 17$ Mpsi, $\nu = 0.220$
- Belt geometry: $d = 3$ in, $D = 9$ in
- Belt thickness: $t = 0.003$ in

Design decisions

- Belt loop periphery
- Belt width b

Preliminaries:

$$H_d = H_{\text{nom}} K_s n_d = 5(1.25)(1.1) = 6.875 \text{ hp}$$
$$T = \frac{63\,025(6.875)}{1125} = 385.2 \text{ lbf} \cdot \text{in}$$

Decision #1: Choose a 60-in belt loop with a center-to-center distance of 20.3 in.

$$\theta_d = \pi - 2\sin^{-1}\left[\frac{9-3}{2(20.3)}\right] = 2.845 \text{ rad}$$
$$\theta_D = \pi + 2\sin^{-1}\left[\frac{9-3}{2(20.3)}\right] = 3.438 \text{ rad}$$

For full friction development:

$$\exp(f\theta_d) = \exp[0.32(2.845)] = 2.485$$
$$V = \frac{\pi d n}{12} = \frac{\pi(3)(1125)}{12} = 883.6 \text{ ft/min}$$
$$S_f = 56.67 \text{ kpsi}$$

From selection step 3, p. 897,

$$a = \left[S_f - \frac{E_t}{(1-\nu^2)d} \right] t = \left[56.67(10^3) - \frac{17(10^6)(0.003)}{(1-0.22^2)(3)} \right](0.003) = 116.4 \text{ lbf/in}$$

$$\Delta F = \frac{2T}{d} = \frac{2(385.2)}{3} = 256.8 \text{ lbf}$$

$$b_{\min} = \frac{\Delta F}{a}\left[\frac{\exp(f\theta_d)}{\exp(f\theta_d)-1} \right] = \frac{256.8}{116.4}\left(\frac{2.485}{2.485-1} \right) = 3.69 \text{ in}$$

Decision #2: $b = 4$ in

$$F_1 = (F_1)_a = ab = 116.4(4) = 465.6 \text{ lbf}$$

$$F_2 = F_1 - \Delta F = 465.6 - 256.8 = 208.8 \text{ lbf}$$

$$F_i = \frac{F_1 + F_2}{2} = \frac{465.6 + 208.8}{2} = 337.3 \text{ lbf}$$

Existing friction

$$f' = \frac{1}{\theta_d}\ln\left(\frac{F_1}{F_2}\right) = \frac{1}{2.845}\ln\left(\frac{465.6}{208.8}\right) = 0.282$$

$$H = \frac{(\Delta F)V}{33\ 000} = \frac{256.8(883.6)}{33\ 000} = 6.88 \text{ hp}$$

$$n_{fs} = \frac{H}{5(1.25)} = \frac{6.88}{5(1.25)} = 1.1$$

F_i can be reduced only to the point at which $f' = f = 0.32$. From Eq. (17-9)

$$F_i = \frac{T}{d}\left[\frac{\exp(f\theta_d)+1}{\exp(f\theta_d)-1} \right] = \frac{385.2}{3}\left(\frac{2.485+1}{2.485-1} \right) = 301.3 \text{ lbf}$$

Eq. (17-10):

$$F_1 = F_i\left[\frac{2\exp(f\theta_d)}{\exp(f\theta_d)+1} \right] = 301.3\left[\frac{2(2.485)}{2.485+1} \right] = 429.7 \text{ lbf}$$

$$F_2 = F_1 - \Delta F = 429.7 - 256.8 = 172.9 \text{ lbf}$$

and $\quad f' = f = 0.32$

17-16 This solution is the result of a series of five design tasks involving different belt thicknesses. The results are to be compared as a matter of perspective. These design tasks are accomplished in the same manner as in Probs. 17-14 and 17-15 solutions.

The details will not be presented here, but the table is provided as a means of learning. Five groups of students could each be assigned a belt thickness. You can form a table from their results or use the table given here.

	t, in				
	0.002	0.003	0.005	0.008	0.010
b	4.000	3.500	4.000	1.500	1.500
CD	20.300	20.300	20.300	18.700	20.200
a	109.700	131.900	110.900	194.900	221.800
d	3.000	3.000	3.000	5.000	6.000
D	9.000	9.000	9.000	15.000	18.000
F_i	310.600	333.300	315.200	215.300	268.500
F_1	439.000	461.700	443.600	292.300	332.700
F_2	182.200	209.000	186.800	138.200	204.300
n_{fs}	1.100	1.100	1.100	1.100	1.100
L	60.000	60.000	60.000	70.000	80.000
f'	0.309	0.285	0.304	0.288	0.192
F_i	301.200	301.200	301.200	195.700	166.600
F_1	429.600	429.600	429.600	272.700	230.800
F_2	172.800	172.800	172.800	118.700	102.400
f	0.320	0.320	0.320	0.320	0.320

The first three thicknesses result in the same adjusted F_i, F_1 and F_2 (why?). We have no figure of merit, but the costs of the belt and pulleys are about the same for these three thicknesses. Since the same power is transmitted and the belts are widening, belt forces are lessening.

17-17 This is a design task. The decision variables would be belt length and belt section, which could be combined into one, such as B90. The number of belts is not an issue.

We have no figure of merit, which is not practical in a text for this application. It is suggested that you gather sheave dimensions and costs and V-belt costs from a principal vendor and construct a figure of merit based on the costs. Here is one trial.

Preliminaries: For a single V-belt drive with $H_{\text{nom}} = 3$ hp, $n = 3100$ rev/min, $D = 12$ in, and $d = 6.2$ in, choose a B90 belt, $K_s = 1.3$ and $n_d = 1$. From Table 17-10, select a circumference of 90 in. From Table 17-11, add 1.8 in giving

$$L_p = 90 + 1.8 = 91.8 \text{ in}$$

Eq. (17-16*b*):

$$C = 0.25\left\{\left[91.8 - \frac{\pi}{2}(12 + 6.2)\right] + \sqrt{\left[91.8 - \frac{\pi}{2}(12 + 6.2)\right]^2 - 2(12 - 6.2)^2}\right\}$$
$$= 31.47 \text{ in}$$

$$\theta_d = \pi - 2\sin^{-1}\left[\frac{12 - 6.2}{2(31.47)}\right] = 2.9570 \text{ rad}$$

$$\exp(f\theta_d) = \exp[0.5123(2.9570)] = 4.5489$$

$$V = \frac{\pi d n}{12} = \frac{\pi(6.2)(3100)}{12} = 5031.8 \text{ ft/min}$$

Table 17-13:

$$\text{Angle}\,\theta = \theta_d \frac{180°}{\pi} = (2.957 \text{ rad})\left(\frac{180°}{\pi}\right) = 169.42°$$

The footnote regression equation of Table 17-13 gives K_1 without interpolation:

$$K_1 = 0.143\,543 + 0.007\,468(169.42°) - 0.000\,015\,052(169.42°)^2 = 0.9767$$

The design power is

$$H_d = H_{\text{nom}} K_s n_d = 3(1.3)(1) = 3.9 \text{ hp}$$

From Table 17-14 for B90, $K_2 = 1$. From Table 17-12 take a marginal entry of $H_{\text{tab}} = 4$, although extrapolation would give a slightly lower H_{tab}.

