CHAPTER 36
STATISTICAL QUALITY CONTROL

Magd E. Zohdi
Department of Industrial Engineering and Manufacturing
Louisiana State University
Baton Rouge, Louisiana

36.1 MEASUREMENTS AND QUALITY CONTROL

The metric and English measuring systems are the two measuring systems commonly used throughout the world. The metric system is universally used in most scientific applications, but, for manufacturing in the United States, has been limited to a few specialties, mostly items that are related in some way to products manufactured abroad.

36.2 DIMENSION AND TOLERANCE

In dimensioning a drawing, the numbers placed in the dimension lines are only approximate and do not represent any degree of accuracy unless so stated by the designer. To specify the degree of accuracy, it is necessary to add tolerance figures to the dimension. Tolerance is the amount of variation permitted in the part or the total variation allowed in a given dimension.

Dimensions given close tolerances mean that the part must fit properly with some other part. Both must be given tolerances in keeping with the allowance desired, the manufacturing processes available, and the minimum cost of production and assembly that will maximize profit. Generally speaking, the cost of a part goes up as the tolerance is decreased.

Allowance, which is sometimes confused with tolerance, has an altogether different meaning. It is the minimum clearance space intended between mating parts and represents the condition of tightest permissible fit.

36.3 QUALITY CONTROL

When parts must be inspected in large numbers, 100% inspection of each part is not only slow and costly, but does not eliminate all of the defective pieces. Mass inspection tends to be careless; operators become fatigued; and inspection gages become worn or out of adjustment more frequently. The risk of passing defective parts is variable and of unknown magnitude, whereas, in a planned sampling procedure, the risk can be calculated. Many products, such as bulbs, cannot be 100%

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inspected, since any final test made on one results in the destruction of the product. Inspection is costly and nothing is added to a product that has been produced to specifications.

Quality control enables an inspector to sample the parts being produced in a mathematical manner and to determine whether or not the entire stream of production is acceptable, provided that the company is willing to allow up to a certain known number of defective parts. This number of acceptable defectives is usually taken as 3 out of 1000 parts produced. Other values might be used.

36.3.1 $\bar{X}$, R, and σ Charts

To use quality techniques in inspection, the following steps must be taken (see Table 36.1).

1. Sample the stream of products by taking m samples, each of size n.
2. Measure the desired dimension in the sample, mainly the central tendency.
3. Calculate the deviations of the dimensions.
4. Construct a control chart.
5. Plot succeeding data on the control chart.

The arithmetic mean of the set of n units is the main measure of central tendency. The symbol $\bar{X}$ is used to designate the arithmetic mean of the sample and may be expressed in algebraic terms as

$$\bar{X}_i = (X_1 + X_2 + X_3 + \ldots + X_n)/n \tag{36.1}$$

where X_1, X_2, X_3, etc. represent the specific dimensions in question. The most useful measure of dispersion of a set of numbers is the standard deviation σ. It is defined as the root–mean–square deviation of the observed numbers from their arithmetic mean. The standard deviation σ is expressed in algebraic terms as

$$\sigma_i = \sqrt{\frac{(X_1 - \bar{X})^2 + (X_2 - \bar{X})^2 + \ldots + (X_n - \bar{X})^2}{n}} \tag{36.2}$$

Another important measure of dispersion, used particularly in control charts, is the range R. The range is the difference between the largest observed value and the smallest observed in a specific sample.

$$R = X_i(\max) - X_i(\min) \tag{36.3}$$

Even though the distribution of the X values in the universe can be of any shape, the distribution of the $\bar{X}$ values tends to be close to the normal distribution. The larger the sample size and the more nearly normal the universe, the closer will the frequency distribution of the average $\bar{X}$'s approach the normal curve, as in Fig. 36.1.

