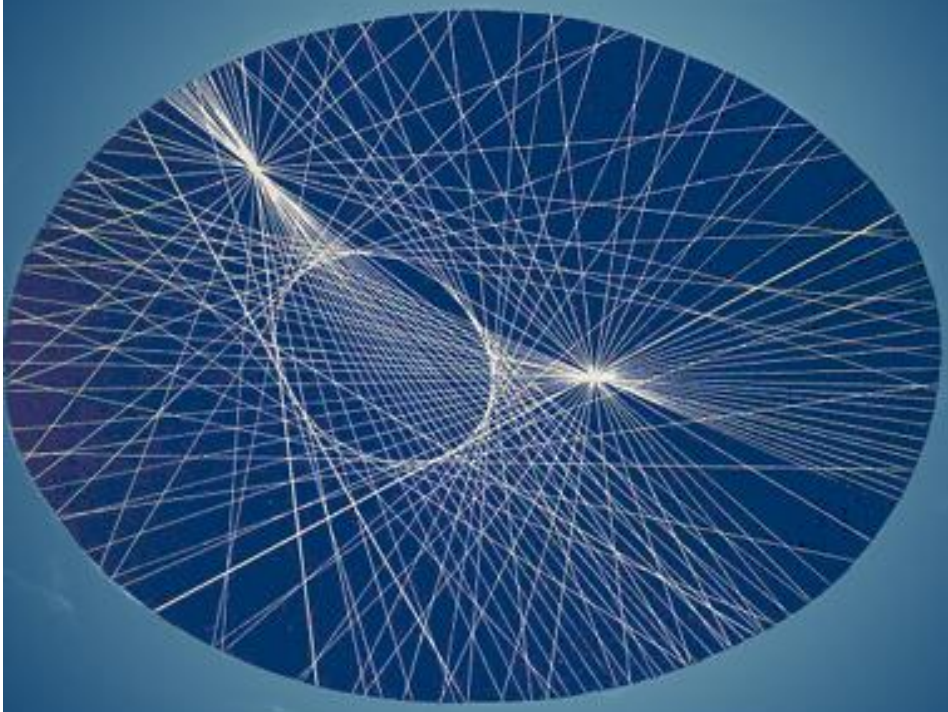
Physical and Ethereal Spaces



George Adams

PHYSICAL AND ETHEREAL SPACES

GEORGE ADAMS M.A. *(Cantab.)*

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Foreword

> Think on it: how the point becomes a sphere and yet remains itself. Hast thou understood how the infinite sphere may be only a point, then come again, for then the Infinite will shine forth for thee in the Finite. (*The original German: ***Sinne nach: wie der Punkt zur Sphare wird und doch er selbst bleibt, Hast du erfasst, wie die unendliche Sphare doch nur Punkt ist, dann komme wieder, denn dann wird dir Unendliches in Endliches scheinen***.)
>
> ***Rudolf Steiner***

George Adams had long hoped to republish his essay of 1933, which appeared (in German) in *Natura,* published by the Goetheanum, Switzerland, and in *Anthroposophy,* journal of the Anthroposophical Society, London and New York. This essay put forward an initial statement of a theory of forces in living nature based on the mathematical conception of space and morphology derived from modern projective geometry. It was, however, only very shortly before his death on the 30th March, 1963, that we began to prepare the new German edition. George Adams intended to include illustrations and annotations and to write a substantial preface, referring not only to his own further work, but also to the work of others - particularly that of Professor Louis Locher-Ernst of Switzerland - in which he would have set this work in its rightful place in the history of scientific thought and endeavour.

I have not tried to write a preface such as only he, with his deep insight into the scientific spirit of modern time, would have done; I have simply made small revisions in the text, inasmuch as I knew his intentions, and have added illustrations and annotations. *Von dem aetherischen Raume* appeared in its new form in 1964, published by the Verlag Freies Geistesleben, Stuttgart; the present volume, in the English language, is uniform with it.

Researching in physical chemistry at Cambridge at the time of the outbreak of the first world war, George Adams saw that modern physical science and mathematical theory, having shaken the foundations of nineteenth-century materialism, was nevertheless clinging to analytic and atomistic concepts and pursuing a path ever more abstruse and far removed from realms of human experience and life. In his search for a less one-sided, more universal approach, he was led to study a stream of

mathematical thought existing side by side with the analytic and symbolic school of Riemann, to which Einstein and his followers have chiefly turned, namely, modern projective - or as it is sometimes called - synthetic geometry and the higher algebra which is related to it. With the creation of this method in geometry are connected such names as those of the Frenchmen Poncelet (friend of Descartes and teacher of Blaise Pascal) and Chasles; Felix Klein and von Staudt in Germany; the Swiss geometrician Jacob Steiner; Cayley, Sylvester and Clifford in England and many other great mathematicians. George Adams saw that the ideas from this realm of mathematics, which came to its height in the last century, had had considerable influence on philosophical thought as represented above all by Professor Whitehead, but that, because of the abstract way in which it had been formulated and its seeming lack of points of application to Newtonian physics, it had received little or no attention from the physicists. He was convinced that by this path fresh avenues of thought would be opened up to widen and complement the one-sidedly analytical approach to the universe and the forms of life within it.

Fundamental to this conviction was his recognition of another modern school of spiritual thought and philosophy, namely, that of Rudolf Steiner. In George Adams these two paths united; he spent his life in the service of them both. Confirmed by Rudolf Steiner in the knowledge that the *quality of thought* prevailing in the new geometry is in reality indispensable both to the scientist in his quest of world-reality and to the individual on a path of spiritual development, it was his life's task to make this school of mathematics ever more accessible and to develop it further, that it should be at the disposal, in scientific research and in education, of those whose impulse it is to dissolve the veils of theory and of unbelief created on the one hand by the symbolism of present-day science and on the other by orthodox religion. The metamorphic character and wonderful quality of organic synthesis of modern projective geometry are reflected in Rudolf Steiner's philosophy of *spiritual activity,* whereby, through the activity of pure thinking, man may reach to the spiritual in the universe and in himself.

George Adams wanted to quote in his preface the words which I have set at the head of this foreword. They are old words given to a pupil by Rudolf Steiner in 1903 and hitherto unpublished. Concerning these words, George Adams wrote, in a letter to a friend in February 1963:

"The whole field with which my article deals is really one of those places where the exoteric science of our time comes hard upon the threshold of the eternal occult truths and asks to be complemented, to be directed and confirmed by the science of spiritual initiation. The formal mathematics used in the idea of ethereal or negative space - Professor Locher sometimes calls it polar-euclidean space - has long been known to pure mathematicians, but apart from their interest in its beautiful form, they have attached no further importance to it. Its significance lies, however, not in its mere form, but in the fact that its discipline, created by the mathematicians of the nineteenth century, gives insight into a quite new conception of space which exactly corresponds to what has always been described by occultists as the secret of the spatial aspect of the etheric - of life.

"It was in particular Rudolf Steiner's deed, in this as in many other fields, to make accessible to the modern scientifically inclined mind the ancient truths of occultism, so crucial to the future culture of mankind, and to give his followers the necessary indications towards the further development of the scientific task. It was in this sense that my article in *Natura* was written, firstly with readers in mind who are familiar with Rudolf Steiner's teaching."

Olive Whicher

Goethean Science Foundation,
Clent, Stourbridge, Worcestershire,
July 1964

I Ancient and Modern Geometry

It was often pointed out by Rudolf Steiner how important it would be for the sciences which are today so specialised to interpenetrate each other in a more living way. The classification which is customary in our universities will in some respects undergo far-reaching changes. "The present grouping of the sciences," said Dr. Steiner, "is in effect inadequate for the attainment of a real world-conception." As an example, he went on to say on one occasion, it would be necessary to have qualified doctors of medicine, well-versed in higher mathematics. For there were fundamental discoveries concerning the metamorphoses of the human organism, which would only be attainable by such combination.

This interplay will, of course, involve changes of spirit in the different sciences themselves. Such an abstruse and formal world of thought as modern mathematics seems to be, will not apparently have much significance for those who primarily have to do with life. Indeed, the different branches of knowledge will only free themselves from arid specialism inasmuch as they are permeated, far more than is the case in the elaborate intellectual systems of today, with an immediate sense of reality. This living contact will above all be stimulated by anthroposophical spiritual science. The intellect alone will scarcely find it.

In the present essay I hope to bear out what has here been said, for one particular domain of knowledge, which from some points of view is fundamental. I refer to the modern school of Geometry (Synthetic or Projective Geometry), to the significance of which the great teacher of Spiritual Science repeatedly drew attention. Far-reaching results could follow if some biologists and doctors would enter thoroughly into the thought-forms of this Geometry. There are important questions in the air - scientific questions which were also pointed out by Rudolf Steiner - which will perhaps only be solved upon this basis. Unhappily, modern Geometry, like most of contemporary mathematics, is as a rule propounded in a highly abstract form, and therefore many, even mathematically gifted scholars, do not ever find their way into the heart of the matter, where the significance of this Geometry for their especial tasks would dawn upon them. They are put off by the too abstract form which they encounter to begin with.

Modern Geometry takes its start from quite other points of view than the familiar "Euclid" of the schools. I will make some attempt to explain it here, not in the usual academic forms, but from the outset in the more living light in which it appears with the help of Anthroposophy. This will make many things intelligible to human thought and feeling, which if put forward in the customary abstract form would after all only appeal to specialists. That it is justified by the real content of the new Geometry to place it in the context which will here be shown - this is a point which I have fully argued and set forth in other writings.[1]

Geometry in the last resort must always have to do with the great Wisdom-teachings of mankind. For with its help we experience how space is formed by the Divine Light of the World. Through the marvellous way in which the human frame is poised in space both inwardly and outwardly - through the indwelling of the conscious spirit in this frame - we human beings have the faculty to reproduce this Divine creation in the light of thought. In Geometry, we have an inner enjoyment of the creation of space. Its truths are not conveyed to us empirically, as it were compulsorily by the data of the outer senses; we apprehend them by an inner insight. Geometry is, therefore, a first step upon the way of spiritual knowledge.

Precisely in this respect, however, the Geometry of our time is in the very opposite position from that of classical antiquity. The Geometry of Euclid and the Greeks was like

a last and distant echo from the ancient Mysteries - those above all of the Egyptian, Hebrew and Chaldean epoch. There it had been one of the final lessons of the "ancient Wisdom," to perceive how the Elohim - the "Spirits of Form" - had placed the human being in his physical body out into cosmic space, into that three-dimensional space which is expressed so wonderfully in the main structure of the human skeleton and which in Masonry becomes the architecture of the tomb, the sacred monument, the Church or Temple.[2] This architectural, mathematical and masonic phase, which had indeed been like a closing chapter of the pre-Christian Mysteries, was for the Graeco-Latin age translated into the more abstract forms of dialectic and practised as a means of training for the mind. This is the real significance of the Geometry of Euclid, as to its source and content on the one hand and its scholastic method on the other. It is the architecture of space - space in its final and most rigid, earthly form - which we experience above all in this Geometry.

Modern Geometry is in a different position, both as to form and content. It originated for the most part in the nineteenth century, at a time when the great majority of men ignored the very existence of a higher spiritual knowledge, or, if they thought of it at all, doubted its possibility for man. And yet the new Geometry is related to the Mysteries, scarcely less intimately than the ancient school. For it arises at the dawn of a new era, wherein all branches of life and knowledge will gradually merge into a newly acquired Wisdom of Mankind. Modern, synthetic Geometry, as Rudolf Steiner pointed out, is in fact one of the few really creative deeds of our epoch in the forming of pure thought as such. But it will only prove its fruitfulness in a wider aspect when we are ready to perceive its fundamental notions in the concrete light which is already now made possible by Spiritual Science. Just as the old Geometry was once abstracted, out of the living substance of the Mysteries into the thinner forms of intellect and dialectic, so shall we have to sublimate the new, from the quite abstract form in which it has begun its course, into the substance of a new-born spiritual knowledge, which as the Mystery-Wisdom of the future will receive nourishment from all aspects of human life and endeavour.[3]

Strangely enough, this Geometry does not take its start from the forms of thought which one might easily regard as the first essential of all spatial notions. I mean the thought-forms fundamental to the architecture of our earthly space, which find classical expression in the following passage from the history of Solomon's Temple in the Book of Kings: "...And there was cedar on the house within, carved with knops and open flowers: all was cedar; there was no stone seen. And he prepared an oracle in the midst of the house within, to set there the ark of the covenant of the Lord. And within the oracle was a space of twenty cubits in length, and twenty cubits in breadth, and twenty cubits in the height thereof; and he overlaid it with pure gold ..."(I Kings, vi. 19-20).

We are here confronted with two notions, namely those of *length* ("so many cubits") and of the *right angle.* Out of these two ideas the cube is formed; it is the archetypal form of earthly space. Here we not only take for granted that there is such a thing as a measure of length or distance in any given direction, but we assume that the measures of length in all directions can be compared, so that it has significance as in this instance to say that they are equal in the three main directions of the cube. Moreover, by combining the ideas of *length* and of the *right angle* we get the measure of areas ("square yards") and of spatial volumes (so many "cubic feet"). The very words we here employ, as "square" or "cubic feet", bear witness to the underlying form of the right angle. What we obtain along these lines is in the last resort (to use a phrase of Rudolf Steiner's) the idea of space as a potential container of physical things, physical phenomena and objects.[4] It is the space in which we measure out our quarts, our cubic volumes in the ordinary sense. The other archetypal form of this space (beside the cross or right angle and the cube) is the sphere, or its plane image - the circle. Presuming in

effect that we may compare measures of length in all directions, the immediate consequence is the idea of a surface whose points are equidistant on every hand from a given centre. That is the sphere. In the plane picture it becomes the circle.

Now the important thing in the new Geometry is precisely this, that we learn to conceive space not *only* in this aspect in which it serves as the potential container of things physical. When we concerve it in this latter aspect, what we experience in fact is but the final stage of a long space-creative process; just as the solid bony man, the rigid skeleton, is but the final stage in the long process of man's evolution as an earthly being. The true ideal archetypes of space are not, however, bound to the *rigid* forms of the cube or of the three perpendicular directions. Modern Geometry gets nearer these ideal archetypes; it penetrates the space-creative *process;* it apprehends "space in becoming." Hence the significance we must assign to it for a more spiritual knowledge also of outer Nature. For we must learn to see in Nature too not only what is ready-made (and therefore dying) but what is new-becoming in her life. We have to liberate imagination from the bondage of the finished forms of space. Yet we should not attain this goal merely by leaving space and spatial forms aside. We need a bridge in knowledge: an understanding of the "becoming spatial" and of the passing-away out of the spatial existence into the purely spiritual once again. We need a real transition from the inner essence to the spatial appearance. The new Geometry provides this bridge, or an essential part thereof, when once we summon it to life with the touch of Anthroposophia.[5]

It should scarcely be necessary to add that the new Geometry too, in the last resort, gives us ideal insight into the finished forms of earthly space: measures of length and volume; sphere and right angle, circle and cube and so forth. But unlike Euclid, it does not start from these. What it begins with, is the idea of a polarity wherein the earthly or physical aspect of space is in fact only one of two. Over against the latter is the ethereal or etheric aspect, as I propose to call it. Speaking in terms of Spiritual Science, we have the true ideal foundations of this Geometry when we bespeak the contrast of the physical and the ethereal, when we take hold of the cosmic polarity of Heaven and Earth, or even Sun and Earth.[6]

This, in effect, is the original conception of all space - far more original, far deeper and more potent than the finished shapes of earthly space with its cube-forms and contents. We can describe it roughly to begin with, from one point of view, as a polarity of "inner and outer." We are reminded of the primal epoch of our evolution, when the celestial body of "Old Saturn" was first encompassed by the heavenly spaces. Into this Saturn-body the spiritual beings of surrounding spheres poured down their forces. That which had been engendered by the Thrones ("Spirits of Will") as an inward essence, as it were a fiery core of the new world, was for the heavenly Spirits of the surrounding spaces the immediate goal of their activity.[7] Though we must use these spatial terms for want of other language, we must not think in this connection of a space so formed and thoroughly configured as is the space of the present Universe. Yet in the very contrast of inner and outer, or of the core of Will and the surrounding spiritual sphere, there were already the first rudiments of space in ancient Saturn.

