Constructing The Universe Activity Books
Volume 1

A Voyage From 1 to 5

Explore Harmony in Mathematics, Nature and Art

Michael S. Schneider

Author of

A Beginner's Guide To Constructing The Universe:
The Mathematical Archetypes Of Nature, Art And Science
(HarperPerennial)

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Create and Explore
Geometric Patterns
of Nature and Art

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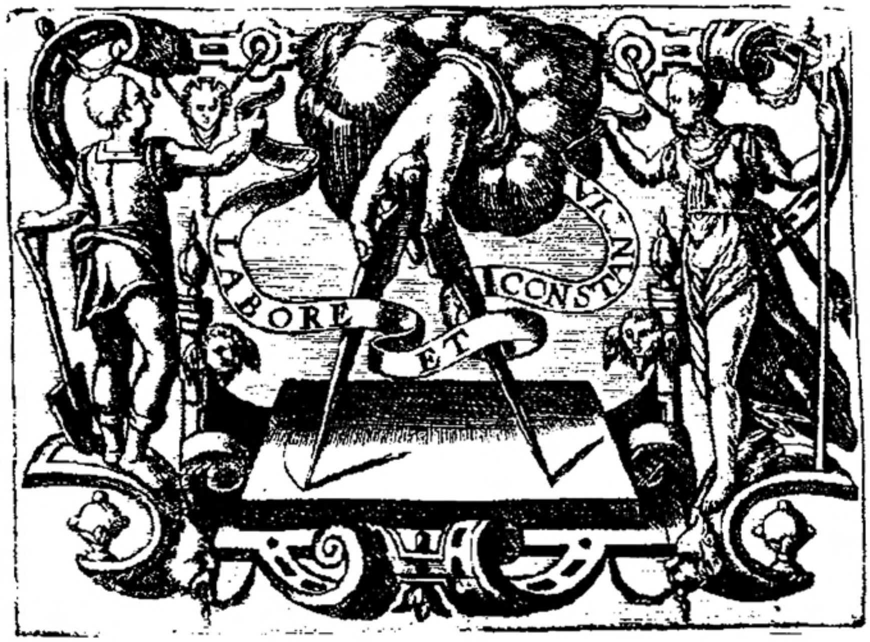

Constructing The Universe Activity Book

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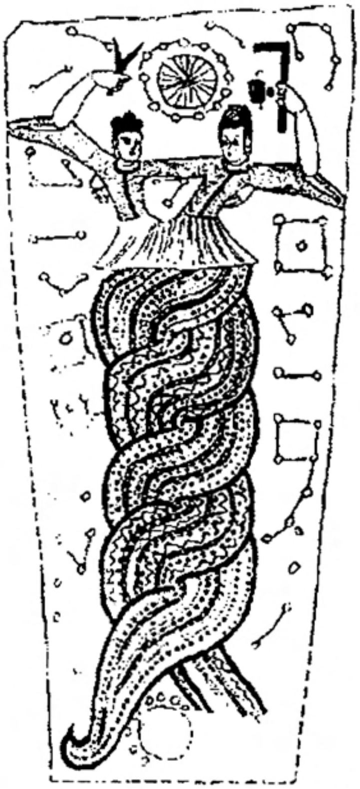

Astana Banner depicting the Chinese "Adam and Eve" intertwined among the starry constellations. They hold a compass and straightedge.

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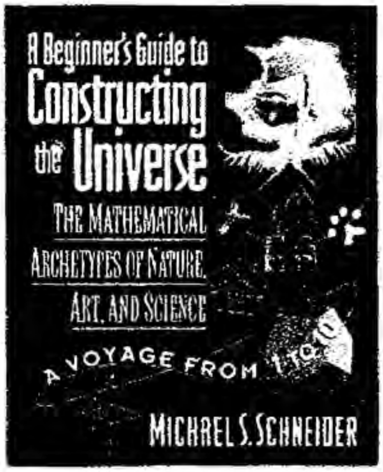
Introduction

This Activity Book (in two volumes) is a hands-on companion to the author's book *A Beginner's Guide To Constructing The Universe* (HarperPerennial 1994) which explored the first ten numbers and shapes as they appear in nature, art, science and symbolism. Many people have said that they enjoyed doing the book's geometric constructions and want more. This Activity Book shows various ways to do geometric constructions for the numbers one through twelve, Circle through Dodecagon, and invites readers to apply them to images of nature and art beyond those found in *A Beginner's Guide*. For more detailed information about any number and shape in this Activity Book, please see the same chapter number in *A Beginner's Guide To Constructing The Universe*.



Geometric construction is usually only a very small part of mathematics education today, if taught at all. But it plays a big part in the natural universe and the art of every culture, and so is well worth learning about. As Galileo wrote, geometric shapes are the alphabet in which The Book Of Nature is written. This Activity Book gives the reader hands-on experience with this language. It provides an introduction to the subject for those who might not otherwise be exposed to it, and opportunities to explore mathematics in nature and art for those familiar with geometric construction. This Activity Book is not intended to teach all or even most of what can be taught about geometric construction, or geometry in nature and art, but the author hopes it makes the reader comfortable and confident with the basic skills of geometric construction, and feel enthusiastic enough to seek and create more. Beginners to geometry will get started doing powerful constructions. People of more advanced interests will see directly how a geometric language appears in nature and has been applied to worldwide art for symbolic reasons. Artists and creative readers may apply to their own works what they learn here from ancient masters.

You'll need these tools:

* **A geometric compass**. In classic times, collapsible dividers were used instead of the fixed position compass common today. The modern compass can do steps not as easily done with dividers, which did not hold the same position, allow lengths to be transferred, "walk around the Circle" or other steps possible with a compass. But since most readers will probably have easiest access to a fixed compass from an office or art supply store, the constructions are described with them. A generic compass/divider with its point and scribe (pencil) sides is shown in illustrations simply to make clear where to place them.

* **A straightedge**. No markings or measurements are required. It's called a "straightedge" because that's all we're interested in, just a *straight edge* to guide a pencil.
* **A pencil, colored pencils, and a sharpener**. Use colored pencils to make lines and patterns more visible in the constructions and on the pictures of nature and art, and to shade in your constructions.
* **Blank paper or a sketchbook**. You may wish to record the geometric constructions you do, and make sketches of geometry and objects you

notice around you, and to glue shapes and patterns collected from magazines, newspapers, advertisements, corporate logos and elsewhere.
* **Scissors and tape** will also be helpful.


Detail from *The Measurers*

Both volumes are bound so that they can be opened to lie flat for best construction and drawing. The pages of these volumes are single sided so that compass and pencil marks don't effect activities on their opposite side. When doing constructions, place a blank sheet of paper under the page so that your compass point and pencil marks don't go through to other pages.

Doing geometric constructions can be a very satisfying activity. Constructing Circles by hand with a compass on paper is much better than doing it electronically on a computer screen with a mouse or stylus and keyboard. Developing your delicate eye-brain-fingers tactile connections using a compass, straightedge, pencil and paper will bestow benefits in other areas of your life. Along with the practical benefits of knowing geometric constructions, a kind of transformation occurs within the geometer who works with the compass and straightedge. You'll just have to do it for yourself and see. Keep learning, experimenting, creating and developing original geometric constructions on your own.

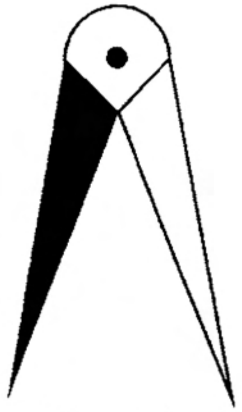
The names of geometric shapes like Circle, Triangle and Square are capitalized as a form of respect.

Occasionally some information is labeled a "Curious Truth". The word "Truth" is used instead of "Fact" because the facts of mathematics are timeless and always true, and something to which everyone agrees, in contrast with the "small t" truths or changing facts about the transitory events and objects of sensory experience. This is why ancient people around the world used geometric design so prevalently in their works great and small. Some, like Egypt and China, modeled their entire civilizations around these timeless Truths. I hope that you enjoy constructing and pondering them.

Michael S. Schneider
4 June 2003
San Anselmo, California


Euclid demonstrating a geometric construction.
Detail from *The School Of Athens* by Raphael.

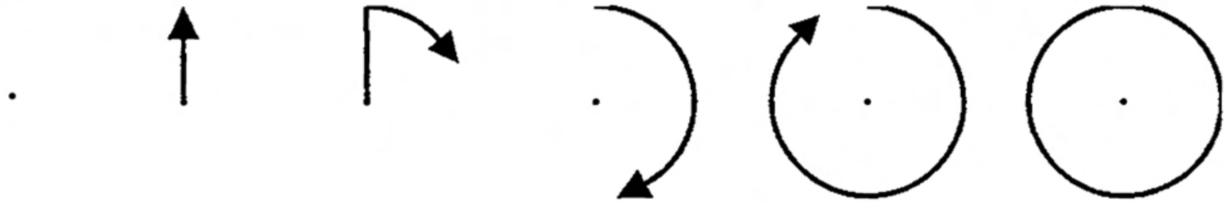
1 The Circle



The Circle and the straight line are the two "parents" which produce geometric constructions. A compass turns a Circle around a point. A straightedge draws straight lines between points. Circles and lines cross to make new points, which can be centers of new Circles and the ends of new line segments. Turning Circles, drawing lines and creating points where they cross are the most important activities to master for geometric construction.

Turning Circles

Since Circles are so important, we'll take a moment to notice them. If you pay attention to each movement of your hand and the compass you'll see that turning a Circle is the result of a series of small steps:



1) First, press the point of your compass into the middle of a blank sheet of paper. This is the center of the universe of your construction. You might press the pencil point in it, or shade lightly over it with the side of your pencil to make the point more easily visible. The blank page is filled with infinitely many points. The compass, straightedge and pencil identify the important ones for each construction.

2) Open the compass to a size that will fit on the paper. This allows you to create through the power of opposites: the point of the compass stands still while the scribe or pencil leg moves around.

3) Beginning: Hold the compass with one hand, and only at the top. Begin to turn your wrist and the compass, leaning it forward and around. Don't hold a leg and don't turn the paper!

4) Continuing: Lean the compass slightly forward and ahead, turning your wrist in a smooth motion.

5) Ending: Don't slow down as you approach the beginning.

6) Lift the compass off the paper. The end of the Circle should seamlessly overlap its beginning. The complete Circle should appear to have no start or finish.

* Don't leave your Circle too quickly but look at each one. Does the line change its thickness? Is it continuous, or can you see a beginning-ending blip?

* Keep your pencil point sharp! Use a sharpener, or a piece of fine sandpaper on the "lead" so that the Circle and lines don't get too thick.

* Keep practicing! Fill a page with Circles of many sizes, all sharing the same center.

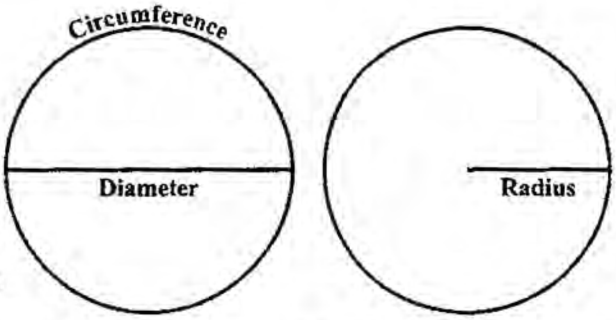
* Then make Circles of different sizes with different centers on the same page.

* Notice where the Circles cross. Practice drawing lines with your straightedge to connect crossing points (see page 5).

* Use colored pencils to shade the different sections of your construction.

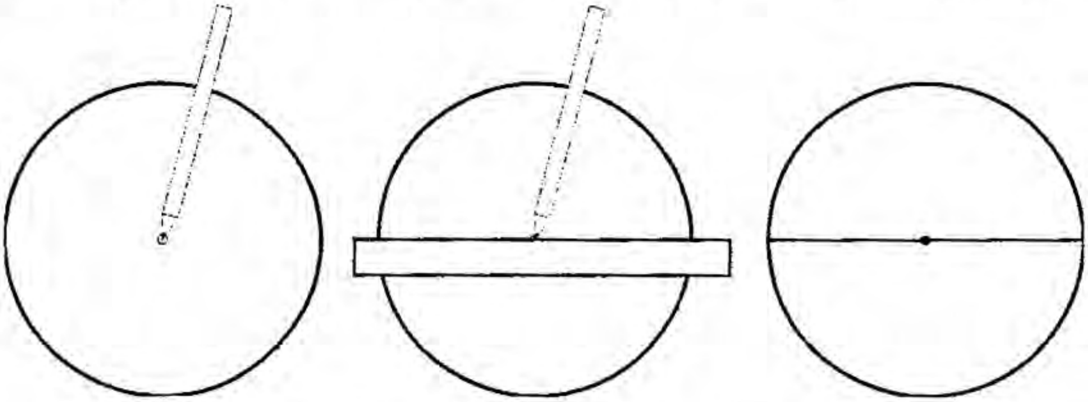
When you get good at turning Circles, learn to do it with your other hand too.

Tip For Drawing Diameters



When you've made some circles, practice drawing their diameters (Greek: "across measure"). A diameter is a line segment going across a circle *through its center*. A radius (from "reed", "rod" and "radiate") is half the diameter from the Circle's center.

The best way to draw a diameter is to *first put your pencil on the center point*, holding it upright, and then bring the straightedge to it. This way you can be sure that the line will go through the center.



Turn circles on a blank sheet of paper and practice drawing their diameters. See whether the diameters go directly through the point at the center.

Connecting Two Points Accurately

Egyptian "rope stretchers" connecting two points on the ground in outdoor geometry.

Drawing lines accurately between two points is very important in a construction. Notice in your own constructions how accurately each line you draw meets its points, and you will improve.

Here are some tips. Practice drawing lines between the crossing points of the intersecting circles on the next page.

* First, choose two points. Then put your pencil on any one of the crossing points. (If lines are thick, choose their middle.)

* Then slide the straightedge next to it until it taps the pencil. (Don't try to bring the straightedge first to the point and pencil last. Our eye can't see accurately where the pencil point will fit.)

* Hold the pencil straight up. This way it will always be the same distance from the edge of the straightedge.

* Hold the pencil firmly on the point while pivoting the straightedge near to the second point.

* Now hold the straightedge down firmly and lift the pencil from the first point. Put it directly on the second point.

* Carefully pivot the straightedge from the first point next to the pencil at the second point. Try not to move it away from the first point.

* Lift the pencil and put it on the first point again to make sure that the straightedge didn't move away from it. Adjust the straightedge if necessary.

* When both points are next to the edge, hold your pencil upright on one point and confidently draw the line segment along the straightedge to the other point.

* Practice by connecting pairs of points of the circles on the following page.

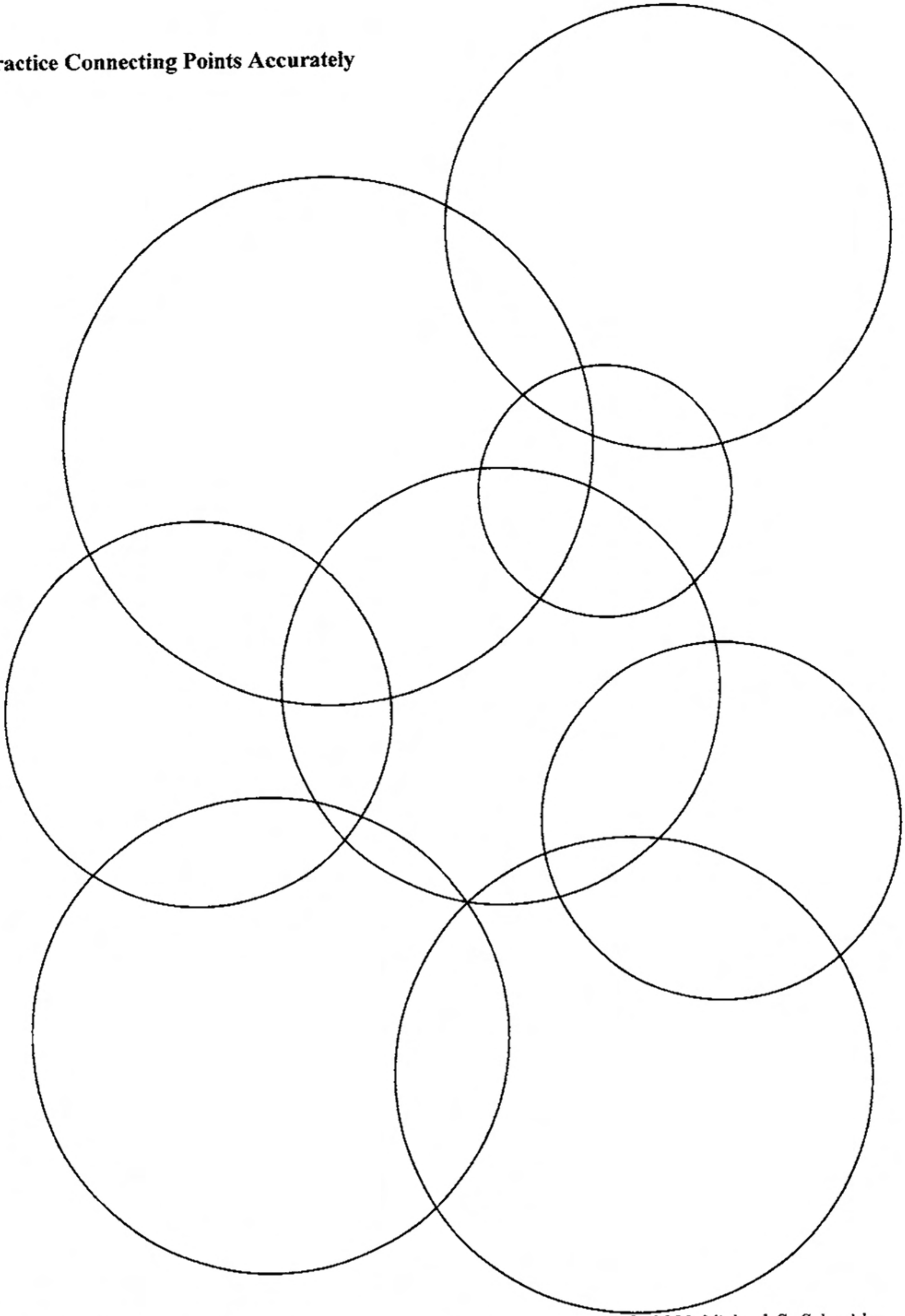
* Draw lines with colored pencils.

* The thickness of your pencil point will effect the accuracy of your construction, so keep it sharp.

* Always look to see how accurately your line segments meet their end points. Pay attention, keep practicing and keep improving.

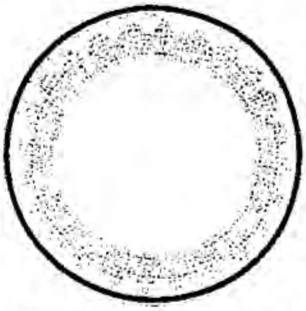
* The more accurate you are are each small step of the construction, the more accurate and satisfying the final result will be.

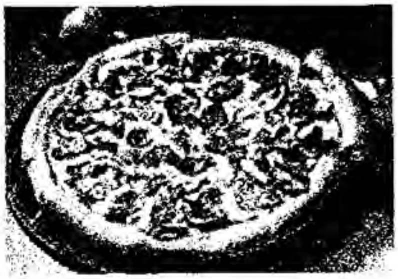
Practice Connecting Points Accurately



A Curious Truth About Circles

A Circle encloses more space inside it than any other shape having the same circumference or perimeter (distance around it)!





Round plates, cups, cans, buckets and bowls *hold more inside them* using less material than any other shape with the same perimeter. And that's why Circles, cylinders and spheres appear in the designs of many familiar objects!

This means that a round pizza will hold more toppings than a Square pizza (with the same length of crust).

Is this really true?
Do the next activity to find out!

Word Origin: The circle's name comes from the ancient Roman word *circus* which means a "ring".

Shapes And Areas

Which shapes enclose the *most and least* areas with the same string?

You'll need:

* String or yarn;
* Graph paper, or a checker board or checkered tablecloth.

Do this:

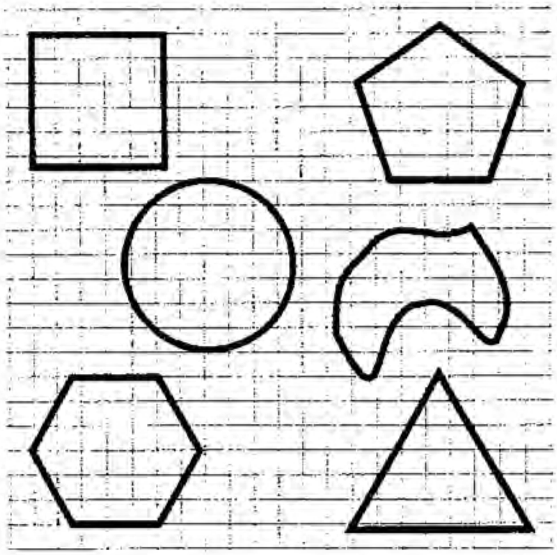
* Tie the string into a loop.

* Form it into many different shapes, including a Circle, ovals, Triangles, Square, rectangles, shapes with different numbers of sides, and blobs.

* Place each shape on the graph paper. Then count the number of Squares (the area) each shape encloses.

* Draw each shape and write its area.

* List the shapes in order from most area to least.



Don't count these -- they're not drawn to scale! Do the activity!

Although each shape uses the same loop and has the same distance (perimeter or circumference) around it, you'll find that only the Circle covers the most area.

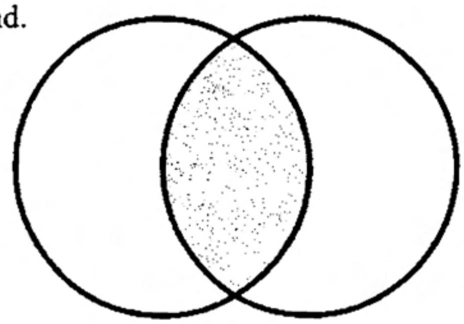
Which shape covered the *least* area?

2 The *Almond*

Almond husk, shell and nut

When two Circles *of the same size* touch each others' center, they cross at two points and create an "almond" shaped space between them, like two friends sharing something in common.

Many great geometric constructions grow from this simple Almond.

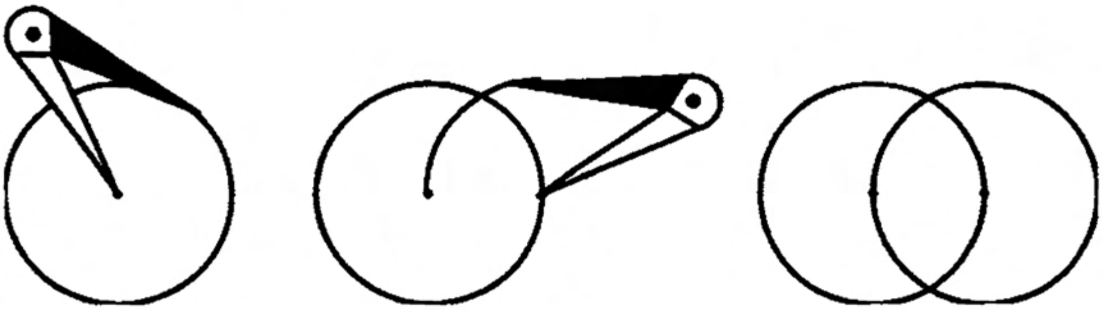


The Almond Construction

Almond Words

In India and Italy, the almond nut and the geometric construction are both called a *Mandorla.* Throughout Europe this construction was called a *Vesica Piscis*, Latin for the "fish's bladder" it resembles.

Construct The Almond



1 2 3

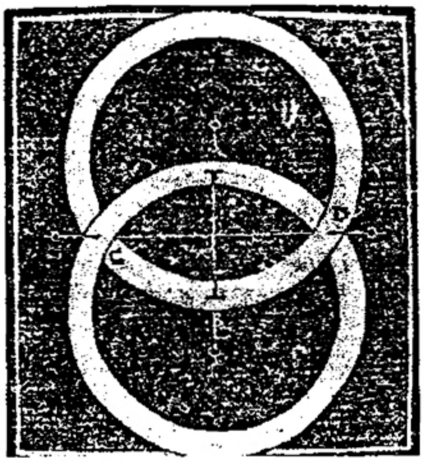
1) Make a point on the paper and turn a Circle with your compass.

2) Lift the compass and reverse it, placing its point anywhere on the Circle. Any point is fine, but the four directions (top, bottom, left, right) are usually chosen.

3) Turn another Circle the same size.

Each Circle should touch the others' center and create the Almond between them.
Do the Almond construction here:

Use The Almond To Divide Any Line Segment In Half



Almond Construction by
Giordano Bruno, 16th century.

The Almond construction is useful for dividing any line into two equal parts.

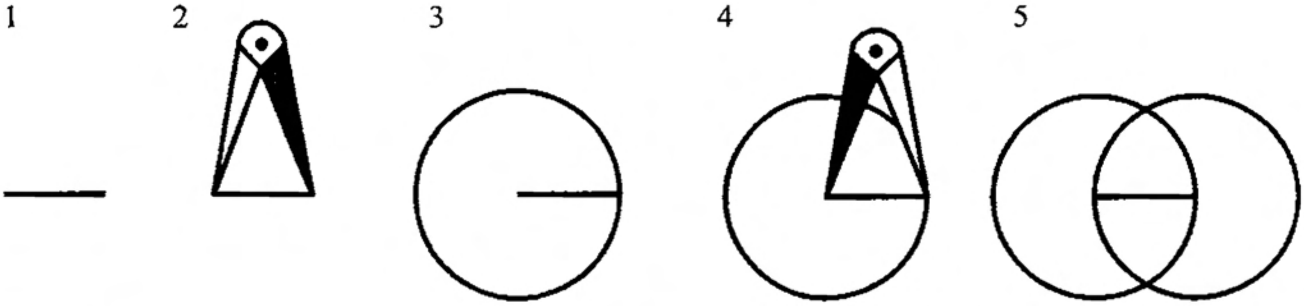
1) Use your straightedge to draw any line segment. It should be no longer than your compass can reach across.

2) Open the compass so that its point and scribe touch the line segment's two ends.

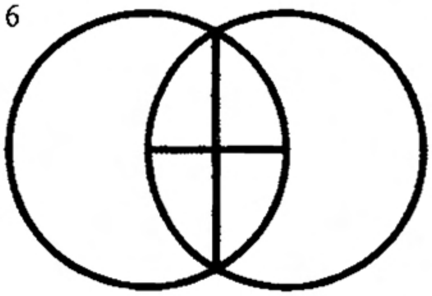
3) Turn a Circle.

4) Switch the compass point and scribe to the other ends of the line segment.

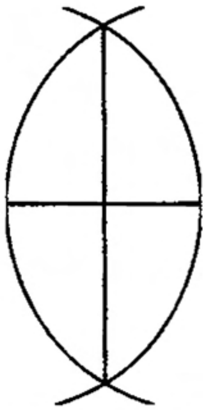
5) Turn another Circle and create the Almond space between them.



6) Use your straightedge to draw a line connecting the points where the Circles cross above and below the line. This divides the original line into two equal halves. The vertical line is called a "perpendicular bisector" of the original line segment.



Practice dividing this line in half.
Then divide each *half* into halves.



Space Saving Tip:
You need not complete the full Circles, but turn just enough to make the arcs of the Almond cross like this.

Replicate These Almond Constructions

Here are some constructions made with the two Circles of the Almond. Replicate them in the Almond Circles provided below or on a blank sheet of paper. You can develop each construction further.

Replicating a construction is like solving a puzzle. You must first look carefully and understand the order of its steps. Some points, lines, arcs and Circles must appear before others can. You have to decide what to do first, second and third.

An important rule: only place the ends of the compass *upon points which already exist*, like a Circle's center and crossings. Don't approximate by eye where a point might be! Any Circles or lines which cross create new points. Every point can be the center of a new Circle, and the end of a new line segment.

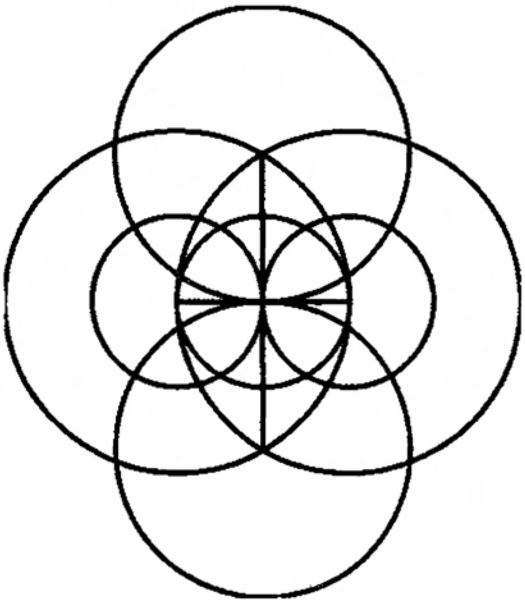
At each step, ask yourself:

* Where should I place the compass point? Where the center of this Circle (or part of a Circle)?

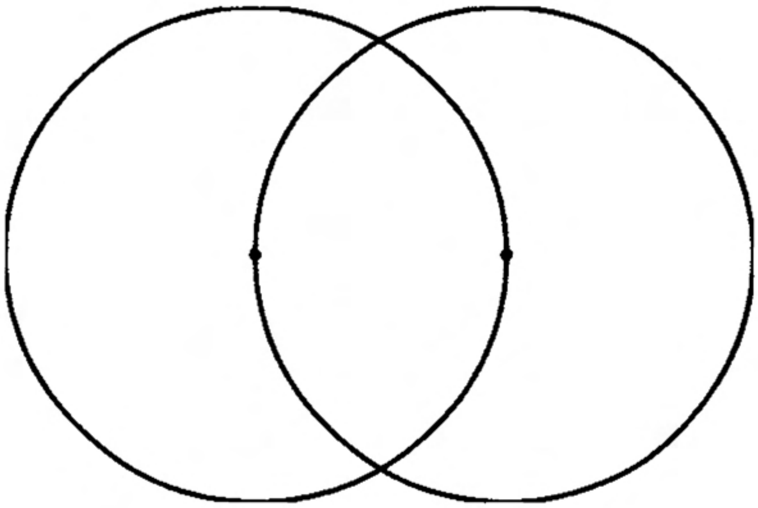
*To what point should I open the compass scribe?

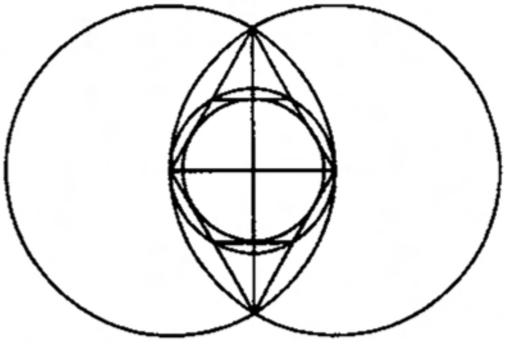
* Which points should I connect to make a new, useful crossing point?

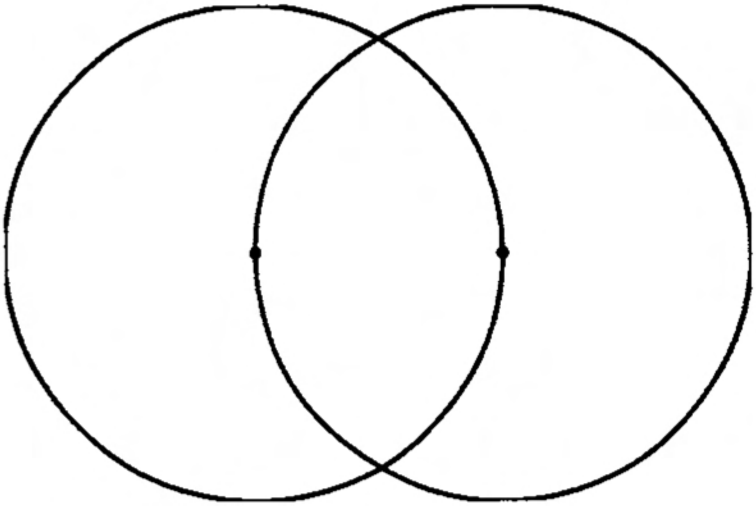
Only draw lines *between two points* or crossings. Geometers usually don't just draw lines away from one point, or extend a line beyond a point.

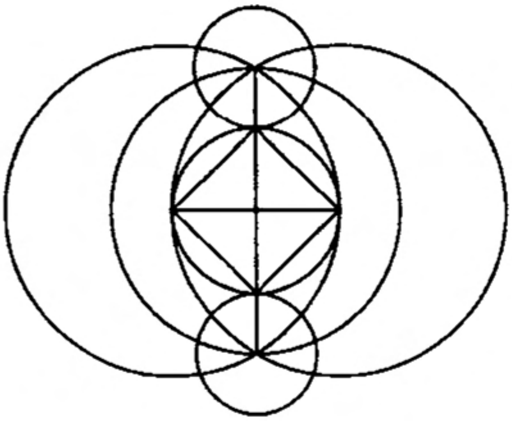


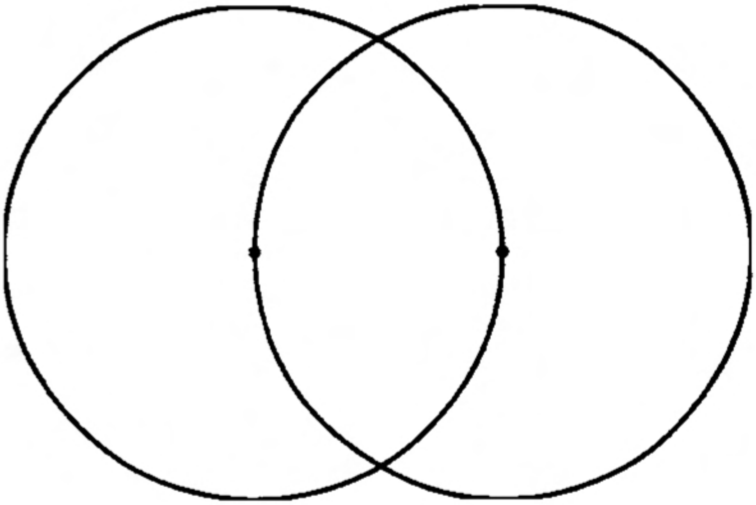
Hint: Start with the four points you already have and draw the two line segments between them.

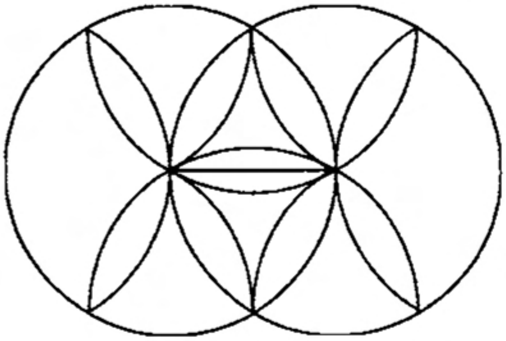


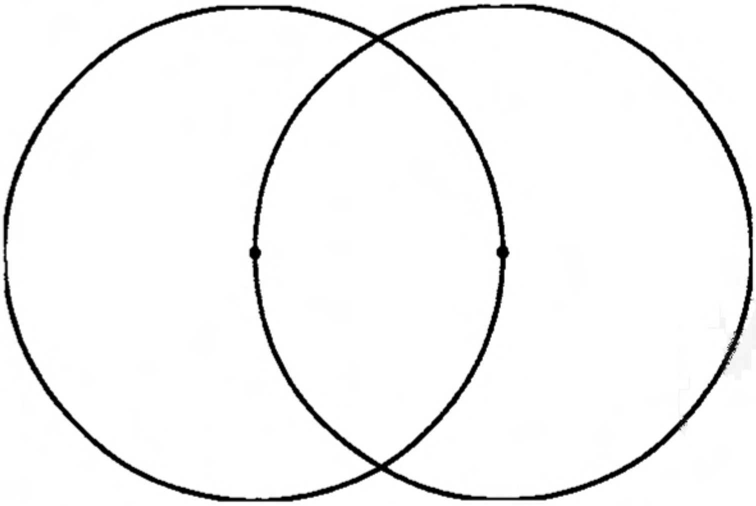




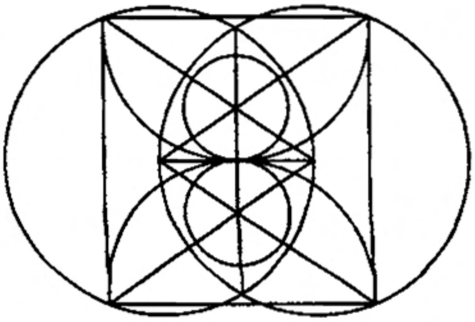


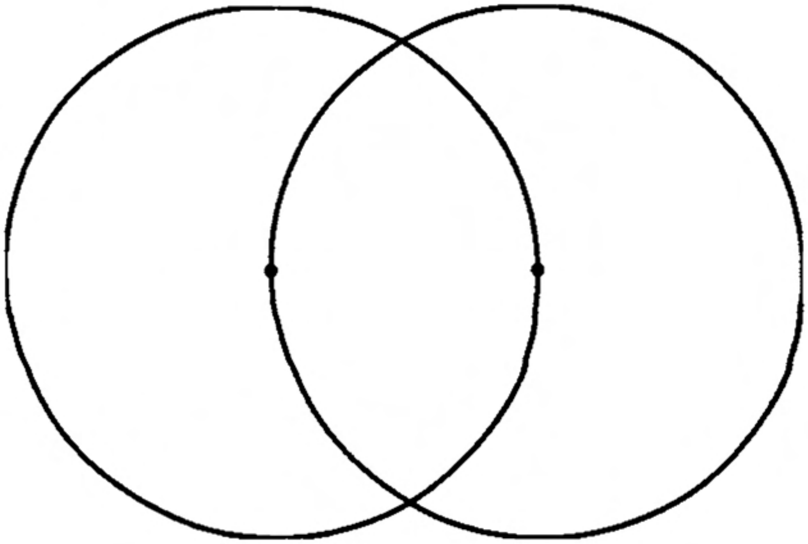




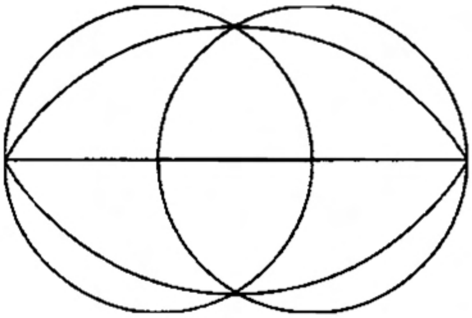


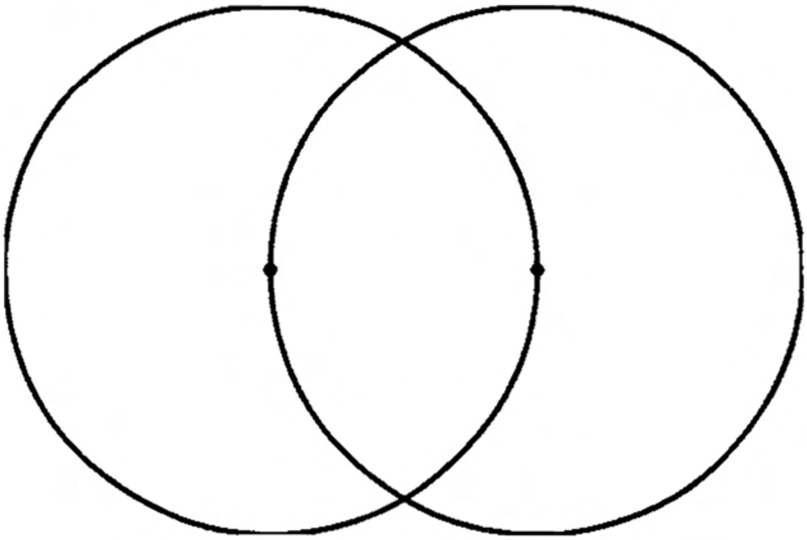
Hint: Keep the compass open the same distance. Use it to find new center points around the circles.

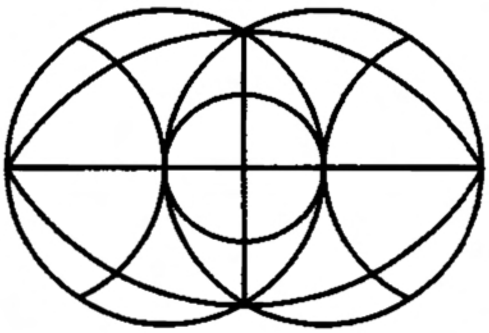


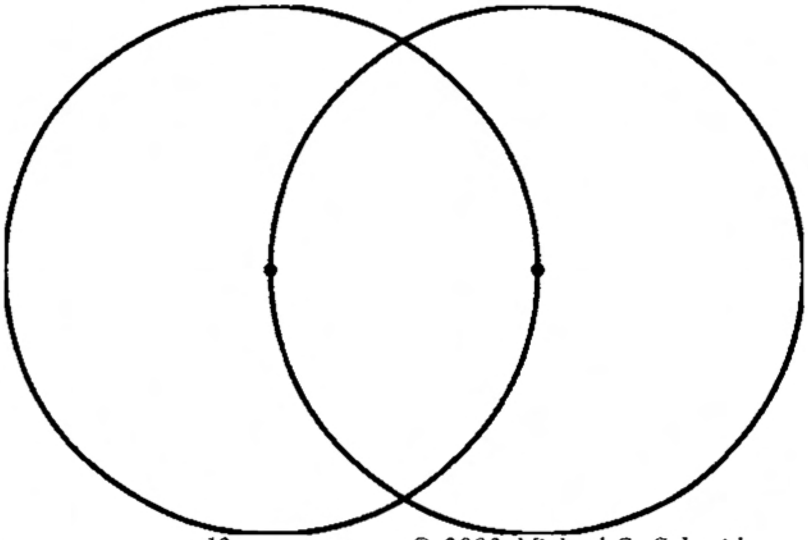


Below: To draw one long diameter *across both Circles*, don't extend the line segment beyond the two center points. Instead, open your compass between the Almond's upper and lower crossing points and turn wide arcs. They should both meet at the extreme ends of the Circles to show two points you can connect by the straightedge.





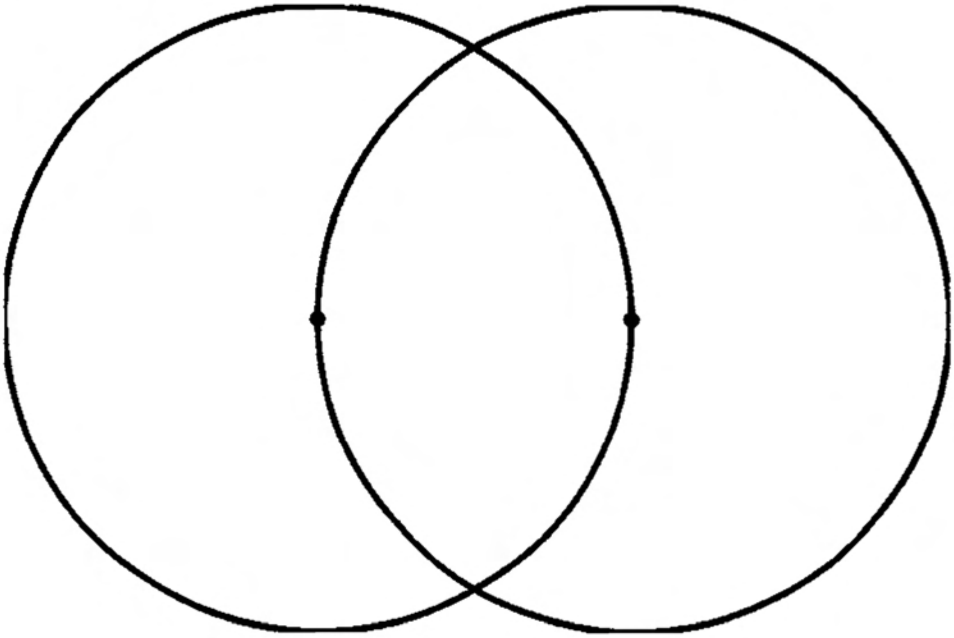


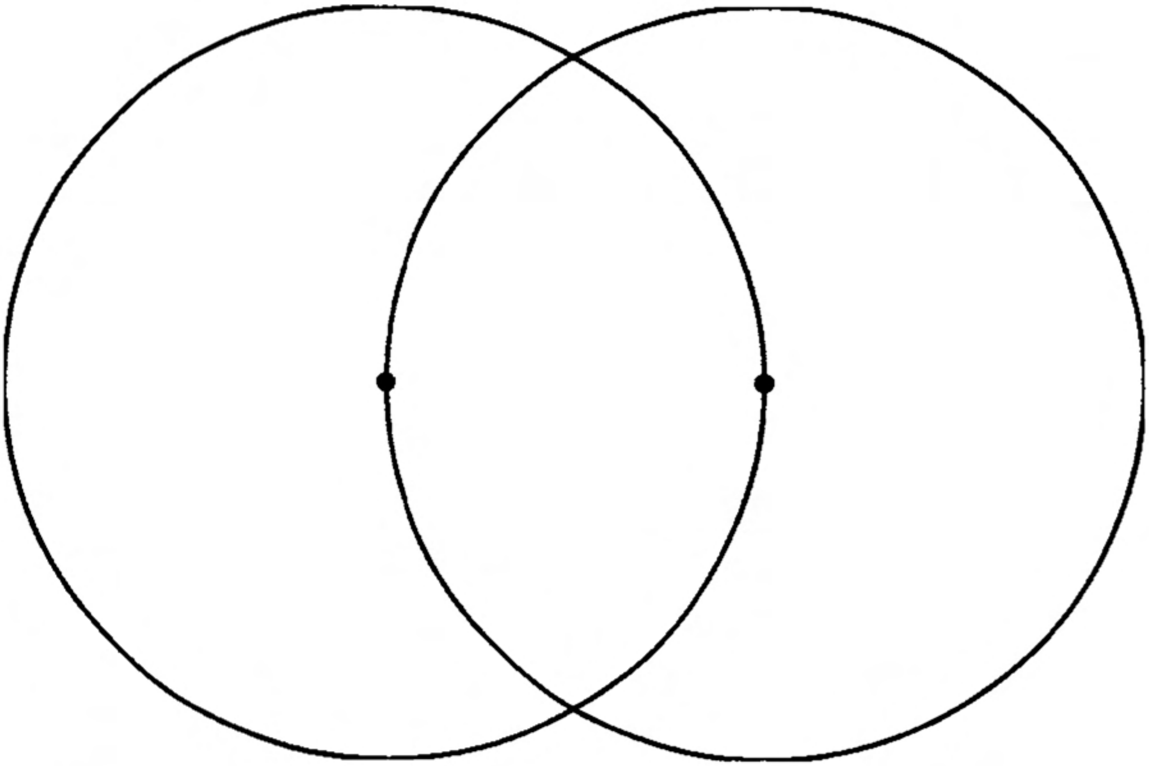


Create Your Own Constructions In The Almond

Invent designs of your own starting with the Almonds below, or on a blank sheet of paper.

Follow the rule: only put your compass on actual points you find.

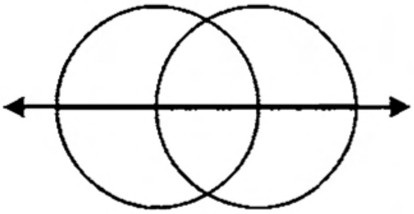




Arithmetic, Geometry And Music

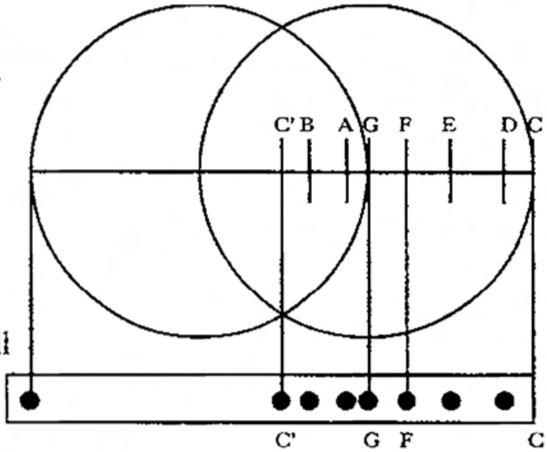
"There is geometry in the humming of strings."
— Attributed to Pythagoras

The Almond and its Circles hold a secret about music. Think of the two Circles as pulling a string in opposite directions.



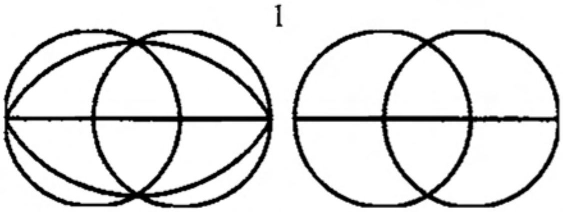
It has long been known that pressing or tapping *certain places* on a vibrating musical string, like frets under a guitar string will produce pleasing musical intervals (relationships between notes). These are called harmonious intervals or *concords* (literally "hearts together"). The Almond can help us find these harmonious intervals on a string without any measuring.

With a knowledge of how to find the points on a string which make concords, you can build a *monochord* (a one string instrument) and can understand the placement of notes on string instruments like the guitar and violin, as well as the placement of holes on wind instruments like flutes.

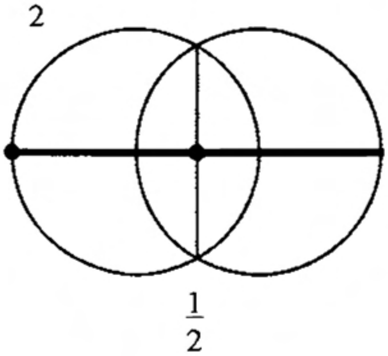


To do the geometric construction which locates the harmonious notes on a musical instrument, start with the Almond.

(1) **Fundamental Tone**. Construct a line across both Circles (see page 12). This length represents the whole string. The longer the string, the lower the note. The shorter the string, the higher the note.



(2) **Find the Octave note**. Draw a vertical line between the Almond's crossings and divide the string in half. Pressing or tapping *this point* on the vibrating string will produce the same note as plucking the whole string, but one octave (eight notes) higher. The octave interval is common to all musical systems around the world.

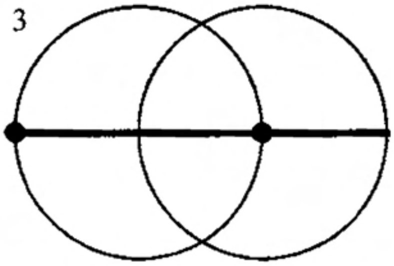


The Octave

Dividing the string in half applies to other instruments as well: the hole halfway along the flute will produce a higher version of the Fundamental note of the whole flute, or tapping a glass half filled with water produces the note an octave higher than the note produced by tapping the full glass (see the picture at the bottom of the next page).

(3) **Find the Musical Fifth**. The next harmonious interval is already shown to us at the center of one Circle. Pressing the string at this point, 2/3 along the entire distance, produces the note at the interval called the "musical fifth" because it is five notes higher than the note of the whole string. The interval between these two notes has been called the "chord of triumph" for its uplifting feel.

3



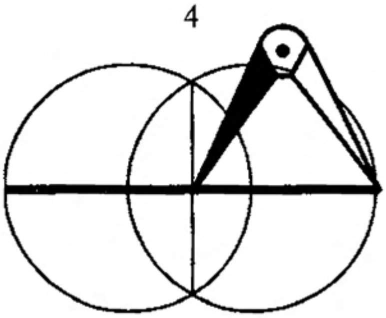
Musical Fifth $\frac{2}{3}$

(4) Find the Musical Fourth. Open the compass across half the string.

(5) Turn an arc (or the whole Circle if you have space).

(6) Keep the same compass opening and reverse the compass *across the same half the string* and turn an arc to create an Almond.

4



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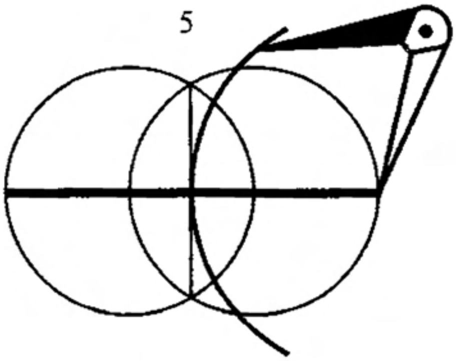

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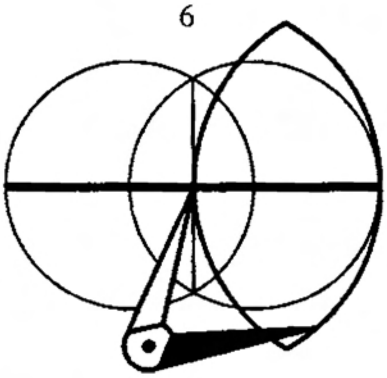
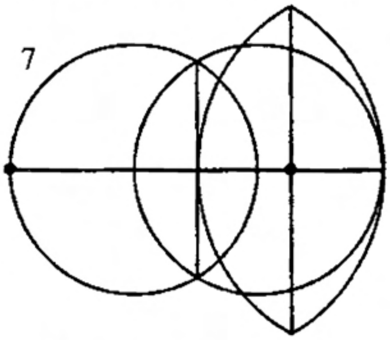


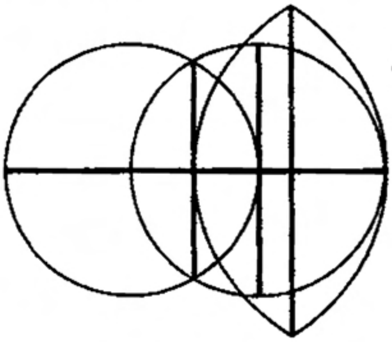
(7) Connect the crossings of this larger Almond. The line segment will cross the whole string at the three-fourths point. Playing this length of string produces the interval of the "musical fourth" or four notes above the tone made by the whole string.

7



Musical Fourth $\frac{3}{4}$

8



$\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$ 1

C' G F C

These lines show the "concord" fractions along a string, holes along a flute, or fullness of glasses of water (try tapping them and listen!)

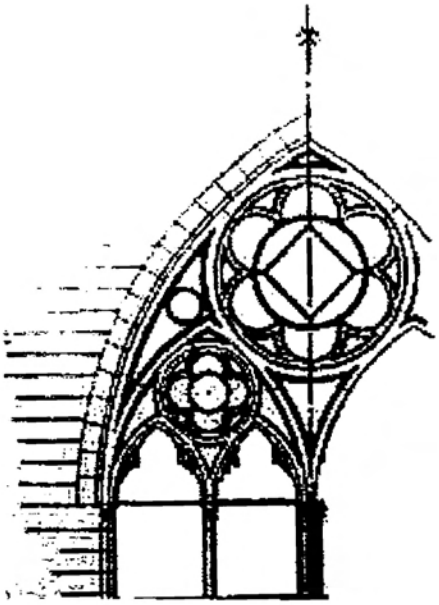
(8) We've found the harmonious concord fractions of the whole string.

They're the frame on which the remaining notes of the scale may be determined.

Music As Architecture

When we look at architecture, at buildings and their details, we don't usually think of music. But some architecture around the world was designed with musical harmony in mind. The geometric construction we did to find the musical concords was also used to design buildings where harmonious songs were sung. This window "tracery" from Notre Dame cathedral in Paris is a good example of musical architecture. It can be found elsewhere in many variations.

To understand its musical design, continue the previous construction where it left off.

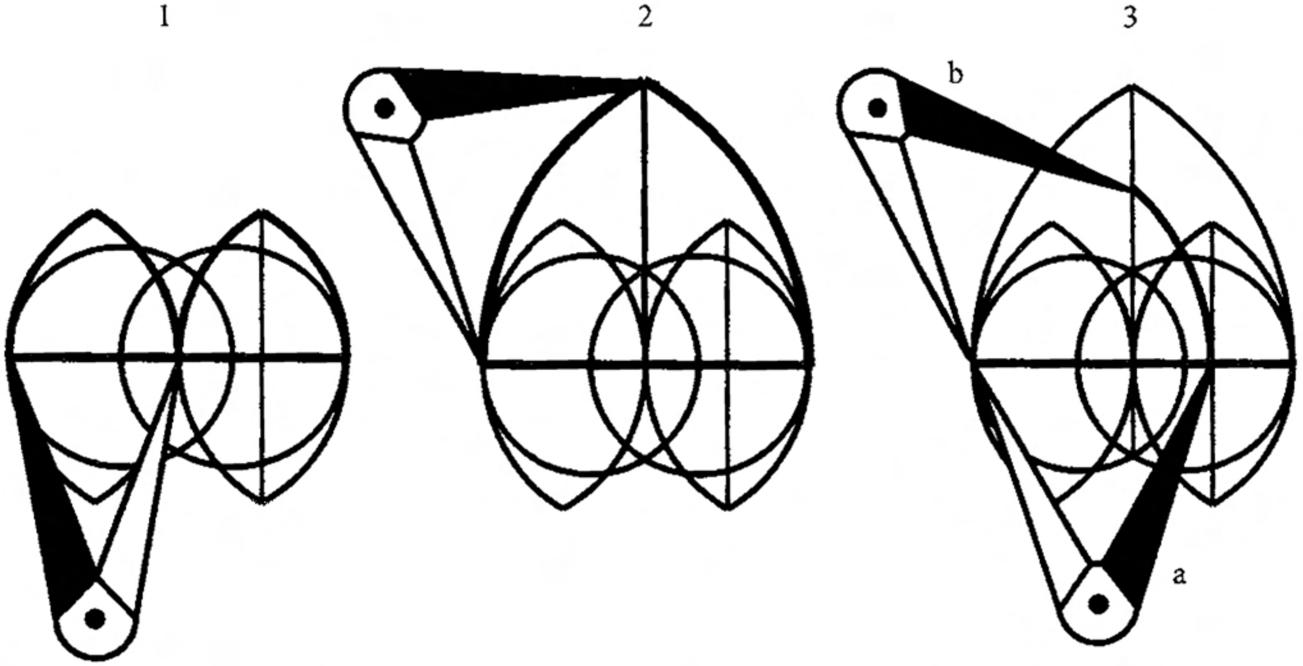


"Architecture is frozen music."
-- Goethe

(1) Repeat the "musical fourth" Almond construction on the other half of the "string".

