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CHAPTER

6

Variable Stresses in Machine Parts

6.1 Introduction

We have discussed, in the previous chapter, the stresses due to static loading only. But only a few machine parts are subjected to static loading. Since many of the machine parts (such as axles, shafts, crankshafts, connecting rods, springs, pinion teeth etc.) are subjected to variable or alternating loads (also known as fluctuating or fatigue loads), therefore we shall discuss, in this chapter, the variable or alternating stresses.

6.2 Completely Reversed or Cyclic Stresses

Consider a rotating beam of circular cross-section and carrying a load W, as shown in Fig. 6.1. This load induces stresses in the beam which are cyclic in nature. A little consideration will show that the upper fibres of the beam (*i.e.* at point A) are under compressive stress and the lower fibres (*i.e.* at point B) are under tensile stress. After

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half a revolution, the point *B* occupies the position of point *A* and the point *A* occupies the position of point *B*. Thus the point *B* is now under compressive stress and the point *A* under tensile stress. The speed of variation of these stresses depends upon the speed of the beam.

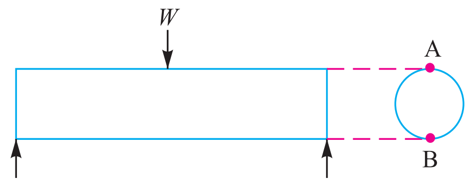


Fig. 6.1. Reversed or cyclic stresses.

From above we see that for each revolution of the beam, the stresses are reversed from compressive to tensile. The stresses which vary from one value of compressive to the same value of tensile or ***vice versa,*** are known as ***completely reversed*** or ***cyclic stresses.***

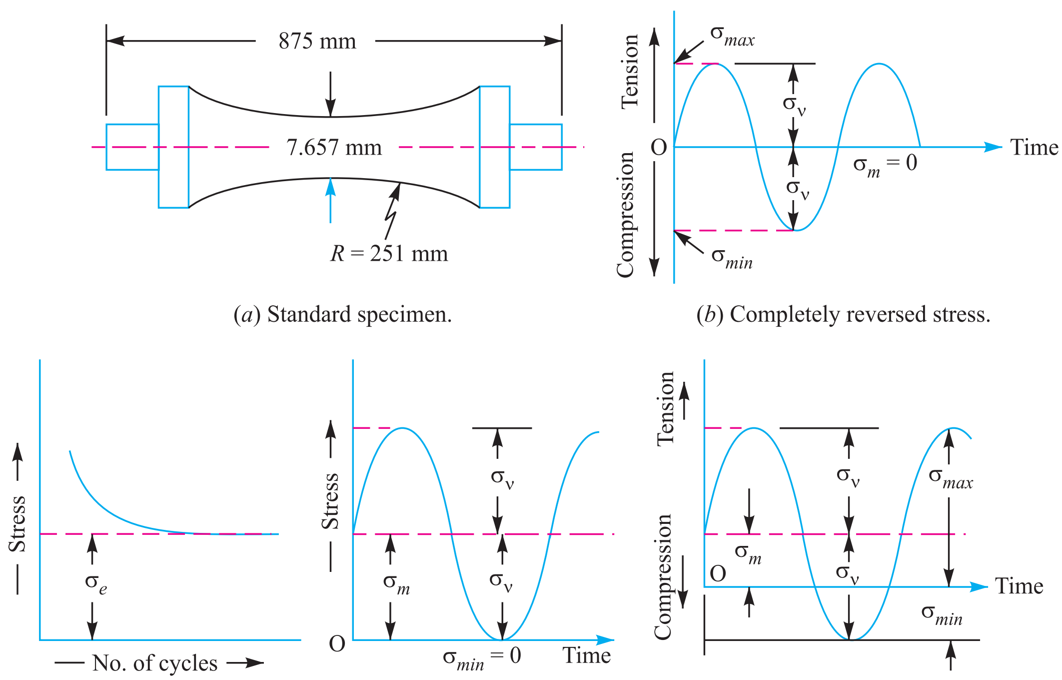
Notes: 1. The stresses which vary from a minimum value to a maximum value of the same nature, (*i.e.* tensile or compressive) are called ***fluctuating stresses.***

2. The stresses which vary from zero to a certain maximum value are called ***repeated stresses.***

3. The stresses which vary from a minimum value to a maximum value of the opposite nature (*i.e.* from a certain minimum compressive to a certain maximum tensile or from a minimum tensile to a maximum compressive) are called ***alternating stresses.***

6.3 Fatigue and Endurance Limit

It has been found experimentally that when a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as **fatigue.** The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. The failure may occur even without any prior indication. The fatigue of material is effected by the size of the component, relative magnitude of static and fluctuating loads and the number of load reversals.



(*a*) Standard specimen.
(*b*) Completely reversed stress.
(*c*) Endurance or fatigue limit.
(*d*) Repeated stress.
(*e*) Fluctuating stress.

Fig. 6.2. Time-stress diagrams.

In order to study the effect of fatigue of a material, a rotating mirror beam method is used. In this method, a standard mirror polished specimen, as shown in Fig. 6.2 (*a*), is rotated in a fatigue

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testing machine while the specimen is loaded in bending. As the specimen rotates, the bending stress at the upper fibres varies from maximum compressive to maximum tensile while the bending stress at the lower fibres varies from maximum tensile to maximum compressive. In other words, the specimen is subjected to a completely reversed stress cycle. This is represented by a time-stress diagram as shown in Fig. 6.2 (*b*). A record is kept of the number of cycles required to produce failure at a given stress, and the results are plotted in stress-cycle curve as shown in Fig. 6.2 (*c*). A little consideration will show that if the stress is kept below a certain value as shown by dotted line in Fig. 6.2 (*c*), the material will not fail whatever may be the number of cycles. This stress, as represented by dotted line, is known as ***endurance*** or ***fatigue limit*** (σ_e). It is defined as maximum value of the completely reversed bending stress which a polished standard specimen can withstand without failure, for infinite number of cycles (usually 10^7 cycles).

A machine part is being turned on a Lathe.

It may be noted that the term endurance limit is used for reversed bending only while for other types of loading, the term ***endurance strength*** may be used when referring the fatigue strength of the material. It may be defined as the safe maximum stress which can be applied to the machine part working under actual conditions.

We have seen that when a machine member is subjected to a completely reversed stress, the maximum stress in tension is equal to the maximum stress in compression as shown in Fig. 6.2 (*b*). In actual practice, many machine members undergo different range of stress than the completely reversed stress.

The stress ***verses*** time diagram for fluctuating stress having values σ_{min} and σ_{max} is shown in Fig. 6.2 (*e*). The variable stress, in general, may be considered as a combination of steady (or mean or average) stress and a completely reversed stress component σ_v. The following relations are derived from Fig. 6.2 (*e*):

1. Mean or average stress,

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

2. Reversed stress component or alternating or variable stress,

$$\sigma_v = \frac{\sigma_{max} - \sigma_{min}}{2}$$

Note: For repeated loading, the stress varies from maximum to zero (*i.e.* $\sigma_{min} = 0$) in each cycle as shown in Fig. 6.2 (*d*).

$$\therefore \qquad \sigma_m = \sigma_v = \frac{\sigma_{max}}{2}$$

3. Stress ratio, $R = \dfrac{\sigma_{max}}{\sigma_{min}}$. For completely reversed stresses, $R = -1$ and for repeated stresses, $R = 0$. It may be noted that R cannot be greater than unity.

4. The following relation between endurance limit and stress ratio may be used

$$\sigma'_e = \frac{3\sigma_e}{2 - R}$$

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where σ'_e = Endurance limit for any stress range represented by R.

σ_e = Endurance limit for completely reversed stresses, and

R = Stress ratio.

6.4 Effect of Loading on Endurance Limit—Load Factor

The endurance limit (σ_e) of a material as determined by the rotating beam method is for reversed bending load. There are many machine members which are subjected to loads other than reversed bending loads. Thus the endurance limit will also be different for different types of loading. The endurance limit depending upon the type of loading may be modified as discussed below:

Let K_b = Load correction factor for the reversed or rotating bending load. Its value is usually taken as unity.

K_a = Load correction factor for the reversed axial load. Its value may be taken as 0.8.

K_s = Load correction factor for the reversed torsional or shear load. Its value may be taken as 0.55 for ductile materials and 0.8 for brittle materials.

Shaft drive.

$\therefore$ Endurance limit for reversed bending load, $\sigma_{eb} = \sigma_e.K_b = \sigma_e$ ...($\because K_b = 1$)

Endurance limit for reversed axial load, $\sigma_{ea} = \sigma_e.K_a$

and endurance limit for reversed torsional or shear load, $\tau_e = \sigma_e.K_s$

6.5 Effect of Surface Finish on Endurance Limit—Surface Finish Factor

When a machine member is subjected to variable loads, the endurance limit of the material for that member depends upon the surface conditions. Fig. 6.3 shows the values of surface finish factor for the various surface conditions and ultimate tensile strength.

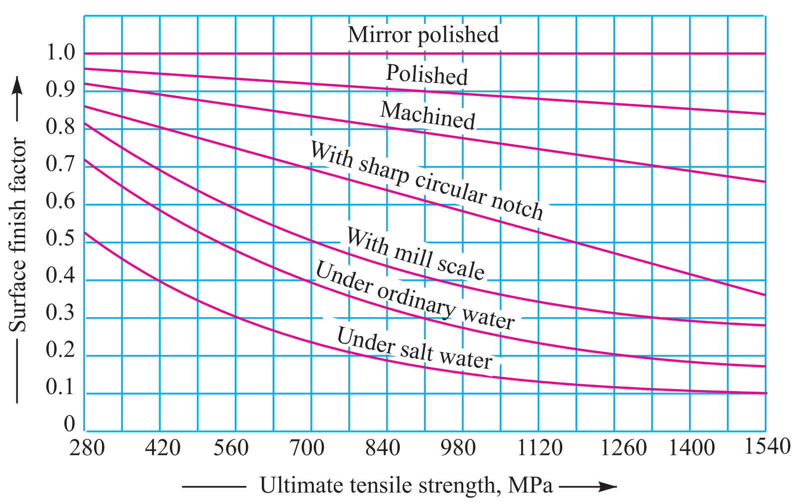

Fig. 6.3. Surface finish factor for various surface conditions.

When the surface finish factor is known, then the endurance limit for the material of the machine member may be obtained by multiplying the endurance limit and the surface finish factor. We see that

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for a mirror polished material, the surface finish factor is unity. In other words, the endurance limit for mirror polished material is maximum and it goes on reducing due to surface condition.

Let K_{sur} = Surface finish factor.

$\therefore$ Endurance limit,

$$\sigma_{e1} = \sigma_{eb}.K_{sur} = \sigma_e.K_b.K_{sur} = \sigma_e.K_{sur} \quad ...(\because K_b = 1)$$

...(For reversed bending load)

$$= \sigma_{ea}.K_{sur} = \sigma_e.K_a.K_{sur} \quad \text{...(For reversed axial load)}$$

$$= \tau_e.K_{sur} = \sigma_e.K_s.K_{sur} \quad \text{...(For reversed torsional or shear load)}$$

Note : The surface finish factor for non-ferrous metals may be taken as unity.

6.6 Effect of Size on Endurance Limit—Size Factor

A little consideration will show that if the size of the standard specimen as shown in Fig. 6.2 (*a*) is increased, then the endurance limit of the material will decrease. This is due to the fact that a longer specimen will have more defects than a smaller one.

Let K_{sz} = Size factor.

$\therefore$ Endurance limit,

$$\sigma_{e2} = \sigma_{e1} \times K_{sz} \quad \text{...(Considering surface finish factor also)}$$

$$= \sigma_{eb}.K_{sur}.K_{sz} = \sigma_e.K_b.K_{sur}.K_{sz} = \sigma_e.K_{sur}.K_{sz} \quad (\because K_b = 1)$$

$$= \sigma_{ea}.K_{sur}.K_{sz} = \sigma_e.K_a.K_{sur}.K_{sz} \quad \text{...(For reversed axial load)}$$

$$= \tau_e.K_{sur}.K_{sz} = \sigma_e.K_s.K_{sur}.K_{sz} \quad \text{... (For reversed torsional or shear load)}$$

Notes: 1. The value of size factor is taken as unity for the standard specimen having nominal diameter of 7.657 mm.

2. When the nominal diameter of the specimen is more than 7.657 mm but less than 50 mm, the value of size factor may be taken as 0.85.

3. When the nominal diameter of the specimen is more than 50 mm, then the value of size factor may be taken as 0.75.

6.7 Effect of Miscellaneous Factors on Endurance Limit

In addition to the surface finish factor (K_{sur}), size factor (K_{sz}) and load factors K_b, K_a and K_s, there are many other factors such as reliability factor (K_r), temperature factor (K_t), impact factor (K_i) etc. which has effect on the endurance limit of a material. Considering all these factors, the endurance limit may be determined by using the following expressions :

1. For the reversed bending load, endurance limit,

$$\sigma'_e = \sigma_{eb}.K_{sur}.K_{sz}.K_r.K_t.K_i$$

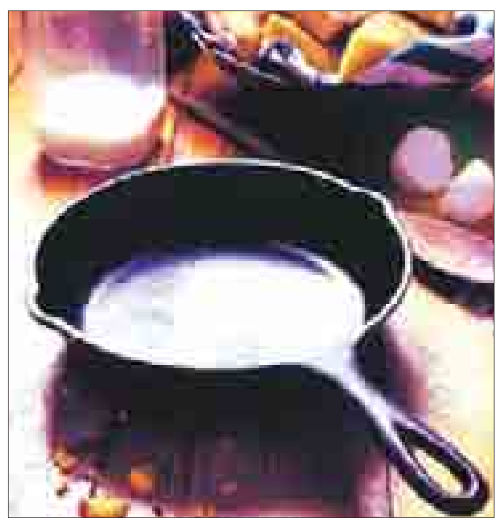
2. For the reversed axial load, endurance limit,

$$\sigma'_e = \sigma_{ea}.K_{sur}.K_{sz}.K_r.K_t.K_i$$

3. For the reversed torsional or shear load, endurance limit,

$$\sigma'_e = \tau_e.K_{sur}.K_{sz}.K_r.K_t.K_i$$

In solving problems, if the value of any of the above factors is not known, it may be taken as unity.



In addition to shear, tensile, compressive and torsional stresses, temperature can add its own stress (Ref. Chapter 4)

Note : This picture is given as additional information and is not a direct example of the current chapter.

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6.8 Relation Between Endurance Limit and Ultimate Tensile Strength

It has been found experimentally that endurance limit (σ_e) of a material subjected to fatigue loading is a function of ultimate tensile strength (σ_u). Fig. 6.4 shows the endurance limit of steel corresponding to ultimate tensile strength for different surface conditions. Following are some empirical relations commonly used in practice :

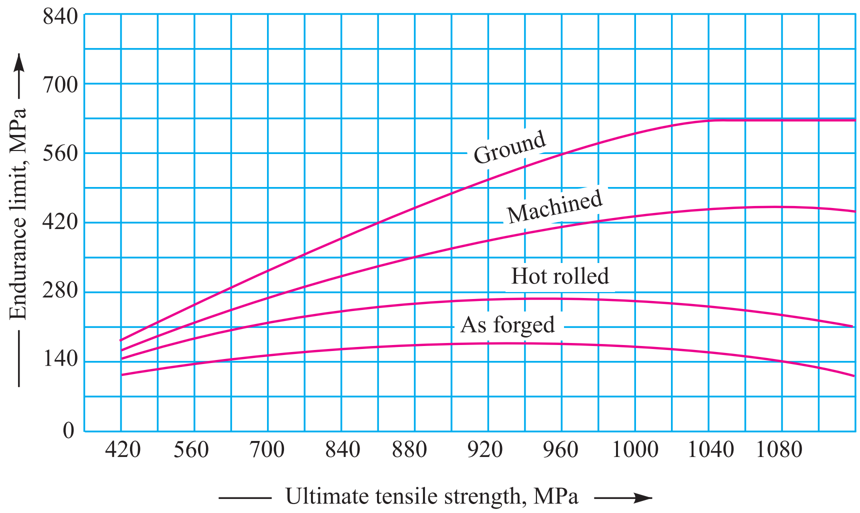


Fig. 6.4. Endurance limit of steel corresponding to ultimate tensile strength.

