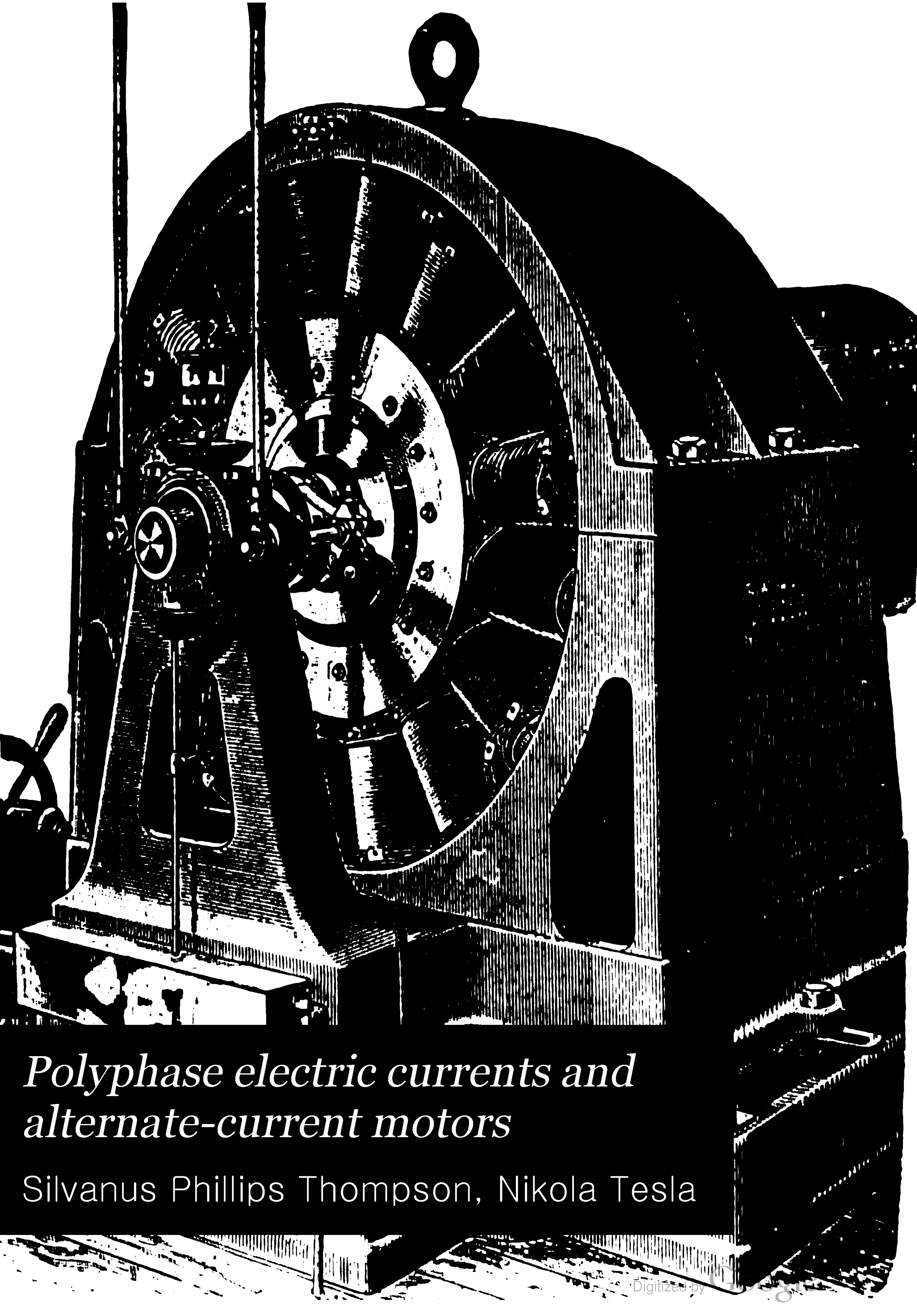

Polyphase electric currents and alternate-current motors

Silvanus Phillips Thompson, Nikola Tesla

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VOLUME SIX

POLYPHASE ELECTRIC CURRENTS

AND ALTERNATE-CURRENT MOTORS

BY

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By NIKOLA TESLA

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PREFACE.

When the course of four Lectures on the subject of Polyphase Currents, delivered by the author at the Technical College, Finsbury, in the autumn of 1894, was completed, representations were made by persons who had attended the Lectures, and by others, to induce him to publish them in permanent form.

In preparing the lectures for the press a good deal of matter has been added. No attempt has been made either to preserve the colloquial form of the discourses or to give to them any pretence to literary style. They are put together in their present shape for the use of students and engineers, and introductory matter has been added to make the relations of polyphase currents to ordinary single-phase currents more clear. In all this work the author has been aided by Mr. Miles Walker, whose assistance is willingly acknowledged, and on whom the task of reducing to written form much of the work has devolved. The graphic method of treating monophase motors, on pp. 165 to 169, is due to Mr. Walker.

The author is also indebted to various firms and designers for valuable information afforded as to recent progress and modern types of machine, and he desires to express his thanks to the following: the Allgemeine Elektricitäts-Gesellschaft, of Berlin; the Helios Company, of Cologne; the Elektricitäts-Aktiengesellschaft (Messrs. Schuckert & Co.), of Nürnberg; the Oerlikon Maschinen-Fabrik (and to Mr. Emil Kolben) of Zürich; and most of all to Brown, Boveri & Co. (and to Mr. C. E. L. Brown), of Baden, Switzerland.

A full Bibliography of the subject of Polyphase Currents and Induction Motors has been appended at the end of the volume.

S. P. T.

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POLYPHASE
ELECTRIC CURRENTS AND ALTERNATE-CURRENT MOTORS.

CHAPTER I.

POLYPHASE GENERATORS.

INTRODUCTORY REMARKS.

No apology is needed for devoting special attention at the present time to the subject of polyphase electric currents. There seems to be no doubt that in the problem of the electric transmission of power a very important part will, in the future, be played by alternating currents combined in systems of two or three different phases. Already a number of examples exist; and some very large works are now in course of construction. The undoubted advantages possessed by polyphase systems over either (*a*) continuous current systems, or (*b*) ordinary single-phase alternate currents, for the special service of power transmission, are beyond question; but it remains to be seen how far the complications thereby inevitably introduced are, in practice, sufficiently great to militate against polyphase distribution for the purpose of general electric lighting supplies.

The comparative novelty of polyphase methods, and the circumstance that the greater part of that which has already been achieved has been done in foreign countries, are reasons why the topic should receive some attention from English engineers.

In these pages the subject will be dealt with under the following subdivisions :—Generators for Polyphase Currents; the Properties of the Rotatory Magnetic Field, with some account of its historical development; the Theory, Construction and Performance of Polyphase Motors; the Theory and Construction of Motors operated by ordinary single-phase Alternate Currents; together with some account of Polyphase Transformers, and of the measurement of power in polyphase systems.

ALTERNATE CURRENTS.

It will be assumed at the outset that we are already acquainted with the general principles of alternate-currents, and with the general features[1] of alternators or alternate-current generators.

Nevertheless, a recapitulation of the main points about alternate currents may be useful as a preliminary.

Faraday's discovery of the induction of currents in wires by moving them across a magnetic field, so as to cut the magnetic lines, suggested the construction of magneto-electric machines to generate currents by mechanical power. If a coil of suitable form is placed, as in Fig. 1, between the poles of a magnet, and spun around a longitudinal axis, it will have currents generated in it which at each semi-revolution die away and then reverse. In the figure the coil of wire is supposed to be so spun that the upper portion comes towards the observer. In that case, the

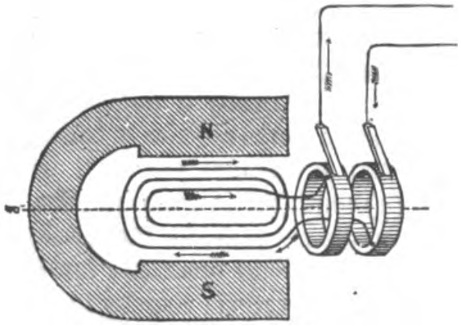

FIG. 1.—SIMPLE ALTERNATE-CURRENT GENERATOR (SINGLE-PHASE).

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[1] For a simple outline of the subject see Chapters IX. and X. of the 1894 edition of the author's *Elementary Lessons in Electricity and Magnetism;* or, for a fuller account, consult the author's larger treatise on *Dynamo-electric Machinery.*

arrows show the direction of the induced currents delivered to the circuit through the agency of two contact rings (or slip-rings) connected respectively to the ends of the coils. In the position shown, the current will be delivered to the left-hand ring, and returns from the circuit to the right-hand ring; but half a turn later it will be flowing to the right-hand ring and returning from the circuit back to the left-hand ring. Fig. 1 is, in fact, a primitive form of alternator, generating a simple periodically reversed or alternating current; and is, in fact, the kind of alternator known as a "magneto-ringer," used for bell service in telephone sets. In alternate current working, the current is rapidly reversed, rising and falling in a set of pulses; the electric currents being set oscillating forwards and backwards through the line and around the circuit with great rapidity—scores or hundreds of times a second—under the influence of a rapidly reversing electromotive force. As is well known, the properties of alternate currents differ somewhat from those of continuous currents. They are affected not only by the resistance of the circuit, but also by its electro-magnetic inertia or self-induction (in other words, by the magnetic field which it sets up around itself), the inductance of the circuit having a choking effect on the alternate currents, diminishing their amplitude, retarding their phase, and smoothing down their ripples.

In Fig. 1 the revolving armature was a simple coil, and the magnet of simple 2-pole horse-shoe form. But for reasons mentioned later, the majority of alternate-current generators are multipolar. Fig. 2 illustrates a frequent form introduced by the Westinghouse Company, having a multipolar field-magnet consisting of a number of radial poles pointing inwards, whilst the revolving coils are grouped upon the periphery of a drum or cylinder built up of iron core-disks.

In order to study the combinations of wires, we must devote a moment to the directions of the currents induced in them.

Consider first Fig. 3, which is a partial sketch of a 4-pole machine laid on its sides. The core, to receive hereafter its appropriate winding, lies between four poles of alternate polarity. If a copper rod *a b* is placed parallel to the axis to represent one of the armature conductors, and is supposed to move along the gap-space right-handedly past the S pole, it will cut the magnetic lines entering that pole. By the rule given below, the induced electromotive-force in it will be upwards. Another conductor *c d* passing the N pole will have induced in it a downward electromotive force. If one was to attempt in a picture such as this to show twenty or

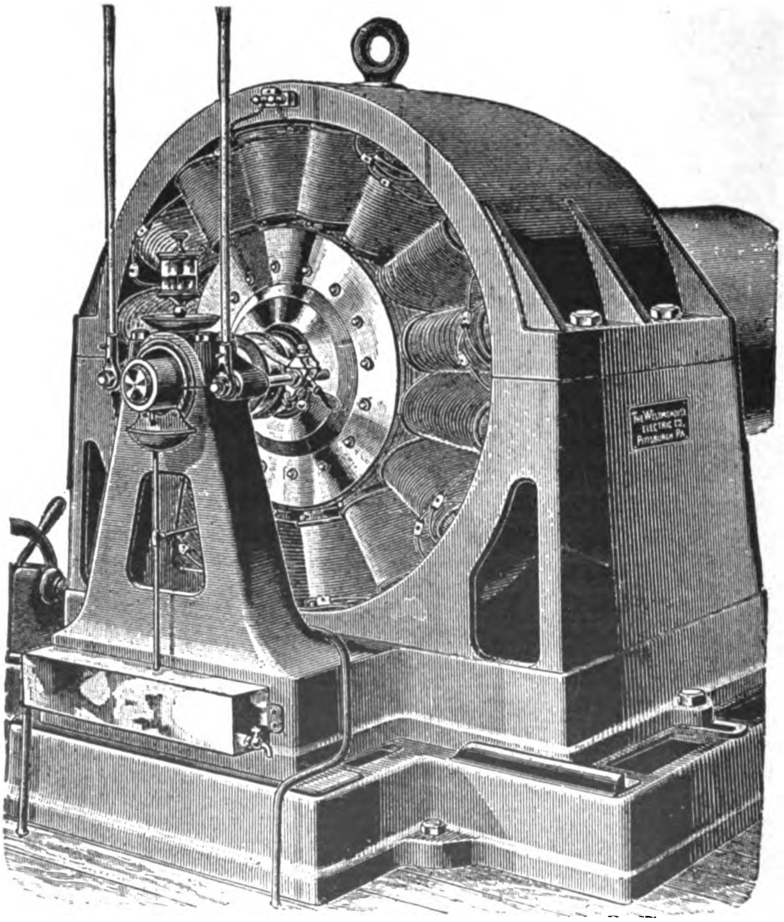

FIG. 2.—THE WESTINGHOUSE CO.'S ALTERNATOR (SINGLE-PHASE).

more conductors and their respective connections, the drawing would be unintelligible. Accordingly we have to imagine ourselves placed at the centre, and the panorama of the four poles around us to be then laid out flat, as in Fig. 4. It will be noticed that the faces of the N and S poles are shaded obliquely for distinction.

The oblique lines are used for the following purpose. If instead of the line *a b* (representing a conductor), a narrow slit in a piece of paper were laid over the drawing of the pole-face, and moved as the dotted arrows show towards the right, the slit in passing over the oblique lines will cause an apparent motion in the direction in which the current tends in reality to flow. It is easy to remember which way the oblique lines must slope; for those on a north pole-face slope parallel to the oblique bar of the letter N.

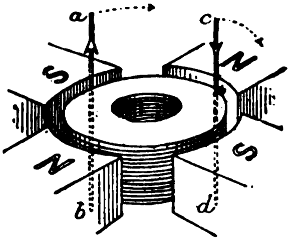

FIG. 3.—SKETCH OF FOUR-POLE FIELD.

Now in an actual machine there are many armature conductors spaced symmetrically around, and these have to be grouped together by connecting wires, or pieces. In the case of ring-wound armatures the connecting conductor goes

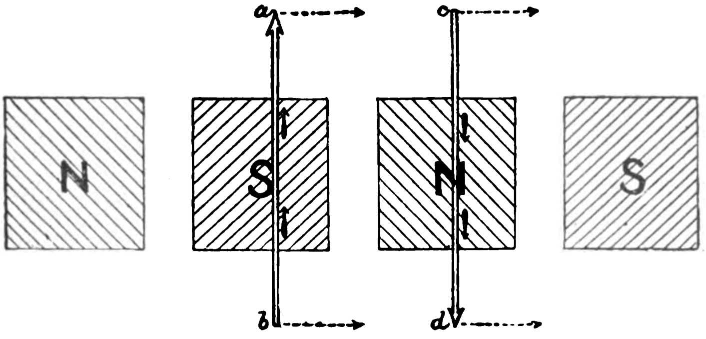

FIG. 4.—FOUR-POLE FIELD DEVELOPED FLAT.

through the interior of the ring-core, thus constituting a ***spiral winding.*** When we go on to those cases in which the winding is entirely exterior to the core, as for drum armatures and disk armatures, we find that there are two distinct modes of procedure, which we may respectively denote as *lap-winding* and *wave-winding.* The distinction arises in the following manner. Since the conductors that are passing a north pole generate electromotive forces in one direction, and those that are passing a south pole generate electromotive forces in the opposite direction, it is clear that a conductor in one of these groups ought to be connected to one in a nearly corresponding position in the other group, so that the current may flow down one and up the other in agreement with the directions of the electromotive forces. So after having passed down opposite a north pole face, the conductor may be connected to one that passes up opposite a south pole face, and the winding evidently may be arranged either to lap back, or to zigzag forward.

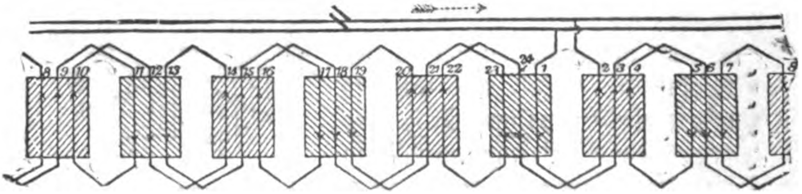

Fig. 5.—Alternate-current Machine: Lap-winding.

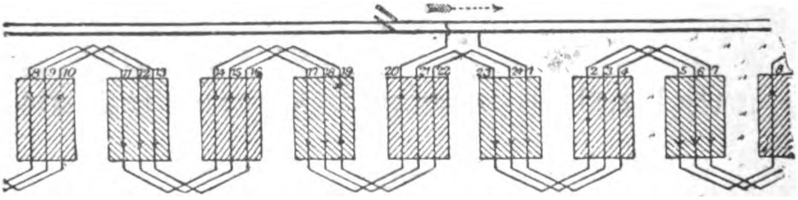

Fig. 6.—Alternate-current Machine: Wave-winding.

This distinction between lap-windings and wave-windings as applied to alternate current machines, is illustrated in Figs. 5 and 6. Fig. 5 represents an 8-pole alternator with lap-winding, each "element" or set of loops extending across the same breadth as the "pitch" or distance from centre to centre of two adjacent poles. Only 24 conductors have been drawn; and it will be noticed that the successive loops are alternately right-handed and left-handed. In Fig. 6 is shown the same alternator with a wave-winding. The electromotive force of the two machines would be precisely the same; the choice between the two methods of connecting is here purely a question of mechanical convenience in construction and cost. In cases where the armature revolves, the beginning and end of the winding are connected to two slip-rings, which in these developed drawings are represented by two parallel lines.

Returning to the simple revolving coil represented in Fig. 1, we have seen above how the coil, by cutting the lines of the magnetic field, sets up periodic electromotive forces, which change at every half-turn, giving rise to alternate currents. In each whole revolution there will be an electromotive force which rises to a maximum and then dies away, followed immediately by a reversed electromotive force, which also grows to a maximum and then dies away. The wave-form depicted in Fig. 7 serves to illustrate this.

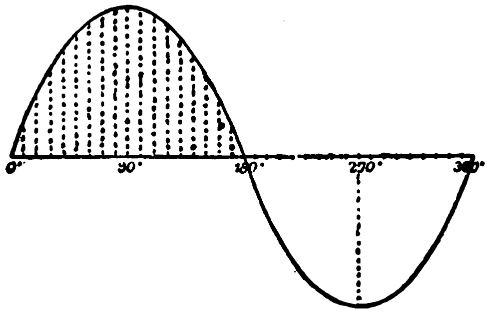

FIG. 7.—CURVE OF INDUCED ELECTROMOTIVE FORCE IN AN ORDINARY OR SINGLE-PHASE ALTERNATOR.

The heights of the curve above the horizontal line represent the momentary values of the electromotive forces: the depths below, in the second half of the curve, represent the inverse electromotive forces that succeed them. Each such complete set of operations is called a *period*, and the number of periods accomplished in a second is called the *frequency* or *periodicity* of the alternations, and is symbolized by the letter n. In 2-pole machines n is the same as the number of revolutions per second; but in multipolar machines n is greater, in proportion to the number of pairs of poles. Thus, in an 8-pole field with 4 north poles and 4 south poles around a centre there will be produced 4 complete periods in one revolution. If the machine revolves 15 times a second (or 900 times a minute) there will be 60 periods a second, or the periodicity will be 60. By revolving in a uniform field the electromotive forces set up are proportional to the sine of the angle through which the coil has turned from the position in which it lay across the field. If in this position the flux of magnetic lines through it were N, and the number of spirals in the coil that enclose the N lines be called S, then it can be shown that the value of the induced electromotive force at any time t when the coil has turned through angle $\theta = 2 \pi n t$ will be

$$E_\theta = 2 \pi n S \mathsf{N} \sin \theta \div 10^8,$$

or, writing D for $2 \pi n S \mathsf{N} / 10^8$, we have

$$E_\theta = D \sin \theta.$$

In actual machines the magnetic fields are not uniform, nor the coils simple loops, so the periodic rise and fall of the electromotive forces will not necessarily follow a simple sine law. The form of the impressed waves will depend on the shape of the polar faces, and on the form and breadth of the coils. But in most cases we are sufficiently justified in assuming that the impressed electromotive force follows a sine law, so that the value at any instant may be expressed in the above form, where D is the maximum value or *amplitude* attained by E, and θ an angle of *phase* upon an imaginary circle of reference. Consider a point P revolving clockwise round a circle (Fig. 8). If the radius of this circle be taken as unity, P M will be the sine of the angle θ, as measured from 0°. Let the circle be divided into any number of equal angles, and let the sines be drawn similarly for each. Then let these sines be plotted out at equal distances apart along the horizontal line, as in Fig. 8, giving us the sine curve.

In Fig. 8 one revolution of P around the circle of reference corresponds to one complete alternation or cycle of changes. The value of the electromotive force (which varies between +D and – D as its maximum values) may be represented at any moment either by the sine P M or by projecting P on to the vertical diameter, giving O Q. As P revolves, the point Q will oscillate along the diameter.

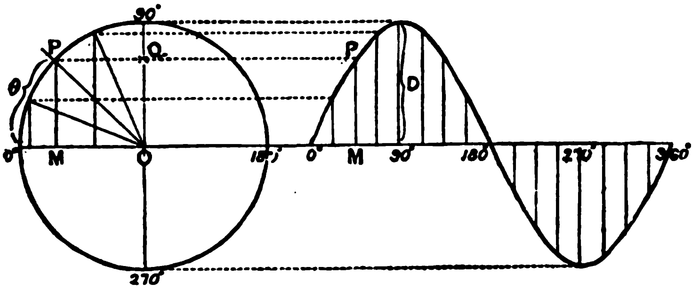

FIG. 8.

The currents which result from these periodic or alternating electromotive forces are also periodic and alternating; they increase to a maximum, then die away and reverse in direction, increase, die away, and then reverse back again. If the electromotive force completes 100 such cycles or reversals in a second, so also will the current.

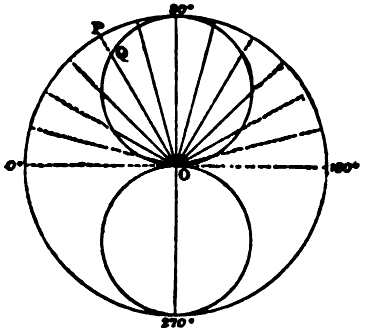

FIG. 9.

There is yet another way of representing periodic variations of this kind—namely, by a diagram akin to that used by Zeuner for valve-gears. Let the outer circle (Fig. 9) be, as before, a circle of reference around which P revolves. Upon each of the vertical radii describe a circle. Then the lengths such as O Q, cut off from the radii, represent the corresponding values of the sine of the angle. If a card with a narrow slit cut radially in it were made to revolve over this figure, the intersection with the two inner circles would show the varying electromotive forces in various positions.

The reader who desires to pursue the graphic study of these matters further should consult the excellent treatise of Prof. Fleming,[1] or that of Mr. Blakesley,[2] and sundry papers by Mr. Kapp.[3]

An application of this construction to a 3-phase system is shown in Fig. 10, where the three lines 120° from one another are supposed to revolve behind the two circular openings. The lengths of the three lines visible at any instant represent respectively the values of the 3 electromotive forces of the 3 currents.

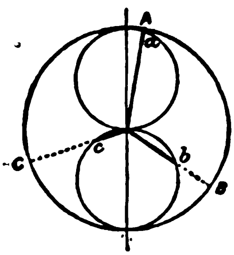

FIG. 10.

The ordinary measuring instruments for alternate currents, such as electrodynamometers, Cardew voltmeters and electrostatic voltmeters, do not measure the arithmetical average values of the amperes or volts. The readings of these instruments, if first calibrated by the use of continuous currents, are the square roots of the means of the squares of the values. They measure what are called *virtual amperes* or *virtual volts*. The mean which they read (if we assume the currents and voltages to follow the sine law of variation) is equal to 0·707 of the maximum values, for the average of the squares of the sine taken over either 1 quadrant or a whole circle is $\frac{1}{2}$; hence the square-root-of-mean-square value is equal to $1 \div \sqrt{2}$ times their maximum value. If a voltmeter is placed on an alternating circuit in which the volts are oscillating between maxima of + 100 and — 100 volts, it will read 70·7 volts; and 70·7 volts continuously applied would be required to pro-

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[1] Fleming, *The Alternate Current Transformer*, London, 1889.

[2] Blakesley, *Alternating Currents of Electricity*, London, 1889.

[3] Kapp on 'Alternate Current Machinery,' *Proc. Inst. Civil Engineers*, 1889, pt. iii.

duce an equal reading. If an alternate-current ampere meter reads 100 amperes, that means that the current really rises to + 141·4 amperes and then reverses to − 141·4 amperes; but the effect is equal to that of 100 continuous amperes; and therefore such a current would be described as 100 virtual amperes.

Alternating currents do not always keep step with the alternating volts impressed upon the circuit. If there is inductance in the circuit the currents will *lag;* if there is capacity in the circuit they will *lead* in phase. Fig. 11 illustrates the lag produced by inductance. The curve marked V represents the alternating volts; that marked C is the current curve. Distances measured from O along the horizontal line represent time. The impulses of current, represented by the blacker line, occur a little *later* than those of the volts. But inductance has another effect of more importance than any retardation of phase; it produces reactions on the electromotive force, choking the current down. While the current is increasing in strength the reactive effect of inductance tends to prevent it rising. To produce a current of 40 amperes in a resistance of 1½ ohms would require — for continuous currents — an E.M.F. of 60 volts. But an alternating voltage of 60 volts will not be enough if there is inductance in the circuit reacting against the voltage. The matter is complicated by the circumstance that the reactive impulses of electromotive force are also out of step: they are, in fact, exactly a quarter period behind the current. If an alternate current of C (virtual) amperes is flowing with a frequency of *n* cycles per second through a circuit of *induct*

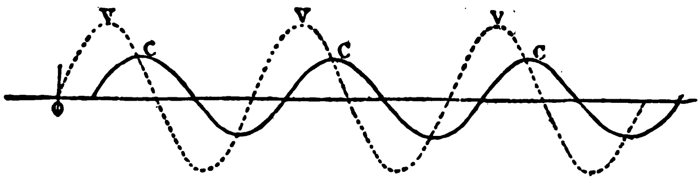

FIG. 11.—CURVE OF CURRENT LAGGING BEHIND CURVE OF VOLTS.

ance L, the reactive electromotive force[1] will be $2\pi n L C$ (virtual) volts. If, for example, $L = 0{\cdot}002$ henry, $n = 50$ periods per second, and $C = 40$ amperes, the reactive electromotive force will be 25·1 volts. Now, if we wish to drive the 40 (virtual) amperes not only through the resistance of 1½ ohms but against this reaction, we shall require more than 60 volts. But we shall not require 60 + 25·1 volts, since the reaction is out of step with the current. Ohm's law is no longer adequate. To find out what volts will be needed we have recourse to geometry.

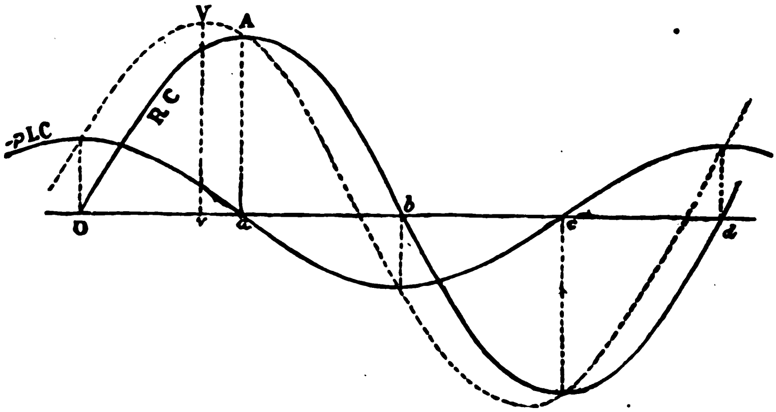

FIG. 12.

Plot out (Fig. 12) the wave-form O A *b d*, to correspond to the volts necessary to drive the current through the resistance if there were no inductance. The ordinate *a* A may be taken to scale as 60. This we may call the current curve. Then

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[1] This is calculated as follows. By definition, L, the coefficient of self-induction, or inductance, represents the amount of self-enclosing of magnetic lines by the circuit when the current has unit value ; hence when current has value C the actual self-induction is C times L. And, as the self-induced electromotive-force is proportional to the rate of change of this quality, we may write $E = L \, . \, dC / dt$. Now C is assumed to be a sine function of the time having instantaneous value $C_0 \sin 2\pi n t$; where C_0 is the maximum value of C. Differentiating this with respect to time we get $dC / dt = 2\pi n C_0 \cos 2\pi n t$. The "virtual" values of cosine and sine being equal we have for E the value $2\pi n L C$, but differing by ¼ period from the current in phase.

plot out the curve marked $-p\,L\,C$ to represent the volts needed to balance the reaction of the inductance. Here p is written for $2\,\pi\,n$. The ordinate at O is 25·1: and the curve is shifted back one-quarter of the period: for when the current is increasing at its greatest rate, as at O, the self-inductive action is greatest. Then compound these two curves by adding their ordinates, and we get the dotted curve, with its maximum at V. This is the curve of the volts that must be impressed on the circuit in order to produce the current. It will be seen that the current curve attains its maximum a little after the voltage curve. The current lags in phase behind the volts. If O *d* is the time of one complete period, the length *v a* will represent the time that elapses between the maxima of volts and amperes. In Fig. 13 the same facts are represented in a revolving diagram of the same sort as Fig. 9. The line O A represents the working volts R × C, whilst the line A D at right angles to O A represents the self-induced volts p L C. Compounding these as by the triangle of forces, we have as the impressed volts the line O D. The projections of these three lines on a vertical line while the diagram revolves around the centre O give the instantaneous values of the three quantities. The angle A O D, or ϕ, by which the current lags behind the impressed volts, is termed the *angle of lag*. However great the inductance or the frequency, angle ϕ can never be greater than 90°. If O A is 60 and A D is 25·1, O D will be 65 volts. In symbols, the impressed volts will have to be such that $E^2 = (R\,C)^2 + (p\,L\,C)^2$. This gives us the equation:—

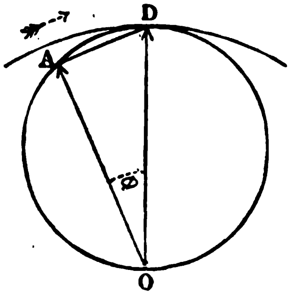

FIG. 13.

$$C = \frac{E}{\sqrt{R^2 + p^2 L^2}}.$$

The denominator which comes in here is commonly called *the impedance*.

In Figs. 14 and 15 the angle of lag is seen to be such that $\tan \phi = p\,\mathrm{L\,C} / \mathrm{R\,C}$ or $= p\,\mathrm{L} / \mathrm{R}$. And it is evident that the effect of the inductance is to make the circuit act as if its resistance, instead of being R, was increased to $\sqrt{\mathrm{R}^2 + p^2\,\mathrm{L}^2}$. In fact, the alternate current is governed, not by the resistance of the circuit, but by its impedance. At the same time, the current is lagging as if the angle of reference were not θ but $\theta - \phi$, so that the equation for the instantaneous values of C, when $\mathrm{E} = \mathrm{D} \sin \theta$, is

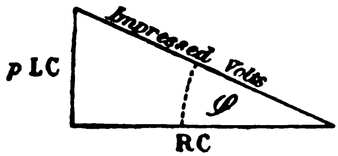

FIG. 14.

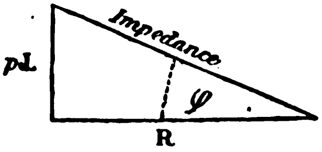

FIG. 15.

$$\mathrm{C} = \frac{\mathrm{D} \sin (\theta - \phi)}{\sqrt{\mathrm{R}^2 + p^2\,\mathrm{L}^2}}.$$

This is Maxwell's law for periodic currents as retarded by inductance. As instruments take no account of phase but give virtual values, the simpler form preceding is usually sufficient.

The effect of capacity introduced into an alternate-current circuit, as by the introduction of a condenser, is to produce a *lead* in the phase of the current. For when the volts are changing most quickly (as at O in Fig. 11) from negative to positive, the current running into the condenser is greatest; the maximum point of the current curve being thus nearly

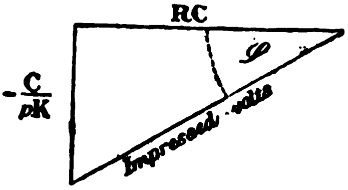

FIG. 16.

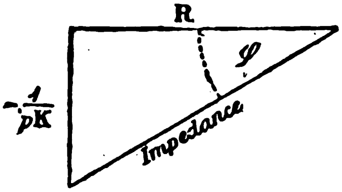

FIG. 17.

90° in advance of that of the curve of volts. The reaction of a condenser, instead of tending to prolong the current, tends to drive it back, and cause it to reverse its direction before the volts have reversed. The reactance is therefore written as $-1/pK$, and the angle ϕ will be such that $\tan\phi = -1/pKR$. The impedance will be $\sqrt{R^2 + 1/p^2K^2}$.

If both inductance and capacity are present,

$$\tan\phi = (pL - 1/pK)/R,$$

the reactance will be

$$pL - 1/pK;$$

and the impedance

$$\sqrt{R^2 + (pL - 1/pK)^2}.$$

Since capacity and inductance produce opposite effects, they can be used to neutralize one another. They exactly balance if $L = 1/p^2K$. In that case the circuit is non-inductive and the currents simply obey Ohm's law.

It will be seen that if in a circuit there is little resistance and much reactance, the current will depend almost exclusively on the reactance. For example, if $p\ (= 2\pi n)$ were, say, 1000 and $L = 10$ henries, while R was only 1 ohm, the resistance part of the impedance would be negligible, and the law would become

$$C = \frac{E}{pL}.$$

Self-induction coils with large inductance and small resistance are sometimes used to impede alternate currents, and are called *choking coils*, or impedance coils.

If the current were led into a condenser of small capacity (say $K = \frac{1}{10}$ microfarad, then $1/pK = 10{,}000$), the current running in and out of the condenser would be governed only by the capacity and frequency, and not by the resistance, and would have the value—

$$C = EpK.$$

The measurement of alternate-current power needs careful consideration. If to measure the power supplied to a motor or other part of an alternate current circuit, we measure separately with an amperemeter and voltmeter the amperes and volts, and then multiply together the readings, we obtain as the *apparent watts* a value often greatly in excess of the *true watts*, owing to the difference in phase, of which the instruments take no account. The true power (watts) is in reality $W = C\ V \cos \phi$, where C and V are the virtual values, and ϕ the angle of lag. But the latter is usually an unknown quantity. Hence recourse must be had to a suitable wattmeter; the usual form being an electrodynamometer specially constructed so that the high-resistance circuit in it shall be non-inductive.

Whenever the phase-difference (whether lag or lead) is very large, the current, being out of step with the volts, is almost *wattless*. This is the case with currents flowing through a choking-coil or into a condenser, if the resistances are small.

POLYPHASE GENERATORS.

We are now in a position to consider the question of polyphase generators. Briefly, the principle of polyphase working consists in providing the armature of the alternator with coils grouped in sets of two, three, or more, which come successively into action in each period.

Up to this point it has been assumed that the field-magnets of the alternator are stationary, whilst the armature revolves. But this is not necessarily so. Indeed, in the majority of modern alternators, whether single-phase or polyphase, the reverse arrangement is adopted. The field-magnets revolve, whilst the armature is fixed. The preference given to this arrangement arises from the greater facility with which insulation of the windings can be insured if the armature is stationary; and this becomes of great importance when, for the purpose of transmission to a distance, high voltages are used.

Suppose, then, we consider a very simple case of a stationary armature—a ring with two coils wound upon it at opposite parts—and a revolving field-magnet of simple bipolar form.

Here in Fig. 18 two such elementary machines are represented as connected by a couple of lines for a transmission of power, one serving as generator, and requiring to be driven by a steam engine or turbine, the other running as a synchronous motor. As is well known, such a motor will not be self-starting. It must be started by hand or in some other way, and run up to speed before it is thrown into the circuit; and when so started, runs in absolute synchronism with the generator, its electromotive force being in almost exact opposition of phase.

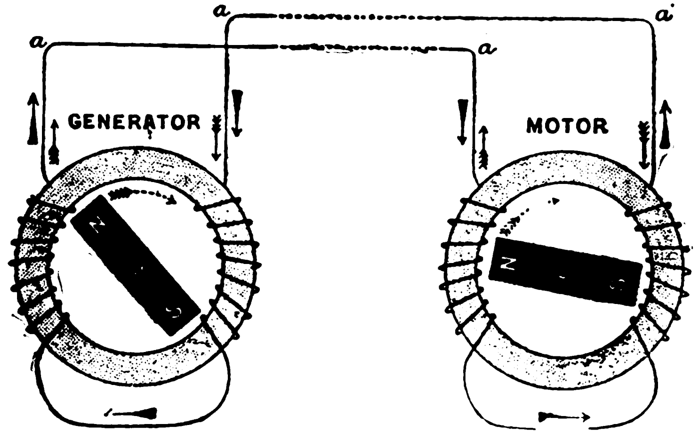

FIG. 18.—TRANSMISSION FROM A SIMPLE SINGLE-PHASE ALTERNATOR TO A SIMPLE SYNCHRONOUS MOTOR.

Some of the very earliest alternators—those of Lontin and Gramme, had revolving multipolar field-magnets with external stationary armatures. Gramme's alternators were built about 1877 for the purpose of supplying alternate currents for working Jablochkoff candles. The diagram, Fig. 19, of this machine shows that it had eight revolving poles, alternately north and south poles. The armatures, having a ring core of laminated iron, received the windings of copper wire in which the alternating currents were to be induced. Now it was found (as will be considered presently) that it was no use making the individual coils very broad. In fact, the closer together the coils in any one group can be huddled together, the more effective are they. Hence, in this machine had there been only eight narrow coils—one opposite each pole—there would have been much idle space on the machine. Gramme, therefore, filled up the idle space with other coils. The sections of the winding of this machine were, in fact, four times as numerous as the poles, and might have been coupled to feed four separate circuits. It is clear that the revolving poles would come past the four adjacent sections successively, so that the four alternating currents generated would differ *in phase* from one another. Gramme knew or discovered that it would not do to join all the coils together. He only joined together those that at any one instant were opposite the poles. So there were four separate circuits each consisting of eight coils joined up in series. And these four separate windings were led off to four entirely separate circuits, each supplying a number of Jablochkoff candles with current. Gramme's alternator was unquestionably a polyphase generator; but there is not the slightest evidence that he at any time attempted to combine the currents of separate phases for any useful purpose, or that he

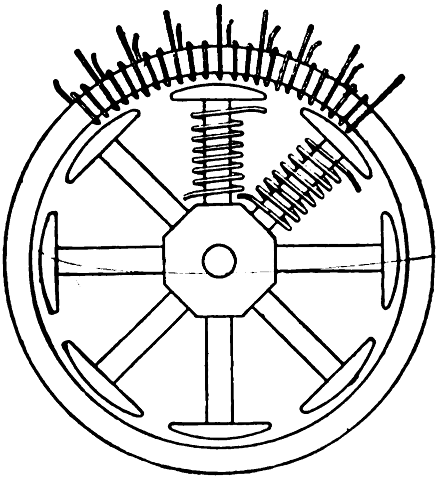

FIG. 19.—GRAMME ALTERNATOR.

knew that they could be so combined. On the contrary, he always kept the circuits separate because the several currents in them were not in phase with one another.

The large two-phase alternators at Paddington, designed by the late Mr. Gordon, have been running ever since 1883.

In 1886 F. Wynne proposed a system of distributing circuits, "the alternating currents in which are so ordered that while the rate of alternation is the same in all of them, the instant at which the alternation takes place is different."

It may be remarked, in passing, that in every type of alternator there will be idle space between the groups of coils if they are wound for single-phase working. Returning to Fig. 18, we note that between the two coils on the ring there was idle space which might advantageously be filled up. Suppose, then, that beside the two coils A A′ on each machine there are wound other two B B′ between the former pair, and that these are connected through a new pair of lines *b b* and *b′ b′*, Fig. 20. It is clear that a second set of alternating currents will be set up in B B′ which will be exactly a quarter-period in phase behind those in A A′. In fact, the two currents will be represented by the two waves of Fig. 21. The electromotive force in A will be greatest just when the pole of the magnet is passing its middle, for at that instant

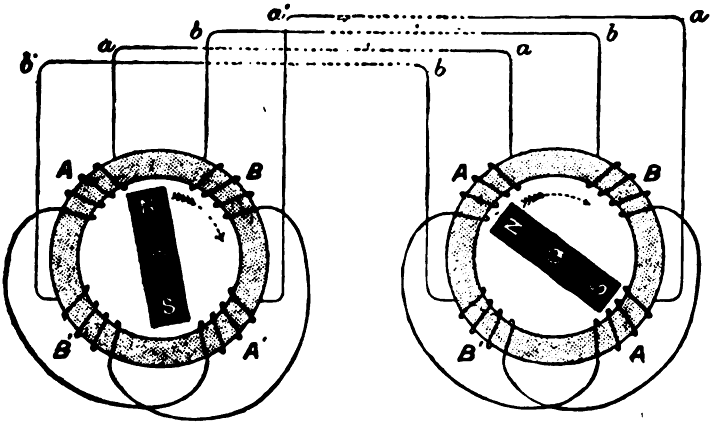

FIG. 20.—ILLUSTRATION OF TWO-PHASE TRANSMISSION.

the rate of change in the magnetization of its core is a maximum. And the maxima for the B coils will correspond to the zeros for the A coils and *vice versâ*. Two alternate currents differing in this manner by a quarter period are said to be "in quadrature." The currents in the A coils of the motor, tending to drag forward the pole of the field-magnet, will not have died down to zero before the currents in the B coils will have already begun; so that there is no dead-point. It is easy to see that in the motor there will be a regular displacement around the ring of the resultant poles. At the moment when the current in A A′ is at a maximum that in B B′ will be zero, and the magnetizing action of A A′ will be to produce two double-poles in the ring at opposite ends of a diameter right under the middle of the B B′ coils. As the current in A A′ dies down that in B B′ begins and increases, and therefore shifts the pole forward. When the currents in A A′ and B B′ have become equal, A and B will act together as one coil, while A′ and B will act together as another coil, the resulting poles lying now between B and A′ on the right and between B′ and A on the left. When the B current is at its maximum the poles will lie right under the middle of the A coils. A pair of travelling poles are therefore produced in the motor ring by the currents coming from the generator, and the magnet in the motor is continually trying to catch up these travelling poles. There are no dead-

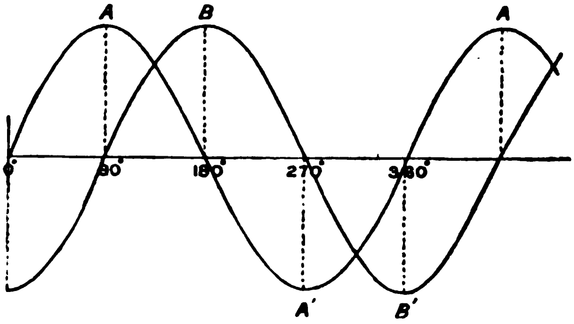

FIG. 21.—TWO ALTERNATE CURRENTS DIFFERING BY A QUARTER PERIOD.

points. The motor will be self-starting if its magnet is not too powerful, and will run up in speed until synchronism is attained. This is, indeed, the great advantage of polyphase currents—they enable motors to be self-starting. But this is not by any means the sole advantage of polyphase generators. We see that they enable a machine to be made which by doubling merely the quantity of copper in the armature will serve as a machine of double power.[1] It will take twice as much horse-power to drive, and will give out twice as much horse-power. But it will not cost twice as much, nor will it take up any more space. It is worthy of remark, too, that *the armature reactions for a two-phase generator are no greater than those of the same machine used as a single-phase alternator.*

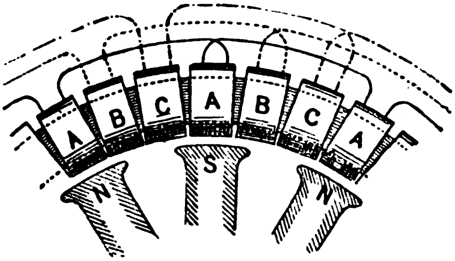

FIG. 22.—A THREE-PHASE GENERATOR.

Suppose that, instead of using two separate groupings of coils we had used three, as indeed Gramme employed in some of his smaller machines. We should then have three currents in three separate successive phases. If these were grouped as in Fig. 22, we might join up the A coils together into one circuit (the coils being wound or connected alternately right-handedly and left-handedly); the B coils being similarly joined up into a second circuit, and the C coils being joined into a third. It is clear that in each set the electromotive-forces would rise and fall in regular succession, and that the

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[1] H. Goerges : "The Comparative Output of the Continuous, Alternating and Drehstrom Armature," *Elektrotechnische Zeitschrift*, vol. xiii. p. 236. Herr von Dobrowolsky mentioned in the discussion of the above paper, a multipolar continuous-current machine which gave 11,000 watts ; the same field magnet with a three-phase armature gave an output of 30,000 watts.

electromotive force in B would not rise to its maximum until after that in A had passed its maximum and was falling. In fact, the differences of phase might be represented by the three curves of Fig. 23. Since the angular distance around the machines from one north pole to the next north pole corresponds to one whole "period" (p. 7), or to one complete revolution of 360° on the imaginary circle of reference (Fig. 8), we see that these three currents will differ in phase from one another by 60°. If we had a separate outgoing and return wire for each of the three circuits we should need no fewer than 6 lines from the machine to the (3-phase) motor which it supplied. But as will be seen, by adopting proper methods of grouping, this complication is unnecessary, the number of lines being capable of being reduced to four or to three. If an earth return were admissible, the number of actual line wires might even be reduced to two.

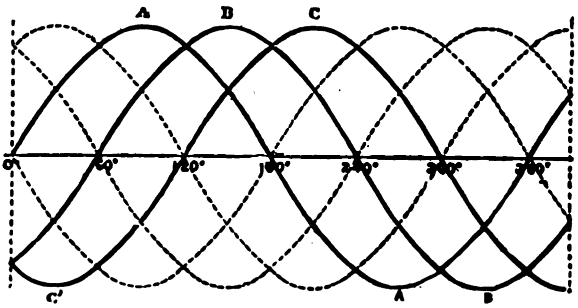

FIG. 23.—THREE-PHASE CURRENTS DIFFERING 60° IN PHASE.

Before we pass to the consideration of any modern polyphase generators we must devote a little attention to the effect of breadth of the windings in the coils of the armature. Consider a multipolar revolving field-magnet such as Fig. 24, in which we will assume that the pole-pieces have been so shaped that the magnetic field in the gap-space between poles and armature cores is distributed in a manner so as to give a regular and smooth wave-form for the curve of electromotive force induced in any conductor placed in the gap. We will represent electromotive forces which act upwards, or towards the reader, by a dot, and those which act downwards, or from the reader, by a cross placed in the section of the conductor. Then, as in Fig. 25, there will be induced electromotive forces acting upwards in those conductors in front of which the south pole is moving to the right, and downwards in those which the north pole is passing. But these electromotive forces will not be equal at the same instant amongst themselves: they will be greatest in those conductors which are most active—that is to say, in those which are passing through the strongest magnetic field. Each conductor will go through an equal cycle of inductive action, but it is clear that they come to their maximum one after the other. For convenience we will suppose this maximum to occur in each conductor as the middle of the pole passes it. Now suppose (as is usual in construction) that a number of these conductors are connected up, as in Fig. 26, to form a coil; their electromotive forces will be added together. If a view is taken, as in Fig. 26, where we are supposed to

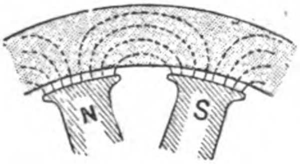

FIG. 24.

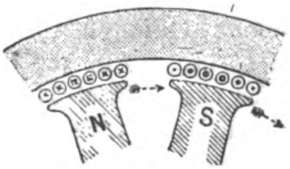

FIG. 25.

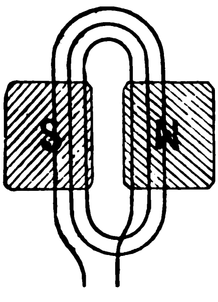

FIG. 26.

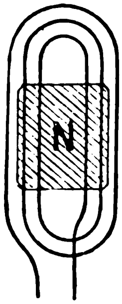

FIG. 27.

be looking back at the poles passing from right to left, we shall understand this a little more plainly. A moment later the north pole will come right behind the coil as in Fig. 27. This figure shows that there can be no advantage in having the inner windings of the coil much nearer together than the breadth of the pole-face, since at this instant their electromotive-forces are opposing one another. There is some advantage in filling up the coil a little narrower than the actual pole-face because of the disposition of the magnetic field. But the actual electromotive-force generated by a coil of a given number of turns would be greater if they could be all of the same size, so that all should reach their maximum action at the same instant.

This point may be further elucidated by the use of a clock-diagram. Suppose the maximum electromotive-force generated in one conductor to be represented by the pointer O A in Fig. 28. Then the projection of O A upon the vertical line O P gives the value of the electromotive-force at the instant when the angle A O P corresponds to the phase of the induction that is going on in the period. Let there be two other conductors situated a little further along, so that their electromotive-forces would be represented separately by O B and O C. We have to find what the effect will be of joining them all in series. By the rules for compounding vector quanties, we shall find their resultant by drawing from A the line A B′ equal and parallel to O B, and from B′ the line B′ C′ equal and parallel to O C. Then O C′ is the resultant; and its projection O Q upon the vertical line gives the instantaneous value of the united electromotive-force of the three conductors. Had they all been placed close up to one another at A without any difference of phase between them, the resultant would have been O A‴, and this projected upon the vertical line gives O P‴ as the instantaneous value.

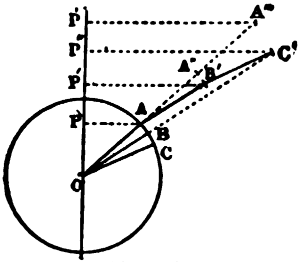

FIG. 28.

A numerical way of considering the matter may be useful. Suppose each conductor to generate an electromotive-force, the virtual value of which is 1 volt: then if three such conductors are connected up in series their total electromotive-force cannot be 3 volts unless they lie so close together that they all receive their maximum values at the same time. Any spreading out of the coils *must* lower the value of the resultant electromotive-force.

It is therefore worth while to calculate a breadth coefficient for a coil of any particular angular breadth. Let the symbol ψ stand for the difference of phase between the centre of any coil and its outermost conductor on either side. If the machine has a two-pole magnet the value of ψ is simply half the angular breadth (in radians) subtended by the coil. If the machine is multipolar, having p pairs of poles, then the angle ψ of the phase difference will be equal to half the angular breadth (as measured on the machine) multiplied by p. Or, if the linear breadth of the coil measured along the circumference be called b, and the diameter of the machine is d, the angle ψ of the phase difference corresponding to the half-breadth will be $= b\,p \div d$. Now the average value of the virtual electromotive-force in all the conductors comprised within this breadth will be given by the formula

$$\frac{1}{\psi}\int_0^{\psi} e\,.\cos\gamma\,.\,d\gamma\,;$$

where e is the virtual value electromotive-force in any one conductor and γ is the angle of difference of phase between the E.M.F. in any conductor of the coil and the E.M.F. in the central conductor of the coil. If we call the part of this expression which depends on ψ the breadth coefficient, and denote it by q, then performing the integration we have

$$q = \sin\psi \div \psi.$$

In order to give some numerical values we may anticipate some of the constructions later shown. For instance, in a ring wound with four coils each covering one quadrant (as in some 2-phase motors, see Fig. 49),

$$\psi = 45^\circ = 0{\cdot}785 \text{ radians}\,;\; q = 0{\cdot}90.$$

In the case of a ring wound with three coils, each covering 120° (see Fig. 54),

$$\psi = 60^\circ = 1{\cdot}05 \text{ radians}; \; q = 0{\cdot}82.$$

In the case of a ring wound with six coils each covering 60° (as in Fig. 57),

$$\psi = 30^\circ = 0{\cdot}523 \text{ radians}; \; q = 0{\cdot}95.$$

As an example, consider a multipolar two-phase generator, having armature conductors carried through holes in the core disks, and having 12 equally spaced holes in the repeat from one N-pole to the next N-pole. In this case six of the conductors belong to one phase, six to the other, and each group will consist of three up and three down. The three in a group occupy one-fourth the whole breadth, or are equivalent to 90° on the circle of reference: but as the conductors are confined within holes, the virtual angular distance between the two outer conductors of the three is 60°, and the half-distance 30°; whence $q = 0{\cdot}95$.

Before leaving this question of the compounding of two electromotive-forces that are in different phases with one another we may note that the principle of vector summation used above leads to a very simple result where two electromotive-forces are concerned. Let O P represent one of these electromotive-forces, O Q the other, the angular phase difference between them being P O Q or ϕ. Compounding them in the usual way, by drawing P R equal and parallel to O Q we get the resultant O R, which represents the magnitude and relative phase of the resultant electromotive-force. Here by ordinary geometry $OR = \sqrt{OP^2 + OQ^2 + 2\,OP \cdot OQ \cos\phi}$. This is obviously a maximum when the phase-difference is zero.

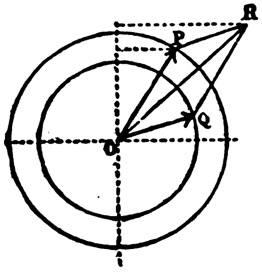

FIG. 29.

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[1] Compare Thomson and Tait's *Treatise on Natural Philosophy*, vol. i. § 58.

MODERN POLYPHASE ALTERNATORS.

We are now prepared to examine some modern examples of polyphase machines.