Eq. (17-17): $H_a = K_1 K_2 H_{\text{tab}} = 0.9767(1)(4) = 3.91 \text{ hp}$

The allowable ΔF_a is given by

$$\Delta F_a = \frac{63\,025 H_a}{n(d/2)} = \frac{63\,025(3.91)}{3100(6.2/2)} = 25.6 \text{ lbf}$$

The allowable torque T_a is

$$T_a = \frac{\Delta F_a d}{2} = \frac{25.6(6.2)}{2} = 79.4 \text{ lbf} \cdot \text{in}$$

From Table 17-16, $K_c = 0.965$. Thus, Eq. (17-21) gives,

$$F_c = K_c\left(\frac{V}{1000}\right)^2 = 0.965\left(\frac{5031.8}{1000}\right)^2 = 24.4 \text{ lbf}$$

At incipient slip, Eq. (17-9) provides:

$$F_i = \left(\frac{T}{d}\right)\left[\frac{\exp(f\theta) + 1}{\exp(f\theta) - 1}\right] = \left(\frac{79.4}{6.2}\right)\left(\frac{4.5489 + 1}{4.5489 - 1}\right) = 20.0 \text{ lbf}$$

Eq. (17-10):

$$F_1 = F_c + F_i\left[\frac{2\exp(f\theta)}{\exp(f\theta)+1}\right] = 24.4 + 20\left[\frac{2(4.5489)}{4.5489+1}\right] = 57.2 \text{ lbf}$$

Thus, $F_2 = F_1 - \Delta F_a = 57.2 - 25.6 = 31.6$ lbf

Eq. (17-26): $$n_{fs} = \frac{H_a N_b}{H_d} = \frac{(3.91)(1)}{3.9} = 1.003 \quad \textit{Ans.}$$

If we had extrapolated for H_{tab}, the factor of safety would have been slightly less than one.

Life Use Table 17-16 to find equivalent tensions T_1 and T_2 .

$$T_1 = F_1 + (F_b)_1 = F_1 + \frac{K_b}{d} = 57.2 + \frac{576}{6.2} = 150.1 \text{ lbf}$$
$$T_2 = F_1 + (F_b)_2 = F_1 + \frac{K_b}{D} = 57.2 + \frac{576}{12} = 105.2 \text{ lbf}$$

From Table 17-17, $K = 1193$, $b = 10.926$, and from Eq. (17-27), the number of belt passes is:

$$N_P = \left[\left(\frac{K}{T_1}\right)^{-b} + \left(\frac{K}{T_2}\right)^{-b}\right]^{-1}$$
$$= \left[\left(\frac{1193}{150.1}\right)^{-10.926} + \left(\frac{1193}{105.2}\right)^{-10.926}\right]^{-1} = 6.72(10^9) \text{ passes}$$

From Eq. (17-28) for $N_P > 10^9$,

$$t = \frac{N_P L_p}{720V} > \frac{10^9(91.8)}{720(5031.8)}$$
$$t > 25\,340 \text{ h} \quad \textit{Ans.}$$

Suppose n_{fs} was too small. Compare these results with a 2-belt solution.

$$H_{\text{tab}} = 4 \text{ hp/belt}, \quad T_a = 39.6 \text{ lbf}\cdot\text{in/belt},$$
$$\Delta F_a = 12.8 \text{ lbf/belt}, \quad H_a = 3.91 \text{ hp/belt}$$
$$n_{fs} = \frac{N_b H_a}{H_d} = \frac{N_b H_a}{H_{\text{nom}} K_s} = \frac{2(3.91)}{3(1.3)} = 2.0$$

Also, $F_1 = 40.8$ lbf/belt, $F_2 = 28.0$ lbf/belt

$$
\begin{aligned}
F_i &= 9.99 \text{ lbf/belt}, & F_c &= 24.4 \text{ lbf/belt} \\
(F_b)_1 &= 92.9 \text{ lbf/belt}, & (F_b)_2 &= 48 \text{ lbf/belt} \\
T_1 &= 133.7 \text{ lbf/belt}, & T_2 &= 88.8 \text{ lbf/belt} \\
N_P &= 2.39(10^{10}) \text{ passes}, & t &> 605\,600 \text{ h}
\end{aligned}
$$

Initial tension of the drive:

$$(F_i)_{\text{drive}} = N_b F_i = 2(9.99) = 20 \text{ lbf}$$

17-18 Given: two B85 V-belts with $d = 5.4$ in, $D = 16$ in, $n = 1200$ rev/min, and $K_s = 1.25$

Table 17-11: $L_p = 85 + 1.8 = 86.8$ in

Eq. (17-17*b*):

$$C = 0.25\left\{\left[86.8 - \frac{\pi}{2}(16 + 5.4)\right] + \sqrt{\left[86.8 - \frac{\pi}{2}(16 + 5.4)\right]^2 - 2(16 - 5.4)^2}\right\}$$

$$= 26.05 \text{ in} \quad \textit{Ans.}$$

Eq. (17-1):

$$\theta_d = 180° - 2\sin^{-1}\left[\frac{16 - 5.4}{2(26.05)}\right] = 156.5°$$

From table 17-13 footnote:

$$K_1 = 0.143\,543 + 0.007\,468(156.5°) - 0.000\,015\,052(156.5°)^2 = 0.944$$

Table 17-14: $K_2 = 1$

Belt speed: $V = \dfrac{\pi(5.4)(1200)}{12} = 1696 \text{ ft/min}$

Use Table 17-12 to interpolate for H_{tab}.

$$H_{\text{tab}} = 1.59 + \left(\frac{2.62 - 1.59}{2000 - 1000}\right)(1696 - 1000) = 2.31 \text{ hp/belt}$$

Eq. (17-17) for two belts: $H_a = K_1 K_2 N_b H_{\text{tab}} = 0.944(1)(2)(2.31) = 4.36 \text{ hp}$

Assuming $n_d = 1$,

$$H_d = K_s H_{\text{nom}} n_d = 1.25(1) H_{\text{nom}}$$

For a factor of safety of one,

$$
\begin{aligned}
H_a &= H_d \\
4.36 &= 1.25H_{\text{nom}} \\
H_{\text{nom}} &= \frac{4.36}{1.25} = 3.49 \text{ hp} \quad \textit{Ans.}
\end{aligned}
$$

17-19 Given: $H_{\text{nom}} = 60$ hp, $n = 400$ rev/min, $K_s = 1.4$, $d = D = 26$ in on 12 ft centers.

Design task: specify V-belt and number of strands (belts). *Tentative decision*: Use D360 belts.

