THE STRUCTURAL BASIS OF ARCHITECTURE

THIRD EDITION

BJØRN N. SANDAKER, ARNE P. EGGEN
& MARK R. CRUVELLIER

ROUTLEDGE

THE STRUCTURAL BASIS OF ARCHITECTURE

This is a book that shows how to "see" structures as being integral to architecture. It engages a subject that is both about understanding the mechanical aspects of structure as well as being able to relate this to the space, form, and conceptual design ideas that are inherent to the art of building.

Analyzing the structural principles behind many of the best-known works of architecture from past and present alike, this book places the subject within a contemporary context. The subject matter is approached in a qualitative and discursive manner, illustrated by many photographs and structural behavior diagrams. Accessible mathematical equations and worked-out examples are also included so as to deepen a fundamental understanding of the topic.

This new, color edition's format has been thoroughly revised and its content updated and expanded throughout. It is perfect as either an introductory structures course text or as a designer's sourcebook for inspiration, for here two essential questions are addressed in parallel fashion: *"How do structures work?"* and *"What form do structures take in the context of architecture – and why so?"* A rich, varied and engaging rationale for structural form in architecture thus emerges.

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Our traveling globe in galactic endlessness is divided into latitude and longitude.

With help of this grid, every point on the earth's surface has its number.

At the grid's intersections each plant, each creature receives
its individual technology – its structure formed and created
by the clouds' movements, the wind's strength, and the shifting positions of the sun.

On this organic mat, the acrobat (builder) attempts, with the help of instruments,
to deceive gravity and challenge death with every leap.

And when the perplexities of thought within your soul is provided space on earth,
arises a duel with substance. Amidst brutality's heat,
beauty is born…

Sverre Fehn
(1924–2009)

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Contents

We wish to dedicate this book to two groups of people without whom none of this would have been possible or worthwhile:

To our immediate families – Wenche, Victoria, Nicolay, and Sophie; Sigrid, Sune, Dan, Aron and Maia; and Patrick and Lauren – we are ever grateful for your longstanding support and sacrifices,
and

To our many students over the years as well as those yet to come.

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Preface

This is a book about structures, more specifically about structures and architecture; it is certainly not the first such book, nor will it be the last. It does represent, however, our view of how to engage a subject that is both about understanding the mechanical aspects of structure as well as being able to relate this to the space, form, and conceptual design ideas that are inherent to the art of building – in other words, how to "see" structures as being fully integral to architecture. It is at once a book that deals with the subject matter in a qualitative and discursive manner, that illustrates this discussion by means of many photographs of architectural projects and structural behavior diagrams, and yet that also doesn't shy away from the relatively accessible mathematical equations and calculations that can be used to reinforce and extend a nascent understanding of the fundamentals of the topic – indeed, there are many ways to learn about and from structures. The lessons about structural forms and behaviors can be derived from building designs that span the course of time, and are here drawn from both the architectural canon as well as recent projects from around the world. Beyond this, we also briefly engage with art and furniture design, among other related fields of interest, as a means of connecting structural principles to a broader cultural context and vastly different physical scale.

Much has happened in the world of architecture since the publication of the first edition of *The Structural Basis of Architecture* in 1989. Stylistic periods such as those of High-Tech, Postmodernism, Deconstructivism, Starchitecture and Blob Architecture have waxed and waned, while Parametric and Computational Design are currently in vogue, as is architecture that is strongly influenced by Sustainability concerns and objectives. The range of examples that are featured in this third edition partially reflects these ongoing changes while at the same time not losing sight of the remarkable designs of earlier periods, most of which still serve as frequent and useful references for building designers today.

In terms of developments in the understanding of structural mechanics, on the other hand, it can be argued that things have been much more stable and that not much is new: statics is still what it was, and beams and domes span space in the manner that we have come to know and understand for hundreds of years, let alone the past 30. And while it is certainly true that computer methods for analyzing structures' forces and stresses are much more prevalent and efficient today than they were three decades ago, nevertheless these programs have not really changed our fundamental understanding of the subject matter as much as sped up its application. Indeed, it has been recognized in both academia and in practice that there can be a certain danger in depending too much on the "black box" of analysis programs without a strong understanding of basic structural behavior. And so, while we recognize and in several places include projects that demonstrate the results of structural analyses derived from such computational advances, it will become evident throughout this book that we still firmly believe in an engagement of the subject matter using simple algebraic formulas and mathematics as well as discussing it in terms that are familiar to us from our everyday living experience. Not only do we see this approach as a means of developing an intuitive basic understanding of how structures work and how their forms make sense, but also that it enables more conceptual thinking on the part of architects and structural engineers alike for extrapolating into uncharted territory. That being said, it can legitimately be argued that where digital technology has had its biggest impact recently is in challenging the age-old building design adage that keeping things simple and repetitive and rectilinear is necessary in order to make construction economically viable. Today, buildings with seemingly infinite variations of member lengths and geometric details can be relatively easily accomplished because of remarkable advances in integrated digital fabrication technologies; some examples of this approach are included in the following chapters, right alongside the more "traditional" – but no less exceptional – forms of building structures.

This third edition of *The Structural Basis of Architecture* shares its title, vision, and basic organization with the original book, although even a cursory comparison will reveal that the contents have been completely revised and the scope substantially expanded since that earliest version. And whereas the second edition involved a comprehensive overhaul of the original, from rewriting the text to expanding and updating the range of illustrated examples, this third edition can perhaps better be characterized as a significant evolutionary step in terms of the development of the book's contents. In that sense, those familiar with the previous edition will recognize and find comfort in numerous similarities. That being said, there are also substantial changes in this new edition that are worth drawing attention to here:

- A new Chapter 2 Introducing Structural Systems serves right from the start to identify fundamental structural actions, consider the basic types of structural elements that can respond to these actions (skeletal vs. surface), and then project how such elements can be combined into three-dimensional building structural systems of various configurations, each having implicit spatial qualities and distinctive forms.

- A completely revamped Chapter 10 The Frame and the Shear Wall greatly expands on the previous treatment of lateral load resisting systems, which we felt in retrospect had been somewhat short-changed in the second edition given their relative importance in the design of buildings – whether from a structural or spatial or conceptual point of view.
- An extended treatment of selected topics in several other chapters, including fleshed-out sections on beam grids, slabs, retaining walls, space frames, etc.
- The addition of many new examples (and the replacement of others) in order to refresh the contents, although without making change just for its own sake; i.e., what we thought served the purpose well in the previous editions has largely been retained.
- And perhaps most obviously at first glance, changes have been made to the layout format: e.g., most illustrations are now in color, more emphasis has been placed on the explanatory structural behavior diagrams, and the running text now has direct call-outs to corresponding illustrations and figures – the better to allow the reader to directly connect images to text commentary. Also, the page layout for this third edition has been changed to a two-column format that more frequently enables text passages to be placed adjacent to related images.

Finally, for those who would like to extend their exposure to the structural basis of architecture, it should also be noted that since the publication of the previous edition of this book two of the present authors – Cruvellier and Sandaker – have co-authored along with colleague Luben Dimcheff the companion book *Model Perspectives: Structure, Architecture, and Culture* (Routledge, 2017). That book's reproductions of many short, insightful essay extracts as well as large-format photos of constructed model studies are intended to be complementary ways of addressing the essential questions at hand in the pages that follow: i.e., *"How do structures work?"* as well as *"What do structures look like in the context of architectural design – and why so?"*

Acknowledgments

Of course, there have been many people who have contributed in one way or another to the contents and production of this book. We are particularly thankful for the excellence, dedication, and patience of several student assistants, whether for the collection of illustrations and rights permissions and the production of line diagrams. At Cornell: Jeremy Bilotti, Ainslie Cullen, Lucy Flieger, Vanille Fricker, Raksarat Vorasucha as well as Patricia Brizzio and Asdren Matoshi. At AHO: Oda Nybø. We have also certainly benefited from the strong support of the administrative leadership of both our respective institutions, and wish to extend our gratitude for this. In particular, at AHO we wish to thank: Dean Ole Gustavsen and Department Chair Thomas McQuillan. At Cornell: former Dean Kent Kleinman and current Dean J. Meejin Yoon and Department Chair Andrea Simitch. And at both institutions, informal discussions and other collaborations with many faculty colleagues both past and present have significantly contributed to this work in various ways, whether by express intention or fortuitous circumstance. In particular, Assistant Professor Solveig Sandness at AHO deserves special mention; her detailed feedback about the second edition's contents has been invaluable.

The patience and support of Routledge Publishers has once again been truly remarkable – in particular, Senior Publisher Fran Ford and Senior Editorial Assistant Trudy Varcianna have helped carry us along and through to the finish line. We are also greatly indebted to the skill and vision of the production team at Routledge, namely Senior Production Editor Alanna Donaldson and layout designer Alex Lazarou for giving this third edition of the book a fresh and compelling new look, but also the copy editors, printers, etc. without whose assistance and commitment this work would not have come to fruition.

Structuring Space

CHAPTER 1

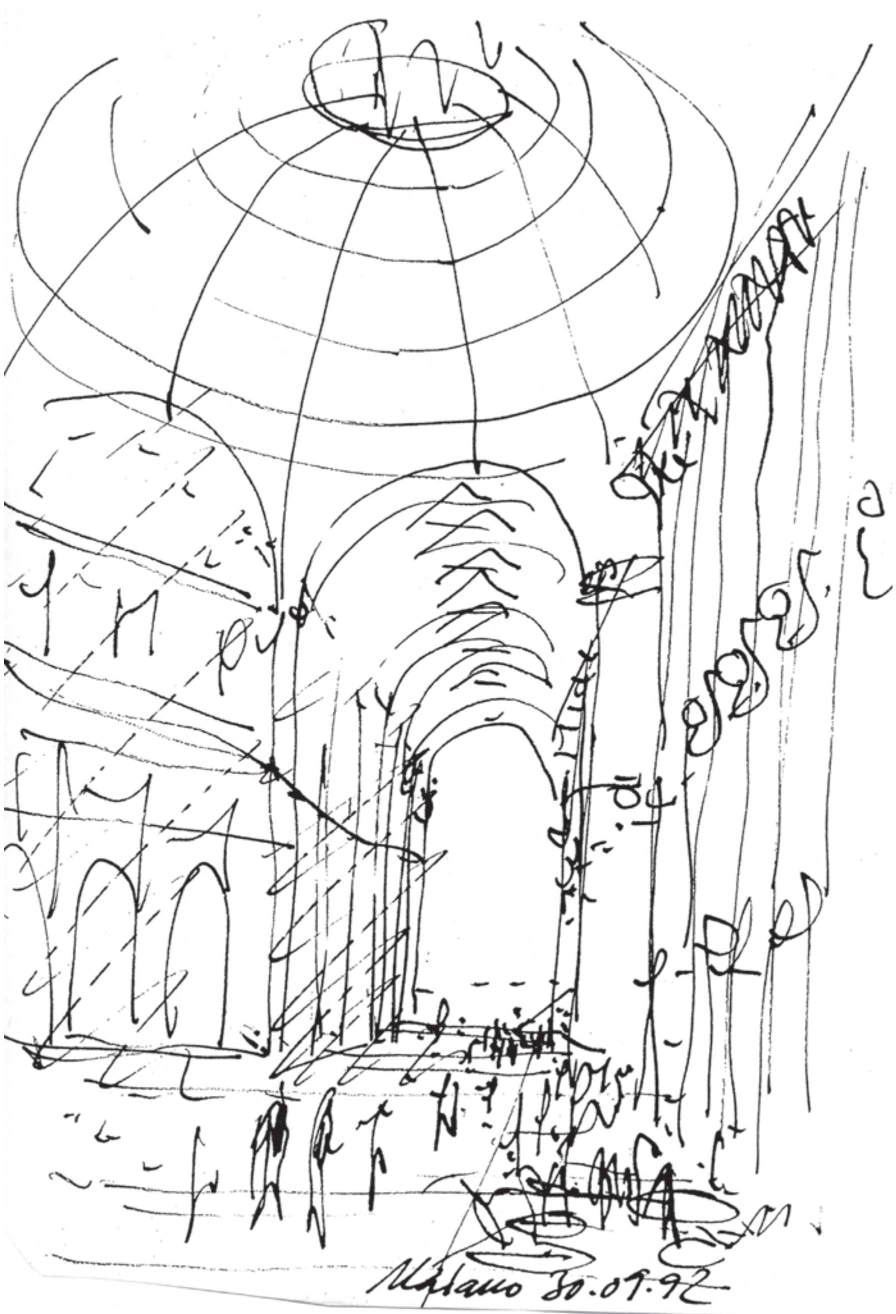

Illustration 1.1
Galleria Vittorio Emanuele II, Milan, Italy (1865–1877).
Prominently sited on the northern side of the Piazza del Duomo, this galleria is a covered double arcade formed by two glass-covered vaults at right angles to each other and intersecting in a domed, octagonal central space.
Architect: Guiseppe Mengoni.

1.1 Structure as Spatial Generator and Mechanical Object

While it is easy to imagine structures without architecture, there can be no architecture without structures. Examples of the first category include construction cranes and transmission towers – structures whose sole purpose it is to keep loads lifted up off the ground. In architecture, the design of buildings commonly includes roofs, floors, and walls whose weight must also be borne and balanced by the help of structures. But beyond that, these elements are typically informed by requirements and conceptual ideas for their interior spaces and exterior forms. Structural issues, therefore, are inherently deeply embedded in architecture. The specific relationship between architecture and structure, however, whereby the one encompasses the other, may vary greatly from one architectural epoch to the next, or even from one building to another within the same time period. Today we are likely both to encounter buildings whose structures are of minor interest for architectural expression as well as others that display a particularly close correlation between structural form and its negative imprint, architectural space.

In order to shed some light on the particular connections that exist between structures and architecture, we first need to establish what we consider to be basic structural functions. Toward this end, we may ask: What purpose does the structure serve? What requirements govern the conditions establishing its overall and detailed form, and in what way do these conditions relate to one another? Addressing such questions allows us not only to develop a broad overview of the technical subject matter but also fosters a deeper understanding of what structures really are and how they can be assessed within the context of architectural design.

A fundamental point to be established from the beginning is that structures in architecture are conceived – and perceived – differently from structures in other contexts, and so they should be evaluated differently. In reflecting on the integral relationship that exists between structures and architectural spaces, forms, and ideas, certain issues arise that differentiate the structures of architecture from structures of other kinds. The most obvious and basic function of a structure is its capacity to keep something above the ground by bearing loads, and the practical use gained from that capacity is to keep floors, walls, and roofs in an elevated position, thereby establishing inhabitable spaces. In many cases in architecture, however, structures are not solely associated with such load-bearing functions. And while engineering is able to solve the necessary safety requirements, the door is luckily left wide open for making the structure even more deeply considered conceptually. Ideally, a close relationship is established between structure, space, and formal expression so that describing and characterizing a structure solely in terms of its load-bearing function is clearly insufficient. To understand structures in a wider sense as being part of an architectural context also means seeing their forms as space-defining elements, or as devices that modulate the amount and quality of daylight, or that reflect today's sustainability concerns, or any number of other assigned functions. Structures can serve many purposes simultaneously to carrying loads, therefore, and we need to keep this in mind not only to enable a more profound understanding of the development of structural forms but also to undertake an appropriate and informed critique of structures within an architectural context.

How can one go about establishing a conceptual model for such a holistic understanding of structures? As a starting point, we can observe that structures play a role both as a provider of necessary stiffness and strength (which are the basic mechanical prerequisites for carrying load safely), and as an instrument for creating architectural spaces that embody certain other qualities. This notion of a dual function, both mechanical *and* spatial, proves rewarding when it comes to understanding and appreciating the multifaceted design of structures in various architectural settings. Structures range from those conceived of as pure force systems that follow a logic of maximum strength for a minimum of materials (i.e., structural efficiency), to those designed to act iconographically as visual images. On the one hand there is a load-bearing function, which helps to explain structural form from the point of view of technology and science, as objects required to supply stiffness, strength, and stability, while on the other the structure may take part in the organization of architectural spaces and the establishment of an architectural expression. Moreover, these dual aspects of structure are not typically wholly separate from one another, but instead tend to mix and their divisions to blur so that certain formal features of a structure may both be explained by mechanics and also be understood in light of their spatial functions. (e.g., Ill. 1.2 and Ill. 1.3, 1.4.)

Illustration 1.2

Eames House (Case Study House No. 8), Pacific Palisades, CA, USA (1949).
Contrasting rather than adapting to the building site, the Eames House was intended to exploit off-the-shelf, prefabricated, industrial building components made of steel and make these applicable to residential design. Partly exposed, the steel structure orders the plan in modular bays of 2.4 by 6.4m (7.5 by 20ft). Quoting the architect: "In the structural system that evolved from these materials and techniques, it was not difficult to house a pleasant space for living and working. The structural approach became an expansive one in that it encouraged use of space, as such, beyond the optimum requirements of living." And: "it is interesting to consider how the rigidity of the system was responsible for the free use of space and to see how the most matter-of-fact structure resulted in pattern and texture."[1]

Architect: Charles and Ray Eames. Structural engineer: MacIntosh and MacIntosh Company. Photographer: Julius Schulman. Title/date: [Eames House (Los Angeles, CA): exterior], [1950] © J. Paul Getty Trust.

Illustration 1.3
The Bordeaux House, Bordeaux, France (1998).
"Contrary to what you would expect," the disabled client told the architect, "I do not want a simple house. I want a complex house, because the house will define my world."[2]
The house consists of three distinct levels: the lowest is cave-like – a series of spaces carved out from the hill for the most intimate life of the family. The highest level is divided into an area for the parents and another for the children. The most important level is almost invisible, sandwiched in between the other two: a glass room – half inside, half outside – that is used for living.
Architect: OMA/Rem Koolhaas. Structural engineer: Arup/Cecil Balmond.

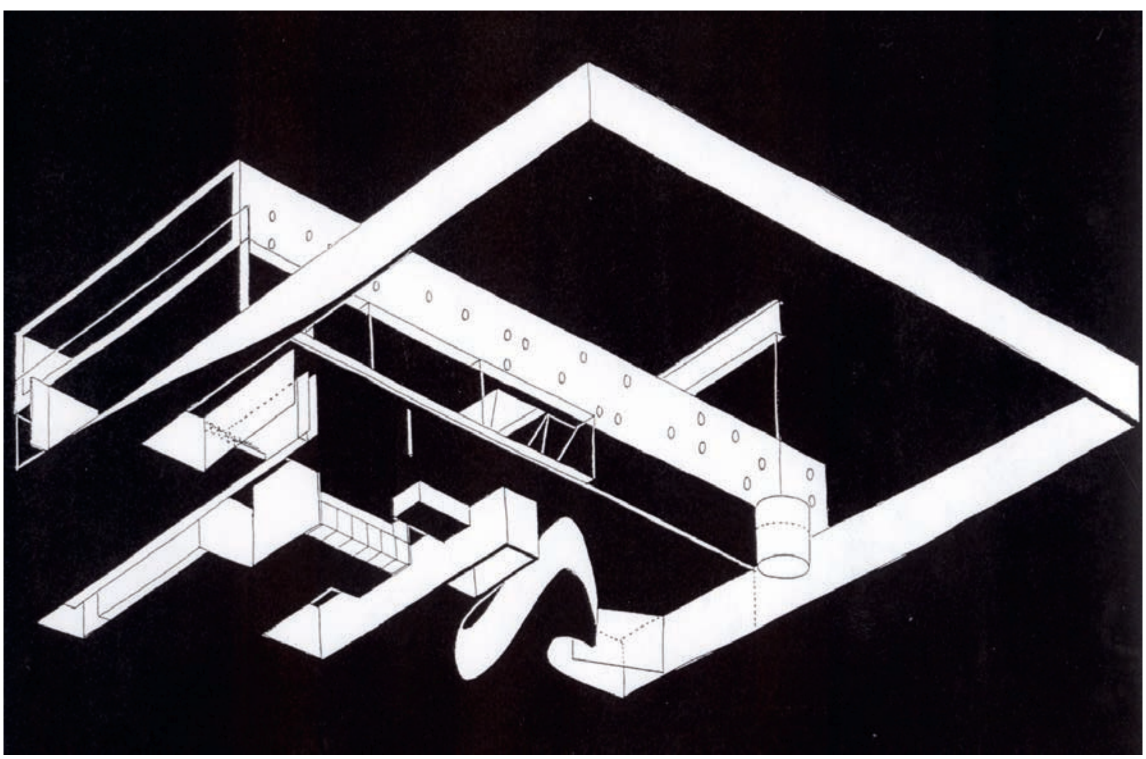

Illustration 1.4
The Bordeaux House. A worm's-eye view diagram showing material elements and structural principles. Moving the supports outside the plan contributed to an opening up of the space.

Illustration 1.5
The Grande Arche de la Défense, Paris, France (1989).
The large Vierendeel beams enable utility functions, accommodating people and their through-passage within the overall structural depth.
Architect: Otto von Spreckelsen. Structural engineer: Erik Reitzel.

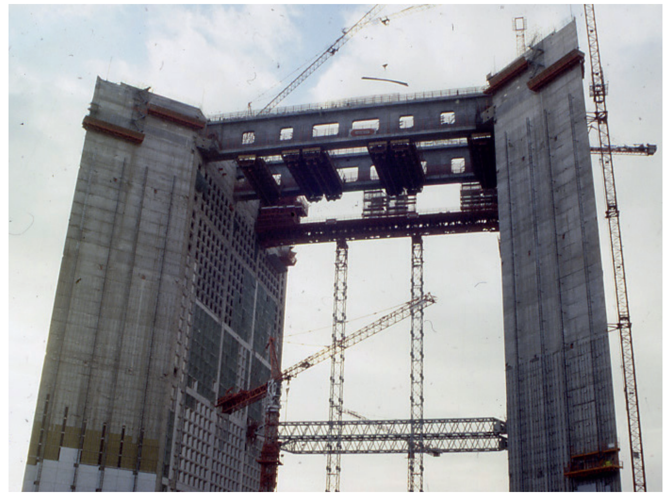

Illustration 1.6
The Grande Arche de la Défense.
Vierendeel beams can be seen at the top during construction.

This object/space duality can serve as a starting point but, as is the case with most conceptual models, it may simplify too much the world of real structures. Nevertheless, as long as we keep in mind that theoretical models of this kind can act as catalysts for increased insight while not necessarily being able to embrace absolutely every possibility, it will be found to be rewarding to identify both *spatial function* and *mechanical function* as the two prime concepts that establish the basis for a holistic understanding of structures in the context of architecture.

1.2 Spatial Aspects

The primary reason for the existence of structures is, of course, the practical purpose that they serve. Structures support loads from their location of application down to the ground, although typically not by means of the shortest possible "route" between those points since open and structure-free spaces of various sizes and shapes are needed in order to inhabit a building. This is the natural order of the relationship between the "why" and the "how," of reason and consequence: practical purpose comes first, and physical necessity follows. The practical purpose that the structure is assigned, its *utility* aspect, is fairly straightforward to accept and appreciate: in the case of bridges, for example, this is made clear by acknowledging the fact that the principal utility function, its "raison d'être" so to speak, is typically that of transporting people and goods across a valley, a river, or even an expanse of sea; i.e., it is all about establishing a transport line from one bank to the other. The straight line of communication that this link commonly results in will most likely suggest a certain structural configuration, either as a construct that becomes an integral part of the structural system, or else as setting up the conditions for how this line should be supported. The utility function provides in either case highly important input for how a structure is actually designed as well as an understanding of the form of bridge that is possible.

The same thing is generally true with the structuring of architectural spaces: the choice of a structural system and its particular articulation is highly dependent on the practical function that is associated with it. For example, in the case of the large beams at the top level of the Grande Arche de la Défense in Paris by architect Johan Otto von Spreckelsen (1929–1987) and engineer Erik Reitzel (1941–2012), there is no way to fully understand the choice of that particular beam type without also recognizing that the structure is actually accommodating human activity within its structural depth, and enabling people to walk freely in the large space within and between these beams, all the while looking at art exhibitions. (Ill. 1.5, 1.6.) This relationship is made possible because the beams are of a type that have large, rectangular openings in them, termed Vierendeels. Hence, what we experience in the interior spaces of this upper level is actually the horizontal and vertical parts of these huge beams that span an impressive 70m (219ft) over the open public plaza located far below.

Illustration 1.7
Museum for the Menil Collection, Houston, Texas (1983). In addition to providing a load-bearing function, the lower part of the spanning elements for the roof are shaped to act as light reflectors; these are precisely spaced apart so as to prevent direct sunlight from entering the museum galleries, however. The lower part of each of these composite structural elements is a curved ferrocement form, while their upper part (unseen in this image) is trussed. Mechanical requirements for the combined strength and stiffness of these elements meet the demands of a particular type of spatial utility function.

Architect: Renzo Piano Building Workshop. Structural engineer: Arup by Peter Rice.

With the Grande Arche it is relatively simple to point out the use-of-space utility function as a factor that offers design constraints and therefore has the ability to influence the chosen structural form. A second, perhaps somewhat more subtle, example of such a utility function may be in a situation where there is a central concern with the diffusion of natural light, which in the case of the Museum for the Menil Collection in Houston, Texas, resulted in a unique design for its roof trusses/reflectors that were made from a combination of different materials. (Ill. 1.7.) Generally, then, it can be said that for people to be able to do whatever they are meant to do in a particular architectural space, or so as to enable a certain non-load-bearing performance on the part of the structure, structural form

Illustration 1.8
The Cabaret Tabourettli, Bern, Switzerland (1987).
(a) Ceiling beams having iconographic function, designed to hint at the musical activities that take place in the room. (b) End-of-beam connection detail.
Architect and structural engineer: Santiago Calatrava.

may sometimes be shaped and configured in very particular ways. Without knowledge of the broader scope of such architectural utility functions in a building, therefore, a complete understanding of a particular structural configuration is not possible.

Beyond such variations of practical "utility," there are other performance functions that are also frequently associated with structures in architecture. In some cases we may find that structures are designed to make observers see something else in them, representing an object outside of itself, or something that is not really there. And in certain of these instances, architects have chosen to design structures in a manner that gives their form a certain similarity to other objects. One reason for this design approach is to bring the imagination of the observer into the visual experience, and to strengthen the perception of a particular presence that is thought to enhance a structure's architectural qualities. We may thus think of these structures as having *iconographical* functions. Among the numerous examples of this type are architect and engineer Santiago Calatrava's "musical" beams for the Cabaret Tabourettli concert hall in Bern, Switzerland, and the lively structures of architect Zaha Hadid's (1950–2016) Vitra Fire Station in Weil-am-Rhein, Germany. Neither of the structures used for these buildings can be fully understood without invoking the concept of mimicry. In the case of the concert hall, beams are given a shape and a materiality that closely resembles that of instruments like violins and cellos, making the observer acutely aware of the type of room one is experiencing; indeed, the thin steel ties that are secured to each beam have an unmistakable likeness to the strings of musical instruments. (Ill. 1.8.) And at the Vitra Fire Station, sharp angles activate the whole composition of structural elements of columns, walls, and slabs alike, creating an unmistakably hyper-active, kinetic image that makes one think of flickering and dancing flames. (Ill. 1.9.)

Illustration 1.9
Vitra Fire Station, Weil-am-Rhein, Germany (1993).
Structural composition of elements in a design that takes the lively flickering of flames as a point of departure. Eventually, there was no longer need for a separate fire station at the Vitra industrial complex, and the building was repurposed to house lectures, concerts, exhibitions, and social events.
Architect: Zaha Hadid. Structural engineer: Sigma Karlsruhe Gmbh and Arup by John Thornton.

Illustration 1.10
Experience Music Project, Seattle, Washington State, USA (2000). Structural form adapts to the overall, formal concept, letting the architectural context and conceptual ideas act as a form generator.
Architect: Frank O. Gehry. Structural engineer: Hoffman Construction Company.

In yet other cases, structures are so closely tied to a particular idea that the *architectural context* is seen to strongly suggest their shape and organization. Structures of this "type" are designed with a primary concern for their ability to enhance an overriding theoretical concept – or at least their design is guided by a certain logic that makes their structural form dependent on formal or conceptual imperatives. Although not necessarily so, the result of such a contextual design approach may well be a structural form in which the "traditional" load-bearing logic that dictates an efficient use of materials and manufacturing methods is significantly disturbed. Some of the work of the architect Frank O. Gehry might be seen to promote structures of this type: the EMP project in Seattle, for example, displays steel beams of varying and not-particularly-efficient shape in order to accommodate the highly intricate external forms of the building, and can be said to be designed "from the skin-in." (Ill. 1.10, 1.11.) Such a close link between this type of architectural

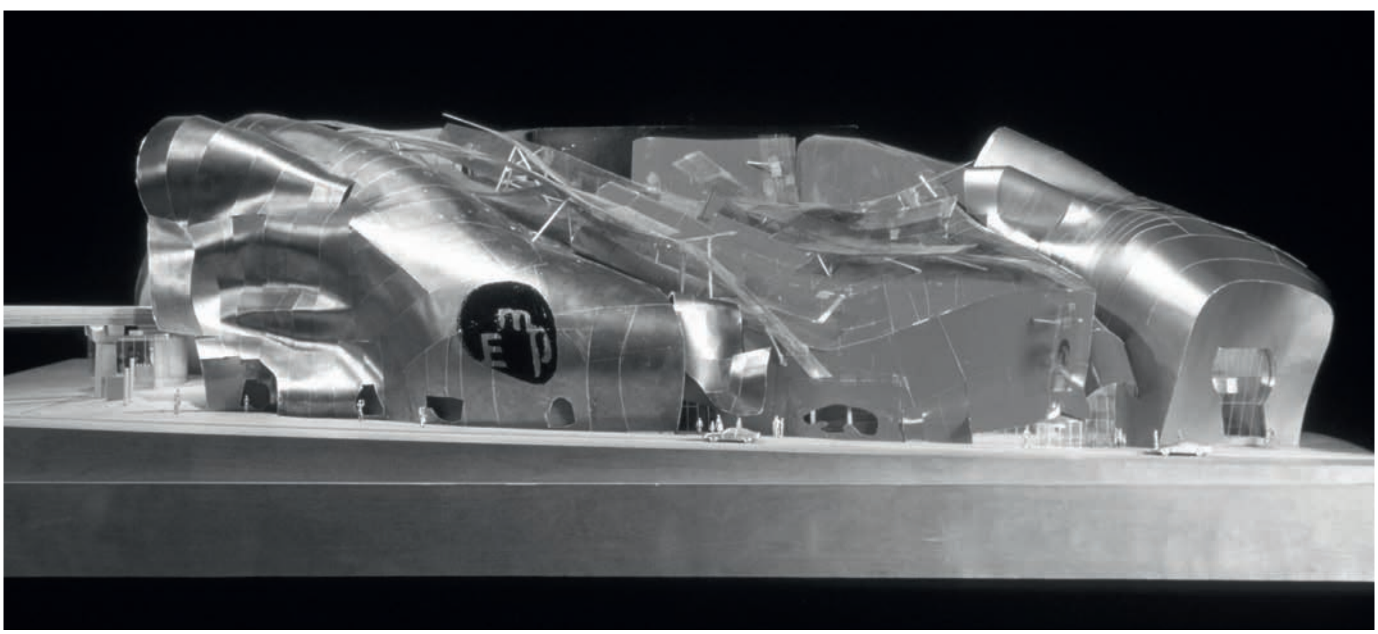

Illustration 1.11
Experience Music Project.
Model. Design concepts and exterior form establish rationale for structural frames' curving profile seen in Ill. 1.10.

Illustration 1.12
The Blur Building, Yverdon-les-Bains, Switzerland (2002).
Blurring the presence of a building with the help of 11 000 fog nozzles spraying water from the lake.
Architect: Diller Scofidio + Renfro. Structural engineer: Passera and Pedretti.

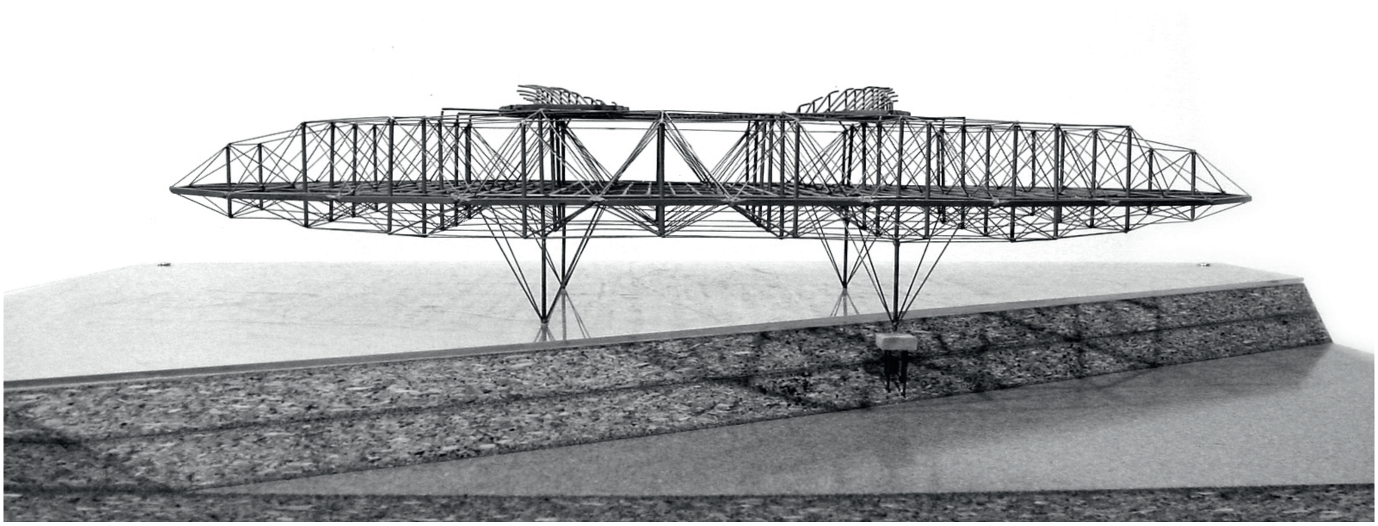

Illustration 1.13
The Blur Building.
A filigree trussed structure made possible the desired light appearance of the building.
Cornell model by Adam LeGrand.

expression and the structural form calls for a different attitude toward evaluating structure than that which is appropriate when confronted by structures that have a more obvious technological basis. In these cases, structural forms cannot properly be understood in isolation as force systems that "purely" meet specific functional demands, but should instead be assessed within the framework of the governing design concepts and ideas. In other words, a "conventional" evaluation of such structures strictly in terms of concepts like strength and stiffness and the most efficient production methods, while not to be ignored, will be completely inadequate to fully explain and appreciate their design.

Of course, the various spatial aspect categories that we have so far identified need not exist in isolation from one another. The Blur Building, erected as a temporary media pavilion for the Swiss Expo 2002 and designed by architects Diller Scofidio + Renfro is an example of a work of architecture in which the structure is part of a design that features both iconographic and contextual qualities, while also maintaining an efficient load-bearing strategy. This is a "both/and" rather than an "either/or" condition. The pavilion was characterized as "an inhabitable cloud whirling above a lake": set on pillars in Lake Neuchatel in Switzerland, it was enveloped in a fine mist created by a huge number of fog nozzles spraying water from the lake and creating an artificial cloud. (Ill. 1.12.) To further strengthen this image, the architects and engineers took care to design a structure that could be considered to have a similarly blurred image. The lightweight structural system was composed of a multitude of the thinnest possible structural members, all arranged according to a strategy of efficient resistance to loads; these structural members were clearly meant to visually disappear into the cloud. (Ill. 1.13.)

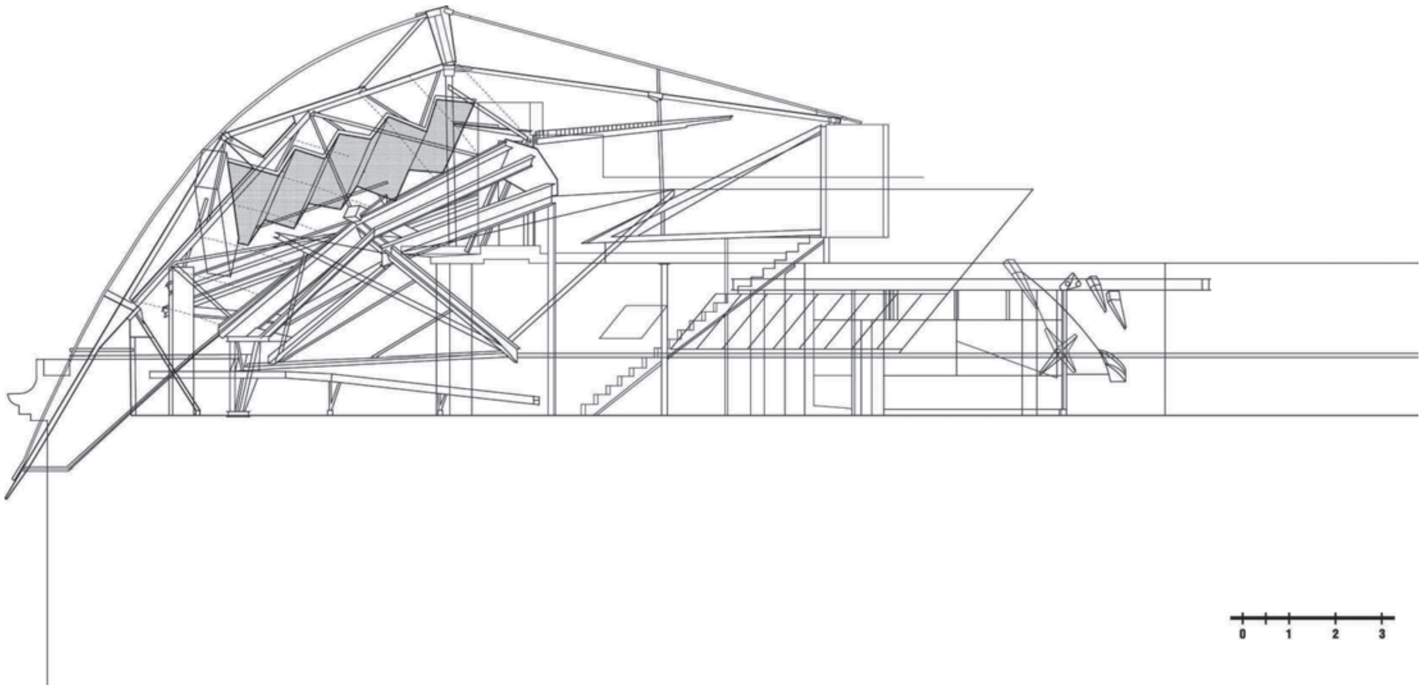

Illustration 1.14
Roof-top Remodeling, Falkestrasse, Vienna (1988).
The structural spine with a distorted and complex look enhances the high-energy character of the architecture.
Architect: Coop-Himmelb(l)au. Structural engineer: Oskar Graf.

As a general observation from examining many other buildings besides the Blur pavilion, it can be stated that design requirements which primarily address the spatial aspects of structures are frequently found to also be in strong agreement with the requirements of a more mechanical nature. In other cases, however, structures that are meant to bring about particular spatial qualities may seem to cause their purely load-bearing and material-efficiency logic to "suffer." At the extreme, a seeming incompatibility between spatial and mechanical requirements may even be seen to exist, lending the structure a certain ambiguous character, but this is still not necessarily to be considered a negative feature of structural form. On the contrary, such a condition can contribute to visual interest and to a clarification of a certain conceptual approach to the architecture/structure relationship. And we should not forget that even structures of this type are inevitably designed to be both safe and sound.

As an example, we can consider architect Coop-Himmelb(l)au's Roof-top Remodeling intervention in Vienna which precisely represents this delicate balance between spatial ambition for structural form on the one hand, and a somewhat-less-than-common mechanical logic on the other. (Ill. 1.14.) Far from being randomly designed, the former qualities can be seen to have led the design and the latter to have become of less importance. One can quickly spot what might be termed a spine in the form of a complex assemblage of steel sections aligned in a skewed plane that cuts right through the project, forming a line of symmetry or, rather, something that resembles symmetry. This is obviously an important structural element. The most spectacular feature of this spine is the thin curving line formed by a steel rod that binds the different members together. In fact, because of their standard structural profiles, all of the steel members seem to have a certain load-bearing function except for that thin, curving rod which is used to establish a visual demarcation line around the whole structural composition. The rod also projects out from the edge of the roof, hovering over the street below where it connects with other steel profiles in order to terminate the whole visual/structural composition. We might ask: Is this apparent complexity of structural pathways and the absence of a clear structural system a negative feature in this design? To which we would answer: No, based on the rationale that both the great intensity of the lines and the ambiguous character of the structure add to the experience of a "high energy" work of architecture. Wolf Prix once said that "structures, although metaphors for forces, follow another force, not of weight, but of energy."[3] We experience the structure of this Viennese rooftop addition, as distorted as it is, as being highly appropriate for such an equally distorted spatial configuration;

Illustration 1.15
The Copenhagen Opera House, Copenhagen, Denmark (2004).
The variation of the thickness of the projecting roof form follows the changing magnitude of forces within its (hidden) beam structure.
Architect: Henning Larsen. Structural engineers: Rambøll, Buro Happold.

indeed, a regular and geometrically simpler structure would have significantly weakened the desired spatial quality.

As we have seen throughout this section, the particularities of structural form can be closely related to spatial functions and to conceptions of space. We can thus interpret structure as being part of an integrated design approach in which we cannot completely explain, understand, or appreciate structural form without recognizing its strong co-dependence on the particular character and use of the architectural space. It is of importance to note, however, that any gross deviation from what can be considered to be a reasonable concern for mechanical requirements should not be the result of random, uninformed, or thoughtless design, but rather of carefully considered ideas related to other design imperatives.

1.3 Mechanical Aspects

We now turn to what can be considered to be the basic mechanical function of structures: that of being load-bearing objects that possess and display specific physical properties. As has been previously mentioned, among such properties is their ability to withstand loads and forces imposed by nature and derived from human activities, qualities that are typically embodied in the physical concepts of strength, stiffness, and stability. All of these latter concepts will be thoroughly explained in the chapters that follow; at this stage, however, it is sufficient to say that they all relate to how structures perform when loads act on them, and that these concepts address the way nature works and lend themselves readily to *scientific analysis* which may involve mathematics and physics. This means that there is a direct relationship that can be demonstrated between structural form, the direction and magnitude of loads, the properties of the materials, and the response of structures. We can illustrate the point in question by referring to one example among many where structural form is revealed or explained by referring to this relationship: i.e., the steel beams that are hidden within the roof of the Copenhagen Opera House clearly have varying structural depth. (Ill. 1.15.) There are no supports at the outer end of the roof cantilever, and so the beams must therefore carry the loads inward toward their line of support, collecting more and more loads along the way and needing to get progressively deeper in order to accommodate this.

Furthermore, there are architectural examples where the connection between form and nature's laws is no longer just intuitively grasped but clearly depends on scientific analysis for their design, not merely for a confirmation of structural dimensions (while also that),

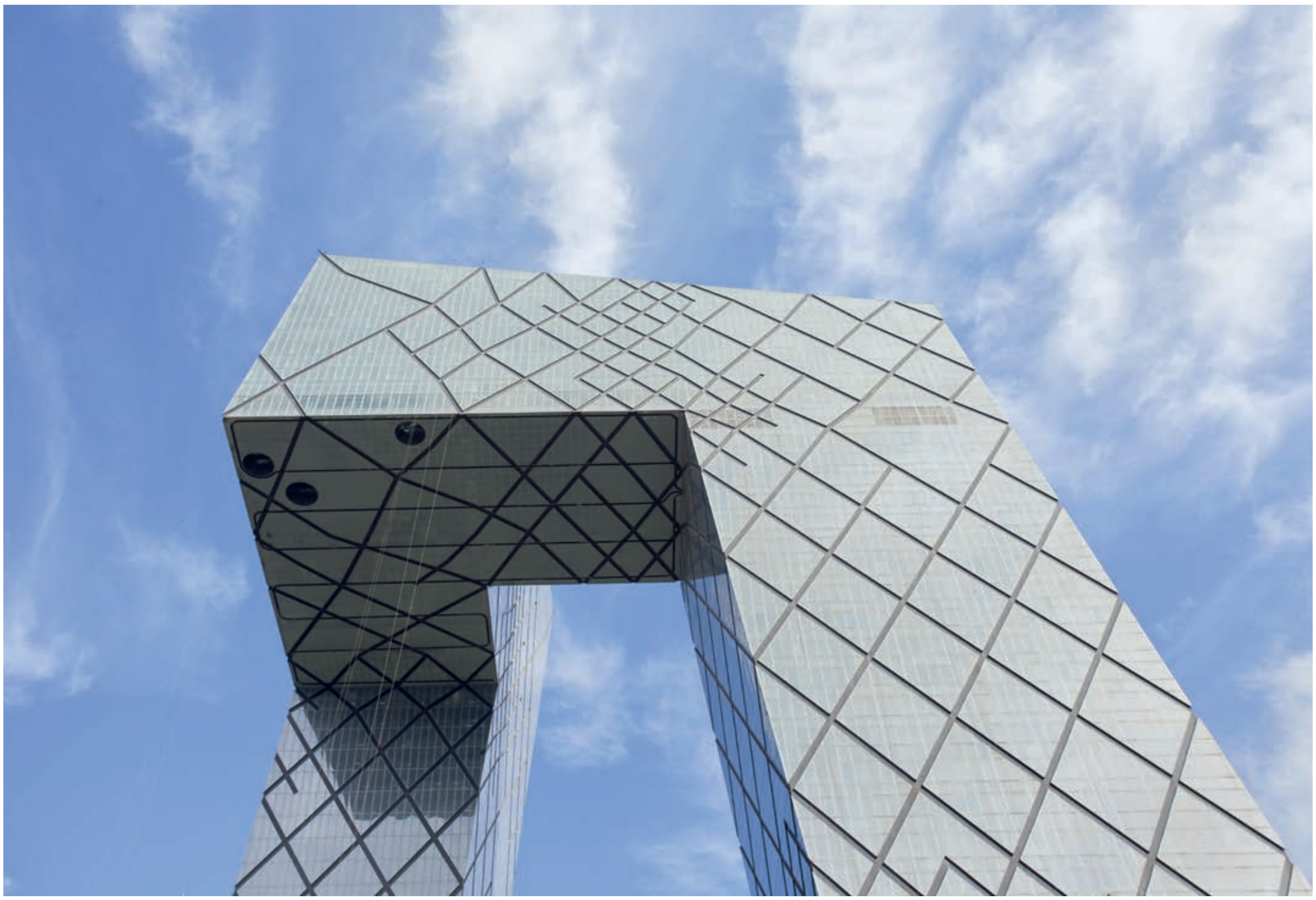

Illustration 1.16
CCTV Tower, Beijing, China (2008).
The diagonal pattern of structural members exposed in this building's façades is irregular, closely following the stress pattern that results from the building's particular shape and loading conditions. Where the intensity of these stresses increases, more structural members are inserted, thus tightening the "web" of structural lines needed to accommodate this.

Architect: OMA/Rem Koolhaas. Structural engineer: Arup by Cecil Balmond.

but more explicitly that their shape cannot be properly explained without addressing theoretical knowledge of the strength/stiffness/stability relationship. Among the many possibilities to illustrate this particular observation is the CCTV Tower in Beijing by architects OMA/Rem Koolhaas and structural engineer Arup/Cecil Balmond, where structures that are exposed in the façade are configured so as to follow a logic of structural sub-optimization that puts its distinctive mark on the character of the building;[4] i.e., the pattern of diagonal lines is noticeably denser where the structure is more highly stressed. (Ill. 1.16.)

Historically, of course, the planning and construction of large objects and structures had nothing to do with science. Such constructs most certainly obeyed scientific laws, regardless of what their builders were aware of, but science played an insignificant role in explaining at the time just how they worked and why they were designed the way they were. Architecture, for its part, had for much of its existence been perfectly happy employing certain building technologies without benefiting from the input of science. For example, even the most advanced Gothic cathedrals were built without theoretical knowledge of mass, gravity, forces, and stability. Their builders employed available construction technologies, but did not command science as a tool for analysis. Today, we may explain the shapes of Gothic cathedrals by invoking scientific concepts, but at that time forms were arrived at following craft-based traditions and by trial and error; consequently, failures happened and these have been duly recorded.

For the past 150 years, however, architecture has become ever more dependent upon and intertwined with the development of scientific knowledge. Part of the reason for this has to do with the sheer size of many architectural projects and that the consequences of construction failures are so grave that mistakes cannot afford to be made, whether for reasons of moral, financial, or legal responsibility. Of course, scientific knowledge also helps to bring about an efficient use of materials, enabling the fewest natural resources to be used. And, finally, we should also remember that architecture is typically concerned with developing "one-off" designs for buildings that explore and account for site specificity and individual programming and conceptual designs that make each building unique. In order to be able to cope with the inherent uncertainties of such new and untried designs, we take advantage of one of the natural sciences' most wonderful abilities: the possibility of predicting the outcome by means of theories developed for material and structural form behavior. Architectural projects can thus be analyzed scientifically as the physical objects that they are, or are about to become, and the behavior of their masses of stones or skeletons of steel can be foretold in advance of construction. Physics, obviously, is the prime instigator in that respect, aided by mathematics.

Looking at structures from a mechanical point of view is not restricted to a study of behavior based on scientific principles, however. It also involves a consideration of what we may think of as being structures' *technological* aspects; i.e., how their parts are manufactured and how they are actually built. Decisions about how structures and structural components are produced and erected also make their imprint on structural form, especially at the detailing level. Consequently, technological matters should also be brought up for consideration when seeking to understand and critique structural form. It is particularly important when we study structures that they are considered not only as finished products, but also as manifestations of certain manufacturing and construction processes. Therefore, we need to look upon the mechanical aspects of a structure from both a scientific *and* a technological point of view, recognizing that there is a difference between the two that enables us to observe and understand the different qualities that these may bring to a design.

Building technology deals with the "making" processes. As such, it simultaneously addresses several production and manufacturing issues, from the production of building materials and structural elements, to their adaptation to suit a particular situation, and, finally, to the actual construction phase of a building. Technology thus involves operations like casting and rolling of metals to form components, sawing of timber boards and gluing them into laminated elements, as well as casting concrete into formwork made of various materials to produce different shapes and surface textures. To understand building *technology*, therefore, means to know how buildings are made. And to understand architecture and structures from *a technological point of view* means to look upon form, shape, and texture as the response of materials and components to their being processed, trimmed, outfitted, and assembled for a particular purpose, namely that of constituting an occupiable building volume. We may thus think of structural form and its articulation as testifying to the manufacturing and construction processes.

As an example we can consider the church Chiesa Mater Misericordiae designed by architects Angelo Mangiarotti (1921–2012) and Bruno Morassutti (1920–2008) with engineer Aldo Favini

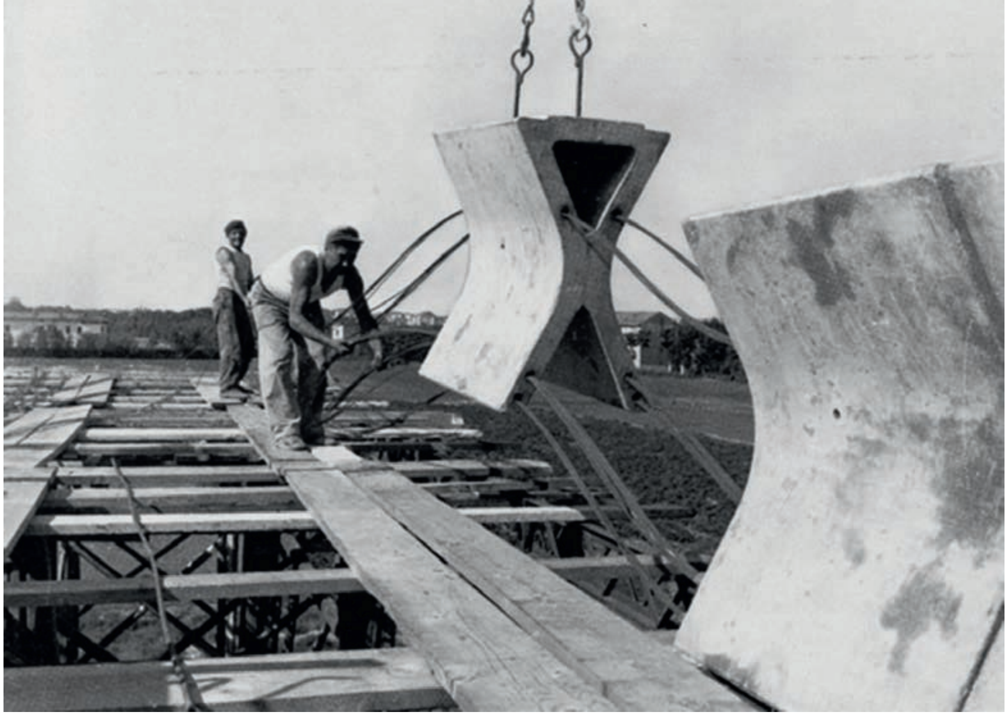

Illustration 1.17
Chiesa Mater Misericordiae, Baranzate, Milan, Italy (1957). Construction technology, or the way the beams are actually built, becomes an important design factor. Here, post-tensioning cables are run through X-shaped precast concrete segments in order to be able to create long-span roof beams.

Architect: Angelo Mangiarotti and Bruno Morassutti. Structural engineer: Aldo Favini.

Illustration 1.18
Chiesa Mater Misericordiae.
Long-span beams seen in ceiling open up the interior space; these beams also project beyond the line of column support. The alternating open and closed-off bottom of these X-shaped beams reflects the variation of their internal compression and tension stresses according to the behavior of continuous beams.

Illustration 1.19
IAA "Dynaform" Pavilion, Frankfurt, Germany (2001). Undulating structural frames reflect the overall architectural context as well as attest to the technological methods used to manufacture them.

Architect: ABB Architects with Bernhard Franken. Structural engineers: Bollinger + Grohmann.

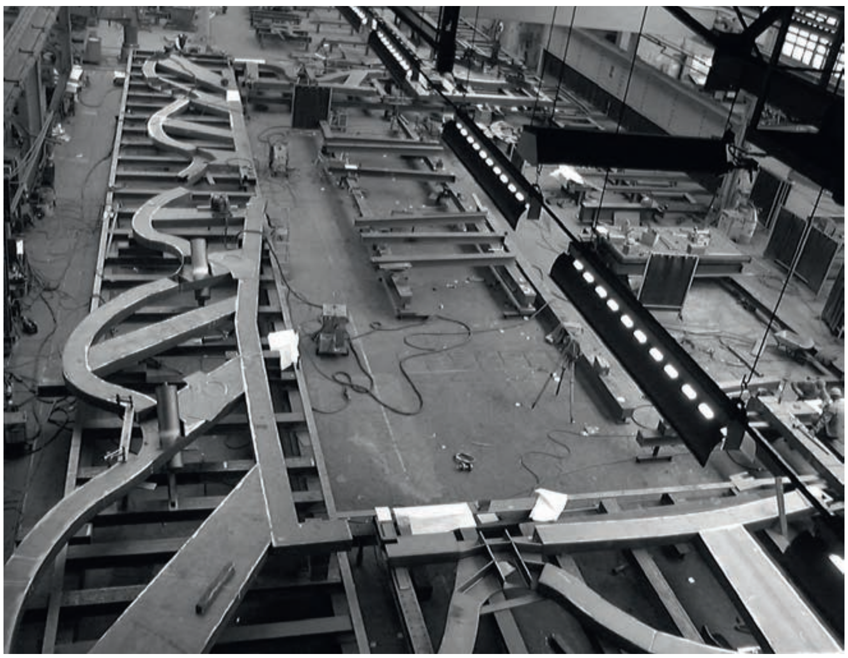

Illustration 1.20
IAA "Dynaform" Pavilion.
CNC laser-cutting of steel plates that are then welded together to create the structural frames.[5]

(1916–2013), in which the roof beams consist of a large number of precast reinforced concrete sections or elements that are poured in a factory, transported to the building site, and then connected together by means of (post-tensioned) cables that run along the length of the beams. (Ill. 1.17.) The discrete component character of these beams stands as "proof" of how the structure is actually built, displaying simultaneously the technology of manufacture and construction that was employed. Beyond this, the church structure is also a good example of the value of invoking the scientific analysis perspective that relates form and strength: each element of the beams basically forms the letter X in cross-section, but with one side (upper or lower, depending on location in the span) closed off with a concrete slab that acts like the lid of a box. This extra material provides a greater resistance to compressive force on the side of the beam that it is on, and such extra capacity alternates from the top to the bottom of the beam along its length according to the behavior of continuous beams. Thus, by keeping in mind both technological and scientific matters, in this case we can better explain and understand the reasons for the particular structural form in the context of the working of the overall system, and of the desired spatial intentions. (Ill. 1.18.)

A second example requiring a technological approach to understanding structure can be found in the IAA pavilion built for BMW exhibitions that was designed by Bernhard Franken of ABB Architects and engineers Bollinger + Grohmann. The roof and walls of this building have an undulating form, with irregular ridges running along its length, while the structure is composed of a series of steel frames that cut transversely across it. Reflecting the overriding architectural design concept and geometry, these frames take on the curving, wave-like shape of the exterior of the building. (Ill. 1.19.) The complex curves of the frames had to be created by using technologically advanced manufacturing methods: they are built up from discrete pieces that are machined out of steel plates using computer-controlled cutters, and then these components are welded together. (Ill. 1.20.) The relatively thick and multiply curved profiles of the structural members making up these frames

Illustration 1.21
ICD-ITKE Pavilion, University of Stuttgart, Germany (2014).
Biomimetic form of this domed, double-layered fiber structure was inspired by the protective shells of beetles' wings, and it is composed of 36 modules, each having unique 3D geometry.

Architects and engineers: ICD-ITKE University of Stuttgart. Prof. Achim Menges and Prof. Jan Knippers.

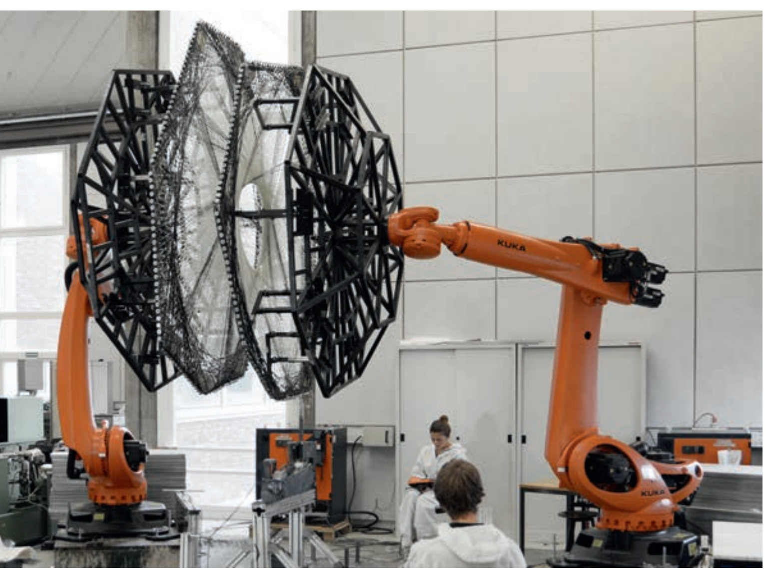

Illustration 1.22
ICD-ITKE Research Pavilion 2013–14.
Seemingly "dueling" 6-axis robots in fact work together in tandem in a highly precise digital choreography, with resin-impregnated fibers spun together according to the results of advanced structural analyses.

would have been impossible to produce by any other method, and acknowledging these structures' particular technological basis and resolution becomes a precondition for gaining an understanding of and appreciation for their overall design.

Advanced technological fabrication methods are taken several steps further with the 2013–14 ICD-ITKE Research Pavilion, designed by teams from the University of Stuttgart's Institute for Computational Design and Institute of Building Structures and Structural Design led by Profs. Achim Menges and Jan Knippers, respectively. (Ill. 1.21.) Inspired by a close study of the structure of beetles' wings and shell abdomens and built as an exquisite adaptation of biomimicry, the distinctively domed structure for this pavilion covered $50m^2$ ($540ft^2$), enclosed a volume of $122m^3$ ($4300ft^3$) and yet weighed only 593kg (1300lbs), with the whole of it dependent on resin-impregnated glass and carbon fibers that were woven together by a pair of carefully synchronized 6-axis industrial robots. (Ill. 1.22.) A highly irregular overall geometry results in the end, taking its cues from specific site conditions, but that was able to be composed and easily erected from 36 prefabricated, double-layered, doubly curved modular units, each one unique in form and size, and each one completely dependent for form and strength on its dense web of woven fibers connecting the inside and outside layers. Moreover, the highly specific layout of these fibers was established by the forces anticipated for the overall structure by means of advanced finite element analyses. In the end, quite a pleasant place to sit and gather with others was created, one which highlighted an essential and creative interaction between innovative material selections, design objectives, structural system configuration and logic as well as the application of state-of-the-art fabrication technology.

These last three examples have shown that building technology is a body of knowledge that helps to bring about the transformation of raw materials into works of architecture, but we also know that scientific principles and mathematical analysis are necessary to make sure that the buildings we design perform according to our expectations and our basic need for safety and efficiency. Thus, both technological decisions *and* scientific reasoning become critical design factors, and while each, on its own terms, puts its imprint on the finished design, only when considered together do they allow for a complete understanding of structures as mechanical objects.

We will stress throughout this book the importance of taking a truly holistic approach to structures by considering *all* the different aspects that we have discussed in this chapter and that may influence structural form in one way or another, from those that relate to mechanical requirements to those that are derived from overall spatial ambitions. (Ill. 1.23.) This broadly based approach allows for the engagement of conceptual ideas that inform the design of structures, and provides an instrument for an informed evaluation of structures as the basis of architecture. Admitting structural issues into the more general architectural assessment of a building project is unfortunately as rare today as it is important; our explicit ambition in communicating structural knowledge is to discuss mechanical issues as an integral part of an overall consideration of architectural spaces, ideas, and forms.[6]

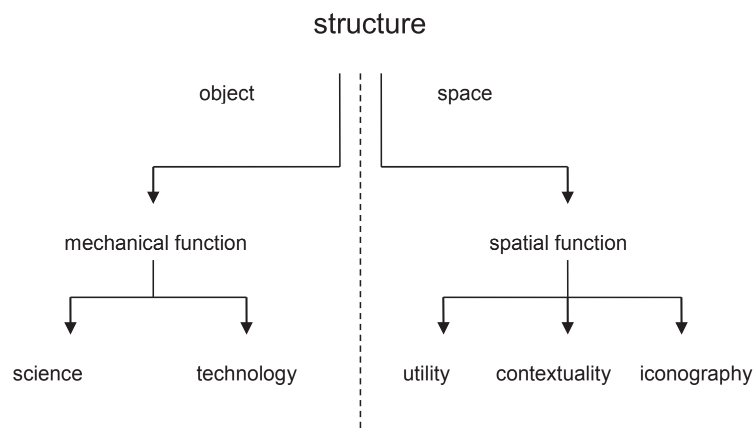

Illustration 1.23
A chart of various aspects of structural form based on a space/object duality.

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CHAPTER 2

Introducing Structural Systems

Illustration 2.1
Streetscape in Arles, France. Prominent in the city's historic urban core: the two-tiered, exposed stone arcade of the Arènes d'Arles, a Roman amphitheatre built in about AD 90 based on the Coliseum in Rome, and the pointed Gothic-style Cordeliers steeple erected in 1469 and restored in 1993.

2.1 Revealing Structures

Our first impression of a building, whether looking at it from a city street or a country road, is usually of its volume; i.e., its height and width and overall shape. Next, we will probably notice its surface, identified by the texture, color, and material nature of the building façade. As we gradually take in the situation, this particular building's relationship to its immediate surroundings will begin to register; for example, whether it is larger or smaller than its neighbors, has similar or different window openings, whether its precise orientation aligns with that of other buildings in the area or perhaps with certain landscape elements instead – or else none of these, as the case may be. We are likely to quickly notice whether this building we are concerned with "blends in" with other buildings, or represents a "contrast" to those, or maybe it stands alone in relative isolation. To learn more about it, we will at this point need to enter the building and investigate its interior spaces – their size and shape and daylight conditions, for example. The main purpose that the building fulfills will probably become clear at this point, if it was not already made evident from the outside. It is also at this stage that we often begin to notice the way in which the building is constructed; i.e., we may see columns and beams or other traces of the building's load-bearing structure, and perhaps also observe a certain pattern or hierarchy that these structural elements follow in order to create the form and size of the different rooms and spaces within the building, and that enable these to be kept up in the air and in specific relation to each other in spite of the forces of gravity that are trying to bring them down to the ground in a heap.

It is also the case, however, that a building structure's form and the material of which it is made may not be evident at all, whether the building is seen from the outside or from within; i.e., in some cases the structural elements are completely hidden from view. This could be for aesthetic reasons according to which an architect does not wish to have structure impart a certain type of character and atmosphere to the building façade nor to its internal spaces, whether as part of her/his general design approach or perhaps it is only in a particular instance for very specific conceptual reasons. Or, perhaps, the covering of structural elements may be for more pragmatic reasons such as shielding them from exterior temperature variations, or due to fire-protection regulations, or perhaps because of a desire to hide what may be considered to be, in certain situations, rather unsightly ventilation ducts, plumbing pipes, electrical conduits, etc., that are often attached to and running alongside the structural components. The question of whether to expose or hide structural elements and systems can be debated, and there is no right or wrong answer. Indeed, there are enough compelling examples at both ends of this spectrum to demonstrate that a building design can be considered to be successful according to one approach or the other, or to one that lies somewhere in the middle. What is irrefutable and what all buildings have in common, however, is that an overall structural system and its component elements must be present somewhere, and for our purposes here in this book it is simply a matter that this structure needs to be revealed in order for us to be able to study it. We shall begin this chapter by doing just that for the Pavilion Suisse, designed by the architect Le Corbusier and completed in 1932, and then for the Kunsthaus Bregenz by Peter Zumthor, which opened in 1997.

The Pavilion Suisse was designed as a facility that would house students from Switzerland at the Cité Internationale Universitaire in Paris. The building has three distinct volumes that essentially each accommodate a different function: there is a low, one-story portion containing the common meeting room for all residents, there is a tower-like middle part incorporating stairs and bathrooms, and finally there is a four-story vertical dormitory block where the students live. (Ill. 2.2.) Each volume has its own separate and different structural system, but it is the one for the dormitory which we will focus on here. We see from the outside that this building block is raised on thick, exposed concrete pillars, called "pilotis" in the vocabulary of Le Corbusier. These are placed in rows along both sides of the long, central axis of the building and support a pair of longitudinal beams, which in turn carry on top of them a slab of substantial thickness – all of which are made of reinforced concrete. As we will see, there is quite a different structural system arrangement for the dormitory levels above, one which is supported on this thick concrete transition slab.

Looking at the south façade of the building we see that glass is the dominant material, and that this exterior wall is visually organized by a grid of horizontal and vertical lines; these lines demarcate the positions of floor levels and interior room-partition walls, respectively. We do not actually see the structural components, but nevertheless we do get a strong indication of where these are located. The north façade, however, shows no such trace of the

Illustration 2.2
Pavilion Suisse, Paris (1932).
Exterior view of south façade of dormitory block.
Architect: Le Corbusier.

structural system. Here we see a uniform wall surface made of prefabricated concrete cladding panels, the only relief to which are square openings for windows. It may come as a surprise, then, when it is revealed that behind these façade walls and throughout the whole of the volume of the dormitory block there is actually a three-dimensional structural grid of steel columns and beams. (Ill. 2.3.) It can be said by analogy, therefore, that there is within this building volume a hidden skeleton that enables it to stand up just as is the case in nature with human beings and animals. Moreover, and also in common with these biological bodies, this structural skeleton can be seen to have a close functional and formal relationship to the internal spaces/organs of what it is supporting as well as to the overall external shape of its enveloping enclosure/skin. For example, in the Pavilion Suisse we find that the distance between the steel columns along the south façade is the same as the width of each student's room and that the height of the rooms is defined by the vertical distance between the steel beams of the frame. But at the same time as the dimensions of the structural grid can be seen to have a clear spatial relationship and visual impact, it is also true that its columns and beams themselves are in fact mostly hidden from direct view by the exterior cladding and by being wholly absorbed within room partition walls and covered over by floor slabs.

In contrast to the situation at the Pavilion Suisse, the exterior of the art gallery building in Bregenz, Austria, is even less revealing: here there are no external indications of a structural assembly

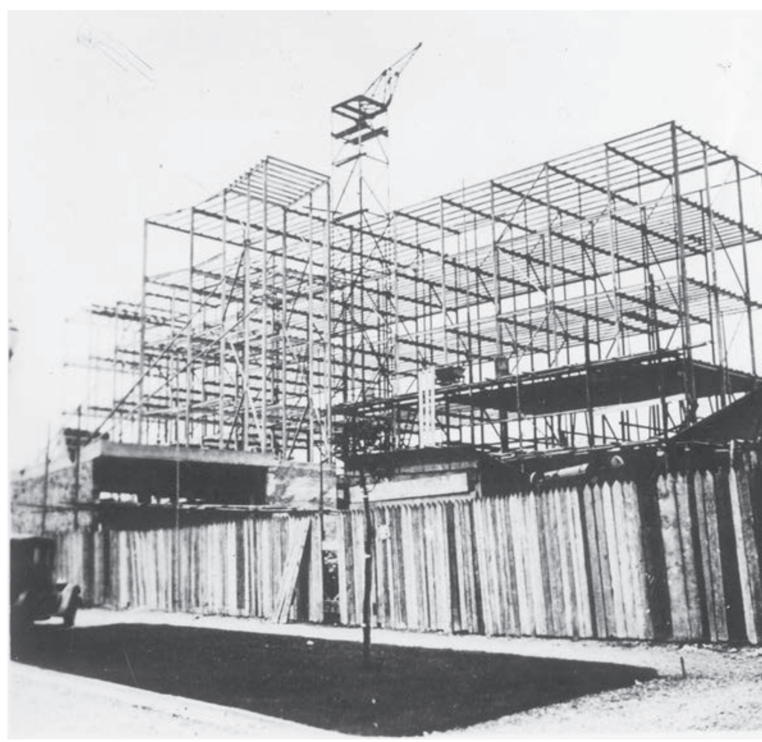


Illustration 2.3
Pavilion Suisse.
Steel skeletal structure is used to support the dormitory floor levels, as seen during construction.

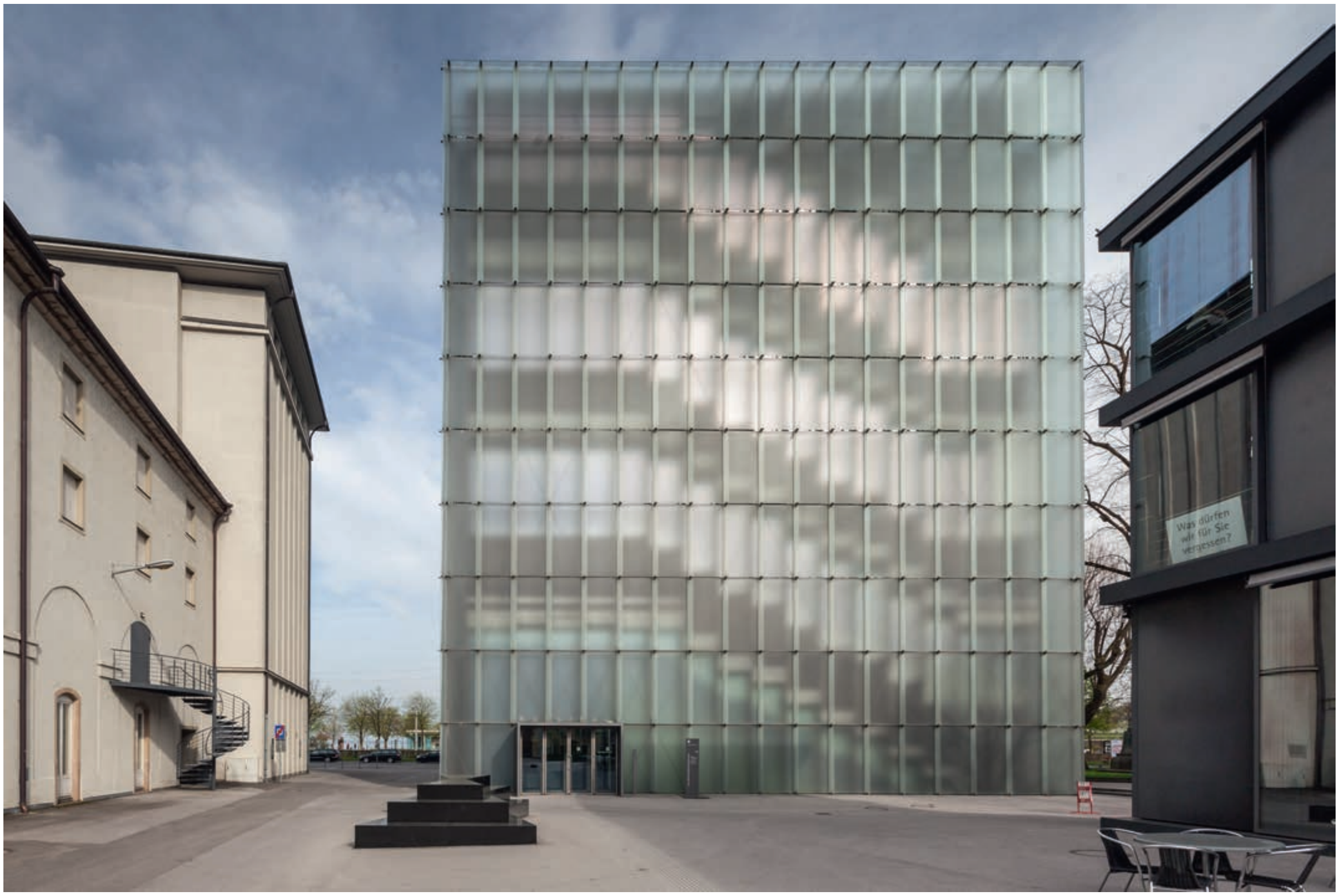


Illustration 2.4
Kunsthaus Bregenz, Bregenz, Austria (1997).
Exterior view; overlapping, etched glass panels cover the entirety of the outside of the building.
Architect: Atelier Peter Zumthor & Partner. Structural engineer: Robert Manahl.

that could begin to suggest, let alone explain, how this particular building is constructed. (Ill. 2.4.) The Kunsthaus is completely clad on all four sides with slightly angled, overlapping, semi-transparent etched glass panels through which we can get a glimpse of the outline of this façade's steel support structure. The glass diffuses the light that enters the building during the daytime, and at night the building is artificially lit from within, turning the whole of the cubical volume into a large urban lantern. We can also see through the façade the blurred outlines of several mysteriously hovering thick horizontal and inclined bands, but there is no hint of what may be holding these up nor of what they may be, or even any recognizable features that would give them scale.

Immediately upon entering the building, however, the load-bearing structure is completely revealed to us: three huge reinforced concrete walls support the accumulating gravity loads at each floor level while also forming the stabilizing system against wind and earthquake lateral loads. (Ill. 2.5, 2.6.) Moreover, these three walls help to organize the building functions and arrange the space according to the daylighting strategy devised by the architect. Contrary to the open skeletal system of the Pavilion Suisse, the structure of the Kunsthaus does not merely indicate where room partition walls might be located, but instead the extensive surfaces of these three load-bearing walls themselves establish the large-scale barriers that isolate the main gallery spaces at each floor level from the circulation stairs and elevators and from the secondary service areas that are located along the outside edges between these walls and the glass façade. The concrete walls are left exposed and, indeed, they delimit space itself.

On the inside of this building, then, the structural system has a clear spatial and visual presence that is not the case for the system of the Pavilion Suisse, at least not to the same extent. On the outside, however, perhaps the opposite could be said, although in neither case is the structural system clearly legible. These two examples show fundamentally different ways of organizing the relationships between structure and architectural form and space, and we will repeatedly return to this way of looking at and considering these various relationships throughout the rest of this book.

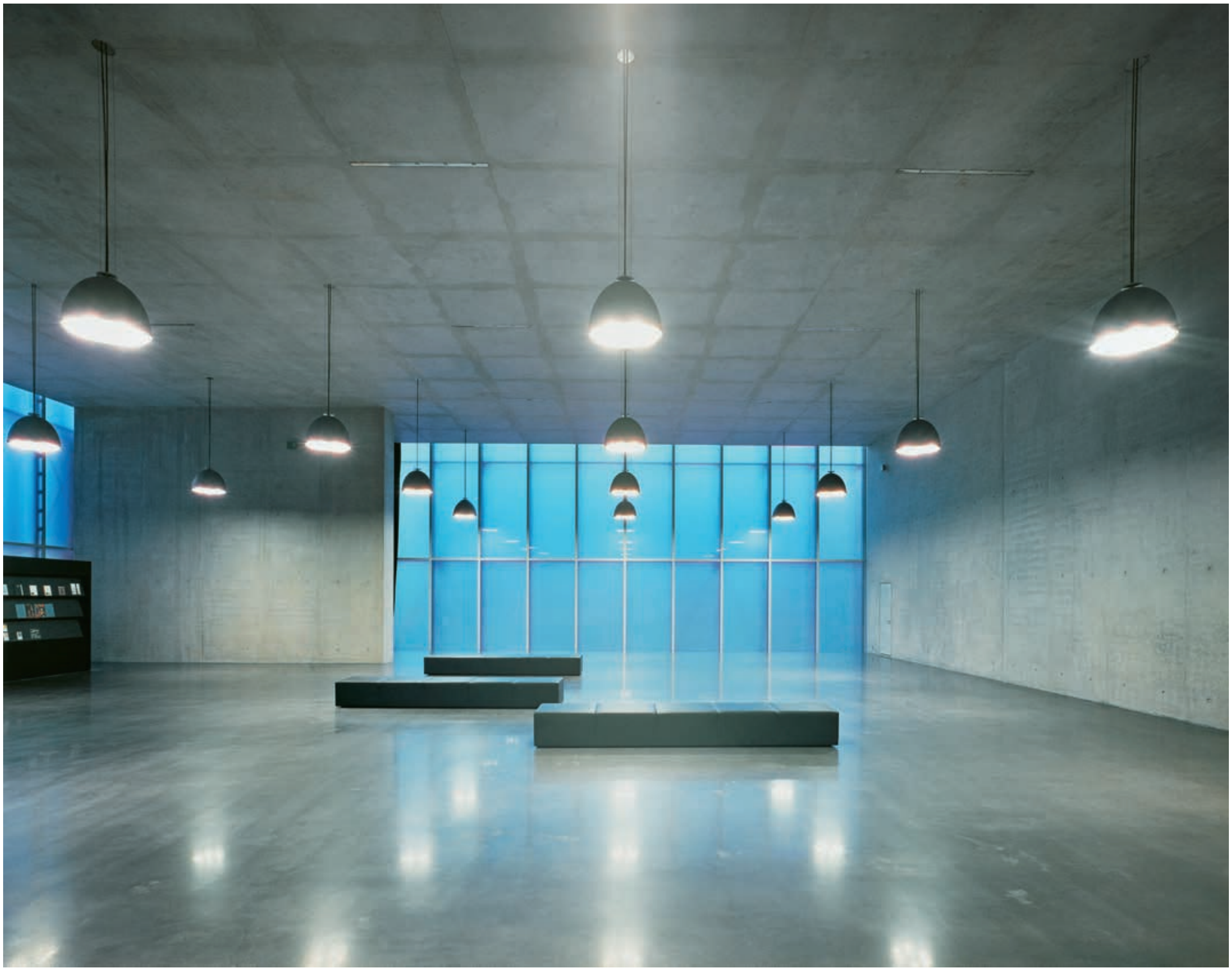


Illustration 2.5
Kunsthaus Bregenz.
Building's vertical structure consists of concrete load-bearing walls; these are in full view in the interior spaces.

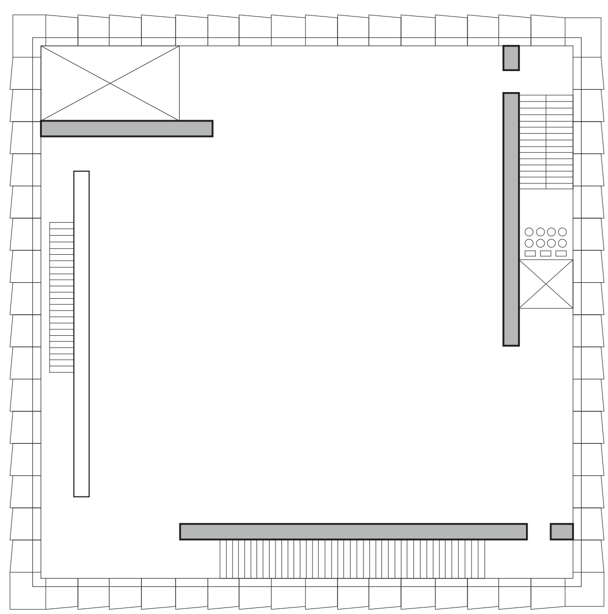


Illustration 2.6
Kunsthaus Bregenz.
Floor plan showing location of the building's three reinforced concrete walls, which are the only vertical structural elements in the building.

2.2 Basic Structural Elements and Systems

Basic Functions and Terms

As has been discussed in the introductory Chapter 1 as well as in the previous section, buildings need a physical structure to keep them standing up. The materials that we use to construct our buildings, whether for the structure itself but also for all the other building components including partition walls and façade claddings and insulation materials, etc., generally constitute considerable weights that are lifted up from the ground and that need to stay there. This also applies to the weight of all the additional things that we put into buildings, including our own weight as building occupants as well as that of furniture and equipment. Moreover, buildings are obviously exposed to the weather and so they need to be able to resist loads caused by such things as wind and snow (perhaps) and in the parts of the world that are prone to earthquakes building structures need to be designed to withstand seismic forces. All this will be covered in much more detail in Chapter 3 Loads. In order to be able to withstand all of these various forces and their effects over long periods of time we have to provide physical *structural elements* in the form of beams and columns and/or walls or, perhaps, and as we will see later, arches or cables or frames or other basic structural components that have as one of their primary functions that of providing our buildings with the physical robustness needed to make them stand up. All of these individual elements considered together as one is known as the building's *structural system*.

We established in Chapter 1 that in addition to providing adequate resistance to weight and other loads, a building structure is frequently called upon to perform other functions such as organizing internal spaces, defining external forms, controlling daylight, establishing circulation paths, etc. A structural system thus frequently also plays a part, to a greater or lesser degree as the case may be, in what might be characterized as the aesthetic and/or functional and/or conceptual agenda influencing the design of a building and, therefore, it may affect the visual expression of the architectural work as a whole. Yet even while acknowledging and even highlighting such a holistic approach to the design of buildings, it remains that the present book is one that is centrally concerned with the physical mechanics of structural behavior as well as how various aspects of construction and material technologies need to be observed in order to ensure that a structural system is able to provide its essential resistance to collapse. In order to do this, we need to first go back to the fundamentals of structural response and discuss what actually happens within structural components when loads are acting on them. Indeed, even before we are able to do that, it is useful here to take one further step back by trying to describe more precisely just what a structure actually is.

A *structure* is commonly thought to be a material element or a number of such elements working together, providing strength, stiffness, and stability in order for loads to be held aloft. The reason, of course, that we need to organize physical matter in particular ways is to satisfy our basic need for shelter. To protect us from the natural elements while at the same time providing inhabitable spaces of various sizes within that shelter calls for an instrument of a sort, otherwise known as a structure, whose function it is to make sure that all loads remain right where they are applied and that these do not cause the shelter to collapse upon us. The loads will nonetheless cause various parts of the structure to respond with smaller-scale deformations, explainable as the result of internal member forces that are established within the structural system in response to the loads that are applied to it. Moreover, these internal forces and the structure's deformations will be of a magnitude and type that is largely established by the structure's overall configuration. Summarizing all this, we can say that for a structure to be functional it needs to be made of sufficiently strong and stiff materials, and that the way it works is heavily influenced by its geometry – which, admittedly, may still seem to be a somewhat vague statement at this point, but it nevertheless establishes the defining principles that will be returned to and refined throughout the rest of this book.

Line vs. Surface Structural Elements

What kinds of structures exist? This is a big question that may be answered in very different ways. We could speak of spanning structures having as their primary function the "transport" of loads over horizontal distances, and of vertical support structures doing the same for loads acting over a building's height.[1] These two groups of structures are identified according to their spatial orientation. We could also identify structures by their physical response characteristics, applying terms like rigid or flexible structures. Furthermore, we might speak of *skeletal structures versus massive structures*, identifying structures by how much space

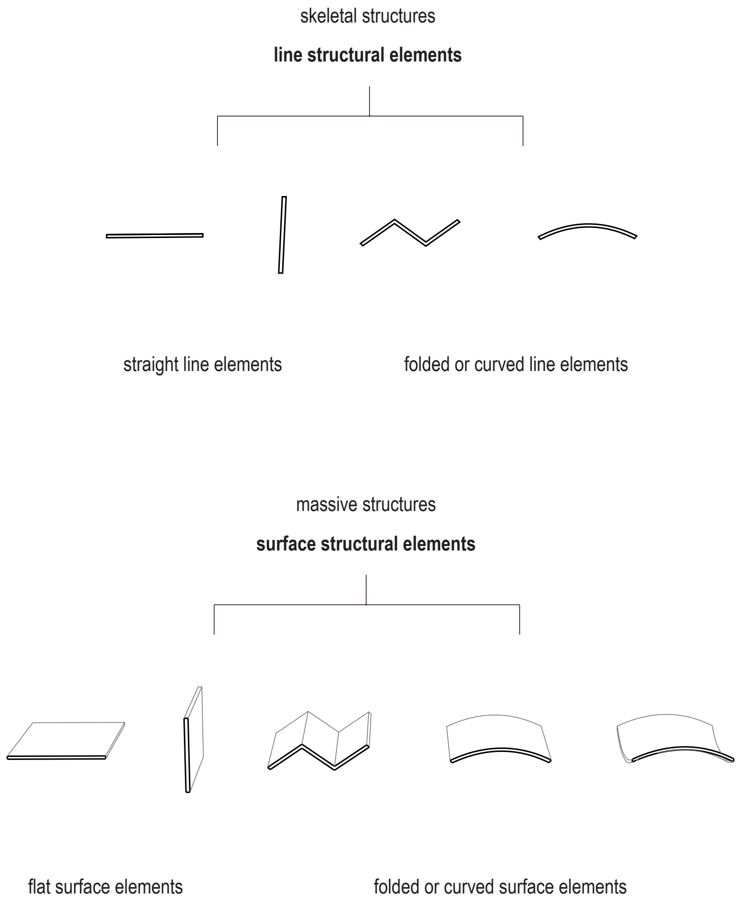


Figure 2.1
Skeletal structures' line structural elements versus massive structures' surface structural elements.

Illustration 2.7
"Construction Work" (1989).
A composition of skeletal structural elements.
Painting by Tom Slaughter.

they occupy and which correspond to *line structural elements versus surface elements,* respectively.[2] (Fig. 2.1; e.g., Ill. 2.7, 2.8.) There are many more ways to make such distinctions between structures, of course, but for now we will elaborate a bit more on this last classification and then go on to discuss the ways in which these two main groups of structural forms relate differently to the architectural spaces that they help create.

The line elements that make up skeletal structures may be classified according to their geometry as straight line elements and folded/curved line elements. *Straight line elements* typically form ties, columns, and beams, and on a more detailed level they also make up trusses that, geometrically speaking, are aggregations of many straight line elements. *Folded or curved line elements* typically form frames, arches, and cable structures. We shall discuss all of these basic structural types in much more detail later in the book.

If we take a closer look at skeletal structural systems that are built up of linear elements we will usually find that the different parts are arranged according to a *system hierarchy.* (Fig. 2.2.) To be able to actually construct the building envelope needed to seal off interior space from the exterior environment, for example, we frequently need a secondary system of linear structural elements attached to

Illustration 2.8
"Torqued Ellipses," *The Matter of Time* Exhibition (2005), Guggenheim Museum, Bilbao, Spain.
Surface elements can be considered structural just as much as they are sculptural.
Sculptures by Richard Serra.

Figure 2.2
System hierarchy; primary and secondary structural elements.

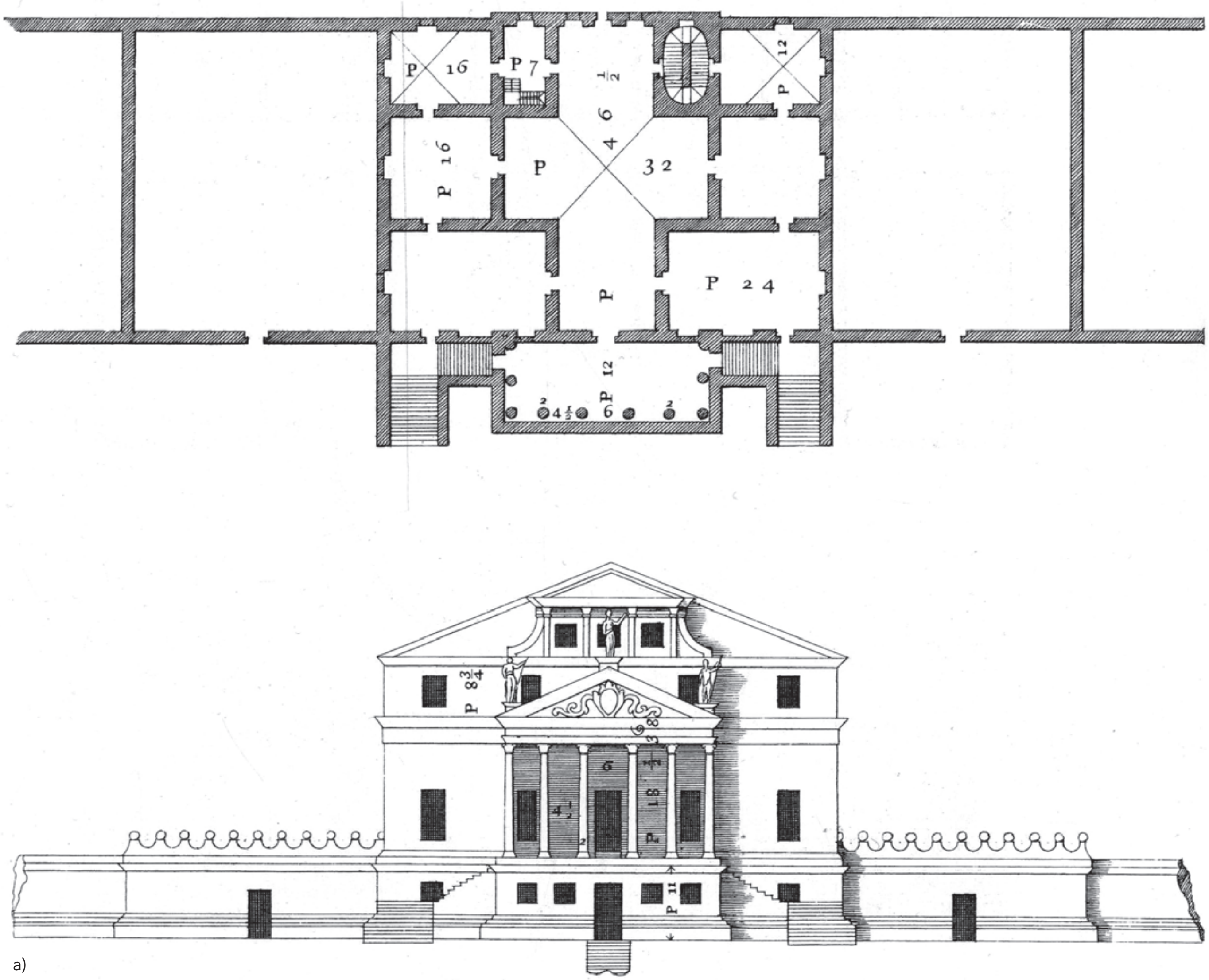
FACING PAGE
Illustration 2.9
Ground floor plans of three houses that represent both the massive structural system with load-bearing walls, and the skeletal structural system with columns that carry vertical loads.
(a) In the Villa Foscari, Malcontenta, Italy (1560) by the architect Andrea Palladio the load-bearing walls throughout also clearly establish the interior spaces. This is true for traditional building systems in which masonry of one sort or another was the most likely choice for structural materials.
(b) In the Tugendhat House, Brno, Czech Republic (1930) by the architect Ludwig Mies van der Rohe, the skeletal structure enables the limits of the space to be independent of the support structure.
(c) In the brick country house (1923) also designed by Mies in which load-bearing walls do not form closed rooms as they do in the Villa Foscari, but rather create open spaces where movement is relatively free and uninhibited, and yet where they still suggest room zones and to a certain extent also control view sightlines.

the primary structure. As a particular example of this condition, we might find that spanning between large inclined roof beams (also known as rafters) there are a number of smaller transverse beams (called purlins) running parallel to each other which, in turn, directly support a wood sheeting material that is used to make the roof surface. Structural elements such as these purlins are likely to span orthogonally with respect to their supporting elements, and to have a shorter spanning distance and therefore also be smaller; these are then typically termed *secondary structural elements* as opposed to the main beams that are the *primary structural elements*. In some cases there can even be a third layer of structural elements called *tertiary structural elements*.

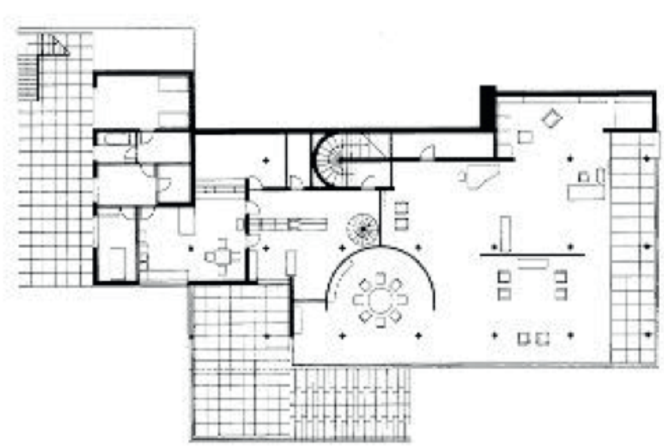
Looking now at the other broad group of structures that we have called *surface elements*, we will find that these can generally be characterized as being essentially two-dimensional, with significant dimensions of both length and width, while having a thickness that is typically much smaller than the other two dimensions. As we did with line elements, we can also classify surface elements geometrically into two groups as flat surface elements and folded/curved surface elements. *Flat* or *planar surface elements* form walls, slabs, and plate structures, while *folded or curved surface elements* in buildings may refer to the components of folded plate structures or else to singly curved arched vaults and cylindrical shells or to doubly curved tension membranes and domes and rigid shells. We will also find undulating surface elements within this last grouping, in the form of roof or floor slabs having varying curvatures, for example. For the time being, however, there is no need to worry about all of these new terms and structural forms; the later chapters of this book will eventually discuss just how all these different surface elements are shaped and how they behave when loads are applied to them.

As was previously discussed, structural systems have broader implications in the context of architecture than "simply" that of carrying and resisting loads. For example, one can observe the differences in terms of the *spatial qualities* produced by the two distinct vertical load-carrying systems of skeletal/line structures (columns) and massive/surface structures (walls) that were introduced above. Let us first consider, for example, the spaces within two well-known residences: the Villa Foscari at Malcontenta in Italy dating from 1560, and the Tugendhat Haus in Brno in the Czech Republic completed in 1930. The house from the Renaissance period designed by Andrea Palladio (1508–1580) represents a traditional building type in which masonry walls carry all the roof and floor loads and self-weight of the walls themselves down to the ground. (Ill. 2.9a.) These surface-type wall elements also very clearly establish the dimensions and sense of enclosure of the interior spaces of the house. It can be said that there is, therefore, an intimate relationship here between the functional aspect and quality of the architectural space on the one hand and the dimensions and geometrical arrangement of the load-bearing structure on the other. This has been the most common condition throughout building history when brick and/or stone structures were dominant and it continued to be the most important structural system until the twentieth century.

In contrast to this, within the 1930 Modernist period Tugendhat House by Ludwig Mies van der Rohe (1886–1969) line structural elements in the form of steel columns carry the vertical loads, and in doing so these hardly interfere with the open space all around them. (Ill. 2.9b.) Indeed, in this house the limits of the different functions within its large room occur in ways that are totally independent of the grid that the columns set out, and these are infinitely changeable. This is an example, then, of the so-called "free plan" advocated by the architect Le Corbusier early in his career, and which is made possible here by the steel column grid; the relationship between the vertical support structure and the space of the house is one that is very free and open.



a)



b)

c)

Looking just at these two examples might lead to the conclusion that load-bearing wall structures belong in the past. But that is not the case. In fact, in just the preceding section we saw that within the Bregenz Kunsthaus from 1997 there are three massive reinforced concrete walls that are the only means of support for the loads of the multilevel art gallery and that these walls also organize the plans and help to define the spaces of the museum, control their lighting, etc., and by doing so clearly demonstrating that the wall has not lost its place in contemporary architecture. In fact, in a similar vein it is interesting to note that early in his career Mies also worked with load-bearing walls as a way of establishing room zones within a basically open living space, as exemplified by his project for a brick country house from 1923. (Fig. 2.9c.) Both of these examples exploit the spatial potential of load-bearing walls in a different way than does the traditional building type in which walls completely enclosed and defined interior spaces. Instead, in the more contemporary examples, overall spaces are much more open and movement is relatively unconstrained in spite of the presence of structural walls.

But at the same time it should also be noted that while the Modernist architectural style mostly developed from the early 1920s onward based on an exploration of new open spatial concepts and of structural systems involving skeletal frameworks, both of these innovations can not-so-coincidentally also be connected with the significant material advances that have occurred over the past 100 years or so; i.e., the industrial production of structural steel and of high-strength concrete – but this is yet another topic that we will come back to repeatedly throughout this book, and especially in Chapter 5 Materials. Of course, Modernism has had a lasting legacy well into our time, with much of what we build today being based at least on some level on its fundamental principles, even as enclosed spaces still find their place and raison d'être today and as surface elements continue to be with us in the form of contemporary load-bearing walls, slabs, folded structures, vaults, and shells. In fact, these structural forms can be said to be experiencing a renaissance of sorts in our age of computer-aided design and computer-assisted manufacturing, and we shall encounter some interesting examples of them in the chapters to come.

Structural System Categories: Long span vs. Low-to-mid-rise vs. Tall Building

It should be pointed out that so far in this section we have primarily been discussing the differences between *vertical* structural elements and the impact of these on certain architectural design objectives. The reason for this is that the majority of buildings around us are relatively *low-to-mid-rise multistory buildings* intended for common purposes; i.e., most are probably residential while a significant percentage will be commercial office buildings. An essential aspect of knowing about structural systems, therefore, must necessarily involve knowing how stories can be stacked up one on top of another and what the structural implications are when this takes place, both spatially and physically.

In this very common building type, horizontal spans for the floors and the roof are typically relatively modest. This means that the structural logic and behavior of these spanning subsystems does not need to vary very much from one case to the next, and that these are thus of lesser importance at this very early stage of the discussion about structural element choices and their overall spatial consequences. The horizontally spanning structure in such buildings could be a flat concrete slab or a slab strengthened by underlying steel beams or else a timber beam system with a walking surface layer of wooden boards, etc. – and the typical spanning range for all of these falls within 3–10m (10–30ft), i.e., certainly enough to cover a typical room's plan dimensions. Because floors generally need to be flat and uniformly solid in order for people to be able to occupy a space and circulate within it, aside from any resultant surface textures and visual patterns (e.g., beam spacing, material choices, etc.) there will be relatively little difference among these horizontal subsystem alternatives that would strongly affect an overall sense of space within this building category. Horizontal spans start to be more structurally challenging and of significant spatial and visual interest, however, when the spans go beyond this, and so we will return to this topic a bit later in the book to discuss the various options that are available for this purpose.

So if we think of the low-to-mid-rise multistory building as a "core" building category, we may start to be able to see that the long-span building and the tall building are both "extensions" of this, one in the horizontal direction and the other in the vertical. (Fig. 2.3.) At one extreme of this range, one-story buildings may be asked to provide large, open spaces that are uninterrupted by structural

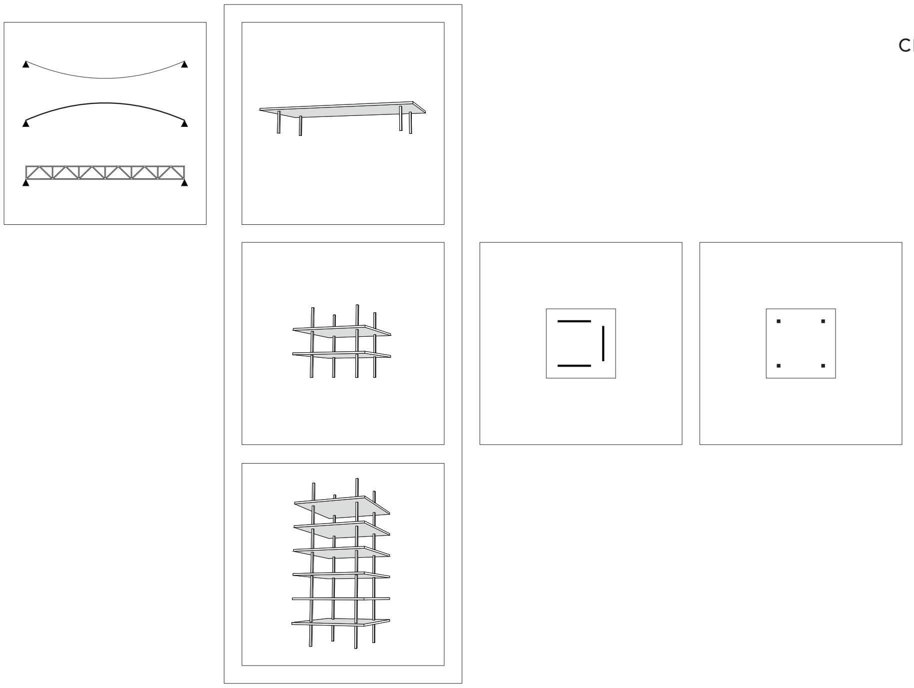


Figure 2.3
With the *low-to-mid-rise building* having a limited number of stories established as the "core" building category, by extending the vertical load-bearing system we may also identify a different building category, namely that of the *tall building.* Whether low or tall, a vertical structural system in a building is typically based on variations of the two fundamental structural element types: the column-based skeletal/line structure or the wall-based massive/surface structure. Likewise, at the other end of the spectrum, by extending the horizontal structural system of a building a large, horizontal span emerges. This third category of the *long-span building* typically leads to a discussion of alternative structural forms for coping with these large spans such as beams, arches, or cable structures.

elements. This calls for structures having *long horizontal spans*, which can be considered to be its own particular building category with its own set of structural and spatial considerations, such as strategically shaped beams, trusses, cable-supported structures, as well as vaults, domes, folded plates, and shells. At the other end of the spectrum, however, we have *tall buildings* in which both vertical gravity loads and lateral forces due to wind pressures and seismic conditions can become very substantial, and these impose new challenges on the structural system, including overall stability and dynamic movement. In our discussion of structural forms throughout this book, we will encounter examples associated with all three of these categories of (highly simplified) building types; i.e., the long-span building, the low-to-mid-rise building, and the tall building.

Locating and Arranging Vertical Structural Elements

Quite often the vertical supporting elements in a building are *located* according to the intersection points of a regular grid, with the horizontal distances between these structural elements found to be similar over most of the building plan. This regularity has the advantage of allowing for a standardized construction process and the eventual flexibility of occupancy arrangements.

Different overall building plan configurations may lead to other ways of positioning the various vertical support elements, however. In a rectangular building, for example, columns and walls may, in a similar way to that which we have just described, be more or less uniformly distributed in each orthogonal direction according to a square grid overlaid over the building plan, resulting in roughly equal floor-beam or floor-slab spans in each direction. Or, perhaps, the grid is not symmetrical and these vertical supporting elements may be more closely spaced in rows that run parallel to the long sides of the building, leading to different span lengths for the beams or slab in the two directions. Or else yet again, columns and walls may be concentrated at certain points in the plan while still maintaining a certain overall geometrical regularity; there are, indeed, numerous ways of doing this within a floor-plan layout's "spacing rules" that can be established by such a grid.

Whether vertical structural elements are located according to positions established by a grid or not, however, we also need to consider their many possible combinations or *arrangements* over a building plan – keeping in mind, of course, that these elements are intended to support the many types of loads, both vertical and horizontal, that act on an overall building structure. To begin this discussion, we will once again start for simplicity's sake and ease of classification with the basic premise that we will distinguish between arrangements that are made up of skeletal/line and massive/surface structural elements, the two basic element form categories that we have described above.

Four basic variations of the many possible plan arrangements for these structural elements are shown in Figure 2.4. We can easily recognize the case of a "pure" skeletal system composed of line-element columns and beams, with a variant of this being a system in which such columns support horizontal surface structural elements in the form of floor slabs. For our limited purposes here, however, in which we are only concerned with the form and arrangement of vertical structural elements, we will label both of these systems as belonging to the "skeletal structure" type. Instead of columns for the vertical structure, however, we may have massive/surface elements in the form of load-bearing walls that are located in the plan either as isolated planar elements, or else several of these may be arranged together in such a way that they form more-or-less-closed "boxes." This latter grouping arrangement of intersecting walls effectively form vertical structural members having a hollow prismatic space in the middle, and these are known to have significant load-bearing capacity while at the same time possessing distinct spatial qualities. All of these four basic vertical structure arrangements can obviously be reconfigured in many different ways according to programmatic needs, design intentions, loading demands, etc. – some examples of which are shown in Figure 2.4.

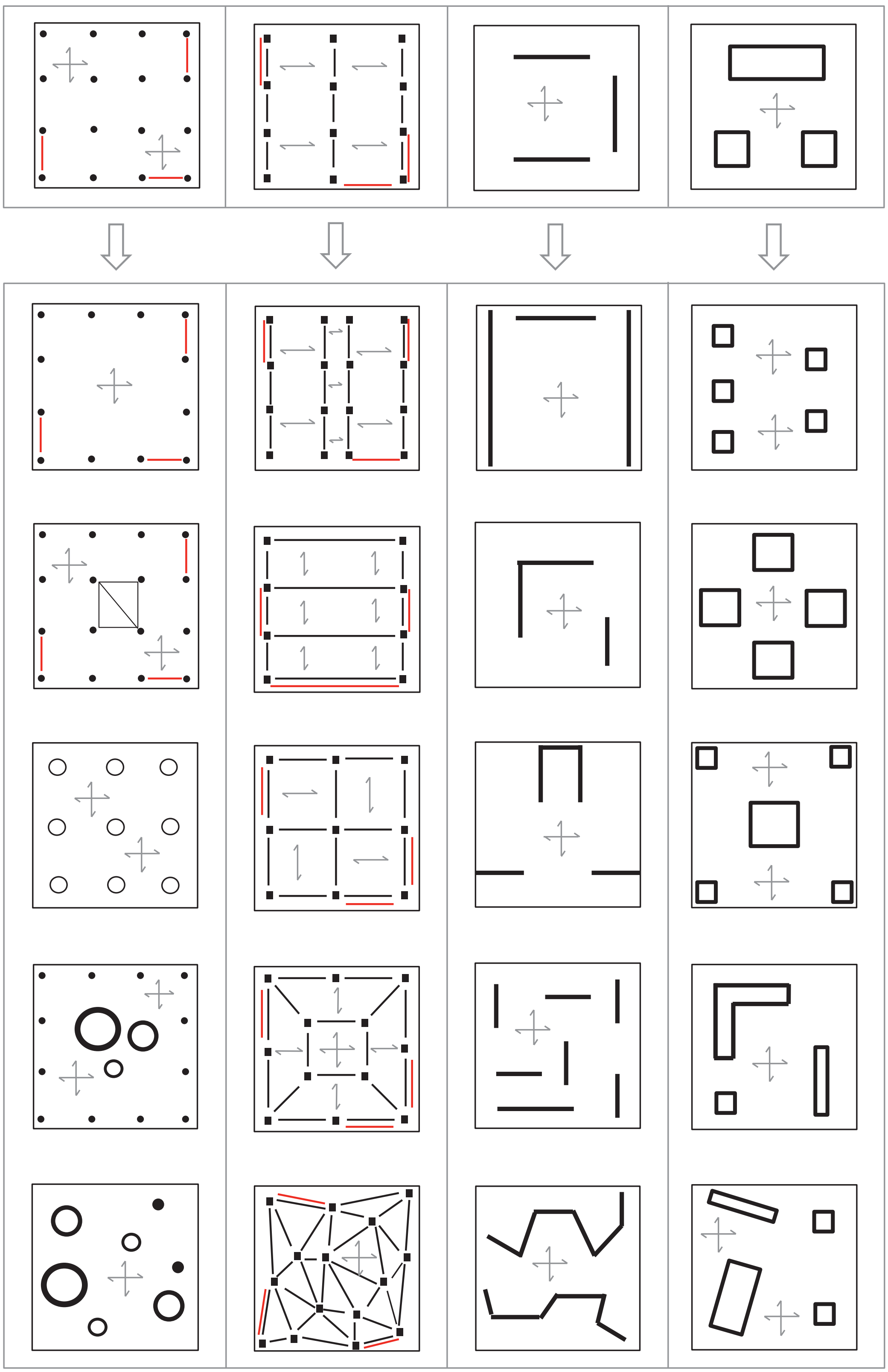


Figure 2.4
Four basic plan variations of vertical structural element arrangements are shown, while a large number of other related arrangements are possible, only some of which are represented. It should be noted that these are merely schematic suggestions and that they do not reflect all there is to a real-life building plan.

Dots indicate column positions and thick black lines indicate the locations of load-bearing walls. Thin black lines between the dots (columns) represent beams, and arrows point out the spanning direction of the floor structure. Where no beams are indicated by straight lines between columns, the horizontal (floor) structure is being thought of as a flat slab of reinforced concrete. Where arrows cross, two-way action of the floor slab is being suggested. Red lines represent the need for some sort of lateral bracing in the vertical structural system.

