# Journal of Theoretics

Journal Home Page


## The Classical Problem of a Body Falling in a Tube Through the Center of the Earth in the Dynamic Theory of Gravity


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### Abstract

There is a new theory gravity called the dynamic theory, which is derived from thermodymical principles in a five dimensional space. In this theory we will examine the classical problem of a body falling in a tube through the earth's center. For simplicity and to an idealized scenario the earth is assumed to be a sphere of constant density equals to the mean density of the Earth. The derived equation of motion will be solved for a variety of initial conditions, and the results will be compared to those of Newtonian gravity.



**Key words:** Dynamic theory of gravity, general relativity, energy-momentum tensor.


### 1 Introduction

There is a new theory called the Dynamic Theory of Gravity (DTG). It is derived from classical thermodynamics and requires that Einstein's postulate of the constancy of the speed of light holds. [1]. Given the validity of the postulate, Einstein's theory of special relativity follows right away [2]. The dynamic theory of gravity (DTG) through Weyl's quantum principle also leads to a non-singular electrostatic potential of the form:

$$V(r) = -\frac{K_o}{r} e^{-\frac{\lambda}{r}}. \qquad (1)$$

where $K_o$ is a constant and $\lambda$ is a constant defined by the theory. The DTG describes physical phenomena in terms of five dimensions: space, time and mass. [3] By conservation of the fifth dimension we obtain equations which are identical to Einstein's field equations and describe the gravitational field. These field equations are similar to those of general relativity and are given below:

$$K_o T^{\alpha\beta} = G^{\alpha\beta} = R^{\alpha\beta} - \frac{g^{\alpha\beta}}{2} R \,. \tag{2}$$

Now $T^{\alpha\beta}$ is the surface energy-momentum tensor which may be found within the space tensor and is given by:

$$T^{\alpha\beta} = T_{sp}^{\alpha\beta} - \frac{1}{c^2}\left[ F_4^{\alpha} F^{4\beta} - \frac{h^{\alpha\beta}}{2} F^{4\nu} F_{4\nu} \right] \tag{3}$$

and $T_{sp}^{\mu\nu}$ is the space energy-momentum tensor for matter under the influence of the gauge fields and is given by:[4]

$$T_{sp}^{ij} = \gamma u^i u^j + \frac{1}{c^2}\left[ F_k^i F^{kj} + \frac{1}{4} a^{ij} F^{k\lambda} F_{k\lambda} \right] \tag{4}$$

which further can be written in terms of the surface metric as follows:[4]

$$T_{sp}^{\alpha\beta} = \gamma u^{\alpha} u^{\beta} + \frac{1}{c^2}\left[ F_k^{\alpha} F^{k\beta} + F_4^{\alpha} F^{4\beta} + \frac{1}{4}\left(g^{\alpha\beta} - h^{\alpha\beta}\right)\left(F^{\mu\nu} F_{\mu\nu} + F^{4\nu} F_{4\nu}\right)\right] \tag{5}$$

since:

$$u^4 = \frac{dy^4}{dt} \Rightarrow \frac{\partial y^4}{\partial t} + \bar{\nabla} \bullet \left( y^4 \bar{u} \right) = 0 \tag{6}$$

is the statement required by the conservation of the fifth dimension, and the surface indices ν, α, β. = 0,1,2,3 and space index i, j, k, l = 0,1,2,3,4, and $g_{\alpha\beta} = a_{ij} y_{\alpha}^i y_{\alpha}^j = a_{\alpha\beta} + h_{\alpha\beta} = a_{\alpha\beta} + 2a_{\alpha 4} y_{\beta}^4 + a_{44} y_{\alpha}^4 y_{\beta}^4$ where the surface field tensor is given by: $F_{\alpha\beta} = F_{ij} y_{\alpha}^i y_{\beta}^j$ and $y_{\alpha}^i = \frac{\partial y^i}{\partial x^{\alpha}} = \delta_{\alpha}^i$ for $i = 0,1,2,3$ and $y_{\alpha}^4 = \frac{\partial y^4}{\partial x^{\alpha}}$.

(7)

$$F_{ij} = \begin{bmatrix} o & E_1 & E_2 & E_3 & V_o \\ -E_1 & o & B_3 & -B_2 & V_1 \\ -E_2 & -B_3 & o & B_1 & V_2 \\ -E_3 & B_2 & -B_1 & o & V_3 \\ -V_o & -V_1 & -V_2 & -V_3 & o \end{bmatrix}. \tag{8}$$

It was shown by Weyl that the gauge fields may be derived from the gauge potentials and the components of the 5-dimensional field tensor $F_{ij}$ given by the 5×5 matrix given in (8). [4]

Now the determination of the fifth dimension may be seen, for the only physically real property that could give Einstein's equations is the gravitating mass or it's equivalent, mass [5]. Finally the dynamic theory of gravity further argues that the gravitational field is a gauge field linked to the electromagnetic field in a 5-dimensional manifold of space-time and mass, but, when conservation of mass is imposed, it may be described by the geometry of the 4-dimensional hypersyrface of space-time, embedded into the 5-dimensional manifold by the conservation of mass. The 5 dimensional field tensor can only have one nonzero component $V_0$ which must be related to the gravitational field and the fifth gauge potential must be related to the gravitational potential.