Table 17-11: $L_p = 360 + 3.3 = 363.3$ in

Eq. (17-16*b*):

$$
C = 0.25\left\{\left[363.3 - \frac{\pi}{2}(26+26)\right] + \sqrt{\left[363.3 - \frac{\pi}{2}(26+26)\right]^2 - 2(26-26)^2}\right\}
$$

$= 140.8$ in (nearly 144 in)

$$
\theta_d = \pi, \quad \theta_D = \pi, \quad \exp[0.5123\pi] = 5.0,
$$

$$
V = \frac{\pi dn}{12} = \frac{\pi(26)(400)}{12} = 2722.7 \text{ ft/min}
$$

Table 17-13: For $\theta = 180°$, $K_1 = 1$

Table 17-14: For D360, $K_2 = 1.10$

Table 17-12: $H_{\text{tab}} = 16.94$ hp by interpolation

Thus, $H_a = K_1 K_2 H_{\text{tab}} = 1(1.1)(16.94) = 18.63$ hp / belt

Eq. (17-19): $H_d = H_{\text{nom}} K_s\, n_d = 60(1.4)(1) = 84$ hp

Number of belts, N_b

$$
N_b = \frac{H_d}{H_a} = \frac{84}{18.63} = 4.51
$$

Round up to five belts. It is left to the reader to repeat the above for belts such as C360 and E360.

$$\Delta F_a = \frac{63\,025 H_a}{n(d/2)} = \frac{63\,025(18.63)}{400(26/2)} = 225.8 \text{ lbf/belt}$$

$$T_a = \frac{(\Delta F_a)d}{2} = \frac{225.8(26)}{2} = 2935 \text{ lbf} \cdot \text{in/belt}$$

Eq. (17-21):

$$F_c = 3.498\left(\frac{V}{1000}\right)^2 = 3.498\left(\frac{2722.7}{1000}\right)^2 = 25.9 \text{ lbf/belt}$$

At fully developed friction, Eq. (17-9) gives

$$F_i = \frac{T}{d}\left[\frac{\exp(f\theta)+1}{\exp(f\theta)-1}\right] = \frac{2935}{26}\left(\frac{5+1}{5-1}\right) = 169.3 \text{ lbf/belt}$$

Eq. (17-10): $$F_1 = F_c + F_i\left[\frac{2\exp(f\theta)}{\exp(f\theta)+1}\right] = 25.9 + 169.3\left[\frac{2(5)}{5+1}\right] = 308.1 \text{ lbf/belt}$$

$$F_2 = F_1 - \Delta F_a = 308.1 - 225.8 = 82.3 \text{ lbf/belt}$$

$$n_{fs} = \frac{H_a N_b}{H_d} = \frac{18.63(5)}{84} = 1.109 \quad \textit{Ans.}$$

Life From Table 17-16,

$$T_1 = T_2 = F_1 + \frac{K_b}{d} = 308.1 + \frac{5\,680}{26} = 526.6 \text{ lbf}$$

Eq. (17-27):

$$N_P = \left[\left(\frac{K}{T_1}\right)^{-b} + \left(\frac{K}{T_2}\right)^{-b}\right]^{-1} = 5.28\left(10^{-9}\right) \text{ passes}$$

Thus, $N_P > 10^{-9}$ passes *Ans.*

Eq. (17-28): $$t = \frac{N_P L_p}{720V} > \frac{10^9(363.3)}{720(2722.7)}$$

Thus, $t > 185\,320$ h *Ans.*

17-20 Preliminaries: $D \doteq 60$ in, 14-in wide rim, $H_{\text{nom}} = 50$ hp, $n = 875$ rev/min, $K_s = 1.2$, $n_d = 1.1$, $m_G = 875/170 = 5.147$, $d \doteq 60/5.147 = 11.65$ in

(a) From Table 17-9, an 11-in sheave exceeds C-section minimum diameter and precludes D- and E-section V-belts.

Decision: Use $d = 11$ in, C270 belts

Table 17-11: $L_p = 270 + 2.9 = 272.9$ in

Eq. (17-16*b*):

$$C = 0.25\left\{\left[272.9 - \frac{\pi}{2}(60+11)\right] + \sqrt{\left[272.9 - \frac{\pi}{2}(60+11)\right]^2 - 2(60-11)^2}\right\}$$
$$= 76.78 \text{ in}$$

This fits in the range

$$D < C < 3(D+d) \;\Rightarrow\; 60 < C < 3(60+11) \;\Rightarrow\; 60 \text{ in} < C < 213 \text{ in}$$

$$\theta_d = \pi - 2\sin^{-1}\frac{60-11}{2(76.78)} = 2.492 \text{ rad} = 142.8°$$

$$\theta_D = \pi + 2\sin^{-1}\frac{60-11}{2(76.78)} = 3.791 \text{ rad}$$

$$\exp(f\,\theta_d) = \exp[0.5123(2.492)] = 3.5846$$

For the flat on flywheel, $f = 0.13$ (see p. 900), $\exp(f\,\theta_D) = \exp[0.13(3.791)] = 1.637$.

The belt speed is

$$V = \frac{\pi dn}{12} = \frac{\pi(11)(875)}{12} = 2520 \text{ ft/min}$$

Table 17-13:

$$K_1 = 0.143\,543 + 0.007\,468(142.8°) - 0.000\,015\,052(142.8°)^2 = 0.903$$

Table 17-14: $K_2 = 1.15$

For interpolation of Table 17-12, let x be entry for $d = 11.65$ in and $n = 2000$ ft/min, and y be entry for $d = 11.65$ in and $n = 3000$ ft/min. Then,

$$\frac{x - 6.74}{11.65 - 11} = \frac{7.17 - 6.74}{12 - 11} \;\Rightarrow\; x = 7.01 \text{ hp at } 2000 \text{ ft/min}$$

and

$$\frac{8.11 - y}{11.65 - 11} = \frac{8.84 - 8.11}{12 - 11} \;\Rightarrow\; y = 8.58 \text{ hp at } 3000 \text{ ft/min}$$

Interpolating these for 2520 ft/min gives

$$\frac{8.58 - H_{tab}}{8.58 - 7.01} = \frac{3000 - 2520}{3000 - 2000} \;\Rightarrow\; H_{tab} = 7.83 \text{ hp/belt}$$

Eq. (17-17): $H_a = K_1 K_2 H_{tab} = 0.903(1.15)(7.83) = 8.13$ hp

Eq. (17-19): $H_d = H_{\text{nom}} K_s n_d = 50(1.2)(1.1) = 66$ hp

Eq. (17-20): $N_b = \dfrac{H_d}{H_a} = \dfrac{66}{8.13} = 8.1$ belts

Decision: Use 9 belts. On a per belt basis,

$$\Delta F_a = \frac{63\,025 H_a}{n(d/2)} = \frac{63\,025(8.13)}{875(11/2)} = 106.5 \text{ lbf/belt}$$

$$T_a = \frac{\Delta F_a d}{2} = \frac{106.5(11)}{2} = 586.8 \text{ lbf} \cdot \text{in per belt}$$