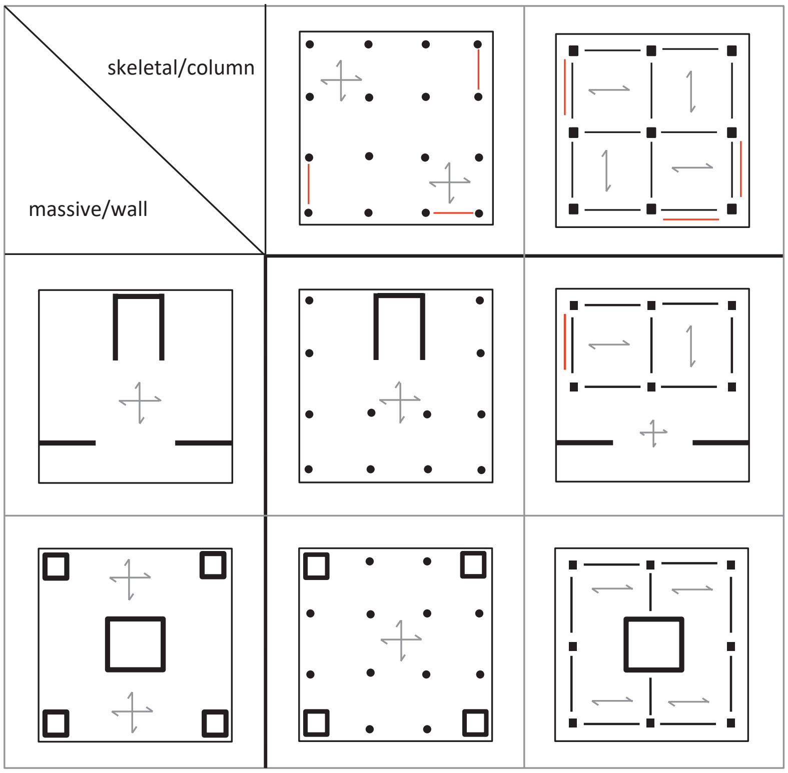


Figure 2.5
Diagram of hybrid structural systems that combine both the skeletal/line and massive/surface structural element types.

Illustration 2.10
White Forest Pavilion, KAIT, Atsugi, Japan (2007). Non-uniform interior spaces of a student workshop are created by having a large number (305) of irregularly placed, randomly oriented, small planar columns that suggest room zones and act as visual screens of variable densities aside from carrying loads.
Architect: Junya Ishigami + Associates. Structural engineer: Yasutaka Konishi. Cornell model by Jordan Berta and Henry Gao.

As an aside, it should be noted that while regular grids are certainly the most common arrangement for vertical structural elements within a building plan, this need not be a necessity. In fact, in contemporary architecture a number of unique buildings are constructed with a much "looser" arrangement of column locations, whereby these elements may themselves establish different sub-spaces and multiple circulation paths within an overall interior space. (e.g., Ill. 2.10.) Such column arrangements obviously correspond to a very different notion of "freedom" of organization and space and movement than that which the Modernists developed according to their regular orthogonal grids.

A structural system that is composed solely of columns as vertical load-bearing elements will need additional lateral bracing in order to maintain a building's lateral stability. This is indicated in Figure 2.4 by red lines that represent bracing strategies such as cross-bracing, single-diagonal bracing, or rigid frames. There is much more discussion to come about lateral stability concerns in buildings and bracing options, as will be found in this chapter in Section 2.5 and then later in the book when a full chapter is dedicated to the topic, i.e., Chapter 10 The Frame and the Shear Wall.

It should also be pointed out that although we have repeatedly distinguished in this section between skeletal/line element structures and massive/surface element structures, in reality it is only in certain cases that only one or the other of these basic structural element types is used exclusively throughout in a building. In most cases we will find that overall structural systems will be something of a *hybrid system*, with both skeletal and massive structural elements utilized together. (Fig. 2.5.) Of course, various factors will influence the choice of a particular structural system and its unique configuration, such as architectural plan layout, desired visual appearance, spanning distances, building material selection, construction techniques, among many others.

Notwithstanding the basic *structural element geometrical form classification* that we have introduced in this section (e.g., column vs. wall), it will be found that the chapters of the present book have been organized according to basic *structural element types that are identified by their behavior when carrying loads.* This means that discussion about the structural response of corresponding line element and surface element forms can take place side by side and within the same context, irrespective of their geometrical

Illustration 2.11
Shibaura Office Building, Tokyo, Japan (2011).
Exterior strongly expresses skeletal/line element structural system of columns, beams, and diagonal braces.
Architect: Kazuyo Sejima & Associates. Structural engineer: Sasaki Structural Consultants.

classification; e.g., beams and slabs are treated as two variants of the same structural problem, namely *bending*, and columns and walls share a chapter where *compression and buckling* are the main structural behavioral issues that one needs to be concerned with.

Before being able to do any of that, however, we will first need to consider much more explicitly just what are these fundamental structural element behaviors/responses to carrying load that have been mentioned, and which we call *basic structural actions*. And this is just what we will do, after a brief interlude comparing the implications of and rationales for the vertical structural systems used in two Tokyo buildings.

2.3 Contrasting Systems in Tokyo

Within the low-to-mid-rise "core" building category, we have seen that numerous structural system arrangement options are possible, all of which are variants and permutations derived from the two fundamental types of vertical structural elements, namely those that are of the skeletal/linear type, or columns, and those of the massive/surface type, or walls. In this section, we will directly compare and contrast two such buildings, both located in central Tokyo, built within a few years of each other, and similar in many other respects as well – such as by their overall modest height and overall size and intended office-use program function. Nonetheless, and in spite of all these similarities, these two buildings have rather dramatically contrasting structural systems that give them vastly different architectural resolutions. We are referring here to the Shibaura House, an office/workshop/community space building designed by the architect Kazuyo Sejima & Associates (Ill. 2.11, 2.12), and to the R4 Office Building by Florian Busch Architects. (Ill. 2.13, 2.14.)

While the Shibaura House can be characterized as being open and expressively extroverted, the R4 Building is much more closed in upon itself. Such overall spatial readings and visual appearances are very much reflected in these buildings' contrasting vertical structural systems – or vice versa. In the former case this structure is composed of a skeletal/line element system of exposed steel columns and beams and diagonal braces whereas in the latter it consists of massive/surface structure in the form of nearly continuous load-bearing reinforced concrete walls all around the perimeter.

Illustration 2.12
Shibaura Office Building.
Skeletal/line element system minimizes member dimensions, creating a sense of openness from the inside toward the surrounding urban context.

Illustration 2.13
R4 Building, Tokyo, Japan (2015).
Exterior expresses the massive/surface structural system of a perimeter concrete wall that is pierced by irregular window openings.
Architect: Florian Busch. Structural engineer: Akira Suzuki / ASA.

Illustration 2.14
R4 Building.
Massive/surface structural system creates shelter and enclosure in a very tight urban condition, while strategic openings allow for limited but selective views toward the outside.

The Shibaura House is completely enveloped in glass and its exterior columns and braces appear only as thin vertical lines that do not obstruct the daylight from entering the building in any significant way. Here the external surroundings are effectively made a part of the interior spaces because of the completely transparent nature of the line-element structural system and of the building's glass cladding. And at night, the Shibaura House becomes a distinctive and active urban lantern. In the case of the R4 Building, only very limited and discrete square window openings are inserted into the perimeter wall at carefully selected locations in order to allow sufficient daylight into the inside and to provide tightly controlled views toward the exterior. Overall, these openings also form a seemingly random and arbitrary pattern on the building face, especially at night, and in so doing carefully camouflage the building's scale and any notion of the predictable floor levels of a typical office building.

The somewhat open public/private program of the Shibaura House invites people to observe the internal activities from the outside and to enter inside. There are also three large open-air terraces that are carved out of the volume of the building, thereby merging internal and external activities and combining the sense of interior/exterior spaces being simultaneously occupied. In contrast, the tight and awkward urban context of the curved and narrow site conditions for the R4 Building called for a much more insular and less transparent building. The activities taking place within its multiple leased office spaces are also inherently of a more private nature than at the Shibaura House, and so these need to be much more sheltered. In this respect as well, then, the reinforced concrete perimeter wall surface structure of the R4 Building works especially well.

These two very different choices of structural system arrangements, then, can be seen to each have their place in contemporary Tokyo. Each system relates in its own way to contrasting architectural design intentions that address visual appearances, spatial experiences, site conditions, and programmatic functions, while simultaneously adding new "life" to their urban environment. All things considered, these are no small tasks that the structural systems are being asked to perform – certainly much more than "just" carrying loads.

2.4 Fundamental Structural Actions

In order to have a more informed and detailed understanding of the mechanics of structural behavior and about structural elements' and systems' response to loading, however, we must at this point take a big step back and establish certain fundamental principles.

Types of Deformations and Structural Actions

To start with, we can identify the different types of *deformations* that result in structural elements when they carry loads, and the name that we give to the *forces* that are associated with those deformations. (Fig. 2.6.) Although we may not see them because of their typically small magnitudes, it must be understood that such *deformations are always present in structural elements that carry load*. But we also take care to design structural elements in such a way as to prevent any especially large deformations that clearly would not be acceptable for aesthetic or physical comfort reasons, or else that might cause cracking of floor and wall finishes, for example. Depending on the direction of the applied load with respect to the structural element, the different types of deformations that can result are as follows:

- *Elongation* or stretching of the element, which we explain as being the result of a *tension* force acting within the element. Tension force is the effect of two loads pulling away from each other in opposite directions.
- *Shortening* of the element, resulting from a *compression* force acting within the element. Compression force is created when two loads are pushing against each other.
- *Flexing* or curving of the element, caused by *bending* action within the element. Bending is the effect of transverse loads being applied to the element.[3]
- *Twisting* of the element, which is seen as a result of forces acting that cause the two ends of an element to twist in opposite directions (torsion).[4]
- *Wracking* or *"shearing"*; a parallel movement of one plane within an element relative to another. This is understood to be the result of *shear* forces acting. Shear force is the effect of two loads of opposite direction acting in two different planes within the structural element.

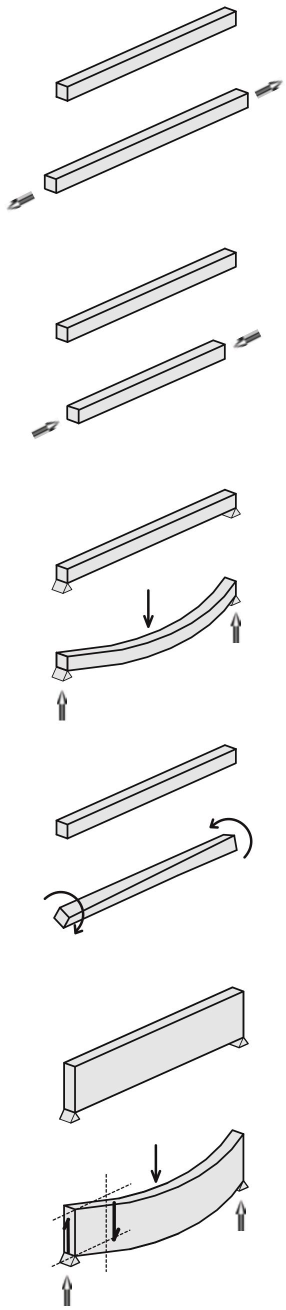


Figure 2.6
Small deformations and accompanying forces in structural elements: elongation/tension, shortening/compression, flexing/bending, twisting/torsion, wracking/shear.

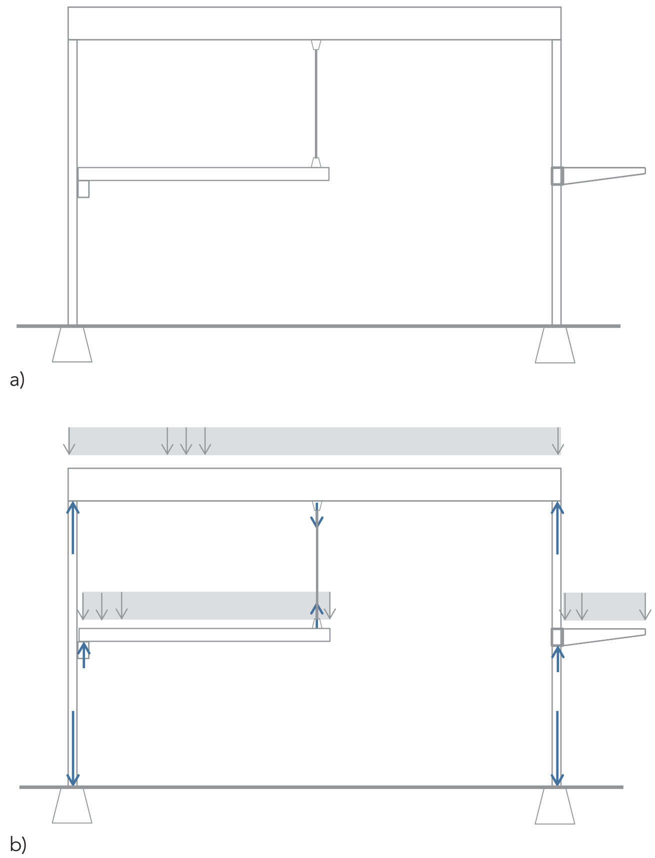


Figure 2.7
Representative building structure and loading.
(a) Building section taken through a large double-height room with a mezzanine. The roof beams rest on columns, while the mezzanine floor beams hang from above at one end and are connected to the columns at the other. A canopy structure projects outwards at the right side without being supported at the far end.
(b) Building section with loads depicted. Arrows symbolize the loads and (reaction) forces involved.

Envisioning Fundamental Deformations and Structural Actions

In order to help visualize these structural actions in an architectural context, we will now closely examine the responses to loading of the elements (mostly columns and beams) that make up a representative building structure whose overall section is drawn in Figure 2.7. This example is particularly designed to demonstrate the different deformations and forces that we presented in isolation above and which we may encounter in any specific building. We see a large room with a ground floor and a mezzanine level over half of the available floor space. Columns carry both ends of the roof beams while the mezzanine at one end is suspended from those beams by help of vertical tension elements, called hangers, and at the other end rests on brackets fastened to the columns. An entrance at the right is protected by a canopy constructed with small beams projecting outwards without support at the outer end, but that are fixed to a beam that we will shortly see runs in what is the longitudinal direction of the building; i.e., into the plane of the paper in this two-dimensional representation. The weight of the structural elements themselves, plus the loads that they are supposed to carry such as snow load on the roof and canopy and live loads from the occupants and furnishings on the mezzanine level, are all represented in Figure 2.7b by a series of arrows that are evenly spread out over the length of the beams. This graphic depiction shows that gravity loads like these act vertically downward

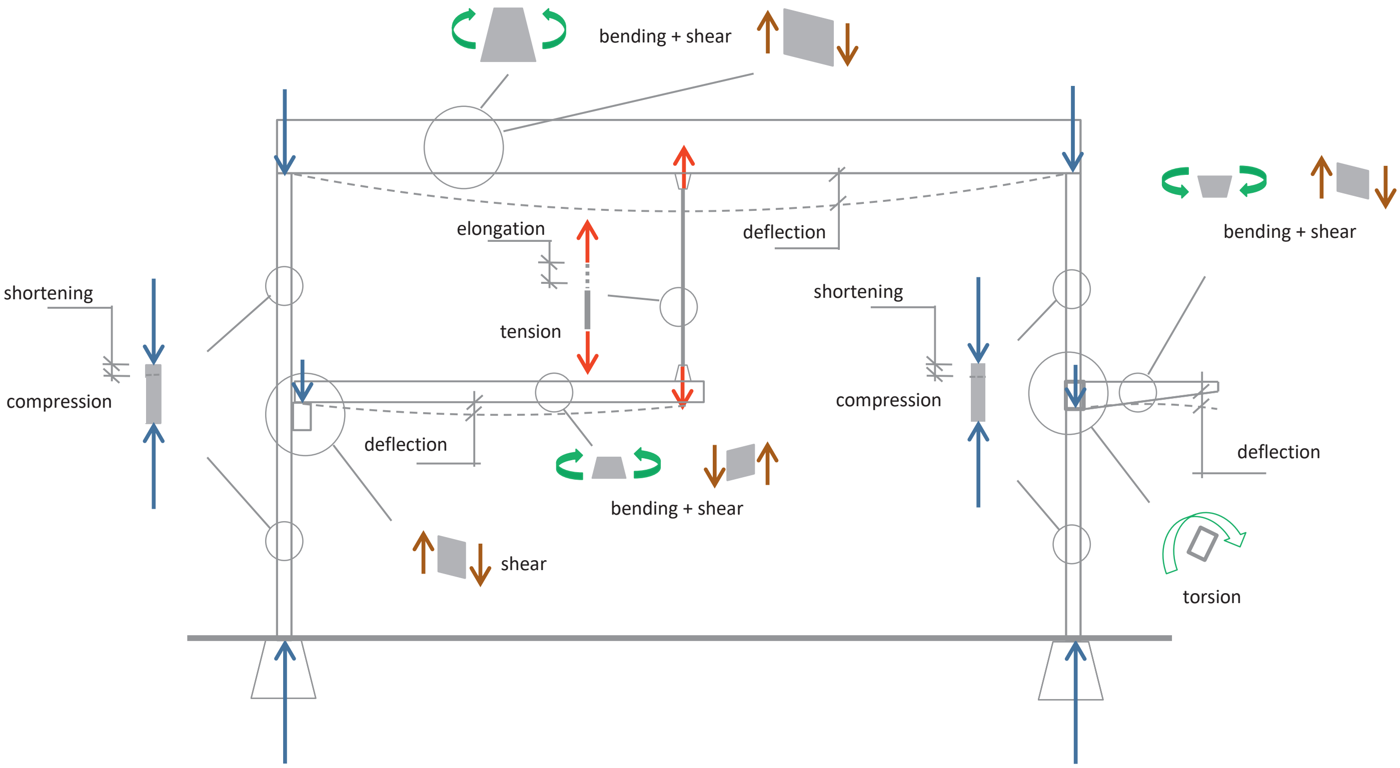


Figure 2.8
Building section depicting the deformations, forces, and bending moments that act in the various structural elements; i.e., in the beams and columns, and in the hanger and support bracket at the ends of the beam for the mezzanine.

and that they are so-called uniformly distributed loads (UDL) along their length, at least in the present case. There will be much more discussion about loads on structures in Chapter 3. What we wish to know for now, however, is primarily what will happen to the different structural elements when these loads act on them. We can work our way through the whole structural system with the goal of identifying the structural actions that result in all the different parts of this building section when the loads act as stated. The result of such an analysis is drawn in Figure 2.8, and all five structural actions defined at the start of this section can be found to be represented.

Starting with the roof beam resting as it is on a column at each end, both the uniformly distributed load it carries as well as the (concentrated) load it receives from the hanger that supports part of the mezzanine level below make the beam bend slightly downwards over its length in a curved fashion. This is an example of flexing, caused *by bending action* within the beam. Such bending can only take place if one part of the beam (the upper, in the present case) is shortened while the other part (the lower) is elongated, but above we have associated shortening and elongation with compression forces and tension forces, respectively. Therefore, *bending action actually results in compression forces and tension forces acting on the cross-section of a structural element that is flexed*. These are forces acting within the beam element: i.e., the upper part of the beam will experience an inner compression force while the lower part of the beam will experience an inner tension force. (Fig. 2.9.) We call the effect of these two internal forces a *bending moment*.

Moreover, although it is not as easily noticed, the beam actually simultaneously deflects in another manner. If we imagine that the beam consists of a series of short planar rectangular segments that somehow are attached to one another side-by-side to make up the whole beam length, we will find that as a result of the loading on the beam there is a tendency for such a segment to "wrack" or deform vertically from one side to the other into a parallelogram shape. This is the type of deformation that is depicted in the element at the bottom of the preceding Fig. 2.6. We understand this deformation to be the result of *shear forces* acting between these two ends of the segment of the beam. Therefore, *when there are bending moments in a beam element there are also shear forces present*.

If we now direct our attention to the columns we can imagine that they will yield a little to the weight of the beam and all of the other loads it supports. (See Fig. 2.8.) The columns are pushed downwards along their lengths in the direction of the foundations and respond by being slightly shortened. This means that internal compression forces are at work according to what we have established in Figure 2.6.

The structure of the mezzanine floor beam reacts in a similar way to the roof beam. (Also see Fig. 2.8.) The mezzanine beam will

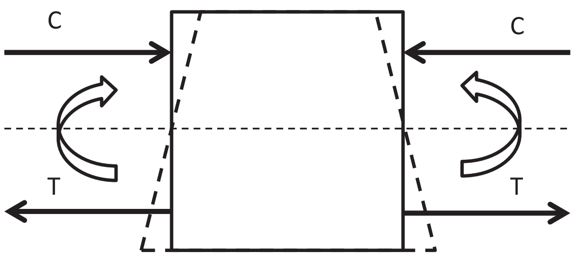


Figure 2.9
Detail of a cut-out of the beam that is subject to bending; the upper part is shortened as a result of the bending and thus experiences a compression force in the cross-section while the lower part is elongated and experiences a tension force.

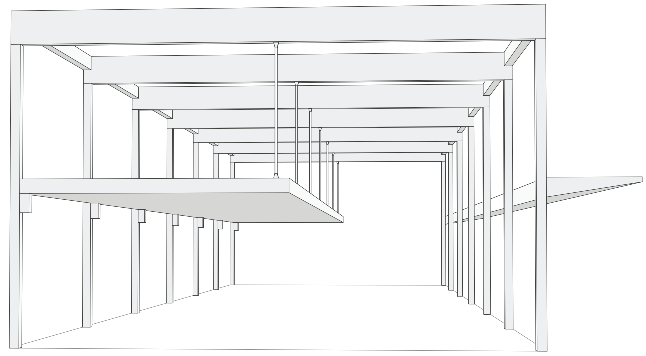


Figure 2.10
Perspective of the building as a volume, where the building section of Fig. 2.7 is successively repeated to form a space. The structural system is shown without any bracing that we will see is actually needed in order to secure its stability.

deflect downward due to the uniformly distributed loads that it is subjected to and so it will be subject to bending moments and, as explained above, simultaneously to shear forces.

At its right-hand end we find that the mezzanine beam is suspended from above by a so-called hanger, commonly in the form of a steel rod or other types of slender steel profiles. The hanger is attached to the underside of the roof beam. A portion of the total weight of the mezzanine is therefore carried by the hanger, which stretches in response.[5] In Figure 2.6 we have associated the elongation of a structural element with *tension force*, and conclude that the hanger experiences tension along its length.

The left end of the mezzanine structure rests on a bracket connected to the column. This short and stubby structural element has a more or less square elevation and if, so as to exaggerate the response to loading, we imagine this bracket to be made of a material that could deform easily, we can more easily visualize that the left-hand face of it which is fastened to the column is unable to move while its right-hand end is able to displace downwards somewhat. The square thereby deforms so that its elevation becomes lozenge-shaped and turns into a parallelogram. Again, this wracking deformation is the kind that we have earlier described as being caused by *shear forces*. Shear force is the dominant structural action for this short structural element.

Finally (at least as far as this two-dimensional representation of the structure is concerned), let us examine the canopy beam on the outside of the building. (See Fig. 2.8.) This beam, while attached to the building at its left end, is seen to be extending out into the air without support at its outer, right-hand end, and we call this condition a *cantilever*, or a cantilevering beam. Transverse loads on the canopy beam will cause it to progressively deflect more and more toward its outer end; with concave-downward curvature. The form of this bending behaviour implies that the *upper part* of the canopy beam in this case is stretched as a result of the inner tension forces that are acting, while the *lower part* is shortened as a result of compression forces at work. And, once again, there will simultaneously be a slight deformation of the beam caused by shear forces.

At this point, however, let us imagine that the building continues into the paper plane of the section drawing, so that three-dimensional space is created and constructed, by repeating the same structural section every so often in what becomes the building's longitudinal direction. (Fig. 2.10.) Let us then take another look at how this cantilevering canopy beam is attached to the rest of the building structure: somehow it is either bolted or welded (if made of steel) or cast (if concrete) to a longitudinal beam running along the building façade between adjacent columns. This condition is repeated at regular intervals all along the building's length, and thereby helps to form an exterior shelter all along this face of the building. The canopy support beams tend to rotate clockwise when gravity loads act on them, but because each of their left ends is rigidly attached to the beam that is spanning longitudinally between columns, this will tend to cause this latter beam to twist. This means that two adjacent cross-sections of the longitudinal beam will rotate relative to one another. (Fig. 2.11.) We thus have a case of *torsion forces* acting in that beam, a structural action that was also briefly introduced at the start of this section as well as in Figure 2.6.

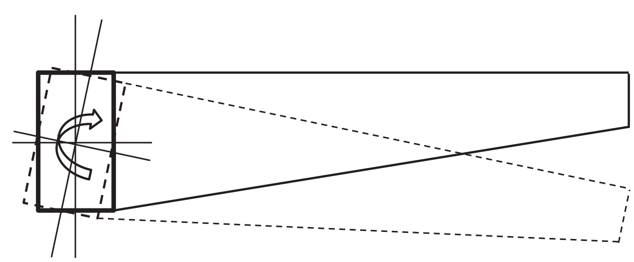


Figure 2.11
Diagram of the cantilevered canopy beam fixed to a beam in the longitudinal direction. This latter beam will be subjected to torsion forces; i.e., twin forces that together make up a particular kind of moment which causes the beam to twist slightly. The amount of twisting shown here is vastly exaggerated.

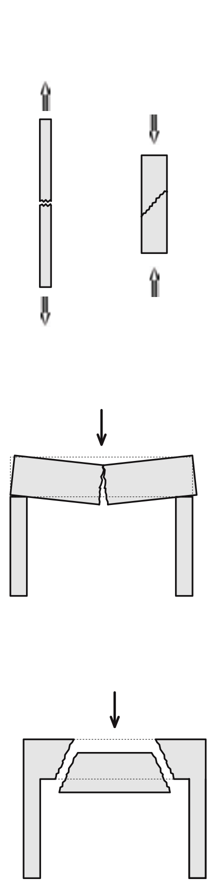


Figure 2.12
Failure mechanisms related to structural strength: tension failure, compression failure, bending failure, and shear failure.

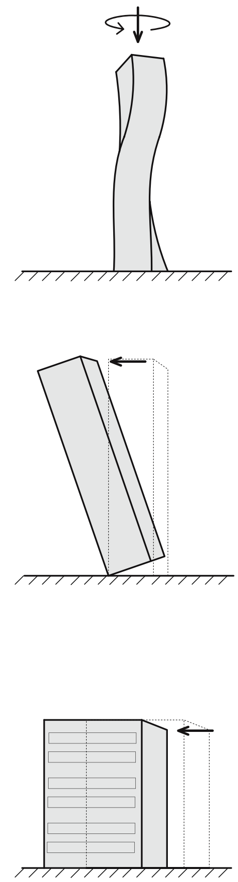


Figure 2.13
Failure mechanisms related to structural stability. Overall stability failures from twisting, overturning, and sliding.

Basic Concepts of Strength, Stiffness, and Stability

The overriding goal of structural design is to make sure that a building structure performs well and safely during its lifetime while preferably also contributing to an enhancement of its other architectural design objectives. This certainly implies that one needs to design the structure in such a way that at the very least strength, stiffness, and stability are all satisfactorily sustained for all possible loads that will be acting on it. By the *strength* of the structure or structural element, we basically refer to the point at which the forces reach the limit of what the material of which it is made can resist. This limit may be different for each type of force (i.e., tension strength, compression strength, etc.), and certainly it will vary quite substantially according to material choice. When such a limit is reached, we say that the structure or structural element fails. *Failure* is intimately linked with structural strength, and there are several ways in which failure can occur. If the tension force has reached the limit of what the element is able to resist for that type of structural action, then we say that a tension failure occurs, etc. (Fig. 2.12.)

By *stiffness*, we mean the ability of a structure to resist deformations, enabling it to function in the way that we expect and allowing us to comfortably occupy a building. We may refer on occasion to the stiffness of each structural element and in other cases to the stiffness of the structural system as a whole. A certain proportional deformation, given as a number, is commonly established as a limit for deformations; e.g., for a beam it might be a deflection of 1/300 of its length, and for a tall building a side-sway of 1/400 of its height may be considered the maximum permitted.

Last, but not least, *stability* concerns need to be addressed; if not, these may also be considered to be the cause of the failure of a structure. We can distinguish between the overall stability of a system and the internal stability of individual structural components. Indeed, a whole building's structural system may twist about its base, or may overturn, or else slide on its foundations when subjected to horizontal loads caused by wind or earthquake. We refer to such failures as a lack of overall stability. (Fig. 2.13.) Moreover, individual structural elements need to hold their position in space relative to other elements; if not, the intended structural cooperation between them may be prevented and unacceptable large deformations may result. Abrupt and uncontrollable sideways deflection of a slender strut subjected to compression forces, for example, may exemplify a lack of stability of a structural member. (We will discuss this in Chapter 8.) The remedy for such concerns, obviously, is to ensure that through a knowledge of predictable structural behaviour we design stable structural elements according to the anticipated loading.

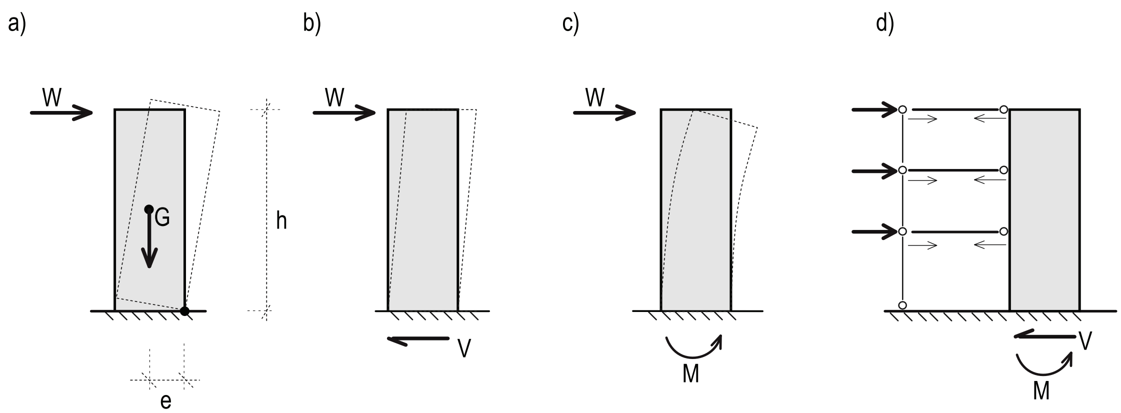


Figure 2.14
A basic overall stability problem is that of rotational equilibrium. (a) The overturning moment produced by the horizontal loads acting at a certain height above the ground must not exceed the resisting moment produced by the dead load of the structure about the potential point of rotation, with the latter increased by a factor of 50 percent to assure safety. (b, c) Both shear forces and bending moments are typically produced and need to be resisted within the laterally stiff structural subsystems in a building. (d) The transfer of lateral loads acting on the building's façade occurs through floor-level beams or slabs to the laterally stiff building subsystems, such as the walls around elevator and stair cores, for example, but that may be located elsewhere.

2.5 Overall Stability – Taking a Bird's-eye View

In this section, we will consider the different overall stability failures that may result from lateral loads acting on building structural systems, and thereby develop certain general principles for how we should organize and distribute structural subsystems within a building plan.

Overall Stability Concerns

For a building that is subjected to lateral loads, there are a number of basic requirements which must be fulfilled so that stability can be maintained. First, when seen in elevation, the building must be prevented from rotating on its edge when it is subjected to lateral loading or, to exaggerate the point, from rolling over like a tumble weed. (Fig. 2.14a.) This is most clearly envisaged for a tall building and especially one that has a particularly low weight. The critical aspect here is to make sure that the weight of the building is sufficient to keep the whole width of the building securely connected to the ground when maximum lateral wind loads are acting on it, and within an additional safety margin of perhaps 50 percent for good measure; i.e., the objective is to be well assured of preventing the building from rotating as a rigid body about its potential axis of rotation, which is the line formed where the building structural system meets the ground on the leeward side, opposite to the one on which the wind acts. If this is not possible because the building is too tall and therefore it is subject to wind loads that are too large to prevent potential rotation, then we must mechanically anchor the building structure to the ground on the windward side. Of course, since wind forces are unpredictable and may reverse direction, the same would likely to have to be done in the opposite direction as well.

Second, the overall structural system itself must be designed so that it is sufficiently stiff as well as strong enough to safely carry all lateral and gravity loads down the building from top to bottom. Among other things, this means that both lateral deformations (deflections) must be kept within acceptable limits, and that the overall horizontal shear forces and bending response of the structure that are produced as a result of the loading conditions will not be such as to cause failure of the structural material anywhere throughout the structural system. (Fig. 2.14b, c.)

It is worth noting here that lateral-load-resisting structural systems do not necessarily have to take up a whole building's length or width; indeed it is typical that they do not. In fact, the lateral-load-resisting

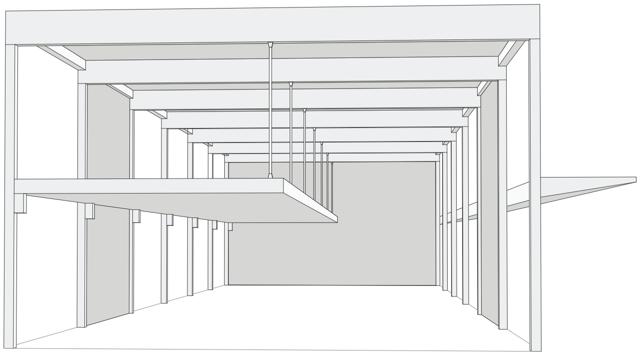


Figure 2.15
Perspective of the building as a volume. The structural system is shown with walls as bracing elements to secure its lateral stability. There are three walls altogether, and the stability of the building is thus covered in both directions. The roof is also shown as a stiff diaphragm to enable the wind loads to be transferred to the bracing walls.

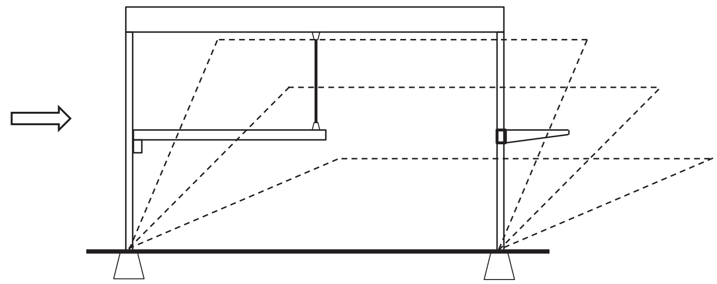


Figure 2.16
Lateral instability: with no walls or other measures to act as bracing in the transverse direction, wind loads will cause the structural framework to lean over and collapse sideways.

subsystem is typically incorporated into an overall three-dimensional building structure that is often much more extensive. These adjacent structural elements are then stabilized by means of direct connection to the laterally stiff walls or other types of subsystems. (Fig. 2.14d.) Such an overall structural system assumes that all lateral forces are able to be transferred through the non-laterally stiff parts to the structure of the stiffer subsystems. In other words, the lateral loads must be channeled through floor beams, slabs, or roof diaphragms (structures that have large in-plane stiffness), and these therefore need to be designed for the resulting compression and/or tension forces *as well as* for their more easily anticipated transverse, out-of-plane bending caused by vertical gravity loads that are acting on these same floor beams, slabs, and roof diaphragms at the same time.

Envisioning and Organizing a Lateral Stability Subsystem

Let us now take another look at the building perspective that we discussed in the previous section. (See Fig. 2.10.) If we "extrude" the planar building section that we initially considered into the third dimension, as we have done in this perspective view, we will have defined an occupiable architectural space. In fact, this is actually how many buildings are constructed, namely by letting the planar structure of a building section be repeated along a linear axis and thus, by help of external cladding fastened to the spaced-apart structural elements, enclosing a volume of space and creating the three-dimensional external form of a building. We can decide on an appropriate distance between the columns that recede into the space and let the same structural framework replicate itself as long as we need to, thereby establishing the building length and its overall size.

At this stage, however, we may become aware of a new problem: the series of planar frameworks of beams and columns may, if we do not take measures to prevent them from doing so, collapse when wind loads act along the *length* of the building. In fact, like a row of huge, open domino tiles that stand on end one after another, these frameworks are prone to topple over on to each other, causing a so-called domino effect. We therefore need to provide some structural stabilizing elements in the third dimension between these frames that will prevent this from happening. Placing two beams that run in the longitudinal direction and that join the tops of all the columns at their connection to the ends of the roof beams is a good start. (Fig. 2.15.) These will tie all of the frameworks together, but that is not enough: at some point along the length of the building we need to make sure that we insert a structural stabilizing subsystem that does not easily give way to the horizontal (lateral) loads that act on the building along its length.

A wall of a certain limited length may be used as what we call a *stabilizing subsystem* that is able to provide lateral resistance, since a wall is very stiff and unyielding along its length. Indeed, it would intuitively seem to be quite difficult to bend such a wall when horizontal forces act in the plane of the wall. So, by putting such a structural wall segment on each side of the building along its length between adjacent columns, and having those columns fastened to these walls, we will have provided stabilizing elements that under normal circumstances would not allow the structure to collapse in this direction. When we use a wall in this way as a stabilizing subsystem we usually call it a *shear wall,* since its dominating structural action for low buildings is shear force resistance. The two extended longitudinal beams that we just described connecting the ends of the transverse roof beams and the tops of the columns will actually enable *all* the frameworks to now be stabilized and prevented from overturning; i.e., they can

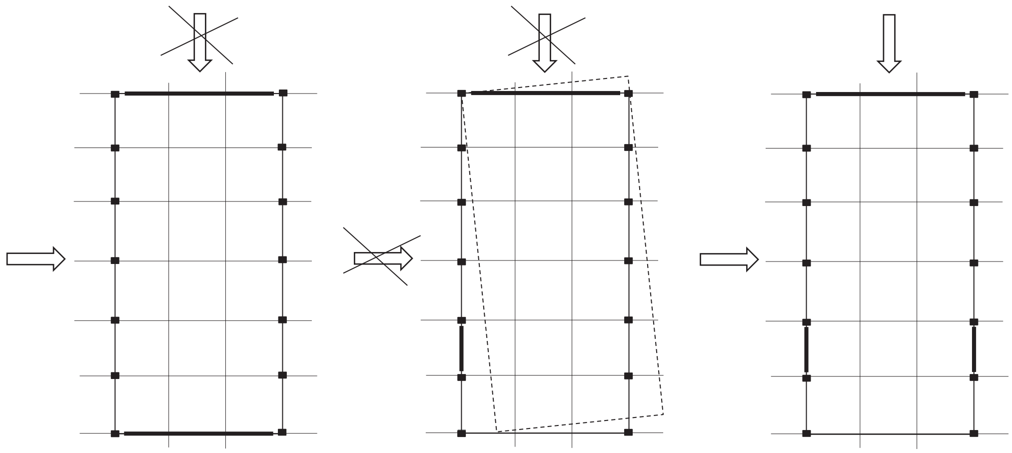


Figure 2.17
Effects of different distributions of lateral-load-resisting wall elements in a building plan: (a) two walls oriented in one direction leads to instability in the other direction; (b) one wall in each direction leads to rotational instability about the point where the walls meet (whether they do so physically or simply by projecting their alignments); (c) three walls oriented in such a way that they do not all meet in the same point provide overall stability. Such a stabilizing strategy depends on having floor and roof structures that can act as stiff diaphragms to connect together the three different walls and enable the wind loads to be distributed amongst them.

all thereby "lean" on to the two stabilizing walls oriented in this direction, through the longitudinal beams transferring of forces from one beam-and-column framework to the next. If the building is not too long, one such stabilizing wall element along both rows of columns should be adequate.

It should be clear at this point, then, that it is *not* sufficient that every building has structural frameworks of sufficient strength and stiffness to withstand only vertical (gravity) loading; i.e., lateral loads will inevitably act on a building and they must be resisted in some way by the structural system. Moreover, we obviously do not live in a two-dimensional world and we must therefore expand our discussion to consider what it takes to have *overall stability* in a building's structural system. To stabilize a building along one axis as we just did is not enough. Neither is it sufficient to have laterally stable subsystems arbitrarily scattered within the building plan. Instead, the overall structural system must comprise a stable structural arrangement that prevents the building as a whole from collapsing when subject to both vertical and lateral loads in *any direction*.

Referring again to the "extruded" building section in the example we have been considering, we can illustrate these observations by putting them into context. It seems reasonable to think that this building may also tend to collapse sideways as a result of wind loads acting on the cladding that we have provided to enclose the sides of the building. Unless some new structural members or elements are introduced to prevent it, we can imagine that wind load coming from the side will cause the relatively thin columns to deform excessively, or that the connections between the columns and the roof beam, as well as those between the columns and the foundations, do not have the strength or stiffness to keep these elements together, but instead will allow the elements to rotate relative to one another. The result of this would be that the whole building leans over and falls down sideways. (Fig. 2.16.) Introducing stabilizing subsystems in the form of walls (or stabilizing subsystems of another type) in this transverse direction is therefore needed to prevent this from happening.

However, we would not be likely to put such walls into the middle of the building plan in the lateral direction as these would tend to interrupt the flow of open space, might interfere with internal circulation, or could limit options for rearranging the organization of internal spaces. Instead, we may choose to close off the ends of the building with transverse walls located there. These should be sufficient to resist the lateral forces acting on the whole building, especially since they will be resisting loads in their plane; i.e., in their longest and stiffest direction. But while these walls will clearly stiffen the end frameworks against sideways collapse, we need also to account for the stability of all the other frameworks along the length of the building. We can take care of this by constructing the roof as a stiff diaphragm (which acts somewhat like a table top), which would then be fully able to transfer wind loads acting along the side walls of the building to the laterally stiff end walls; i.e., all the internal frameworks would then be able to lean on to the roof, which in turn is connected to the stiff end walls. This transverse system strategy, together with the two short shear walls oriented along the length of the building that we discussed earlier, should plausibly be able to provide a complete stabilizing system that will prevent the building from falling over in any direction as a result of lateral loads.

Beyond this specific example, it can more generally be stated with reference to Figure 2.17 that *a minimum of three laterally stable subsystems* must be introduced into a building plan, and that these must be arranged in such a way they can prevent the building from collapsing when subject to vertical and lateral loads acting in any direction.

Basic Types of Lateral-load-Stabilizing Subsystems

The issue of lateral stability is necessarily of central concern for the design of a building structure, just as is the carrying of gravity loads. This is hardly surprising, as we are well accustomed in everyday life of thinking about buildings having to withstand wind and earthquake lateral forces; by implication, therefore, we understand that buildings must inherently contain structural systems that are able to resist such forces.

In our introductory discussion about overall building systems in the preceding sections of this chapter, walls have been described as likely stabilizing elements, since these are easily recognized as being quite stiff in the direction of their planar surface. A *shear wall* is the name given to a wall acting in this manner, which in its most basic form is a flat surface element to which lateral loads are applied over its height *in the direction of its plane*. (Fig. 2.18a.) Shear walls can be made of any number of different materials, including masonry, reinforced concrete, sheathed timber stud walls, steel, etc. In addition to providing lateral stability, a shear wall usually also carries vertical gravity loads in compression. A clear example of a building that utilizes a shear wall structure for all of its gravity and lateral load carrying needs is the Museum for Architectural Drawing in Berlin, in which reinforced concrete walls surround the perimeter of the building, providing a quiet, secure, light-controlled space on the inside and an outer surface that is subtly but very appropriately covered with enlarged inscriptions of certain architectural drawings found in the museum archives. (Ill. 2.15, 2.16) Also, shear walls were instrumental for securing the stability of the Kunsthaus Bregenz, a project that we examined at the start of this chapter. (See Ill. 2.5, 2.6)

Aside from the shear wall, there are two other main groups of lateral-load-resisting subsystems that can be used to stabilize buildings. One is the *braced frame*, which is essentially a column-and-beam assembly of elements provided with diagonals in order to prevent the assembly from wracking sideways when lateral loads are applied to it. (See Fig. 2.18b–e.) Using the braced frame as a lateral-load-stabilizing subsystem is an effective way of not having the major visual obstruction of the plane of the shear wall while maintaining virtually the same degree of lateral stiffness. Aside from the classic-look single- and X-diagonal braced frames, several variations of form can be identified that also belong to this subsystem category, including lattices that are created by diagonal

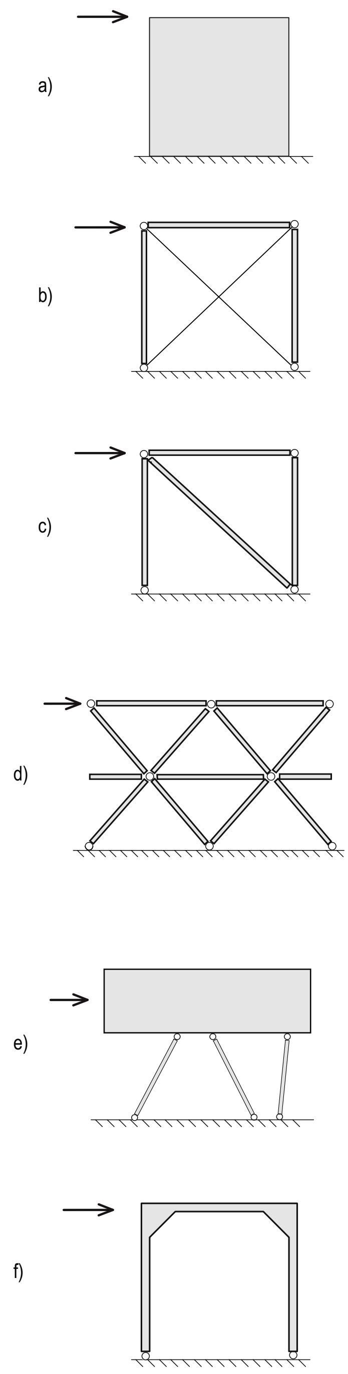


Figure 2.18
Stabilizing elements (from top); (a) shear wall, (b) braced frame with cross-bracing, (c) braced frame with single diagonal, (d) diagrid, (e) inclined columns, and (f) rigid frame.

Illustration 2.15
Museum for Architectural Drawing, Berlin, Germany (2013).
Concrete shear walls act as stabilizing subsystems all along this building's perimeter, while also creating a quiet, secure, light-controlled space for the museum archives. At the top level, however, a steel skeletal structure opens things up for a meeting and reception room with spectacular views to the outside.

Architect: Tchoban Voss Architekten. Structural engineer: PPW Dipl.-Ing. D. Paulisch.

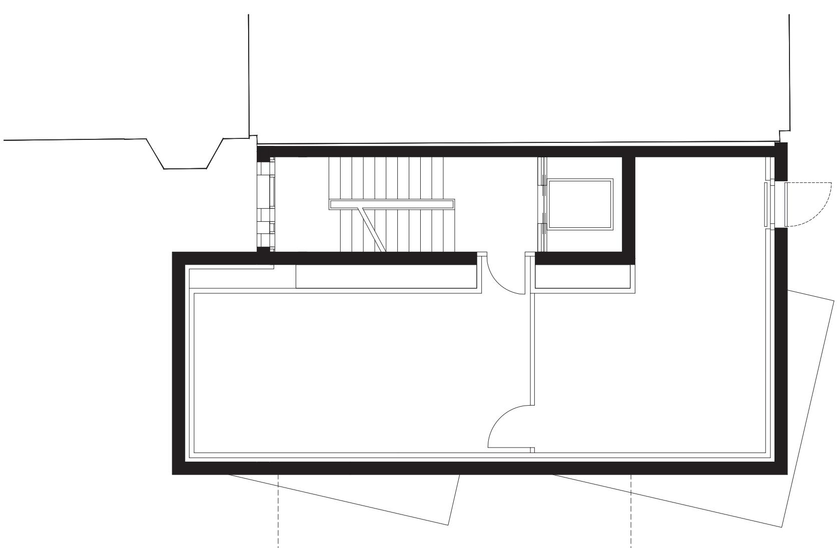


Illustration 2.16
Museum for Architectural Drawing.
Plan of third floor level; the configuration of the perimeter concrete walls changes at the floor levels below.

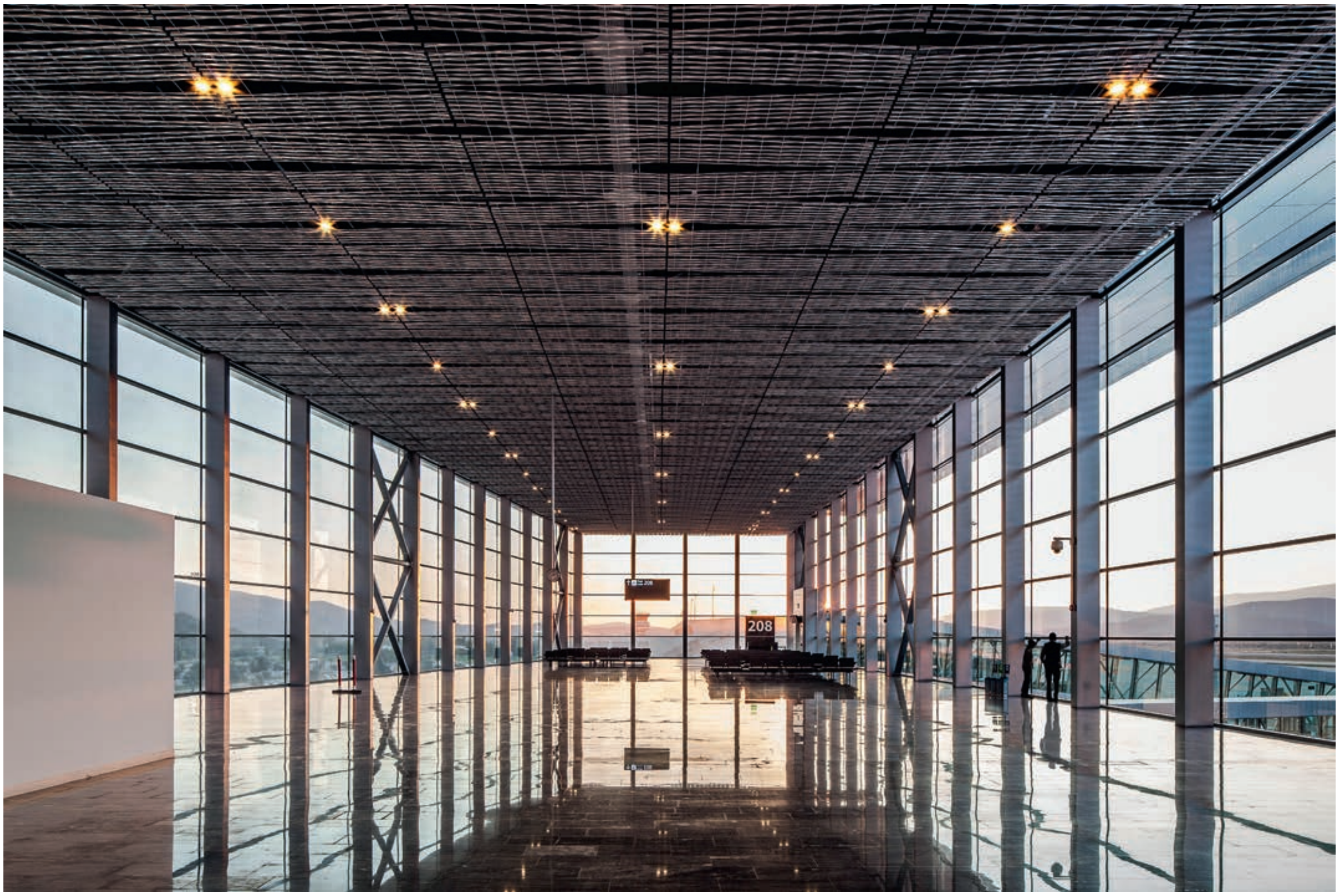


Illustration 2.17
Milas–Bodrum International Airport, Bodrum, Turkey (2012).
High ceilings of terminal building allow for expansive views of surrounding landscape. X-braced frames provide for lateral stability of this space while minimizing any obstruction of views.
Architect: Tabanlıoğlu Architects. Structural engineer: Arup.

and horizontal members in a triangular grid (these are also called diagrids), as well as inclined columns, which can act as both vertical-load-carrying and stability-bracing elements simultaneously. (e.g. Ill. 2.17.)

The third basic lateral-load-resisting subsystem is the so-called *rigid frame*. (See Fig. 2.18f.) The rigid frame can be characterized as simply an assembly of columns and beams that are connected together by means of *rigid* joints (and thus, its name). The rigid frame represents a structural assembly that offers not only support of gravity loads but also good lateral stability, all the while providing an almost complete visual and circulation openness within its interior space; i.e., the system is relatively free of either the obstructing solid plane of the shear wall or the diagonal member(s) of the braced frame. A "classic" example of such a lateral-load-resisting system can be observed in the open, one-story building for the Modern Art Glass Warehouse in Thamesmeade, UK. (Ill. 2.18.) Here, transverse stability is provided by a series of steel rigid frames that have stiffened, rigid connections between the side columns and the ends of the roof beams. This series of frames allows the interior space of the building to be completely free of obstruction.

We will return to these stabilizing subsystems much later on, in Chapter 10 The Frame and the Shear Wall, and there we will go into much more detail about how they work and can be alternatively configured. Before that, however, a more precise account of the *loads* that act on buildings is needed as it would seem meaningless to discuss how individual structural elements and overall systems react to loads without specifying more precisely just what those loads are; i.e., what causes them, how they can be determined, and what design considerations might affect their overall impact. This is the topic for Chapter 3.

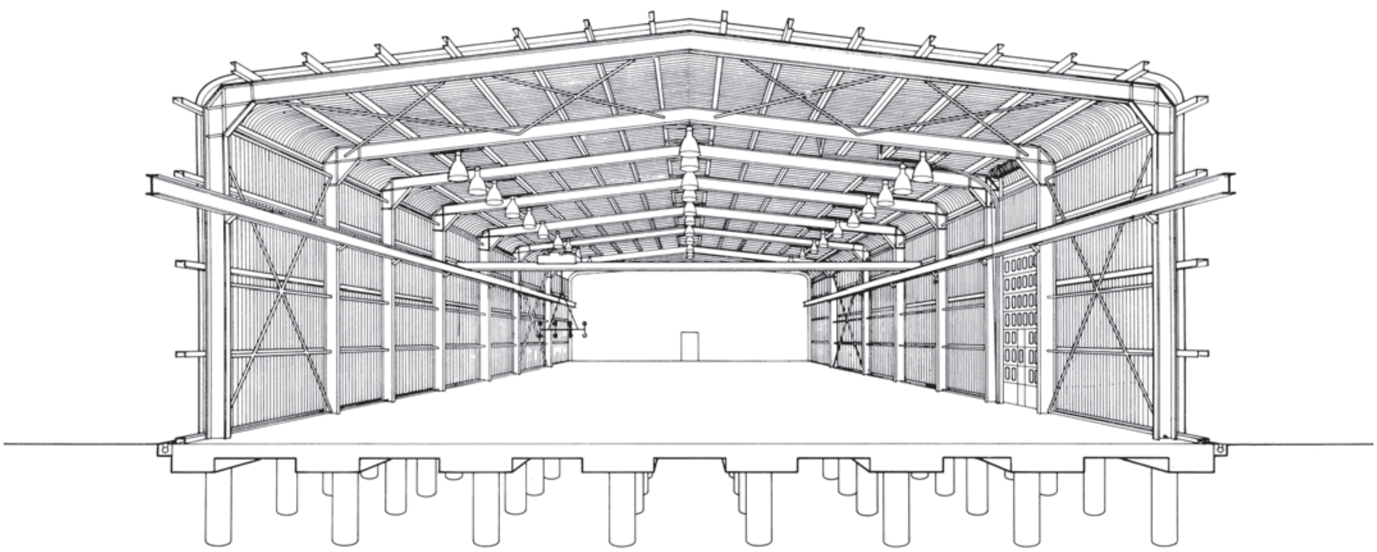


Illustration 2.18
Modern Art Glass Warehouse, Thamesmead, UK (1973).
Transverse stability is provided by a series of steel rigid frames, whose rigid connections between the side columns and the ends of the roof beams are evident. This allows the interior space to be free of obstruction. Resistance to lateral loads on the building in the longitudinal direction, however, is provided by cross-bracing in a few bays along both sides of the building.

Architect: Foster + Partners. Structural engineer: Anthony Hunt Associates.

CHAPTER 3

Loads

Illustration 3.1
Wells Cathedral, Wells, Somerset, England, UK (1239; tower repairs, 1338–1348). Built in response to uneven tower pier settlements, uniquely shaped arches provide structural stability and spatial character. (See also, Ill. 5.14.)

Master mason for tower repairs: William Joy.

Illustration 3.2
CaixaForum, Madrid, Spain (2008).
Three very different wall surface finishes around the entrance courtyard result in large variations of dead loads that need to be considered.
Architect: Herzog & de Meuron. Structural engineer: WGG Schnetzer Puskas Ingenieure. Planted wall designer: Patrick Blanc.

3.1 CaixaForum – Loads to Consider

In central Madrid, an 1899 electrical power station has been transformed into a multipurpose art gallery, music concert hall, film screening center, and conference venue. The Swiss architects Herzog & de Meuron's innovative strategy for the reuse of this industrial building is at once to keep and preserve it, but also to unexpectedly and wholly lift its brick façade up off the ground so as to make it appear that the building is floating in the air. (Ill. 3.2.) The masonry wall thereby becomes truly a "curtain" wall hanging off a completely new structural framing system inside the building – in fact, the old brick walls are lined by new reinforced concrete walls that effectively act as deep beams bracketing off interior vertical concrete cores. The ground floor is left open, so that access to the entrance and spectacular ceremonial staircase is achieved by rather disconcertingly having to walk underneath the bottom edge of the newly "floating" building.

The new cultural program for CaixaForum required a five-fold increase in floor area from that which had previously existed in the industrial building. This radical expansion was achieved both by building beneath the building – notably, a large auditorium is built under the main courtyard (Ill. 3.3) – but also by adding significant building volume above the "roofline" of the historical building. For this upper portion, however, a completely different enclosing material from that of the historical masonry wall is used, namely: rusting cast iron plates. The overall profile and shape of this upper part of the building references the dimensions and roof-scape of the surrounding neighborhood, and the color of the oxidizing metal establishes a dialogue with that of the brick below. Finally, the plates are perforated by many small openings whose overall patterns mimic at a greatly magnified scale that produced by rusting action itself. These openings at once give the plates, when seen from the exterior, a textural scale that relates to the bricks below but also, when experienced from the inside, a certain lightness and transparency that allows some direct light to reach "secret" roof-top terraces adjacent to the upper level café and administration offices.

A third and distinctive cladding system is used on an adjacent blank party wall that frames the museum's main entrance courtyard: a planted wall made up of 15 000 individual plants and 250 different species that was designed by the French botanist and artist Patrick

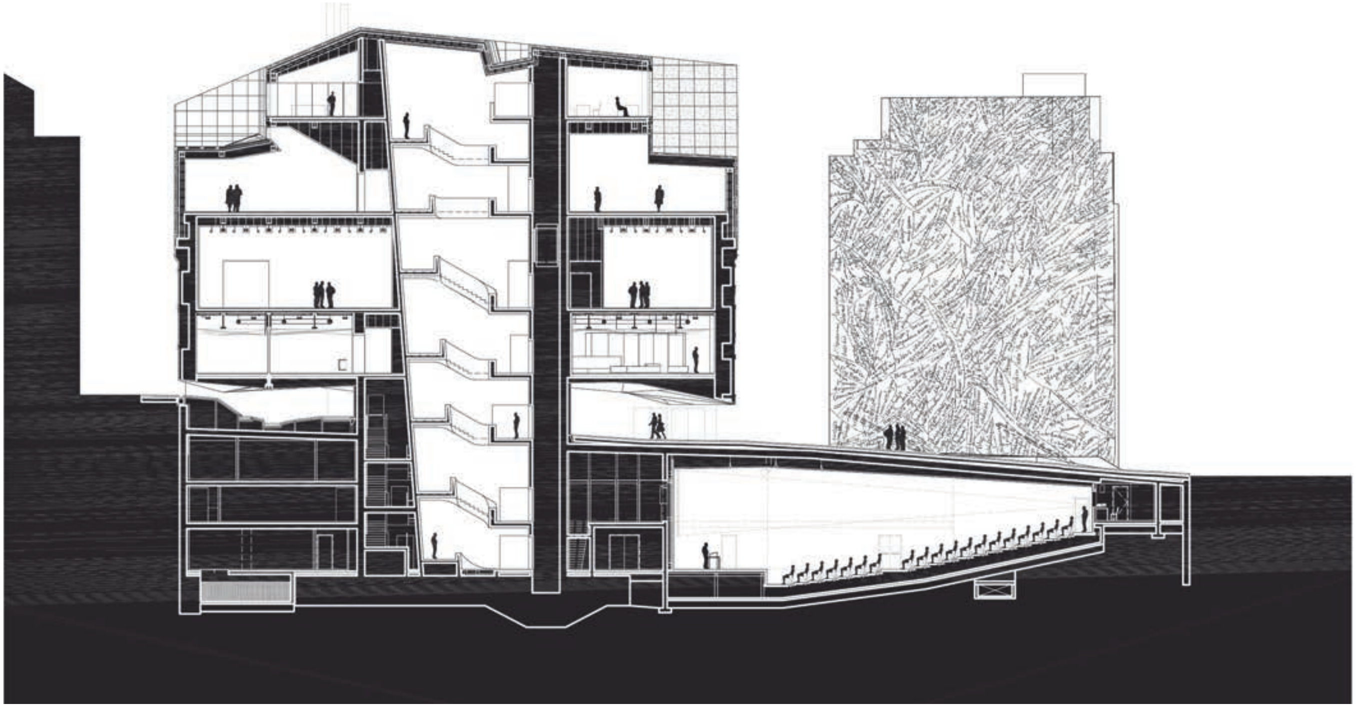
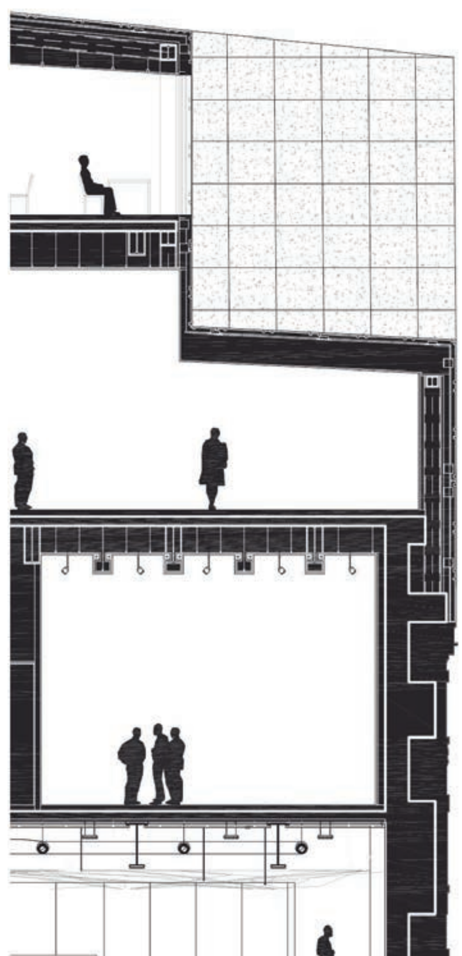


Illustration 3.3
CaixaForum
Building section, including below-ground auditorium under courtyard. (right) Enlarged detail, showing exterior wall changing from brick-lined concrete to perforated cast iron plates above.



Blanc. The structure for this green wall consists of a light metal trellised framework placed in front of and stabilized by the wall of the adjacent building. A thin vertical layer of felt is used for anchoring the plant roots, and nutrient-rich solution is pulled downward by gravity in order to water the plants by means of capillary action. Aside from providing some cooling and relaxing natural vegetation to a very tight urban space, this wall can also be seen to be a direct reference to the Royal Botanical Garden located only a block away.

These three different wall surfaces at CaixaForum (and their associated weights) begin to illustrate the great variability of loads that can act on buildings according to the design decisions that are made, but this example can also serve to suggest the much broader range of loads that typically must be considered in the design of structures and that will be the focus of this chapter. Aside from the exterior surfaces, the supporting structural system of concrete walls, columns, and beams have significant mass and weight themselves that must be carried. Different parts of this building – from the auditorium and galleries to the stairways and restaurants – will be occupied by people in infinitely varying distributions and densities over the course of a single day, while the art exhibits on display will also change, albeit over somewhat longer periods of time. Below ground level, the structure will have to be able to deal with lateral pressures from the earth's natural tendency to fill the void that has been created to serve the expanded program, while above it must be able to respond to the constant variations of wind forces and the potentially devastating effect of any possible earthquake action. CaixaForum, like any other building, is constantly facing an onslaught of loads – and it must be designed to be up to whatever challenge presents itself.

Figure 3.1
Some causes and conditions of loading on buildings.

3.2 Loads on Buildings – Dead or (a)Live?

We use the term *load* to describe any influence that causes forces and deformations in a structure. This reflects common usage of the word and implies a general understanding by the lay person of what it means in the context of building structures. A load can result in compression forces applied to a column that is holding up a roof or in tensile forces pulling on a steel cable that is suspending a walkway. A load can be the weight of a grand piano on a floor beam in a home or the crowds assembled on stadium seating for a World Cup soccer match. The effects of a temperature change on a beam that is fastened at both ends and has no room for expansion can also be considered as being caused by a load acting on the structural member. These examples are obviously but a few of the rather intimidatingly extensive range of different load conditions that can have an effect on structures. (Fig. 3.1.) Fortunately, many of the load types indicated in this figure are rather rare and are only occasionally found to be acting on an individual structural element or on a whole building's structural system at any one time, if ever at all. We will focus in this chapter, therefore, on only that handful of loading types that are most commonly found in the architecture world.

From a conceptual and organizational point of view, loads on buildings are considered to be divided into two separate categories: *dead loads* and *live loads*. Those that are labeled as *dead* are ones that are considered to be *constant over time and not capable of moving or being moved*. The most obvious examples of this type are the self-weights of the columns, beams, floor slabs, walls, and other elements of a building's structural system as well as of a building's material finishes – such as that of the floors and ceilings and exterior wall enclosure systems that were discussed with regard to CaixaForum in the previous section, or that are also evident for the Poli House. (Ill. 3.4, 3.5.) All such loads are caused by the gravitational pull of the earth and have magnitudes, therefore, that depend on specific material densities (Chapter 5) and direction that is vertically downward.

Given the preceding definition for dead loads, we can conclude that loads that are instead going to be considered to be live must be those which are known to *vary with time and that are easily capable of moving or being moved about on a structure*. The most commonly encountered examples of this type of loading are *occupancy loads* and the *environmental loads* produced by snow, earth, water, wind, and seismic activity. Occupancy loads are particularly self-evident as to why they are included in this category: i.e., they include the weight of "live" people that occupy and move about a building space. (e.g., Ill. 3.6.) Perhaps less obviously but also to be included as occupancy loads are the weights of inanimate objects such as furniture, warehouse inventory, museum artwork, book stacks, etc.; i.e., items that over the typical life of a building have the possibility of being moved about a building space, however frequently or infrequently that may occur. Natural phenomena such as snow, wind, and earthquakes also all vary significantly with time and so are considered to be part of the general live load category. We will take a more detailed and specific look at each of these load types in the pages that follow.

Before doing so, however, it is worthwhile making a few general comments about the importance of load calculation within the overall process of designing building structures. In fact, it is difficult to

Illustration 3.4
Poli House, Coliumo Peninsula, Chile (2005).
Dead loads include the weight of the structural system, which here consists of seemingly massive concrete walls. These walls also create the external form and define the internal spaces of this house, and they impart both a sense of solidity and "rootedness" of the building to its rocky outcrop overlooking the Pacific Ocean.

Architect: Pezo von Ellrichshausen. Structural engineer: Cecilia Poblete.

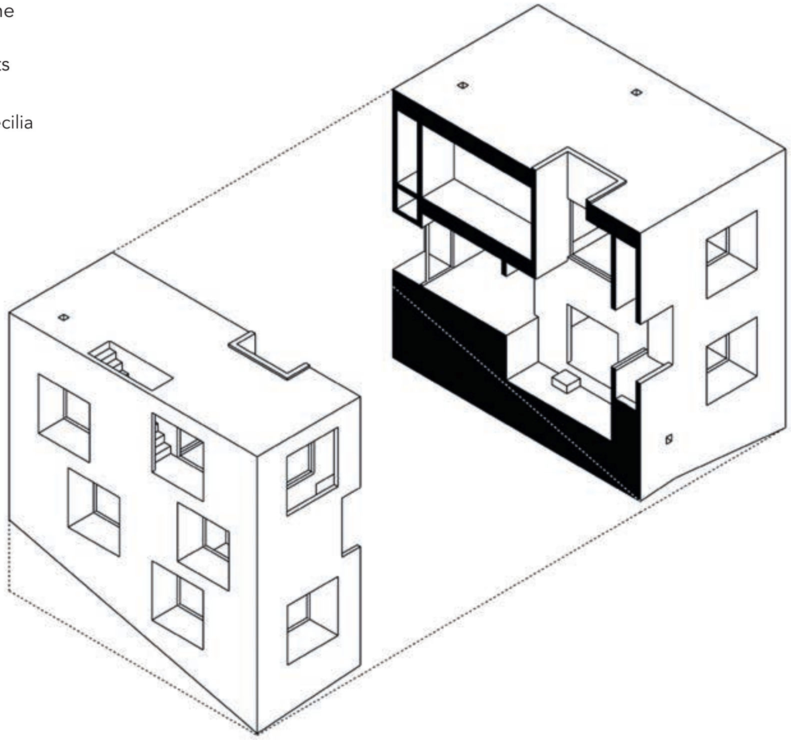


Illustration 3.5
Poli House.
Cut-away axonometric drawing reveals double-layer form of the exterior walls, which incorporate the house's stairs and other service functions.

Illustration 3.6
National Opera and Ballet, Oslo, Norway (2008).
Live loads include the weight of human occupants, wherever these may occur.
Architect: Snøhetta. Structural engineer: Reinertsen Engineering AS.

overstate the critical nature of this seemingly obvious step, for actual building failures can be just as easily attributable to an incorrect anticipation of loading as to an erroneous selection of adequate member sizes after loads have been established. Moreover, the word "failures" here should be understood broadly to include anything that prevents the timely or safe occupancy of a structure, whether this is characterized by an actual collapse caused by loadings that exceed the capacity of structural members and their connections, or by significant instability and user comfort issues, or by some other major problem.[1]

3.3 Dead Loads – Weights of Immovable Things

The most obvious and inescapable of dead loads is the self-weight of the structural elements that make up a building's framing system. (e.g., Ill. 3.7, 3.8.) When a structural system's dimensions and constituent material are known from the start (such as in the case, for example, of a building renovation project) these loads can be determined quite precisely by calculating each element's geometric volume and multiplying by the material's mass density and the gravitational acceleration constant, g, as will be discussed further in Chapter 4. This process can be accomplished simply by old-fashioned hand methods for relatively small-scale projects, but for large structures it quickly becomes a tedious algebraic exercise; fortunately, today these calculations can also be taken care of

Illustration 3.7
Eames House, Pacific Palisades, CA, USA (1949).
Light structural elements such as open web steel trusses and narrow tubular columns help to minimize dead loads caused by self-weight.
Architect: Charles and Ray Eames. Structural engineer: MacIntosh and MacIntosh Company; also Edgardo Contini (for first version of house's design in 1945 – and for which structural components were ordered).

Illustration 3.8
SESC Pompéia, São Paulo, Brazil (1986).
Concrete walls and walkway beams produce substantial dead loads from their own self-weight.
Architect: Lina Bo Bardi. Structural engineer: Figueiredo Ferraz.

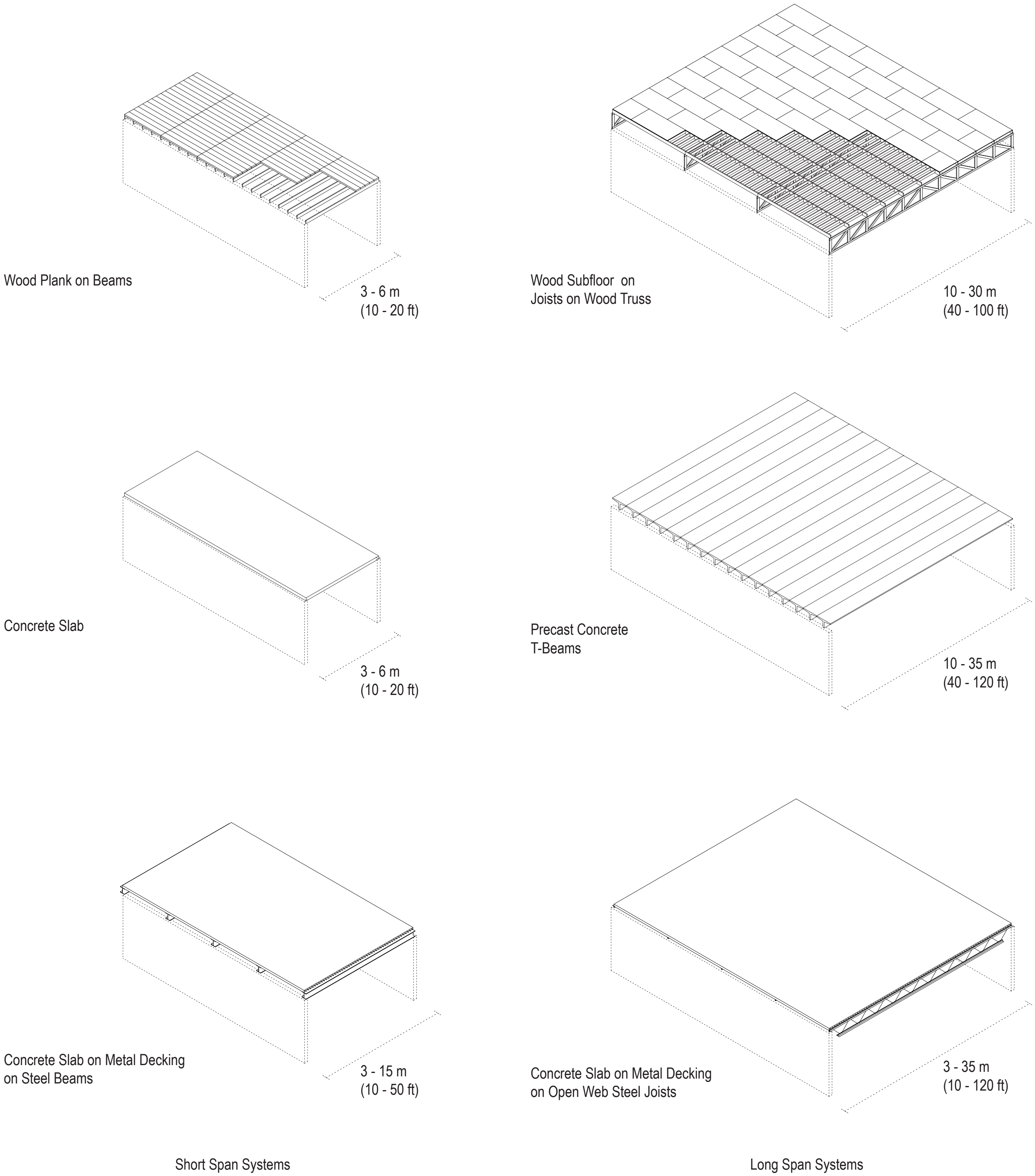


Figure 3.2
Spanning distance ranges for some typical floor and roof structural systems.

EXAMPLES OF DEAD LOADS
FOR FLOOR/ROOF SYSTEMS

Type	kN/m^2	lb/ft^2
Wood subfloor and beams	0.5	10
Space frame	0.5	10
Concrete slab on metal deck on open-web joists	1.5	30
Lightweight concrete slab on steel beams	3.0	60
Concrete slab (200 mm / 8 in thick)	5.0	100
Concrete waffle slab	5.0	100

Figure 3.3
Examples of dead load estimates for various floor and roof structural systems.

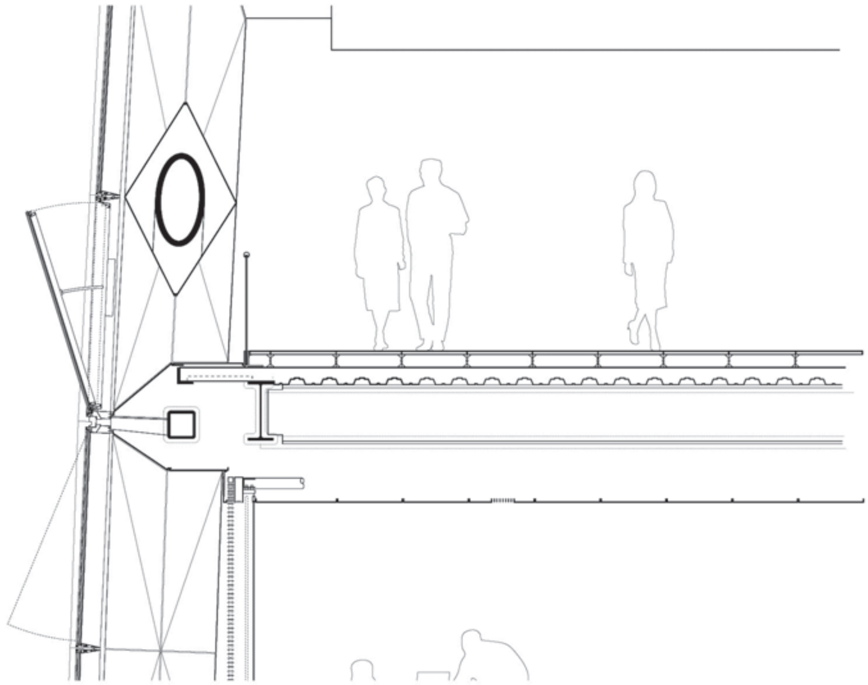


Illustration 3.9
30 St Mary Axe, London, England UK (2003).
Section drawing through floor system demonstrates layered aspect of dead loads produced by structure and various finishes.
Architect: Foster + Associates. Structural engineer: Arup.

automatically by the most basic of structural analysis computer programs.

In the early stages of a building's design process, however, when things are still in the formative and schematic phase, a structural system's configuration, including even spanning distances and specific material choices, may be uncertain. To get things going one must at that point rely on preliminary and very approximate estimates of typical spans and dead loads for various types of building systems. (Fig. 3.2, 3.3.) Starting with such general estimates and applying the lessons of the chapters that follow for designing individual structural members, one can through an *iterative* process relatively quickly reach a point where a more precise determination can be made of the necessary structural member dimensions and the dead loads that result from them.

Besides the structure's self-weight, other dead loads are sometimes just as significant and cannot be ignored. Such loads are attributable to things like floor and ceiling finishes, MEP (mechanical, electrical, and plumbing) service ducts, conduits and pipes, a building's exterior cladding, etc. – all of which are physically fastened to the structure of the building and, therefore, cannot move or be moved relative to it. The need to account for the dead loads of finishes remains true today despite the fact that the overall trend in architecture over time has been to reduce the weights of such "secondary" aspects of finished buildings: i.e., sweeping generalizations can be made that we have gone from thick masonry enclosure systems to thin and light glass curtain walls, and from heavy marble floor veneers and plaster ceilings covering structural framing to simply polishing raw concrete floor slabs and having beams and corrugated metal decking left exposed to view from below.

The weights of typical finishing and enclosure systems are usually defined either in terms of material densities (there will be more on this topic in Chapter 5) or weights per unit surface area according to standard dimensions that result from a particular manufacturing process. A detailed calculation of dead loads for particular floor or wall surface areas is often a matter of accounting for several "layers" of materials – that is, the structure itself plus multiple layers of various finishes. An example of such accumulations is approximated here for a typical floor at 30 St. Mary Axe where, as can be seen in the adjoining section, the floor's framing system consists of a reinforced concrete slab cast on to a corrugated metal deck that is supported by an underlying steel beam framing system. (Ill. 3.9.) Several layers of additional, non-structural finishes such as a raised floor, air handling ductwork, and a suspended ceiling also need to be accounted for above and below the floor system itself. A reasonably accurate estimate of the dead loads that need to be considered for such a floor, therefore, may be calculated from various material self-weights and manufacturers' product estimates as follows:

Lightweight concrete slab on steel beam floor system estimate:	3.0kN/m^2
Raised floor system estimate:	0.75kN/m^2
Air handling ductwork estimate:	0.5kN/m^2
Suspended ceiling estimate:	0.25kN/m^2
Total dead load estimate:	4.5kN/m^2 (90lbs/ft^2)

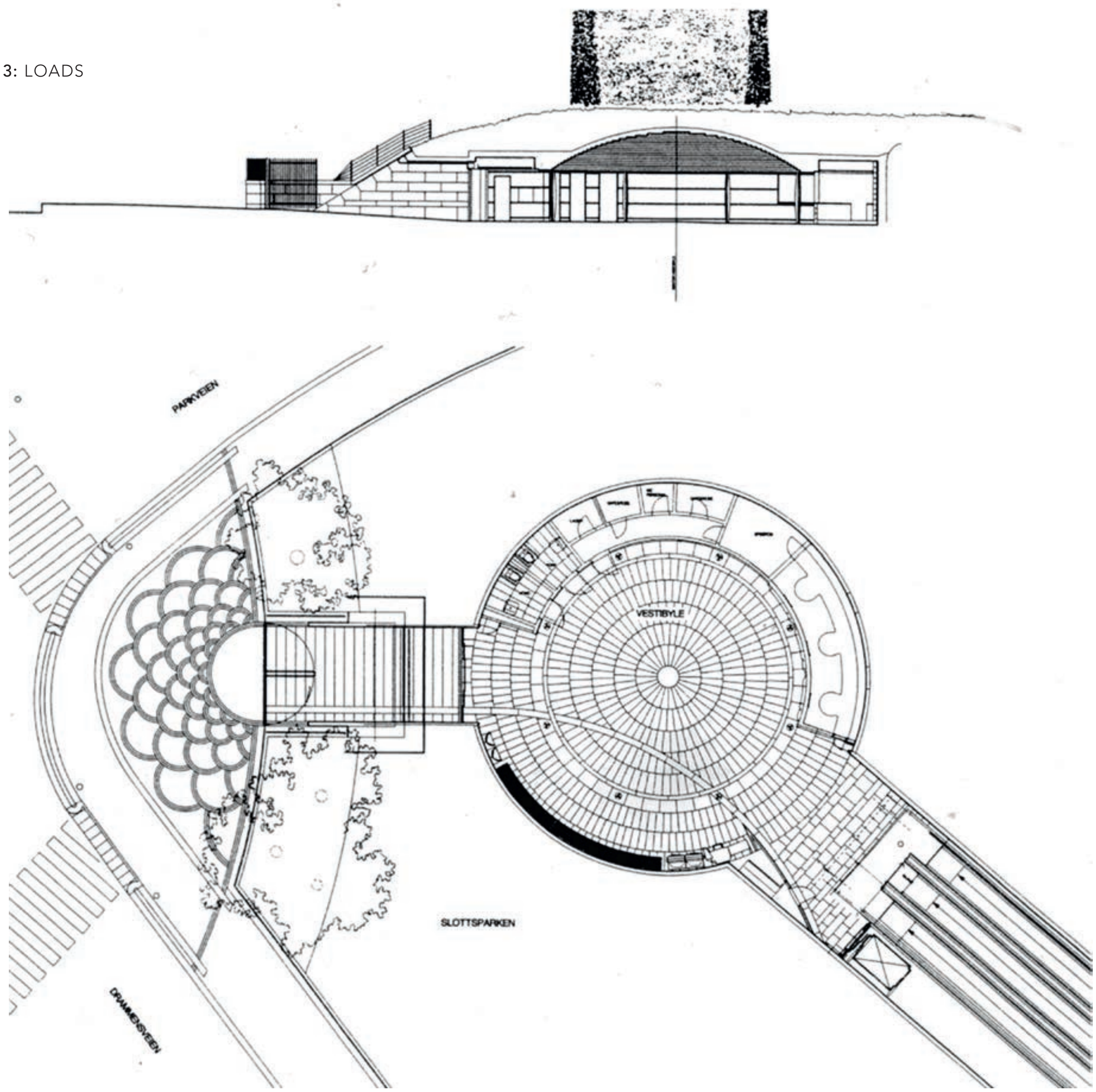


Illustration 3.10
National Theater Railway Station Entrance, Oslo, Norway (1998).
Section and plan drawings highlight underground aspect of station entrance as well as its circular and domed configuration.
Architect: Arne Eggen Arkitekter. Structural engineer: Rambøll Norway AS.

If the weight of such floor structural systems and their finishes is an obvious source of dead load, then so too is that of the exterior wall cladding enclosing virtually all buildings to keep us protected from the vicissitudes of the weather. For example, the section drawing just considered at 30 St. Mary Axe also shows a lightweight, double-glazed, and climate-controlling cladding system. The design of such systems, both in terms of aesthetics and of active and passive climate controlling features, is a long-recognized avenue for architectural expression and, as such, the dead loads of cladding systems can vary greatly. A case in point is that of the CaixaForum in Madrid described earlier in Section 3.1, in which three purposefully very different cladding types help to accent the reprograming of the previously existing building and define its new urban courtyard space.

3.4 National Theater Railway Station's Underground Entrance

The National Theater Railway Station entrance designed by architect Arne Eggen is situated underground, beneath the sloping landscape of the Royal Park in Oslo; the train platform has two means of access from the world above, one at each end. (Ill. 3.10.) From the western entrance vestibule one is able to catch a glimpse of the distant Oslofjord, which acts as an essential point of reference when emerging from the disorientation of the subterranean realm. The vestibule's form, circular in plan and with a vaulted, dome-like ceiling, is inspired by the natural forms of the Park's ridges and rolling landscape. Also, the circular form resolves the different alignments of the passage to the outside world and the tunnel of escalators leading down to the train platform.

Illustration 3.11
National Theater Railway Station Entrance.
Interior view, showing stepped rings of domed ceiling; also, columns that are thickened at mid-height.

In order to deal with both the downward and lateral soil pressures that result from being buried deep underground, the architecture of the station entrance is generated from curved geometric forms like that of the circle, the oval, the cylinder, and the sphere (we will explore these forms' relationship to loading in Chapters 12 and 13). But it is the vestibule's domed ceiling that will mostly be the focus of discussion here; in order to carry the weight of the earth above it, a spherical cap of some sort was considered desirable. (Ill. 3.11.) However, the preparation of formwork for such a doubly curved ceiling surface made out of concrete is not a straightforward matter. In this case, the problem was resolved by transforming the smooth curvature of a spherical surface into one with vertical steps created by a series of horizontal concrete rings of varying diameter. The vertical edges of these rings were formed as short lengths of single-curvature cylinders with decreasing radii as one moves up, whereas the bottom sides of the rings are all purely flat, horizontal surfaces. Because the vertical steps all have the same height, the width of the rings increases toward the zenith point, with the overall composition somehow looking like the rings of the planet Saturn. By constructing the vestibule in this manner, and because the formwork could be built with free access to the excavation site from above before the earth was eventually pushed back to cover the completed structure, the vestibule ended up being relatively the easiest part of the station to build. By comparison, the vaulted tunnels and passages of the station farther below had to be cut out of solid rock and have their curved concrete walls and ceilings cast against it, something not so easily accomplished.

The vestibule of the station also has a couple of other interesting design features that warrant mention. First, in order to give the space lightness and an impression that the ceiling is "floating," vertical support is provided around the perimeter by eight slender steel columns. The main design criterion for the design of these columns was that they have minimum weight of material yet maximum bearing capacity. Toward this end, their cross-section is three-pronged (similar to the star on the grille of a Mercedes) in a manner that we will see makes sense in Chapter 8. Entasis, or the thickening of the column shaft at mid-height, can also be seen here although this is mainly for aesthetic effect rather than structural benefit: a column with a straight profile, although effectively having the same bearing capacity, might have been considered to have looked too frail.

It should also be mentioned that a domed space is by nature considered to be a very active one acoustically. And in the vestibule for the National Theater Railway Station, with its granite floor and the flat-bottomed surfaces of the ceiling's stepped concrete rings, the space acts as a resonator with a sound focal point at its middle. "Flutter echo" is the term used to describe the phenomenon when the sound of footsteps and conversation is reinforced and keeps vibrating within a space, and this is a quality which is distinctly noticeable in the vestibule, giving even further life and interest to the underground circulation space. This observation reminds us of the long history of building designers exploring the relationships between acoustics and the shape and volume of space, and it may bring to mind the Byzantine church of St. Mark's in Venice, which was built over the plan of a Greek cross and thus has five domes – one in the middle and one over each cross arm; in the sixteenth century, the composer and organist Andrea Gabrieli composed music that exploited the special acoustics of this multi-domed space.

Illustration 3.12
Seattle Public Library, Seattle, WA, USA (2004).
Occupancy live load conditions vary within a building according to anticipated function, but also with time: (a) people circulate, and furniture and shelves can be moved, (b) chairs may or may not be used at any given moment in time.
Architect: Office for Metropolitan Architecture (OMA). Structural engineer: Arup and Magnusson Klemencic Associates.

3.5 Occupancy Live Loads – Animate Objects, but Inanimate Too

Amid all the hyperbole that often surrounds architectural design, it is sometimes seemingly forgotten that the primary purpose of buildings is, after all is said and done, to create sheltered space for people and their myriad activities. As has been suggested already, there is an essential variability to the human occupation of buildings that doesn't lend itself to as precise an accounting of loads as we have discussed with dead loads, even when one is at the point of final design for a structural element. (e.g., Ill. 3.12.) For example, an auditorium may be either unoccupied, sparsely populated, or exceed official seating capacity all within a 24-hour cycle, and may be repeatedly so. Or, within the living room of a home, furniture such as bookcases and couches and cabinets may be moved around every now and then as one tires of a particular arrangement. Moreover, every apartment in a building will be furnished differently depending on various individuals' aesthetic tastes and interests. (Ill. 3.13.) And over the longer term, a building may eventually be "re-programed" as it gets reused. Buildings that were once designed as an automotive manufacturing plant

Illustration 3.13
Highline 23, New York City, NY, USA (2009).
Individual preferences for furniture styles and apartment layouts need to be accounted for as part of occupancy live load allowances.
Architect: Neil M. Denari Architects. Structural engineer: DeSimone Consulting Engineers.

or slaughterhouse may eventually become a hotel or library, for example; the permutations and possibilities of such reuse of buildings are virtually endless.

Trying to account for such variability could potentially cause designers to throw up their hands and give up before even beginning, but that is demonstrably not the case as buildings surround us. Instead, practical experience gained over time and corroborating scientific experiments have helped establish the means to develop today's building codes that define load allowances according to different types of inhabitation. (Fig. 3.4.) As can reasonably be expected, the lowest such load allowance, 1kN/m^2 (20lbs/ft^2), may be for an attic space with no headroom and where entry can only be accomplished by means of a trap door (presumably severely limiting one's ability to store especially heavy objects), whereas the larger occupancy loads, 5kN/m^2 (100lbs/ft^2) or more, are associated with building lobby areas, library book stacks, and industrial building spaces. In order to help give a better "feel" for the magnitude of these load allowance numbers, they can be compared to an easily

EXAMPLES OF OCCUPANCY LIVE LOADS

Category	kN/m^2	lb/ft^2
Residential - typical	2	40
Attics (limited access)	1	20
Balconies	3	60
Garages	3	60
Office - typical	3	60
Entrance lobby level	5	100
Stairways/exitways	5	100
Public plazas	12	250
Athletic facilities - typical	5	100
Gymnasia	5	100
Playing surfaces	5	100
Stadium seating	5	100
Hospitals - private rooms	2	40
Operating rooms, labs	3	60
Libraries - reading areas	3	60
Book stacks	7	150
Manufacturing - typical	6	125
Heavy equipment	12	250
Restaurants - typical	5	100
Schools - classrooms	2	40
Hallways/exitways	5	100
Stores - typical retail	4	80
Ground floor	5	100
Warehouses - typical	7	150
Heavy storage conditions	12	250

Figure 3.4
Typical live load allowances for various types of occupancy.

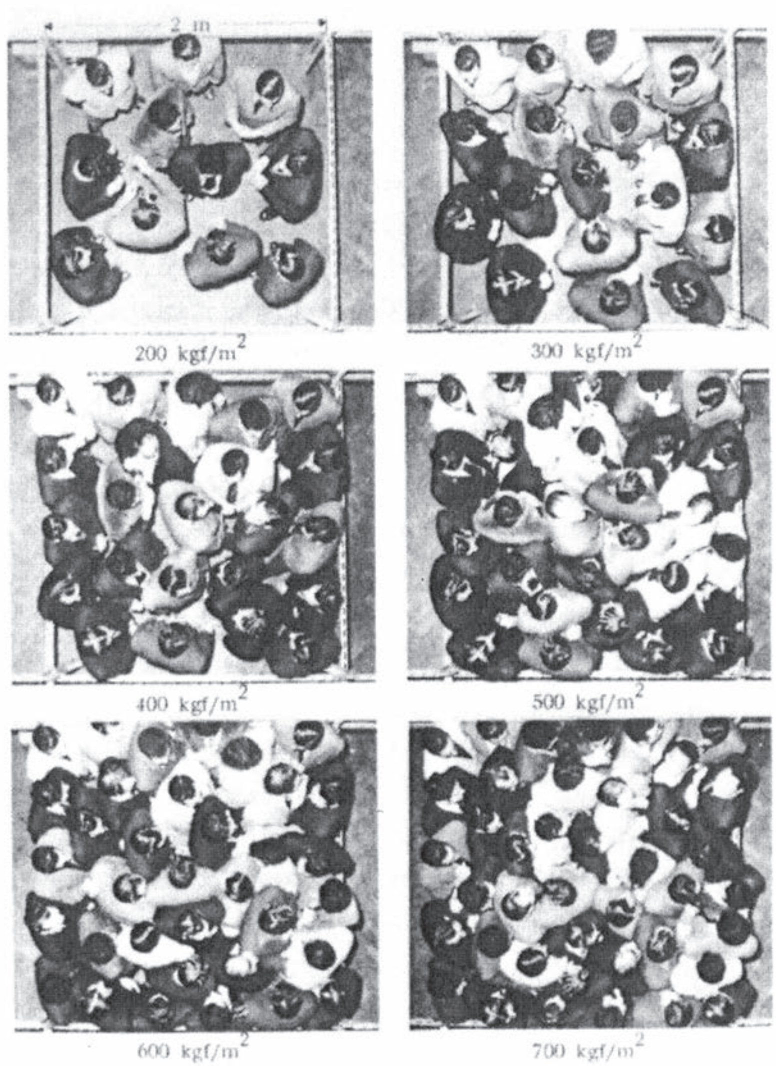


Illustration 3.14
Visualizing occupancy live loads by means of varying densities of people in an elevator.
Top left 200kgf/m² or about 2.0kN/m² (40lbs/ft²), bottom right 700kgf/m² or about 7.0kN/m² (140lbs/ft²).
Source: National Laboratory for Civil Engineering, Portugal, 1971.

imagined condition: if people weighing on average 70kg (154lbs) are standing shoulder to shoulder and front to back with each thereby occupying roughly 0.25m² (2.7ft²), this condition translates roughly into an average loading of 2.7kN/m² (57lbs/ft²); i.e., such a load condition essentially mimics the design loading for typical office floor occupancy. (e.g., Ill. 3.14.) Like the dead loads previously considered, occupancy loads are caused by the earth's downward gravitational pull and they act, therefore, vertically downward.

It is important to understand that building code allowances are almost assuredly different from that of a precise and specific loading situation at any given moment in time; rather, they are meant instead to allow for the infinite variations of how people will occupy a certain space over time as well as for the random and changing placement of such things as filing cabinets and desks according to typically flexible furnishing arrangements. Conceptually such live load allowances can be thought of as though all human occupants and their associated belongings are being converted into a uniformly thick layer of equivalent weight spread over the entire floor area (Fig. 3.5), and these represent an estimate of the maximum live load conditions that can be anticipated for any particular space occupancy; i.e., whether residential, office use, assembly hall, etc.

As useful as such a code-defined approach is in simplifying the definition of occupancy live loads to be considered, it must also

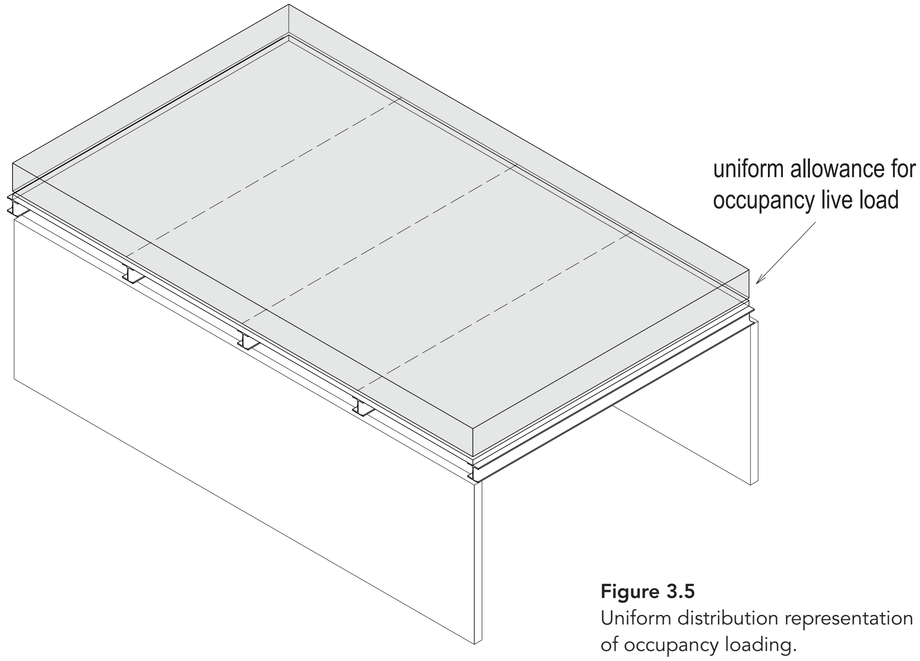


Figure 3.5
Uniform distribution representation of occupancy loading.

Illustration 3.15
The Broken Kilometer (1979).
Art installation that happens to closely mimic building codes' uniformly distributed live loads allowances.

Artist: Walter De Maria. Long-term installation at Dia Art Foundation, 393 West Broadway, New York City, NY, USA. Photographer: Jon Abbott. © Dia Art Foundation.

be recognized that things change with time. In the example of 30 St. Mary's Axe (see Ill. 3.9) the standard office space allowance of 3kN/m² (60lbs/ft²) would not be considered sufficient if the space was to be used as an atypical heavy storage area with a sea of very tall and tightly packed filing cabinets. Judgment and common sense need to be exercised, therefore, both in the original design in anticipating how a space is realistically and legally intended to be used, and then afterward in recognizing when a change of occupancy and/or loading condition might go beyond what the structure was originally designed for.

Illustration 3.16
The New Museum, New York City, NY, USA (2007).
Shifting volumes produce a distinctive building profile and suggest different program spaces within. Structure is not visible, but plays an integral role in creating these.

Architect: SANAA. Structural engineers: Guy Nordenson and Associates; associate: Simpson Gumpertz and Heger; consulting: Sasaki Structural Consultants.

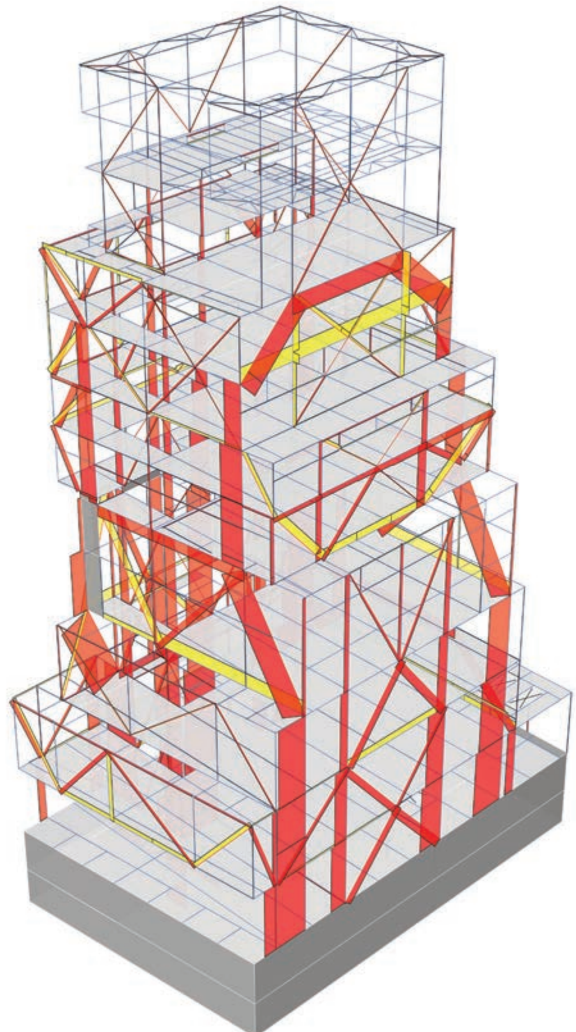


Illustration 3.17
The New Museum.
Structural system shifts in concert with building profile and interior spaces. Diagram highlights increasing forces in columns due to gravity load accumulations over the height of the building.

3.6 Loading Diagrams – Abstractions of Reality

To summarize the preceding discussion, both dead loads and occupancy live loads on buildings are largely determined in terms of load per unit surface area – whether the surface is a structural floor or a wall system. This is a reflection of the largely Cartesian planar geometric reality of our built environment, a phenomenon very strongly dictated by the direction of the force of gravity, but also by economy of means and efficiency of space usage. In very general and simplistic terms, it can be said that we occupy buildings on floors that are typically flat horizontal surfaces and whose exterior and interior walls vertically enclose and subdivide interior space, respectively. To say that buildings are the equivalent of houses of cards may be pushing the point, but as a simplistic approximation with which to begin it is not that far off the mark.

The reality of construction and of structural systems is typically more complicated (and stable, fortunately) than is such a precariously balanced and loose-fitting stack of cards. A common development in buildings is that floors are often supported by a system of beams and occupiable space is opened up by the use of columns instead of walls. Since both beams and columns are linear structural elements as opposed to planar ones, i.e., each such member can basically be defined by a certain cross-sectional shape that is extruded along an axis, a refinement of our simple model of structures is to consider that they consist of an open three-dimensional grid of beams and columns to which planar floor and wall surfaces are attached. (e.g., Ill. 3.16, 3.17.) Such a simplifying notion and vision of structure was instrumental in the development of Modern architecture.

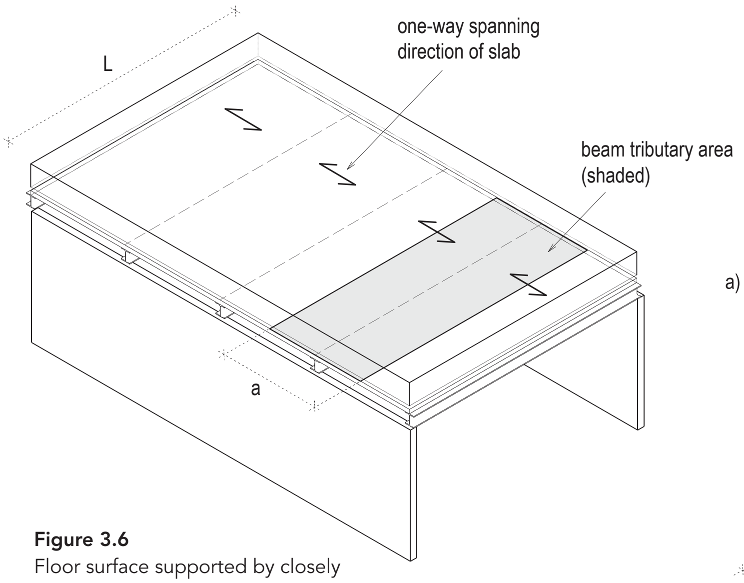


Figure 3.6
Floor surface supported by closely spaced parallel beams; one-way spanning direction between beams is shown; shaded area indicates tributary area for single beam.

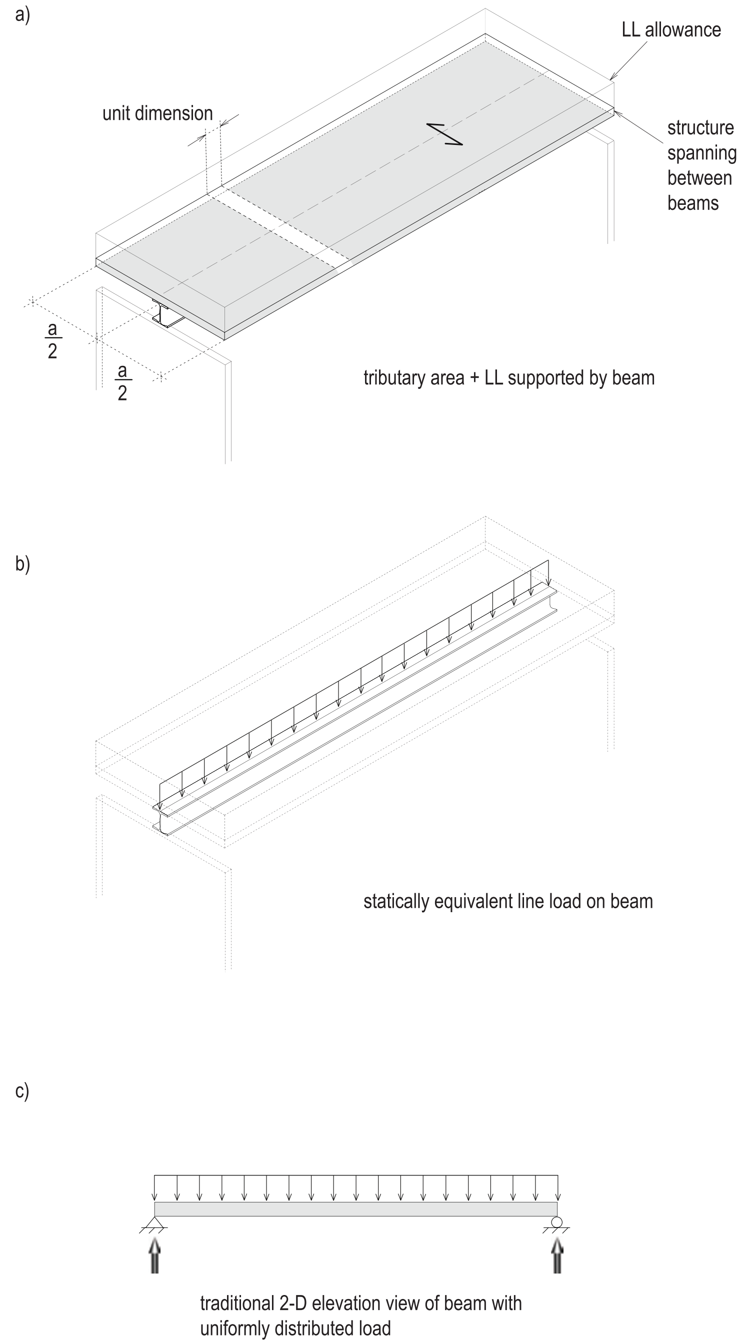


Figure 3.7
(a) Single beam and its tributary area, topped by live load allowance; non-shaded area indicates "tributary strip" carried by unit length of beam, (b) equivalent loading along beam produced by "squashing" together actual 3-D surface loads, and (c) corresponding 2-D representation of uniformly distributed load on beam.

Uniformly Distributed Loads

If we set aside, for the time-being, the overall stability issues of such gridded building frames in response to loading (we will return to discuss this topic in Chapter 10) we can limit our introductory discussion here to being about how planar surface loads are supported on linear structural elements. For example, an individual beam can be seen to be supporting a discrete part of a floor, and such an area is commonly referred to as its *tributary area*.[2] (Fig. 3.6.) Establishing exactly how much of a floor or wall surface is supported by an individual structural member can become somewhat complex depending on particular circumstances, but most of the time what would seem to be intuitively obvious is quite close to reality: e.g., between two relatively closely spaced and parallel supporting beams the floor surface is assumed to span transversely from one to the other (this is known as a one-way spanning system) and the floor load being carried can be considered to be equally divided between the two beams. This condition is often represented graphically by means of arrows indicating the floor's spanning direction and lines drawn midway between adjacent supporting beams indicating the division between supported areas. For visual clarity, the tributary area that is carried by an individual beam can be distinguished by some form of shading.

At this point we are still envisioning a three-dimensional condition of dead and live loads acting on a planar floor surface that is being carried on a supporting linear beam. This situation can be more conveniently represented in two dimensions, however, by notionally "squashing" the surface load perpendicular to the axis of the beam into a statically equivalent linear load along its length. (Fig. 3.7.) At that point, it becomes convenient to draw the beam in a two-dimensional elevation view and the loading condition is known, for self-evident reasons, as a *uniformly distributed load*

Illustration 3.18
"New York Construction Workers Lunching on a Crossbeam" (1932). Nonchalant gathering of workers during construction of RCA Building in Rockefeller Center conceptually represents an example of a uniformly distributed load acting along part of the length of a beam.
Photographer: Charles C. Ebbets. © Bettman/Getty Images.

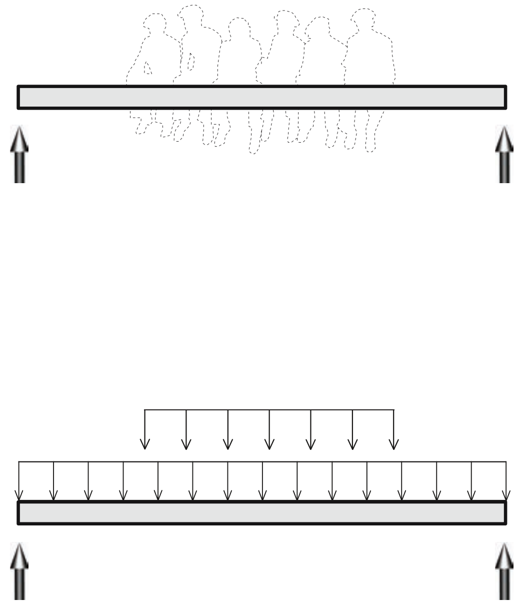


Figure 3.8
Conceptual loading diagram matching unusual condition of the "New York Construction Workers Lunching on a Crossbeam" image in Ill. 3.18: partial-length uniformly distributed load (UDL) corresponds to extent of workers, UDL over full length corresponds to beam's self-weight.

(UDL) acting along the length of the member. The magnitude of the uniformly distributed load on the beam can simply be calculated by multiplying the surface loads (kN/m^2, lbs/ft^2) by the width (m, ft) of the tributary area *perpendicular* to the spanning direction of the beam, thus establishing the load per unit length along the beam as having units of kN/m or lbs/ft.