First the three-phase generators at Lauffen. These are driven by turbines in the river Neckar, and were erected in 1891 for supplying current to the town of Heilbronn, six miles distant, They were, however, at first employed in the now famous historical transmission of power from Lauffen to Frankfort, a distance of 110 miles, on the occasion of the Frankfort Exhibition. They were constructed by the Oerlikon Co. of Zürich from the designs of Mr. C. E. L. Brown; and have revolving internal field-magnets with an external armature with zigzag arrangements of conductors passing through holes in the core-rings. Fig. 30 gives a general view, whilst Fig. 31 shows the field-magnet after the armature has been slid away for inspection. The machine generates three currents, each of 1400 amperes, at a pressure of about 50 volts; taking 300 horse-power when running at 150 revolutions per minute. The armature has an external diameter of 189·4 cm. (nearly 6 feet) and an internal diameter of 176·4. The total thickness of core-rings, parallel to the shaft, is 38·0 cm. Around the inner periphery of the core-rings are 96 circular holes 33 mm. in diameter, at distances of 60 mm. apart. Each of these holes is lined with a tube of asbestos, and through each passes a solid copper rod 29 mm. in diameter. The core-rings, built up of segmental stampings, are assembled in a strong cast-iron frame. The winding, if such it can be called, is in three independent zigzags of 32 conductors each connected according to the following scheme:—

Set A, 1, 4, 7, 10, ... 91, 94.
Set B, 95, 92, 89, 86, ... 5, 2.
Set C, 93, 90, 87, ... 3, 96.

The ends of Nos. 94, 2, 96, are connected to a common junction J, while Nos. 1, 95, and 93 are severally brought out to three external terminals. This constitutes a star-winding

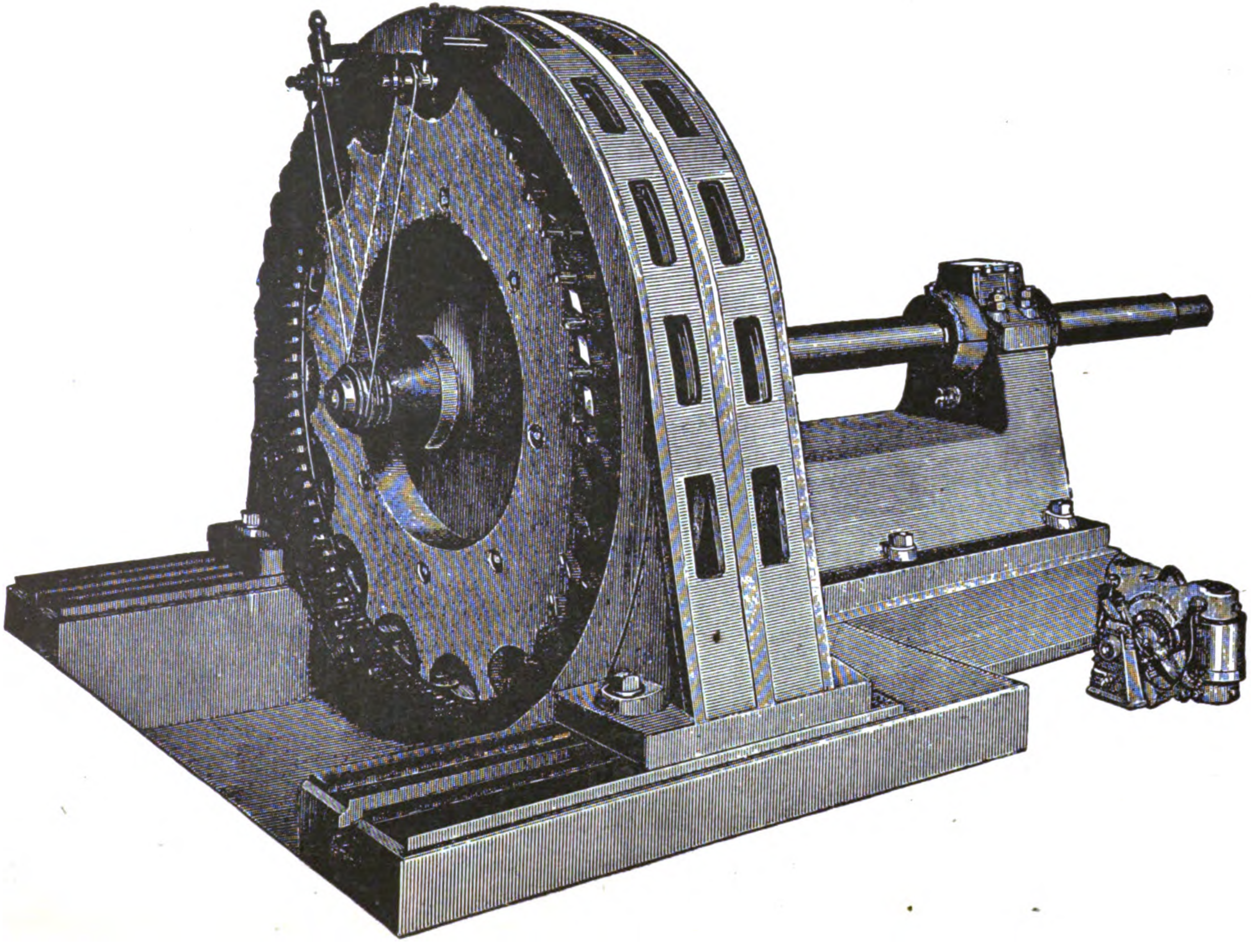


FIG. 30.—BROWN'S THREE-PHASE ALTERNATOR, USED AT LAUFFEN.

(p. 43); the connections of the circuits are shown in Fig. 32, the general arrangements of the windings being illustrated in Fig. 33.

The gap-space between the armature core-ring and the pole-faces of the field-magnet is 6 mm. This field-magnet has 32 poles. It is of great solidity and simplicity, having but a single magnetic circuit. The exciting coil is wound in a channel on the periphery of a sort of pulley of cast iron, to which are bolted two steel rims, each carrying 16 polar expansions or horns. Each of the polar faces has an area of 36 × 16 sq. cm. The channel is 18 cm. wide and 9 cm. deep. In it lie 496 windings of copper wire 5 mm. diameter. A section of this channel is given in Fig. 34; and Fig. 35 illustrates the way in which the polar horns project inwardly, the N-poles between the S-poles over the exciting coil. This

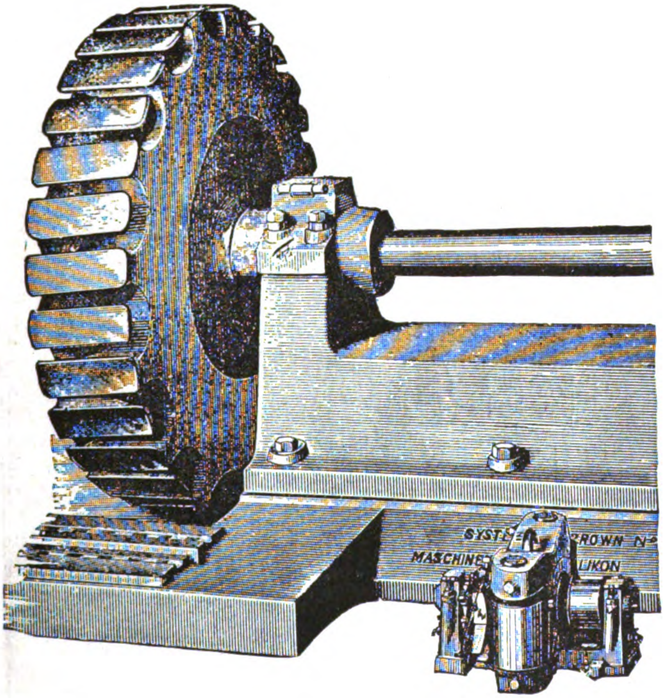


FIG. 31.—FIELD-MAGNET OF THREE-PHASE ALTERNATOR AT LAUFFEN.

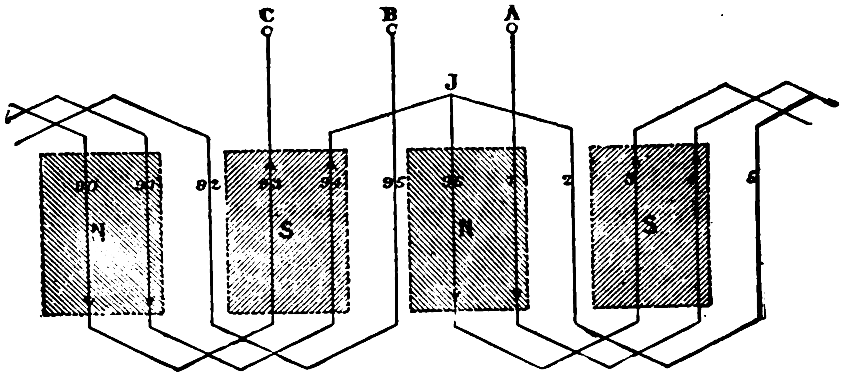


Fig. 32.—Developed Diagram of Winding of Three-phase Alternator.

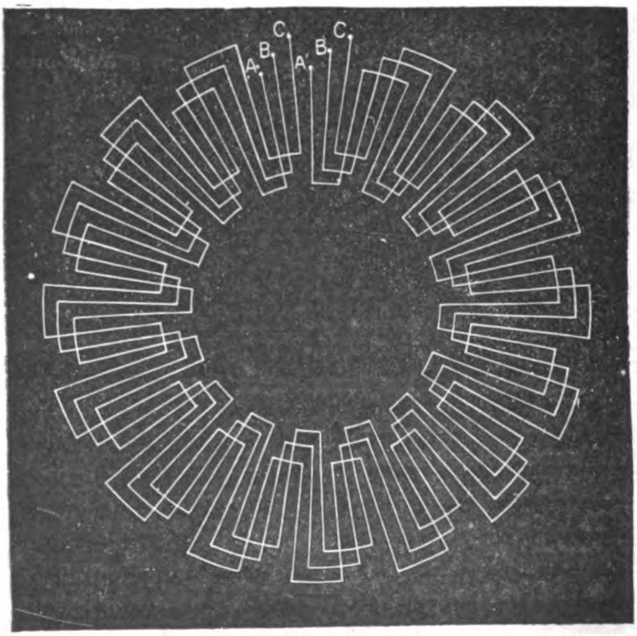


Fig. 33.—Arrangement of Windings of Three-phase Alternator.

arrangement reduces the cost of construction and of excitation to a minimum. In fact, on open circuit only 100 watts are spent on excitation—one-twentieth of one per cent. of the output; and at full load, when the armature reaction is a maximum, it is still far less than one per cent. This excitation is furnished by a small separate dynamo. The exciting current is conveyed to the rotating part by means of flexible metallic cords running over insulated pulleys, in lieu of the usual contact-rings and brushes. At full speed and normal voltage, the loss by friction and hysteresis is 3600 watts, or under 1·7 per cent. of the maximum output. The loss of resistance of armature windings at full load is 3500 watts, making total loss about 4 per cent., and commercial efficiency over 95 per cent. The heating is, in the total absence of eddy-currents, quite negligible. The weight is 4½ tons. As there are sixteen pole-pairs, and the speed is 150 per minute, the frequency is 40 periods per second. The electromotive-force generated in each of the three windings, as measured between the common junction J and the outer terminal, could be increased up to 55 volts.

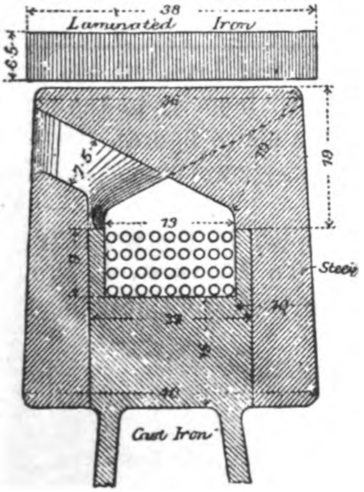


FIG. 34.—SECTION OF FIELD-MAGNET

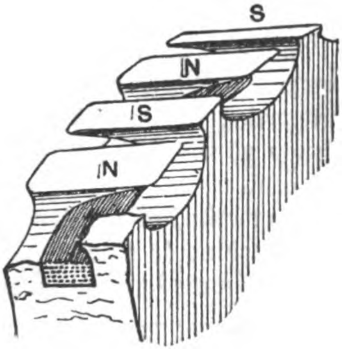


FIG. 35.—SKETCH OF FIELD-MAGNET.

The following are some of the measurements made on this machine by the official jury under Prof. H. F. Weber in 1891:—

Horse-power of Turbine.	Electrical horse-power yielded by Alternator.	Horse-power lost.	Efficiency. per cent.	Current in one circuit.	Volts between common junction and terminals.	Speed. Revolutions per minute.
87·4	75·1	12·3	88	336	54·7	150
120·1	107·5	12·6	90	470	56·1	150
154·7	142·2	12·5	92	644	54·2	149·7
167·2	154·4	12·8	92·6	677	55·9	149·5

The tests were not carried up to full load, but the jury remarked that, assuming the losses to increase in the same proportionality as indicated by the above figures, the efficiency at the full load of 300 horse-power would be 95·4 per cent.

The method of construction adopted in the armature of this machine is worthy of note. To Mr. C. E. L. Brown belongs the credit of introducing the practice of embedding the conductors in holes pierced through the core disks. In nearly all polyphase machines, whether generators or motors, one finds, in fact, the conductors either threaded thus through holes or else imbedded in slots between deeply cut teeth. Toothed cores, as in Fig. 36, are now almost universal in American dynamos and motors; but the pierced cores are distinctively Swiss. At the Oerlikon works circular holes are largely used. Messrs. Brown, Boveri & Co., of Baden (Canton Aargau), sometimes employ circular perforations, but their standard style is, as in Fig 37, an oblong hole about 50 mm. long and

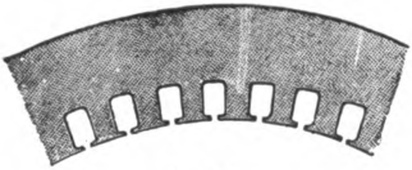


FIG. 36.

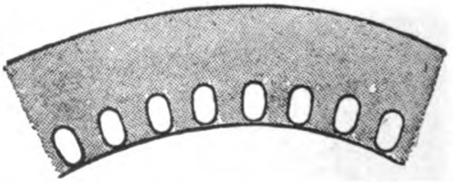


FIG. 37.

20 mm. wide, the insulating lining being a strong tube of corresponding section made of a preparation of paper.

There are several advantages accruing from the bedding of the conductors in iron. The mechanical construction is improved, for the conductors are securely held and driven without the need of any binding wire. Centrifugal force does not displace them, and the clearance between the revolving and the stationary parts may be greatly reduced, thereby economizing exciting power. But more important than these are two other advantages. There are no useless eddy-currents generated in conductors so bedded in iron, so they need not be laminated or stranded, but may be made of solid copper rods. Also there is no tangential drag of the magnetic field upon conductors so bedded: the drag comes on the iron instead of coming upon the copper.

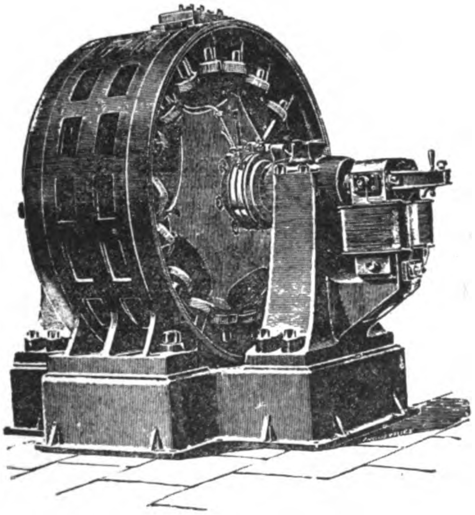


FIG. 38.—POLYPHASE GENERATOR OF THE OERLIKON CO.

A more recent (1894) 3-phase generator, built by the Oerlikon Company, having the same general features of field-magnet and of armature, but with various improvements in detail, is described and depicted in the third (1894) edition of

Kapp's 'Electric Transmission of Power.' A general view is shown in Fig. 38.

As a second example of a polyphase generator we select the large 1000 horse-power machines shown at the Chicago Exhibition by the Westinghouse Co. of Pittsburgh. One of these is shown in Fig. 39, which should be compared with Fig. 2. It is virtually a double machine, having side by side two similar field-magnets, and within two similar armatures upon the same shaft. But the armatures are "staggered"; that is to say, they are so mounted that one of them has an angular advance over the other equal to one-half the angular breadth from a N-pole to a S-pole. By merely shifting the second armature the same machine might be used as one single-phase alternator. In this case the adoption of a

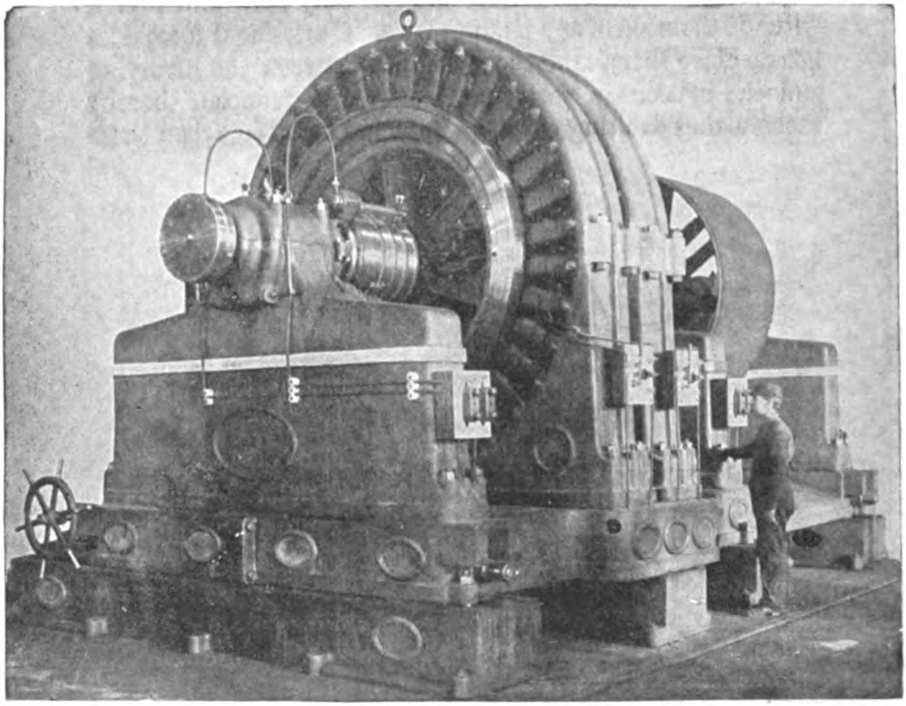


FIG. 39.—WESTINGHOUSE TWO-PHASE GENERATOR.

2-phase system is not accompanied by any economy of space or material in the machine. These alternators are of 750 kilowatt output, running at 200 revolutions a minute, and having a frequency of 60 periods per second. The field-magnets each consist of 36 poles of laminated mild steel cast solidly into the outer yoke. Each armature has 36 teeth; and between these are slipped in and secured the previously-wound armature coils.

Some years ago Mr. William Stanley, of Pittsfield (Mass), introduced a 2-phase alternator of the "inductor type," in which the whole of the copper windings, both primary and secondary, are fixed, the only moving part being of iron.

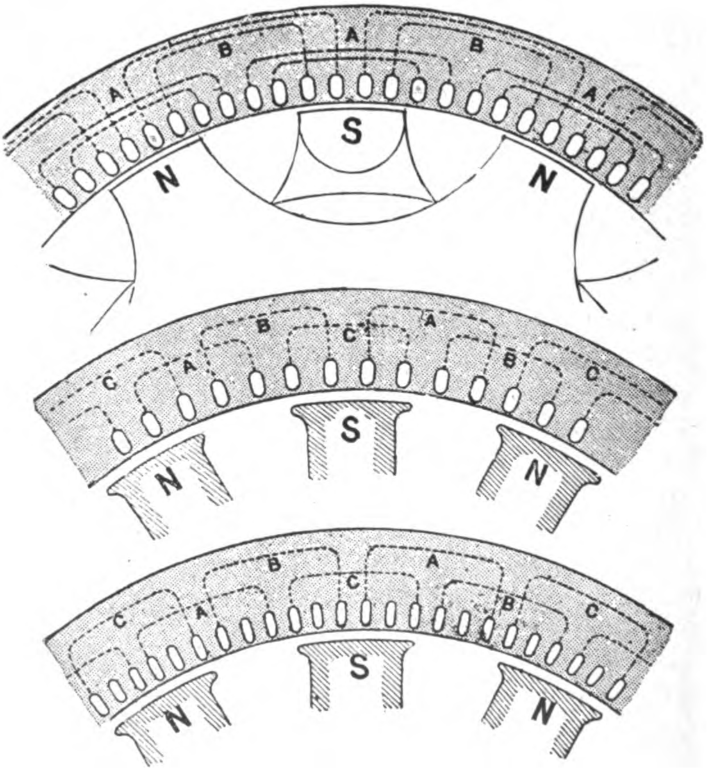
The Brush Electrical Engineering Company, of London, has produced 2-phase alternators on Mr. Mordey's well-known type of construction by the following modification of the armature. In order to allow half the coils to be displaced through the breadth of half a coil, two coils are removed at opposite ends of a diameter. A similar modification makes the machine applicable for 3-phase work. Mr. G. Kapp's recent alternators are capable of being similarly adapted.

In recent years various forms of polyphase alternators have been introduced by Messrs. Brown, Boveri & Co., of Baden, Switzerland, the designs being worked out by Mr. C. E. L. Brown, formerly of the Oerlikon Company.

Some of these machines present no special feature to distinguish them from ordinary alternators beyond having the coils of the armature arranged in sets of twos or threes to correspond to 2-phase or 3-phase work.

Recently Mr. Brown has adopted a form of revolving field-magnet having a series of outward-pointing radial poles, with the peculiarity that only alternate poles are wound with exciting coils, the intermediate ones being simply projections of cast iron of larger cross section than the intermediate cylindrical cores that receive the coils.

Another feature introduced by Brown in the winding of alternators, whether for single-phase, 2-phase or 3-phase work, and applicable to motors as well as generators, is the arrangement of the connecting wires where they emerge outside the core-rings in two sets in different planes. This construction may be noticed in Fig. 171 and Plate II. Though a detail it is of great use in obviating risks of short-circuit. In Fig. 40 this construction is diagrammatically displayed, showing how both the A set and B set of windings in a 2-phase generator may be grouped so as to utilize for each lap two sets of holes side by side. This has some advantages over using single holes of very large size. These would interfere



Figs. 40, 41, 42.

more with the magnetic circuit, and tend to set up greater heating in the polar parts of the field-magnet. Single holes, being of greater depth radially, would, moreover, cause greater magnetic leakage.

Fig. 41 shows an adaptation of the method of arranging the windings of a 3-phase generator, so that the loops of coil can still be situated in two planes. The A coils will of course be connected together in series, though they lie alternately in the inner and in the outer positions; and so likewise the B and the C coils (see Plate II.).

Fig. 42 shows how the core-rings may be utilized for a 3-phase generator (or motor) with a winding in which all the holes are not employed. This winding was used to save the necessity of making a fresh set of stamps for the core-disks. The magnetic reactions are less, when the unused holes are left in the spaces as shown, than would be the case if the core-rings at these parts were not pierced.

To Mr. Brown is due the introduction of the vertical-shaft type of generator so admirably adapted for running direct from a turbine. A large number of these machines are now in operation. One of these machines, a 3-phase generator, has for some years done excellent work at Schönenwert near Aarau in Switzerland, furnishing current for motors in a large shoe-factory. More recently the town of Aarau has been provided with a central station which derives its power from the waters of the river Aar by means of turbines.

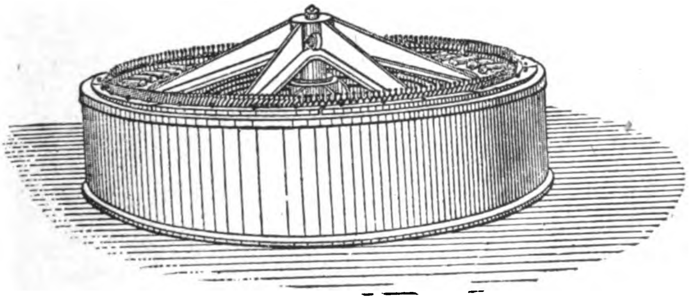
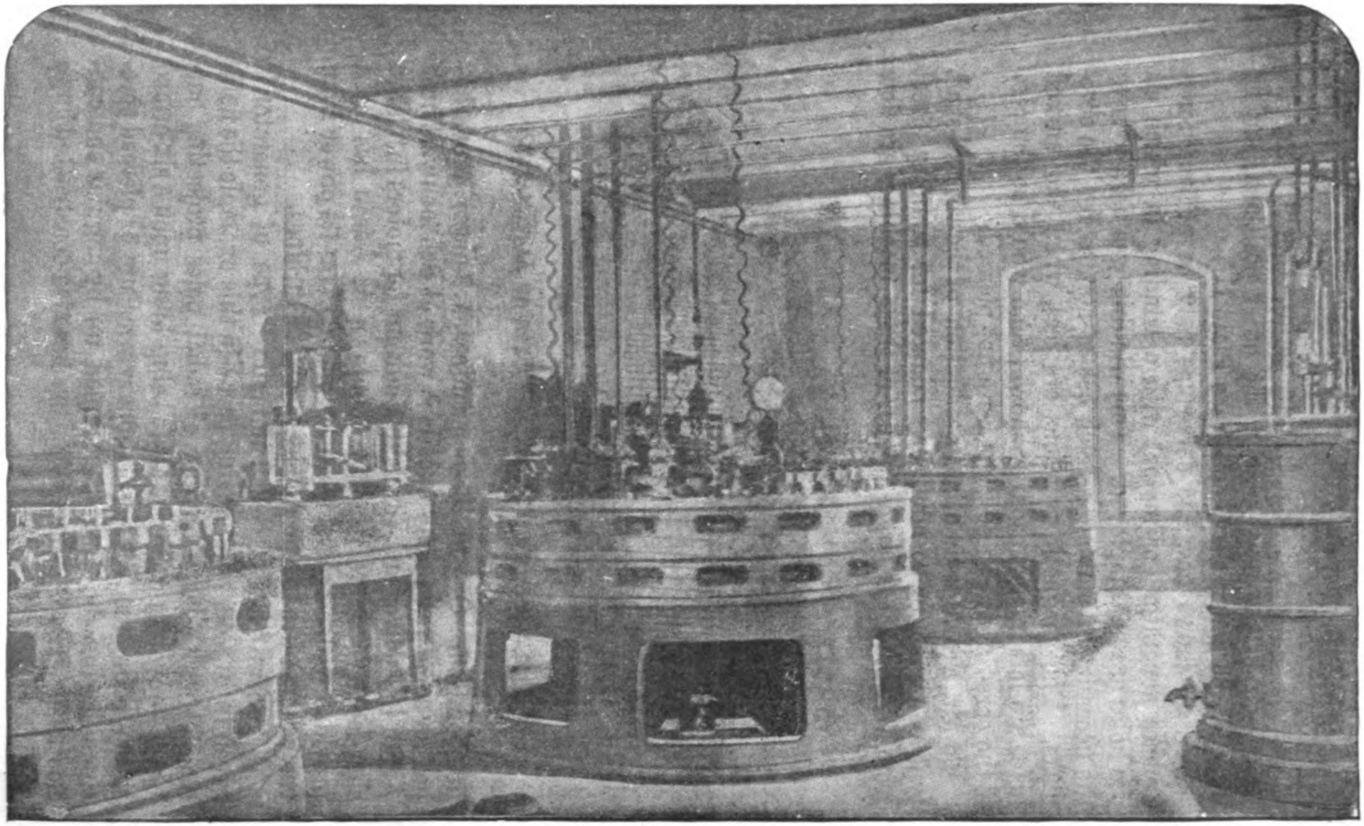


Fig. 43.—Brown's "Umbrella" Type of Alternator.

A general view of this "umbrella" type of generator is given in Fig. 43. Its moving part—in this example the field-magnet with a large number of radial poles projecting outwards—revolves upon the shaft being hung upon a six-armed spider. Outside it is the stationary armature of pierced core-rings, having the windings coupled up by angular end connections. The Oerlikon Company has also constructed large



Three-phase Generators at the Power Station at Hochfelden (Switzerland).

alternators of the umbrella type, for the power stations at Bellegarde, Bremgarten, and at Hochfelden. Fig. 44 gives a view of the latter station, showing the three generators, which were designed by Mr. C. E. L. Brown in 1890. They are 3-phase machines, each of 200 horse-power, running at 180 revolutions per minute. Excepting in having the vertical shafts directly above the turbines by which they are driven, they closely resemble the Lauffen generators. They give 86 volts pressure between the terminals. To raise the voltage each is connected to a 3-phase transformer immersed in oil; one of these transformers being visible on the right hand of the cut. The pressure is raised to 13,000 volts, at which pressure the currents are conveyed by three wires, each 4 millimetres in diameter, to the Oerlikon Works (a distance of 24 kilometres, or about 15½ milles), where by means of step-down transformers of similar construction the pressure is lowered to 190 volts, and the currents are distributed for lighting and power at this pressure.

The Niagara Alternators.—When the project of utilizing the water-power of Niagara by turbines was taking shape the Cataract Construction Company invited many different manufacturers in Europe and in America to submit plans. The machines were to be of 5000 horse-power, driven by turbines making 250 revolutions per minute. Many of these designs were extremely good; nevertheless it was determined to have the machines manufactured in America, owing to the high tariff charged on imported goods, and to the cost of transsport. Some of the designs (including those of Mr. Brown) were of the "umbrella" type, but for various reasons (turning mainly upon the constructive difficulties arising from size and speed) Professor Forbes and Mr. Coleman Sellers were instructed in May 1893 to get out further plans for alternators of the proposed type. Professor Forbes fixed upon an externally-revolving umbrella field-magnet, with inwardly-pointing poles held together by an external annulus of steel, as possessing both great strength and a large fly-wheel action. At first he prepared designs for a 2-phase machine, having the low frequency of 16⅔ periods per second, with 8 poles. Eventually, after the Westinghouse Company had been selected as manufacturers, it was decided to fix the frequency at 25, and to wind the armatures for 2000 volts. The drawings published by Professor Forbes[1] relate to the earlier design, and have certain complications about the armature which became unnecessary when it was decided to keep the voltage at 2000, instead of working at 30,000 volts.

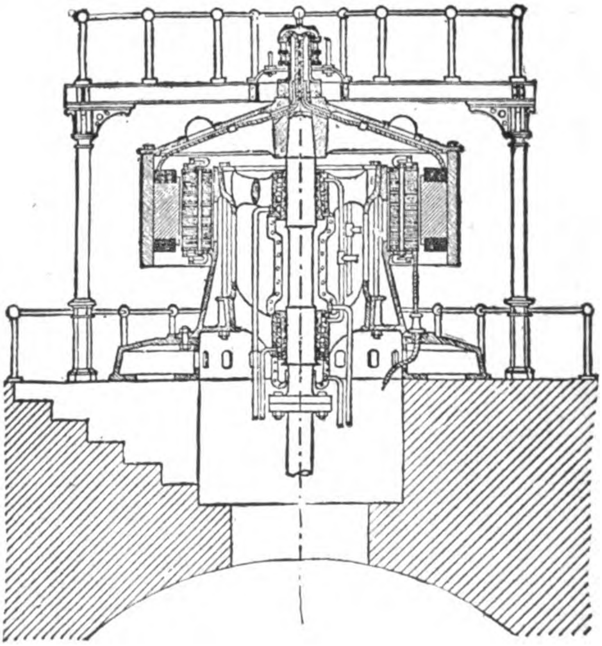


FIG. 45.—SECTION OF THE NIAGARA 5000 H.P. GENERATOR.

Fig. 45 gives a sectional drawing[2] of the Niagara machine as built. Its outer rotating field-magnet consists of a wrought-steel ring, to which are bolted internally twelve inward pointing cast-iron pole-cores. It is hung to the vertical shaft by a six-arm cast-steel spider. The shaft passes up through a bronze bearing, which is supported by four arms projecting inwardly from a cast-iron ring. The latter is itself adjustably fixed within an outer cylindrical mantle of cast-iron, which stands on the foundation ring and carries the stationary in-

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[1] *Journal of the Institution of Electrical Engineers*, November 1893.

[2] From the *Electrical Engineer* (N.Y.) of January 16, 1895.

ternal armature. The core of this is built up of thin sheet iron segmented disks, clamped together by eight bolts of nickel-steel. There are 187 nicks or grooves in the outer face of the core to receive the copper conductors. These are 374 in number, rectangular, being 32 × 8 millimetres in section, two such bars lying side by side in each groove between the armature teeth. The insulation is of mica. They are coupled up by bent end connectors in two independent circuits. The actual voltage at the speed of 250 revolutions per minute is 2250, the output in each of the two circuits being 775 amperes. As there are twelve poles the frequency at this speed is 25 periods per second. The field-magnet windings are

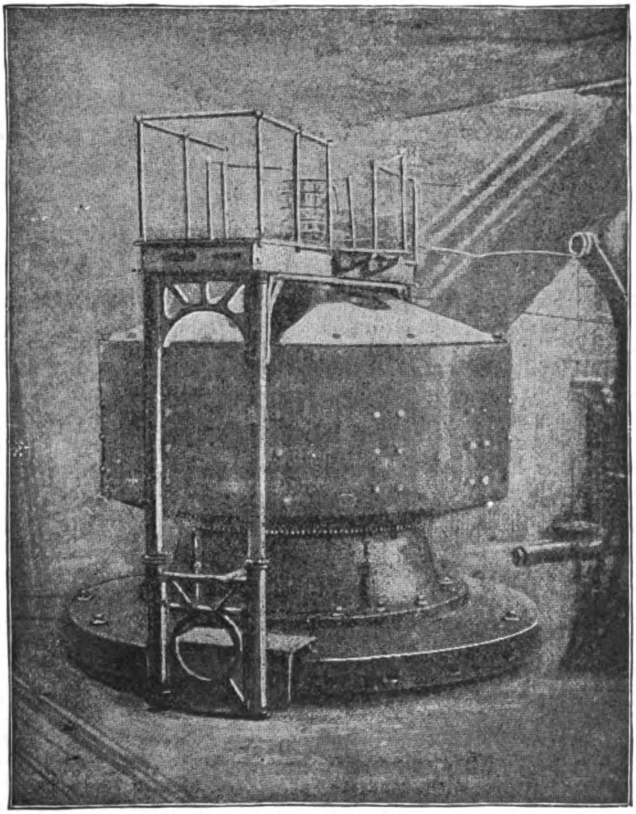

FIG. 46.—ONE OF THE NIAGARA GENERATORS.

supplied with continuous current (derived from a rotating transformer) through two slip-rings fixed on the top of the shaft. Fig. 46 gives an external view of one of the machines. The total height is about 13 feet.

The Strassburg Generators.—The 3-phase generators recently constructed by the Allgemeine Company, of Berlin, for the City of Strassburg are of the "inductor" type without any moving copper.

Asynchronous Generators.—It has been found by several experimenters independently—amongst them Mr. C. E. L. Brown, and the Engineers of the General Electric Company, at Schenectady, New-York—that the asynchronous motors, whether polyphase or monophase, can act as generators provided they are mechanically driven at a slightly higher speed than that of synchronism (see p. 145). But it is not possible to work a circuit with only one such machine to be used as a generator—it is not self-exciting. There must be an alternate or polyphase current already supplied to the mains or terminals. It would probably be convenient in those central stations where the load is apt to show very sudden increase, to use one or more asynchronous generators along with other alternators, as the asynchronous generator might be kept turning as a non-loaded motor at a speed just below synchronism until required. On merely quickening up the speed of its engine (without waiting to "synchronize") it will begin to work as a generator, its electromotive impulses synchronizing perfectly with those of the circuit, though its speed is not synchronous.

In some experiments made in Sweden [1] by Mr. Danielson in 1892 a 3-phase asynchronous motor was coupled with a synchronous 3-phase generator. The former was then driven as generator, and the latter used as motor, running on a brake. It was found that the asynchronous machine would not generate if the circuit included only resistances (lamps) or resistance with self-induction.

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[1] *Electrical World* (N. Y.), Jan. 1893, p. 44; *Electrical Review*, xxxii. p. 169, Feb. 1893.

CHAPTER II.

COMBINATIONS OF POLYPHASE CURRENTS.

NOT until after the growth of the idea of combining currents in different phases for driving motors did any one suggest ways of combining into regular systems the separate groups of coils in which induction was occurring in different phases. The combining of currents of two or three phases has been usually considered in relation to motors; it may, however, be equally usefully discussed in relation to generators.

There are two general ways of combining polyphase circuits, which may be characterized respectively as *star-groupings and mesh-groupings.* In the star-grouping, the coils in which the power is generated or absorbed are joined to a common junction from which they branch star-wise each to its own line. The comparison may be made in the concrete case of a 3-phase system. Fig. 47 is a diagram of a star-grouping of three coils, *a*, *b* and *c*, designed to receive currents in regular rotatory order, the current flowing in toward the centre first through *a* (and passing out by one or both of the other coils), then through *b*, then through *c*. Fig. 48 shows three coils, *p*, *q* and *r*, grouped in mesh fashion. Here the

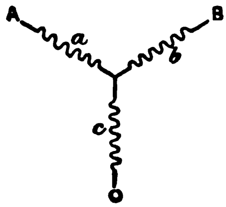

FIG. 47.

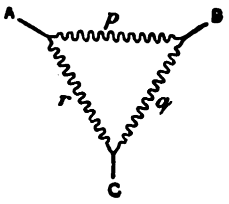

FIG. 48.

coils form a closed mesh connected to the circuit at the corners.

There are also several more complex groupings in which both these features are used at the same time. Fig. 60, on p. 48, shows a combination of the two systems.

A further illustration is afforded by a simple 2-phase system, in which the several possible groupings may be separately considered.

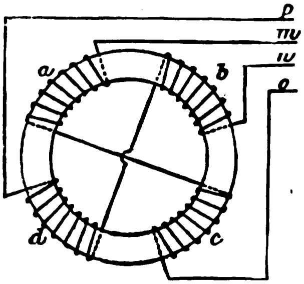

Fig. 49.

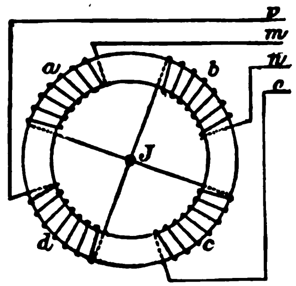

Fig. 50.

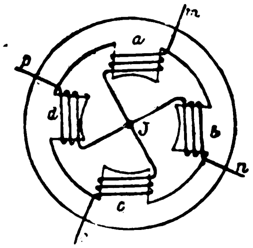

Fig. 51.

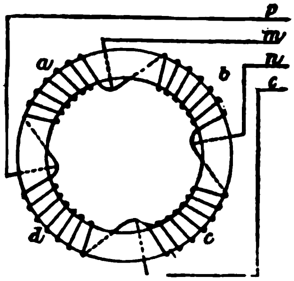

Fig. 52.

(1) Independent windings. The coils on the generator which are in the same phase may be connected together in any methods used in single-phase machines, and the coils belonging to the other phase may be likewise so connected; so that we have two totally independent circuits as shown in Fig. 49, wherein the coils *a* and *c* belong to one circuit, connected to the lines *m* and *o*, and the coils *b* and *d* constitute an entirely separate circuit joined to the lines *n* and *p*.

(2) Star-grouping. The coils or groups of coils may each

have one terminal joined to a common junction J forming a star and the free terminals joined to the line-wires, as shown in Figs. 50 and 51. In Fig. 50 the coils are represented as wound upon a ring; whereas in Fig. 51 they are wound upon polar projections. They differ in their magnetic relations; but, considered simply as circuits, they are identical.

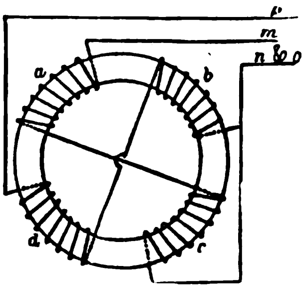

Fig. 53.

(3) Mesh-grouping. The coils may be connected together so as to form a closed circuit and the line-wire attached to the points of junction between the coils, as shown in Fig. 52, which is an ordinary 4-part Gramme ring, having, however, its connections joined to four lines instead of being provided with a 4-part commutator.

(4) In case No. (1) where the coils are otherwise independent, two of the terminals belonging to different phases may be connected together and a single return-wire employed as shown in Fig. 53, instead of having four line-wires.

COMBINATION OF ELECTROMOTIVE-FORCES.

It is necessary to consider in each of these cases how the electromotive-forces of the separate coils are combined, and the effect of such combinations upon the electromotive-forces between the line-wires. Let us say that the E.M.F. of the coil *a* follows the law $v \sin \theta$ where v is the maximum value attained in each period, as calculated from previous considerations (see pp. 8 and 25).

Two-phase Systems.—If the coils are joined up independently, as in Fig. 49, or in a star-grouping, as in Fig. 50, the E.M.F. between the terminals *m* and *o* will be $2\,v \sin \theta$. In the case of the star winding there is also a pressure of $\sqrt{2}\,v \sin (\theta + 45°)$ between the terminals of *m* and *n*; that is to say, the pressure between the two line-wires of different phases is 1·4 times the pressure at the terminals of one coil and is 45° of phase in advance of the foremost coil.

When the coils are joined up in a mesh, as in Fig. 52, the E.M.F. between m and p is of course the E.M.F. generated in a, namely $v \sin \theta$, while the pressure between n and p follows the law $\sqrt{2}\ v \sin (\theta - 45°)$; that is to say, it is 1·4 times the pressure at the terminals of one coil and is midway in phase between coils a and b.

Where a common return is used the E.M.F. between each outgoing wire and the return will be simply double the E.M.F. of one coil, but the E.M.F. between the two outgoing wires will be 1·4 times this or $2 \sqrt{2}\ v \sin (\theta + 45°)$.

Three-phase Systems.—In order to find how the pressure varies between the line-wires of a 3-phase system when the coils of the generator are joined up in star fashion, let us consider Fig. 54. Coil a, as before, may be taken as a standard of reference, the pressure at its terminal being $v \sin \theta$. Consider the E.M.F. outward from the common junction as positive. The E.M.F. in b is $v \sin (\theta - 120°)$.

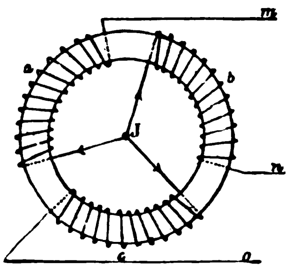

FIG. 54.

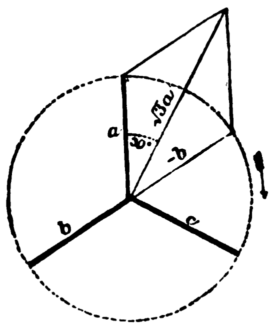

FIG. 55.

The pressure between m and n is the differences of the electromotive-forces in a and b, or $v \sin \theta - v \sin (\theta - 120°)$ $= \sqrt{3}\ v \sin (\theta + 30°)$.

Example: If $v = 141$, the virtual volts generated by a are 100. The pressure between the lines m and n

$= \sqrt{3} \times 100 = 173$ virtual volts. The phase of this pressure is 30° in advance of phase of *a*.

The clock diagram given in Fig. 55 shows the matter clearly. The lines *a*, *b* and *c* represent the E.M.F. in the respective coils. To subtract *b*, produce it backwards as shown $-b$, and then completing the parallelogram we get the resultant E.M.F. 30° in advance of *a*, and $\sqrt{3}$ times as great.

If the coils of a 3-phase system are joined up in mesh fashion, as shown in Fig. 56, the E.M.F. between *o* and *m* is simply the E.M.F. generated in *a*.

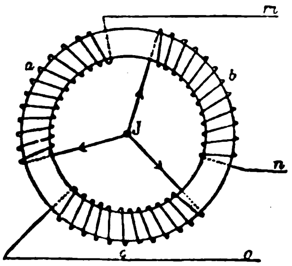

FIG. 56.

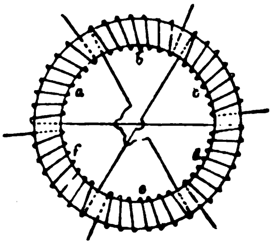

FIG. 57.

A 3-phase generator or motor is not generally built with the simple arrangement of three coils as shown in Fig. 54. There may be six coils or sets of coils, as in Fig. 57, in which case those pairs which are opposite to each other in phase are joined in series, so as to act like one coil of double the E.M.F.

Fig 57 is in fact a diagrammatic representation of the arrangement of coils in Fig. 22, only the coils are spaced round an entire circle instead of merely spanning the space between one N-pole and the next. The coils being joined in pairs, we have virtually only three coils. Taking them in the order

$$a + d$$
$$c + f$$
$$e + b$$

we see that the pairs are 120° apart, and can be treated in the same way as the three coils in Fig. 54, that is to say, we may join them up star fashion, as in Fig. 58, in which case the E.M.F. between m and o will be $2\sqrt{3}\,v \sin(\theta + 30^\circ)$, or they may be joined up in mesh fashion, as in Fig. 59, in which case the E.M.F. between k and s will be $2\,v \sin\theta$.

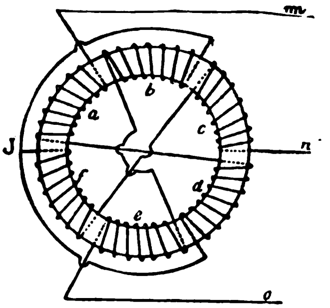

Fig. 58.

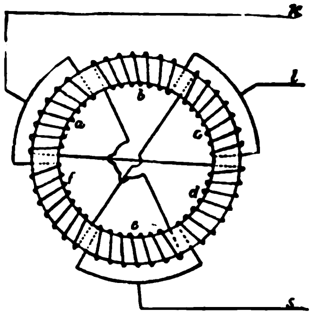

Fig. 59.

A combination of the star and mesh groupings is shown diagrammatically in Fig. 60. Fig. 61 shows how six coils wound right-handedly on a ring might be so connected. In this case the E.M.F. between any two terminals, for example A and B, would follow the law $2\,v \sin(\theta - 60^\circ)$ where the E.M.F. in a is $v \sin\theta$.

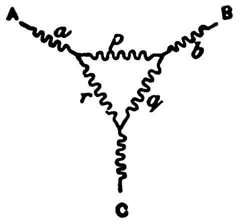

Fig. 60.

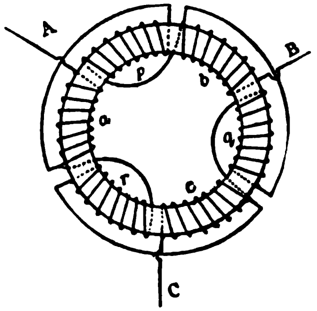

Fig. 61.

Such combinations were first suggested by Dobrowolsky with the object of gaining in a motor a more uniform torque at different points of the revolution than would be attained by a set of coils in three phases only.

COMBINATIONS OF CURRENTS.

It is necessary also to consider the relative values of the current in the different conductors of a polyphase system when they are joined up in star or in mesh. In the first place there are a few general rules which are of service in determining these values in any given arrangement.

1. Where any number of wires meet at a common junction the algebraical sum of the instantaneous values of all the currents (taking, say, the direction outward from the junction as positive) is equal to zero.

2. In the case of alternating currents this rule is only applicable to instantaneous values and not to the effective values of the currents, unless they are all in the same phase.

3. Where two currents differing in phase meet in a common conductor, their resultant can be found by the graphic method given in Fig. 29, where O P and O Q may represent the two currents and O R their resultant. Or the following formula, which follows at once from the graphic construction, may be employed:—If $a \sin (\theta + \phi_{\prime})$ is one of the currents and $b \sin (\theta + \phi_{\prime\prime})$ is the other, then their sum is $\sqrt{a^2 + b^2 + 2\,a\,b \cos (\phi_{\prime\prime} - \phi_{\prime})} \sin (\theta + \phi_{\prime\prime\prime})$
where

$$\tan \phi_{\prime\prime\prime} = \frac{a \sin \phi_{\prime} + b \sin \phi_{\prime\prime}}{a \cos \phi_{\prime} + b \cos \phi_{\prime\prime}}.$$

4. It is necessary to adhere strictly to some notation indicating the directions taken as positive and negative. For instance, in considering a star-grouping of coils in a generator, it is convenient to take the direction outward from the common junction as positive, and in the line-wires the positive direction will then be from the generator to the lamps or motors. In a mesh-grouping the direction clockwise round the mesh may be taken as positive.

In applying these rules to determine the relative values and phases of current in any system, we first of all observe that the currents will be dependent upon the impedances of the various circuits. We can only lay down general rules where we have a symmetrical system symmetrically loaded.

Taking then a 2-phase generator with mesh-grouped coils whose two circuits are equally loaded, the current in the line *m* (Fig. 52) is the sum at every instant of currents in the coils *a* and *b*. If the current in $a = \mathrm{C} \sin \theta$, and that in $b = \mathrm{C} \sin (\theta - 90°)$, the positive direction of the current in *b* being away from the junction, we must write

$$\mathrm{C} \sin \theta - \mathrm{C} \sin (\theta - 90°),$$

from which we get the current in $m = \sqrt{2}\, \mathrm{C} \sin (\theta + 45°)$.

In the case of star and independent windings, the currents in the coils are necessarily the same as in the line-wires.

When in a 2-phase system, one return-wire is used as in Fig. 49, even though the load on each phase may be the same the difference of phase between the two currents is increased to a little more than 90°, that is to say, the current in the leading phase rises to a maximum a little earlier than it would if the currents were independent, and the other current rises to a maximum a little later; but this departure from 90° difference of phase may be made as little as we like by decreasing the resistance of the line. Even in a line wasting 15 per cent. of the total power, the difference in phase is only increased by about 6°,[1] so that for practical purposes we may combine the currents as in the case given above for the mesh winding, and say that the current in the return-wire is $\sqrt{2}$ or 1·4 times as great as in the other wires and midway between them in phase.

The currents in the coils of a 3-phase generator grouped in a mesh can be combined in the same manner as in the case of two phases.

We have from Fig. 55, p. 46,

$$\sin \theta - \sin (\theta - 120°) = \sqrt{3} \sin (\theta + 30°).$$

That is to say, that the line-wire current is $\sqrt{3}$ or 1·73 times as great as the current in the coils. Comparing these results with those obtained for the electromotive-forces we see that

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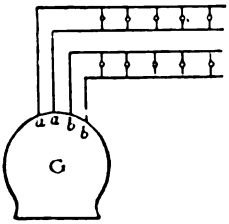
[1] Rodet et Busquet, *Les Courants Polyphasés*, p. 19.

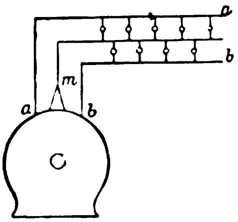
in star grouping the electromotive-force between the line-wires is greater than at the terminals of the coils, and the current remains the same, while with the mesh-grouping, the current in the line-wire is greater than in the coils, but the electromotive-force remains the same.

GROUPING OF LAMPS IN A POLYPHASE SYSTEM.

The foregoing ideas may be illustrated by the various ways of grouping lamps in polyphase circuits.

Lamps on Two-phase Circuit.—Suppose a generator G, Fig. 62, supplies two currents in quadrature. These may be used, as in this figure, as two independent services to supply lamps, while for motor purposes all four wires can be brought into operation. But, as has been already pointed out, three wires only need be used, a middle wire *m*, Fig. 63, serving as a common return. For carrying equal numbers of lamps on the two circuits the middle wire must be thicker than the two outer wires, but need not be of double section as the currents are out of phase; the maximum current in the middle wire being $\sqrt{2}$ times as great as that in the other wires. The voltage across from *a* to *b* will not be double of the voltage from *a* to *m* or from *b* to *m*; but will be $\sqrt{2}$ times as great. In fact, if the lamps in the two rows were 70-volt lamps, a third row might be added of 100-volt lamps connected from *a* to *b*.





A mesh system of lamps may be arranged as in Fig. 64, using four line-wires. In this case if the lamps were 100-volt lamps, the voltage from *a* to *a′* would be 141·4 volts, and that from *b* to *b′* also 141·4 volts. With equal numbers of lamps the current in any one of the four lines will be 1·41 times that required for any one row of lamps.

If the lamps are arranged in a star system of grouping, as in Fig. 65, there will be some advantage in connecting the common junction J to earth (i. e. to a common return, which need not be insulated), provided the coils of the generator are also grouped starwise, so that they can also be earthed. This is really equivalent to a 4-phase system having the phases in two coincident pairs. If the lamps are 100-volt lamps there must be 200 volts from *a* to *a′* or *b* to *b′*; and there will be 141·4 volts from *a* or *a′* to *b* or *b′*.

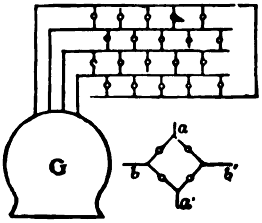


FIG. 64.

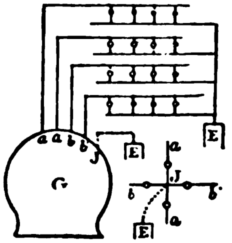


FIG. 65.

Lamps on Three-phase Circuit.—Three-phase circuits akin to the two last-mentioned, are illustrated in Figs. 66 and 67. Fig. 66 represents a mesh-grouping. With equal numbers of lamps in the three rows, the current in any one of the three lines will be equal to 1·73 times that required for any one row of lamps.

