CHAPTER 21

LUBRICATION OF MACHINE ELEMENTS

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By the middle of this century two distinct regimes of lubrication were generally recognized. The first of these was hydrodynamic lubrication. The development of the understanding of this lubrication regime began with the classical experiments of Tower,[1] in which the existence of a film was detected from measurements of pressure within the lubricant, and of Petrov,[2] who reached the same conclusion from friction measurements. This work was closely followed by Reynolds' celebrated analytical paper[3] in which he used a reduced form of the Navier–Stokes equations in association with the continuity equation to generate a second-order differential equation for the pressure in the narrow, converging gap of a bearing contact. Such a pressure enables a load to be transmitted between the surfaces with very low friction since the surfaces are completely separated by a film of fluid. In such a situation it is the physical properties of the lubricant, notably the dynamic viscosity, that dictate the behavior of the contact.

The second lubrication regime clearly recognized by 1950 was boundary lubrication. The understanding of this lubrication regime is normally attributed to Hardy and Doubleday,[4,5] who found that very thin films adhering to surfaces were often sufficient to assist relative sliding. They concluded that under such circumstances the chemical composition of the fluid is important, and they introduced the term "boundary lubrication." Boundary lubrication is at the opposite end of the lubrication

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spectrum from hydrodynamic lubrication. In boundary lubrication it is the physical and chemical properties of thin films of molecular proportions and the surfaces to which they are attached that determine contact behavior. The lubricant viscosity is not an influential parameter.

In the last 30 years research has been devoted to a better understanding and more precise definition of other lubrication regimes between these extremes. One such lubrication regime occurs in nonconformal contacts, where the pressures are high and the bearing surfaces deform elastically. In this situation the viscosity of the lubricant may rise considerably, and this further assists the formation of an effective fluid film. A lubricated contact in which such effects are to be found is said to be operating elastohydrodynamically. Significant progress has been made in our understanding of the mechanism of elastohydrodynamic lubrication, generally viewed as reaching maturity.

This chapter describes briefly the science of these three lubrication regimes (hydrodynamic, elastohydrodynamic, and boundary) and then demonstrates how this science is used in the design of machine elements.

SYMBOLS

A_p	total projected pad area, m^2
a_b	groove width ratio
a_f	bearing-pad load coefficient
B	total conformity of ball bearing
b	semiminor axis of contact, m; width of pad, m
$\bar{b}$	length ratio, b_s/b_r
b_g	length of feed groove region, m
b_r	length of ridge region, m
b_s	length of step region, m
C	dynamic load capacity, N
C_l	load coefficient, F/p_aRl
c	radial clearance of journal bearing, m
c'	pivot circle clearance, m
c_b	bearing clearance at pad minimum film thickness (Fig. 21.16), m
c_d	orifice discharge coefficient
D	distance between race curvature centers, m
$\tilde{D}$	material factor
D_x	diameter of contact ellipse along x axis, m
D_y	diameter of contact ellipse along y axis, m
d	diameter of rolling element or diameter of journal, m
d_a	overall diameter of ball bearing (Fig. 21.76), m
d_b	bore diameter of ball bearing, m
d_c	diameter of capillary tube, m
d_i	inner-race diameter of ball bearing, m
d_o	outer-race diameter of ball bearing, m
$\bar{d}_o$	diameter of orifice, m
E	modulus of elasticity, N/m^2
E'	effective elastic modulus, $2\left(\dfrac{1-\nu_a^2}{E_a}+\dfrac{1-\nu_b^2}{E_b}\right)^{-1}$, N/m^2
$\tilde{E}$	metallurgical processing factor
$\mathcal{E}$	elliptic integral of second kind
e	eccentricity of journal bearing, m
F	applied normal load, N
F'	load per unit length, N/m
$\tilde{F}$	lubrication factor
$\mathcal{F}$	elliptic integral of first kind
F_c	pad load component along line of centers (Fig. 21.41), N
F_e	rolling-element-bearing equivalent load, N
F_r	applied radial load, N
F_s	pad load component normal to line of centers (Fig. 21.41), N
F_t	applied thrust load, N

f race conformity ratio
f_c coefficient dependent on materials and rolling-element bearing type (Table 21.19)
G dimensionless materials parameter
$\tilde{G}$ speed effect factor
G_f groove factor
g_e dimensionless elasticity parameter, $W^{8/3}/U^2$
g_v dimensionless viscosity parameter, GW^3/U^2
H dimensionless film thickness, h/R_x
$\tilde{H}$ misalignment factor
H_a dimensionless film thickness ratio, h_s/h_r
H_b pad pumping power, N m/sec
H_c power consumed in friction per pad, W
H_f pad power coefficient
$H_{\min}$ dimensionless minimum film thickness, $h_{\min}/R_x$
$\hat{H}_{\min}$ dimensionless minimum film thickness, $H_{\min}(W/U)^2$
H_p dimensionless pivot film thickness, h_p/c
H_t dimensionless trailing-edge film thickness, h_t/c
h film thickness, m
$\bar{h}_i$ film thickness ratio, h_i/h_o
h_i inlet film thickness, m
h_l leading-edge film thickness, m
$h_{\min}$ minimum film thickness, m
h_o outlet film thickness, m
h_p film thickness at pivot, m
h_r film thickness in ridge region, m
h_s film thickness in step region, m
h_t film thickness at trailing edge, m
h_0 film constant, m
J number of stress cycles
K load deflection constant
$\bar{K}$ dimensionless stiffness coefficient, cK_p/p_aRl
K_a dimensionless stiffness, $-c\ \partial\bar{W}/\partial c$
K_p film stiffness, N/m
K_1 load-deflection constant for a roller bearing
$K_{1.5}$ load-deflection constant for a ball bearing
$\bar{K}_\infty$ dimensionless stiffness, cK_p/p_aRl
k ellipticity parameter, D_y/D_x
k_c capillary tube constant, m^3
k_o orifice constant, $m^4/N^{1/2}$ sec
L fatigue life
L_a adjusted fatigue life
L_{10} fatigue life where 90% of bearing population will endure
L_{50} fatigue life where 50% of bearing population will endure
l bearing length, m
l_c length of capillary tube, m
l_r roller effective length, m
l_t roller length, m
l_v length dimension in stress volume, m
l_1 total axial length of groove, m
M probability of failure
$\bar{M}$ stability parameter, $\bar{m}p_ah_r^5/2R^5/\eta^2$
m number of rows of rolling elements
$\bar{m}$ mass supported by bearing, N sec^2/m