For steel, $\sigma_e = 0.5\,\sigma_u$;

For cast steel, $\sigma_e = 0.4\,\sigma_u$;

For cast iron, $\sigma_e = 0.35\,\sigma_u$;

For non-ferrous metals and alloys, $\sigma_e = 0.3\,\sigma_u$

6.9 Factor of Safety for Fatigue Loading

When a component is subjected to fatigue loading, the endurance limit is the criterion for faliure. Therefore, the factor of safety should be based on endurance limit. Mathematically,

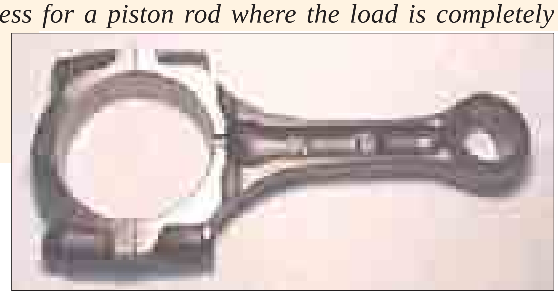
$$\text{Factor of safety } (F.S.) = \frac{\text{Endurance limit stress}}{\text{Design or working stress}} = \frac{\sigma_e}{\sigma_d}$$

Note: For steel, $\sigma_e = 0.8 \text{ to } 0.9\,\sigma_y$

where σ_e = Endurance limit stress for completely reversed stress cycle, and

σ_y = Yield point stress.

Example 6.1. *Determine the design stress for a piston rod where the load is completely reversed. The surface of the rod is ground and the surface finish factor is 0.9. There is no stress concentration. The load is predictable and the factor of safety is 2.*



Piston rod

Solution. Given : $K_{sur} = 0.9$; $F.S. = 2$

The piston rod is subjected to reversed axial loading. We know that for reversed axial loading, the load correction factor (K_a) is 0.8.

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If σ_e is the endurance limit for reversed bending load, then endurance limit for reversed axial load,

$$\sigma_{ea} = \sigma_e \times K_a \times K_{sur} = \sigma_e \times 0.8 \times 0.9 = 0.72\ \sigma_e$$

We know that design stress,

$$\sigma_d = \frac{\sigma_{ea}}{F.S.} = \frac{0.72\ \sigma_e}{2} = 0.36\ \sigma_e$$ **Ans.**

6.10 Stress Concentration

Whenever a machine component changes the shape of its cross-section, the simple stress distribution no longer holds good and the neighbourhood of the discontinuity is different. This irregularity in the stress distribution caused by abrupt changes of form is called ***stress concentration.*** It occurs for all kinds of stresses in the presence of fillets, notches, holes, keyways, splines, surface roughness or scratches etc.

In order to understand fully the idea of stress concentration, consider a member with different cross-section under a tensile load as shown in Fig. 6.5. A little consideration will show that the nominal stress in the right and left hand sides will be uniform but in the region where the cross-section is changing, a re-distribution of the force within the member must take place. The material near the edges is stressed considerably higher than the average value. The maximum stress occurs at some point on the fillet and is directed parallel to the boundary at that point.

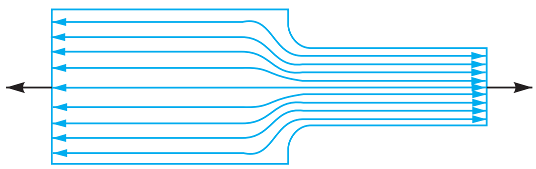

Fig. 6.5. Stress concentration.

6.11 Theoretical or Form Stress Concentration Factor

The theoretical or form stress concentration factor is defined as the ratio of the maximum stress in a member (at a notch or a fillet) to the nominal stress at the same section based upon net area. Mathematically, theoretical or form stress concentration factor,

$$K_t = \frac{\text{Maximum stress}}{\text{Nominal stress}}$$

The value of K_t depends upon the material and geometry of the part.

Notes: 1. In static loading, stress concentration in ductile materials is not so serious as in brittle materials, because in ductile materials local deformation or yielding takes place which reduces the concentration. In brittle materials, cracks may appear at these local concentrations of stress which will increase the stress over the rest of the section. It is, therefore, necessary that in designing parts of brittle materials such as castings, care should be taken. In order to avoid failure due to stress concentration, fillets at the changes of section must be provided.

2. In cyclic loading, stress concentration in ductile materials is always serious because the ductility of the material is not effective in relieving the concentration of stress caused by cracks, flaws, surface roughness, or any sharp discontinuity in the geometrical form of the member. If the stress at any point in a member is above the endurance limit of the material, a crack may develop under the action of repeated load and the crack will lead to failure of the member.

6.12 Stress Concentration due to Holes and Notches

Consider a plate with transverse elliptical hole and subjected to a tensile load as shown in Fig. 6.6 (*a*). We see from the stress-distribution that the stress at the point away from the hole is practically uniform and the maximum stress will be induced at the edge of the hole. The maximum stress is given by

$$\sigma_{max} = \sigma\left(1 + \frac{2a}{b}\right)$$

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and the theoretical stress concentration factor,

$$K_t = \frac{\sigma_{max}}{\sigma} = \left(1 + \frac{2a}{b}\right)$$

When a/b is large, the ellipse approaches a crack transverse to the load and the value of K_t becomes very large. When a/b is small, the ellipse approaches a longitudinal slit [as shown in Fig. 6.6 (*b*)] and the increase in stress is small. When the hole is circular as shown in Fig. 6.6 (*c*), then $a/b = 1$ and the maximum stress is three times the nominal value.

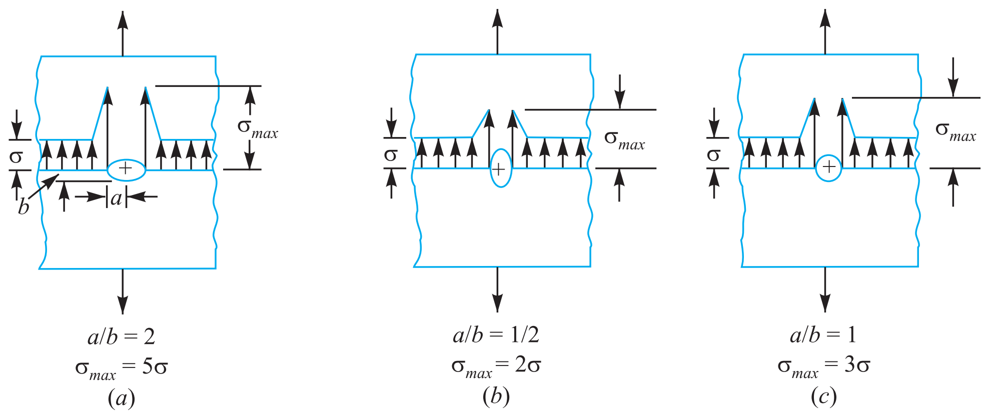

Fig. 6.6. Stress concentration due to holes.

The stress concentration in the notched tension member, as shown in Fig. 6.7, is influenced by the depth a of the notch and radius r at the bottom of the notch. The maximum stress, which applies to members having notches that are small in comparison with the width of the plate, may be obtained by the following equation,

$$\sigma_{max} = \sigma\left(1 + \frac{2a}{r}\right)$$

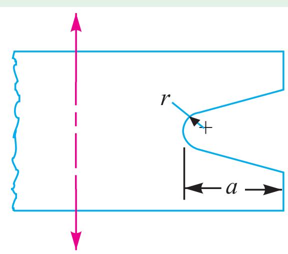

Fig. 6.7. Stress concentration due to notches.

6.13 Methods of Reducing Stress Concentration

We have already discussed in Art 6.10 that whenever there is a change in cross-section, such as shoulders, holes, notches or keyways and where there is an interference fit between a hub or bearing race and a shaft, then stress concentration results. The presence of

Crankshaft

stress concentration can not be totally eliminated but it may be reduced to some extent. A device or concept that is useful in assisting a design engineer to visualize the presence of stress concentration

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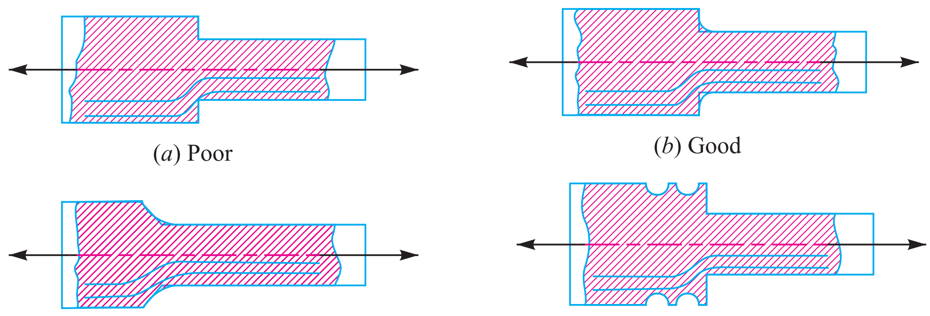
and how it may be mitigated is that of stress flow lines, as shown in Fig. 6.8. The mitigation of stress concentration means that the stress flow lines shall maintain their spacing as far as possible.



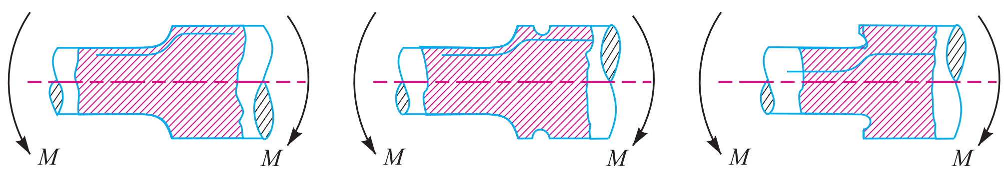
(*a*) Poor (*b*) Good

(*c*) Preferred (*d*) Preferred

Fig. 6.8

In Fig. 6.8 (*a*) we see that stress lines tend to bunch up and cut very close to the sharp re-entrant corner. In order to improve the situation, fillets may be provided, as shown in Fig. 6.8 (*b*) and (*c*) to give more equally spaced flow lines.

Figs. 6.9 to 6.11 show the several ways of reducing the stress concentration in shafts and other cylindrical members with shoulders, holes and threads respectively. It may be noted that it is not practicable to use large radius fillets as in case of ball and roller bearing mountings. In such cases, notches may be cut as shown in Fig. 6.8 (*d*) and Fig. 6.9 (*b*) and (*c*).

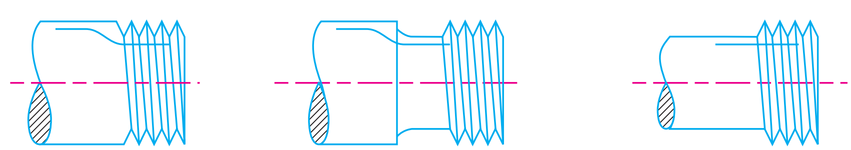


(*a*) Poor (*b*) Good (*c*) Preferred

Fig. 6.9. Methods of reducing stress concentration in cylindrical members with shoulders.

(*a*) Poor (*b*) Preferred

Fig. 6.10. Methods of reducing stress concentration in cylindrical members with holes.



(*a*) Poor (*b*) Good (*c*) Preferred

Fig. 6.11. Methods of reducing stress concentration in cylindrical members with holes.

The stress concentration effects of a press fit may be reduced by making more gradual transition from the rigid to the more flexible shaft. The various ways of reducing stress concentration for such cases are shown in Fig. 6.12 (*a*), (*b*) and (*c*).

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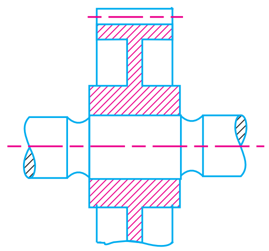
6.14 Factors to be Considered while Designing Machine Parts to Avoid Fatigue Failure

The following factors should be considered while designing machine parts to avoid fatigue failure:

1. The variation in the size of the component should be as gradual as possible.
2. The holes, notches and other stress raisers should be avoided.
3. The proper stress de-concentrators such as fillets and notches should be provided wherever necessary.



(*a*) (*b*) (*c*)

Fig. 6.12. Methods of reducing stress concentration of a press fit.

4. The parts should be protected from corrosive atmosphere.
5. A smooth finish of outer surface of the component increases the fatigue life.
6. The material with high fatigue strength should be selected.
7. The residual compressive stresses over the parts surface increases its fatigue strength.

6.15 Stress Concentration Factor for Various Machine Members

The following tables show the theoretical stress concentration factor for various types of members.

Table 6.1. Theoretical stress concentration factor (K_t) for a plate with hole (of diameter *d*) in tension.

$\frac{d}{b}$	0.05	0.1	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55
K_t	2.83	2.69	2.59	2.50	2.43	2.37	2.32	2.26	2.22	2.17	2.13

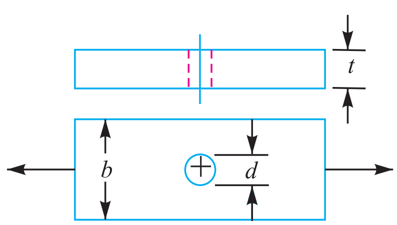


Fig. for Table 6.1

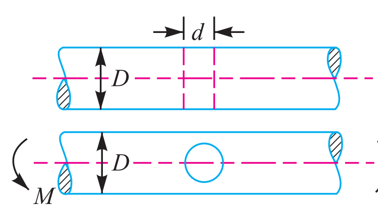


Fig. for Table 6.2

Table 6.2. Theoretical stress concentration factor (K_t) for a shaft with transverse hole (of diameter *d*) in bending.

$\frac{d}{D}$	0.02	0.04	0.08	0.10	0.12	0.16	0.20	0.24	0.28	0.30
K_t	2.70	2.52	2.33	2.26	2.20	2.11	2.03	1.96	1.92	1.90

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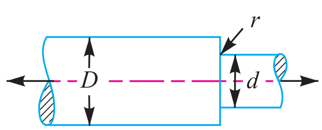
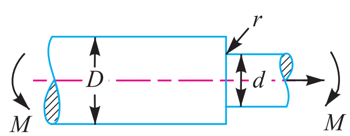
Table 6.3. Theoretical stress concentration factor (K_t) for stepped shaft with a shoulder fillet (of radius r) in tension.

$\frac{D}{d}$	*Theoretical stress concentration factor (K_t)*									
	r/d									
	0.08	*0.10*	*0.12*	*0.16*	*0.18*	*0.20*	*0.22*	*0.24*	*0.28*	*0.30*
1.01	1.27	1.24	1.21	1.17	1.16	1.15	1.15	1.14	1.13	1.13
1.02	1.38	1.34	1.30	1.26	1.24	1.23	1.22	1.21	1.19	1.19
1.05	1.53	1.46	1.42	1.36	1.34	1.32	1.30	1.28	1.26	1.25
1.10	1.65	1.56	1.50	1.43	1.39	1.37	1.34	1.33	1.30	1.28
1.15	1.73	1.63	1.56	1.46	1.43	1.40	1.37	1.35	1.32	1.31
1.20	1.82	1.68	1.62	1.51	1.47	1.44	1.41	1.38	1.35	1.34
1.50	2.03	1.84	1.80	1.66	1.60	1.56	1.53	1.50	1.46	1.44
2.00	2.14	1.94	1.89	1.74	1.68	1.64	1.59	1.56	1.50	1.47



$$A = \frac{\pi}{4} \times d^2$$

Fig. for Table 6.3



$$Z = \frac{\pi}{32} \times d^3$$

Fig. for Table 6.4

Table 6.4. Theoretical stress concentration factor (K_t) for a stepped shaft with a shoulder fillet (of radius r) in bending.

$\frac{D}{d}$	*Theoretical stress concentration factor (K_t)*									
	r/d									
	0.02	*0.04*	*0.08*	*0.10*	*0.12*	*0.16*	*0.20*	*0.24*	*0.28*	*0.30*
1.01	1.85	1.61	1.42	1.36	1.32	1.24	1.20	1.17	1.15	1.14
1.02	1.97	1.72	1.50	1.44	1.40	1.32	1.27	1.23	1.21	1.20
1.05	2.20	1.88	1.60	1.53	1.48	1.40	1.34	1.30	1.27	1.25
1.10	2.36	1.99	1.66	1.58	1.53	1.44	1.38	1.33	1.28	1.27
1.20	2.52	2.10	1.72	1.62	1.56	1.46	1.39	1.34	1.29	1.28
1.50	2.75	2.20	1.78	1.68	1.60	1.50	1.42	1.36	1.31	1.29
2.00	2.86	2.32	1.87	1.74	1.64	1.53	1.43	1.37	1.32	1.30
3.00	3.00	2.45	1.95	1.80	1.69	1.56	1.46	1.38	1.34	1.32
6.00	3.04	2.58	2.04	1.87	1.76	1.60	1.49	1.41	1.35	1.33

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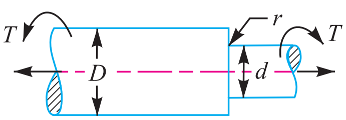
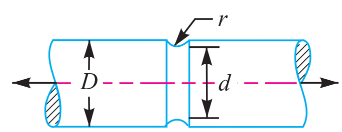
Table 6.5. Theoretical stress concentration factor (K_t) for a stepped shaft with a shoulder fillet (of radius r) in torsion.