The theory makes its predictions for red shifts by working in the five dimensional geometry of space, time, and mass, and determines the unit of action in the atomic states in a way that can be calculated with the help of quantum Poisson brackets when covariant differentiation is used: [6]

$$\left[x^{\mu}, p^{\nu}\right]\Phi = i\eta g^{\nu q}\left\{\delta_{\mu q} + \left|\Gamma^{\mu}_{s,q}\right| x^{s}\right\}\Phi \, . \tag{9}$$

In (9) the vector curvature is contained in the Christoffel symbols of the second kind and the gauge function $\Phi$ is a multiplicative factor in the metric tensor $g^{\nu q}$, where the indices take the values $\nu, q = 0,1,2,3,4$. In the commutator, $x^{\mu}$ and $p^{\nu}$ are the space and momentum variables respectively, and finally $\delta_{\mu q}$ is the Cronecker delta. In DTG the momentum ascribed, as a variable canonically conjugated to the mass is the rate at which mass may be converted into energy. The canonical momentum is defined as follows:

$$p_4 = m v_4 \tag{10}$$

where the velocity in the fifth dimension is given by:

$$v_4 = \frac{\dot{\gamma}}{\alpha_o} \tag{11}$$

and gamma dot is a time derivative and gamma has units of mass density ( $kg/m^3$) and $\alpha_o$ is a density gradient with units of $kg/m^4$. In the absence of curvature (8) becomes:

$$\left[x^{\mu}, p^{\nu}\right]\Phi = i\eta\delta^{\nu q}\Phi \quad . \tag{12}$$

## 2 The equation of motion in the dynamic theory of gravity

To proceed let us assume that a test body of mass m is falling through a tube that passes through the center of the earth. The test body is at a distance r away from the center of the earth. The force that acts on the mass m is associated only with the mass M' of the earth that lies within a sphere of radius r. Thus the shell of the earth that lies outside this sphere exerts no force on the body. Therefore we can write:

$$M'(r) = \rho_o V'(r) = \frac{4\pi\rho_o}{3} r^3 \tag{13}$$

where $\rho_o$ is the density function assumed to be constant and equal to the mean density of the earth material, and V' is the volume of the sphere of mass M'. The gravitational potential in the theory of dynamic gravity can be described as some short of modified Newtonian potential and is given by the relation below: [ ]

$$V(r) = -\frac{GM}{r} e^{-\frac{\lambda}{r}} \tag{14}$$

a solution of the following differential equation, an equation that is derived from Weyl's quantization principle and has the form:

$$r^2\frac{dV(r)}{dr}-(\lambda-r)V(r)=O. \qquad (15)$$

Next the force acting on the body of mass m now takes the form:

$$g(r)=-\nabla V(r)=-\frac{GM}{r^2}\left(1-\frac{\lambda}{r}\right)e^{-\frac{\lambda}{r}} \qquad (16)$$

which can be further written as follows:

$$g(r)=-\frac{4\pi G\rho_o}{3}r\left(1-\frac{\lambda}{r}\right)e^{-\frac{\lambda}{r}} \qquad (17)$$

finally the differential equation of motion in the tube becomes:

$$\frac{d^2r}{dt^2}+\frac{4\pi G\rho_o}{3}r\left(1-\frac{\lambda}{r}\right)e^{-\frac{\lambda}{r}}=O \qquad (18)$$

which is some kind of a non-linear harmonic oscillator equation. The parameter of the theory $\lambda$ depends on the total mass of the body M(R) and is equal to $\lambda = G M_{\oplus}/c^2 = 4.43\times10^{-3}$ m. Therefore during the motion across the tube through the center of the earth $r > \lambda$. Expanding the exponential term to second order and keeping only first order terms in 1/r we obtain the following differential equation of motion:

$$\frac{d^2r}{dt^2}+\frac{4\pi G\rho_o}{3}r-\frac{8\pi G\rho_o}{3}\lambda=O, \qquad (19)$$

which has the following solution:

$$r(t)=2\lambda+c_1\sin\sqrt{\frac{4\pi G\rho_o}{3}}t+c_2\cos\sqrt{\frac{4\pi G\rho_o}{3}}t \qquad (20)$$

and $c_1$ and $c_2$ are two constants to be determined by the initial conditions.

## <u>3 Applying different initial conditions</u>

Applying the initial condition indicated below that we obtain the corresponding solutions, if $\omega=\sqrt{K}=(4\pi G\rho_o/3)^{1/2}$:

i) Initial conditions: r(0)=r'(0)=0

<u>Newtonian gravity solution</u>:

$$r(t)=0 \qquad (21)$$

<u>Dynamic gravity</u>:

$$r(t)=2\lambda\left(1-\cos(\sqrt{K}\ t)\right) \qquad (22)$$

ii) Initial conditions: r(0)=0, r'(0)=$V_o$

<u>Newtonian gravity solution</u>:

$$r(t)=\frac{V_o}{\sqrt{K}}\sin\left(\sqrt{K}\ t\right) \qquad (23)$$

Dynamic gravity solution:

$$r(t) = 2\lambda\left(1-\cos(\sqrt{K}\ t)\right)+\frac{V_o}{\sqrt{K}}\sin(\sqrt{K}\ t) \tag{24}$$

iii) Initial conditions: $r(0)=r_o$, $r'(0)=V_o$

Newtonian gravity solution

$$r(t) = r_o\cos(\sqrt{K}\ t)+\frac{V_o}{\sqrt{K}}\sin(\sqrt{K}\ t) \tag{25}$$

Dynamic gravity solution

$$r(t) = 2\lambda+\left(r_o-\lambda\right)\cos(\sqrt{K}\ t)+\frac{V_o}{\sqrt{K}}\sin(\sqrt{K}\ t) \tag{26}$$

iv) Initial conditions: $r(t_o) = r_o$, $r'(t_o)=V_o$

Newtonian gravity solution

$$\begin{aligned} r(t) &= \sin(\sqrt{K}\ t)\left(\frac{V_o\cos(\sqrt{K}\ t_o)}{\sqrt{K}}+r_o\sin(\sqrt{K}\ t_o)\right) \\ &+\cos(\sqrt{K}\ t)\left(r_o\cos(\sqrt{K}\ t_o)+\frac{V_o\sin(\sqrt{K}\ t_o)}{\sqrt{K}}\right) \end{aligned} \tag{27}$$

Dynamic gravity solution

$$\begin{aligned} r(t) &= 2\lambda+\frac{\sin(\sqrt{K}t)}{\sqrt{K}}\left(V_o\cos(\sqrt{K}\ t_o)+\sqrt{K}\left(r_o-2\lambda\right)\sin(\sqrt{K}\ t_o)\right) \\ &+\cos(\sqrt{K}\ t)\left(\left(r_o-2\lambda\right)\cos(\sqrt{K}\ t_o)-\frac{V_o\sin(\sqrt{K}\ t_o)}{\sqrt{K}}\right) \end{aligned} \tag{28}$$

v) Initial conditions $r(t_o)=r_o$ ,$r'(t_o)=0$

Newtonian gravity solution

$$r(t) = r_o\left(\cos(\sqrt{K}\ t)\cos(\sqrt{K}\ t_o)+\sin(\sqrt{K}\ t)\sin(\sqrt{K}\ t_o)\right) \tag{29}$$

Dynamic gravity solution

$$r(t) = 2\lambda-\left(2\lambda-r_o\right)\left(\cos(\sqrt{K}\ t)\cos(\sqrt{K}\ t_o)+\sin(\sqrt{K}\ t)\sin(\sqrt{K}\ t_o)\right) \tag{30}$$

vi) Initial conditions $r(t_o)=0$, $r'(t_o)=V_o$

Newtonian gravity solution

$$r(t) = \frac{V_o\sin\sqrt{K}\ t_o\sin\sqrt{K}\ t}{\sqrt{K}}-\frac{V_o\cos\sqrt{K}\ t\sin\sqrt{K}\ t_o}{\sqrt{K}} \tag{31}$$

Dynamic gravity solution

$$r(t) = 2\lambda + \sin\sqrt{K}\, t\left(\frac{V_o \cos\sqrt{K}\, t_o}{\sqrt{K}} - 2\lambda \sin\sqrt{K}\, t_o\right) - \cos\sqrt{K}\, t\left(2\lambda\cos\sqrt{K}\, t_o + \frac{V_o \sin\sqrt{K}\, t_o}{\sqrt{K}}\right) \tag{32}$$

vii) Initial conditions $r(t_o)=0$, $r'(t_o)=0$

Newtonian gravity solution

$$r(t) = 0 \tag{33}$$

Dynamic gravity solution

$$r(t) = 2\lambda\left[1 - \left(\cos\sqrt{K}\, t \cos\sqrt{K}\, t_o + \sin\sqrt{K}\, t \sin\sqrt{K}\, t_o\right)\right] \tag{34}$$

viii) Initial conditions $r(t_o)=r_o$, $r'(t_o)= 0$

Newtonian gravity solution

$$r(t) = r_o\left(\cos\sqrt{K}\, t \cos\sqrt{K}\, t_o + \sin\sqrt{K}\, t \sin\sqrt{K}\, t_o\right) \tag{35}$$

Dynamic gravity solution

$$r(t) = 2\lambda - (2\lambda - r_o)\left(\cos\sqrt{K}\, t \cos\sqrt{K}\, t_o + \sin\sqrt{K}\, t \sin\sqrt{K}\, t_o\right) \tag{36}$$

## 4 Velocity and acceleration functions

In particular the expressions for the velocity and acceleration of the body moving under Newtonian and dynamic gravity as relater to equations (25), (26) and also (27) and (28). From equations (25) and (26) we obtain the velocity and acceleration functions under Newtonian gravity:

$$V(t) = \dot{r}(t) = V_o \cos\left(\sqrt{K}\, t\right) - r_o\sqrt{K}\sin\left(\sqrt{K}\, t\right) \tag{37}$$

and next the acceleration function to be:

$$a(t) = \ddot{r}(t) = -Kr_o \cos\left(\sqrt{K}\, t\right) - V_o\sqrt{K}\sin\left(\sqrt{K}\, t\right), \tag{38}$$

next in the case of motion under dynamic gravity we obtain:

$$V(t) = \dot{r}(t) = V_o \cos\left(\sqrt{K}\, t\right) - \sqrt{K}(r_o - 2\lambda)\sin\left(\sqrt{K}\, t\right). \tag{39}$$