Table 17-16: $K_c = 1.716$

Eq. (17-21): $F_c = 1.716\left(\dfrac{V}{1000}\right)^2 = 1.716\left(\dfrac{2520}{1000}\right)^2 = 10.9$ lbf/belt

At fully developed friction, Eq. (17-9) gives

$$F_i = \frac{T}{d}\left[\frac{\exp(f\theta_d)+1}{\exp(f\theta_d)-1}\right] = \frac{586.9}{11}\left[\frac{3.5846+1}{3.5846-1}\right] = 94.6 \text{ lbf/belt}$$

Eq. (17-10):

$$F_1 = F_c + F_i\left[\frac{2\exp(f\theta_d)}{\exp(f\theta_d)+1}\right] = 10.9 + 94.6\left[\frac{2(3.5846)}{3.5846+1}\right] = 158.8 \text{ lbf/belt}$$

$$F_2 = F_1 - \Delta F_a = 158.8 - 106.7 = 52.1 \text{ lbf/belt}$$

$$n_{fs} = \frac{N_b H_a}{H_d} = \frac{9(8.13)}{66} = 1.11 \quad \textit{O.K.} \quad \textit{Ans.}$$

Durability:

$$(F_b)_1 = K_b / d = 1600/11 = 145.5 \text{ lbf/belt}$$

$$(F_b)_2 = K_b / D = 1600/60 = 26.7 \text{ lbf/belt}$$

$$T_1 = F_1 + (F_b)_1 = 158.8 + 145.5 = 304.3 \text{ lbf/belt}$$

$$T_2 = F_1 + (F_b)_2 = 158.8 + 26.7 = 185.5 \text{ lbf/belt}$$

Eq. (17-27) with Table 17-17:

$$N_P = \left[\left(\frac{K}{T_1}\right)^{-b} + \left(\frac{K}{T_2}\right)^{-b}\right]^{-1} = \left[\left(\frac{2038}{304.3}\right)^{-11.173} + \left(\frac{2038}{185.5}\right)^{-11.173}\right]^{-1}$$

$$= 1.68\left(10^9\right) \text{ passes } > 10^9 \text{ passes} \qquad \textit{Ans.}$$

Since N_P is greater than 10^9 passes and is out of the range of Table 17-17, life from Eq. (17-27) is

$$t = \frac{N_P L_p}{720V} > \frac{10^9(272.9)}{720(2520)} = 150\left(10^3\right) \text{ h}$$

Remember: $(F_i)_{\text{drive}} = 9(94.6) = 851.4$ lbf

Table 17-9: C-section belts are 7/8 in wide. Check sheave groove spacing to see if 14 in width is accommodating.

(b) The fully developed friction torque on the flywheel using the flats of the V-belts, from Eq. (17-9), is

$$T_{\text{flat}} = F_i D\left[\frac{\exp(f\theta) - 1}{\exp(f\theta) + 1}\right] = 94.6(60)\left(\frac{1.637 - 1}{1.637 + 1}\right) = 1371 \text{ lbf} \cdot \text{in per belt}$$

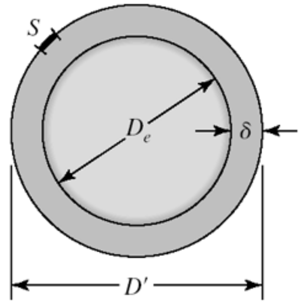
The flywheel torque should be

$$T_{\text{fly}} = m_G T_a = 5.147(586.9) = 3021 \text{ lbf} \cdot \text{in per belt}$$

but it is not. There are applications, however, in which it will work. For example, make the flywheel controlling. Yes. *Ans.*

17-21

(a)



S is the spliced-in string segment length
D_e is the equatorial diameter
D' is the spliced string diameter
δ is the radial clearance
$S + \pi D_e = \pi D' = \pi(D_e + 2\delta) = \pi D_e + 2\pi\delta$

From which

$$\delta = \frac{S}{2\pi}$$

The radial clearance is thus *independent* of D_e.

$$\delta = \frac{12(6)}{2\pi} = 11.5 \text{ in} \quad \textit{Ans.}$$

This is true whether the sphere is the earth, the moon or a marble. Thinking in terms of a radial or diametral increment removes the basic size from the problem.

(b) and **(c)**

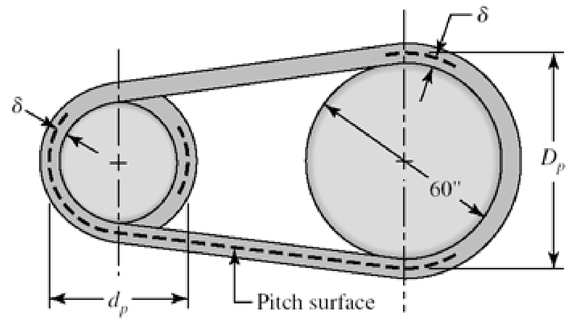
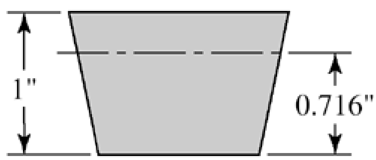


Table 17-9: For an E210 belt, the thickness is 1 in.



$$d_P - d_i = \frac{210 + 4.5}{\pi} - \frac{210}{\pi} = \frac{4.5}{\pi}$$

$$2\delta = \frac{4.5}{\pi}$$

$$\delta = \frac{4.5}{2\pi} = 0.716 \text{ in}$$

The pitch diameter of the flywheel is

$$D_P - 2\delta = D \quad \Rightarrow \quad D_P = D + 2\delta = 60 + 2(0.716) = 61.43 \text{ in}$$

We could make a table:

Diametral Growth	Section				
	A	*B*	*C*	*D*	*E*
2δ	$\frac{1.3}{\pi}$	$\frac{1.8}{\pi}$	$\frac{2.9}{\pi}$	$\frac{3.3}{\pi}$	$\frac{4.5}{\pi}$

The velocity ratio for the D-section belt of Prob. 17-20 is

$$m'_G = \frac{D + 2\delta}{d} = \frac{60 + 3.3/\pi}{11} = 5.55 \quad \textit{Ans.}$$

for the V-flat drive as compared to $m_a = 60/11 = 5.455$ for the VV drive.
The pitch diameter of the pulley is still $d = 11$ in, so the new angle of wrap, θ_d, is

$$\theta_d = \pi - 2\sin^{-1}\frac{D + 2\delta - d}{2C} \quad \textit{Ans.}$$

$$\theta_D = \pi + 2\sin^{-1}\frac{D + 2\delta - d}{2C} \quad \textit{Ans.}$$

Equations (17-16*a*) and (17-16*b*) are modified as follows

$$L_p = 2C + \frac{\pi}{2}(D + 2\delta + d) + \frac{(D + \delta - d)^2}{4C} \quad \textit{Ans.}$$

$$C_p = 0.25\left\{\left[L_p - \frac{\pi}{2}(D + 2\delta + d)\right] + \sqrt{\left[L_p - \frac{\pi}{2}(D + 2\delta + d)\right]^2 - 2(D + 2\delta - d)^2}\right\} \quad \textit{Ans.}$$

The changes are small, but if you are writing a computer code for a V-flat drive, remember that θ_d and θ_D changes are exponential.