(2) Now open the compass to the full length of the string and make an arc upwards. Then, turn the compass around on the string and make another arc which meets the first one above. (If you have space, you can make the full Circles.) Draw a straight line from the crossing point at the top to the middle (octave) point of the string.

(3) (a) Open the compass from the beginning of the string to its 3/4 "musical fourth" point.
(b) Turn an arc so that it meets the vertical line.
You can repeat this with the compass point at the other end of the string to verify this crossing point on the vertical line.

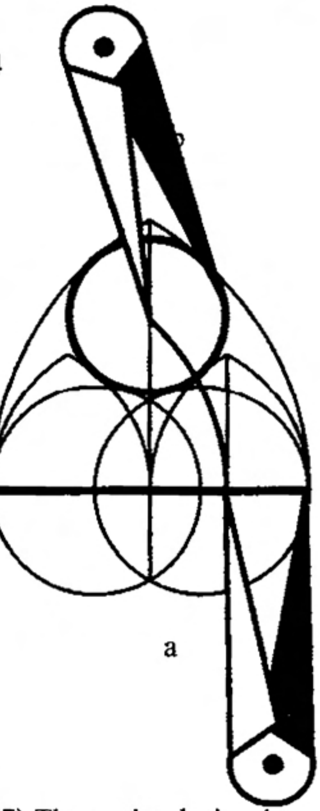


(4) (a) Open the compass between the remaining 1/4 of the string. (b) Lift the compass and put its point where the arc (made in step 3) crosses the vertical line. Use this opening to turn a Circle.

(5) If you were careful along the way, the Circle should just fit in the large arcs, and rest on top of the small arcs, and tangent to the Circles of the original Almond. Shade them in with colored pencils.

(6) The parts of this "window" are related to each other by the simple fractions of musical harmonies along a string. This way, a window or a whole building makes visible the pleasing music sung there.

4



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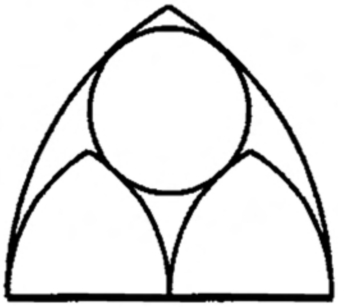

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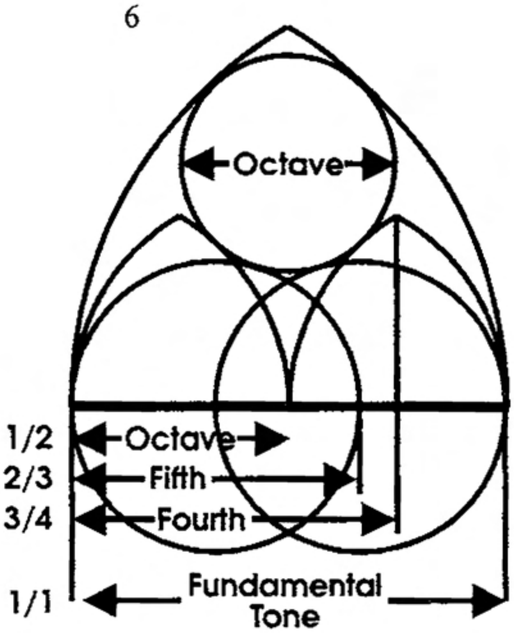
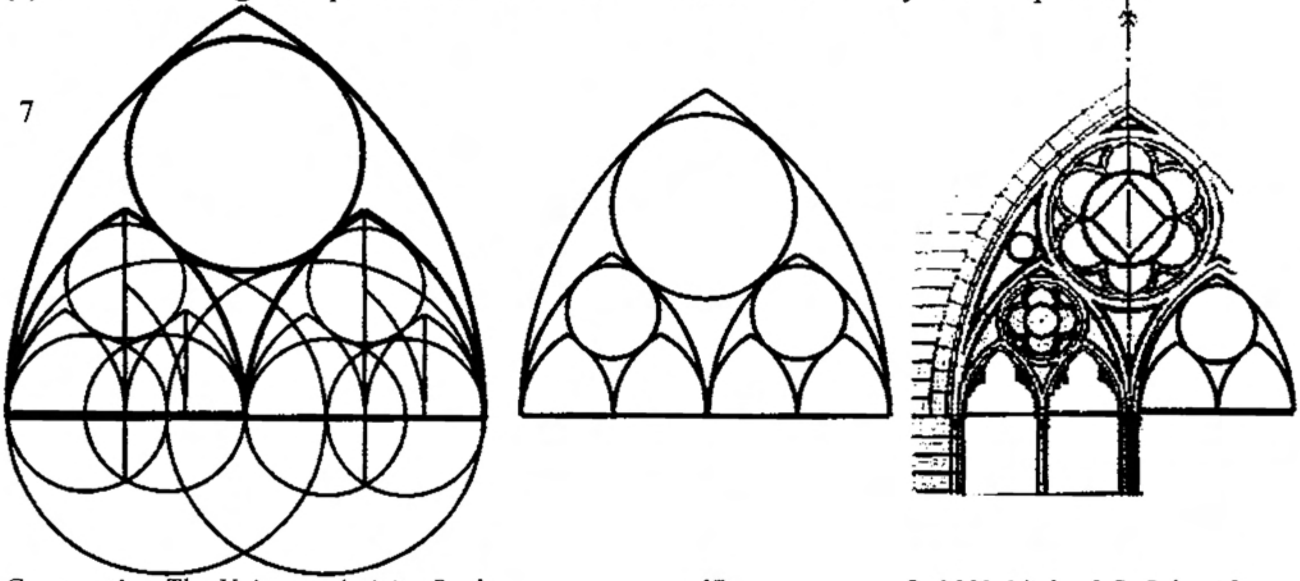
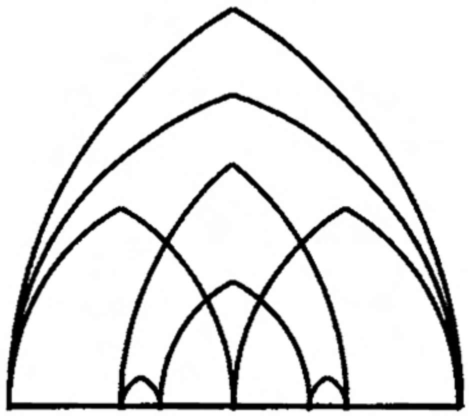


(7) The entire design is repeated in miniature in the smaller arcs. See if you can replicate it.

7

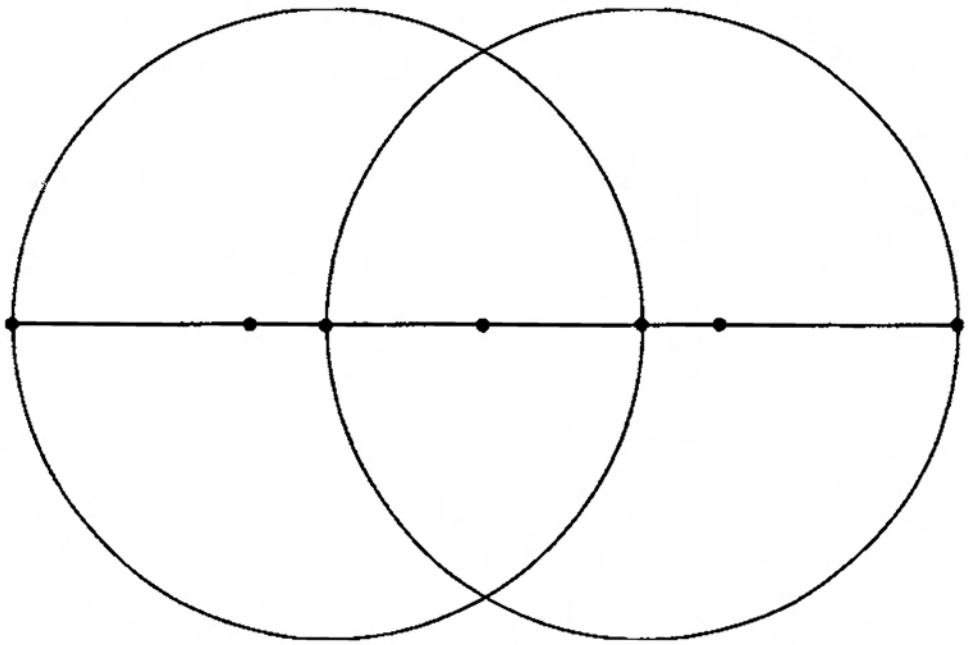


Design Your Own Musical Art, Crafts And Architecture



You can use the geometry of the musical concords as the basis for your own designs.

Seven points, including the ends, are shown along the line (string) across the Almond Circles below. They represent the points of musical concord along a string. Use these points, and the Almond's crossings, as places to open your compass between to turn Circles and draw connections. Open your compass between different pairs of points. Turn arcs and circles. See what designs you discover. Imagine what your constructions would sound like. Color them. Create more of these "musical designs" on blank paper.



3 The Triangle

The geometry of the Almond leads us naturally to the Triangle.

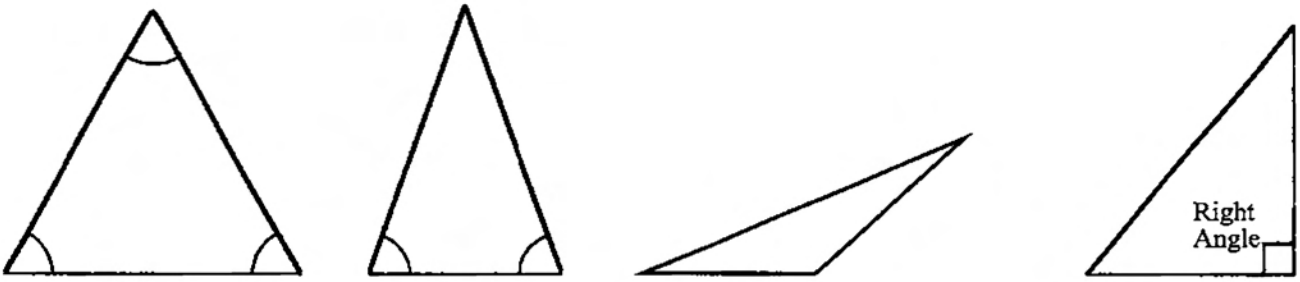
A Triangle is the simplest shape to enclose space with straight lines.
The Greeks called it a "trigon". *Tri-* is the Greek prefix for "three" and *-gon* means "angles".

Actually, the ancient Greek word *gonia* meant "knees". To the Greeks, shapes have bent *knees* instead of corner angles. A regular Triangle (a Greek *trigon*) has three equal angles or knees. Shapes made of many corners, and straight lines which enclose space, are called "polygons" ("many knees"). The Triangle, Square (or *tetragon*), Pentagon and Hexagon are the names of some polygons.

The word "angle" is a cousin of the words "ankle" and "anchor".
The *ang-* or *ank-* sound in a word means it is "bent".

Angle **Ankle** **Anchor**

There are four kinds of plane (flat) Triangles.



An **Equilateral Triangle** has:
3 equal angles
3 equal sides

An **Isosceles Triangle** has:
2 equal angles
2 equal sides

A **Scalene Triangle** has:
0 equal angles
0 equal sides

A **Right Triangle** has:
1 Right (90°) Angle and can be Isosceles or Scalene.

Our constructions here will concentrate on the equilateral (regular) Triangle.

If you did the loop experiment in Chapter 1 (page 7) you might have found that a Triangle enclosed the *smallest* space with the loop, making it the opposite of a Circle.

The word "trinity" means "tri-part unity" or "three-part oneness". Three become one the way that three strands of hair can weave together to become one strong braid.

Three points and Triangles appear in nature and inventions for their great strength and balance.

Microscopic Diatom plant

Clothes Hanger

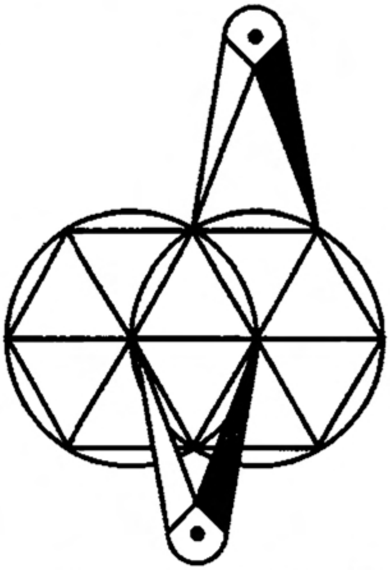
Tripod

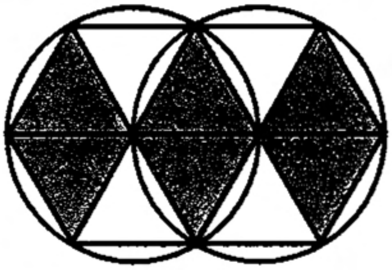
Finding Triangles

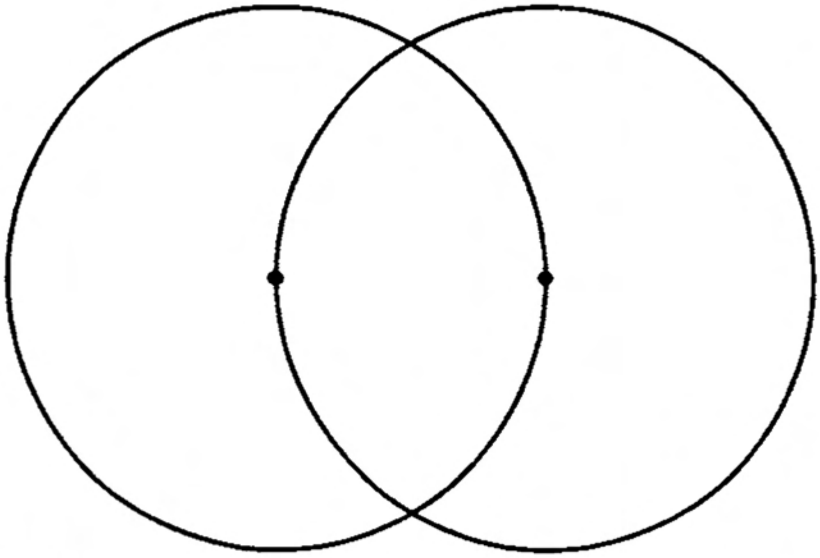
Ten regular Triangles are already hidden in the Almond.

Replicate the construction in the Almond Circles below.

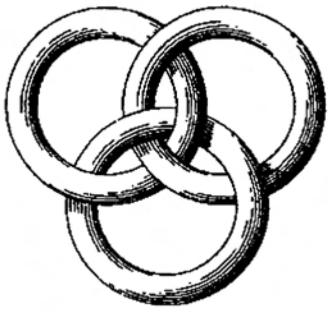
Hint: keep the same compass opening.







Constructing Triangles



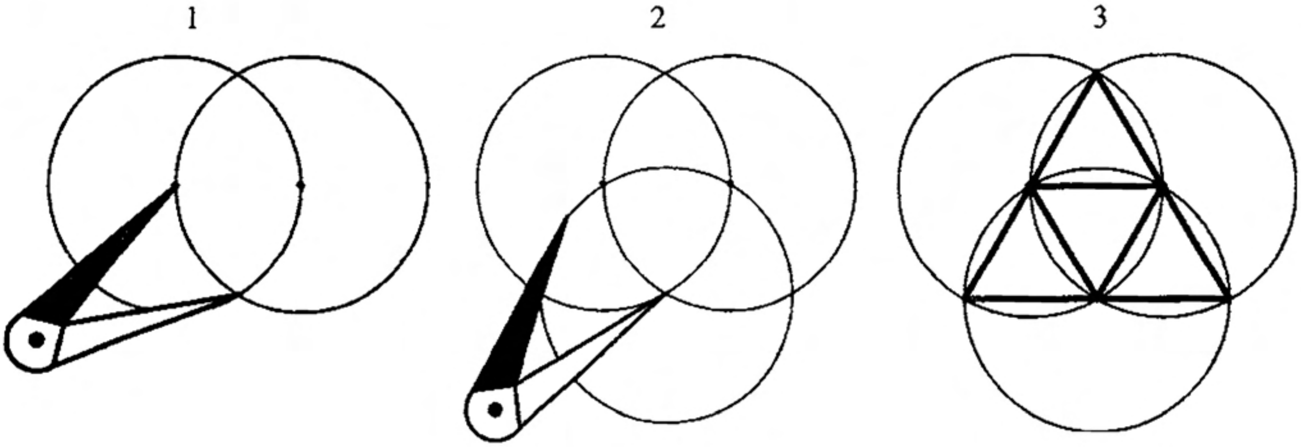
Borromean Rings
No two are linked, but all three lock together.

Here's a useful way to construct one Triangle made of smaller Triangles.

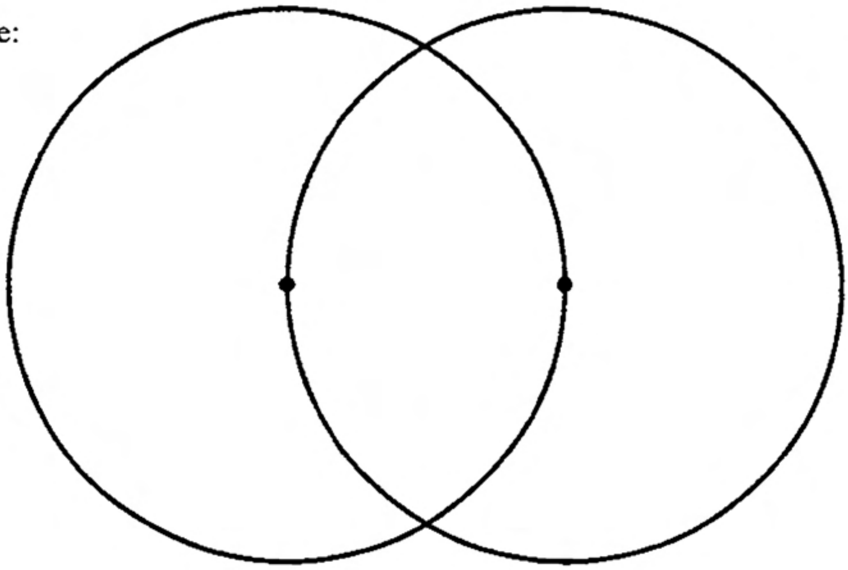
(1) First, do the Almond construction.

(2) Keep the compass open the same size and place its point where the Circles cross below. Turn a third Circle below.

(3) Use your straightedge to connect the circles' center points and crossings. This creates a large Triangle made of four smaller Triangles. If you did it accurately, the Triangle's bottom side should go through the Almond's lower crossing.



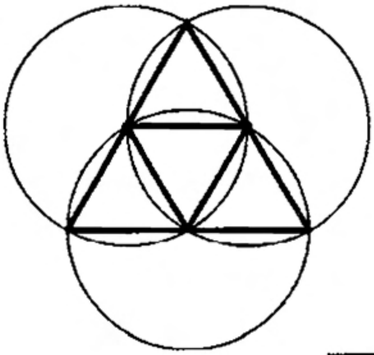
Practice it here:

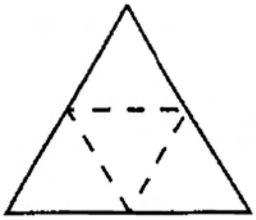


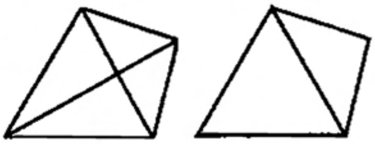
Fold A Tetrahedron

This method of Triangle construction also creates the first three-dimensional structure made of straight lines, the "Tetrahedron". *Tetra-* means "four" and -*hedron* means "face" or "seat". A Tetrahedron has four corners, six edges and four triangular faces. Every way you turn a Tetrahedron it looks the same because it has the same edge lengths, angles and faces. It is the simplest way to enclose the three dimensions of space with straight lines.

(1) It's best to do this construction larger, on a sturdy piece of paper or poster board. (To save space and make the Triangle larger, you need not turn the full Circles.)
(2) Cut out the large Triangle.
(3) Crease and fold up along the dashed edges of the inner Triangle. Then carefully tape the outer edges together.



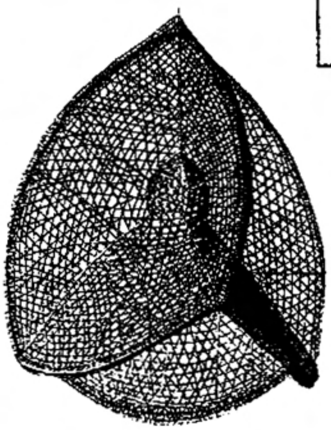




Tetrahedra

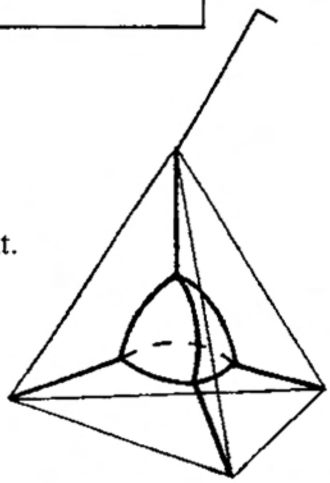
Stacking four spheres (like oranges) builds a Tetrahedron.

A Curious Truth About The Tetrahedron:
As the Triangle is the opposite of a Circle (page 12), a Tetrahedron is the opposite of a sphere. A sphere holds the *most* inside it, with the smallest surface area wrapping around it. With the same surface area as a Sphere, a Tetrahedron holds the *least* inside it. This small, tight space made of Triangles is what makes the Tetrahedron the strongest shape in three dimensions.



TetraBubble

If you make a wire tetrahedron, dip it in soap bubble liquid to see a surprising sight. Then catch a bubble in it and observe the curved tetrahedron bubble inside.



This microscopic radiolarian is a creature which lives in a glassy skeleton which resembles a captured Tetrahedral soap bubble.

What Are Dimensions?

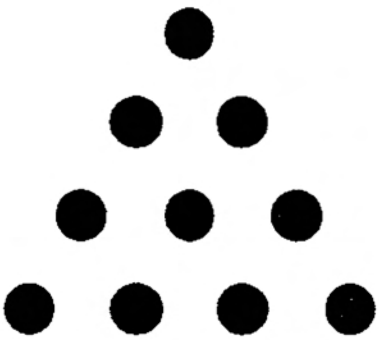
Since we'll be working with different geometric dimensions, we should know what the dimensions are. They can be understood as coming from one, two, three and four points, making the dimensions called point, length, area and volume.

One Point = Zero Dimension.
One point has no dimension.
A point is a location without any size.

Two points = One Dimension
Two points allow one dimension: length.
Two points are the ends of a line segment, a one-dimensional path, a trajectory between two points.

Three points = Two Dimensions
Three points allow two dimensions: length and width.
No matter where you place three points, connecting them always forms a flat "plane" surface. Three points make the corners of a Triangle, the simplest two-dimensional form (having straight lines).

Four Points = Three Dimensions
Four points allow three dimensions: length, width and depth (or height).
The three points of a triangle, plus one point above or below them, provides the four points of a volume which encloses space. A Tetrhedron is the simplest three-dimensional form (having straight lines). Everything you can touch is three-dimensional.



Tetraktys

A Triangle of ten points, the *Tetraktys* ("four levels") was one symbol of the School of Pythagoras (c 520 BCE). It shows the one, two, three and four points representing the dimensions.

These numbers (combined as fractions) also show us the harmonious intervals of a musical string (page 15):

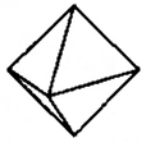
$$\frac{1}{2} \quad \frac{2}{3} \quad \frac{3}{4} \quad \frac{4}{4}$$

The Octave, Musical Fifth, Musical Fourth and Whole string. This coincidence of the dimensions of space and the musical concords led ancient philosophers to believe that the universe is a big musical composition.

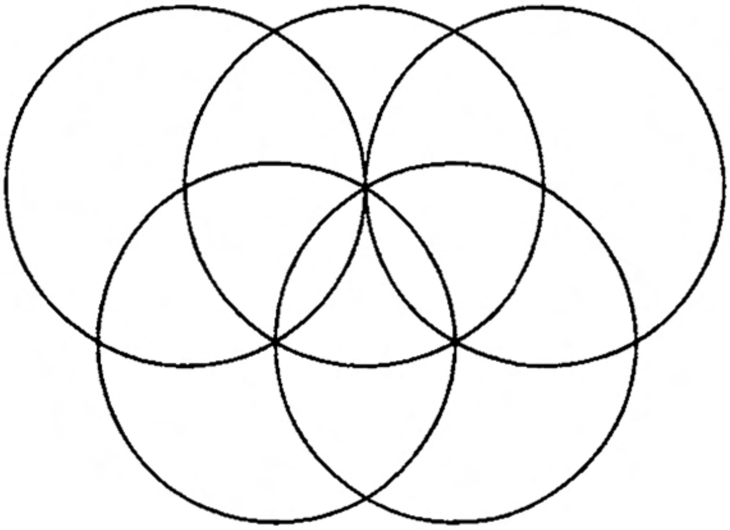
Pythagoras

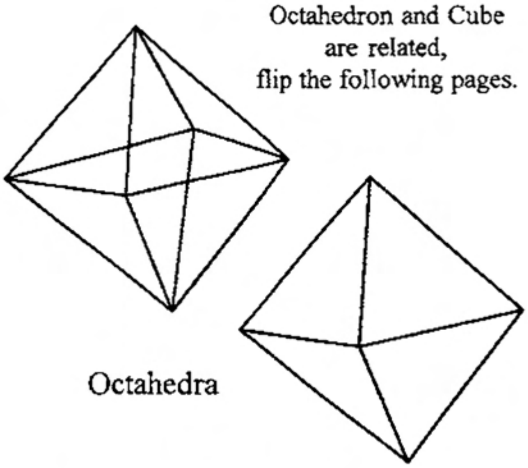
Fold An Octahedron

Another space enclosing volume made only from regular Triangles is the Octahedron. An Octahedron has eight identical triangular faces (or seats), six corners and twelve edges. It looks like two pyramids sharing a Square base. Many natural crystals including gold and diamond grow as Octahedra.



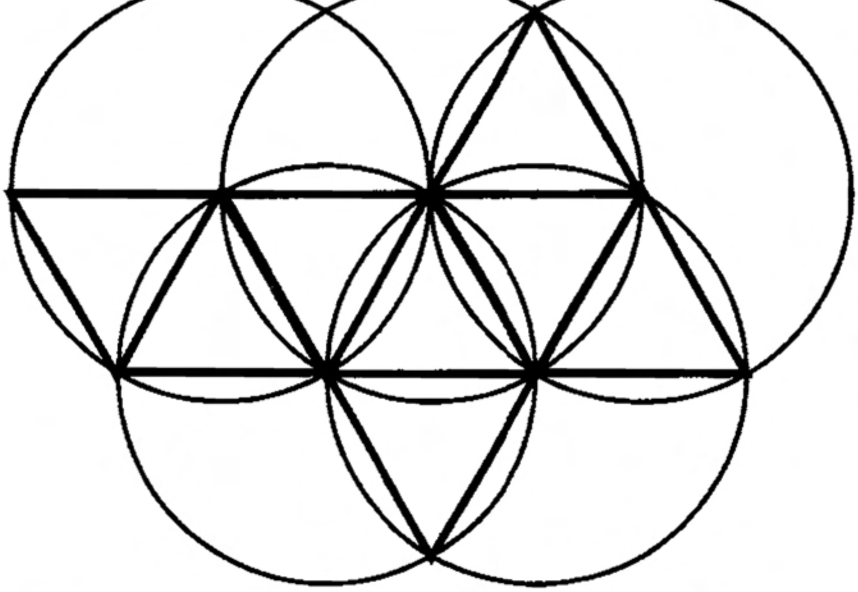
Octahedron To Cube Flip Movie
To see how an Octahedron and Cube are related, flip the following pages.

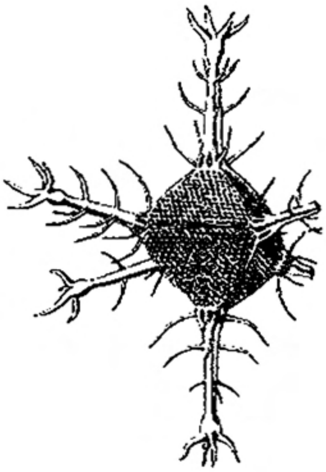




Octahedra

There are many ways to construct them. Try this on a blank sheet of paper. Construct five overlapping Almond Circles touching each others centers (shown above). (Use crossing points as the centers of new Circles.) Then connect corners with a straightedge to draw Triangles (shown below). Color or draw in them. Cut out around all the Triangles.
Crease and fold the Triangles along the edges where they come together.
Tape the edges together and you'll have an Octahedron.

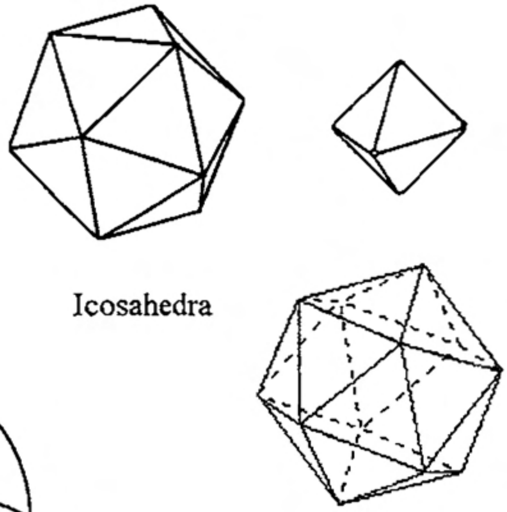




Microscopic radiolarian

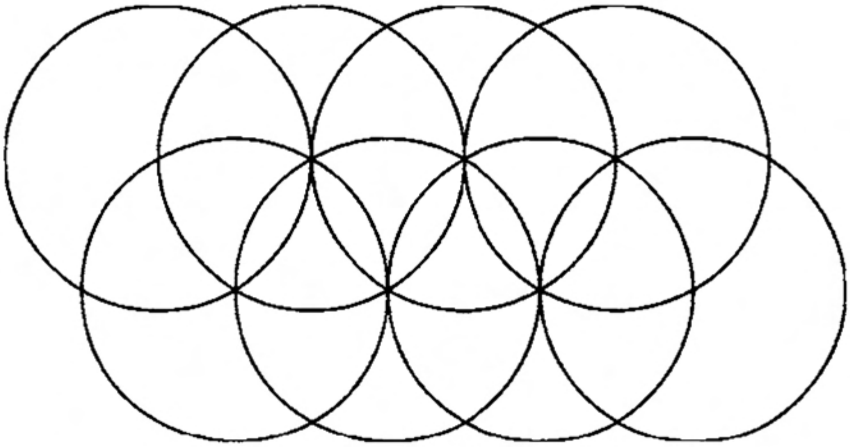
Fold An Icosahedron

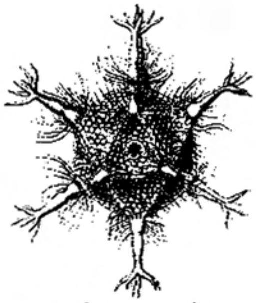
There is a third space enclosing structure made only with regular Triangles called an Icosahedron ("twenty seats"). It has twenty triangular faces, twelve corners and thirty edges.



Icosahedra

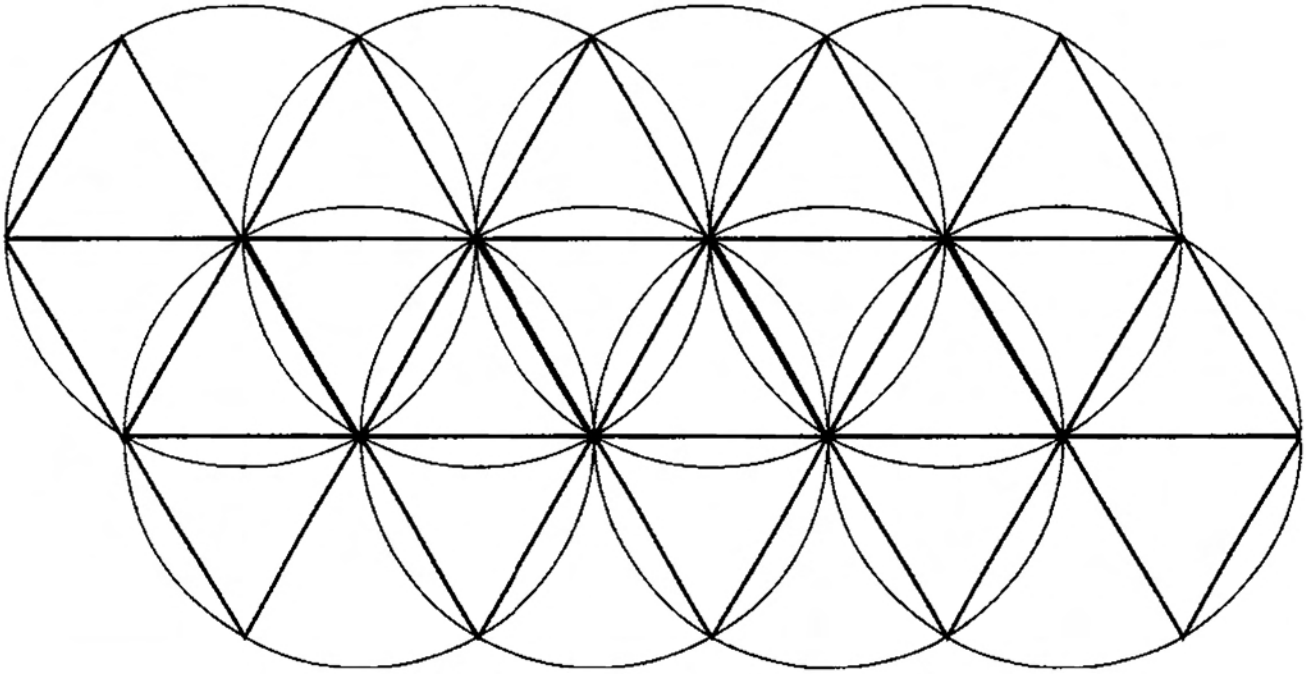
To build one, construct it beginning with eight overlapping Almond Circles like this:





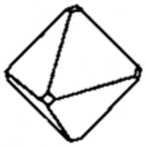
Microscopic Radiolarian

Connect points with a straightedge to create twenty Triangles like this:



Cut out around the outside of all the Triangles.
Crease and fold all the edges.
Carefully tape the edges together to create an Icosahedron.

Subdivide A Triangle



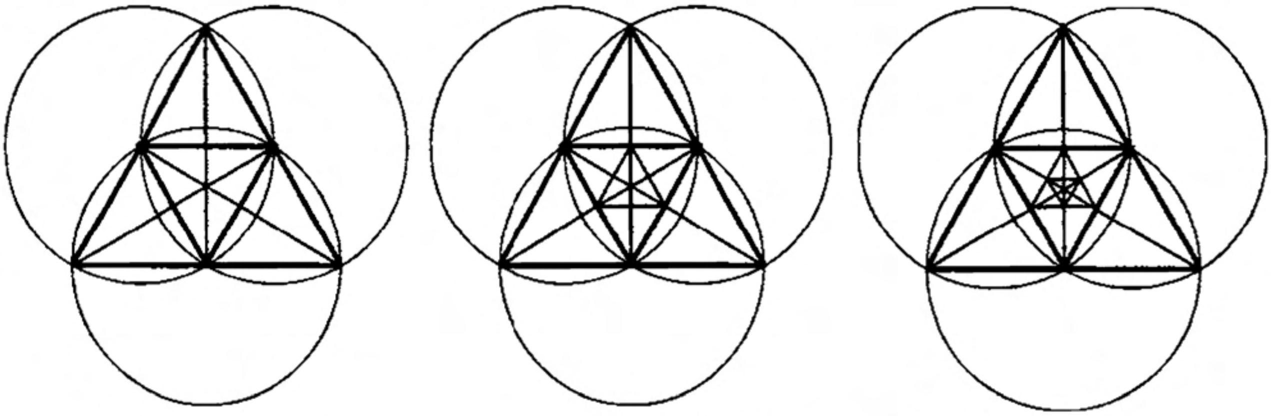
Being able to subdivide a Triangle into smaller Triangles can help us understand designs in nature and art.

(1) First, construct the Triangle in three Almond Circles. Draw lines across the Triangle connecting the corners with the midpoints of the opposite sides, where the Circles cross.

(2) Draw a small Triangle where the lines cross the inner Triangle.

(3) Keep drawing smaller Triangles where these lines cross each previous Triangle. Draw Triangles as sharp as your pencil will allow. Notice how the Triangles alternate pointing down and up.

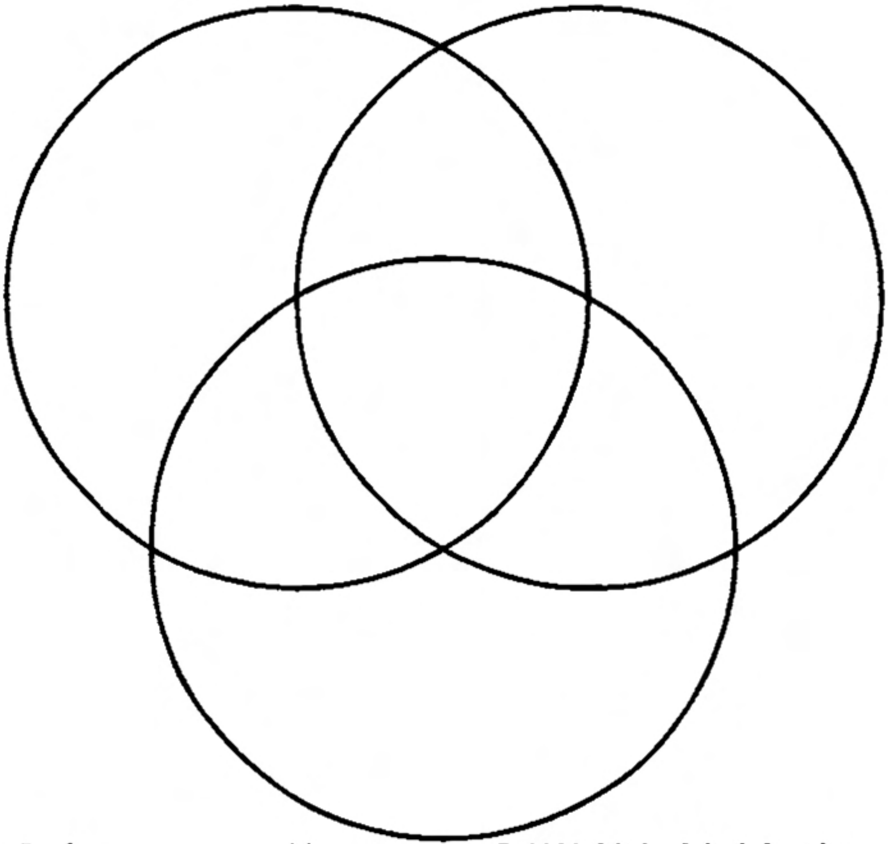
1 2 3



Practice Subdividing A Triangle

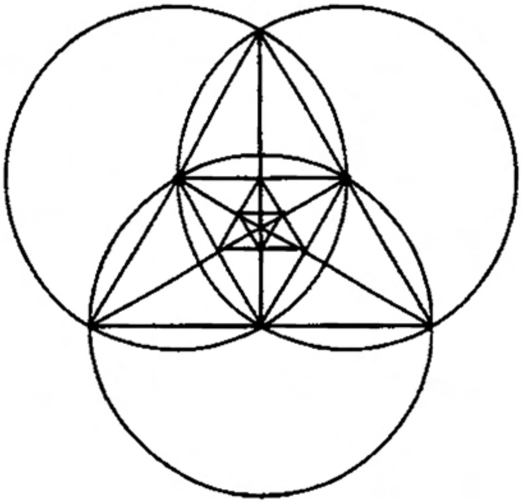
Three Circles touch each others' centers.

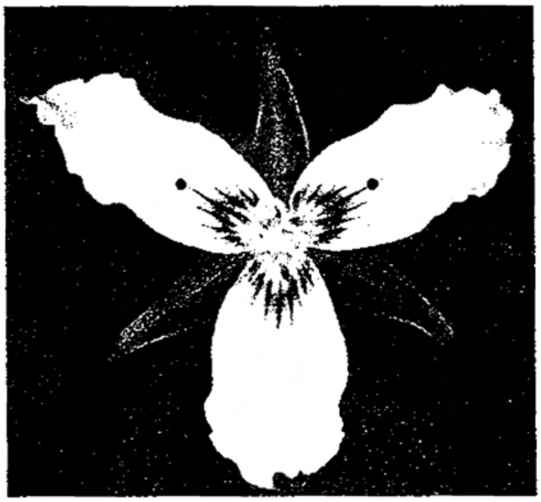
Draw and subdivide a Triangle in them.



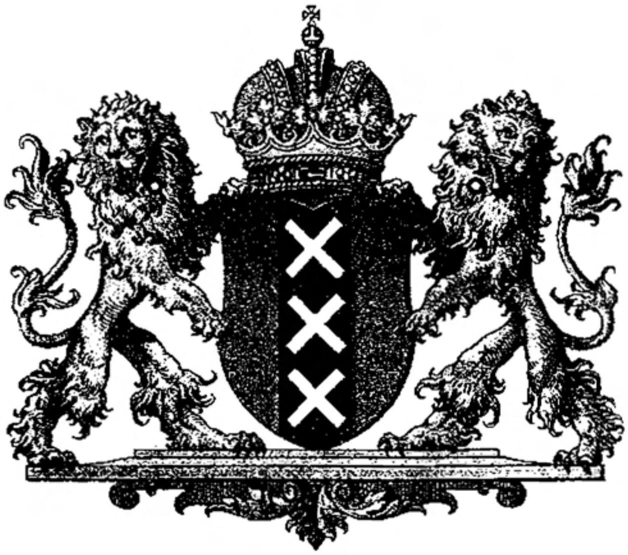
Triangle Design In Nature And Art

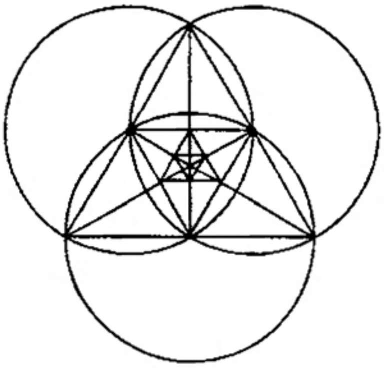
On each picture, look for two points. Open your compass between them and use this distance to turn a third Circle.
Construct a Triangle. Draw lines across the Triangle and subdivide it into smaller Triangles.
This will reveal something about the object's design.



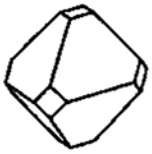


Pharaoh Amenemhat III
Twelfth Dynasty, 1843-1798 BCE

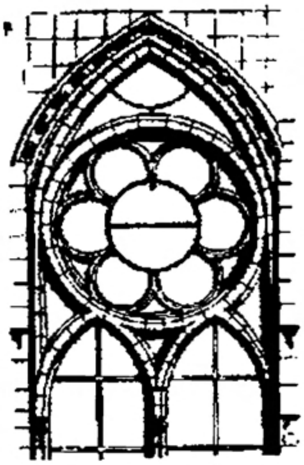


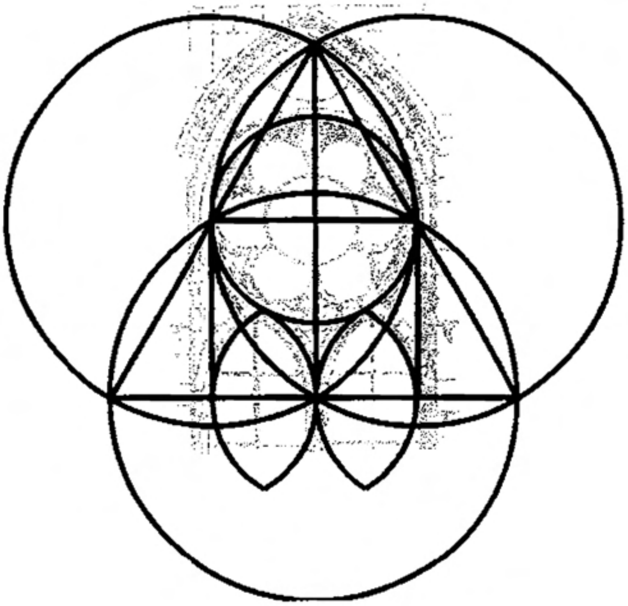


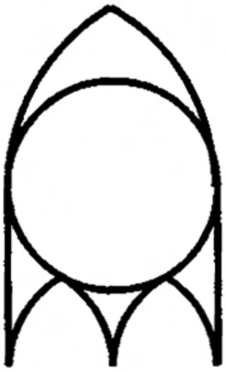
Replicate Another Window Tracery

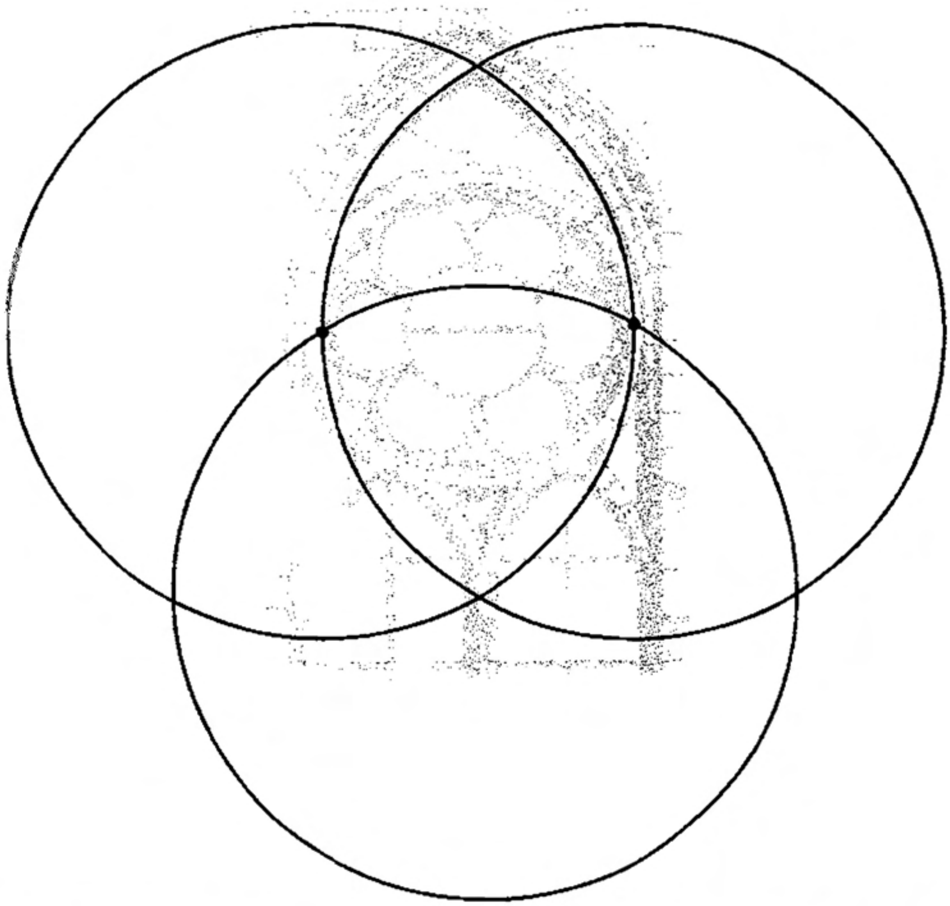


The geometric design of this window is a variation of the musical window design seen before (page 16). It is built on a triangular frame. See if you can replicate its geometric scheme on the image below.

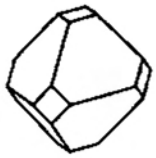




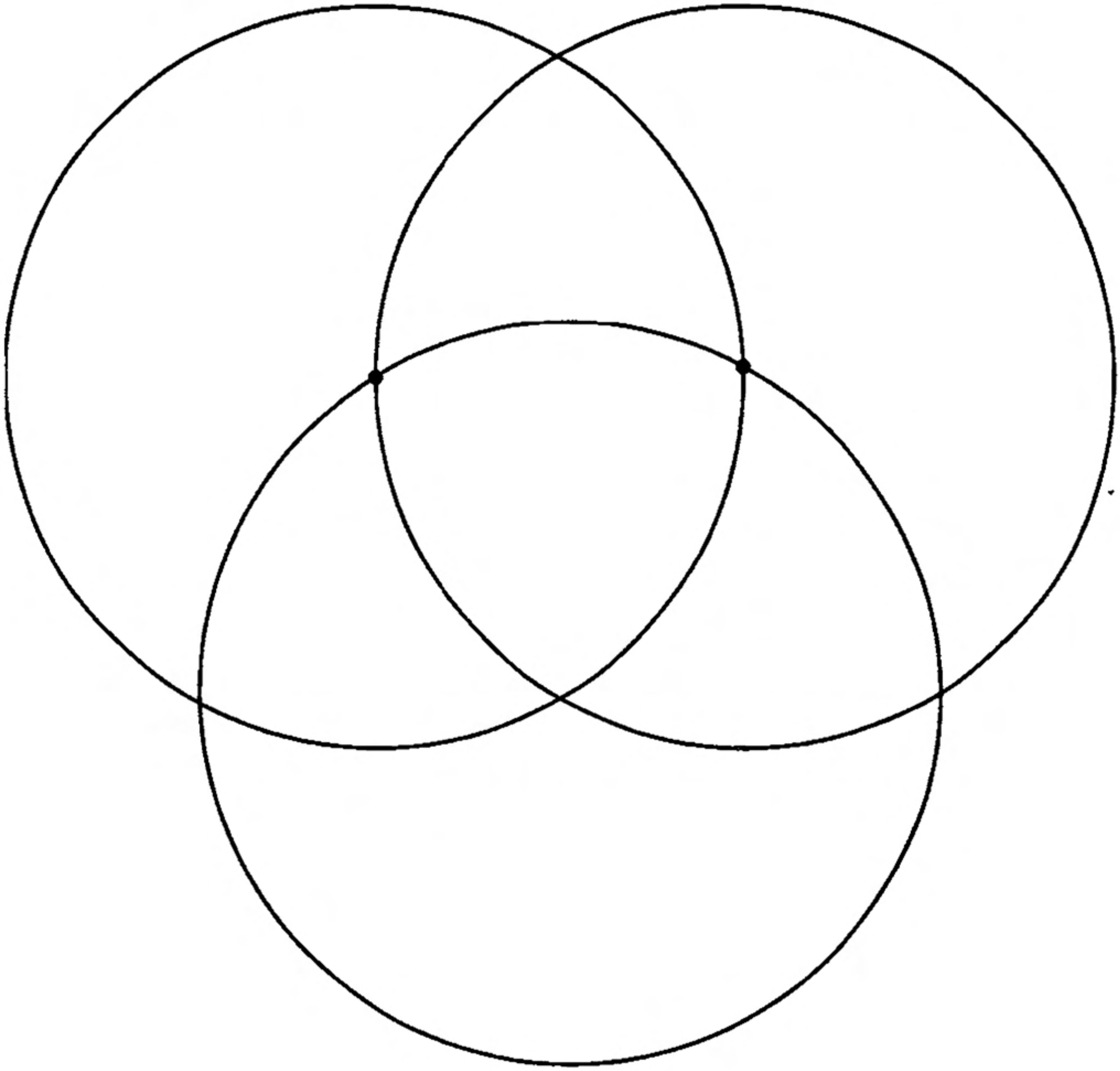




Design Your Own Art

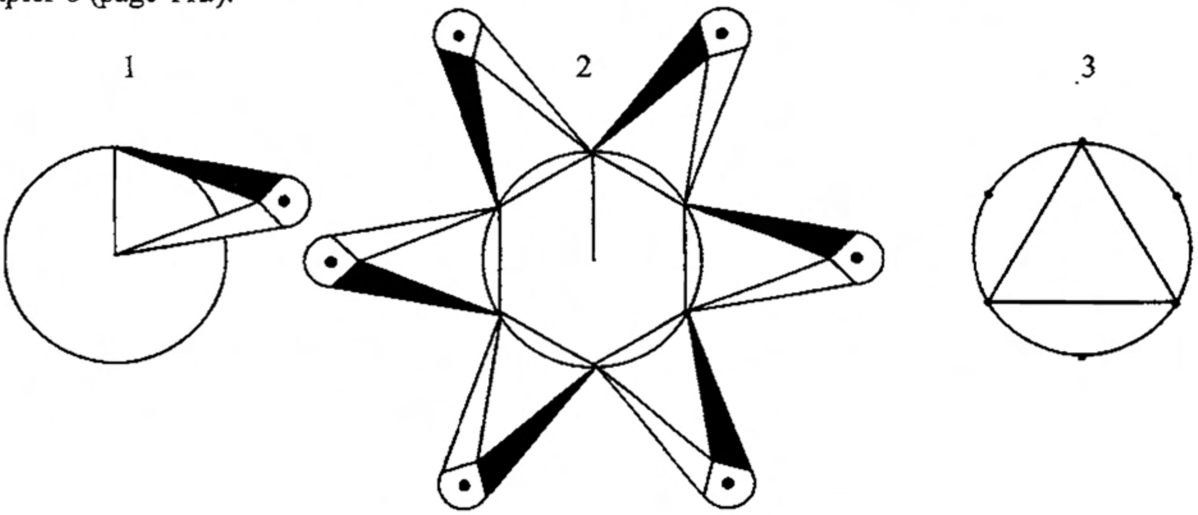


Use these Circles, or make them on a blank sheet of paper, to draw and subdivide a Triangle. Use your artistic imagination and colored pencils to design something guided by the geometry.



Construct A Triangle By "Walking Six-Around-One"

It is a curious Truth that a compass open to the radius of a Circle will "walk" around that Circle exactly six times. ("Walking" can be done with the modern compass but not with classic collapsible dividers.) We can use this Truth to construct and subdivide a Triangle *within just one Circle*. This is also known as "walking the radius" around its Circle. We'll learn more about this construction in Chapter 6 (page 112).

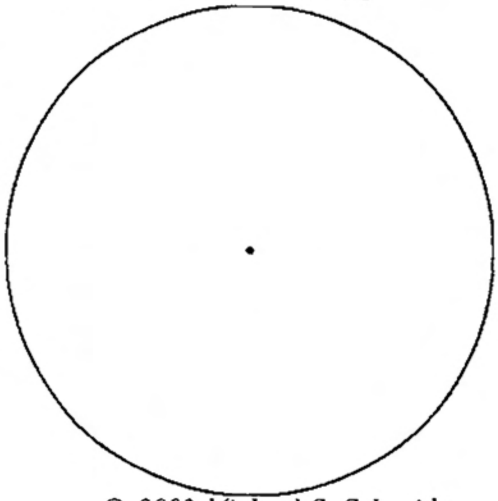


(1) Turn a Circle on a blank piece of paper.

(2) Mark a point at the top of the Circle, then lift the compass and put its point on this new mark. Swing an arc to mark where it crosses the Circle. Lift the compass again and put its point on the next mark on the Circle. Repeat this process of marking points around the Circle until you have arrived, after six steps, to the point where you started.

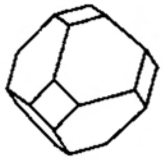
If your ending isn't exactly at the beginning point, then you made slight errors along the way. The remedy is to begin at the original starting point and *repeat the process in the opposite direction*. That is, "walk" the compass around in *both* directions. The actual six points will be found in the *middle of each pair* of marks you've made going in both directions. This technique of "walking" the compass around the Circle *in both directions* from the starting point will be useful for each new polygon construction we learn.

(3) Connect every other point to draw a regular Triangle.



Open your compass to the radius of this Circle and construct a regular Triangle in it by this method. See how accurate you can be finding six points around different size Circles.