In a later epoch - in the "Old Moon" existence of the Cosmos - conditions grew more complicated. From the one central body a duality arose - the duality of Moon and Sun. For when a certain stage was reached, the old Moon-body became separated from the Sun. From that time forth the celestial forces worked down upon the Moon planet not only from the surrounding sphere of Heaven as a whole, but to a large extent from the Sun itself. About this stage Rudolf Steiner writes in his *Outline of Occult Science:*

> "In consequence of having separated from the Sun, the Moon is now related to the Sun in the same way as once was Saturn to the whole of the surrounding cosmic evolution. Saturn was formed out of

the body of the Spirits of Will (Thrones). From the Saturn substance rayed back into cosmic space all that was experienced in consciousness by the Spirit-beings in its environment. And through the events that followed, this raying-back gradually awoke to independent life. Such is the essence of all evolution. Independent being is first separated out from the life of the environment, then the environment engraves itself - as it were, by reflection - upon the separated being, and then the latter evolves further, independently. So did the Moon body sever itself from the Sun, and, to begin with, simply reflect the life of the Sun body."[8]

We are here touching one of the primal secrets of evolution, which is indeed deeply connected with the origin of space itself and of all spatial things. In our age of the cosmos, too, the contrast of the sunlike and ethereal as against the earthly-physical is at work in all living things, nay even in the "dead" mineral, so-called, if once we understand the latter in a deeper way. Speaking of "Sun" in this connection, we have not only to think of a heavenly body occupying an apparent place in the cosmic spaces; we have in mind the whole of the light-filled sphere of Heaven which from all sides is working towards the Earth in a sunlike way - with celestial as distinct from earthly forces.

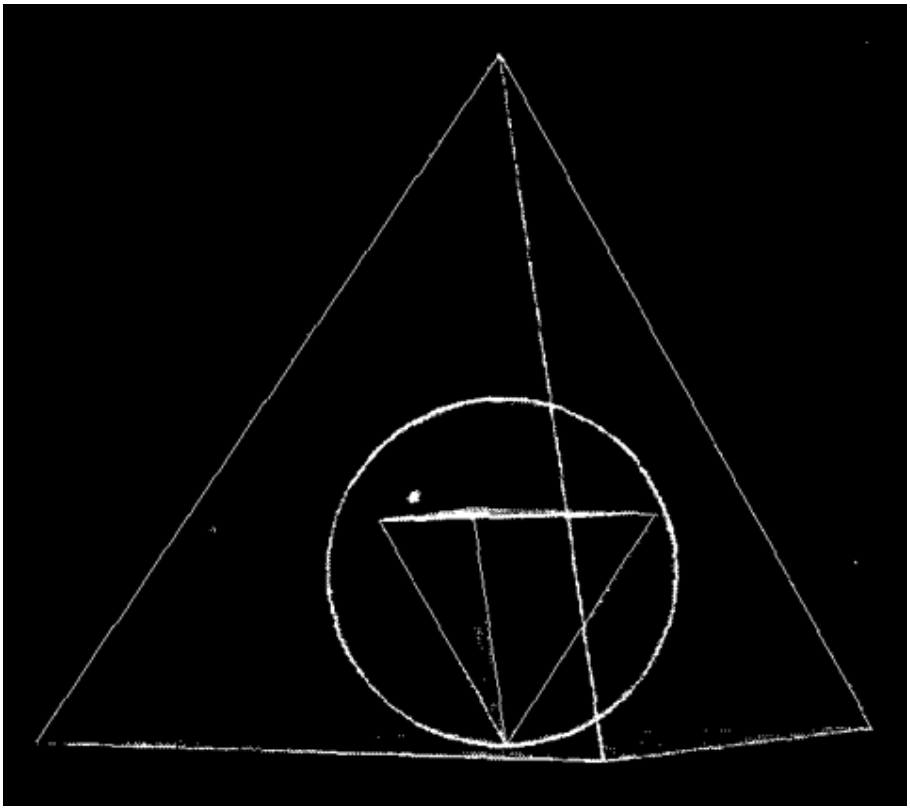

Figure 1: Tetrahedra

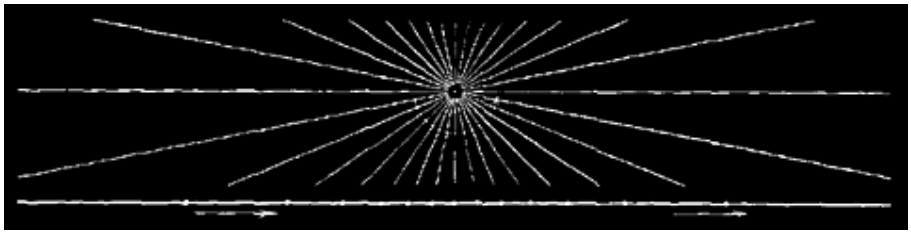

Figure 2: Point of lines and line of points

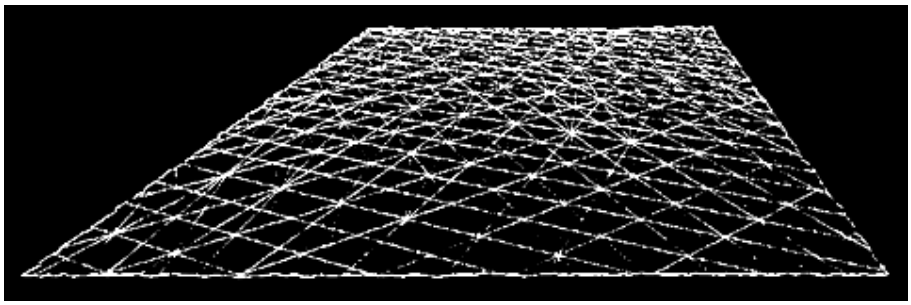

Figure 3: Plane, woven of lines and points

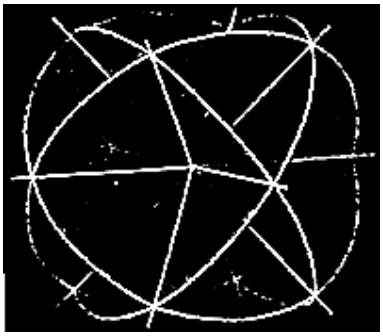

Figure 4: Point, formed of lines and planes

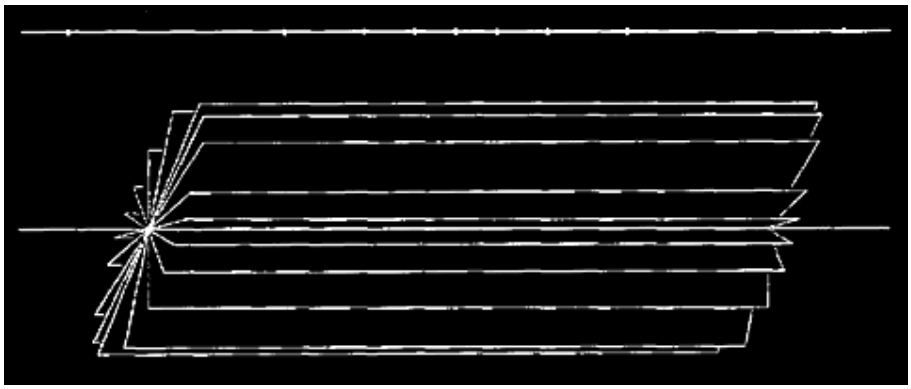

Figure 5: Line of points and line of planes

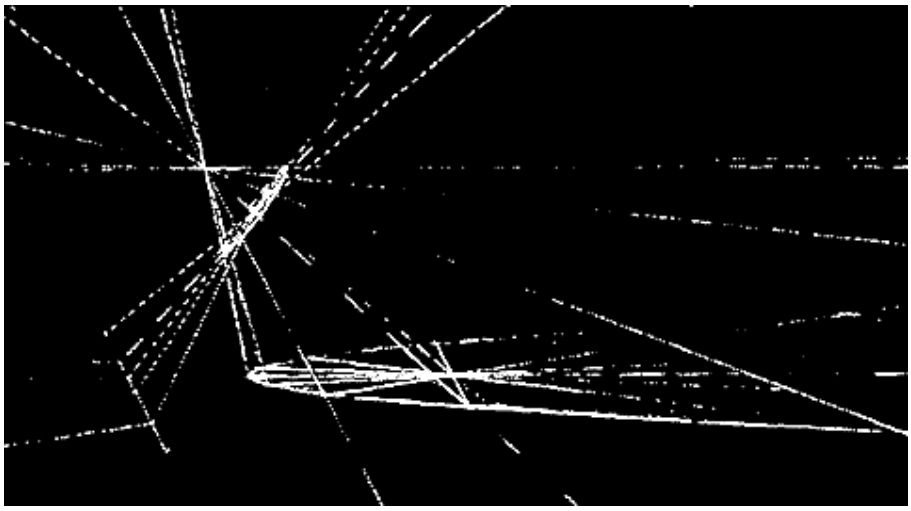

Figure 6: Circle into parabola

We bear this contrast of the sunlike element and of the earthly-physical in our own human nature too, and we experience it naturally in geometrical imagination. *This* is the basic experience on which modern Geometry is founded. It finds expression to begin with in the polarity of Point and Plane. Point is the earthly pole, Plane the celestial. (In Figure 1, the contrast of point and plane as called forth by the sphere is shown beautifully by the tetrahedra, the one circumscribing a sphere, the other inscribed within it.) It is quite true that point and plane appear at first as purely abstract forms, and we may not immediately recognise in them this very real polarity of cosmic Nature. Yet, that it is so, becomes increasingly clear the more we find our way into the matter. To begin with, certain inherent difficulties must be overcome. They have already been surmounted to a great extent in the new Geometry; everyone who begins to learn it has to go through the process for himself. What still remains to be achieved will perhaps only be discovered when Geometry is lifted out of the purely abstract realm and placed in its real context in a spiritual science of the world and man, as we are here endeavouring to do.

The difficulties arise out of the fact that in our natural earthly life we experience space in a one-sidedly physical aspect. In truth, all space is founded on a pure and absolutely "fair" polarity: Heaven and Earth, ethereal and physical, or even "light and darkness" - call it as we will. We, however, experience it to begin with from the standpoint of the *physical* body. True, we have also our ether-body as the essential architect and builder of the physical; the latter, too, is therefore permeated by the celestial light. Hence in all spatial conceptions, even the most rigid and most earthly, the ethereal light is originally there, though it be quite unrecognised.

But the forms of spatial imagination which we unfold to begin with are in fact those that correspond to the realm of earthly space in its most finished state. It is a space that has fallen, as it were, out of its heavenly connection, which we experience in ordinary life and in Geometry of the old-fashioned kind, as the potential vessel or container of physical things and phenomena.

Yet in our active thought we have the latent power to free ourselves from this one-sided, physically spatial bondage. This is precisely what the new Geometry has done to a very great extent, though to do justice to it we must begin quite consciously to think it in this spirit. We usually think of a *plane,* for example, as a particular kind of surface, and we conceive the latter - obviously - as an *extended* entity. It has an area, a magnitude capable of division into smaller parts. When we apply this form of thought

in practice, we divide the surface into square feet or inches, or in the integral calculus into still smaller and at the last "infinitely small" surface elements. So we imagine the surface or plane to consist of extended parts which, added, constitute the whole. It is this extensive way of thinking which we must now be capable of overcoming. We must be able to imagine the plane in its totality; fluidly as it were, in effect ethereally - not as consisting of so many points or partial areas centred around points, but in its infinite extent as a single whole. When this experience is gained, we shall describe the plane quite honestly as undivided and indivisible - nay, as *the* indivisible entity above all, for so it is in the *ethereal* experience of space, just as the point is the indivisible for physical experience.

It goes without saying that we here have in mind not any bounded portion of a plane, such as a table or a piece of paper, but in each case the plane in its totality, continuing as it does into the infinity in all directions. 'This brings us to another difficulty which must be surmounted. It is the difficulty of conceiving what from the physical point of view we should describe as the "infinitely distant". Within the bounds of the physical we can never come to clarity upon this question. The physical mind imagines the infinitudes of space staring into the empty void in all directions (the phrase is Rudolf Steiner's, see *The Course of my Life,* page 45). Truth is that through the so-called "infinite distances" of physical space another, an etheric world begins to shine into our physical imagination. From the purely physical point of view this gives rise to inevitable paradoxes, for we here take leave of the more tangible and sensory ideas and begin to touch the supersensible.

The plane in its totality, as it extends into the infinite in all directions, is in fact a self-contained and almost cyclic entity, returning into itself through the infinite on all sides. Go out into the infinite in one direction, and you come back from the infinite from the opposite direction. For at infinity every straight line of space has a single point. There are not *two* points at infinity, one to the left for example and one to the right; nor can we say that there is none at all. There is just *one* (Figure 2). It is the same infinitely distant point which you attain as you go out to the left and to the right. Go out into the infinite towards the left; you will return from the right and so complete the cycle. Go out towards the right and you will return from the left once more to your starting-point. The same applies to a vertical line. Go upward into the infinite and you will return from the infinite below you. Zenith and nadir are a single point in the pure space of mathematical imagination.

When we apply these notions to the plane in its totality, we perceive that in the "infinite" it has as its periphery, not as one might suppose an immense circle. but a straight line. Though it surrounds us on all sides, this "line at infinity" is straight. And for the whole of space we recognise that in the infinite it has as its circumference - as the totality of infinitely distant points - not as one might suppose a "sphere" of infinite dimensions, but a *plane*.[9] Mathematicians therefore truly speak of the "plane at infinity" of space. It is significant that the infinitely distant. *i.e.* the most cosmic portions of this space are not polarised into the pairs of opposites which belong to the finite and physical world. The circle and the sphere are always polarised into the diametrically opposite pairs of points or antipodes in each direction. But in the infinite of space the two antipodes merge always into one. When we perceive this truly we begin to feel space no longer merely as an unending vast but as a self-contained, organic whole. And we can then find the true transition from the earthly-physical to the ethereal aspect.

As an example one may make the following experiment in thought. Imagine a horizontal plane in any given position, and above it a fixed point in space. Now let there pass through the point a second, mobile plane. Let the latter move quite freely through all possible positions, subject to the one condition that it never takes leave of

the fixed point, which indeed acts like a pivot for the moving plane. Both planes, the moving one as well as the fixed horizontal one below, are, of course, to be conceived in their infinite extent. Somewhere they will have a line - a straight line in common. (Here and throughout the sequel, I shall always use the single word *line,* meanning "straight line".) Try to envisage how this line, in which the two planes meet, moves in the horizontal plane below. It will be recognised at once that the smaller the inclination between the two planes becomes, the farther will the line go out into the distance. ;. Now there is one position where the planes are parallel - where, in effect, the mobile plane also becomes horizontal. Let it come very near this position without quite reaching it, and you will see the common line of the two planes move outward with increasing speed into the infinite horizon. Moreover, this will take place from every side. If the mobile plane was, to begin with, tilted downward to the left and is then made to approach the horizontal, the line will move out to the left. It will move outward to the right, forward or backward or in any direction we please, according to the inclination of the plane from which we begin to approach the parallel position. In the moment when the latter is attained, the common line of the two planes vanishes apparently. Yet in the very next moment, if we overshoot the mark, however little, it will appear again in the opposite quarter. If the line vanishes towards the left, it will return from the right. Or it will sweep around in the horizon, at a great distance and with enormous speed, if we keep the mobile plane very near the parallel position, moving it round and to and fro however slightly. We perceive at length that in the ideal sense the common line of the two planes *is there* in the parallel position also, only it has moved out into the infinite on all sides. It still remains a *straight* line; the common line of two planes cannot conceivably be any other than straight. So we perceive that the infinitely distant portion of every plane is a straight line. Parallel planes are such as have their infinitely distant lines in common, even as other pairs of planes have other lines in common. We can now state without exception: any two planes of space have a straight line in common. We need no longer make exception (as the older Geometry had to do) of planes that are parallel.

As to the infinitely distant part of a straight line itself, we reach a like conclusion. Imagine in a given plane a horizontal line and a fixed point above it, as in Figure 2. Passing through the point and in the plane, think of a second, mobile, line, rotating like a ray about the point. The common point of the moving ray with the horizontal line below moves outward into the infinite, to the right or left as the case may be. As the two lines grow parallel, it disappears for a moment, yet the next instant it emerges again on the horizon from the opposite direction. Here we perceive the quite continuous and quick transition from the unending distance on the left hand to the unending distance on the right, or vice versa. The exactly parallel position corresponds to the "infinitely distant" point, which is to the right and left at once. Therefore the points of a straight line form a *cyclic* series.