It should be noted that in addition to the surface-load-derived UDL there is, strictly speaking, always going to be a second uniformly distributed load that the beam must be designed to carry: that of its own self-weight. (Fig. 3.8.) The magnitude of this load (in units of kN/m or lbs/ft) can be either looked up directly in tables for particular beam sizes or else it can be calculated from the member's cross-sectional dimensions and constituent material density. Depending on the choice of materials, the self-weight of the *supporting* structure may in fact not be very significant compared to that of the *supported* surface loads, and if so it is sometimes conveniently ignored at the preliminary stages of member size selection. Certainly, any final design of a structural element, however, should always account for the structure's self-weight.

While the parallel-beam floor framing condition we have just looked at and the resulting uniform load distribution is a very common situation, it is by no means the only condition that exists; for various aesthetic and practical reasons, not all floors in buildings are supported by beams arranged in such a straightforward manner. In situations where beams are equally spaced in both orthogonal directions (essentially in a "grid" condition), the load is then shared between the beams in the two directions (more about this in Chapter 7). Yet further complexity arises if beam arrangements are chosen that are irregular and non-orthogonal, although the fundamental principles of what we have just discussed here will essentially remain the same.

Point Loads

A second load type can be identified that occurs very frequently in the world of architecture and buildings: that of the *point load* that is highly concentrated over a relatively short length or surface

Illustration 3.19
Hemeroscopium House, Madrid, Spain (2008).
Stone at top and beam-upon-beam construction (made of precast concrete) represents a point load condition.
Architect: Ensamble Studio. Technical architect: Javier Cuesta.

area. (Fig. 3.9.) A point load is not so difficult to conjure: perhaps it is the weight of an exceptionally large and permanently situated sculpture or else a heavy piece of equipment fastened to the floor of the mechanical room in a building;[3] maybe it is the total weight of a lantern at the top of a dome or of a large built-in tree planter on a roof terrace; it may also be the force from a column that is being picked up on a beam in order to open up the occupiable space below; or, perhaps most commonly, it may be the result of the action/reaction support condition where one beam transversely frames into another beam, or connects into a truss, a column, etc. A point load may also be used to represent the statically equivalent gravity load on the surface on a large tributary floor area or, as we will see shortly, of wind pressure acting on the side of a building. Whatever the cause, a load of this type is effectively considered to be acting at a single "point" on a structure and is typically represented in a loading diagram by means of a force arrow showing its magnitude, direction, and location (more about this in Chapter 4.)

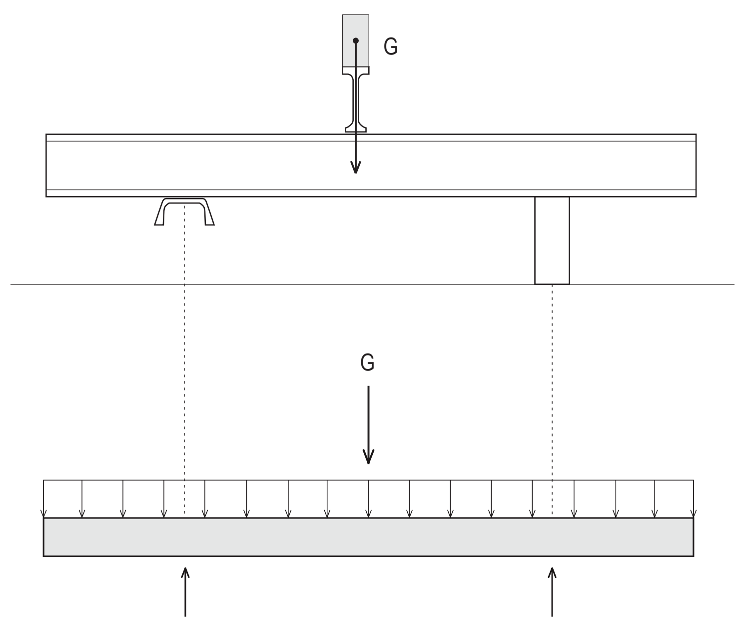


Figure 3.9
Conceptual loading diagram corresponding to the point loading condition illustrated in the photo of the Hemeroscopium House in Ill. 3.19: the large stone block at the top as well as the load from the end of the transverse beam immediately under it produce a large point load on the precast concrete beam seen in the foreground. The self-weight of the precast beam, on the other hand, contributes a uniformly distributed load along its full length.

Illustration 3.20
The Rolling Huts, Mazama, WA, USA (2007).
Snow accumulations can be significant on flat-roofed structures, while wind effects can cause some parts to be swept clean, causing pattern loading.
Architect: Olson Kundig. Structural engineer: Monte Clarke Engineering, Inc.

3.7 Loads from Nature – Earth, Wind, and More

Although the focus of this chapter so far has been on gravity loads caused by the weights of structure and various finishes as well as on code-defined allowances for occupancy, we now turn our attention to the effects of other load-producing natural phenomena. Mother Nature has quite a wide range of "weapons" in her arsenal to throw at the structures we build ever so defiantly; there is no attempt here to deal with them all. Perhaps most critically in terms of developing a primary understanding of overall structural behavior and configuration, however, will be the recognition that some of these natural elements cause significant *lateral* forces to be applied to structures in addition to the gravity loads that we have just considered. Especially important in this regard are the effects of wind pressures and seismic activity, and these will be considered shortly. Before doing so, however, we will first look briefly at the somewhat particular impact of three other frequently encountered natural live load types: snow, earth, and water.[4]

Snow

In temperate and colder climates, the loads produced by the weight of snow accumulations always need to be considered for the design of roofs and other exposed surfaces. As our general experience would suggest, such loads are highly dependent upon, among other things, geographic location, topographic elevation, particular local physical and climatic conditions, and the angle of inclination of a roof surface. The numerical value of the snow load to be used will typically need to be looked up in a local building code that will in one way or another account for these and perhaps other variables as well. Such a number will include, for instance, an allowance for the depth of snow that statistically has been determined to be expected to accumulate at a certain location. Also, as anyone who has shoveled a driveway can attest from first-hand experience, the weight of snow can vary greatly according to its water content – either because of a typically humid local climate or because of the inevitable water-logging of snow in spring or from a winter thaw – and this must be accounted for. The slope of a roof is also a critically important factor in determining design snow loads, with the steeply pitched roofs of Swiss chalets no accident (nor due to a cultural proclivity for a particular architectural style, at least originally). Indeed, the snow-shedding capabilities of pitched roofs are such that if they have a slope steeper than 60 degrees they typically need not be designed for any snow loading whatsoever, whereas flat surfaces in the same location will need to be designed for significant accumulations. Wind may also cause snow to pile up on certain parts of a roof while sweeping it off completely in other areas (e.g., Ill. 3.20), which even for simple

Illustration 3.21
Eames House, Pacific Palisades, CA, USA (1949).
A 60m (200ft) long, full-story-high earth-retaining wall allows the Eames House to be nestled into a hillside despite lateral earth pressures. (See also, Ill. 1.2.)
Architect: Charles and Ray Eames. Structural engineer: MacIntosh and MacIntosh Company; also Edgardo Contini (for first version of house's design in 1945 – and for which structural components were ordered).

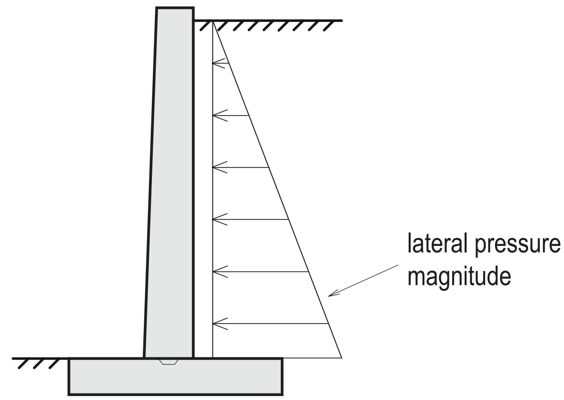


Figure 3.10
Lateral pressure distribution caused by soil against retaining wall.

building shapes can lead to having to design a roof for particular patterns of loading (e.g., with half the roof considered to be loaded with snow and the other half not), whereas for more complex and unusually shaped structures it can lead to the need for complicated computer and physical wind tunnel modeling to predict just how much snow will accumulate on a roof and where.

In order to give at least some sense of the relative magnitudes of typical snow loads, and to provide a starting point for the preliminary design of roof structures, it should be considered that these may range from 1.0kN/m^2 (20lbs/ft^2) to 8.0kN/m^2 (150lbs/ft^2) or more. Regarding the minimum, it should be borne in mind that even for predictably snowless regions a roof surface still needs to be designed for at least a minimum live load allowance of perhaps 1.0kN/m^2 (20lbs/ft^2) in order to allow for roof construction and repair. For most locations roof snow loads will be significantly less than the typical occupancy loads for the floors of the building it covers, with a consequent likelihood of a lighter structural framing system being possible. In certain locations that are particularly susceptible to huge snowfalls in short periods of time, however, this condition can easily be reversed.[5]

Earth and Water

Loose soil or rock also produces its own distinctive type of loading on structures. Although much of architecture is implicitly considered to be about buildings that are above the ground surface, there is a long history of carving inhabitable space out of the earth. Even if a building is not completely below ground, however, any natural slope or incline will require the designer to either "float" the building up on posts on the downhill side or dig it out of the ground on the uphill side (thus requiring the unstable earth to be held back against falling into the living space (e.g., Ill. 3.21)) – and often both of these strategies are employed on the same building. Terracing by means of a series of earth-retaining walls is also a well-known strategy for occupying sloped ground. (e.g., Ill. 3.22.) For buildings on flat terrain there is typically the need in temperate and colder climates for the base of foundations to be kept well below the ground surface level in order to prevent frost upheaval, thereby providing the reason for typical below-grade perimeter basement walls against which loose earth is typically backfilled. And more recently there is renewed interest in taking advantage of the long-recognized energy-saving thermal benefits of building into and against the ground. So, all things considered, loads that are caused by loose earth and stone are not of insignificant interest in architecture after all.

Illustration 3.22
Machu Picchu, near Cuzco, Peru (fifteenth century).
Numerous stone retaining walls create the terraced landscape used to support farming on the steep slopes of this mountaintop Incan royal estate.

We may understand from our own gardening experience that soil is far from weightless, so there is the expectation that accumulations of it will cause significant downward gravity load to be applied to any structure carrying it – and such loads can become very significant or even critical for planted rooftops, as we saw for the underground entrance to the National Theater Railway Station in Section 3.4. In addition, when there is any substantial depth of a "loose" and compressible material like earth (as well as water, for that matter) and when it is "contained" or prevented from expanding sideways as it gets compressed from above, we need to be concerned with more than just vertical loading: also present in this situation will be sideways pressure exerted against the restraining structure – which in the case of architecture is likely to be a foundation or retaining wall of some sort. Of course, as may be familiar from common knowledge about water depths and pressures, the greater the depth of material the more pressure is exerted, and this leads directly to the representation of triangular lateral pressure diagrams acting on any vertical structure that is holding back or containing earth or water. (Fig. 3.10.) Anyone who has walked past an overburdened retaining wall that is uncomfortably leaning outward into empty space, or who has remarked upon the need for the substantial thickness of the transparent walls of deep aquarium pools will have been the unwitting and perhaps uncomfortable observer of such lateral loading. (e.g., Ill. 3.23, 3.24.)

Illustration 3.23
AquaDom, Sea Life Centre, Berlin, Germany (2004).
Contained within a 16m (52ft) tall vertical aquarium made of bonded acrylic panels are 940 000 litres (250 000 US gallons) of saltwater and over 1500 fish of 50 different species. The water is held between two cylinders, one within the other, thus enabling a transparent central elevator. The weight of the water produces a large gravity load at the base; in addition, lateral pressures increase linearly with the water depth, acting radially upon the cylinders. Their circular form in plan is ideal for resisting such loads by means of circumferential hoop stresses, as will be discussed later in Chapter 13.

Designer and constructed by: International Concept Management.

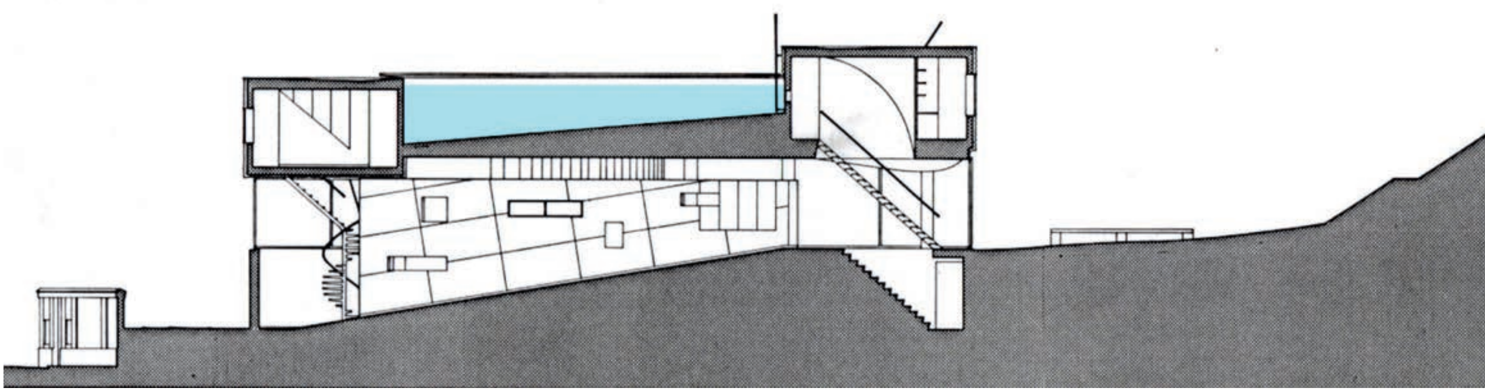


Illustration 3.24
Villa dall'Ava, Saint-Cloud, Paris, France (1991).
Water in roof-top swimming pool causes significant gravity loads as well as lateral pressures to be applied to pool's supporting and enclosing structure, respectively.

Architect: Office for Metropolitan Architecture (OMA). Structural engineer: Marc Mimram.

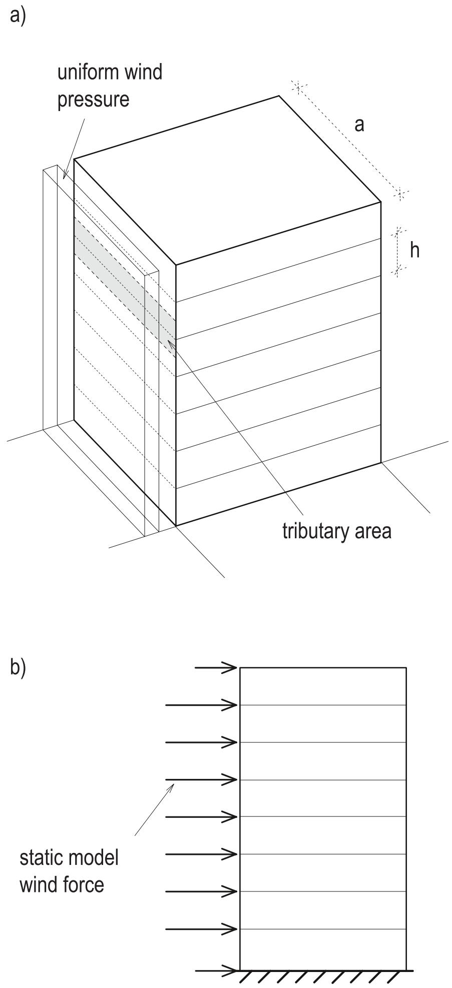


Figure 3.11
Conceptualizing wind forces: (a) wind pressure over building face, tributary area for single floor (shaded), and (b) conversion into point loads at each floor level.

Illustration 3.25
Northwest Corner Building, Columbia University, New York, NY, USA (2010). Wind imposes sideways pressure acting over the surface of building façades, as depicted in Fig. 3.11.

Architect: Rafael Moneo + Moneo Brock Studio and Davis Brody Bond. Structural engineer: Arup.

Wind

The fact that buildings are affected by wind should be self-evident to anyone who has walked outdoors on even a slightly gusty day, while those who have experienced a hurricane or tornado first-hand can attest to its very real capacity for doing serious damage to structures. But acknowledging that the wind will have an effect on buildings is one thing, while finding accurate ways to quantify and design for its highly erratic behavior is quite another matter.

To begin, one can readily recognize that air has both density d (granted, this is relatively small, but it exists nonetheless) and velocity v (potentially quite large) and then apply the basic relationship that wind pressure is proportional to these two quantities in the following manner:

$$\text{pressure} \propto d \times v^2$$

For sea level air density and in metric units this equation becomes approximately

$$\text{pressure} = v^2/1.6$$

where the pressure is measured in N/m^2 and the velocity in m/s. (The equivalent in American Standard Units is pressure = $0.00256\ v^2$, where pressure and velocity are in units of psf and mph, respectively.) From daily weather forecasts, we are used to hearing about calm breezes of 3m/s (5mph), gales of 30m/s (60mph) and hurricane force storm conditions of 45m/s (100mph); in order to give a sense of the typical range of resulting lateral wind pressures acting on buildings, these speeds can easily be converted into corresponding pressures of 5, 480, and 1230N/m^2 (0.10, 9.9, and 24.7lbs/ft^2), respectively.

With wind understood in this way as sideways pressure, it is evident that building shape – especially the structure's "sail" or transverse-to-the-wind-direction surface area – becomes of critical importance in establishing magnitude of loads. Everything we have previously discussed in the previous section in terms of establishing tributary areas and converting uniform surface pressures into equivalent two-dimensional line loading diagrams can be applied here again, with the difference now being that the pressure is lateral and acting on the vertical building face (Fig. 3.11) instead of gravity's downward direction acting over horizontal floor surface. Loads

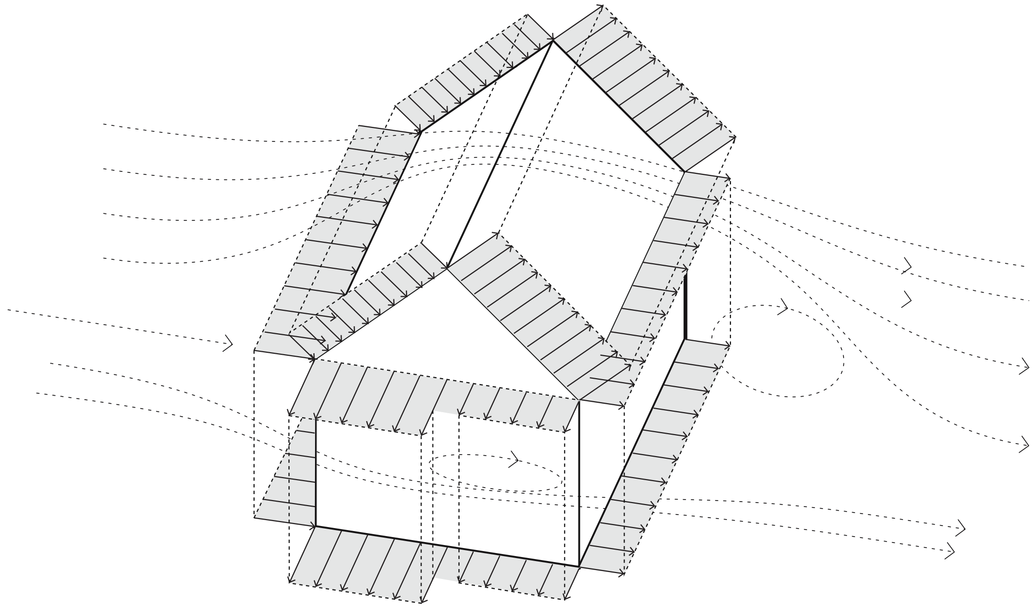


Figure 3.12
Diagram of wind flow around building and code-defined variations of pressures and suctions on various building surfaces.

Illustration 3.26
Akers Mechanical Workshop and Factory, Oslo, Norway (1841). The sideways-displacement effect of wind pressures and suctions acting on a building has long been known, as is indicated by the presence of diagonal bracing members within frameworks of vernacular timber construction. The brickwork here is infill that is unconnected to the frame and is used only to create enclosure, rather than contributing to lateral load resistance.

can thus be determined whether for individual vertical mullions supporting part of a curtain wall or for investigating equilibrium conditions for the building as a whole. In the latter case, determining equivalent point loads for wind at each floor level typically ends up being a matter of multiplying the appropriate wind pressure (roughly, this will be something in the neighborhood of 1.0kN/m^2 (20lbs/ft^2) except for taller buildings) by the vertical tributary area for each floor (i.e., the building's transverse width times the story height).[6]

In reality, of course, the situation is quite a bit more complicated than this simplistic load representation of uniform lateral pressure. In order to help better visualize what is actually going on it is common to make the analogy of a building within an invisible flow of air being like a rock in a fast-flowing stream of water. We easily observe and understand that there is a significant push or pressure exerted by the water on the upstream side of the rock in the direction of the stream flow. On the downstream side immediately behind the rock, the interrupted flow produces a partial vacuum effect and suction force. And on the rock's sides and top (if it is covered by the water), there is an increase in the velocity of the water as it flows around the obstruction – generally also producing negative pressures or suctions on these faces of the rock.[7] Sophisticated computer simulations and the typical diagramming of wind flow over and around buildings will be found to closely follow the general characteristics of this familiar analogy. (Fig. 3.12.)

Perhaps because wind is identified in everyday life by a single number describing its speed, it is common to think of it as being a uniform "laminar flow," that is, that the speed will be the same for every vertical layer, or "lamina," of wind. In reality, however, this mental image is significantly inaccurate. There will be considerable friction or drag right along the surface of the ground produced by the irregularities and "roughness" of the terrain caused by trees, hills, buildings of various heights, etc.[8] The characteristic wind velocity profile, therefore, is one which is close to constant only above an elevation of a few hundred meters but which is considerably reduced from this maximum speed as one gets closer and closer to the ground. Such a varying speed profile is confirmed by measurements on building façades, and is often translated into a corresponding set of building-code-defined static pressures whose

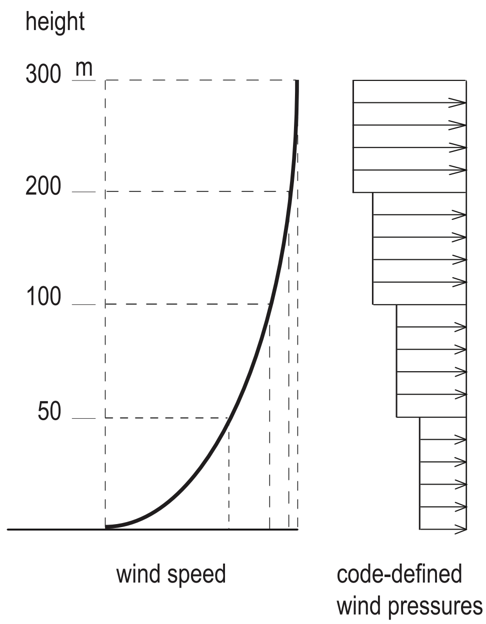


Figure 3.13
Variation of wind speed with height; typical representation in building codes by stepped function of wind pressures.

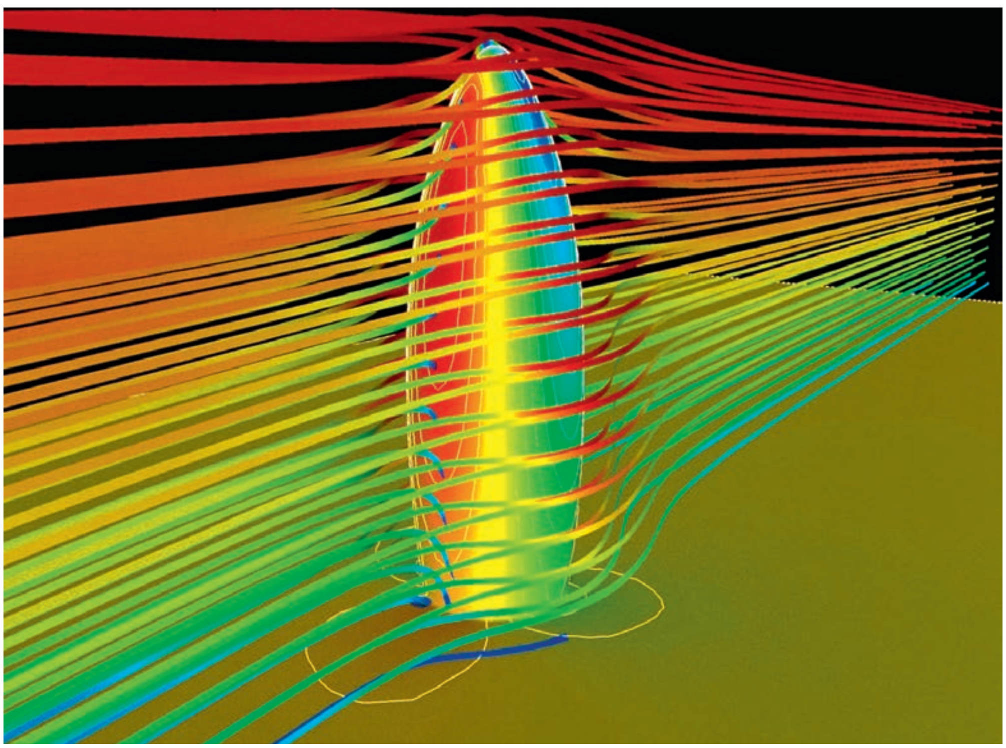


Illustration 3.27
30 St. Mary Axe, London, UK (2003). Relatively smooth wind flow pattern is produced around tapered, rounded shape of building.

Architect: Foster + Partners. Structural engineer: Arup. Wind tunnel consultant: ChapmanBDSP.

magnitudes (in units of kN/m^2 and lbs/ft^2) typically increase in a step-like fashion with elevation. (Fig. 3.13.)

Numerous other characteristics of both the wind and the buildings upon which it acts will further influence the magnitudes of the pressures and suctions that need to be considered. Some of these factors are natural phenomena that are beyond an architect's control on a given project, such as global geographic location and local prevailing wind patterns and directions, but others are well within a designer's capacity to influence, such as a building's shape (streamlined vs. blunt; e.g., Ill. 3.27), surface texture (smooth vs. rough), stiffness of the structural frame (flexible vs. rigid), and the building's height, placement on a site, and relationship to its surrounding context. These effects may be more familiar in other design fields, such as how the streamlining of the shape of cars enhances air flow in order to make them more fuel efficient and how the surface texture of ski and swim suits can significantly enhance the performance of top Olympic athletes, but these characteristics are just as applicable, if perhaps underutilized, in the context of building design.

Illustration 3.28
Carnegie Hall Tower and Metropolitan Tower, New York City, NY, USA (1991, 1987).
Tall buildings' surrounding urban context and local wind conditions as well as their particular geometric form all affect the magnitude and distribution of the wind loads that need to be designed for. In the case of these Manhattan towers of exceptional slenderness ratio (i.e., height vs. width parallel to wind direction), wind-tunnel testing and computer modeling was the only way to properly anticipate the wind loading. (Carnegie Hall Tower is tallest at the center of this photo, Metropolitan Tower is the triangular, black-glass-prism volume to its left.)

Architect: Pelli Clarke Pelli Architects and SLCE Architects, respectively. Structural engineer: Rosenwasser Grossman Consulting Engineers (for both); one of the present co-authors, Mark Cruvellier, worked extensively on their structural modeling, analysis, and design.

It should be noted that buildings that are especially large, unusually shaped, or particularly sensitive cannot be dealt with adequately by the static pressure loading model discussed so far and will have to be otherwise investigated, perhaps using physical testing in a wind tunnel facility. (e.g., Ill. 3.28.) This involves building a scale model of not only the building being designed but also of an extensive portion of the surrounding urban context or natural terrain. This model is then placed on a turntable at the opposite end of a long room from a large high-powered fan – the revolving table allowing the effects of every angle of incidence of wind to be considered. The scaling of readings from several pressure gauges inserted into the model of the building allows for quite accurate predictions of actual wind forces – and includes the well-known but otherwise very-difficult-to-account-for effects of adjacent buildings and landforms on the wind flow.

Finally, we must consider the dynamic response of buildings to wind. The static load model previously discussed, with its implied assumption of never-changing wind speed on a building of standard shape, produces a predictable and singular lateral building deflection. This represents greatly simplified conditions and assumptions for both the wind's behavior and the building's response (although such a model has fortunately been found to be perfectly safe and economical for the design of most low- and medium-scale buildings). Contrary to the stasis of this model, however, our everyday experience with wind reminds us that its speed is endlessly variable, with gusts and lulls constantly occurring. As a result, we need to recognize that a building's general overall response to wind is (a) to deflect sideways roughly based on the average wind speed and (b) to oscillate back and forth about this primary deflected shape because of the irregularities of the wind. (Fig. 3.14.) It is important to recognize, however, that the to-and-fro motion of the building will be according to its own inherent period of vibration and typically not according to wind gust frequencies, although there are a few famous examples of the potentially catastrophic results of having these match.[9] Such behavior is confirmed by careful measurements that have been taken of building movements and by even more dramatic recordings that exist of buildings swaying very regularly back and forth in the wind, sounding for all the world like the creaking wooden hulks of tall-mast sailing ships rolling in the waves. Although a somewhat uncomfortable reality, building motions are indeed an undeniable fact of life that needs to be contained. Fortunately for typical low-rise buildings they are barely

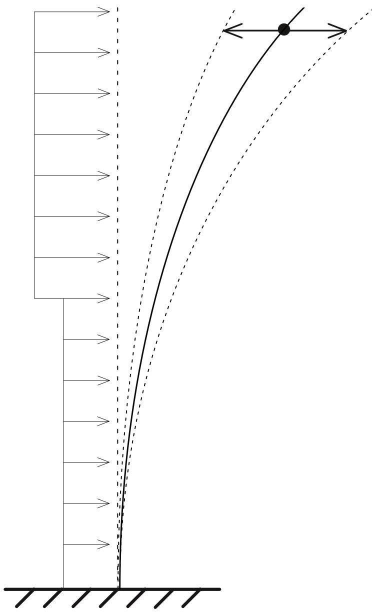
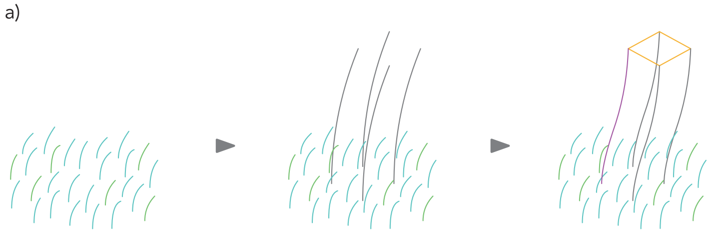
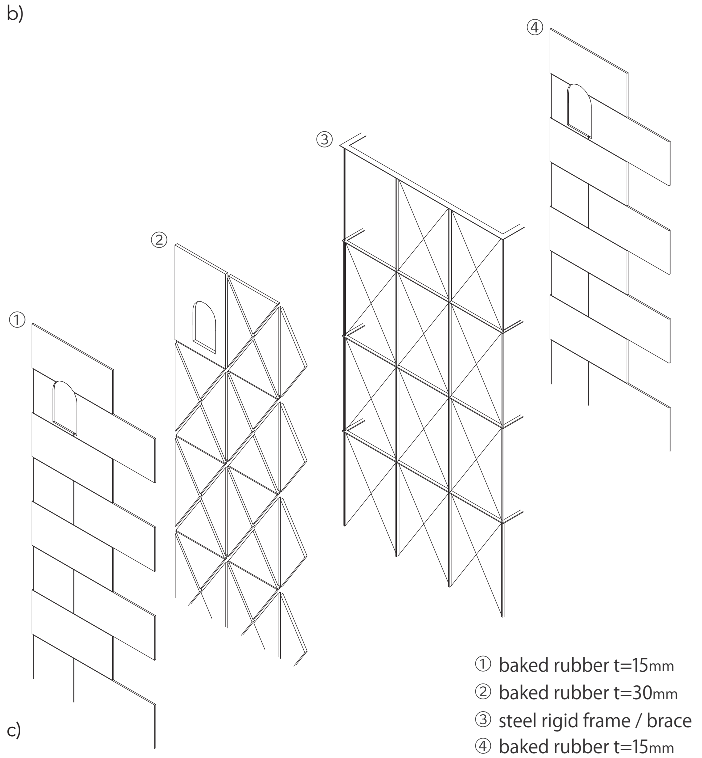


Figure 3.14
Average deflected profile of building (at exaggerated scale) caused by lateral wind pressures; typical back-and-forth oscillations.

a)



b)



c)

Illustration 3.29
Little Hilltop Wind Tower, Yamagushi Prefecture, Japan (2011).
(a, b) Designed to enable visitors to a wind farm to "see" the wind, this tower is deliberately designed to visibly sway back and forth: for a 8m/s (18mph) wind, the top deflects 150mm (6in). (c) The structure is made of particularly light, thin steel components that incorporate specially detailed sliding connections; this is covered by a flexible skin made of vulcanized rubber sheets so as to allow for large displacements without damage.

Architect: Shingo Masuda and Katsuhisa Otsubo, with Yuta Shimada. Structural engineer: Jun Sato Structural Engineers.

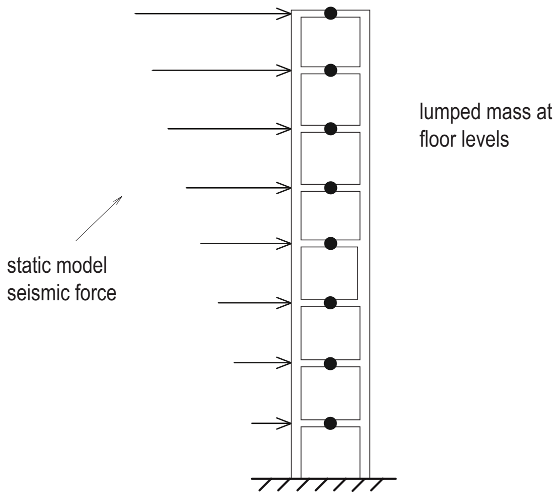


Figure 3.15
Static load model used to represent the effects of earthquake action on a building; masses lumped at floor levels, forces are correspondingly concentrated at those levels.

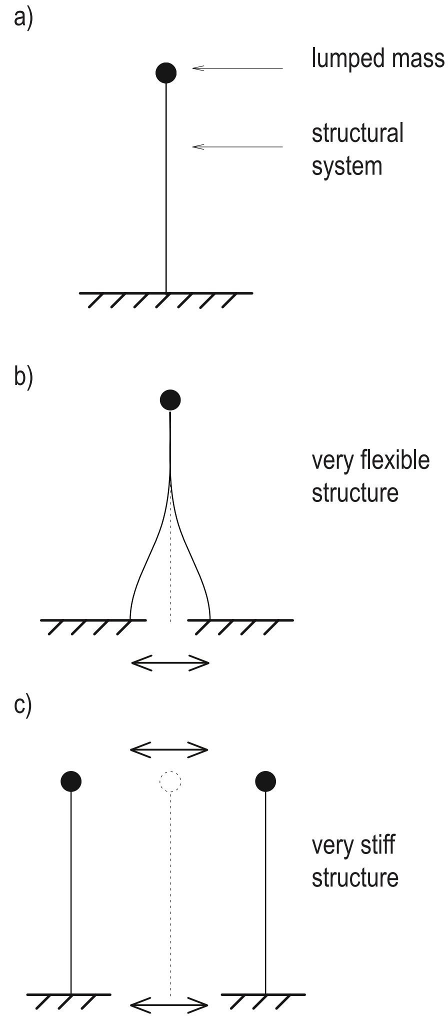


Figure 3.16
Variations of building response to ground shaking caused by earthquakes: (a) conceptually simplified building representation as a single lumped mass and a uniform structural system; (b) with a very flexible lateral-load-resisting system, the mass moves little; (c) a system having very large lateral stiffness causes the mass to move along with the ground.

perceptible, but if not carefully controlled as we build ever lighter, more efficient and flexible structures, dynamic movement may all too readily be heard, seen, or felt to the point of discomfort and alarm. As a result, the past quarter century has seen the rapid development of various damping systems whose objective it is to attenuate building motions, in an analogous fashion to car shock absorbers that quickly reduce the bouncing up and down of a vehicle after it has passed over a bump in the road. This broad topic, while noteworthy and of considerable interest, is generally considered to be beyond the scope of the present book, however.

Earthquake

Accounts of the destructive power of seismic tremors on buildings abound, from such examples as the Great Lisbon Earthquake of 1755 written about by Goethe and Voltaire to the widely publicized events of the more recent past such as at Loma Prieta, Kobe, Bam, Port-au-Prince, Concepción, Amatrice, Christchurch, Kathmandu, Mexico City, etc. – all helping to ensure that there is a general human consciousness about the potentially catastrophic effects of seismic motion on buildings. Moreover, simply spending a few moments observing a seismograph capable of recording every earth tremor around the globe will make one come away convinced about just how unstable is this ever-moving earth's crust upon which we construct our lives.

If the danger of seismic action is well appreciated, what is not so clearly self-evident is just how the earth's "quaking" causes forces to be applied to buildings. Whereas with the other types of loads that we have considered, whether gravity dead loads from material self-weights or occupancy live loads or the lateral pressures produced by wind or soil, it is relatively straightforward to visualize their direct conversion into statically equivalent force vectors applied at particular locations on a structure, in the case of earthquake action this is certainly not the case. The only external action happening to a building during an earthquake is the shaking of the ground on which it sits, and yet the simplest and most common code-defined representation used to account for seismic action on a building is a set of horizontal "earthquake forces" applied at each floor level of the structure. (Fig. 3.15.) It needs to be explained why this makes sense.

In order to better understand the logic of this earthquake force model, we begin by considering a simplistic representation of a building as conceptually consisting of a single lumped mass supported by a structural framing system (represented by a vertical dowel-like line) that is connected to the ground through its foundations. (Fig. 3.16.) Such a framing system is, of course, essential for any building regardless of any earthquake action in

Illustration 3.30
Seattle Public Library, Seattle, WA, USA (2004).
Live loads on buildings include lateral wind and earthquake forces that must be resisted – in this case by means of expressed steel diagonal bracing systems. In relation to Figure 3.16, such bracing is an example of a relatively stiff lateral-load-resisting system that in this case connects one floor level of the building to the next and that, at the lowest level, connects the building to any ground displacements that may occur in an earthquake.

Architect: Office for Metropolitan Architecture (OMA). Structural engineer: Arup and Magnusson Klemencic Associates.

order both to carry gravity loads to the ground as well as to resist the ever-present wind forces that we discussed in the preceding section. Any sideways movement of the ground caused by an earthquake can be thought of as having the model base displacing sideways; the building mass, however, will react differently depending on the lateral stiffness of the framing system. If it is (hypothetically) considered to be infinitely flexible, then the base would move back and forth while the lumped building mass would remain completely stationary above it. If, on the other hand, the framing system's lateral stiffness were (again hypothetically) infinitely rigid, then all of the building's mass would be forced to displace sideways an equal amount to and in unison with the base/ground.

Real building frames, of course, lie somewhere between these two imaginary extremes; i.e., any structural system has a certain lateral stiffness that is neither infinitely rigid nor completely flexible. (e.g., Ill. 3.30.) A lateral displacement of the ground in an earthquake, therefore, also necessarily brings about a certain lateral movement of the building as a whole due to the structural frame "dragging" the building along with it in some fashion. And when the mass "m" of the building that started out at rest is no longer stationary but is instead caused to be moving at some velocity, we can recall from elementary physics that an acceleration "a" must have taken place and, therefore, can begin to think in terms of a *conceptually equivalent earthquake force (since F = ma, see Chapter 4)* being applied to the building. Carrying this line of reasoning a little farther, because for most buildings built today approximately 90 percent of the total mass can be considered to be concentrated at the floor levels (the occupiable space between floors being mostly air), the basis for representing earthquake loading by a set of horizontal forces applied at each floor level becomes more evident.[10]

The detailed procedure for calculating such building-code-defined earthquake forces can become quite tedious, and is something that we will avoid here. It is useful, however, to consider the parameters of typical code equations that are used for this purpose in order to highlight the factors that most strongly affect earthquake force magnitudes. The total horizontal earthquake force V that a building must be designed for is established by an algebraic equation something along the lines of

$$V \propto ZICWR$$

in which Z is a code-defined earthquake zone factor that varies by a factor of four or more according to geographic location, I is a so-called "importance" safety factor that helps to insure that "essential" buildings (e.g., hospitals, etc.) are more likely to remain standing and functional after an earthquake, C is a numerical coefficient that accounts for a building's height, its natural period of vibration, as well as local ground conditions that might amplify initial motions, W is the total *dead load* of a building, and R is a factor that accounts for the relative lateral stiffness of one structural system vs. another.[11]

Let us consider what this proportional relationship can tell us about how to design a building for earthquake forces. Aside from the obvious impact of choosing a strategically advantageous location for a building site – something over which the designer typically has little or no choice, as people will always choose to live in such places as California or Japan or Italy – there are obviously other factors over which the designer can exercise significant judgment at the earliest stages of design so as to preemptively limit earthquake forces that must be dealt with. Reducing a building's mass through the judicious selection of building materials, whether that of the structural system itself or of the attached cladding and finishes, is an obvious case in point. In this regard, the general historical trend away from heavy, massive stone and brick as structural materials and toward lighter and more open metal frames is undeniably advantageous, but even among today's building materials and finishes there are important decisions for the designer to make: e.g., brick or stone exterior cladding panels vs. woven metal mesh? Or an extremely light fabric membrane roof vs. a heavy beam system? These selections will have a very significant and obvious impact on the seismic force that needs to be designed for, aside from being determinant of a building's appearance.

A building structure's lateral stiffness also plays a critical role in determining the magnitude of the earthquake forces that need to be designed for, but this aspect is somewhat counterintuitive: the greater the lateral stiffness of the system, the greater will be the earthquake forces and the more will have to be done to counter them. In life one is much more used to the notion that more is better, and certainly that is the usual expectation in structures; i.e., a bigger column will carry more load, as will a deeper beam. But here we have the opposite effect: i.e., the stiffer one makes a lateral-load-resisting system, the more one increases the earthquake force that must be designed for – something of a self-defeating situation. We can explain this apparent contradiction by going back to the conceptualized stiff-versus-flexible structural systems that we considered previously in Fig. 3.16. In the rigid case, when the ground displaces back and forth, the mass of the building is dragged along closely with it, thus maximizing accelerations and forces.[12] A completely flexible system, on the other hand, would in theory allow the horizontal ground displacements to occur without entailing any sideways movement of the building whatsoever – meaning that there would be no accelerations to speak of and, therefore, no resulting earthquake forces acting on the building. Of course, even if it were possible to build such a completely flexible lateral-load system it would be useless for dealing with the wind forces that also must be resisted by the structural frame on an everyday basis, so some lateral stiffness is, in fact, always necessary. Escaping the development of earthquake forces is thus never possible, although interesting base support isolation strategies and detailing can be

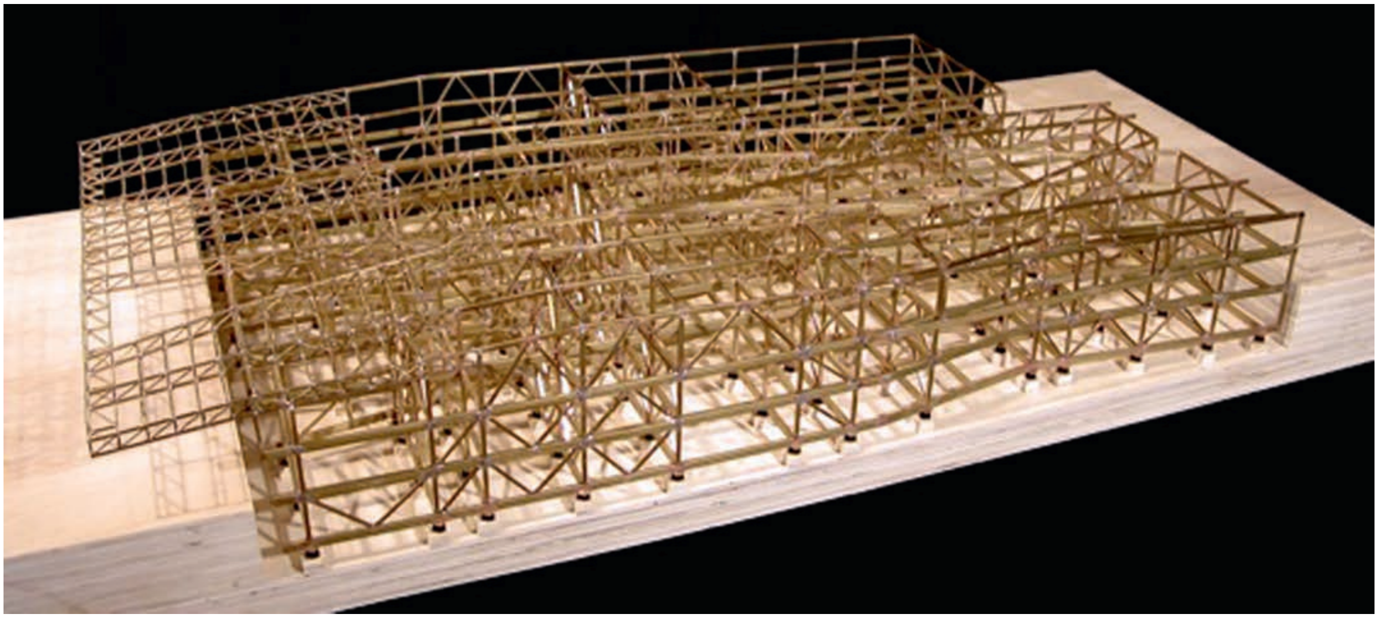


Illustration 3.31
de Young Museum, San Francisco, CA, USA (2005).
Detail in model of overall structural system suggests column base-isolation method used in actual building to reduce earthquake loads; further physical isolation is provided by providing a gap at the lower level between building and surrounding ground.
Architect: Herzog & de Meuron. Structural engineer: Rutherford & Chekene. Cornell model by Reilly Hogan and Kumar Atre.

developed that work toward this objective. (e.g., Ill. 3.31.) We will come back to further discussions of frames and lateral stability issues in Chapter 10; for now, however, the implications of this discussion for building designers in earthquake-prone regions are clear – there is distinct structural advantage to making buildings both light and flexible, assuming that this fits with other architectural objectives.

Finally, it must be recognized that the static earthquake load representation discussed so far can hardly be taken to be an accurate reflection of what is in reality a highly dynamic situation. Not only does the ground's motion vary greatly during an earthquake but also the building's ensuing response will consist of back-and-forth vibrations whose frequencies are themselves independent of the earthquake's shaking.[13] As a simple analogy to the condition of a single-story building, we can think of a tennis ball skewered on to a thin metal vertical rod that is rigidly attached to a base. If the base is sharply displaced laterally (crudely mimicking the ground motion of an earthquake) the ball starts swinging back and forth with its own characteristic period of vibration that continues even when the ground motion has stopped. The same happens in a building, and as the mass swings to and fro it accelerates and decelerates from rest at the extremes of the oscillation to a maximum velocity at mid-vibration, effectively producing constantly varying load conditions and deformations that need to be accounted for in the design of the building's structure. And in a multistory building the situation becomes even more complex, with several modes of vibration occurring simultaneously and superimposing themselves upon each other. Yet further adding to the intricacies of this highly time-dependent situation, the back-and-forth swinging of the building will gradually diminish as the earthquake's imparted energy is dissipated. Fortunately, computer modeling can simulate all this dynamic behavior relatively accurately and be accomplished relatively quickly and economically, and this is done for any structures that venture outside of the norms of conventional construction. (e.g., Ill. 3.32.)

Illustration 3.32
Century Tower, Tokyo, Japan (1991).
Double-story-height eccentrically braced frames expressively reflect the need to design for large lateral wind and seismic forces in Tokyo; the eccentric bracing configuration does not form rigid triangles in the central part of the frame, thereby providing a desirable measure of flexibility to the lateral-load-resisting structural system with regard to large earthquake forces.

Architect: Foster + Partners. Structural engineer: Arup.

Statics

CHAPTER

4

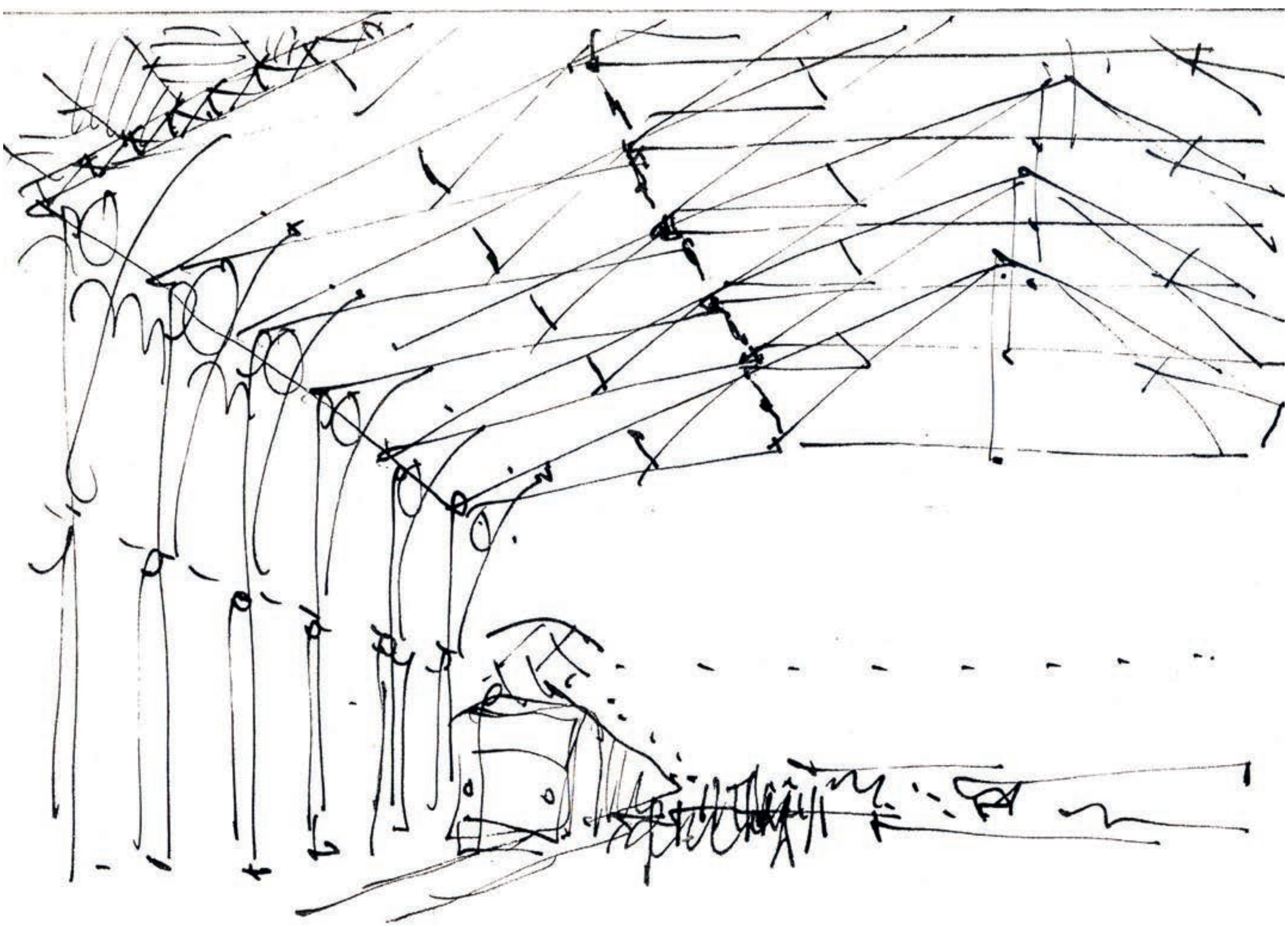


Illustration 4.1
La Gare d'Austerlitz, Paris, France (1869).
The so-called Polonceau truss that was introduced to several of Paris' railway terminals in the mid-nineteenth century spans over the tracks and platforms.
Architect: Louis Renaud. Structural engineer: Sévène.

Illustration 4.2
La Gare St-Lazare, Paris, France (1852).
Traces of Polonceau trusses are evident in the roof structure.

Sketch by Claude Monet, 1877.
Architect: Alfred Armand. Structural engineer: Eugène Flachat.

4.1 Polonceau – Past and Present

The French engineer and constructor Camille Polonceau (1813–1859) exploited the principle of how a slender beam can be reinforced by adding a small transverse compression member below its center (or three, as the case may be) and tying the lower end of this strut by means of tension rods to the ends of the beam. By inclining two such reinforced beams toward each other and further connecting the ends of the two compression struts with a horizontal member in tension, Polonceau designed a very effective structure for large roofs. (Fig. 4.1.) In this way the two beams were held in position and at the same time he gained greater spatial height than could be achieved with a traditional roof structure having a horizontal lower flange. The structure made use of the available materials of that time in an optimal way; for the beams he mostly used wood while the compression struts were made of cast iron and the tension ties of wrought iron. This so-called Polonceau truss, well suited for long spans, was used extensively in large hall projects during the mid-1800s. From a contractor's point of view, the structure had the advantage of consisting of two symmetrical parts; each could be assembled separately on the ground, then lifted up and connected at the top and across at middle height by means of the horizontal tie.

The roofs of many large railway terminals in Paris were structured by means of such Polonceau trusses. In 1877 the painter Claude Monet made several studies, sketches, and paintings of La Gare Saint-Lazare depicting the lively atmosphere of modern travel; through the steam from the locomotives we get a glimpse of the roof structure. (Ill. 4.2, 4.3.) The Polonceau system, with its slender, balanced members and efficient distribution of forces in tension and compression, has gathered many admirers over the decades since then.

Illustration 4.3
La Gare St-Lazare.
Close-up view of part of a Polonceau truss, with visible distinction between its tension and compression components.

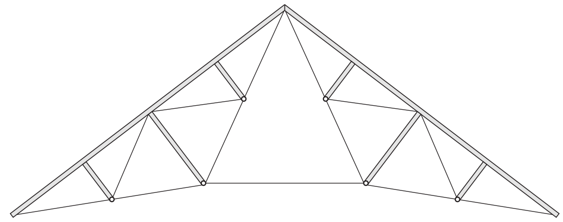


Figure 4.1
Overall Polonceau truss configuration. At each side along the roofline, a slender inclined beam is reinforced by angled tension rods between the beam ends and the tips of three transverse compression struts. Two such inclined reinforced beams, one at each side and meeting at the central ridgeline, are also connected to each other by means of a horizontal tension tie.

Illustration 4.4
Riding Hall, Flyinge, Sweden (2005).
Architect: AIX Arkitekter. Structural engineer: J. Riebenhauer.

Illustration 4.5
Riding Hall, Flyinge.
Detail of joint with intersecting tension and compression members.

On the plains of Skaane in southern Sweden, we find the National Stud of Flyinge, world famous for horse breeding since 1661, where riders trot among buildings in a seventeenth-century aristocratic setting. Hidden behind solid red brick façades, however, is a small riding hall with an innovative roof sporting a contemporary version of the Polonceau truss, simply detailed but still honest and trustworthy in appearance. (Ill. 4.4.)

In 2005, a design competition was won by AIX Arkitekter for the large space of the riding hall. To span this space, several changes have been made to its Polonceau roof structure from that of the original system of 150 years ago. The wooden beam is replaced by inclined slabs of solid wood. The tension part of the structure consists of paired steel rods, enabling a better and more evenly distributed support of the wooden slabs (i.e., the single tie has here been subdivided). And the ties consist of simple steel reinforcing bars with typical ribbing, which here have found a most simple and elegant aesthetic purpose. (Ill. 4.5.)

There is no need for insulating the roof in addition to the wood since the horses and light fixtures provide the necessary warmth for the daily use of the hall. With no need for a ventilation system of air ducts running here and there, and with lighting fixtures attached directly to the wooden ceiling, the result is a crisp and elegant structure. The hall has glazed walls toward the north and east as well as skylights of frosted glass. Careful studies of the sun path have been done to avoid glare that might disturb the horse and rider.

4.2 Isaac Newton and the Mechanical Basis of Structures

Studying the load-bearing properties of structures means to look at form from a mechanical point of view. The Polonceau truss that we just considered, for example, is an uncommonly clear load-bearing structure where a group of structural elements together and in a visually expressive manner provide the necessary resistance to the loads that are acting on it. We are able to grasp how it works since we have previously introduced the various basic types of load-bearing elements and described their corresponding structural actions in Chapter 2 (Section 2.4). But in order to more fully be able to understand and communicate just what is actually taking place within structures more broadly we first need to refresh our memories about some fundamental mechanical principles.

Mathematics and the particular branch of physics called *mechanics* enable us to analyze structural forms. (Fig 4.2.) Mechanics deals with motions and forces, with *statics* being the branch of mechanics concentrating on studying forces acting on rigid bodies at rest. The very word statics comes from the Greek word "staticos," which means "to make something stand still." This is precisely the request we make of structural elements in the context of architecture; i.e., that they maintain their position in space even when loads are acting on them. This basic demand enables us to analyze structural forms and structural systems in order to find out to what extent they are stressed and what types of stresses are acting in the system.

We have again just used the rather abstract term of "force", as we did in Chapter 2. We commonly think that forces appear in material bodies when loads are imposed on them. In reality, though, the physical concept of force is a bit vague; nobody has

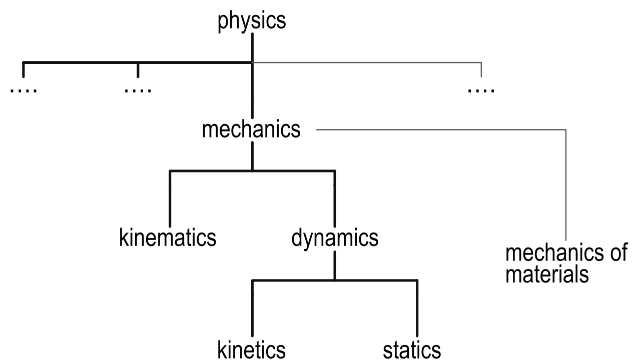


Figure 4.2
The mechanical sciences represented as a branch of physics. *Kinematics* deals with pure geometrical description of movement and was pioneered by Galileo Galilei (1564–1642). *Dynamics* study the laws governing motion, founded by Sir Isaac Newton (1642–1727),[1] with *kinetics* operating on force systems in motion and *statics* on force systems at rest. *Mechanics of materials*, or the strength of materials, is an extension of mechanics into the study of stresses and strains in material bodies.

ever seen a force. What we are able to observe, however, are the results of forces acting in the form of movements or deformations of a body. The latter effect of forces was described in Section 2.4. One way of defining a force is therefore to claim that it is a physical influence, caused by a load which changes, or tries to change, the state of rest in a body. Moreover, as we have mentioned, a force may also deform or deflect a material body.

It was among Newton's remarkable achievements that he linked the concept of force to the state of rest. He observed that a body will continue to move at a constant velocity, or be at rest, if no net force is acting on it.[2] We call this observation Newton's first law. This does not necessarily mean that no forces are acting, but rather that the *sum* of forces must be zero. If there are a number of forces, they must effectively cancel each other out in order for the body or element to remain still, or resting. We depend on this rest for a load-bearing structure to do its job, where the sum of forces is zero and the structure remains still. The energy corresponding to the forces involved instead causes the structure to deform. If, on the other hand, a net force *is* acting, this will accelerate the body. The net force, also called the *resultant force*, will set the body in motion, and the acceleration will be proportional to the applied force. The proportional constant is the body's mass; the larger the mass, therefore, the more force must be applied to obtain the same acceleration. We call this statement Newton's second law. In mathematical terms, we may write

$$F = ma$$

where F = the resultant force, a = the acceleration, and m = the body's mass. If, in the equation, F is zero, then a is also zero. This means that if no resultant force is acting on a body, then there is no acceleration. No acceleration implies constant velocity or the body being at rest. Hence we can observe that Newton's first law is a special case of his second.

Acceleration is defined as a change of velocity per unit of time. Velocity is most familiar to us in terms of kilometers per hour (miles per hour). In scientific terms, however, velocity is typically measured in meters per second (m/s) or feet per second (ft/s). Acceleration, then, is expressed in units of meters per second per second, or m/s^2 (ft/s^2).

Mass is measured in kilograms, kg (slugs).[3] If we multiply acceleration by mass we will have a unit for force written as

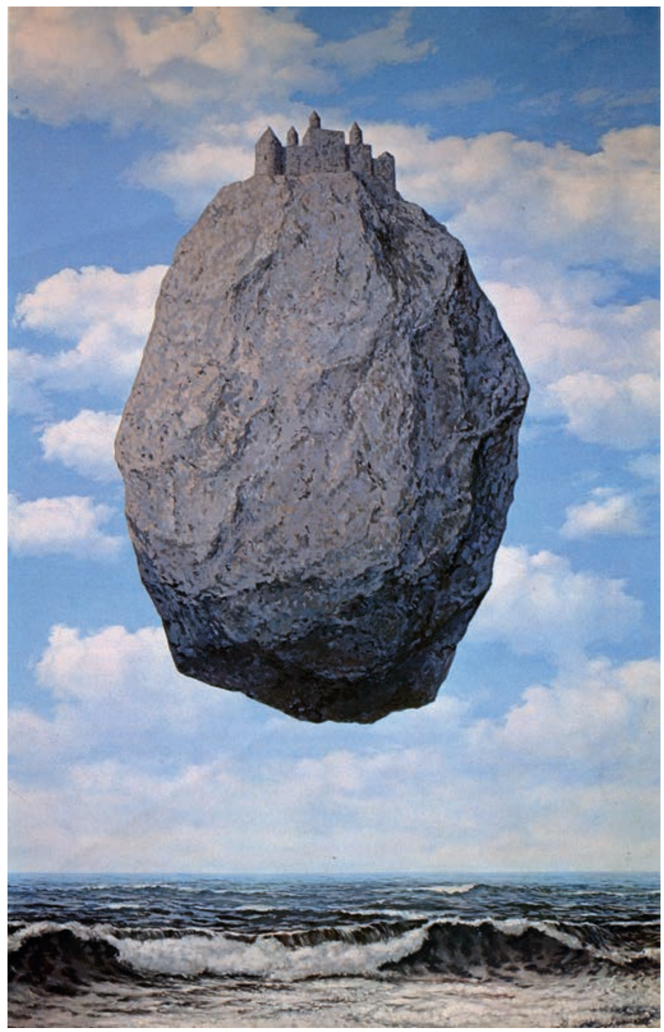


Illustration 4.6
"Le Château des Pyrénées" (1959).
Imagined free-fall motion.
Painting by René Magritte.

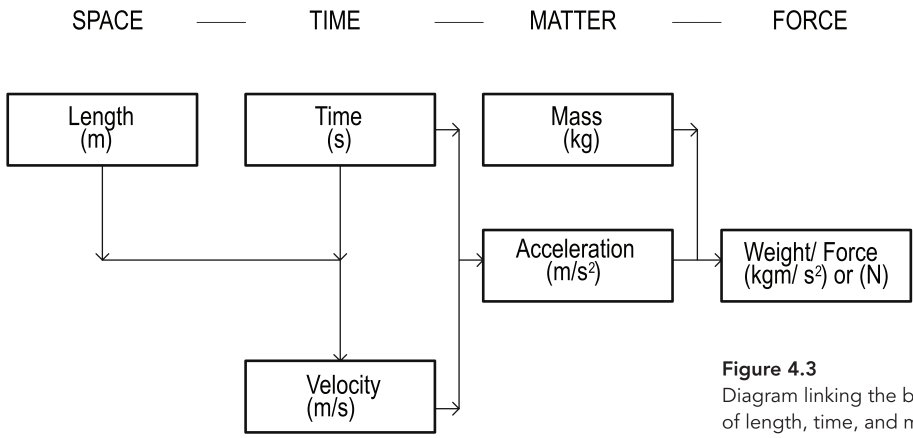


Figure 4.3
Diagram linking the basic physical concepts of length, time, and mass to those of velocity, acceleration, and weight/force.

kgm/s^2 ($slug\text{-}ft/s^2$). This force unit in the Système International (SI) aptly is called Newton, N. Trying to grasp what this unit represents physically, we may think of the weight of one fairly large apple, linking the unit to the legend of Isaac Newton having an apple fall on to his head. Here we acknowledge the most common force of all, the force of gravity, also known as weight, which is the force that pulls all material bodies in the direction of the center of the earth. For this force we can write

$$W = mg$$

where W = the force of gravity acting on a body having a mass of m, with the acceleration in this case being the gravitational constant $g = 9.81m/s^2$ ($32.2ft/s^2$) applying in the context of the gravity of the earth. According to Imperial or American standards, the force unit ($slug\text{-}ft/s^2$) is called pound, abbreviated lb.[4] One pound is approximately 4.45N. Since 1N is a fairly small force, it is convenient to also operate with 1000N as a unit for force; this unit is called kiloNewton (kN). Parallel to this we find in the Anglo-American tradition the force unit called kip, which is the same as 1000lb.

While in the equation for Newton's second law acceleration is a familiar concept, *mass* tends to be more evasive. Unlike weight, the *mass* of 1kg (or 1 slug) of steel is the same in all gravitational systems. The mass is a constant throughout the universe, whether we measure mass on earth or on the moon. The *weight* of this mass, however, will vary according to the "strength" of the gravitational field which, in the case of the moon, is about one-sixth of the value for the earth. We may think of mass, then, as a measure of the quantity of matter.

All of these various concepts, quantities, terms, and the relationships among them are depicted in Figure 4.3.

There is also a third "law" attributed to Newton. This one introduces us to the idea of forces having directions, as well as to the most fundamental observation of equilibrium: if a body is at rest on a horizontal plane, it quite certainly exerts a pushing force on the surface of that plane, the force being the gravity force. We have learnt from the second law, however, that if there is a net force acting, the body will accelerate, in this case downward. But this is contrary to our observation of the body being at rest on the plane, so there must in fact be another force acting on the body which maintains equilibrium.[5] That force has to be of the same magnitude as the gravity force for the resultant force to be zero, and also to act in exactly the opposite direction. (Fig. 4.4.) Newton called this force a reaction force and stated that all forces have reaction forces which are of the same magnitude as the "action" forces but are oriented in the opposite direction. Or, in the Latin words of Newton's time: "actio = reactio."

Having introduced the basic concepts of statics, we will in Sections 4.4 and 4.5 look more closely into the ways we represent and analyze forces theoretically; we will also study the conditions for equilibrium. First, however, we will reflect a bit more on the concept of weight, this time from an architectural perspective.

4.3 Pyramidal Contrasts – Weight vs. Lightness

Mass is of particular importance for the structures of architecture. As we have just seen, mass is associated with weight, and a tendency today is to free architecture from as much weight as possible, with the objective of lighter, more delicate structures. This has not always been the case, however; mass has also been seen as a particular quality of value expressing monumentality, endurance, and power.

In the last period of the European Stone Age, mighty, heavy monuments were raised in the Mediterranean area and its surrounding continents. We meet them at Stonehenge in England, in circular forms in Bretagne, France, and farther north in Europe as well in the form of huge stone blocks forming chamber tombs. However, this common urge toward erecting stone massifs saw its fulfillment in the architecture and arts of ancient Egypt where hundreds of

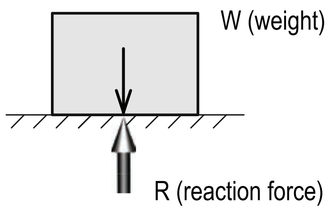


Figure 4.4
The weight W of the body held in equilibrium by a reaction force R from the surface that the body is at rest on.

Illustration 4.7
"Levitated Mass" (2012).
Motion of 340-ton boulder is arrested by steel brackets attached to the concrete side walls of a trench through which visitors may walk.
Sculpture at Los Angeles County Museum of Art (LACMA) by Michael Heizer.

generations of slave labor produced the crystalline expressions of mass and weight reflective of that society's hierarchy and enduring stability: the pyramids.

The Cheops Pyramid

In the group of royal tombs dating from the Fourth Dynasty (2723–2563 bc) and located at Giza in the vicinity of Cairo, Egyptian architecture found its most refined and impressive realization. The original experience of powerful and durable masses of stone figures had been developed and symbolized in terms of absolute and determining stereometric relationships to each other. Father of all later historians, the Greek Herodotus, traveling about the ancient world and taking notes, visited the pyramids around 500 bc. The Cheops Pyramid, at that time already in place for 2000 years, is the largest and oldest of pyramids; it is oriented exactly according to its celestial latitude and longitude, while its square plan measures 230 by 230m (754 by 754ft) and it rises to an impressive height of 147m (482ft).

According to Herodotus, Pharaoh Cheops ordered "all Egyptians" to work for him. They numbered 100 000 at a time, all toiling continuously for three months each year for 20 years. Some were ordered to the stone quarries in the Arabic mountains, while others dragged the stones by ropes on wooden sleighs up to the building site after they had been carried down the Nile on boats. The workers' tools were simple chisels and picks made of copper that enabled the piling up of some 2.5 million blocks of stone weighing on

Illustration 4.8
Cheops Pyramid, Giza, Egypt (third millennium BC).

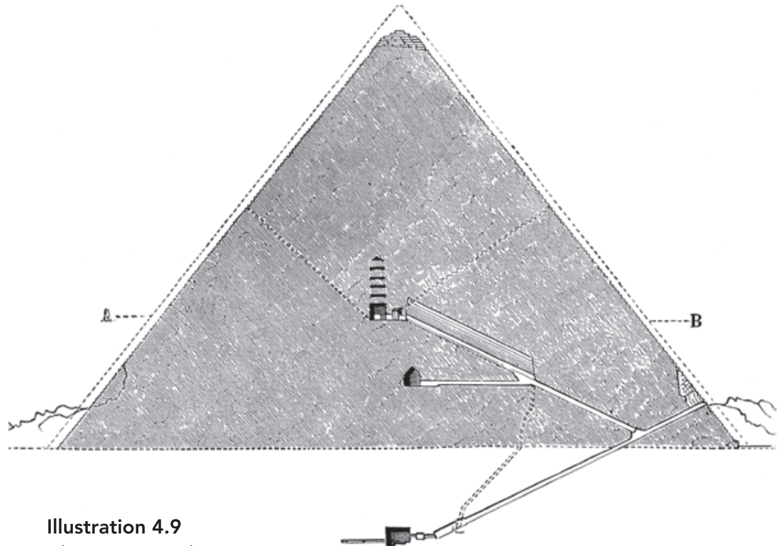


Illustration 4.9
Cheops Pyramid.
Section drawing, depicting narrow passages and small burial chambers within mass of built-up stone blocks.
Drawing courtesy of the Florida Center for Instructional Technology.

average 2.5 metric tonnes. (Ill. 4.8.) The whole massive volume of the pyramid was originally covered with precisely polished Tura limestone that reflected the rays of the sun; ornamentation and detailing were omitted on the outside so as to strengthen the effect of the basic form and its smooth surfaces. The pyramids, representing the Egyptian cosmos, were made to last for eternity. The observant visitor at Giza today notices that the old Egyptians were careful in choosing the sites for their vast monuments: the pyramids are built to last, situated as they are on solid rock.

Within these solid masses of stone, narrow passages led to three burial chambers conceived of as small cells. (Ill. 4.9.) Above the voids of these internal pathways, mighty tilted stone slabs lean in against each other, forming a pitched roof in section and leading the tremendous weight of the stone mass above on to the long sidewalls of the cell.

Contrasting with this ancient quest for weight and solidity, our era has seen a search for minimal structures and material economy, an ambition of "zero weight and infinite span" in the words of the French engineer Robert le Ricolais (1894–1977). In this spirit of lightness and transparency, supported by advanced computer technology and a refined building process industry, a large glass pyramid was built in Paris in the late twentieth century.

La Pyramide du Grand Louvre

Contrary to the heavy mass at Cheops, another famous pyramid exists at one of the largest museums in the world, the Louvre; this pyramid, however, is a relatively lightweight structure made of glass and thin stainless steel bars and rods, designed by the architect I.M. Pei. In addition to being the main entrance and a source of natural light to the museum's spacious underground vestibule, this large pyramid is situated along the great Parisian axis of Le Louvre–L'Étoile–La Défence; clearly strategic pyramid positioning was not something restricted to the Egyptians. With a height of 21.5m (70ft) and a side length of 35m (115ft), the glass pyramid is placed like a finely cut diamond in the Cour Napoléon, surrounded by the Louvre's eighteenth-century stone buildings. (Ill. 4.10).

Let us take a closer look at what it takes to make a pyramid with 612 rhombus-shaped glass panels. Each triangular side of the pyramid is supported by two sets of 16 intersecting, inclined trusses of different lengths, the top edges of which lie flush with the pyramid's surface. These trusses' compression members, primarily their top chord and the perpendicular struts, are built of hollow, circular-shape members, while the tension members at the bottom chord of the trusses and its diagonals are solid steel rods or cables. The glass panels are fastened at the intersecting points of the top chords of the trusses by extension bolts, allowing their weight to

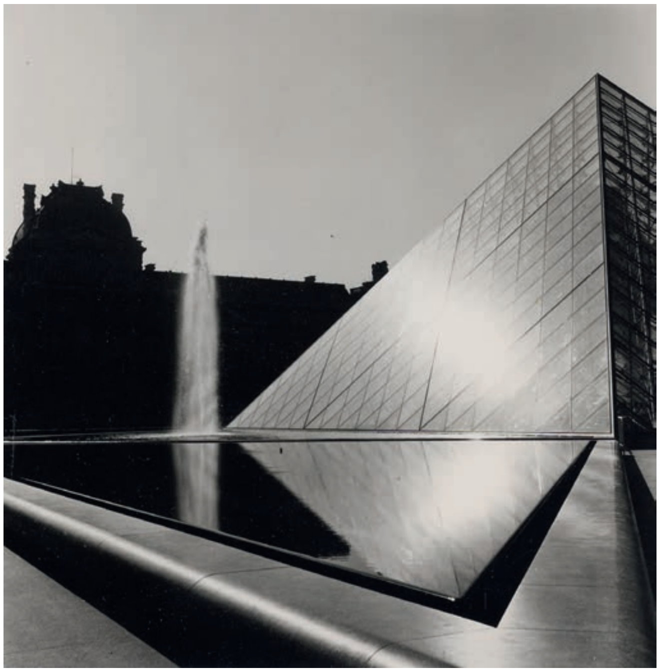


Illustration 4.10
La Pyramide du Grand Louvre, Paris, France (1989).
Architect: I.M. Pei. Structural engineers: Rice Francis Ritchie (RFR) and Nicolet Chartrand Knoll Ltd.

Illustration 4.11
La Pyramide du Grand Louvre.
Interior, with very evident structural forms and linear elements.

be carried but keeping the glass free from direct contact with the main load-bearing system. (Ill. 4.11.)

Aside from carrying their own self weight and the dead loads of the glass, these trusses also resist the inward wind pressures applied to the surface of the pyramid. The negative surface pressures, or suction, which can also result from wind action (see Section 3.7) is countered by yet another set of tension rods/cables with opposing curvature that are also connected to the joints of the truss network.

According to this description, the structure of the Pyramide can be seen to be very much in keeping with the French tradition that starts with Polonceau's achievements in the nineteenth century of incorporating subtle variations between compression and tension elements in steel structures and that we saw earlier in this chapter in Section 4.1. Another important aspect of this pyramid is hinted at by its cast stainless steel joints. (Ill. 4.12.) We recognize in these the turnbuckle and clevis (or shackle) that we find in the rigging of sailboats and yachts, suggesting that what we have in this pyramid is actually a minimal structure that is at least partly designed to withstand large tension forces, in a manner similar to the way that the rigging of sailboats holds the mast. Thus, with the help of outstanding "seamanship" and the successful rigging and stiffening of all the joints under the precise glass surfaces of this remarkable structure, a contemporary pyramid was made possible.

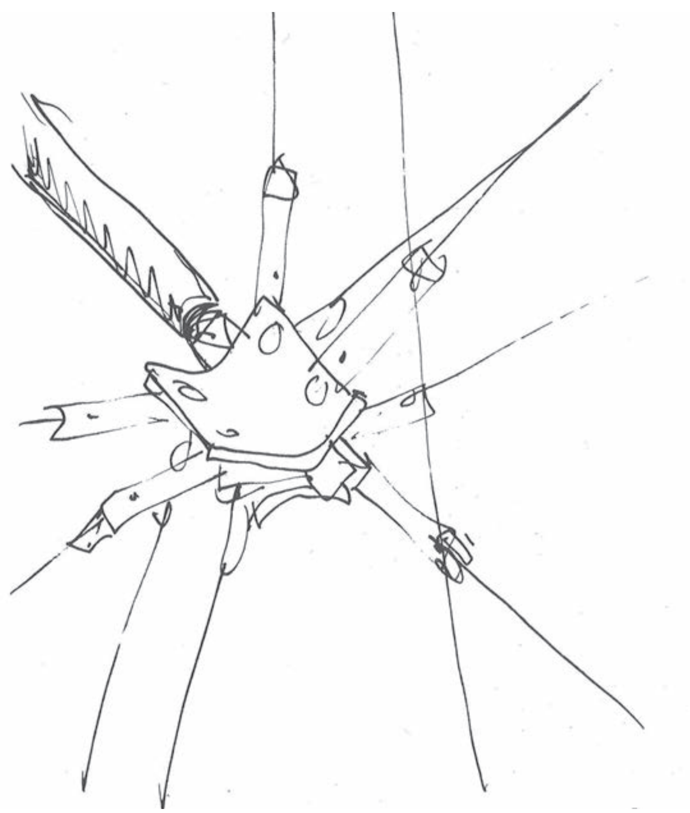


Illustration 4.12
La Pyramide du Grand Louvre.
Sketch of connection detail for intersecting tension elements.

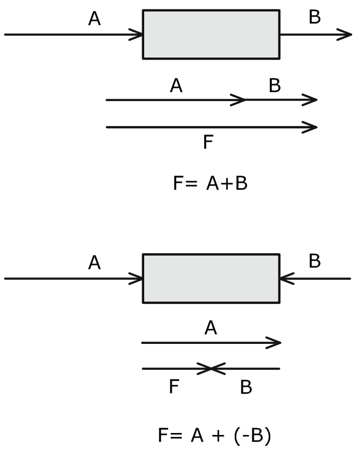


Figure 4.5
The sum of two force vectors acting along the same line of action. When acting in the same direction the resultant force will be the sum of the magnitude of the forces. If the forces act in opposite directions, then one should be subtracted from the other.

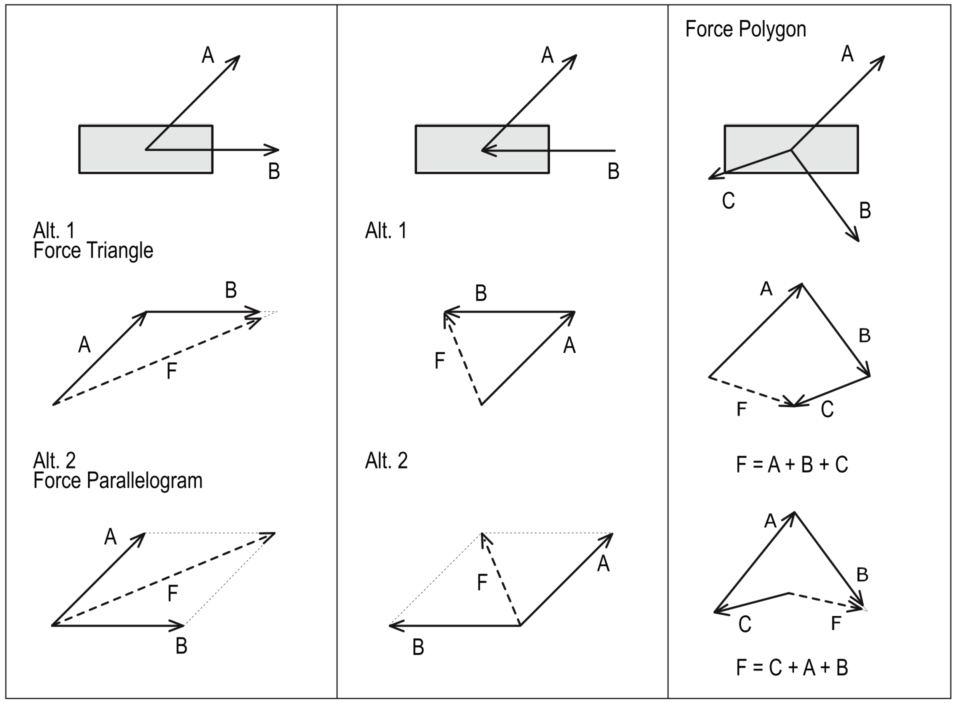


Figure 4.6
The sum of force vectors acting in different, but intersecting directions.

4.4 Forces and Moments – Concepts to Explain Movement and Deformation

Mass is a quantity that is exactly defined by a number establishing its magnitude, as are length and time. These are *scalars*. Forces, however, cannot be precisely defined without stating both their magnitude and their direction. The same is true for velocities and accelerations. This latter group of physical phenomena that are also defined by directions are called *vectors*. Graphically, we usually let arrows represent vectors, where the length of the arrow sometimes stands for its magnitude and the direction of the arrow indicates the direction of the vector.[6]

If two or more forces act along the same line, as they do in the well-known tug-of-war game, we say that they have the same *line of action*. The combined result of such forces acting on a body can be found by simple arithmetic. When acting in the same direction the resultant force will be the sum of the two forces. If they are acting in opposite directions, then one should be subtracted from the other. Graphically, the resultant force vector is found by setting the beginning of one vector after the end of another, observing their magnitude and direction. (Fig. 4.5.) The specific succession of the vectors is unimportant, and their sum – the resultant force or net force – is the vector force which may be drawn from the tail of the first vector to the tip of the last vector in the sequence. This is the principle of vector addition along a straight line.

If, on the other hand, force vectors are combined which have different directions while their lines of action are passing through the same point (i.e., the vectors are intersecting), the resultant force vector can always be found graphically. (Fig. 4.6.) We set one vector after the other in such a way that the resultant force vector completes a figure in the form of a triangle (in the case of two forces) or another polygon (in the case of more than two forces). The resultant force is, as before, the vector running from the tail of the first to the tip of the last force.

Such insight about summing the effects of force vectors is not new: as far as we know, the graphical method for finding the sum of forces was first used by Leonardo da Vinci (1452–1519) (e.g., Ill. 4.13), but Stevin from Brügge (1548–1620) was the first to publicize the method formally in 1586.

Whether for intersecting forces or for forces acting along the same line, we can therefore find a resultant force, which is the force having the same effect on the body as do the sum of all the separate forces acting on it simultaneously. We may say that the resultant force is *statically equivalent* to the system of forces it is derived from. As an example to help visualize this conclusion as well as the graphical abstractions of Figure 4.6, one can consider the situation of the many intersecting tow-lines and evident resultant movement of the Condeep oil platform shown in Ill. 4.14.

At this point, we can consider all this from another perspective. Since we have established that we can combine two or more forces into one resultant or net force having the same effect as all the others, we may also do just the opposite: i.e., it is possible to resolve a force into its component forces in such a way that their sum is statically equivalent to the original force. What we do in

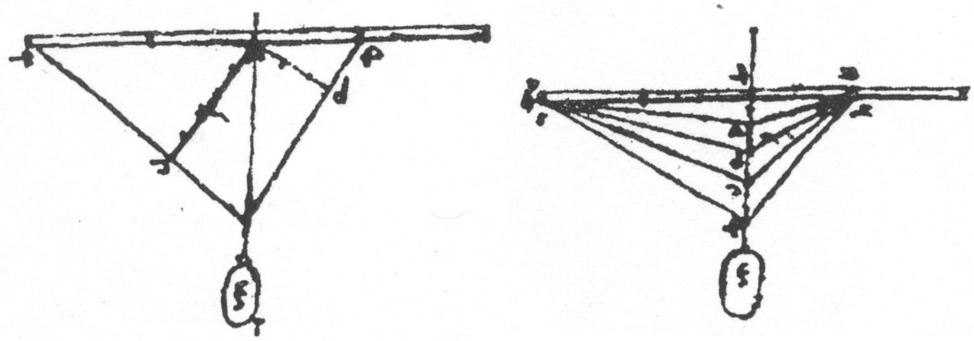


Illustration 4.13
Leonardo da Vinci's sketch demonstrating his early insight into vector analysis of forces.

Illustration 4.14
Pulling of the Condeep oil platform in the North Sea (1987).
The tugboats' lines of forces will sum up to help establish the
speed and direction of the platform's movement through the water.

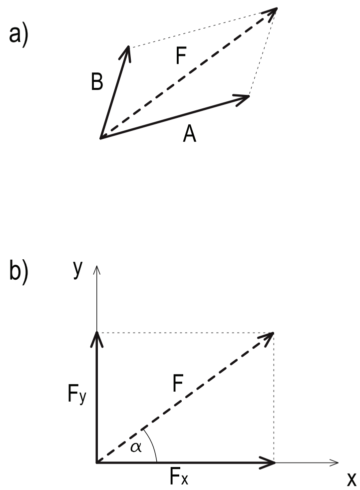


Figure 4.7
(a) Arbitrary resolution of force F into components A and B by help of a force parallelogram. (b) Resolution of force F into horizontal and vertical component forces F_x and F_y.

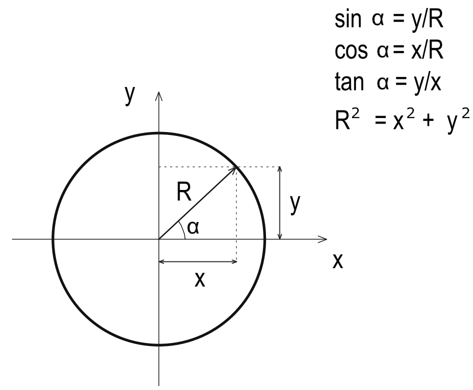


Figure 4.8
Definition of the trigonometric relationships of sine, cosine, and tangent.

this case is called *resolving a force into components*. Forces may be resolved in countless ways as long as the principle of closed triangles or polygons is observed. The force F in Figure 4.7a has the components A and B along with innumerable other possibilities and combinations. The number of component forces we might wish to resolve a singular force into, and the directions we choose for this new group of forces, will depend on the geometry of the case in question and what function we would want the component forces to accomplish. Often, the resolution of a force is made by splitting it into horizontal and vertical components, which is the same as asking for the effect of the force in the horizontal and vertical directions. This is a common way of organizing forces for the sake of clarity and ease of calculation. Besides, in practical instances of force resolution, it is actually the case that most beams and floors are horizontal and the majority of columns or walls are vertical.

Considering the force F in a Cartesian coordinate system, we may resolve the force into components aligned with the x- and y-axes.[7] Those components are commonly called forces F_x and F_y. (Fig. 4.7b.) And while a graphical approach to the study of force vectors has been convenient for conveying the basic principles until this point, actual practice and computer programs favor the use of simple mathematics for calculating vector sums. By using trigonometry, therefore, we can determine the horizontal and vertical components of force F to be

$$F_x = F \cos\alpha$$
$$F_y = F \sin\alpha$$

where α = the angle between force F and the x-axis. In the context of Cartesian coordinates, it is also possible to keep track of the directions of the forces and force components, as positive forces can be taken as those which point in the positive directions of the x- and y-axes while negative forces point in the opposite direction. In Section 4.7 and throughout the rest of this book we will see the great advantage of resolving forces into components parallel to the horizontal and vertical axes, especially when many forces are involved and we need to know their combined effects.

Before proceeding with that, however, there is still one common and critical effect of forces that we have not yet looked into and to which we will now turn our attention: i.e., when net forces act on a body they not only tend to cause it to displace along a straight line but also to rotate. In this case, both the magnitude of the force as well as its distance from the point (or axis) about which the rotation takes place are relevant parameters that help establish the action. We thus need to define a new concept called the *moment* of a force, M, in which both the force and a distance are involved; i.e.,

moment = force × distance

Since moments M are products of forces and distances, they are commonly defined in units of (kilo)Newton meters, kNm or Nm, respectively. Similarly, Imperial or American Standard Units give moments in pound-inches (lb-in), pound-feet (lb-ft), or kip-feet (kip-ft). The distance is called the *moment arm*.

When one thinks about it, we are not even able to do a simple thing like opening a door without experiencing the effect of moments. (Fig. 4.9.) We push or pull the door open by exerting a force on the door surface. When pushing open the door, we instinctively apply a force at the longest possible distance to the vertically hinged axis about which the door swings; this will ensure that the moment is pleasantly and usefully large. If we try to open the door by pushing it closer to the hinges, we find that we need to apply more force in order to do so.

The *lever principle* perfectly illustrates the effect of moments.[8] To lift a heavy boulder vertically up out of the ground, for example, we may use a stiff bar acting as a lever with one end under the boulder and then we apply a downward force at the other end. (Fig. 4.10.) The lever will rotate about a strategically placed smaller stone on the ground (a fulcrum) and produce a pushing force acting on the heavy boulder from below which hopefully will be sufficient to lift it up. The whole idea of the level principle, though, is that in

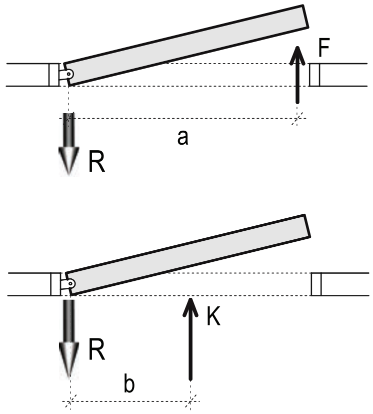


Figure 4.9
Moments at work in the simple operation of pushing open a door. The moment arm is always the perpendicular distance between the line of action of the force and the point or axis about which rotation may take place. If the magnitude of the moment (taken about the hinge) necessary to open the door is M = Fa = Kb, then since b < a, producing this moment requires that K > F.

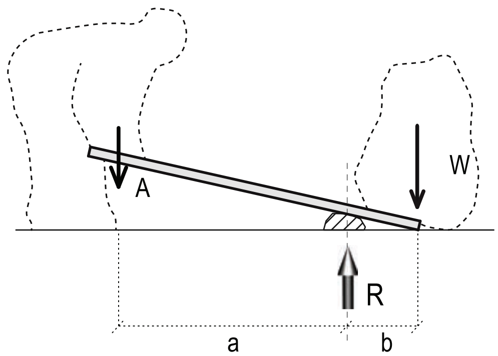


Figure 4.10
The lever principle is illustrated. Because of the differences between the lengths of the moment arms a and b, a being much longer, the magnitude of the necessary applied force A will be significantly smaller than the weight W to be lifted. The forces involved are proportionally related as the inverse ratio of the corresponding lengths.

order to do this we actually only need to apply a moderate amount of downward force at our end because we do this at a considerable distance from the point of rotation of the bar. This distance is called the *lever arm*. If the force pushing on the boulder from below is greater than its weight, rotation of the lever will take place and the boulder will be lifted. This will happen if the moment of the applied force is greater than the moment of the weight of the boulder that tries to prevent rotation. If the magnitude of the applied force at the very instant when rotation about the fulcrum occurs is designated as A, and the weight of the stone is W, then

$$Aa = Wb$$
$$A = Wb/a$$

where a = the lever arm, which is the perpendicular distance between the line of action of the applied force and the fulcrum axis about which the rotation takes place, and b = the corresponding distance from the line of action of the stone's weight. If "a" is four times the distance "b", for example, then A = W/4 which means that applying a force of a little more than a quarter of the weight of the stone is all that is necessary to lift it. Beyond this specific example, we can say that moments that are created by applying forces at a distance from a point of rotation multiplies the effect of these forces, whether for good (as for the lifting of this boulder) or for bad (as when considering the effect of lateral wind forces acting on a building at certain heights above ground; e.g., for the common situation shown in Fig. 3.11.)

There is another thing to note and extrapolate from in the preceding lever example: not only do the forces cause the bar to experience in-plane rotation about the so-called fulcrum point, but in addition they produce moments that will result in the flexing or bending of the bar, since this is made out of real-life materials and is not infinitely stiff. This is in essence the effect of transverse loads acting on structural elements that was generally introduced in Chapter 2 (Section 2.4); more explicitly, transverse forces produce what are very aptly called *bending moments* in beams – a topic that will be much further discussed and elaborated upon in Chapter 7. Analogous to our observation that if the sum of forces is zero then structures will not move, but will deform/deflect instead, in the case of a zero moment sum structures will not rotate, but will bend instead.

Finally, in this very introductory discussion about forces and moments, we should also be aware that we have so far only discussed moments that tend to rotate a structural element as a rigid body in 2-D planar space. While this is generally a perfectly adequate simplification for dealing with the analysis and design of most structural elements of real-life buildings (more about this shortly), we should at this stage point out yet another potential effect of forces that are acting at a certain distance from elements but in 3-D space: i.e., force-produced moments may in fact cause structural elements to not only bend but also to *twist* about the axis along their length. Consider a locked door handle, for example. (Ill. 4.15.) By applying a downward force to the one-piece handle, the part of it that is

Illustration 4.15
Forces and moments on a door handle: when the lock is set, a downward push on the one-piece handle will cause a twisting deformation of the part of it that is protruding at right angles from the door, whereas the part that we grip will tend to bend.

Illustration 4.16
"Locus of Lines."
Visualizing balance and movement in dynamic equilibrium. After a period of oil painting, the Japanese artist Sūsūmū Shingu developed an interest in the third dimension and started making objects that moved in the wind. His works become one with the natural energy of water and wind and seem to breathe with a life of their own.

Sculpture by Sūsūmū Shingu. Model in painted aluminum by architecture students at AHO; Kristin C. Braut, Karen Sletvold, and Emelie Tornberg.

sticking out perpendicularly from the door surface tends to be twisted about its axis; i.e., the force magnitude multiplied by its distance to the axis about which the handle twists produces a moment, but now which is called a *torsional moment*. The part of the handle that we grip, however, will be subject to the more usual bending deformations and corresponding bending moments.

4.5 Equilibrium – A Fundamental Structural Requisite

As we next consider the effects of these forces and moments that we have just described, it is helpful to visualize and specify just what it is that these actions will tend to do to the objects that they act upon: i.e., to displace and rotate in space. The "Locus of Lines" constructed art piece, for example (Ill. 4.16), is delicately crafted and articulated so as to be able to move quite freely with the wind, but always in such a way that it also remains balanced with respect to the gravity loads acting on it. One can clearly envision in this case the various downward force vectors from the weights of the individual rods and the sideways force vectors from the wind all acting at the same time – and, moreover, that the lines of action of these forces will be at certain distances from various axes of rotation of the sculpture, thereby producing moments. Not only are the anticipated displacements and rotations of the different parts of this artwork clear to imagine, but it is also obvious that this is a highly dynamic situation, i.e., one that changes easily with time. Even from a loose, layperson's understanding of the concept of equilibrium, then, it should be clear why one would refer to this as an example of *dynamic equilibrium*. In a similar manner, Alexander Calder's "mobiles" also come to mind, and these can then be contrasted with the equilibrium conditions of his "stabile" sculptures. (Ill. 4.17, 4.18.)

Or, as another way to broach this broad topic of equilibrium, we might also choose to contemplate the words of the French author André Gide (1869–1951):

> *This state of equilibrium is only attractive when we walk*
> *a tightrope;*
> *sitting on the ground there is nothing marvelous about it.*[9]

But, of course, "sitting on the ground" is just what the buildings of architecture do – and yet, as we shall see, that doesn't make equilibrium any less marvelous!

In order to begin to develop a more mathematical/scientific understanding of equilibrium, we need to go back to statements of first principles. For obvious reasons, neither large-scale translational movements nor rotations are acceptable in a building structure, unlike in the "Locus of Lines" or "Little Janey Waney" sculptures. Also, forces and moments resulting from loads acting in one part of the system must be balanced by forces and moments acting elsewhere so that structural elements or systems are always kept at rest. We need to develop the conditions, then, that ensure these statements are true by making sure that forces and moments really are in equilibrium.

Recalling from the previous section our consideration of *forces* having the same line of action, we stated that their force resultant is

Illustration 4.17
"Little Janey Waney" (1976), Louisiana Museum of Modern Art, Humlebaek, Denmark.
A so-called "mobile" sculpture by Alexander Calder (1898–1976).

Illustration 4.18
"Big Sail" (1966), MIT Campus, Cambridge, MA, USA.
A so-called "stabile" sculpture by Alexander Calder.

the sum of the magnitudes of the forces, observing their positive or negative directions. If two or more such forces acting on a body are to be kept in equilibrium, there has to be a force or forces present having a magnitude which is the same as the resultant force, but acting in the opposite direction. (Fig. 4.11.) In this way the total sum of forces will be equal to zero, which is one requisite for maintaining equilibrium. We may call such a condition *translational equilibrium*; this can be described as the necessary condition to prevent a body from starting to move along the line of action of the forces. Since forces are balanced a body is compressed or stretched rather than set into motion.

We have also looked at bodies subjected to intersecting force components A and B whose combined resultant is force F. (Fig. 4.12.) Unless there is another force acting on it, the body will accelerate in the direction of the force F according to Newton's first law. But since we cannot allow structural parts to move, we must be certain that the resultant force is met by an equally large but directly opposite force, R, in order to maintain equilibrium. We have shown that the resultant force F makes up a triangle with A and B as the other two sides, where the direction of force F is

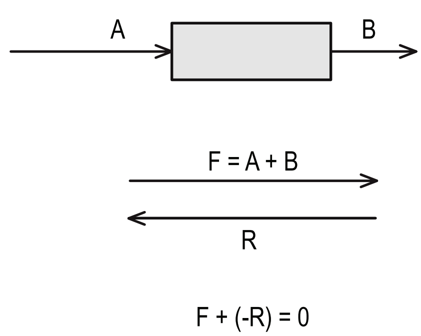


Figure 4.11
Translational equilibrium of forces acting along the same line. To be at rest, there must be a force R present which is equal to the sum of A and B (A + B = F), but acting in the opposite direction.

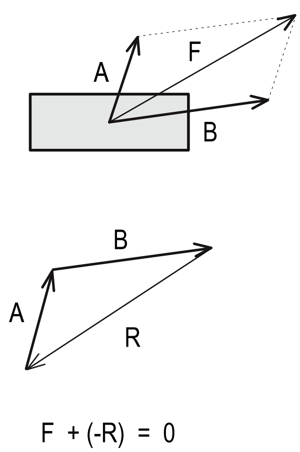


Figure 4.12
The body acted on by forces A and B, which together produce the resultant force F, but that can be held in equilibrium by a balancing force R.

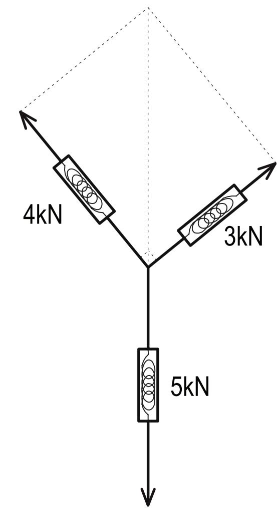


Figure 4.13
Demonstration of the equilibrium of forces using springs.

from the tail of one component to the tip of the other. The force R which is to hold the vector sum of A and B at rest, however, is the "reactio" of F and is directed oppositely to it. To have equilibrium between the three forces (A, B, and R), therefore, their vectors should make up a closed triangle where all forces are linked from tail to tip. Likewise, a system of more than two forces is in equilibrium if all force vectors comprise a polygon where the "last" force ends up tip to tail with the "first" force. This is the graphical depiction of forces whose net result is zero, written

$$\Sigma (A + B + R) = 0$$

where the Greek letter Σ (capital sigma) means "the sum of." When all three forces are present, *translational equilibrium* is maintained. This means that no net resulting force is acting and that the structural element or system is at rest. We will return to considering this condition of equilibrium later in this section.

Considering now the equilibrium of *moments*, we can observe through an example that in order to prevent rotation, moments acting on a structure must necessarily cancel one another out. The Buvette de la Source Cachat (Ill. 4.19) is primarily known because Jean Prouvé (1901–1984) was involved in its design and construction.[10] The structure for this particular building serves as a convenient example for illustrating the principle of equilibrium of both moments and forces. A series of somewhat Y-shaped steel structural elements support the roof at the tip of its two "arms" that project outward from a central column or pillar. Such "arms" or beams that have support at only one end (their other end is free of support) are called cantilevers (see Chapter 7). Since the two cantilevers in this case support different portions of the roof area and, therefore, different amounts of vertical load, there is a real risk that the Y-shaped steel structure would overturn by rotating about the point where it meets the ground. As can be seen, the column tapers in width to almost a point at its base, which would enable such rotation to occur easily. To prevent this, the tip of one of the projecting beams (the one shown on the right side in Fig. 4.14) is tied down to a foundation in the ground by means of a vertical steel rod. The whole structure is kept in equilibrium because the moment produced by the roof load on the tip of the (left-side) cantilever is equally as large as the moment resulting from the tension force in the rod.[11] Both moments are considered to act about the potential point of rotation at the base of the Y-shaped column.

When doing a mathematical summing of moments we must take care to observe their potential direction of rotation. Moments either *tend* to rotate a body clockwise or counterclockwise, and we may, as a *sign convention, define moments acting clockwise as positive and moments acting counterclockwise as negative*. In the diagram of the Buvette structure shown in Fig. 4.14, then, the moment of the tension force T about the column base is taken to be positive and the moment of the roof load P is negative. Formally, we seek the magnitude of these moments whose sum is zero in order to have rotational equilibrium. In mathematical terms this means that

$$+Te - Pa = 0$$

where e and a are the respective moment arms. This is the condition for maintaining equilibrium between the two moments, both considered to act about the point of support of the structure. We call this a *moment equilibrium equation*, and have, therefore, that

$$Te = Pa$$

which means that a state of equilibrium is found if the two moments produced are equal in magnitude to each other but have opposite directions. We can state as a *general rule that the sum of moments*

Illustration 4.19
Buvette de la Source Cachat, Évian, France (1956).
A line of distinctive Y-shaped steel columns supports the roof; to give a sense of scale to this image, the distance between the support structures in the longitudinal direction of the building is 6m (20ft).
Architect: Maurice Novarina. Structural engineer: Serge Ketoff. Designer and craftsman: Jean Prouvé.

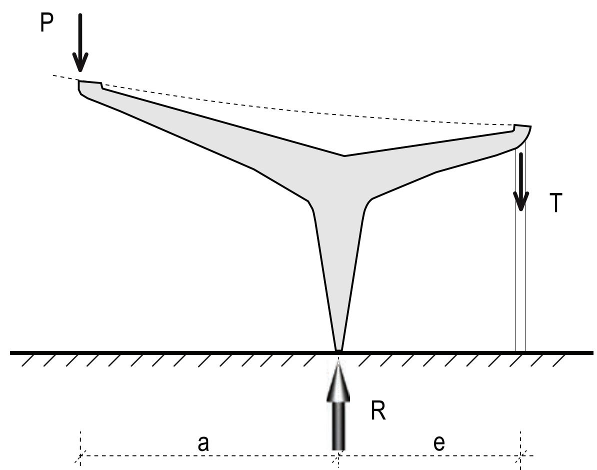


Figure 4.14
Buvette de la Source Cachat.
Sketch of Y-shaped steel structural component. Positive moments act clockwise (+Te), balancing negative counterclockwise (–Pa).

taken about any point in a structure must be equal to zero if the structure is to be at rest; i.e., if it is not to rotate. Mathematically, we write this as

$$\Sigma M_z = 0$$

where M is the sum of all the moments acting on a structure about some arbitrarily chosen point "z" in its 2-D plane. (Technically, it actually doesn't matter whether this point is within the structure or not, although it is often convenient to consider it to be.) So, in addition to the general need for equilibrium of forces that we described before, another requirement for maintaining overall equilibrium of a structure is that the sum of positive and negative moments acting on it should always nullify one another. This is called *rotational equilibrium*.

It should be pointed out that in this example we have been considering forces that are not acting along the same line nor intersecting, as we had been doing until now, but rather that are parallel to each other. Nevertheless, these forces' combined effect on a structure must sum up to zero in every way in order to have equilibrium; i.e., not only rotational equilibrium but also translational. For the Buvette structure just considered, we have seen that summing moments can help us guarantee rotational equilibrium, but this will not inform us about any tendency of the structure to move up or down in a vertical direction. To prevent this, we must introduce the mathematical condition that the sum of forces present is always equal to zero. Referring again to Fig. 4.14, we may write this as

$$+R - P - T = 0$$

where R is taken as a positive force acting upward and P and T as negative forces acting downward. R is in fact a reaction force supporting the combined forces of P and T, and for equilibrium this necessarily has to act upward. Solving the equation will give us

$$R = P + T$$

Not surprisingly, R will be of the same magnitude as the sum of P and T, but is acting in the opposite direction. We may, therefore, state as a *general rule that the sum of all forces in a structure must be equal to zero if the structure is to be in translational equilibrium.* Mathematically, we write this as

$$\Sigma F = 0$$

where F represents *all* forces acting. In the case of the Buvette, the forces we have been looking at all act vertically and we may indicate this by adding the subscript "y" to the group of forces, such that F_y refers to vertical forces oriented in a Cartesian coordinate system. Likewise, any forces that may act horizontally would be given the subscript "x". If we have resolved forces into components acting vertically and components acting horizontally, we can write the requirement for translational equilibrium in terms of these two directions. Thus, if in any planar system of forces the three requirements for equilibrium applying to moments and to forces acting in the two directions of x and y are observed, then the structure is in both rotational and translational equilibrium. The three corresponding equilibrium equations are, then

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_z = 0$$

which we will return to and make use of over and over again throughout the rest of this book. Equilibrium is indeed a fundamental and powerful concept.

4.6 Intermezzo Italiano

A couple of examples in the central Italian city of Gubbio can help to shed some light on Newton's laws. Along the street leading to Piazza della Signoria stands a masonry dwelling of five stories. Over the course of many years, the façade of this building has had the tendency to bulge farther and farther out in certain places, to the point that it was necessary to take precautionary measures to prevent it from tumbling into the street.

The solution was a set of vertical, two-story-high bracing systems mounted to the façade with bolts. (Ill. 4.20.) A vertical piece of timber, attached to secure elements in the building structure at its top and bottom, is connected at its middle to a horizontal compression member projecting outward and that is secured at its tip by means of angled tension rods, recalling the basic elements of the Polonceau system we have seen earlier in this chapter in Section 4.1, but here oriented vertically. With the help of turnbuckles in the rods in order to keep them tightly stretched, this bracing system can keep the façade in check and control the forces causing its outward deformation. The situation is now stable, having achieved equilibrium between the forces pressing the façade outward and those of the support system – all according to Newton's first law, which states that for a system of forces to be at rest, no net resulting forces can act on it. Another result of these emergency arrangements, as we may call them, is that they happen to have enriched the visual character of the streetscape.

In the same street, a bit closer to the piazza, an antique store offers crossbows for sale. The tension system of this weapon presents a situation similar to that of the support system bolted to the bulging wall façade. When the crossbow is cocked, the system is in equilibrium in accordance with Newton's first law. But the moment we pull the trigger and fire, the tension is unleashed and the arrow flies, fulfilling Newton's second law. (Ill. 4.21.)

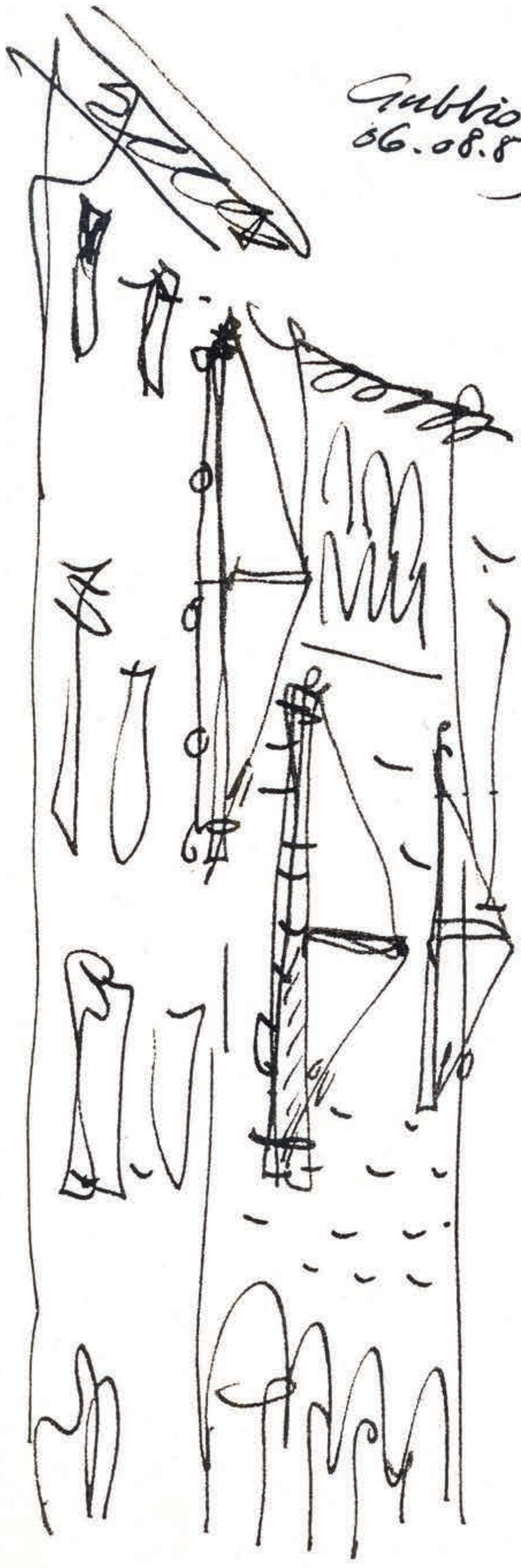


Illustration 4.20
Tension and compression façade-stabilizing bracing assemblies in a street in Gubbio, Italy.

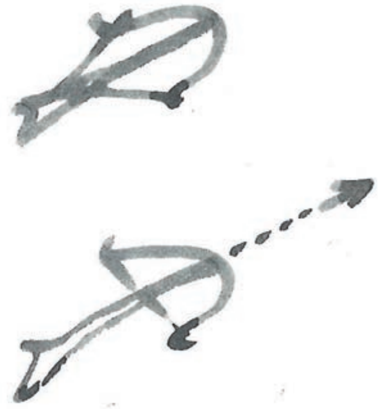


Illustration 4.21
A crossbow: when cocked, the components are in equilibrium and at rest; when the trigger is pulled, stored energy is released and the arrow is put into motion.