17-22 This design task involves specifying a drive to couple an electric motor running at 1720 rev/min to a blower running at 240 rev/min, transmitting two horsepower with a center distance of at least 22 inches. Instead of focusing on the steps, we will display two different designs side-by-side for study. Parameters are in a "per belt" basis with per drive quantities shown along side, where helpful.

Parameter	Four A-90 Belts	Two A-120 Belts
m_G	7.33	7.142
K_s	1.1	1.1
n_d	1.1	1.1
K_1	0.877	0.869
K_2	1.05	1.15
d, in	3.0	4.2
D, in	22	30
θ_d, rad	2.333	2.287
V, ft/min	1350.9	1891
$\exp(f\theta_d)$	3.304	3.2266
L_p, in	91.3	101.3
C, in	24.1	31
H_{tab}, uncorr.	0.783	1.662
$N_b H_{\text{tab}}$, uncorr.	3.13	3.326
T_a, lbf · in	26.45(105.8)	60.87(121.7)
ΔF_a, lbf	17.6(70.4)	29.0(58)
H_a, hp	0.721(2.88)	1.667(3.33)
n_{fs}	1.192	1.372
F_1, lbf	26.28(105.2)	44(88)
F_2, lbf	8.67(34.7)	15(30)
$(F_b)_1$, lbf	73.3(293.2)	52.4(109.8)
$(F_b)_2$, lbf	10(40)	7.33(14.7)
F_c, lbf	1.024	2.0
F_i, lbf	16.45(65.8)	27.5(55)
T_1, lbf · in	99.2	96.4

T_2, lbf · in	36.3	57.4
N', passes	$1.61(10^9)$	$2.3(10^9)$
$t > h$	93 869	89 080

Conclusions:
- Smaller sheaves lead to more belts.
- Larger sheaves lead to larger D and larger V.
- Larger sheaves lead to larger tabulated power.
- The discrete numbers of belts obscures some of the variation. The factors of safety exceed the design factor by differing amounts.

17-23 In Ex. 17-5 the selected chain was 140-3, making the pitch of this 140 chain14/8 = 1.75 in. Table 17-19 confirms.

17-24 **(a)** Eq. (17-32): $H_1 = 0.004 N_1^{1.08} n_1^{0.9} p^{(3-0.07p)}$

Eq. (17-33): $$H_2 = \frac{1000 K_r N_1^{1.5} p^{0.8}}{n_1^{1.5}}$$

Equating and solving for n_1 gives

$$n_1 = \left[\frac{0.25(10^6) K_r N_1^{0.42}}{p^{(2.2-0.07p)}}\right]^{1/2.4} \quad \textit{Ans.}$$

(b) For a No. 60 chain, $p = 6/8 = 0.75$ in, $N_1 = 17$, $K_r = 17$

$$n_1 = \left\{\frac{0.25(10^6)(17)(17)^{0.42}}{0.75^{[2.2-0.07(0.75)]}}\right\}^{1/2.4} = 1227 \text{ rev/min} \quad \textit{Ans.}$$

Table 17-20 confirms that this point occurs at 1200 ± 200 rev/min.

(c) Life predictions using Eq. (17-40) are possible at speeds greater than 1227 rev/min. *Ans.*

17-25 Given: a double strand No. 60 roller chain with $p = 0.75$ in, $N_1 = 13$ teeth at 300 rev/min, $N_2 = 52$ teeth.

(a) Table 17-20: $H_{tab} = 6.20$ hp
Table 17-22: $K_1 = 0.75$
Table 17-23: $K_2 = 1.7$
Use $K_s = 1$
Eq. (17-37):

$$H_a = K_1 K_2 H_{tab} = 0.75(1.7)(6.20) = 7.91 \text{ hp} \quad \textit{Ans.}$$

(b) Eqs. (17-35) and (17-36) with $L/p = 82$

$$A = \frac{13 + 52}{2} - 82 = -49.5$$

$$C = \frac{p}{4}\left[49.5 + \sqrt{49.5^2 - 8\left(\frac{52 - 13}{2\pi}\right)^2}\right] = 23.95p$$

$$C = 23.95(0.75) = 17.96 \text{ in, round up to 18 in} \quad \textit{Ans.}$$

(c) For 30 percent less power transmission,

$$H = 0.7(7.91) = 5.54 \text{ hp}$$

$$T = \frac{63\,025(5.54)}{300} = 1164 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

Eq. (17-29):

$$D = \frac{0.75}{\sin(180^\circ/13)} = 3.13 \text{ in}$$

$$F = \frac{T}{r} = \frac{1164}{3.13/2} = 744 \text{ lbf} \quad \textit{Ans.}$$

17-26 Given: No. 40-4 chain, $N_1 = 21$ teeth for $n = 2000$ rev/min, $N_2 = 84$ teeth, $h = 20\,000$ hours.

(a) Chain pitch is $p = 4/8 = 0.500$ in and $C \doteq 20$ in.

Eq. (17-34):

$$\frac{L}{p} \doteq \frac{2C}{p} + \frac{N_1 + N_2}{2} + \frac{(N_1 - N_2)^2}{4\pi^2 C/p}$$

$$= \frac{2(20)}{0.5} + \frac{21 + 84}{2} + \frac{(84 - 21)^2}{4\pi^2(20/0.5)} = 135 \text{ pitches} \quad \text{(or links)}$$

$$L = 135(0.500) = 67.5 \text{ in} \quad \textit{Ans.}$$

(b) Table 17-20: $H_{tab} = 7.72$ hp (post-extreme power)

Eq. (17-40): Since K_1 is required, the $N_1^{3.75}$ term is omitted (see p. 914).

$$\text{constant} = \frac{(7.72^{2.5})(15\,000)}{135} = 18\,399$$

$$H'_{tab} = \left[\frac{18\,399(135)}{20\,000}\right]^{1/2.5} = 6.88 \text{ hp} \quad \textit{Ans.}$$

(c) Table 17-22:

$$K_1 = \left(\frac{21}{17}\right)^{1.5} = 1.37$$

Table 17-23: $K_2 = 3.3$

$$H_a = K_1 K_2 H'_{\text{tab}} = 1.37(3.3)(6.88) = 31.1 \text{ hp} \quad \textit{Ans.}$$

(d)

$$V = \frac{N_1 pn}{12} = \frac{21(0.5)(2000)}{12} = 1750 \text{ ft/min}$$

$$F_1 = \frac{33\,000(31.1)}{1750} = 586 \text{ lbf} \quad \textit{Ans.}$$

17-27 This is our first design/selection task for chain drives. A possible decision set:

