On the nature of the force acting on a charged classical particle deviated from its geodesic path in a gravitational field

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23 October 2001

Abstract

In general relativity the gravitational field is a manifestation of spacetime curvature and unlike the electromagnetic field is not a force field. A particle falling in a gravitational field is represented by a geodesic worldline which means that no force is acting on it. If the particle is at rest in a gravitational field, however, its worldline is no longer geodesic and it is subjected to a force. The nature of that force is an open question in general relativity. The aim of this paper is to outline an approach toward resolving it in the case of classical charged particles which was initiated by Fermi in 1921.

General relativity provides a consistent no-force explanation of gravitational interaction of bodies following geodesic paths. However, it is silent on the nature of the very force we regard as gravitational - the force acting upon a body deviated from its geodesic path due to its being at rest in a gravitational field.

In both special relativity (in flat spacetime) and general relativity (in curved spacetime) a particle offering no resistance to its motion is represented by a geodesic worldline. As the non-resistant motion of a particle is regarded as inertial a particle whose worldline is geodesic is moving by inertia. In both special and general relativity a particle whose worldline is not geodesic is prevented from moving by inertia and therefore is subjected to an inertial force. Hence a particle supported in a gravitational field is deviated from its geodesic path (i.e. prevented from moving by inertia) which means that the force acting on it is not gravitational but inertial in origin.

The mass causing the spacetime curvature determines the shape of the particle's geodesic worldline, and in general which reference frames are inertial [7], but the force arising when the particle is deviated from its geodesic path originates neither from that mass nor from the distant masses (as Mach proposed). This force has the same origin as the force acting on a test particle prevented from following a geodesic path in an empty spacetime. It should be stressed that in general relativity the force acting on a particle deviated from its geodesic path due to its being at rest in a gravitational field is non-gravitational in origin. As Rindler put it "ironically, instead of explaining inertial forces as gravitational... in the spirit of Mach, Einstein explained gravitational forces as inertial" [8]. This is the reason why "there is no such thing as the force of gravity" in general relativity [9].

Here it will be shown that a corollary of general relativity - that the propagation of light in a gravitational field is anisotropic - in conjunction with the classical electromagnetic mass theory [1]- [6] sheds some light on the nature of the force acting on a classical charged particle deviated from its geodesic path.

Consider a classical electron [10] at rest in the non-inertial reference frame N^g of an observer supported in the Earth's gravitational field. Following Lorentz [4] and Abraham [5] we assume that the electron charge is uniformly distributed on a spherical shell. The repulsion of the charge elements of an electron in uniform motion in flat spacetime cancels out exactly and there is no net force acting on the electron. As we shall see below, however, the average anisotropic velocity of light in N^g (i) gives rise to a self-force acting on an electron deviated from its geodesic path by disturbing the balance of the mutual repulsion of its charge elements, and (ii) makes a free electron fall in N^g with an acceleration **g** in order to balance the repulsion of its charge elements. No force is acting upon a falling electron (whose worldline is geodesic) but if it is prevented from falling (i.e. deviated from its geodesic path) the average velocity of light with respect to it becomes anisotropic and disturbs the balance of the mutual repulsion of the elements of its charge which results in a self-force trying to force the electron to fall. This force turns out to be equal to the gravitational force $\mathbf{F} = m^g\mathbf{g}$, where $m^g = U/c^2$ represents the passive gravitational mass of the classical electron and U is the energy of its field. As the coefficient m^g in front of $\mathbf{g}$ is exactly equal to U/c^2 (without the 4/3 factor) it turns out that the mass of the *classical* electron is purely electromagnetic in origin when the average anisotropic velocity of light in a gravitational field is taken into account.

In 1921 Fermi [11] studied the nature of the force acting on a charge at rest in a gravitational field of strength g in the framework of general relativity and the classical electromagnetic mass theory. The potential

$$\varphi = \frac{e}{4\pi\epsilon_0 r}\left(1 - \frac{1}{2}\frac{gz}{c^2}\right) \tag{1}$$

he derived, however, contains the 1/2 factor in the parenthesis which leads to a contradiction with the principle of equivalence when the electric field is calculated from this potential: it follows from (1) that the electric field of a charge supported in the Earth's gravitational field coincides with the instantaneous electric field of a charge moving with an acceleration $\mathbf{a} = -\mathbf{g}/2$ (obviously the principle of equivalence requires that $\mathbf{a} = -\mathbf{g}$). Now we shall show that the average anisotropic velocity of light in a gravitational field gives rise to a Liénard-Wiechert-like contribution to the potential φ which removes the 1/2 factor in (1).