If the lamps are grouped star-wise (Fig. 67), the mid-point may be earthed, provided the corresponding point on the generator (or transformer) is also earthed. This was done in the case of the lamps at Frankfort on the 3-phase transmission from Lauffen in 1891, and is carried out in the 3-phase distribution at Heilbronn (p. 217). If the lamps are 100-volt lamps, the pressure between any two of the three circuits will be 173 volts. Neither the star nor the mesh-grouping of itself secures absolute independence of the various parallels of lamps, though the star methods are, on the whole, more nearly so. When lamps are turned on or off in any one row the pressure of the other rows is always more or less affected thereby; but the use of the common return from the centre of the star system greatly reduces this.

Amongst the curiosities of polyphase work may be mentioned incandescent lamps with three wicks meeting in a point,

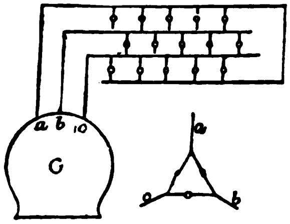


Fig. 66.

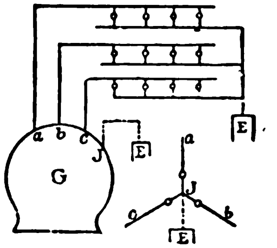


Fig. 67.

and three outer terminals. Such were first shown in 1891 by Dobrowolsky. Others having three spirals were constructed by the Edison-Swan Company for the author for his lecture at the Royal Institution in February 1894. At the same lecture a 3-phase arc-light was shown with three carbons meeting at 120° to one another. The arc or flame showed a gyratory movement.

ECONOMY OF COPPER.

It has been claimed that by the adoption of polyphase systems, as compared with single-phase systems, there is effected a saving in the total weight of copper needed for the transmission of a given amount of power to a given distance. Of the correctness of this view in the main there can be no question; the conflict of opinion which has arisen being due to the circumstance that the various disputants have taken different criteria as the basis of comparison. The economy in copper—which is the most important factor in the cost of long-distance transmission—depends, as every electrical engineer knows, upon the electric pressure at which the current is transmitted; so that, in comparing together different systems, the comparison, to be fair, should be made upon the basis of employing equal voltage. But the question arises, Between what points is the voltage of the system to be taken in the comparison?

But it must be remembered that while high voltage is the secret of economy of copper in electric transmission and distribution, the voltage at which a system can be operated is determined by different considerations in different cases. In low-pressure systems of distribution the voltage is determined by the glow-lamps ; and as these are not practicable for voltages over 100 to 110, it follows that this is the limit for the voltage of the system. On the other hand, where the distribution is for power only and not for lighting, or where transformers can be used, the limit to the voltage is fixed by the entirely different consideration of the insulation which can be safely relied upon.

Hence, to answer the question just raised we must distinguish broadly between the two classes of systems and consider each on its own merits.

1. *High-pressure Systems.*—In a high-pressure system it is the difficulty of devising an insulation that will not break down which practically limits the voltage. Therefore, in comparing polyphase and single-phase we must take cases that are on a par from this point of view of insulation. It has been common in the case of single-phase (and also in that of continuous currents) to keep one line at the potential of the earth, and to insulate the other sufficiently, having regard to the pressure between the two lines. In this case it is clearly the maximum pressure between the lines that forms what we here are calling *the voltage of the system.* If, however, both the lines are independently insulated from earth so as to withstand safely the maximum pressure occurring between line and earth, then the lines may have between themselves a voltage equal to double that maximum pressure without fear of a breakdown, provided always the lines, and the respective circuits into which they lead, are so well insulated from one another as to obviate risks in this respect. The question then arises whether, in comparing the advantages of various systems, we shall take as the basis of com-

parison the pressure between any two lines or the pressure between line and earth. If we take the maximum pressure between any point of the line and earth as the basis of comparison, then there is no saving in copper by the employment of polyphase currents; for each line of *any* system, carrying a certain current at the maximum allowable pressure above the earth may be considered as dependently transmitting a certain amount of power; and therefore the total power is simply proportional to the number of line-wires, to which the total weight of copper is also proportional.

For instance, a 3-phase system joined up in star fashion with the common junction to earth, and having a pressure of 1000 volts between each line and earth (and therefore 1732 volts between line and line), has no advantage so far as the insulation of the line is concerned, over a single-phase system having a pressure of 1000 volts between each line and earth (and therefore 2000 volts between line and line). To transmit equal power, with equal loss in the lines, each of the two wires of the single-phase system must be 1½ times as heavy as each of the three wires of the 3-phase system. The two systems will require equal total weights of copper.

If, on the other hand, we take the maximum pressure between any two lines as the basis of comparison, we are now equating not the risks of breakdown of the lines, but the risks of breakdown of the apparatus, machines, transformers, etc., in which the goodness of the insulation must be considered equal. And on this basis of comparison there is a decided economy of copper by the employment of 3-phase currents, as can be seen by the following considerations.

Taking first an installation connected in mesh fashion (Fig. 56, p. 47), if the distribution is symmetrical the current a in one limb of the mesh (see Fig. 160, p. 187) is $\frac{1}{\sqrt{3}}$ of the current p in the line (see p. 50). Therefore the power absorbed in one limb is $\frac{1}{\sqrt{3}}\,p\,V$, where V is the pressure between the lines (p and V being measured in virtual amperes and virtual volts respectively). The total power is therefore $\sqrt{3}\ p\ V$. Taking instead a star connection, the pressure between the ends of one branch of the star is $\frac{1}{\sqrt{3}} V$, and the current in each branch is the same as the current in the line. Therefore the total power is as before, viz. $\sqrt{3}\, p\, V$. Let the resistance of one line be r, then the total loss of power in the three lines is $3\, p^2 r$. Now consider a single-phase system to transmit the same amount of power $\sqrt{3}\ p\ V$. Let x be the resistance of one line, such that there shall be the same loss $3\ p^2 r$. The total resistance of the two lines will be $2\ x$. The current will be $\sqrt{3}\, p$, and the loss $6\, p^2 x$. Hence $6\, p^2 x = 3\, p^2 r$; or $x = \frac{1}{2} r$. The resistance of one of the two single-phase lines will have to be one-half the resistance of one of the three-phase lines. Or, to put it in another way: the single-phase system requires two wires, each of double cross-section, as against the three wires of the 3-phase system. The weight of copper required on the 3-phase system is only three-fourths of that required by any single-phase system.

A 2-phase system with four wires is exactly on a par with a single-phase system, so far as the economy of copper is concerned.

If in a 2-phase system only three wires are used, one wire acting as a common return, the pressure between the two outgoing lines is about $\sqrt{2}$ times the pressure V between each line and the return; we must therefore regard the voltage of the system as $\sqrt{2}\ V$. More copper is required in this case than for a single-phase transmission at $\sqrt{2}\ V$.

2. *Low-pressure Systems.*—Here the pressure is limited by the requirements of incandescent lamps. What we wish to attain is to have the voltage between the lines that transmit the power as high as possible, consistently with keeping the pressure at the lamps terminals at the right amount. Putting aside for the moment the so-called three-wire and five-wire method of distribution in which balancing wires are used, and comparing 3-phase using three wires with single-phase using two wires, we see, from the considerations on p. 50, that with the lamps joined in mesh the 3-phase system

has the advantage of using only 75 per cent. of the copper required for the single-phase system. If the lamps are connected in star fashion, we have the pressure between the lines $\sqrt{3}$ times the pressure on the lamps, with a result that the copper employed is only one-fourth of that required for a single-phase system with two wires; but the system could not be regulated without having a return-wire from the common junction, and it would be comparable thus rather to a three-wire than a two-wire single-phase system.

Mr. Goerges, in a paper read before the Elektrotechnischen Verein, reprinted in the *Elektrotechnische Zeitschrift*, January 17, 1895, gives the following data for weight of leads for equal power, drop and voltage:—

For single-phase, 2 wires	100
" single-phase, 3 wires (assuming the third wire half the section of the other)	31·35
" 2-phase, 4 wires	100
" 2-phase, 3 wires (reckoning the voltage between lines and common return)	72·8
" 3-phase, 3 wires (mesh)	75
" 3-phase, 4 wires (neutral wire from common junction) ..	29·2

Another way of putting the matter is to consider the voltage to which the wires of a system would have to be raised in order, with equal total weights of copper in the lines, that equal power may be transmitted with equal loss. If a 3-phase system is arranged in star fashion with four wires, the voltage between any of the three lines and the neutral wire being reckoned as 1000, the voltage across the lines of a single-phase system to be equally efficient must be 1850. It is here assumed that the system is balanced so that no current goes by the fourth wire. The extreme voltage from wire to wire in the 3-phase plant will be only 1732. If there is no fourth wire used in the 3-phase system (as need not be where motors only or transformers are to be supplied), the voltage of the single-phase system must, if it is to be equally efficient, be raised to 2000 volts.

COMBINATION OF MAGNETIC FIELDS.

As the main object of polyphase working is to produce rotatory magnetic fields by the combining of alternating magnetic fields in different phases, it is appropriate now to consider how currents which are different in phases can be combined to produce resultant magnetic fields.

We may take it that when a simple alternating current is carried in a coil around a core, the magnetism along that core will be an alternating magnetism. If the core is merely air, we shall have an alternating field. If the core is of iron, the flux of magnetic lines through it will be an alternating flux; that is to say, a flux which begins, increases to a maximum, then dies away, reverses in direction, increases to a reversed maximum, and dies away to begin the cycle over again. The frequency of this alternating flux will be the same as that of the impressed magnetomotive-force; that is to say, of the current. If the iron is properly laminated, and there are no secondary circuits to perturb by reactions, the rise and fall of the magnetic flux will be practically in phase with that of the surrounding current. Any eddy-currents in the core, and any secondary currents induced by the core in neighboring conductors, will inevitably have the effect of retarding the phase of the alternating magnetic flux, and of causing it to lag. Such reactions by induced currents in closed secondary circuit plays a vitally important part, as we shall see, in the modern polyphase motor.

It is self-evident that (in the absence of secondary reactions) a magnetizing force which alternates along a fixed direction will produce an alternating magnetic flux; whereas a magnetizing force which is constant in value, but is continuously changing in direction—rotating in space—will tend to produce a rotating magnetic flux. Whether this resulting rotating magnetic flux will have a constant value or a uniform speed of rotation, will depend not only upon the uniformity of the impressed rotatory magnetizing force, and on the absence of secondary currents, but will also depend on the shape of the magnetic masses, as to whether they also are magnetically symmetrical around the axis of rotation of the magnetizing forces.

For the present, to gain simplicity in grasping the subject, we will consider the problem of the combinations of magnetizing forces to produce a resultant magnetizing force.

If the direction and intensity of a magnetic field may be represented by the direction and length of a line, then we may apply the ordinary parallelogram rule for the compounding of vectors, and find the resultant of two magnetic fields that differ in direction and magnitude by compounding the vectors that represent them, and drawing the diagonal. In cases where the two components have values that vary in a regular periodic manner, the questions arise whether they have the same period of variation, and what is the difference between their phases. Consider for example two components A and B, the directions of which are fixed, but of which the magnitudes can vary. Take the case first (Fig. 68) in which they vary together without any difference of phase. When component A has the small value O A_1 and component B the small value O B_1, the resultant will be O R_1. When A grows to O A_2 and B to O B_2 the resultant will be O R_2; and it is evident that if the magnitudes of O A and O B increase and decrease together, the resultant O R will also vary in the same phase, but will remain fixed in its own direction. In brief, two alternating vectors of equal period and in identical phase have as their

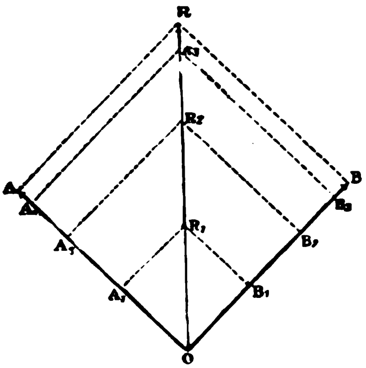


FIG. 68.

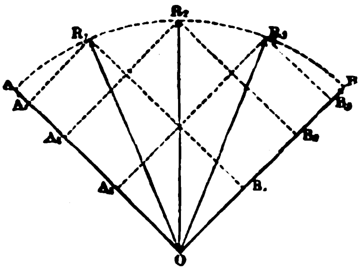


FIG. 69.

resultant another alternating vector of equal period, identical phase, and of fixed direction.

If, however, as in Fig. 69, the two components go through their periodic changes with a difference between their phases, not increasing and decreasing together, the resultant will no longer have a fixed direction. Let the variations of A and B be such that when O A is large, O B is small, and that while O A decreases O B increases. Then it is evident that the resultant will change, as in the figure, from O R_1 to O R_2, O R_3 &c.; and, *if the variations of the two components follow the proper law*, the resultant may be caused to change continuously in direction without changing in magnitude; or, in other words, two alternating vectors may be arranged to have as their resultant a rotating vector of constant value. What the law must be to produce this effect we must next see.

In 1883[1] Marcel Deprez communicated to the *Académie des Sciences* an important theorem on the production of a true rotatory magnetic field, by the combination of two alternating magnetic fields having as their difference of phase a quarter period.

It is well known that a uniform circular motion can be decomposed into two rectilinear harmonic motions at right angles to one another, the two having equal amplitude, equal period and a phase difference of one-quarter period. Let P be a point uniformly revolving around centre O. The projections of the radius O P upon the two axes (Fig. 70) are O M and O N. If the radius O P be called r we have O N $= r \sin \theta$, and O M $= r \cos \theta = r \sin (\theta + 90^\circ)$. While P revolves the point N will oscillate up and down the line Y Y′; the amplitude of its motion being equal to the radius of the circle. Also the point M will oscillate along the line X X′ with equal amplitude and in equal time; but O N will be at its maximum when O M has zero value, and *vice versa*. It follows kinematically that a uniform circular motion may be produced out of two straight-line motions, by combining them at right angles, provided they are harmonic, of equal period, of equal amplitude and differing by an exact quarter period.

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[1] *Comptes Rendus*, 1888, II. 1193.

Mechanically this motion is equivalent to that of two pistons having equal travel, working by two connecting rods upon the same crank pin, but placed at right angles to one another (Fig. 71). If motion of rotation is given to the shaft it will be decomposed into two rectilinear motions; the apparatus then acting as a two-throw pump. If on the other hand the cylinders are made to produce two rectilinear motions one ahead of the other by a quarter period in time, the apparatus will combine these motions into a true circular motion, and becomes equivalent to a two-crank engine with parallel cylinders.

Deprez saw that a similar combination can be magnetically effected. If an alternating current is led around a coil so as to produce an alternating or oscillating magnetic field along the line O X, and a second alternating current is led round a second coil so as to produce a second alternating magnetic field along the line O Y, then the result will be a *rotatory* magnetic field, provided these two magnetic fields are of equal period and amplitude, and differ exactly a quarter in phase. If they are of equal period, but not of exactly equal amplitude, the result will be equivalent to an *elliptically*-rotating magnetic field; that is to say, one in which the strength and direction of the field is represented by the successive values of the radius vector drawn to an ellipse from its centre, and sweeping over equal areas in equal times. An elliptically rotatory field will

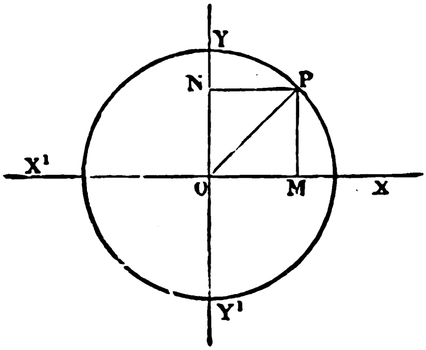

FIG. 70.—DEPREZ'S THEOREM.

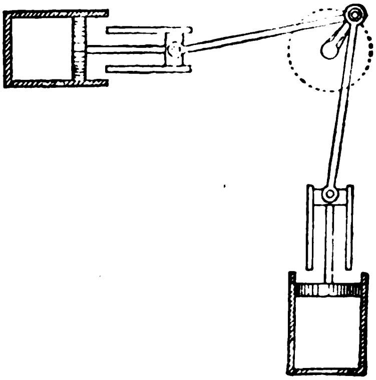

FIG. 71.

also be produced if the two component magnetic fields, though equal in period and amplitude, do not differ by exactly a quarter period. For a perfect rotatory field, corresponding to uniform circular motion, the two components must vary precisely as the sine and the cosine [1] of an angle respectively.

This is not by any means the only combination that will produce a rotatory magnetic field. The mechanical analogues of the three-crank engine, and of the three-throw pump, at once suggest other solutions. In the former instance three cylinders are used, with three pistons which operate in successive phases differing by one-third of a period from one another. If the three cylinders are set (as in a Brotherhood's engine) at 120° to each other, their connecting-rods may actuate a single crank. If the three cylinders are set parallel side by side, then there must be three cranks spaced out in angular positions 120° from one another. If the angular positions of the cranks were not exactly 120° apart, the phase-differences of the motions will not be exactly one-third of the period. The phase-difference of the motion must correspond to the peripheral spacing of the mechanism that combines them. It is a kinematic principle that in combining harmonic motions to produce rotation, the space-phase of angle in the combining mechanism must be the supplement of the angle which represents the time-phase of motion, otherwise the resulting motion will not be a *uniform* rotation.

The famous three-phase system of currents (or *drehstrom*) for producing a rotatory magnetic field, is the electrical analogue of the three-crank mechanism. In dealing with such combinations of magnetic fields, we may proceed analytically. We have three coils, or three pairs of coils, each producing a component magnetic field which alternates along a fixed direction, and we want to find the resultant field when they are combined together. When the coils are placed at an angle to each other we have to take into account not only the strength of each component field determined by the phase of the current, but also the direction of it. This is most easily done by splitting up the field produced by each circuit into

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[1] See also Ferraris, "Rotazioni elettrodynamiche," *Turin Acad.*, March 1888.

components along two axes X and Y. For instance, taking the coils on the ring in Fig. 58, the coils *b* and *e* will together produce a horizontal flux in the direction of O *b* along the axis of X in Fig. 72, which will change in value following the law $H \sin\theta$. The coils *d* and *a* will produce a field in the direction of O *d* which will follow the law $H \sin(\phi - 120°)$. Similarly the coils *f* and *e* will produce a flux in direction O*f* following the law $H \sin(\theta - 240°)$.

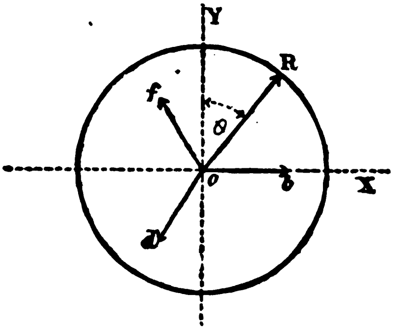

FIG. 72.

Adding together the components of these along the axis of X we get

$$H \sin\theta - H \sin(\theta - 120°)\cos 60° - H \sin(\theta - 240°)\cos 60$$

$$= \frac{3}{2} H \sin\theta.$$

And taking the components along the axis of Y

$$H \sin(\theta - 240°)\cos 30 - H(\sin\theta - 120°)\cos 30 = \frac{3}{2} H \cos\theta.$$

If we draw a line O R representing $\frac{3}{2} H$ to scale, making the angle θ with the axis of Y we shall see that as θ increases with time and R revolves round O, the projection of O R upon the axis of X is $\frac{3}{2} H \sin\theta$, and on the axis of Y it is $\frac{3}{2} H \cos\theta$. Therefore O R represents the direction of the resultant of the field at any moment. The field has, therefore, a constant value equal to one and one-half times the field produced by one pair of coils, and rotates at a constant angular velocity.

Generally we may say that with a symmetrical grouping of coils, if the number of phases is called m, the ratio of the resultant field to the field produced by one phase $\frac{m}{2}$.

We see, then, what are the time and space relations in simple 2-phase and in simple 3-phase working. To produce a uniformly rotating magnetic field, we may have as components either two equal fields differing by a quarter-period in time, and set at 90° (i. e. a quarter circumference) to one another in space; or, instead, three equal fields differing by a third of a period from one another in time, and set mutually at 120° (i. e. a third of a circumference) to one another in space. Obviously other cases might arise. Reference to Fig. 71, p. 61, will show that by simply putting the two cylinders at some other angle than a right angle, uniform circular motion might be resolved into two simple harmonic motions of equal period which did not differ by a quarter period. It was stated on p. 62 that a uniform circular motion may be recompounded out of two equal simple harmonic motions that do not differ by a quarter period, provided the space-phase of their angular positions is equal to the supplement of the time-phase of their motions.

We may very briefly discuss those cases where the time-phase of the components does not correspond to the space-phase of their angular positions. In such cases the result is not uniform circular rotation; it becomes, in fact, elliptical. The resultant, while revolving, does not revolve at a uniform angular speed, nor maintain an invariable magnitude. The cases of elliptical resultant rotation may be classified under several heads. If the two component simple-harmonic motions are equal in amplitude and of equal period, but not set in such relative positions that the angle between them in space is the supplement of the phase-difference between them in time, the resultant motion will be elliptical, not circular. If the two components, of equal period, and having angular positions supplemental to the phase-difference between them in time, are not equal in amplitude, there will result elliptical motion, and the major axis of the ellipse will coincide with the direction of the component of greater amplitude. If the phase-relations are not supplemental, and the amplitudes of the two components are unequal, yet the resultant motion will still be elliptical if the periods of the harmonic components are equal. Hence it follows that the resultant of any three (or more) simple harmonic components of equal periods, whatever their amplitudes and whatever the relations between their mutual angles and their phase-relations in time, will be an elliptical rotation (including in that general case the two special cases of rectilinear simple-harmonic motion, when the time-phases of all components are alike, and of uniform circular motion when the components are equal in amplitude and in time-phase relations corresponding to their angular differences in space). This is why unsymmetrical polyphase systems, such as the so-called " monocyclic" system (which is a distorted three-phase system) will drive motors about as well as a symmetrical system will do.

Returning to the case of 2-phase and 3-phase combinations properly so-called, it may be remarked that even though the amplitudes of the components are equal and their phase relations are properly adjusted, the result cannot be a uniform circular motion unless the individual components are truly harmonic—that is to say, follow exact sine functions. Now we know that in many cases[1] the form of the curves of electromotive-forces, currents and magnetomotive-forces in the actual alternating current systems in use differs considerably from that of a simple sine curve. It is easy to see in general what will be the effect of any departure from the simple sine form. Taking a 2-phase combination, if the component curves are of the peaked type, the resultant curve will have the general form of Fig.73, while if the component curves are of the broad topped round-backed variety, the resultant will have the

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[1] **See for a recent example, the curves given by Fleming in *The Electrician* of February 22d, 1895, for some alternators used in the City of London.**

general form of Fig. 74. If one or both the curves present a rippled outline owing to the presence of a sub-component of higher order of periodicity, the contour of the resultant curve will also be rippled. This corresponds to the entirely unimportant case of a motor designed to work with a rotatory magnetic field with the amplitude of the resultant magnetic force undergoing more rapid periodic fluctuations.

In the paragraphs immediately preceding, the combination of component vectors has purposely been discussed from the rather abstract or kinematic point of view. The rotatory phenomena in the polyphase motors are both more concrete and more complex. In them the impressed magnetic field seldom has a simple uniform circular rotation. They are mostly multipolar; they have projecting poles, teeth and other discontinuities of structure, all of which must have a tendency to cause the magnetic field to rotate more or less by jumps, and with variations in its magnitude from point to point. This is, however, of minor importance, for, as we shall see, the necessary tendency of the induced effects in the conducting revolving masses is to react against all departures from simple and uniform rotation. Further, in the ideal case, what is sought is not a uniformly rotating magnetic field, but such a combination of rotating magnetic field with a set of

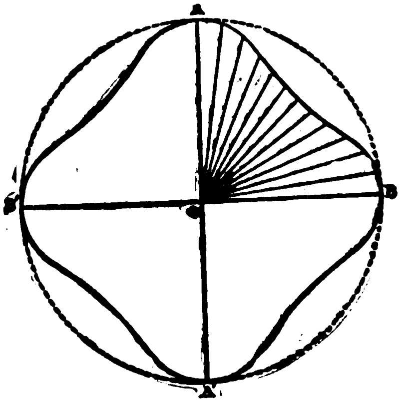

FIG. 73.

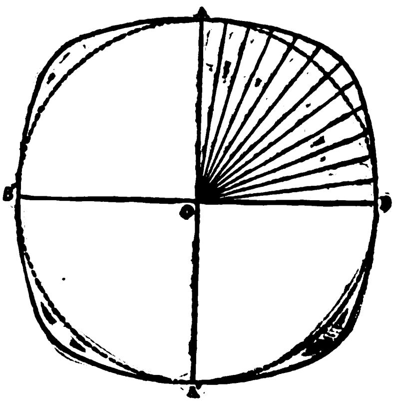

FIG. 74.

induced currents that the conductors carrying the latter (or the iron core in which they are embedded) shall be urged around its axis with a sufficient and sufficiently uniform torque. The torque at different points of the revolution is not uniform in steam engines, even in those provided with two and three cranks. But even in the worst polyphase motor the torque is much more uniform than in the best steam engine. No polyphase motor, no single-phase motor even, needs any fly-wheel to regularize the irregularity of the turning effort.

Lastly, it may be well to remind the student that the principle of vector combination (such as in the well-known parallelogram of vectors) is only applicable to magnetomotive-forces, magnetic fluxes, and electric currents when we are considering these quantities as *vectors*, that is to say when their actual direction in space is being taken into account, and, therefore, obviously cannot be employed in dealing with quantities of a circuital nature such as the total magnetomotive force, or as the total magnetic flux in a circuit, or in combining currents flowing from several wires into a common wire. There the quantities in question have merely a *scalar* value; their directions varying throughout their circuit. If we are considering the magnetic force at a point, we have something with a perfectly definite direction, and may, therefore, combine it with another magnetic force at the point. Similarly, when we are considering magnetic fields whose directions at a particular instant are uniform over the space we are considering, as in the case of the magnetic fields in the theorem of Marcel Deprez on p. 60, and the theorem which follows it, as to the resultant field in a particular three-phase motor, the principles of vector combination are applicable. But in a multipolar motor, where the flux is along curved paths, as shown in Figs. 24 and 125, the flux as a whole cannot be considered as a vector, and it is for this reason that in Chapter V., in the discussion of the progression of the magnetic field, the diagrams have been drawn to show how the circuit magnetomotive-forces progress peripherally along the motor.

The student must clearly distinguish between the application of the polygon of vectors in the case where *vector* quantities are being added, and the application of the same geometrical construction when *scalar* quantities, following a sine function of the time, are being added. In the latter case the quantities have their phase relations represented by the inclination of lines to one another; the legitimacy of the process depending solely upon the peculiar properties of the sine functions.

CHAPTER III.

PROPERTIES OF ROTATING MAGNETIC FIELDS.

CONSIDERING how much attention has been devoted to magnetic fields and to the combinations of the fields of two or more magnets, it is surprising how little thought has been given to the properties of rotating magnetic fields. The essence of the modern polyphase motor is the production of rotating magnetic fields within which masses of metal are set into forcible rotation by reason of the currents induced in them. These rotating magnetic fields are generated by the artifice of combining together two or more oscillating magnetic fields by the use of alternate currents in different phases, as already shown. But the principal properties of rotatory fields can be investigated and demonstrated without any such artifices, by very simple means. All that is required is an apparatus for spinning a magnet, the field of which is then investigated while it revolves at some known speed.

The subject appears to have first attracted notice about the year 1825, in the discussion of the phenomenon of Arago's rotations, of which, accordingly, some is here given.

Arago's Rotations.—As has so often occurred in other branches of science, the discovery of the magnetic rotations was made nearly simultaneously by several persons, for all of whom priority has been claimed. About 1824, Gambey,[1] the celebrated instrument-maker of Paris, had made the casual observation that a compass-needle, if disturbed, and set oscillating around its pivot, comes to rest sooner if the bottom of the compass-box is of copper than if it is of wood or other material. Barlow and Marsh,[2] at Woolwich, had at the same

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[1] See Jamin, *Cours de Physique*, iii. 296, 1869, and Verdet, *Conférences de Physique*, i., 415 1872. [2] *Edinburgh Philos. Journal*, xiii. 122, 1825.

time been observing the effect on a magnetic needle of rotating in its neighborhood a sphere of iron. Arago,[1] the astronomer, who is said to have learned of the phenomenon from Gambey, but who is also said[2] to have independently discovered it in 1822, when working with Humboldt at magnetic determinations, was beyond question the first to publish an account of the observation, which he did verbally before the *Académie des Sciences* of Paris, on November 22d, 1824. He hung a compass-needle within rings of different materials, pushed the needle aside to about 45°, and counted the number of oscillations made by the needle before the angle of swing decreased to 10°. In a wooden ring the number was 145, in a thin copper ring 66, and in a stout copper ring it was only 33.

The effect of the presence of the mass of copper is to damp the vibrations of the needle. Each swing takes the same time as before, but the amplitudes are lessened; the motion dying down, as though there were some invisible friction at work. Arago marked that it gave evidence of the presence of a force which only existed whilst there was relative motion between the magnet-needle and the mass of copper. He gave the phenomenon the name of magnetism of rotation. In 1825 he published a further experiment, in which, arguing from the principle of action and reaction, he produced a reaction on a stationary needle by motion of a copper disk (Fig. 75). Suspending a compass-needle in a glass jar closed at the bottom by a sheet of paper or of glass, he held it over a rotating disk of copper. If the latter turns slowly the needle is simply deviated out of the magnetic meridian, tending to turn in the sense of rotation of the disk, as though invisibly dragged by it. With quicker rotation the deviation is greater. If the rotation is so swift that the needle is dragged over 90° a continuous rotation ensues. Arago found, however, that the force was not simply tangential. Suspending a needle vertically from the beam of a balance over the revolving disk he found that it was repelled when the disk was

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[1] *Annales de Chimie et de Physique*, xxvii. 363, xxviii. 325, xxxii. 213.

[2] *Arago, Œuvres complètes*, iv. 424.

revolved. The pole which hung nearest the disk was also acted upon by radial forces tending, if the pole was near the edge of the disk, to force it radially outward, but if the pole was nearer the centre, tending to force it radially inward.

Poisson, steeped in Coulomb's notions about magnetic action at a distance, essayed to build up a theory of magnetism of rotation, affirming that all bodies acquire a temporary magnetism in the presence of a magnet, but that in copper this temporary magnetism took a longer time to die away. In vain did Arago point out that the theory failed to account for the facts. The so-called "magnetism of rotation" threatened to become a fixed idea.

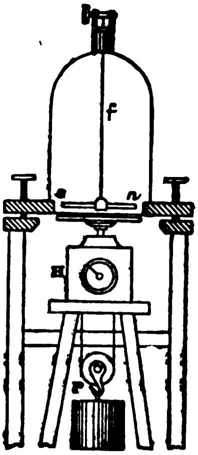

Fig. 75.—Arago's Spinning Disk.

At this stage the phenomenon was investigated by several English experimenters, by Babbage and Herschel, by Christie, and, later, by Sturgeon and by Faraday. Babbage and Herschel measured the amount of retarding force exerted on the needle by different materials, and found the most powerful to be silver and copper (which are the two best conductors of electricity), after them gold and zinc, whilst lead, mercury and bismuth were very inferior in power. In 1825 they announced the successful reversal of Arago's experiment; for by spinning the magnet underneath a pivoted copper disk (Fig. 76) they had caused the latter to rotate briskly. They also made the notable observation that slits cut radially in a copper disk (Fig. 77) diminished its tendency to be rotated by the spinning magnet. If the rotatory force of the unslit disk be reckoned as 100, one radial slit reduced it to 88, two radial slits to 77, four to 48, and eight to 24. Ampère, in 1826, showed that a rotating disk of copper also exercises a turning moment upon a neighboring copper wire through which a current is flowing. Seebeck in Germany, Prévost and Colladon in Switzerland, Nobili and Bacelli in Italy, confirmed the observations of the English experimenters, and added others. Sturgeon showed that the damping effect of a magnet pole upon a moving copper disk was diminished by the presence of a second magnet pole of contrary kind placed beside the first. Five years later he returned to the subject and came to the conclusion that the effect was an electrical disturbance, "a kind of reaction to that which takes place in electro-magnetism," when the publication of Faraday's brilliant research on magneto-electric induction, in 1831, forestalled the complete explanation of which he was in search. Faraday, in fact, showed that relative motion between magnet and copper disk inevitably set up currents in the metal of the disk, which, in turn, reacted on the magnet pole with mutual forces tending to diminish the relative motion—that is, tending to drag the stationary part (whether magnet or disk) in the direction of the moving part, and tending always to oppose the motion of the moving part. In fact, the currents go eddying round in the moving disk, unless led off by sliding contacts. This, indeed, Faraday effected, when he inserted his copper disk edgeways (Fig. 78) between the poles of a powerful magnet, and spun it, while against edge and axle were pressed spring contacts to take off the currents. The electromotive-force, acting at right angles to the motion, and to the lines of the magnetic field, produces currents which

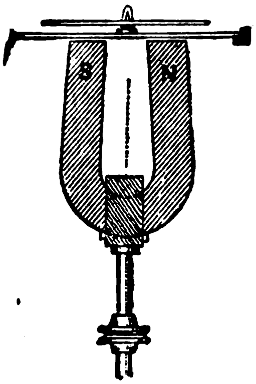

Fig. 76.—Barbage and Herchel's Experiment.

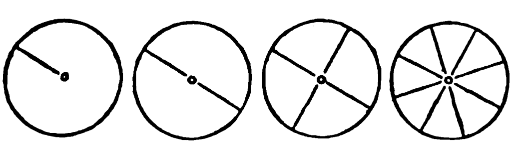

Fig. 77.—Slit Disks used by Babbage and Herschel.

flow along the radius of the disk. If no external path is provided, the currents must find for themselves internal return paths in the metal of the disk. Fig. 79 shows the way in which a pair of eddies is set up in a disk revolving between magnet poles. These eddies are symmetrically located[1] on either side of the radius of maximum electromotive-force (Fig. 80). The direction of the circulation of eddy-currents is always such as to tend to oppose the relative motion. The eddy-current in the part receding from the poles tends to attract

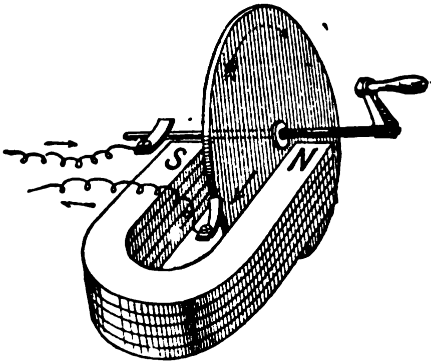

FIG. 78.—FARADAY'S DISK MACHINE.

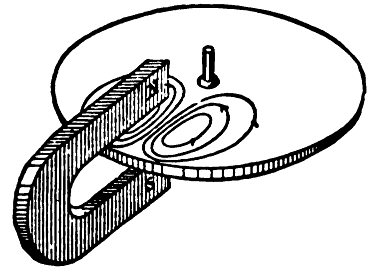

FIG. 79.—EDDY-CURRENTS IN SPINNING DISK.

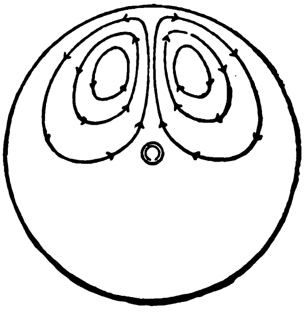

FIG. 80.—PATHS OF EDDY-CURRENTS.

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[1] Unless the speed of the rotation is *very* great; in which case the self-induction of the eddy-circuits will cause a time-lag shifting the position of the radius of maximum current ahead of the radius of maximum electromotive-force.

the poles forward or to drag this part of the disk backwards. The eddy-current in the part advancing toward the poles tends to repel those poles and to be repelled by them. It is obvious that any slits cut in the disk will tend to limit the flow of the eddy-currents, and by limiting them to increase the resistance of their possible paths in the metal, though it will not diminish the electromotive-force. In the researches of Sturgeon[1] a number of experiments are described to ascertain the directions in which the eddy-currents flow in disks. Similar, but more complete researches were made by Matteuci. The induction in rotating spheres was mathematically investigated by Jochman, and later by the lamented Hertz.

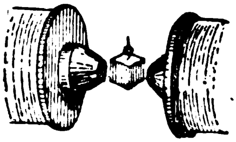

FIG. 81.

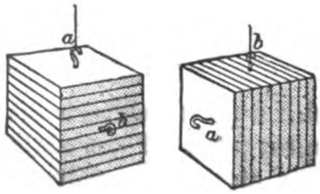

FIG. 82.

Faraday showed several interesting experiments on eddy-currents. Amongst others he hung from a twisted thread a cube of copper in a direct line between the poles of a powerful electromagnet (Fig. 81). Before the current was turned on the cube, by its weight, untwisted the cord and spun rapidly. On exciting the magnet by switching on the current, the cube stops instantaneously ; but begins again to spin as soon as the current is broken. Matteucci varied this experiment by constructing a cube of square bits of sheet copper separated by paper from one another. This laminated cube (Fig. 82) if suspended in the magnetic field by a hook *a*, so that its laminæ were parallel to the lines of the magnetic field, could not be stopped in its rotation by the sudden turning on of the current in the electromagnet ; whereas if hung up by the hook *b*, so that its laminations were in a vertical plane, and

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[1] *Edinburgh Philosophical Journal*, July 1825 ; and *Philosophical Magazine*, April and May 1832. See also Sturgeon's *Scientific Researches*, p. 211.

then set spinning, was arrested at once when the electromagnet was excited. In the latter case only could eddy-currents circulate; since they require paths in planes at right angles to the magnetic lines.

With the explanation given by Faraday of the Arago rotations, as being merely due to induced eddy-currents, the peculiar interest which they excited whilst their cause was unknown, seems almost to have died out. True, a few years later some interest was revived when Foucault showed that they were capable of heating the metal disk, if in spite of the drag the rotation was forcibly continued in the magnetic field. Why this observation should have caused the eddy-currents discovered by Faraday as the explanation of Arago's phenomenon to be dubbed Foucault's currents is not clear. If any one is entitled to the honor of having the eddy-currents named after him, it is obviously Faraday or Arago, not Foucault. A little later, Le Roux produced the paradox that a copper disk rotated between concentric magnet poles is not heated thereby, and does not suffer any drag. The explanation of this is as follows. If there is an annular north pole in front of one face of the disk, and an annular south pole in front of the other face, though there is a magnetic field produced right through the disk, there are no eddy-currents. For if all round the disk there are equal electromotive-forces directed radially inwards or radially outwards, there will be no return path for the currents along any radius of the disk. The periphery will simply take a slightly different potential from the centre; but no currents will flow because the electromotive forces around any closed path in the disk are balanced.

In 1884, Willoughby Smith published[1] an investigation on rotating metal disks in which he found iron disks to generate electromotive-forces superior to those generated in copper disks of equal size.

Guthrie and Boys in 1879,[2] hung a copper plate over a rotating magnet by means of a torsion thread, and found that

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[1] Lecture at Royal Institution: "Volta and Magneto Electric Induction," June 1884.

[2] *Proc. Physical Soc.*, iii. pt. iii. 127, and iv. 55.

the torsion was directly proportional to the velocity of rotation. They pointed out that such an instrument was a very exact one for measuring the speed of machinery. They also made experiments upon varying the distance between the copper plate and the magnet, and varying the diameter and thickness of the copper disk.

Experiments were made upon various metals, and the torque was found to vary as the conductivity of the metal as far as the latter could be judged after being rolled into the form of plate. Messrs. Guthrie and Boys then applied the method to the measurement of the conductivity of liquids.

In 1880, De Fonvielle and Lontin observed that a lightly pivoted copper disk could be maintained in continuous rotation—if once started—by being placed, in presence of a magnet, within a coil of copper wire wound on a rectangular frame (like the coil of an old galvanometer), and supplied with alternate currents from an ordinary Ruhmkorff induction coil. They called their apparatus an electromagnetic gyroscope.

But it does not seem to have occurred to any one that the Arago rotations could be made use of in the construction of a motor prior to the year 1879 (see p. 84).

Experiments in a Rotating Magnetic Field.—With the simple apparatus of Fig. 76 a number of interesting and easy experiments can be shown. A pivoted compass-needle placed over the magnet revolves synchronously. If a number of small "charm" compasses are placed close together over the revolving magnet, they all turn together in unison. Any pivoted disk of thin sheet iron (ferrotype plate or tinned-ware), also rotates synchronously. An iron nail or a steel pen laid on a sheet of glass over the magnet begins to revolve as soon as the magnet is turned, and will acquire a great speed, turning always synchronously with the magnet. So will a small iron key; but if the magnet is first spun very quickly and the iron nail or key is then laid down on the glass, it fails to fall into step, and does not turn. If iron filings are sprinkled from a pepper-box upon the glass sheet (a sheet of mirror glass is preferable) over the slowly revolving poles, a very curious and beautiful effect is produced. Owing to the magnetic fields proceeding from the poles having vertical components, each tuft of filings rises up on end as the poles pass under, and turns a complete somersault at each rotation of the magnet. By each somersault the tufts of filings are shifted a little in a sense contrary to that of the rotation; giving, as the speed is increased, the effect of particles waltzing round in a flock, and gradually tending either to heap together in the centre or to drift out of the field at the edges of the sheet of glass. Iron bullets or buttons revolve synchronously with the magnet. Bullets or egg-shaped pieces of copper or brass revolve quite slowly however, and do not keep pace with the revolving magnet, as do bodies of magnetic material. Pivoted disks of copper, brass, zinc, or, best of all, of aluminium, placed over the magnet, also take up a slow non-synchronous revolution, being driven by the eddy currents generated in them.

If the pivoted disks, compass needles and the rest are however, not placed centrally over the moving magnet, but are set at some distance laterally, quite outside the sweep of the moving poles, the rotation produced in the pivoted disks is in a sense opposite to that of the rotation of the magnet. If the pivoted disk is set centrally over the revolving magnet at a height of 6 or 8 inches above its poles, and is gradually moved laterally away from this central situation, a point may be found at some distance where the disk does not tend to turn in either direction. Inside the zone of such neutral points the tendency is to revolve in the same sense as the magnet; outside that zone, to revolve in the opposite sense. The neutral zone widens as the vertical distance is increased. If a pivoted disk is set at a neutral place, it may be made to revolve by placing in the interspace pieces of iron or even strips or hoops of copper in positions which either distort the field or convey by eddy-currents a new revolving field. If a conducting cage of copper strip, made up like Fig. 83, is set up over the revolving magnet, and a well-pivoted disk of aluminium is placed over the top of it at *a*, this disk may be set in rotation, though the distance between the magnet and the disk is far too great for rotation to be set up without this adjunct. The effect is improved by inserting a mass of iron at *b* to increase the inductive effect of the revolving magnet.

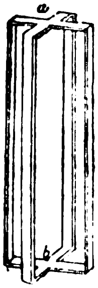

FIG. 83.

Another instructive experiment, shown by the author at the Royal Institution in February 1894, is afforded by cutting out a piece of sheet copper in the form shown in Fig. 84, having two long slits running nearly the whole length. This may be several feet long, and three or four inches broad. If the strip is laid down horizontally, with the point A centrally over the revolving magnet, with a block of iron over it to enhance the inductive action, eddy-currents are set up in the strip (in reality 3-phase eddy-currents) which are able to turn a nicely-poised metal disk placed on a pin at the distant end B. The disk used in this experiment consisted of a copper disk having a thickened rim, with a smaller iron disk laid within the rim, the whole being provided with a jewelled centre to diminish friction. All these effects can be produced with much greater power by substituting for the mechanically-turned steel magnet any apparatus producing a rotatory field by the combination of true polyphase currents as described later on.

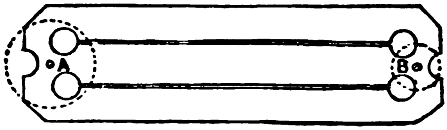

FIG. 84.

For those who have no true polyphase apparatus at their command, but who have batteries capable of furnishing 5 to 10 amperes at a pressure of 10 to 20 volts or more, it may be convenient to describe a method of artificially imitating true polyphase currents by means of a hand-driven commutator.

Fig. 85 shows a very simple form of commutator, by which a rotating field can be produced if the terminals A, B and C are connected to the terminals *m*, *o* and *n* of a ring wound as in Fig. 58, p. 48, and a battery is connected by one pole to the return terminal R of the commutator, and the other pole to the common junction J of the coil. On turning the handle rapidly the three contact springs receive currents at successive intervals, which may be said to differ in phase by 120°, though of course there is no reversal. It will be seen that the intervals overlap by an angle of 60°; that is to say, the current in B is switched on $\frac{1}{6}$ of a period before the current in A is switched off, and for $\frac{1}{6}$ of a period current in B only is on, after which currents in B and C are both on for $\frac{1}{6}$ of a period, and so on. The bearing surface of the teeth is one-half of the pitch, and the tips of the contact springs virtually tri-sect the pitch. This commutator can be cut out of a single sheet of brass, and though soon spoilt by the sparking is easily repaired.

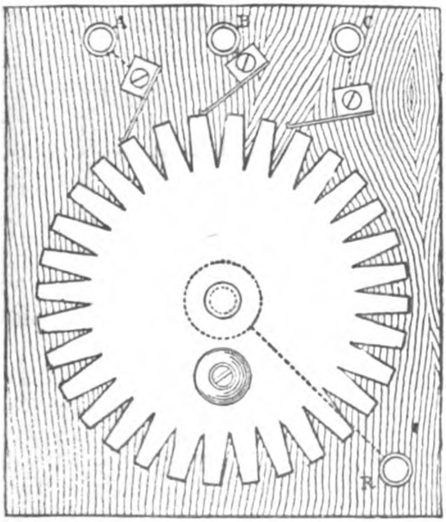

Fig. 85.—Hand Commutator for Imitating Three-phase Currents.

A more elaborate commutator which *reverses* the currents in the three lines in proper succession, and does not require a fourth line as a return-wire, is shown in Fig. 86. It represents a wooden barrel about 2 inches in diameter and about 5 inches long, upon which are screwed two properly spaced contact-pieces. Against this barrel press springs; three of these being terminals for the three lines; two to be joined to the terminals of the battery. A developed drawing of this commutator, showing how the contact pieces are spaced out, is given, exactly half the actual size, above the figure.

By carefully following the order of operations during one revolution, it will be seen that the current is successively

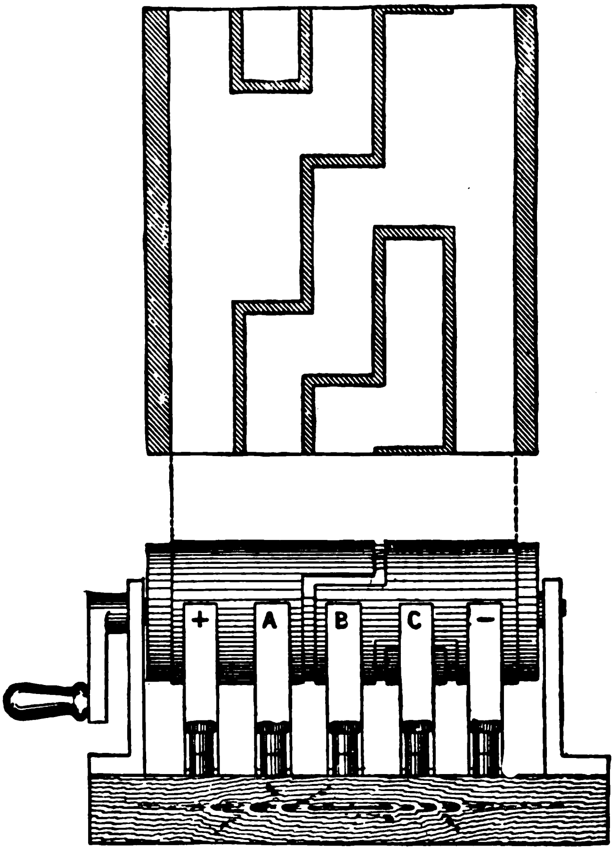

FIG. 86.—HAND COMMUTATOR FOR PRODUCING THREE-PHASE CURRENTS.

reversed in each line and that while the current is flowing from a + terminal through the A line, the return current to a – terminal is shifted from the B line to the C line, and so forth in regular order.

A similar device (but, of course, with different spacing of the parts) may be used to imitate 2-phase currents, using four lines. This will require six springs, unless a common return is used. Indeed, the commutator just described works fairly well for 2-phase apparatus if the terminal C is used with a common return-line for the two circuits that go out from A and B.

To show simple rotatory-field experiments with this three-phase commutator, all that is necessary is a ring electromagnet properly wound. Take a ring-core made either of iron wire or of iron core-rings stamped from sheet iron, having an external diameter of 3 or 4 inches and an internal diameter of 2 or 3 inches. Its depth may also be half an inch or so. After insulating it with tape, or paper varnished, wind carefully upon it six equal coils of No. 16 S.W.G. covered copper wire (or stranded wire of 7 No. 23 S.W.G. for greater flexibility). Let the three coils each cover 60°. Their ends may be furnished with terminals, so that if desired they may be joined either in star fashion or in mesh fashion. Each coil should have at least 100 turns. If a finer wire is used (and it has some advantages) there must be a proportionately greater number of turns wound upon the ring. With a small ring such as this almost all the above experiments can be shown.

For experiments on a larger scale, polyphase currents can be readily procured by those who have access to an electric lighting supply of continuous current. For it is easy to adapt a small motor to serve the purpose of a running transformer. Suppose the supply is at 100 volts. Then a small motor of 1 horse-power, or even of ½ or ¼ horse-power, can readily be adapted to the purpose, provided there is room on its shaft at the end opposite to the commutator to adapt to it three insulated slip-rings which are connected to three symmetrical points on the armature winding. From these three slip-rings three contact-brushes take off the 3-phase current (p. 183).

One of the most fascinating experiments which can readily be shown with such an apparatus is the spinning of a copper egg. For this purpose, a somewhat larger ring electromagnet is required than that described above for producing the rotating field. An 8-inch ring, wound in 6 or 12 sections, and connected up as in in Fig. 58 or Fig. 59, serves excellently. A ring wound in 12 sections (see Fig. 157, p. 180) is very convenient, since it can also be used for 2-phase currents. The ring is laid upon a table, and a common china plate may be set upon it. An egg of copper, solid or hollow, or, better still, of copper filled with iron filings, revolves rapidly when the current is turned on. As its speed of rotation increases it finally rises up and spins on its end. An aluminium egg revolves even better. A stout disk of copper or aluminium, if slightly convex on the face, and rounded at the edges, spins and gradually rises up until it spins on its edge like a coin.

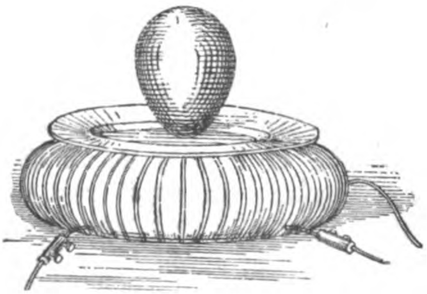

FIG. 87.

MECHANICAL ILLUSTRATIONS OF POLYPHASE TRANSMISSION.

The analogies between polyphase current apparatus and machines in which two cranks or three cranks are used so as to avoid dead-points, have been several times alluded to. Mechanical models corresponding to any particular case of polyphase currents can easily be designed, and are very instructive. A very simple model designed by the author to illustrate a simple 3-phase transmission of power, may be worthy of record.

Three cords are attached to a pin P on a small crank, rotating about centre O in the middle of a fixed board

(Fig. 88). The three cords are led off through three equidistant holes A, B, C, furnished with porcelain eyelets to diminish friction. The three cords pass over three pulleys *a*, *b*, *c* to a distant point, where they are brought down to a similar board and reunited at a common junction *p*, to which a pencil is fastened. On imparting a circular motion by hand to the handle *h*, the point *p* also performs a circular motion, though there is no crank to guide its motion, and traces an (approximate) circle on the board.

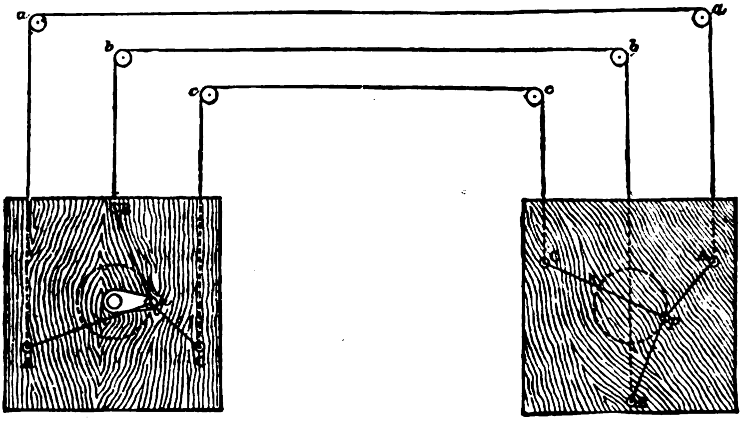

FIG. 88.

Another method of mechanical illustration, using cords and pulleys, was devised by Mr. P. A. N. Winand (see *Journal of the Franklin Institute*, October, 1892).

CHAPTER IV.

EARLY DEVELOPMENT OF THE POLYPHASE MOTOR.

THE notion of producing rotation by using several magnet poles which should come into operation successively, and so attract an armature forward, is of no recent date. Multipolar motors are to be found in some of Wheatstone's earliest patents, whilst several of Pacinotti's motors of about 1861 to 1865 embody the same idea. In none of these, however, was there any suggestion that the shifting poles should operate by inducing currents in the rotating part.

