CHAPTER 71
INVESTMENT ANALYSIS

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71.1 ESSENTIALS OF FINANCIAL ANALYSIS

71.1.1 Sources of Funding for Capital Expenditures

Engineering projects typically require the expenditure of funds for implementation and in return provide a savings or increased income to the firm. In this sense an engineering project is an investment for the firm and must be analyzed as an investment. This is true whether the project is a major new plant, a minor modification of some existing equipment, or anything in between. The extent of the analysis of course must be commensurate with the financial importance of the project. Financial analysis of an investment has two parts: funding of the investment and evaluation of the economics of the investment. Except for very large projects, such as a major plant expansion or addition, these two aspects can be analyzed independently. All projects generally draw from a common pool of capital funds rather than each project being financed separately. The engineering function may require an in-depth evaluation of the economics of a project, while the financing aspect generally is not dealt with in detail if at all. The primary reason for being concerned with the funding of projects is that the economic evaluation often requires at least an awareness if not an understanding of this function.

The funds used for capital expenditures come from two sources: debt financing and equity financing. Debt financing refers to funds that are borrowed from outside the company. The two common sources are bank loans and the sale of bonds. Bank loans are typically used for short-term financing, and bonds are used for long-term financing. Debt financing is characterized by a contractual arrangement specifying interest payments and repayment. The lender does not share in the profits of the investments for which the funds are used nor does it share the associated risks except through the possibility of the company defaulting. Equity financing refers to funds owned by the company. These funds may come from profits earned by the company or from funds set aside for depreciation allowances. Or, the funds may come from the sale of new stock. Equity financing does not require any specified repayment; however, the owners of the company (stockholders) do expect to make a reasonable return on their investment.

The decisions of how much funding to secure and the relative amounts to secure from debt and equity sources are very complicated and require considerable subjective judgment. The current stock market, interest rates, projections of future market conditions, etc., must be addressed. Generally, a company will try to maintain approximately a constant ratio of funding from the different sources. This mix will be selected to maximize earnings without jeopardizing the company's financial well

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being. However, the ratio of debt to equity financing does vary considerably from company to company reflecting different business philosophies.

71.1.2 The Time Value of Money

The time value of money is frequently referred to as interest or interest rate in economic analyses. Actually, the two are not exactly the same thing. Interest is a fee paid for borrowed funds and is established when the loan is made. The time value of money is related to interest rates, but it includes other factors also. The time value of money must reflect the cost of money. That is, it must reflect the interest that is paid on loans and bonds, and it must also reflect the dividends paid to the stockholders. The cost of money is usually determined as a weighted average of the interest rates and dividend rates paid for the different sources of funds used for capital expenditures. The time value of money must also reflect opportunity costs. The opportunity cost is the return that can be earned on available, but unused, projects.

In principle, the time value of money is the greater of the cost of money and the opportunity cost. The determination of the time value of money is difficult, and the reader is referred to advanced texts on this topic for a complete discussion (see, for example, Bussey[1]). The determination is usually made at the corporate level and is not the responsibility of engineers. The time value of money is frequently referred to as the interest rate for economic evaluations, and one should be aware that the terms interest rate and time value of money are used interchangeably. The time value of money is also referred to as the required rate of return.

Another factor that may or may not be reflected in a given time value of money is inflation. Inflation results in a decreased buying power of the dollar. Consequently, the cost of money generally is higher during periods of high inflation, since the funds are repaid in dollars less valuable than those in which the funds were obtained. Opportunity costs are not necessarily directly affected by inflation except that inflation affects the cash flows used in evaluating the returns for the projects on which the opportunity costs are based. It is usually up to the engineer to verify that inflation has been included in a specified time value of money, since this information is not normally given. In some applications it is beneficial to use an inflation-adjusted time value of money. The relationship is

$$1 + i_r = \frac{1 + i_a}{1 + f} \tag{71.1}$$

where i_a is the time value of money, which reflects the higher cost of money and the higher opportunity cost due to inflation; f is the inflation rate; and i_r is the inflation-adjusted time value of money, which actually reflects the true cost of capital in terms of constant value. The variables i_r and i_a may be referred to as the real and apparent time values of money, respectively. i_r, i_a, and f are all expressed in fractional rather than percentage form in Eq. (71.1) and all must be expressed on the same time basis, generally an annual rate. See Section 71.3 for additional discussion on the use of i_r and i_a. Also see Jones[2] for a detailed discussion.

The time value of money may also reflect risks associated with a project. This is particularly true for projects where there is a significant probability of poor return or even failure (e.g., the development of a new product). In principle, risk can be evaluated in assessing the economics of a project by including the probabilities of various outcomes (see Riggs,[3] for example). However, these calculations are complicated and are often dependent on subjective judgment. The more common approach is to simply use a time value of money which is greater for projects that are more risky. This is why some companies will use different values of i for different types of investments (e.g., expansion versus cost reduction versus diversification, etc.). Such adjustments for risk are usually made at the corporate level and are based on experience and other subjective inputs as much as they are on formal calculations. The engineer usually is not concerned with such adjustments, at least for routine economic analyses. If the risks of a project are included in an economic analysis, then it is important that the time value of money also not be adjusted for risks, since this would represent an overcompensation and would distort the true economic picture.