Why the average velocity of light between two points in a gravitational field is not equal to c can be most clearly shown by considering two extra light rays parallel and anti-parallel to the gravitational acceleration $\mathbf{g}$ in the Einstein thought experiment involving an elevator at rest in the Earth's gravitational field (see Figure 1) [12].

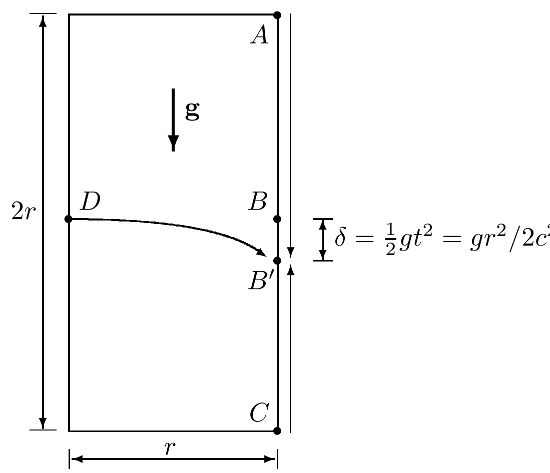

Figure 1. Three light rays propagate in an elevator at rest in the Earth's gravitational field. After having been emitted simultaneously from points A, C, and D the rays meet at B' (the ray propagating from D toward B, but arriving at B', represents the original thought experiment considered by Einstein). The light rays emitted from A and C are introduced in order to determine the expression for the average velocity of light in a gravitational field. It takes the same coordinate time $t = r/c$ for the rays to travel the distances $DB' \approx r$, $AB' = r + \delta$, and $CB' = r - \delta$. Therefore the average velocity of the downward ray from A to B' is $c_{AB'} = (r + \delta)/t \approx c(1 + gr/2c^2)$; the average velocity of the upward ray from C to B' is $c_{CB'} = (r - \delta)/t \approx c(1 - gr/2c^2)$.

Three light rays are emitted simultaneously in the elevator (representing a non-inertial reference frame N^g) from points A, C, and D toward point B. The emission of the rays is also simultaneous in a reference frame I (a local Lorentz frame) which is momentarily at rest with respect to N^g. At the moment the light rays are emitted I starts to fall in the gravitational field. At the next moment an observer in I sees that the elevator moves upward with an acceleration $g = |\mathbf{g}|$. Therefore as seen from I the three light rays arrive simultaneously not at point B, but at B' since for the time $t = r/c$ the elevator moves at a distance $\delta = gt^2/2 = gr^2/2c^2$. As the simultaneous arrival of the three rays at the point B' in I is an absolute event (the same in all reference frames) being a *point* event, it follows that the rays arrive simultaneously at B' as seen from N^g as well. Since for the *same* coordinate time $t = r/c$ in N^g the three light rays travel different distances $DB' \approx r$, $AB' = r + \delta$, and $CB' = r - \delta$ before arriving simultaneously at point B' an observer in the elevator concludes that the *average* velocity of the light ray propagating from A to B' is slightly greater than c

$$c^g_{AB'} = \frac{r+\delta}{t} \approx c\left(1+\frac{gr}{2c^2}\right).$$

The average velocity $c^g_{CB'}$ of the light ray propagating from C to B' is slightly smaller than c

$$c^g_{CB'} = \frac{r-\delta}{t} \approx c\left(1-\frac{gr}{2c^2}\right).$$

It is easily seen that to within terms $\sim c^{-2}$ the average light velocity between A and B is equal to that between A and B', i.e. $c^g_{AB} = c^g_{AB'}$ and also $c^g_{CB} = c^g_{CB'}$:

$$c^g_{AB} = \frac{r}{t-\delta/c} \approx c\left(1+\frac{gr}{2c^2}\right) \tag{2}$$

and

$$c^g_{CB} = \frac{r}{t+\delta/c} \approx c\left(1-\frac{gr}{2c^2}\right). \tag{3}$$

As the average velocities (2) and (3) are not determined with respect to a specific point and since the *coordinate* time t is involved in their calculation, it is clear that the expressions (2) and (3) represent the average *coordinate* velocities between the points A and B and C and B, respectively.