4.7 Support Conditions and Reactions

We have examined the requirements for equilibrium of forces and moments in Section 4.5 and established that for a planar structure there are three equations that mathematically express its essential state of rest; i.e., that there will be no overall displacements and no rotation of the structure. Until this point, though, we have arrived at these equilibrium conditions by studying only a few carefully selected examples with given force vector magnitudes and directions and that, especially, were strategically isolated from having to deal with certain aspects of built reality. In particular, there was no explicit consideration of the effect on equilibrium of a structure's support conditions – where one structural element is connected to another or where one meets the ground – and how such support conditions may in fact help to ensure equilibrium. And so it is to these issues that we now turn our attention.

In order to develop an understanding of the importance of structural support conditions in relation to equilibrium, and vice versa, we will start by considering the situation of the very basic case of a horizontal beam spanning between two supports with a downward point load P acting on it at mid-span. (Fig. 4.15.) An obvious question in this situation quickly comes to mind: What forces must be generated at the end supports of such a beam in order for it to be kept in equilibrium?

To answer this question, we will first need to understand and describe in some detail the physical behavior of the beam in response to the loading as well as in relation to the specifics of the support conditions. With respect to the latter in particular, this actually

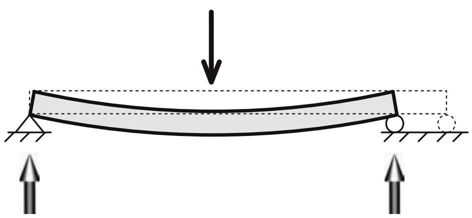


Figure 4.15
When loads act on a beam it will bend. (The amount of bending, as depicted here, is greatly exaggerated.) The new curved line takes a longer path between the supports, and to accommodate this without stretching the beam, the supports will need to move relative to one another, thus making the span slightly shorter. Hence, one of the supports (it doesn't matter which) should be a pinned connection while the other is of the roller type.

means establishing whether or not the connections allow for some rotation and/or for some translation of the beam ends relative to the supporting structures. In this case, we will set the connections to be such that the beam is restricted from horizontal movement at the left support, but is allowed a slight sideways movement at the right one. At both supports, however, vertical movements are prevented. Also, we will imagine the connections are of a type that allows both left and right beam ends to be able to rotate slightly. We call the type of support/connection at the left end a *pinned connection*, and that at the right end is called a *roller connection*.

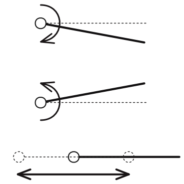
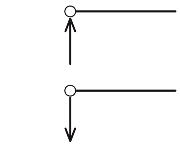
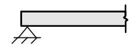
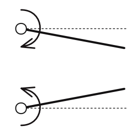
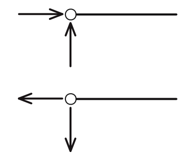
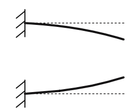
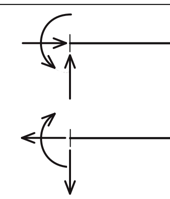
The logic behind these support conditions is as follows. When the load acts on the beam it will surely deflect downward, however slightly. By becoming curved, it will need to contract laterally; i.e., the actual distance between the supports would have to become smaller. To accommodate this, one of the supports needs to be able to move horizontally in order to prevent elongation of the beam itself. In the end, it does not matter which of the two ends is allowed to be pulled inward; it is the relative movement between the two ends that matters. It would not be wise, however, to allow both supports to move, because then the overall horizontal stability of the beam would be lost. In actual building practice where the span is modest, we will commonly observe that the two support connections are similar and it is not possible to identify which is the pinned connection and which is of the roller type. With a large horizontal movement of the beam thus prevented, the idea is that the theoretical shortening of the beam to accommodate its small vertical deflection can nonetheless take place within the imprecision of the connections themselves, thus eliminating the need for an explicit roller connection. When the span is larger, however, a true roller connection is more likely to be necessary.

Figure 4.16 shows the symbols used to graphically represent these two different types of support conditions, and the potential translations and rotations as well as the corresponding forces and moments that each type of support/connection condition is able to provide. A third possible support/connection type, this one called *fixed*, is also included and will be discussed a bit later; for now, though, it is enough to point out that at this type of connection condition neither translation nor rotation is permitted. A specific example of each of these three types of connections is provided in Illustration 4.22, but it should be borne in mind that these are very particular cases that have been chosen because they clearly express their respective translational/rotational freedoms and constraints; in reality, the way connections "look" can and does vary quite widely. Regardless of their physical appearance, however, what is critical to establish is how a particular connection (or pairs of end-connections considered together, as discussed in the preceding paragraph) addresses the possibility of translation and rotation of the structural element.

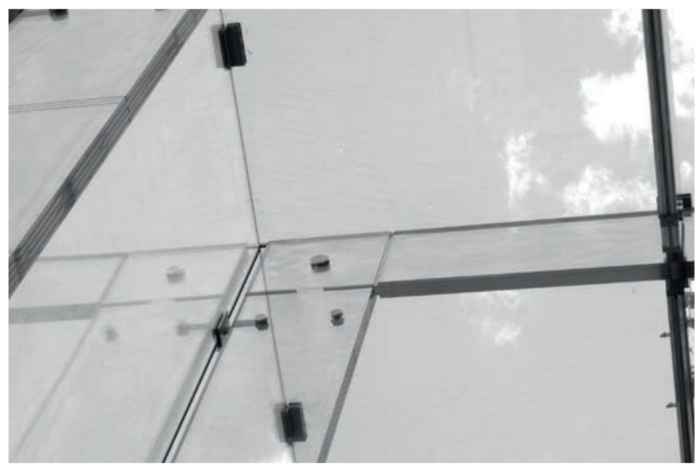
We are now in a position to address the equilibrium of the end-supported beam we introduced in Figure 4.15. One end of this beam is taken to have a pinned support and the other a roller support, a situation that is so common in building construction that it is given its own name and is explicitly called a *simply supported beam*. The forces at supports resulting from the applied load are called *support reactions*, and their magnitude and direction can be found by seeking equilibrium for the whole beam structure, as will be done presently.

Let us consider the simply supported beam AB having an inclined force P acting on it at mid-span, as shown in Figure 4.17. Where the loaded beam is restrained from possible movements, support reactions are generally created. Hence, in this example three unknown support reactions are established. These are the horizontal and vertical force components at support A, called A_x and A_y, and the vertical component at support B, called B_y. We will now study the beam's behavior when looked at as a rigid body. If we consider the beam from the point of view of support A, it seems as if the force B_y and the vertical component of the external

Figure 4.16
Three types of support conditions and their accompanying force reactions. These are idealized representations for the roller support, pinned support, and fixed support.

TYPE OF CONNECTION	SYMBOL	POTENTIAL ROTATION AND TRANSLATIONS AT THE CONNECTION	FORCES AND MOMENTS THAT CAN BE DEVELOPED AS RESULTS OF RESTRICTED MOTION OF THE CONNECTION
ROLLER SUPPORT			
PINNED SUPPORT			
FIXED SUPPORT			

a)



b)

Illustration 4.22
Examples of the three basic types of support conditions.
(a) Roller support condition, literally so here for the support of one end of a long-span beam; beam end is free to rotate and the steel cylinder permits free movement in the direction of the span while still preventing vertical displacement.

(b) Apple Store, Fifth Avenue, New York, NY, USA (2006).
Pinned support condition, here for a glass beam-to-column connection; beam end is free to rotate about the central steel pin, but neither lateral nor vertical displacements are allowed.
Architect: Bohlin Cywinski Jackson. Structural engineer (glass): Eckersly O'Callahan.

(c) Stratford Regional Station, Jubilee Line, London, UK (1999).
Fixed support condition for the end of a curved roof beam; anchoring bolts over full width of connection mean that rotation of the beam end is prevented, as are displacements in any direction.
Architect: Wilkinson Eyre. Structural engineer: Hyder Consulting Limited.

c)

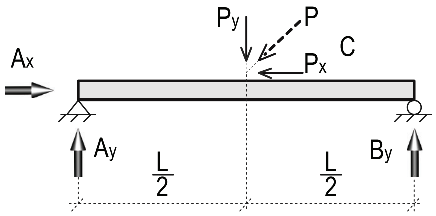


Figure 4.17
A beam simply supported at A and B with support reactions A_x, A_y, and B_y. The detailing of the connections at A and B informs us about the support conditions.
Support A is imagined to be a pinned connection which restricts both horizontal and vertical movements. The connection is thus able to mobilize support reactions in both directions to keep the beam at rest. At support B, small horizontal movements are possible. This means that no horizontal support reaction can be established; only a vertical reaction exists that keeps the support from moving downwards.
With these support conditions, we have three unknown force support reactions which can be calculated by applying the three equations for equilibrium. We say that such a beam is statically determinate.

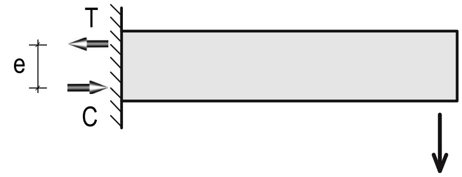


Figure 4.18
A cantilever has one fixed support.
When loads act on the beam a couple (forces having the same magnitude but opposite directions, acting with a distance between them) is created at the support which can provide equilibrium with moments resulting from the loads. (In addition – but not shown here – a vertical reaction force must also balance the net transverse force acting on this cantilever.)

load, P_y, might each cause the beam to rotate around the support. Both forces act at a distance to support A. Each creates a moment where the actual perpendicular distances from support A to the respective forces' lines of action constitute moment arms. The forces acting at support A, however, are directed through the center of the connection and create no moment about A since there is no moment arm. For the beam to be in equilibrium, therefore, the two moments of the forces B_y and P_y, acting in opposite directions, must be equal in magnitude. Since the sum of moments taken about support A must be zero ($\Sigma M_A = 0$) we have

$$+P_yL/2 – B_yL = 0, \text{ or}$$
$$B_y = P_y/2$$

This equation shows B_y to be one half of the magnitude of the load P_y. Having found one vertical support reaction, we may now look at the requirement for vertical equilibrium. If $\Sigma F_y = 0$, then

$$A_y + B_y – P_y = 0, \text{ or}$$
$$A_y = P_y – B_y = P_y – P_y/2 = P_y/2$$

This shows A_y to be equal to B_y. Furthermore, the condition for horizontal equilibrium yields

$$A_x – P_x = 0$$
$$A_x = P_x$$

(Note that if force P is vertical, then $P_x = 0$ and therefore $A_x = 0$, so there would be no horizontal reaction force at A in that case.)

All three unknown forces have thus been found by applying the three equations of equilibrium. Neither of the supports in the example is able to resist moments by itself; i.e., one support depends on the cooperation of the other in order to prevent the beam structure from rotating. There are ways to design support connections, however, in such a way that one support on its own may prevent rotation. As mentioned previously, we call such a connection a *fixed support* (see Fig. 4.16, Ill. 4.22c), and a beam with a support of this type at one end is called a *cantilever*.

The way in which a cantilever works is by effectively establishing two lines of force within the dimensions of the connection itself, in such a way that there is a distance, or moment arm e, between them. (Fig. 4.18.) When moments act about this support, two oppositely directed force reactions will develop (T will pull and C will push at the beam end) and create a moment ($M = Te = Ce$) at the end of the beam which can provide equilibrium with the moments resulting from the loads acting on the beam. The two horizontal force reactions at the fixed support have equal magnitude, but opposite directions. We call such a set of forces a *couple*, and recognize that a couple produces a moment.

We noted that all *three* support reactions in the example of the simply supported beam above were found by applying equilibrium equations. This is logical, since there are *three* conditions for equilibrium for plane structures. So if we are able to establish three independent equations in such conditions, then obviously three unknown forces will be able to be found by solving those equations.

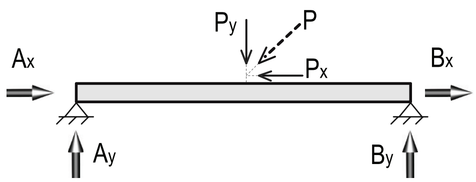


Figure 4.19
A statically indeterminate beam with four support reactions. With two pinned supports, there are four unknown forces. We are unable to calculate the magnitude of these forces by methods from statics alone and we refer to such a system as being statically indeterminate externally.

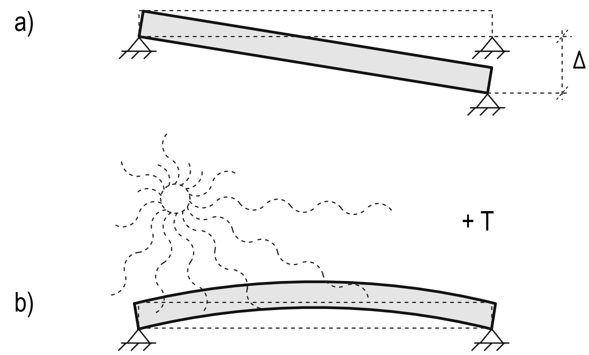


Figure 4.20
Both beam supports are pinned and prevented from moving sideways.
(a) If one support were to sink, the vertical movement will result in the beam becoming slightly longer, which means that a tension force is created within the beam.
(b) A rise of temperature in a statically indeterminate beam that is unable to expand horizontally leads to internal compression forces in the beam (and eventually to its bending). A statically determinate system, on the other hand, would compensate by moving horizontally.

But what if the support conditions for the beam were different than simply supported? Let us say, for example, that we support a beam on two pinned supports. (Fig. 4.19.) If the loads on the structure are resolved into both horizontal and vertical components, then horizontal and vertical force reactions will be created at both supports. We will then be dealing with four unknown force reactions, not three. How would we be able to calculate those by applying three equations? The answer is that we will not! To calculate the forces in such a system, we need to go beyond statics and apply a knowledge of how the system deflects. The theory for this, though, is outside the scope of this book. What we should know, however, is that a plane structural system (comprising one rigid body) where there are more than three unknown support reactions is referred to as being *statically indeterminate externally*. We also say that such a structure is *redundant*, a label that expresses that there are more support reactions present than the minimum that is necessary for maintaining equilibrium. In the case of the situation cited above with two pinned supports for the structure, there is one redundant horizontal force in the system. On the other hand, a system that is statically *determinate* with respect to its exterior supports (such as the simply supported beam that we were able to *determine* the reactions for) has exactly the minimum number of required support reactions for the structure to be in equilibrium (three for a planar system, six for a three-dimensional spatial system).[12]

Now imagine what would happen if one of the supports in the statically indeterminate beam with two pinned ends were to sink. (Fig. 4.20a.) This can happen, for example, if foundations give way slightly when the supporting soil at one end is soft and the width of the foundation is inadequate. Since any horizontal movement of the supports is assumed to be prevented, a vertical movement will cause the beam to actually have to become physically longer, and tension forces will necessarily develop along the length of the beam. An unforeseen vertical support movement will, therefore, create new and unforeseen forces in the structure. The same is true if an increase in temperature results in a tendency for a beam to expand. (Fig. 4.20b.) Since the beam is prevented from moving horizontally to accommodate this added length, horizontal compression forces will be created within the beam which will push against the supports. The pinned supports will allow each end to rotate, though, and so the compression force will result in the beam curving out from its initially straight, horizontal axis, causing it to bend. Hence, in redundant or statically indeterminate systems additional forces and moments are developed internally, whereas this is not the case for structures that are statically determinate, a fact that is considered to be an advantage in many cases.

Historically, the innovative large spans in iron and steel that were developed during the nineteenth century were to a large extent designed to be statically determinate structures; their engineers felt that such structures' behavior was easier to control as well as to calculate, and so those types were preferred. Are statically indeterminate systems entirely less desirable, then? Not at all. In fact, many structural systems built today are highly redundant. The very complicated calculations that are necessary to predict their behavior, however, no longer represent the same obstacles in this era of computers. Besides, having more means of support than is strictly necessary may also mean greater safety: if one support gives way, the structure may remain in equilibrium by means of those that are left.

In the following chapters, we will apply our knowledge of the requirements for equilibrium introduced in this chapter to study in more detail how the most common of structural elements function, as well as how much more complicated structural systems work to resist the loads that are applied to them.

4.8 Nordic Expressions of Forces and Moments

Structures made from all materials face the same elementary challenges: How should elements be connected? This is a question that has tortured the minds of architects ever since Antiquity. Many requirements must be fulfilled by the connections, but certainly an overriding one is the necessity for force transmission. In that process, forces commonly change directions. In this section we will look at a few architectural details and systems that involve some form of force distribution between two or more structural elements, starting with column-to-beam connections in concrete and then moving on to study some complex joining of elements in wood, all the while within a Nordic architectural context.

The Gothenburg Law Courts from 1937 is one of the most admired works of modern architecture. It stands shoulder to shoulder with the nineteenth-century neoclassic Town Hall and faces Gustav Adolf Square in the center of old Gothenburg. Inside, a large open hall extends through three floors and radiates a remarkable spaciousness, partly by making use of glass walls facing the inner courtyard, effectively connecting exterior and interior into one large space. Also within this hall, the architect Gunnar Asplund (1885–1940) created a fine cantilevered mezzanine level with a beautiful open staircase.

To achieve a high degree of transparency for the interior space, Asplund decided to use a minimal and open steel frame, although due to fire regulations the steel had to be encased in concrete. The structural honesty is nevertheless evident in the free-standing columns, where the concrete form reveals the shape of the hidden H-rolled steel profile as well as that of the tapered profiles of the beams under the mezzanine and roof. (Ill. 4.23.)

Illustration 4.23
Gothenburg Law Courts, Gothenburg, Sweden (1937). Column-to-beam connections in central hall. Steel structural elements are encased in concrete. This is an example of a fixed connection.
Architect: Gunnar Asplund.

In particular, close to the glazed façade, the daylight from the courtyard brings life to the beautifully designed connections between columns and beams.

Paustian is a furniture dealer in Copenhagen with a marvelous stock of classic modern furniture, certainly Danish but also international in origin. Somewhat away from the city center, Jørn Utzon (1918–2008) was asked to design a new furniture store in a formerly run-down part of the Copenhagen harbor area, thus initiating some much-needed urban redevelopment. For this, Utzon opted for a type of precast concrete system that might be found in any industrial building. With a pitched roof covering one large hall with mezzanine floor levels, the building offers a variety of spaces, both high and low. Daylight from above brings the interior scenery alive; one of Utzon's recurring inspirations is the characteristic space and light of the Danish beech woods. The structural system here is based on continuous columns with brackets for carrying the beams that support the floor decks and roof, the whole system together forming frames along the length of the building. (Ill. 4.24.) Rigid frame connections are achieved between columns and beams by shaping the top of the brackets into large triangular gusset plates; besides securing the stability of the building, this column-beam system and its distinctive connections form beautiful frames for viewing the various furniture departments on the mezzanine levels. Furthermore, double precast concrete T-beams, with their ribbed appearance on the underside, add to the structural quality of the hall. All concrete work is bright white; the only color introduced is by the dark ultramarine glazed ceramic tiles on top of the balustrades and the handrails in the stairs.

Leaving the subject of beam-to-column connections, we will next take a look at four architectural examples featuring structural systems that involve the cooperation and connection of a number of structural elements. It is true that advances in technology have made it possible for us to enclose large spaces with long and deep simple beams, some of which result in uniform and unarticulated architectural expressions. Yet long spans can alternatively be broken up and a shorter-span structural system can be deployed to create a varied and rich architectural expression. Talented Nordic architects and builders through time have incorporated these principles in the design of structures made of wood.

The Nes stave church in Hallingdal, Norway, dating from the 1100s, was at one time one of the few examples of a so-called middle-masted church in wood: after a long period of deterioration,

Illustration 4.24
Paustian Furniture Showroom, Copenhagen, Denmark (1986). Overall framework made with precast concrete elements connected together. Triangular brackets create fixed connections between columns and beams.

Architect: Jørn Utzon. Structural engineer: Johs Jørgensen AS.

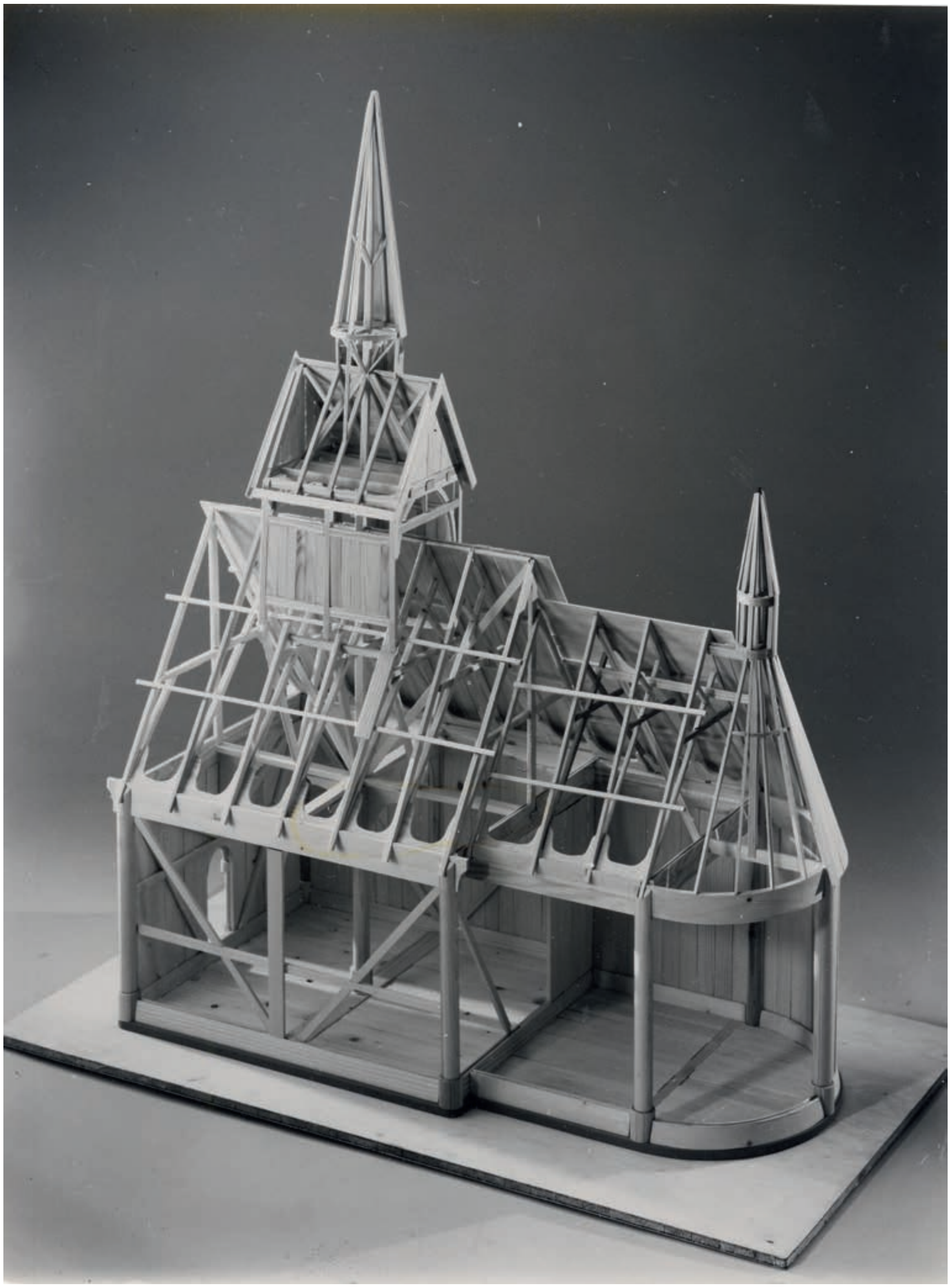


Illustration 4.25
Nes stave church, Hallingdal, Norway (twelfth century).
Structural system recreated from measured drawings done before the church was demolished in 1864.
Model by architecture students at AHO: Olav Dalheim, Svein Hoelseth, and Jan Petter Seim.

however, the church was demolished in 1864. Fortunately, detailed measured drawings were done during the church's very last hours, giving us good insight into the structural system. (Ill. 4.25.) The floor plan was quite simple and consisted of a rectangular nave with four corner posts and a mast in the middle, plus an apse. The middle-mast extended up to the rafters and was connected to four beams stiffened by half arches. Diagonal struts from here supported the peaks of the gables and the roof 's spire. A series of scissor-trusses supported the roof over the nave. A more thorough examination of this church is beyond the scope of this book, but we can begin to imagine how such a 900-year-old structural system worked and gave the building its particular character. Its design and construction is unified with the creation of its architectural expression.

Alvar Aalto's (1898–1976) courthouse in Säynätsalo, Finland, was inaugurated in 1952. The multiform complex in red brick is deployed around a central courtyard that is elevated with respect

Illustration 4.26
Town Hall at Säynätsalo, Finland (1952).
Compression struts fan up to support the ceiling of the council chamber. The lower ends of these struts connect together at two locations in the middle of the space, but instead of having columns come down from there and interrupt the open space below, these connection points are supported by two angled wood elements that bring the loads back up to the top of the brick walls surrounding the room.
Architect: Alvar Aalto.

to the surrounding terrain. The main hall of the courthouse is a cubic form, with its height matching its width. The sloping roof structure over this space spans almost 10m (33ft) by having two wood tension elements drop down to meet at an angle, forming a prominent connection point from which a bundle of compression struts fan up and out to support the roof above. (Ill. 4.26.) The depth of the roof beams is thereby reduced, since the bundles of struts greatly shorten the distance that the roof beams have to span. The base of the strut fan could have obviously been supported by a column that would have transferred the load on downward, but Aalto chose instead a hanging tension system that brings the roof loads back up to the top of the load-bearing walls at the perimeter. The actual connection collecting the 16 individual strut members is carefully designed as a single steel trough.

In the unbuilt project for an indoor swimming pool at Peblingesø, a lake in central Copenhagen, the Danish architect Jørn Utzon

Illustration 4.27
Indoor swimming pool facility, Peblingesø, Copenhagen, Denmark (designed 1979; unbuilt).
Roof was to be carried by angled wooden compression struts that would have branched out from two lines of columns along the sides of the pool. This number of struts would have minimized the span of the roof beams, keeping their dimensions to a minimum.
Architect: Jørn Utzon.

(1918–2008) shows how he mastered a theme and gave it his own special interpretation. The building design is light and open, giving the observer an impression of the lake as continuing in through the building to form the swimming pool. (Ill. 4.27.) Structural order characterizes the building plan, and the roof above is carried by two parallel rows of wooden compression struts. The struts branch out to support the rafters which consequently are subdivided and can be very slender in relation to their length. Massive foundation pillars with wide bases combined with the triangles formed by the arrangement of compression struts and rafters make sure that the building is fully stabilized in both directions. The reflection of the branching structure mirrored in the water surface underlines the main idea of visual integration of the lake and the pool.

Finally, the Metla Building from 2005 houses the Finnish Forest Research Institute at Joensuu, a university town in the forest-rich area close to the Russian border. An innovative use of wood was the natural starting point for the design by SARC architects. Appearing as a cubic volume, the building is organized around a courtyard that gives access to the vestibule and laboratories. The structural system is a regular and flexible fir-laminated post-beam-slab system based on a module of 7.2m (24ft); such a system allows for a change in internal partitions and even external façades as needed.

While the layout for laboratories and office follow a straightforward and economical pattern, the most impressive part of the building is the three-story-high vestibule area with reception and cafeteria. The distinctive structure here comprises a row of four bundles of columns. (Ill. 4.28.) Each bundle in turn consists of four inclined timber members jutting out from one unified steel joint on the floor. To prevent these members from buckling (they have, after all, a length of 12m (39ft) up to the ceiling), these columns become spatial; i.e., they are each composed of four subparts having square cross-section that are spaced apart. At equal intervals along the height of the columns these parts are kept apart by means of steel-spacers, in this way producing a fine curved appearance and an increased resistance against buckling. (There will be more about this in Chapter 8). Dramatized by the shifting daylight, the bold and thoroughly detailed structure creates a memorable space.

Illustration 4.28
Metla Building, Finnish Forest Research Institute, Joensuu, Finland (2005).
Bundled groups of angled columns support the roof in the three-story-high vestibule area. To increase their load-carrying capacity given their exceptional length, each column element is made up of four subparts that are spaced apart.
Architect: SARC. Structural engineer: Olof Granlund Oy.

Materials

CHAPTER 5

Illustration 5.1
Admiral Hotel, Copenhagen, Denmark (eighteenth century).
Pomeranian pine was used for the structure of this old warehouse, now converted into a modern hotel at the Copenhagen waterfront.

FACING PAGE
Illustration 5.2
St. Paul's Cathedral, London, UK (1708).
Perspectival section drawing shows St. Paul's dome to be composed of three geometric surfaces: an inner masonry dome that is most visible from the inside and so has paintings on its underside, above that a straight-sided conical surface also made of masonry blocks and that bears the weight of the heavy lantern, and the outer, lead-sheet-clad dome surface that creates the form seen from the outside – which can be seen to be supported on timber framework also carried by the masonry cone.
Architect: Sir Christopher Wren.
Engraving by Samuel Wale and John Gwynn (1755).

5.1 St. Paul's to Tate Modern – A Material Walkabout

In central London, one is easily able to span the course of time in relatively few steps. There is one such short walk in particular, going from St. Paul's Cathedral across the Millennium Bridge to the Tate Modern art gallery, along which various building materials that have been used over the past 350 years are conveniently concentrated and put on display. In observing these, one can also contemplate just how material selection plays a key and integral role in the design of structural systems supporting diverse architectural design objectives, irrespective of date of construction.

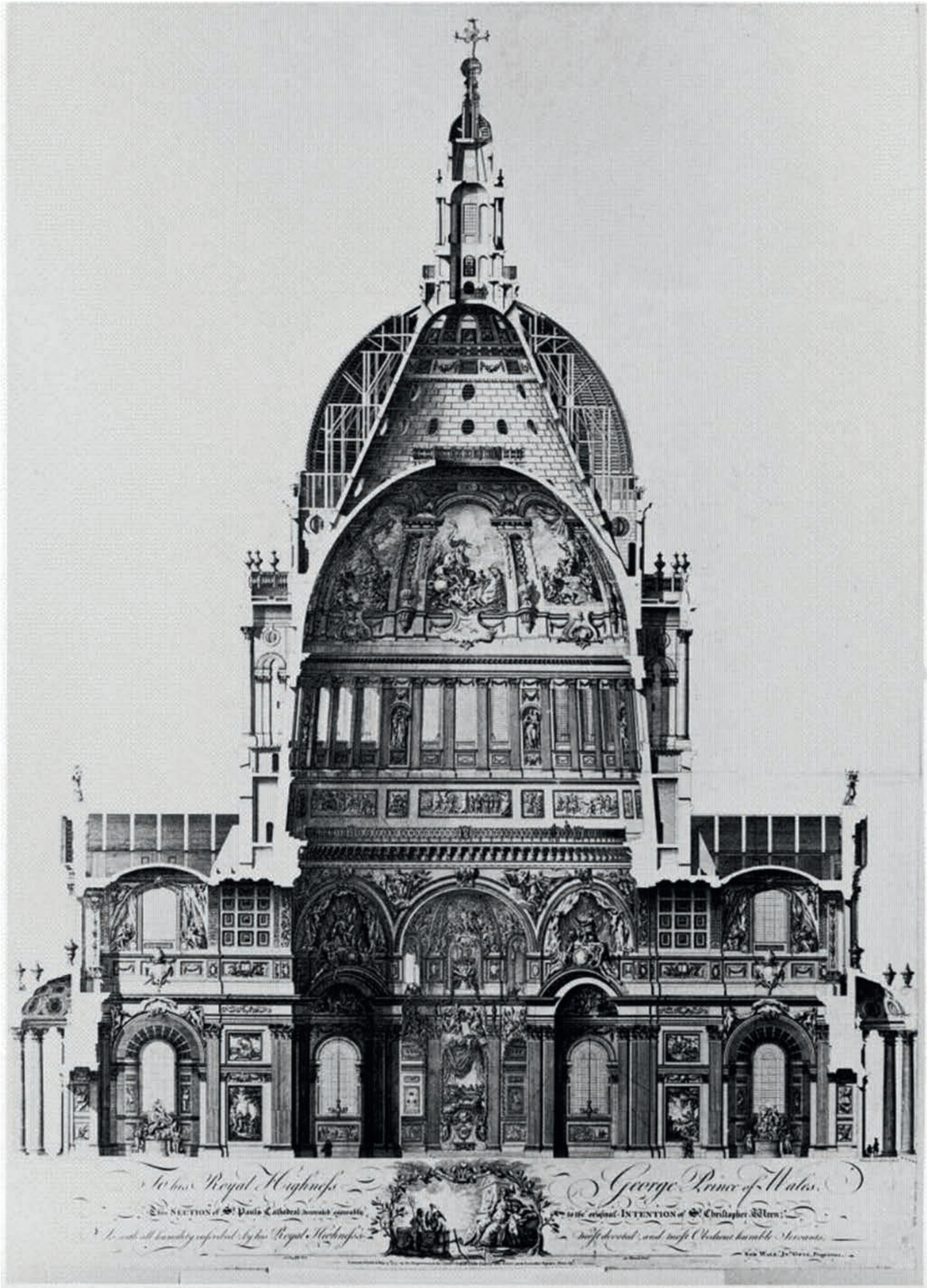
St. Paul's Cathedral

Following London's Great Fire of 1666 and the destruction of the previous St. Paul's Cathedral with its wooden roof construction and its spectacularly tall Gothic spire (this part was actually destroyed a century earlier in 1561 due to another fire, thought to have been caused by a lightning strike), Sir Christopher Wren (1632–1723) was commissioned to design a replacement cathedral that would be no less dominant on the London skyline. In developing the design for the new St. Paul's, Wren was influenced by his travels to continental Europe (and especially by the domed Pantheon and Baroque-style St. Peter's in Rome) as well as by medieval English church designs and, also, by a then rapidly evolving scientific understanding of the workings of structural systems. The exterior walls of the Cathedral were built of Ashlar masonry – grayish-white rectangular-cut limestone blocks from Portland in Dorset – which was considered at the time to be the finest stone masonry but was becoming increasingly rare, which likely made it seem all the more appropriate for this landmark building. Above these walls at the crossing, a large dome was erected that is supported by eight stone piers – two along each side and four corner bastions – rather than the more typical four supports because of what were known to be unstable ground conditions on the site, thus enabling the vertical loads to be more distributed and thus reduce the risk of differential settlement.

This dome of St. Paul's is a most remarkable structure: it is actually composed of three geometric surfaces with differing profiles, as can clearly be seen in the perspectival section drawing in Ill. 5.2. (See also Ill. 13.15.) A mostly hidden brick structural cone is in the middle and is the workhorse of the three; it is only 46cm (18in) thick and yet reaches up the desired height of 111m (365ft) in order to support the large ornate stone lantern at the top of the dome that weighs 850 tonnes (1 900 000lbs). Carrying such a large concentrated load on a conical structure, which is straight-line in section, is remarkably efficient with most of the load able to be carried in direct compression while also minimizing the outward bulging that a curved-in-section structural dome would be subjected to. (There will be more on this topic in Chapters 12 and 13.) Any limited tendency for the cone to bulge outward due to gravity or lateral wind loading is resisted by circumferential iron chains located at intervals up the height of the cone, as well as by a set of radial stone buttress walls around the base of the cone (which are hidden from view behind a perimeter colonnade).

But this brick cone also supports an open timber framework necessary to carry the traditionally curved shape of the lead-sheet-clad outer dome surface, a profile that would have been expected on such a prominent city and ecclesiastical landmark during the seventeenth century – notwithstanding any scientific advances being made at that time. Also to be expected would have been a correspondingly domed-shape interior ceiling above the crossing – not the steep, straight sides of the interior of a cone. And so, within the structural cone, we indeed find the third of St. Paul's dome surfaces: this one also made of brick, also 46cm (18in) thick, but having a strategic catenary shape to support its self-weight (again, see Chapters 12 and 13) and with a large opening at the center. The paintings on this inner dome surface can thus be better brought into sight of the congregation below, while the opening at the center allows the sense of space to project further upward, with strategic openings in the conical surface illuminating the apex of its underside.

In summation, then, we see here at St. Paul's dome that the use of traditional heavy masonry materials like stone and brick replaced earlier and more vulnerable timber construction, and moreover that these were shaped and deployed in innovative ways so as to achieve a relatively light and efficient overall structural system given the building's remarkable height and open volume; i.e., the choice of materials enables and supports the architectural design intentions.


To his Royal Highness
George Prince of Wales.
This SECTION of St. Pauls Cathedral
to the original INTENTION of Sr. Christopher Wren;
most devoted, and most Obedient humble Servants.


DISPLAYS
FUTURISM
KIRKEBY

FACING PAGE
Illustration 5.3
(a) Millennium Bridge, London, UK (2000).
(b) Tate Modern creates backdrop when looking southward. Lighting reveals details of this distinctive footbridge structure: its two Y-shaped steel and concrete piers, shallow-profile steel suspension cables, and closely spaced steel cross beams whose geometry changes along the span. St. Paul's dome looms directly on axis with the bridge.
Architect: Foster + Partners with Anthony Caro (sculptor).
Structural engineer: Arup.

Millennium Bridge

As its name implies, this landmark footbridge was built to mark the celebrations of the year 2000. Its alignment connects visually on the north side of the Thames with the open urban axis up Peter's Hill to St. Paul's dome and on the South Bank to the Tate Modern located within a renovated and newly expanded power station building, which is a city landmark of its own right but from a different era. (Ill. 5.3, 5.4.) In contrast to needs-to-be-seen guidelines set for St. Paul's Cathedral dome just discussed, for the Millennium Bridge the design objective was also to have a signature structure but in this case one which was very low and slender so as to preserve existing views to and from the urban surroundings. This goal was accomplished through a collaboration of the architects Foster + Partners, sculptor Anthony Caro, and Arup structural engineers.

The Millennium Bridge is a 320m (1050ft) 3-span steel suspension cable structure (we will discuss this structural type in detail in Chapter 11) that has an exceptionally low profile, with the draping cables running closely alongside the pedestrian deck level – i.e., they go from just below the deck level at mid-span to only 2.3m (7.5ft) above it at the piers. This very shallow cable profile works well to allow maximum views of the surrounding cityscape and minimize the physical presence of the bridge, but with a profile six times as shallow as for a conventional suspension bridge the magnitude of the resulting cable forces are greatly magnified over what they might otherwise be. As a result, eight relatively large suspension cables are needed – two groups of four 120mm diameter (4.7in) on each side, each one a locked coil strand; i.e., these are made of many galvanized, shaped, high-strength steel wires that are wound helically around a core, with this particular material and technology being used to minimize structural dimensions while also seeking to prevent rust corrosion.

The two Y-shaped piers that provide intermediate cable support are made of folded, welded steel box sections for their upper arms that are connected together at the tops of tapering elliptical reinforced concrete piers, whose shape accounts for water flow and barge/ship impact as well as for rising water levels due to global warming. The 4m (13ft) wide deck is supported on steel box-section cross beams located 8m (26.5ft) apart that connect the deck to the cables; the walkway deck itself is made of extruded aluminum panels that span side-to-side between two steel tube edge beams.

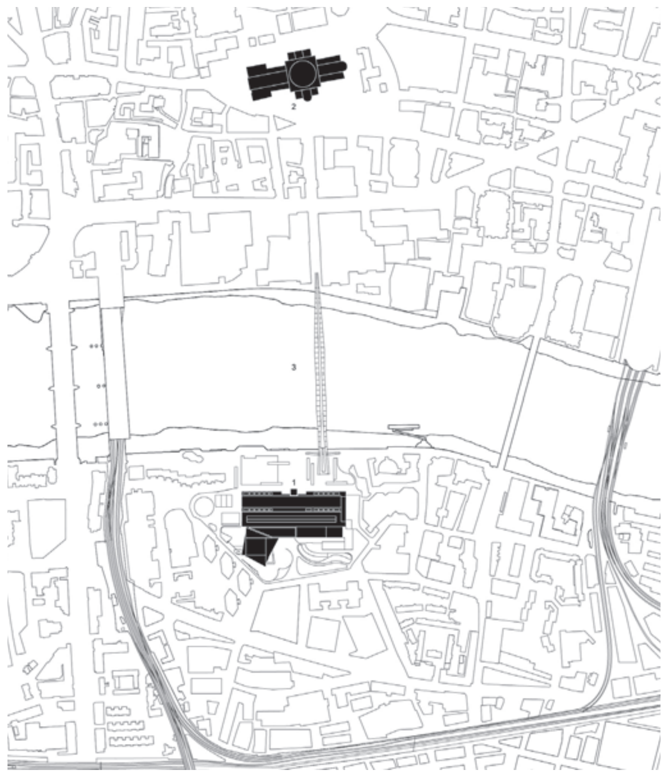


Illustration 5.4
Site plan establishes urban context and relationships between St. Paul's Cathedral, the Millennium Bridge, and Tate Modern, London, UK.
Drawing by Herzog & de Meuron.

At the bridge abutments, a 3m (10ft) thick reinforced concrete pile-cap is anchored to the ground by means of groups of 2.1m (7ft) diameter reinforced concrete piles – 12 at the north bank with concrete shear walls aligned with the bridge axis to transfer the cable forces, and 16 at the south bank due to tighter site constraints, necessitating a shorter pile-cap and, therefore, less moment arm to work with to resist the overturning moment produced by the cables being anchored above ground level. The cables pull with a force of 2000 tonnes (4 400 000lbs) against the abutments at each end, which is enough to support 5000 people on the bridge at one time. At the south end, the site constraints have been turned to design advantage, with the pedestrian pathway splitting apart

Illustration 5.5
Bankside Power Station, London, UK (1953, 1963 for two phases of construction).
Conversion to Tate Modern (2000).
Turbine room of former power station was converted into a large-scale public art gallery. The column-and-beam steel framing on both sides of this central open space was largely retained in the repurposed building.
Architect: Herzog & de Meuron (for conversion); Sir Giles Gilbert Scott (for previous power station).
Structural engineer: Arup (for conversion).

before doubling back on itself into a north-pointing staircase. The view of St. Paul's dome from this vantage point is very fine indeed!

Of course, the Millennium Bridge was made famous/infamous because of its instability problems on opening day, when 90 000 people visited (about 2000 at a time, or about 1.5 people/m^2 (1.25 people/yd^2)). The result was greater-than-expected side-sway due to people intuitively locking step with the natural oscillations of the bridge, thus multiplying the effect with discomforting results. The bridge was closed for two years of study and research of the phenomenon, with a retrofit of dampers being added below the deck to lesson movements – 37 fluid-viscous energy-dissipating dampers to control horizontal movement and 52 tuned mass inertial dampers to control vertical movement. The bridge has since then been open and performing as intended. Pushing the limits of design obviously can have its consequences, but at the same time this often leads to remarkable advances in the understanding of structural behavior, with longstanding and more widespread benefits that ensue.

Tate Modern – Turbine Hall

Located just beyond the Millennium Bridge on the South Bank, and balancing the physical presence of St. Paul's Cathedral to the north, is the large brick bulk of a building that is the Tate Modern, an art gallery also opened in the year 2000 for displays of contemporary art and large-scale public art installations. The galleries inhabit the shell of the former mid-twentieth-century electricity-generating Bankside Power Station which, in turn, was built on the site of an earlier building that used to burn coal to produce electrical power, but this process was considered too polluting for the city center and so a change was made to burning oil instead. It is interesting in the context of our current discussion that the height of the power station's prominent chimney tower was limited to being less than that of St. Paul's dome, with the result that air pollution in the surrounding neighborhoods was greater than it otherwise might have been. The structure for this power station was basically that of an open steel framework, with the perimeter of the building enclosed by a brick skin; the 4.2 million bricks used for this challenged the availability of this building material at the time. The power station building was organized into three parts running east–west, with a boiler house to the north, the full-height open turbine hall in the middle, and the switch house and underground oil storage tank facilities on the back, southern side. This basic plan and volumetric organizing principle was retained by the architects Herzog and de Meuron for their recent reincarnation of this building as the Tate Modern, whose central public space is the Turbine Hall that is used for large-scale art installations, and with multilevel art galleries and service spaces being inserted in and added to the north and south sides, respectively.

The original steel column-and-beam framing system is most easily observed along the long sides of the open space of the original Turbine Hall, as are the trusses spanning across the roof skylight from north to south and the gantry crane now (re)used to move large artwork. (Ill. 5.5.) This steel framing also is used today to provide anchorage points for various large-scale art installations, some of which require elevated points of attachment. Contrasting subtly with this is a completely new steel structural frame that has been inserted into the northern, boiler house portion of the preexisting building to support seven new levels of galleries and support service facilities. All along the edge adjacent to the Turbine Hall, these new steel columns are situated just "behind" the existing steel columns – so that the new structure is barely evident from the main hall. (Ill. 5.6.) Upon closer examination, however, the differences between the old and new columns are quite evident: the former are larger, made up of multiple steel sections that are riveted together while the latter are smaller-dimensioned but thicker-walled contemporary rolled steel sections. This subtle contrast exemplifies the design intentions of Herzog & de Meuron to respect and be influenced by the preexisting structure and building design but not be bound by it; theirs is an intent to blend old and new design, as well as space and structure alike.

A steel frame in this northern portion of the repurposed building was also used for other, less-evident reasons: it was easier to fit it into the tight space of the existing building envelope, and its relatively light self-weight helped to address some of the difficulties of inserting new foundations within those that were previously existing. Also, the steel column grid could be better coordinated with the retained building envelope so that the vertical "lancet" windows would remain unobstructed. Finally, the relatively slender dimensions of the new steel structure could be mostly hidden within the new galleries' double partition walls that also incorporate lighting and mechanical and other building service functions. Here at the Tate, the old and new steel frame structure have been deployed purposefully in support of the design: made evident in the Turbine Hall where the dimensions and desired qualities of the space demanded it, but hidden away in the smaller gallery levels where this it is not beneficial, while being strategically and somewhat perplexingly contrasted at the boundary between these two programmatic and spatial areas. There is nothing self-evident here, just as was the case in the dome of St. Paul's.

Illustration 5.6
Tate Modern.
Contrasting, yet echoing, material technologies: bolted splice for end-to-end connection of rolled steel column segments in art gallery (left), next to built-up, riveted steel plate columns of previous power station (right).

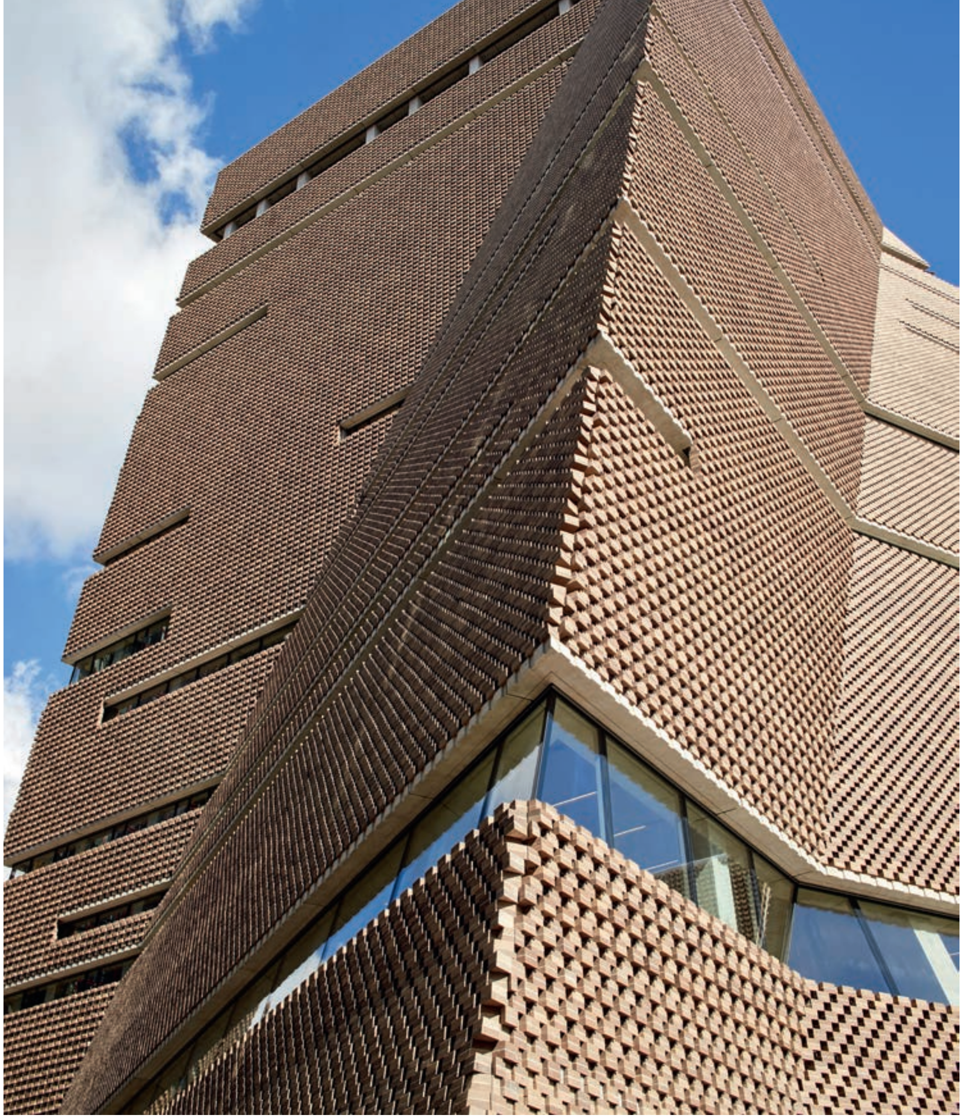


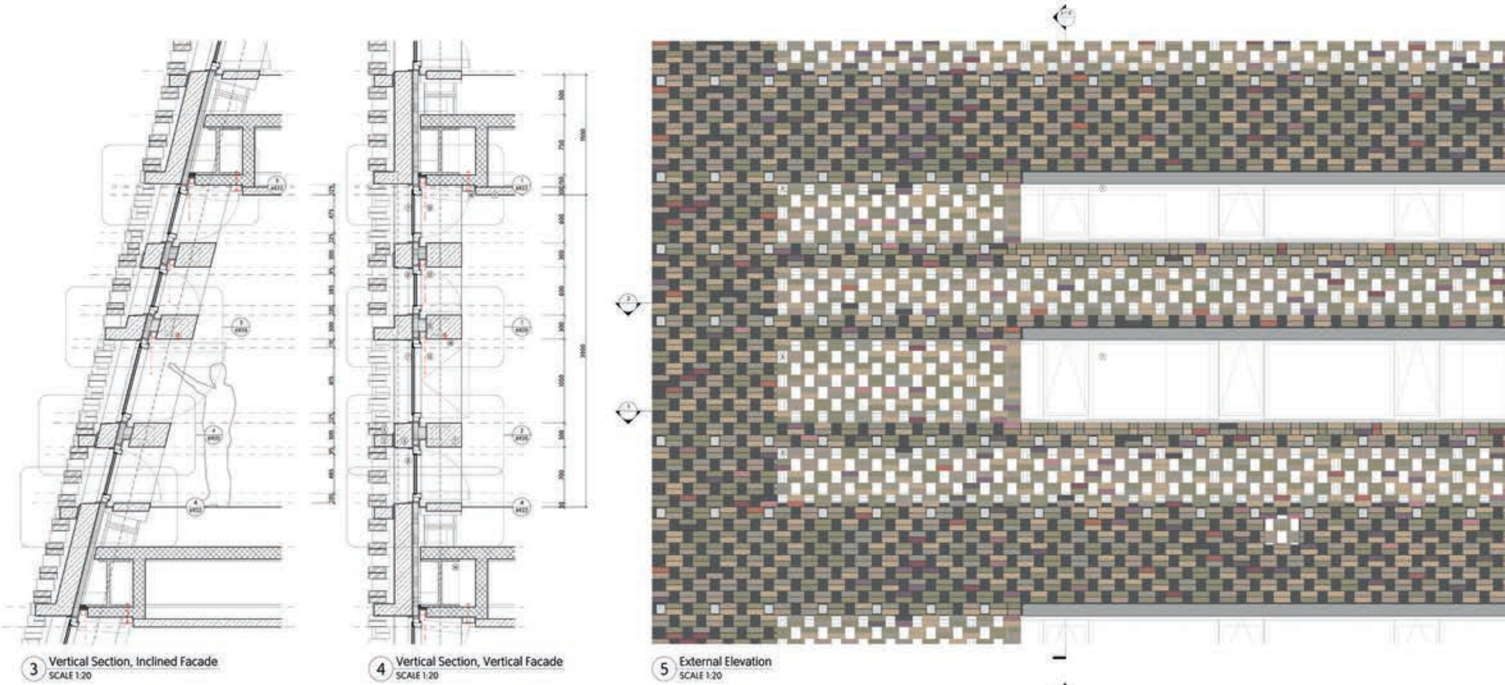
Illustration 5.7
Switch House extension to Tate Modern, London, UK (2016).
Angled, warping geometry of outer walls required development of innovative lattice-like brick-stacking technique that relied on digital technology to implement. Unique visual effects result, whether seen from inside or out, or during the daytime or when lit at night. Long horizontal window openings reveal edges of precast concrete floor elements that support the weight of the brickwork at regular vertical intervals.

Architect: Herzog & de Meuron. Structural engineer: Rambøll.

Tate Modern Extension – Switch House

Expanding the Tate Modern, as had been originally planned, into the southern portion of the original building's tripartite division is the more recently completed (2016) Switch House. This addition builds upon the preexisting underground concrete cylinders that had once been used for oil storage for the power station, but it is essentially a new 10-story building volume that adds 60 percent more space to the heavily visited museum. Also designed by Herzog & de Meuron, the unique cut-off pyramidal, ziggurat-like twisting outer form of this building creates its own iconic addition to the London city skyline, all the while addressing the disjuncture between the long rectangular bulk of the power-station-turned-museum and the much more eclectic architectural forms and styles that have mushroomed around it. On the inside, the Switch House can be considered to be organized as an ascending circulation promenade that links together a variety of galleries, education rooms, public spaces, and bar, going from dark, interior basement galleries up to a panoramic sky terrace, with strategically located intermediate stop-off points along the way. Continuing the promenade near the top, a link to the Turbine Hall is reestablished by means of an elevated bridge that gives a new and different vantage point to the undisputed central space of the museum.

The main material used for the structural system of this building is concrete, from a core of reinforced concrete shear walls seemingly emerging from the cylindrical concrete tank walls in the basement to an open concrete framework of columns, beams, and slabs. Of particular interest are the precast concrete perimeter columns that slope and twist at different angles according to the cranked building geometry; their relative slenderness is made possible by having a core made of structural steel, while their cruciform profile provides projecting arms to support precast concrete panels, glazing, and brickwork. The floor system consists of a concrete beam and slab system, with long-span continuous beam ribs supporting the concrete floorplate. In strategic places at the lower levels connecting to the power station, steel trusses and beams are introduced to further open up the spans, with 18m (59ft) clear spans achieved in certain galleries; moreover, echoing the "blurring" of old/new systems that we saw for the steel structural systems of the northern galleries, some of these steel beams span from the Switch House's concrete core walls to new steel columns inserted adjacent to the preexisting Turbine Hall's riveted steel columns. All in all, then, what we have



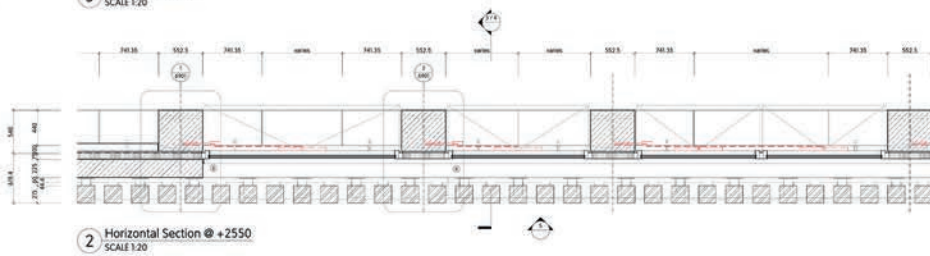


Illustration 5.8
Switch House extension to Tate Modern.
Detail drawing of brickwork façade and its connection to supporting precast concrete structure.
Drawing by Herzog & de Meuron.

here in the Switch House is an opportunistic wall-frame structural system made mostly of concrete but that in the end is made of various structural material combinations and technologies that best suit structural capacity needs and architectural design intentions.

As for the unique and arresting exterior brickwork skin that encloses this concrete structural skeleton: it is a non-load-bearing perforated lattice made up of 336 000 bricks that allows light to filter into the building during the day and turn it into a glowing form at night. Of course, such a brick curtain wall serves to unify the Switch House addition to the original brick-clad power station building. But upon closer examination, there is something very different and contemporary about this new brick exterior. Because of the steeply angled and warping slope of the outer walls of the building, the bricks could not be placed by the usual bricklaying techniques with a standard mortar connection; instead, preassembled pairs of bricks bear on each other through small neoprene washers held in place by vertical stainless steel rods that are inserted through carefully positioned predrilled holes in the brick pairs. Given the varying geometry of the building façade, all this needed to be very carefully coordinated, with the predrilled holes and ever-changing overlaps digitally calculated. (Ill. 5.7, 5.8.)

Of course, the fact that the window openings in the upper part of the Switch House are horizontal instead of vertical as they are in the power station is a dead give-away that its brickwork is not carrying its self-weight down to the ground, let alone supporting anything else; nonetheless, this orientation of the openings makes possible spectacular views outward to the cityscape. Such viewpoints also provide a fitting terminus to our short material walkabout in London, where we went from brickwork being strategically shaped into a cone configuration to do all the heavy lifting at St. Paul's in order to support a "fake" exterior dome surface to the opposite here in the Tate Modern Switch House, with an exuberant and very visible brickwork skin being supported on a concrete structural framework. Among other things, this establishes an interesting material dialogue across the centuries, with the slender steel cables of the Millennium Bridge connecting the two design approaches.

Illustration 5.9
Gando Primary School Extension, Burkina Faso (2008).
Locally made and readily available materials are used for this structural system: hand-made compressed earth bricks for the walls as well as the curved ceiling vaults spanning across the classrooms from side-to-side; a wide reinforced concrete beam at the top of the walls resists the outward thrust of these vaults; and many steel reinforcing bars are welded together into a trussed/space frame-like structure that supports a shading canopy above, which also allows air circulation.
Architect: Kéré Architecture. Structural engineer: Prof. Dr.-Ing. Eddy Widjaja.

5.2 The Mechanical and Physical Properties of Materials

To successfully design structures, a basic knowledge of the most common structural materials is needed. The shapes and dimensions of structural members are heavily influenced by the various properties of the materials used, and a familiarity with how materials are produced and processed greatly helps the designer to make appropriate decisions. (e.g., Ill. 5.9, 5.10.) In addition, a good knowledge of materials may inspire new design ideas. Within a building's life span, materials may also undergo changes that we need to understand and foresee at the time of construction in order to prevent unpleasant surprises as the structure ages. Moreover, and from a broader perspective, the impact materials may have on the environment, whether when they are produced or when they are used in buildings (including the energy consumption associated with their production and manufacture as well as shipping) has today become an especially important material characteristic that we need to pay more and more attention to.

Of obvious primary interest for us in the present context is knowledge about how materials respond when forces are applied to them. We want to know what it takes to break or crush a particular material, and how it deforms. Material properties that inform us about such things are called the *mechanical properties*

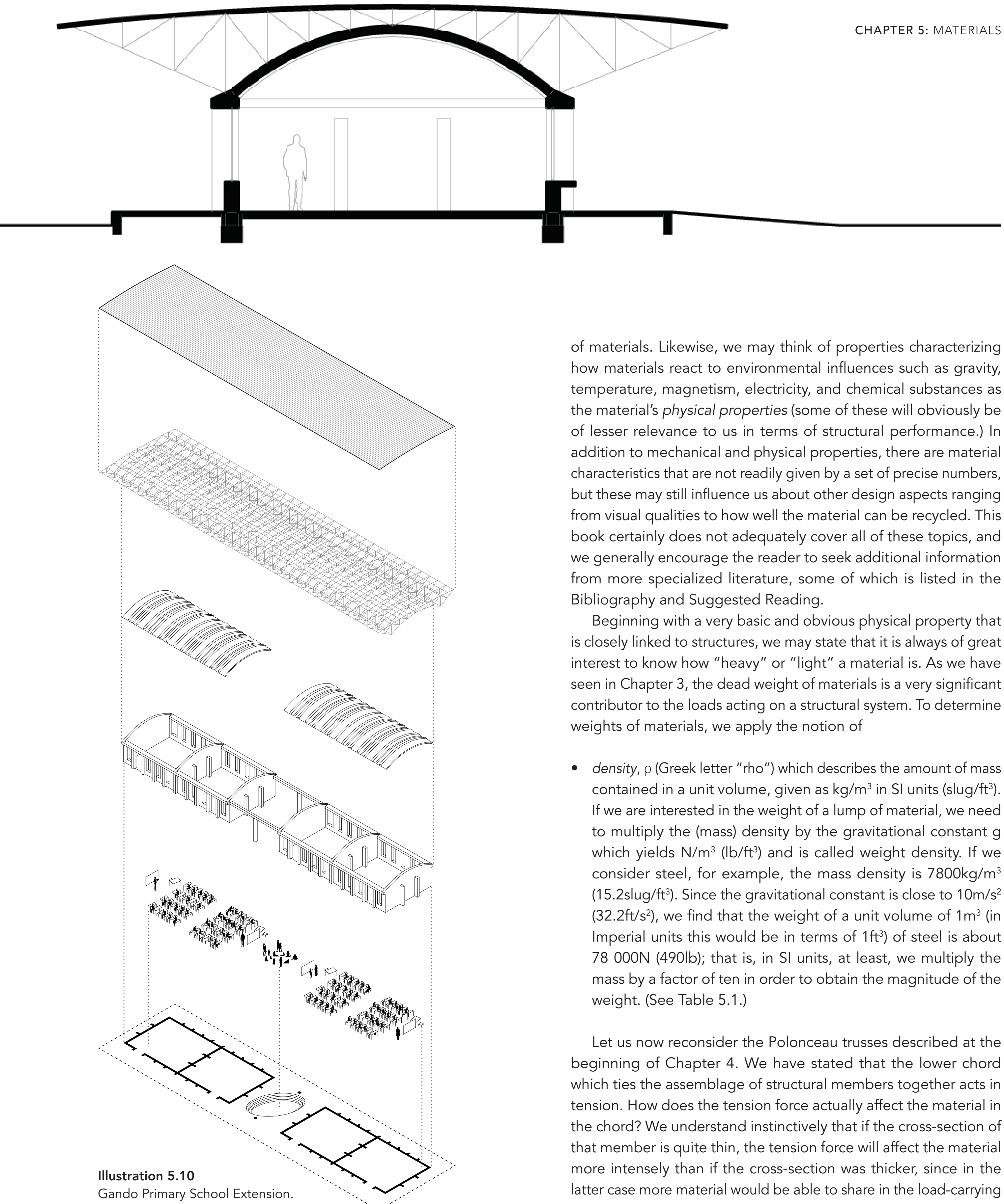


Illustration 5.10
Gando Primary School Extension.
Exploded axon and section drawings.

of materials. Likewise, we may think of properties characterizing how materials react to environmental influences such as gravity, temperature, magnetism, electricity, and chemical substances as the material's *physical properties* (some of these will obviously be of lesser relevance to us in terms of structural performance.) In addition to mechanical and physical properties, there are material characteristics that are not readily given by a set of precise numbers, but these may still influence us about other design aspects ranging from visual qualities to how well the material can be recycled. This book certainly does not adequately cover all of these topics, and we generally encourage the reader to seek additional information from more specialized literature, some of which is listed in the Bibliography and Suggested Reading.

Beginning with a very basic and obvious physical property that is closely linked to structures, we may state that it is always of great interest to know how "heavy" or "light" a material is. As we have seen in Chapter 3, the dead weight of materials is a very significant contributor to the loads acting on a structural system. To determine weights of materials, we apply the notion of

- *density*, ρ (Greek letter "rho") which describes the amount of mass contained in a unit volume, given as kg/m^3 in SI units ($slug/ft^3$). If we are interested in the weight of a lump of material, we need to multiply the (mass) density by the gravitational constant g which yields N/m^3 (lb/ft^3) and is called weight density. If we consider steel, for example, the mass density is $7800kg/m^3$ ($15.2slug/ft^3$). Since the gravitational constant is close to $10m/s^2$ ($32.2ft/s^2$), we find that the weight of a unit volume of $1m^3$ (in Imperial units this would be in terms of $1ft^3$) of steel is about 78 000N (490lb); that is, in SI units, at least, we multiply the mass by a factor of ten in order to obtain the magnitude of the weight. (See Table 5.1.)

Let us now reconsider the Polonceau trusses described at the beginning of Chapter 4. We have stated that the lower chord which ties the assemblage of structural members together acts in tension. How does the tension force actually affect the material in the chord? We understand instinctively that if the cross-section of that member is quite thin, the tension force will affect the material more intensely than if the cross-section was thicker, since in the latter case more material would be able to share in the load-carrying duty. What is of interest for the material, then, is the intensity of

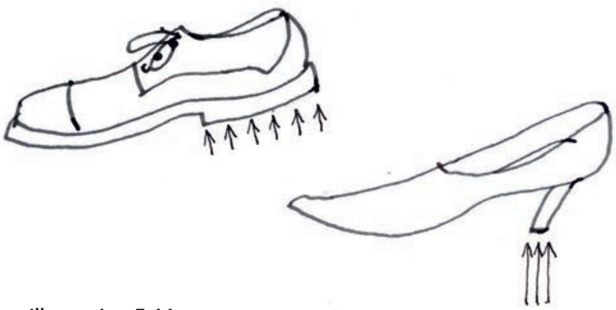


Illustration 5.11
Drawing depicting the relative stress magnitudes under a high-heel shoe versus that under a flat-soled shoe.

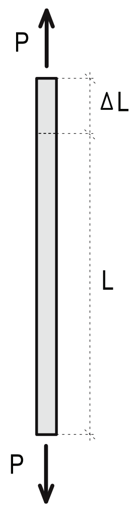


Figure 5.1
Tension element with tensile axial stresses.
Since the material is elastic, some elongation ΔL takes place in the element. If the length of the element is L before the force is applied, it will be L + ΔL after the tensile force has acted. The relative elongation ΔL/L (or shortening if compression stresses are acting) is called the strain, ε.

the force, or the force per unit of area that the force acts on. We call this force intensity

- *stress*, and measure stress in N/mm^2 (lb/in^2). (N/mm^2 is also called MPa, megapascals.) Mathematically, we write

  $$\sigma = P/A \qquad (5.1)$$

  where σ = the material stress (Greek letter "sigma"), in this case a tension stress, and P = the tension force (N, lb) acting on the cross-sectional area A (mm^2, in^2). (e.g., Ill. 5.11.) We are aware of the fact that structural members may break if the force that is acting on it becomes too large. What actually happens is that the stress in the material reaches a value where the molecules are no longer able to withstand the amount of tension (or compression) that they are being subjected to, and the bonds between them break. The stress level at which this occurs differs from material to material and is, therefore, an important mechanical property of the material. We call this stress level the material's
- *ultimate stress*, σ_u, or material *strength*, defined once again by the stress units of N/mm^2 (lb/in^2). A material which has a high ultimate stress is subsequently called a strong material, while a material which fails at a low stress level is commonly thought of as a weak material. Many materials will show great differences in ultimate stresses when subjected to tension forces as opposed to compression forces or shear forces. (See Table 5.1.) Materials like stone, clay bricks, unreinforced concrete, and cast iron are relatively strong in compression, but quite weak in tension. In such cases, then, we need to identify the type of force we are referring to when we give figures for ultimate stresses. It is common to speak of the material's ultimate stress in tension, in compression, or in shear; these are also referred to as the *tensile strength*, the *compression strength*, and the *shear strength* of the material, respectively. When the material reaches its ultimate stress in a structural member, we say that the *member strength* is reached. This is the load-carrying capacity of the structural member.

When stresses develop in a structural member subjected to tension or compression forces, the member deforms, by an amount ΔL. If the stresses are tensile, the member becomes longer whereas if they are compressive, the member becomes shorter. Structural materials are, to a larger or lesser degree, *elastic*; some elongation or shortening will always take place when they are loaded, even if the deformations are so small that they can only be observed by the use of precise measuring instruments. (Fig. 5.1.) Let us once again refer to the lower chord of the Polonceau truss discussed in Section 4.1. Since the tension force acting in the chord follows the direction of the chord's axis along its length, so-called *axial stresses* develop. These stresses have a direction which is perpendicular to the cross-section over which they act, and consequently the axial tension stresses are also called *normal* stresses. Furthermore, these stresses are uniformly distributed over the entire area of the cross-section. Obviously, normal stresses may also be compressive in a member subject to compression forces.

When the force is increased, the elongation ΔL will increase. This direct correlation between force and deformation is a measure of the stiffness of the material. If the length of the chord is L before the force is applied, it will be L + ΔL after the tension force has acted on it. The relative elongation (or shortening if compression stresses are acting) is called the *strain*, ε:

$$\Delta L/L = \varepsilon$$

where L = the original length of the member, ΔL is the change in length and, ε = the strain resulting from the applied force (Greek letter ε, "epsilon", mm/mm (in/in)). Since strain is a relative number, or ratio, it is given as a percentage (%) of the original length. Moreover, if force and strain are proportional, that is, if an increase of the force by a factor of two leads to an increase of the strain by a matching factor of two and so on, then the material is called *linearly elastic*. There is in that (very common) case a linear correlation between applied force P and the resulting strain ε. We can write:

$$\Delta L/L = \varepsilon = \text{constant} \times P \qquad (5.2)$$

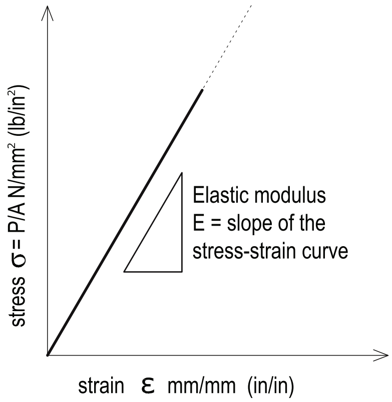


Figure 5.2
The relationship of stress and strain in a linearly elastic material. The slope of the straight line, σ/ε, is the modulus of elasticity, E.

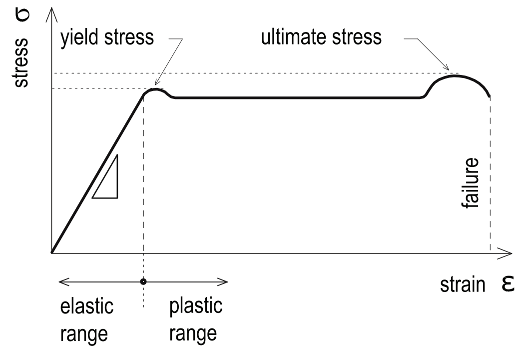


Figure 5.3
Simplified depiction of the stress/strain diagram for steel in tension. The diagram is not to scale. While the yield stress may be reached at a strain of 0.002 mm/mm (in/in), or 0.2 percent, failure is only expected to occur at 12–15 percent strain.

Which parameters influence the relationship between force and relative elongation? Obviously, the larger the cross-section of the member, the smaller will be the elongation caused by a specified force. The cross-sectional area A is, therefore, inversely proportional to the strain. Furthermore, the elastic properties of the material naturally also play an important part, since a very elastic material like rubber will experience a much larger deformation than a very stiff material like steel when the two materials have the same force applied to them and if their cross-sections are identical. We therefore need to introduce a parameter which is a measure of how *stiff* materials are, enabling us to compare materials and to calculate deformations exactly. This is the

- *modulus of elasticity* of materials, E, also called Young's modulus.[1] (See Table 5.1.) It represents a very important mechanical property of structural materials: the modulus is large for very stiff materials and small for more deformable materials. Since a stiffer material (high E) will experience smaller deformations, the modulus of elasticity is also inversely proportional to the strain. The preceding equation (5.2) thus becomes:

$$\Delta L/L = \varepsilon = P/EA \qquad (5.3)$$

where E = the material's modulus of elasticity, having units of N/mm^2 (lb/in^2 or psi), P = the applied force (N, lb), and A = the area of the cross-section (mm^2, in^2). But we have previously defined P/A as the stress in the member, and so it is convenient to write the equation above as:

$$\varepsilon = \sigma/E, \text{ or}$$
$$\sigma = \varepsilon E \qquad (5.4)$$

The modulus of elasticity in fact "regulates" the relative values of stress and strain in a material. This very important equation (5.4) is called *Hooke's law* after the British scientist Robert Hooke (1635–1703), who was the first to observe scientifically how forces and deformations in materials relate to one another. It is valid for linearly elastic materials, and offers an understanding of the elastic behavior of structural members. If we graphically depict the stress/strain relationship, letting the y-axis represent the stress and the x-axis the strain, the slope of the straight line is precisely the elastic modulus. (Fig. 5.2.)

For quite small axial deformations, meaning as little as a few thousandths of a meter, it is reasonable to think of most structural materials as linearly elastic. As stresses increase, however, so do deformations, and we might no longer find their relationship to be directly proportional. In fact, some materials, particularly metals, show a strange but nonetheless desirable behavior when stressed. Long before reaching their ultimate stress, the relationship between stress and strain changes dramatically. A so-called *plastic* range replaces the elastic range, in which large deformations happen without the stresses increasing significantly. (Fig. 5.3.) We say that at this level of stress the material *yields*. If an applied force is removed while a material is within the elastic range, the deformations will go back to the original condition. Once stressed to the point where the material is in the plastic range, however, the deformations will not go back to the original even when a force is removed; the structural member will in this case exhibit a permanent deformation. For steel, the potential plastic deformation is extremely large compared to the maximum elastic deformation, and in this plastic range the concept of a material having a constant modulus of elasticity is no longer valid.

Materials that experience large plastic deformations are called *ductile*. Ductile materials have distinct advantages as structural materials because, if unduly stressed, they will deform significantly before ever reaching their ultimate stress, and this deformation can likely be observed, warning of possible collapse. The plastic range also works as a reserve whereby the material is able to carry loads long after the elastic limit is reached, a phenomenon that can be taken advantage of in the design of buildings, having particular importance in seismic regions. Steel, in particular, has quite a precise and easily definable limit where the material moves from an elastic state into a plastic state. The stress level associated with this limit is called the material's

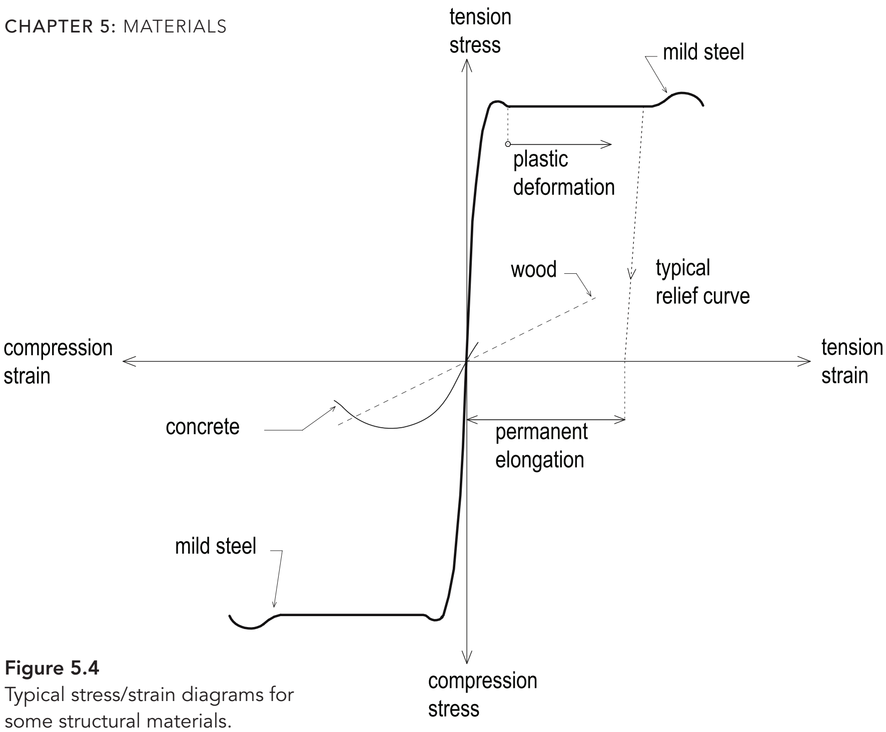


Figure 5.4
Typical stress/strain diagrams for some structural materials.

- *yield stress*. The yield stress represents an important material property which conveys much about our understanding of the behavior of some materials, particularly metals, when subjected to loads.

Conversely, materials that show no significant plastic range when stressed, but instead fail abruptly when the stress has reached a critical level, are classified as *brittle*. Brittle materials are in an elastic state until very close to the point of their ultimate stress, and since the elastic deformation commonly is quite small no visual forewarning of an oncoming material failure is typically observed. Glass is a typical brittle material, as is historically important cast iron. Great care should be shown when brittle materials are used to carry load.

The typical stress/strain diagrams for common structural materials are shown in Figure 5.4.

Aside from the material properties relevant to mechanical loads, other characteristics can be defined in relation to a wide range of physical influences; in this context, we need to recognize that the effect of temperature can be particularly important. All materials expand when the temperature rises, and having a clear notion of what actually happens when they do is a precondition to avoiding cracking and unwarranted deformations and stresses in materials and structural assemblies. Various materials' change of dimension when subjected to temperature change can be expressed by their

- *coefficient of thermal expansion*, α (Greek letter "alpha"). (See Table 5.1.) Materials that experience a substantial change of volume when the temperature changes are said to have a high coefficient of thermal expansion. If not greatly influenced by temperature changes, on the other hand, the material has a low such coefficient. For a structural element of length L subjected to a positive temperature change of ΔT (temperature rise), we can calculate the (increased) relative length $\Delta L/L$ as follows:[2]

$$\Delta L/L = \alpha \Delta T, \text{ or}$$
$$\alpha = \Delta L/(L\Delta T) \qquad (5.5)$$

The unit α for thermal expansion is, therefore, given as mm/(mm °C), or 1/°C (1/°F), which can also be written as $°C^{-1}$ ($°F^{-1}$). Incidentally, the reason why it is possible to reinforce concrete with steel bars without causing significant distress to the material when temperature changes occur is that the two materials have very similar coefficients of thermal expansion. If this were not the case, temperature changes would lead to the materials expanding or contracting very differently, introducing stresses that might result in cracking or other material distress.

From an ecological perspective, structural materials usually provide relatively few negative *environmental effects* per unit weight compared with a number of other building materials.[3] Nevertheless, since structural materials account for a large part of the total weight of a building, the choice of materials remains an important factor in a building's environmental profile. Of particular relevance are energy consumption and greenhouse gas pollution (CO_2). The primary energy consumption is the amount of energy required to first produce a unit weight of the material. This parameter is an indication of the energy that is effectively stored in different structural materials; recycling of materials, however, makes the absolute measure of the stored energy less precise and often far lower than what the primary energy consumption parameter indicates. Structural materials may represent approximately 30–40 percent of the total primary energy consumption needed to build a house, including transport,[4] but we should also recognize that from the lifecycle perspective, buildings have traditionally consumed far more

	Weight Density		Ultimate Stress				Modulus of Elasticity		Coefficient of Thermal Expansion	
			Tension		Compression					
	kN/m^3	(lb/ft^3)	N/mm^2	(10^3 psi)	N/mm^2	(10^3 psi)	10^3 N/mm^2	(10^6 psi)	10^{-6}/°C	(10^{-6}/°F)
Acrylic Glass (PMMA)	12	(77)	80	(12)	120	(17)	3	(0.4)	110	(61)
Aluminium	27	(172)	270	(39)	270	(39)	70	(10)	24	(13.3)
Clay Brick Masonry	19	(120)	-	-	10	(1.5)	10	(1.5)	5	(2.8)
Concrete	23	(144)	-	-	20-140	(3-20)	30-50	(4-7.5)	10	(5.6)
Glass Fiber Reinf. Polymer (GFRP)	19	(121)	500	(72)	500	(72)	45	(6.5)	25	(13.9)
*Glass Fiber Fabric, PTFE coat.**	*0.012 kN/ m^2*	*(0.25 lb/ft^2)*	*16N/mm*	*(91 lb/in)*	-	-	-	-	-	-
Float Glass	25	(160)	30	(4.5)	200	(29)	70	(10)	8.5	(4.7)
Polycarbonate (PC)	12	(77)	65	(9)	80	(12)	2.4	(0.3)	65	(36)
Steel - Structural (typical)	77	(490)	400	(58)	400	(58)	210	(27)	12	(6)
Steel - High Strength (typical)**	77	(490)	600	(87)	600	(87)	210	(27)	12	(6)
Stone	25	(160)	-	-	60-130	(9-19)	20-100	(3-15)	12.5	(6)
Wood (softwood, fiber direction)	5	(32)	30	(4.5)	30	(4.5)	11	(1.6)	5	(2.8)

* *Fabric thicknesses vary according to type and weight, but in most cases will be of the order of only a few millimeters.*

** *Steel wire used for winding into cables has an even higher tension strength, e.g. of the order of 1600 N/mm^2 (230 psi).*

Table 5.1
Examples of mechanical properties of a number of materials.

Note: Numbers given for stress and elasticity, especially, are highly approximate; in reality these vary substantially according to the quality/type of the material. For a more precise and detailed account, see specialized literature.

operational energy than what was embodied in the manufacture of their structure, and that is something that is unlikely to change very significantly for the foreseeable future.

We will in the following sections of this chapter discuss the mechanical and physical properties of the most common structural materials, describing each in some detail. For convenience's sake and for ease of comparison, however, some specific numerical values that can be associated with these materials have been collected and presented together in Table 5.1. Within the individual sections we will also discuss some other important material characteristics, any number of which may be relevant to their being selected and used in a particular building project. Since architecture is by nature

Illustration 5.12
Fondazione Querini Stampalia, Venezia, Italy (1963).
The Fondazione Querini Stampalia in Venice was renovated and reorganized in 1963 by that city`s famous architect Carlo Scarpa (1906–1978). The palazzo is accessed from the adjacent piazza by a lightweight stepped bridge. Beyond overcoming the relatively short distance 6.5m (21.3ft) between the ends of the bridge, the two steel support arches allow sufficient height for gondolas to pass underneath. The railings complete the very delicate detailing of the small bridge: they are composed of flat steel bars supporting two kinds of handrails. One is an oval-shaped lacquered teak railing for leaning on that has brass end plates; these are attached to the other handrail made of tubular steel and used for holding on to while crossing. The steel tube is closed at its ends with a polished hemisphere. Brass fittings to hold the teak handrail are secured with copper bolts, with the metal connectors carefully detailed to be flush with the teak. When Le Corbusier passed over this bridge while conceiving his unrealized city hospital for Venice, he is reported to have remarked: "Who is this fine craftsman?"
Architect: Carlo Scarpa.

Illustration 5.13
La Cathédrale de Beauvais, Beauvais, France (begun 1225; largely completed by 1272; major collapse in 1284; reconstruction continued through to mid-1500s).
The soaring height and remarkable light admission qualities of this structure are self-evident; its tall, open, internal space is made possible by an armature of transverse stone walls and flying buttresses on the outside.

holistic, a successful design will often result from a thoughtful consideration of many different aspects of material properties. Among these important concerns are appearance and sensory characteristics, for while the mechanical and physical properties of materials describe how *materials* react to the influence of the environment, their visual and tactile qualities involve how *we*, in turn, react to them. (e.g., Ill. 5.12.)

5.3 Lessons from History and Nature

Buildings were long built using traditional materials such as wood, stone, and clay brick and following assembly methods that were based on historical experience. Builders learned from past successes and perhaps especially from failures, and tried to correct for the latter by further experimenting and developing new construction methods and systems for each particular building type and specific design condition. In this way it can be said that within certain socio-geo-political realms, at least, there has been continuity in the development in the art of building over the centuries.[5]

The master builders of the Gothic period, for example, were skilled craftsmen in architecture, engineering, and detailed stonework. They were equally qualified as designers and technicians; sketchbooks and notes from that period show that they were also well traveled, and we can be sure that they kept an attentive eye open for new solutions.[6] La Cathédrale de Beauvais (begun in 1225) today stands as a symbol of the Gothic period's heaven-aspiring world view and consequent structural experimentation and expertise. (Ill. 5.13.) Without taking anything away from the remarkable aspect of this building, it is worth noting in the present context that the designers initially pushed the link between heaven and earth further than had been previously attempted for the 60m (198ft) height of the choir, exceeding the limits of the structure's

Illustration 5.14
Wells Cathedral, Wells, Somerset, England, UK (1239; tower repairs, 1338–1348).
Across-the-nave "scissor"-arch that was built to retroactively stabilize the original tower structure.
Master mason for tower repairs: William Joy.

capacity. A few of the columns failed, leading the choir to collapse spectacularly as is described in Erik Lundberg's book *The Visual Language of Architecture*.[7] After thorough examinations of these ruins at the time, it was concluded that the original columns needed to be strengthened and the builders made a fresh start; after many years the cathedral was reconstructed and it stands to this day as it was rebuilt.

Sometimes the onset of a collapse could be anticipated in advance of such catastrophic failure by observing increasing deformations and visible cracks in a building's masonry, and the problem could be dealt with before calamity struck. One of the more remarkable examples of such a reactive and inventive solution to loading problems can be seen, also in the Gothic motif, at Wells Cathedral. In this case the builders went too far by adding a weighty spire on to the central tower, whose four supporting pillars started to settle unevenly due to differing soil conditions under the tower legs' foundations. The spectacular and very specific solution that master mason William Joy introduced over the next ten years was to construct unique "scissor"-arches on three of the four sides of the crossing under the tower. (Ill. 5.14.) Supplemented by other semi-hidden buttresses, these scissor-arches (which effectively are a form of X-bracing that will be discussed in Chapter 10) simultaneously prevented the pillars from failing, redistributed the forces more evenly amongst the piers, and braced the tower against lateral wind forces. This dramatic feat of retroactive strengthening not only has kept the tower stable in the intervening 650 years without giving further cause for anxiety but it also proved to be an instant visual success and architectural attraction.

Aside from such experimentation and after-the-fact correction, the designers of structures over time have also speculated and theorized on structural capacity prior to construction. In his discussion of different types of bridges, Andrea Palladio (1508–1580) wrote that all beam bridges could have an unlimited span, as long as their internal proportions remained constant.[8] (Ill. 5.15.) While this proposal at first sounds quite logically appealing and, indeed, might have roughly worked for him within the limited range of spans that he was involved in designing, Palladio was actually wrong: beyond a certain limited span range, bridges designed by this rule will collapse. The reason why? Let us imagine starting with a freely supported beam having a cross-section of 1m x 1m (3.3ft x 3.3ft) and a length of 3m (9.8ft), but then doubling the beam dimensions so that the cross-section becomes 2m x 2m (6.6ft x 6.6ft) and increasing its length to 6m (19.6ft). The weight of the beam increases according to its change in material volume

$$1\text{m} \times 1\text{m} \times 3\text{m} = 3\text{m}^3 \qquad \text{vs.} \qquad 2\text{m} \times 2\text{m} \times 6\text{m} = 24\text{m}^3$$

i.e., the beam must carry roughly eight times the weight of the initial condition and now over twice the original distance. The maximum stresses that result from the bending of beams that carry only their own weight, however, are proportional to the cross-sectional dimensions and the square of the span and are inversely proportional to the beam width multiplied by the square of its depth (we will explain all this later on in Chapter 7), or quantitatively:

$$[1 \times 1][3]^2 / [1 \times (1)^2] = 9 \qquad \text{vs.} \qquad [2 \times 2][6]^2 / [2 \times (2)^2] = 18$$

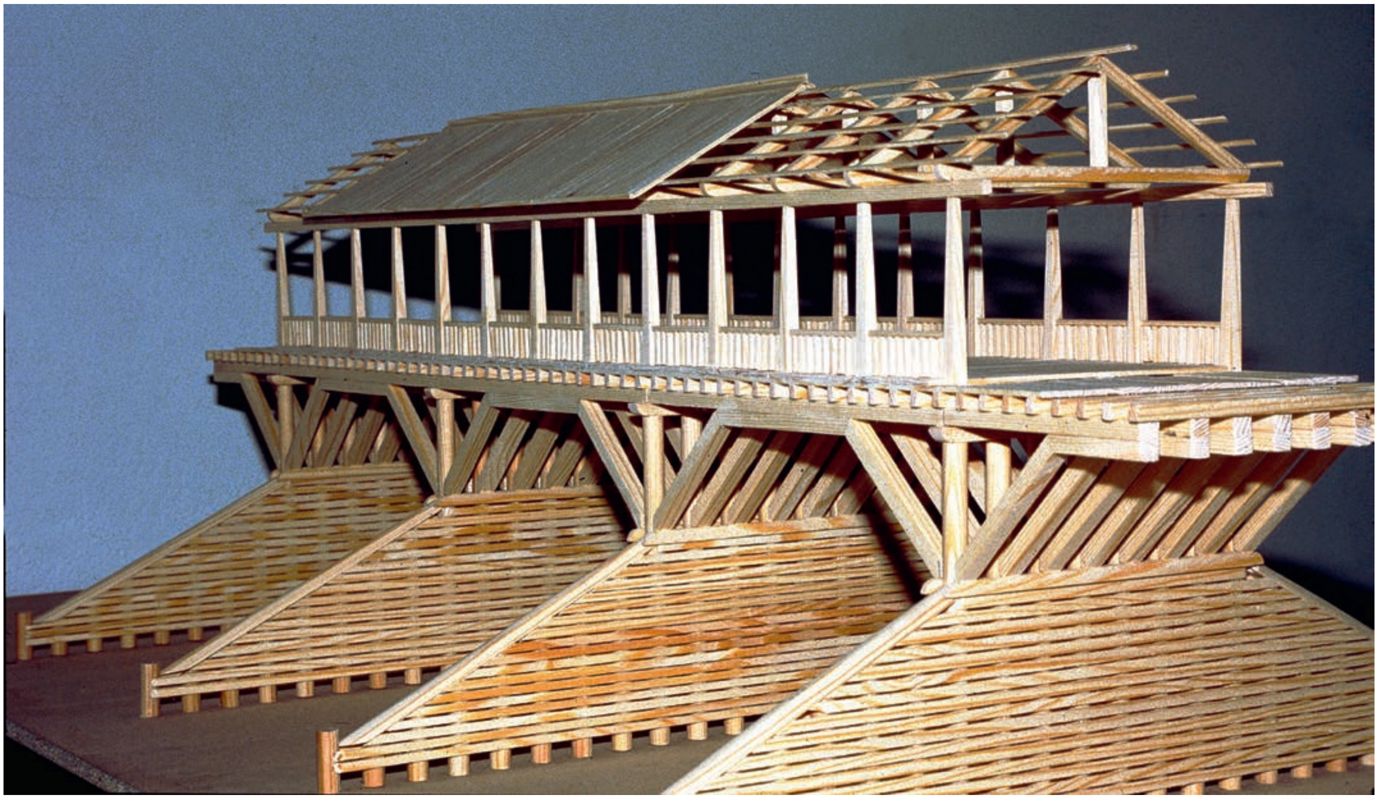


Illustration 5.15
Ponte degli Alpini, Bassano della Grappa, Vicenza, Italy (originally built 1569).
Designer: Andrea Palladio. AHO model.

i.e., "the maximum stresses in the larger, longer beam will be twice as large as in the smaller one" – clearly Palladio's rule about proportional increases to the dimensions of beams, when taken beyond a limited range of extrapolation, would have been a highly dangerous one to follow.

In fact, it was not that long after Palladio that Galileo Galilei (1564–1642) became the first person to formally propose that maximum spans for particular beam sizes do in fact exist, as he clearly demonstrated in his *Dialogues Concerning Two New Sciences*.[9] In this work, his "new science" is written as a dialogue between three men, Salvati, Sagredo, and Simplicio, who discuss a number of examples that show that the size of an object or a building has an important influence on the efficient use of construction materials; also, the point is made that certain types of construction materials are only applicable within a certain range of sizes. One of these well-known examples shows what the bone of a small animal would look like if it were to try and fulfill the same load-bearing function in an animal three times as large. (Ill. 5.16.) One might at first expect the bone simply needing to be three times bigger, but as with Palladio's beams the increase in load from the change in the volume and weight of the animal would be much greater than the load-carrying capacity of a bone having triple its cross-sectional width. The bone, therefore, would need to be disproportionately enlarged to withstand the actual load increase. Similar changes would

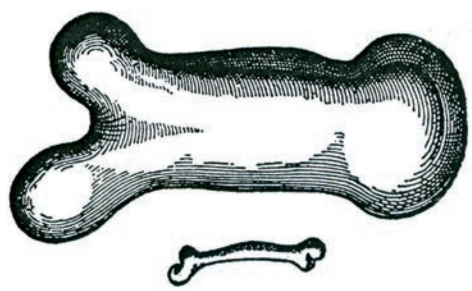


Illustration 5.16
Disproportionate bone size comparison sketches by Galileo Galilei in *Due Nuovo Scienze* (1638).

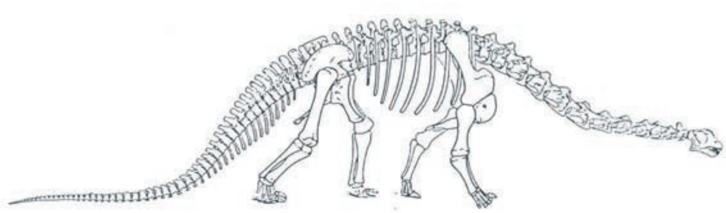


Illustration 5.17
Skeleton of a Camarosaurus dinosaur from Jurassic period (200–145 million years ago).

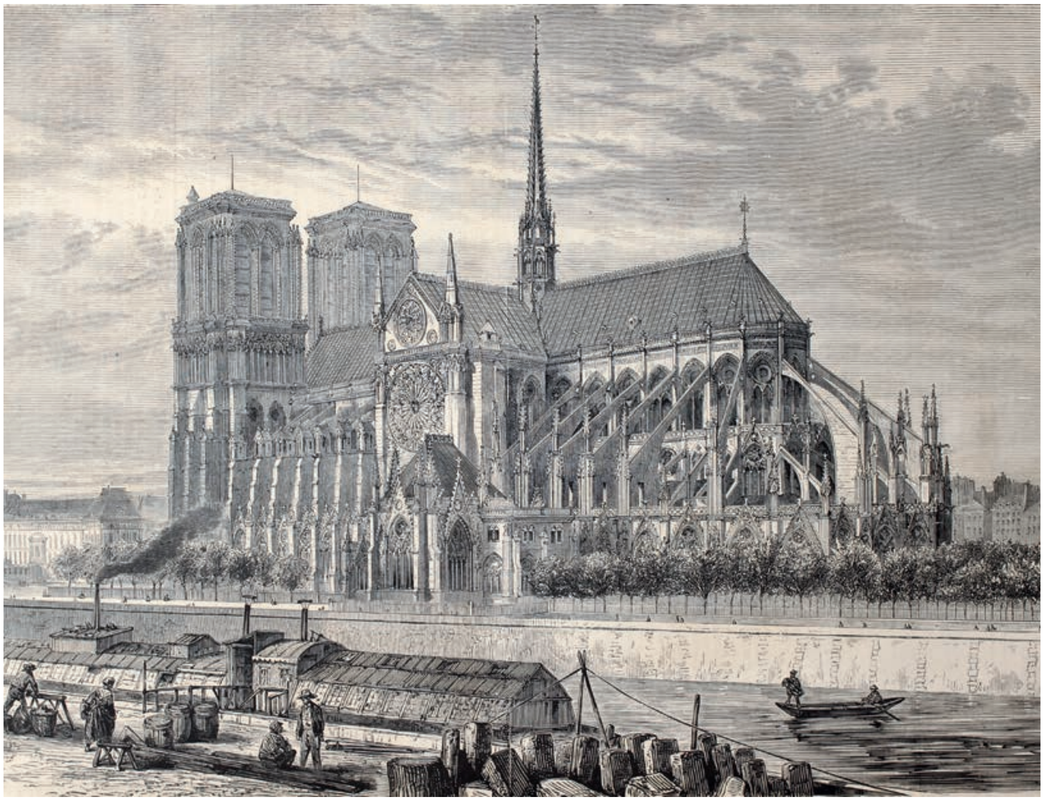


Illustration 5.18
Cathédrale Notre-Dame de Paris, France (1163–1345).
Among the first cathedrals to feature flying buttresses, these were added as a retrofit as the tall, thin walls of the nave and choir showed signs of cracking and distress.

Engraved illustration from drawing of Fichot and Gaildrau, published in "L'Illustration, Journal Universel", Paris, 1860.

occur with all the joints in the animal, and we can then begin to imagine a resulting different type of creature, considerably sturdier and heavier than was the original. This phenomenon has clearly manifested itself in the natural world: dinosaurs and their colossal bones are long since extinct, perhaps because they became too heavy, too slow, and lost the battle for survival against smaller and quicker species. (Ill. 5.17.) Large and heavy animals, such as elephants, have massive bone structures and move slowly, while antelopes with their spindly bone structure are remarkably fleet-footed. And on the weight-to-strength relationship, Salvati notes that a small animal will have a greater relative strength than a larger one, which he illustrates by pointing out that "a small dog probably could carry on his back two or three dogs of his own size, but a horse could not carry even one of his own size."

Given this discussion, it is interesting to consider that many buildings in the past were planned and built with the help of small-scale models on which certain simple load tests and evaluations were conducted. While these may have obviously been helpful in developing a first order of understanding of primary structural actions such as tension and compression forces, the above anecdotal discussion and modern-day theories and experiments warn us that the structural member proportions that work quite well for a model should not be directly and proportionately applied to a building that will be many times larger. In Hagia Sophia in Istanbul, for example, where we can be almost certain that a model was used in planning and construction during the sixth century, relatively recent investigations and calculations conclude that the existing foundations challenge the limits of capacity.[10] But such a lesson is not just about historical structures: even today building designers must take care not to rely too literally on simply being able to scale up physical model measurements and then expect the corresponding life-size building to function without distress.

To summarize our model-to-structures scaling discussion so far, one can obviously say that the proportions of a given structural member need to be carefully considered relative to the loads it is expected to carry, and that it may not be enough to know that a certain structural system works at one scale to know that it can safely be applied to another. Beyond this, though, the "brute force" approach of overcompensation for increases in scale may not be at all appropriate from an architectural point of view. It is at this stage that the material of which a structure is made may have to be completely changed or else the structural configuration as a whole may need to be revised. For instance, the architectural equivalent of the dinosaur bones discussed above would clearly have been at odds with the literal and figurative *admission-of-light* objectives of the Gothic designers as they built taller and taller cathedral naves, leading them to instead develop and use the highly innovative flying buttress system in order to dramatically *reduce* the bulk of the side walls. (Ill. 5.18.)

Today we have clearly moved beyond the scale model approach for trying to responsibly establish member sizes.[11] We regularly make use of computer structural analysis programs that are at our ready disposal and that can with amazing speed and accuracy determine the loads on a structure and the member dimensions that are needed to safely carry such loads. Nevertheless, despite such advances and the highly scientific, mathematical, and automated approach toward structural dimensioning that this process involves, it is important that structural design not become merely relegated to an isolated evaluation of such quantities as member forces, directions, and sizes. The design of structural members and the overall configuration of structural systems should still be seen in relation to a building project's expressive and programmatic objectives; moreover, the choice of member materials and system configurations can do much to support, both literally and figuratively, a designer's architectural intent. We should take heed from the Gothic masters' versatility and comprehensive vision.

5.4 Concrete, Stone, Earth, and Clay Bricks

Concrete

Concrete is a construction material that has been used by people for thousands of years. The first large-scale users of the material were the Romans, who employed it throughout their empire during the period 300 BC–AD 476. The primary reason for this was their incorporation of pozzolanic ash into the mix of materials that typically made up concrete – the name derives from the town, Pozzuoli, where the main deposits of ash were found. In the presence of water, the ash chemically reacts with other elements in the mix at room temperature to produce insoluble compounds that eventually harden and bind materials together. This property freed the Romans from the restrictions of stone and brick materials, and enabled the arches, vaults, and domes of their large-scale constructions, from the Colosseum and Pont du Gard (both largely concrete with stone masonry facing; for the latter, see Ill. 12.8) to the Pantheon (which remains today the world's largest unreinforced concrete dome, see Ill. 13.20.)

After the fall of the Roman Empire, however, the use of burned lime and pozzolana was all but forgotten from AD 500 to the eighteenth century. The extent of what had been forgotten in terms of this building material can begin to be appreciated when it is considered that a patent was granted in 1824 to Joseph Aspdin (1778–1855) for Portland cement, named after the color of the limestone that was quarried for this purpose on the Isle of Portland in Dorset, England; his son, William Aspdin (1815–1864), is regarded as the inventor of "modern" Portland cement due to his further development of the material in the 1840s. The French gardener Joseph Monier (1823–1906) received a patent in 1867 for the introduction of iron bars into flower pots and tubs made of concrete as a means to reinforce these against cracking and falling apart; Monier would quickly go on to apply this technological advance to the design of beams and even a bridge in 1875. He had exhibited his reinforced flower pots at the 1867 Paris World's Fair, however, and there they were seen by the French engineer and self-taught builder François Hennebique (1842–1921). Hennebique began experimenting with how this composite material could be applied to building construction, and by 1892 he too was granted a patent, this time for the first truly monolithic structural system in which columns and beams could work together as one structural entity. This technological advance spread quickly: during the decade that followed several thousand structures are said to have been built using the Hennebique system, from buildings to water towers to bridges, etc.

Today, by volume concrete is the most utilized building material. It is produced by mixing cement, water, and aggregate, the latter usually made up of crushed stone or gravel and sand. (Ill. 5.19.) *Aggregate* comprises approximately 70 percent of concrete's total volume, critically contributing to the material's hardness and compression strength. Lightweight concretes can be made by substituting light (e.g., expanded clay) aggregate for the crushed stone that is typically used. *Cement* is a fine, gray powder manufactured from a number of raw materials that are dominated by lime and gypsum, and what are called *hydraulic cements* set and harden after combining with water, thereby becoming an effective "glue" that binds the different materials together. The compression strength of concrete is highly dependent on the *ratio of water to cement* in the mix, which can be readily set according to specific needs. Beyond a certain minimum water content needed to ensure that all of the cement will chemically react and harden, it can be stated as a general rule that the less water is added to the concrete mix the higher will be its resulting strength; a typical

Illustration 5.19
Église Saint-Pierre de Firminy, Firminy, France (1973–2006). Polished cut surface of concrete displays stone aggregate distributed within its solidified binding "matrix" of water-activated cement and sand.
Architect: Le Corbusier, José Oubrerie.

water-to-cement weight ratio is approximately 1:2. There are also a large number of additives available that can be added to the base mix of cement, water, and aggregate in order to improve or alter the characteristics of concrete.

In terms of type of structural load-carrying capability, concrete on its own is strong in compression but very weak in tension. (See Table 5.1 and Fig. 5.4.) In structural elements, therefore, concrete is typically *reinforced* with steel bars so as to provide the resulting composite material with tensile *as well as* compressive strength. There are two primary ways of producing structural components: cast-in-place or precast concrete. *Cast-in-place concrete* (also called "*in situ*" concrete) is poured directly on site and allows for monolithic structural systems in a wide variety of shapes. By creating the appropriate formwork, concrete has a remarkable sculptural potential, so that establishing shape, size, texture, color, etc. can be significant design factors. *Precast concrete*, on the other hand, is generally made in a factory, where the mixing of concrete and casting of elements take place in a controlled environment and the finished structural element is later transported to the building site. Common precast concrete components are beams, columns, slabs, and wall panels, as well as unreinforced products like concrete masonry blocks. Notably, reinforcement of such precast elements may be of the *pre-stressed* type, whereby compression forces are introduced into the concrete cross-section by the *pre-tensioning* of steel strands before the concrete is cast and hardens in the manufacturing plant. Such a strategy is typically used to anticipate and counter the loading that will eventually be applied to the structural element by partly or totally eliminating tensile stresses in the cross-section. Pre-stressing of cast-in-place concrete can be done by means of the *post-tensioning* technique, whereby steel strands that are threaded through channels within the concrete are stressed after the concrete has hardened. Pre-stressed concrete is generally a more efficient material than is conventionally reinforced concrete, resulting in the opportunity for a lighter, more slender structure, or one that spans greater distances or carries greater loads.

Cast in a formwork of lumber, plywood, metal, fiberboard, or polymers, concrete will yield different surface textures. (e.g., Ill. 5.20, 5.21.) Pigmenting admixtures can also be used to add color to the material, while white concrete is made by using white cement and aggregate of white minerals. Form-ties help to prevent the two sides of the formwork from separating due to the outward pressure of the wet concrete when it is poured; the imprint of

Illustration 5.20
Faculty of Architecture and Urbanism, University of São Paulo, Brazil (1968).
A rough concrete finish left by formwork boards reflects a certain “honesty” of expression about the liquid-to-solid making of this material, as does – in this case – the sculpted/plastic form of the column supports for the large elevated volume. Their common surface finish also serves to unify these very different elements of the building structure.
Architect: João Vilanova Artigas and Carlos Cascaldi. Structural engineer: Escritorio Figueiredo Ferraz.

Illustration 5.21
The Royal Library’s “Black Diamond” extension, Copenhagen, Denmark (1999).
The smooth finish of the concrete structure that is exposed in certain places within this building works with the sleek glass and polished black granite surfaces of the exterior to offer a contemporary contrast with the traditional architecture and materials of the surrounding urban context.
Architect: Schmidt/Hammer/Lassen Architects. Structural engineer: Moe & Brødsgaard A/S.

Illustration 5.22
Cube House, Ithaca, NY, USA (2000).
A contemporary two-story residence whose perimeter walls can be clearly distinguished as being made of concrete masonry blocks.
Architect: Simon Ungers; Matthias Altwicker (project architect).

their anchoring will remain visible on the surface of the hardened concrete structure, however. For exposed concrete surfaces it is therefore important that both the location of the form-ties, as well as the configuration of the formwork panels, are thoroughly considered as part of the design process.

Fiber-reinfored concrete (FRC) contains short discrete fibers instead of steel reinforcing bars. The fibers are uniformly distributed and randomly oriented throughout the concrete. The "fibers" themselves can be made of very short steel strips or glass or synthetic filaments, all of which have the ability to change the characteristics of the hardened material. The use of fibers in lieu of steel reinforcing bars can enable the manufacturing of remarkably thin structural cross-sections, such as prefabricated shells, for example. A related material is called *ferrocement*. This is a composite material made by a plastering technique whereby mortar is put by hand over several layers of wire mesh; the result is a dense matrix of mortar filling the spaces of the reinforcing mesh grid. This technique allows for thin, delicate structural elements with a hard, dense surface texture. The Italian engineer Pier Luigi Nervi (1891–1979) is perhaps the best known among the pioneers of this particular material.

Concrete masonry units (CMU) are industrially produced, standard-sized rectangular blocks cast from concrete whose aggregate is typically sand or fine gravel. Low-density blocks may use industrial waste such as fly ash as the aggregate instead, which are thus known as cinder blocks in the United States. Typical block sizes vary somewhat from country to country but are approximately 410 x 200 x 200mm (16 x 8 x 8in) when used for structural purposes and these can be built up into walls in masonry fashion, with layers/courses of staggered blocks stacked one on top of another. (e.g., Ill. 5.22) The blocks usually have two hollow cavities which, if oriented vertically, can allow for the insertion of steel reinforcing bars to tie the blocks together once the voids are filled with concrete grout – something that is especially critical in regions that are prone to seismic activity. The cavities in the block also greatly reduce this material's weight per unit of volume. CMU can be made to have different finishes, opening patterns, and colors, and can be useful in various instances for providing thermal mass, fire safety, sound insulation, and visual screening; these qualities make them frequently used in dense, low-rise urban environments around the world, as well as in buildings with repetitive floor plans such as dormitories, hospitals, apartments, hotels, etc.

The production of cement is the main contributor to the negative *environmental effects* of concrete. Cement production releases large amounts of the greenhouse gas carbon dioxide, and requires a substantial amount of energy. Today, so-called low carbon concrete is also available, where measures have been taken along the whole production chain to minimize the carbon footprint. For reinforced concrete, environmental issues concerning the production of the steel reinforcing bars will also come into play. On the other hand, the durability of concrete structures is a positive environmental factor; since they can last for decades, if not centuries, any negative environmental factors should be considered in the context of the material's impressive life span.

Illustration 5.23
Inca stone wall, Peru (fifteenth century).
Large stones, with their inherent irregularities, were finely worked to fit and dry stacked without mortar.

Illustration 5.24
SGAE (General Society of Authors and Publishers) Central Office, Santiago de Compostella, Spain (2008).
Contemporary stone wall, forming outer edge of covered walkway. Seemingly "random" arrangement is actually carefully balanced, then held together by steel rods.
Architect: Ensamble Studio.

Stone

Stone is a natural building material that has been used by humankind for thousands of years, and it is clear from that that it is typically a strong and durable one. Pyramids persist, early temples still shelter, Gothic cathedrals soar ever skyward, while myriad forts, palaces, stadia, and amphitheaters alike remain, just as do infrastructural bridges and aqueducts built exclusively of this remarkable material. (e.g., Ill. 5.23.) Moreover, stone has many different colors and textures and it can be sculpted into exquisite artistic forms, giving the material many additional qualities for designers to consider in an architectural context aside from structural capacity – especially so historically, although not necessarily so. (e.g., Ill. 5.24.) Much depends on how the stone being used was naturally "made"; i.e., whether by igneous processes (e.g., granite), sedimentary (sandstone, limestone), or metamorphic (marble, slate), with granite, sandstone, and limestone often having been used for building walls because of their relatively widespread and abundant availability from surface quarries. The strength of stone can vary from certain types of relatively weak marble up to tremendously strong granite – with a numerical range that essentially matches concrete's, which should not be surprising given that the latter's strength is largely established by that of its stone aggregate. The earliest stone walls were composed by so-called dry stacking methods, in which irregular stones were carefully selected and fit together, although such walls typically lack long-term stability. Cut, shaped, and smoothed stones

Illustration 5.25
Guggenheim Museum, Bilbao (1997).
Thin sheets of stone cladding are attached to a braced steel structure that is responsible for carrying all the loads to the ground.
Architect: Frank O. Gehry and Associates. Structural engineer: Skidmore Owings & Merrill (SOM).

provide much better fit and stability, but were considerably more intensive in terms of the labor needed to produce them. Masonry stone walls improved on that yet again with stacked stones fixed together using mortar – a cement or lime and water mixture layered between courses to act as a binding agent. True solid stone walls are rarely built today because of the expense of the quarrying, cutting, transporting, and intensive labor involved to erect them, even as stone remains a favored material for building façades – but these are usually only a thin veneer that has been glued or mechanically fastened to other supporting elements that do the structural load-carrying work. (e.g. Ill. 5.25.)