According to the statistical theory, (the Central Limit Theory) in the long run, the average of the $\bar{X}$ values will be the same as μ, the average of the universe. And in the long run, the standard deviation of the frequency distribution $\bar{X}$ values, $\sigma_{\bar{x}}$, will be given by

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \tag{36.4}$$

where σ is the standard deviation of the universe. To construct the control limits, the following steps are taken:

Table 36.1 Computational Format for Determining $\bar{X}$, R, and σ

Sample Number	Sample Values	Mean $\bar{X}$	Range R	Standard Deviation σ'
1	$X_{11}, X_{12}, \ldots, X_{1n}$	$\bar{X}_1$	R_1	σ'_1
2	$X_{21}, X_{22}, \ldots, X_{2n}$	$\bar{X}_2$	R_2	σ'_2
.	...	.	.	.
.	...	.	.	.
.	...	.	.	.
m	$X_{m1}, X_{m2}, \ldots, X_{mn}$	$\bar{X}_m$	R_m	σ'_m

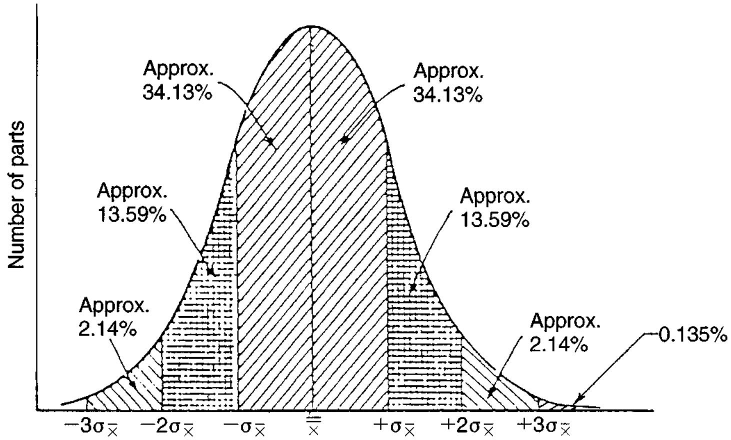

Fig. 36.1 Normal distribution and percentage of parts that will fall within σ limits.

1. Calculate the average of the average $\bar{\bar{X}}$ as follows:

$$\bar{\bar{X}} = \sum_{1}^{m} \bar{X}_i/m \qquad i = 1, 2, \ldots, m \tag{36.5}$$

2. Calculate the average deviation, $\bar{\sigma}$ where

$$\bar{\sigma} = \sum_{1}^{m} \sigma_i'/m \qquad i = 1, 2, \ldots, m \tag{36.6}$$

Statistical theory predicts the relationship between $\bar{\sigma}$ and $\sigma_{\bar{x}}$. The relationship for the $3\sigma_{\bar{x}}$ limits or the 99.73% limits is

$$A_1\bar{\sigma} = 3\sigma_{\bar{x}} \tag{36.7}$$

This means that control limits are set so that only 0.27% of the produced units will fall outside the limits. The value of $3\sigma_{\bar{x}}$ is an arbitrary limit that has found acceptance in industry.

The value of A_1 calculated by probability theory is dependent on the sample size and is given in Table 36.2. The formula for 3σ control limits using this factor is

$$CL(\bar{X}) = \bar{\bar{X}} \pm A_1\bar{\sigma} \tag{36.8}$$

Once the control chart (Fig. 36.2) has been established, data ($\bar{X}_i$'s) that result from samples of the same size n are recorded on it. It becomes a record of the variation of the inspected dimensions over a period of time. The data plotted should fall in random fashion between the control limits 99.73% of the time if a stable pattern of variation exists.

So long as the points fall between the control lines, no adjustments or changes in the process are necessary. If five to seven consecutive points fall on one side of the mean, the process should be checked. When points fall outside of the control lines, the cause must be located and corrected immediately.

Statistical theory also gives the expected relationship between $\bar{R}$ ($\Sigma R_i/m$) and $\sigma_{\bar{x}}$. The relationship for the $3\sigma_{\bar{x}}$ limits is

$$A_2\bar{R} = 3\sigma_{\bar{x}} \tag{36.9}$$

The values for A_2 calculated by probability theory, for different sample sizes, are given in Table 36.2. The formula for 3σ control limits using this factor is

$$CL(\bar{X}) = \bar{\bar{X}} \pm A_2\bar{R} \tag{36.10}$$

In control chart work, the ease of calculating R is usually much more important than any slight

