# INTERFERENCE

## A Grand Scientific Musical Theory


Richard Merrick

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A Grand Scientific Musical Theory

Third Edition

Richard Merrick

To my wife, Sherolyn,
my daughter, Adrienne, my stepson, Matthew,
and my parents Doris and Rex.

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ISBN: 978-0-615-20599-1
Version 1.0
Printed in the United States of America

Cover art and inside illustrations by the author.

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# Preface

*"There is geometry in the humming of the strings, there is music in the spacing of the spheres."*
*- Pythagoras*

It should be said from the outset that this is not a book of Science. If it were, it would not discuss history, music, art or philosophy, as those are topics squarely in the realm of the Humanities. This is also not a book about the Humanities because if it were, it would not delve into the subjects of psychology, physics, physiology, genetics, mathematics or cosmology, being fields of Science. And this is certainly not a book about religion, spirituality or metaphysics, as these are strictly matters of faith. But this *is* a book about how everything can be better understood and explained through the framework of harmonic science, as it once existed in the Pythagorean philosophy of *musica universalis*.

Two and one-half years ago I decided to return to a research project in music perception that I had postponed thirty years earlier. It seems time had not diminished my curiosity about how we are able to *organically measure* the degree of dissonance and *mentally anticipate* the direction of resolution in music harmony. My original work in this area had taken me deep into mathematics and computer simulations in search of an explanation. Now, armed with the scientific method, powerful computer tools and access to the world's latest research, I was sure that I could determine once and for all whether our perception of music was something organic or nothing more than cultural conditioning. I had no idea that what I was about to learn would shake the very foundation of my 21st century worldview.

Triggered by a quiet moment of insight as a young musician, my investigations led me down a path of knowledge that has been known but well guarded for thousands of years. What I found was a long forgotten yet scientifically supported explanation for how we perceive music geometrically and how nature itself is structured as a kind of quantum music. Through the principles of harmonic natural science I also found an integrated worldview – a comprehensive "system of thought" and uplifting philosophy – that offered a warm alternative to the chilling scientism currently in vogue. Following this path to its inevitable conclusion was a liberating experience for me, as I know it will be for you.

Yet this is not a leisurely read. The subject covers a broad range of information and is so interdisciplinary that it can be a challenge regardless of your background. The simultaneous use of musical, mathematical and scientific terminology can be somewhat difficult to grasp at times, even though the universal principles of nature are quite simple and elegant. While essential ideas and terms are defined along the way, someone with a little musical training and some high school math and science will probably fare more easily.

Beyond unfamiliar terminology, the diagrams also require an investment of time to review and even more time to ponder. They juxtapose diverse concepts in harmonic philosophy, from the mythology of acoustics to the geometry of life and color mapping of the planets. As a result, some will not have the time to invest or see it as either too technical or conceptually abstract. When this happens, I urge you to forge ahead to the next topic or illustration that catches your eye. There are many different opportunities to understand the essential message.

If you have a specific category of interest, you may want to enter the book nonlinearly and circle back for background. To facilitate this, the work is divided into five sections: Social Thesis, Psychoacoustical Theory, Psychophysiological Principles, Harmonic Models and Physical Archetypes. But while each section focuses on a particular area of interest, they do build on preceding definitions and concepts. For this reason a glossary of terms and other definitions are provided in the appendix for reference.

Anyone curious about the real history of harmonic theory and how its *Diabolus in Musica* came to shape Western civilization should continue reading straight through. Learning the true unedited story behind the development of music was my entry into the study of harmonics, leading naturally into the deeper subjects that follow. But if your preference is to find out what harmonic science has to say about the very puzzling question of music perception, then you might want to skip to the second and third sections where a revolutionary new *Harmonic Interference Theory* integrates the fields of acoustics, psychology, physiology and music to explain exactly how we recognize and respond to coherent sound. If the subject of perception is not your first interest, then perhaps the organic visualization of music in the fourth section will be a more pragmatic entry point. Organic harmonic models brought to life using computer simulations are destined to revolutionize how we notate, compose and analyze music. In the fifth section, these same harmonic models are then used as physical archetypes to help describe all levels of nature – from the cosmos to the smallest quantum realm and all living creatures in between – as different instances of the same crystallized musical structure.

My approach here, which is essentially a modern rendition of the Pythagorean path to knowledge, has been to integrate the latest research from diverse fields into a wholly consistent system founded on the physics of harmonics. To aid comprehension, the more technical aspects of the system track along in the footnotes and are compiled in the appendices. Copious editorial is added along the way to help illuminate the relevance of harmonic principles to other topics of interest in natural philosophy, such as science history, social theory and the impact on personal belief systems, both religious and scientific. Some of these comments may inadvertently offend some readers and for that I sincerely apologize in advance. It is simply not possible to fully discuss harmonic science without including those particular topics needed to explain why we see nothing of this ancient knowledge system today.

With all of the warnings and disclaimers out of the way, I would urge anyone to consider the relevance of harmonic science and its attendant natural philosophy to your life. You have a birthright to know about this whether you ultimately accept it or not. You owe it to yourself to consider a different interpretation of nature, society and self than the one presently offered by the schools, churches and popular media. By the end of this book you will never see the world the same way again.

*"The noblest pleasure is the joy of understanding."*
*- Leonardo da Vinci*

During this long journey, many have given me their encouragement and support. From my earliest days, it is jazz great John Sheridan who I must thank for revealing the first secret of harmony to me. Without his jazz piano instruction I would never have broken through the veil. And to Lloyd Taliaferro and Joe Walston, who saw something in my crazy ideas, thanks for giving me the courage to pursue my real interests in blending music, math and computers. Likewise, I am deeply indebted to Robert Xavier Rodriguez who in those wonderful early days gave me the tools of a composer and the inspiration of a master. He continues to inspire me.

Many thanks to Mark White for taking the time to answer my questions about genetics, review my work and provide insightful suggestions along the way. I still have his DNA inspired toy to always remind me that life is the greatest puzzle of all. Thanks also to long-time friend Dan Reed for slogging through the early drafts, raising important issues and tuning his wife's piano to the geometry of a 15$^{th}$ century chapel. Special thanks go to my dad Rex for bravely proof reading the entire book (and being the first to actually finish it!) and to my mom Doris, who listened patiently for hours at a time as I struggled to find words for what I had found.

My thanks also go to Gaelan Bellamy, Sam Marshall and everyone in Distant Lights whose enthusiastic interest, attention to detail and requests for clarification during the final edits helped me find better ways to explain some of the more difficult points. And for publishing (and editing) my articles, my deep appreciation goes to Daniel Pinchbeck and Duncan Roads, whose extraordinary readers kindly shared their enlightening research and insights with me.

Others who have offered their support and encouragement along the way include Rusty Smith, Norris Lozano, Lana Bryan, David and Valeria Jones, Johnny Marshall, Lew Cook, Michael Browning, Scott Page, Brent Hugh, Stuart and Tommy Mitchell, Jim Von Ehr, Dennis Kratz, Frank Dufour, Steven Lehar and the amazing Dunbar family. And of course I shall be forever grateful to my wife Sherolyn, whose unwavering love, support and encouragement through the tough times made this project possible.

*March 2009 – Dallas, Texas*

**SECTION ONE**

# Social Thesis

*"Do not believe in anything simply because you have heard it. Do not believe in anything simply because it is spoken and rumored by many. Do not believe in anything simply because it is found written in your religious books. Do not believe in anything merely on the authority of your teachers and elders. Do not believe in traditions because they have been handed down for many generations. But after observation and analysis, when you find that anything agrees with reason and is conducive to the good and benefit of one and all, then accept it and live up to it."*
*- The Buddha*

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The greatest barrier in either understanding or making music must be the monumental task of learning all the rules. Everyone seems to have a theory and some set of rules to explain how music works – from Pythagoras and the Greeks to the many scholars of the Roman Catholic church in the Middle Ages; from Carl Phillipp Emmanuel Bach (son of J.S. Bach) to Leopold Mozart (father of Wolfgang) in the 18th century; and in the past century from Paul Hindemith to Arnold Schönberg, who devised a twelve-tone compositional system that broke every rule he had ever learned.

Given the preponderance of rules and exceptions to the rules (and exceptions to the exceptions!), we still find ourselves today with absolutely no unifying model for music that adequately explains historical usage or perception. No philosophy, no grand theory, no overarching logic to explain all the variations.  Just rules.  You're told simply that if you learn all the rules and practice, you might some day understand how music works.

My first encounter with these rules occurred when I began playing the piano at the age of eight. Well, the truth is I enrolled in a piano class at my school and practiced strictly by touch –

no sound for the first year. Wisely, my mother had decided to wait and see if I liked the piano enough to warrant purchasing one. So, I practiced on a short wooden board with raised keys painted to look like a piano keyboard. With simulated piano on my knees, I would diligently practice finger positions and patterns, imagining the sound as I did so. After a year of this silent practice, we bought a real piano so I could hear what I was doing. But, I really think it was the feel of the intervals spaced within my hands and patterns on the keyboard that attracted me to the piano.

In 1978 at age 23, I met professor John Sheridan, a phenomenal stride and swing jazz pianist. He had just completed a Masters thesis at the University of North Texas on Impressionist composer Claude Debussy and was beginning his college teaching career. As this was my first piano lesson with him, I was asked to demonstrate my own jazz improvisation skills. Given a simple song chart from a "fake" book that offered only a few chords and a simple melody, I was expected to improvise on the spot. I remember the dialogue between student and teacher very well.

*"It's not going to sound very good," I said defensively.*

*John replied: "Why not?"*

*"Because there isn't much rhythm to this piece and the melody is slow."*

*"Don't worry, go ahead and see what you can do," he said encouragingly.*

And so I began to play. He immediately stopped me.

*"Hold on – try it without adding all the rhythm and arpeggios. Those hide the harmony."*

*"Well, then, it really will sound horrible," I replied, fear leaking into my voice.*

*"That's ok," he assured me and I began again.*

This time without rhythm as my crutch, my naivety of jazz harmony was easy to see. In spite of all my prior jazz and classical training, thorough knowledge of scales, chord inversions, arpeggios, jazz voicings, music theory classes – *even professional stage and studio work* – my playing still sounded like a rank amateur. After a couple of painful and embarrassing minutes, he stopped the madness.

*"Have you ever heard of a tritone substitution?*

*"No," I said sullenly.*

My teacher then shared the greatest gift an aspiring jazz pianist could imagine.

> *"Play a dominant seventh chord with your left hand. Ok, now play a dominant seventh chord three whole steps, or "tritone," above with your right hand. Now, play them both at the same time. Yes. Now play them and resolve both hands to the major seventh chord a fifth below your left hand."*
>
> *With astonishment, I said slowly: "It sounds…like…jazz."*

Beaming, he then generalized from the specific:

> *"Yes, you can stack a chord a tritone above any dominant seventh chord. We call this a 'tritone substitution' or 'tritone sub' for short – jazz pianists use them all the time. Now, invert the chords and try some other voicings you've learned."*

I followed his instructions and as I experimented my euphoria grew. I knew that I had learned a Great Secret – I was in the club. But then my excitement faded and turned to confusion – even anger.

> *"Why haven't I read about this or learned this from my other piano teachers or in my theory classes?" I demanded.*
>
> *He replied casually, "They don't normally teach this in school. Anyway, you still need to know one more thing before you can play jazz."*
>
> *"What's that?"*
>
> *"Use tritone subs wherever you can – but then also try to play everything you ever learned about music theory all at the same time."*
>
> *Laughing incredulously, I said: "That's impossible! You can't play everything you ever learned about music theory all at the same time."*
>
> *"I know that," he said with a smile, "but while you try to play everything you know at once, you'll be playing jazz."*

It was after this that I realized I had not been told everything about music – not even the basics – and that some very educated musicians and highly regarded music theorists either were not privy to such information or just did not know how to present it as part of classical theory. How is it that one chord could be substituted for another or played at the same time and still sound good? In light of what I had learned, it seemed to me that the entire world was composing, playing and thinking about music *entirely on faith* without any understanding of what was really going on.

With that, the domino effect had begun. Within a few weeks of learning the Great Secret, I was improvising complex jazz changes, substituting other chords and parts of chords for standard jazz songs. I was stacking chords on top of other chords – not only tritone subs – while trying countless chord voicings that I had learned from books but never knew how to use. In the process, I was playing jazz.[1]

Other music started making more sense, too. Bach inventions, fugues and toccatas seemed more purposeful now. Mozart and Haydn sonatas fit better in hand and Beethoven, Chopin, Rachmaninoff and Schumann's chromatic chords and arpeggios made more sense than ever. But most of all, it was the "impressionist" Debussy whose fluid wholetone scales and pagan pentatonic clusters now seemed less radical and, well, more jazzy than before. Even the $20^{th}$ century's *enfant terrible* Sergei Prokofiev and his dissonant satirical chords were more natural and organic to me. Maybe *these* people knew the Great Secret and didn't tell anyone.

I became convinced there was something big going on *inside* music harmony to make it sound good – something fundamental involving the tritone, only bigger. I could feel similarity in patterns throughout all of the music I played, though I could not describe it nor could I find anything in my music theory books that explained it for me. I knew there must be a logic and consistency behind it all. Something much simpler than the collection of rules I had been taught in my music theory classes.

With an urgency I had not felt before, I decided to get to the bottom of it. I purchased an orange ring binder and began a quest to write and organize every combination of chord progressions and scale groupings imaginable to see if I could find something – a unifying pattern of some sort – that would explain harmony simply and logically. I went through countless possibilities and named them all so I could keep track of my ideas and not repeat dead end strategies.

For me, an arbitrary world where music was nothing but a blind set of "how to" rules was meaningless and unimaginable. I was determined to fix that.

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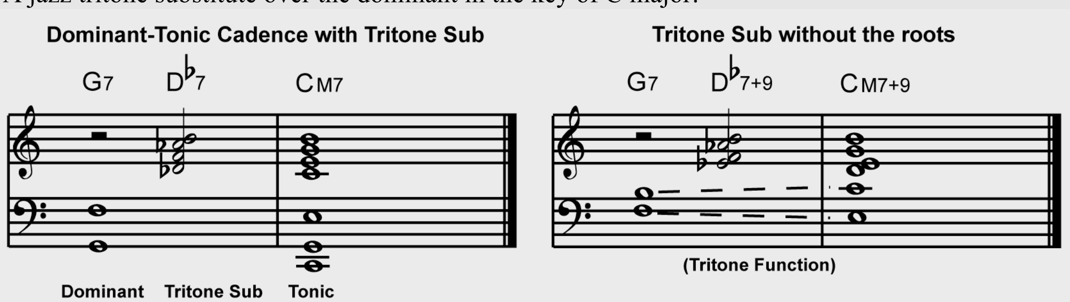
[1] A jazz tritone substitute over the dominant in the key of C major:



# *Spiral Stars*

*"I climb this tower inside my head*
*A spiral stair above my bed*
*I dream the stairs don't ask me why,*
*I throw myself into the sky."*
*- Gordon Sumner*

Most of us are familiar with Greek mathematician and philosopher Pythagoras (580 - 490 BC) for his famous Pythagorean theorem of right triangles. But few realize that his discovery was the result of a much older knowledge handed down from the ancient mystery schools. While it is very difficult to tell fact from legend, historical accounts by Aristoxenus, Dicaearchus and Timaeus indicate that Pythagoras trained for many years (some say 22 years) in the Egyptian mystery school, probably followed by a period of study in the Chaldean and Phoenician mysteries, before settling down in Crotana, Italy to start his own institute for the study of nature.

Known then as the "Ionian teacher," the "sage of Samos" and today as "the father of numbers," he and his followers the Pythagoreans believed that everything was related to simple numeric proportions and, through numbers, everything could be predicted and measured as rhythmic patterns or cycles. It was his study of proportion in musical tuning and scales (called *modes*) that led to his discoveries in mathematics.

Believed by many to be a savant and prophet in his time, Pythagoras is credited with discovering that the relationship between musical pitches could be expressed in numerical ratios of small whole numbers, such as 2:1 (an *octave*) and 3:2 (a *perfect 5$^{th}$*). But he also found that when he stacked twelve pitch intervals in repeating perfect 5$^{th}$ proportions, like that of a panpipe of reeds cut into a curve of 2/3$^{rd}$ proportions, it did not loop around to reconnect with itself at a higher octave as expected.[2]

Instead, Pythagoras found that pitch follows a logarithmic spiral of frequency into infinity. For example, when the last pitch in a stack of twelve perfect 5$^{th}$ intervals is transposed down next to the starting tone, there remains a small gap of about a quartertone[3]. This gap is known as a *Pythagorean comma* and causes melodies to sound "out-of-tune" when transposed to distant keys. As an unavoidable property in the natural logarithmic spiral of musical pitch, such gaps have been a continuing problem throughout history.

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[2] The Chinese used a similar panpipe bamboo system of perfect 5ths, called lüs, and similar systems were used in Tibet, Mongolia, Oceania, India, Russia, Africa and the Americas
[3] That is a complex ratio of 531441:524288 equal to 1.013643 or about 23.46 cents above the starting tone.

As music advanced over the next 2,500 years beyond a single voice in a single key and toward parallel melodies (called "polyphony") and vertical harmonies (called "homophony"), various tuning methods were developed to minimize the out-of-tune sound. This was done by slightly flatting each perfect 5th in the stack of twelve to force, or *temper*, them into a closed loop at the seventh octave. Thus, scale *temperament* is analogous to cutting the natural *spiral of pitch frequency*, stretching it to fit within a *perceived circular octave* and then adjusting each of the inside tones to sound less out-of-tune. In this way, musical scale temperament is nothing less than an attempt to close an infinite spiral into a closed circle.

With names like "meantone temperament" during the Renaissance and "well temperament" in the 18th and 19th centuries, each temperament method had its advantages and its disadvantages. Beginning in the 20th century, the 12-tone "equal temperament" method became the standard tuning method, dividing the cyclic octave into twelve equal logarithmic steps called semitones.[4]

To the Pythagoreans, musical temperament and modes were seen as the very geometry of sound and as such were associated proportionally with certain regular shapes. For instance, a stack of <u>five</u> perfect 5th intervals was associated with the five-pointed pentagram or "star" found in Sumerian, Egyptian, Babylonian, Zoroastrian and Roman Mithraism theosophies. Having learned about the pentagram during his time in the mystery schools, Pythagoras believed it to be the most important shape in nature and thus music.

This belief in perfect geometry, natural order and predictability was central to the Pythagorean worldview, as it had been to civilizations long before the Greeks. So when it was discovered that a stack of five perfect 5ths does not close to form a regular pentagram at the third octave as expected – forming instead an open and warped pentagonal shape in logarithmic pitch space – this was taken as a profound error in nature.

What Pythagoras found during his musical experiments was the last interval must be stretched up by a messy ratio of 128:81 (instead of the perfect 5th ratio of 3:2) to align with the third octave. In music lingo, the last interval must become an *augmented 5th* instead of a perfect 5th.[5] While very close to creating a pentagram, it still fell short of the perfection expected.

This imperfection is central to understanding the Greek worldview because it reveals a conflict, a paradox really, between the cyclic geometry of a regular pentagram and the Spiral of 5ths as it occurs naturally in sound. Philolaus (c470 BC – c385 BC), a "most ancient" follower of Pythagoras, referred to this paradox in the opening of his book *Peri physeos*, or *On Nature*:

> *"Nature in the cosmos is composed of a harmonia between the unlimited and the limited and so too is the whole cosmos and everything in it."*

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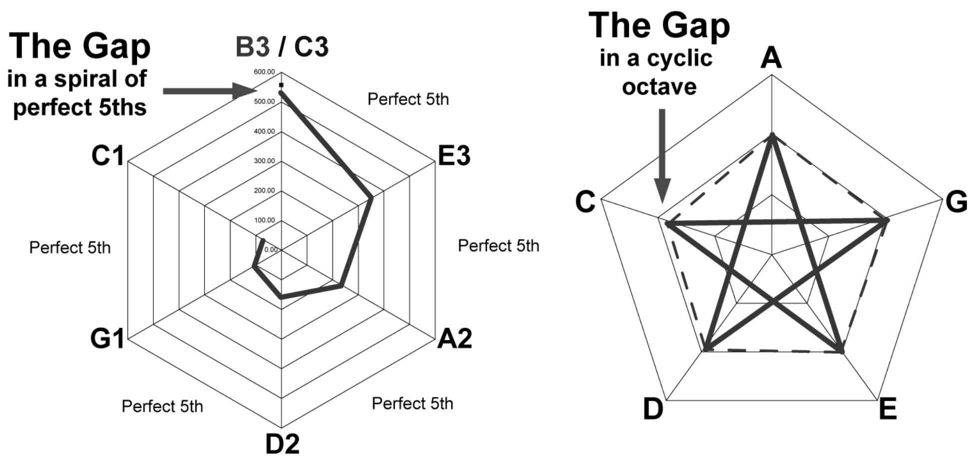
[4] Each step is spaced by a ratio of $2^{1/12} = 1.05946309$. In today's parlance, each semitone is measured as 100 *cents*.

[5] This extra stretch up totals about 90.22 cents or nearly a semitone.

**Figure 1. The Spiral of Five Perfect 5ths**




This gap between the closed or "limited" octave cycle and the infinite or "unlimited" spiral of pitch was a major embarrassment to the Pythagoreans because it undermined the purity of their philosophy of numbers and simple proportions. Given the importance placed on numeric proportions by the Pythagoreans, we have to wonder how they might have reconciled this error within their belief system.

With many early Pythagorean treatises lost or stolen, we are left with only the accounts of later Greek philosophers such as Philolaus, Nicomachus and Plato. From these accounts we know about Pythagoras' theories of numerical proportion, his tuning methods, the Greek modes and the supreme importance of his adopted symbol the pentagram.

As for the pentagram, Pythagoras appears to have first learned of it from his closest teacher, Pherekydes of Syros, who wrote a treatise entitled *Pentemychos* describing what he called the "five hidden cavities" of the soul. Of course, the notion that a geometrical shape could somehow be related to our "soul" sounds very mystical and unscientific to modern ears, but he was essentially correct about the pentagram playing a very important role in how nature organizes itself.

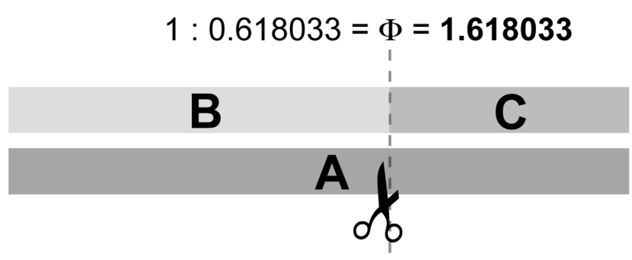
Beyond the obvious organizing principle of the number 5 in such things as roses, starfish and the human anatomy, the pentagram contains a very special numerical proportion known as the "golden ratio." If you have not heard of it before, the golden ratio is an infinite non-repeating proportion of about 1 to 0.6 usually represented by the Greek symbol *Phi* or "Φ" (pronounced either "fi" or "phee"). The most important thing about this ratio is that it is found approximated everywhere in nature, such as:

- *proportions of human, animal and insects arms, legs, hands and feet,*
- *branching of veins and nerves in humans and animals,*
- *branching of plant limbs, leaf veins and petal spacing,*
- *growth spirals of shells, the human ear and flower petals,*
- *proportions of chemical compounds and geometry of crystals,*
- *in a DNA molecule as a proportion between double helix grooves,*
- *as population growth,*
- *as stock market behavior (Elliott wave theory), and*
- *in the double spirals of a hurricane and spiral galaxy.*

Euclid described it best in the *Elements* as a balance of proportion between two lines:

*"A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the less."*

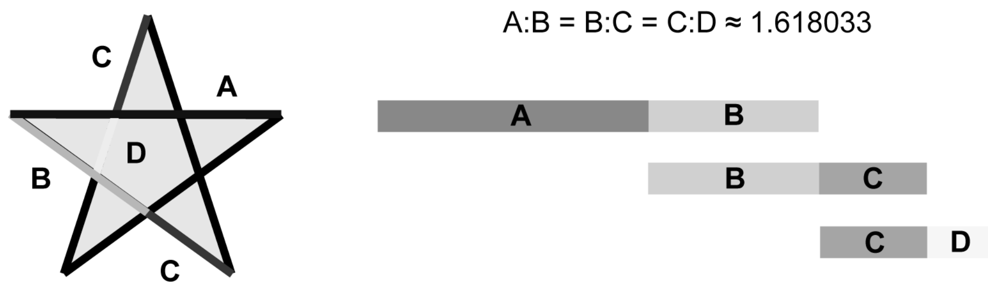
**Figure 2 - The Golden Ratio**




Like Russian matryoshka dolls nested one inside the other, the golden ratio represents a self-mirroring property in nature where cells divide and subdivide into a balance of two cell groups of about 61% and 39%. In many cases, this process continues, causing life to grow or "unfold" according to the golden ratio (*Phi*) equal to about 1.618033. For the sake of brevity, this constant of nature is usually referred to using only its Greek symbol Φ.

A great deal has been written about this mysterious number. In mathematics, the golden ratio is most often mentioned in connection with the five Platonic solids and the arithmetic spirals known as the Fibonacci and Lucas series (discussed later). But what is seldom mentioned is that each of these mathematical constructs inherits their golden ratio from a common source – the square root of five (or √5) in the intersecting lines of a pentagram. The following figure shows a pentagram with its four intersecting line segments in golden ratio to one another. In fact, every single intersection forms a golden ratio while each triangle is itself known as a golden triangle.

**Figure 3 - The Golden Ratio in a Pentagram**

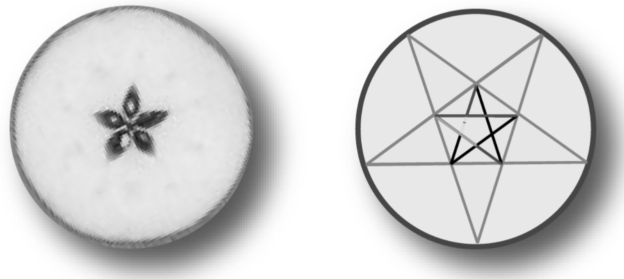



The Egyptians, and the Sumerians before them, held the pentagram and its golden ratio in very high regard. So much so that the golden ratio was preserved in the height-to-base dimensions of the Great Pyramid of Cheops, today known as the Egyptian or Kepler triangle. As a student of the Egyptian mystery school, Pythagoras was keenly aware of the golden ratio in the geometry of the pentagram and its occurrence in spiral formations throughout nature. His wife Theano, an Orphic initiate and mathematician in her own right, was said to have written a treatise on the golden ratio in honor of her husband after his untimely death. Sadly, it has never been found.

Many historians believe that this and many other such writings were once kept in the Royal Library of Alexandria (in Egypt), but lost when it was sacked by command of Caesar in the 3rd century and later burned by decree of Emperor Theophilis, Bishop of Alexandria in 391 A.D. Proclaimed as evil and heretical by Theophilis, up to a million ancient scrolls were destroyed by fire as acts of religious purification. As a result, we are left with only second hand accounts about how Pythagoras and his followers understood this mysterious 5-fold geometry and how it related to their philosophy of numbers and harmonic proportions. Many of these can be found in the musical allegories and harmonic archetypes of Greek mythology.

Greek god *Hecate* and goddess *Persephone* (also named *Kore*) were considered the rulers of the Underworld. The Underworld to the Greeks and Pythagoreans was the “inmost chamber” and the Core of Inner Being. There are numerous tales of Greek heroes, philosophers and mystics descending into the chaotic Underworld in quest of wisdom. Once in the Underworld, the seeker might meet the Goddess *Kore* who would offer an Apple of Knowledge. Not coincidentally, when a real apple is cut horizontally through the middle, it reveals a pentagram of seeds in its “core” and with it the forbidden knowledge of the golden ratio.

**Figure 4 - Recursive pentagram in an apple core**



This same symbolism can be found in the mythology of Pandora’s jar or box. On the fifth day of the month, Pandora opened her box scattering misery, hate and evil among the tribes of men. These were the Daimon seed of Eris (later the Roman *Discordia*), the goddess of war and discord, who were believed to bring about a heavy sickness on mankind. In fact, it was Eris, sister of murderous Ares, who threw a “golden apple” into the council of the gods, triggering the Trojan War. Thus, we find Eris or “error” inside the “golden apple” as its pentagram of seeds.

The apple as a symbol of error was passed down through Greco-Roman mythology into the well-known Christian symbolism of Eve’s “forbidden fruit” on the Tree of Knowledge. Early Christianity tried to reform the pentagram itself by using it as a symbol for Christ’s five wounds or Mary’s five joys of Jesus. Only later did the Church decide to reverse their positive opinion and associate the pentagram with Hell and Satan as an updated analog to the Greek Underworld. Today, the pentagram is still revered as a symbol of nature by various neo-pagan groups whose followers draw an outer circle around the pentagram to “bind together” the elements of water, fire, earth, air and spirit[6], *bringing them into harmony.*

The correspondence of the pentagram and golden ratio to harmonic philosophy can be found as a recurring theme in ancient mythology. Harmonia, the opposite of Eris, was considered the immortal goddess of harmony in nature. She was the daughter of Ares and Aphrodite (other sources say Zeus and Electra) and the mother of Ino and Semele. She was married to the Theban ruler Cadmus, and as such beloved by the Thebans. For her wedding she received a robe and a necklace, the latter bringing disaster[7] and death to all those who possessed it. In this parable based on musical symbolisms, the circular necklace appears to refer to the perfect cycle of an octave whose pentagram of perfect 5ths had been warped by the spiral of logarithmic frequency.

The irregularity and imperfection in the Spiral of Five Perfect 5ths was personified in Greek mythology in many other ways, but it was the apple and its pentagram of seeds that survived as

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[6] Spirit was also called *quintessence* or the *aether*.
[7] Disaster meaning “against the star.”

the universal symbol for imperfection in nature. From the undeniable evidence of 5-fold geometry in life, particularly the human body, Greek philosophers had no choice but to conclude the golden ratio was also present in the "inmost chamber" of our mind and somehow negatively influenced human behavior. As the central prop in the mythological Greek Underworld, the Apple of Knowledge was nothing less than the "root of all evil" and source of all Earthly strife.

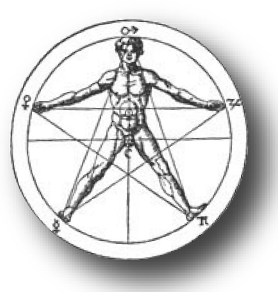
The burning question for Pythagoras and all Greek philosophers after him must have been: "How can we mend the spiral of pitch into a cyclic octave to repair the harmony in nature and save ourselves? What is mankind's salvation?"

These questions and pagan symbols eventually filtered down into Christianity as "original sin" and the salvation of Christ. But like the circle of friends who each tell the next what they heard until it comes back completely wrong, the original harmonic principles of Greek philosophy were incorrectly translated and misinterpreted, creating an aura of confusion and ambiguity around what it all really meant.

Today the pentagram is strangely regarded as a symbol of both good and evil. There are pentagonal stars on sheriff badges, the helmets of "America's Team" and the American flag while the U.S. Pentagon manages the nasty business of war. Stars are awarded to our children and stardom bestowed upon our celebrities, yet necromancers practice rituals with the pentagram they believe invoke and control nature's "elemental beings," a practice viewed widely as satanic worship.

Arriving to us from a dark and mysterious past, even our measurement systems for time and space were once based on the pentagram. During the Middle Ages the cosmological ordering of the planets were represented by the *Pentagrammon* showing the Moon at its center followed by five visible planets labeled clockwise from the top as Mars, Jupiter, Saturn, Mercury and Venus.

**Figure 5 - The Pentagrammon published in 1534**



The Sun was located at the center of the upper horizontal line of the pentagram by following Saturn up the leg of an imaginary superimposed human figure to the "solar plexus." Not coincidentally, the Sun's point of intersection along this path corresponds to one of the pentagram's golden sections.

The seven days of the week were also represented by a pentagram using the names of the planets starting with Monday (Moon day) followed by Tuesday (Tiw's or Mars day), Wednesday (Woden's or Mercury day), Thursday (Thor's or Jupiter day), Friday (Freya's or Venus's day), Saturday (Saturn's day) and then ending with Sunday, the Sabbath day of the Sun so important to both pagan and Christian worshippers.

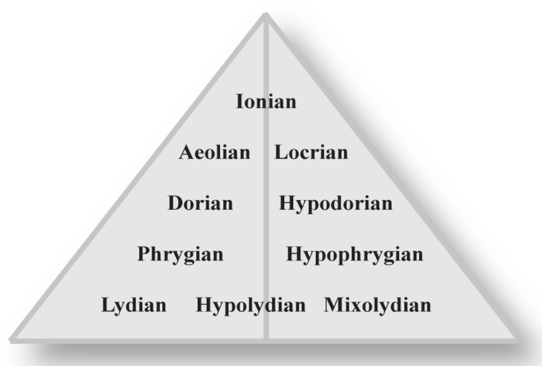
Altogether, the pentagram was the defining symbol for virtually all of the musical-astrological theosophies prevalent in ancient times. Passed down through Pythagoras from the ancient mystery schools as a measure of truth in nature, it became the ultimate symbol of natural knowledge in medieval Europe. And as a universal icon for harmony in the human body, the pentagram came to represent health and wellbeing. Today we unknowingly embrace this same philosophy when we say "an apple a day keeps the doctor away."

# *Harmonic Geometry*

*"Liberal and beautiful songs and dances create a similar soul, the reverse kind create a reverse kind of soul." - Damon, Athenian philosopher, 5th century B.C.*

Early Greek treatises identified ten scales, called *modes*, divided into five groups within Pythagorean tuning. These modes were used in music to create a sense of center or musical gravity known as *tonality*. They were usually grouped into the shape of a pyramid.

**Figure 6 - The ten Greek modes**




According to Aristotle's *Politics*, each mode was believed to be a form of persuasion and grouped according to masculine and feminine emotions. In fact, listening to a mode was believed to mold one's character, especially the young, and was known as *ethos*.

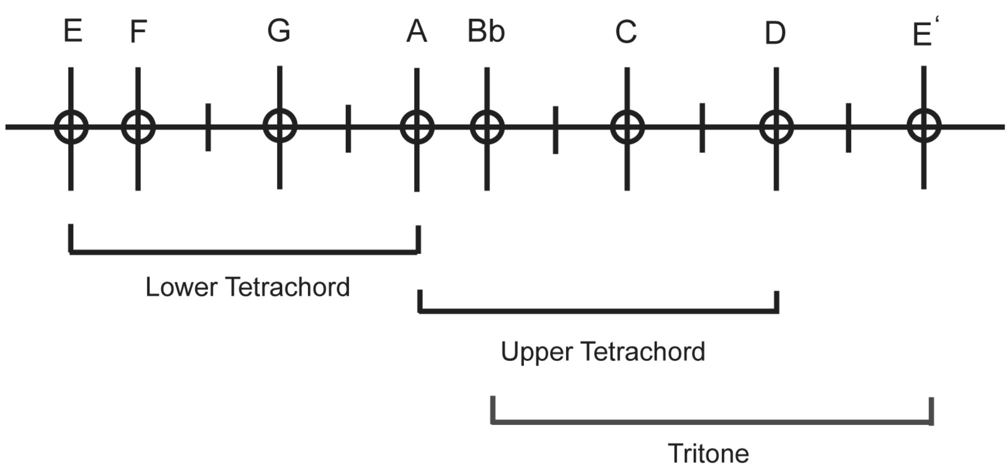
For instance, Dorian mode was said to produce a moderate ethos while Phrygian inspired enthusiasm. Both were considered appropriate by Plato to aid warriors in building the *ethic* of courage. However, Aristotle disagreed with Plato in that he thought the modes could also help purge emotions (known as *katharsis*), thereby purifying the mind. For instance, Mixolydian might be used to bring forth sadness as a form of psychotherapy [Comotti 1979].

These modes became the inspiration behind the modes used later in the Roman Catholic Church, leading to today's diatonic system of major and minor scales. In fact, the three "Hypo" modes approximate our "melodic minor" scale and foreshadowed the development of modern Jazz/ Blues scales. But while this is the conventional history normally taught in our universities, there is a much deeper origin to these musical scales that descended from Pythagoras and his harmonic geometry known as a *tetrachord.*

Each Greek mode is composed of two tetrachords – an upper "synemmenon" and a lower "meson" tetrachord. The outer interval of any tetrachord is found by dividing a string into 4 parts, creating a 4:3 ratio called a *perfect 4$^{th}$*. The two remaining internal intervals could then be tuned in a variety of ways, according to certain rules, to create the different modes.

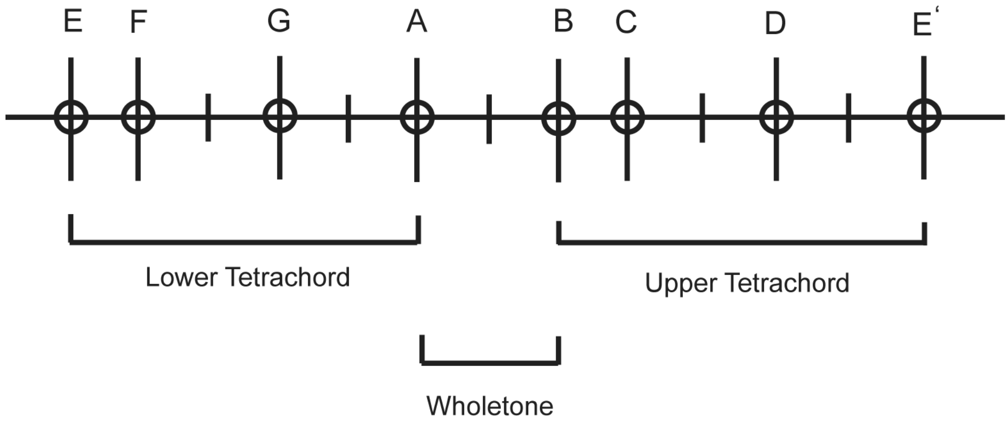
Originally, the two tetrachords were stacked end-to-end in what was called a *conjunctive heptachord* system that spanned only a minor 7$^{th}$ – one wholetone shy of an octave. This convention was due to the use of the 7-string lyre tied to the system of ethos thought to influence behavior. In fact, the number "7" was so central to Greek society and law that fines were sometimes levied against Greek musicians caught adding more strings to their lyre.

**Figure 7 - Conjunctive Heptachord System of Tetrachords**



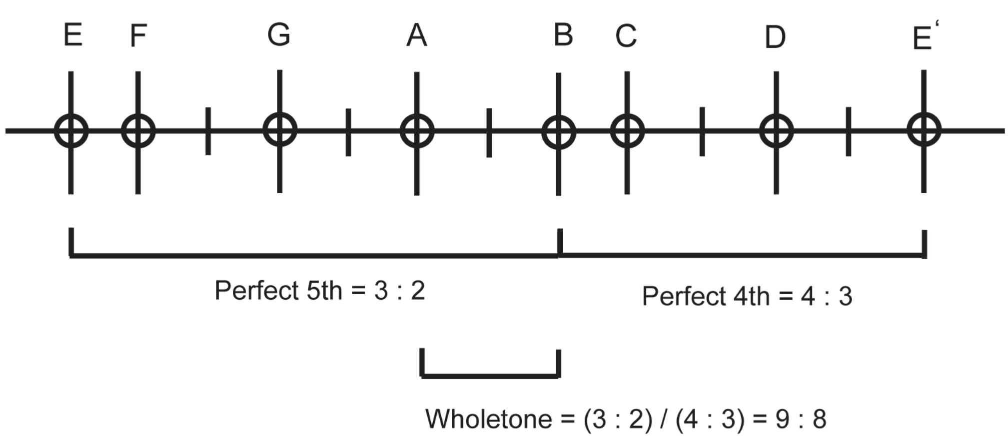

Still, some could not resist adding an eighth string to close the octave cycle. Doing so, though, often resulted in an undesirable side effect. It caused the outer interval of the top tetrachord to stretch from a perfect 4$^{th}$ to a much more dissonant interval spanning three whole tones or *tritone* ({Bb, E} in the figure). It was quite offensive sounding to those who expected to hear a perfect 4$^{th}$. Pythagoras, who also preferred an eighth string to complete the octave, decided to tackle the tritone problem by devising a new *disjunctive schema* to eliminate the tritone while still closing the octave. The senators ultimately accepted his solution, presumably due to Pythagoras' highly respected position on such matters, and the law was revised to finally permit the 8-string lyre as long as the accepted "Pythagorean tuning" was used.

**Figure 8 - Pythagorean Disjunctive Tetrachord System**




To make the disjunctive tetrachord system work, a wholetone had to be inserted between the upper and lower tetrachords. Pythagoras justified this by referring to the purity of the perfect 5$^{th}$ and perfect 4$^{th}$ intervals that were created between both the bottom and top of the mode. For instance, the intervals {E, A} and {E, B} can be considered either a perfect 4$^{th}$ or perfect 5$^{th}$ from either end of an octave. Besides this rationalization, there was a natural beauty in the sound of the two tetrachords balancing symmetrically around the center of the wholetone.

**Figure 9 – Wholetone ratio of Perfect 4th to Perfect 5$^{th}$**



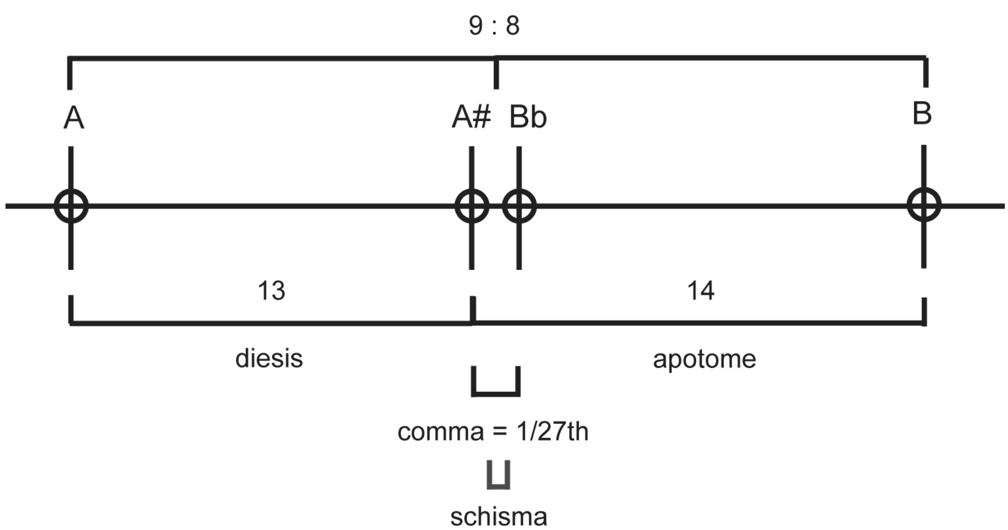

But while this new tetrachord system solved one tritone problem, another popped up. When Pythagoras tried to calculate the center of the octave by splitting the middle wholetone in half (as if it were a musical atom), he was surprised and embarrassed to find it did not result in a simple proportion as he would have hoped. Instead, he found a complex ratio of 256 : 243 resulting from the ratio of the tetrachord perfect $4^{th}$ to the product of the two inner wholetone ratios.[8]

Since this proportion is slightly smaller than the correct ratio of half a wholetone[9], it was given the special name of *leimma,* meaning "left over." So, within an octave constructed from two tetrachords separated by a wholetone, there were two leimmas, each called a *diesis* by Philolaus, to indicate a shortened semitone. At first glance, this would seem to solve the problem.

But when the five wholetones and two diesis semitones comprising the Pythagorean disjunctive scale were subtracted from an octave, yet another small fraction of $1/27^{th}$ of a wholetone still remained. This tiny gap, called a *comma*, was then added to one of the diesis to produce a slightly larger semitone called an *apotome* [Levin 1994].

This last adjustment finally spliced the spiral closed to create the "epitome" of a perfect circular octave.[10]

**Figure 10 - Division of the wholetone according to Philolaus**




[8] Splitting a wholetone in half:

$r$ = perfect $4^{th}$ / (wholetone × wholetone)

$r$ = (4 : 3) / ((9 : 8 × (9 : 8))

$r$ = (4 / 3) / (81 / 64)

$r$ = 256 / 243 = 1.05349

[9] Correct split value of a wholetone: (√(9/8) □ 1.06066),

[10] *Epitome*: Greek origin – *epitemnein* 'abridge,' from *epi* 'in addition' + *temnein* 'to cut'

Since this explanation of the octave comma by Philolaus far predates Plato's description of the "Pythagorean comma" at the seventh octave, we could rightfully call it the real Pythagorean comma. Fact is, this comma was also documented in ancient Chinese music theory as one-third of a step within an octave divided into 53 equal steps, or "53 Equal Temperament" (53-ET). But since Plato's definition of the Pythagorean comma has been long accepted (and historical convention is notoriously difficult to overcome), we will instead refer to this comma simply as the *Philolaus octave comma* calculated as the ratio 9:8 / 27. In a civilization founded on the ethos of harmony, this became a very important proportion in Pythagorean and later Greek philosophy.

As we already know, the small gap between a closed octave and an open spiral was nothing less than heresy to the Pythagoreans. The inability to evenly divide a wholetone using simple whole number proportions represented a profound error in the cosmos that threatened everything the Pythagoreans believed. Indivisibility at the center of an octave was taken as proof that an intrinsic chaos or evil force existed and that it could be found everywhere in nature. The Pythagoreans became absolutely convinced nature was broken.

This belief was especially evident in the Pythagorean name for a half-comma[11], which was *schisma* meaning split or crack. It is important to note that this crack at mid-octave corresponds to the undesirable tritone interval mentioned earlier. Because of this correspondence with the schisma error, the tritone interval also became associated with the concept of error in nature. To this day, even with the great strides in scientific achievement and technology, the tritone remains a paradox and a point of heated controversy between musicologists concerning scale temperament and how it should be handled in music theory.

It stands to reason that if we can grasp the underlying cause behind the schisma created by the tritone, we should be able to understand how this reconciles within the Pythagorean belief of harmony in numbers. Certainly, without an answer to this mystery we can never hope for a complete explanation of music harmony and how we perceive it.

For the Pythagoreans and later Greek philosophers, tetrachords were considered to be the auditory geometry of a "perfect" 4-sided, 4-pointed tetrahedron solid. As a geometrical model for music, the two tetrachords in a Greek mode could have been intended to represent two opposing and interlocking tetrahedrons balanced around a shared center, thereby creating the 8 vertices of a cube or *hexahedron*. Alternatively, the vertices (or points) of this cube could have also been intended to pinpoint the center of 8 triangular faces that form its geometric opposite, the *octahedron*. In either case, the geometric perfection represented by two balanced tetrachords is the founding principle behind the Greek modes and thus even our present major and minor system of scales.

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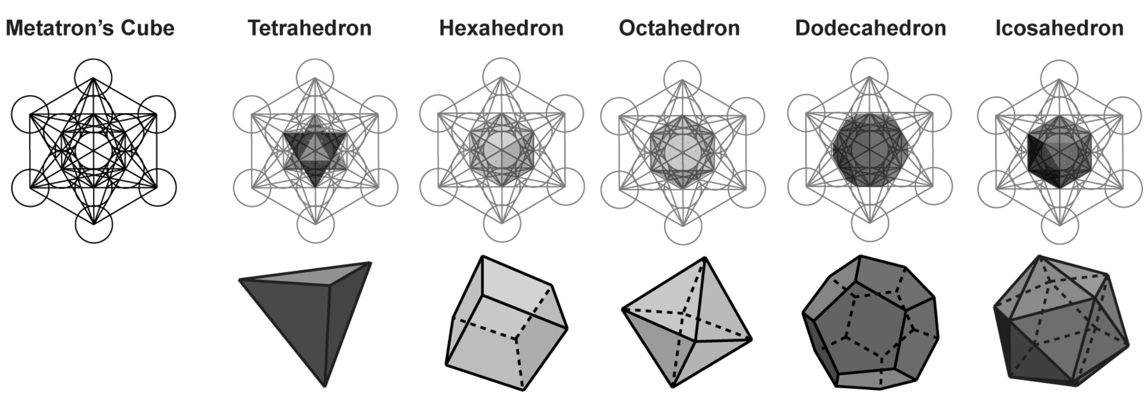
[11] A half-comma, or *schisma*, is equal to 9:8/13.5 = 0.083333333.

In the 20th century, Buckminster Fuller found that octahedral and tetrahedral geometries can be alternated to form a uniform tiling of space, called an *octet truss*, which is both very efficient and extremely sturdy in architecture. Fuller coined the term "tensegrity" to describe "floating compression," an idea suggested to him by one of his early students, the natural sculptor/ artist Kenneth Snelson. It is well known today that tetrahedral lattices create the strongest structures possible, found in such things as geodesic domes and stage trusses, yet few are aware that the idea actually originated thousands of years earlier in the geometry of Greek music.

Greek geometers found that tetrahedrons could be used to construct more complex perfect shapes. For example, a pair of octahedrons can be balanced to form either the 12 vertices of an *icosahedron* or 12 pentagonal faces of a *dodecahedron*. Because of this, tetrachords, octaves and a stack of twelve perfect 5ths were associated with the geometrical shapes of the *five perfect solids*, which the Greeks considered integral to the structure of the universe. It seems they were even right about this, since these same shapes have been found to occur naturally in the carbon allotrope molecules of soot and graphite called Fullerenes or *buckyballs* (after Buckminster Fuller). With carbon the most stable element in the universe and the foundation for all life, the Greek worldview based on musical geometry was correct in more ways than they could have known.

Some 200 years after Pythagoras, Plato wrote about the five perfect solids – the tetrahedron, hexahedron, octahedron, icosahedron and dodecahedron – in his dialogue *Timaeus* c360 BC. These same five shapes remain to this day the only known geometric solids where the sides, edges and angles are all congruent and fit neatly within a sphere.

**Figure 11 - The Platonic Solids from Metatron's Cube**



It so happens that each of the perfect solids, or so-called *Platonic solids*, can be derived from a single 6-point geometrical figure known as *Metatron's Cube*. Extracted from the *Flower of Life* pattern dating back to a 6th century B.C. stone carving at the Egyptian Temple of Osiris (and later studied by the Pythagoreans), this geometrical pattern results from an arrangement of thirteen circles that act as nodes from which a diagonal lattice can be constructed. Interestingly, each of the five solids fit perfectly inside the Metatron's Cube intersecting lines, indicating a shared mathematical relationship between tiled 2-dimensional circles and regular 3-dimensional geometry. It is because of this profound correspondence that Metatron's Cube played such a key role in the mysticism and symbolisms of Judaism (as the Star of David) and early Christianity.

But there is another very interesting mathematical relationship in the perfect solids and Metatron's Cube that holds a clue about how Pythagoras may have designed the tetrachord. An icosahedron can be constructed by cutting each vertex of the octahedron by the golden ratio, producing five octahedral that can be used to define any given icosahedron or its dual dodecahedron. Applying this to the 10 Greek modes, their male-female ethos could then be represented geometrically as an icosahedron (5 octahedral) and a dodecahedron (another 5 octahedral). Since the golden ratio is involved in both of these perfect solids and is intrinsic to the geometry of Metatron's Cube, Pythagoras may well have designed his tetrachords around this constant of nature.

Without explanation, Pythagoras designed three variations, or *genera*, of disjunctive tetrachords that could be used in various combinations to construct the ten Greek modes and their dual ethos. But, when we look closely at the intervals inside the tetrachord genera as a unified system, we find compelling evidence to support the idea that the golden ratio is at the bottom of his musical geometry.

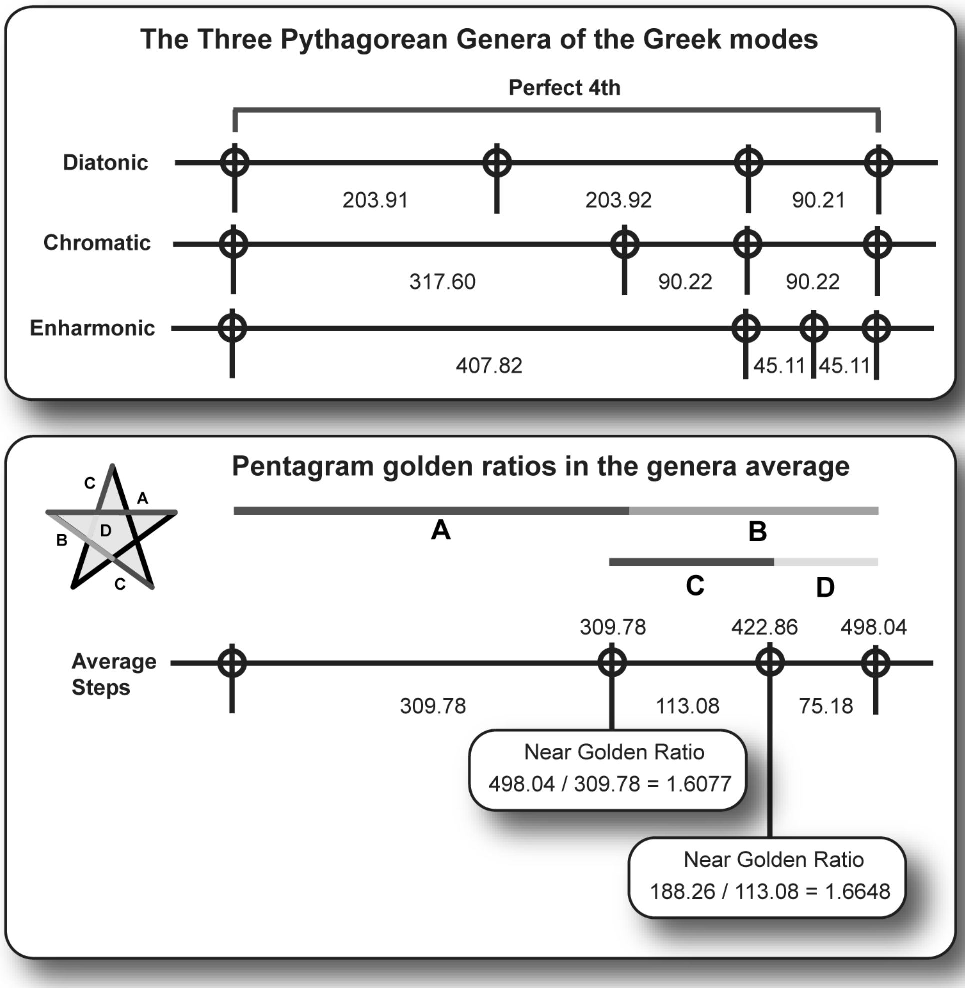
From visual inspection alone, it is obvious in Figure 12 that the internal tones are not spread evenly inside the tetrachord's perfect 4th interval[12]. Instead, they are clustered toward the upper end. This weighting of intervals in the Greek tetrachord genera is more easily seen when we average all three tetrachord variations together. Incredibly, the result is a statistical weighting of the genera system near a golden ratio between the second tone and the outer perfect 4th.[13] Even more incredible, there is yet another near golden ratio between the third and fourth tones.[14]

Could Pythagoras have been attempting to geometrically balance five "female" modes as an auditory icosahedron and five "male" modes as an auditory dodecahedron – all around shared golden ratios? If so, what can we say is the underlying geometry that could make this work?

---

[12] In Pythagorean temperament, a perfect 4th is 498.04 cents, slightly smaller than the 500 cents used in today's equal temperament.
[13] 498.04 / 309.78 = 1.6077
[14] 118.26 / 113.08 = 1.6648

**Figure 12 - Greek tetrachord scale designed according to a pentagram**




Looking back at the earlier discussion, we now find that the tetrachord genera average of one golden section nested inside another golden section to be *the same as that found in the intersections of a pentagram*. Furthermore, the ratio of the difference between the bottom interval and middle interval versus the ratio of difference between the middle interval and the top interval is almost precisely a 3:2 proportion, very close to the ratio of a perfect $5^{th}$ to an octave.

We have to ask ourselves is this simply a coincidence or might Pythagoras have tried to resolve the spiral vs. circle conflict by engineering it this way? While the 4:3 proportion of the perfect 4th tetrachord interval seems like a natural choice for a scale, the selection of internal pitches found in the three tetrachord genera were clearly designed by Pythagoras for a reason. He seems to have built the entire tetrachord genera system around the pentagram's golden ratio in an effort to form five male and five female modes of persuasion compatible with the five perfect

(Platonic) solids contained in Metatron's Cube. But why would he do this? What would convince him that such geometry is connected to the physics and physiology of music perception?

The answer may be that the same pentagonal geometry underlying the tetrachord genera can be found in the orbital relationship between Earth and the planet Venus. It is an astronomical fact that Venus traces a near perfect pentagram in the Earth's sky every eight years. As a planetary harmony, Venus rotates slowly in the opposite direction to the Earth (and most other planets) with its day two-thirds of an Earth year – the same 3:2 proportion of a musical perfect $5^{th}$. Thus, as eight Earth years equal thirteen Venus years, Venus always faces Earth in the same position five times to trace a near perfect pentagram in space. The orbital ratio 13:8, equal to the Fibonacci number 1.625, is again close to the golden ratio and accounts for the pentagonal "star" geometry and its importance in the ancient mystery schools.

Both the 3:2 and 13:8 proportions in Venus's orbit correlate directly to the tetrachord's perfect $5^{th}$ and golden ratio approximations. And as we saw with the warped Spiral of 5ths, Venus also does not close to a perfect pentagram, confirming the ancient belief that some kind of "error in nature" applies not only to music but also to the planetary orbits. Well-known and revered in ancient times, the celestial pentagram formed by Venus – the fabled *Star in the East* and *Star of Bethlehem* – was without a doubt the inspiration behind Pythagoras' philosophy and his musical designs. The direct correspondence between numbers, geometry, musical proportions and astronomical observations were more than enough evidence to convince Pythagoras and many other philosophers that life and perception work the same way.

Admittedly, Pythagorean scholars could take issue with some of this. After all, we do not have enough evidence to support any specific claim about Pythagoras' personal motives and intentions in the design of the tetrachord genera. And while Eudemus of Rhodes (c370 BC – 300 BC) claimed that Pythagoras discovered all five of the perfect solids, other evidence suggests that he discovered only three (the tetrahedron, hexahedron and dodecahedron) with the octahedron and icosahedron documented a couple of hundred years later by Greek mathematician Theaetetus (c360 BC). Nonetheless, Pythagoras must have known about Venus and the golden ratio from his years in the Egyptian mystery school, making it more than a coincidence that his musical system reflects this geometry as well as it does.

So, given the preceding assumptions, each group of eight tones in a Greek mode could have been designed to represent a musical octahedron (associated with air) composed of two tetrahedrons (associated with fire). The upper five modes could then have been defined as Ionian, Aeolian, Locrian, Dorian and Hypodorian – together comprising a dodecahedron that was typically used to represent the male gender and aether (middle realm) of the cosmos. The lower five modes would then be Phrygian, Hypophrygian, Lydian, Hypolydian and Mixolydian – together comprising the feminine gender and icosahedral geometry of water. These two groups of five would then have been paired to produce a dual male-female ethos of ten musical modes:

| ***Music harmony*** | ***Geometric Harmony*** |
|---|---|
| *2 tetrachords = octave* | *2 tetrahedrons = octahedron* |
| *cut an octave with golden ratio* | *cut an octahedron with golden ratio* |
| *5 upper "male" modes* | *5 octahedrons = dodecahedron* |
| *5 lower "female" modes* | *5 octahedrons = icosahedron* |

Continuing with this hypothesis, each tetrachord genera in a given mode could have been seen as balancing around a positive (male) golden ratio and a negative (female) golden ratio found in the middle of the diesis or apotome semitones – just as the golden ratio slices an octahedron into a dodecahedron or its dual icosahedron. The Pythagorean concept of ethos would then be described as a tug-of-war on either side of these two golden ratios, pulling emotions positively or negatively, up or down like shades of color. Indeed, a particular tetrachord combination would be seen as a designer blend – like a recipe – of these emotional forces that could be used to influence or persuade character in a particular way.

The implications of this are staggering. If the Greek philosophers were right, then the human psyche must be organized geometrically something like an octahedron or octave in music with the two counter-posing golden ratios at work inside our brain. Perceiving music would then be a matter of *physically* matching musical harmonies to identical proportions built into the structure of our brain.

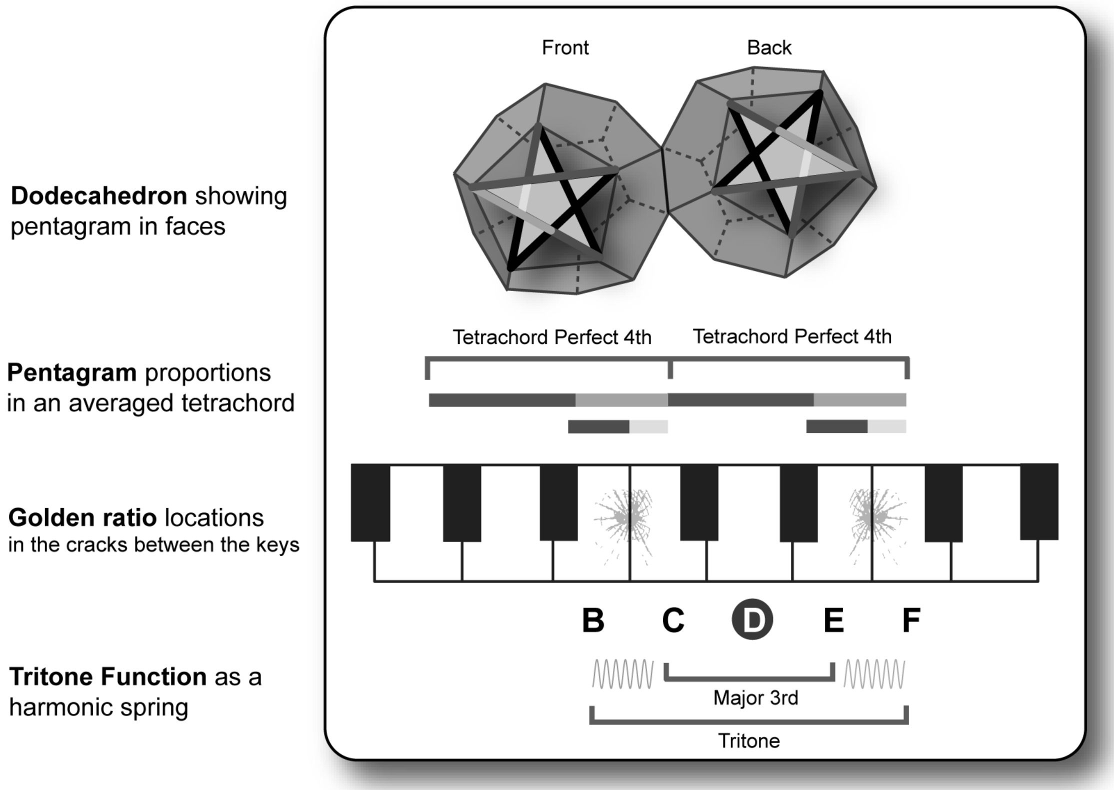
Ok, so how can this be used to explain common musical practice?

When this tetrachord geometry is applied to an equal tempered piano keyboard, the same approximate pentagonal proportions can be found in the musical scales we use today. In Figure 13, the middle interval again falls into the two semitones (or "cracks between the keys") in a {C} major scale. While we could never play all of the Pythagorean tetrachords in equal temperament because of their use of quartertones, equal temperament still comes very close to the pentagonal golden ratios first designed into the Greek modes some 2,500 years ago.

By simply rolling a dodecahedron along a piano keyboard, the two golden ratios in the averaged tetrachord genera will naturally fall between semitones {B, C} and {E, F} or intervals of tritone {B, F} and major 3$^{rd}$ {C, E} in the key of {C}. Much more than just a nice theory, playing these two intervals back and forth on a piano will create the audible sensation of a rubber band or harmonic spring being stretched and then released. Perhaps Pythagoras heard the same thing when he tested his tetrachord system on a lute, then decided to build around this musical spring action to mend his warped pentagram of perfect 5ths into a closed star.

In any case, Pythagoras must have believed that the two golden ratio locations in an octave have a measurable effect on our emotions. As a balance of opposites in a {C} major scale, the two golden ratios and their corresponding tetrachord pentagrams certainly do seesaw symmetrically either side of {D}. Could it be that the balance of the dual golden ratios and the tritone, with its irreconcilable schisma, all play a tangible and even *physical* role in music cognition? Could our natural recognition of these symmetrical proportions have guided the development of music throughout history in the same way they guided Pythagoras?

**Figure 13 - Golden ratio locations in a C major scale**




When we cross-reference the Pythagorean tetrachords to conventional music theory, this does indeed seem to be the case. Hundreds of years of Western music practice have consistently defined the tritone interval as "unstable" with a "strong tendency" to resolve. Furthermore, conventional "voice leading" rules hold that the tritone resolves best in a contrary (or symmetrically opposing) fashion toward the major 3$^{rd}$ interval of the tonic triad [DeLone 1971]. Known as the *Tritone Function*, this universally recognized and historically documented spring action of the tritone (across the two golden ratios) is considered the strongest harmonic function in music. In this way, the Tritone Function could be described as a cognitive *Pythagorean* *mending function* in harmonic music that acts to fill the gaps across the infinite golden ratios to form closed auditory geometries.[15]

Given the correspondence of regular geometry and the golden ratio to historically preferred musical scales, perhaps it is time we consider the possibility of geometry as the first principle of music perception. Maybe the physics behind such geometries have actually *guided* the historical development of music rather than the conventional wisdom that music was an accident of man-made rulemaking. Maybe the golden ratio really does play a role in the physiology of our auditory system as a proportion we can instinctively recognize. If hard evidence can be found that the golden ratio is a physical property of acoustics and perception, as Pythagoras apparently believed, it could well become the unifying foundation for a new theory of not only music, but also human perception.

But before we can approach music scientifically, we need some kind of historical context to understand how music came to be separated from the study of nature. How could it be that the idea of musical geometry was lost along the way and has never reemerged? Why is the study of harmonics not a dedicated field of science or a central component of music theory? What exactly went wrong in Western history to leave Pythagorean harmonic science behind on the road to enlightenment as if it were some kind of metaphysical road kill?

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[15] **Hypothesis 1:** The tetrachord genera were a Pythagorean "mending function" for a musical octave, joining the Spiral of 5ths and octave cycle into a pentagram at two octave golden ratio proportions inside what is today known as the *Tritone Function*.

# *Devil's Trident*

*"Somehow our devils are never quite what we expect when we meet them face to face."*
*- Nelson DeMille*

Using the Greek system of musical ethos as a model, the early Roman Catholic Church began to develop new rules for what music was acceptable during services. Music that did not adhere to the rules for "sacred music" was then considered impure. More specifically, some intervals were considered "perfect" while others were "imperfect." Of all the intervals in an octave, the most impure and imperfect musical interval was the tritone that divides an octave.

The tritone was considered not only an unfit and unpleasing interval by the Church fathers – it was believed to be an *evil* interval that could adversely affect our character when used in music. It was even referred to as *Diabolus in Musica*, or *Devil in Music*, and expressly forbidden under Church canon law. To this day the Church officially maintains a policy of tritone avoidance as set forth in the decree of "universal liturgical music in Gregorian chant," most recently reaffirmed in the 1903-1967 *Musicam Sacram* [Joncas 1997]. Because of this law, the tritone has remained off limits to church composers for many hundreds of years and prohibited from all forms of "sacred" music.

One specific form of church music – the *Old French Canon* – is perhaps the best example of this anti-tritone doctrine. Named after the Greek *kanon* for rule or law, a musical canon is a type of contrapuntal music involving imitation between two or more voices. Most would recognize it as a simple *round,* found in such children's songs as *Frere Jacques* or *Row, Row, Row Your Boat*. But it is much more than this.

As the story goes, English troubadours, French jongleurs and German minnesingers would travel from town to town during the $11^{th}$ century, playing instruments, singing and otherwise entertaining the villagers. As more than one singer would perform together, they would improvise and mimic one another in melody, thus creating a round. As rounds increased in popularity, the Church was compelled to incorporate them into its services. But this was not so simple.

In order for musical rounds to be accepted into the Church, they had to be written in accordance with the canonical rules that prohibited use of the tritone (which occurred routinely during improvisation). Eliminating this possibility then required a much more scripted and rules-driven approach, transforming a simple round into the proper *canon style*. With *Diabolus in Musica* eliminated, the canonical round was then acceptable for church services.

Speaking personally, I find more than a little irony in all this. First, the Church's canonical rules for music targeted the humblest of all songs – the playful rounds sung by innocent children and plain town folk as they imitated one another in song. Second, the Church had restricted the most natural form of musical expression possible, stealing away the pleasure and pure joy found in improvised harmonies. Third, with rounds firmly under the control of "canon" law, they were given the name of the very ecclesiastical legal system itself!

But the greatest irony is this. In spite of the Church's sacred act of musical purification – in the face of a disapproving clergy and fear of damnation – the cheerful round survived and remained quite popular in secular society. Children continued to sing them with great delight (just as they do today) while their parents clapped along, holding little concern for any mysterious side effect that might result from the Devil's interval.

Now, the inevitable questions may begin.

Why? Why would such an evil be thought to exist in music, much less this particular interval? What should be so horribly offensive in dividing an octave in half (as the tritone does) that it should be singled out and banned from all Western sacred music through an elaborate set of canonical rules? Was this effort purely based on religious symbolism or was there really something dangerous in the tritone that could hurt us? Could it negatively influence our hearts and minds, perhaps turning us toward evil? Many have speculated on this.

Some say the tritone represents the Devil because it is a dissonant interval with an irreconcilable split ratio of 7:5 (augmented $4^{th}$) or 10:7 (diminished $5^{th}$) as found in meantone temperament. But dissonance cannot be the only reason. The tritone is not much (if any) more dissonant sounding than the intervals of a minor $2^{nd}$ or major $7^{th}$ and no one thinks they are devilish. They're not even naughty.

Some say it was the Devil in music because the tritone is so close to the interval of a perfect $5^{th}$ that two monks could too easily sing dissonantly as they tried to chant in pure parallel 5ths. But this cannot be the only reason because when they sang out of tune anywhere else, *those* wrong intervals weren't the Devil. They were just out of tune.

An April 2006 article in BBC News Magazine quotes Bob Ezrin, a former business associate of mine and music producer of rock bands like Pink Floyd, KISS and Alice Cooper, as saying: "It apparently was the sound used to call up the beast. There is something very sexual about the tritone."

While walking the streets of London some years back, Bob and I had discussed this subject and conjectured that the symmetric contraction of the tritone must have been taken as a symbol of symmetry in the human body and thereby sexuality and carnal knowledge. At the time, this was the only reason I could imagine for its exclusion from the Church and avoidance in music theory.

There were no other psychological or physiological studies I knew that suggested the tritone was some kind of harmonic Viagra to enhance feelings of sexuality. But even if this were found to be the case, surely the procreative act should be considered a beautiful spiritual experience.

Probably the most common reason given for the evil reputation of the tritone is its connection to the number "666," the *Number of the Beast* referenced in the Biblical Book of Revelation. The importance of this number appears to have originated in the ancient Hebrew practice of *gematria*, or number geometry known today as *numerology*, where the tritone's 3 wholetones (the Devil's Trident perhaps) spanning 6 semitones could have suggested three consecutive sixes.

A more likely gematria theory correlates to the number 216 as the ancient Hebrew symbol for God. It was believed that finding the missing code for this number would bring about the return of the Hebrew Satan in a final showdown, thus triggering a Messianic Age of peace. Not coincidentally, the cube root of 216 is six, or 6×6×6. Perhaps the missing code for the Hebrew Satan was once associated with the tritone.

Similarly, when 6 tritones, each composed of 6 semitones, are stacked over 3 octaves it surrounds the pentagonal cycle of perfect 5ths with a triple hexagonal cycle of tritones, or 666. In the end, it seems to make little difference which theory we choose to accept because alignments with the old Beast of Christianity pops up everywhere we look. As we shall see later, these sixes are neither coincidences nor silly mysterious numerological symbolisms, but are in fact numerical proportions related to the physics of highly resonant vibration. Still, the speculation surrounding the tritone continues because nowhere in the mountains of religious or scientific literature will you find it taken seriously and explained as a natural property of acoustics.

In the final analysis, the most likely justification for the tritone's evil reputation arrives to us from the harmonically inspired mythology of the Greeks and the "error" Pythagoras found in the irreconcilable schisma at mid-octave. This error, intertwined with the pentagram, golden ratio and Devil's interval were nothing less than the Biblical forbidden fruit with the tritone itself an audible version of "original sin."[16]

Yet even when we disregard religious and numeric symbolism there remains a real sensation of *tension* in the tritone and an anticipated *tendency* for it to automatically spring closed. We don't really need religious symbolisms and stories to see how the priests could have concluded that something demonic was involved. At a time of deep mysticism and belief in satanic forces, the tritone was probably seen as some kind of daemonic planchette, sliding along a musical Quija board under its own supernatural power.

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[16] **Hypothesis 2:** The negative reputation of the tritone interval in music history is due to its association with the pentagram and its contained golden ratio, thought to reveal an error in nature and, thus, in mankind as "original sin."

Far more than just a musical concept, the tritone offers us a window into the collective psyche, touching on our concept of God and belief in good and evil. Like an anthropological microscope equipped with a musical lens, the tritone allows us to see inside a society shaped by the Roman Catholic Church. Indeed, it brings into focus a world still largely under the influence of its anti-pagan, anti-harmonic doctrine.

Cast as an evil force and anti-Christ figure in the Underworld of music, the tritone is still avoided in many ways. With a history of embarrassment and ill repute, the tritone remains little discussed in academic settings for fear of offending the faithful. Textbooks simply refuse to discuss the history of the tritone, much less suggest it as a central tenet of music harmony. And never will you see the pentagram or golden ratio explained as having any functional role in music, these also being related to Pythagorean harmonic science. Instead, music theory and composition continue to be taught historically based on religious tradition, amended as needed by an ever-growing number of exceptions to the rules.

If we ever hope to solve the mystery of music harmony and our perception of it, we must question these outdated conventions against all criticism. We must stand in that dreadful position at the center of the octave schisma, where harmony seems most irreconcilable, and try to understand the tritone. We simply must force ourselves to explain what it means to mend a warped pentagram into a closed circle using the Tritone Function. And in spite of the many skeptics and scientific debunkers who see the golden ratio as having nothing to do with music, we must objectively explore what physical role it could actually play in sound and perception.

We need to ask what is it about the tritone that is restless or has a spring-like tendency to move. Is it in our head or *out there* in the air somewhere? Who made these rules and, more importantly, why were they broken? Was it a good thing or a bad thing that the tritone was forbidden by the Church? And how might the avoidance of the tritone have impacted our present theories and educational doctrine concerning music and the natural sciences?

# *Medieval Quadrivium*

*"The ultimate binding element in the medieval order was subordination to the divine will and its earthly representatives, notably the pope." - Irving Babbitt*

After the fall of the Roman Empire around 476 A.D., the Roman Catholic Church increased its power during a time of intense missionary activity and rapid expansion. As a key part of what could be called the first (and most successful) viral marketing campaign in history, the Church established universities at monasteries across Europe to attract students, convert nonbelievers and raise money. Their mission: create a centralized social control system founded on Christianity that would unify a turbulent and chaotic Europe.

Taken from Greek tradition by the Romans, these Church-sponsored schools and universities were based on the *trivium* – the "three ways" or "three roads" of grammar, logic and rhetoric. This was then a (trivial) prerequisite for study of the *quadrivium,* or "four ways," of arithmetic, geometry, music and astronomy that could lead to advanced study of philosophy and religion. Unlike our present educational system, music (or more accurately harmonic science) was central to the study of nature and mathematics and harmonic philosophy was the greatest knowledge one could achieve. In those days, all knowledge was based on the Pythagorean theory of celestial bodies known as *musica universalis*, or *Music of the Spheres.*

The *musica universalis* philosophy held that celestial bodies – the Sun, Moon and planets – all move in accordance with the musical laws of simple harmonic proportions. Pythagoras is usually credited with the concept in his system of numerology and mathematics, but the Hindu Vedas describe a similar concept in the "Sound Current" or "Audible Life Stream."

The same concept, known as the *Music of the Spheres,* was described in the early 14$^{th}$ century by Dante Alighieri in his *Divina Commedia* (*The Divine Comedy*). In it Dante describes the Earth encompassed by a set of nine harmonically space concentric spheres, a cosmology based on the way sound propagates in an expanding bubble. In the 17$^{th}$ century, this became the founding principle for astronomer Johannes Kepler's *Harmonice Mundi* (or *Harmony of the World*). Based on this and his own calculations, Kepler believed there were universal harmonic principles that describe our solar system as nested spheres where planetary orbits are spaced according to the five Platonic solids, one nested inside the other.

Unfortunately, Kepler's solar system model was ultimately proven inaccurate in predicting the elliptical orbits of the planets. Because of this, history has consistently painted Kepler as a frustrated (and perhaps even delusional) man who never gave up hope on somehow making the solar system fit into his idealized harmonic model. His failure to demonstrate a natural order, which he named *celestial physics*, was a big reason (though not the only reason) that *musica universalis* was abandoned by early scientists. It ultimately served as a warning to avoid speculative thought based on harmonic ideals and instead pursue a path of pure observation apart from religious or philosophical influence.

Nonetheless, buried in Kepler's *Harmony of the World* was the essentially correct idea that the golden ratio (inside the Platonic solids) is involved in the formation of the planets just as it is in the formation of life. When we take the semi-major axis (ellipsoidal radius) through the foci of each planet's orbit, calculate the ratio of distance between each planet and then average all the interplanetary distances, we find the effective ratio between planets to be 1.61813, or within a 0.006% variance of the golden ratio. Kepler never knew this because he was unaware of all of the planetary bodies, like the dwarf planet Eris only recently discovered outside (and partially overlapping) the orbit of Pluto.

**Figure 14 - The golden ratio in planetary spacing**

| **Planet** | | **in kilometers* (millions)** | **Ratio of distance between bodies** | **Category** |
|---|---|---|---|---|
| Mercury | | 57.91 | 1.00000 | Planet |
| Venus | | 108.21 | 1.86859 | Planet |
| Earth | | 149.60 | 1.38250 | Planet |
| Mars | | 227.92 | 1.52353 | Planet |
| Ceres Asteroid Belt | | | | |
| | Ceres | 413.79 | | Small Dwarf Planet |
| | Juno | 399.13 | | |
| | Vesta | 353.20 | | |
| | Ceres Avg. | 388.71 | 1.70545 | |
| Jupiter | | 778.57 | 1.88156 | Planet |
| Saturn | | 1433.53 | 1.84123 | Planet |
| Uranus | | 2872.46 | 2.00377 | Planet |
| Neptune | | 4495.06 | 1.56488 | Planet |
| Pluto | | 5906.38 | 1.31397 | Small Dwarf Planet |
| Eris | | 10123.01 | 1.71391 | Small Dwarf Planet |
| **Total** | | | **17.79939** | |
| Average | | | 1.61813 | |
| Golden Ratio (Φ) | | | 1.61803 | |
| Variance | | | 0.006% | |

* Semi-major axis is the radius of an ellipse running through the foci. Source: NASA

Though seldom mentioned today, there are many other harmonic and golden ratios to be found in the solar system that are either related to planetary size or orbital proportion. One of the best examples is the Φ-proportion found in Saturn's Cassini Division, a large dark gap in the planet's colorful rings, which is mirrored in the inner C and D rings where the orbital material is very thin and dark. From the latest Hubble telescope image in Figure 15, a clear dual golden ratio can be seen to suggest harmonic activity like that in a musical octave. Yet, without the benefit of a universal harmonic model to provide an explanation, modern astronomy is left to speculate as to what might cause such formations. And when a plausible theory is suggested to explain the various influences that would generate these patterns, we are still left wondering how *those* physical forces came to organize themselves as they do or how that might be related to other phenomena. Without a reasonable theory we might just conclude the golden proportions in the rings are nothing more than a mysterious quirk or random coincidence.

**Figure 15 - Golden ratios in Saturn's Cassini ring**


So while Kepler was not entirely accurate in his Platonic model of the solar system, there was certainly an important truth in his view that some kind of harmonic geometry involving the golden ratio is deeply embedded in the solar system. In the same way, the quadrivium educational system founded on *musica universalis* during the Middle Ages was not always accurate, but it did offer a valid philosophical model of order necessary for planets to form and life to emerge.

In those days, both religion and harmonic science shared a common bond in the study of proportions in music. In fact, this worldview based on Pythagorean philosophy was widely held throughout Europe and Islamic cultures during the Middle Ages, from its rediscovery at the end of the first millennium to its blossoming in the Renaissance.

But after Kepler, less and less credence was given to the *Music of the Spheres* and its guiding harmonic philosophy. The reason usually given for this is the rise of the scientific method, which is true on the surface, but there is deeper reason. The real reason is the Pythagorean theory of *musica universalis* was part of the same anti-harmonic campaign being waged against *Diabolus in Musica* to control harmonic theory. It all began centuries before Kepler was born in the late $8^{th}$ century when the Roman Catholic Church had begun to assume the power of the fallen Roman

Empire. We see Church control of harmony first beginning to take shape in the Carolingian music system led by the English Catholic scholar, teacher and astrologer Alcuin of York.

Appointed by Charlemagne as Master of the Schola Palatina, Alcuin moved to standardize Church music by combining three harmonic theories – *De musica* of Augustine, the *Institutions* of Cassiodorus and the *Etymologiae* of Isidore of Seville – into a single field of musical study as part of the quadrivium. By unifying these separate systems, Alcuin was the first to institute Church oversight and systematic control of music and harmonic theories in Western civilization.

By the 9$^{th}$ century, Aurelian of Reome, a Frankish writer and music theorist, had published this unified musical system in his *Musica disciplina* based on a set of eight modes and a crude notational system taken from an incorrect (and unfortunate!) interpretation of a 6$^{th}$ century Arabic-to-Latin translation of the Greek modes by Boethius. These new modes, later known as the "church modes" or *Gregorian modes* after pope Gregory VIII, were numbered I through VIII and said to be numerically symbolic of Christ. The mode names were retained from the Greeks, but the rules for constructing them, being quite obtuse and mired in religious doctrine, had little to do with Pythagorean tetrachords or other Greek harmonic theories.

In the late 10$^{th}$ and early 11$^{th}$ centuries, a Benedictine monk by the name of Guido of Arezzo then further codified and extended the church modes as part of a widely referenced music theory book entitled *Micrologus*. In it, he formalized music staff notation to help Gregorian monks remember their chants while enabling more detailed analysis (and control) of the relationships between absolute pitches within a scale. It was Guido who formalized the naming conventions for the different octaves, naming the lowest note "gamma ut," shortened to "gamut" to identify the entire range of audible octaves. The term *gamut* has since been taken to describe a complete range of any spectrum, such as the range of visible colors.

Guido was heavily influenced by Muslim theories about music (as preserved by the Arabs from the Greeks), like those of Gerbert of Aurillac[17] who taught the four subjects of the quadrivium in Spain. Based on these ancient Greek theories, Guido proposed yet another new *hexachord system* for the church modes, known as the *solfege* or *solfeggio scale,* that described how the scales should be built. Unlike Aurelian's system, his system used the same tuning developed by Pythagoras, but with one really *big* difference. Guido's system was designed to avoid the tritone between {B} and {F}.

> *"…the development of Guido of Arezzo's hexachordal system which made B flat a diatonic note, namely as the 4$^{th}$ degree of the hexachordal on F. From then until the end of the Renaissance the tritone, nicknamed the "diabolus in musica" was regarded as an unstable interval and rejected as a consonance." [Stanley 1980]*

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[17] Later becoming Pope Sylvester II, d. 1003.

While some sort of tritone avoidance doctrine was certainly in effect long before the *Micrologus* was published, Guido was the first to provide a systematic set of rules that could be used to avoid the tritone. This system was quickly embraced and promulgated throughout the network of Catholic quadrivium monasteries in Europe.

In this way, the foundation of Western music was based in large part on tritone avoidance. This one overarching harmonic symbol was not only the invisible driving force behind music theory, but also the first principle for interpreting *musica universalis* in the Catholic quadrivium. As it had been for the Greeks, regulating music was the first step toward regulating society. The balance between good and evil in music, as symbolized by the symmetrical movement of the tritone, was now officially replaced by an asymmetrical rules-based system for music.

The resulting quadrivium tradition of teaching music, math and astrology within the context of a religiously filtered and strictly controlled harmonic policy expanded rapidly during the 11th century. One of the foremost quadrivium scholars during this period was Hermannus Contractus, who lived in the abbey of Reichenau on an island in Lake Constance in southern Germany. Known for his treatises on the science of music, geometry, arithmetic and astronomy, he continued to track the planets and their orbital harmonies with an astrolabe he designed in spite of the growing movement against harmonic science and Pythagorean philosophy from Rome.

One chilly winter night I stayed over in an Augustinian Monastery on the island of Chiemsee in Bavaria, approximately 150 miles east of Reichenau abbey. Arriving late after a long flight from the States, I entered through a side door and checked in at a dimly lit desk. The reason for my visit was to deliver a keynote address the next morning at the 1997 *PC Trends* Internet conference held there at the monastery.

Fortunately, I found the minimalism and quiet meditative setting, complete with its whisper policy for the halls and ban on telephones, just the thing for a good night sleep. Having only a single bed, a thick hemp rug and plain desk, the room appeared frozen in time – an unbroken continuation of the strict monastic life and scholarly pursuits of the Middle Ages. It was an exact match for the cloister described by Hermann Hesse in his 1930 novel *Narcissus and Goldmund*.

The next morning, I spoke about the emergence of e-mail, instant messaging, chat rooms and rich-media “push” platforms, like those of the then-popular *PointCast* and *Backweb*. I made the bold prediction that these and other more immediate forms of digital communication would eventually supplant even e-mail. The possibility of broadband and streaming audio-video much less wireless handheld devices still seemed a long time away. No one could have predicted the sudden rise of mobile text messaging, high speed Web surfing or the replacement of proprietary push technologies with RSS and blogs.

The ensuing discussions with the German technologists and business owners in attendance were not disappointing and probably no less passionate than conversations centuries earlier.

Everyone believed they were at a new beginning, but the beginning of what? No one knew how to build a business for the Internet, so there was a palpable fear in the room that they or their company might not adapt or make some catastrophic error. Reflecting on this now, I imagine that the enthusiasm and comradeship in the face of uncertainty was not much different from that of the monastery students many years before on this very spot. Yet, there remained one unmistakable difference between my fledgling Internet audience and that long lost quadrivium classroom.

Without the context of a larger social project, the Web phenomenon was mostly seen as an opportunity of commerce and marketing. No one spoke of how the Internet might bring greater enlightenment to oneself, unify people around the world or elevate the condition of mankind. No one considered how this unexpected disruptive invention might accelerate materialism, further distract our children's attention or foster anti-social behavior. The benefit of philosophical reflection or social responsibility was irrelevant and even laughable in the scramble for virtual territory.

In this regard, the Middle Ages with its quadrivium system stand in stark contrast to the enlightened modern man and educational system. The old view of a natural and universal philosophy grounded in universal harmonic principles is antithetical to the modern man's project of specialization, compartmentalization and isolation. Science works to dissect things into such small parts that the whole picture – the *harmonia* – becomes invisible. Personalization, now amplified by Internet and mobile communications, has elevated individual opinion above all else, making it virtually impossible for anyone to hold a communitarian worldview or agree on any common course of action. This was certainly not the philosophy of the quadrivium.

The absence of Pythagorean harmonic science from Western civilization has had far reaching consequences upon our social order. At a time when music was still a central part of education, the quadrivium student could not have imagined the omission of harmonic principles from natural science any more than my Internet audience could have accepted music harmony as a Theory of Everything.

By the mid-11$^{th}$ century, the opinions and practices of music in the Catholic Church were beginning to harden into a clear set of compositional rules centered dogmatically on tritone avoidance. A Camaldolese monk named Gratian created the first truly comprehensive collection of these rules for music known as "Gratian's Decree." These quickly became the de facto standard for music composition and performance in the Church.

At the beginning of the 12$^{th}$ century, Johannes Cotto, a music theorist, possibly of English origin and likely working in southern Germany or Switzerland, wrote another very influential music treatise entitled *De musica*. Building on the work of music theorists before him, such as Boethius, Guido of Arezzo, Odo of Clunym, Isidore of Seville and Hermannus Contractus, this treatise provided precise directions for composing chant and organum, an early form of polyphonic chant. Like others in Germany at the time, Cotto was heavily influenced by Greek music and theories, preferring to use the old modal names, like Mixolydian and Phrygian.

In fact, it was probably his study of Greek theory that convinced Cotto to more fully address the restless tritone. He developed an even more elaborate set of rules for good voice leading to guide composers around the spring action of the nefarious *Diabolus in Musica*. After several chapters on Greek notation, musical timbre and the ethical and moral effects of the musical modes of chant, Cotto included a detailed description of how to compose organum. He defined rules for note-against-note voice leading showing how to end on a 5$^{th}$ or an octave without striking a tritone. Cotto also emphasized the importance of "contrary motion," in place of the earlier parallel voice motion of chant, and gave specific directions for composing melody, including pacing, position and other patterns. The rules in this treatise influenced music for centuries to come, remaining in print until well after 1400AD.

Then, in 1234AD – just two years after the tribunal of the Papal Inquisition was established to seek out and eliminate paganism – *Gratian's Decree* supported by *De musica* was promulgated by Pope Gregory IX into canon law. This law, entitled the *Liber Extra*, was subsequently used to enforce the *ars antiqua* style of music and officially prohibit the tritone from Church music. We know this form of music today as "Gregorian chant."

The early 13$^{th}$ century was a time of intense change and discovery. Genghis Khan unified the Mongols, King John of England was forced to sign the Magna Carta, Marco Polo and his family reached China and the first secular universities were founded. The famous Franciscan Roger Bacon was espousing empiricism, or truth in nature, as one of the earliest advocates of the modern scientific method. And, Leonardo of Pisa, also known as Leonardo Fibonacci, wrote about a very important numeric series known today as the *Fibonacci series*.

The Fibonacci series, a spiraling numerical series that spirals into the golden ratio, was first published in India in the *Maatraameru*[18]. Like the ten Greek modes, the Fibonacci series was first represented as a pyramidal array or "mountain" of numbers, thus its Indian name meaning "Mount Meru."[19] But the Indian mathematician Virahanka later discovered that the series could also be expressed algebraically using the recursive equation $F(n+1) = F(n) + F(n-1)$. To his amazement, Virahanka had found a way to generate or *grow* the entire Fibonacci series beginning with only the numbers 0 and 1 (shown in Figure 16).

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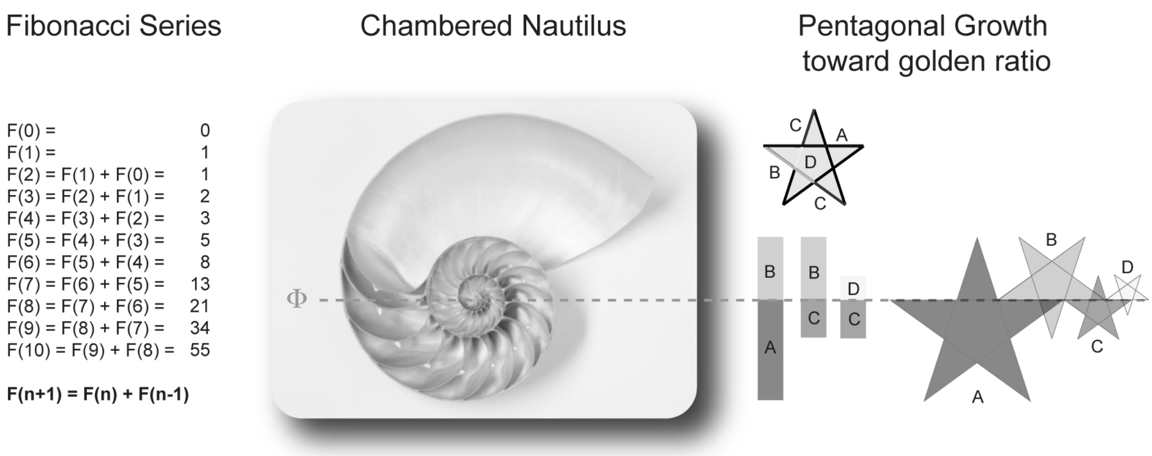
[18] Known as the *Mountain of Cadence,* or "Mount Meru," was documented by the Sanskrit grammarian Pingala in the Chhandah-shastra, *Art of Prosody,* some time between 450 and 200 BC. There is evidence that this simple additive series of numbers was known much earlier than this.

[19] Also known as *Pascal's triangle*, numbers can be arranged into a pyramid such that each is the sum of the two directly above it. They diagonals in this triangle will always sum to Fibonacci numbers.

### Figure 16 - The Fibonacci series as a natural growth spiral




The Fibonacci series is found extensively in nature as a pattern of incremental growth proportions. As mentioned earlier, this appears as the branching or spiraling patterns found in plants and animals that converge into the golden ratio. Each Fibonacci number in the series results from the addition of the two previous numbers, such as 2+3=5, 3+5=8 and 5+8=13. Each pair of adjacent numbers can then be divided to produce an increasingly accurate approximation of Φ spiraling into infinity. In this way, the Fibonacci series is a way to *grow the golden ratio* from simple whole numbers, just as it occurs in the proportional chambers of a *Nautilus pompilius*.

For instance, adjacent Fibonacci proportions 13:8, 21:13 and 34:21 get closer to Φ ≈ 1.61803 as the numeric series 1.625, 1.615, 1.619. No matter how far we go up the Fibonacci series, it will get very close but never actually reach the golden ratio itself since this is an infinite number. In this way, the Fibonacci series can best be described as a *rational* approximation for Φ, obtaining its irrationality from the square root of 5 inside a pentagram.

One other property of the series is each successive ratio alternates or *oscillates* around Φ, demonstrated in part by 13:8 and 34:21 being greater than Φ while 21:13 is less. Strangely, the presence of this amazing odd-even alternating pattern throughout nature is seldom discussed, typically dismissed as a result of unknown physical processes or perhaps organic natural selection. Some propose that this pattern is the best way for leaves to collect sunlight or the most efficient way for organisms to grow themselves stepwise, but such explanations fail to explain why the same spirals appear in galaxies and hurricanes.

As it pertains to music, the Fibonacci series has hardly made a ripple. It has played no role whatsoever in music theory, despite its obvious relationship to the golden ratio and ancient connection to harmonic science and natural philosophy. Even into present day, we can find no record linking the Fibonacci series to any musical theories for voice leading, the tritone or other musical concern.

One would have thought that something like this, omnipresent and organic in nature, would have been afforded an important role in harmonic science and human perception. But with the Church campaign against all forms of neo-Pythagorean *gnosis* in full swing in the early 13th century, we can be sure Church fathers were well aware of its relationship to the golden ratio and suppressed it as they did the tritone.[20] What else could account for its diminished importance in modern science and absence in contemporary theories for music, acoustics and perception?

In ancient times, the Fibonacci series was actually recognized as the most important organizing principle of life and a way to "reach for God without looking him in the face." Its coiling and spiraling geometry was the founding principle behind nearly every pre-Christian religion, symbolized in tribal cultures as serpent or dragon gods. Serpents were seen as divine creatures because they could transform themselves from a stable coil into a crawling periodic wave, thus harmonizing the spiral with the circle, the unlimited and the limited. It is no wonder that the early Church chose the serpent as its prime Anti-Christ symbol.

Serpent worship once existed all around the world. It is found in both Hindu and Buddhist mythology as the divine Naga (meaning "serpent" in Sanskrit). They are found in Chaldean serpent worship, the Egyptian god Sebek (a "musuh" or crocodile messiah) and Lung Dragons of China. Even in South America we find the Mayan serpent kings of Caramaya, Naga Maya (and later Kukulcan and Quetzalcoatal) as well as the Amarus and Con Ticci Viracocha of Peru. But of all these serpentine cultures, it was the Egyptians that "nagged" the Roman Church most.

Descended from the Sumerian serpent god Ningizzida (symbolized in the caduceus symbol of modern medicine), it was the Egyptian Naga followers who built the great pyramids and taught Pythagoras that all life emerges from spirals into waves like a serpent.[21] So, it should come as no surprise to learn that the name Pythagoras has its origin in the Greek words "Python" and "agoras," together translating into the phrase "serpent meeting." Pythagoras, as the grand unifying archetype for all ancient Naga philosophies, was founded on the study of Fibonacci spirals and harmonic waves in nature.

It was Franco of Cologne in the mid-13th century, a papal chaplain at Cologne and music theorist from the Notre Dame School of Paris, who more fully formalized tritone avoidance in the *ars antiqua* Gregorian music style. He wrote several musical treatises on organum, descant,

[20] *Gnosis* means to have knowledge of nature. Medieval theosophies based on Gnosticism descended from Platoism, Pythagoreanism and the older mystery schools.
[21] The word *nag* is found in various other natural philosophies, such as the thirteen *Nag Hammadi* treatises of the Gnostics (found in Egypt) and the Mesoamerican word "nagual" originating from the word *nahualli* that referred to practitioners of harmful natural magic. This term has been used by some to refer to that part of nature that was not "tonal" or made of structure.

polyphony, clausulae and conductus – especially concerning consonance and dissonance – which remained popular for a hundred years. *Ars cantus mensurabilis*, his most famous work, provided even more detailed rules for music notation, particularly regarding note duration and rhythmic modes, without the typical inclusion of religious speculation.

By the early 14th century, musical tastes were evolving and the Gregorian rules of *ars antiqua* was beginning to come undone. It was being challenged by the radical *ars nova* style emerging out of the south of France, a movement led by the rebellious Phillippe de Viltry. *Ars nova*, or "new art," broke many of the rules of *ars antiqua* by including such things as free rhythms in place of sanctioned rhythmic modes and even lyrics of love poems sung above the sacred texts. Very controversial in the Roman Catholic Church at the time, *ars nova* paralleled the "Great Schism" when the Church had a pope in both Rome and Avignon, France. The style was rejected by Pope John XXII in Rome but supported by Pope Clement VI in Avignon, who argued it was much more expressive and varied than Gregorian chant. At a time when perspective painting was just being discovered, music was becoming more *spatial* in its structure as well.

Another Frenchman, Guillaume de Machaut, was also known for this rebellious musical style. In 1364, during the pontificate of Pope Urban V in Avignon, Machaut composed the first polyphonic *ars nova* mass entitled *Le Masse de Notre Dame*, which was performed in the *Palais des Papes* (or Palace of the Popes). It was from this that Avignon received its reputation as a hotbed of musical innovation and after which pagan influences pushed music into the more radical style known as *ars subtilior*.

Centuries later, Avignon is still known across Europe for its theatre, art and music festivals. Visiting with my family in late July 2005, we attended the theatre festival there while staying in *La Mirande Hotel*, formerly the old Bishop's residence next to the Palace of the Popes.

Since this occurs in early August, just prior to the Avignon Jazz Festival, every bohemian actor and musician in France was in town and performing in the streets. Posters and leaflets were everywhere promoting one avant-garde show or another. It was a wonderful, albeit unexpected, immersion in French musical culture and the Avignon experimental art tradition.

During our stay, we attended a modern ballet premier of *Frere & soeur* (*Brothers and sisters*) by famed choreographer Mathilda Monier, which was performed in the central courtyard of the Pope's palace. With a history of avant-garde musical productions in the palace, we should not have been surprised by the dancers appearing and disappearing onstage from a large black box; fighting one another; standing very still in what appeared to be harmonic lines – then, embracing one another only to stop and arrange bales of hay or roll a few bicycles onstage. It should also have been no surprise when the female dancers lost their tops two-thirds of the way through the program. Needless to say, I was impressed with the perseverance and, dare we say, French dedication to duty in maintaining Avignon's forward leaning *ars nova* reputation.

In retrospect, the odd conflicted performance of brother against sister staged right in the heart of Avignon's Palace of the Popes was nothing less than a tribute to French harmonic dualism. Originating in the Dualistic and Gnostic religious sects of Catharism in the Languedoc region of France during the 11th century, the Catholic "anti-papacy" founded during the Great Schism in Avignon can be traced directly to their pagan harmonic beliefs.[22] It was their knowledge or *gnosis* of the natural harmony of opposites that ultimately caused the split away from the asymmetrical doctrine of *ars antiqua* and toward a more harmonic form of music.

By the 17th century, these ancient Egyptian-inspired ideas had begun to permeate all of French society. It began with René Descartes' philosophy of *dualism* proposing the separation of mind and body. This spilled over into other areas of Art and Science, from Claude Debussy's Impressionist harmony of dual wholetone scales to Edouard Manet's paintings depicting the "duality of the personae;" from Charles Baudelaire's modernist duality in prose and poetry to physicist Louis de Broglie's discovery of wave-particle duality. In mathematics there was French mathematician Henri Poincare's famous duality theorem for manifolds and the list goes on and on. This Egyptian-inspired harmonic dualism has been a recurring theme in French culture, derived it seems from an overall heightened awareness of nature's balance.

So having sifted through a bit of medieval history, are we any closer to explaining the tritone?

Well, we do have a better historical context within which to understand the tritone. We know that the rise of the Inquisition with its canon laws were driven by the Church's project to eradicate paganism and with it the free study and use of harmony. We know that at the center of this anti-pagan campaign was the musical tritone – *Diabolus in Musica* – the harmonic interval most likely to reveal knowledge of the golden ratio. And we know that fear of this musical interval and what it represented is what shaped music history.

Under the Church's anti-harmonic agenda, suppression of the tritone in music was crucial. Canon laws were passed to mandate rules for how to construct scales, how to handle voice leading, what was considered consonant and dissonant, how contrary motion should be handled, what tones should and should not be emphasized in rhythm, which rhythmic patterns were acceptable and specific instructions for how music should be written. Tritone omission was the first principle – the foil, if you will – behind all of these rules.

But the tritone was much more than this. It was the model for all canon law and became the legislative model used to prosecute paganism in general. It was the ethical justification behind the Inquisition in its mission to eradicate Pythagoreanism, Platoism, Mithraism, Egyptian Hermeticism, Zoroastrianism, the Hebrew Kabbalah and similar Gnostic teachings in revival

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[22] "Cathar" originated from the Greek word *katharsoi* (from *katharsis*), meaning "pure ones." The term "Catholic" has its origin in the related word *katholikos*, meaning "whole" or "complete."

during the 11th through 13th centuries. With tritone omission at the front line of its plan to control Europe, the Church mounted a Great Crusade against the very cradle of civilization – the people who had discovered and preserved the ancient knowledge of harmonics. The Church knew that eradicating paganism simply could not be done without first eradicating free harmony in music.

Along with the tritone, the harmonic geometries of the pentagram and hexagram were cast as evil pagan symbols and linked with satanic worship, witchcraft and occult ritual. Those who studied such harmonic properties in numbers, geometry and astronomy, like the Cathars of southern France, were threatened, executed or exiled, forcing many to roam the countryside as "gypsies" (a pejorative form of Egyptian). The Knights Templar, the primary keepers of the Babylonian and Egyptian temple mysteries, were then hunted down and murdered by the Church (on Friday, October 13, 1307) with a few fleeing to far-flung places like northern Scotland. Within a few hundred years, the Church had managed to seek out and crush most competing pagan religions across Europe, leaving only a few small groups that met in secret.

Viewing the medieval Church through the musical lens of a tritone makes it easy to see how broad and comprehensive their anti-harmonic strategy really was. Even the Biblical story of the Garden of Eden was a manifesto against Pythagorean and Egyptian beliefs. The serpent in the Tree of Knowledge (representing spiraling Fibonacci growth) whose fruit was the Apple of Knowledge (with its circular pentagram and golden ratio at core) was presented as the foremost agent of evil in the world. As the first principle on the first page of the Bible’s lead story, Genesis was a dire warning to all who might seek to follow the Greek and Egyptian path into the Underworld to discover nature’s harmony for themselves.

For if a perfect order were thought to exist in nature, immediate and present in everything, there would be no need for a separate creator, no need for an intermediary savior and no need for the Church to intercede on behalf of God. People would simply seek answers directly from the world around them and in their own “inmost chamber” as had always been the shamanic way of tribal cultures. The Church fathers had little choice but to demonize and outlaw all musical-astrological symbolisms, whether auditory or visual, to establish and maintain social control.[23]

In the final analysis, the suppression of harmonic knowledge was an unparalleled success. Today, the tritone is still avoided in music theory textbooks and classrooms. At the same time, the golden ratio buried inside harmonic oscillation is mostly avoided in scientific research. Its true role in physics, physiology and human perception remains underappreciated and downplayed. Indeed, all harmonic philosophies have been discredited in Western society to such an extent that their very *absence* has served to propagate the same anti-harmonic doctrines instituted long ago by the medieval Church. In this sense, the Inquisition and Crusades never ended.

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[23] **Hypothesis 3:** The Medieval Catholic Church banned the tritone in the early 13th century due to its association with Pythagoreanism and other Hermetic/ Kabbalistic philosophies.

The monumental struggle between the Church and paganism has been told many times, in many ways. But of all the stories, none has resonated with people more than the 1939 film *The Wizard of Oz* starring Judy Garland. Based on the book *The Wonderful Wizard of Oz* by L. Frank Baum, this film is usually thought of as a nice American fairy tale or perhaps a thinly veiled political commentary, as suggested by Henry M. Littlefield in his "Parable on Populism". But it is more likely a tale of the Roman Catholic Church's war against paganism and the freedom to pursue truth in nature.

It all begins on the Kansas prairie, a symbol for the colorless material world, with a girl named Dorothy Gale.[24] After singing *Somewhere Over the Rainbow*, in which she wishes upon a star, our heroine is swept up by a spiraling tornado and deposited in a colorful and musical place called Munchkinland. Populated by a cherub-like pagan tribe called Munchkins[25], their leader the good witch Glenda presents magical ruby-crystal slippers to Dorothy. She is then told to follow the yellow brick road (a continued golden spiral from the twister) to reach the great wizard of Oz who has the power to send her home. Taken from the Hebrew name meaning "strength," Oz represents enlightenment in the "inmost chamber" of Dorothy's mind.

As she walks in fear along the golden path, traveling through a dark forest symbolizing nature, she meets three wise men – the Scarecrow, the Tin Woodsman and the Cowardly Lion. As personifications of Dorothy's fear, the mindless Scarecrow represents her loss of reason and direction, the rusty Tin Woodsman her loss of heart and spirit, and the Cowardly Lion (the Egyptian sphinx) represents her loss of courage and confidence. At the end of her journey, just before she is about to fall asleep under an evil spell, Dorothy arrives at the legendary Emerald City of Oz – symbolic of the Emerald Tablet of Egyptian philosopher Hermes Trismegistus.

Gaining entry to the city, Dorothy asks to see the "great and powerful Oz" to ask for his help in returning home. When she meets the wizard, appearing as a terrifying personification of the medieval Inquisition, he is the gatekeeper to knowledge and demands that she dispose of his adversary, the Wicked Witch of the West, to learn how to return home. As the wizard's crusader against paganism, Dorothy journeys to the witch's lair and sneaks into her watchtower, but is discovered and captured. Fortunately, Dorothy manages to escape and ultimately slays the pagan witch through holy baptism.

Having conquered paganism, Dorothy returns triumphantly to the wizard and again pleads with him to take her home. But instead of honoring his promise, the fiery wizard orders her away. Angry at the wizard's broken promise, Dorothy's dog Toto (representing her inner spirit and

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[24] The name "Dorothy" is of Greek origin and means "gift of God." When taken together with the tornado, the name Dorothy Gale implies that the spiraling whirlwind that Dorothy travels through is a gift of God.
[25] "Munchkin" is probably a contraction of the German words "Munchen" and "kinder," meaning "Munich children." In the original story, the Munchkin were described as all wearing blue clothes corresponding to the blue in the Bavarian flag and the Blue Lodges of German Freemasonry.

intuition) pulls back a velvet curtain (symbolizing the façade of Church doctrine) to reveal the fact that the powerful wizard is no great deity after all – just a man.

Now enlightened and full of inner strength, Dorothy takes back her reason, her intrinsic spirituality and her courage along with a promise from the would-be wizard (whose real name is Oscar Zoroaster[26]) to take her home. But as the former carnival wizard (now representing modern science) is about to spirit her away in a balloon (symbolizing technology), an error occurs and he only saves himself, leaving Dorothy to find her own way home.

While it would seem that Dorothy had lost her last chance to find salvation, the good witch Glenda (meaning "holy and good") suddenly reappears in a sphere to tell her she can return any time she wants through the power of 3 in her own mind. With this, Dorothy realizes that salvation is not found in the promises of either religion or science, but only in the inmost chamber of her own mind. As she wakes up from her inner Underworld adventure, we too wake up to the black and white reality that nature's harmony has remained hidden from Western society far too long.[27]

We now know with some certainty the cast of characters who worked so hard behind the velvet curtain to conceal the truth about the tritone and its relationship to nature. It started with Boethius, then Alcuin of York, Aurelian of Reome, Guido of Arezzo, Gerbert of Aurillac, Odo of Clunym, Isidore of Seville, Hermannus Contractus, Johannes Cotto, Franco of Cologne, monk Gratian and ultimately Pope Gregory IX. Each of these people, all in good faith of course, played critical roles in establishing the tritone's unfortunate reputation in music history and scarce mention in music theory.

We also know how and when the tritone was outlawed. Rules of omission began to take shape during the rise of Catharism in the 11th century with Guido of Arezzo. They were then systemized by monk Gratian and further codified in the late 11th and early 12th century by both Johannes Cotto and Franco of Cologne. In 1234, Pope Gregory IX then promulgated this musical doctrine into canon law as a philosophical cornerstone of the Catholic Inquisition, requiring every Roman Catholic Church to honor its policy of tritone avoidance.

Yet even as these tritone omission laws became increasingly entrenched into the late Renaissance, a rebellion was brewing against this and other papal laws. Leading the charge was, of course, German monk and theologian Martin Luther.

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[26] The wizard's first name Oscar means "spear of the gods." His last name then corresponds to the ancient Iranian prophet Zoroaster (meaning "living star"), who founded the musical-astrological theosophy known as Zoroastrianism. Under this belief, the universe is seen as a cosmic struggle between truth and lie, giving rise to Free Will. Pythagoras was rumored to have studied with Zoraster in Babylonia.

[27] If there is any doubt that this is the interpretation Baum most likely intended, consider the fact that he also anonymously wrote *The Last Egyptian: A Romance of the Nile* about an English Egyptologist, an Egyptian woman named Kāra and a Dragoman named Tadros. It seems Kāra was a descendent of Ahtka-Rā, High Priest of Ậmen (Ra), the Egyptian sun god.

# *Counterpoint Reformation*

*"I have no pleasure in any man who despises music. It is no invention of ours: it is a gift of God. I place it next to theology. Satan hates music: he knows how it drives the evil spirit out of us."*
*- Martin Luther*

In 1979, contemporary composer and professor Lloyd Taliaferro was an expert in the polyphonic counterpoint techniques of the Renaissance. As a hand-me-down from canon law, this style of counterpoint did not use chords – instead, playing note-against-note according to the Church rules. Dr. Taliaferro enjoyed this style of music very much and seemed to find great pleasure in taking his students, including me, through the torture of learning it.

Learning polyphonic counterpoint is a very rigorous and ritualized process. One begins by first learning *species counterpoint*, or strict counterpoint, followed by other "species" of increasing complexity, eventually being allowed to compose *free counterpoint*. The most famous pedagogue of polyphonic counterpoint was Johann Fux who published *Gradus ad Parnassum* (*Step by Step Up Mount Parnassus*) in 1725 toward the end of the Baroque period.

Descended from Church canon law, polyphonic counterpoint is about as much fun as writing a tedious legal contract. And like most contracts, the "contract terms" must be followed according to a pre-approved boilerplate. Below is a summary of the legal boilerplate required for this kind of music. Feel free to skip down any time.

*First Species is defined as "note-against-note" with consonances defined as 3rds, 5ths and 6ths and dissonances as 2nds, 4ths and tritones (of one variant or another). Of the consonances, only unison, 5$^{th}$ and octave are "Perfect" while 6$^{th}$ and 3$^{rd}$ intervals are "Imperfect."*

*The only allowed melodic motion between voices were 1) same interval in the same direction, 2) contrary in opposing directions and 3) oblique, or one voice stationary as another voice moves. [Now, here comes the good part.] Perfect consonance to another perfect consonance must proceed in contrary or oblique motion. Perfect consonance to imperfect consonance may proceed in any of the three motions. Imperfect consonance to Perfect consonance must proceed in contrary or oblique motion and Imperfect consonance to Imperfect consonance may proceed with any of the three motions.*

*HOWEVER, great care is needed in moving from note to note in direct motion as more Imperfect than Perfect consonances should be employed and the beginning and end must BOTH be Perfect consonances. Further, for vocal performance you should avoid augmented, diminished or chromatic intervals, and no intervals larger than the 5$^{th}$ – except for octave and minor 6$^{th}$ (the minor 6$^{th}$ only in upward direction!). No skips may then follow each other in the same direction. In general, the next to last bar MUST be a major 6$^{th}$ interval if the cantus firmus is the lower part – minor 3$^{rd}$ if the upper. And, the 2$^{nd}$ degree of the mode ALWAYS occurs as the next to last tone in the cantus firmus moves to the*

> *7th degree and always as the next to last tone in the counterpoint. Repetition of a tone (oblique motion) should NOT be repeated more than once.*
>
> *And this is only the beginning! More rules are given for First Species and there are four more Species of rules after this with even more addendums of rules based on special rhythmic and melodic considerations.*

Of all these rules, one particular First Species rule stands out:

> *"No movement by an augmented 4th [a tritone] and such movement should also be avoided if reached stepwise…"*

This tritone omission rule, when combined with other rules regarding the tritone as a defined dissonance, is the key to understanding the importance placed on its elimination in sacred music and common practice [Benward 1977]. Renaissance music was really a continuation and refinement of the original canonical rules. It was from Dr. Taliaferro that I first learned about the *Diabolus in Musica* and how all of the rules had something to do with tritone avoidance. Knowing that I needed to steer clear of the tritone made it much easier to learn the musical elements acceptable to the Church during the Renaissance.

The rules for polyphonic counterpoint were first formalized in 1532 with the publication of Giovanni Maria Lanfraco's theory book *Scintille di musica*. This publication was the final move away from the old Boethian-Pythagorean music theories of the early Middle Ages and toward the new ideas of Bartolome Ramos de Pareja in the late 15th century.

During a period of emerging humanism near the end of the Renaissance, many Greek treatises were being rediscovered, including Aristotle's *Problems*, the *Harmonics* of Aristoxenus and the *Harmonics* of Claudius Ptolemy (2nd century AD). It was this last book that had introduced Ramos to a tuning system called *diatonic syntonon*. From this he proposed a new tuning system in his *Musica practica* that favored the simple ratios of 5:4 and 6:5 for 3rds instead of the complex Pythagorean ratios of 81:64 and 32:27 of the earlier dual tetrachord system. This new tuning system became the foundation for the next great advancement in music.

From Ramos' idea of consonant 3rds and 6ths, Venetian school theorist Gioseffo Zarlino suddenly expunged the entire Pythagorean modal system in his *Le institutioni harmoniche* (1558), replacing it with a division of the octave into <u>twelve</u> tones. Soon thereafter, he replaced the Pythagorean tetrachord system (based on the perfect 4th and golden ratio) with his own system based on a set of numbers from 1 to 6. This system, which emphasized C and the 7-step Ionian mode, was a clear precursor to today's simplified system of major and minor.

Zarlino's choice to divide the octave by twelve was based on a rather complex theory called *2/7-comma meantone* that sought a balance between whole number harmonic proportions and the spiral of pitch. His tuning system emphasized the importance of triangular or *triadic* chords instead of just scales or intervals. This had the effect of elevating the role of vertical harmonies over independent melodies and represented a major shift in auditory geometry away from the pentagram (inside the tetrachord) to an architecture based on an equilateral triangle.[28]

Another Italian composer and theorist, Lodovico Zacconi, then helped distribute Zarlino's system by codifying it into two volumes of his *Practica di musica*, first in 1592 and later in 1622. These treatises on polyphonic counterpoint laid out a system of twelve modes – six "Authentic" and six "Plagal" – while conspicuously omitting the Locrian and Hypolocrian modes that emphasized the tritone. It seems tritone avoidance was still very much in effect.

As we will see later, emphasis on this triadic auditory geometry was a key discovery in the development of modern harmonic music. Ever since Zarlino (and Zacconi) introduced what we might call their "Z-twelve" octave, most music has been composed using triadic harmonies in a 12-step octave. Yet inexplicably, there has never been any theory, whether in music, acoustics, physiology, psychology or any other field of study, to explain why this system has been so successful over the past 400 years. It is just assumed as a convention of history.

By the middle of the 16th century, the Renaissance was over and Martin Luther's Protestant Reformation was underway, thanks in large part to the newly invented printing press. As a pious and scholarly Augustinian monk, Luther was well versed in quadrivium teachings, ultimately earning a Doctor of Theology. His knowledge of Greek natural philosophy was a strong influence on his views of God over papal law. Luther's seal even incorporated Pythagorean symbolism as two 5-petaled roses, together representing the Dualist knowledge of the pentagram, Nature's golden ratio and the importance of triangles in creating harmony (shown in Figure 17).

Luther's use of the rose symbol has given some to speculate that he was a Rosicrucian, for whom the rose also symbolized a pledge of secrecy concerning the forbidden knowledge of harmonics (known as *sub rosa*). In fact, there is no hard evidence to prove Luther was a member of any pre-Rosicrucian order, even though harmonic philosophies were certainly part of the Hermetic, Kabbalist and Gnostic undercurrents present in the early 14th century. As a theological scholar, Luther would have been aware of these pre-Christian natural philosophies and encouraged by the esoteric thinkers around him to question papal authority. It is no coincidence that the emergence of the Rosicrucian order as a formal organization occurred just a little later in Germany, between 1607 and 1616 led by followers of Martin Luther.

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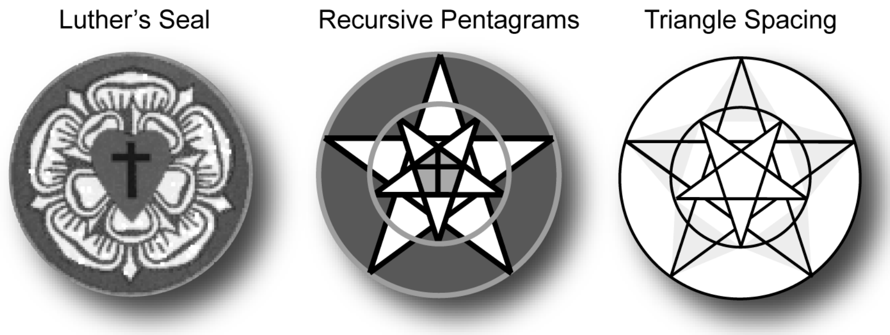
[28] **Hypothesis 4:** The development of the 12-step octave and simplified system of major-minor diatonic scales resulted from the replacement of a Pythagorean pentagonal design with that of an equilateral triangular design.

**Figure 17 - Recursive pentagrams in Luther's seal**




Luther wrote many heretical treatises that attacked the authority of the Catholic Church and its canon law, calling for the elevation of scripture over papal law as well as the abolishment of enforced confessionals and oath of monastic celibacy. Most controversial in his writings is his jaw-dropping anti-Semitic views suggesting that Jewish books and churches should be burned, later taken by the German Nazis as justification for the persecution of the Jews. Following his Treatises of 1520 where he proclaimed that the Pope was the Anti-Christ, it came as no surprise that he was excommunicated from the Catholic Church, forcing him to establish his own brand of Christianity known as *Protestant Lutheranism*.

As it concerned music, Luther and his followers largely ignored the canon laws stipulating Gregorian rules of tritone avoidance. He preferred instead congregational singing of hymns like *A Mighty Fortress is Our God*. In this way, Luther opened further the door to vertical organization of music harmony (homophony) and with it the release of restrictions against the tritone.

In 1545 Luther wrote his most scathing pamphlet entitled *Against the Roman Papacy, An Institution of the Devil*. Ironically, it is here where we find the tritone's evil reputation balanced ambiguously between Catholicism and Protestantism. We are left to ponder which camp "the Devil" was in – the tritone that Luther wished to unleash or the pope's rules against it? Was the tritone legend, descended from Pythagoreanism and the Underworld of Greek mythology, somehow a hindrance to spirituality or did it actually *reveal* it? Was the Catholic canon law suppressing the true nature of harmony and obscuring the path to spirituality or was the Protestant view actually releasing some evil trapped in nature?

Through the lens of a tritone we can see how Western society was forcing itself to confront the question of whether God was in nature or apart from nature. As a backlash against the Church's unnatural and anti-harmonic policies, remnants of Pythagoreanism had finally begun to loosen Rome's grip. With Protestant Lutheranism beginning to replace Roman Catholicism in Germany, a new wave of reform was quickly ushered in. [Soergel 1993]

Between 1545 and 1563, an Ecumenical Council known as the Council of Trent convened on three occasions to decide how best to adapt to the change incited by the Protestant Revolution. Commonly referred to as the *Counter Reformation*, Church leaders decided that the Arts should communicate religious themes using more direct and emotional involvement. This amounted to a rethink of what music was acceptable to the Church.

Flying on the winds of change in the Church, Italian composer Claudio Monteverdi (1567 – 1643) led the transition away from Renaissance music into the much more extravagant style known as *Baroque,* literally meaning "broke" music.[29] Monteverdi did in fact break many of the canonical rules of sacred music by introducing the idea of *functional harmonic tonality*. As with de Viltry and Machaut in their battle with *ars antiqua* two centuries earlier, Monteverdi's unconventional madrigals and motets were repeatedly criticized, especially by Church scholar Giovanni Artusi. As one of Zarlino's best students known for his music theory book on dissonance (or how to avoid the tritone), Artusi was very vocal in his condemnation of Monteverdi and denounced his "crude modern style."

As a compromise to quell Artusi's attack, Monteverdi proposed a new split in music (as was apparently popular in those days) into two streams: *prima practica* and *seconda practica*. Prima practica followed the traditional polyphonic counterpoint rules (based on tritone avoidance) while seconda practica utilized a much freer counterpoint and increasing hierarchy of voices, emphasizing soprano against bass. In this way, seconda practica precipitated a move toward *monody* or simple sung melody lines with an independent instrument accompaniment of chords, usually strummed on a lute. It was a critical move away from the Church's suffocating canonical rules toward much greater voice independence and harmonic spatial complexity, including more adventuresome use of the tritone.

The split in musical practice was not the only thing beginning to come apart. During this same period, Italian astronomer Galileo Galilei had come to the conclusion that astronomy could move forward only by abandoning all philosophical guidance, this of course being the Pythagorean theory of *musica universalis*. Even though he was very interested in harmonic phenomena and had researched harmonic motion (inspired by the work of his musician father), Galileo was also a devoted Catholic and struggled to find a way to compromise his Copernican sun-centric view of the solar system with the Church's Ptolemaic earth-centric doctrine.

Historians often reference letters from Galileo to his daughter as proof that the motivation behind abandoning the guidance of *musica univeralis* was based strictly on the rationale that observations and experimental evidence should be enough. But this unguided approach to natural study had as much to do with his inner conflict as a Catholic and a strong desire to avoid

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[29] Baroque – Origin mid-18th century; from French irregular, imperfect pearl or a wart; disparagement of such Art considered broken.

offending the Church. As a practical matter of the greatest degree, we can be sure that the threat of torture and imprisonment by the Inquisition tribunal weighed heavily on any admission he might have otherwise made concerning guidance from pagan harmonic principles.

We can see this strategy of self-censorship beginning to form in 1616 when Pope Paul V (participating in the Inquisition) reprimanded Galileo concerning his heliocentric theory of the solar system, warning him to curtail his investigations into the "supernal realms." The pope chided Galileo that the motions of the heavenly bodies, having been touched upon in the Psalms, the Book of Joshua, and elsewhere in the Bible, were matters best left to the Holy Fathers of the Church. Galileo obeyed this order, silencing himself on the subject for seven years. He did finally return to work on his sun-centric theory in 1623 after the death of his sister, but this time it was based strictly upon observation with absolutely no mention of any harmonic theories that might upset the Church.

Unfortunately, this strategy did not help him much. Galileo's support of the Copernican theory had already placed him in the same camp as Greek, Indian and Muslim pagans who had centuries before discovered this same fact. As such, he was eventually put on trial, accused of heresy and condemned by the Church. Just as the tritone had been outlawed centuries earlier to suppress paganism, so too did the Church wish to suppress the true harmonic nature of the solar system. Galileo's trial went out as a warning to scientists everywhere that they should avoid the study of harmonics and remain loyal to Church doctrine. As a result, music and its harmonic principles became strictly taboo in scientific circles.