These expressions for the average coordinate velocity of light in N^g can be also obtained from the coordinate velocity of light at a point in a *parallel* gravitational field. If the z-axis is antiparallel to the elevator's acceleration $\mathbf{g}$ the spacetime metric in N^g has the form [13]

$$ds^2 = \left(1+\frac{2gz}{c^2}\right)c^2dt^2 - dx^2 - dy^2 - dz^2 \tag{4}$$

from where the coordinate velocity of light at a point z in a parallel gravitational field is immediately obtained (for $ds^2 = 0$)

$$c^g\left(z\right) = c\left(1+\frac{gz}{c^2}\right). \tag{5}$$

Notice that (4) is the standard spacetime interval in a *parallel* gravitational field [13], which does not coincide with the expression for the spacetime interval in a spherically symmetric gravitational field (i.e. the Schwarzschild metric) [14, p. 395]

$$ds^2 = \left(1-\frac{2GM}{c^2r}\right)c^2dt^2 - \left(1+\frac{2GM}{c^2r}\right)\left(dx^2+dy^2+dz^2\right). \tag{6}$$

The metric (4) can be written in a form similar to (6) if we choose $r = r_0 + z$ where r_0 is a constant

$$ds^2 = \left(1-\frac{2GM}{c^2\left(r_0+z\right)}\right)c^2dt^2 - \left(dx^2+dy^2+dz^2\right). \tag{7}$$

As $g = GM/r_0^2$ and for $z/r_0 < 1$ we can write

$$ds^2 = \left(1-\frac{2GM}{c^2r_0}+\frac{2gz}{c^2}\right)c^2dt^2 - \left(dx^2+dy^2+dz^2\right). \tag{8}$$

As the gravitational potential is undetermined to within an additive constant we can choose $GM/r_0 = 0$ in (8); more precisely, when calculating the gravitational potential we can set the constant of integration to be equal to $-GM/r_0$. With this choice of the integration constant (8) coincides with (4). Although similar (7) and (6) have different values for g_{ii} $(i = 1,2,3)$: $g_{ii} = -1$ in (7), whereas $g_{ii} = -\left(1+2GM/c^2r\right)$ in (6). This reflects the fact that in a parallel gravitational field proper and coordinate times do not coincide (except for the proper time of an observer at infinity) whereas proper and coordinate distances are the same [15].

Throughout the paper we will be concerned only with a parallel gravitational field. As all effects we will be studying involve distances of the order of the classical electron radius ($\sim 10^{-15}$ m) that approximation is an excellent one. It is also clear that the acceleration g will have the same value in all points of a region of such dimensions.

Using the coordinate velocity (5) we obtain for the average coordinate velocity of light propagating between A and B (Figure 1)

$$c_{AB}^{g} = \frac{1}{2}\left(c_{A}^{g} + c_{B}^{g}\right) = \frac{1}{2}\left[c\left(1 + \frac{gz_{A}}{c^{2}}\right) + c\left(1 + \frac{gz_{B}}{c^{2}}\right)\right]$$

and as $z_A = z_B + r$

$$c_{AB}^{g} = c\left(1 + \frac{gz_{B}}{c^{2}} + \frac{gr}{2c^{2}}\right). \tag{9}$$

For the average coordinate velocity of light propagating between B and C we obtain

$$c_{BC}^{g} = c\left(1 + \frac{gz_{B}}{c^{2}} - \frac{gr}{2c^{2}}\right) \tag{10}$$

since $z_C = z_B - r$. When the coordinate origin is at point B ($z_B = 0$) the expressions (9) and (10) coincide with (2) and (3).

There exists a third way to derive the average coordinate velocity of light in N^g. As the coordinate velocity $c^g(z)$ (5) is continuous on the interval $[z_A, z_B]$ in the case of weak parallel gravitational fields one can calculate the average coordinate velocity between A and B:

$$c_{AB}^{g} = \frac{1}{z_B - z_A}\int_{z_A}^{z_B} c^{g}(z)\,dz = c\left(1 + \frac{gz_{B}}{c^{2}} + \frac{gr}{2c^{2}}\right).$$

As expected this expression coincides with (9).