Table 36.2 Factors for $\overline{X}$, R, σ, and X Control Charts

	Factors for $\overline{X}$ Chart		Factors for R Chart		Factors for σ' Chart		Factors for X Chart		$\sigma = \overline{R}/d_2$
Sample Size n	From $\overline{R}$ A_2	From $\overline{\sigma}$ A_1	Lower D_3	Upper D_4	Lower B_3	Upper B_4	From $\overline{R}$ E_2	From $\overline{\sigma}$ E_1	d_2
2	1.880	3.759	0	3.268	0	3.267	2.660	5.318	1.128
3	1.023	2.394	0	2.574	0	2.568	1.772	4.146	1.693
4	0.729	1.880	0	2.282	0	2.266	1.457	3.760	2.059
5	0.577	1.596	0	2.114	0	2.089	1.290	3.568	2.326
6	0.483	1.410	0	2.004	0.030	1.970	1.184	3.454	2.539
7	0.419	1.277	0.076	1.924	0.118	1.882	1.109	3.378	2.704
8	0.373	1.175	0.136	1.864	0.185	1.815	1.054	3.323	2.847
9	0.337	1.094	0.184	1.816	0.239	1.761	1.011	3.283	2.970
10	0.308	1.028	0.223	1.777	0.284	1.716	0.975	3.251	3.078
11	0.285	0.973	0.256	1.744	0.321	1.679	0.946	3.226	3.173
12	0.266	0.925	0.284	1.717	0.354	1.646	0.921	3.205	3.258
13	0.249	0.884	0.308	1.692	0.382	1.618	0.899	3.188	3.336
14	0.235	0.848	0.329	1.671	0.406	1.594	0.881	3.174	3.407
15	0.223	0.817	0.348	1.652	0.428	1.572	0.864	3.161	3.472
16	0.212	0.788	0.364	1.636	0.448	1.552	0.848	3.152	3.532
17	0.203	0.762	0.380	1.621	0.466	1.534	0.830	3.145	3.588
18	0.194	0.738	0.393	1.608	0.482	1.518	0.820	3.137	3.640
19	0.187	0.717	0.404	1.597	0.497	1.503	0.810	3.130	3.687
20	0.180	0.698	0.414	1.586	0.510	1.490	0.805	3.122	3.735
21	0.173	0.680	0.425	1.575	0.523	1.477	0.792	3.114	3.778
22	0.167	0.662	0.434	1.566	0.534	1.466	0.783	3.105	3.819
23	0.162	0.647	0.443	1.557	0.545	1.455	0.776	3.099	3.858
24	0.157	0.632	0.451	1.548	0.555	1.445	0.769	3.096	3.895
25	0.153	0.619	0.459	1.540	0.565	1.435	0.765	3.095	3.931

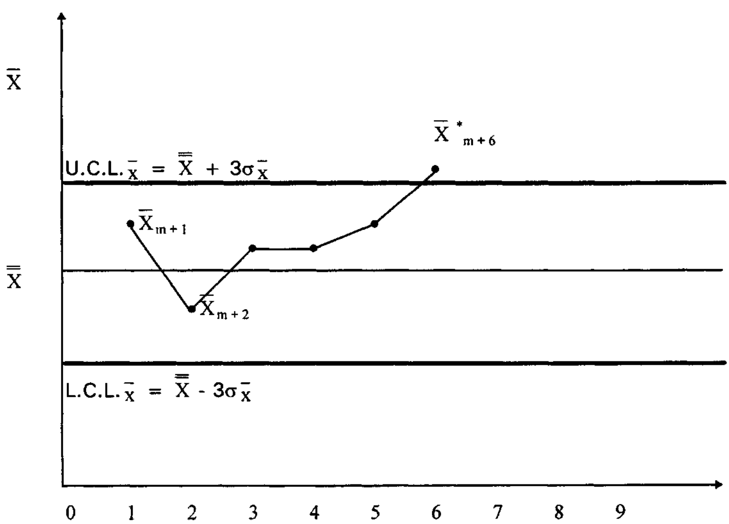

Fig. 36.2 Control chart $\overline{X}$.

theoretical advantage that might come from the use of σ. However, in some cases where the measurements are costly and it is necessary that the inferences from a limited number of tests be as reliable as possible, the extra cost of calculating σ is justified. It should be noted that, because Fig. 36.2 shows the averages rather than individual values, it would have been misleading to indicate the tolerance limits on this chart. It is the individual article that has to meet the tolerances, not the average of a sample. Tolerance limits should be compared to the machine capability limits. Capability limits are the limits on a single unit and can be calculated by

$$\text{capability limits} = \bar{\bar{X}} \pm 3\sigma \qquad (36.11)$$
$$\sigma = \bar{R}/d_2$$