71.1.3 Discounted Cash Flow and Interest Calculations

For the purpose of economic analysis, a project is represented as a group of cash flows showing the expenditures and the income or savings attributable to the project. The object of economic analysis is normally to determine the profitability of the project based on these cash flows. However, the profitability cannot be assessed simply by summing up the cash flows, owing to the effect of the time value of money. The time value of money results in the value of a cash flow depending not only on its magnitude but also on when it occurs according to the equation

$$P = F \frac{1}{(1 + i)^n} \tag{71.2}$$

Table 71.1 Discounted Cash Flow Calculation

Year	Estimated Cash Flows for Project	Discounted Cash Flows[a]
0	−$120,000	−$120,000
1	− 75,000	− 68,200
2	+ 50,000	+ 41,300
3	+ 60,000	+ 45,100
4	+ 70,000	+ 47,800
5	+ 30,000	+ 18,000
6	+ 20,000	+ 11,300

[a]Based on i = 10%.

where F is a cash flow that occurs sometime in the future, P is the equivalent value of that cash flow now, i is the annual time value of money in fractional form, and n is the number of years from now when cash flow F occurs. Cash flow F is often referred to as the future value or future amount, while cash flow P is referred to as the present value or present amount. Equation (71.2) can be used to convert a set of cash flows for a project to a set of economically equivalent cash flows. These equivalent cash flows are referred to as discounted cash flows and reflect the reduced economic value of cash flows that occur in the future. Table 71.1 shows a set of cash flows that have been discounted.

Equation (71.2) is the basis for a more general principle referred to as economic equivalence. It shows the relative economic value of cash flows occurring at different points in time. It is not necessary that P refer to a cash flow that occurs at the present; rather, it simply refers to a cash flow that occurs n years before F. The equation works in either direction for computing equivalent cash flows. That is, it can be used to find a cash flow F that occurs n years after P and that is equivalent to P, or to find a cash flow P that is equivalent to F but which occurs n years before F. This principle of equivalence allows cash flows to be manipulated as needed to facilitate economic calculations.

The time value of money is usually specified as an annual rate. However, several other forms are sometimes encountered or may be required to solve a particular problem. An interest rate* as used in Eq. (71.2) is referred to as a discrete interest rate, since it specifies interest for a discrete time period of 1 year and allows calculations in multiples (n) of this time period. This time period is referred to as the compounding period. If it is necessary to change an interest rate stated for one compounding period to an equivalent interest rate for a different compounding period, it can be done by

$$i_1 = (1 + i_2)^{\Delta t_1/\Delta t_2} - 1 \qquad (71.3)$$

where Δt_1 and Δt_2 are compounding periods and i_1 and i_2 are the corresponding interest rates, respectively. Interest rates i_1 and i_2 are in fractional form. If an interest rate with a compounding period different than 1 year is used in Eq. (71.2) then n in that equation refers to the number of those compounding periods and not the number of years.

Interest may also be expressed in a nominal form. Nominal interest rates are frequently used to describe the interest associated with borrowing but are not used to express the time value of money. A nominal interest rate is stated as an annual interest rate but with a compounding period different from 1 year. A nominal rate must be converted to an equivalent compound interest rate before being used in calculations. The relationship between a nominal interest rate (i_n) and a compound interest rate (i_c) is $i_c = i_n/m$, where m is the number of compounding periods per year. The compounding period (Δt) for i_c is 1 year/m. For example, a 10% nominal interest rate compounded quarterly translates to a compound interest rate of 2.5% with a compounding period of ¼ year. Equation (71.3) may be used to convert the resulting interest rate to an equivalent interest rate with annual compounding. This later interest rate is referred to as the effective annual interest rate. For the 10% nominal interest above, the effective annual interest rate is 10.38%.

Interest may also be defined in continuous rather than discrete form. With continuous interest (sometimes referred to as continuous compounding), interest acrues continuously. Equation (71.2) can be rewritten for continuous interest as

*The term *interest rate* is used here instead of the time value of money. The results apply to interest associated with borrowing and interest in the context of the time value of money.

$$P = F \times e^{-rt} \tag{71.4}$$

where r is the continuous interest rate and has units of inverse time (but is normally expressed as a percentage per unit of time), t is the time between P and F, and e is the base of the natural logarithm. Note that the time units on r and t must be consistent with the year normally used. Discretely compounded interest may be converted to continuously compounded interest by

$$r = \frac{1}{\Delta t} \ln(1 + i)$$

and continuously compounded interest to discretely compounded interest by

$$i = e^{r\,\Delta t} - 1$$

Note that these are dimensional equations; the units for r and Δt must be consistent and the interest rates are in fractional form.

It is often desirable to manipulate groups of cash flows rather than just single cash flows. The principles used in Eq. (71.2) or (71.3) may be extended to multiple cash flows if they occur in a regular fashion or if they flow continuously over a period of time at some defined rate. The types of cash flows that can be readily manipulated are:

1. A uniform series in which cash flows occur in equal amounts on a regular periodic basis.
2. An exponentially increasing series in which cash flows occur on a regular periodic basis and increase by a constant percentage each year as compared to the previous year.
3. A gradient in which cash flows occur on a regular periodic basis and increase by a constant amount each year.
4. A uniform continuous cash flow where the cash flows at a constant rate over some period of time.
5. An exponentially increasing continuous cash flow where the cash flows continuously at an exponentially increasing rate.

These cash flows are illustrated in Figs. 71.1–71.5.

Any one of these groups of cash flows may be related to a single cash flow P as shown in these figures. The relationship between a group of cash flows and a single cash flow (or a single to single cash flow) may be reduced to an interest factor. The interest factors resulting in a single present amount are shown in Table 71.2. Derivations for most of these interest factors can be found in an introductory text on engineering economics (see, for example, Grant et al.[4]). The interest factor gives the relationship between the group of cash flows and the single present amount. For example,

$$P = A \cdot (P/A, i, n)$$

The term $(P/A, i, n)$ is referred to as the interest factor and gives the ratio of P to A and shows that it is a function of i and n. Other interest factors are used accordingly. The interest factors may be manipulated as if they are the mathematical ratio they represent. For example,

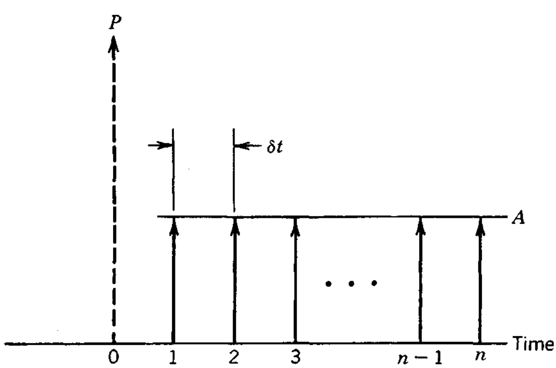

Fig. 71.1 Uniform series of cash flows.