The First Induction motor.—Amidst the crowd of modern inventions little note has been taken of the modest beginnings of the polyphase motor, the birth of which dates from 1879. Fig. 89 illustrates the elementary motor which Mr. Walter Baily exhibited to the Physical Society of London on June 28, 1879, on the occasion of his reading a paper entitled, "A Mode of Producing Arago's Rotations."

Down to that date the only mode of producing the Arago-rotations of a copper disk had been by rotating beneath it a steel magnet. Baily, instead of rotating any material magnet below the disk used a fixed electromagnet, but caused its magnetism to shift progressively between four successive poles, thus producing in the copper disk pivoted above them eddy-currents, which by their reaction gave the disk a mechanical motion in the direction of the progression of the poles.

The disk in this primitive model is about $2\frac{3}{8}$ inches in diameter; the four magnet cores are about 4 inches high, joined to a common yoke; and each is wound with about 150 turns of insulated copper wire 2·5 mm. in diameter. The coils are connected two and two in series, like two independent horseshoe magnets set diagonally across one another. The two circuits are brought down separately to an ingenious revolving commutator built up of a simple arrangement of springs and contact strips mounted on a bit of wood, with a wire handle by which it is turned. On rotating it, the currents from two batteries are caused to be reversed alternately in the two circuits, giving rise to the following successions of polarity in the four poles:—

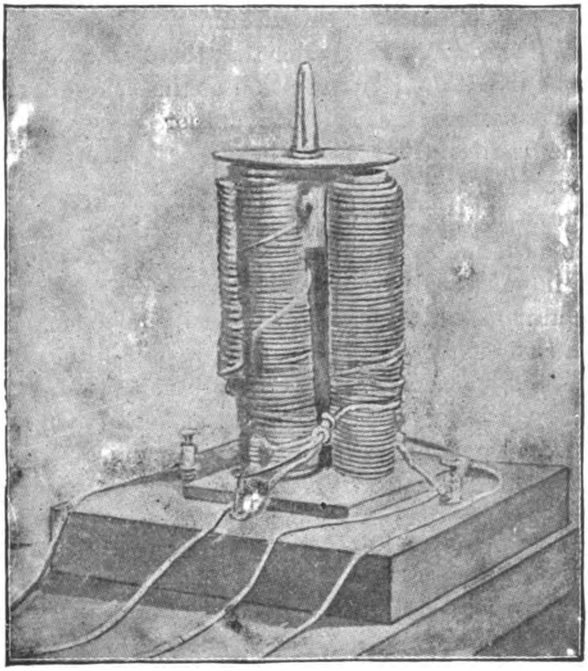

FIG. 89.—WALTER BAILEY'S POLYPHASE MOTOR (1879).

N O	N N	O N	S N	S O
↘	↓	↙	←	↖
O S	S S	S O	S N	O N

and so forth. Mr. Baily had very clear views as to how far this really represented a rotatory magnetic field. His own words are as follows:

"The rotation of the disk is due to that of the magnetic field in which it is suspended, and we should expect that if a similar motion of the field could be produced by any other means the result would be a similar motion of the disk.

"Possibly the rotation of the magnet may be the only practicable way of producing a uniform rotation of the field; but it will be shown in this paper that the disk can be made to rotate by an intermittent rotation of the field effected by means of electromagnets." The author then goes on to discuss the result of the increase in strength, of a pole while a neighboring pole of the same sign decreases in strength, and suggests that if a whole circle of poles were arranged under the disk, and successively excited in opposite pairs, the series of impulses all tend to make the disk move in one direction around the axis; adding: "In one extreme case, viz. when the number of electromagnets is infinite, we have the case of a *uniform* rotation of the magnetic field, such as we obtain by rotating permanent magnets." He then returns to the case of his actual model with two pairs of poles *a a′* and *b b′*, and points out that if the *b b′*, pair are arranged to be reversed *before* the *a a′* pair, the rotation will be in one direction; whilst, if the *b b′* pair are reversed *after* the *a a′* pair the rotation will take place in the other direction. He goes on to show how the reversal of the direction of rotation may be effected either by reversing the action of the commutator, or by reversing the connections of one of the two batteries. The diagram accompanying the original paper suggests that the cores should be of laminated iron; but those of the actual model are solid. In a final paragraph the author remarks that the effect on the disk might be much increased by placing four other electromagnets above the disk, each opposite one of the lower magnets, and connected with it so as to present an opposing polarity.

The model runs exceedingly well when four dry cells are used to excite the electromagnets.

On the occasion, now fifteen years ago, when the paper was read, and the model shown, the late Prof. Guthrie asked jokingly how much power it was expected that the motor would give. To which Mr. Baily modestly replied that at present he could only regard it as a scientific toy.

Researches of M. Marcel Deprez.—In 1880, M. Marcel Deprez brought before the Société Française de Physique a paper upon the electric synchronization of rotations, in which artificially produced 2-phase currents were transmitted from a rotating commutator to a synchronous motor consisting of two shuttle-wound armatures set on one shaft, each one lying between the poles of a horseshoe magnet; one of them being given an angular lead of 90° relatively to the other, so that there could be no dead points. Fig. 90 shows how the currents were transmitted from the battery to the two armatures.

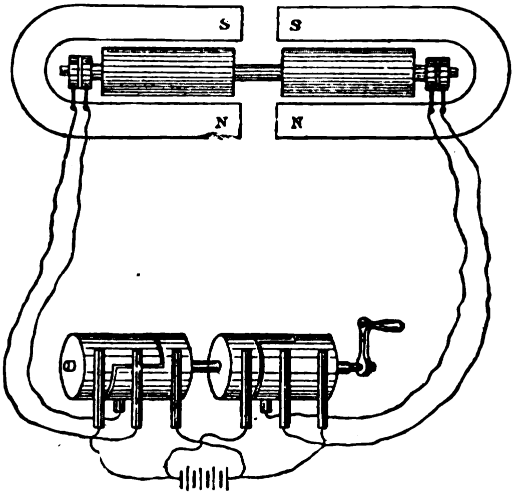

FIG. 90—MARCEL DEPREZ'S APPARATUS.

This apparatus resembles that of Baily only in requiring a 2-phase system of currents to operate it. Both will operate either with the artificial 2-phase currents produced by such commutators from a battery, or with 2-phase currents produced inductively in a periodic manner. They differ, however, totally in operation. Deprez's is a mere combination of two ordinary motors at right angles, so as to have no dead-point. There is nowhere embodied in it the principle of the rotatory magnetic field. Whereas Baily's motor possesses as its main feature the progressive shifting of a magnetic field in regular order round a centre, and develops currents by induction in a rotating metal mass without sliding contacts or commutator.

Three years later Deprez laid down the important theorem which we have discussed on p. 60, as to the production of a true rotatory magnetic field by the combination of two alternating currents, having as their difference of phase a quarter period.

Deprez's theorem bore no fruit: it remained a geometrical abstraction.

Researches of Professor G. Ferraris.—In 1887, several investigators were independently at work.

Professor Galileo Ferraris,[1] of Turin, had already in 1885, arrived at the same fundamental ideas as those of Baily and of Deprez. But the result was more fruitful, inasmuch as he, without knowing of the work of either, united both sets of ideas. Like Baily he proposed to produce rotation of a copper conductor by means of eddy-currents induced in it by a progressively shifted magnetic field; and this progressively shifted magnetic field he proposed to generate as a true rotatory field by combining at right angles to one another two alternate currents which differed by a quarter-period from one another.

In 1885, Professor Ferraris constructed the motor depicted in plan in Fig 91, which was not, however, publicly shown till 1888. It was exhibited in 1893 at the World's Fair at Chicago. It consisted of two pairs of electromagnets A A and B B′, having a common yoke made by winding iron wire around the exterior. Two alternate currents differing in phase were led into these two circuits, and the pivoted central body was observed to revolve.

Ferraris's first publication was in March, 1888, entitled *Electrodynamic rotations produced by means of alternate currents.* After expounding the geometric theory of the rotatory magnetic field, he suggested that a simple way of procuring the desired phase-currents would be to branch the circuit of

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[1] Ferraris, "Rotazioni elettrodynamiche," *Turin Acad.*, March, 1888.

an alternate current into two parts, into one of which should be inserted a resistance without self-induction, into the other a coil of much self-induction but of small resistance. The two windings of the motor should be respectively introduced into these two branches. The difference of phase thus produced would be sufficiently near to 90° to be effective. He expressed the opinion that in this way one might obtain all the effects that can be obtained by the rotation of a magnet. He then described the following experiments which were made in the autumn of 1885.

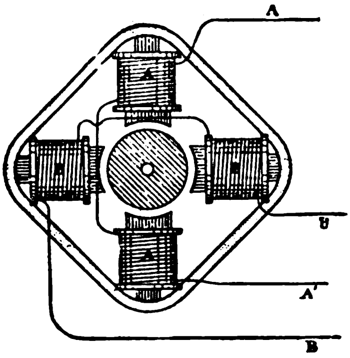

Fig. 91.—Ferraris' Motor (1885).

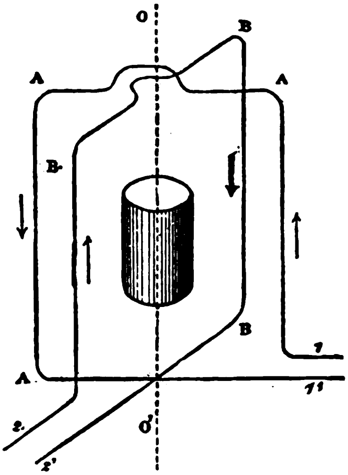

Fig. 92.

Two flat coils, one of thick wire, the other of thinner wire, represented diagrammatically at A A and B B of Fig. 92, were set at right angles to one another. Into the first was brought a current from the primary of a Gaulard's transformer, and into the second the current from the secondary, with more or less non-inductive resistance. In the central space was suspended a small hollow closed cylinder of copper. If the current was turned on in one only of the two windings the cylinder remained immovable, but on turning on the second current it at once began to rotate. The sense of the rotation could be reversed by simply changing, with a reversing-switch, the connections of the second coil. The same results were found to follow when a cylinder of iron was substituted for that of copper. A laminated iron cylinder built up of insulated disks also turned. Then followed suggestions for constructing alternate current motors on this principle but of modified form; for, as Professor Ferraris remarked, it was evident that a motor thus made could not have any importance as a means of industrial transformation of power. He therefore designed a larger model, having as its rotating part a copper cylinder weighing 10 lbs., having a length of 18 cm. and a diameter of 8·9 cm., borne on a horizontal shaft 1 cm. in diameter. It was surrounded by two sets of coils A A and B B at right angles to one another, as in the Fig. 93. It was, however, of but small power. Ferraris discussed the elementary theory of the apparatus, pointing out that the inductive action would be proportional to the *slip*, that is to say to the difference between the angular velocity of the magnetic field and that of the rotating cylinder, that the induced current in the rotating metal would also be proportional to this; and that the power of the motor is proportional jointly to the slip and to the velocity of the rotating part. Ferraris also suggested measuring instruments for alternate currents based on this principle. Lastly he succeeded in producing rotation in a mass of mercury placed in a vessel in the rotatory field. In 1894 Ferraris published a further discussion of the theory of these motors, which is dealt with in Chapter VIII.

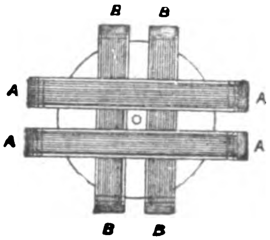

FIG. 93.

Borel's Motor.—In 1887 M. Borel devised an alternate-current motor for use in a supply meter, which was brought out as the Borel-Paccaud meter. It was, in reality, a two-phase motor, in which the difference of phase was produced from a single alternating current by using two circuits with different time-constants. Upon the two sides of an iron frame were wound two coils A A, to give an alternating magnetic field in the direction from right to left. Outside the frame were wound two other coils B (one of them is removed from

Fig. 94 to show the interior) tending to produce a second alternating field at right angles to the first. In the centre of the whole was pivoted an iron wheel, which was set into rotation by the combined rotatory field.

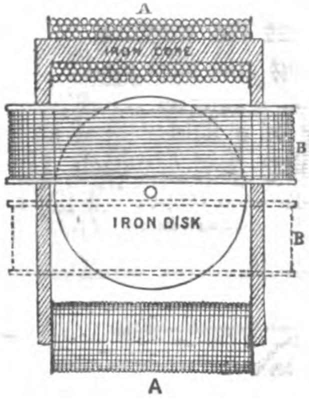

FIG. 94.—BOREL'S MOTOR.

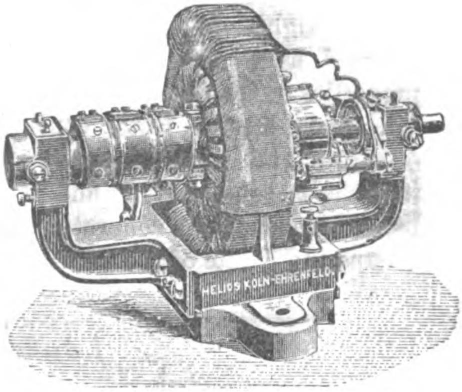

FIG. 95.—COERPER'S MOTOR.

Early Motors of the Helios Co. of Cologne.—In 1887 the Helios Co. constructed in accordance with a patent of Mr. Coerper,[1] some small motors, of which some were for monophase currents, synchronous and asynchronous, while another was the 3-phase motor depicted in Fig. 95. It had three slip-rings on the revolving part to receive a 3-phase current. As the motor required three leads, and as at that time all efforts to obtain a satisfactory working with two leads were not successful, the Helios Co. dropped the patent in 1890. A later patent of 1891 described a monophase motor with an additional winding which acts only on the iron of the revolving part, and is introduced only during the operation of starting.

Bradley's Motors.—Amongst the early American pioneers of polyphase work was Charles S. Bradley, whose work dates from early in 1887. His U. S. patent, filed May 8th, 1887 (No. 390,439), describes a generator with a Gramme ring, having four radial connectors (Fig. 96), led off at four

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[1] **Specification of Patent No. 9013 of 1887. See also D. R. P. 43538 of 1887, and 70084 of 1891. Compare *Elektrotechnische Zeitschrift*, 1893, p. 82.**

symmetrical points to four slip-rings. He thus obtained two alternate currents differing 90° in phase. The object of this arrangement was stated to be to obtain a larger output —which is, indeed, true, since the output of a polyphase machine is considerably higher than that of any other of equal weight. It was also stated that the machine could be used as a motor, though nothing was said about the properties of the rotatory field. Claim 9 runs as follows:— "A rotary electric motor consisting of a field-magnet and armature and pairs of current-leading devices—such for instance as contact rings and brushes—the respective pairs being independently connected into the armature winding at alternating points of the same, and arranged for connection with two independent external circuits." Here was, then, in 1887, a polyphase motor unmistakably described. In October 1888 (patent No. 404,465) comes an asynchronous motor, driven by means of directed eddy-currents in a stationary external mass of iron. The rotating inductor received 2-phase currents through four slip-rings. The whole principle of magnetic slip is fully explained.

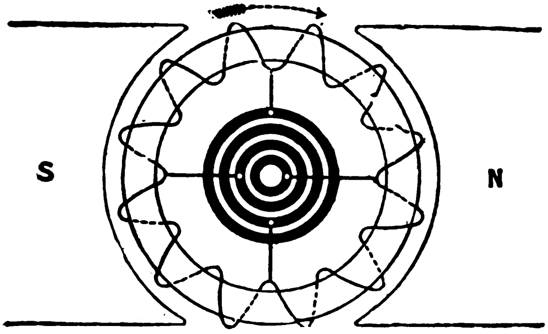

FIG. 96.

In a patent (No. 409,450), published August 20th, 1889, Bradley describes a similar armature tapped at three equidistant points and connected to three slip-rings, thus constituting a 3-phase system. This machine also was for use as either generator or as motor. In another patent of same date, Bradley indicates a method of splitting a single-phase alternate current into two of different phases by use of his machines.

Researches of Nikola Tesla.—The work done by Nikola Tesla between the years 1887 and 1891, is of itself sufficient had no other workers been occupied in the same field of research, to have established the rotatory-field motor upon a solid basis. He arrived in 1886 at the firm conviction that some method must exist of driving an armature by currents induced within it, instead of driving it by currents brought into it (as in the ordinary electric motors), through the agency of metallic contacts, commutators and brushes. By October 1887, Tesla's work was sufficiently advanced for him to apply to the United States Patent Office for patents covering numerous points of a more or less fundamental character. Other applications for patents followed in November and December of the same year, but none were issued from the Patent Office until May, 1888, when a whole batch of them were granted.

The first of these specifications set forth the general scope of Mr. Tesla's ideas. He says, "Though I have described various means for the purpose, they involve the same main principles of construction and mode of operation, which may be described as follows: A motor is employed in which there are two or more independent circuits through which alternate currents are passed at proper intervals, in the manner hereinafter described, for the purpose of effecting a progressive shifting of the magnetism or the "lines of force" in accordance with the well-known theory. and a consequent action of the motor. It is obvious that a progressive shifting of the lines of force may be utilized to set up a movement or rotation of either element of the motor, the armature or the field-magnet, and that if the currents directed through the several circuits of the motor are in the proper direction no commutator will be required; but to avoid all the usual commuting appliances in the system I prefer to connect the motor circuits directly with those of a suitable alternate-current generator." He then proceeds to describe by a diagram (Fig. 97 which is taken from Fig. 9 of the specification) how a generator is wound with two separate coils, the free ends of which are connected to insulated contact rings on the shaft. From four brushes that press on the rings four wires are led away to the motor. This is, in fact, a simple 2-phase generator, inducing two-currents in quadrature. The motor is shown as a ring built up of core-sheets, having wound upon it four coils, two of which are connected in circuit with one pair of wires, the other two being in the other circuit. They tend to co-operate in pairs to produce magnetic poles on diametrically opposite parts of the ring. Within the ring is pivoted as rotor a disk D of iron, preferably cut away at its sides so as to form an elongated body; and turns so as to convey from side to side of the ring the greatest number of magnetic lines. It was found that this form was not essential to rotation, since a circular disk of iron was also set revolving. This phenomenon Mr. Tesla attributed to a certain magnetic inertia or resistance to shifting of the magnetic lines; and deemed this view confirmed by the observation that a circular disk of steel is more effectively rotated than one of soft iron. In a series of eight diagrammatic figures Mr. Tesla explained the successive phases through which the coils of the generator pass during one revolution, and the corresponding and resulting changes of magnetism produced in the ring of the motor.

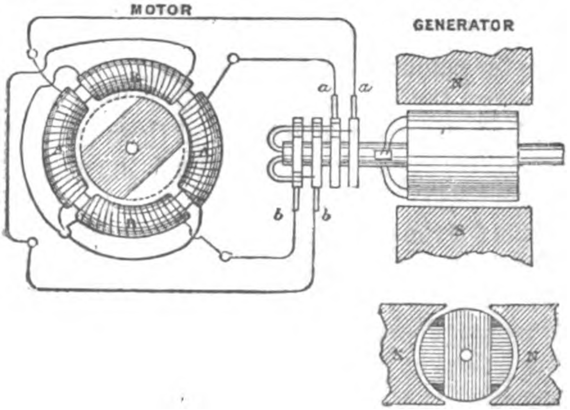

FIG. 97.

The resultant direction of the magnetic field shifts progressively round (Fig. 98), dragging the iron disk with it.

This combination amounts then to a 2-phase synchronous motor not operating by induced currents in the rotor, but by magnetic reactions, together with a suitable 2-phase generator for supplying the current.

Other forms were described at the same time. A motor had a drum armature wound with two coils at right-angles, to which the currents were brought by four slip rings. This armature revolved between the two parts of an exterior shell of iron or steel, which to *prevent* eddy-currents (!) was preferably to be laminated. It was not wound, being magnetized solely by the polarity of the armature. Then followed a 3-phase generator and motor on similar lines to the 2-phase generator and motor first described. The generator had three revolving coils and six slip-rings. It was connected by six line-wires to the ring of the motor, which had six coils wound upon six inward pointing poles, constituting a 2-pole field with three phases. The rotor was as before a disk or cylinder of iron cut away on two sides to form an elongated body. The next form described was a 2-phase combination, having in the generator a revolving magnet and two pairs of fixed armature coils, while the motor had as before a cut-away disk of iron as rotor, surrounded by two fixed coils set at right angles to one another. A form of motor was shown having arrangements for bringing the 2-phase currents to its revolving windings as well as to windings on a fixed external ring. It was found to be advantageous in the case where an external iron shell or fixed magnet was employed to give this a fixed magnetic

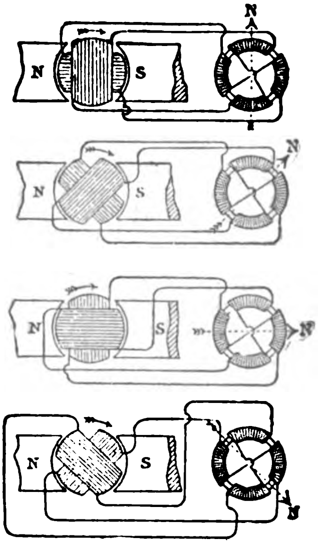

FIG. 98.

polarity by separately exciting it with a continuous current. These motors were of course synchronous. Transformers for currents such as were used in these systems were made by winding a set of primary and a set of secondary wires upon the same ring of laminated iron, in which the magnetism underwent a progressive shifting of polarity. In November came the first suggestion of a real induction motor. Hitherto Mr. Tesla had produced and maintained the rotation by the "direct attraction" of the magnetic elements of the motor. "I have discovered," he says, "that advantageous results may be secured in this system by utilizing the shifting of the poles primarily to set up currents in a closed conductor located within the influence of the field of the motor so that the rotation may result from the reaction of such currents upon the field." He placed within the ring that was to generate the rotatory magnetic field, a soft iron cylinder or disk, carrying two coils of insulated wire wound at right angles to one another, and having their respective ends joined so that each formed a separate closed circuit; this was placed on an axis mounted on bearings. In another form the rotor was formed of an iron core, built up of disk to prevent eddy-currents, and enclosed within external coils or conductors, "applied to the cylinder longitudinally," formed into one or more independent circuits around the core. If copper plates were thus employed they were to be slotted longitudinally. This construction, using induced closed circuits on the revolving part of a motor wound for a progressive shifting of the magnetic polarity, was broadly claimed. The still wider claim of the discovery of a new method of electrical transmission of power must be given in Mr. Tesla's own words:—

"I am aware that it is not new to produce the rotations of a motor by intermittently shifting the poles of one of its elements. This has been done by passing through independent energizing coils on one of the elements, the current from a battery or other source of direct or continuous currents, reversing such current by suitable mechanical appliances, so that it is directed through the coils in alternately opposite directions. In such cases, however, the potential of the energizing currents remains the same, their direction only being changed. According to my invention, however, I employ true alternating currents; and my invention consists in the discovery of the mode or method of utilizing such currents.

"The difference between the two plans and the advantages of mine are obvious. By producing an alternating current, each impulse of which involves a rise and fall of potential, I reproduce in the motor the exact conditions of the generator, and by such currents and the consequent production of resultant poles, the progression of the poles will be continuous and not intermittent. In addition to this, the practical difficulty of interrupting or reversing a current of any considerable strength is such that none of the devices at present could be made to economically or practically effect the transmission of power by reversing in the manner described a continuous or direct current. In so far, then, as the plan of acting upon one element of the motor is concerned, my invention involves the use of an alternating as distinguished from a reversed current, or a current which, while continuous and direct, is shifted from coil to coil by any form of commutator, reverser, or interrupter. With regard to that part of the invention which consists in acting upon both elements of the motor simultaneously, I regard the use of either alternating or reversed currents as within the scope of the invention, although I do not consider the use of reversed currents of any practical importance.

"What I claim is—

"The method herein described of electrically transmitting power, which consists in producing a continuously-progressive shifting of the polarities of either or both elements (the armature or field-magnet or magnets) of a motor by developing alternating currents in independent circuits, including the magnetizing-coils of either or both elements, as herein set forth."

In April 1888, Tesla finds he can use a common return in a 2-phase system, and so reduces the four wires to three. He also shows how to take off 2-phase currents from an ordinary continuous current dynamo, by providing it with four slip-rings which are severally joined to four symmetrical points on its commutator. Passing on to generators which (like the well-known Thomson-Houston arc-light dynamo) have three coils united at a common joint, with their free ends connected to the segments of a commutator, Tesla shows that by connecting each of the three ends to a separate slip-ring with collecting brushes, three alternating currents can be taken off. These will be in three symmetrical phases. He suggests that in this case the motor or transformer should also be furnished with three energizing coils placed symmetrically.

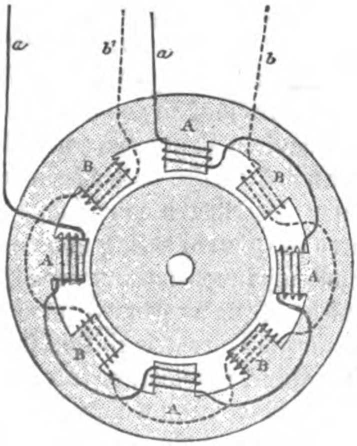

Fig. 99.—Multipolar Design.

From an early period in his researches Tesla seems to have grasped the importance of multipolar designs in reducing the speed. In May, 1888, he already had multipolar synchronous motors, and later this feature developed. Fig. 99 shows a design of a 4-pole field having four poles in the A circuit (alternately N and S poles), and four intermediate poles in the B circuit. In such a case the progression of the field is not a uniform rotation. The field of a pole at A does not shift round to the next pole at B. What happens is that the magnetism of the A pole dies out, while fresh magnetism grows in the neighboring B pole.

In April 1889, Tesla describes methods of operating two-phase motors from an ordinary (single-phase) alternate current, by using the device of *splitting the phase*, for starting synchronous motors; putting the two sets of coils in parallel with a non-inductive resistance in one branch (Fig. 100), and a self-inductive resistance (or choking-coil) in the other branch. When the motor has started these are cut out; but the motor continues to revolve as a synchronous motor. This device was not claimed generally by Tesla, and wisely, since it had already been used by Ferraris (p. 89); but he claimed it as a means of starting a synchronous motor. His words are:—"I believe I am the first to operate electromagnetic motors by alternating currents . . . . by producing a progressive movement or rotation of their poles or points of greatest magnetic attraction by the alternating currents until they have reached a given speed, and then by the same currents producing a simple alternation of their poles, or, in other words, by a change in the order or character of the circuit connections to convert a motor operating on one principle to one operating on another for the purpose described." None but synchronous motors appear to be contemplated.

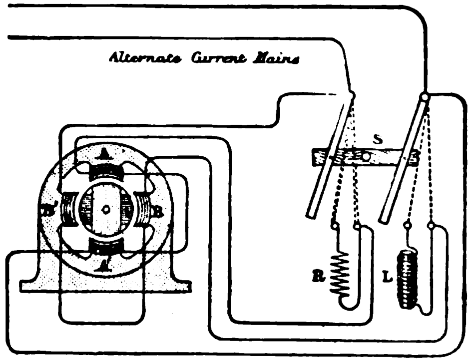

FIG. 100.—PHASE-SPLITTING DEVICE.

This was followed by other patents for various species of split-phase motors, including one illustrated in Fig. 101, in which there are two sets of coils to be united in parallel to ordinary alternate-current mains. The coils of one set were wound of thick wire on long iron cores, having much self-induction and small resistance; the others were wound on very short poles with wire of high resistance. The result is to retard the currents through the former as compared with the latter, and so establish a progressive shifting of polarity. Sundry other forms were devised between 1889 and 1891, when the series closed with a form of six-pole motor, in which the desired difference of phase was produced in one set of coils by the use of a condenser excited by currents in a secondary winding. This important series of patents passed into the possession of the Westinghouse Company. For fuller accounts of Tesla's work see his lecture of May, 1888, before the American Institute of Electrical Engineers; also the volume on Tesla's Inventions by Mr. T. C. Martin.

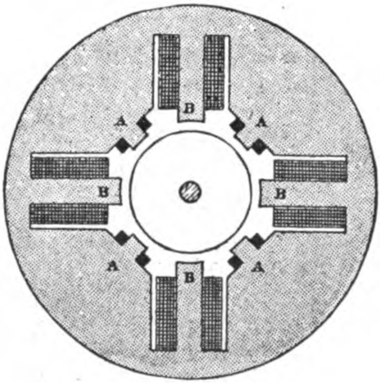

FIG. —101.—SPLIT-PHASE MOTOR.

Haselwander's Motors.—In the summer of 1887, Haselwander, an engineer of Offenburg (Baden), constructed a 3-phase machine of about 10 horse-power, having a stationary ring-armature 40 centimetres in diameter, wound with 12 coils, and an internal revolving 4-pole field-magnet. It had also a commutator to excite its own field-magnet. It was exhibited in 1891 at the Frankfort Exhibition. Haselwander's leading idea was as follows:—Every ordinary dynamo or motor for continuous currents really generates in its successive groups of coils alternating electromotive-forces in different phases; and the commutator serves to change these polyphase currents into an overlapping succession of uni-directional currents. In the transmission of power by means of continuous currents, two such continuous-current machines are joined together by two conducting lines. The pulsating continuous current given off by the primary machine (or generator) is again resolved into its components by the commutator of the secondary machine (or motor), and returns to the form of a series of polyphase alternating currents. So now the idea occurred to the inventor to suppress the two similar but converse operations of first uniting and commuting, and then of commuting back and resolving the polyphase currents generated in the separate sections of the armature. Thus one arrives at polyphase transmission of power and suppresses the commutator and brushes, except so far as these may be used in an auxiliary way to divert a small fraction of the currents to excite the field-magnets. The grouping of the coils was that of a star, but the coils were provided with terminals which enabled the

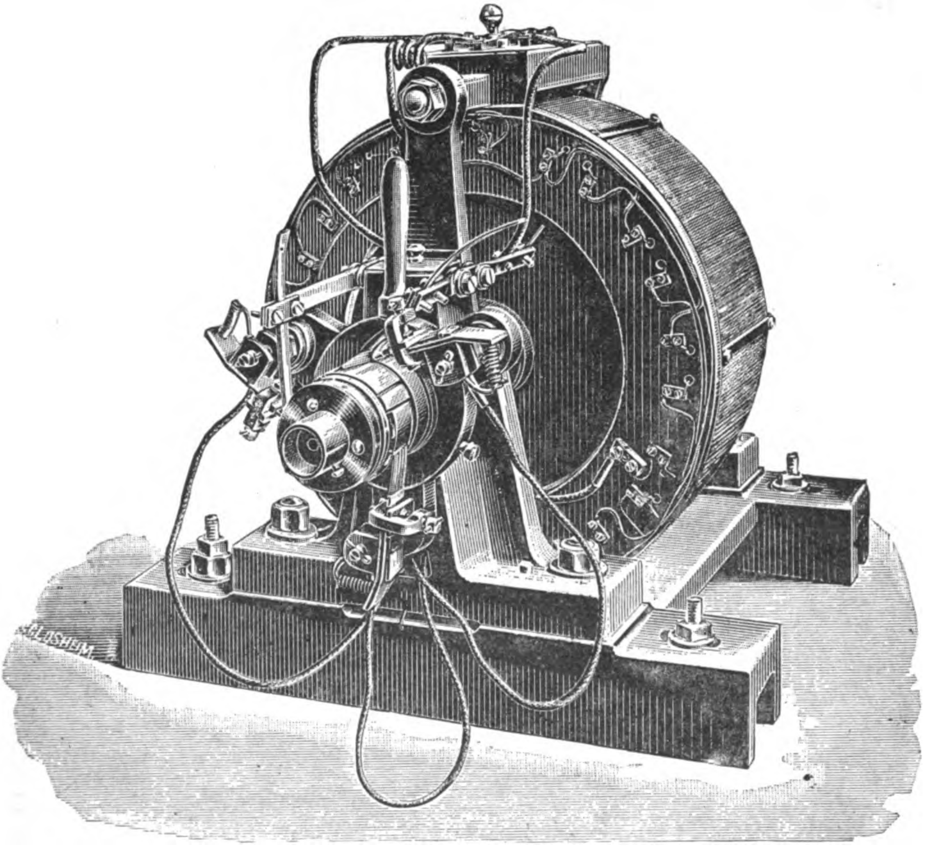

FIG. 102.—HASELWANDER'S MOTOR (1887).

individual coils in each of the three phases to be grouped either in series or in parallel. Each of the 12 coils had 52 windings of 1·52 millimeter wire. A current of 24 amperes at 100 volts could be taken off in each phase, at 960 revolutions per minute. This machine is described in a lecture by Dr. J. Epstein, in the *Elektrotechnische Anzeiger*, 1891.[1]

Wilson's Motor.—In a patent specification (No. 18525 of 1888) E. Wilson describes a 2-phase motor having an armature of ring or drum type, with commutator. Two-phase currents were supplied to both field-magnets and armature, the direction of rotation being controlled by the position of the brushes.

Wenström's Motors.—The late Mr. Wenström in 1890 took out a British patent (No. 5423 of 1890) for a 3-phase system. He describes, and gives a remarkably clear winding-diagram of a 3-phase generator. He proposed to join up the three windings in star fashion. A 3-phase transformer and a 3-phase motor are also included in the specification.

M. von Dolivo Dobrowolsky's Researches.—M. von Dolivo-Dobrowolsky is one of the chief electricians of the Allgemeine Elektricitäts-Gesellschaft of Berlin. To him we owe the term "Drehstrom" (originally applied to a 3-phase system), to denote a polyphase system of currents.

The first of Dobrowolsky's British patent specifications (No. 10933 of 1889) relates to the rotors of polyphase machines, and refers specifically to the production by Ferraris of rotatory fields in which conducting bodies are acted upon by eddy-currents induced in them. The proposal was to use as rotor an iron body in which there are set conductors or veins of copper, bars or strips, arranged so as to be transverse both to the direction of rotation and to the lines of the field; these conductors or strips being short-circuited at their ends. The drawings show simple forms of short-circuited rotors (including a "squirrel-cage") with solid iron bodies.

The next two patents (19554 and 19555 of 1889) relate to a form of polyphase generator and to a 3-phase trans-

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[1] See also the official report of the Electrotechnical Exhibition at Frankfort of 1891 (pub. 1893), p. 251; also *Elektrotechnische Zeitschrift*, 1891, pp 540 and 609.

former. The latter had a 3-branched core; the magnetic circuit forming a star-combination.

In August 1890, comes specification No. 13260 of that year, with the device of adding to the common junction of a 3-phase (or *n*-phase) system a common return, so as to render the three (or more) circuits independent of one another, and with regulating apparatus to control the pressures in each of the circuits. Two 3-phase adjustable auto-transformers are described, one of them being for long-distance work; and a combination of three separate transformers is also described.

Specification No. 20425 of 1890 describes a laminated rotor wound with insulated coils; and after pointing out how, at starting, the reaction of the rotor currents interferes with the field produced by the primary currents and diminishes the torque, proposes the method of introducing into the rotor circuit resistances capable of regulation. Liquid resistances are shown in the drawings.

In specification No. 3191 of 1891, Dobrowolsky shows polyphase transformers for changing currents of any number of different phases into a 3-phase system, together with methods of transforming 3-phase currents into a larger number of phases. And in No. 13503, of the same year he describes his method of obtaining currents of intermediate phase by combining mesh and star systems. For instance, he showed how in a 3-phase system six phases of currents could be produced from the three line-wires by six coils, three of which were in series severally with the three lines, and the other three joined as shunts across the lines; all six coils being spaced out properly on the inductor-core. By the introduction of these intermediate phases, Dobrowolsky proposed to make the torque (which in the absence of reactions from the rotor would fluctuate between certain maximum and minimum values in each complete period) more constant. In a large number of figures these various modes of concatenation of circuits and phases were elaborated.

Polyphase Work at the Frankfort Exhibition, 1891.—No record of the development of polyphase currents would be complete without a reference to the Electrotechnical Exhibition at Frankfort on the Main in the summer of 1891. Though nominally an International Exhibition the exhibits were mainly by German firms; and the feature of greatest interest were the polyphase apparatus contributed by many firms. The official report[1] gives many illustrations of these, together with the tests carried out during several months by the jury of experts. The following notes as to the exhibits of this class are extracted from this report.

Messrs. W. Lahmeyer & Co., of Frankfort, sent out from their model central station in the Machine-hall a 3-phase current at 75 volts, which worked several 3-phase motors, including the historic machine of Haselwander (Fig. 102, p. 101), a 10 horse-power synchronous motor of the ordinary 4-pole Lahmeyer type, but provided with three slip-rings instead of the usual commutator, and a number of smaller motors.

Messrs. Schuckert & Co. had two large 2-phase generators with armatures of their well-known flat-ring type, provided with slip-rings. One of these machines was in the Machine-hall and furnished power to the pumping station on the Main; the other, situated more than a mile away, at the Palm-garden, supplied power to the Distribution-hall in the Exhibition. As the ring-winding of these armatures was joined up in a mesh (Fig. 52, p. 44), it was necessary to employ two independent circuits with four lines in total; but by the introduction of transformers (compare Fig. 155, p. 179), it was possible to use a 3-wire system of transmission to a distance. Similar machines, with constantly-excited field-magnets, were used as motors. They ran synchronously, and with a greater output than if used as asynchronous motors without separate excitation. The 25 horse-power machine used as motor in the Exhibition had, indeed, an auxiliary commutator to enable it to excite its own magnets. The 50 horse-power motor at the pumping station was separately excited. The transformers employed were also of flat-ring form, the coils being wound in grooves planed out from a core built up of hoop-iron wound up in a close spiral,

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[1] *Allgemeiner Bericht über die Internationale Elektrotechnische Ausstellung in Frankfurt am Main*, 1891, 2 vols., published Frankfort, 1893

Messrs. Siemens & Halske showed some small 3-phase motors, one type having a closed-circuit rotor; another having a rotor provided with a commutator into which the 3-phase currents were led by three equidistant brushes after having traversed the three circuits of the windings on the stator. They also exhibited two 3-phase generators; one resembling their ordinary alternator, having as armature a set of 24 bobbins (in three series of eight bobbins each) revolving between two crowns of 16 alternate poles; the other on the lines of their ordinary continuous-current dynamos, having a drum armature connected at three equidistant points of the winding to three slip-rings.

The Allgemeine Elektricitäts-Gesellschaft, of Berlin, had, in conjunction with the Oerlikon Machine Company, of Zurich, undertaken the striking demonstration of long distance transmission of power, at high voltage, from Lauffen to Frankfort, which is further described below. This was a 3-phase (or so-called *Drehstrom*) transmission. The 100 horse-power motor in the Exhibition, which received current from Lauffen 110 miles away, was employed to pump water to supply an artificial waterfall. It is depicted in Fig. 104, p. 108. A smaller 3-phase motor[1] of about 3 horse-power, used to drive a small continuous-current dynamo with a load of lamps, had a construction the inverse of that now usual in induction motors. The currents were led by three slip-rings into a revolving armature, whilst a stationary external part, consisting of iron core-rings furnished with closed circuit winding, constituted an induced field-magnet. A still smaller motor with induction rotor without contacts served to drive a small fan. Other motors exhibited at the same place by the Oerlikon Company, constructed from Brown's designs, had the now usual construction of a fixed external armature built of core-rings pierced to receive the windings; whilst the rotor was also built of pierced core-rings with a simple copper circuit of bars short-circuited with two end-rings like a squirrel-cage. One of these, of 20 horse-power, at 1200 revolutions per minute, weighed only 420 kilogrammes.

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[1] Now in the laboratory of the Technical College, Finsbury.

THE LAUFFEN-FRANKFORT TRANSMISSION.

At Lauffen, near Heilbronn, the River Neckar has a fall of about 12 feet. The power had for some years been partially utilized for a cement factory; of the 1500 available horse-power about 1200 was taken up by turbines, but enough remained to furnish 200 or 300 horse-power, and it was proposed to utilize this for lighting the town of Heilbronn 6 miles distant. While this project was under consideration, came the suggestion, in the autumn of 1890, to seize the opportunity afforded by the Frankfort Exhibition to show what could be done in the way of transmitting power to a long distance at high voltage, and at the same time to demonstrate the advantages of the *Drehstrom* or polyphase system. Lauffen is 110 miles from Frankfort. To transmit, as was proposed, 100 horse-power through three copper wires, each only 4 millimetres thick, and with an efficiency of at least 75 per cent., necessitated the employment of a pressure of no less than 8000 volts. This *tour de force* was nevertheless accomplished. The engineer of the line and of the Lauffen generating station was Mr. Oskar von Miller, of Munich. With him were associated in harmonious action two great commercial firms, the Allgemeine Elektricitäts-Gesellschaft, of Berlin, and the Oerlikon Maschinen-Fabrik, of Zürich. They had the cordial co-operation of the Imperial German Post Office in the difficult task of laying out and constructing the line.[1] The copper wire was lent for the purpose by the firm of Hesse, in Hedderheim. The two generators, designed by Mr. C. E. L. Brown, and constructed by the Oerlikon Company, are described on p. 27. Each was capable of furnishing three currents, each of 1400 amperes at about 55 volts, the frequency being 40 periods per second. At each end of the line 3-phase transformers were used: at Lauffen to raise the pressure to 8500 volts, at Frankfort to reduce it back to about

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[1] A map of the route, together with detailed descriptions of the machinery and line, and of the tests made by the experts of the commission under Prof H. F. Weber, of Zürich, will be found in the volumes of the Official Report, published at Frankfort in 1893.

65 volts. These transformers (some built in Berlin, others in Oerlikon) were immersed, for better insulation, in oil. Their outward form resembled that of the Hochfelden transformers in Fig. 44, p. 38. The connections of both low-pressure and high-pressure windings were star-wise, the common junctions being earthed in every case. Fig. 103 gives a diagram that is self-explanatory. The lines were carried on about 3000 poles at a height of about 25 feet, each pole supporting three porcelain insulators with internal rims for holding oil. It crossed territories of four governments, Würtemberg, Baden, Hesse and Prussia, following generally the route of the Neckar Railway, but avoiding the long tunnel through the Odenwald at Krählberg, by going over the mountain. The total weight of copper in the lines was about sixty tons. The construction of the line was carried out under the direction of Mr. Ebert, Telegraph Inspector of the Imperial Post Office, which co-operated with the Würtemberg Royal Post and Telegraph Service in this undertaking. The Post-master General of the German Empire, Dr. Von Stephan, took a great personal interest in the work, and by his influential support contributed much to bring it to a successful issue. On August 24, 1891, the line was handed over by the officials to the Oerlikon and Allgemeine Companies, and the following day lamps in the Frankfort Exhibition were lit up by the power from Lauffen. In the exhibition there was a 100 horse-power 3-phase motor (Fig. 104), designed by von Dolivo-Dobrowolsky, and constructed by the Allgemeine Company,

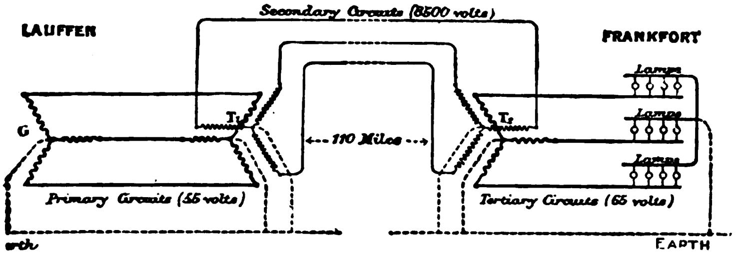

FIG. 104.—CIRCUITS OF THE LAUFFEN-FRANKFORT TRANSMISSION.

and other smaller motors, to which allusion has already been made above. This motor worked a centrifugal pump, taking about 60 horse-power, raising water for an artificial waterfall

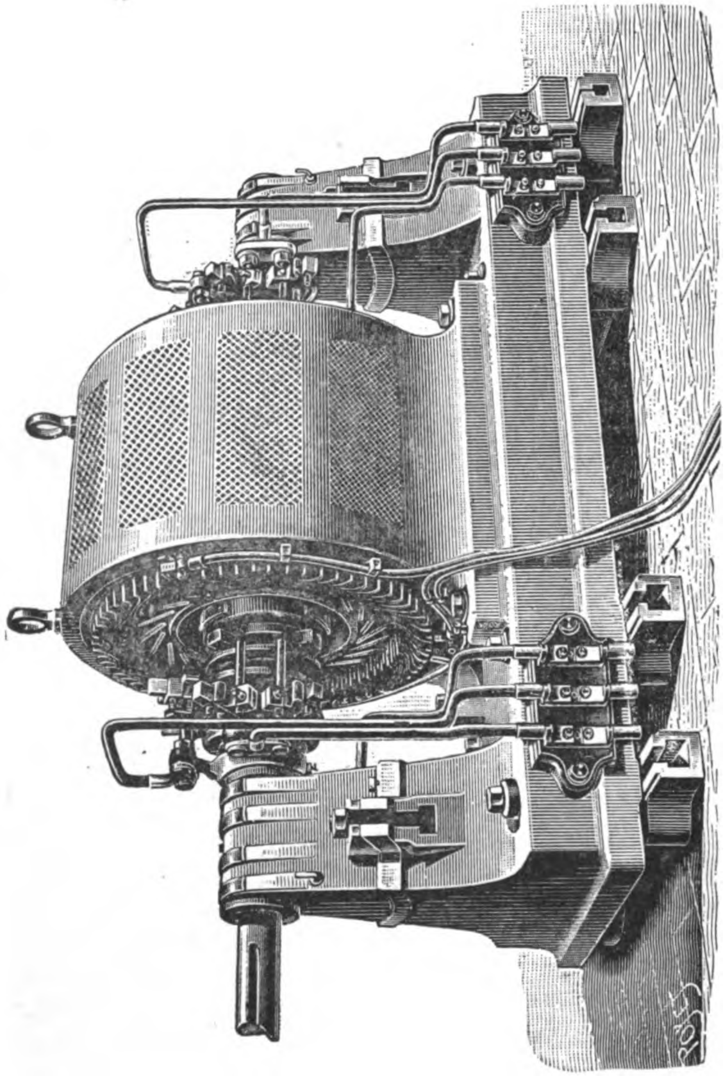

Fig. 104.—Dobrowolsky's 100 H. P. Three-phase Motor.

about 33 feet high in the grounds of the exhibition. In addition to these motors, there were about 1000 glow-lamps operated by the current so transmitted.

Great scepticism prevailed at first as to the probable result of the transmission under the novel conditions of using such very high voltages over so long a line, and with polyphase currents. It was anticipated by some that the efficiency would be greatly reduced by possible disturbances due to the capacity of the lines acting as condensers, or to leakage over the 10,000 insulators on which the line was supported. In private, some who were very closely connected with the enterprise expressed their fears lest the efficiency should fall below 50 per cent., and at one time there was some apprehension lest the jury would not be allowed to make full tests. However, as experience was gained these fears were found to be empty. The elaborate tests carried out by the commission in the autumn months, mostly at about 8000 volts pressure, showed that the energy given out electrically at Frankfort was as much (on the average) as 74 per cent. of the energy given by the turbines at Lauffen to the generator. The various sources of loss were ascertained and carefully measured, and the results of the various tests embodied in Professor Weber's report. It concludes with the following summary:—

1. In the Lauffen-Frankfort plant for the electric transmission of energy over a distance of 170 kilometres, by means of a system of alternating currents, with a pressure of 8500 to 7500 volts, and bare copper conductors insulated by oil and porcelain, the lowest output in the tertiary circuit at Frankfort was 68·5 per cent., and the highest output was 75·2 per cent., of the energy given out by the turbine at Lauffen.

2. In this transmission to a distance the only cause of loss measurable by the instruments was that due to the resistance of the circuit (Joule's effect).

3. Theoretical considerations showed that the influence of capacity upon long aerial bare conductors for transmission of energy to a distance by alternate currents, under the

conditions employed, and with use of a frequency of 30 to 40 periods per second, is of so entirely subordinate a magnitude that it need not be considered in designing electric transmissions.

4. As the expression of our experience during the foregoing measurements for the determination of the efficiency of the Lauffen-Frankfort transmission of energy we add, as a fourth result:—The electrical running with alternate currents of 7500 to 8500 volts in conductors of more than 100 miles in length, insulated by means of oil, porcelain and air, proceeds just as regularly, safely, and as free from disturbances as does running with alternate currents of a few hundred volts' pressure over conducting wires of a few metres' length.

In some further researches made later in the year by Dr. Kittler and Mr. W. H. Lindley,[1] extra high pressures, exceeding in some cases 28,000 volts, were obtained by putting two transformers in series at each end of the line, with the following summary result:—The transmission of power from Lauffen to Frankfort, with a high pressure of 25,000 volts (from line to line, or at 14,000 to 15,000 volts between lines and earth), and with a frequency of 24 periods per second, gave an efficiency of about 75 per cent. with a load of about 180 horse-power.

The Lauffen-Frankfort transmission was much more than a mere experiment. It was a daring and successful demonstration not only of the utility of high voltages in the transmission of power, but of the success of polyphase currents. As such it marked an epoch in the commercial development of electricity. It evoked an extraordinary interest throughout the continent of Europe, and in Germany in particular. One evidence of this is to be found in the circumstance that early in the history of the project the German Emperor himself made a contribution of 10,000 marks toward the cost of the scheme.

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[1] Official Report of the Frankfort Exhibition, ii. 451.

CHAPTER V.

STRUCTURE OF POLYPHASE MOTORS.

A POLYPHASE motor has been considered above as an apparatus in which a rotatory magnetic field produces Arago rotations in a moving mass of metal. But it may equally justly be regarded as a sort of revolving transformer, having primary and secondary circuits wound upon an iron core; the latter being so designed as to permit one of the two copper windings to revolve.

If the portion of the motor into which the polyphase currents are led in order to produce the rotatory field is regarded as the primary or inductor, the other portion, whether revolving or fixed, must be considered as the secondary or induced circuit.

In effect, the primary currents induce currents in the secondary windings, which are then acted upon by the magnetic field in which they find themselves, and are accordingly driven mechanically. Regarded thus, it becomes evident that to produce the best effects, the induced currents, whether called eddy-currents or not, should be provided with paths or conductors which will utilize them to the greatest mechanical advantage. If, for example, the current might be led through either of two paths, one of which lay in a weak magnetic field where its mechanical effect in aiding the rotation is small, the other lying in a position where at the moment when the current is strongest there is a strong magnetic field tending powerfully to aid the rotation, then obviously it will be advantageous to direct the current into the latter of the two paths.

Again, the primary or inductor may stand still, whilst the secondary or induced circuit rotates; or the machine may be designed in the inverse fashion, having a primary arranged to revolve while the induced or secondary part stands still, and by its reaction drives the primary part. The former of these two methods has the great advantage, that, since in all polyphase motors, except those of the largest sizes, the secondary circuit need consist of nothing more than a simple short-circuited winding, and therefore the machine will need no commutator, slip-rings, sliding contacts, or flexible connections—a result eminently tending to mechanical simplicity. The latter method of design requires that slip-rings and contact brushes be provided to bring the currents into the rotating part; but enables the resistance of the secondary closed winding to be readily altered—which, however, is no great advantage. To the latter class belong a small 3-phase machine Fig. 95, constructed in 1887 by the Helios Company; and one of the two motors of the Allgemeine Company shown at Frankfort in 1891, built from the designs of Dobrowolsky. Few motors are, however, now made thus.

In those machines which have a stationary inductor, the magnetic field revolves rapidly, and the rotating part runs up to synchronism, or near to synchronism with it, the magnetism induced in the rotating part tending to preserve a fixed direction relatively to the metal mass. In motors of the other sort, with revolving inductor, the inductor itself tends to revolve in the opposite sense to that of the magnetic field which it itself generates; and so tends to produce in the stationary secondary mass surrounding it magnetism in a fixed direction. Only, however, in the case of actually attaining the speed of synchronism does the magnetism in the induced part attain to fixity of direction relatively to the mass of metal under induction. In all other cases the magnetism slowly revolves relatively to the induced masses, with a frequency equal to the difference between that of the impressed currents and the frequency of the actual motion.