$\frac{D}{d}$	Theoretical stress concentration factor (K_t)									
	r/d									
	0.02	*0.04*	*0.08*	*0.10*	*0.12*	*0.16*	*0.20*	*0.24*	*0.28*	*0.30*
1.09	1.54	1.32	1.19	1.16	1.15	1.12	1.11	1.10	1.09	1.09
1.20	1.98	1.67	1.40	1.33	1.28	1.22	1.18	1.15	1.13	1.13
1.33	2.14	1.79	1.48	1.41	1.35	1.28	1.22	1.19	1.17	1.16
2.00	2.27	1.84	1.53	1.46	1.40	1.32	1.26	1.22	1.19	1.18



$$\frac{J}{d/2} = \frac{\pi d^3}{16}$$

Fig. for Table 6.5



$$A = \frac{\pi}{4} \times d^2$$

Fig. for Table 6.6

Table 6.6. Theoretical stress concentration factor (K_t) for a grooved shaft in tension.

$\frac{D}{d}$	Theoretical stress concentration (K_t)									
	r/d									
	0.02	*0.04*	*0.08*	*0.10*	*0.12*	*0.16*	*0.20*	*0.24*	*0.28*	*0.30*
1.01	1.98	1.71	1.47	1.42	1.38	1.33	1.28	1.25	1.23	1.22
1.02	2.30	1.94	1.66	1.59	1.54	1.45	1.40	1.36	1.33	1.31
1.03	2.60	2.14	1.77	1.69	1.63	1.53	1.46	1.41	1.37	1.36
1.05	2.85	2.36	1.94	1.81	1.73	1.61	1.54	1.47	1.43	1.41
1.10	..	2.70	2.16	2.01	1.90	1.75	1.70	1.57	1.50	1.47
1.20	..	2.90	2.36	2.17	2.04	1.86	1.74	1.64	1.56	1.54
1.30	..	..	2.46	2.26	2.11	1.91	1.77	1.67	1.59	1.56
1.50	..	..	2.54	2.33	2.16	1.94	1.79	1.69	1.61	1.57
2.00	..	..	2.61	2.38	2.22	1.98	1.83	1.72	1.63	1.59
∞	..	..	2.69	2.44	2.26	2.03	1.86	1.74	1.65	1.61

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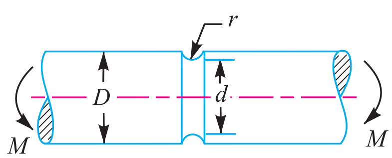
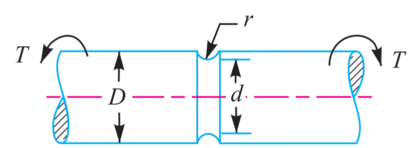
Table 6.7. Theoretical stress concentration factor (K_t) of a grooved shaft in bending.

$\frac{D}{d}$	Theoretical stress concentration factor (K_t)									
	r/d									
	0.02	0.04	0.08	0.10	0.12	0.16	0.20	0.24	0.28	0.30
1.01	1.74	1.68	1.47	1.41	1.38	1.32	1.27	1.23	1.22	1.20
1.02	2.28	1.89	1.64	1.53	1.48	1.40	1.34	1.30	1.26	1.25
1.03	2.46	2.04	1.68	1.61	1.55	1.47	1.40	1.35	1.31	1.28
1.05	2.75	2.22	1.80	1.70	1.63	1.53	1.46	1.40	1.35	1.33
1.12	3.20	2.50	1.97	1.83	1.75	1.62	1.52	1.45	1.38	1.34
1.30	3.40	2.70	2.04	1.91	1.82	1.67	1.57	1.48	1.42	1.38
1.50	3.48	2.74	2.11	1.95	1.84	1.69	1.58	1.49	1.43	1.40
2.00	3.55	2.78	2.14	1.97	1.86	1.71	1.59	1.55	1.44	1.41
∞	3.60	2.85	2.17	1.98	1.88	1.71	1.60	1.51	1.45	1.42



$$Z = \frac{\pi}{32} \times d^3$$

Fig. for Table 6.7



$$\frac{J}{d/2} = \frac{\pi d^3}{16}$$

Fig. for Table 6.8

Table 6.8. Theoretical stress concentration factor (K_t) for a grooved shaft in torsion.

$\frac{D}{d}$	Theoretical stress concentration factor (K_{ts})									
	r/d									
	0.02	0.04	0.08	0.10	0.12	0.16	0.20	0.24	0.28	0.30
1.01	1.50	1.03	1.22	1.20	1.18	1.16	1.13	1.12	1.12	1.12
1.02	1.62	1.45	1.31	1.27	1.23	1.20	1.18	1.16	1.15	1.16
1.05	1.88	1.61	1.40	1.35	1.32	1.26	1.22	1.20	1.18	1.17
1.10	2.05	1.73	1.47	1.41	1.37	1.31	1.26	1.24	1.21	1.20
1.20	2.26	1.83	1.53	1.46	1.41	1.34	1.27	1.25	1.22	1.21
1.30	2.32	1.89	1.55	1.48	1.43	1.35	1.30	1.26	—	—
2.00	2.40	1.93	1.58	1.50	1.45	1.36	1.31	1.26	—	—
∞	2.50	1.96	1.60	1.51	1.46	1.38	1.32	1.27	1.24	1.23

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Example 6.2. *Find the maximum stress induced in the following cases taking stress concentration into account:*

1. A rectangular plate 60 mm × 10 mm with a hole 12 diameter as shown in Fig. 6.13 (a) and subjected to a tensile load of 12 kN.

2. A stepped shaft as shown in Fig. 6.13 (b) and carrying a tensile load of 12 kN.

Stepped shaft

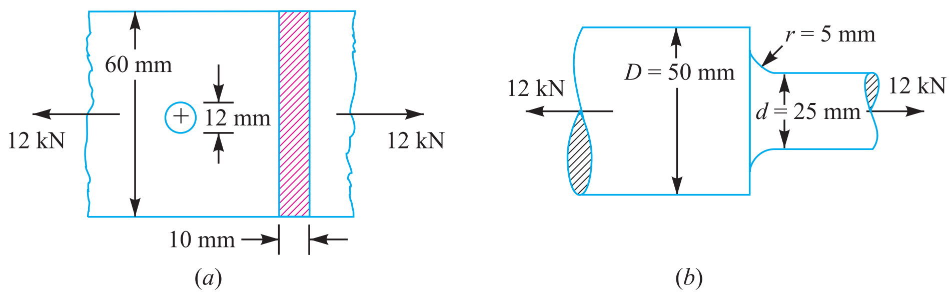

Fig. 6.13

Solution. Case 1. Given : b = 60 mm ; t = 10 mm ; d = 12 mm ; W = 12 kN = 12 × 10^3 N

We know that cross-sectional area of the plate,

$$A = (b - d)\, t = (60 - 12)\, 10 = 480 \text{ mm}^2$$

$$\therefore \quad \text{Nominal stress} = \frac{W}{A} = \frac{12 \times 10^3}{480} = 25 \text{ N/mm}^2 = 25 \text{ MPa}$$

Ratio of diameter of hole to width of plate,

$$\frac{d}{b} = \frac{12}{60} = 0.2$$

From Table 6.1, we find that for d / b = 0.2, theoretical stress concentration factor,

$$K_t = 2.5$$

$$\therefore \quad \text{Maximum stress} = K_t \times \text{Nominal stress} = 2.5 \times 25 = 62.5 \text{ MPa}$$ **Ans.**

Case 2. Given : D = 50 mm ; d = 25 mm ; r = 5 mm ; W = 12 kN = 12 × 10^3 N

We know that cross-sectional area for the stepped shaft,

$$A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4}(25)^2 = 491 \text{ mm}^2$$

$$\therefore \quad \text{Nominal stress} = \frac{W}{A} = \frac{12 \times 10^3}{491} = 24.4 \text{ N/mm}^2 = 24.4 \text{ MPa}$$

Ratio of maximum diameter to minimum diameter,

$$D/d = 50/25 = 2$$

Ratio of radius of fillet to minimum diameter,

$$r/d = 5/25 = 0.2$$

From Table 6.3, we find that for D/d = 2 and r/d = 0.2, theoretical stress concentration factor, K_t = 1.64.

$$\therefore \quad \text{Maximum stress} = K_t \times \text{Nominal stress} = 1.64 \times 24.4 = 40 \text{ MPa}$$ **Ans.**

6.16 Fatigue Stress Concentration Factor

When a machine member is subjected to cyclic or fatigue loading, the value of fatigue stress concentration factor shall be applied instead of theoretical stress concentration factor. Since the determination of fatigue stress concentration factor is not an easy task, therefore from experimental tests it is defined as

Fatigue stress concentration factor,

$$K_f = \frac{\text{Endurance limit without stress concentration}}{\text{Endurance limit with stress concentration}}$$

6.17 Notch Sensitivity

In cyclic loading, the effect of the notch or the fillet is usually less than predicted by the use of the theoretical factors as discussed before. The difference depends upon the stress gradient in the region of the stress concentration and on the hardness of the material. The term ***notch sensitivity*** is applied to this behaviour. It may be defined as the degree to which the theoretical effect of stress concentration is actually reached. The stress gradient depends mainly on the radius of the notch, hole or fillet and on the grain size of the material. Since the extensive data for estimating the notch sensitivity factor (q) is not available, therefore the curves, as shown in Fig. 6.14, may be used for determining the values of q for two steels.

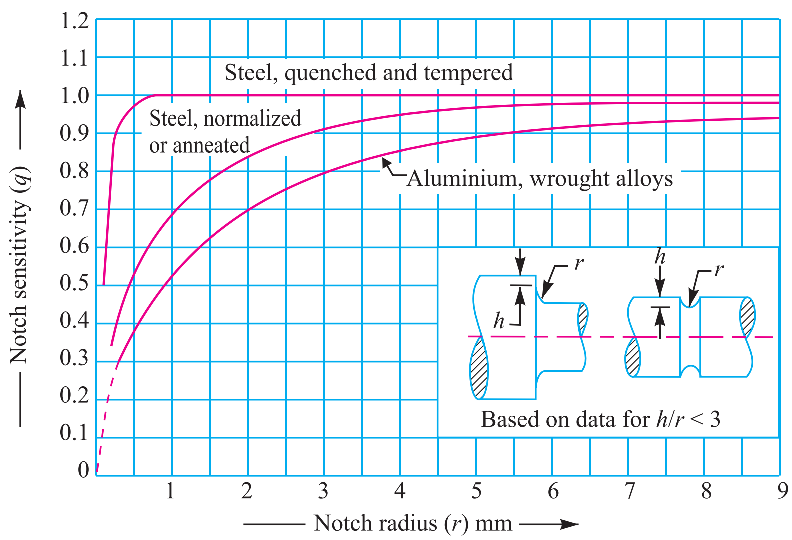

Fig. 6.14. Notch sensitivity.

When the notch sensitivity factor q is used in cyclic loading, then fatigue stress concentration factor may be obtained from the following relations:

$$q = \frac{K_f - 1}{K_t - 1}$$

or $\quad K_f = 1 + q\,(K_t - 1)$ ...[For tensile or bending stress]

and $\quad K_{fs} = 1 + q\,(K_{ts} - 1)$ ...[For shear stress]

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where K_t = Theoretical stress concentration factor for axial or bending loading, and

K_{ts} = Theoretical stress concentration factor for torsional or shear loading.

6.18 Combined Steady and Variable Stress

The failure points from fatigue tests made with different steels and combinations of mean and variable stresses are plotted in Fig. 6.15 as functions of variable stress (σ_v) and mean stress (σ_m). The most significant observation is that, in general, the failure point is little related to the mean stress when it is compressive but is very much a function of the mean stress when it is tensile. In practice, this means that fatigue failures are rare when the mean stress is compressive (or negative). Therefore, the greater emphasis must be given to the combination of a variable stress and a steady (or mean) tensile stress.

Protective colour coatings are added to make components it corrosion resistant. Corrosion if not taken care can magnify other stresses.

Note : This picture is given as additional information and is not a direct example of the current chapter.

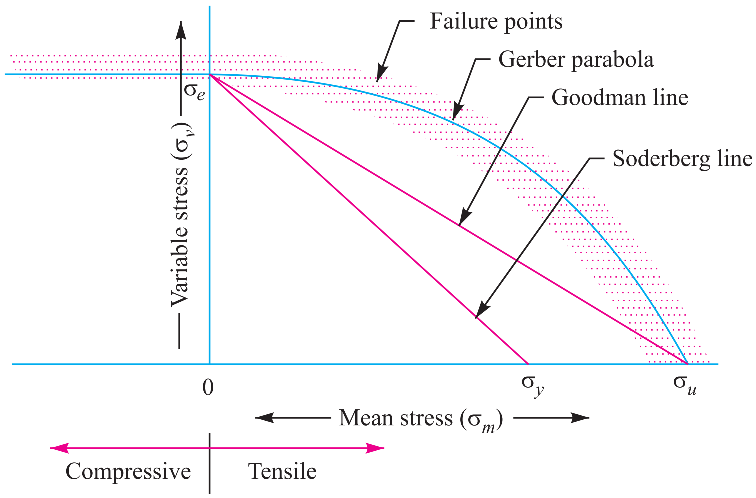

Fig. 6.15. Combined mean and variable stress.

There are several ways in which problems involving this combination of stresses may be solved, but the following are important from the subject point of view :

1. Gerber method, **2.** Goodman method, and **3.** Soderberg method.

We shall now discuss these methods, in detail, in the following pages.

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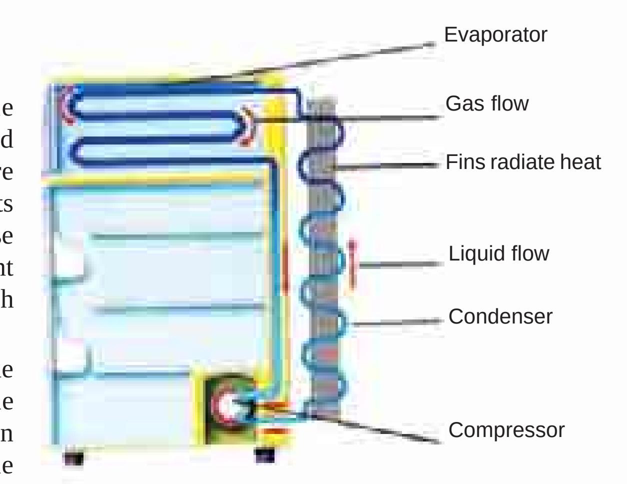
6.19 Gerber Method for Combination of Stresses

The relationship between variable stress (σ_v) and mean stress (σ_m) for axial and bending loading for ductile materials are shown in Fig. 6.15. The point σ_e represents the fatigue strength corresponding to the case of complete reversal ($\sigma_m = 0$) and the point σ_u represents the static ultimate strength corresponding to $\sigma_v = 0$.

A parabolic curve drawn between the endurance limit (σ_e) and ultimate tensile strength (σ_u) was proposed by Gerber in 1874. Generally, the test data for ductile material fall closer to Gerber parabola as shown in Fig. 6.15, but because of scatter in the test points, a straight line relationship (*i.e.* Goodman line and Soderberg line) is usually preferred in designing machine parts.



Liquid refrigerant absorbs heat as it vaporizes inside the evaporator coil of a refrigerator. The heat is released when a compressor turns the refrigerant back to liquid.

Note : This picture is given as additional information and is not a direct example of the current chapter.

According to Gerber, variable stress,

$$\sigma_v = \sigma_e \left[\frac{1}{F.S.} - \left(\frac{\sigma_m}{\sigma_u} \right)^2 F.S. \right]$$

or

$$\frac{1}{F.S.} = \left(\frac{\sigma_m}{\sigma_u} \right)^2 F.S. + \frac{\sigma_v}{\sigma_e} \quad \ldots(i)$$

where $F.S.$ = Factor of safety,

σ_m = Mean stress (tensile or compressive),

σ_u = Ultimate stress (tensile or compressive), and

σ_e = Endurance limit for reversal loading.