A priori decisions
- Function: H_{nom}, n_1, space, life, K_s
- Design factor: n_d
- Sprockets: Tooth counts N_1 and N_2, factors K_1 and K_2

Decision variables
- Chain number
- Strand count
- Lubrication type
- Chain length in pitches

Function: Motor with $H_{\text{nom}} = 25$ hp at $n = 700$ rev/min; pump at $n = 140$ rev/min; $m_G = 700/140 = 5$
Design Factor: $n_d = 1.1$
Sprockets: Tooth count $N_2 = m_G N_1 = 5(17) = 85$ teeth–odd and unavailable. Choose 84 teeth. *Decision*: $N_1 = 17$, $N_2 = 84$

Evaluate K_1 and K_2
Eq. (17-38): $H_d = H_{\text{nom}} K_s n_d$
Eq. (17-37): $H_a = K_1 K_2 H_{\text{tab}}$

Equate H_d to H_a and solve for H_{tab} :

$$H_{\text{tab}} = \frac{K_s n_d H_{\text{nom}}}{K_1 K_2}$$

Table 17-22: $K_1 = 1$

Table 17-23: $K_2 = 1, 1.7, 2.5, 3.3$ for 1 through 4 strands

$$H'_{\text{tab}} = \frac{1.5(1.1)(25)}{(1)K_2} = \frac{41.25}{K_2}$$

Prepare a table to help with the design decisions:

Strands	K_2	H'_{tab}	Chain No.	H_{tab}	n_{fs}	Lub. Type
1	1.0	41.3	100	59.4	1.58	B
2	1.7	24.3	80	31.0	1.40	B
3	2.5	16.5	80	31.0	2.07	B
4	3.3	12.5	60	13.3	1.17	B

Design Decisions
We need a figure of merit to help with the choice. If the best was 4 strands of No. 60 chain, then

Decision #1 and #2: Choose four strand No. 60 roller chain with n_{fs} = 1.17.

$$n_{fs} = \frac{K_1 K_2 H_{\text{tab}}}{K_s H_{\text{nom}}} = \frac{1(3.3)(13.3)}{1.5(25)} = 1.17$$

Decision #3: Choose Type B lubrication
Analysis:
Table 17-20: H_{tab} = 13.3 hp
Table 17-19: p = 0.75 in

Try C = 30 in in Eq. (17-34):

$$\frac{L}{p} \doteq \frac{2C}{p} + \frac{N_1 + N_2}{2} + \frac{(N_2 - N_1)^2}{4\pi^2 C / p}$$

$$= 2(30 / 0.75) + \frac{17 + 84}{2} + \frac{(84 - 17)^2}{4\pi^2(30 / 0.75)}$$

$$= 133.3$$

$L = 0.75(133.3) = 100$ in (no need to round)

Eq. (17-36) with p = 0.75 in: $A = \dfrac{N_1 + N_2}{2} - \dfrac{L}{p} = \dfrac{17 + 84}{2} - \dfrac{100}{0.75} = -82.83$

Eq. (17-35):

$$C = \frac{p}{4}\left[-A + \sqrt{A^2 - 8\left(\frac{N_2 - N_1}{2\pi}\right)^2}\right]$$

$$= \frac{0.75}{4}\left[-(-82.83) + \sqrt{(-82.83)^2 - 8\left(\frac{84 - 17}{2\pi}\right)^2}\right] = 30.0 \text{ in}$$

Decision #4: Choose $C = 30.0$ in.

17-28 Follow the decision set outlined in Prob. 17-27 solution. We will form two tables, the first for a 15 000 h life goal, and a second for a 50 000 h life goal. The comparison is useful.

Function: $H_{nom} = 50$ hp at $n = 1800$ rev/min, $n_{pump} = 900$ rev/min, $m_G = 1800/900 = 2$, $K_s = 1.2$, life = 15 000 h, then repeat with life = 50 000 h
Design factor: $n_d = 1.1$
Sprockets: $N_1 = 19$ teeth, $N_2 = 38$ teeth
Table 17-22 (post extreme):

$$K_1 = \left(\frac{N_1}{17}\right)^{1.5} = \left(\frac{19}{17}\right)^{1.5} = 1.18$$

Table 17-23: $K_2 = 1, 1.7, 2.5, 3.3, 3.9, 4.6, 6.0$

Decision variables for 15 000 h life goal:

$$H'_{tab} = \frac{K_s n_d H_{nom}}{K_1 K_2} = \frac{1.2(1.1)(50)}{1.18 K_2} = \frac{55.9}{K_2} \qquad (1)$$

$$n_{fs} = \frac{K_1 K_2 H_{tab}}{K_s H_{nom}} = \frac{1.18 K_2 H_{tab}}{1.2(50)} = 0.0197 K_2 H_{tab}$$

Form a table for a 15 000 h life goal using these equations.

K_2	H'_{tab}	Chain #	H_{tab}	n_{fs}	Lub
1	55.90	120	21.6	0.423	C'
1.7	32.90	120	21.6	0.923	C'
2.5	22.40	120	21.6	1.064	C'
3.3	16.90	120	21.6	1.404	C'
3.9	14.30	80	15.6	1.106	C'
4.6	12.20	60	12.4	1.126	C'
6	9.32	60	12.4	1.416	C'

There are 4 possibilities where $n_{fs} \geq 1.1$

Decision variables for 50 000 h life goal
From Eq. (17-40), the power-life tradeoff is:

$$(H'_{\text{tab}})^{2.5} 15\,000 = (H''_{\text{tab}})^{2.5} 50\,000$$

$$H''_{\text{tab}} = \left[\frac{15\,000}{50\,000}(H'_{\text{tab}})^{2.5}\right]^{1/2.5} = 0.618\, H'_{\text{tab}}$$

Substituting from (1),

$$H''_{\text{tab}} = 0.618\left(\frac{55.9}{K_2}\right) = \frac{34.5}{K_2}$$

The H'' notation is only necessary because we constructed the first table, which we normally would not do.

$$n_{fs} = \frac{K_1 K_2 H''_{\text{tab}}}{K_s H_{\text{nom}}} = \frac{K_1 K_2 (0.618 H'_{\text{tab}})}{K_s H_{\text{nom}}} = 0.618[(0.0197) K_2 H_{\text{tab}}]$$
$$= 0.0122 K_2 H_{\text{tab}}$$

Form a table for a 50 000 h life goal.

K_2	H''_{tab}	Chain #	H_{tab}	n_{fs}	Lub
1	34.50	120	21.6	0.264	C'
1.7	20.30	120	21.6	0.448	C'
2.5	13.80	120	21.6	0.656	C'
3.3	10.50	120	21.6	0.870	C'
3.9	8.85	120	21.6	1.028	C'
4.6	7.60	120	21.6	1.210	C'
6	5.80	80	15.6	1.140	C'

There are two possibilities in the second table with $n_{fs} \geq 1.1$. (The tables allow for the identification of a longer life of the outcomes.) We need a figure of merit to help with the choice; costs of sprockets and chains are thus needed, but is more information than we have.