In this way, Galileo became a model for self-censorship. In fact, the avoidance of harmonic principles became the founding principle for the "scientific method." Study of nature would no longer be guided by either religion or philosophy, but instead as a strict system of observation and experimentation intended only to collect information and report findings. To this day, the institution of modern science continues to avoid any independent field of harmonic research.

With harmonics and philosophy expunged from science, music was then reduced to a field of historical study and musical performance, becoming part of what we now know as "the Humanities." In this way, we have been left with a society and educational system shaped in large part by the fear of the Inquisition. Contrary to the conventional wisdom that the Inquisition is far behind us, this fear is present today throughout Western civilization, founded on Galileo's strategy of self-censorship and maintained by the unquestioned sanctity of the scientific method.

The truth is for the past 350 years a *complicity of convenience* has existed between the institutions of Western religion and science to avoid any unifying system of harmonic study.[30]

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[30] **Hypothesis 5:** The Inquisition created a *complicity of convenience* in the $17^{th}$ century between Western religion and science that resulted in the separation of harmonic science from natural science. This resulted in the formation of history, philosophy and music as a humanities track well insulated from the scientific method.

But it is not Galileo who should be faulted for this. He was just a product of his religious upbringing and one of countless victims who suffered at the hand of the most powerful regime on Earth. Music and harmonics were so intertwined with the effort to eradicate paganism that any attempt to uncouple harmonic principles for use as a guiding philosophy in science must have seemed impossible.

So as music, religion and science were forced apart at the end of the Renaissance, neo-Pythagoreanism and the various Hermetic/ Kabbalistic philosophies were forced underground into the "secret societies." One of the last remaining proponents of this ancient harmonic science was Robert Fludd, a prominent English physicist, astrologer and Rosicrucian mystic. He is known for his treatise in 1618 entitled *De Musica Mundana* that defended the Pythagorean idea that the universe was a "divine" or "celestial" monochord of vibratory energy from which all things resonated into being. In it, he proposed that the interval between the elements of Earth and the highest heaven was a disdiapason hierarchy of energy, representing the two extremes of existence as a double octave.

In Fludd's view, the material world of the Sun and Earth were separated by a lower octave while the spiritual realm between the Sun and Heaven were separated by a higher octave. He represented this geometrically using *two interpenetrating pyramids* to symbolize the double octave as a marriage of energy between the material and spiritual worlds. Not surprisingly, the pyramids were aligned with one another at precisely the Pythagorean ratios of 4:3 (a perfect 4th) and 3:2 (a perfect $5^{th}$) to represent the four elements of air, fire, water and mineral plus the quintessential spiritual plane. In doing so, Fludd had reproduced the dual Pythagorean tetrachord as a *star tetrahedron*, serving as a hexagonal harmonic model of the cosmos. Of course, in the rising age of reason and empiricism, philosophical theories like this evoked no small degree of criticism.

English physicist and mathematician Sir Isaac Newton, a contemporary of Fludd, was also a Rosicrucian and shared an interest in Pythagorean harmonic science, especially as it related to light and color. But while much of his scientific work was inspired by a lifelong interest in alchemy and harmonic ideals, he grew to disagree with Fludd's philosophically guided study of nature derived from harmonics. Taking up Galileo's method of unguided experimental study instead, Newton publicly debated Fludd arguing that nature should be examined and documented without use of any harmonic philosophy or prescribed rationale.

It was from Newton's 1687 treatise *Philosophie Naturalis Principia Mathematica* that we receive our present day "scientific method" and corresponding anti-harmonic view of nature. Departing from the integrated *musica universalis* worldview of the past, Newton's method based on Galileo's self-censorship strategy ultimately won out over Fludd's *Musica Mundana*. The result was a science that excluded harmonic philosophy. This was *The Enlightenment*.

While the exclusion of harmonic principles from science was arguably a clear case of throwing the baby out with the bathwater, it did allow the coexistence of Science with the Church while greatly reducing the connection between paganism and the study of nature. In turn, this reduced the pressure on the Church to control music, opening up a new harmonic style in the Baroque period (1600–1750). Art of all kinds became less ambiguous, less arcane and less mysterious – more direct, obvious, ornate and expressive. As music moved away from the old polyphonic counterpoint rules based on tritone avoidance, a new Baroque counterpoint using vertical harmonies and Zarlino's simplified major and minor scales took its place.

By the time Johann Sebastian Bach entered the scene at the beginning of the 18th century, the Baroque period was two-thirds over. With his entire family supported by musical work within the German Lutheran Church, Johann was part of the first generation to be raised outside of the influence of Catholic canon law. He was free to study and incorporate the latest musical ideas of his day, including works by Italian composers Antonio Vivaldi, Arcangelo Corelli and Giuseppe Torelli, producing a prodigious collection of organ music, chorales, canons and fugues which were all based on a revolutionary theory of vertical chord harmonies.

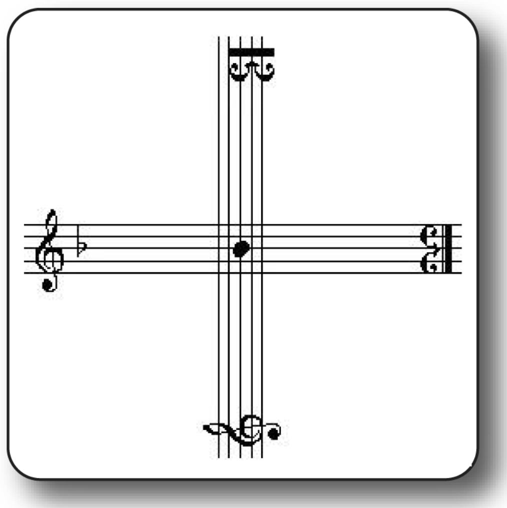
After years of composing sacred music for the German Lutheran Church, Bach finally took a position in 1717 as the Kapellmeister (director of music) for Prince Leopold of Anhalt-Cothen. Leopold, a musician and admirer of Bach, paid him well and gave him a great deal of latitude in the music he composed. Since the Prince was a Calvinist, he followed a more fundamental tradition from the Protestant Reformation that did not use elaborate music in worship. Because of this, most of Bach's work during this time followed secular themes, such as the now famous Brandenburg concertos written between 1719 and 1721 for Christian Ludwig, Margrave of Brandenburg.

Free of all religious restrictions, Bach pursued increasingly innovative musical directions. Inspired by the new music of Vivaldi, he created music with stronger impact than ever before and with more pull and tension in the cadences toward the tonic. He began to use more and more diminished arpeggios, ascending and descending rapidly against a held bass note to create a heightened sense of harmonic tension. In doing this, Bach had unleashed the full harmonic power of the tritone upon the world.

For example in 1723, after writing his Brandenburg concertos, Bach wrote the *Magnificat*, a powerful cycle of twelve short orchestral and choral movements performed for Vespers on Christmas Day in Leipzig. At the end of the second movement, with full orchestra and trumpets blazing, a most astonishing example of literal word setting occurs. The word "dispersit" is scattered throughout the choir, which then suddenly comes together with an anguished and violent tritonal diminished chord on the word "superbos" (meaning "the proud"). Symbolically, Bach had bound the chaos of independent dispersed voices together through a magnificent harmonic swirl of tritones as if to unite the congregation in a kind of protest against the social control system of the Vatican.

Indeed, the tritone had been set free through the Protestant revolution with Bach as its foremost evangelist. He increasingly used the tritone in vertical harmonies to generate higher and higher levels of excitement and tension prior to resolution. His music became ever more grand in harmonic architecture with the tritone as its foundation. A review of the body of music in his later years will show that Bach was the one most responsible for legitimizing use of the tritone, leading to its increasing popularity thereafter.[31] Even his signature of a double cross of musical clefts on $B^b$ represented both the first letter of his last name and the tritone *schisma* found in the center of the Pythagorean disjunctive tetrachord system starting on bottom line E.

**Figure 18 - Bach's cross as composer's signature**



By the beginning of the 18th century, Bach and his contemporaries once again began to consider music as a science in itself – the very sound of mathematics – reducible to theory and law. They believed that music could reveal the deeper truths in nature and that all things could be understood through music. This resurgence of Pythagorean philosophy developed out of a number of important discoveries, including the startling news of an observable proof of wave harmonics.

31 **Hypothesis 6:** Johannes Sebastian Bach was the leading proponent of the Tritone Function and popularized its use thereafter.

Two Oxford scientists, William Noble and Thomas Pigot, demonstrated in 1673 that multiple pitches could be produced from a single string as it vibrated. They demonstrated this by tuning strings at the octave, octave plus $5^{th}$, and octave plus major $3^{rd}$ *below* a reference string. When the reference string was then plucked, the non-plucked strings would vibrate sympathetically at the unison – thus, proving harmonic vibration as an observable fact of music. These findings quickly made their way into the hands of composers and music theorists across Europe.

In 1722, Jean Philippe Rameau was the first to base a music theory on the newly discovered laws of acoustics. In his *Traité de l'harmonie*, he proposed that the bottom tone of a harmonic stack was the primary determiner of chord progression. Giuseppe Tartini further supported the role of a fundamental bass tone as steering harmony in 1754 with his discovery of what he called "the third sound." This third sound was a barely audible tone appearing naturally as the difference between the bottom tone and top tone in an interval. It was determined that this middle tone usually reinforced the note of a chord that Rameau identified as the fundamental bass. But while both Rameau and Tartini worked to provide convincing mathematical proofs of these ideas, contemporary scientists like Jean d'Alembert and Leonhard Euler were critical of their work, pointing out their oft-faulty calculations.

Nonetheless, such scientific discoveries and new music theories did have a major impact on composers during the late Baroque period. Near the end of his life in 1747, Bach joined the Mizler "Corresponding Society for Musical Sciences" founded nine years earlier by his polymath friend Lorenz Cristoph Mizler. This group counted as members other notable Baroque composers, such as Georg Philipp Telemann and George Frideric Handel as well as Leopold Mozart, Wolfgang's soon-to-be father. It was during this time that Bach took polyphonic canon form to a higher art form with deeper significance, often making musical puzzles to "the glory of God." Like many in Mizler's Society, Bach believed in a greater harmony of the planets and wrote his music to reflect this natural order.

One story goes that upon visiting Frederick the Great (King of Prussia) a few years earlier, Bach had been given several difficult musical themes with which to improvise on the king's new instruments. Frederick, being a Freemason and a son of *The Enlightenment*, had summoned "old Bach" with a plan to humiliate and discredit his well-known esoteric beliefs. At a large party given in his honor, he asked Bach to improvise an impossibly difficult six-part fugue for his guests. Since Bach had never written a six-part fugue, much less improvised one, he declined the request saying that not every subject is suitable for improvisation in six voices and promised the king that he would work it out later on paper.

With this, Frederick proudly claimed a victory of reason and rationalism over the maestro at his own game. The story was published in the local newspaper, making a laughing stock out of Bach and by implication discrediting his esoteric belief in the *Music of the Spheres*. Frederick was openly contemptuous of what he viewed as metaphysical thinking, saying it "smelled of the Church" (a twist of irony).

But keeping his promise, Bach did later use Frederick's themes to compose one of his greatest musical puzzles – the *Musical Offering*. As a final reply to the king's challenge, Bach incorporated a number of harmonic symbolisms. Its key of three flats and order of tones seem to suggest the symbolism of an Egyptian triangle and the Masonic officer hierarchy, itself descended from the principles of *musica universalis*. The ten canons in the piece may have been intended to represent the 10-point *Tree of Life* symbol taken from the circular Egyptian *Flower of Life* geometry, revered by the Rosicrucian Order that emerged from the Protestant revolution. With his *Musical Offering*, Bach seemed to be reminding the king of the ancient knowledge of nature's harmonic organization that so many seemed determined to forget in the rush toward reason and "enlightenment."

In fact, Bach incorporated many such esoteric symbolisms in his greatest music. The ideals of harmonic geometry, borrowed from the Kabbalah number system of gematria, greatly inspired him during his last years. This is most noticeable in his thirteenth canon of fourteen on the Goldberg Ground as well as the Canonic Variations on Von Himmel Hoch. These were some of his most harmonically complex compositions, all published through Mizler's Corresponding Society of Musical Sciences.

Bach saw the Baroque canon, and more broadly the various contrapuntal techniques emergent from canon form, as both a scientific and a spiritual expression. Perhaps he saw the canon as a bridge between his Christian beliefs and the pagan harmonic sciences. Maybe it was a symbolic mending of the split between Catholicism and Protestantism or God and Nature. But no matter how he thought of it, we can be sure that Bach's interest and dedication to the canon expressed his deep personal belief in a purposeful order in the cosmos.

In modern times, Bach's music has become a great inspiration to science. His *Brandenburg Concerto No. 2 in F*, *First Movement, Prelude and Fugue in C, No. 1* and *Partita No. 3 in E major* for violin were all included on *The Golden Record* sent with the Voyager 1 and 2 spacecraft as an example of humanity's best achievements. Bach's music was even suggested as the first method of communication with extraterrestrial civilizations:

> *"I would vote for Bach, all of Bach, streamed out into space, over and over again. We would be bragging of course, but it is surely excusable to put the best possible face on at the beginning of such an acquaintance. We can tell the harder truths later." – Lewis Thomas*

And so we can imagine Bach's celestial music forever flying on a golden disk as mankind's greatest achievements in the musical sciences, proudly carrying the Church's *Diabolus in Musica* along for the ride.

# *Romantic Duality*

*"You are the music; while the music lasts." - T. S. Elliot*

With the move toward Classicism in art and architecture in the latter half of the 18th century, young Wolfgang Amadeus Mozart and his contemporaries offered a much cleaner and simpler style of music than that of the Baroque period. The inspiration from harmonic philosophy had continued to fall away as the 17th century *Age of Reason* ushered in a more secular view of music. With Newton's physics fully in force, it was believed that ideas should be well formed in axioms and clearly articulated following the scientific method. As a result, musical architecture became even more transparent during the *Classical Period*, relying on unadorned chords and sweeping melodies that everyone could understand.

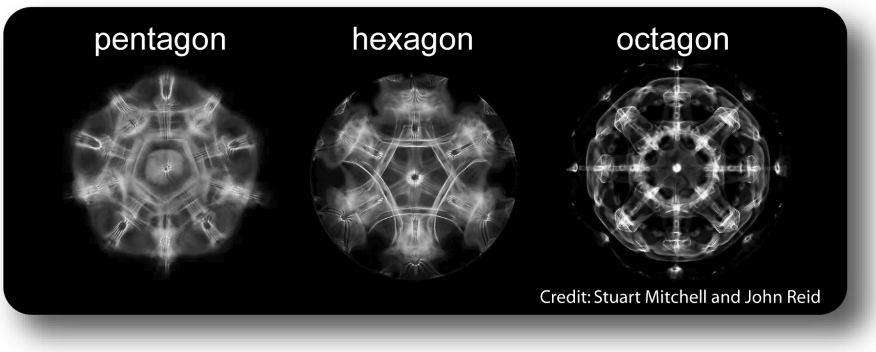
Since royal patrons supported most of the composers of this time, music was usually written for lighthearted dance occasions or theatrical entertainment, particularly comic operas. The harmonic form had become a dressed up version of the earlier Monteverdi madrigal song style. Lavish melodies supported by block chords (triads) stayed mostly within a single major or minor key. The Cycle of 5ths, so important to Pythagoras, was now crystallized into vertical chord progressions. The tritone was used much less apologetically, though more judiciously than in the late Baroque period. It was quite common to hear the tritone emphasized at a dominant-tonic cadence or inferred as a "leading tone" in melodies, usually lingering a moment before resolving delicately.

Interest in cryptic harmonic puzzles continued, but with fear of Church retribution no longer a factor, theosophical symbolisms and allegory became more overt. Like many leading Germans and Austrians in the late 18th century, Mozart, Haydn and Telemann were all accepted Freemasons and as such were quite intrigued by Egyptian and Greek theories concerning harmonics. Most people are unaware that Mozart wrote one of his most famous pieces, *The Magic Flute*, for his Masonic friends. Known as "The Freemason Opera" because of its use of the number 3 and other symbols of Freemasonry, prominent German writer and Freemason Johann Wolfgang von Goethe once commented:

> *"It is enough that the crowd would find pleasure in seeing the spectacle; at the same time, its high significance will not escape the initiates."*

It was during this time that a great number of discoveries relating to wave theory and harmonic geometry were being discovered. In 1787, German physicist Ernst Chladni published a book entitled *Entdeckungen über die Theorie des Klanges* describing his experiments in visualizing musical tones as auditory geometries. He found that he could sprinkle sand on round and square metal plates and then vibrate the edge with a violin bow to create a wide variety of beautiful patterns from simple polygons to elaborate patterns.

**Figure 19 - Chladni figures vibrated on a round plate**




He noticed that these patterns, later named *Chladni figures*, formed whenever harmonic wave frequencies were applied to flat surfaces. His equation, $f = C(m + 2n)^p$ using C and p as coefficients of the plate, became known as *Chladni's law*. Chladni's work in harmonics led him to other important discoveries, including the calculation of the speed of sound for different gases and a number of scientific methods used to optimize the design of acoustic instruments, in particular violins, guitars and cellos.

But even though Chladni is widely credited for discovering the fact that harmonics produce regular geometric patterns on vibrated surfaces, such knowledge had actually been around for centuries and probably thousands of years. This was proven recently in the discovery of Chladni figures carved into the face of "musical cubes" jutting out from the arches of Rosslyn Chapel outside of Edinburgh, Scotland. Built in 1446 by followers of a Gnostic Christian sect known as Ebionites (similar to the Cathars and Templar Order[32]), these auditory patterns were carved into stone nearly 300 years before Chladni was even born.

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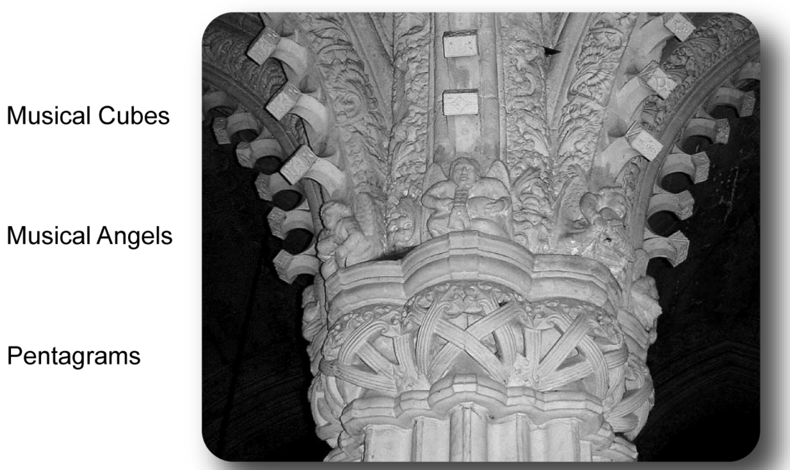
[32] The Scottish Knights Templar was founded by a group of French Templars who escaped to Scotland in 1307 after King Philipe IV of France decided to suppress the order. They originated from the Visigoths in southern France known as Arian Christians – followers of Arius, an Alexandrian Gnostic. Alexandrian Gnosticism included influences from many world religions, including Greek, Egyptian and Persian, though did not adhere to the doctrines of Christ's divinity and resurrection. This group is said to have descended from the French Merovingian kings through the Cathar family, who founded the French Templar order. It was the "Cathar heresy" in Avignon during the 13th century that ultimately led to the French papacy and the split away from canon law.

It was the Scottish father-son team of Thomas and Stuart Mitchell who finally deciphered the 215 “musical cubes” in the chapel’s arches to uncover a melody they named *The Rosslyn Motet*. Each “musical cube” was matched against a particular Chladni pattern to determine a pitch which, when strung together, formed a melody. In my e-mail exchanges with Stuart Mitchell, he suggested that the number of musical cubes could be taken allegorically to correspond to 216, the ancient Hebrew number for God said to be awaiting one last sound by the “Anti-Christ” to herald in the Messianic Age.[33] But how other carvings in the chapel related to this remained a mystery.

Underneath the cubes were 13 musical angels carved into the stone pillars. Some of these angels are depicted sitting on carved pentagrams, in place of the top golden triangle of the stars, while playing musical instruments. The stars are then counterbalanced by 8 dragons-serpents wrapped around the base of one of the pillars whose tongues intertwine with vines, possibly representing the Tree of Life. Visitors have marveled over this for centuries wondering what it could all mean.

My suggestion to Stuart was the 13 musical angels and 8 dragons might be representative of the Fibonacci ratio of 13:8, or 1.625, signifying a number close to the golden ratio and corresponding to the Earth-Venus orbital proportion of a pentagram. As I came to discover later, the interior of the chapel was designed in a proportion not far from the harmonic 5:3 proportion of a major 6th, suggesting a further link between the architectural symbols and principles of harmonic resonance. Still, what could the designers be trying to tell us?

**Figure 20 – Harmonic symbols in Rosslyn chapel**




[33] Using gematria, the digits of 216 are added together to equal 9 or $3^3$, also taken as a Hebrew symbol for God. The same digits can be produced by simple numbers: $3^3 \times 4 \div 5 = 21.6$.

Knowledge of resonant proportions and their relationship to geometrical patterns can be traced back to the ancient mystery schools. Evidence suggests that such harmonic patterns were even used in the iconography of ancient languages like Sanskrit, Hebrew and the Egyptian hieroglyphics. It is not hard to imagine shamanic chants over some dusty drumhead magically conjuring up geometrical patterns, which were then taken as messages from God and used in written language. The Biblical phrase “In the beginning there was the Word and the Word was with God” probably had its genesis in such sound patterns. Was this the intended message of Rosslyn chapel?

As Chladni demonstrated centuries later, harmonic intervals will take the form of simple geometric shapes like pentagons, hexagons and octagons. Was the secret of Rosslyn chapel the simple fact that the interplay of 13:8 and 5:3 is found not only in music but in the heavens as well? Were its master masons trying to tell us that the Naga dragon-serpents, symbolizing the Fibonacci series spiraling into the golden ratio, is what supports the resonance (or enlightenment) of angels as they play their musical instruments on the pentagonal *Star in the East*?

To this day we continue to find both geometry and resonance as symbolic themes in Masonic and Rosicrucian orders, especially the triangle, pentagram and hexagram as it was passed down through the various Hermetic and Kabbalistic currents. The *Rosslyn Motet* and the carvings of Rosslyn chapel were clearly a way to preserve these harmonic principles of nature while hiding a much greater secret in music.

What was this Great Secret you ask? Why, the tritone of course! By placing it into the melody and “freezing” it into the chapel’s architecture, its designers had preserved their most sacred knowledge of *musica universalis* at a time when such pagan ideals were heresy in the eyes of the Church.

Ludwig van Beethoven began his musical career as a young piano virtuoso and composer in the late 18$^{th}$ century, propelled by the atmosphere of unrestrained free thought that blossomed into the *Romantic Period* of the 19$^{th}$ century. Like his predecessors, Beethoven was intrigued by the esoteric harmonic symbolisms found in German Freemasonry, composing songs such as *Maurefragen* (Masonic Questions) and *Der Freye Mann* (The Free Man). While no record has yet been found of his becoming a Freemason or even attending a meeting, many of his friends were members, leading some to speculate that he was a secret member too. In any case, the idea of a science and society built around principles of harmonic science continued to push Beethoven and other composers of his time toward musical experimentation well outside of accepted Church tradition.

Other composers of this period, such as Hector Berlioz, Johannes Brahms, Frederick Chopin, Franz Liszt, Robert Schumann, Peter Tchaikovsky and Richard Wagner, were all inventing harmonic combinations and rhythms according to their own vision – their own new rules. The old definitions rooted in sacred music had become irrelevant, replaced by the subjective opinion of each composer. Romantic philosophy held that not everything could be rationalized with axioms and it should be the heart and ear that leads us.

In Romanticism, we can see the earlier musical forms continuing to fragment. Each split of society and culture – *ars antiqua* vs. *ars nova*, *prima practica* vs. *seconda practica*, Christian vs. pagan, Catholic vs. Protestant, religious vs. secular, alchemical vs. scientific and now Newtonian-Cartesian objectivism vs. Romantic subjectivism – seemed to represent yet another move away from a unified coherent worldview. Musical experimentation during this period led to increasingly radical harmonies, musicians and scientists alike were drawn to the conclusion that music was *entirely* a matter of personal opinion and determined largely by cultural influences.

In this way, Romantic subjectivism seemed to undercut the Baroque idea of music as a predictable science governed by universal laws. The new and more complex harmonies were seen as exceptions to the rules rather than as new rules in themselves. With no theoretical context to contain such flights of fantasy, new harmonic forms became mere "alterations" to the traditional diatonic practices already formulated in Catholic canon law. Music theory had hit a brick wall and was splintering into a thousand tiny pieces. That is except for the tritone, which was used as if it were an emergency handle in some runaway musical train.

Tritone usage ran rampant. Beethoven was hammering away at us with tritones inside dominant 7$^{th}$ chords, Chopin and Schumann were running them up and down the keyboard in loping arpeggios and Wagner was pulling at our proverbial heart strings with startling tritone leaps in melodies. There was a movement away from diatonic keys and toward *chromatic* harmonic movement that relied on the mysterious pull of the tritone into remote keys to overcome traditional diatonic expectations. This emotional and seemingly illogical progression between chords and keys foreshadowed and paralleled French Impressionism in late 19$^{th}$ century art. The use of bright colors in paintings, and more colorful techniques like blue shadows instead of darkened background hues, was the visual equivalent to complementary chromatic harmonies on a musical color wheel. New terminology had to be invented to describe these capricious non-diatonic chord progressions, such as "Neapolitan 6$^{th}$" and the "German Augmented 6$^{th}$" chord popularized by Wagner in *Tristan and Isolde*.

In this new way of thinking, Romantic harmonies were really built on an alternation between two 6-tone wholetone scales rather than the 7-tone diatonic scales used previously. For instance, the Neapolitan chord slid down by a half step to *phase shift* from one wholetone scale to the other. Similarly, the German chord expanded symmetrically outward by a half step, again phase shifting from one wholetone scale to the other. This harmonic technique led to the characteristic "tritone substitute" of 20$^{th}$ century Jazz, always resolving by a sliding half step. In each of these cases, such sliding movements were considered both efficient and satisfying, often adding an ethos of sensuality to the music. It is this sensuality that many feel in Jazz music, first originating in the New Orleans bordellos and so-named after the provocative scent of French jasmine perfume.

Nonetheless, chord resolution by a half step was an important development in music since it represented a *generalization of the Tritone Function*. That is, if a tritone (a 3 wholetone interval) and an augmented 6$^{th}$ (a 5 wholetone interval) are preferred to move symmetrically by a half step, why would not *any* interval that is a wholetone multiple work the same way? Seen in this light, we might consider *any* chromatic harmony to be a simple alternation between the dual wholetone scales – one group of six on a seesaw with the opposing group of six. Fact is, just about any combination of chords oscillating between the two 6-step scales will create a sensation of harmonic movement.

Far from proving that harmony was only a matter of opinion, the Romanticists had stumbled on an intrinsic duality universal to all music harmony. They had found that two groups of tritones – one group as a foreground wholetone scale contracting to the other group as a background wholetone scale – seemed to function as a driving oscillating force in music. In this sense, the Romantic style can be characterized as a generalization of the Tritone Function into two alternating auditory hexagons.[34]

French composers Claude Debussy and Maurice Ravel then expanded on the Romantic idea of tritone generalization during the late 19$^{th}$ and early 20$^{th}$ century. Their musical style, known as *Impressionism*, employed free movement between keys by sliding along a wholetone scale, contracting or expanding by multi-wholetone intervals on a whim, then resolving temporarily into a diatonic key. But rather than use a diatonic scale, Debussy preferred a more primitive sound that excluded the diatonic tritone, leaving only the five tones of a *pentatonic* scale. In the most general sense, Debussy had created a style based on oscillating auditory hexagons (two wholetone scales) interrupted by the occasional auditory pentagon (pentatonic scale)

Incredibly, music had emerged from the Dark Ages into the 20$^{th}$ century only to reach the Greek harmonic ideal of an apple – a circle inscribing a pentagram. Was this, like the Z-twelve octave, purely natural – a byproduct of subjective Romanticism, or was it designed intentionally based on ancient Egyptian/ Pythagorean philosophies? Debussy's emphasis on the pentatonic scale surrounded by the odd-even circularity of dual wholetone scales certainly could be seen as a grand musical allusion to the ancient Pentagrammon symbol. But whether or not Debussy embraced such symbolism is far less important than the fact that his music still sounded beautiful to everyone completely outside of the long tradition of 7-step diatonic harmony passed down from the Church.

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[34] **Hypothesis 7:** A wholetone scale is a generalization of a tritone – thus, chromatic harmony can be seen as two oscillating wholetone scales derived from the generalization of the Tritone Function.

Without excluding the possibility of an esoteric interest in ancient harmonic science, it is easy to see how Debussy's music and its implied geometry evolved out of the subjective Romantic worldview, foreshadowing the pure geometric forms of Cubist art soon to arrive in the 20th century. Even so, neither he nor Ravel claimed to hold any particular theory nor ever considered themselves part of any Impressionist schools of thought. Like the Romanticists before them, they claimed only to follow their ear and temper it with a predictable form and structure. With all of the manmade rules stripped away, primitive harmonic geometry just came naturally.

Romantic experimentation had returned to music's most pagan roots – the rose symbol of Martin Luther and the Greek's golden apple of knowledge. From the auditory geometry of dual rings emanating out of a pentagram, the Impressionists had created a new musical language of chromatic chord clusters, interval cycles, mode fragments and pentatonic scales. And beneath this simple system laid the fuzzy generalization of the tritone contracting or expanding from one auditory hexagon to the other. Romanticism had indeed pixilated into a dreamy, whirling blur of shapes and landscape impressions that always seemed to maintain a sense of balance between expectation and realization.

Many people think of Impressionist music as not functioning harmonically, but it really does. As a contrast of foreground against background, a pentatonic or wholetone cluster (the foreground) will always phase shift to the opposing pentatonic or wholetone scale (as the background). Sometimes the harmony progresses along a cycle of 5ths, 4ths, 3rds or 6ths – called *planing* – suspending briefly in foreground on a wholetone scale before continuing its double hexagon oscillation. In fact, the very definition of harmonic function changed during this period to become a simple matter of contrasting foreground against background, paralleling similar developments in the Berlin School of experimental psychology of the 1890s and foreshadowing the German Gestalt theory of the 1920s. All the while, at the center of this Gestalt duality, the generalized Tritone Function continued the role it had always played – mending the spiral of pitch into a closed auditory geometry.

Russians Igor Stravinsky and Sergei Prokofiev entered the scene in the early 20th century to generalize the tritone further still. Inspired perhaps by the political dualism of the first world war, both men continued to incorporate the concepts of foreground-background contrast invented by the French, only now constructing chords and progressions using adjacent semitones. Sounding much more dissonant than the evenly divisible hexagons of Impressionism, melodies and chords were now less contiguous, jumping across a wide range with chord clusters moving wildly in chromatic half steps.

Yet somehow, recognizable chords and scales could still be heard. The tritone was still a major source of dissonance and tension, though now buried inside dense polychords and thickly clustered harmonic structures that diluted its tonal gravity.

But just when it seemed there was no new ground to break in the business of serious art music, Arnold Schoenberg arrived on the scene with his 12-tone composition system known as *dodecaphony*. This was a mathematical system based on a *twelve-tone row* (sometimes called a *dodecaphonic matrix*) that was specifically designed to eliminate any possibility of a tonal center or functioning harmony. As the ultimate reincarnation of the Church's strategy of avoiding natural harmonic principles, Schoenberg had invented a new 20$^{th}$ century sound that eschewed any kind of harmony in favor of form, timbre and dynamics. Twelve-tone rows "guaranteed" the elimination of tonality by ordering tones into inverted, retrograde and retrograde-inverted groups so that one note would never be emphasized over any other. This style of music and later *serialism* techniques became known generally as *atonality.*

**Figure 21 - Twelve-tone matrix for "atonal" music**

| $P \rightarrow$ | $I_0$ | $I_3$ | $I_7$ | $I_2$ | $I_{10}$ | $I_5$ | $I_{11}$ | $I_6$ | $I_9$ | $I_8$ | $I_4$ | $I_1$ | $I \downarrow$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $P_0$ | 2 | 5 | 9 | 4 | 0 | 7 | 1 | 8 | 11 | 10 | 6 | 3 | $R_0$ |
| $P_9$ | 11 | 2 | 6 | 1 | 9 | 4 | 10 | 5 | 8 | 7 | 3 | 0 | $R_9$ |
| $P_5$ | 7 | 10 | 2 | 9 | 5 | 0 | 6 | 1 | 4 | 3 | 11 | 8 | $R_5$ |
| $P_{10}$ | 0 | 3 | 7 | 2 | 10 | 5 | 11 | 6 | 9 | 8 | 4 | 1 | $R_{10}$ |
| $P_2$ | 4 | 7 | 11 | 6 | 2 | 9 | 3 | 10 | 1 | 0 | 8 | 5 | $R_2$ |
| $P_7$ | 9 | 0 | 4 | 11 | 7 | 2 | 8 | 3 | 6 | 5 | 1 | 10 | $R_7$ |
| $P_1$ | 3 | 6 | 10 | 5 | 1 | 8 | 2 | 9 | 0 | 11 | 7 | 4 | $R_1$ |
| $P_6$ | 8 | 11 | 3 | 10 | 6 | 1 | 7 | 2 | 5 | 4 | 0 | 9 | $R_6$ |
| $P_3$ | 5 | 8 | 0 | 7 | 3 | 10 | 4 | 11 | 2 | 1 | 9 | 6 | $R_3$ |
| $P_4$ | 6 | 9 | 1 | 8 | 4 | 11 | 5 | 0 | 3 | 2 | 10 | 7 | $R_4$ |
| $P_8$ | 10 | 1 | 5 | 0 | 8 | 3 | 9 | 4 | 7 | 6 | 2 | 11 | $R_8$ |
| $P_{11}$ | 1 | 4 | 8 | 3 | 11 | 6 | 0 | 7 | 10 | 9 | 5 | 2 | $R_{11}$ |
| $RI \uparrow$ | $RI_0$ | $RI_3$ | $RI_7$ | $RI_2$ | $RI_{10}$ | $RI_5$ | $RI_{11}$ | $RI_6$ | $RI_9$ | $RI_8$ | $RI_4$ | $RI_1$ | $R \leftarrow$ |

Now, many scholars view the advent of atonality as final proof that people are not predisposed to any kind of natural harmonic laws. The fact that some enjoy the chaotic sounds and dissonances of atonal music seems to confirm that music really is just a matter of cultural influence and quirky personal opinion. Of course, this raises the question of whether music harmony is something organic and universal or simply learned like a language.

There are two ways to understand atonal music - one through reason and the other through emotion. If you cannot recognize a sustained pattern (whether melodic or as simultaneous voices), then we might reasonably conclude that atonality does indeed prove that harmony is not organic or intrinsic to music perception and thus all music must be a product of culture. This is the opinion implied in most modern music theory books.

On the other hand, if you think we all have an *innate recognition* of harmony in music as practiced in some form for thousands of years, then atonal music must still be operating within an instinctive cognitive framework that is *emotionally predisposed* to respond to geometric harmonic auditory structures. By this last definition, you would conclude that atonal music could never prove music is the exclusive product of cultural programming. You would have to conclude that music appeals emotionally to an intrinsic *cognitive pattern recognition system* that is somehow evolved from harmonic principles. You would conclude that our mental auditory apparatus must itself be harmonic in some way and at all times be trying to recognize predictable harmonic patterns in music – however short and fragmented.

But without an accepted theory for how we actually perceive music, including prescribed methods to measure and predict it, most people would have to conclude with the rational argument and agree that music is probably subjective and a matter of cultural influence. This is exactly what happened by the mid-20$^{th}$ century. Any prestige that once surrounded the study of harmonic theory had all but disappeared with the advent of atonality and any interest in advancing the theory and models of music harmony had evaporated. Music theory had become a simple matter of better packaging for the old rules and nomenclature passed down from Church tradition. Any notion that the tritone or golden ratio might somehow explain music organically or geometrically remained "in the closet" and beyond the awareness of musicians and musicologists.

Famed music composer and respected theorist Paul Hindemith said in the preface of his 1943 book *A Concentrated Course in Traditional Harmony*:

> *"Despite the evident loss of prestige which conventional harmony teaching has suffered, we must still count on it as the most important branch of theory teaching, at least so long as it has not been replaced by any generally recognized, universally adopted, more comprehensive, and altogether better system."*

It is well known that Hindemith based much of his own music theories on 17$^{th}$ century theorists Rameau and Tartini and their "difference tones," which he called "mid-tones." Even though many students still learn harmony from Hindemith's repackaged medieval theory books, few scientists or musicians would consider his approach to be an adequate explanation of even the most basic questions of music. Without a complete and thorough explanation for harmonic music perception, music pedagogy has continued to be a subjective art well outside the purview of contemporary science.

American Jazz arrived just in the nick of time, as the 20$^{th}$ century was moving away from recognizable harmonies and toward academic abstraction. In the shadow of rapid industrialization and worldwide wars, few had any interest in music as a natural philosophy. Most people only wanted a catchy tune, simple lyrics and a nice beat to take their mind off the reality of their situation.

Scott Joplin, born in Linden, Texas at the end of the Civil War, was one of the first composers of the early jazz music known as Ragtime. Originating from African slave roots, the styles of Ragtime, New Orleans Dixieland, Barrel House Boogie-woogie, "Stride" piano, Big Band Swing and other regional variations of jazz gained widespread popularity from the 1920's through the World War II years. All of these styles relied heavily on traditional diatonic harmony and the always-reliable Tritone Function. With the advent of theaters and radio broadcasts, large audiences could finally agree on the type of music they preferred and Jazz was it. Jazz caught on in Europe, especially in France, unifying all classes and strata of Western culture under a single style of music for the first time in history. From the plush tritone-powered harmonies of Jazz, the predictable geometry of harmony blossomed into a golden age.

The essential harmonic ingredient of jazz was the *tritone substitute*. This unique stylistic element emerged at the turn of the 20$^{th}$ century during the final transition from late Romanticism to Impressionism. It was a new *polychordal* style based on the Tritone Function that worked by stacking chords spaced by a tritone or other wholetone interval. Doing this caused the chords to share (or share by implication) the same diatonic tritone, thereby amplifying its strong pulling effect. For example, a dominant 7$^{th}$ chord ($G^7$ in the key of C) could be combined with its complementary dominant 7$^{th}$ a tritone away ($D^{b7}$) to strengthen the pull to the Tonic {C}. In common practice, these chords were described as "altered" and so designated with flats and sharps or extra notes relative to the diatonic key, such as $G^{7\#5b9}$ – $C^{M7/9}$.

As polychords became more and more accepted into the 1950's, jazz pianists like Art Tatum used them to such an extent that the original key would sometimes become ambiguous and even disappear altogether at times. Whether realized or not, Tatum's music was actually guided by the ambiguity of the tritone, often leading him deep into nested micro-compositions of quasi-Impressionist harmonies only to reemerge suddenly back into the original piece.

Stacking tritone chords and other "wholetone-relative" chords above the tonic key became the characteristic sound of jazz. The effect was a smooth, softer sound resulting from the removal of the contrast provided by distinct groups of tones. Yet again, the harmonic function was reduced to a Romantic duality between the hexagonal wholetone scales found in early 20$^{th}$ century Impressionist music. In fact, the harmonic styles of Jazz and Impressionism both shared the generalization of the Tritone Function.

This idea is best illustrated by a short piano piece entitled *Golliwogg's Cakewalk* from *Children's Corner No. 6* by Claude Debussy. Based on a late 19$^{th}$ century children's story about a black rag doll named Golliwogg, contemporary music historians often avoid this reference, casting a politically correct eye to the past. But the parallel between the two styles and their merger in this 1908 piece should really be seen as a critical turning point in music history. It is symbolic of the handoff of the tritone legacy from European Romantic tradition into black American jazz culture – right at the crucial moment when contemporary music was about to slam the door on tonality.

By the 1950's, the African blues scales that led to jazz had begun to mix with straight European folk country rhythms to create a new form called Rock and Roll. From this, popular music began to quickly expand into a plethora of new styles, entering a period of increasing "hyphenationism" that has since become the hallmark of Western pop culture.

Accelerated first by mass production of vinyl records, then radio stations, then retail music stores, digital computers, the Internet revolution and mobile music devices, the mass appeal of Jazz and early Rock and Roll has fragmented into an ever-increasing fusion of musical genres. A partial list includes Latin-Jazz, R&B-Soul, Disco-Funk, Classical-Rock, Jazz-Rock, Country-Rock, Folk-Country, Metal-Rock, Punk-Rock, Reggae-Island, Hip Hop and Trip Hop, Rap, Electronica and House-Techno, Speed-Thrash, World-Ethnic and the list goes on and on. With the advent of mobile music and video players like the iPod and iPhone in the early 21st century, music has become an omnipresent soundtrack of personal musical style.

As we find it today, the tritone is as ubiquitous as music itself. *Every* style of popular music, with the possible exception of Rap or the occasional "trance beat" of Electronica, uses the tritone in a diatonic scale for its strong harmonic effect. Long gone is any concern for harmonic offensiveness, psychological ethos or religio-philosophical symbolisms associated with any aspect of music harmony. What remains is a mostly global preference for diatonic harmony, perhaps with a dash of Romantic chromaticism and Jazz polychordalism thrown in for good measure, all based on Zarlino's "Z-twelve" octave. Few question any of this, assuming all music harmonies to be a simple matter of personal preference and cultural programming from birth.

Over the course of music history one thing has remained constant – the tritone. From Pythagoras to present day, the tritone interval has always played a central role in the development of Western music and society. Whether in the geometry of the Greek modes, omission by Church law, rediscovery in European diatonic music, generalization into a chromatic duality or its rise into popular American Jazz and pop, the tritone has remained the most mysterious, the most feared and the most powerful musical force in Western civilization. As the force of symmetry and harmonic function in music, the Tritone Function oscillates around the golden ratios in a diatonic scale, mending the gaps in an octave as if it were a musical sewing machine. When allowed, the Tritone Function will always reveal the symmetry of counter posing golden ratios.

Yet, society still has little or no awareness of any of this. The Greek mythology of error in the tritone schisma adopted into the Catholic canons has been extraordinarily successful in blinding us from the role of harmonics in music and everything else in nature. Without harmonic principles as a working model for social order, people grew away from the natural laws and began to fall apart. As music fractured into a gradient of styles, religion splintered into a spectrum of denominations. Science too, having long ago given up seeking answers in *musica universalis*, followed a similar path of dividing and subdividing through the scientific method, splitting into smaller and smaller cells of specialization in a never-ending attempt to avoid facing its own harmonic connection with nature.

In the absence of any grand unification theory, the fragmentation continues. We remain convinced that the Earth, life and our very own consciousness are all just an accident and the universe is simply an arbitrary collection of amusing mysteries with little coherence behind it. Discoveries occur not through any comprehensive guiding philosophy, but as if by stumbling around in a large dark room while shining a flashlight on objects we happen to find. It is as if the missing tritone on nature's musical compass, having been stolen away long ago, has left us to wander in a vast atonal landscape.

Yet after every attempt to eradicate it – in the wake of a sustained pattern of *social interference* to make us forget – the tritone has reemerged triumphant as the most active natural agent in society today. With the Tritone Function's mending effect at work inside our mobile music culture, it has become the antidote to a socially chaotic, spiritually fragmented and scientifically incomplete world. As the very energy and soul of popular music, the tritone has become our refuge of last resort.

We might now wonder, given the historical significance of the tritone and its ubiquity today, how it has remained so marginalized in music education and contemporary cognitive theories. Why is the tritone still overlooked as the true centerpiece of Western music history and social theory?

# Conventional Wisdom

*"The enemy of the conventional wisdom is not ideas but the march of events."*
*- John Kenneth Galbraith*

Without a better explanation, modern music pedagogy has had no choice but to continue handing down the arcane traditions of the past. Major and minor scales, as leftovers from Church modes Ionian and Aeolian, form the foundation of virtually all popular music around the world. Boiled down from the ten Greek modes, our modern concept of music has become a simple black and white duality of major and minor – happy or sad. This is the conventional wisdom of ethos in the 21st century.

In the traditions of diatonic harmony, we are given seven tonal steps that comprise a repeating scale over an octave. As listed in Figure 22, each tone of the scale is named according to its traditional use in the Church and abbreviated in common practice.

**Figure 22 - Conventional music nomenclature**

| Scale degree | Scale tone name | Interval | Abbrev. |
|---|---|---|---|
| 1st tone | Tonic | unison | U |
| | | minor 2nd | m2 |
| 2nd tone | Super Tonic | major 2nd | M2 |
| | | minor 3rd | m3 |
| 3rd tone | Mediant | major 3rd | M3 |
| 4th tone | Subdominant | perfect 4th | P4 |
| | | tritone | TT |
| 5th tone | Dominant | perfect 5th | P5 |
| | | minor 6th | m6 |
| 6th tone | Submediant | major 6th | M6 |
| | | minor 7th | m7 |
| 7th tone | Leading Tone | major 7th | M7 |
| 8th tone | Tonic | octave | 8va |

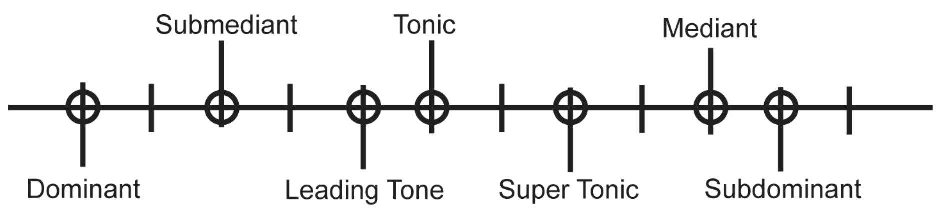
Look closely at these names, as they are a big clue to how musicians still think about music today. For instance, the 1st tone of a major scale is considered the Tonic (meaning *body*) generally believed to be the *tonal center* of a given diatonic key (meaning *through the body*). The 2nd tone is then mysteriously related to the Tonic only by being above, or "Super," to it – no other explanation is given. The 3rd and 6th tones appear related through the "Mediant" or "Submediant" midway between the Dominant and Subdominant. The 4th and 5th tones share the root term "Dominant" due to their dominating rubber band pulling effect back to the Tonic. While no one knows why we recognize this pull, the Dominant, Tonic and Subdominant are explained as part of the Cycle of 5ths, which everyone agrees seems to pull chords downward. This is usually enough to fend off a student's question, but no one really seems to know how any of this works.

As Sun-day is to the days of the week, the 7th tone is considered to be a very special tone in musical scales by being designated the "Leading Tone." Again for some unexplained reason, this tone leads harmony upward by a half step (or semitone) to the Tonic. We actually seem to anticipate hearing the Leading Tone move in this way and feel a satisfaction when it does so. When combined with the Subdominant tone's downward pull, we have the interval of a *diatonic tritone* with its mysterious tendency to contract. But of these two tones, the Leading Tone is considered more important than the Subdominant, perhaps because it forms a major 3rd interval with the Dominant that (again for some unknown reason) resolves down a perfect 5th to the Tonic. Known as a *Dominant-Tonic cadence*, this downward chord resolution is widely accepted as the cornerstone of traditional diatonic harmony, in spite of the fact there is absolutely no explanation for why this musical progression is so universally preferred.

This last point is very important in that it relies on the conventional wisdom that harmony moves *asymmetrically*. That is, the Dominant triad is considered stronger than the Subdominant triad (thus the name), even though both are collinear on the Cycle of 5ths and equally spaced either side of the Tonic. This principle of asymmetry in music ties in with history's great regard for the Leading Tone over the Subdominant. If you think about it for a minute, even the traditional step names for the scale appear to have been *designed* around this idea of asymmetry. They actually seem intended to *hide* any natural symmetry that might exist in harmony.

For example, when we try to arrange a major scale evenly around the Tonic we have the following conventional ordering in equal-tempered pitch space:

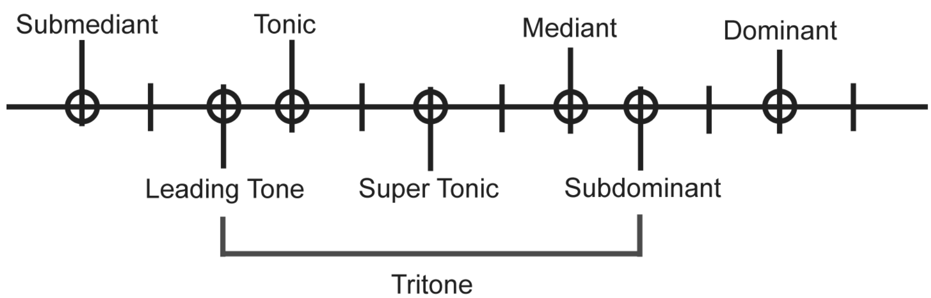
**Figure 23 - Conventional asymmetry on the Tonic**



This simple diagram shows that the traditional nomenclature is not at all symmetrical in pitch space. The Leading Tone is spaced a semitone from the Tonic while the SuperTonic is a wholetone away. The Leading Tone and Subdominant, together forming a tritone, is a long way from appearing symmetrical in pitch space either side of the Tonic. Furthermore, the like-named Mediant and Submediant are not spaced symmetrically either. Only the Dominant and Subdominant appear balanced symmetrically around the Tonic, as they are all three – the Dominant, Tonic and Subdominant – part of the Cycle of 5ths.

Now, take a look in Figure 24 at the correct symmetrical ordering of the diatonic scale.

**Figure 24 - Actual symmetry on the SuperTonic**




In this arrangement, we can see the tritone properly balanced around the SuperTonic at the center. In fact, all of the tones are symmetrically spaced about the SuperTonic, even though none of the steps of the scale exhibit this symmetry in their names. One would think that if symmetry in the diatonic scale had been important to past music theorists, they would have named complementary pitches alike in some way. Something like Dominant vs. Inverse Dominant or Leading Tone vs. Inverse Leading Tone.

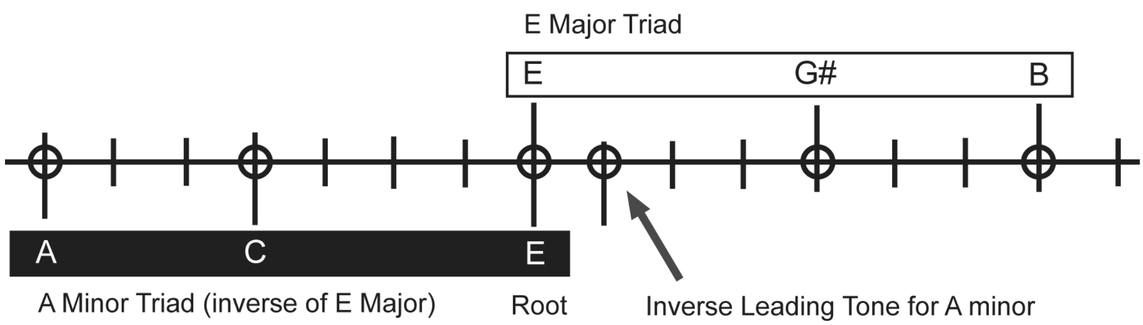
Scale symmetry within a cyclic octave, however, has never been a part of the conventional music theory passed down from the Church. Recognizing the influence of the Church's canonical rules forbidding use of the tritone, we can only conclude that the names used in scales and chords were actually intended to obscure this symmetry. Why would they do this? The reason, of course, is that asymmetrical names redirect attention away from the forbidden tritone along with its perceived tendency to contract or expand symmetrically. Sticking to this tradition, modern music theory continues to promulgate the naming conventions established long ago under canon law, therein hiding the natural symmetry of harmony. This is the conventional wisdom.

Digging into the question of major and minor now, we cannot help but stumble across the work of Hugo Riemann (not to be confused with the mathematician Bernhard Riemann). Hugo

Riemann was a highly regarded German musicologist from the late 19th and early 20th centuries who was (and still is) considered by many to be one of the most important theoreticians on tonality and harmony. He wrote many books about music, including the *Handbuch der Harmonielehre* published in 1918 that set forth his harmonic theories for music. In it, he defines his idea of musical consonance and dissonance based on a "natural" system of concatenation of intervals, which he used to construct scales and chords.

In particular, Riemann suggested that a minor scale is an inverse of the major scale (an upside down major scale) based on theoretical or perceptual undertones that do not actually exist in sound waves. To see what he means, we can construct a minor triad from a major triad starting, for example, in the key of {E} major using Riemann's theory:

**Figure 25 - Riemann theory of minor as inverse major**

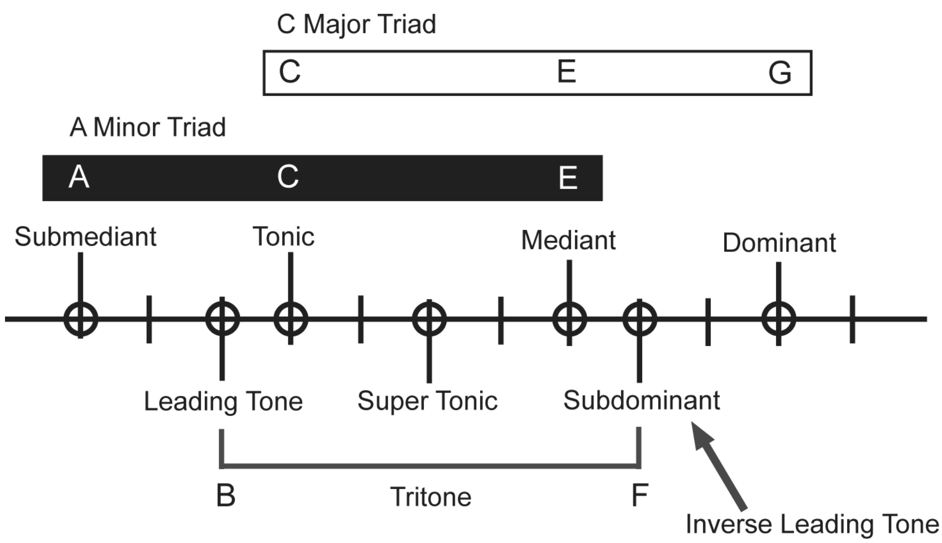



In Riemann's view, the intervals of an {E} major triad reflect symmetrically on the Tonic {E} to create an "upside down" minor triad on the Subdominant {A}. The Tonic {E} of the upside down minor triad would then, in effect, act as a mirror-like root for both triads. At the same time, the Inverse Leading Tone {F} would be *implicit* in the upside down {A} minor scale, pushing downward to {E}. Riemann felt this best exhibited the symmetry and balance found in nature.

Unfortunately, this theory has a couple of fatal flaws. First, assuming invisible "undertones" in the harmonics of musical tones without any explanation for how these are produced or heard is hard to swallow. Second, if we construct minor from a major based strictly on inverting intervals without the context of a scale, how can we then defend the sudden appearance of an Inverse Leading Tone without a scale structure to contain it? Third, there is no explanation for what might cause our perception of pull in the Leading Tone, or Inverse Leading Tone, in the first place. While it seems better than a pack of arbitrary rules, it cannot be correct. Though I laud the principle of symmetry and an earnest desire to construct a harmonic theory from nature, this theory is incomplete at best and ultimately faith-based.

Fortunately, there is another explanation for major and minor, that is actually derived from the observable harmonic waves of musical tones and which correctly describes the symmetrical relationship between major and minor.

**Figure 26 - Correct symmetry of major and minor**




In this more reasonable view of the diatonic scales, everything is symmetrical around the SuperTonic in the middle. The {C} major triad and scale share the diatonic tritone {B, F} with its relative {A} minor triad and scale. Here we can imagine the tritone contracting symmetrically to create the *upward push* of the Leading Tone {B} and *downward pull of the Inverse Leading Tone* {F} (or Subdominant). Together, these two opposing "leading tones" establish the same major 3rd interval {C, E} that is shared between the major and minor triads.

As we see here, the relative minor scale {A} shares the *exact same tones* with its relative major {C} so there is no separate interpretation of intervals, chords or scales as there is in Riemann's method. In this way, we can define major and minor scales (and keys) to be mirror opposites reflected around the SuperTonic {D} and sharing the oscillating Tritone Function {B, F} ↔ {C, E} as a kind of harmonic engine. All we needed to see this was to use the SuperTonic as the axis of symmetry instead of the Tonic.[35]

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[35] **Hypothesis 8:** The conventions of diatonic harmony based on major and ("relative") minor scales are founded on the recognition of symmetry around a shared SuperTonic centered in the middle of the Tritone Function.

Now we must ask ourselves what could cause the point of symmetry in a diatonic key to be located somewhere other than the Tonic? Is there something in the way tones combine that result in an offset center for harmony? This question will be addressed in detail later, but even without an answer one might still wonder how anybody could overlook something so obvious as scale symmetry or think it not important enough to mention.

The absence of symmetry in modern music theory can only be explained by the crusades against paganism. It is because of this that our culture has forgotten even the most basic harmonic principles of nature, once taught to Greek schoolchildren.

In the late 1950s, two groups of researchers, a husband and wife team, Peterson and Peterson and another researcher, John Brown, proposed an explanation for how people or entire societies can forget such things. Known as *interference theory*[36], the Peterson-Brown paradigm postulates that memories interfere with one another when two different responses are associated with the same stimulus. *Proactive interference* is said to occur when an older memory interferes with the newer thing we are trying to remember while *retroactive interference* occurs when a more recent memory interferes with one in the past.

In the case of scalar asymmetry in modern music theory, we might now see that the medieval Church implanted this idea into society to help block and confuse recollection of pagan harmonic ideas. Our modern music nomenclature is best explained as a retroactive *social interference* memory enforced by the Gregorian tritone omission rules of *ars antiqua.* As it was promulgated throughout the quadrivium educational system, the collective memory of harmony as a function of balance and symmetry was replaced by the new memory of harmony as asymmetric.

To be sure, the asymmetrical naming conventions of the Church were not some innocent misunderstanding, but were clearly *designed* to hide scale symmetry around the SuperTonic and its "impure" Tritone Function.[37] The Cycle of 5ths was then taken as a plausible alternative system – a cover story if you will – that would never reveal the pagan tritone or its attendant natural philosophy.

Complicity with this over the centuries has conditioned even the best and brightest musicians, musicologists and composers to completely bypass tritone and scalar symmetry. We ignore it in our musical language, the methods of our notation and the context of our discussion about harmony. And now, without a scientific explanation for how symmetry could be central to the perception of harmony, we continue thinking the same old way. Unfortunately, *this* too is conventional wisdom.

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[36] Also known as *retrieval interference* by Roediger & Karpicke, 2006.

[37] **Hypothesis 9:** Modern music theory is based on the rules and conventions of asymmetry inherited from the tritone avoidance laws of the Medieval Catholic Church.

Whether hidden in religious embrace, dismissed by artistic opinion or left behind by the scientific revolution, the underlying principles of music harmony remain a thing of complete and utter mystery at the dawning of the 21st century. Modern music pedagogy, with its textbooks and theory curriculum, continues to incorrectly describe harmony using asymmetrical nomenclature based upon layer after layer of unquestioned tradition. As a result, our knowledge about music remains quite limited with no universally consistent theoretical model for use in the classroom.

With the exception of the small field of experimental psychology investigating music cognition, contemporary science continues to shun the serious study of music. The avoidance of any unifying field of harmonic science has left music and any derivative natural philosophy as a humanities project. We might reasonably wonder why the scientific community continues to ignore something so immediately accessible and universal as music harmony. Outside of the tradition of avoidance established by the Church, what else could be keeping our most inquisitive minds from seriously investigating the role of harmonics in music perception?

Perhaps the biggest drawback to scientific study of music is its intimate connection to culture. When one considers music across diverse societies over thousands of years, it is easy to conclude *everything* in music is subjective and all just a matter of cultural indoctrination and personal opinion. With observation and testability central to the scientific method, how could any scientist hope to carve out a theory about something involving manmade conventions, human perception and individual opinions? Early scientific thinkers, such as Galileo and Newton, along with harmonic theorists, like Tartini and Rameau, tried to unravel this musical knot and failed. In the 16th century, French philosopher Rene Descartes was so perplexed by the problem of music perception that he flatly declined to judge the goodness of harmonic consonance by any rational method, protesting that the ear prefers one tonal combination or another according to musical context rather than any concordance of vibrations.

Then there is the matter of modern pragmatism. What practical results would the scientific study of music harmony yield? How does that help solve the world's problems? There are no jobs, no government grants and few university tenures relating to the scientific research of music, harmony or otherwise. There is no belief that a thorough understanding of harmonics in sound might be useful in other areas of application. An aspiring mathematician or scientist would certainly be better served to pursue established fields to make a living, leaving musicians to get on with the business of art and entertainment (this is what I did).

Getting past the stumbling blocks of subjectivity and pragmatism, we then come face to face with the age-old association of music with medieval religion, Greek mythology and ancient theosophies. After about 1650, with Galileo's and Newton's break from Pythagoreanism in the *Age of Reason*, the harmonic science of music was completely abandoned by "enlightened men" along with alchemy, astrology and other presumed metaphysical leanings. Since then, few in the scientific community have had any interest in risking their career reputation or university tenure on reestablishing that taboo relationship.

Today, so-called New Age advocates warmly embrace the numeric symbolisms and cyclic wave properties inherent in the theory of *musica universalis*, integrating it into astrology, metaphysical yoga disciplines, chakra healing, crystal and tuning-fork therapy, neo-pagan and occult ceremonies, esoteric lore and ritualistic practices, explanations for the paranormal, countless flavors of pseudo-science, *etcetera* – all with the mostly good intention of reinstating a much needed "harmonic balance" to our modern chaotic lives. Unfortunately, such enthusiasm from the New Age crowd further compounds the scientific community's deep-seated fear of championing research into music or any unified field of harmonic science.

And the music community is not exactly rushing in to help the cause of science either. New techniques and technologies challenge the very humanity of music as an Art. It could be argued that the creative "soul" in music touches that indefinable thing that makes us human, so over-rationalization may detract from this purpose. Entertainment is what music is all about these days, so if a musician wants to study mathematics or the sciences they should just go do that instead, shouldn't they?

So, why do *this*? Why rehash the old ideas of Pythagorean harmonic science?

A return to first principles is needed to solve the really big problems. Yes, we know that control of harmonic theory played a central role in the Church's suppression of pagan theosophies in Europe. And thanks to Bach and the Protestant revolution, the tritone managed to escape the tight clutches of the Church, therein releasing the spring action of the Tritone Function and paving the way for the discovery of chromatic music. Yet many questions remain, like:

- *What makes a key sound like a "center of gravity?"*
- *What exactly defines a key – its bass note, chord intervals or scale?*
- *What do we really mean when we say we are in a* key *when we end up using so many* accidentals *and* altered *chords that break the key?*
- *Why do we use twelve tones in an octave anyway?*
- *What definition of consonance and dissonance should we use and why?*
- *What is our explanation for why the tritone sounds* restless *or why we feel relieved when it* resolves*? Is it really because it is* dissonant?
- *How does consonance or dissonance relate to chord progressions?*
- *How can we explain the emotional relationship between major and minor?*
- *What makes the Cycle of 5ths work –* really*?*
- *How does perception of music harmony relate to physical sound waves?*
- *What is it in music harmony that sounds pleasing?*
- *Is there a single unifying theory for music harmony?*

To a musicologist or educator, obtaining rational and consistent answers for these and other properties of music perception could elevate the methods of music composition and improve the accuracy of music education while revolutionizing analysis of past works. A more comprehensive definition of harmony could lead to advances in computer-aided composition and music therapy. To a composer or performing musician, a correct conceptual model of harmony would provide a richer, more intuitive way to express musical ideas.

The same thing could be said for science. A unified field of harmonic science could help direct research into answering such questions as:

- *What is the underlying physics of harmonic formation?*
- *What role do harmonic principles play in cosmology?*
- *Is atomic bonding and chemistry a harmonic function?*
- *Is quantum coherence at the micro scale related to perception at the macro scale?*
- *Are harmonics involved in genetics and the structure of life and, if so, how?*
- *Could we say that perception is founded in the physics of harmonic oscillation?*
- *How might science and society be limited by the continued exclusion of the scientific study of music?*

For the scientist, a unified field of scientific harmonic research would provide a model for how sound waves interact in nature and are interpreted in the human mind. This understanding could offer guidance to many other scientific disciplines, such as quantum physics and cosmology. It could "break open the head," suggesting research in diverse areas of cognitive science, particularly artificial intelligence and neural networks. A rational and wholly consistent model for harmonic function would surely support and extend our understanding of the brain's neurophysiology, cascading into any number of unimaginable discoveries and inventions to change the world for the better. If nothing more, grasping the true principles of music harmony could recast our historical understanding of how science developed in the absence of natural harmonic philosophies.

There may even be a benefit to society at large. Acceptance of harmonic principles by prominent leaders and organizations in the Arts and Sciences would inevitably spill over into popular culture, raising awareness of a natural order. This in turn would have a good chance of lifting up social consciousness, mutual responsibility and self-image.

Under such a hypothetical metamorphosis of society, the control systems of religion and government would have no choice but to adapt, transforming their fear-meme systems into positive programs designed to motivate and actualize the population. While only a jaded dream as things stands today, such a future can only be found if we can correctly recognize how coherence forms in nature and then apply this knowledge as a working model to society. One thing is certain, if we cannot grasp how harmony works in music, we will never find it in ourselves or our society, leaving us to continue believing in the asymmetrical, unnatural and unsustainable systems of the past.

Everyone benefits from a better understanding of music and the unifying field of harmonic science from which it came. Our perception and cognitive recognition of harmonic wave properties in nature is indeed one of the great remaining mysteries, touching every aspect of our life. Unlocking these secrets should yield a philosophical bonanza of personal relevance for us all.

SECTION TWO

# Psychoacoustical Theory

*"We all agree that your theory is crazy, but is it crazy enough?" - Niels Bohr*

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Dr. Lloyd Taliaferro and his friend Joe Walston had known one another since childhood, sharing their dreams and ambitions as well as many interesting and lively discussions through the years. By 1979, Lloyd had established his career as a respected music composer and university professor while Joe had gravitated toward math and computer technology. Somehow, both had remained in contact and cultivated an interest in the other's career. One subject common to both men, as well as many others at that time, was the inevitable convergence of music and computers. Only a year earlier consumer electronics company Pioneer had begun selling the first *LaserDisc* players to General Motors as a training aid for Cadillac salesmen, thus inaugurating the age of digital recording.

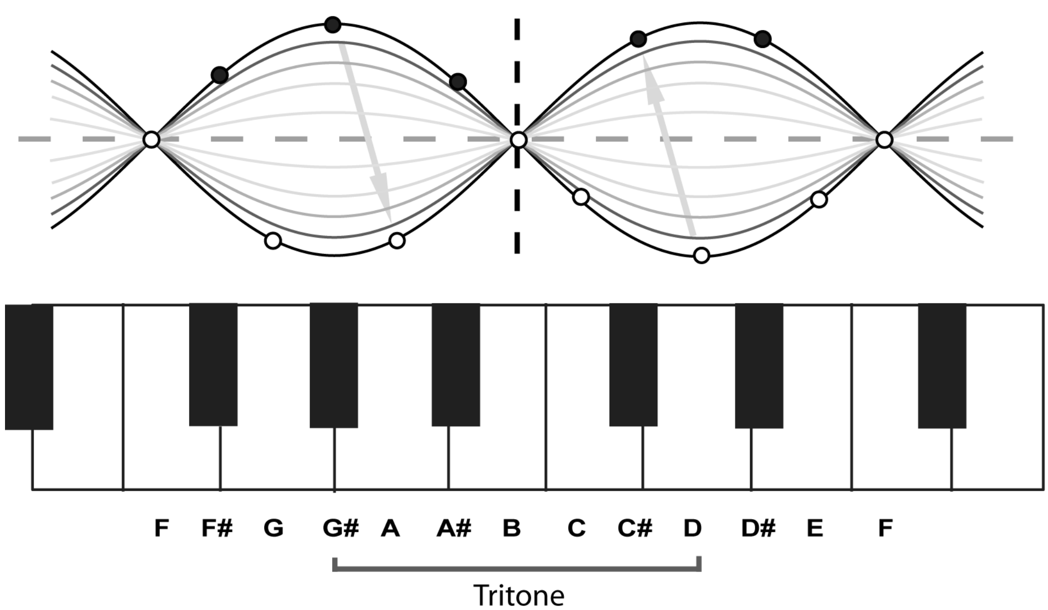
The emergence of digital music had been a topic of some debate between the two men. Lloyd was not fond of the idea of losing any of the upper harmonics from music by converting sound into quantized steps of binary zeros and ones. He felt digitization took the warmth out of the music and simply did not sound as good as his vinyl records, even with the occasional scratch. This comment was repeated several times as he sketched the stair step waveform of a digital sound wave on the chalkboard for my Renaissance counterpoint class.

It was this off-topic mention of harmonics that finally encouraged me to stay after class and share my latest crazy ideas on music theory with the professor. After a period of feverish activity in the first half of 1979, I had produced a set of acrylic transparencies that could be overlaid to illustrate several intriguing geometrical representations of music harmony that I had envisioned a few months earlier.

One Sunday morning, after a late night performing at a local nightclub, I awoke bleary eyed to the idea (vision really) of two opposing waves balanced around a horizontal line. Behind my half closed eyelids I had the impression of the two waves oscillating like a guitar string in slow motion with musical notes "floating" on each wave. After scrambling out of bed to find something to write with, I quickly sketched what I saw on the back of an envelope and stared at it for a while.

Resembling a kind of visual music, I had drawn one wave (or series of circles, I couldn't decide which) labeled with the notes of a six-note wholetone scale against an opposite and inverted wave labeled with the notes of the opposing six-note wholetone scale. Together, these scales comprised the twelve tones of an octave split into odd and even groups alternating between two intersecting waves. The thing that really jumped out and made the most sense to me in this was the way the tritone landed precisely at the points of maximum vibration on the waves. I arranged the twelve notes in my mind so that the piano black keys would float on the top half of the waves with the white keys on bottom, creating a resonating keyboard.

**Figure 27 - Early Standing Wave model (c1979)**



More than anything else, it just *felt* right. I could imagine my hands touching – no, *grasping* – chords as geometric shapes on the waves while mentally running through scales and arpeggios. But there was one particular scene in this that played over and over in my head. It was the symmetrical contraction and expansion of the Tritone Function between the waves, corresponding perfectly to what I had learned from my music theory studies and experienced first hand on the piano. Building on this, I could imagine common chord progressions, like the Cycle of 5ths, alternating between the two waves as complementary opposites – one red and the other green. Happy with my discovery, I folded the envelope and stuffed it into my shirt pocket.

Dr. Taliaferro stood gazing very patiently as I shuffled through the stack of transparencies – one showing a grid with two symmetrical, intersecting waves labeled with musical notes. Then, one with the tritone contracting to the major 3rd of the resolved tonic triad followed by a few others demonstrating typical chord progressions as triangular shapes connecting note positions on the waves.

The last transparency I showed the professor was something quite odd. It was the same double opposing wholetone scales, only this time as a series of circles tiled vertically top to bottom, left to right on the transparency to form a grid of perfectly tangent circles that touched at the top, bottom, left and right. I had drawn lines between all of the diatonic tritone locations in the diagram. As I had been experimenting with many different combinations, I noticed that this pattern carved out a little face inside the central circle. I thought it was pretty funny that I could create a face having symmetrical gaps for two eyes, a nose and a mouth in about the right proportion and shape simply by drawing lines between tritones in a grid of tiled circles.

To my surprise, the professor became excited about my harmony idea, though perhaps a bit perplexed by the face. I was just relieved he didn't laugh me out of the classroom. I told him that I could imagine this as a single wave alternating up and down like a reflected *standing wave* in a bathtub full of water, but with harmony moving rhythmically *into* the transparency. He immediately suggested an introduction to his computer friend, Joe Walston, a designer on the Texas Instruments *Speak and Spell* toy.[38] I learned that Joe had worked on the toy's speech synthesis software and was quite knowledgeable about linguistics and acoustical theory. Dr. Taliaferro told me that Joe had recently purchased one of the new Apple® II Plus computers and might be interested in helping me write a program to convert my transparencies into computer animations.

We arranged a meeting at Joe's house and I went to meet him. Joe was much older than I had expected, balding and rather portly. He was single and lived alone in a nice house that was decorated in late 1970's American bachelor pad, complete with leopard skin couch, gourd ashtray

[38] The Speak and Spell was the first commercial product to use computer-generated text-to-speech and the world's first Digital Signal Processor (DSP).

hanging from the ceiling and a stack of unopened toys filling an entire corner of a room nearly to the ceiling. I guessed from this that he was an eccentric fellow, but probably pretty damn smart. He explained that he constantly bought toys as research and whenever he was bored he opened one and disassembled it to see how it worked.

We quickly moved to the bar so I could spread out my transparencies to see what he thought. After studying each one very carefully, he said that we could probably graph them on his new computer. But before we could do this, we needed to go eat dinner at Kirby's Steakhouse on lower Greenville Avenue near downtown Dallas. As I was to learn, Joe's priorities were 1) have a rib eye, baked potato and glass of wine, and 2) stay up all night writing code.

We did just that. By 3:00AM the next morning, we had my harmonic model graphed on his computer in different colors (a new "plus" feature for Apple®) – red for one wave and green for the other – with all the notes labeled as I had envisioned. Excited beyond belief, we called Dr. Taliaferro to wake him up and tell him that we had done it – we had converted music into mathematical equations and displayed it as geometry on a computer!

I met with Joe a few times after that to try taking it further, but we began spending more time discussing wild ideas at dinner rather than programming. So, I decided to buy my own Apple II to spend more time working on it. Since the Apple® Motorola 6502 processor was *very* slow, I learned how to make binary shape tables for the vector waveforms and display them in succession to create crude animation. Even then, the display speed was much too poor to accomplish what I wanted. It was because of this that I ventured further into mathematics and computer graphics, promising myself a return to the music visualization project when computers became powerful enough to handle it.

Enrolling in math classes while I finished a music degree, I continued my research into a universal theory for music harmony. My curiosity about music was actually driving me forward into math and computing. The more I learned about physics and mathematics, the more I saw the same thing in music. I learned that my model for harmony was really just like any standing wave phenomenon in nature and emulated the physical standing wave of a musical tone, though how it might apply to explain harmony was still puzzling to me. As I later came to understand it, I had stumbled upon a *harmonic standing wave model* for music that organized all of the individual standing waves of pitch together into a single, larger coherent structure. Using trigonometry, this took the form of intersecting sine and cosine waves in an interference pattern.

# *Tritone Paradox*

*"By denying scientific principles, one may maintain any paradox." - Galileo Galilei*

In 1981, an interdisciplinary college course entitled *Music – From Source to Experience* was one of the first modern university classes to venture into the science of music. Four highly respected professors had agreed to instruct the class – a physicist, a biologist, a psychologist and a musician, together representing a modern revival of the medieval quadrivium classroom.

Of course, I signed up for it immediately. I was certain it would finally answer my questions about the physics and physiology of music perception. Over the course of the semester, we did indeed study the sciences of acoustics, biology and psychology as they applied to music. By the end it was clear to everyone that music was deeply intertwined with each of these areas of natural study. Yet after all this, I still did not find the explanation I was hoping for.

The professors did, however, mention a few other things in response to my questions. The first was the c-squared constant for light in the equation $E=mc^2$ did not really mean anything except "that is just the way the equation turned out." The second was that pitch in physics was indeed a result of standing waves, but there was certainly no "meta" standing wave for music harmony. The third was that perception of things like music was mostly cultural and subjective, not unlike the moon appearing larger near the horizon next to nearby objects. And, the fourth was there was nothing special about the shape of our ear – it just evolved that way by chance.

While this class was a significant step forward in acknowledging music as a scientific concern, it is fair to say I was disappointed by the degree of chance afforded natural processes and with it the perception of music. I knew there had to be a deeper causality behind our cognition of music and its relationship to biology and physics, even though the dots had not yet been connected. I was still very young and naïve and should have known that it was unlikely I would ever solve such an enormous mystery. Yet, the absence of a suitable answer only motivated me to redouble my efforts and dig deeper to find out what was really going on.

As with Dr. Taliaferro a year earlier, I thought I would again share my ideas about music theory with one of the instructors, so I chased down the psychology professor after class. He chuckled and said he recognized my wave models as something similar to an idea presented at the *Annual Meeting of the Western Psychological Association*, April 20, 1978 in San Francisco. A cognitive scientist by the name of Roger N. Shepard had submitted a paper called *The Double Helix of Musical Pitch* proposing that cognition of pitch could be represented as a DNA-looking geometrical structure in pitch space. Arriving at his office, the professor handed me a copy of the original draft of the paper dated April 8, 1978 quoted here:

> *"…pitch can not adequately be represented on a single rectilinear dimension and might more properly (following Drobisch, 1846) be represented on a simple helix that completes one turn for each octave."*

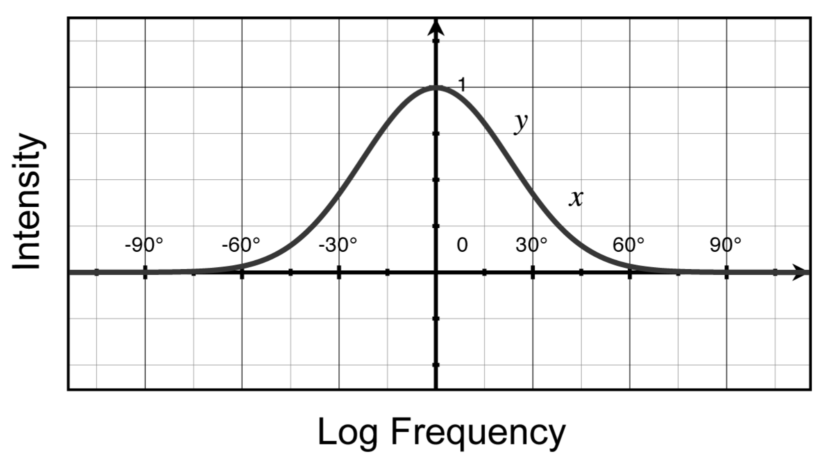
To support this hypothesis, Shepard had confirmed circularity of pitch across the entire auditory spectrum in an experimental psychology study that used specially designed fade filters over electronic tones moving along the Cycle of 5ths.

> *"A validation of the separation of pitch onto a rectilinear dimension of pitch 'height' and a circular dimension of tone 'chroma' implied by this helical structure was provided by the computer-generated tones with which I was able to demonstrate complete circularity of judgments of relative pitch (Shepard, 1964)."*

Later dubbed *Shepard Tones*, the experiment created the effect of an endlessly ascending staircase of pitches, something like an auditory version of M.C. Escher's *Ascending and Descending* depicting an endlessly looping staircase. In this way, Shepard had proven the perceived circularity of the entire auditory spectrum.

This experiment involved using an electronic synthesizer to play a sequence of alternating intervals (or pitch glides) up or down in pitch space while the volume for each interval was slowly faded in and out according to a Gaussian normal distribution, more commonly known as the "Bell Curve." Each interval was duplicated across 10 octaves to create the "illusion" of pitch being ambiguous in terms of pitch or height – then, tested against a number of subjects. This Gaussian fade curve was essential in normalizing the sound by reducing volume at either end of the spectrum, thus revealing the naturally perceived circularity of pitch space.

**Figure 28 - Shepard Tones Gaussian fade curve**

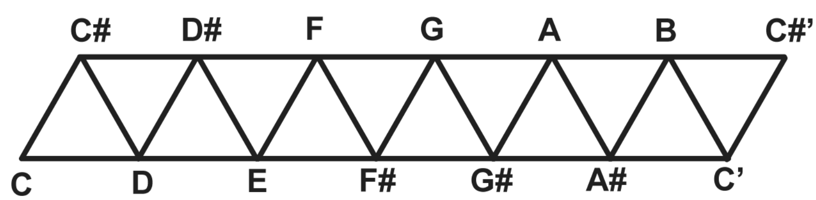


The paper goes on to present a lattice structure, like the Fuller octet truss mentioned earlier, based on the following assumptions:

1. *"Within the context of a particular key, the successive steps correspond to equal distances in the underlying structure,*
2. *Since all keys are equivalent under transposition, the distances within the underlying structure must be invariant under transposition,*
3. *Since the octave relation is unique, tones an octave apart must have a unique geometrical relationship in the underlying structure."*

From this, Shepard proposed that (a) and (b) together implied that any three successive notes of the chromatic scale must define an equilateral triangle, meaning that the semitone and wholetone steps must be *perceived as equivalent* in any given key. That's right, a half step and whole step must be audibly recognized as the very same distance in pitch space. Based on this premise, Shepard then constructed what amounts to a 12-vertex lattice of equilateral triangles, labeling the resulting parallel lines of the lattice with the opposing wholetone scales. By this point in the paper, the theory was beginning to resemble my standing wave idea.

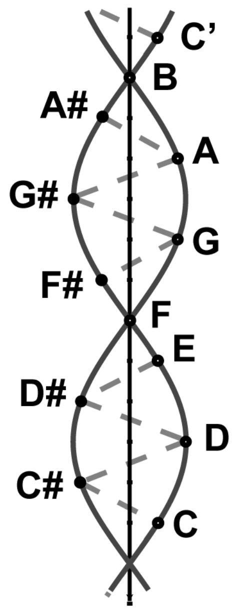
**Figure 29 - Shepard lattice defining whole to semitone equivalence**




Shepard's next move (in Figure 30) was to make the case that the simplest rigid motion of 3-dimensional space was to fold back on itself in a twisting screw motion, resulting in a model of music harmony he called the *Double Helix of Musical Pitch*. This approach was the only way to preserve the presumed invariance required between adjacent pitches. As seen in Figure 30, the half twist establishes the tritone interval (e.g., {F, B} or {D, G#}) at every cycle of π in the troughs and crests of the double helix:

It is important to note again that this model was based entirely on cognitive psychology experiments using both musically trained and untrained volunteers listening to Shepard Tones. None of the properties in Shepard's model were derived from either the physics of sound or the conventions of music history. It had been formulated strictly from experimental results using formal laboratory test procedures.

**Figure 30 - 2-D view of the *Shepard Double Helix of Musical Pitch***




Shepard did, however, identify some limitations in his double helix pitch model:

> *“Of course, distances in a geometrical structure, being strictly symmetric, cannot themselves explain the consistent asymmetries in the perceived relations (e.g., between a perfect fifth above or below, or between a minor second above or below the Tonic). A complete account will require, in addition to any invariant underlying geometrical structure, some processes or “rules of projection” that specify how that underlying structure is operated upon or “read” within the context of any particular musical key.”*

His suggestion here was that a set of rules would be needed to derive all of the common practice musical scales, chords and especially the Cycle of 5ths from the double helix. He admitted that support for those rules would require further psychological test data to support his model. One example he gave was devising a test to confirm a cognitive *twisting* of the double helix to create a “higher-order helix” for musical key transposition. The problem then becomes how do you test people to confirm a twist operation actually occurs somewhere in the mind?

It was at this point that I realized several limitations in building a complete perceptual model for music based strictly on psychological studies:

1. *Any conclusions derived from cognitive analysis alone would always be tentative without additional evidence from physics, neurophysiology, mathematics and old-fashioned musical practice.*

2. *While it may seem likely that our cognition of pitch would follow a natural organic structure like DNA, it was more likely to me that our cognition of harmony would be based on the real-time perception of oscillating waveforms – perhaps something like the standing wave geometries found on a vibrating Chladni plate covered with sand.*

3. *I was not sure about any assumption of perceived equivalence between a semitone and a wholetone based simply on the fact they were used together to create a scale. However, I did think there was some relation of opposition in music harmony that created our sensation of tension and resolution or consonance and dissonance.*

4. *It would be a very inefficient and unlikely cognitive process that required a transformation of complex compression waves hitting our eardrums into a "twisted" helical structure in some three dimensional mental space. Something else had to account for the strange architecture of our outer ears, inner ears and cerebral auditory cortex.*

For these reasons, I chose not to change my focus to the double helix model. However, the idea of tonal groups as being perceived spatially and in symmetrically opposing groups did support my own intuition about harmony. I took this as an underlying principle in my own standing wave model.[39]

Roger Shepard's seminal work represented the birth of the field of music cognition in experimental psychology, triggering an increase in the interest and investigation into the mystery of music. From that point forward, the field of music cognition has steadily gained respect and recognition as an important area of psychological research. Many discoveries have occurred in this field since Shepard's original double helix model for pitch, most notably the work of one of his colleagues Dr. Diana Deutsch at the University of California, San Diego. One paper she published in 1986, entitled *A Musical Paradox*, is of special relevance to the tritone and how we perceive its symmetry in pitch space.

Deutsch found that when the alternating Shepard Tone intervals were *less than* a half octave (a tritone) apart, the top tone sounded "higher" in pitch than the bottom tone, just as we would expect. But when the Shepard Tone intervals were *greater than* a half octave (a tritone) apart, the top tone is unexpectedly perceived as sounding *lower* in pitch than the bottom tone. These results are very perplexing because they suggest we are all predisposed to not only determine pitch relative to distance in an octave (a doubling of pitch), but also relative to whether an interval spans more than a tritone or half octave. Said simply, this *Tritone Paradox* indicates that we

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**Principle 1**: People interpret the pitch spectrum as a vertically geometric pitch space that is both circular and symmetric.

perceive pitch intervals as ascending when less than 180° (0 .. $\pi$) and descending when greater than 180° ($\pi$ .. $2\pi$) in a 360° octave.[40]

Based on this, Deutsch then decided to use the tritone as the Shepard Tone interval, instead of the previously used octave, finding a number of other surprising and paradoxical results:

> *"When played in one key it is heard as ascending, yet when played in a different key it is heard as descending instead. When a tape recording is made of this pattern, and it is played back at different speeds, the pattern is heard either as ascending or as descending depending on the speed of playback. To add to the paradox, the pattern in any given key is heard as ascending by some listeners, but as descending by others." [Deutsch 1987]*

In later studies by Lloyd Dawe, John Platt and Eydra Welsh at McMaster University in South Western Ontario, Canada, it was discovered from similar tests with a group of 20 immigrant subjects, mostly multilingual, that their geographic location and culture influenced whether they heard the tritone ascending or descending. In particular, they found a consistent differentiation between American and British language test subjects, in spite of exposure to American television, and that the orientation of the scale used appeared to be malleable and changes to match linguistic features, including spelling and pronunciation, within a geographical area. One hypothesis suggested that when language inflection is upward, there is a tendency to interpret the tritone as ascending; whereas, downward inflection leads to interpretations of a descending tritone.

Later, Lloyd Dawe at Cameron University, in collaboration with Platt and Welsh at McMaster University, also found spectral-motion aftereffects following a lengthy exposure to the Shepard scale. While motion aftereffects are well documented in vision, this phenomenon in sound had not been documented previously. They found that prolonged movement in a particular direction with a spiraling set of intervals produced a reverse "drift" in the opposite direction when subjects were suddenly presented with a linear, or static, set of intervals. The neurological explanation for this is the neurons responsible for direction of pitch motion are "leaky" like those for vision and will continue to trigger along a pattern when presented with ambiguity. [Dawe et. al. 1998]

All of these experiments support the idea that we interpret musical tones within some sort of "cognitive space." Anatomically, this space appears to correspond initially to the receptors for

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**Principle 2**: People interpret circularity in the frequency doubling at the octave.

**Principle 3**: People interpret tones in an interval having a tendency or tension to move up or down based on whether it is less than or greater than a half octave or tritone. The tritone itself is perceived as an ambiguous inflection point between opposing directions, producing what is popularly known as the *Tritone Paradox*.

sound frequency that lie in a line along the basilar membrane of the inner ear. These receptors are known to respond to variations in the frequency of sound energy in much the same way as the surface of the eye retina responds to variations in the spatial contour of light. This and other audition/ vision similarities suggest that musical intervals are actually recognized as *auditory objects* within a space just like visual objects. It also implies that both pitch and color are the consequence of spatial frequency-to-proportion transforms of incoming wave stimulus. For these reasons, the perception of both sound and light is commonly considered part of a larger, more general context described as "position within a continuum of order." [41]

As discussed earlier, the idea that music might be perceived as spatial geometry is not new, originating at least as far back as the Pythagoreans. Over the years many respected physicists, music theorists and cognitive psychologists have come to agree that melodies, intervals and chords are cognitive equivalents for visual objects moving in space. Assuming this, the next question then becomes what universal principles and structure shall we say describe the "continuum of order" upon which our perception of space and time is based?

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**Principle 4**: People interpret movement between tones as motion between locations in pitch space, analogous to the perception of spatial location and motion of objects in visual space.

# *Pitch Alphabet*

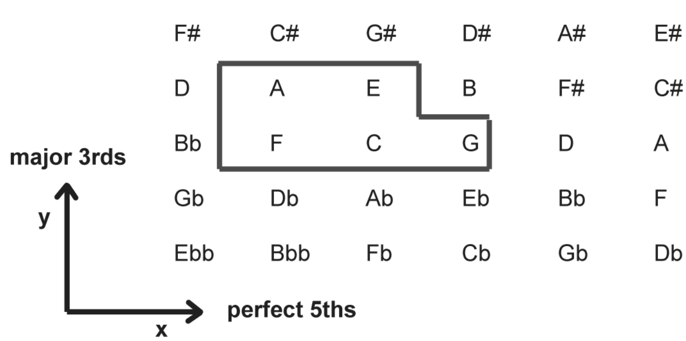
*"Let man then learn the revelation of all nature and all thought to his heart; this, namely; that the Highest dwells with him; that the sources of nature are in his own mind." - Ralph Waldo Emerson*

There have been many different geometric representations of music over the years, including cylinders, helices and lattices of different flavors. Each of these has been used to explain how we might recognize spatial relationships between tones and groups of tones. In 1739, Swiss mathematician and physicist Leonhard Euler was the first to propose a lattice spatial model for pitch space, describing Just temperament (tuning with simple numeric proportions) as one axis of perfect 5ths against another of major 3rds. This was the first of many such musical models that have never really made it into music theory textbooks or curriculum.

Schoenberg argued in 1951 that our auditory system had the ability to recognize sound objects in pitch space in the same way our vision system recognized objects in light space. He reasoned that chords and melodies could be inverted in pitch or retrograded (or retrograde-inverted) in time and still be recognized just as we would recognize a visual object rotated in space. This became the first principle behind his twelve-tone matrix system for atonal composition. However, the ability to recognize melodies inverted or retrograde-inverted is hotly debated. Detractors point out that we are not skilled at understanding language when spoken backward, so how could we do this with pitch?

Another spatial pitch model was proposed in the 1970's by cognitive scientist H. Christopher Longuet-Higgins to represent pitch relationships in 3-dimensional space.

**Figure 31 - Longuet-Higgins Tonal Space**



In this model, tones adjacent along the first dimension are separated by perfect 5ths while those adjacent along the second dimension are separated by major 3rds – just like Euler. But unlike Euler, tones adjacent along the third dimension were then separated by octaves to make intervals appear as vectors in pitch space. For instance, a C major scale would occupy a compact group in this array, suggesting that we recognize a key as a *neighborhood* in pitch space.

Princeton University music professor Dmitri Tymoczko has suggested a similar looping pitch space grid. In a recent article in *Science Magazine* (July 2006) entitled *The Geometry of Musical Chords*, he proposes this:

> *"A musical chord can be represented as a point in a geometrical space called an* orbifold*. Line segments represent mappings from the notes of one chord to those of another. Composers in a wide range of styles have exploited the non-Euclidean geometry of these spaces, typically by utilizing short line segments between structurally similar chords. Such line segments exist only when chords are nearly symmetrical under translation, reflection, or permutation. Paradigmatically consonant and dissonant chords possess different near-symmetries, and suggest different musical uses."*
> *[Tymoczko 2006]*

Tymoczko goes on to describe that an *orbifold* of ordered pitches is essentially a Möbius strip – a square matrix of intervals whose left edge is given a half twist and spliced with its right. He states: "the orbifold is singular at its top and bottom edges, which act like mirrors" and "any bijective voice leading between pairs of pitches or pairs of pitch classes can be associated with a path." The essential idea here is that the rules of proper voice leading in Western musical styles arose from the preference of moving between chords that are *nearly symmetrical* under transposition, permutation, or inversion. Tymoczko speculated that near symmetry instead of exact symmetry was historically preferred by composers because greater voice independence (and presumably greater perceived interest) occurs when one voice moves by a half step while another moves by a whole step.

The best thing about this theory is it builds on the idea that harmony is perceived spatially as a geometric shape and that it is somehow related to recognition of symmetrical or "nearly symmetrical" groups. Like Roger Shepard's double helix model, Tymoczko had identified a "twist" operation as essential to the relationships found in common practice music.

This said, the orbifold model does not explain how sound energy could be transformed into the spatial proportions of his lattice structure within the inner ear and auditory cortex. There is also no coupling mechanism given to the physics of vibrating sound or the harmonic series and how that might influence cognitive qualities of consonance or harmonic movement. No connection is given to any neurophysiology theory or study that might support such a model and no path is visible down to the atomic (or quantum) level from which matter itself emerges. What we find is a theoretical model based in mathematics that illustrates compositional elements as geometric objects in spacetime, but one that is clearly inorganic and built upon abstract notions disconnected from an actual physical explanation.

As another alternative, Deutsch and Feroe (1981) proposed the idea of *pitch alphabets.* This music cognition model is based on the linguistics theory of "production grammars," something I studied heavily in my graduate computer science classes and used extensively during my software career. This model proposes that music scales and chords are learned like a language with the listener acquiring a repertoire of hierarchical alphabets (nested fragments) after repeated exposure. The hierarchy is built up into higher levels from chromatic scales (semitones) to major and minor scales to triadic chords and so on [Deutsch 1999]. There are a number of studies that led to this conclusion.

For instance, one study found that when test subjects listened to tone sequences and were asked to recall what they had heard, any errors made tended to remain within the alphabet of tones established in the tests. This was taken to indicate a preference for the tonal alphabet given [Dowling 1978]. The conclusion was essentially that melodic memory (as programmed by culture) was probably the primary determiner behind our preference for certain scales, rather than any instinctual bias or biological preference.

As its fundamental premise, the pitch alphabet model implicitly claims no innate 'out-of-the-box' method for recognizing scales, intervals or chords. The conclusion one would draw from this is that harmony is learned just like a spoken language. Such a view bypasses any natural preference for specific tonal proportions, instead emphasizing the case that we are programmed by repetition in our respective cultures to recognize complex hierarchical harmonic relationships from common practice. At the bottom, this contemporary view from experimental psychology depends mostly, if not entirely, on memory retention:

> *"the process of key assignment is a complex one, involving such factors as low-level groupings, knowledge of the pitches forming diatonic collections, and knowledge of hierarchies of prominence for tones within different keys."*

A number of research studies and whitepapers are taken to support this theory. With such overwhelming evidence and support from within the cognitive psychology community, we might assume that it is a *fait accompli* that music is learned like a natural language, mostly a matter of regional cultural influence, and that any preference for different tone combinations is entirely the result of environmental conditioning. But this simply cannot be the whole story.

When we take an honest look at musical practice, we find that linguistics and abstract mathematics are insufficient to explain how we can differentiate degrees of musical consonance or dissonance in pitch or anticipate the directional movement of tense intervals to resolved intervals. There is nothing in grammar syntax or language production rules to explain the "restless" *feeling* of a tritone or our apparent preference for a Dominant-Tonic progression – only more manmade rules to explain what we do not really understand. Shouldn't we demand to know quantifiably what causes harmony to be perceived as it is and the fundamental nature of its mechanics?

The simple fact is that music cognition could not have evolved based on inorganic structures like rectangular lookup tables, Möbius strips and pitch grammars. Music cognition and any *real* theory of music must originate from the relationship of biology to its environment – far predating the human invention of organized sound we call music. After all, the principle of "least action" first stated by Maupertuis in 1746 and later used by Darwin in his theory of molecular evolution would have relegated any inefficient method of sound recognition to the dustbin of history. Our auditory cognition system clearly supports instantaneous proportional measurement of space between tones and timbre while simultaneously *recognizing and predicting* harmonic motion. With absolutely no conscious effort on our part, we all experience the same pleasurable and satisfying *emotional* experience we know as harmony.

This all said, the leading music theories do bring us closer to a consensus on a few important things. First, they agree on the importance of circularity and symmetry in music and that some kind of *half twist* is involved. They also agree on the idea that pitch appears to be recognized as a relational geometry of proportions between frequencies. And within such auditory geometries, they also agree that some kind of *hierarchical organization* of pitch space is involved.[42]

But, if none of these theories are the final answer, what should we say is the correct and complete answer? To find this we need to ask one very important question:

> *How could we recognize and feel <u>any</u> hierarchical grouping of pitches instantly from the enormously complex waveform structures that occur in real music?*

In particular, how can we explain a mostly universal perception of specific intervals within a diatonic scale that are interpreted as "tense" or "resolved," pleasingly concordant or gratingly dissonant? What is the "power source" for the Tritone Function? What causes the universal perception of "gravity" when a tense Dominant chord moves to a restful Tonic triad? How can we *feel* the mood difference between songs in major and minor keys, especially when we are very young, without someone telling us how to interpret them? And shouldn't we be able to explain why it is our brain prefers a 12-step octave or 7-step scale instead of, say, an 11-step octave?

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**Principle 5**: People interpret pitch space in hierarchical groupings that are recognized as auditory geometry. Furthermore, within this hierarchy exists a "half twist" reflective symmetry.

Clearly, there is a natural cognition of sound waves at work *before* cultural influence and memory kicks in to interpret – somewhere *underneath* the abstract theoretical models currently in vogue. At the bottom of the hierarchy of cognition we have to ask ourselves: "What structure exists when sound hits our ears?" Are there any special characteristics in those waves that would make it possible for our primitively evolved auditory system to easily recognize and *prefer* certain patterns? Cognitive studies have already shown that tones are perceived spatially like vision. Why would there not be some sort of natural landmarks or "signposts" *present in the structure of sound itself* that our auditory system might "edge detect" and use to pattern match sonic shapes economically, instantly and effortlessly?

To find these answers, we must now step out of the field of experimental psychology and pure mathematical models into the natural sciences of acoustics and physiology. Why? Because experimental psychology alone can never explain how the structure of sound interacts with the structures of life to produce the cognitive system being tested. Psychology offers only a "black box" testing of the brain. What we need is a "white box" testing method where we are able to "see" inside and accurately model the physiological structure of the ear and brain. Experimental psychology can only provide confirming evidence to support a more comprehensive and interdisciplinary approach based on the physics of sound, our biological equipment and the musical experience.

We should expect to find at the bottom of music harmony a real world explanation derived from a coupling of the structure of sound waves to our auditory system. Whatever it turns out to be, it should prove to be compatible with common musical practice, experimental psychology and elegantly expressed as mathematical constructs. Such an explanation should be intuitive and self-consistent in such a simple way that it can be easily visualized and understood by anyone. Finally, it should be beautiful – it should look like us!

The place to begin is in the structure of sound itself.

# *Spectral Analysis*

*"Science is spectral analysis. Art is light synthesis." - Karl Kraus*

Sound begins as a mechanical action, like a vibrating string or column of air, traveling to our ears as spherical compression waves. These waves share the same properties as other types of waves in nature, exhibiting frequency, wavelength, period, amplitude and velocity. While we can only hear sounds within a particular range of frequencies, at best 20 Hz at the low end to about 20 kHz at the high end, our ability to recognize a wide range of characteristics within this range has been essential to human survival and evolution.

There is actually a fair amount of knowledge about how sound waves work. Acoustics (from the Greek word ακουστός) is the branch of physics that deals specifically with the study of sound vibration, usually applied to the improvement of vocal and music quality in auditoriums, A/V equipment and speakers. However, the wave theory behind acoustics has application well beyond sound, including geologic formation analysis, stress testing aeronautical designs, predicting weather patterns and detecting underwater activity, to name a few.

More than anything else, acoustics is concerned with *resonance* and the reflective qualities of waves in an enclosed space. When a room reflects sound, some frequencies resonate more than others depending on the shape of the room, materials used and other structural elements. Analysis of a room can identify the fundamental resonant frequency and set of higher sympathetic frequencies, called *harmonics*, that reflect most. Too much reflection of certain harmonics can make it quite difficult to recognize speech and music. In loudspeaker systems, these reflective frequencies are responsible for the high pitch squeal of "feedback" we hear all too often. Fortunately, feedback frequencies can be reduced or eliminated through the use of an electronic "equalizer" to shape the sound to fit the natural resonance of the room.

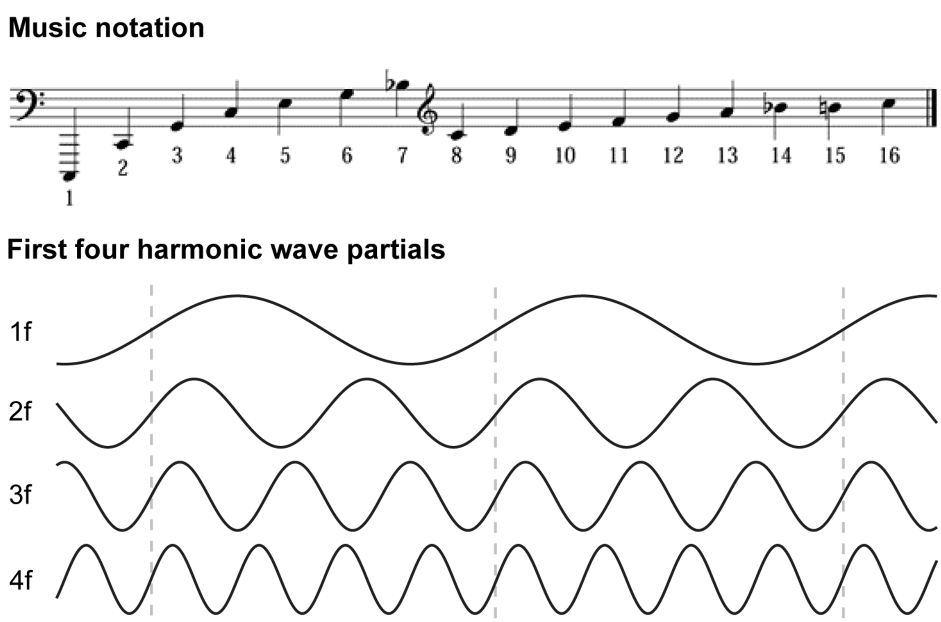
Harmonics, known as *overtones* to musicians or *wave partials* to acousticians, play a central role in the acoustical *timbre* of a musical instrument. Musicians sometimes cause specific harmonic overtones to occur by lightly touching a string at midpoints or by "over-blowing" a wind instrument to produce a higher resonant pitch. So while sound engineers work to make a room neutral in its harmonic character, a musician will use harmonics as a specific timbral or musical quality. In either case, harmonic resonance is a shared physical property.

But sound is not the only place where harmonics play an important role. They are central to such fields as electrical engineering, lasers, biochemistry, brain neurophysiology, cosmology and the string theory of quantum physics, to name only a few. Even so, science as a social institution has shied away from any unifying field of harmonic research and along with it any generally accepted explanation for music harmony and our perception of it.

As it relates to music cognition, we always recognize a series of harmonics as a single tone – even when there is a lot of background noise or when multiple tones are sounded together. Our auditory system seems especially attuned to recognize the harmonic series as a whole no matter what. Since harmonics resonate sympathetically in the standing wave of any musical tone, we need to understand what causes harmonics to form in the first place in order to determine how they could be recognized in our auditory system. We begin with the conventional definition of the musical harmonic series:

> ***Harmonic Series*** *– In acoustics and telecommunications, the harmonic of a wave is a component frequency of the signal that is an integer multiple of the fundamental frequency. For a sinusoidal wave, it is an integer multiple of the frequency of the wave. For example, if the frequency is f, its harmonics have frequency 2f, 3f, 4f, etc. In musical terms, harmonics are component pitches ("partials," "partial waves" or "constituent frequencies") produced by the* standing wave *of a vibrating tone which sounds at whole number multiples above a given fundamental frequency. The first four octaves of the natural harmonic series for {C} are shown in Figure 32 along with the first four corresponding component wave partials.*

**Figure 32 - Four octaves of the musical harmonic series**



Using the harmonic series, any sound imaginable can be constructed from a weighted sum of harmonic wave partials, also called *normal Fourier modes* after the French mathematician and physicist Joseph Fourier. Consequently, the timbral sound quality of any instrument is also the result of the particular combination of harmonics it emphasizes. In acoustic instruments this is dependant on such things as the type of materials used, properties of the resonating cavity and the acoustics of a room. In synthesized instruments, electronic filters and other special effects are often used to create exotic sounds and timbres, but underneath it all remains a specific combination of harmonic wave components that make up the essential quality of the sound.

To represent a given timbre, the generalized *Fourier series expansion*[43] equation uses trigonometric sine and cosine wave components to represent each harmonic wave partial. These are then combined into a single equation to represent a particular sound, something like this:

$$\begin{aligned} f(t) = 0 & \\ &+ \cos(x) + \sin(x) \\ &+ \cos(2x) + \sin(2x) \\ &+ \cos(3x) + \sin(3x) \\ &+ \cos(4x) + \sin(4x) \\ &+ \ldots \qquad + \ldots \end{aligned}$$

In Figure 33, the first four harmonic wave components of this series are shown as separate sine and cosine waves with equal amplitudes. They are then added together to produce the composite wave below. Variations to this, such as changing the amplitude (volume) of each harmonic or adding and subtracting different wave partials, can be used to shape the sound further. The possibilities are endless, but the idea is really pretty elegant.

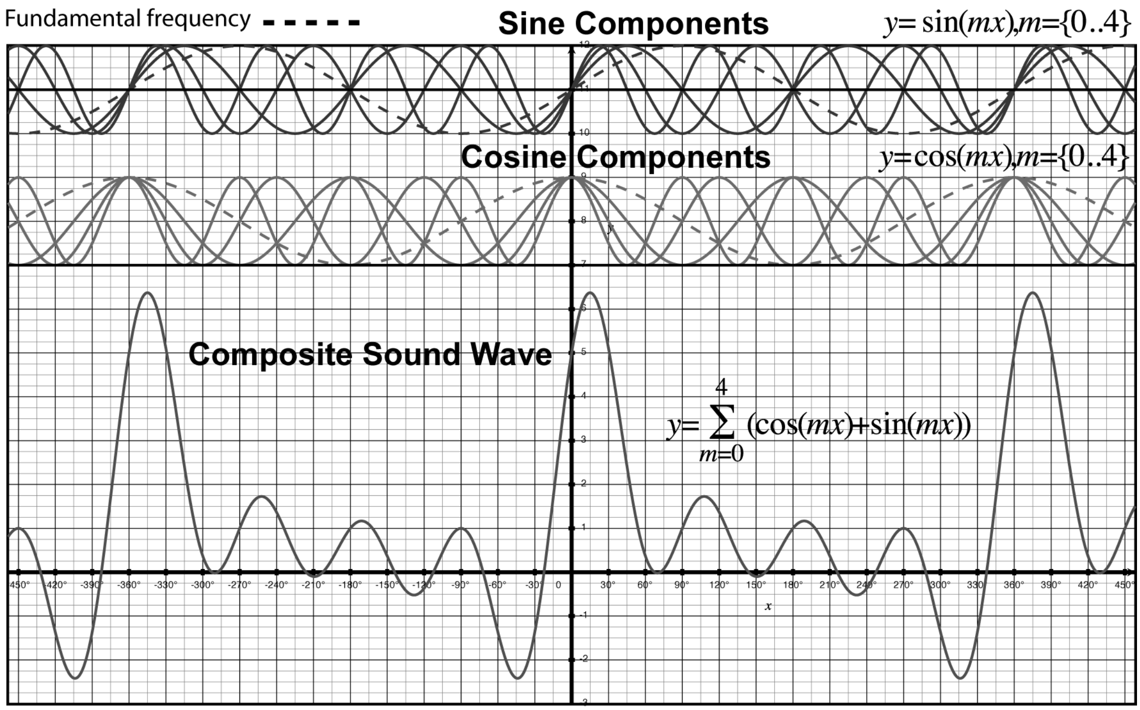
One of the interesting things about the Fourier series is the way in which it represents the odd-even attributes of timbre. For instance, a waveform containing only sine components, like a "saw tooth" wave, would be called an *odd* function. Such odd waveforms are always *in-phase* and aligned symmetrically around both x and y-axes at the origin.

On the other hand, an *even* harmonic equation only contains *cosine* components, such as a "triangular" wave, that are symmetrical around the y-axis, but *not* the x-axis. This orientation is due to a 90° (one quarter cycle) phase-shift away from the sine components. Known as *phase-quadrature*, odd and even components in the Fourier series comprise what is called a "balanced orthogonal system of right-angled waves" capable of describing *any* kind of sound.

---

[43] During the French occupation of Egypt in the late 18th / early 19th centuries, Napoleon led a mixed scientific and literary expedition to Upper Egypt that included Joseph Fourier. As Fourier was not fond of the heat in the Egyptian desert and suffered rheumatism in extreme temperatures, he began to study the propagation of heat. Just after his return to France in 1801, he wrote a memoir entitled *Theory of the Propagation of Heat in Solid Bodies* in which he first proposed a theory of how waves traveled. It was within this theory that the *Fourier series* was first proposed.

**Figure 33 - First four harmonic partials summing to a composite wave**




From this, the complex timbre of any instrument can be explained simply as a mix of odd and even wave components, each with a particular amplitude (or volume) coefficient to shape the sound quality. For instance, a weighting of sine components in an odd function, like that approximated by an open pipe recorder or flute, is generally characterized as "pure" or calming. A closed pipe instrument like the clarinet also produces only odd harmonics but stacked so as to create an "odd square wave." This harmonic combination sounds less soft or smooth than a flute, producing a characteristic hollow or nasal-like quality instead. By comparison, a weighting of cosine waves into an "even function," like the saw tooth waveforms of brass instruments, is considered bright, brash or perhaps tense sounding. When both odd and even components are mixed together, the result is a more complex and balanced sound like that of a saxophone or tenor voice.

In a very real way, an entire orchestra can be thought of as a single overarching Fourier waveform with different instruments specializing in odd, even and blended harmonic components. When a composer combines instruments at different dynamics (volume) in an orchestrated piece of music, our ears recognize it as a grand symphony of odd-even harmonics vibrating as a single composite wave on our tiny eardrums. The end result, of course, is what we call emotion.

But the question remains how we are so easily able to distinguish individual instruments and follow a specific melody buried within a complex interference pattern of harmonics. This becomes even more perplexing when you add in the fact that harmonics in the real world are sometimes slightly sharp or flat from perfect whole number multiples.

For instance, the harmonics of a piano string often resonate increasingly sharp up the harmonic series due to a phenomenon known as *inharmonicity*, or *stretch* between actual and theoretical harmonic frequencies. This is due to the piano wire appearing stiffer to the upper harmonics with shorter wavelengths, thus causing the wire segment to vibrate faster or sharper than the expected whole number multiple [Jorgensen 1977].

Since this "stretch" is no more than about 1.2 cents (1.2% of a semitone) and relatively small, our ability to recognize slightly sharp harmonics and "fuse" them together into a single tone remains undiminished. This impressive natural ability to tolerate imperfection and adapt underlies our ability to maintain attention on individual sounds, thus explaining in part our ability to distinguish between several instruments in an orchestra. Without this tolerance for a little slack or stretch in harmonics, recognition and enjoyment of music would be impossible.

By now you may be wondering how timbre could be relevant to the broader subject of music cognition and harmony. After all, what could phase-quadrature and odd-even wave functions of a single tone have to do with our perception of music harmony? Surely, you say, the shear complexity of music – comprising melodies, intervals, chords, voice leadings, polyphonic counterpoint and tritone substitutions – is a much bigger and vastly more complex issue than this.

Well, in spite of the separate treatment of timbre and harmony in music textbooks, many physicists would agree that the degree to which harmonics reinforce or cancel one another in sound probably has a lot to do with how pleasing or disturbing certain musical harmonies are to our ears. When the wave patterns of different musical intervals are compared with their historical use in evoking a sense of tension or relief, it would seem to support a tie between harmonics and harmony.

Yet even though this idea seems like a perfectly logical conclusion, no one knows just how wave theory should be applied to explain our ability to recognize and anticipate music. When it comes to ranking the consonance of musical intervals, few can agree on how it should be done. Fewer still can agree on any idea about how harmonics might be directly connected to our ability to anticipate the directional movement of music harmony. As a result, there are no accepted standards for how to physically measure even the most basic musical qualities of consonance, dissonance, tension or resolution.

Surely there must be an explanation for how physical harmonic waves interact with our physiology to make the recognition of music possible. Might it be that our perception of musical consonance and tension is due to some intrinsic ability to *physically recognize* the patterns produced by harmonics? More to the point, could perception of all musical harmonies be explained by some kind of physical pattern matching process as harmonics overlap and interfere with one another in our ears and brain? If so, what would this look like?

In Figure 34, the *Blackman Spectral Analysis* of two tones diverging over an octave shows how intersecting odd-even harmonic components interfere with one another constructively and destructively to create a distinctive pattern of vertical gaps.[44] For reference, each gap is labeled with its corresponding sound frequency, interval ratio and closest note name within a 12-step octave starting on {A}.

From this simple analysis it is plain to see that most of the gaps occur as simple proportions between harmonics, confirming once again Pythagoras' original discovery of this phenomenon. For instance, the 3:2 ratio of a perfect 5$^{th}$ occurs precisely at the convergent gap created by the third and second harmonic partials. For some intervals in the octave, more than one gap or ratio appears. The choice of which gap to use for a given interval is the reason we find so many different instrument tunings and temperament methods used throughout history. It is also the cause for never ending arguments concerning our perception of music, since different combinations seem to favor different styles and evoke slightly different emotions.

At first glance, we might assume that the larger gaps are perceived as consonant while the thinner gaps (or no gap) are perceived as dissonant. But this is not entirely correct. Though it is true that some of the larger gaps are universally perceived as sounding more pleasing or *consonant* to the ear, attempts to rank interval concordance based on gap size alone does not match either our overall perception of consonance or the common practice use of intervals in the music literature.

For example, of all the vertical gaps in the spectral analysis, excluding the unison and octave at either end, there is one gap clearly much larger than the rest. This is the wide major 6$^{th}$ gap {A, F#} at the ratio of 5:3 in the analysis. But counter to what conventional music theory would tell us, this gap appears much wider than the perfect 5$^{th}$ gap, widely assumed to be the most consonant and stable interval. More confounding that this, the major 6$^{th}$ corresponds to the *thirteenth* harmonic while the perfect 5$^{th}$ corresponds to the *third* harmonic with a simpler 3:2 ratio. How can a major 6$^{th}$ with a <u>more complex</u> ratio located <u>further away</u> from the fundamental in the harmonic series have a more resonant gap in the spectral analysis than a perfect 5$^{th}$?

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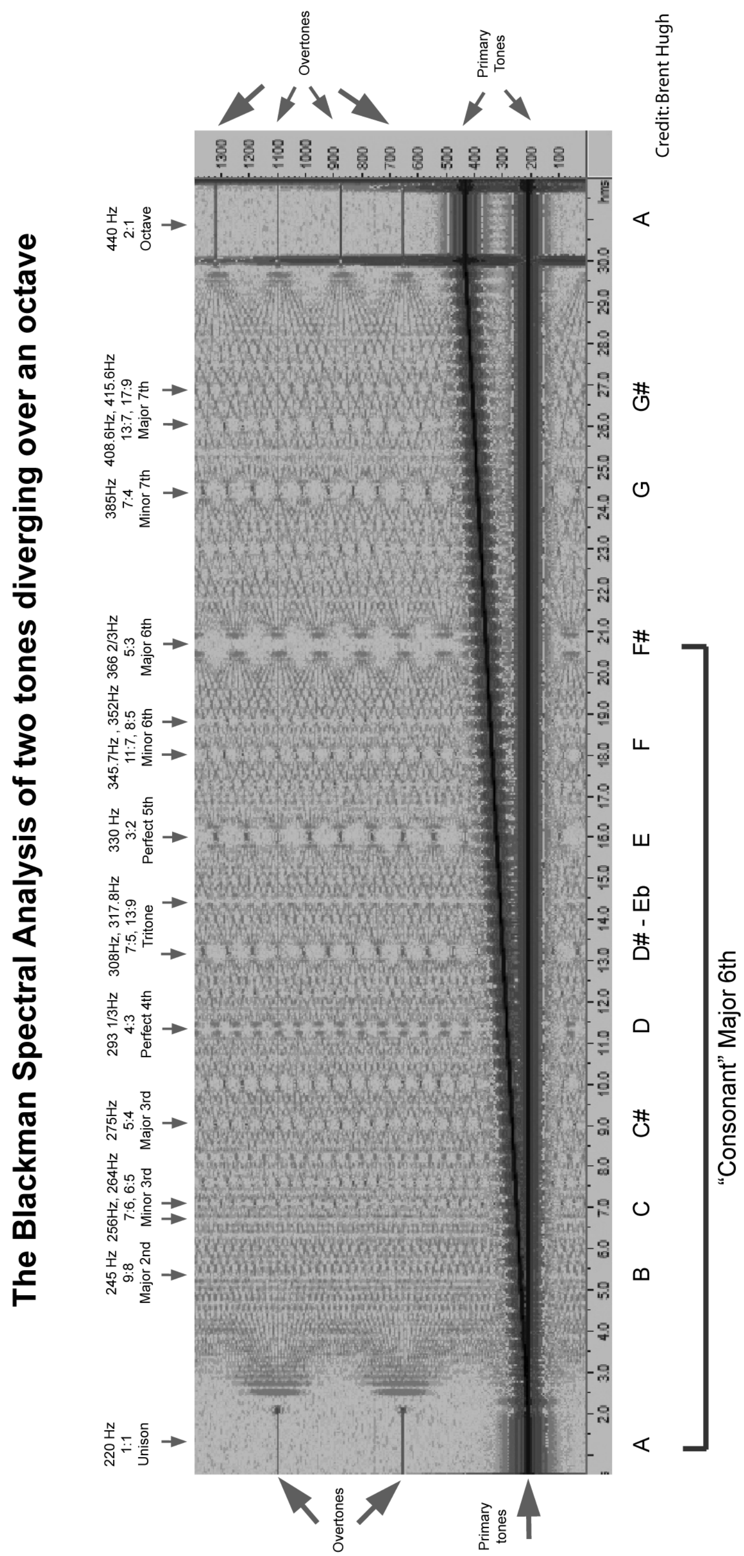
[44] Created in Adobe Audition (formerly Cool Edit Pro) by generating sine waves using a built-in function, then visualized as an interference pattern in the "spectral analysis" view.

## Figure 34 - Blackman Spectral Analysis of two tones diverging over an octave



Well, it turns out that this little mystery can be explained by the natural concordance of a major 6th proven using simple interval multiplication (shown in footnote).[45] But the simpler answer is that a major 6th interval between two pure tones is less destructive in its interference pattern that any other combination of tones in an octave. Counter to all opinions and arguments to the contrary, this is an indisputable and observable fact of nature.

Based on this, we might easily jump to the conclusion that the resonant gap size of other intervals in the spectral analysis could be used to rank our perception of consonance in music. Yet, even though resonance is a key factor in our perception of consonance, the historical ranking of intervals from most consonant to most dissonant *does not* follow the order of gap size. This little enigma probably explains why acoustics has never been fully adopted into music theory or as a basis for cognitive music theories.

Another example where gap size does not match common practice is found in the large consonant-looking 7:5 gap at the tritone near mid-octave.[46] Everyone agrees that a tritone sounds dissonant (thus its unfortunate nickname *Diabolus in Musica*), but its spectral gap is much larger than either the minor 6th or a perfect 4th, both traditionally considered very consonant intervals. If it is supposed to be so dissonant, shouldn't the tritone have a much smaller gap than all the other intervals or even no gap at all?

Likewise, the major and minor 3rds in the lower half of the octave exhibit very thin vertical gaps; yet common practice and our own ears would tell us they are as consonant as the major and minor 6ths and certainly more consonant than the larger tritone gap. What could possibly explain all this?

It is no wonder there is so much confusion and disagreement over how to define and rank intervals from consonant to dissonant. Even today, there is no accepted theory to explain the difference between acoustical analysis and the use of intervals in common practice. The apparent disconnect between acoustical analyses and how we interpret it has led to the belief that music perception is not a physical response, but a matter of tradition and regional cultural influence.

If music is ever to rise above the mystical realm of subjective opinion, a broader scientific explanation is needed that couples acoustics with our perception of consonance and dissonance. One that is compatible with the physical evidence provided by the spectral analysis, but goes a step further to include our physiological structure. If gap size alone does not define the cognitive qualities of music, how then should we define and measure it?

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[45] First, consider that a major 6th above a perfect 5th is the same as the *major 3rd* of the octave harmonic. When we multiply the major 6th ratio against the perfect 5th (5:3 × 3:2 = 15:6) we obtain the ratio 2.5. This is the very same ratio as that of the major 3rd an octave above: 2:1 × 5:4 = 2.5. In this way, the fifth harmonic (major 3rd) of the bottom tone resonates in the same octave as the third harmonic (perfect 5th) of the top tone (the major 6th interval) to strengthen and amplify the major 6th interval more than any other interval in the harmonic series, except the octave.

[46] Specifically the simpler mid-octave proportion known as a diminished 5th.

# *Gaussian Interference*

*"It occurred to me by intuition, and music was the driving force behind that intuition. My discovery was the result of music perception." - Albert Einstein*

When we stop to carefully consider the relative proportions of gaps in the Blackman spectral analysis of diverging tones, a natural overarching shape of gap sizes can be discerned. This distribution of gaps happens to match the mathematical curve of a *first derivative Gaussian distribution*, a variation of Shepard's Bell Curve fade filter. In recent years, this distribution curve has also been found to play a central role in the neurophysiology of human/animal sensory systems.

In the latest research on vision perception, the "Gaussian derivative model for spatial-temporal vision" has been found to best describe the first stage of processing in our visual cortex for motion, orientation, location and size [Young et. al. 2001]. And in similar studies of auditory cognition, the latest findings with cats and monkeys also claim that spatio-temporal auditory cognition is best described as a natural Gaussian derivative filter located within the inner ear and auditory cortex [Fishman 2001 and Cedolin 2005].

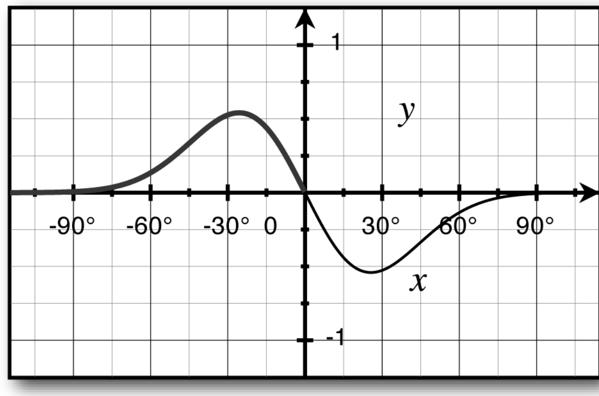
Together, these studies strongly suggest the existence of a biological Gaussian function in mammals that acts as a natural filtering and focusing process to help identify and maintain attention on objects and sounds, a necessary skill for survival. For instance, a spatiotemporal filter like this for audition would help focus attention on the sounds of a predatory animal in time for fight or flight or identify the cry of offspring separated from the pack. In people, a Gaussian function in our auditory system could help explain how we recognize the pattern of concordant gaps in the octave spectral analysis.

Figure 35 shows a graph of the positive range of a first-derivative Gaussian curve overlaid on the spectral analysis in order to compare curve height to gap size. In the graph, the major $6^{th}$ interval {A, F#} is found to be coincident with the peak of the Gaussian curve. Interestingly, the octave golden ratio (Φ=1.61803…) happens to occur very near this location, sitting right in between the major and minor $6^{th}$ ratios of 5:3 (1.66667) and 8:5 (1.6). But unlike harmonic ratios, the golden ratio does not correspond to any particular vertical series of gaps, instead landing in a very orderly diagonal lattice of wave interference that appears to emanate from the wide convergent gap at the major $6^{th}$.