Subdivide A Triangle

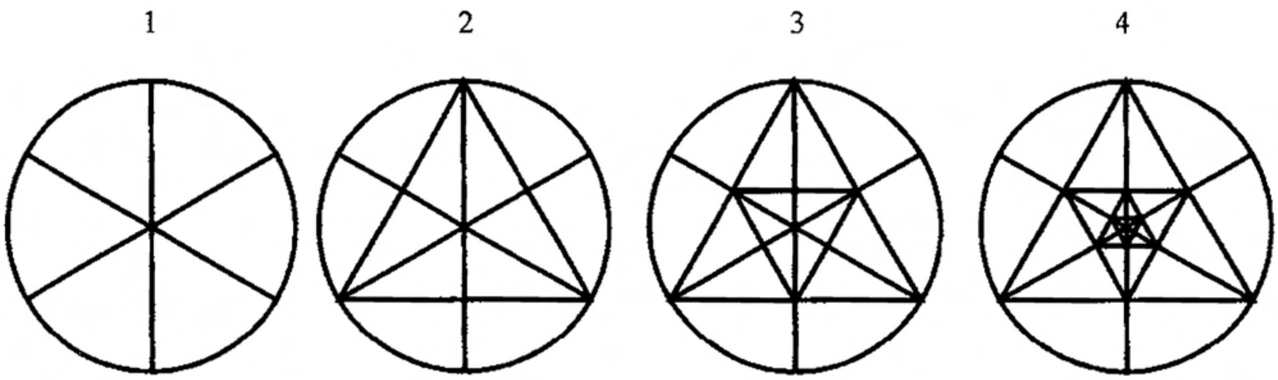


(1) Once you have found six equally spaced points around the Circle, connect the opposite pairs of points and draw three diameters as shown.

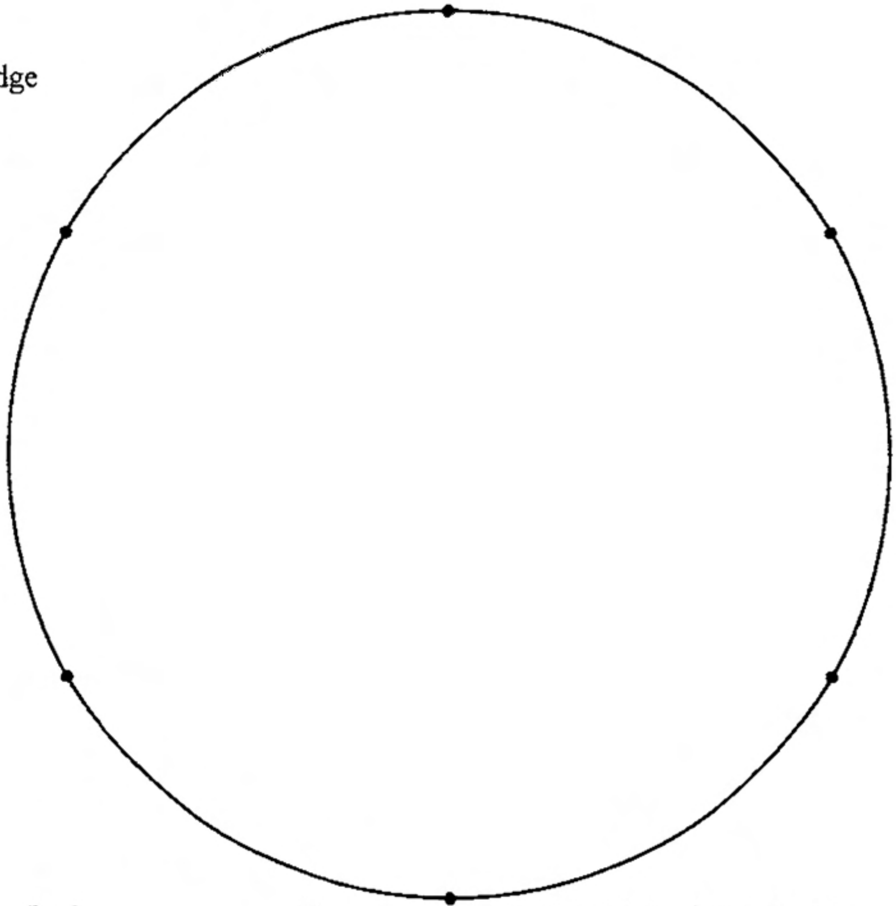
(2) Construct a Triangle by connecting every other point.

(3) Subdivide this Triangle by connecting the points where the diameters cross each new Triangle.

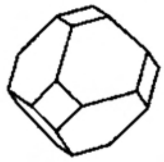
(4) You can do this as small as your pencil point allows.



Use your pencil and straightedge to inscribe and subdivide a regular Triangle in this circle.



Another Subdivision Of The Triangle



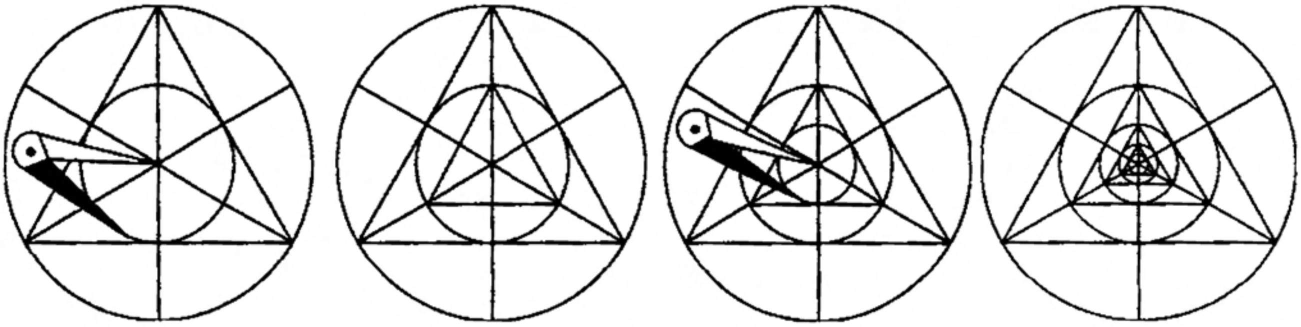
Another way to subdivide a Triangle creates smaller Triangles which always point upward.

(1) Inscribe a Triangle in a Circle. Place the compass point at the center and open it to where the diameter crosses the Triangle. Turn a Circle.

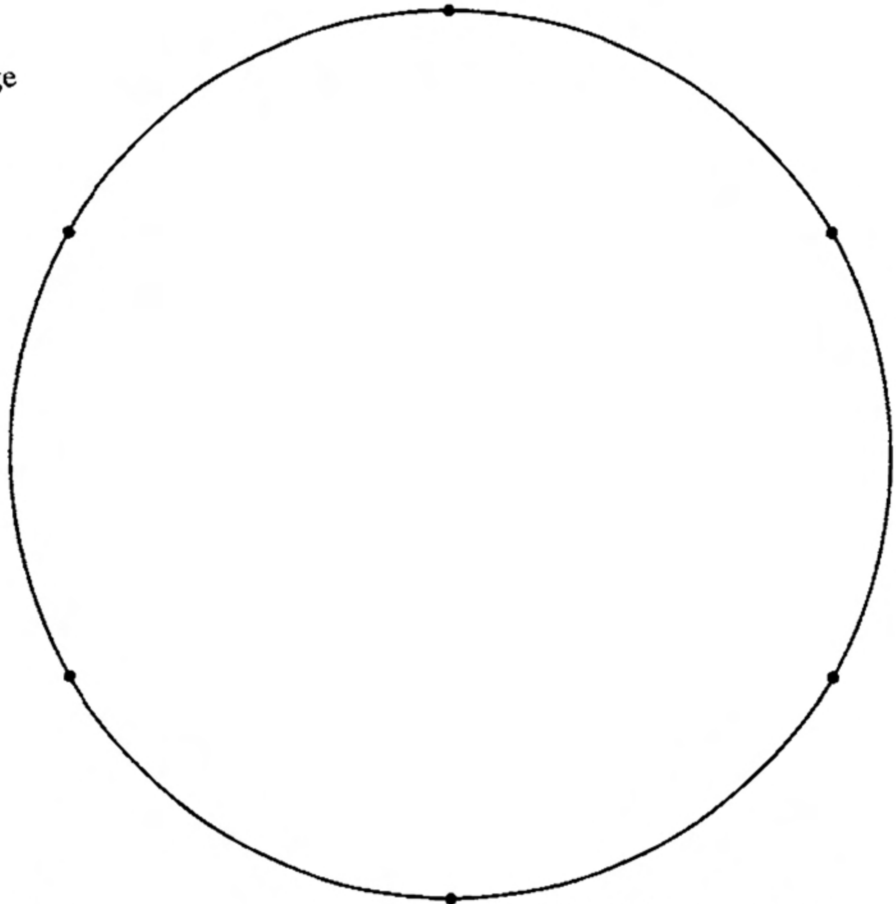
(2) Connect the points where the diameters cross the Circle and make a Triangle.

(3) Repeat the process turning a Circle inside in each new Triangle.

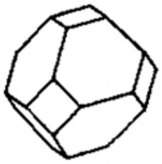
(4) Make a new Triangle inside each Circle. Do this as small as your pencil point will allow.



Use your pencil and straightedge to inscribe and subdivide a regular Triangle in this Circle.



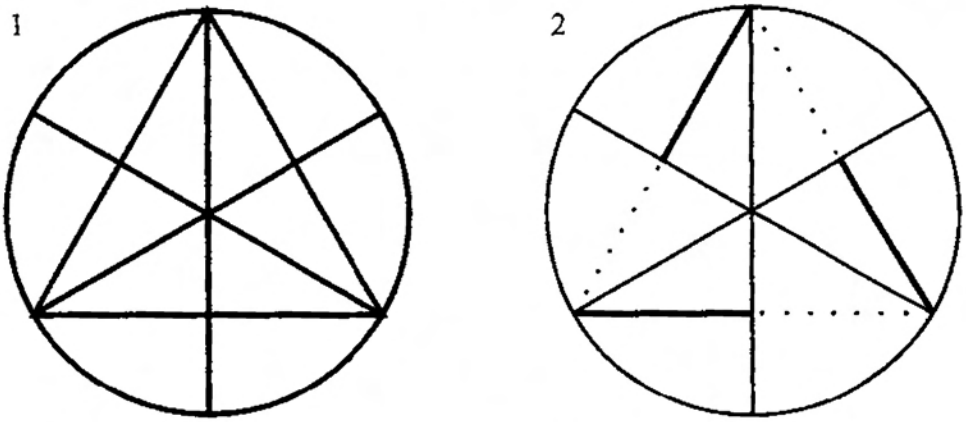
Another Way To Build The Space Enclosing Forms



We saw on page 34 how to build three space enclosing forms with Triangle faces: the Tetrahedron, Octahedron and Icosahedron. Here's another way to make them on sturdy paper or poster board by cutting out Circles containing Triangles.

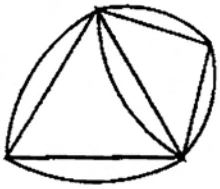
(1) Construct a Triangle with three diameters in one Circle by the "Walk Six-Around-One" method.

(2) Use scissors to carefully cut out the circles and cut slots *halfway* along each side of the Triangle (as shown).



Use this as a template to easily make more of them. These "Triangles within Circles" will join together by their slots to build the three space enclosing forms. They will appear to be inscribed in a sphere.

Four of these Triangles within Circles will slot together to build a Tetrahedron.

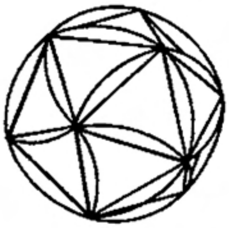


Tetrahedron

Eight of these Triangles within Circles will slot together to build an Octahedron.

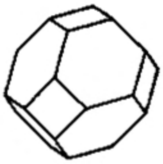
Octahedron

Twenty of these Triangles within Circles will slot together to build an Icosahedron.



Icosahedron

Replicate These Constructions

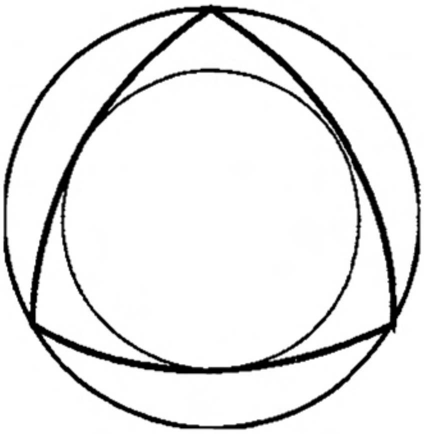


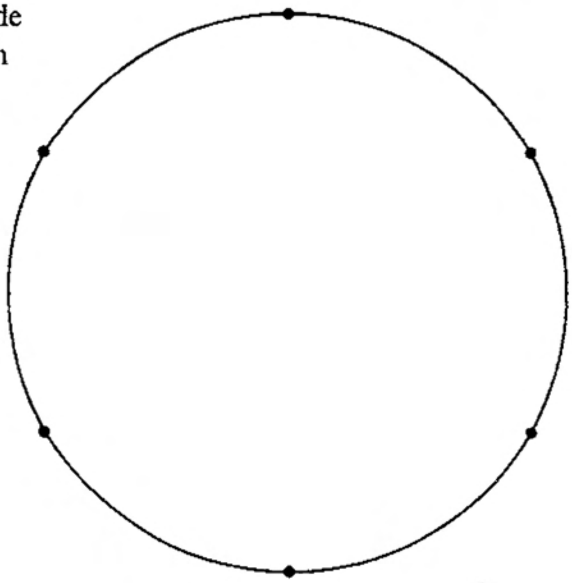
Circles with six equally spaced points are already provided.

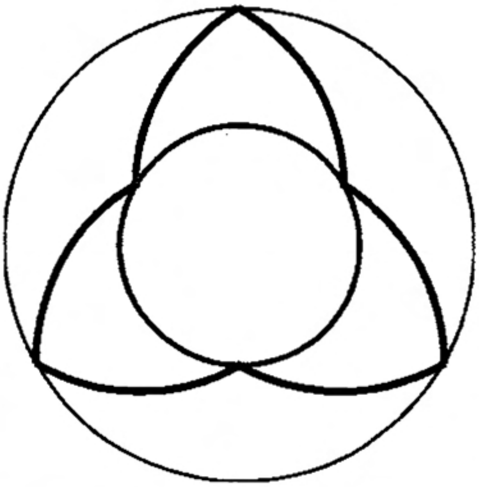
To replicate any construction, you must ask yourself some questions:

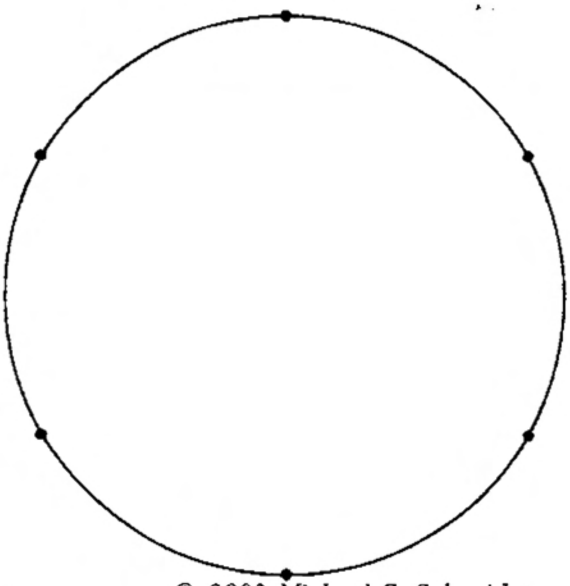
* What is the sequence of steps for this construction? (Certain steps must occur before others can be done.)
* At each step, where should the compass center point be placed?
* Where does the compass scribe (pencil) open to?
* Which points should I connect to make new crossing points?

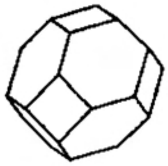
For most of these constructions, your first step will be to draw three diameters across the Circle. The second step may be to inscribe a Triangle in the Circle. You'll have to decide what you need to do next by looking ahead to where each step must lead. Not all the lines needed are shown.

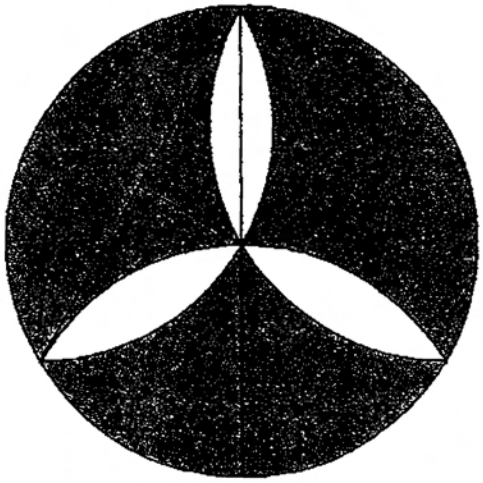




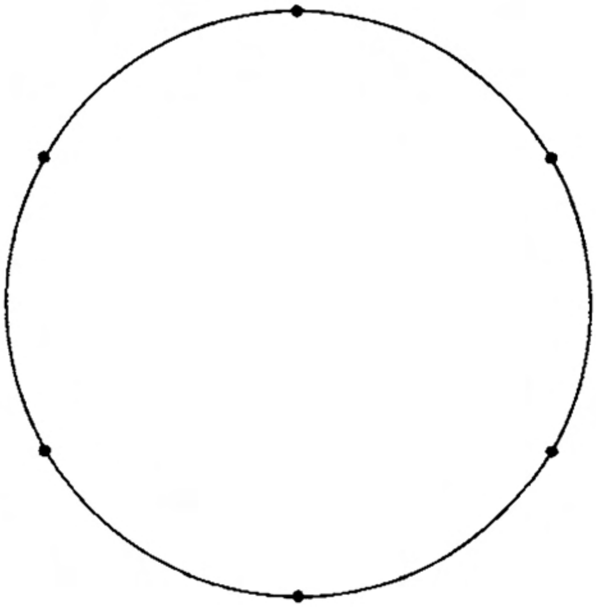


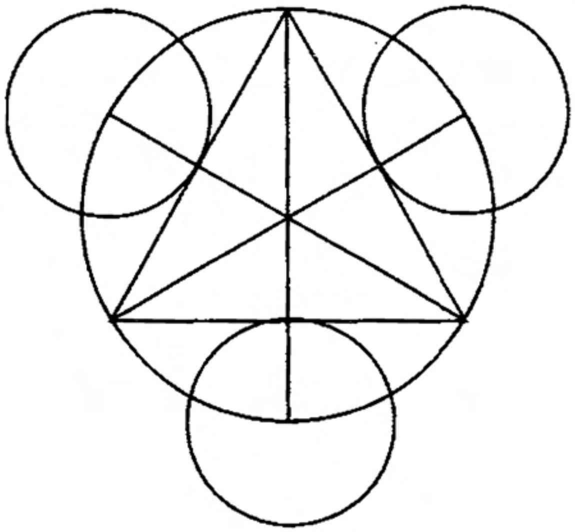


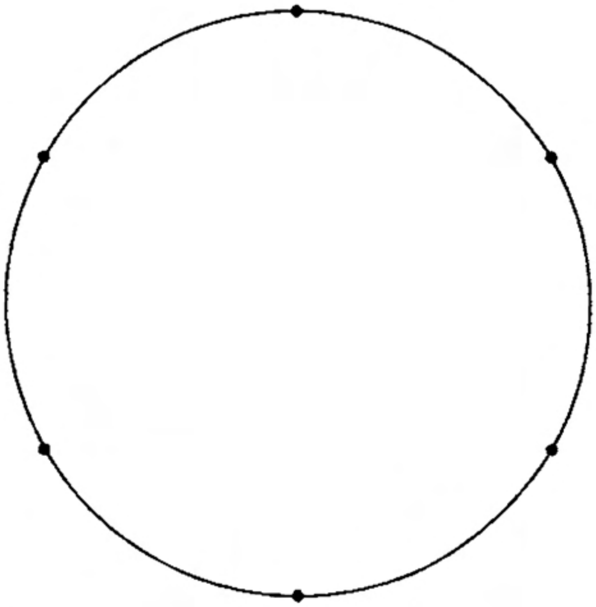




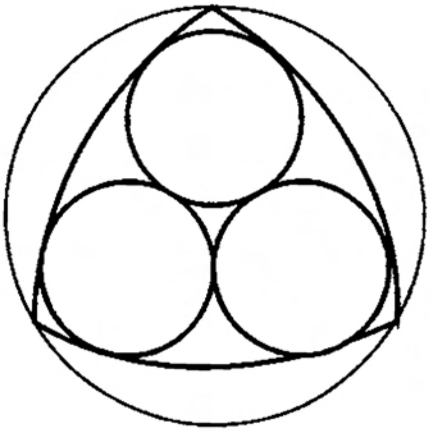
Cut along the three straight lines and rearrange
only the shaded pieces into a rectangle!

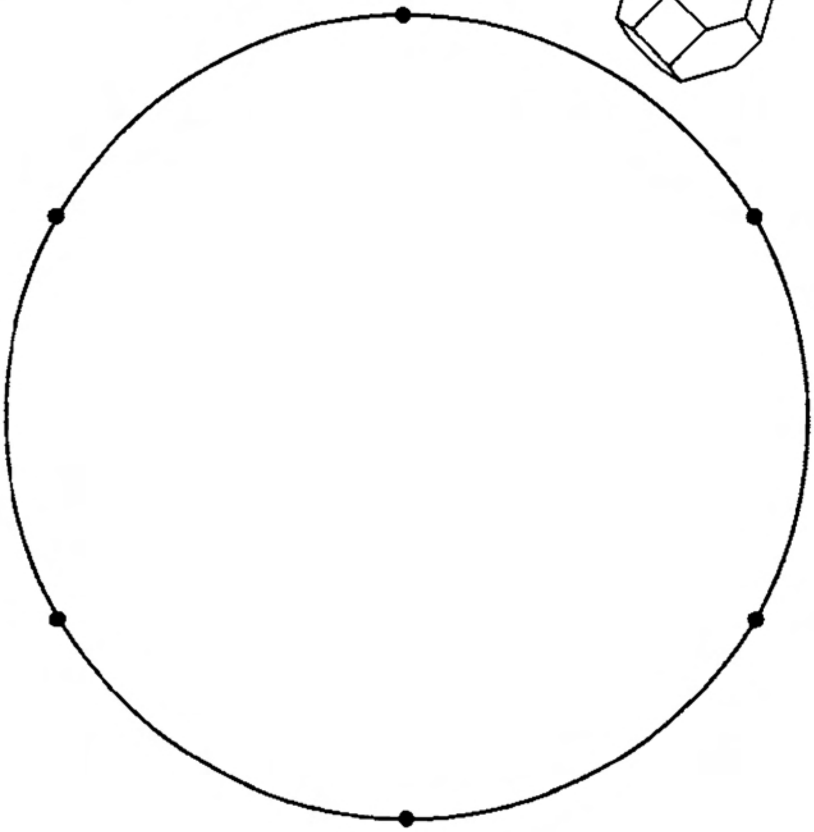




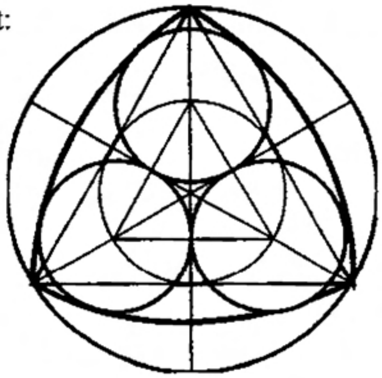


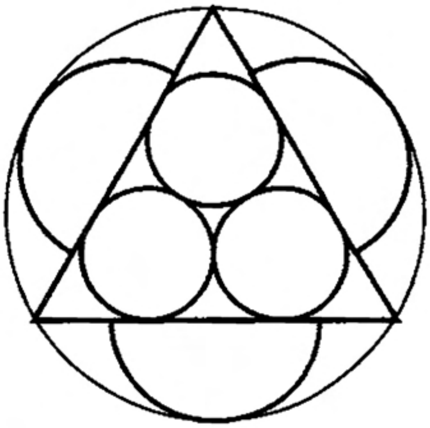
These are a little more difficult,
so hints are provided for each.

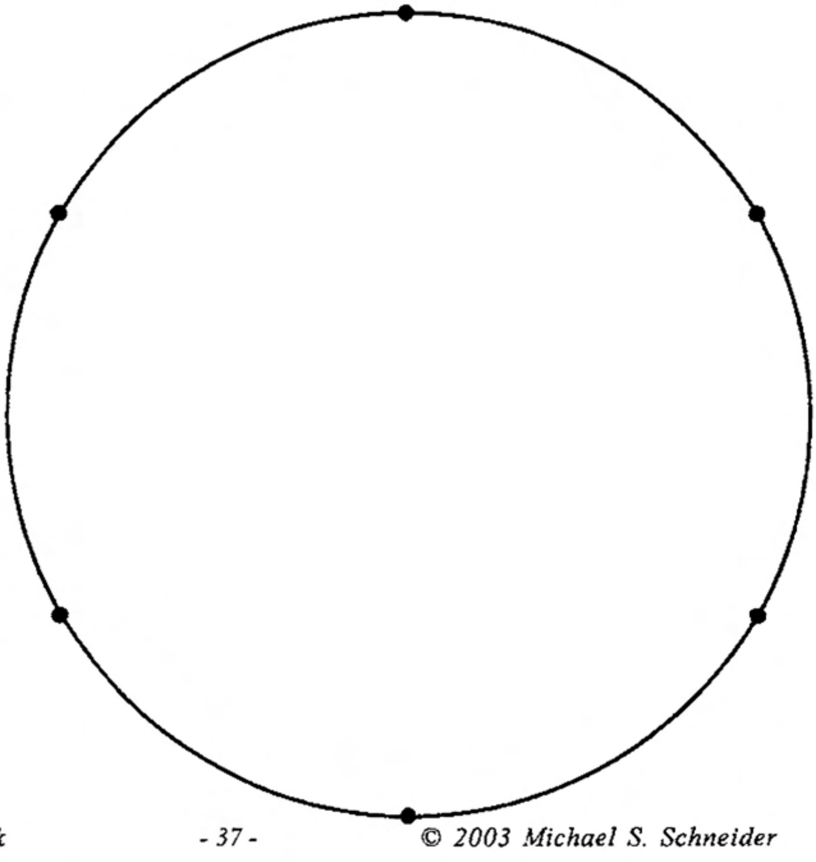




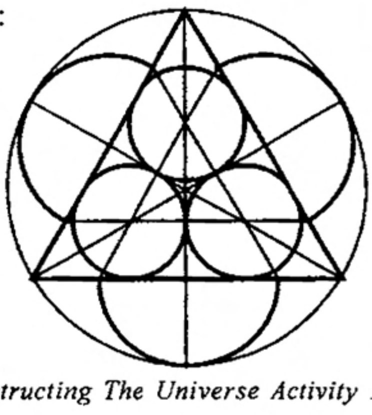
Hint:



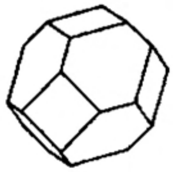




Hint:



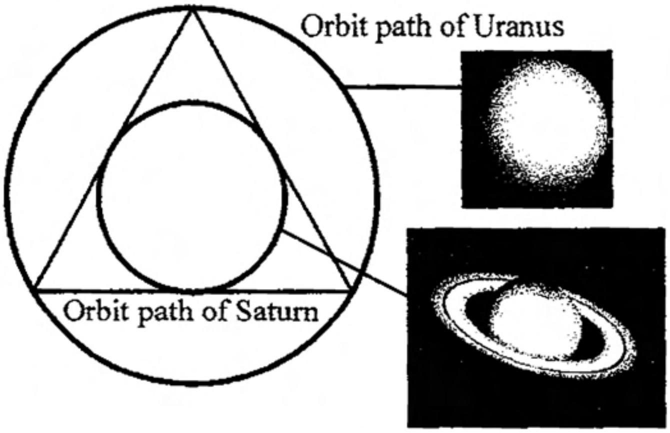
The Mean Orbits of Planets

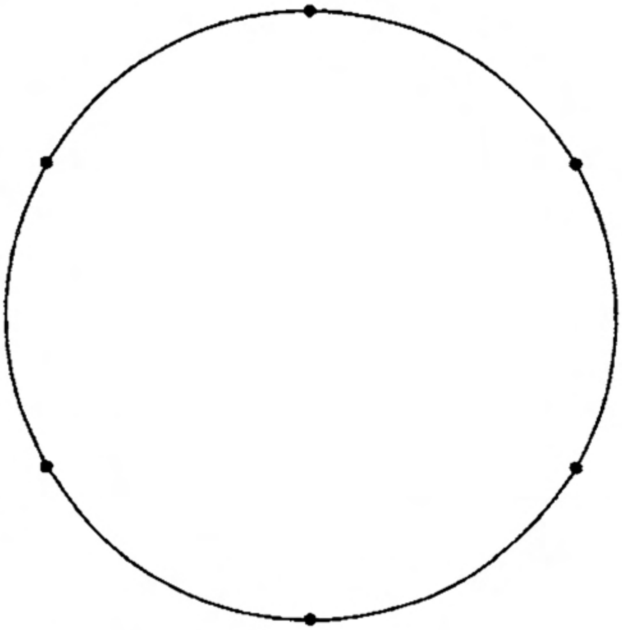


The planets of our solar system orbit the sun along paths which are ellipses. But if they are adjusted to become actual Circles we see their *mean* or average orbits. Simple geometric constructions can show their relative distances from each other and the sun at the center with 99.9% accuracy. These constructions were derived and combined from *A Little Book Of Coincidence* by John Martineau, and *The Harmony Of The Spheres* by Ofmil C. Haynes.

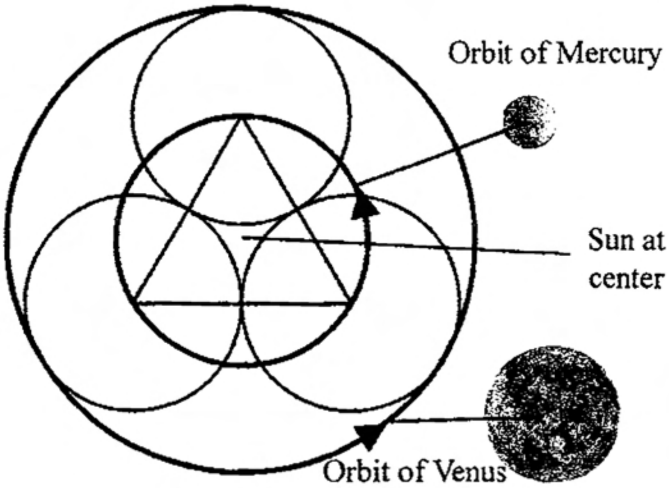
The sizes of the planets are not drawn to scale, only their orbits are.

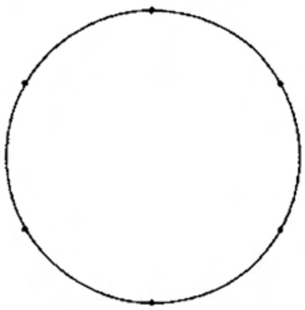
Saturn and Uranus





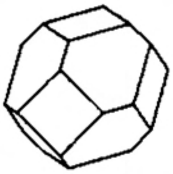
Mercury and Venus



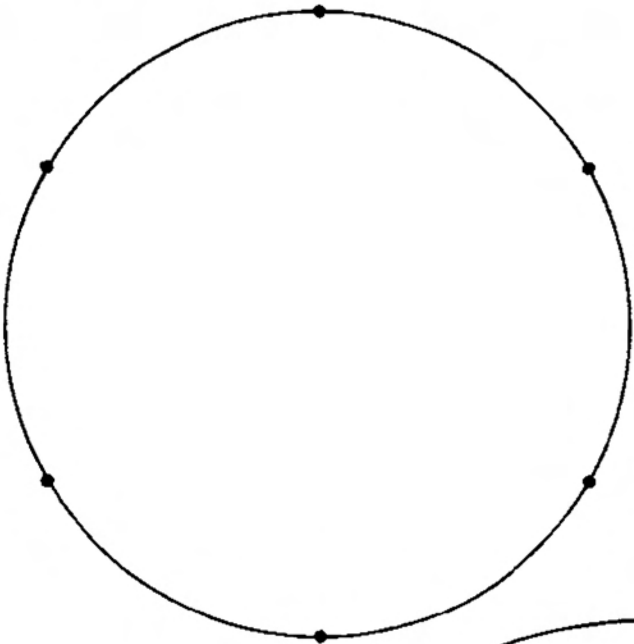


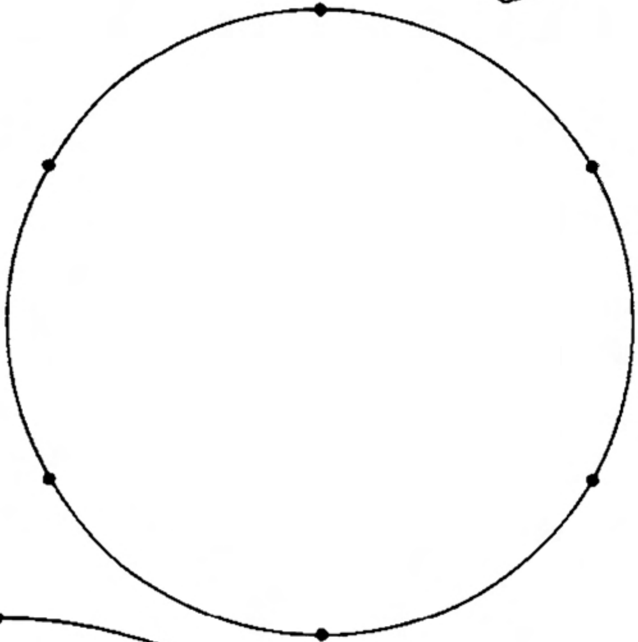
Hint: Start with the inner Circle and Triangle.

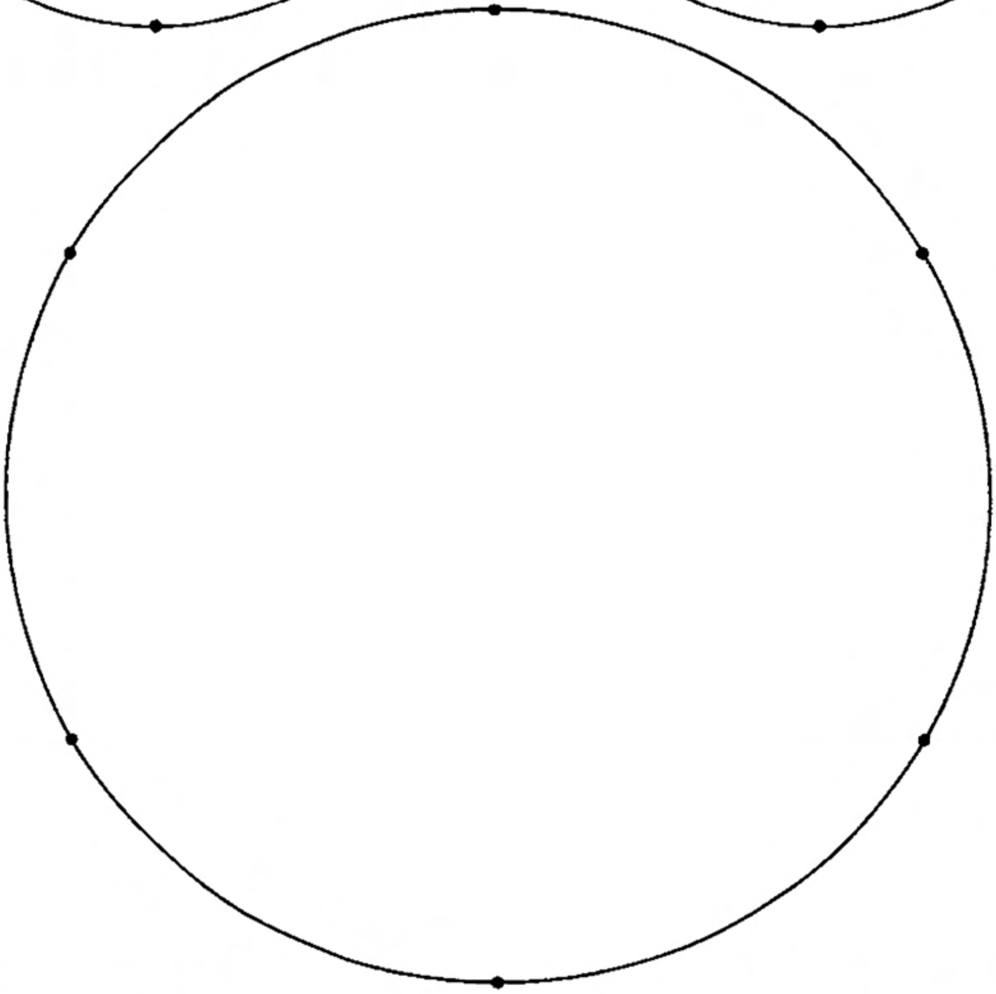
Create Your Own Patterns



Use the six points around these Circles, or do your own on blank paper, to create your own constructions. Use colored pencils to shade them in.

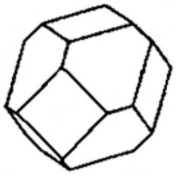




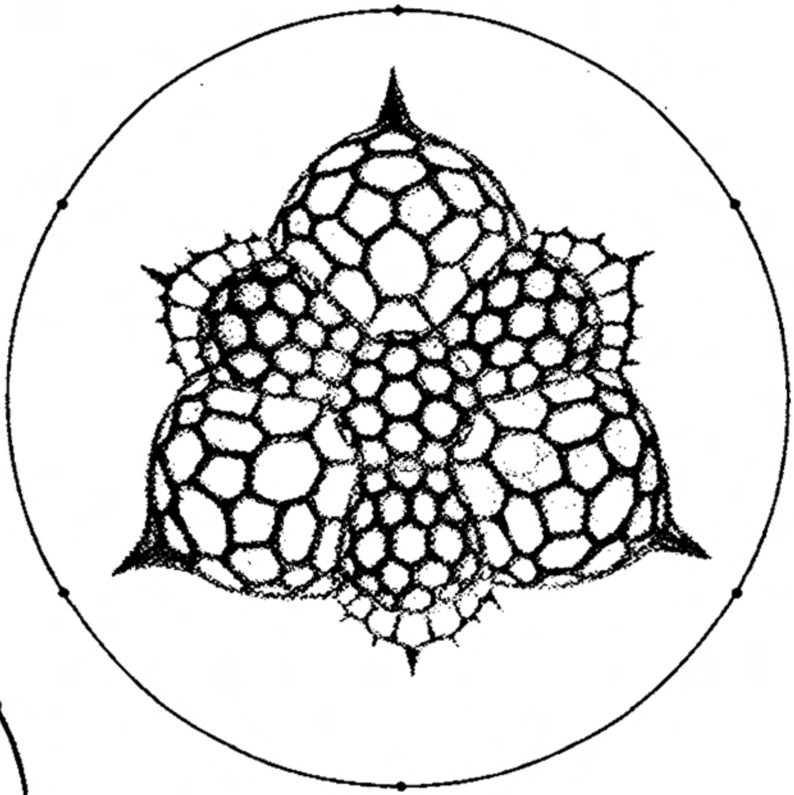


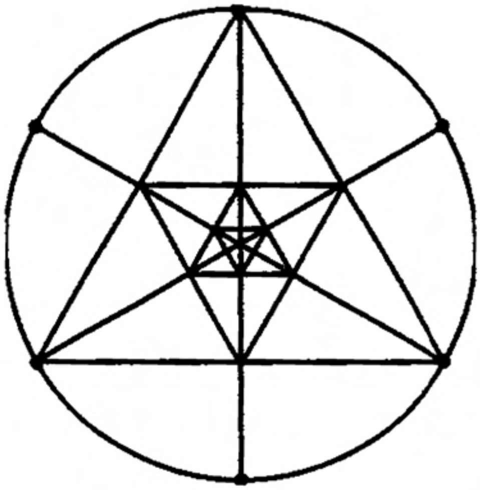
Triangular Designs In Nature

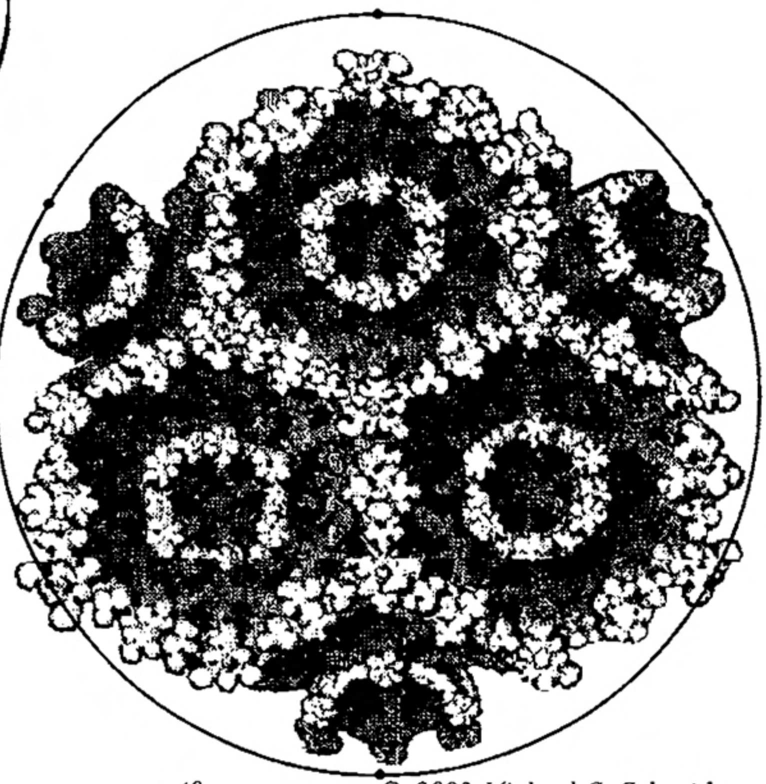
Use the six points around each Circle to subdivide a Triangle and see the geometric schemes of these natural forms.



Microscopic Radiolaria



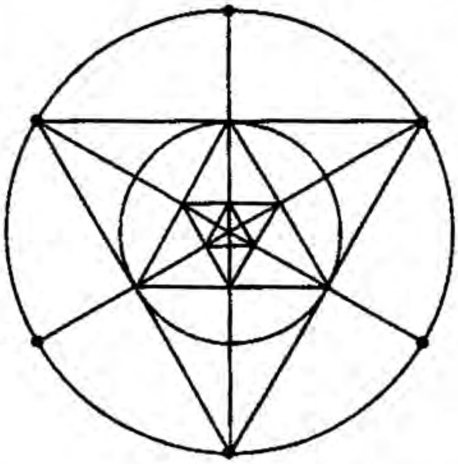


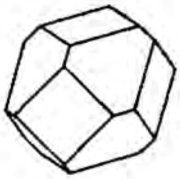


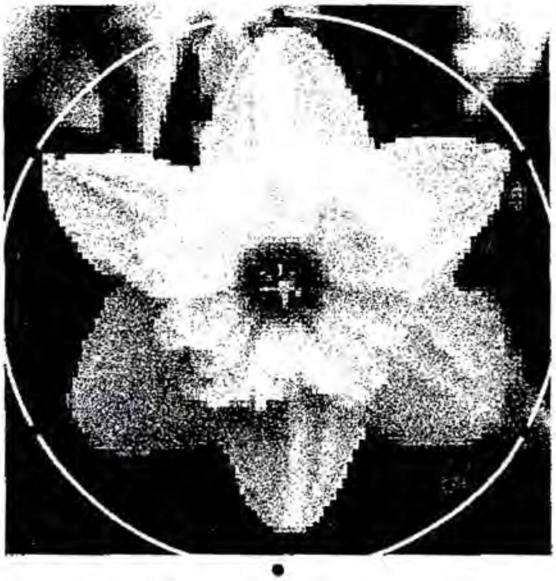
Flowers and Fruits (seed packages)

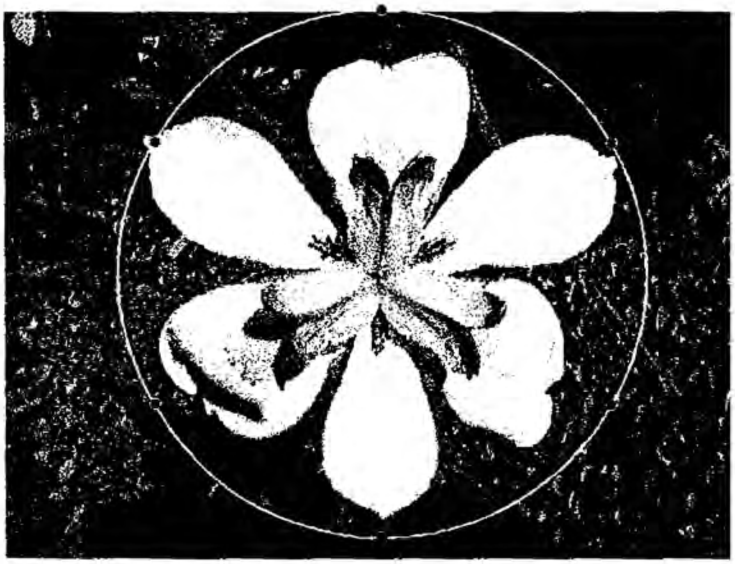
Do this construction on each picture with the large Triangle pointing downward.

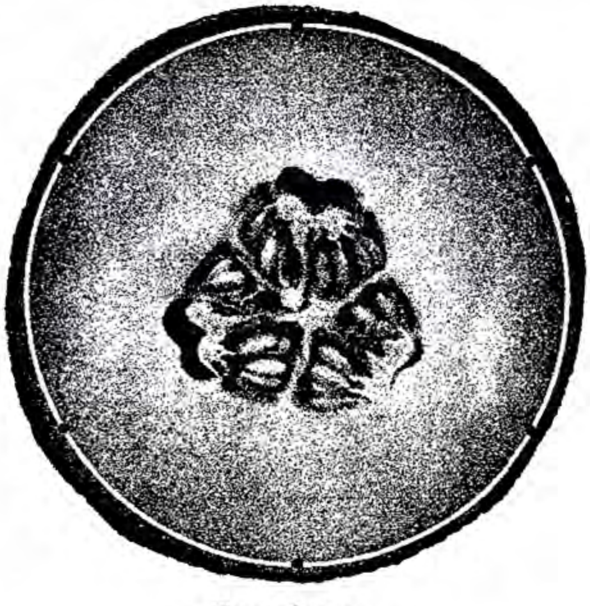
Notice flowers and the geometry of round slices of fruits and vegetables, like tomatoes, green peppers and watermelon.



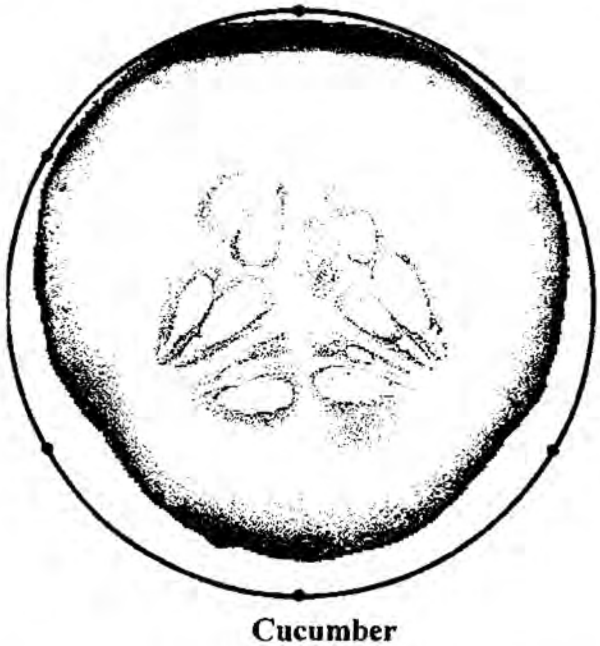








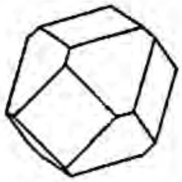
Cantaloupe



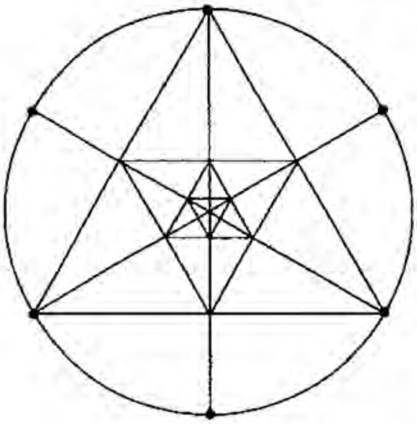
Cucumber

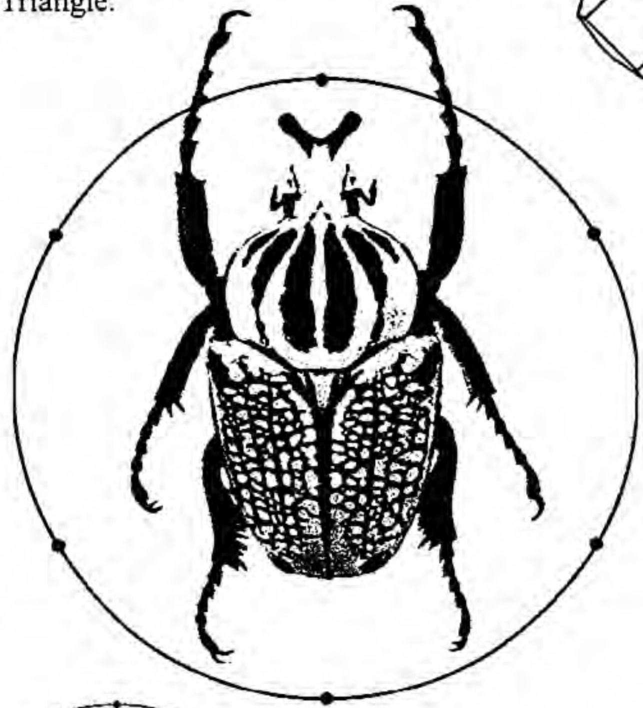
Insects

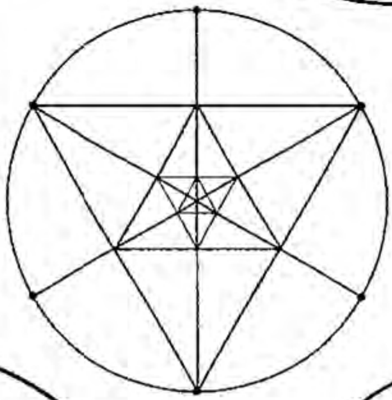
All insects have three body sections and six legs so it is natural to look at them through the construction of a subdivided Triangle.



Beetle

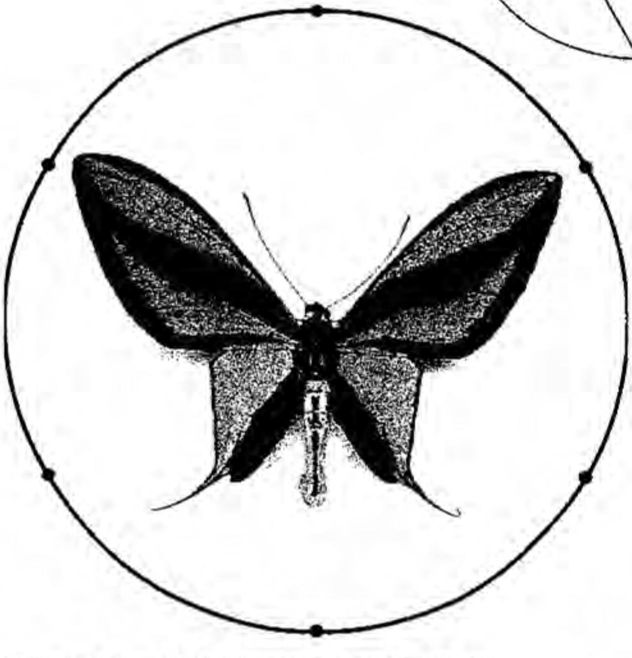


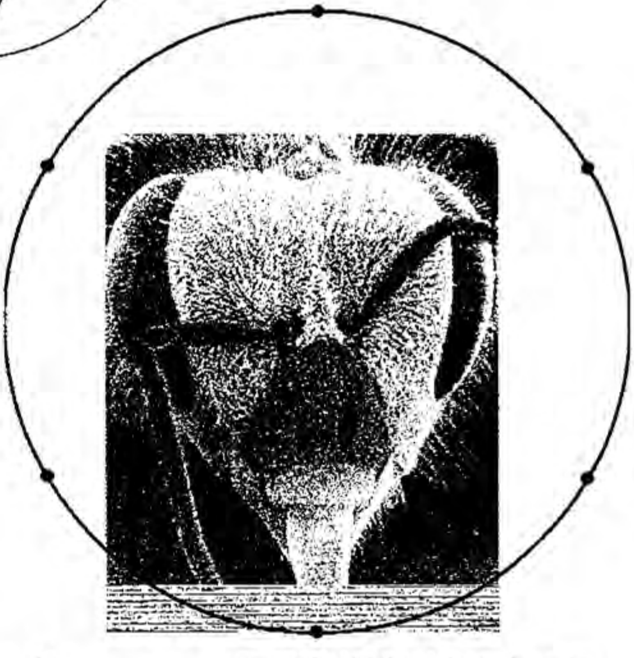




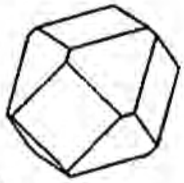
Moth

Bee's Face



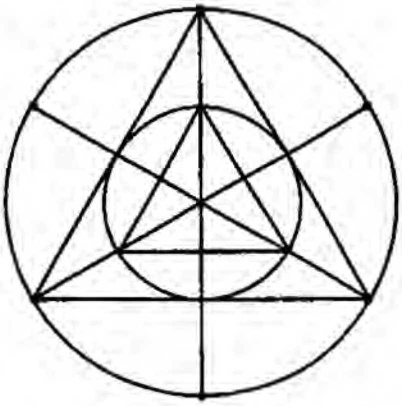


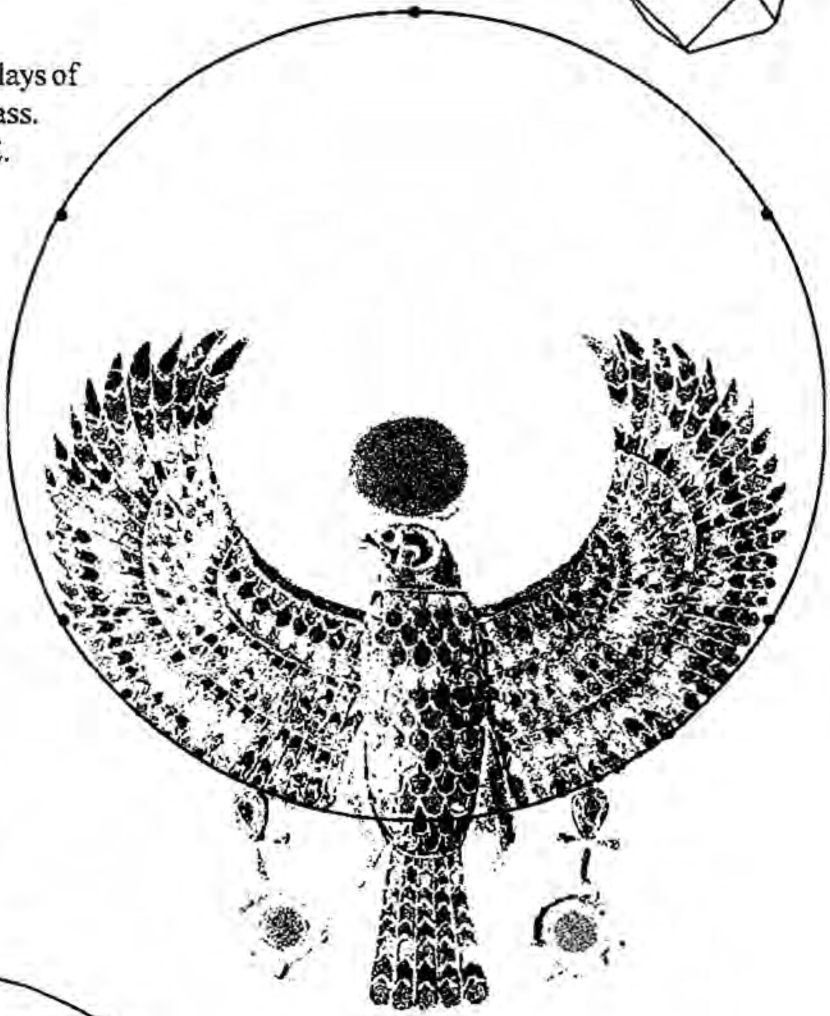
Triangles In Arts, Crafts And Architecture



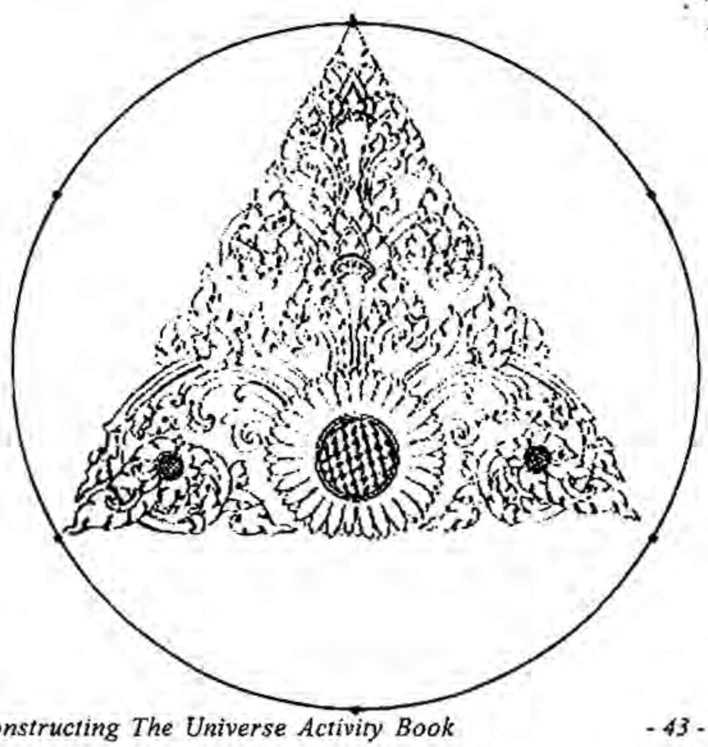
Egyptian Horus Hawk Gold Pectoral of Pharaoh Tutankhamen

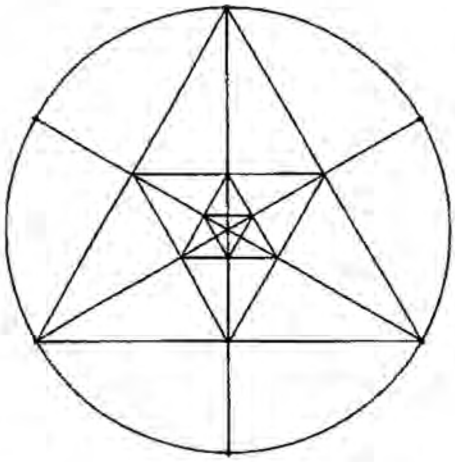
Cloisonne technique of Gold with inlays of lapis lazuli, cornelian and blue glass. 18th Dynasty, 1336-1327 BCE.