m_p	preload factor
N	rotational speed, rps
N_R	Reynolds number
n	number of rolling elements or number of pads or grooves
P	dimensionless pressure, p/E'
P_d	diametral clearance, m
P_e	free endplay, m
p	pressure, N/m^2
p_a	ambient pressure, N/m^2
p_l	lift pressure, N/m^2
$p_{\max}$	maximum pressure, N/m^2
p_r	recess pressure, N/m^2
p_s	bearing supply pressure, N/m^2
Q	volume flow of lubricant, m^3/sec
$\overline{Q}$	dimensionless flow, $3\eta Q/\pi p_a h_r^3$
Q_c	volume flow of lubricant in capillary, m^3/sec
Q_o	volume flow of lubricant in orifice, m^3/sec
Q_s	volume side flow of lubricant, m^3/sec
q	constant, $\pi/2 - 1$
q_f	bearing-pad flow coefficient
R	curvature sum on shaft or bearing radius, m
$\overline{R}$	groove length fraction, $(R_o - R_g)/(R_o - R_i)$
R_g	groove radius (Fig. 21.60), m
R_o	orifice radius, m
R_x	effective radius in x direction, m
R_y	effective radius in y direction, m
R_1	outer radius of sector thrust bearing, m
R_2	inner radius of sector thrust bearing, m
r	race curvature radius, m
r_c	roller corner radius, m
S	probability of survival
Sm	Sommerfeld number for journal bearings, $\eta N d^3 l/2Fc^2$
Sm_t	Sommerfeld number for thrust bearings, $\eta u b l^2/F h_0^2$
s	shoulder height, m
T	tangential force, N
$\overline{T}$	dimensionless torque, $6\,T_r/\pi p_a/(R_1^2 + R_2^2)\,h_r \Lambda_c$
T_c	critical temperature
T_r	torque, N m
U	dimensionless speed parameter, $u\eta_0/E'R_x$
u	mean surface velocity in direction of motion, m/sec
v	elementary volume, m^3
N	dimensionless load parameter, $F/E'R_x^2$
$\overline{W}$	dimensionless load capacity, $F/p_a l(b_r + b_s + b_g)$
$\overline{W}_\infty$	dimensionless load, $1.5G_f F/\pi p_a(R_1^2 - R_2^2)$
X, Y	factors for calculation of equivalent load
x,y,z	coordinate system
$\overline{x}$	distance from inlet edge of pad to pivot, m
α	radius ratio, R_y/R_x
α_a	offset factor
α_b	groove width ratio, $b_s/(b_r + b_s)$
α_p	angular extent of pad, deg
α_r	radius ratio, R_2/R_1
β	contact angle, deg

β'	iterated value of contact angle, deg
β_a	groove angle, deg
β_f	free or initial contact angle, deg
β_p	angle between load direction and pivot, deg
Γ	curvature difference
γ	groove length ratio, l_1/l
Δ	rms surface finish, m
δ	total elastic deformation, m
ϵ	eccentricity ratio, e/c
η	absolute viscosity of lubricant, N sec/m^2
η_k	kinematic viscosity, ν/ρ, m^2/sec
η_0	viscosity at atmospheric pressure, N sec/m^2
θ	angle used to define shoulder height, deg
$\overline{\theta}$	dimensionless step location, $\theta_i/(\theta_i + \theta_o)$
θ_g	angular extent of lubrication feed groove, deg
θ_i	angular extent of ridge region, deg
θ_o	angular extent of step region, deg
Λ	film parameter (ratio of minimum film thickness to composite surface roughness)
Λ_c	dimensionless bearing number, $3\eta\omega(R_1^2 - R_2^2)/p_a h_r^2$
Λ_j	dimensionless bearing number, $6\eta\omega R^2/p_a c^2$
Λ_t	dimensionless bearing number, $6\eta u l/p_a h_r^2$
λ	length-to-width ratio
λ_a	length ratio, $(b_r + b_s + b_g)/l$
λ_b	$(1 + 2/3\alpha)^{-1}$
μ	coefficient of friction, T/F
ν	Poisson's ratio
ξ	pressure–viscosity coefficient of lubricant, m^2/N
ξ_p	angle between line of centers and pad leading edge, deg
ρ	lubricant density, N sec^2/m^4
ρ_0	density at atmospheric pressure, N sec^2/m^4
σ_{max}	maximum Hertzian stress, N/m^2
τ	shear stress, N/m^2
τ_0	maximum shear stress, N/m^2
ϕ	attitude angle in journal bearings, deg
ϕ_p	angle between pad leading edge and pivot, deg
ψ	angular location, deg
ψ_t	angular limit of ψ, deg
ψ_s	step location parameter, $b_s/(b_r + b_s + b_g)$
ω	angular velocity, rad/sec
ω_B	angular velocity of rolling-element race contact, rad/sec
ω_b	angular velocity of rolling element about its own center, rad/sec
ω_c	angular velocity of rolling element about shaft center, rad/sec
ω_d	rotor whirl frequency, rad/sec
$\overline{\omega}_d$	whirl frequency ratio, ω_d/ω_j
ω_j	journal rotational speed, rad/sec

Subscripts

a	solid a
b	solid b
EHL	elastohydrodynamic lubrication
e	elastic
HL	hydrodynamic lubrication
i	inner

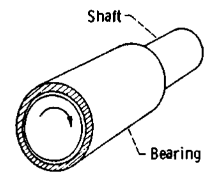

Fig. 21.1 Conformal surfaces. (From Ref. 6.)

iv isoviscous
o outer
pv piezoviscous
r rigid
x,y,z coordinate system

21.1 LUBRICATION FUNDAMENTALS

A lubricant is any substance that is used to reduce friction and wear and to provide smooth running and a satisfactory life for machine elements. Most lubricants are liquids (like minerals oils, the synthetic esters and silicone fluids, and water), but they may be solids (such as polytetrafluorethylene) for use in dry bearings, or gases (such as air) for use in gas bearings. An understanding of the physical and chemical interactions between the lubricant and the tribological surfaces is necessary if the machine elements are to be provided with satisfactory life. To help in the understanding of this tribological behavior, the first section describes some lubrication fundamentals.