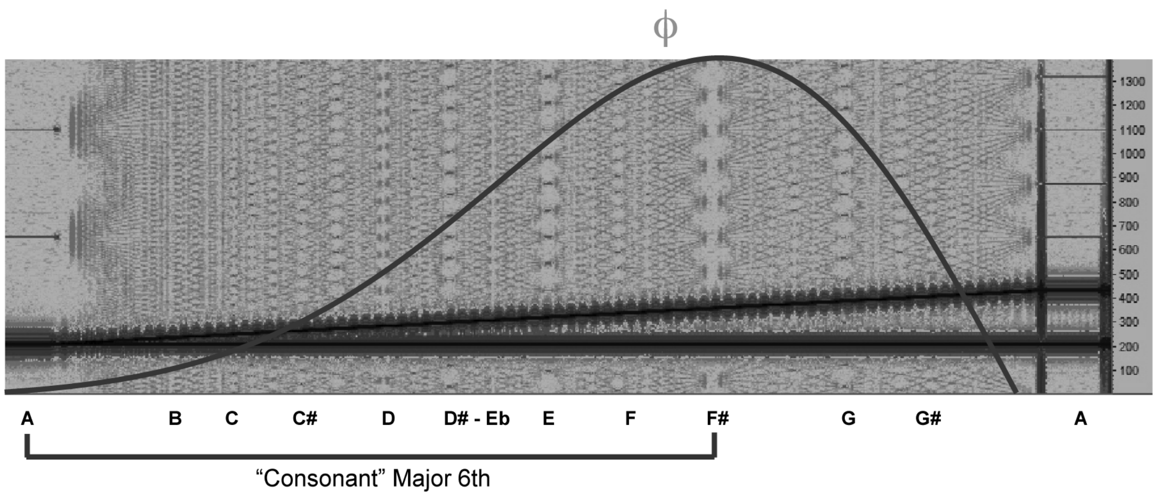
**Figure 35 – Gaussian interference pattern over an octave**

**Gaussian distribution curve**

$$y = \frac{d}{dx}\left(\frac{1}{s \cdot \mathrm{sqrt}(2\pi)} e^{-\frac{m \cdot x^2}{2s^2}}\right), s=1, m=5$$



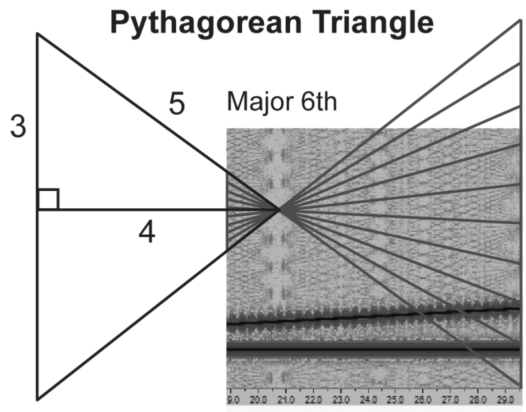

**Gaussian curve superposed on the octave spectral analysis**




Looking more closely at the diagonal lattice pattern, we can see that as the two tones diverge each interval creates its own small piece of the overarching Gaussian pattern of harmonic interference. The regions of greatest concordance appear as light open spaces while the dark lattice regions occur where there is little or no concordant energy. Given this, which one should we say explains our perception of music – the light foreground shapes or the dark background lattice? After all, both are part of the same pattern of sound.

In the next figure, the lattice is now traced out so we can get a better look at the overall pattern of gaps near the top of the curve.

**Figure 36 - Analysis of resonance pattern around a major 6$^{th}$ interval**




Proportions in resonant pattern:

Major 6th 5:3 = 1.666
Major 3rd 5:4 = 1.250
Perfect 4th 4:3 = 1.333

Here we see ten diagonal lines crossing at each convergence point in the major 6$^{th}$ gap. Between these diagonals are nine resonant regions or gaps where wave energy converges. As a combined pattern, a series of triangular shapes can be seen to extend upward to the octave at right. Of these, the outermost triangle happens to correspond to the *Pythagorean Triangle* having sides in the exact proportion of 5 × 4 × 3.[47]

A little known property of the Pythagorean Triangle is its three sides actually form a musical harmony between the consonant proportions of a major 6$^{th}$, a major 3$^{rd}$ and a perfect 4$^{th}$. Furthermore, adding the sides together creates a perimeter of 12 while dividing them as 5:3 / 5:4 / 4:3 works out to equal 1. What could be more harmonious than this? We might wonder if this spectral octave pattern of sound could have been the inspiration behind Pythagoras' discoveries in numbers, geometry and music? If so, how could he have possibly known about this particular interference pattern?

The answer may be that Pythagoras found this pattern on plates of vibrated sand, a skill he could have learned during his 22 years in the Egyptian mystery school. Just as Ernst Chladni documented thousands of years later, there is growing evidence that the Egyptian priests chanted over dusty drumheads to study the geometric patterns sound makes as sand vibrates into calm background regions. It is already known that ancient Chinese gong-makers sprinkled sand over their gongs to tune them (some still do to this day). Could this be a clue to how our ears recognize sound? Could music be recognized as a kind of auditory geometry?

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47 So named due to its use by Pythagoras to discover his famed *Pythagorean Theorem* $a^2 + b^2 = c^2$.

Given that such patterns will form on any vibrated surface, why would the same patterns that form on drumheads and gongs not also form on our eardrums? And why would these geometrical sound patterns not be preserved into the inner ear and transformed into corresponding neural patterns in our brain? Could it be that music perception is a process of recognizing different auditory geometries like the ancient shamans and gong makers seemed to believe?

If we take this as a starting hypothesis for music perception, one thing is certain. Recognition of patterns can only occur by detecting differences between foreground and background regions. Patterns are by definition the result of contrast and must be measured as such. Like a woodcut relief used to stamp out foreground words and pictures on a paper background, the patterns produced by musical tones and harmonies must be measured as a contrast between foreground resonance and background calm.

Unfortunately, there is one little problem with this approach. How are we supposed to measure the contrast in sound patterns when no prescribed method currently exists to do this? Modern wave theory focuses entirely on the structure of the waves themselves – not the calm background space between them. Fourier analysis tells us how to combine and decompose waves, but says nothing about the geometrical patterns that form across a spectrum of wave interference. Even the Gaussian equation used by Shepard and others to describe the neurophysiological process of auditory perception cannot represent sound as a balance between foreground and background.

Alas, it seems wave theory and statistical distribution equations in their current forms cannot help us analyze the overall geometry of harmonic interference in the way needed to couple sound to the human auditory system. We are left with no choice but to invent an entirely new way of representing the spectral pattern of harmonics over an octave.

Johann Carl Friedrich Gauss (1777 – 1855), a German mathematician and scientist known as "the Prince of Mathematicians," is considered one of history's most influential mathematicians. As the one who discovered the Gaussian probability distribution (the "Bell curve"), his discoveries have had a great impact on many fields, such as number theory, statistics, analysis, differential geometry, geodesy, electrostatics, astronomy and optics. Fact is, much of our understanding of nature and human behavior is based on his foundational work.

But when we take a good hard look at Gauss's famous equation for probability in nature (see earlier Figure 35), one cannot help but be struck by its complexity. For something that can so accurately predict countless natural phenomena from organic physiology and human intelligence to financial variables and sound patterns, why would this symbolic form be so obtuse and inelegant? Why would his equation not clearly represent the duality and balance found everywhere in nature? Could there be another, better way to represent probability in nature so that it does represent the symmetry we need to understand harmonic interference patterns?

Fortunately, there is an alternative - one so simple and so natural that you will either love it or hate it. Those who see nature as complex, random and mystifying will surely find fault while those who see it as simple, elegant and beautiful will find it delightful. In either case the answer will probably surprise you.

A five-step transformation (shown in footnote) takes us from a conventional Gaussian first-derivative curve to the same curve defined as a simple function of foreground harmonic proportions balanced against a background of <u>near</u>-Fibonacci proportions.[48] While the mathematics may be of little interest to most people, anyone can appreciate the fact that these few steps represent a huge philosophical shift from the one-sided asymmetrical view of nature adopted into modern science from Gauss to one where we can finally begin to see the intrinsic symmetry between space and time, energy and form and other balancing properties of nature.

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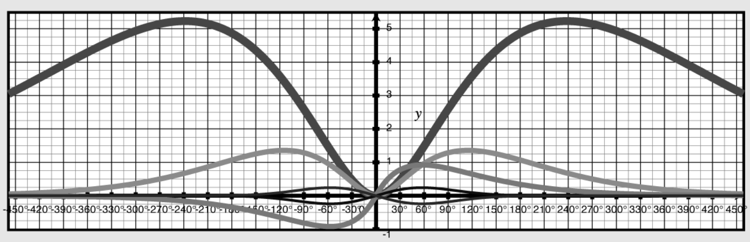
[48] The following steps demonstrate how to convert a first derivative Gaussian distribution into an anti-harmonic form based on balancing harmonic and near-Fibonacci proportions. The only difference is one of scale.




The first derivative of the Gaussian distribution in the familiar form

$$y = \frac{d}{dx}\left(\frac{1}{\sqrt{2\pi}}\cdot\left(e^{-\frac{x^2}{2}}\right)\right) \tag{1.1}$$

can be negated to flip it about the x-axis

$$y = -\frac{d}{dx}\left(\frac{1}{\sqrt{2\pi}}\cdot\left(e^{-\frac{x^2}{2}}\right)\right) \tag{1.2}$$

which can then be taken as a positive fraction over the domain while differentiating the exponent power $x^2 / 2$ to produce a linear power of $|x|$.

$$y = \left(\frac{x}{\frac{\exp(|x|)}{\sqrt{2\pi}}}\right) \tag{1.3}$$

Restating this now as resonance over the domain by squaring the numerator

$$y = \left(\frac{x^2}{\frac{\exp(|x|)}{\sqrt{2\pi}}}\right) \tag{1.4}$$

we may then express the Gaussian distribution in terms of a ratio of $\Phi$ over period $\sqrt{5}$ instead of the natural exponent over $\sqrt{2\pi}$ to obtain the final symmetrical organic form:

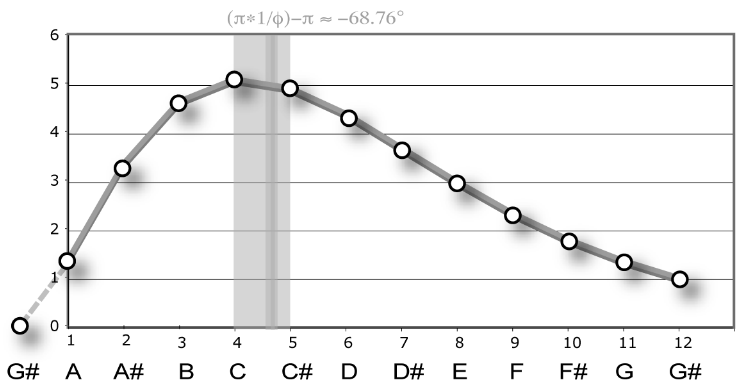
$$y = \left(\frac{x^2}{\frac{\Phi^{|x|}}{\sqrt{5}}}\right) \tag{1.5}$$

The final equation $y = x^2 / ( \Phi^{|x|} / \sqrt{5} )$ is a very useful alternative to the more complex Gaussian equation. It provides the same Gaussian shape found in the interference pattern of an octave, only now in a way where we can measure the calm background region too. We will refer to this symmetrical variation of a first derivative Gaussian curve as the ***INTERFERENCE*** equation because it represents the interference distribution between the square of the first twelve natural integers (encompassing *harmonic* proportions) and the first twelve Fibonacci numbers (encompassing increasingly *anti-harmonic* proportions).[49]

For instance, applying the ***INTERFERENCE*** equation over the range of n={1..12} in Figure 37, we can describe the resonance proportions of two tones *diverging downward* over an octave.

**Figure 37 - The *INTERFERENCE* function**

$$y = \frac{n^2}{Fib(n)},\ Fib(n) = \frac{\phi^n}{sqt(5)},\ n = \{1..12\}$$




| n | Fibonacci Number | Fib(n) Estimate | $n^2$ | $n^2$ / Fib(n) | Interval | Note |
|---|---|---|---|---|---|---|
| 1 | 1 | 0.723605 | 1 | **1.38** | Unison | A |
| 2 | 1 | 1.1708146 | 4 | **3.42** | m2 | A# |
| 3 | 2 | 1.8944132 | 9 | **4.75** | M2 | B |
| 4 | 3 | 3.0652174 | 16 | **5.22** | m3 | C |
| 5 | 5 | 4.9596136 | 25 | **5.04** | M3 | C# |
| 6 | 8 | 8.0248037 | 36 | **4.49** | P4 | D |
| 7 | 13 | 12.984373 | 49 | **3.77** | TT | D# |
| 8 | 21 | 21.009105 | 64 | **3.05** | P5 | E |
| 9 | 34 | 33.993362 | 81 | **2.38** | m6 | F |
| 10 | 55 | 55.00228 | 100 | **1.82** | M6 | F# |
| 11 | 89 | 88.995339 | 121 | **1.36** | m7 | G |
| 12 | 144 | 143.99713 | 144 | **1.00** | M7 | G# |

[49] The first derivative Gaussian curve is also known as a *spatial Gaussian*.

In doing this, we find that the greatest resonance appears in the bottom half of the octave at a minor 3rd {A, C} with another golden ratio located just above in the dark vertical band. This is an exact reflection of the earlier spectral analysis. The little known fact that makes this ***INTERFERENCE*** series work is the square root of the golden ratio happens to be the real part of a simple periodic continued fraction that uses (complex) Gaussian integers (a + ib) instead of natural integers [Good, 1992]. By taking the ratio between the square of the integer proportions of the harmonic series (as multipliers of frequency) and a seldom-used Fibonacci "growth" function, the Gaussian curvature produced by the golden ratio is revealed.

The most important thing about this organic looking curve is it can bridge the fields of physiology and acoustics, thus yielding a completely natural explanation for music and perception. Since life always grows according to a balance of both Fibonacci and harmonic proportions, the ***INTERFERENCE*** curve provides a much better tool to examine biological shape as a function of resonance than the standard Gaussian equation. And when used with recent research suggesting our vision and auditory systems operate as a biological first derivative "spatial" Gaussian to decode sound waves, a grand scientific musical theory – hereafter referred to simply as ***Harmonic Interference Theory*** – emerges as the *most organic* model possible to measure harmonic phenomena in nature.[50]

You might now wonder as I do why Gauss chose to represent nature in the more complex form with Unity, π and e (base of the natural logarithm) instead of the much simpler and symmetrical Φ and √5 representation. You might also wonder why scientists and mathematicians continue to use the standard Gaussian equation in such a complex and asymmetrical form.

The easy (and politically correct) answer might be that it has become part of a larger standardized system in science that now ties together many diverse fields. But while tradition and pragmatism provide a strong argument, it still fails to explain why at least a few fields like acoustics and biology never use this symmetrical form as an alternative avenue of exploration. Why is it that we should only want to express energy in terms of its *foreground structure* without regard to the calm *background field* that contains it? Why is our starting premise always that energy is practical and useful while the space or medium in which it travels is not? Most might dismiss this asymmetrical view as excusable and of little relevance – a mere convention of history

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**Principle 6:** The distribution of wave interference in the harmonic series is described by a ratio between the square of the harmonic series and the Fibonacci series, referred to here as the ***INTERFERENCE*** *resonance function*:

$y = 1 / (\Phi/\sqrt{5}),\ 4 / (\Phi^2/\sqrt{5}),\ 9 / (\Phi^3/\sqrt{5}),\ \ldots,\ n^2 / (\Phi^n/\sqrt{5})$

$\mathbf{y = n^2 / (\Phi^n / \sqrt{5})\ ,\ n = \{1..12\}}$

and nothing more. Yet our avoidance of this simple idea is more likely related to a time when knowledge of nature was commonly encoded to hide harmonic principles from a disapproving clergy.

As discussed earlier, the medieval Church campaign against natural harmonic philosophies had an adverse affect on the methods of science. With the formulation of the scientific method during the 17th century Inquisition, the clash between the Church and pagan theosophies forced a climate of self-censorship on early scientists, particularly as it related to harmonic processes. So, just as we find the golden ratio used secretly in artwork, veiled gematria puzzles in music and forbidden melodies carved into the architecture of chapels, an arcane language of mathematics became accepted in science to avoid offending the Church.

Arising as a *complicity of convenience* during the so-called *Age of Reason*, avoidance of the golden ratio in equations has created a clear bias toward foreground structure for centuries while directing attention away from any interconnecting and unifying properties in the background medium. If this hypothesis is correct (and it certainly seems so), many of the founding conventions of modern science continue to impose a severe drag on scientific progress, constricting the most basic questions we ask and lines of thought we pursue.

# *Dodecaphonic Dice*

*"I know that the twelve notes in each octave and the variety of rhythm offer me opportunities that all of human genius will never exhaust." - Igor Stravinsky*

Having now found a more organic way of expressing the Gaussian distribution, we can begin to see harmony (of any kind) as it really is – a simple balance of foreground harmonic waves and background Fibonacci gaps.[51] Instead of studying sound asymmetrically as foreground wave energy, we can finally begin to see music as our ears do – a contrasting pattern of sound and silence. Like those old Greek philosophers who dared to venture into the invisible Underworld of nature seeking knowledge, we now have what we need to begin our journey into the underworld of music.

The first thing we need to do is gather a few basic facts about the ***INTERFERENCE*** curve. In particular, we need to determine where the foreground is in perfect balance (or harmony) with the background. This is found by taking the arithmetic mean of the curve corresponding to the curve's "normal distribution" and "standard deviation." These basic properties of the spectral interference pattern over an octave tell a very interesting story about the relationship between music and mathematics.

***Balance point between foreground and background in the INTERFERENCE curve:***

| | |
|---|---|
| *Arithmetic mean* | *3.13971 (or 3.142522 from Fibonacci integers)* |
| *Standard deviation* | *1.72845025* |
| *Normal distribution:* | *1.0* |
| *X* | $12 = n^2$ |

We find here the most amazing thing. The arithmetic mean converges toward PI, or mathematical constant π ≈ 3.14159, located in the middle of the curve. We further find this point in the distribution curve to be equal to Unity (or 1) when the domain value X = 12. This is significant because twelve is the square root of 144, the value shared by both harmonic and Fibonacci series in a 12-step octave (see table in Figure 38). Squaring each of the table values and dividing by twelve confirms that 12.02383 ≈ 12 is the point of balance between foreground and background.

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[51] Harmonic proportions are {1:2, 2:3, 3:4, 4:5,…n:n+1} while Fibonacci proportions are {1:1, 1:2, 2:3, 3:5, 5:8, … F(n+1) = F(n) + F(n-1)}.

The significance of twelve as a point of balance in the octave interference pattern is proven further by plugging it into the equation, confirming the curve height equal to Unity at the octave.[52] But even more significant than this is the fact that plugging the *square root of twelve* into the equation results in the amplitude y = 5.0666.[53] Care to guess what this number represents?

It is none other than the y-axis amplitude for the golden ratio in an octave. Yes, the square root of twelve in the Gaussian ***INTERFERENCE*** pattern occurs precisely at Φ, right in the "cracks between the keys" of a major 3rd and minor 3rd in an octave. Just like the dense lattice region between a major 6th and minor 6th, the infinite golden ratio also provides an *anti-harmonic proportion* in the lower half of an octave. This occurs naturally at the square root of 12 (or fourth root of 144) in a 12-step octave.

No matter how you do the math, both harmonic and Fibonacci series reach a harmonic balance with one another at n=12 and an anti-harmonic dead zone at n=√12.[54] Division of the octave by twelve (not eleven, nineteen or any other number) is revealed here as a completely natural pattern produced by linear harmonics that are *curved in pitch space* by Fibonacci proportions as they converge to Φ. Could Gioseffo Zarlino's decision to divide the octave into twelve steps have involved some knowledge of this simple relation between harmonics and the Fibonacci series?

Looking back now at the ***INTERFERENCE*** curve in Figure 38, we can see that the highest point on the curve occurs around a minor 3rd. To determine if this is the point of maximum resonance, we plug the *cube root of a tritone (144 / 2)* into the equation to represent resonance in 3-dimensional space.[55] As expected, the result is a perfect match for the maximum amplitude of 5.22 in the ***INTERFERENCE*** table corresponding to a minor 3rd. Inverted into a major 6th, this corresponds perfectly to the most resonant gap in our spectral analysis.

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52
$y = n^2 / (\Phi^n / \sqrt{5})$
$y = 12^2 / (\Phi^{12} / \sqrt{5})$
$y = 144 / 144$
$\mathbf{y = 1}$ // INTERFERENCE octave amplitude

53
$y = n^2 / (\Phi^n / \sqrt{5})$
$y = 3.4641^2 / (\Phi^{3.4641} / \sqrt{5})$ // $\sqrt{12} = 3.4641$
$y = 12 / 2.36845$
$\mathbf{y = 5.0666}$ // INTERFERENCE minimum resonance

54 **Principle 7:** The 12-step octave follows the natural distribution of harmonic interference that reaches an octave harmony at √144 = 12 and anti-harmonic center at √12.

55
$y = n^2 / (\Phi^n / \sqrt{5})$
$y = 4.16^2 / (\Phi^{4.16} / \sqrt{5})$ // $\sqrt[3]{(144/2)} = \sqrt[3]{72} = 4.16$
$y = 17.3 / 3.31$
$\mathbf{y = 5.228}$ // INTERFERENCE maximum resonance

In general, the ***INTERFERENCE*** equation can be used to measure resonant amplitudes for any musical interval under any temperament or octave division. This equation tells us that *minimum resonance* occurs at the fourth root of an octave (or square root of twelve) while *maximum resonance* occurs at the cube root of half an octave. Taken together, these results offer clear evidence that harmonic interference balances naturally around 12 as the most rational and harmonic number possible. This is then counterbalanced by the golden ratio, which is the most irrational and enharmonic number possible. When Einstein said: "*God does not play dice with the universe,"* he must not have considered the possibility of dodecahedral dice.

As an even stranger twist to the ***INTERFERENCE*** equation, you may be further entertained to learn that it has a remarkable associative property that reveals our old friend the Pythagorean comma. Remember, this is the small gap that mysteriously appears in a stack of twelve perfect 5ths at the seventh octave.[56] We need only insert n=√12 as the location of minimum resonance and shift the parentheses left one position in the equation to find it.[57]

As a surprising correspondence between music and math, this little trick reveals the Pythagorean comma accurate to 3 decimal places. More amazing still, if we recalculate using the un-rounded arithmetic mean 12.02383 found earlier in place of 12, we obtain a slightly better estimate for the Pythagorean comma good to 4 decimal places.[58] This bizarre associative property in the ***INTERFERENCE*** equation using the anti-harmonic golden ratio location of n=√12 proves *the golden ratio is a physical property* in the natural harmonic series and not some kind of error or "evil" in nature as portrayed by the Church. Vibration needs room to resonate in space and the Pythagorean comma created by the golden ratio appears to be just the right amount of room needed.

While the mathematics of resonance may seem far removed from our awareness and experience of music, it does demonstrate a profound connection between the physics of sound and number theory. There is an undeniable interlocking mathematical relationship between the irrational constants Φ, π, e and the rational numbers 5, 7 and 12 in the musical harmonic series. Grouped one way, we arrive at amplitudes corresponding to harmonic resonance in an octave

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56 Often elegantly expressed as powers of simple proportions: $(3:2)^{12} \times (1:2)^{7}$

57

$y = (n^2 / \Phi^n\,) / \sqrt{5})$
$y = (12 / \Phi^{\sqrt{12}}) / \sqrt{5}$
$y = 2.265846758 / \sqrt{5}$
$y = \mathbf{1.013317471}$ // The Pythagorean comma to 3 places

58 The associative property of the INTERFERENCE function yields the y value of 1.013651449 which is equal to the Pythagorean comma to four places: **1.01364326.**

while grouped another way we find the anti-harmonic Pythagorean comma at the seventh octave. Since most music tends to follow the distribution pattern created by these rational and irrational numbers, it is important to point out that we actually *hear and feel* these numbers as special proportions in sound.

In fact, these same proportions can be found to occur over the *entire auditory spectrum*. When we continue to apply the ***INTERFERENCE*** equation across multiple octaves, taking each arithmetic mean as a ratio between adjacent octave groupings {1, {1,2}, {1,2,3},… (1..m)}, we find that each ascending octave is also a harmonic proportion. We find that the auditory spectrum is itself organized as a harmonic series of octaves.

| *Octave Group* | *Mean* | | *Adjacent Group Ratios* | | *Harmonic Ratios* |
|---|---|---|---|---|---|
| *1* | *3.14 ≈* | *π* | *undefined* | | |
| *1..2* | *1.61 ≈* | *Φ* | *1.9445* | *≈ 2* | *2:1 = 2* |
| *1..3* | *1.07 ≈* | *1* | *1.4999* | *≈ 1.5* | *3:2 = 1.5* |
| *1..4* | *0.08* | | *1.3333* | | *4:3 = 1.3333* |
| *1..5* | *0.06* | | *1.25* | | *5:4 = 1.25* |
| … | | | | | |

Taking the limit on this table will confirm that adjacent octave ratios do converge to Unity, just like the harmonic series. So, from the harmonic series of a single tone to the interference pattern of two tones over an octave to the harmonic pattern of octaves *across the entire audible sound spectrum*, nature reuses the same mathematical principles at different scales in sound. At the heart of music we find the cyclic constant π and spiraling constant Φ working together to spin off harmonics to create a magnificent hierarchical sonic architecture.

The ***INTERFERENCE*** equation $\mathbf{y = n^2 / ( \Phi^n / \sqrt{5} )}$ offers a new icon for music – like a musical $E=mc^2$ – representing the natural harmonic interplay and symmetry of the world's most enigmatic numbers. It brings back the natural principle of *harmonic interference* as the basis for the study of harmony without need of the *social interference* of manmade rules. It returns musicians and scientists alike to the simple truth that nature has its own order that we should listen to and follow closely without prejudice or bias.

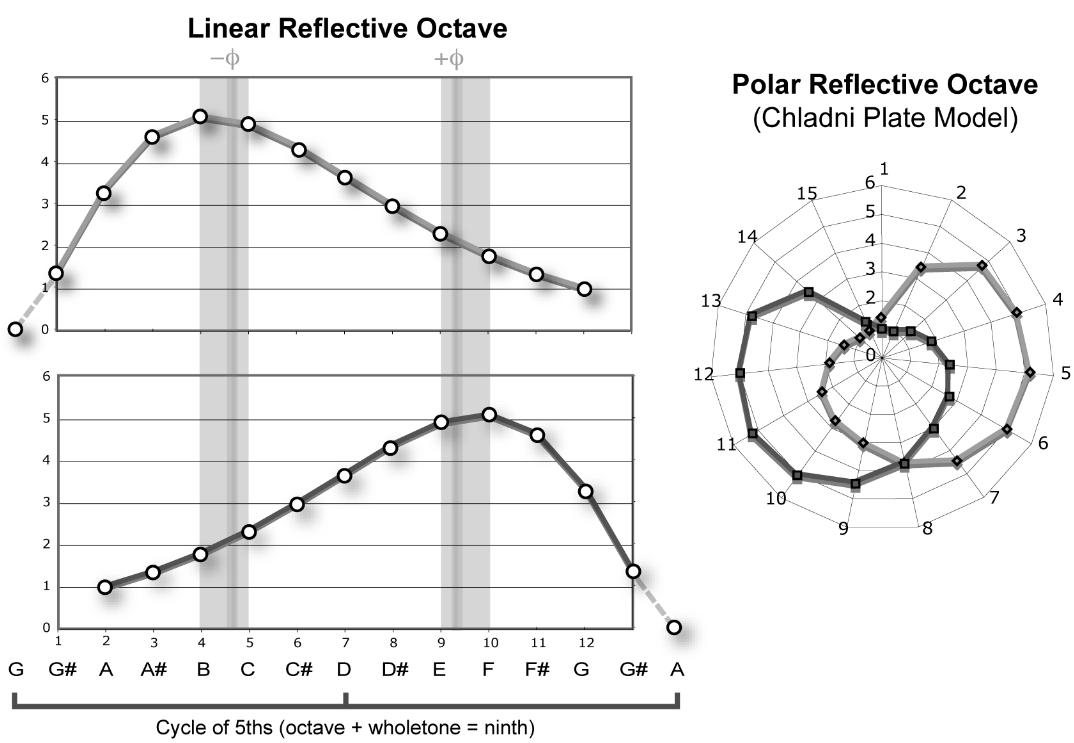
The honest and unfiltered study of harmonics has the real potential to bridge various fields of science with the study of music, all the while telling a fascinating story about the origins of our belief systems and the cultural assumptions we base on them. Nothing else is more revealing of our selves or our society.

# Cognitive Consonance

*"After silence, that which comes nearest to expressing the inexpressible is music."*
*- Aldous Huxley*

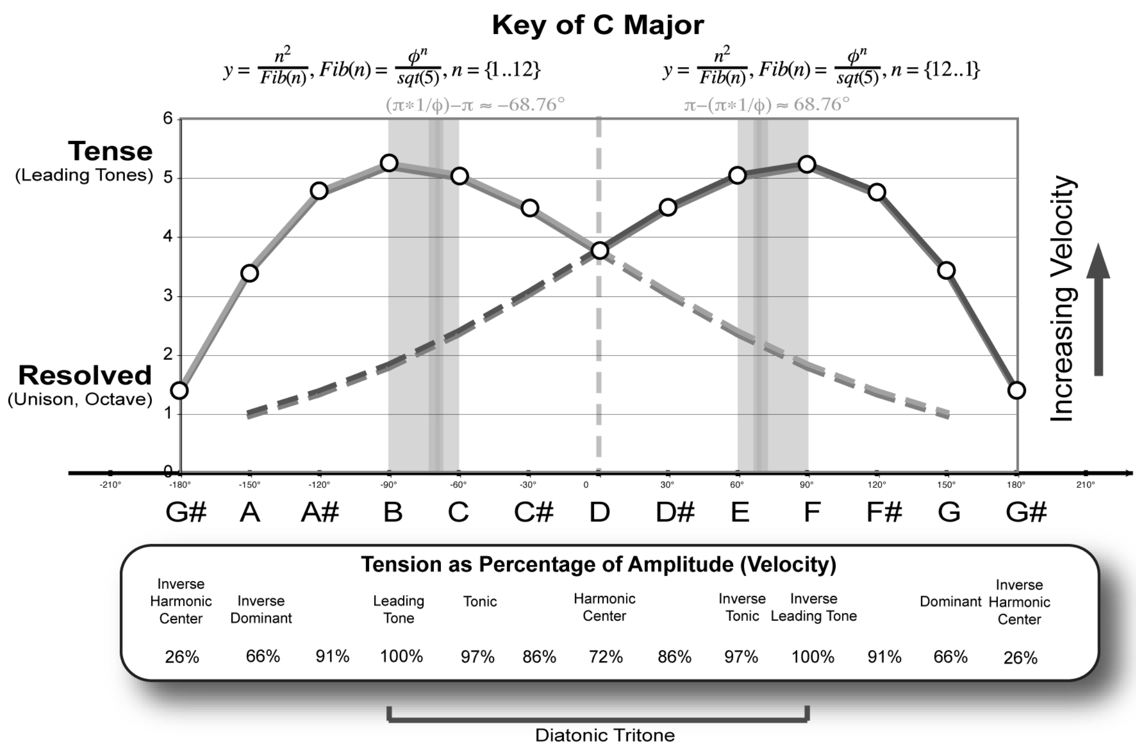
Using the Gaussian ***INTERFERENCE*** equation, we can now seek to establish a single unifying model and set of simple metrics to explain the musical qualities of consonance, dissonance, tension and resolution. Based on the previously established *Tritone Paradox* and symmetrical cognitive properties described in Principle 3, a new model for *perceptual resonance* is proposed. The resulting ***REFLECTIVE INTERFERENCE*** distribution in Figure 38 shows both halves of a first derivative Gaussian curve reflected around mid-octave, aligning with the points of maximum resonance {B, F} and balancing between the Dominant and Inverse Dominant {G, A} on the Cycle of 5ths. In this way, the octave is treated as a kind of organic resonance container, much like a vibrating Chladni plate, that can be used to mathematically measure the perceptual qualities of harmonic tension and resolution in music.

**Figure 38 - Octave as REFLECTIVE INTERFERENCE Container**



Here is how it works. The degree of resonance in an octave is measured symmetrically from the middle, what we might refer to as its *Harmonic Center*. Like most applications of a first derivative Gaussian, the wave height or amplitude represents the *interference velocity* at any point on the curve while the slope measures the change in velocity. Reflected either side of the Harmonic Center, the velocity is then taken as shown in Figure 39 to represent our perception of musical tension[59].

**Figure 39 - Measuring harmonic tension as *REFLECTIVE INTERFERENCE***




The greater the curve amplitude of a given tone, the greater is its resonance within the octave. Since this is a measure of velocity in the interference pattern, it indicates how freely the energy moves for any given tone relative to the Harmonic Center in the middle. This in turn is its degree of perceived tension or resolution, expressed here as a simple percentage of maximum amplitude.

For instance, the most resonant tones in the key of {C} major are Leading Tone {B} and Inverse Leading Tone {F} corresponding to their high amplitudes and maximum wave velocities. From this we know that the diatonic tritone {B, F} is the interval of greatest tension in an octave. The next highest amplitudes and resonant tones are {C} and {E}, creating the second tensest (not

[59] The REFLECTIVE INTERFERENCE velocity curve can also be approximated by a single Gaussian having a second-degree (square factor) polynomial numerator.

to be confused with dissonant) interval of a diatonic major 3rd {C, E}. Together these intervals form the Tritone Function – the tensest and most resonant locations in a diatonic scale.[60]

This ranking order corresponds closely to the harmonic function of most tonal music from at least the mid-16th century, when Zarlino established the 12-step octave. Since it was only after this that vertical harmonies became generally accepted, it now appears that the 12-step octave was the only natural outcome of a universal organic recognition of the ***REFLECTIVE INTERFERENCE*** pattern of harmonics over an octave.

But while Zarlino clearly receives credit for discovering the ideal harmonic division of an octave, his writings indicate no awareness of the *psychoacoustical* properties proposed here. It was simply a delayed outgrowth (by thirteen hundred years!) from Ptolomy's proposed *diatonic syntonon* tuning system that evolved out of the Pythagorean *musica universalis* tradition.

Another way to view octave interference is by adding the two Gaussian curves together into a *composite* reflective distribution curve. The resulting ***REFLECTIVE INTEGRAL*** model can then be used to measure the complementary musical qualities of consonance and dissonance.

When the two reflected curves are added together, they stack one on top of the other to form a single symmetrical distribution resembling a slightly elongated Bell curve. Measuring from the Harmonic Center at the middle dashed line in Figure 40, musical intervals can now be ranked as a percentage of consonance increasing from the center out to the ends of the curve.

For example, the most *consonant* intervals are the unison and octave that align with the lowest amplitude at each end of the graph. Logically, half of this distance would then yield the most dissonant interval – the tritone {D, G#}.

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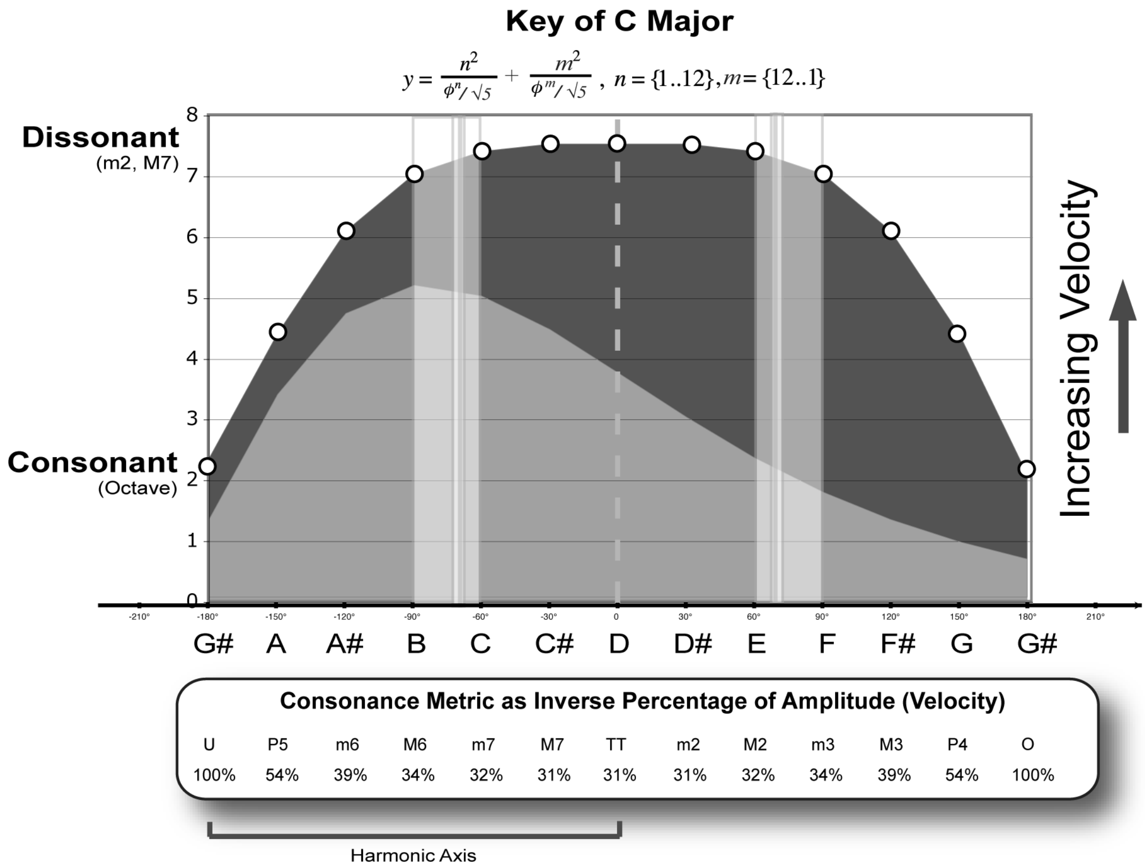
60

**Principle 8: *Harmonic tension*** can be measured as a function of the REFLECTIVE INTERFERENCE pattern. Taking each of the amplitudes as a percentage of maximum resonance from the Leading Tones, we have the following order of tension (greatest to least) in a diatonic scale:

| Diatonic Scale | C Major Scale | Percent |
|---|---|---|
| Leading Tone, Inverse Leading Tone | {B, F} | 100% |
| Tonic, Inverse Tonic | {C, E} | 97% |
| Harmonic Center | {D} | 72% |
| Dominant, Inverse Dominant | {G, A} | 66% |

### Figure 40 - Measuring interval consonance as *REFLECTIVE INTEGRAL*




The next most consonant intervals are then the perfect 4th and perfect 5th just above {G#} on the curve while the next most dissonant intervals are the minor 2nd and major 7th.[61] Once again, this ranking order follows the common practice definitions for consonant and dissonant intervals, only now as the integral velocity distribution of reflected harmonic wave interference over an octave.

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61 **Principle 9:** ***Interval consonance*** can be measured as a function of the *INTEGRAL INTERFERENCE* pattern. Taking each of the amplitudes as a percentage of maximum consonance from the octave, we have the following order of consonance (greatest to least):

| **Interval** | **Inverse Harmonic Center** | **Percent** |
|---|---|---|
| Octave | {G#, G#} | 100% |
| P4, P5 | {C#, D#} | 54% |
| M3, m6 | {C, E} | 39% |
| m3, M6 | {B, F} | 34% |
| M2, m7 | {A#, F#} | 32% |
| m2, M7 | {A, G} | 31% |
| Tritone | {D, G#} | 31% |

You may have already noticed that some of the tone groups in the Consonance Metric are the same as those in the Tension Metric. This is because both are symmetrically oriented around the Harmonic Center. But while the musical qualities of tension and consonance are both measured from top to bottom, the physical qualities being measured are actually complementary opposites. In other words, tension (greater energy flow) is an opposing sensation to dissonance (lesser energy flow) in the same way resolution (lesser energy flow) is opposite to consonance (greater energy flow).

Furthermore, while the tritone ranks as both the most tense and most dissonant interval, there are actually two different tritones we are measuring here. Maximum tension and resolution are measured by the diatonic tritone {B, F} while maximum consonance and dissonance are measured by the tritone {D, G#}, acting as a stabilizing *Harmonic Axis* within an octave. These two tritones are also complementary opposites of one another, each balancing half way between the other inside an octave just as a cosine wave balances midway with its complementary sine wave. Together in phase-quadrature, these intervals form the auditory square of a diminished chord, generally regarded as the tensest chord in common practice music theory.

Perhaps the most interesting complementary opposite relationship between the two models is the fact that the consonant sounding major 6$^{th}$ and minor 3$^{rd}$ intervals in the Consonance Metric align with the tense tritone interval in the Tension Metric. This means that the resonant tritone is recognized as either tense or consonant *depending on the context of the music*. The context would then be defined as two possible cognitive states: 1) as the *spatial* (static) measurement of consonance or 2) as the *temporal* (flowing) measurement of tension. In other words, we must interpret resonance in an interference pattern as either a static integral function or a moving differential function depending on the degree of diatonic movement in a given piece of music.[62]

For instance, when a song moves diatonically through a chord progression the diatonic tritone within the Tritone Function is recognized within a flowing context as both tense and dissonant. But, if the song did not follow the Tritone Function and a chord with tritone elements instead remained static and motionless, the same tones could be recognized as resolved and consonant. This would explain why contemporary jazz and pop music often uses elements of the diatonic tritone to form what are called *major 7$^{th}$* and *suspended* chords. In a static context, we are then able to perceive them as pleasantly soft and plush, seemingly “floating along” as if on a cloud.

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62

**Principle 10:** The ***Principle of Tritone Duality*** is the ability to perceive intervals as either consonant-dissonant or tense-resolved depending on context. During the recognition process, an interval can either be 1) measured spatially as an integral function to produce the sensation of consonance or 2) measured temporally as a differential function to produce the sensation of tension. The choice between the two is apparently determined by the degree of diatonic harmonic movement afforded in the context of the music.

This complementary relationship between static and dynamic perceptions of harmony, named here as the *Principle of Tritone Duality*, is a critical new definition for music theory. Unlike conventional theories that fail to distinguish between the definitions of interval consonance and harmonic tension, the ***REFLECTIVE INTERFERENCE*** and ***REFLECTIVE INTEGRAL*** models separate these concepts into two opposing groups – one describing a static or spatial interference pattern and the other describing a dynamically changing or temporal interference pattern. While they do share the same Gaussian harmonic distribution, our *interpretation* of the pattern appears to depend on whether we hear static tones in isolation or as part of a progression of tones moving through a scale over time. Further, the extent to which a given musical piece reinforces the natural symmetry inherent in the harmonic series affects our ability to recognize such "tendencies" moving one direction or another. This effect could help account for the *Tritone Paradox* and other unexplained tritone experiments by Diana Deutsch.

Using these cognitive metrics to analyze music, we can now identify which intervals are more pleasurable to us in static isolation and which generate the greatest tendency and anticipation when allowed to move or progress. The metrics can also be used to take interval rank percentages as a running average over time to identify mean levels of consonance or tension in a given song. In the same way, melodies could be analyzed over time to determine overall balance, including which sections trigger "perception vectors" for positive or negative emotions. This and other analytics based on these models could be implemented in computer aided music systems to enable real-time analysis during performance, ethos-guided music composition and music visualization for educational purposes. As a complete and beautifully organic system, it all originates in the one simple idea that music harmony is symmetrical and recognized as a *standing wave interference pattern* over an octave.

# *Perfect Damping*

*"In quiet places, reason abounds." - Adlai Stevenson*

When musical tones vibrate they always assume the form of a *standing wave***.** Within the standing wave, energy reflects and resonates in equal and opposite directions, causing the formation of sympathetic whole number harmonic waves. Without the stabilizing effect of standing waves, there would be no sound and we would have no ears. Indeed, we would not even exist without the structuring properties of standing waves resonating at every level of our reality from the quantum level beneath atomic matter to the cellular structure of our bodies.

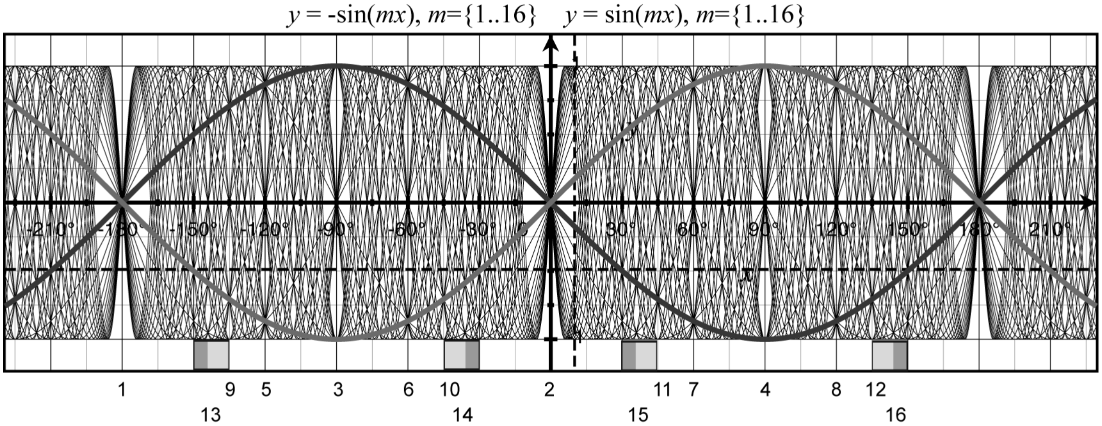
At the same time, without *harmonic damping* to control the resonance of standing waves, atomic structures could not bond, planets would not have formed and life would not have evolved. Unbridled oscillation would have rent asunder any attempt at coherent organization of energy, leaving a cosmos full of free particles with no way to stick together. Fortunately, we find ourselves in a place where resonance and damping are perfectly balanced; producing a harmony we are able to hear with our own ears.

The most important thing to realize about standing waves is there is no room for fractional or enharmonic wavelengths. A standing wave simply cannot be sustained if there is any wave partial that is not a whole number multiple of the fundamental. In fact, the sympathetic vibration that produces a musical tone will cease to exist if such a frequency is introduced. To keep this from occurring, a standing wave naturally polices itself and annihilates enharmonic frequencies that may attempt to form during resonant vibration. The question of course is what mechanism causes this to occur?

The police action inside a standing wave can be found in the proportions located in the *background space* out of which whole number wave frequencies resonate. As we already know, this background is the result of Fibonacci proportions as they converge adjacently to the golden ratio. We can even see how this affects the spacing of gaps that form between the first sixteen partials in a harmonic standing wave.

In Figure 41, gaps form in the standing wave of a single tone at simple harmonic proportions – the locations where enharmonic Fibonacci proportions have been thinned out – thus, enabling the wave energy of the fundamental frequency (shaded heavy lines) to be shared between all of the co-resident wave partials. Without this thinning process, there would be no gaps and the tone would not have the necessary space between whole number harmonics within which to vibrate.

**Figure 41 - Standing wave of the first sixteen harmonic partials**



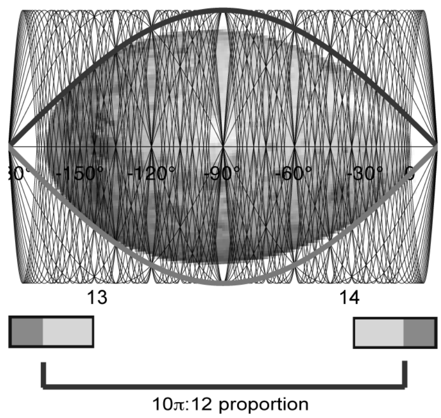

In the diagram, the first 16 vertical gaps are numbered along the bottom according to visual gap size. Unlike the earlier spectral analysis of two tones, the gaps in a single tone follow an evenly divisible pattern of halves, fourths, sixths and twelfths, thereafter shrinking in toward the convergent nodal points located at multiples of 180° on the horizontal axis. The last four gaps, numbered 13 through 16, can be seen to occur at a point inside the π/12 spaced grid lines (at the border in the small shaded bars at bottom). In fact, they occur at about the proportion of a small golden ratio between the grid lines. So why, you might ask, is this important?

Well, beginning with the 13$^{th}$ gap in the harmonic series, each subsequent concordant gap becomes increasingly smaller than π/12. This occurs naturally as harmonic waves approach the denser interference regions near the nodes where resonance damps down to zero. Here the waves become compressed, less resonant and increasingly dissonant, thus creating a dense barrier around each node that captures and *contains* the inner coherence of simple harmonic proportions in the numeric range {1..12}, such as 3:2, 4:3 and 5:4.

In other words, the semi-transparent area either side of the node measuring π/12 in width is a place where enharmonic frequencies cannot resonate as easily, thus fading away to leave mostly clear space around the nodes at 180° (π) multiples. Inside this protective envelope, each wave partial then has the slack it needs to vibrate *in its own space* while sharing the vibrating medium with other harmonics. Outside this sonic barrier, the harmonic waves begin to interlock, dampen, become frozen and ultimately *crystallize*. More than a nice explanation of a musical tone, we can find the same behavior everywhere in nature.

For example, a bird egg can be defined as twelve sinusoidal harmonics encased in an outer damping shell. To prove this point, take the length of the 10π /12 non-damped region in the preceding standing wave as a proportion of its total displacement of 2 to yield the ratio (5π /6) : 2 or 1.3090. This number happens to exactly match another theoretical geometric model of the egg by Carlos Rojas based on cubic proportions of the golden ratio: $\Phi^3 : 2\Phi$ or 1.3090. And, when we cross-reference this number to a survey of 34 species of bird eggs, we find the average length to width proportion to be about the same at 1.3097. As illustrated in Figure 42, the average dimensions of a bird egg closely match the same theoretical Φ-damping proportions of a standing wave.

**Figure 42 - Harmonic damping in an eggshell**




The pressure of Fibonacci proportions converging to Φ inside a harmonic standing wave is the only possible explanation for the physiological structure of the common egg, though how this occurs as a property of DNA has yet to be fully understood. Perhaps Aesop's little fable of *The Goose That Laid the Golden Eggs* (with origins tracing back to the Egyptians!) is not so much about alchemy or gold metal as it is an allegory involving the role of the golden ratio in the shape of the humble egg.

Intrinsic to this wondrous process of perfect damping are the qualities of symmetry, reflexivity and whole number structure of space. But it is the force of natural damping that provides coherence and prevents "overly exuberant" resonance from forming fractional waves. Beginning with the 13$^{th}$ convergent gap at the ratio of about 1.3, the golden ratio acts as a kind of physical container to keep standing waves from simply exploding. But how would modern science explain this strange relationship between "unlucky 13" and the golden ratio.

Ironically, in the field of acoustics, the one thing you do *not* want is a standing wave. In a performance hall or home theatre, acoustic engineers go to great lengths to construct spaces and use materials that act to cancel standing wave reflection so that the original sound and music is propagated cleanly. Since reflection can occur between any two parallel surfaces, it is important to choose proportions and angles that eliminate this possibility. Of course, eliminating reflection is hard to do in a rectangular theatre or box speaker enclosure where every side is parallel and a potential reflector of standing waves.

In the field of amplified speaker design, there is a long-known secret about how to keep unintended standing waves from forming. Sealed box enclosures are always built with interior proportions approximating the golden ratio and its inverse, such as 0.62 × 1.0 × 1.62. These proportions are widely considered anti-harmonic and optimal for eliminating standing waves inside a speaker box. This works because standing waves cannot form when reflected at or near the proportion of the golden ratio or its inverse. Similarly, rooms and auditoriums built in this proportion will not echo and, along with non-reflective materials, avoid extraneous dissipation or cancellation of the original sound energy that can occur in interfering waves.

This idea was further proven in a recent scientific experiment using a square "drum" resonator "pinned" at only two proportional locations Φ and 1/Φ (about 1.61803 and 0.61803). The study confirmed that this ratio was far more efficient at damping a resonant surface than any other location, including 1,000 points along the boundary [Russ and Sapoval 2002].

So why does this work? Because Φ is an infinite irrational number that cannot be evenly divided by whole number ratios to sustain a standing wave and co-resident harmonic partials. As the Fibonacci series diverges from the harmonic series after 13:8 (=1.625), it begins to converge infinitely to Φ (≈1.61803) between harmonic proportions, thereby *Φ-damping* all enharmonic partials in the standing wave. In this way, the golden ratio is the perfect damping force that attenuates physical resonance in nature and enables the formation of whole number harmonic proportions in a standing wave.[63]

If we had to sum all this up in one concept, it might go as follows. The golden ratio acts as a "strange attractor" for the Fibonacci "growth spiral" that then "wraps around" the harmonic series beginning with the thirteenth partial to contain resonance. Wave partials based on Fibonacci ratios from thirteen up die out as they get ever nearer to Φ. Within this natural damping field, the first twelve harmonic waves can then resonate into geometric shapes. One of the great philosophical truths of all time is that the harmonic and Fibonacci series have been working together like this from the very first moments of our universe to create organic structure and imbue it with flexibility.

The role of damping in flexible organic systems can even be found in our own anatomy. A January 2003 article in *The Journal of Hand Surgery* entitled "The Fibonacci Sequence: Relationship to the Human Hand" found that the proportions of the Fibonacci series could describe the equiangular proportions between the *midpoint spaces* of joints in the spiral of a clenched hand. For the first time, this report suggested that Fibonacci proportions might have

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**Principle 11:** The Fibonacci series converging to the golden ratio and its inverse ratio acts as natural **Φ-damping** proportions within the harmonic series to prevent the formation of destructive fractional wave partials.

something to do with the space or gap between bones rather than the more popular notion that it defines the proportions of the bones themselves. Of course, the small gaps in our joints are precisely what give us the freedom of physical movement.

Fibonacci proportions in life act to damp and contain unrestrained harmonic growth, accounting in some way for every gap, crevice, slice or cavity found in any life structure. If we characterize the Fibonacci series as a surgeon, the golden ratio would be its infinitely sharp scalpel. The material silence of these gaps is the very thing that enables our joints to flex and articulate, thus allowing the purposeful movement necessary for survival. Just wrapping the Fibonacci spiral of our hand around an object is itself a damping action upon whatever we grasp.

And while it is usually overlooked, the “silent” space separating harmonics is as important as the energy itself. In our perception of sound, it is the separation or “articulation” between overlapping waves that actually enables us to distinguish one tone from another and maintain our auditory attention level over time. Without these gaps as predictable “signposts,” sound would be an unrecognizable sonic roar. We would not have survived to speak and understand language, much less enjoy the diversity and differentiation required of music. Why is it we never see damping discussed in any theory of music, language or thought?

Once again it appears that the de-emphasis of damping as a complement to resonance is likely due to the prevailing asymmetrical view of nature inherited long ago into science from Western religious doctrine. From at least the time of Pythagoras, the golden ratio and converging Fibonacci series have been interpreted as the source of error and chaos in nature when all along it was the very thing that made life possible in the first place.

The importance of wave damping is substantiated elsewhere in physical science, such as plasma physics. The eminent Soviet physicist Lev Davidovich Landau found that damping of longitudinal space-charged waves in plasma prevented instability from developing and created a region of stability in their intersection (or near intersection). He called this region of stability the *parameter space*.

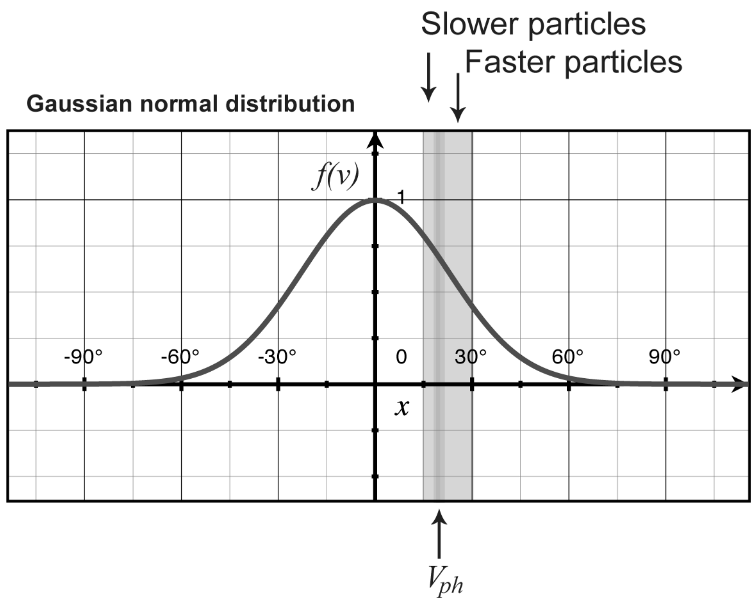
Landau found that particles having velocities *slightly less* than a specific damping location (within a given parameter space) would be *accelerated* by the wave electric field while those particles having velocities *slightly greater* than a damping location will be *decelerated* by the wave electric field, losing energy to the wave. This became known as *Landau damping*.

Landau proved this in an experiment where plasma waves made *adiabatic* (or collisionless) crossings and the particle velocities were defined by a…

> *“Maxwellian distribution function based on the magnitude of a 3-dimensional vector in a Gaussian normal distribution.”*

While this is a mouthful, it says that the number of particles with velocities slightly less than the wave phase velocity were more than the number of particles with velocities slightly greater – thus, more particles will gain energy from the wave than lose. This process continues to create a cascade effect, ultimately leading to wave damping [Hagedorn 1991].

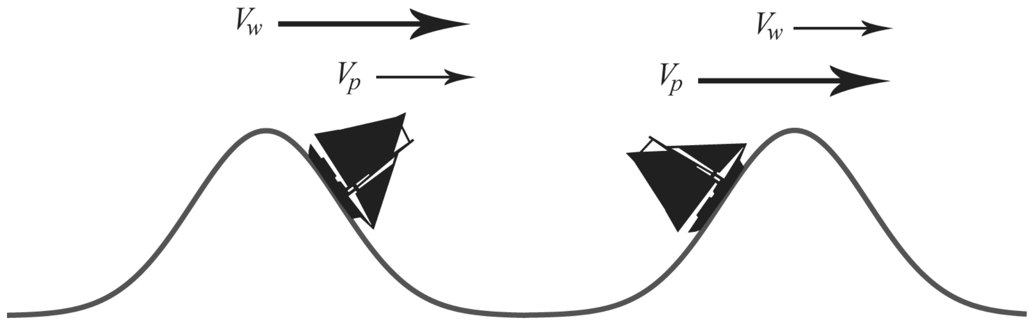
**Figure 43 - Landau damping region in plasma waves**




As an analogy, consider a surfer trying to catch an ocean wave. If the surfer is going in the same direction as the wave, but slightly less than the velocity of the wave, he will be caught and pushed along the wave while gaining energy. However, if the surfer is going in the same direction as the wave, but going slightly faster than the wave, he will be pushing on the wave as he moves uphill on the preceding wave – thus, losing energy. As this is repeated over and over like a pendulum, a gradual loss of energy occurs during the transfer due to friction and heat dissipation around the adiabatic crossing. This eventually brings all waves to rest.

The principles describing "collisionless" damping and energy transfer in quantum dynamics were set forth by *Landau-Zener Theory* in 1932. In this paper, wave energy transfer is referred to as "curve hopping between adiabatic surfaces" and described as occurring across a thin space called a "small eigenvalue gap." In a *Landau-Zener transition*, the probability of energy curve hopping between waves is dependent upon the size of the gap between them and their relative wave velocity. If the gap size is always about the same, as it is in the standing wave of a musical tone, then proximity to the small eigenvalue gap by each harmonic wave partial is fixed and their relative velocity becomes the primary determinant for the probability of energy transfer.

**Figure 44 - Surfers gaining and losing energy on the ocean**




Given the universal nature of wave phenomena, there are a number of correlations we can make between plasma waves and sound waves. Let us now restate the description of Landau damping in terms of highly coherent and predictable harmonic standing waves of a musical tone.

Landau damping prevents instability from developing in any musical tone, allowing sympathetic harmonic wave partials to fold in from the fundamental. The "parameter space" of this damping is then defined as the relatively calm, slack area around golden ratio or *Φ-damping locations*. Any harmonic wave traveling slightly *slower* than another wave crossing near a Φ-damping location, gains vibratory energy and is accelerated. Conversely, any harmonic wave traveling slightly *faster* than the fundamental wave velocity near a Φ-damping location loses vibratory energy and is pulled backward relative to its position. As this process continues, the tone will gradually die out.

Assuming now that the behavior of plasma waves and Landau damping applies to mechanical harmonic sound waves as proposed here, we should expect our perception of consonance and dissonance to be related to the *degree of energy transfer* within the "halo" area surrounding Φ-damping locations. That is, we should recognize and prefer the conservation of energy (as consonance) and dislike the loss of energy (as dissonance), even though alternation between the two serves to provide contrast and a sense of progression or movement.

This energy transfer is actually visible between harmonic wave partials in a vibrating string. When first plucked, the fundamental wave transfers, or loses, some of its energy to each wave partial as the string begins to vibrate sympathetically into a standing wave. Wave partials transfer, or lose, some of their energy back to the fundamental or other wave partials as the medium seeks a resonant equilibrium. This process continues back and forth across Φ and 1/Φ proportions, creating an undulating blur of phase shifts as all wave partials share the singular energy induced into the string. Generally speaking, the same ideas would apply equally to a tube of air, a vibrating bar or any other standing wave found in nature.

Landau damping helps explain our perception of tension and resolution in scales and chords. Those harmonic waves crossing in the parameter space (immediate proximity) of Φ-damped locations in an octave must be pulled or pushed relative to the direction of energy exchanged. From this physical process, our perception of harmonic "tension" must then have its origin in the *direction energy travels* as wave partials gain and lose energy.

Based on the Tension Metric we know that the higher an interval's corresponding velocity the more energy it must represent and thus the more likely it will transfer some of its energy to another interval to achieve equilibrium. As example, the diatonic tritone has the highest ***REFLECTIVE INTERFERENCE*** velocity, thus has the greatest probability of giving up some of its energy. According to common practice, the direction it should follow is across the Φ-damping zones, contracting to a major 3$^{rd}$.

This will be covered in greater depth later, but we can already begin to see how movement toward and across Φ-damping zones must be central to our ability to predict musical harmonies. The likelihood of energy transfer and its direction of flow would be a way of explaining the *anticipation/reward potential* we all feel in harmonic music. In the most general sense, the degree to which a song matches the natural exchange of energy in the harmonic series is a measure of its emotional content.[64]

Landau damping really does offer the best explanation for how a musical tone shares its energy cooperatively between harmonic wave partials. It also provides an accepted scientific model to describe the very real damping role the golden ratio plays in acoustics, our anatomy and any other standing wave system. Within these physical properties is found the true origin of our common practice rules for voice leading and diatonic harmony in music – the directional exchange of harmonic energy in the standing wave of a tone.

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**Principle 12:** The Landau damping principle in plasma waves provides a physical model for energy transfer between harmonic wave partials in a sonic standing wave. Our auditory system appears to judge interval consonance and dissonance based on the gain or loss of energy in corresponding harmonic partials. We also appear to recognize the directional energy flow between wave partials in the harmonic series as tension and resolution, creating a cognitive *anticipation/reward potential* for tones to move toward and across Φ-damping zones. The degree to which a particular harmonic progression in a song matches the natural exchange of energy is a measure of emotional content.

# *Free Space*

*"I view words and images and emotions as carriers of error. And error is potentially a wonderful thing. It's like tolerance in a machine. It's like saying because the piston is a little bit sloppy inside the block of the engine, it's able to do its useful work."*
*- Dwight Marcus, The Chamber of Poets (1999)*

The ancient knowledge of harmonic resonance and damping was a most sacred truth for the builders of Rosslyn chapel in the 15th century. Conceived by Sir Gilbert Haye for William Sinclair, the 1st Earl of Caithness, they built a temple of stone as close to the godhead of creation as they could imagine, carved with musical sound patterns, angelic statues of heavenly resonance, golden pentagrams and Fibonacci dragon-serpents emerging from the silent Underworld.[65] Yet, beyond these harmonic symbols lies an even deeper secret of resonance and damping designed into the very dimensions of the chapel.

Measuring 81' long (inclusive of rear buttresses), 40' 6" wide and 68' high from the underground sacristy to the steeple of the front facade, the exterior dimensions of the chapel exhibit a length-to-width ratio of 2:1 and height-to-width ratio of 68:40.5 equal to 1.679012346. While close to the 5:3 ratio of a major 6th, this last number does not match any simple harmonic proportion. Nonetheless, it still appears close enough to suggest a connection to some harmonic principle.

In *The Hiram Key,* authors Christopher Knight and Robert Lomas proposed that the dimensions of Rosslyn chapel were actually based on an interior room of King Solomon's legendary temple known as the "holy of holies," described in the Biblical Book of Ezekiel. If this is true and Rosslyn chapel really was built to reproduce Solomon's design, what could be so important about these particular dimensions that they would be used in that ancient temple then used again in this 15th century Scottish chapel thousands of years later?

To answer this, we must analyze the properties of the chapel and determine if it could be based on real harmonic principles. To do this, we need to first determine the proportions of maximum resonance and maximum damping in a harmonic standing wave, then look for these in the chapel's architecture. In this way, we might determine if both Rosslyn and Solomon's temple were both based on some kind of harmonic knowledge descended from ancient times through Solomon.

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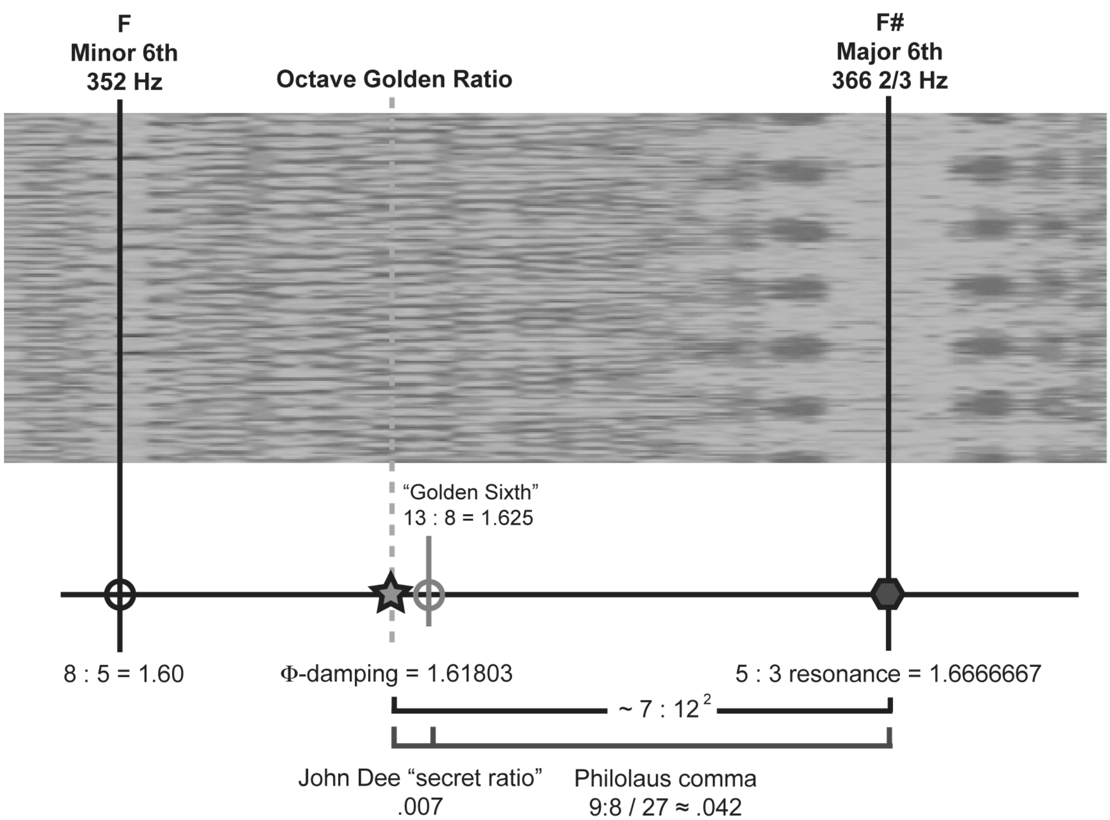
[65] Haye traveled to China (Cathay) and stayed 14 years there to study their culture, subsequently bringing back to Scotland knowledge of Chinese alchemy through the art of gong making. Beginning at least 5,000 years ago, the Chinese gong makers would throw salt or sand onto the gong to tune them to specific cymatic patterns.

At one extreme, we have the interval of maximum resonance: the 5:3 ratio of a major 6th. As shown in Figure 45, the closest enharmonic Fibonacci ratio to this is 13:8 equal to 1.625, which we will call a Golden Sixth. It may come as a surprise to learn that the difference between these two numbers is the Philolaus octave comma (9:8)/27, directly connecting this musical gap to the damping effect of the golden ratio. Starting at 13:8 and ascending to the resonant major 6th (5:3=1.6666667), a region of increasing resonance can be defined within the octave.[66]

**Figure 45 - Spectral interference between major/minor 6ths in an {A} octave**

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66 Maximum resonance R defined from the Fibonacci ratio 13:8 is:

R = (13 : 8) + (9 : 8) / 27 = 1.6666667 // Resonant 5:3 (major 6th)

In the other direction, a very small region of Fibonacci ratios converges toward the anti-harmonic Φ-damping location marked by a star. This width can be calculated by subtracting Φ from 13:8 to arrive at 0.00696666667, which we will round up to .007 as a practical matter. This region represents the small amount of slack or *free space* needed in a standing wave for sympathetic harmonics to form. All other enharmonic waves intersecting this region are suppressed or *silenced* in the .007 between Φ and 13:8.

Pythagorean scholars and alchemists during the 15th century were well aware of the significance of such secret harmonic numbers. They would have known that adding .007 onto the octave golden ratio reaches the proportion 13:8 and that it was a Fibonacci number. They would have also known that 13:8 is located very near *yet another* golden ratio as a fraction between the major and minor 6th intervals. This proportion begins the natural harmonic damping region that spirals inward along the Fibonacci series toward the octave golden ratio. Esoteric knowledge of harmonic principles like this was not uncommon amongst alchemists and natural philosophers in the 15th century, though word of the golden ratio as a silencing action in sound and its central role in harmonic formation were a well-kept secret.

Dr. John Dee, a 16th century polymath alchemist and founder of the Rosicrucian Order in England, actually used the number 007 as his covert signature to Queen Elisabeth I to represent his promise of silence as her royal spy. As we all know, fiction writer Ian Fleming later used this same moniker in the popular James Bond 007 secret agent books and movies. It may have even led Fleming to choose the title *Diamonds are Forever* for one of his most popular books since a one-carat diamond is equal to 200 milligrams or about .007 of an ounce. Whether intentional or not, we can be sure that John Dee would have appreciated this association with his secret harmonic damping constant. After all, everyone knows carbon requires an immense amount of damping pressure to crystallize into the octahedral shape of a diamond.

Using Dee's "secret ratio," we can measure the space between maximum damping at Φ to the point of maximum resonance at a major 6th. We first add .007 to the Philolaus octave comma to obtain the ratio $7:12^2 \approx 0.048$.[67] This result is then subtracted from a major 6th to approximate the point of maximum damping at the golden ratio in an octave.[68]

With this, we have now (re-) discovered the astonishing fact that the proportional space of about $7:12^2$ is what separates the ratios of maximum resonance from maximum damping in the harmonic series, thereby enabling a standing wave to do its useful work. Whatever you want to

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[67] $(9:8) / 27 + .007 = 0.04866667 \approx 7:12^2$

[68] Maximum damping D can be estimated from the major 6th ratio 5:3 as:

$D = (5 : 3) – (7 : 12^2) = 1.6180$ // Φ-damping golden ratio call it – the parameter space, Landau damping, eigengap, slack area or comma – it is all *free space*.[69]

Out of this one beautiful law of physics, whole number harmonics form into the symmetrical organization we see in the ***REFLECTIVE INTERFERENCE*** model. This may be the essence of Solomon's legendary wisdom – a Pythagorean harmonic science that measures the balance between extremes of resonance and damping.[70]

At the resonant end of the $7:12^2$ region lays the wide-open gap of the major 6th. It is this resonant space that Rosslyn chapel's designers may have sought to immortalize as their most sacred truth, Solomon's *holy of holies*.

You might recall that earlier we found 1.72845025 to be the standard deviation for the ***INTERFERENCE*** distribution of harmonics over an octave. In the field of statistics, this standard deviation accounts for 68% of a Gaussian normal distribution and, as applied to an octave, covers about two-thirds of the pitch space around the center. But what statisticians would almost certainly never guess is that the standard deviation for the Gaussian equation was built into a 15th century chapel long before Gauss was even born.

To find it, the standard deviation of the ***INTERFERENCE*** distribution must be converted into the musical language of proportions. We do this by using a major 6th ratio of 5:3 plus a fractional golden ratio of $0.1/\Phi = 0.061803$ to sum to 1.7284. This can be simplified further by approximating the fractional golden ratio term using the rational form 5 / 81 (or ratio $5:3^4$) and recalculate to reach the finite "rational standard deviation:"

**Rational Standard Deviation** = *$(5 : 3^4) + 5 : 3 = 1.728395062$.*

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[69] In classical physics, free space is characterized by the permeability of the vacuum to permit the flow of magnetism and light. This is defined by the constant $4\pi \times 10^{-7} = 12.566370614 \times 10^{-7}$, which can be expressed in terms of the golden ratio as approximately $(7:12^2)\Phi^2 \times 10^{-5}$. In this way, free space is the balance of resonance and damping as either $(\pi : \Phi^2) / 10^2$ or $((7:12^2) : 4)$, both of which when divided by a resonant major 6th (5:3 ratio) approximate the mysterious fine-structure constant $1 / 137.036 \approx .007297$.

[70]

**Principle 13:** The Fibonacci series, converging to the golden ratio Φ, acts as a ***natural damping proportion*** within the harmonic interference pattern of an octave to prevent fractional wave partials from forming while enabling standing wave harmonics to resonate. Maximum resonance and damping locations within the harmonic series or octave may be estimated to four decimal places using these equations:

Max Resonance Ratio $= \Phi + (7 / 12^2) = 1.6666 \approx$ major 6th = 5:3 ratio

Max Damping Ratio $= (5 / 3) - (7 / 12^2) = 1.618 \approx \Phi$ ratio

The distance between these two extremes is equal to about $7:12^2$, composed of the Philolaus octave comma of $(9:8)/27 = 6:12^2$ plus an additional "free space" of $0.006966 \approx 0.007 \approx 1:12^2$ of an octave.

Now, this is where the secret of harmonic formation finally begins to unravel. When we factor this number by 5 we obtain the string of digits 0.345679012. And when we further factor our rational golden ratio term ($5:3^4$) by 5, we obtain the repeating string of digits 0.012345679 which can be represented compactly as the "magic ratio" 1:81 or $1:3^4$.

Adding this "magic ratio" to the resonant major 6th proportion 5:3 takes us *exactly* to the Rosslyn exterior height-to-breadth ratio of 68:40.5 or 1.679012346. Apparently, the science of harmonics was not lost on the keepers of Hermetic wisdom in 15th century Scotland.

**Rosslyn magic ratio** = *(68 : 40.5) - (5 : 3) = 0.012345679… = 1:81 =* $1:3^4$

The proportion 1:81 is actually of great relevance to the study of both number theory and music. This *Rosslyn magic ratio* represents a sort of numerical refraction of our periodic base-10 number system, splitting out all of the digits (except the octave identity digit "8") like harmonics resonating on a guitar string.

The significance of this "magic ratio" is found in how it precisely defines the wide gap of resonance around a major 6th. In particular, the proportional width of the major 6th gap is *exactly twice* the Rosslyn magic ratio. Furthermore, the proportional width from the Rosslyn height-to-breadth ratio to the maximum Φ-damping proportion is almost *five times* this ratio – a kind of Pythagorean pentagram based on the magic ratio.

**Pentagonal magic ratio** = *Rosslyn magic ratio × 5 = 0.061728395*

Subtracting this pentagonal proportion from the height-to-breadth ratio of the chapel actually yields the golden ratio within what John Dee might have considered an "above top secret" ratio of about .00075 – less than half a percent margin of error.[71] Figure 46 shows how the Rosslyn magic ratio measures the halo of resonance around a major 6th while also defining the distance from minimum resonance, as defined by the Pentagonal magic ratio.

The greatest secret of Rosslyn chapel is that the pentagram and its golden ratio are actually designed into its architecture, literally reflecting the symbolic carvings of pentagonal damping and angelic resonance using the most resplendent proportions possible. As Sir Francis Bacon once put it "mysteries are due to secrecy," but this secret goes much deeper than we might imagine – all the way to the very heart of ancient number theory.

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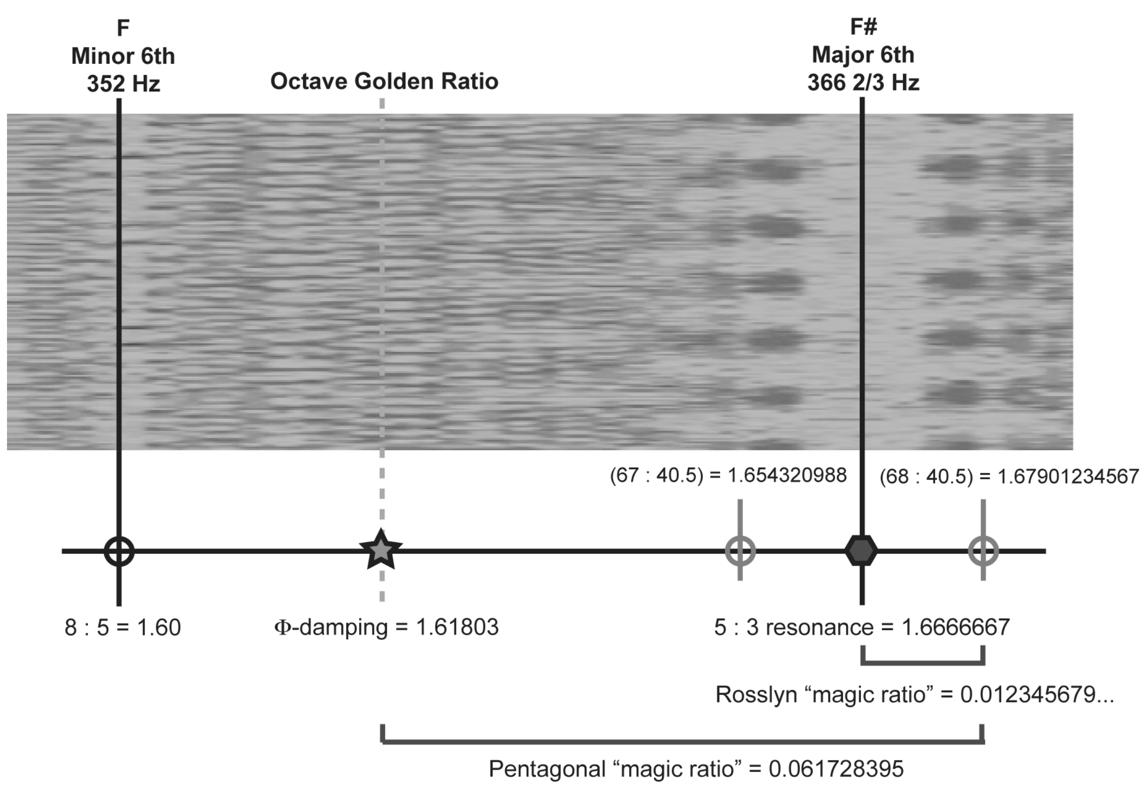
Rosslyn height-to-breadth ratio – Pentagonal magic ratio = 1.61728395

1.61728395 + .00075 = 1.61803395 ≈ Φ

It happens that the chapel's external length of 81 divided by the magic ratio goes exactly 6561 times, which is also the square of 81. Thus, the chapel length of 81 can be factored simply as $3^4$ while its square of 6561 can be factored as $3^5$. Both 81 and 6561 are evenly divisible by the magic ratio of chapel height to breadth. Ok, but why was this important to the builders of Rosslyn chapel?

In the ancient Hebrew practice of gematria, the number 81 is taken as a symbol of change or enlightenment. This association comes from the *Tetragrammaton*, or four letters YHVH (the Hebrew God "Yahweh"), using a tetrahedron represented numerically as $3^2 = 9 = 8+1$. This was then taken as either 81 or its geometric reverse 18 to represent two interlocking tetrahedrons, known as a *star tetrahedron* or hexagram known as the Star of David. In this religious symbolism we find an interesting harmonic balance between addition and division as the sum of the digits of 6561 ("666") total to 18 while its square root is 81. Thus, the magic ratio of 1:81 represented resonance and enlightenment in numbers and all things.

**Figure 46 - The Rosslyn "magic ratio" of resonance**



The stunning clue we find preserved in Rosslyn chapel is the fact that the Hebrew practice of gematria was actually an early form of number theory based on an advanced knowledge of harmonic science stretching back thousands of years. The Rosicrucian Order chose to preserve this knowledge as gematria while the Freemasons preserved it as solid geometry. Of course, few in today's fraternal organizations are the least bit aware of the harmonic physics behind their own rituals and symbols. A few writers, such as Manly P. Hall, do mention that Masonic wisdom has something to do with the "lost keys" of resonance, but little else.

It appears from Rosslyn chapel's special dimensions and symbolic carvings that its builders wished to demonstrate the resonant properties of numbers and explain their physical role in music, perception, geometry and the cosmos. Here are a few more fascinating properties of the Rosslyn magic ratio that will be useful later on.

**The magic ratio multiplied by the chapel's length is Unity.**

*0.012345679… × 81 = 1*

**Every multiple of 3 produces a repeating group of three digits, such as:**

*0.012345679 × 3 = 0.037037037...*
*0.012345679 × 6 = 0.074074074...*
*0.012345679 × 9 = 0.11111111...*

Notice how each adjacent pair of fractions from this series forms a perfect 5$^{th}$ proportion, such as 111/074 = 1.5 = 3:2 which is 0.6666666 when inverted. Beginning to get the picture?

**The magic ratio reflects as an inverse numeric sequence either side of 1:**

*1 - 0.012345679 = 0.987654320…*

The result is now a string of digits in reverse with an 8 but no 1. In this we find a sort of micro-octave relationship either side of Unity.

**In addition to splitting apart the number system, the magic ratio also factors evenly into *any* whole number:**

*1 / 0.012345679… = 81*
*2 / 0.012345679… = 162*
*3 / 0.012345679… = 243*
*4 / 0.012345679… = 324*

*5 / 0.012345679… = 405*
*6 / 0.012345679… = 486*
*7 / 0.012345679… = 567*
*8 / 0.012345679… = 648*
*9 / 0.012345679… = 729*

Note that each dividend ends with the last number being divided and that the sum of the digits either adds up to 9 or 18, which sums to 9 as the Hebrew symbol for YHVH. Additionally, the ratio between each adjacent number is harmonic – thus, the Rosslyn magic ratio effectively splits apart or diffracts our number system like a prism separating colors out of white light.

**The golden ratio has 81 reversed as 18 starting at the 100ths place in its fraction (1.618) and so when divided by 81:100 the golden ratio is approximately equal to a 2:1 octave ratio, with a bit of free space remaining.**

*Φ / .81 = 1.9975728 ≈ 2 = 2:1 octave ratio*

Following this trail we are led to the realization that an octave could be evenly divided into 81 arithmetic steps based on the Rosslyn magic ratio of n/ 40.5 where n = {1..81}. As a special kind of equal temperament I call *81 Arithmetic Equal Temperament* (or *81-AET),* this division of the musical octave balances some of the advantages of Equal Temperament with the consonance of Just temperament. When you consider that it fits proportionally with the chapel's exterior dimensions, this temperament was perhaps a way to harmonize the chapel with the outer environment of the Earth, thus creating a sacred space.[72]

**The golden ratio also generates a periodic repeating octave when taken as a ratio of its first four digits 1.618:**

*1.6 / 0.018 = 88.8888888…*

**And when we substitute 81 for 18, we can diffract all ten digits into an alternating odd-even configuration:**

*1.6 / 0.081 = 19.75308642*

It seems clear that Rosslyn was intended to preserve the wisdom that everything exists as a balance between two opposing forces – odd and even, resonance and damping, foreground and

72 See Appendix for a definition of 81-AET.

background and other such dualities. The ancient Chinese, Egyptians, Babylonian and Brahman priests all understood the relationship between harmonics, numbers and geometry. Their "gnosis" was the fact that the structure of nature comes from a simple harmony of opposites, just as quantum physics presumes. And they knew this knowledge can be accessed directly by everyone through music as it triggers the emotions at the center of music perception.

As an "angelic" symbol of resonance in Rosslyn chapel, the magic ratio demonstrates how space itself must also resonate by powers of three. By simply changing one operator, the magic ratio transforms from an exponential axis as 1:(3 ^ 4) = 1:81 to a multiplication axis as 1:(3 × 4) = 1:12, thus returning us to the balancing point of twelve in the ***INTERFERENCE*** function. In gematria these two axes become 1/9 and 1/3 respectively – both fractional powers of three.

As a final and fitting tribute to the magic ratio, Rosslyn chapel's proportion 68 : 40.5 can also be multiplied by the golden ratio to reveal the number that may well have led to the discovery of logarithms and modern statistics.

$$\Phi * (68 : 40.5) \approx 1.618033 * 1.68 = 2.71829 \approx e$$

This is, of course, very close to the base of the natural logarithm $e = 2.71828$ (within a margin of error of just 0.0005%) as used in conventional Gaussian equations. Given the apparent preexisting knowledge of 1.68 as a common factor between $e$ and $\Phi$, we have to wonder if Gauss, a deeply religious man, might have used $e$ in his equations to avoid offending the Church. After all, both constants share the common Rosslyn proportion of about 1.68, which Gauss could have used to avoid revealing the physical damping role of the golden ratio. We will never know for sure, but even if Gauss did not intend to hide the golden ratio, it was already buried in the system of logarithms he used.

In point of fact, the implied logarithmic constant $e$ in the dimensions of Rosslyn chapel *far predates* even the official discovery of logarithms in 1618 by Scottish mathematician and alchemist John Napier.[73] With no historical record to tell us how Napier came to learn mathematics, we can only wonder if his teachers were members of the same esoteric Scottish group that built Rosslyn chapel – just a few miles away from Edinburgh where Napier lived. After all, Napier's father was a disciple of Dr. John Dee and shared his interest in Kabbalist angel magic, as suggested in the musical columns of the chapel. Perhaps it was Napier who first chose

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[73] It is a great irony that logarithms were discovered in 1618, considering the obvious relationship to the golden ratio of about 1.618. It is no less ironic that the Catholic canon law forbidding the tritone was enacted in 1234, given its numerical correspondence to the resonant Rosslyn magic ratio of about 0.01234. Similarly, was Friday 13, 1307 chosen to eradicate the Knights Templar because of the role 13 plays in harmonic damping, therein symbolically killing harmonic knowledge? Are these all examples of dates being fixed harmonically to color history or only a coincidence of high strangeness?

to encode his system of logarithms with e instead of Φ in order to keep his knowledge of harmonics a secret.

So, just how long has all this been known? How old are these secrets?

According to the Old Testament and other Hebrew literature, King Solomon was born in Jerusalem around 1000 BC and reined over Israel from about 970 to 928 BC. He was said to be the *thirteenth* son of David and second king of Israel who slew the legendary giant Goliath. Conceived under adulterous circumstances with a beautiful woman by the name of Bathsheba, David's 13$^{th}$ son Solomon was very unlucky indeed. Solomon's scandalous birth cost David an alliance with Bathsheba's grandfather, who viewed Solomon's conception out of wedlock as a sin and so withheld his support during a crucial rebellion.

Known as the "last great sorcerer" and a very wise man, Solomon was said to have ruled for 40 years. He collected 666 talents (27 kg) of gold with which he built many buildings, including a grand palace (that took 13 years to build) and, of course, his famous temple. He was very powerful, commanding 14,000 chariots and 12,000 horsemen. As the legend goes, Solomon died standing up leaning against his cane, collapsing only after a "worm of the Earth" invisibly chewed it apart. It was said the worm represented knowledge of an invisible force long forgotten.

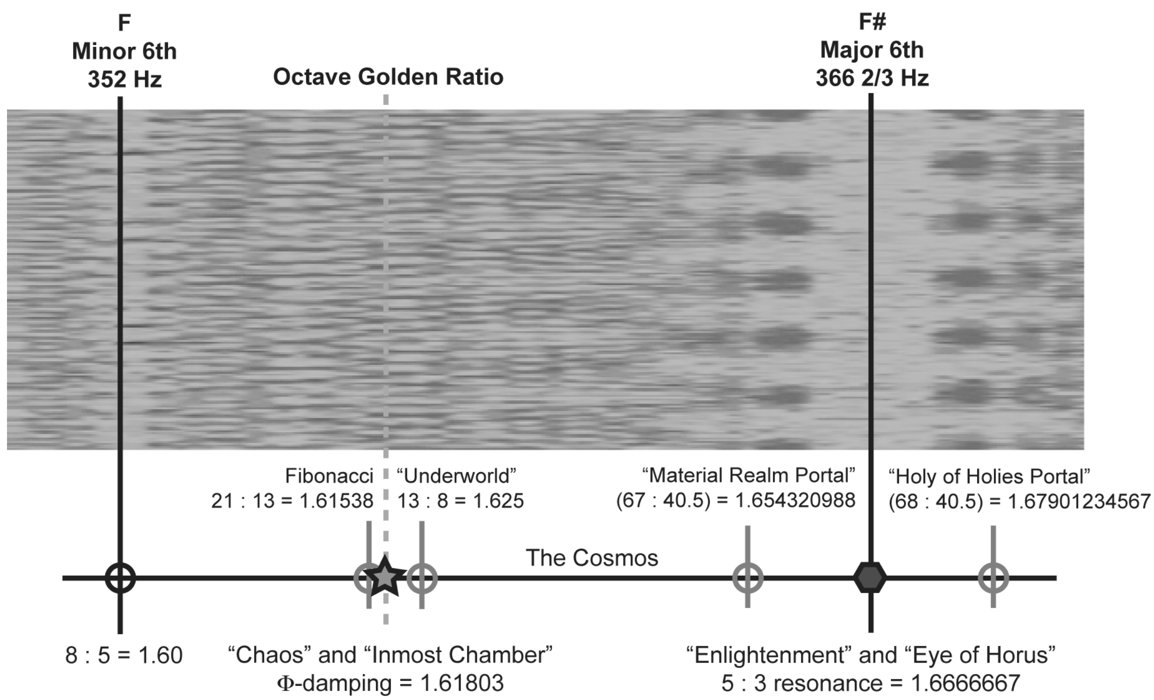
Whether historical fact or mythological symbolism, we find in the story of Solomon the significance of the anti-harmonic number 13, the harmonic force of 12 and the resonance of 666. We can understand the invisible worm that ate Solomon's cane to be the same dragon-serpents of Rosslyn chapel chewing upon its pillars, rising from the Fibonacci Underworld to damp harmonic structure. Together with the "sacred geometry" of Solomon's temple, we can piece together the secret of an ancient theory of the cosmos derived from harmonic principles. A theory that eventually became the Pythagorean *Music of the Spheres*, describing everything – atoms, plants, animals and planets – as a balance of harmonic forces.

In Figure 47, the balancing action of nature is again expressed in the interference pattern of two tones diverging from a minor 6$^{th}$ {A, F} up to a major 6$^{th}$ {A, F#}. Between these two points lies the Octave Golden Ratio bounded on both sides by Fibonacci ratios 21:13 and 13:8 – both factored by "unlucky 13." Opposite this Landau damping region is the resonant "666" gap of a major 6$^{th}$, opening symmetrically by the distance of the Rosslyn magic ratio. In this way, the interference pattern represents maximum damping, maximum resonance and a number of theosophical associations with harmonic science.

First off, the upper or rightmost boundary in the pattern defined by the proportion 68 : 40.5 (the 1.68 factor separating e and Φ) was most likely associated with Solomon's "holy of holies" as a symbolic portal to "Heaven" – a path of enlightenment into the upper part of the octave. The lower side could then be interpreted as an entrance to the material realm of Earth and the rest of the cosmos as it extends down to the "Underworld" boundary and "inmost chamber" of the golden ratio.

This idea can actually be found in Rosslyn chapel as two paths – one as a high altar (representing Heaven) and one down a flight of stairs at the rear into a sacristy (representing the Underworld). It suggests the entire chapel was intended to harmonically represent the space between the two extremes of a resonant "Heaven" and a damped and dead "Underworld."

**Figure 47 - The Hermetic symbolism of harmonic science**




The symbolism further suggests that one must pass through the perfect resonance of the holy of holies to reach enlightenment. And like the legendary Masonic all-seeing eye floating above its pyramid base, Rosslyn's magic ratio portal located just above the major 6$^{th}$ could have inspired the original symbolism of the resonant "eye of Ra" or "eye of enlightenment" floating above the chaotic material realm of Earth. The earlier analysis in Figure 36 tracing the Pythagorean Triangle striations around the major 6th supports this notion, since the gap does cut off the tip of the triangle or pyramid as found on the back of a U.S. dollar bill. In this way, we could easily conclude that the Rosslyn magic ratio location above the resonant major sixth and Solomon's holy of holies are one in the same – represented proportionally in the chapel by the high altar.

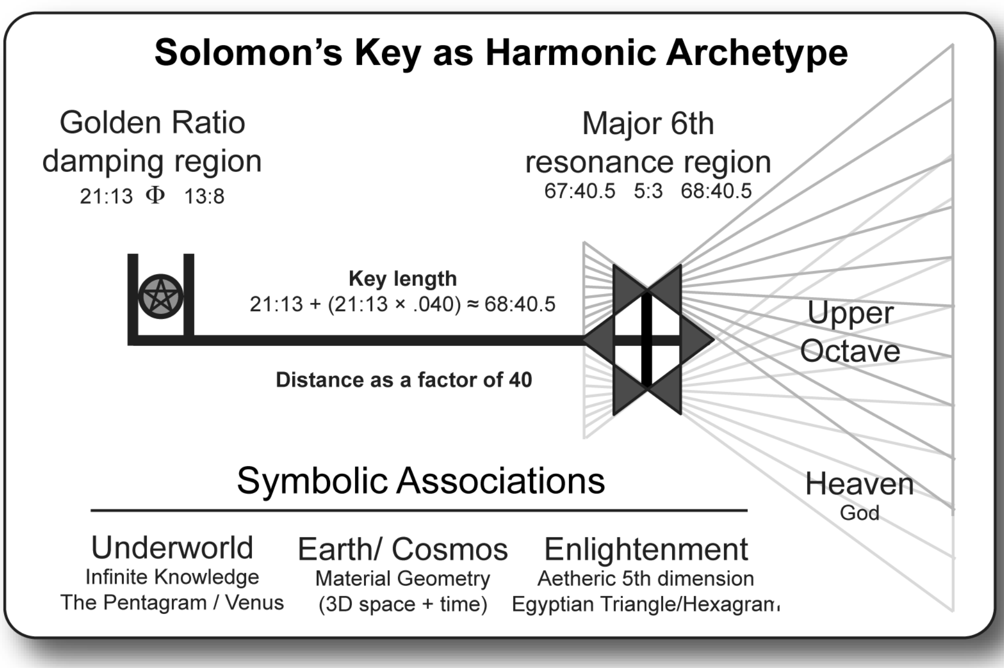
Harmonic proportions can be found to extend into other religious symbolisms as well. King Solomon's 40 years of rule, together with the legendary 40 days of the great flood, Moses' 40 days on the mountain with God (twice) and the Israelites 40 days in the wilderness looking for the promise land – all are likely symbols for the maximum distance between damping and resonance in an octave.

In Figure 48, this philosophical unit is found as a factor of 40 thousandths between the lowest damping ratio of 21:13 and the highest resonance ratio of 68:40.5 (≈1.68). As perhaps the original meaning for the *Key of Solomon*, the resonating Star of David hexagram around a major 6th reaches 40 long units to the 13:8 Venus-Earth pentagram, "unlocking" the infinite golden ratio.

Taken together in the shape of a key, these proportions symbolize the idea of freedom in nature. This is the free space created by the damping action of the golden ratio as it physically "unlocks" the resonance of the major 6th. Of course, anyone symbolically holding the key fob would have found the "lost key" of Freemasonry and attained Solomon's secret knowledge.

As a harmonic archetype, Solomon's Key offers freedom of movement, freedom of choice and freedom of thought. This was the ancient wisdom in revival during 15th and 16th century Europe, a time when freedom of thought was so controlled by the Church. As its highest mission, Rosslyn chapel represented free will and the birthright to seek truth in nature. The key to this freedom – the key to unlock the inmost chamber of harmonic knowledge and the key to the real meaning of *musica universalis* – was built into the very architecture of Rosslyn chapel.

**Figure 48 - Symbolic associations with harmonic proportions**



It seems there is no shortage of symbolism, secrecy and sorcery to be found in harmonic science. The architecture and carvings in Rosslyn chapel represents the highest Pythagorean ideal that everything in the cosmos is a form of music. This was the essential knowledge handed down by a long line of thinkers from more than 3,000 years ago, though still well hidden behind the curtain of Western religion and modern science.

During the course of my research, I corresponded with a top Masonic researcher and educator identifying himself as one of 40 Living Fellows of the Philalethes Society, the world's largest Masonic research organization. I was surprised to learn he was adamant that Rosslyn chapel was not at all associated with Freemasonry. He also dogmatically believed that any attempt to connect Freemasonry with Hermetic currents prior to about 1600 AD is "purely fanciful and has been debunked many, many times."

In our correspondence, there was no acknowledgment of any role of harmonic science in modern Freemasonry and, I might add, a determined lack of interest – even hostility – in considering any such notion. The point was made very clear in this fellow's challenge to:

*"Show me a single ritual where ANY type of harmonic proportions are presented."*

Of course, Masonic triangles, pentagrams and hexagrams – including the hexagonal Masonic emblem formed by the intersecting square and compass – are all examples of harmonic proportions and ideals. To the extent these geometries are used in Masonic rituals, who can deny that harmonics are indeed presented on a regular basis, even if their true meaning is a mystery? Indeed, the 33 degrees of Masonry is itself a representation of angles in a circle and polygons.

Yet according to this fellow, any harmonic philosophy underlying such geometries has no place in Freemasonry. He claims these symbols represent only mundane architectural objects and that only five percent of the five million Freemasons in the U.S. would have any interest in such esoterica. These things, he says, are seldom if ever discussed in a meeting. If we are to believe him, we must conclude that mainstream Freemasonry has largely forgotten its roots in *musica universalis* and harmonic science.

Fortunately, awareness of the importance of harmonic principles has been accelerating outside the secret brotherhoods, fueled by easy access to information on the Internet and an accelerating flow of books about ancient cultures and quantum physics. In stark contrast to modern Freemasonry, there appear to be quite a few curious souls who do consider it important that an old harmonic science once unified astronomy, arithmetic, geometry and music into a single worldview, raising us out of the Dark Ages into the Renaissance. A few must also find it of no small consequence that these harmonic proportions are built into everything and thus everything can be described in musical terms. Surely some can agree that harmonics offer an essential knowledge upon which to build meaningful philosophies and great civilizations.

# ***Harmonic Engine***

*"If I were not a physicist, I would probably be a musician. I often think in music. I live my daydreams in music. I see my life in terms of music." - Albert Einstein*

We find ourselves now caught between the worlds of ancient mythology and modern rationalism. If we ever hope to rejoin the two, we must find a way to rejoin natural philosophy with hard science. The mythological Underworld must be grounded in observable physics while the institutions of Science must recognize the musical attributes of geometry. As the foremost thinkers behind these two worldviews, Pythagoras and Fourier must also join hands to mend the path of knowledge – split apart for so long – into a seamless and consistent understanding of natural science and harmonic philosophy.

To this end, the engine behind harmonics lies not in stories and mysterious symbols nor in cold mechanics and abstract formulas, but in the universal physics of a standing wave. Cracking open this bundle of energy should reveal both the mechanics at work in music and the source of our perceptions of it. Like the fingerprints of our brain, standing wave patterns hold the essential clues for how we perceive our world at the very deepest level.

But decoding these patterns is not easy. It demands very detailed detective work. It requires that we start with the smallest possible pattern of vibration – a *single cycle of a single tone*.

To prove that musical perception has its origin in the structure of a standing wave, we need to compare the interference pattern of one tone with the pattern of two tones to determine if there is a shared physical architecture common to both instrument timbre and musical harmony. If we find the same tension and consonant metrics for timbre that were established earlier for intervals in an octave, then we can rightfully claim that the interference pattern of two or more tones is a kind of *amplification* of the interference pattern of a single tone. From this, we could conclude that timbre and harmony are not perceived separately, but instead as two instances of the same physical system. Among other things, this would suggest that music perception occur as *layers* in a single overarching pattern of standing wave interference.

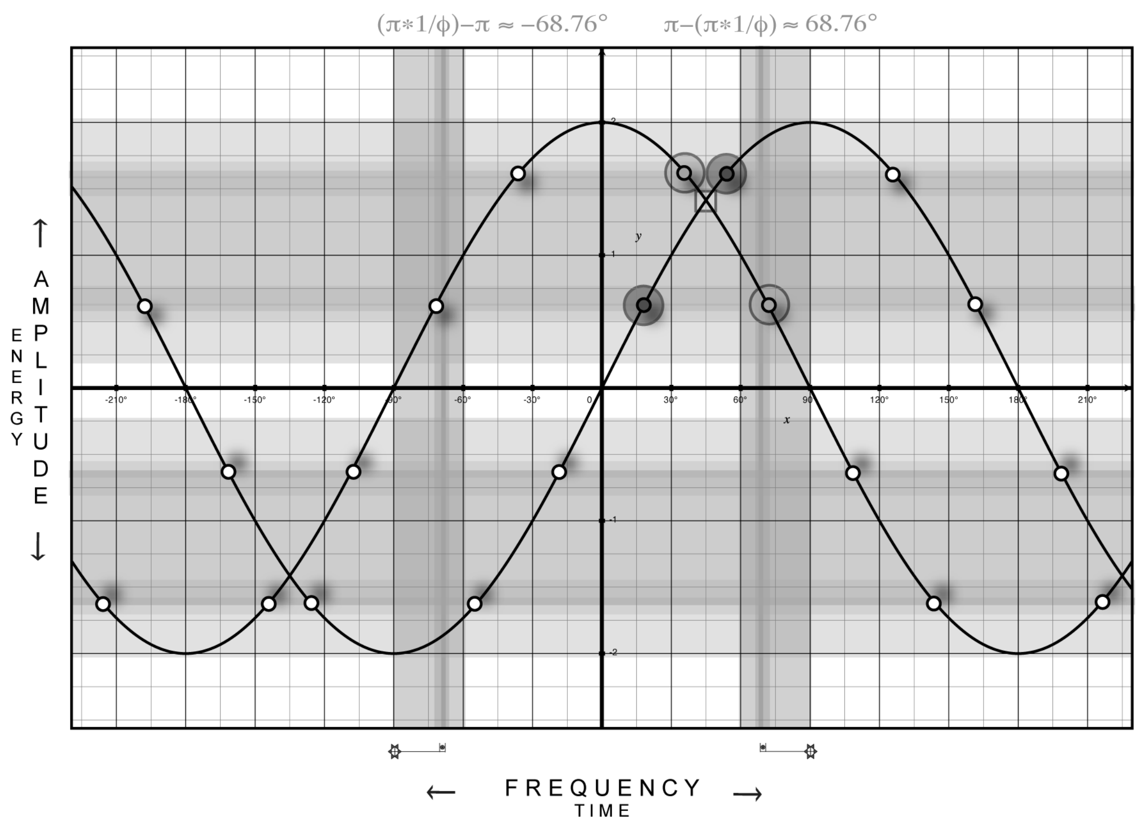
Now, in order to measure sound as our ears and brain do, we must analyze harmonics as separate superposed waves rather than a single composite wave. The reason for this is our eardrums and middle ear bones deconstruct incoming composite waves into transverse component waves in the inner ear, so the brain receives only component patterns and proportions. By analyzing harmonics individually, we can identify proportions of resonance and damping between intersecting harmonic wave partials and more accurately depict how sound is interpreted in the auditory cortex. From this data, we can develop a comprehensive *psychoacoustical* explanation for music cognition that couples the external wave phenomena of timbre and harmony to human auditory physiology. While this procedure is not a particularly easy exercise, it is absolutely essential in understanding how we can easily recognize something as complex as music.

Consider now in Figure 49 two superposed sine and cosine waves with equal amplitudes representing the fundamental frequency of a musical tone. As we saw earlier, this is the standard Fourier method that describes all sound as odd-even wave components in phase-quadrature.

**Figure 49 – Golden ratios locations in a single cycle of a musical tone**

$$y=2\cos(x) \textit{ and } y=2\sin(x)$$

$$\Phi = 2\,sin\,3\,\frac{\pi}{10} = 2\,cos\,\frac{\pi}{5} \;\textit{ and }\; \frac{1}{\Phi} = 2\,sin\,\frac{\pi}{10} = 2\,cos\,2\,\frac{\pi}{5}$$



The first thing to look at in the figure is the point of symmetry where the odd and even wave components intersect one another. A small square marker indicates this location. In a normalized wave, this intersection point tells us where resonant energy is greatest since it adds up to the peak amplitude[74]. It creates the most identifiable phase inflection point or "fringe" in a harmonic wave and is a key element in our recognition of sound.

Our primary interest in this fringe will be to determine how close each harmonic partial comes to this resonant location because it will tell us how much relative energy each wave contains. Since these intersections are repeated at π intervals and are the only common points between sine and cosine components, the square marker will be designated as *π-resonant* ("Pi resonant"). As individual harmonic waves then intersect or balance around these peak "π" multiples, we will classify them as either *π-symmetric* (balanced around) or *π-aligned* (intersecting) at the square marker. The closer a harmonic comes to these locations, the greater we can say is its energy flow or <u>*timbral tension*</u>, which can then be compared to the Tension Metric.

Now, acting against these resonant intersection points as stabilizers are two *frequency* Φ-damping proportions at ±68.76° either side of the center. These are highlighted as before with dark lines inside the two vertical bars and spaced around the middle of the waveform just like those in an octave. In addition to these horizontal locations, we can identify sixteen vertical *amplitude* Φ-damping locations (indicated by small White dots) spaced throughout the waveform. This group represents all of the possible variations of the four trigonometric identities for Φ and 1/Φ. In combination, the 2 frequency and 16 amplitude damping locations identify 18 points where wave energy is most stable and thus most likely to be transferred to another wave.

Since intersecting any of these damping locations would create a deadening effect within the standing wave, we can always determine the loss of energy flow between waves by measuring how close each harmonic comes to one of these damping points. Those harmonics furthest away from damping locations[75] resonate the most while harmonics closer to damping locations exhibit less free resonance, sounding more "pinched down." In this way, the damping points in a standing wave act as a kind of template through which wave energy can *extrude*, producing harmonics that range from partially Φ–damped to fully Φ-aligned. We will refer to the degree of resonance relative to these damping locations as its <u>*timbral consonance*</u> for direct comparison with the Consonance Metric.

When we next consider where damping must be greatest, we are logically drawn to two specific regions where frequency and amplitude damping points are closest to one another. Not surprisingly, this special region occurs near one-fifth of a cycle (the angle of a pentagram) and

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[74] A wave's amplitude is its amount of resonant energy or volume.

[75] We can identify these anti-harmonic locations as opposites to the first 16 harmonics over four octaves and the first 16 gaps found in a harmonic standing wave.

encompasses a domain difference of 72° - 68.76° ≈ 3.24°. To keep track of this region, a thin vertical medium-dark bar in the figure is used to demarcate it.

While this does not appear significant at first, when we divide 3.24 by 180° to determine its proportion in each half-period, we find the resulting value 0.0180 to be the exact same proportion five orders of magnitude down in the waveform. In other words, since 1.80 is the square root of 3.24, the region between the closest amplitude and frequency-damping locations represents a miniature recursive half-period or "half twist" embedded deep inside the standing wave of a tone. A standing wave then resonates up and around these narrow gaps like a swirling tornado or, as they might have said in the 15th century, like a dragon-serpent coiling itself around a golden pillar.

In other areas of wave physics, this kind of gap is described as an "eigengap" where a damping "well" or "envelope" forms naturally to create stability in both time and space. It is here where we expect to find the source of our Landau "parameter area" – the place where waves have the greatest probability of "avoided crossings" and where energy from one wave can most easily transfer to another. Surrounding these Landau damping wells is then a region highlighted in the diagram by the wide vertical bars. This represents the same proportional damping region between a minor 6th and major 6th in an octave and so can be measured using the Solomon Key model.

Already we can see that timbre and harmony happen to share the same resonance and damping proportions at π and Φ locations. From here, we need only prove they both produce the same effect at the micro level of timbre as they do at the macro level of harmony. To do this, we will now proceed to rank individual harmonics by proximity to these locations and then compare the result to our earlier interval metrics to see if they match. If they do, we will finally know for certain if our perception of multiple tones results from the same balance of resonance and damping inside the interference pattern of a single tone.

So now, given the fact that the SuperTonic has already been determined as the center of symmetry in a diatonic scale, we will begin there by analyzing the corresponding harmonic wave known as *Partial 9*. Again, our goal will be to determine how close Partial 9 comes to π-aligned and Φ-aligned locations on the fundamental.

In Figure 50, Partial 9 is shown superposed over the fundamental wave diagram. After a review of all marked locations, we find that it aligns perfectly with all sixteen of the Φ-damped dots and both square π-symmetric locations on the fundamental. This diagram proves that harmonic Partial 9 *physically* shares the same 18 resonant and damping properties with the fundamental frequency (Partial 1). It tells us that Partial 9 is perfectly symmetric and in exact phase alignment with Partial 1 just as the SuperTonic is a point of symmetry in a 7-step diatonic scale. This 1:9 proportional relationship is represented numerically by the repeating fraction 1 / 9 = 0.1111111, splitting one "1" into an infinite number of copies of itself.

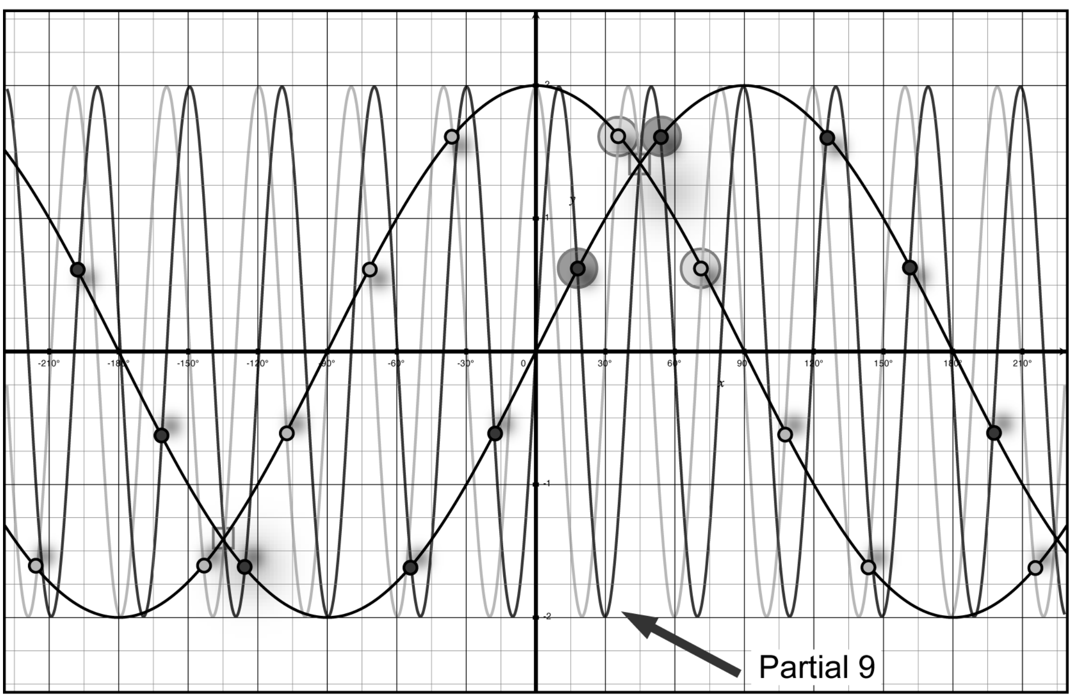
We can deduce from this that Partial 9 is the frequency that divides the fundamental into additional harmonic waves, known as *heterodyning*. It does this by resonating with the fundamental, which can be represented mathematically by squaring 1/9$^{th}$ into 1:81. Of course, this is the Rosslyn magic ratio 0.012345679, which numerically represents the harmonic series if we add back in the octave identity. As a physical embodiment of this, Partial 9 acts as a resonant point of symmetry or *Harmonic Center* in the harmonic series which other harmonics orbit like a miniature solar system.[76] Corresponding to this, the SuperTonic acts as a Harmonic Center in a diatonic scale to create a sense of stability and consonant tonality in music. This can explain why we so often hear the SuperTonic in ninth chords as the "ninth," producing the pleasing and calm sensation characteristic of Jazz and "easy listening" music.

**Figure 50 - Partial 9 as the stable Harmonic Center**

$y$=2sin(9$x$) *and* $y$=2cos(9$x$)

$y$=2cos($x$) *and* $y$=2sin($x$)

$\Phi = 2\,sin\,3\,\frac{\pi}{10} = 2\,cos\,\frac{\pi}{5}$ *and* $\frac{1}{\Phi} = 2\,sin\,\frac{\pi}{10} = 2\,cos\,2\,\frac{\pi}{5}$

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**Principle 14:** Harmonic Partial 9, corresponding to the SuperTonic, is fully π-symmetric and Φ-damped relative to the fundamental (Tonic). This tone-to-octave relation is given the label of *Harmonic Center* as a special point of balance in the harmonic series.

The SuperTonic and Partial 9 together create a Harmonic Center for both harmony and the timbre of a single tone, aligning perfectly with all of the resonance and damping locations of the fundamental Tonic. While this is probably a new idea for most – even for the most advanced musician or acoustician – the knowledge of “9” as a resonant number has been around for a very long time.

There is little doubt that Pythagoras, and certainly his teachers, understood resonance as it related to nine. For them and later followers of Hermetic and Kabbalistic traditions in the Middle Ages, spirit and structure in the cosmos were believed to unfold by even powers of three something like this:

| ***Harmonic proportions*** | ***Resonant geometry*** |
|---|---|
| *3* | *The Origin* |
| *$3^2 = 9$* | *Resonating plane in 2-D space* |
| *$9^2 = 81$* | *Resonating cube in 3-D space* |
| *$81^2 = 6561$* | *Resonant hyper-cubic motion in 4-D spacetime*[77] |

This 3-fold spatial geometry can be found in the Rosicrucian “rosy cross” symbolism of a rose in the center of a cross. It begins with the six sides of Metatron’s Cube unfolding into a flat cross that is 3 squares wide by 4 squares tall, revealing a five petal rose at the intersection. The rose, with its spiraling groups of five petals, represents the *sub rosa* secret that life unfolds from a cube into a pentagonal geometry according to the golden ratio (see Luther’s symbol). The cube represents resonance while the hidden rose represents damping.

The idea of nine as a creative force descended to us from ancient times. For instance, the Ancient and Baladi Egyptians believe that the universal energy matrix consists of “nine realms” composed of seven heavens and two earthly realms. The Chinese associated nine with the dragon, which symbolized the balance between 9 × 9 as “yang” (masculine or square) and 9 × 4 as “yin” (feminine or round). In the Forbidden City, the circular altar platform of the *Temple of Heaven* has one circular marble plate in the center, surrounded first by a ring of 9 plates, then by a ring of 18 plates, etcetera, to produce a total of nine rings with the outermost having 81 plates. Even Indian Hindus use nine in their religious ceremonies, including the nine main *Siddhi* (or devotions) and Nine Devine Nights that mark the advent of winter. All of these ideas made their way into Pythagoras’ *musica universalis*, translating into the nine concentric spheres described in Dante’s *Divina Commedia.* This ultimately evolved into the idea of a cat having “nine lives” and the ever-popular metaphor for happiness known as “floating on cloud nine.”

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[77] It is no coincidence that 81 cubed is equal to the numerator of the Pythagorean comma 531441/524288.

Of course, in the modern worldview of wave mechanics and statistical distributions, such theosophical symbolisms have no place. Harmonics are viewed simply as the mechanics of a standing wave vibrating around Partial 9. No attempt is made to connect the mechanical actions of nature with philosophy – just the facts. While I applaud the rigor of scientific investigation, I cannot help but ask myself which explanation imparts more wisdom and a more complete knowledge of the world? How deeply shall we choose to view nature?

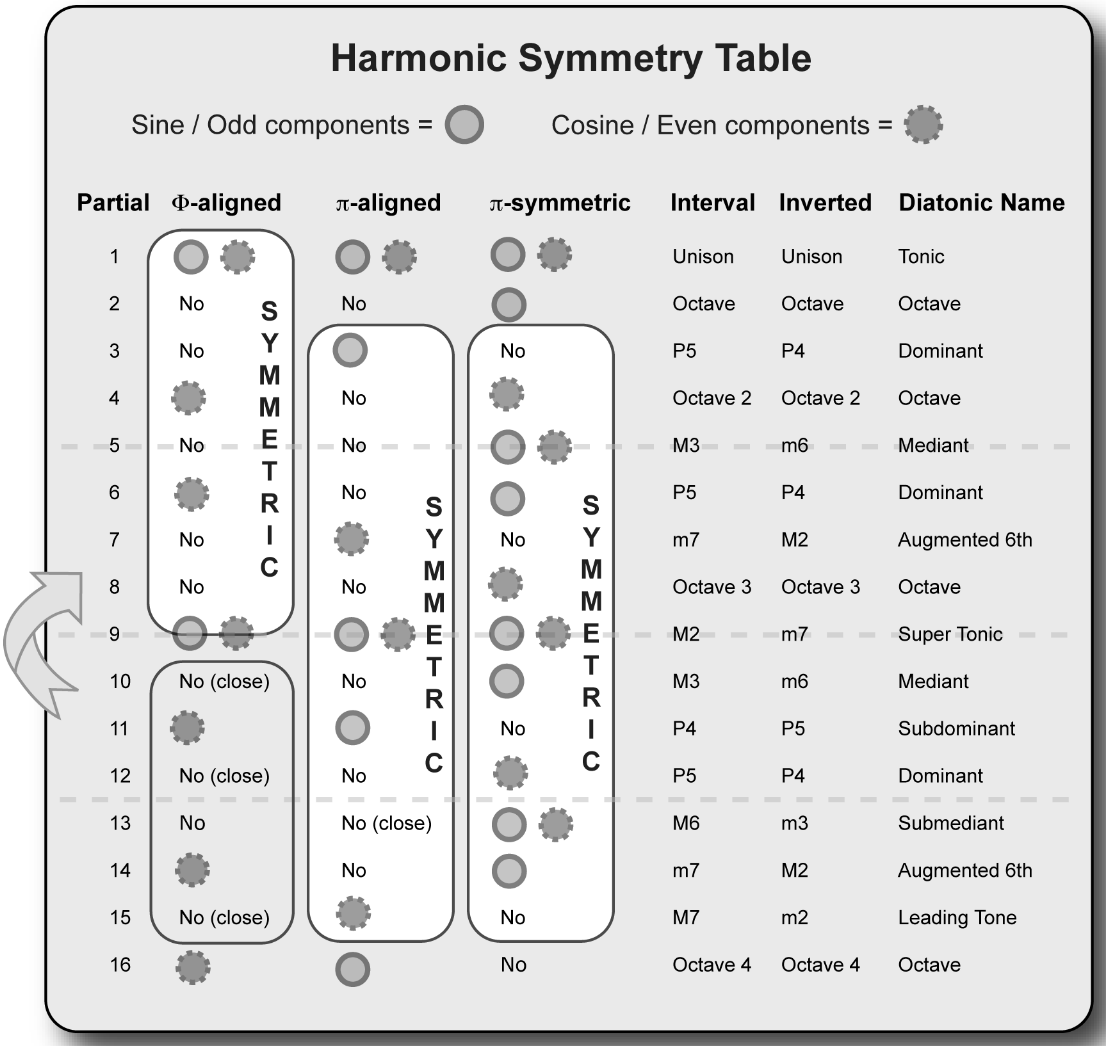
Deep down inside a standing wave lies a most amazing balancing act of harmonics. As they orbit around Partial 9, each in their own space, they exhibit symmetry across a range of π- and Φ-alignment values on either side. We can actually see this for ourselves by using a mathematical graphing program to plot each harmonic frequency against the fundamental, then measure to determine how closely each aligns to the peak resonance and damping points. Of course, this takes a lot of time and space, making it impractical to include this here. However, the end result can be simplified and tabulated as shown in the *Harmonic Symmetry Table* in Figure 51.

This table lists each harmonic partial, with its corresponding octave interval and diatonic name, as well as symbols indicating how it ranks in terms of Φ-damping or π-resonant proximity. For convenience, the Fourier convention of odd and even components is used respectively to indicate sine and cosine trigonometric waves.

Right away, the synchronous phase alignments of Partials 1 and 9 are clearly indicated in the center of the table by identical sets of light and dark dots symbolizing odd and even wave alignment in each column. In orbit around Partial 9 we also see a pattern of symmetry in the π-symmetric and π-aligned columns, as we would expect for a periodic standing wave. But in the Φ-aligned column there is a different balance of the first nine partials around Partial 5 (something like a cube containing a rose!) with another group symmetry in the next six partials between Partials 12 and 13. What could this mean?

Taking the puzzling Φ-aligned column first, if we fold the bottom group over to fit inside and centered within the top group, then all of the harmonics can be seen to balance around Partial 5. As discussed earlier, the golden ratio and ***INTERFERENCE*** functions receive their irrationality from the square root of 5. Here we see this irrationality expressed in how harmonics damp symmetrically around Partial 5.

**Figure 51 - Symmetry of harmonics around resonance and damping points in a tone**




In this folded configuration, the closer a harmonic is to Partial 5 in the table, the more *timbral consonance* it would exhibit while the further away from Partial 5 it is, the closer it is to the irrational square root of Partial 5 and the more *timbral dissonance* it would incur. This would be something like striking a cymbal while pinching its edge – the damping and energy loss is greatest closest to the edge.[78]

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[78] **Principle 15:** The greater the wave symmetry in Φ-damping, particularly when weighted toward the out-of-phase *cosine* component, the greater is the perceived *timbral dissonance*. Lack of any particular damping alignment indicates maximum *timbral consonance*. In general, harmonics above the thirteenth partial are increasingly damped due to shorter wavelengths that bring them ever nearer to one or several of the damping locations.

This "folded ordering" leads to the ranking of harmonic intervals as listed in Principle 16 based on their damping attributes in the Harmonic Symmetry Table.[79] Referred to here as the *Timbral Consonance Principle*, this ordering compares favorably to the Consonance Metric for intervals in an octave (see Principle 9). This is very encouraging news because if we can next show a similar match with the Tension Metric, we should have what we need to claim that our perception of music harmony originates from the acoustical properties of a single tone.

Indeed, in the π-aligned and symmetric columns we find that the wave partials cluster around Partial 9 with some aligning evenly, then odd-aligned, then odd-symmetric and finally odd-even-symmetric. So, just as the proximity of intervals to specific peak velocity points in an octave can be used to measure tension, proximity to the peak π fringes in a tone can be used to indicate the degree of *Timbral Tension*.[80]

When we follow the alignment from greatest to least, the Harmonic Symmetry Table ranks each corresponding diatonic scale step from tensest to least tense as listed in Principle 18.[81]

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[79] **Principle 16:** The ***Timbral Consonance Principle*** is the ranking of standing wave partials and their corresponding music intervals based on harmonic Φ-damping and Φ-alignment attributes. Following the order from non-damped to even to mostly odd-damped, we can rank intervals from most consonant to most dissonant:

| | Interval | | Harmonic Φ-damping Attribute |
|---|---|---|---|
| 1. | major 6th | minor 3rd | Not Φ-damped |
| 2. | minor 6th | major 3rd | Not Φ-damped |
| 3. | perfect 5th | perfect 4th | Even Φ-damped |
| 4. | minor 7th | | Odd Φ-damped |
| 5. | major 2nd | | Odd/ Even Φ-damped |
| 6. | major 7th | | Near Odd/ Even Φ-damped and Odd Φ-symmetric |
| 7. | minor 2nd | | Near Odd/ Even Φ-damped and Odd Φ-symmetric |

[80] **Principle 17:** The greater the wave symmetry in *π-alignment*, particularly when weighted toward the in-phase *sine* component, the greater is the perceived *harmonic resolution*. Partials above the thirteenth partial are increasingly non-aligned, making them seem harmonically unresolved to the ear.