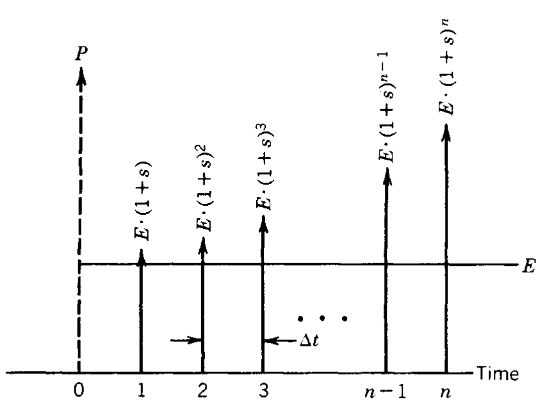

Fig. 71.2 Exponentially increasing series cash flows.

$$(A/P, i, n) = \frac{1}{(P/A, i, n)}$$

Thus for each interest factor represented in Table 71.2 a corresponding inverse interest factor may be generated as above. Two interest factors may be combined to generate a third interest factor in some cases. For example,

$$(F/A, i, n) = (P/A, i, n)(F/P, i, n)$$

or

$$(F/C, r, t) = (P/C, r, t)(F/P, r, t)$$

Again the interest factors are manipulated as the ratios they represent. In theory, any combination of interest factors may be used in this manner. However, it is wise not to mix interest factors using discrete interest with those using continuous interest, and it is usually best to proceed one step at a time to ensure that the end result is correct.

There are several important limitations when using the interest factors in Table 71.2. The time relationship between cash flows must be adhered to rigorously. Special attention must be paid to the time between P and the first cash flow. The time between the periodic cash flows must be equal to the compounding period when interest factors with discrete interest rates are used. If these do not match, Eq. (71.3) must be used to find an interest rate with the appropriate compounding period. It is also necessary to avoid dividing by zero. This is not usually a problem; however, it is possible

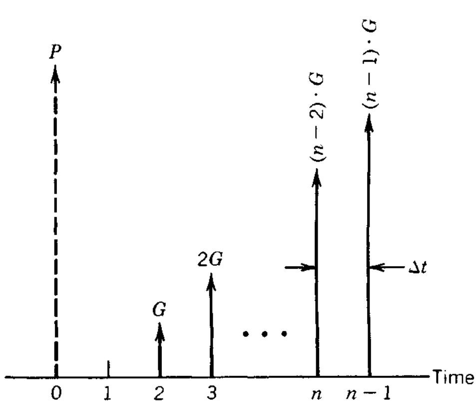

Fig. 71.3 Cash flow gradient.

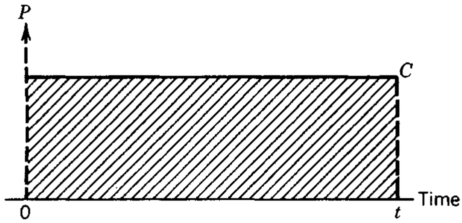

Fig. 71.4 Uniform continuous cash flow.

that $i = s$ or $r = a$, resulting in division by zero. The interest factors reduce to simpler forms in these special cases:

$$(P/F, i, s, n) = n, \qquad s = i$$
$$(P/C, r, a, t) = c \cdot t, \qquad a = r$$

It is sometimes necessary to deal with groups of cash flows that extend over very long periods of time ($n \to \infty$ or $t \to \infty$). The interest factors in this case reduce to simpler forms; but, limitations may exist if a finite value of P is to result. These reduced forms and limitations are presented in Table 71.3.

Several of the more common interest factors may be referred to by name rather than the notation used here. These interest factors and the corresponding names are presented in Table 71.4.

71.2 INVESTMENT DECISIONS

71.2.1 Allocation of Capital Funds

In most companies there are far more projects available than there are funds to implement them. It is necessary, then, to allocate these funds to the projects that provide the maximum return on the funds invested. The question of how to allocate capital funds will generally be handled at several different levels within the company, with the level at which the allocation is made depending on the size of the projects involved. At the top level, major projects such as plant additions or new product developments are considered. These may be multimillion or even multibillion dollar projects and have major impact on the future of the company. At this level both the decisions of which projects to fund and how much total capital to invest may be addressed simultaneously. At lower levels a capital budget may be established. Then, the projects that best utilize the funds available must be determined.

The basic principle utilized in capital rationing is illustrated in Fig. 71.6. Available projects are ranked in order of decreasing return. Those that are within the capital constraint are funded, those that are outside this constraint are not. However, it is not desirable to fund projects that have a rate of return less than the cost of money even if sufficient capital funds are available. If the size of individual projects is not small compared to the funds available, such as is the case with major investments, then it may be necessary to use linear programming techniques to determine the best set of projects to fund and the amount of funding to secure. As with most financial analyses, a fair amount of subjective judgment is also required. At the other extreme, the individual projects are

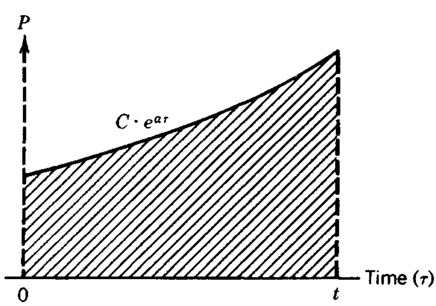

Fig. 71.5 Exponentially increasing continuous cash flow.