21.1.1 Conformal and Nonconformal Surfaces

Hydrodynamic lubrication is generally characterized by surfaces that are conformal; that is, the surfaces fit snugly into each other with a high degree of geometrical conformity (as shown in Fig. 21.1), so that the load is carried over a relatively large area. Furthermore, the load-carrying surface remains essentially constant while the load is increased. Fluid-film journal bearings (as shown in Fig. 21.1) and slider bearings exhibit conformal surfaces. In journal bearings the radial clearance between the shaft and bearing is typically one-thousandth of the shaft diameter; in slider bearings the inclination of the bearing surface to the runner is typically one part in a thousand. These converging surfaces, coupled with the fact that there is relative motion and a viscous fluid separating the surfaces, enable a positive pressure to be developed and exhibit a capacity to support a normal applied load. The magnitude of the pressure developed *is not* generally large enough to cause significant elastic deformation of the surfaces. The minimum film thickness in a hydrodynamically lubricated bearing is a function of applied load, speed, lubricant viscosity, and geometry. The relationship between the minimum film thickness $h_{\min}$ and the speed u and applied normal load F is given as

$$(h_{\min})_{\mathrm{HL}} \propto \left(\frac{u}{F}\right)^{1/2} \tag{21.1}$$

More coverage of hydrodynamic lubrication can be found in Section 21.2.

Many machine elements have contacting surfaces that *do not* conform to each other very well, as shown in Fig. 21.2 for a rolling-element bearing. The full burden of the load must then be carried by a very small contact area. In general, the contact areas between nonconformal surfaces enlarge considerably with increasing load, but they are still smaller than the contact areas between conformal

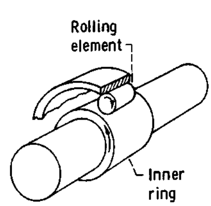

Fig. 21.2 Nonconformal surfaces. (From Ref. 6.)

surfaces. Some examples of nonconformal surfaces are mating gear teeth, cams and followers, and rolling-element bearings (as shown in Fig. 21.2). The mode of lubrication normally found in these nonconformal contacts is elastohydrodynamic lubrication. The requirements necessary for hydrodynamic lubrication (converging surfaces, relative motion, and viscous fluid) are also required for elastohydrodynamic lubrication.

The relationship between the minimum film thickness and normal applied load and speed for an elastohydrodynamically lubricated contact is

$$(h_{\min})_{\mathrm{EHL}} \propto F^{-0.073} \tag{21.2}$$

$$(h_{\min})_{\mathrm{EHL}} \propto u^{0.68} \tag{21.3}$$

Comparing the results of Eqs. (21.2) and (21.3) with that obtained for hydrodynamic lubrication expressed in Eq. (21.1) indicates that:

1. The exponent on the normal applied load is nearly seven times larger for hydrodynamic lubrication than for elastohydrodynamic lubrication. This implies that in elastohydrodynamic lubrication the film thickness is only slightly affected by load while in hydrodynamic lubrication it is significantly affected by load.
2. The exponent on mean velocity is slightly higher for elastohydrodynamic lubrication than that found for hydrodynamic lubrication.

More discussion of elastohydrodynamic lubrication can be found in Section 21.3.

The load per unit area in conformal bearings is relatively low, typically averaging only 1 $\mathrm{MN/m^2}$ and seldom over 7 $\mathrm{MN/m^2}$. By contrast, the load per unit area in nonconformal contacts will generally exceed 700 $\mathrm{MN/m^2}$ even at modest applied loads. These high pressures result in elastic deformation of the bearing materials such that elliptical contact areas are formed for oil-film generation and load support. The significance of the high contact pressures is that they result in a considerable increase in fluid viscosity. Inasmuch as viscosity is a measure of a fluid's resistance to flow, this increase greatly enhances the lubricant's ability to support load without being squeezed out of the contact zone. The high contact pressures in nonconforming surfaces therefore result in both an elastic deformation of the surfaces and large increases in the fluid's viscosity. The minimum film thickness is a function of the parameters found for hydrodynamic lubrication with the addition of an effective modulus of elasticity parameter for the bearing materials and a pressure–viscosity coefficient for the lubricant.

21.1.2 Bearing Selection

Ball bearings are used in many kinds of machines and devices with rotating parts. The designer is often confronted with decisions on whether a nonconformal bearing such as a rolling-element bearing or a conformal bearing such as a hydrodynamic bearing should be used in a particular application. The following characteristics make rolling-element bearings *more desirable* than hydrodynamic bearings in many situations:

1. Low starting and good operating friction
2. The ability to support combined radial and thrust loads
3. Less sensitivity to interruptions in lubrication
4. No self-excited instabilities
5. Good low-temperature starting

Within reasonable limits changes in load, speed, and operating temperature have but little effect on the satisfactory performance of rolling-element bearings.

The following characteristics make nonconformal bearings such as rolling-element bearings *less desirable* than conformal (hydrodynamic) bearings:

1. Finite fatigue life subject to wide fluctuations
2. Large space required in the radial direction
3. Low damping capacity
4. High noise level
5. More severe alignment requirements
6. Higher cost

Each type of bearing has its particular strong points, and care should be taken in choosing the most appropriate type of bearing for a given application.

The Engineering Services Data Unit documents[7,8] provide an excellent guide to the selection of the type of journal or thrust bearing most likely to give the required performance when considering the load, speed, and geometry of the bearing. The following types of bearings were considered:

1. Rubbing bearings, where the two bearing surfaces rub together (e.g., unlubricated bushings made from materials based on nylon, polytetrafluoroethylene, also known as PTFE, and carbon).
2. Oil-impregnated porous metal bearings, where a porous metal bushing is impregnated with lubricant and thus gives a self-lubricating effect (as in sintered-iron and sintered-bronze bearings).
3. Rolling-element bearings, where relative motion is facilitated by interposing rolling elements between stationary and moving components (as in ball, roller, and needle bearings).
4. Hydrodynamic film bearings, where the surfaces in relative motion are kept apart by pressures generated hydrodynamically in the lubricant film.