So we perceive that the extensive entities of space - the straight line, the plane - do not go out into vague nothingness but are really self-enclosed, organic wholes. Such insight will already prepare us to recognise them as single, indivisible entities. We must be able to feel them in this latter aspect too, if we are to find the way through Geometry into the more ethereal aspect of space and of the cosmos. We then no longer merely have the feeling that the plane consists of infinitely many points; we feel it as a single whole. Even as Euclid defines the point: "A point is that which has no parts", so now we feel about the plane: "A plane is that which has no parts". Strange as it may sound, this is the very truth for the *ethereal* aspect of space, just as the other is for the more obvious - the physical. And now to compensate for this we must develop on the other hand the faculty to feel the point - which is so obviously the indivisible for physical experience - as a thing manifoldly organised and membered, *i.e.,* divisible. As

Rudolf Steiner said on one occasion, we must learn to experience what is "extensive" intensively, and what is "intensive" extensively. To feel the "extensive" intensively is to experience the plane as a single, indivisible whole. To feel the "intensive" extensively, is to see the point composed (in spite of its "no extension"!) of many parts and members. What are the parts or members of the point? They are the planes that pass through it in all directions. The relation is mutual. Just as the plane is divisible into points or point-centred regions, so is the point divisible into planes and "planar regions" - using the latter word, admittedly, in an unaccustomed sense but none the less exactly. Just as the plane is in one aspect an organism of infinitely many points and of infinite diversity (for the points of a plane do not lie chaotically side by side, but are manifoldly organised, as we experience at once when we begin to draw geometrical patterns on a piece of paper *(e.g.,* Figure 3), so does the point in space now represent an organism of infinitely many planes, no less wisely and diversely membered (Figure 4)).[10]

The purely geometrical idea we have here unfolded (it is the so-called "Principle of Duality" of modern Geometry, carried to its logical conclusion) is the ideal counterpart of a cosmic fact of Nature. Recognise this, and we have gone half-way - more than half-way in one direction - to bridge the gulf between occultism and modern science. The physical experience of space and spatial things asserts with obvious conviction that "the whole is greater than the part" - Euclid once more! This proposition answers, for example, to the division of a plane into its points or point-centred regions. For the ethereal world the very opposite is true. In the ethereal world the whole is by no means greater than the part; it is on the contrary smaller. So Rudolf Steiner told us how the formation of an etheric organ - say the etheric liver - appears to supersensible consciousness. Manifold streams and influences are flowing together from the cosmos. At the place where they interpenetrate, there arises by their interplay (it is a *qualitative* interplay but its effect is at the same time spatial) the etheric organ as a whole. These currents from the universe are the cosmic parts, the etheric members of the organ. The organ as a whole is therefore smaller than its parts. This is an absolutely real process, perceptible to supersensible consciousness. In the world of pure thought, *i.e.,* in Geometry, the corresponding truth is in its most ideal form the conception of the point as an organism of many planes - inwardly membered and composed of the planes that weave through it in all directions.

Between the polarities of point and plane, the straight line mediates. In its relations it is evenly balanced between them. (Two planes, for example, have a straight line in common; so have two points, namely the line that "joins" them. Or again: a *point* and a line in space outside it generate a *plane;* so do a *plane* and a line outside it generate a *point* - their point of meeting.) The line inclines on the one hand to the physical and on the other to the ethereal aspect of space. It rays through the points of space, and it weaves in the planes, creating many forms and pictures. Seen as the ray that issues from the point, we experience the straight line more with the quality of Will; when it weaves forms and pictures in the plane, we follow it more with our life of Thought.

The line itself can be divided in a twofold way (Figure 5). First it can obviously be divided into the infinitely many points that lie "along it." This is the obvious, the physically spatial aspect. But we can also divide the line ethereally, for we can recognise it as organically formed of all the planes that circle round it. Think of a line as the common axis of its planes. Try to experience the picture, not in the usual extensive way, but as far as possible feeling the extensive things intensively. One will then recognise the planes quite naturally as the members, nay, as the *parts* of the line, just as in physical space one feels the points as its parts or members. And if we now remember that the latter too - the points of a straight line - constitute a *cyclic* series, we see that the polarity is perfect. Just as the planes as the etheric members of a line circle around it, so do the points as its physical members circle "along" it - out through the infinite

and back again. This is the beautiful polarity we recognise when in pure thought we overcome the one-sidedness of the naive imagination, which arises out of the fact that in our earthly life the physical is the familiar aspect.

Closely connected with the polarity of point and plane in space, is an activity of thought which was in fact the historic origin of the new Geometry. It is a form of thought which has in some way to do with our vision of the spatial world, and therefore with the art and science of perspective, or "projection" as it is called. Hence, too, the name, "Projective Geometry," by which modern Geometry is sometimes known. The world of space around us is spread far and wide, filled with the light of the Sun. Our eye by contrast - seen as a physical organ - has a contracting, point-like function. It receives this widespread space-of-light, as it were, into a single focus. True, with our ether-body we are living *in* this light-filled world. Etherically, as Rudolf Steiner once described it, we are swimming in the light and with it.[11] None the less,we only bring it home to physical and earthly consciousness by exercising the necessary contraction, by means of the physical organ. This contracting process, this gathering of the field of vision into a point-like focus and its projection on to the tiny surface of the retina, is described in the familiar way by physiological optics. Though there is much illusion in the familiar explanations, what is quite evidently true is that this interplay of contraction and expansion in the relation of the eye to the surrounding world is somehow fundamental to our vision of the sun-filled spaces.

The purely spatial aspects of the process are studied in the theory of perspective, as it was practised also by the great artists of the Renaissance time - the dawn of the modern naturalistic era. It was in this that the new Geometry was really born. W begin by imagining every point of space as a potential eye-point able to receive into itself all forms and pictures of surrounding space. This was the very natural outcome, for pure Geometry, of the whole theory of optical perspective. Misleading as these forms of thought have been in the materialistic theories of optics - with their quasimaterial light-rays, flying corpuscles and other hypothetical constructions - they are no less significant and fruitful for pure thought. Imagine, for example, in a plane some geometrical picture, woven of lines and points. Over against the plane, conceive an eye-point, *i.e.*, any point in space. The forms and figures which appear extensively before us in the plane *picture* are received by the eye-point as a cone of rays, which is more like a *seed* or germ of light. Speaking in detail, the lines of the plane picture become so many planes in the eye-point inasmuch as every line in space, taken together with a point outside it generates a plane. The points of the plane picture become rays, that is to say, straight lines in the eye-point. Whatever picture or pattern was woven of the lines and points in the plane, is received as a corresponding configuration of planes and lines in the eye-point. But this configuration will not be outwardly imaginable as the plane picture was. The picture in the plane, as an extended entity, has been received into the point intensively. From an expanded and visible form, it has become, as it were, a seed-of-light, no less specifically formed. Truly we can describe it as a seed of light, if by the "light" we connote the formative, form-giving principle of space. For if we now confront the point which contains this latent configuration by any other plane of space, there will arise in the latter a fresh picture, answering in every detail to the original, only in corresponding metamorphosis, according to the relative positions of the two planes and of the eye-point.

To take a simple case, imagine in a plane a circle, an opposite the centre of circle any point in space (Figure 6). There will ray forth in the latter - not only towards the circle but in both directions from the point - a cone (an ordinary "right circular cone," as it is called). If we confront this cone by another plane, we obtain a fresh curve. It is a so-called "conic" or conic section; it will be circle or ellipse, parabola or hyperbola as the case may be. It is a metamorphosis of the original circle. We must remember that the

original picture, *i.e.,* the circle, really consists not only of points but at the same time of lines within its plane, namely the tangent lines, which, as the saying goes, "envelop" it. It is formed in other words not only by the physical aspect of space (radially, point by point as from its centre), but at the same time ethereally - peripherically. The points of the circumference give rise to the rays, the so-called "generators" of the cone; the tangent lines give rise to the tangent planes of the cone. The cone too is formed not only of the generating lines which lie so outwardly and visibly before us, but consists no less organically of the tangent planes by which it is "enveloped.[12]

Continuing this rhythmic interplay of expansion into outward pictures and contraction into the seed-like quality of points, we create the basic metamorphoses of spatial form. Moreover, by this means we are not only able to transform once given forms as in the last example, but to bring forth fresh ones, stage by stage, in an ever-growing process of enhancement. This rhythm does in fact contain the creative quality of space, and it is in this - the most original idea of space - that we apprehend the great polarity which underlies the evolution of the world itself. We may call it a polarity of seed and form, or *seed and picture*. In our own human being too, we are thus polarised. For we can come to recognise that this polarity of Space is deeply related to the life of Time, the play of past and future-birth and death and resurrection of the world. We have the two poles in our human being. Out of the past we carry in us in our head-nature manifold cosmic pictures, whose dying shadows we experience in our life of Thought; while raying on into the future we carry seeds of future worlds within us in our Will, in the inner spiritual forming of the limb-man. From the past we are fashioned out of the primal cosmic archetypes, of which only a faint echo is visible to us in the constellations, spread out in the apparent infinitude of Heaven. Out of the infinite periphery of space the cosmic picture die into our being. This is the secret of the human head-formation. But in the realm of Will the cosmic archetypes arise to a new life, out of those depths of Earth into which we descend with our I as into the fire-kernel of the world. "The past throwing its shadows," Rudolf Steiner writes, "the future holding seeds of reality, meet in the human being. And the encounter is the human life of present time."[13]

Now we may name the spiritual Mystery to which the new Geometry has all unconsciously been drawing near, first in the pure idea of space. It is the Mystery of Persephone - the evolution of the Earth and Man out of the past and on into the future. It is the same deep process which is enacted in the world of plants, time and again in the shorter rhythms of the year. To recognise this fact is of untold significance, for we thereby begin to take hold of space no longer merely in its rigid, finished form, but in its evolution out time. Time in its essence is fundamental to the archetypal thoughts of space. In the spiritual progress of science we must grow capable of referring space and all spatial relationships once more to the element of time, for by this means in our life of knowledge we begin to lift the outer spatial world into the spiritual.[14]

During his lectures on the relation of the different sciences to Astronomy (Stuttgart, in January, 1921), Rudolf Steiner spoke in the following terms about the world of plants:

> "Through the plant-world the life of the Earth as it were opens itself to the Universe. The plant-life, covering a given region of the Earth, is indeed a kind of sense-organ, sensitive to all that is revealed towards the Earth out of the Cosmos. At seasons when the interplay is more intense between a portion of the Earth's surface and the Universe, it is as though a human being were opening his eyes to the outer world to receive sense impressions. And when the interplay is less intense between the Earth and the Cosmos, the consequent decline and inward closure of the vegetative life is like a closing of the eyes to the Cosmos. It is more than a mere comparison to say that through its vegetation a given territory opens its eyes to the Universe in spring and summer and shuts its eyes in autumn and winter. And as by opening and closing of our eyes we do in a way

converse with the outer world, so too it is a kind of information or revelation from the Universe which the Earth receives by the opening and closing of its eyes through the life of plants.[15]

"And to describe it a little more precisely, we may consider the vegetation of a given region of the Earth when it is exposed, as it were, to the most vivid interplay with the solar life, and we may then turn our attention to the state of vegetation in this region when it is not thus exposed. The winter, I need hardly say, does not interrupt the vegetative life of the Earth. It goes without saying that the vegetative life continues through the winter. But it expresses itself in quite another way than when exposed to the intensive working of the Sun's rays - or, shall we say, of the Cosmos. Under the influence of the solar life the vegetative life of the Earth shoots outward into form. The leaves unfold and grow more complex; flowers and blossoms develop. But when this is followed by the closing of the eyes to the Universe, if we may call it so, the vegetative life goes back into itself - into the seed. Withdrawing from the outer world, it no longer shoots into outward form; it concentrates, if I may put it so, into a point; it becomes centred in itself.

"We may describe this contrast truly as a law of Nature. The interplay between the earthly and the solar life reveals itself in the Earth's vegetation. Under the solar influence the vegetative life shoots outward into form; under the influence of the earthly life it closes up into a point; it becomes the seed or germ. In all this there is a quality of expansion, and of contraction or gathering into a centre. Here we begin to apprehend the relationships of space itself in a directly qualitative aspect. This is the very thing which we must practise in the development of our ideas, if we would make essential progress in this sphere."

What Rudolf Steiner here describes, leading up to the demand that we take hold of the relationships of space from a *qualitative* aspect - this is indeed the cosmic background of the root-ideas of space which are revealed in the new Geometry. We may quite consciously apply, in this purely geometrical context, the words "expansion and contraction" taken from Goethe's theory of plants. Goethe himself does not intend them in a trivially spatial sense, but with a deeper and more qualitative meaning.

II. Ethereal Space and its Forces

However, it is not only a question of these more general relationships. A great variety of new and very concrete possibilities is hereby opened out to science. Rudolf Steiner in his special scientific lectures spoke of such openings quite definitely. To understand the spatial working of the ethereal and sunlike forces, we must develop among other things the idea of a kind of counter-space or antispace, as against the one-sided physical space of normal consciousness. Here are quite definite tasks, both for the mathematical thinker and for the investigator of external Nature. For there are many things in Nature which we fail to recognise - though they may be near us all the time - because in fact they take their course in such a counter-space, while the consciousness of to-day, with its one-sided physical orientation, has its attention fixed in the opposite direction and therefore does not see the processes in question. That the ethereal forces have to do with an opposite kind of space or spatial working, is expressed by Rudolf Steiner even in the elementary chapters of his spiritual science. The ordinary, phyical aspect of space is such that we nearly always experience it as from a relative center. This is true both as regards the purely geometrical structure of this space and as regards the physical forces and substances which it contains. Typical of this kind of space is for example the idea of "polar co-ordinates", as they are called, where we measure distances in all directions radially from a chosen centre and at the same time mark fhe angles between the different radii. We thereby obtain a precise record of the relationships of form and position in this kind of space. So much for pure Geometry. And when we think of the substances and forces in outer physical space, we experience this centric quality still more intensely. All physical effects ray out from centres - centres of gravity, magnetic poles, electric charges, sources of radiation and the like. They ray out from centres and themselves on all sides in the periphery of space. Physical forces may therefore truly be described as "centric forces." In their totality they constitute the earthly pole of Nature's workings. Orthodox science hitherto has thought of these, broadly speaking, as the only kind of forces, and has endeavoured to explain Nature in terms of them alone. Dr. Steiner on the other hand sets over against them another kind of force, which he describes as "peripheral" or "cosmic forces." These are precisely the "etheric forces" of anthroposophical science.[16] They always work, he says, as from the world-circumference. This does not mean (as one might all too readily conclude, following physical analogies) that they work from single points of the circumference of space. Every "etheric force" works from the *whole* circumference. That is to say, its spatial source is of precisely the opposite nature from what one would imagine physically. Its source is not a point at all, but the very opposite, namely an "infinite periphery". The reader may refer to the scientific introductory chapters in the medical handbook *Fundamentals of Therapy* by Rudolf Steiner and Dr. Ita Wegman. We read for example in the third chapter, of the "Phenomena of Life":

> "Observation shows ... that the phenomena of life have an altogether different orientation from those that run their course within the lifeless realm. Of the latter we shall be able to say, they reveal that they are subject to forces radiating outward from the essence of material substance. These forces radiate from the - relative - centre to the periphery. But in the phenomena of life, the material substance appears subject to forces working from without inward - towards the relative centre. Passing on into the sphere of life the substance must withdraw itself from the forces raying outward and subject itself to those that radiate inward."
>
> "Now it is to the Earth that every earthly substance, or earthly process, owes its forces of the kind that radiate outward. It has these forces in common with the Earth. It is, indeed, only as a constituent of the Earth-body that any substance has the nature which Chemistry discovers in it. And when it comes to life, it must cease to be a mere portion of the Earth; it leaves its community with the Earth

and is gathered up into the forces that ray inward to the Earth from all sides - from beyond the . earthly realm. Whenever we see a substance or process unfold in forms of life, we must conceive it to be withdrawing from the forces that work upon it as from the centre of the earth, and entering the domain of others, which have, not a centre, but a peripherry ... "

Dr. Steiner not only speaks of *forces* working in an oppositely spatial manner; in many passages he actually speaks of an opposite kind of space itself. (To students of contemporary science this will not seem altogether surprising, for, as the recent developments of physics have suggested, we can no longer so abstractly distinguish a space from the totality of forces which are working in it.) Now the whole question is: Where shall we find the *point d'appui* for the idea of such a counter-space - i.e., a space opposite in quality - to the familiar space of physical experience? Reality can no doubt be reached by the human spirit in many ways. Our mathematical way - the way of pure thought - is only one way, albeit an important one in this connection. The fact is that modern Geometry provides the clue in the most natural manner, if once again we venture to take Rudolf Steiner's words in earnest, however paradoxical they appear at first sight. It is significant how at this moment pure mathematical thinking and supersensible investigation lead in the same direction.

To begin with we must realise quite clearly how the freely metamorphic, evenly balanced space of "projective Geometry" - the space whose foundations are revealed in the pure polarity of point and plane, expansion and contraction, manifest form and seed-of-form - is related to the rigid and already earthly space which we are dealing with in the old Geometry of Euclid, of the schools. The men of genius who brought the new Geometry to its fulfilment during last century became fully clear about this relationship;[17] yet a fresh riddle emerged out of the very clarity of their perception. Only the insight of spiritual science into the nature of the cosmos will at this point afford a real solution.