The average coordinate velocities (9) and (10) correctly describe the propagation of light in N^g yielding the right expression $\delta = gr/2c^2$ (see Figure 1). It should be stressed that without these average coordinate velocities the fact that the light rays emitted from A and C arrive not at B, but at B' cannot be explained.

As a coordinate velocity, the average coordinate velocity of light is not determined with respect to a specific point and depends on the choice of the coordinate origin. Also, it is the same for light propagating from A to B and for light travelling in the opposite direction, i.e. $c_{AB}^{g} = c_{BA}^{g}$. Therefore, like the coordinate velocity (5) the average coordinate velocity is also isotropic. Notice, however, that the average coordinate velocity of light is isotropic in a sense that the average light velocity between two points is the same in both directions. But as seen from (9) and (10) the average coordinate velocity of light between different pairs of points, whose points are the same distance apart, is different. As a result as seen in Figure 1 the light ray emitted at A arrives at B before the light ray emitted at C.

The average coordinate velocity of light explains the propagation of light in the Einstein elevator, but cannot be used in a situation where the average light velocity between two points (say a source and an observation point) is determined with respect to one of the points. Such situations occur, as we shall see, when one calculates the potential, the electric field, and the self-force of a charge in a gravitational field. As the local velocity of light is c the average velocity of light between a source and an observation point depends on which of the two points is regarded as a reference point with respect to which the average velocity is determined (at the reference point the local velocity of light is c). The dependence of the average velocity on which point is chosen as a reference point demonstrates that that velocity is anisotropic. This anisotropic velocity should be regarded as an average *proper* velocity of light since it is determined with respect to a given point and its calculation involves the proper time at that point.

Consider a light source at point B (Figure 1). To calculate the average proper velocity of light originating from B and observed at A (that is, as seen from A) we have to determine the initial velocity of a light signal at B and its final velocity at A both with respect to A. As the local velocity of light is c the final velocity of the light signal determined at A is also c. Its initial velocity at B as seen from A is

$$c_{B}^{g} = \frac{dz_B}{d\tau_A} = \frac{dz_B}{dt}\frac{dt}{d\tau_A}$$

where $dz_B/dt = c^g(z_B)$ is the coordinate velocity (5) at B

$$c^{g}(z_B) = c\left(1 + \frac{gz_{B}}{c^{2}}\right)$$

and $d\tau_A$ is the proper time at A

$$d\tau_A = \left(1 + \frac{gz_{A}}{c^{2}}\right)dt.$$

Since $z_A = z_B + r$ for the coordinate time dt we have

$$dt = \left(1 - \frac{gz_A}{c^2}\right) d\tau_A = \left(1 - \frac{gz_B}{c^2} - \frac{gr}{c^2}\right) d\tau_A.$$

Then for the initial velocity c_B^g at B as seen from A we obtain

$$c_B^g = c\left(1 + \frac{gz_B}{c^2}\right)\left(1 - \frac{gz_B}{c^2} - \frac{gr}{c^2}\right)$$

or keeping only the terms $\sim c^{-2}$

$$c_B^g = c\left(1 - \frac{gr}{c^2}\right).$$

For the *average proper* velocity $c_{BA}^g = (1/2)(c_B^g + c)$ of light propagating from B to A *as seen from* A we have

$$c_{BA}^g\,(as\ seen\ from\ A) = c\left(1 - \frac{gr}{2c^2}\right). \tag{11}$$

As the local velocity of light at A is c it follows that if light propagates from A toward B its average proper velocity $c_{AB}^g\,(as\ seen\ from\ A)$ will be equal to the average proper velocity of light propagating from B toward A $c_{BA}^g\,(as\ seen\ from\ A)$. Thus, as seen from A, the back and forth average proper velocities of light travelling between A and B are the *same*.