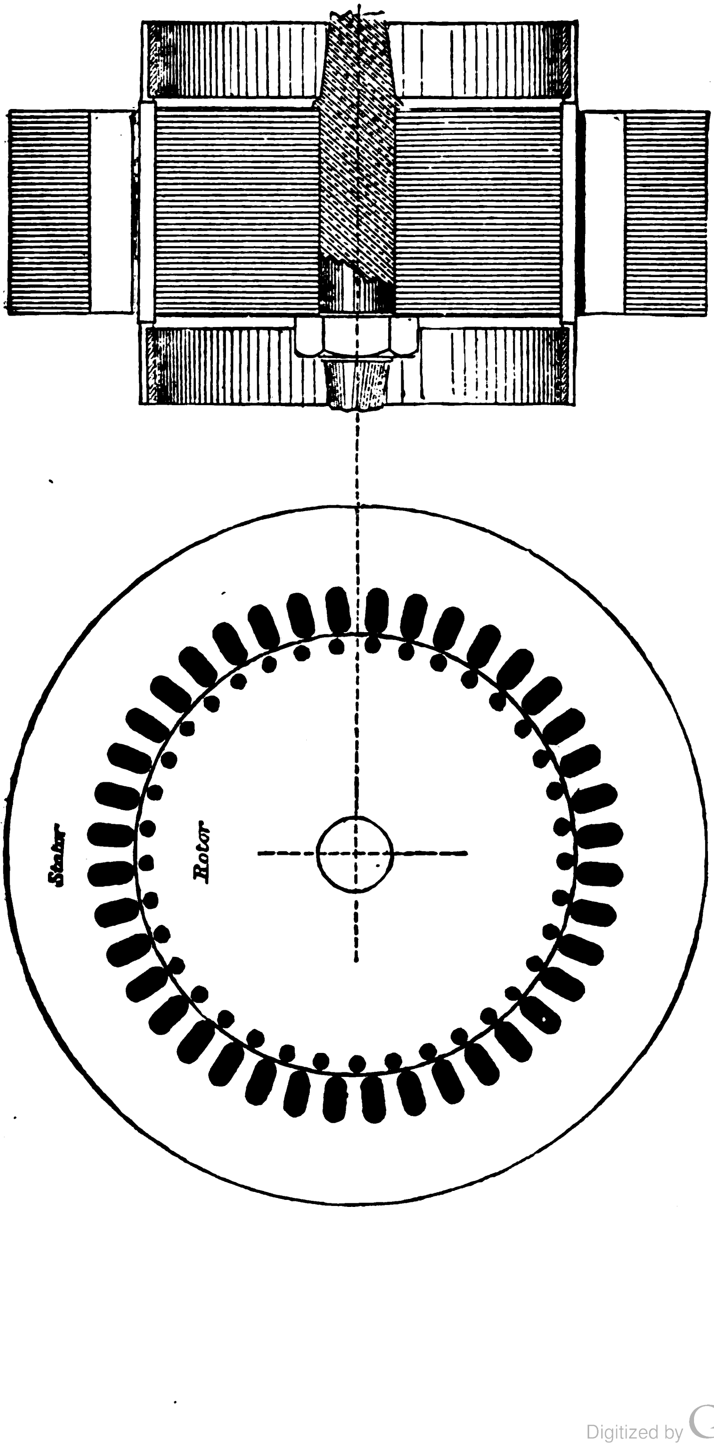
Rotor and Stator.—These considerations raise the question whether either part, the inductor or the induced mass, can be truly called an armature or a field-magnet. In ordinary dynamos and alternators we know that this question is settled not by the accidental circumstance whether the part revolves or stands still, but by the criterion whether the magnetism preserves an invariable direction or not with respect to the metal mass. In the field-magnet of every dynamo, motor and alternator the magnetism is fixed in direction. In the armature part of every dynamo, motor and alternator the magnetism changes rapidly with respect to the metal mass: the armature of a motor is moreover that part which receives the incoming current from the line.

Hence we may regard that part of the polyphase motor which receives the current as corresponding to the armature, whilst the other part, in which the magnetism is nearly fixed in direction with respect to the metal masses, corresponds to the field-magnet: it is, in fact, a field-magnet which is not magnetized by any separate currents or by any commuted part of the current, but is magnetized by the eddy-currents which are induced in it.

However, since the workman has got the notion that the revolving part must be called an armature, it is quite common to find the rotating part of polyphase motors so described. Yet in reality in almost all polyphase motors—for example Figs. 167, 169 and 170—the true armature is the part that stands still and surrounds the rotating part.

To avoid all confusion on this head we shall generally avoid the use of the terms armature and field-magnet in describing the parts of polyphase motors, and shall call the rotating part the *rotor*, and the stationary part the *stator;* the stator winding is usually the primary, the rotor winding secondary.

Both stator and rotor are commonly built up of soft sheet-iron stampings, pierced with holes to receive the windings. Fig. 105 shows the stampings for the 4-pole, 6-horse-power, 2-phase motor, the full drawing of which is given in Plate I. It will be observed that the holes are punched extremely near to the external periphery of the rotor and the internal periphery of the stator, so that, after machining, no more than a mere shred of iron remains, across which little magnetic leakage can take place. Other forms are shown in Fig. 36 and



FIGS. 105, 106.—STATOR AND ROTOR DESIGNED BY BROWN.

Plate II. Fig. 106 is a section of the motor parallel to the shaft, showing the stampings built up. The rotor is flanked at each end with a stout plate of metal; in large motors bolts are passed from end to end at some distance from the shaft. The figure shows the copper rods passed through paper tubes, and short-circuited at their ends by wide hoops of copper, which present a large cooling surface.

The above remark as to synchronism must not be taken to mean that the number of revolutions of the rotor tends to become equal to the number of periods in the frequency of the currents. That would be the case if the field was bipolar. But nearly all polyphase motors are multipolar; and the actual speed of rotation is reduced in inverse proportion to the number of pairs of poles in the revolving field. For example, if currents of frequency 60 per second are given to a motor so wound that its stator produces a revolving field of six alternate poles, i. e. 3 pairs of poles, the polarity will pass 60 times a second through one-third of the circumference; or the 6-pole field will complete 20 revolutions in a second, and *this* the speed of the rotor will tend to raise. The advantage of multipolar designs is, then, the attainment of slow speeds without the use of gearing.

Structure of the Rotor.—It was remarked on p. 111 that for the best mechanical effect the currents induced in the rotor ought to be led through paths which are so situated as to contribute best to the driving forces.

Consider the most elementary case—that of the cylinder of copper situated in a rotatory field, as in Ferraris's early motor, Fig. 92. The effect is equivalent to that produced by a pair of magnet poles placed at opposite sides of the cylinder, and revolving around it. Suppose the north pole to be in front of the cylinder (Fig. 107) and to be moving past it from right to left (or clockwise as viewed from above). The inductive action will be the same as if the pole stood still while the cylinder revolved from left to right. This will (by the principle explained on p. 5) set up electromotive-forces in the part which is passing under the pole in a direction shown by the arrows, upward; and there will be set up as the result a pair of eddy-currents as indicated in the sketch. Now the mechanical force which a conductor carrying a current experiences when in a magnetic field, is always in a direction at right angles both to the lines of the magnetic field and at right angles to the line of flow of the current. That portion of the copper which carries the upward current across the field, will be urged laterally to the left, whilst those parts in which the current is flowing horizontally will simply be urged up or down and will contribute nothing to the torque. On the other hand the parts of the copper in which the currents are flowing downwards will—if they lie in the same magnetic field—experience forces tending to turn in the other sense. Clearly, then, a better result will ensue if the downward returning currents are led into some path where they will return across a field of opposite polarity from that across which they flowed up. Then they will doubly tend to produce rotation.

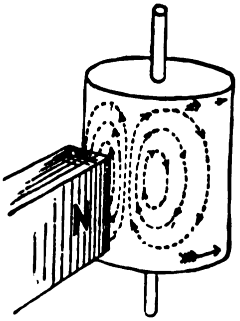


FIG. 107.—EDDY-CURRENTS INDUCED IN A COPPER CYLINDER.

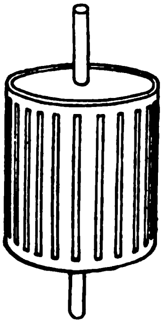


FIG. 108.

As a first stage to this, it will obviously be an improvement to make in the copper cylinder a number of parallel slits, which extend nearly to the ends of the cylinder as in Fig. 108, or to build it up of a number of parallel bars all joined together by a ring at each end. Dobrowolsky, who appears to have been the first to introduce the latter construction under the name of *Schluss-anker*, seems to have thought that the insulation of these bars from the iron core was of little importsnce. He regarded the bars as merely veins of copper lying buried in a solid mass of iron.

An iron core to the cylinder is obviously a great improvement over a mere copper shell or solid mass of copper, since it greatly improves the magnetic circuit and strengthens the field; thereby not only increasing the inductive action of the stator, but increasing also the mechanical effect of the currents in producing a torque. A solid cylinder of iron will of course serve as a rotor, as it is magnetically excellent; but the high specific resistance of iron prevents the flow of induced currents from taking place sufficiently copiously; and a solid cylinder of iron is improved by surrounding it with a mantle of copper, or by a squirrel-cage of copper bars, or (like Fig. 109) by imbedding rods of copper (short-circuited together at their ends with rings) in holes just beneath its surface. But since all eddy currents that circle round, as those sketched in Fig. 107, are less advantageous in their mechanical effect than currents confined to proper paths, and as they, whether mechanically advantageous or not, consume power and spend it in heating effects, it is still better, as found by Brown, to adopt a still more careful method of construction—namely to build up the iron core of thin disks or rings of soft sheet iron, to insulate them (lightly) from one another, and to insulate them (fully) from contact with the copper bars which constitute the conducting circuit. So we arrive at the form, Fig. 109, of rotor which has been so generally employed for small motors, and

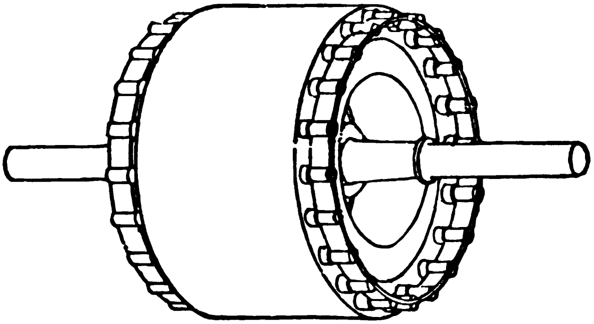


FIG. 109.—MODERN SHORT-CIRCUITED ROTOR.

even for quite large ones, of the *squirrel-cage* of copper rods imbedded in a laminated and insulated iron core, and provided with a short-circuited ring of copper (or in some cases of German silver) at each end.

But this simple form was not arrived at without experiment. Some hitherto unpublished researches of C. E. L. Brown, made early in 1890, are of great interest on this point. Mr. Brown had a number of rings constructed, all of the same internal diameter, wound in different ways, but all adapted to receive about an equal excitation by 3-phase currents, and also a number of different rotors of 199 cm. external diameter adapted to run in any of the rings. He was thus able to experiment upon the performance of a large variety of combinations, to test their torque when allowed to run at various speeds, and to measure their respective outputs of power. Fig. 110, which is copied from the drawing used in the con-

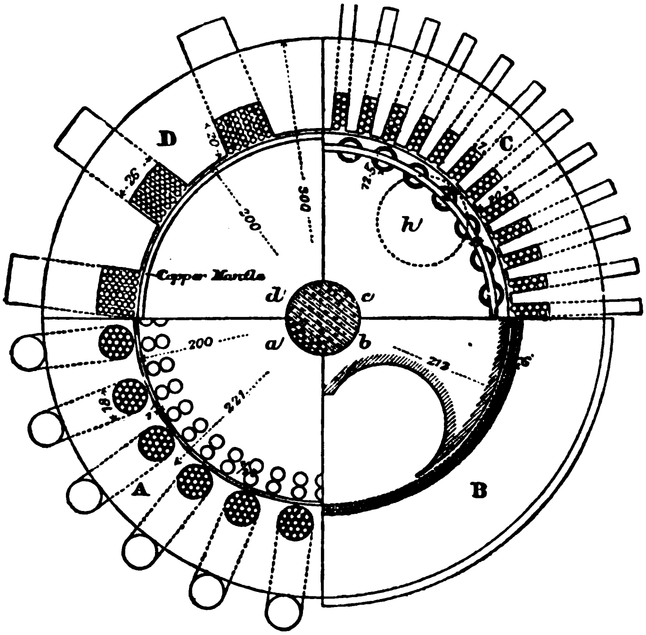


FIG. 110.—EXPERIMENTAL FORMS USED BY BROWN.

struction of this apparatus at the Oerlikon works, shows in four quadrants four of these ring structures, and four of the experimental constructions used for the rotor. The four rings used were as follows,—

A *Hole-ring*, pierced with 24 holes each 18 mm. in diameter, through each of which passed 21 wires. B. *Smooth ring*, an ordinary plain core-ring with winding in two layers, there being 24 groups of coils with 19 turns in each coil. C. *Fine-tooth ring*, having 48 teeth with slots between them, 9 wires being wound in each slot. D. *Coarse-tooth ring*, having 12 wide slots, each holding 36 windings of wire. The length of these ring-cores parallel to the shaft was 150 centimetres.

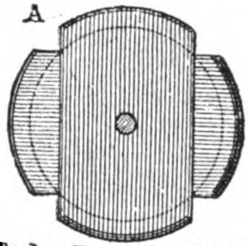
The four different rotors indicated in Fig. 110 were as follows:—*a.* A solid cylinder of wrought iron pierced with 44 pairs of holes. *b.* A massive wrought-iron double-T form, like a Siemen's shuttle armature, but without any windings upon it. *c.* A laminated iron cylinder built up of core-disks pierced with 30 holes just within the periphery, and furnished with stout copper rods 10 millimetres in diameter, all short-circuited at the ends by two copper rings to form a squirrel-cage. *d.* A massive wrought-iron cylinder surrounded by an outer cylindrical mantle of copper 4 millimetres thick. Besides these four were six others of the same size:—*e*, a simple massive cylinder of wrought iron; *f*, a massive cylinder of cast-iron; *g*, a massive cylinder of steel; *h*, a cylinder of wrought iron having four large holes bored through it (as indicated by the dotted circle at *h* in Fig. 110); *j*, a steel casting shaped as a cylinder with two parallel faces cut away; *k*, a double-T like *b*, but built up of laminæ of sheet iron; and lastly, a cylinder pierced with holes like *a*, but of massive wrought iron, and furnished with short-circuited copper conductors through the holes, but without insulation.

Of all these various rotors the laminated double-T proved the worst—it refused to run under any load. The solid cylinder of wrought iron was much better than that of cast iron; whilst the cylinder with the copper mantle surpassed both, whichever ring was used externally. Brake-tests were applied, as well as the test of using the motor to drive a small dynamo of which the output was electrically measured and controlled. The best form of rotor, whichever ring was employed, was found to be the laminated cylinder having the copper squirrel cage.

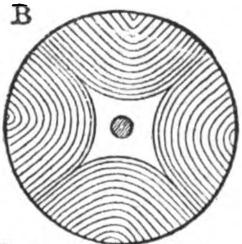
Of the four rings that with smooth core was found to be the least effective. The fine-tooth and coarse-tooth slotted rings gave larger torque than the hole-wound ring, but both of these, especially that with coarse teeth, gave rise to considerable heating of all the solid rotors, and caused a singing noise. Using the hole-wound ring, the massive rotors heated less strongly, but the squirrel-cage rotor, with insulated conductors and laminated core, remained quite cool as to its iron parts, and the copper parts heated but little. These results decided the issue in favor of the hole-windings, both for rotor and stator, a construction which has since abundantly justified itself.

Doubtless other pioneers have gone through similar experiments. It is of some interest to compare together various forms of winding suggested by different inventors at different times. Most of the forms of rotor depicted in Fig. 111 are taken from patent specifications, of which the dates are added.

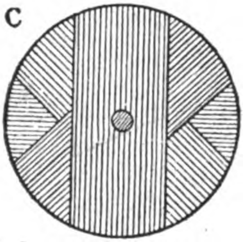
A is Tesla's form of 1888, with two closed coils wound over a laminated core at right angles to one another, and is suitable for a 2-pole field. B is a year and a-half later in date, and presents a winding suitable for a 4-pole field. C is described also for a 4-pole field, but is in reality only suitable for a 2-pole or a 6-pole field, as the coils are wound across diameters. It is only suitable, in fact, for a field in which there is a travelling S-pole diametrically opposite a travelling N-pole. For converse reasons the form D which Tesla describes for use in a 2-pole field is really adapted for a 4-pole field. E is a rational winding for a 2-pole, 6-pole, or 10-pole field, but not for a 4-pole or an 8-pole. The form shown at F is a simple Gramme ring wound with a series of coils forming a closed circuit or series of circuits. This would be extremely ineffective if merely coupled up in one closed



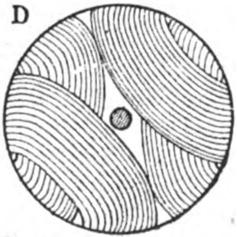
Tesla Two-pole Field
May 1888.



Tesla. Four-pole Field
Dec 1889.

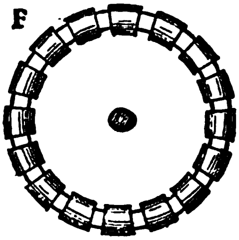


Tesla Four-pole Field
Dec. 1889.

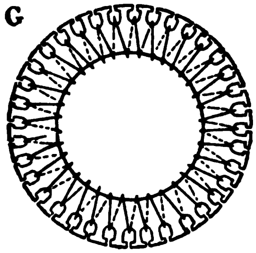


Tesla Two-pole Field
Dec. 1889.

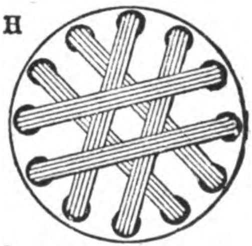
Tesla. Two-pole Field.
Jan. 1891



Tesla Four-pole Field
Dec. 1889



Helios Co Four-pole Field
1887



Dobrowolsky Two pole Field
1889

FIG. 111.—VARIOUS FORMS OF ROTORS.

circuit, as the induced electromotive-forces would oppose one another. The same defect is to be noted in the form G, which is that proposed by Mr. C. Coerper, of the Helios Company, in 1887 (see p. 91). The form H is one suggested by von Dolivo-Dobrowolsky in 1889.

Winding of the Rotor.—It will be convenient here to continue the consideration of the best modes of coupling up the conductors of the rotor, or of winding it, in case it is furnished with an actual winding of wires. As remarked above, it is obvious that the best effect will be obtained by so connecting the conductors that the currents flowing downward across a field of one polarity should return upward across a field of the opposite polarity. In a 2-pole machine, then, the loops of winding should span across, or nearly across, a diameter; whilst in 4-pole machines the span should be 90° or so, and in 6-pole machines 60° or so. This condition admits of many groupings of the connections, and is not, in the case of small machines, inconsistent with the short-circuited or squirrel-cage form. But there is another consideration to be taken into account, particularly in larger machines—namely, the advisability of adopting such a grouping or winding as will permit of the introduction, at the time of starting the motor, of an auxiliary resistance for the double purpose (see pp. 143, 197) of obtaining a larger starting torque, and of preventing too great a rush of current when the motor is switched on.

To give definiteness to the argument, let us consider the case of a rotor having 24 conductors carried through 24 holes in the periphery of the core-disks, and placed in a rotatory 6-pole field. If the field is revolving right-handedly with respect to the rotor, and a N-pole is just passing conductor No. 1, inducing an upward electromotive-force in it (i. e. one tending to send a current toward the spectator), there will be equal and similar electromotive-forces in Nos. 9 and 17, and equal but opposite electromotive-forces in Nos. 5, 13 and 21. To produce the best effect, these six conductors should be connected together, and there are several ways of doing this. We will regard these six as "similars."

Method No. 1.—*All similars in series.*—If the six conductors are all joined in series, with a sort of zigzag or wave-winding, they will constitute a closed circuit. In that case there would be four such closed circuits in the winding, Nos. 2, 6, 10, 14, 18 and 22, constituting a similar closed circuit; and the others similarly.

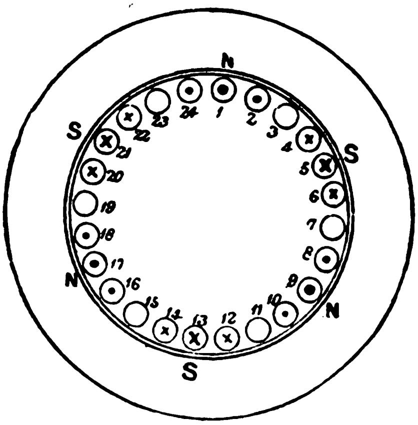


FIG. 112.

Method No. 2. Similars connected in diametral groups.—Let each be connected into a closed loop with its fellow at the opposite end of a diameter. This gives three independent closed circuits for the six similars, as in Fig. 114. Or, for the whole rotor, 12 separate circuits. But if the electromotive-forces acting up on one side and down on the other, are equal, there is not the slightest reason why the separate loops should not be connected at their crossing point, as in Fig. 115. Applying the same argument to the rest of the con-

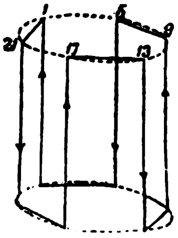


FIG. 113.

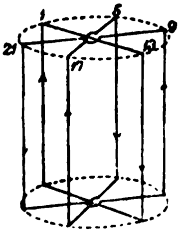


FIG. 114.

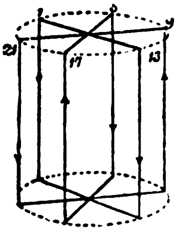


FIG. 115.

ductors, this leads to a simple bunching of all the twenty-four conductors together at both ends—they will be all short-circuited.

Method No. 3. *Similars connected in neighboring pairs.* —Let each conductor be paired off in a closed circuit with its nearest "similar." The result will be, as in Fig. 116, to give three independent closed circuits, or twelve separate circuits for the whole rotor. Obviously it will make no difference whether No. 1 is paired with No. 5 or with No. 21. Hence Fig. 117, which shows them all joined together at their ends by hexagonal connectors, will be electrically just as effective. The legitimate conclusion of this construction is the squirrel-cage structure short-circuiting all the conductors. Methods Nos. 2 and 3 are electrically equivalent to one another, though No. 3 is obviously of greater mechanical convenience.

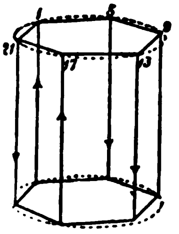


FIG. 116.

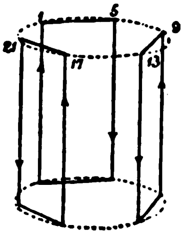


FIG. 117.

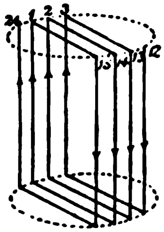


FIG. 118.

Meteod No. 4. *Neighboring conductors grouped as similars.* Adopting the less advantageous plan of ignoring slight differences of phase, and treating neighboring conductors as though they were simultaneously acted upon, we may combine groups of neighboring conductors as though they were similars. For instance, in Fig. 118, we may deal with Nos. 24, 1, 2 and 3, along with Nos. 12, 13, 14 and 15, and combine them in several ways. We may join them into four independent closed circuits, No. 24 with No. 15; No. 1 with No. 14; No 2 with No. 13; and No. 3 with No. 12. Or we may put them all in parallel, Nos. 1, 2, 3 and 24, being bunched together and joined at both ends to Nos. 12, 13, 14 and 15, also all bunched together. Or lastly we may join them in series in one single closed circuit in order, and finally joining the last to the first. In that case the whole of the 24 conductors would constitute three independent groups, each consisting of eight conductors in series.

Another mode of grouping, electrically equivalent to the preceding, is to combine Nos. 1, 2, 3 and 4, with Nos. 5, 6, 7 and 8 as returns in a group. In this case also, if the mode of combination were in series, the whole of the 24 conductors would constitute three independent groups.

It will be evident that in the case of the combining of "similars" in closed circuits, however many of them are joined in series or in parallel, the current in each conductor will be the same independently of the grouping, because as they are joined up in series of 2, 4, 6 or more, the total resistance and the total electromotive-force increase in the same proportion. From this point of view, so long as "similars" are being dealt with, it makes not the slightest difference to the action of the motor whether the grouping is in independent circuits, or in series, or in parallel. But when it is desired to provide arrangements for introducing into the rotor circuits a starting resistance, it then becomes obligatory to use series methods of combining, so as to simplify the number of slip-rings and brushes that must be used, to reduce the quantity of current that must be handled, and also to minimize the influence of the resistance of the contact brushes, etc., after the additional resistance has been cut out.

Method No. 5. Grouping for insertion of starting resistance.—It is usual in cases where starting resistances are to be introduced into the circuits of the rotor for the purpose of increasing the initial torque, to provide means for leading the current out of the rotor by slip-rings and contact-brushes. To avoid complications, it is usual to group the windings starwise in three series, having a common junction, each free end being led to a slip-ring on the shaft. From the three contact-brushes wires are led off through three appropriate resistances (frequently a liquid resistance, such as water containing carbonate of soda, with carbon plates as electrodes, is used) to a common junction. This construction, which may be observed in Plate II., is used even when the currents supplied to the stator are 2-phase as well as when they are 3-phase. But, for this grouping in three series it is preferable that the number of conductors per pole of the revolving field should be divisible by three. This is not the case in the example just considered where we had a 24-part rotor in a 6-pole field. With an 18-part or 36-part rotor, this could be done. True, the winding can be divided into three symmetrical series (as in method No. 4 above), but the three series could not be grouped as a star unless a fourth slip-ring were added to provide for a common return.

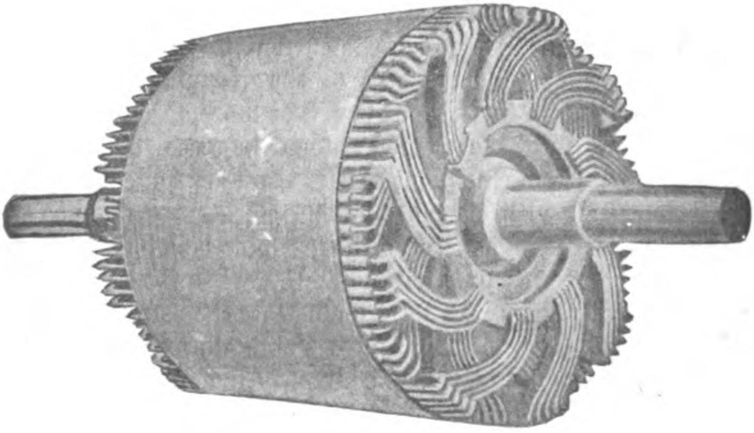


Fig. 119.—Wound Rotor of the Oerlikon Company.

Fig. 119 illustrates a wound rotor constructed by the Oerlikon Company.

One detail relating to the squirrel-cage form of construction is not without importance. In all the cases where the number of conductors on the rotor is such as to have a common factor with the number of poles in the rotating field, there is (more particularly at starting) a tendency for the rotor to operate unduly as a mere transformer. Such a form of apparatus as Fig. 120 would form an excellent 3-phase transformer without moving, the rotatory field simply inducing synchronous 3-phase currents in the windings of the central part. Now the tendency to turn in this case would be extremely small. In all induction motors something of the same kind would tend to occur if the number of conductors or groups of conductors on the rotor corresponded with those of the stator. To avoid this it is usual to design the stator and rotor with different numbers of groups or of conductors, and in the case where the windings are all short-circuited together this is carried so far as that the numbers chosen shall not even have a common factor. As an example see the 2-phase motor of Brown, Fig. 105, p. **114**, which has the stator pierced with 40 holes to receive the primary winding, and has as the rotor a squirrel-cage of 37 bars.

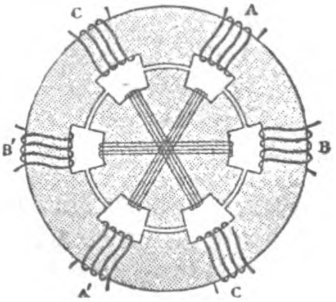


FIG. 120.

Structure of the Stator.—The winding of the stator of a motor is commonly of the nature of a drum-winding with the conductors passed through holes in the iron as previously shown in Figs. 37, 41 and 42, though for small motors it is sometimes similar to the winding of a Gramme ring as shown diagrammatically in Figs. 49 and 57. In considering the theory of a motor it is usual for the sake of simplicity to take a simple winding like that of Fig. 49, with the field diametrically across the rotor. It is well therefore to follow the connection between such a diagram and a multipolar drum-wound stator such as is commonly found in practice. Fig. 121 is a diagram similar to Fig. 49, with only one turn on each coil passed through holes in the iron. Fig. 122 shows the same iron wound in drum fashion, the directions of the current in the active conductors remaining unchanged. The dots in the holes indicate a current coming upwards, the cross a current going downwards through the plane of the paper.

Now imagine the stator cut through at the line C D, and straightened out into an arc of much greater radius, and a number of these straightened pieces put end-on to each other. We should then have what is shown in Fig. 123, which in principle is the same as the winding of Brown's 2-phase motor shown in Fig. 171.

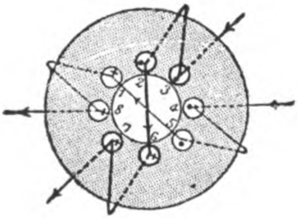


FIG. 121.

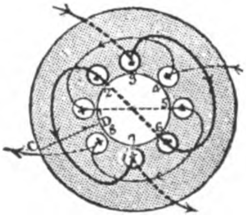


FIG. 122.

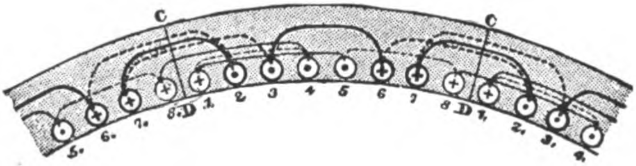


FIG. 123.

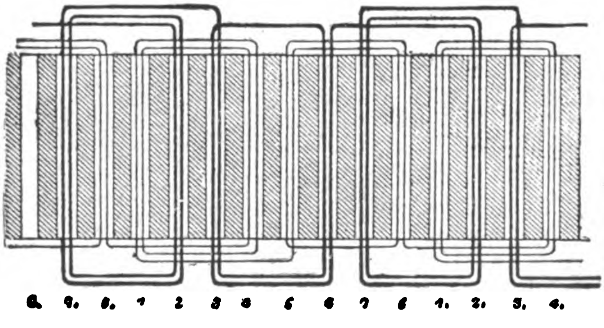


FIG. 124.

The wire in each coil, such as that through the holes 3 and 6, may go several times round before passing on to the next one. For instance, the winding in the particular Brown motor mentioned comes up 3 and down 6, then up 3 and down 6 again many times, and then passes up 2, being wound many times again through 2 and 7 before it passes on to $3_{/}$, and so on.

This winding is shown developed in a chart, Fig. 124, with only two windings on each coil.

Progression of the Field.—The way in which the magnetic poles progress along the face of a multipolar stator with a winding of this kind can be seen by reference to the Figs. 125 to 128, in which the stator surface is drawn straight instead of curved.

The holes through whichthe stator conductors pass are represented by the upper row of circles, the lower row represent the holes for the rotor conductors (but of currents in the rotor no account is here taken). An air space is shown between the stator and rotor. To distinguish the coils belonging to each circuit, the holes for one circuit (which we will call circuit No. 1) are drawn with thicker circles than the other. In the clock diagram on the left of each figure the thick crank A shows the phase of the current in circuit No. 1 and the thinner crank B shows the phase in circuit No. 2. In Fig. 125 the current in circuit No. 1 is at its maximum and circuit No. 2 is carrying no current. The magnetic lines will circulate through the iron in paths similar to those shown by the arrows. The *magnetomotive-force* exerted by the stator coils at each point along the surface of the stator is shown by the square-cornered curve below each figure. This is arrived at in the following manner:—In the space between the holes 7_0 and 2_0 there is a certain magnetomotive-force due to the current in the conductors 6_0, 7_0, 2 and 3. Its amount at any moment may be represented by the projection of the crank A on a vertical line (the current in the coils being proportional to this projection). We may therefore draw the line L M equal to the projection of A; then M N represents approximately the magnetomotive-force at each point along the surface between the holes 7_0 and 2. Between 2 and 3 the magnetomotive-force is nil, the conductors

FIG. 125.

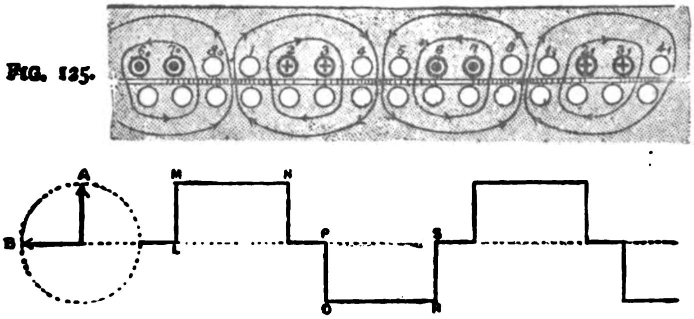


FIG. 126.

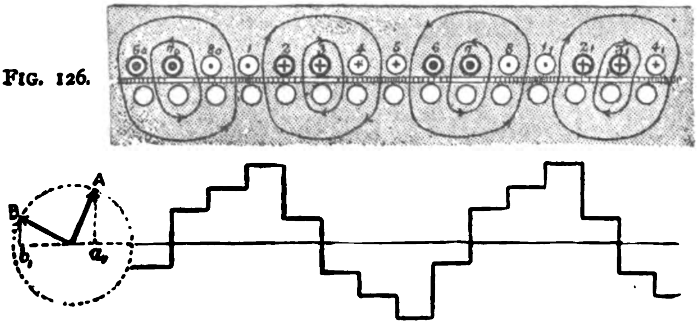


FIG. 127.

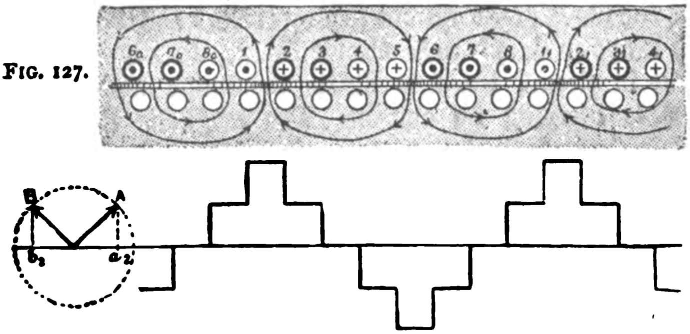
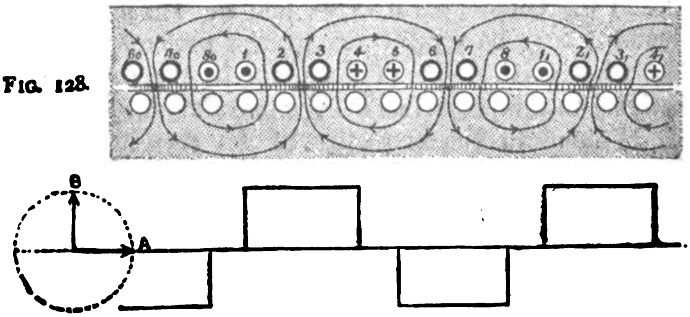


FIG. 128.



on one side neutralizing the effect of the conductors on the other. Between 3 and 6 the force is reversed and is represented by the curve P Q R S below the zero line.

In Fig. 126 the phase is advanced by $\frac{1}{16}$ of a period. If we draw for the coils of circuit No. 1 a curve similar to that of the last figure, taking A *a*, as the magnetomotive-force and also a similar curve for the coils of circuit No. 2 taking B *b*, as the magnetomotive-force, then the sum of these curves is that shown below, Fig. 126.

In Fig. 127 the phase is advanced another $\frac{1}{16}$ of a period so that the currents in the two circuits are equal. The sum of their magnetomotive-forces is shown by the curve in this figure. After another $\frac{1}{16}$ of a period the curve would be of irregular form again like that in Fig. 126, but is not shown in the figures. After a further $\frac{1}{16}$ of a period the current in circuit No. 1 has sunk to zero and in circuit No. 2 is a maximum, so that the curve (Fig. 128) is similar to that of Fig. 125 but is shifted forward through the space of two holes. At a quarter period later it is shifted on past another two conductors and after another half period has passed all eight holes and is in the position shown Fig. 125 relatively to the next set of coils. The poles in fact have gone through a complete cycle.

Though these curves approximately represent the distribution of the magnetomotive-force they do not represent the distribution of the magnetic flux, for there can be no sudden changes such as are represented by the corners in the above curves. The natural spreading of the lines of force and the fact that they are moving forward cutting conductors, tend to wipe out the effect of the corners in the magnetomotive-force curve, so that the curve representing the density of the flux at each point will be a round even curve not unlike a sine curve; in fact any deviation from a sine curve would be an irregularity strongly opposed by currents in the conductors, and we may therefore, without falling into any great error, assume that the density of the flux follows a sine curve in its distribution.

It will be noticed that the maximum ordinate in the curve in Fig. 127, is greater than the maximum ordinate in Fig. 125, the ratio being as $\sqrt{2} : 1$. Thus the maximum value of the flux-density tends to change in value. Any change is however opposed by the conductors of the rotor, so that very little change will actually take place.

M. von Dolivo-Dobrowolsky in 1891 gave the curve shown in Fig. 129, in which the effect of the two magnetizing currents I and II in a simple 2-phase motor are added together, giving rise to a field of varying intensity shown by the upper curve F F, the variation amounting to 40 per cent. In Fig. 130 are given his curves for a 3-phase motor in which the variations only amount to 14 per cent. The greater the number of phases the less does this variation become. M. Dobrowolsky was then of opinion that such fluctuations of the field were disadvantageous in a motor and lessened the torque, but in the modern light of the monophase motor it does not seem that they are harmful. The fluctuations, so far as they occur despite the counteracting effects of conductors in the vicinity, will act as though there were superimposed upon a constant rotating field a number of stationary alternating fields having poles at every point where a group of conductors belonging to one circuit lie next to a group of conductors belonging to another circuit and having a frequency the same as the frequency of supply.

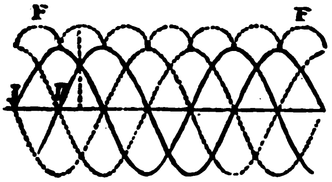


FIG. 129.

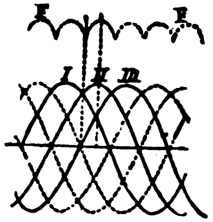


FIG. 130.

Flux-density.—The maximum flux-density B permissible in the iron of these motors depends upon the frequency of the supply. The loss of energy per cycle in the iron cores owing to hysteresis increases disproportionately to the flux-density. If the latter is increased from 4000 to 8000 the loss by hysteresis increases nearly fourfold; the law discovered by Steinmetz being that the loss is proportional to the 1·6 power of B. At high densities the iron cores would consequently overheat, unless the frequency of alternations were reduced. Or, conversely, with low frequencies flux-densities are permissible which cannot be used with high frequencies. Kolben gives the following values:—

For	40 cycles	.. .. ..	B =	6500 to 5500
"	50 "	.. .. ..		6000 " 5000
"	60 "	.. .. ..		5000 " 4500
"	80 "	.. .. ..		4500 " 4000
"	100 "	.. .. ..		4000 " 3500
"	120 "	.. .. ..		3500 " 3000

In the iron between the holes the flux-density is often greater than is represented by these figures, and may be carried up to 11,500 for a frequency of 40.

The breadth of the iron of the stator (C D in Fig. 123) may be a little less than one-half the polar pitch so as to afford an easy path for one-half of the lines from one pole which distribute themselves as shown in Figs. 24 and 25.

CHAPTER VI.

ELEMENTARY THEORY OF POLYPHASE MOTORS.

IN considering the theory of polyphase motors it is proposed first of all to give the general relations between the speed of revolution of the magnetic field, the speed of the revolving part of the machine, the resistance of the circuits, the torque, and efficiency of the machine. Afterwards an analytical method of treating the theory will be given. For the sake of simplicity we will take a bipolar machine, the iron in whose stator is of the general shape shown in Fig. 105 and the air space so small that magnetic leakage may be neglected. Suppose that a rotatory magnetic field is produced by either 2-phase or 3-phase currents in the stator. The currents in the rotor (as we shall see more exactly in the next chapter) also produce a magnetic field, which, compounded with that of the stator, gives rise to a resultant rotating field. It is to this resultant field that the electromotive forces in the conductors and the torque are due. We will consider that it consists of a uniform flux flowing diametrically through the rotor and cutting the conductors of both stator and rotor as it revolves

Let Ω stand for angular speed of the rotatory magnetic field $= 2 \pi n$ in a bipolar machine, where n is the frequency of period. If the machine is multipolar having m pairs of poles then $\Omega = 2 \pi n \div m$.

Let ω stand for the angular speed of the rotating part, or *rotor* of the machine, $= 2 \pi n_2$, where n_2 is the actual number of turns per second.

Let T be the torque between the stator and the rotor.

Let W stand for the power (total watts) communicated by the stator to the rotor.

Let w stand for the power (useful watts) actually used in turning the rotor.

$\Omega - \omega$ is the *slip*[1] of the rotor with respect to the field, or is the difference of their angular speeds. If the field has an angular speed $\Omega - \omega$ greater than that of the rotor, it is clear that the inductive action on the circuits of the rotor will be exactly the same as if the rotor were revolved backwards with a speed $\Omega - \omega$ while the field stood still.

$W - w$ is the power wasted in heating the conductors and iron of the rotor, since it is the difference between the total power supplied to the rotor and the power it utilizes.

Now W is proportional to T and to Ω, and, therefore, by choosing suitable units may be written $W = T\,\Omega$. And w is proportional to T and ω, and may be written $w = T\,\omega$.

Hence, dividing the last equation by the preceding.

$$\frac{w}{W} = \frac{\omega}{\Omega}.$$

From this we see that the efficiency of the *rotor* is the same as the ratio of the two speeds. The efficiency of the stator will be considered presently.

Further, the rotatory field motor is simply a sort of running transformer, of which the stator and rotor windings constitute respectively the primary and secondary. Now, if ω were made $= \Omega$ there would be no induced currents in the rotor conductors, the stator would then simply act as a choking coil; hence, it follows that if the condition of supply of the primary currents is that of constant voltage, the magnetic flux through the machine, rotating with speed Ω, will have an approximately constant value at all loads, just as the flux in the core of an ordinary transformer has. This, of course, is only true when the current in the stator coils is unrestricted; it is not true, for instance, if a resistance is put in series with the stator coils, or when the motor is starting without any resistance in its rotor circuit, as will be seen hereafter. Further, if there is

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[1] Some writers apply the word "slip" to the ratio of the two speeds, $\frac{\omega}{\Omega}$ This is conveniently distinguished by the word "slippage."

very little magnetic leakage in the gap between stator and rotor (as is indeed the case in well-designed motors), the only electromotive-forces in the rotor conductors will be those produced by the resultant magnetic field, and therefore the maximum currents in them will occur when the conductors are in that part of the field where the flux-density is a maximum. And as the flux is constant at all loads (subject to the above conditions), it follows that the torque will be proportional to the currents in the rotor. But these are proportional to the slip $\Omega - \omega$: hence, also, it follows that T will be proportional to $\Omega - \omega$, and may be written $T = b\ (\Omega - \omega)$, where b is a constant depending on the strength of the field, the radius of the rotor, and the length and resistance of the conductors of the rotor.

We may now write:—

$$\text{Useful watts } w = b.\ \omega\ (\Omega - \omega)\ ;$$
$$\text{Total watts } W = b.\ \Omega\ (\Omega - \omega)\ ;$$
$$\text{Wasted watts } W - w = b.\ (\Omega - \omega)^2.$$

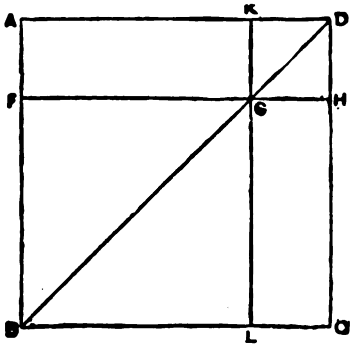

Fig. 131.

Hence we may at once apply the now well-known diagram of motor efficiencies, by drawing a square A B C D (Fig. 131), having its side A B numerically equal to Ω, and cutting off a piece B F equal to ω. The area A F H D represents the total watts supplied, the area A F G K, or G L C H, the watts utilized, and the square K G H D the watts wasted in heating the conductors of the rotor. The efficiency will approach unity as F moves up towards A; and, as with continuous-current motors, if it were not for the weakening of the field by armature reaction, the output would be a maximum when $\omega = \frac{1}{2}\ \Omega$, the efficiency being then only 50 per cent. We shall see presently that when the motor is running at much below its proper speed, magnetic leakage and other causes play such an important part that the torque is actually less than at a higher speed. Fig. 131 is, however, applicable to cases of normal running, and shows how these rotatory-field motors behave in an exactly similar manner to continuous-current motors.

In good modern rotatory-field motors the slip is only, at the most, about 4 per cent., except for very small sizes of machine, where it may be 10 per cent. at full load.

In the above investigation no account has been taken of the loss due to heating in the conductors of the primary or stator circuit. This, like the ordinary C^2R loss in the exciting circuit of any dynamo, is but a small percentage of the whole energy supplied, and can be readily calculated from resistance of the stator coils. Neither has any account been taken of hysteresis losses in the iron of the stator, which also have to be supplied, as it were, by additional excitation, but are small in a well-designed machine. Besides losses by hysteresis or by eddy-currents in the iron, the friction of the journals will deduct from the available power.

2. RESULTANT MAGNETIC FLUX IN MOTOR.

It was pointed out above, from consideration of transformer analogies, that the magnetic flux in the motor is of approximately constant value at all normal loads. And we have seen (p. 131) that in the air gap between rotor and stator the flux-density varies approximately as a sine function around the periphery from point to point. Let the density of this flux in the direction in which it is a maximum be called B. This flux-density, like the flux-density in a transformer core, is the result of the magnetizing actions of both the primary and the secondary windings. Kapp has given[1] a discussion of the reaction, which may be summarized as follows:—

Take a line B, to represent (Fig. 132) the maximum of the flux-density in the motor; in a bi-polar machine it may be considered as revolving clockwise around O as a centre,

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[1] Gisbert Kapp, "Electric Transmission of Energy," 1894, p. 310.

with an angular speed Ω. This field is due to the joint action of the impressed field excited by the primary currents in the stator, and of the induced field excited by the secondary currents in the rotor. These rotor currents are in phase with the resultant field (if there is no magnetic leakage), and proportional to it, and to the slip. They tend to produce a cross-magnetizing reaction. They may be represented by a length *c*, set off along the line B. This current *c* tends to produce a cross-magnetizing field proportional to itself. Let the line *b* at right angles to B represent this cross field. Here $b = k\, c$ where *k* is a coefficient depending on the reluctance of the magnetic circuit and the number of windings on the rotor. Complete the triangle B *b* *a* by drawing the line *a*. Then *a* represents in magnitude and phase the magnetic field that must be impressed by the primary currents in the stator, since B is the resultant of *a* and *b*. The angle β is the angle by which the current in the rotor lags behind the impressed field.

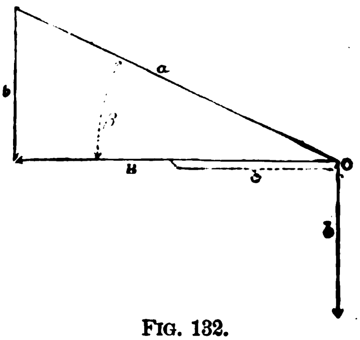

FIG. 132.

Further, since the torque is proportional to both B and *c*—that is to B and *b*—the area of the triangle *a* B *b* will represent the torque.

Moreover, since *c* depends on the electromotive force in the rotor conductors, it is proportional to the slip, and to B, and to a constant depending inversely on the resistance R in the rotor circuit, we may write

$$c = \frac{\mathrm{B} \times \text{slip}}{\mathrm{R}}$$

$$\text{or slip} = \frac{c\,\mathrm{R}}{\mathrm{B}};$$

and substituting $b + k$ for c,

$$\text{slip} = \frac{b}{\mathrm{B}} \times \frac{\mathrm{R}}{k}:$$

but $b \div \mathrm{B}$ is $\tan \beta$, hence slip is proportional to R $\tan \beta$. That is to say, if the slip is great the angle of lag β will be great.

3. CONDITIONS OF OPERATION.

There are three chief stages of operation to be considered; and for the present we will consider the supply voltage constant, and the machine devoid of magnetic leakage.

(i.) *Starting.*—Here $\omega = 0$, and slip $= \Omega$. Rotor currents enormous, primary currents also enormous. Therefore, β the angle of phase-difference between primary currents and resultant field very large. Torque enormous if no magnetic leakage.

(ii.) *Running at Light Road.*—Here ω is very nearly equal to Ω; and as slip is small, rotor currents will be small, and their reaction small. Angle β will be small, and a will not be much larger than B.

(iii.) *Running with Heavy Loads.*—Here $\Omega - \omega$, the slip, must be considerable enough to allow of the generation in the rotor of currents considerable enough to produce the necessary torque at the actual speed of rotation.

In addition to the above, if the speed is artificially brought up to synchronism by supplying from without power to overcome friction, etc., there will be no rotor currents and no torque. If the speed is artificially increased beyond this, so that the rotor runs faster than its field, power will be consumed in driving it, and it will act as a generator, pumping back current into the supply network, as we shall see presently.

4. STARTING TORQUE.

In the above we have considered a motor working under nominal conditions, so that the rotor currents are not excessive and the effect of magnetic leakage has been neglected. When, however, the motor is being started, the slip is so great that enormous currents would be generated in the rotor circuit if of low resistance. These currents would call for very large currents in the primary coils to keep up the magnetic flux just as in a transformer. The effect would be threefold. In the first place, a considerable fraction of the pressure of supply would be lost upon C^2R losses in the stator coils. Secondly, the ampere-turns of the stator and rotor coils, opposing each other with very great magnetomotive-forces, would force a number of lines along paths which do not thread through both sets of coils (for example, leakage would appear along the air-gap), and these lines would be the cause of electromotive-forces in the stator and rotor coils, in addition to the electromotive-forces produced by the common resultant field, and have a choking effect upon the currents in these coils. Thirdly, not only is the true resultant field B diminished by the above causes, but the little that remains is out of phase with the current in the rotor circuit, so that the torque is very much reduced instead of being increased by excessive slip when the rotor circuit is of low resistance. This is very simply exhibited in Mr. Kapp's construction. When the slip is great, the triangle *a* *b* will become of the form of Fig. 133; for if slip is proportional to R tan β, and R is small, tan β must be very great, β will be nearly 90°, the impressed field *a* is limited by the foregoing considerations, so the torque (represented by the area) will be very small. If we increase R we necessarily decrease tan B, making B greater and the area greater, and so we get a greater starting torque. Thus, introducing a non-inductive resistance into the rotor circuit at starting enables the machine to start with a greater torque.

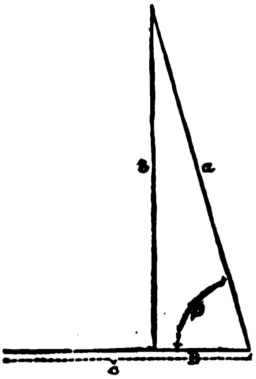

FIG. 133.

5. RELATION BETWEEN TORQUE AND SLIP.

In order to get an equation for the torque in terms of the slip and the resistance of the rotor, we note that from Fig. 132 it follows that

$$b = a \sin \beta,$$

and

$$\mathrm{B} = a \cos \beta.$$

Now, from the equation $\text{slip} = \frac{b}{\mathrm{B}} \times \frac{\mathrm{R}}{k}$ we get $\frac{\text{slip}}{\mathrm{R}} \times k = \frac{b}{\mathrm{B}}$. Therefore, by merely altering the scale of Fig. 132, we can rename the sides of the triangle as shown in Fig. 134, where s stands for the slip.

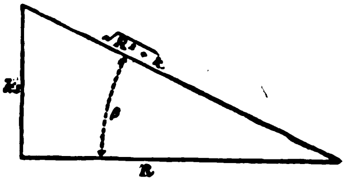

FIG. 134.

From this we see that $\sin \beta = \frac{k\,s}{\sqrt{\mathrm{R}^2 + k^2 s^2}}$, and $\cos \beta = \frac{\mathrm{R}}{\sqrt{\mathrm{R}^2 + k^2 s^2}}$.

Therefore the torque T, which is proportioned to $b \times \mathrm{B}$, is proportional to $a^2 \sin \beta \cos \beta$; and therefore, writing q as a quantity involving a^2 and constants depending on construction, we have

$$\mathrm{T} = q . \frac{s\,\mathrm{R}}{\mathrm{R}^2 + k^2 s^2}.$$

Here we are assuming that a, the impressed field, is constant (see p. 135).

If we wish to see graphically what this equation means, we may plot out the relation between T and s as a curve, assuming a definite value for R.

Take the line O X (Fig. 135) to represent the speed of rotation of the magnetic field, and cut off from it a part O Q to represent the speed of the motor. Then the remainder, Q X, represents the slip. This is equivalent to plotting the slip backwards from X. The vertical ordinates then represent the values of the torque as calculated from the equation. For example, when Q X is taken as s; P Q is plotted to represent the corresponding value of T. Thus, beginning at X where the slip is zero, we get a curve X P t_1, which rises steeply, comes to a maximum, and dies away to the value

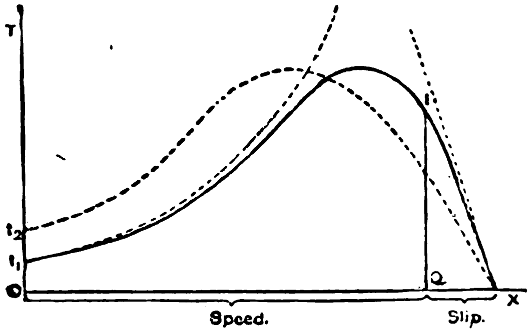

FIG. 135.