Since $\sigma' = \sqrt{n}\,\sigma_{\bar{x}}$, the capability limits can be given by

$$\text{capability limits } (X) = \bar{\bar{X}} \pm 3\sqrt{n}\,\sigma_{\bar{x}} \qquad (36.12)$$
$$= \bar{\bar{X}} \pm E_1\bar{\sigma} \qquad (36.13)$$
$$= \bar{\bar{X}} \pm E_2\bar{R} \qquad (36.14)$$

The values for d_2, E_1, and E_2 calculated by probability theory, for different sample sizes, are given in Table 36.2.

Figure 36.3 shows the relationships among the control limits, the capability limits, and assumed tolerance limits for a machine that is capable of producing the product with this specified tolerance. Capability limits indicate that the production facility can produce 99.73% of its products within these limits. If the specified tolerance limits are greater than the capability limits, the production facility is capable of meeting the production requirement. If the specified tolerance limits are tighter than the capability limits, a certain percentage of the production will not be usable and 100% inspection will be required to detect the products outside the tolerance limits.

To detect changes in the dispersion of the process, the R and σ charts are often employed with $\bar{X}$ and X charts.

The upper and lower control limits for the R chart are specified as

$$UCL\,(R) = D_4\bar{R} \qquad (36.15)$$
$$LCL\,(R) = D_3\bar{R} \qquad (36.16)$$

Figure 36.4 shows the $\bar{R}$ chart for samples of size 5.

The upper and lower control for the T chart are specified as

$$UCL\,(\sigma) = B_4\bar{\sigma} \qquad (36.17)$$
$$LCL\,(\sigma) = B_3\bar{\sigma} \qquad (36.18)$$

The values for D_3, D_4, B_3, and B_4 calculated by probability theory, for different sample sizes, are given in Table 36.2.

36.4 INTERRELATIONSHIP OF TOLERANCES OF ASSEMBLED PRODUCTS

Mathematical statistics states that the dimension on an assembled product may be the sum of the dimensions of the several parts that make up the product. It states also that the standard deviation of the sum of any number of independent variables is the square root of the sum of the squares of the standard deviations of the independent variables. So if

$$X = X_1 \pm X_2 \pm \ldots X_n \qquad (36.19)$$
$$\bar{X} = \bar{X}_1 \pm \bar{X}_2 \pm \ldots \bar{X}_n \qquad (36.20)$$
$$\sigma(X) = \sqrt{(\sigma_1)^2 + (\sigma_2)^2 + \ldots + (\sigma_n)^2} \qquad (36.21)$$

Whenever it is reasonable to assume that the tolerance ranges of the parts are proportional to their respective σ' values, such tolerance ranges may be combined by taking the square root of the sum of the squares:

$$T = \sqrt{T_1^2 + T_2^2 + T_3^2 + \ldots + T_n^2} \qquad (36.22)$$

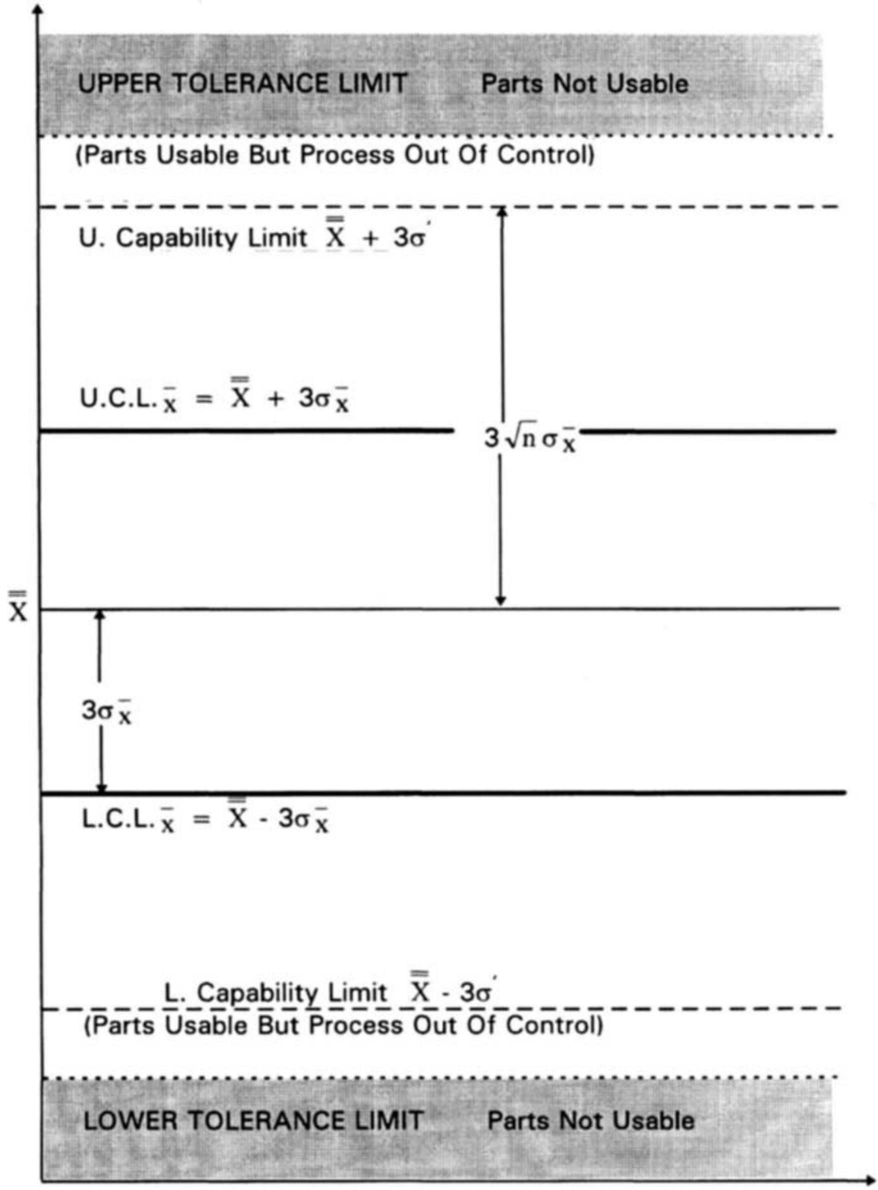

Fig. 36.3 Control, capability, and tolerance (specification limits).

36.5 OPERATION CHARACTERISTIC CURVE (OC)

Control charts detect changes in a pattern of variation. If the chart indicates that a change has occurred when it has not, Type I error occurs. If three-sigma limits are used, the probability of making a Type I error is approximately 0.0027.

The probability of the chart indicating no change, when in fact it has, is the probability of making a Type II error. The operation characteristic curves are designed to indicate the probability of making a Type II error. An OC curve for an $\bar{X}$ chart of three-sigma limits is illustrated in Fig. 36.5.

36.6 CONTROL CHARTS FOR ATTRIBUTES

Testing may yield only one of two defined classes: within or outside certain limits, acceptable or defective, working or idle. In such a classification system, the proportion of units falling in one class may be monitored with a p chart.

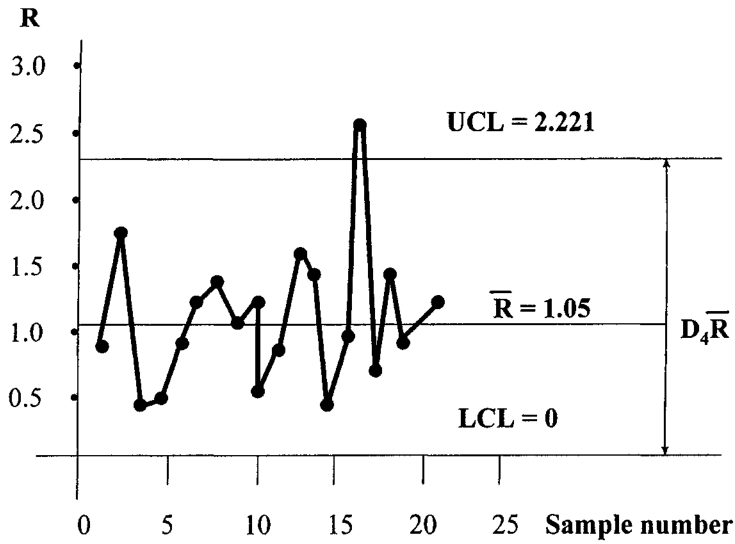

Fig. 36.4 *R* Chart for samples of 5 each.