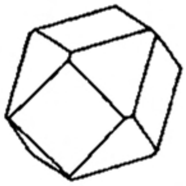


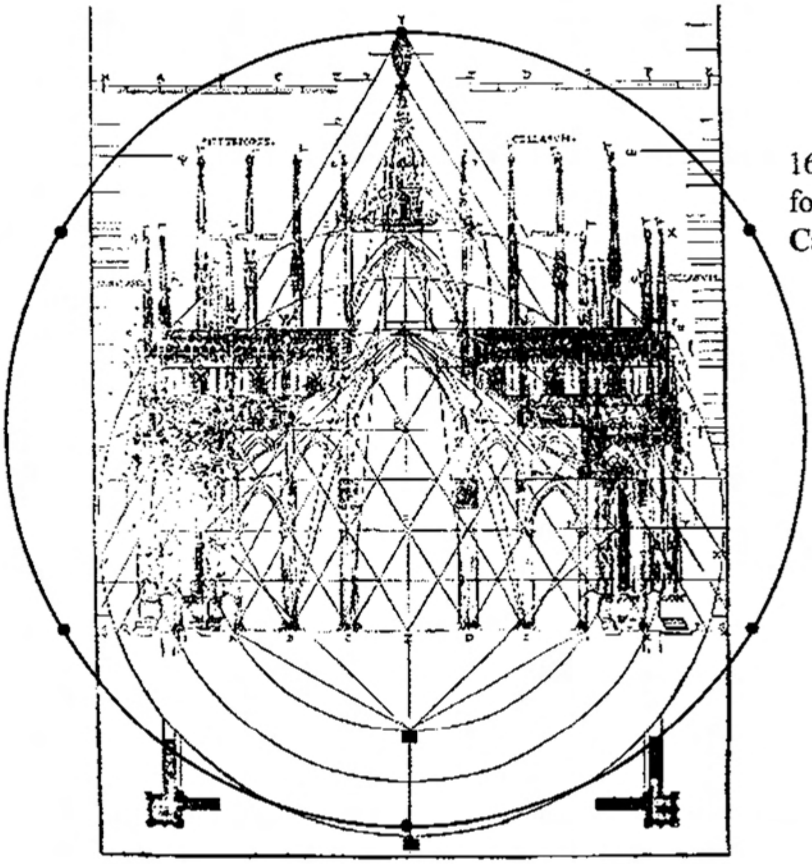


Classic design from Thailand

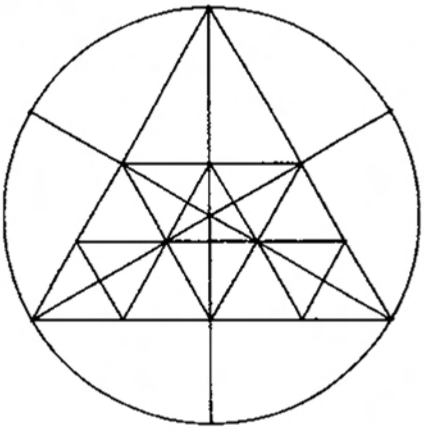


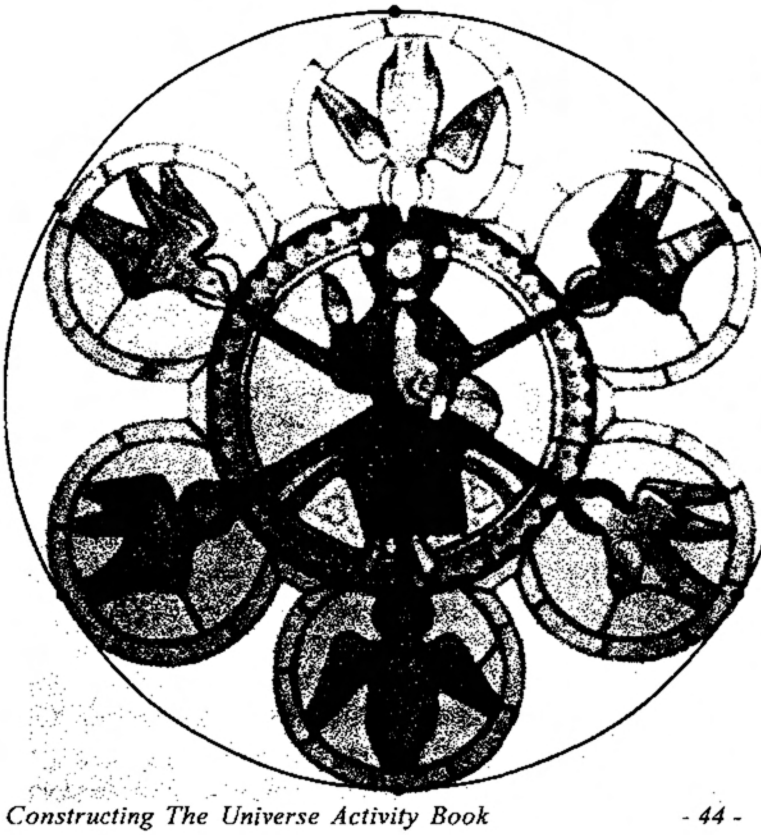




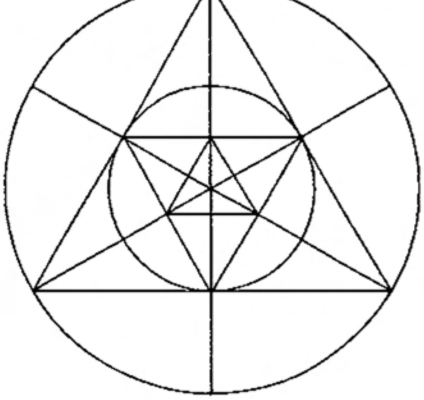


16th century plan
for he Milan, Italy **Cathedral by Caesarino**, which was never used.

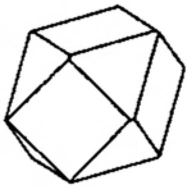




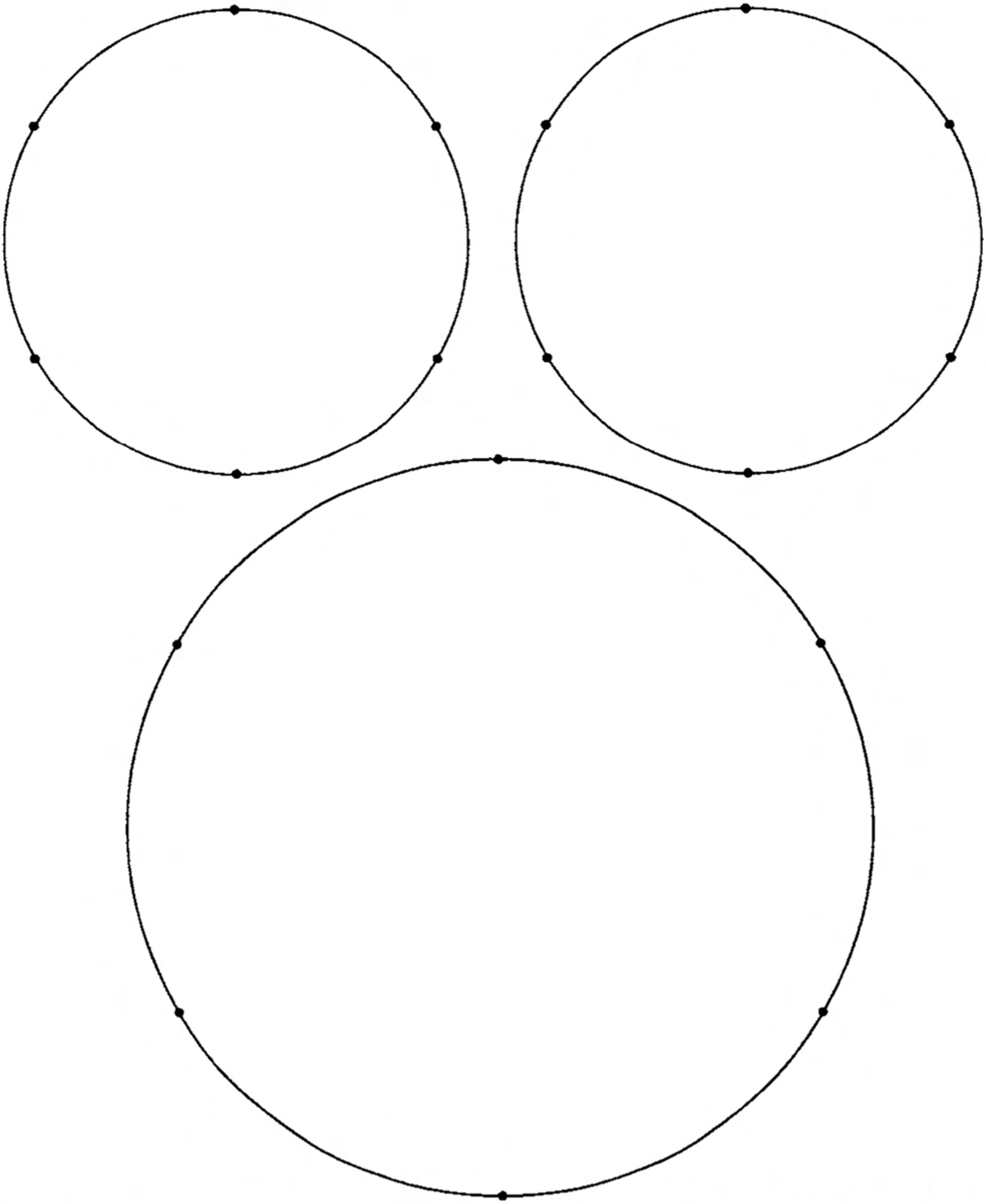
Church Window:
"Seven Gifts Of The Holy Spirit"



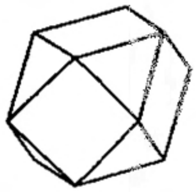
Design Your Own Art, Crafts And Architecture



Use these six-pointed circles (and ones you create on blank paper) to subdivide a Triangle as a frame to guide your designs. Of course, use colored pencils.

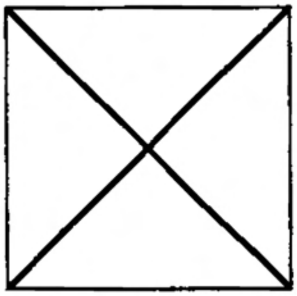


4 The Square



The Square is our next shape to explore. It is a friendly creature, stable, sure and reliable. It treats others fair and Square, makes Square deals and eats Square meals.

Every Square has four equal sides and four right angles. All Squares are the same shape but different sizes.

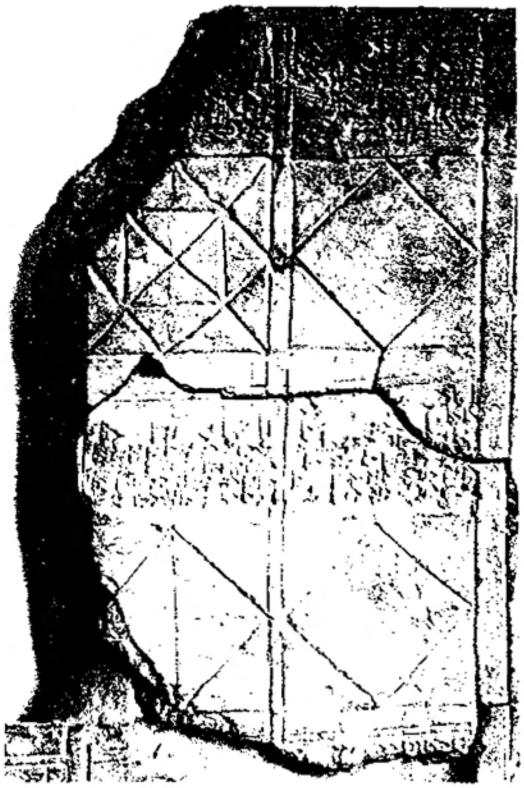


The Square is the simplest shape to have equal diagonal lines stretching between opposite corners. Its diagonals cross at right angles to show us the center point where a Square balances and where our attention automatically goes.

"Four" is the only number word in English which has as many letters as its name says.

It is the only number which equals a number which can be added to itself or multiplied by itself (2+2 = 2x2).

The ancient Greeks called the Square a *tetragon* ("four knees").

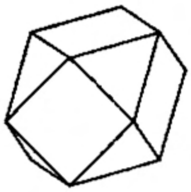


English words about Squares and Fours
often have the Roman (Latin) root
-QUA- in them:

Quarter: one-fourth part of anything.
Quadrant: one-fourth section of a Circle
Quart: one-fourth of a gallon; i.e., four quarts make a gallon.
Quartet: four musicians playing together.
Quarantine: isolation for 40 days
Quadruplets: four babies born together
Squad: a group of four people
Squadron: a group of four jets

Babylonian clay tablet showing the construction and subdivision of the Square (c 1500 BCE)

Constructing A Square Around A Circle



There are many ways to construct a Square. The method you choose depends on the situation and your purpose for creating it.

Some constructions of the Triangle were based on a Circle. The Square's construction is also based on the Circle, but in different ways.

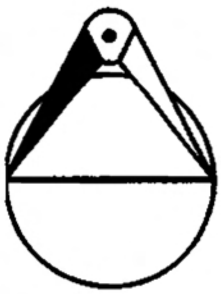
Here is perhaps the oldest method for constructing a Square, dating back to Babylon, India and elsewhere. It is useful for constructing a Square when you know where its center is.

(1) First, turn a Circle and draw its diameter. Open the compass to the ends of the diameter.

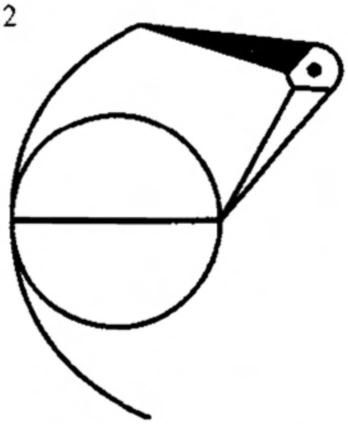
(2) Turn a Circle (if you have the space) or just an arc (as shown).

(3) Reverse the compass on the diameter and turn another Circle (or arc) to make an Almond. Draw a vertical line between the crossings which cuts the Circle at two points, top and bottom.

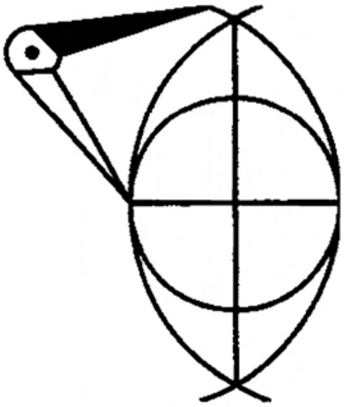
1



2



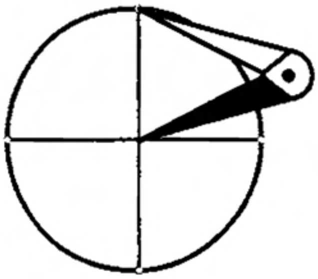
3



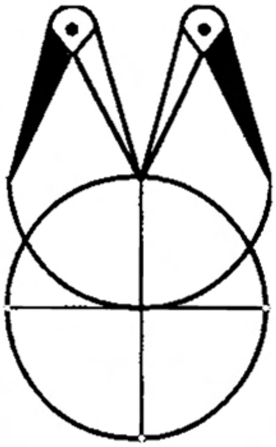
(4) Place the point of the compass on one of the four points (top, bottom, left, right) around the Circle and open the scribe to the center.

(5) Turn a full (or half) Circle.

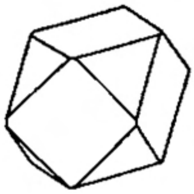
4



5



(6) Repeat this by placing the compass point on each of the four directions of the Circle and turn a Circle (or arc).

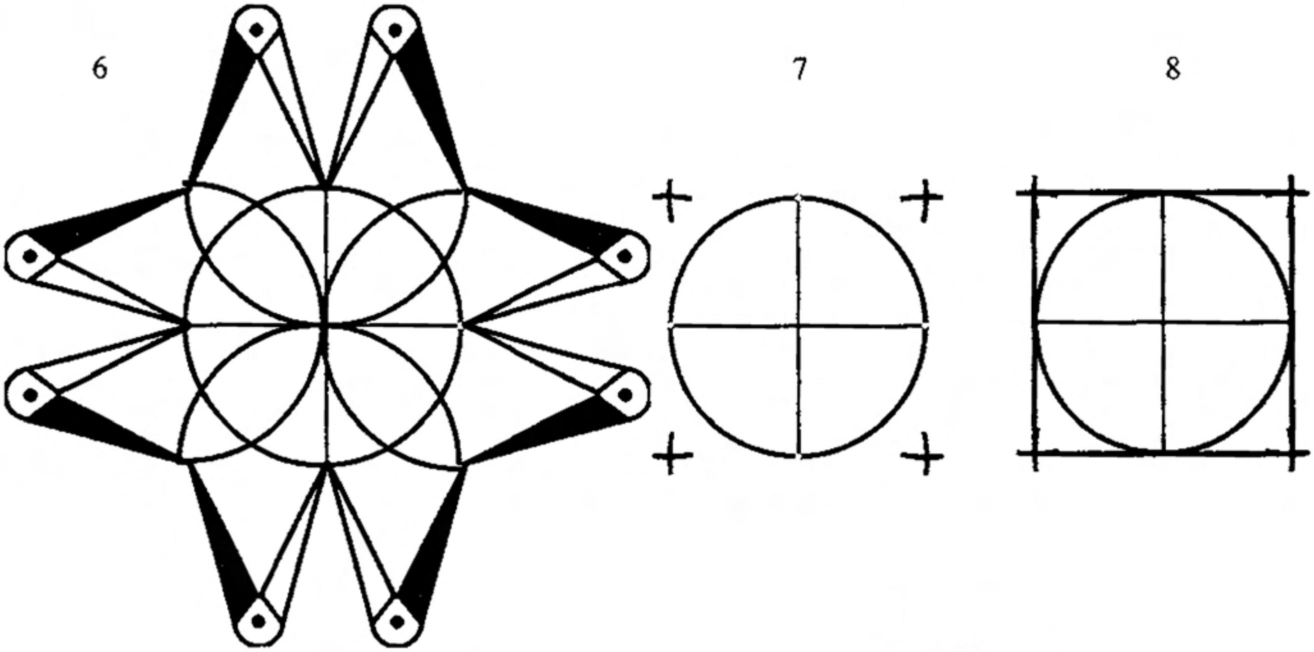


(7) We're interested in the four points made where the arcs cross.

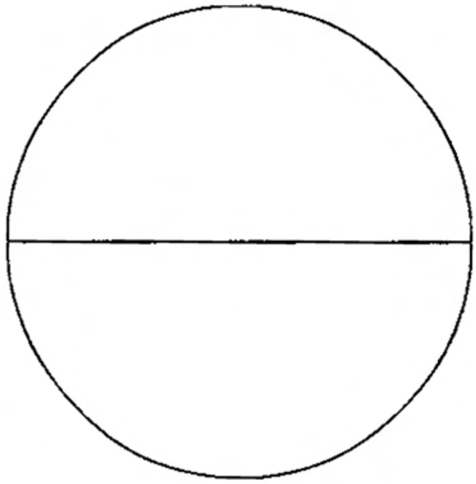
(8) Connect these four points to construct a Square. If you've done it accurately, the sides of the Square will be tangent to (just touching) the sides of the Circle.

This construction will also be useful for constructing the Octagon (Chapter 8) and the Dodecagon (Chapter 12).

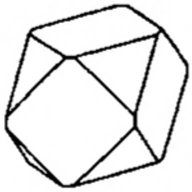
Practice this construction in the Circle below (or larger on a blank sheet of paper). Learn to construct this Square without looking at these pages.



Construct a Square around this Circle.



Construct A Square Upon A Line Segment

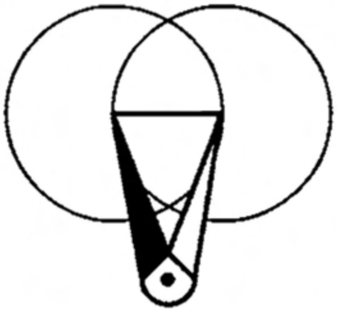


This next construction also brings a Square through the Almond, but it has a different beginning. It is used to construct a Square if you already know the length of one side.

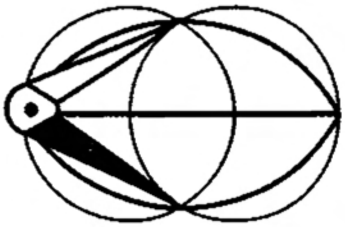
(1) Open your compass between the two ends of the line segment and construct the Almond.

(2) Open the compass between the Almond's two crossing points and turn arcs to find their end points (page 12). Use them to draw the long diameter across both Circles.

1



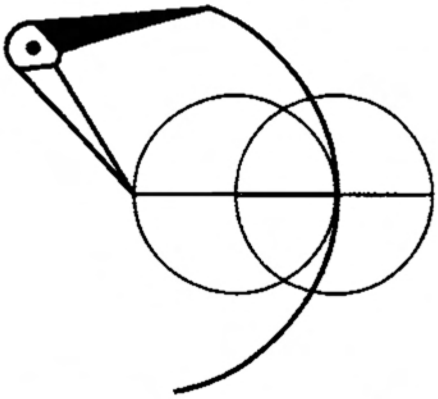
2



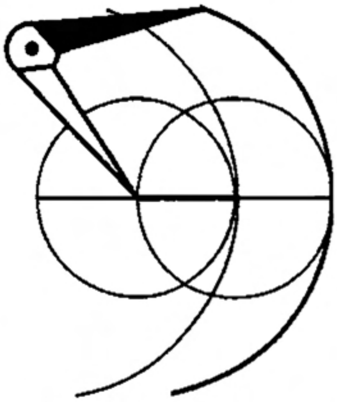
(3) Open the compass across the diameter of one Circle, and draw and arc as shown (or full Circle).

(4) Move the fixed compass opening across the diameter of the other Circle and draw another arc.

3



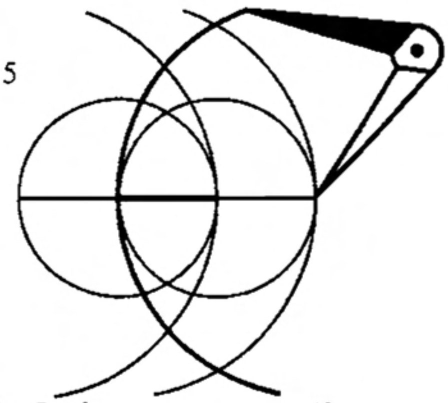
4



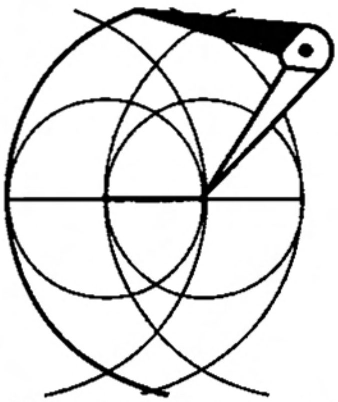
(5) Reverse the compass to the other end of the long diameter (as shown) and mark an arc.

(6) Move the fixed compass to the diameter of the other Circle and draw another arc. We have really created two large Almonds.

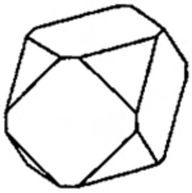
5



6

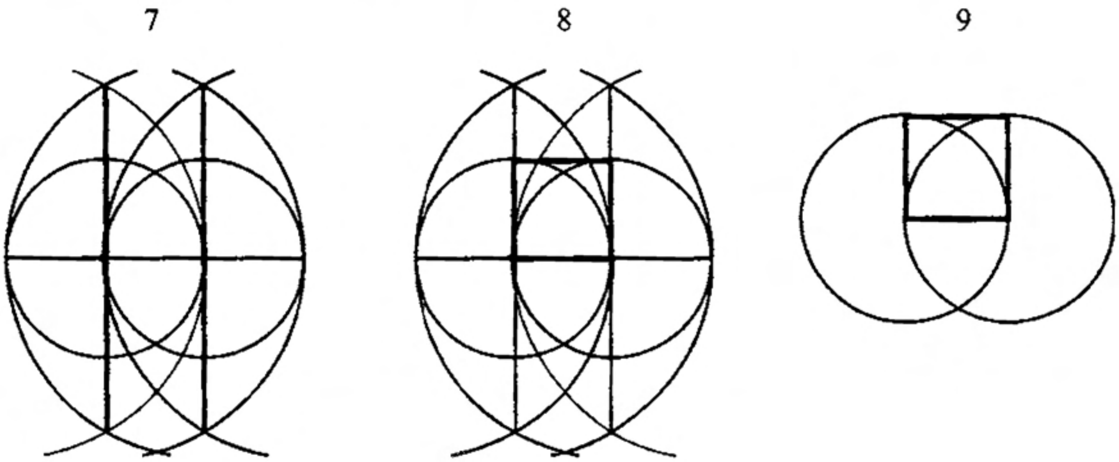


(7) Use your straightedge to connect the crossings in each of these two large Almond crossings as shown.



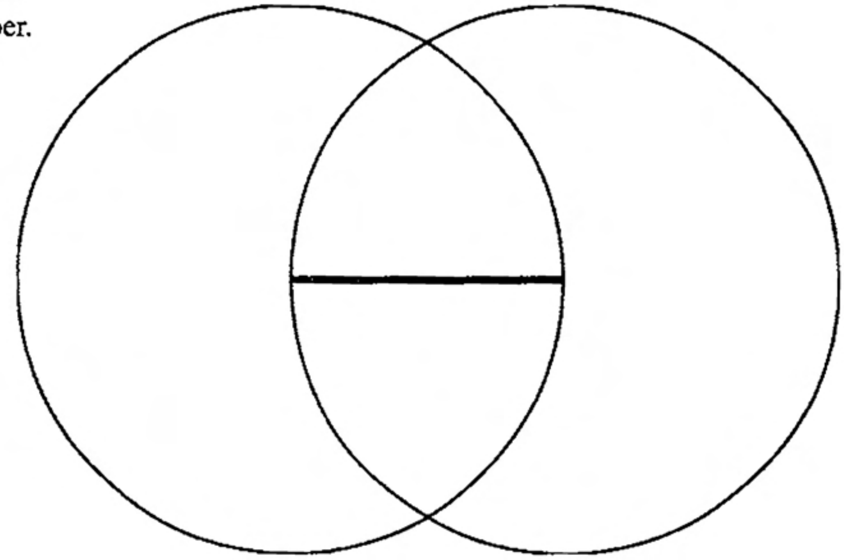
(8) Draw a line connecting the two points where the lines cross the top of the Circles.

(9) This completes the Square. It has the same side lengths as the original line segment. Another Square can also be drawn below.

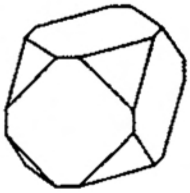


Construct A Square

Construct a Square upon the line segment in this Almond, or larger on a sheet of blank paper.



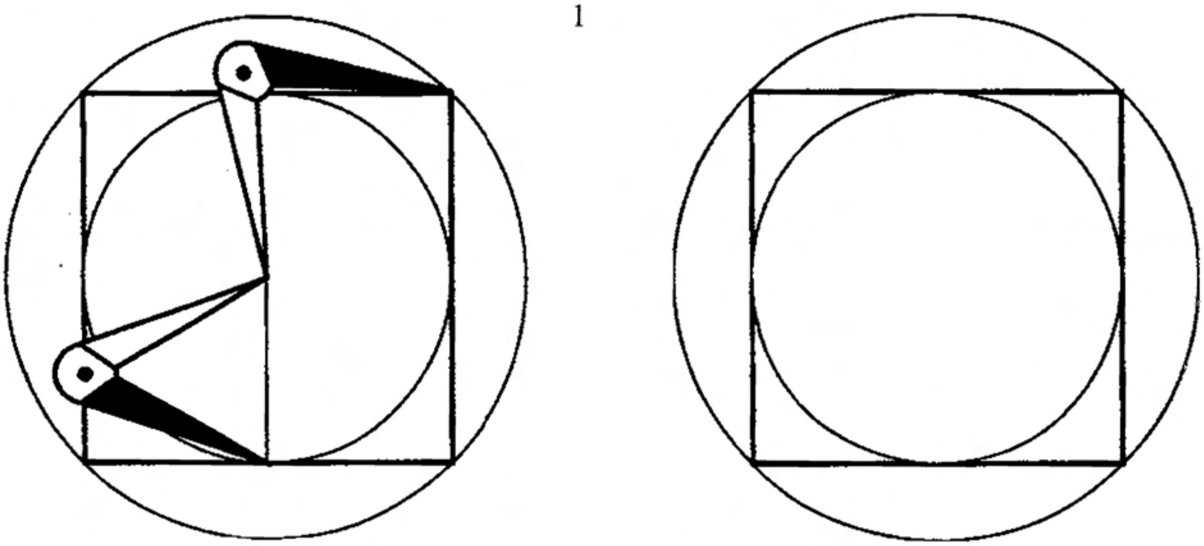
A Curious Truth About The Square



(1) When a Circle is circumscribed *around* a Square and also inscribed *within* the Square, the area of the large Circle is divided exactly in half.

(2) The area in the shaded ring is exactly equal to the area of the white inner Circle.

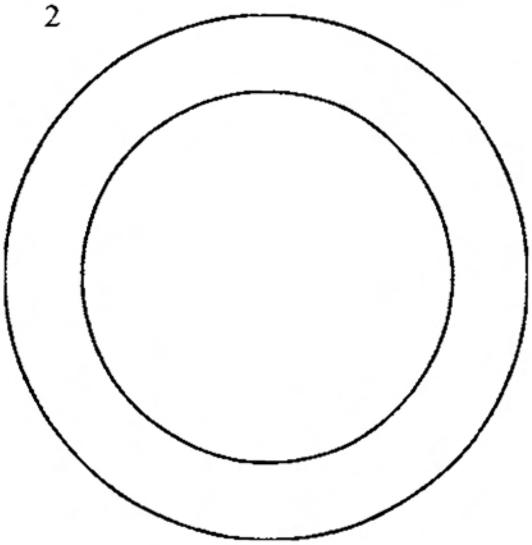
1



This equality of areas gives a feeling of balance to the whole even if we don't see the Square.

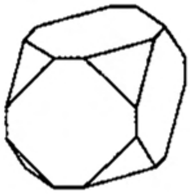
Many circular objects have been designed this way to be visually pleasing, even when we don't see the Square, like this ancient Greek plate (white Square added).

2



Perhaps you can design something this way.

Subdivide A Square



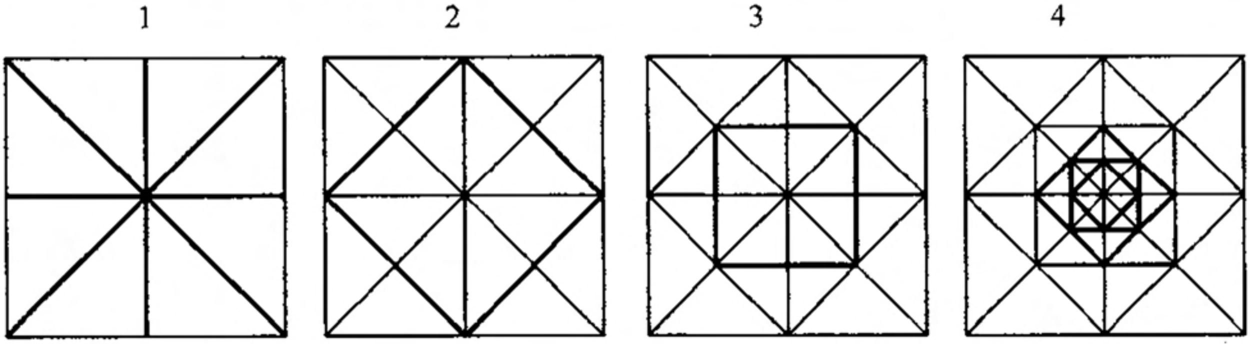
If we construct a Square around a Circle (page 47) then we already know where the midpoints of the four sides are. We can use them with the diagonals to subdivide the Square into smaller Squares all sharing the same center. We only need a straightedge and pencil.

(1) Construct a Square around a Circle. Draw its diagonals and also the lines connecting the midpoints of its opposite sides.

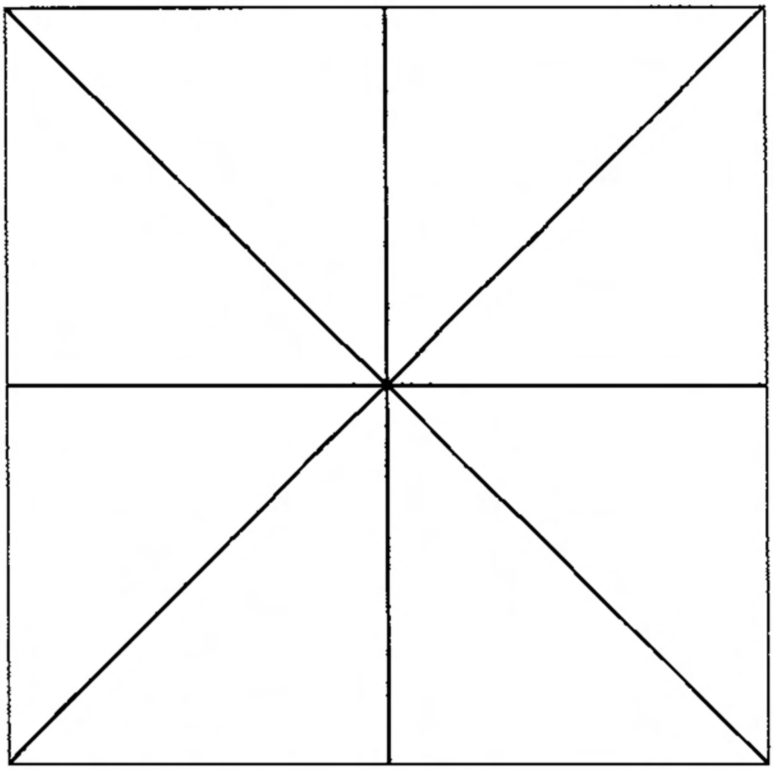
(2) Connect the midpoints of the sides to draw a tilted Square.

(3) Connect the points where the diagonals cross this tilted Square to make a smaller Square.

(4) Draw a new Square wherever the diagonals or midpoint lines cross the smallest Square. Do this as small as the point of your pencil allows.

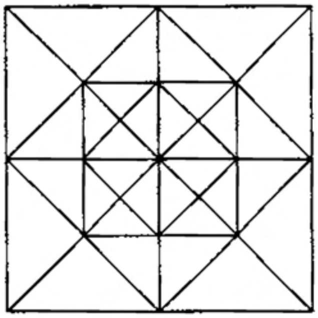


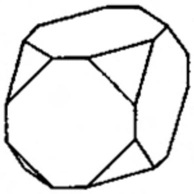
Subdivide This Square



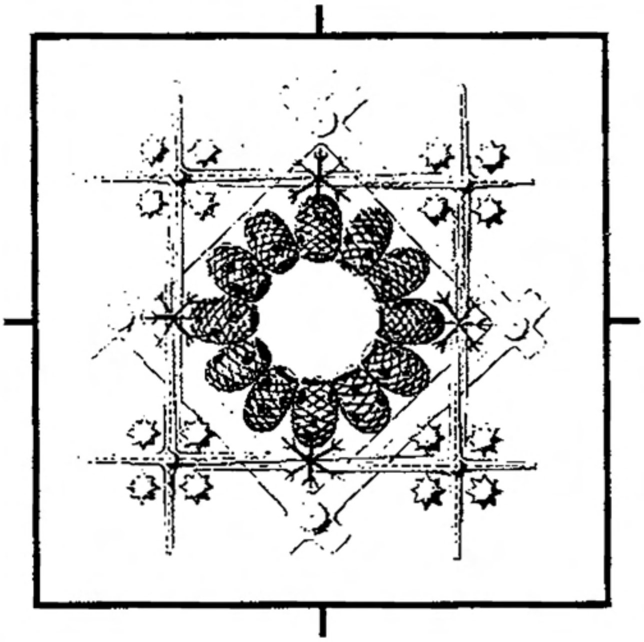
Subdivide Square Designs In Nature

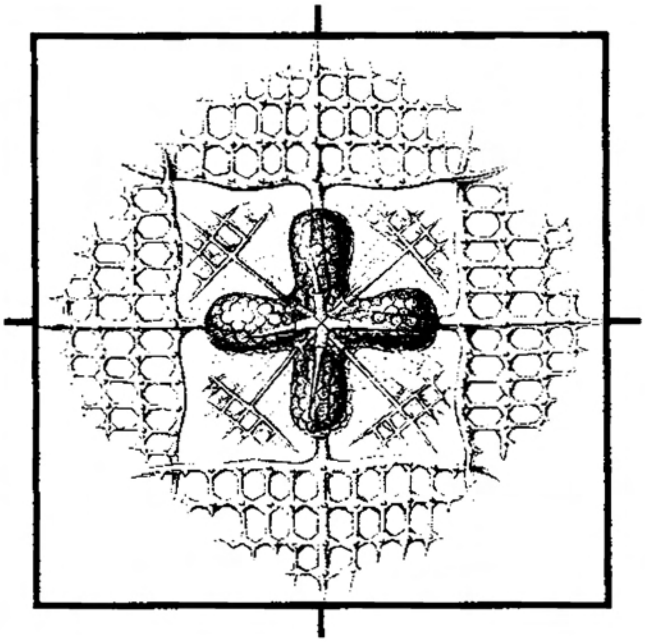
A Square has been constructed around these drawings of natural forms. The midpoint of each side has been marked. Subdivide the Square upon each of them to see how their parts align with it.



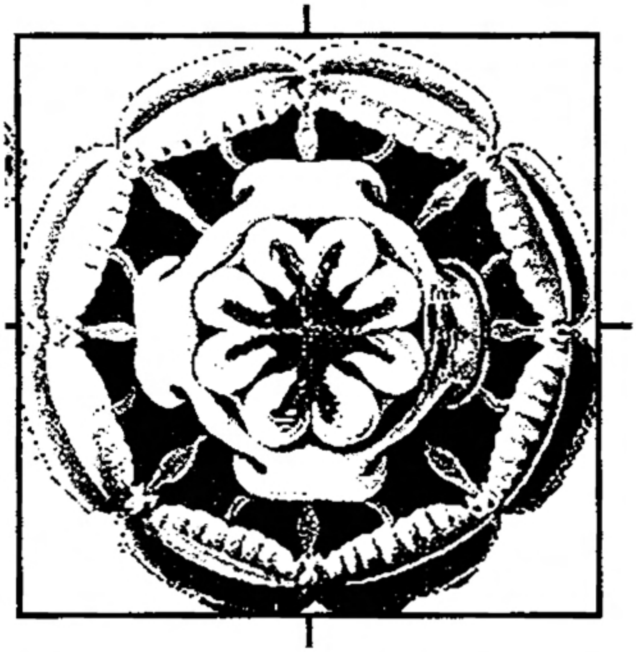


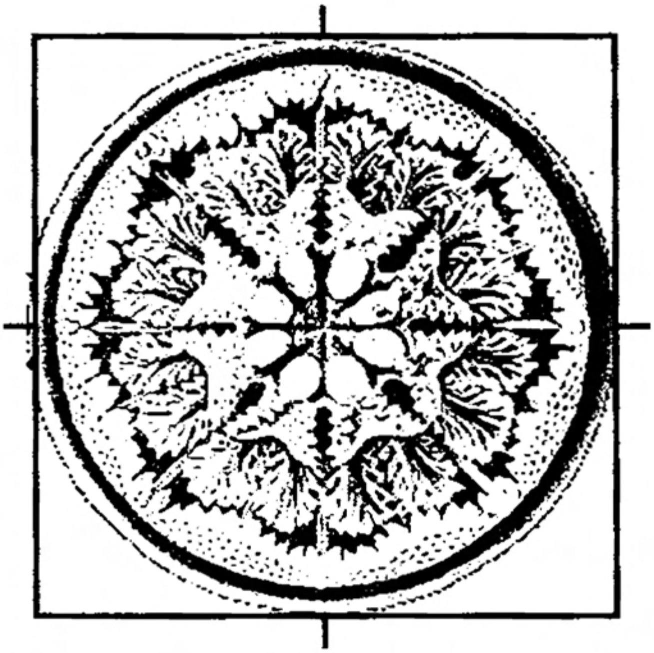
Microscopic **Radiolarian** Skeletons



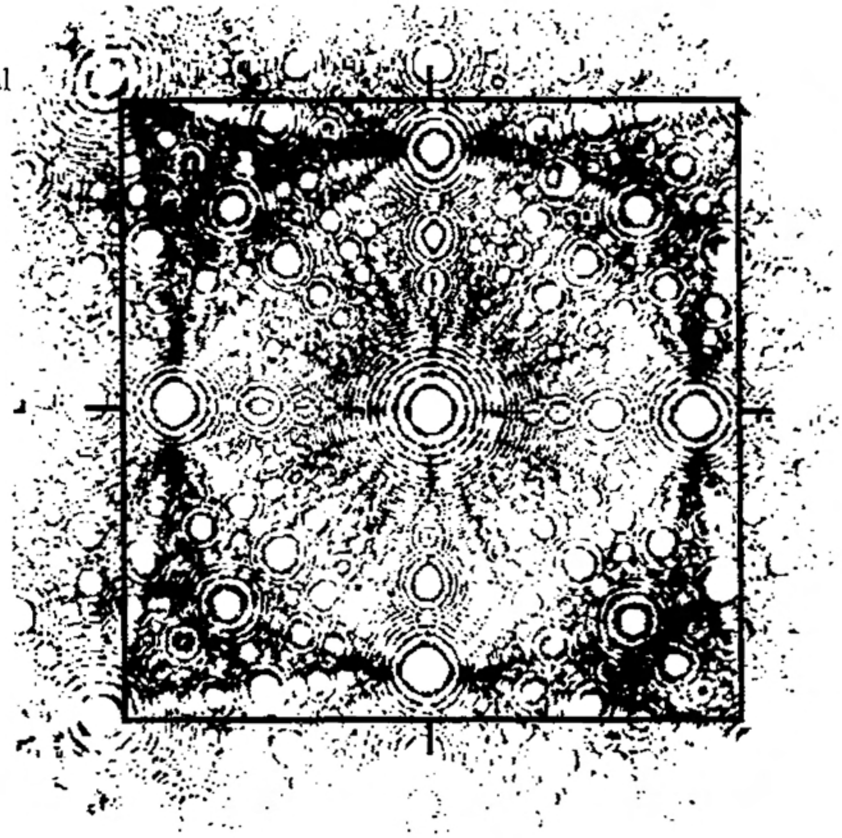


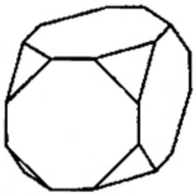
Jellyfish seen from below





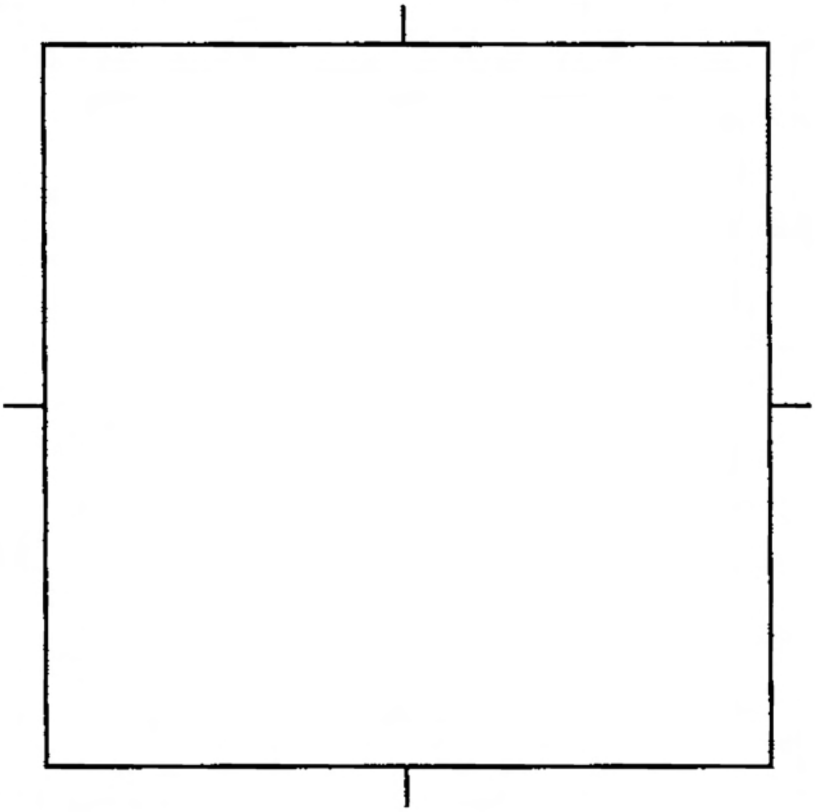
Tungsten Metal
Crystal



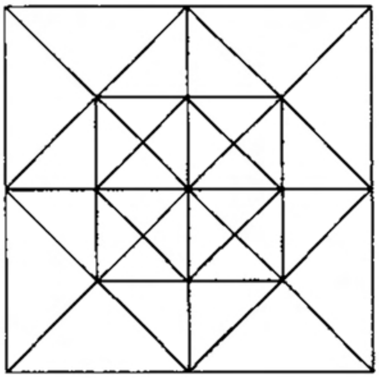


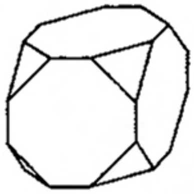
Design Your Own Natural Form

Subdivide this Square and use its points, lines and areas to design a natural form.



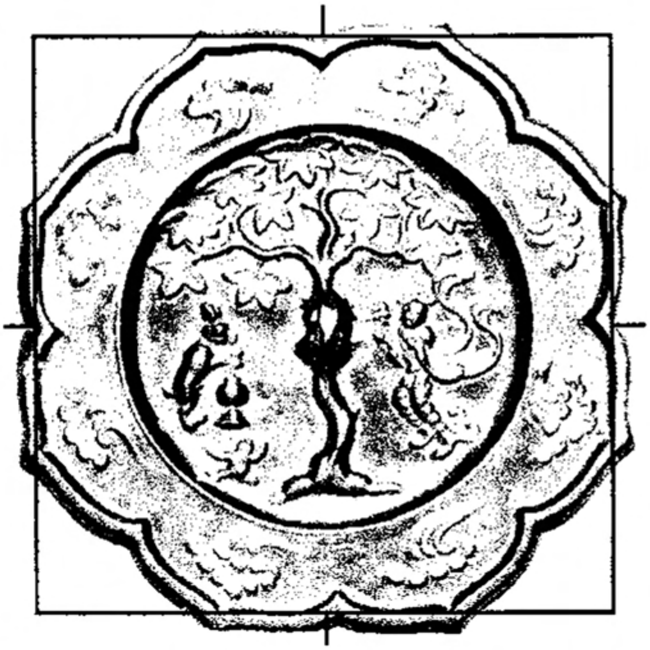
Subdivide Square Designs In Art

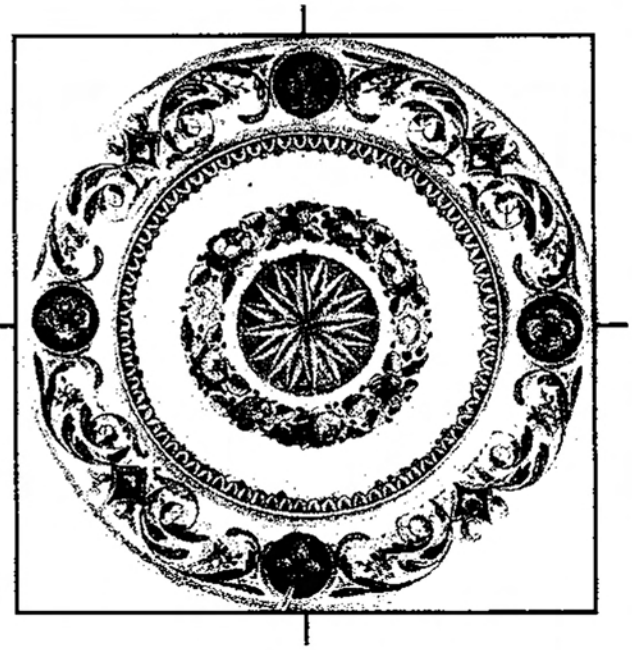




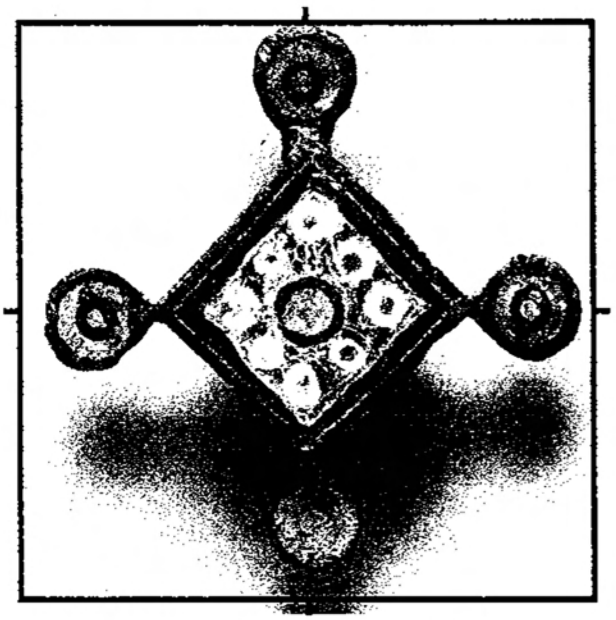
A Square has been constructed around each image. The midpoints of each side are marked. Subdivide it into smaller Squares to see how different cultures used the same geometry in different ways.

Chinese Plates

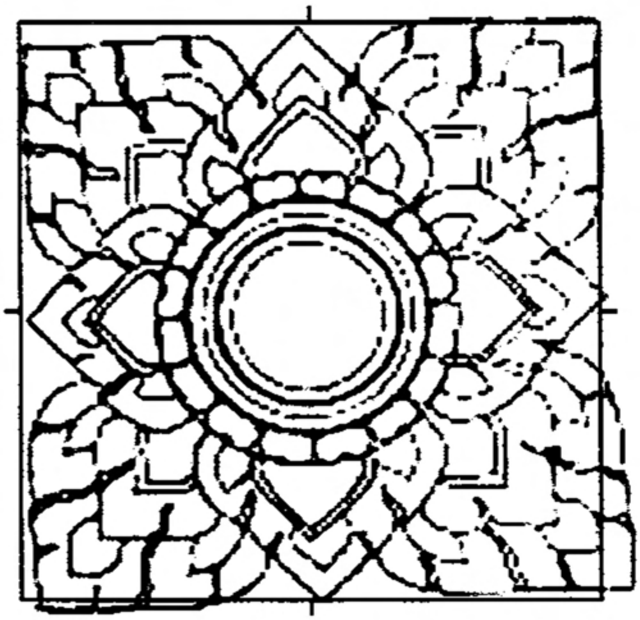




Celtic Brooch 2nd century

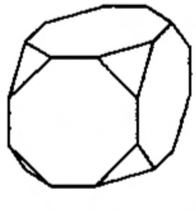


Classic Design from Thailand

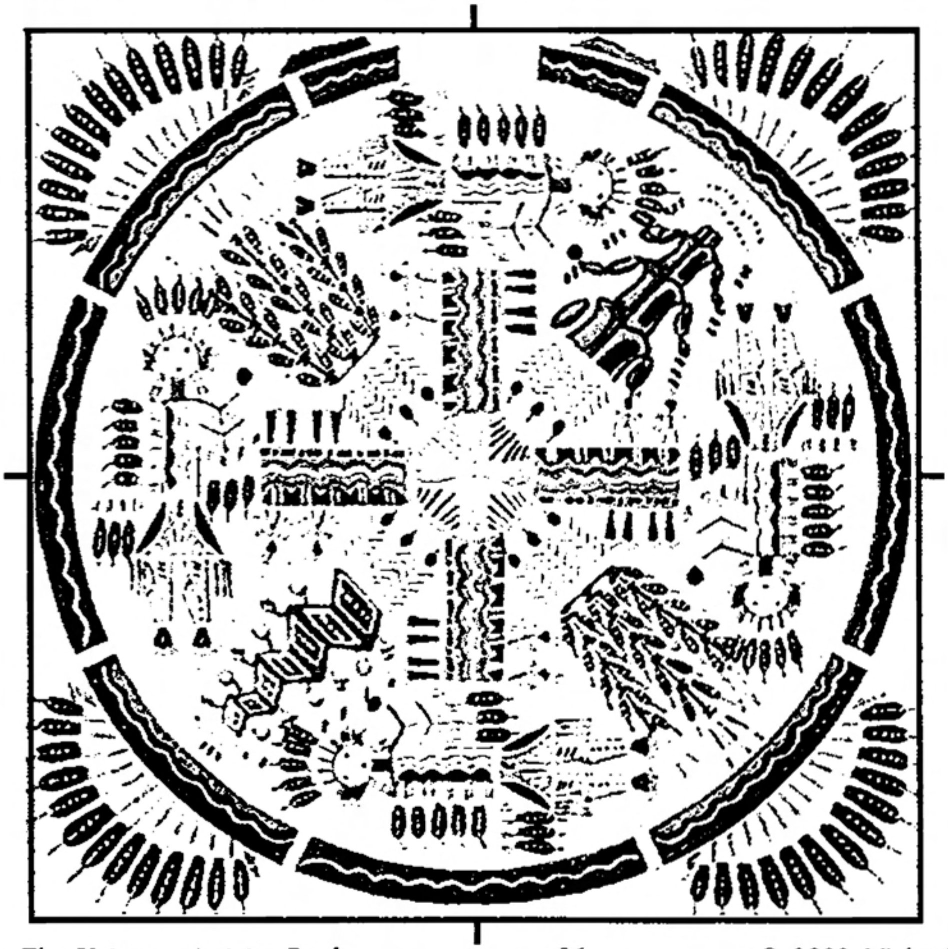


Jewish Holiday Plate

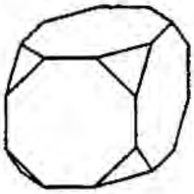
Greek painting in a kylix bowl. How does the geometry guide the runner's body?