Table 71.2 Mathematical Expression of Interest Factors for Converting Cash Flows to Present Amounts[a]

Type of Cash Flows	Interest Factor	Mathematical Expression
Single	$(P/F, i, n)$	$= \frac{1}{(1+i)^n}$ $n = \frac{t_2 - t_1}{\Delta t}$
Single	$(P/F, r, t)$	$= e^{-rt}$ $t = t_2 - t_1$
Uniform series	$(P/A, i, n)$	$= \frac{1}{i}\left[1 - \frac{1}{(1+i)^n}\right]$ n = number of cash flows
Uniform series	$(P/A, r, n, \Delta t)$	$= \frac{1}{e^{r\Delta t} - 1}(1 - e^{-rn\Delta t})$ Δt = time between cash flows n = number of cash flows
Exponentially increasing series	$(P/E, i, s, n)$	$= \frac{1+s}{i-s}\left[1 - \left(\frac{1+s}{1+i}\right)^n\right]$ n = number of cash flows s = escalation rate
Exponentially increasing series	$(P/E, r, s, \Delta t, n)$	$= \frac{1+s}{(e^{r\Delta t} - s - 1)}\left[1 - \left(\frac{1+s}{e^{r\Delta t}}\right)^n\right]$ n = number of cash flows Δt = time between cash flows s = escalation rate
Gradient	$(P/G, i, n)$	$= \frac{1}{i}\left[\frac{(1+i)^n - 1}{i(1+i)^n} - \frac{n}{(1+i)^n}\right]$
Continuous	$(P/C, r, t)$	$= \frac{1 - e^{-rt}}{r}$ t = duration of cash flow
Continuous increasing exponentially	$(P/C, r, a, t)$	$= \frac{1 - e^{-(r-a)t}}{r-a}$ t = duration of cash flow a = rate of increase of cash flow

[a] See Figs. 71.1–71.5 for definitions of variables n, Δt, s, a, E, G, and C. Variables i, r, a, and s are in fractional rather than percentage form.

relatively small compared to the capital available. It is not practical to use the same optimization techniques for such a large number of projects; and little benefit is likely to be gained anyway. Rather, a cutoff rate of return (also called required rate of return or minimum attractive rate of return) is established. Projects with a return greater than this amount are considered for funding; projects with a return less than this amount are not. If the cutoff rate of return is selected appropriately, then the total funding required for the projects considered will be approximately equal to the funds available. The use of a required rate of return allows the analysis of routine projects to be evaluated without using sophisticated techniques. The required rate of return may be thought of as an opportunity cost, since there are presumably unfunded projects available that earn approximately this return. The required rate of return may be used as the time value of money for routine economic analyses, assuming it is greater than the cost of money. For further discussion of the allocation of capital see Grant et al.[4] or other texts on engineering economics.

71.2.2 Classification of Alternatives

From the engineering point of view, economic analysis provides a means of selecting among alternatives. These alternatives can be divided into three categories for the purpose of economic analysis: independent, mutually exclusive, and dependent.

Independent alternatives do not affect one another. Any one or any combination may be implemented. The decision to implement one alternative has no effect on the economics of another alter-

Table 71.3 Interest Factors for $n \to \infty$ or $t \to \infty$

Interest Factor	Limitation
$(P/A, i, n) = \dfrac{1}{i}$	$i > 0$
$(P/A, r, n, \Delta t) = \dfrac{1}{e^{r\,\Delta t} - 1}$	$r > 0$
$(P/E, i, s, n) = \dfrac{1 + s}{i - s}$	$s < i$
$(P/E, r, s, \Delta t, n) = \dfrac{1 + s}{e^{r\,\Delta t} - s - 1}$	$s < e^{r\,\Delta t} - 1$
$(P/G, i, n) = \left(\dfrac{1}{i}\right)^2$	$i > 0$
$(P/C, r, t) = \dfrac{1}{r}$	$r > 0$
$(P/C, r, a, t) = \dfrac{1}{r - a}$	$a < r$

native that is independent. For example, a company may wish to reduce the delivered cost of some heavy equipment it manufactures. Two possible alternatives might be to (1) add facilities for rail shipment of the equipment directly from the plant and (2) add facilities to manufacture some of the subassemblies in-house. Since these alternatives have no effect on one another, they are independent. Each independent alternative may be evaluated on its own merits. For routine purposes the necessary criterion for implementation is a net profit based on discounted cash flows or a return greater than the required rate of return.

Mutually exclusive alternatives are the opposite extreme from independent alternatives. Only one of a group of mutually exclusive alternatives may be selected, since implementing one eliminates the possibility of implementing any of the others. For example, some particular equipment in the field may be powered by a diesel engine, a gasoline engine, or an electric motor. These alternatives are mutually exclusive since once one is selected there is no reason to implement any of the others. Mutually exclusive alternatives cannot be evaluated separately but must be compared to each other. The single most profitable, or least costly, alternative as determined by using discounted cash flows is the most desirable from an economic point of view. The alternative with the highest rate of return is not necessarily the most desirable. The possibility of not implementing any of the alternatives should also be considered if it is a feasible alternative.

Dependent alternatives are like independent alternatives in that any one or any combination of a group of independent alternatives may be implemented. Unlike independent alternatives, the decision to implement one alternative will affect the economics of another, dependent alternative. For example, the expense for fuel for a heat-treating furnace might be reduced by insulating the furnace and by modifying the furnace to use a less costly fuel. Either one or both of these alternatives could be implemented. However, insulating the furnace reduces the amount of energy required by the furnace thus reducing the savings of switching to a less costly fuel. Likewise, switching to a less costly fuel reduces the energy cost thus reducing the savings obtained by insulating the furnace. These alternatives are dependent, since implementing one affects the economics of the other one. Not recognizing dependence between alternatives is a very common mistake in economic analysis. The dependence can occur in many ways, sometimes very subtly. Different projects may share costs or cause each

Table 71.4 Interest Factor Names

Factor	Name
$(F/P, i, n)$	Compound amount factor
$(P/F, i, n)$	Present worth factor
$(F/A, i, n)$	Series compound amount factor
$(P/A, i, n)$	Series present worth factor
$(A/P, i, n)$	Capital recovery factor
$(A/F, i, n)$	Sinking fund factor