Figure 21.3, reproduced from the Engineering Sciences Data Unit publication,[7] gives a guide to the typical load that can be carried at various speeds, for a nominal life of 10,000 hr at room temperature, by journal bearings of various types on shafts of the diameters quoted. The heavy curves

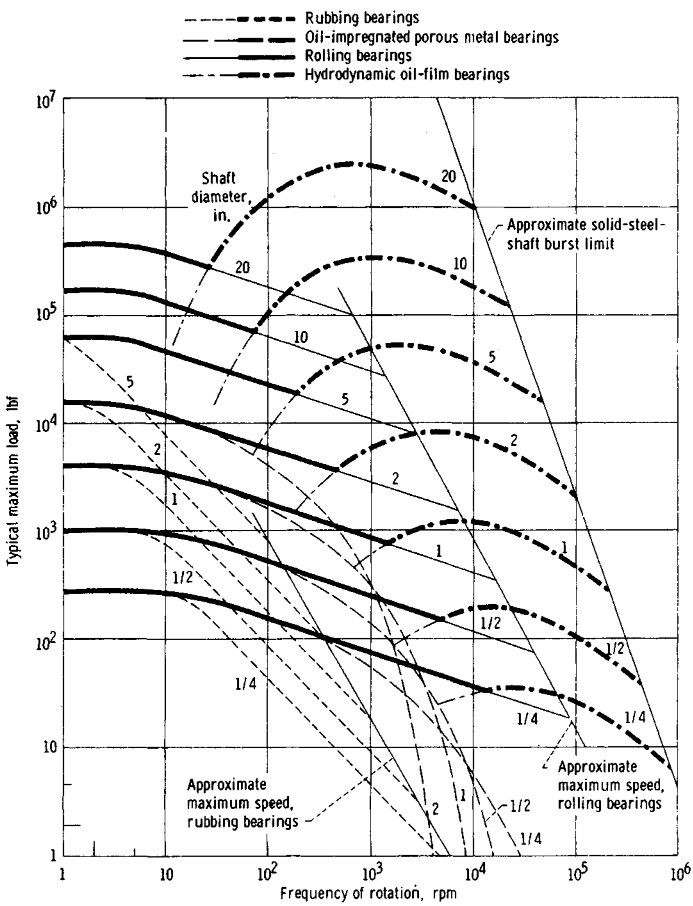

Fig. 21.3 General guide to journal bearing type. (Except for roller bearings, curves are drawn for bearings with width equal to diameter. A medium-viscosity mineral oil lubricant is assumed for hydrodynamic bearings.) (From Ref. 7.)

indicate the preferred type of journal bearing for a particular load, speed, and diameter and thus divide the graph into distinct regions. From Fig. 21.3 it is observed that rolling-element bearings are preferred at lower speeds and hydrodynamic oil film bearings are preferred at higher speeds. Rubbing bearings and oil-impregnated porous metal bearings are not preferred for any of the speeds, loads, or shaft diameters considered. Also, as the shaft diameter is increased, the transitional point at which hydrodynamic bearings are preferred over rolling-element bearings moves to the left.

The applied load and speed are usually known, and this enables a preliminary assessment to be made of the type of journal bearing most likely to be suitable for a particular application. In many cases the shaft diameter will have been determined by other considerations, and Fig. 21.3 can be used to find the type of journal bearing that will give adequate load capacity at the required speed. These curves are based upon good engineering practice and commercially available parts. Higher loads and speeds or smaller shaft diameters are possible with exceptionally high engineering standards or specially produced materials. Except for rolling-element bearings the curves are drawn for bearings with a width equal to the diameter. A medium-viscosity mineral oil lubricant is assumed for the hydrodynamic bearings.

Similarly, Fig. 21.4, reproduced from the Engineering Sciences Data Unit publication,[8] gives a guide to the typical maximum load that can be carried at various speeds for a nominal life of 10,000 hr at room temperature by thrust bearings of the various diameters quoted. The heavy curves again indicate the preferred type of bearing for a particular load, speed, and diameter and thus divide the graph into major regions. As with the journal bearing results (Fig. 21.3) at the hydrodynamic bearing is preferred at lower speeds. A difference between Figs. 21.3 and 21.4 is that at very low speeds

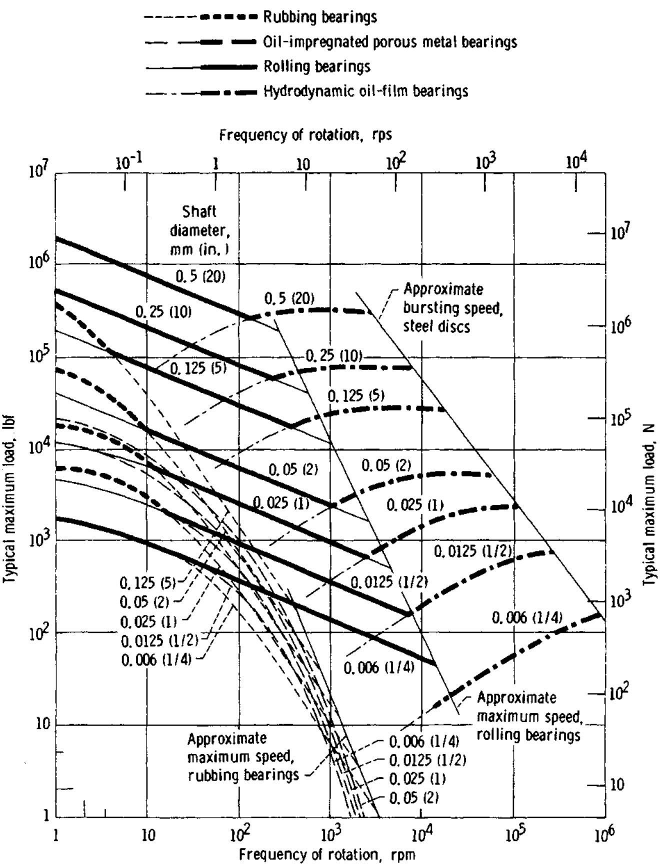

Fig. 21.4 General guide to thrust bearing type. (Except for roller bearings, curves are drawn for typical ratios of inside to outside diameter. A medium-viscosity mineral oil lubricant is assumed for hydrodynamic bearings.) (From Ref. 8.)

there is a portion of the latter figure in which the rubbing bearing is preferred. Also, as the shaft diameter is increased, the transitional point at which hydrodynamic bearings are preferred over rolling-element bearings moves to the left. Note also from this figure that oil-impregnated porous metal bearings are not preferred for any of the speeds, loads, or shaft diameters considered.

21.1.3 Lubricants

Both oils and greases are extensively used as lubricants for all types of machine elements over wide range of speeds, pressures, and operating temperatures. Frequently, the choice is determined by considerations other than lubrication requirements. The requirements of the lubricant for successful operation of nonconformal contacts such as in rolling-element bearings and gears are considerably more stringent than those for conformal bearings and therefore will be the primary concern in this section.