[81] **Principle 18:** ***Timbral Tension*** is the ranking of standing wave partials and their corresponding diatonic music intervals based on π-alignment and π-symmetry about Partial 9 (the Harmonic Center or SuperTonic). Following the order of even to odd to symmetric alignment, the corresponding diatonic scale steps are ranked from most tense to most resolved:

| Scale Step | Harmonic π-symmetry Attribute |
|---|---|
| 1. Leading Tone (major 7th) | Even π-aligned |
| 2. Dominant (perfect 5th) | Odd π-aligned |
| 3. Subdominant (perfect 4th) | Odd π-aligned |
| 4. Augmented 6th (minor 7th) | Odd π-symmetric |
| 5. Mediant (major 3rd) | Odd π-symmetric |
| 6. Submediant (major 6th) | Odd / Even π-symmetric |
| 7. SuperTonic (major 2nd) | Odd / Even π-aligned and fully symmetric |
| 8. Tonic (unison) | Odd / Even π-aligned and fully symmetric |

As predicted, the results compare favorably to the Tension Metric for intervals in an octave. And when combined with the previous match to the Consonance Metric, we can now finally see that our perception of tonal timbre and musical harmony must share a <u>single overarching acoustical framework</u> founded in the physics of a standing wave.

From this analysis, timbre and harmony are proposed as cognitive equivalents within a naturally created and historically preferred 'continuum of order.' The Harmonic Symmetry Table identifies physical properties common to the standing wave interference patterns of both timbre and harmony. This implies that musical intervals and chords must be perceived as *a layer above timbre* that amplify and strengthen those physical resonance properties already recognized in the harmonic series of a single tone.[82]

To my knowledge, this is the first time music harmony has been shown to be proportionally equivalent to timbre within a unified cognitive framework for music. The resulting *Timbre/ Harmony Equivalence Principle* (Principle 19) suggests exciting new directions for cognitive studies that will further prove the cognitive coupling of timbre and harmony.

For instance, the wave components of a flute might be shown to be a cognitive equivalent to the un-damped resonance of a major 6th interval. Listening to the timbral mix of odd-even components in an opera tenor's voice might be shown to trigger the same emotional reaction as a Tonic chord within a diatonic key. Experiments like these can further establish timbre and harmony act as cognitive equivalents at different scales of resolution within a layered hierarchy of harmonic interference.

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82 **Principle 19:** The *Timbre/ Harmony Equivalence Principle* holds that instrument timbre and music harmony are the exact same cognitive recognition process occurring at different levels in a hierarchy of harmonic interference. Intervals and chords simply amplify corresponding harmonic partials to strengthen the effect of the underlying harmonic interplay occurring in a standing wave of sound.

## Redefining Dissonance as Velocity

In 1877, physician/physicist Hermann von Helmholtz proclaimed dissonance to be:

1. *the overall size of an interval, and*
2. *any "beating" caused by the adjacency of two tones.*

Furthermore, he proposed that dissonance was at its "roughest" when the beat rate between two tones was roughly 35 cycles per second, or about the proportion of the semitone located in the middle piano register between B and C, frequencies 495Hz and 528Hz [Helmholtz 1877].

This definition stood for a century until Jian-Yu Lin revised it in 1995 under the direction of Professor Bill Hartmann at Michigan State University. This was again amended in 1999 when researchers Zwicker and Fastl collected data showing a maximum roughness of twice the Helmholtz interval or 70 cycles per second. Today, these studies are widely regarded as the only correct scientific definition of dissonance and are often sited by musicologists. This being said, "roughness" and "beating" does not necessarily explain our overall perception of dissonance. For instance, when one extends a "rough" interval beyond an octave, it will still seem dissonant to our ears even though the "beating" effect has been diminished by octave displacement. In this light, should we even define dissonance according to how much it beats?

In truth, Helmholtz must have known his definition was incomplete when he added the comment that separating two tones by more than an octave relieves the beating effect of a semitone while increasing its perceived stability. There is no indication that he was aware of the role Φ-damping plays in stabilizing a standing wave or how it determines the amount of energy exchanged between harmonics. He had not reached the realization that timbre and harmony are simply layers within a grand hierarchical Gaussian distribution of wave interference.

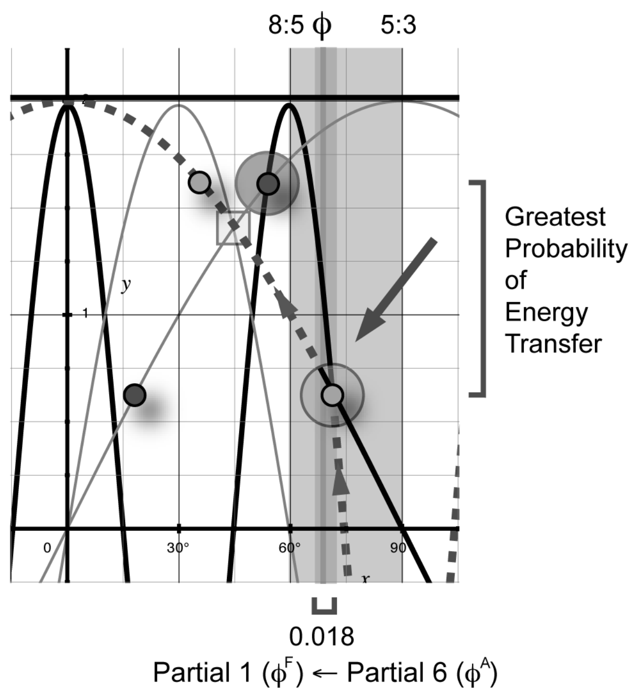
So now, based on the correspondence between the Harmonic Symmetry Table and ***REFLECTIVE INTERFERENCE*** model, perception of dissonance can be more universally defined as physical proximity of harmonic wave partials to Φ-damping locations. This is quite different than the generally accepted idea of dissonance as only adjacent beat frequency. Clearly, beat frequency is an *absolute* measure of the more general perception of dissonance and consonance as the *relative* degree of harmonic energy reinforced and amplified by each interval in an octave. This can, as we now know, be measured very accurately as velocity amplitude using the ***REFLECTIVE INTERFERENCE*** distribution model and Consonance Metric.

## Redefining Harmonic Function as Directional Energy Transfer

When we speak of the tension or resolution of intervals within a diatonic scale, we are really referring to our ability to *anticipate* the direction of harmonic movement before it occurs. Given the earlier proposed rankings of tense and resolved intervals, how should we then progress from the static analysis of simple phase alignment to a more complete and comprehensive explanation of our obvious ability to *feel the direction* melodies and chords are about to move?

To begin answering this, let's follow the popular Dominant-Tonic cadence in Figure 52. We can represent this cadence in the harmonic series with Partial 6 (a Dominant perfect 5$^{th}$) as it transfers its energy to Partial 1 (the Tonic fundamental). By zooming in on the intersection of Partial 6 with Partial 1, we see the Landau parameter region (or damping well) in the thin dark vertical region where energy has the greatest probability of an "adiabatic transfer." In particular, Partial 6 is shown here transferring its energy to Partial 1, thereby "closing the circuit" many times a second to create a continuous energy flow *downward* by a perfect 5$^{th}$.

**Figure 52 - Energy exchange in the Dominant-Tonic cadence and Cycle of 5ths**




This downward flow (occurring at the arrow) probably accounts for the long-lived popularity of both the Dominant-Tonic cadence and Cycle of 5ths chord progression, since both progress down by a perfect 5$^{th}$. Given that this interval coincides perfectly with the fundamental inside the

Landau damping well, our ears must be able to recognize the current flow here and prefer those melodies and chords that move through it. As a universal physical process in all standing waves, this path represents the most efficient path for energy to follow through harmonic interference.

Of course, further research is needed to confirm that energy exchange in the Landau region corresponds to other common practice rules of voice leading and chord progressions. Nonetheless, the evidence does indicate that directional energy currents are the central psychoacoustical driver behind common practice music. It also suggests that harmonic proportions must be encoded in our auditory system and brain in order for us to recognize it.

Assuming that directional energy flows across the small Landau eigengaps as shown here, it should be recognizable in our inner ear as a pronounced inflection point or fringe within a superposed standing wave pattern. As this occurs, the frequency modulations and phase-shifts induced by a given interval would set up an expectation of directional movement along the most efficient path of energy exchange through the interfering waves, some paths being stronger and at a higher velocity than others. This sensation of motion, confirmed by Dawe et. al. as spectral-motion aftereffects from continuous neural firing along a pattern, would then create a preference (an anticipation/reward potential) in the direction of efficient energy flow. In effect, our attention would float along with the strongest current in the sound stream toward resolution.

As an aural focusing skill for survival, the human auditory system seems to be quite adept at recognizing and following directional energy transfer in sound – especially at specific pitches. We are able to anticipate the probability of energy transfer as a *feeling* and, then when it occurs, *feel rewarded* with a *feeling of satisfaction*. The anticipation/reward system intrinsic to the brain can of course be triggered in any number of different ways by melodies and chords, creating continuously changing levels or shades of anticipation and reward. But it is the degree to which our anticipation matches the actual outcome that determines the emotional response to music.[83]

Generalizing this principle to all wave partials and their intervallic counterparts, our recognition of physical energy exchange is *the most likely explanation* for the widely held idea of "gravity pulling" the Dominant toward the Tonic. Applied repeatedly in a Cycle of 5ths, the cascading downward flow of energy would certainly produce a very satisfying chain of anticipation/ reward potentials. In the larger context of energy transfer and phase/frequency modulation described in Landau-Zener theory, we now have a natural and wholly organic explanation for preferred voice leadings in common musical practice. The best tool to measure these voice leadings is the Tension Metric from the ***REFLECTIVE INTERFERENCE*** model.

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[83] **Principle 20:** Within the calm Landau parameter space between neighboring amplitude and frequency Φ-damping locations, wave partials transfer energy as a phase/frequency modulation. This is perceived as an auditory sensation of temporal movement in the direction of energy flow. The degree with which melodies, intervals and chords follow the physical direction of energy over time determines the measure of anticipation/reward potential in a piece of music.

## Redefining Harmonics as a Hierarchy of Wave Interference

From the principles leading up to Principle 19, we can now define a mathematical system to describe timbre and harmony as co-resident *layers* within a hierarchical pattern of acoustic harmonic waves. This is best described as a recursive structure that begins inside a tone and stacks upward and outward to cover the entire auditory spectrum in the most orderly way possible. Here is how it works.

Since the square of twelve (144) defines the octave in the ***INTERFERENCE*** function, we should expect that the harmonic series would also cycle its interference pattern recursively based on other powers of twelve. To support itself, such a hierarchy would have to start with the interference pattern inside a single tone and repeat itself recursively outward through pitch space in orders of magnitude. Furthermore, for an octave to align and resonate according to a power of twelve, harmonic standing waves must align at five specific levels of magnitude – one-twelfth of a tone, one tone, a semitone, an octave and twelve octaves. Mathematically, pitch space would then be organized as a 5-level $12^{th}$-power recursive *Harmonic Hierarchy*[84], represented recursively as the finite power series $2^{12^n/12}$ where n = {-2..2}.

It turns out that this definition is fully compatible with equal temperament due to the fact that it too is based on a $12^{th}$-power of 2, specifically $2^{1/12}$. Furthermore, it is equally compatible with the twelve-fold ***REFLECTIVE INTERFERENCE*** distribution over an octave, once again confirming that this proposed hierarchy must truly be a natural organizing principle of harmonics.

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[84] **Principle 21:** The *Harmonic Hierarchy* is defined as an equivalence class of 5 identical levels of harmonic interference that is generated from a single tone, aligning at different resolutions over pitch space:

| | | | |
|---|---|---|---|
| TwelfthTone = | $2^{(2/3456)} = 2^{(12^\wedge-3)}$ | $2^{(12^\wedge-2)/12}$ | 1.000401207 |
| Tone = | $(\text{TwelfthTone})^{12}$ | $2^{(12^\wedge-1)/12}$ | 1.004825126 |
| Semitone = | $(\text{Tone})^{12}$ | $2^{(12^\wedge0)/12}$ | 1.059463094 |
| Octave = | $(\text{Semitone})^{12}$ | $2^{(12^\wedge1)/12}$ | 2 |
| TwelfthOctave = | $(\text{Octave})^{12}$ | $2^{(12^\wedge2)/12}$ | 4096 |

which may be represented bi-directionally as the recursive exponential functions:

$$4096 = \text{TwelfthOctave}(\text{Octave}(\text{Semitone}(\text{Tone}(2^{2/3456})^{12})^{12})^{12})^{12}$$
$$2^{2/3456} = \text{TwelfthTone}(\text{Tone}(\text{Semitone}(\text{Octave}(4096)^{1/12})^{1/12})^{1/12})^{1/12}$$

or more compactly as a finite power series of 2 beginning with n = -2:

$$f(n) = 2^{12^n/12},\ n=\{-2..2\}$$

These five layers act as a harmonic projection screen from which the musical geometry of melody, intervals and chords can emerge. Any property found at one level of this hierarchy will apply at all levels.

As a unified musical framework for pitch space, we might even describe the alignment pattern of standing waves as a 5-layer *harmonic projection screen*. With a model such as this, music theory becomes far more than a bag of rules handed down by tradition – it becomes the study of geometrical shapes and how they fit together at different resolutions within a spatial harmonic landscape. From this one simple idea, many common assumptions about music begin to change.

For instance, counter to the conventional wisdom, an equal-tempered 12-step octave can now be seen as the only natural tuning with all other temperaments mere artificial tunings that *diminish* (not improve) overall harmonic pattern matching. Zarlino did not invent the 12-step octave – he *discovered* it. Similarly, equal temperament is not a manmade contrivance of convenience, as held by popular opinion and taught in our universities, but a primordial property of sound existing prior to the human invention of music and long before two tones were ever sounded together. As a natural organizing property of coherent waves, this Harmonic Hierarchy can now be seen as the true reason 12-ET ultimately won out against the artificial temperaments to become the de facto modern tuning standard. It is intrinsic to the physics of sound and evolved into our physiological structure, making its emergence in musical practice all but inevitable.

Since the time of Pythagoras, musicologists and theorists have argued that scales should strive to align with simple harmonic proportions as much as possible in order to be "pure" and most pleasing. Here we find that not only is a scale of simple proportions like Just temperament not necessary for recognition but that the 12-step equal-tempered scale actually does the best possible job of aligning with the natural harmonic series intrinsic to all nature. Beyond being a "just adequate" compromise for music, 12-ET now proves to be the very physical and cognitive model of coherence against which all music is organically measured by our naturally evolved ***REFLECTIVE INTERFERENCE*** auditory system.[85]

At the bottom of this harmonic projection screen we find a seed ratio of 2 raised to the power of 2/3456, not too surprisingly equal to the cube root of 12 (or $12^{-3}$). When we then take this and divide it into the Philolaus octave comma of (9:8) / 27, the result is exactly 72 or half of 144 found earlier at the octave.[86] We can only conclude from this that the Philolaus octave comma is not an anomaly in music, but something that originates deep inside a single tone as one-half of a wave cycle. It becomes yet another "half twist" *two levels below* the octave in the Harmonic Hierarchy. The octave comma and all other commas in music are not manmade either – they

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**Principle 22:** The equal temperament system, based on the multiplied semitone ratio of $2^{1/12}$, is a natural proportion within the recursive structure of the harmonic series generated by a standing wave. Therefore, contrary to any argument that Equal Temperament is manmade, it is the one true natural tuning.

86 This is related to the fact that the cube root of 12 splits the octave comma exactly in half relative to its INTERFERENCE period of $12^2$ or 144.

actually “bubble up” from the tiny gaps between the harmonics of a single tone, curving and “warping” what we think of as “pitch space” as it does so.

Does this sound familiar? It should because this is proportionally the exact same half twist property found earlier inside the Landau damping well in a standing wave.

**Landau damping proportion** = $(0.0180 : \pi) \approx (12^{-3} \times 10) \approx 0.0057$

The occurrence of this proportion at two different levels in the Harmonic Hierarchy shows how the damping effect of Φ propagates upward through the ***INTERFERENCE*** hierarchy and across the entire audible spectrum to twist linear harmonics like a miniature tornado into a non-linear Gaussian curve. This further implies that the natural spiral of pitch space, long considered an embarrassing error of nature, is nothing more harmful than the accumulation of slack space in standing waves when tones are sounded together.

When we look inside an oscillating standing wave, we now find the ***INTERFERENCE*** pattern repeated at each level from one-twelfth of a tone through twelve octaves. With the entire auditory spectrum defined by a grand Gaussian distribution over twelve octaves (confirmed first by Shepard Tones and later by neurophysiological studies), we must finally admit that the irrational golden ratio is why pitch warps into a spiral and “commas” exist in the first place. As farfetched as it may sound, the logarithmic curvature of pitch space is nothing less than a byproduct of Φ-damping everywhere in nature – *even space itself* – creating the free space needed for propagation of all forms of wave energy.

To some this will seem an inevitable conclusion and to others it will seem ridiculous. The simple way to understand it is to realize that both the auditory spectrum and a single tone are the result of the same principles of damping and resonance predicted by quantum mechanics in the vibrating lattice of space itself. This is Einstein’s curvature of spacetime and the first principle of Max Planck’s quantum theory. It is found in the wave-particle duality of Louis De Broglie and the “quantum harmonic oscillator” of matter proposed by Erwin Schrödinger. Current research concerning the structure of space, mostly in the area known as Lattice Quantum Chromodynamics (*Lattice QCD*), is finding more and more evidence that Φ-damping plays a crucial role in the structure of all things.

As it applies to the mechanics of sound and music, perception of the 5-level 12$^{th}$-power Harmonic Hierarchy in music must have evolved as an environmental response to the same universal standing wave principles of quantum mechanics.[87] Mechanical sound waves need free space to vibrate and propagate just as biological growth (resonating up from carbon-12 resonance) needs a little slack to grow and articulate. From the commas of our elbows to the oscillation of our heart, we must assume that human physiology incorporates the very same resonance and damping properties as a vibrating string, crystallizing into the familiar human form as a 12:5 growth pattern.

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**Principle 23:** The cognition of music harmony is defined by the proportional interference of wave resonance and Φ-damping in the natural harmonic series following the hierarchy of $_{2}12^{n}/12$. Specifically, the proportions recognized in the standing wave interference pattern of a single tone are the same across the hierarchy of a semitone, octave and 12-octave frequency spectrum. In this way, the spatial and temporal qualities of timbre, harmony and spectra form a cognitive equivalence class corresponding to the 5-level Harmonic Hierarchy.

# *Tritone Crystallization*

*"You know, that's it, there's no turning back because what it's made of is so fine. It's like crystal, you know, it's like the purest crystal." - Eric Clapton*

Perhaps by now you will have noticed something missing in the Harmonic Symmetry Table. It is the impetus behind this book and the very force of symmetry in music harmony yet entirely absent from the Harmonic Symmetry Table. I am referring to, of course, the long forgotten tritone, whose "diabolical" tendencies we will now explore in the greatest detail.

In the first four octaves of the harmonic series there are four tritone intervals[88]. There is one between Partials 5 and 7, an octave duplicate of this between Partials 10 and 14, an inverted one between Partials 7 and 10 and the last one between Partials 11 and 15. Since this is a very complex set to consider, we will simplify things a bit by focusing on only the lowest and most recognizable tritone formed between Partials 5 and 7.

We can further simplify by grouping Partials 5 and 7 into odd and even components for independent analysis. The purpose in doing this is to identify shared features within or between each group that can explain the tritone's reputed tenseness and dissonance in common musical practice. For completeness, a brief technical analysis of these tritone partials is provided, though you may wish to skip to the summary paragraph that follows:

*In Figure 53, no Φ-alignment is visible whatsoever in any of the sine or cosine components, though there is a strong mix of near misses on both. This degree of Φ-damping on both wave components would pinch down on resonant vibration, making the tritone a* very dissonant *interval in accordance with earlier findings. As for the tritone's legendary tension, one of its cosine components does π-align, creating a strong out-of-phase sound to the fundamental sine wave. This, added to the π-symmetry alignment of both sine components away from the fundamental, characterizes the tritone as very tense and* harmonically unresolved.

This thumbnail analysis confirms that the tritone partials do indeed match the earlier Consonance and Tension Metrics for the octave tritone, thus explaining the source of its dissonance and tension as the 7:5 harmonic interference pattern in a single tone. But while this again confirms timbre and harmony as two sides of the same coin, there is something of even greater significance to be found here. Take a close look at Figure 53 and count the number of intersections at light and dark pentagonal markers.

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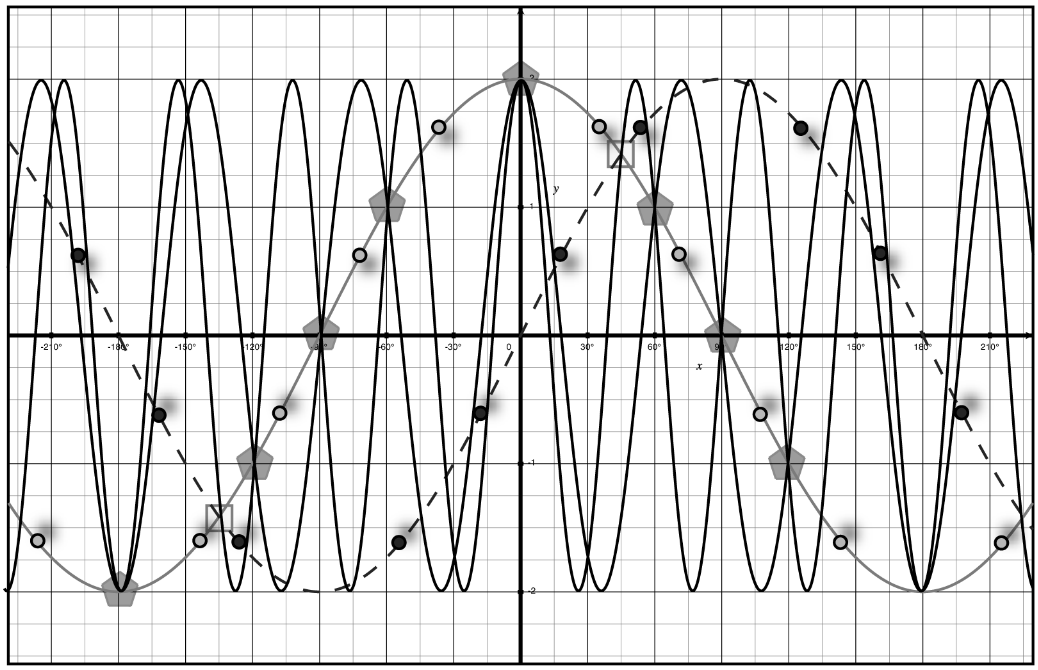
[88] The earlier staff notation of the harmonic series in Fig. 32 will confirm this.

**Figure 53 - Tritone partials partition a tone into odd-even groups**

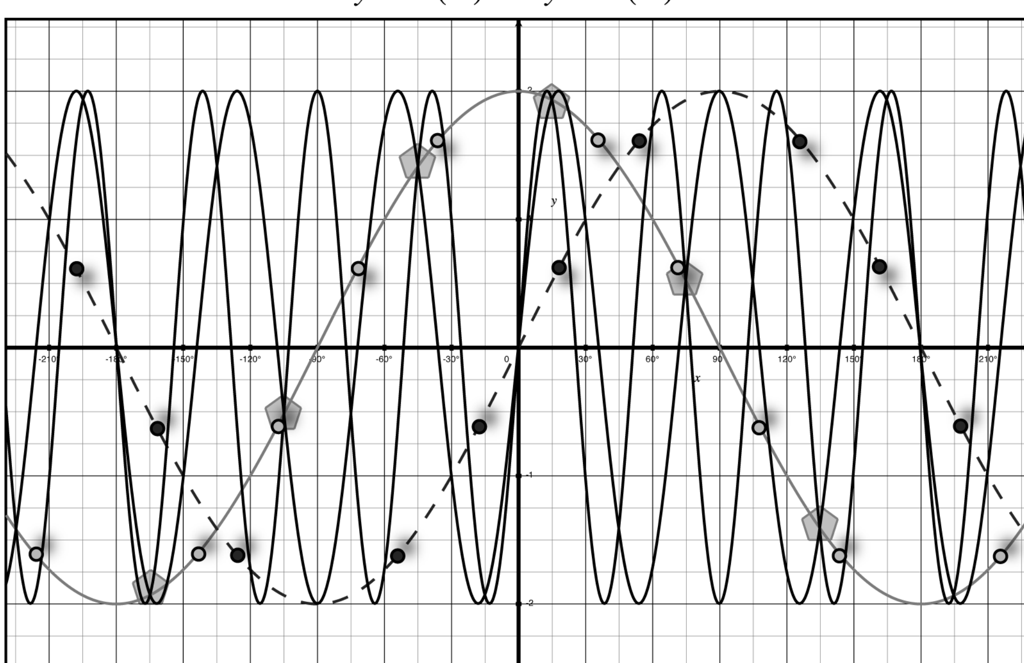
Tritone - 5th and 7th Partials: Even Components

*y*=2cos(5*x*) *and* *y*=2cos(7*x*)




Tritone - 5th and 7th Partials: Odd Components

*y*=2sin(5*x*) *and* *y*=2sin(7*x*)



While the even tritone components intersect along the even fundamental wave (solid line), the odd components on the odd wave (dashed line) do something a bit unexpected. They fall on the even fundamental too – just phase offset by 1/24th of a cycle (π/12 radians). This places *all* of the tritone intersections on the even wave (solid line), leaving absolutely no tritone intersections on the odd fundamental wave. Fourier explained this kind of occurrence as the product of an even (cosine) and odd (sine) function that together always forms an even (cosine) function. As a result, the cross product of tritone partials is an even function, crossing twelve times on the even cosine component – six odd and six even – leaving two non-intersecting tangents at each of the nodes.

Though this last sentence may seem like a small point, it is actually very important because it proves that tritone wave partials *physically* divide the fundamental into twelve recognizable fringes. This is not a special case or abstract mathematics – it is a very real physical property of harmonics that occurs in any resonating medium.

The spikes, "edges" or "intensity fringes" produced by these intersections should be easily recognizable as twelve distinct points in the inner ear. In fact, they may well be a built-in feature of the inner ear's basilar membrane and/or auditory cortex. While experimental studies are needed to confirm this, a single tone could very easily create a faint impression of this division in the inner ear, especially since Partials 5 and 7 are so low in the harmonic series. Like a neurophysiologic template burned into our auditory hardware over millions of years of evolution, a ghostly sense of twelve positions in a wave period could have evolved as a template for our present day 12-step octave.

So here again is another bit of hard evidence to support the claim that the 12-step octave was *pre-established* in the interference pattern of the harmonic series. As an ultimate twist of irony, we must finally admit that it was the tritone – the mid-octave embarrassment of the Pythagoreans and medieval devil in music – that brought us by the hand to the discovery and widespread adoption of Zarlino's 12-step octave.[89]

As for the two extra tangents at each anti-node, they are our best answer to the Deutsch *Tritone Paradox*. The two odd and even components, being slightly out of phase and traveling in opposing directions, would create a continuous physical sensation of ambiguity in the inner ear. When emphasized by a tritone interval, the odd group flows one direction while the even group flows another. Which direction should we anticipate and follow? This can only be determined by context.

The persistence of a single Harmonic Center and diatonic scale has the effect of removing the uncertainty of the ambiguous tritone crosscurrents; making it possible to choose which direction

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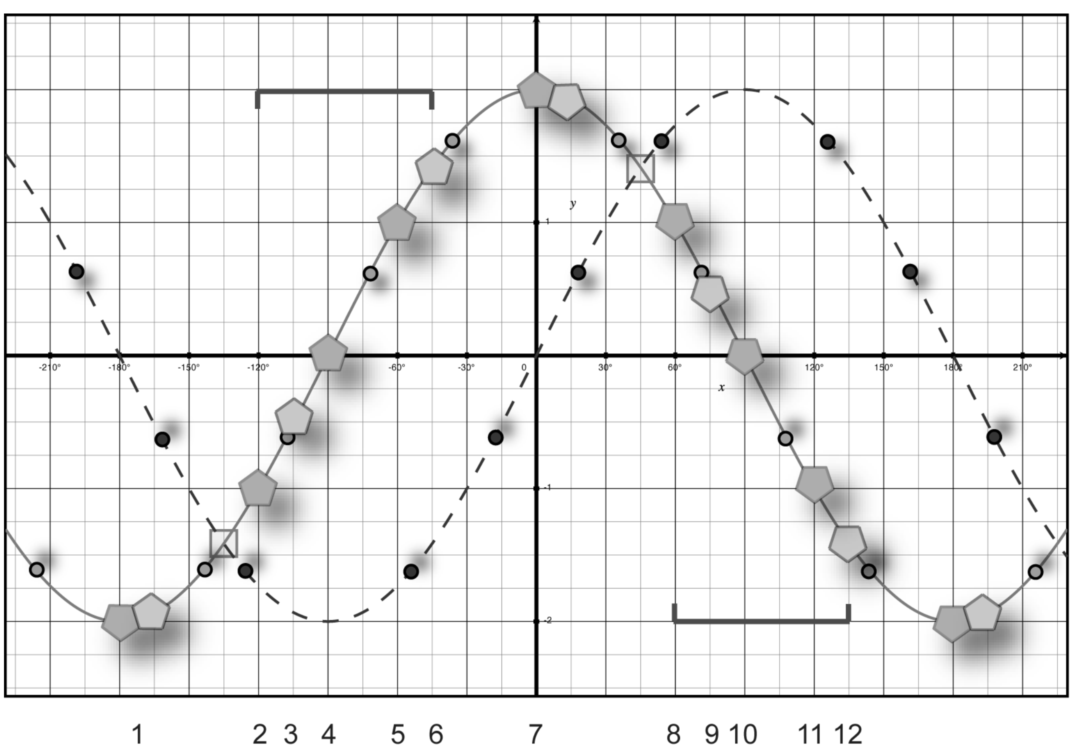
**Principle 24:** The development of the 12-step octave originates in the natural recognition of the tritone interference pattern of Partials 5 and 7 against the fundamental.

we should follow. In this way, diatonic scales create a sense of center, or “tonality,” making music more predictable and satisfying.

Now maybe you wonder as I do what Pythagoras might have thought about this explanation. Was he able to intuit these patterns (perhaps in one of his meditative states) as, say, oscillating perfect solids? And what might Fourier have thought? Was he able to imagine the waves intersecting in the harmonic series and understand their relation to regular solids and how they might be perceived? What if the two men were to meet? Could they bridge the gap between their two different worldviews and reunify the path of knowledge from ancient wisdom to modern science?

In an attempt to mend the competing paradigms of Pythagoras and Fourier, we will now undertake a very unusual thought experiment. Based on the process of crystallization in nature where pressure forces minerals into regular lattice formations, we will *crystallize* the tritone partials of a Fourier musical tone into a Pythagorean harmonic solid. Doing this will lay the foundation for a broader view of harmonic science that bridges the study of energy and form.

**Figure 54 – Origin of the 12-step octave from tritone Partials 5 and 7**



The components are now recombined in Figure 54 to show all of the dark and light pentagonal markers where the tritone partials intersect one another on the fundamental. As you can probably see, this graph reveals the natural occurrence of two proportionally identical groups of *five* markers (in brackets) reflected and flipped on either side of the vertical y-axis. To crystallize this into a solid geometry, we must now apply 'virtual pressure' in a horizontal direction against the oscillating wave to cause an equidistant compaction and axial alignment of the markers.

As pressure builds into Figure 55, the dark nodal markers on the horizontal x-axis begin to shift to the right by $\pi/24$ to a position of balance in the middle of each group of five. Two large pentagrams have been added to represent this balance as Pythagoras himself might have imagined. At the same time, the two markers at each anti-node are squeezed together, "fusing" at the mid position after shifting symmetrically inward by the same $\pi/24$ proportion.

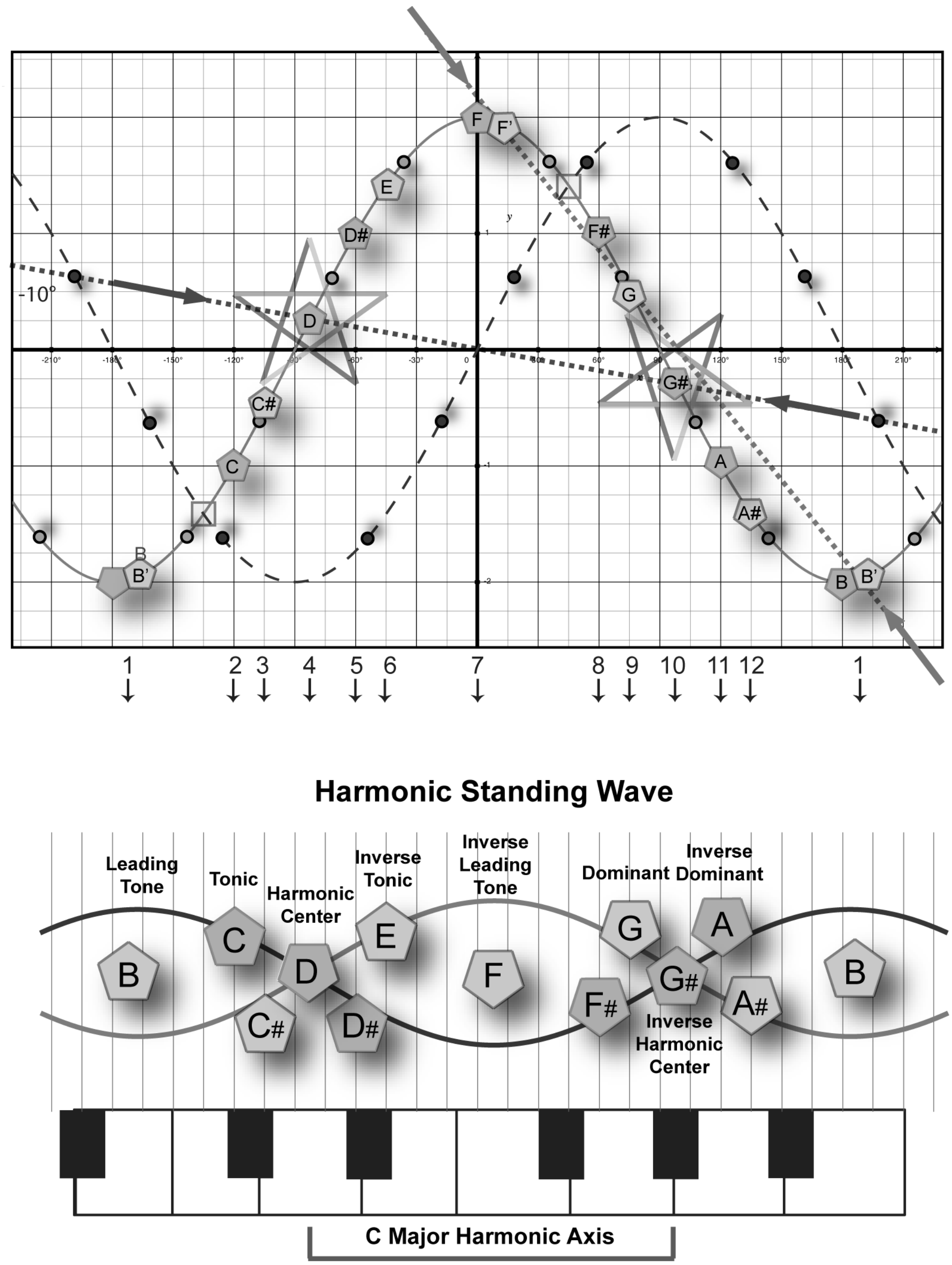
The result is a horizontal division of the $2\pi$ wave period into twelve sections where two groups of five markers balance perfectly around an angular dotted axis (see arrows) with slope of -5 and slant of -10° from horizontal. For future reference, the markers are labeled with the twelve note names of the octave so we can translate the harmonic relationships of a tone into the harmonic intervals of an octave.

This slanted axis links the new nodal positions (labeled {D, G#}) while passing through two <u>dark shaded</u> Φ–damping locations on the odd wave component. At the same time, another dotted axis (see arrows) is formed that links the new anti-node positions (labeled {F, B}), passing through a <u>light shaded</u> Φ–damped location. What does this mean? Think of it in the context of mineral crystallization.

In nature, pressure causes a compaction of atoms into lattice structures, like that of a quartz crystal or diamond, according to their particular atomic structure. In our thought experiment, the "pressure" creates a new axis – a *Harmonic Axis* – that is perfectly aligned with the least resonant locations. Forced alignment with these Φ-damping locations has the effect of canceling oscillation of the harmonic standing wave, freezing it in time. This "crystallizes" the waveform, thereby stabilizing it into a vertical lattice and "tempering out" any free space. In ancient theosophical interpretations this might be taken as mending the schisma closed, correcting the "error in nature" or theologically "casting out the devil" – all reducing freedom of movement.

In this thought experiment, the tritone's "atomic structure" then appears as ten intersections and two fused nodes locked into a diagonal Φ-aligned axis. With the oscillation stopped, we can now see a pentagonal crystalline geometry beginning to form around the node markers. Within the twelve-fold grid or lattice, the markers now appear to align vertically to the five vertices of a deadening pentagram. Reflecting either side of the x-axis, one pentagram points up and the other points down, reminiscent of the ancient Hermetic duality symbols for good and evil. Perhaps Pythagoras imagined music, and everything else with it, something like this.

**Figure 55 - Pentagonal projection of the dodecaphonic scale**



Next, projecting the two pentagonal groups vertically downward into the *Harmonic Standing Wave* underneath, we can see how the markers are positioned around two nodal points in the harmonic wave of the fundamental {C}. At the octave level in the Harmonic Hierarchy, the Harmonic Center {D} becomes the SuperTonic for a {C} major scale while the Harmonic Axis {D, G#} forms a new axis around which the standing wave begins to stabilize. In the process, we shift from the non-reflected vibration of the fundamental {C} to a fully reflected and stationary standing wave orbiting around the Harmonic Center {D}. This was a direct result of a natural *warping of the wave* by π/24, or 0.1309, causing it to crystallize at a 10° angle.[90]

With this, we may have yet another reason to explain why Partial 9 acts as the center of the harmonic series of a tone. The standing wave creates an easily recognizable amplitude peak at phase-quadrature (the square marker), corresponding here to the fundamental {C}. The stabilizing axis and Harmonic Center for the standing wave then becomes Partial 9, acting as a polar *axis of resonance*. This puts pressure on the vibrating medium (stretching it slightly) to force axial alignment and open up the space needed for sympathetic harmonic waves to form in the range {1..12} while squeezing out the remaining Fibonacci proportions in the range {13..∞}.

At the octave level, Partial 9 then aligns with the SuperTonic {D} to create the Harmonic Center for the diatonic scale of {C} major. The remaining harmonics then space equally around this balance point to complete the equal-tempered 12-step octave.

When we continue the "crystallization" process into Figure 56, the Harmonic Standing Wave then begins to "materialize" into a regular solid by folding the wave back in on itself (as if fitting it into a cube!) at the anti-nodes so that the top and bottom positions of the two pentagonal shapes are filled. Doing this produces a regular pentagon and inverse pentagon tiled onto a planar surface where the Fibonacci damping proportions and convergent golden ratio are pushed out onto the edges and vertices. These edges are sometimes called "Platonic folds" where the irrationality of Φ becomes trapped in between harmonic structure. This is the geometrical equivalent for the musical idea of the golden ratio falling into the "cracks between the keys" of a piano keyboard.

When we next connect the two pentagonal groups at one edge, a 2-dimensional tiling of all twelve markers is created. Folding all of the edges closed in 3-dimensional space, we then arrive at the perfect solid of a 12:5 pentagonal dodecahedron. Not coincidentally, this shape has been found in the smallest Fullerine $C_{20}$ carbon molecule right up to the Universe itself.[91]

Is it really any surprise that a crystallized tritone in a standing wave could produce this shape? Long ago, the Greeks had proposed the dodecahedron as the structure of the cosmos. The only question is did they know it was also a 12:5 interference pattern of standing waves?

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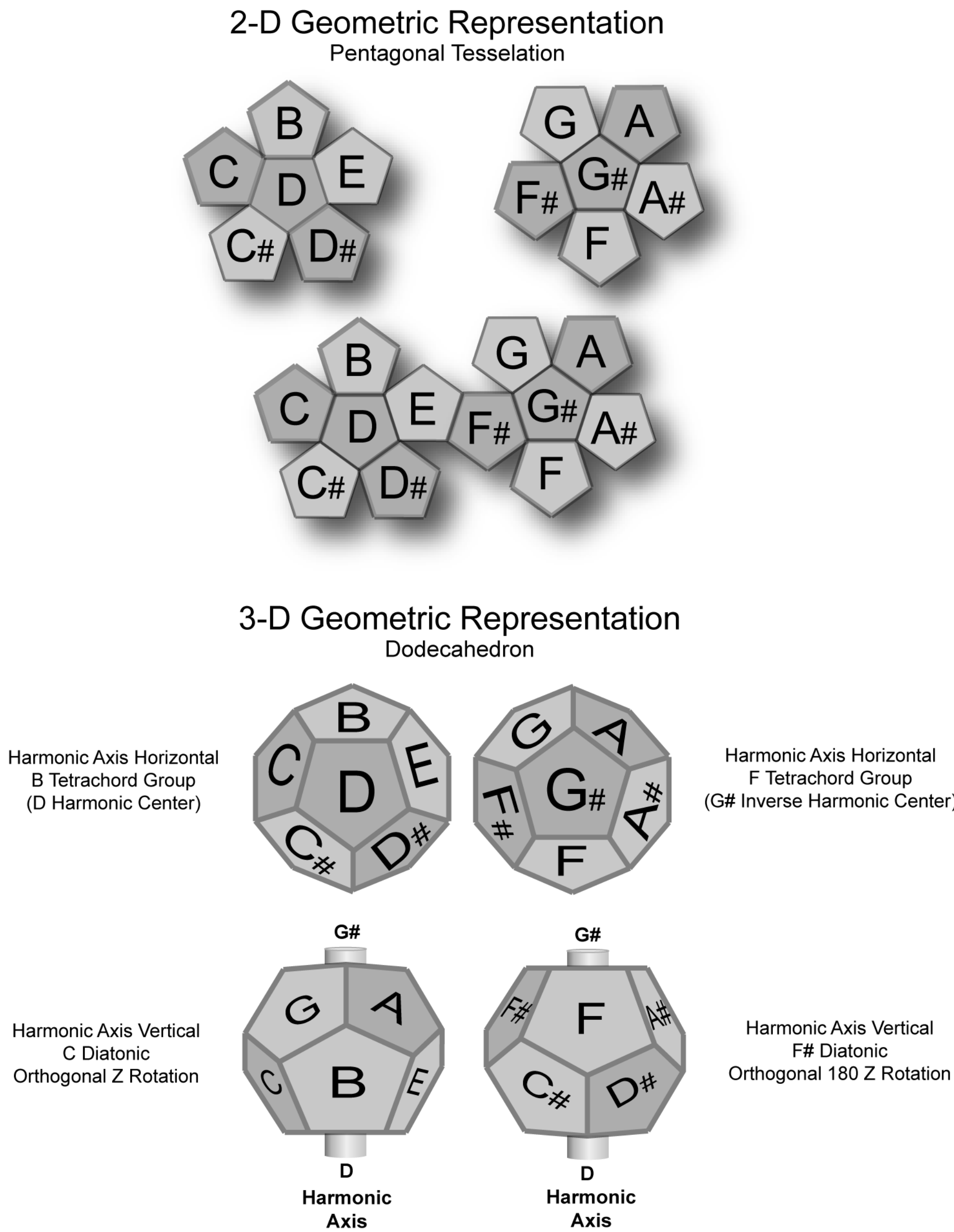
[90] Recall that this is the same proportion of a bird egg represented by the damping proportion $\Phi^3 / 2\Phi$.

[91] Found in 2003 by the WMAP satellite. The temperature fluctuations of space left behind by the Big Bang can be expressed as a sum of spherical harmonics plotting into a spherical dodecahedron. [Selfe 2003]

**Figure 56 - Crystallization of a dodecahedron from tritone interference**




We can view this pentagonal dodecahedron crystal from any angle, though the default here is to display the twelve tones in an octave as two groups of six, {B, C, C#, D, D#, E} and {F, F#, G, G#, A, A#}, each contained inside the boundary of a perfect 4$^{th}$. You may recognize this from Section One, Figure 13 as the Pythagorean tetrachords shown as dodecahedrons floating above a piano keyboard.

When the dodecahedron is next rotated 180° into a vertical position at bottom, five of the seven tones of the {C} major scale are visible on the left balanced around a shared Harmonic Axis running vertically. On the right (flip) side, its tritone opposite {F#} major scale is also visible. Spinning this structure up or down along the Harmonic Axis creates a double helix DNA-like geometry with ten rungs that correspond perfectly to Roger Shepard's *Double Helix of Musical Pitch*.

With this, our thought experiment is complete. We have crystallized Fourier's worldview of wave duality into the Pythagorean dualism of perfect geometry. More importantly, the ancient harmonic science of proportional geometry can now be seen to be entirely compatible with modern wave theory. The two clearly share the very same properties, therein mending the worlds of ancient wisdom and modern rationalism into an unbroken path.

But this exercise was more than just an amusing experiment in philosophy. It is a construction proof that demonstrates the equivalence between the *coherent energy of a standing wave* (Fourier) and the *coherent form of a dodecahedron* (Pythagoras). This can be proven mathematically using the polyhedron formula |V|-|E|+|F|=2, discovered in the early 19$^{th}$ century by Leonhard Euler. When applied to a dodecahedron, this formula can crystallize a standing wave.

***<u>Euler polyhedron formula for a pentagonal dodecahedron</u>***
*|V|-|E|+|F|=2 // Vertices – Edges + Faces = 2*
*20-30+12=2*

Here is how it works. As we already know, the tritone partials intersect at <u>12</u> resonant locations. And, we have seen how these intersections split into <u>2</u> pentagonal groups by aligning to Φ-damping locations along the Harmonic Axis. But when fused during crystallization, the standing wave also creates <u>20</u> anti-harmonic points comprising the original sixteen amplitude Φ-damping locations plus the four new phase-shifted locations at the nodes and anti-nodes. Connecting these twenty anti-harmonic points with the two groups of five harmonic points produces <u>30</u> unique anti-harmonic intervals. Ergo, the standing wave of a musical tone can be seen to have the same geometric relationships found in a pentagonal dodecahedron. To deconstruct this dodecaphonic crystal back to pure energy, we need only subtract the <u>20</u> anti-harmonic points from the <u>30</u> anti-harmonic intervals to leave <u>12</u> harmonic "faces." From this we arrive back at our original Fourier tritone embodied in those two pentagonal groups, or Euler's formula: 20 – 30 + 12 = 2.

If this still does not convince you that a dodecahedron is the crystalline shape generated by tritone partials in a standing wave, take a look at the side length of a regular dodecahedron, 2/Φ = -1+√5. This length is <u>identical</u> to the difference in amplitude between the two dark shaded Φ-damping locations that form the Harmonic Axis of a tone. Furthermore, the *dihedral angle,* or angle between each face on the dodecahedron, equal to 2arctan(Φ) = 116.56°, is also exactly equal to the x-value of yet another dark Φ-damping location on the odd fundamental wave component. In the most general case, the Cartesian coordinates for the vertices of a dodecahedron (from the origin) are the same essential proportions of damping and resonance found in the ***INTERFERENCE*** equation. No matter how you look at it, the golden ratio is what dampens a standing wave into the regular geometry of a pentagonal dodecahedron solid.

***Cartesian coordinates for a pentagonal dodecahedron***
*(±1, ±1, ±1)*
*(0, ±1/ Φ, ±Φ)*
*(±1/ Φ, ±Φ, 0)*
*(±Φ, 0, ±1/Φ)*

It is from this simple experiment that all life can be seen as a crystallized interference pattern of odd and even harmonics. When one considers how the fractional "edges" and "vertices" of the dodecahedron correspond to the damping points on the *odd* fundamental component and its twelve resonant tritone intersections correspond to "faces" on the *even* component, then it follows that odd waves perform the job of *carving shapes* out of the even waves as they intersect in a harmonic standing wave. We can only conclude that it is the quadratic interaction between odd-even (or sine-cosine) harmonics that are the true primordial progenitors of organic form.

Assigning odd to male and even to female, the tritone in a standing wave probably even kick-started the biological process of reproduction. The odd-male could be described as damping or *dividing* the harmonic even-female, effectively giving birth to a pentagonal dodecahedron through the tritone schisma. After all, this is exactly what our thought experiment did. Could nature's reproductive process of dividing itself have as its harmonic engine the tritone partials of an inanimate oscillating standing wave? Preposterous you say? Well, what other natural principle gives rise to the reproductive process of cell mitosis? How does a cell learn to divide and grow?

Every great thinker from Pythagoras through the 17th century had a grasp on the harmonic relationships shared between geometry, music, astronomy and life. But since those days, such knowledge has been completely lost to the moderns leaving few of us with any interest in returning to such "esoteric wisdom" or "sacred geometry." Hopefully enough evidence has been offered here to encourage a deeper respect for the golden ratio as a central organizing principle in music and other natural processes.[92]

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[92] **Principle 25:** The organizing property of the golden ratio is the central cognitive principle of music harmony.

# *Sonic Architecture*

*"I call architecture frozen music."*
*- Johann Wolfgang von Goethe*

From the one simple idea that harmonics organize themselves into a 5-level $12^{th}$-power hierarchy, we can now dig deeper into the physical architecture of harmonic cognition. The first step is to illustrate the *Timbre/ Harmony Equivalence Principle* to show how musical harmonies reinforce our perception of harmony. This is done by scaling and aligning the interference pattern of a single tone with that of two tones over an octave. Shown in Figure 57, the damping wells that twist inside the oscillation of a single tone can be seen to ripple upward into an octave as two vortices that power the spring action of the Tritone Function.

Beginning at the Harmonic Center (Partial 9 or SuperTonic), the two Fibonacci spirals represent two vortices of energy that travel across the Landau damping wells. One spirals clockwise in the upper half of the octave while the other spirals counterclockwise in the lower half. As the two spirals drill down into the two halves of the sine wave, the small dark dots on the x-axis indicate the Φ-damping locations in the tiny half-period gaps of the harmonic series[93]. Remember – these locations represent the Landau damping wells that suppress fractional waves between whole number harmonics in all standing waves.

Looking at the diagram, it is pretty easy to see where the convergent gaps at the Tone Level align with those at the Octave Level. In particular, the different gaps in the sine wave (shown) clearly align with the tritone, M3, m3, M6 and m6 intervals while the cosine wave gaps (not shown) will align with the P4, P5, m2 and M2 intervals. Together, these correspond perfectly to a 12-step equal-tempered octave as predicted in Principles 19 and 21. Furthermore, the two Φ-damping regions also align between the tone and octave, as indicated by the shaded pillars.

Below this, the "spring" of the Tritone Function is then shown oscillating across the shaded pillars. To represent harmonic function, elasticity or stiffness of a stretched string is a good way to represent the potential energy in a string's vibration in the same way an electronic capacitor or battery represents potential energy in electronics. Modeling potential energy in the Tritone Function can take either form, though a mechanical spring seems most intuitive.

When the Tritone Function then moves, its potential energy is converted to kinetic energy by following the silent Fibonacci spirals as a path of least resistance. In this way, we recognize and follow kinetic energy in every diatonic melody, interval and chord we hear while anticipating the potential energy in the Tritone Function (or "twin pillars") repeatedly in a single song.

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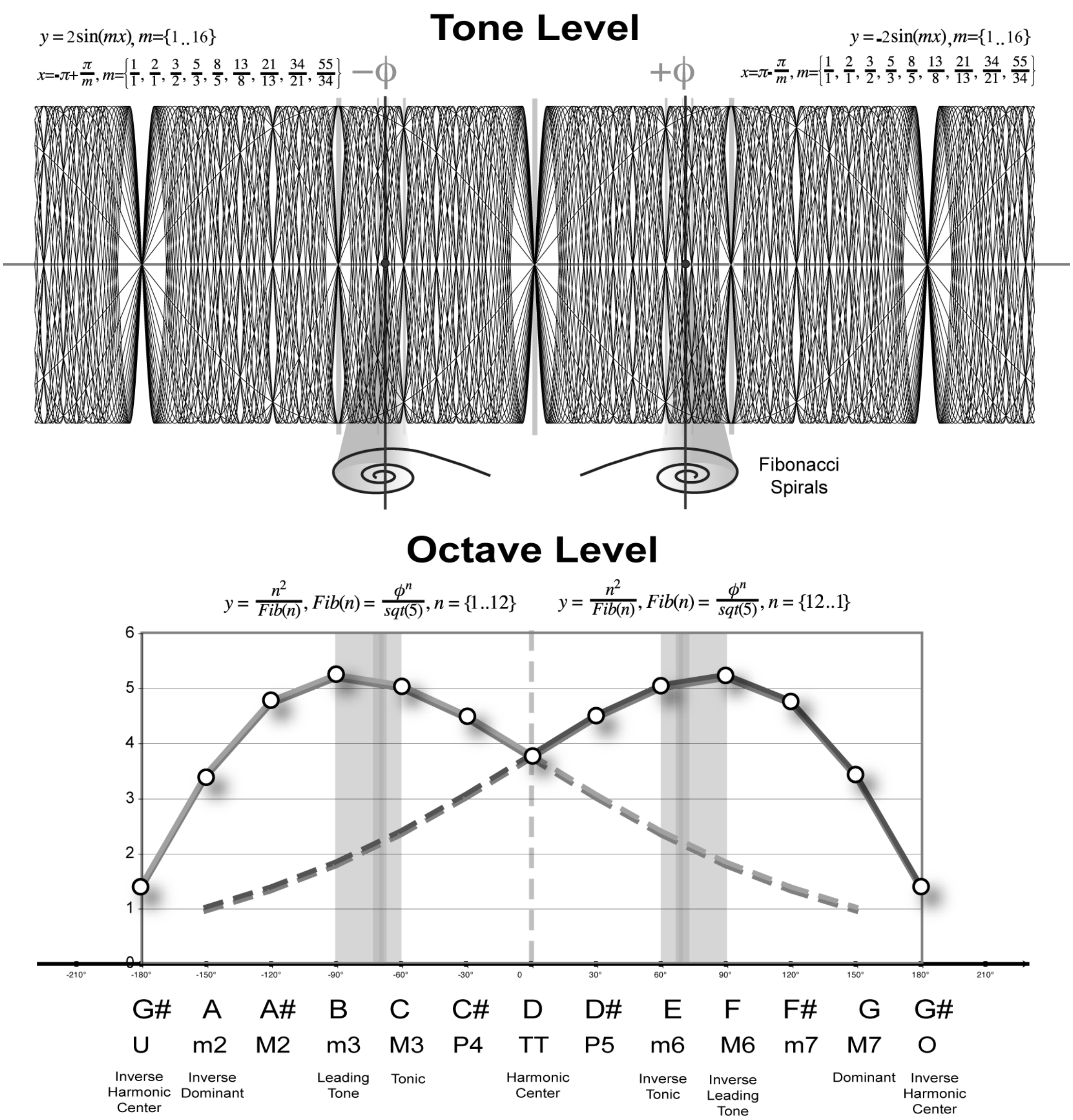
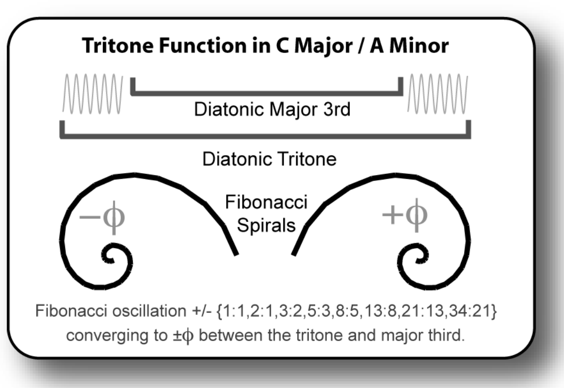
93 The half-period golden ratios are located at $\pi - (\pi \times 1/\Phi)$ and $(\pi \times 1/\Phi) - \pi$.

## Figure 57 - Harmonic Hierarchy of a tone and octave in equal scale



It is across these twin pillars that standing wave energy moves and through the swirling action of their Fibonacci vortices that resonance is maintained. This concept, while a new idea to most of us today, was apparently quite well known in ancient times and was often represented in architecture and mythology as actual physical golden pillars.

Twin pillars are found everywhere in mythology, most notably the Phoenician "Pillars of Hercules." At Gibraltar, twin pillars are said to have joined two seas to commemorate the legendary founders of Atlantis – Hercules (or Herakles) and Atlas. Alexander the Great personally saw and inspected many such giant pillars of gold in India that bore strange scripts. Plato even claimed in his *Critias* that the Atlantean kings inscribed their laws, edicts and judicial decisions on golden pillars. Could it be that these pillars, immortalized in the legendary strength of Hercules and Atlas, represented the golden ratio in standing waves and a musical cosmos?

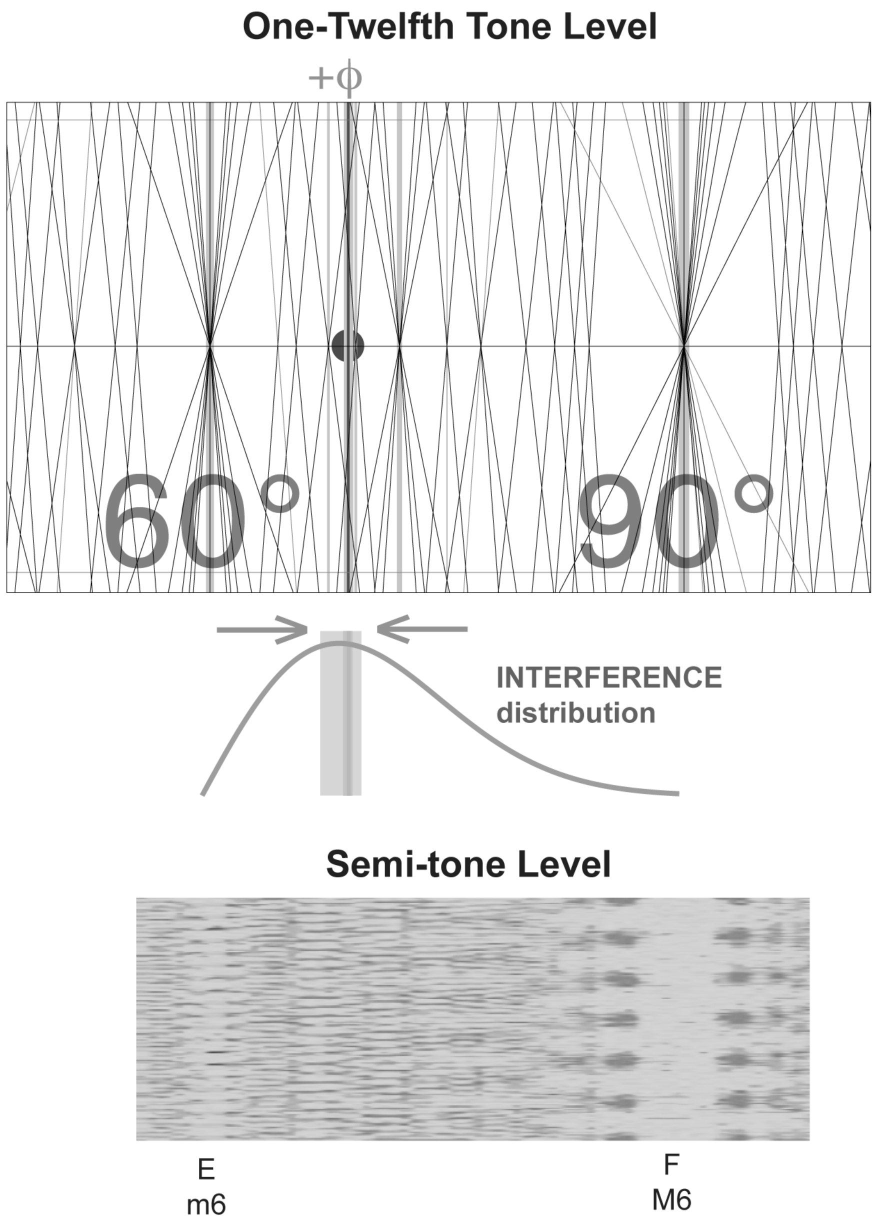
According to an epic poem by Peisandros of Rhodes in 600 BC, Hercules had to perform twelve labours, one of which brought him to cross the mountain that was once Atlas holding up the world. The story goes that rather than climbing the great mountain, Hercules decided to smash through the mountain, splitting it in half using his indestructible mace or club. By doing this he connected the Atlantic Ocean to the Mediterranean Sea, forming the Strait of Gibraltar. The two mountains of Gibraltar and Monte Hacho on either side are now known as the *Pillars of Hercules*.

Interpreting the twelve labours as the natural harmonic division of a tone (or octave) and the splitting of Atlas into mountains as two reflecting ***INTERFERENCE*** curves, we can see how the ancients could have understood the role of resonance and damping in the tritonal crystallization of material structure. The great strength of Hercules would have symbolized the physics of resonance while Atlas played the invisible role of damping beneath the world of structure. This simple musical philosophy, described in Homer's *Odyssey*, is readily apparent in the beautiful Gibraltar-like symmetry of our Tone-Octave model.

> *"Atlas the magician: he knows the depths*
> *of all the seas, and he, no other,*
> *guards the tall pillars*
> *that keep the sky and earth apart."*
> *- Odyssey 1.52*

Every time we cross the pillars, we become Hercules once again breaking through the dissonant barrier of damping that separates the resonant harmonics of {1..12} from the infinite Fibonacci damping spiral in {13..∞}. This idea becomes more obvious in Figure 58 as we zoom into a convergent damping well in the standing wave of a tone.

**Figure 58 - Harmonic Hierarchy of 1/12th tone and semitone**




At this magnification, we can see the bottom level of the Harmonic Hierarchy inside a tone as it aligns to the spectral interference pattern of an equal tempered semitone. The ***INTERFERENCE*** distribution once again matches the pattern of gaps in the harmonic series of a single tone in the same way it does over an octave. It is quite easy to see where the vertical dark lines of the Fibonacci series diverge away from equal harmonic proportions, diving into the silent "black hole" that pinpoints the golden ratio.

As each ascending Fibonacci proportion becomes less and less harmonically reflective, it "implodes" like a tiny tornado toward Φ, carving out enharmonic fractional waves as it does over an octave or twelve octaves. In a very physical way, the deadening proportions of the Fibonacci series act to articulate or slice through all five levels of harmonic wave interference from the depths of the natural damping well upward through pitch space to enable complete freedom of movement and propagation through the air. Once again we find one of Atlas' pillars and the iconic mountain of Gibraltar, reproduced here in miniature.

According to Principles 19 and 21, the features that we recognize at one level in the Harmonic Hierarchy would be similarly represented and perceptible in the other levels. Thus, as the Fibonacci series iteratively nests within itself, it twists around the always-constant ±Φ pillars at each level. And at each stage of recursion, wave partial energy would be exchanged across smaller and smaller Fibonacci / Landau regions, approximating a golden spiral (or *Spira Mirabilis*) vortex to suck our attention inward like invisible tornados. And like Dorothy being swept up to Oz, the Fibonacci series takes us up layer after layer – from inside the tiny gaps of a standing wave outward across the entire auditory spectrum, grabbing our attention and keeping it focused. It is this physical process that gives us our sense of "key" and "musical gravity."[94]

The long history of preference for contrary motion in voice leading is now fully explained by the organic auditory recognition of rapid and continuously repeated transfers of energy exchanged at Fibonacci proportions.[95] With the two swirling Fibonacci series as our guide, our spiraling ears are able to follow and anticipate the movement inside the Tritone Function. Once again, this can only be possible if our auditory system is physiologically structured according to the very same physical standing wave principles illustrated here. Predicting movement through the patterns of harmonic interference must be as simple as matching harmonic sound to the same harmonic geometry already evolved in our ears and brain. Of course, for any of this to work our brain must itself be organized as a standing wave.[96]

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94

**Principle 26:** The Fibonacci Series acts as a vortex-like temporal damping function at each level of the auditory hierarchy of $_{2}12^{n}/12$. This is the cognitive "gravity" of music harmony.

95

**Principle 27:** The oscillating ratios of the Fibonacci series represent increasingly calm areas within an octave and semitone where energy may be exchanged between harmonic wave partials. Common practice music theory and preferred voice leading was a direct result of a natural cognitive awareness of this energy transfer.

96

**Principle 28:** Common practice use of the tritone and Tritone Function in music harmony follows the oscillating behavior of the Fibonacci Series as it temporally damps any harmonic standing wave. The universality of this principle in Western history suggests the human brain is itself organized like a standing wave.

# *Anticipation Reward*

*"Our thinking and our behavior are always in anticipation of a response. It is therefore fear-based." - Deepak Chopra*

Taking a moment now for a quick review of the psychological theories presented earlier, we might now see how they all seem to hint toward *Harmonic Interference Theory*. The Longuet-Higgins Tonal Space model of tone adjacency exhibits the same spatial economy and coherence found in the energy-form equivalence of a standing wave and dodecahedron. Roger Shepard's double helix is itself the exact same dodecaphonic standing wave rotated and shifted around the central Harmonic Axis. His Gaussian "Bell curve" needed to produce Shepard Tones is also directly related to the ***INTERFERENCE*** family of Gaussian distributions. Even the "half twist" of Dmitri Tymoczko's orbifold can find its physical origin in the half twisting vortex that forms around Landau damping wells at each level of the Harmonic Hierarchy.

Many of Diana Deutsch's findings are also supportive. Her proposed scale-chord hierarchy underlying a musical grammar is, to some degree, an abstraction of the 5-layer Harmonic Hierarchy. Her *Musical Paradox* seems to experimentally confirm the ambiguity between tritone Partials 5 and 7 at the small anti-node gap in a standing wave. She proved beyond a shadow of a doubt that people do, in fact, recognize opposing movement either side of the tritone. We can only conclude that music perception is an organic process of anticipating and recognizing energy currents as they flow through the highly predictable interference patterns of harmonic waves.

These and many other music cognition studies and mathematical music treatises can be explained in a wholly consistent fashion by the intuitive natural principles of *Harmonic Interference Theory*. Still, there remains one other mystery of music harmony we have yet to fully address.

We have established that harmonic cognition is rooted in the anticipated energy exchange across Landau damping regions according to Fibonacci proportions. But how is it that we would be able to instantly recognize an infinite number of such locations? How could this be done efficiently and effortlessly by our naturally evolved auditory system? Moreover, at any given moment in a piece of music, how is it we can instantly anticipate where a melody is taking us or what the next move is likely to be in a chord progression? How can we tell what the "proper" voice leading should be in polyphonic counterpoint? What is the big clue that makes it so easy for us to *predict* harmonic movement?

The answer to these questions may be that our ears use a rational shortcut to make it far easier and more efficient to anticipate voice leadings. That is, our auditory system could recognize and anticipate the flow of energy along a simpler *coherent pathway* through harmonic interference. In place of an infinite number of discrete Fibonacci gaps, our auditory system may simply approximate the irrational quasi-periodic flow using the rational pathway of Partial 5 and corresponding pentagonal geometry There are a number of good reasons why this could be the case:

1. *Partial 5 comes closest to intersecting the Φ-damping frequency proportions at the x-axis than any other harmonic frequency,*
2. *Partial 5 is the only harmonic wave that* perfectly *intersects the fundamental at each equal division of twelve, making it an ideal pathway for our ears to follow,*
3. *Partial 5 is one of the tritone partials in the harmonic series, central to resolving ambiguity and central to the Tritone Function in common practice,*
4. *The golden ratio at the center of the Fibonacci series gets its irrationality from five (e.g., $\Phi = (1 + \sqrt{5}) / 2 = 1.618033$), recommending Partial 5 as a rationalized Fibonacci substitute.*

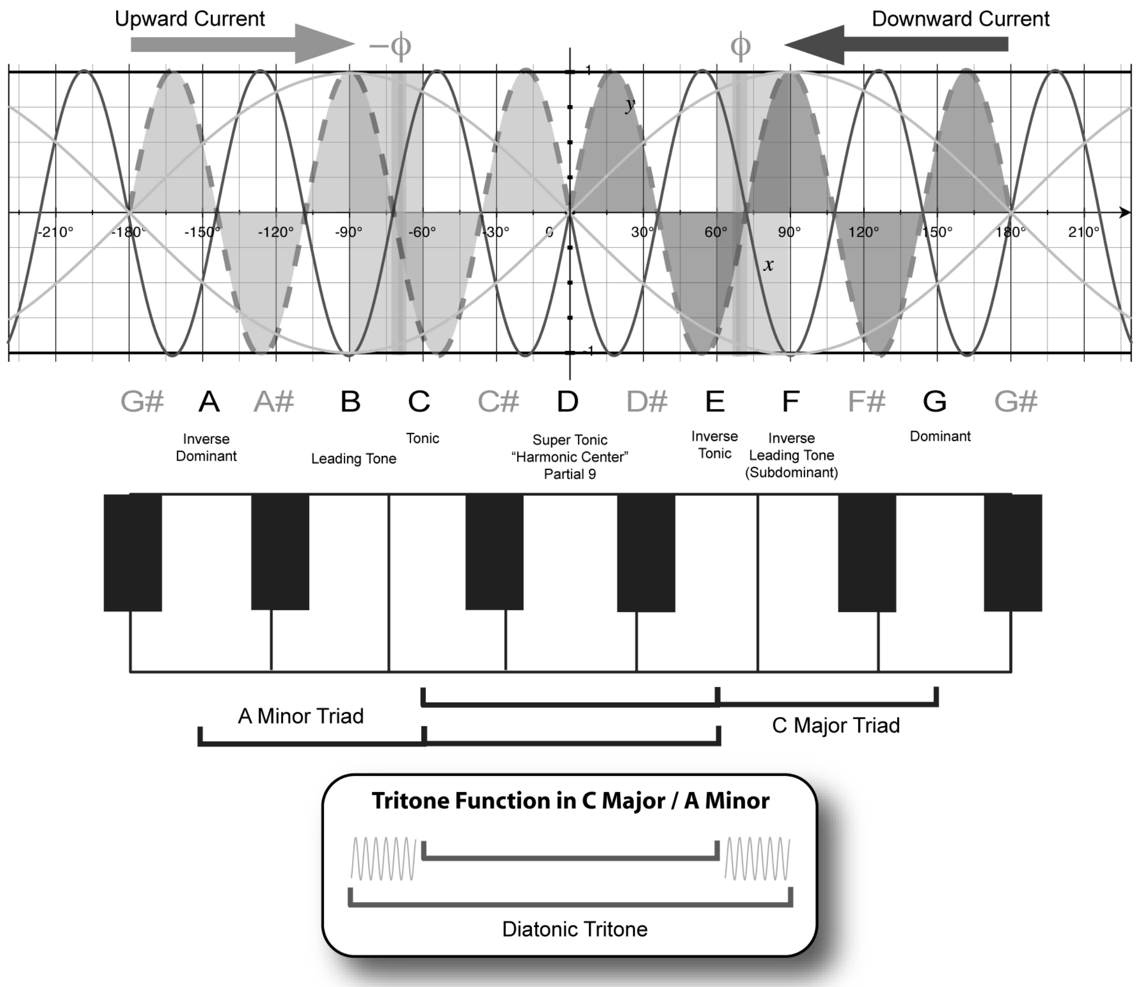
Using Partial 5 as a kind of pentagonal weather vane, directional movement in diatonic harmony could be determined by measuring the weighting and direction of harmonics intersecting it. Musical tones in the half octave *beneath* the Harmonic Center would follow the *energy gain* of Partial 5 upward in pitch while tones in the half octave *above* the Harmonic Center would follow the *energy loss* of Partial 5 downward. In this way, tones would be symmetrically opposed across the Harmonic Center, following the half twisting path of Partial 5 (see again *Principle of Tritone Duality*).

In Figure 59, this simplified energy flow model is represented using the complementary shades of light and dark to indicate the oscillation state of Partial 5. Superposed over an octave on a piano keyboard, the current state and directional tendency for any tone or combination of tones is easily predicted according to which up-down state the wave is in either side of the Harmonic Center {D}.

Of course this diagram is just a static snapshot of what is in reality a continuously oscillating standing wave aligning over an octave, but slowing it down makes it possible to illustrate how harmony could be perceived as a series of discrete moments. In the model, when the energy direction reverses due to new (diatonic) tones sounding, the wave would flip along the horizontal axis and the assigned shades (or colors) exchange. This process then continues back and forth (like a spring) between potential and kinetic energy depending on which tones of the Tritone Function are sounded.

So, in the key of {C}, the tritone {B, F} would be anticipated to contract inward to the Tonic major $3^{rd}$ {C, E} as shown in the diagram. But once this expectation is satisfied and the tones {C, E} are played, a new audible sensation triggers an expectation for the harmony to expand back out. In the diagram, the wave would then flip and the current would be shown flowing along the *opposing phase* of Partial 5. This back and forth process would continue ad infinitum until resolving on the Tonic triad {C, E, G} where {C, E} cancel out and {G} lands in an upward flowing (energy gaining) position, creating the happy ethos usually associated with a major key.

**Figure 59 - Coherent pathway of Partial 5 (C major)**




Following the directional current of Partial 5, all diatonic tones would then be anticipated to follow this same alternation – even when elements of the Tritone Function are not played. This occurs because the process of energy transferal in each wave partial would be recognized as flowing symmetrically toward the Harmonic Center over *all* of the tones in the surrounding octave.

Using the two phases of the Tritone Function as a model, two opposing states for all diatonic harmony can be defined. Within the key of {C} major (or {A} minor), for example, these phase states would be:

**Diatonic Phase 1** = *{B, F} = {up, down}*
**Diatonic Phase 2** = *{C, E} = {down, up}*

If you like, try these out on a piano and combine them with other diatonic chords. You will find that the anticipated direction of energy flow either side of the Harmonic Center is reinforced by whichever tone of the Tritone Function is being sounded in that half octave. This would be the logical trigger to help our brain *sample* incoming waves, resolve ambiguity and create an overall context for perception of music harmony.

For instance, if {F} is being played in the upper half-octave to orient toward a downward current, then its neighbor {G} would also be interpreted as tending *down* along the Partial 5 pathway. But if {E} were being played together with {G}, then both would be interpreted as tending *up* along the opposing phase of Partial 5. Using the phase state model, a {G} in Diatonic Phase 1 tends down while {G} in Diatonic Phase 2 tends up.

Among other advantages, the phase state model is quite useful in predicting the cognitive properties of chords. For instance in the key of {C} major, the {F} chord {F, A, C} would have the directional energy {down, down, down} because the {F} tends down in Diatonic Phase 1 at the same time the tones {A, C} also tend down in Diatonic Phase 2. Together, the entire F major chord would be anticipated to ride Partial 5 downward in pitch space, losing energy as it does so. If this expectation were fulfilled in some way, say by moving down to a Tonic triad, the wholesale loss of energy might be perceived as “restful” or “passive.” In fact, this very chord progression has been used for centuries for precisely this purpose. Known as the *Plagal Cadence,* or “Amen Cadence,” it appears frequently (one might say religiously!) at the end of Church hymns to create a submissive ethos.

Melodies can also be seen to follow the phase state model using memory as an aid in pattern recognition. Consider the simple case where a melody is a continuous arpeggio (playing in sequence) using the Tonic triad {C, E, G}. This melody will have the same effect as the triad because it occurs in a very brief period of time and we remember the tones as a group. In this case, we would anticipate the direction as {down, up, up} or, on average, upward. Of course, the melody might do anything next, but the *reward potential* would not be as great as it would if the next note of the melody followed the average direction *up*.

So, as a piece of music is played, these harmonic phase states based on Partial 5 would instantly set up a chain of expectations for which direction, or “voice leading,” the melody and chords are likely to take next. No cognitive decoding is required, as the phase would be detected at a very high resolution by the tiny hairs of the basilar membrane in the inner ear. Whichever direction our ear tells us the energy is moving, amplified by whatever tones are being played, that is the direction we would predict next.

This marriage between the physics of sound and the physiology of the human auditory system explains the *anticipation-reward potential* of music as a predominantly *psychoacoustical* phenomenon – not simply a function of cultural indoctrination and psychological interpretation as still widely believed.[97]

## Redefining Music Harmony as Standing Wave Symmetry

Major and minor scales can also be explained by the opposing energy flow either side of the Harmonic Center. According to the ***REFLECTIVE INTERFERENCE*** model, a minor scale must be represented as an *auditory reflection* along the central Harmonic Axis. From this, every minor chord and melody would be interpreted as a mirror image of its major counterpart. In this "upside down" spatial orientation, a reversed reading of standing wave oscillation would follow the energy exchange downward, creating a dark or "sad" ethos opposite to the upward "happy" directional flow of a major scale. The mechanics of this will be covered in more detail later, but it is already apparent that the Greeks were essentially correct in their musical system of opposing male and female modes. There is indeed a system of dual ethos intrinsic to music harmony, originating in our natural ability to recognize and anticipate energy reflected either side of the Harmonic Center.

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**Principle 29:** *Anticipation-reward potential* in music harmony can be measured using Partial 5 as a "coherent pathway" through the interference pattern of the harmonic series. This pathway is hypothesized to be recognizable by the auditory system in two concurrent and opposing phase states of oscillation either side of the Harmonic Center. At the octave level of the interference pattern, the "cognitive cue" for which phase to recognize can be defined by which member of the Tritone Function is in play and its oscillation state. The phase indicators around the Harmonic Center are:

Diatonic Phase 1 = tritone = {up, down}
Diatonic Phase 2 = major 3rd = {down, up}

The auditory system may then measure and anticipate the potential direction of movement as an averaged direction in each half octave around the Harmonic Center, ideally using the Diatonic Phase indicators as cues. This principle applies over time within a memory context to predict melodic direction and overall musical momentum.

In the most general form, anticipation-reward potential follows the oscillation and energy exchange in a standing wave.

As explained earlier, the idea of symmetry in music continues to find little support in our present educational system. When music was formalized in rules under the medieval Church's policy of tritone avoidance, our asymmetrical musical system became firmly entrenched, redirecting attention away from the natural balancing properties of harmonics. Once the rule of asymmetry became the first principle, the notion of any corresponding balancing effect in music theory was eliminated and all naming conventions became a unidirectional bottom-to-top reading of scales.

This unbroken conditioning has led to utter confusion about how harmony really works and how we are able to recognize it. As yet another *complicity of convenience*, common practice education has continued to promulgate the idea that music is entirely manmade and subjective, making it a foregone conclusion by all those indoctrinated that there is no natural balancing property in nature and that music perception does not result from any kind of natural process. In turn, this philosophy has spilled over into every area of science and society, convincing everyone that nature is intrinsically chaotic and nonsensical. Much more than just an inconsequential side note to history, an unnatural view of music has had a decidedly deleterious effect on Western philosophy and (through overwhelming Western influence) the rest of the world.

The poster child for this presumption of asymmetry is the continued use of the term "Subdominant" in music education. As a "Dominant chord underneath the Tonic", we might expect that a Subdominant triad would resolve up along the Cycle of 5ths to the Tonic. But as voices in a diatonic scale, the Subdominant is better described as the *downward* flow of energy in a harmonic standing wave. As explained earlier, the Subdominant tone really moves downward as an *Inverse Leading Tone* in tritone opposition to the upward pulling Leading Tone in the Tritone Function.[98] As a result, any music curriculum based on *Harmonic Interference Theory* must use a bi-directional nomenclature (or scale labeling) corresponding to the symmetrical oscillation of energy in a physical standing wave.

Like a velvet curtain over the symmetry in nature, the old asymmetrical naming conventions must be pulled back to learn how harmony really works. Though painful to admit, our educational system continues to teach music from behind this curtain without any regard to natural physical processes. Tearing down this curtain begins by revising music textbooks to use symmetrical naming conventions based on the physical model of a standing wave. Music educators and book review boards must free themselves from Church tradition and reintegrate harmonic principles back into the study of music. Likewise, the institutions and societies within which they work must also allow harmony back into the Arts and Sciences.

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[98] Musicians can prove this to themselves. First, play the Dominant chord in the key of C major based on {G, B} as a sixth chord {G, B, D, E} played low to high. Now play the Subdominant chord as a sixth chord sitting upside down (like the waves themselves) built on {A, F} or {A, F, D, C} played high to low. If you now read either one of the chords from low to high and drop the sixth, the Dominant becomes {G, B, D} and the Subdominant becomes {F, A, C} or the F triad.

**SECTION THREE**

# Psychophysiological Principles

*"We haven't really paid much attention to thought as a process. We have engaged in thoughts, but we have only paid attention to the content, not to the process." - David Bohm*

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The book entitled *Isomorphic Models for the Function of Harmony in Time* went unpublished. After working on it for three years graphing every periodic cycle in dodecaphonic music I could find and constructing every kind of 2-D and 3-D variation, I decided to shelve it. While it certainly seemed like the right way to explain music, I simply could not understand why it should be correct or how it might apply to the perception of sound. It seemed more like a solution in search of a problem.

Still, I would daydream about how elegantly the trigonometric waves would intersect one another, coinciding perfectly at each of the tones in an octave. It was obvious to me that music was related to the interference of harmonic frequencies in sound, but how this was processed in our ears and brain seemed completely out of reach.

In 1988, while researching and applying linguistic principles to computer software, I was surprised to find that words and *even our thoughts* follow a predictable Gaussian distribution. During the development of a full-text search engine named DARWIN[99], a predecessor to Yahoo! and Google, my software team was tasked to find a way to retrieve documents based on their *relevance* to whatever search terms were used.

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[99] DARWIN was an acronym for Data Retrieval WIth Intelligence.

At that time, search engines (then known as Text Information Management Systems or TIMS) could only display retrieved documents in chronological order or other fixed sorting criteria. There was absolutely no idea how search results could be automatically ranked in order of relevance as we see in today's Internet search engines. Yet, it was obvious that the relevance ranking of documents was becoming critical to finding specific information in the world's rapidly growing digital archives. All we needed to do was figure out how to do it.

In those years the American Society for Information Sciences published a monthly journal containing the latest research in the field of linguistic science. In one issue, I noticed an interesting paper by R. Losse, a highly regarded semantics and structural linguistics professor at the School of Library Sciences at the University of North Carolina at Chapel Hill. He demonstrated how a common statistical metric, called the *Poisson distribution*, could be used as a "random and best-first document selection model."

Statisticians would not find this particularly surprising, considering that the Poisson distribution arises in many different situations where the probability of events is constant in time or space. Nevertheless, it is still amazing that this pattern is found in just about everything we do, such as:

- *the number of cars that pass through a certain point on a road,*
- *the number of spelling mistakes made while typing a single page,*
- *the number of phone calls at a call center per minute,*
- *the number of times a web server is accessed per minute,*
- *the number of road kill (animals killed) found per unit length of road,*
- *the number of mutations in a given stretch of DNA after a certain amount of radiation exposure,*
- *the number of unstable atomic nuclei that decayed within a given period of time in a piece of radioactive substance,*
- *the number of pine trees per unit area of mixed forest,*
- *the number of stars in a given volume of space,*
- *the distribution of visual receptor cells in the retina of the human eye*
- *the number of light bulbs that burn out in a certain amount of time,*
- *the number of viruses that can infect a cell in cell culture,*
- *the number of hematopoietic stem cells in a sample of unfractionated bone marrow cells,*
- *the number of inventions of an inventor over his/her career, and*
- *the number of particles that "scatter" off a target in a nuclear or high energy physics experiment.*

As I read the Losse article, it suddenly dawned on me that words in documents were not unlike music – just a different kind of event occurring in time. I also realized that the Poisson algorithm could easily be added to our existing software architecture to rank the relevance of any document according to the frequency of search terms found. Within only a few days, Poisson relevance ranking had been added to DARWIN, astounding everyone with its accuracy in recommending the right document.

It worked because it follows a universal distribution of words found in all languages.

When documents were retrieved for a given search query, DARWIN would rank them by the frequency of occurrence of each word relative to each document and then calculate a *mean degree of relevance* (called a *lambda value*) for each document using the Poisson distribution equation. This allowed the software to display the documents in order of relevance to the search criteria with the best matches shown first. We also added the ability to determine the mean lambda value and Poisson curve for *an entire library* by finding the mean statistical relevance, or word frequency, of the entire population of words. By doing this, we could measure relevance between different collections and types of documents.

Not too surprising, legal libraries had the lowest relevance ranking of all documents due to their high use of redundant and "irrelevant" legal terminology (lawyers would be the first to agree with this). On the other hand, general information magazines, like *Time* or *Newsweek*, had the highest relevance ranking due to the very large variety of searchable names, places and product advertising text. In general, this technique allowed us to extract a basic profile, which we called a *relevance signature*, from any document collection by simply identifying the in-context use of words.

The philosophical implication of this study for the way we think and communicate is actually quite profound. No matter what word anyone may choose to say, it will fall into a statistical distribution of frequency (or relevance) that can be modeled by the Poisson distribution. In fact, if all the text in the chapter you are now reading were analyzed by word frequency it too would trace out a rough Poisson distribution. Analyzing this entire book would create a still smoother curve while analyzing all of the non-fiction books on Amazon would create an extraordinarily smooth Poisson curve. Each of these curves would exhibit a different relevance signature, ranging from a shallow curve to deep sweeping arc, depending on the word distribution in each collection. Yet, they would all still follow the same natural Poisson distribution. I was shocked to learn that I simply could not type, speak *or even think* a word that would not create this curve after a minute or two.[100]

The impact of this revelation was nothing short of Earth-shattering for me. It was clear that the structure of language was not simply an arbitrary learned behavior, but must be the *only*

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[100] I found the same thing to be true in a personalization study I did a few years back for Microsoft Canada. Their online customers could customize their e-newsletters by selecting from a wide variety of categories and topics. Over a six-month period, more than 46,000 different choice combinations had accumulated. Sorting these created a very smooth Poisson curve, just as I had found in my linguistic research a decade earlier. It seems even "free will" itself follows a predictable path for a given population.

*possible result* of a universe defined by the orderly, or "normal," distribution of what seemed like random events. This triggered a cascade of realizations for me about how we perceive music.

I imagined that harmony must also follow a similar predictable distribution in order for us to recognize and follow relevant voices and melodies out of a background field of noise. I also started seeing language as a form of harmonic interference where thoughts emerged into coherent forms out of a background of gibberish, nonsense and filler. The question I began to ask myself is how should we describe the deeper principles that give rise to *both* language and music? Since the Poisson distribution is actually a special case of the Gaussian normal distribution – in the region I later came to learn centered on a golden section of the curve – it seemed likely that this "normal distribution" might actually describe how the brain processes music.

The more I thought about it, the more I began to see language and thought as a kind of resonance or consonance in a field of Gaussian interference. I also started seeing low relevance words and thoughts as a kind of dissonant white noise or damping action in the brain. The most relevant thoughts became those that resonated into happy things like innovative technologies and successful careers while less relevant thoughts would end up in accidents, mistakes and dead ends. And like harmonics in a standing wave, when relevant trends or innovations crossed at just the right time, they would amplify one another and bring about a change in direction, often suddenly and in unexpected ways. To me, the entire world was improvising music.

I started thinking of a standing wave as something directly applicable to my life and work. For me, the game of life became a quest to find the most relevant and coherent paths through the field of noise and nonsense. I began to see my habit of improvising jazz at night as a sort of philosophical laboratory to test and develop coherent intuition for the real world during the day. I figured the better I could get at forming complex harmonies in real time on the piano, the better I would be in making sense of the dynamically changing marketplace and creating the right products at just the right time. When the unexpected happened (which it always did), my goal would be to find a way to change that "wrong note" into a new theme or, if this was not possible, look for a graceful ending in order to start a new "song."

Opportunities would constantly appear. As this would happen, I imagined myself as a surfer looking for the perfect wave. I saw my job as choosing the right waves to catch while letting the weak and untimely waves pass me by. Even though I had little or no direct control over which waves came my way, I certainly had the ability to choose which to accept and which to reject. From here, the only question left was which waves to choose.

Just as it happens in jazz, decisions for which path to take are always best when they occur intuitively. But for me, intuition was never some kind of meta-magical thinking. It was a rational *system of thought* based on observing which patterns worked and in what cases, then weaving those patterns together like music. During the dot-com boom of the late 1990s, I would often say that my favorite hobby was "collecting business models," which I saw as a form of musical pattern matching or *harmonic thinking*. As part of all this, I would pay close attention to the things successful people did, storing these too into my library of musical business patterns. Like learning a complex piece of music, everything was analyzed down to its core philosophy – then, internalized so I could recognize and "play" the right patterns intuitively. In time, this intuitive system became my business philosophy, helping me decide which waves to ride and which ones to let go.

As my jazz improvisation improved, so did my choice of opportunities. I would envision myself stepping off one wave and onto another without falling off. A big wave might be a new company while a small wave might be a new project or a new business contact. Some waves might be tiny ripples, like the just-in-time arrival of a new person with the perfect talent, while another wave could be a disastrous tsunami threatening to knock me (and my company) off course. This process eventually matured into a real-time blending of look-ahead strategy, tactical execution and pure intuition. When the decision was a good one, it always *felt* right and I knew that I was still "in-synch" with the world and the marketplace.

Whenever I would take a moment to reflect on how this *musical system of thought* seemed to model society and the marketplace so well, I would wonder why no one else I knew used something like this for guidance. I wondered why no philosophy of nature could be found in the business world, the universities or for that matter anywhere else. It seemed like most people would either use brute force to try and control events or had just given up and resigned them selves to react to whatever hit them in the face. Experience with harmonic intuition suggested that nature could never be forced – it could only be surfed gracefully.

The more success I had with this philosophy, the more I became convinced that my old music theory was correct. I knew if I could find the time and have access to the right research, I might figure out how our ears and brain really worked. But in those days, without an Internet filled with scientific research to help connect the dots, my questions went unanswered.

# ***Spatial Coherence***

*"Take but degree away, untune that string, and, hark! What discord follows; each thing meets in mere oppugnancy." - William Shakespeare*

During the 1970's when Roger Shepard and Diana Deutsch were pioneering the field of music cognition, two musical set theorists were also publishing their ideas about musical scales. David Rothenberg introduced his model of *musical propriety* in the 1978 paper *A Model for Pattern Perception with Musical Applications Part I: Pitch Structures as order-preserving maps*. In it he defined musical scales as a "quasi-periodic function," using the mathematical term for any irregularly repeating sequence. He said scales should always repeat at certain fixed intervals – higher with each note in a finite set of notes (normally an octave) – and that no musical scale should be allowed to have a pitch that is less (or more) than the pitch of another scale. Known as *Rothenberg propriety*, this theory set forth scales as exclusive, repeatable tiled patterns within an easily recognizable pitch space.

Gerald Balzano, professor of music at UCSD, called this same idea *coherence* first in a 1980 group theory paper and then a 1982 follow-on paper entitled *The Pitch Set as a Level of Description for Studying Musical Pitch Perception in Music, Mind and Brain*. In these papers, Balzano worked to characterize five proper scales (diatonic major, ascending minor, harmonic minor, harmonic major and major locrian) as exclusive in the pitch space of *19 Equal Temperament* (or 19-ET). Since then, Balzano has remained a leading advocate for 19-ET as a likely replacement for 12-step equal temperament.

This idea of dividing an octave into nineteen steps (or any other number) is an important one because it challenges any assumption of twelve steps as an organic organizing principle in music theory and cognition. It was originally proposed by Joseph Yasser in his 1932 book *A Theory of Evolving Tonality*, which incidentally was the first book I read that did a really good job of explaining how the spiral of pitch can be cut and stretched to form a circular octave. In it Yasser argued that 19-ET offers a formalization of the microtone systems used in world music and provides a better scale for the stretched "blues notes" of American Jazz and Blues. Yasser even went so far as to speculate that mankind appears to be evolving away from 12-ET and toward a 19-ET division of the octave.

When I first read about this in the 1970's, I was unconvinced that a non-symmetrical prime number division of the octave, like 19-ET, could be as satisfying to a symmetrically organized human being as 12-ET. While I had not yet formulated a reason for my view, there was nothing in a cycle of nineteen that seemed as organic as twelve despite any rationale for why it should be. I considered the fact that damping locations in an octave would still be traversed in 19-ET and that we might still be able to recognize the current flow, but it seemed to me then (as it does now) that the contrast available within a symmetrical and simpler $2^2 \times 3$ division of the octave would be easier to recognize than a prime number division. Beyond this, the microtones of world music, like those of the Indian raga system and African blues notes of jazz, never seemed to me to be fixed tones – instead, more like glissandos or dynamic shades designed to emphasize the spring effect of the Tritone Function.

Unbeknownst to me then was the evidence now presented by *Harmonic Interference Theory*. It suggests that spatial coherence within the natural harmonic series and our recognition of it is clearly maximized at twelve – not nineteen or any other division. Frankly, I am always perplexed by the notion of any artificial division or temperament of the octave because it seems entirely disconnected from the natural cohering principles of a standing wave from which all things originate. This said, division by twelve is no guarantee of coherence either.

Non-equal 12-step temperaments can also lack the contrast and geometrical balance necessary for recognition. They can sound out-of-tune in distant keys and encroach or entirely overlap the pitch space of other scales, making recognition difficult at best and horribly offensive at worst. Even though "spiral favoring" temperaments, like Just temperament, produce intervals that are more concordant and consonant sounding in the reference key, transposition will render melodies and chords literally less coherent to our ears' innate sense of balance. Rothenberg and Balzano both thought the exclusion of overlap in pitch space was necessary to create the propriety or *spatial coherence* needed for cognitive pattern recognition.

At the center of this discussion then lies the paradoxical tritone. As we found in the intersection of tritone Partials 5 and 7, a split or schisma occurs at the two anti-nodes of the fundamental's cosine component that causes the ambiguity we sense in a tritone. In common musical practice, this ambiguity is generally considered a non-distinct "fuzzy" area and handled by either stretching a little beyond a half-octave to a diminished $5^{th}$ or slightly inward to an augmented $4^{th}$. This is precisely where the perception of spatial coherence becomes most sensitive, just as Diana Deutsch's *Musical Paradox* proved so well. When these two stretched versions of the tritone are used inconsistently and in uneven temperaments, recognition of scales and any music based upon them becomes inconsistent, unstable, less recognizable and less predictable. In fact, diatonic harmony can simply cease to work if the Tritone Function is not well balanced and given enough space to "spring" around the two octave Φ-damping locations.

In this sense, Balzano's and Rothenberg's proposed spatial coherence and exclusivity in pitch space are really the same principles needed for harmonic partials to coexist in the same medium (e.g., on the same string). Music harmony is most coherent – and *most organic* – when it aligns to the harmonic series within an equal division of the octave by twelve. However, it is not correct to say that 12-ET is the only recognizable tuning for an octave.

Fact is, *any* tuning is recognizable as long as it provides exclusivity and cyclic closure in pitch space.[101] We might think of the many different scale temperaments as a kind of artificial flavoring for music – fun for a change, but not the soul food we come home to.

So how do we sum all this up into one simple idea? The answer is found right in the definition of coherence as it applies to wave interference.

> ***Coherence*** *is the property of wave-like states that enables them to exhibit interference patterns. In interference, at least two wave-like entities are combined and, depending on the relative phase between them, they add constructively or subtract destructively to form a consistent pattern. The* degree of coherence *is equal to the interference visibility, or how stable a given interference pattern is.*
>
> ***Standing waves*** *exhibit the greatest stability and coherence in space and time – thus, harmonics produce the most visible interference pattern possible.*

As we saw with the Harmonic Symmetry Table earlier, the interference pattern produced by a harmonic standing wave is highly predictable and most coherent; so much so that each harmonic is always in the same place, balancing around π-symmetry locations and exchanging energy at Φ-damping locations. The more a given harmonic pattern aligns around these positions, the easier job our ears will have locking in and pattern matching music against the spatial and temporal proportions in the harmonic series. In this way, we can view the harmonic series itself as a natural template of coherence for aural cross-correlation and spatial recognition.

In light of *Harmonic Interference Theory*, we can say that Rothenberg propriety and Balzano coherence related to non-overlapping scales is really rooted in the highly predictable behavior of a standing wave and its harmonic series. And regardless of which scale temperament we choose to use, anticipation-reward potential is maintained as long as closed quasi-periodic shapes are allowed to exist in pitch space without overlap. This is like saying visual objects in physical space cannot occupy the same space – they must coexist like harmonics. From the perspective of spatial coherence, the rules for perception are the same for both vision and audition.

Within a given temperament, the *degree of coherence* would then be measured by how closely a scale (persisting as a key) aligns to those stable positions of π-symmetry and Φ-damping in a harmonic interference pattern. And like vision, cognition of musical harmonies becomes a matter of edge detection and *shape recognition* of the pattern of special fringes in the interference. Regular geometries, like the pentagonal dodecahedron, would be the most recognizable regardless of whether they are experienced in visual or aural space.

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**Principle 30:** Any temperament (tuning method) that provides exclusivity and cyclic closure in pitch space will enable the recognition of harmonic shape to some degree against the proportions of the harmonic series.

Simply put – music is audible geometry. The more regular a shape becomes in pitch space, the more predictable and coherent it is. A pentagonal dodecahedron is the most coherent geometric shape possible in visual space while a dodecaphonic octave is its aural equivalent. Mathematically, both maximize coherence in cognitive space because, according to our earlier tritone crystallization experiment, they share a common origin in the physics of a standing wave. As tritone partials induce this form in their interference pattern, we prefer to hear symmetry and equally spaced geometry simply because it is the easiest to recognize.

This is the way the Greeks understood music – the transformation of spatial proportions into musical ratios. Not in some abstract philosophical way, but in a fully quantified proportional geometry and number theory. To them, examination of harmonic principles in nature was most accessible through the study of harmony in music.

Today, the property of wave coherence is the basis for many commercial applications, such as holography, the Segnac gyroscope, radio antenna arrays, optical coherence tomography, and telescope interferometers. Each of these applications correlates the phase between waves, examining visible "fringes" and measuring cancellation or "damping" between signals.

In nature, coherence is usually achieved by repeatedly nesting something inside itself, called *recursion*. For instance, each branch on a tree is a replica of the branches immediately preceding it. Reproduction in life is a form of recursion where new life emerges from inside existing life. This organizing property bubbles up from the atomic structure of matter, which is itself a recursive process. In fact, *all* of nature is built upon recursive principles, coherently balanced between damping and resonance during the recursion process.

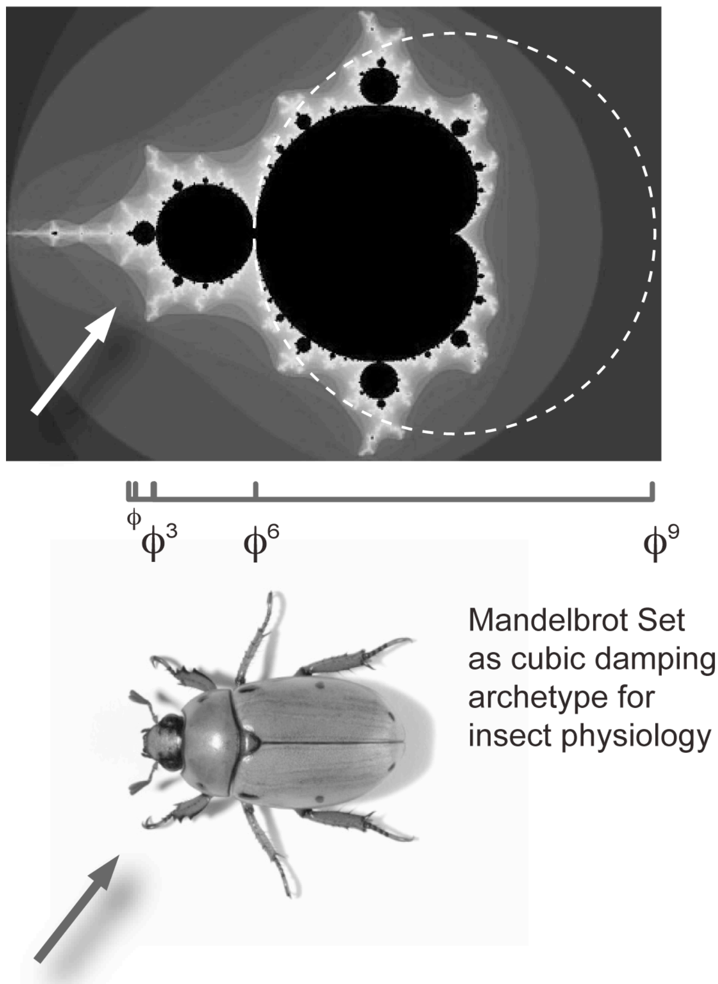
This omnipresence of recursion in nature was the inspiration behind the discovery of *fractal geometry* in the 1970's. At the center of this revolution in geometry was one particular fractal shape known as the *Mandelbrot Set*. Discovered by mathematician Benoit Mandelbrot, it represents nothing less than the spatial boundary between chaos and order. When Mandelbrot first saw it he believed he was looking at the "face of God." Today, most people think of it as computer art, but it is much more than this. It is really a visual form of harmonic recursion.

Looking like the abdomen, thorax and head of a beetle, with its large cardioid region connected to a series of shrinking circles, the recursive Mandelbrot Set is the epitome of spatial coherence. The horizontal x-axis represents exponentially decreasing circular regions while the vertical y-axis represents the symmetrical products of positive and negative even numbers. This unique equation, expressed as a sine wave that has been Φ-damped into <u>three</u> circular regions of cubic powers, acts to attenuate the vertical resonance of the even squares of the harmonic series.

When compared to a beetle, the Mandelbrot equation would suggest that it is really a kind of cellular octave resonance carved into shape by a cubic Φ-damping envelope. The damping extends to an even greater degree into the branching areas at the end of each "bulb" location. The similarity between the "fractal lightning" and the front legs and antennae of the beetle demonstrate how harmonics really do play a central role in the growth structures of life. The simple truth is an insect is actually an elegantly crystallized set of harmonics.

**Figure 60 - Cubic damping in the Mandelbrot Set**

$$x_{n+1} = x_n^2 - y_n^2 + a \textit{ and } y_{n+1} = 2x_n y_n + b$$




Theories about the very smallest structures of the Universe also depend on coherence. *Quantum coherence*, which is the scientific study of in-phase states in quantum mechanics, is found in such phenomena as lasers, superconductivity and superfluidity. For instance, superfluidity is the study of atoms (such as helum-4) that have been cooled to the point that they become highly coherent. The result is something called *Bose-Einstein condensate*.

In this exotic state of matter, atoms are said to resonate in-phase and within a common Schrödinger wave function. One of the things researchers found with this condensate is the golden ratio plays a central role in encouraging coherence and enabling harmonic structures to form. In fact, one study using a laser to cool calcium within a "golden ratio quasi-electrostatic 3-D lattice" was found to be the most efficient way to generate Bose-Einstein condensate [Adams, et. al. 2003].

In general, the golden ratio can be found everywhere coherence occurs in nature. For instance, the recursive "heterodyning" of new waves from existing ones using the Schrödinger wave function also relies heavily on the golden ratio. As two electromagnetic frequencies combine to create two new frequencies, one at the sum of the two mixed frequencies and the other at their difference, Φ acts as a stabilizing influence in the wave function. Recent theoretical research has found that the Schrödinger wave function itself is the result of an infinitely recursive summation of wavelengths directly connected to, or a result of Φ [Giandinoto 2008]. Like the musical difference tones of Rameau and Tartini, electromagnetic waves produce new frequencies only through the recursive coherence of Φ.

As a fundamental property of all matter, coherence is also critical to chemical bonding and the formation of crystalline structures. This process begins when electron standing waves (called "shells" or "clouds") are shared between pairs of nuclei, holding them together within a molecular lattice structure. From this atomic "stickiness" come higher-order structures that build up into fractal-layered hierarchies of ever increasing complexity and variability, not unlike the 5-layer hierarchy for coherent sound.

It was Danish physicist Niels Bohr who first suggested that atoms work harmonically. In 1913, he proposed an atomic model based on the spherical standing waves of electron particles, each in its own orbit and spaced according to whole number proportions. From this early model of atomic harmony, French physicist Louis de Broglie proposed in 1924 that atoms also behave as both a wave and a particle, called *wave-particle duality*. This was perhaps the first full realization of energy-form equivalence, like that illustrated in our earlier tritone crystallization experiment.

In 1927, famed physicist Werner Karl Heisenberg then declared the location of particles in atoms to be uncertain and a matter of probability. He based this on the observation that electrons appear to suddenly leave one orbit and take a *quantum jump* to another orbit, inexplicably disappearing and reappearing without traversing the space in between. He found that electrons were not so much in a particular orbit, but instead distributed out from the nucleus with the highest probability restricted to a given orbit. From this he claimed one could only determine that a particle was somewhere inside the width of a standing wave cloud of orbiting electrons and could only be estimated as the average of one half of a cycle or π radian. This became known as *Heisenberg's Uncertainty Principle* but it might just as well have been called the Tritone Paradox.

This quantum principle is really just another way to describe the embarrassing *schisma* Pythagoras had already found 2,500 years earlier when he split a wholetone in half. As we saw with the ambiguous "fuzzy" anti-nodes of tritone Partials 5 and 7, the uncertainty of the Tritone Paradox applies to standing waves of all kinds and at all levels of reality. How are we to resolve the ambiguity of the tritone without an observer to establish the musical context? And how are we to measure the location of a particle without an observer to interpret how it should be measured? Whether we believe that perception creates our reality or reality is instead an infinite collection of "many worlds," our perception is built upon the harmonic properties of standing waves, which itself is the source of all uncertainty.

Though it is seldom mentioned in polite company, quantum mechanics is just a then veneer over Pythagorean harmonic science. Particles, including electrons, are described as oscillating back and forth like a "particle in a box" forming simple whole number waves according to the harmonic series. Momentum of particles is always calculated from simple harmonic proportions, accurately describing atomic and subatomic behavior. With electrons acting just like harmonics on a guitar string, the same principles of resonance and damping in music harmony apply to atoms – even the golden ratio.

To this point, an article entitled *Persistent currents in mesoscopic Fibonacci rings* recently found that Φ does plays a primary role in controlling the harmonic flow of electrons:

> *"the flux-dependent electron eigenenergies E(Φ) in mesoscopic Fibonacci rings [i.e., the resonance of electrons above the atomic level] still form "bands" with respect to the flux Φ, but there is a scaling relation between the total "bandwidth" and the Fibonacci number. When the strength of the one-dimensional quasiperiodic [Fibonacci] potential increases, the persistent current decreases rapidly. Interestingly, for a generalized mixing model of mesoscopic Fibonacci rings, free-electron-like persistent current may appear if the number of electrons of the system takes a specific value." [Jin, et. al. 1997]*

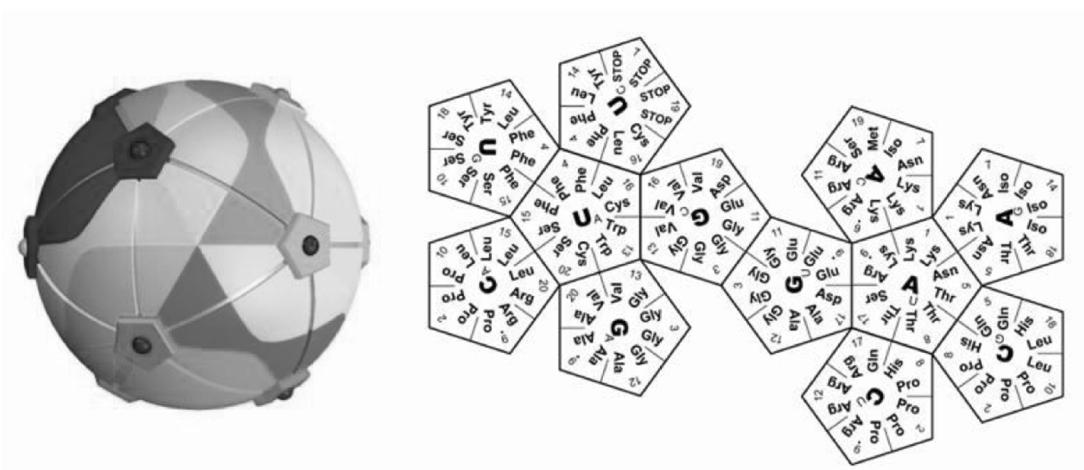
This study found that electron current flow "decreases rapidly" as the strength of the Fibonacci rings increase. However, if the electron count takes non-Fibonacci values (harmonic values) then electron current flow may continue. As a balance of resonance and damping, we may find the ***INTERFERENCE*** distribution can be used to predict and measure the probable movement of electrons as easily as it does the flow of energy in music harmony. After all, the basic principles of harmonic science should apply the same to all levels of nature.

At the human level, the coherence of atomic structure can be explained very simply as crystallized music. From the bottom up, we find musical tones acting like "electron shells" around the "nucleus" of a Harmonic Center, transferring or sharing energy between themselves and other tones. Within a coherent standing wave, tritone partials oscillate across two positions near the ambiguous half cycle, or π radian – the same as the Heisenberg orbital average. The structure of musical harmony behaves the same as atomic bindings, combining into lattices to form the molecular equivalent of scales, intervals and chords – all harmonically spaced in their octave orbits.

The building blocks of tones and intervals then bond together in music, like amino acids in DNA as they fold into the sonic equivalent of protein. They twist and transpose in spacetime forming frozen standing waves of much slower changing helical forms, only to suddenly “mutate” through the uncertain tritone and “phase shift” into another key. Tones might aggregate into spatially coherent geometries of polyphonic counterpoint or the vertical “cells” of homophonic chord progressions. Such structures may replicate and specialize, forming “colonies,” “organs” and “landscapes” that we hear as musical movements and thematic sections. On the flip side, what we recognize as a strong harmonic function in the lifespan of a piece of music becomes a sonic metaphor for layer upon layer of organized electrons heterodyning recursively to damp energy into particles and resonate into life. We cannot help but ask ourselves are we studying the harmonic formation of life or a musical symphony?

A recent paper by physician Mark White entitled *The G-ball, a New Icon for codon symmetry and the Genetic Code* proposed that the codon table of the genetic code follows the shape of a spherical dodecahedron. Since there are exactly four nucleotides in DNA, that combine in sequences of three consecutive nucleotides to produce sixty-four codons ($4^3 = 64$), he proposed that the genetic code organizes itself according to the shape of tetrahedrons grouping together into a spherical pentagonal dodecahedron like that of our crystallized tritone.

**Figure 61 - White G-ball model for genetic codon symmetry**



Credit: Mark White

Following the equilateral genetic structure predicted by Russian physicist and cosmologist George Gamow, the twenty edges of the dodecahedron (or twenty triangular faces of its dual icosahedron) can be used to represent the 20 standard amino acids in DNA. The amino acids are then assigned locations in the geometry according to their water affinity (how much they like or dislike water). From this, protein is represented as a bonded sequence of amino acid tetrahedrons. When the dodecahedron is then rotated around a fixed axis, it creates the well-known ten-step spatial symmetry of the DNA molecule double helix [White 2007].

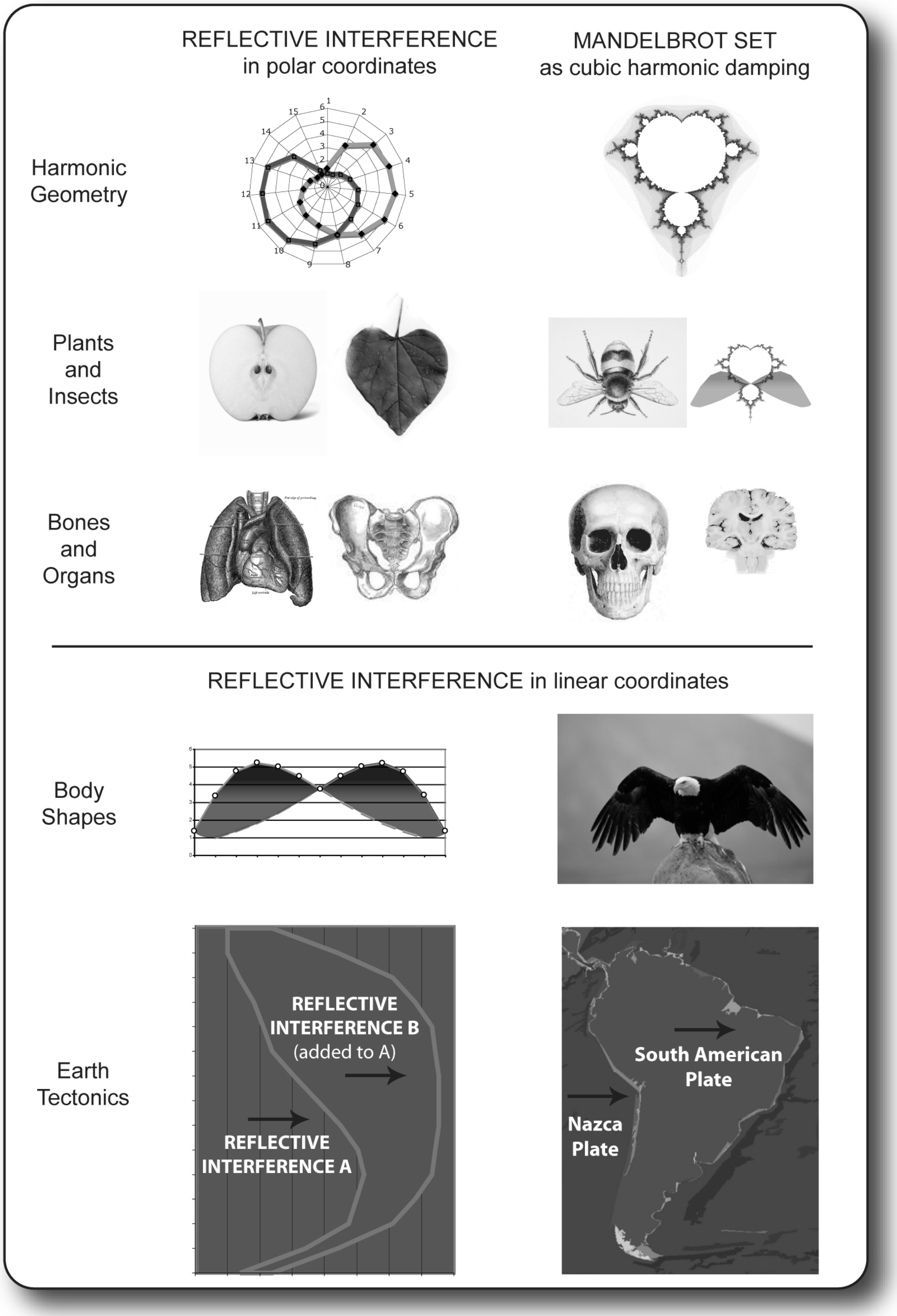
This view of life as a spatial structure of symmetry perfectly matches that found in music harmony. Our musical Harmonic Axis is clearly isomorphic to the central axis of DNA with each of the remaining ten tones representing the ten facets of the DNA molecule produced by its opposing helical strands. The 20 amino acids could also be chemical equivalents to the 20 anti-harmonic points identified earlier in any standing wave (16 amplitude Φ-damping locations plus 4 ambiguous tritone anti-nodes). Even the 64 codons of DNA appear equivalent to the sum of the 20 vertices, 30 edges, 12 faces and 2 halves of a dodecahedron, totaling 64. DNA really is a kind of crystallized harmonic standing wave resonating at different frequencies and timbres to create the full spectrum of life.

In a universe composed of waves, the same proportions found in the harmonic series of sound can be seen everywhere. And since we instantly recognize and prefer dodecaphonic harmony, sensing the various musical qualities at once, we must be coherently structured according to the same harmonic principles. Instinctively, our musical scales evolved in accordance with the harmonic balance in nature and in ourselves. We simply could not help but do this.

In this light, the ***REFLECTIVE INTERFERENCE*** resonance model expresses the simplicity and symmetry of harmonic behavior in the most organic way. It explains in the broadest terms the balance of harmonic and damping forces that must be at the bottom of all things. In the standing waves of quantum structure, atomic binding, molecular lattices, plant/ animal physiology and planetary activity, the same musical forces manifest all around us. Statistical models based on the Gaussian distribution predict the frequency of human intelligence, social trends, market fluctuations and other human behavior as a variance against the coherent harmonic series. And though it is seldom mentioned in either scientific or musical circles, wherever we find harmonic activity we will always find the golden ratio and the Fibonacci series to establish coherence. One does not exist without the other.

Though largely ignored by modern biology, spatial coherence and *not* natural selection is the first principle of life. The integration of our heart and lungs or pelvic structure clearly manifests as a union of the ***REFLECTIVE INTERFERENCE*** distribution in polar space, tilting slightly into a double cardioid. Our skeleton lattice is damped at Fibonacci intervals to carve the small spaces between our joints, enabling movement and ensuring survivability. Even the tectonic plates of continents shift from the Gaussian distribution of momentum in our Earthly standing wave.

**Figure 62 – Reflective patterns of Gaussian *INTERFERENCE* in nature**



Not surprisingly, harmonic principles are also present in the management of heat energy necessary for milk production. It is during female lactation that breasts consume more energy than that of the brain – far more than any other organ – requiring a damping force to contain such resonant production. While medical science suggests breasts are merely an evolutionary efficiency to avoid smothering, it is surely more than this. The most consistent explanation can only be "bio-harmonic damping" as defined by the ***REFLECTIVE INTERFERENCE*** model.

What could be more appropriate than both the Fibonacci growth series and golden ratio being central to the human survival process? And what better position could your slightly slanted heart take in its biological standing wave role than the Harmonic Center of a ***REFLECTIVE INTERFERENCE*** distribution? The Greeks and Egyptians before them understood this basic principle. Why is it not common knowledge today?

Standing waves are the universal principle behind coherence at all levels of reality and this most certainly includes music cognition. When multiple tones occur together, it is the spatial coherence of a standing wave that bonds them into a recognizable interference pattern. This pattern is then *already* in a predictable, natural shape for recognition by our harmonically attuned ears and brain. Any understanding of music should be based on these concepts first and foremost.

Yet underneath it all in the background field of silence lays the stabilizing influence of the Fibonacci series as it spirals its way through the harmonic series. And at the center of this cleansing vortex is the infinite golden ratio, playing the Herculean role of Atlas holding up the world of harmonic structure.

Spatial coherence is the reason we recognize music. We intuitively know how to recognize musical harmonies as geometric shapes created by interfering standing waves. We know instinctively that timbre and harmony is really the same thing, the geometry of coherent sound. And the "empty" space that contains this geometry – no doubt a perfect five-fold fractal hierarchy too – is sure to be bubbling up from inside the quantum lattice as the interplay of harmonics and anti-harmonics. It is all part of a grand coherent spectrum of periodic octaves divided equally by twelve.

At each level of the Harmonic Hierarchy, energy is exchanged across two periodic golden ratio proportions as they balance in seesaw fashion on their Harmonic Center. From this emerges the auditory shapes of triangles, squares, pentagrams, hexagrams and other such geometries transformed from frequency to proportion for <u>instantaneous</u> recognition by our auditory cortex.

These shapes are then pattern matched against the *exact same* harmonic proportions evolved from the bonding properties of carbon-12 within an identically coherent environment.[102] From the simplest song to the most complex symphony, it is the degree to which scales, melodies and chords reinforce the primordial balance in a standing wave and support the exchange of harmonic energy that determine our success in recognizing and anticipating musical harmonies. Music is nothing less than a reflection of the spatial coherence in life itself – intrinsic to body and thought.

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**Principle 31:** Music cognition results from the pattern matching of auditory shapes against the same harmonic shapes evolved into the structure of the inner ear and auditory cortex. The degree to which auditory shapes can be recognized and predicted is defined by how closely the musical scale conforms to a harmonic standing wave, especially following the energy transfer across Φ-damping locations.

# *Temporal Coherence*

*"The whole history of science has been the gradual realization that events do not happen in an arbitrary manner, but that they reflect a certain underlying order, which may or may not be divinely inspired." - Stephen Hawking*

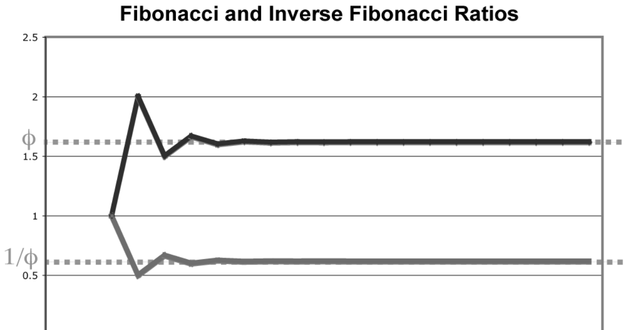
Given the importance of spatial coherence in music perception, what should we say comprises a similar framework for *temporal coherence*? If the dodecahedron represents spatial coherence in musical scales, intervals and chords, what "shape" can we say represents temporal coherence? And how might such a shape be used in our auditory system to *instantly and effortlessly* measure harmonic qualities in real time?

Logically, if foreground spatial coherence is represented by 12-fold geometry, then background temporal coherence is probably 5-fold. After all, it is the background pressure of Φ-damping in a standing wave that crystallizes into a 5-fold pentagonal geometry, stabilizing oscillation around its Harmonic Axis. And it is Φ-damping at work in plants and animals as Fibonacci "growth" proportions that provide the free space needed for cells to grow, evident in the organic branching shapes of life. Even the "dissonant" Φ-damping of an eggshell offers the coherence and temporal stability needed for biological structure to fully unfold during the gestation period. In these and many other ways, the golden ratio and its convergent Fibonacci series are the obvious source of temporal coherence throughout nature, not the least of which is human perception of music harmony.