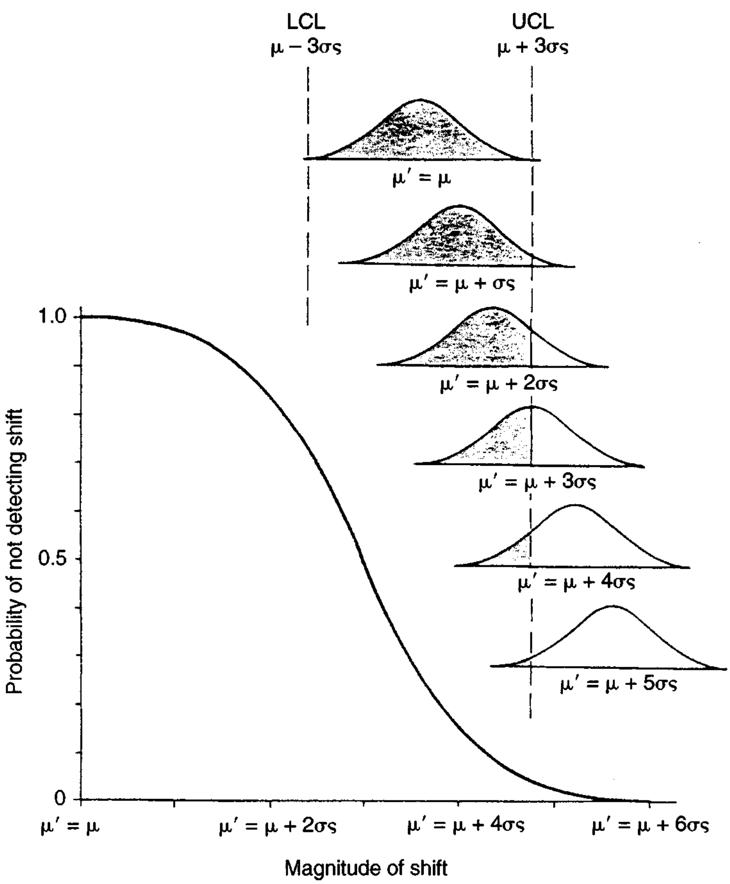

Fig. 36.5 Operating characteristic curve for 3σ limit.

In other cases, observation may yield a multivalued, but still discrete, classification system. In such case, the number of discrete observations, such as events, objects, states, or occurrences, may be monitored by a c chart.

36.6.1 The *p* and *np* Charts

When sampled items are tested and placed into one of two defined classes, the proportion of units falling into one class p is described by the binomial distribution. The mean and standard deviation are given as

$$\mu = np$$
$$\sigma = \sqrt{np(1 - p)}$$

Dividing by the sample size n, the parameters are expressing as proportions. These statistics can be expressed as

$$\bar{p} = \frac{\text{total number in the class}}{\text{total number of observations}} \tag{36.23}$$

$$s_p = \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}} \tag{36.24}$$

The control limits are either set at two sigma limits with Type I error as 0.0456 or at three-sigma limits with Type I error as 0.0027. The control limits for the p chart with two-sigma limits (Fig. 36.6) are defined as

$$CL(p) = \bar{p} \pm 2S_p \tag{36.25}$$

However, if subgroup size is constant, the chart for actual numbers of rejects np or pn may be used. The appropriate model for three-sigma control limits on an np chart is