Now let us determine the average proper velocity of light between B and A with respect to the source point B. A light signal emitted at B as seen from B will have an initial (local) velocity c there. The final velocity of the signal at A as seen from B will be

$$c_A^g = \frac{dz_A}{d\tau_B} = \frac{dz_A}{dt}\frac{dt}{d\tau_B}$$

where $dz_A/dt = c^g\,(z_A)$ is the coordinate velocity at A

$$c^g\,(z_A) = c\left(1 + \frac{gz_A}{c^2}\right)$$

and $d\tau_B$ is the proper time at B

$$d\tau_B = \left(1 + \frac{gz_B}{c^2}\right) dt.$$

Then as $z_A = z_B + r$ for c_A^g we obtain

$$c_A^g = c\left(1 + \frac{gr}{c^2}\right)$$

and the average proper velocity of light propagating from B to A *as seen from* B becomes

$$c_{BA}^g\,(as\ seen\ from\ B) = c\left(1 + \frac{gr}{2c^2}\right). \tag{12}$$

If a light signal propagates from A to B its average proper velocity $c_{AB}^g\,(as\ seen\ from\ B)$ will be equal to that from B to A $c_{BA}^g\,(as\ seen\ from\ B)$. Comparing (11) and (12) demonstrates that the two average proper velocities between the same points are not equal and depend on from where they are seen. As we expected the fact that the local velocity of light at the reference point is c makes the average proper velocity between two points dependant on where the reference point is.

In order to express the average proper velocity of light in a vector form let the light emitted from B be observed at different points. The average proper velocity of light emitted at B and determined at A is given by (11). As seen from point C the average proper velocity of light from B to C will be given by an expression derived in the same way as (12)

$$c_{BC}^g\,(as\ seen\ from\ C) = c\left(1 + \frac{gr}{2c^2}\right).$$

As seen from a point P at a distance r from B and lying on a line forming an angle θ with the acceleration $\mathbf{g}$ the average proper velocity of light from B is

$$c_{BP}^g\,(as\ seen\ from\ P) = c\left(1 + \frac{gr\cos\theta}{2c^2}\right).$$

Then the average proper velocity of light coming from B as seen from a point defined by the position vector $\mathbf{r}$ originating from B has the form

$$\bar{c}^g = c\left(1 + \frac{\mathbf{g}\cdot\mathbf{r}}{2c^2}\right). \tag{13}$$

As evident from (13) the average proper velocity of light emitted from a common source and determined at different points around the source is anisotropic in N^g - if the observation point is above the light source the average proper velocity of light is slightly smaller than c and smaller than the average proper velocity as determined from an observation point below the source. If an observer at point B (Figure 1) determines the average proper velocities of light coming from A and C he finds that they are also anisotropic - the average proper velocity of light coming from A is greater than that emitted at C. However, if the observer at B (Figure 1) determines the back and forth average proper velocities of light propagating between A and B (or between B and C) he finds that they are the same.

One deduces from (13) that $|\mathbf{g}\cdot\mathbf{r}/2c^2| < 1$ in order that $\bar{c}^g$ be positive. In fact, that restriction is always satisfied in all cases involving the principle of equivalence since it is weaker than the one imposed by that principle which requires that only small regions in a gravitational field where the field is parallel are considered [16]. In the case of light travelling a large distance h between two points A and B ($r_A > r_B$) along the radial direction in a gravitational field it can be shown that the average proper velocity of light can be expressed in terms of the gravitational potentials of the source and observation points. When propagating along the radial direction a light signal does not "feel" the spacetime curvature and the coordinate velocity (5) is used in the calculation of the average proper velocity. For instance, the average proper velocity

$$c^g_{BA}\,(as\ seen\ from\ A) = c\left(1 - \frac{gh}{2c^2}\right)$$

of light propagating from B to A as seen from A (which means that the local light velocity at A is c) can be written as

$$c^g_{BA}\,(as\ seen\ from\ A) = c\left(1 + \frac{GM}{2c^2 r_A} - \frac{GM}{2c^2 r_B}\right).$$

The velocity (13) demonstrates that there exists a directional dependence in the propagation of light between two points in a non-inertial frame of reference N^g at rest in a gravitational field. This anisotropy in the propagation of light has been an overlooked corollary of general relativity. In fact, up to now neither the average coordinate velocity nor the average proper velocity of light have been defined. However, we have seen that the average coordinate velocity is needed to account for the propagation of light in a gravitational field (to explain the fact that two light signals emitted from points A, and C in Figure 1 meet at B', not at B). We will also see below that the average proper velocity of light is necessary for the correct description of electromagnetic phenomena in a gravitational field.