Traditional Navajo Sand Painting: "Whirling Logs"

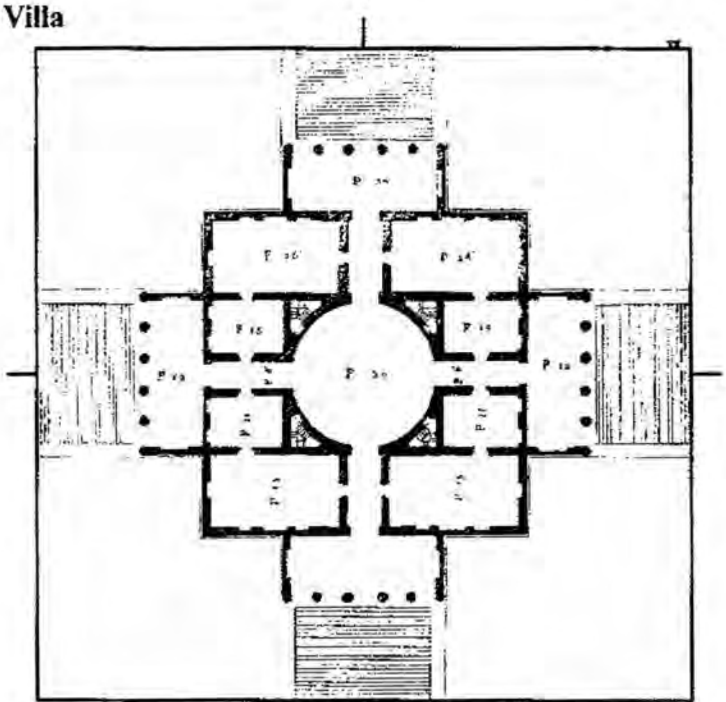


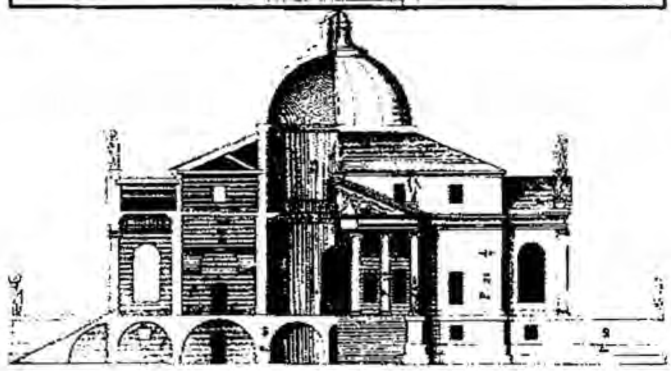
Byzantine Dome Ceiling



Why are the Circles the sizes they are?

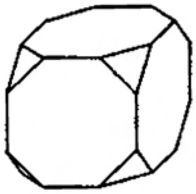
Floor Plan for an Italian Villa by the architect Andreas Palladio (1508-1580).

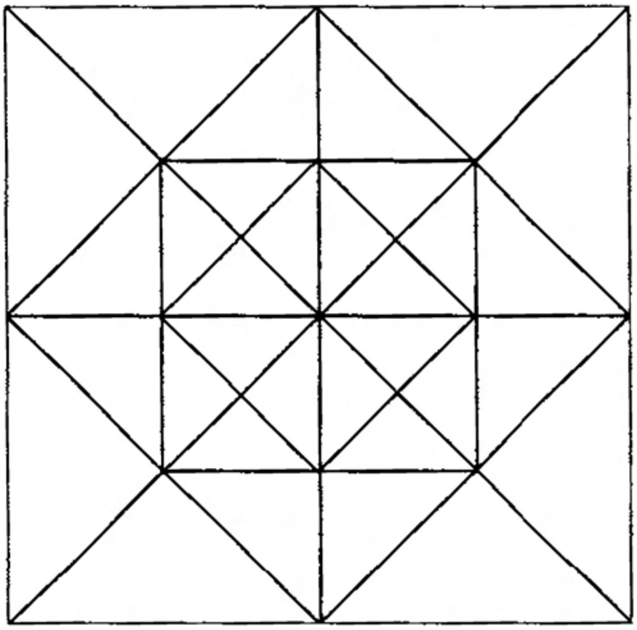


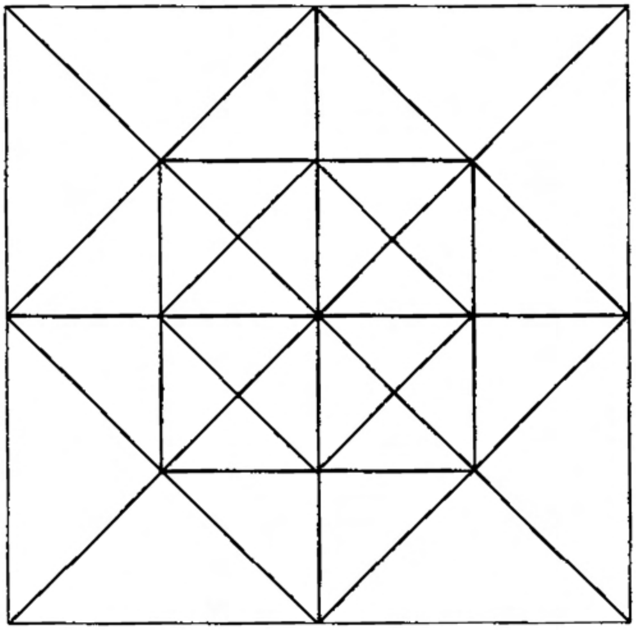


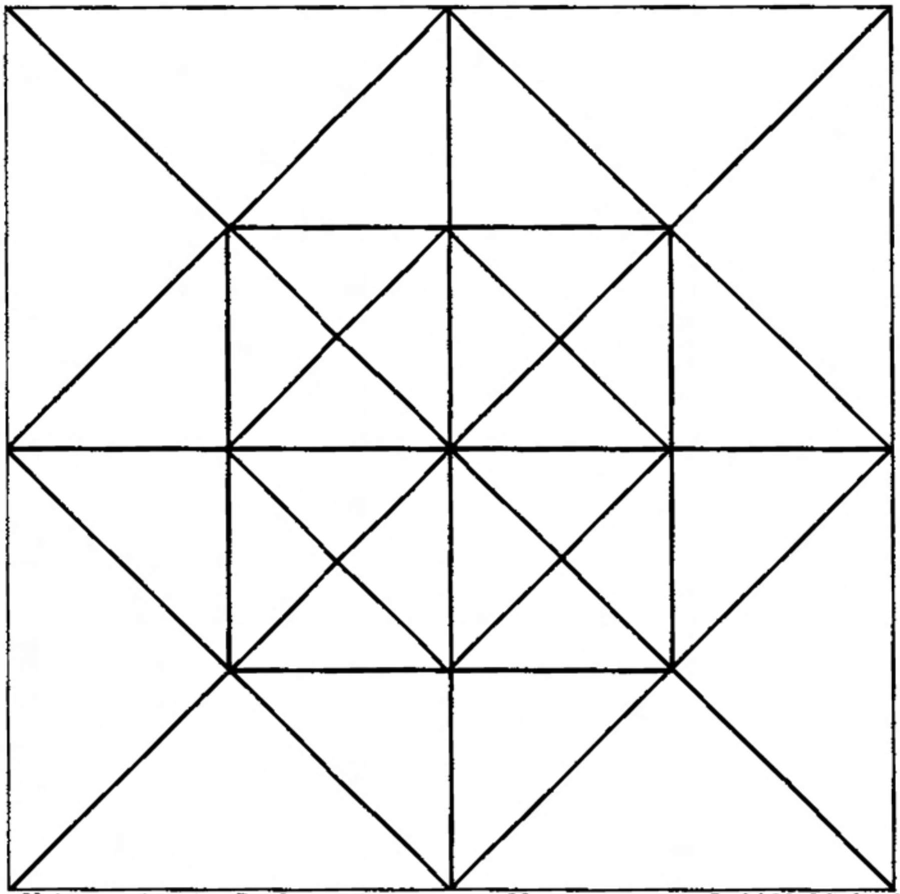
Design Your Own Square Art

Use the lines of these subdivided Squares as guides for your original art, crafts and architectural designs.



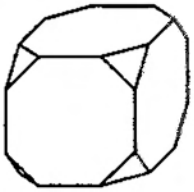


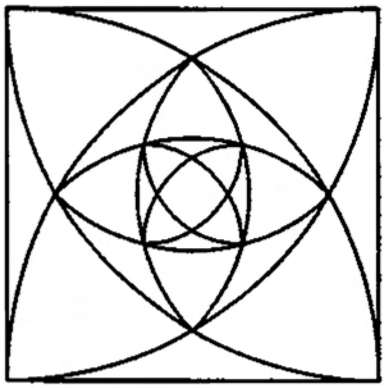


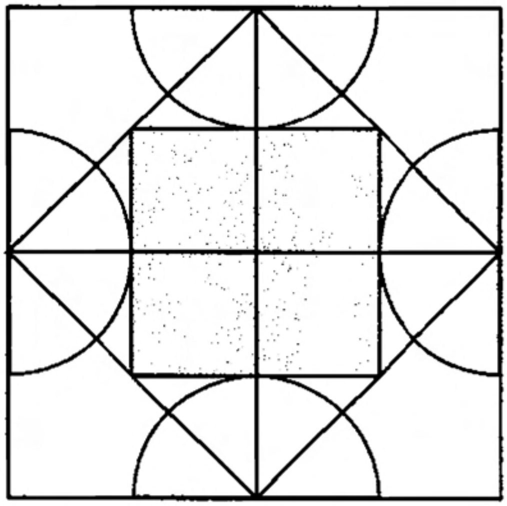


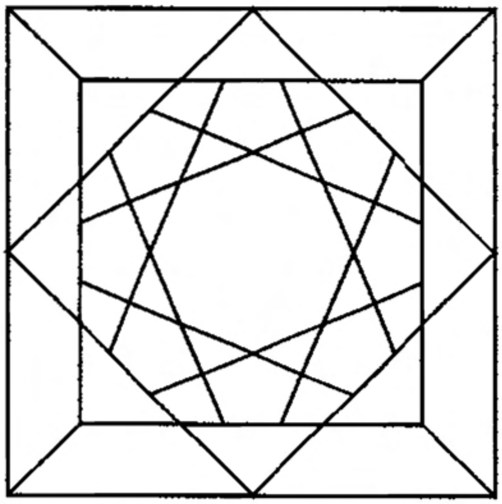
Replicate These Constructions

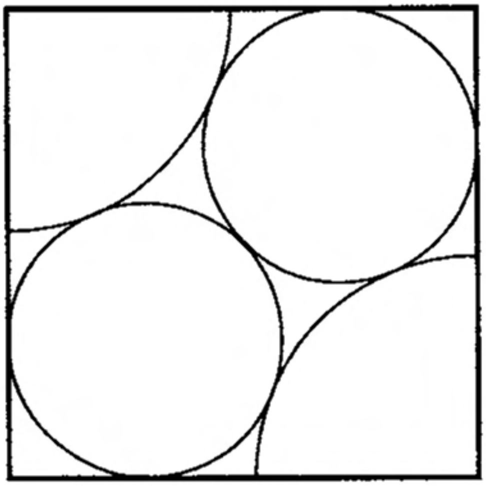
(All points, lines and arcs needed may not be shown.)

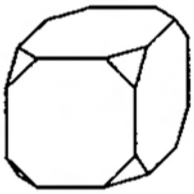


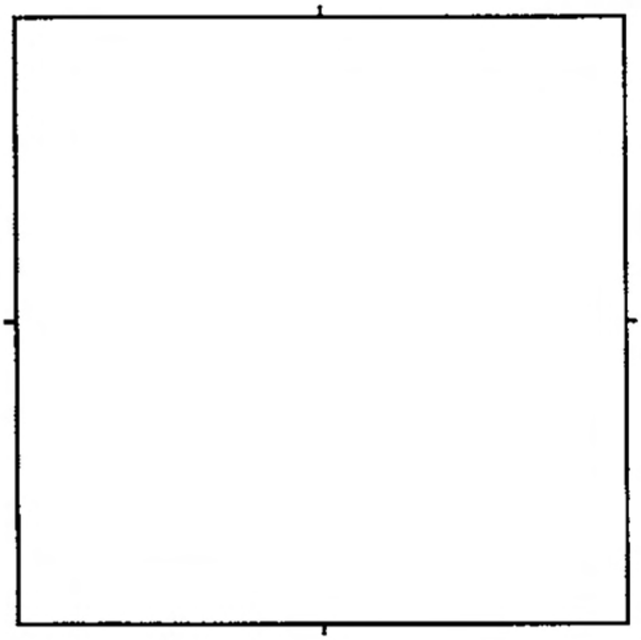




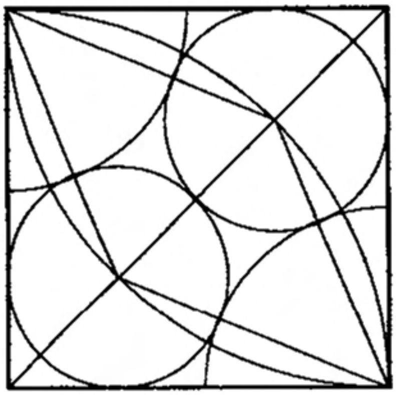


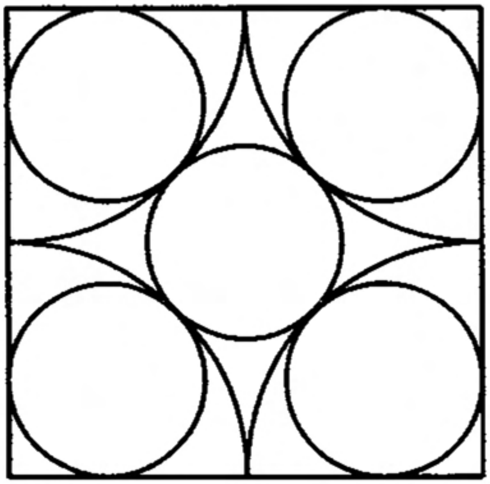




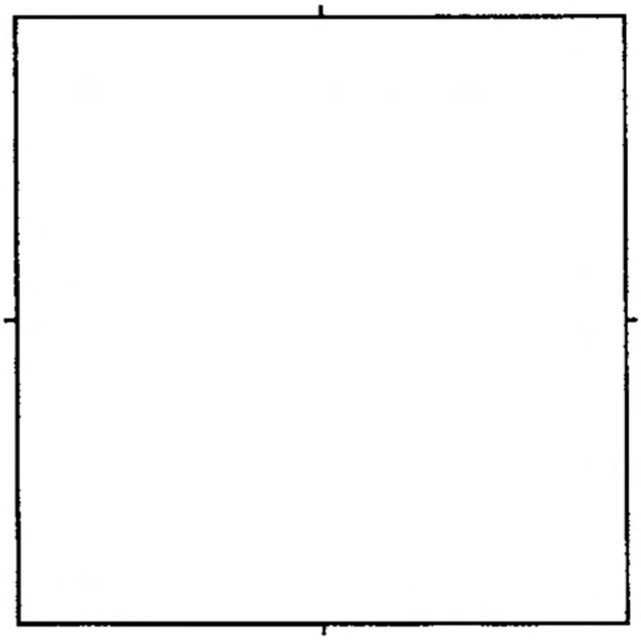


Hint:

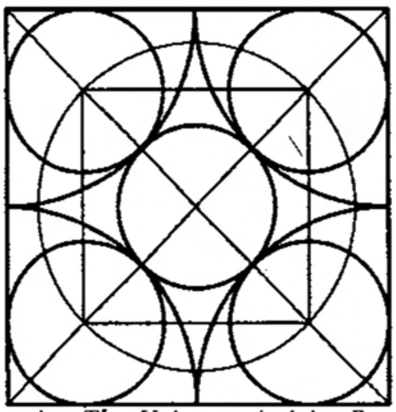




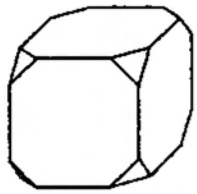
All five Circles are the same size. Which one must appear first?

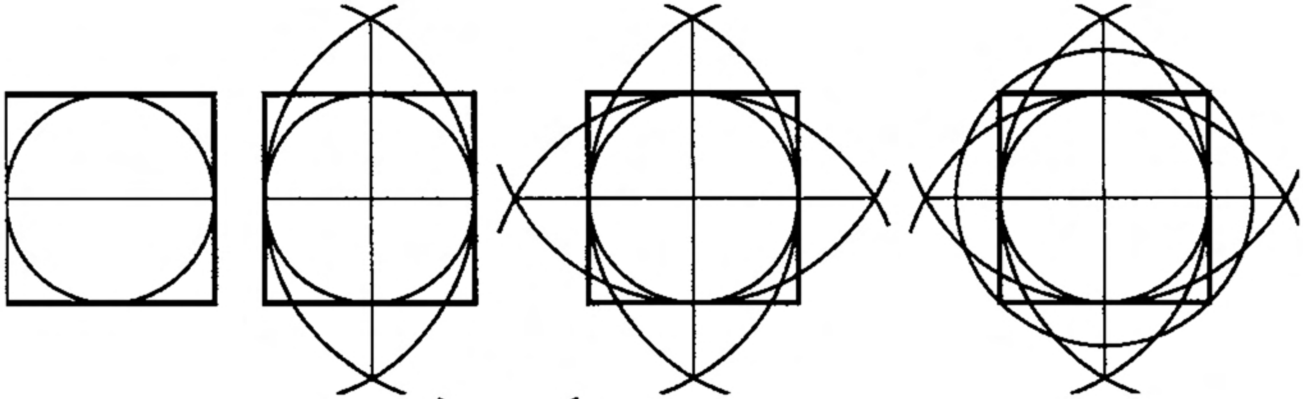


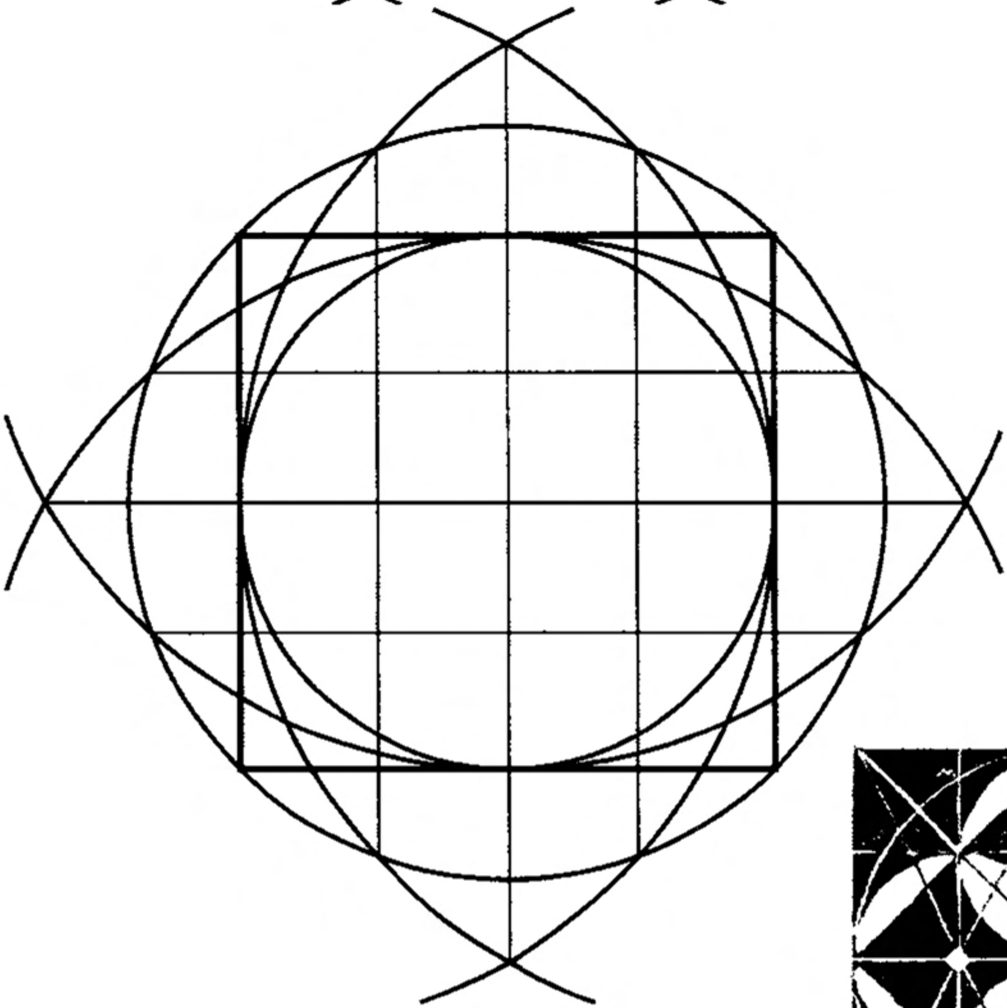
Hint:

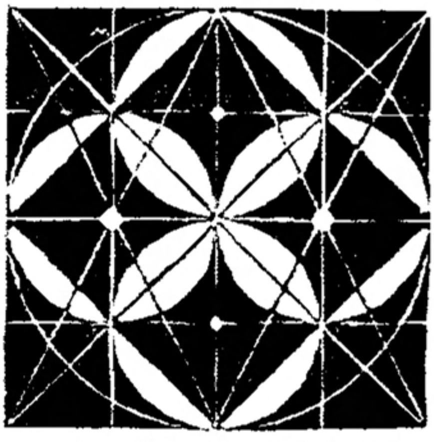


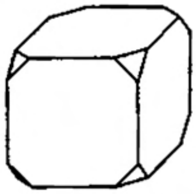
On a blank sheet of paper construct this 4x4 Square grid. Then use it to replicate the construction (below) by Giordano Bruno (16th century).

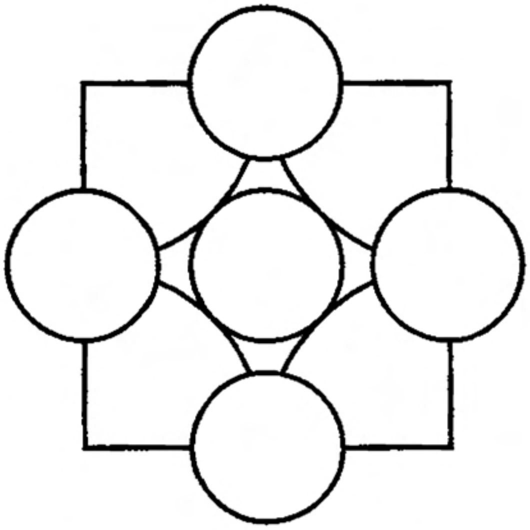


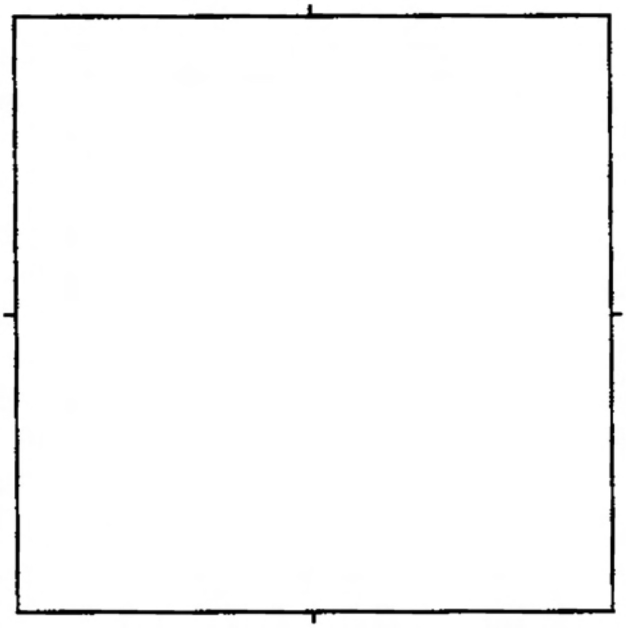




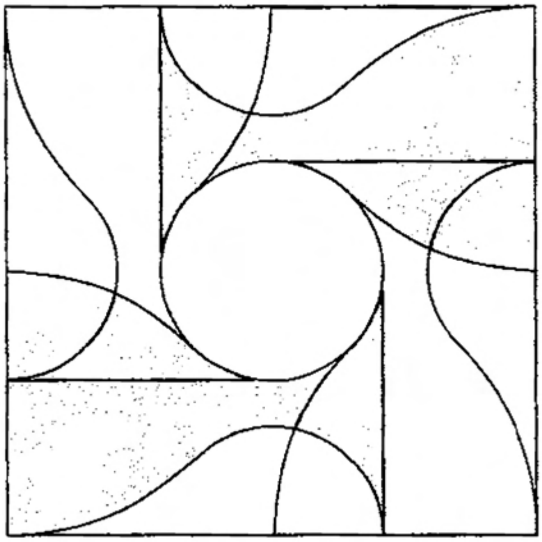




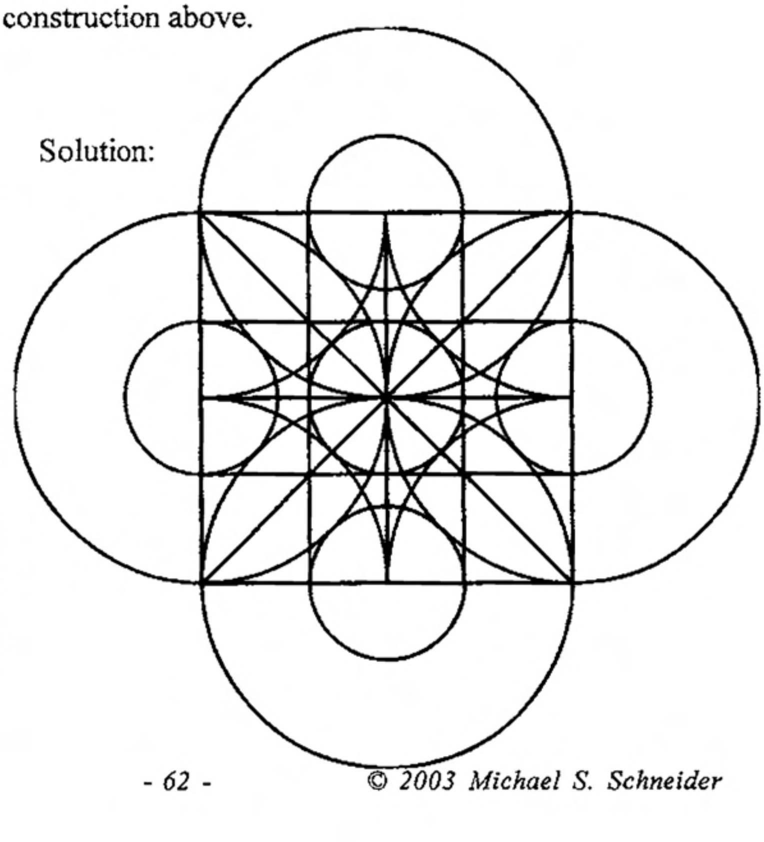




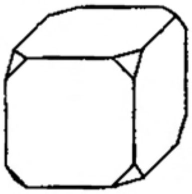
Do this ***Rotating Goblets*** **Construction** in a Square on a separate piece of paper. It is related to the construction above.



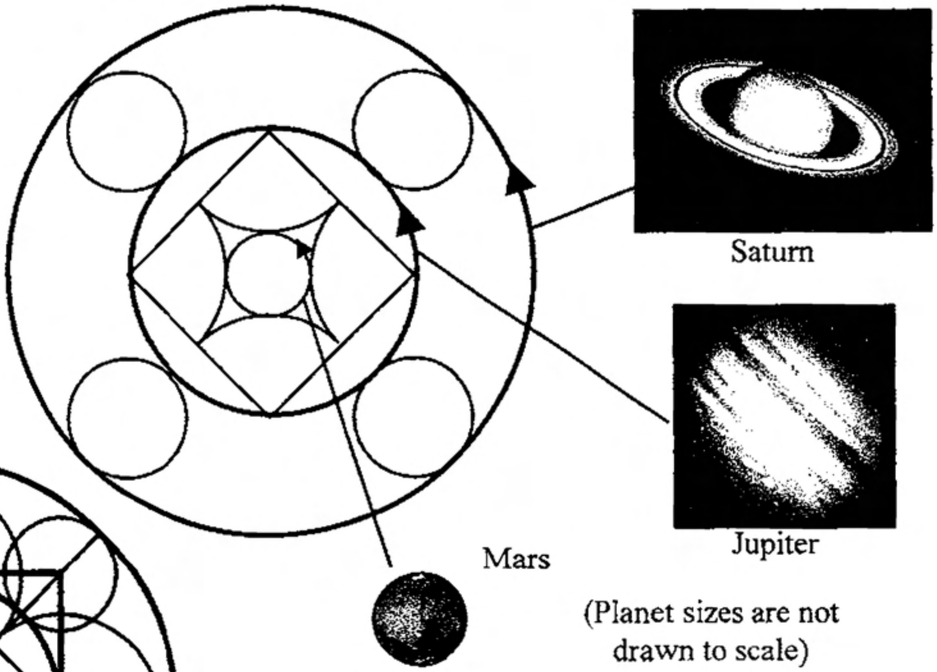
Solution:



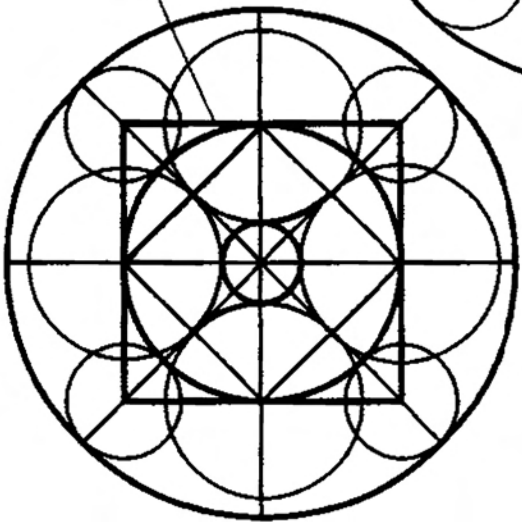
The Orbits Of Mars, Jupiter And Saturn



Do this construction of the mean (average or circular) planetary orbits (to 99.9% accuracy) on a blank sheet of paper, or in the Square below.

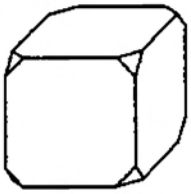


Hint: the original Square is around the orbit of Jupiter.



(Planet sizes are not drawn to scale)

Rearrange A Square Into A Triangle



Knowing how to construct a Square will let you solve this puzzle:

"How can a Square be cut into four pieces which can be rearranged into a regular Triangle?"

This puzzle was proposed and solved by Henry Ernest Dudeney, the self-taught mathematician and puzzle inventor in England in 1907. His construction is too complicated to present here, but the puzzle can be more easily solved if the Triangle is isosceles, having two equal sides (page 19).

Construct a paper Square and cut it out. Then see if you recognize how to cut it.

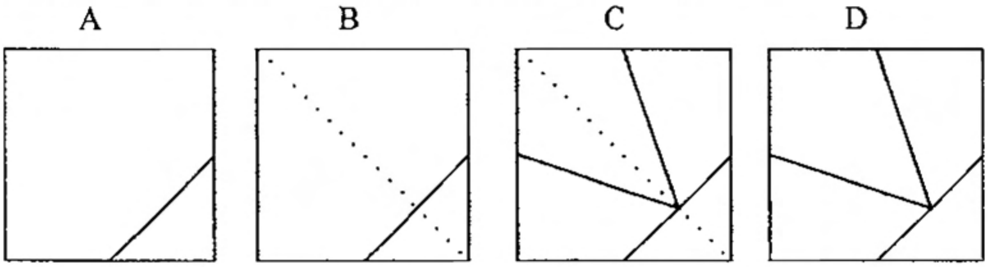
Here's one way to solve it:

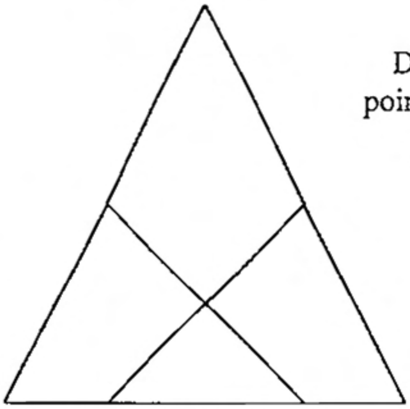
(1) Draw a line connecting the midpoints of two adjacent sides of the Square.

(2) Lightly draw the diagonal of the Square through that line.

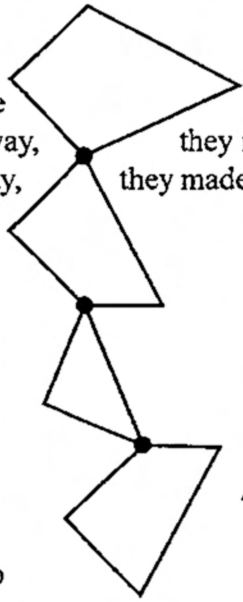
(3) Notice the point where the diagonal crosses the line drawn between midpoints in Step 1. Draw lines to the midpoints of the Square's other two sides.

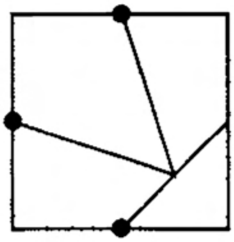
(4) Cut out these four pieces and rearrange them into a Triangle.

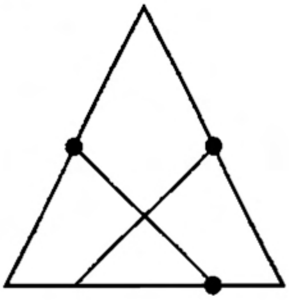




Dudeney built a wooden set with hinges at the points shown here. This let the pieces hang as a chain. When swung one way, they made a Square. When swung the other way, they made a Triangle.





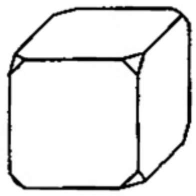


The solution seems simple once you see it. The solution for a regular Triangle is not too different!

(For the numerically inclined: Show that this Triangle's two side lengths are in the ratio Square-Root-of-5 to 2.)

Construct And Fold A Cube

A cube, or *hexahedron*, has six Square faces, 8 corners and 12 edges. All corners meet at right angles. A cube is the only way to neatly enclose three dimensions of space using only Squares.



Notice how the "Octahedron" (page 24) is becoming a Cube!

There are many ways to build a cube. Here's a popular one which results in a cubic box:

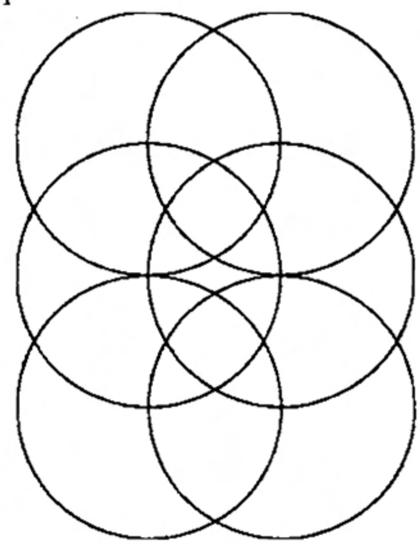
(1) Use your knowledge of constructing a Square upon a line segment (page 49) to construct six overlapping Almond Circles as shown.

(2) Draw straight lines between appropriate crossing points.

Cut out around the outer straight lines.

(3) Fold along the creases and carefully tape the edges as they come together.

1



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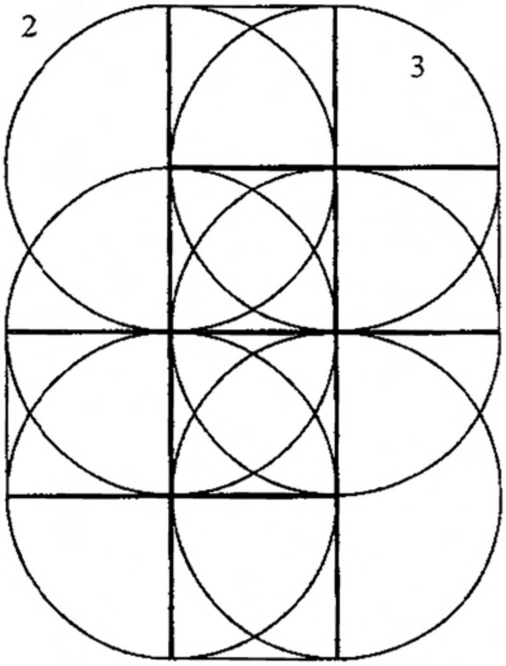

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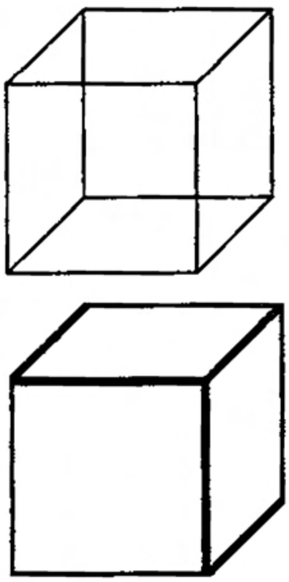
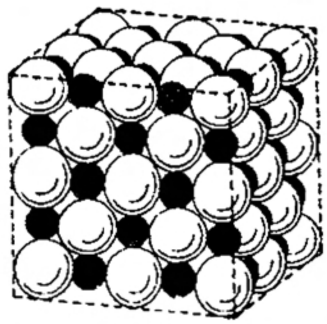


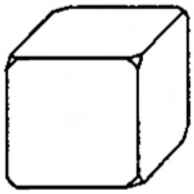
Many crystals, including common salt, are cubes. Look at them with a magnifying glass.

The numbers on honest cubic dice have *opposite sides* which always add to 7.



Salt is a Cubic package of Sodium and Chlorine atoms.

Build A Cube With Circles



(1) Construct a Square on sturdy paper or poster board.

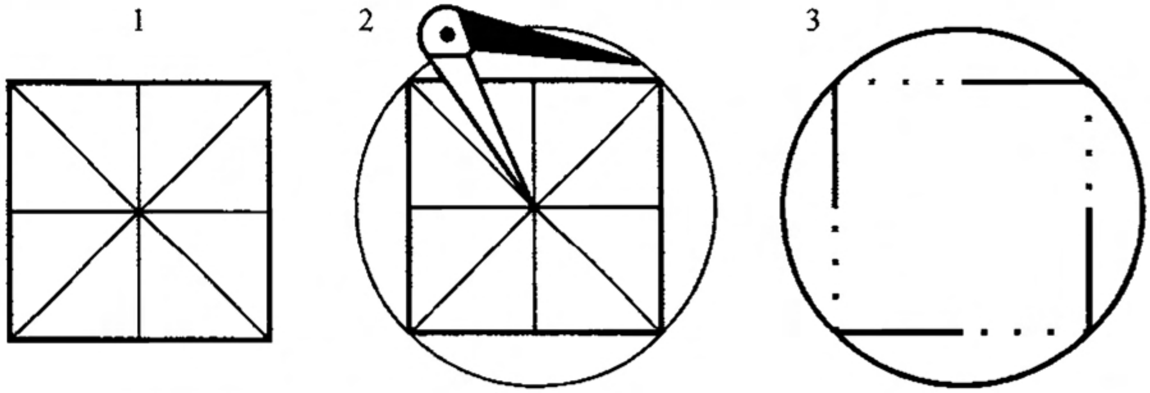
(2) Circumscribe a Circle around the Square; that is, place the compass point at the Square's center and open the scribe to a corner, and turn.

Mark the four corners and use it as a template to trace a total of six Circles.

(3) Cut halfway along each side of the Squares, always in the same direction (as shown).

(4) The Circles will slot together and build a cube which appears to be cut from a sphere.

1 2 3



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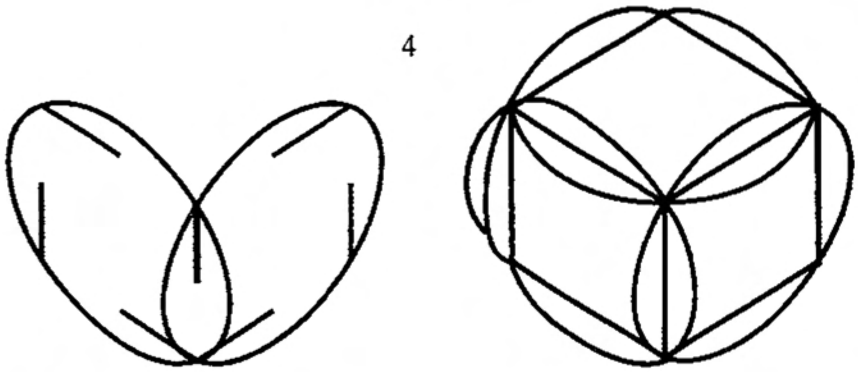
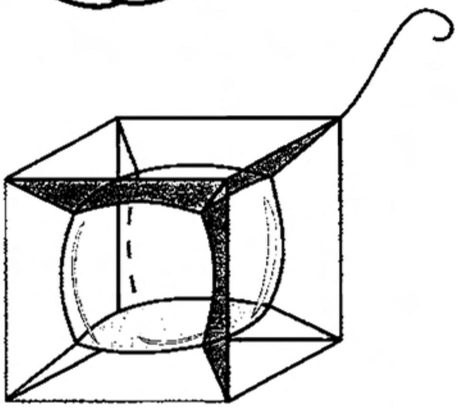
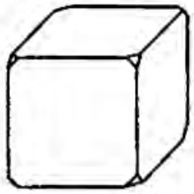


Make Soap Bubble Cubes

Make a wire cube with a handle and dip it in soap bubble solution. The soapy film will show you the forces pulling on that shape. Capture a bubble inside to see it stretch into a rounded cube.

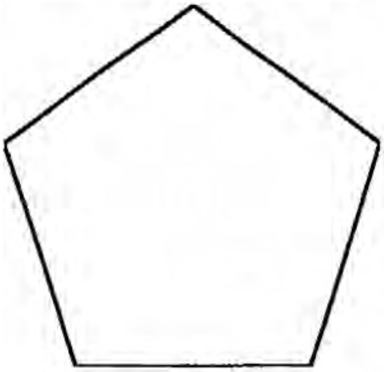


5 The Pentagon

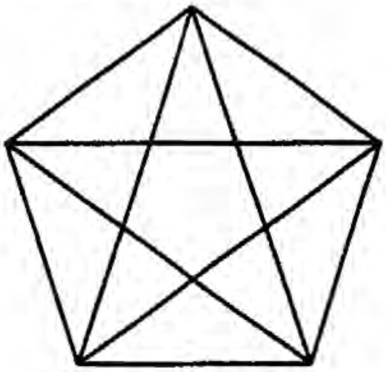


Our next polygon, the regular Pentagon, has five corners, five equal angles and five equal sides.

The diagonals drawn between the corners of a Pentagon make a Pentagram star. The star contains a downward pointing Pentagon. The Pentagon is the simplest shape which lets us draw all its diagonals *without lifting our pencil.*

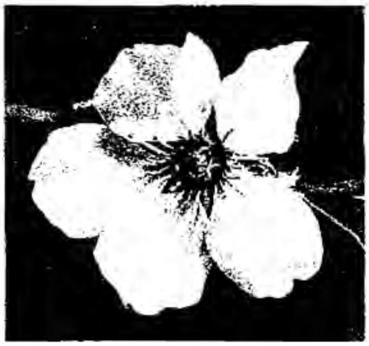


Regular Pentagon



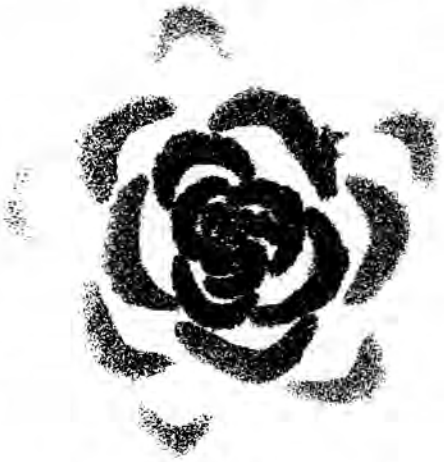
Pentagon with its Pentagram star

The Pentagram star was another symbol of the School of Pythagoras (see page 23) and a topic of their study. It was also associated with the Greek goddess of health, *Hygeia*, and her Roman counterpart *Salus*. Traditional Chinese medicine is based on cycles of five elements of nature. Indeed, five in nature is associated with health and life. Notice the geometry of fruits and vegetables. The flower of every edible fruit has five petals. The five pointed star is also associated with excellence, authority and humanity. Many cultural symbols into modern times are shaped like Pentagons and Pentagram stars. Do you know any?



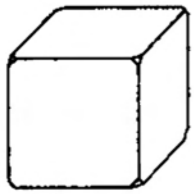
Apple Flower
The geometry of the flower determines the geometry of the fruit.

A Slice of Apple
shows its Penta package of seeds



The Whirling Star of a **Celery Bunch**

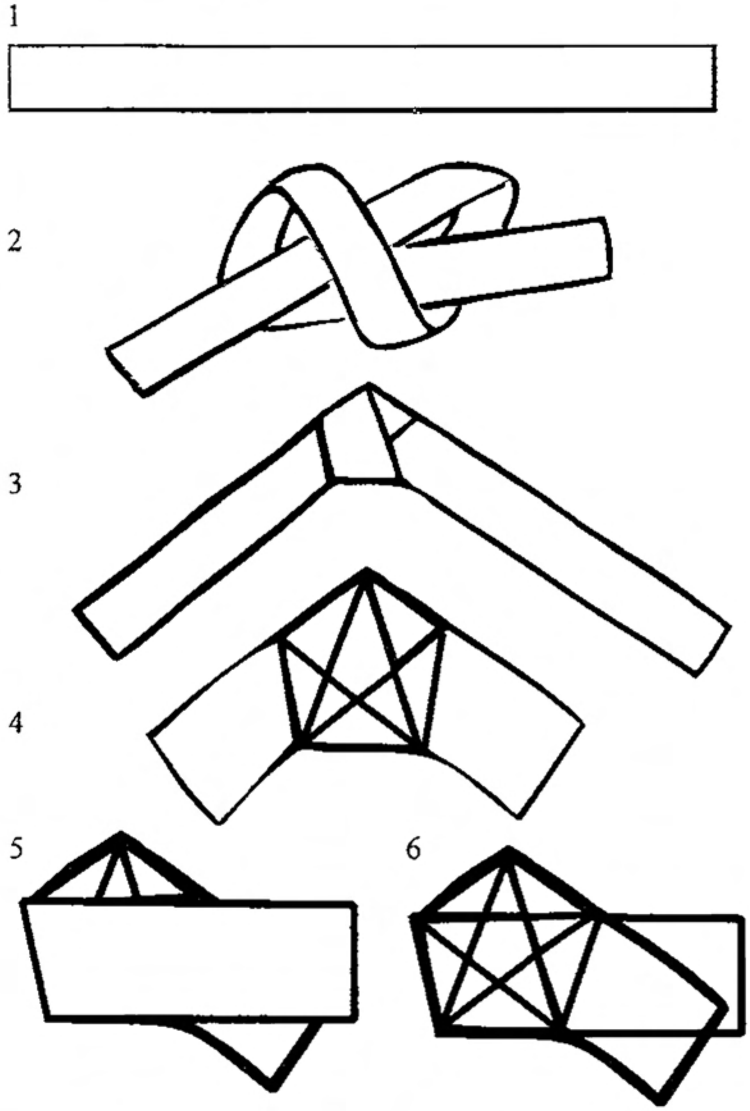
Tie A Penta Knot



The simplest construction of a Pentagon and Pentagram star doesn't require a compass or straightedge. It automatically happens when we tie a knot in a flat strip of paper.

(1) It's best to start with a strip of translucent tracing paper (say 1 1/4 inches by 14 inches).

(2) Carefully tie the strip into a simple knot, over and through.



(3) *Gently* pull the ends tighter until all sides are equal. But don't pull too tightly and curl or bend any of it! When all the corners meet and the edges are equal length, press the edges flat. It should make a regular Pentagon.

(4) Hold it up to the light. Do you see the Pentagon and Pentagram star?

(5) But the Pentagram is missing one line. To add it, bend one end of the strip back and insert it through the knot as shown.

(6) Press it's edge flat and hold it up to the light again.

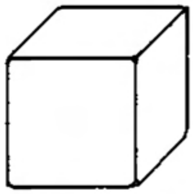
You should now see a lovely, complete Pentagram star inscribed in a Pentagon.

If you have a *very* long strip of paper, you might want to tie a *series* of these knots (but not the final step).

If all the knots are tied *in the same direction* (say, right over left) an equal distance apart, you'll come back to the beginning after the fifth knot and create a large Pentagon, with a penta-knot at each corner.

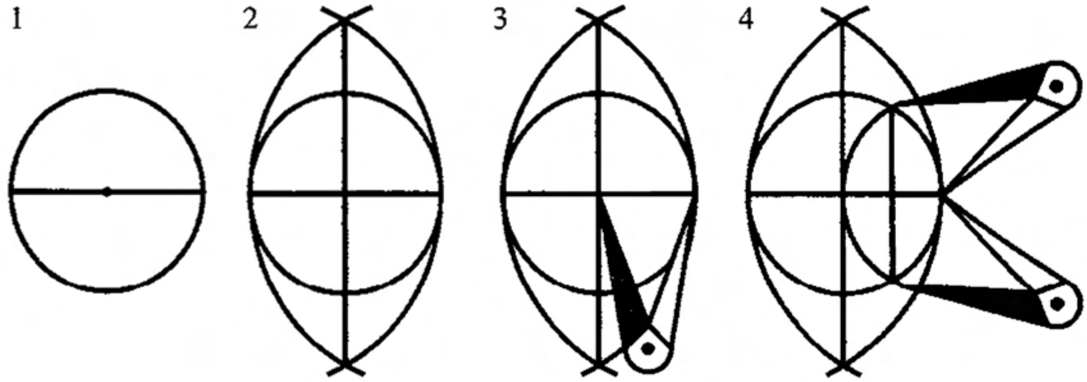
But if you *alternate* directions as you tie the knot (left over right, then right over left, repeat) you'll model the zigzag pattern made by ivy leaves on a vine, and the wavy jet stream wind around the Earth.

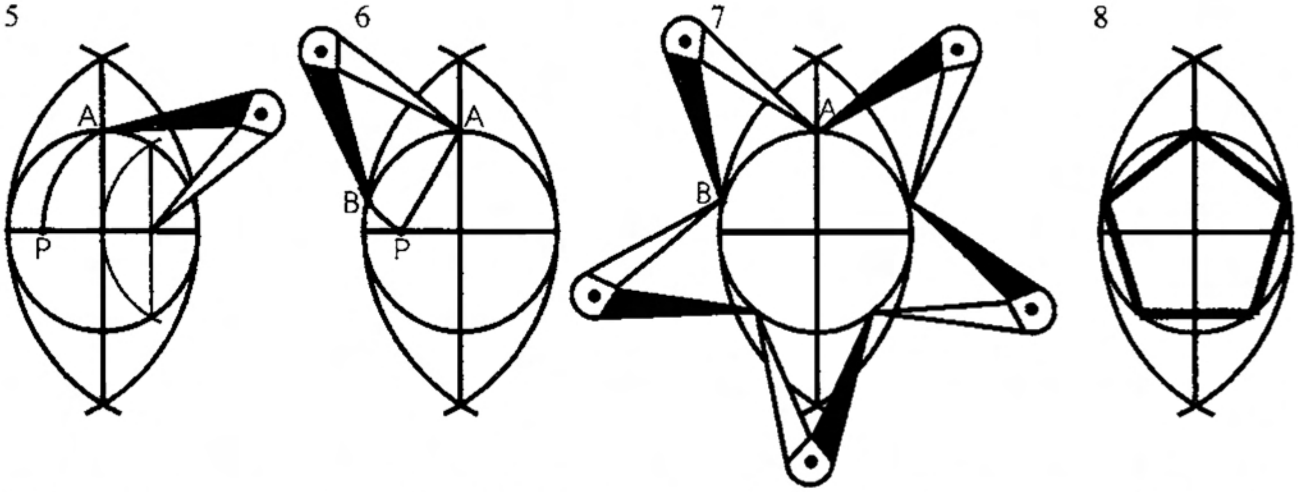
Construct A Pentagon In A Circle



The construction of the regular Pentagon was kept secret for centuries until 1509 when it was published in a book by Fra Luca Pacioli, the mathematics teacher of Leonardo da Vinci and others.

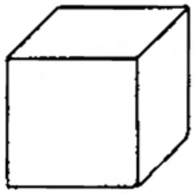
(1) This construction begins with a Circle and its diameter.
(2) Open the compass across the full diameter and make the arcs of the Almond. Connect its crossings with a straight line. (If you have enough space, you may turn full Circles for all arcs.)
(3) Open the compass to the radius of the Circle again. Place the compass point on the end of the diameter (as shown).
(4) Turn an arc until it crosses the Circle. Draw a line connecting this Almond's crossings. We have divided the radius of the Circle in half. Notice its midpoint.

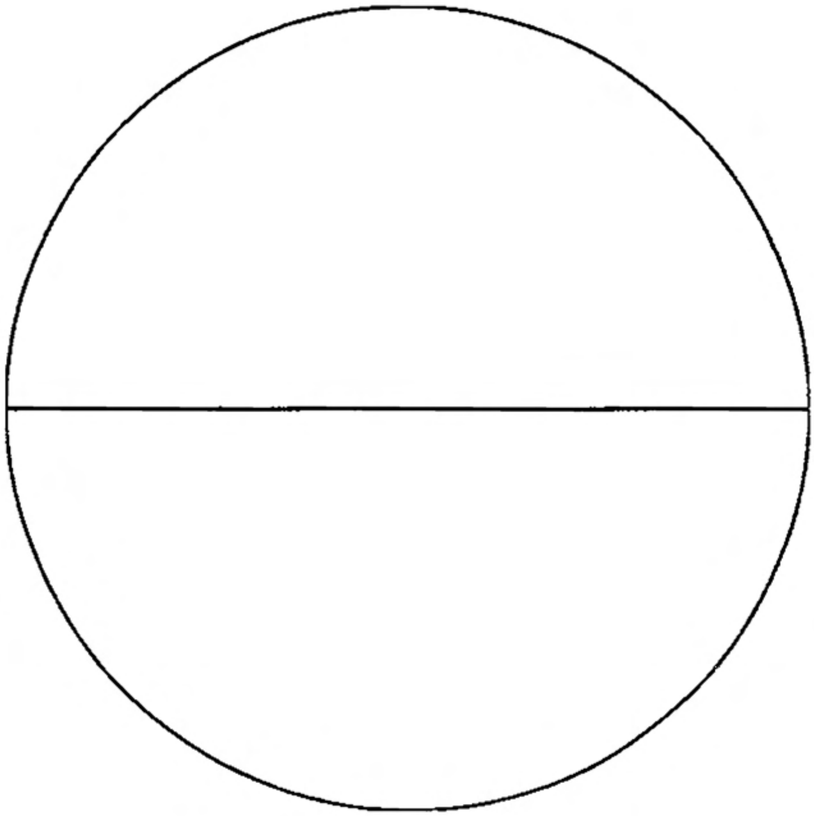




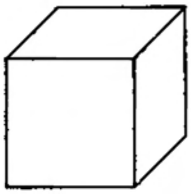
(5) Place the compass point on this midpoint and open the scribe to the top of the Circle (point A). Then, draw an arc down to cross the Circle's diameter at point P.
(6) Place the compass point at the top of the Circle and open the scribe to point P. Make an arc upward until it crosses the Circle at point B.
(7) This opening of the compass should "walk" around the Circle exactly five times. If you don't come back to the beginning point, then "walk" the compass *in the other direction* from the top starting point A. The true five points of the Pentagon will be found halfway between each pair of marks.
(8) Connect the points with straight lines to create a regular Pentagon.

Construct a Pentagon By This Method



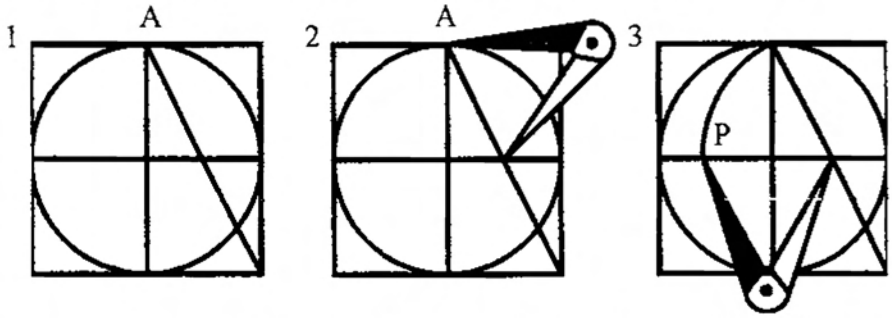


Construct A Pentagon In A Square

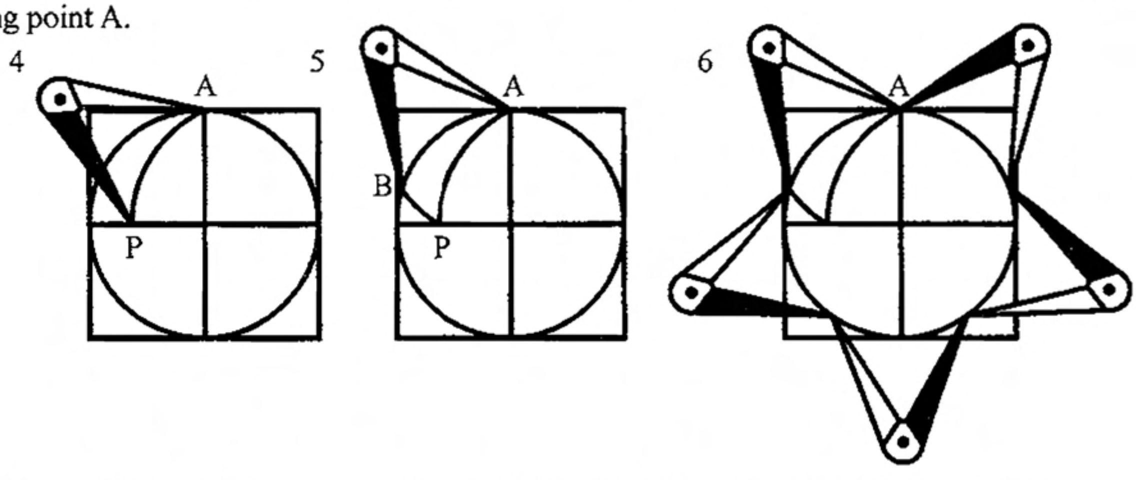


This is similar to the previous construction but is useful if you are comfortable with making a Square, or find one in a construction. You might compare the two constructions.