$$CL(np) = n\bar{p} \pm 3\sqrt{n\bar{p}(1 - \bar{p})} \tag{36.26}$$

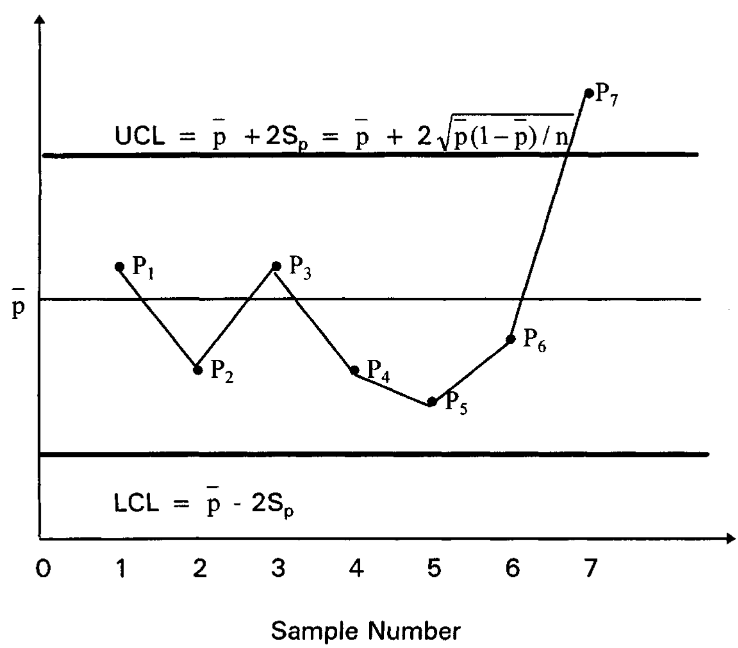

Fig. 36.6 *P* charts.

36.6.2 The c and u Charts

The random variable process that provides numerical data that are recorded as a number c rather than a proportion p is described by the Poisson distribution. The mean and the variance of the Poisson distribution are equal and expressed as $\mu = \sigma^2 = np$. The Poisson distribution is applicable in any situation when n and p cannot be determined separately, but their product np can be established. The mean and variable can be estimated as

$$\bar{c} = S_c^2 = \frac{\sum_{l}^{m} C_i}{m} = \frac{\sum_{1}^{m} (np)_i}{m} \tag{36.27}$$

The control limits (Fig. 36.7) are defined as

$$CL(c) = \bar{C} \pm 3S_c \tag{36.28}$$

If there is change in the area of opportunity for occurrence of a nonconformity from subgroup to subgroup, such as number of units inspected or the lengths of wires checked, the conventional c chart showing only the total number of nonconformities is not applicable. To create some standard measure of the area of opportunity, the nonconformities per unit (c/n) or u is used as the control statistic. The control limits are

$$CL(u)\bar{u} \pm 3\frac{\sqrt{\bar{u}}}{\sqrt{n_i}} \tag{36.29}$$

$$\text{where } \bar{u} = \frac{\sum C_i}{\sum n_i} = \frac{\text{total nonconformities found}}{\text{total units inspected}}$$

$c = nu$ is Poisson-distributed, u is not

36.7 ACCEPTANCE SAMPLING

The objective of acceptance sampling is to determine whether a quantity of the output of a process is acceptable according to some criterion of quality. A sample from the lot is inspected and the lot is accepted or rejected in accordance with the findings of the sample.

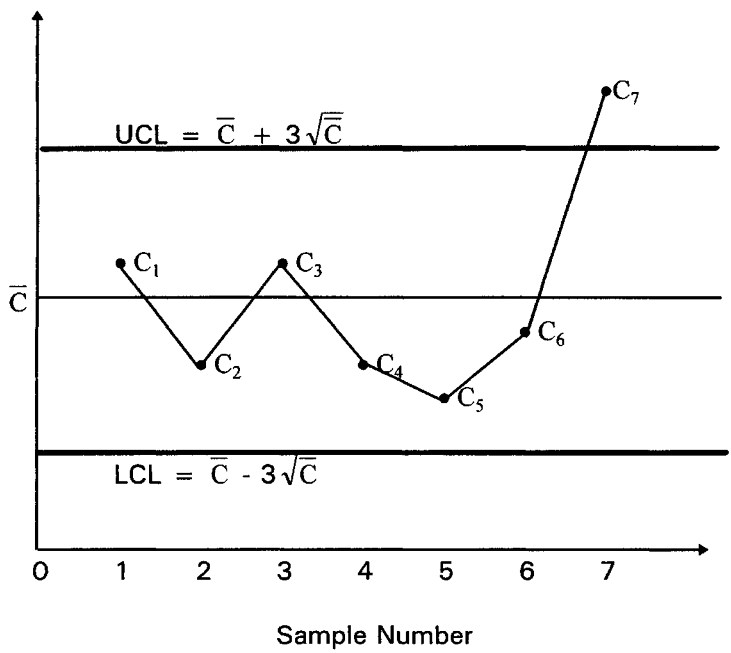

Fig. 36.7 *C* charts.