The anisotropic velocity of light (13) leads to two changes in the scalar potential

$$d\varphi^g = \frac{1}{4\pi\epsilon_0}\frac{\rho dV^g}{r^g} \tag{14}$$

of a charge element of an electron at rest in N^g; here ρ is the charge density, dV^g is a volume element of the charge and r^g is the distance from the charge to the observation point determined in N^g.

First, analogously to representing r as $r = ct$ in an inertial reference frame [17, p. 416], r^g is expressed as $r^g = \bar{c}^g t$ in N^g. Assuming that $\mathbf{g}\cdot\mathbf{r}/2c^2 << 1$ (weak gravitational fields) we can write:

$$\left(r^g\right)^{-1} \approx r^{-1}\left(1 - \frac{\mathbf{g}\cdot\mathbf{r}}{2c^2}\right). \tag{15}$$

Substituting $\left(r^g\right)^{-1}$ in (14) gives the potential (1) obtained by Fermi.

The second change in (14) is a Liénard-Wiechert-like (or rather anisotropic) volume element dV^g (not coinciding with the actual volume element dV) which arises in N^g on account of the average anisotropic velocity of light there. The origin of dV^g is analogous to the origin of the Liénard-Wiechert volume element [17, p. 418] $dV^{LW} = dV/\left(1 - \mathbf{v}\cdot\mathbf{n}/c\right)$ of a charge moving at velocity $\mathbf{v}$ with respect to an inertial observer I, where $\mathbf{n} = \mathbf{r}/r$ and $\mathbf{r}$ is the position vector at the retarded time. This can be explained in terms of the "information-collecting sphere" of Panofsky and Phillips [18, p. 342] used in the derivation of the Liénard-Wiechert potentials (similar concepts are employed by Griffiths [17, p. 418], Feynman [19, p. 21-10], and Schwartz [20, p. 213]). The Liénard-Wiechert volume element dV^{LW} of a charge appears greater than dV (in the direction of its velocity) due to its greater contribution to the potential since it "stays longer within the information-collecting sphere" [18, p. 343] sweeping over the charge at the velocity of light c in I. By the same argument the anisotropic volume element dV^g also appears different from dV in N^g: in a direction opposite to $\mathbf{g}$ the velocity of the information-collecting sphere (which propagates at the velocity of light (13) in N^g) is smaller than c since for light propagating against $\mathbf{g}$ we have $\mathbf{g}\cdot\mathbf{r} = -gr$ in (13); therefore an elementary volume dV^g of the electron charge stays longer within the sphere and contributes more to the potential in N^g.

Consider a charge of length l at rest in N^g placed along $\mathbf{g}$. The time for which the information-collecting sphere sweeps over the charge in N^g is

$$\Delta t^g = \frac{l}{c^g} = \frac{l}{c\,(1+\mathbf{g}\cdot\mathbf{r}/2c^2)} \approx \Delta t\left(1-\frac{\mathbf{g}\cdot\mathbf{r}}{2c^2}\right),$$

where $\Delta t = l/c$ is the time for which the information-collecting sphere propagating at speed c sweeps over an inertial charge of the same length l in its rest frame. When the information-collecting sphere moves against $\mathbf{g}$ in N^g its velocity is smaller than c. Therefore the charge stays longer within the sphere since $\Delta t^g > \Delta t$ ($\mathbf{g}\cdot\mathbf{r} = -gr$) and its contribution to the potential is greater. This is equivalent to saying that the greater contribution comes from a charge of a larger length l^g which for the same time Δt^g is swept over by an information-collecting sphere propagating at velocity c:

$$l^g = \Delta t^g c = l\left(1-\frac{\mathbf{g}\cdot\mathbf{r}}{2c^2}\right).$$

The anisotropic volume element which corresponds to such an apparent length l^g is obviously

$$dV^g = dV\left(1-\frac{\mathbf{g}\cdot\mathbf{r}}{2c^2}\right). \tag{16}$$

Substituting (15) and (16) into (14) we obtain the scalar potential of a charge element ρdV^g of the electron

$$d\varphi^g = \frac{1}{4\pi\epsilon_0}\frac{\rho dV^g}{r^g} = \frac{1}{4\pi\epsilon_0}\frac{\rho dV}{r}\left(1-\frac{\mathbf{g}\cdot\mathbf{r}}{2c^2}\right)^2$$

or if we keep only the terms proportional to c^{-2} we get

$$d\varphi^g = \frac{\rho}{4\pi\epsilon_0 r}\left(1-\frac{\mathbf{g}\cdot\mathbf{r}}{c^2}\right)dV. \tag{17}$$