I must here make some effort to explain how the space of Euclid - one-sidedly physical space, as I have called it - is related to the more mobile and, as we know, more original space of projective Geometry. Set forth in a purely abstract form, these things appeal only to trained mathematicians; yet in themselves they are not so difficult, and it is important for wider circles to become acquainted with them. [18] I may therefore borrow from another realm of knowledge certain ideas which will assist in explaining what is meant. Those who know Goethe's *Metamorphosis of Plants*[19] will be familiar with an idea which is indeed significant, not only in botany but in all the sciences of life and for that matter in aesthetics also. There is a living entity whose whole formation bears evidence of a certain underlying principle of form. The formative idea does not express itself in an external, causal manner, as by the putting-together of such or such external factors. It reveals itself intrinsically, livingly. It may be of a plastic nature or more musical and rhythmical - as a motif, an underlying theme. Unfolding as a living process, as an indwelling idea, it comes to expression in all parts and organs of the entity in question. In every plant for instance, such a formative idea or hidden archetype is living, and with the eye of imagination we can recognise the same in constant metamorphosis, true to itself in leaf and bract, petal and capsule. But the same formative principle which the several parts thus variedly reveal - we find it manifested also in the form and character of the whole. The hidden principle which underlies all organs and all vital functions, emerges once again as in a single picture in the shape of the entire organism. Moreover this is true not only in a spatial sense but in the life of time. That which we recognise for example in each single plant of the Earth as the essential rhythm of its life - "expansion and contraction" - we find it again in the relation of the living Earth - body to the Sun, in the great cosmic interplay of the seasons (compare the above-quoted passage from Rudolf Steiner).

The beautiful idea, that in the whole is manifest what as a process works in every part, applies also to the formation of space as such. In the new Geometry we begin to experience space more as a living organism; hence these ideas of metamorphosis apply. We have already seen what is at work as the original idea of space: the polarity of point and plane. In this polarity we have to do with infinitely many points and infinitely many planes. What matters is their potential interplay by virtue of the line that rays and weaves between the two kinds of entity creating what we have described as the "pictures" and the "seeds" of form. What now if we discovered, in the formation of cosmic space as a whole, one single archetypal plane, one archetypal point? We should be able to say that in the forming of the whole of space the very idea of polarity which is at work in all its parts and processes, is revealed once more as in a cosmic form in its totality. This possibility here opens out to thought.

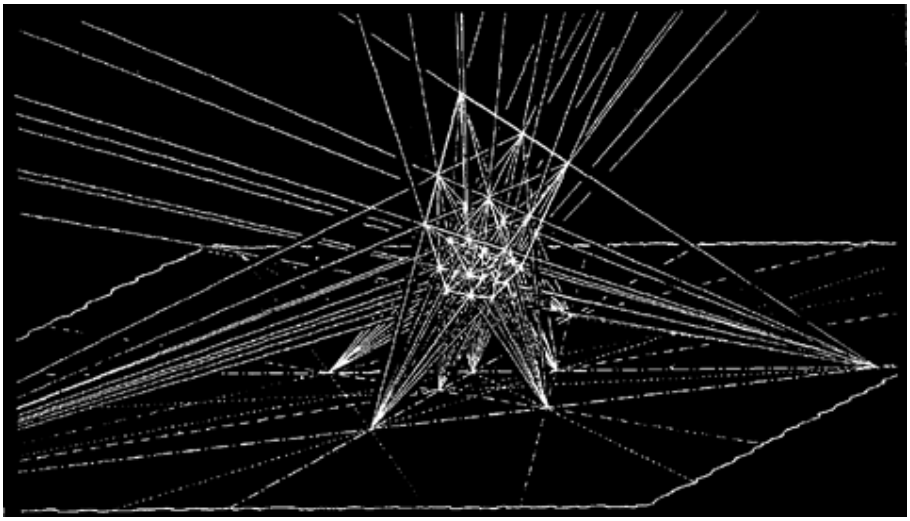

Figure 7: Light-woven space of the crystal

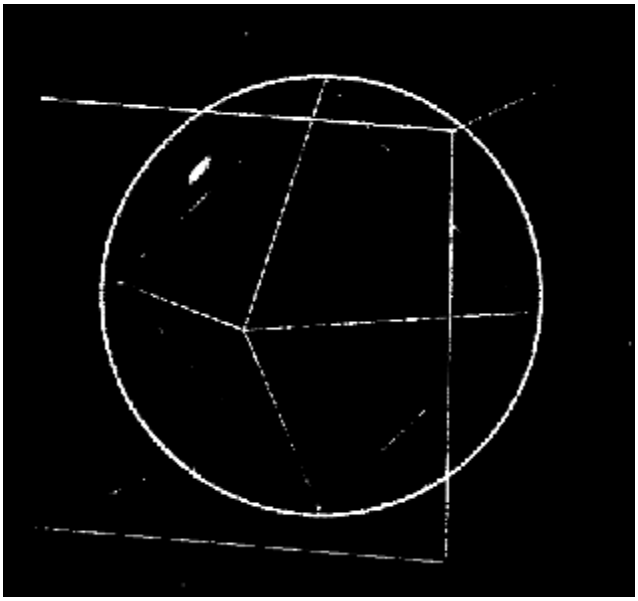

Figure 8: Cube and octahedron

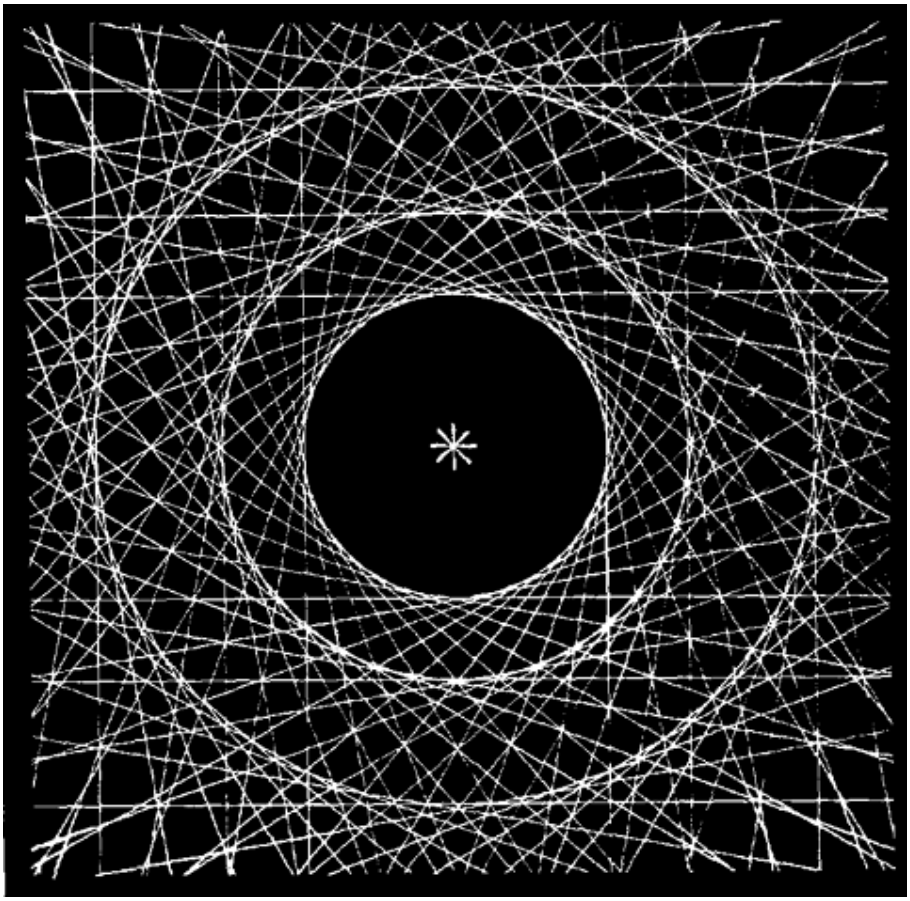

Figure 9: Circles about a "cosmic centre"

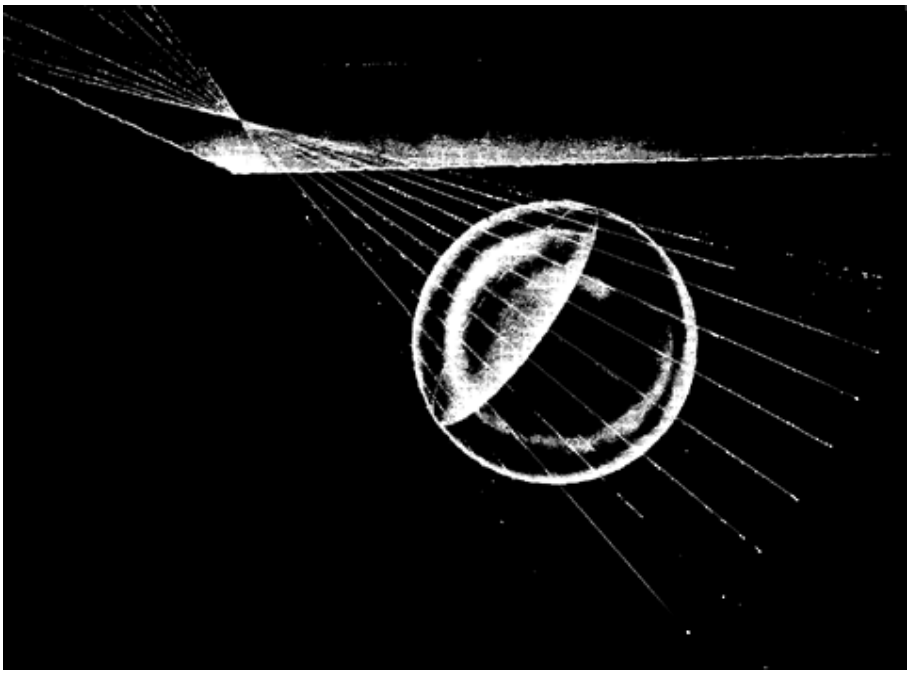

Figure 10: Poles and polar planes

Figure 11: Reciprocation between centre and infinite periphery

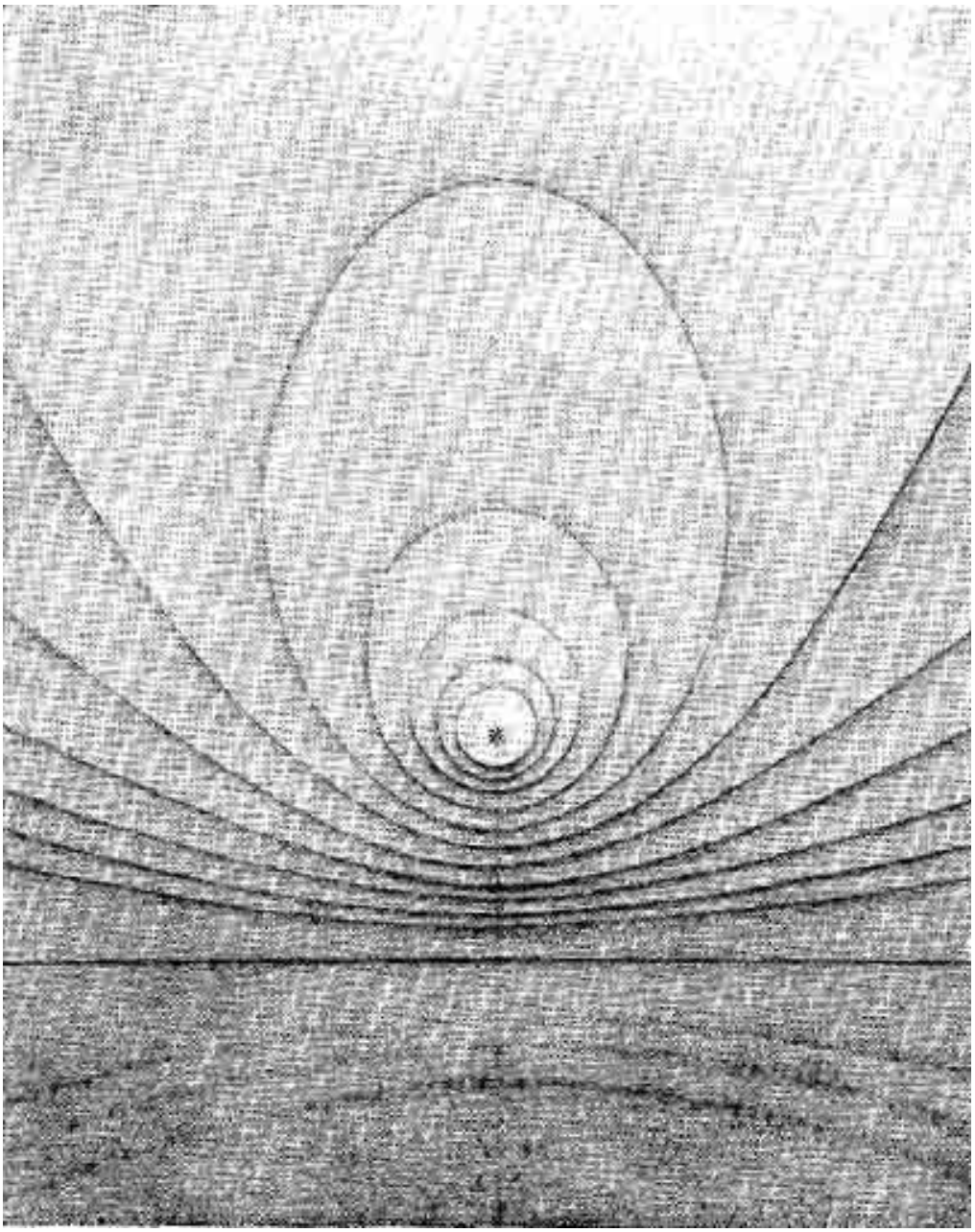

Figure 12: "Ethereal concentric Spheres"

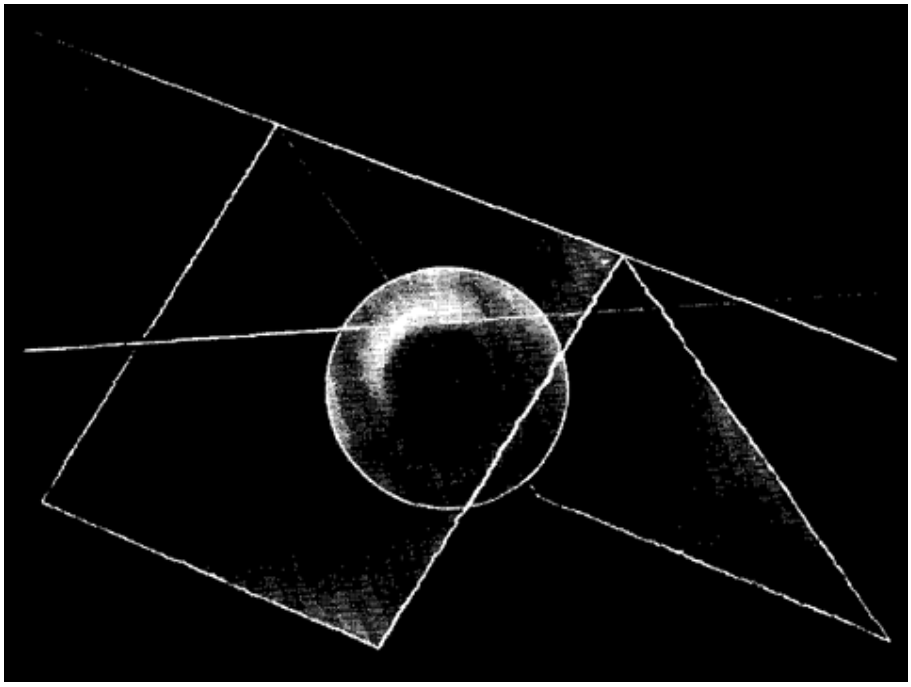

Figure 13: Line-line polarity with respect to the sphere

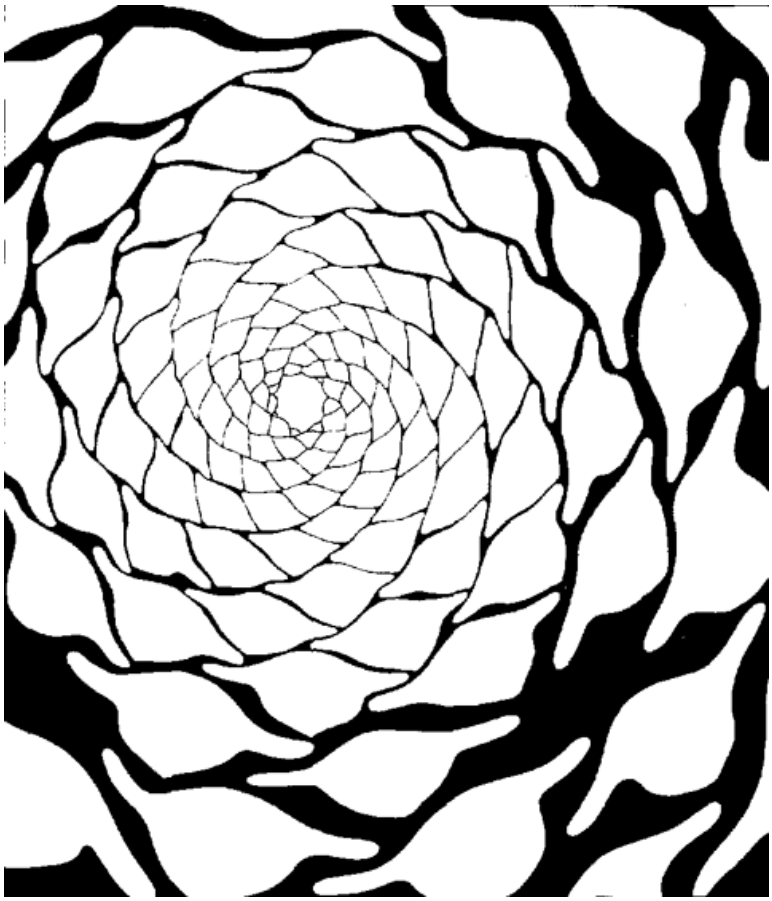

Figure 14: Leaf-primordia at growing-point (Euphorbia Wulfenii)