O t, which is the torque at starting. The torque has a certain maximum value for which $\beta = 45°$. It will be noted that the steep end part of the curve is nearly straight, being an asymptote to a straight line, which would represent the relation between torque and slip if the magnetic field were constant and there were no magnetic leakage. In fact, this line corresponds to the expression $T = b(\Omega - \omega)$ on p. 136. Or if in our present equation we consider values of s, which are small compared with R, the equation might be written $T = q\frac{s}{R}$. At the other end of the curve where slip is great, the curve is hollow. Here we may approximate by supposing that s is very great compared with R, or that R^2 is small compared with s^2; in which case the equation reduces to $T = q \frac{R}{s}$. This is the equation to a hyperbola (also shown in dot). When the motor is at rest $s = \Omega$, or O Q = zero, giving at O t_1 the value $T = q \frac{R}{\Omega}$. That is to say, *at starting*, the torque is proportional to the resistance of the rotor. If we then assign a higher value to R, and plot out a new set of ordinates, we obtain a new curve (shown in dotted line) which also starts at X, rises to a maximum of the same height as before, and then falls, but this time to t_2. The effect, then, of introducing more resistance is to raise the torque at starting; but it also has the effect of causing the maximum torque to occur when the slip is greater. The motor gives out practically the same power as before, but runs with a greater difference of speed between its speed at light load and its speed at full load. And the efficiency at full load is diminished. If, with a 5 per cent. slip and a 95 per cent. efficiency, we do not get a sufficient starting torque we can get it by introducing resistance, and contenting ourselves (at full load) with, say, a 10 per cent. slip, and a 90 per cent. efficiency. And one understands the reason for the modern device of constructing the rotor so that a resistance can be put in at starting, and then short-circuited as soon as the rotor has got up a fair speed.

In the various theories of the rotatory field motor[1] the subject is attacked from many different points of view, but,

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[1] L. Duncan, 'Alternate Current Motors,' *Elec. World*, (N.Y.), xvii. 341, 357.

Hutin and Leblanc, *La Lumière Électrique*, xl. 373.

Dr. J. Sahulka, 'Ueber Wechselstrom-Motoren mit Magnetischem Drehfelde,' Leipzig, 1892.

R.-V. Picou, 'Les Moteurs Électriques à champ magnétique tournant,' Paris, 1892,

E. Arnold, 'Theorie und Berechnung der asynchronen Wechselstrom-Motoren'; and see articles by same author in *Elec. World* (N.Y.), 1893-4.

G. Ferraris, 'A Method for the Treatment of Rotating or Alternating Vectors, &c.,' *Electrician*, 1894, xxxiii. 110, 129, 152, 184.

Reber, 'Theory of Two and Three-phase Motors,' Amer. Inst. Elec. Engineers, Oct. 1894.

Steinmetz, American Inst. Elec. Engineers, December 1894, p. 803.

through whatever mathematical intricacies it has passed, the expression for the torque is of the general form

$$T = q \frac{s R}{R^2 + k^2 s^2}.$$

The above method of deducing the formula, though incomplete in so far as it does not contain symbols for all the quantities concerned, perhaps has the advantage of keeping clearly in view the main principle, and enabling the student to follow the physical meaning of the expressions throughout. The quantity k, it will be remembered, is a constant, depending upon the reluctance of the magnetic circuit and the number of windings on the rotor. It is, in fact, the self-induction of one complete turn of conductor on the motor. The quantity q involves a^2 and total number of complete turns upon the rotor. In comparing with the formula given by other writers, it must be remembered that s is an angular speed, and is equal to $2 \pi (n - n_2)$ (see p. 134).

Steinmetz gives the formula for finding the torque in pounds at 1 ft. radius in the form

$$T = \frac{f e^2 g^2 s R}{R^2 + k^2 s^2}$$

to use our own symbols; g being the ratio of the secondary turns to the primary turns, and

$$f = \frac{550}{746 \pi p n}$$

where n is the frequency, and p the number of poles. Steinmetz's theory is very complete in this respect, that he takes into account both leakage and hysteresis, and gives an expression for e, the counter-electromotive-force in the stator conductors, in terms of the impressed volts and of an expression involving these quantities. Plotting values for torque at different amounts of slip he gives the curve shown in Fig. 136, which is of the same character as that given in Fig. 135, only extended in both directions. If the speed of the motor is run up by mechanical means beyond synchronism, the torque becomes negative and the machine acts as a generator, giving the lower branch of the curve (see p. 42). If, on the contrary, the motor is turned in the sense opposite to the rotation of the field, the torque decreases as shown on the left of the figure.

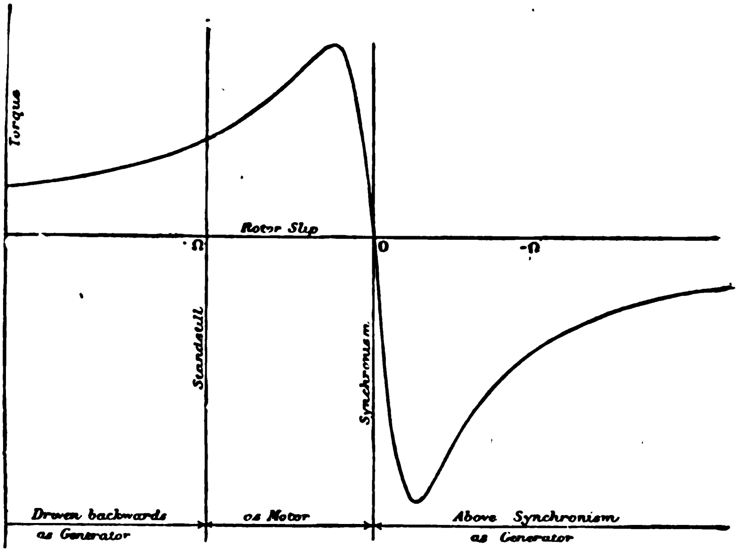

FIG. 136.

CHAPTER VII.

ANALYTICAL THEORY OF POLYPHASE MOTORS.

THE following method of treating the theory of polyphase motors is a modification of that due to M. Potier.[1] Instead of considering the rotating field as a constant flux cutting the conductors of the rotor as it revolves, we may resolve it into two alternating fluxes in the constant directions of the axes X and Y (Fig. 137) at right angles to each other; then all motions of the rotor and the fluxes and currents therein can be referred to these common axes.

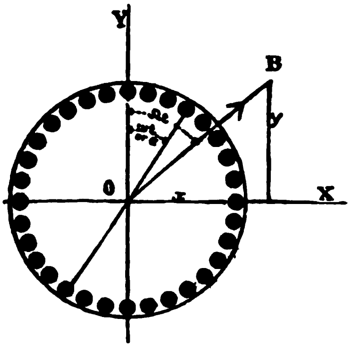

FIG. 137.

Let the line O B represent the direction and amount of the rotating magnetic flux (that is to say, the resultant of the impressed field due to the current of the stator and the field due to the currents in the rotor). Then x and y show the direction and intensity of its horizontal and vertical components respectively at any instant.

If the field revolves at a uniform speed of Ω radians per second, and remains constant in intensity, then

$$x = x_m \sin \Omega t \text{ and } y = y_m \cos \Omega t$$

where x_m and y_m are the maximum values of the horizontal and vertical components, and represents the magnitude of the revolving flux.

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[1] A. Potier, "Sur les Moteurs à induit fermé sur lui-méme," *Bull. de la Soc. Internationale des Electriciens*, May 1894, p. 248.

If, however, we wish to be perfectly general, and include elliptically rotating fields such as are found in monophase motors, we will write

$$x = x_m \sin \Omega t \quad \text{and} \quad y = y_m \sin (\Omega t + \phi), \qquad (1)$$

where x_m and y_m are not necessarily equal.

We will consider the general case first.

Suppose that there are Z conductor bars in the rotor, one can always consider them as $\frac{Z}{2}$ spires wound drum fashion, each spire having a certain resistance r.

If the rotor is revolving at a speed of ω radians per second the angle which the plane of one of the spires makes with the vertical may be written ωt (see Fig. 137).

The flux through this spire is $x \cos \omega t - y \sin \omega t$. The electromotive-force in the spire will be due both to the fact that it is changing its inclination to the fields x and y, and to the fact that x and y are changing in value.

Let us write $\dot{x}$ and $\dot{y}$ for the rate of change of x and y with respect to time, then we have the instantaneous electromotive force

$$e = -x \omega \sin \omega t - y \omega \cos \omega t + \dot{x} \cos \omega t - \dot{y} \sin \omega t.$$

$$e = \cos \omega t (\dot{x} - y \omega) - \sin \omega t (\dot{y} + x \omega) = c r, \qquad (2)$$

where c is the current in the spire.

In order to obtain an expression for the torque due to all the spires, it is necessary to integrate the expression for the torque due to one of the spires between the limits of half a revolution; and for this purpose it is more convenient to signify by a, the angle which any particular spire makes with the axis of Y. Then the torque due to one spire $= c (y \cos a + x \sin a)$. Filling in the value of c given above, and integrating between the limits $a = 0$ and $a = \pi$, and remembering that the angle π contains $\frac{Z}{2}$ spires so that

$$\Sigma \sin^2 a = \Sigma \cos^2 a = \frac{Z}{4}, \text{ and } \Sigma \sin a \cos a = 0$$

we get the torque

$$\text{T (for the instant)} = \frac{Z}{4r}\Big[y(\dot{x} - y\,\omega) - x(\dot{y} + x\cdot\omega)\Big].$$

Taking the general case where $x = x_m \sin \Omega\, t$ and $y = y_m \sin(\Omega\, t + \phi)$ and substituting the values for x, y, $\dot{x}$ and $\dot{y}$ we get, after integrating between the limits $\Omega\, t = 0$ and $\Omega\, t = 2\,\pi$ and then dividing by $2\,\pi$,

$$\text{the mean torque} = \tfrac{1}{2}\,\frac{Z}{4r}\Big[2\,x_m\,y_m\,\Omega \sin\phi - \omega\,(x^2_m + y^2_m)\Big]. \quad (3)$$

This general expression for the torque is applicable to every form of polyphase motor with elliptical rotating fields. If the field revolves at a uniform speed and remains constant in value so that $\phi = 90°$ and $y_m = x_m$ this expression for the torque simplifies to

$$\frac{Z}{4r}\,x^2_m\,\Big[\Omega - \omega\Big].$$

If the stator is one of the ordinary type, such as is shown in Fig. 105, so that the reluctance of the magnetic circuit is practically independent of the direction of the field, the expression for the components along X and Y of the cross magnetization due to the current in the rotor is very simple; for, if we denote by F the flux produced normally to the plane of a spire by a unit current in that spire, the components $x_1\ y_1$ of the cross field will be

$$x_1 = -\,\mathrm{F}\,\Sigma\, c \cos a \quad \text{and} \quad y_1 = \mathrm{F}\,\Sigma\, c \sin a.$$

Substituting for c its value

$$\frac{1}{r}\Big[(\dot{x} - y\,\omega)\cos a - (\dot{y} + x\,\omega)\sin a\Big],$$

and integrating between the limits $a = 0$ and $a = \pi$ so as to include all the spires, we get

$$x_1 = -\,\frac{Z\,F}{4\,r}\Big(\dot{x} - y\,\omega\Big) \quad \text{and} \quad y_1 = -\,\frac{Z\,F}{4\,r}\Big(\dot{y} + x\,\omega\Big). \quad (4)$$

For $\dfrac{Z\,F}{4\,r}$ we will write $\dfrac{1}{u}$, so that

$$u\,x_1 = -\,(\dot{x} - y\,\omega) \quad \text{and} \quad u\,y_1 = -\,(\dot{y} + x\,\omega).$$

To find an expression for the components of the impressed field, it is only necessary to subtract the cross field from the resultant field; thus if Φ_x and Φ_y be the components along the axes X and Y of the impressed field we have

$$\Phi_x = x - x_1 \quad \text{and} \quad \Phi_y = y - y_1,$$

or

$$\begin{aligned} u\,\Phi_x &= u\,x + \dot{x} - y\,\omega, \\ u\,\Phi_y &= u\,x + \dot{y} + x\,\omega. \end{aligned} \tag{5}$$

If we know the components of the impressed field we can find x and y and *vice versa.*

Let us now apply these formulæ to the case of a simple motor whose rotary field has the horizontal and vertical components.

$$x_m \sin \Omega\,t \quad \text{and} \quad x_m \cos \Omega\,t.$$

We have from equations (5)

$$u\,x_m \sin \Omega\,t + x_m\,\Omega \cos \Omega\,t - \omega\,x_m \cos \Omega\,t = u\,\Phi_x,$$

from which we get the component of the impressed field along the axis of X

$$\Phi_x = \frac{\sqrt{u^2 + (\Omega - \omega)^2}}{u} \sin\,(\Omega\,t - \beta),$$

where

$$\beta = \tan^{-1} \frac{\Omega - \omega}{u}.$$

Along the axis of Y it has the same value, but varies as the cosine instead of as the sine.

The torque, as we have seen, p. 148, is

$$\frac{Z}{4\,r}\,x_m^2\,(\Omega - \omega).$$

To obtain the power yielded by the rotor, multiply by the angular velocity ω

$$P = \frac{Z}{4\,r}\,x_m^2\,\omega\,(\Omega - \omega).$$

Next consider the heat produced in the rotor. In one spire heat is being produced at the rate of $r\,c^2$ joules per second. Filling in the value of c, and integrating as before for the whole of the spires, we get

$$\frac{Z}{4\,r}\left[(\dot{x} - y\,\omega)^2 + (\dot{y} + x\,\omega)^2\right]$$

as the rate for the instant.

Now taking the average of this throughout a complete period we find H, the heat produced per second,

$$= \tfrac{1}{2}\,\frac{Z}{4\,r}\left[(x_m^2 + y_m^2)\ (\Omega^2 + \omega^2) - 4\,x_m\,y_m\,\Omega\,\omega\,\sin\,\phi\right].$$

In the case of the uniformly rotating field $x_m = y_m$, and $\sin\,\phi = 1$, so that

$$H = \frac{Z}{4\,r}\,x_m^2\,(\Omega - \omega)^2.$$

Therefore,

$$P + H = \frac{Z}{4\,r}\,x_m^2\left[(\Omega - \omega)^2 + \omega\,(\Omega - \omega)\right]$$

and the efficiency of the rotor

$$\frac{P}{P + H} = \frac{\omega}{\Omega},$$

which is the result arrived at from other considerations (see p. 135).

In considering the loss in a 2-pole stator we will denote the total number of active conductors on it by Z_1 (so that the number in one coil is $\frac{Z_1}{4}$) and the total resistance of these if joined in series by R. The difference of potential at the terminals of one of the circuits—for instance, that which produces the horizontal flux (the stator being joined up as shown in Fig. 49—can be found as follows:—

We have seen that the horizontal impressed flux

$$= \frac{\sqrt{u^2 + (\Omega - \omega)^2}}{u}\,x_m\,\sin\,(\Omega\,t - \beta),$$

and this must be equal to

$$\frac{Z_1 F}{4} c_1,$$

where c_1 is the current in the stator coil, therefore,

$$c_1 = \frac{4 F}{Z_1} \frac{\sqrt{u^2 + (\Omega - \omega)^2}}{u} x_m \sin (\Omega t - \beta),$$

and the difference of potential at the terminals of one of the circuits for the instant

$$e_1 = \frac{R}{2} c_1 + \frac{Z_1}{4} \dot{x}.$$

Filling in the values of c_1 and $\dot{x}$ and writing λ for $\frac{R}{Z_1^2} \frac{Z}{r}$, this gives us

$$e_1 = \frac{Z_1}{4} x_m \Omega \left[2 \lambda \frac{u}{\Omega} \sin \omega t + \left(1 + 2 \lambda \frac{\Omega - \omega}{\Omega} \right) \cos \Omega t \right];$$

If we write this in the form

$$e_1 = e_m \sin (\Omega t - \gamma),$$

we find that the tan γ

$$= \frac{\Omega - \omega}{\omega} + \frac{\Omega}{2 \lambda u}.$$

The lag of the current was β where $\tan \beta = \frac{\Omega - \omega}{u}$; therefore the difference in phase between current and electromotive-force ϕ_1 is such that

$$\tan \phi_1 = \frac{u \Omega}{\Omega (\Omega - \omega) + 2 \lambda \ u^2 + (\Omega - \omega)^2}.$$

The heat per second generated in the stator $= R c_1^2$

$$= \tfrac{1}{2} R \Phi^2_m \left(\frac{4}{Z_1 F} \right)^2 = \tfrac{1}{2} \frac{R}{Z_1^2} \frac{Z}{r} \left(\frac{4 r}{Z F} \right)^2 \frac{Z}{r} \Phi^2_m = \tfrac{1}{2} \lambda u^2 \Phi_m \frac{Z}{r} = H_1;$$

or substituting the value for Φ_m,

$$H_1 = \frac{Z}{4 r} x^2_m \, 2 \lambda \left[u + (\Omega - \omega)^2 \right].$$

To summarize these results write $K = \frac{Z}{4\,r}\,x^2_m$, then we have

$$\begin{aligned} &\text{The torque T} &&= K\,(\Omega - \omega), \\ &\text{The power P} &&= K\,\omega\,(\Omega - \omega), \\ &\text{The rotor heat H} &&= K\,(\Omega - \omega)^2, \\ &\text{The stator heat } H_1 &&\doteq K\,2\,\lambda\,[u^2 + (\Omega - \omega)^2]. \end{aligned}$$

The maximum value of the back electromotive-force of one circuit of a two-phase stator

$$E_m = \frac{Z_1}{4}\,x_m\,\Omega\left(1 + 2\,\lambda\,\frac{\Omega - \omega'}{\omega}\right) \text{ nearly.}$$

The maximum value of the current in the stator when the motor is running normally loaded,

$$c_1 = \frac{4\,F}{Z_1}\,\frac{\sqrt{u^2 + (\Omega - \omega)^2}}{u}\,x_m.$$

The difference of phase is given above.

In the above it has been assumed that the whole of the flux passing through the rotor cuts the stator conductors. This, under normal conditions of load, is very nearly true; but at starting, for the reasons stated on p. 140, it is not true; a large fraction of both stator and rotor fields being closed along independent paths. To make the theory applicable to all speeds and loads, the self-induction of each circuit as well as the mutual induction between the circuits must be taken into account. The expressions involving all these terms become so complex that they are of no practical utility even if the quantities involved could be determined from a given design. The author has therefore thought that the elementary theory given in Chapter VI., and the indication of a method of treating the subject analytically given in this chapter, will be more acceptable to the student than a reiteration of the more extended theories, reference to which will be found on p. 143.

CHAPTER VIII.

MONOPHASE MOTORS.

Motors for use with a monophase—that is to say, with an ordinary—alternate current have the obvious advantage that they only need two lines instead of three or four wherewith to supply them with currents, and can be run from alternate-current lightning mains, as existing in many of our towns. Prior to the invention of polyphase motors, the only alternate-current motors in use were the ordinary alternate current machines (designed as generators) having separately-excited magnets. These only ran in perfect synchronism, and were not self-starting. But as soon as the polyphase asynchronous motor had reached the stage of practical success, it became evident that monophase motors might be constructed on analogous lines.

When an alternate current is passed through one group of coils of a polyphase motor, it produces a magnetic flux in a certain fixed track through the rotor. This flux rises to a maximum, dies down to zero, changes in sign, and rises to a negative maximum and so on, but it does not change in direction as a rotating field. Very powerful currents will be produced in those rotor bars which enclose this flux, but there will be no more tendency to turn in one direction than in another. Just as in a steam engine with a single cylinder, the forces are harmonic and rectilinear; with the crank at a dead point it is impossible to start without the interference of some outside force to upset the balance. Once give the rotor a good start and it will go on increasing its speed until synchronism is nearly reached and will exert a great torque The reason of this will appear from the theory of monophase motors presently to be considered.

If we take a solenoid (Fig. 138) with a bundle of iron wire for a core, and pass round it an alternating current, a simple alternating field is produced. If in this field we suspend a copper ring, as shown in the figure, we find [1] that it tends to turn until its plane is parallel to the direction of the flux, so that it does not enclose any magnetic lines. It is only when the ring is in an oblique position that it tends to turn. If it be placed with its plane directly at right angles to the direction of the magnetic lines, it will remain stationary; if ever so little displaced to the right or left, it will turn until its plane is parallel to the lines. If β be the angle between the plane of the ring and the direction of the magnet field (Fig. 139), the electromotive-force, and therefore the current induced in it by the alternation in strength of the field, will be proportional to sin β. Now the turning moment acting upon the ring is proportional to the current in it, to the strength of the field, and to the cosine of the angle β. Hence it is proportional to the product sin β cos β. The tendency to turn is zero both at 0° and at 90°; in the former case because there is no current, in the latter because the current has no leverage. It is a maximum when $\beta = 45°$.

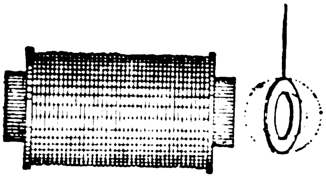

Fig. 138.

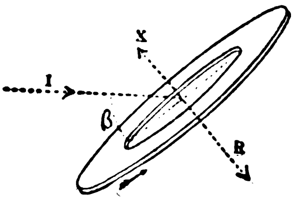

Fig. 139.

Even in this position there would be no torque if there was no lag of the currents in the ring; for the phase of the induced E.M.F. is in quadrature with the phase-state of the field. When the field is of maximum strength there is no E.M.F., and when the E.M.F. reaches its maximum there would be no field. But if there is self-induction in the ring

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[1] See Elihu Thomson' "Novel Phenomena of Alternating Currents," *Electrical World* (N. Y.), May 28, 1887.

causing the current to lag, there will be a net turning moment tending to diminish β. The largest torque will be obtained if the self-induction and resistance of the ring are such, relatively to the frequency, that $2\,\pi\,n\,L = r$; or when the lag of current in the ring is 45°.

This phenomenon may be explained by saying that the current induced in the ring produces a cross-field, which, being out of phase with, and inclined to, the field impressed by the primary alternate current, causes a rotary field; and this in its turn reacting on the conductor, a turning moment results.

A more precise way[1] of stating what occurs is the following. Suppose that the ring is inclined at an angle β to the direction of the field impressed by the solenoid. The actual flux passing through the ring will be the resultant of (1) the component of the impressed flux in the direction normal to the plane of the ring, and (2) the cross flux due to currents in the ring. This resultant flux will follow a sine law, and may be represented by the curve R R (Fig. 140). The current in the ring will be at right angles to this in phase, and may be represented by the curve CC, where the positive sense is taken in the direction of the arrows in Fig. 139. The cross field will then be represented by the curve X, and the normal component of the impressed field will be represented by the curve I I, which is obtained by subtracting the ordinates of X X from those of R R. It will be seen that the impressed field, which is of course proportional to its normal component, is partly in phase with the current, so that their product is, upon the whole, positive, and applying Fleming's rule to Fig. 139, it will be seen that the torque is in the direction tending to lessen β.

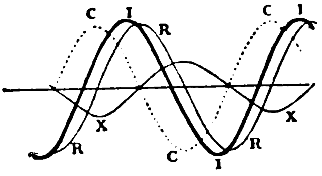

FIG 140.

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[1] For complete analytical treatment of this subject, see G. T. Walker, "Repulsion and Rotation produced by Alternating Electric Currents." *Phil. Trans. Royal Society*, 1892, A, 279. See also J. A. Fleming, on "Electromagnetic Repulsion," *Proc. Royal Institution*, xiii. 296, March 6, 1891, and *Journal of the Society of Arts*, May 14, 1890.

Elihu Thompson took an ordinary continuous-current armature placed in an alternating field, and having short-circuited the brushes, placed them in an oblique position with respect to the direction of the field. The effect was to cause the armature to rotate with a considerable torque. The conductors of the armature acted just as an obliquely placed ring, but with this difference, that the obliquity was continuously preserved by the brushes and commutator, notwithstanding that the armature turned, and thus the rotation was continuous (see p. 172).

This tendency of a conductor to turn from an oblique position was thus utilized by him (p. 172) to get over the difficulty of starting a monophase motor.

The way in which monophase motors are commonly started is to superimpose upon the alternating field an oblique field differing in phase. This is usually done by having additional coils on the stator fed by a current that is out of step with the current of the main coils, and it is necessary to have some device which will cause a difference in the phase of the currents of the two branches. This operation of *splitting the phase* may be performed in many ways. We have seen (p. 14) that in circuits possessing resistance and self-induction the tangent of the angle of lag of the current behind the electromotive-force is equal to $\frac{pL}{R}$. If, therefore, we have a comparatively large self-induction in one branch of the circuit, and comparatively large resistance in the other, the phases of the currents will differ by nearly 90°. This difference in the self-induction of the branches may be caused either by the difference in the number of turns of wire in the coils on the stator and the arrangement of the iron around them, as in the case of Tesla's motor (Fig. 101), or it may be caused by putting in series with one of the branches a coil of iron on an iron core. A non-inductive resistance may be introduced into the other branch.

A difference in phase can also be produced by giving one of the branches capacity by means of a condenser, capacity having the effect of giving the current a lead. The kind of condenser usually employed for this purpose is an electrolytic condenser, consisting of a number of iron plates with a solution of carbonate of soda between them.

There are also many ways of producing a difference of phase by means of special transformers. These are considered under the head of phase-transformation, Chapter IX.

A simple single-phase motor with closed-coil rotor was described in a specification of patent filed in the English Patent Office in November 1892 (No. 20,505), being a communication from the Oerlikon Machine Company of Zürich. The cause of the torque after the motor is started is there given in these words:—"On rotary motion being imparted to the machine in any suitable manner, currents will be induced in those conductors of the armature which are approaching one pole of the exciting coils and moving away from the opposite neighboring pole, these currents being less strongly repelled by the first pole than by the second pole because, in consequence of the rotation, a given conductor will assume a phase which, in the position of rest, would belong to a conductor located further back." A diagrammatic view of a two-pole motor is shown in Fig. 141. The windings of the stator are reversed so as to produce consequent poles at the top and bottom. This specification describes a method of starting the motor by means of an additional winding on the stator, carrying a current differing from the main current in phase in the manner described on p. 201.

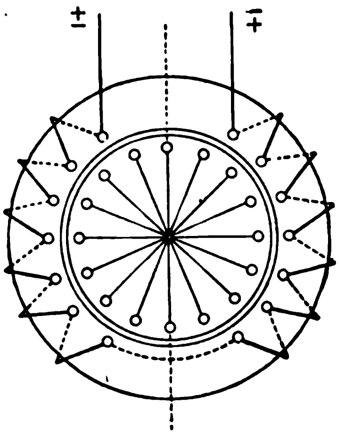

FIG. 141.

In specification of patent No. 23,902, filed by Mr. C. E. L. Brown in December 1892, some monophase induction-motors are described [1] with rotors of the squirrel-cage type.

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[1] See also *Elektrotechnische Zeitschrift*, xi. 81, Feb. 17, 1893; *Industries*, xiv. 89.

Since this date many monophase motors have been constructed by both these firms; the main differences consisting in the methods adopted in the starting-gear, and in the use of toothed core-rings as against core-rings pierced with holes. The dispute as to priority[1] which arose between these two firms in 1893 does not concern us. It appears to have been closed by the letter of Mr. Brown in the *Elektrotechnische Zeitschrift* of July 14th, 1893.

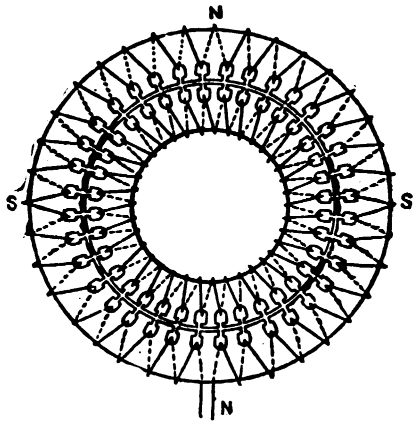

FIG. 142.

A closely-kindred form of motor was described in May 1891 by the Helios Co. It is depicted in Fig. 142.

THEORY OF THE MONOPHASE MOTOR.

The difficulty in following the action of a monophase alternate-current motor, lies in the fact that while there is passing through the rotor an impressed alternating field of a certain frequency, inducing currents with their opposing fields, there is at the same time a turning of the rotor causing the phenomena of currents and magnetic fields at another frequency. Any theory which comprises all these things, and takes account of the magnitude and phase of each, necessarily contains a complex grouping of symbols, whose physical

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[1] *Elektrotechnische Zeitschrift*, xi. 81, 178, 283, 285, 411, 1893; *Industries*, xiv. 89, 327, 425, 522, 1893.

[2] D. R. Patent 70084.

meaning is not easily grasped, while a theory which for the sake of simplicity neglects one or more of these things is too incomplete to be satisfying.

It is proposed here to give in the first place a comprehensive analytical theory as given by M. De Bast,[1] secondly the graphic method which is due to Professor Ferraris,[2] and thirdly to translate these into a mental picture of what actually goes on in the rotor.

Assume in the first place that a two-pole motor is started, and running at a constant speed of m revolutions per second, and that it is being supplied with an unvarying alternate current which follows a simple sine law with a frequency of n periods per second; then the flux-density B of the field impressed by the stator coils at any instant is

$$B = B_0 \sin 2 \pi n t,$$

B_0 representing the maximum flux density reached in each period, and μ assumed constant. If A is the area enclosed by a conductor embracing the rotor about a diameter which makes an angle a with the plane normal to the direction of the impressed field, the total magnetic flux

$$N = A \cos a \, B_0 \sin 2 \pi n t,$$

the flux-density being assumed to be uniform.

As the rotor turns m revolutions per second,

$$a = 2 \pi m t.$$

The E.M.F. in the conductor is

$$E = - \frac{d N}{d t}$$

$$= - A B_0 [- 2 \pi m \sin 2 \pi n t . \sin 2 \pi m t + 2 \pi n \cos 2 \pi m t \cos 2 \pi n t]$$

$$= - \frac{A B_0}{2} [2 \pi (n + m) \cos 2 \pi (n + m) t + 2 \pi (n - m) \cos 2 \pi (n - m) t].$$

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[1] De Bast, *Bull. de l'Assoc. des Ingénieurs Electriciens*, Aug. 1898.

[2] G. Ferraris, "A Method for the Treatment of Rotating or Alternating Vectors, etc." *The Electrician*, xxxiii, 110, 129, 152 and 184.

Thus the electromotive-force is the sum of two simple harmonic electromotive-forces of the frequencies $(n + m)$ and $(n - m)$.

If we represent the resistance of the conductor by r, and its coefficient of self-induction by L, the impedances I_1 and I_2 to the two electromotive-forces respectively will be

$$I_1 = \sqrt{r^2 + 4\pi^2 (n + m)^2 L^2},$$
$$I_2 = \sqrt{r^2 + 4\pi^2 (n - m)^2 L^2}.$$

The instantaneous value of the current in the conductor will be

$$C = -\frac{A B_0}{2}\left[\frac{2\pi(n + m)}{I_1}\cos\left\{2\pi(n+m)t - \phi_1\right\} + \frac{2\pi(n - m)}{I_2}\cos\left\{2\pi(n - m)t - \phi_2\right\}\right],$$

the angles of lag being ϕ_1 and ϕ_2, of which

$$\cos\phi_1 = \frac{r}{I_1},$$
$$\cos\phi_2 = \frac{r}{I_2}.$$

The potential energy of the conductor is

$$W = -C N = -C A B_0 \cos a \sin 2\pi n t,$$

and in moving through the small angle da the work done is (neglecting the sign)

$$dW = C A B_0 \sin 2\pi n t \sin a \, da.$$

Substituting the value for C given above, and $2\pi m \, dt$ for da, we get

$$dW = \frac{A^2 B^2_0}{2}\left[\frac{2\pi(n + m)}{I_1}\cos\left\{2\pi(n+m)t - \phi_1\right\} + \frac{2\pi(n - m)}{I_2}\cos\left\{2\pi(n - m)t - \phi_2\right\}\right]$$
$$\times \left[2\pi m \sin 2\pi n t \sin 2\pi m t\right] dt.$$

Integrating this between the limits $t=1$ and $t=0$, we get the work per second, or, in other words, the mean power for one conductor.

$$P = \frac{2\pi m A^2 B^2}{8}\left[\frac{2\pi(n-m)}{I_2}\cos\phi_2 - \frac{2\pi(n+m)}{I_1}\cos\phi_1\right]$$

$$= \frac{2\pi m r A^2 B^2}{8}\left[\frac{2\pi(n-m)}{I_2^2} - \frac{2\pi(n+m)}{I_1^2}\right].$$

The total mean power is obtained by multiplying by the number of conductors Z, and the torque is obtained by dividing the total mean power by the number of radians per second $= 2\pi m$.

$$\text{Torque} = \frac{Z P}{2\pi m};$$

therefore we obtain as the final expression[1] the following formula:

$$\text{Torque} = \frac{r Z A^2 B^2 \pi}{4}\left[\frac{(n-m)}{r^2 + 4\pi^2(n-m)^2 L^2} - \frac{(n+m)}{r^2 + 4\pi^2(n+m)^2 L^2}\right].$$

Professor Ferraris has given[2] a method of treating the subject in which the alternating magnetic field is regarded as being resolved into two magnetic fields rotating in opposite directions. It is a familiar point in mechanism that any simple harmonic rectilinear motion may be resolved into two equal circular motions in opposite directions. Fig. 143 illustrates one way of doing this, the mechanism being well known to engineers. The amplitude of the original motion is equal to the diameter of each of the circular motions. Ferraris deals, however, with the problems of the alternating magnetic

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[1] Compare Hutin and Leblanc, *La Lumière Electrique*, xl. 418 (1891).

[2] Galileo Ferraris, "A Method for the Treatment of Rotating or Alternating Vectors, with an Application to Alternate-current Motors." *The Electrician*, xxxiii, 110, 129, 152, 184 (1894).

field quite generally, applying the geometrical notion of rotating vectors.

If we represent by the vector b_1 (Fig. 144) which rotates clockwise uniformly about O, the magnitude and direction of a rotating magnetic field, and by b^2 the magnitude and direction of another field of the same strength rotating in the opposite sense with the same frequency n, it will be seen that the direction of the resultant field is always along the line B, and the magnitude of the resultant field will alternate between the values $+ 2b$ and $- 2b$ following a sine function of the time, so that we may write $B = 2b \sin 2 \pi n t$.

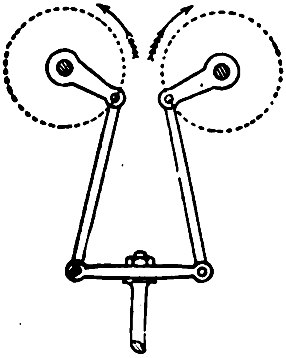

FIG. 143.

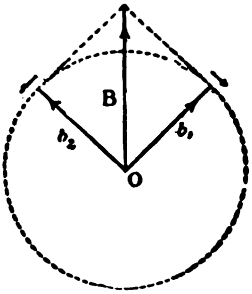

FIG. 144.

Conversely, if we have an alternating field following the law $B_0 \sin 2 \pi n t$ as in a monophase motor, we may resolve it into two oppositely rotating fields of the same frequency n, and consider the effect of each field separately upon the rotor.

If the rotor turns clockwise with a frequency m, the frequency of rotation of the clockwise field with respect to the rotor will be $n - m$, and the frequency of rotation of the counter-clockwise field with respect to the rotor will be $n + m$.

Each field may be considered as generating currents in the rotor, and the torque due to such currents flowing through conductors in the field may be ascertained by the formulæ employed in the case of rotary field motors.

It was found above (see p. 144) that a field rotating with a speed s relatively to the rotor produced a torque

$$T = q \frac{r s}{r^2 + 4 \pi^2 L^2 s^2}.$$

The torque produced by the two oppositely rotating fields will be

$$\text{Torque} = q r \left[\frac{n-m}{r^2 + 4 \pi^2 L^2 (n-m)} - \frac{n+m}{r^2 + 4 \pi^2 L^2 (n+m)^2} \right],$$

which is the same as the expression deduced above (p. 161), where

$$q = \frac{Z A^2 B^2 \pi}{4}.$$

It is not necessary to consider the torque exerted by reason of the fact that the currents due to one rotating field flow in conductors that are immersed in the *oppositely* rotating field, because the frequency of these currents differs by 2 m from the frequency of the opposite field, and consequently this torque is rapidly reversing in direction.

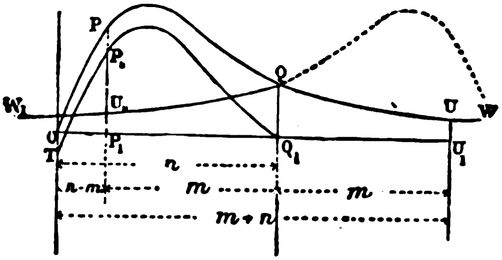

FIG 145.

In order to find the torque due to the field rotating clockwise with the frequency $n - m$, we draw the curve O P Q W (Fig. 145) (see p. 142 where the curve is reversed) showing the relation between slip and torque obtained by the formulæ.

$$T = q \frac{r s}{r^2 + 4 \pi^2 L^2 s^2}.$$

Let O Q, represent the speed of rotation of field of frequency n; then measuring backwards from Q a distance Q

$P_{\prime} = m$ (= speed of rotor) we get the abscissa $O\,P_{\prime} = n - m$, and the ordinate $P_{\prime}\,P$ represents the torque in question.

To find the torque due to the counter-clockwise rotating field, we measure off forwards from $Q_{\prime}$ the distance $Q_{\prime}\,U_{\prime} = m$ and get $O\,U_{\prime} = n + m$, then $U\,U_{\prime}$ represents the torque due to a slip $n + m$. This being in the opposite sense to the torque $P\,P_{\prime}$ we can cut off from $P\,P_{\prime}$ a part $P\,P_{\prime\prime} = U\,U_{\prime\prime}$ and obtain $P_{\prime\prime}\,P_{\prime}$ which represents the actual torque on the rotor.

For convenience in deducting the torques due to counter-clockwise field we may draw $Q\,W_{\prime}$ symmetrical with $Q\,W$, and then deduct the intercepted parts such as $U_{\prime\prime}\,P_{\prime}$ from the ordinates such as $P\,P_{\prime}$. Doing this for all the ordinates between O and $Q_{\prime}$ we obtain the new curve $T\,P_{\prime\prime}Q_{\prime}$, the ordinates of which represent the actual torque for various values of m.

When $m = O$, that is to say, when the rotor is stationary, the two opposite torques balance one another; as m is increased the torque rises to a maximum, and then falls to zero before m is quite as great as n, and further increase in m produces an opposing torque.

This argument assumes that B_0 remains fixed, which is only true so long as the motor is supplied with the same current. The curve cannot therefore be taken as the true characteristic of the monophase motor supplied at constant voltage, but is useful as a simple indication of its general behavior. When load is thrown on to the motor its speed decreases a little, more current flows through the stator, and the impressed field is correspondingly increased, so that the quantity denoted by q is by no means a constant quantity, but increases with the load. The theory given here merely explains how the alternating flux is capable of producing rotation.

In order to get a mental picture of what is going on in the rotor, let us apply the construction given on p. 138, for finding the direction of the resultant field and current to the case of the two oppositely rotating vectors. In Fig. 146*a* let O P represent by its length and direction the magnitude and direction of one (b_1 in Fig. 144) of the rotating magnetic fields which together make up the alternating field $B_0 \sin 2\pi n t$. Suppose that the rotor is revolving clockwise, making m revolutions per second, and that O P is revolving in the same direction at a slightly faster speed (n revolutions per second). As O P cuts across the conductors of the rotor at a speed of $(n-m)$ revolutions per second, it will generate currents whose intensity will vary at different points around the circumference of the rotor very nearly according to a sine law. This current, whose maximum intensity we will denote by C_1, will produce a cross magnetic field at right angles to the direction of its flow, whose intensity may be denoted by X_1. This field, compounded with the impressed field, gives the resultant field, and we may find the direction of all three by setting off O A, making the angle ϕ_1 (which is known, see below) with O P, and dropping the perpendicular P A upon it, than O A indicates the resultant field B_1, and P A indicates the cross-field X, in direction and magnitude. The angle ϕ_1 is known because the ratio between the cross-field to the resultant field is known from considerations of the speed $n - m$, the resistance R, and the magnetic reluctance of the path, and this gives $\tan \phi_1$, (see p. 138). For instance, if the cross-flux is equal to $k\ C_1$, then as C_1 is equal to

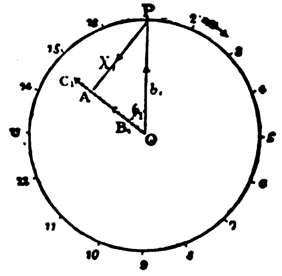

FIG. 146*a*.

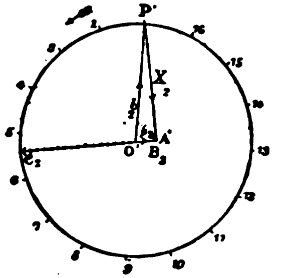

FIG. 146*b*.

$$\frac{B_1\, 2\pi(n-m)}{R},$$

$$\tan \phi_1 = \frac{2\pi(n-m)k}{R}.$$

ϕ_1 at full load ought to be a little greater than 45°. We can go through the same construction with regard to O P′, Fig.

146*b*, which represents the other rotating field (b_2, in Fig. 144); but this time, as

$$\tan \phi_1 = \frac{2 \pi (n + m) k}{R},$$

it is about 40 times as great as tan $\phi_{\prime}$, because *m* only differs from *n* by about 5 per cent.

ϕ_2 is therefore very nearly a right angle, and B_2 about one-twentieth the magnitude of $B_{\prime}$. It will be seen that the area of the triangle P O A is much greater than the triangle P′ O′ A′, which, according to the argument on p. 138, tells us that the torque clockwise is much greater than the torque counter-clockwise.

In order to indicate the directions of the current C_1 and C_2, the following convention may be employed. Suppose the rotor bars to be short-circuited at each end by a copper disk extending over the whole of the end of the rotor. A uniformly distributed current flowing across the disk, parallel to any diameter, would produce a sine distribution of current in the rotor bars, which bring the current at one side and take it away at the other, the maximum intensity being in the two bars joined by the diameter which is parallel to the direction of the current. We may, therefore, in a clock diagram, indicate the direction and magnitude of such a current across the end of the rotor, by a line drawn from the centre, whose length is proportional to the maximum value of the current in the rotor bars, and this method is applicable to motors in which no such copper disk exists, so long as we understand the distribution of current which it is intended to indicate. In Figs. 146*a* and 146*b*, the dotted lines C_1 and C_2 represent in this way the distribution of the current at any moment. They are drawn in a line with B_1 and B_2, the current being always in phase with the resultant magnetism, and they are made equal to X_1 and X_2 respectively, those lines being proportional to the two currents. The arrowheads show the sense which the reader may ascertain by Fleming's rule, noting that the arrowheads on the lines indicating magnetic flux point in the direction in which a N-pole would move.

Having ascertained the direction and amount of the resultant fields and currents due to the oppositely rotating vectors O P and O P′, in Figs. 146*a* and 146*b*, let us place one figure over the other and recombine them so as to obtain one field and one current. For this purpose the circle may be divided into a number of equal parts, say 16, which represent fractions of the period of one cycle. At the position 1 we have O P coinciding with O P′, their sum being a vector of double the length representing the maximum value of the impressed field shown by the line 0 1 in Fig. 147.

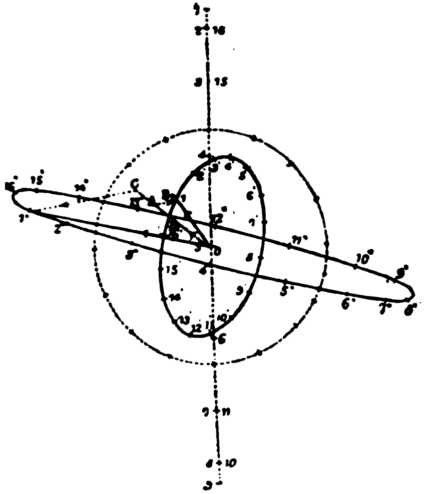

FIG. 147.

If we now add the vectors B_1 and B_2, taking note of their sense we get the vector O 1′, shown in Fig. 147, and adding C_1 and C_2 we get the vector O 1″. Then (reverting to Figs. 146*a* and 146*b*) move P and P′ to the position 2, triangles P O A and P′ O′ A′ being correspondingly slewed round. The addition of O P and O P_1, B_1 and B_2, and C_1 and C_2 will now give the vectors 0 2, 0 2′ and 0 2″ respectively; and so as we go round the circles in Figs. 146*a* and 146*b* we shall get the various points 1 2 3 4, etc., 1′ 2′ 3′ 4′, etc., and 1″ 2″ 3″ 4″, etc. in Fig. 147, through which we can draw the line and the ellipses shown.

The two ellipses show at a glance what occurs in the rotor during each alternation. That numbered 1′ 2′ 3′ etc., which really differs very little from a circle (its eccentricity being exaggerated in the figure to show how it is tilted), shows that there is a rotating magnetic field of slightly varying intensity which has the same frequency as the impressed field (given by the line 1 2 3 etc.), the latter being wiped out, or rather transformed into a rotary field, by the currents in the rotor. The sense of rotation is the same as that of the rotor. The other ellipse (numbered 1″ 2″ 3″ etc.) shows that there is a rotary current which varies in value between very great limits. At the moment 1″ it is just past its maximum, and is flowing from right to left across the end of the rotor upon which we are looking, at the instant 4″ it is near its minimum value and is flowing downwards. At 8″ it is very great and flows from left to right. It will be seen that it rotates in the opposite sense to the rotor and the field. To see how such a current and rotary field can produce a torque, we must see, in the first place, the relation of their phases. When the current is at its maximum near the instant 16″ the field is in phase with it—that is to say, both are represented by lines in the same direction, with the arrowheads pointing the same way. This produces a great torque in the sense of the rotor motion. At the instants 1″ and 2″ the current is diminishing and getting out of phase with the field, but the torque remains positive. As soon as the angle between the two becomes greater than a right angle the torque becomes negative, but by that time the current has become small and the angle changes very rapidly, so that it is only during the instants 3″ 4″ and 5″ that there is a small negative torque; during the instants 6″ 7″ 8″ 9″ and 10″ the torque is positive again, and is very great at 8″. During the period of one alternation the torque is twice positive and twice negative, but the positive torque greatly exceeds the negative, and the interval of time occupied by it is greater.

It may not at first sight be apparent how a rotating field which varies so little in value can produce a current in the rotor of such great eccentricity as that shown in the ellipse. It must be remembered, however, that the speed of the field relatively to the conductors is only $(n - m)$, while the little variation which takes place in the value of the rotatory field has a frequency of n periods per second.

The inclination of the major axis of the ellipse 1′ 2′ 3′ etc. to the direction of the impressed field increases as the speed of the motor increases. The angle between the two is $\frac{1}{2}(\phi_2 - \phi_1)$.

The counter electromotive-force in the stator conductors is produced by the rotation of the resultant field we have been considering. Its phase is therefore shown by the position of the vector 01′, 02′, 03′, etc., which we have seen is practically the radius of a circle. The counter electromotive-force is at its instantaneous maximum value when this vector is at right angles to the central line which represents the direction of the impressed alternating flux. The current in the stator coil is at a maximum at the instant 1, because the impressed flux is then greatest. We see, therefore, that the inclination of the vector 01′ at this instant, or, in other words, the angle which it has passed through since it was perpendicular to the central line, gives the lag of the current in the stator coils behind the counter electromotive-force. For instance, suppose that the motor is running nearly synchronously, ϕ_2 will be almost nothing, so that B_1 may be taken equal to and in phase with b_1. $B,_2$ on the other hand, is practically wiped out altogether. At the instant 1, when the current is at its maximum, the angle which B_1 has passed through since it was at right angles to the central line is about 90°. This is the angle of lag of the current behind the E.M.F. As load is put upon the motor ϕ_1 increases, and the angle of lag decreases. It must not be supposed, however, that B_1 decreases, as it would seem to do in our figure. We must increase the size of our figure so as to keep the vector representing the resultant magnetism nearly constant.[1] It would thus be possible for any given motor to go through the construction for different amounts of slip, and plot out the characteristic from the differences of the areas of the triangles A O P and A′ O′ P′.

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[1] The resultant magnetism is proportional to the counter E.M.F., which within practical working limits does not alter 2 per cent. For the exact relation between impressed E.M.F. and counter E.M.F. see Steinmetz, *Amer. Inst. Elec. Engineers*, Dec. 1894, p. 803.

CHAPTER IX.

MISCELLANEOUS ALTERNATE-CURRENT MOTORS.

ALTERNATE-CURRENT motors may be classified as follows:—

A. *Single-phase Synchronous.*—Ordinary alternate-current machines, in fact, used as motors. They are not self-starting.

B. *Polyphase Synchronous.*—Mentioned below.

C. *Polyphase Asynchronous.*—The main topics of this work.

D. *Single-phase Asynchronous.*—The monophase motors already considered, and which require a starting-gear of some sort to split the phase.

E. *Series-wound Laminated Motors.*—For small sizes any form of continuous-current motor with ordinary commutator and brushes, provided the field-magnet is thoroughly laminated. They are not altogether satisfactory, as their self-induction is generally too great.

Synchronous Polyphase Motors.—A polyphase system of distribution, while giving great facility in the use of self-starting motors, does not sacrifice the possibility of installing synchronous motors in cases where perfect uniformity of speed is desired. A synchronous motor for a polyphase system may consist of an ordinary alternator placed across two of the mains; but preferably it is identical in construction to the polyphase generators described in Chapter I., and connected to all the lines. It differs from an asynchronous motor mainly in the fact that instead of a rotor it has a field magnet separately excited by means of a continuous current; and as the poles always keep the same position relatively to the iron of the magnet when once they are run up to the speed of the revolving poles of the armature, the North and South poles take hold of each other and the magnet is dragged round in perfect synchronism. For the principles which govern synchronous running the reader is referred to the author's treatise on Dynamo-Electric Machinery, and to other works[1] on the subject. The ordinary single-phase synchronous motor must be run up to speed by some independent source of power; but in a polyphase system the rotatory field acting upon eddy-currents in the broad unlaminated poles of field-magnets is sufficient to start the motor. It is thus possible to so far combine the principle of a polyphase asynchronous motor with a truly synchronous motor, that it shall be capable of starting itself, and after running up to speed, will keep its speed at all loads as constant as the periodicity of the supply. It is to be noted that while a polyphase generator will always act as a synchronous motor, it is not necessarily self-starting. Its design should facilitate the generation of currents in the polar projections if it is intended to be self-starting. A very good instance of an installation of synchronous motors of this kind is at the Ponemah Cotton Mills, Taftville, Conn., U.S.A.[2] Six hundred horse-power is transmitted from a mill 3 miles distant, where water power is available, at a pressure of 2500 volts. The system is a 3-phase one. The motors are the same in construction as the generators, and while being able to start themselves, run under load with perfect synchronism. The efficiency of the complete transmission from the power

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[1] Dr. J. Hopkinson, "On the Theory of Alternating Currents, particularly in reference to Two Alternate-Current Machines connected to the same Circuit." *Journ. Soc. Tele. Engineers*, vol. xiii. p. 496, 1884.

W. M. Mordey, "On Parallel Working, with Special Reference to Long Lines," *Inst. of Elec. Engineers*, xxiii. 260, 1894.

Blondel, "Couplages et Synchronisation des Alternateurs," *La Lumière Élec.*, xlv. 351, 1892.

Steinmetz, "Theory of a Synchronous Motor," *Amer. Inst. Elec. Engineers*, Oct. 17, 1894.

Picou "Transmission de Force par Moteurs Alternatifs Synchrones," *Soc. Internationale des Electriciens*, Feb. 1895.

Bedell and Ryan, "Action of a Single-Phase Synchronous Motor," *Journ. Franklin Inst.*, March 1895.

Rhodes, "Theory of the Synchronous Motor," *Proc. Physical Society*, 1895.

[2] *Elec. Rev.* (N. Y.) xxiv. p. 210, May 2, 1894.

applied to the dynamo pulley to that delivered to the motor pulley is reported to be 80 per cent,

A number of alternate-current motors have been devised which do not come under any one of the preceding classes, and yet are hardly susceptible of classification.

Elihu Thomson's Motor.—In the course of his observations on the effects of alternate currents,[1] in 1886–7, Elihu Thomson observed that a copper ring placed in an alternating magnetic field tends either to move out of the field or to return so as to set itself edgeways to the magnetic lines. It follows that if an ordinary armature (say drum-wound) is placed in an alternating field, and the brushes are given an angular lead in either direction and then short-circuited together, the armature will turn, and yield considerable power. When once so set turning, the armature will continue to rotate, even if the brushes are disconnected or thrown off. Following up this plan, he then constructed motors in which the use of commutator and brushes was restricted to the work of merely starting the armature, which when so started was then entirely short-circuited on itself, though disconnected from the rest of the circuit. It thus operated purely as a monophase induction motor. A motor of this kind was shown at the Paris Exposition of 1889. In 1892, Elihu Thomson patented an alternate-current motor, for use with single-phase circuits, in which a rotatory effort was produced by the use of auxiliary condensers shunting the coils wound on alternate poles.

The Ferranti-Wright Motor.—If one end of a laminated bar of iron is placed in a magnetizing coil supplied with an alternate current, it will undergo an alternating magnetization. But if at a point further along it is surrounded by a stout copper ring or ferrule, the eddy-currents induced in the latter, being out of phase with the primary current, will react locally on the alternating magnetization, and retard the phase of the magnetic polarity at all points beyond. Consequently, if two or three such closed rings or bands of copper surround the iron core at different distances along, the effect will be the same as if the poles

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[1] Elihu Thomson, "Novel Phenomena of Alternating Currents," *Elec. World* (N. Y.), ix. 258, May 28, 1887; xiv. 231, October 5, 1889.

travelled along the iron at a finite speed, a north pole being followed by a south pole, and again by a north pole, each travelling toward the tip, and there dying out. On this plan the Ferranti-Wright motor is based. It is used in Ferranti's alternate-current meters. A pivoted iron disk is placed between two curved pole-pieces of laminated iron, each of which is furnished with retarding-rings of copper, as shown in Fig. 149.

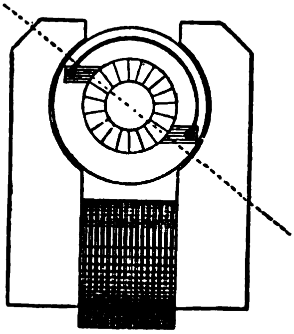

FIG. 148.

