Zero-point field induced mass vs. QED mass renormalization

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Abstract

Haisch and Rueda have recently proposed a model in which the inertia of charged particles is a consequence of their interaction with the electromagnetic zero-point field. This model is based on the observation that in an accelerated frame the momentum distribution of vacuum fluctuations is not isotropic. We analyze this issue through standard techniques of relativistic field theory, first by regarding the field A_μ as a classical random field, and then by making reference to the mass renormalization procedure in Quantum Electrodynamics and scalar-QED.

A general property of vacuum fluctuations in any relativistic theory is the Lorentz invariance of their frequency spectrum. This is true for quantum field theories and also for "stochastic" field theories – those models in which the use of commuting fields supplemented by a suitable stochasticity principle allows to re-obtain most results of the full quantum field theories [1]. In any case, it is required that all observers, independently of their relative motion, see the same fluctuations spectrum.

However, it has been known for a long time (since the work by Unruh and Davies [2]) that an accelerated observer will see a different spectrum of the vacuum fluctuations, corresponding to an apparent non-zero temperature. The temperature is a function of the acceleration: $T_a = a/(2\pi ck)$. This effect offers a method of principle for detecting absolute acceleration.

Since the acceleration has a definite direction and versus, one also expects that the accelerated observer will notice an anisotropy in the distribution of the vacuum fluctuations. For the electromagnetic field such anisotropy corresponds to a Poynting vector which does not vanish on the average, and to a radiation pressure which is not exactly balanced in all directions. This physical interpretation is well justified in Stochastic Electrodynamics, where the zero-point field is regarded as a real random field, whose effects can only be observed in the presence of some

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cause of inhomogeneity or anisotropy. As a consequence of the unbalanced radiation pressure, any accelerated observer feels a resistance to its acceleration. It has been shown by Haisch and Rueda [3] that this resistance is proportional to acceleration and has therefore the typical property of inertia. To this end, they considered a sequence of Lorentz transformations between the accelerated system and local co-moving inertial systems.

The idea that the inertia of matter could be the result of its interaction with vacuum fluctuations is appealing from the logical point of view, because in this approach it is not necessary to start from an equation of motion for the observer which already contains an inertial term.

In other words, while any classical Lagrangian and any wave equation (or field equation in the case of second quantization) is an evolution of Newton's second law $\mathbf{F} = m\mathbf{a}$, in the approach mentioned above inertia follows from the anisotropic radiation pressure. In this way, inertia appears as a consequence of the third law, i.e., of momentum conservation, and ultimately of translation invariance.

There is, of course, a problem with the size of the accelerated particle. The part of the vacuum fluctuations spectrum acting upon the particle depends on its size; only for finite (non-zero) size can the "inertial effect" be finite. The standard approach to this kind of vacuum effects in QED is to consider point-like, structureless particles, and re-absorb infinite contributions to their self-mass into the "bare mass" through the renormalization procedure.

Now, it is interesting to compare the results of the "radiation pressure" approach, based only upon momentum conservation, with those of standard relativistic field theories with "built in" second Newton law. What happens if we introduce finite cut-offs in the field theoretical expressions for the self-mass Σ? One finds that the result depends much on the spin of the particles. For scalar particles, it is possible to introduce a cut-off in Σ, set the bare mass to zero and interpretate somehow the physical mass as entirely due to vacuum fluctuations – except for the problem that the "natural" cut-offs admitted in QFT (supersymmetry scale, GUT scale, Planck scale) all correspond to very large masses. For spin 1/2 particles (QED with fermions) one obtains a relation between bare mass and renormalized mass which is compatible both with the observed electron mass and with a finite cut-off, but only if the bare mass is not zero. Below we shall give the explicit expressions for the scalar and spinor case. Before that, however, let us make a short premise and consider a semiclassical approximation.

Effective mass of charged particles in a thermal or stochastic electromagnetic field.

Let us consider charged particles with bare mass m_0 immersed in a thermal or stochastic background $A_\mu(x)$. For scalar particles described by a quantum field ϕ, the Lagrangian density is of the form

$$L = \frac{1}{2}\phi^*(P^\mu - eA^\mu)\phi(P_\mu - eA_\mu) - \frac{1}{2}m_0^2|\phi|^2 \tag{1}$$

and contains a term $e^2\phi^* A_\mu A^\mu \phi$, which after averaging on A_μ can be regarded as a mass term for the field ϕ. Take, for instance, the Coulomb gauge: The effective squared mass turns out to be equal to $m^2 = m_0^2 + e^2\langle|\mathbf{A}(\mathbf{x})|^2\rangle$.

For homogeneous black body radiation at a given temperature T, the average is readily computed [4]. One has

$$\langle|\mathbf{A}|^2\rangle = \int_0^\infty d\omega \frac{u_\omega}{\omega^2} \tag{2}$$

where u_ω is the Planck spectral energy density. By integrating one finds that the squared mass shift is given by $\Delta m^2 = const.\sqrt{\alpha}kT$ (the constant is adimensional and of order 1). This mass shift can be significant for a hot plasma. If the plasma is dense, the average $\langle|\mathbf{A}(\mathbf{x})|^2\rangle$ will be itself defined in part by fluctuations and correlations in the charge density [5]. Therefore the mass shift for one kind of particles in the plasma depends in general on which other particles are present.