Rammed Earth

The ancient techniques of employing earth in building structures have worked their way into contemporary architecture, albeit in modified form. Roughly, we can say that there are two main techniques available in this context: making un-fired earth bricks and blocks is one option; the other is the production of monolithic structural elements by using a ramming technique. *Adobe* in its modern form involves the manufacturing of load-bearing bricks or blocks made of tightly compacted earth, clay, and straw. So-called *CEB*, or compressed earth blocks, contain no straw, but add lime or cement as a stabilizer to hold the material together. In the *rammed earth* technique, the soil is mixed with cement, water, and waterproofing

Illustration 5.26
Windhover Contemplative Center, Stanford University, Stanford, CA, USA (2014).
Rammed earth walls made from soil from the site compose the structure as well as the interior and exterior spaces of this one-story nondenominational center intended for relaxation and silent contemplation on this university campus; distinctively uneven horizontal layering is a visual pattern that contributes to the relaxing atmosphere of this building.
Architect: Aidlin Darling Design; Andrea Cochran Landscape Architecture. Structural engineer: Rutherford & Chekene.

additives so as to form primary structural wall elements that are manufactured *in situ*.

It is perhaps the *rammed earth* technique which has the most interesting potential in a contemporary structural and architectural context, with its unexpected ability to form an earth-based material into a firm, hard vertical wall surface that also has significant compression strength. (e.g., Ill. 5.26.) Stabilized rammed earth uses the natural subsoil (free of humus) or crushed stone in a (damp) mixture with 6–7 percent cement as a stabilizer; its compression strength is moderate, but certainly adequate for low-rise structures. When hydraulically compacted in removable formwork, the finished surface of the wall usually has no need for additional protection. Rammed earth can be made very compact if the particles are of the right size and there is a proper distribution with particles of different sizes. The color of the finished material is basically that of the earth from which it is made, which can lead to a visually interesting layered appearance, to say nothing of its obvious visual connection to the ground upon which it sits; the use of white cement, on the other hand, can lighten all colors.

The materials of rammed earth construction and its low-tech manufacturing process are environmentally friendly, with quite low embodied energy; the material also has a high potential for eventually being recycled.

Clay Bricks

Clay materials that are dried and fired are called ceramics. As a group, these materials can generally be characterized as being hard, brittle, and heat resistant. Clay bricks are made in a series of steps involving the preparation of the raw material, the extrusion of the soft clay into long strips, before cutting these into short pieces that will later become individual bricks. After drying, these bricks are fired in an oven at well over 1000 °C (1832 °F).

Clay bricks are one of the oldest of building materials, especially if we are considering materials that involve a certain degree of human intervention in order to make them. Historic architecture in most cultures depended to a large extent on clay-brick masonry and its particularities for the making of built form. It is impossible to think of Roman architecture, to take one example, without recognizing the dependence of many vaults and arches as well as walls and pillars on the particular strength properties of the clay brick. In fact, structural principles and the shape of structural elements in Roman classical architecture are testimonies to the low tension strength and the good compression strength of clay-brick masonry.

Clay bricks today are produced in different sizes and shapes, whether perforated or solid (e.g., Ill. 5.27, 5.28, and see also Ill. 5.9). The density of the material depends on the composition

Illustration 5.27
Robie House, Chicago, USA (1909).
The essential low-slung horizontal form of the house is accented by (a) the continuous concrete bands at the tops of the walls and parapets as well as by the low, cantilevering roofline. (b) the extra-long bricks that were used for the exterior walls. Also, the horizontal mortar joints are slightly recessed and form continuous lines while the vertical joints are filled flush with the brick and blend in with its red/brownish color, thus preventing vertical lines from being established.

Architect: Frank Lloyd Wright.

Illustration 5.28
Church of Christ the Worker, Atlantida, Uruguay (1960).
Flat clay bricks are used in different ways and orientations in the curving sidewalls, the double-layered undulating roof surface with concrete mortar and integrated steel reinforcing bars, as well as the angled light-screening façade elements.

Designer and structural engineer: Elaudio Dieste.

of its raw material, and clay bricks with the highest densities are those with the highest strength. More important than the strength of individual bricks, however, is the strength of the brick *masonry* that these bricks are built up into. When bricks are laid in mortar, the material becomes anisotropic, having different properties in different directions. Furthermore, the ultimate compression stresses for the combined brick/mortar masonry material is typically significantly lower than that of the individual clay bricks themselves.

The natural colors of clay bricks vary according to regional differences of the chemical compositions of the clay. Clays with a high content of iron become naturally red after firing, while clay with high lime content tends to have a yellow color. In addition, the duration in and temperature of the oven will affect their final color: the longer the time and higher the temperature of drying, the darker and browner the brick will become. Metal oxides can also be mixed in with the clay before burning in order to make other color variations.

Because of their considerable density and particular material composition, masonry made of clay bricks will have very good fire-resistance and sound insulation properties. Clay bricks also exhibit exceptionally long durability and low maintenance requirements, resulting in an environmentally friendly material over the long term, in spite of the considerable energy consumption needed initially for the high-temperature firing process.

5.5 Steel, Iron, and Aluminum

Steel and Iron

Iron alloys constitute the most important metals in contemporary architecture, and foremost among them is steel.[12] Common to these metals is their small carbon content which, in spite of its relatively modest weight in comparison to iron, heavily influences metals' properties. The first major breakthrough for metals into the structures of architecture was by means of the introduction of *cast iron*, a material that was able to be manufactured in large quantities when coke-fired ovens were introduced toward the end of the eighteenth century. The resulting metal is hard and strong in compression, but is brittle and performs poorly when subjected to tension and bending.[13] Historically, *wrought iron* is the precursor of modern steel. By being a metal that is easily forged while hot and also a ductile material, wrought iron found interesting applications in structures in the nineteenth century; a case in point is the Eiffel Tower, made of wrought iron in 1889 instead of steel, which was considered too expensive at the time.

Historically, Sir Henry Bessemer is credited with being the first to introduce the manufacturing of *steel* by modern methods in about 1856. Steel is the end product of a process that begins with the raw material that is found in nature called iron ore. This material in the form of pellets is mixed with coke[14] and limestone and fed into a blast furnace, a process that isolates the iron from the ore. Two alternative methods are used to produce steel from iron; either by means of a converter process or an electric arc furnace technique. The resulting metal of the converter process is mostly so-called mild carbon steel. Alternatively, recycled iron and steel is fed into an electric arc furnace where the metal is transformed into high-quality special steels.

The steel products of primary interest for architecture are rolled profiles, tubes, and steel plates or sheets. (e.g., Ill. 5.29, 5.30, 5.31) Structural profiles are hot rolled or cold formed, with the latter type used for thin sections manufactured from sheet metal. *Hot-rolled profiles* are made by deforming the steel while red-hot. This is done by a series of rollers working on the metal in a number of cycles, gradually making the cross-section thinner and smaller, resulting in a prescribed section of standard measurements. Other hot-rolled products include *steel plates* of different thicknesses and *steel bars*. *Tubes* and rectangular hollow sections are manufactured from folded sheets and are welded after shaping, or else are made "seamless" by means of a process in which the material in the center of a solid section is punched out along the length. Steel *wires* are manufactured by repeated drawing of a rod through progressively smaller dies or, traditionally, through holes in special draw plates, thus reducing the cross-section to the desired diameter. *Wire strands* are made by twisting together several wires like a helix, and several strands together in turn make up a *wire rope* by employing a similar twisting process. (Wire strands and ropes are particular to tension cable structures, and will be discussed further in that context in Chapter 6, Section 6.8.)

Carbon steel for construction purposes is highly ductile, easily forged, and has excellent welding properties; in addition, it works

Illustration 5.29
"Midday" (1960).
Short segments of different profiles of rolled structural steel are displayed, with the classic shape of an I-beam standing up in the foreground.
Sculpture by Anthony Caro.

Illustration 5.30
Hotel Arts, Barcelona, Catalonia, Spain (1992; built as part of the Vila Olímpica for the 1992 Olympic Games).
Detail of external structure made of carbon steel rolled profiles. Differently shaped sections as well as various bolted and welded connections can be seen.
Architect and structural engineer: Skidmore, Owings & Merrill (SOM).

Illustration 5.31
Tondonia Winery Pavilion, Haro, La Rioja, Spain (2006).
Sides of wine-flask-shaped tasting pavilion are made of rolled steel plates precisely welded together and stiffened with orthogonally arranged ribs. Steel is painted, giving interior a light, bright atmosphere that sets off the winery's *fin-de-siècle* style wooden pavilion that had been brought to the 1910 Brussels World Fair. (See also Ill. 7.15.)
Architect: Zaha Hadid Architects. Structural engineer: Jane Wernick Associates.

Illustration 5.32
Stuttgart Airport Terminal, Stuttgart, Germany (1998).
Cast steel joint in the structural "trees" of the terminal building. Cast steel has higher manganese and silicon content than carbon steel, as well as high carbon content. This provides this type of steel with a good form-filling ability. In addition to lending itself to casting, the best of cast steels have strength and ductility that are comparable to those of carbon steel. It can be welded, including to carbon steel.

Architect von Gerkan, Marg und Partner. Structural engineer: Schlaich Bergermann und Partner; Weidleplan Consulting Gmbh.

Illustration 5.33
Tate Modern, London, UK (2000).
Painted surface of riveted built-up steel column was originally part of Bankside Power Station, London, UK (1953, 1963), now converted to Tate Modern. (See also Ill. 5.6.)

Architect: Herzog & de Meuron (for conversion); Sir Giles Gilbert Scott (for previous power station). Structural engineer: Arup (for conversion).

very well in compression as well as in tension. Other steels or iron alloys of interest for architecture are so-called *cast steel* (e.g., Ill.5.32) and *ductile iron* (spheroidal graphite iron). In ductile iron, the molecular form in which the carbon occurs reduces the brittleness characteristic of normal cast iron, and this results in an iron having higher strength and ductility. Compared to and unlike cast steel, which has to be reheated after casting, ductile iron can be made into finer and more delicate shapes. We should note, however, that unlike cast steel, ductile iron cannot be welded.

Ferrous metals are highly susceptible to corrosion since iron oxidizes easily.[15] As a result, steel structures that are left exposed must have their surfaces protected, and the most common form of protection is provided by *painting* it (e.g., Ill. 5.30, 5.33.); this is especially necessary in a wet or aggressive environment. Paint provides a barrier that restricts the transport of water, oxygen, and ions, all of which cause corrosion to occur. *Stainless steel*, on the other hand, is an example of an alloy with a higher amount of the metals chromium and nickel as well as a higher carbon content than has carbon steel, all of which are helpful to prevent corrosion. *Cor-Ten steel* is a particular weathering steel type that is an alloy of iron, carbon, copper, chromium, silicon, and manganese; the surface oxidizes quickly and forms a dense, passive barrier against further corrosion. The surface becomes textured with colors ranging from brown to orange/red or purple. (e.g., Ill. 5.34.) Yet another important way of protecting steel from rusting is by means of a galvanizing process. *Hot-dip galvanizing* involves dipping steel components into a bath of molten zinc. A thin coat of an iron/zinc alloy is created on the steel surface, while a topcoat of pure zinc is exposed to the environment. Galvanized steel has a characteristically reflective, crystalline, or speckled surface pattern which oxidizes to a self-protecting matte gray color.

Illustration 5.34
"Shaft", Oslo, Norway (1989).
Cor-Ten weathering steel's oxidized surface can take on striking coloration and patterning.
Sculpture by Richard Serra.

Steel exhibits a very significant reduction in strength and stiffness at higher temperatures and so in most cases of architectural application steel structural members will need fire protection of one sort or another. The most common types are either a fire-resistant paint or else a protective cladding made of fire-resistant material such as gypsum board, cementitious coatings, or sprayed fire-resistive material (SFRM) coatings. Oversized cross-sections will also increase the time it takes for the steel to reach critical temperatures. A more unusual protection method of steel tubes consists of letting water fill the hollow structural profiles in an effort to keep the steel temperature down. Hollow profiles filled with reinforced concrete is also an option, where the idea is that the concrete core becomes structurally active as soon as the steel around it loses strength and stiffness as a result of the higher temperature. Encasing steel structural members in reinforced concrete is also a well-established method of fire protection. Many of these methods have their aesthetic implications, of course, which need to be taken into account by the designer.

From an environmental standpoint, steel production and manufacturing create significant amounts of greenhouse gases. Moreover, the amount of energy needed for their initial manufacture is substantial. Steel is, however, one of the construction materials that lends itself best to recycling, and the energy required to produce steel members, as well as the carbon footprint, are significantly lowered when it is being produced from recycled materials. And although a detailed consideration of this topic is beyond the scope of this book, it is worth noting that a focus on required energy is only meaningful if also we take into consideration the actual work (or load-bearing function) that we can demand from a specific amount of material. It should be clear that steel, with its high load-bearing capacity and the resulting minimization of the amount of material required, benefits from incorporating this perspective into overall ecological considerations.

Aluminum

Commercial production of aluminum started in the 1890s, but the then newly available metal was primarily employed for kitchen hardware. It took as long as the 1930s before this material found its way into the building sector, with aluminum presented as a new option for window frames. Aluminum is actually the most abundant metal element in the Earth's crust, but it is typically bound in natural mineral compounds such as bauxite ore; the metal is made by means of an electrolytic process that extracts aluminum from these mineral compounds, a process that is highly energy intensive.

Today aluminum is the second most commonly used metal in the construction industry after steel, although it still remains unusual for use in primary structural members. (e.g., see Ill. 6.11, 8.18.) It is a silvery white metal that is easily forged. It is very light and has a favorable strength-to-weight ratio. Pure aluminum is too soft for structural use, however, and for construction purposes it is commonly alloyed with copper, manganese, zinc, silicon, and magnesium. The particular mix can be designed to suit specific purposes, such as improving casting abilities or adding strength. Aluminum remains quite soft, however, and has an elastic modulus E that is about one-third that of steel, indicating that it is a material that is inherently much less stiff. Therefore, if deflection is a critical

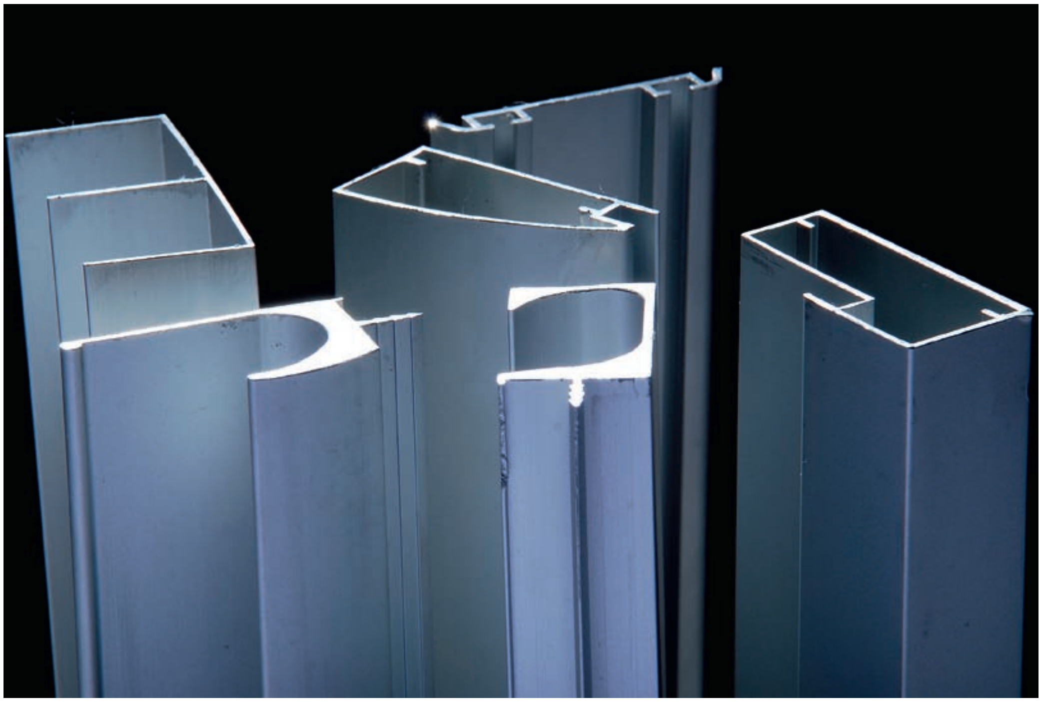


Illustration 5.35
A distinctively shaped extruded aluminum profile. Great complexity and precision of cross-section are possible with this manufacturing technology. The material appears with its well-known grayish, silvery color which is neither shiny nor dull.

design issue in a particular instance, an aluminum cross-section may turn out to need to be quite a bit larger than that made of steel, in spite of the metal's relatively low weight-per-unit-volume (which reduces self-weight due to dead load.)

The shaping of aluminum components employs some of the same methods that are used for working steel, including casting, hot rolling, and cold forming. In addition to those methods *extruding* the metal through a die allows for more complex shapes and forms to be created. (e.g., Ill. 5.35.) The die tool is a cylinder of steel with a hole in the shape of the desired profile. A massive aluminum bar, heated to 500–550 °C (932–1022 °F), is forced through the tool, extruding profiles in continuous lengths of up to 40m (130ft). The working of aluminum takes place at a much lower temperature than steel because its melting point is lower. The actual production of structural elements from bulk aluminum is therefore less energy-consuming and far cheaper than working steel.

There are other pros and cons to the metal that need to be considered. Some disadvantages, for example, are greater thermal expansion and lower fire resistance than steel; on the other hand, aluminum has excellent corrosion-resistance properties, although not when in direct contact with other, more noble metals – a situation that can result in so-called galvanic corrosion.[16] Also, as we've already stated, the primary extraction of aluminum from bauxite is an extremely energy-consuming process; yet again, aluminum recycles very well, resulting in large reductions of required energy when the material is reused.

5.6 Wood and Cardboard

Wood

Wood is a "natural" material, which means that very little processing of the material itself is needed before using it for structural members. Wood is basically ready for use in the state we find it in nature, which goes a very long way to explaining its long-standing and widespread use in various contexts and for different purposes. The most commonly used tree species for structures are *softwoods* such as spruce, pine, and fir – which are all characterized by being light and relatively strong materials.[17]

However, in spite of its being so familiar and of such common origin, wood is in fact quite a complex material. To begin with, it is anisotropic, which means that it has different visual and physical properties in its two main directions. At a micro-scale, wood's tube-like cells can be considered to be a structure in and of themselves. (Ill. 5.36.) In fact, an analogy to the microstructure of wood could be a bundle of long, thin, hollow drinking straws that are bound together side by side and that follow the direction of the tree trunk. Because of this particular cellular structure, the material has very different properties along the length of the wood grain (or straw!) than perpendicular to it; as a general rule the stiffest and strongest direction by far is parallel to the grain, in the so-called longitudinal direction.

The density of softwoods is less than that of water; which is the reason why wood typically floats. Wood expands (swells) as

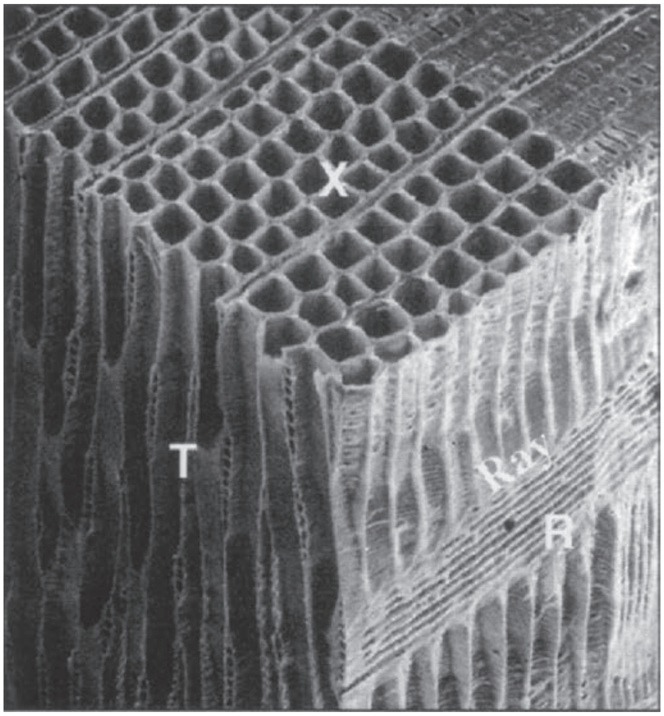


Illustration 5.36
The microstructure of wood. Cross-section features long tube-like cells.

Illustration 5.37
Tautra Maria Convent, Tautra Island, Norway (2006). Laminated wood members are used for the main roof support structure. Smaller sawn lumber used to create a diagrid to carry the glass roof. Angled ends of the glulam members are connected with hidden steel plates. The resulting material quality as well as light and shadow effects in this space are remarkable.

Architect: Jensen & Skodvin. Structural engineer: Dr.techn. Kristoffer Apeland AS.

its moisture content rises and shrinks as it lowers, and more so in the plane of a typical member cross-section; i.e., perpendicular to the grain. The *coefficient of thermal expansion* also varies relative to the direction of the grain, with the largest dimensional change from temperature also to be expected perpendicular to the wood grain.

In spite of all this, wood is a most efficient structural material in the sense that it has much to offer in terms of strength relative to its weight. Perhaps surprisingly, the tensile strength-to-weight ratio for softwoods like spruce, pine, and fir is quite significant, easily competing with that of steel. Tension and compression capacities along the length of a member are comparable, although the tensile strength when tested on faultless wood specimens is somewhat higher than that of the corresponding compression stress, since the cellular tube structure in compression is susceptible to buckling failure. (There will be much more on this mode of failure in Chapter 8.) The micro-tube cellular structure, however, leads to very different compressive capacities for wood whether being considered parallel or perpendicular to the grain; with the latter being considerably lower due to the tendency for the tubular cells to flatten and crush when transverse loads are applied to a wood structural element. We should also be aware that wet lumber can be expected to have 25 percent lower strength than dry lumber.

The basic method for manufacturing most wood products for use in the construction industry includes sawing the log longitudinally, producing what is then called lumber. *Structural lumber* is judged on its density and strength, including the effect of knots, splits, and other natural deformities, all features that compromise the structural integrity of the material. With respect to cross-sectional dimensions, wood members are cut from the original log into a variety of different and commonly used sizes; those that are larger than nominally 125 x 125mm (5 x 5in) are commonly called *timber* or structural solid wood. Taken together, sawn members of all sizes still comprise a large percentage of all wood structural elements that are used today; in North America, for example, small-dimensioned sawn lumber is the most basic unit of construction for the platform framing technique that is used for standard residential home construction.

To provide greater strength and length in structural timber members as well as larger structural dimensions than are typically possible today from one log, and thereby offer other architectural design possibilities, the use of *laminated wood* is an attractive option. Structural members in laminated wood (also called *glulam*) are built up of layers of wood that are glued together to form rectangular cross-sections of specific dimensions suitable for use as beams, columns, trusses, etc. (e.g., Ill. 5.37.) The glue provides full static

Illustration 5.38
Hedmark Museum, Hamar, Norway (1971).
Glulam beam supporting roof in the northwest corner of the museum curves upward toward the roof rafters to give headroom clearance for the elevated platform of the exhibition space just below.
Architect: Sverre Fehn. Structural engineer: Terje Orlien.

interaction between the wood layers, guaranteeing the strength of the cross-section as if it were made from one, homogeneous piece of material. Indeed, since the different layers of 30 to 40 mm (roughly, 1.25 to 1.5in) thicknesses of wood used in the glulam are made of graded lumber in which knots and splits have been carefully avoided, the quality and strength of laminated wood is generally higher than that associated with timber of the same dimensions. Laminated wood can also easily be curved to form bent beams or arches; the hardened glue will cause the member to maintain its shape. (e.g., Ill. 5.38.)

Wood panel products are another interesting and efficient use of the material that have a variety of applications. Among the different wood panels that are specified either by their thickness or a span rating, *plywood* is structurally among the most interesting. This manufactured product is made up of multiple wood veneers (thin sheets) that are glued together, commonly in such a way that adjacent layers in the panel have alternating grain directions at 90ğ to each other. This provides the panel with nearly identical strength and stiffness properties in both orthogonal directions.

Structural insulated panels (SIPs) are industrial products that consist of a sandwich of two layers of wood panels, usually OSB (*oriented strand board*) or plywood, with an insulating layer of foam in between them. The rigid insulation core and the facing panels perform as web and flanges respectively, securing adequate strength and stiffness for SIPs to find applications in relatively short height/span walls, floors, and roof surfaces.

Beams may also be produced from glued veneers. So-called *laminated veneer lumber* (LVL) uses multiple layers of thin veneers or OSB cut into rectangular strips that are glued together. Such beams have a grain orientation parallel to their length, and are less likely to warp, twist, or shrink than conventional lumber. By being more uniform, they are also stronger.

Other engineered wood products include the *timber I-joist*, a built-up wood beam with enlarged flanges and relatively thin web whose overall profile resembles that of a rolled steel section. Such beams may replace conventional sawn lumber for floor structures involving long spans. The flanges may be manufactured from lumber or glulam, with a web made of plywood or other wood-based panels.

A more recent structural wood product is the thick planar *solid wood panel* element that can be used to make a floor or roof slab or else wall elements capable of carrying both vertical loads as well as in-plane and out-of-plane horizontal loads. (e.g., Ill. 5.39.) A number of types of such elements are produced, but common for all is the use of boards arranged in layers and that are bonded by glue, or by wooden dowels. Elements can be produced in a factory by a process similar to that used for laminated wood, whereby a

Illustration 5.39
Pulpit Rock Mountain Lodge, Strand, Norway (2008).
Solid wood panels are used to create a series of transverse walls, which works spatially to subdivide space into individual rooms; in this communal space, however, these walls are partially opened up to become distinctive frames.
Architect: Helen & Hard. Structural engineer: Wörle Sparowitz Ingenieure.

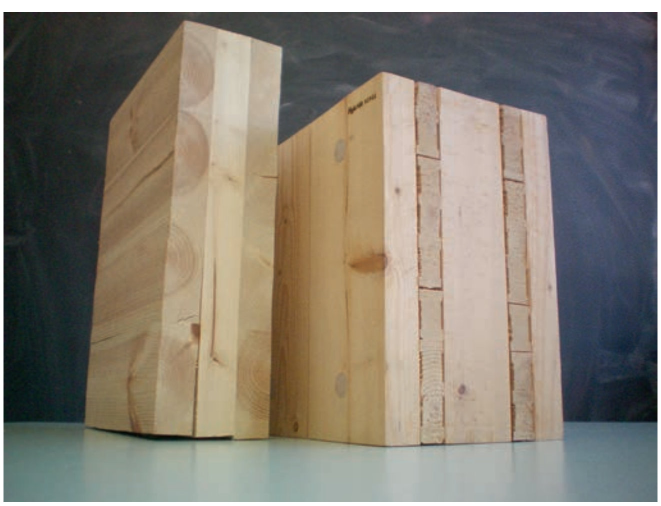


Illustration 5.40
Solid wood samples. Two samples formed using different manufacturing technologies: glued and doweled.

large number of boards are stacked one on top of another until a large, thick solid wood panel element is formed. Alternatively, layers of boards can be stacked with alternating layers oriented in orthogonal directions, forming a structural element having the same strength in both directions, which recalls the manufacture of plywood but in this case using boards instead of thin sheets of wood. (Ill. 5.40.)

While it seems self-evident that wood may catch fire and burn, depending on the member dimensions wood structures actually can perform relatively well when subjected to fire. One reason for this is that a layer of charcoal is produced on the external wood surface when it burns, and this has the ability to slow down the burning of the remainder of the wood member. Second, wood burns at a predictable speed which makes it possible to calculate the time it will take before the member cross-section is reduced to a size that will no longer be capable of resisting the loads being carried. Member sections can thus be sized to withstand fire for a prescribed duration of time (that is established by building codes in order to allow for evacuation.)

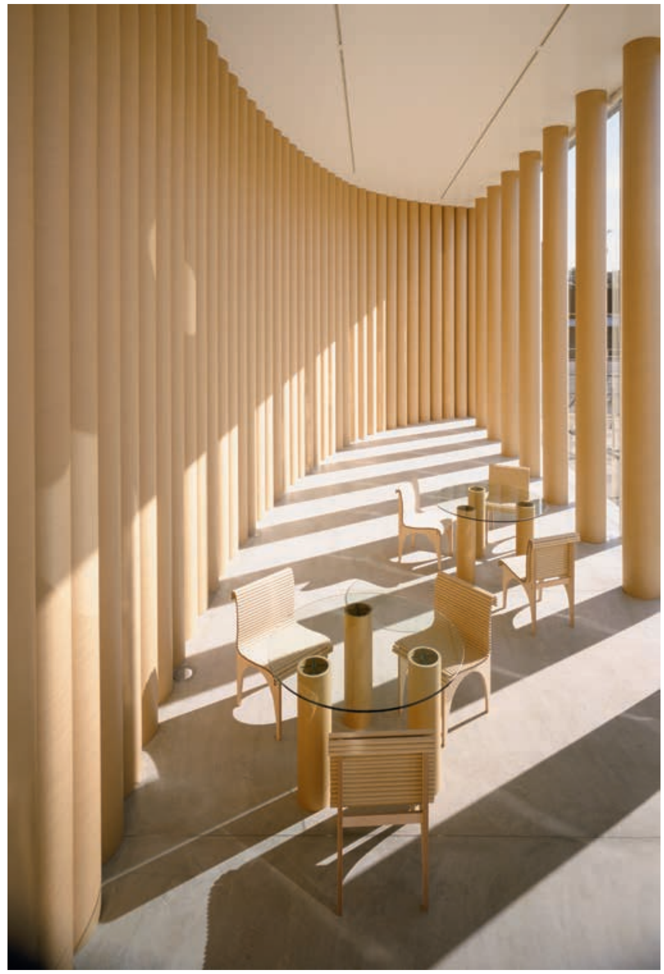


Illustration 5.41
Miyake Design Studio Gallery, Shibuya-Ku, Tokyo, Japan (1994). Side-by-side cardboard tube columns effectively form a curved backdrop to this temporary gallery, while spaced further apart they open the space to the outside; smaller diameter tubes serve structurally at a furniture scale as table legs (columns) and chair seats (beams).
Architect: Shigeru Ban.

Wood is obviously a renewable structural material and, moreover, is one that traps its embodied carbon dioxide (CO_2), which makes it doubly environmentally friendly. Also, if the material is not transported across large distances, the energy consumption associated with its production, manufacture, and delivery to the building site is fairly low compared with a number of other structural materials. The use of glue to produce glue-laminated wood, plywood, and solid wood panels will, however, affect this very favorable energy and carbon footprint balance to a smaller or larger degree, as will the extensive use of steel detailing that often accompanies larger wood structures.

Cardboard

Cardboard is the end product of a multi-step manufacturing process in which wood is the originating raw material; i.e., it is made from a number of layers of recycled paper that are glued together. In recent years cardboard has found interesting applications beyond its conventional packaging use by being formed into load-bearing structural elements that are used in several high-profile architectural projects. Certainly the architect best known for much of the pioneering development of cardboard as a structural material that can be used for building purposes is Shigeru Ban. (e.g., Ill. 5.41, see also Ill. 6.25, 6.26, and Ill. 9.14.)

We typically find cardboard that is used structurally to be in the form of hollow tubes (Ill. 5.42); these can be effectively used as individual columns and other straight-axis elements, such as angled struts and beams. Lines of tubes placed side by side can form walls of various configurations in plan, as in the Miyake Picture Studio Gallery (seen in Ill. 5.41). The cardboard tubes themselves cannot be curved along their longitudinal axis, however, so arches and other inherently bent structural forms need to be created as a connected series of many short, straight-line segments. Cardboard panels can also be created, either as a solid made up of many flat sheets of the material glued together or else in a mostly composite hollow form, with a honeycomb core made up of short transverse tube segments with multilayered cardboard sheets making up the panel's outer surfaces. These can act as flat panel segments of a folded roof structure, for example; an added benefit of this arrangement is that the trapped air within the honeycomb will significantly improve such a panel's insulation properties.

Tests show that the ultimate compression and tension stresses of cardboard have about the same value, with ultimate stresses roughly one-quarter to one-third of that of softwood's for compression in the direction of the grain.[18] The material is considerably more elastic than wood, though, bending and flexing far more when subjected to load if members with equal dimensions are compared. The mechanical properties of cardboard strongly decrease with increased moisture content, and so techniques for ensuring that the material stays dry must be included in the design.

We are well aware that paper burns easily and so we should logically be concerned about this aspect of structural cardboard, but as does wood in a fire, the surface of thick cardboard panels char and this creates a protective layer for the underlying material,

Illustration 5.42
Samples of cardboard hollow tubes.

restricting further damage. By applying a varnish to the cardboard surface, improved resistance to the spread of flames can also be achieved. On the other hand, since the material's thicknesses may be relatively small, as is the case for hollow cardboard tubes, additional protections must be taken in certain cases where severe fire classifications must be met.

Since cardboard is made from recycled paper products, this suggests that it is likely to be a material of considerable interest from an environmental conservation point of view. It should be borne in mind, however, that its manufacturing process does consume quite a lot of energy.

5.7 Glass

Although known and highly valued as a material at least since the time of the Phoenicians around 5000 BC, and used in subsequent millennia for everything from jewelry and artwork to drinking containers and storage vessels to small windows and spectacular piecework stained glass window rosettes in Gothic cathedrals, it was not until the mid-1600s that significant plate glass processes were devised. Industrial-scale production of glass can be said to have been heralded by the Crystal Palace for the World's Fair in London in 1851 and thereafter the material took on more significance in terms of its use in an architectural context for opening up interior space to the outside and vice versa. But even so, its use as a structural material, beyond limited capabilities for resisting wind pressures by spanning across a window frame and vertically from the outside edge of one floor level to the next for floor-to-ceiling glass, was very limited. Over the past few decades, however, the structural properties of glass have increasingly been explored and developed, and today it is a material that has its place among the viable palette of options for designing certain structural elements, albeit still a limited one since it is fundamentally a brittle material and so special care must be taken to design load-bearing structures that are made of it.

Glass is an inorganic, transparent material that has become effectively solid and rigid without crystallizing. The production of glass starts with melting together (mainly) quartz sand (silica), sodium carbonate, and lime.[19] A controlled cooling process (*annealing*) produces an amorphous (i.e., not crystalline) material which is solid at room temperatures, even though the microstructure resembles that of liquids. The faintly green color of glass is due to small amounts of impurities in the glass from iron and chrome oxides.[20]

By far the most common form of this material in architecture is the glass sheet made using the *float glass* method.[21] In this process, a continuous ribbon of glass is formed by using a bath of molten tin, on to which the molten glass spreads laterally, controlled by gravity and surface tension. The molten glass forms a floating ribbon on the tin surface having a perfectly smooth glossy surface on both sides. The thickness of the glass is controlled by the speed of the flow, and the continuous glass ribbon is cut after controlled cooling. A standard maximum size for a finished sheet glass is

Illustration 5.43
Time Warner Center, New York City, NY, USA (2003).
The reflective quality of flat sheet glass seen from an angled point of view is observed here for a very large, cable-supported glass wall made up of many individual panels. The transparent visual quality of glass when viewed transversely can be observed for this same glass wall in Ill. 6.2, 6.3.
Architect: Skidmore, Owings & Merrill, and, for glass wall, James Carpenter Design Associates.
Structural engineer: WSP Cantor Seinuk and, for glass wall, Schlaich Bergermann und Partner.

3210 × 6000mm (10ft × 20ft) with thicknesses ranging from 1mm to 25mm (0.04in to 1in). (e.g., Ill. 5.43.)

It is difficult to give the strength of sheet glass as fixed material properties, since flaws such as microscopic cracks (so-called Griffith flaws) that develop all over the surface will concentrate and magnify any applied stress and thus significantly limit the material's ability to withstand tensile stress.[22] Compressive stress, on the other hand, tries to close rather than open any crack; consequently, glass is considered to have a higher compressive strength than tensile strength. Nonetheless, glass is being used with increasing frequency in bending (for panels and beams) where the tensile strength is decisive for establishing the necessary structural dimensions (e.g., Ill. 5.44, 5.45, see also Ill. 7.13, 7.14), and it can even be found being used as a primary tension element in hanging glass façades in which the glass sheet is made to carry not only its own weight but that of all glass panels hung below it. (e.g., see Ill. 11.26.) The ultimate stresses of glass are usually given as design values according to the direction of load, and they are statistically determined.

The strength of glass may be increased by subjecting it to another process. *Toughened glass* is heat treated after the initial manufacturing process, in order to leave the outer surfaces of the glass with large compression stresses that are balanced by tension

Illustration 5.44
Apple Store, Fifth Avenue, New York, NY, USA (2006).
Detail showing curved glass in combination with titanium bolts. While glass structures in most cases have connections made of stainless steel bolts, in certain cases titanium bolts are used instead. The reason is that titanium and glass are much more compatible as far as thermal expansion is concerned than are stainless steel and glass. The risk of damage due to temperature changes when the two materials are in direct contact with each other is thus reduced.
Architect: Bohlin Cywinski Jackson. Structural engineer (glass): Eckersly O'Callahan.

Illustration 5.45
Casa da Música, Porto, Portugal, (2005).
Undulating structural glass for walls situated at both ends of the main auditorium; shape allows glass to span greater distance vertically without secondary support system.
Architect: OMA. Structural engineer (for the glass walls): Rob Nijsse.

stresses in the core (resulting from different rates of cooling). The locked-in compression stresses will prevent the surface cracks from opening, and thus make the glass significantly stronger. Starting with annealed float glass, heat treatment can result in two kinds of toughened glass: *heat strengthened* and *fully tempered*, with the latter being the strongest. When toughened glass breaks, it characteristically shatters into many small fragments.

Two or more glass layers may be laminated into one thick sheet by the help of thin plastic interlayers, typically of polyvinyl butyral (PVB). The interlayered plastic film may be colored or otherwise printed. In the case of breakage of this so-called *laminated glass*,

Illustration 5.46
Imagination Headquarters Building, London, UK (1990). PTFE-coated glass-fiber tensioned fabric covers five-story atrium, admits light into former gap in this inventive building conversion project.
Architect: Ron Herron. Structural engineer: Buro Happold.

the outer layers stick to the plastic interlayer, thus reducing the risk of injury from falling glass splinters or shards. As a result, laminated glass and toughened glass are both considered and labeled as *safety glasses*.

From an ecological point of view, in spite of the intensive manufacturing process just described, glass is able to at least partly counter this with an almost unmatched resistance to deterioration. Although it is important to note that it must be protected from direct contact with concrete, cement, and lime mortars as water-containing substances from those materials is strongly alkaline and may damage the glass surface. Glass is also certainly a material that can easily be recycled. And, more indirectly, it must be acknowledged that the transparency of glass may present great ecological benefit from frequently being used in architecture projects that exploit solar energy for thermal gain, although the downside of this is the likelihood of overheating when large glass areas are left exposed to the sun. The balancing of all these environmental pros and cons is not an obvious matter, however.

5.8 Fibers and Fabrics

Mineral and synthetic fibers are materials of crucial importance in contemporary lightweight tensile membranes and in polymer (plastic) composites. Their tensile strength is exploited in pre-stressed membrane structures, in which woven fabrics commonly form doubly curved structural shapes. (e.g., Ill. 5.46, see also Chapter 11, Section 11.8.) In rigid fiber/polymer composites, the fibers lend strength to the polymer that envelops and holds them in place; these will be discussed in the next section.

Natural fibers like cotton and wool are by no means stiff and strong enough to be of much help in contemporary fabric structures; neither do they have appropriate aging and weathering properties. Instead, the mineral *glass fibers* and synthetic *polyester fibers* (e.g., Dacron) are now the two most common in structural textile fabrics. Well-known among yachtsmen are also the very stiff *aramid fibers* (e.g., Kevlar) that are used in sails. An important feature of fibers is that their strength may far exceed that of the same material in another form. The case of glass fibers is particularly illustrative: as a fiber, glass contains surface cracks infinitely smaller than those

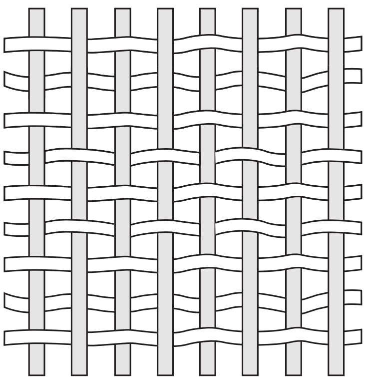


Figure 5.5
Diagram showing weaving of threads for manufacture of structural fabric; warp and weft directions, straight and undulating, respectively.

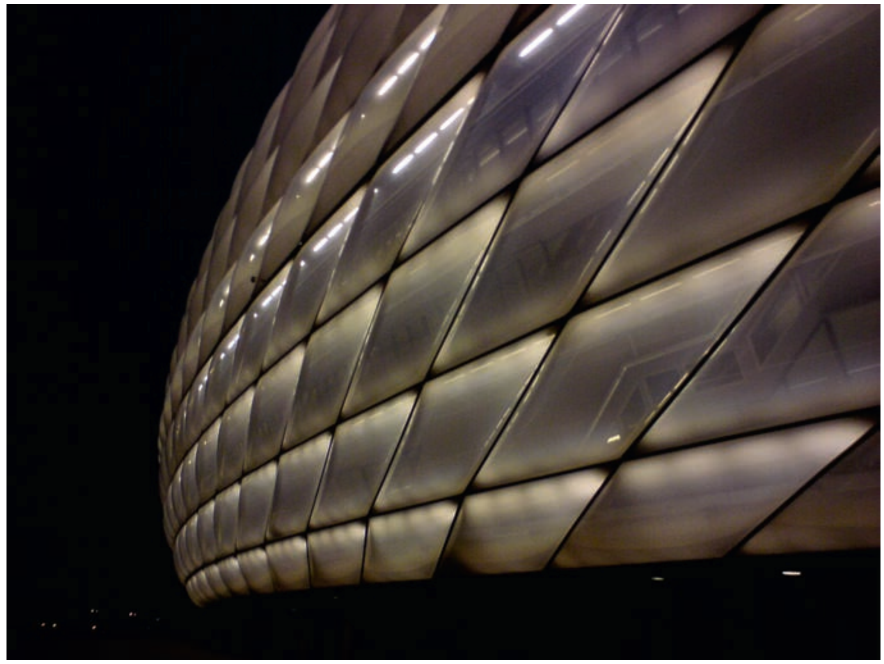


Illustration 5.47
Allianz Arena, Munich, Germany (2004).
Enclosure created by numerous air-inflated "pillows" made of layers of thin ETFE foil. These can be lit and colored for various effects and events.
Architect: Herzog & de Meuron. Structural engineer: Arup; Sailer Stephan and Partner; R+R Fuchs.

found in a sheet of glass. This means that the micro cracks are far less critical and do not reduce the tensile strength of glass fibers by the same proportion that they do in sheet glass. *Carbon fibers*, invented in the 1960s, is a highly interesting material for rigid composites.[23] Carbon fibers are exceptionally stiff (i.e., a substantial tension force results in a very small elongation) and also quite strong.

One *thread* or *filament* usually consists of a large number of fibers; in turn, many threads are woven together to form *fabrics*. The initially straight threads running in the direction in which the fabric is manufactured are called *warp* threads, while the threads that are woven orthogonally under and above these are called the *weft* (or fill) threads. Since the straightened warp threads are pre-stressed during manufacturing, the resulting fabric material has more stiffness and strength in this direction, with less elongation before failure. (Fig. 5.5.)

To protect the woven fabric from moisture, UV radiation, or fungus or microbe attack, it is usually coated on both sides. *Coatings* also influence the fabric's resistance to becoming dirty, and affect its working life. The most common coating material is *PVC* (polyvinylchloride), often applied to polyester-fiber fabrics, and *PTFE* (poly-tetra-fluoro-ethylene) for protecting glass-fiber fabrics. PTFE-coated glass-fiber fabrics are non-combustible and are generally thought to have longer life than PVC-coated polyester-fiber fabrics, and are therefore commonly used in "permanent" membrane structures. In addition, the PTFE coating provides a surface on which dirt does not collect easily, and in combination with the glass-fiber fabric it has a high degree of translucency.

In recent years, *foils* have also become common as structural tensile membranes. Foils are, unlike the anisotropic woven fabrics discussed above, materials that have the same strength and stiffness properties in all directions. The most important foil is the polymer *ETFE* (ethylene-tetra-fluoro-ethylene). Its tension strength, however, is far lower than what can be achieved in fabrics, meaning that it is more appropriately applied to much smaller spans. ETFE foil is mostly used for double-membraned air-inflated "pillows" or "cushions" that are attached in modular fashion to some sort of multicellular structural framework (e.g., Ill. 5.47), but it can also be a material option for mechanically pre-tensioned membranes if these are used for quite small spans. Since it has a very high translucency and an extremely high permeability to UV radiation, ETFE even presents distinct advantages over glass for enclosing greenhouses (e.g., see Section 13.1) and swimming pools. (e.g., Ill. 9.40.) It is also an almost fully recyclable material.

Illustration 5.48
Kunsthaus Graz, Graz, Austria (2003). Unusual form and coloration of art gallery building with acrylic glass roof contrasts with surrounding historical urban context.

Architect: Spacelab Cook-Fournier; Architektur Consult. Structural engineer: Bollinger + Grohmann.

Illustration 5.49
Kunsthaus Graz.
Curvature of acrylic glass also echoes that of the Baroque spires of Mariahilferkircke.

5.9 Plastics and Composites

Plastics

Polymers are large chain-like molecules that are based on carbon atoms, and are present in such substances as *plastics*, rubbers, and adhesives. The most important base material for all polymers is oil. We have already introduced various plastics that are used as fibers (polyester, aramid), coatings (PVC, PTFE), and foils (ETFE). There are two main groups of plastics; thermosetting plastics and thermoplastics. *Thermosetting plastics* (or thermosets) have a complex molecular structure that resists being reshaped by heating; once set, thermosetting plastics retain their shape. Among the more common thermosetting plastics are epoxy and polyester.[24] While holding their shape under normal temperatures, *thermoplastics*, on the other hand, will deform under heat and pressure, and can thus be given new shapes multiple times. Thermoplastics are recyclable and regain their properties after cooling. They include materials like PVC, PTFE, ETFE, as well as acrylic glass and polycarbonate.

Transparent thermoplastics like acrylic glass (polymethylmethacrylate, PMMA) and polycarbonate (PC) are sometimes used as substitutes for glass.[25] *Acrylic glass* is also known as Perspex and Plexiglass. It has the best optical properties of all the polymers, and its weight density is only about half that of glass. Corrugated sheets find interesting applications as cladding material and light transmission varies according to color. (e.g., Ill. 5.48, 5.49.) PMMA is permanently weather and UV resistant, which means that dyed elements hold their color even outdoors. Acrylic glass presents the advantage over glass of having roughly two to three times its tension strength; another advantage is that, unlike glass, thermoplastics experience both elastic and plastic deformation when subjected to stresses, i.e., they are not brittle. These materials are much less stiff, however, with an elastic modulus only about *one-twentieth* that of glass, which means that where deformations are a critical issue, much larger structural thicknesses are needed.

Polycarbonate has particularly good strength to resist impact loads, better than PMMA and far better in this respect than float glass of the same thickness. While being permanently weather resistant, PC discolors if left untreated; for outdoor uses, therefore, polycarbonate sheets are co-extruded with UV protection layers on both sides. Polycarbonate has a weight density that is close to that of acrylic glass, but the material is less transparent.

Plastics in general are durable and degrade very slowly; moreover, both PMMA and PC are 100 percent recyclable. On the other hand, burning plastics may release toxic fumes, and the manufacturing of plastics can create undesirable chemical pollutants.

Illustration 5.50
Chanel Mobile Art Container (2008).
Cladding provided by 400 uniquely shaped panels made of fiber-reinforced plastic composite.
Architect: Zaha Hadid Architects. Structural engineer: Arup.
Manufactured by Stage One.

Composites

Composites consist of two or more different material components which are joined to give a combination of properties that cannot be attained by the original materials independently. Although this strategy will be seen to have broad relevance in structures, in the context of this section we will restrict the discussion of composites to being about *fiber-reinforced plastics*. (e.g., Ill. 5.50.)

Fiber-reinforced plastics come in different forms, but obviously always involve fibers, which to a large extent define the mechanical properties of the composite, as well as a so-called matrix which surrounds the fibers to protect them and fix them in position. The matrix is commonly a thermosetting plastic like polyester or epoxy, with the latter being the more expensive of the two. Depending on how the fibers are distributed in the composite, fiber-reinforced plastics may be isotropic (having the same properties in all directions) or anisotropic; this is up to the choice of the designer according to how the finished component is required to act structurally.

The two reinforcing fiber materials of greatest interest in the context of composite materials are glass and carbon. Carbon fibers are used when very high stiffness and strength are needed, in combination with low weight. Carbon often acts as reinforcement in an epoxy matrix, and finds application in elements where the stiffness-to-weight ratio is seen as crucial. *Carbon fiber-reinforced plastics* (CFRP), however, are rarely used in architecture. Glass fibers, on the other hand, are much cheaper and also have significant stiffness and strength properties. *Glass fiber-reinforced plastics* (GFRP) – otherwise known as *fiberglass* – commonly employ polyester as a matrix, where the glass fibers may be introduced into the matrix in a number of ways and with different orientations as needed or deemed desirable.

Among the particular characteristics of such composites are their low weight, high strength-to-weight ratio, and exceptional corrosion and weather resistance. Since two materials are being merged, fiber-reinforced plastics actually come into being only once the combination of materials actually acquires its final form; this means that design can, if desired, have a great influence on establishing material properties such as strength and stiffness.

Illustration 5.51
Wassily chair (1925).
The very first chair made from steel pipes combines a springy metal frame strength with leather straps for the back, seat, and armrests.
Designer: Marcel Breuer.

5.10 The Case of Chairs – Exploiting Material Properties

Several architects that we consider to be pioneers of modern architecture shared a common obsession: universality. Their driving force was the dream of shaping humankind's environment, whether from their great visionary city plans, individual building designs, or down to the smallest objects of everyday function. In the last 100 years the evolution of the chair, in particular, has in many ways signaled the development of groundbreaking ideas in design and new material applications.

The steel tube chair can be considered as important a step in design development as was the introduction of the free plan and the glass curtain wall in modern architecture. Several architects of the period tried their hand at this kind of chair. Marcel Breuer (1902–1981), a teacher at the Bauhaus school in Weimar in the 1920s, used to bicycle to school; he saw that the steel pipe of the handle bars also could be used in furniture design. (Ill. 5.51.) This was the first steel tube chair not intended for use in the kitchen or the dentist's waiting room, but rather for the living room. His Wassily chair from 1925 combines the light, springy strength of the metal tube structure with the taut leather straps of the seat, its back, and armrests. The whole chair ensemble is complex in form and construction, but beautifully subtle in its elegance.

The Barcelona Pavilion was Germany's and architect Mies van der Rohe's (1886–1969) contribution to the World's Fair in 1929. Placed on a terrace of travertine marble, the pavilion consists of a horizontal roof surface supported by eight free-standing cruciform steel columns. Mies had also carefully placed within the pavilion a number of his now-famous Barcelona chairs that were specially designed for this purpose. The structural concept consists of two pairs of intersecting, chromed flat steel bars (that could be seen to be modified pieces of the pavilion's cruciform columns) joined

Illustration 5.52
Barcelona chair (1929).
Bent, intersecting, chromed, flat steel bars support and provide "springiness" for padded leather cushioned chair that was famously displayed at 1929 World's Fair in Barcelona.
Designer: Ludwig Mies van der Rohe.

Illustration 5.53
Landi chair (1939).
Early example of industrial furniture design made entirely from aluminum.
Designer: Hans Coray.

by three horizontal flat bars at the top, middle, and front of the chair. (Ill. 5.52.) A number of broad leather straps support the back and the seat of the chair, which are padded cushions made of natural-colored leather. The curvature of the chromed pieces, the elegant cushion work, and the beautiful proportions have all combined to make the chair a timeless classic.

Aluminum was the preferred material of the Swiss designer Hans Coray (1906–1991) when designing the Landi chair for the Swiss National Fair in 1939. This chair, forerunner of the modern aluminum chair that gained such widespread use, is an early example designed for industrial mass production. (Ill. 5.53.) Distinctive features are a pair of chair legs that are bent over to form the armrests. The seat and back are formed as a single piece – a curved aluminum shell surface manufactured using a high-pressure shape-forming technique. The chair, including its perforated shell, weighs less than 3kg (6.6lb), is impervious to virtually any kind of weather, can be stacked up and is maintenance free. Its "good form" in silver anodized aluminum has won many admirers and is represented in design museums worldwide.

We do not know who built the first Windsor chair. Its simple form and light structure has nevertheless certainly fascinated many: Siegfried Giedion mentions it in "Space, Time and Architecture" and makes comparisons of it to the development of the "balloon frame" for house construction. In 1949, the Danish designer Hans

Illustration 5.54
Peacock chair (1949).
A landmark in Danish furniture design. Produced in ash, birch, or oak.
Designer: Hans J. Wegner.

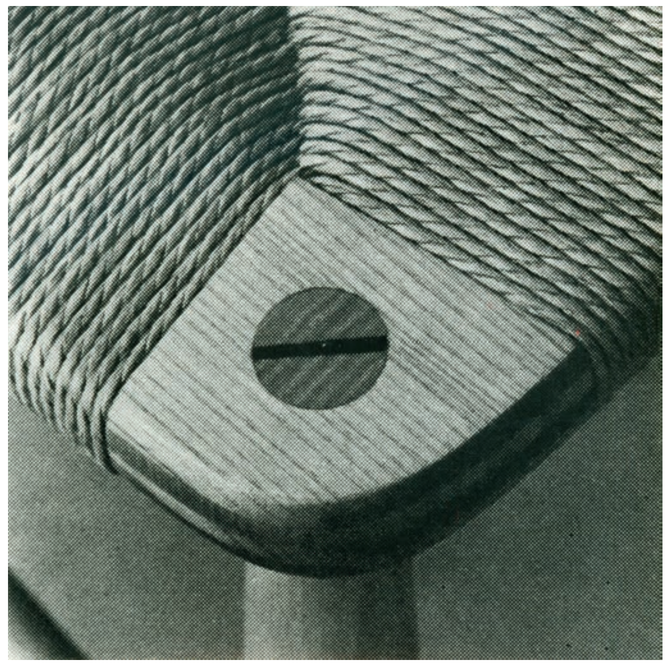


Illustration 5.55
Peacock chair connection detail.

J. Wegner (1914–2007) presented his interpretation of this classic design as "The Peacock Chair." (Ill. 5.54) With its high arched back and fine detailing, the chair stands as a landmark in Danish furniture design. The corners of the seat expose a rounded dovetail joint of critical importance; the chair leg is pushed up into a hole in the seat and then locked into place by means of a hardwood wedge that is pounded down into the leg from the top. This fine little construction detail is beautifully expressed by an ash circle and a teak diagonal; moreover, the wedge guarantees a solid bond between leg and seat. (Ill. 5.55.) For lightweight and frequently moved furniture such as wooden chairs, much of the challenge lies in solving the connection between leg and seat. While Alvar Aalto "bent around" the corners in his famous chairs from the 1930s (see Ill. 10.59), Wegner took on the problem head-on and thereby demonstrated his mastery at finding solutions for traditional joint details.

US designers Ray and Charles Eames (1912–1988 and 1907–1978, respectively) were always interested in the potential of new materials, and they saw the possibility with plastic to be able to form an organic seat shell that conforms to the body's shape. (Ill. 5.56.) Based on molding techniques developed during World War II, their DAR chair seat shell is made of glass-fiber reinforced polyester that is connected to a metal-rod base with rubber shock mounts. First presented at the Museum of Modern Art in 1948, this chair has been in mass production ever since and has found a wide application in contemporary projects.

Illustration 5.56
DAR chair (1948).
Seat made of molded fiber-glass reinforced polyester is carried on contrasting thin metal rod base.
Designer: Ray and Charles Eames.

Illustration 5.57
Graphite chair (1989).
Chair is especially lightweight and folds into a flat, compact shape.
Designer: Richard Horden.

The English architect and yachtsman Richard Horden has been interested in transferring the elegance of sailboat construction and the beauty of modern aircraft design into architecture, always with the aim of light prefabricated buildings and components. His "graphite chair," introduced in 1989, is inspired by the lightweight quality of the modern carbon-fiber tennis racket. The intention is to achieve a high design folding chair for use in home, office, or café; i.e., something that is especially light and compact. The chair belongs to a series of products titled "aerospace group" because the early prototypes were developed with engineers from Britain's Concorde and Rolls-Royce aerospace factories. The café chairs are produced with a silver frame and a seat and back made of vinyl fabric. (Ill. 5.57.)

The Hanger and the Tie

CHAPTER 6

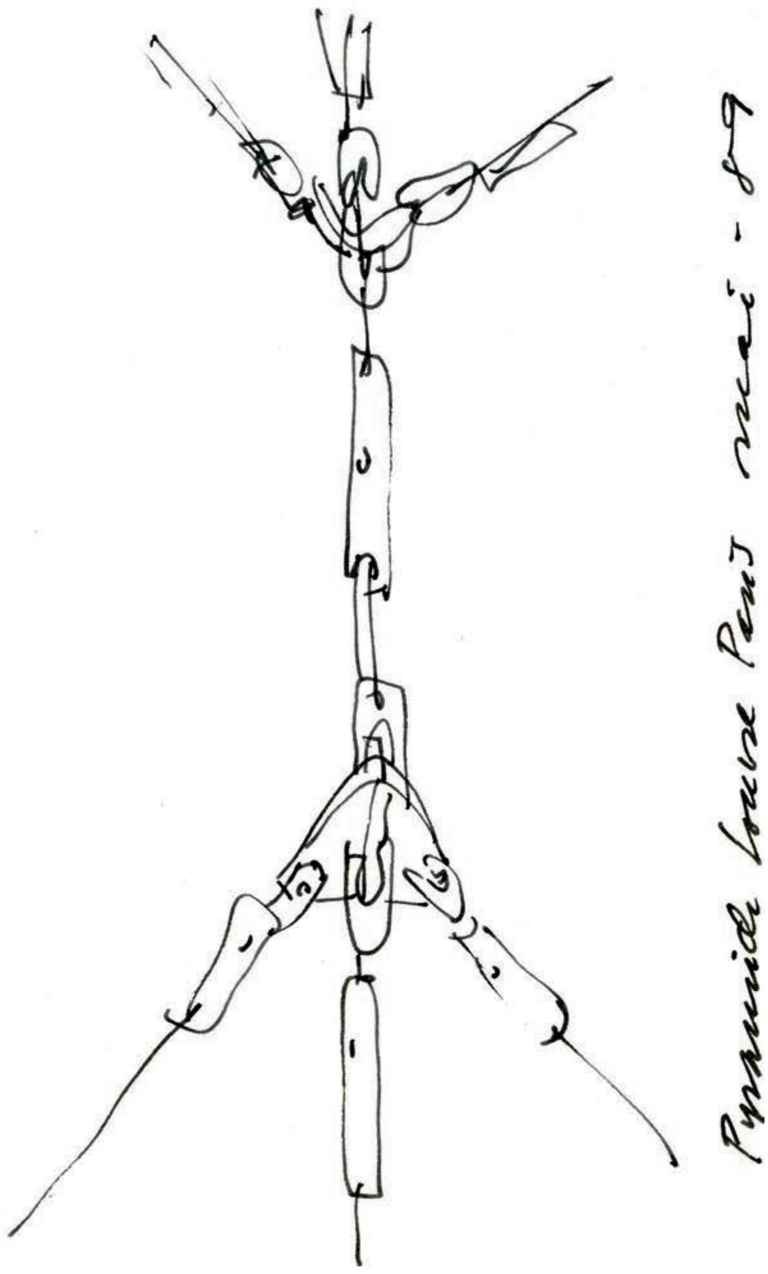


Illustration 6.1
La Pyramide du Grand Louvre, Paris, France (1989).
Signature detail for the connection of multiple tension elements used to help support the iconic glass structure. (See also Section 4.3.)

Architect: I.M. Pei. Structural engineer: Nicolet Chartrand Knoll Ltd. and Rice Francis Ritchie (RFR).

Illustration 6.2
Time Warner Center and the Allen Room of Jazz at Lincoln Center, New York City, NY, USA (2003). View through hanging glass wall and double cable support systems on to Central Park South and Midtown.

Architect: Skidmore, Owings & Merrill; for glass wall, James Carpenter Design Associates. Structural engineer: WSP Cantor Seinuk; for glass wall, Schlaich Bergermann und Partner.

6.1 Jazz at Lincoln Center – A Hanging Glass Wall

At the southwest corner of Central Park, where the famously regular Manhattan grid of streets meets the diagonal Broadway Avenue at the landmark statue of Columbus Circle, the Time Warner Center creates one of the city's largest mixed-use development projects with 251 000m^2 (2 700 000ft^2) of space divided among retail, hotel, office, cultural, and residential uses. Rather than having this be one overpowering building, however, architect Skidmore, Owings & Merrill took cues from the urban context and broke down the massing into two 230m (750ft) towers that bracket 59th Street, thus allowing a very strong visual axis to be extended westward. At the podium-level connecting the towers, this idea is further reinforced by means of a glass-walled atrium of street-matching width that provides access to retail stores and to what is the complex's visual and acoustic focal point: the Allen Room performance space for Jazz at Lincoln Center. Lifted 25m (82ft) up into the air and with a full height glass wall as a backstage, the auditorium simultaneously makes a spectacle to be seen from the outside and provides the audience with unparalleled views of the south end of Central Park where it meets the busy streets of Midtown. (Ill. 6.2, 6.3.) It is on the remarkable transparency of this glass wall as made possible by the minimal dimensions of its tensioned-cable support structure that we will focus our attention here.

The history of glass walls and of architectural interest in bringing light and exterior space into buildings or, conversely, of extending inside spaces to the outside is filled with varied and creative inspiration, from the intricate stained glass windows of medieval Gothic cathedrals to the pure fluidity of space found in both Mies van der Rohe's Barcelona Pavilion and Philip Johnson's Glass House in New Canaan, Connecticut. If there has been a common theme to this story over time, however, it has been to make use of contemporary technological developments in order to try and minimize as much as possible the intervening structure that is needed to support the very thin and fragile sheets of glass that are manufactured to maximize transparency.

At the Time Warner Center, the 46m (150ft) high and 25m (82ft) wide glass wall designed by James Carpenter Associates together with Schlaich Bergermann und Partner is supported on a two-way pre-tensioned cable net that is connected around the four sides of its perimeter. (Ill. 6.4.) (We will discuss more fully the behavior of cable nets in Chapter 11.) The gravity loads of the wall consist mostly of the dead weight of the glass, which is actually made of two layers of 11.5mm (0.450in) thick heat-strengthened glass sheets laminated together for safety reasons.[1] However, rather than the gravity loads being transferred straight down to the ground as one is accustomed to with conventional walls, here they are carried up by means of the vertical steel cables and transferred to the top of the transverse and inclined truss that spans across the top of the glass wall. These 28mm (1.1in) diameter cables, made of cold-drawn steel wires that have been helically twisted together into what are called "strands," are also attached at their bottom end at the basement level. A second set of cables connects horizontally across the width of the glass wall and is connected to the structure at the two sides of the opening. Any lateral (i.e., out-of-plane) deflections of the

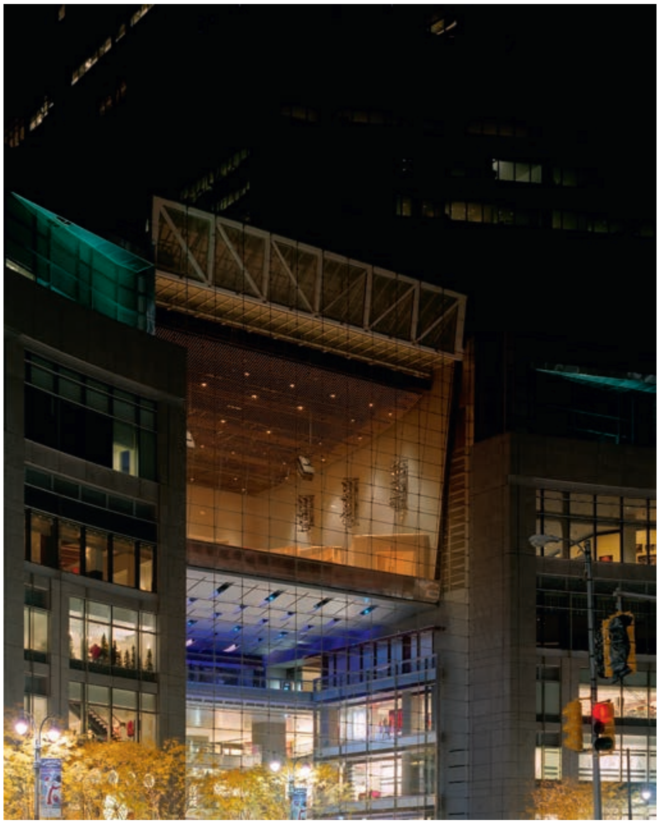


Illustration 6.3
Time Warner Center.
Glass wall is suspended from truss above performance space; exterior cable net is also anchored at sides and bottom edges.

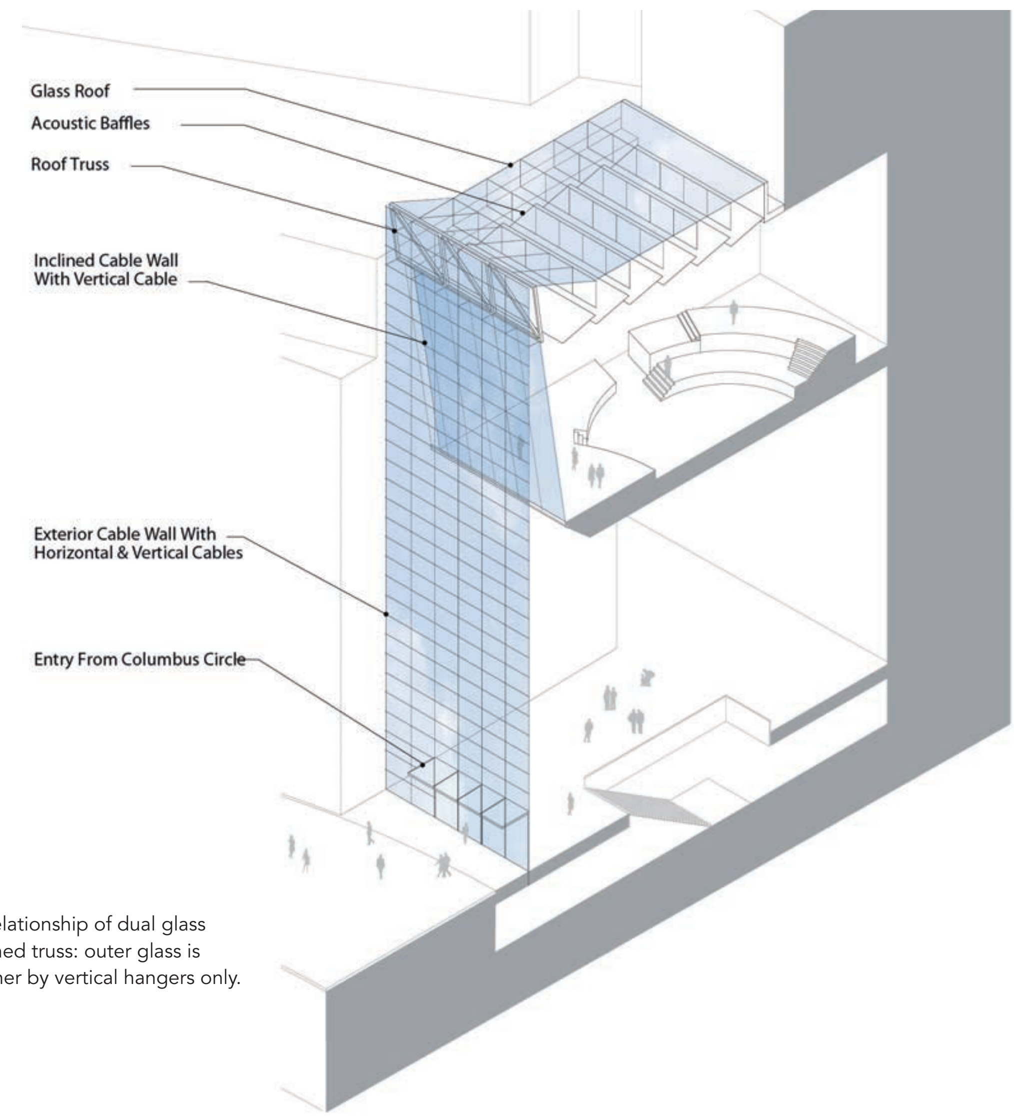


Illustration 6.4
Time Warner Center.
Axon drawing illustrating relationship of dual glass walls suspended from inclined truss: outer glass is supported by cable net, inner by vertical hangers only.

wall that are caused by wind are resisted by the strong tendency of both the horizontal and vertical sets of cables, which are highly pre-tensioned upon installation, to return to their initial straight alignments. Typical deflection limits for such a cable net are in the range of L/40 or L/50, both to protect the glass from damage and to minimize the occupants' discomfort due to perception of motion: in this particular case, provision has been made for the glass to be able to deflect laterally up to 550mm (22in), which is quite significant but nevertheless is within the permissible range. Such flexibility obviously also depends upon the careful detailing of the connectors at the intersection points of the cable grid to which the glass panels are attached: stainless steel nodal clamps here accommodate the lateral deflections of the cable net by allowing up to 10° of relative rotation between the glass plate and the fastener.

This is not the end of the story at Time Warner Center, however, for there is also a second, inner glass wall that encloses the east end of the auditorium facing 59th Street. One function of this additional wall is quite obvious: it provides the necessary acoustic isolation for the jazz room from the unwanted sounds in the public spaces below while maintaining the virtually complete transparency needed for the Allen Room's audience to have direct views of Central Park and the Manhattan skyline. More subtly, this second glass wall has been inclined so as to distinguish the auditorium volume within the overall atrium space and at night to capture the reflections from the street traffic's headlights and tail lights moving silently up and down the backdrop of the performance space. For the inner glass wall, only inclined vertical cables have been provided to carry the glass gravity loads; in contrast to the outer wall, here there are no wind pressures to worry about as it is a completely interior environment and the horizontal cables of the net are not needed. The tops of these cables are anchored to the lower chord of the same (inclined) truss that is used by the outer wall cables, while their lower ends are connected to springs attached to the jazz room's floor beams in order to allow for the changing vertical deflections of the floor produced by audience live loads.

Jazz in New York City thus can be seen to have come full circle, from its roots hidden away in the cellars of the speakeasies of the early 1900s to nearly a century later being put out on full display and engaging one of the most dynamic views of the city; in achieving this transformation, the straight tension element can be said to have been instrumental.

6.2 Floating Space

Although much of architecture is about connection to and engagement with the ground, in this chapter we begin our study of the fundamentals of structural element behavior by examining the tension member, which is frequently associated with quite the opposite physical condition; i.e., spaces and occupiable surfaces that seemingly hover and float in mid-air and that seek in some way to defy our everyday experience of being earthbound by gravity. This perceptual condition is quite often the result of the remarkable thinness – and, therefore, the tendency to disappear and be invisible – of the simplest and most efficient of structural elements: the tension rod.

Historically, the reasons to elevate space perhaps began by mostly being strategic: e.g., in order to gain an elevated position from which to view an enemy. Certainly, the elevated vantage points of medieval defensive towers embody this purpose although their supporting masonry structures typically exemplify the "grounded," massive, and compression architecture of the period. Later, military hot-air balloons were devised that attended to some of the same strategic objectives but did so in a dramatically different physical manner: by relying on the lightness and thinness of tension elements hanging a basket for human occupancy – and thereby producing an obvious and literal disengagement of the occupied space from the ground. (e.g., Ill. 6.5.)

Leaping ahead yet again in time, a contemporary structure that carries forward the observation balloon's spatial attributes and fundamental reliance on key members in tension is London's Millennium Wheel. (Ill. 6.6.) Enclosed oval pods carry visitors high above the south bank of the Thames, affording spectacular and unprecedented views of the city center. And although at first glance the structure resembles a traditional Ferris wheel, the so-called London Eye relies on a set of highly tensioned steel members connecting the circumferential trussed rim to the central axle. The dimensions of these rods are so small that when seen from any distance they tend to disappear, causing the disconcerting and sensational impression of the disengagement of the rim from the rest of the supporting structure. The advantage of this thin-element system is that it both reduces to a minimum any visual obstruction of the London cityscape, while also heightening the sense of awe and disquiet that one often associates with being lifted high up in the air.

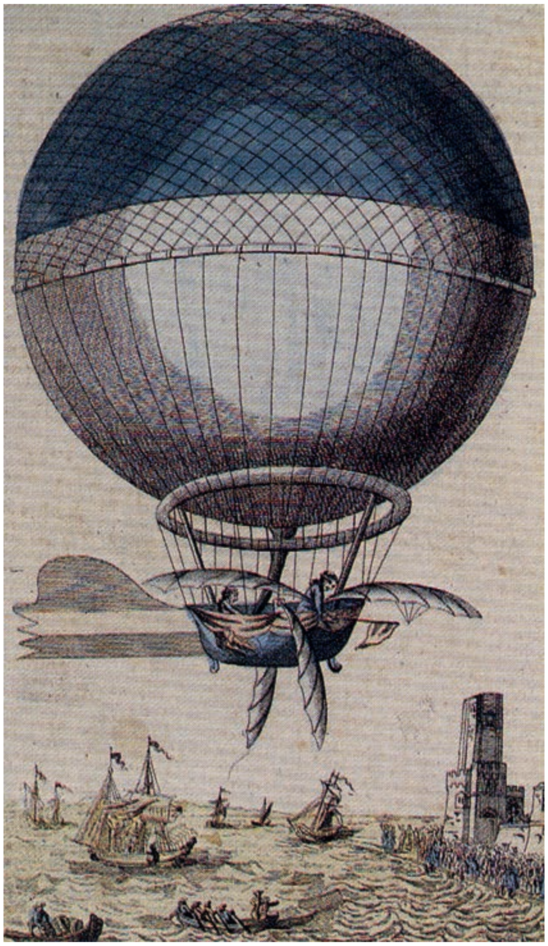


Illustration 6.5
Blanchard and Jeffries crossing the Channel in January 1785.

Illustration 6.6
Millennium Wheel, London, UK (2000).
Tension rods provide only means of support for outer trussed ring.

Architect: Marks Barfield Architects. Structural engineer: Jane Wernick, then at Arup.

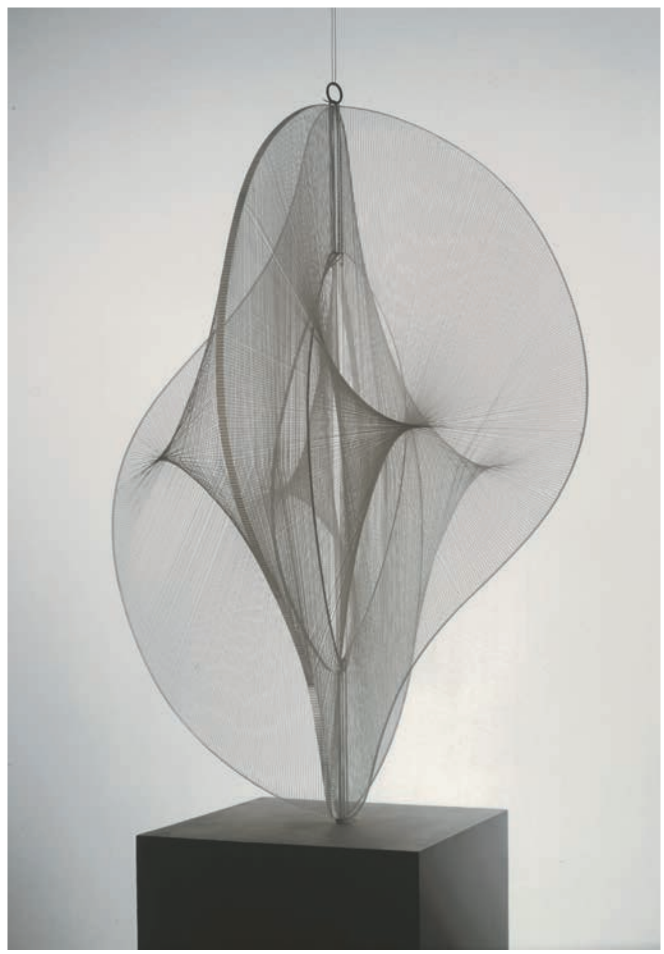


Illustration 6.7
"Linear Construction in Space No. 2" (1949). Plastic nylon, 30 × 20 × 20in.

Artist: Naum Gabo (American, born in Russia; 1890–1977). Gift of Florene May Schoenborn, 1971.879, The Art Institute of Chicago.

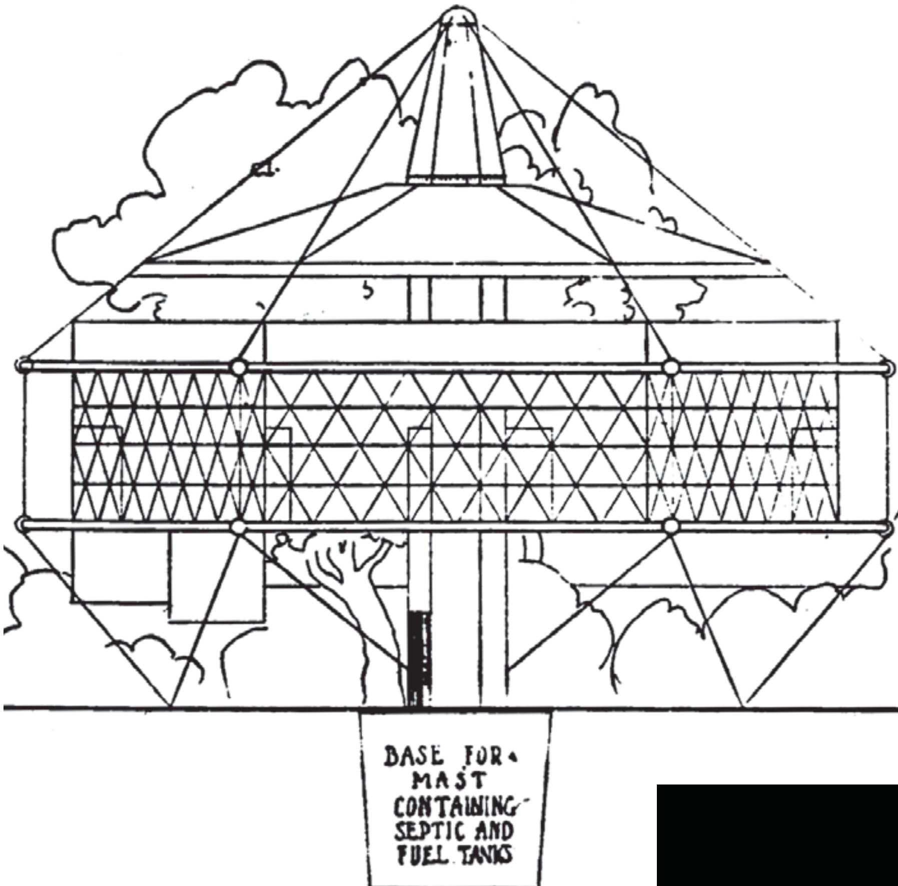


Illustration 6.8
Dymaxion House project (1929, revised in 1945). Prototype for efficient kit-of-parts housing unit, with perimeter of roof hung from top of central stainless steel mast by sloped tension rods.
Architect: Buckminster Fuller.

Illustration 6.9
Deep Purple, Milan, Italy (2008). Architects' model for Casa per Tutti Triennale proposal for emergency housing unit. Corners of floors suspended from top of central steel pole.
Architect: Massimiliano + Doriana Fuksas.

The Millennium Wheel system is certainly unique from many points of view, but there are plenty of other structures in which an impression of "floating" is even more directly and conventionally provided by vertical and slanted tension hangers. The Dymaxion House by Buckminster Fuller (Ill. 6.8) and the Casa per Tutti project by Massimiliano and Doriana Fuksas (Ill. 6.9) are but two of many examples of inhabitable building projects to which have been applied the structural and spatial strategy of tension hangers and seemingly "levitating" space.

Illustration 6.10
Vertical hanger supports PH 4/3 pendant lamp from the Targetti/Louis Poulsen corporation.
Design: Poul Henningsen.

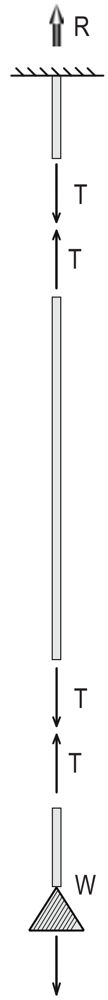


Figure 6.1
Equilibrium diagrams for bottom, middle, and top segments of vertical hanger.

Illustration 6.11
Microcompact unit (2005).
Vertical tension hanger plays key role in this rendering of the intended delivery of a prefabricated 2.6m (8.5ft) cube dwelling into remote locations.
Architect: Richard Horden of Horden Cherry Lee Architects. Consultants: Dipl. Ing. A. Uehlein Drees and Sommer GmbH.

6.3 The Vertical Hanger

Without doubt the most obvious of structural elements in terms of fundamental load-carrying mechanism and behavior is the vertical hanger, and there are plenty of familiar examples in everyday life – from children's swings to chandeliers to construction cranes – that help us visualize and instinctively recognize the distinguishing characteristics of the typical hanger. (Ill. 6.10.) The basic situation for the vertical tension member in each of these common instances has the following generic qualities: (a) a significant weight or load of some type is being carried at the bottom, (b) a typically remarkably thin, long vertical element is connected to this load in some fashion, and (c) a support structure of one type or another is located at the top end to which the hanger is anchored.

Free body diagrams for the different segments of this system (Fig. 6.1) lead to the obvious conditions of vertical equilibrium: i.e., the downward gravity weight, W, of whatever is being hung must be balanced by an equal and opposite upward force provided by the hanger, or T = W. The hanger is in uniform tension (of magnitude T = W) from one end to the other, stretched between the weight being hung and the support structure above. The downward pull of the hanger is balanced at the top by an equal and opposite upward support reaction, R, also necessarily of the same magnitude as the total load, W, being carried. We are making the simplifying assumption here that the weight of the hanger is calculated beforehand and included in the total load W being supported; although for preliminary estimating purposes the relatively small physical dimensions of typical hangers are such that one will not be far off the mark to consider the weight of hangers to be relatively negligible in magnitude when compared to that of the typical loads being supported.

Figure 6.2
Uniform tension stresses acting over hanger's cross-sectional area balance total tension force.

The tension force, T, in the hanger itself results in the stretching of the material from which it is composed, and it is not difficult to envision the consequent set of tensile stresses, σ_t, acting over this member's cross-section. (Fig. 6.2.)

The typically long, thin proportions of the hanger insure that these stresses are uniform over the entire cross-sectional area, A, resulting in the following very simple equation of equilibrium:

$$T = \sigma_t \times A$$

This equation is the basis for designing all hangers, indeed all purely tensile structural elements, and as such is worth spending a few more moments discussing.

If a certain material whose ultimate tension stress capacity is known and cross-sectional dimensions are established for a hanger, the present form of the equation will easily define the maximum load that this hanger/tension element can carry:

$$T_{max} = \sigma_{t\,(ultimate)} \times A$$

Alternatively, in a preliminary design phase where decisions about member sizes and materials are having to be made for a given load that must be carried, this equation can also be reorganized and then applied to determine the hanger dimensions that are needed for a particular selection of material; i.e.,

$$A_{required} = T/\sigma_{t\,(ultimate)}$$

It is to be noted that the result of this equation for the cross-sectional area required will be the same anywhere along the length of the hanger; i.e., no matter whether it is 3m (10ft) or 10m (33ft) long (at least as long as we once again quite reasonably ignore or make an allowance for the relatively small variations caused by the hanger's own self-weight). This means that a tension member is a highly efficient way in which to carry load, since each and every bit of material over its entire cross-section and over all sections from one end of the member to the other is equally stressed; in other words, there is no underutilized material. We will see in the next chapter that this is not at all the case when we come to other types of very common load-carrying elements – such as beams.

Beyond meeting the fundamental structural requirement, at a more conceptual level the basic tension member equilibrium equation also establishes the potentially remarkable visual impact of using a material such as steel that has a very high tensile capacity; i.e., cross-sectional dimensions can purposefully be made very small, which can in turn make hangers almost disappear when they are seen from any distance. And as we saw in the preceding sections, it is this very basic and fundamental consequence of equilibrium and material capacity that is fully exploited by architects to "float" roofs or inhabitable spaces for myriad practical reasons and conceptual or visual effects. We will now look at one such example located in France where the choice of a minimal hanger system is clearly and integrally connected to the building's design concept and fundamental *raison d'être*.

Chaix Morel and Associates' design for an archeology museum consists of a building that is built directly above the ruins of the ancient Roman city of St.-Romain-en-Gal, not far south of Lyon in central France. (Ill. 6.12.) In order to minimize, as much as possible, the new building's intrusion into the actual ruins, an unconventional approach to the design and supporting structure was required, and this resulted in the floors of the building being hung from its roof.

The roof structural system consists of a series of deep steel beams that span transversely across the width of the museum; these in turn are supported by four rows of columns along the building's length. From each of the roof beams the main museum floor and the mezzanine walkways are hung by means of a series of vertical steel rods. Clearly, bringing gravity loads down to the ground on a standard grid of more closely spaced columns would not have been acceptable in terms of preserving the ruins, whereas hanging the floors from the long-spanning roof beams enables the building to "float" over these with significantly less disruption. (Ill. 6.13, 6.14.)

In order to get an idea of the size of one of the main hangers, let us assume that some work has previously been done along the lines of what we have covered in the preceding chapters; i.e., structure and finish material details are sufficiently known to estimate dead

Illustration 6.12
Musée Gallo-Romain de St.-Romain-en-Gal, Vienne, France (1996).
Museum building next to and above ongoing excavations of Roman ruins.
Architect: Chaix Morel et associés. Structural engineer: Arcora.

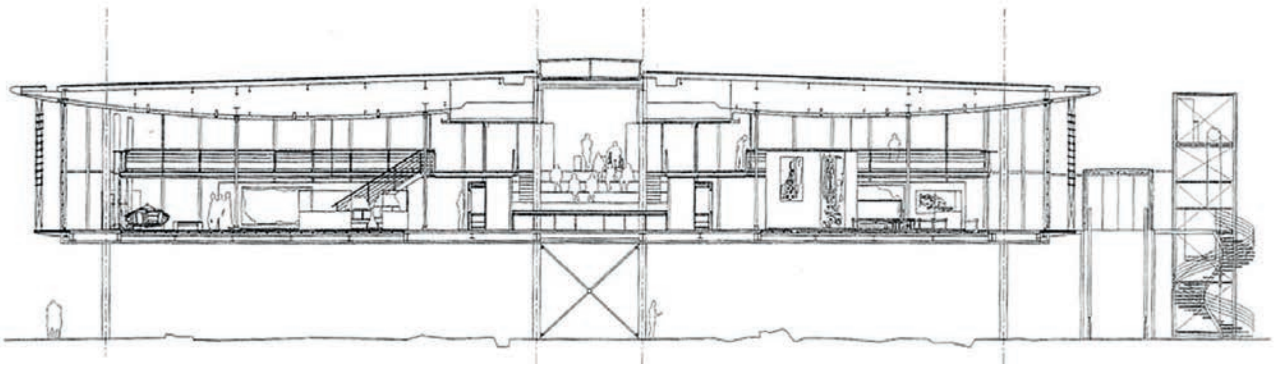


Illustration 6.13
Musée Gallo-Romain.
Section drawing showing basic strategy of hanging museum's main floor and mezzanine levels from the large transverse steel beams located at roof level.

Illustration 6.14
Musée Gallo-Romain.
Multiple vertical hangers are evident that support the museum floor area as well as its mezzanine walkways. Loads are carried up the underside of the transverse roof beams, hidden in this view above the hung ceiling.

Illustration 6.15
Former Central Bank of Ireland building, Dublin, Ireland (1979).
The number of external vertical hangers increases up the height of the building as each successive floor level is carried.
Architect: Stevenson, Gibney & Associates.
Structural engineer: Arup.

loads, and the occupancy live loads for the museum's main and mezzanine floors have been established. The tributary floor areas supported by a hanger can also be determined. With loads and tributary areas thus known, the load needing to be carried by a hanger is easily determined; for example, it may be calculated that a load of 380kN (85.5kips) needs to be carried up in a hanger to the underside of the transverse beam. Assuming that the steel of the hanger cannot be permitted to exceed a stress level of 320N/mm^2 (48kips/in^2), its required cross-sectional area can be established:

$$A_{required} = T/\sigma_{t\ (ultimate)}$$
$$A_{required} = (380\ 000N)/(320N/mm^2)$$
$$A_{required} = 1188mm^2$$

Steel manufacturers' tables of section properties for various structural shapes might then lead one to select perhaps a 20mm x 75mm (¾in x 3in) flat bar, which has a cross-sectional area of 1500mm^2 (2.25in^2) – which is safely larger than that which has been found to be needed, but is nonetheless quite a small member size given the load that needs to be carried. It should be noted that in actual building practice some safety factors are added that increase the load and reduce the stress capacity in the calculation. There can be no question that such a minimalist result for the hanger dimensions is very much in keeping with the architects' design intent to create a very open and flexible museum space and, moreover, one that perceptually and physically hovers above the preexisting ruins.

Even more explicitly expressed is the vertical hanger rod system used to carry and "float" the floors of the former Central Bank of Ireland building in Dublin, where the number of rods can be seen to increase *up* the sides of the building as the loads of each successive floor are carried to the roof level. (Ill. 6.15.) At that point, angled tension members anchor the hanger system to the top of a central, upwardly projecting concrete core. Aside from being so visually expressive, this hanger system was used as part of an unusual construction approach: after the full extent of the central core was built each floor level was erected on the ground and then successively lifted up in to place, with the building therefore quite exceptionally taking shape from top to bottom.

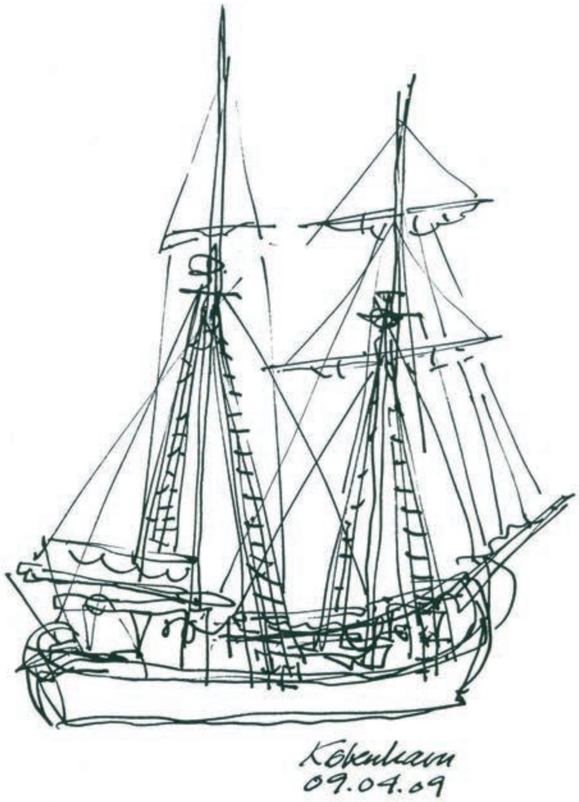


Illustration 6.16
Masts of the Danish schooner *Havet* of Helsingør are stayed by means of angled ropes in tension.

Illustration 6.17
VM Husene building, Copenhagen, Denmark (2005).
Ends of triangular balconies are supported by angled tension rods anchored back into building structure.
Architect: Julien de Smedt and Bjarke Ingalls of PLOT Arkitekter.
Structural Engineer: Moe & Brødsgaard A/S.

6.4 Inclining the Hanger – The Stayed System

As we have already seen in Sections 6.1 and 6.2, tension hangers are not always purely vertical but often are inclined for either pragmatic or conceptual design reasons. The term "stayed" is typically applied to such structures – inspired, no doubt, from their association with the similarly angled ropes and mast-stabilizing cords found in the rigging of historical sailing vessels. (e.g., Ill. 6.16.)

While such an inclined condition does not change the basic behavior of a tension member, it does present a subtly different overall equilibrium-of-forces situation that warrants attention.[2] Consider, for example, the spectacularly triangular balconies of the apartments of the VM Husene buildings in Copenhagen designed by PLOT Arkitekter. (Ill. 6.17.) Support is provided to each balcony by a couple of inclined tension rods that are anchored back to the building. These tension rods are very small in cross-sectional dimension, thereby simultaneously minimizing the potential for overall visual clutter in the appearance of the building and diminishing any obstruction to the views of the surrounding park that they otherwise might present. At the specific point where a sloped hanger connects with the horizontal floor, we can draw a simple free body diagram of the forces that are acting. (Fig. 6.3.)

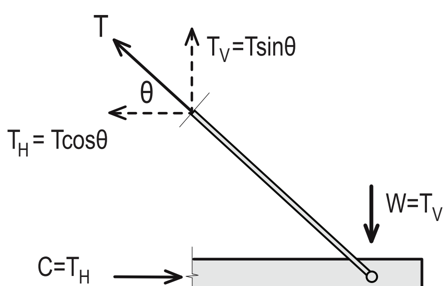


Figure 6.3
Equilibrium-of-forces diagram corresponding to end of VM Husene balcony.

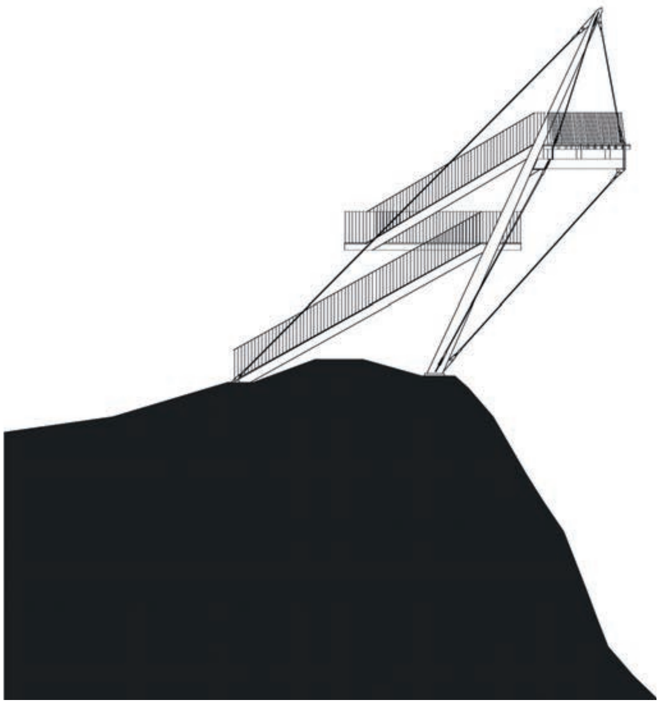


Illustration 6.18
Conn Viewing Platform, Flims, Switzerland (2006).
Platform projects forward into open space of valley, held up and back by angled tension elements.
Architect: Corinna Menn. Structural engineer: Prof. Dr. Christian Menn and Bänziger Partner AG.

Illustration 6.19
Conn Viewing Platform.
Angled tension rods support the ends of the platform and others anchor the structure down against uplift, all the while minimizing visual obstruction. The platform acts as equilibrating compression strut, and is notably thicker. (The platform also supports vertical gravity loads and its thickness is also due to that fact.)

Two things immediately stand out: (a) the downward gravity force W at this point that is caused by the dead and live loads on the balcony must be balanced by the upward vertical component of the tension force in the inclined hanger, and (b) the fact that the hanger is sloped means that it necessarily will also have a horizontal component which will be pulling inward on the connection point. Obviously, this is a force that also must be balanced, this time by an axial compression force in the horizontal structure of the balcony being supported – something that we do not have to consider or deal with if the hanger is vertical.

From our review in Chapter 4 of vector forces and their components, the first of these observations leads to the requirement that the force T in an inclined hanger is going to need to be larger than the vertical gravity force W that is being carried. The sloped member will, as a result, have to be larger in cross-section than it would need to be if it were vertical. This effect will accentuate itself the more inclined the hanging member is, although because of the efficiency of typical steel tension members such changes will tend not to have a very strong visual impact. The second observation, that there is a compression force C in the horizontal member of a stayed system, has the consequence that the structure that is supported by angled tension rods must be designed not only as a typical horizontally spanning beam structure but now in addition as a simultaneous compressive strut. This is an inevitable consequence that is common to all floors and roofs and bridge decks that use the inclined hanger/stayed configuration. And as we will see in the succeeding chapters, since the size of structural members needed both for beams and for resisting compression forces is

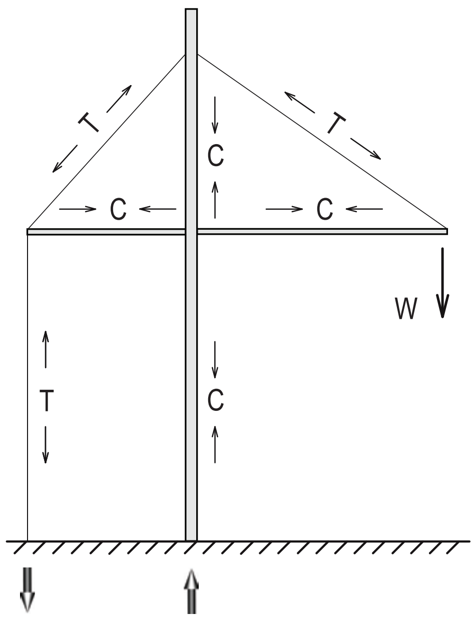


Figure 6.4
"Classic" arrangement of tension and compression forces in members of stayed system.

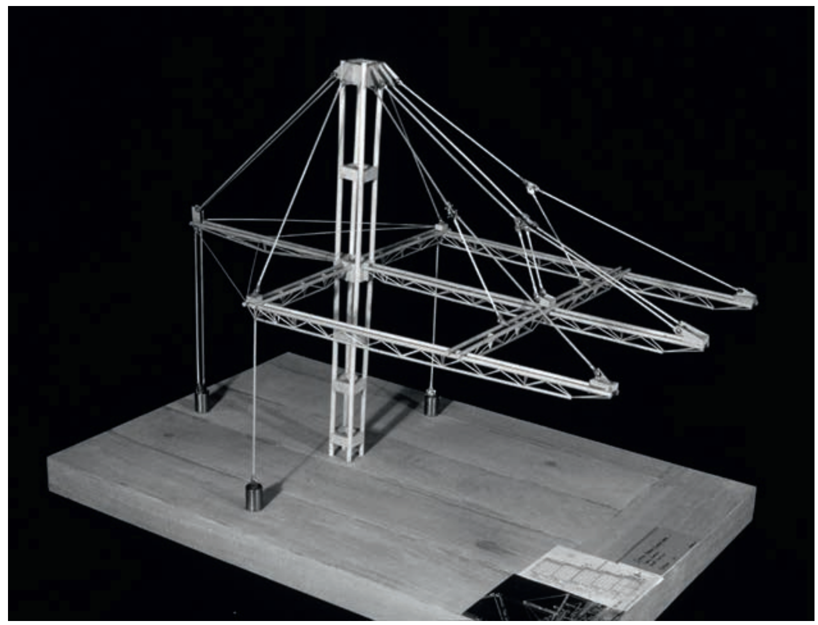


Illustration 6.20
Darling Harbour Exhibition Centre, Sydney, Australia (1988). Relative proportions differentiate parts of stayed system in tension (thin) and compression (thick).

Architect: Phillip Cox and Partners. Structural engineer: Arup. Cornell model: Bryant Lu (1994).

considerably larger than that necessary for carrying tension, it is typical for the horizontal elements of a stayed structural system to be significantly greater in cross-sectional dimensions than is the inclined hanger. (Ill. 6.18, 6.19.)

At the top of a sloped tension member there is also a need for equilibrium, and a similar equilibrium force analysis for that point will result in the conclusion that (a) the outward pull of the tension rod produced by the horizontal component of that member's force will have to somehow be resisted (often by an anchoring backstay), and (b) the downward vertical component of the tension force in the hanger will need to be balanced by an upward vertical (compressive) force from the support structure (frequently a vertical, or near-vertical, mast). The consequence of all of these observations is that the classical configuration of the stayed-mast system is that of two intersecting compression members – one vertical, the other horizontal, or approximately so – having a certain thickness of form in order to prevent their buckling (as we will discuss in Chapter 8) and quite thin tension elements connecting the ends of this cruciform shape. (Fig. 6.4.) Such relative differences in the proportioning of members in bridge or roof structures is typically quite evident, and should be anticipated by the architect even at the most preliminary stages of design. Two examples that illustrate clearly these fundamental relative proportions are the configuration for the supporting elements of the basic module of the cable-stayed roof system of the Darling Harbour Exhibition Centre (Ill. 6.20) and the multistory floor support module of the High Tech classic HongKong Shanghai Banking Corporation Headquarters building. (Ill. 6.21, 6.22.)

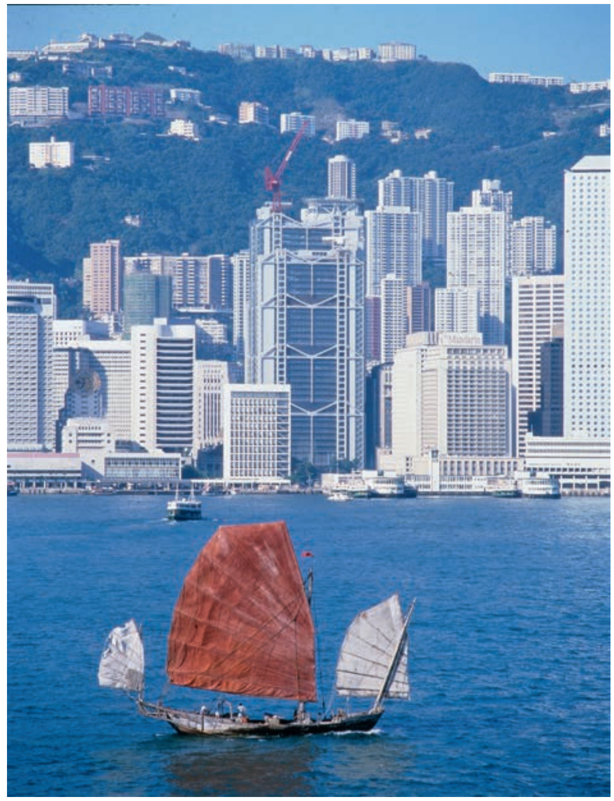


Illustration 6.21
HongKong Shanghai Banking Corporation (HSBC) Headquarters, Hong Kong, China (1985). Iconic view of elevation reflects multilevel modular tension hanger structural system.
Architect: Foster + Partner. Structural engineer: Arup.

Illustration 6.22
HSBC Headquarters.
Relative proportions differentiate parts of system in tension (thin; i.e., central vertical hanger, angled tension members, and external vertical anchoring tie-downs) vs. compression (thick; i.e., horizontal balancing struts where tension members change direction, and built-up vertical masts).

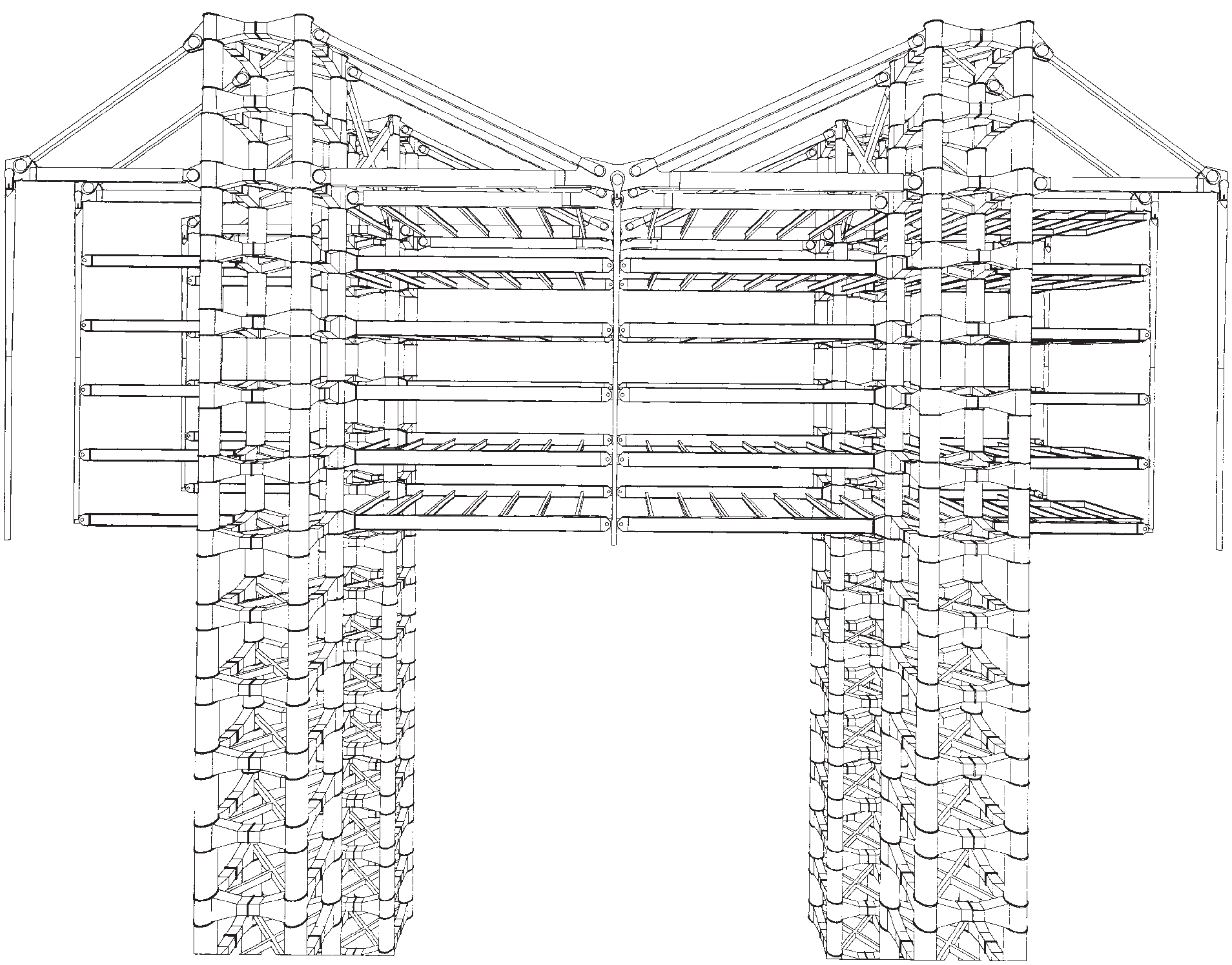


Illustration 6.23
Ypsilon Footbridge, Drammen, Norway (2008).
Asymmetrical cable-stayed system, including alternate pathways at north end.
Architect: Arne Eggen. Structural engineer: Knut Gjerding-Smith.

In order to reinforce and elaborate upon these concepts in slightly more depth, we will examine a stayed bridge in Drammen, Norway, in the next section. Before doing so, however, it bears emphasis that we are discussing in this chapter only straight-line tension elements with loads applied along their axis and not those with the curved suspension-cable profile typical of cables that are supporting transverse loads; we will encounter the latter in due course in Chapter 11.

6.5 Ypsilon – An Asymmetrical Cable-Stayed Footbridge

The waterway of the Drammenselva River was used for many years to float logs to the numerous saw- and paper-mills of the Norwegian town of Drammen; in fact, this was the basis for the region's settlement. Today, however, with the traditional timber industry gone, cultural institutions, offices, and residential buildings face the river while new parks and promenades are being developed alongside it in order to attract people to the area once again.

Crossing the broad river that previously divided the town, a new footbridge has been built that links the network of pedestrian paths on the two banks. (Ill. 6.23.) Y-shaped in plan, the Ypsilon cable-stayed bridge was designed by the architect Arne Eggen in collaboration with the structural engineer Knut Gjerding-Smith. The structure has a main span of 90m (295ft), while the two shorter arms of the bridge each span half that distance. By dividing the bridge into two as it reaches toward the north bank (which at this location is characterized by a very small bay) the designers added extra length and also elevation to the walkway; in doing so, they addressed two important elements contained in the project brief – allowing the required clearance for boats to pass beneath the structure and providing the gentle slope needed to insure wheelchair accessibility for the pathway.