Decision #1: #80 Chain (smaller installation) *Ans.*
$n_{fs} = 0.0122 K_2 H_{\text{tab}} = 0.0122(8.0)(15.6) = 1.14$ *O.K.*
Decision #2: 8-Strand, No. 80 *Ans.*
Decision #3: Type C′ Lubrication *Ans.*
Decision #4: $p = 1.0$ in, C is in midrange of 40 pitches

$$\frac{L}{p} \doteq \frac{2C}{p} + \frac{N_1 + N_2}{2} + \frac{(N_2 - N_1)^2}{4\pi^2 C / p}$$
$$= 2(40) + \frac{19 + 38}{2} + \frac{(38 - 19)^2}{4\pi^2(40)}$$
$$= 108.7 \Rightarrow 110 \text{ even integer} \quad \textit{Ans.}$$

Eq. (17-36):

$$A = \frac{N_1 + N_2}{2} - \frac{L}{p} = \frac{19 + 38}{2} - \frac{110}{1} = -81.5$$

Eq. (17-35):
$$\frac{C}{p} = \frac{1}{4}\left[-(-81.5) + \sqrt{(-81.5)^2 - 8\left(\frac{38 - 19}{2\pi}\right)^2}\,\right] = 40.64$$

$$C = p(C/p) = 1.0(40.64/1.0) = 40.64 \text{ in (for reference)} \quad \textit{Ans.}$$

17-29 The objective of the problem is to explore factors of safety in wire rope. We will express strengths as tensions.

(a) Monitor steel 2-in 6 × 19 rope, 480 ft long.

Table 17-2: Minimum diameter of a sheave is 30d = 30(2) = 60 in, preferably 45(2) = 90 in. The hoist abuses the wire when it is bent around a sheave. Table 17-24 gives the nominal tensile strength as 106 kpsi. The ultimate load is

$$F_u = (S_u)_{\text{nom}} A_{\text{nom}} = 106\left[\frac{\pi(2)^2}{4}\right] = 333 \text{ kip} \quad \textit{Ans.}$$

The tensile loading of the wire is given by Eq. (17-46)

$$F_t = \left(\frac{W}{m} + wl\right)\left(1 + \frac{a}{g}\right)$$
$$W = 4(2) = 8 \text{ kip}, \quad m = 1$$

Table (17-24):
$$wl = 1.60d^2 l = 1.60(2^2)(480) = 3072 \text{ lbf} = 3.072 \text{ kip}$$

Therefore,
$$F_t = (8 + 3.072)\left(1 + \frac{2}{32.2}\right) = 11.76 \text{ kip} \quad \textit{Ans.}$$

Eq. (17-48):
$$F_b = \frac{E_r d_w A_m}{D}$$

and for the 72-in drum

$$F_b = \frac{12(10^6)(2/13)(0.38)(2^2)(10^{-3})}{72} = 39 \text{ kip} \quad \textit{Ans.}$$

For use in Eq. (17-44), from Fig. 17-21

$$(p/S_u) = 0.0014$$
$$S_u = 240 \text{ kpsi}, \quad \text{p. 920}$$
$$F_f = \frac{0.0014(240)(2)(72)}{2} = 24.2 \text{ kip} \quad \textit{Ans.}$$

(b) Factors of safety

Static, no bending:

$$n = \frac{F_u}{F_t} = \frac{333}{11.76} = 28.3 \quad \textit{Ans.}$$

Static, with bending:

Eq. (17-49): $$n_s = \frac{F_u - F_b}{F_t} = \frac{333 - 39}{11.76} = 25.0 \quad \textit{Ans.}$$

Fatigue without bending:

$$n_f = \frac{F_f}{F_t} = \frac{24.2}{11.76} = 2.06 \quad \textit{Ans.}$$

Fatigue, with bending: For a life of $0.1(10^6)$ cycles, from Fig. 17-21

$$(p/S_u) = 4/1000 = 0.004$$
$$F_f = \frac{0.004(240)(2)(72)}{2} = 69.1 \text{ kip}$$

Eq. (17-50): $$n_f = \frac{69.1 - 39}{11.76} = 2.56 \quad \textit{Ans.}$$

If we were to use the endurance strength at 10^6 cycles ($F_f = 24.2$ kip) the factor of safety would be less than 1 indicating 10^6 cycle life impossible.

Comments:

- There are a number of factors of safety used in wire rope analysis. They are different, with different meanings. There is no substitute for knowing exactly which factor of safety is written or spoken.
- Static performance of a rope in tension is impressive.
- In this problem, at the drum, we have a finite life.
- The remedy for fatigue is the use of smaller diameter ropes, with multiple ropes

supporting the load. See Ex. 17-6 for the effectiveness of this approach. It will also be used in Prob. 17-30.

- Remind students that wire ropes do not fail suddenly due to fatigue. The outer wires gradually show wear and breaks; such ropes should be retired. Periodic inspections prevent fatigue failures by parting of the rope.

17-30 Since this is a design task, a decision set is useful.

A priori decisions

- Function: load, height, acceleration, velocity, life goal
- Design Factor: n_d
- Material: IPS, PS, MPS or other
- Rope: Lay, number of strands, number of wires per strand

Decision variables:
- Nominal wire size: *d*
- Number of load-supporting wires: *m*

From experience with Prob. 17-29, a 1-in diameter rope is not likely to have much of a life, so approach the problem with the *d* and *m* decisions open.

Function: 5000 lbf load, 90 foot lift, acceleration = 4 ft/s^2, velocity = 2 ft/s, life goal = 10^5 cycles
Design Factor: $n_d = 2$
Material: IPS
Rope: Regular lay, 1-in plow-steel 6 × 19 hoisting

Design variables
Choose 30-in D_{min}. Table 17-27: $w = 1.60d^2$ lbf/ft

$$wl = 1.60d^2 l = 1.60d^2(90) = 144d^2 \text{ lbf, each}$$

Eq. (17-46):

$$F_t = \left(\frac{W}{m} + wl\right)\left(1 + \frac{a}{g}\right) = \left(\frac{5000}{m} + 144d^2\right)\left(1 + \frac{4}{32.2}\right)$$

$$= \frac{5620}{m} + 162d^2 \text{ lbf, each wire}$$

Eq. (17-47):

$$F_f = \frac{(p/S_u)S_u Dd}{2}$$

From Fig. 17-21 for 10^5 cycles, $p/S_u = 0.004$. From p. 920, $S_u = 240$ kpsi, based on metal area.

$$F_f = \frac{0.004(240\,000)(30d)}{2} = 14\,400d \text{ lbf each wire}$$

Eq. (17-48) and Table 17-27:

$$F_b = \frac{E_w d_w A_m}{D} = \frac{12\left(10^6\right)0.067d\left(0.4d^2\right)}{30} = 10\ 720d^3 \text{ lbf, each wire}$$