Because of its fluidity oil has several advantages over grease: It can enter the loaded conjunction most readily to flush away contaminants, such as water and dirt, and, particularly, to transfer heat from heavily loaded machine elements. Grease, however, is extensively used because it permits simplified designs of housings and enclosures, which require less maintenance, and because it is more effective in sealing against dirt and contaminants.

Viscosity

In hydrodynamic and elastohydrodynamic lubrication the most important physical property of a lubricant is its viscosity. The viscosity of a fluid may be associated with its resistance to flow, that is, with the resistance arising from intermolecular forces and internal friction as the molecules move past each other. Thick fluids, like molasses, have relatively high viscosity; they do not flow easily. Thinner fluids, like water, have lower viscosity; they flow very easily.

The relationship for internal friction in a viscous fluid (as proposed by Newton)[9] can be written as

$$\tau = \eta \frac{du}{dz} \tag{21.4}$$

where τ = internal shear stress in the fluid in the direction of motion
η = coefficient of absolute or dynamic viscosity or coefficient of internal friction
du/dz = velocity gradient perpendicular to the direction of motion (i.e., shear rate)

It follows from Eq. (21.4) that the unit of dynamic viscosity must be the unit of shear stress divided by the unit of shear rate. In the newton-meter-second system the unit of shear stress is the newton per square meter while that of shear rate is the inverse second. Hence the unit of dynamic viscosity will be newton per square meter multiplied by second, or N sec/m^2. In the SI system the unit of pressure or stress (N/m^2) is known as pascal, abbreviated Pa, and it is becoming increasingly common to refer to the SI unit of viscosity as the pascal-second (Pa sec). In the cgs system, where the dyne is the unit of force, dynamic viscosity is expressed as dyne-second per square centimeter. This unit is called the poise, with its submultiple the centipoise (1 cP = 10^{-2} P) of a more convenient magnitude for many lubricants used in practice.

Conversion of dynamic viscosity from one system to another can be facilitated by Table 21.1. To convert from a unit in the column on the left-hand side of the table to a unit at the top of the table, multiply by the corresponding value given in the table. For example, η = 0.04 N sec/m^2 = 0.04 × 1.45 × 10^{-4} lbf sec/in.^{2} = 5.8 × 10^{-6} lbf sec/in.^{2}. One English and three metric systems are presented—all based on force, length, and time. Metric units are the centipoise, the kilogram force-

Table 21.1 Viscosity Conversion

To Convert From—	To— cP	kgf s/m^2	N s/m^2	lbf s/in^2
	Multiply By—			
cP	1	1.02×10^{-4}	10^{-3}	1.45×10^{-7}
kgf s/m^2	9.807×10^3	1	9.807	1.422×10^{-3}
N s/m^2	10^3	1.02×10^{-1}	1	1.45×10^{-4}
lbf s/in^2	6.9×10^6	7.034×10^2	6.9×10^3	1

second per square meter, and the newton-second per square meter (or Pa sec). The English unit is pound force-second per square inch, or reyn, in honor of Osborne Reynolds.

In many situations it is convenient to use the *kinematic viscosity* rather than the dynamic viscosity. The kinematic viscosity η_k is equal to the dynamic viscosity η divided by the density ρ of the fluid ($\eta_k = \eta/\rho$). The ratio is literally kinematic, all trace of force or mass cancelling out. The unit of kinematic viscosity may be written in SI units as square meters per second or in English units as square inches per second or, in cgs units, as square centimeters per second. The name stoke, in honor of Sir George Gabriel Stokes, was proposed for the cgs unit by Max Jakob in 1928. The centistoke, or one-hundredth part, is an everyday unit of more convenient size, corresponding to the centipoise.

The viscosity of a given lubricant varies within a given machine element as a result of the nonuniformity of pressure or temperature prevailing in the lubricant film. Indeed, many lubricated machine elements operate over ranges of pressure or temperature so extensive that the consequent variations in the viscosity of the lubricant may become substantial and, in turn, may dominate the operating characteristics of machine elements. Consequently, an adequate knowledge of the viscosity–pressure and viscosity–pressure–temperature relationships of lubricants is indispensable.

Oil Lubrication

Except for a few special requirements, petroleum oils satisfy most operating conditions in machine elements. High-quality products, free from adulterants that can have an abrasive or lapping action, are recommended. Animal or vegetable oils or petroleum oils of poor quality tend to oxidize, to develop acids, and to form sludge or resinlike deposits on the bearing surfaces. They thus penalize bearing performance or endurance.

A composite of recommended lubricant kinematic viscosities at 38°C (100°F) is shown in Fig. 21.5. The ordinate of this figure is the speed factor, which is bearing bore size measured in millimeters multiplied by the speed in revolutions per minute. In many rolling-element-bearing applications an

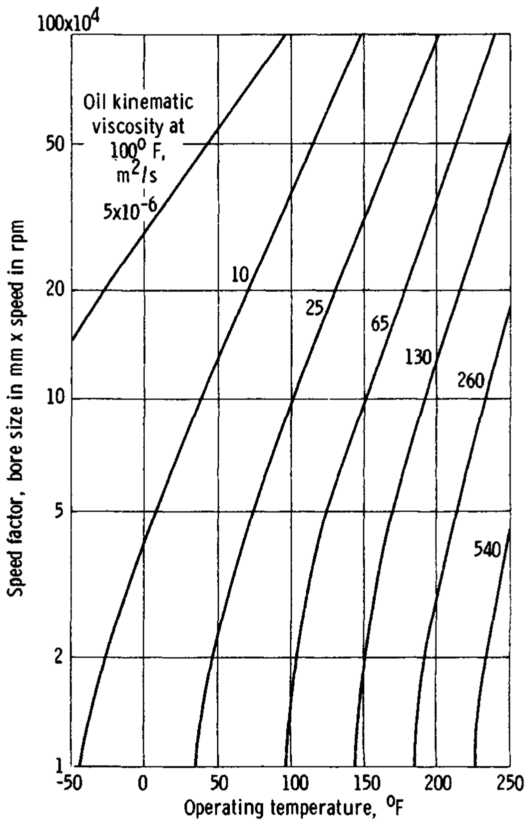

Fig. 21.5 Recommended lubricant viscosities for ball bearings. (From Ref. 10.)

oil equivalent to an SAE-10 motor oil [4×10^{-6} m^2/sec, or 40 cS, at 38°C (100°F)] or a light turbine oil is the most frequent choice.