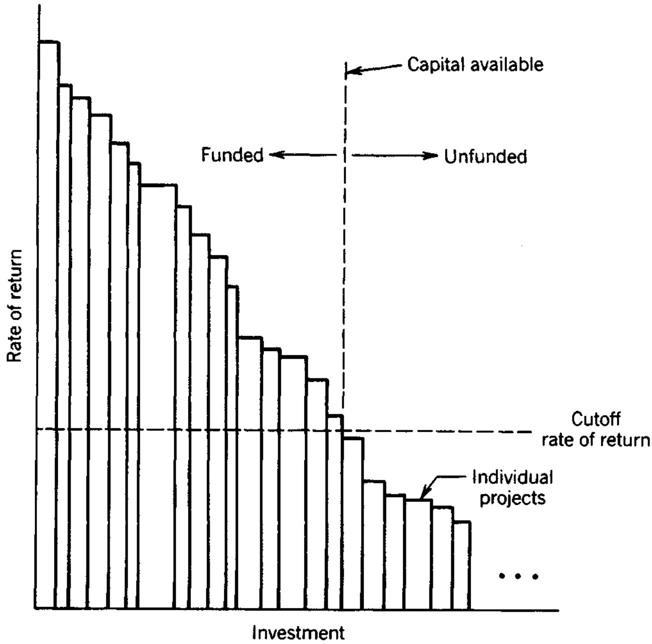

Fig. 71.6 Principle of capital allocation.

other to be more costly. They may interfere with each other or complement each other. Whenever a significant dependence between alternatives is identified, they should not be evaluated as being independent. The approach required to evaluate dependent alternatives is to evaluate all feasible combinations of the alternatives. The combination that provides the greatest profit, or least cost, when evaluated using discounted cash flows indicates which alternatives should be implemented and which should not. The number of possible combinations becomes very large quite quickly if very many dependent alternatives are to be evaluated. Therefore, an initial screening of combinations to eliminate obviously undesirable ones is useful.

Many engineering decisions do not deal with discrete alternatives but rather deal with one or more parameters that may vary over some range. Such situations result in continuous alternatives, and, in concept at least, an infinite number of possibilities exist but only one may be selected. For example, foam insulation may be sprayed on a storage tank. The thickness may vary from a few millimeters to several centimeters. The various thicknesses represent a continuous set of alternatives. The approach used to determine the most desirable value of the parameter for continuous alternatives is to evaluate the profit, or cost, using discounted cash flows for a number of different values to determine an optimum value. Graphical presentation of the results is often very helpful.

71.2.3 Analysis Period

An important part of an economic analysis is the determination of the appropriate analysis period. The concept of life-cycle analysis is used to establish the analysis period. Life-cycle analysis refers to an analysis period that extends over the life of the entire project including the implementation (e.g., development and construction). In many situations an engineering project addresses a particular need (e.g., transporting fluid from a storage tank to the processing plant). When it is clear that this need exists only for a specific period of time, then this period of need may establish the analysis period. Otherwise the life of the equipment involved will establish the project life.

Decisions regarding the selection, replacement, and modification of particular equipment and machinery are often a necessary part of many engineering functions. The same principles used for establishing an analysis period for the larger projects apply in this situation as well. If the lives of the equipment are greater than the period of need, then that period of need establishes the analysis period. If the lives of the equipment are short compared to the analysis period, then the equipment lives establish the analysis period. The lives of various equipment alternatives are often significantly different. The concept of life-cycle analysis requires that each alternative be evaluated over its full life. Fairness in the comparison requires that the same analysis period be applied to all alternatives that serve the same function. In order to resolve these two requirements more than one life cycle for the equipment may be used to establish an analysis period that includes an integer number of life cycles of each alternative. For example, if equipment with a life of 6 years is compared to equipment with a life of 4 years, an appropriate analysis period is 12 years, two life cycles of the first alternative and three life cycles of the second.

Obsolescence must also be considered when selecting an analysis period. Much equipment becomes uneconomical long before it wears out. The life of equipment, then, is not set by how long it can function but rather by how long it will be before it is desirable to upgrade with a newer design. Unfortunately, there is no simple method to determine when this time will be since obsolescence is due to new technology and new designs. In a few cases, it is clear that changes will soon occur (e.g., a new technology that has been developed but that is not yet in the marketplace). Such cases are the exception rather than the rule and much subjective judgment is required to estimate when something will become obsolete.

The requirements to maintain acceptable liquidity in a firm also may affect the selection of an analysis period. An investment is expected to return a net profit. The return may be a number of years after the initial investment, however. If a company is experiencing cash flow difficulties or anticipates that they may, this delay in receiving income may not be acceptable. A long-term profit is of little value to a company that becomes insolvent. In order to maintain liquidity, an upper limit may be placed on the time allowed for an investment to show a profit. The analysis period must be shortened then to reflect this requirement if necessary.

71.3 EVALUATION METHODS

71.3.1 Establishing Cash Flows

The first part of any economic evaluation is necessarily to determine the cash flows that appropriately describe the project. It is important that these cash flows represent all economic aspects of the project. All hidden cost (e.g., increased maintenance) or hidden benefits (e.g., reduced downtime) must be included as well as the obvious expenses, incomes, or savings associated with the project. Wherever possible, nonmonetary factors (e.g., reduced hazards) should be quantified and included in the analysis. Also, taxes associated with a project should not be ignored. (Some companies do allow a before-tax calculation for routine analysis.) Care should be taken that no factor be included twice. For example, high maintenance costs of an existing machine may be considered an expense in the alternative of keeping that machine or a savings in the alternative of replacing it with a new one, but it should not be considered both ways when comparing these two alternatives. Expenses or incomes that are irrelevant to the analysis should not be included. In particular, sunk costs, those expenses which have already been incurred, are not a factor in an economic analysis except for how they affect future cash flows (e.g., past equipment purchases may affect future taxes owing to depreciation allowances). The timing of cash flows over a project's life is also important, since cash flows in the near future will be discounted less than those that occur later. It is, however, customary to treat all of the cash flows that occur during a year as a single cash flow either at the end of the year (year-end convention) or at the middle of the year (mid-year convention).