Eq. (17-45):

$$n_f = \frac{F_f - F_b}{F_t} = \frac{14\ 400d - 10\ 720d^3}{(5620 / m) + 162d^2}$$

We could use a computer program to build a table similar to that of Ex. 17-6. Alternatively, we could recognize that 162 d^2 is small compared to 5620 */ m*, and therefore eliminate the $162d^2$ term.

$$n_f \doteq \frac{14\ 400d - 10\ 720d^3}{5620 / m} = \frac{m}{5620}(14\ 400d - 10\ 720d^3)$$

Maximize n_f ,

$$\frac{\partial n_f}{\partial d} = 0 = \frac{m}{5620}[14\ 400 - 3(10\ 720)d^2]$$

From which

$$d^* = \sqrt{\frac{14\ 400}{3(10\ 720)}} = 0.669 \text{ in}$$

Back-substituting

$$n_f = \frac{m}{5620}[14\ 400(0.669) - 10\ 720(0.669^3)] = 1.14 \text{ m}$$

Thus n_f = 1.14, 2.28, 3.42, 4.56 for *m*=1, 2, 3, 4 respectively. If we choose *d* = 0.50 in, then *m* = 2.

$$n_f = \frac{14\ 400(0.5) - 10\ 720(0.5^3)}{(5620 / 2) + 162(0.5)^2} = 2.06$$

This exceeds $n_d = 2$

Decision #1: *d* = 1/2 in

Decision #2: *m* = 2 ropes supporting load. Rope should be inspected weekly for any signs of fatigue (broken outer wires).

Comment: Table 17-25 gives *n* for freight elevators in terms of velocity.

$$F_u = (S_u)_{\text{nom}} A_{\text{nom}} = 106\ 000\left(\frac{\pi d^2}{4}\right) = 83\ 252d^2 \text{ lbf, each wire}$$

$$n = \frac{F_u}{F_t} = \frac{83\ 452(0.5)^2}{(5620/2) + 162(0.5)^2} = 7.32$$

By comparison, interpolation for 120 ft/min gives 7.08 - close. The category of construction hoists is not addressed in Table 17-25. We should investigate this before proceeding further.

17-31 Given: 2000 ft lift, 72 in drum, 6 × 19 MS rope, cage and load 8000 lbf, accel. = 2 ft/s^2.

(a) Table 17-24: $(S_u)_{\text{nom}}$ = 106 kpsi; S_u = 240 kpsi (p. 920); Fig. 17-21: $(p/S_u)10^6$ = 0.0014

Eq. (17-44):

$$F_f = \frac{(p/S_u)S_u dD}{2} = \frac{0.0014(240)d(72)}{2} = 12.1d \text{ kip}$$

Table 17-24: $wl = 1.6d^2\, 2000(10^{-3}) = 3.2d^2$ kip

Eq. (17-46):

$$F_t = (W + wl)\left(1 + \frac{a}{g}\right)$$

$$= (8 + 3.2d^2)\left(1 + \frac{2}{32.2}\right)$$

$$= 8.5 + 3.4d^2 \text{ kip}$$

Note that bending is not included.

$$n = \frac{F_f}{F_t} = \frac{12.1d}{8.5 + 3.4d^2}$$

d, in	n	
0.500	0.650	
1.000	1.020	
1.500	1.124	
1.625	1.125	← maximum n *Ans.*
1.750	1.120	
2.000	1.095	

(b) Try m = 4 strands

$$F_t = \left(\frac{8}{4} + 3.2d^2\right)\left(1 + \frac{2}{32.2}\right)$$
$$= 2.12 + 3.4d^2 \text{ kip}$$
$$F_f = 12.1d \text{ kip}$$
$$n = \frac{12.1d}{2.12 + 3.4d^2}$$

d, in	n	
0.5000	2.037	
0.5625	2.130	
0.6520	2.193	
0.7500	2.250	← maximum n *Ans.*
0.8750	2.242	
1.0000	2.192	

Comparing tables, multiple ropes supporting the load increases the factor of safety, and reduces the corresponding wire rope diameter, a useful perspective.

17-32

$$n = \frac{ad}{b/m + cd^2}$$

$$\frac{dn}{dd} = \frac{(b/m + cd^2)a - ad(2cd)}{(b/m + cd^2)^2} = 0$$

From which

$$d^* = \sqrt{\frac{b}{mc}} \quad \textit{Ans.}$$

$$n^* = \frac{a\sqrt{b/(mc)}}{(b/m) + c[b/(mc)]} = \frac{a}{2}\sqrt{\frac{m}{bc}} \quad \textit{Ans.}$$

These results agree closely with the Prob. 17-31 solution. The small differences are due to rounding in Prob. 17-31.

17-33 From Prob. 17-32 solution:

$$n_1 = \frac{ad}{b/m + cd^2}$$

Solve the above equation for *m*

$$m = \frac{b}{ad/n_1 - cd^2} \qquad (1)$$

$$\frac{dm}{dd} = 0 = \frac{\left[(ad/n_1) - ad^2\right](0) - b\left[(a/n_1) - 2cd\right]}{\left[(ad/n_1) - cd^2\right]^2}$$

From which $d^* = \dfrac{a}{2cn_1}$ *Ans.*

Substituting this result for *d* into Eq. (1) gives

$$m^* = \frac{4bcn_1}{a^2} \quad \textit{Ans.}$$

17-34 **Note to the Instructor**. In the first printing of the ninth edition, the wording of this problem is incorrect. It should read " For Prob. 17-29 estimate the elongation of the rope if a 7000 lbf loaded mine cart is placed in the cage which weighs 1000 lbf. The results of Prob. 4-7 may be useful". This will be corrected in subsequent printings. We apologize for any inconvenience encountered.

Table 17-27:

$$A_m = 0.40d^2 = 0.40(2^2) = 1.6 \text{ in}^2$$
$$E_r = 12 \text{ Mpsi}, \quad w = 1.6d^2 = 1.6(2^2) = 6.4 \text{ lbf/ft}$$
$$wl = 6.4(480) = 3072 \text{ lbf}$$
$$\gamma \doteq wl/(A_m l) = 3072/[1.6(480)12] = 0.333 \text{ lbf/in}^3$$

Treat the rest of the system as rigid, so that all of the stretch is due to the load of 7000 lbf, the cage weighing 1000 lbf, and the wire's weight. From the solution of Prob. 4-7,

$$\delta_1 = \frac{Wl}{AE} + \frac{\gamma l^2}{2E}$$
$$= \frac{(1000 + 7000)(480)(12)}{1.6(12)(10^6)} + \frac{0.333(480^2)12^2}{2(12)(10^6)}$$
$$= 2.4 + 0.460 = 2.860 \text{ in} \qquad \textit{Ans.}$$

17-35 to 17-38 Computer programs will vary.