For a number of military applications where the operational requirements span the temperature range −54 to 204°C (−65 to 400°F), synthetic oils are used. Ester lubricants are most frequently employed in this temperature range. In applications where temperatures exceed 260°C (500°F), most synthetics will quickly break down, and either a solid lubricant (e.g., MoS_2) or a polyphenyl ether is recommended. A more detailed discussion of synthetic lubricants can be found in Bisson and Anderson.[11]

Grease Lubrication

The simplest method of lubricating a bearing is to apply grease, because of its relatively nonfluid characteristics. The danger of leakage is reduced, and the housing and enclosure can be simpler and less costly than those used with oil. Grease can be packed into bearings and retained with inexpensive enclosures, but packing should not be excessive and the manufacturer's recommendations should be closely adhered to.

The major limitation of grease lubrication is that it is not particularly useful in high-speed applications. In general, it is not employed for speed factors over 200,000, although selected greases have been used successfully for higher speed factors with special designs.

Greases vary widely in properties depending on the type and grade or consistency. For this reason few specific recommendations can be made. Greases used for most bearing operating conditions consist of petroleum, diester, polyester, or silicone oils thickened with sodium or lithium soaps or with more recently developed nonsoap thickeners. General characteristics of greases are as follows:

1. Petroleum oil greases are best for general-purpose operation from −34 to 149°C (−30 to 300°F).
2. Diester oil greases are designed for low-temperature service down to −54°C (−65°F).
3. Ester-based greases are similar to diester oil greases but have better high-temperature characteristics, covering the range from −73 to 177°C (−100 to 350°F).
4. Silicone oil greases are used for both high- and low-temperature operation, over the widest temperature range of all greases [−73 to 232°C (−100 to 450°F)], but have the disadvantage of low load-carrying capacity.
5. Fluorosilicone oil greases have all of the desirable features of silicone oil greases plus good load capacity and resistance to fuels, solvents, and corrosive substances. They have a very low volatility in vacuum down to 10^{-7} torr, which makes them useful in aerospace applications.
6. Perfluorinated oil greases have a high degree of chemical inertness and are completely nonflammable. They have good load-carrying capacity and can operate at temperatures as high as 280°C (550°F) for long periods, which makes them useful in the chemical processing and aerospace industries, where high reliability justifies the additional cost.

Grease consistency is important since grease will slump badly and churn excessively when too soft and fail to lubricate when too hard. Either condition causes improper lubrication, excessive temperature rise, and poor performance and can shorten machine element life. A valuable guide to the estimation of the useful life of grease in rolling-element bearings has been published by the Engineering Sciences Data Unit.[12]

It has recently been demonstrated by Aihara and Dowson[13] and by Wilson[14] that the film thickness in grease-lubricated components can be calculated with adquate accuracy by using the viscosity of the base oil in the elastohydrodynamic equation (see Section 21.3). This enables the elastohydrodynamic lubrication film thickness formulas to be applied with confidence to grease-lubricated machine elements.

21.1.4 Lubrication Regimes

If a machine element is adequately designed and lubricated, the lubricated surfaces are separated by a lubricant film. Endurance testing of ball bearings, as reported by Tallian et al.,[15] has demonstrated that when the lubricant film is thick enough to separate the contacting bodies, fatigue life of the bearing is greatly extended. Conversely, when the film is not thick enough to provide full separation between the asperities in the contact zone, the life of the bearing is adversely affected by the high shear resulting from direct metal-to-metal contact.

To establish the effect of film thickness on the life of the machine element, we first introduce a relevant parameter Λ. The relationship between Λ and the minimum film thickness h_{min} is defined to be

$$\Lambda = \frac{h_{\min}}{(\Delta_a^2 + \Delta_b^2)^{1/2}} \tag{21.5}$$

where Δ_a = rms surface finish of surface a
Δ_b = rms surface finish of surface b

Hence Λ is just the minimum film thickness in units of the composite roughness of the two bearing surfaces.

Hydrodynamic Lubrication Regime

Hydrodynamic lubrication occurs when the lubricant film is sufficiently thick to prevent the opposite solids from coming into contact. This condition is often referred to as the ideal form of lubrication since it provides low friction and a high resistance to wear. The lubrication of the contact is governed by the bulk physical properties of the lubricant, notably viscosity, and the frictional characteristics arise purely from the shearing of the viscous lubricant. The pressure developed in the oil film of hydrodynamically lubricated bearings is due to two factors:

1. The geometry of the moving surfaces produces a convergent film shape.
2. The viscosity of the liquid results in a resistance to flow.

The lubricant films are normally many times thicker than the surface roughness so that the physical properties of the lubricant dictate contact behavior. The film thickness normally exceeds 10^{-6} m. For hydrodynamic lubrication the film parameter Λ, defined in Eq. (21.5), is an excess of 10 and may even rise to 100. Films of this thickness are clearly also insensitive to chemical action in surface layers of molecular proportions.

For normal load support to occur in bearings, positive pressure profiles must develop over the length of the bearing. Three different forms of hydrodynamic lubrication are presented in Fig. 21.6.

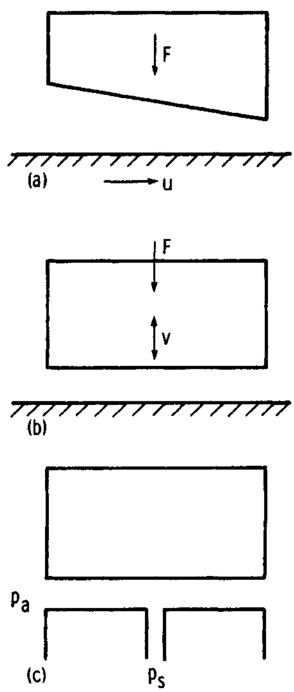

Fig. 21.6 Mechanisms of load support for hydrodynamic lubrication. (*a*) Slider bearing. (*b*) Squeeze film bearing. (*c*) Externally pressurized bearing.

Figure 21.6*a* shows a slider bearing. For a positive load to be developed in the slider bearing shown in Fig. 21.6*a* the lubricant film thickness must be decreasing in the direction of sliding.

A squeeze film bearing is another mechanism of load support of hydrodynamic lubrication, and it is illustrated in Fig. 21.6*b*. The squeeze action is the normal approach of the bearing surfaces. The squeeze mechanism of pressure generation provides a valuable cushioning effect when the bearing surfaces tend to be pressed together. Positive pressures will be generated when the film thickness is diminishing.