Estimates of the cash flows for a project are generally determined by establishing what goods and services are going to be required to implement and sustain a project and by establishing the goods, services, benefits, savings, etc., that will result from the project. It is then necessary to estimate the associated prices, costs, or values. There are several sources of such information including historical data, projections, bids, or estimates by suppliers, etc. Care must be exercised when using any of these sources to be sure they accurately reflect the true price when the actual transaction will occur. Historical data are misleading, since they reflect past prices, not current prices, and may be badly in error owing to inflation that has occurred in recent years. Current prices may not accurately reflect future prices for the same reason. When historical data are used, they should be adjusted to reflect changes that have occurred in prices. This adjustment can be made by

$$p(t_0) = p(t_1)\,\frac{PI(t_0)}{PI(t_1)} \tag{71.5}$$

where $p(t)$ is the price, cost, or value of some item at time t; $PI(t)$ is the price index at time t, and t_0 is the present time. The price index reflects the change in prices for an item or group of items. Indexes for many categories of goods and services are available from the Bureau of Labor Statistics.[2] Current prices should also be adjusted when they refer to future transactions. Many companies have projections for prices for many of their more important products. Where such projections are not available, a relationship similar to Eq. (71.5) may be used except that price indexes for future years are not available. Estimates of future inflation rates may be substituted instead:

$$p(t_2) = p(t_0)(1 + f)^n \tag{71.6}$$

where f is the annual inflation rate, n is the number of years from the present until time t_2 ($t_2 - t_0$

in years), and t_0 is the present time. The inflation rate in Eq. (71.6) is the overall inflation rate unless it is expected that the particular item in question will increase in price much faster or slower than prices in general. In this case, an inflation rate pertaining to the particular item should be used.

Changing prices often distort the interpretation of cash flows. This distortion may be minimized by expressing all cash flows in a reference year's dollars (e.g., 1990 dollars). This representation is referred to as constant dollar cash flows. Historic data may be converted to constant dollar representation by

$$Y^c = Y^d \frac{\overline{PI}(t_0)}{\overline{PI}(t)} \tag{71.7}$$

where Y^c is the constant dollar representation of a dollar cash flow U^d, $\overline{PI}(t)$ is the value of the price index at time t, t_0 is the reference year, and t is the year in which Y^d occurred. An overall price index such as the Wholesale Price Index or the Gross National Product Implicit Price Deflator is used in this calculation, whereas a more specific price index is used in Eq. (71.5). Future cash flows may be expressed using constant dollar representation by

$$Y^c = Y^d \frac{1}{(1+f)^n} \tag{71.8}$$

where f is the projected annual inflation rate and n is the number of years after t_0 that Y^d occurs. It is usually convenient to let t_0 be the present. Then n is equal to the number of years from the present and, also, present prices may be used to make most constant dollar cash flow estimates. The use of constant dollar representation simplifies the economic analysis in many situations. However, it is important that the time value of money be adjusted as indicated in Eq. (71.2). Additional discussion on this topic may be found in Ref. 2.

Mutually exclusive alternatives and dependent alternatives often yield cash flows that are either all negative or predominantly negative, that is, they only deal with expenses. It is not possible to view each alternative in terms of an investment (initial expense) and a return (income). However, two alternatives may be used to create a set of cash flows that represent an investment and return as shown in Fig. 71.7. Alternative B is more expensive to implement than A but costs less to operate or sustain. Cash flow C is the difference between B and A. It shows the extra investment required for B and the savings it produces. Cash flow C may then be analyzed as an investment to determine

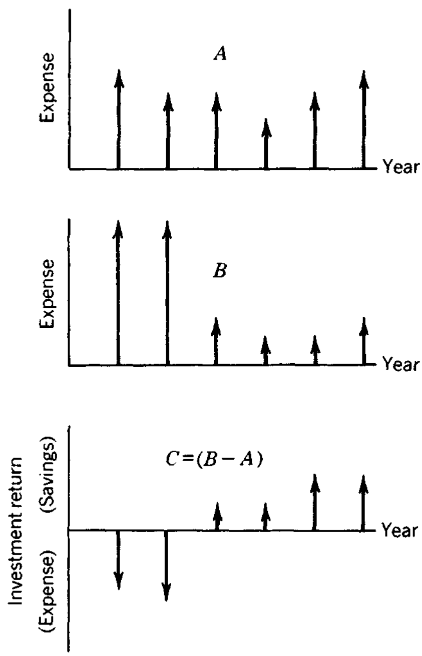

Fig. 71.7 Investment and return generated by comparing two alternatives.

if the extra investment required for C is worthwhile. This approach works well when only two alternatives are considered. If there are three or more alternatives, the comparison gets more complicated. Figure 71.8 shows the process required. Two alternatives are compared, the winner of that comparison is compared to a third, the winner of that comparison to a fourth, and so on until all alternatives have been considered and the single best alternative identified. When using this procedure, it is customary, but not necessary, to order the alternatives from least expensive to most expensive according to initial cost. This analysis of multiple alternatives may be referred to as incremental analysis, since only the difference, or incremental cash flow between alternatives, is considered. The same concept may be applied to continuous alternatives.

71.3.2 Present Worth

All of the cash flows for a project may be reduced to a single equivalent cash flow using the concepts of time value of money and cash flow equivalence. This single equivalent cash flow is usually calculated for the present time or the project initiation, hence the term present worth (also present value). However, this single cash flow can be calculated for any point in time if necessary. Occasionally, it is desired to calculate it for the end of a project rather than the beginning, the term future worth (also future value) is applied then. This single cash flow, either a present worth or future worth, is a measure of the net profit for the project and thus is an indication of whether or not the project is worthwhile economically. It may be calculated using interest factors and cash flow manipulations as described in preceding sections. Modern calculators and computers usually make it just as easy to calculate the present worth directly from the project cash flows. The present worth PW of a project is

$$PW = \sum_{j=0}^{n} Y_j \frac{1}{(1+i)^j} \tag{71.9}$$

where Y_j is the project cash flow in year j, n is the length of the analysis period, and i is the time value of money. This equation uses the sign convention of income or savings being positive and expenses being negative.