The compression pylon for this cable-stayed system is made of a pair of "cigar-shaped" masts (i.e., they are thicker at mid-height than at their ends – a refinement of form that will be discussed in Chapter 8) whose varying cross-sectional dimensions are achieved in this case by welding together a series of differently truncated steel cones. The two 47m (154ft) high masts sit atop a concrete base that momentarily splits the river channel into two. The masts are hinged at the foundation and connected together at the top

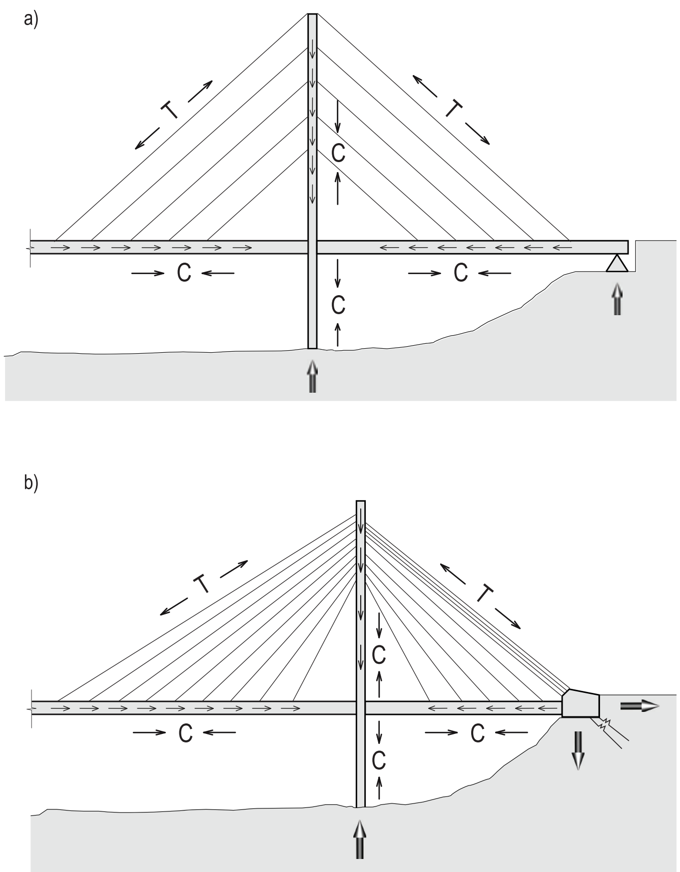


Figure 6.5
Symmetrical (a) and asymmetrical (b) stayed system configurations; force balance and imbalance, respectively, results in different end support conditions.

with two horizontal cables. The main span is made up of two parallel steel tubes that are supported by eight pairs of stay cables attached on either side of the walkway. The structure of each side span, on the other hand, consists of a single steel tube that is carried by four cables and that has a deck cantilevering sideways from it. (The torsional response that this latter eccentrically supported walkway produces in causing the tube to twist will be discussed in Chapter 7. (See Ill. 7.27.)) The stay cables themselves have a diameter of 45mm (1.77in) and have a capacity of about 2200kN (495kips). The overall configuration and the specific details of all of these components of the bridge serve to emphasize both the local landscape form of the bay and the equilibrium of forces that are at play within the structure.

In its cross-river elevation, the Ypsilon Footbridge has a form that is asymmetrical about the compression mast; i.e., it has a main span on one side of the mast and a shorter back-span on the other. This is a configuration that has been adopted many times during the past half century in order to avoid having a pier located in the middle of a river channel, and visually it presents a compellingly dynamic appearance with one side of the bridge seeming to reach out toward the opposite bank. This asymmetric form, however, also has a fundamental static equilibrium problem that must be dealt with, and it is to this end that we now turn our attention.

The basic module of structural elements for a stayed bridge consists of the following: a cable that runs from its main-span beam connection up to the mast, which point is in turn connected by means of a backstay cable to the side-span beam; the vertical and horizontal components of the tensile forces in the two cables can thus be seen to be neatly balanced both top and bottom by the compression forces in the mast and the deck. If the main and side spans have the same length, there can be an equal number of cables and spacing in the two spans and the deck compressive forces would then naturally balance each other at the mast. (Fig. 6.5a.) However, with one span shorter than the other there is a strong imbalance to the system that is produced by dead load considerations alone, to say nothing of the variations caused by live loading. In order to deal with this problem, it is typical to have several of the outermost stays from the main span anchored directly back to the side span's abutment (the bridge end's connection to the ground); this also means that the unbalanced compression force from the main span will now have to be transmitted all the way through the side-span structure to the abutment. (Fig. 6.5b.)

The Ypsilon Footbridge takes a similar approach to solving this equilibrium problem, except that the balancing of the forces from the main span is shared between the two angled side spans and their abutments. This plan configuration has the added benefit of also providing significant stiffness and stability to the bridge for resisting lateral loads such as wind. The two concrete abutments on the north riverbank (Ill. 6.24) thus have several functions to fulfill: they act as anchors for the main span's four outer stay cables, they

Illustration 6.24
Ypsilon Footbridge.
Detail of cable anchorage and angled geometry of bridge abutment.

transmit the compressive and torsional forces from the bridge's structure to the ground, and, from a conceptual design perspective, they can be seen as key transition elements between the bridge and the riverbank where the man-made structure meets the natural ground. Their inclined geometries are defined by the bridge's stay cables, and with one abutment on each side of the small bay these structures become triangular concrete bastions for the local precinct; moreover, the direction and pattern of their concrete formwork also follows the cable angles, further visually reinforcing the tension forces at work.

Like a huge harp with pointed masts and made of white-painted steel, the Ypsilon Footbridge provides new opportunities for viewing and experiencing the river and its landscape and it is playing a central role in Drammen's ongoing urban renewal.

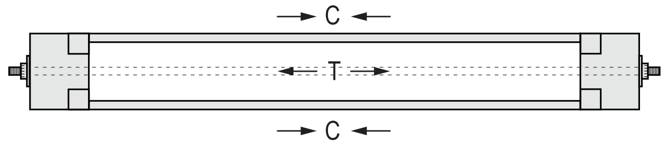


Figure 6.6
Self-equilibrating condition for basic component of Shigeru Ban's Library of a Poet structure: tension in central rod balanced by compression in surrounding cylinder.

Illustration 6.25
Library of a Poet, Zushi, Kanagawa, Japan (1991).
View of glass-enclosed library addition reveals overall frame structure composed of multiple short cardboard tube segments.

Architect: Shigeru Ban Architects. Structural engineer: Gengo Matsui (Hoshino Architect and Engineer).

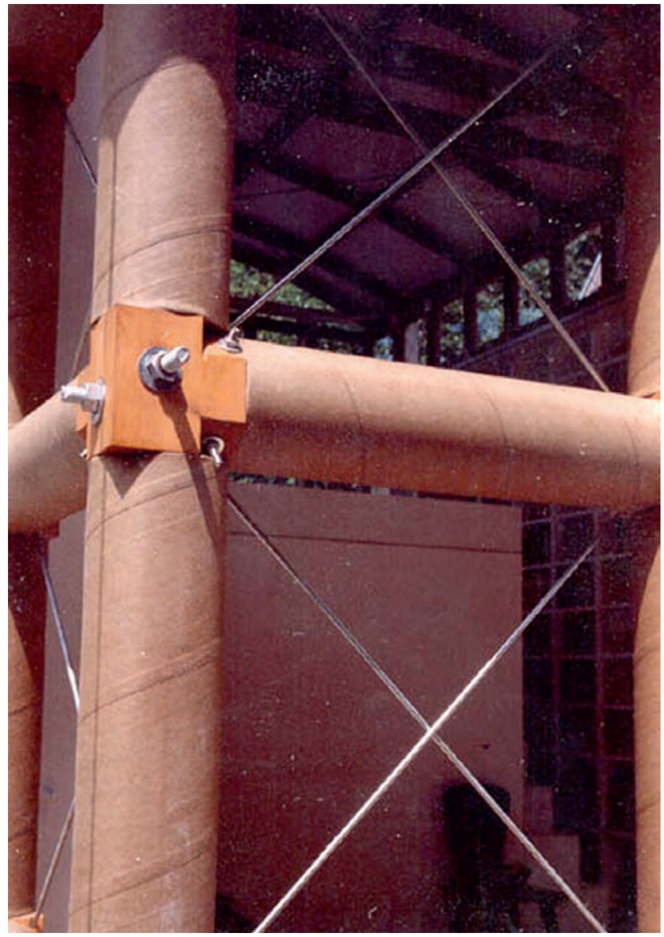


Illustration 6.26
Library of a Poet.
Wood block connection detail, transition element between compression cardboard tubes and the tensioned steel rods threaded inside them as well as anchorage for the diagonal bracing rods.

6.6 Ties and Guys

The notion of using tension in order to hold together things that naturally want to fall apart is not a new idea nor is it divorced from our everyday experience. We use and observe this principle all the time: in the elastic rubber bands stretched around a set of documents, for example, and also with the tightened cords or wires that stabilize either a camping tent or one of today's omnipresent cell-phone network transmission towers.

In an architectural context, one can also find clear examples of tension "ties" that in a similar essential fashion link together a set of discrete structural components that could not otherwise hope to stand up nor carry the applied loads. Shigeru Ban, for example, relied on numerous tension rods to hold together the many individual components of one of his early examples of innovative cardboard cylinder structures. (Ill. 6.25, 6.26.)

It needs to be recognized that the tension elements on which this structure relies have been put into a state of tension before the inescapable gravity and lateral loads are considered to be applied to the overall structure; i.e., they have been *pre*-stressed. What

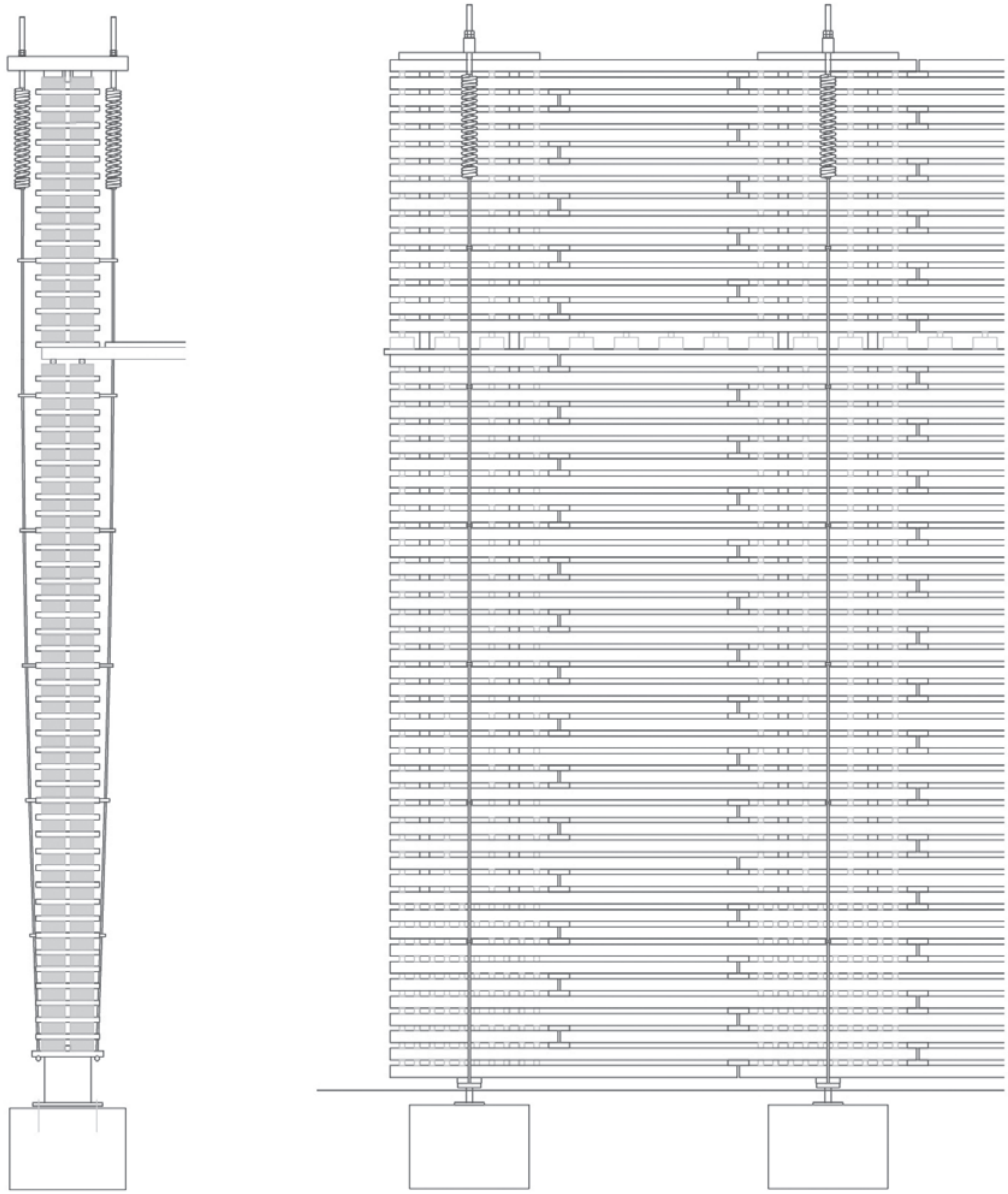


Illustration 6.27
Swiss Pavilion, Hanover, Germany (2000).
External vertical tension rods anchor together otherwise loosely stacked pieces of lumber; use of springs at top allows for tension to be maintained as wood dries and shrinks.
Architect: Peter Zumthor. Structural engineer: Conzett, Bronzini, Gartmann AG.

would otherwise be an impossibly loose-fitting set of cardboard tube cylinders is stabilized and held together by means of the tightening of the nuts at the ends of the steel tie rods that are threaded inside the tubes. (Fig. 6.6.) The face of such a nut bears against a steel washer or plate that bears in turn upon the wooden block that forms the junction point, and these finally push upon the ends of the cardboard cylinder. Exactly the opposite happens in the reverse direction at the other end of the tube member, thus tightening the otherwise loose-fitting collection of elements. The tension rod is thus being pulled apart between its endpoints while the cylinder is subject to an equal but opposite shortening and compression force.

This same fundamental strategy was rendered even more visibly obvious for the walls of Peter Zumthor's temporary Swiss Pavilion structure for the Hanover World's Fair, where stretched springs anchored the external vertical tie rods located on either side of otherwise loose stacks of lumber. (Ill. 6.27.)

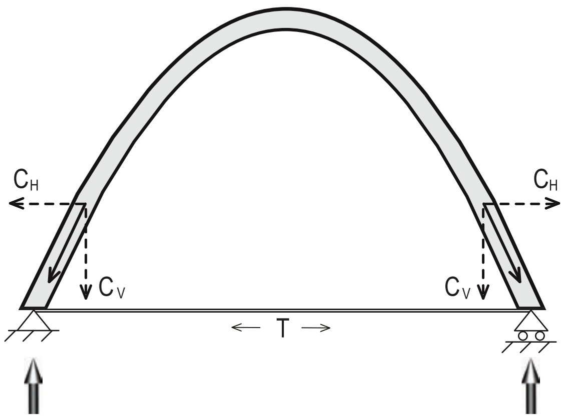


Figure 6.7
Self-equilibrating system: tension tie links opposing outward compressive thrusts at arch base.

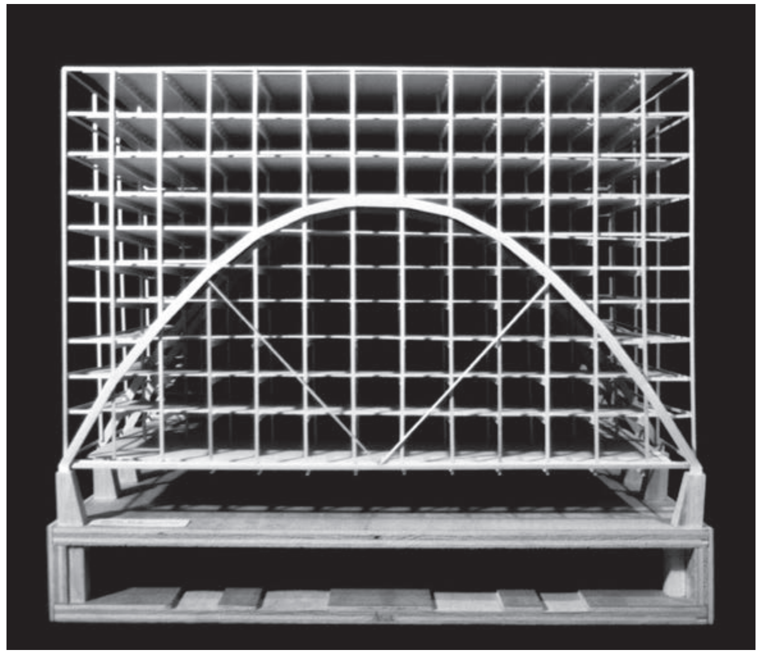


Illustration 6.28
Broadgate Exchange House, London, UK (1990).
Multistory office building is supported by arches that span over underground railway tracks, with horizontal tension tie anchoring the opposing outward thrusts of the inclined arch legs. Also, whereas below the arch all loads are carried up to this curved compressive structure by means of vertical tension hangers, above the arch loads are carried down to it on vertical columns.

Architect: Skidmore, Owings & Merrill (SOM). Structural engineer: SOM. Cornell model by Jennifer Miller.

Also, without getting too far ahead of ourselves, it is relevant to recognize that tension ties are often found linking together the base supports of arches, a structural form that we will examine more closely in Chapter 12. At this point, though, we can apply our common-knowledge understanding of arches working in compression to carry load and the fact that the outwardly angled sides of the arch will inevitably cause outward forces to be present where the arch meets the ground. As we will see, there are several ways of dealing with this thrust of the arch, but one viable alternative relevant to the current discussion is to balance the outward thrust from one leg of the arch with that of the other by means of a tension tie across the base of the arch. (Fig. 6.7.) In a vaulted structure made from a series of side-by-side arches, a set of tension ties may be seen that link the two sides (Ill. 6.28, 6.29, see also Ill. 12.32, 12.33), although such ties may not always be clearly obvious if they occur within the level of a connecting floor slab. And developing this strategy even farther, we will see in Chapter 13 that domes rely for their stability on hooping rings of tension around their base. But that is for later; for now, let us get back to straight tension elements.

With the inclined tension rods or guy cables[3] that are used to stabilize structures against wind or other lateral forces we once again encounter a strategy of pre-stressing, in which members

Illustration 6.29
Broadgate Exchange House.
Detail of arch base support showing horizontal tension tie that counters arch's outward thrust.

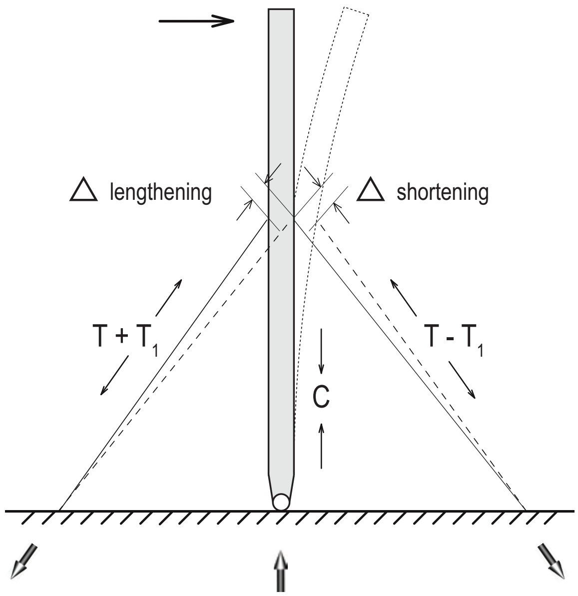


Figure 6.8
Effect of lateral deflection on inclined guys: windward member lengthens, increasing tension force; opposite for those on leeward side.

are put in a high state of tension before any anticipated external loads are ever applied. (Fig. 6.8.) This should not be surprising when we remind ourselves of our common experience of erecting a camping tent and the immediate need to stretch and tighten the angled tent-to-ground elastic cords – not only to stabilize the kit-of-parts structure but also in anticipation of the effects of the first windstorm. Sufficiently pre-stressing such tension members in different directions around the structure will insure that any lateral side-sway will be immediately resisted and countered – i.e., there will be no "slack" or sag in the guying cables that will need to be taken up before a taut cable can act to resist the lateral displacement.

How such a guyed system works can be described a little more precisely. A lateral force on the structure being supported inevitably causes it to try to displace sideways in the direction of loading. The geometry of the situation will tend to cause a lengthening and, therefore, the development of a tension force in the guy that leans in the direction of lateral movement and a shortening, and thus a compression force, in that which is inclined against it. But we know from our own intuitive experience that a very long and thin rod will not be able to carry any significant load in compression before failing by buckling out of alignment (more on this in Chapter 8). If in anticipation of just such a failure we sufficiently pre-tension all of the lateral stabilizing guy wires, we can strategically avoid this situation. The guy on the windward side simply has its tension force increased – something that can efficiently be designed for with minimal increase in cross-sectional area – while the guy on the leeward side, if it has been pre-tensioned sufficiently to offset the compressive force that is anticipated, will remain in a net condition of pure tension even with the lateral displacement of the structure taking place. Thus, both windward and leeward stabilizing guys all around the structure will continue to be in tension regardless of their orientation. And from a visual perspective, the inherently very thin dimensions of these guying members mean that such a stabilizing system will virtually disappear when seen from any distance, allowing quite tall structures to seemingly and spectacularly stand on end.

6.7 A Tale of Tension in Two Towers

Towers, with their very presence set against the skyline, affect and fascinate us – whether they are towers of Italian medieval towns demonstrating the power of ruling families or today's high-voltage electricity masts marching through a natural landscape. Minimizing the structure and refining the detailing along with dealing with the reality of side-sway due to wind gusts and turbulence presents opportunities for the cooperation of architects and engineers and occasionally this effort can result in elegant landmarks.

The Festival of Britain at the London South Bank was held to celebrate the centenary of the 1851 exhibition that had featured Joseph Paxton's revolutionary Crystal Palace. The intentions 100 years later were similarly to stimulate good design, advertise British products, and attract foreign orders and tourists. The festival should also be seen in the light of the optimism of the years after World War II; among the many projects designed for the festival, a competition for "a vertical feature" was enthusiastically received and generated 157 entries for a design brief that suggested an abstract approach and a demonstration of the originality and inventiveness of British designers.

The winning "Skylon" project by architects Philip Powell (1921–2003) and Hidalgo Moya (1920–1994) with engineer Felix Samuely (1902–1959) was a cigar-shaped structure some 90m (300ft) tall that was supported at the bottom by a system of tension hangers and guys. (Ill. 6.30.) The vertical support was provided by three twin

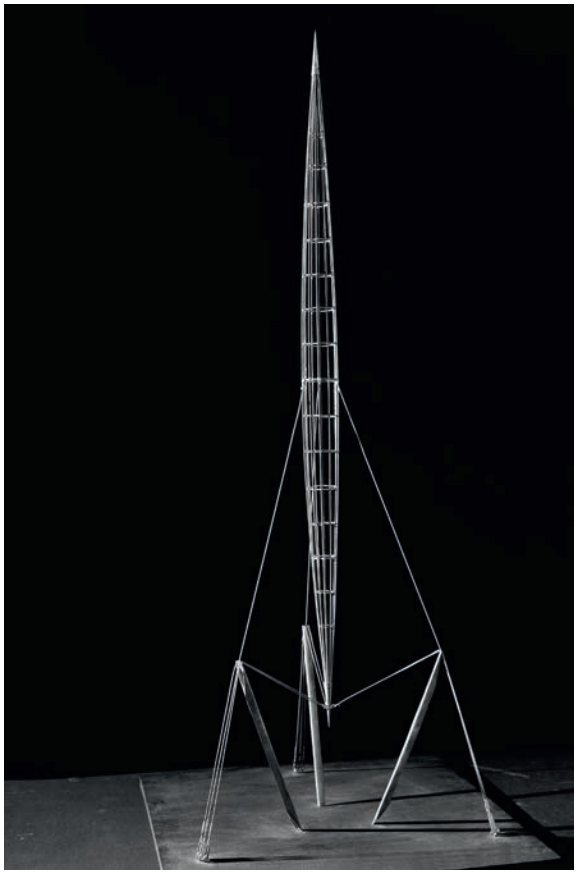


Illustration 6.30
Skylon, London, UK (1951).
Tension elements supported lower end of "flying column" and provided lateral stability to overall structure.
Architect: Philip Powell and Hidalgo Moya. Engineer: Felix Samuely.
AHO model by Nicolaj Zamecznik and Tarjei Torgersen.

Illustration 6.31
Torre de Collserola, Barcelona, Spain (1991).
While being almost invisible, "guy" cables efficiently anchor the tower against lateral side-sway.
Architect: Foster + Partners. Structural engineer: Arup and Ingeniería CAST.

cables that formed a cradle to lift the structure 15m (49.2ft) off the ground; these cables passed over the top of three outwardly slanted pylons spaced 120^0 apart in plan, and then ran to their anchor points in the ground. Pre-stressed guy cables also extended from the top of the three pylons to a point about two-thirds of the way up the levitated tower structure in order to give it lateral stability. The Skylon was made of lattice steel segments and was lit from the inside; at night (and even during the day from a certain distance) the structure seemed to float freely in the air with no visible means of support.[4] Such a central "flying column" supported only by cables can also be labeled as a tensegrity system – which we will more fully discuss in Chapter 9.

Preparing for the Summer Olympic Games in 1992, Barcelona launched an extensive rebuilding program in order to signal the city's importance within the newly united Europe. Concerned that in doing so the hill behind Barcelona would not bristle with a multitude of telecommunication towers, city planners sought instead to have a single elegant structure that all companies could share. Architect Norman Foster + Partners together with engineers at Arup won the competition with their design for the Torre de Collserola, a tower 288m (944ft) tall with a spectacular viewing platform 135m (443ft) above the ground. (Ill. 6.31.) Unlike the Skylon, the central part of this tower consists of a concrete core that reaches all the way down to the ground, and so it can quite conventionally deal with all gravity loads. In order to keep the core's profile as slender as possible on the very windy hilltop, however, the structure is laterally supported by eight pre-stressed guys that splay from the corners of the bottom of the tower's triangular antenna pod to multiple anchorages in the rock of the hillside. The Torre de Collserola is an iconic structure that is on full display from the city far below, but from that distance the thin guys vanish completely from view; indeed, it is telling in this regard that the official branding icon for this structure omits the presence of these key structural elements altogether.

6.8 Tension Elements and Connections

Several materials can be used for tension elements, although by no means all. The differing material properties that we examined in the preceding chapter play a critical role in establishing which can be employed for this purpose and which cannot. Steel, with its equally high capacity to sustain either tension or compression stresses, is excellent in this regard and is commonly used. Wood also is naturally capable of resisting both types of loading when applied parallel to the grain, even if only to a much lower stress level than steel and so should be used in situations where tension stresses are comparatively modest. Among other materials that work well in tension: natural and synthetic fibers that historically have been used for ropes in boat rigging and cords for hoisting loads and staying unstable structures, and that are found in the contemporary fabrics and tensile membranes to be discussed in Chapter 11.

It should be noted, however, that some common construction materials such as brick and stone and concrete are essentially useless in tension, and it is best to ignore them for this purpose. The only way to address tension stresses in structures made of these materials is to have them incorporate a second material, typically steel in the form of reinforcing bars, that has no problem handling tension. (Such a strategic combining of materials in structural elements will be especially relevant to bear in mind when we come to discussing the behavior of reinforced and pre-stressed concrete beams in the next chapter, in Section 7.8.)

Timber tension elements are typically made of standard-dimension sawn lumber or manufactured wood products. As we have discussed earlier in this chapter, a state of pure tension in a structural member is not a difficult condition to design for: a simple algebraic equation relates the force to be carried and the tensile material's stress capacity to the cross-sectional area that is required. Moreover, the typically flat sides of wood components ensure that their connections to other members can be fairly easily accomplished; for example, by means of steel bolts passing through pre-drilled holes, although care must be taken to ensure that the full tension force can be carried by the reduced cross-sectional area of the element at the bolt-hole locations. Steel plates or washers are typically needed to prevent crushing of the wood fibers as bolts are tightened. An example of timber tension hangers can be seen being used to

Illustration 6.32
Kube Hus, Bygdøy, Oslo, Norway (1977).
One-family house with two upper-level rooms and a glass façade "wall" that are hung from roof beams.
Architect: Terje Moe. AHO model by Ida Gjerde, Jenny Rognli Mohn, Sindre Fredriksen, Jonas Løland.

Illustration 6.33
Kube Hus.
Relatively small dimensions of timber tension hangers ensure the spatial openness of the main living space on the ground floor level.

support the "hanging" rooms and glass façade of Terje Moe's "Kube Hus." (Ill. 6.32, 6.33.)

For the much more common tension elements made of steel, two basic types are used: rolled sections and cables. Essentially any rolled section will work as long as the cross-sectional area is sufficient to safely match the design load and the shape can be conveniently connected at the ends. The second criterion tends to favor the use of flat plate sections or flat-sided structural angles, compact rolled sections, or hollow tubes and these are commonly used for hangers and the tension components of truss

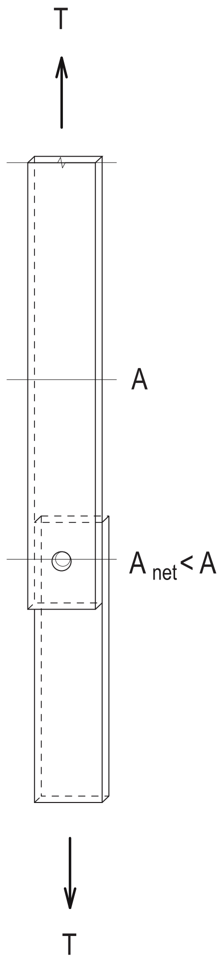


Figure 6.9
Member cross-sectional area reduced by hole for bolt or pin.

members, for example. Welding at the ends of such members can enable a direct transmission of tension forces to the rest of a steel structural system. However, if bolted or pinned connections are in some way necessary or desirable, the ends must be enlarged in order to account for the fact that at the bolt-hole section only the net area of the member (i.e., its total area minus that of the bolt-holes) is available to carry the tension force. (Fig. 6.9, e.g., see Ill. 6.29.) In certain cases, the full length of the member may be enlarged to what is needed at the bolt-holes so as to have consistent dimensions from one end of the member to the other, but this relinquishes at least some of the high degree of efficiency of a tension structural element.

Round steel rods are also commonly used as tension elements, especially since this minimizes their cross-sectional dimensions and, thus, their visual presence. It is also the form which from everyday experience is the most expressive of a material's being in tension; e.g., a very malleable material such as putty or chewing gum, when stretched, will take this basic cross-sectional shape. And from the opposite perspective, most people have experienced that a thin rod of any significant length is essentially unable to resist being bent or compressed (because of negligible flexural stiffness and the buckling phenomenon, respectively, which will be discussed in more detail in Chapters 7 and 8.) The high strength of steel in tension also means that round rods are usually of remarkably small diameter. But such minimal cross-sectional dimensions can cause their own set of problems: e.g., sloped or horizontal tension rods may noticeably sag in an unsightly manner due to their own self-weight and stays and hangers may display unwelcome tendencies to vibrate when wind blows over them. A common rule of thumb to prevent such problems, regardless of loading demand, is for a rod to have a diameter of at least 1/500 of its length – so that a 6m (20ft) tension rod, for example, would need to be at least 12mm (1/2in) in diameter. In other situations, especially long horizontal tension ties may need intermittent vertical support along their length to prevent excessive sagging, while excessive vibrations in tension elements can be countered by means of attaching small dampers to them. (e.g., see Ill. 6.37.)

The simplest of end connections for a round steel rod is for it to be welded along a short segment of its length, but this may not be the most aesthetically pleasing solution. Earlier in this chapter, we have seen with the Library of a Poet project an example of an alternative and conceptually clear end-connection detail for a steel tension rod; i.e., its "enlargement," in that particular case by means of a nut screwed on to the end of a threaded rod, that transfers the tension force in the member by bearing in compression against an opposing structure of some sort, such as perhaps a steel plate or wood block. (e.g., see Ill. 6.26.) A more detailed examination of the load transfer mechanism at the end of a threaded-rod tension hanger used to support an elevated walkway in the BMW Welt in Munich is illustrated in Ill. 6.34 and Fig. 6.10. In basic mechanical functioning, these are no different conceptually from the strategy employed when one is mending

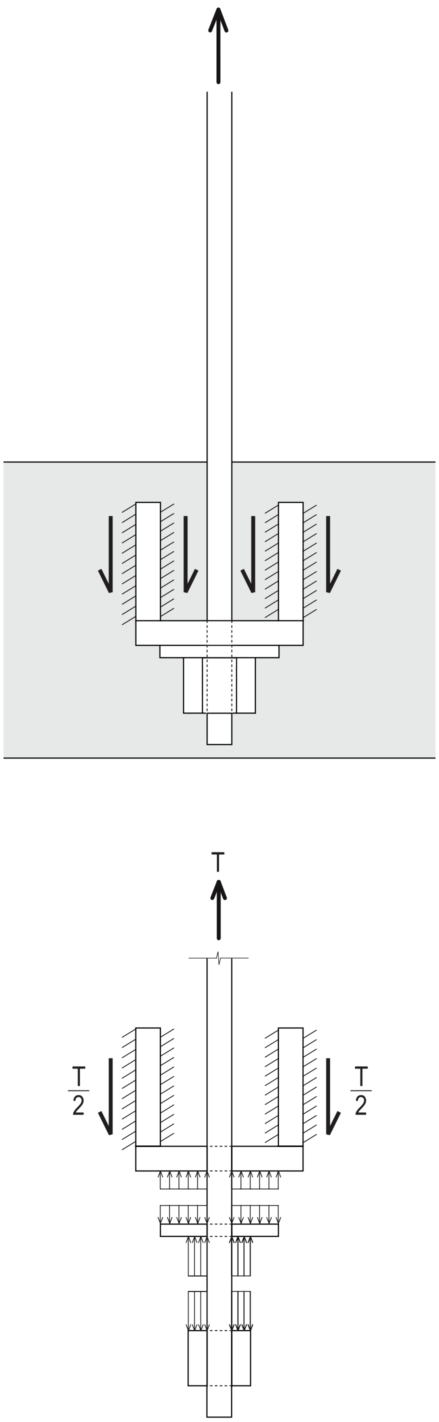


Figure 6.10
Transfer of forces mechanism at end of threaded steel rod: tension in rod balanced by plates bearing in compression against one another, then countered by shear in welds along anchoring plates.

Illustration 6.34
BMW Welt, Munich, Germany (2007).
Typical tension rod connection detail, including threaded nut bearing against load-transferring steel plates.
Architect: Coop Himmelb(l)au. Structural engineer: Bollinger + Grohmann and Schmitt, Strumpf, Frühauf + Partner.

Illustration 6.35
Turnbuckle with opposite direction threading at its ends allows for tightening of tension rods.

Illustration 6.36
Brooklyn Bridge, New York, NY, USA (1883).
Detail of cables of this famous bridge clearly shows that they are made up of steel wires helically wound together.
Designer and structural engineer: John A. Roebling, Washington Roebling, Emily Warren Roebling.

clothes; i.e., the tying of a knot at the end of a thread prevents it from being pulled through the fabric, and the thread can thereby be stretched and tightened.

Depending on the situation and the level of adjustment necessary, both ends of a tension rod may be adjustable in this fashion or, perhaps, one end is simply anchored against "pull-through" while the other has the threaded end needed for putting the member into tension. A third option is by means of a turnbuckle located somewhere along the length of the member. (e.g., Ill. 6.35.) This type of connector works by having opposite direction threading on the ends of the two rod segments being connected; the twisting of the turnbuckle thereby allows the two segments to be pulled together and the rod thus straightened by being put into a certain amount of *pre-tensioning* before any external loads are applied.

Aside from steel rolled section members, cables are also commonly used as tension elements, typically when there are high load requirements such as for supporting bridges and suspending long-span roof structures, but they also can exist at a much smaller scale where a particularly "light" aesthetic effect is desired. Generally speaking, cables are made up of many wires of high-strength steel that are helically wound together – analogously to the way natural and synthetic fiber strings and ropes are made. (e.g., Ill. 6.36.) Each thin steel wire is produced by drawing a rolled steel rod through a succession of tapered holes of diminishing diameter in dies made from an especially hard material such as tungsten carbide, in the end reducing the original cross-section by as much as 90 percent

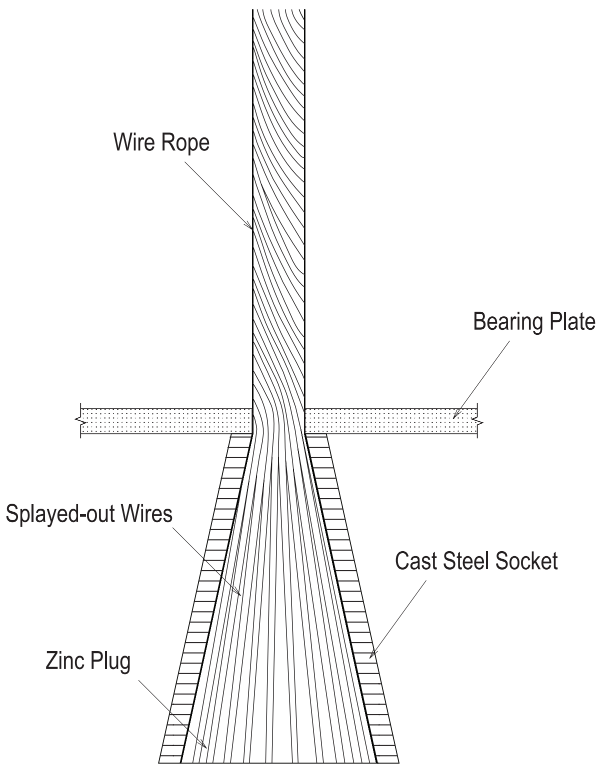


Figure 6.11
Member cross-sectional area reduced by hole for bolt or pin.

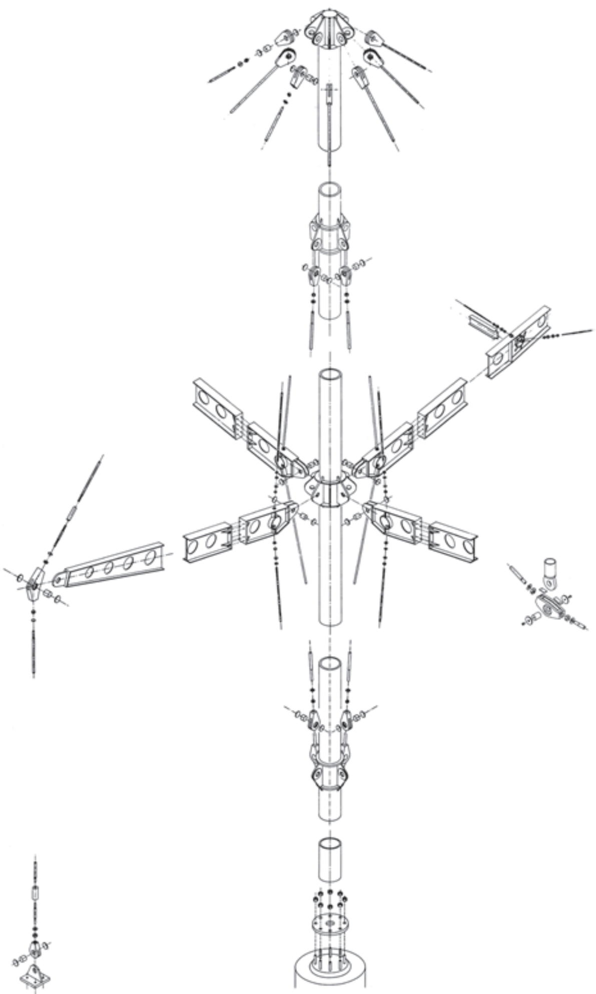
and resulting in a wire that may be ten times as long as the original rod was. Such wires are then helically wound around a core wire – usually six wires are wound around the core to form what is called a *strand*, and a number of strands (usually six) can be helically wound around a core strand to form a *rope*. Even larger cables can in turn be formed by winding together several ropes, and specialized *in-situ* cable-laying techniques have been developed for the especially large suspension cables needed for long-span bridges.

One problem with cables in comparison to rods is that their end connections can be somewhat more difficult to accomplish, such that a wide range of specialized attachments have been developed for these. The basic principles of tension anchorage that we have previously described remain the same but the detailed resolution must be adjusted since the steel wires of cables cannot be welded, threaded, or bolted. The "enlargement" needed to secure the end of a cable is instead typically made by means of a socket-type anchorage attachment in which the many wires of the rope or strand have been splayed apart within a conical void in a cast steel connector before molten zinc is poured into the spaces between the wires, which upon hardening creates a solid three-dimensional cone-shaped "plug." (Fig. 6.11.) The socket can then be used to transfer the tension force to the supporting structure in the same way as before, such as by having it bear against an opposing steel plate of some sort. (e.g., Ill. 6.37.)

There are yet other pros and cons to consider when comparing solid tension rods and wire cables: for example, since cables will often be used in exterior conditions (e.g., for bridges, stadia roofs, etc.), protecting them from corrosion is a common concern, in particular because of their being made up of many wires, a situation that lends itself well to the danger of water infiltration and internal rusting that may be especially hard to detect. Typically, such protection is done by means of subjecting the steel cable to a zinc coating/galvanizing process and/or by having a synthetic sheathing surrounding the cable, such as a nylon or PVC tubular covering. In contrast, a solid steel rod only presents a single outer exterior surface to the elements – and as long as regular painting is done it doesn't present the same potential problem. Rods are also cheaper than cables, and so tend to be used for smaller tension elements whose forces can be easily accommodated.

Another issue that cables have to contend with in comparison to rods is a direct result of their being made up of many wires; i.e., they tend to be fairly "stretchy" and have a relatively low modulus

Illustration 6.37
Millennium Wheel, London, UK (2000).
Anchorages for tensioned cables illustrate the basic principal of enlarging their ends so as to have these bear against a steel plate that is part of a custom-designed load-transfer connection. Small attachments along cables are dampers used to minimize vibrations. (See also Ill. 6.6.)
Architect: Marks Barfield Architects. Structural engineer: Jane Wernick of Arup.



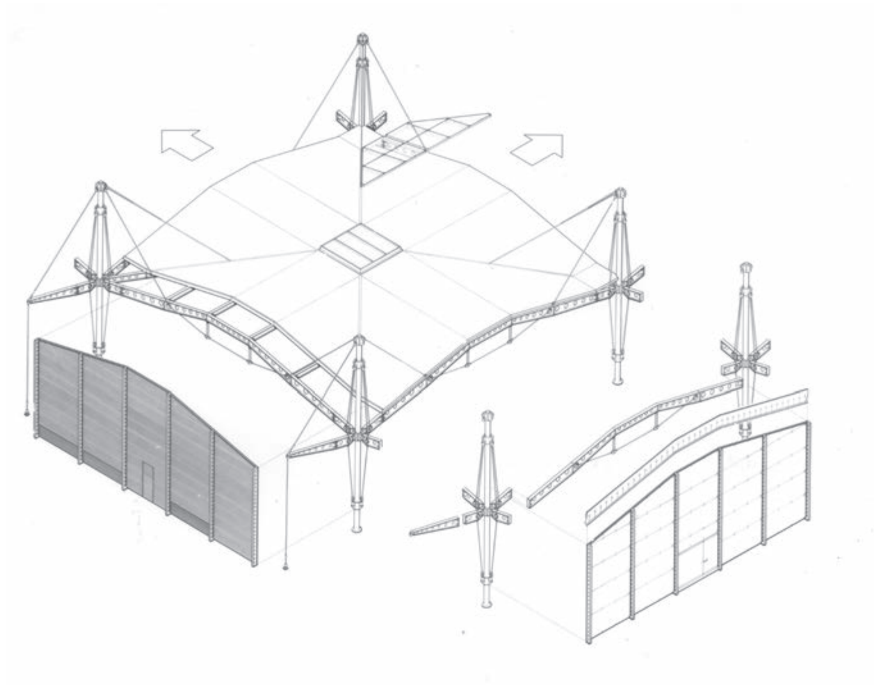


Illustration 6.38
Renault Distribution Centre, Swindon, UK (1980).
Expressive/iconic tension connection details.
Architect: Foster + Partners. Structural engineer: Arup.

of elasticity. Because cable wires are not straight but rather are wound together in helical fashion this gives them some geometric "slack," and when a significant load is applied that uses the full strength of the cable the strain can be 3–4 times what it would be if a solid steel rod were used.

Finally, there is a particular design aspect to remark upon that is common to both round steel rods and cables: their inherently difficult shape to grip or attach to means that connection details in tension structures are typically somewhat more substantial and considerably more geometrically complex than the members themselves. In fact, these joints are often of such visual interest and complexity that they are frequently highlighted in the design of tensile structures. In this sense, then, it can perhaps be said that tension members have a very conflicted role to play in architectural design: i.e., on the one hand they are associated with minimalism (in the sense of using as little material as possible) but on the other they often have very expressive and even flamboyant form – to the point where these members have effectively become iconic symbols for the whole of the buildings that they serve. (e.g., Ill. 6.38.)

The Beam and the Slab

CHAPTER

7

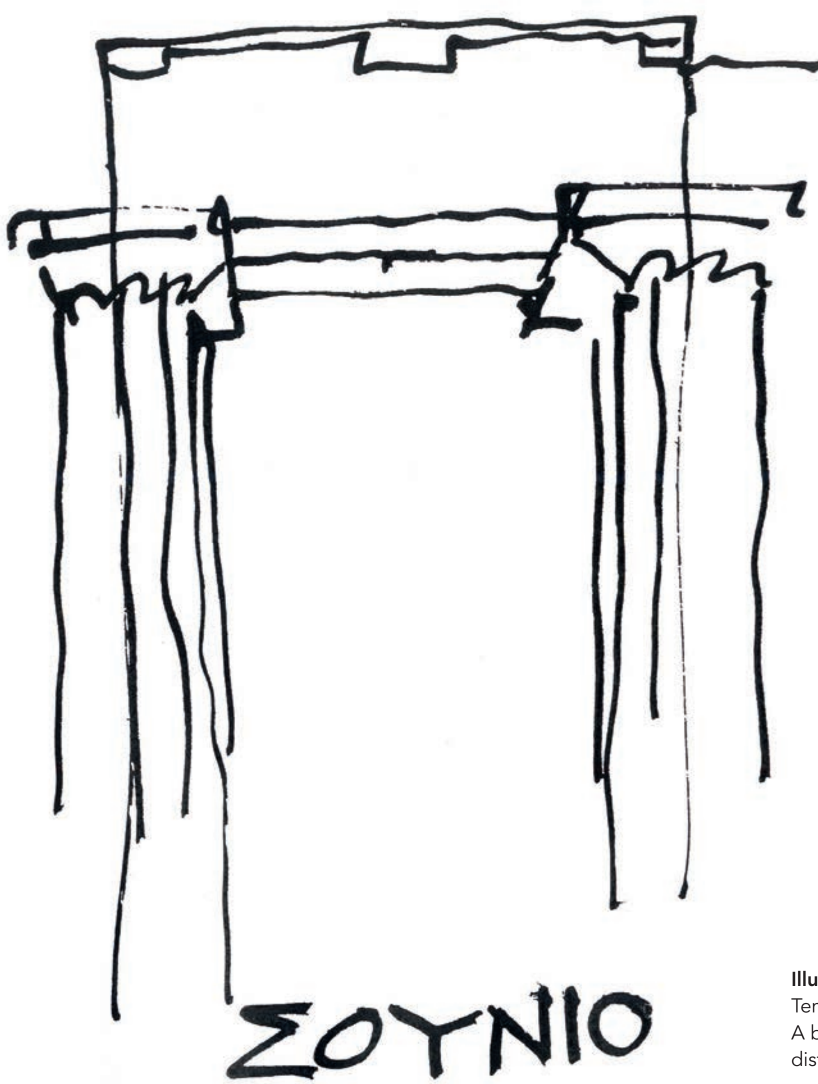


Illustration 7.1
Temple of Poseidon, Sounion, Greece (fifth century bc). A beam of classical proportions spans the short distance between adjacent columns.

Illustration 7.2
The Nordic Pavilion, Venice, Italy (1962).
Pavilion extends park space inside, with glass walls on two sides and preexisting trees left in place and piercing through roof structure. This side elevation view shows ends of one of two sets of orthogonal concrete beams placed one on top of the other.
Architect: Sverre Fehn. Structural engineer: Arne Neegård.

7.1 Nordic Pavilion and Jewish Museum – Contrasting Beam Patterns

Venice's Nordic Pavilion

The Nordic Pavilion in Venice by the Norwegian architect Sverre Fehn (1924–2009) is essentially an art gallery consisting of one room. The space measures about 470m² (5059ft²) and has no interior vertical supports. On two adjacent sides there are concrete walls closing off a more or less square plan, while the other two edges permit an almost invisible transition between interior and exterior space, achieved by means of sliding floor-to-ceiling glazing. (Ill. 7.2.) This visual openness brings the surrounding park into the building; the only element indicating the boundary between inside and out is the limit of the stone tile floor.

"Building a museum for the visual arts," Fehn said, "is the story of the struggle with light." One of the basic ideas of the Nordic Pavilion's roof structure design is to protect the art on display from direct sunlight. This is done by devising a roof structure made up of two orthogonal layers of closely spaced thin concrete beams that create an interior atmosphere of diffused light, recalling the light of "the shadowless world of the Nordic countries." (Ill. 7.3.) The art works are thus exhibited in a lighting environment intended to resemble that of the countries in which they were made. To maintain as much of the intensity of light as possible the concrete is cast in a mixture of white cement, white sand, and crushed white

Illustration 7.3
The Nordic Pavilion.
Column-free interior space, with two-layered beam grid evident in the ceiling.

marble. The beams follow a structural spacing module of 523mm (20.6in) – of ancient Egyptian origin, according to Fehn – while their height and thickness are 1000 by 60mm (39.4 by 2.7in). These dimensions relate exactly to the height of the sun at the Venetian summer solstice (64 degrees from the horizontal) so as to ensure the blocking out of any direct sunlight. The span of the bottom layer beams is about 18m (59ft), not counting the more than 4m (13.1ft) of cantilevering overhang. In between the beams of the upper layer translucent gutters of glass-fiber reinforced plastic sheets are hung to keep out the rain while fully admitting light.

This two-way orthogonal beam system, so devised to control interior light, also works very well to accommodate preexisting trees within the Pavilion, an important feature in helping the interior space being perceived as an extension of the only park in Venice. The trees actually penetrate through openings in the roof that dramatically interrupt certain beam spans, something made possible only by means of the two-way sharing of load characteristic of beam grids, as will be discussed in Section 7.10. The Nordic Pavilion demonstrates with great clarity the value of considering structural systems not merely as mechanical assemblages but also as architectural compositions that affect natural light and perceptions of space.

Illustration 7.4
Glass Courtyard, Jewish Museum, Berlin, Germany (2007).
Aerial view shows relationship of glass courtyard to enveloping U-shaped Baroque Kammergericht, to sharply angled plan of Libeskind's museum building, and to gardens.
Architect: Daniel Libeskind.
Structural engineer: GSE Ingenieur-Gesellschaft mbH.

Illustration 7.5
Glass Courtyard, Jewish Museum. Irregular and skewed beam grid picks up on geometry of museum. Columns also branch out in multiple directions and unconventional forms, reducing beam grid spans and potential stress concentrations.

Berlin's Jewish Museum Glass Courtyard

The Jewish Museum in Berlin by architect Daniel Libeskind, as completed in 1999, still needed a multifunctional space that could provide additional room for receptions, lectures, and concerts. The addition, also designed by Libeskind, is located in the courtyard of the historical building that is part of this museum complex, the former Baroque Kammergericht built in 1735 and now serving as an entrance to the contemporary museum. (Ill. 7.4.) The fully glazed addition offers an unobstructed view to the garden and can be used throughout the year while still preserving the sense of the original courtyard space: sliding doors in the glazed façade can be opened to transform the enclosure into a covered outdoor terrace.

The addition to Libeskind's original zinc-clad and zigzagging museum appears at first glance from the outside to be a relatively simple fully glazed cubic volume. Upon closer examination, however, it quickly becomes evident that things are anything but ordinary: the roof structure consists of a completely irregular grid of intersecting steel beams that is carried by four free-standing bundles of columns. (Ill. 7.5.) Each of these vertical supports consists of three column elements that branch out in multiple directions to meet the roof beams above; the columns have rectangular hollow cross-sections made from steel plates welded together, with one column in each bundle filled with concrete so as to be able to carry the full load of the roof in case of fire. The design concept was inspired by the Sukkah, the Hebrew word for a hut made of branches used for gatherings during the ceremonial Sukkot; here the bent and twisted structural elements can be seen to be like the tree trunks and branches of the traditional hut.

The structure as a whole displays a wonderful spatial quality that can be seen at once to be linked to traditional heritage and to the sharp-angled geometry of Libeskind's adjacent museum building. But while the spatial sequences of the museum building represent a closed structure with an atmosphere depicting the darker side of European history, in contrast the Glass Courtyard addition is full of light and the structure casts a lively and ever-changing pattern of shadows on the yellow-ochre walls of the surrounding Kammergericht building. For the invitation to the opening ceremony of the new space, the museum chose an appropriate title: "Wohin mit dem rechten Winkel?" which roughly translates to "What has happened to the right angle?"

Illustration 7.6
Valley Temple of Chefren, Giza, Egypt (Fourth Dynasty, 2723–2563 bc). Openings between lines of columns are spanned by pink granite beams of relatively short length and large depth.

7.2 Beam Origins

The classic example of a beam in a historical building context is a horizontal stone resting on two vertical columns. This simple structural configuration would have allowed people to pass or look through an obstructing or enclosing masonry wall. Moreover, if this basic form is made three-dimensional it leads to the primordial inhabitable space – four columns and a roof – and thus represents perhaps the beginnings of architecture. Beyond such practical purpose, however, a beam with two supports can also serve universal existential needs and it is both these aspects that characterize two structures of monumental historical character at the Valley Temple in Egypt and at Stonehenge in England.

Serving as part of a processional portal to the mysteries of the Pyramid and Great Sphinx tomb complex at Giza, Pharaoh Chefren's Valley Temple includes the well-preserved remains of a ceremonial hall built up of 16 monolithic pink granite pillars, each weighing roughly 100tons (220kips) or more. Spanning short distances between these pillars are horizontal stone blocks of the same material, themselves every bit as massive and heavy as the vertical elements. (Ill. 7.6.) In keeping with the Egyptians' preference

Illustration 7.7
Stonehenge, Salisbury, England, UK (approximately 2500 bc).
Three great “trilithons”; mystical power as defined by massive proportions of stone beams and columns.

for clear geometric forms, this temple is composed of a set of simple building blocks that are carefully balanced and very precisely cut and polished.

Stonehenge is also a cult building site that symbolizes power and endurance and was built at roughly the same time as the Valley Temple, although it is also generally understood to have been constructed in several phases over a period of many centuries. The complex originally comprised several concentric circles of rock formations, the alignments and orientations of which make it reasonable to conclude that the site was likely used for worshiping the sun and for making astronomical calculations that predicted the changing of the farming seasons; i.e., the complex likely functioned at least partly as some kind of celestial calendar. What we see today (Ill. 7.7) represents Stonehenge in ruin since many of the great stones have fallen or have been used by intervening generations for nearby home construction or road repair. From what remains, however, we can still imagine what Stonehenge once looked like. For example, the evidence suggests that an outer ring of 30 carefully shaped and massive upright stones were capped with horizontal lintel beams linked end to end into a continuous circle of stone propped high above the ground. (Today the most complete section of this circle consists of only

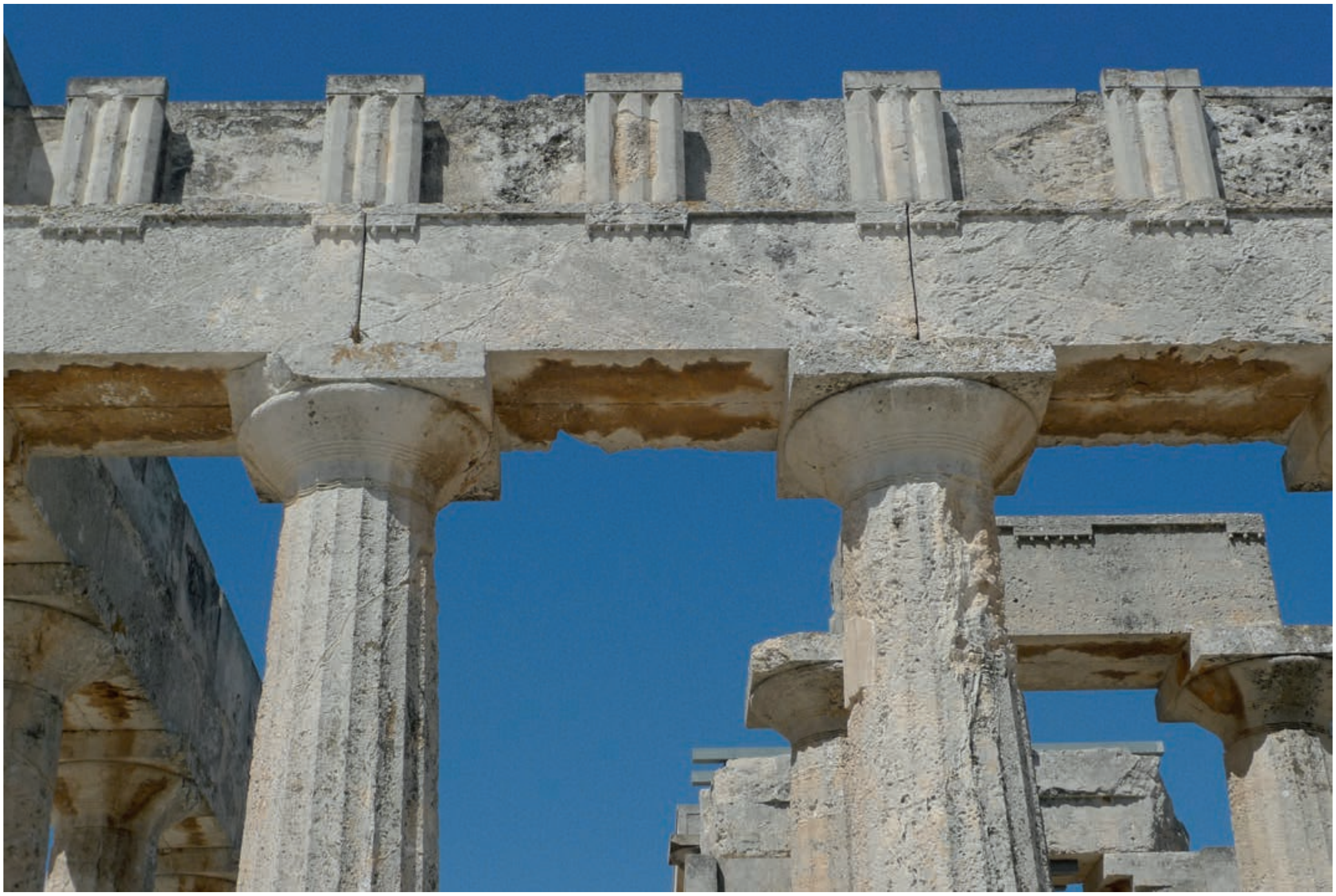


Illustration 7.8
Temple of Aphaia, Aegina Island, Greece (c.500 BC).
Stone beam segments able to span only short distances between column supports; relative proportion of beam depth to spanning distance is quite large by standards of contemporary construction materials and methods.

three beams that are still in place.) Also, the grandest and most impressive part of the whole arrangement, the sanctum, was an open-ended oval incorporating three great "trilithons" (derived from Greek and meaning "three stones"): two massive uprights capped by a horizontal beam spanning element. The bottom ends of these 40ton (88kip) upright stones, which extend 6.5m (21ft) into the air, are partially buried in order to give them lateral stability. All the stones are quite roughly carved and portray much of the natural character of their constituent material; i.e., a particular type of sandstone called sarsen, in which grains of sand are bound together by silica.

Over the course of time, there have obviously been many other structures, both large and small, mundane and monumental, that have incorporated similarly massive stone beams. As heralded as such monuments are in our cultural history, however, today from a structural perspective they display an almost absurd massiveness; i.e., their stone beams' dimensions seem grossly over-scaled when taking into account the relatively short distances that they span. (e.g., Ill. 7.8.) As we will see shortly, these no-longer-familiar proportions clearly express some of the underlying problems with beam structures, problems that are only accentuated when made of stone.

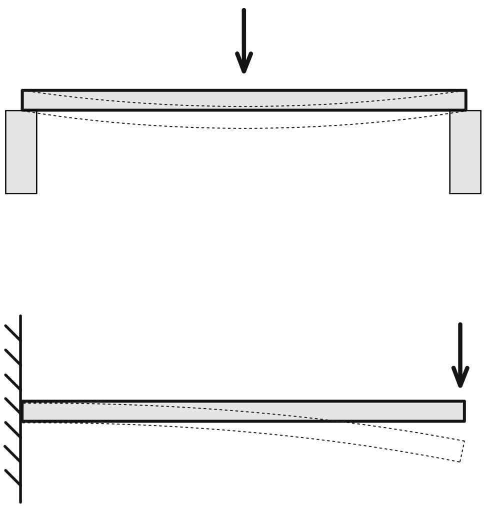


Figure 7.1
Load applied perpendicular to beams' spanning direction axis; resulting deflected profiles.

7.3 Equilibrium from Internal Actions

The Simply Supported Beam

Perhaps it is not unreasonable to argue that a beam was the first load-bearing structural element exploited by humans: somewhere in a prehistoric forest a tree that had fallen across a raging stream would have made it possible to cross the water without getting washed away – thus likely becoming, quite by accident, the world's first beam bridge.

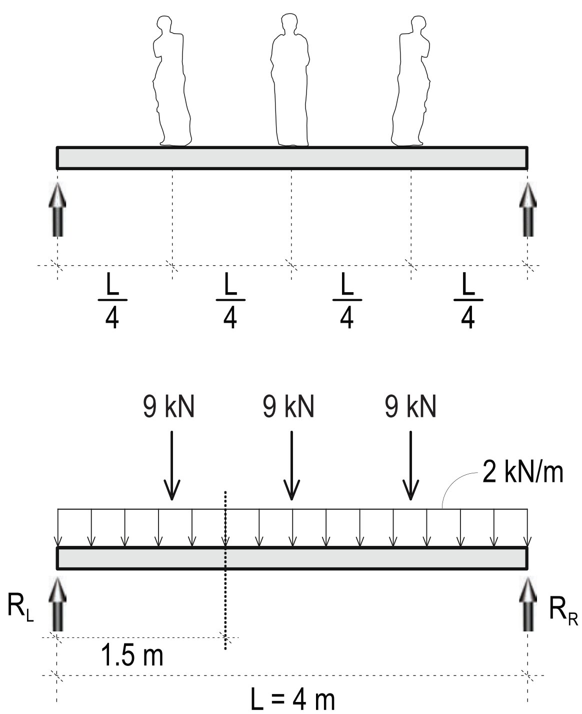


Figure 7.2
Beam loading diagram and end support reactions for situation similar to sculpture display at Parma's Galleria Nazionale shown in Ill. 7.9.

Illustration 7.9
Galleria Nazionale, Palazzo della Pilotta, Parma, Italy (1583; renovation: 1986). Steel beam supports classical sculptures as part of museum renovation.
Architect (of renovation): Guido Canali.

In contemplating the possibility of such origins, one is also able to recognize that the primary task of a beam is to bear loads that are applied *perpendicularly* to its longitudinal axis and spanning direction. As is commonly experienced with this type of load condition, whether it is from the stacking of one's shelves with reams of books or venturing out on to a diving board in preparation for a swim, the beam reacts by deflecting in the direction of the transversely applied load; i.e., the initially straight longitudinal axis of the beam in the unloaded condition is no longer so when load is applied.[1] (Fig. 7.1.) But despite our everyday experience with this characteristic transverse-deflection behavior of the beam, it is only through a detailed equilibrium consideration of this seemingly simple response that we can arrive at a fundamental understanding of how beams work.

This objective is perhaps most clearly accomplished by means of a numerical example. Consider as an arbitrarily chosen representative condition a beam that is simply supported at its two ends and has three concentrated point loads as well as a uniformly distributed load applied to it. (Fig. 7.2.) It may help to visualize a real-life condition that would be quite similar to this situation, such as by considering the sculpture-supporting beam at the National Gallery in Parma. (Ill. 7.9.)

We know from Chapter 3 how to determine such applied loads; e.g., the weight of sculptures can be converted into equivalent point loads, and a beam's self-weight can be represented by a uniformly distributed load. We also know from Chapter 4 how to use equilibrium considerations that are applied to the overall beam structure in order to determine the magnitude and direction of the support reactions at the two ends of the beam. For the symmetrical example under consideration here, if each of the three point loads is taken to be 9kN and the uniformly distributed load is determined to be 2kN/m, then the support reactions at each end of the 4m span are:

$R_L = R_R = [(3 \times 9\text{kN}) + (4\text{ m} \times 2\text{kN/m})]/2$
$R_L = R_R = 17.5\text{kN}$

But this equilibrium analysis so far only tells us what is happening at the external supports and will not be of direct help in establishing the size and shape that is necessary for the beam that is carrying the load to the ends. In order to be able to accomplish the latter, we need to find out what is going on internally in the beam, and this will be done by once again applying the fundamental principles and equations of equilibrium. In this instance, however, we will consider the equilibrium of only a *portion* of the beam structure rather than the *whole*. That we can selectively apply equilibrium principles equally well to parts of the structure as well as to its entirety is understandable when we consider what the equilibrium conditions actually imply in a physical sense; i.e., just as the beam as a whole is not going to be displacing vertically under load nor rotating in space (recalling that these are the actual physical meanings of the equilibrium equations $\Sigma F_y = 0$ and of $\Sigma M = 0$), these same truths obviously must also be valid for any segment or portion of the beam, as these certainly will not be displacing nor rotating any more than is the beam of which they are a part.

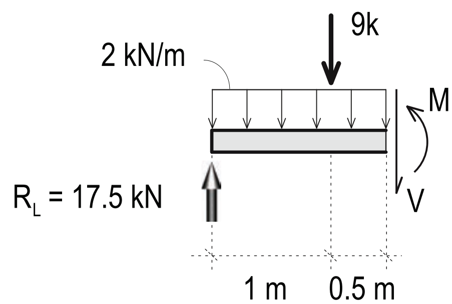


Figure 7.3
Free body diagram for left-hand portion of beam when imaginary cut is made 1.5m from left-hand support.

For example, if we want to find out what is happening in the beam just described at a distance of 1.5m from the left-hand support, we can make a purely imaginary and conceptual "cut" through the beam at that location and then draw the free body diagram of the geometry, the externally applied forces, and the support reactions that are acting on the beam on *either* side of this "cut." (Fig. 7.3.)

Summing, for instance, the vertical forces seen to be acting on the left-hand segment of the beam, where forces acting upwards are taken as to be positive and forces acting downwards as negative, leads to:

ΣF_Y = 17.5kN – 9kN – (2kN/m × 1.5m)
ΣF_Y = 5.5kN

which is a sum that is clearly not equal to zero, but indicates an unbalanced (positive) force resultant that acts upwards. Yet we know that for vertical equilibrium to be present in this part of the beam in order for it not to be translating vertically, the sum of the forces acting on this beam segment *must* be equal to zero. The only way for this to be true is if there is present at the location of the imaginary cut an equal but opposite-direction transverse force (V, acting downwards) that will in fact make the summation equal to zero; i.e.,

ΣF_Y = 0
17.5kN – 9kN – (2kN/m × 1.5m) – V = 0
V = 5.5kN

This necessary balancing transverse force V is known as the *shear force* in the beam, and it is an internal force that is developed within the beam itself. As is obvious by considering what would result from making such imaginary "cuts" through the beam at other locations along its length, the magnitude and direction of the balancing internal shear force will necessarily vary; this is something that we will be discussing again shortly.

A beam's other equilibrium requirement has to do with moment summations and recognizing that there must be rotational equilibrium for any beam segment. Summing moments about the "cut" for the external forces acting on the free body in Fig. 7.3 produces the following equation (recalling from Chapter 4 that clockwise moments are taken to be positive, while counterclockwise moments are assigned a negative value):

ΣM = (17.5kN) (1.5m) – (9kN) (0.5m) – [(2kN/m) (1.5m)] [(1.5m)/2]
ΣM = (26.25kNm) – (4.5kNm) – (2.25kNm)
ΣM = 19.5kNm

which is, once again, not summing to zero as we know it must in order for equilibrium to be present. Clearly what we are establishing this time is that there must also be present at the "cut" an internal moment, termed a *bending moment* and labeled M, that is going to have to be equal in magnitude and opposite in direction (i.e., counterclockwise, with a negative value) to the net sum of moments produced by the external forces acting on the beam segment; i.e.,

ΣM = 0
(19.5kNm) – M = 0
M = 19.5kNm

As for the shear force, the magnitude of the bending moment that is needed for equilibrium is going to depend on the location of the "cut" that is considered; these varying results along the length of the beam will shortly be plotted in order to better be visualized.

It is also worthwhile pointing out before going any further that both the *internal* shear force V and bending moment M that are found to be necessary at a particular location along the length of the beam are exactly what are needed in order to have equilibrium there; i.e., their magnitudes are exactly equal to the *external* net vertical force and *external* net moment of forces acting on the beam at that location, and their directions necessarily opposite to them.

The Cantilever

Although behaviorally an integral part of the beam family of structures, the cantilever is afforded special status by its support condition, profile, and nomenclature. Here we are talking about a beam that projects outward into the air, fixed against any deflection and rotation at its root but dramatically unsupported and unrestrained at its free end. Common examples of this situation abound in everyday life: e.g., both vertical tree trunks and their horizontal branches, diving boards, wings on an airplane fuselage, etc. And in the context of buildings, the cantilever, like the "regular" beam, has its own long history of development in terms both of scientific understanding and design approaches that have been applied to canopies, balconies, vertical towers, etc. (e.g., Ill. 7.10.)

Although fundamentally different in support condition from that of the typical beam, the cantilever is nonetheless still carrying load applied transversely to its longitudinal axis and the same beam-like bending behavior will result from it. We can apply the same equilibrium analysis process to cantilevers in order to predict shear force and bending moment magnitudes and their variations along

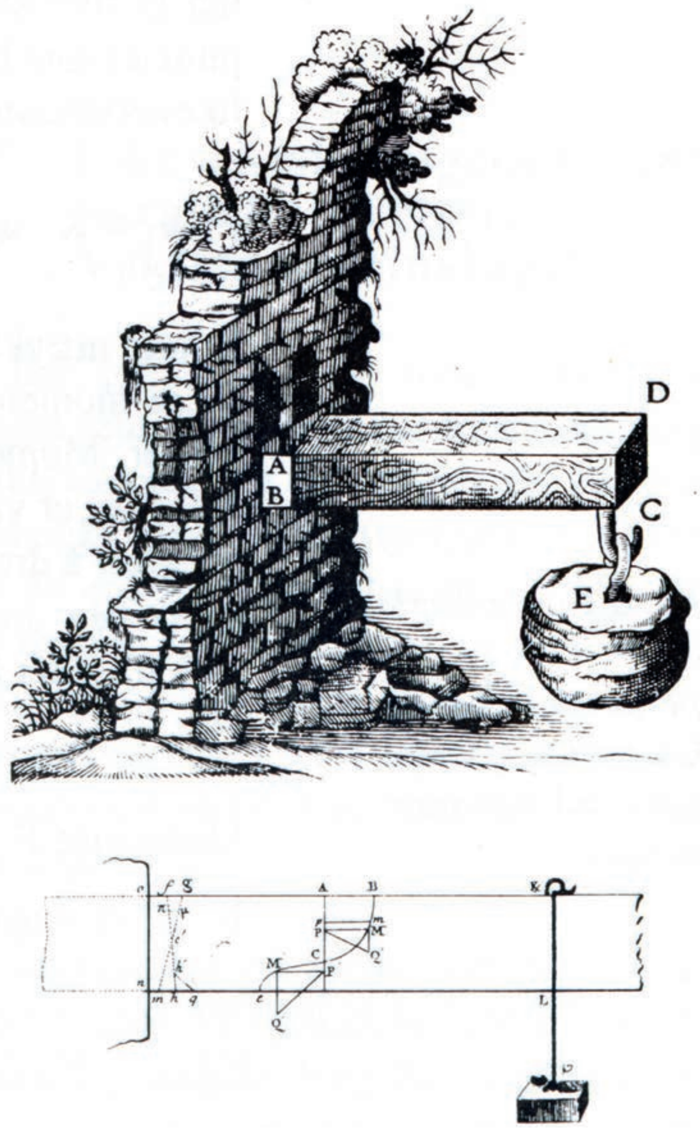


Illustration 7.10
"Galileo's problem."
Seventeenth-century scientist's experiment investigating cantilevered wooden beam behavior: correctly established that structural demand increases with square of projecting length; somewhat incorrectly predicted stress distribution over depth of beam.
Drawing from: *Due Nuove Scienze* 1638.

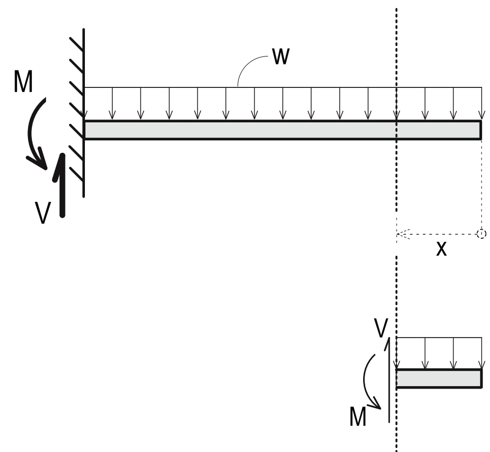


Figure 7.4
Cantilever beam subject to uniformly distributed load; free body diagram for beam segment to right of an imaginary cut at a distance "x" from the end.

the length of the member. Consider, for example, a cantilever beam to which we assume a uniformly distributed load is applied. (Fig. 7.4.)

Based on the corresponding free body diagram and equilibrium considerations, the following equations establishing the shear and bending moment can be written (in terms of the distance x from the end of the cantilever):

$\Sigma F_Y = 0$
$V - wx = 0$
$V = wx$

and

$\Sigma M = 0$
$(wx)(x/2) - M = 0$
$M = wx^2/2$

Here, the total load represented by the uniformly distributed load on the beam segment is (wx), and the moment that results from it is found by imagining all of that load to be acting as a point load at its center, which is in the middle of the length of the distributed load; hence, the moment arm is (x/2).

The variations in V and M defined by these equations are, like those of the simple beam, quite amenable to being plotted in diagrams and this will be addressed in the following section. Variable depth beam and cantilever profiles can then be devised corresponding to these plots; e.g., as is evident for the cantilever canopy shown in Ill. 7.11. Before getting into this more deeply, however, it is worthwhile noting for future reference that the direction of the internal bending moment M in a cantilever is opposite to that in a corresponding simply supported beam. We also saw that

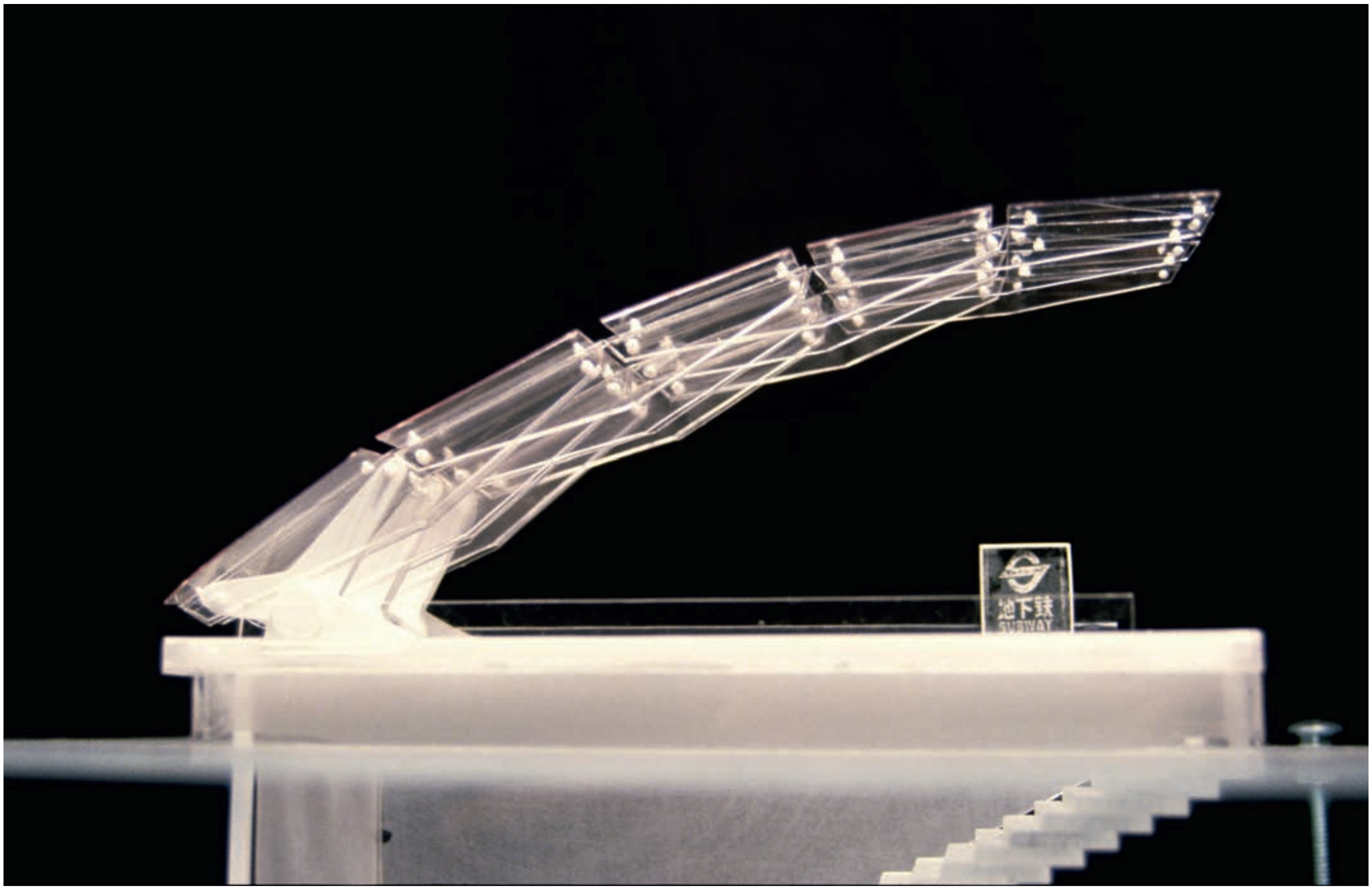


Illustration 7.11
Yūrakuchō Subway Station Canopy, Tokyo, Japan (1996).
Overlapping glass plates gradually increase in depth toward cantilevered canopy's base support; this is associated with the variation of the internal bending moments and shear forces along the length of the cantilevered structure.
Architect: Rafael Viñoly Architects. Structural engineer: Dewhurst Macfarlane and Partners. Cornell model by Maki Kawasaki.

in Chapter 2. This clearly is related to the opposite curvature of these two structures under the same transverse load; i.e., under gravity loading the simple beam is concave upward, whereas the cantilever is convex, as we saw previously in Fig. 7.1. We also intuitively understand from common experience that the top part of a gravity-loaded horizontal cantilever is stretched in tension while the bottom is compressed – just the opposite of what we anticipate takes place in a simply supported beam. For now, though, it is enough to have made these general observations; we will come back to them more specifically in Section 7.7 in the context of precisely determining the sets of stresses that are associated with the behavior of beams.

7.4 Fallingwater – Cantilevering Terraces

Certainly among the most architecturally well known of cantilevered structures ever built is Fallingwater, one of Frank Lloyd Wright's (1867–1959) most famous and admired works. (Ill. 7.12.) It was completed in 1937 as a weekend house for Edgar J. Kaufmann and is located not far from Pittsburgh, Pennsylvania, on a natural site that is characterized by deciduous forest, wild rhododendrons, and rapids. Built on a sandstone embankment, the house was designed as a series of projecting terraces that directly overhang the water and its falls. The architect described the house as

> *an extension of the cliff beside a mountain stream, making living space over and above the stream upon several terraces upon which a man who loved the place sincerely, one who loved and liked to listen to the waterfall, might well live.*[2]

Illustration 7.12
Fallingwater, Mill Run, Pennsylvania, USA (1937; restoration 2002).
Cantilevered concrete floors strengthened by upturned concrete edge beams that simultaneously form railings. System now further stiffened by hidden post-tensioned cables.
Architect: Frank Lloyd Wright. Structural engineer: Metzger-Richardson. Restoration structural engineer: Robert Silman Associates.

For Wright, the principle of cantilevering was a very personal solution, as natural as a branch that grows from the trunk of a tree or an outstretched arm: used with insight and ingenuity, this type of structure had many possibilities – it could make column-free spaces and create independently shaped and sized floor plates one on top of another. Fallingwater's main terrace is made of reinforced concrete and it was at that time a highly advanced structure with a cantilever of about 5m (16.4ft). The concept was based on the interaction between the beams in the deck and the upwardly folded concrete edges. Donald Hoffman's book *Frank Lloyd Wright's Fallingwater and its History* furnishes a good insight into the difficult and at times dramatic planning stages and building process for this house. Several times, the daring and visionary Kaufmann expressed serious doubt about the ability of the cantilevers to properly carry the load and he had his engineer independently check Wright's dimensions; furthermore, he had him measure the bending deflection of the terraces at regular intervals as long as he lived.

Evidently, Kaufmann intuitively knew something about cantilevers. By 1985 the projecting structure was noticeably sagging by up to 175mm (7in) and the concrete parapets were cracking badly, requiring temporary shoring to be installed that completely undermined the floating essence of the building. To rectify the problem, structural engineers Robert Silman Associates devised a clever post-tensioning cable system (more on the logic of this method later in this chapter in Section 7.8) that was threaded unobtrusively into the cantilevering floor system.[3] The house thus once more stands as originally designed and remains as one of the major works of twentieth-century architecture.

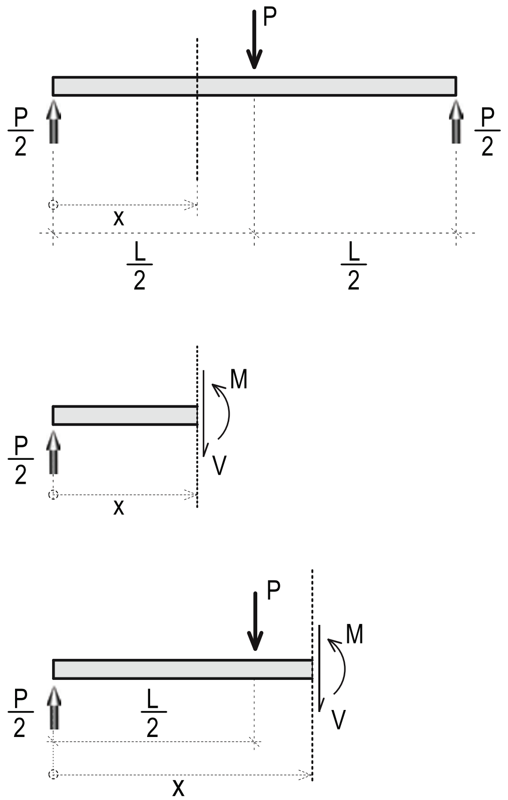


Figure 7.5
Simply supported beam with concentrated load P at center. (a) and (b): free body diagrams for beam segments cut to the left and the right side of P, respectively.

	x	V	M
	0	P/2	0
	L/4	P/2	PL/8
Just less than	L/2	P/2	PL/4
Just more than	L/2	-P/2	PL/4
	3L/4	-P/2	PL/8
	L	-P/2	0

Figure 7.6
Relative magnitudes of shear force, V, and bending moment, M, in beam at different distances, x, from left-hand end.

7.5 Visualizing Beam Actions – Shear and Moment Diagrams

In the earlier Section 7.3 we have established that beams must have at *each and every* cross-section along their length both an internal shear force V and a bending moment M in order to counterbalance the net effects of the external loading at that location. It was also discussed that, in general, the magnitude of these quantities that needs balancing changes from one location to another along the length of a beam. As will be demonstrated presently, it is conventional to graphically represent the variation of these quantities in what are called shear force and bending moment diagrams. The advantage of this method of visual presentation goes beyond mere convenience and mathematical convention, however; we will see eventually that it also has a far-reaching impact in suggesting to the designer the potential for the shaping of beam structures.

First let us consider a *simply supported beam with a concentrated point load P* applied midway along a span of length L. (Fig. 7.5.) The symmetry of the condition means that the upward support reactions at each end are equal to P/2. To find the shear force and bending moment at an arbitrary section located at a distance x from the left-hand end, a free body diagram for the cut portion of the beam can be drawn in a manner that will be appropriate for $0 \leq x \leq L/2$. (Fig. 7.5a.)

If we now write the $\Sigma F_Y = 0$ and $\Sigma M_{cut} = 0$ equations for the translational and rotational equilibrium of this beam segment, we will have:

$$\Sigma F_Y = 0$$
$$P/2 - V = 0$$
$$V = P/2$$

and

$$\Sigma M_{cut} = 0$$
$$(P/2)(x) - M = 0$$
$$M = (P/2)(x)$$

Substituting different values of x into these expressions yields the magnitude of the shear force and bending moment at those respective locations, as have been given in Figure 7.6.

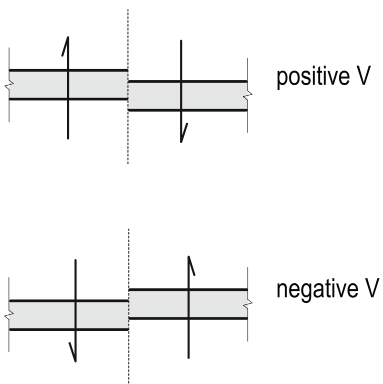


Figure 7.7
Sign convention for shear force, V, in terms of tendencies to effectively cause clockwise vs. counterclockwise rotation.

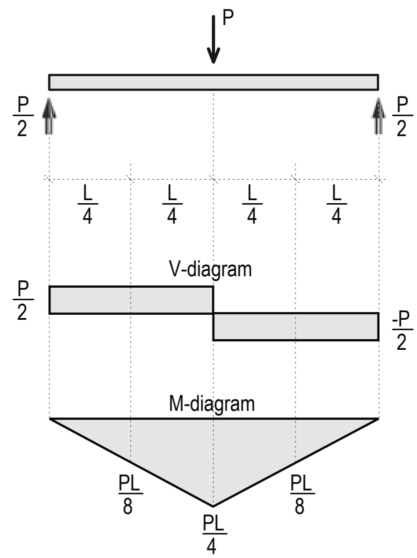


Figure 7.8
Shear force and bending moment diagrams for simply supported beam subject to concentrated load at mid-span.

Similarly, for the part of the beam beyond the midpoint (i.e., where $L/2 \le x \le L$), a different free body diagram must be drawn (Fig. 7.5b) and the following expressions that are developed from it for shear force and bending moment enable the completion of the remainder of the table in Figure 7.6:

$\Sigma F_Y = 0$
$P/2 – P – V = 0$
$V = –P/2$

and

$\Sigma M_{cut} = 0$
$(P/2)(x) – (P)(x – L/2) – M = 0$
$M = PL/2 – Px/2$

The results can then be plotted graphically along the length of the beam in what are known as shear force and bending moment diagrams. Before fully being able to do so, however, sign conventions need to be established for this purpose.

- *Bending moment diagram sign convention:*
  The bending moment is drawn on the tension side of the beam.
- *Shear force diagram sign convention:*
  If the balancing shear forces would tend to rotate the beam segment in a clockwise direction, the shear is termed positive; the contrary is called a negative shear condition. (Fig. 7.7.)

From the V and M diagrams incorporating these sign conventions for the point-loaded simply supported beam (Fig. 7.8), the following patterns that are specific to this load condition can be observed:

- The magnitude of the shear force in the beam is a constant V = P/2 from one end of the beam to the other, although this action changes direction at mid-span.
- The magnitude of the bending moment in the beam varies linearly from zero at either end to a maximum value of PL/4 at mid-span.

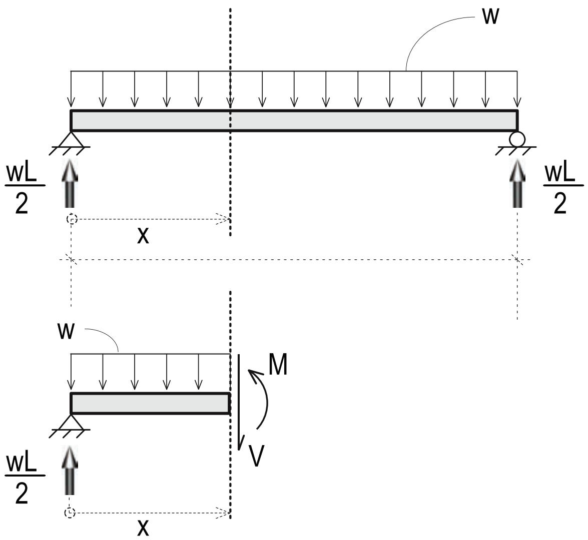


Figure 7.9
Simply supported beam with uniformly distributed load; free body diagram for segment to left of imaginary cut at distance "x" from left end.

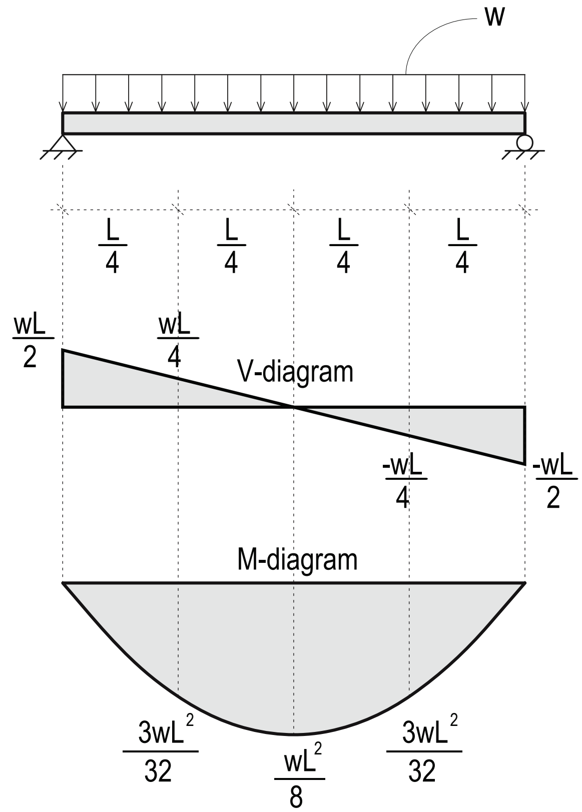


Figure 7.10
Shear force and bending moment diagrams for simply supported beam subject to uniformly distributed load.

A similar analysis can be applied to perhaps the most common situation of a *simply supported beam with uniformly distributed load w* applied to it. (Fig. 7.9.) In this case a single free body diagram that is drawn in terms of a variable distance x will suffice and the following expressions for shear force and bending moment emerge:

$F_Y = 0$
$wL/2 - wx - V = 0$
$V = w (L/2 - x)$

and

$M_{cut} = 0$
$(wL/2) (x) - (w x) (x/2) - M = 0$
$M = w [(Lx - x^2)/2]$

Once again, the results of these equations can be plotted. (Fig. 7.10.) For this load condition, some of the key patterns and observations that emerge are that:

- the magnitude of the shear force in the beam varies linearly from a maximum at one end to an equal but opposite maximum at the other end, with zero magnitude at mid-span;
- the magnitude of the bending moment in the beam varies parabolically from zero at one end to a maximum of $wL^2/8$ at mid-span and back to zero again at the opposite end.

Likewise for the *cantilever with a point load P at its free end, or for the cantilever with uniformly distributed load w* the results for shear and moment at different locations along the beam can be determined and plotted. (Fig. 7.11.) The clear patterns that emerge in these cases are that:

- for the point load condition, the magnitude of the shear force remains constant throughout at V = P, while the magnitude of the bending moment increases linearly from zero at the free end to M = PL at the support;
- for the uniformly distributed load condition, the magnitude of the shear force increases linearly from zero at the free end to a maximum of V = wL at the support, while the magnitude of the bending moment increases parabolically from zero at the free end to $M = wL^2/2$ at the support.

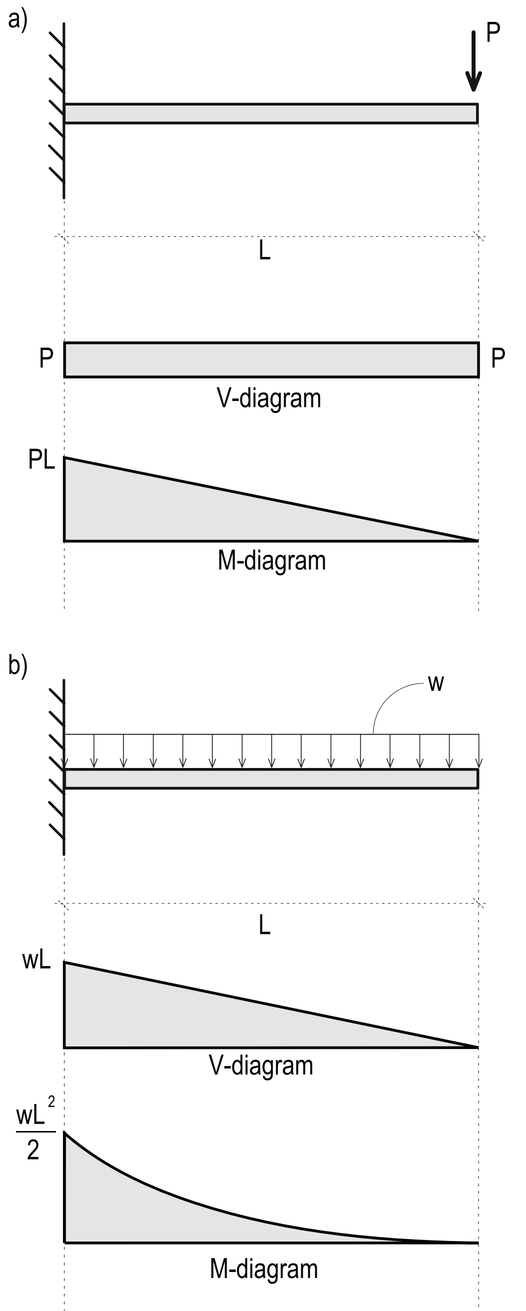


Figure 7.11
Shear force and bending moment diagrams for cantilever beams subject to (a) concentrated load at free end, (b) uniformly distributed load along entire length.

We have thus considered four of the simplest (yet also among the most common) of beam support and load case situations – and have used these to illustrate the graphic means that is typically employed to present the variations in magnitude and direction of shear forces and bending moments along the lengths of beams. More complex situations can readily be dealt with in exactly the same manner by carefully considering the equilibrium of appropriate segments of beams, and more intricate V and M diagrams will inevitably result. By plotting these quantities in this graphical manner, however, the variations and maximum values of the internal beam actions become easily legible, and this is something that will prove to be of critical value for the sizing and shaping of beams, as we shall see in the following sections.

7.6 Form Follows Diagram, Or Not …

The types of algebraic formulas derived in the preceding section for calculating bending moments and shear forces will prove very useful when it comes to selecting the beam sizes and cross-sectional shapes that are necessary to carry loads, but for now let us focus on how the overall shapes of the V and M diagrams provide an opportunity for informing the design of beams in terms of their elevational profile.

We have seen in each of the load conditions that we have looked at so far that the bending moment in a beam varies more quickly along the span than does the shear force; e.g., for a uniformly distributed load the bending moment changes with the square of the distance x from the end whereas the shear force varies linearly, while for a concentrated load the bending moment changes linearly and the shear force remains constant along the entire length of the beam. What this implies generally is that with increasing span there is a dramatically greater increase of bending moment than there is of shear force. Consequently, if structural efficiency is required, or if simply structural expression is desired, it is the bending moment diagram that is typically reflected in a beam's physical form and, most noticeably, in the variation of its vertical dimension (beam depth).

One example of this can be seen in the support beams for the enclosed glass pedestrian bridge designed by Dirk Jan Postel in Rotterdam to link the otherwise separated second-floor offices of a

Illustration 7.13
Glass bridge for Kraaijvanger Urbis, Rotterdam, the Netherlands (1994). Curved floor beam profile mimics its bending moment diagram.
Architect: Dirk Jan Postel/Kraaijvanger Urbis. Structural engineer: ABT/Rob Nijsse.

single architecture studio. (Ill. 7.13, 7.14.) This bridge is unusual in that all of its structural framing and enclosure system utilizes structural glass technology – not only the floor plate, the side walls, and the ceiling, but also the two laminated glass support beams carrying the bridge's dead and live loads. Moreover, the support beams' dramatically curved bottom-edge profile can quite obviously be understood to be influenced by the parabolically shaped bending moment diagram for such a uniformly loaded simple span.

A second example of this relationship between beam depth and bending moment diagram, this time for a cantilever, can be seen in the roof canopy that the architect Zaha Hadid designed for the Tondonia Winery in Spain. (Ill. 7.15.) Since the vertical dead and live loads on the roof canopy results in what can be approximated to be a uniformly distributed load on each of the cantilevering ribs, these will have, as we saw in the previous section, internal bending moments that increase toward their "root." The resulting shape of the bending moment diagram is generally reflected by the variation in the depth of the roof's structural steel ribs. The cantilever aspect is highlighted even further in this project by having the roof structure supported on one side by vertical ribs that are themselves cantilevered from the ground. This overall cantilever-upon-cantilever configuration serves not only to shelter the flask-shaped wine shop and tasting room but also to highlight and provide a visual backdrop for the distinctive pavilion within a very tight and eclectic agglomeration of buildings.

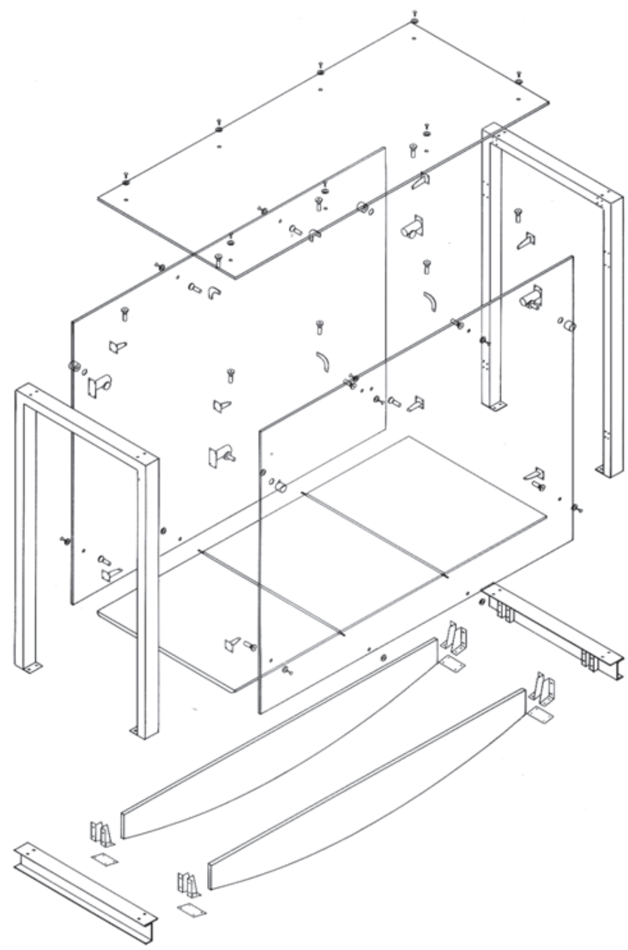


Illustration 7.14
Glass bridge for Kraaijvanger Urbis.
Exploded axon drawing of walkway components.

Illustration 7.15
Tondonia Winery, Haro, Spain (2006).
Horizontal projecting steel ribs of cantilevered canopy increase in depth toward support, following cantilever beam's bending moment diagram. (See also Ill. 5.31.)
Architect: Zaha Hadid Architects. Structural engineer: Jane Wernick Associates.

Illustration 7.16
Kunsthal, Rotterdam, the Netherlands (1992).
Roof beam with constant depth in spite of variations in bending moment demand.
Architect: Office for Metropolitan Architecture (OMA). Structural engineer: Arup.

It is immediately surprising once one becomes cognizant of this potential link between the shape of bending moment diagrams and beam depths to find the extent of reproduction of this relationship in the built world. Very many long span beams in stadia roofs and bridges regularly exploit this form-making potential (see also Ill.1.15), but it can also be found in smaller scale projects; e.g., the varying depths of the glass plates for the sheltering canopy of Yūrakuchō Subway Station seen in Illustration 7.11 can be reexamined in this context. This is a topic that we will see has very broad application, and we will come back to it once again in the context of both trusses and arches in Chapters 9 and 12, respectively.

But as compelling as these examples are, we will end this section by completely undermining the suggested design direction presented here so far. For there should be no misconception that all beams must follow the shape of their bending moment diagrams; indeed, the typical condition is anything but like this. In fact, it is quite normal for practical and economical and sometimes aesthetic reasons as well for beams in buildings to retain the same depth and geometric profile over their entire length. Manufacturing techniques for rolled steel members ensure that these are constant in section along their length, and milling practices do the same for sawn lumber. Clearly, however, the structural demands indicated by the shear and bending moment diagrams are not going to go away and these must still be attended to. In the case of constant section beams, only the maximum value of the bending moment and shear force acting on the member is deemed critical, wherever this occurs – and *the member is sized only for that largest value*. This means that everywhere else along the length of the beam an oversized section is being provided! This very common situation begins to explain why typical beams are, from the point of view of material usage, extremely *inefficient* structures – and there is more to come on this score as we shall see in the next section.

Clearly in the case of constant-depth beams there is at work another design agenda rather than the one of pure structural efficiency. More often than not it will simply be a matter of pure economics – it is far cheaper to mass produce members of constant sectional profile rather than to custom manufacture each and every member according to the specific demands placed on it. But it may also be a matter that sometimes a certain design aesthetic is desired, such as that which we saw earlier at Sverre Fehn's Nordic Pavilion (see Ill. 7.2) or as is also evident with the roof beams of the Kunsthal in Rotterdam. (Ill. 7.16.)

Another design approach provides the opportunity for a variation on this theme of constant-depth beams as can be seen at the Madrid–Barajas Airport developed by the partnership of architects Richard Rogers and Antonio Lamela. Here the series of steel beams still have roughly constant depth, but, instead of being straight, their elevational profile undulates strongly up and

Illustration 7.17
Madrid–Barajas Airport, Madrid, Spain (2006).
Continuously undulating roof beams of near constant depth.
Architects: Richard Rogers Partnership and Estudio Lamela.
Structural engineers: Anthony Hunt Associates, OTEP Internacional, TPS.

down several times across the full width of the terminal building. (Ill. 7.17.) Despite the very different look, however, in such cases the statics of the problem with respect to vertical gravity loading do not change significantly from that of a perfectly horizontal beam, and the shear force and bending moment demands and diagrams will be essentially the same. (Fig. 7.12.)

7.7 Deformations and Internal Stresses

In order to further understand basic beam behavior, to help make sense of the various beam cross-sectional shapes that exist as well as to develop the structural theory that will eventually enable beam sizes to be determined, we need to extend our discussion of internal shear forces and bending moments to defining the sets of internal beam stresses that produce these actions.

We will begin by considering a beam with a compact, rectangular cross-section and that has a concentrated point load at mid-span that induces downward deflection in the beam. (Fig. 7.13.) From common experience, we know that in such a condition the beam material will experience tension stresses caused by stretching along the bottom while at the top the material will shorten and be in compression. A simple experiment with a piece of foam or rubber will reconfirm this statement, with the flexible material being useful for the model in order to exaggerate the necessarily

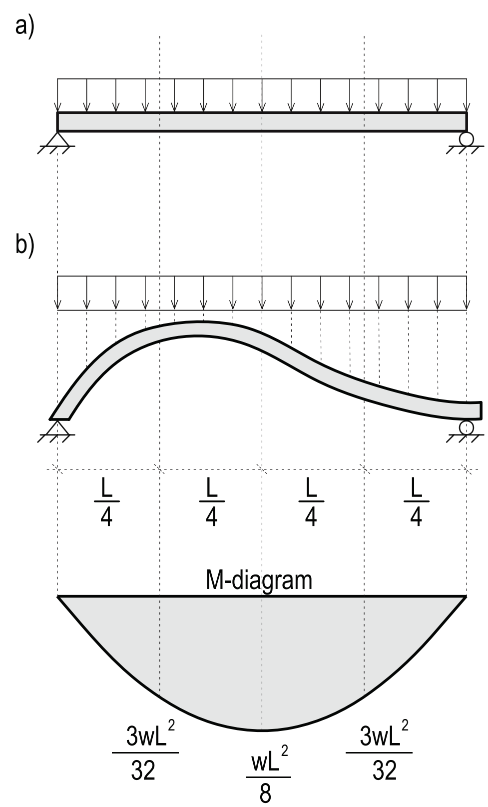


Figure 7.12
Simply supported beams with different elevational geometry – one straight, the other curved – but subject to identical loading have common bending moment diagram.

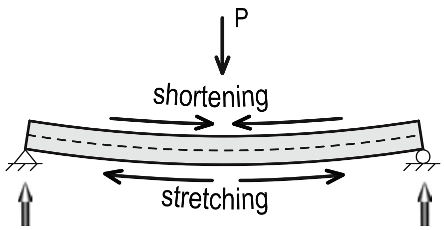


Figure 7.13
Deformation along the span of a beam with transverse loading. The beam responds by being shortened and stretched, in its top and bottom halves, respectively.

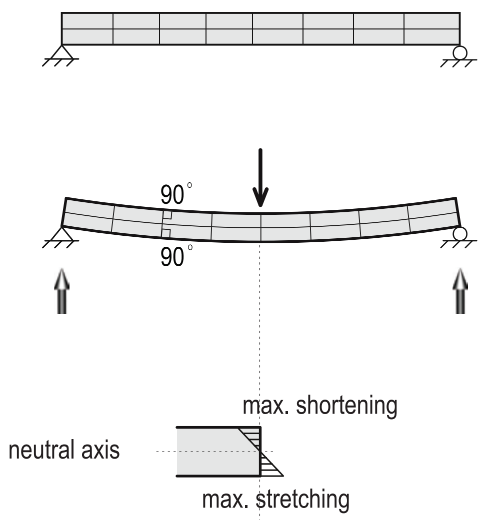


Figure 7.14
Lines drawn on flexible foam beam (representing arbitrary sections through the beam) rotate when beam is loaded, but still remain straight; this corresponds to a linear distribution of deformation tendencies over depth of beam.

much smaller deformations that take place in a real load-carrying beam in a building.

To reinforce and expand upon what has just been described, imagine that we were to draw evenly spaced parallel vertical lines on the outside of the piece of foam or flexible rubber in the unloaded condition. When such a "beam" is loaded, the shortening of its top side and elongation along the bottom that we anticipate is immediately evident from the rotation of the lines. (Fig. 7.14.) Moreover, the experiment reveals that the distance between the originally parallel lines is linearly smaller and larger in proportion to the distance from the mid-depth of the beam, the level – called the *neutral axis* – at which the distance between the lines that were drawn remains equal to what it was originally.[4] Described another way, this experiment reveals that the originally vertical lines – which conceptually represent cross-sectional planes cut through the three-dimensional beam – remain straight even in the loaded condition, although they do rotate so as to remain perpendicular to the top and bottom of the deflected beam.

If this description of fundamental beam behavior and deformation may seem relatively simple, it is worth noting, perhaps gratifyingly, that historically it has not come easily. Leonardo da Vinci (1452–1519) hypothesized about beam behavior (among other things …) (Ill. 17.18) and as we saw in Section 7.3 Galileo Galilei (1564–1642) worked on the problem to the point that he presented the first (erroneous) formal analytical theory on the subject in 1638.[5] Over the next 200 years a series of French mathematician/scientists modified Galileo's hypothesis, culminating in 1826 with Claude Louis Marie Henri Navier (1785–1836) publishing what is widely credited today as being the correct solution for the bending behavior of beams.[6]

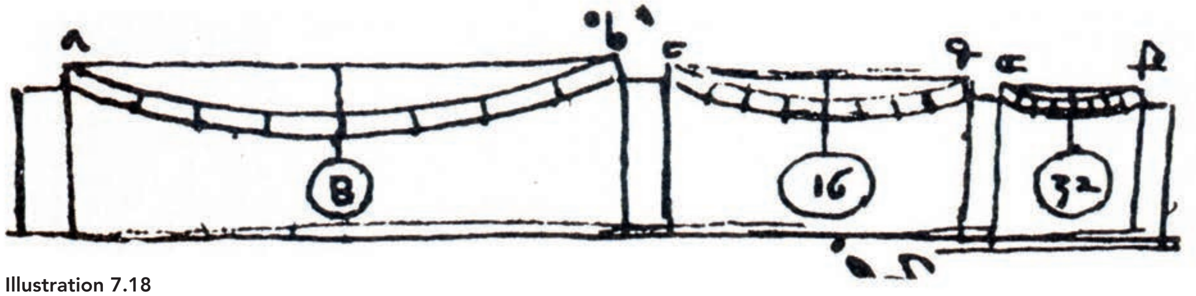


Illustration 7.18
Leonardo da Vinci's diagrams of relative deflections for various beam spans.

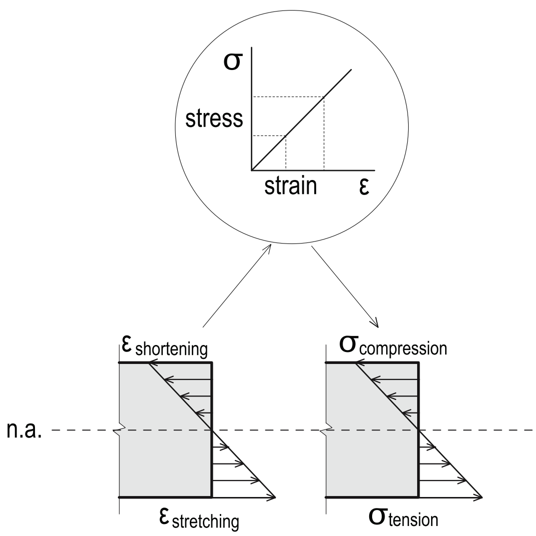


Figure 7.15
Strain and stress diagrams over beam depth, related by means of Hooke's Law for elastic materials.

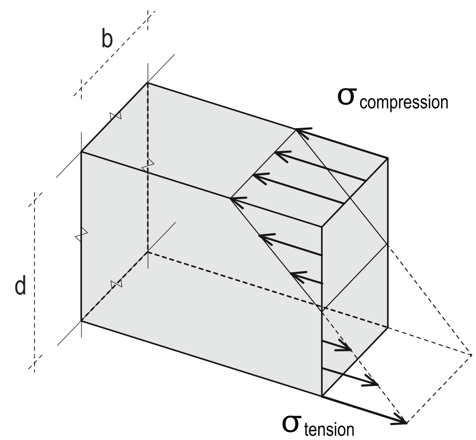


Figure 7.16
Corresponding triangular wedges of compression and tension stresses in 3-D view of beam.