Now making use of equations (27) and (28) we obtain for Newtonian gravity:

$$V(t) = \dot{r}(t) = \sqrt{K}\cos\left(\sqrt{K}\, t\right)\left[\frac{V_o \cos\left(\sqrt{K}\, t_o\right)}{\sqrt{K}} + r_o \sin\left(\sqrt{K}\, t_o\right)\right] - \sqrt{K}\sin\left(\sqrt{K}\, t_o\right)\left[r_o \cos\left(t\sqrt{K}\right) - \frac{V_o \sin\left(t_o\sqrt{K}\right)}{\sqrt{K}}\right] \tag{40}$$

and finally

$$a(t)=\ddot{r}(t)=-K\sin\left(t_o\sqrt{K}\right)\left[\frac{V_o\cos\left(t_o\sqrt{K}\right)}{\sqrt{K}}+r_o\sin\left(t_o\sqrt{K}\right)\right]$$
$$-K\cos\left(t_o\sqrt{K}\right)\left[r_o\cos\left(t_o\sqrt{K}\right)-\frac{V_o\sin\left(t_o\sqrt{K}\right)}{\sqrt{K}}\right] \qquad (41)$$

## 5 Plotting the solutions of the differential equations

To obtain an idea between motion under Newtonian gravity and motion under dynamic gravity some numerical parameters should be calculated. First constant K has the value:

$$\omega=\sqrt{K}=\sqrt{\frac{4\pi G\rho_o}{3}}=1.241\times 10^{-3}\ \sec^{-1} \qquad (37)$$

where the mean density of the earth $\rho_o$ has been taken equal to $\rho_o$ = 5.52 g/cm$^3$ [7]. Next four equations of all eight cases will be chosen, namely (25) ,(26), (27) and (28) and their graphs will plotted and compared for Newtonian and dynamic gravity. Taking $r_o$ = 1km = $10^3$ m, and $V_o$ = $10^2$ m/sec we obtain the graphs below for a number of a thousand points plotted We actually observe that there is a difference between dynamic gravity and Newtonian gravity displacement amplitude The Newtonian amplitude appears to be slightly larger than the dynamic one in both cases where relations have been derived for the corresponding velocities and accelerations.

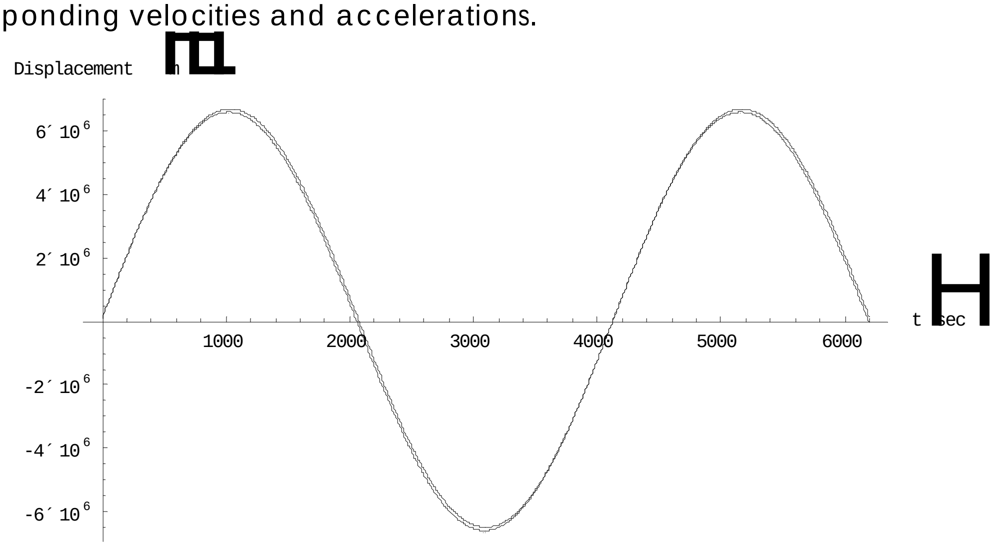


Fig 1 Displacement versus time graph of the Newtonian and dynamic gravity solutions with initial conditions r(0)=V(0)=0.