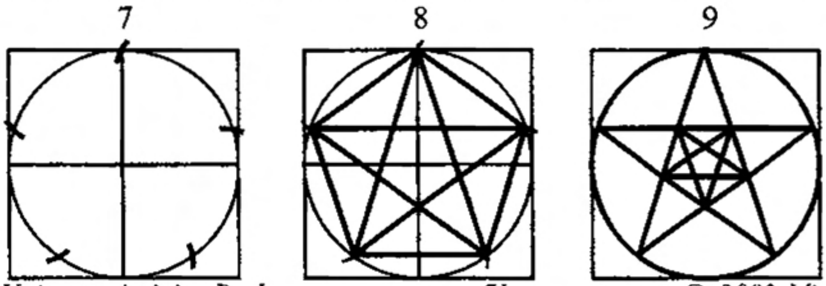
(1) Start with the construction of the Square around a Circle (page 47). Draw a diagonal across two small quarter-Squares as shown.
(2) Place the compass point where the diagonal crosses the Circle's diameter, and open the scribe to the top of the Circle point A.
(3) Make an arc downward until it crosses the Circle's diameter at point P.



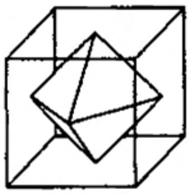
(4) Now place the compass point at the top of the Circle and open the scribe to the point P where the arc crosses the diameter.
(5) Make an arc upwards until it crosses the Circle at point B.
(6) This opening of the compass should "walk" around the Circle exactly five times. If you don't come back to the beginning point, then "walk" the compass in the other direction from the top starting point A.



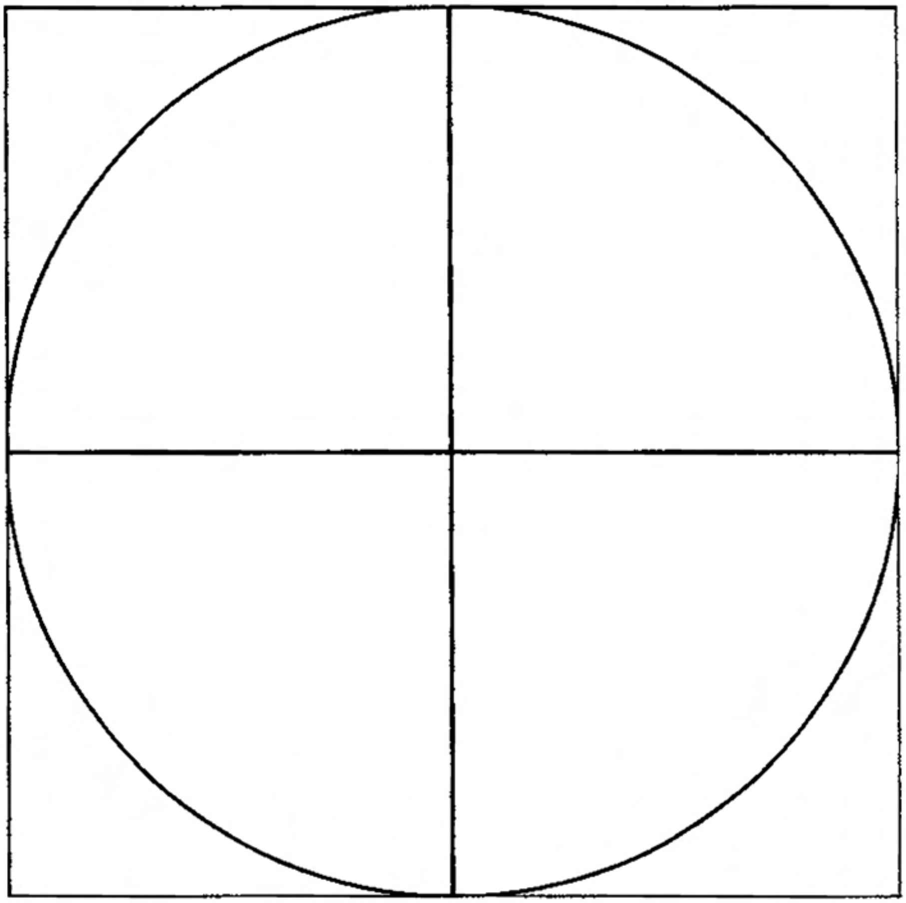
(7) The true five points of the Pentagon will be found halfway between each pair of marks.
(8) Connect the points with straight lines to create a regular Pentagon and Pentagram star.
(9) The Pentagram contains infinitely endless smaller Pentagons and Pentagrams.



Construct a Pentagon In This Square

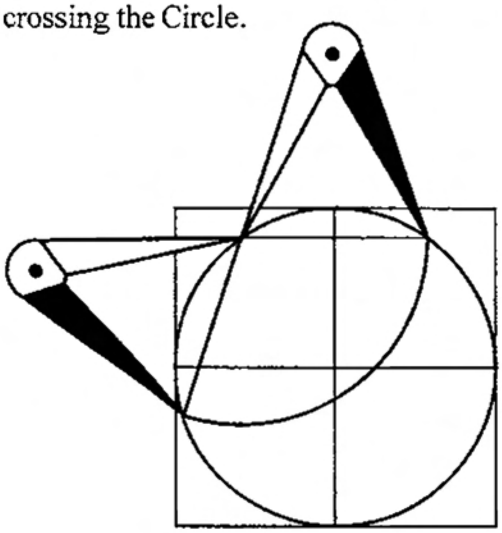


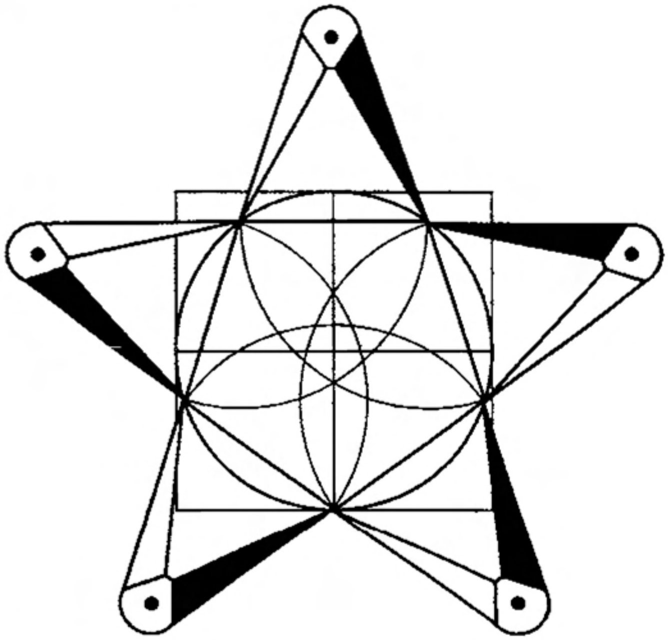
The six corners of the Octahedron touch the middle of each of the six faces of the Cube.



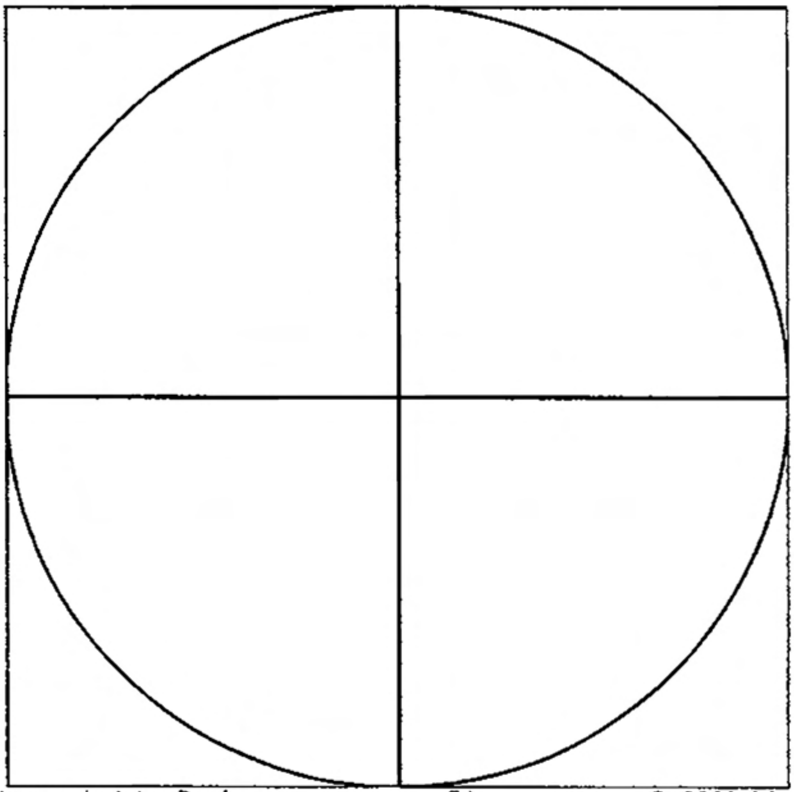
(7) Swing an arc to cross the Circle. The line segment between the crossings reveals the second side of the Pentagon.

(8) Continue to "walk" the compass around the Circle, swinging arcs if you wish to see a rounded Pentagram star within. If the final crossing doesn't exactly meet the first crossing, then "walk" the compass in the opposite direction from the starting point. The true five corners of the Pentagon are found between the arcs crossing the Circle.





Construct a Pentagon By This Method



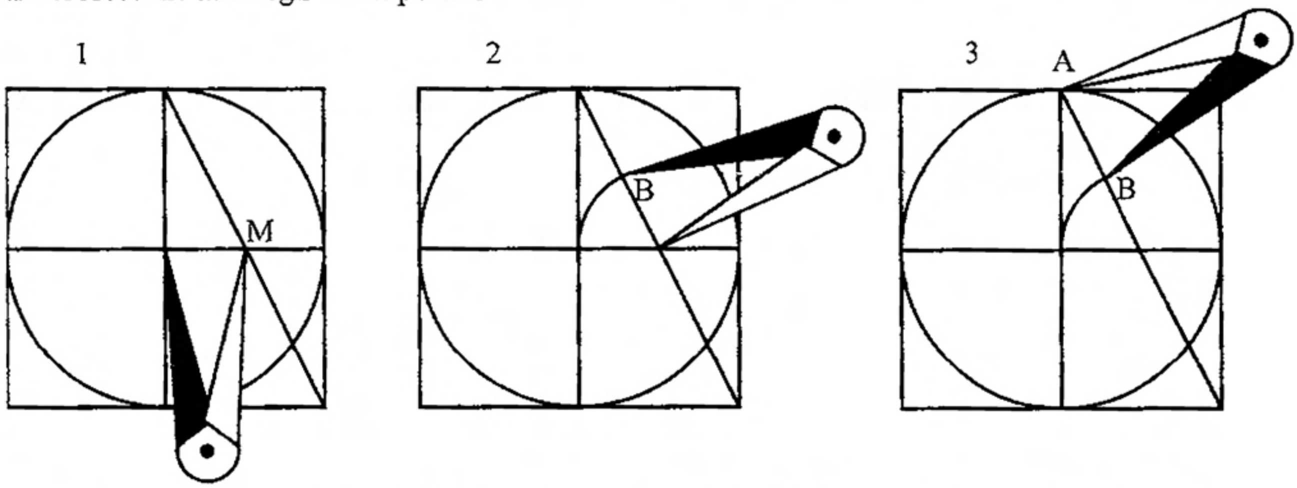
Another Pentagon Construction In A Square

This construction begins as the previous two do: finding the midpoint of the radius of the Circle. Pick up the construction here:

(1) Place the compass point at the center of the radius (point M), and open the scribe to the center of the Circle.

(2) Turn an arc as shown, crossing the half-diagonal at point B.

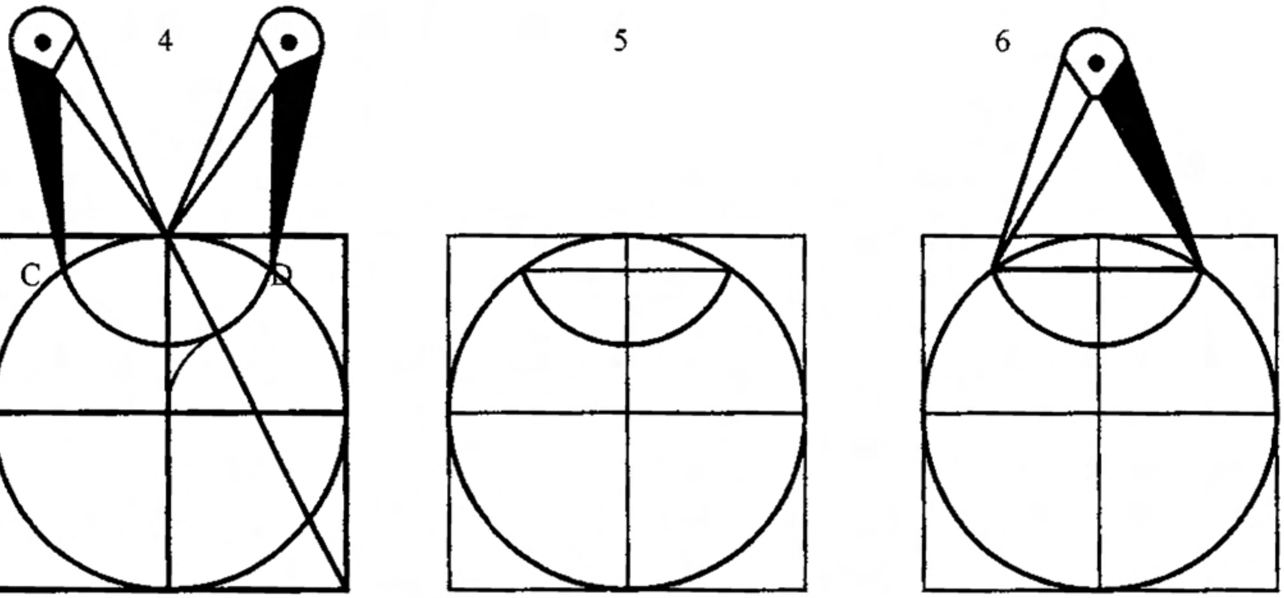
(3) Now place the compass point at the top of the Circle (point A) and open the scribe to where the arc crosses the line segment at point B.



(4) Swing an arc from the top center (point A) to where it crosses the Circle at points C and D.

(5) Draw the line segment between these crossing points C and D. This is one side of the Pentagon.

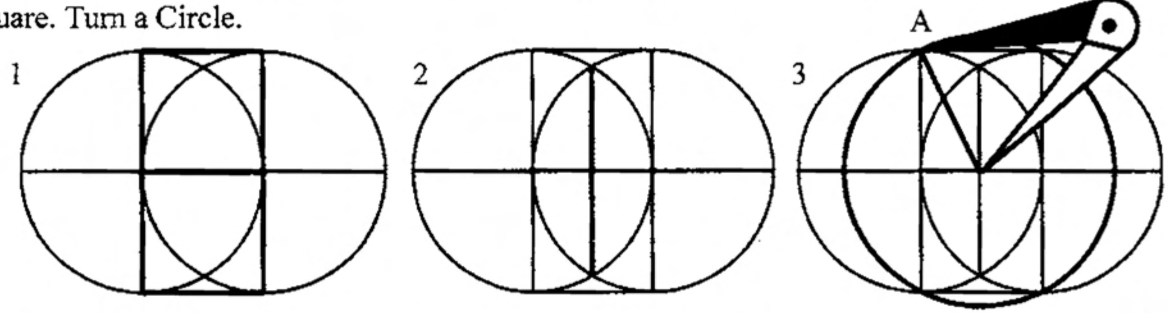
(6) Open the compass between the ends of this line segment.



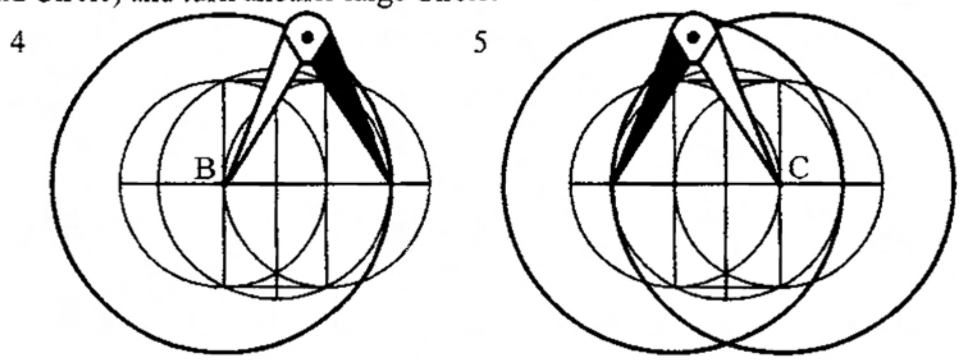
Construct A Pentagon On A Line Segment

If you already have an existing line segment, you can construct a Pentagon upon it as its base. First, you must do the construction of the Almond and Square upon the line segment (page 49).

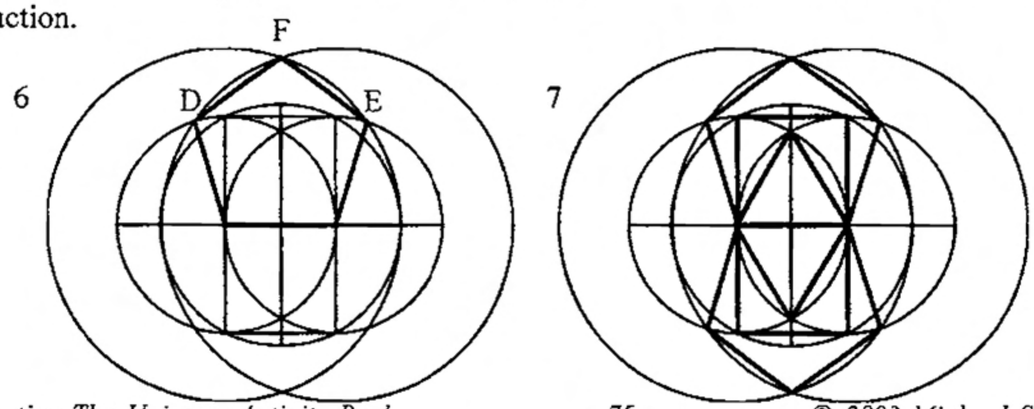
(1) Draw the construction's double Squares, and extend the diameter across both Circles (page 12).
(2) Connect the crossings of the Almond with a vertical line to find its center.
(3) Place the compass point on this center and open the scribe to point A at one corner of the upper Square. Turn a Circle.



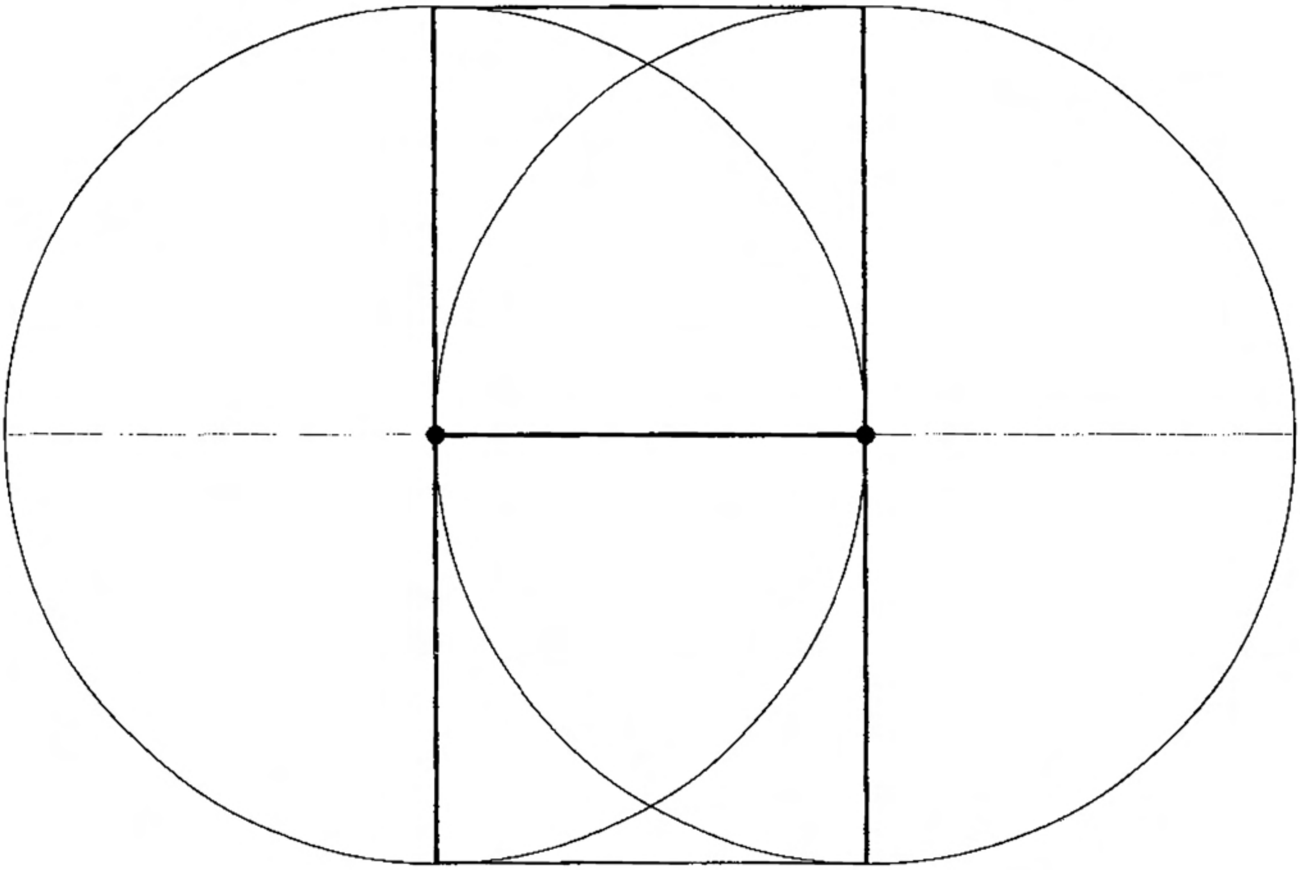
(4) Now place the compass point on the lower corner of the Square at point B (the center of one of the Almond Circles) and open the scribe to the point where the Circle just drawn crosses the diameter. Turn a large Circle.
(5) Reverse the compass, place its point on the Square's other lower corner point C (the center of the other Almond Circle) and turn another large Circle.



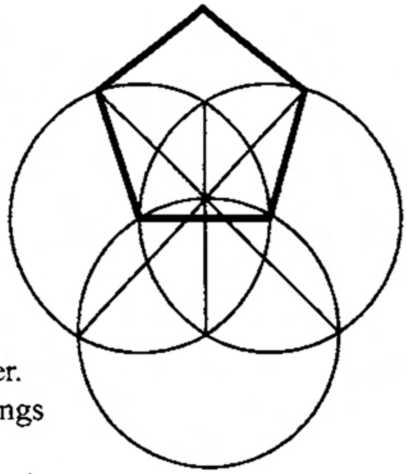
(6) Connect the ends of the original line to the points where the large Circles cross the Almond Circles points D and E. Then connect these points with the crossing of the two large Circles at point F. These are the five sides of a Pentagon.
(7) Actually, mirror-image pairs of the Triangle, Square and Pentagon can be found reflected in this construction.



Construct a Pentagon On This Line Segment



Construct Dürer's Pentagon

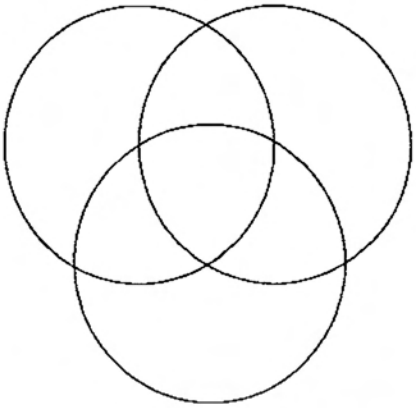


The artist, mathematician and writer Albrecht Dürer (1471-1528) published the construction of a Pentagon, but it should not be confused with being a *regular* Pentagon, which it is not. It is an approximation which only appears to be regular.

Here's how it's done:

(1) Construct the three Almond Circles which touch each others' center.
(2) Draw the line connecting the centers of two Circles, and the crossings of one Almond (as shown).
(3) Draw a line from the lower (left) crossing of the Almond Circles (point B) to the point where the vertical line crosses the top of the middle Circle (point A) and *extend* it until it reaches the Circle. Do the same from the other (right) lower crossing (point B) as shown.

1



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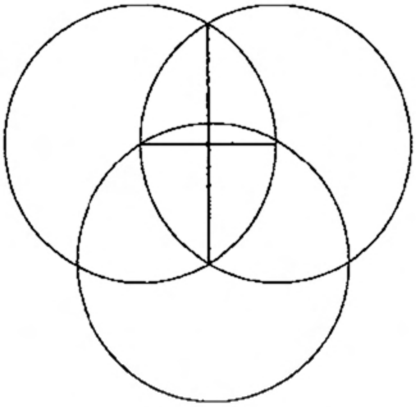

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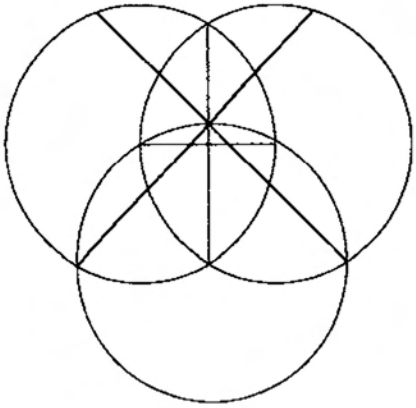
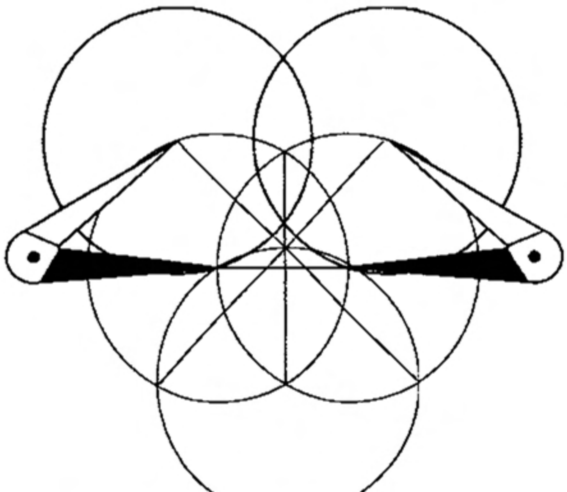
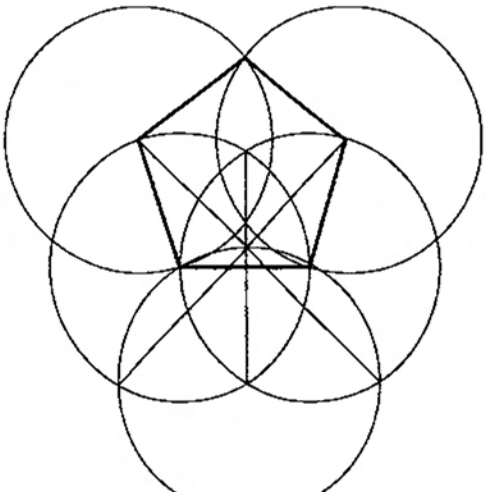


(4) Place the compass point where this extended line crosses the upper Circle and open it to the center of one of the original Almond Circles (as shown). Turn a Circle. Repeat this with the compass point on the end of the other line segment and original Circle center.
(5) Connect the five points as shown to draw Dürer's (non-regular) Pentagon.

4



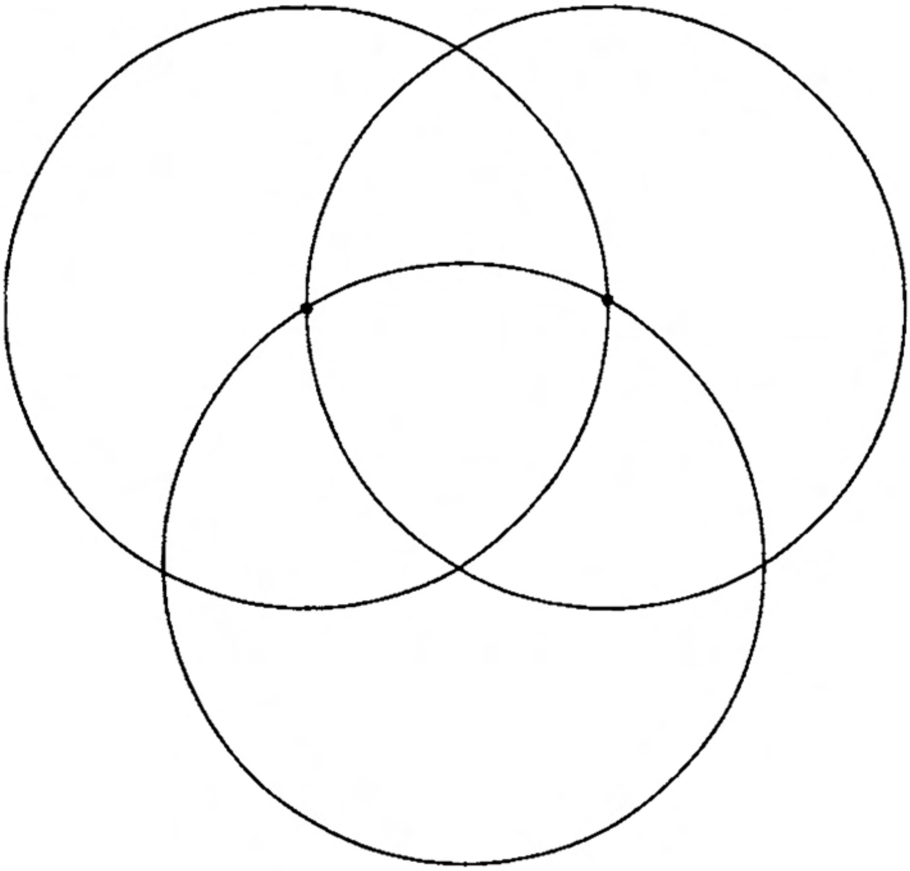
5



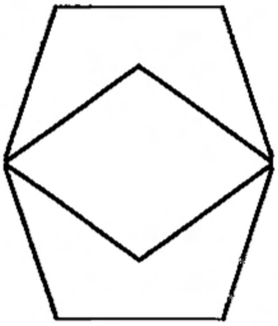
Replicate Durer's Pentagon

Follow the instructions on the previous page.
Can you see that it isn't exactly a regular Pentagon?

Albrecht Dürer's Self-Portrait at 28
1500

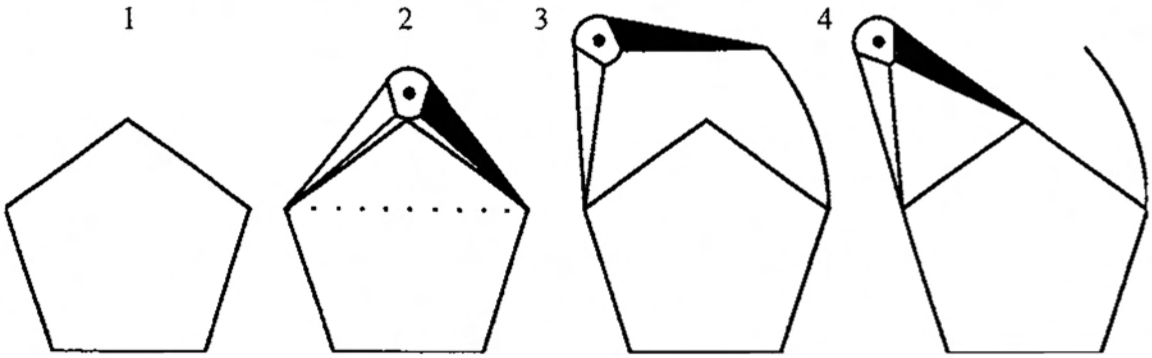


Married Pentagons

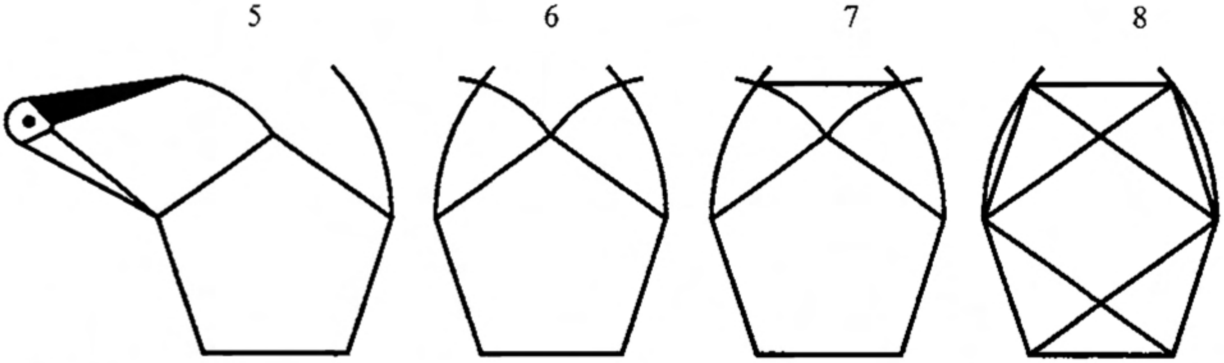


When two Pentagons come together head to head and penetrate each other, they create a non-regular Hexagon called "Married Pentagons" or a "Pentagonal Hexagon". It's a very interesting figure rich with construction possibilities.

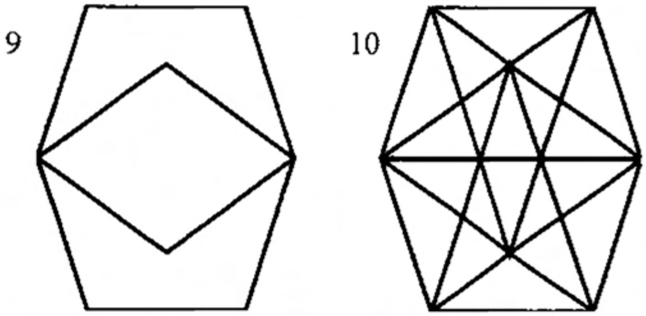
(1) To construct Married Pentagons, first construct a single Pentagon.
(2) Open your compass between the Pentagon's widest corners.
(3) Turn an arc upward (or full Circle if you have space).
(4) Keep the compass point at the same place and open the scribe to the Pentagon's top.



(5) Turn an arc upward.
(6) Place the compass point on the opposite side of the Pentagon and repeat steps 2, 3, 4 and 5.
(7) Connect the arc's crossing points as shown.
(8) Draw straight lines from the side corners of the Pentagon to the other points of the construction.

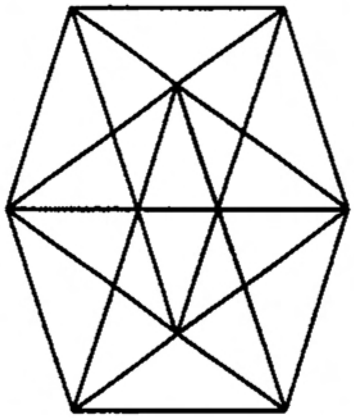


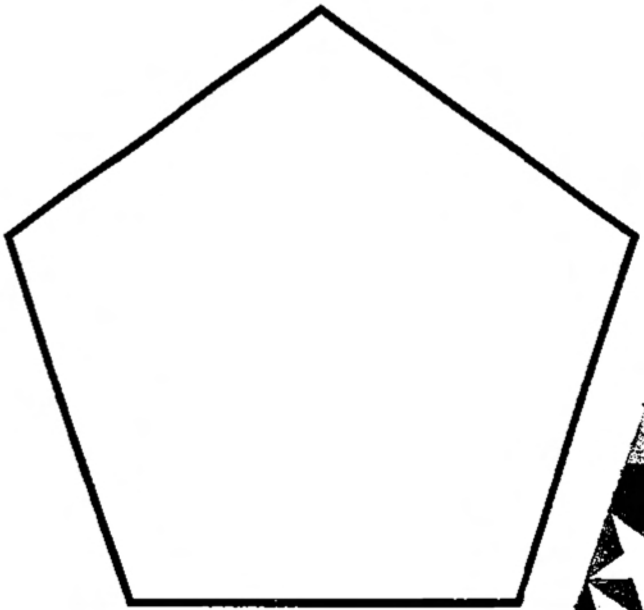
(9) Two Married Pentagons share a common space.
(10) Married Pentagons with their Pentagram stars.

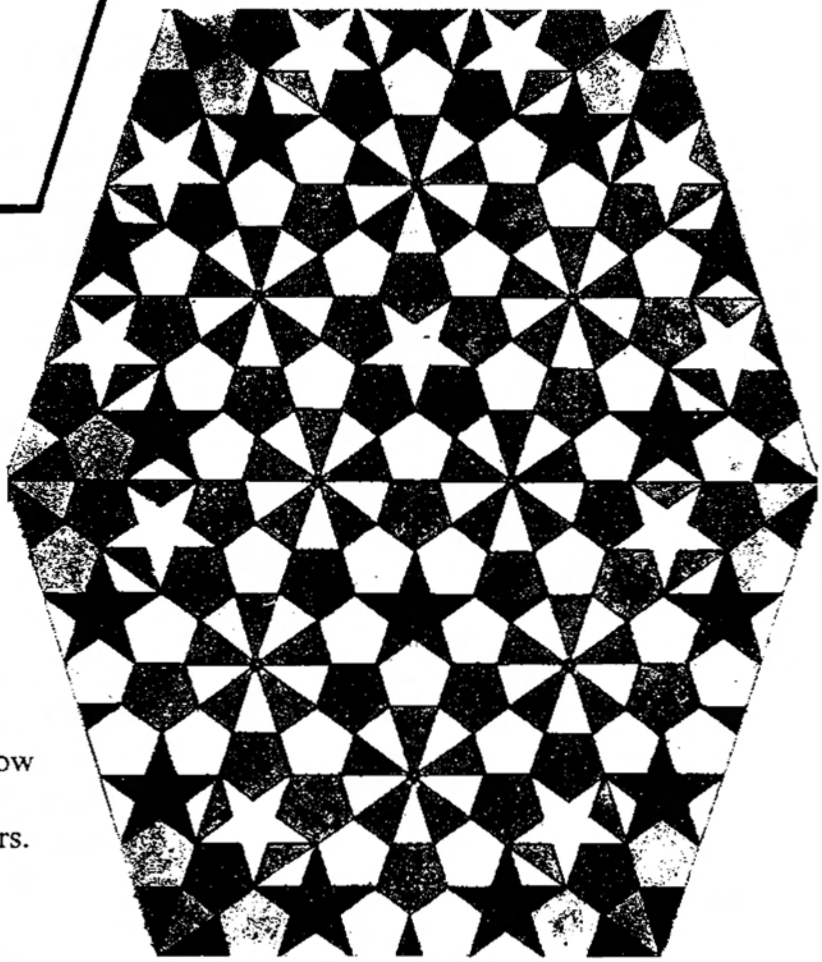


Construct The Married Pentagons

Start with this Pentagon or one you construct on blank paper.







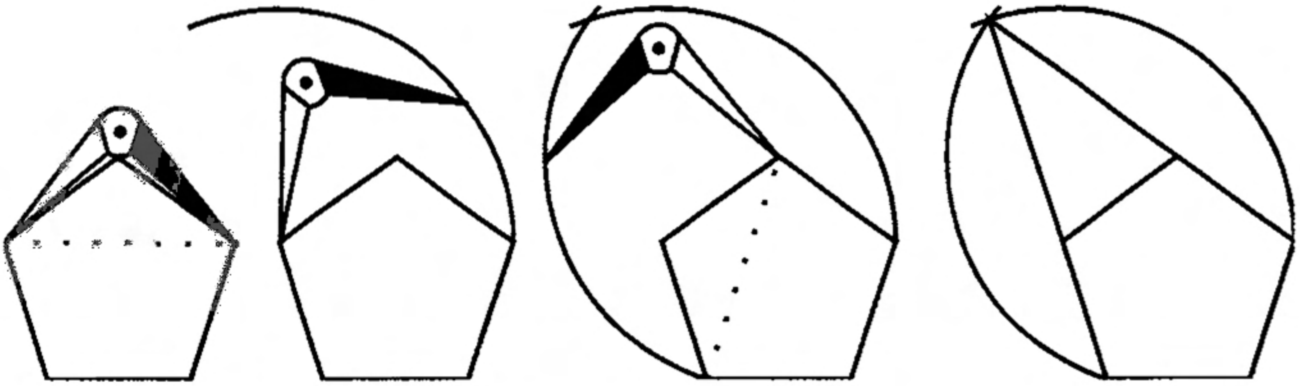
Pentagonal Hexagon 2
Watercolor by John Michell

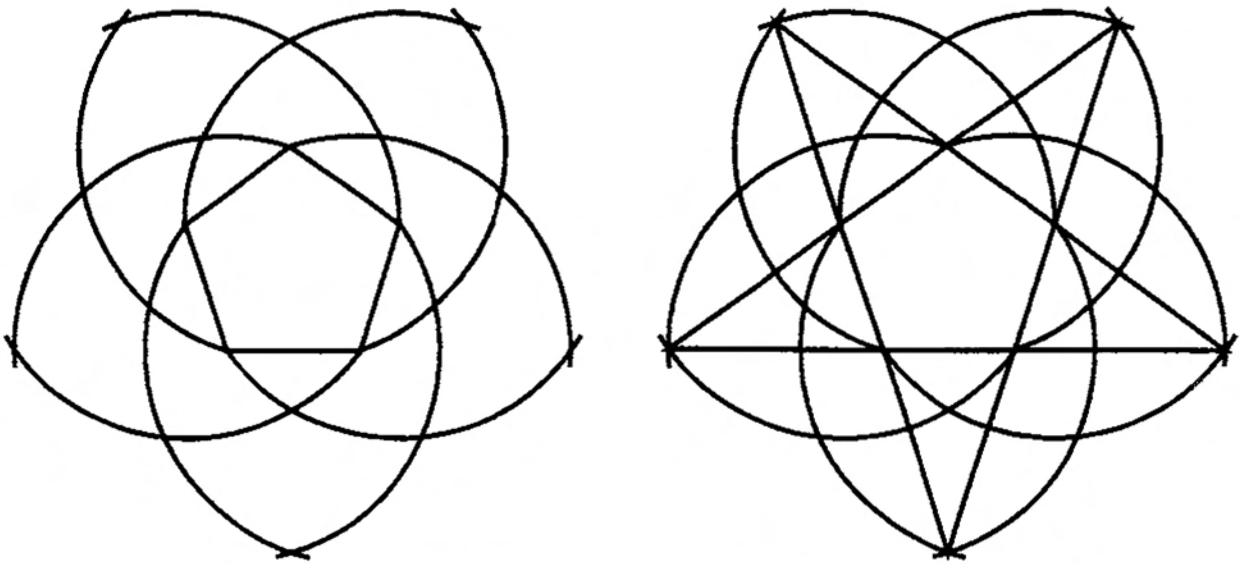
The following pages will show you how to subdivide Pentagons endlessly into smaller Pentagons and Pentagram stars.

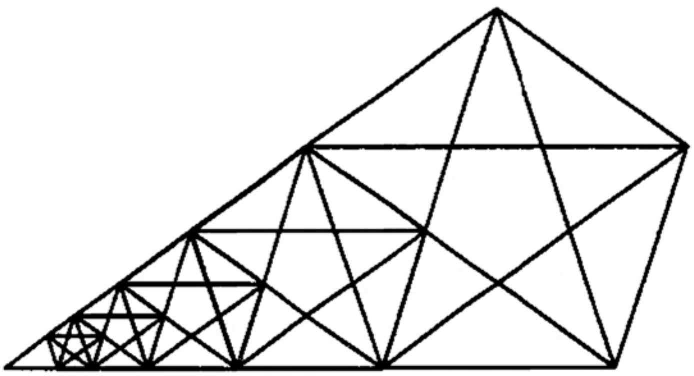
Expanding A Pentagon

This construction will expand any regular Pentagon into a larger Pentagon and Pentagram star pointing in the opposite direction.

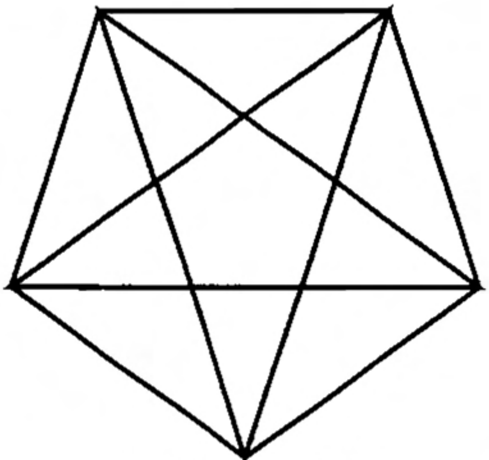
See if you can follow the construction just by looking at the pictures.





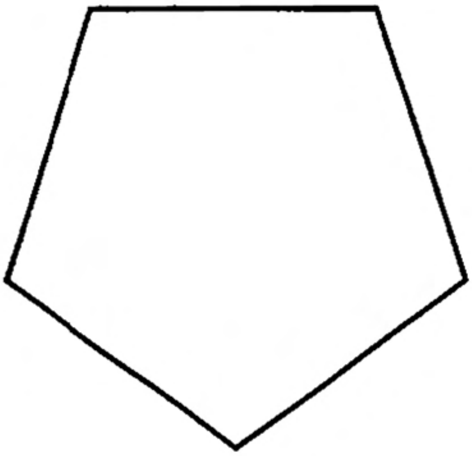


Pentagram stars in the arm of an extended Pentagon



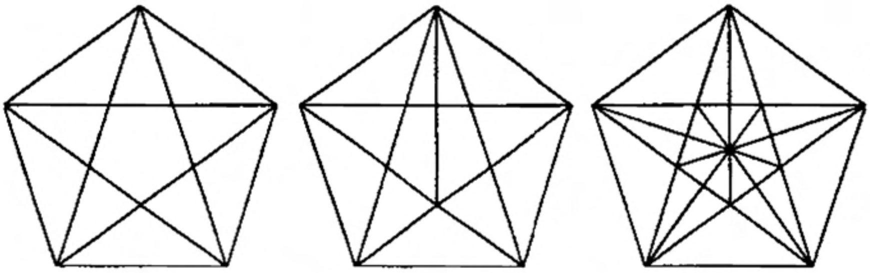
Each new, larger Pentagon may be expanded this same way.

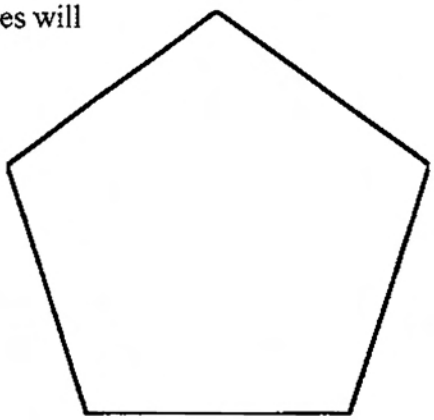
Expand This Pentagon



How To Find The Center Of A Pentagon

Draw a Pentagram star in the Pentagon. Then draw straight line segments between each corner and the opposite crossing of the star. All the lines will cross at the center.

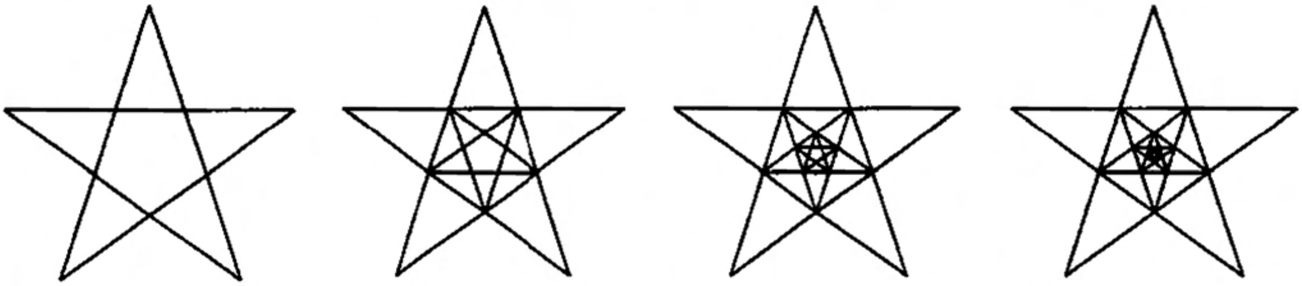




Find the center of this Pentagon

Subdivide A Pentagram Star

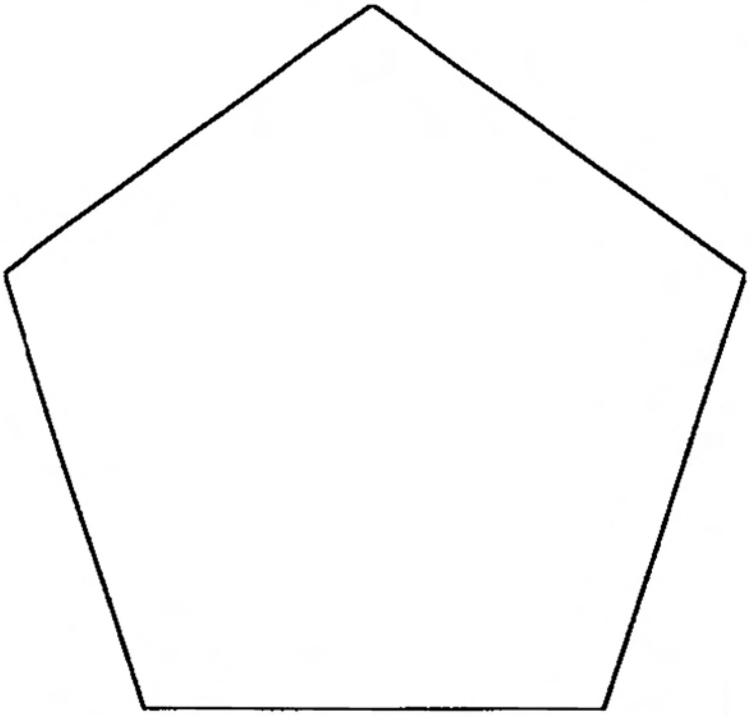
At the center of a regular Pentagram star is a small, opposite pointing regular Pentagon. Another Pentagram star may be drawn within it. This process can continue indefinitely, limited only by the sharpness of the point of your pencil.



Use your straightedge to draw a Pentagram star in this Pentagon.

Keep constructing Pentagrams in the central Pentagon as small as your pencil point will allow.

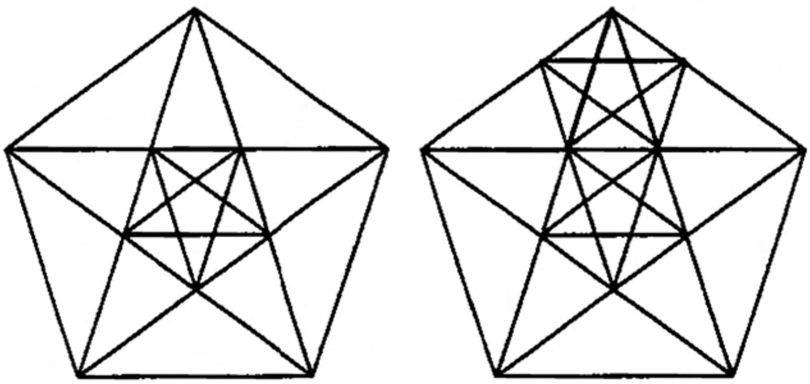
Color them any way you wish.



Subdivide A Pentagon Into Smaller Pentagram Stars

One of the many remarkable characteristics of the Pentagon and Pentagram star is that they are filled with infinitely many smaller versions of themselves.

We saw how lines connecting the Pentagon's five corners cross themselves to make the Pentagram Star. A smaller Pentagon in the middle can contain a Pentagram star too. This downward pointing Pentagram star can be "flipped" to point upward on the large Pentagram star's triangular arm because they are the same size and same star.

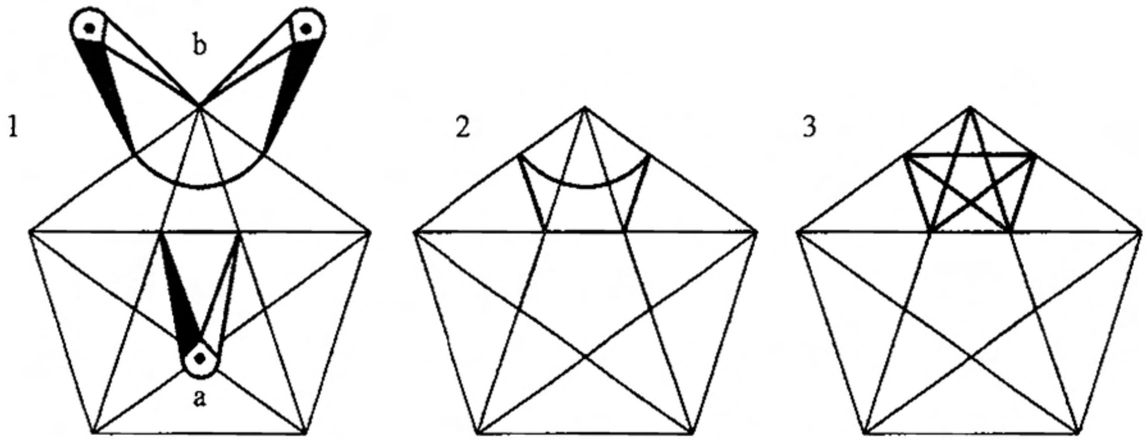


You can do this construction as follows:

(1) Open the compass (position a) between the ends of one side of the Pentagram star's inner Pentagon. Then move the compass point to the top of the Pentagon (position b) and mark points on each side of it as shown.

(2) Draw straight lines connecting these new points with the two original two points of the small central Pentagon to make a Pentagon on the upper arm.

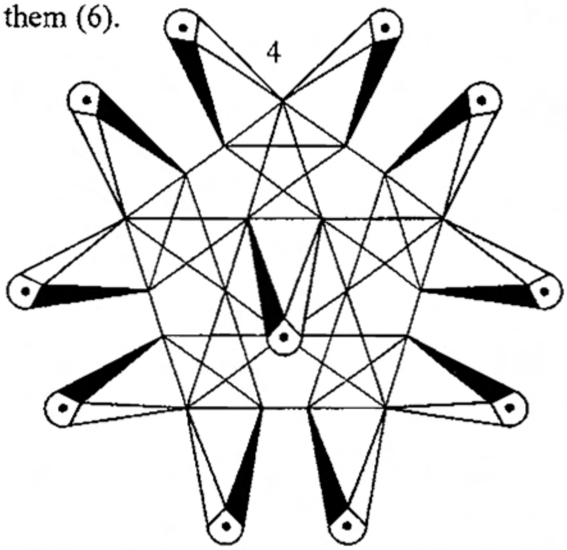
(3) Connect its corners to draw a Pentagram star in this Pentagon.



(4) With the same compass opening, place the compass point on each of the five corners of the large Pentagon and mark points on each side of it.

(5) Connecting these points will allow you to create Pentagons containing Pentagram stars, with even smaller Pentagons and Pentagram stars between them (6).

4



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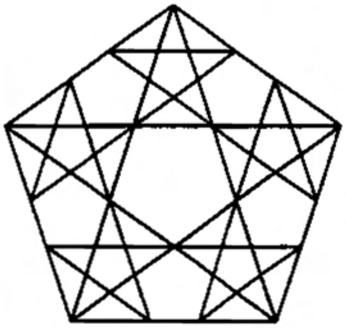

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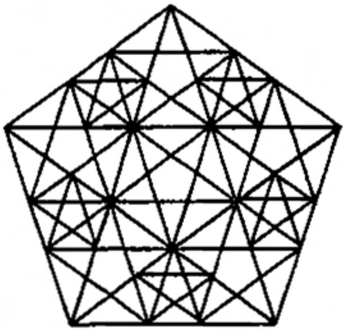
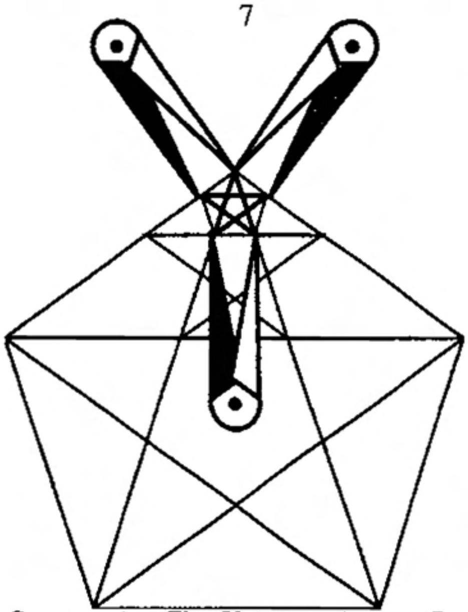


(7) Each of these stars can have Pentagons and Pentagrams constructed beyond and within them by repeating this method: measure the side of the central Pentagon within the Pentagram star and mark this distance from the top of the large Pentagon. Then connect the lines to create Pentagons and Pentagram stars. You can do this as small as your compass and pencil point allow to construct as many Pentagram stars as you wish.