We can also insert into eq. (2) the Lorentz-invariant spectrum of vacuum fluctuations (see [6, 7]; this is valid both for Stochastic Electrodynamics and QED). In this case the integral diverges, and a frequency cut-off is needed (compare our discussion above). Alternatively, one can introduce in the integral an adimensional weight function $\eta(\omega)$ peaked at some "resonance" frequency ω_0. In this way, one finds that $\Delta m \sim \omega_0$ in natural units ($\hbar = c = 1$). Haisch and Rueda have proposed [8] to set $\omega_0 \sim \omega_{Compton}$ for the electron, so that $\Delta m \sim m_e$.

In conclusion, field theory offers a quite straightforward method to evaluate the influence of a classical random background on effective mass. This appears to work, however, only for scalar particles. The Dirac Lagrangian is linear with respect to the field A_μ, therefore it is impossible to obtain a mass term for spinors by averaging over the electromagnetic field.

The case of QED and relation to mass renormalization.

For a quantized electromagnetic field the simple considerations above are not applicable. One must compute the propagator of the charged particles taking into account QED corrections. It is found that the pole of the propagator (giving the physical mass) is shifted, due to the effect of vacuum fluctuations.

Before mass renormalization, the full *scalar* propagator has the form

$$\Delta_0(p) = \frac{1}{p^2 - m_0^2 - \Sigma(m^2) + i\varepsilon} \tag{3}$$

where m_0 is the "bare" mass and $\Sigma(p^2)$ represents the sum of all possible connected vacuum polarization diagrams. In the Feynman renormalization procedure, which appears to be the most suitable for our purposes, $\Sigma(p^2)$ is Taylor-expanded and the mass renormalization condition

$$m_0^2 + \Sigma(m^2) = m^2 \tag{4}$$

is imposed, where m_0 and $\Sigma(m^2)$ are diverging quantities, but their difference is finite.

An alternative view, corresponding to the idea of inertia as entirely due to vacuum fluctuations, is the following: set $m_0 = 0$, impose a physical cut-off M in Σ and compute m from (4). For scalar QED, one finds in this way that Σ is of the order of the cut-off.

In QED with *spinors* the mass renormalization condition involves a logarithm [9]:

$$m - m_0 = \left[\Sigma(p^2, M)\right]_{\sqrt{p^2}=m} = m_0 \frac{3\alpha}{4\pi}\left(\ln\frac{M^2}{m_0^2} + \frac{1}{2}\right) \tag{5}$$

Therefore m_0 must be non-zero. For an estimate, let us rewrite the cut-off in natural units as $M = 10^\xi\ cm^{-1}$ and set $m/m_0 \equiv k > 1$ (i.e., vacuum fluctuations increase the mass by a factor k). Eq. (5) becomes

$$k - 1 = \frac{3\alpha}{2\pi}\left(\xi \ln 10 - \ln m + \ln k + \frac{1}{4}\right) \tag{6}$$

Let us apply this to the electron ($m \sim 0.5 \cdot 10^{10}\ cm^{-1}$), taking the Planck mass as cut-off ($\xi \sim 33$). Solving with respect to k we find that $k \sim 1.19$ – a quite moderate renormalization effect, after all. For smaller cut-offs, k turns out to be closer to 1.

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References

[1] L. de la Pena and A. Cetto, *The quantum dice: an introduction to Stochastic Electrodynamics*, Kluwer, Dordrecht, 1996.

[2] P.C.W. Davies, J. Phys. **A 8** (1975) 609. W.G. Unruh, Phys. Rev. **D 14** (1976) 870.

[3] B. Haisch and A. Rueda, *Inertia as a reaction of the vacuum to accelerated motion*, Phys. Lett. **A 240** (1998) 115-126; *Contribution to inertial mass by reaction of the vacuum to accelerated motion*, Found. of Phys. **28** (1998) 1057-1108.

[4] G. Modanese, *Inertial mass and vacuum fluctuations in quantum field theory*, report hep-th/0009046.

[5] A.I. Akhiezer et al., *Plasma Electrodynamics*, Pergamon, New York, 1975.

[6] P.W. Milonni, *The quantum vacuum*, Academic Press, New York, 1994.

[7] B. Haisch and A. Rueda, *The zero-point field and inertia*, in *Causality and Locality in Modern Physics*, 171-178, G. Hunter, S. Jeffers and J.-P. Vigier eds., Kluwer, Dordrecht, 1998.

[8] Y. Dobyns, B. Haisch and A. Rueda, *Inertial mass and the quantum vacuum fields*, report gr-qc/0009036, to appear in Annalen der Physik.

[9] C. Itzykson and J.-B. Zuber, *Quantum field theory*, McGraw-Hill, New York, 1985.