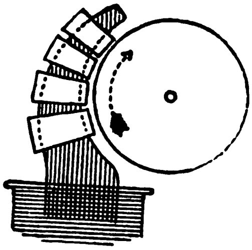

FIG. 149

Shallenberg's Motor.—This motor, which is used in an alternate-current meter, produces the rotation of an iron disk by a very neat method of splitting the phase of the alternate current. The disk is placed between two coils with a rectangular opening, within which, and also passing over and under the disk, is a closed coil, or rather stamping of copper, set obliquely at about 45° with the main coil, from which it receives induced currents. If it lay parallel to the main coil it would receive stronger induced currents, but would produce no rotatory effect. If it lay at right angles, it would receive no currents, and therefore also have no rotatory effect. As its currents are a little retarded behind complete opposition of phase, its oblique position gives a component to the resultant field producing rotation; but the resultant rotatory field is in reality an elliptical one (see p. 64 above).

Atkinson's Motors.—In 1888[1] Mr. Llewellyn B. Atkinson (of Messrs. Goolden & Co.) devised some alternate-current motors which had the feature of two rotors (or amatures) side

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[1] Specifications of Patents, 16,852 of 1888, and 7895 of 1889.

by side, interconnected in their (closed) windings, and two separate stationary parts with windings into which the alternate current was brought. Each rotor served alternately as a transformer to send current into its neighbor's windings, so producing a rotatory effort, though there is no rotatory field.

The Stanley Kelly Motors.—Mr. William Stanley, of Pittsfield (Massachusetts), who was associated with Mr. Westinghouse from 1886 in the development of alternate-current machinery, has devised a two-phase system,[1] in which the generator is of the "inductor" type, the only revolving part being a steel wheel with polar projections of laminated iron. The Stanley-Kelly-Chesney motor used with this system differs radically from the majority of those described in this work; for in it there is, in truth, no rotating field at all. The stator into which the 2-phase currents are led consists of two entirely separate parts, each of which is separately supplied by one of the two currents. There are therefore produced two simply alternating independent magnetic fields with a 90° difference of phase between them. Within these two stators, which are fixed side by side, there revolve two rotors mounted side by side on the same shaft. The windings of these rotors are so interconnected that the wire which lies directly under the poles on one armature is in series with the wire that lies between the poles in the other. So connected each rotor acts alternately as a motor to receive current and be driven by it, and as a transformer to send current to the other. The windings on the two rotors together are closed, having no external connections by slip-rings or commutator. It is claimed that these motors will give a torque at starting from 1½ to 2 times as great as when running at full load. Condensers are used in parallel with the stator circuits to furnish at starting the idle current, and prevent the inductive drop in the pressure. A resistance is inserted in the circuit at starting.

T. Duncan's Motor.—This is a form intermediate between Shallenberger's and Ferranti's, the oblique coil of the former

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[1] *Electrical World*, p. 325, 1893.

being replaced by an oblique iron core surrounded near its extremities with throttling circuits of copper. It is applicable to three-phase circuits, and is intended for use in meters.

Mordey's Motors.—Mr. W. M. Mordey has designed various forms of alternate-current motor. In one of these—a motor with laminated iron throughout—he proposes to pass part of the alternate current through a commutator on the shaft for the purpose of exciting the field-magnet, so that as the motor acquires speed the frequency of the alternations of current in the motor itself is reduced until synchronism is attained, when the current, so far as the magnet itself is concerned, becomes unidirectional.

Ganz's Motor.—A similar proposal emanated from Messrs. Ganz and Co., of Buda-Pesth.

W. Landon-Davies's Motor.—This is a form of split-phase motor having two or more sets of coils placed at different angles; the windings being calculated so as, while producing equal ampere-turns, to have phase-angles which are the supplements of the position angles of their respective coils.

CHAPTER X.

POLYPHASE TRANSFORMERS.

THE principles underlying the transformation of polyphase currents to currents at a higher or lower pressure, hardly differ from those involving single-phase currents. The law which states that the ratio E_1 / E_2 between the electromotive-forces in the primary and secondary is equal to the ratio between the number of winding S_1 / S_2, is of course applicable to every case of coils wound upon the same magnetic circuit, and the laws relating to the losses in the copper and iron are applicable to polyphase and single-phase currents alike. Indeed, the transformation of polyphase currents might be carried out entirely by means of ordinary single-phase transformers, it being only necessary to place a transformer in each of the polyphase circuits to raise or lower the tension to any desired extent. It is, however, convenient to have one transformer for all the circuits, and a saving in material is effected by so doing. In the case, for instance, of 3-phase working, just as three of the six wires originally employed can be dispensed with and a saving in copper affected by joining the three circuits to a common junction, so there is a corresponding saving in iron by having a common junction at each end of the cores wound with the various circuits of a transformer. Fig. 150 shows diagrammatically a transformer in which the cores A B C are so joined to a common junction at each end. In order that the cores may be properly laminated, it is easier to build them of iron stampings of the forms shown in Figs. 151 and 152. If the coils are wound around A, B and C, the flux in these cores will follow a law similar to that of the circulating currents; that is to say, it will be a 3-phase flux, there being a difference of 120° between the flux-phase of each limb. It will be seen that the portions D′, D″ and D‴ in Fig. 152 form a mesh-connection of the paths A, B and C; there will therefore be a difference of 120° between their respective flux-phases; and generally it may be said that Fig. 55 (p. 46) which shows the relations both of magnitude and phase of the currents in a mesh-circuit is equally applicable to the fluxes in the various parts of the core shown in Fig. 152, where A, B and C take the place of the line wires, and D′, D″ and D‴ the phase of the limbs of the mesh. We may take D′ D″ and D‴ as the cores of transformer instead of A, B and C, or we may wind both or either set with primary or secondary coils, and as such coils may themselves be joined up in star or mesh, a great number of combinations and permutations are possible.

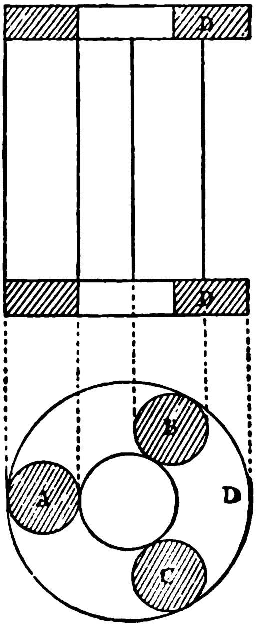

Fig. 150.

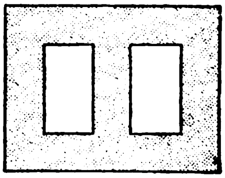

Fig. 151.

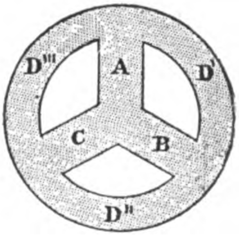

Fig. 152.

The transformers actually employed in 3-phase working usually consist of three vertical columns of laminated iron, having common yokes across the ends; the primary and secondary coils being wound in the usual manner over the vertical parts of the core. Fig. 153 illustrates a 3-phase transformer made by Messrs. Siemens and Halske of Berlin. The transformers used in the famous Lauffen-Frankfort transmission of 1891, and still used in the supply of 3-phase current to the town of Heilbronn, are illustrated on p. 386 of the Official Report referred to above (see p. 106). They were designed for the purpose of transforming from 15,000 volts to 100 volts, or *vice versâ*, but admitted of various groupings. The arrangements of the circuits was illustrated in Fig. 103, p. 107. The common junction of both the high-pressure and low-pressure windings was in every case put to earth.

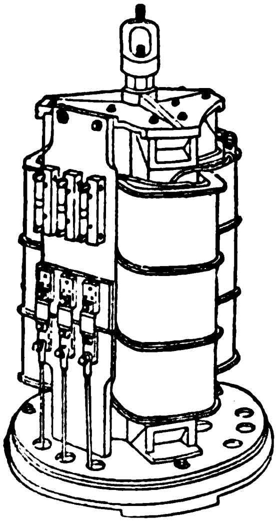

FIG. 153.—THREE-PHASE TRANSFORMER CONSTRUCTED BY SIEMENS AND HALSKE.

For 2-phase transformation two separate transformers might be used, one in each circuit. But just as in the circuits themselves there is some saving of copper by combining them so as to employ three lines instead of four (one of slightly greater section serving as a common return for the other two); so there is a saving in combining the two iron circuits in one, having in one part a common core. The appropriate arrangements and connections of the windings are shown in diagram in Fig. 154,

The proper cross-section for the common core is $\sqrt{2}$ times that of the separate cores, if the same maximum of flux-density in the iron is to be attained.

In those cases where a mesh-grouping is adopted in a two-phase generator the circuits are not capable of being used with a common return; two separate lines must be used for each circuit. But if transformers are used at both ends of the transmission, three lines only are needed. This disposition, which is shown in Fig. 155, was used by Schuckert & Co. at Frankfort in 1891 in one of their transmissions of power.

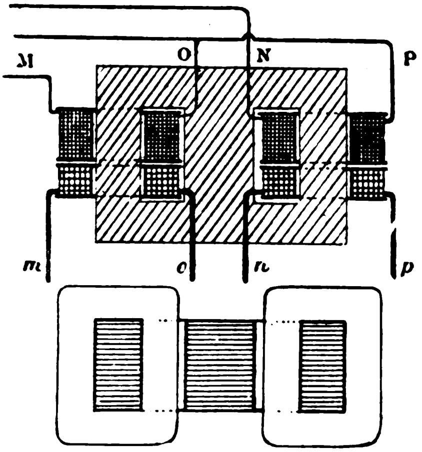

FIG. 154.

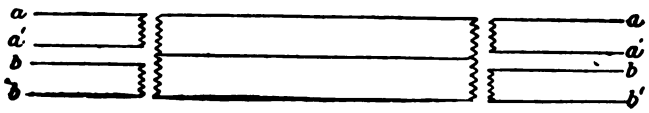

FIG. 155.

Phase-transformation.—So far we have dealt with the problem of transforming the voltage of a given system of currents. But there is another problem needing solution, namely, that of transforming 2-phase currents into 3-phase, or *vice versâ.*

The simplicity with which this problem can be resolved will easily be understood by further developing the ideas unfolded above.

When a transformer of the type shown in Fig. 152 is in action, the field is in the nature of a rotatory one. The core is in the shape of a wheel with three spokes. If we increase the number of spokes as shown in Fig. 156 we get a more uniformly rotating field.

We may have as many spokes as we like wound with primary coils, and we may divide the rim of the wheel into as many portions as we like, each wound with a secondary coil, and thus we may transform from a system of currents of any number of phases to a system of any other number, provided we start with something more than a single phase, so as to obtain a rotatory field, and not a merely alternating one.

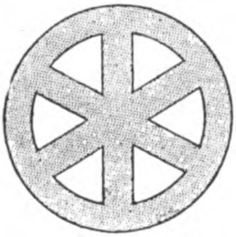
FIG. 156.

The same result can also be effected by dividing the rim into a certain number of parts wound with primary coils, and into a different number of parts wound with secondary coils. Instead of spokes, in such a case the centre would be filled up with iron, leaving gaps just sufficient for the windings. It is not even necessary to have primary and secondary coils. If there be one closed winding on the rim, like the winding of a Gramme ring, and wires of one system of currents be connected successively at equal intervals all round to this winding it is possible to draw off a system of currents of any other number of phases by merely joining the required number of wires to the winding in the same manner. The first suggestion for transforming from 3-phase to 2-phase on this plan was made by the author at a lecture delivered at the Royal Institution, Feb. 23, 1894, on the transformations of electric currents. On that occasion a ring-transformer having twelve coils in closed series was joined to a 8-phase supply at three equidistant points, A, B and C. An alternate current could be taken off from the two ends of any diameter across the windings, as from A $A_{\prime}$, whilst at the same time another alternate current, of voltage equal to the former, could be taken off at the ends of a diameter

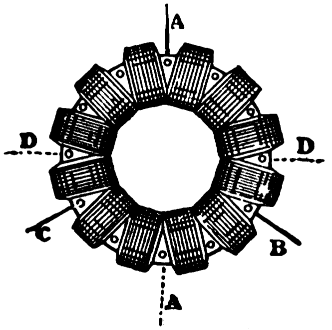

FIG. 157.

D $D_{\prime}$ at right angles to A $A_{\prime}$. As in this case the 2-phase coils subtend 180°, while the 3-phase subtend 120°, the relative voltages will be as 1 : 0·75, being proportional (if the distribution of the magnetic flux follows a sine-function around the periphery) to $1 - \cos \beta$; where β is the angular breadth.

By such an apparatus any desired phase-transformation might be effected. The magnetic circuit is greatly improved by providing an iron centre-piece, properly laminated, whether stationary or revolving.

A few days later, on March 1st, 1894, at a meeting of the National Electric Light Association, at Washington, Mr. C. F. Scott, chief electrician of the Westinghouse Company, proposed a different solution of the same problem, requiring the use of two transformers. As arranged for transforming from 2-phase to 3-phase it is described as follows:—"The primaries of two transformers are connected to a generator giving 2-phase current. The secondary electromotive-forces, therefore, differ 90°. One secondary is made equal to 100 turns, and a loop is brought out at its middle point, giving 50 turns at each side. The second secondary has 87 turns, which is approximately 50 multiplied by the square root of 3. One end of the secondary circuit is connected with the middle point of the secondary of the first transformer, as shown, and the three free terminals will then deliver electromotive-forces differing in phase 120°. If the electromotive-force on each primary be 1000 volts, and on one secondary

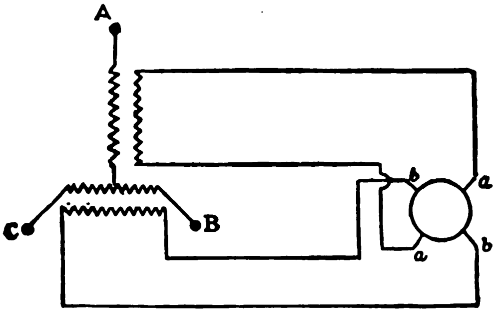

FIG. 158.

100 volts, and on the other 87 volts, then the electromotive-force measured between any two secondary terminals will be 100 volts."

Methods of Transforming (Single-phase) Alternate Currents into Two-phase or Three-phase.—The following method of obtaining 3-phase currents out of 2-phase is due to M. Désiré Korda. It consists in principle of a transformer with three cores, and of a movable self-induction coil. The circuit carrying the monophase current, $c = C \sin pt$, is divided into two branches I. and II. having the same ohmic resistance. A self-induction coil is inserted into branch II. in order that

$$\frac{Lp}{R} = \sqrt{3} = \tan 60^\circ \quad . \quad . \quad . \quad (1)$$

The current in branch I. may be expressed by

$$c_1 = \frac{E}{R} \sin pt.$$

The current in branch II. is expressed by

$$c_2 = \frac{E}{\sqrt{R^2 \times p^2 L^2}} \sin (pt - \varphi)$$
$$= \frac{E}{2R} \sin (pt - 60^\circ).$$

That is to say, c_2 will be one-half of current c so long as equation (1) is satisfied. If the branch II. contains n turns on one of the cores of the transformer, the branch I. must contain $\frac{1}{2} n$ turns wound on the second core, and the directions of the windings are opposed to each other, so as to produce an equal flux in each core, differing in phase by 120°. The third core of the transformer is wound with both branches, the direction being such as to produce a third flux, differing again by 120° from the other two. Three-phase currents are then obtained from secondary currents wound on the three cores.

Methods of Transforming Continuous Current into Polyphase Alternating Currents, or vice versâ.—The methods of transforming continuous currents into simple alternating currents, or *vice versâ*, are applicable also to polyphase currents. For brevity we may write A C for alternating currents, C C for continuous currents, and P C for polyphase currents.

The oldest method of transforming C C to A C or *vice versâ*, is by coupling together two machines, one of each kind, the one as motor to drive the other as generator. An example of this exists at the town of Cassel, which has a continuous current three-wire supply (with accumulators) fed from C C dynamos which are driven by A C synchronous motors receiving single-phase alternate currents at a high voltage from a distance.

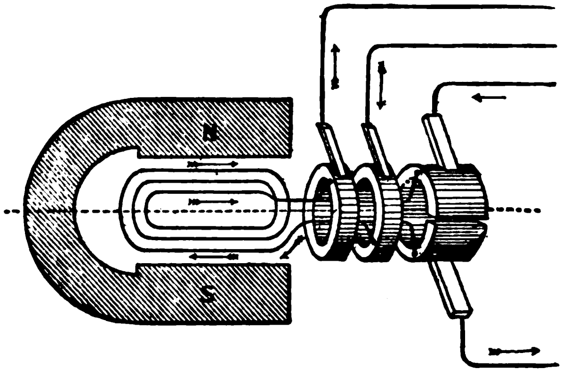

Fig. 159.

In the second method, an armature revolving in a magnetic field receives C C to drive it as a motor, and gives out A C by means of slip-rings suitably connected to the same windings. Fig. 159 illustrates this mode of transformation, which can also be used for the converse change of A C into C C. In the drawing the revolving armature is shown as a simple coil with a two-part commutator for the C C connections; but in practice a more complex armature with a many-part commutator is employed. For example, an ordinary Gramme ring is used, with the addition of two slip-rings, which are conducted to two points 180° apart. Such a machine has been in use at the Technical College, Finsbury, since 1885, when the rings were added by Dr. Walmsley. It will serve as a transformer either way, or, if driven by power, will furnish either kind of current, or both at once. In 1887, a patent was taken out by the Helios Co. for this very combination: and in 1889, Mr. Bradley and Mr. Tesla patented similar devices. For producing 3-phase currents from C C, three slip-rings must be connected on at three symmetrical points. For two-phase currents four slip-rings are connected at points 90° apart. In a recent apparatus of Hutin and Leblanc[1] a row of eighteen slip-rings are connected at as many symmetrical points, giving rise to eighteen alternate currents, each differing in phase by 20° from its next neighbor.

A simple revolving combined commutator, like that of Fig. 159, would suffice to convert C C into A C, or to rectify A C into C C, without any field-magnet, were it not for the practical difficulties arising about sparking. The use of the field-magnet is to balance the electromotive-forces in the different parts of the windings, as well as to maintain the proper rotation.

At the Frankfort Exhibition of 1891 many revolving transformers based on this plan were shown. The firms of Lahmeyer and Schuckert, in particular, displayed many very interesting forms of polyphase apparatus, in which this feature was prominent (see p. 104).

Messrs. Schuckert & Co. showed a six-pole ring-wound machine, capable of transforming from a continuous current or single-phase, 2-phase, or 3-phase currents to currents of any one or all of the other three kinds. It consists of an ordinary ring armature with a 144-part commutator, whose windings in front of the different pairs of poles are cross-connected in parallel (Mordey's well-known method). As there are 144 sections in the winding, and six poles, the number of sections that lie between any pole and the next pole of the same sign, will be 48. From Nos. 1, 17 and 33, that is to say, at points equally spaced out at distances of one-third of the extent of the winding between any pole and the next pole of same sign, are attached three wires which are

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[1] See an article in *l'Électricien* of April 21st, 1894.

brought down to three slip-rings, from which brushes supply 3-phase currents. To four points also equally spaced along the same section of the winding (namely Nos. 1, 13, 25 and 37), are attached four wires, which going to four other slip-rings, supply both single and 2-phase currents.

Messrs. Schuckert have installed at Budapesth a station outside the town, from which power to the amount of 1000 kilowatts is transmitted by a 2-phase system, at a pressure of 2000 volts, to several sub-stations in the town, and is there transformed into a continuous current. Each transformer is a double machine consisting of an alternate-current motor and a continuous-current dynamo mounted on the same shaft. The efficiency of transformation is 85 per cent. An installation has been erected by the same firm at Bilboa, in which a 3-phase generator is coupled direct with a turbine, and transmits 46 kilowatts to a station at a distance of two miles, where it is transformed into a continuous current.

An eight-pole revolving transformer on a similar principle, but having a wave-wound drum armature, was shown at Frankfort by the Allgemeine Company. It could receive continuous current at about 100 volts, and transform this into 3-phase current at about 70 volts. This transformer is now in the laboratory of the Technical College, Finsbury.

M. Hospitalier has drawn up a general classification[1] of apparatus for effecting transformations of currents of one species to those of another, and designates such as *polymorphic* machines.

At Dublin, an electric tramway using continuous currents at 500 volts is being worked by power transmitted by a 3-phase system at 3500 volts. In this instance the transformation is effected by motor-dynamos placed in sub-stations; each machine consisting of a synchronous 3-phase motor rigidly coupled to a continuous-current generator. The machinery is supplied by the British Thomson-Houston Co.

The subject of transformers would be incomplete without

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[1] *Société française de Physique*, 1894, p. 203.

a reference to the *auto-transformer* used sometimes when a smaller electromotive-force is required for a short time, as in the case of the starting of a motor. The auto-transformer (or "one-coil" transformer) merely consists of a coil of wire wound on an iron core, and connected across the mains. To some point in it, at a greater or less distance from one end, according to the voltage required, a branch wire is attached and current is drawn off between this branch and one end. It will be seen that a much greater current can be drawn off in this way than is actually supplied by the mains, as the piece between the branch and the end in use acts as the secondary of a transformer.

Polyphase Choking-Coils.—Polyphase choking-coils can be constructed upon the plan of polyphase transformers by using one set of windings (in two or in three phases) upon the cores in lieu of the primary and secondary windings. The ordinary rules for winding choking-coils apply, due regard being had to the groupings of the phases.

CHAPTER XI.

MEASUREMENT OF POLYPHASE POWER.

As is well known, the power given by an alternate current to any part of a circuit may be measured in several ways: by the use of a watt-meter; by the method of the three voltmeters; or by several analogous methods.[1]

In the case of 2-phase and 3-phase systems there is some complication. In cases where the two or the three circuits are kept separate, a suitable watt-meter in each suffices; and the total power supplied is the sum of the amounts measured separately. For example, in a three-phase system, arranged in either star or mesh, a separate measurement may be made of each limb of the circuit.

It is obvious that in the case of 3-phase motors such a method of measurement would be highly inconvenient; and it is easily shown that a simplification may be effected.

In the case where there is perfect symmetry in the three circuits, it is obviously sufficient to measure with a watt-meter the power consumed in any one of the circuits, and multiply by 3 to ascertain the total power. But in any general distribution no such equality of consumption can be assumed to exist.

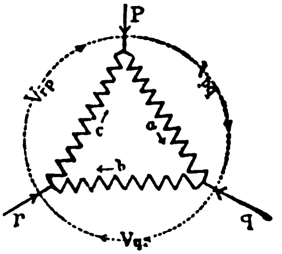

FIG. 130.

Three-phase Power Measurement.—When we have three currents in three conductors, one of which is the resultant of

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[1] For these the reader is referred to such works as Fleming's *Alternate-current Transformer*, or Blakesley's *Alternating Currents of Electricity*.

the other two, and we have a corresponding relation between the pressures across the three conductors, it is evident that these six quantities are not independent of each other, and therefore it should be possible to measure the power without having to measure all six.

Take the simple case of a 3-phase circuit of incandescent lamps joined in mesh-fashion, as in Fig. 160, where a, b and c are the lamp-circuits. Denoting by a, b and c the currents in these circuits, and by V_{pq}, V_{qr} and V_{rp} the pressures between their ends, we have the total watts

$$W = a\,V_{pq} + b\,V_{qr} + c\,V_{rp}.$$

Taking the positive sense as indicated by the arrows in the figure, we have at any instant

$$V_{pq} + V_{qr} + V_{rp} = 0; \quad \therefore V_{pq} = -V_{qr} - V_{rp},$$

substituting in above

$$\begin{aligned} W &= -a\,V_{qr} - a\,V_{rp} + b\,V_{qr} + c\,V_{rp} \\ &= V_{qr}\,(b-a) + V_{rp}\,(c-a). \end{aligned}$$

If p, q and r are the currents in the mains leading into the mesh, then $(b-a) = q$, and $(c-a) = -p$; therefore

$$W = V_{qr}\,q - V_{rp}\,p.$$

This expression appears to be the difference of two quantities of power on account of the sense chosen as positive in the figure if we reverse the sense of the pressure between r and p; then as $V_{rp} = -V_{pr}$,

$$W = V_{qr}\,q + V_{pr}\,p.$$

That is to say, if we pass the current q through the series coil of a watt-meter whose shunt coil is connected between q and r, and pass the current p through the series coil of another watt-meter whose shunt coil is connected between p and r, then the sum of the watts registered by the instruments is the total power absorbed in the circuits a, b and c.

If the circuits form a *star*-grouping a similar formula can be deduced. Employing the letters of Fig. 54 (p. 46) we have the total watts

$$W = V_{jm}\,a \times V_{jn}\,b \times V_{jo}\,c.$$

Taking now the currents, instead of the pressures as in the case of the mesh,

$$a + b + c = 0;\ \therefore\ a = -b - c.$$

Substituting as before and noting that

$$V_{jn} - V_{jm} = V_{mn} \text{ and } V_{jo} - V_{jm} = V_{mo}$$

we get

$$W = V_{mn}\,b + V_{mo}\,c.$$

Two watt-meters of suitable construction suffice, therefore, to measure the power. Dr. H. Aron[1] has constructed a meter suitable for measuring the consumption, the design of the instrument being a modification of his well-known pendulum meter with differential gearing. The 3-phase meter has its second pendulum accelerated by the two movable coils of the two watt-meters, each moving coil lying within its corresponding fixed coil.

Other forms of polyphase meter have been proposed by T. Duncan,[2] and by Shallenberger.[3]

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[1] *Elektrotechnische Zeitschrift*, xiii. 193, April, 1892.

[2] Specification of British Patent 6241, of 1893.

[3] Specification of British Patent 143, of 1895.

CHAPTER XII.

NOTES ON DESIGN OF ROTATORY-FIELD MOTORS.

As with every class of machinery, so with polyphase motors, only practical experience can lead to excellence in design. All that can be given here is the fundamental argument upon which the general dimensions and winding of a motor designed for a particular purpose are based.

The problem we have to consider is: Required, a motor of a given number of phases for a given voltage of supply, what shall be the dimensions of its parts and the number of its windings in order that it may yield a certain prescribed power?

Let us begin with the stator. The duties of the conductors of the stator (so far as the purposes of design are concerned) are (1) to provide a back electromotive force equal to V, the voltage of supply; (2) to carry a current in each circuit equal to $\frac{W}{V\,h \cos\phi}$, where W stands for total watts taken in at full load by the machine, and h is the number of circuits. The cosine of the angle of lag ($\cos\phi$) may be taken at 0·85. For instance, in the six horse-power 2-phase motor, shown in Plate I., intended for 100 volts, particulars of which are given on p. 211, taking the efficiency at 80 per cent. the watts absorbed at full load will be 5600. The current in each circuit will be

$$\frac{5600}{100 \times 2 \times 0{\cdot}85} = 33 \text{ amperes.}$$

The smallness of the stator is limited by the fact that we have to get into it (preserving the proportions between iron space and copper space specified below) a certain total length of active conductor which will perform these duties. The total length of active conductor in each circuit can be found from the equation.

$$V - v = \frac{q\,B\,\lambda\,s}{10^8},$$

where V = pressure of supply in virtual volts.
v = volts lost in resistance of stator conductors (see below).
q = factor depending on the angular breadth of the coils (see p. 25), and may be taken as 0·9 for a 2-phase motor like Fig. 97, and as 0·95 for a 3-phase motor like Fig. 57.
B = virtual flux-density (see below).
λ = total length of active conductor required.
s = linear speed of magnetic field in cm. per second.

The lost volts v may be taken between 0·05 V for small motors and 0·02 V for motors of 100 horse-power and over.

We have denoted by B the $\sqrt{\text{mean sq.}}$ value of the flux density in the air space. As the maximum value of the flux density ought not to exceed 6000 lines per sq. cm. (which means over 11,000 in the iron between the holes), we may take $6000 \div \sqrt{2}$, say 4200, as the value of B in the above equation.

With regard to s it is difficult to lay down any rule, as it depends a great deal upon the purpose for which the motor is intended. The linear speed of the periphery of a rotor may be carried to a much higher point than is desirable in the case of an ordinary dynamo armature. While 1500 cms. per second[1] is an ordinary speed for the periphery of a 100 horse-power armature, a rotor of this power may be run at 2400 cms. per second with equal safety. The linear speed of the periphery alters very little with the size of the machine, in fact 2000 cms. per second is a good useful speed for machines from 10 to 100

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[1] Since 1 foot approximately equals 30 cm., it follows that a speed of 1 cm. per second is approximately equal to 2 feet per minute. Hence 1500 cm. per second is about equal to 3000 feet per minute.

horse-power, though it may be increased considerably beyond this for machines of very great radius. The speed s of the magnetic field is from 2 to 5 per cent. higher than this, according to the amount of slip. The radius of the rotor, for reasons which will appear later, varies very nearly in proportion to the square root of the horse-power. From a comparison of the dimensions of well designed polyphase motors it appears that the formula

$$r = 200 \sqrt{\frac{\text{H.P.}}{s}}$$

gives us in centimetres the length of the radius suitable to form the basis of the design. This formula is based on the comparison of motors intended for a frequency of about forty or fifty periods a second. For higher frequencies sufficient data are not available, but as theoretically the frequency does not affect the size of the motor, it would seem that the general method of design indicated here is applicable to frequencies as high as 100 per second. Of course a wide departure from the radius calculated as above would be admissible. If on roughly calculating out the dimensions the length of the rotor in the direction parallel to the shaft came out too long, it is easy to assume a larger radius than the formula gives, and recalculate from this point onwards. A suitable number of revolutions per second (here denoted by n_1) will be about $\frac{320.}{r}$ The frequency of alternation of the supply we will denote by n. The number of pairs of poles produced by one circuit of the stator (carrying one of the polyphase currents) will be $\frac{n}{n_2}$ where n_2 is the number of revolutions of the field per second. This ratio $\frac{n}{n_2}$ must be a whole number, and we can therefore take it as the whole number which will make n_2 as nearly as possible equal to $1 \cdot 03\, n_1$ (allowing a slip of three per cent.). The linear speed s will then be $2 \pi n_2 r$. Of course a maker may modify this calculation by other considerations, as, for instance, when he has already iron stampings in stock which will do for the motor.

Having fixed s we have all the data from which to calculate λ, and upon this and the cross section of the wire the breadth of the stator depends.

The diameter of the conductor may be chosen so that the current density amounts to between 250 and 300 amperes per square cm., according to the mode of insulation and facility for cooling. We will denote by a the area occupied by the cross section of a conductor, including insulation and space lost in packing.

Next we have to consider into how many pieces the total active conductor in one circuit shall be divided. The total space available for the stator conductors will depend upon the radial depth d of the winding (see Fig. 105).

It is desirable to make d as small as possible, for the greater it is the greater will be the amount of magnetic leakage. In general it may be taken as twice the breadth of a hole, though this will be subject to modification to suit other matters in the design. We have seen that $\frac{n}{n_2}$ is the number of pairs of poles, and from this and the number of phases the number of holes can be settled. For instance, if the winding is to be like that shown in Fig. 171, and the number of phases is two, taking the frequency n at 50 and n_2 at 5, the number of coils in each circuit will be 20, that is 40 coils in all. The question how many holes we shall allow for each coil (or for each wave if a wave winding is adopted) depends upon the diameter of the stator and the number of coils or waves. We cannot do better than follow the example of such designers as Mr. C. E. L. Brown, and make the iron between the holes about the same breadth as the holes themselves. This will mean that where there are only a few coils, as in the four-pole motor, Plate I., a number of holes (four in the figure) will be assigned to each coil; where, on the other hand, the number of coils is great, as in Fig. 171, two holes only will suffice for each. The object in view is to keep the winding as near to the surface of the stator as possible. Having ascertained the most suitable number of holes (g) the breadth of each is $\frac{\pi r}{g}$, and taking d as twice the breadth, we have the area nearly $= \frac{2\pi^2 r^2}{g^2}$. From this we must allow a certain area for the paper tube or other insulation, and then we have left the available area A of each hole.

We then have $\frac{A}{a}$ as the number of conductors through each hole, and $\frac{g}{h} \times \frac{A}{a} =$ total number of conductors in one circuit. Therefore the length of each conductor must be

$$l \text{ in centimetres} = \lambda \frac{h\,a}{g\,A}.$$

This gives us the breadth of the stator face. It will be seen that the last variable that we have taken, the one in fact which finally adjusts the machine to the desired voltage, is the length of each active conductor, or, what is the same thing, the measurement across the stator face parallel to the axis. This is a measurement which can be varied considerably in its relation to r without interfering either with the cost per horse-power or the efficiency of the machine. It will be found, however, that in large machines it is always made smaller in proportion to r than in small machines. As the stator is built of laminated iron clamped together, there are mechanical reasons why its breadth should not be great as compared with its radial depth. It is partly on account of this that we find the radius of the rotor varies as the square root of the horse-power rather than as the cube root. The radial depth is about one-half of the breadth of one pole. The fact that the radial depth d of the stator holes decreases in its relation to the radius as the radius is increased, causes the stator breadth to increase a suitable amount as the size of the machine is increased, for the power of the machine is roughly proportional to the weight of copper in the stator.

The dimensions of the stator having been fixed, those of the iron work of the rotor immediately follow. The total breadth of laminated iron in the direction parallel to the axis is the same in each. The air space between the two is made as small as possible, allowing just enough room for the rotor to run clear under practical working conditions. In the motor shown in Fig. 105 the air-space is only 0·5 mm. across, that is to say, the outside diameter of the rotor is 1 mm. less than the inside diameter of the stator. In small motors of 4 or 6 poles the rotor is built up of disks, such as that shown in Fig. 105. When the poles are numerous, the centre portion of such disks would be inoperative; the laminated portion of the rotor is therefore in the form of a ring, which may be built up of pieces of sheet iron, interleaved at the junctions, and bolted together on the rim of a cast iron support, which may be in the form of a wheel such as shown in Fig. 170.

We have seen that it is desirable, though not absolutely necessary, to have the number of the conductors on the rotor incommensurable with the number of holes in the stator, so that there may be no tendency either at starting or at any speed below synchronism for the two to cog magnetically into one another. When the rotor bars are merely short-circuited at their ends with a solid ribbon of copper, no difficulty is experienced in choosing a suitable number. For instance, in Fig. 105 the number of stator holes is 40, and the number of rotor bars 37.

When it is intended to connect the rotor bars in a regular winding, either for the purpose of inserting a resistance at starting or for the purposes discussed on p. 122, some discretion must be used in the selection of their number. If, for instance, we intend to form three circuits to bring down to slip rings on the shaft, for the purpose of introducing resistance at starting, we may divide the space occupied by one magnetic pole into three divisions 1, 2 and 3. All the conductors in division 1 in front of a North pole may be connected to conductors in division 1 in front of a South pole, and so on, forming a wave winding round the rotor; and the conductors in divisions 2 and 3 forming similar wave windings. If there are the same number of conductors y in each division, then the total number of conductors will be $3\,y\,p$ where p is the number of single poles. This number will ordinarily have a common factor with the number of holes in the stator, but that in itself will not prevent the motor from starting when the conductors are sufficiently numerous, and particularly if the numbers within the breadth of one pole are incommensurable. For instance, in the motor shown in Fig. 170 the number of holes in the stator is 80 and the number in the rotor 180: the numbers in the breadth of a pole are 4 and 9 respectively.

In fixing the cross section of the rotor bars it will be remembered that the greater it is, the greater will be the efficiency of the rotor, provided proper iron space is also allowed. There is nothing to be gained by making the total cross section greater than the total cross section of the stator windings, and in practice it is generally a little less. The current, per centimetre of periphery, in the one, is (neglecting magnetizing current) equal to the current per centimetre of periphery in the other. The conductors being of solid copper, and only lightly insulated, can be put into much less space than the stator conductors; and for this reason the holes in the rotor are usually smaller than one-half the size of those in the stator.

CHAPTER XIII.

MECHANICAL PERFORMANCE OF POLYPHASE MOTORS.

THE three chief requirements in the mechanical performance of a motor are (1) it shall exert a good torque at starting; (2) it shall be capable of running at nearly constant speed at all loads; (3) it shall yield in mechanical power a high percentage of the power put into it.

The Starting of Polyphase Motors.—The conditions under which a great torque at starting can be obtained in a polyphase motor have been considered in Chapter VI. The actual torque obtained of course depends upon the current which is passed through the stator coils. It may amount to four or five times the torque at full load. In the case of large motors it is undesirable to draw such a large current as would, if unrestrained, flow through the motor while it is getting up speed. The resistance inserted in the rotor circuit has the effect of keeping down the stator current by allowing the stator to act as a choking coil; its self-induction not being wiped out by the currents in the rotor, as would be the case if no resistance were inserted. At the same time a much greater torque is obtained, as shown in Chapter VI., than if the current were kept down by a resistance in the primary circuit. Fig. 161 shows a resistance composed of three vessels containing liquid to which the three wires from the rotor slip-rings are attached. The common junction consists of three plates, which can be raised and lowered in the liquid so that the resistance can be altered at will. In some cases the resistance is carried in the body of the rotor itself, and a device is mounted on the shaft for short-circuiting it after the motor has got up speed. For all small motors, and indeed for all sizes up to 10 horse-power, this arrangement is to be preferred, as it dispenses with all complications of slip-rings, brushes, and the like. Large motors are generally started light, and the load is gradually put on by belt-shafting or friction-pulley devices. For cranes, elevators, &c., special motors are made without collecting rings and brushes or any other means of inserting a resistance in the rotor circuits. These have a large armature slip (up to as much as 12 per cent.) and a low power factor, but they start with an initial torque equal to two or three times their normal load torque. The following table, supplied by Mr. Kolben, gives the current and voltage at the terminals of a standard 9 horse-power crane motor, made by the Oerlikon Company, at starting with various loads. The pull between armature and field is not constant in all positions of the armature; the minimum and maximum torque is therefore indicated.

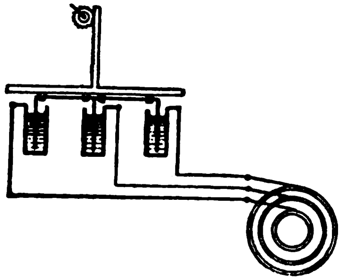

FIG. 161.—DOBROWOLSKY'S STARTING-RESISTANCE.

STARTING TORQUE OF 9 HORSE-POWER THREE-PHASE CRANE MOTOR.

Volts between neutral point and each terminal.	Amperes in each branch at starting.	Pull in kilogs. on lever of 13 cm. at rest.
48·5	60	15– 30
58	83	30– 60
69	100	50– 90
75	105	60–100
80	115	90–140

The same motor, when running light at a speed of 1000 revolutions per minute, with 110 volts pressure, took 20 amperes. At normal load the current was 39 amperes and the speed 890, developing 8·5 horse-power.

Dr. Louis Bell, in a paper read before the American Institute of Electrical Engineers, January 17, 1894,[1] gives a number of useful experimental data as to ability of motors to start under load, the current required at starting, and various other matters. The curves in Fig. 162 are taken from his paper; they show the starting torque of a triphase 5 horse-power motor. B_2 shows the variation of torque with current for a given fixed resistance in the rotor circuit, the voltage being varied, and the resistance being such as to give a heavy torque. B_3 is the same as B_2, except that the resistance was such as to give very moderate starting currents. The full normal load torque was 17·5 lb.-ft. It will be seen that the motor when starting will develop full load torque on considerably less than full load current. With full load current, in fact, the torque at starting is 50 per cent. more than the running torque. Similar curves are given for a 10 horse-power motor, and also curves showing effect of varying the resistance.

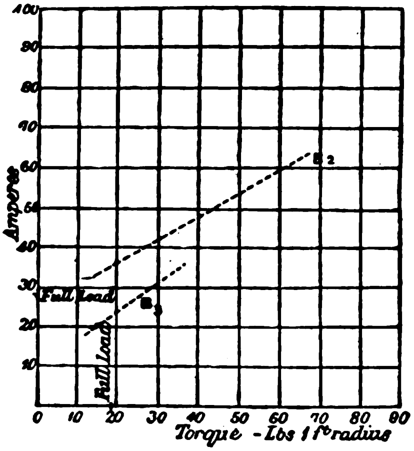

Fig. 162.

The Starting of Monophase Motors.—The methods by which a monophase motor may be started divide themselves into

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[1] *Electrical World,* xxiii. 334–367, 400 (1894).

two classes—(1) those in which a rotatory field is produced by an auxiliary winding on the stator carrying a current differing in phase from the current in the main winding; (2) those in which the conductors of the rotor are connected (as by brushes on a commutator) so that the currents in them produce a polarity inclined to the polarity of the stator.

The difference of phase in the currents in the two windings of the stator may be caused by the windings having themselves unequal coefficients of self-induction, or by putting resistance or capacity in series with one, or inductance in series with the other; or any combination of these may be employed, as described on p. 156.

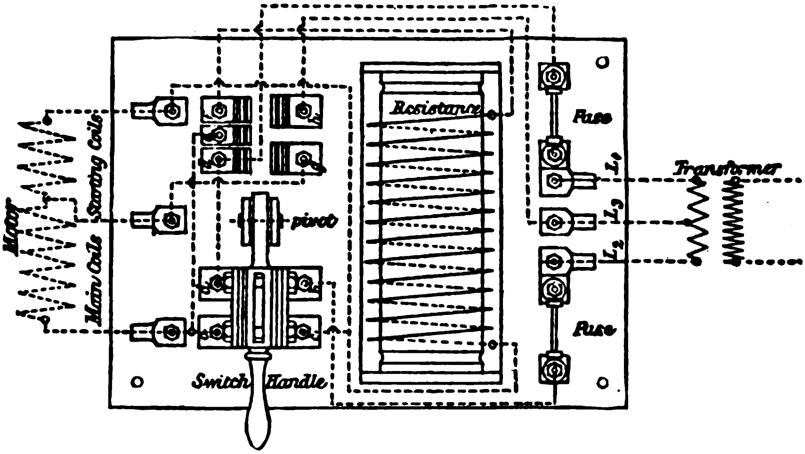

FIG. 163.—KOLBEN'S STARTING-GEAR FOR MONOPHASE MOTOR.

Fig. 163 is a drawing of switch arrangements made by the Oerlikon Machine Company for the purpose of starting a motor. It is shown with the "chopper" of the switch in the full-speed position. When in the starting position the blades of the chopper connect together the pieces *d*, *e* and *f* and also *g* with *h*. The points L_1 and L_2 receive the full pressure of the transformer, while L_3 is connected to the middle point of the transformer, so that between L_1 and L_3 there is a lower voltage than between L_1 and L_2, but more current can be drawn without making a great demand on the mains. Considering now the chopper in the starting position, the current after going from L_1 to *d*, has two paths—one through *e* to the main coils, the other through *f* to the resistance coil and starting coils, returning by the pieces *g* and *h* to L_3. After the motor has got up speed the chopper is thrown over, joining *a* to *d* and *c* to *b*. It will then be seen that the two windings are in series, and take the full voltage of supply.

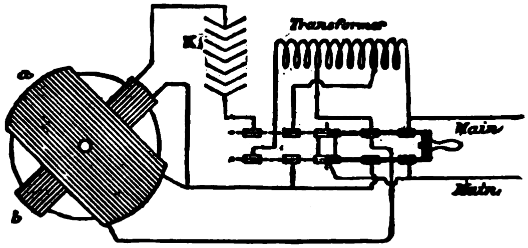

Fig. 164.—Brown's Starting-gear.

In Fig. 164 is shown diagrammatically Mr. Brown's method of starting a motor by means of an electrolytic condenser, (see p. 157) marked K in the figure. The fine dotted line shows the connections of the chopper switch at starting, and the thicker line shows the same at full load. When the motor is not in use the chopper stands vertically. The coil *a* represents the ordinary working winding, and *b* the auxiliary winding, which in this case is cut out altogether after the motor is started. An auto-transformer (see p. 185) is shown in the figure.

In the specification of patent No. 24,098, filed December 1892, Mr. Brown describes a number of methods of starting monophase motors, including the methods using auxiliary windings with self-induction and capacity, and also including some methods which fall under the second class described above. In these the rotor is wound as a Gramme or Siemens armature, with connections to a commutator, just as in the case of a continuous-current machine. Two opposite points of the winding are also connected to two slip-rings. When it is required to start the motor a resistance is put between the brushes on these slip-rings, and the brushes on the commutator are placed so as to short-circuit a few of the windings of the rotor which lie obliquely across the direction of the alternating flux produced by the stator. The large current in the short-circuited coils makes them turn so as to become parallel to the alternating flux, and the brushes retaining their position, a continuous torque is produced. As the motor gets up speed the brushes may be drawn further apart, until diametrically opposite. The brushes on the slip-rings are then also short-circuited. Mr. Brown also describes some methods in which the alternating current from the mains is supplied to the rotor through the commutator for the purposes of starting.

Constancy of Speed.—As to the question of constant speed, we have seen that in a well designed motor the slip does not exceed more than 5 per cent. at full load, so the speed can only vary 3 or 4 per cent. between light load and full load. In a case cited by Dr. Louis Bell, an installation of 17 rotatory-field motors in Columbia, S. C., showed a maximum variation in speed from an output of 75 horse-power to friction load of the motors of only 2·2 per cent., individual motors showing slighter variations down to 1½ per cent. It is, however, possible to vary the speed of a polyphase motor at will by inserting a rheostat in the main circuit.

Efficiency.—The high efficiency of polyphase and monophase asynchronous motors will be seen from the following collection of data.

Mr. Kolben has supplied the table opposite relating to some three-phase induction motors built by the Oerlikon Company. They have no collector-rings or brushes, no resistance being used in the rotor circuit. All motors have a drum-winding with 3, 5, 7 or 11 phases short-circuited in themselves.

Mr. Kapp, in his book "Electric Transmission of Energy," gives some data of two three-phase motors tested on a brake by Mr. Kolben, which are reproduced in the table on p. 204.

RESULTS OF BRAKE TESTS ON STANDARD OERLIKON THREE-PHASE MOTORS.

	Type No.	351	352	353	354	355	356	357	358	359	360	361	362	363	364	365	366	367	368
	Effective horse-power at the pulley.	¼	½	¾	1½	2	3	4½	6	9	12	18	24	36	48	60	75	100	125
	Revolutions per minute running free	1500	1500	1500	1500	1500	1500	1500	1000	1000	1000	1000	1000	1000	750	750	600	600	500
	Revolutions per minute at half load.	1460	1470	1470	1475	1475	1480	1480	985	985	988	990	990	990	740	742	595	596	496
	Revolutions per minute at full load.	1410	1420	1435	1440	1445	1450	1450	960	960	970	970	970	970	725	730	585	588	488
	E.M.F. between terminals (star connection) volts	190	190	190	190	190	190	190	190	190	190	190	190	190	190	190	190	190	190
	Frequency	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
per branch	Current running free (amperes) ..	0·85	1·3	2·2	2·8	3·6	6	7·5	12	14	19	21	25	27	40	48	65	75	90
	Current at half load (amperes) ..	1	1·4	2·6	3·7	4·8	8	11	16	19	25	32	41	53	73	90	115	143	180
	Current at full load (amperes) ..	1·5	2·3	3·3	5·8	7·5	11	14·5	20·5	28	35·5	53	70	100	130	160	200	265	330
	Efficiency at full load (per cent.) ..	55	65	68	72	75	78	80	82	87	88	90	91	92	92	93	93	93	94
	Cosine of lag between E.M.F. and current (power factor)	0·67	0·75	0·75	0·8	0·8	0·8	0·86	0·8	0·82	0·85	0·85	0·85	0·88	0·9	0·9	0·9	0·91	0·91
	Number of poles..	4	4	4	4	4	4	4	6	6	6	6	6	6	8	8	10	10	12
	Current at starting with tight belt and light load (amperes)	4	6	11	16	30	36	55	65	75	80	85	90	100	105	112	120	150	220
		Switched on without resistance.				Started with common resistance in the primary circuit.								Special starting device with auto-transformer.					

KOLBEN'S TESTS OF THREE-PHASE MOTORS.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	
Metric horse-power at the brake of armature pulley.	Watts equivalent to effective horse-power 736 × effective horse-power.	Current in each branch Amperes.	Volts between the neutral and each terminal.	Apparent Watts 3 × volts × amperes.	Real Watts intake measured at motor terminals by Watt-meter.	Power Factor = cosine of angle of lag.	Per cent. slip at load 50 ~	Efficiency. $\frac{\text{Watts II.}}{\text{Watts VI.}}$	
60	44,160	318	60	57,240	48,300	0·844	2	0·91	Motor I. Theoretical speed 750. Squirrel cage armature. A.E.G. make.
42	30,910	252	60	45,360	36,800	0·81	1·3	0·84	
20	14,720	150	60	27,000	17,700	0·655	..	0·83	
0	0	125	60	22,500	..	..	..	..	
53·8	39,600	180	93	50,220	42,100	0·84	4	0·94	Motor II. Theoretical speed 750. Drum armature with 11 divisions, each division short-circuited in itself. Oerlikon make.
46	33,900	158	95	45,000	37,440	0·83	3	0·905	
0	0	40	98	11,760	1,710	0·145	0	..	

The following are particulars of three-phase motors of various sizes built by the Allgemeine Elektricitäts-Gesellschaft:—

Types.	D. R. 1.	D. R. 5.	D. R. 10.	D. R. 50.	D. R. 100
Normal horse-power	$\frac{1}{8}$	$\frac{1}{2}$	1	2	50
Number of poles	2	4	4	4	8
Weight in kilogrammes	18	65	94	245	1200
Current in each circuit at starting	..	..	20	50	400
Current in each circuit at full load	1·4	4	8	36	280
Current in each circuit at no load	..	..	4·5	15	150
Kilowatts absorbed at full load..	0·23	0·52	0·985	4·38	40·2
Revolutions per minute at full load	2300	1400	1375	1395	725
Ditto at no load	2380	1490	1490	1490	745
Ditto of field	3000	1500	1500	1500	750
Slip-page at full load	23°/₀	8°/₀	66°/₀	7°/₀	3·3°/₀
Commercial efficiency..	..	0·71	0·75	0·84	0·91
Torque in *m k g*	..	..	0·52	2·6	49·4

The following interesting comparison between an 80 horse-power synchronous alternate-current motor installed in a corn mill, and a 100 horse-power asynchronous motor driving a spinning mill, was made by Mr. Kolben. The results are shown graphically in Figs. 165 and 166.

The synchronous motor is of the Kapp type, with a flat disk armature, constructed by the Maschinen-Fabrik Oerlikon. It is wound for 2000 volts, with its shaft direct-coupled to its exciter. The E.M.F. curve is almost a sine curve. The asynchronous motor shown in Fig. 167 is an 18-pole high-pressure three-phase motor built by the same firm, for 1730 volts pressure at 50 periods per second, and arranged for rope driving at the low speed of 320 revolutions a minute.

It will be seen from the curves that the power-factor of the synchronous motor is more favorable for all loads than that of the asynchronous motor, although the difference at full load is small, being 0·94 as compared with 0·86. The difference, for instance, with a loss on the line of 5 per cent., would scarcely bring about ½ per cent. additional drop in the conductor. On the other hand the efficiency of the

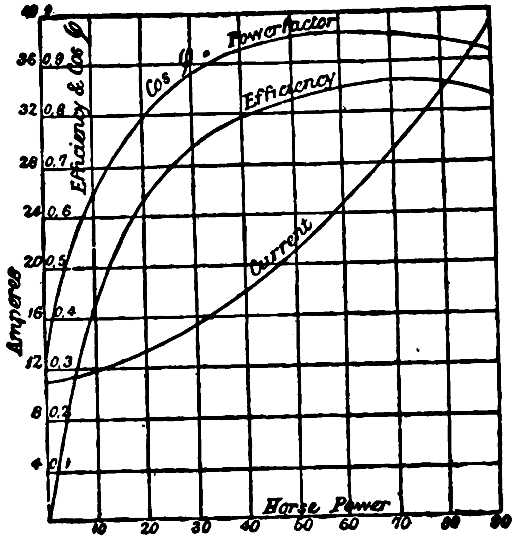

FIG. 165.—SYNCHRONOUS MOTOR.