Navier based his theories on the assumption just described that originally "planar sections in a beam will remain planar" in the deflected condition under loading, a statement that has since been proven experimentally to be valid. Today's building code specifications concerning beam behavior are still fundamentally based on this hypothesis developed nearly 200 years ago.

Bending Stresses

By applying Hooke's Law (which, it will be recalled from Section 5.2, relates strain – and thus deformation – to stress for elastic materials), the linearly varying shortenings and elongations over the beam depth that we have just described can be associated with a corresponding straight-line variation of compression to tension bending stresses, typically designated by the Greek letter σ (sigma). (Fig. 7.15.) In three dimensions, this state of stress in the beam can be visualized as triangular wedges of compression and tension stresses acting over the upper and lower halves of a beam's cross-section. (Fig. 7.16.)

The effects of this stress distribution acting on, for example, the rectangular cross-section of a wood beam having width b and depth d can now be studied. The stresses on the compression and tension sides of the beam acting over their respective cross-sectional area halves effectively produce an equal but oppositely directed compression force in the upper part of the beam and a tension force in the lower section; i.e., an oppositely directed pair of forces separated by a distance "a" (together these are known as a force couple) is established within the depth of the beam. (Fig.7.17.)

This force pair produces a moment about the neutral axis level whose magnitude is defined by

$$M = (C \times a/2) + (T \times a/2)$$

Since C = T, we can also write

$$M = C x a = T x a$$

It is this internal moment in the beam that is resisting whatever external moment imbalance exists at that location from the applied forces and support reactions (Fig. 7.18). Clearly, with such a small lever arm "a" limited to something less than the beam's depth, in order for this moment to be significant it will be required that the magnitudes of the C and T forces in the beam (and, therefore, of the bending stresses that produce them) be quite large. This conclusion begins to suggest the fundamental problem with the way in which beams carry load – but there will be more on this topic later.

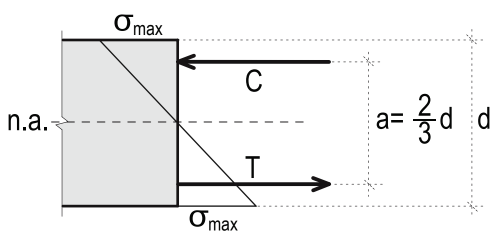


Figure 7.17
Compression (C) and tension (T) forces, statically equivalent to stresses in corresponding top and bottom halves of beam, produce an internal resisting couple, which is defined as a bending moment caused by two equal but opposite forces located a certain distance apart from each other.

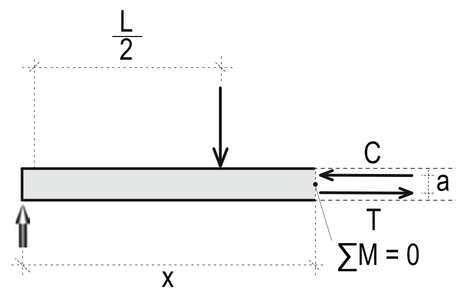


Figure 7.18
Internal bending moment produced by C and T must balance the sum of moments produced by external forces and reactions.

Getting back to our rectangular beam example, the magnitude of the forces C and T in the beam will be equal to the volume of the stress triangles acting over their respective beam halves; i.e.,

$$C = T = [1/2]\ [\sigma_{max} \times b \times d/2] = \sigma_{max} \times b \times d/4$$

For a triangular stress distribution the distance *a* between forces C and T is equal to (2/3)d. (Fig. 7.17.) With the appropriate substitutions, therefore, the bending moment produced can be rewritten as:

$$M = (\sigma_{max} bd/4)\ (d/3) + (\sigma_{max} bd/4)\ (d/3)$$
$$M = \sigma_{max}\ (bd^2/6)$$
$$M = \sigma_{max} \times S$$

The result of this derivation indicates that the internal bending moment in a beam is directly proportional to the magnitude of the maximum stress produced by bending – an observation that will shortly be shown to have a direct bearing on the methods used for the sizing and selection of beam sections. Moreover, it can be seen that the constant of proportionality between bending moment and maximum bending stress is dependent only on the dimensions of the cross-section; this constant is called the *section modulus* S having units mm^3 (in^3). For the rectangular cross-section that we have been examining, for example, S is equal to $bd^2/6$, a quantity that can easily be calculated algebraically or obtained from standard manufacturers' tables.

This same equation can be rearranged into what is called the *bending* or *flexure formula*:

$$\sigma_{max} = M/S \qquad (7.1)$$

that clearly establishes the maximum bending stress in a beam as being solely dependent on M, the external moment at a section (itself a function of loading and the geometry of structural framing), and on S, a quantity established by the beam's cross-sectional geometry.[7] Such an equation tends to lead to the conclusion that the design of beams is a purely scientific matter, but this ignores the architectural design choices that are implied by it. As we have seen, moments are a function of loads and of the choice of structural system in terms of materials, spanning distances, spacing, orientation, and support conditions – all of which are well within the control of the designer. Moreover, the choice of beam cross-sectional dimensions and shapes are also completely a matter of a designer's intentions – to be established not only by material capacity but also in terms of visual effect. The types of decisions that led to the selection of very different beams for the Nordic Pavilion and the Jewish Museum's courtyard roof seen

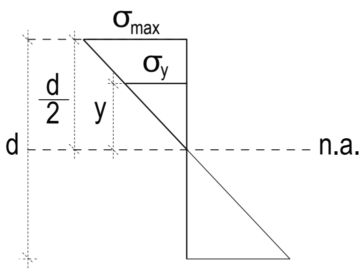


Figure 7.19
Similar triangles relate stress magnitudes at different levels of beam.

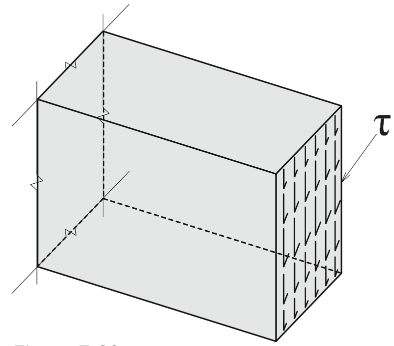


Figure 7.20
Notional set of shear stresses acting over the depth of a beam's cross-section.

at the beginning of this chapter obviously have to do with many things besides a mechanical and unimaginative application of the flexure formula. It will be good to bear this in mind as we proceed a bit further with the algebraic development.

The formula as presented so far defines the maximum bending stress occurring at the top and bottom of a beam. Because of the linear variation of these stresses over the depth of a beam, however, it is also relatively simple to establish what will be the magnitude of the stress σ_y at any distance y above or below the neutral axis. (Fig. 7.19.) By similar triangle geometry, it can be seen that

$$\sigma_{max}/(d/2) = \sigma_y/y$$
$$\sigma_{max} = \sigma_y\,[(d/2)/y]$$

Now equating the two expressions for σ_{max} results in:

$$M/S = \sigma_y[(d/2)/y]$$
$$\sigma_y = My/S(d/2)$$
$$\sigma_y = My/I$$

which establishes that the magnitude of the stress at any level of a beam is a function of a modified cross-sectional constant, I, that is called the cross-section's *moment of inertia* and is equal to S(d/2) and thus has units mm^4 (in^4).

Since $\sigma_{max} = M/S$ and $S = I/(d/2)$, an alternate form for the bending formula for maximum stress in the beam in terms of moment of inertia is, therefore,

$$\sigma_{max} = (M \times c)/I \qquad (6.2)$$
$$\sigma_{max} = (M \times d/2)/I$$

in which c is the beam's half-depth; i.e., c = d/2.

For the rectangular cross-section, where as we have previously seen $S = bd^2/6$,

$$I_{rect} = S\,(d/2)$$
$$I_{rect} = (bd^2/6)\,(d/2)$$
$$I_{rect} = bd^3/12$$

A more detailed derivation than is appropriate for this text allows the moment of inertia to be defined more generally for any cross-sectional shape by the integral equation

$$I = \int y^2 dA$$

where y is the distance from the neutral axis for an elemental bit of cross-sectional area dA.[8] Both the general formula for I and that more specifically for the simple rectangular section clearly establish that the distance of beam cross-sectional material from the neutral axis is critical to just how much it can contribute to developing the internal resisting moment; i.e., the farther beam material is located from the beam's neutral axis the more effective it is in helping the beam carry load, and exponentially so. This will shortly be seen to have important implications when we consider making cross-sectional shapes more efficient than simple rectangular ones are.

For now, however, it is sufficient to remind ourselves of the essential of what has just been established: the necessity of equilibrium between external and internal moments allows us to predict the maximum bending stresses that loading imposes on beam material. And, generally speaking, if we are to avoid failure of that material, we must obviously ensure that such bending stresses are less than those the material is deemed capable of carrying; i.e.,

$$\sigma_{max} \le \sigma_{ultimate}$$

Shear Stresses

We shall now investigate how shear force, the other internal action that we found necessary in order to provide equilibrium in beams, produces a second set of stresses acting over a beam cross-section. The condition of shear can be thought of as a tendency for one portion of a beam to try to "slide" transversely past the rest of it as a result of the imbalance of external forces. To prevent this from occurring, it has been established that an internal shear force must be present, and this can be thought of as being produced by some distribution of shear stresses designated by the Greek letter τ (tau) acting over and in the plane of a beam's cross-section. (Fig. 7.20.) This can alternatively very loosely be likened to a set of "friction" stresses acting over the plane of the cross-section that is preventing one part of the beam from sliding transversely past the other.

Based on the same "plane sections' remaining plane" behavior we have previously discussed, it can be derived that shear stress magnitudes are not uniform but rather vary parabolically over a cross-section's depth; i.e., τ has its greatest magnitude at the neutral

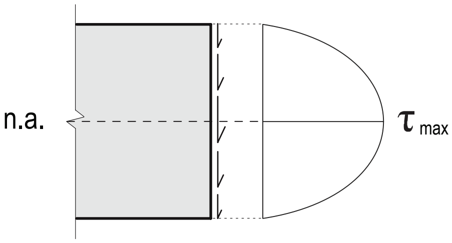


Figure 7.21
Parabolic distribution of relative shear stress magnitudes over beam depth.

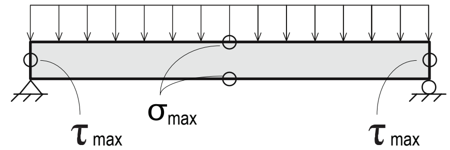


Figure 7.22
Locations of maximum bending and shear stresses in a simply supported beam subject to uniformly distributed loading.

axis level and is equal to zero at the top and bottom edges of the beam. (Fig. 7.21.)

This is obviously a very different stress distribution pattern than that which describes bending stresses, and one must be cognizant of this difference in sizing and shaping a beam. For bending, the highest demand on the structural material was found to be at the top and bottom of the beam, with zero demand at the neutral axis, whereas for shear it is quite the opposite, with the largest demand at the level of the neutral axis and zero demand at the top and bottom. And since the locations along the length of the beam that have maximum bending moment and maximum shear force are typically not one and the same, this will lead to quite different locations of critical demand. (Fig. 7.22.)

For rectangular beam cross-sections made out of a single material, such as standard-sized timber having width b and depth d, there exists the following relationship between shear force V and maximum shear stress, τ_{max}:

$$\tau_{max} = (3/2)V/bd$$

For the flanged shape that is common to many steel beams, however, where the width of the top and bottom flanges b_f is much larger than that of the central web t_w, the shear *stresses* end up being so much *greater* in the web than in the flanges that the common approximation for such beams is to ignore any contribution from the flanges; i.e.,

$$\tau_{max} = V/(t_w \times d)$$

In either case, whether for standard timber or steel shapes, the maximum shear stress must notionally be kept to within the material's shear capacity in order for the beam to be safely designed; i.e.,

$$\tau_{max} \leq \tau_{ultimate}$$

But transverse stresses acting in the plane of the cross-section are not the end of the story when it comes to shearing response, as is suggested by imagining the following experiment: lay two planks on top of each other and let them span freely between two supports. With a drill, bore holes through both of the planks, not too far from the supports. Insert a pencil into the hole and then have a colleague sit on the planks. The pencil will likely snap in two as the planks slide *horizontally* past one another, demonstrating the presence of a second set of shearing forces in beams that are supplemental to the transverse shear forces previously considered.

The presence of such so-called complementary shear stresses can be further confirmed by considering the equilibrium of a very small element imagined to be cut out of a cantilevered beam. There will be acting on the imaginary cut face of such an element a transverse shear stress that balances the net external vertical load at that point. To maintain vertical equilibrium of the small element, there is also a transverse shear stress at the other face of the element, acting in the opposite direction. But together these transverse stresses create a force pair that left to its own devices would rotate the element. To counterbalance this rotation, an equal but opposite force pair must be found: this is provided by shear stresses that are acting at the top and bottom of the element; i.e., in the direction of the length of the beam. (Fig. 7.23.) In the plank–beam example of the previous paragraph, it was these complementary shear stresses that were the ones trying to break the pencil in two. By the necessity of equilibrium, the magnitude and distribution

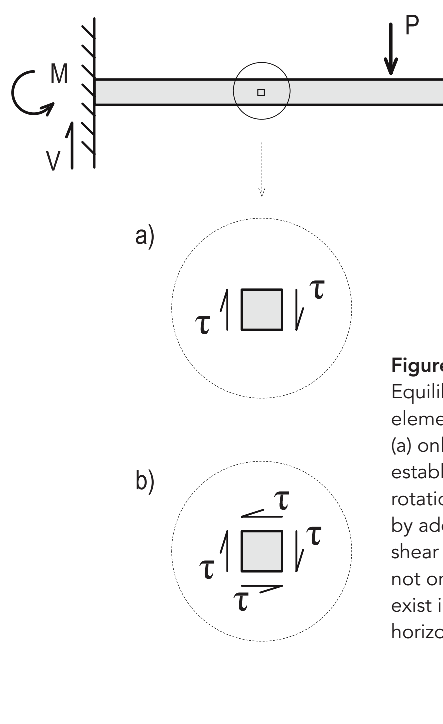


Figure 7.23
Equilibrium study for a small element within a beam:
(a) only vertical balance established, (b) vertical and rotational balance established by addition of complementary shear stresses. This shows that not only vertical shear stresses exist in a beam, but also horizontal shear stresses.

of the complementary set of shear stresses must be identical to that which we have established for transverse shear. Perhaps the most tangible illustration of the presence of complementary shear stresses in beams is provided by the mode of failure sometimes observed at the ends of overloaded timber members; i.e., by *horizontal splitting at mid-depth of the beam at the supports*. The location and direction of this failure confirms the location of maximum shear stress in a simply supported beam, as well as wood's relatively weaker shear stress capacity in the direction of the wood fiber grain rather than transverse to it.

Shear resistance is conceptually somewhat more complicated for reinforced concrete beams, where the limited capacity of concrete subject to shearing action is typically supplemented by a series of steel reinforcing bar hoops, called stirrups, that are placed in cross-sections every so often along a beam's length. As shear force demand would suggest, stirrups typically become more closely spaced the closer one gets to a support. But if carefully considered according to our discussion so far, this approach would at first seem to be a rather puzzling strategy: if anything, the addition of transverse bars would only seem to help the shear capacity of the beam at the cross-sections where stirrups are being located, and to do nothing between them. The detailed consideration and explanation of this apparent shortcoming is beyond the objectives of the present book, but generally involves conceptualizing shear being carried in a reinforced concrete beam by means of the development of a zigzagging truss mechanism within the depth of the beam, with short diagonal compression struts forming in the concrete that are balanced by tension in the transverse stirrups.[9]

Beam Deflections

To this point we have described material strength constraints for a beam; i.e., whether the bending and shear stresses that are produced when transverse loads act on a structural member are within material capacities. But in designing beams we must also be equally concerned with their deflections under load. For example, a roof beam that sags too much may not appear safe when seen from below, or it may be the cause of cracking of ceiling finishes or, more seriously, of water ponding on the roof leading to an increased and dangerous load condition. Such psychological and practical considerations have led to building codes adopting criteria to limit the vertical deflections Δ of beams; e.g., among other limits that have to be checked, building codes typically state something along the lines of

$$\Delta_{max} \leq L/(200 \text{ to } 400)$$

which is simply limiting transverse beam deflections to some small fraction of their spanning distance L.

We commonly say that a beam with only a small downward deflection is stiffer than a second beam of equal length that has larger downward deformation caused by the same load. The amount of beam deflection can reasonably be expected and rigorously shown to be inversely dependent on the cross-sectional property of moment of inertia, I, that we have just linked to a beam's bending behavior, as well as on the beam's material stiffness, or modulus of elasticity, E. Generally speaking then, when we talk about beam deflection stiffness we are referring to the inverse relationship:

$$\Delta = \text{constant}/EI$$

But beam stiffness also depends on length: in the case where E and I are equal for two beams but their length varies, we expect from common experience that the shortest one will be stiffer under loading.

When examined more closely, the rate of variation of stiffness according to these different factors is not the same. Consider, for example, the equations for maximum deflection for several different beam support and loading conditions in Figure 7.24. As can be seen, in each one of these cases Δ_{max} is linearly proportional to load (P or w), inversely linearly proportional to material and cross-sectional stiffnesses (E and I, respectively), but exponentially proportional to spanning distance L. Rather emphatically, the relation between Δ_{max} and L is to the third or fourth power rather than one to one. For example, doubling the length of a simply supported beam with uniformly distributed load will cause it to deflect vertically 16 times as much.

In a beam, therefore, there will be not only bending and shear stresses that must be designed for, but also deflections that must be checked to ensure that such a structural member is adequately designed. In some cases, and especially if long-spanning distances become of central interest, the deflection criterion will often be the limiting beam-sizing factor. (e.g., Ill. 7.19, 7.20.)

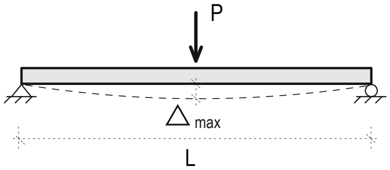
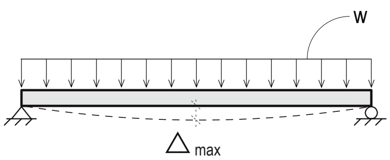
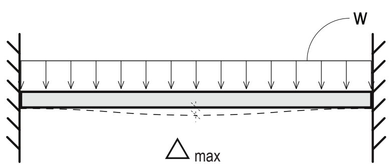
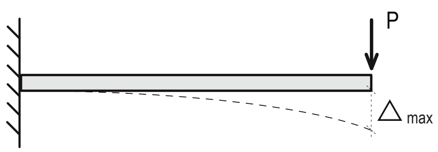
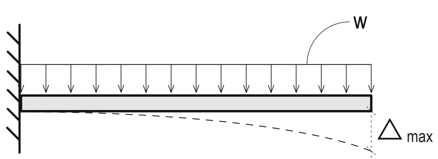
Load and Support Condition	Δ_{max}
	$\frac{PL^3}{48EI}$
	$\frac{5wL^4}{384EI}$
	$\frac{wL^4}{384EI}$
	$\frac{PL^3}{3EI}$
	$\frac{wL^4}{8EI}$

Figure 7.24
Beam deflection formulae for different loading and support conditions. Note that under similar conditions, a beam with two fixed ends deflects significantly less (i.e., only 1/5 as much) compared to a simply supported beam. Also, cantilevers deflect much more than do beams supported at both ends.

Illustration 7.19
UniFor Headquarters Building, Turate, Italy (1982).
Long span roof beams are prone to significant deflection, but pre-stressing of concrete greatly reduces the problem. Compare the beam span-to-depth proportions here to those of historical stones seen in Section 7.2.
Architect: Studio Mangiarotti. Structural engineer: Vintani Alberto.

Illustration 7.20
UniFor Headquarters Building.
For Mangiarotti's so-called U70 constructive system, long-spanning, precast concrete beams span side by side in one direction into precast concrete beams that are in turn carried on precast columns – all of which are configured and integrated with one another into a whole structural system of remarkable qualities.

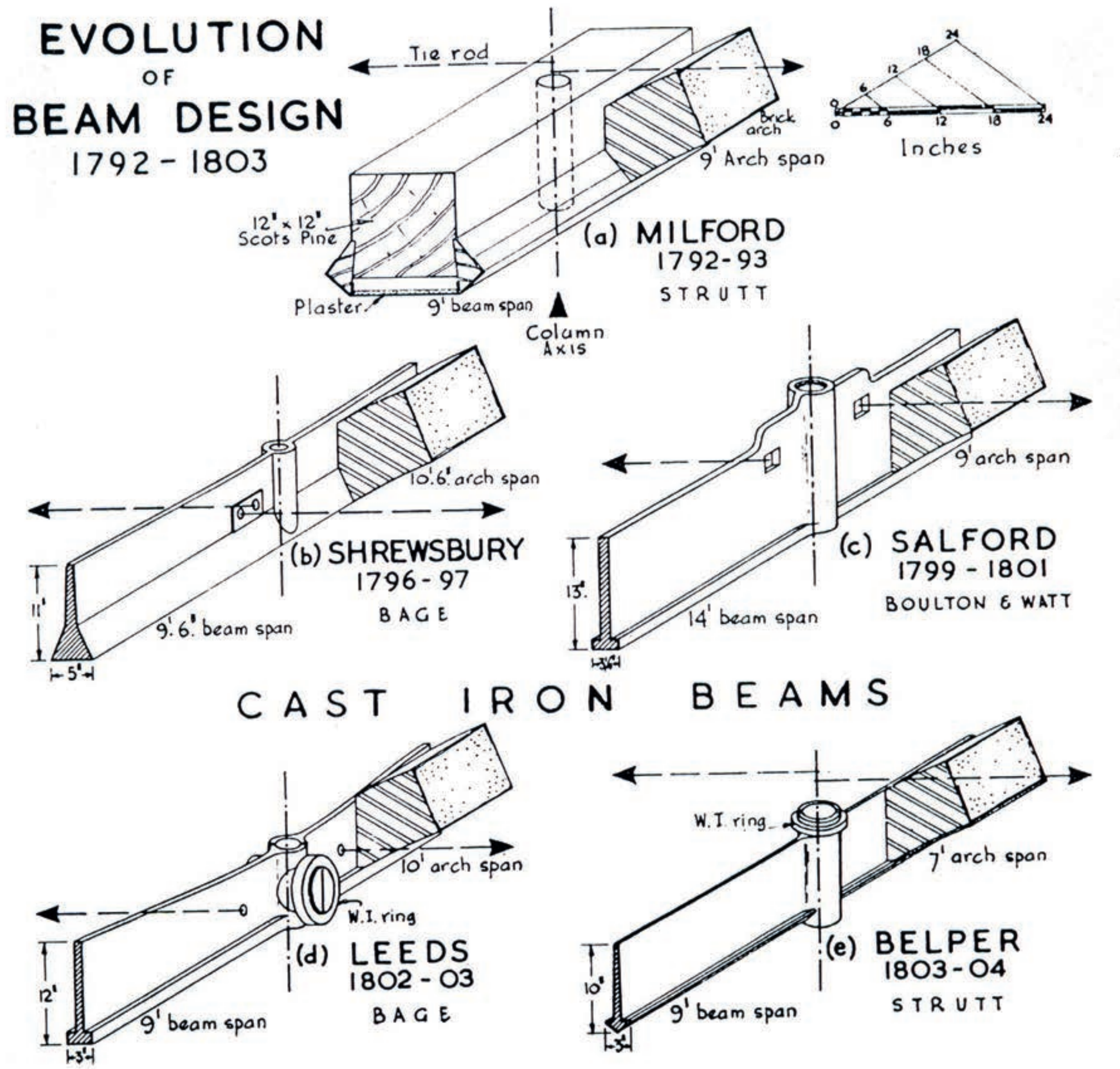


Illustration 7.21

Decade of evolution of the cast iron beam from the Industrial Revolution. Metal significantly reduces sectional dimensions of the earlier timber beam, but various iron sections shown are almost uniform in width, with only small projections at bottom mostly there to support the transverse brick arch spans between adjacent beams; also, more material on the tension side of the beams helps compensate for the low tension strength of cast iron.

Drawing from Newcomen Society Transactions, vol. 30, after Johnson and Skempton, ca. 1950.

7.8 The Trouble with Beams, and Shape or Material Responses

We have so far established a basic understanding of beam behavior and of the factors that control their design, yet this same understanding also contributes to furthering an earlier observation: that this structural behavior represents a very *inefficient* utilization of material for carrying load. We have already discussed how bending moments and shear forces vary over the length of beams, and yet how we often only size them for the peak values on these diagrams. Compounding this “inefficiency problem” is the linearly varying-from-top-to-bottom bending stress distribution diagram that we described in the previous section; i.e., at all levels of a beam section except at the very top and bottom the material will be under-stressed, and at the levels of the beam near the neutral axis the material will be especially lightly challenged.

Despite these shortcomings, beams have obviously been and continue to be one of the two most common structural elements (the other being the column) so clearly ways to at least partly offset these inefficiencies have long been sought. Historically, this “trouble” with beams was surely understood from first-hand experience; e.g., the

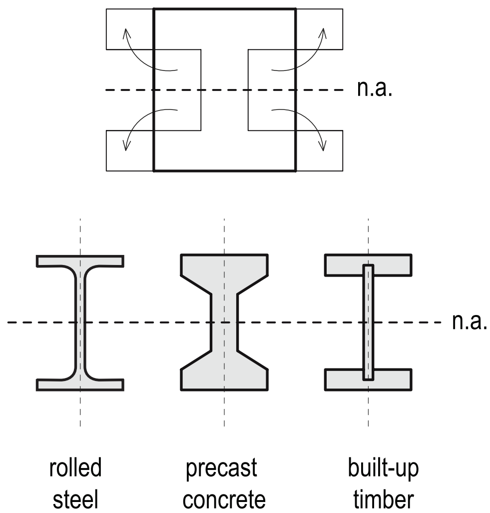


Figure 7.25
Conceptual moving of cross-sectional area away from neutral axis of rectangular beam results in an "I-beam" configuration, corresponding to rolled steel, precast concrete, and built-up timber sections shapes.

Illustration 7.22
"Les Constructeurs" (1950).
I-beam shapes are evident in this painting of construction workers in action erecting a building's steel frame, with material moved to top and bottom flanges of the beam cross-sections. Although the circular holes evident in one of the beams are not very commonplace, the fact that these are depicted at mid-depth of the beam corresponds to the low level of bending stresses there. (See also Ill. 7.23.)
Artist: Fernand Léger (1881–1955).

difference in stiffness and load-carrying capability between a wood plank spanning between two supports when laid flat horizontally versus standing vertically on its narrow edge is something that could readily be observed and experienced. This understanding is also evident in the orientation of the stone beam segments of the Temple of Aphaia (seen earlier in Ill. 7.8) and the essentially vertical-plate shape of cast iron beams developed during the early years of the Industrial Revolution. (Ill. 7.21.)

In order to try and counteract some of the inefficiency of beams, we can conceptually think of removing material from the middle region of a rectangular beam's cross-section and moving it to the upper and lower parts of the section (Fig. 7.25) where it can be more highly stressed in bending and, therefore, be put to better use; i.e., the effective distance between the force pair C and T will get significantly larger (referring back to Fig. 7.17 and Fig. 7.18) which leads to a substantially larger internal resisting moment. In this way the "flanged" beam is able to develop much greater load-carrying capability for the same amount of material used, and this is the logic behind the frequent use of such beam shapes in everyday construction, such as with the ever-so-common steel I-beam (e.g., Ill. 7.22, see also Ill. 5.29), for instance, or similarly shaped manufactured timber beam sections can thus also be properly understood.

The way in which this modified cross-sectional shape makes the beam more effective is, of course, through the effect of a section's moment of inertia $I = \int y^2 dA$ or its associated section modulus S

Illustration 7.23
Mechanized cutting of typical rolled steel members. Low levels of shear stress may allow cut halves to be reconfigured and welded together to produce circular holes in web of so-called "castellated" beams.

Illustration 7.24
Hemeroscopium House, Madrid, Spain (2008).
Stacked set of especially large precast, pre-stressed concrete beams establish identity and define spaces of house. Various cross-sectional shapes express the strategy of moving material away from neutral axis, whether flanged (seen end-on at right side and transversely across top) or U-shaped (seen end-on, conveniently shaped for a lap-pool) or inverted-U-shaped (running transversely across the middle of the image).
Architect Ensamble Studio. Technical architect: Javier Cuesta.

and their use in the previously established flexure formula σ_{max} = (Md/2)/I or σ_{max} = M/S. The beneficial effect of the exponential factor contained in the definition of I or S for different shapes reinforces what has just been discussed; e.g., doubling the depth of a rectangular beam will multiply by a factor of eight the moment of inertia, and so dramatically reduce the maximum bending stress that has to be designed for in the beam. Adding flanges that project sideways from the top and bottom of a beam section applies the same logic, placing a substantial amount of material as far away from the beam neutral axis as possible. The consequence of this effect is to reduce necessary beam weights and to make stiffer beam cross-sections, thereby permitting significantly longer spans to be achieved or else much more load to be carried. In one way or another, a very effective means has been found to respond to the fundamental shortcoming of the beam's load-carrying mechanism. (e.g., Ill. 7.24, see also Ill. 7.16.)

The strategy of moving material away from the neutral axis in order to increase the effective moment of inertia can also be seen to have been applied in at least two specialized beam types: the box beam and the truss. Long spans and heavy loads on bridge structures frequently result in the use of box-beam sections, whereby, as the name implies, the middle of the beam section is hollowed out.

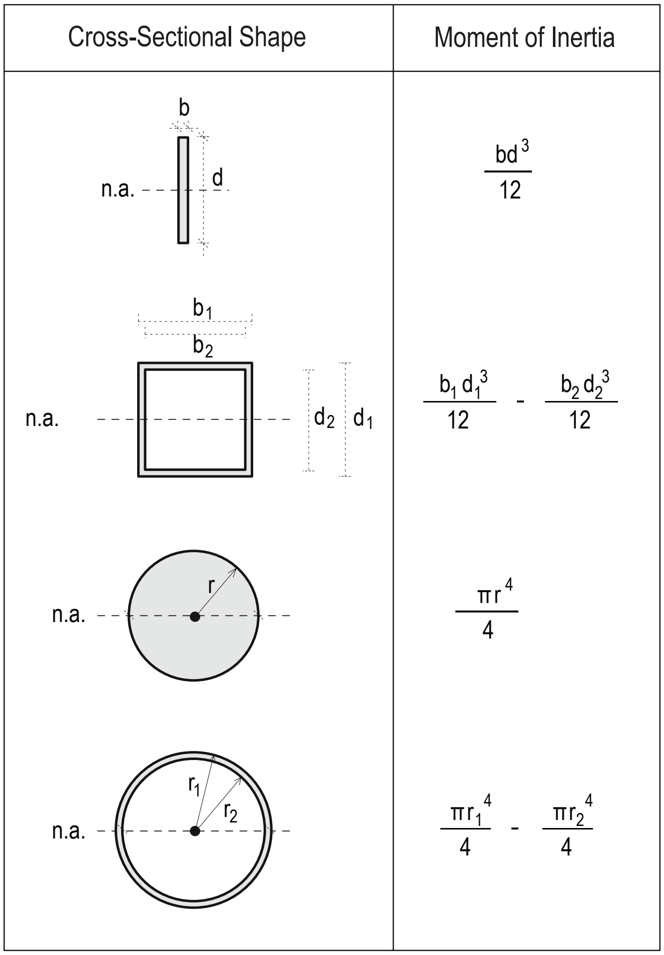


Figure 7.26
Moment of inertia formulae for some different cross-sectional shapes.

Illustration 7.25
Brazilian Museum of Sculpture, São Paulo, Brazil (1988).
Deep, wide concrete beam spans across, unifies, and shelters exterior exhibition space. Multilevel "ground" actually roofs over sunken museum interior spaces. This beam is in fact of hollow cross-section – see Ill. 7.26.
Architect Paulo Mendes da Rocha. Structural engineer: Escritório Técnico Júlio Kassoy and Mário Franco Engenhieros.

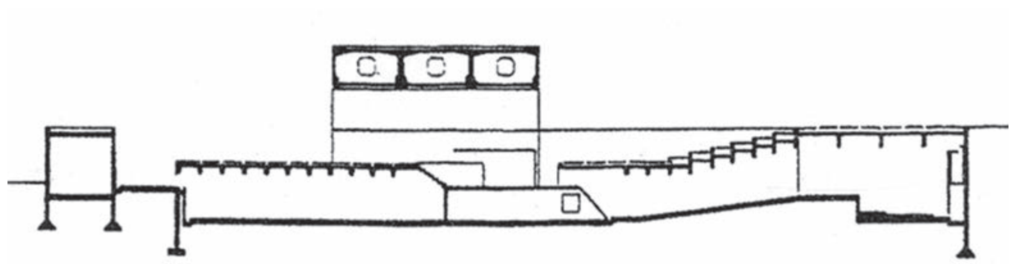


Illustration 7.26
Brazilian Museum of Sculpture.
Section through overriding beam reveals hollow, cellular configuration to maximize moment of inertia.

(Fig. 7.26.) This strategy can also be found in certain long-spanning building structures, such as at the Brazilian Museum of Sculpture designed by Paulo Mendes da Rocha (Ill. 7.25, 7.26), and in the vertical cantilevered walls surrounding many buildings' elevator cores. (See Section 10.4.) The sectional shape in these cases is clearly all about placing as much material as possible away from mid-depth so as to increase the bending moment capacity of the section. The top and bottom flanges of the beam must obviously remain connected to each other, however, in order for the shear force still to be carried and this is done in the box beam by means of vertical or sloped side walls that act as the "web" elements of this large cellular section.

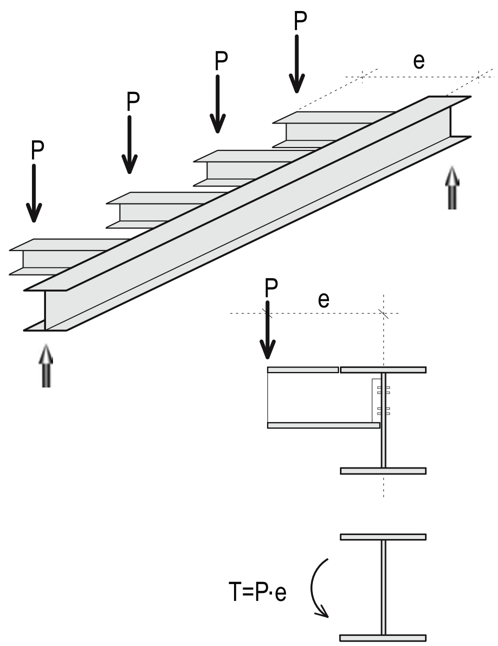


Figure 7.27
Example of torsion loads applied to a beam: one-sided cantilevering projections with loads P at their ends tend to cause twisting along supporting beam axis. An I-beam or H-beam as shown here has little torsional stiffness and will be expected to twist significantly.

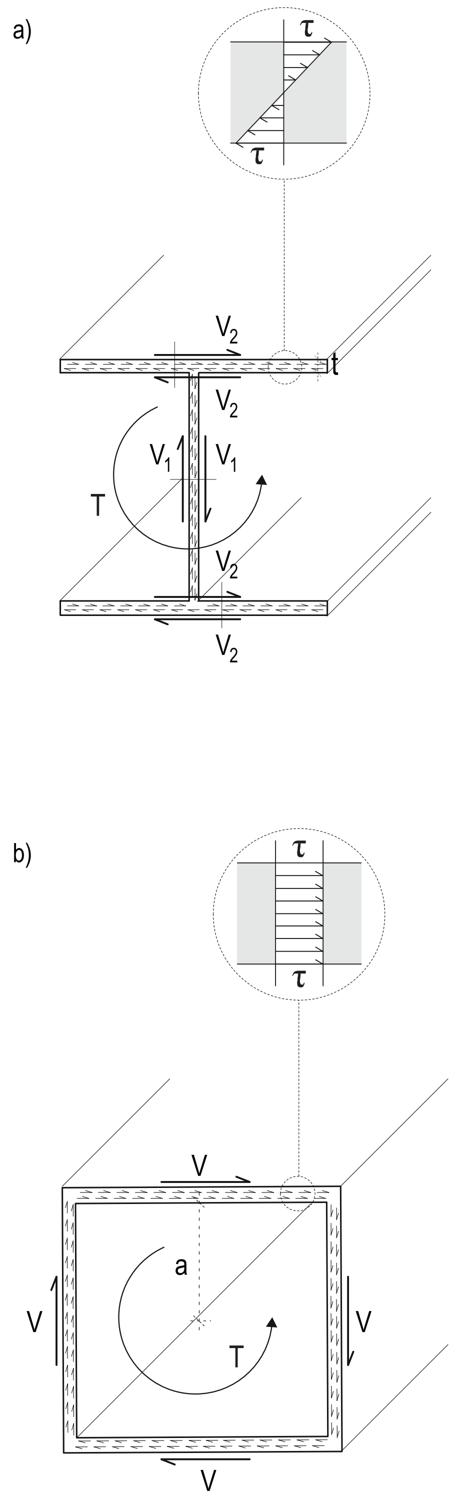


Figure 7.28
Torsional resistance mechanisms for (a) I-shape and (b) hollow-cell beam sections.
In (a) beam resists torsion by producing only linearly varying shear stresses acting across plate thicknesses, whereas in (b) a much larger torsional resistance is produced by additional uniform shear stresses that circulate about the cell and act about its center. A closed cross-section is therefore much better to resist torsion than an open cross-section.

It should also be mentioned in passing that the hollow cross-sectional shape is particularly adept at resisting one of the primary structural actions that we encountered in Chapters 2 and 4, but that we haven't yet seen many examples of: namely, torsion. Torsion in a beam may be as a result of eccentric loads being applied to a beam section that causes it to want to twist about its longitudinal axis (e.g., Fig. 7.27), or perhaps the loads are symmetrical but the beam itself curves in plan in three-dimensional space.

The standard I-shape of the steel beam is particularly weak in resisting such torsion: the countering internal torsional moment necessary for equilibrium (i.e., $\Sigma T = 0$; therefore $\Sigma T_{external} = \Sigma T_{internal}$) must be provided by the twisting of the flanges and web individually about each of their own longitudinal centroidal axes. Shear stresses can be envisioned that effectively circulate around the cross-section to create the resisting torques; these stresses will act in opposing directions across each of the relatively narrow thicknesses of the web and flanges. (Fig. 7.28a.) Needless to say, such stresses have rather small effective lever arms to work with and the result is that torsional stresses in any I-shaped beam easily become quite large and the torsional capacity of the member is quickly reached.

In contrast, hollow box-beam sections resist torsion by having the whole sectional shape rotate about a single common central axis. The shear stresses associated with this behavior are almost constant in magnitude through the thickness of the cell walls, but more importantly these circulate in a single direction around the

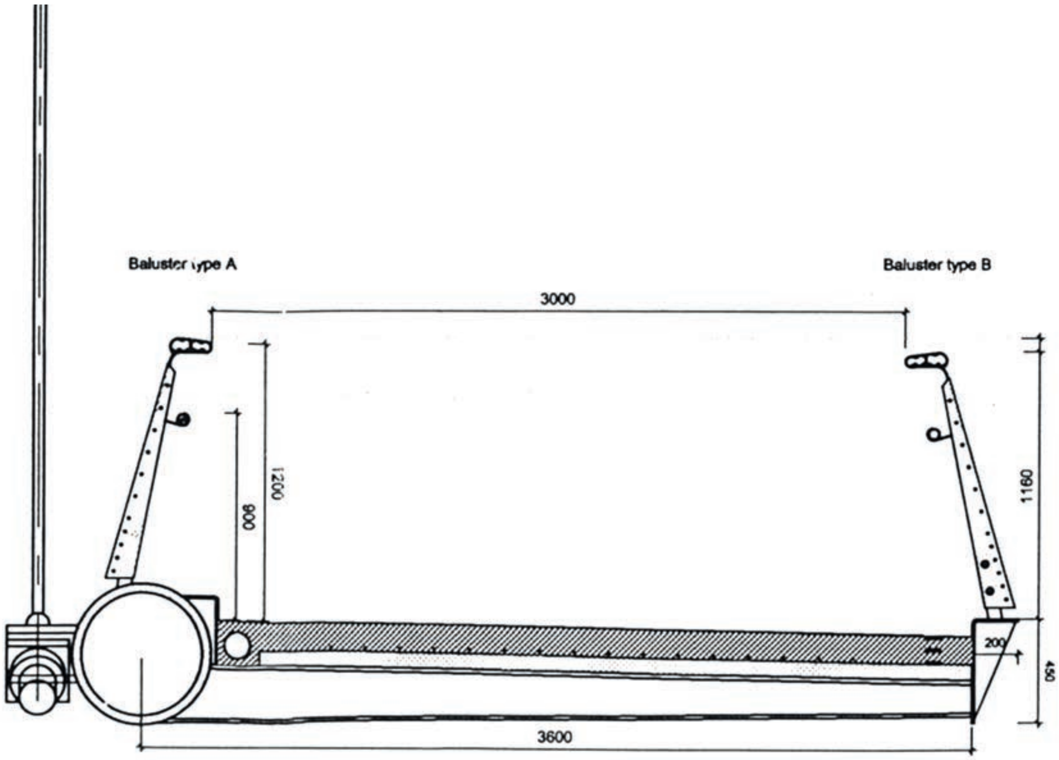


Illustration 7.27
Ypsilon Footbridge, Drammen, Norway (2008).
Section of asymmetrical bridge deck at north end; single round tubular beam needed to resist torsion caused by eccentric walkway loads and stay-cable anchorage. See also Section 6.5 for other images of bridge.

Architect: Arne Eggen Arkitekter. Structural engineer: Knut Gjerding-Smith.

whole of the hollow cell. (Fig. 7.28b.) Such stresses thus have a much larger lever arm to work with than in the I-shape, and the result is a sectional type that is particularly appropriate for resisting torsional loads on beams. Small or large torques are regularly handled by the use of round, square, rectangular, and trapezoidal hollow-section structural members. An example of this can be seen in the circular steel tube that is supporting from one side the walkway of the Ypsilon Footbridge (Ill. 7.27) in Drammen, Norway, a structure that was more fully described in Section 6.5.

As has been mentioned, a second example of extrapolating from the beneficial principle of moving material away from a beam's neutral axis can be found in the seemingly very different structural form of the truss. Given the frequent presence of trusses in structures, we will spend considerable time in Chapter 9 looking at this structural type in much more detail. But one way of introducing them in the current context is to think of a truss as starting out as a beam with large top and bottom flanges but in which a series of triangular holes have been cut out of the web. (Fig. 7.29.) Conceptually this can be thought of as similar to moving material away from a beam's neutral axis, even if in the case of the truss the resulting visual effect is quite different and distinctive. The truss pushes this strategy to an extreme, whereby almost nothing is left of the web – only a diagonal member to carry shear – and the section is able to resist relatively large bending moments due to the relatively large distance (almost the full sectional depth) between the tension and compression force couple in the top and bottom chords. Whether this concept is applied to small-scale members in order to maximize their openness and transparency or to heroic mega-structures that span enormous distances, we can see that the basic strategy for why a truss is shaped the way it is is fundamentally linked to the beam behavior mechanism for carrying load.

Finally, in concluding this section entitled the "trouble with beams," we must address a somewhat different problem: that posed by the fact that not all materials have the same capacity to carry load and, in particular, that some materials behave quite differently whether they are subject to tension or compression. Most notably among these are stone and concrete, both of which have

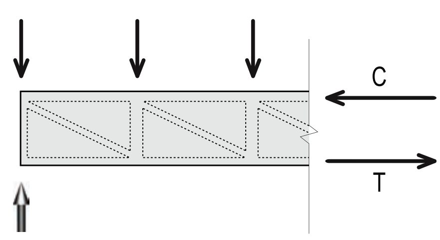


Figure 7.29
Beam with triangular holes cut out of it can be conceived of as prototypical truss.

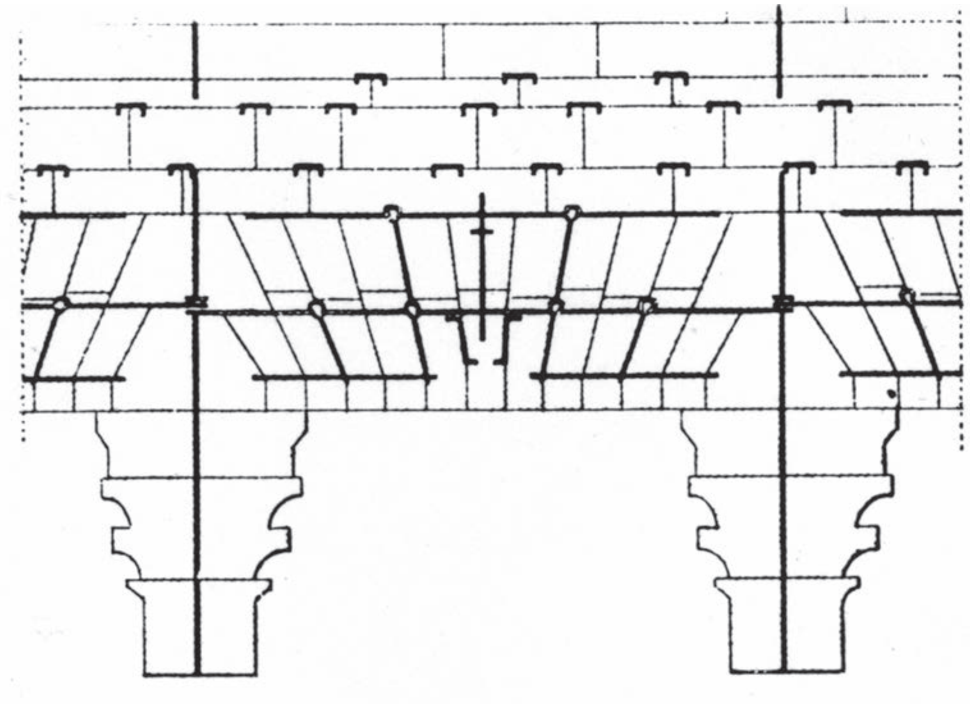


Illustration 7.28
Metal clamps and bars have long surreptitiously held together stone structures – in this case that of the Louvre Museum in Paris, France, designed by the architect Claude Perrault around 1670.

Drawing by Jean Rondelet (1743–1829), French architect, constructor, and theorist; *Traité théorique et pratique de l'art de bâtir*, pl. 150.

Illustration 7.29
Louvre Museum detail.
Corresponding exterior view of stonework, whose surfaces are delicately carved and decorated.

essentially negligible capacity to resist tension stresses (Chapter 5). This widely known fact ominously undermines what we have established so far as the basis for describing how beams work; i.e., that stresses vary linearly from the top to the bottom of a beam and that this results in counterbalancing triangular stress wedges that together create the resisting moment needed for equilibrium. When a stone beam is subject to very light loading, exactly this behavior occurs and loads can be carried in "pure" flexure as we have discussed so far. But a critical change occurs when the tension stresses at the bottom of the beam reach the level of stone's relatively small tensile capacity. At that point the stone cracks and an abrupt failure of the beam structure will occur. Given stone's very significant self-weight to begin with, the spans that are possible in stone therefore typically remain very small. Even so, stone beams must be of disproportionately large depth (in order to compensate for their weakness in tension by generating a large enough moment of inertia so as to produce very small bending stresses) – all of which brings us back to a convincing explanation for the beam dimensions that we saw at Stonehenge, the Valley Temple of Chefren, and the Temple of Aphaia in Section 7.2 near the start of this chapter and, more broadly and without overstatement, to an understanding of the fundamental proportions of beam elements in classical architecture.

Dealing with this critical weakness of stone and similar materials in tension (and thus in bending) has been resolved over time by strategically reinforcing them with another material – usually a metal

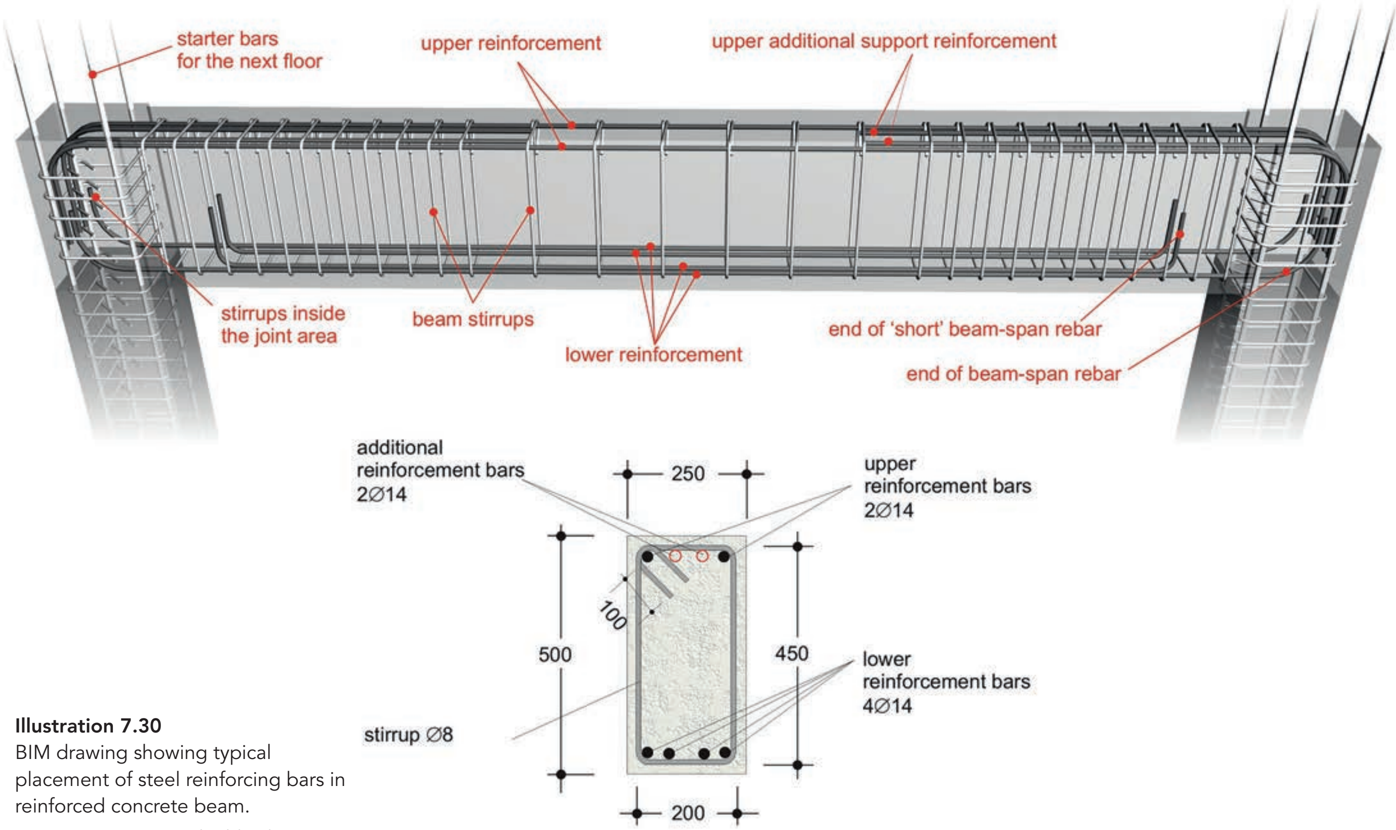


Illustration 7.30
BIM drawing showing typical placement of steel reinforcing bars in reinforced concrete beam.
Drawing source: www.buildinghow.com

such as iron and later steel – that is well capable of carrying tensile stress. In fact, it is quite remarkable to realize just how extensively metal reinforcing has been used surreptitiously in what we have often presumed to be purely stone structures. (e.g., Ill. 7.28, 7.29.)

This discussion quickly leads us to examine just how the modern reinforced concrete beam works to carry load. To start with, and for very light loads, it functions like any other beam as previously described. But almost immediately, and for any significant applied load, cracks will develop in the lower part of the beam where tension needs to be present in order to develop the resisting couple – and those cracks will tend to open up. Effectively preventing them from doing so, however, are steel reinforcing bars placed near the bottom of the concrete section (or near the top in a cantilever beam because of the reversal of internal moment direction). (e.g., Ill. 7.30.) The stress diagram for the reinforced concrete beam, therefore, looks different from what we have seen so far. There are still compression stresses in the compression portion of the beam but now these are balanced by much higher magnitude tension stresses that are confined to acting only over the cross-sectional area of the steel reinforcing bars; no stresses, on the other hand, will be present in the cracked, tension part of the concrete. (Fig. 7.30.) By multiplying the stresses that exist in the compressive concrete and the tensile steel by the sectional areas corresponding to these parts, the situation can be seen to thereby be converted back into the expected force couple needed to produce the essential internal resisting bending moment.

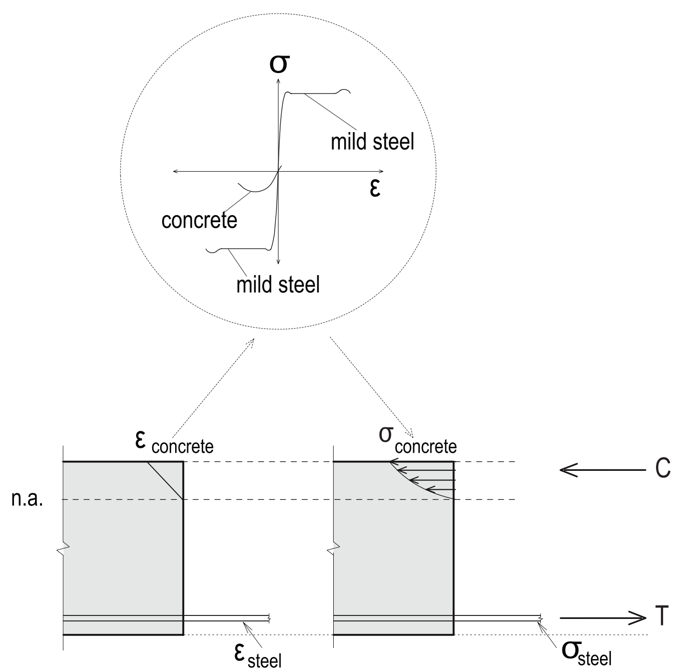


Figure 7.30
Typical relationship between strains and stresses in reinforced concrete beams, as produced by differing material properties in tension (upper right quadrant of σ–ε graph) and compression (lower left). Steel is well able to resist both tension and compression, whereas concrete resists only compression stresses.

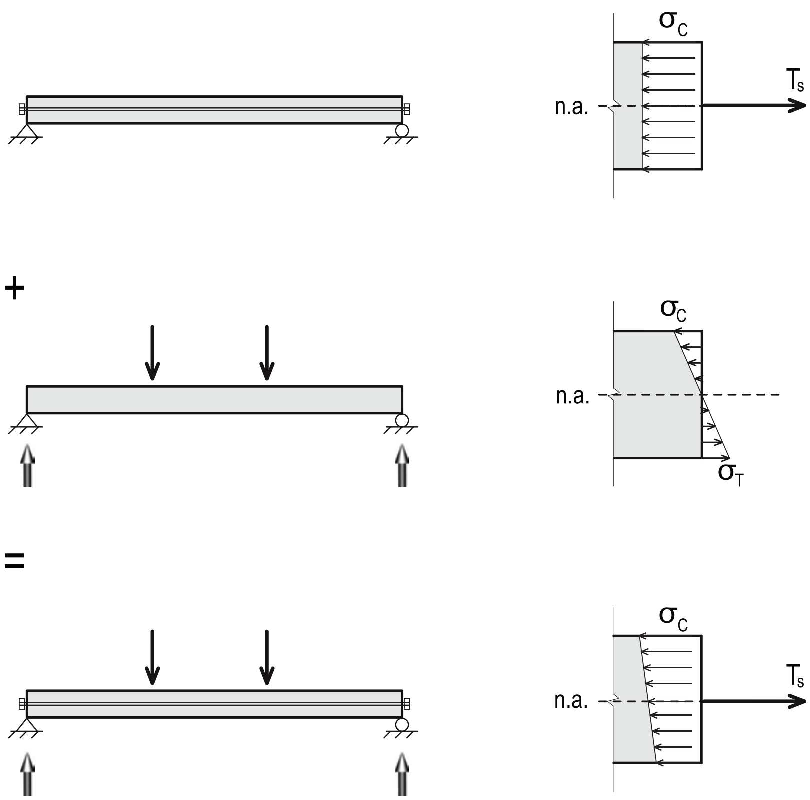


Figure 7.31
States of stress in representative pre-stressed concrete beam.
Top: uniform compression in concrete due to centered pre-stressing strand anchored at ends. Middle: tension-to-compression variation due to transverse loads only. Bottom: combination of both.

So concrete on its own, like stone and masonry before it, is virtually useless as a contemporary material for beams, but by combining it with steel the composite material of *reinforced concrete* can be made to be highly effective in resisting flexure, as its omnipresence in building structures around the world today would suggest. And unlike typical steel and timber sections that more likely than not will have the same section from one end of the beam to the other based on maximum moment demand occurring within the span, the number and size of steel bars in a reinforced concrete beam can be varied along the member length according to varying bending moment demand, and this is regularly done as standard building practice. The result produces a beam structure that more efficiently varies in strength along its length – even though this attribute will not typically be visible to the naked eye as the reinforcing bars will have been cast within the subsequently hardened concrete.

Having considered all this, and as commonly used as is this type of reinforced concrete beam in construction, it is important to recognize that we have not completely resolved "the problem" of this beam type. For although we have ingeniously taken care of the need for tension capacity with reinforcing, we are left with a situation where much of the concrete of the beam (i.e., the part that is not in compression) is effectively useless for helping to carry the load and is instead simply dead weight having to be brought along for the ride. This is certainly not a terribly efficient state of affairs, and it leaves the reinforced concrete beam vulnerable to being limited to spanning relatively short distances.

We have previously encountered the general technique by which this problem can be resolved: i.e., by pre-stressing, or pre-loading, a structure in anticipation of actual loading so that the final condition is advantaged. (See Section 6.6 for how this concept was introduced for maintaining cables in tension.) In the case of the concrete beam the basic problem is to find a way to keep the whole of the beam material in compression rather than allowing it to crack because of tension – and the classic solution is to pre-stress the entire beam into a state of sufficient axial pre-compression so that no part will ever go into tension when bending stresses inevitably develop.

Although there are many different ways and sequences of construction to achieve this objective, in fundamental concept they are all alike. Before transverse loads are applied to the concrete beam structure, there is a pre-stressing steel rod or cable that is stretched tightly and anchored at the two ends of the beam. The anchorages push inward on the beam ends but in opposite directions, thus putting the beam into a column-like state of compression along its longitudinal axis. Then, when transverse loads are applied

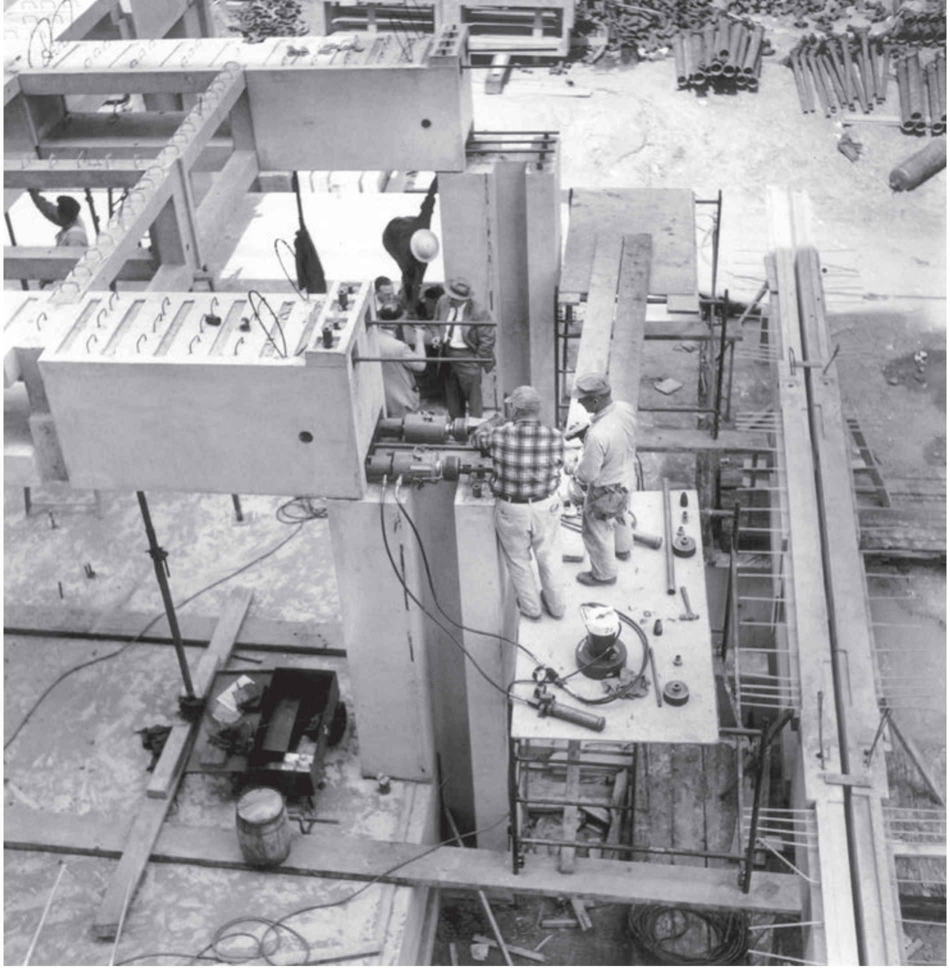


Illustration 7.31
Richards Medical Research Laboratories, Philadelphia, PA, USA (1961).
Stressing of post-tensioning strands threaded through precast concrete beams.
Architect: Louis Kahn. Structural engineer: August Komendant.

to the beam and bending stresses develop, rather than working off a state of zero stress as in the case of the conventional beam, these stresses are instead added to and subtracted from the initial state of pre-compression axial stress. (Fig. 7.31, e.g., Ill. 7.31, 7.32) If this has all been anticipated correctly the end result will be one where the net state of stress will still be one that varies from top to bottom of the section but now these stresses will *all* be compressive – meaning that the usual tension reinforcing for bending is conceptually no longer essential.

A further refinement to this basic strategy occurs if the pre-stressing strand is placed in the lower part of the beam, as now the beam can be pre-stressed into both pre-compression and into upward pre-bending (otherwise known as camber) that is opposite to what is anticipated to be going to happen from the subsequent downward transverse loading of the beam. This strategy is employed repeatedly in concrete structures in order to carry especially heavy loads, or to span long distances, or to reduce the depth of the concrete beam that is needed, or all of these together, as the case may warrant and according to design intentions. And as a further refinement still, placing a pre-stressing strand in a curved line in response to the varying bending moment diagram along the length of the beam further increases the effectiveness of the pre-stress.

Illustration 7.32
Richards Medical Research Laboratories.
Open spaces and double-cantilevered corners at each floor level that are made possible by the long span pre-stressed beams, as revealed in photo taken during construction before building was enclosed.

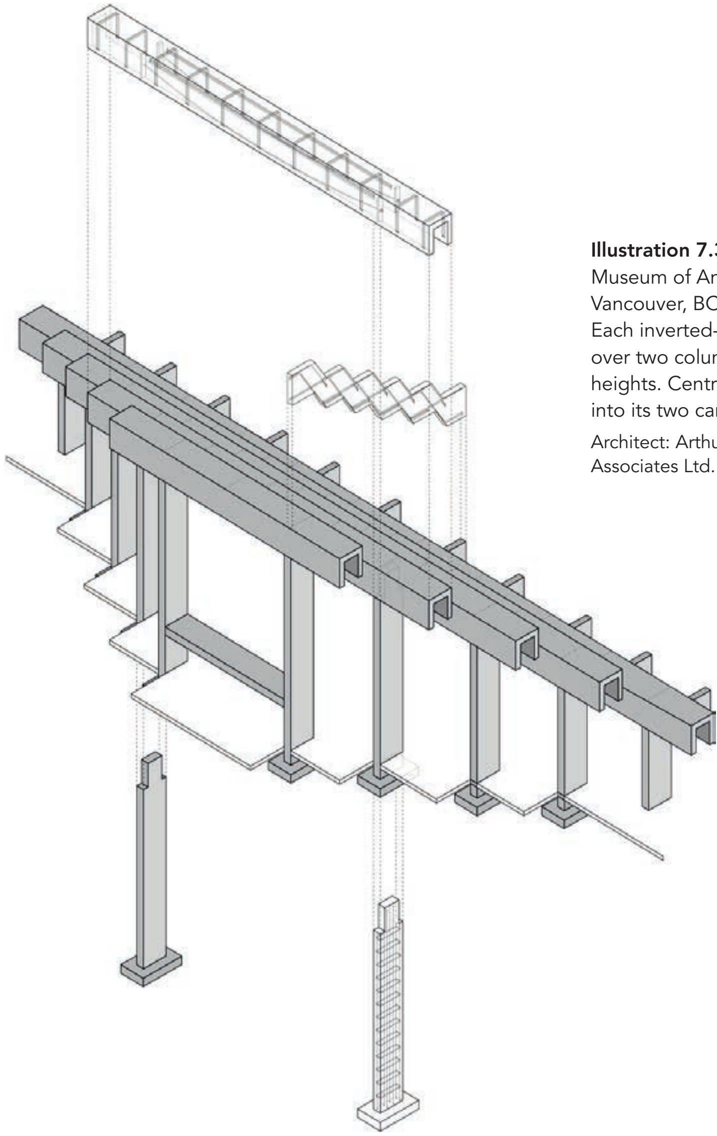


Illustration 7.33
Museum of Anthropology, University of British Columbia, Vancouver, BC, Canada (1976).
Each inverted-U-shaped precast concrete beam spans over two columns, creating portals of varying widths and heights. Central span of each beam runs continuously into its two cantilevered ends.

Architect: Arthur Erickson. Structural engineer: Bogue Babicki Associates Ltd.

Illustration 7.34
UBC Museum of Anthropology.
Progressively taller portal frames open up views to surrounding landscape, extending spatial sense of main gallery space outward.

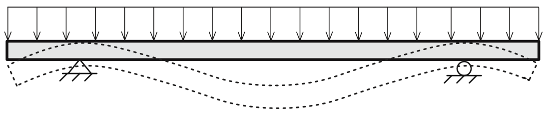


Figure 7.32
Deflected shape of single-span continuous beam with projecting ends; "inward" rotation tendency in central span is countered by opposing rotations in cantilevering segments.

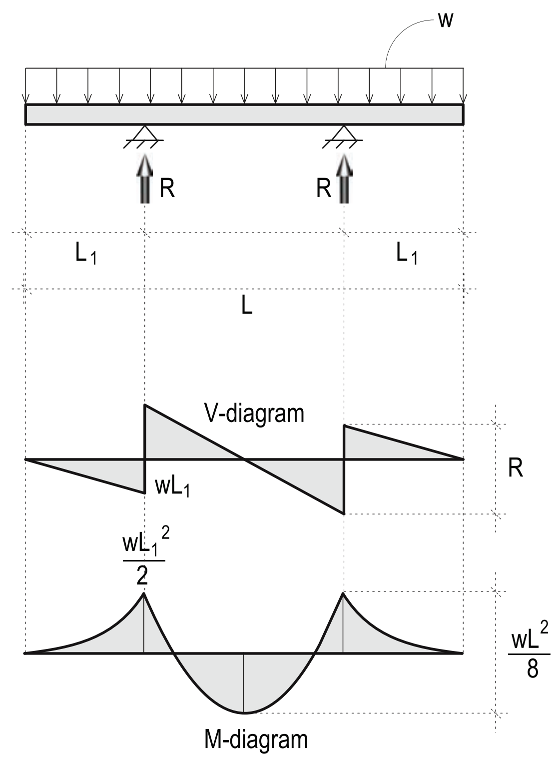


Figure 7.33
Shear force and bending moment diagrams for single-span continuous beam subject to uniform loading. Peak values of bending moment in the continuous beam are reduced from the simple span condition without end projections; i.e., less than $wL^2/8$.

7.9 The Virtues of Continuity

Until this point we have mostly been talking about beams that are either simply supported at their two ends or else cantilevered from one of them. Now we shall look at the behavior of a beam that combines these aspects by spanning between two supports but that also has overhanging ends; i.e., a beam that runs in a *continuous* fashion over its supporting structural elements, as do those of the great portals of the Museum of Anthropology at the University of British Columbia (Ill. 7.33, 7.34) designed by Arthur Erickson (1924–2009).

When a uniformly distributed load is applied to a continuous beam we can anticipate its deformations. (Fig. 7.32.) Assuming that it is long enough we expect the middle section between the supports to sag downward, with the usual tension stresses developing along the bottom side and compression stresses at the top. The overhanging ends, however, will also tend to deflect downward in the way that cantilevers do, with tension developing along the top side and compression on the bottom. We have until now considered the behaviors of such beam segments independently of each other, but here the critical difference is that they are connected as one continuous structural element – so whereas the ends of a simply supported beam are completely free to rotate into the span, such rotation now is countered by the opposing tendency for the beam to rotate oppositely into the cantilevering ends.

One of the obvious beneficial effects of this situation is the tendency to significantly reduce the maximum downward deflection of a simply supported beam, as the reduction and even elimination of any rotation at the supports will necessarily "lift up" the original sagging tendency in the span. The exact amount by which it does so will be dependent on the loads' location and magnitude as well as on the relative lengths of the different segments of the beam. In fact, taken to an extreme, if the load is especially large on the overhanging ends and the lengths of these ends are quite significant in relation to that of the middle span, the beam can even be lifted up in the middle so that it has tension along its top side over the whole length of the beam.

But the latter case is certainly highly atypical and is mentioned only to make a point; let us go back instead to look a little more comprehensively at the behavior of a more typically proportioned continuous beam having a uniformly distributed load w applied to it. For example, the dimensions and the loads acting on such a beam and the balancing upward reactions at the two supports can be calculated by what are now familiar methods. We can also then determine the shear forces and bending moments at various key points along the beam using exactly the same equilibrium techniques that we have described in Sections 7.5 and 7.7 in order to draw the corresponding V and M diagrams that reflect the variations of these quantities along the length of the continuous beam. (Fig. 7.33.)

Some important observations ensue from this process, mostly with respect to the beam's bending moment diagram: moments associated with tension at the top of the beam develop as expected at the two ends in the cantilevered segments, but these continue in the beam past the support and into the central span; moments that produce tension at the bottom develop in the middle of the span, but the portion of the beam with such moments is shorter than the full span between supports. In fact, the total mid-span moment needing to be designed for in order to meet overall statics requirements for the span, $M = wL^2/8$, can be seen in the continuous beam to be *shared* by the numerically smaller opposite direction moment peaks in the beam at the supports and at mid-span. The conclusion that emerges from this last observation is that beams that have some continuity over their supports can be designed

Illustration 7.35
Farnsworth House, Plano, IL, USA (1951).
Multi-span continuous floor and roof beams supported by regularly spaced columns.
Architect: Ludwig Mies van der Rohe. Collaborating architects: Myron Goldsmith, William Dunlap, Gane Summers.

for smaller moments than would be necessary if the span were of the simply supported type and that, therefore, such beams can span longer distances, carry greater loads or, conversely, the beam depth can be reduced to less than would otherwise be necessary. A particular case of interest here is where the peak moments over the supports have the same magnitude as the moment at mid-span, making a beam of constant depth as efficient as possible. Calculations show that this will happen if a beam with a uniformly distributed load on it has a ratio of side-span length to main-span length of about 1:3:1.

Extending the benefits of this strategy further still, we will now look at a beam that spans continuously over three or more supports, such as is the case for the floor and roof edge beams at Mies van der Rohe's Farnsworth House. (Ill. 7.35.)

Notwithstanding the exceptional qualities of the Farnsworth House, having beams run continuously over supports is in fact not an atypical situation: in fact, for ease and economy of construction alone it is not unusual to let a beam run over several supporting columns without cutting and splicing it. A continuous beam of many equal spans supporting a uniformly distributed load will deflect in an undulating up-and-down fashion. (Fig. 7.34.) Once again, we can anticipate and observe a sagging profile occurring between the supports whereas over these the beam will have an oppositely "hogging" profile owing to the fact that the load to one side of a support will be working against that on the other. As a result of the symmetry of the situation in this case of equal spans, the beam will be effectively prevented from rotating one way or the other right over each interior support – which we should recognize

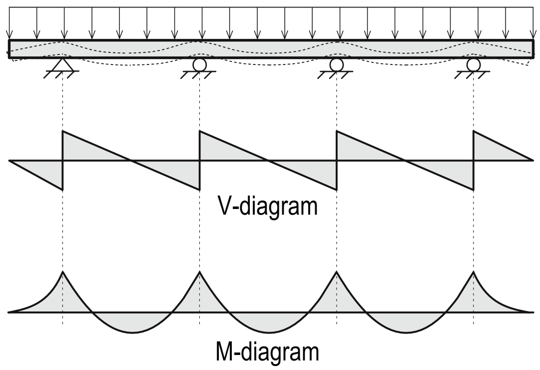


Figure 7.34
Multi-span continuous beam; general form of deflected shape, shear force, and bending moment diagrams.

as effectively mimicking fixed-end support conditions. Thus, the majority of the interior spans of such a continuous beam can, from the point of view of statical equivalency, be considered as a series of beams with fixed ends.[10] A comparison between the responses of beams with both ends fixed versus one with simple supports shows that the greatest bending moment needing to be designed for will be 50 percent larger in the simple beam and the deflection deformation five times as much. (Fig. 7.35.) The advantages of making beams continuous thus quickly become quite obvious.[11]

But all is not necessarily advantageous with continuous construction. Consider once again the condition of a simply supported beam, but this time one having a column support at one end that is set on an unstable foundation. If this end at some point in the life of the building sinks because of the settling of the ground, the beam will be able to rotate because of the simple support conditions and easily reposition itself in a new state of equilibrium. (Fig. 7.36a.) Nothing will have changed in terms of the bending demand on and behavior of the beam as it will now be slightly sloped but otherwise undeformed as a result of the ground settlement. If the same thing happens to a multi-span continuous beam, however, the effectively fixed nature of the connections will cause the beam to have to bend in order to assume its new elevation at the settled support. (Fig. 7.36b.) This bending will cause additional internal moments to be introduced into the beam that will be supplemental to those resulting from the original gravitational dead and live loads. Determining just how large these supplemental moments are, however, is not a simple matter as the continuous beam is statically indeterminate – a category of structure that can be described as involving more unknown quantities to be solved for than there are equations of equilibrium available to do it. We would also need to account for the beam's deflection in order to calculate the forces and moments, and this is a very laborious process. Fortunately, today we can rely on computer structural analysis programs to quickly and accurately predict the behavior of such structures; unfortunately, this relative

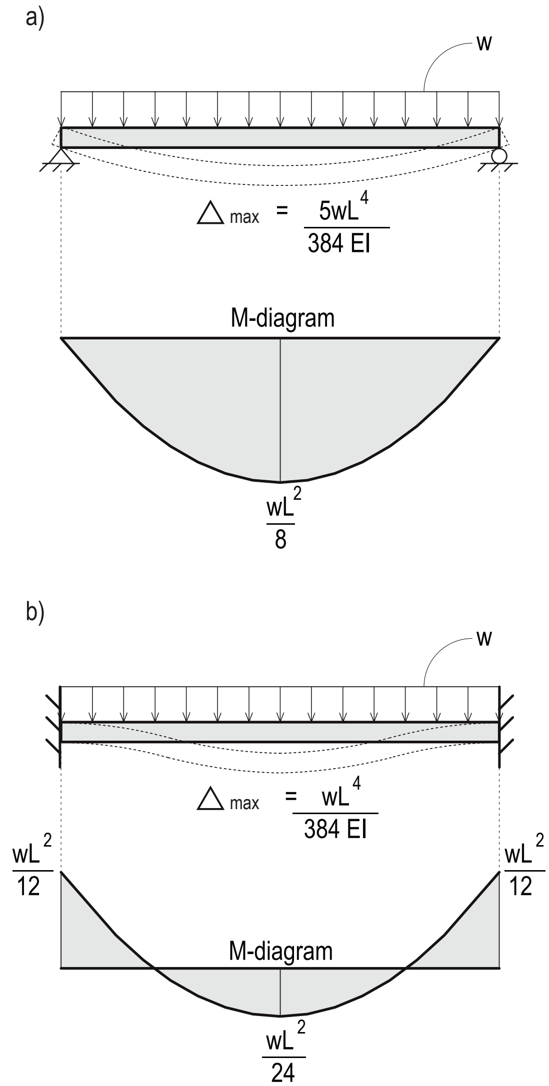


Figure 7.35
Comparison of maximum deflection and bending moment responses for simply supported vs. fixed-ended beams. (See also Fig. 7.24.)

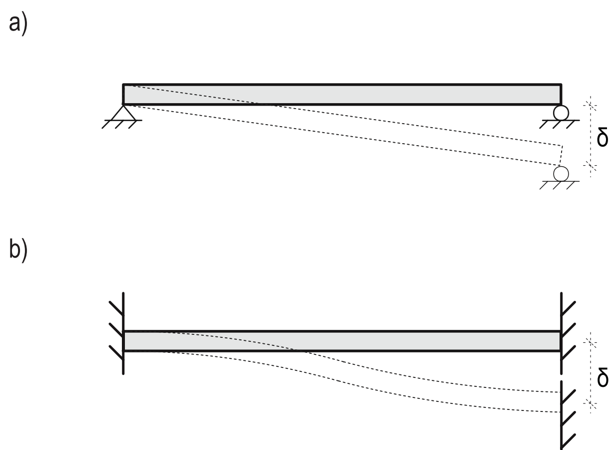


Figure 7.36
Effects of differential support settlement: (a) ends of simply supported beam rotate freely, allowing beam to be inclined without having to flex, (b) for fixed-ended beam, opposite is true.

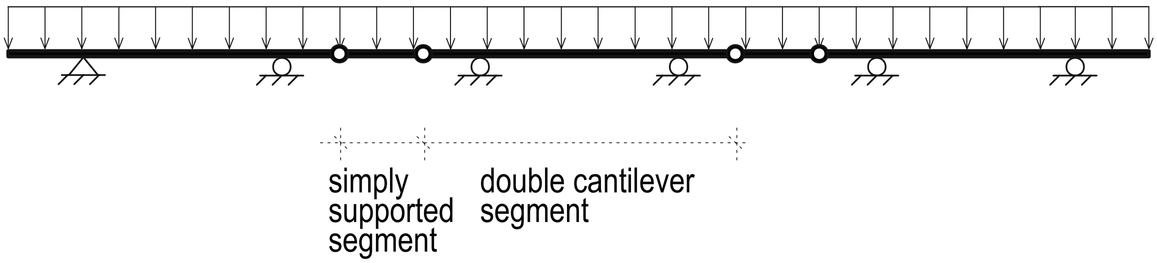


Figure 7.37
Multi-span Gerber beam configuration alternates double-cantilever segments with simple spans that are said to be "suspended" between ends of cantilevers.

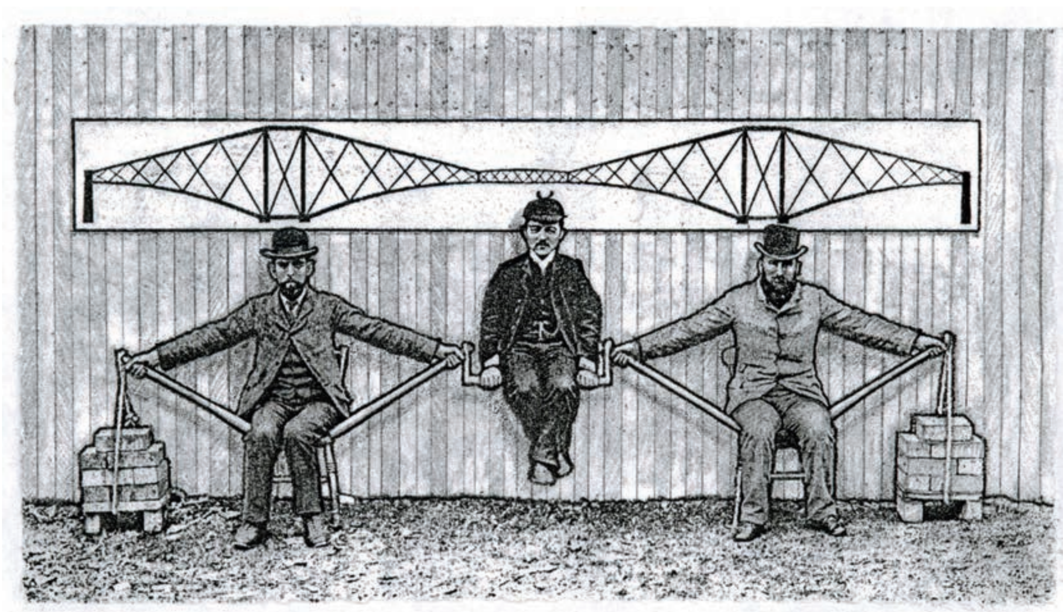


Illustration 7.36
Forth Railway Bridge, near Edinburgh, Scotland, UK (1890).
Famous example of Gerber beam construction – and expression of structural actions by human analogy.
Designers: Sir Benjamin Baker and Allan Stewart.

easing of the calculation difficulty does not diminish the negative effects of foundation settlement in producing additional stresses in a continuous structure.

To conclude this section, we will look briefly at a particularly ingenious solution to this problem of the statical indeterminacy of continuous beams that was developed well before computers were available. This technique was especially devised to reap the benefits of continuous beam construction for multi-pier bridges while anticipating the negative effects caused by the frequent settling of such supports. This double objective was the basis for a very particular type of continuous beam developed by the German engineer Heinrich Gerber (1832–1912); the eponymous Gerber beams are created by introducing hinges in alternating spans of a continuous beam structure. (Fig. 7.37.)

Gerber beams can thus be understood and treated as follows: they are statically determinate beams with cantilevering side spans (such as we examined at the start of this section) that are connected by simply supported beam segments spanning between the adjacent cantilevered ends, thus placing upon these ends only vertical loads (for evident reasons, these are often termed "suspended" spans). The most famous illustration of these basic Gerber beam principles, applied to the trusses of the mega-scale Firth of Forth Bridge, is surely that of three human structural "actors," in which the tension and compression forces acting on/in their various body parts can easily be intuited. (Ill. 7.36.) Many years later, the advantages of this clever strategy remain relevant, whether applied to the appropriately named "gerberettes" and long-spanning trusses of the Pompidou Center (see Ill. 9.20) or to the clever reuse of a large timber beam for the renovation of the home of one of the present authors. (Ill. 7.37.)

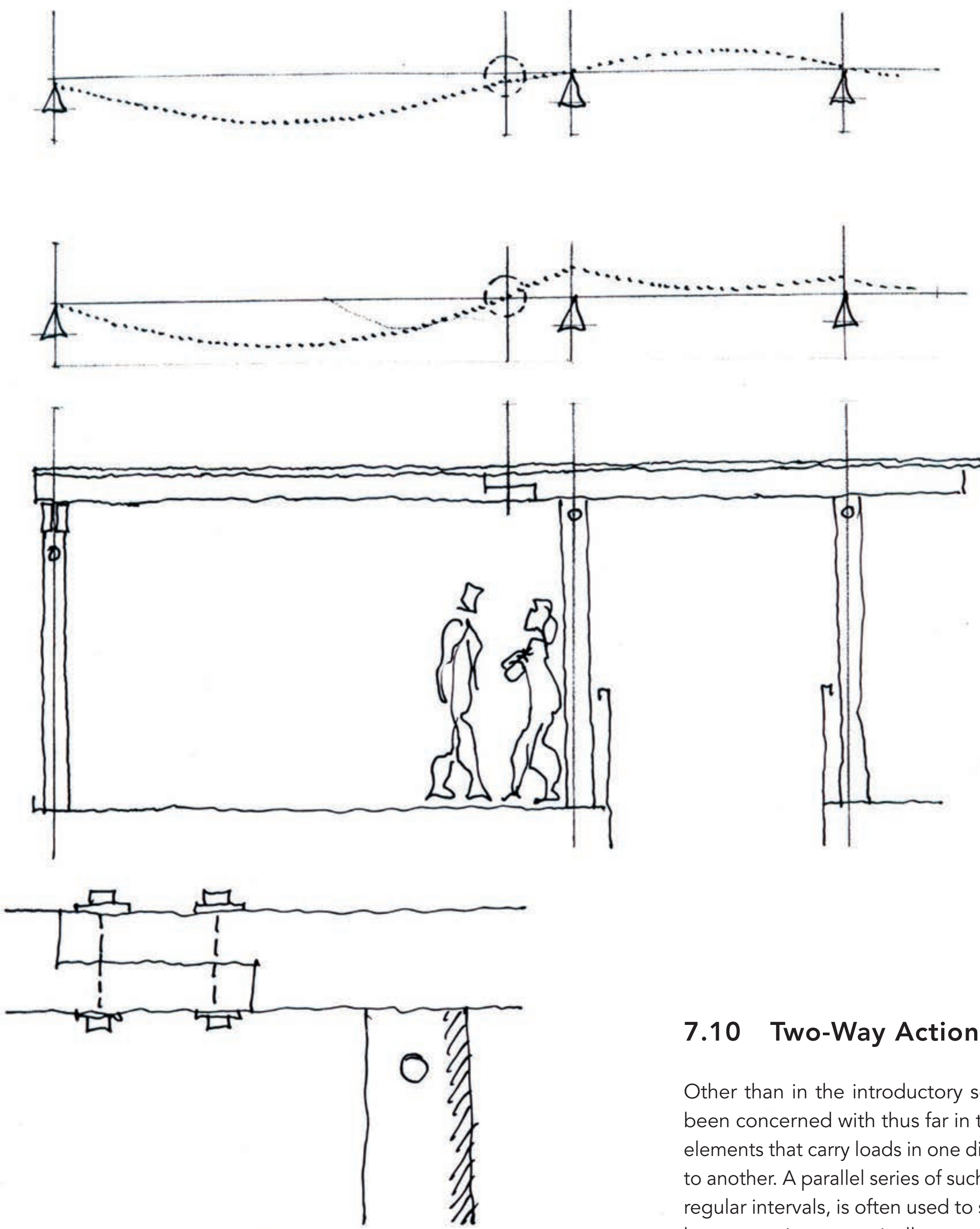


Illustration 7.37
Eggen House, Oslo, Norway (1985).
Gerber construction ensures zero bending moment at point where reused timber beam is spliced together; also reduces magnitude of maximum moment to be designed for when UDL is applied to whole length of beam. Deflection diagram (top), bending moment diagram (bottom).
Architect: Arne Eggen Arkitekter.

7.10 Two-Way Action and Beam Grids

Other than in the introductory section, the beams that we have been concerned with thus far in this chapter have been structural elements that carry loads in one direction from one point of support to another. A parallel series of such *one-way* beams, spaced apart at regular intervals, is often used to support a floor or roof, and these beams are in turn typically supported by transverse structures of some sort, whether larger beams (often termed girders) or trusses or lines of closely spaced columns or even solid walls. (Fig. 7.38a.) There is a clear hierarchy of structural elements in such a system: loads can be considered to be carried by the floor surface in the short direction between parallel beams, which then in turn carry the loads to their own supporting structures which, if these are also beams or trusses, span in one-way fashion between their own supporting elements, etc. In order to make the most of this system to support a rectangular-in-plan surface area, the parallel series of beams will typically be oriented to span in the shortest direction between lines of support since, as we have seen, beams'

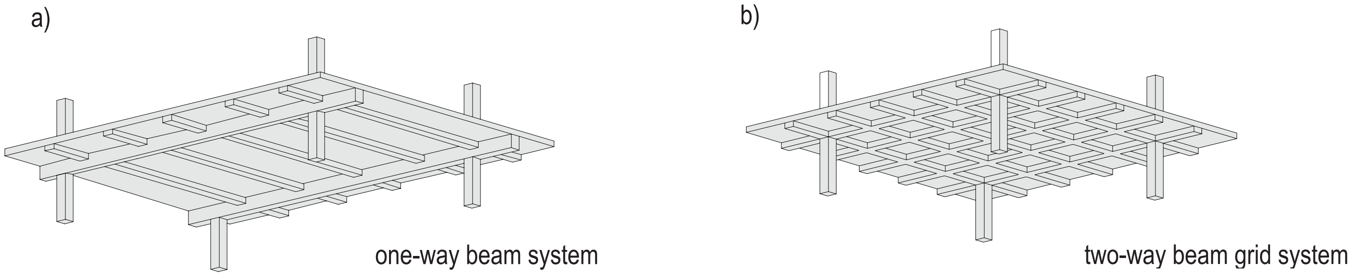


Figure 7.38
Differing structural arrangements and visual patterns for (a) one-way beam system, (b) two-way beam grid.

Illustration 7.38
Casa El Mirador, Valle de Bravo, Mexico (2013). One-way spanning system of parallel wooden beams, supported at their ends by much higher capacity flanged steel beams. The one-way system gives a certain texture, rustic character, and sense of orientation to the space of this horse stable – one that is unusually situated directly above the owner's residence. (See also Ill. 10.9.)

Architect: CC Arquitectos. Structural engineer: Miguel Campero, Jorge Soto, Pedro de la Fuente.

load-bearing capacity is inversely proportional to the square of the span. Visually, such an arrangement of beams naturally lends a certain directional orientation to the space directly below it. (e.g., Ill. 7.38, see also Ill. 7.19, 7.20.)

Quite a different arrangement of beams is sometimes used (and contrasting visual effects obtained) when spanning over a space that is nearly square, where support is provided equally around the perimeter or in the four corners. In this situation, it may be found that two sets of parallel beams are used, one running transversely to and intersecting with the other, thus forming what is typically called a regular beam grid.[12] (Fig. 7.38b.) The basic notion here is that the load to be carried is shared simultaneously by the two sets of beams, producing what is logically termed a two-way structure, and thus essentially putting only half the demand on each individual beam as would be the case if it were a one-way system. A beam grid having a two-way structure also could be considered for more distinctly rectangular spaces by accounting for the differences in bending stiffness in the two directions, but the span difference should not be too large for this configuration to be effective; a span length ratio of 1:1.5 is typically considered to be the maximum. The obvious advantage of using a beam grid is to enable greater load-bearing capacity and/or to have smaller deflections for a set maximum beam depth or, conversely, to have a shallower set of beams than would otherwise be possible. Visually, the effect of the beam grid is to render the space non-directional, as can be observed for the classically orthogonal steel beam grid roof of the Neue Nationalgallerie in Berlin by Mies van der Rohe (Ill. 7.39) or the reinforced concrete equivalent of the long-spanning roof of the Faculty of Architecture and Urbanism Building in São Paulo. (Ill. 7.40.)

While the general beneficial effect of sharing load between intersecting sets of beams in a grid is clear, other aspects of their interaction are not so obvious without a bit more discussion. Consider, for example, a load that is applied to an intersection point of the grid. Because of the interconnectedness of all the members, the gridded surface will deform into an overall upwardly curved "dished" shape, resulting in the vertical deflection and bending of many beams of the structure, not just the two that intersect directly under the load. (Fig. 7.39a.) A beam grid is thus many times redundant and statically indeterminate, while loads are carried with great efficiency because of this sharing of responsibility. Furthermore, if all the beam connections are rigid, the surface

Illustration 7.39
Neue Nationalgallerie, Berlin, Germany (1968).
Mies van der Rohe built several famous projects in which the two-way grid of beams is the point of departure for elegant roof structures over long spans.
Architect: Ludwig Mies van der Rohe. Structural engineer: Ingenieurbüro Prof. Dr.-Ing. H. Dienst und G. Richter.

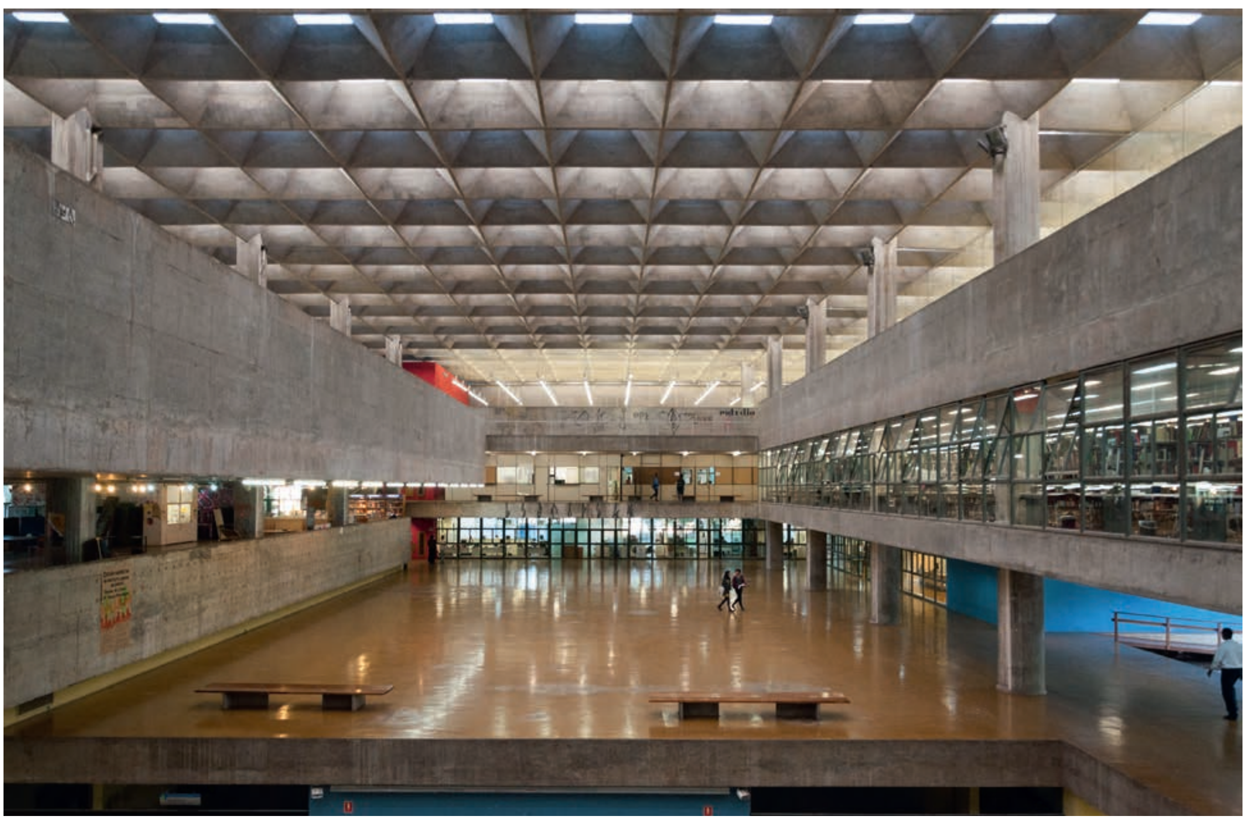


Illustration 7.40
Faculty of Architecture and Urbanism Building, University of São Paulo, São Paulo, Brazil (1968).
Disparate parts of architecture school are gathered around a large interior column-free space that is covered by a two-way concrete beam grid, unifying the sense of space while also admitting natural light through the grid openings. (See also Ill. 5.20.)
Architect: João Batista Vilanova Artigas and Carlos Cascadi. Structural engineer: Escritorio Figueiredo Ferraz.

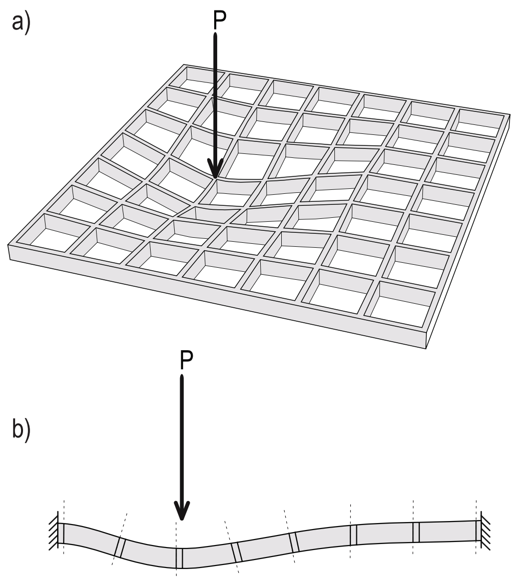


Figure 7.39
Deformation depictions for two-way beam grid under single point load suggest (a) sharing of load among several beams in both directions, (b) simultaneous bending and torsional response of all beam segments of system. This shows the response if all beam connections are rigid, when torsion also becomes involved, which will increase the stiffness of the grid compared to a grid having hinged beam connections, thereby resulting in less deformation and/or larger capacity for load bearing.

Illustration 7.41
Telcel Theater, Mexico City, Mexico (2012).
Distinctive two-way grid of angled steel plate beams acts as an urban marker for the location of a completely underground theater; also acts as a sun-shading canopy for the entrance.
Architect: Ensamble Studio. Structural engineer: Colinas de Buen.

Illustration 7.42
Telcel Theater.
Section drawing showing relationship of beam grid to the underground theater.

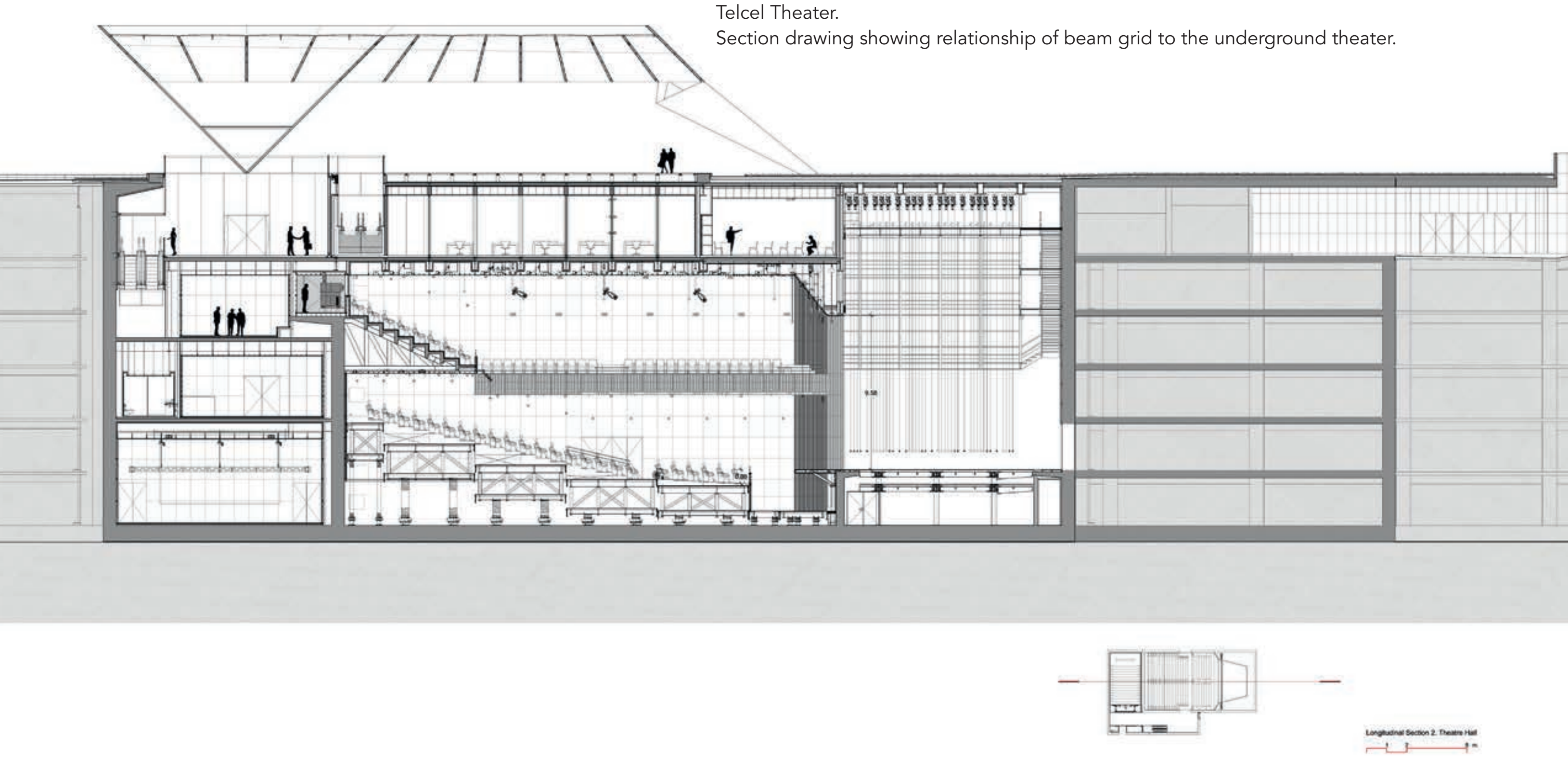


Illustration 7.43
Serpentine Gallery Pavilion, London, UK (2002). Irregular, skewed, multi-directional beam grid made of steel plates, with resulting triangular and trapezoidal shapes intermittently covered, glazed, or left open. At edges, system folds over in continuous fashion into similar side "walls."
Architect: Toyo Ito Associates. Structural engineer: Arup.

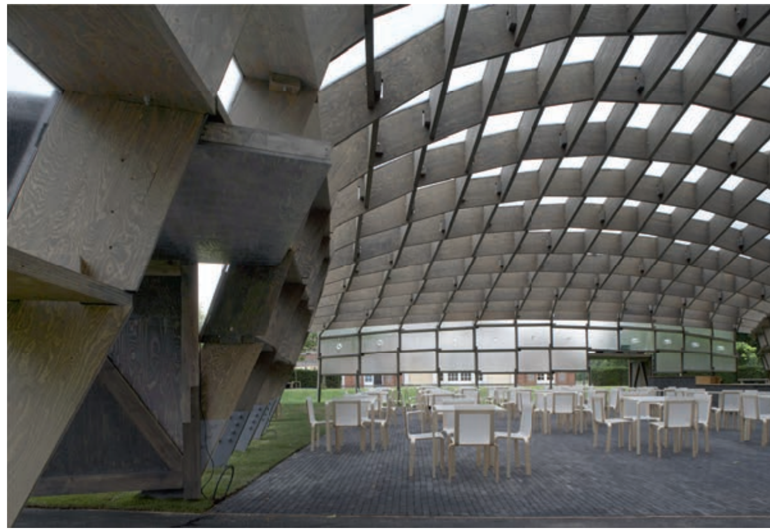


Illustration 7.44
Serpentine Gallery Pavilion, London, UK (2005).
Remarkable interior space created by irregularly curved two-way grid of short plywood elements.
Architect: Álvaro Siza and Eduardo Souto de Mora. Structural engineer: Cecil Balmond of Arup.

deformation will cause virtually all of the beams to be subject *both* to bending action caused by their vertical deflection *and* to torsional behavior due to their simultaneously having to twist about their individual longitudinal axes. (Fig. 7.39b.) If these beams are box-like and have hollow cross-sections, they will have substantial torsional stiffness themselves, and this will even further contribute to the sharing-of-load-carrying mechanisms and the high-degree of statical indeterminacy of the beam grid and, therefore, to its relatively remarkable load-carrying and spanning capabilities. Finally, whether the vertically supported edges of the grid are prevented from rotating or twisting (perhaps by having the grid run continuously or cantilever over the supports) or should instead be considered hinged will add yet another layer to the whole complexity of beam grid behavior – while also providing yet more opportunity for magnifying its structural advantages.

Of course, all this is to say nothing about the possibility of varying the beam grid itself; i.e., the grid need not be orthogonal, as we shall see shortly, nor do the beams themselves need to be of the typical variety. It stands to reason that a grid composed of intersecting box beams would be extremely stiff, for example, because of the high degree of both bending and torsional stiffness; of course, connecting such intersecting hollow box-beam grid members is not an easy task, and so is rarely done. Another example of atypical beams in a grid are the variously angled steel plates of the elevated canopy for the underground Telcel Theater in Mexico City, which not only serves as a distinctive place marker within a visually busy urban environment, but also as an effective sun-shading device for covering of the entrance to the otherwise unable-to-be-seen cultural spaces. (Ill. 7.41, 7.42.)

Another distinctive beam grid made of thin steel plates was the temporary Serpentine Pavilion of 2002 designed by architect Toyo Ito. (Ill. 7.43.) Unlike for the preceding example, however, the plates were all oriented vertically in this case. But instead it was the plan geometry of the grid that was highly irregular (or seemingly so, as there was indeed a graphic logic to the development of the geometry). Here the vertical steel plate segments (with little torsional stiffness) intersect to form seemingly arbitrary triangular and trapezoidal shapes of various sizes, and these are either glazed or made solid by covering panels, resulting in a lively exterior appearance and interior lighting experience, with the gridded structure acting almost as large-scale foliage to cover the café in the park. Of course, notable as well here is that the beam grid does not stop at the building perimeter but rather folds down to become all four sidewalls of the structure, thus providing structural continuity for the roof grid all around its edges, to say nothing of the visual uniqueness and formal unity that is simultaneously achieved by this means.

And in one final example of the richness of possible beam grid variations, we will briefly examine another Serpentine Pavilion, this one built in 2005 and designed by the architects Álvaro Siza and Eduardo Souto de Mora together with the engineer Cecil Balmond of Arup. (Ill. 7.44.) The architects' conceptual sketches formed the basis for a project that ended up highlighting the form-making possibilities of contemporary computing and robotic manufacturing

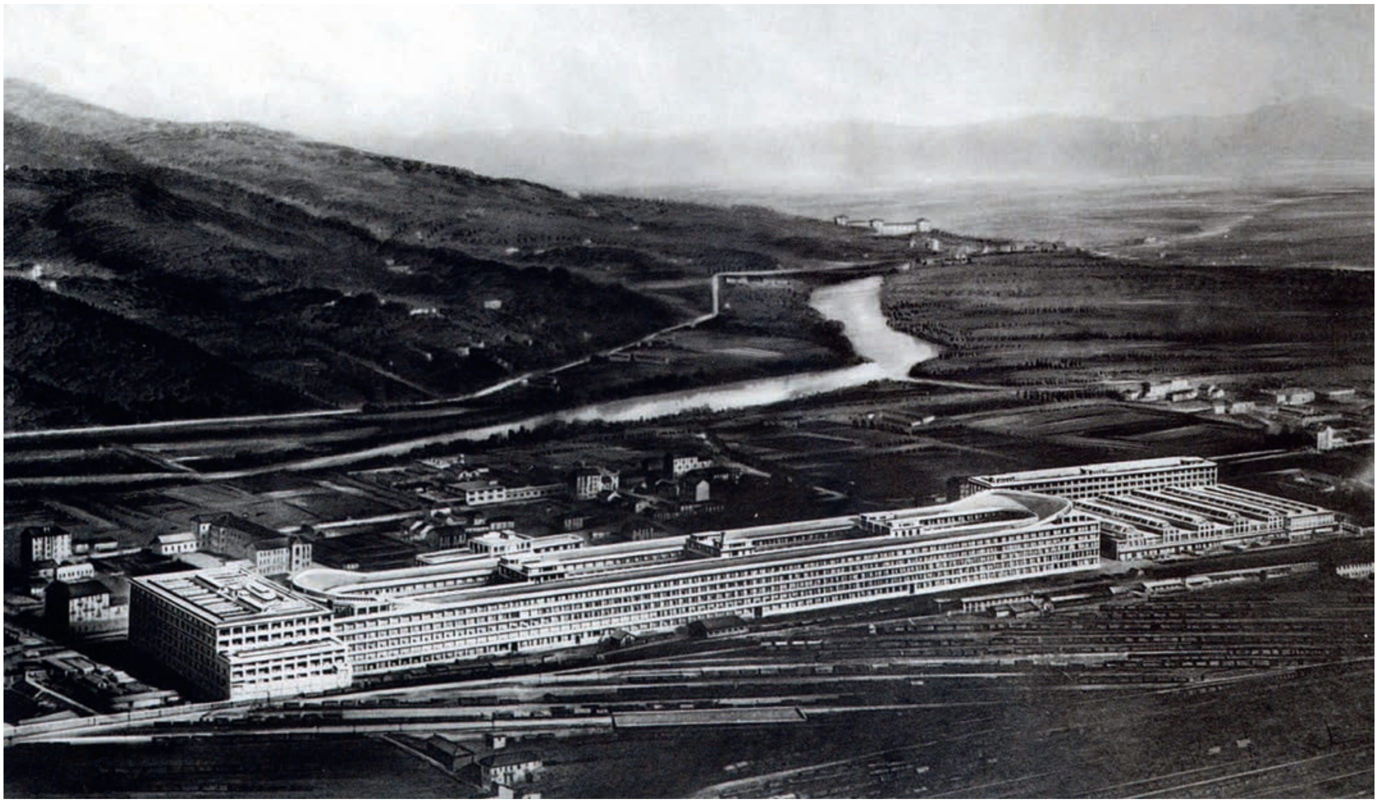


Illustration 7.45
FIAT Lingotto Factory, Turin, Italy (1926).
Rural historical setting contrasts sharply with present urban situation. Monumental scale of overall building is nonetheless apparent, as is basic structural grid module of concrete columns and beams.
Building designer and structural engineer: Giacomo Mattè-Trucco. Image from "il Lingotto Storia e Guida" by Umberto Allemandi.

technologies. Covering some 400m^2 (4306ft^2), the roof surface was free of interior columns and shaped into an undulating, alignment-offset grid of plywood beams. Actually, upon closer examination it can be seen that the entire structure was built up from 427 relatively short and thin beam segments, each one having a different length and inclination and linked to the adjacent pieces using mortice-and-tenon connections. The ever-changing geometry of the 500mm (20in) deep, 39mm (1.5in) thick spruce plywood segments was digitally defined by Arup into a format that could directly be communicated to the manufacturer, the German construction firm Finnforest Mark. Using robotic technology, all of these differently shaped elements could thereby be produced with precision within a rather remarkable two-week turn-around period. The pavilion was clad with 248 translucent polycarbonate panels, each one of which incorporated a solar-powered fixture. The requirements and the simple open-space program for this Serpentine Pavilion proved to be well suited to beam-grid experimentation in combining contemporary architectural design with innovations in structural analysis and material fabrication techniques.