Now the remarkable thing is that modern Geometry gives us the answer precisely in this direction; yet it is only half the answer! For in effect the space of Euclid - the form of space we normally experience - emerges from the free and archetypal space above described, when from among all planes of the latter we single out one with a special, as it were a cosmic function. It is of course the "infinitely distant plane" (page 19 above ("... "infinite distances" of physical space... ")). In the original space of projective Geometry, the polarity of points and planes works as an underlying process of "expansion and contraction". From this original space, the space of Euclid emerges when we postulate one single plane, as it were the absolute, the infinite periphery of such a space. It is in relation to this, the unique or absolute or cosmic plane of ordinary space, that we then find the deepest explanation of the symmetries and measures which are so familiar to us. In Figure 7 this unique plane is pictured; from it the crystal forms of cube and octahedron are drawn projectively. For the cube and octahedron in Figure 8, this plane would be the infinitely distant plane of physical, Euclidean space. (It should be added that not only this "absolute plane" or "plane at infinity," but a kind of archetypal circle-form within it, the "imaginary circle at infinity" is needed to complete the transition from free projective space to the space of Euclid. This fact, as I have elsewhere shown, fits in entirely with the conceptions here unfolded; for the idea of the circle, not in its rigid form but in its free and mobile aspect as the conic or quadric, is the immediate outcome of the original polarities of point and line and plane. What then more natural than that among such forms one should emerge as a kind of archetypal circle, imprinted in the cosmic form of space in its totality?)[20]

With *physical* imagination we can go no farther! We can at most distort this space to some extent, as has been done in the so-called "non-Euclidean Geometries". But we can now entertain the idea that there exists, over against the space of physical imagination, another space where the relationships are precisely opposite - a space wherein we are confronted not with a cosmic plane but with a cosmic point: not with an infinite periphery but with an "infinite centre", if the paradox may be permitted. In such a space the relationships of form and measure, instead of raying out from central points (relative "origins" as they are called in analytical Geometry) into the infinite periphery on every hand, would be working inward from the wide spaces, *i.e.,* from relative peripheries towards a single "absolute point" or cosmic centre. In other words, we are to conceive a space which receives its stamp and measure, not from a unique cosmic plane as "plane at infinity", but from a unique cosmic point. In such a counter-space or anti-space a point and not a plane, a quality of centre and contraction, not expansion, would be the "infinitely distant". All the conditions of this space, all the activities therein, would gradually spend themselves inward towards this central point, just as in physical space they gradually lose themselves outward into the wide expanse, into the cosmic circumference. The linewise circles in Figure 9 (albeit only two-dimensional) call to mind a peripheral space-formation of this type.[21]

It is significant that Rudolf Steiner describes precisely in these terms the space in which the *solar* activities of the universe take place, as for example in the ether-body of a living being and above all in the "heavenly body," as we call it, of the Sun itself. I quote once more a passage from the aforementioned lecture-course on the different domains of Science in relation to Astronomy.

> "That which is commonly thought of as the physical constitution of the Sun will never really be understood with the ideas which we derive from earthly life. We must of course begin with the results of pure observation, which indeed are eloquent up to a certain point; but we must seek to penetrate them with ideas such as are true to their real nature...

"The following attempt must here be made. We may begin by imagining some process that takes place in earthly life. We see it take its course as we follow the directions outward from some central point. It takes its course in a *radial* direction. It may be a kind of outbreak, such for example as a volcanic eruption, or the direction of some deformation, as in an earthquake or the like. We follow such a process on the Earth in the direction of a line that goes outward from the given centre. And now in contrast to this you may conceive that the inside of the Sun, as we are wont to call it, is of such nature that its phenomena are not driven from the centre outward but on the contrary, that they take their course from the corona via the chromosphere towards the photosphere - not from within outward but from without inward. You are to conceive, once more ... that the processes go *inward* and, so to speak, gradually lose themselves towards the central point to which they tend, just as phenomena that issue from the Earth lose themselves outward in expanding spheres, into the wide expanse. You will thus gain a mental picture which will enable you to bring some kind of order into the empirical results....

"Only when we enter thus into the qualitative aspect, only when we are prepared, in the widest sense of the word, to unfold a kind of *qualitative mathematics,* shall we make essential progress... Here I should only like to add that there is a possibility, notably for pure mathematicians, to find the transition to a qualitative mathematics. Indeed this possibility exists in a high degree in our time. We need but consider Analytical Geometry with its manifold results in relation to Synthetic Geometry - to the real inner experience of Projective Geometry. True, this will only give us the beginning, but it is a very, very good beginning..."

Knowledge derived from Spiritual Science is in no case abstract and remote; it always has to do with some reality of life. So in this instance. We shall come nearer to the idea of a "peripheral space" or counter-space which has here been suggested, if we call to mind how we ourselves in certain stages of our life before birth or after death live in another kind of space than on the Earth. For we go out into the cosmic ether and into the realms of Sun and Stars. As we here live in a point-centre - in a space which takes its start, for our experience, from the given centre where our earthly body is - so we there live in the circumference, in the periphery. In a lecture given on the 21st October, 1921,[22] Rudolf Steiner describes the inner experience of the human being after death in some such words as these: "The cosmic world that was formerly at the periphery surrounding us - therein we feel ourselves to be *within;* while on the other hand, what was formerly the earthly world on which we stood - we feel it henceforth as our *centric outer world....*" If this be so, there is indeed, even in real experience, such a thing as a "centric outer world" - an outer world, not spread around us as a circumference, but such that we ourselves indwell the whole circumference and look towards a centre as our outer world. We look into the infinite, not as into the wide expanse of a circumference, but rather inward as into a cosmic centre.

But within earthly Nature too, this anti-space, this sunlike quality of space is active. Above all for a detailed knowledge of the ether-body of man and other living creatures, it is important to develop, not only the ideas of such a space but the corresponding spatial feeling. In many processes of Nature, it is this kind of space and spatial force which is at work. Moreover for the human body - even the physical body in its anatomy and physiology - this polarity of space and anti-space becomes important when we seek to understand in detail those metamorphoses, those "turnings insideout," which lead from the organs of the metabolic and limb-system to the corresponding organs of the head-man. In this connection too, Rudolf Steiner pointed out how necessary it would be to approach the problems with the help of "qualitative mathematics". (Hence his remark concerning doctors and the higher mathematics which we quoted at the outset.)

Now from the standpoint of modern Geometry it is possible to work out the Geometry of such an anti-space with full exactness. It is well known that Euclidean Geometry has already been modified into the non-Euclidean Geometries, so-called. Such modifications were however mainly a kind of distortion of the space of Euclid, without any *very*

radical change in its fundamental qualities. Here on the other hand, speaking in mathematical terms, we should have to develop the direct inverse of Euclidean Geometry.[23] We should thereby be filling in a gap of which mathematicians of modern time have been quite well aware - sometimes even painfully aware. They had experienced to begin with, in projective Geometry (what I have here called the geometry of original or archetypal space - in German: *Urraum)* the harmoniously balanced polarity of point and plane. From this Geometry, the ordinary space of Euclid was derived as I explained above, by the ideal assumption of a unique or cosmic plane - the "plane at infinity". So far so good. But the full harmony of mutual relations was thereby imparied; for in Euclidean space one always had to do with the relations to this unique plane, and there was no such thing as a unique point in space which would ideally have balanced it. A certain one-sidedness - quite unaccountable, to begin with, from the aspect of pure thought - thereby crept into the beautifully balanced harmony which one had first experienced in projective space.[24] Why this one-sidedness? the searching mind will ask.

The solution of this problem will only be found with the help of anthroposophical science. We must return once more, in effect, to the human being, for it is he after all who is experiencing all these thoughts and imaginations.[25] Man in his present normal consciousness is placed into the world of space in such a way that he experiences it in an earthly-physical, that is to say in a centric, point-like way. In pure thought, modern Geometry has to some extent transcended this one-sided aspect with the discovery of projective or archetypal space, wherein point and plane, *i.e.,* the earthly and the heavenly ideal poles, are fully balanced. And it will be a further step in overcoming the one-sided earthly standpoint, now to unfold the notions of that space which is precisely opposite to the physical; which is one-sidedly celestial in other words, even as this one is earthly.[26]

It is important to perceive that the relationships are interlaced. If I may have recourse once more to Goethe, we have not merely to do with a polarity as of "light and darkness", where the polarities were mingled in a merely outward sense. (This would result in an indifferent grey!) It is when darkness works intrinsically, inwardly into the light, and likewise light into the darkness, that the living colours - yellow and orange-red, violet and blue - are born. So too we must understand the polarities of space. It is precisely characteristic of *physical* space - that is, the space which we experience in a more centric, point-like way - that it receives its stamp and measure from a cosmic *plane,* from a periphery in other words, from an *ethereal* and cosmic entity. This is the very secret of our world. That which resides in the depths - namely the earthly stone, the mineral, the crystal - has its source of form in the far spaces of the cosmos, in the celestial light. Or in the technical language of Anthroposophia: it is the mineral, residing in the "depths", which has its etheric body or body-of-formative-forces in the "heights", in the celestial circumference. (Even pure thought discovers this essential fact in the notion of the infinitely distant plane of the space of Euclid; for the Euclidean, Cartesian space is quite essentially a crystal thought-form.[27])

Once more then: physical or centric space has an essential source of its formation in the world-periphery. And for the heavenly effects, that is, for the ethereal or peripheral forces (page 36 above ("Knowledge derived from Spiritual Science... ")) the opposite is true. The *ethereal* forces are precisely those that direct their activity to something *physical*; speaking more definitely, to some germinating point or seed, whether it be the Earth itself in the more macrocosmic workings, or any one of the myriad living germ-points which develop in the earthly realm and here receive the celestial forces.

The ethereal is ever *active* in the world, pouring in from cosmic spheres and directing its activity towards some physical germ or seed-point - towards a relative world-centre. The physical on the other hand *exists* in finished form, having received its formation

more or less recently or in primeval times from the ethereal periphery of heaven, out of the "infinitely distant plane" of the physical or earthly space it occupies. This latter process will direct our thought towards the past. The physical falls out of the domain of "life" and becomes *merely* physical existence, precisely inasmuch as the livin ether-body withdraws into the "infinitely distant". The former process on the other hand concerns the future. The celestial spaces direct their ether-forces towards the physical germ-point which is *their* infinitude; they nurture and sustain it from all sides, for they perceive in each living germ-point something potential of the cosmic future.

If we call "archetypal space" or *Urraum* the idea of space wherein the elements of past and future, of the celestial periphery and the earthly centre, *i.e.,* of plane and point, hold perfect balance - in other words the idea of "projective space", described at the beginning - we have the following relationships:

We must remember in applying these ideas that in reality we have to do not with one space alone but with an untold number of interpenetrating spaces - spaces ethereal and physical. Even within the physical, we have not one space but as Rudolf Steiner once explained an untold number. For in reality every crystal form or crystal type in the mineral world has its own space, filling the Universe.[28] Pure thought itself will recognise that this is so, if the known laws of Crystallography are once interpreted as they deserve to be, in the light of the new Geometry. Here too, the supersensible experience of the mineral world in its true essence will confirm what is suggested by the empirical evidence and the clear thinking of Geometry. Thus in the physical we have not only one but an infinitude of spaces, all of them physical in type. In the ethereal it will be likewise. Wherever in the world a seed arises *(i.e.,* a seed or germ-point in the widest sense) - wherever in the element of living brooding warmth a new seed of physical existence is preparing to receive the formative ideas of Heaven, there an ethereal space will shape itself about this central point as about *its* infinitude, its "Absolute." For the formation of the living Earth in macrocosmic evolution, the fire-core of the Earth must have acted in this way in pristine ages. True to herself the living Earth brings forth, in the repeated rhythm of the seasons, myriads of seed-points of her living kingdoms. The fire-process of new birth is re-enacted in all germination.[29]

III. Polarity with respect to the Sphere

We now have to develop, to begin with in pure thought, the possibilities that are implicit in the idea of ethereal space or antispace. In the further course it will be most essential to preserve the proper balance as between thought and experience. We have not to create another branch of "applied mathematics" in the accustomed sense. *The point is that in developing the pure mathematical thought-forms of ethereal space we shall acquire a new spatial feeling, and as we do so, we shall begin to see things newly. We shall again he able to awaken in ourselves the inner experience of the etheric, thus overcoming the old mental bondage of the merely physical space-aspect.* It is in this way that the new line of thought will fertilise our knowledge of external Nature - and of the human being. We shall, once more, begin to see things newly - and new things.

Our first task, therefore, is to develop the pure Geometry of negative or ethereal space. We may begin with the most simple and natural example - namely, the surface of a sphere. We try to envisage it in its ethereal as well as in its physically spatial aspect. (I shall describe normal Euclidean space, in what follows, as "physical space". This is not meant to imply that we experience it physically, sensually; it is the *form of space which corresponds to the idea of the physical.* The opposite kind of space, which we are now about to study in more detail, is the "ethereal".)

To begin with we must call to mind - in the light of modern Geometry - the familiar properties of the sphere such as it is in ordinary space. The first thing obviously is that it has a centre. The moment we think of a "sphere", our spatial thought is centred in this point. But the significance of this point, for physical space, is only relative - relative to the given sphere. Another sphere will have another centre. That which has absolute significance for physical space as such is not a point but a plane - namely, the "plane at infinity". Indeed the centre of the given sphere only arises by the relation of the latter to the said "plane at infinity", which is the "absolute plane". The centre is the "pole" of the absolute plane with respect to the given surface. We may arrive at it as follows. From any chosen point of the plane at infinity - in other words from any infinitely distant point - we draw all possible lines to the (?) surface. These lines will evidently form a cylinder, touching the sphere in a great circle, or relative (?). The plane of the great circle passes through the centre. If we do this for all the infinitely distant points of space, we obtain all the planes through the centre. In other words, from all the points of the infinitely distant plane we obtain all the planes of the central point: from the plane-of-points, as it were, a point-of planes. The sphere engenders in this way a thorough turning inside-out of space - not only in the local sense, as that the outer is transformed into the inner or the periphery into the centre, but in a still more qualitative sense: points into planes, planes into points. For in like manner every plane of space will by relation to the sphere give rise to a point as its specific "pole" (Figure 10). Points relatively near the centre will arise from planes far away out in space. If we imagine a point travelling along a radius from the centre outward to the circumference, the corresponding plane will travel inward from the celestial periphery to meet it (Figure 11). Beginning in the infinite plane - even as the point begins in the centre - the plane will move inward parallel to itself, at right angles to the radius in question. The point in its outward, the plane in its inward movement, will at length merge into one another as they reach the surface of the sphere itself, from within and from without respectively. Here now the point becomes a "point of the surface", while the plane becomes the corresponding "tangent plane". The point and the tangent plane belong organically to one another and to the organism of the sphere. Indeed the sphere - or any other surface, for that matter - only becomes a plastic surface inasmuch as it consists not only of so many points, namely the end-points of its radii in this instance, but at the same time of so many planes. It is not only formed from within, radially and as it were

atomically as with a pair of compasses, but it is also moulded plastically from without by the tangent plane, which coming inward from all sides from the plane of heaven, like a moving hand of light, envelops it and models it out of the cosmos.

If as was indicated at the beginning (page 18 ("... But the forms of spatial imagination...")) we have learned to experience the plane in its ethereal totality, we shall no longer make objection, as we might be apt to do with purely physical imagination, that the tangent plane after all "only belongs to the surface in a single point", whereas the point in its entirety lies in the surface. We shall perceive that *the plane, etherically speaking, belongs to the surface every whit as wholly as the point does physically.* We shall be able to follow up these thoughts to their logical conclusion in all directions, and we shall find them the more justified by every new test we apply. We can of course also conceive the plane in its physical, extensive aspect as made up of infinitely many points, but if we do so we shall likewise have to think of the point as made up of its etheric parts, namely its infinitely many planes (page 22 ("... The purely geometrical idea we have here unfolded...")), and we shall find the corresponding answer. One of the points of the extensive plane "belongs" to the surface; so too does one of the planes of the point.