In the case of independent alternatives, the PW of a project is a sufficient measure of the project's profitability. Thus, a positive present worth indicates an economically desirable project and a negative present worth indicates an economically undesirable project. In the case of mutually exclusive, dependent, or continuous alternatives the present worth of a given alternative means little. The alternative, or combination of dependent alternatives, that has the highest present worth is the most desirable economically. Often cost is predominant in these alternatives. It is customary then to reverse the sign convention in Eq. (71.9) and call the result the present cost. The alternative with the smallest present cost is then the most desirable economically. It is also valid to calculate the present worth of the incremental cash flow (see Figs. 71.7 and 71.8) and use it as the basis for choosing between alternatives. However, the approach of calculating the PW or PC for each alternative is generally much easier. Regardless of the method chosen, it is important that all alternatives be treated fairly; use similar assumptions about future prices, use the same degree of conservatism in estimating expenses, make sure they all serve equally well, etc. In particular, be sure that the proper analysis period is selected when equipment lives differ.

Projects that have very long or indefinite lives may be evaluated using the present worth method. The present cost is referred to as capitalized cost in this application. The capitalized cost can be used

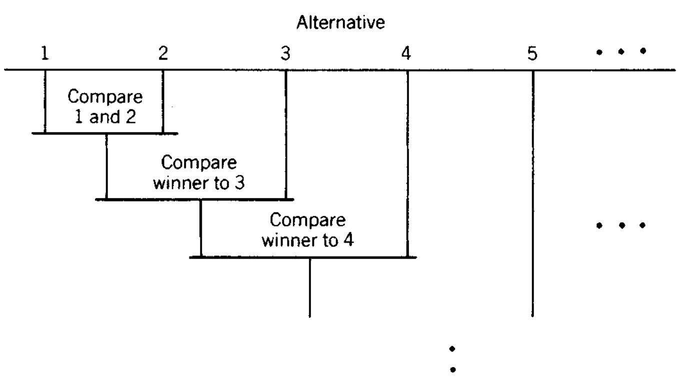

Fig. 71.8 Comparison of multiple alternatives using incremental cash flows.

for economic analysis in the same manner as the present cost; however, it cannot be calculated using Eq. (71.9), since the number of calculations required would be rather large as $n \to \infty$. The interest factors in Table 71.3 usually can be used to reduce the portion of the cash flows that continue indefinitely to a single equivalent amount, which can then be dealt with as any other single cash flow.

71.3.3 Annual Cash Flow

The annual cash flow method is very similar in concept to the present worth method and generally can be used whenever a present worth analysis can. The present worth or present cost of a project can be converted to an annual cash flow, ACF, by

$$ACF = PW \cdot (A/P, i, n) \tag{71.10}$$

where $(A/P, i, n)$ is the capital recovery factor, i is the time value of money, and n is the number of years in the analysis period. Since ACF is proportional to PW, a positive ACF indicates a profitable investment and a negative ACF indicates an unprofitable investment. Similarly, the alternative with the largest ACF will also have the largest PW. The PW in Eq. (71.10) can be replaced with PC, and ACF can be used to represent a cost when that is more appropriate. The ACF is thus equally as useful for economic analysis as PW or PC. It also has the advantage of having more intuitive meaning. ACF represents the equivalent annual income or cost over the life of a project.

Annual cash flow is particularly useful for analyses involving equipment with unequal lives. The n in Eq. (71.10) refers to the length of the analysis period and for unequal lives that means integer multiples of the life cycles. The annual cash flow for the analysis period will be the same as the analysis period for a single life cycle, as shown in Fig. 71.9, as long as the cash flows for the equipment repeat from one life cycle to the next. The annual cash flow for each equipment alternative can then be calculated for its own life cycle rather than for a number of life cycles. Unfortunately, prices generally increase from one life cycle to the next due to inflation and the cash flows from one life cycle to the next will be more like those shown in Fig. 71.10. The errors caused by this change from life cycle to life cycle usually will be acceptable if inflation is moderate (e.g., less than 5%) and the lives of various alternatives do not differ greatly (e.g., 7 versus 9 years). If inflation is high or if alternatives have lives that differ greatly, significant errors may result. The problem can often be circumvented by converting the cash flows to constant dollars using Eq. (71.8). With the inflationary price increases removed, the cash flows will usually repeat from one life cycle to the next.

71.3.4 Rate of Return

The rate of return (also called the internal rate of return) method is the most frequently used technique for evaluating investments. The rate of return is based on Eq. (71.9) except that rather than solving for PW, the PW is set to zero and the equation is solved for i. The resulting interest rate is the rate of return of the investment:

$$O = \sum_{j=0}^{n} Y_j \frac{1}{(1+i)^j} \tag{71.11}$$

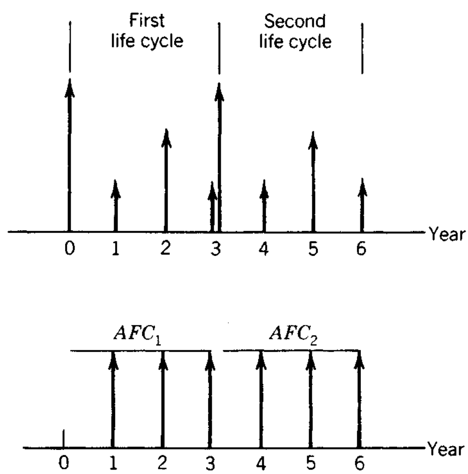

Fig. 71.9 Annual cash flow for equipment with repeating cash flows (3-year life).