Considering the fatigue stress concentration factor (K_f), the equation (*i*) may be written as

$$\frac{1}{F.S.} = \left(\frac{\sigma_m}{\sigma_u} \right)^2 F.S. + \frac{\sigma_v \times K_f}{\sigma_e}$$

6.20 Goodman Method for Combination of Stresses

A straight line connecting the endurance limit (σ_e) and the ultimate strength (σ_u), as shown by line *AB* in Fig. 6.16, follows the suggestion of Goodman. A Goodman line is used when the design is based on ultimate strength and may be used for ductile or brittle materials.

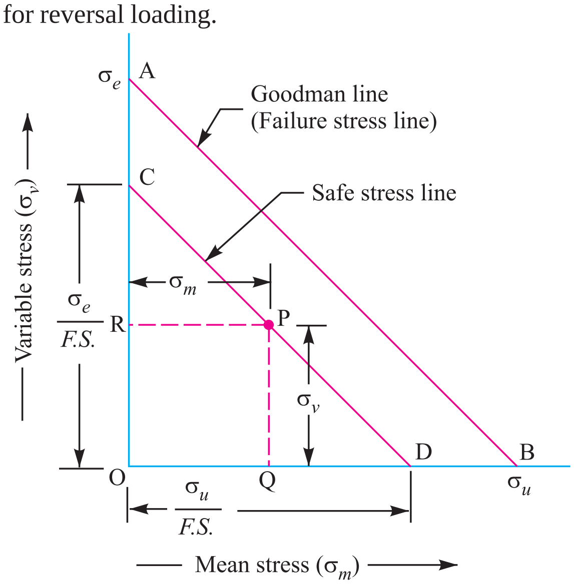


Fig. 6.16. Goodman method.

In Fig. 6.16, line *AB* connecting σ_e and

σ_u is called ***Goodman's failure stress line.*** If a suitable factor of safety (*F.S.*) is applied to endurance limit and ultimate strength, a safe stress line *CD* may be drawn parallel to the line *AB*. Let us consider a design point *P* on the line *CD*.

Now from similar triangles *COD* and *PQD*,

$$\frac{PQ}{CO} = \frac{QD}{OD} = \frac{OD - OQ}{OD} = 1 - \frac{OQ}{OD} \qquad ...(\because QD = OD - OQ)$$

$$\therefore \qquad \frac{{}^{*}\sigma_v}{\sigma_e / F.S.} = 1 - \frac{\sigma_m}{\sigma_u / F.S.}$$

$$\sigma_v = \frac{\sigma_e}{F.S.}\left[1 - \frac{\sigma_m}{\sigma_u / F.S.}\right] = \sigma_e\left[\frac{1}{F.S.} - \frac{\sigma_m}{\sigma_u}\right]$$

or
$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v}{\sigma_e} \qquad ...(i)$$

This expression does not include the effect of stress concentration. It may be noted that for ductile materials, the stress concentration may be ignored under steady loads.

Since many machine and structural parts that are subjected to fatigue loads contain regions of high stress concentration, therefore equation (*i*) must be altered to include this effect. In such cases, the fatigue stress concentration factor (K_f) is used to multiply the variable stress (σ_v). The equation ***(i)*** may now be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e} \qquad ...(ii)$$

where
$F.S.$ = Factor of safety,
σ_m = Mean stress,
σ_u = Ultimate stress,
σ_v = Variable stress,
σ_e = Endurance limit for reversed loading, and
K_f = Fatigue stress concentration factor.

Considering the load factor, surface finish factor and size factor, the equation ***(ii)*** may be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_{eb} \times K_{sur} \times K_{sz}} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_b \times K_{sur} \times K_{sz}} \qquad ...(iii)$$

$$= \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}} \qquad ...(\because \sigma_{eb} = \sigma_e \times K_b \text{ and } K_b = 1)$$

where
K_b = Load factor for reversed bending load,
K_{sur} = Surface finish factor, and
K_{sz} = Size factor.

---

* Here we have assumed the same factor of safety (*F.S.*) for the ultimate tensile strength (σ_u) and endurance limit (σ_e). In case the factor of safety relating to both these stresses is different, then the following relation may be used :

$$\frac{\sigma_v}{\sigma_e / (F.S.)_e} = 1 - \frac{\sigma_m}{\sigma_u / (F.S.)_u}$$

where
$(F.S.)_e$ = Factor of safety relating to endurance limit, and
$(F.S.)_u$ = Factor of safety relating to ultimate tensile strength.

Notes : 1. The equation ***(iii)*** is applicable to ductile materials subjected to reversed bending loads (tensile or compressive). For brittle materials, the theoretical stress concentration factor (K_t) should be applied to the mean stress and fatigue stress concentration factor (K_f) to the variable stress. Thus for brittle materials, the equation ***(iii)*** may be written as

$$\frac{1}{F.S.} = \frac{\sigma_m \times K_t}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_{eb} \times K_{sur} \times K_{sz}} \qquad \text{...}(iv)$$

2. When a machine component is subjected to a load other than reversed bending, then the endurance limit for that type of loading should be taken into consideration. Thus for reversed axial loading (tensile or compressive), the equations ***(iii)*** and ***(iv)*** may be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_{ea} \times K_{sur} \times K_{sz}} \qquad \text{...(For ductile materials)}$$

and

$$\frac{1}{F.S.} = \frac{\sigma_m \times K_t}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_{ea} \times K_{sur} \times K_{sz}} \qquad \text{...(For brittle materials)}$$

Similarly, for reversed torsional or shear loading,

$$\frac{1}{F.S.} = \frac{\tau_m}{\tau_u} + \frac{\tau_v \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}} \qquad \text{...(For ductile materials)}$$

and

$$\frac{1}{F.S.} = \frac{\tau_m \times K_{ts}}{\tau_u} + \frac{\tau_v \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}} \qquad \text{...(For brittle materials)}$$

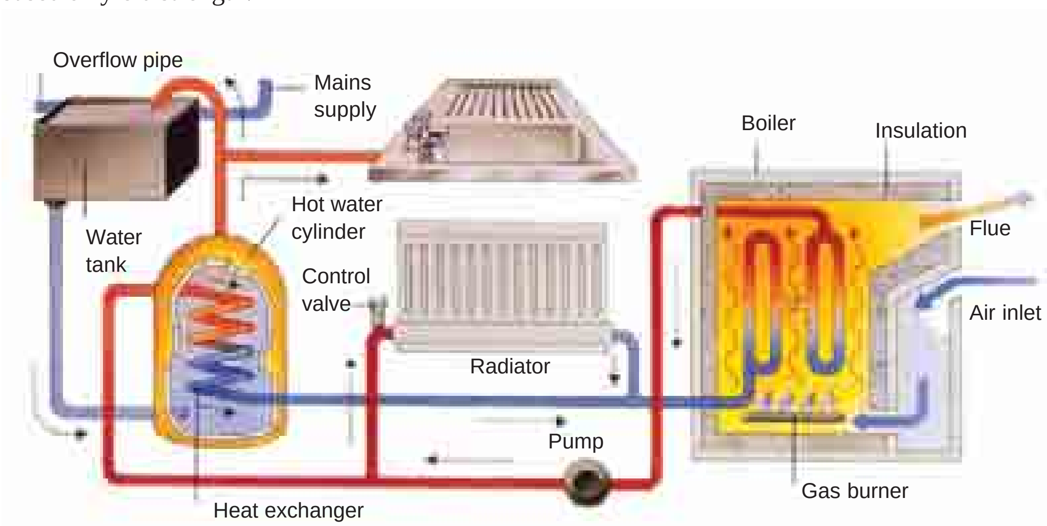
where suffix '*s*' denotes for shear.

For reversed torsional or shear loading, the values of ultimate shear strength (τ_u) and endurance shear strength (τ_e) may be taken as follows:

$$\tau_u = 0.8\,\sigma_u; \text{ and } \tau_e = 0.8\,\sigma_e$$

6.21 Soderberg Method for Combination of Stresses

A straight line connecting the endurance limit (σ_e) and the yield strength (σ_y), as shown by the line *AB* in Fig. 6.17, follows the suggestion of Soderberg line. This line is used when the design is based on yield strength.



In this central heating system, a furnace burns fuel to heat water in a boiler. A pump forces the hot water through pipes that connect to radiators in each room. Water from the boiler also heats the hot water cylinder. Cooled water returns to the boiler.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Proceeding in the same way as discussed in Art 6.20, the line AB connecting σ_e and σ_y, as shown in Fig. 6.17, is called ***Soderberg's failure stress line***. If a suitable factor of safety ($F.S.$) is applied to the endurance limit and yield strength, a safe stress line CD may be drawn parallel to the line AB. Let us consider a design point P on the line CD. Now from similar triangles COD and PQD,

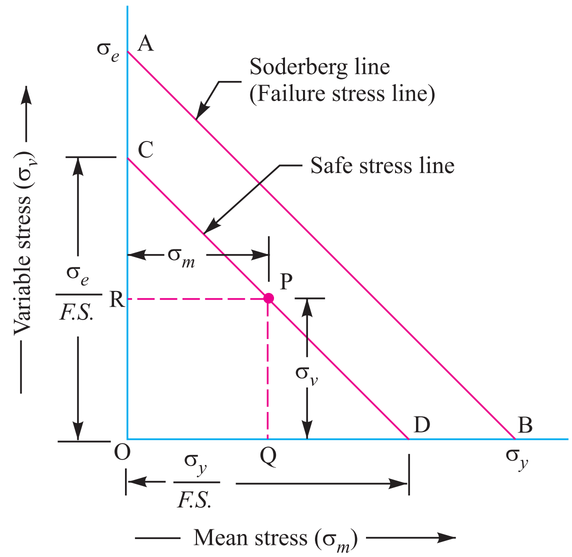


Fig. 6.17. Soderberg method.

$$\frac{PQ}{CO} = \frac{QD}{OD} = \frac{OD - OQ}{OD}$$

$$= 1 - \frac{OQ}{OD}$$

$$...(\because\ QD = OD - OQ)$$

$$\therefore \quad \frac{\sigma_v}{\sigma_e / F.S.} = 1 - \frac{\sigma_m}{\sigma_y / F.S.}$$

or $$\sigma_v = \frac{\sigma_e}{F.S.}\left[1 - \frac{\sigma_m}{\sigma_y / F.S.}\right] = \sigma_e\left[\frac{1}{F.S.} - \frac{\sigma_m}{\sigma_y}\right]$$

$$\therefore \quad \frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e} \qquad ...(i)$$

For machine parts subjected to fatigue loading, the fatigue stress concentration factor (K_f) should be applied to only variable stress (σ_v). Thus the equations ***(i)*** may be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e} \qquad ...(ii)$$

Considering the load factor, surface finish factor and size factor, the equation ***(ii)*** may be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_{eb} \times K_{sur} \times K_{sz}} \qquad ...(iii)$$

Since $\sigma_{eb} = \sigma_e \times K_b$ and $K_b = 1$ for reversed bending load, therefore $\sigma_{eb} = \sigma_e$ may be substituted in the above equation.

Notes: 1. The Soderberg method is particularly used for ductile materials. The equation ***(iii)*** is applicable to ductile materials subjected to reversed bending load (tensile or compressive).

2. When a machine component is subjected to reversed axial loading, then the equation ***(iii)*** may be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_{ea} \times K_{sur} \times K_{sz}}$$

3. When a machine component is subjected to reversed shear loading, then equation ***(iii)*** may be written as

$$\frac{1}{F.S.} = \frac{\tau_m}{\tau_y} + \frac{\tau_v \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}}$$

where K_{fs} is the fatigue stress concentration factor for reversed shear loading. The yield strength in shear (τ_y) may be taken as one-half the yield strength in reversed bending (σ_y).

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Example 6.3. *A machine component is subjected to a flexural stress which fluctuates between + 300 MN/m² and – 150 MN/m². Determine the value of minimum ultimate strength according to 1. Gerber relation; 2. Modified Goodman relation; and 3. Soderberg relation.*

Take yield strength = 0.55 Ultimate strength; Endurance strength = 0.5 Ultimate strength; and factor of safety = 2.

Springs often undergo variable stresses.

Solution. Given : $\sigma_1 = 300$ MN/m² ; $\sigma_2 = -150$ MN/m² ; $\sigma_y = 0.55\ \sigma_u$; $\sigma_e = 0.5\ \sigma_u$; $F.S. = 2$

Let σ_u = Minimum ultimate strength in MN/m².

We know that the mean or average stress,

$$\sigma_m = \frac{\sigma_1 + \sigma_2}{2} = \frac{300 + (-150)}{2} = 75 \text{ MN/m}^2$$

and variable stress,

$$\sigma_v = \frac{\sigma_1 - \sigma_2}{2} = \frac{300 - (-150)}{2} = 225 \text{ MN/m}^2$$

1. ***According to Gerber relation***

We know that according to Gerber relation,

$$\frac{1}{F.S.} = \left(\frac{\sigma_m}{\sigma_u}\right)^2 F.S. + \frac{\sigma_v}{\sigma_e}$$

$$\frac{1}{2} = \left(\frac{75}{\sigma_u}\right)^2 2 + \frac{225}{0.5\sigma_u} = \frac{11\,250}{(\sigma_u)^2} + \frac{450}{\sigma_u} = \frac{11\,250 + 450\,\sigma_u}{(\sigma_u)^2}$$

$$(\sigma_u)^2 = 22\,500 + 900\,\sigma_u$$

or $$(\sigma_u)^2 - 900\,\sigma_u - 22\,500 = 0$$

$\therefore$ $$\sigma_u = \frac{900 \pm \sqrt{(900)^2 + 4 \times 1 \times 22\,500}}{2 \times 1} = \frac{900 \pm 948.7}{2}$$

$= 924.35$ MN/m² **Ans.** ...(Taking +ve sign)

2. ***According to modified Goodman relation***

We know that according to modified Goodman relation,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v}{\sigma_e}$$

or $$\frac{1}{2} = \frac{75}{\sigma_u} + \frac{225}{0.5\,\sigma_u} = \frac{525}{\sigma_u}$$

$\therefore$ $\sigma_u = 2 \times 525 = 1050$ MN/m² **Ans.**

3. ***According to Soderberg relation***

We know that according to Soderberg relation,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e}$$

or $$\frac{1}{2} = \frac{75}{0.55\,\sigma_u} + \frac{255}{0.5\,\sigma_u} = \frac{586.36}{\sigma_u}$$

$\therefore$ $\sigma_u = 2 \times 586.36 = 1172.72$ MN/m² **Ans.**

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Example 6.4. *A bar of circular cross-section is subjected to alternating tensile forces varying from a minimum of 200 kN to a maximum of 500 kN. It is to be manufactured of a material with an ultimate tensile strength of 900 MPa and an endurance limit of 700 MPa. Determine the diameter of bar using safety factors of 3.5 related to ultimate tensile strength and 4 related to endurance limit and a stress concentration factor of 1.65 for fatigue load. Use Goodman straight line as basis for design.*

Solution. Given : W_{min} = 200 kN ; W_{max} = 500 kN ; σ_u = 900 MPa = 900 N/mm^2 ; σ_e = 700 MPa = 700 N/mm^2 ; $(F.S.)_u$ = 3.5 ; $(F.S.)_e$ = 4 ; K_f = 1.65

Let d = Diameter of bar in mm.

$$\therefore \quad \text{Area, } A = \frac{\pi}{4} \times d^2 = 0.7854\, d^2 \text{ mm}^2$$

We know that mean or average force,

$$W_m = \frac{W_{max} + W_{min}}{2} = \frac{500 + 200}{2} = 350 \text{ kN} = 350 \times 10^3 \text{ N}$$

$$\therefore \quad \text{Mean stress, } \sigma_m = \frac{W_m}{A} = \frac{350 \times 10^3}{0.7854\, d^2} = \frac{446 \times 10^3}{d^2} \text{ N/mm}^2$$