In our conception of the sphere so far, two distinct elements have played a part. First, the ethereal and physical polarity - the "principle of duality" which prevails in archetypal space, as we called it. This does not yet determine the physical (Euclidean) type of space. The fact that the sphere relates to every plane a specific point and vice versa, is a fact of archetypal space. The root-idea of space, namely the polar relationship of points and planes, comes to expression by virtue of the sphere in a specific form and quality. To recognise it we need not yet be thinking in Euclidean terms; indeed, we shall scarcely understand it fully until we have freed ourselves to some extent from the ideas of Euclidean space, as we do in projective Geometry. But there was also a second element in our considerations, namely the absolute or infinitely distant plane which is the characteristic of physical space. From the uniqueness of this plane we derived the point which for the given sphere is correspondingly unique, namely the centre as the pole of yonder plane.

Turning from physical to the ethereal type of space, we shall preserve unchanged all ideas that were due to archetypal space pure and simple. Only the last-named element - that which determined the one-sided physical variety of space - we shall now have to turn into its opposite. Instead of an infinitely distant plane or cosmic plane, we shall now have to conceive as our infinitude a cosmic point. For simplicity, let us imagine it to begin with in the centre of the sphere. This being given, we may now develop the idea of the sphere from the periphery inward, *i.e.,* in the sense of etheric space. However at this point a preliminary explanation must be given, as to the method of description. When we describe in detail the aspects of etheric space, it is at first a little difficult to make ourselves understood without confusion. For in our mental pictures, to begin with at any rate, we still have the forms of physical space. And the accustomed names of spatial forms and positions are all of them derived from this latter aspect. Yet we desire to evolve in pure thought the idea of a space which is precisely opposite in quality. We can only do so by overcoming the physical aspect which is more or less automatically given to us in our mental pictures. In spite of the accustomed semblance of these pictures, we now begin to develop a quite different spatial feeling. (We may describe it as "dinamic" for what we feel as a dynamic quality in our aesthetic sense of forms in Nature and in Art, is often very near to this etheric aspect. Yet all the while we are obliged to use expressions borrowed from physical space, which for etheric space have not the meaning that the names imply. We must refer, for example, to the "infinitely distant plane." In the organic totality of archetypal space it is a plane among other planes. Yet for etheric space its significance is not that it is infinitely distant; quite on

the contrary, it is more often in the very midst - the peripheral midst, if we may use this paradox - the peripheral resting-place; from which we start. Nevertheless we cannot but describe it to begin with as the plane at infinity in the space of our imagination. How else should we refer to it?

The difficulty lies in the nature of the case. As we are thinking in terms of Geometry and we do not mean it in a merely abstract or symbolic sense, we cannot but call forth definite mental pictures. We experience these pictures to begin with in their physically spatial aspect: right angles as right angles, parallels as parallels, equal distances as equal, and so on. It is in overcoming these obvious aspects that we shall often be led to see what is ethereal. Yet we derive the names from the very aspect which we have to overcome, for in this aspect our accustomed spatial terms originate. With everyday consciousness we live once and for all in physical space; the other aspect we must gain by inner effort. To avoid misunderstanding, I shall describe as the "space of our imagination", or as the "apparent" aspect, what we experience in our mental pictures to begin with. Thus the expression "the infinitely distant plane of our imagination" will refer us to the plane in question, while at the same time the qualifying words will suggest that for the argument at the given moment it is perhaps by no means "infinitely distant", but very near and most accessible.

As an example of the dynamic experience of forms, we may imagine a circle (that is, an apparent circle) in an etheric space whereof the cosmic point is not quite in the centre. To our immediate visual imagination it is a circle with an eccentric point. For etheric space it is no longer a circle at all; we shall experience it dynamically as an ellipse. This is quite literally true; it is not merely an ellipse in theory, but we shall feel it as such once we have entered into the ideas of etheric space for the form in question. That which in physical space was the mere difference in position, namely the eccentricity, without affecting the form as such, for dynamic space becomes an inner qualitative change of form once the eccentric point is acting as the cosmic point.

Conversely, that which to physical imagination may appear eccentric or unsymmetrical, will under certain conditions in the dynamic forming of etheric space become concentric and symmetrical (Figure 12). The Kepler ellipses of the planetary orbits may serve as an example. If we conceive the planetary path to be in an ether-space - the corresponding planetary sphere - whereof the cosmic point is the sun's centre, the path of the planet for such a space is a pure circle, though physically it appears as an ellipse. As an historical aside, this is not uninteresting when we call to mind how even Kepler took his start from the traditional idea that the paths of the heavenly bodies must be based on the pure circle.[30]

In the organic world we often have to do with forms arranged in layers more or less eccentrically about some nucleus or kernel. If we imagine such a process to have been brought about etherically - from the periphery instead of from within - quite new possibilities are opened out for the interpretation of such living forms.[31]

Now to return to the example of the sphere in ether-space: as we have placed the cosmic point in the apparent centre, the sphere will appear as such in our etheric space as well as in the physical. In fact we have chosen an etheric space related to the space of our imagination in the simplest possible way with regard to the given surface. The plane which to our physical imagination is the "infinitely distant", is with respect to the surface in polar relation to the point which is the infinitely distant of the chosen ether-space.

In physical space the "interior" of the sphere represents a finite volume such as we might fill with any substance. In the ethereal space it has become infinitude. This infinitude reaches inward to the cosmic point, just as in ordinary space the infinite expanse goes outward to the celestial periphery. Moreover we now form the sphere, not from the centre outward as with a pair of compasses, but from without -

peripherally, spherically. To do so, we begin not from a central point but from the corresponding plane - the infinitely distant plane of our imagination. This plane has now become the "peripheral centre" of our surface, just as a point was its physical centre. And as in physical space we might imagine a point going radially outward from the centre in all directions till it attains the surface, so do we now conceive a plane, travelling inward from the celestial "middle plane" until it touches the surface. If as a plane we take our start from the periphery (the "plane at infinity" of physical imagination), we can approach the sphere from every direction: from above and from below, from the right and from the left, and so on. The plane moves inward, parallel to itself: this is the etheric counterpart of the point moving radially outward from the centre. For every parallel direction there are two diametrically opposite ways inward from the celestial periphery to the surface. A horizontal plane, for example, will begin "at infinity" either above or below (it is the same "plane at infinity" above us and below us!) and approach the surface, moving downward in the one case, upward in the other. This corresponds to the evident fact of physical space, that a point which travels vertically outward from the centre can reach the surface in either of two opposite directions, upward or downward. We obtain diametrically opposite pairs of planes, or points, of the sphere (see Figure 11).

The plane as it comes inward from the celestial circumference from all directions, models and moulds the sphere out of etheric space as it envelops it on every hand. We have described the typical etheric forming of the sphere, as against its physical construction. One of the deepest secrets of existence underlies the full analogy yet fundamental difference of quality between the two. It has to do with the relationship of macrocosmic and microcosmic man. What is it in ethereal space that corresponds to the radii in the physical? A twofold infinitude of such lines rays outward from the centre in all directions. Each of these lines meets the surface in two diametrically opposite points. So too in the etheric middle plane (the infinitely distant plane of physical imagination) we have a twofold infinitude of lines. We see them interweave throughout the sphere of heaven. Each of them is the common line that joins the middle plane of heaven with two diametrically opposite parallel planes of the surface. (Figure 13 may help in forming the necessary geometrical picture. We see pictured the common line of two planes which meet in the finite. Allow these two planes to turn into the vertical position until they become parallel and their common line will move outward and merge into the plane at infinity. Keeping their contact with the sphere, the planes will then determine two diametrically opposite points on the sphere, whose common line is a diameter of the sphere. Over against this diameter or radial line of the sphere is the common line of the two planes; it is now in the infinite. This is the line which joins the plane at infinity with the two diametrically opposite parallel planes of the sphere.)

Thus, for example, the horizontal line in the celestial plane is the common line or "ethereal path", leading from yonder plane to all horizontal planes - downward or upward, as we please. Once more, from the celestial middle plane there are two possible ways of reaching the surface horizontally. We here recall what was said above (page 23 ("...Think of a line as the common axis of its planes...")): each line in space is membered not only physically into its infinitely many points, but ethereally into its planes. Thus in etheric space the line is essentially a "path", not for the points that run along it, but for the planes that have in it their common axis.[32] And if as in our last example the line itself is physically speaking infinitely distant, the movement of its planes appears as parallel.

But we have not yet dealt with one essential question: How do we know that we are modelling a sphere precisely, not an ellipsoid for example, or a more complicated surface? In other words, what is there in ethereal space to correspond to the idea of

equal distances along all radii from the centre, by which idea we differentiate the sphere from all other forms in ordinary space? The answer to this question would acquaint us with another supersensible and mathematical form-principle, which is at work not only in physical or etheric space but already in the archetypal (projective) space which underlies them. It is impossible to expound it fully within the limits of this essay.Some explanation will be found in the second chapter, "Music of Number" in *Space and the Light of the Creation,* and in *Strahlende Weltgestaltung.*[9] Here I can only give the following brief indication. In Euclidean space, the principle of form to which we now refer appears in intimate connection with the Number Three (the three dimensions), with the right angle, and with that comparison of distances in radial directions by which the sphere is normally defined. We may remind ourselves how intimately the forms of the circle and the sphere are related to the form of the right angle; witness the theorem of Pythagoras, the elements of Trigonometry, the circular functions, etc. An infinitely distant point of space, by its relation to the sphere, gives rise as we saw to a plane through the centre. For the sphere, this plane is *always at right angles* to the infinitely distant point. That is what differentiates the sphere from the ellipsoid and all other forms. Thinking this out to its conclusion, we see the radii in threes. Proceeding from the centre of the sphere, we behold ever so many sets of three radii at right angles to one another. Each of the lines is at right angles to the plane contained by the other two: These are the well-known Cartesian systems of axes - for physical space, the most natural of all systems of reference - and we can draw them in every conceivable direction from the given centre. The sphere is indeed the perfect image, the all-round and finished prototype of this threefold cross of the dimensions of space. It is related to the three axes as the skull is to the human frame.

Now for ethereal space the essence of the matter is that the sphere likewise determines trinities of lines - not radially in a point-like centre, but as celestial lines in the plane of heaven, in the "peripheral centre." In effect the sphere engenders ever so many right-angled triangles in the plane of heaven. Every such triangle is composed of three lines and points in the celestial plane, just as each system of Cartesian axes is composed of three lines and planes in the earthly centre. Not only the radii in the earthly centre but the celestial lines in the periphery are thus related to each other by the sphere. (In practical Astronomy we measure out the apparent heavens by means of such right-angled triangles, as is indeed well known.)

In modern Geometry we have to understand this principle of form already as it works in archetypal space: that is to say, not yet in the rigid form of the right angle but in the underlying number-rhythm; not in the finished form of the circle but in circling movements and relationships; not in the sphere as a specific form but in the rounding and enclosing potency which archetypal space likewise contains.[33] In effect, the root-ideas of space not only lead to an eternal in-and-outbreathing polarity as between the earthly and heavenly qualities of point and plane; they also lead to a rounding and enclosing principle which comes to birth and maintains itself between these poles, and finds its typical expression in the perfect sphere. Mathematically speaking, this rounding and encircling principle is connected with the great riddle of "imaginary numbers" (see square root of [-1]). It has its concrete spiritual background in what is called the astral. It is a principle more arithmetical and musical than spatial, but it plays formatively into space through all the rhythms of number, which are most beautifully expressed in circling forms. In the real ether-world this principle reveals itself in the so-called sound-ether or chemical ether; "ether of numbers" it is sometimes called.

The supersensible reality to which we here refer lives likewise in the number Three. The trinity that finds expression in the three polar axes at right angles, or in the corresponding right-angled triangle in the plane of heaven, is in the spiritual world-foundation the very same that reveals itself in the soul of man: Thinking and Feeling

and Willing. We here refer to Rudolf Steiner's explanations about the three dimensions of the body and of space itself in their relation to the three forces of the soul. The corresponding triangle in the celestial sphere is described in a lecture given at Dornach on the 11th April, 1920. This is the mystery of macrocosmic and microcosmic Man, towards which we are reaching. Just as in pure Geometry we do not fully grasp the sphere till we are able to create it not only from the centre outward as with an earthly instrument, but from the cosmic periphery inward, so is it with the spiritual origin of these spatial forms. The underlying trinity is at work "as above, so below"; in the cosmic sphere of stars as in the realm of earthly-radial directions.[34]

The plastic modelling of the surface by a moving plane, which we described above in terms of pure ethereal Geometry - this too is the ideal shadow of a supersensible reality. That does not mean that the etheric forces are to be made the subject of a calculating science. We must distinguish here the qualitative and ideal aspect of mathematics, in which the spiritual essences of Form and Number find expression, from the mere quantitative aspect. In lectures given at the Hague on the 8th and 9th April, 1922,[35] on the relation of Anthroposophy to the sciences and to the plastic arts, Rudolf Steiner speaks of the very contrast of physical and ethereal space with which we are here concerned. He describes the latter as a kind of "plastic space." It is the space a sculptor must experience: in truth, as Rudolf Steiner shows, a very different space from the Euclidean. He has been speaking to begin with of the three dimensions of physical space. He tells how we derive the Cartesian form of the three axes or dimensions from our own bodily experience: as up-and-down, right-and-left and forward-backward. Then he goes on to describe how we can presently expcrience a corresponding though more qualitative form in the Heavens, in the sphere of the stars. And he now speaks of a space of forces - forces proceeding not from points but from the infinite sphere.

"It is the secret of this other kind of space that we cannot take our origin in a point and relate everything to this point as in Cartesian Geometry; we must begin from the very opposite. What is the opposite of a point? It is an infinitely distant sphere - a sphere to which we should look upward approximately as we do to the blue firmament of Heaven (supposing that the latter were really there): Suppose that instead of a point I here had a hollow sphere, in the midst of which I was, and that I now related all that is inside it to this hollow sphere. Instead of relating things by co-ordinates to a point as origin, I should determine everything in relation to this hollow sphere..." Here Dr. Steiner indicates in a more or less pictorial form how the powers of this latter kind of space work inward. We must imagine him making a sketch on the blackboard as he continues:

"The kind of space we thus obtain cannot be properly described in terms of the mere three dimensions. Finding our way thus inward from the starry heavens to the conception of another kind of space, we obtain a space which I can only indicate in picture-form. Just as I had to symbolise the former kind of space with the three perpendicular lines, so I should have to indicate this latter space by drawing on all hands some such configurations; *it is as though there were surfaces or planes of forces coming inward from the Universe from all sides, drawing near the Earth, and from without working plastically upon the different regions of the Earth's surface.*

"We do indeed arrive at such a notion when we begin to get beyond what can be seen of living beings, and above all man, with physical eyes - *i.e.,* by means of the physical senses - seeing the physical body. When we have got beyond this and have reached what I am now describing as 'Imagination', when in place of the physical man the Universe begins to open out to us in picture-form and to endow us with a new kind of space - when we advance to this, we recognise what is in fact a second body of the human being, which an old-time intuitive clairvoyance called the 'ether-body' and which we ourselves, as I said just now, may rather call the 'body of formative forces'. It is a supersensible body, made up of fine ethereal substantiality and permeating the physical body of man. *We study the physical body quite truly when we look within the space it occupies for the forces that pervade it. But in this type of space we can never study the etheric body, or body of formative*

forces, which none the less works through and through the human being. We can only study it when we perceive how it is formed from the entire Universe; when we perceive how from all sides these surfaces or planes of living forces are drawing near the Earth, drawing near the human being too, and from without are plastically forming the etheric body."

It is remarkable how Rudolf Steiner speaks, in the passages I have here italicised, of surfaces or planes of forces: of a plastic, plane-like working of the ethereal from the celestial periphery. This is precisely the quality we found in the pure mathematical idea of etheric space. "Plane" is the mathematical idealisation of this ethereal quality, working in cosmic surfaces of force even as "point" is the mathematical ideal form of the physical and earthly. Having regard to the world of plants, we may say: Plane is leaf, while point is seed or eye.[36] When the plant opens and expands into the leaf, it reveals the sun-like element; when it contracts into the point or seed, it reveals the earthly (compare the other passage quoted from Rudolf Steiner, page 27 ("...Through the plant-world the life of the Earth...")). Many organic forms and shapes in living Nature - in embryology and other spheres - will lend themselves to far more penetrating explanation once we are thoroughly pervaded with the truth that space as such originates not only in the point, but in the plane; that it is built not only outward and extensively as from a physical and earthly starting-point, but inwardly, intensively, from the ethereal and celestial.