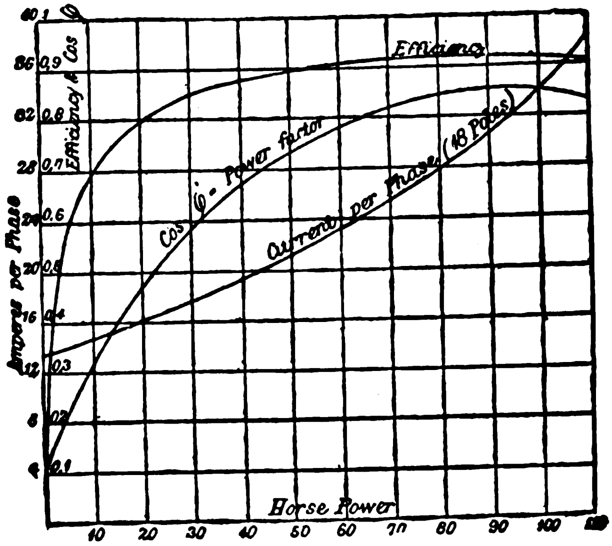

FIG. 166.—THREE-PHASE MOTOR.

asynchronous motor is higher at all loads; its curve is similar to those of a good transformer: it amounts at full load to 91 per cent., as compared with 86 per cent. of the synchronous motor. The total exciting energy, including losses in the

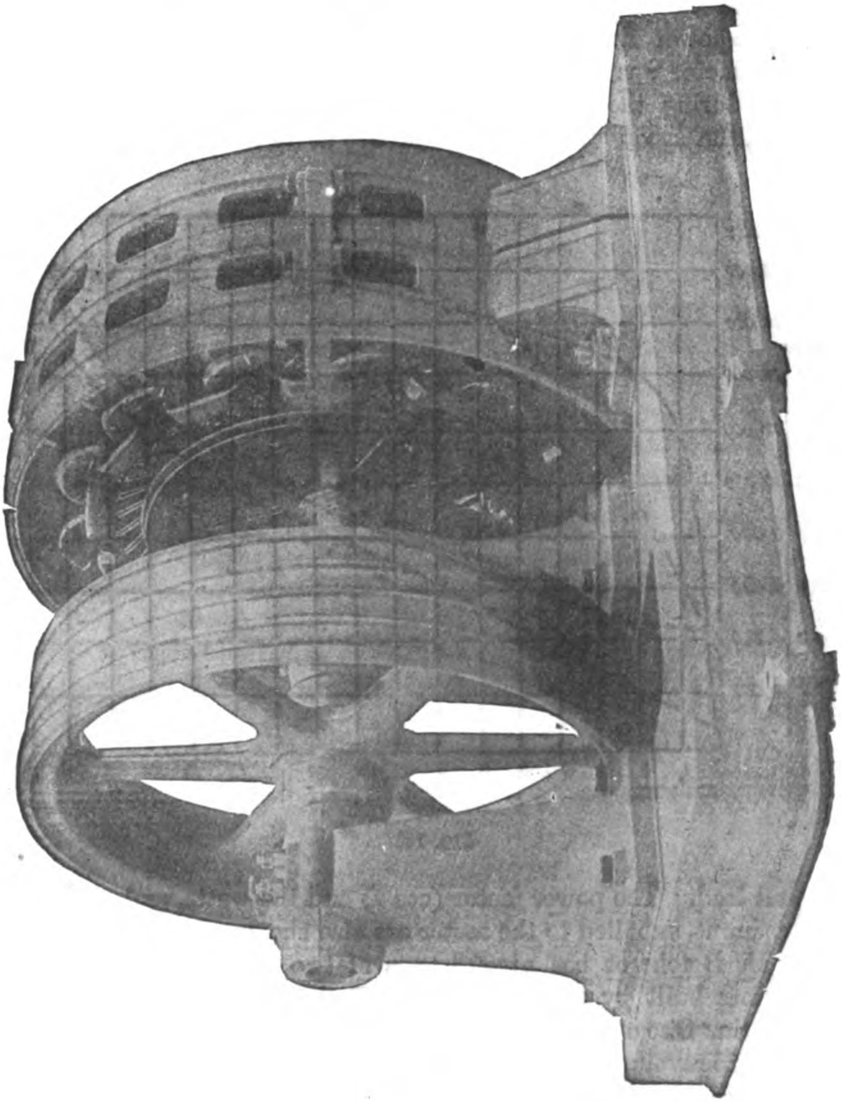

Fig. 167.—18 Pole High-pressure Three-phase Motor.

exciter, is comprised in this, and, especially at small loads, reduces the efficiency.

The efficiency of a monophase asynchronous 6-pole 15 horse-power Brown motor, was tested by Ricardo Arno.[1] The motor was built for a frequency of 40 cycles per second, but during the test the frequency was a little higher than this, the speed varying between 876 at no load, and 850 at full load. The power factor (cos φ) and the watts, real and apparent, supplied to the motor are also shown.

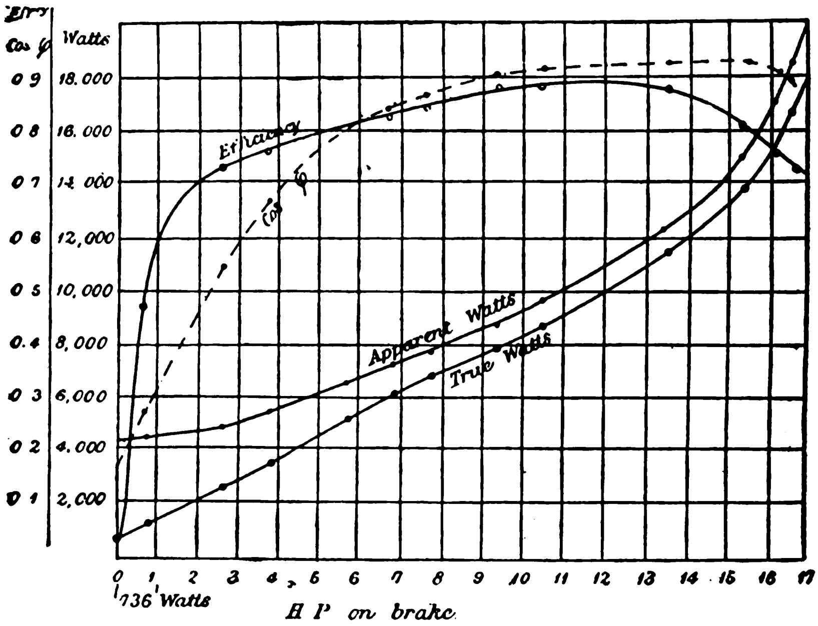

FIG. 168.

M. Botcherot has given[2] tests of two Brown two-phase motors built by the Weyher and Richemond Company at Pantin. One of these, of 2–3 horse-power, weighing 120 kilos., at 1125 revolutions per minute its efficiency was 76 per cent. A larger motor, of 17–20 horse-power, weighing 520 kilos.,

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[1] *L'Elettricista*, iii. No. 7, p. 149.

[2] *Bulletin de la Société Internationale des Électriciens*, xi. 482, Dec. 1894.

had an efficiency of 90 per cent. at 770 revolutions per minute.

A 50 horse-power Tesla 2-phase motor, built[1] and tested at the works of the Westinghouse Company, ran at 750 revolutions per minute, on 220 volts mains with frequency of 25 periods. The speed falls only 2 per cent. from no load to full load. The efficiency is 84 at quarter load, and 89·5 at full load. The maximum starting torque is 2·5 times the torque at full load; or, with a resistance in the secondary, 1·5 times. As the light-load efficiency is so high, this would be a most economical motor for all-day work on a variable demand for power. When running unloaded there is a nearly watt-less current of 62 amperes.

Dr. Louis Bell, in his paper mentioned above, gives the following data as to weights per horse-power of rotatory field motors.

Horse-power.	Weight in lbs. per horse-power.
5	103
10	66
15	68
20	73 6-pole
100	66 8-pole

The following figures give the relation between weight and horse-power of standard motors of European make :—

Horse-power.	Weight in lbs. per horse-power.
2	120
6	100
13	88
50	70
70	66
100	58

These weights compare extremely well with those of continuous-current motors; an ordinary 100 horse-power continuous-current motor seldom weighing less than 80 lbs. per horse-power.

Mr. Kapp has pointed out that as between a 2-phase motor and a 3-phase motor, the plant-efficiency of the latter is the better, its output being 111 as against 100 for a 2-phase motor of the same weight.

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[1] *Electricity*, (U.S.A.), May 15, 1895.

CHAPTER XIV.

SOME EXAMPLES OF MODERN POLYPHASE MOTORS.

By the kindness of two of the firms which have been foremost in the development of the polyphase motors, the author is enabled to describe several recent examples of this class of machine.

Motors of the Oerlikon Machine Company (Zürich).—From the autumn of 1891 the Oerlikon Machine Company has continued to develop the rotatory field motor, and has made many hundreds of different sizes. In all small sizes, whether 3-phase, 2-phase or monophase, the rotor is of the simple squirrel-cage construction, while for the larger sizes wound rotors are used so as to enable resistances to be inserted at starting. So far back as July 1892, the engineers of this firm had succeeded by detailed improvements of construction in producing a 3 horse-power, 4-pole, 3-phase motor, with an efficiency of 71 per cent. The firm has adopted a standard frequency of 50 periods per second in its machines. The larger motors are generally arranged to start light on a loose pulley to avoid any great rush of current, since a high efficiency at full load, with small percentage of slip, involves a small starting torque; but for cranes, elevators and the like, special motors are made (also without slip-rings or brushes) with a slip (at full load) of as much as 12 per cent. Their power-factor is consequently low, but they start with an initial torque equal to two or three times their torque at normal load. Some data of a crane motor were given on p. 198.

A reference to this firm's monophase motors, and the starting-gear used for them were also given above.

The Oerlikon Company's own works are driven by electric power transmitted some 14½ miles from a waterfall at Hochfelden, near Bulach. The 3-phase machines by which this is accomplished were the first of their kind. There are three 3-phase generators, each of 200 horse-power. They are depicted in Fig. 44, p. 38. They were designed by Mr. C. E. L. Brown, in the autumn of 1890, at the same time as the machine used in the famous Frankfort experiment of 1891. There is a similar transmission of 300 horse-power in Zürich from a waterfall at Killwangen, 12½ miles distant. The power is distributed by overhead lines to numerous small motors. At St. Etienne, in France, there is a similar transmission of 1000 horse-power, and at Wangen, in Würtemberg, of about 350 horse-power.

Motors of MM. Brown, Boveri & Co.—Some mention was made on p. 118 of the earlier work of Mr. C. E. L. Brown. His firm has, since 1892, turned out a large amount of polyphase plant, including generators and motors. By the courtesy of the firm several motors of modern design are here described in considerable detail.

In Plate I. is shown, one-sixth full size, the elevation of a 2-phase motor capable of yielding 6 horse-power. A sectional elevation is also shown in the plate, and the rotor and stator stampings are shown in Fig. 105. The winding of the stator is carried out in the manner described on p. 35, with the ends of the coils alternately bent sideways and arched over. In this particular motor, which is intended for a pressure of 100 volts and a frequency of 40 periods per second, there are 9 wires of 3·8 mm. diameter passed through each of the 40 slots in the stator. There are 37 round copper rods on the rotor, each 9 mm. in diameter, all short-circuited together at each end by a broad copper hoop, which, besides serving as a good conductor, presents a large cooling surface to the air. The air gap between rotor and stator is only 0·5 mm. in breadth. The maximum value of B in the iron between the slots is 11,500, and in the iron behind the slots 7500. The same carcase can be wound as a single-phase motor, and will then yield 4 horse-power. It will be seen from the drawings that the bearings are of the self-oiling type. The following are some of the principal dimensions:

	Centimetres.
Diameter of rotor	24·9
Inside diameter of stator	25
Radial depth of stator	7
Breadth of stator face	11·5
Radial depth of slots	2·5
Width of slots	1
Average width of iron between slots	1
Diameter of holes in rotor	1
Diameter of rods in rotor	0·9
Area of rods of rotor, in sq. cm.	0·64
Area of conductor in stator, in sq. cm.	0·13
Number of conductors per hole	9

Plate II. and Fig. 169 relate to a form of motor which has been constructed with different windings for different purposes. In the plate the windings are those of a 3-phase motor taking current directly from high-pressure mains at 5000 volts, with a frequency of 40 periods per second and a speed of 600 revolutions per minute. Its output is then 100 horse-power. Its height is 120 cm. or about 4 feet, and its length from

FIG. 169.—BROWN'S TWO-PHASE MOTOR OF 120 HORSE-POWER.

outside of bearings is a little less. The diameter of the rotor is 75 cms., and its length, parallel to the shaft, is under 45 cms. The rotor has 96 holes through which insulated copper conductors are threaded, to be joined up in a wave-winding constituting a three-branched star, of which the three outer ends are led down through the central hole bored in the shaft to three slip-rings, so as to allow of an external starting resistance being applied. There are 48 holes in the stator core-rings, and through these the coils are wound, being protected by strong tubes of prepared paper. The mode of arrangement of these coils to produce a 4-pole field is shown in Fig. 40. This motor starts under full load, taking less than full load current.

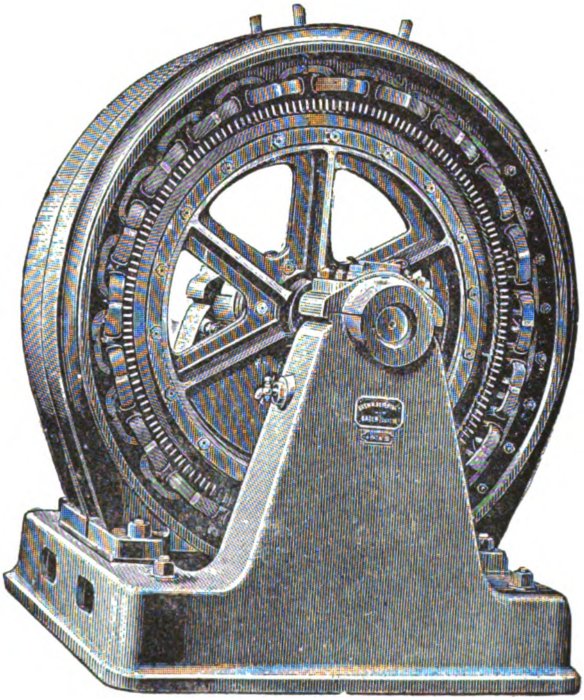

FIG. 170.—BROWN'S SLOW-SPEED TWO-PHASE MOTOR OF 100 HORSE-POWER.

The same framework, wound as a 2-phase machine at 2000 volts, is shown in Fig. 169. It has an output of 120 horse-power. The starting resistance in this case is placed inside the rotor, with a simple mechanism projecting out through the end of the shaft to short-circuit it when the motor has started. This device is seen on the right-hand end of the shaft in Fig. 169.

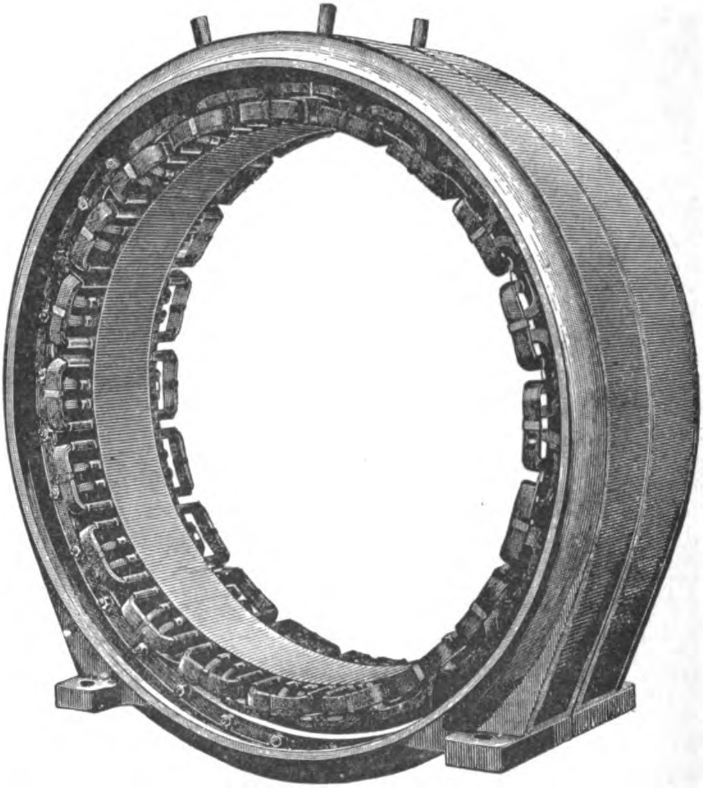

FIG. 171.—STATOR OF BROWN'S SLOW-SPEED TWO-PHASE MOTOR.

In Figs. 170 and 171, is shown another 100 horse-power 2-phase motor of different design, built by the same firm for running at a slower speed. This motor, supplied at a pressure of 2000 volts (and frequency of 38 periods per second), runs at a speed of 200 revolutions per minute. Tested on a brake. it gave up to 200 horse-power before stopping. The stator coils, and the way in which the end connections are arranged, can be seen from Fig. 171. The plan of winding is the same as that shown in Fig. 124, there being 28 turns in each coil. There are 40 coils (20 in each phase-circuit) threaded through 80 holes. In the rotor there are 180 conductors connected in three circuits, the ends of which are brought to slip-rings for the purpose of introducing resistance at starting. Motors of the same type are made for ordinary monophase supply, and at a pressure of 2700 volts will yield 120 horse-power at 300 revolutions per minute.

Brown's motors are now extensively used for distribution of power in factories, the 3-phase current being particularly applicable to isolated plant of this description. The large installation at Schönenwert, near Aarau, has been referred to on p. 37.

Another example is furnished by the 2-phase distribution in the extensive engine-works of Weyher and Richemond at Pantin, near Paris. In these works there were formerly three separate steam-engines of 120, 80 and 50 horse-power respectively. These have now been replaced by a single horizontal engine of 200 horse-power (capable of working up to 400 horse-power), at 60 revolutions per minute. This engine drives three 2-phase generators, each of 88 kilowatt capacity, having revolving drum-wound armatures aud stationary 8-pole field-magnets. The frequency is 40 periods per second. Usually only two of these generators are run, the third being a reserve machine. Down to the present time the number of motors installed in the different shops is 17, having a total output of 119 kilowatts, or about 150 horse-power. The outputs of these are as follows:—1 of 33 kilowatts, used for coal hoisting, 2 of 22 kilowatts, 1 of 14·5, 1 of 9·5, 1 of 5·8, the rest of 2 kilowatts or under. Two still larger motors are now in course of construction. According to M. Boucherot, who has given [1] a full account of the plant, with views of the shops and machinery, the efficiency of the larger motors is 94

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[1] *Bulletin de la Société Internationale des Électriciens*, xi. 482, Dec. 1894.

per cent., that of the smallest (1·1 kilowatt) 74 per cent.; the average efficiency of the motors taken all together being over 89 per cent. M. Boucherot considers these machines to contrast most favorably with continuous-current machines of equal power. The 2-phase motors, for equal efficiency, cost no more (including starting gear) than continuous-current motors, and run at a slower speed. The generators, for an equal efficiency, cost some 15 per cent. less than continuous-current dynamos of equal output.

In Berlin the Allgemeine Elektricitäts-Gessellschaft has developed the 3-phase motor for many purposes, notably for driving machine tools, centrifugal machinery and elevators. They have made a speciality of centrifugals in sizes varying from 1 to 7 horse-power. The largest are used in bread factories, whilst the smaller are employed in sugar refineries. For example, the sugar refinery of the firm of P. Schwenger's Söhne, at Uerdingen on the Rhine, is fitted up throughout with electric motors to the number of 91, employing a total of about 490 horse-power. At the Breitenburger Cement Works at Lägerdorf are two 3-phase generators, each of 110 horse-power, for transmitting power to elevators, pumps, stampers, and the like. The machine works of Colomna, in Russia, has a 3-phase plant of 600 horse-power for driving the machine-tools and cranes in its shops. Of the three-phase machines made by the Allgemeine Co. for the new central station at Strassburg some mention is made in the next chapter.

The machinery in the workshops of the Westinghouse Co. at Pittsburg, U.S.A.,[1] is driven by Tesla 2-phase motors; the installation, consisting of 39 motors varying between 10 and 80 horse-power, having an aggregate of nearly 800 horse-power, Sixteen larger motors will shortly be added, increasing the capacity to double. The generators are of a new type, superior to that shown in Fig. 39, both circuits being connected to one winding after the manner shown in Fig. 96. The lighting of the shops is from the 2-phase circuits.

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[1] *Electricity* (U. S. A.) vol. viii. 169, 185 (1895). See also the same journal, May 15, 1895, for efficency test of one the Tesla motors.

CHAPTER XV.

DISTRIBUTION OF POLYPHASE CURRENTS FROM CENTRAL STATIONS.

FOR the mere purpose of electric lighting, there is no great advantage in the use of polyphase currents as distinguished from ordinary single-phase alternate currents. But where other purposes are contemplated in an electric supply, particularly the distribution of electric power, then the advantages of polyphase systems begin to appear.

For many months the only example of a general distribution of polyphase currents from a central station was that of the town of Heilbronn, which derives its 3-phase supply from the generating station at Lauffen, on the river Neckar, about 9 miles distant. The engineer who laid out the system is Mr. Oskar von Miller, of Munich, by whose courtesy the following information is supplied.

The generators at Lauffen, the same that were used in the famous Frankfort transmission (Fig. 30), give each about 4000 amperes at 50 volts. By a step-up transformer this supply is transformed into a current of 40 amperes at 5000 volts, at which high pressure the currents are transmitted through three copper wires 6 millimetres in diameter, carried overhead on oil-insulators supported on timber poles. At Heilbronn these three currents are received by a step-down transformer, and transformed to about 133 amperes at the intermediate pressure of 1500 volts, at which pressure the currents are distributed to the various quarters of the town. As a matter of fact there are three turbines (one for reserve), two generators, two step-up transformers, and two step-down transformers. The final transformation at Heilbronn, from 1500 volts to 100 volts, is accomplished by small transformers of 5 kilowatts and 10 kilowatts capacity, which are placed in overground transformer-pillars at about twenty-five convenient points, and feed the low-voltage network that distributes the current to the lamps and motors of the consumers. Triple-concentric armored cables distribute current over about 5 miles of streets. Arc lamps and glow lamps are used upon all three circuits, as well as motors. Down to the end of 1894 there were the equivalent of 11,000 8 candle-power lamps upon the circuits, and 25 motors having a total output of about 53 horse-power. The small motors up to about 3 horse-power are arranged to be switched direct on to the circuits without any special starting gear. They are of the usual 3-phase type with squirrel-cage rotors. The larger motors up to 8 horse-power are provided with starting-gear, including liquid resistances, so that the full current is not taken until some 15 or 20 seconds have elapsed, after which time they have got up their speed and are then switched over direct on to the mains. About half-way between Lauffen and Heilbronn, at the hamlet of Sontheim, a few glow lamps in the streets are supplied by a transformer working direct from 5000 volts down to 100 volts. No trouble is found to arise in the maintenance of proper regulation of the voltage in the three circuits. The motors tend to equalize the pressures and currents between the three circuits, though the numbers of lamps may be unequal.

Amongst other polyphase stations at work are those of Dresden Railway Station, Chemnitz, Buda-Pesth, Strassburg, and Bockenheim.

At Chemnitz, a municipal central station was equipped by Messrs. Siemens and Halske, in 1894, with a 3-phase system. The generators, of the "R" type, have an output of 52 amperes at 2000 volts. They have an outer fixed armature built up of core-rings, and an interior revolving star-shaped field-magnet with 40 poles of alternate kinds. At 150 revolutions per minute the frequency of the alternations is 50 periods per second. The inner periphery of the armature ring is slotted with 120 slots, or 3 slots per pole, to receive the windings. The slots are narrowed at their mouth to receive wooden keys to hold in the windings. The windings are arranged as in Fig. 41, with their bent exterior portions in two planes, and all the coils of each phase are joined in series. One end of each of the three series is carried to a common junction, and the other three ends are brought out to three separate terminals on the machine. The winding is therefore a star winding (Fig. 58). One auxiliary coil on each generator furnishes a current to the synchronizing apparatus at 25 volts. There are three generators, each direct-driven by a triple-expansion condensing engine. If the excitation of the field-magnets is kept constant, the drop of voltage at full load on a non-inductive resistance is about 7 per cent. ; but when motors are being run on the circuit a much greater drop is liable to occur, requiring to compensate it an increase of 20 to 30 per cent. in the exciting current. From the generators the currents are led through fuses and switches and measuring instruments to three omnibus bars on the switchboard, from which the high-pressure feeders go to the distributing mains. Triple-concentric cables, lead-covered and armored, conduct the currents at 2000 volts to transformers dotted about the town at twenty-four different points, where they are transformed to 120 volts for the low-pressure networks. There are about 6 miles of high-pressure cable, 12 miles of triple-concentric low-pressure cable, and about 4 miles of bare conductors. The furthest point is about 2 miles from the generating station, which is itself about 1 mile from the centre of the town. By the end of 1894 there were in use the equivalent of about 11,000 8 candle-power lamps, 160 arc lamps, and 30 motors of an average size of 2 horse-power. The motors have as stator a ring wound with coils in slots, similarly with the generators. The rotor is built up of iron core rings slotted on the external periphery, wound with coils, which are also joined up in a star grouping, and end at three slip-rings. These permit resistances to be inserted at starting and gradually cut out as the motor acquires speed. At full speed the rotor is short-circuited. A full description, with drawings, is given in the *Elektrotechnische Zeitschrift*, February, 1895.

For the city of Strassburg, of which Mr. Oskar von Miller is engineer, a 3-phase system has been adopted. The generators, of the "inductor" type, were built by the Allgemeine Elektricitäts-Gesellschaft of Berlin, and are of 400 horse-power each. These machines are described by Mr. von Dolivo-Dobrowolsky in the *Elektrotechnische Zeitschrift* of February 7, 1895.

At Bockenheim, an important suburb of Frankfort, is a 3-phase station equipped by Messrs. W. Lahmeyer & Co. There are two direct-driven 3-phase alternators of about 150 kilowatts each, of the same type as those used at Lauffen (p. 28), having fixed armatures and revolving field-magnets, but with only eight poles. They work at 80 volts, and their currents are at once led into 3-phase transformers resembling Fig. 153, to be transformed to 660 volts, at which high pressure they are conveyed by cables to the various distributing points. For motor-driving the 3-phase motors are connected direct to the high-pressure mains. For lighting, transformers are interposed which reduce the pressure, and feed into a distributing network. Triple-concentric cables bring the currents into the houses. Asynchronous motors up to 20 horse-power are in use. They start under load, or even with an over-load. Those over 8 horse-power are, however, provided with loose pulleys so as to start on a light load. Water-resistances are used in the starting gear. Aron's polyphase meters are used. There is an independent continuous current supply in use for lighting only. Altogether the total consumption of power for motor purposes exceeds 200 horse-power.

The regulation of voltage in the three circuits is not found to present any difficulty. If a star-grouping (like Fig. 67) is adopted in a 3-phase distribution, the fourth wire, which is brought back to the common junction of the three circuits of the generator, serves to equalize the pressures in case the numbers of lamps in the three branches are unequal. But in discussing the Chemnitz distribution it has been shown by Mr. Görges that this is not necessary. A 3-phase equalizer can be added at some convenient point in the network, in the form of a 3-phase transformer, each limb of which is wound, however, with but a single coil. It is, in fact, a three-phase choking-coil or auto-transformer. These three coils are united in star-fashion, and the fourth wire of the circuits is brought to their common junction. Mr. Görges states in one experiment 100 lamps were introduced in one of the three circuits, 20 in the second, and 1 lamp in the third, causing a very great inequality in the three voltages, until the fourth wire was joined to the mid-point of the equalizer, when at once the voltages became all alike. Three-phase motors inserted in the circuit have similarly an equalizing effect on the three voltages. Some years ago Mr. von Dolivo-Dobrowolsky noted the fact that 3-phase transformers also have a similar equalizing effect.

At Buda-Pesth Messrs. Schuckert have a 2-phase station outside the city, as mentioned on p. 185.

A 2-phase station has now been at work at Pittsfield (Massachusetts) for about two years, on the system of Messrs. Stanley, Kelley and Chesney. Another is in progress of development at Montreal.

A list of 3-phase power stations which have been equipped by the Oerlikon Co. is given in the Table on p. 222.

In the above enumeration no mention has been made of a very large number of isolated plants, in factories and the like, where polyphase methods seem likely to supersede all other modes of transmitting and distributing power. In addition to those mentioned on p. 211 as having been carried out by the Oerlikon Co., and those of Brown, Boveri & Co. on p. 215, it may be noted that in Hellsjön, Sweden, there is a 400 horse-power transmission[1] of 3-phase currents over 8 miles, by machinery designed by the late Mr. Wenström.

The "*Monocyclic*" *System.*—Mr. C. P. Steinmetz has proposed[2] a system of distributing electricity for lighting and power which, although its essence is that more than one cycle or phase is used, is called the "monocyclic" system. By this

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[1] See G. Kapp's 'Electric Transmission of Energy,' p. 418, 4th edition, 1894.

[2] See *Electrical World*, xxv. p. 182, Feb. 1895; also E. Boistel, "Distribution Monocyclique," *L'Éclairage Électrique*, iii. p. 152, April, 1895.

Name of Station.	Horse-power of Units.	Present total capacity of Station in Horse-power.	Distance of transmission in kilometres.	Line Pressure in volts.	Purpose.
Lauffen-Heilbronn	300	600	10	5,000	Distribution for light and power.
Dietikon-Zürich	300	300	15	5,000	“ power.
Hochfelden-Oerlikon	300	900	25	13,000	“ “
Pergine, Tyrol	100 200	300	2	1,800	“ light and power.
Wangen, Würtemberg ..	100	200	11	5,000	“ “
St. Étienne, France	300	600	6–15	5,000	“ “
Florensac, France..	100	200	..	5,000	“ “
Triberg, Germany	150	300	15	5,000	“ “
Bellegarde, France	600	600	0·8	1,000	“ power.
Bremgarten-Zürich	325	1300	18	5,000	“ “
Innsbrück, Tyrol	100	200	2	1,800	“ light and power.
Toledo, Spain..	200	200	..	3,500	“ “
Blies-Schweigen, Germany.	175	350	4	5,000	“ “
Riva, Tyrol	150	450	..	3,500	“ light.

system it is sought to obtain the advantages of a polyphase system for motor purposes, combined with the ease of regulation of a single-phase system. Under normal conditions the whole of the power is supplied by two mains, between which is maintained a constant alternating pressure. To those parts of the district where motive power is required, a third wire is carried, from which a current can be drawn, differing in phase from the main current, and by which motors can be started. The windings of the motor are arranged so that when full speed is attained the back E.M.F. is as great as the pressure from the third wire, so that no more current is drawn from it, the power being supplied by the principal mains. One of the ways of maintaining the difference of phase in the third wire is to wind on the alternator coils displaced in relation to the main coils, so as to generate an E.M.F. differing by 90° from principal E.M.F. One of the terminals of these displaced coils is joined to a middle point of the principal winding of the alternator, and the other to the third wire in question. The number of turns of the displaced coils is so arranged that the E.M.F. resulting from them and the half of the principal coils in series with them shall have the required difference in phase. Where a number of motors are installed, the back E.M.F. of any motor that happens to be running will be sufficient to start another motor, so that in cases where some of the motors are always running it will not be necessary to carry the third wire to the alternator.

The monocyclic system is therefore really a three-phase distribution in which two of the phases are nearly in opposition, while the third phase, nearly at right angles to the other two, is used as an auxiliary for starting motors. It cannot be maintained that this system, because it is not symmetrical, is not a polyphase system.

The author has devised several other ways of attaining a similar end. One method is to use two alternate currents at any phase-difference whatever between 90° and 120°, on an ordinary three-wire system of distribution, keeping constant the difference of potential between each and the middle wire. For motor-starting a third circuit is obtained from the two outer wires, which will yield a current out of phase with the other two. The exact phase and exact potential of this third current is immaterial: the motors used may be either 3-phase, or 2-phase with a common return.

Another unsymmetrical system has been proposed by Mr. Imhoff.

Where ordinary single-phase currents are supplied on a three-wire system, as in the City of London, it is exceedingly easy to introduce a phase-difference into the middle wire, enabling motors to be started and run. In fact, if a single 3-phase motor is started on any such circuit, it will help other motors on the same circuit to start, as it will itself tend to preserve the requisite differences of phase.

An American example of 3-phase lighting, at Concord (N. H.) is described in the 'Western Electrician' of Feb. 16, 1895, and another at Winooski (Vt.) in the 'Electrical Engineer,' June, 1895.

At Baltimore there is now a 2-phase station furnished with four Westinghouse alternators, each of about 1000 kilowatts output. They are direct-driven, and will be largely used for arc lighting as well as for glow-lamps. Tesla two-phase motors will be used.

The largest plant in the world for the supply of electric currents is a polyphase plant—namely, that established at Niagara (see p. 39)—using a 2-phase system of currents. It will begin operations at an early date.

Recent as have been these inventions and rapid as their development has proved, it is already evident that their industrial aspect is settling down upon well-defined lines. Yet finality is far from having been attained. The problems of the conversion of electric currents from the alternating to the continuous varieties are still in progress of solution. What results the newest methods of transformation may bring about none can foresee. The next few years may witness developments at present quite beyond the contemplation of the electric engineer.

CHAPTER XVI.

ELECTRICAL COMMUNICATION WITH THE PLANETS.

By NIKOLA TESLA.

THE idea of communicating with the inhabitants of other worlds is an old one. But for ages it has been regarded merely as a poet's dream, forever unrealizable. And yet, with the invention and perfection of the telescope and the ever-widening knowledge of the heavens, its hold upon our imaginations has been increased, and the scientific achievements during the latter part of the nineteenth century, together with the development of the tendency toward the nature ideal of Goethe, have intensified it to such a degree that it seems as if it were destined to become the dominating idea of the century that has just begun. The desire to know something of our neighbors in the immense depths of space does not spring from idle curiosity nor from thirst for knowledge, but from a deeper cause, and it is a feeling firmly rooted in the heart of every human being capable of thinking at all.

Whence, then, does it come? Who knows? Who can assign limits to the subtlety of nature's influences? Perhaps, if we could clearly perceive all the intricate mechanism of the glorious spectacle that is continually unfolding before us, and could, also, trace this desire to its distant origin, we might find it in the sorrowful vibrations of the earth which began when it parted from its celestial parent.

But in this age of reason it is not astonishing to find persons who scoff at the very thought of effecting communication with a planet. First of all, the argument is made that there is only a small probability of other planets being inhabited at all. This argument has never appealed to me. In the solar system, there seem to be only two planets—Venus and Mars—capable of sustaining life such as ours;

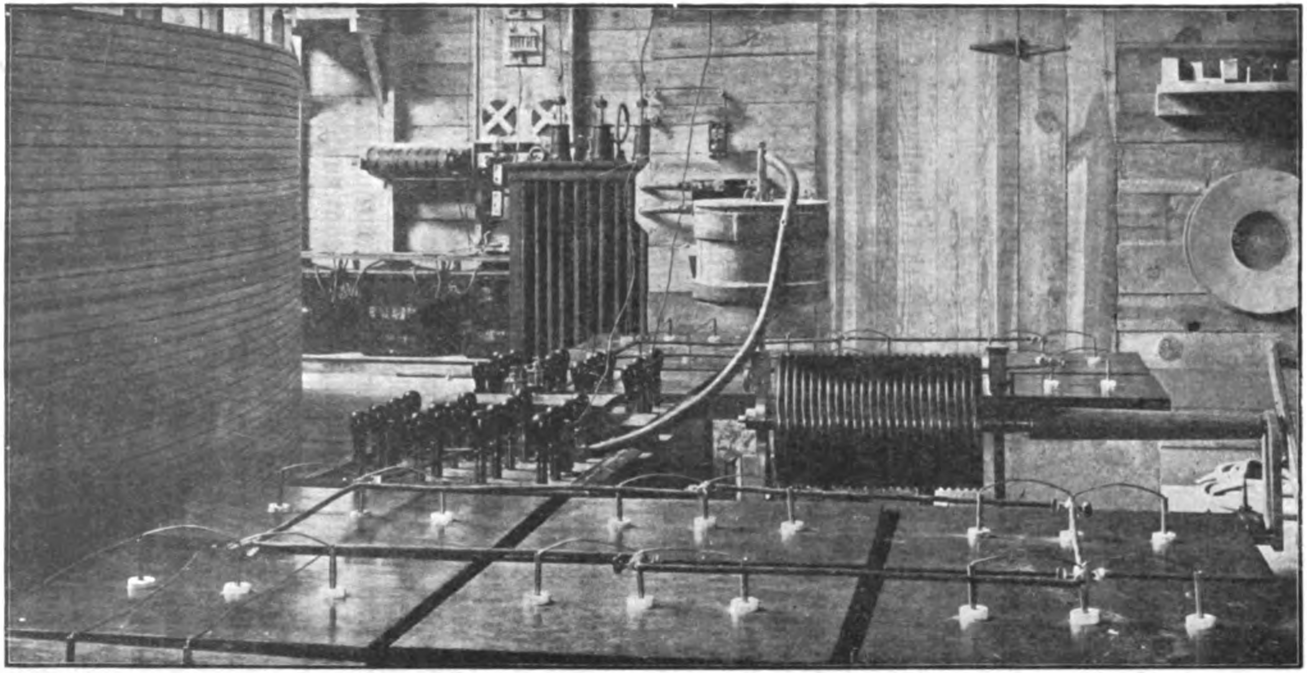

Fig. 172.—Photographic View of the Essential Parts of the Electrical Oscillator Used.

but this does not mean that there might not be on all of them some other forms of life. Chemical processes may be maintained without the aid of oxygen, and it is still a question whether chemical processes are absolutely necessary for the sustenance of organized beings. My idea is that the development of life must lead to forms of existence that will be possible without nourishment and which will not be shackled by consequent limitations. Why should a living being not be able to obtain all the energy it needs for the performance of its life-functions from the environment, instead of through consumption of food, and transforming, by a complicated process, the energy of chemical combinations into life-sustaining energy?

If there were such beings on one of the planets we should know next to nothing about them. Nor is it necessary to go so far in our assumptions, for we can readily conceive that, in the same degree as the atmosphere diminishes in density, moisture disappears and the planet freezes up, organic life might also undergo corresponding modifications, leading finally to forms which, according to our present ideas of life, are impossible. I will readily admit, of course, that if there should be a sudden catastrophe of any kind all life processes might be arrested; but if the changes, no matter how great, should be gradual, and occupied ages, so that the ultimate results could be intelligently foreseen, I cannot but think that reasoning beings would still find means of existence. They would adapt themselves to their constantly changing environment. So I think it quite possible that in a frozen planet, such as our moon is supposed to be, intelligent beings may still dwell, in its interior, if not on its surface.

Then it is contended that it is beyond human power and ingenuity to convey signals to the almost inconceivable distances of fifty million or one hundred million miles. This might have been a valid argument formerly. It is not so now. Most of those who are enthusiastic upon the subject of interplanetary communication have reposed their faith in the light-ray as the best possible medium of such communication. True, waves of light, owing to their immense rapidity of succession, can penetrate space more readily than waves less rapid, but a simple consideration will show that by their means an exchange of signals between this earth and its companions in the solar system is, at least now, impossible. By way of illustration, let us suppose that a square mile of the earth's surface—the smallest area that might possibly be within reach of the best telescopic vision of other worlds—were covered with incandescent lamps, packed closely together so as to form, when illuminated, a continuous sheet of light. It would require not less than one hundred million horse-power to light this area of lamps, and this is many times the amount of motive power now in the service of man throughout the world.

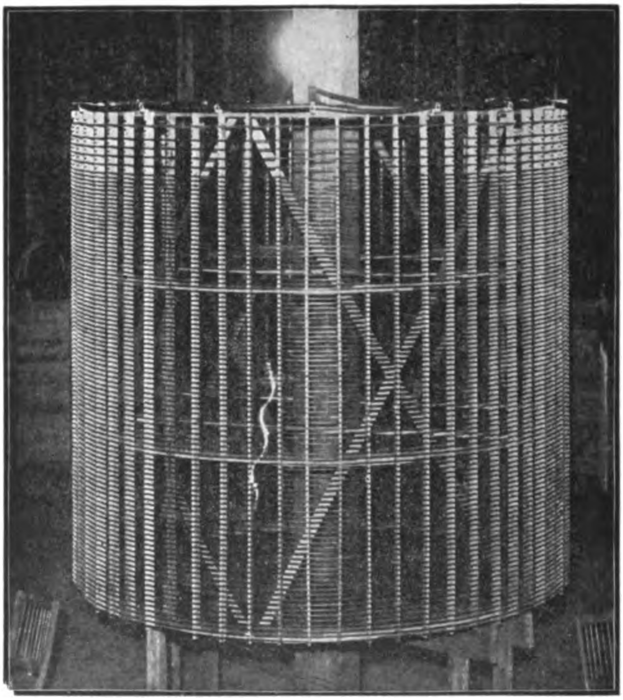

FIG. 173.—SUPPLYING ELECTRICAL ENERGY THROUGH A SINGLE WIRE WITHOUT RETURN

But with the novel means, proposed by myself, I can readily demonstrate that, with an expenditure not exceeding two thousand horse-power, signals can be transmitted to a planet such as Mars with as much exactness and certitude as we now send messages by wire from New York to Philadelphia. These means are the result of long-continued experiment and gradual improvement.

Some ten years ago, I recognized the fact that to convey electric currents to a distance it was not at all necessary to employ a return wire, but that any amount of energy might be transmitted by using a single wire. I illustrated this principle by numerous experiments, which, at that time, excited considerable attention among scientific men.

This being practically demonstrated, my next step was to use the earth itself as the medium for conducting the currents, thus dispensing with wires and all other artificial conductors. So I was led to the development of a system of energy transmission and of telegraphy without the use of wires, which I described in 1893. The difficulties I encountered at first in the transmission of currents through the earth were very great. At that time I had at hand only ordinary apparatus, which I found to be ineffective, and I concentrated my attention immediately upon perfecting machines for this special purpose. This work consumed a number of years, but I finally vanquished all difficulties and succeeded in producing a machine which, to explain its operation in plain language, resembled a pump in its action, drawing electricity from the earth and driving it back into the same at an enormous rate, thus creating ripples or disturbances which, spreading through the earth as through a wire, could be detected at great distances by carefully attuned receiving circuits. In this manner I was able to transmit to a distance, not only feeble effects for purposes of signalling, but considerable amounts of enery, and later discoveries I made convinced me that I shall ultimately succeed in conveying power without wires, for industrial purposes, with high economy, and to any distance, however great.

To develop these inventions further, I went to Colorado in 1899, where I continued my investigations along these and other lines, one of which in particular I now consider of even greater importance than the transmission of power without wires. I constructed a laboratory in the neighborhood of Pike's Peak. The conditions in the pure air of the Colorado Mountains proved extremely favorable for my experiments, and the results were most gratifying to me. I found that I could not only accomplish more work, physically and mentally, than I could in New York, but that electrical effects and changes were more readily and distinctly perceived. A few years ago it was virtually impossible to produce electrical sparks twenty or thirty feet long; but I

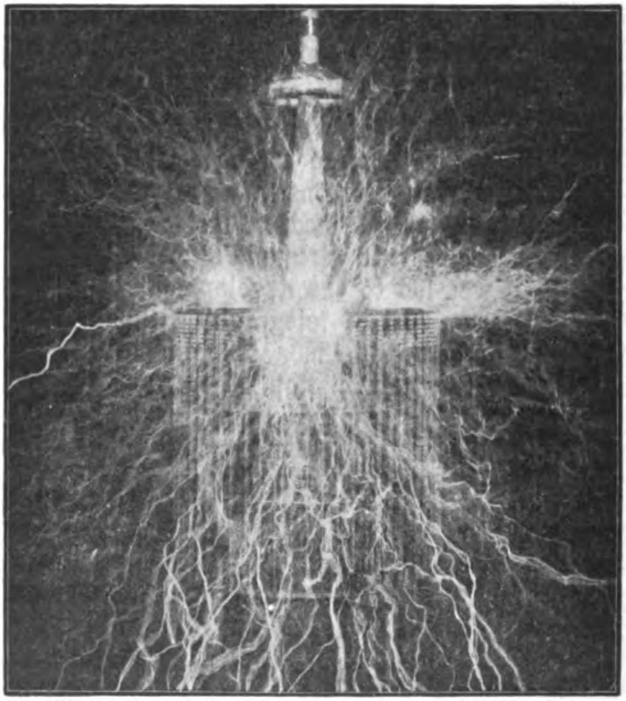

FIG. 174—AN ELECTRICAL OSCILLATOR DELIVERING ENERGY AT A RATE OF 75,000 HORSE-POWER.

produced some more than one hundred feet in length, and this without difficulty. The rates of electrical movement involved in strong induction apparatus had measured but a few hundred horse-power, and I produced electrical movements of rates of one hundred and ten thousand horse-power. Prior to this, only insignificant electrical pressures were obtained, while I have reached fifty million volts.

The accompanying illustrations, with their descriptive titles, taken from an article I wrote for the "Century Magazine," may serve to convey an idea of the results I obtained in the directions indicated.

Many persons in my own profession have wondered at them and have asked what I am trying to do. But the time is not far away now when the practical results of my labors will be placed before the world and their influence felt everywhere. One of the immediate consequences will be the transmission of messages without wires, over sea or land, to an immense distance. I have already demonstrated, by crucial tests, the practicability of signalling by my system from one to any other point of the globe, no matter how remote, and I shall soon convert the disbelievers.

I have every reason for congratulating myself that throughout these experiments, many of which were exceedingly delicate and hazardous, neither myself nor any of my assistants received an injury. When working with these powerful electrical oscillations the most extraordinary phenomena take place at times. Owing to some interference of the oscillations, veritable balls of fire are apt to leap out to a great distance, and if any one were within or near their paths, he would be instantly destroyed. A machine such as I have used could easily kill, in an instant, three hundred thousand persons. I observed that the strain upon my assistants was telling, and some of them could not endure the extreme tension of the nerves. But these perils are now entirely overcome, and the operation of such apparatus, however powerful, involves no risk whatever.

As I was improving my machines for the production of intense electrical actions, I was also perfecting the means for observing feeble effects. One of the most interesting results, and also one of great practical importance, was the development of certain contrivances for indicating at a distance of many hundred miles an approaching storm, its direction, speed and distance travelled. These appliances are likely to be valuable in future meteorological observations and surveying, and will lend themselves particularly to many naval uses.

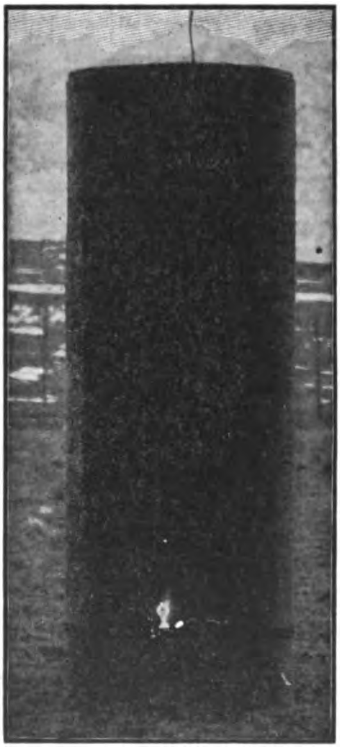

FIG. 175.—TRANSMITTING ELECTRICAL ENERGY THROUGH THE EARTH WITHOUT WIRE.

It was in carrying on this work that for the first time I discovered those mysterious effects which have elicited such unusual interest. I had perfected the apparatus referred to so far that from my laboratory in the Colorado mountains I could feel the pulse of the globe, as it were, noting every electrical change that occurred within a radius of eleven hundred miles.

I can never forget the first sensations I experienced when it dawned upon me that I had observed something possibly of incalculable consequences to mankind. I felt as though I were present at the birth of a new knowledge or the revelation of a great truth. Even now, at times, I can vividly recall the incident, and see my apparatus as though it were actually before me. My first observations positively terrified me, as there was present in them something mysterious, not to say supernatural, and I was alone in my laboratory at night; but at that time the idea of these disturbances being intelligently controlled signals did not yet present itself to me.

The changes I noted were taking place periodically, and with such a clear suggestion of number and order that they were not traceable to any cause then known to me. I was familiar, of course, with such electrical disturbances as are produced by the sun, Aurora Borealis and earth currents, and I was as sure as I could be of any fact that these variations were due to none of these causes. The nature of my experiments precluded the possibility of the changes being produced by atmospheric disturbances, as has been rashly asserted by some. It was some time afterward when the thought flashed upon my mind that the disturbances I had observed might be due to an intelligent control. Although I could not decipher their meaning, it was impossible for me to think of them as having been entirely accidental. The feeling is constantly growing on me that I had been the first to hear the greeting of one planet to another. A purpose was behind these electrical signals; and it was with this conviction that I announced to the Red Cross Society, when it asked me to indicate one of the great possible achievements of the next hundred years, that it would probably be the confirmation and interpretation of this planetary challenge to us.

Since my return to New York more urgent work has consumed all my attention; but I have never ceased to think of

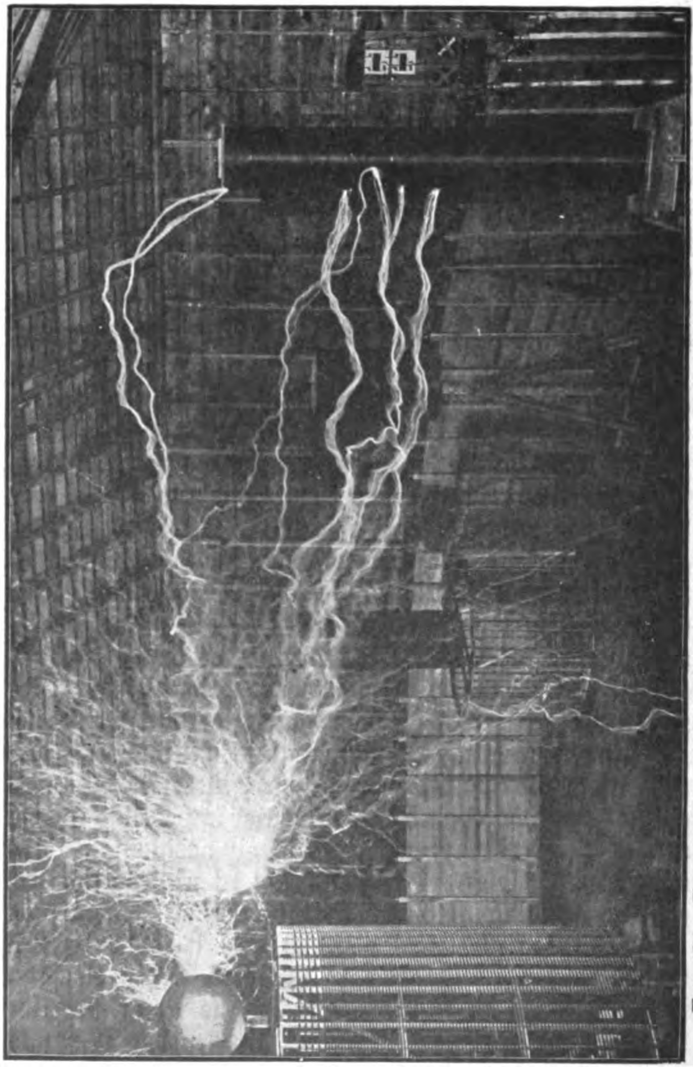

Fig. 176.—Experiment to Illustrate the Capacity of the Oscillator for Producing Electrical Explosions of Great Power.

those experiences and of the observations made in Colorado. I am constantly endeavoring to improve and perfect my apparatus, and just as soon as practicable I shall again take up the thread of my investigations at the point where I have been forced to lay it down for a time.

At the present stage of progress, there would be no insurmountable obstacle in constructing a machine capable of conveying a message to Mars, nor would there be any great difficulty in recording signals transmitted to us by the inhabitants of that planet, if they be skilled electricians. Communication once established, even in the simplest way, as by a mere interchange of numbers, the progress toward more intelligible communication would be rapid. Absolute certitude as to the receipt and interchange of messages would be reached as soon as we could respond with the number "four," say, in reply to the signal "one, two, three." The Martians, or the inhabitants of whatever planet had signalled to us, would understand at once that we had caught their message across the gulf of space and had sent back a response. To convey a knowledge of form by such means is, while very difficult, not impossible, and I have already found a way of doing it.

What a tremendous stir this would make in the world! How soon will it come? For that it will some time be accomplished must be clear to every thoughtful being.

Something, at least, science has gained. But I hope that it will also be demonstrated soon that in my experiments in the West I was not merely beholding a vision, but had caught sight of a great and profound truth.

APPENDIX I.

BIBLIOGRAPHY OF POLYPHASE CURRENTS AND ROTATORY-FIELD MOTORS.

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APPENDIX II.

SCHEDULE OF SOME BRITISH PATENTS BEARING ON POLYPHASE AND ALTERNATE-CURRENT MOTORS.

		No.
1886	Wynne	11391
1887	Coerper	9013
1888	Tesla	6481
	Tesla	6502
	Goolden, Atkinson & Gold	16852
	Wilson	18525
1889	Thomson, E.	6065
	Tesla	6527
	Dobrowolsky	10933
	Tesla	19420
	Tesla	19426
	Dobrowolsky	19555
1890	Wenström	5423
	Dobrowolsky	13260
	Kennedy	14817
	Kennedy	16889
	Dobrowolsky	20425
1891	Hutin & Leblanc	584
	Dobrowolsky	3191
	Siemens & Halske	8151
	Stanley & Kelly	9522
	Siemens & Halske	10612
	Siemens & Halske	10613
	Tesla	11473
	Sahulka	12046

		No.
1891	Dobrowolsky	13503
	Brain & Arnot	13627
	Bradley, Taylor & McDonald	16099
	Tesla	16709
	Siemens & Halske	18890
	Stanley & Kelly	20604
1892	Siemens & Halske	360
	Stanley & Kelly	7504
	Siemens & Halske	13543
	Hutin & Leblanc	13765
	Stanley & Kelly	14056
	Swinburne	16919
	Oerlikon Co.	20505
	Brown	21811
	Arnold	23290
	Brown	23902
	Brown	23961
	Brown	24098
1893	Rice	510
	Bradley	1621
	Lundell & Johnson	5339
	Lundell & Johnson	5340
	Duncan	6241
	Hutin & Leblanc	12139
	Hutin & Leblanc	12458
	Kingdon	14664
	Swinburne	16307
1894	Ries Electric Co.	3986
	Bradley	4170
	Bradley	4171
	Ferranti	6458
	Coerper	7239
	Blondel & others	8638
	Mordey	9605
	Rowland	11059
	Kelly & Stanley	12875
	Langdon-Davies	19364
	Clark	20241
	Siemens & Halske	21141

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