Acceptance sampling plans call for the random selection of sample of size n from a lot containing N items. The lot is accepted if the number of defectives found in the sample $\leq c$, the acceptance number. A rejected lot can either be returned to the producer, nonrectifying inspections, or it can be retained and subjected to a 100% screening process, rectifying inspection plan improves the outgoing quality. A second attribute-inspection plan might use two samples before requiring the acceptance or rejection of a lot. A third plan might use multiple samples or a sequential sampling process in evaluating a lot. Under rectifying inspection programs, the average outgoing quality level (AOQ), the average inspection lot (I), and the average outgoing quality limit (AOQL) can be predicted for varying levels of incoming fraction defective p.

Assuming that all lots arriving contain the same proportion of defectives p, and that rejected lots will be subjected to 100% inspection, AOQ and I are given below:

$$\text{AOQ} = \frac{P_a p(N - n)}{N - pn - (1 - P_a)p(Nn)} \tag{36.30}$$

$$I = n + (1 - P_a)(N - n) \tag{36.31}$$

The average outgoing quality (AOQ) increases as the proportion defective in incoming lots increases until it reaches a maximum value and then starts to decrease. This maximum value is referred to as the average outgoing quality limit (AOQL). The hypergeometric distribution is the appropriate distribution to calculate the probability of acceptance P_a; however, the Poisson distribution is used as an approximation.

Nonrectifying inspection program does not significantly improve the quality level of the lots inspected.

36.7.1 Double Sampling

Double sampling involves the possibility of putting off the decision on the lot until a second sample has been taken. A lot may be accepted at once if the first sample is good enough or rejected at once if the first sample is bad enough. If the first sample is neither, the decision is based on the evidence of the first and second samples combined.

The symbols used in double sampling are

N = lot size

n_1 = first sample

c_1 = acceptance number for first sample

n_2 = second sample

c_2 = acceptance number of the two samples combined

Computer programs are used to calculate the OC curves; acceptance after the first sample, rejection after the first sample, acceptance after the second sample, and rejection after the second sample.

The average sample number (ASN) in double sampling is given by

$$ASN = [P_a(n_1) + P_r(n_1)]n_1 + [P_a(n_2) + P_r(n_2)](n_1 + n_2) \tag{36.32}$$

36.7.2 Multiple and Sequential Sampling

In multiple sampling, three or more samples of a stated size are permitted and the decision on acceptance or rejection is revealed after a stated number of samples.

In sequential sampling, item-by-item inspection, a decision is possible after each item has been inspected and when there is no specified limit on the total number of units to be inspected.

OC curves are developed through computer programs. The advantage of using double sampling, multiple sampling, or sequential sampling is to reach the appropriate decision with fewer items inspected.

36.8 DEFENSE DEPARTMENT ACCEPTANCE SAMPLING BY VARIABLES

MIL-STD-105 A, B, C, D, and then ABC-STD-105, are based on the Acceptance Quality Level (AQL) concept. The plans contain single, double, or multiple sampling, depending on the lot size and AQL and the probability of acceptance at this level P_a. Criteria for shifting to tightened inspection, requalification for normal inspection, and reduced inspection are listed in the tables associated with plan.

MIL-STD-414 plans were developed to reduce inspection lots by using sample sizes compared to MIL-STD-105. They are similar, as both procedures and tables are based on the concept of AQL; lot-by-lot acceptance inspection; both provide for normal, tightened, or reduced inspection; sample sizes are greatly influenced by lot size; several inspection levels are available; and all plans are identified by sample size code letter. MIL-STD-414 could be applied either with a single specification limit, L or U, or with two specification limits. Known-sigma plans included in the standard were designated as having "variability known." Unknown-sigma plans were designated as having "variability unknown." In the latter-type plans, it was possible to use either the standard deviation method or the range method in estimating the lot variability.

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