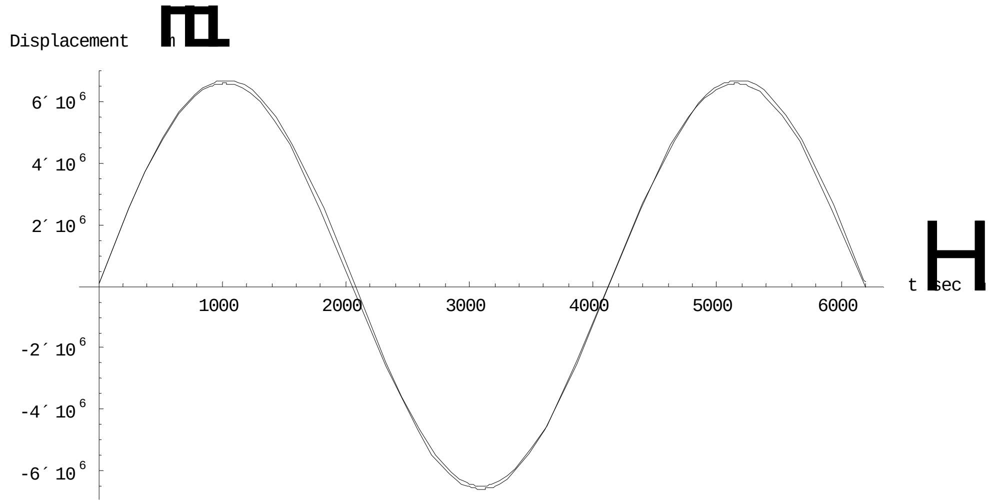


Fig 2 Displacement versus time graph of the Newtonian and dynamic gravity with initial conditions r(10)=1000 m, V(10)=100 m/sec.

Therefore we have the following amplitude relations:

Case 1

Newtonian gravity oscillation amplitude:

$$A_N = \sqrt{r^2{}_o + \left(\frac{V_o}{\omega}\right)^2} \tag{38}$$

Dynamic gravity oscillation amplitude:

$$A_D = \sqrt{(r_o + \lambda)^2 + \left(\frac{V_o}{\omega}\right)^2} = \lambda\sqrt{1 + \frac{2r_o}{\lambda} + \left(\frac{A_N}{\lambda}\right)^2} \tag{39}$$

Case 2

Newtonian gravity oscillation amplitude:

$$A_N = \sqrt{\left(r_o \cos(\omega t_o) + \frac{V_o \sin(\omega\, t_o)}{\omega}\right)^2 + \frac{(V_o \cos(\omega t_o) + r_o \omega \sin(\omega t_o))^2}{\omega^2}} \tag{40}$$

Dynamic gravity oscillation amplitude

$$A_D = \sqrt{\left(2\lambda + (r_o - 2\lambda)\cos(\omega t_o) - \frac{V_o \sin(\omega t_o)}{\omega}\right)^2 + \frac{(V_o \cos(\omega t_o) + \omega(r_o - 2\lambda)\sin(\omega t_o))}{\omega^2}}$$

(41)

## 6. Applying an approximate method for solving the same equation

Observe that equation (18) can be written as follows, if second order terms are kept in the expansion and $\lambda^3/r^2$ are omitted:

$$\frac{d^2r}{dt^2}+\omega^2 r = 2\omega^2\lambda - \frac{3\lambda^2}{2r}. \tag{42}$$

This equation can be classified as one having the general form below:

$$\frac{d^2r}{dt^2}+\omega^2 r + \varepsilon F\left(r,\dot{r}\right) = 0 \tag{43}$$

so if we assume a solution of the form r(t) = A sin($\omega$t+$\phi$) where both A and $\phi$ are assumed functions of t to be determined so that r(t) = A sin($\omega$t+$\phi$) =A sin$\psi$ becomes a solution of (43). This is known as the method of equivalent linearization. Following the analysis in [8] we have that:

$$\frac{dA}{dt} = -\frac{\varepsilon}{\omega}K_o(A) = -\frac{\varepsilon}{2\pi\omega}\int_o^{2\pi} F(A\sin\phi, A\omega\cos\phi)\cos\phi d\phi \tag{44}$$

$$\frac{d\psi}{dt} = \omega + \frac{\varepsilon}{2\pi A\omega}\int_o^{2\pi} F(A\sin\phi, A\omega\cos\phi)\sin\phi d\phi. \tag{45}$$

The above equations give that:

$$\frac{dA}{dt} = 0 \Leftrightarrow A = \mathrm{const} = r_o \tag{46}$$

$$\psi = \omega\left(1-\frac{3\lambda^2}{2A^2}\right)t + \theta_o \tag{47}$$

which makes the first approximation to the solution to be:

$$r(t) = r_o \sin\left[\left(1-\frac{3\lambda^2}{2r_o^{\,2}}\right)\omega t + \theta_o\right], \tag{48}$$

this is a harmonic oscillation with constant amplitude $r_o$ and angular frequency given by the expression $\omega(1-3\lambda^2/2r_0^2)$ which depends on the constant amplitude as well as the dynamic theory parameters $\lambda$ and is itself a constant.