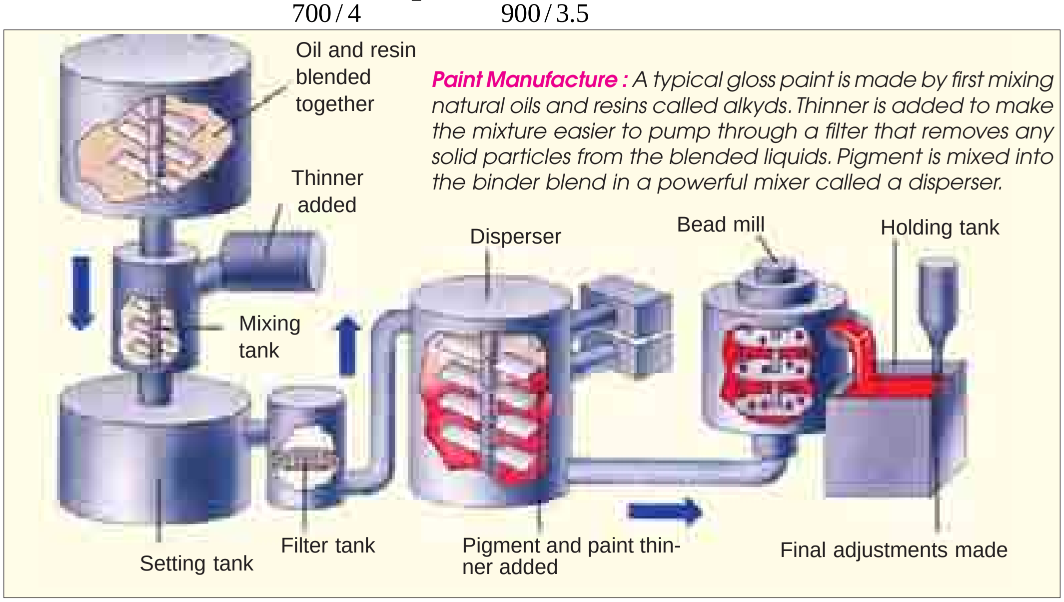
$$\text{Variable force, } W_v = \frac{W_{max} - W_{min}}{2} = \frac{500 - 200}{2} = 150 \text{ kN} = 150 \times 10^3 \text{ N}$$

$$\therefore \quad \text{Variable stress, } \sigma_v = \frac{W_v}{A} = \frac{150 \times 10^3}{0.7854\, d^2} = \frac{191 \times 10^3}{d^2} \text{ N/mm}^2$$

We know that according to Goodman's formula,

$$\frac{\sigma_v}{\sigma_e / (F.S.)_e} = 1 - \frac{\sigma_m . K_f}{\sigma_u / (F.S.)_u}$$

$$\frac{\dfrac{191 \times 10^3}{d^2}}{700/4} = 1 - \frac{\dfrac{446 \times 10^3}{d^2} \times 1.65}{900/3.5}$$



Paint Manufacture : *A typical gloss paint is made by first mixing natural oils and resins called alkyds. Thinner is added to make the mixture easier to pump through a filter that removes any solid particles from the blended liquids. Pigment is mixed into the binder blend in a powerful mixer called a disperser.*

Note : This picture is given as additional information and is not a direct example of the current chapter.

$$\frac{1100}{d^2} = 1 - \frac{2860}{d^2} \quad \text{or} \quad \frac{1100 + 2860}{d^2} = 1$$

$\therefore$ $d^2 = 3960$ or $d = 62.9$ say 63 mm **Ans.**

Example 6.5. *Determine the thickness of a 120 mm wide uniform plate for safe continuous operation if the plate is to be subjected to a tensile load that has a maximum value of 250 kN and a minimum value of 100 kN. The properties of the plate material are as follows:*

Endurance limit stress = 225 MPa, and Yield point stress = 300 MPa.

The factor of safety based on yield point may be taken as 1.5.

Solution. Given : b = 120 mm ; W_{max} = 250 kN; W_{min} = 100 kN ; σ_e = 225 MPa = 225 N/mm^2 ; σ_y = 300 MPa = 300 N/mm^2; *F.S.* = 1.5

Let t = Thickness of the plate in mm.

$\therefore$ Area, $A = b \times t = 120\, t$ mm^2

We know that mean or average load,

$$W_m = \frac{W_{max} + W_{min}}{2} = \frac{250 + 100}{2} = 175 \text{ kN} = 175 \times 10^3 \text{ N}$$

$$\therefore \quad \text{Mean stress, } \sigma_m = \frac{W_m}{A} = \frac{175 \times 10^3}{120\, t} \text{ N/mm}^2$$

$$\text{Variable load, } W_v = \frac{W_{max} - W_{min}}{2} = \frac{250 - 100}{2} = 75 \text{ kN} = 75 \times 10^3 \text{ N}$$

$$\therefore \quad \text{Variable stress, } \sigma_v = \frac{W_v}{A} = \frac{75 \times 10^3}{120\, t} \text{ N/mm}^2$$

According to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e}$$

$$\frac{1}{1.5} = \frac{175 \times 10^3}{120\, t \times 300} + \frac{75 \times 10^3}{120\, t \times 225} = \frac{4.86}{t} + \frac{2.78}{t} = \frac{7.64}{t}$$

$\therefore$ $t = 7.64 \times 1.5 = 11.46$ say 11.5 mm **Ans.**

Example 6.6. *Determine the diameter of a circular rod made of ductile material with a fatigue strength (complete stress reversal), σ_e = 265 MPa and a tensile yield strength of 350 MPa. The member is subjected to a varying axial load from W_{min} = – 300 × 10^3 N to W_{max} = 700 × 10^3 N and has a stress concentration factor = 1.8. Use factor of safety as 2.0.*

Solution. Given : σ_e = 265 MPa = 265 N/mm^2 ; σ_y = 350 MPa = 350 N/mm^2 ; W_{min} = – 300 × 10^3 N ; W_{max} = 700 × 10^3 N ; K_f = 1.8 ; *F.S.* = 2

Let d = Diameter of the circular rod in mm.

$$\therefore \quad \text{Area, } A = \frac{\pi}{4} \times d^2 = 0.7854\, d^2 \text{ mm}^2$$

We know that the mean or average load,

$$W_m = \frac{W_{max} + W_{min}}{2} = \frac{700 \times 10^3 + (-300 \times 10^3)}{2} = 200 \times 10^3 \text{ N}$$

$$\therefore \quad \text{Mean stress, } \sigma_m = \frac{W_m}{A} = \frac{200 \times 10^3}{0.7854\, d^2} = \frac{254.6 \times 10^3}{d^2} \text{ N/mm}^2$$

Variable load, $W_v = \dfrac{W_{max} - W_{min}}{2} = \dfrac{700 \times 10^3 - (-300 \times 10^3)}{2} = 500 \times 10^3 \text{ N}$

$\therefore$ Variable stress, $\sigma_v = \dfrac{W_v}{A} = \dfrac{500 \times 10^3}{0.7854\, d^2} = \dfrac{636.5 \times 10^3}{d^2} \text{ N/mm}^2$

We know that according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e}$$

$$\frac{1}{2} = \frac{254.6 \times 10^3}{d^2 \times 350} + \frac{636.5 \times 10^3 \times 1.8}{d^2 \times 265} = \frac{727}{d^2} + \frac{4323}{d^2} = \frac{5050}{d^2}$$

$\therefore$ $d^2 = 5050 \times 2 = 10\,100$ or $d = 100.5$ mm **Ans.**

Example 6.7. *A steel rod is subjected to a reversed axial load of 180 kN. Find the diameter of the rod for a factor of safety of 2. Neglect column action. The material has an ultimate tensile strength of 1070 MPa and yield strength of 910 MPa. The endurance limit in reversed bending may be assumed to be one-half of the ultimate tensile strength. Other correction factors may be taken as follows:*

For axial loading = 0.7; For machined surface = 0.8 ; For size = 0.85 ; For stress concentration = 1.0.

Solution. Given : W_{max} = 180 kN ; W_{min} = – 180 kN ; $F.S.$ = 2 ; σ_u = 1070 MPa = 1070 N/mm^2; σ_y = 910 MPa = 910 N/mm^2 ; $\sigma_e = 0.5\, \sigma_u$; $K_a = 0.7$; $K_{sur} = 0.8$; $K_{sz} = 0.85$; $K_f = 1$

Let d = Diameter of the rod in mm.

$\therefore$ Area, $A = \dfrac{\pi}{4} \times d^2 = 0.7854\, d^2 \text{ mm}^2$

We know that the mean or average load,

$$W_m = \frac{W_{max} + W_{min}}{2} = \frac{180 + (-180)}{2} = 0$$

$\therefore$ Mean stress, $\sigma_m = \dfrac{W_m}{A} = 0$

Variable load, $W_v = \dfrac{W_{max} - W_{min}}{2} = \dfrac{180 - (-180)}{2} = 180 \text{ kN} = 180 \times 10^3 \text{ N}$

$\therefore$ Variable stress, $\sigma_v = \dfrac{W_v}{A} = \dfrac{180 \times 10^3}{0.7854\, d^2} = \dfrac{229 \times 10^3}{d^2} \text{ N/mm}^2$

Endurance limit in reversed axial loading,

$$\sigma_{ea} = \sigma_e \times K_a = 0.5\, \sigma_u \times 0.7 = 0.35\, \sigma_u \qquad \text{...}(\because \sigma_e = 0.5\, \sigma_u)$$

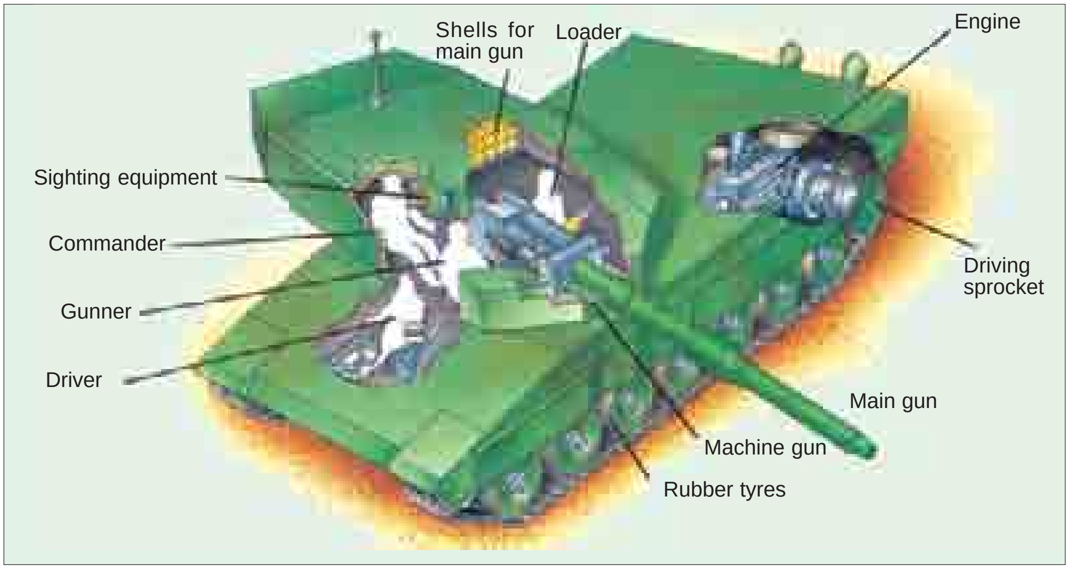
$$= 0.35 \times 1070 = 374.5 \text{ N/mm}^2$$

We know that according to Soderberg's formula for reversed axial loading,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_{ea} \times K_{sur} \times K_{sz}}$$

$$\frac{1}{2} = 0 + \frac{229 \times 10^3 \times 1}{d^2 \times 374.5 \times 0.8 \times 0.85} = \frac{900}{d^2}$$

$\therefore$ $d^2 = 900 \times 2 = 1800$ or $d = 42.4$ mm **Ans.**



Layout of a military tank.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Example 6.8. *A circular bar of 500 mm length is supported freely at its two ends. It is acted upon by a central concentrated cyclic load having a minimum value of 20 kN and a maximum value of 50 kN. Determine the diameter of bar by taking a factor of safety of 1.5, size effect of 0.85, surface finish factor of 0.9. The material properties of bar are given by : ultimate strength of 650 MPa, yield strength of 500 MPa and endurance strength of 350 MPa.*

Solution. Given : l = 500 mm ; W_{min} = 20 kN = 20 × 10^3 N ; W_{max} = 50 kN = 50 × 10^3 N ; *F.S.* = 1.5 ; K_{sz} = 0.85 ; K_{sur} = 0.9 ; σ_u = 650 MPa = 650 N/mm^2 ; σ_y = 500 MPa = 500 N/mm^2 ; σ_e = 350 MPa = 350 N/mm^2

Let d = Diameter of the bar in mm.

We know that the maximum bending moment,

$$M_{max} = \frac{W_{max} \times l}{4} = \frac{50 \times 10^3 \times 500}{4} = 6250 \times 10^3 \text{ N-mm}$$

and minimum bending moment,

$$M_{min} = \frac{W_{min} \times l}{4} = \frac{20 \times 10^3 \times 500}{4} = 2550 \times 10^3 \text{ N-mm}$$

∴ Mean or average bending moment,

$$M_m = \frac{M_{max} + M_{min}}{2} = \frac{6250 \times 10^3 + 2500 \times 10^3}{2} = 4375 \times 10^3 \text{ N-mm}$$

and variable bending moment,

$$M_v = \frac{M_{max} - M_{min}}{2} = \frac{6250 \times 10^3 - 2500 \times 10^3}{2} = 1875 \times 10^3 \text{ N-mm}$$

Section modulus of the bar,

$$Z = \frac{\pi}{32} \times d^3 = 0.0982\, d^3 \text{ mm}^3$$

∴ Mean or average bending stress,

$$\sigma_m = \frac{M_m}{Z} = \frac{4375 \times 10^3}{0.0982\, d^3} = \frac{44.5 \times 10^6}{d^3} \text{ N/mm}^2$$

and variable bending stress,

$$\sigma_v = \frac{M_v}{Z} = \frac{1875 \times 10^3}{0.0982\, d^3} = \frac{19.1 \times 10^6}{d^3} \text{ N/mm}^2$$

We know that according to Goodman's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{1.5} = \frac{44.5 \times 10^6}{d^3 \times 650} + \frac{19.1 \times 10^6 \times 1}{d^3 \times 350 \times 0.9 \times 0.85} \qquad \text{...(Taking } K_f = 1)$$

$$= \frac{68 \times 10^3}{d^3} + \frac{71 \times 10^3}{d^3} = \frac{139 \times 10^3}{d^3}$$

$\therefore$ $d^3 = 139 \times 10^3 \times 1.5 = 209 \times 10^3$ or $d = 59.3$ mm

and according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{1.5} = \frac{44.5 \times 10^6}{d^3 \times 500} + \frac{19.1 \times 10^6 \times 1}{d^3 \times 350 \times 0.9 \times 0.85} \qquad \text{...(Taking } K_f = 1)$$

$$= \frac{89 \times 10^3}{d^3} + \frac{71 \times 10^3}{d^3} = \frac{160 \times 10^3}{d^3}$$

$\therefore$ $d^3 = 160 \times 10^3 \times 1.5 = 240 \times 10^3$ or $d = 62.1$ mm

Taking larger of the two values, we have $d = 62.1$ mm **Ans.**

Example 6.9. *A 50 mm diameter shaft is made from carbon steel having ultimate tensile strength of 630 MPa. It is subjected to a torque which fluctuates between 2000 N-m to – 800 N-m. Using Soderberg method, calculate the factor of safety. Assume suitable values for any other data needed.*

Solution. Given : $d = 50$ mm ; $\sigma_u = 630$ MPa = 630 N/mm^2 ; $T_{max} = 2000$ N-m ; $T_{min} = -800$ N-m

We know that the mean or average torque,

$$T_m = \frac{T_{max} + T_{min}}{2} = \frac{2000 + (-800)}{2} = 600 \text{ N-m} = 600 \times 10^3 \text{ N-mm}$$

$\therefore$ Mean or average shear stress,

$$\tau_m = \frac{16\, T_m}{\pi d^3} = \frac{16 \times 600 \times 10^3}{\pi (50)^3} = 24.4 \text{ N/mm}^2 \qquad \text{...}\left(\because\ T = \frac{\pi}{16} \times \tau \times d^3\right)$$

Variable torque,

$$T_v = \frac{T_{max} - T_{min}}{2} = \frac{2000 - (-800)}{2} = 1400 \text{ N-m} = 1400 \times 10^3 \text{ N-mm}$$

$\therefore$ Variable shear stress, $\tau_v = \dfrac{16\, T_v}{\pi d^3} = \dfrac{16 \times 1400 \times 10^3}{\pi (50)^3} = 57 \text{ N/mm}^2$

Since the endurance limit in reversed bending (σ_e) is taken as one-half the ultimate tensile strength (*i.e.* $\sigma_e = 0.5\, \sigma_u$) and the endurance limit in shear (τ_e) is taken as 0.55 σ_e, therefore

$$\tau_e = 0.55\, \sigma_e = 0.55 \times 0.5\, \sigma_u = 0.275\, \sigma_u$$
$$= 0.275 \times 630 = 173.25 \text{ N/mm}^2$$

Assume the yield stress (σ_y) for carbon steel in reversed bending as 510 N/mm^2, surface finish factor (K_{sur}) as 0.87, size factor (K_{sz}) as 0.85 and fatigue stress concentration factor (K_{fs}) as 1.