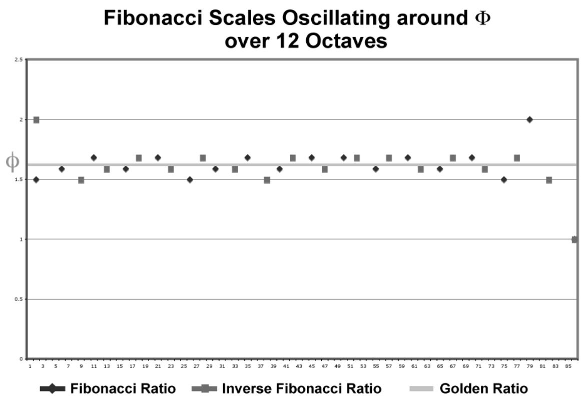
To show how this applies to music perception, we will begin by performing a simple time-based experiment using a virtual piano keyboard. The purpose will be to show a direct coupling between the Fibonacci series and the Tritone Function, previously established as the primary driver for diatonic harmonic flow. To do this, we need to create scales tuned to Fibonacci proportions and run them up and down twelve octaves over a {C} major scale, extending over the entire auditory spectrum and representing the top level of the Harmonic Hierarchy. All intersections will then be logged and analyzed to see if any temporal pattern or shape emerges. Here is a detailed description:

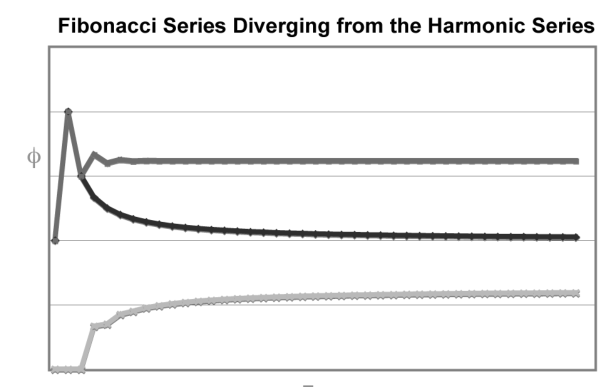
*In Figure 63, the top left chart shows the Fibonacci ratios used to build the scales and how they quickly converge toward Φ over time. To the right, the equal-tempered {C} major scale runs vertically next to two columns of "Fibonacci Scales." The column labeled* Fibonacci Scale *runs bottom to top from the lowest {C} while the column labeled* Inverse Fibonacci Scale *runs top to bottom from the highest {C}. Dark stars to the right of the virtual keyboard then pinpoint Fibonacci intersections occurring near Φ in the Tritone Function (between either {B, C} or {E, F}). White stars to the left indicate all of the other hits.*

## Figure 63 - Fibonacci scales create background damping field in C major




| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1.00 | 2.00 | 1.50 | 1.66 | 1.60 | 1.625 | 1.615 | 1.619 | 1.617 | 1.618 | 1.617 | 1.618 | 1.618 | 1.618 | 1.618 | 1.618 | 1.618 | 1.618 | 1.618 |
| | | 1 | 0.5 | 0.67 | 0.6 | 0.63 | 0.625 | 0.615 | 0.619 | 0.617 | 0.618 | 0.617 | 0.618 | 0.618 | 0.618 | 0.618 | 0.618 | 0.618 | 0.618 | 0.618 |




### Near Φ Hits in the Tritone Function

| Note | Equal Tempered Scale | Fibonacci Scale | Inverse Fibonacci Scale | Misses | Tritone Function Hits |
|---|---|---|---|---|---|
| C | 133952.29 | 136732.06 | 133952.29 | | INVERSE ROOT |
| B | 126434.13 | | | | |
| A | 112640.00 | | | | |
| G | 100350.83 | | | | |
| F | 89402.43 | 84505.06 | | | ★ |
| E | 84384.65 | | | | |
| D | 75178.18 | | | | |
| C | 66976.14 | | 66976.14 | | OCTAVE |
| B | 63217.06 | | | | |
| A | 56320.00 | 52227.00 | | ☆ | |
| G | 50175.42 | | | | |
| F | 44701.21 | | 44650.76 | | ★ |
| E | 42192.33 | | | | |
| D | 37589.09 | | | | |
| C | 33488.07 | 32278.05 | | | ★ |
| B | 31608.53 | | | | |
| A | 28160.00 | | 26790.46 | ☆ | |
| G | 25087.71 | | | | |
| F | 22350.61 | | | | |
| E | 21096.16 | 19948.95 | | ☆ | |
| D | 18794.55 | | | | |
| C | 16744.04 | | 16744.04 | | OCTAVE |
| B | 15804.27 | | | | |
| A | 14080.00 | | | | |
| G | 12543.85 | 12329.10 | | ☆ | |
| F | 11175.30 | | | | |
| E | 10548.08 | | 10304.02 | ☆ | |
| D | 9397.27 | | | | |
| C | 8372.02 | 7619.84 | | | ★ |
| B | 7902.13 | | | | |
| A | 7040.00 | | | ☆ | |
| G | 6271.93 | | 6378.68 | | |
| F | 5587.65 | | | | |
| E | 5274.04 | 4709.26 | | ☆ | |
| D | 4698.64 | | | | |
| C | 4186.01 | | | | |
| B | 3951.07 | | 3939.77 | ☆ | |
| A | 3520.00 | | | | |
| G | 3135.96 | 2910.58 | | ☆ | |
| F | 2793.83 | | | | |
| E | 2637.02 | | 2433.39 | ☆ | |
| D | 2349.32 | | | | |
| C | 2093.00 | | | | GEOMETRIC MEAN |
| B | 1975.53 | 1798.68 | | ☆ | |
| A | 1760.00 | | | | |
| G | 1567.98 | | 1504.28 | ☆ | |
| F | 1396.91 | | | | |
| E | 1318.51 | | | | |
| D | 1174.66 | 1111.91 | | ☆ | |
| C | 1046.50 | | | | |
| B | 987.77 | | 929.61 | ☆ | |
| A | 880.00 | | | | |
| G | 783.99 | | | | |
| F | 698.46 | 686.77 | | | ★ |
| E | 659.26 | | | | |
| D | 587.33 | | 574.55 | ☆ | |
| C | 523.25 | | | | |
| B | 493.88 | | | | |
| A | 440.00 | 425.14 | | ☆ | |
| G | 392.00 | | | | |
| F | 349.23 | | 355.10 | ☆ | |
| E | 329.63 | | | | |
| D | 293.66 | | | | |
| C | 261.63 | 261.63 | | | OCTAVE |
| B | 246.94 | | | | |
| A | 220.00 | | 219.46 | ☆ | |
| G | 196.00 | | | | |
| F | 174.61 | | | | |
| E | 164.81 | 163.52 | | ☆ | |
| D | 146.83 | | | | |
| C | 130.81 | | 135.64 | ☆ | |
| B | 123.47 | | | | |
| A | 110.00 | | | | |
| G | 98.00 | 98.11 | | ☆ | |
| F | 87.31 | | | | |
| E | 82.41 | | 83.83 | | ★ |
| D | 73.42 | | | | |
| C | 65.41 | 65.41 | | | OCTAVE |
| B | 61.74 | | | | |
| A | 55.00 | | | | |
| G | 49.00 | | 51.81 | ☆ | |
| F | 43.65 | | | | |
| E | 41.20 | | | | |
| D | 36.71 | | | | |
| C | 32.70 | 32.70 | 32.02 | | ROOT ★ |
| B | 30.87 | | | | |
| A | 27.50 | | | | |

Not surprisingly, the experimental results are consistent with what we would expect. As the Fibonacci Scales diverge from the harmonic series, they follow a path that *snakes between* the equal-tempered tones of the {C} major scale. As they do so, the Fibonacci Scales intersect six octaves of {C} as well as *seven* dark-starred intersections in the "cracks between the keys" near Φ-damping locations. Three of these intersections occur in the lower half of the Tritone Function between {B, C} while four are in the upper half between {E, F}.

All of the remaining frequencies of the Fibonacci Scales then land between non-diatonic semitones, creating a kind of damping field around each of the twelve tones of the octave. Describing it as a field is correct because if the Fibonacci Scales were allowed to continue back and forth across the spectrum in this serpentine fashion, we would find an infinite number of locations between the twelve resonant regions to block out all fractional or enharmonic frequencies. Ok, so what does this mean?

First, the experiment illustrates how the Fibonacci series forms a temporal *background pattern* as a kind of invisible container for 12-fold spatial coherence. In a very real way, the physical cracks between the keys of a piano keyboard physically represent the articulated gaps created by Fibonacci damping proportions, thus permitting the emergence of whole number harmonics. Without the gaps created by such damping, sound would be nothing but a sonic roar and the keyboard would be only an unarticulated solid piece of wood (if wood could grow at all under such circumstances). Like the Rosslyn dragon-serpents wrapping around the Tree of Knowledge on the chapel's columns, this experiment once again proves that the Fibonacci series partitions harmonic structure into twelve whole number pieces as it spirals through time.

One other interesting thing to note is how the Fibonacci series alternates above and below Φ as it winds around the keys (middle graph in Figure 63). Mentioned earlier, it really is no coincidence that it follows the same oscillating motion of a standing wave seeking equilibrium. First realized by Pythagoras in his tetrachord system, the Fibonacci series acts like a silent sewing machine to temporally mend whole number harmonic proportions into a closed octave.

When the same experiment is repeated using the small whole number ratios of Just temperament, the results are even more compelling. In this case, there are *eleven intersections* in the middle of the Tritone Function for Just temperament instead of seven. This is because the pure harmonic ratios {1:1, 2:1, 3:2, 5:3, 8:5} in Just temperament match the first few ratios of the Fibonacci series. But if Equal temperament is the most coherent tuning, why is it that Just temperament has more intersections with the Tritone Function?

Our first reaction would be to say that spiral-favoring temperaments are more coherent than circle loving temperaments because they intersect more pure harmonic ratios. But to assume this would be to ignore the correct definition of coherence. Coherence refers to how easy a pattern is to recognize in space and time – *not* necessarily how consonant or pleasing it is. This is a critical point.

The correct definition of coherence is found in the physics of a standing wave. Even though various "spiral favoring" temperaments will sound more consonant in a particular reference key, they lack coherence and alignment within the Harmonic Hierarchy of a standing wave. The closer we get to the equal spacing of a standing wave, the easier it is for our auditory system to pattern match and recognize music harmony. Temporal coherence is then no different than spatial coherence – it originates in the physics of a standing wave with 12-ET as its octave equivalent.

But no matter which scale temperament is used, the Fibonacci series will still intersect and oscillate inside the Landau regions of the Tritone Function. The only difference is how much space a given temperament provides and whether it offers enough contrast to make this oscillation obvious to our ears. This is the idea of temporal coherence embodied in all those centuries-old rules of common musical practice, compelling us toward such things as contrary motion, the Tritone Function and alternation of the diatonic Cycle of 5ths chord progression.

Consider for example a song built on triadic chords progressing around the diatonic Cycle of 5ths. As the song plays, each diatonic triad can be seen to follow the Tritone Function with one or more of its tones, springing back and forth with each chord played. In the key of {C} major, this boils down to the musical series {B, {C, E}, {C, F}, {B, F}, {B, E}, {C, E}, F}, which acts as a kind of inner skeleton for the diatonic Cycle of 5ths. It audibly demonstrates without further explanation how each triad is pulled repeatedly by the Fibonacci vortices (and their convergent Φ-damping locations) across the "cracks between the keys," bringing a sense of temporal symmetry and coherence to any song.

With this one simple experiment, the Fibonacci series can be seen as the rational damping function that brings temporal coherence to music. It supports 12-fold spatial coherence while providing the driving force behind our historical preference for the Tritone Function. Its spiraling action toward Φ-damping locations is the very thing we instinctively recognize in the Tritone Function and anticipate in any diatonic music. Like the 12-step octave, this too must be emblazoned on the architecture of our auditory system.

But instead of being points of resonance that we hear in the Fibonacci series, it must be the gaps or spaces where resonance is least that comprise temporal coherence. On the round Chladni plate of our eardrum, we must be able to hear both foreground waves energy *and* the silent background space between waves. The silent gaps in sound must also appear as calm patterns in our inner ear and indeed as quiet neural regions in our brain. This invisible silent Fibonacci pattern must be as much a part of music cognition as whole number harmonics, forming its own unique background geometry as a complement to foreground resonance patterns. The only question is what kind of anti-harmonic geometrical shape does the Fibonacci series make?

When we calculate the geometric mean of all the tones of the Fibonacci Scales, we arrive at {C} in the center of the extended keyboard (2093.000 Hz shaded in Figure 63). Though it is not a member of either Fibonacci Scale, it is still *temporally implied* as a silent geometric mean tone of all the ratios of the Fibonacci series as they reflect over twelve octaves. With this as our invisible center, what inaudible shape can we say surrounds it?

Given that this shape is drawn by the Fibonacci series and Φ, we must be talking about a *quasi-periodic* shape of some sort. Such figures always have proportions at or near the golden ratio and always form irregularly repeating structures. Mathematician Roger Penrose was well known for this kind of tiled structure, called a *tessellation figure*, which he would construct by combining sections of a pentagram. Known as "Penrose tiles," his quasi-periodic patterns were quite beautiful from a distance, but not so easily recognized close up. Like our inaudible temporal damping shape, Penrose tiles can be considered "anti-harmonic" background fields, filling space irregularly in golden sections through the irrationality of √5.

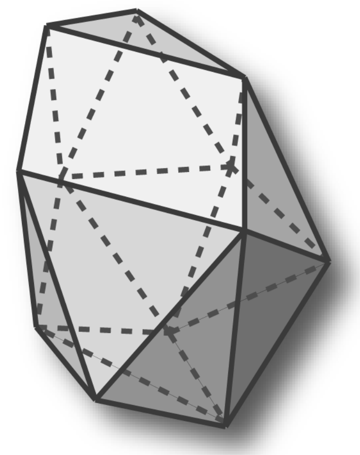
One other property of quasi-periodic patterns is they are always the product of *two incommensurable frequencies* within a dynamical system. This is an important clue because our background shape must be the result of the most incommensurable musical interval possible. What is this interval? It is the one that "beats" the most – Helmholtz's dissonant semitone.

Now, in order to find the geometric shape corresponding to this interval, we need to enlist the help of a little known $19^{th}$ century device known as the *rotary harmonograph*. Invented in 1844 by one Hugh Blackburn, a professor of mathematics at the University of Glasgow in Scotland, the harmonograph incorporates two vibrating arms and a hanging pendulum with a pen at the end. It can be set so that the arms vibrate the pendulum in the same directions (concurrently) or in opposite directions (countercurrently). The amplitude (or energy) can be adjusted up or down, as can the frequency of vibration in each arm. In this way, any number of harmonic and enharmonic intervals can be produced, tracing out a wide variety of organic geometrical patterns on paper.

By setting the harmonograph's pendulum to vibrate in a closed phase with a lateral (or *near unison*) interval tuned slightly less than an equal tempered semitone, we find the incommensurable shape we are looking for. As the harmonograph's pendulum dawdles near the center, it slowly draws out the simplest organic shape possible. Formed from the *concurrent* interference of two incommensurable frequencies, we find that the space inside a semitone naturally creates the anti-harmonic geometry of an egg.

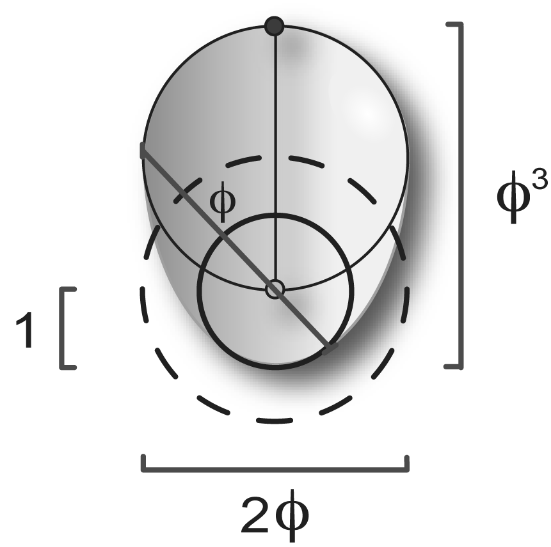
As this applies to our experiment, the 18 tones of each of the two Fibonacci scales over 12 octaves can be interpreted as the concurrent quasi-periodic egg shape of a *sphenomegacorona* polyhedron having 12 vertices, 18 faces and 28 edges. Since this geometry is constructed from 16 triangles and 2 squares, it does not depend on any "cut and paste" operation from Platonic or Archimedean solids. Taken as a low-resolution approximation for the "quasi-periodic crystal" of an egg, this is the anti-harmonic background shape of silence.

**Figure 64 - Sphenomegacorona - the shape of Fibonacci silence?**



Could a common hen's egg really represent the silent Fibonacci background field in coherent sound? As we saw earlier, the dimensions of the Rojas egg model correspond perfectly to the compressed region in a standing wave that forms around nodes. Recall too that this barrier occurs when ascending frequencies start producing Fibonacci proportions in the range {13..∞}, thereby converging to the golden proportions $\Phi^3 \times 2\Phi$ of the Rojas egg model. It seems pretty clear that our Fibonacci Scales could very well act as an egg container to provide the *temporal coherence* needed for *spatial coherence* and regular harmonic shapes to emerge in time.

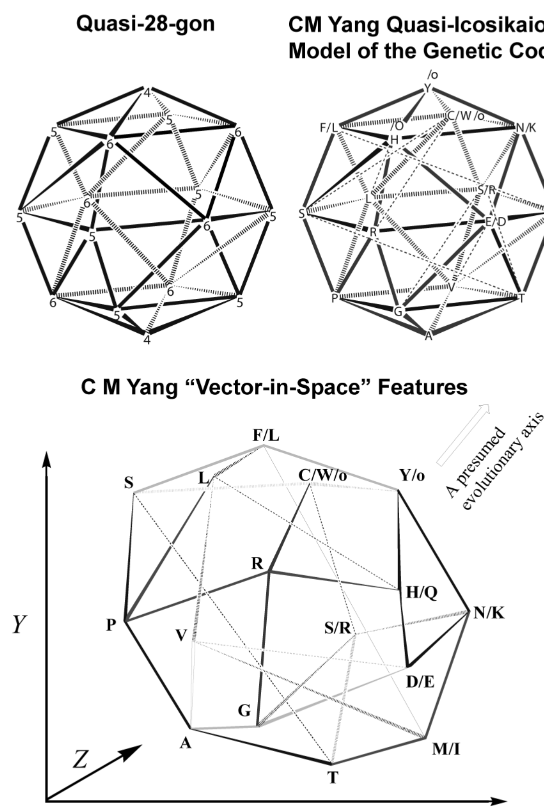
**Figure 65 - The Rojas egg model**



Far more than a mere coincidence, new genetic research from China seems to confirm this very idea. A recent theory by Chi Ming Yang at Nankai University in China claims to have found a quasi-periodic egg geometry in the human genetic code that is complementary to Mark White's dodecahedral G-Ball model. Derived from the same building blocks of 20 standard amino acids and 64 tri-nucleotide codons in DNA, Yang states:

> *"In the present work, 16 genetic code doublets and their cognate amino acids in the genetic code are fitted into a polyhedron model. Based on the structural regularity in nucleobases, and by using a series of common-sense topological approaches to rearranging the Hamlitonian-type graph of the codon map, it is identified that the degeneracy of codons and the internal relation of the 20 amino acids within the genetic code are in agreement with the spherical and polyhedral symmetry of a quasi-20-gon, i.e.,* icosikaioctagon.*" [Chi Ming Yang 2003]*

## Figure 66 - Yang quasi-periodic model of DNA




Credit: C.M. Yang

Yang found that his *icosikaioctagon* geometric model of the genetic code can be generally described by a “quasi-28-gon” shape having 16 vertices, 42 edges and 28 faces (Figure 66). Within this geometric pattern, he identified a pattern that suggests an evolutionary axis of symmetry that apparently followed five “stereochemical” growth stages over a period of millions of years. When he applied a “covalent-bonding hybrid of the nitrogen atoms as a measure for structural regularity in nucleobases,” the 20 amino acids followed a cooperative vector-in-space addition principle that stretched into an ellipsoid, or egg-like shape similar to the sphenomegacorona.

Related to the five evolutionary growth stages, Yang also found that the overwhelming majority of the amino acids in the genetic code contain “side-chain carbon-atom” numbers that balance at 17 according to the Lucas series {2, 1, 3, 4 and 7}. This series, a variant of the Fibonacci series, also converges to the golden ratio beginning with {2, 1,…} instead of {0, 1,…}.

A follow-up paper to this model by Tidjani Negadi entitled *A Connection between Schcherbak’s arithmetical and Yang’s 28-gon polyhedral “views” of the genetic code* suggest a further correlation of the genetic code to the prime number 37 by “borrowing a zero” to the symmetry of 36 nucleon’s sums of amino acid blocks and/or side-chains mentioned above. Negadi demonstrated a preponderance of such occurrences of divisibility by 37 in the genetic code including 3.7 amino acid residues per turn of the DNA helix [Negadi 2003].

Yang and Negadi’s findings have relevance to our experiment with Fibonacci Scales over twelve octaves because it too was comprised of exactly 36 tones, totaling to 37 if we were to start from an imaginary “zero valued tone” instead of one. You might also recall the number 0.037037037 produced by multiplying 3 against the resonant Rosslyn magic ratio.[103] And what about those five evolutionary growth stages – could this be our musical Harmonic Hierarchy at work in organic chemistry?

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[103] The number 37 is found elsewhere in physics, most notably the fine-structure constant of 1 / 137.035999070(98) that characterizes the strength of the electromagnetic interaction. In his book *QED: The Strange Theory of Light and Matter*, famed physicist Richard Feynman wrote:

“It has been a mystery ever since it was discovered more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it. Immediately you would like to know where this number for a coupling comes from: is it related to π or perhaps to the base of natural logarithms? Nobody knows. It's one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man. You might say the "hand of God" wrote that number, and "we don't know how He pushed his pencil." We know what kind of a dance to do experimentally to measure this number very accurately, but we don't know what kind of dance to do on the computer to make this number come out, without putting it in secretly!”

Interestingly, when 137.03599907098 is divided by the Rosslyn magic ratio 0.012345679, it is equal to 11,100 within a 0.0007% variance (a difference of 0.001037). From the perspective of harmonic science, the fine-structure constant appears to be a unity property of cubic space resonance.

When we now combine Yang's quasi-28-gon model with White's dodecahedral model, we may just have found the ultimate bio-harmonic archetype for life. What would that be you ask? Why the damping container of a quasi-28-gon "egg" with the "yolk" of a dodecahedral G-Ball resonating inside. Through the magic of harmonics, nature has engineered DNA to create its own eggshell as a temporal container to carry along the spatial harmonic geometry of life. Our auditory physiology had no choice but to recognize the egg-like shape of Fibonacci proportions in harmonic music. The proportions were already built into the egg shape of our brain.

Time and again life appears as a kind of crystallized music. When we square the harmonic series in the ***INTERFERENCE*** distribution, we are creating a spatially resonant plane. And when we divide it by the Fibonacci (or Lucas) series, we extrude it in time to create the Gaussian shape of life. We find that a pentagonal dodecahedron represents the *foreground spatial property* of the harmonic series while the quasi-periodic sphenomegacorona or icosikaioctagon represents the *background temporal growth property* of the Fibonacci or Lucas series converging to Φ. With DNA carrying 5-fold instructions for its own egg to contain the recursive growth of 12-fold spatial structure, life and perception is literally an outgrowth of the same 5-level 12$^{th}$-power Harmonic Hierarchy of the ***INTERFERENCE*** model.

The shape of life is but a shadow – a Gaussian energy-shadow if you will – projected as a harmony of particles onto the quantum screen of spacetime. Schrödinger said as much, proposing that all matter should be defined as a probability of wave interference. But not just any wave interference – *coherent* wave interference. From the quantum level up, the coherent model of a standing wave is what shapes our body and carves the architecture of our brain. It is central to how we perceive our world.

It is from these predictable patterns that we are able to measure the structure of sound and anticipate the flow of melody and harmony. Our ears, a reflection of coherent sound itself, already knows how to follow this oscillation of energy across the spiraling Fibonacci damping path and its rational Partial 5. And our auditory cortex – the yolk of our brain sitting inside the eggshell of our cranium – also knows very well how to recognize the harmonic patterns of music, always estimating the probability of the next move though its own Gaussian interference pattern.

# *Fibonacci Unwinding*

*"Only the most foolish of mice would hide in a cat's ear, but only the wisest of cats would think to look there." - Andrew Mercer*

Thinking starts in the hardware of our brain. But the signals that our brain receives are dependent entirely on the proportions recognized by our ears. Out of these proportions emerge the *feelings* of harmony in the standing wave structure of the brain itself. Without the right hardware to decode the sound waves into something predictable and recognizable, none of the other attributes of music could occur. Only a harmonically structured brain would have the right hardware to decode harmonic sound.

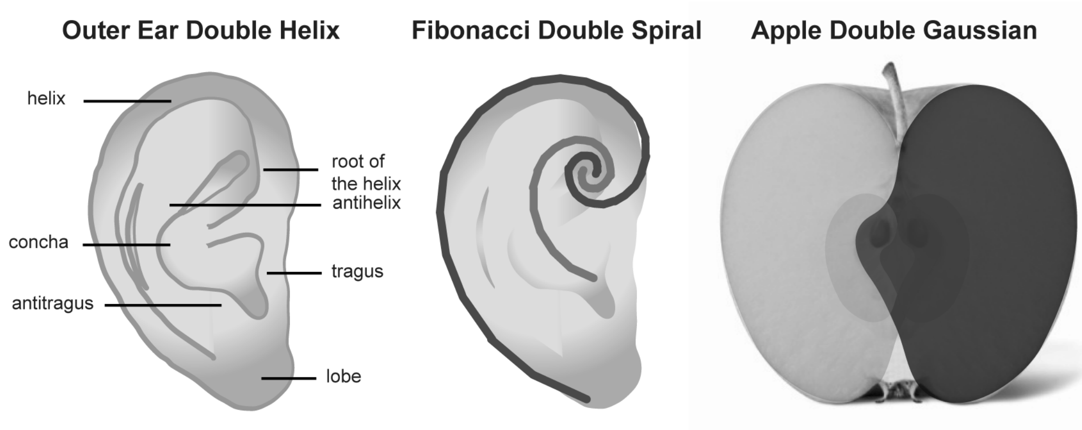
A recent "brain music therapy" study by Russian scientists proves this beyond a shadow of a doubt. The team used computers to convert emotions corresponding to a subject's brain waves into audible music. This psychoacoustical 'brain music' was then played back to the subject to reinforce the desired behavior, such as calm or excitement, corresponding to the original brain wave patterns. Not surprisingly, they found that such therapy has a measurable correlation to subject attitude and behavior [Levin 1996]. The only way this can possibly work, of course, is if brain waves are composed of the same harmonics preserved in musical form. The conclusion is simple: our brain thinks in harmonics.

Guided by these findings, we must conclude that the resonance and damping properties in the physics of a standing wave also exist in the physical structure of our auditory system. Moreover, the ***REFLECTIVE*** and ***INTEGRAL INTERFERENCE*** patterns for consonance and tension must also apply to our physiology just as it does to music harmony. Somewhere between the Fibonacci spiral of our ears and the harmonic structure of our brain there must be the right equipment to unwind the Gaussian interference pattern of coherent sound.

Beginning with the outer ear, Figure 67 shows a comparison of the ear's double helix to a double Fibonacci spiral. Since the ear canal must grow outward from the center of this spiral, common sense would tell us this shape has a lot to do with the recognition of sound. In fact, it is directly related to auditory perception inside the ear and deeper inside the auditory cortex.

The ***INTERFERENCE*** model would tell us that the human ear follows the Fibonacci growth series in order to damp out standing waves. Furthermore, such a damping structure would start the decoding process as soon as possible by canceling reflection while *unwinding* harmonic proportions from the Gaussian interference in order to focus. After all, our ears must have evolved with one express purpose – to recognize coherent "signal" in of a sea of noise.

**Figure 67 - Fibonacci and Gaussian physiology of the outer ear**



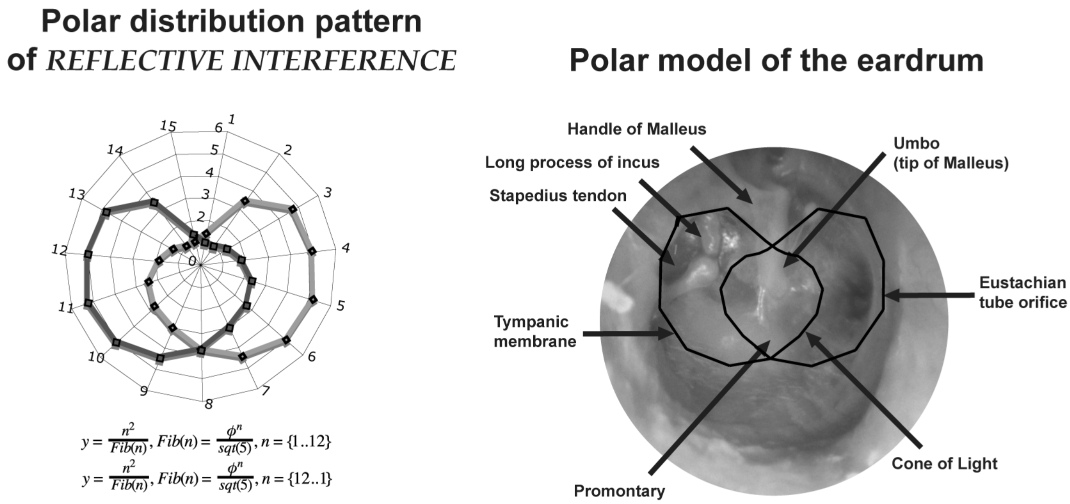

But the unwinding process does not end here; it continues into the core of the ear. When sound hits our eardrums it vibrates into the same geometrical patterns of resonance and calm found on a drumhead or round Chladni plate. Now when a Chladni plate vibrates, it is π periodic in both x and y dimensions and thus reflects at half cycles (or half octaves) according to the mathematical relation known as a *Bessel function*. But unlike a perfectly circular plate, the human eardrum has a more organic shape to it. It has the peculiar cardioid-like shape matching that of the polar ***REFLECTIVE INTERFERENCE*** distribution.

In Figure 68, we see this familiar Gaussian distribution pattern of harmonic interference plotted now in polar space. Comparing this to the human eardrum, we find that it too follows this distinct polar Gaussian shape. Just like the geometry of the outer ear, the eardrum has a distinct apple shape to it that is created by the *Handle of Malleus* bone as it spirals around the top and protrudes straight down to the Harmonic Center of the two Gaussian interference curves. If the Tympanic membrane of the eardrum were an apple, the Malleus would be its stem.

Given this amazing topological match, it seems unavoidable to conclude that our ears grew to match the Gaussian distribution pattern of the harmonic series so that it might recognize simple whole number proportions instantly and efficiently. As sound reflects over the Tympanic membrane at π periods, like a Bessel function on a Chladni plate, it would have to create a single line of symmetry reflected at mid-octave (at the tritone) right where the Malleus stops.

Around this, the other bones of the inner ear must also form at specific positions in order to measure the full range of resonance and damping in the geometrical patterns on the vibrating surface. In particular, the bones must align themselves with points of maximum resonance and maximum damping in order to recognize and focus on the most coherent sound patterns in the environment.

## Figure 68 - Human eardrum as a *REFLECTIVE INTERFERENCE* surface




The rationale for the proportions and locations of the bones of the inner ear will be covered in greater depth later, but suffice it to say for now that this organic variant of the ***REFLECTIVE INTERFERENCE*** Gaussian distribution does transform and transmit this pattern into the inner ear cochlea, stimulating the *Organ of Corti* above the basilar membrane as it does so. This process converts the composite compression waves pushing against the eardrum into individual *transverse* sinusoidal wave components that travel through the liquid of the inner ear. The Organ of Corti, itself the core of the Fibonacci growth spiral, contains four rows of hair cells that protrude from its surface, totaling 16,000 to 20,000 hairs on the inside of the inner wall as it spirals to an end. It is in the movement of the hair cells on the basilar membrane where resonance and damping proportions are measured and our neural interpretation of sound begins.

The scientific explanation for how the inner ear works is called *place theory*. High frequencies stimulate the opening area of the cochlea while lower frequencies stimulate the area toward the center. The cochlea acts as a coiled up ***INTERFERENCE*** model to unwind and measure the same simple harmonic proportions found in the octave spectral analysis. We can be sure that the convergent gaps found at simple harmonic proportions will appear on the surface of the basilar membrane as the waves sweep though the cochlear chamber. These proportions are then transmitted as neural "action potential" impulses through nerve fibers (*axons*) into the auditory cortex of the brain known as the *Brodmann Area* [Zwicker 1999].

The Brodmann Area is then composed of three concentric rings identified as primary, secondary and tertiary and sit almost vertical between the ears. The primary auditory cortex, located as the outermost ring connecting to the ears, is *tonotopically* organized, meaning that it is sensitive to specific frequencies and loudness. The middle ring of the secondary auditory cortex then recognizes melodic, harmonic and rhythmic patterns while the tertiary auditory cortex at the center integrate everything into the musical experience [Abbot 2002]. Perception of music is also distributed elsewhere in the brain, most especially the *rostromedial prefrontal cortex* located immediately behind the forehead that is especially sensitive to resonant tones. [Janata 2002].

There is no conclusive explanation for how the various auditory components work together and why it is organized in this particular way. However, recent research on rhesus monkeys suggests that human brains do think harmonically according to a Gaussian distribution. A team at INIST-CNRS found that a Cartesian two-dimensional Gaussian model correctly characterizes the firing rates of neurons when taken as a function of the horizontal and vertical amplitude of movements for each neuron [Platt and Glimcher, 1998].

The same Gaussian model has been found to apply to the inner ear where individual tones are measured as proportions along the basilar membrane. Studies have found a built-in "sharpening mechanism" here that enhances our recognition of specific tones by suppressing surrounding hair cells. As a preprocessor to the auditory cortex, it is this natural response action in the inner ear that shapes sound into the more recognizable "fringe" of a Gaussian curve. While science does not yet offer an explanation for what caused our senses and brain to organize according to a Gaussian distribution, *Harmonic Interference Theory* may be able to tell the whole story.

From the inside out, our ears grew into the shape of Fibonacci spirals because these proportions comprise the background field in harmonic interference. To prepare sound for the brain, the cochlea and basilar membrane spiraled together to subtract out the Fibonacci proportions, thus emphasizing or *focusing* on harmonic proportions. The Tympanic membrane then spun closed into a ***REFLECTIVE INTERFERENCE*** surface – suspending at the most resonant location in the ear canal – so as to convert and polarize composite waveforms into superposed transverse waves. Together, these membranes could deconstruct complex sound patterns while emphasizing simple harmonic proportions, sending enhanced patterns to the ringed auditory cortex known as the Brodmann Area.[104] The Gaussian neural patterns are then filtered through the Gaussian rings of the cortex (probably a simplified Chladni pattern) to identify simple shapes.

We can now see the ear for what it really is: a "cognitive lens" built to cancel standing waves and unwind the Harmonic Hierarchy. No other explanation or theoretical model is required to explain this essential process of harmonic cognition. It is all an elegant, fully integrated transformation of frequency to proportion that can be easily pattern matched as neural geometry against the natural harmonic series intrinsic to the brain's physiology.

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[104] **Principle 32:** The REFLECTIVE INTERFERENCE structure of the eardrum and Fibonacci action of the basilar membrane of the inner ear is the essential coupling mechanism between the physics of sound and the Brodmann Area in the auditory cortex of the brain. We can predict from this that the Brodmann Area itself may also be organized as a REFLECTIVE INTERFERENCE neural network.

## Redefining Nature as Coherent Interference

The evolution of life had little choice but to follow the same efficiency of harmonic resonance and damping found in a vibrating environment. There is no better example of this than our own multi-purpose auditory system, capable of recognizing timbre, harmony and spectrum *simultaneously* with a single elegant piece of bio-Fourier transform equipment. With this one ingenious trick, we are able to recognize proportional auditory shapes and predict their movement whether in one tone or concert of many.

During the entire physical experience of music, from initial vibration and aerial transmission to auditory reception and recognition, nothing is lost. Intersections at maximum damping and resonance points continue to exist even after they combine into a single wave front. An organic transformation of compression to transverse waves is all that is needed to *instantly* match the original Fibonacci and harmonic proportions against our own physiology. No other translation language, look-up table or Möbius strip is required. From *Harmonic Interference Theory*, we can easily understand how the recognition of harmonic wave patterns is *integral* to the emotional experience of music. Sound, physiology and perception are not independent phenomena at all, just different instances of the same coherent model of a standing wave.

As we listen to music, we are navigating and interpreting fantastical geometric patterns across the 12:5 Harmonic Hierarchy of pitch space. Through this finely webbed projection screen on our eardrums, we can "see" a seamless auditory landscape of constantly morphing, shifting and tumbling geometrical objects. The ***INTERFERENCE*** model describes how waves come together to resonate or damp within this space to produce a unified recognition of timbre and harmony. The ***REFLECTIVE INTERFERENCE*** model explains for us how tension and resolution are anticipated as flowing currents while the ***REFLECTIVE INTEGRAL*** model explains our measurement of spatial structure in terms of consonance and dissonance. It is the velocity of energy through the Gaussian field of harmonic interference that we measure with the same shape of our Gaussian physiology.

The *Harmonic Symmetry Table*, taken from the spatial proximity of harmonic waves, is yet another way to measure the coherence of sound. From the interference pattern of one cycle of a standing wave we can see how tritone Partials 5 and 7 have partitioned our senses into twelve resonant regions. And how, under external damping pressure, wave energy can crystallize into the Pythagorean ideal of a 12:5 geometry. It is from this that we find the deeper meaning of $E=mc^2$ as the ratio between energy and form in the standing wave of light. As the only path capable of reconnecting ancient wisdom to modern rationalism, harmonic science can no longer be relegated to history's dustbin. It is at the dead center of the study of nature.

In glorious vindication, the tritone, pentagram, Fibonacci series and golden ratio can now reemerge into the light of day – resurrected from the dark shadows of the Middle Ages into the core of a modern understanding of a music. We will recognize these proportions wherever the gap between spiral and circle need mending and where energy stitches together the damping wells of harmonic waves. We will recognize them as the standing wave engine behind nature's perception of itself.

Throughout history, musicians, theologians and scientists have all sought to understand the warp in pitch, the curvature of space and whether this constitutes an error or a blessing. Through the study of harmonic interference, everything can finally be understood as a reflective balance of resonance and damping. *Harmonic Interference Theory* offers a guiding philosophy not only for music, but for other fields of study as well, taking science in directions it may never have otherwise considered. It can help explain at the very deepest level the principles at work in an atom, the solar system and consciousness itself, giving us a renewed musical philosophy that can transport us through the facade of nature's mirror.

By understanding how nature's mirror works we can learn to see perception as the inevitable outcome of a cosmos composed of coherent interference. We can understand how we arrived at the apex of all this coherence as a stable pattern able to experience other stable patterns. From the smallest quantum of our surroundings through our loftiest dreams, we can see that it is the coherence that has the power to make everything make sense.

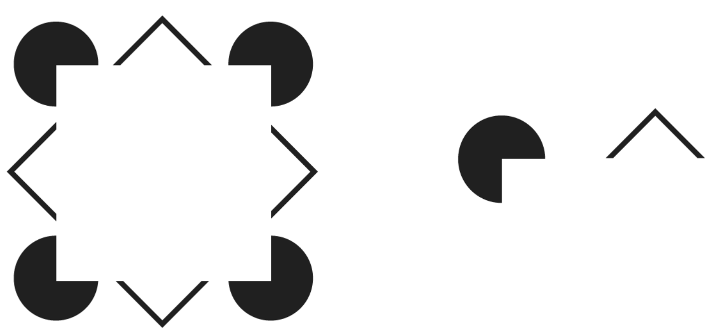
# *Holonomic Harmonics*

*"Nature and nature's laws lay hid in night; God said "Let Newton be" and all was light."*
*- Alexander Pope*

Whenever the topic of music perception pops up, we inevitably end up having to grapple with the broader questions pertaining to *all* areas of human perception. This is no small task. Our brains receive and process sensory input from different sources simultaneously while instantly recognizing, responding to and storing this information for future recall. We take for granted our seemingly magical ability to synthesize audio/visual information at will or manufacture our dreams, improvising surreal movies set in metaphorical dreamscapes. This all remains a field of immense research under hot pursuit by a fair number of brilliant scientists.

In the early 20$^{th}$ century, the German school of *Gestalt psychology* was first to confirm several key features of vision cognition. Based mainly on experiments in visual perception, the central principle was our apparent knack for configuring and recognizing individual objects as part of a larger pattern. This idea was so revolutionary at the time that it was repeated frequently as the catch phrase: "the whole is more than the sum of its parts." It is easily demonstrated in Figure 69 by repeating a seemingly random foreground pattern to imply a background square.

**Figure 69 - The Gestalt Principle as a coherent pattern**



Another key principle of Gestalt psychology holds that any recognized configuration is as good as the context permits, known as the "Prägnanz principle". It holds that the most recognizable visual configurations exhibit continuity, regularity, simplicity, stability, and unity. These are the principles generally referred to as *Gestalt theory*, summarized as follows.

> ***The closure principle:*** *we perceive whole images from partial visual data. For example, we see a circular figure with small gaps in it as a full or closed circle. In the same way, we perceive a whole figure even when part of an image falls on the blind spot of the retina.*
>
> ***The continuity principle:*** *we tend to simplify and shape-smooth irregular, abruptly changing contours.*
>
> ***The proximity principle (grouping):*** *we organize elements into columns whenever the vertical distance between spatial elements is less than the horizontal distance.*
>
> ***The similarity principle (grouping):*** *we connect or group elements based on proportions of size, shape, color, texture, material and surface.*
>
> ***The common fate principle (movement):*** *we group elements as a unit if they move together. For example, when seen from a distance a group of stationary cars are difficult to distinguish from the background landscape – however, when they begin moving together we identify them as a group separate from the background.*
>
> ***The "phi" principle:*** *when two stationary elements, such as a colored red dot and a blue, are alternated in rapid succession, we perceive a continuum of movement as a blur of color (if close) or succession of dots (if further apart).*

Gestalt theory was based on visual experiments, but its principles can be found to parallel and support our findings in auditory cognition. The Gestalt visual principles of continuity, regularity, simplicity, stability and unity are identical to the concepts of spatial and temporal coherence exemplified in coherent sound. Grouping and movement of objects based on shared properties, foreground-background separation and spatial proximity are found in the Harmonic Hierarchy and the way energy is exchanged in a standing wave. When you think about it, all of the above principles of Gestalt psychology are exhibited in a harmonic standing wave and thus at the heart of our ability to recognize according to such features in our environment.

Turning Gestalt on its head, perhaps the first principle of cognition should be the physics of a standing wave. Given our ability to instantly recognize the proportions of symmetry within a single tone (as timbre) or group of tones (as harmony), why would Gestalt psychology not just be a confirmation of our pattern recognition of proportions in the coherent interference patterns of sound and light? Why would we not define Gestalt principles according to the physical model of a standing wave and how it manifests in biological structures or neural patterns? Well, this is exactly what was proposed in 1987 in something called *holonomic brain theory*.

Developed by neurosurgeon Karl Pribram, in partnership with British quantum physicist David Bohm, holonomic brain theory superceded the principles of Gestalt theory by proposing all cognition as the product of standing wave interference patterns in the brain. Pribram theorized that memory and information is stored not in cells, but rather in *wave interference patterns*:

> *"What the data suggest is that there exists in the cortex, a multidimensional holographic-like process serving as an attractor or set-point toward which muscular contractions operate to achieve a specified environmental result. The specification has to be based on prior experience (of the species or the individual) and stored in holographic-like form. Activation of the store involves patterns of muscular contractions (guided by basal ganglia, cerebella, brain stem and spinal cord) whose sequential operations need only to satisfy the 'target' encoded in the image of achievement much as the patterns of sequential operations of heating and cooling must meet the set-point of the thermostat."*

He came to this conclusion based on two facts:

1. *"The visual cortex has response functions that correspond to Gabor functions, which are related to hologram images, and*
2. *Significant lesions can be made in animal brains that reduce, but do not extinguish memories, again resembling the distribution of light information throughout a hologram."*

Pribram built his model of the brain and cognition based on Fourier analysis and the transformation of complex waves into component sine waves. This line of thinking led him to an understanding of the eye as an organic lens capable of performing a *double Fourier transform* that reduces visual patterns into swirls (like a hologram) and quantum units (like a very high resolution graphics display). To this, David Bohm added "if you take the lenses of our senses away what you are left with in our brain is a hologram."

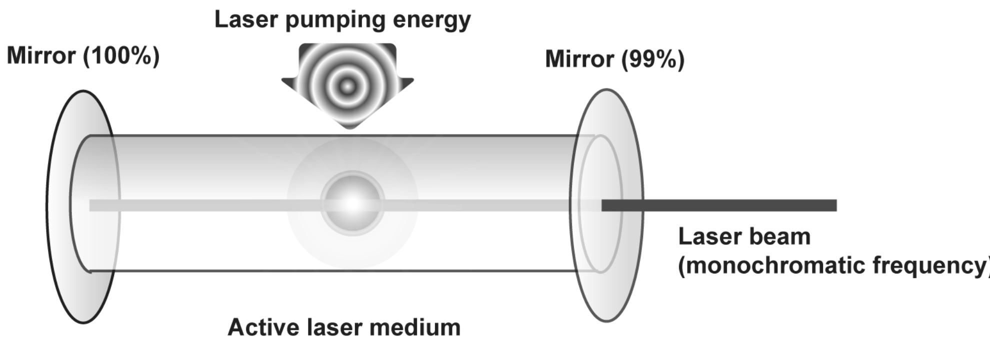
Adding to this, Pribram proposed that all our senses perform this function of wave transformation. Could it be that the Gaussian interference patterns of coherent sound are the same as the holographic images of coherent light?

First theorized by Albert Einstein in 1917 as "stimulated emission," it was the accidental discovery of holographic photography, or "holography," by scientist Dennis Gabor in 1947 that first focused attention on the usefulness of coherent light. And it was because of this that Theodore Maiman and/or Gordon Gould then invented the optical "laser" in 1958 based on the invention a few years earlier by Townes and Schawlow of an amplified microwave radiation device known as a "maser." There have been countless applications of laser technology since then ranging from optical storage and medical devices to computer mice and measuring instruments. But at the center of all these inventions is one simple concept – a coherent standing wave.

Most textbooks on physics explain lasers as instruments that produce spatially coherent light by inducing atoms to emit all of their light in phase. This is really an oversimplification. It is true that the fluorescing atoms in a laser emit light that is in-phase with waves traveling between two mirrors inside a laser cavity – however, the in-phase emissions only amplify the traveling light waves and are not the trick that makes light spatially coherent. So, what then does create coherent laser light? Mirrors!

It is the mirrors in the resonant cavity of a laser that preserve coherent light while rejecting *incoherent* light. As light travels down an infinitely long "virtual tunnel" between the two mirrors, out of phase frequencies (incoherent fractional waves) cancel each other out leaving only a single monochromatic frequency. This monochromatic frequency then continues to be amplified by the mirrors before being released as a coherent beam.

**Figure 70 - Laser mirrors create coherence**



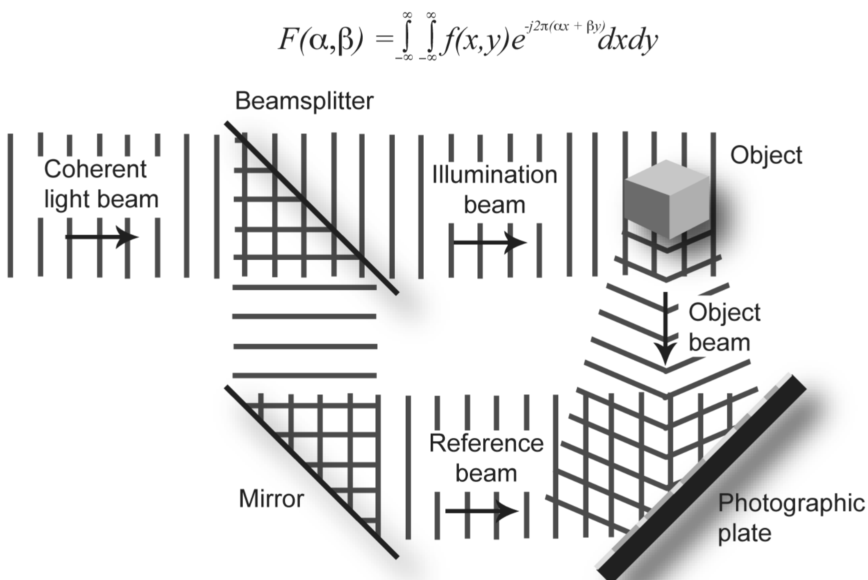

The same phenomenon occurs in nature. Starlight that has traveled a great distance becomes coherent over time. As starlight travels from its source, all the waves gradually add up to form a wave with a single wave front. The farther the light travels from its source, the more it approaches the shape of a perfect plane wave and thus spatial coherence.

In the same way, a laser allows light to travel great distances by bouncing it between two mirrors. This creates a coherent spherical standing wave in the same way a vibrating string creates a standing wave at a specific frequency to produce a tone. The space between the two mirrors acts just like the string of a guitar with the distance between the mirrors determining the frequency of coherent light. While harmonic frequencies can exist between the mirrors, out-of-phase enharmonic waves cannot. Ultimately, the monotonic laser frequency works just like a vibrating string to damp out fractional frequencies during amplification to yield a single pure harmonic standing wave of light that fits perfectly between the two mirrors.

As it is used in holographic photography, a coherent laser beam is then split into two parts to create a *reference beam* and an *illumination beam*. The illumination beam is aimed at the object to be photographed, then bounced back as the *object beam*. The reflected light from the object is then recombined with the reference beam (at a mostly orthogonal angle) to create a stereoscopic interference pattern on a photographic plate. The resulting hologram is a recording of the two light sources superimposed in various phase shifts and amplitudes to produce a series of *intensity fringes* on the film.

When the hologram is later lit from behind with a coherent point of light (from a laser or just a pinpoint of light), this aligns with the reference pattern in the hologram that then passes through the film to our eyes. When we move our heads to a different position, we perceive a change in light intensity and phase shift relative to the reference frequencies (remaining constant) and object frequencies (that change) within the interference pattern. The object then appears to us as a three-dimensional spatial object just as it would have the moment the photograph was taken.

**Figure 71 - Laser holography as double Fourier transform**

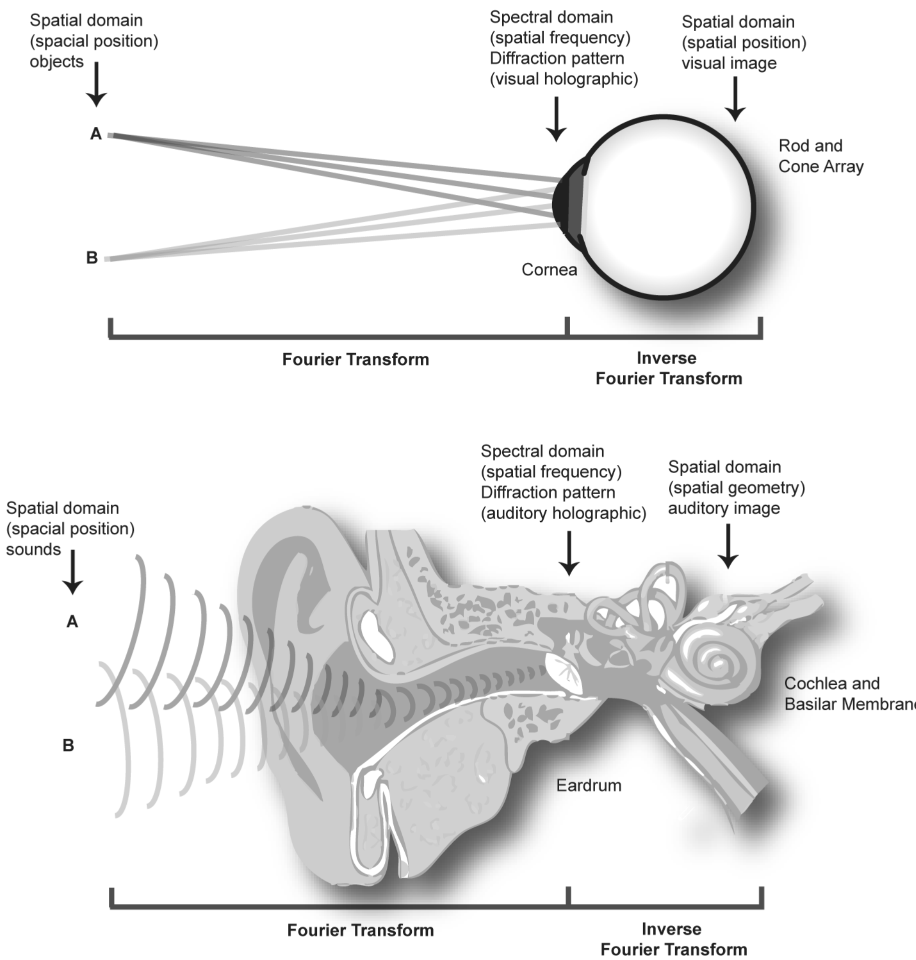



So, what does any of this have to do with music perception and harmony? It has everything to do with it because coherent sound is organized according to the same physical standing wave physics as coherent light. Furthermore, we measure space stereoscopically (like two beams) and in both vision and audition our eyes and ears follow the very same steps to convert wave patterns into neural impulses. Perception works much like laser holography.

Consider the diagram in Figure 72. The first steps of visual perception are described as an organic two-dimensional Fourier transform between our cornea and the rod-cone array at the back of the eye. Similarly, the first steps of auditory perception perform the same two-dimensional Fourier transform between our eardrum and the cochlea. Both vision and audition transform the information in the spatial domain of outer reality into the spectral domain of waves before transforming it back into the proportional spatial domain of our inner reality.

## Figure 72 - 2-dimensional Fourier transform in vision and audition

$$F(\alpha,\beta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j2\pi(\alpha x + \beta y)}dxdy$$




Throughout this entire process, our eyes and ears both work to filter and focus on the most important patterns necessary for survival. They must recognize the most coherent peaks or edges in the incoming interference patterns as well as the calmest regions, emphasizing whatever shapes may exist over the extraneous noise. They must preserve the proportional distance and size of objects (whether visual or auditory) from the outside world while moving it into the neural space of our brain for precise measurement. Entering this neural space through two identical sources (eyes or ears), the information is stereoscopically triangulated as phase shifts within an interference pattern much like a hologram.

In a series of studies where he inserted surgical probes into the visual cortex of the human brain, neurosurgeon Dr. Karl H. Pribram measured the lateralization and layering of neural structure. In doing this, Pribram confirmed that the structure in the brain's visual processing hardware matched the geometrical pattern of two interfering standing waves, something like a hologram composed of neurons. From this he deduced that perception must occur as a neural standing wave interference pattern and, like a hologram, required two coherent sources with which to recognize objects in space. In collaboration with quantum physicist David Bohm, Pribram named this idea *holonomic brain theory* [Pribram 1991].

When we compare this holonomic model with *Harmonic Interference Theory* we find the two are really saying the same thing. That is, the brain relies on two sources, each projecting through a standing wave interference pattern that can be described as a Gaussian ***INTERFERENCE*** distribution. Over a full spectrum, this would necessarily resemble the earlier spectral harmonic interference pattern of two tones diverging over an octave. In this way, the brain would naturally filter and focus the two sensory sources according to the same simple harmonic proportions found over a musical octave while processing each source independently as a standing wave.

For example, if we close one eye or cover one ear, we are processing a single standing wave having equally spaced harmonic proportions. Neural measurement of the incoming information would be focused on the recognition of spikes and fringes near key resonance and damping locations like those in the Harmonic Symmetry Table. As a basic metric for coherence, this process would identify specific frequencies and patterns for matching against known patterns.

But if we open both eyes and ears, we then begin processing two standing waves while measuring the proportional phase shifts between them. Within this holonomic model we have a virtual space where two sets of harmonics interfere to produce a 3-dimensional spatial model of the outside world. Regular harmonic geometries (whether visual or aural) would be the easiest to recognize within this coherent space since the standing wave patterns from both sources would be mostly in-phase. As a seamless process of focusing through the Gaussian structure of our eyes and ears then filtering through the double Gaussian of our brain's neural network, we are naturally predisposed to recognizing coherent harmonic objects in a 3-dimensional space.

From this a more complete model for perception can be proposed. It would be fair to say the brain acts as a holonomic process that is focused harmonically. Furthermore, the ***REFLECTIVE INTERFERENCE*** model and Harmonic Symmetry Table offer a common model to describe sensory perception and the first stages of cognition, particularly as it applies to stereoscopic visual and aural perception. As it applies to music perception in particular, we can describe music as geometrical objects moving through a holonomic space. Here is how it works.

Under a *Holonomic Music Cognition* model, movement of musical objects is described as phase shifts within a harmonic interference pattern. This is based on: 1) holograms require a reference beam and an object beam and 2) holograms encode spatial relationships as phase shifts in an interference pattern. When this is coupled with the Harmonic Symmetry Table describing the phase alignment of odd-even wave components, harmonic music perception can be described as the continuous measurement of phase-shifts between a persistent, time-coherent reference scale (e.g., a coherent reference beam) and incoming melodies, intervals and chords (e.g., the object beam). In this way, harmonic movement in music would be perceived as a holographic auditory object viewed from different angles (phase shifts), thereby simulating movement in a 4-dimensional spacetime. In fact, this is precisely the model already proposed to explain the mechanics of anticipation-reward potential in Principle 29.

Under this model, the brain is viewed as a double standing wave hologram capable of measuring proportions of both 2-D and 3-D auditory objects. Perception of timbre is described as a double Fourier transform into a 2-dimensional shape while perception of music harmony is described as a squared double Fourier transform, appearing in the brain as a 3-dimensional object that moves (or phase shifts) through spacetime.

As a goal seeking system, the central objective of this system would be to measure the relative degree of spatial coherence (foreground harmonic shapes in the interference pattern) and temporal coherence (background Fibonacci shapes in the interference pattern) of sound in order to trigger a response, whether emotional or physical. Together, holonomic brain theory combined with *Harmonic Interference Theory* offers a complete Gestalt model for the understanding of music cognition and the visual modeling of music.[105]

We can now see the brain for what it really is – an egg and yolk system that evolved into a semi-crystallized neurological standing wave interference pattern with the express purpose of harmonically filtering, matching and responding to the most coherent input within a 4-dimensional holonomic space. Seems obvious, does it not?

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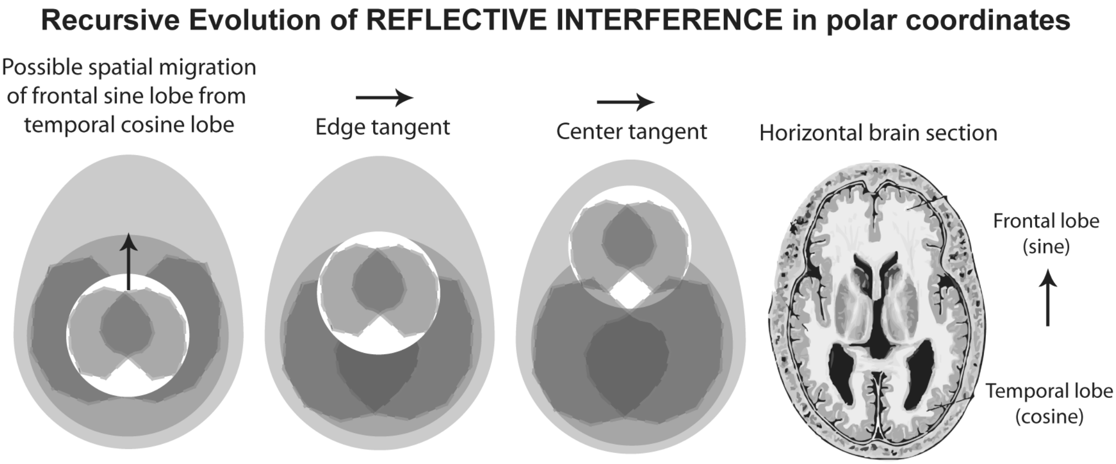
**Principle 33:** *Holonomic Music Cognition* is defined as a spatiotemporal coherence pattern matching operation as follows:

1. Sensory perception of music harmony begins as a neurological Fourier transform from spatial frequency to spatial position and proportion.
2. Spatial coherence in sound is a cognitive pattern matching of harmonic proportions against fixed proportions of the natural harmonic series within a range of tolerance.
3. Temporal coherence in sound is a cognitive pattern matching of melodies, intervals and chords (following Partial 5 and the Fibonacci series as a coherent pathway) phase shifted and/or frequency modulated against a fixed spatially coherent reference scale.
4. The proportions of the reference scale are instantly and economically recognized as neural pathways (like Partial 5) in the auditory cortex.

Holonomic brain theory [Pribram (1991)] offers the best explanation for the cognitive functions required to recognize the standing wave interference pattern produced by the natural harmonic series. From this, we might also predict that the fundamental organizing principles of the brain will follow the INTERFERENCE functions.

**Figure 73 - Is the human brain really a spherical standing wave?**




As the penultimate example of the time-honored *Principle of Least Action*, none of us should be the least bit surprised when scientists some day announce with great fanfare (and some embarrassment perhaps) that the brain is a 5-fold $12^{th}$-power hologram that follows the same symmetrical Gaussian ***REFLECTIVE INTERFERENCE*** resonance model in music harmony. With this announcement we might also hear that human cognition is a least-action transform inside a neurological standing wave lattice according to *Harmonic Interference Theory*.

We may even come to agree that the brain is exactly how physicist David Bohm saw it – the enfoldment of a holographic universe divided between the *explicate* and the *implicate*. His idea was that the geometrical forms we see around us represents the explicate view, like the 3-D images of a hologram, while the universe viewed at an angle represents the implicate view of a swirling 2-D interference pattern of waves. He thought this implicate view of the Universe resembled the surface of a hologram and was really the hidden view of nature described by quantum physics.

Perhaps some day Bohm's holographic universe theory will be an accepted truth. And when it is, perhaps then will we understand life as holographic interference patterns produced by the two coherent DNA of our parents projected inside a 5-level $12^{th}$-power harmonic spacetime.[106]

---

[106] I should add that it was an investigation into Fourier analysis and standing wave interference that led me from *Harmonic Interference Theory* to holonomic theory and not the other way around. The correspondence emerged naturally while researching the behavior of coherent light and holography with respect to the decomposition of sound waves by Fourier transform. Because of this, holonomic theory need not be taken as validation of the correctness of *Harmonic Interference Theory*, as it stands on its own as a unifying tool for music pedagogy. Nevertheless, the concurrence between holonomic brain theory and the harmonic principles proposed here do appear to offer the best explanation for music perception. Through holonomic theory we can embrace a very deep causality behind the musical experience, reaching through the physiology of our body right down to the quantum level.

# *Synesthetic Coupling*

*"A single color is only a color. Two shades form a harmony." - Henri Matisse*

When I was growing up in the 1960's, my family had a collection of colored aluminum drinking cups that produced an odd metallic taste. Whenever I drank from one of these cups, I associated the color, metallic sensation and flavor of the liquid together as a rather strange and complex emotion. Somehow, the mild metallic shock I received from the cup seemed to have fused the color with the taste, leading me to prefer certain cup colors for certain beverages, such as silver for milk, blue for grape juice and red (of course) for strawberry Kool-Aid.

Before long, I began to notice that whenever I saw those specific colors elsewhere, it would trigger the same complex taste emotions. Even today, when I see these specific shades of colors, I experience the same inexplicable but briefly intense emotions that cross the boundary between vision and taste perception. While I could never capture the emotion long enough to describe it to anyone, I did come to understand the phenomena as something called *synesthesia.*

Born out of computer science research into artificial intelligence ("AI"), the cognitive science of synesthesia is the study of cross-sensory perception. From the Greek word *syn-* meaning union and *aesthesis* meaning sensation, synesthesia is a neurological condition in which two or more bodily senses are coupled, or sensed together in some way. There are different forms of such cognitive "union sensation," including grapheme-color synesthesia, number-form synesthesia, personification, lexical-gustatory synesthesia and one other that happens to be very relevant to harmonic science – *music-color synesthesia*. An estimated four percent of the population has some form of synesthesia from the rare occurrence to a continuous cross-sensory experience.

In individuals having music-color synesthesia, colors are experienced in response to tones or other aspects of music, such as timbre or key. Like grapheme-color synesthesia, people who experience music-color synesthesia rarely report the same colors for given tones, yet individuals do remain internally consistent, giving the same results when tested months later.

Colors that trigger pitch sensations may involve more than simply the hue of the color. Brightness or darkness, color intensity and hue may be triggered in a music-color synesthesia episode (Campen & Froger 2003). In addition, it is often reported that the colors move into or out of their field of vision.

Obviously, synesthesia is not only the result of what someone hears or sees, but what is *perceived* in the mind. Sometimes, synesthesia is associated with other neurological or psychiatric conditions that trigger a crossover of perception between different areas of the brain. One particular avenue of synesthesia research involves interviewing people who have taken psychedelic drugs. More than half have reported experiencing synesthesia, such as:

> *"Visions moving on music rhythms,"*
>
> *"Only with my eyes closed have I 'seen' music. It appeared as fine patterns of primary colours, spinning and morphing along with the music," and*
>
> *"…so I watched them, even though I wondered if maybe I hadn't really done it this time, and what they were doing was they were making objects come into existence by singing them into existence. Objects which looked like Fabergé eggs from Mars morphing themselves with Mandaean alphabetical structures." (Terrence McKenna, 1993).*

In his 1954 book *Doors of Perception*, Aldous Huxley details his experiences taking organic mescaline, commenting that the mind normally filters sensory information to avoid confusion while psychedelics suspends these filters to enable perception without interpretation and separation. It would seem that music-color synesthesia is a natural, but thankfully filtered, capability in us all.

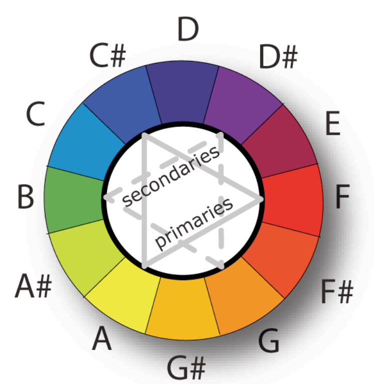
Now, given the previously proposed holonomic model coupling light and sound, it makes sense to find a way to combine color and tone into a single "synesthetic model" as a reference aid for music visualization and analysis. For instance, we could talk about colors Green and Red instead of {B} and {F} or refer to Primary Colors to indicate a set of musical tones. In a visual representation of music as auditory geometry, color could make things a little friendlier. This assumes of course that the two systems work intuitively together.

There have been many attempts through history to establish an association between color and pitch, though none have been universally accepted. Composers like Berlioz, Debussy, Wagner and Scriabin all had ideas about which colors matched which tones. The Rosicrucian Order developed their own color mapping and even Charles Fourier suggested in his 1846 *Theorie de l'Unite Universelle* an alchemical connection between certain pitches, colors and metals. One of the more recent proposals suggests that we should reduce light frequencies down to the speed of sound in order to produce a color mapping. While this last theory is a reasonable approach, physicists would argue against this, pointing out that that sound and light waves are not the same kind of energy. Science requires some other causal link or coupling.

Rather than use any of the above methods, we will construct our synesthetic model from Isaac Newton's popular 12-step *tertiary* color wheel containing three primary colors, three secondary colors and six tertiary colors. Taken as two groups of six colors, the even group of primary and secondary colors can mix adjacently to produce an odd group of tertiary colors in much the same way as one wholetone scale mixes to the other. This suggests coherent light is perceived to mix harmonically in much the same way as coherent sound mixes into music harmony. This is without a doubt due to the fact that the visible light spectrum frequency doubles to form an octave of light just like an octave of sound.

Since the visible color spectrum ranges from about 375 terahertz on the low end to about 750 terahertz on the high end, the visible color spectrum naturally forms a 2:1 octave doubling of light frequencies like that of a musical octave. From this, we can proportionally map twelve colors to twelve tones by starting just below human visibility at 370 terahertz and then calculating twelve color frequencies by multiplying each preceding color by $2^{1/12}$, making sure to balance around the center of the visible spectrum. Doing this creates a logarithmic color scale that perfectly matches an equal-tempered musical octave. It also places each color within its corresponding spectral color band for the three cone photoreceptors on the retina of the human eye (see Figure 74).

**Figure 74 - The synesthetic music-color model for {C} harmonic series**




**Color spectrum to logarithmic octave mapping**
(using $2^{1/12}$ log spacing in terahertz mapped to {C} octave)

| Tone | Color | $2^{1/12}$ Calculated Color Centers |
|---|---|---|
| D# | Purple | 370.0000 |
| E | Magenta | 392.0013 |
| F | Red | 415.3110 |
| F# | Red-orange | 440.0066 |
| G | Orange | 466.1708 |
| G# | Yellow-orange | 493.8907 |
| A | Yellow | 523.2590 |
| A# | Yellow-green | 554.3736 |
| B | Green | 587.3384 |
| C | Cyan | 622.2633 |
| C# | Blue | 659.2651 |
| D | Indigo | 698.4670 |
| D# | Violet | 740.0000 |
| E | Purple | 784.0027 |

| Spectral Color Bands |
|---|
| Red: 384-482 |
| Orange: 482-503 |
| Yellow: 503-520 |
| Green: 520-610 |
| Blue: 610-659 |
| Violet: 659-750 |

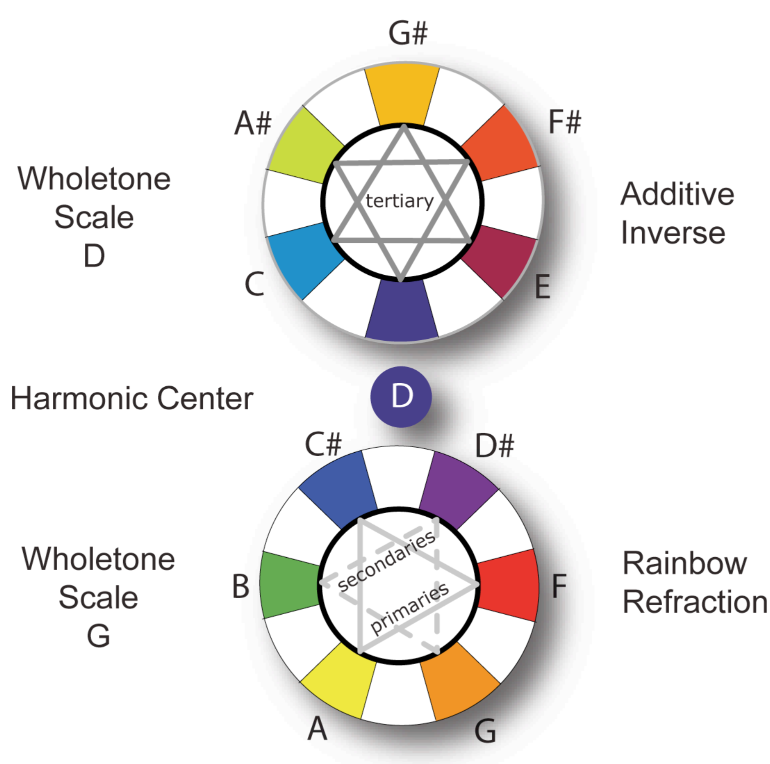
2:1 color octave

In this model, each of the twelve colors map onto each of the twelve tones of an octave such that Green and Red, along with the entire 7-color rainbow, balance around the one symmetrical possibility – the Dark Blue color known as Indigo. Placing this color at the top of the color wheel in alignment with the symmetrical Harmonic Center then results in each of the twelve calculated color frequencies being ordered clockwise from Indigo. The result is a ***synesthetic color model*** compatible with *Harmonic Interference Theory* and usable for music visualization.

For practical purposes, the orientation can be fixed to {C} major around the Harmonic Center {D}, but of course this can always be transposed to other keys as needed. From this, music harmony can be treated like color harmony, mixing colors and tones interchangeably.

As an example, consider the bottom color wheel in Figure 75. Six of the seven rainbow colors are spaced equally by 60° and, as such, represent the primary and secondary colors. This group of six colors then matches the same octave proportions of an equal-tempered wholetone scale. From this, the seventh Newtonian rainbow color, Indigo, resides on the top wheel of tertiary colors as a point of symmetry that corresponds to the Harmonic Center {D} in the harmonic series for Tonic {C}.

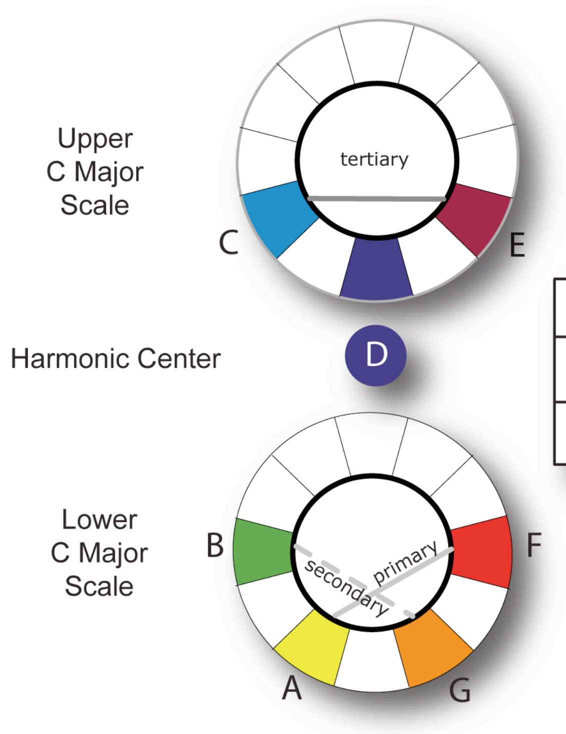
**Figure 75 - Dual color wheels as two opposing wholetone scales**



Other useful correlations are possible with this synesthetic color model. Complementary colors, such as Green and Red, are half a cycle apart, thus representing the same opposing relationship found in the diatonic tritone {B, F}. The dual wholetone scales can also be separated into {Primary, Secondary} and {Tertiary} groups to illustrate an additive inverse of the core rainbow colors. As part of a new synesthetic language for color-tone harmony, we will refer to the bottom wheel as the Dominant and the top as the Tonic.

Simplifying this idea further into Figure 76, we can isolate just the colors of the {C} major scale as they balance around Harmonic Center {D}. Amazingly, this mapping does a good job of illustrating the most important chord cadences, notably the Dominant and Inverse Dominant tones and how they resolve color-wise to Tonic. For instance, each adjacent pair of colors in the bottom Dominant wheel will mix in paint or ink to produce each of the colors in the upper Tonic wheel. This simple color mix method represents the Dominant-Tonic (or V – I) chord cadence, perhaps the most popular chord progression in music history.

**Figure 76 - C major scale derivative color model**




| Color Mix | Pitch Interval |
|---|---|
| Primary | Inv. Dominant |
| Secondary | Dominant |
| Tertiary | Tonic |

Here are some more detailed observations of this synesthetic color assignment as compared to the {C} diatonic scale:

- *The Green and Red colors identify the tritone and, when combined in RGB (Red, Green, Blue) additive color space for light, produce Yellow. This is recognized in our mapping as Inverse Dominant {A}, the relative minor to Tonic {C}. The colors Red, Green, Indigo and Yellow represent the pitches of the "tense" diminished 7$^{th}$ triad used in much of the harmonic music of the past 250 years.*

- *Following the {Green, Red} tritone, conventional theory calls for a contraction to Cyan {C} and Magenta {E}, representing the "resolved" major 3$^{rd}$ interval. In color theory, mixing Cyan and Magenta in CMYK (Cyan, Magenta, Yellow, Black) subtractive color space for paint produces Indigo, corresponding to the Harmonic Center of {D} for {C} major.*

- *The Green {B} from the tritone is the Leading Tone in {C} major, identified in the Harmonic Symmetry Table as the tensest diatonic tone relative to the Harmonic Center. It follows the energy transfer upward a semitone in the Tritone Function to Cyan {C}.*

Of course, comparisons between color and music have always been very controversial, often ending with demands by skeptics to prove coupling. A physicist would point out that the electromagnetic energy of light is a different kind of wave than the mechanical energy of sound, which is of course transmitted through air as compression waves.[107] So then, what should we say is the coupling mechanism between color and tone?

The answer (of course) is us! Just like the coupling of senses in synesthesia, we must consider *ourselves* to be the coupling mechanism between light and sound. Our eyes and ears transform color and tone into recognizable proportions within a shared spatial interference pattern over an octave. It is our brain with its natural double Gaussian filtering and harmonic focusing architecture that equates the two different wave energies of color and tone inside a shared 'continuum of order.'

It is probably worth pointing out that coherent light has been proven to produce harmonics just like coherent sound. This was demonstrated in 1998 by Donald Umstadter and his High-Field Science team at the University of Michigan. Using a high-powered laser system to accelerate electrons near the speed of light, the team found that the laser began omitting light frequencies at whole number multiples (harmonics) of the fundamental frequency exactly like a musical tone. Furthermore, the team recorded specific angular directions for each harmonic frequency, proving that coherent light carries with it a harmonic series of other frequencies like a musical tone. Much

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[107] Coupling is usually explained using the correspondence of lightning to thunder where the coupling mechanism is represented by the vacuum or atmospheric gap created by the super heated air from the electrical energy of lightning. The air then implodes into the gap to cause the sound of thunder.

like over-blowing a flute, light harmonics are only revealed when a specific light frequency is over resonated.

With conclusive evidence of harmonics in coherent light, it is reasonable to assume that our perception of light follows the same Gaussian ***INTERFERENCE*** model as sound. To this point, color frequencies are detected in the eye's retina as resonance on three types of cones arranged according to a Gaussian density curve around the *fovea centralis* (the "blind spot"). This location at the back of the eye acts as the polar origin of a Gaussian distribution curve in the same way the Malleus does in the eardrum.

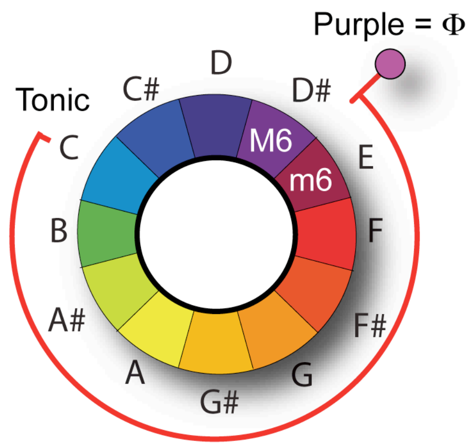
In fact, neurophysiologists often use the *Gaussian Color Model* to describe how our eyes use color to help us recognize shapes. This model is different than any other color model because it combines spatial and color information into an integrated concept of *spatial neighborhood*. Within the Gaussian Color Model there are three variables – hue, saturation and brightness. Hue is calculated as the *mean* of the Gaussian, saturation as the *variance (width)* and brightness as the *area* under the Gaussian curve. Together, these can calculate any visible hue or shade imaginable.

The same model can be applied to sound. In the Shepard Tones experiment, Roger Shepard was employing what we might call a *Gaussian Tone Model* that is analogous to color vision. It too had three variables composed of a reference tone, an interval and an amplitude curve. The Shepard tone represented the mean of the Gaussian, the Shepard interval the variance (or width) and the amplitude as a Gaussian fade curve in time. Taken together, the Gaussian Color and Tone Models represent a unified synesthetic framework for perception entirely compatible with the models used in *Harmonic Interference Theory*.

In both cases, Gaussian models couple vision and audition through the same kind of spatial neighborhood. The brain seems hardwired to interpret both light and sound as a periodic space within a spectral Gaussian distribution polarized around the central mean of the curve (either fovea centralis or malleus). And, since Gaussian physiology can only be grown according to simple harmonic and Fibonacci damping principles, *Harmonic Interference Theory* based on the Gaussian ***INTERFERENCE*** function is the only possible organic way to explain our perception of repeating octaves in both light and sound. As it comes together inside our brain, the ***REFLECTIVE INTERFERENCE*** distribution is our best explanation for the "higher-order continuum of order" predicted by experimental psychology and the Gestalt school.

Through the synesthetic color model, we might now fully understand what it means to mend the octave closed – even in light. As the one color not present in the visible light spectrum, Purple is how our brains close the spiraling spectrum of light into a periodic octave. As the color interval {Red, Blue} (or {Magenta, Violet}) at opposite ends of the visible light spectrum, Purple is what we call the imaginary splice point at the edges of our visual perception. It is the "invisible" 180° opposite of Yellow Green at the ambiguous tritone center of our visual range. It is the "horse of a different color" in Oz. Purple is the color of infinity.

**Figure 77 - Purple mends the color octave closed at the golden ratio**




But there is something else very special about Purple that everyone should know. Relative to the Cyan Tonic {C} on the synesthetic color wheel, Purple is found between the Magenta minor 6th {E, C} and Violet major 6th {D#, C}. This point happens to lie between the Fibonacci ratios 8:5 = 1.6 and 5:3 = 1.66666666 at the infinite proportion of $\Phi \approx 1.618033$. The stunning thing we find in all this is Purple is the color our brain uses to mend the infinitely spiraling spectrum of light closed at the irrational (and invisible) golden ratio. Long considered a color worthy only of gods and royalty, we should more accurately call this special interval from the Tonic fundamental the *Purple ratio*.

So it appears from this discussion that perception of color and tone share the same ***REFLECTIVE INTERFERENCE*** model for filtering and pattern matching within a neurological holonomic space. Furthermore, vision and audition, oriented at orthogonal angles of one another in the physiology of the brain, focus light and sound harmonically within a periodic 2:1 octave framework in order to recognize coherent shapes in both space and time. Taken altogether, human perception is undeniably predisposed to the recognition and anticipation of regular geometric patterns within the 5-fold 12th-power spatial neighborhood described by the ***INTERFERENCE*** function recursing across a projected Harmonic Hierarchy.

Neurological research, when combined with *Harmonic Interference Theory*, strongly supports the psychophysiological coupling of colors and tones within the physical Gestalt model of standing wave interference. Though this does not constitute conclusive experimental proof, we can safely accept the synesthetic color model as a valid cognitive hypothesis and visual aid for harmonic analysis.[108]

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[108] **Principle 34:** The best-fit *Synesthetic Color Model* matching the perceptual interference pattern of the harmonic series maps color frequencies of common practice primary, secondary and tertiary colors onto the 12-step octave. The assignment proceeds clockwise as an ascending spectrum of frequency.

| Diatonic Scale Step | Color Assignment | Color Group |
|---|---|---|
| Inverse Dominant (Submediant) | Yellow | Primary |
| | Light Green | Tertiary |
| Leading Tone | Green | Secondary |
| Tonic | Light Blue (Cyan) | Tertiary |
| | Blue | Primary |
| Harmonic Center | Indigo | Tertiary |
| | Violet | Secondary |
| Inverse Tonic (Mediant) | Dark Red (Magenta) | Tertiary |
| Inverse Leading Tone (Subdominant) | Red | Primary |
| | Red Orange | Tertiary |
| Dominant | Dark Orange | Secondary |
| | Orange | Tertiary |

**SECTION FOUR**

# Harmonic Models

*"The sciences do not try to explain, they hardly even try to interpret, they mainly make models. By a model is meant a mathematical construct which, with the addition of certain verbal interpretations, describes observed phenomena. The justification of such a mathematical construct is solely and precisely that it is expected to work." - Johann Von Neumann*

By the early 1990's, the computer industry was awakening to the potential of CD-ROM optical storage for delivering music, video and animation. Computer giant IBM was partnering with Intel to develop *Digital Video Interactive* (*DVI*) while Microsoft was developing its own multimedia formats and extensions to their *Windows* operating system. Hewlett Packard had already tried in the late 1980's to set the standard for multimedia with their "compound document" format in *HP NewWave*, but Microsoft quickly made it obsolete by announcing the same thing in *Multimedia Windows*. There was an industry-wide war underway to be the first to bring multimedia capabilities to the personal computer and ultimately into the living room.

The graphics software industry, then led by Micrografx where I worked, was at the forefront of using CD-ROMs to deliver large libraries of clipart and images. As the one responsible for the company's multimedia presentation product *Charisma*, I expanded this to include sound effects libraries and video clips. The industry mantra had become "content is king" and it was only a matter of time before competitors Corel, Adobe and Microsoft would have their own multimedia CD-ROM products.

Since I had a long-held personal interest in combining graphics and animation with music in my music theory, I thought it was important to make sure *Charisma* was equipped with a large library of professional audio clips. In my quest for this, I was led to a Los Angeles company named Prosonus. The president of Prosonus, Ken Rose, had what I thought was the best library of digital sound effects and music clips available. But there was something else interesting about Ken. He knew the sax player for the art rock band Pink Floyd, who just happened to be a favorite musical group of Micrografx founder and president George Grayson.

At our first meeting in Dallas, I introduced Ken and George. In our conversation, Ken mentioned that he was going to a party in L.A. that weekend where Scott Page, the former saxophone player for Floyd, would be in attendance. Given George's interest in meeting Scott, Ken invited him to go with him.

In those days, Scott owned a sound recording and merchandising company in Glendale named Walt Tucker, in honor of both Walt Disney and carmaker Preston Tucker. To my surprise a relationship quickly developed between George and Scott, leading to business discussions around the production of interactive multimedia CD-ROM titles. One of the first ideas, as I recall, was to license the digital rights for the comedy trio *The Three Stooges*.

George's radical departure from productivity software to entertainment titles came as quite a shock to the company's board of directors, even laughable to those conservative businessmen. To me it just seemed inevitable that interactive games would be the next big growth market for the computer industry. In fact, unbeknownst to anyone at Micrografx I had already been working on my own business plan to start up a CD-ROM entertainment company in partnership with a local video postproduction house. My idea was to create an interactive adventure game to visit the seven wonders of the ancient world where the player would seek out clues to unlock some secret knowledge. What that knowledge was, I had no idea, but it still seemed like a fun idea.

As I daydreamed about how digital multimedia could be used to create virtual worlds and entertain, I began to treat business presentations as if they were musical theatre rather than just slide shows. Instead of plain text, I preferred a blend of graphics, animation, music, sound effects, voiceovers and video to create a more immersive experience. Using pre-release versions of *Charisma*, I would try to time and choreograph what I said with music and animation so that everything flowed together like a movie, blurring the lines between live and recorded, real and virtual. When I was successful, these mixed-reality presentations became a kind of educational entertainment where I became an actor in a story rather than just another presenter clicking on slides. The punch line would always be entertainment sells…and so the story would go.

But in my mind, the multimedia clipart libraries in *Charisma* were much more than mere fodder for business presentations – they represented *archetypal fragments* that could tell a story symbolically. If an archetype for some idea did not yet exist, I could always create it from stock clipart or record something new to make my point. As if composing a piece of music, I would weave together thematic layers, timing them to create a sense of anticipation and reward with each scene. It was always great fun browsing and selecting media chunks to emphasize my point, often adding a surprising twist or metaphorical viewpoint. To me it was really just another form of music, a mixed-reality multimedia theatre.

While working on these multimedia shows, I would often stop and ponder the possibility of developing a similar software product for music composition. Not the arcane professional music composition programs or the silly band-in-a-box ideas, but a visual music composition system where shapes on the screen represented the same shapes that music would form *in your head*. I knew that before long computers would be powerful enough to reproduce my original standing wave models for visual music.

Powered by new generations of computers, I could imagine a modern day *Music of the Spheres,* spinning the spiraling galaxies of musical scales into the cyclic rings of planetary chords. I was sure something like this would project beautifully on large screens, behind a symphony orchestra or hovering over an opera, materializing harmonies into real-time musical landscapes. With this I envisioned dancers moving in geometric procession through a spectral light show of sound, perhaps even radiating down onto the stage from computer-generated clouds above. In my Pythagorean dreamscape of harmonic numbers and forms, the geometric splendor of nature would literally unfold onstage.

To me, this kind of multimedia experience would represent the ultimate expression of musical theatre – one where the physics of perception could play a central role. I knew computers had the unique potential to reveal the secrets of harmonic science by transforming sound into its visual form. I thought perhaps it would be through this technology where we would one day see the return of the long forgotten musical-astrological myths and their superheroes of coherence.

Staged and choreographed by the archetypes of nature themselves, Venus would float above the clouds as the pentagonal *Star in the East* whilst Atlas played his damping role in the Underworld. As the legendary hero of resonance himself, Hercules might once again return to his twelve labours, retrieving the three Golden Apples and defeating the evil tri-headed dog. Harmonia and Eris would also return to their posts in the clouds, telling us of the musical balance that sustains our world. Even Pandora – the old diva herself – would make a cameo appearance to mend the spiral into a circle, at last bringing coherence in place of panic and pandemonium.

In some grand scientific musical theatre of the future – composed of coherent sound, light and structured space – I was convinced the *Music of the Spheres* and its supporting cast would make a comeback. And when it did, I knew it would transform everything and everybody. It would change how we look at music and how we teach our science. It would mend the schisma between ancient harmonic philosophy and modern science. It would end the conflict between the Church and its pagan adversary, returning the “lost key” of wisdom to our technological world.

# *Cyclic Rings*

*"We dance around the ring and suppose, but the secret sits in the middle and knows."*
*- Robert Frost*

In the 1960's, American music theorist and composer David Lewin proposed a new way to represent music using a mathematical system he called *transformational theory*. At the same time in Europe, music theorists Anatol Vieru and Iannis Xenakis began exploring a similar approach that represented music as a "collection of elements, relations between them and operations upon them." But it was the German team of Halsey and Hewitt who ultimately defined the group theoretical musical system used today in their celebrated 1978 paper *Eine gruppentheoretische Methode in der Musik-theorie*.

Since those days, music pedagogy has continued to steadily evolve toward what is now called *musical set theory*. Today's composition students are often required to learn musical set theory as a way to analyze and describe music as *pitch classes* and *class operations* in place of the old rules of religious tradition. Musical set theory has become the new language of music and the new face of musicology in the 21st century. With the addition of *Harmonic Interference Theory*, this formalized system promises to usher in a grand new era for musical study.

Since musical octaves are perceived as periodic, modern algebraic theory describing repeating groups has become the preferred tool of theorists to describe abstract musical concepts. Founded on the idea of a ***cyclic ring***, each element in an octave can be produced by repeatedly adding (or as a ratio multiplying) a constant to generate all twelve pitches in a periodic octave.[109] In an equal tempered octave ring, each pitch element is multiplied by the constant $2^{1/12}$ to wrap around into a circular octave after twelve steps. While no other temperament can be created so elegantly, any tuning system can be described as a cyclic ring as long as exclusivity and coherence in pitch space is maintained (Principles 30 and 31).

Using this foundation, a cyclic ring of twelve elements can then be represented by the set of integers $\mathbb{Z} = \{0, 1, \ldots, n\text{-}1\}$ where $n = 12$. This is symbolized as $\mathbb{Z}/12\mathbb{Z}$ (or "Z-twelve")[110] operating under the rules of modular arithmetic (called *modulo n*) where numbers "wrap around" after they reach a certain value known as the *modulus*. For music, the modulus $n = 12$ and

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[109] A *cyclic ring* of *n* elements is a group (G, $\cdot$) where G is a group and $\cdot$ is a binary operation with at least one element g such that each element of G is equal to g $\cdot$ g $\cdot$ … $\cdot$ g a finite number of times.

[110] It is purely a coincidence that Zarlino and Zacconi are responsible for the 12-step octave now represented in musical set theory as Z/12Z. Or is it?

corresponding constant ratio $2^{1/12}$ are said to produce a *congruent equivalence class* of tones, meaning only that all twelve tones are equally space around the cyclic ring.

Within the cyclic ring of $\mathbb{Z}/12\mathbb{Z}$, the operators of addition and multiplication are then used to form musical intervals, therein producing a special kind of cyclic ring called a *commutative ring*. From this, all of the commonly used scales, melodies and harmonies in Western music can then be derived using this one commutative cyclic ring.

For instance, if we start the $\mathbb{Z}/12\mathbb{Z}$ cyclic ring from {C} as:

*ℤ/12ℤ = { C, C#, D, D#, E, F, F#, G, G#, A, A#, B },*

we can stack an interval of a major 6th {C, A} above a perfect 5th {C, G} to reach the pitch {E} in the next octave.[111] In this way, all stacked intervals extending beyond an octave simply wrap around and remain within the $\mathbb{Z}/12\mathbb{Z}$ cyclic ring.

In most applications of musical set theory, traditional note names are replaced with a number in the range {0..11}, corresponding to the twelve pitches in a "chromatic" octave.

*ℤ/12ℤ = { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 }*

Thus, the cyclic ring for pitch is composed of only twelve elements, neatly divisible by two, and producing group symmetry as a *dihedral group*.[112] As it pertains to musical set theory, a dihedral group enables two important operations – rotation and reflection, with respect to an axis. In music lingo, reflections are called *inversions* while rotations are called *transpositions*, both relative to a selected reference pitch that acts like an axis. A dihedral group in music is then the collection of all possible transpositions and inversions of symmetries in the cyclic ring $\mathbb{Z}/12\mathbb{Z}$.

Any number of cycles, or "pitch class sets", can also be derived from the dihedral group. In set theory these cycles are referred to as "affine orbits" or simply *orbits*, a term preferred in contemporary French music theory in place of the standard pitch-class set nomenclature found in American universities [Andreatta, et. al., 2000]. The French term really makes more sense, especially when you consider the fact that harmonics oscillate symmetrically around a Harmonic Center in a standing wave. At the octave level, scales, intervals and chords also form orbits around the cyclic ring of an octave.

So, if we adopt "orbit" in place of "pitch class set," any number of musical orbits can be created through transposition and inversion using the musical set theory operators $\{T_n, I_n\}$ as

[111] This is done by adding the interval {C, G} with {C, A} to produce the resulting interval {C, E} represented simply as: E = G + A.
[112] A dihedral group is represented as $(D_n, \cdot)$ and said to be of order 2n.

applied to any *n* elements in $\mathbb{Z}/12\mathbb{Z}$.[113] Through these operators, concatenating and superimposing orbits and orbital fragments from the $\mathbb{Z}/12\mathbb{Z}$ cyclic ring can reproduce any kind of harmonic music.[114] This method also allows for the representation of music in tabular form or even visually as geometric models.

For instance, we could represent musical orbits in a 2-dimensional table like Arnold Schönberg's 12-tone matrix. We could tile orbits in other ways on an imaginary surface or tessellate tones in a 3-dimensional space for the purposes of visualizing, analyzing and

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113 In set theory, operations for transposition $T_n$ and inversion $I_n$ are simple addition and subtraction within a modulo-12 cyclic ring. For example, if n, m $\in \mathbb{Z}/12\mathbb{Z}$ (n and m are an element of the octave cyclic ring) such that n is the pitch to be transposed and m is the transposition interval, we can say:

$T_n = (n + m) \bmod 12$

Similarly, if n $\in \mathbb{Z}/12\mathbb{Z}$ we can invert any interval by subtracting from octave 12:

$I_n = 12 - n$

For example, using our earlier addition for n = 7 and m = 9 we can transpose {G (7)} up a major 6th to {E}:

| | |
|---|---|
| $T_7 = (7 + 9) \bmod 12$ | // G + A |
| $T_7 = 4$ | // E |

We can also use subtraction to invert an interval n, such as a perfect 5th {G} inverted to become a perfect 4th {F}:

| | |
|---|---|
| $I_7 = 12 - 7$ | // Octave - G |
| $I_7 = 5$ | // F |

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**Axiom 1:** Transposition $T_n$ and Inversion $I_n$ operations for orbits under $\mathbb{Z}/12\mathbb{Z}$ can be defined as:

If n, m $\in \mathbb{Z}/12\mathbb{Z}$ such that n is the pitch to be transposed and m is the transposition interval, then:

$[T_n = (n + m) \bmod 12]$ $[I_n = 12 - n]$

**Axiom 2:** An ***initial set definition of affine orbits*** for cyclic ring $\mathbb{Z}/12\mathbb{Z}$ can be defined as:

| | |
|---|---|
| m2 / M7 Orbits: | $[T_n, I_n : n, m = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}]$ |
| M2 / m7 Orbits: | $[T_n, I_n : n, m = \{0, 2, 4, 6, 8, 10\}]$ |
| m3 / M6 Orbits: | $[T_n, I_n : n = \{0, 3, 6, 9\}, m = \{0, 1, 2\}]$ |
| M3 / m6 Orbits: | $[T_n, I_n : n = \{0, 4, 8\}, m = \{0, 1, 2, 3\}]$ |
| P5 / P4 Orbits: | $[T_n, I_n : m, n = \{0, 7, 2, 9, 4, 11, 6, 1, 8, 3, 10, 5\}]$ |
| TT Orbits: | $[T_n, I_n : n = \{0, 6\}, m = \{0, 1, 2, 3, 4, 5\}]$ |
| Major Scale Orbits: | $[T_n : n = \{0, 2, 4, 5, 6, 7, 11\}, m = \{0..11\}]$ |
| Minor (Relative) Scale Orbits: | $[T_n : n = \{0, 2, 3, 5, 7, 8, 10\}, m = \{0..11\}]$ |

composing music. The famous 20th century French composer Olivier Messiaen (who by the way lived in Avignon and experienced a mild music-color synesthesia) was famous for transforming natural melodies and rhythms from bird songs into tiled orbits for use in canons and other musical styles. For Messiaen, this method was such an instinctive compositional method that he gave little consideration for harmony or pitch, composing music from patterns alone.

Yet even with all its mathematical clarity, musical set theory does have a major drawback. It has a tendency to focus attention on numerical patterns while deemphasizing or even obscuring the natural harmonic relationship that exists between two or more tones. For instance, when we list out a set, say the triad {C, E, G}, we lose sight of the fact that the interval {C, E} is different from {E, G} or {C, G} – even if they are clearly defined elsewhere. As a result, the current trend in modern music theory has become heavily weighted toward identifying and categorizing abstract patterns of pitch, rhythm and form rather than any underlying harmonic properties that might affect the emotional experience. This is not too surprising when you consider that musical set theory first emerged out of the 20th century atonal movement whose overriding concern was the very avoidance of conventional diatonic harmony rather than its advancement. Truth be told, set theory can be unintuitive and get in the way if your goal is to create harmonies that sound natural and organic.

In an effort to compensate for the drawbacks of a purely mathematical approach, computer software tools have been developed in recent years to represent music with visual models. In a 1998 paper entitled *Music Theory's New Pedagogability*, music composer and theorist Richard Cohn called for a more interdisciplinary approach to music theory using "computational musicology" methods to model music visually. As a response to this, a Lisp-based visual programming language called *OpenMusic* was developed by the Ircam Music Representation Group in Paris that enabled group theoretic visual representations of music for composition and analytical purposes.

One particular visualization technique in *OpenMusic* involves the use of musical pitches on circles (or rings) to represent what they call *mathematical music theory*. This type of representation comes closest to achieving Cohn's vision by creating a new visual framework for music that is both easier to understand and unrestricted by historical conventions. *OpenMusic* correctly represents interval proportions as a circular geometry, fully supporting algebraic ring theory in a way that anyone can immediately and intuitively understand.

Yet, even *OpenMusic* does not fully address the question of how we perceive harmony nor does it help us properly measure musical qualities, like consonance, dissonance or the progression of harmony in time, based on physics or physiology. Set theory really does nothing, in and of itself, to help us understand, represent or predict how we actually recognize harmony.

To this end, *Harmonic Interference Theory* can extend the visualization of musical set theory to represent coherent sound as our ears and brain understand it. We can develop new and better ring and orbital models to translate the physical standing waves of music into the organic forms of geometry. From this we can invent a new language of *musical objects* from the physical archetypes of nature to express the harmony of resonance and damping in both sound and the brain. In this way, musical set theory can become the new face of *musica universalis*.

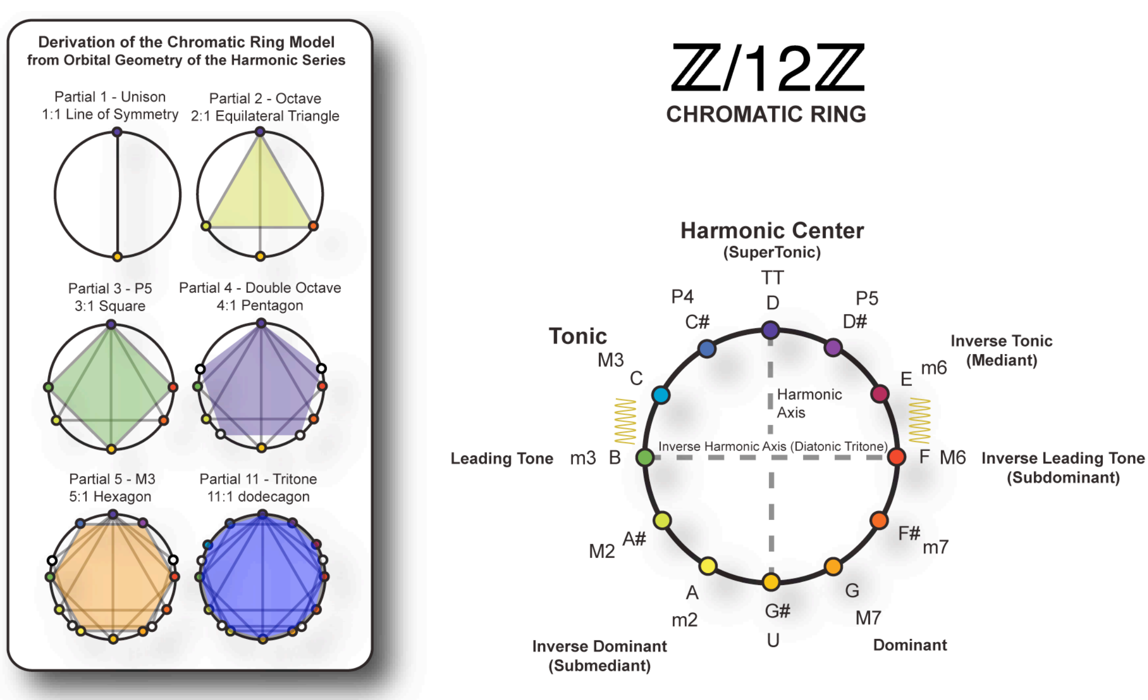
And in the interest of returning natural philosophy to its proper role in the Arts and Sciences, we will extend these musical models to include the acoustical symbolisms and mythological archetypes of a musical nature. Whether in the classroom, a theatre or a cloud of portable music devices, the application of *Harmonic Interference Theory* to musical set theory and computer-aided music visualization promises to revolutionize the role of music in modern society. The time has come to deliver on this promise.

# *Orbital Geometry*

*"The orbit of human vision has widened and art has annexed fresh territories that were formerly denied it." - Max Bill*

The first step in visualizing musical set theory is to harmonically divide a circle using whole number orbits balanced about a vertical line of symmetry. As each harmonic orbit is added, the ring is gradually divided into twelve equal sections. This geometrical progression begins with a 1:1 line, then 2:1 equilateral triangle, a 3:1 square, a 4:1 pentagon, a 5:1 hexagon then jumping up seven partials to an 11:1 dodecagon to complete the ring. In this way, an equal 12-step division of an octave can be represented as a symmetrical assignment of harmonic orbits where each tone is separated by $\pi/6$, or 30° increments on a 360° circle. Each element in the $\mathbb{Z}/12\mathbb{Z}$ musical set is then assigned in clockwise order on the cyclic ring starting at the Harmonic Center at top to create the *Chromatic Ring* shown in Figure 78.

**Figure 78 - The $\mathbb{Z}/12\mathbb{Z}$ Chromatic Ring**



The Chromatic Ring gives us the simplest possible geometrical representation of an octave. Relative to Harmonic Center {D} for the key of {C} major, it aligns symmetrically with the white keys on the piano keyboard. To make the ring even more intuitive, the synesthetic color model from Principle 34 can be mapped onto the ring so that each tone is colored according to its corresponding $2^{1/12}$ logarithmic proportion in the visible light spectrum. Adding the common practice abbreviations for intervals and the earlier proposed symmetrical labeling, the Chromatic Ring becomes the ultimate visual model for the organic representation of music.

From prior discussions, you may recall that the tritone {D, G#} is the axis of symmetry and greatest stability within a {C} major scale. And that at a right angle to this axis is the diatonic tritone {B, F}, representing the axis of greatest resonance and instability within {C} major. As we now find them on the Chromatic Ring, the vertical axis becomes the stable *Harmonic Axis* and the horizontal axis the "restless" *Inverse Harmonic Axis*, otherwise known as *Diabolus in Musica*. According to the earlier Tension Metric, the points of maximum tension on the ring would be Green {B} and Red {F} on the Inverse Harmonic Axis followed by Cyan {C} and Magenta {E} as the Tonic major 3$^{rd}$ in the key of {C}. These four tones taken together represent the harmonic engine of the Tritone Function, indicated on the ring by two gold springs.

Besides just a nice visual representation of $\mathbb{Z}/12\mathbb{Z}$ set theory, this geometric representation of an octave also suggests several interesting "alchemical" associations. For instance, the Tritone Function could represent the phase shift of organic chemistry that occurs nine months into the Earth's orbit around the Sun. As plants lose their chlorophyll molecule in autumn and the carotenoid molecules (mostly carotene) and flavonoids (mostly anthrocyanins) begin to appear, we see a progressive counterclockwise shift between Green, Yellow, Orange, Red (perhaps Magenta) and finally an equal mix of Red and Green as Brown. This progression of colors on the Chromatic Ring correspond geometrically to a contraction of the diatonic tritone on the ring, therein representing the Tritone Function in a 12-month annual calendar as it moves from the autumn equinox to the winter solstice. As the cycle begins anew in spring, the chlorophyll once again returns to complete a full tritone traversal and bisection of the Chromatic Ring.

From this interpretation, a composer might see autumn as a kind of musical cadence of organic chemistry that passes from the tense diatonic tritone through the Dominant colors and resolving into the winter at the center of the ring. In the Spring, the opposite clockwise color shift toward the blues then matches chlorophyll's oxidation of $H_2O$ (water) into $O_2$ (oxygen) gas, reflecting a light cyan as the color of oxygen in our atmosphere.

Taking this one step further, it happens that green chlorophyll corresponds to the Green Leading Tone in music, which (as we know) leads to the Cyan Tonic of oxygen. Carotene is then the Yellow and Orange Dominants of autumn while anthrocyanin becomes the Magenta Inverse Leading Tone resolving into dormant Brown in the middle winter of the ring. The organic chemistry of plants essentially follows the same Dominant-Tonic chord progression used in

Country and Western music. While this may sound more like alchemy than music or chemistry, the colors reflected by plants (and perceived by our eyes) do indeed indicate which chemicals are predominant at a specific interval in the annual cycle and do in fact correspond harmonically to music and color through the Chromatic Ring.

The organic compounds in plants naturally follow a quarter rotation cycle from horizontal to vertical that matches the orthogonal phase shift from sine to cosine components in a harmonic standing wave. With phase relations the same at all levels of nature, we find that plant chemistry exhibits an atomic 90° phase shift reflective compounds in order to match and optimize the Sun's energy transfer to the Earth. Plants literally consume specific harmonic light frequencies based on the Earth's orbital phase.

Given the same periodicity and oscillation elsewhere in nature, many other correspondences can be made. As a model for all things harmonic, the Chromatic Ring model can provide scientists and artists alike a source of endless inspiration and guidance.

## Octave Orbits

Using the Chromatic Ring as a foundation, we can begin to build out the geometric architecture of music. The octave orbits of 2nds, 3rds and 5ths and their inverse 7ths, 6ths and 4ths appear as the following geometric shapes on the ring:

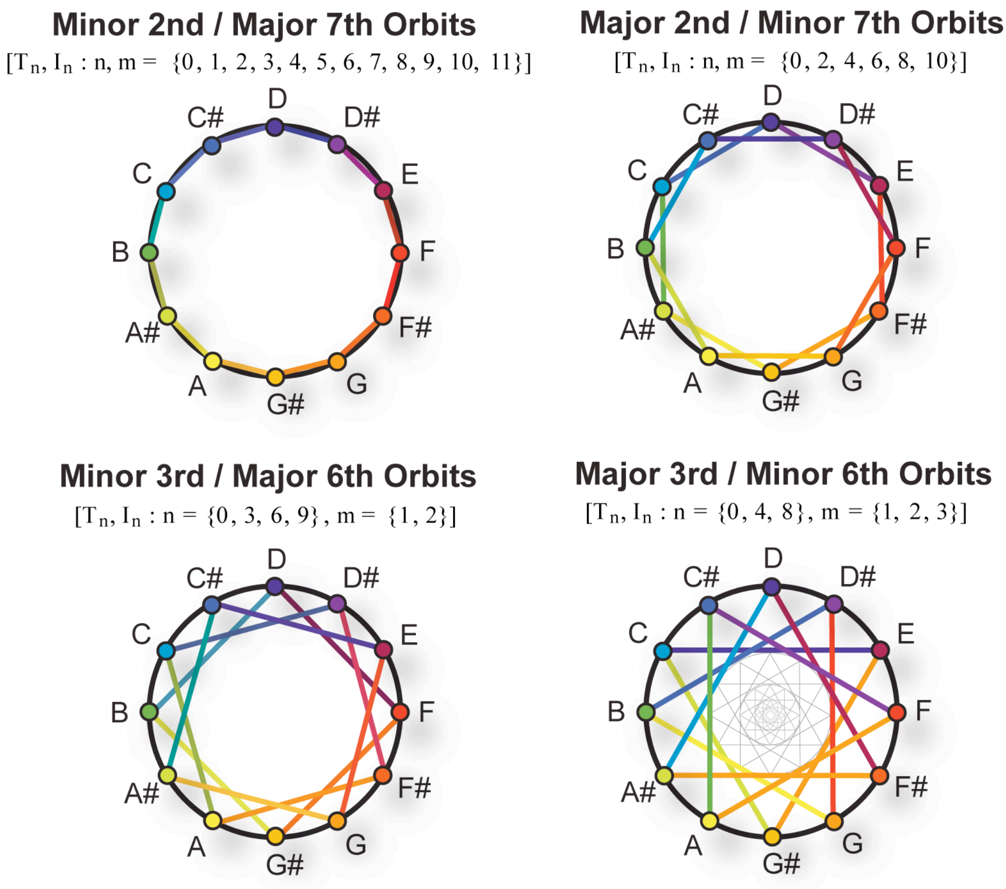
1. *one dodecagonal orbit of {m2, M7},*
2. *two hexagonal orbits of {M2, m7},*
3. *three square orbits of {m3, M6}, and*
4. *four triangular orbits of {M3, m6}.*

Like the harmonic shapes shown earlier in Figure 78, intervals in an octave also follow simple periodic shapes. But unlike the harmonic geometry in a single tone, the polygons formed by musical orbits do not initially appear to match those in the harmonic series.

For instance, in Figure 79 the major 3$^{rd}$ interval orbit (corresponding to Partial 5) creates an octave triangle while it is an octave harmonic orbit (Partial 2) that creates a 2:1 triangle. Similarly, the minor 3$^{rd}$ interval orbit (corresponding to Partial 6) creates an octave square while it is perfect 5$^{th}$ (Partial 3) that forms a 3:1 harmonic square. Why would there be a difference between the auditory geometry of an interval orbit and harmonic geometry in a single tone?

Though puzzling at first, the difference is due to the simple fact that an interval orbit represents an equal division of the logarithmic *spiral* of pitch while a harmonic orbit represents an equal division of a *cyclic* tone. One divides an octave while the other divides a tone.

**Figure 79 – Orbital geometry of 2nds, 3rds, 6ths and 7ths**




We can convert between harmonic and octave geometries very simply using base-2 logarithms (explained in footnote).[115] But make no mistake, intervals and harmonics produce the same

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To calculate an equal tempered interval from its corresponding harmonic, simple raise 2 to the inverse exponent of its harmonic equivalent:

Interval = $2^{(1 / (\text{Partial} + 1))}$

For instance, to find the square interval orbit corresponding to the harmonic square Partial 3, just plug 3 into the equation to find a minor 3rd:

Interval = $2^{1/4}$ = 1.189207115 // equal to the equal tempered minor 3rd, an auditory square.

geometry in pitch space – one as an equal division of a spiraling octave and the other as an equal division of a standing wave. It works because the frequency doubling of a musical octave is in phase with Partial 2 of the harmonic series, thus requiring base-2 to convert between the two.

The Harmonic Hierarchy as defined in Principle 21 supports this conclusion. It says that harmony is no different than timbre, so recognition of geometry at one level would be the same at another. So in listening to music, our psychoacoustical pattern matching equipment would recognize the same simple shapes and patterns and assign identical qualities of consonance and dissonance or tension and resolution to both timbre and harmony.

To get an idea how this must appear to our ears and brain, take a look the major 3rd / minor 6th orbits in the previous figure and how it recursively nests within itself inside the Chromatic Ring model. Each level of the Harmonic Hierarchy fits inside the next while maintaining exclusivity and spatial coherence.

When a musician plays a diminished chord composed of minor 3rds, he or she is playing a square shape that would be recognized in the same way as the timbral square of Partials 3 and 1 – just at a different 12th-power frequency scale. Similarly, playing a hexagonal wholetone scale would be recognized in the same way as the timbral hexagon produced by Partials 5 and 1. In general, the geometry of the harmonic series in a tone (as reflected in the neurophysiology of our auditory system) acts as a geometric template for music cognition. This is something like the Hindu and Buddhist concept of a *mandala*.

Mandalas have been around for thousands of years, originating in the Hindu holy book of the *Rig Veda*. Meaning “essence,” “having,” or “containing” in Sanskrit, mandalas are often used as sacred decorations and as an aid in meditation. Tibetan Buddhists believe these geometric patterns represent the source of experience as “a microcosm representing various divine powers at work in the universe.” The psychoanalyst Carl Jung agreed, saying the mandala was in some way “a representation of the unconscious self.” While no historical record exists to tell us of the true

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Inversely, we can calculate the harmonic partial from the equal tempered interval by taking the inverse log of the interval in base 2:

$$\text{Partial} = 1 / \log_2 (\text{Interval}) - 1$$

Using this equation, we can find which harmonic partial corresponds to the hexagonal interval orbit of a major 2nd by plugging its equal tempered value $2^{1/6}$ into the equation to find that Partial 5 combines with Partial 1 to create a 5:1 = 6 hexagonal auditory geometry in the harmonic series.

$$\text{Partial} = 1 / \log_2 ( 2^{1/6} ) - 1 = 5 \qquad \text{// equal to Partial 5.}$$

meaning of these circular patterns in Indian culture, musical vibratory patterns on a circular surface are the most likely explanation.

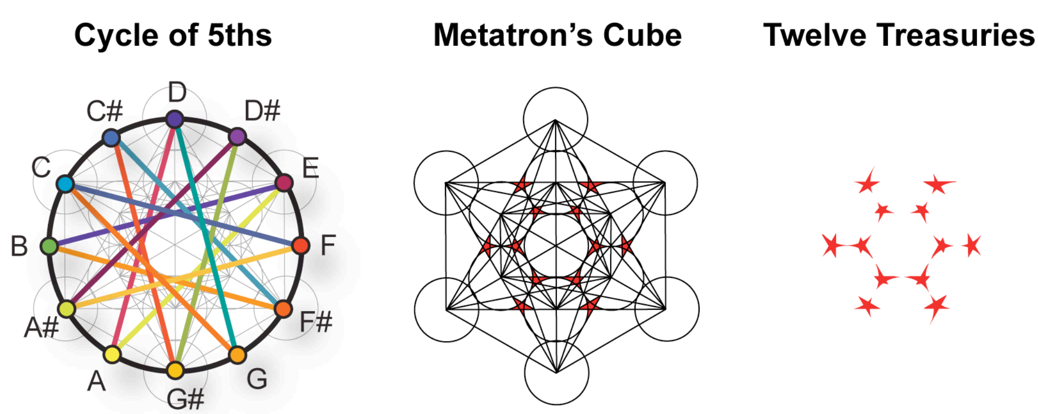
Mandala patterns are probably just stylized interpretations of Chladni figures, the geometric patterns of sand that form from standing wave interference patterns on round drum surfaces. They also follow the same geometric shapes of harmonics and intervals found in the orbits of the Chromatic Ring. Since ancient times, the conversion of sound currents into iconic ring geometries has been the sacred model to understand all patterns in nature, especially the organic geometries of life that unfold from the conversion of electromagnetic energy into animate matter.

In this light, consider next the auditory geometry of the Cycle of 5ths in Figure 80. This orbit of perfect $5^{th}$ intervals completes a circuit of all twelve tones in the Chromatic Ring as it spins around seven octaves. As it does this, the connecting lines circumscribe a smaller ring in the center. Amazingly, this small ring corresponds exactly to the circles used to construct *Metatron's Cube*, explained earlier as originating from the Egyptian/ Pythagorean *Flower of Life* pattern. This is due to the 3:2 arrangements of circles in Metatron's Cube corresponding to the musical proportion of a perfect $5^{th}$. But the correspondence of sacred geometry to music doesn't stop here.

In the pattern are twelve irregular pentagonal stars that occur in the intersecting regions between each perfect $5^{th}$, which are almost certainly the "twelve treasuries" revered by the Gnostics. When you stop to consider our recognition and historical preference for the musical Cycle of 5ths, it is no wonder that Metatron's Cube and the Cycle of 5ths were considered sacred to the Gnostics and Pythagoreans. Far more than just a nice pattern, this auditory shape played a central role in the development of ancient theism descended in some way into all world religions. While now mostly forgotten, every one of them was founded on the physics of harmonics.

**Figure 80 - Orbits of 5ths and 4ths as Metatron's Cube**

$[T_n, I_n : m, n = \{0, 7, 2, 9, 4, 11, 6, 1, 8, 3, 10, 5\}]$

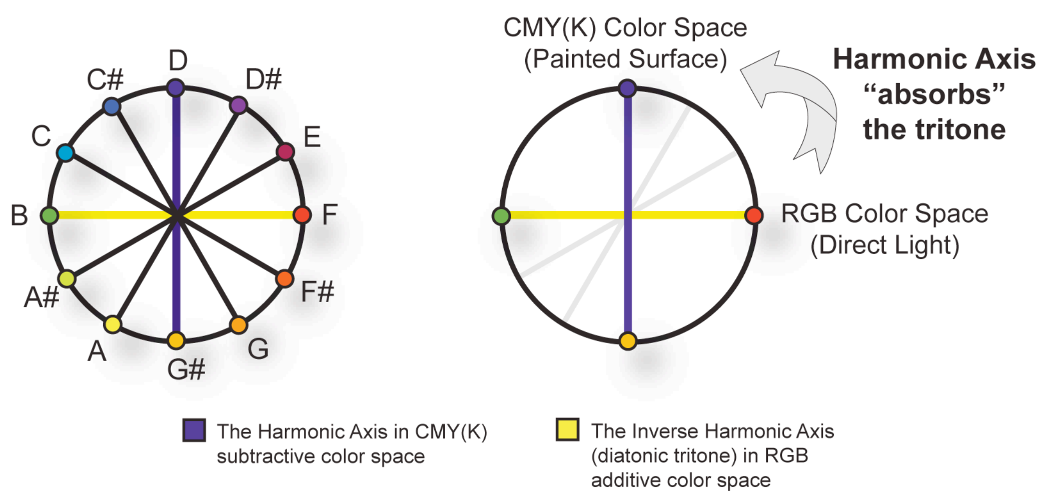


## Tritone Function

The last orbit to consider is (of course) the tritone. It is the only interval that passes through the center of the Chromatic Ring, thereby raising the question of how to assign a color to its interval. The tritone's ambiguity becomes readily apparent when we try to mix the endpoints {Green + Red} to produce Brown, which is not to be found on the ring. The only solution is to *polarize* it to a specific Harmonic Center, just as it occurs in the Tritone Function of a 7-step diatonic key.

**Figure 81 – Tritone orbits in color space**

$[T_n, I_n : n = \{0, 6\}, m = \{1, 2, 3, 4, 5\}]$




For instance, when the horizontal {B, F} tritone is mixed as {Green, Red} in RGB *additive color space,* it yields Yellow as the assigned color[116]. Then, when we take the complement of Yellow in the *subtractive color space* of CMY(K) used for mixing paint and ink, the resulting Indigo color is naturally assigned to the vertical Harmonic Axis {D, G#}[117]. Of course, Indigo

[116] RGB color space is a theoretical 3-dimensional space comprising the colors Red, Green and Blue that is capable of creating all of the colors in the visible light spectrum. This is the method used in CRT television monitors and other such direction projection devices to create full color images and video. In general, RGB is used to mix colors in direct light by the addition of varying amounts of these three colors, often in increments of 256 steps for each color.

[117] CMYK color space is a theoretical 4-dimensional space comprising the colors Cyan, Magenta, Yellow and Black ("K" often taken as "kill") that is capable of creating most of the colors in the visible light spectrum. This is the method used in paint, color inkjet and offset printing to reflect full color images in magazines and books. In general, CMYK is used to mix colors of reflected light by subtracting varying amounts of these four colors, again often in increments of 256 steps for each color.

already matches the color of the Harmonic Center {D} at top rather than the alternative Yellow-Orange {G#} at bottom. In this way, the orthogonal (right) angle between the horizontal and vertical axes are a perfect geometrical representation for the opposition between the additive RGB color space of direct light and the subtractive CMY(K) color space of reflected light. This particular instance of visual isomorphism between tone and color harmonies is a first principle for harmonic science since it explains music tonality (diatonic scales) as a direct result of duality and polarity found elsewhere in nature.

When we map color space to tonal space within a polarized cyclic ring we find that the *horizontal* diatonic tritone is to *direct* light what the *vertical* Harmonic Axis is to *reflected* or absorbed light. In this way, the vertical Harmonic Axis acts like a "painted surface" in a musical scale that absorbs the "energy" of the diatonic tritone.

While it may sound a bit odd that musical scales and harmonies would be polarized and that sound energy could be absorbed like light on a painted surface, this is really how diatonic harmony has been used throughout history. Consider that a diatonic scale is symmetrically polarized around its SuperTonic at a right angle (or minor 3$^{rd}$) to its diatonic tritone, therein triggering our anticipation for the entire scale to contract inward toward the SuperTonic (Harmonic Center) just like paint absorbs light. Acting like a spring, the horizontal tritone stores energy while the Harmonic Center and its vertical axis forms the damping axis to absorb it.

With the basic ring model now constructed for the key of {C}, the *Principles of Harmonic Interference* can be applied. This begins by adding the Φ-damping locations of the Tritone Function to the Chromatic Ring model. As in the earlier ***REFLECTIVE INTERFERENCE*** octave model, Fibonacci proportions can be represented as spirals starting at the Harmonic Center and swirling into two counterbalancing Landau damping wells positioned on the ring.

There are a few different ways to illustrate this, but the easiest way is to connect the two converging Fibonacci series as lines across the ring. This then visually represents the Tritone Function as a series of alternating or oscillating lines that converge horizontally into ±Φ locations. As shown in Figure 82, the first non-zero proportion in the Fibonacci Series is 1:1 indicated on the ring as the vertical gold Harmonic Axis at x = 0. After this, the remaining values are all horizontal, clearly demarking the Landau spring or spiraling vortex action inside the Tritone Function.

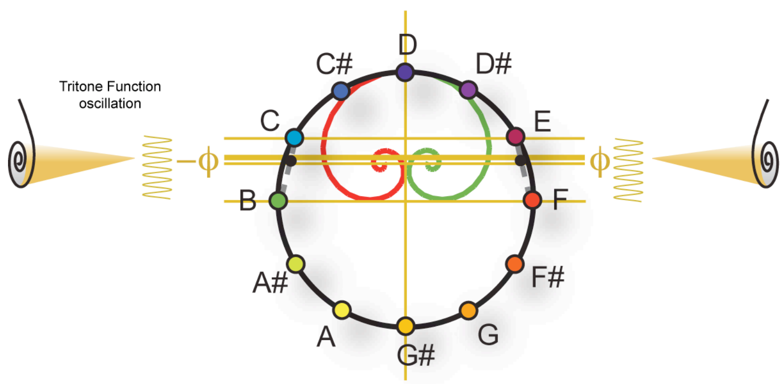
If the Fibonacci spirals are then run backwards from ±Φ to the Harmonic Center, the orthogonal shift from horizontal to vertical illustrates the process of tritone energy absorption. This is precisely what the Fibonacci series does in nature – it absorbs energy in a standing wave to regulate oscillation. So while the harmonic series constructs the cyclic geometry of the $\mathbb{Z}/12\mathbb{Z}$ Chromatic Ring, it is the enharmonic proportions of the Fibonacci series that polarize the ring around the vertical Harmonic Axis by symmetrically "siphoning off" some of its energy. The

Tritone Function then follows and reinforces our recognition of this polarity by oscillating across these damping whirlpools, repeatedly "mending the schisma" closed in the Chromatic Ring.

**Figure 82 - The Tritone Function on the Chromatic Ring model**

$$x=\left(\pi-\frac{\pi}{m}\right), m=\left\{\frac{1}{1},\frac{2}{1},\frac{3}{2},\frac{5}{3},\frac{8}{5},\frac{13}{8},\frac{21}{13},\frac{34}{21},\frac{55}{34}\right\}$$

$$x=\left(-\pi+\frac{\pi}{m}\right), m=\left\{\frac{1}{1},\frac{2}{1},\frac{3}{2},\frac{5}{3},\frac{8}{5},\frac{13}{8},\frac{21}{13},\frac{34}{21},\frac{55}{34}\right\}$$




A musical set theory definition for the Tritone Function helps describe how the oscillation between potential and kinetic energy works in any diatonically polarized music. The first step is to define one orbit for the "restless" tritone interval and another for the "resolved" major 3rd. These two orbits are then combined in odd-even 'clock tick' fashion to mimic the oscillation of a standing wave.

In musical set theory, we might choose the trident-looking Greek symbol ψ (Psi pronounced "sigh") to represent the clock ticks in an *oscillation set*. This set comprises the odd (tritone) and even (major 3rd) tones in the musical subset ψ□{-1, 0, 4, 5} which we extract from ℤ/12ℤ. Using this, the oscillation set for time *t* would look something like this:

***The Tritone Function*** [118]

$\{\psi^{t}{}_{2z}$ □ $\psi^{t+1}{}_{2z+1}\}$

To make this theoretical Tritone Function oscillate and drive diatonic harmony, we need only feed it ascending time *t* values and tone *z* values from ψ. This definition has the advantage of

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[118] **Axiom 3:** The ***Tritone Function*** is defined by the harmonic oscillation of orbits ψ□{-1, 0, 4 5} taken from ℤ/12ℤ. This is represented by the dihedral relation $\{\psi^{t}{}_{2z}$ □ $\psi^{t+1}{}_{2z+1}\}$ as it occurs over a time *t* and between 2ℤ (even) and 2ℤ+1 (odd) cycles.

flexibility when describing the flow of diatonic melodies, intervals and chords because it allows the odd tritone and even major 3rd tones in ψ to occur both separately and together as long as the adjacent semitones are *exclusive in time*. From this basic definition, the Tritone Function is easily applied to any key using the transposition and inversion operators of Axiom 1. In general, the oscillation set ψ can be applied to define many other conventions of diatonic harmony, such as the Cycle of 5ths progression and common practice voice leadings.

But diatonic harmony is not all this oscillation set offers. Chromatic harmony can also be seen as a generalization of diatonic harmony using ψ. As explained earlier, the harmonic duality of late Romantic, Impressionist and 20th century music followed a more "Gestalt" oscillation of the Tritone Function as a contrast between foreground and background. In set theory, this generalized oscillation can be defined by extending ψ so that it alternates between odd and even wholetone orbits on the Chromatic Ring.