IV. Physical and Ethereal Forces and Substances

We will now work out this polarity as regards substances and living forces. We shall thus find a fresh ideal access to the ethereal substances and forces, such as are known to Spiritual Science. Euclidean space is like a vessel or container for physical, *i.e.,* material, substance. It is the essence of physical substance as we know it that it occupies a certain space. When space contains etheric substance on the other hand, our spatial sense (being physical) will feel this rather as an emptying, nay more, a hollowing-out of space. That is precisely what Dr. Steiner describes for the interior of the Sun (see the quotation, page 34: "... That which is commonly thought of as the physical constitution..."). We shall be led to this idea with perfect clarity and evidence, and we shall find confirmed what Rudolf Steiner frequently declared in scientific lectures: that the etheric finds expression when in the physical formulae we transform the positive into a negative, provided we make this substitution not merely quantitative but truly qualitative. We thus arrive at the true concept of negative matter - negative substance. So much for the substantial aspect. Speaking of forces in the world of space, the physical-ethereal polarity will find expression in some sense as "Light and Darkness". Just as each point of matter tends towards the earthly centre, so does each ether-plane tend outward to the plane of heaven. And as we call the former tendency gravity or *weight,* so may we call the latter the inherent *light* of the ethereal. Heaviness, gravity upon the one hand, light-lightness on the other, are the true opposites.[37] These two are held in balance when an ethereal entity - an etheric body - is organically joined to a physical, namely the physical body of a living being. Not only is the physical matter of the body uplifted, in spite of the force of gravity, towards the heavens, the ethereal is at the same time held down within the realm of earthly life, in restraint of its constant hunger for the celestial heights. When the living creature dies, the physical substance of the body disintegrates and falls to earth; the ethereal vanishes into the heights. This vanishing of the ethereal must not be thought of in a physically spatial or point-like way (like an ascending balloon) but in a truly peripheral manner. The ether-body vanishes into the periphery, into the circumference of heaven as a whole. It hovers out into the wide expanse. In this connection, see Rudolf Steiner's deeply significant lectures on light and gravity (9th and 10th December, 1920, printed in the series on the Theory of Colour).[38]

These elemental notions of etheric force and substance may now be illustrated by our example of the sphere. The illustration is most natural, for the sphere of all forms is the one that holds most perfect balance between the physical and the ethereal. Thus Rudolf Steiner once described it as the zero- or null-sphere in the transition from positive to negative space and vice versa.[39] It must be borne in mind, however, in what follows that we have greatly simplified the example by choosing our cosmic point and our cosmic plane concentrically in relation to the given sphere (*cf.* page 45 ("... how even Kepler took his start from the traditional idea that the paths of the heavenly bodies must be based on the circle...")). If we do not do so, the form of the sphere for physical space no longer coincides with the corresponding form for the ethereal. The "peripheral midst" of an ethereal sphere is then no longer the "plane at infinity" of physical space, but we shall see it as some other plane. In other words, the ethereal sphere is then no longer peripherally centred in the absolute periphery of cosmic space but in some other, in a relative periphery. We understand Rudolf Steiner's expression, "forces which have, not a centre, but a periphery" (see the quotation, page 30 ("Observation shows... that the phenomena of life have an ...")) - *i.e.,* an individual, a relative periphery.

It will be most important for the geometry of the ethereal to think out these more complicated cases clearly, for we shall then perceive how the ethereal can build up manifold forms and can thus work creatively into the physical, namely the physical

bodies of living beings, which as we know are moulded out of the ethereal. The highly mathematical form of many flowers, for example, may encourage us to develop this geometry of ether-forms. But it would take too long to do so here; I hope to work out some of these things in a later publicarion.[40]

Let us imagine our sphere once more in physical space, and concentrate this time upon the volume it contains. It is a point-like volume, as we said before. It is so many cubic millimetres; if we are "integrating" it, we build it up of infinitesimal spaces, vanishing ultimately into points. Analogously, the ethereal volume of the sphere - that is, the whole of the rest of space, with the exception of the physical interior which is ethereally "hollowed out" - will be composed of planes, or plane-like elements of volume.[41] Just as a point inside the sphere can freely move from point to point of the physical interior, so can a plane move freely in the remaining outer space, which for ethereal thinking is the interior. The geometrical centre of the sphere is at once the centre of the finite physical volume which is shut off from outer space by the sphere's surface. Likewise, the "plane at infinity" constitutes the peripheral centre for an ethereal sphere, also of finite volume. This ethereal volume extends from without inward. What we are wont to call the inner space is in fact hollowed out. The space extending from infinity (from the infinity of physical space, that is to say) in to the surface is for the ethereal space the finite content; the space extending from the surface inward to the cosmic point - finite to physical appearance - is the infinitude of the ethereal space . So we obtain the precise idea of a space negatively filled. That which is physically the infinite and empty space outside the surface - the entirety of space, *minus* the finite sphere inside - is ethereally the finite volume. Conversely, that which is physically the finite volume is ethereally the hollow empty space; the infinite emptiness towards the cosmic point, which remains over when the finite ether-volume is subtracted.

Let us now think of corresponding processes in this space and anti-space. We think, for instance, of a physical effect raying outward from a centre; some manifestation of physical force or energy having its source in the neighbourhood of the given centre. It will go outward in expanding spheres. The farther out it goes, the more attenuated it becomes. It goes outward from the point and "loses itself into the wide expanse" (page 35 ("... The following attempt must here be made...")), The opposite will be true in ethereal space. In the ethereal midst, *i.e.,* in the infinite sphere of Heaven, we may conceive the source of some ethereal influence. It leaves its celestial home and works inward in ever closer spheres towards the cosmic point. The smaller the radius to physical appearance, the greater becomes the ethereal sphere in reality. The process grows into the infinite as it attains the cosmic point. It is a process that "loses itself towards the central point to which it tends", just as if a physical sphere, growing outward, were to lose itself in the infinite periphery. We apprehend the cosmic point no longer as a zero-point but as infinitude, immensity in Rudolf Steiner's paradoxical but true expression, it is a "point which has the area of an infinite sphere, which it turns inward."

Modern geometry will enable us to develop these ideas in all exactitude. Thus we may indicate the apparent measures of an ethereal, inward growth. For the ethereal space the cosmic point is itself infinity. Therefore a uniform ethereal inward growth will appear physically like a gradual congestion towards this point, just as the trees we see in the perspective picture of an avenue become apparently congested towards the infinite horizon. It is in fact the same type of congestion as at the vanishing-points of a perspective - only from all sides at once. We see the spheres of influence grow inward in ever closer circles; yet with ethereal spatial feeling we shall not feel this as a growing closer, as a congestion, but as an endless growth in *equal* stages. The precise measures are as follows. If, to begin with, a sphere grows *physically* outward in equal steps, and we assign to the first stage a radius of unit length, then at the second stage the length

is 2, at the third stage 3, and so on. So we obtain the uniformly growing series of radial distances as from the centre:

0, 1, 2, 3, 4, 5, ...

the 0, of course, referring to the centre. Take now an ether-sphere growing inward, and let this process too take place ethereally in equal rhythmic stages. If we call 1 the *apparent* radius of the sphere that is attained at the first step (that is the radius from the centre, from the cosmic point outward, not the ideal but the apparent radius), then the apparent radius at the next step of inward growth will be 1/2, at the third step 1/3 and so on. At the ethereal origin of the whole process - at the celestial periphery from which we take our start - the physically apparent radius is infinite. Thus we obtain the sequence of apparent measures:

∞, 1/2, 1/3, 1/4, 1/5,...

But the apparent congestion is even greater. For we must first subtract these figures from one another to obtain the measures of successive *steps*. What for ethereal space - i.e., ideally and truly - are the equal steps (1, 1, 1,...) will in the physical appear as the congested series:

∞, 1/2, 1/6, 1/12, 1/20,...

Incidentally, it may be observed that in this simplest instance the physical type of growth can be transformed into the ethereal by the well-known polar-reciprocal transformation.[42] Each point is transformed with respect to the unit-sphere into the corresponding polar plane and vice versa (*cf.* Page 42 ("...We shall perceive that the plane, etherically speaking, belongs to the surface every whit as wholly as the point does physically....")). Thus, for the 2 we get 1/2; for the 3 1/2; and so on. It is a kind of reflection among numbers, whereof the essence is that the nought is mirrored in the infinite and the infinite in the nought. Spiritually too, this mutual transformation of 0 and ∞ is the true picture of the transition from the infinite sphere of Heaven with its divine and cosmic archetypes, to the Earth-centre as the cosmic seed. The latter to begin with is the nought, but it receives unto new life that which is dying in the cosmic spheres We may recall the well-known words *Faust:" In deinem Nichts hoff' ich das All zu finden".*

Let us now think of an ethereal inward process with a far greater impetus. Instead of growing uniformly inward - for physical appearance congested towards the cosmic point, which by this uniform and finite growth it can never reach - let it grow ever faster inward to the infinite, until at last it reaches its infinitude, the cosmic point itself. We apprehend the cosmic point as an ethereal sphere grown infinite. We feel it no longer in its trivial aspect as a dead physical point, but to quote Rudolf Steiner's words once more, as a point which has the area of an infinite sphere. In its ethereal infinitude it is woven of many planes. As in the heavenly sphere for physical eyes there are the countless points of stars, so in the cosmic point are countless planes, weaving it through and through. If this be ever a real process in external Nature - in other words, if at the cosmic point of such an ether-space a living germ, a physical recipient is there - the moment when the ether-sphere, grown inward, lightens upon this point, will signify for the physical a new beginning. Out of another world - from the ethereal spaces inward - new powers of creation have arisen in it. It is new birth into the physical world. Fire or warmth will ray out for instance in this point outward into the physical surrounding

spaces. And from this moment onwards we shall have once more a physically spatial radiation of the accustomed kind, one which in turn will "lose itself into the wide expanse." This is a process akin to that which takes place at the growing tip of every plant. (Figure 14 is *Euphorbia Wulfenii,* a microscopic picture.)

We can imagine the latter process in its turn going outward into the infinite, till it attains the cosmic plane and is there changed again to an ethereal activity which once again works inward. Where the physical loses itself in the infinite the ethereal is born, and vice versa. We thus arrive at the idea of a rhythm not only outward and extensive but qualitative: ether-space losing itself into the physical, physical thereby arising and in its turn losing itself into the cosmic vast, for the new birth of an ethereal once more. Such possibilities are the quite natural outcome of the idea of space and anti-space. Thought is no longer bound to a once-given rigid space within which only extensive movements would be possible. We conceive processes whose essence lies in the becoming and unbecoming of spaces, in the creation of spaces and their annihilation. Rudolf Steiner once described the essence of warmth or heat as an "intensive movement" in this very sense. When, for example, as in a flame, chemical forces work themselves out and give rise to warmth, we may well have this type of process. Chemical action is not in truth contained within the limits of physical space.[43] It is a temporary makeshift when we represent it with our physically spatial symbols: atoms and molecules, structural formulae and so on.

For the understanding of living warmth as opposed to dead warmth, it will be most significant to conceive this working inward from ethereal spaces and welling forth anew within the point. Manifold qualities and elements of form can be at work in this process; the concept of the sphere is but the matrix, as it were, wherein all manner of effects of this type may be differentiated. With the idea of space and anti-space as here evolved, we have an access to such processes as Rudolf Steiner often described out of his Spiritual Science. How these things are in actual reality, experience alone can tell, whether it be by way of outer experiment or by supersensible investigation. But the right forming of ideas is in either case important.

It is a beautiful exercise in thought, to imagine the aforesaid process: the working inward of an ether-sphere from cosmic distances, the merging of it in the central point as its infinitude, the sudden qualitative reversal, whereat the point is changed in function from ethereal infinitude to the beginning of new physical existence and in its turn sends forth a physical expanding sphere, until the latter once again reaches the sphere of heaven and again a qualitative change takes place - this time in the opposite direction - and a fresh ether-sphere once more grows inward...

According to Dr. Steiner, the processes in real Nature, *e.g.,* the radiations of light and heat and other forces, do not lose themselves in vague infinitude but only go out to a certain sphere and then return into themselves, transformed in quality. From the aspect of space and anti-space we can imagine well this qualitative type of rhythm.

Let us now dwell for a moment on the elementary contrast of physical and ethereal forces or, as we called them typically, "Gravity and Light". Once more the sphere will serve as illustration. A sphere of physical matter reveals its essential nature in forces of pressure, Gravity has a contracting tendency. Each particle of matter tends towards the centre, namely the "centre of gravity" of the whole. Therefore if this force only were at work, the entire sphere would fall at least into the single point. But there is another property, equally characteristic of physical matter, namely its "impenetrability". Each particle maintains the space it occupies. Thus as a counteracting force we have the elasticity, the sustaining power, which "gives" a little but in the end withstands the inward-drawing pressure. According to the specific weight and the specific space-asserting tendency or elasticity of the material in question, a certain balance is created in the play of pressure and counter-pressure.

Over against this we may now evolve, to begin with in pure thought, the idea of an ethereal sphere of substance. We have already seen that it is filled, not from within outward but from the cosmic periphery in to the surface. Its ethereal midst is not a centre of gravity but Heaven's plane of light. Moreover it is formed of planes, not points. Each of its planes tends to return as with a thirst of light to the celestial periphery. The latter is not only the *geometrical* middle; it is the peripheral centre for the *forces* of the ether-sphere. Just as the physical body has its dynamic centre in a "centre of gravity", so the ethereal body has its dynamic centre in a middle "plane of light," or lightness.[37] So we arrive at the concept of "negative weight" - negative gravity. It does not press the sphere of substance inward; it sucks or draws outward. But the substantial nature of the ether-sphere gives it likewise a kind of space-maintaining quality, albeit negative instead of positive. It contains forces that withstand the quick return of all its parts into the plane of heaven. Just as the physical matter maintains the space it occupies by outward-carrying elastic forces, so the ethereal space maintains its space by *inwardly* sustaining forces. It lives in the balance of its negative space-filling and of its negative gravity. We thus obtain with full exactitude the idea of qualitatively suctional forces. (The suction-forces of ordinary mechanics are only metamorphosed pressures. The suctional forces to which we here refer are from the outset qualitatively different, even in their relation to space.)

In such an ether-sphere, if as a consequence of an excessive "negative space-filling" a sudden discharge, as it were an explosion were to come about, this would work inward through the surface of the sphere towards the cosmic point. It would be the very opposite of what occurs in the physical volcanoes of the Earth. Such are the forms of thought which Rudolf Steiner calls for in the interpretation of solar phenomena (page 34 above ("..."that which is commonly thought of as the physical constitution of the Sun...")). But they are also of importance within the earthly kingdoms of Nature, for the sun-quality is at work here too. Thus for example Dr. Steiner told us how a sun-like element is at work in the human being, in all sensation or sense-perception. In the sense of touch for example we become aware of the pressure of physical bodies on our own. Our ethereal body answers the pressure with correspondin forces of suction. Sensation arises in this play of forces. I quote again from the aforesaid lectures on the relation of Astronomy to the different branches of science:

> "... This force of suction, even as it proceeds from the Sun, works also in the human being, permeating his etheric body from above downward. In the human body therefore, two opposite entities are at work - solar and earthly. We should be able to prove in detail that these things are there, and we can do so, once we become aware of the true interpretations... If I press here with my finger, there will arise over this surface the force of pressure whereby this ponderable matter presses against me. The counter-pressure corresponds to the force of the Sun which works through me - namely, through my etheric body. Imagine here a surface pressing against the human being - or against which the human being presses - and you have the working of the ponderable and the imponderable forces. It is the interplay of the ponderable pressure from without inward, and the imponderable from within outward, which gives the conscious sensation of pressure. If we perceive it clearly we may truly say that the polarity of Sun and Earth, into the midst of which the human being is placed, is really felt by us in every sense perception."

Finally, I will draw attention to an aspect of ethereal space which will not be unimportant for the understanding of homeopathic effects.[44] We spoke just now of the physical outward growth and of the ethereal inward growth of a sphere. Let us reverse the thought and imagine a physical sphere decreasing in magnitude - shrinking towards its centre. If it contains physical substance, this latter will grow concentrated, as the same quantity is now distributed over an ever smaller volume. What of the

corresponding ethereal process? The sphere becomes attenuated, it "shrinks outward" (physically, and therefore paradoxically speaking). Externally it grows in size, but the apparent growth is not a real one; in fact the ether-sphere is becoming smaller. It becomes ever nearer to the celestial sphere, its middle plane.

If we have practised the idea of ethereal space, we are soon capable of feeling two processes quite differently though they may be coincident in outward spatial appearance. Externally, for example, there is a spherical space growing larger. If we conceive it physically, we shall feel it truly as a growth. But if we think of it as an ethereal process, we shall feel it as a growing-smaller, a peripheral contraction into the celestial plane, which is ethereally speaking the "sphere of infinitesimal dimensions".

Precisely here it will be seen how needful it is to find the true idea for any process in Nature, instead of blindly applying ideas that may after all be foreign to the essence of it. Think of an actual space, a spherical flask for example, filled with a physical substance, say a watery solution. If we apply the accustomed physical ideas of space, we shall say: "If we make the space smaller, say by evaporating it down, the solution will grow more concentrated, and as we concentrate it the action of the substance will become more intense." It is as obvious as can be.