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Army Tank

Note : This picture is given as additional information and is not a direct example of the current chapter.

Since the yield stress in shear (τ_y) for shear loading is taken as one-half the yield stress in reversed bending (σ_y), therefore

$$\tau_y = 0.5\ \sigma_y = 0.5 \times 510 = 255\ \text{N/mm}^2$$

Let $\quad F.S. =$ Factor of safety.

We know that according to Soderberg's formula,

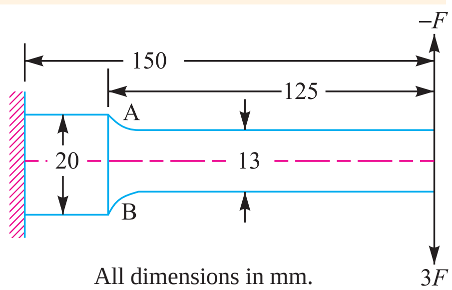
$$\frac{1}{F.S.} = \frac{\tau_m}{\tau_y} + \frac{\tau_v \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}} = \frac{24.4}{255} + \frac{57 \times 1}{173.25 \times 0.87 \times 0.85}$$

$$= 0.096 + 0.445 = 0.541$$

$\therefore \quad F.S. = 1 / 0.541 = 1.85$ **Ans.**

Example 6.10. *A cantilever beam made of cold drawn carbon steel of circular cross-section as shown in Fig. 6.18, is subjected to a load which varies from – F to 3 F. Determine the maximum load that this member can withstand for an indefinite life using a factor of safety as 2. The theoretical stress concentration factor is 1.42 and the notch sensitivity is 0.9. Assume the following values :*

Ultimate stress = 550 MPa
Yield stress = 470 MPa
Endurance limit = 275 MPa
Size factor = 0.85
Surface finish factor = 0.89



All dimensions in mm.

Fig. 6.18

Solution. Given : $W_{min} = -F$; $W_{max} = 3F$; $F.S. = 2$; $K_t = 1.42$; $q = 0.9$; $\sigma_u = 550$ MPa $= 550\ \text{N/mm}^2$; $\sigma_y = 470$ MPa $= 470\ \text{N/mm}^2$; $\sigma_e = 275$ MPa $= 275\ \text{N/mm}^2$; $K_{sz} = 0.85$; $K_{sur} = 0.89$

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The beam as shown in Fig. 6.18 is subjected to a reversed bending load only. Since the point *A* at the change of cross section is critical, therefore we shall find the bending moment at point *A*.

We know that maximum bending moment at point *A*,

$$M_{max} = W_{max} \times 125 = 3F \times 125 = 375\ F \text{ N-mm}$$

and minimum bending moment at point *A*,

$$M_{min} = W_{min} \times 125 = -F \times 125 = -125\ F \text{ N-mm}$$

∴ Mean or average bending moment,

$$M_m = \frac{M_{max} + M_{min}}{2} = \frac{375\ F + (-125\ F)}{2} = 125\,F \text{ N-mm}$$

and variable bending moment,

$$M_v = \frac{M_{max} - M_{min}}{2} = \frac{375\ F - (-125\ F)}{2} = 250\,F \text{ N-mm}$$

Section modulus, $$Z = \frac{\pi}{32} \times d^3 = \frac{\pi}{32}(13)^3 = 215.7 \text{ mm}^3 \quad \ldots(\because d = 13 \text{ mm})$$

∴ Mean bending stress, $$\sigma_m = \frac{M_m}{Z} = \frac{125\ F}{215.7} = 0.58\ F \text{ N/mm}^2$$

and variable bending stress, $$\sigma_v = \frac{M_v}{Z} = \frac{250\ F}{215.7} = 1.16\ F \text{ N/mm}^2$$

Fatigue stress concentration factor, $K_f = 1 + q\,(K_t - 1) = 1 + 0.9\,(1.42 - 1) = 1.378$

We know that according to Goodman's formula

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{2} = \frac{0.58\ F}{550} + \frac{1.16\,F \times 1.378}{275 \times 0.89 \times 0.85}$$

$$= 0.001\ 05\ F + 0.007\ 68\ F = 0.008\ 73\ F$$

∴ $$F = \frac{1}{2 \times 0.008\,73} = 57.3 \text{ N}$$

and according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{2} = \frac{0.58\ F}{470} + \frac{1.16\,F \times 1.378}{275 \times 0.89 \times 0.85}$$

$$= 0.001\ 23\ F + 0.007\ 68\ F = 0.008\ 91\ F$$

∴ $$F = \frac{1}{2 \times 0.008\ 91} = 56 \text{ N}$$

Taking larger of the two values, we have F = 57.3 N **Ans.**

Example 6.11. *A simply supported beam has a concentrated load at the centre which fluctuates from a value of P to 4 P. The span of the beam is 500 mm and its cross-section is circular with a diameter of 60 mm. Taking for the beam material an ultimate stress of 700 MPa, a yield stress of 500 MPa, endurance limit of 330 MPa for reversed bending, and a factor of safety of 1.3, calculate the maximum value of P. Take a size factor of 0.85 and a surface finish factor of 0.9.*

Solution. Given : $W_{min} = P$; $W_{max} = 4P$; L = 500 mm; d = 60 mm ; σ_u = 700 MPa = 700 N/mm^2 ; σ_y = 500 MPa = 500 N/mm^2 ; σ_e = 330 MPa = 330 N/mm^2 ; *F.S.* = 1.3 ; K_{sz} = 0.85 ; K_{sur} = 0.9

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We know that maximum bending moment,

$$M_{max} = \frac{W_{max} \times L}{4} = \frac{4P \times 500}{4} = 500\,P \text{ N-mm}$$

and minimum bending moment,

$$M_{min} = \frac{W_{min} \times L}{4} = \frac{P \times 500}{4} = 125\,P \text{ N-mm}$$

∴ Mean or average bending moment,

$$M_m = \frac{M_{max} + M_{min}}{2} = \frac{500\,P + 125\,P}{2} = 312.5\,P \text{ N-mm}$$

and variable bending moment,

$$M_v = \frac{M_{max} - M_{min}}{2} = \frac{500\,P - 125\,P}{2} = 187.5\,P \text{ N-mm}$$

Section modulus, $$Z = \frac{\pi}{32} \times d^3 = \frac{\pi}{32}(60)^3 = 21.21 \times 10^3 \text{ mm}^3$$

∴ Mean bending stress,

$$\sigma_m = \frac{M_m}{Z} = \frac{312.5\,P}{21.21 \times 10^3} = 0.0147\,P \text{ N/mm}^2$$

and variable bending stress,

$$\sigma_v = \frac{M_v}{Z} = \frac{187.5\,P}{21.21 \times 10^3} = 0.0088\,P \text{ N/mm}^2$$

We know that according to Goodman's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{1.3} = \frac{0.0147\,P}{700} + \frac{0.0088\,P \times 1}{330 \times 0.9 \times 0.85} \qquad \text{...(Taking } K_f = 1)$$

$$= \frac{21\,P}{10^6} + \frac{34.8\,P}{10^6} = \frac{55.8\,P}{10^6}$$

$$\therefore \quad P = \frac{1}{1.3} \times \frac{10^6}{55.8} = 13\,785 \text{ N} = 13.785 \text{ kN}$$

and according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{1.3} = \frac{0.0147\,P}{500} + \frac{0.0088\,P \times 1}{330 \times 0.9 \times 0.85} = \frac{29.4\,P}{10^6} + \frac{34.8\,P}{10^6} = \frac{64.2\,P}{10^6}$$

$$\therefore \quad P = \frac{1}{1.3} \times \frac{10^6}{64.2} = 11\,982 \text{ N} = 11.982 \text{ kN}$$

From the above, we find that maximum value of P = 13.785 kN **Ans.**

6.22 Combined Variable Normal Stress and Variable Shear Stress

When a machine part is subjected to both variable normal stress and a variable shear stress; then it is designed by using the following two theories of combined stresses :

1. Maximum shear stress theory, and **2.** Maximum normal stress theory.

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We have discussed in Art. 6.21, that according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_{fb}}{\sigma_{eb} \times K_{sur} \times K_{sz}} \quad \text{...(For reversed bending load)}$$

Multiplying throughout by σ_y, we get

$$\frac{\sigma_y}{F.S.} = \sigma_m + \frac{\sigma_v \times \sigma_y \times K_{fb}}{\sigma_{eb} \times K_{sur} \times K_{sz}}$$

The term on the right hand side of the above expression is known as ***equivalent normal stress*** due to reversed bending.

∴ Equivalent normal stress due to reversed bending,

$$\sigma_{neb} = \sigma_m + \frac{\sigma_v \times \sigma_y \times K_{fb}}{\sigma_{eb} \times K_{sur} \times K_{sz}} \quad \text{...(}i\text{)}$$

Similarly, equivalent normal stress due to reversed axial loading,

$$\sigma_{nea} = \sigma_m + \frac{\sigma_v \times \sigma_y \times K_{fa}}{\sigma_{ea} \times K_{sur} \times K_{sz}} \quad \text{...(}ii\text{)}$$

and total equivalent normal stress,

$$\sigma_{ne} = \sigma_{neb} + \sigma_{nea} = \frac{\sigma_y}{F.S.} \quad \text{...(}iii\text{)}$$

We have also discussed in Art. 6.21, that for reversed torsional or shear loading,

$$\frac{1}{F.S.} = \frac{\tau_m}{\tau_y} + \frac{\tau_v \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}}$$

Multiplying throughout by τ_y, we get

$$\frac{\tau_y}{F.S.} = \tau_m + \frac{\tau_v \times \tau_y \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}}$$

The term on the right hand side of the above expression is known as ***equivalent shear stress.***

∴ Equivalent shear stress due to reversed torsional or shear loading,

$$\tau_{es} = \tau_m + \frac{\tau_v \times \tau_y \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}} \quad \text{...(}iv\text{)}$$

The maximum shear stress theory is used in designing machine parts of ductile materials. According to this theory, maximum equivalent shear stress,

$$\tau_{es(max)} = \frac{1}{2}\sqrt{(\sigma_{ne})^2 + 4\,(\tau_{es})^2} = \frac{\tau_y}{F.S.}$$

The maximum normal stress theory is used in designing machine parts of brittle materials. According to this theory, maximum equivalent normal stress,

$$\sigma_{ne(max)} = \frac{1}{2}(\sigma_{ne}) + \frac{1}{2}\sqrt{(\sigma_{ne})^2 + 4\,(\tau_{es})^2} = \frac{\sigma_y}{F.S.}$$

Example 6.12. *A steel cantilever is 200 mm long. It is subjected to an axial load which varies from 150 N (compression) to 450 N (tension) and also a transverse load at its free end which varies from 80 N up to 120 N down. The cantilever is of circular cross-section. It is of diameter 2d for the first 50 mm and of diameter d for the remaining length. Determine its diameter taking a factor of safety of 2. Assume the following values :*

Yield stress	= *330 MPa*
Endurance limit in reversed loading	= *300 MPa*
Correction factors	= *0.7 in reversed axial loading*
	= *1.0 in reversed bending*

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Stress concentration factor	*= 1.44 for bending*
	= 1.64 for axial loading
Size effect factor	*= 0.85*
Surface effect factor	*= 0.90*
Notch sensitivity index	*= 0.90*

Solution. Given : l = 200 mm; $W_{a(max)}$ = 450 N; $W_{a(min)}$ = – 150 N ; $W_{t(max)}$ = 120 N ; $W_{t(min)}$ = – 80 N; *F.S.* =2 ; σ_y = 330 MPa = 330 N/mm^2 ; σ_e = 300 MPa = 300 N/mm^2 ; K_a = 0.7; K_b = 1; K_{tb} = 1.44 ; K_{ta} = 1.64; K_{sz} = 0.85 ; K_{sur} = 0.90 ; q = 0.90

First of all, let us find the equivalent normal stress for point *A* which is critical as shown in Fig. 6.19. It is assumed that the equivalent normal stress at this point will be the algebraic sum of the equivalent normal stress due to axial loading and equivalent normal stress due to bending (*i.e.* due to transverse load acting at the free end).

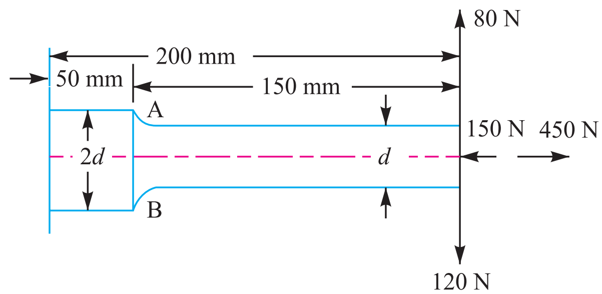

Fig. 6.19

Let us first consider the reversed axial loading. We know that mean or average axial load,

$$W_m = \frac{W_{a(max)} + W_{a(min)}}{2} = \frac{450 + (-150)}{2} = 150 \text{ N}$$

and variable axial load,

$$W_v = \frac{W_{a(max)} - W_{a(min)}}{2} = \frac{450 - (-150)}{2} = 300 \text{ N}$$

∴ Mean or average axial stress,

$$\sigma_m = \frac{W_m}{A} = \frac{150 \times 4}{\pi d^2} = \frac{191}{d^2} \text{ N/mm}^2 \qquad \ldots \left(\because A = \frac{\pi}{4} \times d^2\right)$$

and variable axial stress,

$$\sigma_v = \frac{W_v}{A} = \frac{300 \times 4}{\pi d^2} = \frac{382}{d^2} \text{ N/mm}^2$$

We know that fatigue stress concentration factor for reversed axial loading,

$$K_{fa} = 1 + q\,(K_{ta} - 1) = 1 + 0.9\,(1.64 - 1) = 1.576$$

and endurance limit stress for reversed axial loading,

$$\sigma_{ea} = \sigma_e \times K_a = 300 \times 0.7 = 210 \text{ N/mm}^2$$

We know that equivalent normal stress at point *A* due to axial loading,

$$\sigma_{nea} = \sigma_m + \frac{\sigma_v \times \sigma_y \times K_{fa}}{\sigma_{ea} \times K_{sur} \times K_{sz}} = \frac{191}{d^2} + \frac{382 \times 330 \times 1.576}{d^2 \times 210 \times 0.9 \times 0.85}$$

$$= \frac{191}{d^2} + \frac{1237}{d^2} = \frac{1428}{d^2} \text{ N/mm}^2$$

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Now let us consider the reversed bending due to transverse load. We know that mean or average bending load,

$$W_m = \frac{W_{t(max)} + W_{t(min)}}{2}$$

$$= \frac{120 + (-80)}{2} = 20 \text{ N}$$

and variable bending load,

$$W_v = \frac{W_{t(max)} - W_{t(min)}}{2}$$

$$= \frac{120 - (-80)}{2} = 100 \text{ N}$$

Machine transporter

∴ Mean bending moment at point A,

$$M_m = W_m (l - 50) = 20 (200 - 50) = 3000 \text{ N-mm}$$

and variable bending moment at point A,

$$M_v = W_v (l - 50) = 100 (200 - 50) = 15\,000 \text{ N-mm}$$

We know that section modulus,

$$Z = \frac{\pi}{32} \times d^3 = 0.0982\, d^3 \text{ mm}^3$$

∴ Mean or average bending stress,

$$\sigma_m = \frac{M_m}{Z} = \frac{3000}{0.0982\, d^3} = \frac{30\,550}{d^3} \text{ N/mm}^2$$

and variable bending stress,

$$\sigma_v = \frac{M_v}{Z} = \frac{15\,000}{0.0982\, d^3} = \frac{152\,750}{d^3} \text{ N/mm}^2$$

We know that fatigue stress concentration factor for reversed bending,

$$K_{fb} = 1 + q (K_{tb} - 1) = 1 + 0.9 (1.44 - 1) = 1.396$$

Since the correction factor for reversed bending load is 1 (*i.e.* $K_b = 1$), therefore the endurance limit for reversed bending load,

$$\sigma_{eb} = \sigma_e \,.\, K_b = \sigma_e = 300 \text{ N/mm}^2$$

We know that the equivalent normal stress at point A due to bending,

$$\sigma_{neb} = \sigma_m + \frac{\sigma_v \times \sigma_y \times K_{fb}}{\sigma_{eb} \times K_{sur} \times K_{sz}} = \frac{30\,550}{d^3} + \frac{152\,750 \times 330 \times 1.396}{d^3 \times 300 \times 0.9 \times 0.85}$$

$$= \frac{30\,550}{d^3} + \frac{306\,618}{d^3} = \frac{337\,168}{d^3} \text{ N/mm}^2$$

∴ Total equivalent normal stress at point A,

$$\sigma_{ne} = \sigma_{neb} + \sigma_{nea} = \frac{337\,168}{d^3} + \frac{1428}{d^2} \text{ N/mm}^2 \quad ...(i)$$

We know that equivalent normal stress at point *A*,

$$\sigma_{ne} = \frac{\sigma_y}{F.S.} = \frac{330}{2} = 165 \text{ N/mm}^2 \quad ...(ii)$$

Equating equations (*i*) and (*ii*), we have

$$\frac{337\,168}{d^3} + \frac{1428}{d^2} = 165 \quad \text{or} \quad 337\,168 + 1428\, d = 165\, d^3$$

$\therefore$ $236.1 + d = 0.116\, d^3$ or $d = 12.9$ mm **Ans.** ...(By hit and trial)

Example 6.13. *A hot rolled steel shaft is subjected to a torsional moment that varies from 330 N-m clockwise to 110 N-m counterclockwise and an applied bending moment at a critical section varies from 440 N-m to – 220 N-m. The shaft is of uniform cross-section and no keyway is present at the critical section. Determine the required shaft diameter. The material has an ultimate strength of 550 MN/m² and a yield strength of 410 MN/m². Take the endurance limit as half the ultimate strength, factor of safety of 2, size factor of 0.85 and a surface finish factor of 0.62.*

Solution. Given : $T_{max} = 330$ N-m (clockwise) ; $T_{min} = 110$ N-m (counterclockwise) = – 110 N-m (clockwise) ; $M_{max} = 440$ N-m ; $M_{min} = -220$ N-m ; $\sigma_u = 550 \text{ MN/m}^2 = 550 \times 10^6 \text{ N/m}^2$; $\sigma_y = 410 \text{ MN/m}^2 = 410 \times 10^6 \text{ N/m}^2$; $\sigma_e = \frac{1}{2}\sigma_u = 275 \times 10^6 \text{ N/m}^2$; $F.S. = 2$; $K_{sz} = 0.85$; $K_{sur} = 0.62$

Let d = Required shaft diameter in metres.