As seen from (17) making use of dV^g instead of dV accounts for the 1/2 factor in (1). Now we can calculate the electric field of a charge element ρdV^g of an electron at rest in N^g by using only the scalar potential (17):

$$d\mathbf{E}^g = -\nabla d\varphi^g = \frac{1}{4\pi\epsilon_o}\left(\frac{\mathbf{n}}{r^2} - \frac{\mathbf{g}\cdot\mathbf{n}}{c^2 r}\mathbf{n} + \frac{\mathbf{g}}{c^2 r}\right)\rho dV. \tag{18}$$

The distortion of the electric field (18) is caused by the anisotropic velocity of light (13) in N^g. For the distorted field of the whole electron charge we find

$$\mathbf{E}^g = \frac{1}{4\pi\epsilon_o}\int\left(\frac{\mathbf{n}}{r^2} - \frac{\mathbf{g}\cdot\mathbf{n}}{c^2 r}\mathbf{n} + \frac{\mathbf{g}}{c^2 r}\right)\rho dV. \tag{19}$$

The self-force with which the field of an electron interacts with an element ρdV_1^g of its charge is

$$d\mathbf{F}^g_{self} = \rho dV_1^g \mathbf{E}^g = \frac{1}{4\pi\epsilon_o}\int\left(\frac{\mathbf{n}}{r^2} - \frac{\mathbf{g}\cdot\mathbf{n}}{c^2 r}\mathbf{n} + \frac{\mathbf{g}}{c^2 r}\right)\rho^2 dV dV_1^g. \tag{20}$$

Due to the distorted electric field (19) of an electron at rest in N^g the mutual repulsion of its charge elements does not cancel out. As a result a non-zero self-force with which the electron acts upon itself arises:

$$\mathbf{F}^g_{self} = \frac{1}{4\pi\epsilon_o}\int\int\left(\frac{\mathbf{n}}{r^2} - \frac{\mathbf{g}\cdot\mathbf{n}}{c^2 r}\mathbf{n} + \frac{\mathbf{g}}{c^2 r}\right)\rho^2 dV dV_1^g. \tag{21}$$

After taking into account the explicit form (16) of dV_1^g (21) becomes

$$\mathbf{F}^g_{self} = \frac{1}{4\pi\epsilon_o}\int\int\left(\frac{\mathbf{n}}{r^2} - \frac{\mathbf{g}\cdot\mathbf{n}}{c^2 r}\mathbf{n} + \frac{\mathbf{g}}{c^2 r}\right)\left(1-\frac{\mathbf{g}\cdot\mathbf{r}}{2c^2}\right)\rho^2 dV dV_1.$$

Assuming a spherically symmetric distribution of the electron charge [4] and following the standard procedure of calculating the self-force [21] we get:

$$\mathbf{F}^{g}_{self} = \frac{U}{c^2}\mathbf{g}, \tag{22}$$

where

$$U = \frac{1}{8\pi\epsilon_o}\int\int\frac{\rho^2}{r}dV dV_1 \tag{23}$$

is the electrostatic energy of the electron. The famous 4/3 factor does not appear in (22) since the correct volume element (16) was used in (21). As U/c^2 is the mass associated with the energy U of the electron field, (22) obtains the form:

$$\mathbf{F}^{g}_{self} = m^g\mathbf{g}, \tag{24}$$

where $m^g = U/c^2$ is interpreted as the electron passive gravitational mass which is entirely electromagnetic in origin in the case of the classical electron.

As (24) shows an electron whose worldline is not geodesic (since it is at rest in N^g) is subjected to the self-force $\mathbf{F}^{g}_{self}$ which turns out to be equal to what is traditionally called the gravitational force. The self-force $\mathbf{F}^{g}_{self}$ is electromagnetic since it originates from the unbalanced repulsion of the electron charge elements due to their distorted fields (18) which in turn are caused by the average anisotropic velocity of light in N^g. Thus the classical electromagnetic mass theory in conjunction with the general-relativistic corollary of the average anisotropic velocity of light in a gravitational field provides an insight into the nature of the force acting on the deviated from its geodesic path classical electron - the self-force $\mathbf{F}^{g}_{self}$ is inertial since it resists that deviation and is electromagnetic in origin.