How will it be on the other hand if the said physical substance is united with something ethereal - if for example as a vegetable essence it is connected with life, or as a metal salt with cosmic life from pristine ages of the Earth. In so far as something physical is present as the bearer of ethereal life, it will act like the Earth itself in its relation to the celestial forces. That is to say, it will act in a germinal way - in that wider sense of the term which we explained at the beginning. A sphere of ether-forces has its "cosmic point", its "infinitude", in the domain of this portion of physical substance. Wherever we transport the latter, the ethereal sphere goes with it. The essential thing about this comparatively small physical volume is not the physical as such, but the ethereal sphere it carries with it. The latter is united with the physical substance, which was at one time the goal of its activity. It has not left it yet, for if it had, the substance would be dead and of no more use for the effect in question. What happens now when we dilute the physical substance? We really concentrate (though the expression is contradictory), *i.e.,* we intensify the specific sphere of ethereal force and substance which is united with this physical. Thus it is fully intelligible - nay for ethereal space just as evident as is the opposite for purely physical conditions - that by dint of physical dilution we can generally make the ethereal influence more strong. This is precisely the principle of homeopathic dilutions, which to our physical thinking are for so long a cause of offence. They seem absurd so long as people cherish thought-forms which appear with incontrovertible logic to demonstrate their uselessness. There are of course those others who despite all theoretical objections let the facts of experience convince them.

Here too it is most important that we turn our thoughts in the right direction and develop above all the right spatial feeling. The point is not, by a mere polar transformation automatically to translate all physically spatial or even mechanical ideas into their opposite and to develop the ethereal geometry in the same analytical, indifferent and merely causal manner in which the physical has all too often been treated. The polar transformation does indeed reflect a fact of Nature, nay an archetypal truth of our Universe; of this there can be no doubt. Nevertheless it should be used with care; we need not fall again into the old dogmatic errors. Our aim in this respect, so far as pure thought is concerned, should be that the polarity of physical and ethereal spaces may teach us to regard the former in a more living way, not to develop the latter with all the old intellectualism. The very nature of the case will guide us well, if we do not make too much of mere formal cleverness and speculation. For in this realm we have no longer to do with a ready-made and rigid space, but with the

coming-into-being and the passing-away-again of spaces in the play of opposites. Into this coming and passing of spaces, purely spiritual influences also will work. Space is no longer a self-enclosed system.

Our last example may illustrate this point. For the ethereal effects it is significant that the intensifying action - that is, the apparent physical dilution - should not take place in an indifferent way but with a certain rhythm; what is called "potentising." A certain element of time and number is important.[45] Though we might not foretell by spatial theory that this must be so, yet we can well appreciate the fact, for it reminds us of that deeper insight which to a great extent showed the beginning of this way of thought. The fundamental polarity of Heaven and Earth is the spatial counterpart of a mystery of Time. It is indeed "Time which becomes Space", when we understand the latter in the spirit of the new Geometry. In its becoming and in its passing, Space here receives rhythms of Time and Number from a purely spiritual world. Thus does the cosmic life begin to be revealed, and from the side of pure Geometry the way is also opened into that spiritualising of all spatial knowledge for which the Spirit of our Age is calling.

Notes and References

1. George Adams (who changed his name from George Adams Kaufmann in 1940) wrote his main geometrical work in German: *Strahlende Weltgestaltung: Synthetische Geometrie in geisteswissenschaftlicher Beleuchtung,* published in 1934 by the Mathematical Section at the Goetheanum, Dornach, Switzerland. In 1960 there appeared another main work: *Die Pflanze in Raum und Gegenraum: Elemente einer neuen Morphologie,* by George Adams and Olive Whicher, published by Verlag Freies Geistesleben, Stuttgart. In the former work George Adams set forth the fundamentals of Modern Geometry with many illustrations in a widely accessible form, relating it to the history of culture in general and especially to the teachings of Spiritual Science as imparted by Rudolf Steiner. In the later work the theory of Physical and Ethereal Spaces is brought to bear especially on the phenomena of plant form. Here, the leading ideas of Goethe's *Metamorphosis of Plants* are interpreted and developed in a fresh direction, with the help of scientific methods not yet available in Goethe's time. / In the years between these two works, besides numerous articles and essays, there came the two English works of which *Die Pflanze in Raum und Gegenraum* is a further development: *The Living Plant,* 1949 and *The Plant between Sun and Earth,* 1952, both published by the Goethean Science Foundation, Clent, Worcestershire. / References to the German book, *Die Pflanze in Raum und Gegenraum,* are included here because this work is far more substantial than the two English books. It is hoped in due course to publish an English translation.

2. Rudolf Steiner: *Universe, Earth and Man* (The Rudolf Steiner Publishing Co., London), a lecture course held in Stuttgart in 1908.

3. Of the origin of Geometry in the ancient Mysteries, see Rudolf Steiner's lectures, *Flashlights on the deeper Impulses of History.* See also the book by Ernst Bindel: *Die aegyptischen Pyramiden als Zeugen vergangener Mysterienweisheit,* Verlag Freies Geistesleben, Stuttgart, 1957.

4. Rudolf Steiner's second lecture-course on Science, Lecture 14 (Dornach, 14th March, 1920).

5. This was done in the work on plants and in other fields. See also *The Threefold structure of the World,* in the *Golden Blade* of 1953. (Rudolf Steiner Press, London.)

6. *Die Pflanze in Raum und Gegmraum,* Chapters II and III, especially §46; *The Plant between Sun and Earth,* §24; *Space and Counterspace:* George Adams, M.A., in *The Faithful Thinker, Centenary Essays on the Work and Thought of Rudolf Steiner* (Hodder and Stoughton, London, 1961).

7. Rudolf Steiner: *Occult Science, an Outline* (Rudolf Steiner Press, London, 1963), pages 115-129.

8. Rudolf Steiner: *Occult Science,* page 141.

9. It is no doubt possible to speak of a "sphere (or circle) of infinite dimensions," for we thereby suggest the continued growth of a sphere or circle beyond all finite limits. The point is that when a sphere expands into the infinite, it at last merges in the infinitely distant *plane.* When the radius is infinite, it is no longer a sphere in the strict sense of the word; it is a plane. As it becomes a point when it contracts without limit, so does the sphere become a plane when it expands. See also George Adams Kaufmann: *Space and the Light of the Creation,* pages 31,32, published by the author, London, 1933.

10. Readers acquainted with analytical Geometry will recognise that this is precisely what happens when we pass from "point co-ordinates" to "plane co-ordinates." When we write the "equation of a plane" (in point co-ordinates, in the familiar Cartesians for example) we are envisaging the plane, not as a whole, but as the sum-total of infinitely many points, namely all points that satisfy the equation. On the other hand when we write the point in plane co-ordinates, we are in fact considering the point, not as an indivisible entity, but as made up of infinitely many planes. The idea of plane co-ordinates, discovered by Plucker and others in the nineteenth century, was only made possible by the spirit of modern synthetic geometry.

11. Rudolf Steiner: First Science Course (23rd Dec. 1919-3rd Jan. 1920), Lecture VI.

12. See Louis Locher-Ernst: *Raum und Gegenraum* (Dornach, 1957), Nr. 4, page 24. (Refer also to note 16.)

13. Rudolf Steiner: *The Michael Mystery,* the letters of January 1925 and Christmas 1924; *What is the Earth in reality in the Macrocosm?* and *The Mystery of the Logos.* (Anthroposophical Publishing Co., London, 1956.)

14. See a lecture by Rudolf Steiner (17th December, 1922) about the spiritualising of the knowledge of space in *Man and the World of Stars.* (Anthroposophic Press Inc., New York, 1963.)

15. Gerbert Grohmann: *Die Pflanze* and *Die Pflanze als Lichtsinnesorgan der Erde,* Verlag Freies Geistesleben, Stuttgart, 1962.

16. The working of "cosmic forces" has especially been dealt with by Dr. G. Wachsmuth in his book *Etheric Formative Forces in Universe, Earth and Man* (Anthroposophical Publishing Co., 1932) and in other works. In recent years these forces have been investigated and confirmed in manifold directions by anthroposophical experimenters. In this essay the attempt is made, following certain indications given by Rudolf Steiner, to develop *mathematical* conceptions corresponding to the nature of such forces, and in particular -as an initial step in this direction - the conception of a counter-space or peripheral space. / Other works by George Adams, which aim to develop in this sense the fundamental concepts of physics and physical chemistry, have appeared in the Korrespondenz of the Mathematisch-Physikalischen Institut, Dornach, Switzerland, led by Dr. Georg Unger. See George Adams: *Universalkrafte in der Mechanik* and also works by G. Balaster, M. Martin and G. Unger. / Quite independently, Louis Locher-Ernst developed the mathematical basis for a peripherally formed negative space or "Gegenraum" (he called it "polar-Euclidean space"). See Louis Locher-Ernst: *Projektive Geometrie und die Grundlagen der Euklidischen und Polareuklidischen Geometrie"* (Zurich, 1940) *Zur mathemattschen Erfassung des Gegenraums;* Mathematisch-Astronomische Blatter, Heft 3, Dornach, *1941. Raum und Gegenraum, Eisfuhrungin die neuere Geometrie,* published for the Mathematisch-Astronomieche Sektion, Goetheanum, Dornach,1957.

17. A most important step in metamorphic thinking was achieved in the new algebra (the so-called theory of invariants) in the middle of the nineteenth century by Cayley, Sylvester and others (see *Strahlende Weltgestaltung).*

18. See *Strahlende Weltgestaltung* and *Die Pflanze in Raum und Gegenraum* (see Note 1)

19. An English edition of the *Metamorphosis of Plants* with introduction by Agnes Arber was published by the Chronica Botanica Co., Waltham, Mass. (H. K. Lewis and Co., London), 1946.

20. *Die Pflanze in Raum und Gegenraum,* Chapter II; *The Plant between Sun and Earth,* § 13; *The Living Plant,* Chapter III; *Space and the Light of the Creation.*

21. *Die Pflanze in Raum und Gtgenraum,* Chapter III; *The Plant between Sun and Earth,* §§ 14, 15; *The Living Plant,* §§ 27,28.

22. *Goetheanum Weddy* (1933), Nos. 4 to 6.

23. See Felix Klein's *Lectures on non-Euclidean Geometry* (Berlin, 1928), pages 182, 183, 189. In a short article published in 1910 *(Proceedings of the Edinburgh Mathematical Society,* Vol. 28), Professor D. M. Y. Sommerville enumerates no less than twenty-seven conceivable geometries of threedimensional space. Among them are the Euclidean and the two wellknown non-Euclidean geometries. One of the twenty-four others is the geometry of "anti-space" which is here intended. Somewhere in mathematical literature there may be further developments in this direction, I have not found them. Interest has generally centred on such spaces as are more nearly in accord with the conditions of physical imagination, or else, alternatively, the geometry of abstract spaces of any number of dimensions has been worked out, quite without reference to the imagination or to the forms of nature. The paper here referred to is also mainly analytical in form.

24. W. K. Clifford, the eminent English mathematician, sought refuge from this painful difficulty in the hypothesis that *cosmic* space is not Euclidean at all, but curved that is to say, non-Euclidean. For in the

one type of non-Euclidean geometry (in the so-called "elliptic space") the pure polarity remains unimpaired. "Upon this supposition of a positive curvature, says Clifford *(Lectures and Essays,* Vol. I, page 322), the whole of geometry is far more complete and interesting; the principle of duality, instead of half breaking down over metrical relations, applies to all propositions without exception. In fact I do not mind confessing that I personally have often found relief from the dreary infinities of homaloidal space in the consoling hope that, after all, this other may be the true state of things". (The "dreary infinities" of course, are changed into quite another aspect when one recognises the reality of the ethereal. That was not possible in Clifford's time.)

25. See Rudolf Steiner's letter: *The Michael's Mystery,* letter XXIV.

26. *Die Pflanze in Raum und Gegenraum,* Chapters II and III, in particular §§ 23, 24, 25, 37, 38. See also the two English books (Note 1).

27. See *Strahlende Weltgestaltung,* in particular Chapter VII and *Die Pflanze in Raum und Gegenraum,* § 27. See also Note 13)

28. Rudolf Steiner: *True and False Paths in Spiritual Investigation* (Anthroposophical Publishing Co., 1926), Chapter III. See also *Space and the Light of the Creation,* Figures 10 and 19.

29. *Die Pflanze in Raum und Gegenraum,* §§ 38-40. *The Plant between Sun and Earth,* §§ 23, 27, 28; *The Living Plant,* § 35.

30. Rudolf Steiner, in lecture III of the third science course, (jan. 1st - 18th, 1921.) One aspect of this question is dealt with by Louis Locher-Ernst in *Goetheanum,* 18th October, 1953: "Was sagen uns Keplers Gesetze fur die Welt der Bildekrafte?"

31. *Die Pflanze in Raum usd Gegenraum,* § 57; *The Plant between Sun and Earth,* § 28; *The Living Plant,* §§ 10, 28, 39·

32. *Die Pflanze in Raum und Gegenraum,*§§ 59,60; *The Plant between Sun and Earth,* §§ 29, 3I; *The Living Plant,* §§ 46, 47

33. *Die Pflanze in Raum und Gegenraum,*§§ 33-36; *The Plant between Sun and Earth,* §§ 4, 5, Note 9; *The Living Plant; Space and the Light of the Creation,* Chapter II.

34. See Rudolf Steiner's letter in *The Michael Mystery,* No. XXIV. See also the articles by Olive Whicher on "Human Movement" in *Gymnastic Education,* by Bothmer (published by the Goetheanum, Dornach) and in the *Golden Blade,* 1960 and 1962 (Rudolf Steiner Press, London).

35. Published in translation in the *Golden Blade,* 1961.

36. *Die Pflanze in Raum und Gegenraum,* §§ 38, 43; *The Plant between Sun and Earth,* § 10; *The Living Plant,* §§ 5,24,42,45.

37. The ethereal nature of light is as yet imperfectly understood. It is no mere matter of chance that the word "light" or "lightness" has its apparently double meaning. The wisdom of language is here in advance of our physical understanding. Goethe was more aware of the true polarity of "Light and Darkness," to which idea the science of the future will without doubt find its way back again. In his book *Man and Matter* (Faber & Faber, 1951), Ernst Lehrs writes about "lightness" or "levity" as the opposite pole of gravity and about Space and Counterspace. There appear here the apt expressions coined by him of "all-relating point" and "all-embracing plane." (Chapter XII, page 215.)

38. Rudolf Steiner's Farbenlehre (II). English translation: *Colour* (Rudolf Steiner Publishing Co., 1935)·

39. Second Science Course (March 1st-14th, 1920), lecture VII.

40. *Die Pflanze in Raum und Gegenraum,* Chapter VII; *The Plant between Sun and Earth,* §§ 9-12; *The Living Plant,* Chapter VII.

41. If as an element of physical volume we take the content of an infinitesimal sphere about the point in question, as is often done in mathematical considerations, the corresponding idea in ether-space would be the "etheric volume" of an exceedingly flattened hyperboloid of rotation, enveloping the plane on either side. (The cosmic point is at a focus.) If on the other hand a cube is taken as the physical space-element, the corresponding thing in ether-space would be a kind of octahedron. Physically speaking, the ethereal elements of volume interpenetrate; compare what was said on page 22.

42. This well-known transformation has an important bearing on the relation of ethereal geometry to physical. Deeply related as it is to the archetypal idea of space ("inner and outer", and their mutual reflection), it occurs often and from diverse points of view in anthroposophical mathematical writings. Nevertheless its connection with the ideas we are here setting forth is not quite so far-reaching as might appear from the much simplified example we have chosen. The notion of the "cosmic point" and of the consequent "negative space" is an essentially new one. (The qualitative, polar reciprocal transformation - points into planes, planes into points - which was described on page 42 and to which we also now refer, is not to be confused with the more frequently described "inversion", which only changes inner points into outer *points* and vice versa. Both are important, but it is the former which will aid us most in the transition of thought from physical to ethereal spaces. It is in fact the more elementary of the two; yet it demands stronger activity in qualitative thinking.)

43. Second Science Course, lecture IV; *Die Pflanze in Raum und Gegenraum,* § 48; *The Plant between Sun and Earth,* § 23.

44. The effects of highly potentised dilutions or "smallest entities" have been abundantly tested and proved in a great variety of natural phenomena by L. Kolisko and many others.

45. See Theodor Schwenk: *Uber einige feinere Vorgange bei der Herstellung ¸flüssig potenzierter Heilmittel* (Weleda-Verlag Arlesheim, Switzerland); and George Adams: "Potentisation and the Peripheral Forces of Nature" in *The British Homoeopathic Journal,* Vol. **L,** No.4, October 1961.

The reader is referred for further mathematical and scientific data to the very extensive Notes and References in *Die Pflanze in Raum und Gegenraum* and *The Plant between Sun and Earth.* Good elementary textbooks on Projective Geometry to be recommended are those by Cremona, L. N. G. Filon and J. L. S. Hatton. Unfortunately, most of the easier works still fall back on Euclidean metrical conceptions, instead of deriving these from purely projective notions; for the transition to ethereal space an understanding of the latter process is of course essential. Of the greatest value are: Veblen and Young, *Projective Geometry,* Vols. I and II, Boston, 1910-18; D. M. Y. Sommerville, *Analytical Geometry of Three Dimensions,* Cambridge, 1934; also the various lecture-courses by Felix Klein (Springer, Berlin, 1924-28), English editions of some of which have since been published in the United States.

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