We know that mean torque,

$$T_m = \frac{T_{max} + T_{min}}{2} = \frac{330 + (-110)}{2} = 110 \text{ N-m}$$

and variable torque,

$$T_v = \frac{T_{max} - T_{min}}{2} = \frac{330 - (-110)}{2} = 220 \text{ N-m}$$

$\therefore$ Mean shear stress,

$$\tau_m = \frac{16\, T_m}{\pi\, d^3} = \frac{16 \times 110}{\pi d^3} = \frac{560}{d^3} \text{ N/m}^2$$

and variable shear stress,

$$\tau_v = \frac{16\, T_v}{\pi\, d^3} = \frac{16 \times 220}{\pi d^3} = \frac{1120}{d^3} \text{ N/m}^2$$

Since the endurance limit in shear (τ_e) is 0.55 σ_e, and yield strength in shear (τ_y) is 0.5 σ_y, therefore

$$\tau_e = 0.55 \times 275 \times 10^6 = 151.25 \times 10^6 \text{ N/m}^2$$

and

$$\tau_y = 0.5 \times 410 \times 10^6 = 205 \times 10^6 \text{ N/m}^2$$

We know that equivalent shear stress,

$$\tau_{es} = \tau_m + \frac{\tau_v \times \tau_y\, K_{fs}}{\tau_e \times K_{sur} \times K_{sz}}$$

$$= \frac{560}{d^3} + \frac{1120 \times 205 \times 10^6 \times 1}{d^3 \times 151.25 \times 10^6 \times 0.62 \times 0.85} \quad ...(\text{Taking } K_{fs} = 1)$$

$$= \frac{560}{d^3} + \frac{2880}{d^3} = \frac{3440}{d^3} \text{ N/m}^2$$

Mean or average bending moment,

$$M_m = \frac{M_{max} + M_{min}}{2} = \frac{440 + (-220)}{2} = 110 \text{ N-m}$$

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and variable bending moment,

$$M_v = \frac{M_{max} - M_{min}}{2} = \frac{440 - (-220)}{2} = 330 \text{ N-m}$$

Section modulus, $Z = \frac{\pi}{32} \times d^3 = 0.0982\, d^3 \text{ m}^3$

∴ Mean bending stress,

$$\sigma_m = \frac{M_m}{Z} = \frac{110}{0.0982\, d^3} = \frac{1120}{d^3} \text{ N/m}^2$$

and variable bending stress,

$$\sigma_v = \frac{M_v}{Z} = \frac{330}{0.0982\, d^3} = \frac{3360}{d^3} \text{ N/m}^2$$

Machine parts are often made of alloys to improve their mechanical properties.

Since there is no reversed axial loading, therefore the equivalent normal stress due to reversed bending load,

$$\sigma_{neb} = \sigma_{ne} = \sigma_m + \frac{\sigma_v \times \sigma_y \times K_{fb}}{\sigma_{eb} \times K_{sur} \times K_{sz}}$$

$$= \frac{1120}{d^3} + \frac{3360 \times 410 \times 10^6 \times 1}{d^3 \times 275 \times 10^6 \times 0.62 \times 0.85}$$

...(Taking $K_{fb} = 1$ and $\sigma_{eb} = \sigma_e$)

$$= \frac{1120}{d^3} + \frac{9506}{d^3} = \frac{10\,626}{d^3} \text{ N/m}^2$$

We know that the maximum equivalent shear stress,

$$\tau_{es(max)} = \frac{\tau_y}{F.S.} = \frac{1}{2}\sqrt{(\sigma_{ne})^2 + 4\,(\tau_{es})^2}$$

$$\frac{205 \times 10^6}{2} = \frac{1}{2}\sqrt{\left(\frac{10\,625}{d^3}\right)^2 + 4\left(\frac{3440}{d^3}\right)^2}$$

$$205 \times 10^6 \times d^3 = \sqrt{113 \times 10^6 + 4 \times 11.84 \times 10^6} = 12.66 \times 10^3$$

$$\therefore \quad d^3 = \frac{12.66 \times 10^3}{205 \times 10^6} = \frac{0.0617}{10^3}$$

or $\quad d = \frac{0.395}{10} = 0.0395 \text{ m} = 39.5 \text{ say } 40 \text{ mm}$ **Ans.**

Example 6.14. *A pulley is keyed to a shaft midway between two bearings. The shaft is made of cold drawn steel for which the ultimate strength is 550 MPa and the yield strength is 400 MPa. The bending moment at the pulley varies from – 150 N-m to + 400 N-m as the torque on the shaft varies from – 50 N-m to + 150 N-m. Obtain the diameter of the shaft for an indefinite life. The stress concentration factors for the keyway at the pulley in bending and in torsion are 1.6 and 1.3 respectively. Take the following values:*

Factor of safety	= *1.5*
Load correction factors	= *1.0 in bending, and 0.6 in torsion*
Size effect factor	= *0.85*
Surface effect factor	= *0.88*

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Solution. Given : σ_u = 550 MPa = 550 N/mm^2 ; σ_y = 400 MPa = 400 N/mm^2 ; M_{min} = – 150 N-m; M_{max} = 400 N-m ; T_{min} = – 50 N-m ; T_{max} = 150 N-m ; K_{fb} = 1.6 ; K_{fs} = 1.3 ; *F.S.* = 1.5 ; K_b = 1 ; K_s = 0.6 ; K_{sz} = 0.85 ; K_{sur} = 0.88

Let d = Diameter of the shaft in mm.

First of all, let us find the equivalent normal stress due to bending.

We know that the mean or average bending moment,

$$M_m = \frac{M_{max} + M_{min}}{2} = \frac{400 + (-150)}{2} = 125 \text{ N-m} = 125 \times 10^3 \text{ N-mm}$$

and variable bending moment,

$$M_v = \frac{M_{max} - M_{min}}{2} = \frac{400 - (-150)}{2} = 275 \text{ N-m} = 275 \times 10^3 \text{ N-mm}$$

Section modulus, $Z = \dfrac{\pi}{32} \times d^3 = 0.0982\, d^3 \text{ mm}^3$

∴ Mean bending stress,

$$\sigma_m = \frac{M_m}{Z} = \frac{125 \times 10^3}{0.0982\, d^3} = \frac{1273 \times 10^3}{d^3} \text{ N/mm}^2$$

and variable bending stress,

$$\sigma_v = \frac{M_v}{Z} = \frac{275 \times 10^3}{0.0982\, d^3} = \frac{2800 \times 10^3}{d^3} \text{ N/mm}^2$$

Assuming the endurance limit in reversed bending as one-half the ultimate strength and since the load correction factor for reversed bending is 1 (*i.e.* K_b = 1), therefore endurance limit in reversed bending,

$$\sigma_{eb} = \sigma_e = \frac{\sigma_u}{2} = \frac{550}{2} = 275 \text{ N/mm}^2$$

Since there is no reversed axial loading, therefore equivalent normal stress due to bending,

$$\sigma_{neb} = \sigma_{ne} = \sigma_m + \frac{\sigma_v \times \sigma_y \times K_{fb}}{\sigma_{eb} \times K_{sur} \times K_{sz}}$$

$$= \frac{1273 \times 10^3}{d^3} + \frac{2800 \times 10^3 \times 400 \times 1.6}{d^3 \times 275 \times 0.88 \times 0.85}$$

$$= \frac{1273 \times 10^3}{d^3} + \frac{8712 \times 10^3}{d^3} = \frac{9985 \times 10^3}{d^3} \text{ N/mm}^2$$

Now let us find the equivalent shear stress due to torsional moment. We know that the mean torque,

$$T_m = \frac{T_{max} + T_{min}}{2} = \frac{150 + (-50)}{2} = 50 \text{ N-m} = 50 \times 10^3 \text{ N-mm}$$

and variable torque, $T_v = \dfrac{T_{max} - T_{min}}{2} = \dfrac{150 - (-50)}{2} = 100 \text{ N-m} = 100 \times 10^3 \text{ N-mm}$

∴ Mean shear stress,

$$\tau_m = \frac{16\, T_m}{\pi d^3} = \frac{16 \times 50 \times 10^3}{\pi d^3} = \frac{255 \times 10^3}{d^3} \text{ N/mm}^2$$

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and variable shear stress,

$$\tau_v = \frac{16\ T_v}{\pi d^3} = \frac{16 \times 100 \times 10^3}{\pi d^3} = \frac{510 \times 10^3}{d^3}\ \text{N/mm}^2$$

Endurance limit stress for reversed torsional or shear loading,

$$\tau_e = \sigma_e \times K_s = 275 \times 0.6 = 165\ \text{N/mm}^2$$

Assuming yield strength in shear,

$$\tau_y = 0.5\ \sigma_y = 0.5 \times 400 = 200\ \text{N/mm}^2$$

We know that equivalent shear stress,

$$\tau_{es} = \tau_m + \frac{\tau_v \times \tau_y \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}}$$

$$= \frac{255 \times 10^3}{d^3} + \frac{510 \times 10^3 \times 200 \times 1.3}{d^3 \times 165 \times 0.88 \times 0.85}$$

$$= \frac{255 \times 10^3}{d^3} + \frac{1074 \times 10^3}{d^3} = \frac{1329 \times 10^3}{d^3}\ \text{N/mm}^2$$

and maximum equivalent shear stress,

$$\tau_{es(max)} = \frac{\tau_y}{F.S.} = \frac{1}{2}\sqrt{(\sigma_{ne})^2 + 4\,(\tau_{es})^2}$$

$$\frac{200}{1.5} = \frac{1}{2}\sqrt{\left(\frac{9985 \times 10^3}{d^3}\right)^2 + 4\left(\frac{1329 \times 10^3}{d^3}\right)^2} = \frac{5165 \times 10^3}{d^3}$$

$$\therefore \quad d^3 = \frac{5165 \times 10^3 \times 1.5}{200} = 38\ 740 \text{ or } d = 33.84 \text{ say } 35 \text{ mm} \quad \textbf{Ans.}$$

6.23 Application of Soderberg's Equation

We have seen in Art. 6.21 that according to Soderberg's equation,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e} \qquad \text{...}(i)$$

This equation may also be written as

$$\frac{1}{F.S.} = \frac{\sigma_m \times \sigma_e + \sigma_v \times \sigma_y \times K_f}{\sigma_y \times \sigma_e}$$

or

$$F.S. = \frac{\sigma_y \times \sigma_e}{\sigma_m \times \sigma_e + \sigma_v \times \sigma_y \times K_f} = \frac{\sigma_y}{\sigma_m + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times \sigma_v} \qquad \text{...}(ii)$$

Since the factor of safety based on yield strength is the ratio of the yield point stress to the working or design stress, therefore from equation (*ii*), we may write

Working or design stress

$$= \sigma_m + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times \sigma_v \qquad \text{...}(iii)$$

Let us now consider the use of Soderberg's equation to a ductile material under the following loading conditions.

1. *Axial loading*

In case of axial loading, we know that the mean or average stress,

$$\sigma_m = W_m / A$$

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and variable stress, $\sigma_v = W_v / A$

where W_m = Mean or average load,

W_v = Variable load, and

A = Cross-sectional area.

The equation ***(iii)*** may now be written as follows :

Working or design stress,

$$= \frac{W_m}{A} + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times \frac{W_v}{A} = \frac{W_m + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times W_v}{A}$$

$$\therefore \quad F.S. = \frac{\sigma_y \times A}{W_m + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times W_v}$$

A large disc- shaped electromagnet hangs from jib of this scrapyard crane. Steel and iron objects fly towards the magnet when the current is switched on. In this way, iron and steel can be separated for recycling.

Note : This picture is given as additional information and is not a direct example of the current chapter.

2. *Simple bending*

In case of simple bending, we know that the bending stress,

$$\sigma_b = \frac{M.y}{I} = \frac{M}{Z} \qquad \ldots \left(\because Z = \frac{I}{y}\right)$$

$\therefore$ Mean or average bending stress,

$$\sigma_m = M_m / Z$$

and variable bending stress,

$$\sigma_v = M_v / Z$$

where M_m = Mean bending moment,

M_v = Variable bending moment, and

Z = Section modulus.

The equation ***(iii)*** may now be written as follows :

Working or design bending stress,

$$\sigma_b = \frac{M_m}{Z} + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times \frac{M_v}{Z}$$

$$= \frac{M_m + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times M_v}{Z}$$

$$= \frac{32}{\pi d^3}\left[M_m + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times M_v\right] \qquad \ldots \left(\because \text{For circular shafts, } Z = \frac{\pi}{32} \times d^3\right)$$

$$\therefore \quad F.S. = \frac{\sigma_y}{\frac{32}{\pi d^3}\left[M_m + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times M_v\right]}$$

3. *Simple torsion of circular shafts*

In case of simple torsion, we know that the torque,

$$T = \frac{\pi}{16} \times \tau \times d^3 \text{ or } \tau = \frac{16\,T}{\pi\, d^3}$$

$\therefore$ Mean or average shear stress,

$$\tau_m = \frac{16\,T_m}{\pi\, d^3}$$

and variable shear stress, $\tau_v = \dfrac{16\,T_v}{\pi\, d^3}$

where T_m = Mean or average torque,

T_v = Variable torque, and

d = Diameter of the shaft.

The equation ***(iii)*** may now be written as follows :

Working or design shear stress,

$$\tau = \frac{16\,T_m}{\pi d^3} + \left(\frac{\tau_y}{\tau_e}\right) K_{fs} \times \frac{16\,T_v}{\pi d^3} = \frac{16}{\pi d^3}\left[T_m + \left(\frac{\tau_y}{\tau_e}\right) K_{fs} \times T_v\right]$$

$$\therefore \qquad F.S. = \frac{\tau_y}{\dfrac{16}{\pi d^3}\left[T_m + \left(\dfrac{\tau_y}{\tau_e}\right) K_{fs} \times T_v\right]}$$

where K_{fs} = Fatigue stress concentration factor for torsional or shear loading.

Note : For shafts made of ductile material, $\tau_y = 0.5\ \sigma_y$, and $\tau_e = 0.5\ \sigma_e$ may be taken.