An externally pressurized bearing is yet a third mechanism of load support of hydrodynamic lubrication, and it is illustrated in Fig. 21.6*c*. The pressure drop across the bearing is used to support the load. The load capacity is independent of the motion of the bearing and the viscosity of the lubricant. There is no problem of contact at starting and stopping as with the other hydrodynamically lubricated bearings because pressure is applied before starting and is maintained until after stopping.

Hydrodynamically lubricated bearings are discussed further in Section 21.2.

Elastohydrodynamic Lubrication Regime

Elastohydrodynamic lubrication is a form of hydrodynamic lubrication where elastic deformation of the bearing surfaces becomes significant. It is usually associated with highly stressed machine components of low conformity. There are two distinct forms of elastohydrodynamic lubrication (EHL).

Hard EHL. Hard EHL relates to materials of *high* elastic modulus, such as metals. In this form of lubrication both the elastic deformation and the pressure–viscosity effects are equally important. Engineering applications in which elastohydrodynamic lubrication are important for high-elastic-modulus materials include gears and rolling-element bearings.

Soft EHL. Soft EHL relates to materials of *low* elastic modulus, such as rubber. For these materials the elastic distortions are large, even with light loads. Another feature of the elastohydrodynamics of low-elastic-modulus materials is the negligible effect of the relatively low pressures on the viscosity of the lubricating fluid. Engineering applications in which elastohydrodynamic lubrication are important for low-elastic-modulus materials include seals, human joints, tires, and a number of lubricated elastomeric material machine elements.

The common factors in hard and soft EHL are that the local elastic deformation of the solids provides coherent fluid films and that asperity interaction is largely prevented. Elastohydrodynamic lubrication normally occurs in contacts where the minimum film thickness is in the range $0.1\ \mu\text{m} < h_{\min} \leq 10\ \mu\text{m}$ and the film parameter Λ is in the range $3 \leq \Lambda < 10$. Elastohydrodynamic lubrication is discussed further in Section 21.3.

Boundary Lubrication Regime

If in a lubricated contact the pressures become too high, the running speeds too low, or the surface roughness too great, penetration of the lubricant film will occur. Contact will take place between the asperities. The friction will rise and approach that encountered in dry friction between solids. More importantly, wear will take place. Adding a small quantity of certain active organic compounds to the lubricating oil can, however, extend the life of the machine elements. These additives are present in small quantities (< 1%) and function by forming low-shear-strength surface films strongly attached to the metal surfaces. Although they are sometimes only one or two molecules thick, such films are able to prevent metal-to-metal contact.

Some boundary lubricants are long-chain molecules with an active end group, typically an alcohol, an amine, or a fatty acid. When such a material, dissolved in a mineral oil, meets a metal or other solid surface, the active end group attaches itself to the solid and gradually builds up a surface layer. The surface films vary in thickness from 5×10^{-9} to 10^{-8} m depending on molecular size, and the film parameter Λ is less than unity ($\Lambda < 1$). Boundary lubrication is discussed further in Section 21.4.

Figure 21.7 illustrates the film conditions existing in hydrodynamic, elastohydrodynamic, and boundary lubrication. The surface slopes in this figure are greatly distorted for the purpose of illustration. To scale, real surfaces would appear as gently rolling hills rather than sharp peaks.

21.1.5 Relevant Equations

This section presents the equations frequently used in hydrodynamic and elastohydrodynamic lubrication theory. They are not relevant to boundary lubrication since in this lubrication regime bulk fluid effects are negligible. The differential equation governing the pressure distribution in hydrodynamically and elastohydrodynamically lubricated machine elements is known as the Reynolds equation. For steady-state hydrodynamic lubrication the Reynolds equation normally appears as

$$\frac{\partial}{\partial x}\left(h^3 \frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(h^3 \frac{\partial p}{\partial y}\right) = 12\eta u \frac{\partial h}{\partial x} \tag{21.6}$$

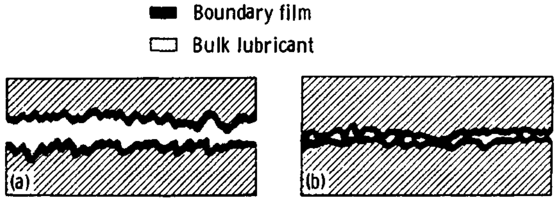

Fig. 21.7 Film conditions of lubrication regimes, (*a*) Hydrodynamic and elastohydrodynamic lubrication—surfaces separated by bulk lubricant film. (*b*) Boundary lubrication—performance essentially dependent on boundary film.

where h = film shape measured in the z direction, m
p = pressure, N/m^2
η = lubricant viscosity, N sec/m^2
u = mean velocity, $(u_a + u_b)/2$, m/sec

Solutions of Eq. (21.6) are rarely achieved analytically, and approximate numerical solutions are sought.

For elastohydrodynamic lubrication the steady-state form of the Reynolds equation normally appears as

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{\eta}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\rho h^3}{\eta}\frac{\partial p}{\partial y}\right) = 12u\frac{\partial(\rho h)}{\partial x} \tag{21.7}$$

where ρ is lubricant density in N sec^2/m^2. The essential difference between Eqs. (21.6) and (21.7) is that Eq. (21.7) allows for variation of viscosity and density in the x and y directions. Equations (21.6) and (21.7) allow for the bearing surfaces to be of finite length in the y direction. Side leakage, or flow in the y direction, is associated with the second term in Eqs. (21.6) and (21.7). The solution of Eq. (21.7) is considerably more difficult than that of Eq. (21.6); therefore, only numerical solutions are available.

The viscosity of a fluid may be associated with the resistance to flow, with the resistance arising from the intermolecular forces and internal friction as the molecules move past each other. Because of the much larger pressure variation in the lubricant conjunction, the viscosity of the lubricant for elastohydrodynamic lubrication does not remain constant as is approximately true for hydrodynamic lubrication.

As long ago as 1893, Barus[16] proposed the following formula for the isothermal viscosity–pressure dependence of liquids:

$$\eta = \eta_0 e^{\xi p} \tag{21.8}$$

where η_0 = viscosity at atmospheric pressure
ξ = pressure–viscosity coefficient of lubricant

The pressure–viscosity coefficient ξ characterizes the liquid considered and depends in most cases only on temperature, not on pressure.

Table 21.2 lists the absolute viscosities of 12 lubricants at atmospheric pressure and three temperatures as obtained from Jones et al.[17] These values would correspond to η_0 to be used in Eq. (21.8) for the particular fluid and temperature to be used. The 12 fluids with manufacturer and manufacturer's designation are shown in Table 21.3. The pressure–viscosity coefficients ξ, expressed in square meters per newton, for these 12 fluids at three different temperatures are shown in Table 21.4.