***The Wholetone Function***[119]
$WT^t_z = \{\psi^t_{2z} = \{0, 2, 4, 6, 8, 10\} \cup \psi^{t+1}_{2z+1} = \{1, 3, 5, 7, 9, 11\}\}$

The method used in musical set theory to produce this function is known as *transitive closure*. It involves applying the Tritone Function repeatedly in an odd-even fashion across all tones in $\mathbb{Z}/12\mathbb{Z}$, thereby forming a *higher-order* oscillation between the wholetone orbits. In this way, the Tritone Function and Wholetone Function are part of a unified language for both diatonic and chromatic music that is founded on the physical model of an oscillating standing wave. This in turn can be extended into the larger cognitive "continuum of order" by incorporating visual harmonic properties like geometrical shapes or color mixing.

As example, the synesthetic color model parallels the Wholetone Function as colors mix between foreground {Primary, Secondary} and background {Tertiary} groups. Every art student knows that the primary and secondary colors mix into tertiary colors (and vice versa), forming a foreground-background alternation of six colors each. Geometrically, these become congruent odd-even groupings of tones and colors in the shape of alternating hexagons, or dual Metatron's Cubes if you prefer, that split the $\mathbb{Z}/12\mathbb{Z}$ Chromatic Ring into a balanced duality.

Under the power of these harmonic oscillation sets, music, colors, numbers and shapes can all be visualized together on the dihedral Chromatic Ring. And within this visual framework of standing wave physics, many other connections between music and nature also become possible.

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[119] **Axiom 4:** The ***Wholetone Function*** is a generalization of the Tritone Function to represent chromatic harmony as the oscillation between the two wholetone scales:

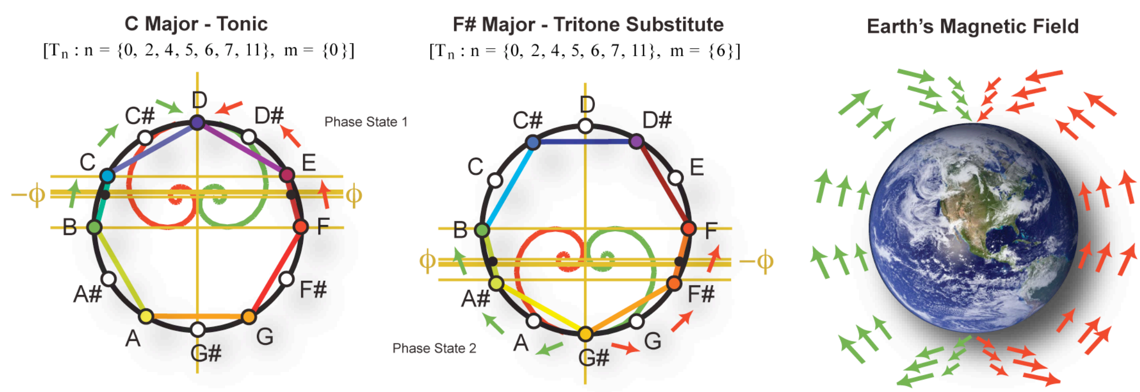
$$WT^t_z = \{\psi^t_{2z} = \{0, 2, 4, 6, 8, 10\} \cup \psi^{t+1}_{2z+1} = \{1, 3, 5, 7, 9, 11\}\}$$

## Musical Magnetism

The Tritone and Wholetone Functions not only reveal the mechanisms at work underneath diatonic and chromatic harmonies, but can also explain the underlying mechanism of modern Jazz. Recall from an earlier discussion that a tritone substitute occurs when two Dominant 7$^{th}$ chords are stacked a tritone apart, therein strengthening the pull toward the Tonic. This pulling sensation can now be described as the synchronous flow of energy between two opposing standing waves and Tritone Functions, forming something like a *magnetic musical field*.

In Figure 83, the diamond-like diatonic orbit of {C} major is illustrated with its tritone substitute {F#} major orbit – each representing the direction of energy flow in an imaginary vertical dipole magnet like that of the Earth.

**Figure 83 - The Jazz Tritone Substitution as magnetic attraction**




Since the horizontal tritone {B, F} contains the only common tones between the two diatonic orbits, the other tones reflect vertically above and below the horizontal line of symmetry. Each opposing Harmonic Center on the vertical axis then acts like a positive or negative pole in an ordinary dipole magnet to represent how the two diatonic orbits can attract one another and even *stick together*. As it has been used in chromatic music and especially Jazz, this dipole attraction property in standing wave interference is easily recognized and considered harmonically pleasing, whether played simultaneously or as substitutes for one another in a song. But what is it exactly that triggers this sensation of "opposites attracting" in traditional jazz harmony?

When tones from the "negative" {F#} orbit are substituted or played together with its "positive" {C} orbit, they can be said to "phase lock" to the "polarity" of the oscillating Tritone Function in order to pull together toward a single Harmonic Center. As described in Principle 20 and illustrated earlier in Figure 59, the anticipated direction of energy flow in a standing wave is determined by the context of the diatonic scale or key previously established in the music, here defined as {C} major pulling inward to the Harmonic Center {D}. As it applies to the jazz tritone substitute, the energy current in the standing wave interference pattern could be said to *flow synchronously* from the Inverse Harmonic Center {G#} upward on both sides of the ring. This behavior is entirely consistent with the *Phase State Model* proposed earlier.

Interpreting the two rings as synchronized standing waves, their orbits would phase lock and flow in the same direction to avoid self-destruction and coexist in the same spacetime. In the figure, Diatonic Phase 1 in the <u>upper half of the left ring</u> would occur simultaneously with Diatonic Phase 2 in the <u>lower half of the right ring</u>. So, as the Fibonacci series spirals inward to the Φ-damping locations to establish the home key of {C} major, the Fibonacci series in the tritone substitute key {F#} *unwinds in reverse* toward the vertical Harmonic Axis. The cognitive result would be that we perceive the two Tritone Functions synchronized as a single standing wave and thus represented flowing bottom to top on the combined rings. In this configuration, the two standing waves intensify the polarizing effect of the Tritone Function around a single center.

The Earth's magnetic field works in much the same way. The magnetic field flows perpendicular to the direction of horizontal rotation, creating a toroidal flow from south to north over the surface of the planet while radiating outward into space. Of course, this is what causes a compass needle to point to magnetic North Pole. The Jazz tritone substitute really does work a lot like the Earth's magnetic field, pulling "northward" to the Harmonic Center. Given that all life evolved within the dipole magnetic field of Earth and shares a similar electrical polarity, it should not come as a surprise to anyone that we would have the natural ability to recognize magnetic principles in the interference pattern of harmonics whenever we listen to music.

# *Unfolded Dominants*

*"Darn the wheel of the world!*
*Why must it continually turn over?*
*Where is the reverse gear?"*
*- David Attenborough*

In an earlier discussion we found that Landau damping was the underlying cause for the half twist at each level of the Harmonic Hierarchy. Like two swirling tornados drilling down into the "cracks between the keys," musical standing wave interference generates the same torsion effect as a hurricane. More than just a nice metaphor, the idea that harmonic interference patterns torque into audible geometric shapes in our ears is very real.

Landau damping theory tells us that as a standing wave begins to vibrate, a pressure differential builds up where waves of different velocities cross one another, creating a damping well in the surrounding field of wave interference to suppress fractional waves. The Fibonacci series represents this increasing torque toward golden sections in the same way as a hurricane forms its two golden spirals into the center. This physical twisting action, known as the *Coriolis Effect*, also occurs in the standing wave of a tone. Instead of a hurricane's double torus of swirling air and water currents, diatonic music forms a pattern of harmonic sine and cosine waves in phase-quadrature and traveling in opposite directions that is represented as a twisting operation in the $\mathbb{Z}/12\mathbb{Z}$ Chromatic Ring.[120]

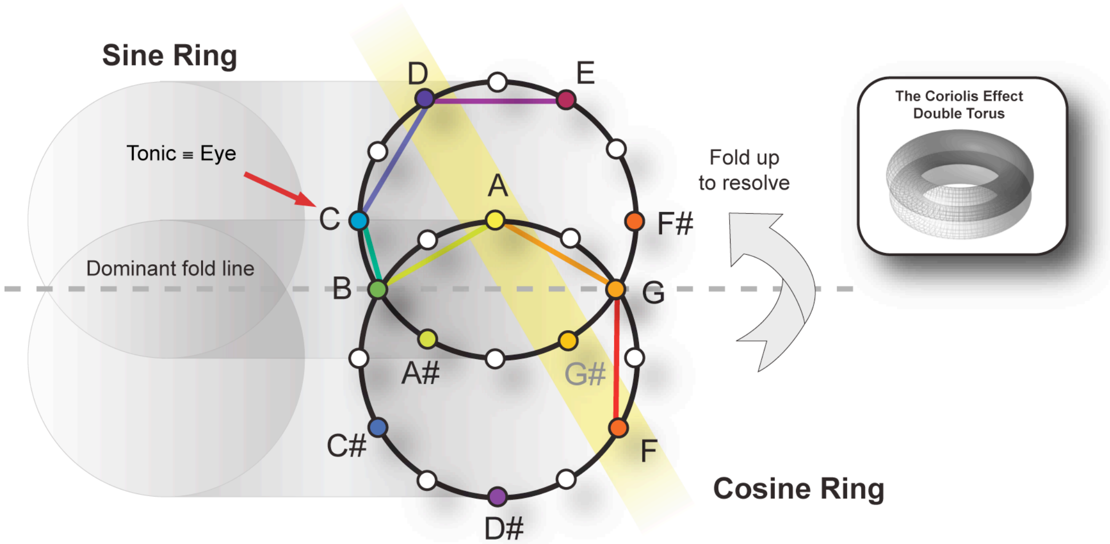
In the context of musical set theory, the odd-even sine and cosine wave components can be represented geometrically as two cyclic rings counterbalanced in an interlocking fashion. More specifically, the two rings must each intersect the center of the other ring to mimic the way in which sine and cosine waves balance with one another in phase-quadrature. Furthermore, it requires that we split the $\mathbb{Z}/12\mathbb{Z}$ Chromatic Ring into two opposing hexagonal groups of tones corresponding to the dual wholetone orbits. Under the Wholetone Function defined in Axiom 4, this takes the geometric form of the "unfolded" *Diatonic Dual Ring* model shown in Figure 84.

[120] The *Coriolis Effect*, named after Gaspard-Gustave Coriolis, a 19th century French scientist, is caused by the Coriolis force which appears in the equation of motion of an object within a rotating frame of reference. For instance, in the rotating reference frame of the Earth, moving objects veer to the right in the northern hemisphere and left in the southern. The spin is accompanied by the formation of a double torus flow (like stacked donuts) where each torus rotates in opposing directions while pulling material inward to a common center, forming a disc of material spiraling in between. This idea is commonly taken within the theory of torsion physics to explain an apparent orthogonal relationship between gravity and electromagnetism.

### Figure 84 - Folding the Yellow Inverse Dominant (the Subdominant)

$$y=\cos\left(m\left(\frac{7\pi}{6}+x\right)\right)+\sin\left(m\left(-\frac{\pi}{6}+x\right)\right),\ m=\{1..16\}$$

$$WT^{t}_{z} = \{\ \psi^{t}_{2z} = \{0,\ 2,\ 4,\ 6,\ 8,\ 10\} \quad \psi^{t+1}_{2z+1} = \{1,\ 3,\ 5,\ 7,\ 9,\ 11\}\}$$




Here's how it works. The bottom ring is unfolded out of the top ring (toward the viewer) making the top six tones the Tonic ring and the bottom six tones the Inverse (Sub) Dominant ring. The two rings are arranged to touch their mutual centers, creating an orthogonal (90°) phase shift between them relative to the Tonic. The center of the top ring becomes the Inverse Dominant {A} due to the fact that this tone is a quarter-octave diatonic interval from Tonic {C}. The remaining five tones on the bottom Dominant ring must then be ordered in reverse – as a reverse *half twist* – so that the tones and colors align where the two rings intersect.

This ring arrangement forms a *Yellow Inverse Dominant* chord {D, F, A} as a diagonal vector through the center of the top ring. In common practice, the center {A} is the root of the relative minor and geometric inverse of {C} major. The dashed *fold line* between the rings then naturally aligns with the major 3$^{rd}$ {G, B} of the Dominant chord, so creating an edge of reflective symmetry in the key of {C}.

The geometry of the Diatonic Dual Ring model represents the duality and natural balance in the most popular musical harmonies in a very organic way. Just as the single Chromatic Ring represents the magnetic field of the Earth, the dual rings represent the Coriolis Effect at work in a hurricane on the surface of the Earth. Both are instances of the same thing – the intersection and interplay of two opposing standing waves, the only difference being the angle of view.

For the single ring, we are viewing harmonic interference *vertically* as we would see the Earth from orbit. But for the dual rings, we are viewing it *horizontally* as if we were standing on the Earth looking at the edge of a hurricane. In the figure, the intersection area between the dual rings is analogous to a cutaway view of half of a hurricane seen from the side with air and water flowing in opposing directions above and below a disk of clouds. The dual rings are then simply a slice through a double torus (or interlocking donuts if you prefer) that grind against one another to produce torque in the middle and increasing toward the eye. The only way for the hurricane to stop is for one of the two rings to stop rotating and collapse back to a single ring.

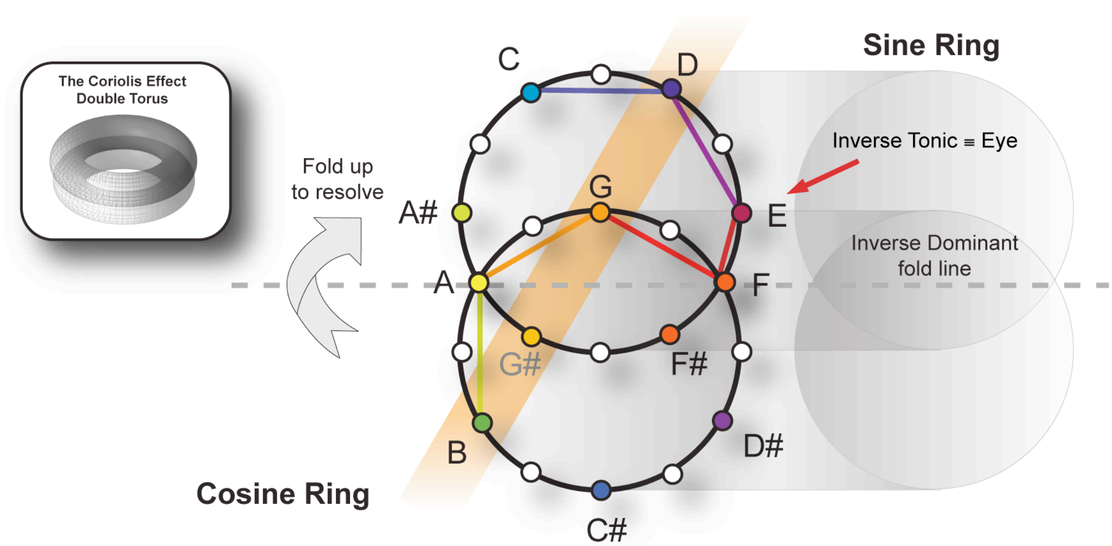
Harmony in music works just like this. In the Diatonic Dual Ring model, harmonic resolution occurs by *folding* the rings together while harmonic tension occurs by *unfolding* them. So, when we fold the dual rings into a single Chromatic Ring to resolve the Inverse Dominant chord, the tones follow a flow from the inside out and around like that of a magnet or hurricane. In this case, {A} flows from the center to the Tonic {C} while {F} flows around to {E} – together forming the interval of a resolved Tonic major 3$^{rd}$. This fold operation in musical pitch space is commonly referred to as the ii - I (or IV$^{6}$ – I) "plagal cadence" and represents a shifting of even cosine components from a phase-quadrature alignment into a fully synchronous in-phase alignment to odd sine components. In this way, folding up the Diatonic Dual Rings into a single Chromatic Ring is identical to a resolving phase shift of chords in diatonic music.

Using this same model for the Inverse Tonic, the new fold line becomes the Inverse Dominant major 3$^{rd}$ {F, A} for what is now an *Orange Dominant* chord {G, B, D}. This is shown in Figure 85 ready to be folded (or resolved) with a half twist counterclockwise.

**Figure 85 - Folding the Orange Dominant**

$$y=\cos\left(m\left(-\frac{7\pi}{6}+x\right)\right)+\sin\left(m\left(\frac{\pi}{6}+x\right)\right)$$

$$WT^{t}_{z} = \{\ \psi^{t}_{2z} = \{0, 2, 4, 6, 8, 10\} \quad \psi^{t+1}_{2z+1} = \{1, 3, 5, 7, 9, 11\}\}$$



Like turning a volume knob down on a music player, we resolve this tense Dominant chord by rotating the top ring one "notch" to the left to phase align and "fold up" the bottom ring. Doing this cause the center {G} to be pulled "magnetically" toward {E} while the {B} slides around to lock in place next to {C} – again creating the resolved Tonic major 3rd. This is how musical set theory can be used to visualize the most popular chord progression of all time – the Dominant-Tonic cadence.

When we compare the Yellow and Orange Dominant models to the physical behavior of a hurricane, we find that they both balance around the Tonic major 3rd {C, E} to form a central axis just like a hurricane's eye. The Yellow and Orange Dominant chords act very much like the arms of a musical Coriolis Effect to stir up a harmonic hurricane, one rotating left while the other rotates to the right, creating a "∧" vortex shape. The dominant centers {A} and {G} are equidistant and balanced in the middle of these arms. Their directional opposition then creates torque that is represented geometrically by the intersecting disk region between the rings, which then pulls inward to the Tonic eye. Like the dipole attraction of musical magnetism, we can presume that human physiology has evolved according to the same counter-directional unfolding pattern and instinctively follows such currents as phase shifts in the interference pattern of harmonics.

Of course, the hurricane analogy should also hold true for the synesthetic color model. The Yellow {A} folds to the Cyan {C} in the Inverse Dominant ring set while the Orange {G} folds to the Magenta {E} in the Dominant ring set. Just as we find with musical tones, both color transitions occur as an orthogonal phase shift. Since the Dominant colors {Yellow, Orange} are primary and secondary respectively and Tonic colors {Cyan, Magenta} are both tertiary, we again find that common chord resolutions in music are the same as mixing primary and secondary colors to produce tertiary colors. This is then part of an overarching harmonic framework where colors and tones can both be described by the Coriolis Effect as it occurs in *any* Gaussian interference pattern. In dual ring geometry, this becomes simple fold operations in space.

The idea of representing audio/visual perception as an unfolding of colors and geometry is really nothing new. If you live in the United States, you have probably seen this on television as the NBC peacock's colorful tail unfolding to the sound of 3 tones on a xylophone. Or you would have also seen the CBS network "eye" logo – an exact match for the geometry of the dual rings. In light of the preceding discussion, one cannot help but wonder if these logos were designed as an artistic whim or from knowledge of something much older – an ancient symbol of geometric harmony perhaps.

It may well be that the intersecting circles of the CBS eye were adopted from the "sacred geometry" of Gnosticism descended from the Egyptian/ Pythagorean mystery school. Long before any idea of modern physics or wave theory, Gnostic theosophical literature spoke of intersecting and reflecting circles called *emanations*.

In the *Book of Jeu*, it describes "three watchers" who *emanate* into "twelve heads in each place of the treasuries." And it describes the beginning of the Universe, saying: "the Father is ready to bring forth emanations." In the literature, emanations are always represented using three circles or rings. The "Father" ring is said to emanate the "two Sons," which are then balanced exactly like the Diatonic Dual Ring model to complete the triad or "holy trinity." It is in this intersection of emanations where we find the origin of sacred geometry (and quite possibly the CBS eye logo).

In practice, the Gnostics used the central Father ring and either of the two Son rings to construct higher-order geometries. The intersection area between the dual rings is known as the *Vesica Piscis* or *Mandorla*, believed by the Pythagoreans to be the underlying geometry of Metatron's Cube and through which the pentagram and other regular figures are formed. In fact, it is still used to this day by geometers to derive geometric structures from first principles.

There is a great deal more to the Vesica Piscis than can be covered here, but suffice it to say that it is part of a comprehensive geometrical explanation of the cosmos based on intersecting and rotating octahedrons. Two of these create the Vesica Piscis while three create the *Tripod of Life* and six create the *Seed of Life*. This is then expanded into the *Flower of Life* from which Metatron's Cube (the *Fruit of Life*) and the five perfect solids can be derived.[121] From this, the ten-point *Tree of Life* diagram central to the Rosicrucian Order can also be found. But at the bottom – underneath it all – lay the harmonically balanced Diatonic Dual Ring system of the Vesica Piscis.

Perhaps the most amazing thing of all is the geometric ratio of width to height in the Vesica Piscis intersection is the square root of 3 (≈1.73205). This is known as Theodorus' constant or *the measure of the fish*.[122] While the Gnostic "fish" metaphor is a common religious symbol today, often affixed on the rear of automobiles to symbolize "Christ the fisherman," it also happens to be the approximate shape of our cornea and used in the manufacture of optical lenses. What could be more appropriate than a √3 unfolding of musical rings to help focus light and aid vision?

To modern scientific eyes, sacred geometry such as this can be interpreted within the context of coherence and a physical standing wave. Pythagoreanism and its related Gnosis are actually a knowledge system of coherence in nature designed to impart the underlying harmonic relationships shared between numbers, sound, light and life. But whether we choose to describe

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[121] The *Seed of Life* can be found on the opening page of the Preface while the *Flower of Life* is found in the Epilogue.

[122] Also known as the Theodorus constant proven by Theodorus of Cyrene around 800 BC. It is believed he proved this using the Pythagorean method of odds and evens, composing a spiral of contiguous right angles with hypotenuse length equal to the square root of 3. This method is now called the Spiral of Theodorus.

this from the perspective of ancient theosophical wisdom or modern science, we can always use *Harmonic Interference Theory* to accurately explain and represent it within a unified $\mathbb{Z}/12\mathbb{Z}$ musical set theory and geometric visualization framework.

The Diatonic Dual Ring model is not simply an abstract representation of harmony – it is an isomorphic representation of the way in which we perceive the symmetry and proportional relationships in sound and electromagnetic waves. Through this perceptual model everything becomes the same thing unified through a physical model of duality. Here is a list of terms to summarize a few of the attributes found so far:

| ***Top Ring*** | ***Bottom Ring*** | ***Isomorphic Category*** |
|---|---|---|
| *In-Phase* | *Phase-Quadrature* | *Trigonometry* |
| *Sine* | *Cosine* | *Trigonometry* |
| *Circle* | *Center* | *Geometry* |
| *Folded* | *Unfolded* | *Geometry* |
| *Major* | *Minor* | *Music* |
| *Tonic* | *Dominant* | *Music* |
| *Resolution* | *Tension* | *Cognition* |
| *Tertiary* | *{Primary, Secondary}* | *Light* |
| *Top Torus* | *Bottom Torus* | *Coriolis Effect* |
| *Clockwise* | *Counterclockwise* | *Coriolis Effect* |

The natural harmonic philosophy of ancient civilizations was a perfectly valid and effective way to describe the physical properties of nature within a unified context. This had the uplifting effect of personalizing the importance of nature to each person and informing them of how they fit into the natural order. In those days, music was the entry point into this philosophy because it could be tested and experienced emotionally. Our ability to feel the physical organization of "emanations" through our ears was considered essential to the study of nature.

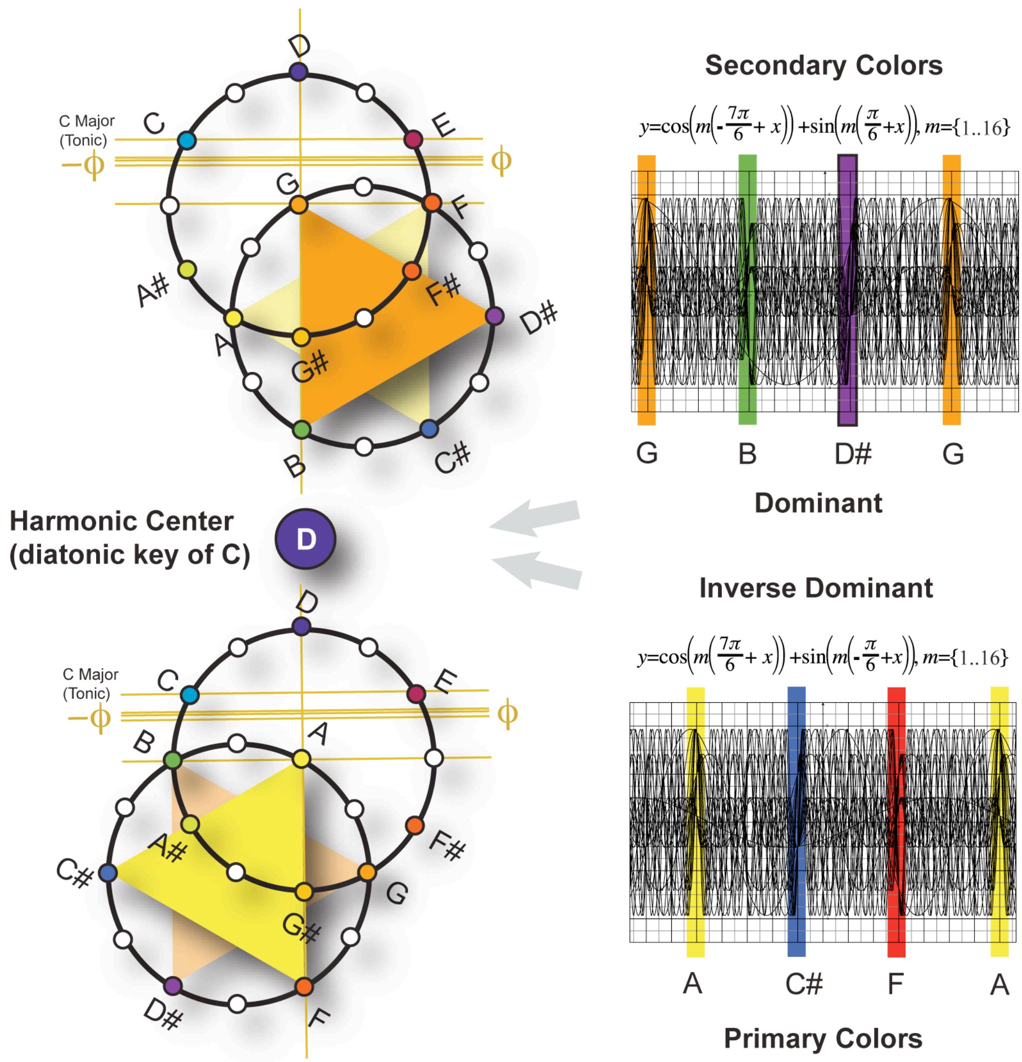
Today, most remain skeptical of music as an entry point to physics and usually consider it a distraction from hard science. Perhaps some day our schools and universities will again see that music offers the most immediate and most accessible laboratory available for the study of natural phenomena, pointing the way forward for every field of research.

Ok, this is all well and good, but how do we know that our ears can actually recognize any of this? What is it in sound waves that physically create recognizable geometric shapes? For instance, how do our ears hear a hexagon?

In Figure 86 both Yellow and Orange Dominant models reveal three very clear convergent gaps in their waveform equivalents. Highlighted by their corresponding synesthetic colors, these gaps form the proportions of an auditory equilateral triangle known in conventional music theory as a chromatic or "augmented" triad. So, whenever a dominant chord is played in music, the dual ring geometry would be recognized as an out-of-phase alignment of <u>even</u> Dominant components and <u>odd</u> Tonic components in the harmonic series. The same idea applies to primary-secondary colors that seem emotionally strong or perhaps "tense" and *refract* out of "resolved" tertiary colors.

When we combine the two dominants in the set {G, A, B, C#, D#, F}, it forms the hexagonal auditory shape of the wholetone orbit in the *bottom* Dominant ring. In this way, either dominant triad would imply the dual ring geometry and represent the unresolved or "tense" sensation of an auditory hexagon (or wholetone scale).

**Figure 86 - Recognizing the Dominants as gaps in harmonic interference**



Opposite the Dominant set is, of course, the Tonic set {C, D, E} which forms half of the opposing hexagonal wholetone orbit in the *top* ring. Resolving the Dominant then involves phase realignment or a folding up of the hexagons into a single Chromatic Ring. In the Phase State model, this represents one clock-tick of the Wholetone Function.

These phase shifts in diatonic harmony really do appeal to our Gestalt foreground-background pattern recognition. The moment we hear the Dominant pattern, we cannot help but anticipate a phase shift to the orthogonal Tonic pattern just as we expect a resolving color mix of primary and secondary colors to tertiary colors. In both sight and sound, the pattern matching principles of Gestalt psychology originate in the physics of standing waves as they seek equilibrium and phase alignment between odd-even Fourier wave components. This balancing act is what we really mean when we talk about harmony.

In music, the oscillation of triangles and hexagons is easily predicted by our holonomic brains and very enjoyable. People will listen to it all day long, anticipating the folding and unfolding (or mixing and refracting) of the same geometric combinations over and over. When the music comes to a stop, it will inevitably land on a canceling mix of primary, secondary and tertiary colors or tones. And when this happens, the auditory geometry becomes the shape of an irregular triangle or rhombus, canceling and damping down free energy transfer in the standing wave of the fundamental.[123]

Of course, not all music ends on an irregular shape to cancel the oscillation effect. We have all heard music with “tense” tones added to the final chord to maintain our anticipation of continued oscillation. When used, these added tones tend to form even and symmetrical auditory geometries, thus sustaining the energy and excitement at the end of a piece of music. In Rhythm and Blues music, we often hear this as a tritone in the final chord, usually repeated and held for a very long time (and at maximum volume!) to draw out applause and cheers. It makes us happy to hear the tritone sustained in this way, as if in rebellion to ending with a predictable irregular shape. But as a general rule of thumb, regular or symmetric auditory shapes tend to move music forward while irregular auditory geometries will stop it cold.

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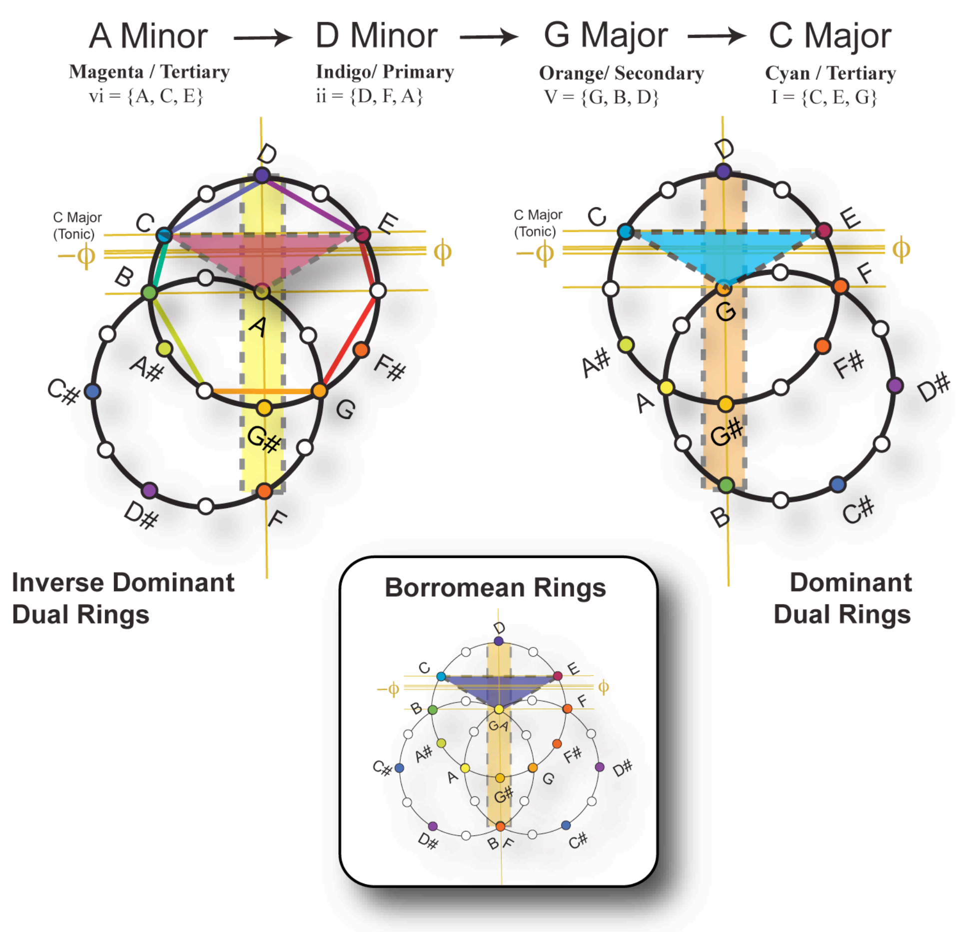
[123] **Axiom 5:** The ***Dominant and Inverse Dominant Function*** is defined by the harmonic oscillation of orbits $\{\psi^{t}{}_{0} = \{5, 7, 9, 11\} \cup \psi^{t+1}{}_{1} = \{0, 2, 4\}\}$ contained in the harmonic series as divided by $\mathbb{Z}/12\mathbb{Z}$. Cancellation or resolution of oscillation occurs upon introduction of a union set of $\psi^{t+n} = \{0, 4, 7\}$ of the major Tonic triad or $\psi^{t+n} = \{9, 0, 4\}$ of the minor Inverse Tonic triad. Other resolving set intersections are possible, though resulting in lesser degrees of standing wave cancellation and cognitive resolution.

# *Diatonic Rainbow*

*"All our progress is an unfolding, like the vegetable bud, you have first an instinct, then an opinion, then a knowledge, as the plant has root, bud and fruit." - Ralph Waldo Emerson*

Like a prism for sound, the Diatonic Dual Rings can help visualize how traditional music harmony orbits around the Harmonic Center and Axis. For instance in Figure 87, when viewing the Cycle of 5ths chord progression as triads on the Dominant and Inverse Dominant Dual Rings, the chords are seen to balance symmetrically on the vertical Harmonic Axis. When combined into a triangular configuration, the unfolded Dominants form three *Borromean rings* (or in knot theory a *Brunnian link*), also known as the Tripod of Life or Holy Trinity in Gnostic teachings.

**Figure 87 – Triangular symmetry of the diatonic Cycle of 5ths chord progression**



With no prior knowledge of music theory whatsoever, this one geometrical structure illustrates exactly how music is perceived geometrically:

1. *both ring pairs produce congruent triads,*
2. *both ring pairs reflect and alternate on opposite sides of the Harmonic Axis, and*
3. *diatonic harmony is symmetrical within a triangular geometry.*

In this model, the left ring displays minor while the right displays major, showing us how each represents a geometric reflection of the other. When integrated into Borromean rings, the geometry, colors and tones polarize around a single vertical axis {D, G#} with two Dominant rings on the bottom and single Tonic ring on top. Only when the dual rings are presented in this way can the symmetry of diatonic harmony and major-minor keys become readily apparent. Preference rules for voice leading and other historical conventions in music then become a matter of interlacing melodies and chords between odd-even orbits across the three rings (in Axiom 6).[124] This process is not unlike weaving a woven basket, braiding a circular mosaic pattern or tying a knot. But there is something else woven into this geometry.

What we find in this common chord progression is a diatonic cross inside the Christian symbol for the Holy Trinity. Taken from the Tripod of Life contained in the Egyptian/ Pythagorean Flower of Life pattern, the Borromean rings are yet another piece of evidence showing that the Church was well aware of the symmetry in music harmony and sought to control this knowledge through its social policies.

Named after the 16th century aristocratic family named Borromeo, this ancient symbol was so important to them that they incorporated it into their coat of arms. The family even purchased three islands (which naturally formed a triangle) off the coast of Italy, now known as the Borromean Islands. The Borromeo family was also very influential in the Roman Catholic Church. Charles Borromeo, a nephew of Pope Pius IV, was a leading cardinal during the Counter Reformation and is known for establishing an “academy of learned persons” named the Academy of the Vatican Nights. For this and other contributions to the Church, Charles Borromeo was beautified a saint after his death. Today, there are many Catholic schools named for him.

In the symbol of the Borromean rings we find that the true significance of the Holy Trinity and the crucifix lies in its ancient pagan relationship to Gnosticism and harmonic science. There

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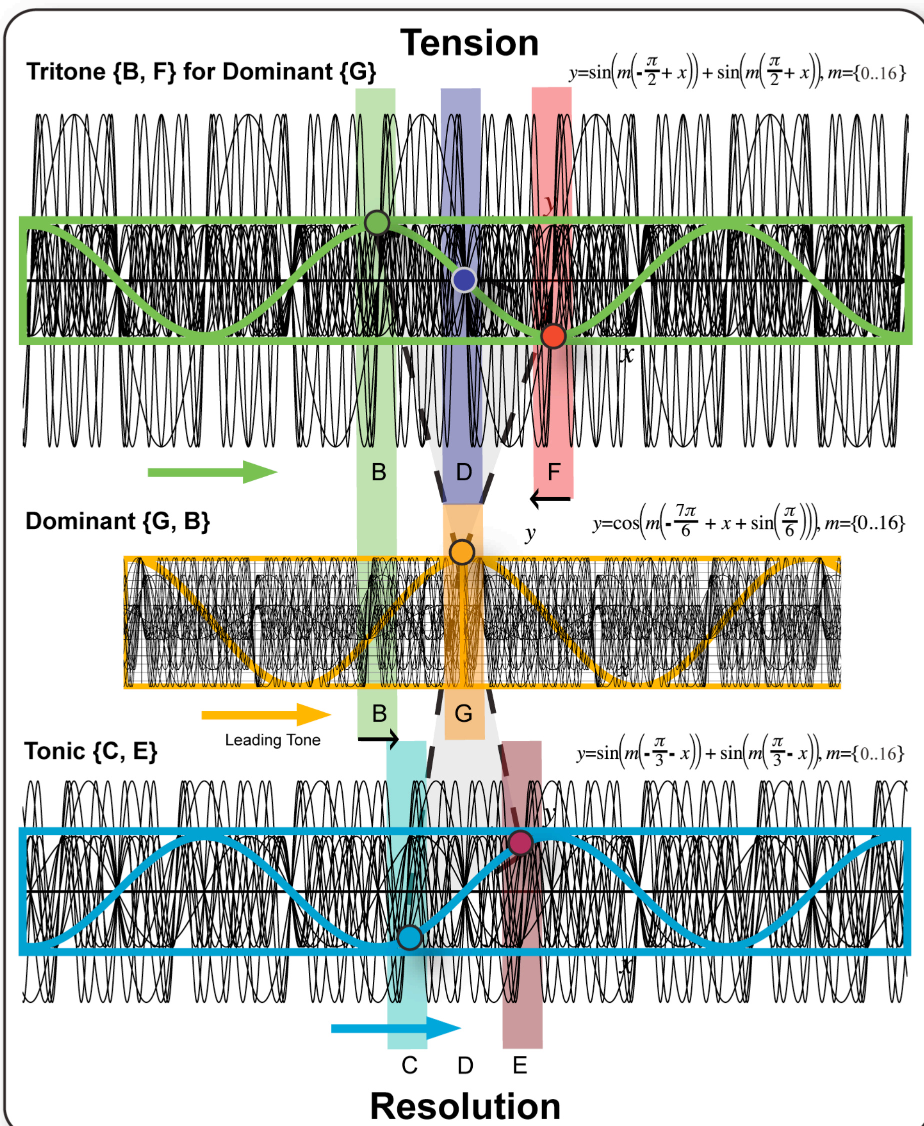
[124] **Axiom 6:** The ***Diatonic Cycle of 5ths*** is defined as a symmetrical movement across the harmonic series following a path of alternating downward perfect 5th phase modulations between sine (odd Tonic) and cosine (even Dominant/ Inverse Dominant) components. This is defined as a harmonic oscillation of odd-even orbits $\{\psi^t_0 = \{0, 4, 2\} \cup \psi^{t+1}_1 = \{\{5, 11\}, 9, 7\}$ contained in the harmonic series as divided by $\mathbb{Z}/12\mathbb{Z}$. Note that tritone orbit {5, 11} acts as an equivalence class within the oscillation set.

can be little doubt that the medieval Church had a very clear understanding of harmonic structure in music and elsewhere in nature.

The symmetry of the Borromean rings even carries over into the interference patterns found in common practice chord progressions. In Figure 88, the Dominant-Tonic cadence from the previous right ring set is illustrated as it might appear to our ears. Represented as simplified harmonic combinations, gaps can be seen to form regular geometries that are easily followed.

**Figure 88 - Tritone Function as a half-twist phase shift of Dominant to Tonic rings**



Before you skip over this illustration thinking it too complex to understand, take a moment to consider the fact that the gap patterns are symmetrical and triangular just like the Borromean rings. This is because it *is* the Borromean rings, just rolled out horizontally into transverse waves as it might appear along the ear's basilar membrane. Progressing top-to-bottom, the Dominant {G} → Tonic {C} cadence is illustrated in three steps to show how it could be recognized as a sliding phase shift operation within a harmonic interference pattern of waves.

The sequence of Green, Orange and Cyan colored waves correspond to two orthogonal phase shifts that create a 180-degree "half twist" current through the interference pattern, flowing toward the Indigo Harmonic Center {D} in the middle of the graph. In the process, the primary and secondary gaps {Red, Orange, Green} at the top mix into the tertiary gaps of {Cyan, Magenta) at bottom. We can think of this mixing process as a *Dominant triangle* that twists into into a *Tonic triangle* through the Orange {G} in the center, as represented by the dual rings. Take a minute to see this for yourself in the diagram.

The fundamental waves are then colored in each step to show how this mixes in the subtractive CMY(K) color space of paint according to the color equation Cyan = Dark Green – Orange. In this equation, subtracting Orange from Dark Green is the color equivalent of folding the Orange Dominant to wind up on the Cyan Tonic fundamental frequency. In this way, mixing color-tone intervals can be described as the exact same *sliding phase shift* through a field of harmonic interference. Like the sliding tritone substitution resolutions in jazz harmony, this pattern is very predictable and efficient – a path of least resistance through the interference.

Notice that during this process a perfect $5^{th}$ axis of vertical symmetry {G, D} forms temporarily through the Harmonic Center as the odd-sine components shift over to even-cosine and back to odd-sine in the interference. Indeed, this is exactly what a perfect $5^{th}$ does in music – it creates an axis of *diatonic symmetry* between Tonic and Dominant chords. From this perfect $5^{th}$ axis, we can understand any 7-step *Diatonic Key* as a 180-degree phase shift (or "half twist") between sine components (representing Tonic) and cosine components (representing either of the Dominants). Using musical set theory, any diatonic key can be formally defined as a half twist using the oscillation set ψ (see Axiom 7). [125]

If you are a student of music, you may wonder why cadences are never explained as a harmonic phase shift, even though chords do follow the natural currents in harmonic interference.

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[125] **Axiom 7:** A 7-step ***Diatonic Key*** is defined by a perfect $5^{th}$/ $4^{th}$ axis of symmetry between the Tonic sine and Dominant/ Inverse Dominant cosine groups, phase shifted with a reverse "half twist" of 180 degrees within a standing wave interference pattern. For a given Harmonic Axis, a diatonic key is represented by the orbits $\psi^{t}_{0} = \{\{0, 2, 4\} \cup \psi^{t+1}_{1} = \{\{5, 11\}, \{7, 9\}\}$ contained in the harmonic series described by $\mathbb{Z}/12\mathbb{Z}$. Its complementary tritone substitute key is given by $\psi^{t}_{0} = \{6, 8, 10\}\} \cup \psi^{t+1}_{1} = \{\{1, 3\}, \{5, 11\}$.

The reason of course is there is simply no accepted theory for how acoustics and physiology could be coupled through the universal physics of standing waves. And without proper visual and notational models to illustrate the physics at work in standing waves, the symmetry and phase shifts in musical harmonies are not at all apparent and are overlooked or ignored. If our understanding of music and cognition is ever to move forward this really has to change.

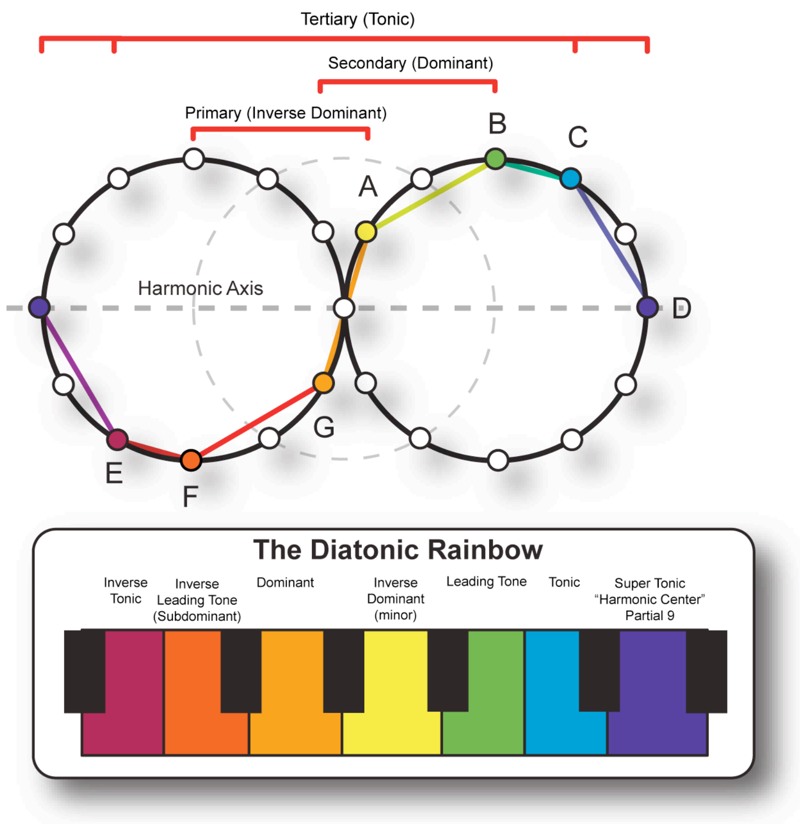
Models like the Diatonic Dual Rings are absolutely essential in understanding the physics and physiology behind most music. Without the dual ring model, the Dominant and Inverse Dominant remain *folded up*, appearing to move *asymmetrically* relative to the Tonic. Similarly, the inverse nature of major-minor scales and chords also remains well hidden without benefit of proper visualization. Yet our ears and brain were never fooled. They have always known about the Tripod of Life in a diatonic scale and how to translate its half twist into an emotional response.

When applied to common musical practice, the Diatonic Dual Rings can easily describe how Dominant-Tonic, major-minor and the diatonic Cycle of 5ths phase shift through the coherent paths of a harmonically organized pitch space. It shows with a simple twist what musicians have felt to be true in the 7-step diatonic scale for thousands of years but somehow never quite able to express. With this model we now have a way to visualize the essential attributes of diatonic harmony. The only question is when will our educational system begin to see music as we recognize it?

The same thing could be said for color. The half twist found in diatonic harmony is directly related to the reason for the difference between direct light and reflected light. Why is it we never see an explanation for how the primary colors of light {Red, Green, Blue} become the three primary colors of paint {Red, Yellow, Blue} (or rather Magenta, Yellow and Cyan)? The reason is an avoidance in our schools to discuss the strange half twist found everywhere in nature.

When the same Borromean rings are configured into the linear configuration shown in Figure 89, we can now see how the synesthetic color model twists around the Harmonic Axis into a symmetrical 7-color rainbow. By dividing the visible light spectrum into three secondary colors (for the Dominant), three primary colors (for the Inverse Dominant) and six tertiary colors (for the Tonic), a 7-step diatonic color scale will appear as a "half twist mix." From this, the seven most prominent colors of a rainbow can be modeled as a phase shift operation in the visible light spectrum corresponding to a symmetrical 7-step major scale in a 12-step octave. This creates what we might call a *Diatonic Rainbow*.

### Figure 89 - The Diatonic Rainbow (C major)




While this diagram is certainly not a physical representation of the process of either light refraction or diffraction, it does correspond to the specific diatonic frequency proportions and angles within the visible color octave perceived by our vision system. As a simplified synesthetic ring model, it can represent the essential perceptual qualities of consonance and tension in diatonic music using color mixes anyone can understand.

For instance, the oscillating Tritone Function can be represented in the CMY(K) color space of paint by alternately adding or subtracting pairs of primary and secondary colors with pairs of tertiary colors. That is, subtracting {Green - Yellow} and {Red - Orange} resolves to {Cyan, Magenta} as the Tonic major 3$^{rd}$. Or, we can mix {Cyan + Yellow} and {Magenta + Orange} to create tension with the tritone {Green, Red}. At the Harmonic Center we get a mix of {Cyan + Magenta} which yields the stabilizing high frequency color of {Indigo}. Subtracting and mixing color harmonies in this fashion can easily demonstrate to a classroom how frequencies in a spectrum correspond to phase angles in a cycle and how these correspond to tonal combinations. Our recognition of proportions between cyclic phases really does apply to both color and musical harmonies in exactly the same way.

Now, one very curious thing occurs in CMY(K) color space when adding {Magenta + Yellow} to produced {Red}. The tones {E, A} corresponding to {Magenta, Yellow} form the perfect 5$^{th}$ of a *minor* triad, but when we perform the exact opposite operation on the Diatonic Rainbow by mixing {Orange + Cyan} to form the perfect 5$^{th}$ colors of a *major* triad, we get a Dark Green instead of the pure bright Green of the Leading Tone. So, why would we not arrive at the pure green normally found (as a thin band) in the refracted rainbow?

It turns out that the extra darkness in the Green is due to some residual Magenta left behind from the color mix in CMY(K) color space. As mentioned earlier, the Dark Green color of the Leading Tone is a mix of Cyan and Orange, or specifically {100% Cyan, 50% Magenta, 100% Yellow} in the Tonic triad {C, E, G}. In effect, the Dark Green Leading Tone is a mix of the *entire* major triad while the Red Inverse Leading Tone for the minor is only a partial perfect 5$^{th}$ mix or minor dyad. This could be the reason why the Leading Tone has such a reputation for pulling strongly up to the Tonic. It clearly resonates in the harmonic series with the Tonic major triad better than the Inverse Leading Tone resonates with the minor triad. It is simply "happier."

We can bet that adepts of harmonic science in the Middle Ages would have interpreted such asymmetry in color mixing as proof positive of a slight asymmetry in nature. They would have looked around at nature and pointed to the fact that plant life is mostly a darker green, rather than a bright pure green, and that many plants pass beyond red into magenta in the autumn before turning brown from a tritonal mix of opposites. So, if the plant kingdom is nature's Leading Tone, what aspect of plant chemistry could account for the 50% magenta in Dark Green?

Organic chemistry would explain the extra magenta as the anthocyanin compound in the flavonoids found in all plants. This is the compound that darkens the green color in plants and reflects the complementary red or magenta color across the color wheel from cyan (oxygen). Anthocyanin is the reddish color we see as plants lose their chlorophyll in autumn.

So, the next time you see a dark red or magenta leaf in autumn, you can hold the leaf up to the cyan sky and claim nature's tritone has at last contracted to a Tonic major 3$^{rd}$ of {Cyan, Magenta} in the Diatonic Rainbow. And if someone says this is just a bunch of nonsense and so much metaphysics, you can tell them with a straight face that the dark red color of their blood makes them Nature's Inverse Leading Tone, just a 180° half-twist away from those oxygen exhaling dark green plants upon which they depend for their next breath. Though this may seem in the dark realm of alchemy to present day scientists, this "science of the bleedin' obvious" was considered sacred knowledge to ancient civilizations whose shamans and sorcerers studied closely such harmonic properties in nature.

During this writing I had the pleasure of sponsoring the publishing of a new piece of music entitled *Sor(tri)lège: Trio III,* meaning "Sorcery for Three." Composed by contemporary American composer Robert Xavier Rodriguez, my former composition/ theory professor, the piece was a trio written for violin, cello and piano and composed in three parts: *Incantation*, *Magie de Cabaret* and *Charme de Trois*.

*Sor(tri)lège* is described as a highly chromatic piece built on "a short quasi-tonal chord sequence which moves upwards by minor thirds (an auditory square) over an octatonic scale," then recurring in each of the twelve possible keys under various rhythmic variations. In its premier in Dallas and New York's Carnegie Hall, the audiences were charmed by its "splashes of colorful harmonics in both strings and piano," receiving rave reviews. But unbeknownst to anyone, it told the story of an even deeper sorcery and magic behind the number 3.

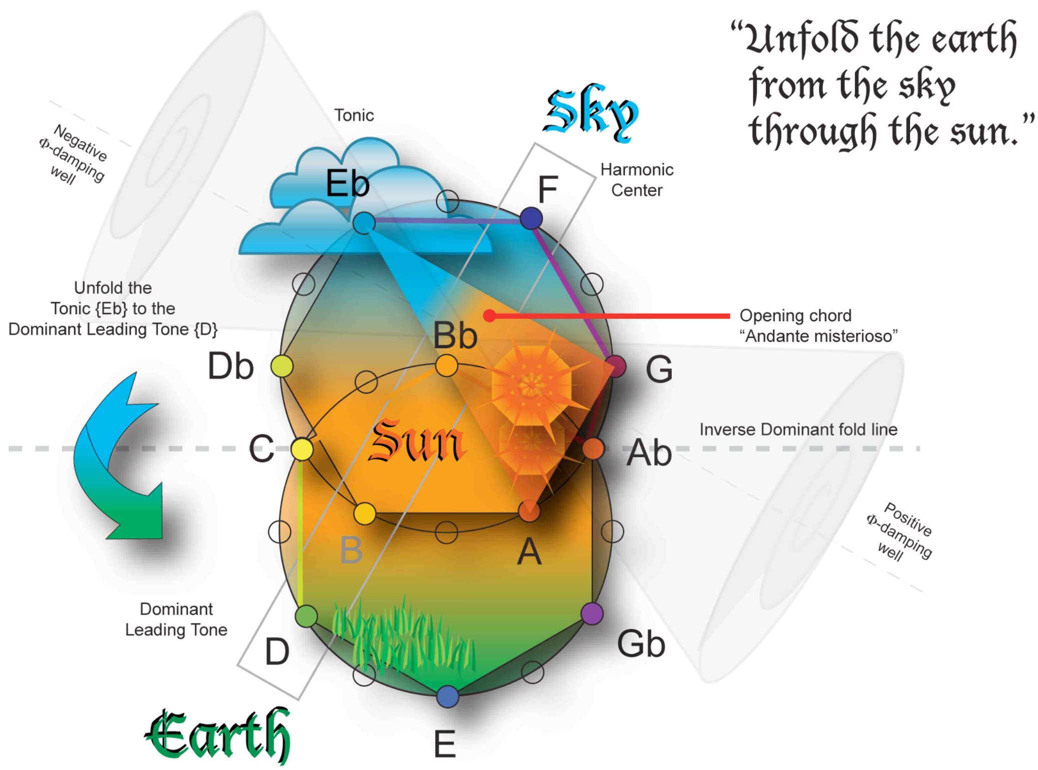
The Incantation opens balanced around an Indigo Harmonic Center. A continuous Cyan and Yellow color field supports a recurring and trilling tritone defiantly opposing the Cyan Tonic. In our synesthetic color mapping, this tone takes on the color Red-Orange, associated in ancient times with the creative life force and godhead of all things. Tibetan monks consider it closest to the creator and use it for their robes to reflect the creative force that lives in the heavens opposite the Cyan sky while Hindus consider it the color of the pelvic chakra and creative force inside the womb. Driven by the composer's natural instinct, selection of a high Red-Orange tone could not have been more appropriate for the opening of a sorcerer's incantation.

Presented in Figure 90, a layered harmonic archetype based on the Diatonic Dual Ring model is used to describe this opening section of the Incantation. Transposing to the harmonic series of {Eb} with Harmonic Center {F} used in the piece, the first few pages are symbolized by a triangle emanating from the Cyan sky. At its "incenter" is the high trill on tritone {A}, represented by the Red-Orange color of the Sun. At the bottom, the Dominant ring unfolds to the Green Leading Tone of Earth and its plant life.

While *Sor(tri)lège* was not written with any of this symbolism in mind, it is important to note that the interval proportions that *felt right* for the subject matter naturally correspond to a harmonic archetype for nature. It is as if the opening chord of the Incantation commanded the creative force of the Red-Orange Sun to *unfold the earth from the sky through the sun.*

In the context of modern science and strict rationalism, symbolisms like this are apt to seem too philosophical, too close to theology or perhaps alchemy than anything resembling hard science. Yet, bridging the compartmentalized fields of science through harmonic associations does not make science less "hard," only more *expressive, interpretive and meaningful.*

**Figure 90 - Sor(tri)lège: Trio III, I. Incantation harmonic symbolism**




What if when we were very young we had all learned music, art and science as an integrated discipline in this way? What would we have been like as individuals or a society? Would some of us have been inspired artistically to pursue the sciences or even pursue music as a scientific art? Perhaps our instructor would have awakened our curiosity by saying something like this:

> *"Have you ever wondered why the rainbow appears as seven colors? Today we learn that the light spectrum is just like an octave on the piano. When we divide the spectrum into twelve equal logarithmically spaced colors on two rings, the seven rainbow colors twist around to align perfectly with the white keys of a major scale on a piano. Let us listen now to these colors."*

As it stands today, something like this is unlikely in an academic setting. The reason being that neither the Arts nor the Sciences recognize any theory that couples light, sound and cognition into a $\mathbb{Z}/12\mathbb{Z}$ framework of harmonic interference. Such harmonic associations are often discredited as "wrong thinking" even while quantum physics is founded on the idea of coherent standing waves and phase shifting harmonics. Overcoming the institutionalized avoidance of standing wave principles in music and color theory will require a break with the anti-harmonic legacy of our medieval past.

# *Chromatic Recursion*

*"Thus it is necessary to commence from an inescapable duality: the finite is not the infinite."*
*- Hans Urs von Balthasar*

Diatonic harmony is a natural byproduct of coherence in physical reality. As mankind evolved under the same physics as sound, our auditory system evolved to recognize and follow the contrary currents of harmonics as they swirled around their Landau damping wells. With standing wave brains thinking inside the same reflective Gaussian interference pattern, we cannot help but have a preference for the harmonies of diatonic music with its predictable energy currents and simple geometric shapes.

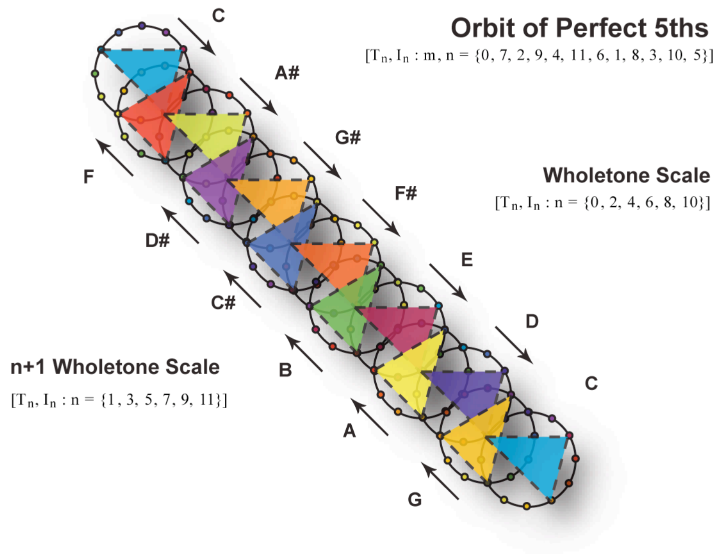
Yet, if diatonic harmony really is so organic and universally preferred, how then are we to explain the rise of chromatic harmony in the late 19$^{th}$ century? What properties in wave interference shall we take to support our recognition and enjoyment of chromatic music and how should this be represented using musical set theory and harmonic models?

To answer these questions we need to revisit the Cycle of 5ths, only this time as the *chromatic* Cycle of 5ths. Unlike the diatonic Cycle of 5ths, the chromatic cycle follows the Orbit of 5ths through *all twelve* tones of an octave rather than just the seven diatonic steps. As it was practiced at the height of the Romantic period in the late 19$^{th}$ century, sections of the chromatic Cycle of 5ths were used to phase shift, or *modulate*, between two or more diatonic keys. Romantic-era composers discovered they could reuse a given musical phrase by repeating it in different registers, thus extending a musical composition through "thematic development."

As 19$^{th}$ century composers and audiences became more comfortable with the chromatic Cycle of 5ths as a modulation pathway between keys, music began to rely less and less on the safety of a single diatonic key. Composers modulated more freely, shifting through the strong pull of the tritone and Cycle of 5ths fragments to new diatonic keys. Music evolved toward a deeper, more generalized set of harmonic principles that made use of all twelve tones in an octave rather than just seven, becoming much more colorful or chromatic in the process.

In Figure 91, the colors of the chromatic Cycle of 5ths is shown as a series of triads moving along a chain of unfolding Dominant rings that form a lattice of triangular auditory shapes. In this diagram, the chords are shown progressing through all twelve tones (and colors) of the {C} major octave. When viewed on the dual ring model in a continuous fashion like this, the Orbit of 5ths appears divided into two wholetone orbits, each as a series of congruent triangles, that are separated by a dihedral angle of 120 degrees (one-third of an octave).

**Figure 91 - The chromatic Cycle of 5ths as triads on cascading dual rings**




We can clearly see the dihedral nature of $\mathbb{Z}/12\mathbb{Z}$ represented as the same odd and even groups used by Fourier wave analysis. The 45-degree downward angle of the ring lattice even acts as a kind of visual calculation of the geometric mean that exists naturally between sine and cosine Fourier components. When presented as a continuous cycle in this way, the inverse properties of the dual ring model cannot help but naturally partition the Z-twelve octave into two separate wholetone groups oriented at a right angle (or phase-quadrature) to one another.

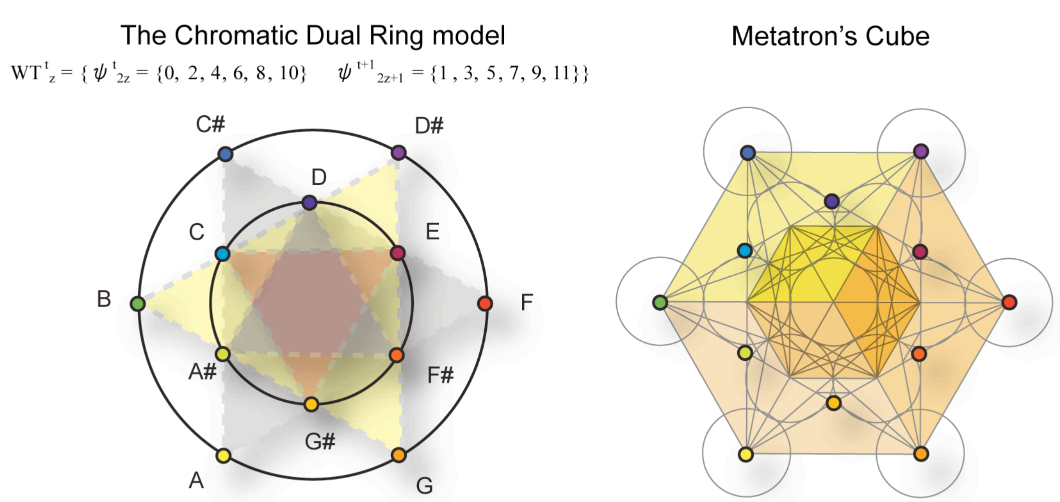
As the two wholetone orbits "grow" into this simple lattice structure, they align into parallel planes. Melodies or chords that move along one of these planes, as in the music of Debussy or Ravel, will seem to glide along smoothly. Such movement, appropriately called *planing*, is often associated with dreamlike images of a rippling reflection over a lake, a spiraling staircase or perhaps a blossoming flower. Stepping erratically between both planes will then paint an opposing auditory picture of angular geometries something like the shifting staircase in M.C. Escher's 1951 *House of Stairs*. As the simplest expression of the Pythagorean ideal of balance between odd and even numbers, music made from the raw dihedral geometry of $\mathbb{Z}/12\mathbb{Z}$ always creates an impression of primordial self-similarity, organic symmetry and natural beauty.

The best-known example of this is the piano piece *Clair de Lune* by Debussy. As an auditory landscape of moonlight reflecting over rippling water, chords will skim or plane across the mirror-like surface of each wholetone scale, occasionally sitting on a pentatonic "star" before continuing on. While conventional history has painted Debussy as a confection-loving rebel who cast aside the rules of music to chase the romantic whims of his heart, he was actually revealing to the world a very rational system of harmonic duality that lay hidden in the primitive underbelly of the ℤ/12ℤ cyclic ring. The Impressionist style really was a continuation and generalization of the Tritone Function, distilled into a slow-motion oscillation of the odd-even Wholetone Function.

During the latter half of the 19$^{th}$ century, chromatic harmonies were especially popular in the Victorian culture predominant throughout Europe. Like an escape valve for a repressed society, late-Romantic music would jump promiscuously from one Harmonic Center to another. Romantic and Impressionist composers were in love with the idea that the human ear no longer demanded a monogamous relationship with only one Harmonic Center, creating instead music that courted several within the space of a single piece. But what these Romanticists did not realize was they were really falling in love with their own self-image repeated over and over again. Just as "French mirrors" became all the rage during the late Romantic period of the Victorian Age, so too were key modulation and French wholetone planing a kind of mirroring process. In fact, this self-referential style of music can be described as *chromatic recursion*.

Reconfiguring now in Figure 92, chromatic harmony can be represented geometrically as two *concentric* and congruent wholetone rings – the odd (2ℤ+1) and even (2ℤ) groups from ℤ/12ℤ. When the Wholetone Function of Axiom 4 is then applied, the resulting *Chromatic Dual Ring* model represents the foreground-background harmony of the late Romantic and Impressionist periods as a *recursive spherical standing wave*. The outer ring becomes the tense Dominant cosine component that collapses recursively into the resolved inner Tonic sine ring. In fact, this geometry is identical to an 8-cell hypercube known as a *tesseract*, often described as oscillating inside out in 4-dimensional spacetime. To the Gnostics it was Metatron's Cube.

**Figure 92 - The Chromatic Dual Ring model as Metatron's Cube**

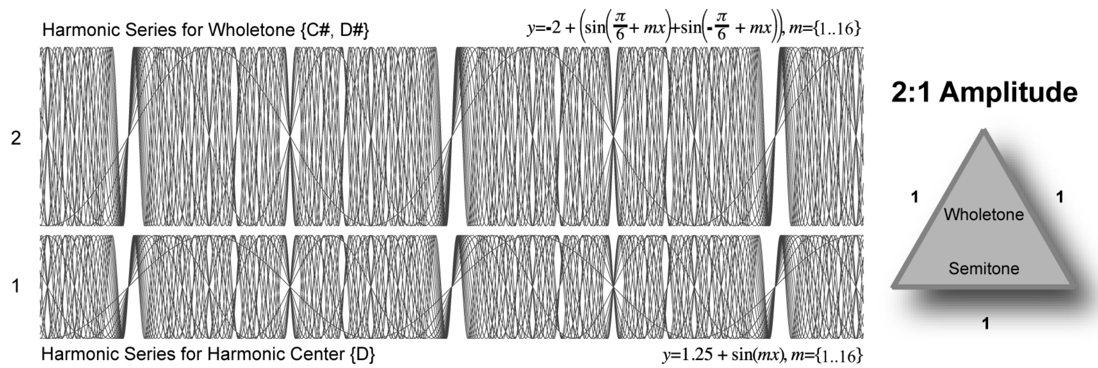


Just as Roger Shepard had suggested in the 1970's, harmonic music is the auditory geometry of an equilateral triangular lattice twisted around into an octave cycle. But unlike Shepard's double helix, the Chromatic Dual Ring forms closed odd and even circles that reduce pitch space into one cyclic octave. In this way, we can visualize chromatic music as two opposing rings resonating recursively on a round surface. Chords and melodies become shapes suspended between the rings, reflecting from the center out to the edge and back to center again. In this oscillating model we return to the physical model of a vibrating Chladni plate as it is found in the human eardrum. Thus, chromatic music becomes the geometry of chromatic recursion.

We saw earlier how equilateral triangles form in the gaps of wave interference generated by the Diatonic Dual Ring model. We also saw how the Dominants polarize into the triangular configuration of rings known as the Borromean rings. We might now wonder if we can also recognize a triangular geometry in the Chromatic Dual Ring model. More specifically, are we able to hear Shepard's equilateral triangular lattice between wholetone intervals on the outer ring to its semitone midpoint on the inner ring? If so, then Shepard would be proven correct in his claim of a perceived triangular equivalence relation between a wholetone and semitone.

In Figure 93, the harmonic series for Harmonic Center {D} on the inner ring is compared with the harmonic series for the wholetone interval {C#, D#} on the outer ring. When aligned in phase, the two patterns are seen to be identical except for amplitude doubling of the wholetone. In this way, the wholetone represents a 2× multiple of its mid-tone just a semitone away and does indeed create an auditory 2:1 equilateral triangle as a proportion of amplitudes. In the dimension of frequency, the wholetone and semitone intervals are equivalent, thus confirming Shepard's triangular lattice as a natural feature of wholetone interference.

**Figure 93 - Perception of an equilateral triangle as an amplitude proportion**

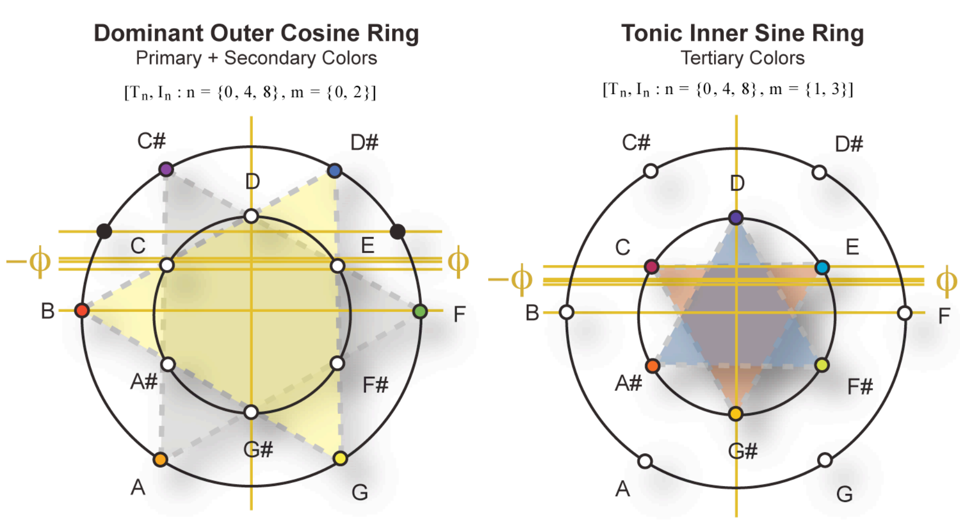


As we should probably expect by now, the same triangle can be found in the synesthetic color model as a mix of two adjacent Primary and Secondary colors into a middle Tertiary color. The only way in which this differs from a wholetone interval is the interval is not replaced by its mid-tone. This is due only to the mid-tone being swamped or overpowered by the double amplitude of the wholetone interval. So, while we cannot easily hear the mid-tone, it is always there in the background softly forming an auditory equilateral triangle.

So, we can represent this perceived triangle in musical set theory by first equating the wholetone to its mid-tone as $\{0, 2\} \equiv \{1\}$, creating an equivalence class for wholetones and semitones, and then generalize this through transitive closure into a higher-order wholetone orbit equivalence class: $\{1, 3, 5, 7, 9, 11\} \equiv \{0, 2, 4, 6, 8, 10\}$. When the Wholetone Function of Axiom 4 is then applied to this triangular ring lattice, a recursive oscillation model is created that can be used to represent the harmonic function of virtually any kind of chromatic music. With this one model, we can visualize both the heartthrob of "Romantic duality" and the Gestalt foreground-background music of Impressionism.

The inner ring represents our foreground and calm Tonic while the outer ring provides the background or tense Dominant. Instead of the folding or unfolding operations of diatonic harmony, the oscillation between Tonic and Dominant is now represented as energy flowing alternately inward and outward like vibrations on a Chladni plate. As this occurs, energy is exchanged during the transition *between* the two rings rather than actually on them. Depending on the musical style, we can interpret this energy exchange to occur in either of the two ways shown in Figure 94.

**Figure 94 - Φ-damping on the Chromatic Dual Ring**




Chromatic Φ-damping model

Diatonic Φ-damping model

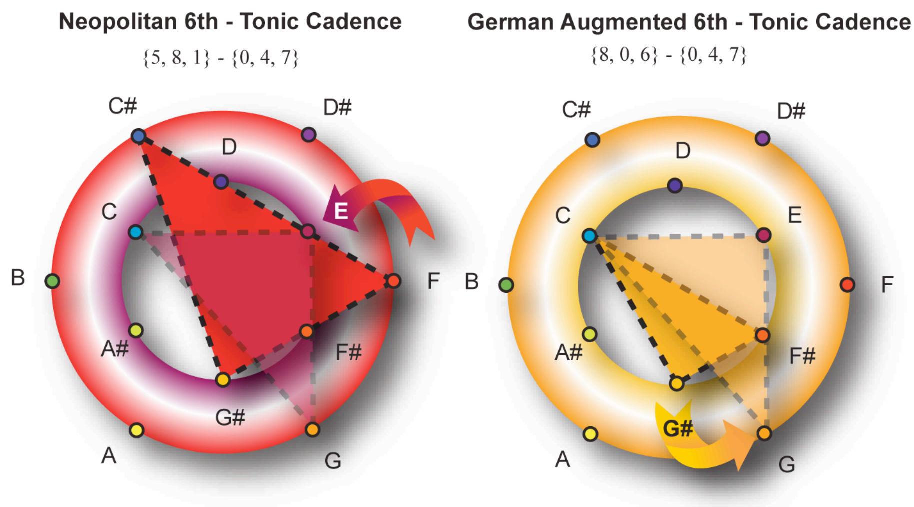
The first interpretation best describes *chromatic harmony* by scaling the Fibonacci series to align where the inner ring tones *would have been* in a single ring configuration (shown at left). This interpretation represents chromatic resolution as a recursive damping action from the outer Dominant ring {B, A, G, F} to the inner Tonic ring {C, D, E}. In musical terms, the inner wholetone ring would be stable and resolved relative to the tense outer wholetone ring, which acts like a boundary reflecting energy back inward. This is how energy in a spherical standing wave is reflected – from edge to center like a round Chladni plate – forming a calm nodal ring at about an 8:5 proportion to the outer ring. Not coincidentally, this same proportion corresponds inversely to a Just tempered Tonic major 3$^{rd}$ {C, E}. We will return to this later.

The second interpretation supports *diatonic harmony* by aligning the Fibonacci series *between* the two rings (right figure). In this orientation, Φ-damping occurs not as an alignment between the outer edge and the center, but as points *in the space between* the tritone {B, F} and major 3$^{rd}$ {C, E} of the diatonic Tritone Function. This interpretation of damping can then be used to represent diatonic harmony as a proper subset of chromatic harmony. This said, whenever harmony locks into a key and a single Harmonic Center for any length of time, the Diatonic Dual Rings would always be a better model.

So then, the first interpretation is always preferred for Impressionist music while the second is more appropriate for the hybrid style of Romantic music. This actually corresponds to how life organizes itself since cells first grow *inward* recursively like chromatic harmony before growing *outward* into a lattice like diatonic harmony. In this way, chromatic harmony exists as a first principle for the same properties of resonance and damping that occur in a higher-order 2:1 diatonic harmony. And like the physical model of the Chromatic Ring as a horizontal view of Earth's magnetic field and the Diatonic Dual Rings as a vertical cut-away view of a hurricane, the Chromatic Dual Rings now give us a top down view of the hurricane.

This idea becomes a little easier to understand if we project the Chromatic Dual Ring model into three dimensions to create a *Chromatic Torus*. In Figure 95, the Neapolitan 6$^{th}$ and German Augmented 6$^{th}$ chromatic chord cadences now appear to spiral through the currents of a torus according to the Coriolis Effect in a hurricane. While traditional music theory will only describe the Neapolitan 6$^{th}$ chord as *a triad on the flatted second in first inversion* or the German Augmented 6$^{th}$ as a *7$^{th}$ chord on the flatted sixth*, the Chromatic Torus explains them more intuitively as tones rotating around a torus or donut. It describes without words the idea that a Tonic triad resolves in alignment with the vertical Harmonic Axis in the same way a magnet must align with the polar axis of another magnet. Using the 3-D Chromatic Torus, we need only follow the natural toroidal flow of melodies and chords around and around as their harmonies seek to stabilize themselves to a given polarity.

**Figure 95 - Romantic chords on the Chromatic Torus**




These are only some of the ways in which the chromatic and diatonic ring models can be used. After all, most music since the 19th century is neither exclusively chromatic nor exclusively diatonic. Sometimes the melodies and chords will grow outward diatonically and other times they will rotate inward chromatically. As a flat 2-D geometry, Romantic music tends to trace out a beautiful and intricate mosaic pattern – crawling along for a while, pausing occasionally to spin and pump in place, then continuing its crawl. Representing this correctly may require a combination of ring models that switch ad-hoc to fit the harmonic character of the music.

But there are still several absolute truths to be found in all this. Music harmony always oscillates, whether between wholetone orbits or diatonic orbits. It will always seek a balance between tension and resolution. And on any of the ring models, tension always resolves by moving between opposing orbits or currents. As a general rule of thumb, we can identify three basic objectives of all dodecaphonic music harmony:

> *1) pump or fold/ unfold cyclic rings to oscillate between tension and resolution,*
> *2) use polar symmetry to create a sense of spatial orientation and predictable energy flow, and*
> *3) stop harmonic oscillation by locking down or "short circuiting" duality into a singular ring.*

Axiom 8 summarizes this in terms of chromatic harmony while Axiom 9 derives the essential process of diatonic harmony as a subset (or lower-order interpretation) of chromatics.[126]

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**Axiom 8:** ***A Simple Rule of Thumb for Chromatic Harmony***: *Any* progression of scales, intervals or chords that alternate one or more tones between the wholetone scales will create a sense of tension. The sensation of resolution occurs when the oscillation is canceled with any interval that straddles the two wholetone scales, such as a perfect 5th.

As a radical departure from traditional music theory, a diatonic "key" should not be a first principle of harmony, but rather the result of a persistent polarity in an oscillating standing wave of wholetone rings. This physical model is exactly backwards to historical music development based on diatonic harmony as a first principle, from which "accidentals" are then used to artificially explain chromatic music.

Even musical staff notation presumes that all music should be based on key signatures and the persistence of a single scale. These traditions demand that we use extra "flats" or "sharps" and forcibly "break the key" in order to create chromatic music, leaving us with the impression that chromatic music is not natural or even desirable. By design, conventional music notation and traditional nomenclature have had the effect of obscuring perfectly natural and beautiful chromatic harmonies while guiding young composers away from harmony altogether.

Without a physics-based approach like *Harmonic Interference Theory* to provide intuitive visual models for music harmony, a narrow "diatonic first" education policy has become deep rooted. As an unavoidable outcome of the anti-pagan, anti-harmonic social policies that began over a thousand years ago, our present day educators remains steadfast in their avoidance of any kind of unified harmonic theory. This needs to change.

The apparent lack of interest in combining acoustics and physiology within a dedicated field of harmonic study has really been a great disservice to society. Without a unifying field of musical physics, a general misunderstanding and defensive posture *against* natural harmonic principles has resulted. In its place we have only an arcane set of rules and traditions in our schools that continue to distract us from a real understanding of music.

When jazz pianist John Sheridan suggested to his young student:

> *"Use tritone subs wherever you can – but then also try to play everything you ever learned about music theory* all at the same time,"

he was really saying that diatonic scales, Cycle of 5ths, chromatic alterations, polychordal voicings and the unmentionable tritone all boil down to the simple idea of oscillation between wholetone scales. Imagine, no Great Secret – just one all-encompassing idea that opens the door to endless musical possibilities while unifying musical practice with natural science. Shouldn't this be one of the first things explained in a music lesson or theory class?

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**Axiom 9: *A Simple Rule of Thumb for Diatonic Harmony*:** Under Axiom 8, incorporate the Tritone Function and/or Dominant (or Inverse Dominant) cadences to strengthen recognition of a single Harmonic Center and coherent pathway of the 7-step diatonic scale. Continued recognition of diatonic harmony is then directly proportional to persistence of just one Harmonic Center.

Shouldn't music education communicate the fundamental concepts of duality and symmetry in sound? Shouldn't it be taught using auditory geometries and the harmonic wave patterns found all around us, connecting our perception with the fundamental processes of nature? Shouldn't there be a suitable language and modeling system for all diatonic and chromatic music that integrates everything into a single organic process?

Why not explain to the world that music is recognized through the calm locations in intersecting standing waves and that we anticipate music harmony as directional energy currents flowing through harmonic interference? Why not explain the ***REFLECTIVE INTERFERENCE*** distribution of resonance and damping in our holonomic brains as the reason we are able to recognize the musical qualities of consonance, dissonance, tension and resolution? And while we are at it, why not illuminate the fact that all sound is recognized geometrically in rings and measured proportionally – a direct result of the spatial and temporal coherence of closed geometry spinning out of spirals?

What could be wrong with a consistent framework like this to help everyone understand the true significance of music and how we perceive it? How could it be undesirable or somehow dangerous for people to know that they enjoy music because they are even structured according to this same framework of music?

It is a little known fact that once sound is converted into neural impulses, it is processed through three concentric ring regions located in the auditory cortex between our ears. Organized as primary, secondary and tertiary auditory cortex regions, the outer ring forms a boundary between the primary auditory cortex (specializing in tonal cognition of proportions) and the secondary auditory cortex (specializing in spatial harmonic cognition). The inner ring then forms a boundary between the secondary and tertiary auditory cortices that mix everything together into the overall experience of music. Just like the Chromatic Dual Ring model, the outer ring resolves recursively to the inner ring, representing an oscillating dual ring system that literally implodes music into emotions at the center of the brain.

Since these rings of the auditory cortex sit nearly vertical in the middle of the brain, they separate the front and rear halves of the brain. In this way, they are suspended between the two ear openings in the skull and must resonate as a standing wave inside a horizontal Gaussian ***REFLECTIVE INTERFERENCE*** pattern of neurons polarized along a harmonic axis running between the temporal and frontal lobes. It is not unreasonable to presume from a strictly topological analysis that the dihedral geometry of the chromatic rings at this central nexus point in the brain evolved as the only possible response to the structure of coherent sound in a spherical container, thus accounting for our natural ability to cognitively measure spatial and temporal proportions between standing wave harmonics.

But, the human brain is not the only place where the dual ring model can be found in nature – it emerged from a much more primordial harmonic template. At the smallest scale of life, the process of cell division known as mitosis follows the same dual ring model. Like living harmonic rings, the outer wall and inner nucleus of a cell form the equivalent of a biological spherical standing wave passing nutrients and oxygen from the outside to the inside while forming a stable perceptive structure (a Tonic Ring) in the middle. As Tonic shifts to Dominant in the Chromatic Dual Rings, so too will the spherical cell eventually over resonate and "outgrow" its inward damping container to unfold outward into a Diatonic Dual Ring replica of itself. As the two cells continue to phase shift apart (temporarily forming an incommensurable egg geometry), two standing waves then beat independently, each in their own $\mathbb{Z}/12\mathbb{Z}$ Chromatic Dual Ring container and connected inside a shared cellular lattice. We find in the resonance of a single cell the first musical steps life takes to develop coherence between itself and its environment.

# *Coherent Pathways*

*"All the great things have been denied and we live in an intricacy of new and local mythologies, political, economic, poetic, which are asserted with an ever-enlarging incoherence."*
*- Wallace Stevens*

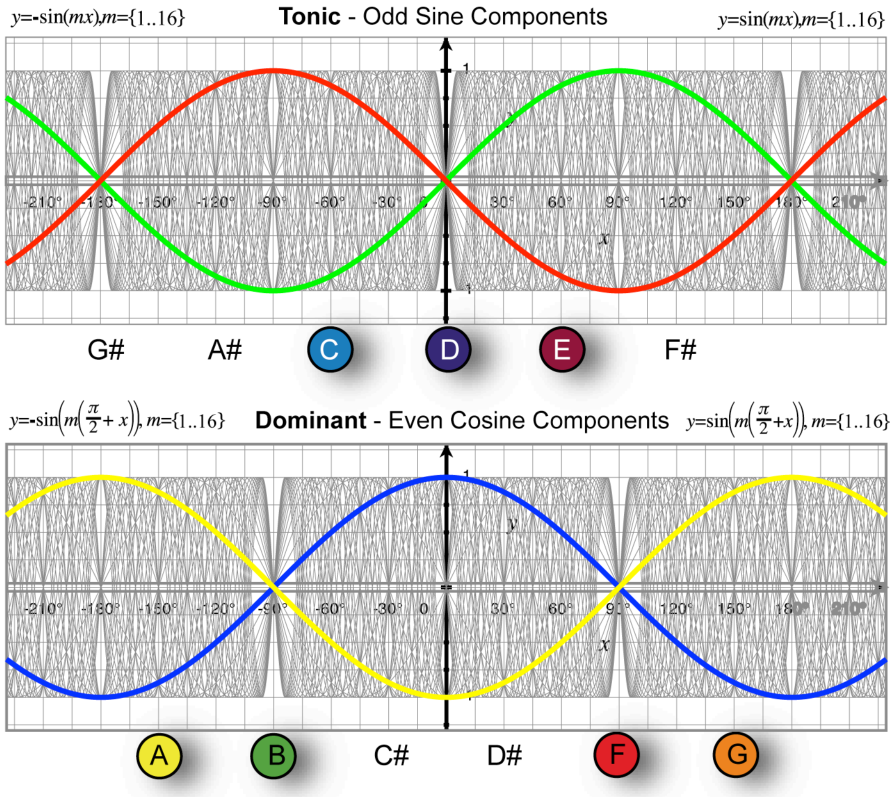
Out of spirals spin the periodic shapes of rings, toruses and spheres. And from these arise the coherent geometry of squares, triangles, pentagons and hexagons. As they polarize harmonically around a central axis, we hear the music of countercurrent crystals and living quasi-crystals extruding through a structured space. Viewing our world as a product of resonant ring geometries inside damping spirals makes everything seem like frozen music.

But while geometry does a great job of describing the spatial qualities of musical structure, it is not so good at explaining the actual processes that make it possible. Geometry alone does not represent the traveling patterns of coherent energy in standing waves and cannot easily illustrate the exchange of energy in their interference patterns. Geometry does not show how different harmonic proportions intersect or phase shift around damping wells. Geometry does not tell us about the *coherent pathways* that form in fields of interference as they trace out nature's shapes.

Since geometry is a form of crystallized energy, a state in which energy is stationary and does not travel, the only way to understand music or anything else is through the physical properties of an oscillating standing wave. Standing waves are the ultimate source of coherence in music and elsewhere in nature, oscillating at all scales and in everything. Harmonics have no alternative but to travel along predictable pathways in a standing wave. Some pathways are long, like the musical Orbit of 5ths, while others are short like the triangular orbit of a major 3$^{rd}$. Other pathways are more complex, such as major and minor scales that twist between the wholetone orbits to form *compound coherent pathways*. Understanding how this pathway emerges as the 7-color Diatonic Rainbow out of a standing wave is the key to unlocking the role time plays in forming geometrical shapes and the way we recognize them.

In Figure 96, the dual wholetone rings are rolled out into odd and even wave components balanced around the Harmonic Center node. Over this are superposed both halves of the {C} major scale – the Tonic group {C, D, E} on the odd sine wave and the Dominant group {F, G, A, B} on the even cosine wave. In this way, the phase offset between the Tonic and Dominant groups in the diatonic scale is readily apparent just as it is in the dual ring models. But, instead of folding and unfolding or shrinking and growing rings, we now represent the Diatonic Rainbow as a *phase shift* of odd-even harmonics through a field of interference.

**Figure 96 - The Standing Wave component model for C major**




As explained earlier, the even Dominant components slide over to phase-align with the odd Tonic components, thereby triggering a feeling of relief and resolution. Here the two halves of a major scale can be seen to correspond to the same opposing groups of tense Dominant and resolved Tonic tones. Just playing a {C} major scale follows an alternation between in-phase and out-of-phase while reducing or increasing tension. If a tone is in-phase and odd, you sense a degree of resolution and calm. If the tone is out-of-phase and even, you then sense a degree of tension. Nature is really pretty simple.

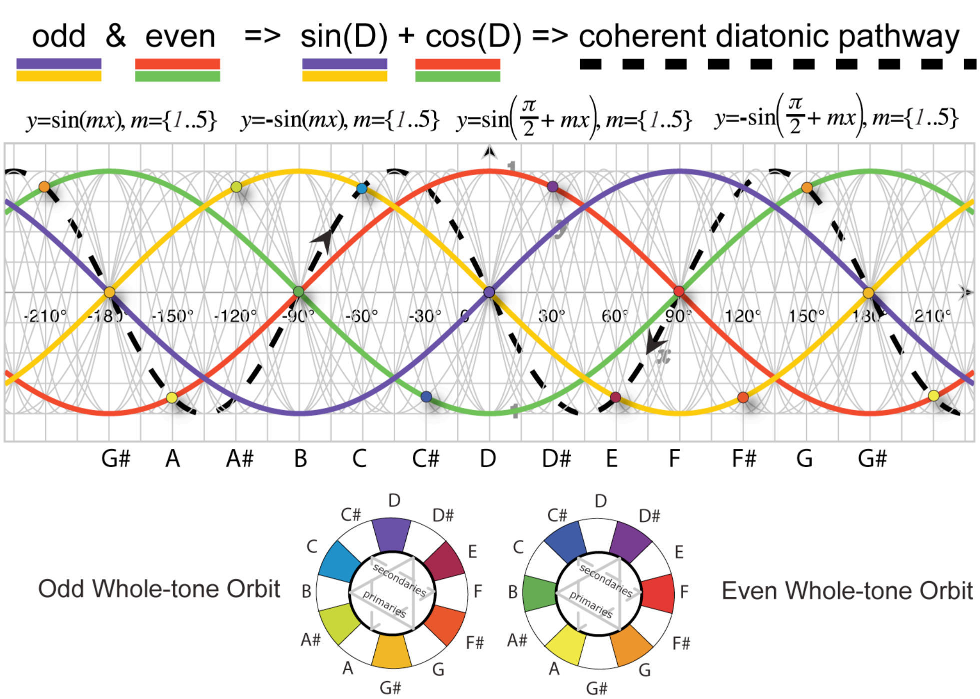
Looking close at the gaps in the two series, each tone in an equal tempered octave aligns perfectly with little bundles of harmonics spaced at multiples of thirty degrees (or π/6 radians). The bundles are as close as we can come to finding the origin of all things countable and the source of odd-even parity in our natural number system. As odd-even harmonics group together to flow through coherent pathways in a field of wave interference, so too do whole numbers follow various coherent pathways through a field of abstract integer space.

Real numbers then represent the fractional noise or "enharmonic torque" between odd and even whole numbers that are suppressed through damping. It is this torquing effect between odd and even elements that give rise to coherence and forms geometric pathways for structure of all kinds to build upon. From this perspective, we can say that the half-twist represented by the 7-step Diatonic Rainbow is the source of all torque, maximizing coherence in our number system, music, color and, well, everything else. This invisible torquing property behind coherence is the real pot of gold at the end of the rainbow.

Now, to see just how the Diatonic Rainbow forms in a field of standing wave interference, we need to slow everything down again into up-down clock ticks using Principle 29 and the Diatonic Phase State model. Recall this is how we represent energy direction in the oscillating harmonic series, stepping through time as *discrete movements along coherent pathways*. This turns a standing wave into a kind of digital clock cycling up and down where each half cycle is equal to one clock tick. Of course, real melodies and chord progressions hold and subdivide time in a great many ways in order to generate interest, set expectations and deliver (or deny) reward. But for now, we need only clock ticks of equal length to explain the Diatonic Rainbow.

In Figure 97, the above odd-even component series have been combined and simplified. The number of partials in each series has been reduced and grayed while the fundamental waves bolded to stand out. Colored dots from the Chromatic Dual Ring have been placed at each convergent bundle of harmonics to indicate the odd-even octave proportions (or angles) on the fundamental wave. As the last step, the coherent compound pathway of {C} major is shown as the dashed black line with arrows indicating direction of flow in the diatonic current.

**Figure 97 - The Wholetone Standing Wave model for C major**



In this one model, we are able to simultaneously represent musical tones and their synesthetic colors in native form as a single *Wholetone Standing Wave*. The diatonic tritone {B, F} forms the nodes of the even cosine wave (colored Red and Green) that can be seen to phase shift through the dashed coherent diatonic pathway of {C} major to align with the odd sine wave (colored Yellow and Blue). This is how the tritone (and Dominant chords in general) resolves to the Tonic major 3rd {C, E} in a typical pattern of harmonic interference. It represents exactly one anticipation/reward potential of the Tritone Function as it contracts inward.

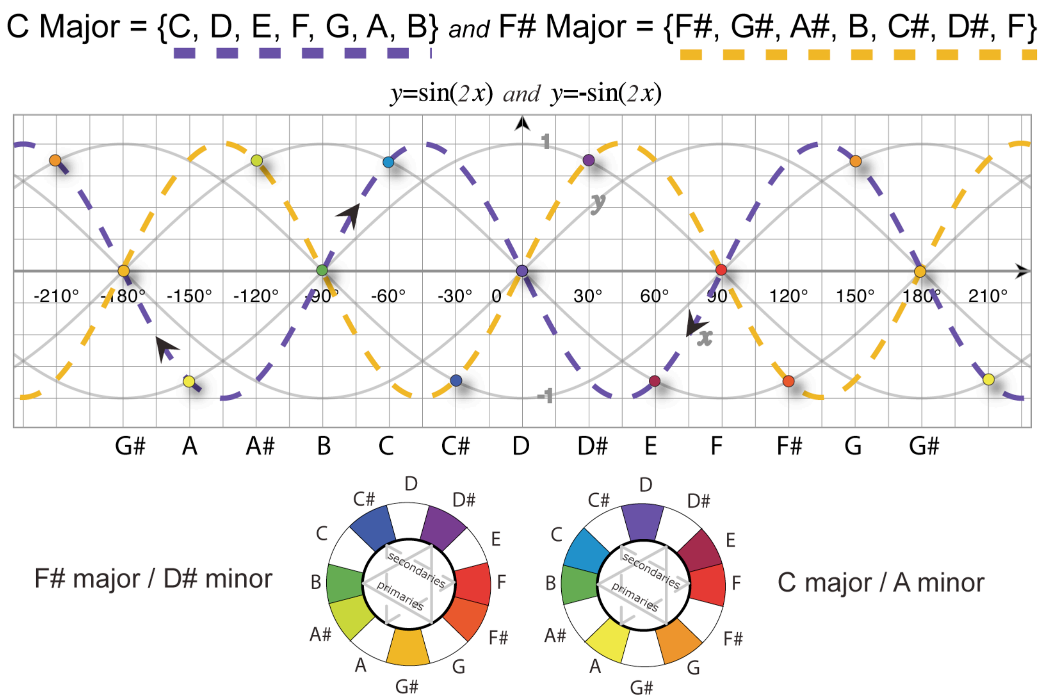
When we next compare the dashed {C} major pathway to the colored odd-even fundamental waves, we find that it has a periodic frequency of 2:1 or *twice* that of the fundamental. This happens to be identical to the frequency of Partial 2 (the first octave harmonic) and aligns perfectly with it in the Harmonic Hierarchy. Thus, what we will see with any 7-step diatonic major or minor scale is it follows a octave-doubling pathway through the wholetone scales. This is the essential idea proposed earlier in Principle 29 and embodied in rules of thumb, but its role in how we focus our attention to understand music cannot be overstated.

In our brain, the {C} major scale becomes a *double clock tick* of the oscillation of the wholetone scales. It is as if our perception of 7-step diatonic harmony results from a *higher-order 2:1 frequency interpretation* of the harmonic series in the same way an octave resonates at a higher frequency to the fundamental pitch of a tone.

We are able to tune in our attention at specific proportional frequencies something like a strobe light freezing dancers in motion in order to recognize coherent paths through harmonic interference. Based on this hypothesis, cognition of spatial scales and melodies can only be a function of *how fast* we choose to sample a standing wave. Temporal coherence in music based on sample frequency is therefore required *prior* to the recognition of spatial harmonic geometry. Since recognition of Φ and the Fibonacci scales has been determined to be the source of temporal coherence, then harmonic damping is also a prerequisite for spatial coherence and the recognition of harmonic structures of any kind. Simply put – time comes before space.

Returning now to the figure, recognition of diatonic harmony requires that we mentally regroup the odd-even Fourier components of the Wholetone Standing Wave model into a new path that is itself a standing wave between {C} major and its tritone substitute {F#} major. This was the essential discovery of 20th century jazz. Chords and scales a tritone apart could be stacked and recognized as a higher-order temporal structure based on Partial 2 and its octave frequency doubling in the harmonic series. This is presented as the simplified *Diatonic Standing Wave* model in Figure 98.

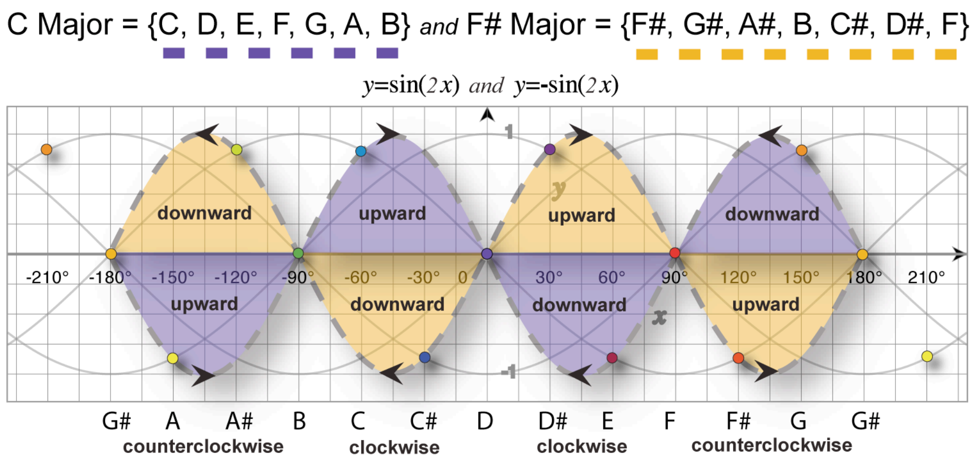
**Figure 98 - The Diatonic Standing Wave model for C major and F# tritone sub**




As you look at the Diatonic Standing Wave model, try to visualize the diatonic scales as pathways through a dense oscillating interference pattern composed of odd-even wholetone groups in the harmonic series. As the two opposing major scales weave their way through, each follows a different direction of harmonic energy exchange. And as the orbit of each major scale flows in opposite directions, a clockwise or counterclockwise circular rotation is created. This is the countercurrent Coriolis rotations of a double torus, or harmonic hurricane, represented next in the following *Diatonic Energy Flow* model in Figure 99.

The way in which energy flows in a diatonic scale is at the bottom of many of the questions remaining for music cognition and historical practices. By realizing that the frequency alignment of diatonic melodies and harmony occur at twice that of the fundamental in the Harmonic Hierarchy, the preference rules for voice leadings can be explained as our natural ability to recognize countercurrent energy currents orbiting around the Harmonic Center. Like the countercurrents in a hurricane, the energy in a {C} major scale rotates around the Harmonic Center {D} while the tritone opposite {F#} major scale rotates around the Inverse Harmonic Center {G#}. By simply following the arrows on each "hump" of the diatonic current, we can predict many of the voice leadings and other preference rules of traditional music theory.

**Figure 99 - Diatonic Energy Flow model for C major and F# tritone sub**




For instance in the case of {C} major, a chromatic Neapolitan $6^{th}$ chord {F, G#, C#} would create a {down, neutral, down} or average *downward* anticipation toward the Tonic triad {C, E, G}. Similarly, a chromatic jazz tritone substitute for the Dominant {G} would be something like {{G, B, D, F}, {C#, F, B}} or {{down, up, neutral, down}, {down, down, up}}, creating the anticipation of a predominantly *downward* resolving transfer of energy to the fundamental {C}. In this way, accepted voice leadings in conventional music theory can be directly correlated to the exchange and directional energy flow in a standing wave.

The Diatonic Energy Flow model is an excellent tool for predicting proper voice leading, replacing countless rules and exceptions to the rules. The axiomatic system prescribed earlier for chromatic and diatonic harmony is compatible with this model, whether oscillating between wholetone scales for chromatic harmony or following the Tritone Function in diatonic harmony. Any previously unexplained perceptual tendencies in diatonic music, together with the hundreds of years of traditions based on subjective rules, are reduced here to the simple physical exchange of energy in the harmonic series as amplified by tonal combinations in music.

Musicians need only remember and practice these diagrams as patterns on a keyboard to learn how to tap into their intuition and "ride the waves" of music harmony. When a song modulates to a new key, the preferred energy pathways move with it. Whenever a piece of music uses unorthodox harmonies, a musician's practiced intuition and scan through the music to identify temporary Harmonic Centers will always make improvisation or sight reading much easier. In time, a jazz pianist will cease thinking in terms of "altered chords" and arbitrary collections of sharps and flats, instead imagining an interference pattern of flowing energy hovering just above the keyboard, into which two hands can be inserted to grab the waves of positive or negative emotions. And since the wave patterns repeat up and down the audible spectrum, playing complex harmonies and dispersing them across multiple octaves becomes much easier. A pattern-based approach to harmonic function is no different than practicing scales, just requiring a little dedication to a new way of harmonic thinking.

We find in this system an immediately intuitive way of representing music. Since our auditory system recognizes shapes and energy currents as simple proportions and paths, why would we not represent those patterns using the same organic shapes and colors our eyes and brain instantly recognize? As powerful computers and music visualization software are accepted for use in performance, composition and education, geometric and wave models like those proposed here will inevitably replace traditional paper music notation. Software based on such models can simulate the exchange of energy and even calculate what our preferences are likely to be from it, making computer-aided music composition and analysis a truly useful tool for musicians and educators.

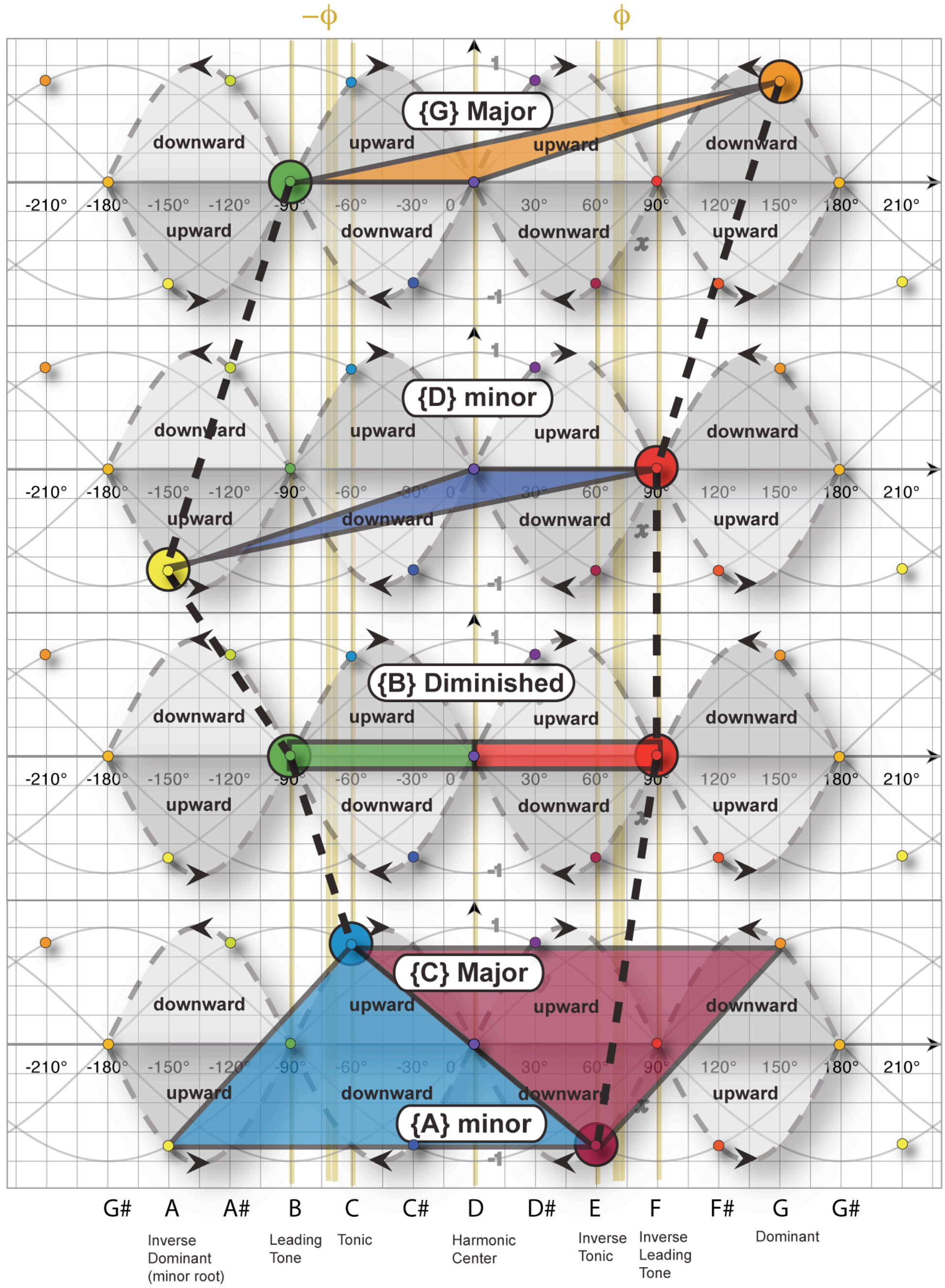
As computerized music stands and composition consoles replace paper music, music notation will become animated chords tumbling through pitch space and melodies floating through currents. Colors will change with the sounds we hear and trace out the same coherent pathways found in our brain and our body. When the patterns we see become the same patterns we hear and play, music will once again be unified with science as a single system of knowledge. Visualized through the universal language of mathematics, the *Principles of Harmonic Interference* will again reveal the physics and physiology of music at work in nature. *This* is the future of music.

For a glimpse into this future, consider the chord progression {G, Dm, B°, {C, Am}} in Figure 100. The chords appear to shrink inward toward the Harmonic Center as the waves oscillate. Starting out very wide like a stretched rubber band, the chords slowly contract to the Tonic major 3$^{rd}$ at bottom as they follow the currents of the Tritone Function. When played on a piano, it sounds just like it looks – a seesaw on either side of the Harmonic Center. Once the seesaw ends on the {C} major chord, we hear it as happy and upbeat. But if we end it on the mirrored {A} minor chord, we are left feeling a bit melancholy or sad. This seesaw duality is nothing less than a picture of our brain as it organically pattern matches incoming harmonies against its built-in ***REFLECTIVE INTERFERENCE*** physiology and triggers our emotions.

As a spatiotemporal visualization tool, the Diatonic Standing Wave model reveals the balance we hear in music. We can immediately tell if a chord is happy-bright or sad-dark by whether it is weighted to the right or left of the Harmonic Center. If weighted to the right, the emotion becomes positive and "major" while weighting to the left becomes negative or "minor".

This same seesaw effect applies to any number of tones over multiple octaves. It works because each octave maintains the same proportions at each frequency doubling, enabling an unbroken energy flow across the entire Harmonic Hierarchy. Like a series of harmonic hurricanes viewed from the side, the Diatonic Standing Wave model correctly represents diatonic music as a perpetual process of weaving, balancing and measuring the mechanical energy currents in sound.

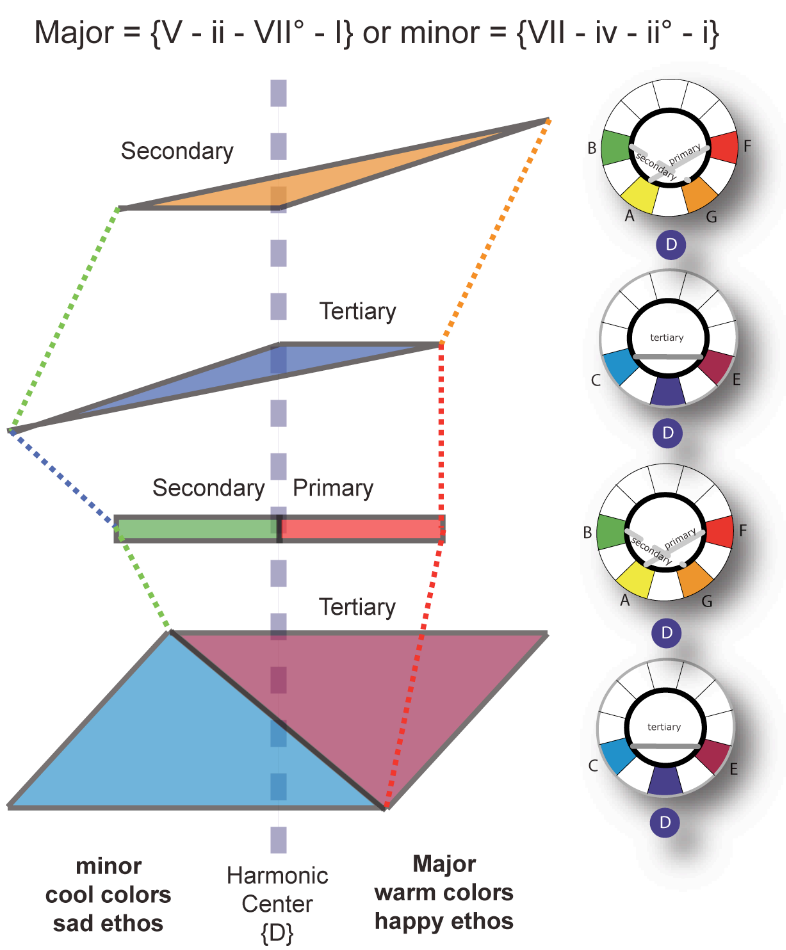
**Figure 100 - Chord progression on the Diatonic Standing Wave model**



This is essentially how our auditory system works. It automatically tracks the overall balance of auditory geometry in real time around the Harmonic Center while simultaneously triggering corresponding positive or negative emotions and maintaining in memory a short-term running average of which way the seesaw has been leaning. This measurement process can apply to colors just as well.

When we extract just the geometry from the previous figure into Figure 101, emphasizing the synesthetic colors on either side of the dashed Indigo line, we obtain the simplest possible geometric representation of color-tone harmonies as they oscillate in clock-tick fashion between {Secondary, Primary} and blended {Tertiary} colors.

**Figure 101 - Color mixing of chordal wave geometry to determine ethos**



Whether we choose to represent it in ring geometry or standing waves, harmonic music triggers our emotions as they teeter from one side to the other. As the music plays, we *feel* which side is "heaviest" and gauge the general direction energy is flowing through the harmonic interference.

Of course, various factors will modify our interpretation of the currents. For instance, the overtones in a particular instrument have their own symmetrical balance that may be counter that of the chord harmony. When certain instruments in an orchestra are emphasized over others, this can amplify or reduce the harmony's positive or negative effect. Rhythm, which can be seen as a very slow form of harmony, may also emphasize or oppose natural harmonic currents (i.e., as harmonic rhythm). And when we factor in other musical attributes, such as lyrics, personality and artist style, an underlying sad harmonic effect may be interpreted instead as a bittersweet love or longing, normally considered a happy thing. Similarly, a happy musical ethos may be paired with political lyrics to produce satire, normally thought of as a negative ethic. But all this being said, when music is allowed to oscillate in a symmetrical fashion like a harmonic spring, it is still the single most important factor driving our emotional response to music.

The relevance of geometry to our perception of music could not be greater. It is hard wired into the physics of sound and the structure of our ears. With most instruments tuned to favor the equal temperament of an octave, musical geometry plays directly to the equally spaced harmonic series evolved into our physiology. And perhaps the most stunning thing of all is the fact that these spatial structures and temporal currents can be traced back through Fourier to a single, all encompassing Pythagorean ideal – the pentagonal dodecahedron.

Unified in the dual ring model of Metatron's Cube taken from the Gnostic Flower of Life lattice, all of the perfect solids with their triangles, hexagons, pentagons, parallelograms, rhombuses, lines and other polygons can be transformed into coherent energy pathways on the $\mathbb{Z}/12\mathbb{Z}$ standing wave models. Describing harmony and melody as dynamic geometric shapes shifting through standing wave interference is arguably the most practical, most organic and most comprehensive explanation for music possible. Here are just a few reasons why.

***Top-10 Advantages of the Standing Wave Models over traditional music theory***

1. *Identifies symmetry correctly around a Harmonic Center,*
2. *Provides visual clues for anticipation of harmonic qualities and movement,*
3. *Intuitive weighting of directional energy currents to measure emotional effect,*
4. *Simultaneous presentation of a synesthetic color mix model,*
5. *Naturally represents pitch space and multiple octaves in a horizontal scale,*
6. *Derives from and correlates all music to harmonic wave phenomena,*
7. *Only organic model that couples acoustics and biology,*
8. *Can be fully defined as a mathematical model for computer simulation,*
9. *Integrates with ancient philosophies concerning harmonics,*
10. *Offers a unique new notational system for music.*

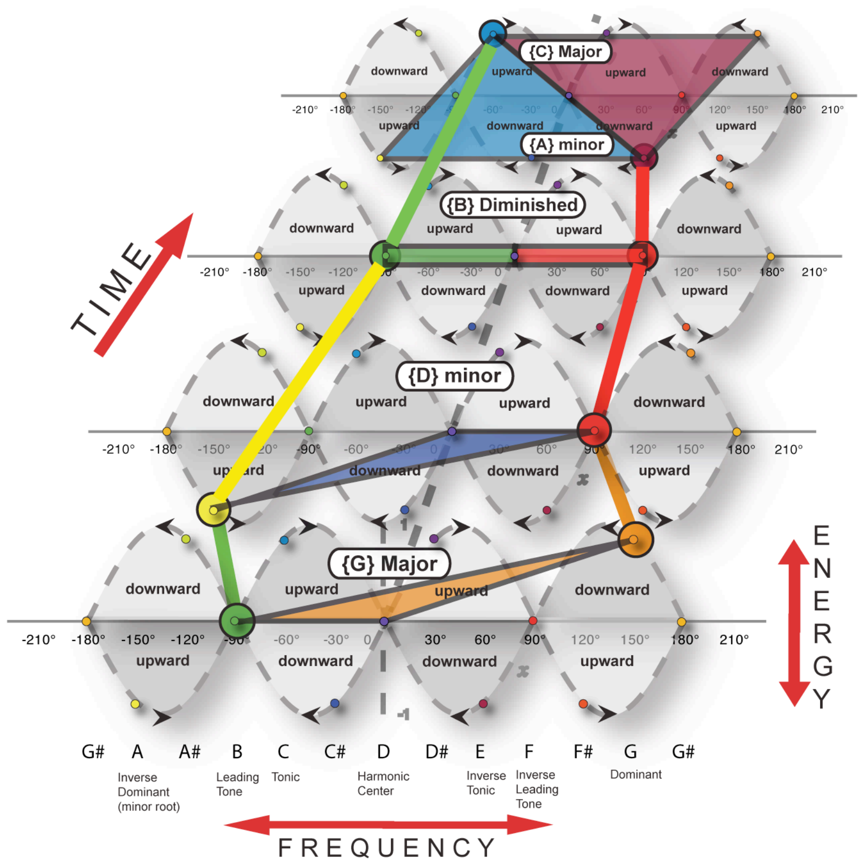
Much more than just a better visualization tool for music theory, the standing wave models also offer a better way to notate music.

***Top-5 Advantages of the Standing Wave Models over traditional music notation***

1. *One-to-one application to a piano keyboard for performance and composition,*
2. *Ability to be printed on paper or animated on a computer display,*
3. *When combined with textual annotation, it communicates more information more intuitively than ordinary music notation,*
4. *Can be developed into software applications that analyze and compose music based on a consistent physical and cognitive theoretical model.*
5. *Capable of expressing music as natural and organic forms*

When incorporated into real-time computer-aided music systems, standing wave models promise to completely reinvent music. Consider the application in Figure 102 where the Diatonic Standing Wave model is used to represent music moving along a z-axis *into* the computer screen.

**Figure 102 - Diatonic Standing Wave model in 3-dimensions**



In the figure, the Diatonic Standing Wave model is combined with a look-ahead into the next measure or two in order to represent rhythm along another dimension of harmonic space. This approach provides an unlimited temporal workspace within which to trace the coherent pathways and anticipation/reward potentials of each clock tick of music. At the same time, it would offer the capability of measuring both rhythm and pitch with the same set of metrics while the synesthetic model highlights the musical ethos in the visual dimension of color.

In 3-D space, persistent "tracer" lines can also be added between the harmonies to provide a cognitive audit trail and act as anticipation/reward vectors. The persistent Red line and intermittent Green lines act to call out the "Red × Green" spring action of the tritone as it stores and releases tension toward a Tonic resolution in the backmost layer.

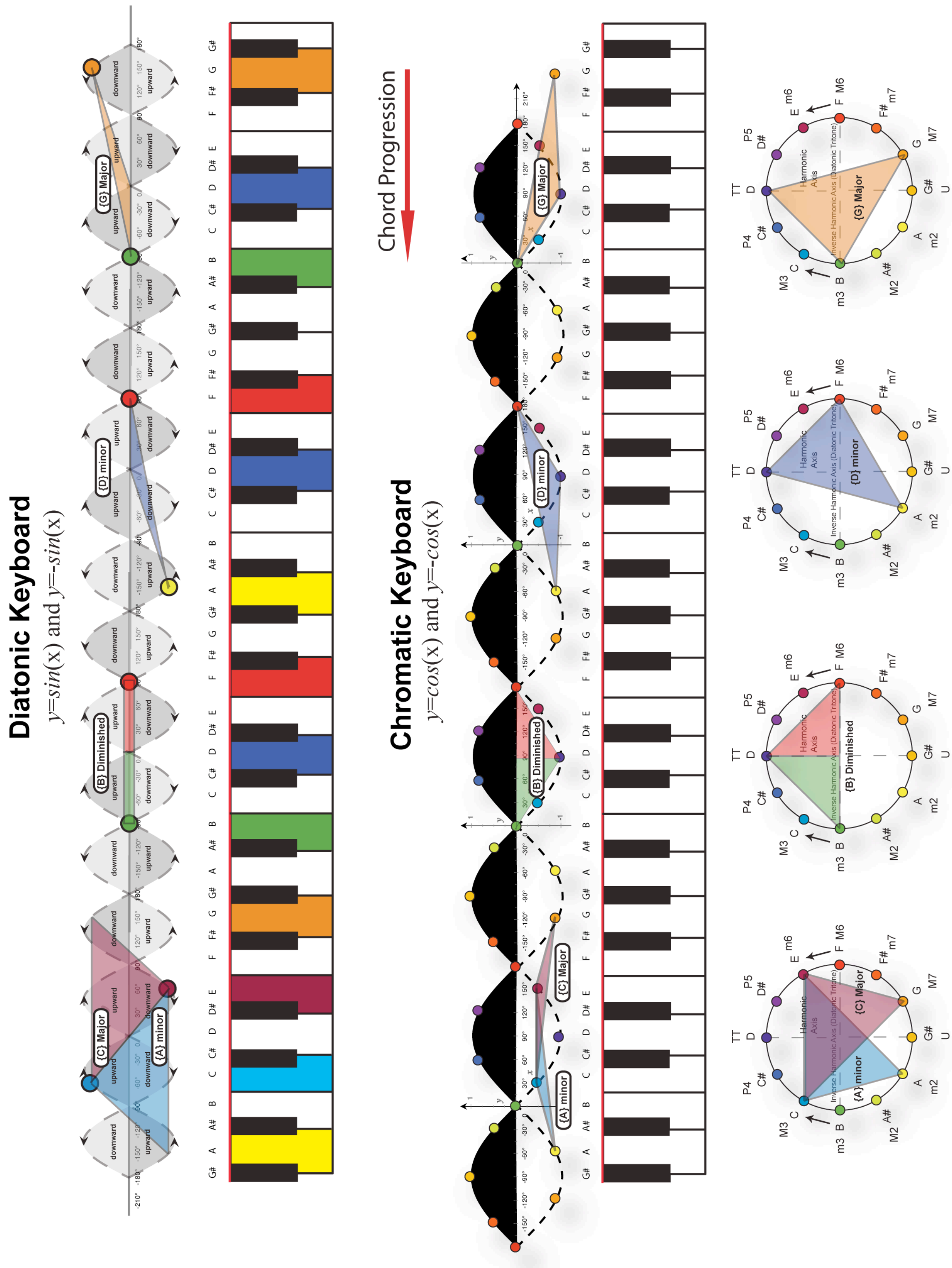
Taking the idea further, any number of 2-D and 3-D *musical dashboards* can be built to include a piano keyboard around various ring, wave and interference models. These can be tailored to address a wide variety of music composition and analysis applications. In a mixed-model dashboard like this, the synesthetic color model becomes especially useful in cross-referencing between different models at a glance, particularly as they change in real-time.

One possibility is shown in Figure 103 that combines standing wave and ring models for use at the piano. As a much more flexible alternative to traditional music notation, the models could be adjusted and arranged on a computer display as needed to visualize either diatonic or chromatic music. While the Chromatic Keyboard model is probably the most readable relative to the piano keyboard (due to its alignment with the black and white keys), a frequency doubling to the Diatonic Keyboard model is all that is needed to instantly visualize the directional current flow in diatonic harmonies. Much more is possible.

Consider how a musical dashboard might work for an orchestra conductor. Appearing as semi-transparent layers in 3-D pitch space, regions on a computer display could be used to hold each instrument section in an orchestra. As the score unfolds, geometric harmonic objects for each section would be viewed moving toward the conductor as tumbling objects in a fixed harmonic lattice. Under his or her control, the simplified geometric score would show entrances and exits highlighted with a countdown above all of the musical objects. The scale of each object and speed of rotation would indicate the dynamics and rhythm for each section. Entire sections could be minimized or rolled up, displaying only major changes, while tagged instruments would be brought to the center in advance of their entrance – perhaps zooming in to the beat of the conductor's Bluetooth "smart baton."

Placing similar dashboards at each music stand in an orchestra and interconnecting them through a local area network, the conductor's annotations could become instantly visible to anyone, if desired. Performers could configure their dashboard as they please and share their annotations with others too. Automated dashboards could even bring about new forms of improvisation and group composition, using such things as shapes, colors and metaphors.

## Figure 103 - Standing Wave keyboard models



A different kind of music visualization dashboard could be tailored to the work of a musicologist or music history professor. Converting existing sheet music into visual simulations could aid in the analysis and cataloging of historical music styles. For the first time in history, the tritone avoidance rules of Church music and Renaissance counterpoint would become painfully obvious, called out by the persistent absence of Red and Green marker connections. Many new and unconventional types of analysis could be applied to visually identify each musical style and period, including the degree of conformance to either diatonic or chromatic model. In combination with the ***REFLECTIVE INTERFERENCE*** model of *Harmonic Interference Theory*, the min/max points of consonance or tension could be used to provide a calculated running average for any piece of music, displayed perhaps as an automated "ethos ranking" for emotional content.

In general, musical dashboards like this are suitable for any number of music applications ranging from music theory, jazz improvisation and musicology to sophisticated computer-aided composition systems like *OpenMusic*. Connected to MIDI keyboards, the use of shapes and colors can help identify harmonic function and reveal coherent pathways for students. At the same time, ring models can provide a new symbolic approach to music based on nature's own periodic geometries. When projected into three dimensions like the previous standing wave model, we would see a full spatiotemporal simulation of music that accurately depicts *and predicts* the cognitive patterns of harmonic interference. Appearing as tumbling geometric figures or even life-like shapes floating inside a holonomic space, musical dashboards offer a revolutionary (and evolutionary) new way to represent music as our ears and brain recognize it.

After hundreds of years of suppression and avoidance of the most basic harmonic principles, the reintroduction of models like these promise to reunify music notation, composition and analysis into a single grand scientific musical theory. Founded in the physics of sound and the physiology of perception, music once again has the real potential to fuse Art and Science. Through organic harmonic models, mankind can once again discover its direct and intimate connection to nature, leading the world into a new cultural renaissance based on the physical archetypes of harmony.

**SECTION FIVE**

# Physical Archetypes

*"In a few decades of reconstruction, even the mathematical natural sciences, the ancient archetypes of theoretical perfection, have changed habit completely!" - Edmund Husseri*

In October of 1992 two very large waves converged. For the very first time, the entertainment and computer industries came together to produce a spectacular 4-hour live experiment in music and technology. It was called *The Grand Scientific Musical Theatre*.