4. *Combined bending and torsion of circular shafts*

In case of combined bending and torsion of circular shafts, the maximum shear stress theory may be used. According to this theory, maximum shear stress,

$$\tau_{max} = \frac{\tau_y}{F.S.} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

$$= \frac{1}{2}\sqrt{\left[\frac{32}{\pi d^3}\left\{M_m + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times M_v\right\}\right]^2 + 4\left[\frac{16}{\pi d^3}\left\{T_m + \left(\frac{\tau_y}{\tau_e}\right) K_{fs} \times T_v\right\}\right]^2}$$

$$= \frac{16}{\pi d^3}\sqrt{\left[M_m + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times M_v\right]^2 + \left[T_m + \left(\frac{\tau_y}{\tau_e}\right) K_{fs} \times T_v\right]^2}$$

The majority of rotating shafts carry a steady torque and the loads remain fixed in space in both direction and magnitude. Thus during each revolution every fibre on the surface of the shaft undergoes a complete reversal of stress due to bending moment. Therefore for the usual case when $M_m = 0$, $M_v = M$, $T_m = T$ and $T_v = 0$, the above equation may be written as

$$\frac{\tau_y}{F.S.} = \frac{16}{\pi d^3}\sqrt{\left[\left(\frac{\sigma_y}{\sigma_e}\right) K_f \times M\right]^2 + T^2}$$

Note: The above relations apply to a solid shaft. For hollow shaft, the left hand side of the above equations must be multiplied by $(1 - k^4)$, where k is the ratio of inner diameter to outer diameter.

Example 6.15. *A centrifugal blower rotates at 600 r.p.m. A belt drive is used to connect the blower to a 15 kW and 1750 r.p.m. electric motor. The belt forces a torque of 250 N-m and a force of 2500 N on the shaft. Fig. 6.20 shows the location of bearings, the steps in the shaft and the plane in which the resultant belt force and torque act. The ratio of the journal diameter to the overhung shaft diameter is 1.2 and the radius of the fillet is 1/10th of overhung shaft diameter. Find the shaft diameter, journal diameter and radius of fillet to have a factor of safety 3. The blower shaft is to be machined from hot rolled steel having the following values of stresses:*

Endurance limit = 180 MPa; Yield point stress = 300 MPa; Ultimate tensile stress = 450 MPa.

Solution. Given: *N_B = 600 r.p.m. ; *P = 15 kW; *N_M = 1750 r.p.m. ; T = 250 N-m = 250 × 10^3 N-mm; F = 2500 N ; $F.S.$ = 3; σ_e = 180 MPa = 180 N/mm^2 ; σ_y = 300 MPa = 300 N/mm^2 ; σ_u = 450 MPa = 450 N/mm^2

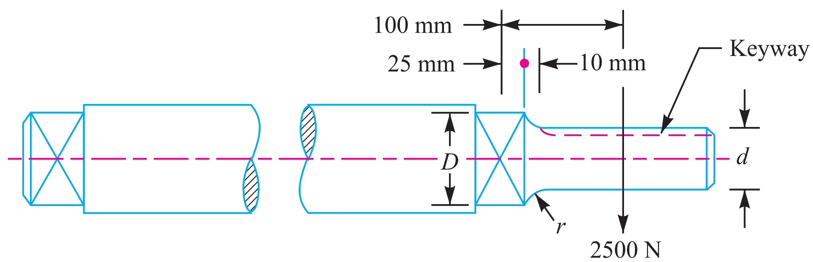

Fig. 6.20

Let D = Journal diameter,

d = Shaft diameter, and r = Fillet radius.

∴ Ratio of journal diameter to shaft diameter,

$$D/d = 1.2 \quad \text{...(Given)}$$

and radius of the fillet,

$$r = 1/10 \times \text{Shaft diameter } (d) = 0.1\, d$$

$$\therefore \quad r/d = 0.1 \quad \text{...(Given)}$$

From Table 6.3, for D/d = 1.2 and r/d = 0.1, the theoretical stress concentration factor,

$$K_t = 1.62$$

The two points at which failure may occur are at the end of the keyway and at the shoulder fillet. The critical section will be the one with larger product of $K_f \times M$. Since the notch sensitivity factor q is dependent upon the unknown dimensions of the notch and since the curves for notch sensitivity factor (Fig. 6.14) are not applicable to keyways, therefore the product $K_t \times M$ shall be the basis of comparison for the two sections.

∴ Bending moment at the end of the keyway,

$$K_t \times M = 1.6 \times 2500\,[100 - (25 + 10)] = 260 \times 10^3 \text{ N-mm}$$

$$\text{...}(\because K_t \text{ for key ways} = 1.6)$$

and bending moment at the shoulder fillet,

$$K_t \times M = 1.62 \times 2500\,(100 - 25) = 303\,750 \text{ N-mm}$$

Since $K_t \times M$ at the shoulder fillet is large, therefore considering the shoulder fillet as the critical section. We know that

$$\frac{\tau_y}{F.S.} = \frac{16}{\pi d^3}\sqrt{\left[\left(\frac{\sigma_y}{\sigma_e}\right) K_f \times M\right]^2 + T^2}$$

* Superfluous data

$$\frac{0.5 \times 300}{3} = \frac{16}{\pi d^3} \sqrt{\left[\left(\frac{300}{180} \times 303750\right)^2 + (250 \times 10^3)^2\right.}$$

... (Substituting, $\tau_y = 0.5\ \sigma_y$)

$$50 = \frac{16}{\pi d^3} \times 565 \times 10^3 = \frac{2877 \times 10^3}{d^3}$$

$\therefore$ $d^3 = 2877 \times 10^3/50 = 57\ 540$ or $d = 38.6$ say 40 mm **Ans.**

Note: Since r is known (because $r/d = 0.1$ or $r = 0.1d = 4$ mm), therefore from Fig. 6.14, the notch sensitivity factor (q) may be obtained. For $r = 4$ mm, we have $q = 0.93$.

$\therefore$ Fatigue stress concentration factor,

$$K_f = 1 + q\,(K_t - 1) = 1 + 0.93\,(1.62 - 1) = 1.58$$

Using this value of K_f instead of K_t, a new value of d may be calculated. We see that magnitudes of K_f and K_t are very close, therefore recalculation will not give any improvement in the results already obtained.

EXERCISES

1. A rectangular plate 50 mm × 10 mm with a hole 10 mm diameter is subjected to an axial load of 10 kN. Taking stress concentration into account, find the maximum stress induced. **[Ans. 50 MPa]**

2. A stepped shaft has maximum diameter 45 mm and minimum diameter 30 mm. The fillet radius is 6 mm. If the shaft is subjected to an axial load of 10 kN, find the maximum stress induced, taking stress concentration into account. **[Ans. 22 MPa]**

3. A leaf spring in an automobile is subjected to cyclic stresses. The average stress = 150 MPa; variable stress = 500 MPa; ultimate stress = 630 MPa; yield point stress = 350 MPa and endurance limit = 150 MPa. Estimate, under what factor of safety the spring is working, by Goodman and Soderberg formulae. **[Ans. 1.75, 1.3]**

4. Determine the design stress for bolts in a cylinder cover where the load is fluctuating due to gas pressure. The maximum load on the bolt is 50 kN and the minimum is 30 kN. The load is unpredictable and factor of safety is 3. The surface of the bolt is hot rolled and the surface finish factor is 0.9.

During a simple tension test and rotating beam test on ductile materials (40 C 8 steel annealed), the following results were obtained :

Diameter of specimen = 12.5 mm; Yield strength = 240 MPa; Ultimate strength = 450 MPa; Endurance limit = 180 MPa. **[Ans. 65.4 MPa]**

5. Determine the diameter of a tensile member of a circular cross-section. The following data is given :

Maximum tensile load = 10 kN; Maximum compressive load = 5 kN; Ultimate tensile strength = 600 MPa; Yield point = 380 MPa; Endurance limit = 290 MPa; Factor of safety = 4; Stress concentration factor = 2.2 **[Ans. 24 mm]**

6. Determine the size of a piston rod subjected to a total load of having cyclic fluctuations from 15 kN in compression to 25 kN in tension. The endurance limit is 360 MPa and yield strength is 400 MPa. Take impact factor = 1.25, factor of safety = 1.5, surface finish factor = 0.88 and stress concentration factor = 2.25. **[Ans. 35.3 mm]**

7. A steel connecting rod is subjected to a completely reversed axial load of 160 kN. Suggest the suitable diameter of the rod using a factor of safety 2. The ultimate tensile strength of the material is 1100 MPa, and yield strength 930 MPa. Neglect column action and the effect of stress concentration. **[Ans. 30.4 mm]**

8. Find the diameter of a shaft made of 37 Mn 2 steel having the ultimate tensile strength as 600 MPa and yield stress as 440 MPa. The shaft is subjected to completely reversed axial load of 200 kN. Neglect stress concentration factor and assume surface finish factor as 0.8. The factor of safety may be taken as 1.5. **[Ans. 51.7 mm]**

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9. Find the diameter of a shaft to transmit twisting moments varying from 800 N-m to 1600 N-m. The ultimate tensile strength for the material is 600 MPa and yield stress is 450 MPa. Assume the stress concentration factor = 1.2, surface finish factor = 0.8 and size factor = 0.85. **[Ans. 27.7 mm]**

10. A simply supported shaft between bearings carries a steady load of 10 kN at the centre. The length of shaft between bearings is 450 mm. Neglecting the effect of stress concentration, find the minimum diameter of shaft. Given that

    Endurance limit = 600 MPa; surface finish factor = 0.87; size factor = 0.85; and factor of safety = 1.6. **[Ans. 35 mm]**

11. Determine the diameter of a circular rod made of ductile material with a fatigue strength (complete stress reversal) σ_e = 280 MPa and a tensile yield strength of 350 MPa. The member is subjected to a varying axial load from 700 kN to – 300 kN. Assume K_t = 1.8 and *F.S.* = 2. **[Ans. 80 mm]**

12. A cold drawn steel rod of circular cross-section is subjected to a variable bending moment of 565 N-m to 1130 N-m as the axial load varies from 4500 N to 13 500 N. The maximum bending moment occurs at the same instant that the axial load is maximum. Determine the required diameter of the rod for a factor of safety 2. Neglect any stress concentration and column effect. Assume the following values:

Ultimate strength	=	550 MPa
Yield strength	=	470 MPa
Size factor	=	0.85
Surface finish factor	=	0.89
Correction factors	=	1.0 for bending
	=	0.7 for axial load

The endurance limit in reversed bending may be taken as one-half the ultimate strength. **[Ans. 41 mm]**

13. A steel cantilever beam, as shown in Fig. 6.21, is subjected to a transverse load at its end that varies from 45 N up to 135 N down as the axial load varies from 110 N (compression) to 450 N (tension). Determine the required diameter at the change of section for infinite life using a factor of safety of 2. The strength properties are as follows:

Ultimate strength	=	550 MPa
Yield strength	=	470 MPa
Endurance limit	=	275 MPa

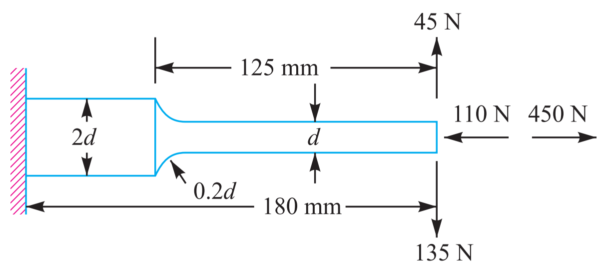

Fig. 6.21

The stress concentration factors for bending and axial loads are 1.44 and 1.63 respectively, at the change of cross-section. Take size factor = 0.85 and surface finish factor = 0.9. **[Ans. 12.5 mm]**

14. A steel shaft is subjected to completely reversed bending moment of 800 N-m and a cyclic twisting moment of 500 N-m which varies over a range of ± 40%. Determine the diameter of shaft if a reduction factor of 1.2 is applied to the variable component of bending stress and shearing stress. Assume

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(a) that the maximum bending and shearing stresses are in phase;

(b) that the tensile yield point is the limiting stress for steady state component;

(c) that the maximum shear strength theory can be applied; and

(d) that the Goodman relation is valid.

Take the following material properties:

Yield strength = 500 MPa ; Ultimate strength = 800 MPa ; Endurance limit = ± 400 MPa.

[Ans. 40 mm]

15. A pulley is keyed to a shaft midway between two anti-friction bearings. The bending moment at the pulley varies from – 170 N-m to 510 N-m and the torsional moment in the shaft varies from 55 N-m to 165 N-m. The frequency of the variation of the loads is the same as the shaft speed. The shaft is made of cold drawn steel having an ultimate strength of 540 MPa and a yield strength of 400 MPa. Determine the required diameter for an indefinite life. The stress concentration factor for the keyway in bending and torsion may be taken as 1.6 and 1.3 respectively. The factor of safety is 1.5. Take size factor = 0.85 and surface finish factor = 0.88. **[Ans. 36.5 mm]**

[**Hint.** Assume $\sigma_e = 0.5\ \sigma_u$; $\tau_y = 0.5\ \sigma_y$; $\tau_e = 0.55\ \sigma_e$]

QUESTIONS

1. Explain the following terms in connection with design of machine members subjected to variable loads:

(a) Endurance limit, (b) Size factor,

(c) Surface finish factor, and (d) Notch sensitivity.

2. What is meant by endurance strength of a material? How do the size and surface condition of a component and type of load affect such strength?

3. Write a note on the influence of various factors of the endurance limit of a ductile material.

4. What is meant by `stress concentration'? How do you take it into consideration in case of a component subjected to dynamic loading?

5. Illustrate how the stress concentration in a component can be reduced.

6. Explain how the factor of safety is determined under steady and varying loading by different methods.

7. Write Soderberg's equation and state its application to different type of loadings.

8. What information do you obtain from Soderberg diagram?

OBJECTIVE TYPE QUESTIONS

1. The stress which vary from a minimum value to a maximum value of the same nature (*i.e.* tensile or compressive) is called

(a) repeated stress (b) yield stress

(c) fluctuating stress (d) alternating stress

2. The endurance or fatigue limit is defined as the maximum value of the stress which a polished standard specimen can withstand without failure, for infinite number of cycles, when subjected to

(a) static load (b) dynamic load

(c) static as well as dynamic load (d) completely reversed load

3. Failure of a material is called fatigue when it fails

(a) at the elastic limit (b) below the elastic limit

(c) at the yield point (d) below the yield point

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4. The resistance to fatigue of a material is measured by
   (*a*) elastic limit (*b*) Young's modulus
   (*c*) ultimate tensile strength (*d*) endurance limit
5. The yield point in static loading is .... as compared to fatigue loading.
   (*a*) higher (*b*) lower (*c*) same
6. Factor of safety for fatigue loading is the ratio of
   (*a*) elastic limit to the working stress
   (*b*) Young's modulus to the ultimate tensile strength
   (*c*) endurance limit to the working stress
   (*d*) elastic limit to the yield point
7. When a material is subjected to fatigue loading, the ratio of the endurance limit to the ultimate tensile strength is
   (*a*) 0.20 (*b*) 0.35
   (*c*) 0.50 (*d*) 0.65
8. The ratio of endurance limit in shear to the endurance limit in flexure is
   (*a*) 0.25 (*b*) 0.40
   (*c*) 0.55 (*d*) 0.70
9. If the size of a standard specimen for a fatigue testing machine is increased, the endurance limit for the material will
   (*a*) have same value as that of standard specimen (*b*) increase (*c*) decrease
10. The residential compressive stress by way of surface treatment of a machine member subjected to fatigue loading
    (*a*) improves the fatigue life (*b*) deteriorates the fatigue life
    (*c*) does not affect the fatigue life (*d*) immediately fractures the specimen
11. The surface finish factor for a mirror polished material is
    (*a*) 0.45 (*b*) 0.65
    (*c*) 0.85 (*d*) 1
12. Stress concentration factor is defined as the ratio of
    (*a*) maximum stress to the endurance limit (*b*) nominal stress to the endurance limit
    (*c*) maximum stress to the nominal stress (*d*) nominal stress to the maximum stress
13. In static loading, stress concentration is more serious in
    (*a*) brittle materials (*b*) ductile materials
    (*c*) brittle as well as ductile materials (*d*) elastic materials
14. In cyclic loading, stress concentration is more serious in
    (*a*) brittle materials (*b*) ductile materials
    (*c*) brittle as well as ductile materials (*d*) elastic materials
15. The notch sensitivity q is expressed in terms of fatigue stress concentration factor K_f and theoretical stress concentration factor K_t, as

    (*a*) $\dfrac{K_f + 1}{K_t + 1}$ (*b*) $\dfrac{K_f - 1}{K_t - 1}$

    (*c*) $\dfrac{K_t + 1}{K_f + 1}$ (*d*) $\dfrac{K_t - 1}{K_f - 1}$

ANSWERS

1. (*c*)	**2.** (*d*)	**3.** (*d*)	**4.** (*d*)	**5.** (*a*)
6. (*c*)	**7.** (*c*)	**8.** (*c*)	**9.** (*c*)	**10.** (*a*)
11. (*d*)	**12.** (*c*)	**13.** (*a*)	**14.** (*b*)	**15.** (*b*)