For a comparable change in pressure the relative density change is smaller than the viscosity change. However, very high pressures exist in elastohydrodynamic films, and the liquid can no longer be considered as an incompressible medium. From Dowson and Higginson[18] the density can be written as

$$\rho = \rho_0\left(1 + \frac{0.6p}{1 + 1.7p}\right) \tag{21.9}$$

where p is given in gigapascals.

Table 21.2 Absolute Viscosities of Test Fluids at Atmospheric Pressure and Three Temperatures (From Ref. 17)

	Temperature, °C		
	38	99	149
Test Fluid	Absolute Viscosity, η, cP		
Advanced ester	25.3	4.75	2.06
Formulated advanced ester	27.6	4.96	2.15
Polyalkyl aromatic	25.5	4.08	1.80
Polyalkyl aromatic + 10 wt % heavy resin	32.2	4.97	2.03
Synthetic paraffinic oil (lot 3)	414	34.3	10.9
Synthetic paraffinic oil (lot 4)	375	34.7	10.1
Synthetic paraffinic oil (lot 4) + antiwear additive	375	34.7	10.1
Synthetic paraffinic oil (lot 2) + antiwear additive	370	32.0	9.93
C-ether	29.5	4.67	2.20
Superrefined naphthenic mineral oil	68.1	6.86	2.74
Synthetic hydrocarbon (traction fluid)	34.3	3.53	1.62
Fluorinated polyether	181	20.2	6.68

The film shape appearing in Eq. (21.7) can be written with sufficient accuracy as

$$h = h_0 + \frac{x^2}{2R_x} + \frac{y^2}{2R_y} + \delta(x,y) \tag{21.10}$$

where h_0 = constant, m
$\delta(x,y)$ = total elastic deformation, m
R_x = effective radius in x direction, m
R_y = effective radius in y direction, m

The elastic deformation can be written, from standard elasticity theory, in the form

Table 21.3 Fluids with Manufacturer and Manufacturer's Designation (From Ref. 17)

Test Fluid	Manufacturer	Designation
Advanced ester	Shell Oil Co.	Aeroshell® turbine oil 555 (base oil)
Formulated advanced ester	Shell Oil Co.	Aeroshell® turbine oil 555 (WRGL-358)
Polyalkyl aromatic	Continental Oil Co.	DN-600
Synthetic paraffinic oil (lot 3)	Mobil Oil Corp.	XRM 109F3
Synthetic paraffinic oil (lot 4)	↓	XRM 109F4
Synthetic paraffinic oil + antiwear additive (lot 2)	↓	XRM 177F2
Synthetic paraffinic oil + antiwear additive (lot 4)	↓	XRM 177F4
C-ether	Monsanto Co.	MCS-418
Superrefined naphthenic mineral oil	Humble Oil and Refining Co.	FN 2961
Synthetic hydrocarbon (traction fluid)	Monsanto Co.	MCS-460
Fluorinated polyether	DuPont Co.	PR 143 AB (Lot 10)

Table 21.4 Pressure–Viscosity Coefficients for Test Fluids at Three Temperatures (From Ref. 17)

	Temperature, °C		
	38	99	149
Test Fluid	Pressure-viscosity Coefficient, ξ, m²/N		
Advanced ester	1.28×10^{-8}	0.987×10^{-8}	0.851×10^{-8}
Formulated advanced ester	1.37	1.00	.874
Polyalkyl aromatic	1.58	1.25	1.01
Polyalkyl aromatic + 10 wt % heavy resin	1.70	1.28	1.06
Synthetic paraffinic oil (lot 3)	1.77	1.51	1.09
Synthetic paraffinic oil (lot 4)	1.99	1.51	1.29
Synthetic paraffinic oil (lot 4) + antiwear additive	1.96	1.55	1.25
Synthetic paraffinic oil (lot 2) + antiwear additive	1.81	1.37	1.13
C-ether	1.80	.980	.795
Superrefined naphthenic mineral oil	2.51	1.54	1.27
Synthetic hydrocarbon (traction fluid)	3.12	1.71	.939
Fluorinated polyether	4.17	3.24	3.02

$$\delta(x,y) = \frac{2}{\pi E'} \iint_A \frac{p(x,y)\, dx_1 dy_1}{[(x - x_1)^2 + (y - y_1)^2]^{1/2}} \tag{21.11}$$

where

$$E' = 2\left(\frac{1 - \nu_a^2}{E_a} + \frac{1 - \nu_b^2}{E_b}\right)^{-1} \tag{21.12}$$

and ν = Poisson's ratio
E = modulus of elasticity, N/m²

Therefore, Eq. (21.6) is normally involved in hydrodynamic lubrication situations, while Eqs. (21.7)–(21.11) are normally involved in elastohydrodynamic lubrication situations.

21.2 HYDRODYNAMIC AND HYDROSTATIC LUBRICATION

Surfaces lubricated hydrodynamically are normally conformal as pointed out in Section 21.1.1. The conformal nature of the surfaces can take its form either as a thrust bearing or as a journal bearing, both of which will be considered in this section. Three features must exist for hydrodynamic lubrication to occur:

1. A viscous fluid must separate the lubricated surfaces.
2. There must be relative motion between the surfaces.
3. The geometry of the film shape must be larger in the inlet than at the outlet so that a convergent wedge of lubricant is formed.

If feature 2 is absent, lubrication can still be achieved by establishing relative motion between the fluid and the surfaces through external pressurization. This is discussed further in Section 21.2.3.

In hydrodynamic lubrication the entire friction arises from the shearing of the lubricant film so that it is determined by the viscosity of the oil: the thinner (or less viscous) the oil, the lower the friction. The great advantages of hydrodynamic lubrication are that the friction can be very low ($\mu \simeq 0.001$) and, in the ideal case, there is no wear of the moving parts. The main problems in hydrodynamic lubrication are associated with starting or stopping since the oil film thickness theoretically is zero when the speed is zero.

The emphasis in this section is on hydrodynamic and hydrostatic lubrication. This section is not intended to be all inclusive but rather to typify the situations existing in hydrodynamic and hydrostatic lubrication. For additional information the reader is recommended to investigate Gross et al.,[19] Reiger,[20] Pinkus and Sternlicht,[21] and Rippel.[22]