(8) With this method, you can discover and construct many Pentagons and Pentagram stars, none interfering with others. A Pentagram star sprouts smaller Pentagrams all over itself! That's why Pentagons and fives appear abundantly in leaves and flowers: the same shape blooms in many sizes at once, all fitting together perfectly on one plant!

7



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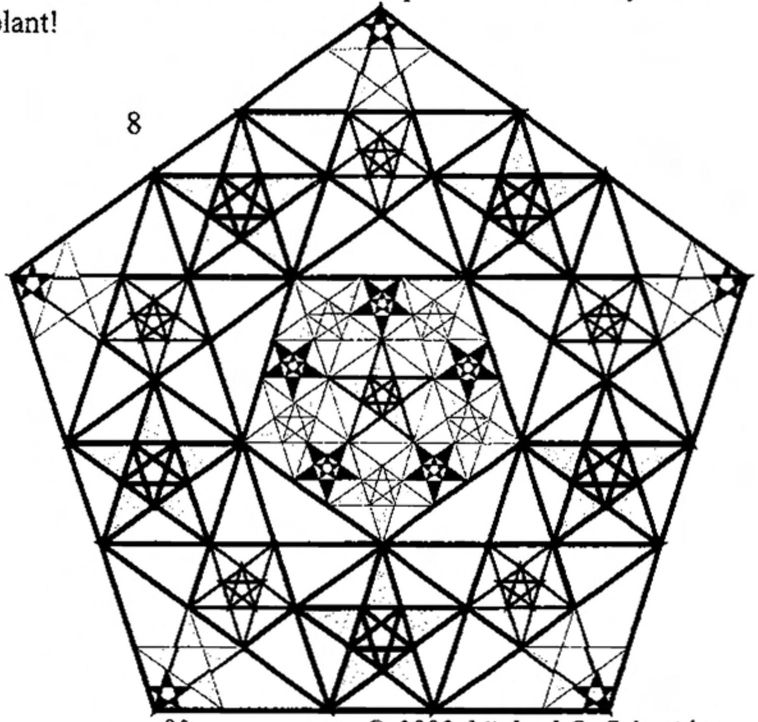
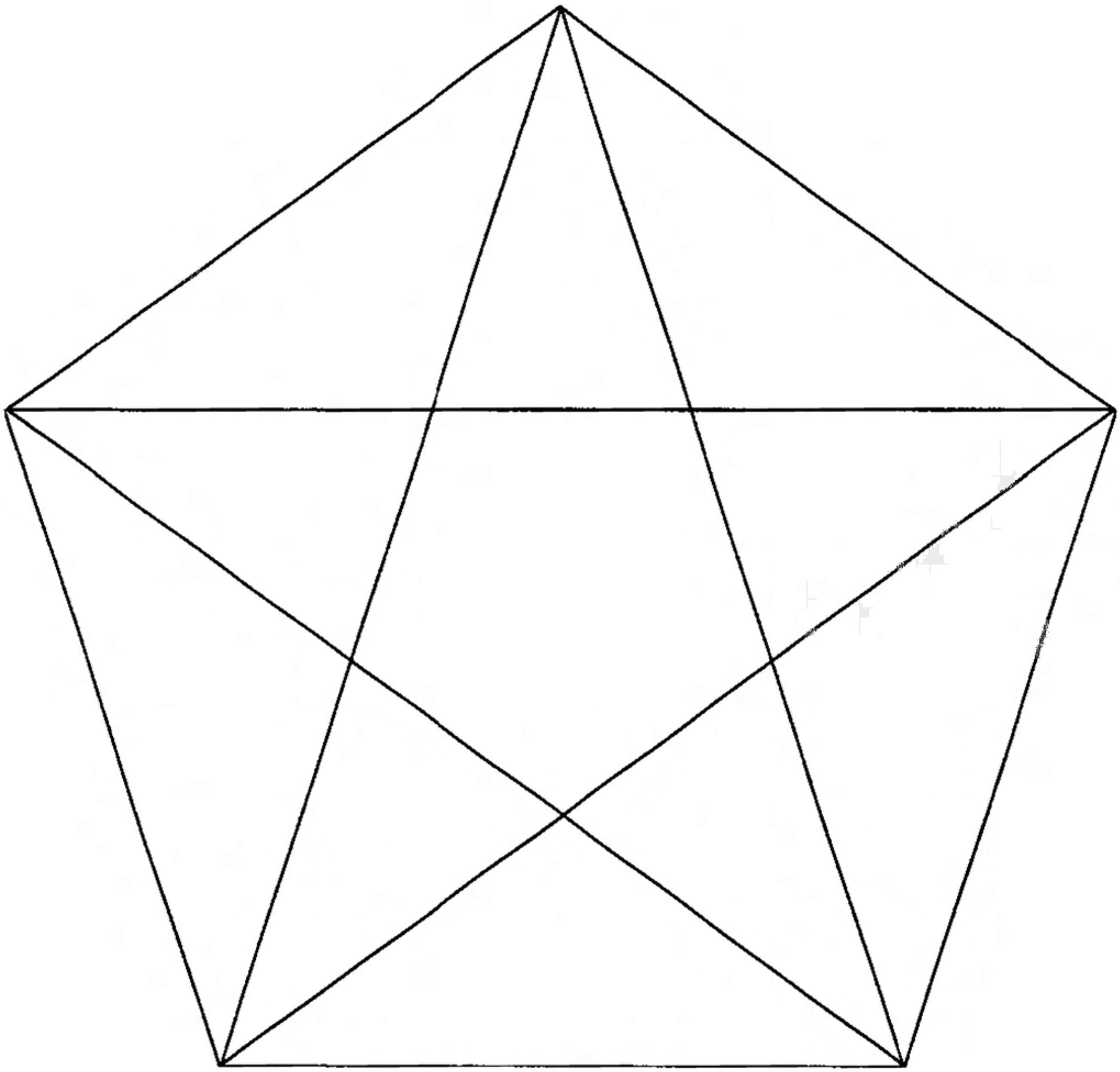


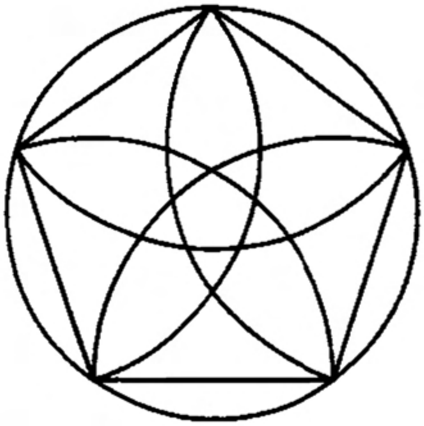
Subdivide This Pentagon

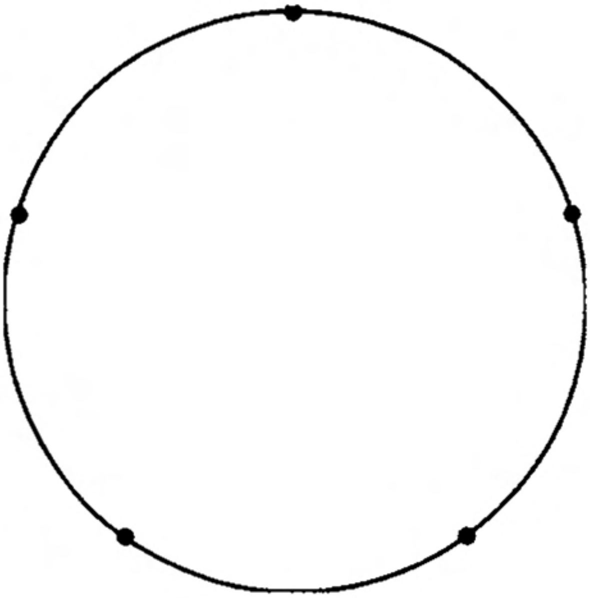
Start with this Pentagon and Pentagram star (or construct your own on blank paper). Use your compass and straightedge to construct many Pentagons and Pentagram stars in the center and around the arms by the method described on the previous pages. Use your imagination and colored pencils to make a delightful design! Perhaps make each Pentagram star into a flower.

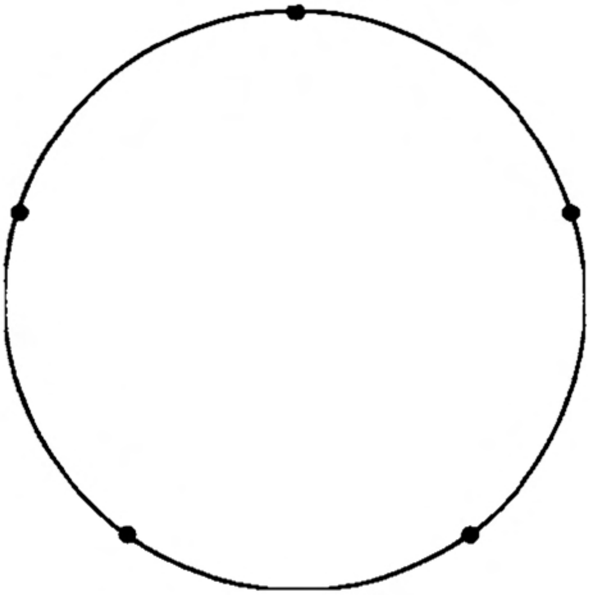


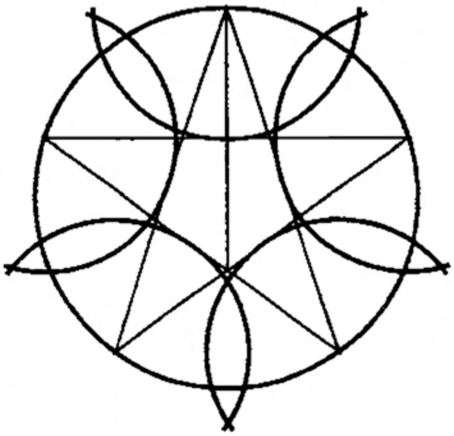
Replicate These Pentagonal Patterns

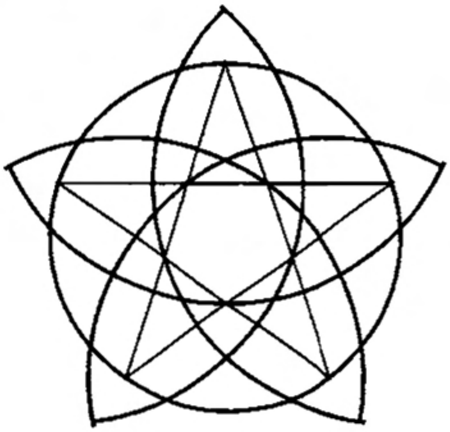
Where is the compass point?
Where is the compass open to?

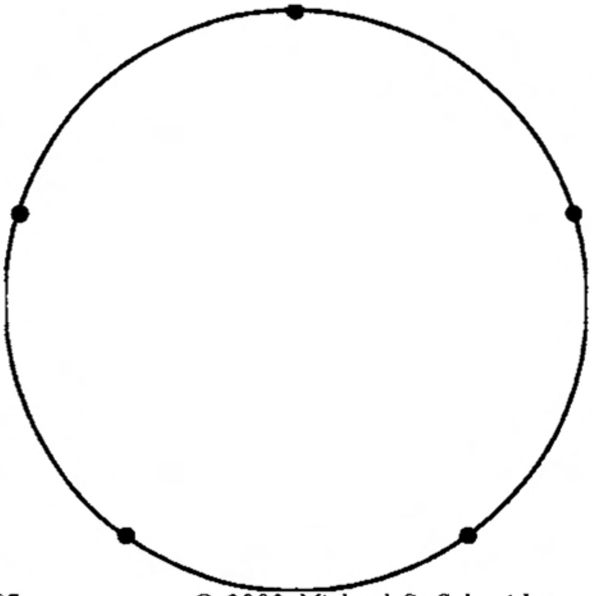


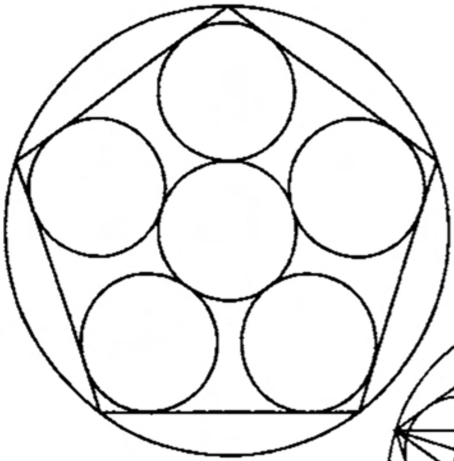


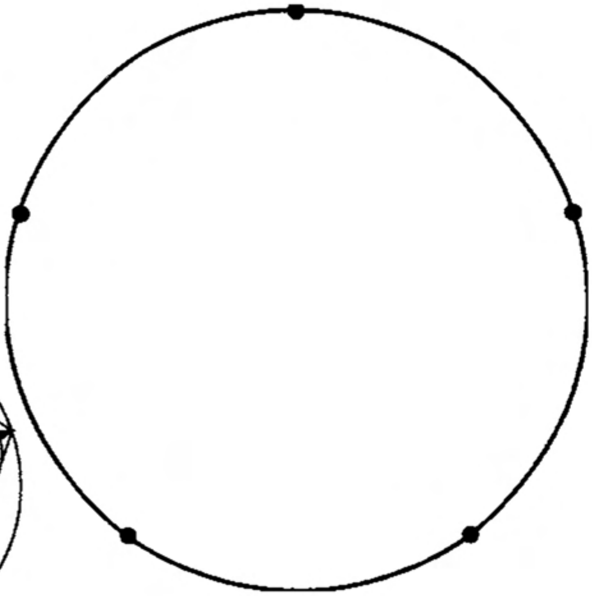


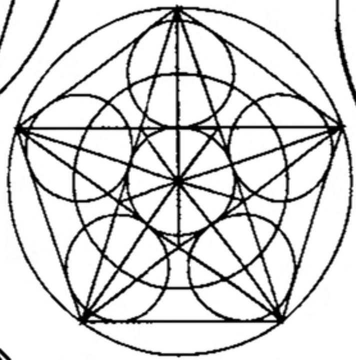


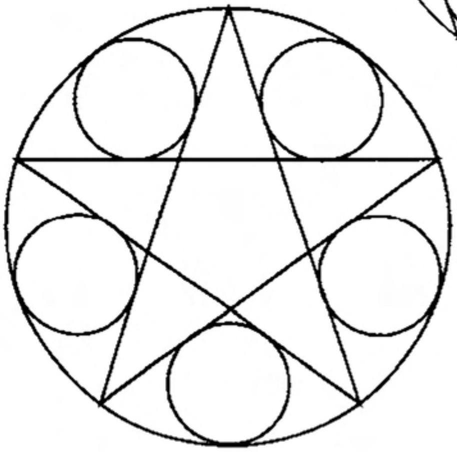


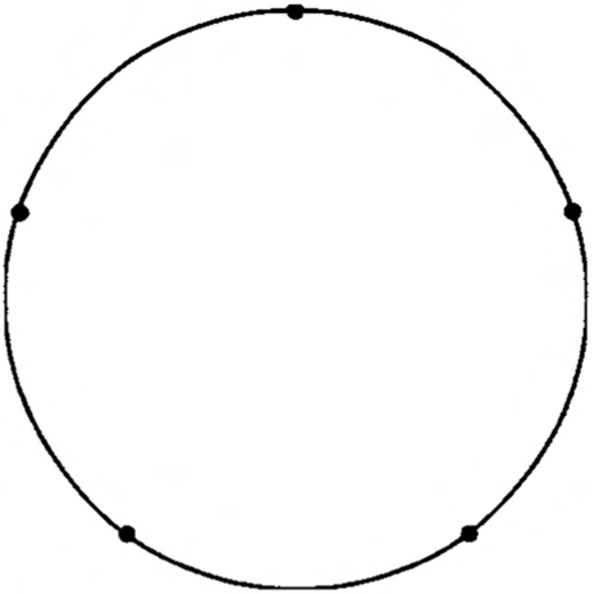


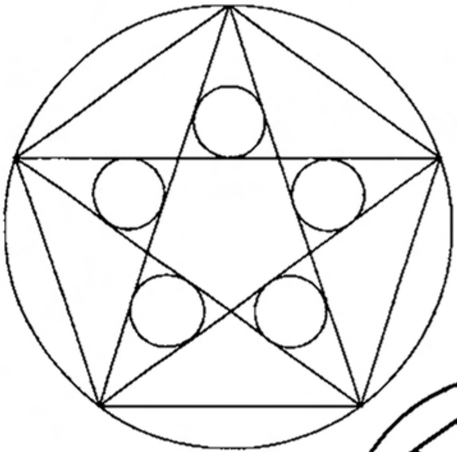


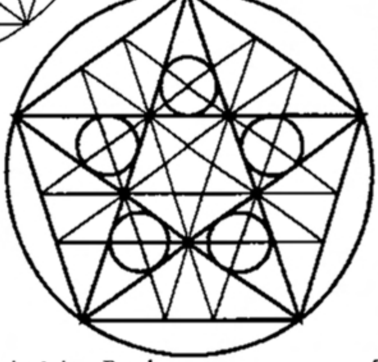


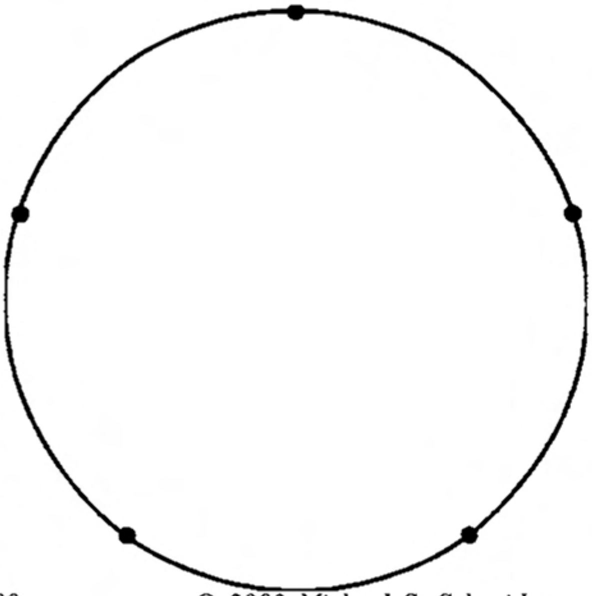


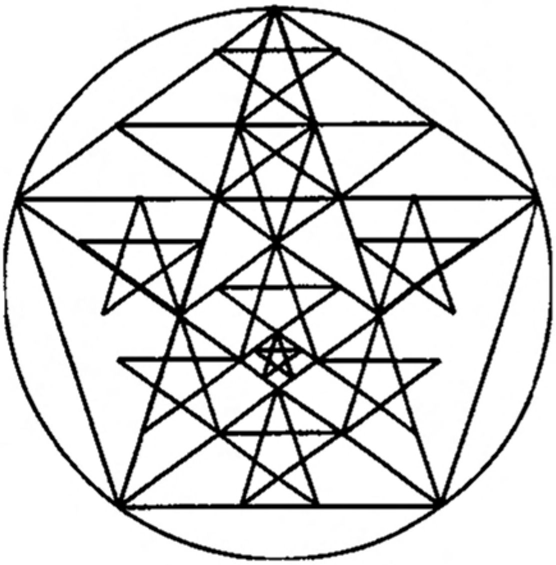


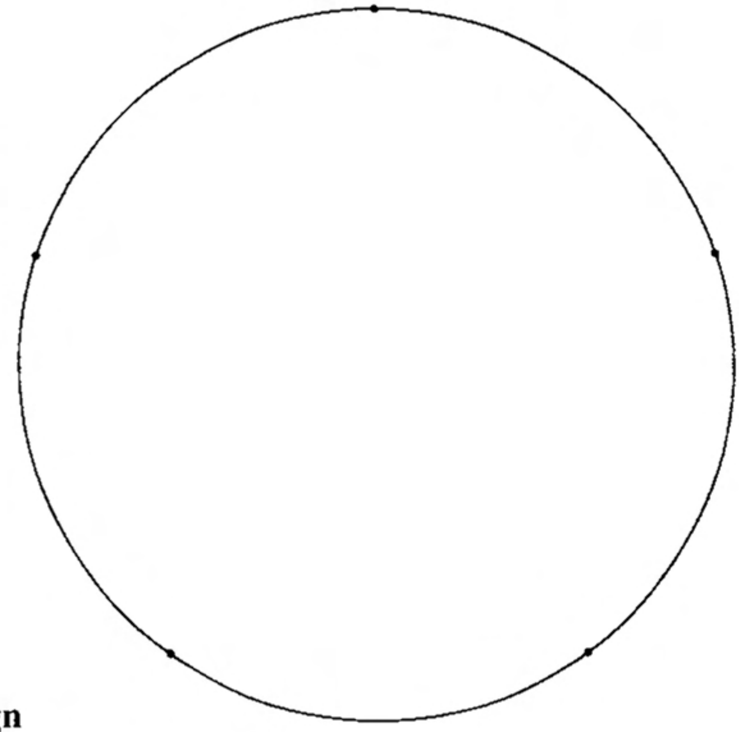




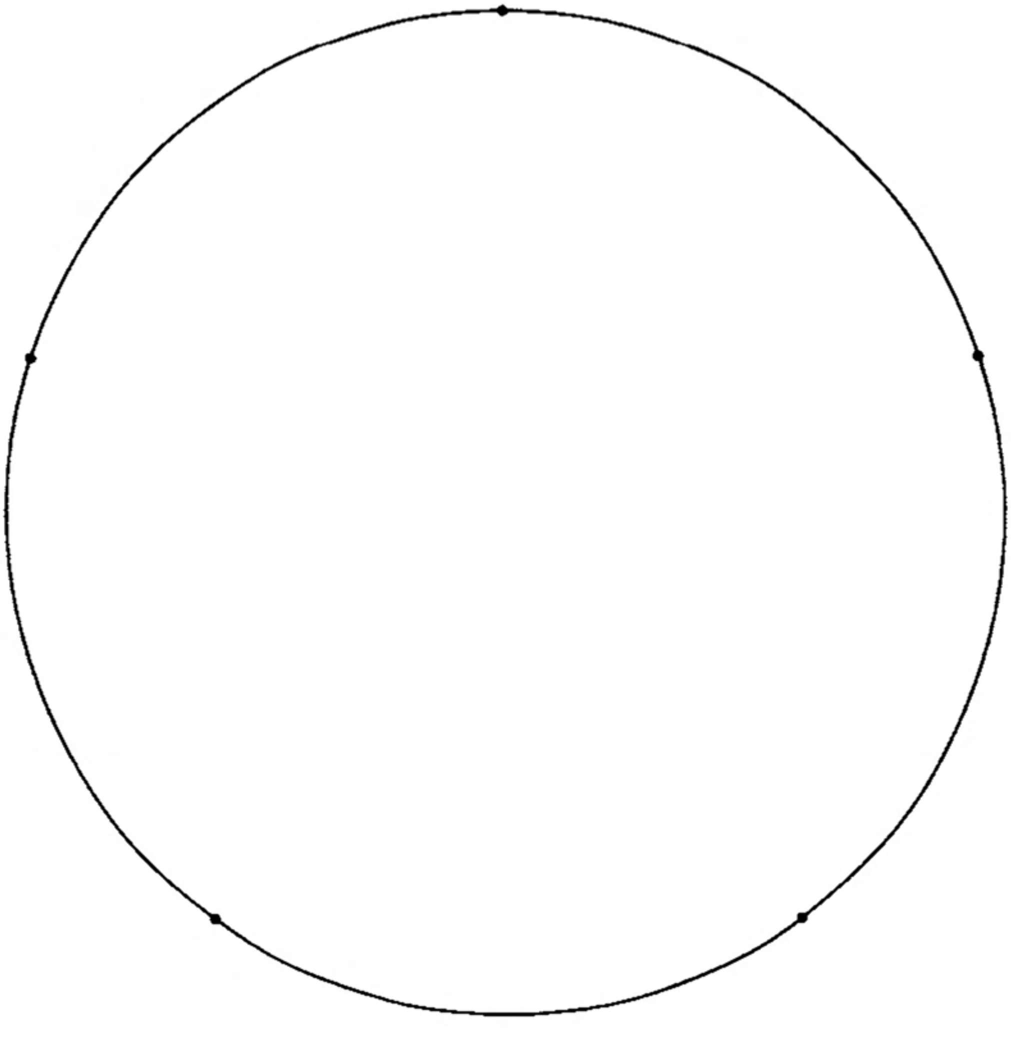




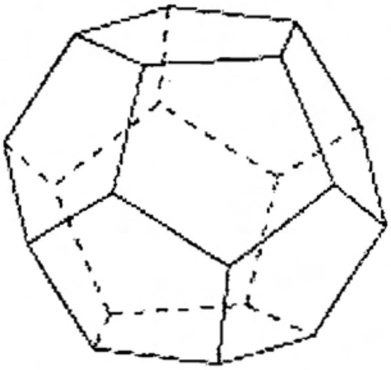




Create Your Own Pentagonal Design

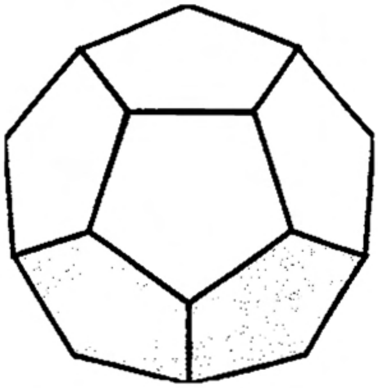


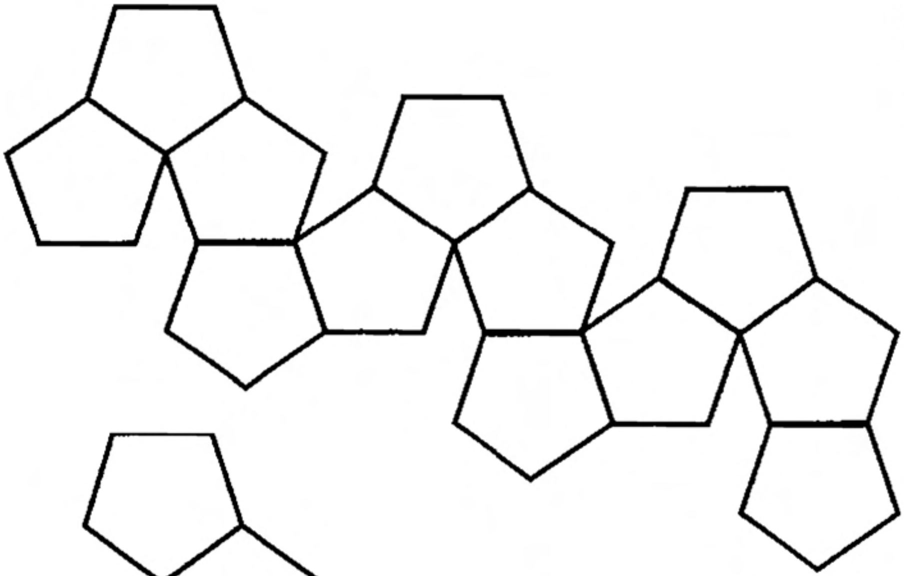
Construct A Dodecahedron With Twelve Pentagons

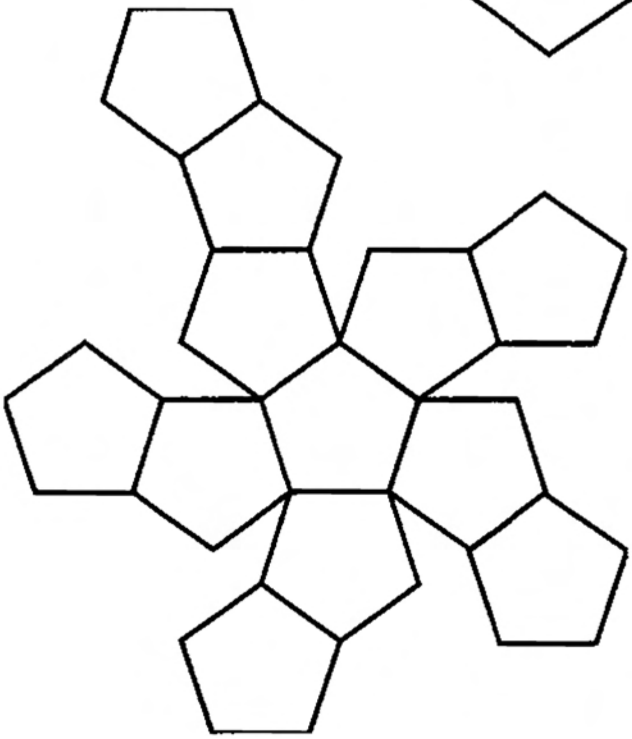


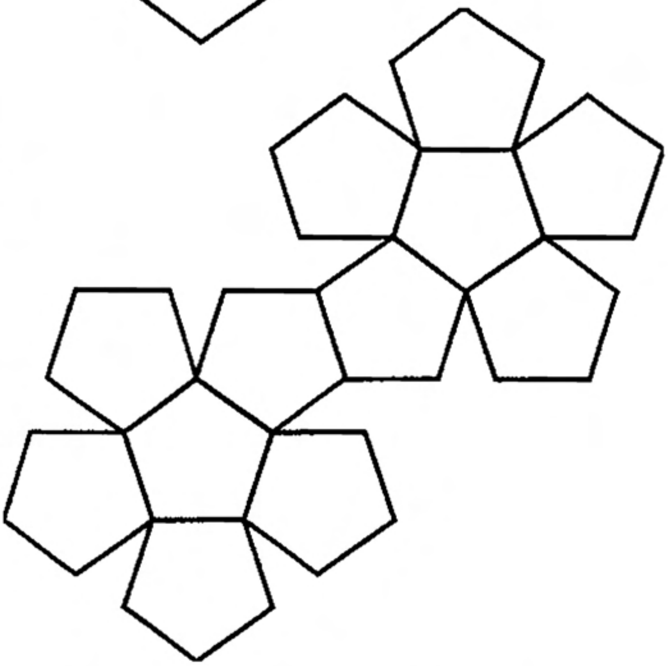
The Dodecahedron (Greek *Do-* = "two" plus *Deka-* = "ten") has twelve faces or seats (*-hedron*) which are regular Pentagons. It is the fifth and final space enclosing form made by one regular polygon. We've seen the Tetrahedron, Octahedron, Icosahedron and Cube. All the Dodecahedron's faces, angles and edge lengths are the same, enclosing three-dimensional space without any overlaps or gaps.

There are many ways to build a Dodecahedron. Three patterns are shown below: a "snake", a "starfish" and a "clam". Just cut out twelve identical regular Pentagons, join them in one of these patterns, and carefully tape the edges together.









Construct A Dodecahedron With Two Pentagons

Another way to build a Dodecahedron can be done with just two Pentagons.

(1) Construct a Pentagon and subdivide it (see page 84) by creating smaller Pentagons around each of the five arms as shown. Small Triangles (shaded) should appear between them. Use this Pentagon as a template to copy another one.

(2) With scissors, make a small cut along *one edge* of each of the small shaded Triangles. The other edge of each Triangle should be folded up.

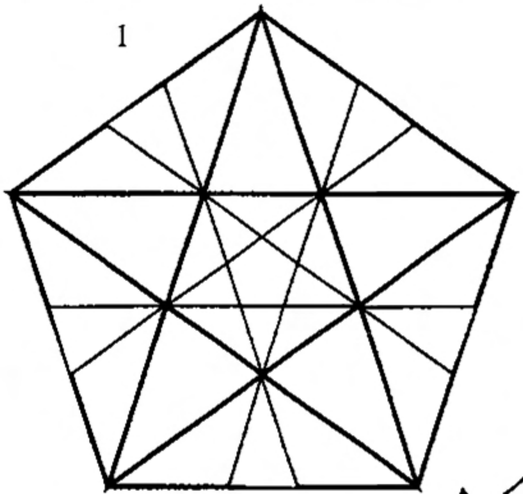
Fold the central dashed edge lines of the inner central Pentagon to fold up the five small Pentagons around it.

Put glue (with a glue stick is easiest) on each of the five shaded Triangles and attach them behind the Pentagon they are each next to. This joins the five small Pentagons around the central one.

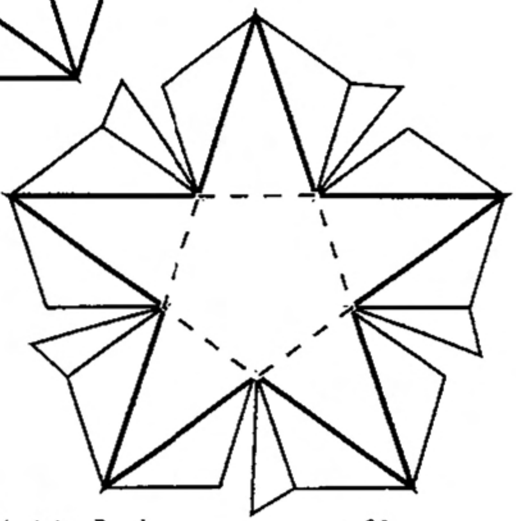
(3) This will make a bowl which is half a Dodecahedron. Make two of these halves and join them together to make one complete Dodecahedron.

Or, twelve of these Pentagonal bowls can be glued back-to-back, three at a time, to build one large Dodecahedron whose faces are twelve bowls.

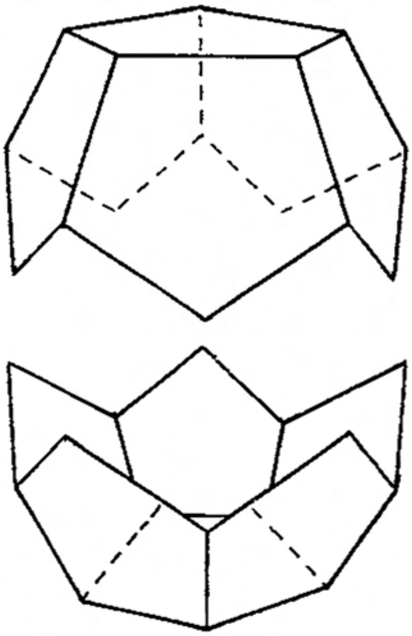
1



2



3



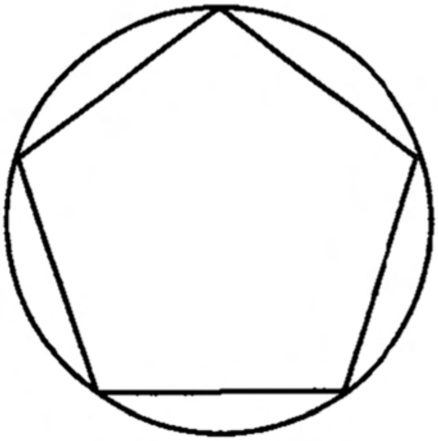
Construct A Dodecahedron With Twelve Circles

(1) Still another method for building a Dodecahedron comes about by inscribing a Pentagon in a Circle on sturdy paper or poster board. Cut around the Circle and use it as a template to make twelve.

(2) Cut slots halfway along each of the five edges of the Pentagon, all in the same direction (along the solid line segments in the illustration).

(3) The twelve Circles will slot together to build a Dodecahedron which appears to have been inscribed in a sphere.

1



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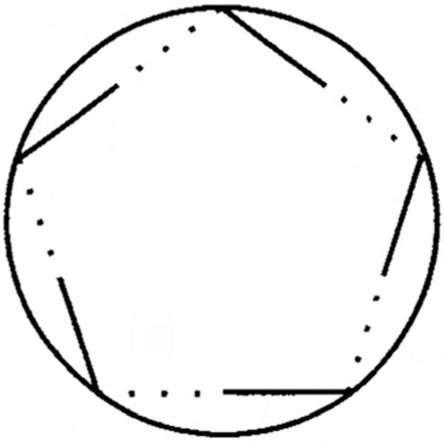

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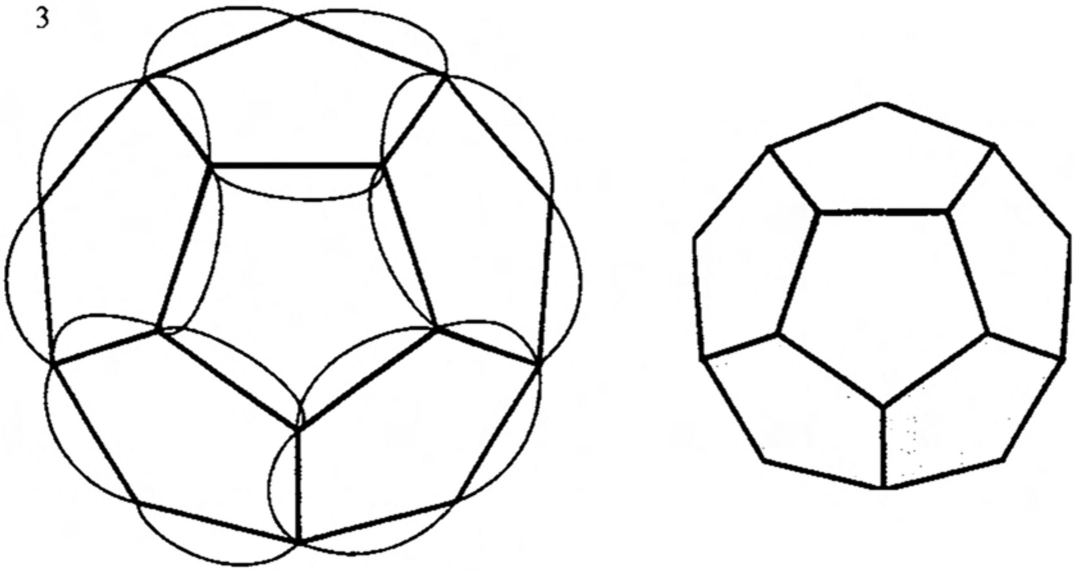
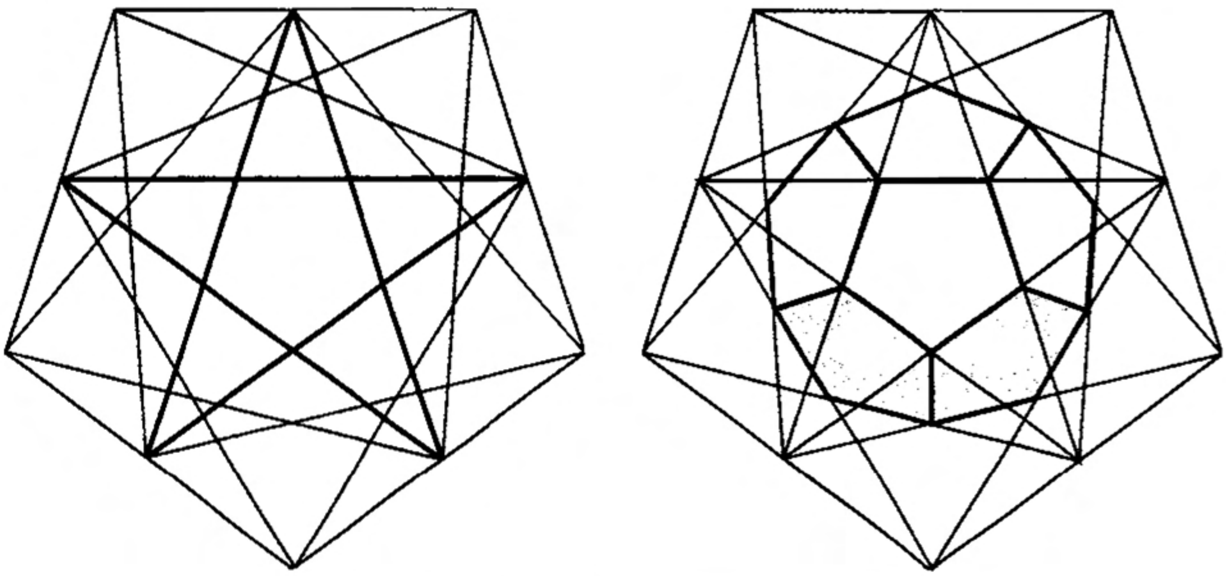
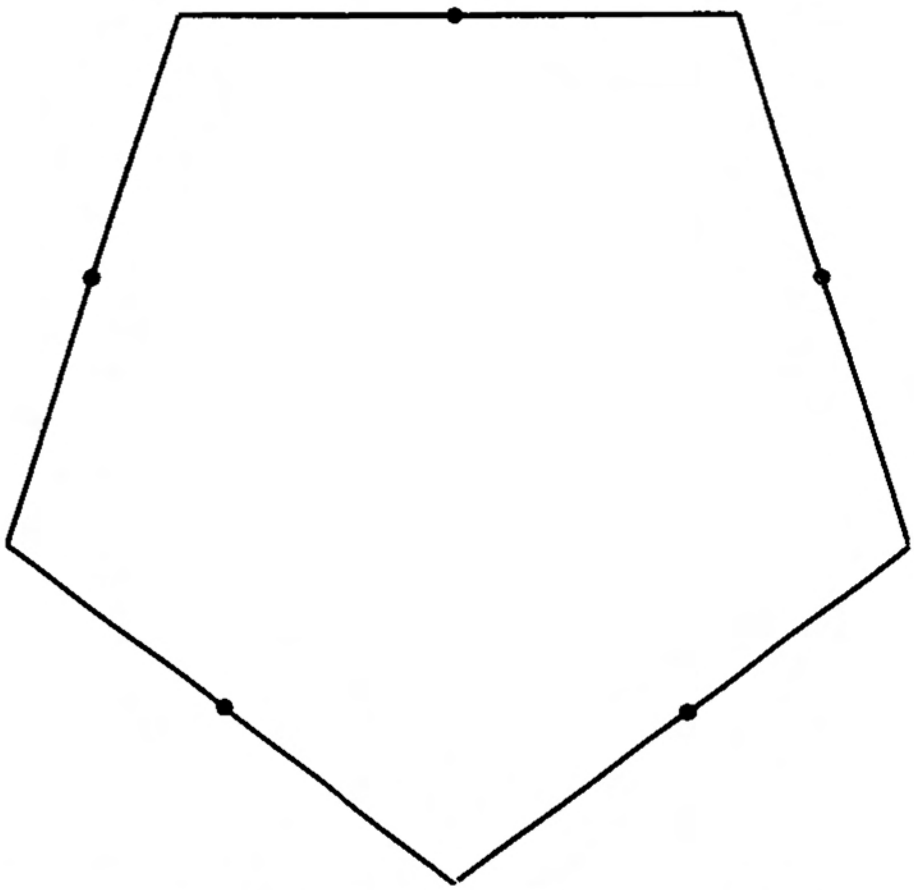


Draw A Dodecahedron

Start with a regular Pentagon and replicate this construction. The midpoints of the Pentagon's sides are given, although you could have bisected each side with an Almond (page 9).

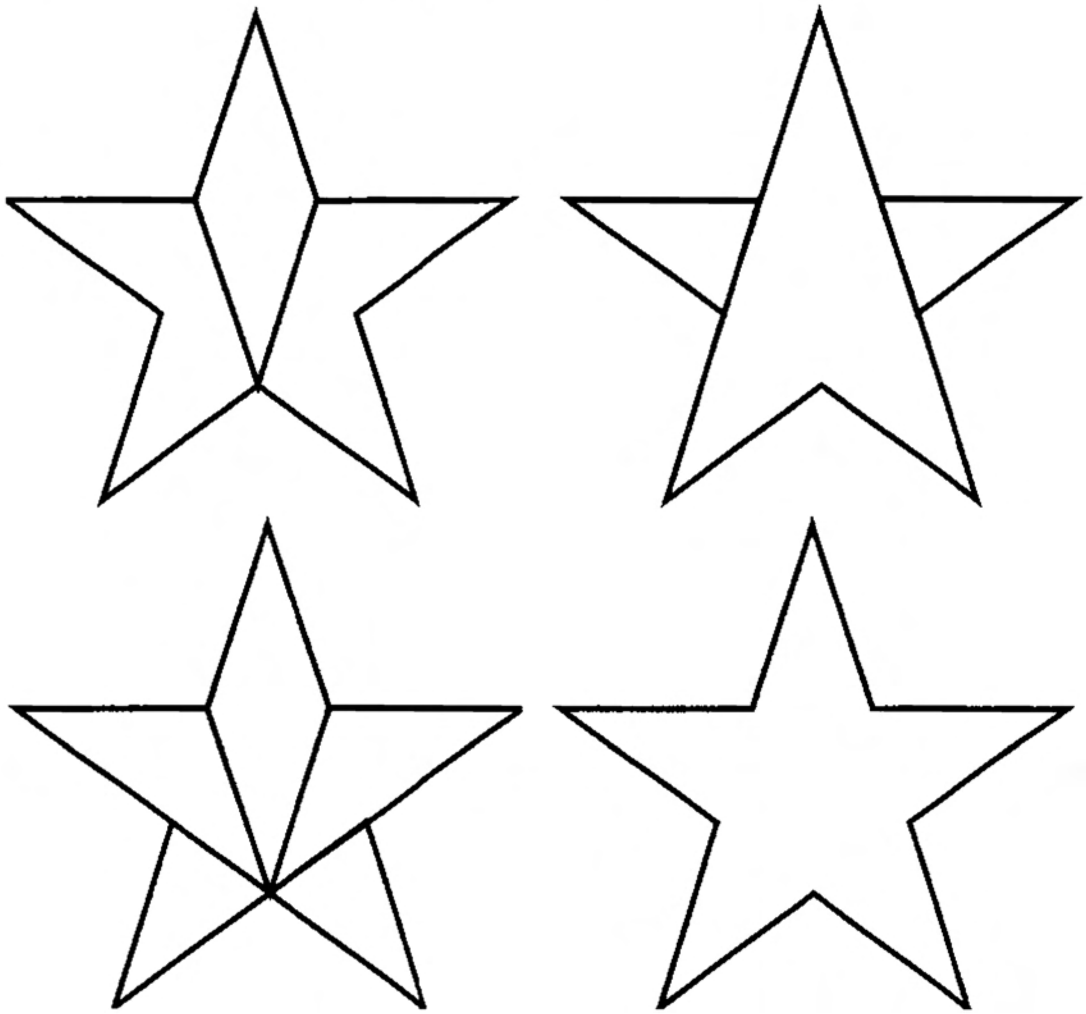
Shade six Pentagonal faces (as shown) to reveal the proportions of a Dodecahedron hidden within.



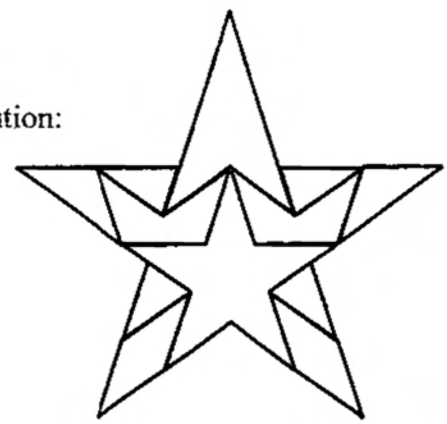


A Pentagram Dissection

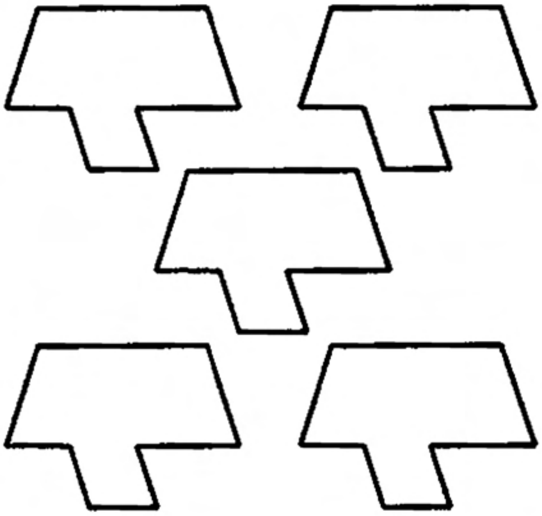
Construct on a sheet of blank paper, or trace, these four Pentagram stars. Cut them along the lines as shown into twelve pieces. They can be rearranged into one large Pentagram star. Try to solve it before studying the solution.



Solution:



A Five Piece Pentagon Puzzle

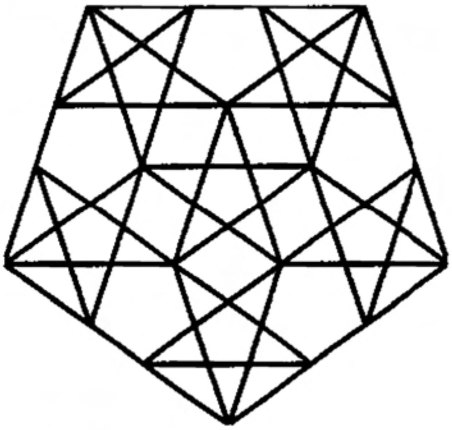


By subdividing a Pentagon into smaller Pentagram stars you can make a puzzle of five identical pieces which fit together to make a Pentagon surrounding a five-pointed Star. You can trace them, but it's better to make them by constructing the geometry.

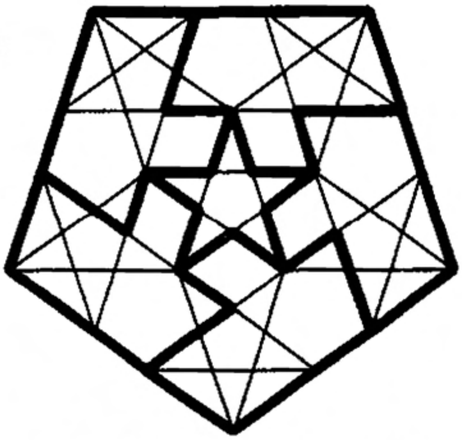
(1) Construct and subdivide a Pentagon into smaller Pentagram stars like this (page 84).

(2) Darken certain line segments as shown here. Carefully cut along these dark lines to have the five identical pieces. (You may or may not wish to include the center star as a sixth piece.)

1



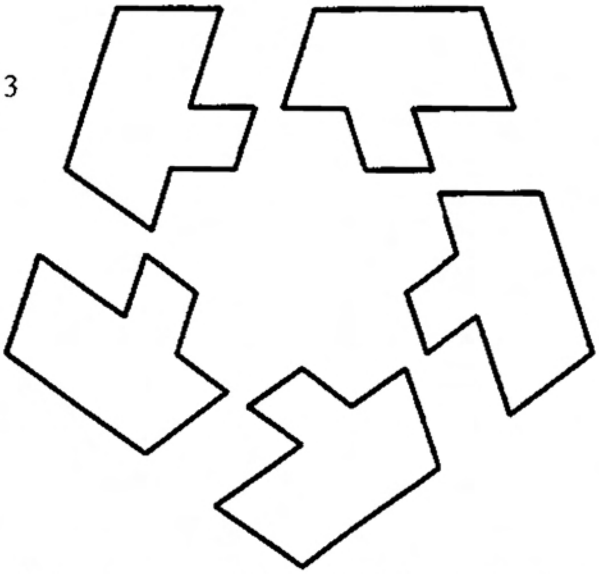
2



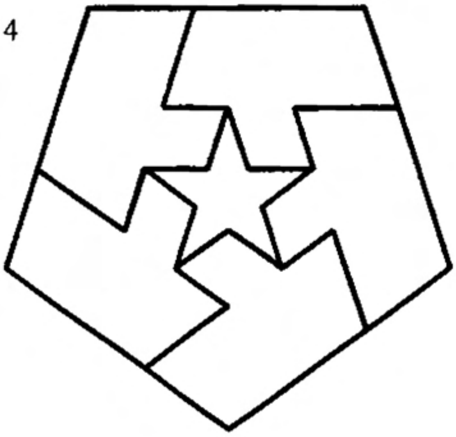
(3) Ask someone if they can arrange the pieces to make a Pentagon and five pointed star. The solution starts by placing them around an invisible center, as shown.

(4) When they all come together a star appears at their center.

3



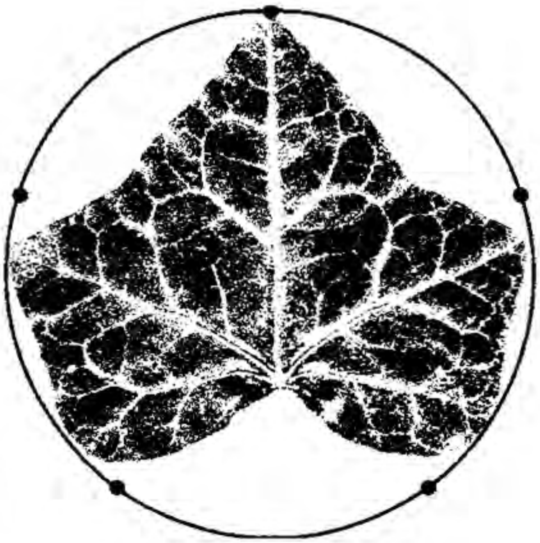
4

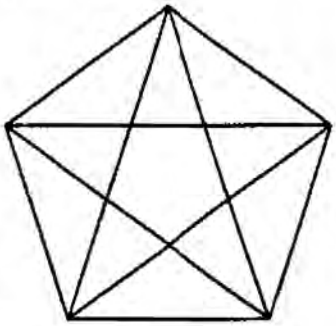


We'll build further upon this construction in Chapter 10 (page 184).

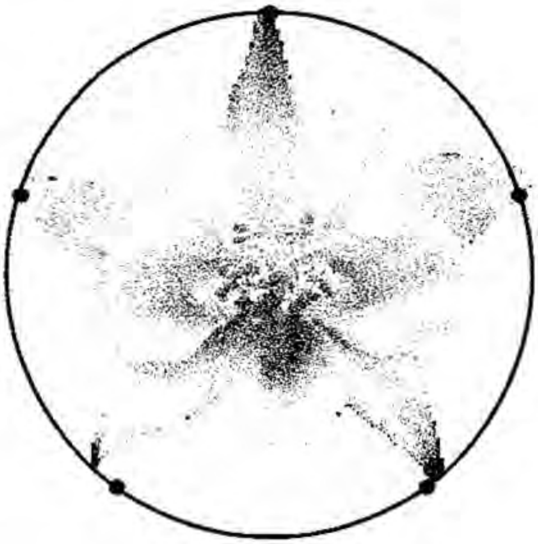
Pentagonal Forms Of Nature

Nature is filled with Pentagonal designs. They're easy to see once you know to look for them. A Circle with five equally-spaced points is constructed around each image. Replicate the geometry next to them to glimpse their Pentagonal pattern. Notice how the parts of each natural form align with your geometric construction.