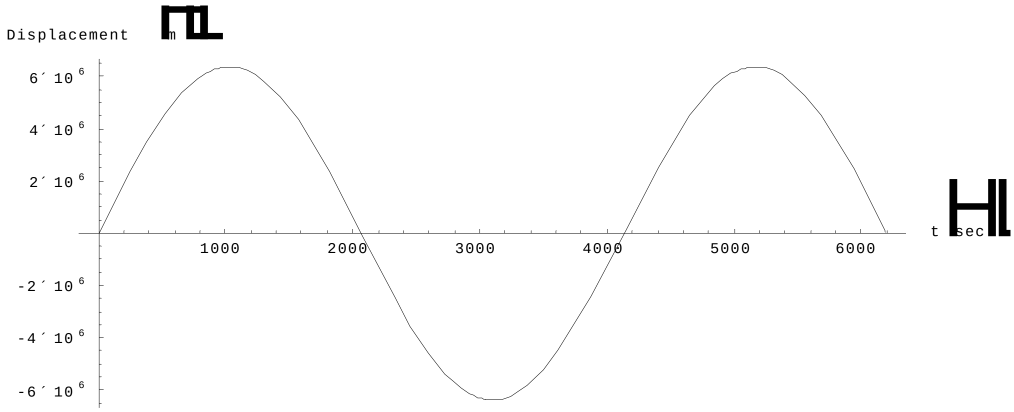


Fig 3 Displacement versus time graph of the linearized solution which has been derived as first approximation to the solution of the non linear harmonic oscillator. The non linear equation is derived from the dynamic gravity potential.

## 7 Trying another density function

We next are going to try the same problem given that the density at a distance r from the center of the earth varies according to the function:

$$\rho(r) = \rho_c\left[1-\left(\frac{r}{R_\oplus}\right)^2\right] \tag{49}$$

where $\rho_c$ is the central density and $R_\oplus$ is the radius of the earth. Taking into account the dynamic gravity acceleration of gravity which now becomes:

$$g(r) = -\frac{4\pi G\rho_c}{3}\, r\left(1-\frac{\lambda}{r}\right)\left(1-\left(\frac{r}{R_\oplus}\right)^2\right)e^{-\frac{\lambda}{r}} \tag{50}$$

we can write down the differential equation for the motion of the mass m inside the tube:

$$\frac{d^2r}{dt^2} + \omega^2 r\left(1-\frac{\lambda}{r}\right)\left(1-\frac{r^2}{R^2_\oplus}\right)e^{-\frac{\lambda}{r}} = 0\,. \tag{51}$$

After expanding the exponential terms as before the first approximate equation describing the motion can be:

$$\frac{d^2r}{dt^2} + \omega^2\left(1-\frac{3\lambda^2}{2R_\oplus^{\,2}}\right)r = 2\lambda\omega^2\left(1-\frac{\lambda^2}{4R_\oplus^{\,2}}\right) \tag{52}$$

which has the following solution:

$$r(t)=\frac{\lambda\left(\lambda^2-4R_\oplus{}^2\right)}{\left(3\lambda^2-2R_\oplus{}^2\right)}+C_1\sin\left[\omega\left(\sqrt{1-\frac{3\lambda^2}{2R_\oplus{}^2}}\right)t\right]+C_2\cos\left[\omega\left(\sqrt{1-\frac{3\lambda^2}{2R_\oplus{}^2}}\right)t\right] \tag{53}$$

If we apply the initial condition r(0)=0, V(0)=0 (53) becomes:

$$r(t)=\lambda\frac{\left(\lambda^2-4R_\oplus{}^2\right)}{\left(3\lambda^2-2R_\oplus{}^2\right)}\sin^2\left[\frac{\omega}{2}\left(\sqrt{1-\frac{3\lambda^2}{2R_\oplus{}^2}}\right)t\right]. \tag{54}$$

Different initial conditions namely r(0)=$r_0$ and V(0)=$V_0$ we obtain:

$$r(t)=\frac{\left(4\lambda r^2{}_o-\lambda^3\right)}{\left(2R_\oplus{}^2-3\lambda^2\right)}+\frac{\left(\lambda^3-3\lambda^2 r_o-4\lambda r_o{}^2+2r^3{}_o\right)}{\left(2R_\oplus{}^2-3\lambda^2\right)}\cos\left[\omega\left(\sqrt{1-\frac{3\lambda^2}{2R_\oplus{}^2}}\right)t\right]$$

(55

$$-\frac{V_o}{\omega\left(2R_\oplus{}^2-3\lambda^2\right)\sqrt{1-\frac{3\lambda^2}{2R_\oplus{}^2}}}\sin\left[\omega\left(\sqrt{1-\frac{3\lambda^2}{2R^2{}_\oplus}}\right)t\right]$$

If now r(0)=0 and V(0) = $V_0$ the solution is:

$$r(t)=\frac{2\lambda\left(\lambda^2-4R_\oplus{}^2\right)}{\left(3\lambda^2-2R_\oplus{}^2\right)}\sin^2\left[\frac{\omega}{2}\left(\sqrt{1-\frac{3\lambda^2}{2R_\oplus{}^2}}\right)t\right]$$

$$-\frac{2V_oR_\oplus{}^2\sqrt{1-\frac{3\lambda^2}{2R_\oplus{}^2}}}{\omega\left(3\lambda^2-3R_\oplus{}^2\right)}\sin\left[\frac{\omega}{2}\left(\sqrt{1-\frac{3\lambda^2}{2R_\oplus{}^2}}\right)t\right]. \tag{56}$$

Next consider possible initial conditions to be r(0)=$r_0$ and V(0)=0, the solution becomes:

$$r(t)=\lambda\frac{\left(\lambda^2-4r^2{}_o\right)}{\left(3\lambda^2-2r^2{}_o\right)}-\frac{\left(\lambda^3-3\lambda^2 r_o-4\lambda r^2{}_o+2r^3{}_o\right)}{\left(3\lambda^2-2r^2{}_o\right)}\cos\left[\frac{\omega}{2}\sqrt{1-\frac{3\lambda^2}{2R_\oplus{}^2}}\right] \tag{57}$$