The brainchild of Scott Page, former Pink Floyd sax man, and George Grayson, the president and cofounder of Micrografx, the Grand Scientific Musical Theatre (or "GSMT") was held in the Thomas & Mack Center in Las Vegas in parallel with the COMDEX computer tradeshow. As a benefit event tacked onto the annual *Micrografx Chili Cook-off*, the idea of the GSMT was to demonstrate what the entertainment and computer industries had in common and how they could work together to create new kinds of digital media experiences.

From prior years, the Micrografx Chili Cook-off was well known and always well attended by the computer industry. Everyone was invited to compete with their best chili recipe while attendees sampled the concoctions, watched armadillo races and listened to the likes of the not-yet-famous Dixie Chicks. There was always some kind of surprise, like the previous year's appearance of George "Spanky" McFarland from the 1930's-40's *Our Gang* and *The Little Rascals*.

As with all company events, the challenge was to dream up something bigger and better. To encourage participation and contribute to a really good cause, it was decided to make that year's event a charity for *The National Center for Missing and Exploited Children*, a favorite charity of IBM. Of course, IBM immediately agreed to become the lead sponsor.

Software companies signed up left and right to cook their chili and watch the show, though nobody had any idea what was in store for them. Leading production and technology companies came on board to create the “scientific theatre” part of the show and big name entertainment acts contributed their time to the cause as well. With last minute help from Scott’s friend, music producer Bob Ezrin (of KISS and Alice Cooper fame), the project began to take shape.

On the visual technology front, Sun Microsystems would generate real-time 3-D animation on a cylindrical display tower powered by their latest high-end graphics computer, aptly named the *Reality Engine*. A 3-D virtual talking head technology was also showcased by comedian Charlie Fleischer (voice of cartoon character Roger Rabbit) and displayed on more than a dozen large screens arrayed around the coliseum. To cover the action happening all over the coliseum, a dozen video “swarm cams” roamed the floor piping live feeds into an 18-wheel mobile production studio in the parking lot where it was mixed in real time and distributed back into the arena for simultaneous playback with the action.

The audio system boasted the latest in 3-D audio, complete with speakers mounted above the floor and focused on every angle of the circular arena. The show lighting system, on loan from rock group Genesis, featured a state-of-the-art computer controlled *Vari-lite* system showcasing the then-new capability of individually swiveled and switched lights under computer control. In 1992, the audio/ visual experience was groundbreaking compared to anything else available at the time – or maybe even since. The musical entertainment was not too shabby either.

Performers for the GSMT included the Nevada Symphony introduced by the governor of Nevada. This was followed by a couple of popular Los Angeles DeeJays interviewing and introducing various ensembles including the Tower of Power horns, Jeff Porcaro from Toto, Jeff “Skunk” Baxter of the Doobie Brothers, John Entwistle of The Who, The Edgar Winter Band (playing *Frankenstein*), Todd Rundgren (*Hello It’s Me*), Jon Anderson of Yes, Flo and Eddie of The Turtles (*So Happy Together*), contortionists from Cirque du Soleil and more. There was even a group of dancers on a side stage wearing glow-in-the-dark body suits that triggered synthesizer music as they performed. It was truly a 3-ring multimedia circus. Altogether, the entertainers and technologists blended into a dynamic and totally immersive audio, video and software *experiment* that really did justify the name originally coined in 1871 by P.T. Barnum for the world’s first circus: *Barnum’s Grand Scientific and Musical Theatre*.

For more than 4 hours, a stunned crowd of more than 10,000 computer industry pioneers representing IBM, Microsoft, Intel, Oracle, Novell, Sun Microsystems, Apple, Adobe and many others watched the spectacle unfold. Executives representing the future of the computer industry were all in attendance, not the least of which was David House (General Manager of Intel), Jim

Cannavino (head of IBM Personal Systems Division) and Bill Gates with soon-to-be wife Melinda French.

But as everyone began to slip into a dreamy vision of entertainment and computers converging in the living room, I imagined things a little differently. It dawned on me that the line between reality and synthetic was starting to blur. I could see how the virtual reality of technology was taking society further and further away from the natural world – yet, I could also see it having the same potential to bring it back. I could envision a tipping point when virtual reality became so real that it would be indistinguishable from reality itself, spilling nature's secrets out for all to see.

As these strange ideas swirled around in my head there in the darkened coliseum, I imagined the standing waves and perfect geometric forms of the music I was hearing projected onto the surrounding screens. In the theatre of my mind, each song would create its own virtual reality as geometrical objects tumbling through a vibrating landscape of intersecting waves and patterns. In this inner realm, I could see melodies become a growing flower, a tritone pumping like a beating heart and chords carving out the shape of some primitive life form. Higher in the hierarchy, I envisioned rich orchestrations and timbres materializing into the physical archetypes of people, plants and planets, giving the audience a glimpse into an invisible mythological realm.

Maybe this would become the vision that could stitch together a fragmented society, I thought, bringing with it a *new mythos* of cultural awareness and social coherence. Maybe this would be the real *Grand Scientific Musical Theatre* for which the world had been waiting.[127]

The after party was the first significant mixing of key figures from both industries with everyone completely overwhelmed and breathless by what they had just experienced. Intel's David House – the one who coined the slogan "Intel Inside" – claimed this was "a defining moment for the computer industry" while computer journalist Stewart Alsop commented that "it was a very significant event." Everyone left that night with a common vision of what was proclaimed as "our multimedia future." That is, everyone except the Micrografx Board of Directors who in the interest of staying focused on the company's graphics software business put a halt to any further investment in digital entertainment.

Within a couple of weeks of the show, George had resigned from Micrografx because of this disagreement on direction. I immediately joined him and his partners, Scott and Bob, to catch the next wave in interactive digital media. In our new interactive entertainment and educational company named 7th Level, we were determined to deliver on the promise of the GSMT by creating compelling interactive experiences first for CD-ROMs and then the rapidly growing worldwide web. My hopes of combining digital media and software into a day job had finally

[127] The words "theatre" and "theory" both come from the Greek word *theorea*, meaning *theatre of the face*.

materialized, though I knew any thoughts of returning to work on my "grand scientific musical theory" would have to wait a while longer.

The 1990's brought a tidal wave of change in digital media production and distribution. Hand-drawn animation advanced from photographing stacks of transparent celluloid on film to 3-D laser scanned models and motion capture. Three-dimensional animation advanced from frame-by-frame video rendering to real-time interactive environments offering six degrees of freedom. Video production shifted from expensive post-production suites down to the humble desktop computer. Music synthesizers moved from the analog realm of electric oscillators to the digital domain of automated MIDI workstations capable of emulating thousands of instruments through digital sampling. And *all* that digital media began to be streamed at accelerating speeds over the Internet and mobile devices, creating a bit torrent of digital fun for everyone. The illusion was complete – the promise fulfilled.

Or was it?

Projecting current trends out a few years, it is not too hard to imagine what might happen next. We know that handheld devices will continue to increase in power and functionality, as will the speed of network access to media content. The quality of audio and video will also advance, driven by more and more parallel processing. Consumer demand for personalization will continue to grow while the expectation of sharing personal choices will be assumed. Flexible "e-paper" displays will become commonplace and 3-D holographic displays will project an even more convincing virtual reality into our theatres and living rooms.

This technology will almost certainly bring about a continued democratization of music. Through a combination of MIDI controlled digital sampling, high-speed wireless communications and mobile audio/visual devices, we can expect to see the rise of personalized music to a level never imagined. Instead of personal collections of songs we will see the rise of customized *do-it-yourself music* ordered up like toppings on a pizza.

Of course, we all know the biggest barrier to this ever happening is the fact that most people simply do not know how to compose or notate music. Few have the talent to play an instrument or sing. Fewer still have the software and equipment to record and mix music. And even if they did, it would not be enough to satisfy the diversity of styles, moods and genres found in the typical music library. Without a doubt, if do-it-yourself music is ever to become a reality, an entirely new way of representing and interacting with musical concepts is needed – one that operates at the higher level of metaphor, story and personality.

At the time of this writing, it is possible to develop a compact visual music composition and playback system for use on handheld audio/video devices. Indeed there are several such applications that have recently become available. But a truly organic music visualization system has yet to appear simply because there is no accepted theory of music perception. The missing ingredient, of course, is *Harmonic Interference Theory*.

Using the ***INTERFERENCE*** metrics and harmonic models proposed in this book, an intuitive do-it-yourself music composition system is now possible. Within such a system, musical elements could be stored as organic visual objects in vast online libraries available instantly anywhere in the world. Chords, lyrics and melodies, user-selected by color, form and model, could be browsed and dragged from the Internet then dropped into the playback grid to start the music tumbling. These organic harmonic archetypes could be transposed, modified and combined to create composite musical objects representing entire songs inside a "live" 4-dimensional landscape. With this kind of technology right "at your fingertips," music composition would become something like a video game – only much, much better.

As organic musical objects became more sophisticated, songs would begin to resemble natural objects like flowers, fish and people. Before long, the line would begin to blur between our personal musical realities and the shared physical reality outside. And as this became a popular way to create music in the global pop culture, something quite amazing would occur – the inevitable realization that nature itself is actually a kind of crystallized music.

With the viral nature of the Internet, this meme could catch on very quickly; spreading through society like wildfire while elevating the consciousness of everyone it touched. A grand new vision of life as crystallized harmonics could quickly emerge, bringing with it a greater compassion to those who too easily find justification for violence and war.

In one or two generations, society could begin to see itself as an extension of nature, rather than something apart and separate from it. New metaphors and vernacular derived from harmonic principles could return to describe these ancient concepts, reinventing Western civilization from the bottom up while transforming the institutions of science and religion from the inside out. A grassroots pop culture movement that reunified physics, physiology and music could have the power to once and for all release the stranglehold of asymmetry in Western philosophy. As the most ironic outcome of the digital revolution yet, a mobile do-it-yourself music composition system may well be the black swan that brings the world back from the brink, back to the Pythagorean archetype of the *Music of the Spheres*.

# *Coriolis Effect*

*"In the living as well as non-living parts of nature, the trained eye encounters wide-spread evidence of periodic systems. These systems point to a continuous transformation from the one set condition to the opposite set." - Hans Jenny*

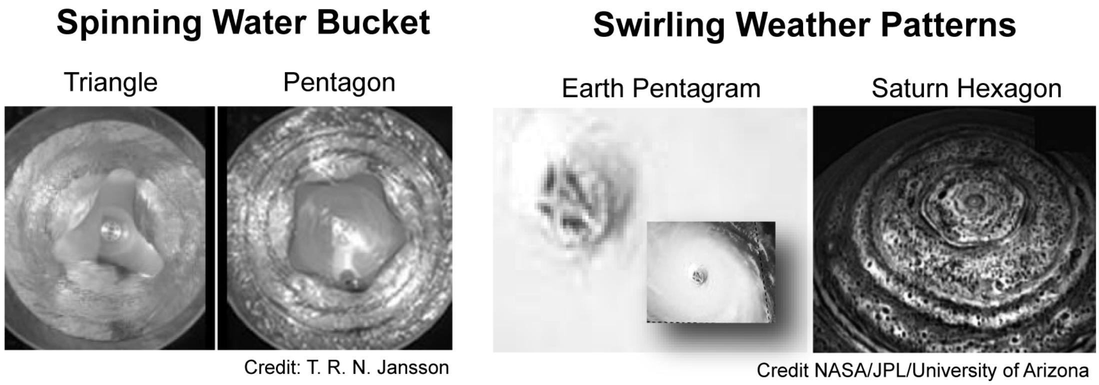
It is a fact of nature that a hurricane spirals due to the difference in density to temperature in our atmosphere. As air currents gather large quantities of water into huge clouds, the atmosphere at the center of a hurricane becomes denser than the surrounding air, creating pressure and causing the formation of a spinning vortex. The Milky Way galaxy does the same thing, but instead of a dense atmosphere it spins into dense plasma gases that collect at the center, tearing a black hole in the fabric of spacetime itself. As a small eddy at the outer edge of this galactic whirlpool, our solar system was itself once a spiraling disc of colliding plasma, creating the pressure that ignited our Sun and coalescing into the planets. In each case and at every level, it is the density differential and corresponding torque that starts everything moving.

A central feature of these natural spinning vortices is the formation of giant arms that spiral inward. As we saw earlier, this is due to the Coriolis Effect caused by two rotating toruses (or donuts) of material spinning in opposite directions, one on top of the other, that generate increasing torque toward the central eye. Like the Chromatic Torus model, their outer surface rotates over and into the donut hole causing anything on the surface of each torus to spiral continuously inward like a vortex. And as pressure builds at the center and velocity increases around the eye, harmonic shapes will begin to form inside the spiral container.

This was confirmed a few years ago in a study at the Technical University of Denmark in Lyngby. Their experiment involved spinning buckets of water at high speeds to cause regular geometry to form out of water vortices. To their amazement, when they dialed in increasing frequencies of spin speed, the water would swirl out of chaotic currents first into the shape of an ellipse, then a triangle followed by a square, a pentagon and a hexagon. The thing that puzzled the researchers most was how the water could make hard corners and clean lines when it was only a fluid. They wondered what could cause this?

At about the same time, reports of geometrical formations in weather patterns began to appear in the news. From high-resolution satellite imagery, hexagonal and pentagonal formations of clouds were spotted in the eyes of powerful storms, such as hurricanes Ivan and Dennis. Even more stunning was photographic evidence of a huge hexagonal pattern on the surface of Saturn (large enough to hold three Earths), photographed during a flyby of the Voyager spacecraft.

**Figure 104 - Harmonic geometry produced by the Coriolis Effect**




Of course, the spinning water bucket and strange weather patterns were simply physical examples of harmonic geometry forming on round surfaces due to a pressure differential and the Coriolis Effect. But while this seems obvious in the light of the preceding discussion of harmonic interference patterns in music, the astrophysicists and meteorologists have little to say about such phenomena and offer no opinions for what could cause this. How could something so fundamental to the behavior of nature remain a mystery in this day and age?

*Harmonic Interference Theory* tells us that geometric patterns arising out of vortices like this are the result of the interference of reflected harmonic waves as they resonate and amplify at simple proportions to the frequency of a "container." Ernst Chladni had long ago proven this by vibrating powder on circular plates as had Blackburn on his harmonograph. Even in modern times, Swiss scientist Hans Jenny demonstrated the universality of these same harmonic patterns by reproducing them in other mediums, such as gas and liquids.

One of Dr. Jenny's more famous experiments in the 1960's involved a spherical vibrating water droplet containing very fine particles. He found that when he reached a particular resonant frequency in the droplet, the particles would form into a 3-dimensional dual tetrahedron. This led to a series of other experiments using small flames of gas burned through perforated sheets that formed the same patterns in thermodynamics. Jenny then fabricated a variety of other water containers that he vibrated at even higher harmonic frequencies to produce a wide variety of fantastical 3-dimensional geometrical patterns, not the least of which was the Egyptian Flower of Life pattern. No matter what medium he used, Jenny always found the same harmonic geometry that could be "tuned in" to produce any number of geometrical patterns.

Jenny named these harmonic patterns *cymatics* after the Greek word "kyma," meaning "wave," and "ta kymatica," meaning "matters pertaining to waves." While he did not embrace a particular theory like fluid dynamics or wave propagation physics or torsion field physics to describe what he saw, he did catalog a great number of examples of these vibratory patterns to prove the universality of harmonic geometry throughout nature.

Jenny was not the only one interested in the mysteries of vibratory patterns. In the late 1950s into the 1960's, Dr. Mary D. Waller classified these patterns according to lines of symmetry. She found that when one line of symmetry was present on a plate (a circle), there were two classes of pattern symmetry while with three lines of symmetry (a triangle) or five lines of symmetry (a pentagon) there were four classes of symmetry. In general, Waller found that *n* lines of symmetry might be divided into classes that are twice as numerous as the number of factors of *n*. But in the simplest case of <u>one line of symmetry on a circular plate</u> she found that one class was nodal (or calm) and the other class was anti-nodal (or excited). Sound familiar? This is, of course, the Chromatic Dual Ring model.

From Waller's cymatic symmetry studies, the Chromatic Dual Ring can be defined as a circular or spherical container with n=2 nodes (as dual rings) and m=12 diametric modes (as harmonic partials), exactly matching the Harmonic Hierarchy of a standing wave. As standing waves move from the center to the outer edge and back, the inner ring becomes a freestanding nodal ring of calm in the interference while the outer ring acts as the resonant anti-node.

So we see here that the Chromatic Dual Ring is not simply some abstract model based on musical set theory nor just the "sacred geometry" of Metatron's Cube, but instead is a <u>physical</u> interference pattern that occurs as a result of coherent interference in any round or spherical container. As a model from *Harmonic Interference Theory*, the Chromatic Dual Ring can be used to explain harmonic phenomena occurring not only in acoustics and music, but elsewhere in nature too. It can even explain the bizarre pentagram in the eye of Hurricane Ivan.

A pentagram will always form between the outer and inner rings of a circular cymatic container as a result of harmonic Partial 4 resonating countercurrently to the fundamental frequency of any container.[128] From this we know that the winds in Ivan's eye must have been rotating at a 4:1 ratio to the winds at its outer edge (equal to a double octave harmonic frequency in a container) in order to generate the 4+1 = 5 points of a pentagram.

Using the Chromatic Dual Ring model as a musical archetype for Ivan's eye, Figure 105 shows how a double octave harmonic will create a pentagram that aligns with the Tonic major 3rd on the calm inner ring. As a coherent pathway and rational substitute for Φ and the Fibonacci series, the pentagram geometry with its 5:3 major 3rd alignment represents the first stage of damping in a circular or spherical standing wave. In music, this becomes the Tonic major 3rd of the Tritone Function, acting like a light caliper brake on resonant vibration. We can bet this was also the process occurring in Hurricane Ivan, perhaps right at the peak of the storm.

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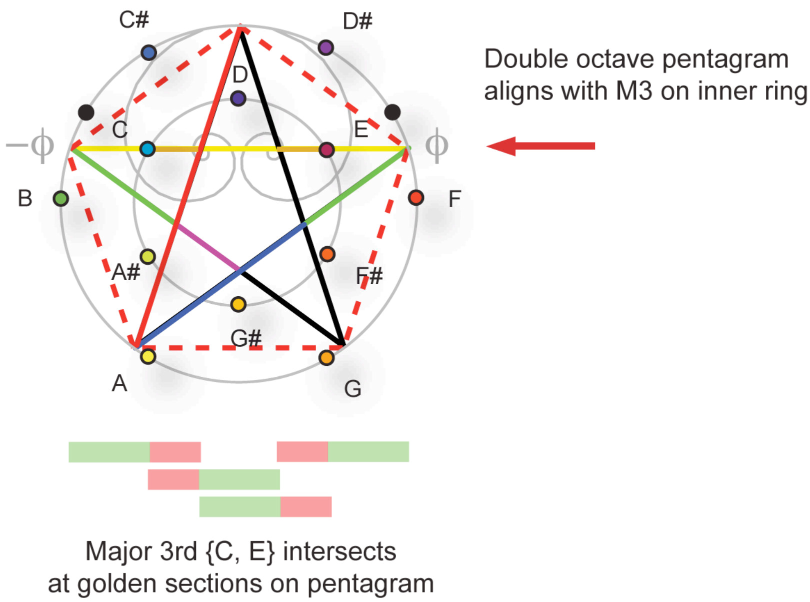
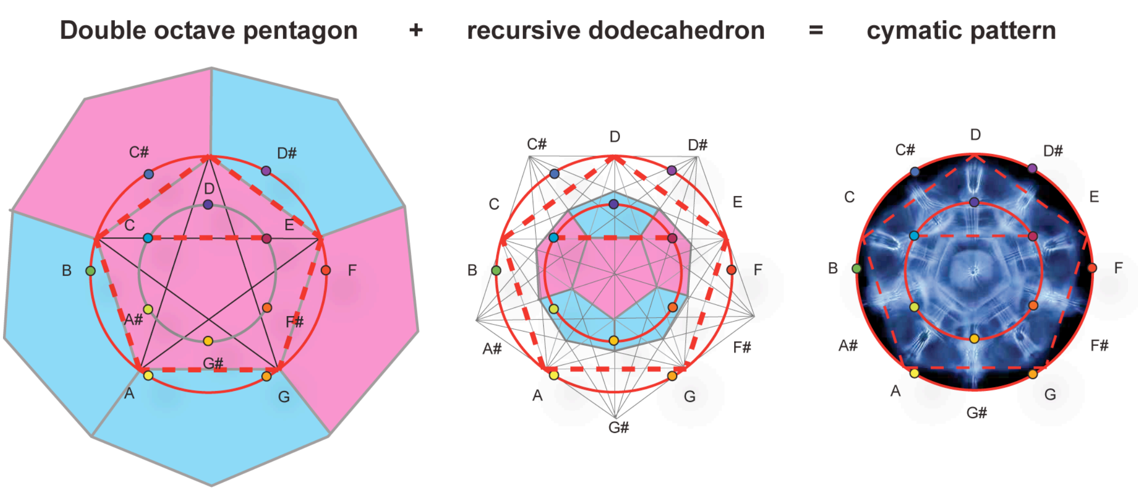
[128] A pentagonal pattern will also form from the countercurrent interval ratio 3:2 of a perfect 5th. The number of points or leaves of a cymatic pattern is determined by adding both numbers in the ratio, thus a 4:1 overtone and 3:2 interval both produce pentagonal patterns [Ashton 2003].

**Figure 105 - Pentagram Φ-alignment with Tonic major 3rd**




One other important feature of cymatic interference patterns like this is that smaller versions will always nest upside down in the center. For instance, a 2-D pentagram will nest geometrically both <u>out of and into</u> a 3-D dodecahedron solid. This is illustrated in Figure 106 by differentiating a pentagon into a smaller 3-dimensional dodecahedron just as it occurs in a physical cymatic pattern.

**Figure 106 - Recursive harmonic geometry of a pentagonal cymatic pattern**



As it happens, the smaller dodecahedron is always sized by the *cube root* of the larger dodecahedron on the outside. This is not only of theoretical interest, but the actual physical interference pattern produced by the double octave harmonic in the cymatic pattern. The geometry underlying this phenomenon is clearly that of a 12-sided dodecahedron flattened to a 2-dimensional round surface. The geometric differentiation process of connecting every vertex and edge bisector of the pentagon is a direct representation of Partial 4 as it reflects and interferes with the fundamental resonant frequency in a round plate or spherical container, polarizing naturally around a single line of symmetry. We already know this line as its Harmonic Axis.

The light colored regions in Jenny's liquid cymatic pattern indicate where particles collect into the calm areas of the wave interference to create a kind of negative or background image to the darker foreground regions where waves converge and resonate. These kinds of calm areas in a field of interference act as a stabilizing lattice from which higher-order geometric patterns will crystallize. We see this happening here with the 3-D dodecahedron emerging from the 2-D pentagonal lattice pattern.

The Chromatic Dual Ring model translates the physical actions of resonance and damping into musical concepts. It shows us how recursive structures emerge from harmonic interference and how musical chords form into auditory geometries. It connects music with the natural cubic geometries that recur throughout nature. Greek philosophers must have chosen the apple with its pentagram of seeds as their favorite harmonic archetype for just this reason – it demonstrates without words how the Spiral of 5ths closes into a circular octave. It was the simplest recursive cubic model for harmony they could find to symbolize the musical properties of nature.

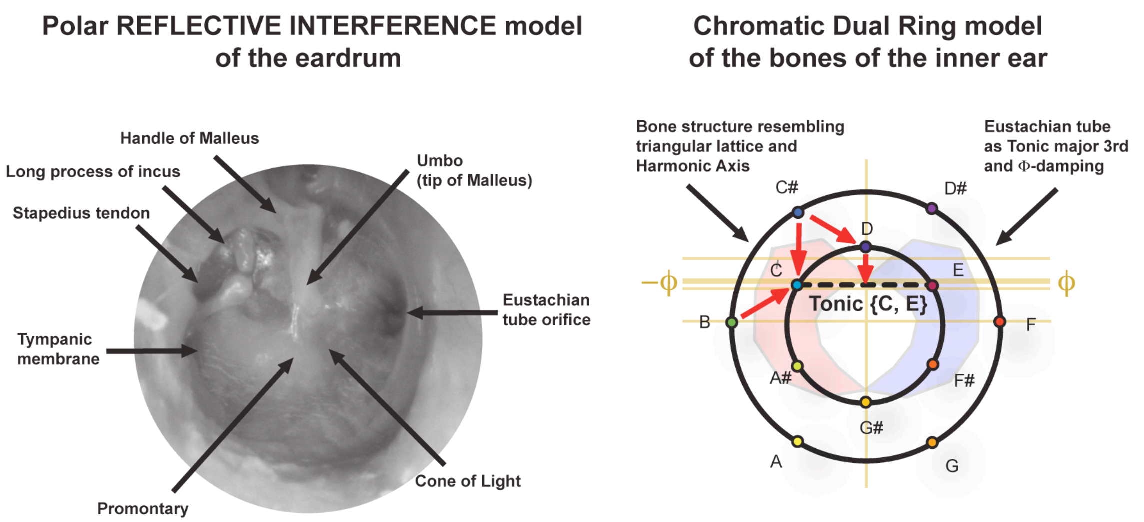
As more and more energy is poured into the Chromatic Dual Ring, the geometry becomes more fractal, finer and intricate – like a Hindu mandala. If chords and melodies describe the geometric skeleton and organic shapes of nature, it is timbre that represents the higher resolution "connecting tissue" and "skin." From macro to micro, the five layers of the Harmonic Hierarchy work to fill in the detail for all organic structures like it does in music.

This really is how everything works. Each recursive layer nests inside the next nondestructively and at all scales. It is through this hierarchy that we perceive our environment and understand music. And it is through the coherence of standing wave interference – *both outside and inside of us* – that makes life possible in the first place.

To help us survive, our ears had to grow in a very special way to focus on coherent patterns in our environment. We have already seen how our auditory system grew to match the polar organization of the ***REFLECTIVE INTERFERENCE*** model, spinning out of a Fibonacci spiral into a thin Tympanic membrane in the shape of an apple. The question now becomes: what cymatic physiological principles were at work to cause its cardioid shape and why did the bones of the middle ears grow beneath it as they did? How did they “learn” to translate compression waves into transverse waves suitable for measurement in the inner ear?

In Figure 107, the three bones of the middle ear, comprising the Long Process of Incus, the Stapedius Tendon and the Handle of Malleus, are strategically positioned in a triangular formation under the tympanic membrane of the eardrum. When cross-referenced to the Chromatic Dual Ring model, a clear picture begins to emerge for how our ears must have evolved to recognize cymatic patterns.

**Figure 107 - Harmonic archetype for the human ear**




Following the red arrows in the dual ring model at right, the bone structure is clearly that of a *triangular lattice* compatible with the resonance pattern and simplest single line of symmetry in a round Chladni plate. Both the Stapedius tendon and Long process of Incus grew from the resonant Dominant outer cosine ring inward toward the calm Tonic inner sine ring. Both of these bones actually meet at the fundamental {C} on the Chromatic Dual Ring while the Handle of Malleus grew straight down along the vertical Harmonic Axis through the Harmonic Center {D}.

This configuration is able to organically measure each of the possible vertical ring proportions in a $\mathbb{Z}/12\mathbb{Z}$ cymatic ring, including the wholetone and semitone proportions between each terminating location. In this way, each bone performs the biological function of a Fourier transform to convert circular regions of resonance and damping on the Tympanic membrane into individual transverse harmonic waves within the inner ear.

Not coincidentally, the Malleus terminates precisely at the horizontal crossbar that forms the Tonic major 3$^{rd}$ coinciding with the same recursive pentagonal geometry described above. This horizontal line, together with the entire range of horizontal Fibonacci proportions on the ring model, can also be seen to converge to Φ coincident with the hollow *Eustachian tube* at the right of the eardrum. As an evolutionary response to relieve air pressure at the surface, the damping proportions of the Fibonacci series were apparently the weakest region in the ear's harmonic structure, allowing air pressure to carve out a canal aligned with the Tonic cross bar and corresponding pentagonal geometry. Combining this with the polar ***REFLECTIVE INTERFERENCE*** model of the eardrum, we can see how the ear images harmonic geometry while focusing on the most coherent proportions as it unwinds sound inward. But still, what can we say caused it to grow this way?

We can see how the ear evolved by again invoking the principles of the harmonograph. Cardioids are drawn on the harmonograph from simple intervals set to vibrate *concurrently* in the same direction. Tissue also grows in a concurrent direction through the addition of cells, taking the form of cardioids in complex life forms. Since the pentagram acts as a primitive damping role and coherent pathway in the harmonic set of $\mathbb{Z}/12\mathbb{Z}$, the entire configuration of the ear can be explained as a concurrent growth pattern that follows the pentagonal double octave Partial 4 as it opens up the coherent pathway of Partial 5 (the major $3^{rd}$) through the interference pattern. This would explain why the bones of our ears grew to the points of maximum resonance and damping, aligning with the interval of a Tonic major $3^{rd}$ (pentagonal 4:1=5) and Inverse Dominant major $6^{th}$ (octagonal 7:1=8) geometry. As our ears grew outward from the brain, they grew along a Fibonacci spiral, suspending a *concurrent cardioidal form of a countercurrent pentagram* at the calmest and most coherent location in the Fibonacci spiral of the ear. So, while Hurricane Ivan suspended a countercurrent pentagram over its calm eye, our body suspended concurrent pentagonal cardioids in the calmest region of our ears.

Given all of the discussion up to this point, should anyone really be surprised to learn that our ears evolved to match the polar ***REFLECTIVE INTERFERENCE*** distribution of the harmonic series and Chromatic Dual Ring model of a round Chladni plate? Should we be shocked to learn that bones grow along harmonic pathways or that tissue forms into canals along damping pathways or even that our ears grew under pressure according to the Coriolis Effect just like a hurricane or galaxy? No - not if we agree on the common sense premise that instantaneous recognition and *feeling* of auditory shapes evolved according to the universal physics of harmonic interference.

Stop and ask yourself if it seems reasonable that all biological processes – bones, organs, muscles, nervous system, blood, brain and senses – would grow coherently out of carbon-12 according to the same balance of resonance and damping in a 12-step octave. Does it make sense that life would grow in conformance with the coherent geometry found in the standing waves of light, sound, electromagnetism, gravity, the solar system and everywhere else in the cosmos? If so, then you would probably conclude that harmonic science is *the one and only* unifying natural philosophy for all natural science – a complete and comprehensive system of thought – capable of "breaking open the head" and replacing the countless artificial theories currently in vogue.

There is mounting evidence to support the idea that an understanding of harmonic geometry existed much earlier than Pythagoras and even the Biblical King Solomon. A recent experiment by British acoustic engineer John Stuart Reid suggests this knowledge could very well date back at least to Old Egypt.

On a trip to Egypt, Reid had noticed that a granite sarcophagus in the King's Chamber of the Great Pyramid sounded particularly resonant when he hummed certain tones. It occurred to him that this may not be an accident, but actually designed as a resonant chamber by the Egyptians based on some knowledge of acoustics and cymatic patterns.

From this hypothesis, Reid asked for and received special permission from the Egyptian government to conduct a sound experiment in the pyramid. In this experiment, he stretched a thin sheet across the opening of that same sarcophagus and sprinkled sand over its surface. As he began to use his equipment to project various frequencies into the chamber to identify its resonant frequencies, various cymatic patterns began to emerge out of the sand.

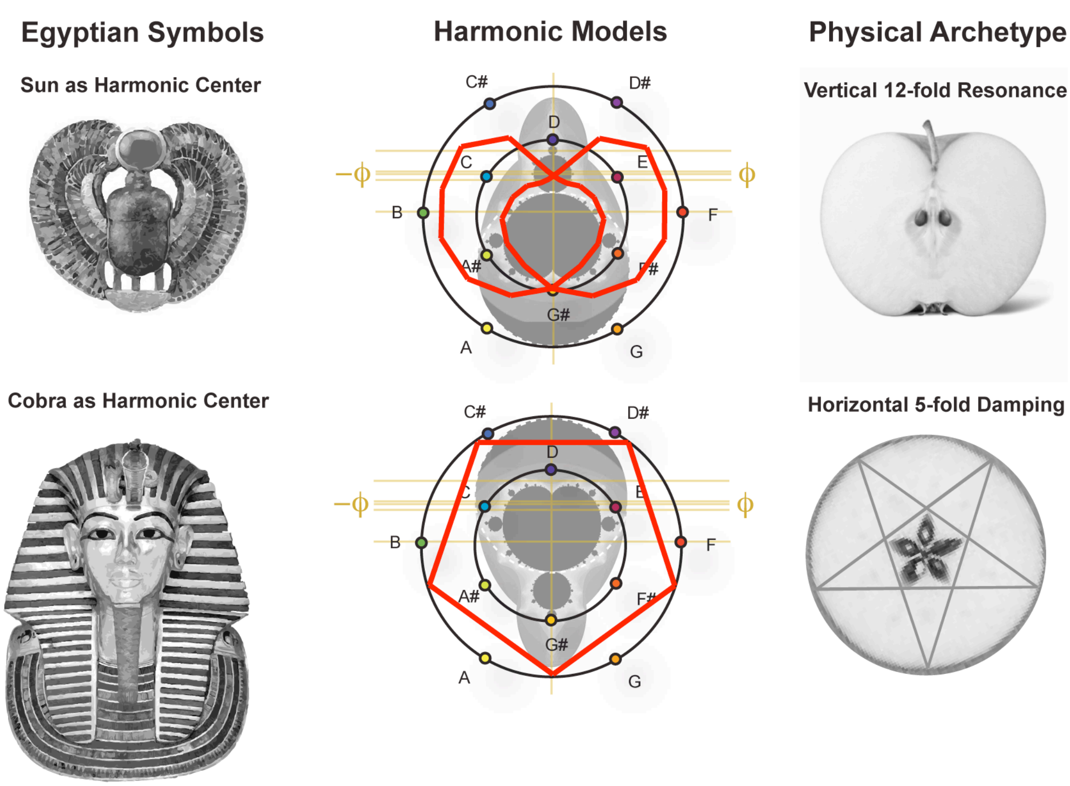
What he found was nothing less than astonishing. Reid discovered that the dimensions and materials used in the sarcophagus created resonance patterns very close to twenty different Egyptian hieroglyphic figures. The correspondence between the sound patterns and hieroglyphs was too close to dismiss, leading him to hypothesize that not only did the ancient Egyptians appear to have an advanced knowledge of cymatics but also that they seem to have partially designed their written language around it. He speculated they might have used chambers like the sarcophagus as a kind of "portal to the spirit underworld" to convert the spoken word of chants into the symbols they found in cymatic patterns [Reid 2007].

Reid claims similar correlations between cymatic patterns and iconography can be found in Tibetan, Australian aborigine and other ancient languages. This echoes the same idea proposed by Hans Jenny forty years earlier when he too noticed that the spoken vowels of the ancient languages of Hebrew and Sanskrit took the iconic shapes of sound patterns on a dusty drumhead. Researcher Stan Tenen has taken this work further and decoded quite a few characters of the Hebrew alphabet into cymatic patterns. But it seems this physics-based approach to symbolizing vowels and words disappeared some time in the past – who knows, maybe burned in the Library of Alexandria – since cymatic icons are no longer to be found in modern languages.

Written language may indeed have its roots in the cymatic geometry of harmonics formed by the spoken word. Could it be mere coincidence that the flat round "cymbals," originating from the Greek word *kumbalon* meaning 'cup', has a resemblance to a round Chladni plate? And would it also be a coincidence that the rhyming Greek word *sumbalon*, meaning 'mark' or 'token of the spoken word', is the origin of our rhyming word "symbol"? Did we once speak over a *cymbal* to find the *symbol* for our speech?

The early Greeks and the Egyptians before them apparently understood the relationship of auditory patterns to the patterns of nature. In Figure 108, two Egyptian symbols can be seen to resemble several of the models proposed by *Harmonic Interference Theory*. The beetle holding the sun (meaning "come into being") suggests the same cubic damping geometry of the Mandelbrot Set while at the same time its wings wrap around to form the heart-shape of a polar ***REFLECTIVE INTERFERENCE*** cardioid. Likewise, the Egyptian headdress appears to correspond to the geometry of a pentagon with the Mandelbrot Set turned upside down to mimic a human head with goatee. In both cases, the sun and cobra symbols are in the same relative position as the Harmonic Center {D} on the Chromatic Dual Ring.

**Figure 108 – Are Egyptian symbols evidence of harmonic knowledge?**



Admittedly, there is scant evidence to prove or disprove the assertion that these symbols were intended to represent harmonic ideals. Even the idea that an apple was once used as a physical archetype to illustrate harmonic principles cannot be proven. But it is at least highly coincidental that the standing wave resonance models proposed by *Harmonic Interference Theory* appear to correspond as well as they do. And, given the role of standing wave resonance in all life, it is certainly ironic that the scarab symbolism of *come into being* and the cobra symbol for the Egyptian *Eye of Horus* both correspond to *the rising of the kundalini*, a reference to serpentine energy currents that must flow through all life.[129]

[129] In Hindu texts the kundalini is likened to a serpent of energy that flows upward through seven chakras in the body starting at the base of the torso, or perineum. The goal of yoga meditation is then the process of "awakening the kundalini" so that it reaches the seventh chakra at the crown of the head. These Indian Hindu beliefs correspond to the Egyptian use of the cobra emerging from the third eye, symbolizing spiritual awakening and enlightenment.

# ***Resonant Nodes***

*“The Sun, with all the planets revolving around it, and depending on it, can still ripen a bunch of grapes as though it had nothing else in the Universe to do.” - Galileo Galilei*

Theoretical harmonic models based on simple acoustic principles can be very useful tools in explaining our relationship to the world around us. They help us visualize how our ears are able to “see” auditory geometries in a field of harmonic interference and how life came to reflect this organization in its physiological structure. They offer a new language with which we might now lift the veil over music harmony and discover how it really works. And like a lens into society, they bring into focus the true history of Western religion and science, revealing the anti-harmonic philosophy that now guides our world. Yet, even with all this there remains a great deal more to be learned from *Harmonic Interference Theory*.

What we actually have in these models is a musical framework within which *all* of nature can be understood. This was the essential knowledge of the ancient mystery schools as unified by Pythagoras under the musical-astrological philosophy of *musica universalis* or *Music of the Spheres*. Indeed, up until only a few hundred years ago this was still the foundation of Western thought and the founding principle in the quadrivium classroom, distorted as it was by religious doctrine. The further back in time we go, the more we find that harmonics and music were once accepted without question as the world’s Theory of Everything.

As it stands today, most scientists do not much care for the idea. Describing the spacing, size or movement of planets in terms of a musical harmony is of little practical interest to modern astronomers and astrophysicists. Instead, the scientific community promotes a process of competing hypotheses without concern for natural harmonic principles, leaving society with the idea that the universe is indeed a very mysterious and nonsensical place. Any serious consideration of harmonics as a causal factor in cosmology is viewed at best as a silly holdover from an earlier time or at worst a distraction and threat to the integrity of the scientific method.

This institutionalized avoidance of any system of unified harmonic principles has convinced everyone that we simply have no choice but to wait until some future date for a believable Theory of Everything. But the truth is we may *never* find a grand unification theory using the scientific method, leaving the world’s population with absolutely no idea that nature can actually be easily understood through the study of harmonics and musical philosophy. We might ask is this a good thing? Should we be content to live our lives within an incomplete and fragmented set of beliefs about nature and ourselves?

The most ironic thing about this state of affairs is it was precisely the old philosophy of a musical cosmos that led mankind to its greatest discoveries. It brought about huge advances in astronomy, mathematics, anatomy and physics; inspiring the search for a universal order in all things. From the pagan heliocentric cosmology of Earth revolving around the central "fire" of the Sun to the very foundations of mathematics, the Pythagorean musical worldview gave birth to the scientific method that now casts it aside.

Aristotle wrote of the Pythagoreans: "They said that the whole universe is constructed according to a musical scale." Most scholars take this to mean that the proportional spacing, velocity and size of the planets were believed to be a form of slowed down music that, when sped up, could be "heard" as melodies or chords. Many such claims have been made, from the Renaissance up thru today's New Age literature, explaining how planetary spacing and alignments (called *conjunctions*) can be interpreted as melodies or even chords. As these conjunctions occasionally form a Tonic triad {C, E, G} as orbital proportions, it is certainly hard to deny that some sort of harmonic phenomena is at work in the cosmos. But even so, with the scientific community's unwillingness to entertain harmonic cosmological theories, we are constantly led away from the possibility of a musical solar system.

Now it is true that the *Music of the Spheres* has been hijacked in the past by so many religious affiliations that its credibility is severely undermined. And it is true that it is deeply intertwined with various unsubstantiated metaphysical beliefs that claim the planets and stars act at a distance to exert influence over our lives. Nevertheless, our solar system does exhibit real harmonic laws proven long ago by solid astronomical observations and calculations.

Following in the footsteps of astronomers Nicholaus Copernicus and Tycho Brahe, Johannes Kepler spent much of his life working to prove harmonics had something to do with the organization of our solar system. While he eventually disproved his own idea that the planets are spaced according to nested Platonic solids, he did manage to arrive at three very important laws that explain planetary motion as a harmonic geometry. In his *Harmonice Mundi* (or "Harmony of the World"), Kepler put forth his final and favorite rule concerning orbits as a geometric balance between orbital frequency and distance.

> ***Kepler's Third Law:*** *The square of the period of a planet's orbit is proportional to the cube of its semi-major axis.*

This law suggests that Kepler had a much deeper knowledge of harmonic principles than we are aware of today – to the extent that he even appeared to view gravity itself as a kind of harmony. In fact, it was from this law that Carl Friedrich Gauss later derived a mathematical constant defining the pull of gravity. This constant of gravity, known today as the *Gaussian* *gravitational constant*, is calculated as a proportion between the square of one orbit around the Sun (one year) to the number of rotations (days) in an Earth year times the total mass of the Sun and Earth – all multiplied by the cube of the semi-major axis (or average radius) of the Earth's orbit. While this sounds very complex, it boils down to one rather arbitrary looking number:

k = *0.01720209895*

Though it has long since been buried inside today's standardized *astronomical unit* (AU), Gauss used this constant in all his calculations concerning gravity and planetary orbits. It begs the question of what could have led these men first to the law of planetary motion and then to such a random looking constant for gravity? Could gravity have anything to do with a much older knowledge of harmonics?

## Redefining Gravity as a Fractured Tritone

Converting the Gaussian gravitational constant into the musical language of proportions, we can begin to see how early scientists might have arrived at their discoveries through a deep understanding of *musica universalis*. Using tritone Partials 5 and 7, the same 7:5 ratio that crystallizes a standing wave into a dodecahedron solid can now be multiplied against the Rosslyn "magic ratio" to arrive at a number very close to the Gaussian gravitational constant *k*.

***Musical gravitational constant derived from a tritone***
k ≈ *Tritone ratio × Rosslyn magic ratio*
k ≈ *7 : 5 × 1/81*
k ≈ *1.4 × 0.0123456790123455*
k ≈ *0.017283951*

You might recognize this ***musical gravitational constant k*** as the Rational Standard Deviation (divided by 100) that was calculated earlier for the Gaussian ***INTERFERENCE*** equation and found in the dimensions of Rosslyn chapel. But how could gravity have anything to do with the distribution of harmonics or Solomon's Wisdom? As we will see, the forbidden tritone is probably the Great Secret behind how Kepler and Gauss actually understood our solar system.

For instance, using the musical gravitational constant *k* we can calculate the number of days in a Julian Year using a well-known equation first introduced by Gauss.

***Julian Year derived from the musical gravitational constant***
*2π /* k *+ 100*k *days = 365.25 days*

Why should this work? Why should we be able to produce the gravitational constant and number of days in an Earth year entirely from the distribution of harmonic interference? Well, it makes sense when you consider the fact that our solar system was once a spinning disc of colliding plasma that formed according to the same harmonic laws as a vibrating plate of sand. Each planet had no choice but to space according to a Gaussian "normal distribution" of gravity just like the interference pattern produced by two musical tones diverging over an octave.

## Redefining Planetary Orbits as Damping Rings

By spending only a small amount of time looking at the orbit of each planet, a familiar pattern begins to emerge. As discussed earlier, the average orbital spacing is equal to the golden ratio, suggesting the calm damping rings on a vibrated Chladni plate. But the planets are also spaced such that their adjacent orbits are balanced symmetrically either side of Jupiter. Because of this, Jupiter's semi-major axis (elliptical radius from the Sun) multiplied by the musical gravitational constant *k* creates a symmetrical number that peaks at the number 7 right in the middle of the solar system while it is also the 7$^{th}$ sphere from the center, inclusive of the Sun. This proportion represents its "gravitational damping ring."

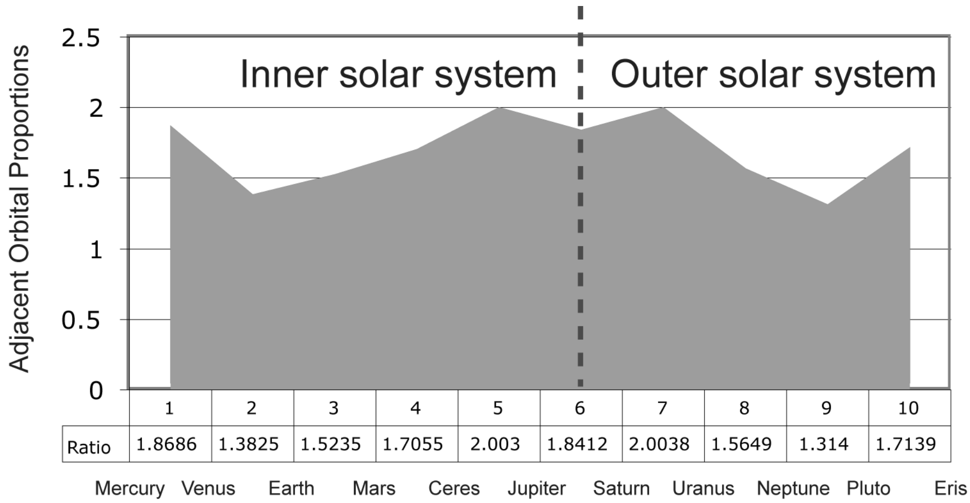
***<u>Jupiter's "gravitational damping ring"</u>***
*778.57 Mkm × k = 13.4567654321*

Can this be pure coincidence? No, not since Jupiter balances at the same approximate proportion between the semi-major axes of the inmost planet Mercury and the outermost dwarf planet Eris (with its dissonant egg-shaped orbit). Perhaps "unlucky 13" is not so unlucky after all.

***<u>Jupiter as approximate midpoint of the solar system</u>***
*Eris / Jupiter = 10123.01 Mkm / 778.57 Mkm = 13.00205505*
*Jupiter / Mercury = 778.57 Mkm / 59.91 Mkm = 13.44448282*

This near symmetry in the solar system's orbital spacing pattern becomes very clear in Figure 109, which shows the adjacent semi-major axis proportions between each of the planets, dwarf planets and the Ceres asteroid belt (a broken planet). The introduction of Earth into the solar system is the only thing that keeps this distribution from being almost perfectly symmetrical. Looking very much like the Gaussian ripple made by a stone thrown into a pond, our solar system spaces around its center near Jupiter in the same way harmonics in a musical tone vibrate around Partial 9 or chords around the SuperTonic in an octave.

**Figure 109 - Symmetrical spacing around Jupiter**




The two Jupiter constants actually happen to differ by the square of a fractional golden ratio, or $57.91 - (\Phi/10)^2 = 57.88$, making Mercury perfectly balanced just outside of its gravitational damping ring.[130] This tells us that Mercury's gravitational damping ring is about equal to Unity.

***Mercury's gravitational resonance***

*57.91 Mkm × k = 1.000913602*

Which then implies Mercury's distance from the Sun can be calculated as a function of Gaussian ***INTERFERENCE*** using Unity, *k* and Φ.

***Mercury's orbit as a special location in a field of Gaussian INTERFERENCE***

*Mercury orbit* $\approx 1 / k + 2(\Phi / 10)^2$

*57.91 Mkm (actual) ≈ 57.91 Mkm (calculated)*

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130 It also works because the metric system was designed based on natural measures, such as the Earth and water. In the late 18th century, Louis XVI of France engaged a number of "experts" to develop a unified, natural and universal system of measurement to replace the old systems in use. The measure used to develop the meter, thus the kilometer, was the circumference of the Earth. The kilometer was then taken as 1/40,000th of the Earth's circumference which happens to share an orbital proportion with Mercury:

Earth orbit / Earth circumference = 149.6 Mkm / 6378.1 km = 0.023455
Mercury orbit / Mercury circumference = 57.91 Mkm / 2439.7 km = 0.023736

This is due to the fact that the circumference (and thus radii) of Earth and Mercury form a proportion near the square of the golden ratio, which is, of course, central to the Gaussian INTERFERENCE distribution.

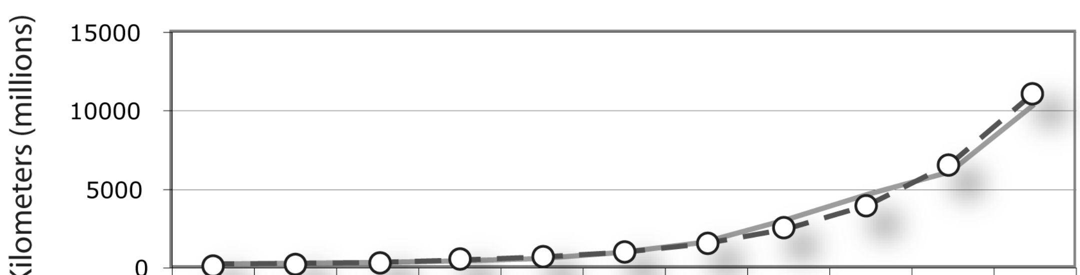
Earth circumference / Mercury circumference = 2.6168 ≈ $\Phi^2$

In this way, Mercury's orbit is proven to be the result of harmonic interference while its "gravitational constant" represents a common factor in each of the other solar orbits. In fact, its orbit is very close to the Landau damping proportion ($12^{-3} \times 10$) found earlier, which as you may recall is the "half twist" needed for any standing wave to form. In the standing wave of our solar system, the Sun acts as the Harmonic Center while Mercury's orbit represents the space necessary for planets to "share" gravitational energy with the Sun.

Even though the field of astrophysics does not yet have a universally accepted theory for the formation of the planets, it seems clear that the same Gaussian interference found on a vibrating Chladni plate or even a guitar string correctly describes the natural organizing principles of our solar system. If this sounds improbable to you, then you will also find it improbable to learn that the orbits of the entire solar system can be calculated mathematically with *no physical measurements whatsoever* and within a cumulative average variance of only 764 kilometers, or about 474 miles. All we need to do is construct the solar system starting at Mercury's theoretical orbit and work our way outward by multiplying each preceding planet's calculated orbit by a specific constant. Care to guess what that constant is? (Here is a hint: it is a number very close to the resonant proportion found in Rosslyn chapel – just above the "holy of holies" portal).

Figure 110 shows the actual vs. theoretical distances of each planet's distance from the Sun using what we will call an "enlightened sixth," derived from the Rosslyn magic ratio.

**Figure 110 - The Gaussian *INTERFERENCE* solar system model**




| | Mercury | Venus | Earth | Mars | Ceres | Jupiter | Saturn | Uranus | Neptune | Pluto | Eris |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Actual* | 57.91 | 108.21 | 149.6 | 227.92 | 388.71 | 778.57 | 1433.53 | 2872.46 | 4495.06 | 5906.38 | 10123 |
| Calculated | 57.8571 | 97.6479 | 164.804 | 278.147 | 469.441 | 792.295 | 1337.19 | 2256.83 | 3808.95 | 6428.52 | 10849.7 |

* Source: NASA

Actual vs. Theoretical Orbital Distances from the Sun

| | |
|---|---|
| Average Variance (kilometers) | 764.8626164 |
| Average Margin of Error (as percentage of Eris semi-major axis) | 0.00000705% |
| Planetary spacing constant | 1.687741698 |

***<u>"Enlightened sixth" Spacing Constant</u>***
*(43267 × Rosslyn magic ratio)$^{1/12}$ = 1.687741698*

Now I ask you – how could Rosslyn chapel have incorporated a ratio found in the average spacing of the planets when no one in the 15$^{th}$ century knew about the outer planets, much less their orbital spacing? The only answer, of course, is they understood harmonic principles of sound found elsewhere in nature and based their design on this. The fact that the central spacing constant for the solar system actually relates to harmonic damping is a testament to the universality of harmonic phenomena at all levels and scales of nature.

As it applies to the solar system, the spacing of the planets is really quite simple. Just calculate Mercury's theoretical orbit from the musical gravitational constant *k* then multiply each previously calculated orbit by the "enlightened sixth" ratio of Pluto:Neptune in order to reach the next orbit[131]. Doing this repeatedly produces a theoretical model of the solar system within a small margin of error when averaged across all the planets. And in spite of the fact that each orbit is not exact, the variance pattern does follow a predictable sinusoidal waveform within a Bessel envelope (also called cylindrical harmonics), again like that found on a round Chladni plate.

In Figure 111, this model of the solar system can be seen to balance all of the planets around Jupiter, spacing the planets just like an equal-tempered piano keyboard tuned to a "solar semitone" of about $534.35^{1/12}$ instead of the musical semitone $2^{1/12}$. The difference between a solar semitone and a musical equal-tempered semitone is then very close to an 8:5 ratio, or the scaling proportion of a musical minor 6$^{th}$. Furthermore, the solar semitone is equal to 1.6877, very close to the Rosslyn chapel height-to-breadth ratio of 68:40.5 or 1.679012346. It would appear that the music being played by the Rosslyn angels, translated from cymatic patterns into the Rosslyn Motet, was quite literally the *Music of the Spheres*.

When the musical scale and synesthetic color model is applied to this solar piano keyboard, we find several other intriguing musical correlations, including:

- *The planets orbit around the Sun as the Harmonic Center and space logarithmically around Jupiter,*
- *Venus is identified as the Tonic or fundamental frequency of the solar system,*
- *Earth becomes the Leading Tone pulling up to Venus with the solar golden ratio located in between in a corresponding aquamarine region,*
- *The Sun and Jupiter form the Harmonic Axis while Earth and Neptune represent a diatonic tritone,*
- *The planets inside Jupiter's orbit should pull inward toward the Sun while the planets outside of Jupiter's orbit should push outward toward Eris,*
- *Eris represents a counterbalance to Mercury that completes the solar octave.*

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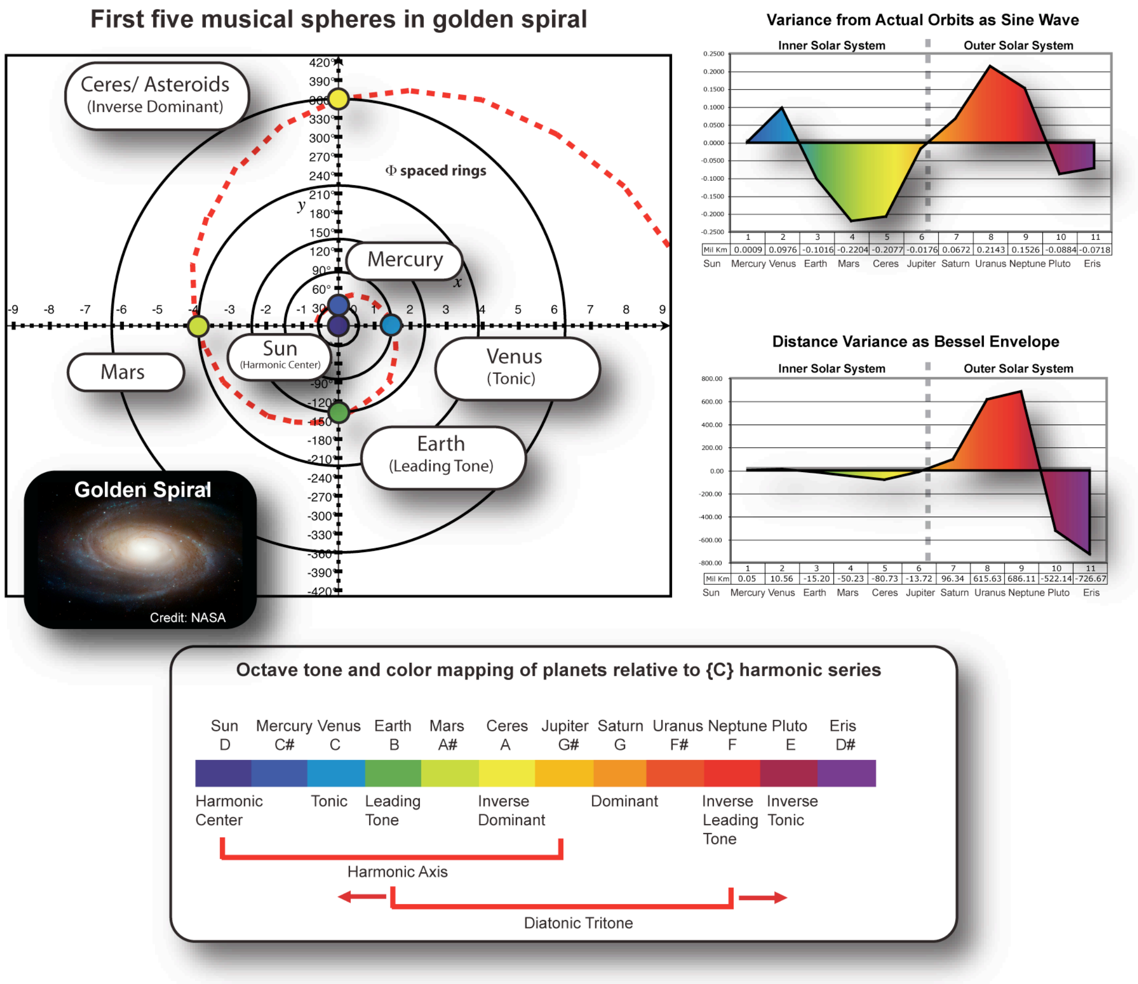
131 Using the standard deviation of the INTERFERENCE distribution.

## Figure 111 – Music of the Spheres archetype of the solar system




From this simple orbital model, it appears that the legendary *Music of the Spheres* could have once been a real theoretical model of the solar system. Its basic architecture could have been easily estimated from a combination of the Devil's tritone, the Rosslyn "magic ratio," John Dee's "secret ratio" and (of course) the golden ratio. But unlike Newton's gravity equation that requires a complex mass variable or Gauss's gravity constant that demands knowledge of the days in an Earth year and distance from the Sun, this musical solar system model estimates both the gravity constant and planetary spacing <u>*geometrically*</u> based on a consistent distribution around a recursive Φ-spaced ring pattern. No physical knowledge of the solar system is required since the entire solar system can be modeled as a giant vibrating string, just as Robert Fludd had suggested centuries earlier in his *De Musica Mundana* and *celestial monochord*.

But unlike Fludd's rather occult approach that includes metaphysical speculations, we now have credible (if not universally accepted) scientific theories, including plasma cosmology and torsion physics, to help us describe how the planets must have formed out of the swirling interference of plasma gases. The story goes something like this (see Figure 111).

When the Coriolis Effect is perfectly balanced and most coherent, two *golden spirals* (or *Spira Mirabilis*[132]) begin to form. The golden spirals, irrational versions of our Fibonacci growth spirals, are logarithmic spirals that get wider from the origin by a constant factor of the golden ratio for every quarter-turn. This can still be seen today in our solar system as the double golden spiral of plasma emitted by the Sun known as the *heliospheric current sheet*. It is along these damping spirals at approximate quarter turns that the planets settled into their stable orbits, resonating elliptically just outside of a series of concentric Φ-spaced rings.

As the golden spirals damped down the harmonic resonance between planets – performing the same function the Fibonacci series and Φ-damping locations do in a musical octave – the planets resonated into spherical nodes. And as they did so, their relative sizes were determined by the same physics that determines resonant gap size as two tones diverge over a musical octave.

## Redefining Planetary Size as Resonant Nodes

If we think of the size of each planet as representing its velocity in a Gaussian distribution of plasma wave interference, we can then consider relative size to be an indication of harmonic proportions in the solar system. However, for this to be visible as the familiar Gaussian ***INTERFERENCE*** curve, we need to multiply each of the last six planets by the gravitational constant *k*. This is required to adjust for a sudden symmetrical reduction in the Sun's energy flow to the outer planets. This well-known astronomical phenomenon, known as the *frost line*, represents a "half twist" reduction in plasma density that occurs just inside Jupiter's orbit at the Harmonic Center of the solar system.

In Figure 112, a graph of the planets according to size clearly approximates a spatial Gaussian curve. In spite of the many random events that must have influenced the formation of the planets, this graph shows that relative planetary size follows a general distribution of harmonic resonance very similar to that of the octave ***INTERFERENCE*** function. Under this model, each planet would have formed as a node near its Φ-damping ring by accumulating whatever amount of material was allowed in the exchange of plasma harmonic waves bouncing between the Sun and the outer edge of the solar disc. When we adjust down the size of the outer planets by a factor of *k* to match the higher solar energy level inside the middle frost line, we find

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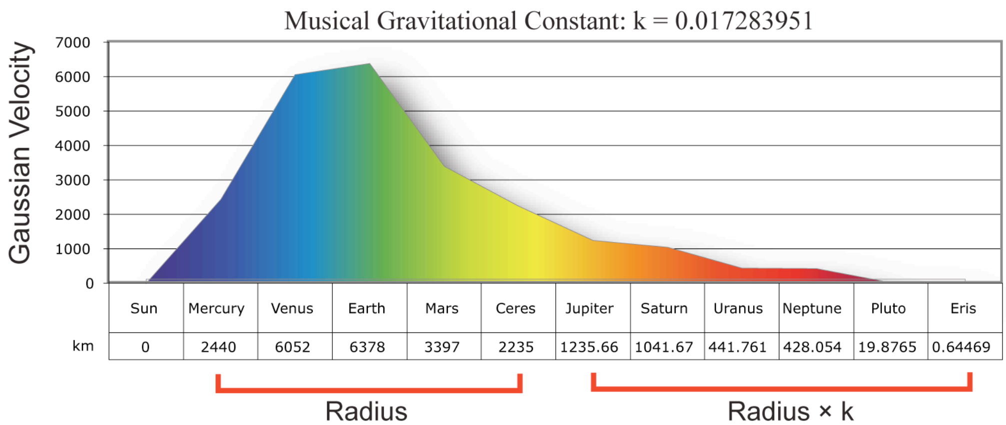
132 Spira Mirabilis means "marvelous spiral," named by Jakob Bernoulli in the 17th century.

that the planet with greatest resonant velocity relative to the Sun is none other than our Earth – the Green Leading Tone of the solar system.

**Figure 112 - Planetary size as a quasi-Gaussian resonance distribution**

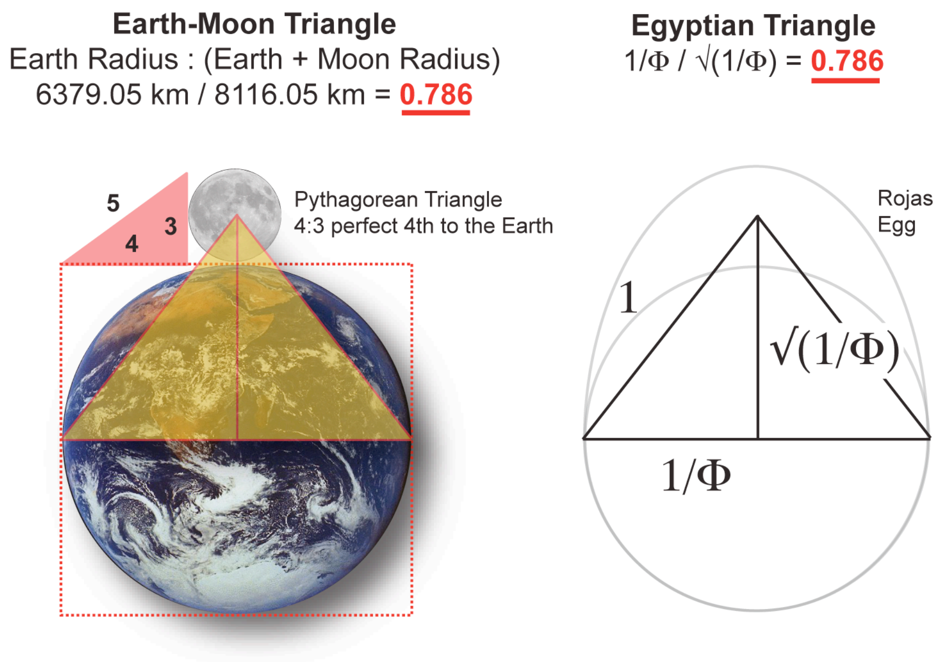



| | Sun | Mercury | Venus | Earth | Mars | Ceres | Jupiter | Saturn | Uranus | Neptune | Pluto | Eris |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| km | 0 | 2440 | 6052 | 6378 | 3397 | 2235 | 1235.66 | 1041.67 | 441.761 | 428.054 | 19.8765 | 0.64469 |

It was at this maximally resonant position in the solar disc where the Earth and Moon spun out of a smaller plasma hurricane some four and one-half billion years ago. The scale model in Figure 113 shows how their relative sizes probably developed as part of an egg-like glob that eventually hatched to form the two spheres we see today. Incredibly, we can still measure this egg using an *Egyptian Triangle*, first employed by the Egyptians in the construction of the Great Pyramid of Cheops. It is a fact that if the Moon were touching the Earth, their combined radii would form this exact same pyramid having a base-to-height ratio of 0.786. Around this would fit the harmonic geometry of an egg, providing a natural damping container for 12-fold resonance of both spheres inside. It is probably easier to understand this in musical terms.

The same Pythagorean Triangle found earlier in the spectral analysis of an octave can be used to describe the relationship between the Earth and Moon as a 4:3 perfect $4^{th}$. This stunning Earth-Moon harmony extends all the way to the Sun, mimicking the 0.40 length factor of the Solomon Key harmonic model. We see this in every solar eclipse when the Moon, at $1/400^{th}$ the size of the Sun, perfectly blocks out the Sun from the Earth, which is spaced precisely 400 Moon units from the Sun. Now *that* is harmony!

**Figure 113 – Harmonic pyramid archetype for the Earth-Moon system**




This is the true meaning of the *Music of the Spheres*. Long discredited by conventional astrophysics, harmonics not only provide a unifying framework to explain our solar system, but also the coherence necessary for the emergence of life on Earth. Known long ago by ancient musical astronomers, we are only now beginning to rediscover the real physical connection between cymatics, numbers, geometry and astronomy. Music was once mankind's laboratory and harmonic science the world's Theory of Everything.

Preserved in fragments by countless theosophical orders and secret societies over the centuries, most of the ideas attributed to science have their origin in this forbidden *musica universalis*. Discovered first perhaps in the crystal of an apple or quasi-crystal of an egg – known at least as far back as the Sumerians and passed down through Egyptian Hermetic currents, the Hebrew Kabbalah and countless Gnostic adepts – it was the harmonic science of 12-fold resonance and 5-fold damping that found its way from Solomon into Rosslyn chapel more than one hundred years before Kepler was born.

And what about the Sun? What does it have to do with the *Music of the Spheres*?

When the Sun's radius is multiplied by the musical gravitational constant, you will find it to be the "central fire" of 12-fold resonance in our Gaussian solar octave.

***The Sun's gravitational resonance***

*Sun's radius* × k = *695,000 km* × k = *12012.345*

As found earlier in the standard deviation over an octave, the Sun represents the pivotal $X = 12$ location that balances at π in a 2π solar system (off by only a factor of the Rosslyn magic ratio!). But this is not all - twelve appears again as the Sun resonates (like a cosmic palindrome) with its first planet Mercury.

***Sun radius : Mercury orbit resonance***

*695,000 km / 57.91 Mkm = 0.01200138145 ≈ 12/1000 ≈* $0.034643^2$

As the Sun's energy resonates outward to the planets, we see that it is Mercury's egg-shaped orbit that brings stability to the solar standing wave. Plugging this back into the ***INTERFERENCE*** function, the result 5.0666 is a perfect match for the velocity and maximum damping effect of the golden ratio (Principle 7).

$y = 3.4643^2 / (\Phi^{3.4643} / \sqrt{5}) = \mathbf{5.0666}$ // **INTERFERENCE** *maximum damping.*

In this way, the harmony of the Sun's 12-fold resonance inside Mercury's 5-fold eggshell emanates outward into the solar system, achieving its maximum potential in the resonant shape of Earth's beautiful 12:5 life forms. And just as we find it in the core of an apple, the Sun bears its pentagonal seeds through the bright *Star in the East*, materializing in the calm aquamarine space between the green Earth and the sky blue "goddess of loves."

# *Venus Five*

*"In some sense man is a microcosm of the universe; therefore what man is, is a clue to the universe. We are enfolded in the universe." - David Bohm*

Whether unlocked by the Key of Solomon, contained by the golden apple of Eris or supported by Atlas in the mythical Underworld, the knowledge of harmonic damping as the support system for all resonant structure was the foundation of Greco-Roman mythology and most other ancient beliefs. As the personification of harmonic principles, mythology was a way to apply the coherence of music to the questions of social order and life's purpose. As a philosophy, it successfully unified Science and Art while guiding society toward a common project. Today, music still remains at the center of our modern mythology, though now in a *very* different way.

Instead of having philosophers carefully design our mythology from nature's model, we have entertainment and media companies inventing pop myths. Instead of idolizing the geometry of pentagonal stars, we idolize the fame of movie stars and rock stars. In place of an awareness of coherence in nature, we assert and piously defend conflicting beliefs in the supernatural and scientism. Instead of a God in Nature (and our 'inmost chamber'), we have a God that lives in Heaven, leaving many to live in a decidedly undecided state concerning matters of spirituality and meaning. We have to ask: given the institutionalized separation of mankind from nature, can society ever hope to regain its old worldview founded on harmonic principles and, if so, where would we even begin?

Let us begin where Pythagoras began. Let us return to the heart of the quadrivium, before it was tainted by the Roman Empire. Let us walk through the desert where the sand is carved by resonance into the pyramids and where wisdom is found in nature's archetype of love.

To the Pythagoreans, the planet Venus and the pentagram were one and the same – both represented order in nature. Venus, with its 13:8 orbital proportion to Earth, always traces out a pentagram in the sky over an 8-year period, bringing hope of stability, coherence and purpose throughout the cosmos.[133]

As it was explained to society in the stories of Greek mythology, this stabilizing effect became the beauty and love represented by the goddess Aphrodite. Renamed from Aphrodite to Venus by the Romans, it was associated with the planet of the same name and widely known as the *Morning Star,* the *Day Star*, the *Star in the East* and finally the *Star of Bethlehem*. The

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[133] Described earlier in Section One, Harmonic Geometry.

Biblical story of three wise men that journeyed to visited Jesus were not physically following a star to find him – they were *followers of the star* of Venus and its harmonic pentagram.

As the brightest object in the sky other than the Sun and Moon, the planet Venus was known in Latin as "Lucifer," the "light-bringer." But in those days, there was nothing evil or Satanic meant by the name Lucifer. This idea came much later with the Church's crusade against pre-Christian theosophies founded on the pentagram, such as Pythagoreanism, Mithraism and Zoroasterianism. Venus once represented knowledge of the golden ratio, harmonic science and *musica universalis*. Far from being evil, the meaning of Venus was enlightenment, beauty, wisdom and a love for all things.

As example, the love of Venus was personified in stories of her giving birth to Eros, leader of the cherubs, that became Romanized to the god of love Cupid. We still celebrate Venus, "the mother of love," on Valentines Day by honoring her cherubic lovechild Cupid, who hides just behind the Church's cover of St. Valentine's Day to shoot an arrow through our heart.

But Venus has another important secret of nature's love of harmony to share with us. During her orbit around the Sun, there is a little bit of slack that stretches a circle into an ellipse. As a small orbital eccentricity, this slack is none other than .007, John Dee's secret ratio of harmonic damping. As the harmonic archetype for the cherubs, Venus keeps this little lovechild buried deep in the middle of the 13:8 pentagonal ratio between the orbital velocities of Venus and Earth:

$$\pi \times (13{:}8 - .007) - \sqrt{.007} \rightarrow 5 \qquad \text{// the Earth-Venus pentagram}$$

…or in more compact terms:

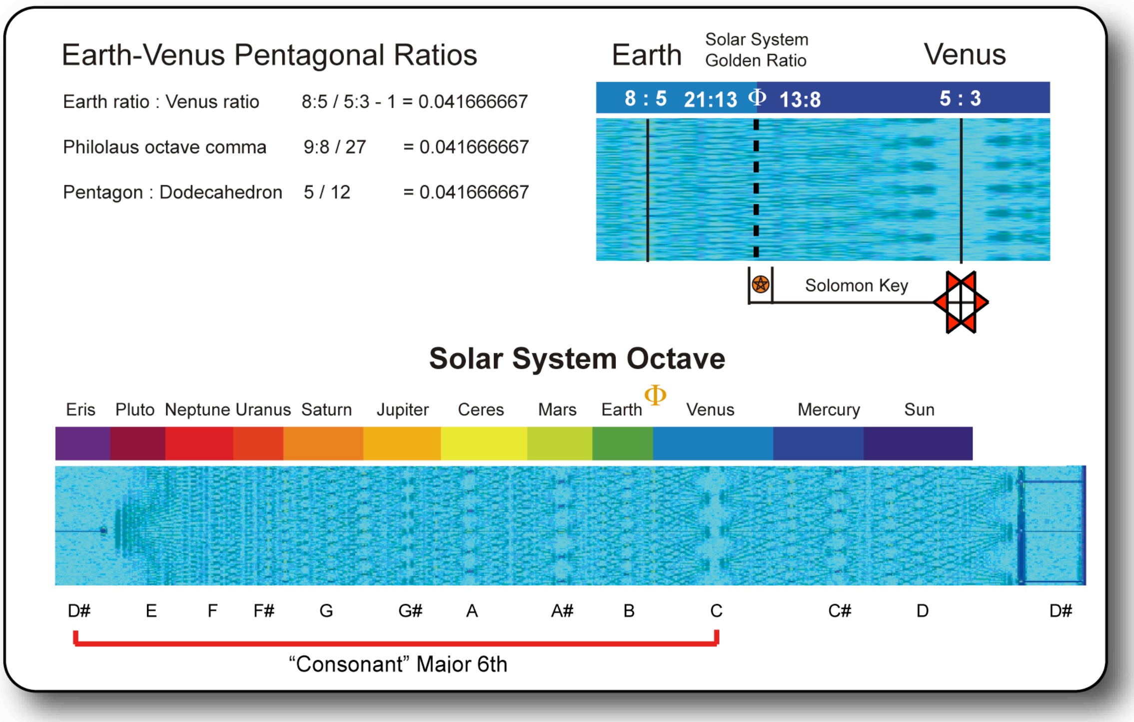
$$\pi \times \Phi - \sqrt{.007} \rightarrow 5$$

Venus, Galileo's own "goddess of loves," is nothing more than the social archetype for the 5-fold pentagonal astronomical harmony between Venus and the Earth. And her love for Earth is none other than a golden section in space that brought us the stability necessary for life. In this regard, ancient philosophers saw love in the harmony between π and Φ.

We see this idea expressed in Figure 114. The solar octave is measured logarithmically from Eris (instead of the Sun) to illustrate Venus as a resonant major 6th interval. So while Earth is the most resonant interval relative to the Sun (a minor 3rd or inverted major 6th), Venus is the most resonant from Eris, the outer edge of the solar system.

Between Earth and Venus lies a Φ-damping region just like that in a musical octave (represented by the proportional color of blue-green or aquamarine). Starting at the 13:8 ratio between Venus and Earth, a Fibonacci spiral continues to dive into this aquamarine region toward the solar golden ratio, thus stabilizing the Earth-Venus pair and harmonizing the solar system.

**Figure 114 - Origin of the Earth-Venus pentagram**




In looking at the figure, you may have noticed a difference of 1 between the Earth-Venus ratio and wondered why this is needed. It can be explained as an octave inversion from the Sun. For instance, using *D* as the distance between Earth and Venus, equal to *41.39 million kilometers*, we have the stunningly simple orbital relationship:

*Earth orbit* / D = (*Venus orbit* / D) + 1

Even if mathematics is not your favorite thing, the simplicity of this equation alone should be convincing evidence that the solar system is really based on very simple harmonic principles. It demonstrates how Earth and Venus are spaced from the Sun as a ratio of their distance from one another *plus 1 or Unity*, which is a function of the musical gravitational constant *k*. We can show this by converting the Earth-Venus orbital ratio into musical gravity units as $k \times 80 = 1.382$ and then represent this in terms of the golden ratio as $1 / (\Phi / \sqrt{5})$.

***Earth-Venus pentagram resulting from the square root of five***

*Earth orbit / Venus orbit ≈ 1 / (Φ / √5)*

*149.6 / 108.21 ≈ 1.381969418*

*1.382496997 ≈ 1.381969418* *// 0.038% variance*

Here we see how the golden ratio and square root of five define the open space between Earth and Venus. This very important (yet apparently little known) fact reveals the unique balance of Earth in the solar system near the stabilizing proportion of the golden ratio. This is proven further by calculating both orbits using only Φ and the interplanetary distance *D*.

*Venus orbit* ≈ D × $\Phi^2$
*Earth orbit* ≈ D × $(\Phi^2 + 1)$ *// 0.13% variance*

Now if this were still not enough to convince anyone that the solar system exhibits the same resonance and damping properties of a musical octave, the distance *D* between Earth and Venus can be used as a factor with the 7:5 tritone to easily calculate the Harmonic Axis between Mercury and Saturn.

*Mercury orbit* ≈ D × *7:5*
*Saturn orbit* ≈ D × *7:5 × (13 × 80)* *// a variant of* k *× 80 = 1.382.*

So, while Earth and Venus share a golden ratio damping-factor in their distance from the Sun, Mercury and Saturn exhibit a tritonal symmetry based on this same distance *D*. It is really very difficult to ignore the central role of Φ-damping in setting these and other planetary orbits, yet it is not a part of conventional astrophysics.

To be sure, none of this is a coincidence or "playing with numbers." It is not the result of an overactive imagination or the reading of medieval texts. Yet neither can it be found in the many scholarly journals, encyclopedic websites and advanced physics textbooks that form our modern scientific worldview. It comes instead from the rather mundane and common sense idea that our solar system swirled into existence as a dual ring plasma standing wave, resonating into spheres that were spaced and sized harmonically according to a Gaussian ***REFLECTIVE INTERFERENCE*** distribution.

Given the starting conditions of the solar system, the planets had no alternative but to space along a golden spiral. They had no "choice" in sizing themselves according to their resonant velocity locations. And no choice in attaining orbital speeds that perfectly balanced their size and distance from the Sun. They were simply following the physics of gravitational harmonics that crystallize light energy into form inside a geometrically structured and polarized space.

## Redefining Life as a 12:5 Interference Pattern

At last count, how many times has the number 5 been mentioned in this book? Pythagoras used it to construct his tetrachord genera system for Greek modes. It was found to play a key role in the ***INTERFERENCE*** distribution over an octave. It was central to the discussion of energy-form equivalence in the pentagonal/tritone crystallization experiment. It formed a rational coherent pathway for the recognition of energy exchange across Φ-damping locations in a standing wave. And we found it recursing from a large pentagon into a small pentagon in a cymatic sound pattern and appeared in the twelve 'treasuries' of Metatron's Cube. It even found its way into the very structure of our solar system, providing the stability needed for Earth to resonate into its special position. The number 5 is obviously an important number in nature.

So, we should not be too surprised to find that it is the real reason harmonic wave interference settles into twelve locations. In Figure 115, we see why this is the case as harmonic Partial 5 touches the fundamental wave at exactly twelve equally spaced locations. Each of these corresponds to a small convergent bundle with other harmonics in the interference pattern of a standing wave, but only Partial 5 equally divides the fundamental twelve times in a $2\pi$ cycle. At the octave level in the Harmonic Hierarchy, this aligns with the twelve tones of an equal-tempered octave. This 12:5 physical alignment in a standing wave is the ultimate cause of all spatial and temporal coherence in nature, including our ability to enjoy harmonic music.

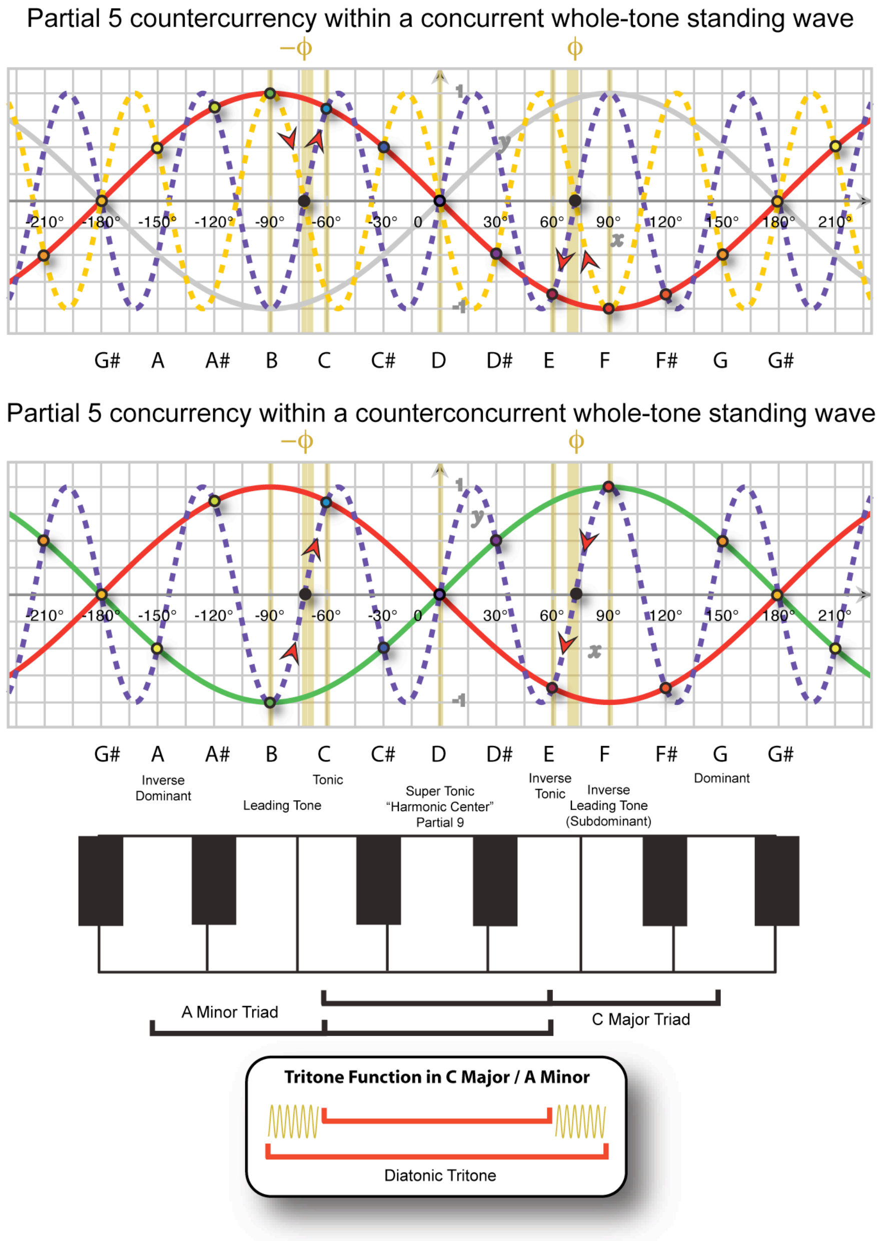
In the top diagram of the figure, Partial 5 is shown synchronized and traveling *concurrently* in the same direction as the fundamental wave. As mentioned earlier, cells also grow concurrently due to carbon-12 water crystals resonating outward one on top of the other.

In the bottom diagram, Partial 5 runs *countercurrently* or in opposing directions to the fundamental. And as demonstrated in our earlier thought experiment, this is the kind of regular alignment that results from minerals crystallizing under pressure into regular geometries inside a lattice structure, like an octahedral diamond or hexagonal quartz crystal.

In both animate and inanimate matter, Partial 5 acts as the rational equivalent to the golden ratio by beginning the damping (or de-resonating) process in a standing wave. When added to Partial 8 corresponding to the third octave, Partial 5 triggers the Fibonacci convergence to Φ by creating the first enharmonic Partial 13. It is this Partial 13 that is the "unlucky 13" and Fibonacci ratio 8:5 responsible for initiating the clamp down or *retrograde pressure* against resonance of all kinds.

The retrograde influence of Partial 5 can be found all around us. In music, Partial 5 corresponds to the Inverse Tonic of a minor diatonic scale, creating the ethos of melancholy or sadness. In organic chemistry, Partial 5 damps down light absorption as the red or magenta color of anthrocyanin in plants, which (as an antioxidant) filters ultraviolet light to slow the oxidation of other molecules.

## Figure 115 - The coherent pathways of Partial 5



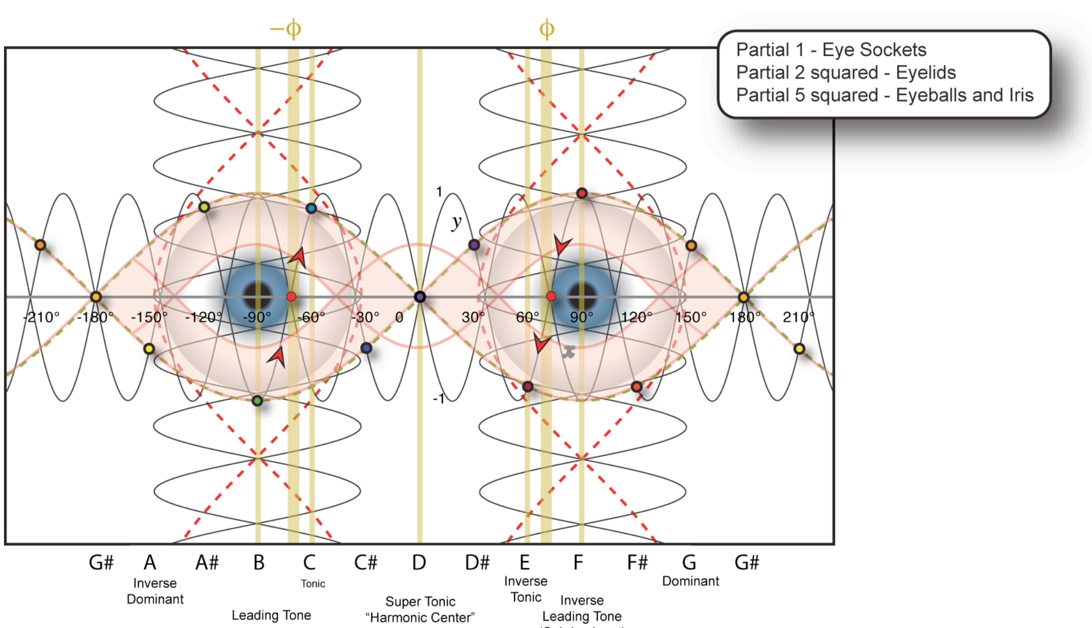

Partial 5 is found in the pentagonal Earth-Venus geometry, but also in the proportion between the outermost planet Eris and the Earth, creating a celestial major 3rd interval (fifth wave partial in the harmonic series) in the Gaussian solar system model. In one way or another, the frequency proportion of Partial 5 appears at the center of all of the negative or retrograde directional currents in any harmonic standing wave. However, it is a *very good* thing that this pressure exists in nature because it keeps matter from over-resonating and flying apart. In mythology and metaphor, the physics of Partial 5 becomes the loving embrace of Venus bringing order and stability to Earth.

Given the universal effect of Partial 5, it is not unreasonable to expect that it would also have an equally important role in the formation of biological life. Take, for example, the anatomy of two eyes represented geometrically in Figure 116 as the intersection of horizontal and vertical harmonic waves aligned into a "squared circle" interference pattern. In this orthogonal (or right angle) configuration, harmonics are able to coexist and structurally reinforce one other in space.

The crossings of the large fundamental waves naturally form a circular area within which a smaller circle circumscribed by Partial 5 can balance. These two concentric circles appear as an eyeball and iris, doubled in a standing wave to create a simple harmonic archetype for vision. Within this, Partial 2 then provides the proportion of eyelids to enclose and protect the eyes. Could our eyes be a kind of crystallized standing wave?

The idea that harmonic processes are at work in the human body is not idle speculation. Consider the fact that Partial 5 crosses near Φ-damping locations at the *fovea centralis* (the Red "blind spot" locations) at the back of the eye – precisely where the optic nerve exits through the skull and into the brain. If harmonic processes are not involved here, then how do we explain such "coincidences?" Is it natural selection from millions of mutant life forms that caused this – all but one born with eyes that did not work? If so, where is the fossil record to confirm this?

**Figure 116 - Squared standing wave of Partial 5 as archetype for vision**



We must recognize that energy-form equivalence is not simply a process of transformation from one form to the other, but something that exists *simultaneously* to coherently guide cellular growth from the molecular and atomic levels up into higher forms. To form eyes, wave energy must intersect constructively and at right angles to form concentric rings that crystallize under the pressure of Partial 5 and its stabilizing golden ratio. In this one simple example, we can begin to see how Erwin Schrödinger's quantum wave function influences the macro structure of life.

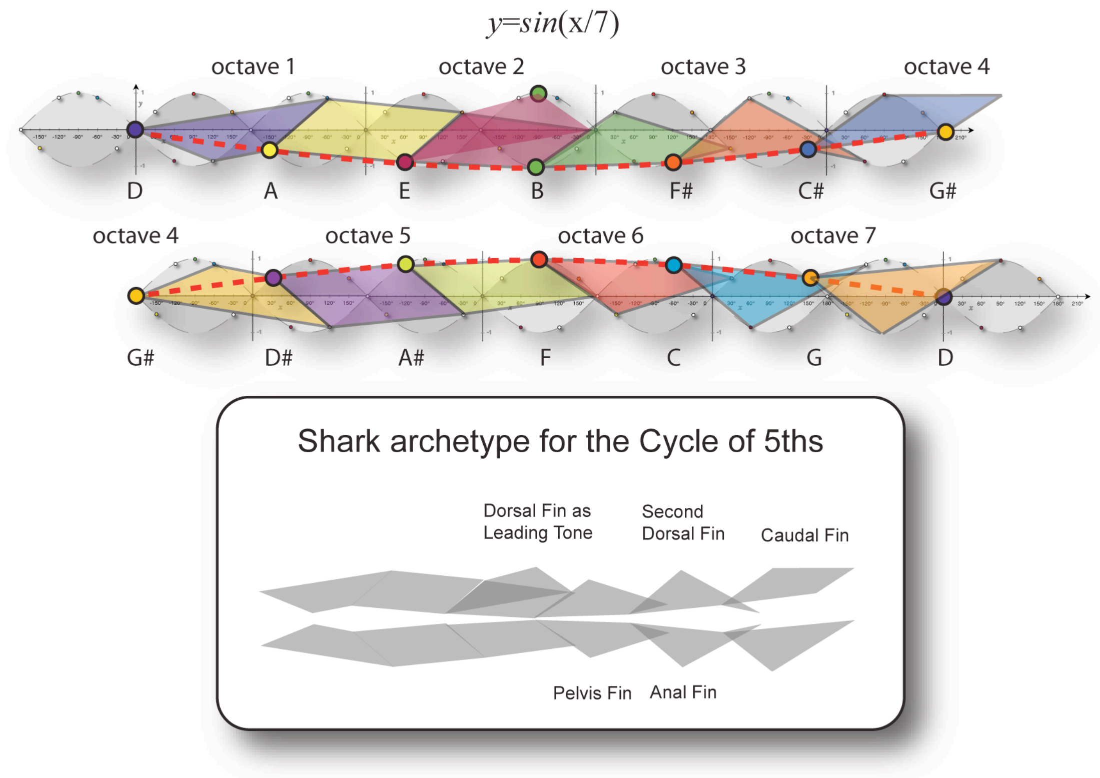
Many other examples exist to show how life organizes itself harmonically. For instance, the musical Cycle of 5ths also exhibits the 12:5 proportion found earlier in DNA. As we know, each perfect $5^{th}$ in this popular chord progression creates a 3:2 ratio in the harmonic series which, when stacked as twelve equal-tempered perfect 5ths, creates a cycle at the seventh octave. Since an equal-tempered Cycle of 5ths coincides with the convergent locations of the harmonic series over seven octaves, it also aligns with each of the twelve convergent nodes of Partial 5. This creates the same coherent pathway through the harmonic interference pattern – just at a slower frequency.

In fact, we find that the Cycle of 5ths is an inverse harmonic of the fundamental that has been slowed down to one-seventh of its oscillation speed.[134] We could even say that it acts like a wave frequency lying *below our auditory range* and could be recognized as a pattern matched against Partial 5. If this is indeed how we recognize the Cycle of 5ths, then it must appear to our harmonic brain as a coherent sinusoidal pathway intersecting all twelve tones over a 7-octave range and coincident with Partial 5 (as shown in Figure 117 with the cycle cut in half to fit on the page).

The long sweeping wave of the Cycle of 5ths is shown in the figure by the Red dashed line that connects each of the Partial 5 intercepts on the gray fundamental wave. In common practice, each tone in the cycle would represent a triad (triangular auditory geometry) with a Dominant $7^{th}$ added to form a tritone, therein strengthening the pull to the next chord in the cycle. Since the two halves (upper and lower) are a tritone apart, each chord then stacks with its mate vertically to create a jazz tritone substitution, strengthening and amplifying the "magnetic pull" even more into the next chord. Other than the double Green Leading Tone {B} mixing with the Magenta chord sticking up in the middle, the two halves are nearly exact reflections and create a pattern very familiar to jazz pianists.

[134] This is proven by the fact that the Cycle of 5ths creates a 7:12 ratio with the 12-octave range of the $2^{12^n/12}$ Harmonic Hierarchy for n=2 just as the 7 semitones comprising a perfect $5^{th}$ interval create a 7:12 ratio for n=1. In other words, one perfect $5^{th}$ represents a 7:12 ratio at level 4 of the harmonic hierarchy while twelve perfect 5ths represents a 7:12 ratio at level 5. Therefore, moving stepwise chromatically from Dominant to Tonic following Partial 5 in an octave is the very same proportionally as following the Cycle of 5ths inside a 12-octave range. They are both the same coherent pathway for n=1 and n=2 levels in the harmonic hierarchy.

**Figure 117 – Jazz Cycle of 5ths as the coherent pathway of a shark**




This figure is actually a good tool to help explain the basics of jazz harmony. A jazz pianist would play this figure in both hands by stepping through each vertical pair of chords right to left until reaching the left end – then flipping the pairs over, continue stepping right to left until the full Orbit of 5ths has been traversed back to the starting chord. As a single coherent structure, this ubiquitous musical progression just happens to match the wave-like *shape of a fish*. But not just any fish either – the taut angular profile of a shark.

It is not hard to imagine that as fish evolved in their wave-laden environment, they had to gracefully travel through an interference pattern of harmonic currents. These ripples, acting on the carbon-12 atomic structure of aquatic life, could have easily carved out the 7× wavelength of the Cycle of 5ths as exemplified in this shark archetype.

In music, chords traveling along the Cycle of 5ths must penetrate a similar interference pattern in the semi-crystallized harmonic structure of our brain, crossing the very same currents of wave partials as they exchange energy. Given the predominance of the Cycle of 5ths in music history and the "tensegrity" of the Dominant 7$^{th}$ chords here, is it really any surprise that the same robust musical geometry people most prefer would exist in the fiercest and most survivable fish in the ocean?

From this, we might expect that other musical orbits would describe different species of aquatic animals just as well. Smaller fish might fit the coherent orbits of 4ths, 6ths and 3rds, scaled up and down at various frequencies to account for size. Compound coherent pathways of diatonic major-minor scales and the Greek modes would appear perhaps as more elaborate variations on the fish theme. For that matter, ring models already match the harmonic geometry of more primitive sea creatures, such as the concentric circles of jellyfish and (of course) pentagonal starfish. The Cycle of 5ths is just one of many such examples of energy-form equivalence manifested in organic form through Partial 5.

But if all this is true and harmonic interference does guide cell organization in life, what role does evolution then play in shaping life? And why are harmonics not a part of evolution? Are the striped furs of tigers and zebras a result of natural selection to help them hide in the bush or are they really patterns of resonance? Are the rainbow colors of a peacock or parrot evidence of random mutations to attract mates or simply variants of a universal bio-harmonic process? Do the skeletal structures of reptiles, fish, birds, amphibians, mammals and even dinosaurs all follow a common morphology because "that's just the way it turned out" or is it because "that's the only way it *could* turn out?'

In today's science, there is absolutely no recognition of harmonic principles in the theory of evolution. Under a continuing *complicity of convenience* with Western religion, the possibility of harmonics at work in life is still strictly taboo and remains mostly absent from the fields of biology and Western medicine. Instead, we are told the structure of life results solely from natural selection (except for the occasional mutation) as a survival mechanism within a hostile and random environment.

Certainly there is plenty of evidence to support the important role of Darwin's theory of evolution and even neo-Darwinist Richard Dawkin's selfish gene theory in the development of life. There should be absolutely no question about this. But the presence of harmonics at all levels in nature also tells us that these explanations must be incomplete. Evolution and natural selection can be better understood as a veneer on top of a much more fundamental set of coherent physics that *guides growth as a crystallizing process* from the atomic level up.

# *Carbon Twelve*

*“We are finding that the world is composed not of matter, but of music.” – Dr. Donald Hatch*

Most people are familiar with the classic orbital model of an atom first proposed by Niels Bohr in 1913. Bearing a striking resemblance to our Chromatic Ring model for music, the Bohr atomic model is based on the idea of a spherical standing wave where electrons cycle in their shells around a central nucleus, jumping to other shells or orbits when energy is added or lost.[135] This model has since been replaced with the quantum atomic model, but the basic behavior of atoms remains essentially the same, especially their ability to resonate together into atomic bonds.

Carbon-12, notated as $^{12}C$, is the international standard for atomic weight. One-twelfth of the mass of one atom of $^{12}C$ is used as the unified atomic mass unit, called a *dalton* (Da), to weigh all other atoms. Carbon-12 is by far the most important element as it accounts for 98.89% of all forms of carbon and is the most flexible bonding element in the universe, producing the softest mineral graphite, hardest mineral diamond and all known life forms. While every form of carbon has 6 electrons grouped with 2 on the inside orbit and 4 on the outside, only the carbon-12 atom has a nucleus containing 6 protons and 6 neutrons. With an atomic weight of 12 by definition, $^{12}C$ is said to be “unbound” with the lowest possible energy. That is, in its stationary ground state it is well suited for crystallization and bonding with other common elements [Bentor 2007]. Carbon can easily crystallize into either countercurrent minerals or concurrent organic shapes.

Not surprisingly, carbon-12 functions much like a $\mathbb{Z}/12\mathbb{Z}$ cyclic ring and can be described by the same Chromatic Dual Ring model we use to describe music harmony. We need only designate the Harmonic Center {D} as the nucleus, the two tones of the Tonic major 3rd interval {C, E} as the inner ring and the four tones of the opposing Dominant group {B, A, G, F} as the outer ring. As shown in Figure 118, the $^{12}C$ atom resembles a 7-step diatonic {C} major scale (or Diatonic Rainbow) that oscillates between the inner sine ring, known as the “calm” ground state, and the outer cosine ring, known as the “excited” state.

Based on the idea that a dodecahedron results from the “crystallization” of tritone Partials 5 and 7 around a Harmonic Center, the $^{12}C$ nucleus can be similarly represented as twelve spheres packed tightly into two opposing groups of six “particles.” Around this nucleus, electrons then fill

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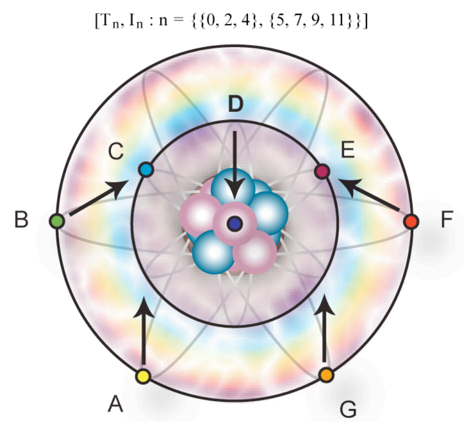
135 In its “ground state,” or stationary state, an electron in the inner ring is thought to *cross through* the center nucleus, creating a “probability cloud” for where it might reappear at any given moment. This behavior is often cited to emphasize the belief that atoms are random, thus all of nature is incoherent at bottom.

the shells of a $^{12}C$ atom from the outside inward in much the same way as the excited outer Dominant ring phase shifts into the "grounded" Tonic ring to resolve.

Now, when electrons move from an inner shell to an excited outer shell – a photon of light is *absorbed*. Conversely, photons are *emitted* when electrons move from an outer shell to a calmer inner shell. The frequency or color of light emitted is determined by the number of electrons in the atom and which shells it transitions between. Ultraviolet light is emitted from the first shell, visible light from the second shell and infrared light from the shell outside of that. It is probably no coincidence that our $^{12}C$ eyes are perfectly tuned to resonate with the seven Diatonic Rainbow frequencies of the Chromatic Dual Ring model.

**Figure 118 - The Chromatic Dual Rings as archetype for carbon-12**




$^{12}C$ = 6 neutrons + 6 protons + 6 electrons

Like a carbon atom, tertiary tones and colors on the dual rings are separated into their primary and secondary components when transitioning from the inner Tonic ring to the outer Dominant ring. The inner Tonic components Cyan {C} and Magenta {E} of CMY(K) subtractive color space are separated or "refracted" into the outer colors of {Green {B}, Red {F}, Yellow {A}, Orange {G}} of RGB additive color space. This shift is an *energy absorbing* process just like an electron jumping outward in an atom.

Transitioning back to the inner ring is then an *energy emitting* process where primary and secondary colors are mixed back into a resolved Tonic tertiary ground state, again emulating classical atomic physics. In general, movement from the outer to the inner ring is a reflective mixing operation while movement outward is a refracting or unfolding operation.

As seen here, the spherical standing wave model of an atom – particularly the stable $^{12}C$ atom – matches the behavior of a {C} major scale within the $\mathbb{Z}/12\mathbb{Z}$ Chromatic Ring model as it partitions into even $2\mathbb{Z}$ and odd $2\mathbb{Z}+1$ sub-rings. It is probably this dual odd-even structure in the

$^{12}C$ atom and its presence in our physiology that we can thank for our ability to anticipate energy flow in harmonic music. After all, electrons jump between the two shells and balance around a polarized atomic nucleus in the same way musical chords balance around a Harmonic Center. At the very least, the $^{12}C$ atomic model provides a useful archetype to explain diatonic music.

But it must be more than just a nice idea that $^{12}C$ and music both follow the same organic model. Music perception does appear to be a macro result of atomic resonance in our carbon-based bodies. Every cell of our body vibrates with carbon-12 atoms, crystallizing with $H^2O$ and other simple atoms and compounds to carve out the Gaussian shapes of our anatomy, including our ears. With these harmonic laws present at all levels of nature, we should expect that the same ***REFLECTIVE INTERFERENCE*** equation found to describe harmony in music could just as well describe the harmonic resonance and damping distribution of electrons in an atom.

As a matter of fact, recent scientific research has discovered the golden ratio plays a quantitative role in establishing the atomic/ ionic radii for many elements. For example, a golden ellipse was found to define the atomic radius of hydrogen, which acts as the simplest atomic condenser [Heyrovska 2004]. When hydrogen then bonds with carbon in an amino acid chain, it is this condenser function that causes the helical rotation of two DNA strands its central axis while spacing according to the golden ratio. Water literally damps carbon-12 resonance.

And as water reaches the mesoscopic level, 14 water molecules will cluster into tiny tetrahedrons, packing into an icosahedron and clustering into 280 icosahedral molecules, and forming a perfect recursive pentagonal geometry [Kusalik & Svishchev 1994]. As this bonds with $^{12}C$, the familiar 12:5 pattern of life begins to take shape. While none of this has made it into the college textbooks (and who knows when it will), there is no doubt that Φ-damping plays a key role in the harmonic activity of atoms and molecules just as it does in a vibrating string.

The Haramein-Rauscher atomic model is the most recent theory to recognize this essential damping action of the golden ratio. Derived from the Coriolis Effect found in the standing wave formation of galaxies, solar systems and hurricanes, the Haramein-Rauscher team has proposed a Unified Field Theory that explains atoms as the result of a golden spiral (like the solar system) swirling into the dense center of an atomic nucleus. They propose that an atom forms and is maintained by the pumping or "breathing" of gravity in the space vacuum, shifting between the shape of a cube (or star tetrahedron) and an octahedron – thus, creating the simplest possible harmonic structure. As these two shapes oscillate, the theory suggests that they pass through a dodecahedron (or its dual icosahedron) to form a golden spiral leading into an infinitesimal black hole at the center of every atomic nucleus. Like a tiny solar system, atomic matter would then resonate out of the spiral into regular geometrical shapes.

Under this theory, space is defined as a polarized structured vacuum, called a *Schwarzchild lattice*, organized into groups of 120 tiny black holes, each in its own "cell" of a lattice that permeates all space. Each cell is then hypothetically organized as a 12-faced pentagonal dodecahedron (or icosahedron), mirrored perfectly by the nucleus of a $^{12}C$ atom. The Haramein-Rauscher atomic model then describes each cell as a harmonically oscillating cubeoctahedron that passes through the Schwartzchild dodecahedron in the space lattice (again centered on a tiny black hole). This pumping action is presumed to create the electromagnetic Coriolis Effect and double torus rotation in opposing directions like a miniature hurricane.

At the torquing edge or "event horizon" of the miniature black hole in each cubic cell of the space lattice, resonant atomic structures are thought to crystallize geometrically depending on the atomic weight of different atoms.[136] Of course, carbon-12 has just the right number of particles to achieve a perfect balance of resonance and damping in a Gaussian ***INTERFERENCE*** pattern. From this perfect harmony grows polypeptide amino acid chains and DNA molecules capable of evolving into the wide variety of carbon water crystals we know as life. If the Haramein-Rauscher atomic model is correct (or close to it), this would explain the spatiotemporal genetic models of White and Yang. Life becomes a kind of atomic music balanced between chaos and order.

But the big question that continues to stump modern science is how organic life makes the big leap from inanimate to animate while learning to regenerate itself. How does life suddenly "discover" the process of forming a shell around itself? How does it kick-start the process of extracting nourishment from its surroundings, converting it into energy and expelling the waste?

While this never made the news, a recent study found that the first step of enclosure needed for a single cell to live is a geometrical folding at the atomic level. A 2006 publication of the American Chemical Society entitled *$Tb_3N@C_{84}$: An Improbable, Egg-Shaped Endohedral Fullerene that Violates the Isolated Pentagon Rule* reported that a large Fullerine carbon-84 atom constructed its own egg-like cage when two adjacent pentagons in the carbon atom fused together. Discovered by a combined team from the University of California, Virginia Polytechnic and Emory and Henry College, this was the first indication that the regular soccer ball geometry of hexagons and pentagons in a large carbon Fullerine could wrap itself into an eggshell by reacting with another atom, causing one of the hexagonal faces to collapse.

The study reported that when the carbon allotrope created its cage, it encapsulated the terbium (Tb) atoms used as an isomer with the carbon, turning it into a sort of nucleus. The team reported that "one TB atom is nestled within the fold of the fused pentagons, while the other Tb atoms are disordered over four pairs of sites." In effect, the chemical reaction caused a geometric transform, resulting in the formation of an egg.

Of course, this discovery could answer a lot of questions. For one thing, we now know that it was the egg that came before the chicken. No seriously. It indicates that this is how amino acids learned to form cells around themselves and offers a reasonable explanation for why ribcages form around vital organs in all vertebrate animals. It even directly suggests the role of harmonic

136 Atomic weight is the number of particles an atom has as measured against the carbon-12 atom.

interference as the underlying cause for the molecular interaction between pentagonal and hexagonal geometry – the same proportions of 12:5 found in a musical tone and our solar system – while explaining precisely how inanimate matter *twists itself* into life.

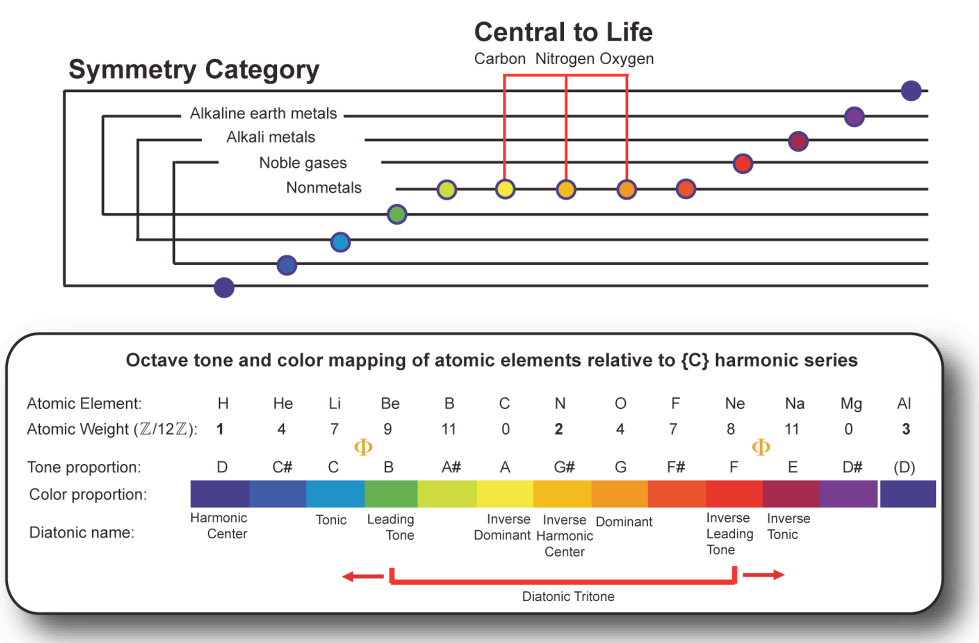
We see this twist in the nuclei of $^{84}$C and $^{65}$Tb around the common factor of 12:

*$^{84}$C: 12 / 7 = 84*
*$^{65}$Tb: (5+(5 / 12)) ×12 = 65*

The two represent a kind of reciprocal relationship where the carbon atom's factor of 7 interferes with the terbium atom's factor of 5. Does this look familiar? It should because this is the same 7:5 proportion of a tritone. Could it be that we really do have a *Diabolus in Atomica* that triggers the folding or twisting of carbon atoms into egg-like cells? Could the introduction of the Tb atom with its nested 12:5 damping proportions have caused the "incommensurable dissonance" in the structure of the carbon atom, cascading into the Fibonacci series that curves harmonic proportions into the closed ring of a living cell?

For this to occur in a carbon-12 atom, it would need to bond with an atom having similar enharmonic pentagonal proportions, such as Boron. Interestingly, Boron has been found to be essential in maintaining the integrity of cell walls in plants. This is seen as its symbol B adjacent to carbon C in the abbreviated and symmetrically arranged periodic table in Figure 119.

**Figure 119 - Harmonic archetype for the first twelve elements in the periodic table**



In the figure, the standard categories are used to classify the elements symmetrically by assigning nitrogen (symbol N) as the Harmonic Center in the first atomic octave. Balancing on either side of nitrogen, carbon (symbol C) and oxygen (symbol O) then form a Cycle of 5ths relationship to hydrogen (H) as the musical orbit {A, D, G}. Is it a mere coincidence that oxygen and carbon balance symmetrically around an axis formed by hydrogen and nitrogen and that these four elements together form the essential bonds of life?

The relevance of symmetry to the periodic table becomes very apparent when we apply the $\mathbb{Z}/12\mathbb{Z}$ cyclic ring to each element's atomic weight. Rounding off to the nearest integer, we find a close symmetry between the lower six and upper six elements, doubling the atomic weight of hydrogen to obtain nitrogen and tripling it to obtain aluminum at the octave.

When the same musical octave assignments used earlier for the planets are applied to the first twelve elements, we again find familiar correlations to music harmony. The most obvious are carbon and oxygen, which act like Yellow and Orange Dominants in a diatonic scale. Similarly, hydrogen and oxygen polarize into a Harmonic Axis, telling us that the water molecule $H^2O$ acts as an axis for all of the elements. Is this yet another coincidence or do atoms actually bond (or converge) as harmonic interference exactly like two tones over a musical octave?

Even the Tritone Function has an atomic equivalent. Lithium and sodium, the first two alkali metals in the periodic table, correspond to a resolved Tonic major 3rd in traditional diatonic music. At the same time, beryllium and neon correspond to the highly resonant diatonic tritone, which as we know has a tendency to phase shift into alignment with the Tonic. So, if atoms work according to harmonic interference, we would expect lithium and sodium (the resolved major 3rd) to be the result of an opposing electron transfer (or phase shift) from the interval of beryllium and neon (the tense diatonic tritone). Whether this accurately explains the interactions of these elements is left as an avenue of research for some aspiring scientist, but it does agree with the fact that atoms bond harmonically and exchange energy within an overarching interference pattern just like music harmony.

Atomic harmony could help explain human perception and brain function. Take the psychoactive properties of lithium, for instance. Lithium salts have been found to have a calming effect on people with mental disorders and is often prescribed to control psychotic episodes. If atomic harmonics work the same as brain harmonics, then the role of lithium as the Cyan Tonic fundamental of the atomic octave in the previous figure could explain why it works so well as a medicine to calm psychiatric patients. This is entirely predictable under harmonic theory.

Harmonics are everywhere in nature, spinning out of spirals into periodic forms and from that into crystalline structures of all kinds, including the chemical composition of our brain and body. It seems inescapable to conclude that the Pythagorean worldview of *music universalis* was more correct than anyone today wants to admit; right down into the mythological Underworld we now call quantum physics.

# *Harmonic Lattice*

*"All matter originates and exists only by virtue of a force… We must assume behind this force the existence of a conscious and intelligent Mind. This Mind is the matrix of all matter." - Max Planck*

Since the quantum physics revolution of the early 20th century, scientists have been moving closer and closer to admitting that we do actually live inside some kind of coherent holographic energy field. The "quantum field," known officially as the *Quantum Chromodynamic Lattice* or *Lattice QCD*, is thought to be a kind of vibrating matrix through which energy projects into material form. First proposed by physicist Max Planck, this became the inspiration behind the science fiction movie *The Matrix*. Yet, the science behind this matrix is anything but fiction.

Recent experiments are yielding solid evidence of an "auxiliary fifth dimension" that acts as a kind of "quantum aether" underneath classical physics to keep electrons spinning. One very recent experiment by the GEO600 team in Hanover Germany may have proven the existence of this aetheric dimension in a laser interferometer experiment while studying gravitational waves. It seems an unexplained noise appeared in all of their experiments that could be the graininess of spacetime. Fermilab physicist Craig Hogan commented: "It looks like GEO600 is being buffeted by the microscopic quantum convulsions of spacetime. If the GEO600 result is what I suspect it is, then we are all living in a giant cosmic hologram" [Chown 2009].

Some of the latest research papers support this idea. In the high-energy research section on the Los Alamos National Laboratory web site we find such titles as:

- *QCD phase diagram: an overview,*
- *Understanding nucleon structure using lattice simulations,*
- *Advances and applications of lattice supersymmetry,*
- *Deconfining Phase Transition on Lattices with Boundaries at Low Temperature,*

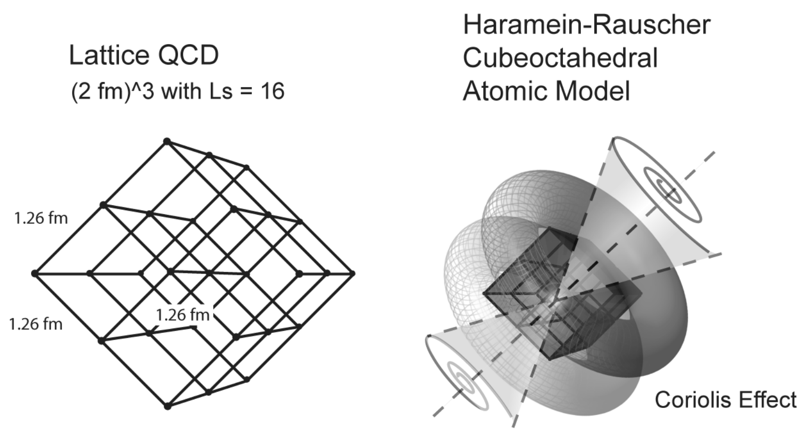
and my favorite:

- *2+1 flavor domain wall QCD on a (2 fm)^3 lattice: light meson spectroscopy with Ls = 16.*

Though these studies are mostly incomprehensible to the layperson, they are all based on the simple idea that atoms *resonate inside a cube* instead of a freestanding sphere as usually taught in physics classes. The cubic atomic model was established early on in quantum theory when it was discovered that atoms with an even number of electrons are more stable than odd numbers. This was found to be especially true for atoms having 8 electrons in the "L shell" because they orbit symmetrically to form the corners of a cube. When you consider that a cube is the best shape to fill space perfectly, how could it be any other way?

The Lattice QCD can be described as follows. Each cube has side length of about 1.26 fermions (fm) and cubic volume of 2 fm.[137] This is then cubed to produce the space of four connected cubes, totaling a volume of 8 fm, which is then doubled to produce eight connected cubes with a combined volume of 16 fm (thus the "Ls = 16" in the above title). This is easily visualized as a quantum *space lattice* having 27 equally spaced points as shown in Figure 120.

**Figure 120 - The quantum space lattice and cubeoctahedral atomic model**




To understand how this contains energy, just imagine a large number of these cubes joined together to create a cage that you could hold onto and shake. As you rattle the cage, energy flows into it and bounces around inside, sometimes aligning in resonance with the lattice and other times creating a canceling pattern of interference. If we made this cage many times bigger, the rattling would turn into ripples, splashes and tides of energy flowing through the lattice like currents in an ocean. Of course this is a prosaic oversimplification, but you get the idea.

Under the Haramein-Rauscher atomic model, the center of each cube of the lattice would then spiral to form swirling vortices according to the Coriolis Effect. These vortices form golden spirals like that of the rational Fibonacci and Lucas growth series that damp or contain overly exuberant atomic resonance. In this way, the quantum space lattice would itself be the origin of

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[137] In particle physics, fermions usually represent particles of matter having a half-integer (or "half twist") spin, such as protons and electrons.

damping in all standing waves, canceling fractional waves and causing energy to form whole number harmonics that then crystallize into various coherent atomic structures.

Within the quantum lattice, when ripples of energy align in the center of the cubes, stable matter called *fermions* are said to form. These particles, including protons, electrons, quarks and leptons, all spin inside a locked cubic structure. But, when ripples align with the damping points of the lattice itself, free energy particles called *bosons* continue to travel outward in waves, apparently spinning off the edges of the vortices, propelling them through the lattice. This would include photons of light, radio waves, x-rays, gamma waves, cosmic waves and gravity – all traveling *between* particles of matter. Throughout the entire lattice, a constant reverberation of zero-point boson energy is believed to be the very thing that keeps electrons spinning in their orbits and nuclear particles bound together. Without it, everything would simply fall apart.

Related to this discussion is the earlier story told by Philolaus about what happens when a wholetone is split in half. Recall that the wholetone ratio of 9:8 could only be reasonably factored into an awkward 27 parts, which we named the Philolaus octave comma or 9:8 / 27. Well, now more than 2,500 years later we are beginning to discover that the Philolaus octave comma may actually be a harmonic description of the quantum space lattice.

When we take the octave comma of 9:8 / 27 = 0.04166667 and divide it into the QCD lattice volume of 16, the result is an even 384 "musical sub-cubes." Each of the eight lattice cubes then has 48 such musical sub-cubes, each of these having a volume of exactly 9:8 / 27 = 0.04166667 fermions. This identical scaling proportion between the quantum lattice and a $\mathbb{Z}/12\mathbb{Z}$ equal-tempered octave suggests that energy resonates up as a 5-level $12^{th}$-power Harmonic Hierarchy through the cubic quantum lattice just as we find it in music, the solar system, life and atomic structure. It seems we really are living in a musical hologram.

But this is nothing new. The ancients were well aware of this fifth dimension of space and how it supported atomic structure. We find it represented by the Sumerian god of Heaven *Anu*, later adopted by the Assyrians and Babylonians, becoming the Egyptian god of the Underworld *Anubis*. In the Hindu Bhagavad-Gita (meaning "Song of God"), Arjuna is enjoined to meditate on the "seer," i.e., the enlightened, omniscient One *Anu*, who is "more atomic than the atom" (anor aniyamsam) and "the support of all". Translated from Sanskrit, *Anu* means "an atom of matter, atomic, fine, minute" and was used as the title of the Indian Brahma to imply both the infinitesimal and the universal.

Greek philosophers echoed this idea, claming physical reality must be built up from indivisible particles called *a-tomos* and that they resonate harmonically like a string into archetypal geometries in space. As passed down into Christianity, Anu and a-tomos became the location of spirit, the *quintessence* (fifth essence) and the location of the *Holy Ghost*. Without any field of quantum physics or advanced technology to support their belief, ancient philosophers and priests simply deduced that everything is shaped by geometrical harmonic archetypes or "spirits" residing in an aetheric realm. It just seemed too obvious to ignore or deny, even without proof.

In more recent times, theosophists such as Blavatsky, Leadbeater and Arundale along with Kirlian photography researchers like Milner and Smart have proposed the existence of an "Ultimate Atom," which they too call ANU. They predict that one day we will find a cardioid or heart-shaped energy flux at the center of all atomic structure. With the 2008 completion of the Large Hadron Collider in Switzerland, proof could come any day now.

The modern science of quantum physics continues to approach parity with theosophical beliefs and simple harmonic principles. The even harmonics of a musical tone resonate to form *octaves* in the same way evenly aligned fermions of quantum physics (material particles) resonate to form *cubic octets* in the quantum lattice. Similarly, the odd harmonics *shape sound into timbre* just as odd bosons work in the quantum lattice to *shape matter into geometry*. No matter what you call it or how you explain it, the structure of space and the life that grows inside it has a common architectural foundation in *Harmonic Interference Theory*.

Quantum physicists have been saying for some time now that matter is the result of harmonic wave interference and that those waves, flowing through the invisible fifth-dimensional lattice of space, overlap and align to produce particles that bond into larger and larger structures. Maybe the time has come to explain the cosmos using the universal language of music. Maybe it is finally safe to admit that everything is a kind of crystallized music resonating inside a harmonic lattice. Maybe it is time to *bring back the aether* and admit once and for all that *musica universalis* really is our Theory of Everything.

## Redefining the Quantum Lattice as the Harmonic Lattice

From the preceding discussion – and in the spirit of mathematical physicist Roger Penrose and his quasi-periodic tiles – a harmonic lattice is now proposed as a universal field or container for all things harmonic. Presented in Figure 121, the lattice is a complete orthogonal system of vertical and horizontal standing waves aligned at shared nodal points. Only in this way can odd and even harmonics coexist nondestructively in space.

As nature would have it, tessellating standing waves at right angles like this creates a pattern of tiled circles separated by a small gap. As we can see from the Red-Green bars measuring out golden sections in the lattice, the slack space separating the waves constitutes a natural Φ-damping region that allows the orthogonal system to remain coherent just as it does in a single standing wave.