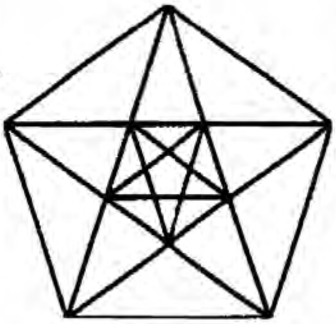


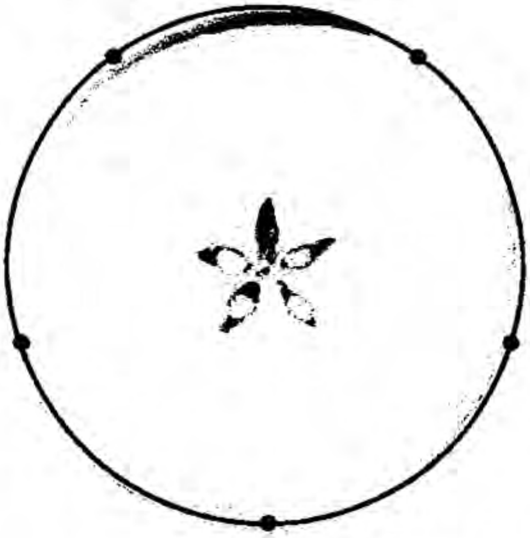


Ivy Leaf

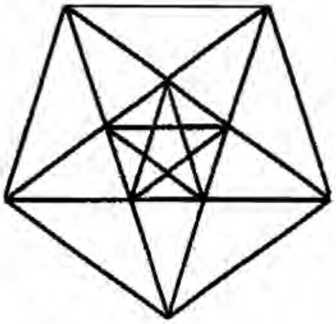


Columbine Flower

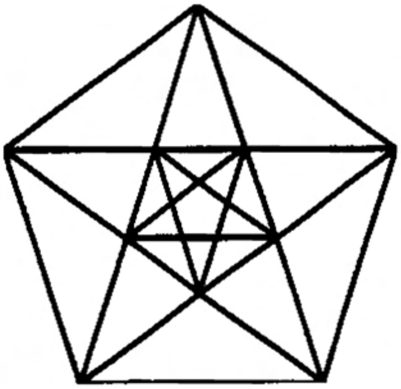


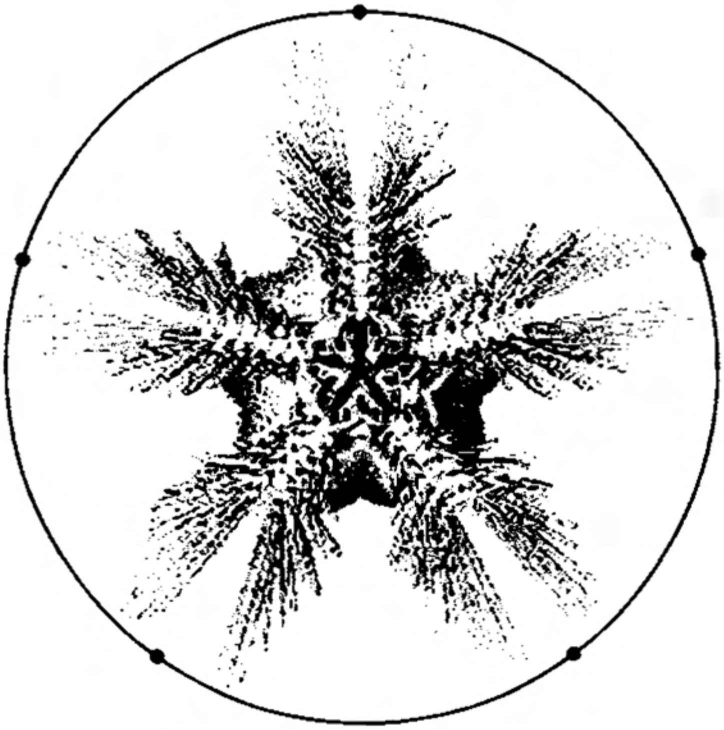


Apple slice

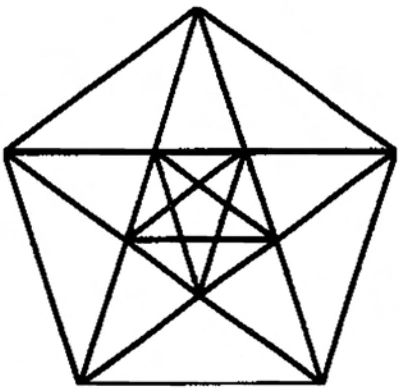


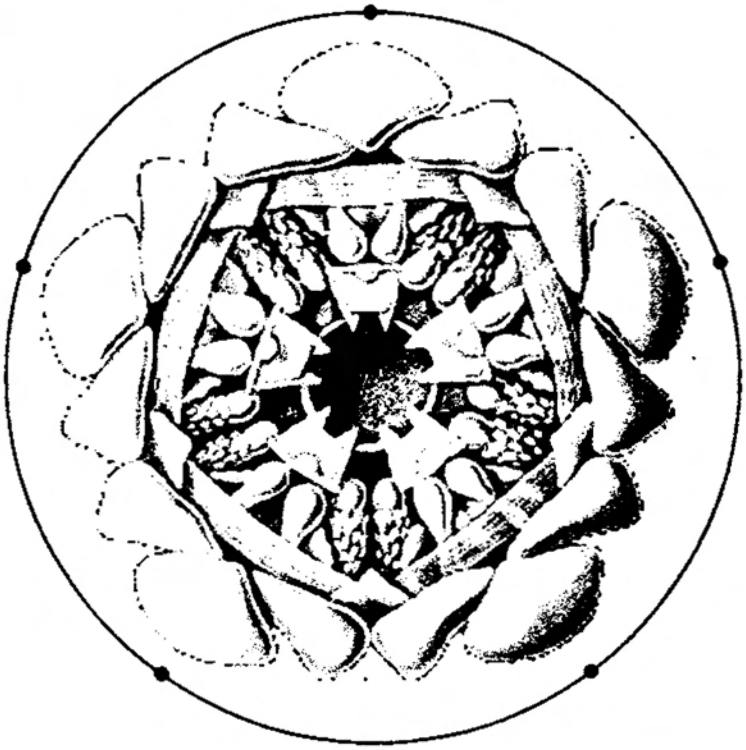
Starfish



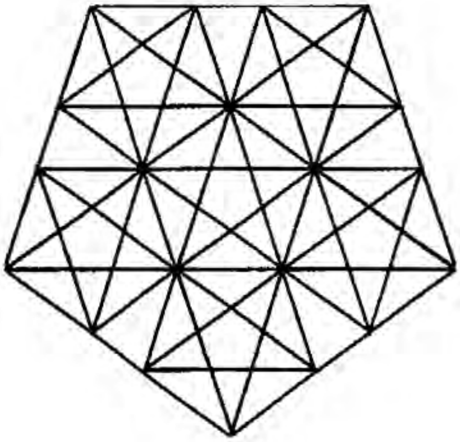


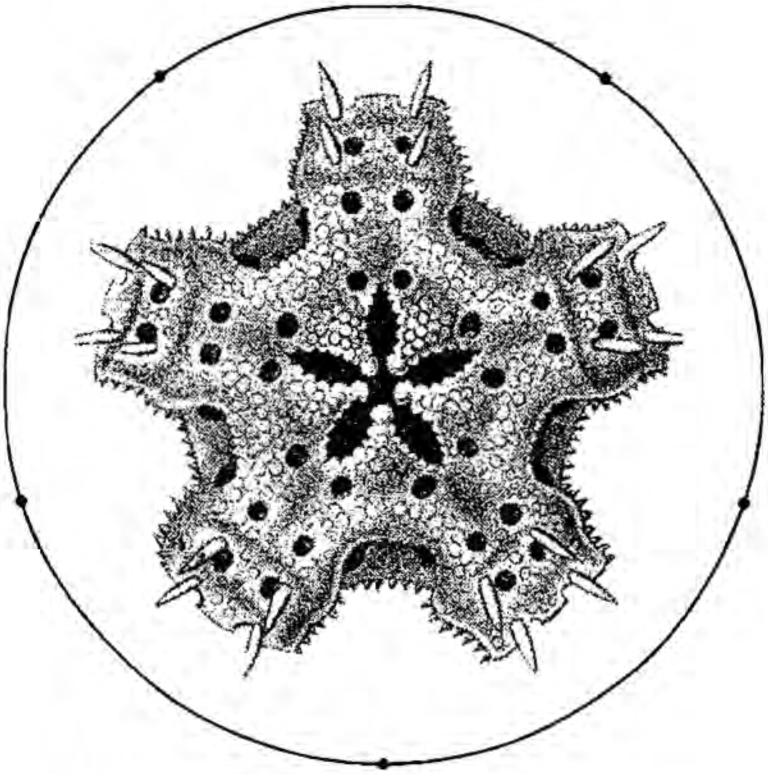
Starfish Mouth
(close-up)



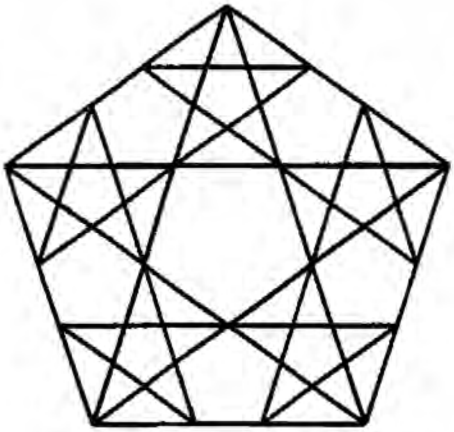


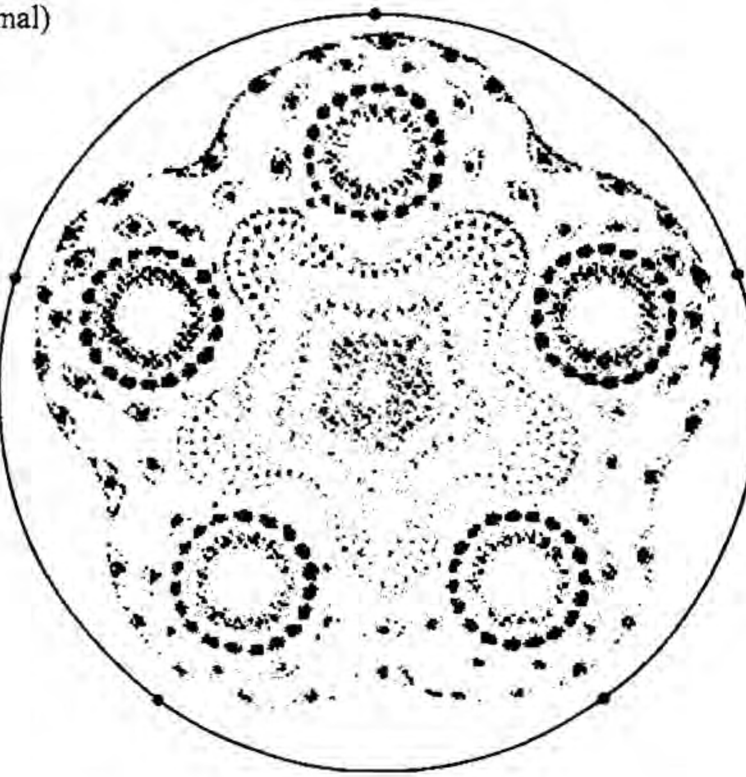
Starfish



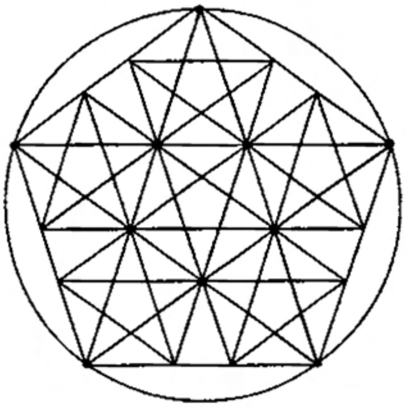


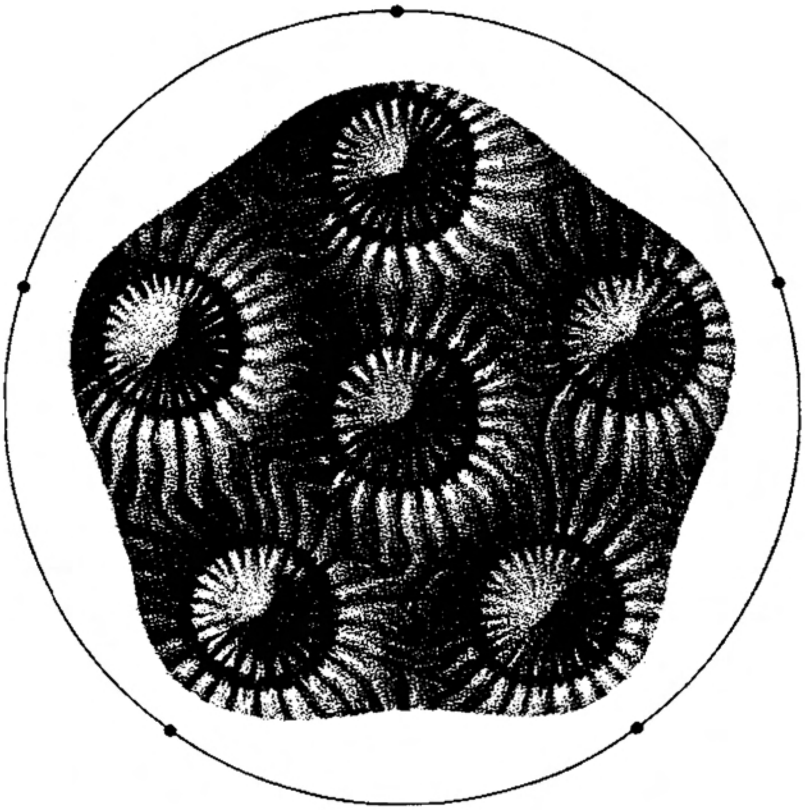
Cross section of a **Sea Cucumber** (an animal)





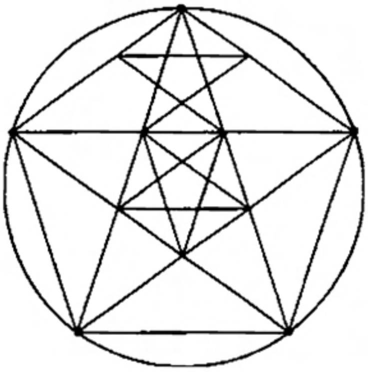
Coral





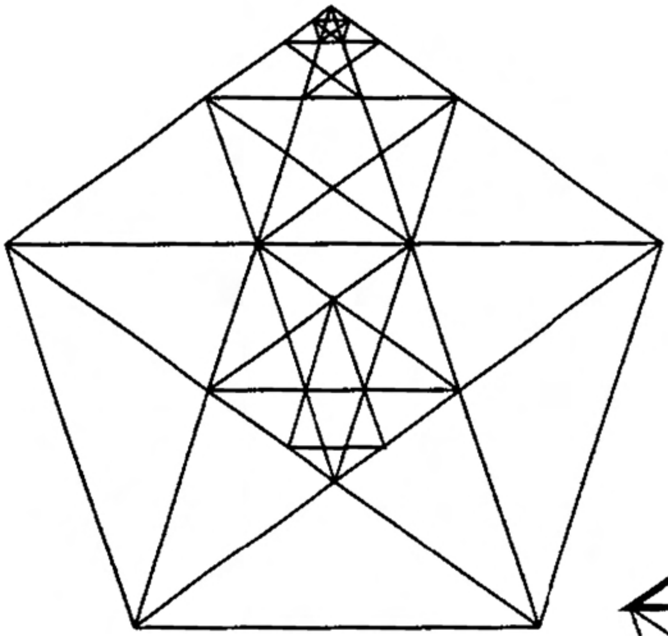
Faces

The faces of many animals can be understood through Pentagonal geometry.



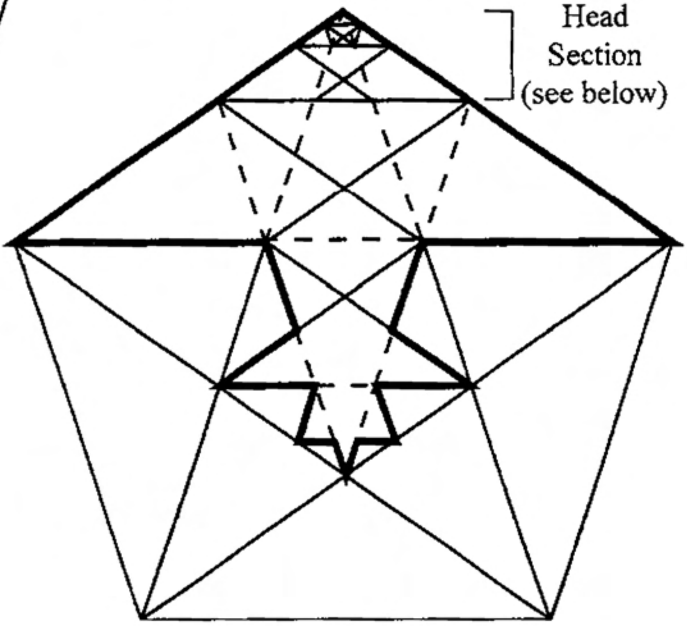
A Pentagonal Bird Pattern

It is astonishing to realize how many living forms display Pentagonal geometry. This can easily be found in the structure of a bird's body. To see it, start with a Pentagon subdivided like this (see page 84) on a sturdy piece of paper or poster board.



Cut out the part of the design which will become the bird (shaded below).

Upper parts of the head should be folded downward ("mountain fold" -- see below). The dashed lines of the lower part of the body should be folded upward. The wings should be folded in both directions (since the bird flaps them when flying).

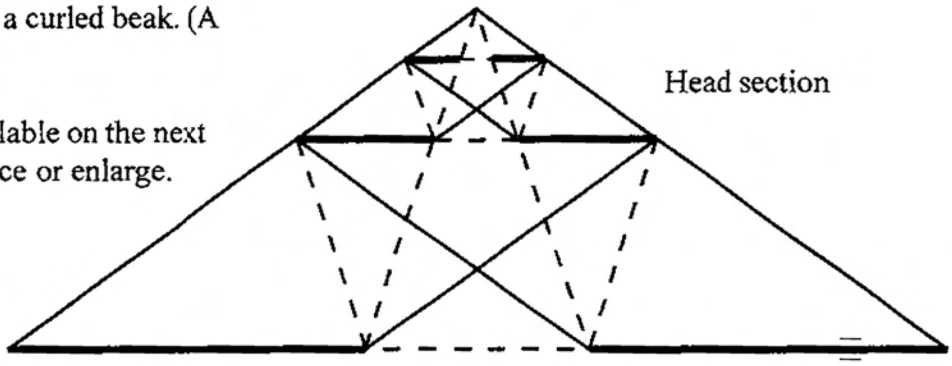


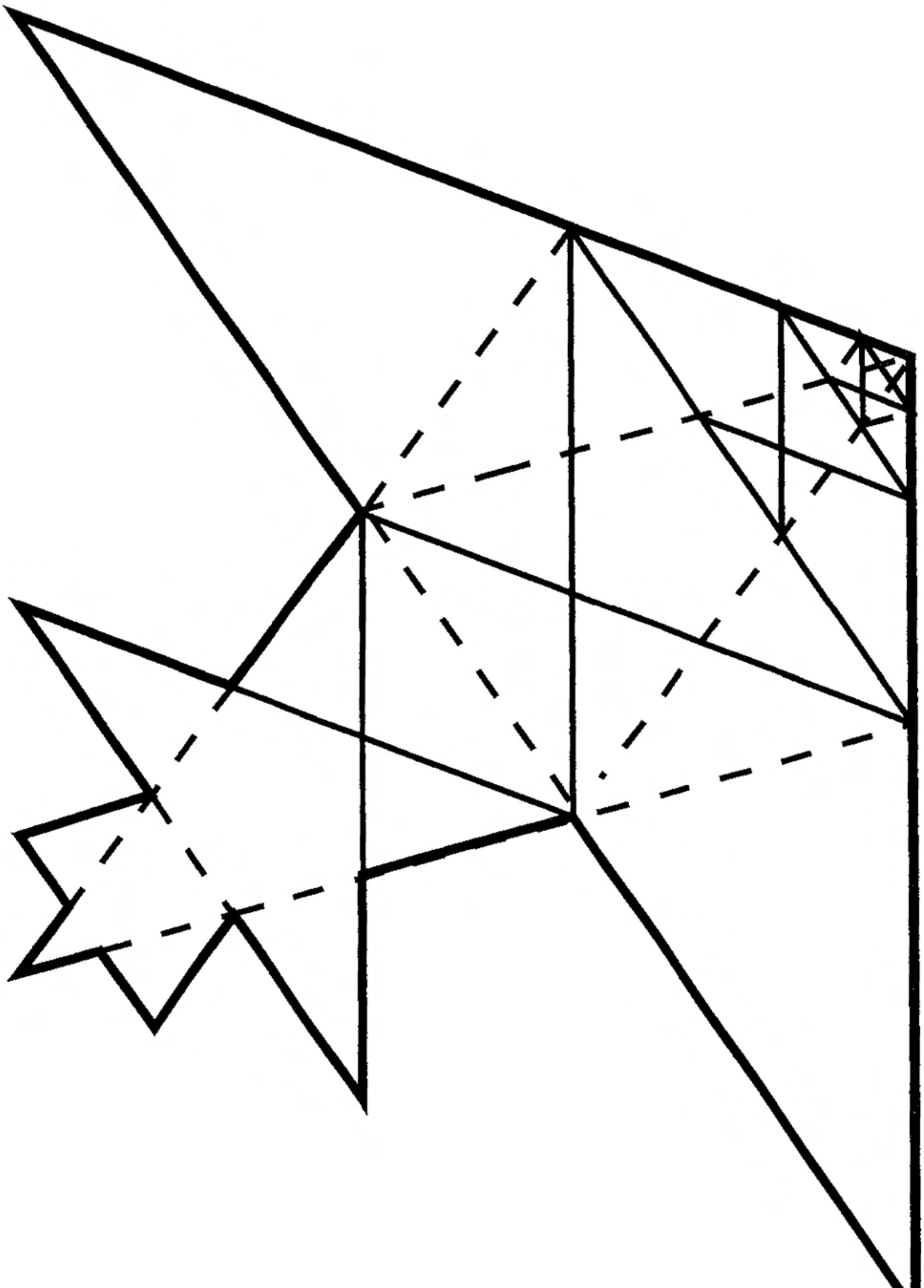
Below is a close-up of the head section. Snip across the thick horizontal lines.

Fold the dashed lines of the opposite pairs of outer Triangles downward. Glue their tips together underneath.

Tuck each smaller section down ito the larger section. This gives the bird a curled beak. (A claw has the same design.)

The whole structure is available on the next page for you to cut out, trace or enlarge.





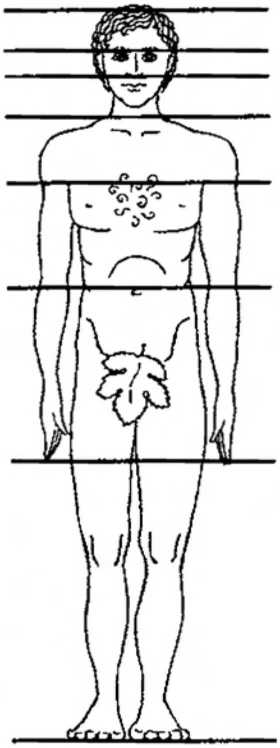
Pentagonal Proportions Of The Human Body

It was commonly believed by the ancient Greeks and others that the human body is not an accidental design but displays balance and beauty because it can be understood as a geometric pattern. The most popular pattern associated with the body's design is called the "Golden Ratio". The Greeks who used it in their sculpture, crafts and architecture simply called it "the section". I've translated its mathematics into a geometric picture not seen before within a Pentagon.

(1) The body can be seen as seven vertical sections between eight levels --

- Crown of Head
- Top of Eyes (eye socket)
- Tip of Nose
- Chin (or bend at neck/shoulders)
- Armpits
- Navel
- Reach of Hands at side
- Soles of Feet

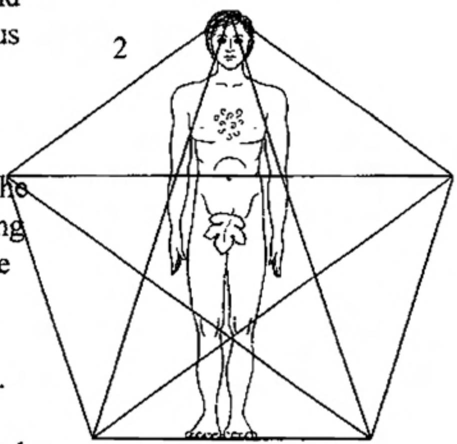
1



Man drawn by Libby Reid

(2) If we start with the body standing in a Pentagon and a Pentagram star inscribed in it, we already see some of these levels -- the Crown and Soles, of course. And the horizontal line of the Pentagram shows us the level of the Navel.

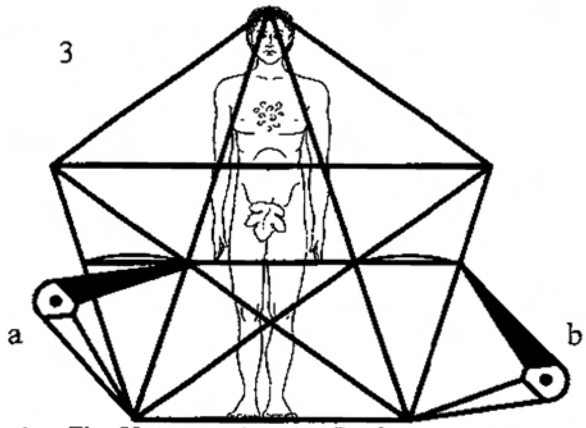
2



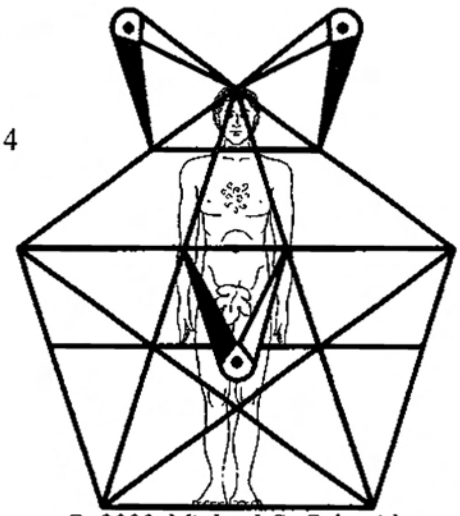
(3) Each of these eight important levels of the body can be found within the measures of the Pentagon and subdivided Pentagram. The downward reach of the hands and fingertips can be found by placing the point of the compass at the Pentagram's bottom corner, and the scribe open to a corner of the central Pentagon (position a). Arcs drawn from there to the sides of the Pentagon (position b) make points which can be connected to show the reach to our fingertips.

(4) The remaining levels can be found from within the Pentagon and marked on its side, creating smaller Pentagons and Pentagrams by the method seen on page 84.

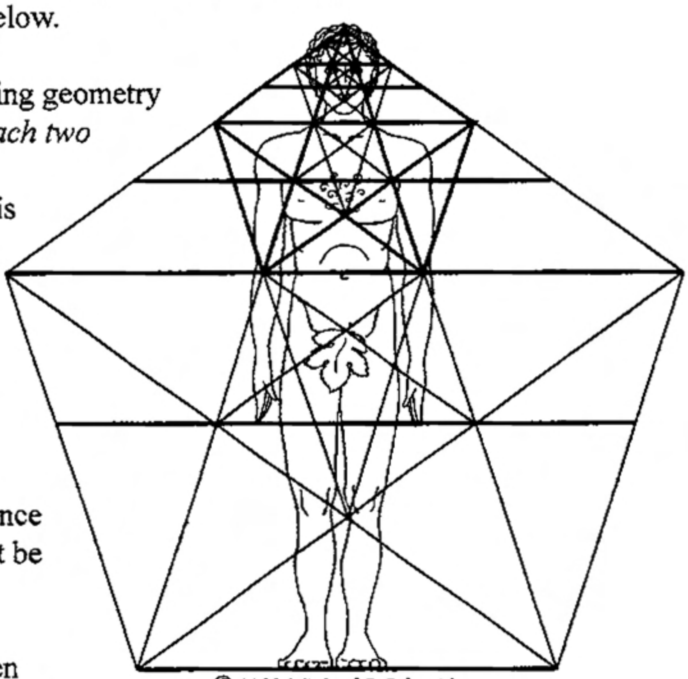
3



4



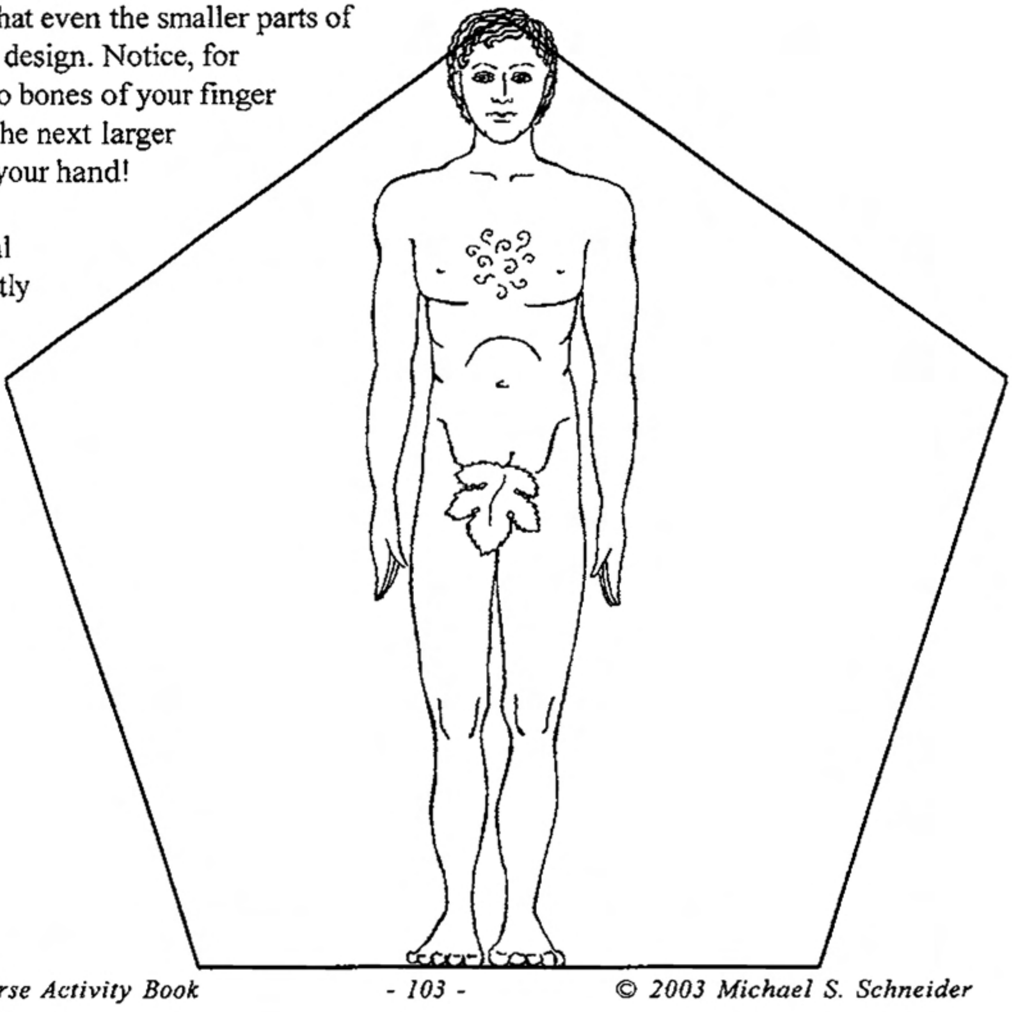
This diagram shows all eight levels of the human body within the Pentagon and Pentagram. Each of the levels can be measured from somewhere *within* this geometry. Explore this picture of the human in the Pentagon with your compass to find all the levels. Then replicate the scheme below.


© 2003 Michael S. Schneider

This design of the body makes use of the interesting geometry of the Pentagon. First, notice that the height of *each two levels added together*, beginning at the eyes downward, *equal the height of the next level*. This is a property of the Golden Ratio ideal, where each level is also 1.618... times taller than the previous one. The completed picture can be cut out and folded along the horizontal levels to form a Golden Spiral which appears in nature as sea shells, whirlpools, and galaxies. Since the human body begins as a spiral embryo, has five fingers on each hand and five senses, the appearance of the spiral and Pentagonal symmetry should not be surprising.

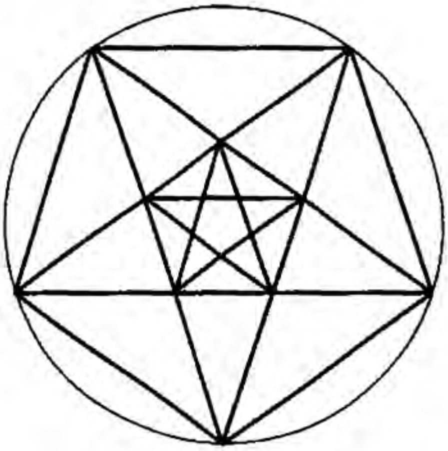
In addition to these seven levels relating in Golden Ratio design, each level may be further subdivided with Pentagrams to reveal that even the smaller parts of our body maintain this design. Notice, for example, how each two bones of your finger add together to make the next larger bone, all the way into your hand!

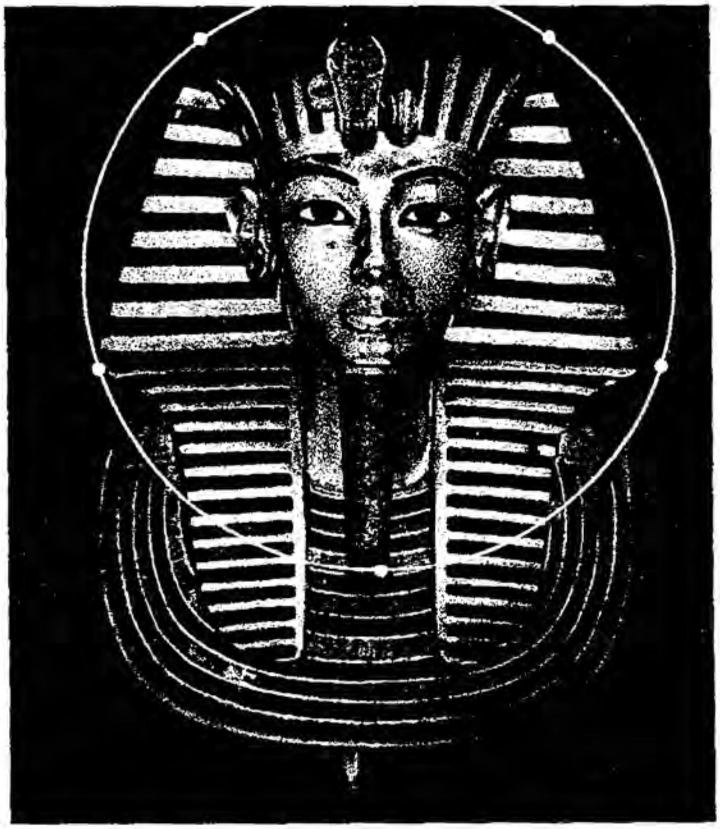
This represents an ideal design. No one is exactly like it. But you might measure yourself and your friends to see how close you come. What about the average of many people?



Pentagonal Design In Arts, Crafts And Architecture

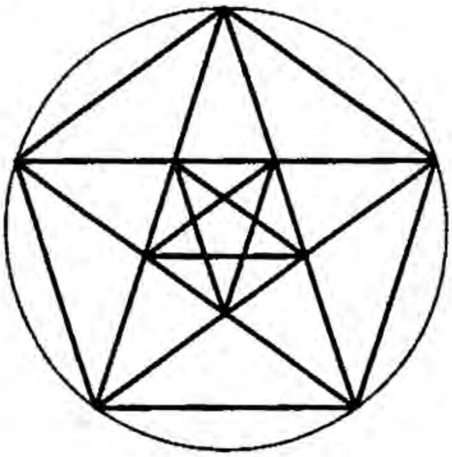
Many works of art worldwide seem to have been designed using Pentagonal geometry. Each of these pictures has a Circle with five equally spaced points around it for you to replicate the geometric scheme seen next to it. Look carefully to see how the geometry guides the parts of the object or scene.



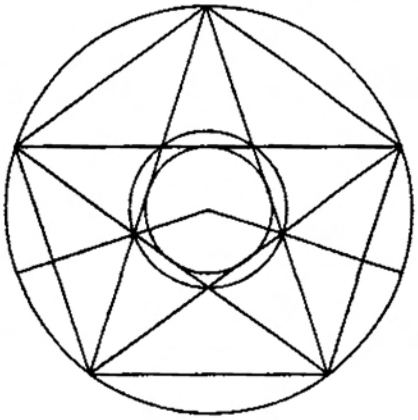


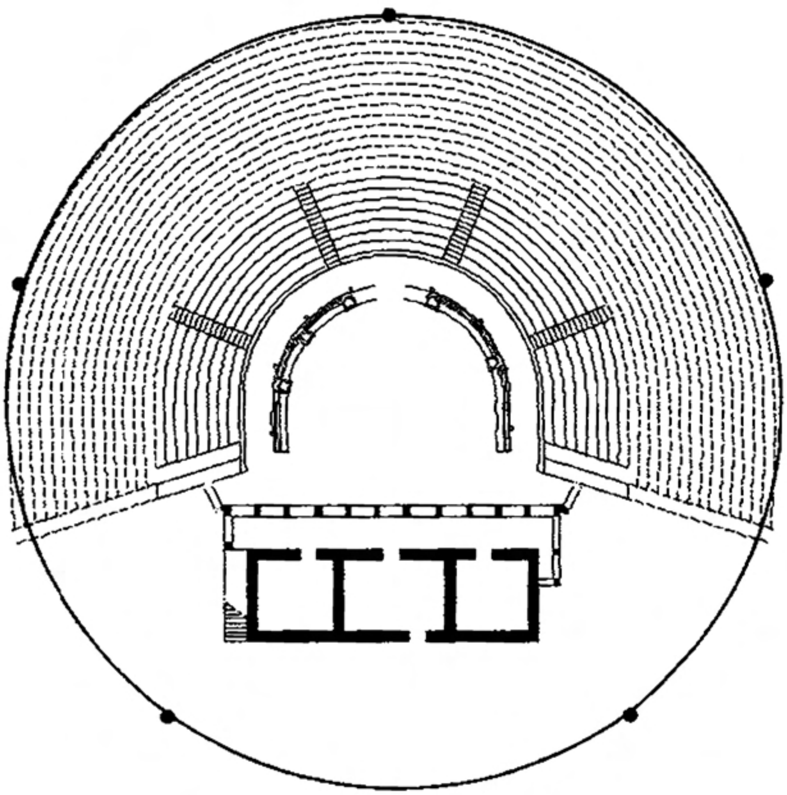
Gold mummy mask of Pharaoh Tut-Ankh-Amen

The Alba Madonna by Raphael, 1511
National Gallery of Art, Washington

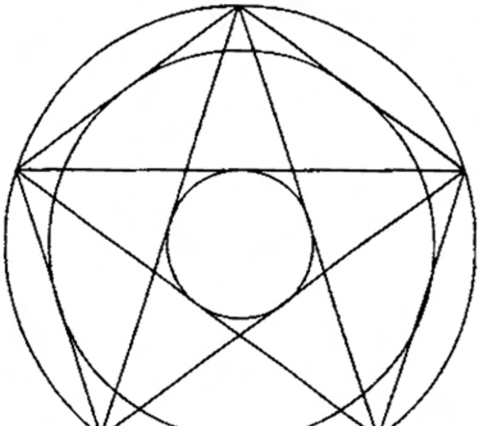


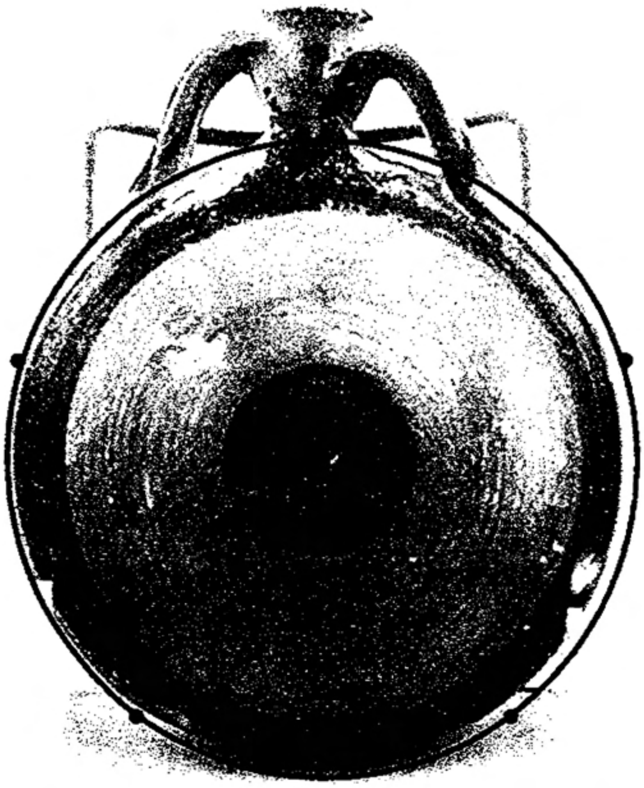
Greek Amphitheater Plan



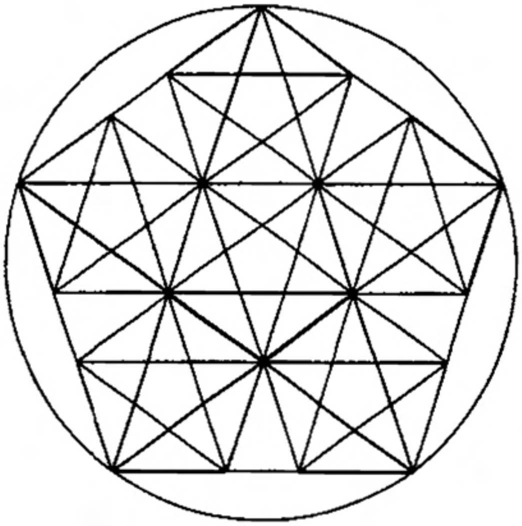


Greek Traveler's Water Canteen





Greek Corinthian Capital



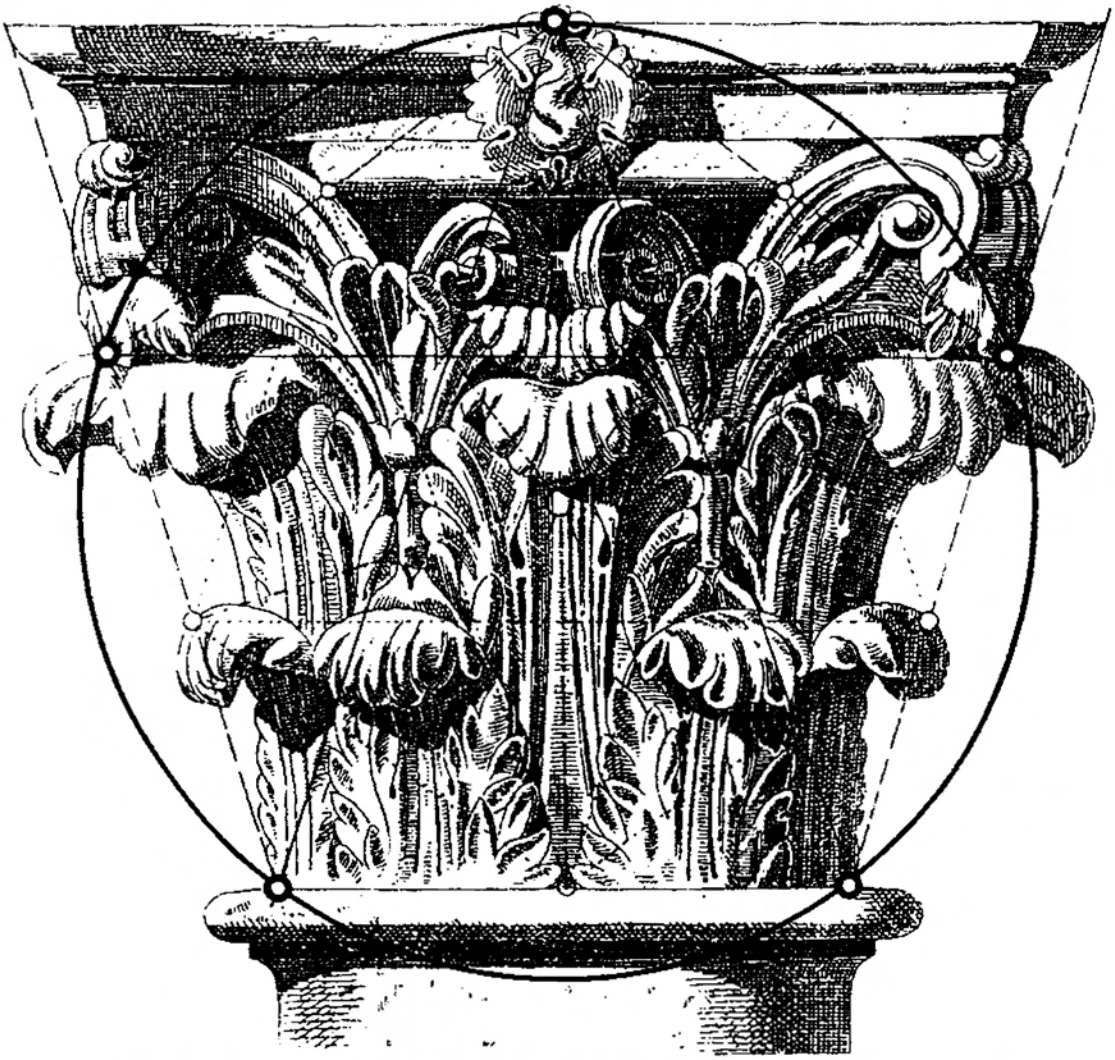


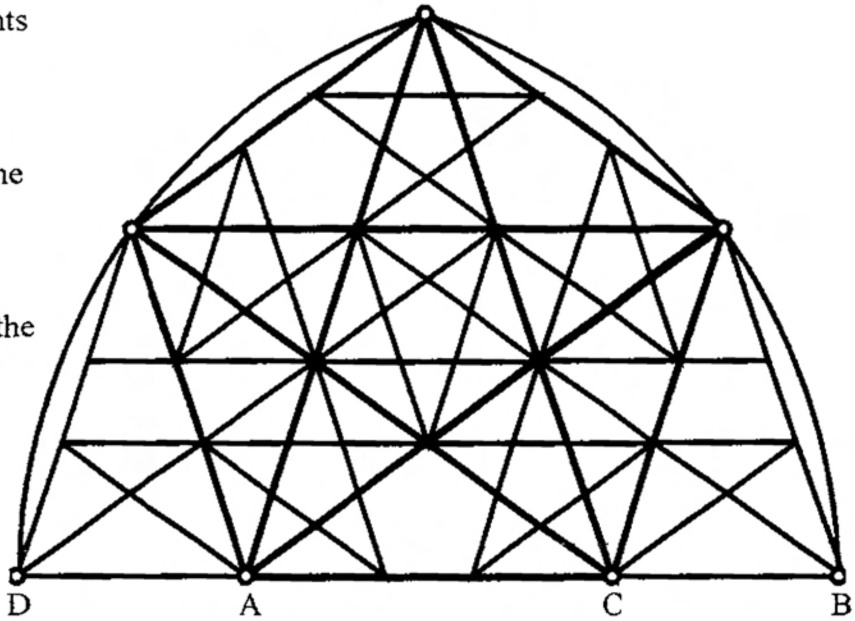
Abb. 115. Kapitäl von der Dominikanerkirche zu Wien
(17. Jahrhundert)

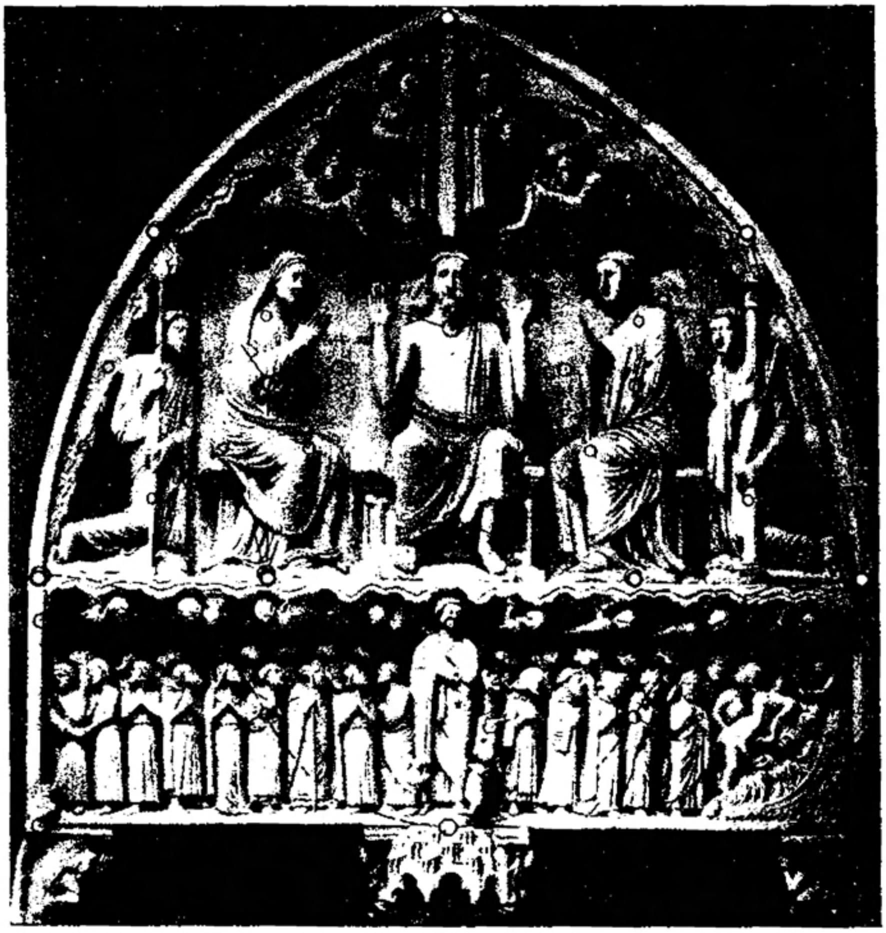
Sculpture From Chartres Cathedral

Draw a Pentagon from the points given and subdivide it into Pentagram stars.

To make the large arcs, place the compass point at point A and open the scribe to point B.

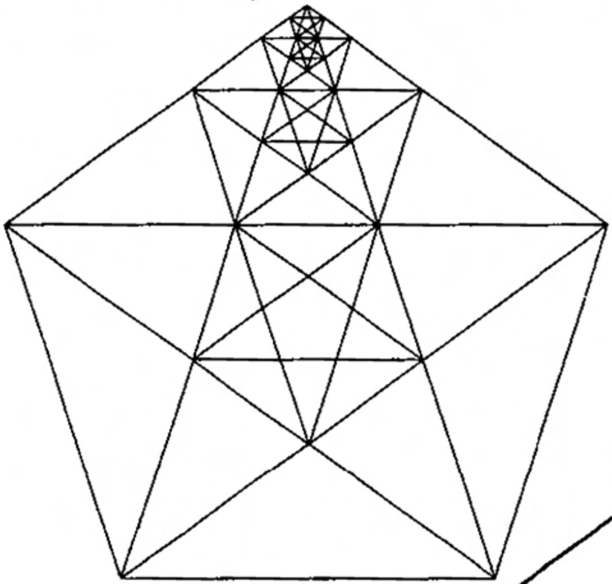
For the other side arc, reverse the compass from C to D.

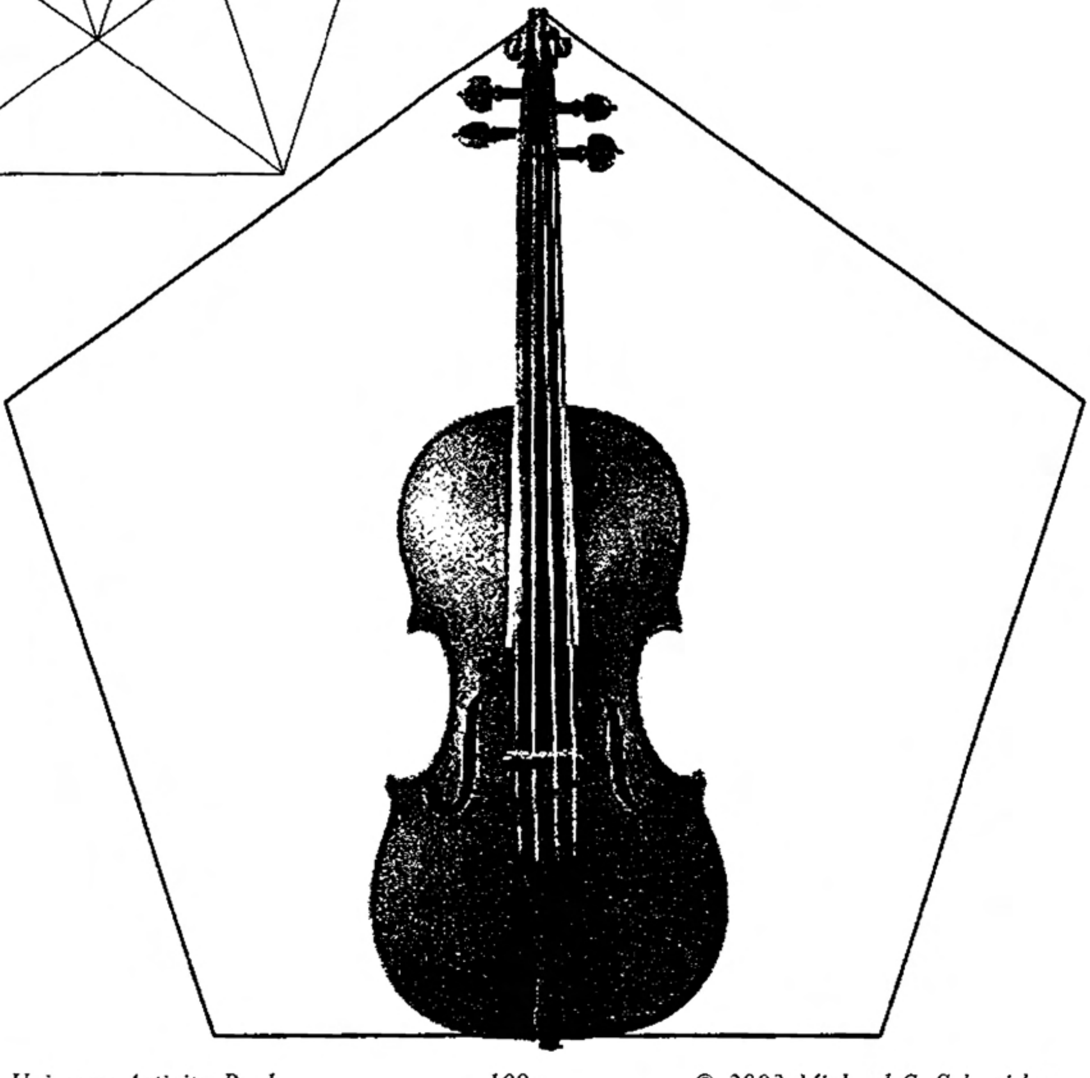




Stradivarius Violin

Violins made by **Antonio Stradivari** (1644-1737) and other masters are still known for the beauty of their sound. Many aspects went into making it successful -- the type of wood chosen, the varnish, skill of the woodworker and many other elements. Of course, the geometric design is very important. No one knows if the proportions of the Pentagon were used, but you can replicate this geometric construction around the violin and judge for yourself whether it is related to the Pentagonal scheme of the human body and other forms of life.



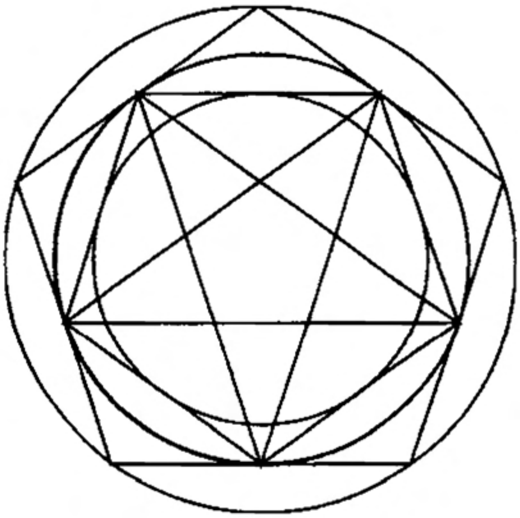


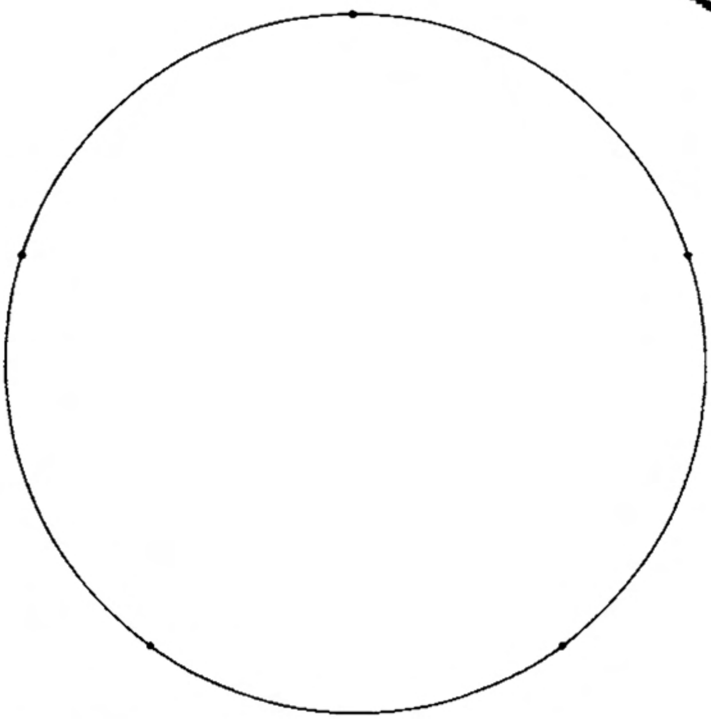
Official Seals

Most official seals, whether of states, counties, offices or countries, are well designed by geometry. Here is one of many examples, which is based on the Pentagon and its proportions.

Replicate the geometry of this Pentagon and Pentagram star to see how it applies to the official seal of Texas, the "Lone Star" state.

The official seals where you live are probably designed by geometry too.





Use Pentagonal geometry to design a Seal for where you live.

Design Your Own Pentagonal Art

Use your imagination and these Circles with five equally spaced points to create your own geometric construction. Then use the lines of the construction to guide your original design of art, crafts or architecture.