## 8 Plotting the solutions

Using equation (54) derived for the given density function we obtain the following graph: Dynamic gravity

Case r(0)=0 and v(0)=0

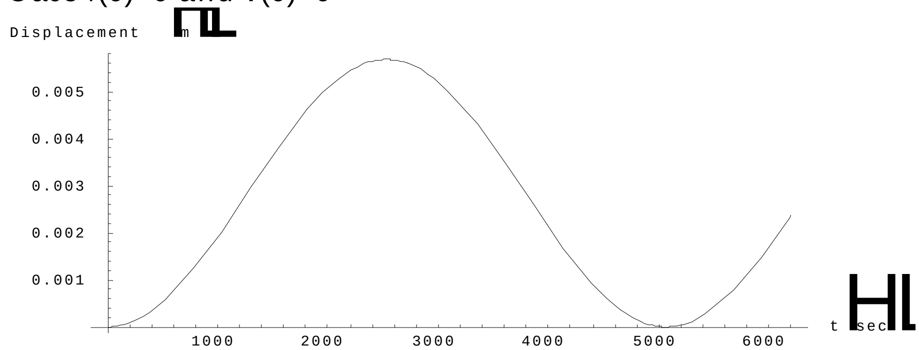


Fig 4 Displacement versus time graph. Solution to the non linear harmonic oscillator equation derived from the dynamic gravity potential and a variable density function.

Dynamic gravity

Case r(0)=$r_0$ and v(0)=$V_0$

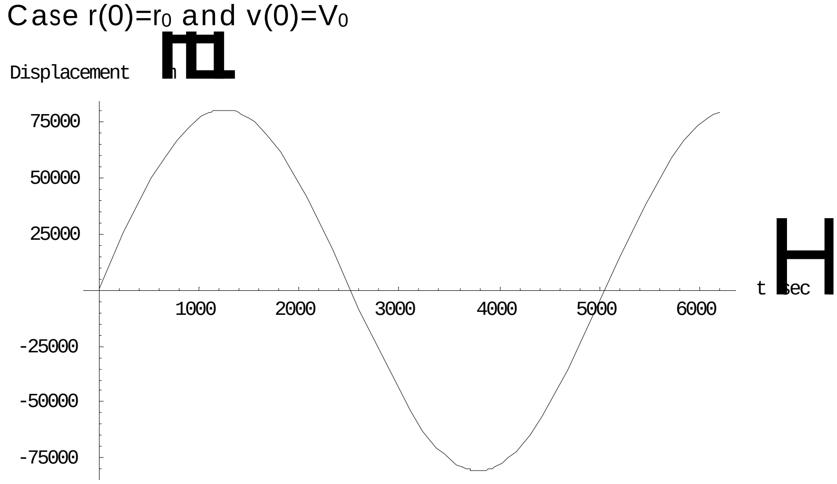


Fig 5 Displacement versus time graph. Solution to the non linear harmonic oscillator equation derived from the dynamic gravity potential and for a variable density function.

Dynamic gravity

Case r(0)=0 and v(0)=100 m/sec

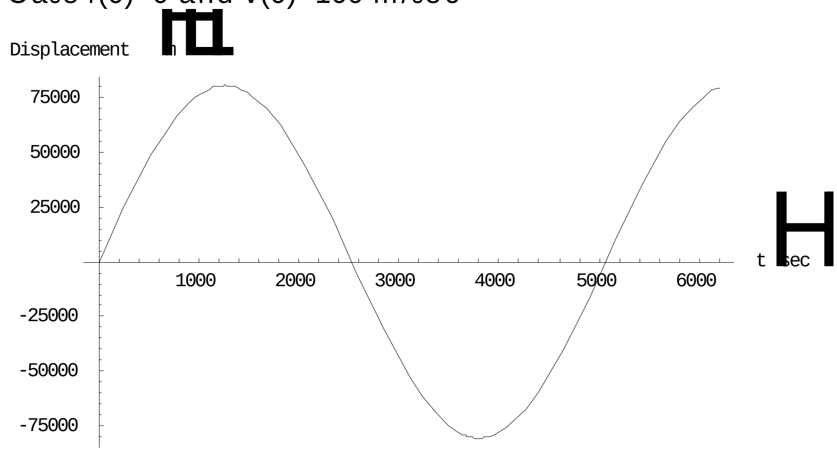


Fig 6 Displacement versus time graph. Solution to the non linear harmonic oscillator equation derived from the dynamic gravity potential and for a variable density function, and for the initial conditions given above.