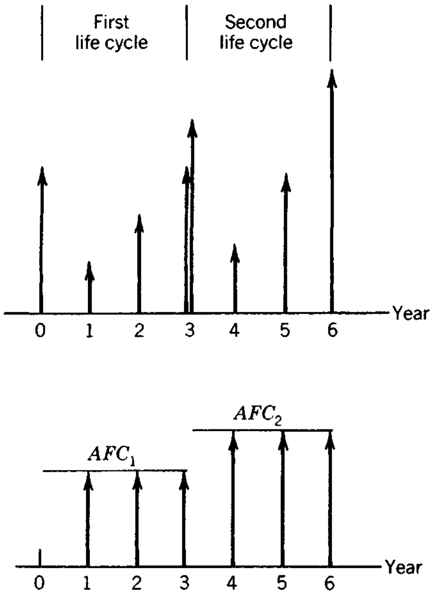

Fig. 71.10 Annual cash flow for equipment with nonrepeating cash flows (3-year life).

where Y_j is the cash flow for year j, i is the investment's rate of return (rather than the time value of money), and n is the length of the analysis period. It is usually necessary to solve Eq. (71.11) by trial and error, except for a few very simple situations. If constant dollar representation is used, the resulting rate of return is referred to as the real rate of return or the inflation-corrected rate of return. It may be converted to a dollar rate of return using Eq. (71.2).

A rate of return for an investment greater than the time value of money indicates that the investment is profitable and worthwhile economically. Likewise an investment with a rate of return less than the time value of money is unprofitable and is not worthwhile economically. The rate of return calculation is generally preferred over the present worth method or annual cash flow method by decision makers since it gives a readily understood economic measure. However, the rate of return method only allows a single investment to be evaluated or two projects compared using the incremental cash flow. When several mutually exclusive alternatives, dependent alternative combinations, or continuous alternatives exist, it is necessary to compare two investments at a time as shown in Fig. 71.8 using incremental cash flows. Present worth or annual cash flow methods are simpler to use in these instances. It is important to realize that with these types of decisions the alternative with the highest rate of return is not necessarily the preferable alternative.

The rate of return method is intended for use with classic investments as shown in Fig. 71.11. An expense (investment) is made initially and income (return) is generated in later years. If a particular set of cash flows, such as shown in Fig. 71.12, does not follow this pattern, it is possible that Eq. (71.11) will generate more than one solution. It is also very easy to misinterpret the results of such cash flows. Reference 4 explains how to proceed in evaluating cash flows of the nature shown in Fig. 71.12.

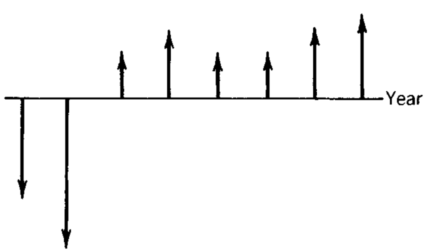

Fig. 71.11 Example of pure investment cash flows.

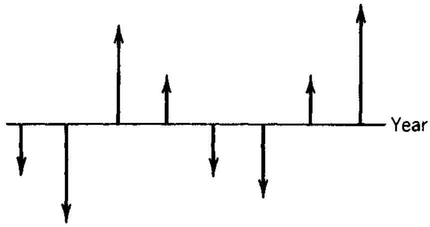

Fig. 71.12 Example of mixed cash flows.

71.3.5 Benefit–Cost Ratio

The benefit–cost ratio (B/C) calculation is a form of the present worth method. With B/C, Eq. (71.9) is used to calculate the present worth of the income or savings (benefits) of the investment and the expenses (costs) separately. These two quantities are then combined to form the benefit–cost ratio:

$$B/C = \frac{PW_B}{PC_E} \tag{71.12}$$

where PW_B is the present worth of the benefits and PC_E is the present cost of the expenses. A B/C greater than 1 indicates that the benefits outweigh the costs and a B/C less than 1 indicates the opposite. The B/C also gives some indication as to how good an investment is. A B/C of about 1 indicates a marginal investment, whereas a B/C of 3 or 4 indicates a very good one. The PC_E usually refers to the initial investment expense. The PW_B includes the income and savings less operating cost and other expenses. There is some leeway in deciding whether a particular expense should be included in PC_E or subtracted from PW_B. The placement will change B/C some, but will never make a B/C which is less than 1 become greater than 1 or vice versa.

The benefit–cost calculation can only be applied to a single investment or used to compare two investments using the incremental cash flow. When evaluating several mutually exclusive alternatives, dependent alternative combinations, or continuous alternatives, the alternatives must be compared two at a time as shown in Fig. 71.8 using incremental cash flows. The single alternative with the largest B/C is not necessarily the preferred alternative in this case.

71.3.6 Payback Period

The payback calculation is not a theoretically valid measure of the profitability of an investment and is frequently criticized for this reason. However, it is widely used and does provide useful information. The payback period is defined as the period of time required for the cumulative net cash flow to be equal to zero; that is, the time required for the income or savings to offset the initial costs and other expenses. The payback period does not measure the profitability of an investment but rather its liquidity. It shows how fast the money invested is recovered. It is a useful measure for a company experiencing cash flow difficulties and which cannot afford to tie up capital funds for a long period of time. A maximum allowed payback period may be specified in some cases. A short payback period is generally indicative of a very profitable investment, but that is not ensured since there is no accounting for the cash flows that occur after the payback period. Most engineering economists agree that the payback period should not be used as the means of selecting among alternatives.

The payback period is sometimes calculated using discounted cash flows rather than ordinary cash flows. This modification does not eliminate the criticisms of the payback calculation although it does usually result in only profitable investments having a finite payback period. A maximum allowed payback period may also be used with this form. This requirement is equivalent to arbitrarily shortening the analysis period to the allowed payback period to reflect liquidity requirements. Since there are different forms of the payback calculation and the method is not theoretically sound, extreme care should be exercised in using the payback period in decision making.

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