In general, if a charged classical particle is prevented from following a geodesic path in a gravitational field, its electric field distorts and a self-force resisting the deformation of the particle's field arises. That force is inertial since it opposes the deviation of the particle from maintaining its non-resistant (inertial) motion and is electromagnetic in origin since it results from the unbalanced repulsion between the charged elements of the particle.

Let us now see whether the approach outlined above predicts that a falling classical electron is subjected to no force as required by general relativity. To verify this let us calculate the electric field of an electron falling in the Earth's gravitational field with an acceleration $\mathbf{a} = \mathbf{g}$. We note that the Liénard-Wiechert potentials must include the correction due to the average anisotropic velocity of light in N^g:

$$\varphi^g(r,t) = \frac{e}{4\pi\epsilon_o r}\frac{1}{1-\mathbf{v}\cdot\mathbf{n}/c}\left(1-\frac{\mathbf{g}\cdot\mathbf{r}}{c^2}\right) \tag{25}$$

$$\mathbf{A}^g(r,t) = \frac{e}{4\pi\epsilon_o c^2 r}\frac{\mathbf{v}}{1-\mathbf{v}\cdot\mathbf{n}/c}\left(1-\frac{\mathbf{g}\cdot\mathbf{r}}{c^2}\right). \tag{26}$$

The electric field of an electron falling in N^g (and considered instantaneously at rest in N^g [22]) obtained from (25) and (26) is:

$$\mathbf{E} = -\nabla\varphi^g - \frac{\partial\mathbf{A}^g}{\partial t} = \frac{e}{4\pi\epsilon_o}\left[\left(\frac{\mathbf{n}}{r^2}+\frac{\mathbf{g}\cdot\mathbf{n}}{c^2 r}\mathbf{n}-\frac{\mathbf{g}}{c^2 r}\right)+\left(-\frac{\mathbf{g}\cdot\mathbf{n}}{c^2 r}\mathbf{n}+\frac{\mathbf{g}}{c^2 r}\right)\right],$$

which reduces to the Coulomb field [23]

$$\mathbf{E} = \frac{e}{4\pi\epsilon_o}\frac{\mathbf{n}}{r^2}. \tag{27}$$

Therefore the instantaneous electric field of a falling electron is not distorted. This demonstrates that the repulsion of its charge elements is balanced, thus producing no self-force. This result sheds light on the question why in general relativity an electron is falling in a gravitational field "by itself" with no force acting on it. As (27) shows, the only way for an electron to compensate the anisotropy in the propagation of light and to preserve the Coulomb shape of its electric field is to fall with an acceleration $\mathbf{g}$. If the electron is prevented from falling its electric field distorts (disturbing the balance of the repulsion of its charge elements), the self-force (24) appears and tries to force the electron to fall in order to eliminate the distortion of its field; if the electron is left to fall its Coulomb field restores, the repulsion of its charge elements cancels out and the self-force disappears [24].

To summarize, the study of the open question in general relativity - what is the nature of the force acting upon a particle deviated from its geodesic path - by taking into account the classical electromagnetic mass theory provides an insight not only into that question (in the case of the classical electron) but also into the question why a falling electron (whose worldline is geodesic) is subjected to no force.

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[22] We consider the instantaneous electron field in order to separate its Lorentz contraction from the distortion caused by the electron acceleration and the anisotropic velocity of light.

[23] It is clear from here that a falling electron does not radiate since its electric field does not contain the radiation r^{-1} terms [12]. If those terms were present in the field of a falling electron this would constitute a contradiction with the principle of equivalence.

[24] The behaviour of the classical electron in a gravitational field as described by general relativity and the classical electromagnetic mass theory can be summarized in the following way: the worldline of an electron which preserves the shape of its Coulomb field is geodesic and represents a free non-resistantly moving electron; if the field of an electron is distorted, its worldline is not geodesic and the electron is subjected to a self-force on account of its own distorted field.