Dynamic gravity

Case r(0)=1000m and v(0)=0

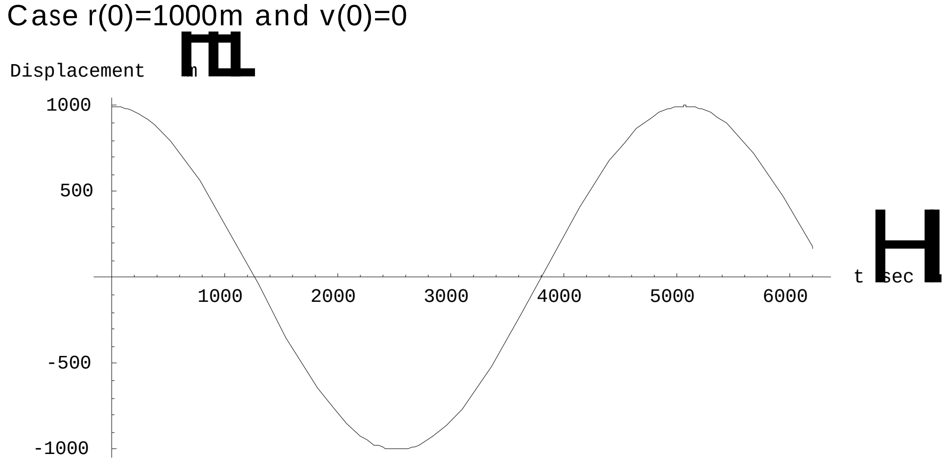


Fig 7 Displacement versus time graph. Solution to the non linear harmonic oscillator equation derived from the dynamic gravity potential and for a variable density function, and for the initial conditions given above.

Next applying the same method as in (7) we can also obtain a first approximate solution to the following non linear oscillator equation below:

$$\frac{d^2 r}{dt^2} + \omega^2 r = \omega^2 \lambda \left( 1 - \frac{r^2}{R_\oplus^{\ 2}} \right) e^{-\frac{\lambda}{r}} \tag{58}$$

the solution can be written as follows:

$$r(t) = r_o \sin\left\{ \left[ 1 + \frac{\lambda^2}{2r_o} \left( \frac{r_o}{R_\oplus^{\ 2}} + \frac{\lambda^2}{3r_o R_\oplus^{\ 2}} - \frac{2}{r_o} \right) \right] \omega t + \theta_o \right\} \tag{59}$$

Dynamic gravity

Plot of the approximate solution

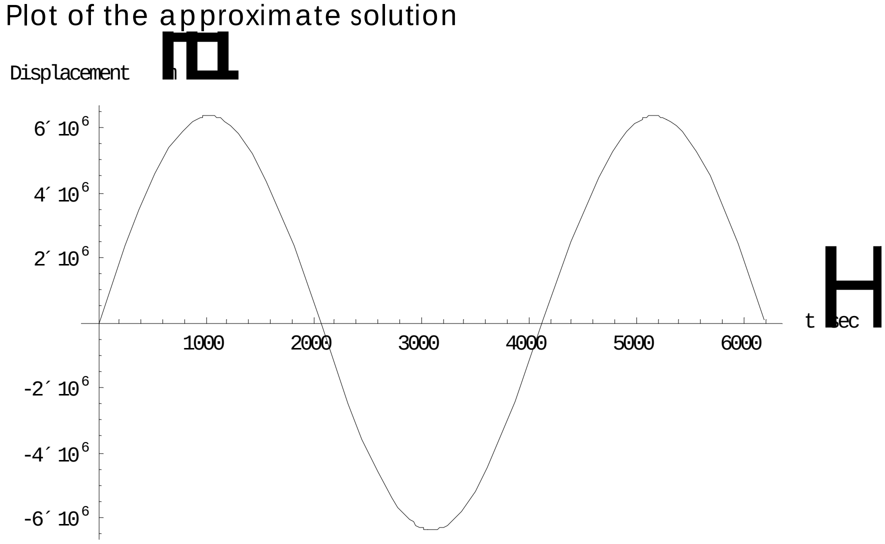


Fig 8 Displacement versus time graph of the linearized solution which has been derived as first approximation to the solution of the non linear harmonic oscillator. The non linear equation is derived from the dynamic gravity potential.

## Conclusions

The gravitational potential of a new theory of gravity namely the dynamic theory of gravity was used to study the classical problem of a mass m falling through a tube at the earth's center. As a first idealization the earth was considered to be a sphere of constant density. The differential equation of the motion derived can be thought as some kind of non linear harmonic oscillator. Next a variety of solutions were obtained for a variety of different initial conditions and some of the solutions were plotted. For the solutions chosen to be plotted we can see that the motion is periodic with an amplitude of oscillation slightly smaller in the case of dynamic gravity when compared to that of the Newtonian gravity. After that and for the solutions which were plotted subjected to the appropriate initial conditions expressions for the amplitudes of the motion were also given. Taking another approach the method of equivalent liberalization was used and a first order approximation for the solution of the non linear equation was obtained and plotted. This plot also demonstrated periodic motion similar to that of figures one and two. Finally a density function was assumed for the interior of the earth and solutions of the new differential equation of motion were obtained subject to four different initial conditions. These solutions were plotted demonstrating again the periodic nature of the motion, except figure four which demonstrates a motion that is periodic but does not cross the center of the earth. Again the linearized solution of the new equation was obtained and plotted demonstrating again the periodic nature of the motion. In closing we conclude that the motion of a body in a tube trough the center of the earth in the case of dynamic gravity resembles that of the periodic motion under Newtonian gravity.

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