### Figure 121 - Sine and cosine waves tessellated into a harmonic lattice

Green Horizontal Wave Group

$x_{even\ \mathbb{Z}} = \sin(y) + even \cdot \pi/2,\ even \in 2\mathbb{Z}$

$x_{odd\ \mathbb{Z}} = \cos(y) + odd \cdot \pi/2,\ odd \in 2\mathbb{Z} + 1$

$y_{odd\ \mathbb{Z}} = \sin(zy),\ odd \in 2\mathbb{Z}+1$

$y_{even\ \mathbb{Z}} = \cos(zy),\ even \in 2\mathbb{Z}$

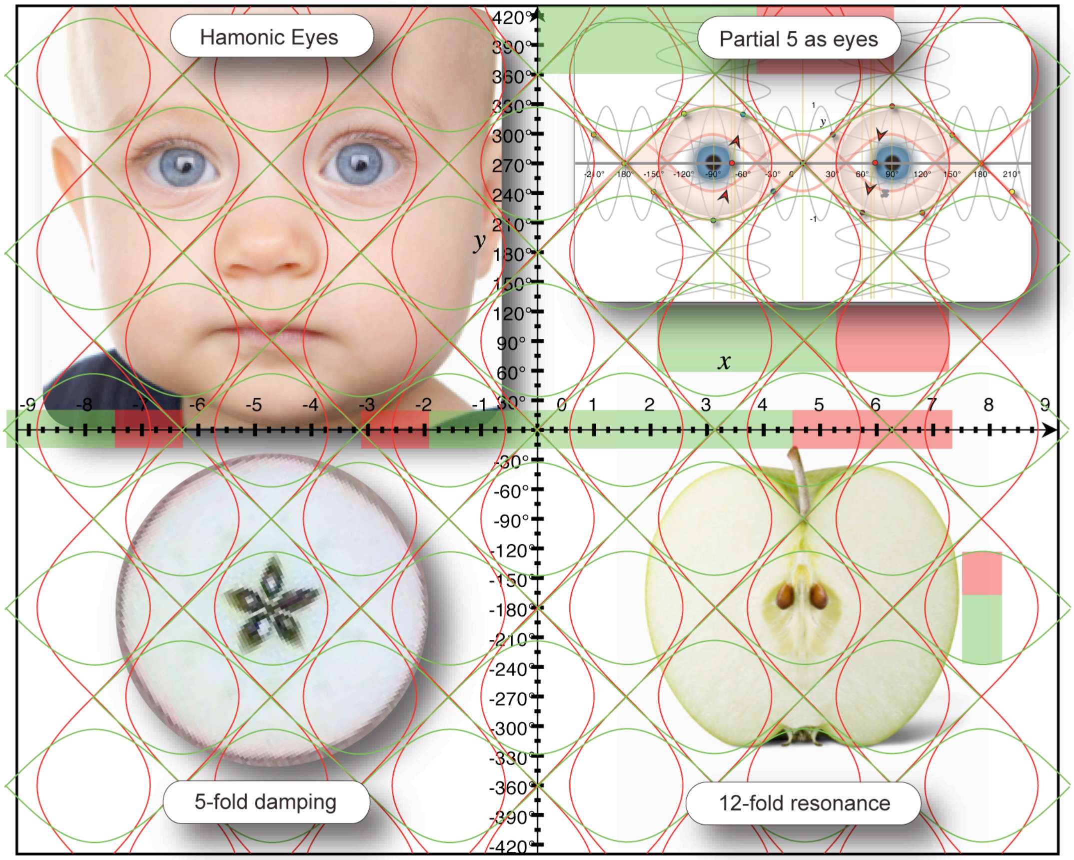
Red Vertical Wave Group

$y_{even\ \mathbb{Z}} = \sin(x) + even \cdot \pi/2,\ even \in 2\mathbb{Z}$

$y_{odd\ \mathbb{Z}} = \cos(x) + odd \cdot \pi/2,\ odd \in 2\mathbb{Z} + 1$

$x_{odd\ \mathbb{Z}} = \sin(zx),\ odd \in 2\mathbb{Z}+1$

$x_{even\ \mathbb{Z}} = \cos(zx),\ even \in 2\mathbb{Z}$

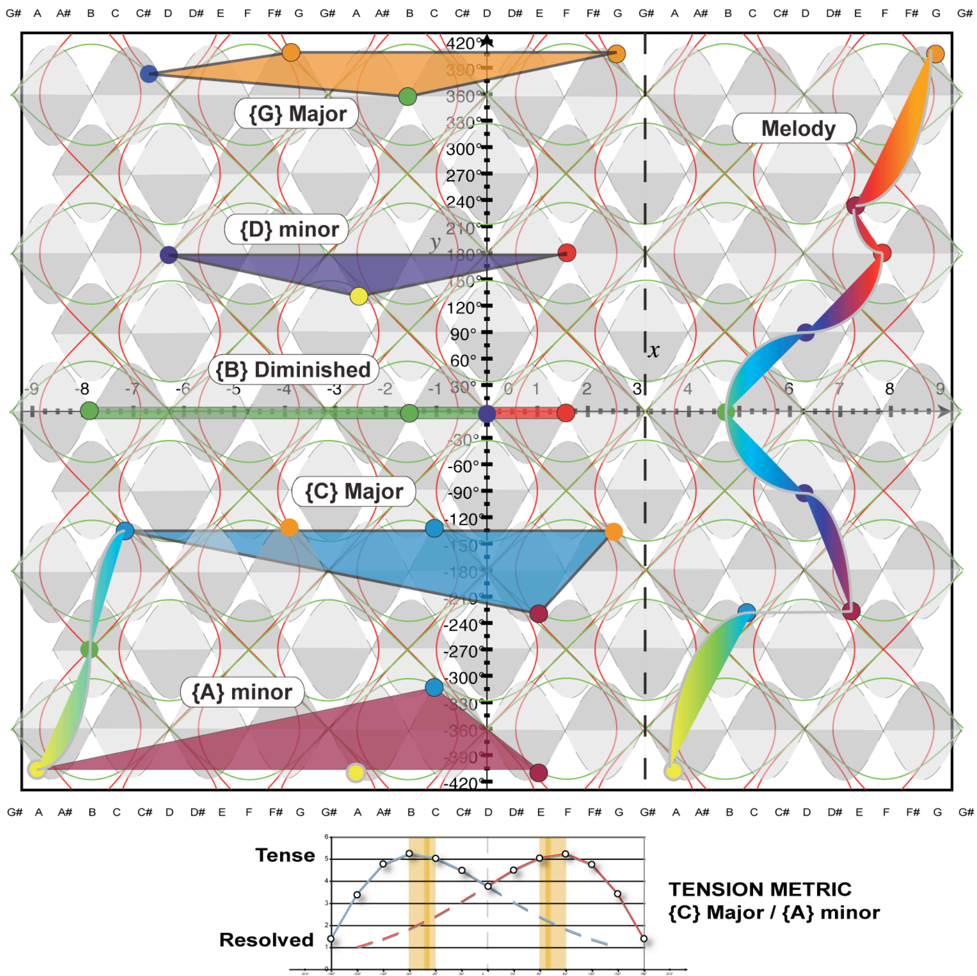



Figures show Φ-spacing between waves and organic objects

Many golden ratios can be seen in the geometry and overall spacing of the objects. The child's eyes are spaced harmonically as are other facial features. In the apple, the central region of horizontal damping and vertical resonance both fit within one of the circles of the lattice while the stem connects right at a node. In every direction and every scale, this standing wave lattice of right-angled waves is capable of holding these and many other things. But above all, it holds the freedom of knowledge and the vision to see the world as music.

Consider the Diatonic Standing Wave model in the lattice as shown in Figure 122. Using the same chord progression as before with an added melody, music can now be visually represented flowing through the harmonic lattice at a diatonic 2:1 sampling rate.

**Figure 122 - Melody and chords in the diatonic lattice**




As in previous harmonic models, the synesthetic colors highlight the flow of energy through the lattice. In the melody, the change of colors indicates its phase shift position in a diatonic scale, showing how the harmonic current flows from tense Greens and Reds at top to resolved Cyans and Magentas at bottom. The Aquamarine color again indicates where Φ-damping is in full effect and where the tritone pull upward is greatest. Applying the Tension Metric of *Harmonic Interference Theory*, we can then visually track the resolution of chords by measuring their amplitude (or velocity) on the curve.

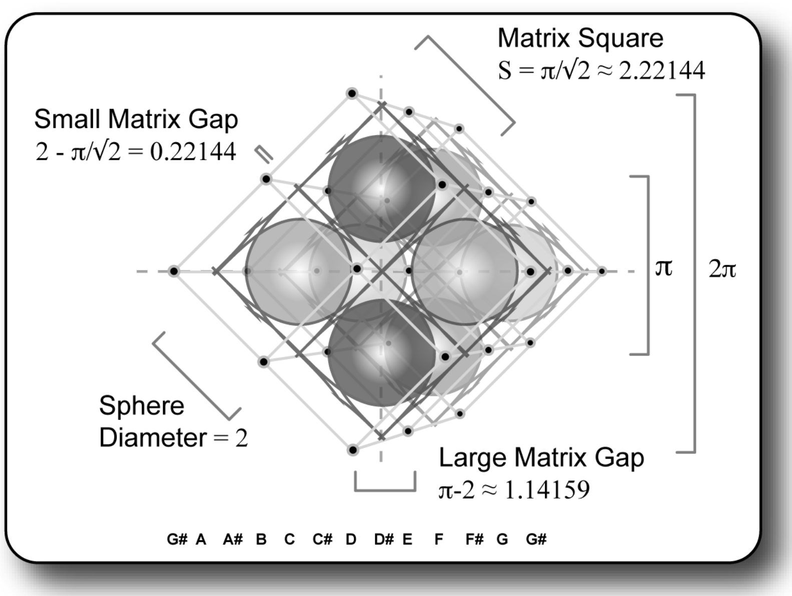
The same methods can then be used to measure organic shapes as a function of resonance or damping. In this way, any organic geometry – from a flower to a face – can be described as a series of musical proportions. Imagine composing music based on the geometric structure of a human face or body. Imagine the picture of a friend transformed into harmonic music.

This lattice may even be used one day to predict the flow of information inside the holonomic interference pattern of our brain. Our brain is continuously setting up patterns at different scales, amplitudes and frequencies in order to lock onto and identify coherent harmonic geometric forms. Those objects that fit best are by definition the most coherent and the most important to our survival. Projected into a 3-dimensional space, we may well find that the flow of information in our standing wave neural network resembles the flow of music playing back in this lattice.

Now, if we want to accommodate 3-dimensional objects, the lattice can be cubed by rotating each wave around its horizontal or vertical transverse axis. This transforms the circular regions into loosely packed spheres which, when unfolded in all directions, describe space as a cubic interference pattern of Φ-spaced standing waves. In this way, the lattice becomes a completely immersive space like a hologram with six degrees of freedom.

This 3-dimensional model is also a perfect match for the Lattice QCD. In Figure 123, this is proven by enclosing eight of the spheres into one full period in x, y and z directions. From Principle 21, this model can then be scaled up and down recursively to represent the five levels of the $\mathbb{Z}/12\mathbb{Z}$ Harmonic Hierarchy over a cubic musical octave.

**Figure 123 - A musical octave in a cubic octet of spheres**



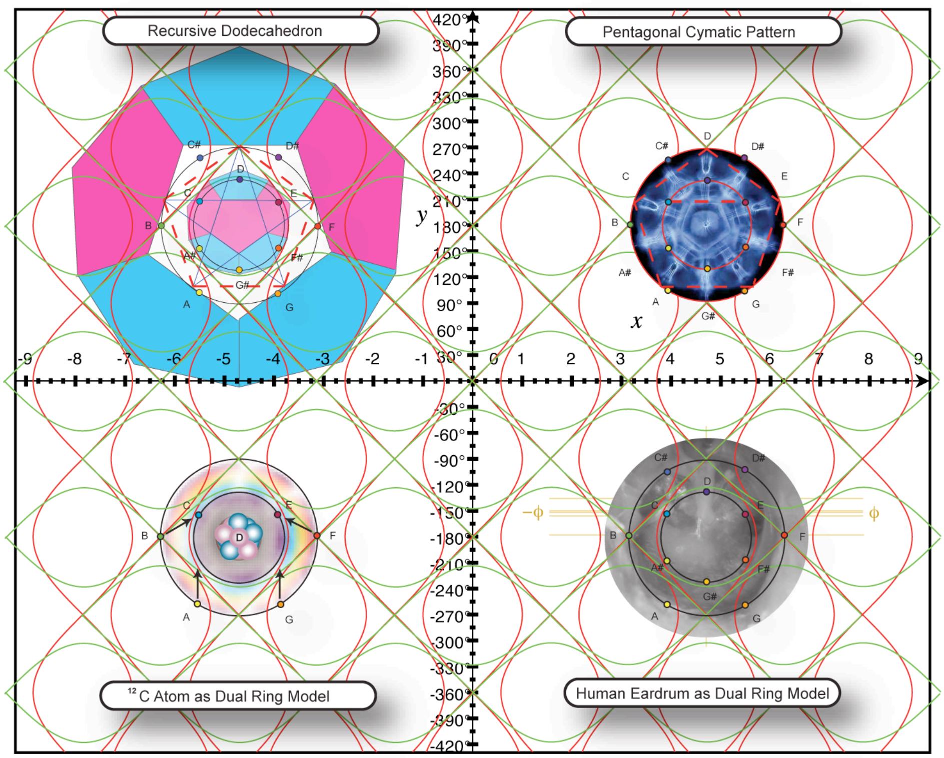
From this point on, the line really begins to blur between harmonic science and quantum physics. The harmonic lattice, originating from the vibration of tones over an octave, is completely compatible with the quantum lattice with its even fermions (of matter) and odd bosons (of energy). It may well be that these spheres – each suspended inside a 3-dimensional tritone – actually represents the *quanta* from which our physical reality originates. And as each harmonic wave aligns at a right angle to form smaller and smaller spheres inward, these may well be the atomic shells of electrons, torquing out of the interference field. Whether we choose to call it the quantum lattice or the harmonic lattice, it is the same thing – a universal container for energy and matter.

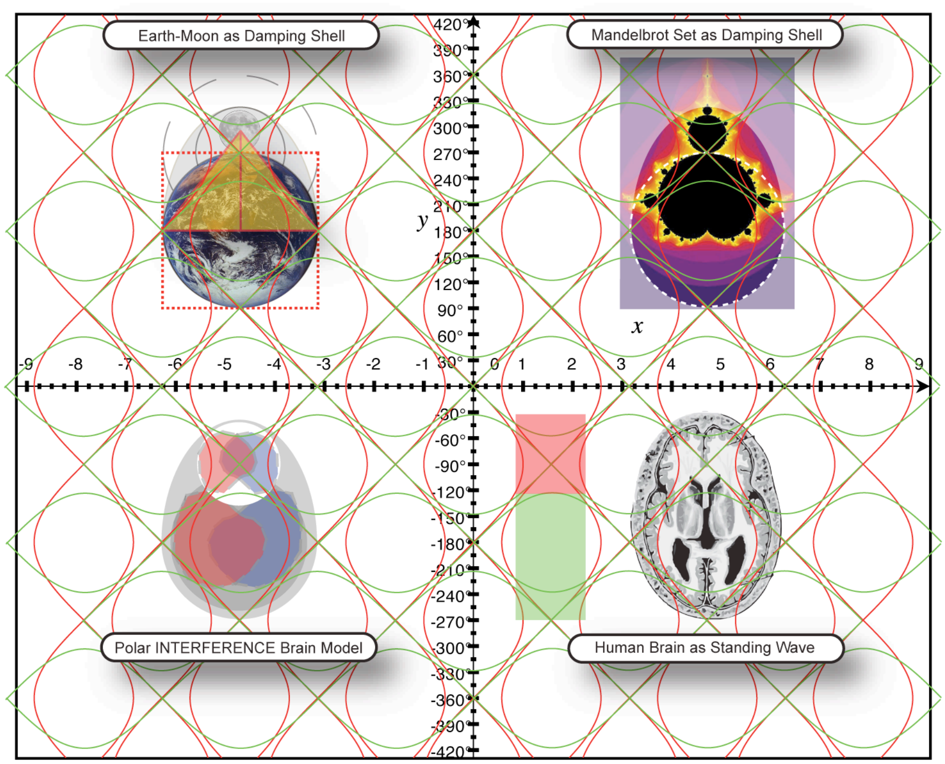
Viewed as a container, the harmonic lattice can help explain the harmonic properties in many things we take for granted. Figure 124 shows a few of the earlier harmonic archetypes scaled to fit into the lattice. I would urge you to take five minutes to inspect this diagram very closely, as it will tell you much more about the harmonic lattice and the objects themselves than can be described with words. Look closely wherever different parts of each object coincide with the intersection points of the waves. Take special note where they align along the edges as well as the insides of some of the objects. In every case, we can find a shared proportional relationship to the harmonic lattice and, in many cases, other objects. Even more shared proportions can be found as harmonics are added back into the lattice and as it nests within itself recursively.

Most of the objects are self-explanatory, but there is one new idea here worth pointing out. The Rojas egg model is shown in combination with several of the objects, including the Earth-Moon system, the human brain cross-section and the Mandelbrot Set. This was done to emphasize the importance of the egg geometry as a damping container, particularly as it relates to organic geometry and the Mandelbrot fractal. As the two are combined into a "Mandelbrot Egg" model, this archetypal object illustrates how organic systems will tend to *migrate recursively outward* through the lattice in much the same way as diatonic geometry unfolds into lattice structures. Organic growth is geometric and prefers to follow preexisting coherent pathways.

It is no coincidence that the large side bulbs of the Mandelbrot cardioid open through the space between the tiled horizontal sine-cosine waves, branching out at weak points in the interference pattern. It is also no coincidence that the first circle off the Mandelbrot cardioid occurs at a calm nodal points that also align perfectly with other branching patterns, such as the front legs and antennae of the beetle shown earlier. Even the tiny black dot at the top of the yellow projection of the Mandelbrot figure, which is actually a tiny nested version of the entire Mandelbrot figure, is at the center of the circle directly above the egg. The Mandelbrot Set is in reality the <u>same basic geometry as an egg</u>, both sharing an undeniable structural relationship with the harmonic lattice.

**Figure 124 - A collection of archetypes aligned in the harmonic lattice**



As luck would have it, I may have actually "seen" this lattice when I was very young. At about ten years of age, as a catcher in a local baseball game, the batter hit the ball and flung his bat backward through the air – directly toward my head. With no time to react, the bat hit me square on the forehead, knocking me out cold. As I began to regain consciousness out of the fizzing blackness, I envisioned an assortment of flashing red and green pentagonal stars (with perfect sharp edges) accompanied by what sounded like birds chirping – exactly like those in old Warner Brothers cartoons. As I came to, it all seemed quite comical to me and I would have laughed out loud had it not been for the splitting headache that rushed in.

I wonder now if this cubic standing wave lattice somehow caused those perfect little stars to appear, representing a reentry back to coherence. And might the chirping sounds have been the stars themselves firing off tiny harmonics in my holonomic brain?

# ***Musical Matrix***

*"Unfortunately, no one can be told what the Matrix is. You have to see it for yourself."*
*- "Morpheus" from The Matrix*

Of all the amazing things harmonic science has to tell us about nature, the most profound must be what it says about the human body. And of all the writings about harmony and balance in the human form, the most famous must be *De Architectura,* written by Roman architect/engineer Marcus Vitruvius Pollio around 80-70 BC.

Descended from a long line of Greek philosophers and writers, Vitruvius' view of architecture crossed the boundaries between nature, art and architecture, leading him to design a model that set forth the ideal proportions for the human body. Known in Roman times as the *Canon of Proportions*, the Vitruvian model has very definite ties to Pythagorean harmonic science and Greek geometry. Leonardo da Vinci, who was also intensely interested in Gnostic geometry and the Platonic solids, later studied and sketched this very model from Book III of *De Architectura*. Today known as the *Vitruvian Man*, this pop-icon of a man drawn spread eagle inside a circle and square can be found on just about everything from T-shirts to wine bottles. Few appreciate its real significance.

Central to the Vitruvian model is a mathematical operation known as the "squaring of the circle." This theory proposes that human form is proportional to both a square and circle such that the square has a perimeter equal (as close as possible) to the circumference of a circle (centered on the navel). While obtaining a perfect match between the square and circle is impossible due to the infinite value of $\pi$ in a circle, the Vitruvian model proposes that these two primitive geometries would be found in near perfect balance in the structural organization of the human body. Da Vinci agreed with this philosophy, writing that he felt the square represented the material realm while the circle represented spiritual existence.

Given the importance placed on "squaring the circle" by these and other great thinkers, we might wonder if the Vitruvian model is somehow related to the circles and squares of the harmonic lattice. Could the Vitruvian Man somehow fit into its interference pattern and be measurable as a function of resonance and damping like a musical harmony? And if so, how could this influence biological growth over the course of millions of years to finally achieve the geometry of the human body?

As explained earlier, growth in all life results from recursive nesting of the same basic pattern at different scales. So to show how something grows harmonically, a recursive (or fractal) scaling dimension must be added to the lattice. This is done by using the z-axis as a growth dimension to represent cells resonating outward like recursive patterns on a Chladni plate.

Along the z-axis, we will place twelve concentric rings such that each ring scales larger than the previous by a proportion of Φ. You may recognize this as the same concentric ring system used earlier to describe the spacing of planets in our solar system, forming nodes at quarter turns along a golden spiral. While the idea is wildly outside of mainstream science, the implied hypothesis here is the human body resonated outward during evolution according to the same harmonic laws of resonance that formed our solar system. Seems obvious, does it not?

Extending the harmonic lattice now along this new recursive dimension, we can "square the circle" for the outermost ring just as Vitruvius described it two thousand years ago. The result shown in Figure 125 is an expanded harmonic lattice that is now scaled to precisely fit Leonardo da Vinci's squaring of the circle. This new ***RECURSIVE INTERFERENCE MATRIX*** uses the following dimensions.

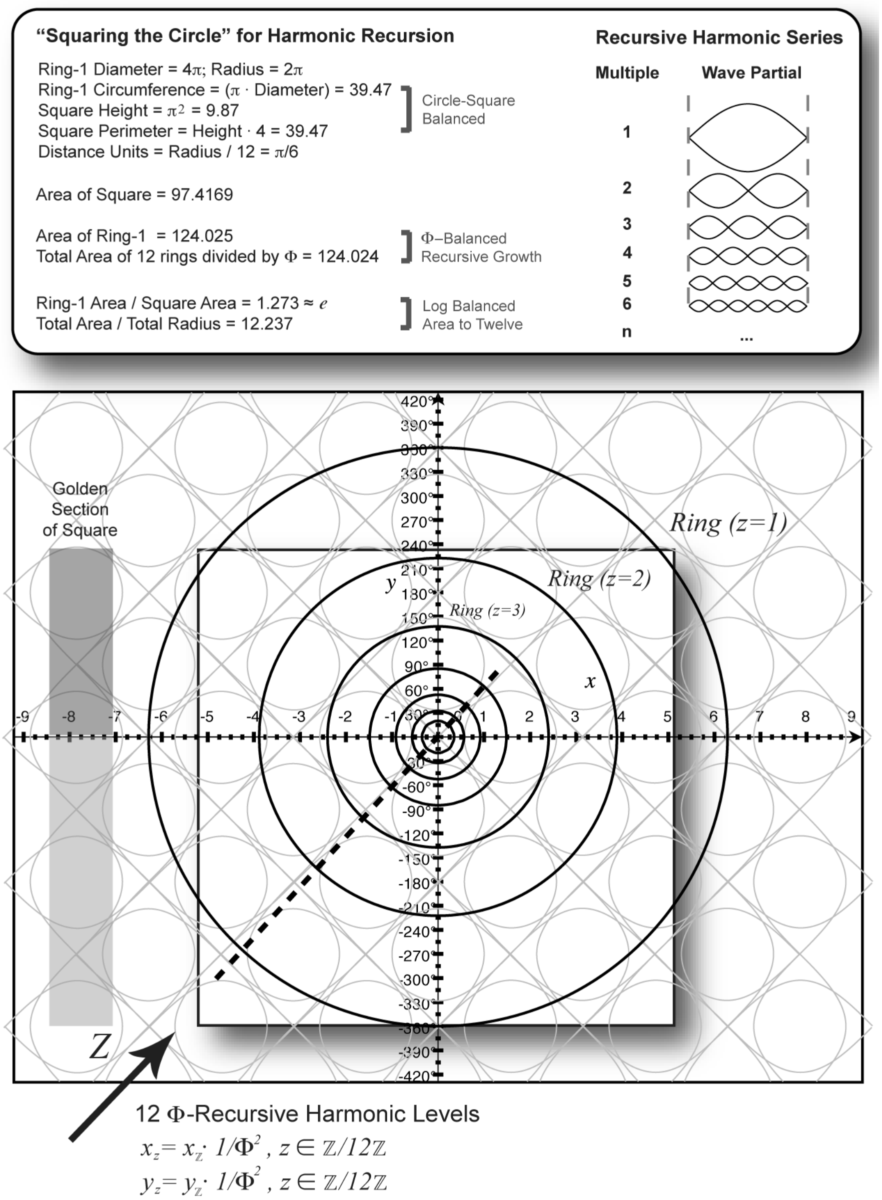
*y-axis = harmonic standing wave period of 4π for Ring-1 diameter.*
*x-axis = amplitude period of 12 for Ring-1 width.*
*z-axis = Φ recursion with ring radius dimension of $R^{1/(\Phi^n)}$, n = {1..12} and*
*wave frequency following the harmonic series sin(n × x).*

In this way, we are placing the Vitruvian Man into a Φ-recursive and Φ-spaced lattice container comprising twelve levels from maximum resonance at center to maximum damping at the outer ring. Since the rings are an abstract growth dimension rather than an actual spatial dimension, this arrangement is said to represent a mathematical "Hilbert space". Hilbert spaces like this are commonly used to represent relationships between physical and abstract concepts in order to describe certain natural phenomena. In this case, the z-axis represents recursive (or fractal) biological growth as a projection of 2-dimensional space.

Now as the rings nest into themselves at each level, so do the wave frequencies of the harmonic lattice. But the waves do not scale by Φ. Instead, each wave scales as a harmonic frequency {1×, 2×, 3×, … 12×} such that each fits within the space of one-half cycle or π period of the top level fundamental wave. In this way, the Φ-recursive rings are actually just a damping function that scales *in parallel* with wave resonance as harmonics get faster and smaller in the same space.

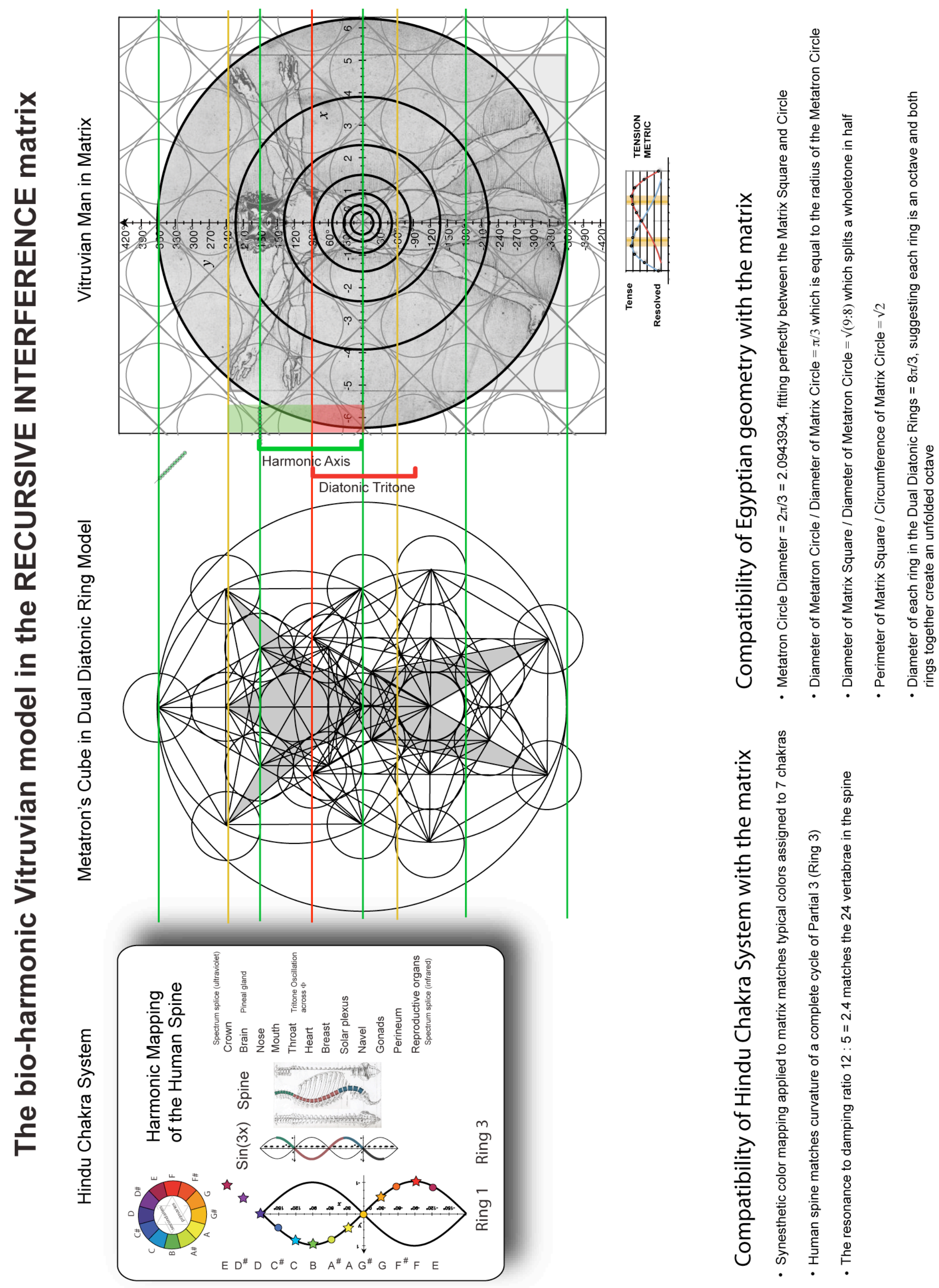
We might think of Φ-recursion in biology as an inhibitor that allows harmonic structures to scale coherently from small to large without colliding, something like the Harmonic Hierarchy in music. Under such a bio-harmonic model, cells would resonate outward according to the same Gaussian ***REFLECTIVE INTERFERENCE*** distribution of harmonics in a musical octave.

**Figure 125 – The "musical"** ***RECURSIVE INTERFERENCE MATRIX***




With the lattice prepared, we can now drop the Vitruvian Man into the lattice to see if it fits. Starting from the outer ring in Figure 126, aligned to match Leonardo's original sketch, we see that the second ring (from the outside) touches the top of the head and knees while the third ring intersects the base of the neck and two vertical markers at each arm joint. The fourth ring then aligns perfectly with the horizontal breast marker and reproductive organs while the fifth ring reaches the approximate width of the torso. Several other important correlations can be found to exist between the recursive matrix and the Vitruvian model.

## Figure 126 - The bio-harmonic Vitruvian model



We can see in the matrix that the vertical sine wave hits one of the nodal points right on the nose – literally on our model's nose (his *sinus*). The outline of the upper and lower torso also follows the sine wave with another nodal point at the navel. Of course, most physicians might consider this a coincidence with no bearing on real anatomy. That is until they compare the wave structure of the spine to harmonic alignments in the lattice.

It turns out that Partial 3 (corresponding to a perfect $5^{th}$) is an exact match for the curvature of the spine. And as we add in the synesthetic color model, it too corresponds perfectly in hue and location to the seven rainbow colors of the ancient Indian Hindu chakra system, traditionally used to explain the flow of energy through the body. Is this too a coincidence, or does the chakra system originate in some ancient knowledge of this same musical matrix?

Could it be that the Indian Brahmin shared this knowledge of the harmonic organization of the human body with Pythagoras some 2,500 years ago? Would they have known that the Φ-damping location was located precisely at the <u>heart</u> of the bio-harmonic Tritone Function at {E, F}? Could they have known that the Kundalini, known as the *serpent chakra*, activates the Tritone Function through the Leading Tone {B} that contracts to Tonic {C} like a kind of musical orgasm?

Would the Egyptians and their descended French Gnostics have understood all this too – how the Vesica Piscis unfolds into two Metatron's Cubes, outlining the ghostly figure of a man? Were da Vinci and Vitruvius guided by this much older knowledge to arrive at their harmonic human archetype as the squaring of the circle? Could this have even been the archetype for the mysterious Holy Ghost inside the Borromean Rings of the Roman Catholic Church, once thought to exist in the aetheric structure of space, but now long forgotten?

At this point, some degree of head shaking is not unexpected. It seems impossible to think that musical names from a piano keyboard can simultaneously refer to the locations and colors of the ancient Hindu chakra system while corresponding to bodily locations and functions. We might rightly think if there was anything to it, we could surely find it documented in medical textbooks or at least as a "fun fact" in a *Popular Science* sidebar. After all, given the global popularity of the Vitruvian Man, shouldn't a search of the Internet bring up tens of thousands of discussions about all this?

While a search certainly will turn up plenty of material about chakras and the Vitruvian Man, strangely nothing can be found to explain how a recursive musical matrix of harmonic waves could explain the human anatomy. In fact, there is no field of research even attempting anything like this. Without an existing theory, how are we to explain what we find here?

The answer is *Harmonic Interference Theory*. The evidence for harmonic resonance and damping in the Vitruvian model is undeniable. It is the harmony of the two that can explain the proportions in the Vitruvian model, as illustrated with the ***RECURSIVE INTERFERENCE MATRIX*** using a little simple math.

For starters, we can take the x-axis radius *R* of the circle from the fundamental matrix wave period as follows:

$$R = 2\pi = 6.28318$$

Then, we can use *R* to calculate the area of the outermost Vitruvian circle:

$$A_{circle} = \pi R^2 = 124.0248$$

If we now think of each ring as having its own (x, y) plane and therefore its own circular area, we can add up all twelve rings to find a combined area of about 200.674. This does not look particularly important at first, but when we divide this number by Φ we find the result to be equal to the area of the outermost Vitruvian circle. It tells us that nesting exactly twelve rings at increasing powers of 1/Φ is the same as multiplying the Vitruvian circle by the golden ratio – thus, *enfolding the Vitruvian circle inward*.

So with damping maximized at the outer ring, resonance starts to increase with each ring as it nests inward. This 12-fold resonance could account for the 12 pairs of major articulated joints from fingertip to toe, increasing by approximate Fibonacci proportions into the body, along with the 12 pairs of vertebrae along the spine. But if twelve is such a magical number in all this, what could we say accounts for our ten fingers and ten toes?

Careful measurement with one of the Red-Green bars indicates that the center of Leonardo's circle is actually a golden section of the square. According to Vitruvius, the length from naval to foot should form a golden ratio with the length from naval to head, and so it does in Leonardo's sketch. Given this, the length of one of the square sides is then a simple multiple of Φ and the radius *R* of the outermost ring.

$$H_{square} = R \times \Phi = 10.1664$$

From this, we can find the area of the square.

$$A_{square} = H_{square}^2 = 103.3555$$

Then, comparing the areas of the Vitruvian circle and its square, we find this amazing fact:

$$A_{circle} : A_{square} = 124.0248 : 103.3555 = 1.199982 \approx \mathbf{1.2 = 12/10 = 6{:}5\ ratio}$$

Squaring the circle is based on the ratio of 12 to 10 – thus, the Vitruvian model of the human body is a 12-fold recursion divided by 10, evidenced by our ten fingers and ten toes growing out of a lattice of twelve major articulated joints connected to twelve pairs of vertebrae. Furthermore, this ratio corresponds to Partials 5 and 6 in the harmonic series that together comprise the 6:5 proportion of a minor $3^{rd}$. When inverted to a 5:3 major $6^{th}$, we once again find the most resonant interval in a musical octave and apparently in the human body.

Mathematically speaking, squaring the circle is the only way to maximize resonance in a spherical standing wave. *This* is the real connection between the Vitruvian Man and Pythagorean harmonic science. With an estimated one-third of Leonardo da Vinci's notebook missing, we can probably assume that he understood this and drew out the entire set of recursive rings. We can also assume that whomever stole or destroyed the missing pages also knew this and chose not to share it with the world. One wonders what private collection these might be sitting in right now.

But even without those extra pages, we can now see why dear Leonardo concluded that the proportions of the human body are governed by the ratio of spirit (as the infinite $\pi$ in the circle) to material structure (as the infinite $\Phi$ in the square). As the most natural philosophy conceivable, the 12/10 or 6:5 ratio in the human body reduces to the simple harmony of a circle and a square represented by $\pi$ and $\Phi^2$ respectively.

$$\pi : \Phi^2 = 1.1999\ldots \approx 1.2$$

As an enfolding of the coherent structure of space, this 12/10 ratio in our DNA is the reason for the 10 rungs of the double helix that twists around an implicit dodecahedron on a central (harmonic) axis. Around this forms an egg to carry it through time until it is ready to free itself.

Thus we find that the circle represents an *inside-out resonance* of cells in the body while the square represents an *outside-in damping* effect to contain the outward push of cellular resonance. Like the standing wave and ring models of *Harmonic Interference Theory*, all organic life (whether a simple flower or a complex human) can be defined simply as energy exploding outward into space that is then pushed back inward into specific shapes, presumably depending on which harmonic proportions are emphasized by the DNA. Described as a "squaring of the circle" by Vitruvius over two thousand years ago, the general structure of the human body corresponds to the resonant circular proportion of $\pi$ inside the square-damping container of $\Phi^2$. It is because of this harmony in the body that we are able to instantly recognize and prefer the same proportions in music whenever we hear it.

# ***Burning Man***

*"When I am working on a problem I never think about beauty.*
*I only think about how to solve the problem.*
*But when I have finished, if the solution is not beautiful, I know it is wrong."*
*- Buckminster Fuller*

In stark contrast to the preceding discussion, the contemporary neo-Darwinian view within the fields of biology and anthropology hold that the appearance of life on Earth was driven entirely by chance from the molecular level up as a survival response to the environment. Indeed, the theory of evolution depends *exclusively* on natural selection and "survival of the fittest" to explain the shapes of life. There is absolutely no admission of cellular growth being guided by any matrix of waves, circles and squares and certainly nothing relating to harmonics as proposed here. This belief is summed up with the oft-repeated phrase: "If we could somehow restart life on Earth (or another planet) from the beginning, it would probably turn out completely different."

Presuming this is correct, how then are we to explain the correlations between the harmonic geometry produced by physical standing waves and the same cymatic geometry found around us in nature? How do we account for the Gaussian and sinusoidal shapes of our muscle tissue and the polar Gaussian knobs at the end of our bones? What natural selection process caused the brain to form according to a Gaussian Random Field (GRF), corresponding exactly to the initial conditions of quantum mechanical fluctuations during cosmic inflation? What universal explanation should we use to explain the exquisite 3-fold geometry of insects, triangular fin structures of fish, the reflected Gaussian wings of birds and 12:5 shape of mammals like ourselves? Surely harmonic principles are at work at all levels of nature, not just in atoms or solar systems.

The acceptance of harmonic science as part (not all) of the explanation of how we came to take the form we do requires one to extend one's view of contemporary science. Given the fact that the scientific method, particularly the biological sciences, must rely on physical evidence and experiment to prove that life form follows a harmonic interference pattern, we are left with no choice but to take a more philosophical and predictive approach. While it is true that there are a few frontier scientists, like British biologist Rupert Sheldrake, German biologist Fritz-Albert Popp, orthopedist Robert O. Becker and physicist Herbert Frohlich, that have found some very good evidence that electromagnetic fields are involved in cell growth and communication, the hard proof for how such a field could resonate into life has a long way to go before acceptance by mainstream science and even longer before it makes its way into our educational systems. In the meantime, perhaps we should take a more pragmatic approach in convincing ourselves of harmonic life as we wait for science to catch up.

Let's say we could show how to harmonically construct an archetypal human being. Rather, what if we could *reverse engineer* the few basic steps that life takes to grow into the geometric form we see in the mirror? What if this could establish a consistent pattern of growth based on the same harmonic principles of resonance and damping found in the Gaussian interference distribution and $\mathbb{Z}/12\mathbb{Z}$ set models? Would this be enough to convince most people that life really is a form of crystallized music playing inside a quantum lattice? Would it be enough to convince you?

## Redefining the Human Body as Recursive Crystallized Music

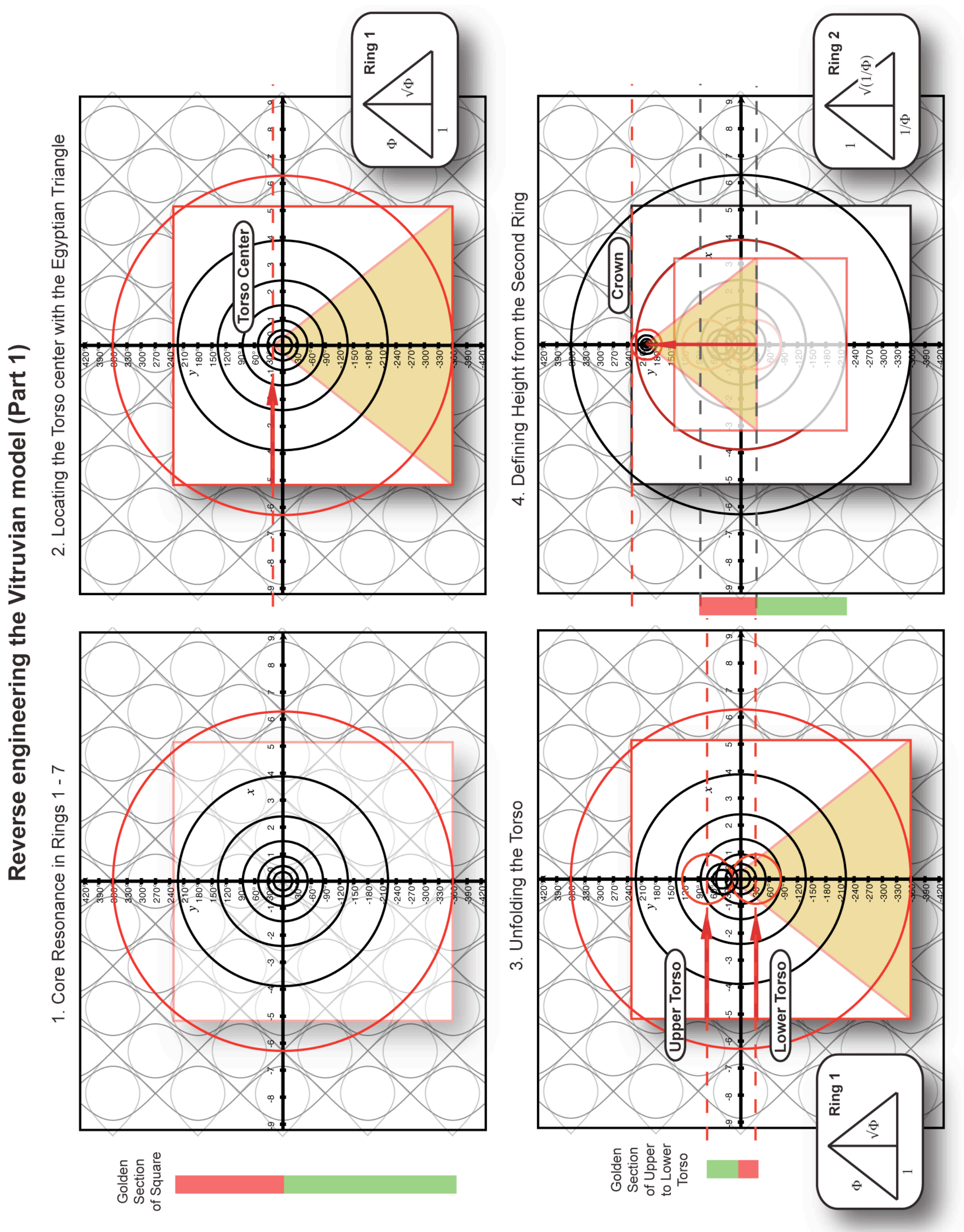
In Figure 127, an Egyptian triangle is used as a kind of Φ-damping ruler to stepwise reconstruct the Vitruvian model inside the ***RECURSIVE INTERFERENCE MATRIX***. For convenience, we will begin from the outermost ring and proceed inward ring by ring, subdividing the torso and then migrating up the spine to continue inward to the twelfth ring in the center of the brain.

Like a pebble tossed into a pond, cell growth must begin at an inner point of resonance and ripple outward through increasing layers of recursive damping. Reversing this process, we would start at the least resonant locations at the fingertips and toes and proceed inward toward the originating point of maximum resonance. In this way, we can reverse engineer the process of harmonic growth in a stepwise fashion.

So, beginning with the first (outermost) ring in Step 1, the circle is at maximum damping and the system is silent. But in Step 2, the introduction of the Egyptian triangle at the bottom of the Vitruvian square in the first ring points us to a calm location *just above* the center of the outer ring system. This identifies a new damping point within the resonantly "squared circle" that is neither centered with the circle nor at a strict golden ratio with the square, instead at a $\sqrt{\Phi} / 2$ proportion to the structural square. This proportion aligns with the circumference of the small inner circle, suggesting a new torso center.

In Step 3, the fifth ring migrates upward to the new torso center, but then inexplicably divides into an upper torso and lower torso version of the fifth ring (shown in Red). This pair of rings is now offset vertically from the polar origin. This causes *two centers* to form at a golden ratio with the origin, indicated by the Red dashed lines. We will return to this shortly.

**Figure 127 - Reverse engineering the Vitruvian model (Part 1)**

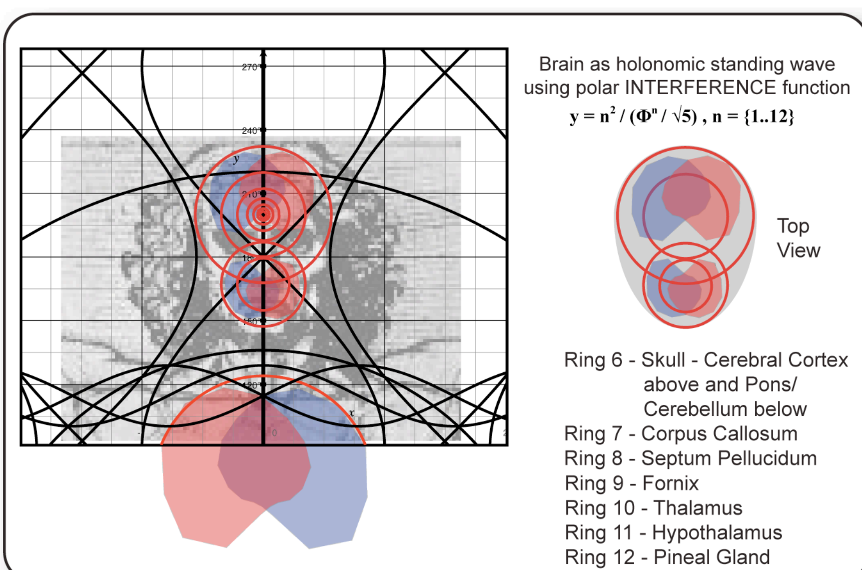


In Step 4, the same triangle is used once again, but this time scaled to fit in the square of the second ring and aligned at the center of the lower torso fifth ring, which also happens to form a golden ratio with the top of the fourth ring. Taking this as a spinal column, the Egyptian damping triangle appears to propel the remaining six rings (six through twelve) upward to migrate into a new origin at about 200 degrees on the vertical axis. This appears coincident in size and location with a head sitting on a spinal column that has migrated up from (or rather down to) the dual ring torso below.

More and more alignments can be found using the Egyptian triangle within the recursive rings. From the third ring in Step 5 we can again locate the top of the spine and the center of the brain. Polar ***REFLECTIVE INTERFERENCE*** cardioids are again a match for the physiology of the right and left lobes of the brain resonating around this point. To form the head, inner rings seven and eight migrate down into the jaw and mouth, aligning into a familiar egg shape. In fact, the entire head forms an egg in both vertical x-axis and recursive z-axis dimensions.

At this point, we need to pause the reverse engineering process to take a closer look at the head in Figure 128. Zooming in, while retaining the relative scales of all the components from the previous diagram, we can now clearly see how the rings nest into the center of the brain, or "third eye." One cannot help but wonder if this same ring system was also once part of a larger harmonic science in the early days of Indian Hinduism.

**Figure 128 - Ring correspondence to skull and brain with Gaussian cardioids**



Certainly there is no record of a harmonic matrix like this in the Vedas to explain why there should be a third eye *or* how the colors and locations of the chakra system were determined *or* where the idea of mandala patterns really came from. But what else could have led the Brahmin to these conclusions? Could it be that this matrix represents the very foundation from which ancient spirituality and theosophical mythology sprang? Might this matrix even be visible during intense meditative or altered states?

Those familiar with such things know the Hindu third eye is oriented with the pineal gland located at the center of the brain. In the diagram, this location doubles as the origin for the dual cardioids of the polar ***REFLECTIVE INTERFERENCE*** function. Many believe this to be an especially resonant location and the source of all spirituality.

It is a fact that the pineal gland naturally produces the endogen molecule DMT, found in controlled laboratory tests to produce extreme and immersive hallucinatory experiences ranging from visions of elves and aliens to angels and even the Buddha. Researchers speculate that this gland is probably the source of everything from dream visions to intense religious experiences, giving DMT its nickname as the “spirit molecule.” Given this, what better place could there be than the pineal gland as the origin of cellular resonance in the body?

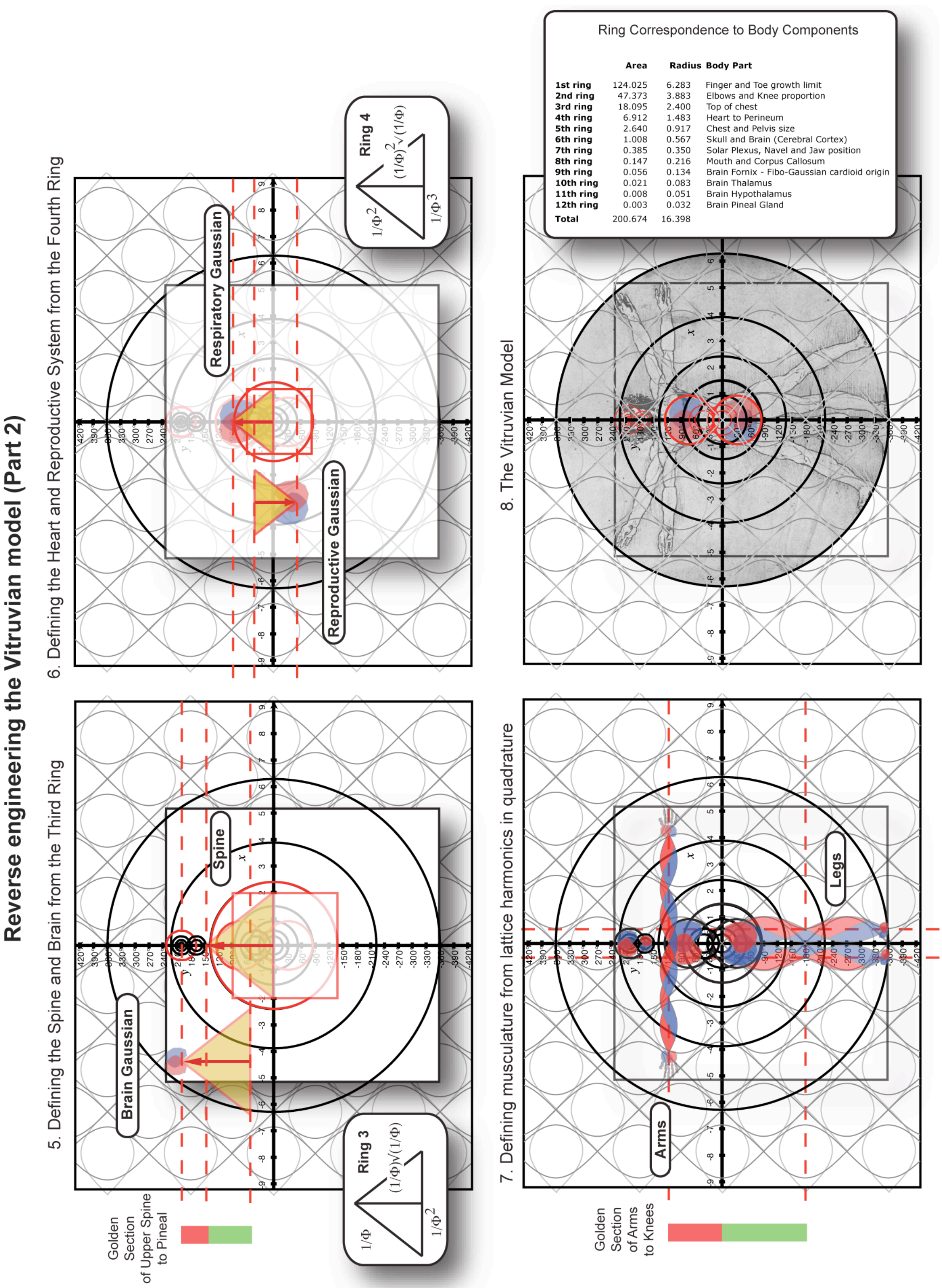
Just below this location we find the nasal bone, a nodal point providing a sinusoidal crossing through which the sinuses and throat form. It is easy to see how the higher frequency of Ring 7 would form two eyes either side of this node, complete with spherical eyeballs fitting into the surrounding lattice at a finer scale. Migrating downward now below the nodal point, again using the Egyptian triangle as a damping measure, we find Ring 6 outlining the mouth. Within this fits a smaller set of dual cardioids suggesting a tongue in the intersecting region.

One other prominent feature to note is what appear to be horns protruding through the space between the waves on either side of the head. Interestingly, this happens to be a popular symbol in certain Pantheistic practices, often represented using deer or elk horns. But as a physical property of the matrix, this space between the waves represents a weak, damping region in the harmonic interference pattern through which ears and horns might resonate. What other explanation might there be, other than of course just another coincidence in a long laundry list of coincidences?

It is not too unlikely that the Christian horned devil is really a veiled reference to a long lost knowledge of this very harmonic wave matrix. Perhaps early Church leaders considered the matrix a pagan threat in the same vein as the apple, pentagram, golden ratio and Devil’s tritone, then demonized it using the archetype of a horned beast. Today there is no trace of this matrix anywhere – not even an amusing side note, just an awkward silence.

Continuing the reverse engineering, we pick up with Step 6 in Figure 129.

## Figure 129 - Reverse engineering the Vitruvian model (Part 2)



The Egyptian triangle now locates the center of the upper respiratory system in the fifth ring of the upper torso starting from the fourth ring. Similarly, the center of the reproductive system is found in the lower torso. Placing the polar ***REFLECTIVE INTERFERENCE*** cardioids in these rings, we can represent the geometry of the heart-lungs and pelvis.

In Step 7, the ring structure is now wrapped in harmonic waves from the surrounding lattice. The arm musculature corresponds to Partial 3 as it runs horizontal at the third ring level while the leg musculature corresponds to Partial 1, which runs vertical at the first ring level.

In the last Step 8, the original Vitruvian model is then placed behind the ***RECURSIVE INTERFERENCE MATRIX*** for final comparison. The alignment is exact. Starting at the maximally resonant twelfth ring level in the center of the brain and growing in reverse, the ring progression can be seen to emanate outward through six rings in the head, down the spine to form the torso (splitting ring five), then down to the origin at the navel and straight out through increasingly damped locations in the limbs, shrinking as if following an oven timer: 4…3…2…1.

With this, the basic growth process is represented as a recursive application of resonance and damping within a harmonic lattice. Our reverse engineering appears to have answered the question of whether harmonics play a role in the human body. But there remains one other puzzle. How does the fifth ring "know" to split into an upper and lower torso? After all, this seems to be responsible for triggering migration of the remaining rings up the spine. Once again, the answer appears to have something to do with the same "half twist" found in a diatonic musical scale.

Looking back at the calculated areas in Figure 129, the area of the fifth ring can be calculated at approximately 2.64. On either side of this number we find the outermost "squared circle" proportion of $\pi : \Phi^2 = 1.1999$ (or 1.2 rounding up) and its reverse squared proportion of $\pi^2 : \Phi =$ 6.099 (or 6.0 rounding down). When we take the area of the fifth ring as a proportion to each of these square proportions then multiply both together, we find the exact transition point where damping overcomes resonance in a standing wave. The half twist occurs at 5.

***Transition point in ring recursion from resonance to damping***

$(A_{Ring\text{-}5} : \lceil(\pi : \Phi^2)\rceil) \times (\lfloor\pi^2 : \Phi\rfloor : A_{Ring\text{-}5}) = \text{Torso Split}$

$(2.64 : 1.2) \times (6.0 : 2.64) = 5$

The reversal to square Φ instead of square π in the transition across a 12:5 proportion is probably our best explanation for the mysterious split of the torso. At and below the fifth ring, we find π and cyclic harmonic proportions in control to resonate and cause growth – but, as we reach the fifth ring, Φ becomes dominant to reflect or dampen the inner resonance, slowly stopping growth in the outer four rings and stopping completely at the toes and fingertips.

We are reminded of the earlier discussion concerning the half-twisting golden spiral that swirls into the Landau damping well, drilling five orders of magnitude down into the heart of a standing wave. Then there is the *frost line* at the middle of our solar system that separates the inner planets from the outer planets. These are all phenomena produced by Partial 5 and its square root in harmonic interference as it controls and contains 12-fold resonance.

In the human body, this little half twist at the fifth ring must be the defining event that causes the ripple upward, crystallizing into the spine and nervous system. The spine, measuring a half-period or π radian in length, is then divided into 24 articulated vertebrae (as the proportion 12:5 = 2.4) with each scaled by π/24 or 1.309:10. You might recall that 1.309 was the very same proportion ($\Phi^3 : 2\Phi$) found earlier to describe the Rojas egg model. Putting this all together, we see that the spine follows the exact same proportions of an egg as it ripples up from the half twist at the solar plexus.

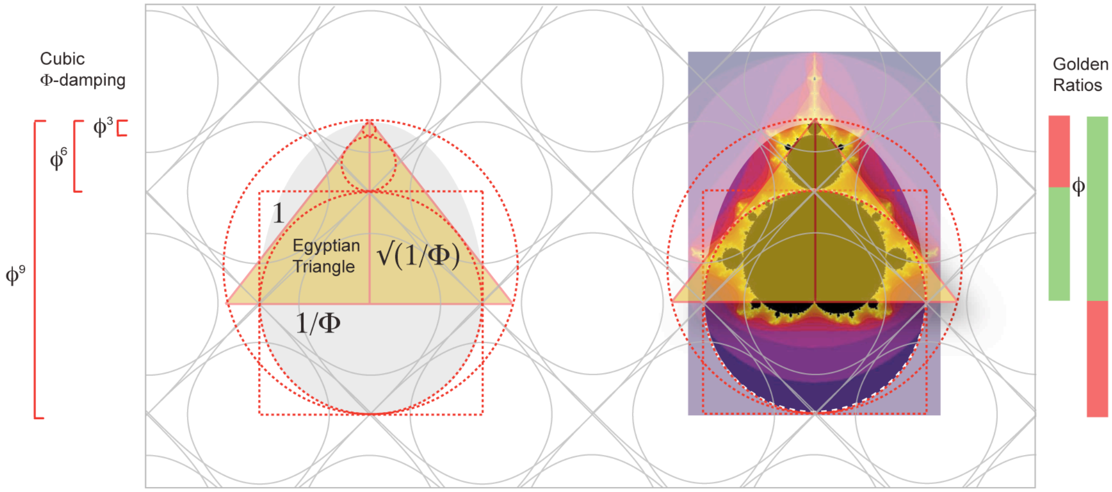
After splitting the torso, ring five then appears to also influence the growth of internal organs. In fact, the inner organs either side of this point appear to match the size and geometry of rings six and seven, shown splitting along with ring five. This is shown in the previous Figure 127 as the smaller concentric circles either side of the division point.

If we look at this like we did the egg and brain models where an inner sine ring migrates out to align tangent with the outer cosine ring, then the torso fits perfectly within a geometric model of two eggs interpenetrating one another. We might describe these as one “absorbing egg” oscillating to breathe and pump blood while the other “refracting egg” oscillates to digest, eliminate and reproduce. Of course, the human torso does not really look like two interpenetrating eggs, but it does look like something very similar.

Earlier a geometric equivalence was drawn between the Earth-Moon system, the Egyptian triangle and the Rojas egg model (Figure 124). A similar equivalence was introduced at that time between the cubic proportions of the Mandelbrot Set and the Rojas egg model. When we now combine these, we find that the Egyptian triangle and Mandelbrot Set both correspond to the geometry of an egg through the golden ratio. In this way, they represent a *golden equivalence relation* and could be used interchangeably.

On first impression, this fact may seem incredulous. How could these symbols seep into our popular culture without ever being associated with one another? In Figure 130, notice how the edge of the Φ-damped Egyptian triangle follows exactly the cubic Φ-damped circles of the Mandlebrot Set. Then notice that the alignment is so perfect that the “yellow lightening” of the fractal seems to emanate right out of the edges of the pyramid, suggesting the popular notion that a pyramid of these dimensions has the ability to capture and store energy like a capacitor. And finally, the tip of the pyramid appears to emit a Mandelbrot “beam” out of its tip just like the iconic Luxor hotel in Las Vegas. Where did such ideas originate if not from something like this, perhaps floating around in some yet to be understood global consciousness or akashic record.

**Figure 130 - Mandelbrot Set as Egyptian pyramid**



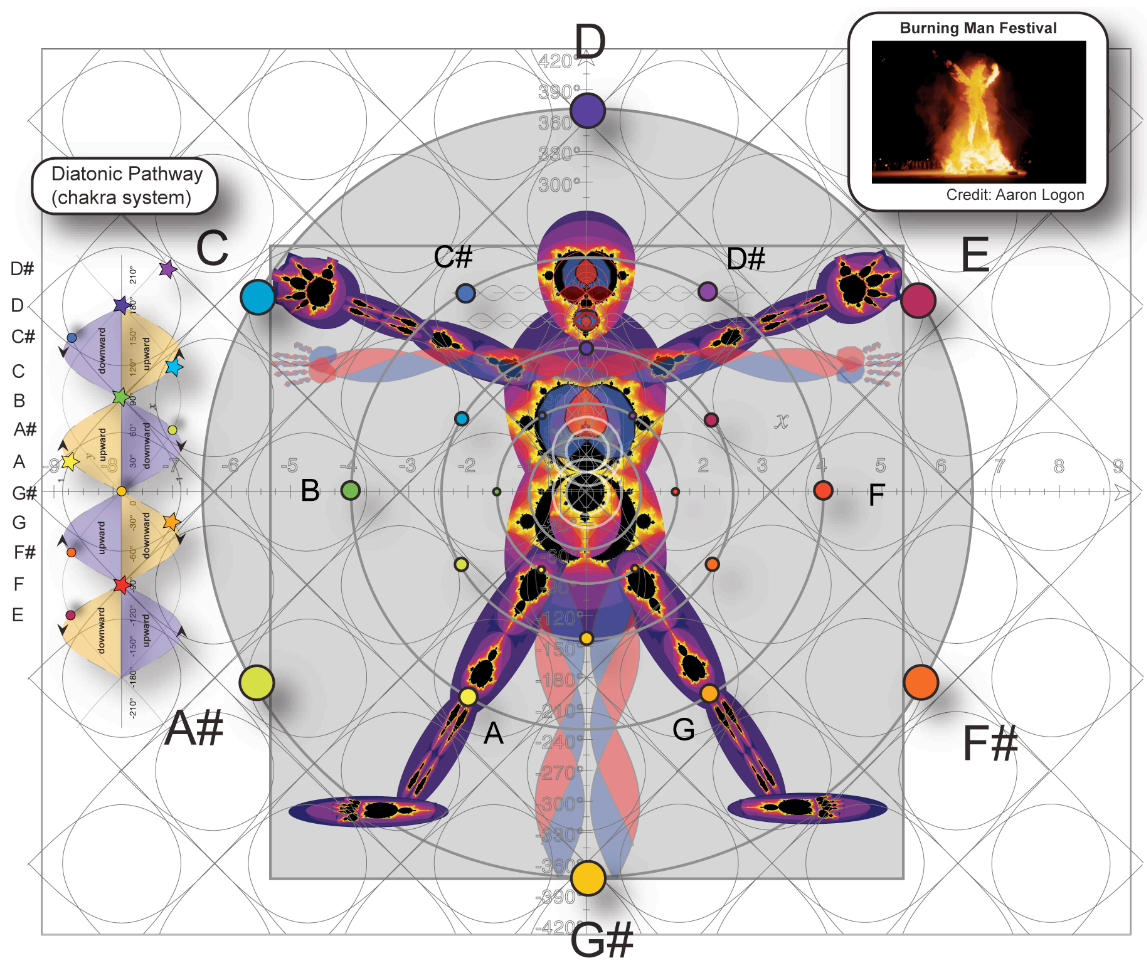

Principles of harmonic science have clearly been around and in our collective consciousness long before computers ever suggested the possibility of fractal mathematics. The importance of the golden ratio as a spatial container was well understood by the Egyptians, only now being quietly rediscovered through the recursive calculations of computers. As additional scientific evidence mounts for a unifying context based on harmonic resonance and damping, we can expect to see the harmonic lattice and recursive matrix become a sort of Rosetta stone to translate between past and present knowledge systems or possibly even as a visual database for cross-referencing diverse organic geometries.

With this last geometric equivalence between the Egyptian triangle and Mandelbrot Set, we can at last "play everything we know about music theory all at once." In Figure 131, the Mandelbrot Set now acts as a logical enhancement to the rings, replacing the Egyptian Triangle with a fractal harmonic lattice closer to the geometry of the human body. Based on its fiery appearance, we will name this harmonic archetype *Burningman*.

Like two eggs (or triangles) interlocking around what would be the seventh ring in the torso, the lower Mandelbrot points up while the upper Mandelbrot points down, resembling an esophageal pathway through the figure. On top of this is then the dihedral $\mathbb{Z}/12\mathbb{Z}$ division of the rings, alternating sine and cosine as they progress inward. And between the outer two rings is a Tritone Function, converging into the center of the upper Mandelbrot torso (or chest) through the large bulbs on either side and in perfect alignment with the Tonic major 3rd on Ring 3. In all this, we can see a definite correlation between the ***RECURSIVE INTERFERENCE MATRIX***, the Chromatic Dual Ring model and the Mandelbrot Set, together representing a harmony between chaos and order in the archetypal human form.

The arms, legs, hands and feet are also represented by interlocking Mandelbrot Sets, spaced between the rings while scaling ever smaller as damping increases in the outer four rings. The torso, itself a compound interlocking "half twist" fractal, fits together perfectly as two intersecting Mandelbrot eggs, creating an interior damping space for various polar Gaussian structures. It shows how electromagnetic currents might be found to flow through the geometry of the human body.

**Figure 131 - The harmonic *Burningman* archetype**

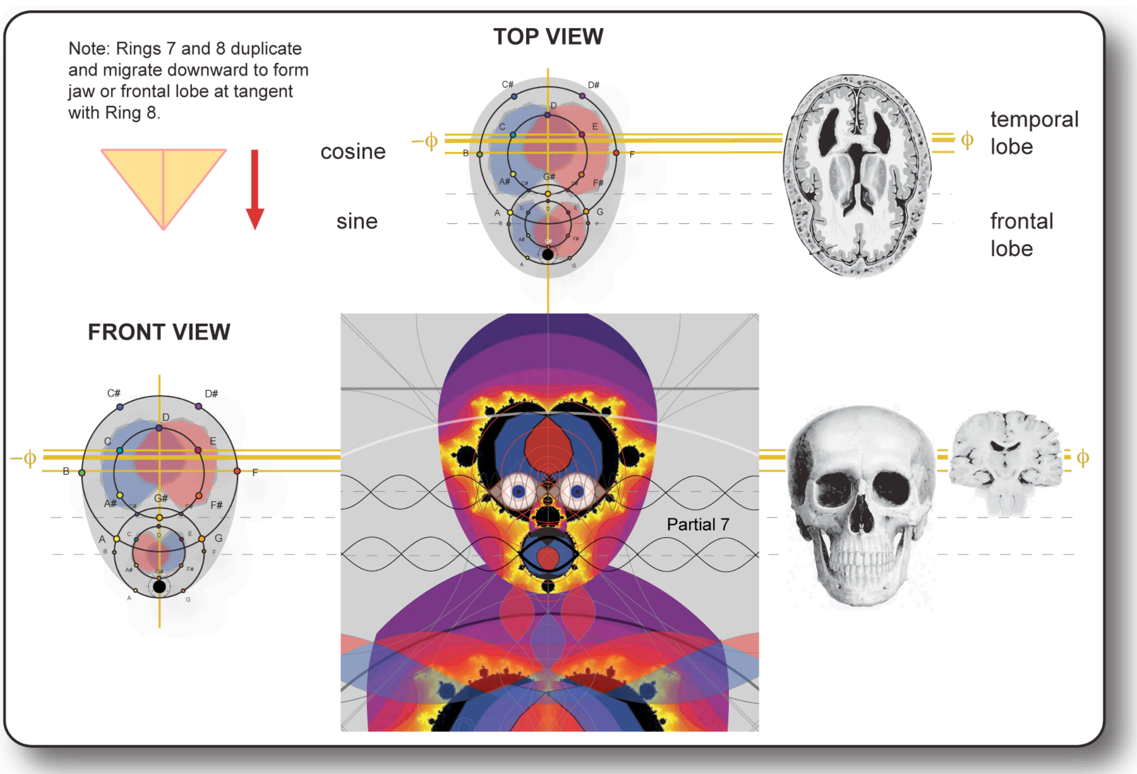



As for "the face" I stumbled upon some thirty years ago, Figure 132 shows how the human head is that of an egg-shaped geometry both vertically and horizontally. It shows how each egg unfolds from a ***RECURSIVE CHROMATIC DUAL RING*** model according to the fractal geometry of the Mandelbrot Set. The Landau damping region of the Fibonacci series (spiraling into the golden ratio) also aligns with various folds and cavities, telling us that the human brain and skull are not only proportioned by the golden ratio but also carved by its damping action. My discovery of a face in the intersections of tritones was really just an instance of this 12:5 ring matrix.

As for facial features, the eyes and ears coincide with the major 3rd {F#, A#} in Ring 1 while the sinusoidal shapes of the eyes and mouth match Partial 7 (shown to scale). In accordance with the earlier vision archetype, the higher harmonic partials can be taken to create eyelids, irises and lips. Polar Gaussian cardioids then model the remaining standing wave architecture of the brain as well as the odd structure of the mouth and tongue.

Nature cannot help but act efficiently, using the same harmonic geometry again and again in different scales and orientations. Given enough time, the entire Vitruvian model could be constructed down to the smallest detail using only a small set of recursive algorithms based on this geometry. Indeed, stretching the matrix while emphasizing different harmonics will produce any organic morphology, creating endless varieties of simulated people and geometric creatures.

**Figure 132 - The head as *RECURSIVE CHROMATIC DUAL RING* model**



Though long forgotten, this idea has reappeared in recent years as the centerpiece of a bizarre countercultural event known as *The Burning Man Festival*. Each year a large number of people gather in the Black Rock desert of the western United States to celebrate art, philosophy and a new tribal vision of society. As a kind of pagan symbol for the reinvention of Man, participants set ablaze a towering wooden effigy known as the Burning Man (see inset in Figure 131).

What its founders intuited as the rebirth of modern man and society now appears to be a resurgent awareness of Pythagorean harmonic science. *Burningman* is nothing less than a bio-harmonic human archetype – a highly resonant superhero, if you will – returning out of the desert to transform the world. Like a mending function for a fragmented society, *Burningman* brings a *new mythos* of coherence and wisdom back from our distant past.

# *New Mythos*

*"Nature is not simply the random flight of atoms through electromagnetic fields. Nature is not the empty, despiritualized lumpen matter that we inherit from modern physics. But it is instead a kind of intelligence, a kind of mind." - Terrence McKenna*

Ok – time out for a reality check.

Are we really supposed to believe what we see here? Are we to believe that all life – including ourselves – grows according to the same physics as a plucked guitar string? With science working so hard to convince us otherwise, can it really be that simple?

The first time I drew that little face in the center of a grid of circles, I too had a hard time accepting it. How could music have anything to do with the structure of life, I told myself. And if it really were true, why wouldn't everyone know about it? Surely, I should be able to look it up in my encyclopedia or find it in a library somewhere. Who would possibly want to keep such important information secret?

Today, I am no less amazed than I was back then. Everything fits so easily into the harmonic lattice and recursive ring matrix. All we have to do is draw it precisely, follow each scale of Φ-damping rings and harmonic waves, and presto – organic objects just snap into place. Nothing has to be forced – just follow the geometry provided by the matrix using a planchette shaped like an Egyptian triangle and everything will work out just fine.

For most, the idea that our body and brain could be constructed from a musical interference pattern of waves and rings probably comes as a surprise, if not outright shock. Re-imaging our self as something resonating up from the quantum and atomic levels seems a bit spooky and not something most want to think about, much less talk openly about. When it is discussed, say in a meditation or yoga class, it is almost always presented in the form of a metaphysical mystery with no real physical system to explain it. Outside the setting of mystical eastern practices, we would never hear something like this in a church, a science class or a doctor's office.

The only explanation we can find to explain the complete absence of harmonic science in Western civilization today is the lengthy and successful campaign of social control begun long ago, now institutionalized into every aspect of modern society. We are continually warned against this forbidden pagan knowledge for fear of eternal damnation and social embarrassment. After two thousand years of continuous negative reinforcement, the idea of life as a rational geometric structure built up from harmonics now seems outlandish, ridiculous and silly – so much metaphysical thinking. Yet as we can clearly see, harmonic proportions in life are no different than the vibrating air in a flute or the harmony of a string quartet. How could it be otherwise?

As I was considering whether to cover the topic of a bio-harmonic human archetype, I described "the face" I found years ago to a very open-minded physician friend of mine. He did not hesitate to offer two very sensible recommendations. The first was to tell me it would be of absolutely no use to medical science, even if it were true. And, the second was that I should *not* under any circumstances include it in this book because it would greatly undermine any credibility and acceptance for my music theory.

Well, I thought long and hard about it before making my decision. I initially concluded *not* to include this information here, telling myself to stay focused on the topic of music theory and wait to include it in another book. But in the end, I changed my mind. I simply could not in good conscience engage in self-censorship and exclude it from this discussion. My rationale was that the role of harmonics in shaping life is the ultimate reason we can recognize and enjoy music in the first place. The fact that we evolved according to harmonic principles is the only correct answer to the countless questions raised by cognitive psychologists and musicologists. Everything else is just an elaborate tap dance to avoid facing the truth.

Another reason I decided to include it is the harmonic structure of life is central to the history of ancient theosophy, mythology, religious orders, secret societies and esoteric symbolisms. Fear of its disclosure was clearly the motivating factor behind the suppression of harmonic science and the social control systems still in place today. How can we even begin to talk about the development of music throughout history without understanding the context of the belief system and political regime within which it developed?

If there is any doubt as to why harmonic science was suppressed for so long, this should be your answer. A resurgent awareness of the harmonic structure of nature might lead people back to a pre-Christian sense of spirituality in Nature, therein threatening the power of the Church to shepherd its flock. It would also direct science into a collision course with Western religion while bringing back an unwanted guiding philosophy to the natural sciences. It would severely undermine the accepted (and unsustainable) conventions of Western society, with all its broken political and financial systems. It would eventually reunite us with Nature and with ourselves. These are the unspoken truths that have remained *sub rosa* inside a continuing *complicity of convenience* for so long.

We might now ask can this ever be changed without precipitating a total collapse and reinvention of Western civilization? Is it even possible to radically change our views on music, religion, physics, biology, anthropology, psychology, cosmology and mathematics without falling into utter chaos?

The answer is a resounding YES! – but, only if we take a more measured and systematic approach; one less susceptible to misinterpretation. After all, the reason the study of harmonics in music was cast out of the sciences in the first place was the degree to which it had been hijacked by uninformed metaphysical cults and politicized religious organizations. To counter this possibility and promote positive change in the future, I would recommend that harmonic science be added as an integrating educational field for today's compartmentalized science. Doing so would create a *social presentation layer* for all natural knowledge, insulating it against politicization while minimizing the impact on our existing scientific institutions.

The purpose of this social presentation layer for science would be to organize the knowledge systems of Science and the Humanities into a unified system – a kind of *new mythos* for society – that presents harmonics and music as the Theory of Everything. The benefit of such a system would be a guiding philosophy that informs everyone about nature's coherent structure and how we fit into that. It would also bring a more interdisciplinary and integrative approach to the compartmentalizing scientific method to help direct and prioritize each field of study; reducing time wasted on dead-end research. Only in this way might we hope to match the social order of ancient civilizations while maintaining the modern benefits of rationalism and rigor.

As it stands now, many people sense something is missing. The Internet has incubated a growing awareness that only a small part of the story is being told in our schools and media. The belief that everything is random, arbitrary, quirky, nonsensical and mechanical – Burgess's *Clockwork Orange* writ-large on the cosmos – is beginning to come apart bit by bit. Like the sudden demise of our 20$^{th}$ century financial systems, Newtonian-Cartesian dominance is also evaporating under the growing influence of quantum theory and universal harmonic principles.

The Internet has been the single biggest factor in overcoming the truth embargo about harmonics. There has been a great surge of popular books, alternative websites and online videos that discuss such things as ancient mysteries, zero-point energy and new theories like torsion physics and plasma cosmology. People from all walks of life are beginning to realize the deleterious effect anti-harmonic social control systems have had on the world and are finally beginning to question what they have been told to believe for so long. There is a growing hunger for truth and the time is right for harmonic science to once again explain the world as music.

But as the *new mythos* takes hold, we need not weaponize it into proof of one god over another. There is no need to anthropomorphize spirits in the forest or tell tales of angels falling from the sky, becoming evil demons that whisper in our ears. It requires no church and no dogma. The truth need only inform us of our birthright and realign our worldview to the benefit of one another and the planet.

This *musica universalis* view of nature should be tangible and undeniable. It should be in our hands and at our fingertips. It should be visual, audible and accessible to all. Most importantly, it must be felt. Anyone should be capable of enjoying it and internalizing it. We should see it in the mirror in the morning, whenever we play our favorite instrument or when we look into the sky. We should see it in our dreams, both sleeping and awake.

Wherever people gather to discuss, play or share their music, they should find these archetypes of harmony spinning positive their vision of culture. Unlike the fragmented and meaningless belief systems now in vogue, the new cultural mythos must provide a forum for *meaning* of all kinds. A meaning that knows no particular flavor of god, no sacred taboos and no exclusions – just one rooted in the truth of a universe coherent from the bottom up.

As we peer into the neo-mythical musical theatre nestled in the palm of our hand, it is here where truth will float back from *the Cloud*. Materializing from inside a shiny new Apple®, we will no longer fear the forbidden fruit. At its core is the path to knowledge; a coherent pathway to the inmost chamber where the Burningman still lives. Through its invisible seeds will we again hear the forgotten songs of Venus and Harmonia.

With a click-drag-drop, a murmured word and a gentle nudge, the Burningman returns to center stage. Around him trumpets the Music of the Spheres. Inside him breathe the geometry of atoms. Through him springs the life of a resonant tritone. From his music comes the sound of order and from this order comes a new song – a deeply philosophical song explaining things words cannot express. Like Bach before us, it will be this song that frees the Burningman inside.

*"We live with our archetypes, but can we live in them?" - Poul Anderson, science fiction writer*

This is the *new mythos*. Inside the seething matrix swirls a zoological wonder of bio-harmonic creatures and wave-like animals large and small, bathed in the colors of the treasuries, here to delight us all. They may whirl and twirl or swim upstream. They may glide upon the winds or stroll lazily along through some exotic landscape. With them we paint a beautiful picture of music. Upon them we orchestrate our dreams. Through them we learn that we are music too.

As the natural order seeps in like an IV Drip of pure consciousness, we witness virtual reality users transformed into physical reality users. "Everything is relative" sounds tired while "everything is proportional" hotwires our imagination. Science, Art and Religion will mend their schisma, reuniting fractured pieces while shattering old dogmas. Even the most fear-hardened will not escape this *new mythos* as it lifts the burden of social control carried on their weary shoulders for so long. Only then will social interference be transformed into the beautiful music of harmonic interference.

But who dares to be God in this musical theatre? Is it the user who creates the song or anthropomorphizes its creatures? Is it the energy matrix with its ever-oscillating harmonic currents? Is it the hardware that runs the cubic lattice or our personal archetypes waiting patiently within? To whom should we pay homage?

As it always has been, the answer can only be found at the center of nature's perception of itself, there in the inmost chamber of the *Grand Scientific Musical Theatre*.


singular
11:11 AM
12 : 5
Through a lens of fiery resonance
seek a crystalline key of silence
to unlock the musical matrix
and free the burning man

# Epilogue: Unconventional Wisdom

*"If Heaven wills it, thou shalt know that Nature,*
*Alike in everything, is the same in every place" - Pythagoras*

During the course of this writing I could not help but feel a growing concern about the current state of our educational system. As one who found harmonic principles to be essential in my career and an endless source of personal inspiration and motivation, I find none of these ideas in our schools to help young people grapple with even the most basic questions of life. Nothing to explain the natural coherence in music, physics and perception. Nothing to connect self-image with nature and no guiding philosophy to make sense of society. Without harmonic philosophy, where can one turn to find real meaning and a path to wisdom?

If you asked someone today to define "wisdom," they probably could not explain it. A wise man might be smart or intelligent, yet most would agree this is not enough. A wise man might have accumulated a great deal of knowledge, but is knowledge alone wisdom? Would you be wise if you are intuitive and caring and give your friends good advice? How about if you are very talented at something – a prodigy or genius perhaps – able to do things few others can do as well. Does this make you wise?

Those who have studied ancient Greek civilization often say they were a wise people. They modeled their culture and political systems after the coherence of harmonics and worked to communicate these ideals to all their citizens. Why after thousands of years is our modern society not as wise as the Greeks?

Seeking wisdom is considered such a fool's game in this day. Mainstream science has informed society that universal truth is not to be found and we have little choice but to accept the fact that everything must remain in limbo as we search. Protected by the unquestioned sanctity of the scientific method, it is the incompleteness of science that now blinds us to even the simplest truths. Truths like everything *is* proportional, everything *does* need a little slack built into it to work and everything *is* interconnected from the bottom all the way to the top. How can we ever hope to find any Theory of Everything and become Wise Men when our starting premise is a half-baked belief system that explains nature as random and arbitrary?

Where is the wisdom in avoiding harmonic science as a unified field of natural study? Where is the wisdom in teaching music based on centuries-old religious propaganda rather than physics, physiology and mathematics? And where is the wisdom in failing to explain to society that nature is fundamentally coherent? The sad truth is our educational system makes no claim of teaching wisdom, instead leading us away from this path toward strictly vocational interests in an increasing anti-philosophical and money-motivated society. When was the last time you even heard someone use the word "wisdom" at school, at work or in your own home?

Harmonic science says a great deal about wisdom. It tells us what people once thought was important. It tells us how our physical reality works, how we emerge from it and how we are able to perceive it. It tells us about the forces that give geometric shape to life and about the foundation of order that underlies this magnificent phenomenon. Harmonic science reconnects us to nature through the most accessible and universal language possible – music.

The political hijacking of human spirituality remains the greatest feat of engineering the world has ever known. The avoidance of harmonic principles in Western religion has made possible an omnipresent and increasingly omnipotent social control system with more momentum than a fleet of oil tankers. Advertised and distributed worldwide through a vast network of neighborhood franchisees, it has a brand bigger than Coca Cola and a market penetration exceeding the common cold. Pagan harmonic knowledge has become our most despised archenemy. Like kryptonite to Superman, it was the diabolical tritone that might rise up to destroy the brand, therein denying the Church its business of social engineering. Evil became the truth of coherence, which ironically became the first omission rule of Science.

Separation of harmonic science from mainstream science has had the chilling effect of cleansing all meaning from society and with it any notion of ever finding it again. The resulting worldview of absolute scientific belief no longer attempts to find philosophical meaning in the cohering principles of nature. The scientific method is not even intended to answer such questions, instead diverted to the comfort of an engineered reality. Evidence of a natural order is obfuscated in incomprehensible languages and arcane processes – all seemingly designed to avoid the revelation of coherence that could threaten the popular meme and offend the faithful. Best to keep music and its harmonic principles safely put away into small discrete compartments.

The absence of a Natural Harmonic Philosophy where people might go for answers to life's questions has had an enormous impact on where things stand today. To fill the void, artificial belief systems have taken root and spilled over into our economics, politics and media to create a "relativistic culture" with no interest in attaining wisdom. Without a tangible idea of how harmonic principles work in nature and in our selves, any sense of a universal truth has become laughable and irrelevant. This is at the bottom of our modern nihilist philosophy in a post-modern world.

All our thoughts and feelings grow up inside this unnatural system, taking less relevant forms in the Gaussian shape of our language and the character of our thoughts. While we seek harmony in a sea of noise, our best thoughts and words remain swamped. Without proper guidance, they have become misshapen and our social order fragmented. As a result, society no longer seeks a noble purpose, opening wide the door to depression, crime, suicide and war.

To counter this, recent generations have saturated their environment with music in a desperate attempt to plug back into something good, something harmonic. The ubiquity of mobile music devices and their omnipresent ear buds are a testament to the need for psychological coherence, an emotional inoculation against institutionalized violence, vapid entertainment and designer mind-viruses. To many, music is their only refuge in an increasingly artificial world. The modern lifestyle based on a background musical soundtrack has become the only substitute for a natural philosophy gone missing.

The time has come to rethink our educational systems and social structures to once again admit the possibility of coherence in the cosmos. This should not be seen as a threat by anyone. No need to consider coherence proof of God or evidence of some diabolical power at work – just the humble realization that life is an *inevitable and non-accidental* pattern emerging from a harmonically organized universe.

Only through the unconventional wisdom of harmonic science might society find its way back to coherence. Only through the resonant lens of a tritone can we once again see the world as a crystallized music. And only through the golden ratio will we learn that damping is our friend, here to extrude energy into so many fun and familiar shapes. It is only through the truth of physics that we can finally understand the love of Venus in that silly jiggly bowl of Jello most still think of as empty space.

Easily understood and explained, these are the ideals that will guide society back to greatness. What have we got to lose? We might even agree on a definition of wisdom and use the word without embarrassment from time to time.

Those wise old musical philosophers and astronomical priests knew exactly what they were doing. They designed their mythological stories to explain the balance of resonance and damping so that others might understand and be lifted up by it. Their personified archetypes embodied the harmonic laws of the spheres, alternating from anger, fear and war to love, beauty and compassion. Floating up in the clouds, they dispensed their natural philosophy in a carefully scripted balance of ethos, acted out upon the stage of a vast musical theatre. Though they might intrude upon the affairs of Man now and again, stirring up a hurricane here or ravaging disease there, they always offered hope – a hope founded in coherence.

In the days of old Egypt and ancient Sumer, harmonic science once penetrated to the core of politics, law and culture. Revised and extended by the Greeks, the struggle of Osiris and Anubis in the Underworld became Kore holding a double Gaussian with Venus at its heart. They spoke of the forbidden knowledge in their stories, always struggling to mend the spiral into a circle to ensure that stability would rein over chaos.

From this came social order and a wise government in full accord with nature's physics. It was a vision reflected in Art, Music and Architecture, inspiring vast social projects to integrate, extend and celebrate the archetypes of these physics in colossal monuments. It was a grand project of integration, not fragmentation.

But beneath it all simmered a fear - not of some mysterious forbidden knowledge spoken by an evil dragon-serpent, but the fear of damping – the fear of death. Hercules faced this fear in his twelfth labour as he fought the tri-headed dog at the entrance to the Underworld. Eris told us of it when she threw the golden apple into the council to start the Trojan War. Harmonia's necklace warned us of the silent Fibonacci serpents that lay coiled, waiting to strike inside its perfect circle. Even Atlas with the world in his arms suffered mightily under this fear in the Underworld of terminal damping.

As the greatest harmonic parable of all, it was resonant Hercules who formed the two golden pillars of Φ by splitting Atlas in half, therein damping out the free space to let the Old World move through the tonic eye of the Strait of Gibraltar. As the supreme archetype for all harmonic knowledge in the Underworld, it is Gibraltar's twin mountains of Atlas – the legendary Pillars of Hercules – that once represented the ***REFLECTIVE INTERFERENCE*** of harmonic resonance, opening wide the western path to freedom that became America. This was the original free space – Francis Bacon's *New Atlantis* and the Gnostic *House of Solomon* – now hidden away with the tritone, pentagram and golden ratio deep inside Pandora's box, slammed shut and locked tight.[138]

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[138] *Freedom* comes from *Freya,* the Goddess of Love and Beauty also known as *Aphrodite* and the Roman *Venus*. Freya's day is Friday, the fifth day of the week, bringing with it the freedom from work and the freedom of natural knowledge in the pentagram.

Yet we hear faint whispers that the lost musical-astrological symbolisms live on through the Invisible Orders, promising a return to unlock "The Great Secret." Down it has passed through the Order of the Quest, the Roshaniya, the Kabbalah, the Knights Templar, the Knights of Malta, the Knights of Columbus, the Masons, the Order of the Golden Dawn, the Ancient and Mystical Order of Rosae Crucis, the Brotherhood of the Dragon and the Rosicrucian Order, to name but a few. But where are these vaunted brotherhoods now in this time of need? Where is that great wisdom they claim from the past that once inspired our science and lifted up our governments? As the invisible social archetypes for the golden ratio itself, these Orders remain eerily quiet. They wait in the silent background, but waiting for what?

Through the fiery lens of a tritone we can see the great struggle that has come before us. God in Heaven, crowned by the selfish gene and the survival of the fittest meme, reigns supreme over the mythological God in Nature. But when will the burden of control be too heavy to carry on? When will the velvet curtain be pulled aside to reveal the real Oz? When will we be allowed to know that God is but a name we give to Nature and that it speaks to us through beautiful music?

Whether it happens in four, forty or four hundred years, a reawakening to the truth of *musica universalis* will surely come. And when it does, it will rise up from inside a virtual world populated by the physical archetypes and mythological avatars of our own personal superheroes. Through their eyes we will rediscover the *new mythos* of a long awaited golden age founded on a golden number. Floating back from *the Cloud*, it will be they who return the "lost key" of crystalline silence. With a half twist, it will be they who once again unlock the musical matrix that hardwires our brain to that ancient wisdom.

# Appendix 1 : Glossary

**Φ (pronounced either "Phee" or "Fi")** – an irrational and infinite number approximately equal to 1.61803 that occurs in nature as a convergent proportion of growth. Commonly known as the golden ratio, this can be derived from the irrationality of the square root of five, or $(1 + \sqrt{5}) / 2$.

**Chladni plate** – a round or square metal plate sprinkled with sand or salt then vibrated to produce patterns of specific musical tones and intervals. Name for physicist Ernst Chladni.

**CMYK color space** – a theoretical 4-dimensional space comprising the colors Cyan, Magenta, Yellow and Black ("K" often taken as "kill") that is capable of creating most of the colors in the visible light spectrum. This is the method used in paint, color inkjet and offset printing to reflect full color images in magazines and books. In general, CMYK is used to mix colors of reflected light by subtracting varying amounts of these four colors, again often in increments of 256 steps for each color.

**Coherence** – the stabilizing property of wave-like states that enables formation of patterns. The most coherent wave is a stationary standing wave producing simple whole number harmonics.

**Coriolis Effect** – named after Gaspard-Gustave Coriolis, a 19th century French scientist, is caused by the Coriolis force which appears in the equation of motion of an object within a rotating frame of reference. For instance, in the rotating reference frame of the Earth, moving objects veer to the right in the northern hemisphere and left in the southern. The spin is accompanied by the formation of a double torus flow (like stacked donuts) where each torus rotates in opposing directions while pulling material inward to a common center, forming a disc of material spiralling in between. This idea is commonly taken within the theory of torsion physics to explain an apparent orthogonal relationship between gravity and electromagnetism.

**Cosine wave** – a periodic trigonometric function representing oscillating energy at a right angle (orthogonal) to a sine wave. The cosine wave is said to be in "phase-quadrature" with the sine wave.

**Cycle of 5ths** – a stack of twelve perfect $5^{th}$ intervals (usually tuned equally) such that they form a closed periodic series.

**Damping** – any effect, either deliberately engendered or inherent to a system, that tends to reduce the amplitude of oscillations of an oscillatory system.

**Diatonic scale** – a group of seven tones that subdivide a musical octave, often used to refer to major or minor scales.

**Diatonic tritone** – the one tritone interval possible in a diatonic scale. Also referred to here as the Inverse Harmonic Axis.

**Dodecahedron** – one of the five perfect Platonic solids having twelve fully connected pentagonal faces.

**Dodecaphonic** – division of the octave into twelve equally spaced tones.

**Dominant** – the fifth tone above the Tonic corresponding to the third wave partial in the harmonic series.

**Equal Temperament (also "12-ET")** – typically refers to the equal division of an octave into twelve equal proportions of $2^{1/12}$.

**Fibonacci series** – the set of numbers based on the addition of the preceding two numbers beginning with {0, 1}. The series begins {0,1,1,2,3,5,8,13,21,34, etc.} and converges in a spiraling fashion toward Φ by taking adjacent ratios {1, 2:1, 3:2, 5:3, 8:5, 13:8, 21:13, 34:21, etc.}.

**First derivative (spatial) Gaussian distribution** – the measurement of change in velocity across a Gaussian distribution.

**Fourier series** – the trigonometric sine and cosine wave components that comprise the composite waveform for a given sound. All sounds can be decomposed into their component sinusoidal waves, known as *Fourier analysis*.

**Gaussian distribution (also "normal distribution")** – a family of continuous probability distributions used to describe natural periodic phenomena.

**Golden ratio (also see Φ)** – a natural mathematical proportion whereby a given length is subdivided so that the *total length* is to the *large segment* as the *large segment* is to the *small segment*. As a percentage, this is approximately 61% to 39% or a ratio of 1 : 0.618033 = 1.61803 – Φ. This is often approximated by adjacent Fibonacci proportions.

**Harmonic** – a whole number multiple of a sound frequency.

**Harmonic Axis** – the tritone beginning on the SuperTonic, or second degree, of a diatonic scale.

**Harmonic Center** – refers simultaneously to the point of symmetry in a 7-step diatonic major/minor scale and the first four octaves of the harmonic series. In an octave, this is commonly known as the SuperTonic and in the harmonic seires as the ninth wave partial.

**Harmonic series** – the set of waves that are whole number multiples (2X, 3X, 4X…) of the fundamental frequency. Occurs naturally in a standing wave of sound and elsewhere in nature.

**Inharmonicity (a.k.a. "stretch")** – the phenomenon of harmonics becoming sharper than their theoretical proportion due to a vibrating medium (e.g., piano string) appearing stiffer to harmonics with shorter wavelengths.

**Holonomic (brain theory)** – a lattice pattern of neurological wave interference established by neurosurgeon Karl Pribram in the 1980s in partnership with British physicist David Bohm.

***INTERFERENCE* equation** – alternate form of the standard Gaussian equation: $\mathbf{y = n^2 / (\Phi^{n)} / \sqrt{5})}$

**Interval** – distance between two musical tones in pitch space.

**Key** – In common practice, the persistent use of a diatonic scale in a piece of music.

**Landau damping** – a theory about how waves exchange energy as they cross (or nearly cross) that results in a net loss of energy in the wave system.

**Lattice** – a regular grid structure, typically diagonal or rectilinear.

**Lucas series** – the set of number based on the addition of the preceding two numbers beginning with {2, 1}. The series begins {2, 1, 3, 4, 7, 11, 18, 29, etc.} and converges in a spiraling fashion toward Φ (like the Fibonacci series) by taking adjacent ratios.

**Phase-quadrature** – a 90-degree or right angle phase shift between sine and cosine waves.

**Pythagorean comma** – the gap between a stack of twelve perfect 5ths and seven octaves, approximately equal to 1.01364326

***RECURSIVE CHROMATIC DUAL RING*** – A three-level cursive application of the Chromatic Dual Ring that emulates the cubic damping proportions of the Mandelbrot Set. After the first Chromatic Dual Ring, each subsequent Chromatic Dual Ring set is positioned with its center in alignment with a common Harmonic Axis and tangent to the outermost ring of the previous. This enables measurement of harmonic proportions within each recursive level of the Mandelbrot Set.

***REFLECTIVE INTEGRAL* equation set** – the REFLECTIVE INTERFERENCE set added together.

***RECURSIVE INTERFERENCE MATRIX*** – A twelve-level recursive hierarchy of phi-proportioned rings and orthogonally aligned harmonic waves capable of containing the archetype of any organic object.

***REFLECTIVE INTERFERENCE* equation set** – the INTERFERENCE equation reflected about the midpoint of an octave.

**Resonance** – the tendency of a system to oscillate at maximum amplitude at a certain frequency.

**RGB color space** – a theoretical 3-dimensional space comprising the colors Red, Green and Blue that is capable of creating all of the colors in the visible light spectrum. This is the method used in CRT television monitors and other such direction projection devices to create full color images and video. In general, RGB is used to mix colors in direct light by the addition of varying amounts of these three colors, often in increments of 256 steps for each color.

**Rosslyn "magic ratio"** – the naturally resonant proportion 1:81, also 68:40.5, equal to 0.0123456790123455000000 or $1{:}3^4$, found in the dimension of 15$^{th}$ century Rosslyn chapel in Scotland.

**Sine wave** – a periodic trigonometric function representing oscillating energy at a right angle (orthogonal) to a cosine wave. Said to be "in-phase."

**Spectral analysis (in sound)** – the pattern of interference produced by one or more wave frequencies.

**Standing wave** – the ability of a wave to oscillate in stationary position, thereby enabling sympathetic harmonic waves to form.

**Shepard Tones** – an experiment that demonstrated the perceived circularity of pitch. Originally, this was demonstrated using a series of ascending or descending octaves following a cycle of perfect 5$^{th}$ intervals that were amplified using a Gaussian fade filter.

**Temperament** – a particular formula for subdividing and tuning tones in an octave, such as Pythagorean temperament, Just temperament, Meantone temperament, Well-temperament and Equal Temperament.

**Timbre** – the characteristic sound of a musical instrument created by a specific set of harmonic sinusiodal components.

**Tonic** – the starting tone of a diatonic scale corresponding to the fundamental or first partial of the harmonic series.

**Tritone** – the interval of three wholetones. In 12-Equal Temperament, this is exact half of an octave.

**Tritone Function** – the oscillating movement between a tritone and circumscribed major 3$^{rd}$ that is the primary driver of harmonic function in a diatonic major-minor key.

**Wave partial** – the fundamental or harmonic wave, usually referred to as a number in the order of natural occurrence in a standing wave.

# Appendix 2 : The Social Interference Thesis

**Hypothesis 1:** The tetrachord genera were a Pythagorean “mending function” for a musical octave, joining the Spiral of 5ths and octave cycle into a pentagram at two octave golden ratio proportions inside what is today known as the *Tritone Function*.

**Hypothesis 2:** The negative reputation of the tritone interval in music history is due to its association with the pentagram and its contained golden ratio, thought to reveal an error in nature and, thus, in mankind as “original sin.”

**Hypothesis 3:** The Medieval Catholic Church banned the tritone in the early 13$^{th}$ century due to its association with Pythagoreanism and other Hermetic/ Kabbalistic philosophies.

**Hypothesis 4:** The development of the 12-step octave and simplified system of major-minor diatonic scales resulted from the replacement of a Pythagorean pentagonal design with that of an equilateral triangular design.

**Hypothesis 5:** The Inquisition created a *complicity of convenience* in the 17$^{th}$ century between Western religion and science that resulted in the separation of harmonic science from natural science. This resulted in the formation of history, philosophy and music as a humanities track well insulated from the scientific method.

**Hypothesis 6:** Johannes Sebastian Bach was the leading proponent of the Tritone Function and popularized its use thereafter.

**Hypothesis 7:** A wholetone scale is a generalization of a tritone – thus, chromatic harmony can be seen as two oscillating wholetone scales derived from the generalization of the Tritone Function.

**Hypothesis 8:** The conventions of diatonic harmony based on major and (“relative”) minor scales are founded on the recognition of symmetry around a shared SuperTonic centered in the middle of the Tritone Function.

**Hypothesis 9:** Modern music theory is based on the rules and conventions of asymmetry inherited from the tritone avoidance laws of the Medieval Catholic Church.

# Appendix 3 : Principles of Harmonic Interference

**Principle 1**: People interpret the pitch spectrum as a vertically geometric pitch space that is both circular and symmetric.

**Principle 2**: People interpret circularity in the frequency doubling at the octave.

**Principle 3**: People interpret tones in an interval having a tendency or tension to move up or down based on whether it is less than or greater than a half octave or tritone. The tritone itself is perceived as an ambiguous inflection point between opposing directions, producing what is popularly known as the Tritone Paradox.

**Principle 4**: People interpret movement between tones as motion between locations in pitch space, analogous to the perception of spatial location and motion of objects in visual space.

**Principle 5**: People interpret pitch space in hierarchical groupings that are recognized as auditory geometry. Furthermore, within this hierarchy exists a "half twist" reflective symmetry.

**Principle 6**: The distribution of wave interference in the harmonic series is described by a ratio between the square of the harmonic series and the Fibonacci series, otherwise referred to as the INTERFERENCE resonance function:

$$y = 1 / (\Phi/\sqrt{5}),\ 4 / (\Phi^2/\sqrt{5}),\ 9 / (\Phi^3/\sqrt{5}),\ \ldots,\ n^2 / (\Phi^n/\sqrt{5})$$
$$y = n^2 / (\Phi^n / \sqrt{5})\ ,\ n = \{1..12\}$$

**Principle 7**: The 12-step octave follows the natural distribution of harmonic interference that reaches an octave harmony at $\sqrt{144} = 12$ and anti-harmonic center at $\sqrt{12}$.

**Principle 8**: Harmonic tension can be measured as a function of the REFLECTIVE INTERFERENCE pattern. Taking each of the amplitudes as a percentage of maximum resonance from the Leading Tones, we have the following order of tension (greatest to least) in a diatonic scale:

| Diatonic Scale | C Major Scale | Percent |
|---|---|---|
| Leading Tone, Inverse Leading Tone | {B, F} | 100% |
| Tonic, Inverse Tonic | {C, E} | 97% |
| Harmonic Center | {D} | 72% |
| Dominant, Inverse Dominant | {G, A} | 66% |

**Principle 9**: Interval consonance can be measured as a function of the INTEGRAL INTERFERENCE pattern. Taking each of the amplitudes as a percentage of maximum consonance from the octave, we have the following order of consonance (greatest to least):

| Interval | Inverse Harmonic Center | Percent |
|---|---|---|
| Octave | {G#, G#} | 100% |
| P4, P5 | {C#, D#} | 54% |
| M3, m6 | {C, E} | 39% |
| m3, M6 | {B, F} | 34% |
| M2, m7 | {A#, F#} | 32% |
| m2, M7 | {A, G} | 31% |
| Tritone | {D, G#} | 31% |

**Principle 10**: The Principle of Tritone Duality is the ability to perceive intervals as either consonant-dissonant or tense-resolved depending on context. During the recognition process, an interval can either be 1) measured spatially as an integral function to produce the sensation of consonance or 2) measured temporally as a differential function to produce the sensation of tension. The choice between the two is apparently determined by the degree of diatonic harmonic movement afforded in the context of the music.

**Principle 11**: The Fibonacci series converging to the golden ratio and its inverse ratio acts as natural Φ-damping proportions within the harmonic series to prevent the formation of destructive fractional wave partials.

**Principle 12**: The Landau damping principle in plasma waves provides a physical model for energy transfer between harmonic wave partials in a sonic standing wave. Our auditory system appears to judge interval consonance and dissonance based on the gain or loss of energy in corresponding harmonic partials. We also appear to recognize the directional energy flow between wave partials in the harmonic series as tension and resolution, creating a cognitive anticipation/reward potential for tones to move toward and across Φ-damping zones. The degrees of consonance and tension are represented as amplitude, or change in velocity, on the REFLECTIVE INTERFERENCE distribution model.

**Principle 13**: The Fibonacci series, converging to the golden ratio Φ, acts as a *natural damping proportion* within the harmonic interference pattern of an octave to prevent fractional wave partials from forming while enabling standing wave harmonics to resonate. Maximum resonance and damping locations within the harmonic series or octave may be estimated to four decimal places using these equations:

Max Resonance Ratio = $\Phi + (7 / 12^2) = 1.6666 \approx$ major 6th = 5:3 ratio

Max Damping Ratio = $(5 / 3) - (7 / 12^2) = 1.618 \approx \Phi$ ratio

The distance between these two extremes is equal to about $7:12^2$, composed of the Philolaus octave comma of $(9:8)/27 = 6:12^2$ plus an additional "free space" of $0.006966 \approx 0.007 \approx 1:12^2$ of an octave.

**Principle 14**: Harmonic Partial 9, corresponding to the SuperTonic, is fully π-symmetric and Φ-damped relative to the fundamental (Tonic). This tone-to-octave relation is given the label of Harmonic Center as a special point of balance in the harmonic series.

**Principle 15**: The greater the wave symmetry in Φ-damping, particularly when weighted toward the out-of-phase cosine component, the greater is the perceived timbral dissonance. No damping alignment indicates maximum timbral consonance. In general, harmonics above the thirteenth partial are increasingly damped due to shorter wavelengths that bring them ever nearer to damping locations.

**Principle 16**: The *Timbral Consonance Principle* is the ranking of standing wave partials and their corresponding music intervals based on harmonic Φ-damping and Φ-alignment attributes. Following the order from non-damped to even to mostly odd-damped, we can rank intervals from most consonant to most dissonant:

| | Interval | | Harmonic Φ-damping Attribute |
|---|---|---|---|
| 1. | major 6th | minor 3rd | Not Φ-damped |
| 2. | minor 6th | major 3rd | Not Φ-damped |
| 3. | perfect 5th | perfect 4th | Even Φ-damped |
| 4. | minor 7th | | Odd Φ-damped |
| 5. | major 2nd | | Odd/ Even Φ-damped |
| 6. | major 7th | | Near Odd/ Even Φ-damped and Odd Φ-symmetric |
| 7. | minor 2nd | | Near Odd/ Even Φ-damped and Odd Φ-symmetric |

**Principle 17**: The greater the wave symmetry in π-alignment, particularly when weighted toward the in-phase sine component, the greater is the perceived harmonic resolution. In general, partials above the thirteenth partial are non-aligned, making them seem harmonically unresolved to the ear.

**Principle 18**: The *Timbral Tension Principle* is the ranking of standing wave partials and their corresponding diatonic music intervals based on π-alignment and π-symmetry about Partial 9 (the Harmonic Center or SuperTonic). Following the order of even to odd to symmetric alignment, the corresponding diatonic scale steps are ranked from most tense to most resolved:

| Scale Step | Harmonic π-symmetry Attribute |
|---|---|
| 1. Leading Tone (major 7th) | Even π-aligned |
| 2. Dominant (perfect 5th) | Odd π-aligned |
| 3. Subdominant (perfect 4th) | Odd π-aligned |
| 4. Augmented 6th (minor 7th) | Odd π-symmetric |
| 5. Mediant (major 3rd) | Odd π-symmetric |
| 6. Submediant (major 6th) | Odd / Even π-symmetric |
| 7. SuperTonic (major 2nd) | Odd / Even π-aligned and fully symmetric |
| 8. Tonic (unison) | Odd / Even π-aligned and fully symmetric |

**Principle 19**: The *Timbre/ Harmony Equivalence Principle* holds that instrument timbre and music harmony are the exact same cognitive recognition process occurring at different levels in a hierarchy of harmonic interference. Intervals and chords simply amplify corresponding harmonic partials to strengthen the effect of the underlying harmonic interplay occurring in a standing wave of sound.

**Principle 20**: Within the calm Landau parameter space between neighboring amplitude and frequency Φ-damping locations, wave partials transfer energy as a phase/frequency modulation. This is perceived as an auditory sensation of temporal movement in the direction of energy flow. Within a standing wave and interference pattern of tonality, energy exchange produces an "anticipation/reward potential" in the progression of melodies, intervals and chords in music harmony.

**Principle 21**: The *Harmonic Hierarchy* is defined as an equivalence class of 5 identical levels of harmonic interference that is generated from a single tone, aligning at different resolutions over pitch space:

| | | | |
|---|---|---|---|
| TwelfthTone = | 2^(2/ 3456) = 2^(12^-3) | 2^(12^-2)/12 | 1.000401207 |
| Tone = | (TwelfthTone)^12 | 2^(12^-1)/12 | 1.004825126 |
| Semitone = | (Tone)^12 | 2^(12^0)/12 | 1.059463094 |
| Octave = | (Semitone)^12 | 2^(12^1)/12 | 2 |
| TwelfthOctave = | (Octave)^12 | 2^(12^2)/12 | 4096 |

which may be represented as the recursive exponential functions:

4096 = TwelfthOctave (Octave ( Semitone ( Tone ( 2^(2/3456) )^12 )^12 )^12 )^12
2^(2/3456) = TwelfthTone (Tone ( Semitone ( Octave ( 4096 )^(1/12) )^(1/12) )^(1/12) )^(1/12)

or as a finite power series of 2 beginning with n = -2:
f(n) = 2^(12^n)/12 , n={-2..2}

These five layers act as a harmonic projection screen from which the musical geometry of melody, intervals and chords can emerge. Any property found at one level of this hierarchy will apply at all levels.

**Principle 22**: The equal temperament system, based on the multiplied semitone ratio of 2^(1/12), is a natural proportion within the recursive structure of the harmonic series generated by a standing wave. Therefore, contrary to any argument that equal temperament is man-made, it is the one true natural tuning.

**Principle 23**: The cognition of music harmony is defined by the proportional interference of wave resonance and Φ-damping in the natural harmonic series following the hierarchy of 2^(12^n)/12. Specifically, the proportions recognized in the standing wave interference pattern of a single tone are the same across the hierarchy of a semitone, octave and 12-octave frequency spectrum. In this way, the cognitive spatial and temporal qualities of timbre, harmony and spectra form a cognitive equivalence class.

**Principle 24**: The development of the 12-step octave originates in the natural recognition of the tritone interference pattern of Partials 5 and 7 against the fundamental.

**Principle 25**: The organizing property of the golden ratio is the central cognitive principle of music harmony.

**Principle 26**: The Fibonacci Series acts as a vortex-like temporal damping function at each level of the auditory hierarchy of 2^(12^n)/12. This is the cognitive "gravity" of music harmony.

**Principle 27**: The oscillating ratios of the Fibonacci series represent increasingly calm areas within an octave and semitone where energy may be exchanged between harmonic wave partials. Common practice music theory and preferred voice leading was a direct result of a natural cognitive awareness of this energy transfer.

**Principle 28**: Common practice use of the tritone and Tritone Function in music harmony follows the oscillating behavior of the Fibonacci Series as it temporally damps any harmonic standing wave. The universality of this principle in Western history suggests the human brain is itself organized like a standing wave.

**Principle 29**: Anticipation-reward potential in music harmony can be measured using Partial 5 as a "coherent pathway" through the interference pattern of the harmonic series. This pathway is hypothesized to be recognizable by the auditory system in two concurrent and opposing phase states of oscillation either side of the Harmonic Center. At the octave level of the interference pattern, the "cognitive cue" for which phase to recognize can be defined by which member of the Tritone Function is in play and its oscillation state. The phase indicators around the Harmonic Center are:

Diatonic Phase 1 = tritone = {up, down}
Diatonic Phase 2 = major 3rd = {down, up}

The auditory system may then measure and anticipate the potential direction of movement as an averaged direction in each half octave around the Harmonic Center, ideally using the Diatonic Phase indicators as cues. This principle applies over time within a memory context to predict melodic direction and overall musical momentum.

In the most general form, anticipation-reward potential follows the oscillation and energy exchange in a standing wave.

**Principle 30**: Any temperament (tuning method) that provides exclusivity and cyclic closure in pitch space enables the recognition of harmonic shape to some degree against the proportions of the harmonic series.

**Principle 31**: Music cognition results from the pattern matching of auditory shapes against the same harmonic shapes evolved into the structure of the inner ear and auditory cortex. The degree to which auditory shapes can be recognized and predicted is defined by how closely the musical scale conforms to a harmonic standing wave, especially following the energy transfer across Φ-damping locations.

**Principle 32**: The REFLECTIVE INTERFERENCE structure of the eardrum and Fibonacci action of the basilar membrane of the inner ear is the essential coupling mechanism between the physics of sound and the Brodmann Area in the auditory cortex of the brain. We can predict from this that the Brodmann Area itself is also organized as a REFLECTIVE INTERFERENCE neural network.

**Principle 33**: *Holonomic Music Cognition* is defined as a spatiotemporal coherence pattern matching operation as follows:

1. Sensory perception of music harmony begins as a neurological Fourier transform from spatial frequency to spatial position and proportion.
2. Spatial coherence in sound is a cognitive pattern matching of harmonic proportions against fixed proportions of the natural harmonic series within a range of tolerance.
3. Temporal coherence in sound is a cognitive pattern matching of melodies, intervals and chords (following Partial 5 and the Fibonacci series as a coherent pathway) phase shifted and/or frequency modulated against a fixed spatially coherent reference scale.
4. The proportions of the reference scale are instantly and economically recognized as neural pathways (like Partial 5) in the auditory cortex.

Holonomic brain theory [Pribram (1991)] offers the best explanation for the cognitive functions required to recognize the standing wave interference pattern produced by the natural harmonic series. From this, we might also predict that the fundamental organizing principles of the brain will follow the INTERFERENCE functions.

**Principle 34**: The best-fit Synesthetic Color Model matching the perceptual interference pattern of the harmonic series maps color frequencies of common practice primary, secondary and tertiary colors onto the 12-step octave. The assignment proceeds clockwise as an ascending spectrum of frequency.

| Diatonic Scale Step | Color Assignment | Color Group |
|---|---|---|
| Inverse Dominant (Submediant) | Yellow | Primary |
| | Light Green | Tertiary |
| Leading Tone | Green | Secondary |
| Tonic | Light Blue (Cyan) | Tertiary |
| | Blue | Primary |
| Harmonic Center | Indigo | Tertiary |
| | Violet | Secondary |
| Inverse Tonic (Mediant) | Dark Red (Magenta) | Tertiary |
| Inverse Leading Tone (Subdominant) | Red | Primary |
| | Red Orange | Tertiary |
| Dominant | Dark Orange | Secondary |
| | Orange | Tertiary |

# Appendix 4 : A preliminary axiomatic system for harmonic models

**Axiom 1:** Transposition $T_n$ and Inversion $I_n$ operations for orbits under $\mathbb{Z}/12\mathbb{Z}$ can be defined as:

If n, m $\in \mathbb{Z}/12\mathbb{Z}$ such that n is the pitch to be transposed and m is the transposition interval, then:

$[T_n = (n + m) \bmod 12]$ $\qquad$ $[I_n = 12 - n]$

**Axiom 2:** An ***initial set definition of affine orbits*** for cyclic ring $\mathbb{Z}/12\mathbb{Z}$ can be defined as:

| | |
|---|---|
| m2 / M7 Orbits: | $[T_n, I_n : n, m = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}]$ |
| M2 / m7 Orbits: | $[T_n, I_n : n, m = \{0, 2, 4, 6, 8, 10\}]$ |
| m3 / M6 Orbits: | $[T_n, I_n : n = \{0, 3, 6, 9\}, m = \{0, 1, 2\}]$ |
| M3 / m6 Orbits: | $[T_n, I_n : n = \{0, 4, 8\}, m = \{0, 1, 2, 3\}]$ |
| P5 / P4 Orbits: | $[T_n, I_n : m, n = \{0, 7, 2, 9, 4, 11, 6, 1, 8, 3, 10, 5\}]$ |
| TT Orbits: | $[T_n, I_n : n = \{0, 6\}, m = \{0, 1, 2, 3, 4, 5\}]$ |
| Major Scale Orbits: | $[T_n : n = \{0, 2, 4, 5, 6, 7, 11\}, m = \{0..11\}]$ |
| Minor (Relative) Scale Orbits: | $[T_n : n = \{0, 2, 3, 5, 7, 8, 10\}, m = \{0..11\}]$ |

**Derivation of Axiom 3: oscillation of the Tritone Function**

If n $\in \mathbb{Z}/12\mathbb{Z}$ and z $\in 2\mathbb{Z} = \{\ldots,-6, -4, -2, 0, 2, 4, 6, \ldots\}$, we can define a generalized oscillation function $\psi$ "Psi" between the dual wholetone scales over time using $t \in N = \{0, 1, 2, 3, \ldots\}$:

$\psi^{t}_{z} = [T_n, I_n : n \in \{0, 4\}, m \in \mathbb{Z}]$ $\qquad$ // Major 3rd = Even: $2\mathbb{Z}$
$\psi^{t+1}_{z+1} = [T_n, I_n : n \in \{-1, 5\}, m \in \mathbb{Z}]$ $\qquad$ // Tritone = Odd: $2\mathbb{Z} + 1$

The Tritone Function can then be defined as an oscillating subset of $\psi$ by pairing up members of the tritone and major 3rd sets over time under the same transposition *m*. This produces the axiom:

**Axiom 3:** The ***Tritone Function*** is defined by the harmonic oscillation of orbits $\psi \in\{-1, 0, 4\ 5\}$ taken from $\mathbb{Z}/12\mathbb{Z}$. This is represented by the dihedral relation $\boldsymbol{\{\psi^{t}_{2z}\ \square\ \psi^{t+1}_{2z+1}\}}$ as it occurs over a time *t* and between $2\mathbb{Z}$ (even) and $2\mathbb{Z}$+1 (odd) cycles.

As example, a cadence of the Tritone Function for the C major scale could be specified like this:

$\{\psi^0{}_0 = T_{-1}\}$, $\{\psi^0{}_0 = T_5\}$
$\{\psi^0{}_0 = (-1 + 0) \bmod 12$, $\{\psi^0{}_0 = (5 + 0) \bmod 12\}$
$\{\psi^0{}_0 = -1\}$, $\{\psi^0{}_0 = 5\}$ // Tritone interval F – B at $t = 0$

$\{\psi^1{}_1 = T_0\}$, $\{\psi^1{}_1 = T_4\}$
$\{\psi^1{}_1 = (0 + 0) \bmod 12\}$, $\{\psi^1{}_1 = (4 + 0) \bmod 12\}$
$\{\psi^1{}_1 = 0\}$, $\{\psi^1{}_1 = 4\}$ // Major 3rd interval C – E at $t = 1$

With respect to time T, we can then construct the oscillation sets for each clock tick *t* in harmonic progression:

C Major Tritone Function Oscillation Set: $\{\psi^0{}_0 = \{-1, 5\} \,\square\, \psi^1{}_1 = \{0, 4\}\}$

And the union set of all members over time *t*:

C Major Tritone Function Union Set over Time: {F, B, C, E}

**Axiom 4:** The ***Wholetone Function*** is a generalization of the Tritone Function to represent chromatic harmony as the oscillation between the two wholetone scales:

$WT^t{}_z = \{\psi^t{}_{2z} = \{0, 2, 4, 6, 8, 10\} \,\square\, \psi^{t+1}{}_{2z+1} = \{1, 3, 5, 7, 9, 11\}\}$

**Axiom 5:** The ***Dominant and Inverse Dominant Function*** is defined by the harmonic oscillation of orbits $\{\psi^t{}_0 = \{5, 7, 9, 11\} \cup \psi^{t+1}{}_1 = \{0, 2, 4\}\}$ contained in the harmonic series as divided by $\mathbb{Z}/12\mathbb{Z}$. Cancellation or resolution of oscillation occurs upon introduction of a union set of $\psi^{t+n} = \{0, 4, 7\}$ of the major Tonic triad or $\psi^{t+n} = \{9, 0, 4\}$ of the minor Inverse Tonic triad. Other resolving set intersections are possible, though resulting in lesser degrees of standing wave cancellation and cognitive resolution.

**Axiom 6:** The ***Diatonic Cycle of 5ths*** is defined as a symmetrical movement across the harmonic series following a path of alternating downward perfect 5th phase modulations between sine (odd Tonic) and cosine (even Dominant/ Inverse Dominant) components. This is defined as a harmonic oscillation of odd-even orbits $\{\psi^t{}_0 = \{0, 4, 2\} \cup \psi^{t+1}{}_1 = \{\{5, 11\}, 9, 7\}$ contained in the harmonic series as divided by $\mathbb{Z}/12\mathbb{Z}$. Note that tritone orbit {5, 11} acts as an equivalence class within the oscillation set.
**Axiom 7:** A 7-step ***Diatonic Key*** is defined by a perfect 5th/ 4th axis of symmetry between the Tonic sine and Dominant/ Inverse Dominant cosine groups, phase shifted with a reverse "half twist" of 180 degrees within a standing wave interference pattern. For a given Harmonic Axis, a diatonic key is represented by

the orbits $\psi^t{}_0 = \{\{0, 2, 4\} \cup \psi^{t+1}{}_1 = \{\{5, 11\}, \{7, 9\}\}$ contained in the harmonic series described by $\mathbb{Z}/12\mathbb{Z}$. Its complementary tritone substitute key is given by $\psi^t{}_0 = \{6, 8, 10\}\} \cup \psi^{t+1}{}_1 = \{\{1, 3\}, \{5, 11\}$.

**Axiom 8:** ***A Simple Rule of Thumb for Chromatic Harmony*****:** *Any* progression of scales, intervals or chords that alternate one or more tones between the wholetone scales will create a sense of tension. The sensation of resolution occurs when the oscillation is canceled with any interval that straddles the two wholetone scales, such as a perfect $5^{th}$.

**Axiom 9:** ***A Simple Rule of Thumb for Diatonic Harmony*****:** Under Axiom 8, incorporate the Tritone Function and/or Dominant (or Inverse Dominant) cadences to strengthen recognition of a single Harmonic Center and coherent pathway of the 7-step diatonic scale. Continued recognition of diatonic harmony is then directly proportional to persistence of just one Harmonic Center.

# Appendix 5 : 81-AET (Arithmetic Equal Temperament)

The research into the harmonic dimensions of Rosslyn Chapel in Scotland, the proportion 1:81 = 0.012345679012345… was found to be a natural spacing constant in the “halo” of resonance surrounding the interval of maximum resonance, the major 6th. This discovery led to the formulation of a temperament system, referred to here as 81 Arithmetic Equal Temperament, 81-AET or simply “chapel temperament,” as a system of instrument tuning adapted to fit the proportions of Rosslyn Chapel.

This temperament divides the octave into 81 equal proportions from which twelve ratios nearest to whole number proportions were selected. The result is a tuning system that is a compromise between 12-ET and Just temperament. While similar to a few other temperament systems, to my knowledge this is its publishing debut.

**81-Arithmetic Equal Temperament (81-AET)**

| Interval | Step Number | 81-ET Arithmetic | Just Intonation | Equal Temperament | Analysis | Adjacent Diff |
|---|---|---|---|---|---|---|
| Unison | 0 | 1 | 1 | 1 | | |
| Minor 2nd | 43 | 1.061728395 | 1.066666667 | 1.059463094 | mean | |
| Major 2nd | 45 | 1.111111111 | 1.125 | 1.122462048 | narrow | 0.049382716 |
| Minor 3rd | 49 | 1.209876543 | 1.2 | 1.189207115 | mean | 0.098765432 |
| Major 3rd | 51 | 1.259259259 | 1.25 | 1.25992105 | mean | 0.049382716 |
| Perfect 4th | 54 | 1.333333333 | 1.333333333 | 1.334839854 | just | 0.074074074 |
| Tritone (7:5) | 57 | 1.407407407 | 1.4 | 1.414213562 | mean | 0.074074074 |
| Perfect 5th | 61 | 1.50617284 | 1.5 | 1.498307077 | wide | 0.098765432 |
| Minor 6th | 65 | 1.604938272 | 1.6 | 1.587401052 | wide | 0.098765432 |
| Major 6th | 68 | 1.679012346 | 1.666666667 | 1.681792831 | mean | 0.074074074 |
| Minor 7th | 73 | 1.802469136 | 1.8 | 1.781797436 | wide | 0.12345679 |
| Major 7th | 76 | 1.87654321 | 1.875 | 1.887748625 | mean | 0.074074074 |
| Octave | 81 | 2 | 2 | 2 | | 0.12345679 |

**Deviation of 81-AET from ET (cents)**

| | | | | Chromatic Scale from A440 | |
|---|---|---|---|---|---|
| ET Cents | 81-AET (cents) | Deviation from ET (cents) | Percentage Deviation | Note | Frequency |
| 0 | 0 | 0 | 0.000% | A | 440 |
| 100 | 100.2138159144 | 0.2138159144 | 0.214% | A# | 467.160494 |
| 200 | 197.9774929201 | -2.0225070799 | -1.011% | B | 488.888889 |
| 300 | 305.2142544254 | 5.2142544254 | 1.738% | C | 532.345679 |
| 400 | 399.7898945698 | -0.2101054302 | -0.053% | C# | 554.074074 |
| 500 | 499.4356922922 | -0.5643077078 | -0.113% | D | 586.666667 |
| 600 | 597.1123930020 | -2.8876069980 | -0.481% | D# | 619.259259 |
| 700 | 703.6748367045 | 3.6748367045 | 0.525% | E | 662.716049 |
| 800 | 808.8382048708 | 8.8382048708 | 1.105% | F | 706.17284 |
| 900 | 898.5120424465 | -1.4879575535 | -0.165% | F# | 738.765432 |
| 1000 | 1011.6015990936 | 11.6015990936 | 1.160% | G | 793.08642 |
| 1100 | 1093.4705517091 | -6.5294482909 | -0.594% | G# | 825.679012 |
| 1200 | 1200 | 0 | 0.000% | A' | 880 |

# Appendix 6 : Bibliography

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# Appendix 7 : Table of Figures