7.11 From Lingotto to Sendaï – Beam Articulations

Returning to Torino in 1915 after a few trips to the United States, the FIAT car company director decided to construct a new American-style factory in the city's Lingotto area in which the entire car-building process would take place in a single 507m (1663ft) long structure. (Ill. 7.45.) The building was designed so that the materials and parts were brought in at the ground floor level and the cars were then put together on giant assembly lines that went up through the five floors. Two spiral ramps at the north and south ends of the workshops led to a test track for the cars on the roof. Old photographs of cars racing around on the banked turns of the roof-top track helped to make the Lingotto factory an icon of industrial modernism; indeed, the structure was hailed by Le Corbusier (1887–1965) in his 1920 manifesto "Vers une Architecture" as a benchmark of modern building technology. The project was designed by the engineer Giacomo Mattè-Trucco (1869–1934) in close collaboration with the founder of FIAT, Giovanni Agnelli, and it was among the first buildings in Italy to introduce reinforced concrete. The design of the ramps, however, is credited to the architect and engineer Vittorio Bonadè Bottino (1889–1979). Although car production at the facility ceased in 1982, Renzo Piano and other architects have since then given new life to the Lingotto Factory by reinventing it as a contemporary civic building having multiple program elements, including a gallery, concert hall, theater, convention center, and hotel.

The plan layout of the original building was based on a regular 6m by 6m (20ft by 20ft) grid module and essentially consisted of two longitudinal 24m (79ft) thick bar buildings separated by a series of open courtyards. Most of the building was built as a "pure," straightforward (and very long) rectilinear structural frame with columns, beams, and slabs made out of cast-in-place reinforced concrete. (Ill. 7.46.) With a combination of large ceiling heights and almost-all-glazed façades, daylight entered freely into the assembly halls from the four long building faces, and the work spaces proved to possess considerable architectonic qualities and potential, as their current reincarnations to serve other purposes attests.

The south ramp in particular, despite being completely encased in the overall rectangular structure, is, of its own right, also considered to be a masterpiece in revealing the remarkable potential of reinforced concrete as a relatively "moldable" material. (Ill. 7.47.) The upwardly ascending ramp revolves around the perimeter of an open well of semicircular shape in plan and a system of concrete ribs that support the ramp radiate out from a central column toward the outer perimeter beam; the arrangement creates quite a remarkable and memorable space.

The structure of the spiraling ramp at the Lingotto factory building anticipates the achievements of the Italian engineer Pier Luigi Nervi (1891–1979) some 30 years later. Nervi, one of the last century's great designers, was a practical visionary who could both design and do the calculations for his own structures. And as he sometimes found no one capable of building what he had designed he formed his own contracting firm, bidding on his very own designs and executing them according to his particular methods. In this way, Nervi introduced a new kind of prefabrication system based on using a series of precast concrete units in the shape of inverted pans as formwork, then nesting reinforcing bars within the voids created by these and finally pouring concrete over the whole ensemble to make the system act as a unit. By thus eliminating traditional wooden forms, the restrictions that straight planks had previously placed on the shaping of reinforced concrete structures were removed. Nervi further refined his methods in a

Illustration 7.46
FIAT Lingotto Factory.
Ceiling of renovated interior space displays regular, rectangular geometry of typical reinforced concrete slab and beam floor system.

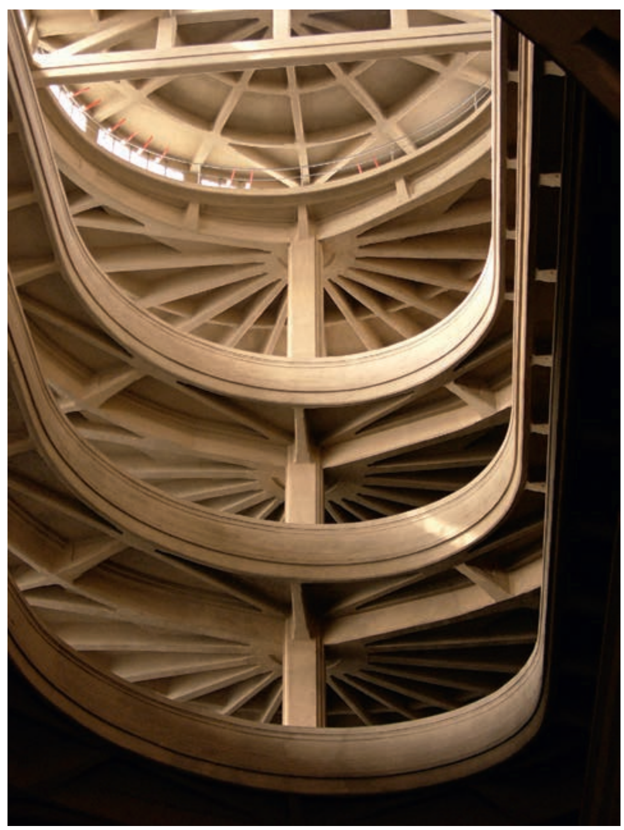


Illustration 7.47
FIAT Lingotto Factory.
South car ramp exhibits radial floor beam system, accenting curve and contrasting with rest of building.
Architect and structural engineer for ramp: Vittorio Bonadè Bottino.

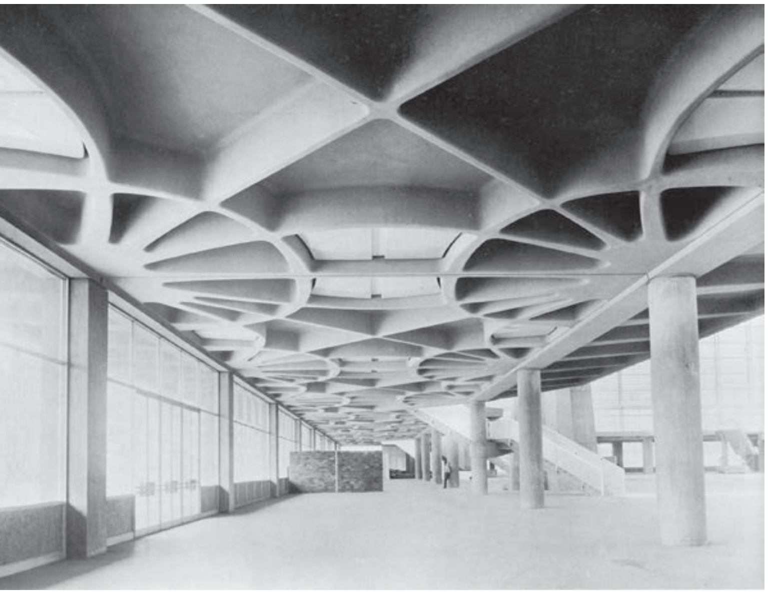


Illustration 7.48
Palace of Labour, Turin, Italy (1961).
Concrete slab's stiffening ribs follow "natural" looking, curving lines of isostatic stress conditions.
Designer and structural engineer: Pier Luigi Nervi.

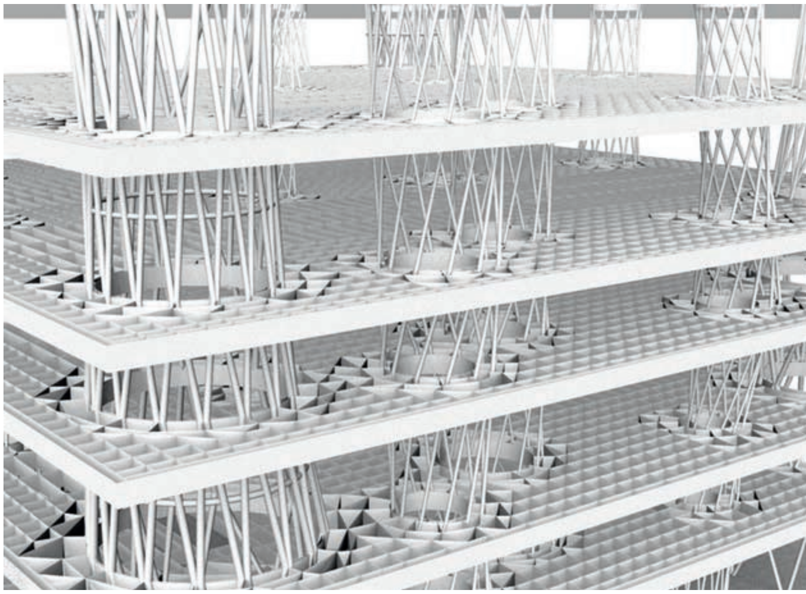


Illustration 7.49
Sendaï Mediatheque, Miyagi Prefecture, Japan (2001).
Orthogonal organization of two-way grid of steel beams cedes to modified, triangulated pattern in vicinity of supporting "column tubes." Whole floor system is connected top and bottom to horizontal steel cover plates to provide huge flexural capacity for long, heavily loaded spans. (See also Ill. 8.20e.)
Architect: Toyo Ito Associates. Structural engineer: Sasaki Structural Consultants.

variety of projects in the 1950s and 1960s, and an interesting example in the current context of beam systems is a portion of the Palace of Labour that he designed and built in Turin in 1961. Based on a column module of 5m by 5m (16ft by 16ft), a system of beam ribs was created for the floor that follows the isostatic lines of stress for a two-way concrete slab, resulting in a beautifully organic pattern that mimics a scientific analysis of its structural behavior.[13] (Ill. 7.48.)

An unusual variation on this beam grid theme was accomplished in 2001 for the Sendaï Mediateque by architect Toyo Ito and structural engineer Mutsuro Sasaki. In this case a flat floor-framing system needed to be devised in order to span the exceptional 20m (66ft) distances between 13 sets of lattice tube-columns (see Section 8.6, Ill. 8.20e). With such long spans and the large gravity and seismic live loads being dictated by the building's program and location, a reinforced concrete slab system would have become much too thick and heavy; instead, the solution was to go to a unique sandwich steel plate system. (Ill. 7.49.) Starting with an orthogonal two-way steel beam grid of 1m (3.3ft) spacing in both directions having 400mm (16in) depth, this grid gets modified in the zones around the tube-columns in accordance with the changing lines of stress that are produced near the vertical supports; the whole of this vertical plate system is then covered and connected both top and bottom to flat steel plates in order to give the system the large bending moment resistance that is needed. Moreover, the top plate is covered with a 70mm (2.8in) concrete slab layer that is made to work compositely with the steel below by means of the provision of shear stud connectors (this technique will be further discussed in the next section, see Fig. 7.44.) The overall simplicity of Ito's architectural vision of thin flat plates spanning freely between waving sets of bundled columns has thus been achieved, but not without the close collaboration of the exceptional engineer Sasaki and his innovative combination of several of the beam grid load-carrying strategies that we have encountered in this chapter.

7.12 The Slab – Beams Stretched Thin

The slab is perhaps the most ubiquitous and yet under-appreciated of all structural elements. Beams, columns, and walls are universally recognized as essential building blocks, yet slabs are also present in virtually all of our buildings as they form the floors upon which we walk and the ceilings or roofs just above our heads. As such, they clearly both carry load and have tremendous space-making architectural impact; how they do this is what we next turn our attention to. It should be understood that by use of the term "slab" we refer to and will mainly consider its most common form in building structures: i.e., that which is made of reinforced concrete. However, the same fundamental principles also apply to slab-like surfaces made of other materials that may be used to make floors and roofs, such as tongue-and-groove sawn wood planks or

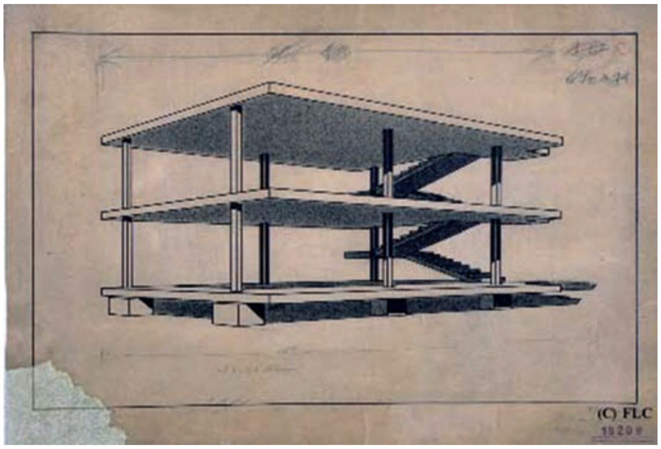


Illustration 7.50
Maison Dom-Ino (1914).
Example of flat plate system; reinforced concrete slab spans directly to supporting columns without any underlying beams.
Architect: Le Corbusier. Structural engineer: Max du Bois.

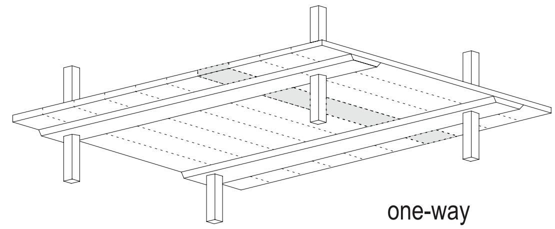


Figure 7.40
One-way slab depicted as series of adjacent beams of limited depth.

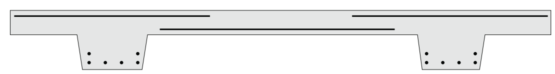


Figure 7.41
Section through slab spanning continuously between and over two adjacent beams; placement of reinforcing bars near bottom of section at mid-span, near top above beam "supports", with zones of overlap.

manufactured products like plywood, pressed-wood sheets, and solid-wood panels, or even steel or glass plates.

Perhaps *the* quintessential illustration of the slab in architecture is the drawing by Le Corbusier of his idea-project "Dom-Ino," (Ill. 7.50) in which the floor and roof slabs can be seen to rest directly on columns without any beam intermediaries, thus establishing his structural concept for the rational building of housing, a vision that very much influenced the development of concrete as a favored load-bearing material during the Modern architectural period and beyond, to say nothing of its role in advancing the case of undifferentiated, non-directional space. The technology to make this in reinforced concrete, however, was not available at that time. But to understand just how such seemingly "simple" structural slabs can work to carry loads first requires some conceptual development.

To begin with, it can be said that the fundamental behavior of the slab is closely linked to that of the beam. Here, once again, we have a structural element in the slab that has loading applied transversely to its length and that under such loading will deform in analogous fashion to the way beams do, sagging downward between vertical supports and curving in the opposite direction when it runs continuously over these or when it cantilevers outward from a supporting structure. In fact, a one-way slab spanning between two beams or walls or other transverse support elements can be likened to a series of one-way beams of relatively small depth that are placed alongside each other. (Fig. 7.40.) The same internal actions, therefore, that we have become so familiar with in beams – namely, bending moments and shear forces and their corresponding sets of stresses – will also be the means by which loads are carried in slabs. And so, in the reinforced concrete slab we will find that steel reinforcing bars need to be aligned in the spanning direction in order to deal with bending behavior – and that these are to be placed both near the bottom or top of the slab section according to location along the span. (Fig. 7.41.)

Of course, if the beam has been previously described as an inefficient means of carrying load because of its (a) relatively small moment arm that is available for producing the internal bending moments needed for equilibrium and (b) typically constant dimensions despite large variations of shear and moment demand from one end of the element to the other, these conditions are also present and even further accentuated in the slab. The very small depth/thicknesses of slabs that make them so attractive in reducing wasted vertical space in multistory buildings, for example, will also be the cause of their being subject to large bending stresses, shear stresses, and deflections that can quickly challenge material capacities and building code limits even under normal loading conditions. And simply increasing a slab's thickness in order to enhance its bearing capacity results in disproportionately large increases in dead load that can quickly "eat up" much of any resultant increase in strength.[14]

Clear-spanning distances for slabs, therefore, are typically quite limited and this can severely impact the often competing architectural desire to maximize the spacing of supporting columns and walls in

Illustration 7.51
"House with One Wall";
Mehrfamilienhaus Forsterstrasse,
Zurich, Switzerland (2003).
Continuous concrete slabs and walls define merged interior and exterior space.

Architect: Christian Kerez. Structural engineer: Aerni + Aerni Ingenieure AG and Dr. Josef Schwartz Consulting AG.

a building. At the same time and notwithstanding such structural drawbacks, however, the slab does have much to offer for other reasons, including the ability to simultaneously provide a necessary walking surface or roofing enclosure as well as to uniquely define and project architectural space – as is powerfully demonstrated by the cantilevering slabs of a remarkable multifamily house in Zurich designed by Christian Kerez. (Ill. 7.51.) It should be noted that in multistory buildings a flat slab of concrete can also have tremendous economic and logistical advantage by providing a smooth finished ceiling, and that this approach makes it easy to mount piping and wiring and air-handling ductwork without the trouble and labor of having to make holes in the right places within a grid of underlying beams.

Given the slab's spanning-distance shortcomings, though, various strengthening techniques that we have previously described for the beam can be especially important to reconsider once again in this context. For example, making a slab run continuously over supports into adjacent spans is greatly advantageous in reducing its bending moments and deflections, and for the slab this is a strategy that certainly also makes perfect sense from an ease-of-construction point of view. Also, the same benefits that we saw for countering the effects of loads on beams by means of pre-stressing can be applied to slabs by running tightly stretched tendons through the surface that are anchored at its outside edges. Yet another approach that we have seen before and that is frequently used to advantage for slabs is to have them configured and working in the two-directional load-sharing manner (Fig. 7.42, e.g., Ill. 7.52) that we previously discussed in the context of beam grids. As long as a slab area is roughly square and that similar support conditions are provided around its perimeter, approximately half of the load will be carried in each orthogonal direction, leading to the designation of such a system as a two-way slab and to the requirement for having orthogonal sets of reinforcing bars carefully placed at the appropriate levels and locations within this thin slab's depth. (Fig. 7.43.) We have already established the clear benefits of using any of these techniques with the beam and the same are even more true for the slab: i.e., larger loads can be carried, spans can be greater, and deflections can be significantly reduced. Moreover, these techniques can be combined for even greater structural benefit.

Yet another approach to enabling longer spans for slabs is to have them work in strategic combination with underlying sets of beams. In this case, rather than having a slab simply resting on top of a supporting beam framework and the two working independently of each other in some sort of preordained structural hierarchy (according to which, let's say, the loads are carried first by the slab to the beams and then independently by the beams to their supporting columns or walls), the two components of slab and beams are instead expressly connected together such that they can work together in unison. When a concrete slab is to be

Illustration 7.52
"From Fracture to Form" (1996).
Glass fracture pattern from the impact of a cannonball allows for a visualization of the multi-directional load-sharing behavior of edge-supported slabs.
From an experiment at AHO by Arne P. Eggen and Nils E. Forsén.
Photographer: Jiri Havran.

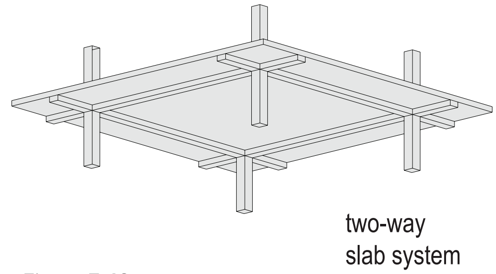


Figure 7.42
Two-way slab system: similar support conditions around roughly square perimeter.

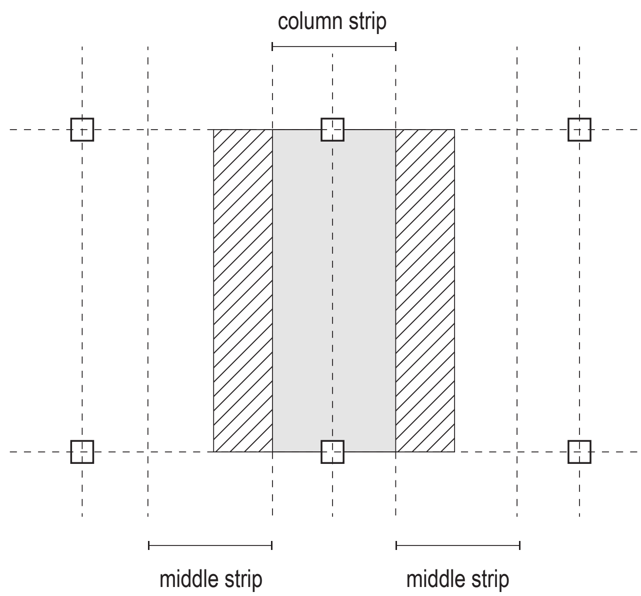


Figure 7.43
Representative drawing of typical design code provisions for "column strips" and "middle strips" used to distribute bending moments for the design of slabs. Corresponding sets of reinforcing bars will be distributed within these bands of slab, strategically placed in layers near the top or bottom of the slab. Shown here for spanning in one direction; a two-way slab will have similar strips of slab to be considered in the orthogonal direction. All this, of course, remains invisible to the eye in the end in the hardened concrete slab.

structurally combined with underlying steel beams, for example, the interconnection between the two is typically made mechanically with what are called shear studs – i.e., short steel bar projections that are welded to the top flanges of the steel beam before they are subsequently embedded into the poured concrete slab – in order to produce what in the end is called a *composite beam*. (Fig. 7.44.) A substantial width of the concrete slab (typically much more than the width of the top flange of the steel beam) is thereby engaged and can be considered to function as a very large compression flange at the top of the composite beam section in places where the overall system sags downward, thereby dramatically increasing the system's effective moment of inertia, decreasing maximum bending stresses, and in turn increasing the system's overall load-carrying capabilities.[15]

In the case of combining a concrete slab with a concrete beam, the two will typically be shaped by common formwork, have an integrated set of reinforcing bars, and be poured as a unit, thereby inherently ensuring composite behavior. The benefits derived from such an individual composite concrete beam construction can in turn be multiplied many times over by having a slab integrally connected to a grid of closely spaced beams in what is known as a *concrete waffle-slab* (see Fig. 7.38b), which clearly derives its name from its underside appearance. Such a structural configuration is typically used where there are especially heavy transverse loads needing to be carried, such as for library book stack areas or gallery spaces

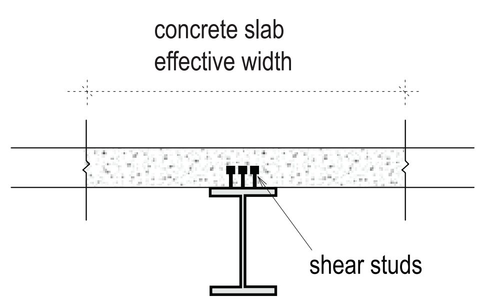


Figure 7.44
"Classic" composite beam section, composed of steel beam topped by concrete slab, connected by means of shear studs.

Illustration 7.53
Le Grand Louvre, Paris, France (1989).
Two-directional waffle-slab system is used for carrying heavy loading of public courtyard plaza above it; on underside, coffering provides visual pattern and scale.
Architect: I.M. Pei of Pei Cobb Freed.

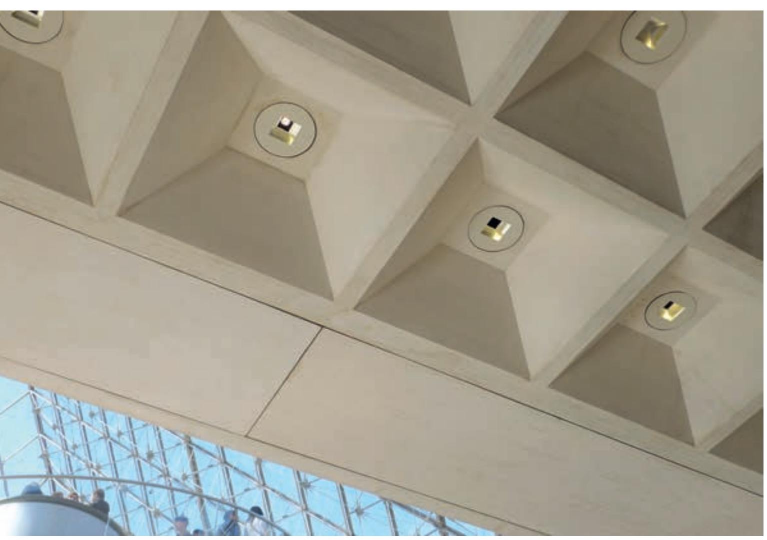


Illustration 7.54
Le Grand Louvre.
Close-up detail of waffle slab with incorporated lighting.

in art museums, or for supporting a ground level public plaza as well as perhaps backfilled earthwork above an underground space. (e.g., Ill. 7.53, 7.54.)

Finally, while the discussion so far concerning the slab's structural behavior has been, in one way or another, mostly about its bending capacity, the simultaneous effects of shear forces cannot be ignored. Perhaps the easiest way to envision the basic problem of shear in the context of the slab is to consider a portion of it supported directly on a column, such as in the Maison Dom-Ino, for example (see earlier Ill. 7.50). For the sake of argument, this can be informally thought of as akin to having a thin sheet of cardboard needing to be carried on a thin dowel-like pencil; it is clear from experience that if one pushes enough on the surface it will not take long for the pencil to push through it, representing a shear failure of the cardboard material. (For obvious reasons, an analogous failure in a real-life structural slab is known as a *punching shear failure*, whereby the column "punches" through the slab, with ensuing catastrophic results from the "pancaking" of slabs one on top of another.)

Fortunately, concrete slabs do not typically present such a dramatic shear problem; i.e., there may be enough slab thickness and effective perimeter distance around a support (and, thereby, enough effective cross-sectional area available) to safely carry the shear force within the relatively low levels of shear stress that are permissible for concrete. There are loading conditions, however, in which as one progressively moves closer and closer to a support a

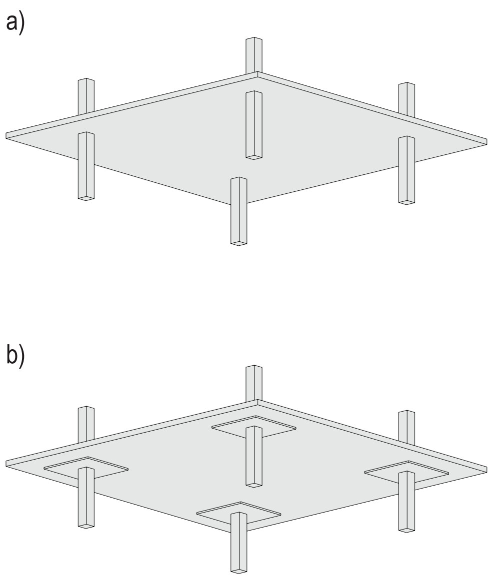


Figure 7.45
(a) "Flat plate" system of uniform depth throughout, (b) "flat slab" system (still no beam projections below slab, but with "drop panels" in area's slab-to-column connections).

shear problem may indeed present itself in a slab, since concrete is not a material that has an especially high strength in this regard (certainly in contrast to steel, which does).[16] The usual means by which this problem is addressed is to increase the slab thickness in the vicinity of the support and create what is called a *drop panel*, looking somewhat like a traditional column capital and thus sometimes being referred to that way instead. (Fig. 7.45.)

To conclude this section about the structural slab, we will briefly consider Miami Beach's 1111 Lincoln Road parking/multi-use structure (Ill. 7.55), a building which is perhaps the best embodiment yet of the possibilities suggested by Le Corbusier's Maison Dom-Ino that we started things out with. At 1111, not only are the beam-less two-way slabs of necessity doing all the work of carrying the loads of its occupants (cars and people alike, in ever-changing fashion) as well as that of its own self-weight, but these slabs are also put on full display since there is no enclosure to this building. Slabs are all there is to be seen of the structure here – or almost so, as there are indeed also many distinctive columns holding the six concrete slabs apart, but considering the logic of form and structural behavior of these elements is the topic of the next chapter. For now, we will let the matter of the beam and the slab rest, although, as will be seen throughout the remainder of this book, the fundamental structural behavior lessons for the beam are quite powerful and will be seen to have repercussions with other structural elements and systems as well.

Illustration 7.55
1111 Lincoln Road, Miami Beach, FL, USA (2010).
Flat plate slabs define varying floor level heights and vertically subdivide the large open structure.
Architect: Herzog & de Meuron. Structural engineer: Optimus Structural Design.

The Column and the Wall

CHAPTER

8

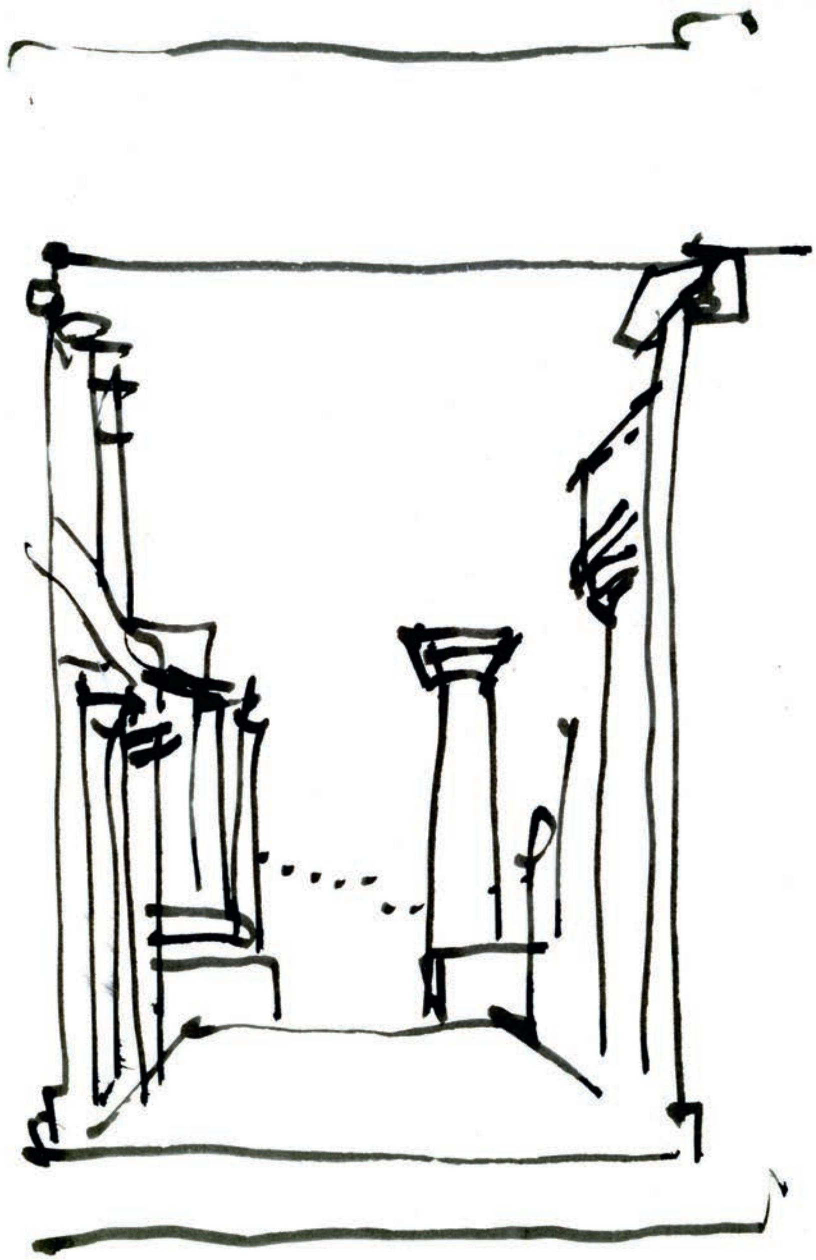


Illustration 8.1
Temple of Aphaia, Aegina Island, Greece (c.500 bc).
Columns and more, built out of gray limestone throughout.

Illustration 8.2
The Maison Carrée (about 16 bc) and the Carré d'Art, Nîmes, France (1993).
Porticos and columns from different eras complement and contrast with each other.
Architect of the latter: Foster + Partners. Structural engineer: Arup.

8.1 Maison Carrée et Carré d'Art – Columns in Dialogue

The city of Nîmes in southern France is host to two cultural buildings that face one another across a square, but that are separated in time by almost 2000 years. (Ill. 8.2.) The older of the two is one of the best preserved temples of the whole of the Roman Empire, aptly named in French the Maison Carrée according to the shape of its rectangular plan and cubic proportions. This building's remarkable preservation is due to the fact that it was rededicated as a Christian church during the fourth century, thus escaping the widespread demolition of temples that followed the adoption of Christianity as the state religion of Roman society. Later, the building served various purposes before finally being turned into a museum in 1823.

Raised on a podium nearly 3m (10ft) above the level of the adjacent square, the Maison Carrée would have dominated the forum of its then-Roman city. Its plan measures 26.4 by 13.5m (87 by 44ft) and is divided into two distinct parts: the cella, with its walls and engaged columns, and the deep portico that takes up almost a third of the building's total length. Three rows of six Corinthian columns each carry the portico with architrave and pediment above. These free-standing columns, made of solid limestone cylindrical segments, are a particularly forceful feature of the classical Greek and Roman architectural styles, and convey an impression of strength, solidity, balance, and lasting endurance. For the cella portion of the building, on the other hand, the regular rhythm of the portico's columns continues, but here perimeter columns are integrated into the enclosing walls, with the one element strengthening the other. The Maison Carrée is a perfect example of classical, Vitruvian architecture, and it has served as direct inspiration for such well-known buildings of the neoclassical period as the Madeleine Church in Paris and the Virginia State Capitol in the United States.

Also now occupying a prominent location on the public square is a thoroughly modern building that establishes a powerful dialogue between old and new. The architects' office of Foster + Partners was commissioned to design a library and art gallery on the site facing the Maison Carrée, an extraordinarily difficult task, and one in which the presence of the classical building became a strong influence on the design of the newcomer. The so-called Carré d'Art, finished in 1993, finds a perfect balance between adaptation and contrast: it is also a building having a roughly rectangular footprint and an enclosed volume (this time clad in steel and glass) as well as a prominent portico. Orthogonally oriented with respect to the front of the Maison Carrée, the modern portico is created by a distinctive row of five free-standing slender steel columns that carry a shading canopy both over the building's main entrance as well as its wonderful elevated exterior café. In contrast to the historic stone columns across the square, these contemporary vertical supports have relatively little mass and cross-sectional dimension; nonetheless, they are clearly picking up on the same architectural language, if in a very different dialect.

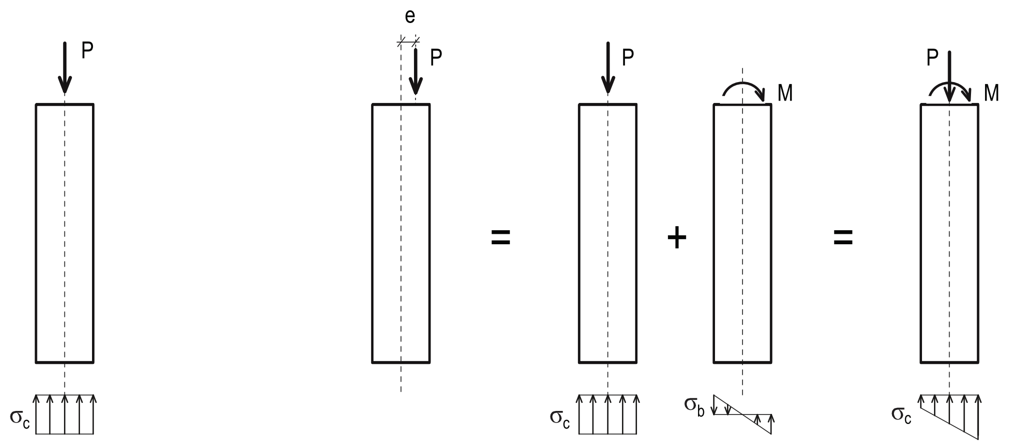


Figure 8.1
Short column with concentrically placed point load (left), and with point load acting eccentrically (right) and the corresponding stress distributions.

8.2 Compression Elements – How They Work

The mechanical function of compression elements is to keep physical bodies apart that want to come together. The existence of such elements is, of course, a necessity for the creation of architectural spaces, the clearest examples of which are the columns and walls keeping floor structures in position and "up in the air" so that we can occupy interior space. What all compressive structural elements have in common is their ability to resist shortening; their stiffness and strength must be sufficient to transmit the forces associated with any deformation tendency for things being pushed together, and we will see that the way that they accomplish this varies according to their shape and dimensions as well as the mechanical properties of the materials of which they are made.

The theory which will be presented in this chapter applies to all compressive elements; i.e., to struts, columns, and walls alike, as long as their dominant load condition is one of compression applied at an element's ends and acting along its longitudinal axis. For an introductory discussion of the topic (and as long as axial compressive loads predominate), the orientation of such elements in space is not considered critical, and their fundamental structural behavior will be taken to be the same regardless of whether the compression element is vertical, inclined, or even horizontal. It is convenient, however, to begin our treatment of this topic by considering the behavior of vertical columns, inasmuch as this not only covers the main and quintessential category of compression elements, but it also reflects what historically triggered an interest in how such elements work to carry load.

We will start with the observation that when it comes to supporting loads (and eventually to failing from overload), columns behave differently according to their height-to-width ratio. As long as a column is quite short relative to its smallest cross-sectional dimension, it will carry load until the compressive strength of the material is reached. The ultimate load-carrying capacity in this case depends on the material's strength and the total amount of material in the cross-section, but not on the column's length. Failure in this situation will typically be characterized by the crushing of the material. A familiar example of this failure mode is what is likely to happen if you press a very soft and unsharpened graphite pencil too firmly against a writing surface.

Let's look more precisely at the behavior of a short column subject to an axial loading P (Newton, pounds) and having cross-sectional area A (mm^2, in^2). When the load is applied to the cross-section, uniform compressive stresses σ_c develop. If we steadily increase the load, the column will eventually fail when the compressive stress exceeds the material's ultimate stress σ_u, where the index u is short for "ultimate," as we acknowledged in Chapter 5.[1] The load-carrying capacity P_c of the short column is, therefore:

$$P_c = \sigma_u A$$

and the compressive stress up to failure is

$$\sigma_c = P/A \leq \sigma_u$$

This formula also suggests that in this situation the actual shape of the column cross-section is of little structural importance; i.e., we are free to design whatever form we like for such a short column without influencing the magnitude of the stresses, as long as we maintain the same sectional area A.

What happens, though, if the load P is not centered axially but instead has a resultant which acts outside the member's central axis? (Fig. 8.1.) In such a case a bending moment will inevitably result which will produce bending stresses in the column. We can think of this load case as being the equivalent of having a couple (moment) M applied to the column in addition to a concentric load P. If the eccentricity of the load is e (mm, in), the bending moment developed is M = Pe (Nm, ft-lb). As was explained in the previous chapter (Section 7.7), the bending stresses produced in the

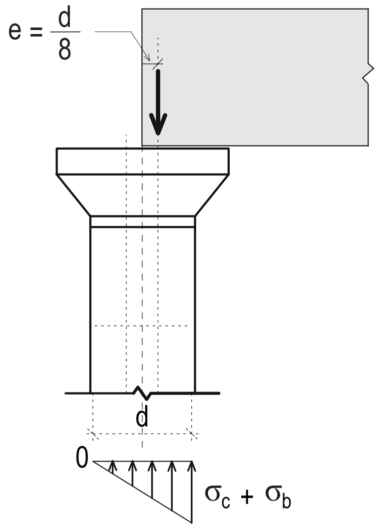


Figure 8.2
The so-called middle-third rule that applies to rectangular cross-sections gets a slightly different expression for circular columns: here the "core" cross-section is within the inner quarter of the column's diameter, as applied to the column of the Aphaia temple at Aegina, Greece.

Illustration 8.3
Drawing of Charlie Chaplin and his slender bamboo cane.

cross-section are σ_b = M/S = Pe/S, where S is the section modulus (mm^3, in^3) and these will interact with the uniformly distributed compressive stresses from the centered load, increasing axial stresses on one side of the column and decreasing them on the other; i.e.,

$$\sigma_c + \sigma_b = P/A \pm Pe/S \leq \sigma_u$$

In this equation, the + sign is valid on the side of the column where the bending stresses add to the uniform compressive stresses, and the – sign is valid where they subtract from it. From this, it can be extrapolated that the decrease of compressive stresses on one side of such a column may reach a point where tensile stresses occur; for a rectangular cross-section this can be shown to be if the resultant load P is applied at a distance exceeding one-sixth of the column's width on either side of the central axis. (Fig. 8.2.) As a result, designing in a manner that the force resultant was always kept in a "core" confined to the middle-third part of the rectangular cross-section was of great importance historically when stone and brick masonry were the favored building materials – as neither of these has significant tensile strength. Setting this particular issue aside, however, it can be generally stated that short columns usually fail by overstressing the material to the point that crushing occurs (or yielding, if made of steel).

In very long columns, however, things are different: when a load is applied to its ends, a column will maintain its straight alignment as long as the load is kept below a certain limit. But increasing the load beyond that point will result in the column deflecting sideways quite suddenly; moreover, this happens uncontrollably and without warning. (Ill. 8.3.) We call this phenomenon *buckling* and say that the column buckles. At the point just before it bows sideways, the column has obviously reached a different maximum load-carrying capacity from that which applies to short columns. The particular magnitude of load at which buckling occurs is called the *critical load* or the *buckling load* for that column. If we further increase the load on the deformed column it will finally truly break (or bow out very strongly), and so we consider its effective point of failure to be the load level at which it first buckled sideways. This buckling phenomenon can easily be tested with a simple plastic ruler or thin wooden stick: as either's thickness is very small compared to its length, applying a load at its end with one's finger will invariably cause it to fail by buckling rather than by crushing of the material.

Buckling is in reality a stability problem. If a long column is ideally (but unrealistically) straight and the load is applied at its central axis, the buckling load may actually be exceeded without anything happening *unless some small disturbances* occur. This condition is a type of equilibrium, but it is a highly unstable one. The smallest sideways push will cause the column to deflect sideways without having the slightest hope of controlling or reversing its deflection. But real columns are not perfectly straight and the load is most likely not applied absolutely concentrically. Generally, therefore, columns will tend to deflect sideways when loaded because of a small bending moment's developing due to the axial load's eccentricities – but as

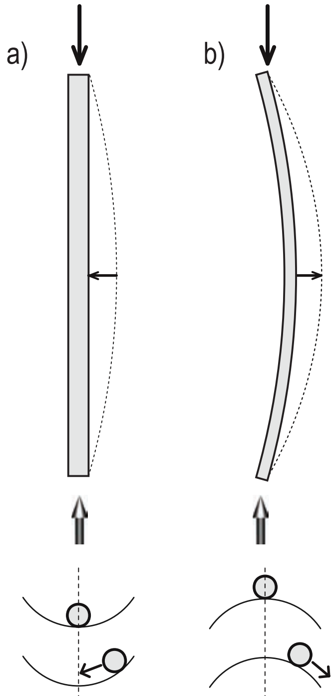


Figure 8.3
(a) Stable and (b) unstable equilibrium.

Illustration 8.4
Temple of Nike at Acropolis, Athens, Greece (453 bc).
View between the Propylaeum's columns.
Architect: Kallikrates.

long as the load is below the buckling load the column's stiffness is able to counteract and control that sideways deflection. If we push the column sideways in this state, the bending will increase but the column will return to its former position once the horizontal load is removed. (Fig.8.3a.) The column is in a state of stable equilibrium.[2] Once the load exceeds the critical load, however, the situation becomes unstable and excessive lateral deflection, also known as buckling failure, can be expected to occur. (Fig. 8.3b.)

Within this context, some interesting questions arise. What parameters determine the buckling load of columns? And what architectural consequences do these parameters have? We have already established that quite short and very long columns behave fundamentally differently, and we can therefore expect those two extreme cases to have quite contrasting design implications. We have also mentioned that stiffness, which is related partly to an element's geometric proportions I (moment of inertia), L (length), and partly to its material properties E (modulus of elasticity), plays an important role in establishing how long columns behave. All of these observations will have consequences in answering our questions about column form and design, but before going farther down this avenue in Section 8.4 we will pause briefly to look more closely at the rich and varied detailing often found at the top of a column and at beam-to-column intersections; these are, after all, where the compressive forces of such interest and consequence are being applied to the column.

8.3 Exploring the Capital

The design of the transition point between column and beam, otherwise known as the capital, has always attracted attention. And while particular material properties and technical innovations may impose limits or create new possibilities in the design of capitals, the basic situation remains the same: structure changes direction in going from column to beam, and perhaps the material changes as well. Also, the reaction force at the end of a beam needed to hold it up must be balanced by an equal and opposite force on to the shaft of the column, and all of this balancing act must take place through the intermediary of the capital. A capital is thus a transition element, and different periods in history have presented us with their interpretation of this loaded theme, as the examples below illustrate.

Standing with one's back to the rock outcropping called the Areopagus, one can look up at the western front of the Acropolis. "There is just one entrance to the Acropolis. No others can be found because the embankment is so steep and all around there is a tremendous wall," wrote the Roman traveler and author Pausanias in the second century ad. Then, as now, a ramp leads up to the Propylaeum that forms the entrance into the Acropolis. From this threshold one can turn right and catch a glimpse, between the Propylaeum's Doric columns, of the little Temple of Nike standing obliquely on top of a protruding bastion. (Ill. 8.4.) The temple, which is dedicated to Athena and identified with the goddess of victory, Nike, was built in 453 bc. The walls of this temple's cell

Illustration 8.5
Temple of Nike at Acropolis, Athens, Greece (453 bc).
Detail of Ionic capital at top of this temple's column.
Architect: Kallikrates.

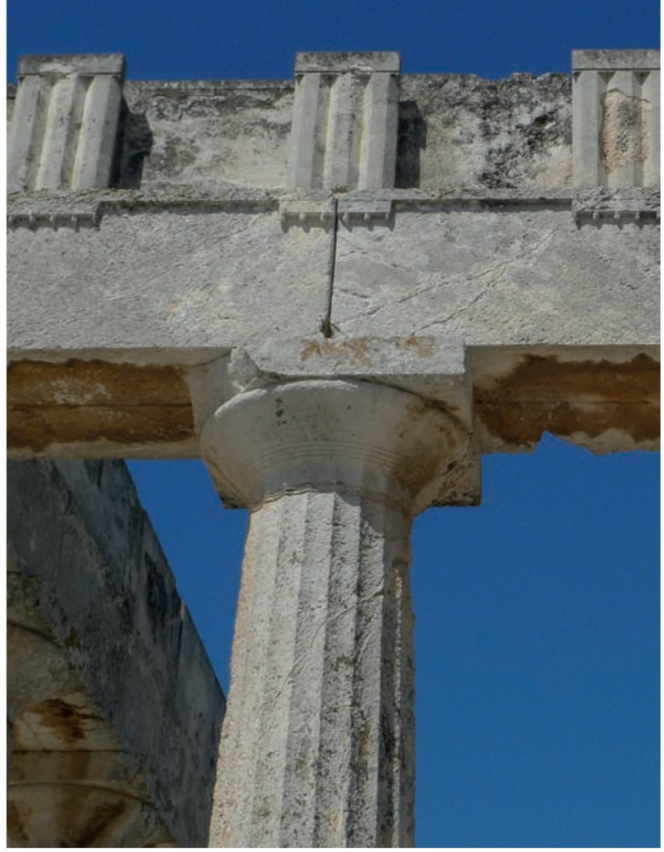


Illustration 8.6
The Temple of Aphaia, Aegina, Greece (fifth century bc).
The Doric capital transfers the weight of the entablature to the column shaft.

structure are pulled back to free its corner columns. These Ionic marble columns, which stand proudly over the modern city of Athens, have all of this structural element's essential and highly developed components; as such, they have stood as an eternal paradigm, both positive and negative, for all of the world's column builders ever since they were first erected.

At the lower end, the temple's columns have an articulated base that mediates the transition between the stylobate (foundation) and the column. The column's shaft consists of several circular stone sections stacked on top of each other, each with vertical fluting around the perimeter that allows light and shadow to enhance an impression of height. The sides of the column are not quite straight in elevation, however, instead forming a slightly convex, bulging curve between base and top. The difference between the straight line and the curved one is called entasis, of which much has been made over the centuries, but which is measured in this case by just a few millimeters. While entasis prevents the undesirable optical illusion of the column looking concave over its height, and thus contributes to suggesting this structural element's power and bearing capacity in resisting compressive force, in reality the column only gains minimal additional load-carrying capability from such thickening at mid-height. (We will come back to this topic in Section 8.6.)

At the top of the column, a capital is formed to accept and transfer the load from the beam above into the shaft of the column below. (Ill. 8.5.)

> *As an expressive measure of compression, the volute piece of the Ionic capital between the beam and the shaft of the column, with its elastically taut stream of lines, reflects the play of forces inside the block – the seemingly springy resistance of the stone's innards here lies open to our eyes.*[3]

The Temple of Aphaia, on the island of Aegina, also dates from the fifth century bc. It is all made of gray limestone, and the temple's columns and beams have a rough appearance and texture, quite different from the finely sanded marble of the Acropolis. The circularly shaped Doric capital here can be described as growing out from the shaft of the column, while the horizontal so-called necklace band captures and concludes the column's vertical fluting. (Ill. 8.6.) The square, flat abacus lies between the rounded echinus and the architrave (which in Latin means "beginning of beam"), its width gladly accepting the load from the beam above. All the stones are very precisely fitted to each other and thoughtfully put together, each piece having a clear role in the transfer of forces. The Doric capital is certainly a beautiful and most expressive gathering of stones.

Jumping forward in time by more than two thousand years, building materials have clearly changed, as have architectural styles – but evidently not without the one influencing the other, as a quick survey of a few column capitals from the past century makes clear.

At the same time as the famous skyscraper experiments in steel framing were taking place with a group of architects now known as the "First Chicago School" in the 1880s, buildings were also being built in that city that were instead constructed with exterior brick walls and interior wooden columns and beams. In Chicago's version of this then-common construction system, the columns ran

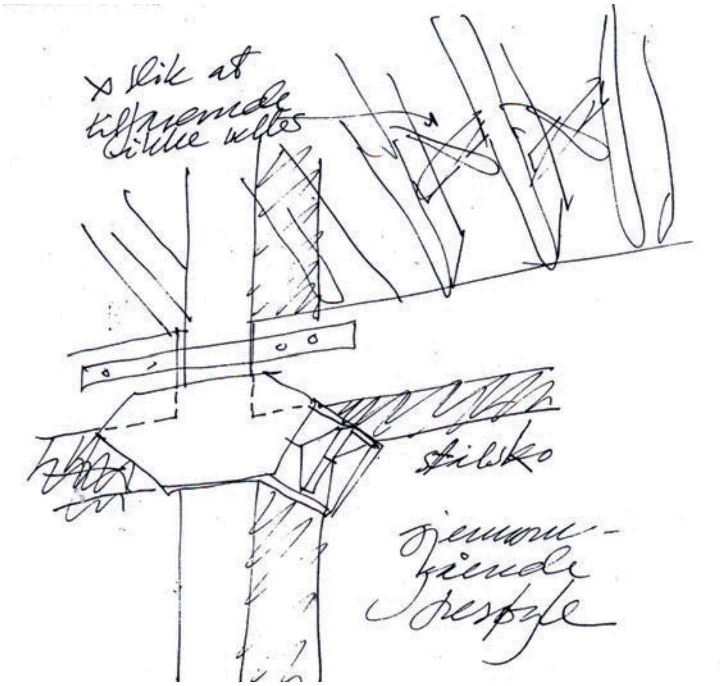


Illustration 8.7
Grace Episcopal Church, Chicago, USA (late 1800s).
An Arts and Crafts building having red brick exterior walls and an interior timber structure. Sketch shows iron brackets embracing the columns and carrying the ends of the main beams.

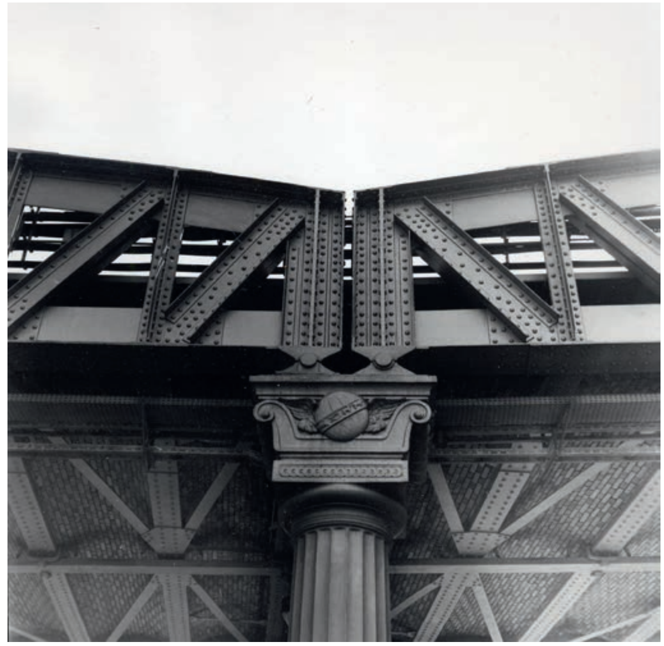


Illustration 8.8
Bridge for Metro Line 2, Paris, France (1910).
Cast iron capital.

continuously up through all the floors of the building and an iron capital (or perhaps a two-sided bracket is a better descriptor) was threaded down the column until a certain point, forming the support for beams on either side. (Ill. 8.7.) By means of this effective load-transferring system, the time-consuming detailed work for wood beam-to-column connections could be avoided and, moreover, the iron brackets could be mass produced. The building industry's demand for quick and effective fittings resulted in simple and robust solutions.

At the beginning of the twentieth century, Paris built its Metro system with Line 2 running above ground and over the boulevards, designed as a bridge system of iron trusses with riveted component elements and joints. At their ends, these trusses rest on ornately molded, reversed-Ionic capitals sitting at the top of Doric columns. (Ill. 8.8.) Here the French freely dipped into history's well, cleverly adapting the retrieved lessons with an elegant resolution of the technical difficulties of material and fabrication tolerances and of the transfer of forces by means of cast iron.

In the mid-twentieth-century design for an exhibition hall for machines in Monza, Italy, the architect Angelo Mangiarotti (1921–2012) displayed the design potential of prefabricated concrete elements. This type of program often requires just one large space, and the conditions were ideal for developing a simple and well-refined structural system: the number of elements present here is very few, only the column, the beam, and the roof deck. (Ill. 8.9.) The capital is integrated into the column and is of hammerhead form; angled tongue-and-groove interlocking joints ensure a rigid connection to the system of precast beam elements.

Illustration 8.9
Exhibition Hall, Monza, Italy (1965).
Interlocking joint with beam supported on the hammerhead capital of the column, all made of prefabricated concrete elements.
Architect: Angelo Mangiarotti.

Illustration 8.10
Eslöv Civic Hall, Eslöv, Sweden (1957).
Main hall with mushroom columns.
Architect: Hans Asplund.

The so-called "mushroom column" is concrete architecture's unique extrapolation on the form of the capital: the top of such a column dramatically enlarges into a conical shape in order to support a thin floor slab above. (Ill. 8.10.) The Civic Hall in Eslöv, Sweden, completed in 1957 by the architect Hans Asplund (1921–1994) has drawn renewed attention due to a careful restoration project. Here, two distinct building volumes – one barrel-vaulted and wedge-shaped (in plan) for a row of auditoria and the other a four-story administration block – emerge from a thick, occupiable horizontal plinth level covering the rest of the site and housing a main hall/vestibule, a restaurant, and other public facilities. The roof slab for this area is supported by a grid of circular columns spaced apart 5 × 5m (16 × 16ft). Each column is dramatically topped by a capital in the shape of a flattened cone, which has a height of only 0.3m (1ft) and an upper diameter of 2m (6.5ft) – thus giving this space much of its unique character. The generously rounded forms are repeated elsewhere in the building, including in the matching circular skylights and a courtyard that is elliptical in plan.

At the other end of the column capital spectrum from the Eslöv Civic Hall's extended supporting projections, is the situation to be found in Berlin with a couple of buildings whose columns seemingly don't have any capitals whatsoever. When Berlin was made the capital of a reunited Germany in 1991, a new parliament was located at Spreebogen, where the Spree River makes a bend in the city. Associated with this, a band of federal buildings, fronted by the Bundeskanzleramt, was designed by the architects Axel Schultes

Illustration 8.11
Bundeskanzleramt, Berlin, Germany (2001). Where the column is expected to connect with the slab above, a hole to the sky lets the daylight in. The non-capital is, however, made possible by thin horizontal crossing brackets.
Architect: Schultes Frank Architekten. Structural engineer: GSE Gmbh; Enseleit und Partner.

Illustration 8.12
Treptow Crematorium, Berlin, Germany (1998). Central gathering vestibule with contemplative space for up to 1000 people is made memorable by the 29 monumental columns with seemingly only light for capitals at their tops.
Architect: Schultes Frank Architekten. Structural engineer: GSE Saar Enseleit und Partner Berlin IDL Berlin.

and Charlotte Frank and completed in 2001. The far western end of this office wing terminates with a covered space supported by a "forest" of free-standing circular concrete columns. Unexpectedly, where these columns meet the roof we are confronted by what can perhaps be best described as non-capitals; i.e., there are seemingly holes at the column tops that are open to the sky (Ill. 8.11), which reprises the memorable detail used by the practice in its earlier design for the sublime central gathering court at the Treptow Crematorium. (Ill. 8.12.) The necessary connections between the top of the columns and the roof structure are, in fact, still being made (or else the roof would not hold up!) but the connections are achieved by means of relatively thin horizontal cross beams and side bracketing attachments, with the remaining opening in the roof covered by an acrylic dome that lets light shine through. Rather quickly at the Bundekanzleramt, nature has further contributed to this unusual experience as Virginia creeper has climbed all the way up the 18m (59ft) tall columns and spread out underneath the ceiling; accented by the daylight from above, a potentially heavy-looking monumental structure has thus been given a touch of romantic lightness and elegance.

8.4 Leonard Euler and the Slender Column

The mathematical problem of describing and predicting how long, thin columns fail by buckling and at what load they will do so was solved by the Swiss mathematician Leonard Euler (1707–1783) and his work was published in two installments in 1744 and 1757. Euler's theory is still valid in its original form and it is perhaps the oldest in structural mechanics that remains in daily use today.

Bending stiffness is the crucial aspect of the column that prevents its excessive sideways deflection and the associated buckling failure, as we discussed in a general, descriptive manner in Section 8.2. We may think of such stiffness as being influenced by three parameters in a compressive element: its cross-sectional shape and dimensions, the elastic properties of its constituent material, and its length. The larger the cross-section and the stiffer the material, the more the element is able to resist its tendency to buckle, and hence the larger will be the load that it can carry before failing. We also understand that the greater the length of a compressive element, the less stiff it is (think, for example, of the behavior of different lengths of the plastic ruler or thin wood stick that we have mentioned before). Thus, while the first two parameters are proportional to the element's overall stiffness, the length is inversely so. Hence,

$$\text{stiffness} \propto EI/L$$

where E = the material's modulus of elasticity, with units of N/mm^2 (psi), I = the moment of inertia of the cross-section – thus indicating both its dimension and shape – in units of mm^4 (in^4), and L = the element length in mm (ft).[4]

Euler showed that the critical buckling load ($P_{cr.}$) of a long, thin, and ideally elastic column that has pinned ends is given by the formula

$$P_{cr.} = \pi^2 EI/L^2 \qquad (8.1)$$

where π is the numerical constant 3.1416.[5] (Fig. 8.4.) This equation is called the Euler buckling formula. We should note that as a consequence of the particular form of this equation (which has column length to the second power in the denominator), long and thin columns will be quite likely to fail by buckling out of alignment. On the other hand, material *strength* (as we have discussed earlier in Section 8.2) is *not* very likely to be of much importance in determining such elements' load-carrying capabilities. Indeed, according to Euler's theory, the load at which a column buckles depends on *material stiffness*, rather than strength. According to his equation, a long column's load-carrying capability can be improved by either choosing a material with a higher elastic modulus E, increasing its cross-sectional dimensions (and thereby affecting I), or by making it shorter. In an architectural context, it should be reinforced that all of these choices are obviously also design choices that directly affect the selection of column size, form, and proportions.

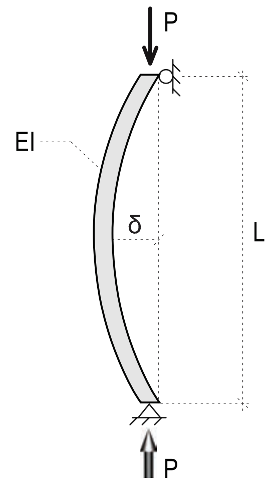


Figure 8.4
The deflection of an ideal elastic column with pinned ends, carrying a compressive load.

Up to this point, we have generally been discussing short and long columns without stating what this means precisely. How short is a short column and how long is a long one? To begin to be more specific, we should instead refer to *squat* or *stocky* columns and to *slender* columns, because what actually matters is not absolute length, but instead is the *comparison of length to cross-sectional dimensions*. Such a notion of relative slenderness obviously involves a ratio in some form, but it is not quite as straightforward as it may seem at first. If all columns were of the same cross-sectional shape, then we could compute the length-to-width ratio directly as a measure of a column's slenderness. But since columns come in many different cross-sectional configurations, such as round hollow tubes, square and rectangular solids, H-shapes, etc., their width alone is hardly a precise measure of their cross-sectional stiffness. This is why Euler needed to use the cross-sectional geometric property of moment of inertia I, which we encountered previously in Chapter 7 on beams, and which precisely deals with these differences of geometric shape and dimension. Furthermore, we can relate the two

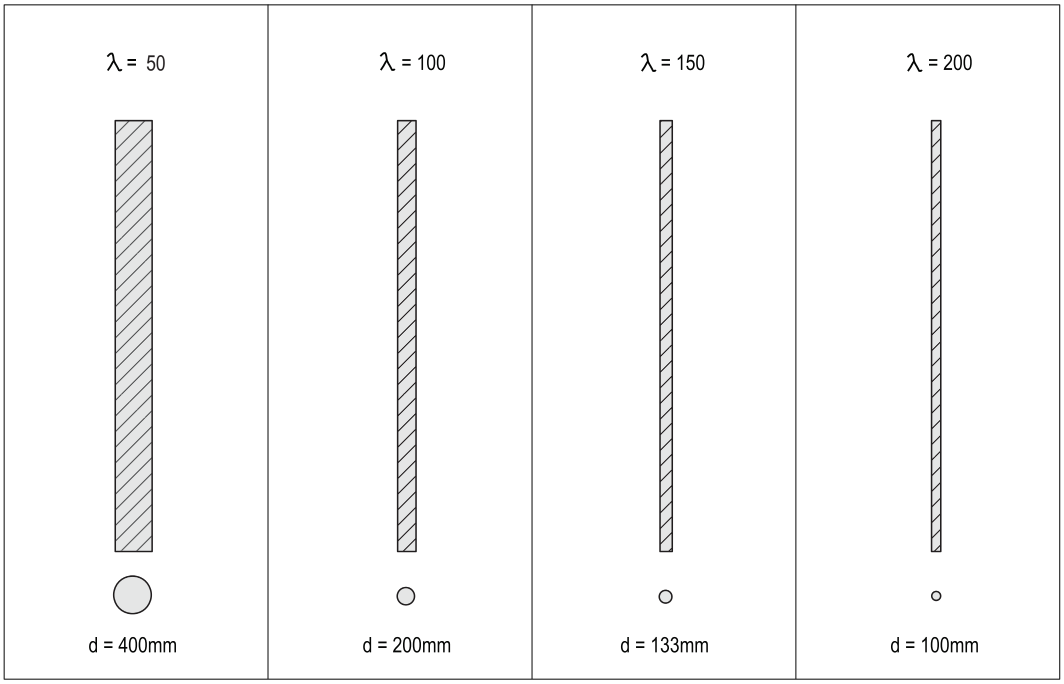


Figure 8.5
Visualizing column slenderness ratios. Physical length (L) of the columns being represented is 5000mm.

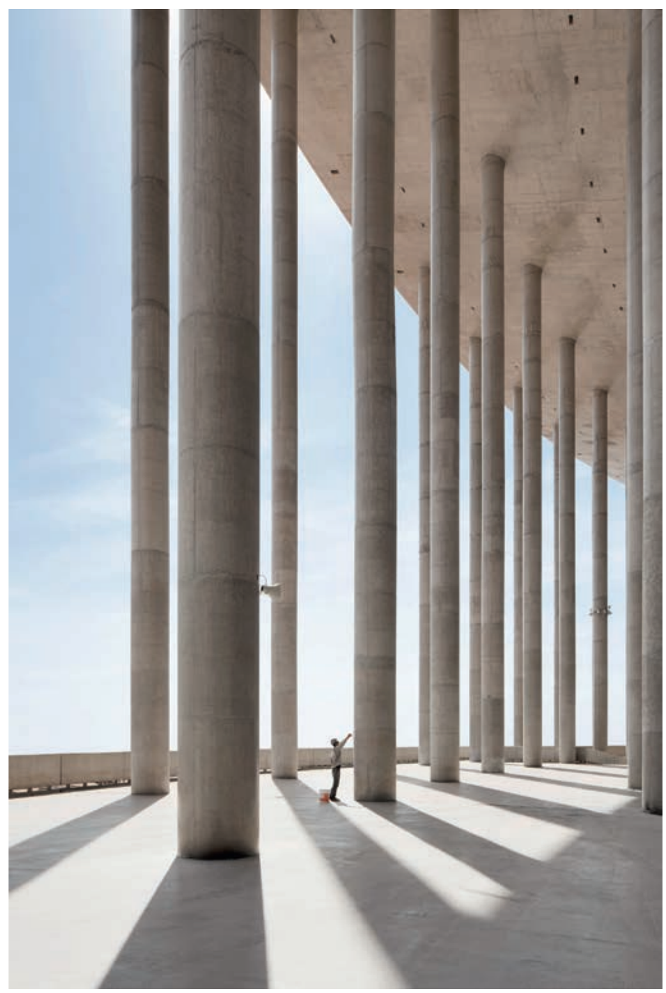


Illustration 8.13
Mané Garrincha National Stadium, Basilia, Brazil (2013). “Forest” of slender, circular concrete columns surrounds the perimeter of this stadium and supports a suspended trussed roof structure covering the playing field.

Architect: gmp Architecten (von Gerkan, Marg und Partner) and Castro Mello Arquitetos. Structural engineer: Schlaich Bergermann und Partner.

cross-sectional values of moment of inertia I and area A by means of another quantity called the cross-section’s *radius of gyration* r (mm, in) according to

$$r = \sqrt{I/A}$$

where r = radius of gyration of the cross-section in mm (in), I = moment of inertia, and A = area.[6]

With this radius of gyration quantity defined, we are now able to present a formula that measures slenderness; i.e., one that accounts not only for the column’s width or thickness, but also for the way that the material is distributed over the cross-section. Hence, all columns can be ascribed a *slenderness ratio* that compares the column length to the radius of gyration:

$$\lambda = L/r$$

where λ = slenderness ratio (Greek letter “lambda”), which is non-dimensional. (Fig. 8.5.) To give a sense of what such slender columns look like in reality, we can observe the examples of the circular concrete columns around the perimeter of the Brasilia National Stadium (Ill. 8.13) or of the steel corner column for the

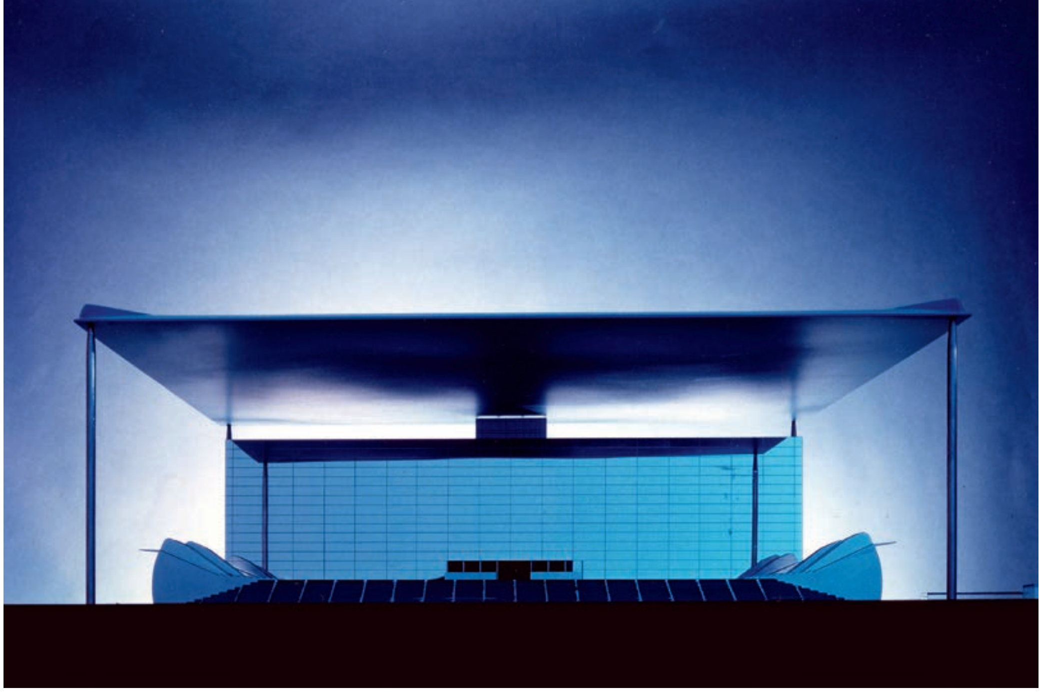


Illustration 8.14
French Pavilion, Expo '92, Seville, Spain (1992).
Slender columns are located at the four corners of a large roof plane covering the pavilion's open public square.
Architect: Francois Seigneur; Viguier and Jodry.

Illustration 8.15
French Pavilion, Expo '92.
Circular steel column tapers at its connection to the roof, accentuating the latter's "floating plane" qualities.

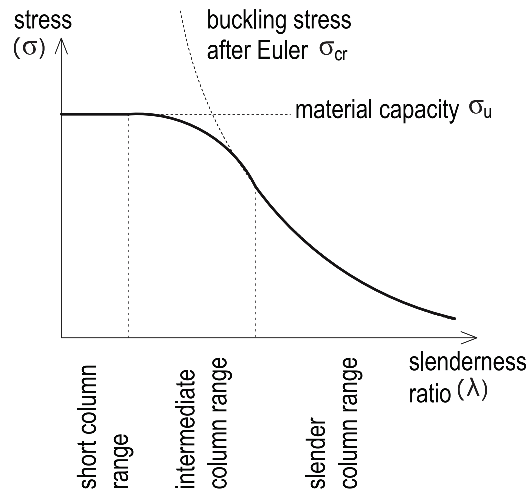


Figure 8.6
Diagram depicting the relationship of buckling stress to slenderness ratio. Also shown is the maximum stress level based on the material capacity. So-called short, intermediate, and slender column ranges are indicated.

French Pavilion at the Expo in Seville in 1992 (Ill. 8.14, 8.15.)

We may at this point go back and write the *Euler buckling load*, equation 8.1, in a different way, this time involving the slenderness ratio of a column. Since $\lambda^2 = L^2A/I$, we can find that

$$P_{cr.} = \pi^2 EA/\lambda^2 \quad (8.2)$$

The buckling load of an ideal elastic column is, therefore, inversely proportional to the square of the slenderness ratio, which in everyday language means that the more slender the column, the (dramatically) less is its ability to support load.

It is useful to take this formula's development a little bit further and to write the Euler equation in yet a different form, this time in a way that introduces the concept of the so-called *Euler buckling stress*. If we divide the buckling load by the area of the cross-section, we will have

$$\sigma_{cr.} = P_{cr.}/A = \pi^2 E/\lambda^2 = \pi^2 E/(L/r)^2 \quad (8.3)$$

where $\sigma_{cr.}$ = Euler or critical buckling stress in N/mm² (psi). This is the average stress on the cross-section at the very moment when the load reaches the critical buckling stage. As seems perfectly reasonable, the higher the slenderness ratio the lower will be the stress level at which the column buckles. Not so predictably, this reduction in column capacity will be proportional to the second power of the slenderness ratio, suggesting that there is a strong (structural) price to be paid for slenderness in columns (even though there may be other design reasons to be willing to pay for it).

This revised form of the Euler expression is very helpful in that it enables us to draw a graph depicting the relationship between the critical stress and the slenderness ratio. (Fig. 8.6.) The critical stress of columns, however, follows Euler's expression only when the columns are quite slender. For small slenderness ratios, the stress at which a column fails by crushing or yielding of the material will be less than Euler's expression indicates the column could be capable of; if this is the case, clearly material strength will establish the maximum load that the column can carry, regardless of its small slenderness. Figure 8.6 also reveals that between the clear-cut cases at either end of the slenderness range there is an intermediate or transition region where the capacity of a column to carry load is influenced by both the ultimate stress of the material (σ_u) *and* the buckling mechanism.

How a column changes from behaving as an "ideal" short column to a slender one will differ from one material to another, and this is something also influenced by the shape of the column's cross-section. Because of the complications of the transition zone, no precise information about this shift can be offered here.[7] It is both possible and instructive, however, to give some sense of the relative magnitudes of the slenderness ratios at the ends of the spectrum. For steel columns, for example, slenderness ratios below 20 may be thought of as being truly short, whereas those above 100 will indicate a slender column where Euler buckling can be expected to occur. Between 20 and 100 we will find steel columns to be of intermediate slenderness and that are influenced by both modes of failure.

To now give these relative numbers some context, we will consider the example of a tubular steel column whose cross-section has diameter d = 114.3mm and wall thickness t = 3.6mm. A list of standard steel cross-sectional properties can tell us that this column has an area A = 1250mm² and a moment of inertia I = 1.92 x 10⁶mm⁴. Therefore, $r = \sqrt{I/A}$ = 39.2mm. For this case, we will take the ultimate stress and the modulus of elasticity of the steel to be σ_u = 400N/mm² and E = 210 000N/mm², respectively. If in case (a) the column length is L_a = 500mm and in case (b) the length is L_b = 5000mm, we find that their slenderness ratios are $\lambda_a = L_a/r$ = 12.7 and $\lambda_b = L_b/r$ = 127, respectively. According to the criteria

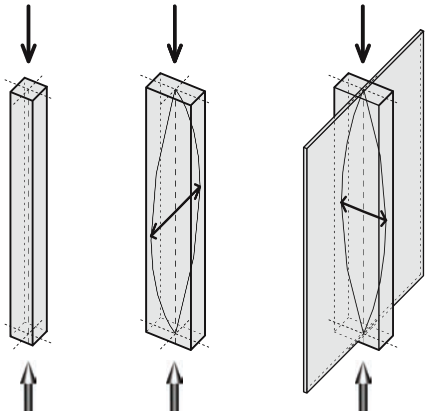


Figure 8.7
The buckling phenomenon as a spatial problem: in an unsymmetrical condition, we should take care to relate the slenderness ratio to the column's weakest axis. In practice, we will frequently find that columns are braced by walls or other building elements in the direction in which they are the least stiff. Typical cases for this are where the columns form part of external walls. To support the wind loading acting on the façade, the columns are most often asymmetrical, having their strongest direction oriented toward the wind load. Because external columns are commonly braced in the plane parallel to the external wall, their potential buckling caused by the vertical load can only take place in the direction of their strongest axis, resulting in a buckling load that is larger than would be possible otherwise. Moreover, columns in multistory buildings are commonly supported horizontally by floor structures at every story, effectively establishing the laterally unsupported column length as that of the story height.

given above, therefore, column (a) is definitely a short column while column (b) is a slender one. Their respective load capacities will be $P = \sigma_u A$ = 500 000N and $P_b = \pi^2 EA/\lambda_b^2$ = 160 000N. We can choose to express the ratio of the maximum loads that the columns can carry as $ß = P_b/P_a = 0.32$; i.e., we have found that the slenderness of column (b) has reduced the column's capacity to carry load to only about one-third of what its constituent material is capable of as a short column – which is obviously quite a significant reduction.[8]

Until this point, we have been considering the (very substantial) effects of slenderness for columns as though these structural elements exist only in two-dimensional space; but we also need to realize that the buckling of columns needs to be considered as a spatial problem. (Fig. 8.7.) What this is alluding to is the fact that a column in a real building may buckle in any direction in three-dimensional space. If a symmetrical-in-plan column is unbraced (that is, it is free to deflect sideways over its entire height) in any direction, then it will be just as likely to buckle with respect to one of its two orthogonal cross-sectional axes as the other. A tubular or square column that is pin-ended at both ends may thus just as easily buckle in one direction as in the other, which is effectively the condition that we have looked at so far. A rectangular cross-section or an H-profile, however, has different stiffness properties in the orthogonal directions and it will first buckle in its "weakest" direction (i.e., the direction with the highest slenderness ratio), thereby establishing the column's critical buckling load. When evaluating column capacities, therefore, we must obviously be careful to compute the Euler formula for the slenderness ratio related to the column axis having the lowest moment of inertia and radius of gyration.

But there is yet another factor to consider in establishing column buckling capacities. So far, we have been discussing slender columns with two *pinned* ends that allow both column extremities to rotate freely when the column tends to deflect sideways over the rest of its height. (e.g., Ill. 8.16.) This is both the simplest case to be considered as well as the worst, since it allows the column to potentially deflect sideways into its buckled shape without being restrained, bowing out along its entire length into the shape of a so-called sine curve. If we are able to restrain one or both ends from rotation (e.g., Ill. 8.17 and see Ill. 8.13), however, the ability of the column to deflect sideways will obviously be significantly reduced from this and the associated load at which the column buckles will correspondingly increase. In the restrained-end condition, only a portion of the column's length can in fact be seen to deflect in just the same way as our pin-ended "reference" column did, and this is generally known as the column's *effective length* L_e. This is the length of the column that establishes the buckling load for the column as a whole; obviously, with this length being less than the column's real length, the buckling load becomes larger, which is what we would expect from providing end restraint. We can, therefore, adapt the Euler formula to be applicable to all kinds of columns having different end conditions (Fig. 8.8.) if we use in it the effective length L_e instead of the real physical length L; i.e.,

$$P_{cr.} = \pi^2 EI/L_e^2$$

As can be seen, for the column with one fixed end we will have an effective length of 0.7L and the Euler buckling load becomes:

$$P_{cr'} = \pi^2 EI/L_e^2 = \pi^2 EI/(0.7L)^2 = \pi^2 EI/0.49L^2 = 2\pi^2 EI/L^2$$

From this expression we can observe that having one end fixed will effectively double (200 percent) the critical buckling load compared to a column with two pinned ends – which, when one thinks about it, is quite a substantial benefit: 100 percent more capacity is being provided to exactly the same column by fixing one end against rotation. For a column having both ends fixed we will find that the effective length L_e is 0.5L and the corresponding critical buckling load will be $P_{cr.} = 4\pi^2 EI/L^2$, which is four times (400 percent) the capacity of a pin-ended column of the same length. At the other extreme, for a column which has one end fixed and the other completely free to translate (move laterally) and rotate, similar to the condition of a flag pole, the effective length is actually 2L and the critical load is $0.25\pi^2 EI/L^2$. With no support at the top, therefore, the column will only be able to carry a mere quarter (25 percent) of the critical load of our reference column of the same size, shape, and length.

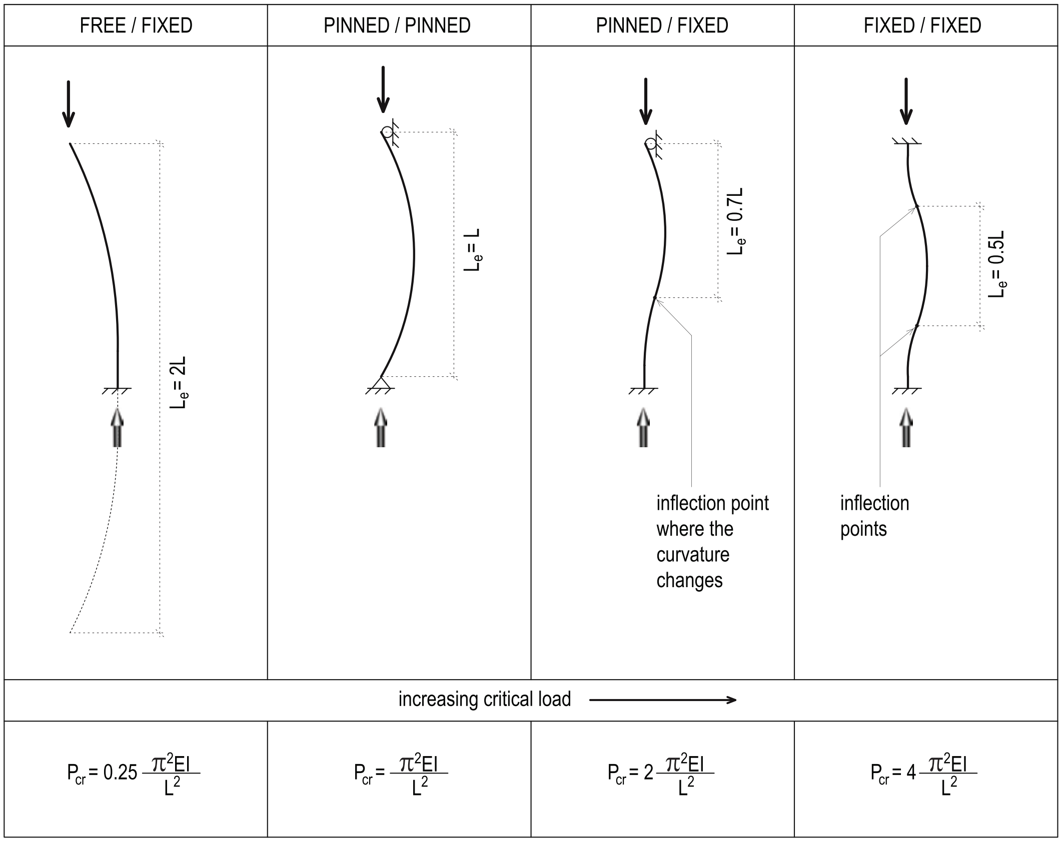


Figure 8.8
The effect of different column end conditions on the critical load.

Illustration 8.16
École d’Architecture de Lyon, Lyon, France (1987).
Example of a pinned-end column base. Rectangular wooden column is attached to a molded cast steel connector that tapers to a minimal dimension and a steel pin/dowel that allows for free rotation.
(See also Ill. 10.11, 10.12.)
Architect: Jourda et Perraudin. Structural engineer: Rice Francis Ritchie.

Illustration 8.17
Schiphol Airport Plaza, Amsterdam, Netherlands (1995).
Example of a fixed-end column base. Tubular steel columns are welded to larger diameter steel baseplate, which is then bolted all around to the base support, preventing any rotation from occurring.
Architect: Benthem Crouwel Architekten.

We may conclude from these examples that any attempt to restrain a slender column from moving laterally or from rotating at its ends will very favorably influence its critical buckling load capacity. We can consider the four cases that we have discussed here as general guides for the analysis of all columns. As columns in actual buildings may only be partially restrained, it may be necessary in reality to interpolate mathematically between the effective lengths of the example cases given here.

8.5 Mikado[9] – A Multitude of Columns

To extend this discussion about slender columns just a bit further, we will briefly consider here the case of buildings that have been designed and built using very many such elements – it fact, in some cases there may seem to be a veritable forest of thin columns, something that has become a noticeable trend over that past decade or so.

Obviously if one wants to lift a roof or occupiable volume up into the air and have it visually and/or conceptually disengaged from the ground, given our discussion about the various factors that affect column buckling there are different ways to go about meeting this objective. One would be to simply have a few, hefty columns (with long-spanning systems in between), whereas another would be to have more columns but that are of smaller cross-sectional dimensions by comparison. The latter approach has been taken to an extreme in the two projects that we will briefly look at here, one in the Netherlands and the other in Spain.

As in a forest, however, we see that at least in these two cases not all of these "trees" stand straight; some are tilted, creating a more "playful" sensation, or perhaps one that is more "natural" than having all purely vertical columns. (This need not always be so – for example, see the Koga Café in Ill. 10.14, 10.15.) Aesthetics aside, however, there are also some structural pros and cons to such tilting of columns: e.g., gravity loads that are being carried result in larger forces in the sloping columns that support them, and so there is a material price to be paid; also, sloping columns can contribute to the lateral stabilization of the building that they are supporting, a beneficial effect of "crooked" columns that may not seem so intuitively obvious at first. (These topics are further discussed in the following section as well as in Chapter 10 in Section 10.5.)

Dutch Pins

A scheme that can perhaps be described as a box sitting on a pincushion was the winning design concept of the architect Micha de Haas in a competition for the Aluminum Center of the Netherlands. (Ill. 8.18.) The Center, located near Utrecht, provides space for the meetings and conferences of people involved with the aluminum industry while also conveying information to the public at large about the wide range of creative possibilities for that material. Consistent with its purpose, the building is mainly built of aluminum and it represents an unusual degree of collaboration between industry and design, with some components being developed specifically for the project while others were specified from a catalog.

Contrary to the common contemporary focus on long span structures, in this project the architect explores the design implications of the opposite: very short spans resulting in a forest of aluminum columns. These number 368 in all and they have diameters that vary from 90 to 210mm (3.5 to 8.25in). While this may seem like many columns, it is a relatively modest number in comparison to earlier versions of the scheme in which up to 1200 were considered. Moving up into the building among these many columns (which are effectively multiplied in number by their reflections in the water at their base) makes for a distinctly vibrant and memorable experience.

Spanish Sticks

This project was one of the last works of the late Spanish architect Enrique Miralles (1949–2001). Situated high up in the rolling hills surrounding the Galician city of Vigo, the campus buildings were planned in connection with a huge landscaping and reforestation project; the elevated single-story constructions follow the slope of the terrain and serve to transform the site into a built landscape.

A long series of small auditoria are supported by a multitude of columns: at the front of the buildings, concrete columns are cast in inverted V-shapes while farther back clusters of tubular steel columns reach up to support the building. (Ill. 8.19.) From a purely mechanical point of view, the use of many slender columns requires more material than having only a few, thicker columns. This is because each slender column has to be sufficiently thick to prevent its buckling, whereas thicker columns that carry larger loads

Illustration 8.18
Aluminum Center, Houten, the Netherlands (2001).
Intended to show the versatility of aluminum, the building is supported on a "forest" of slender aluminum columns.
Architect: Micha de Haas.

Illustration 8.19
Vigo University Campus, Vigo, Spain (2002).
A series of auditoria is carried by a multitude of columns.
Architect: EMBT Miralles Tagliabue.
Structural engineer: Malvar-OHL; Josep Massachs.

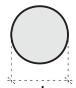
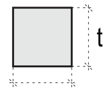
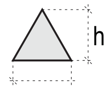
$P_{cr} = \frac{\pi^2 EI}{L^2}$	 d	 t (t)	 b (h)
A = constant	d	t = 0.886d	h = 1.166d b = 1.346d
relative critical load	1.0	1.04	1.20

Figure 8.9
A comparison of solid square, circular, and triangular compressive elements having the same cross-sectional area. Their relative load-bearing capacity (critical load) is given and these show that, theoretically, among these three variations a triangular cross-section makes the best use of the material with an increased capacity of 20 percent compared to the circular column.

are less slender a priori and hence tend to make more effective use of their material. Nevertheless, this odd and fascinating column forest is an essential element in the overall architectural language of the project, just as are the unique arrangement of buildings, their rugged material finishing, and the details of stair railings and sunshades. As a whole these are evidence of the creativity of a remarkable architect. Moreover, there is a tremendous view from the grounds of the campus through this man-made forest of columns out toward the open Galician landscape.

8.6 The Shape of Compressive Elements

Even if we may instinctively identify columns as being the essence of structures in compression, in fact structural elements that are subject to this type of loading condition are much more pervasive. For example, one must also include in this category struts, certain members of trusses, selected diagonal members of lateral bracing systems, arch segments, etc.; i.e., the list quickly begins to get quite long. And even beams, which will always have one of their flanges in compression as a result of their fundamental bending behavior, must be considered to have parts that are a member of the club. The chapters that follow this one and that deal with such things as trusses and shear walls and arches will make clear that in fact all slender compression members may risk failure by buckling – no matter what they are called. So although we will mostly restrict the discussion for now to being about how to strategically shape the more familiar and traditional column compression element, we must always keep in mind that the lessons learned here are equally applicable in many other contexts.

As we have seen and suggested in the preceding sections, there are certain design strategies for avoiding an overall stability failure, also known as the global buckling failure, of a slender, unbraced column. These techniques engage form in two respects: (i) by means of their cross-sectional shape and (ii) according to their elevational profile. If we go back to considering the most basic of slender columns with two pinned ends, we may ask what shape of its cross-section will offer the largest stiffness and result in the column having the highest load-bearing capacity.

Looking at prismatic columns, that is, at columns having the same geometry at every cross-section along their length, we can start by comparing simple solid columns of circular, square, and triangular shapes. Based on the Euler formula, $P_{cr} = \pi^2 EI/L^2$, if we consider the material stiffness E and the length L to be the same for all three columns, then the ratio of their critical buckling loads will necessarily be the same as the ratio of their cross-sectional moments of inertia I. It is possible to calculate this ratio if we take as a precondition that they all are to have exactly the same cross-sectional area A. Contrary to what we may have expected, theoretical values show that the column with a triangular cross-section offers about 20 percent higher critical load capacity than the circular column, while the latter is almost identical to the one with the square cross-section. (Fig. 8.9.) It is clear that some of the most common shapes of columns in architecture are chosen for reasons other than extreme material efficiency.

When dealing with buckling, which is a stability problem that ultimately involves a column's bending, and with the Euler formula incorporating the associated cross-sectional property of moment of inertia I, it is clearly going to be beneficial in terms of a column's load capacity to distribute its material as far away as possible from the central axis, thereby increasing I. This means that hollow columns make better use of their material than do ones with solid cross-sections (assuming that they have the same cross-sectional area), quite simply because the moment of inertia will be larger for the hollow columns, and therefore the slenderness ratio will be less for the same amount of material. To demonstrate this, we can look at two cross-sections having equal areas: A_1 from a 60 x 60mm solid, and A_2 shaped as a 100 x 100mm hollow with a wall thickness of t = 10mm. Both cross-sectional areas are $A_1 = A_2 =$ 3600 mm^2. Their moments of inertia, however, can be determined to be $I_1 = (1/12)(60)^4$mm^4 = 1 080 000mm^4 and $I_2 = (1/12)(100^4 - 80^4)$mm^4 = 4 920 000 mm^4, respectively. This yields a ratio for the columns' critical loads of $P_{cr,2}/P_{cr,1} = I_2/I_1 = 4.92/1.08 = 4.5$. In other words, the slender hollow column with exactly the same mass and weight can theoretically carry 4.5 times as much as the slender solid column before buckling. Clearly there is very distinct advantage to this cross-section-shaping strategy of moving material away from the central axis, and this is a lesson that is regularly applied to compression elements in various ways. (e.g., Ill. 8.20.)


a)


b)


c)


d)

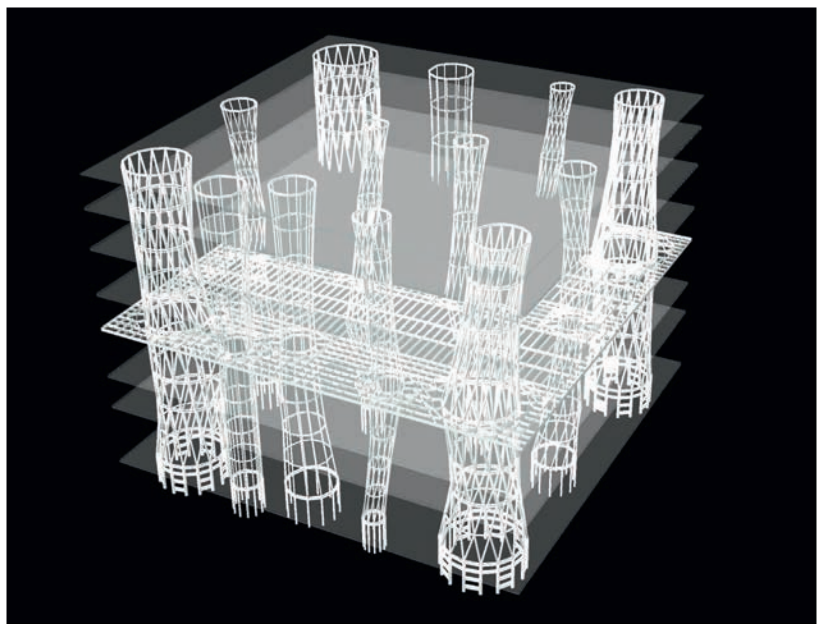

e)

Illustration 8.20
Strategies for increasing the critical load of slender columns.
(a) Maison Louis Carré (1959).
Central concrete column with four timber stiffeners at its sides. (See Ill. 8.21 for detail.)

Architect: Alvar Aalto.

(b) National Opera and Ballet, Oslo, Norway (2008).
Concrete columns with a marked thickening at mid-height.

Architect: Snøhetta. Structural engineer: Reinertsen Engineering ANS.

(c) Neue Nationalgalerie, Berlin, Germany (1968).
Flanged-section rolled steel column with two additional transverse T-sections welded to its web – yielding a cruciform-shaped flanged section. (See Ill. 8.25 for detail.)

Architect: Mies van der Rohe.

(d) The Renault Distribution Center, Swindon, UK (1983).
Central steel column with tension-rod-supported lateral bracing at mid-height. (See also Ill. 6.38.)

Architect: Foster + Partners. Structural engineer: Arup.

(e) Sendaï Mediatheque, Sendaï, Japan (2001).
Large tubular columns made of many smaller steel tubes; i.e., the hollow column at a larger scale. (See also Ill. 7.49.)

Architect Toyo Ito. Structural engineer: Mutsuro Sasaki.

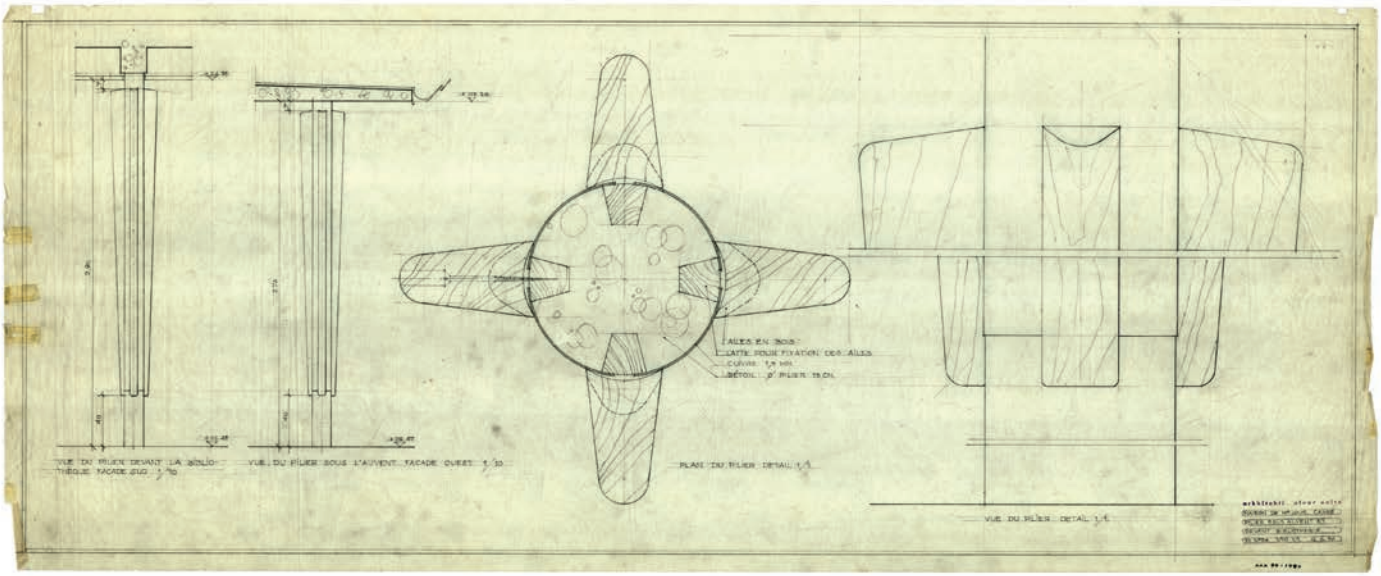


Illustration 8.21
Maison Louis Carré (1959).
Drawing of overall shape and cross-section of timber-stiffened column. (See also Ill. 8.20a.)
Architect: Alvar Aalto

Moving on to considering the effect of column elevational shape or profile, it must be acknowledged that a recurring, if occasional, design strategy for the column over the course of architectural history is making columns that thicken (or get “fatter”) between their ends. (e.g., Ill. 8.20b.) This is certainly a well-known feature, called entasis, of the columns of ancient Greek temples. (See previous Ill. 8.4.) More recently, the architect/engineer Santiago Calatrava and many others have used it extensively and expressively, with the column element’s profile becoming quite obviously “cigar” or “lens” shaped. Certainly if buckling occurs, such a profile does indeed provide a stiffer cross-section over that portion of the compression element that experiences the largest sideways deflection. The operative principle here, once again, would seem to be to distribute the material of the column in such a way that it is of the most benefit, this time by moving it away from the ends of the column toward its mid-height. While this may sound like quite an advantageous strategy, however, in reality relatively subtly varying the dimensions of a column’s cross-section in this particular manner offers only a modest improvement for resisting buckling. In the end, then, even if there is some structural merit to it, this particular shaping of column form may have more to do with optics after all; perhaps its bulging profile is so strongly suggestive of the internal compression forces that designers find visual reasons for using it.

The case of very slender columns having individualized bracing systems would seem to be of related interest here given the likeness of visual profile. But unlike the preceding case of a “simple” fattening at the middle, a column that is laterally supported at mid-height by a system of struts with pre-stressed rods or wires connected to their ends actually proves to be very materially efficient and a highly effective way to improve a column’s capacity. (e.g. Ill. 8.20d.) In this case, the additional material that provides the extra stability works by means of pure axial tension forces, which we know to be highly efficient in terms of material use and stiffness. Furthermore, if such lateral bracing results in a reduction of the central column’s effective length to one half of the original, then we have seen earlier in this chapter that the necessary geometric stiffness, or moment of inertia I, of the column cross-section can be reduced to one-quarter of what it otherwise would need to be. The obvious consequence will be a much smaller cross-section and significant weight reduction for this built-up column system. It must be noted, however, that the added weight of struts and tensile rods will partially offset this material saving; also, the cost of the manufacturing and construction of this kits-of-parts column will not be insignificant. Nevertheless, individually braced columns of this sort are wonderfully expressive and light, and they have been used frequently over the past few decades.

Finally in this context of overall column-shaping strategies, it can be observed with interest and curiosity that as we saw in the preceding section there is a recent tendency in certain projects to employ tilted or inclined columns. (e.g., Ill. 8.22.) We may wonder whether additional forces are produced by not letting vertical loads be supported by vertical structural elements, and, beyond that,

Illustration 8.22
Peckham Library, London, UK (2000).
When columns are tilted, horizontal forces will occur at both ends which need to be counteracted by the building's bracing system as well as the foundations. Also, gravity produces some small bending moments in the column which must be considered. On the other hand, there is a potential lateral stability benefit from tilted columns.
Architect: Will Alsop. Structural engineer: Adams Kara Taylor.

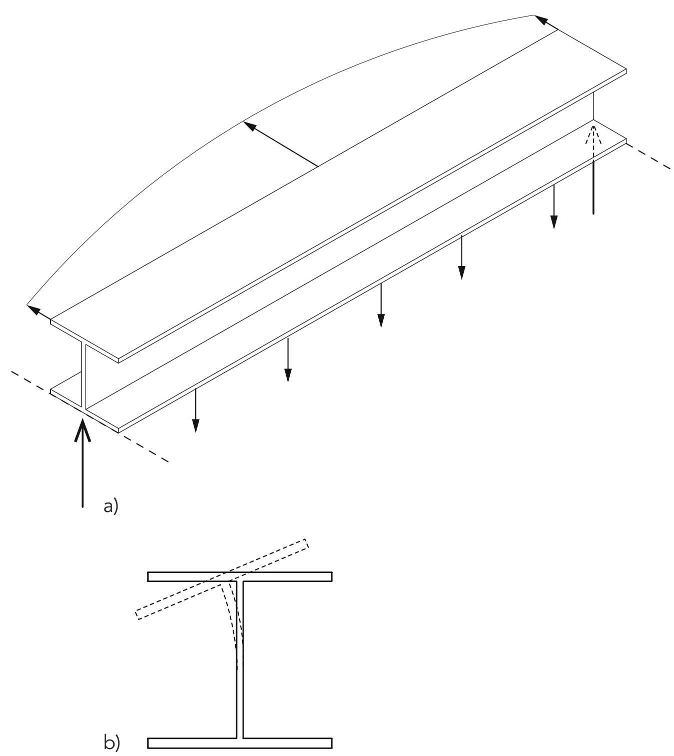


Figure 8.10
Potential lateral-torsional buckling failure caused by compression bending stresses produced as a beam flexes to carry transverse loading.

whether there are any negative effects of this inclination on the column's load-carrying capacity. Since a vertical load sets up an axial compressive force in the column having the same angle of inclination as the column itself, horizontal forces will be necessary at both ends of the tilted column in order to maintain equilibrium. Such lateral forces must be countered by the building's bracing system and by the capacity of the foundations to withstand horizontal forces. Moreover, as was stated in Section 8.5, the inclination of the column will cause the axial force in the column to be larger than that of the vertical force being carried (recall components of forces from Chapter 4). And beyond that still, the weight of the column will no longer follow the column axis, but will instead produce some bending moments in it. Although the latter are typically not very large, they will nevertheless cause bending stresses in the column that will be additive to its usual set of compressive stresses; in principle, this effect will reduce tilted columns' load-bearing capacity. But all is not bad: one advantage of tilted columns is their potential to contribute to the lateral stability of the whole building in resisting wind or earthquake lateral loads; this is a topic that will be taken up again in Chapter 10 (see Section 10.5.) Moreover, the examples that we just saw in the previous section of the Aluminum Centre and at the Vigo University Campus also clearly illustrate this tilted-column strategy at work.

Before concluding this section, we need to consider that buckling can occur in the smaller parts of larger overall structures, a phenomenon that is called *local buckling*. Such buckling is typically addressed by one of two strategies: either designing the local compression part of the structure to have sufficient thickness to greatly reduce its local slenderness ratio, or else supplying it with some form of local lateral bracing.

For example, the portion of a beam that is in compression due to bending stresses is liable to having a sideways buckling failure, also known as the so-called *lateral-torsional* mode of buckling or *warping* of the beam. (See Fig. 8.10.) To counter this effect, the compression flange in a beam is often effectively braced laterally by being regularly connected to something like a secondary, transverse joist system, or to timber floor sheathing, or to a metal sheeting/concrete slab system, etc. – any of which may serve to arrest the tendency toward sideways displacement for that part of the beam that is in compression. It should be noted that the same buckling problem and its remedying strategies also apply to the compression parts of trusses which, as we will see in the next chapter, act quite like beams do in many respects (see Fig. 9.17).

If, for some reason, the plane of such a potentially stabilizing surface is not well positioned with respect to the primary beams, the compression part of the latter may be unbraced, and an alternative strategy will need to be employed. At Crown Hall at IIT in Chicago by the architect Mies van der Rohe, for example, the top compression flange is made wider in steps along the span, with the widest at mid-span, which helps to reduce the (local) slenderness ratio of the flange with regard to sideways instability. (This project is illustrated and further discussed in Chapter 10; see Ill. 10.44, 10.45.) Also to

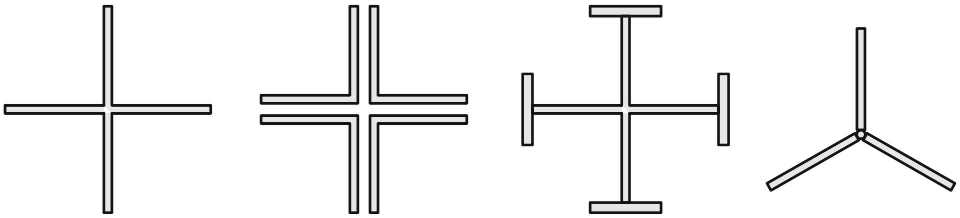


Figure 8.11
Columns with cruciform cross-sections.

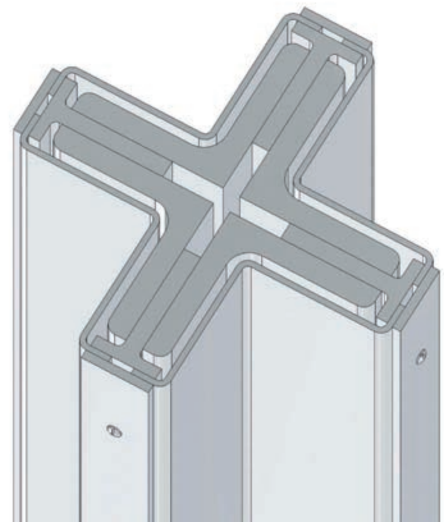


Illustration 8.23
German Pavilion, International Exhibition, Barcelona, Spain (1929).
Cruciform, chromium-clad steel column.
Architect: Ludwig Mies van der Rohe.

be noted through this example: with large steel plate girders we will also commonly find local stiffeners every so often along the beam web whose function it is to prevent the relatively thin sheet of steel from buckling locally as it transfers large compressive forces.

We will conclude this section by examining more closely one last example of local buckling: that which can happen in the unbraced slender parts of columns themselves, especially if the column has a cruciform shape or is designed with other thin, protruding flanges. (Fig. 8.11.) Historically, the cruciform column was born in the era of the wrought iron- and steel-engineered structures of the nineteenth century, but it has reappeared periodically in architectural projects made of steel, reinforced concrete, and wood alike. Because of the column side-protrusions characteristic of this shape, special care must be taken to reduce the risk of these thin extensions buckling locally, perhaps by making them thicker in order to reduce their local slenderness. In larger cross-sections with heavy loads, supplemental stiffeners in the form of transverse plates at the tips of the flanges may be a wise choice in order to avoid local buckling problems.

8.7 The Masters' Cruciform Columns

Columns of cruciform cross-section were highly regarded by some of the pioneers of Modern architecture, not least for their suitability for expressing a freedom of floor plan and spatial organization; i.e., with a cruciform column it can be said that the column remains neutral in its orientation, with no accenting of one direction over the other. The thin and sharp edges of such a column made of steel were particularly admired and explored.

The cruciform columns of Mies van der Rohe's (1886–1969) Barcelona Pavilion, built in 1929, are made of four equal steel angle-sections placed back to back, with the whole ensemble covered by highly polished sheet steel. (Ill. 8.23.) The columns' shiny exterior surface is in keeping with the pavilion's other materials: polished honey-yellow onyx, green Tinos marble, and the many reflecting glass surfaces. An observant visitor to the carefully rebuilt pavilion (1986) will notice one change from the original, however: the chromed sheet steel of 1929 has been changed to polished stainless steel.

A year later, Mies designed the Tugendhat House in Brno in then Czechoslovakia. Here, just as he did in Barcelona, Mies demonstrates his ideas about the free plan and clarity of construction: the house is especially known for its open and sober lounge enclosed with three glass façades. (See Ill. 2.9b.) As in the Barcelona Pavilion, the flat roof is carried by cruciform steel columns; here, however, the polished sheathing is more rounded. (Ill. 8.24.) Perhaps this reflects the curved forms found elsewhere in the house, such as that of the glass wall that surrounds the staircase from the entrance terrace and the semicircular screen wall in the lounge.

Mies' Neue Nationalgalerie in Berlin (1968) is defined by one large open space surrounded by glass walls. Here it is covered by a square roof made up of a beam-grid structure of welded steel plates. (See Ill. 7.39.) This "floating" roof is carried by eight cross-shaped columns that are placed along the perimeter, two on each side, leaving the corners column-free. By this time the chromium-clad supports from Barcelona and Tugendhat have been replaced by four welded T-sections shaped into a modified cruciform configuration. (Ill. 8.25.) As opposed to the earlier simpler versions with flat extensions, the T-shape here provides transverse local bracing to the protruding flanges in order to increase the columns' stiffness and load-bearing capacity; this is a much heavier roof, after all. Cantilevered from the concrete base, the columns are supplied

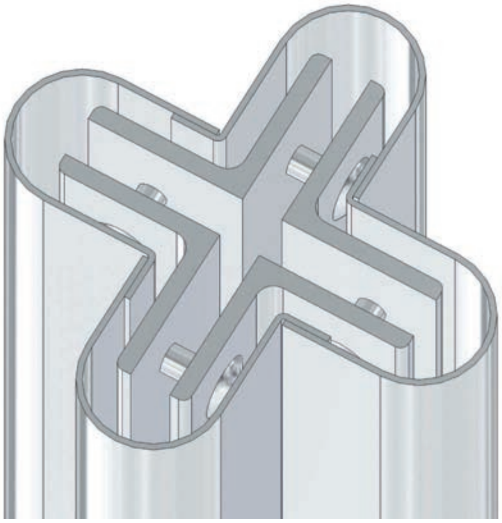


Illustration 8.24
Tugendhat House, Brno, Czech Republic (1930).
Cruciform, chromium-clad steel column.
Architect: Ludwig Mies van der Rohe.

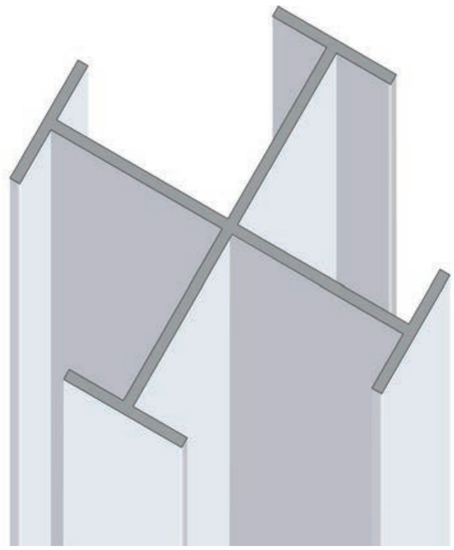


Illustration 8.25
Neue Nationalgalerie, Berlin, Germany (1968).
Cruciform column with flanged ends providing stiffening against local buckling – as well as increasing the column's moment of inertia to counter overall buckling of the column from top to bottom. (See also Ill. 8.20c.)
Architect: Ludwig Mies van der Rohe.

with hinged column heads; a keen eye will also notice that the column is slightly tapered on its way up to the roof.

Le Corbusier's (1887–1965) material of choice was concrete, with its rich plastic potential for expression. However, in his very last work, the Centre Le Corbusier in Zurich (1967), which was inaugurated after his death, he used steel as a building material. A steel-plate canopy covers a light pavilion that is based on cubic volume units measuring 226 x 226 x 226cm (7.5 x 7.5 x 7.5ft) and constructed of steel elements with cruciform cross-sections. (Ill. 8.26.) In contrast to Mies' free-standing columns, Le Corbusier's cruciform sections are employed both vertically and horizontally to create a three-dimensional orthogonal open grid that defines the pavilion's volume and frames the façade elements of glass and enameled steel panels. There is no sheathing of the cruciform elements here; instead they remain completely "honest" and visible, as are the bolts that hold together their four equal steel angle components.

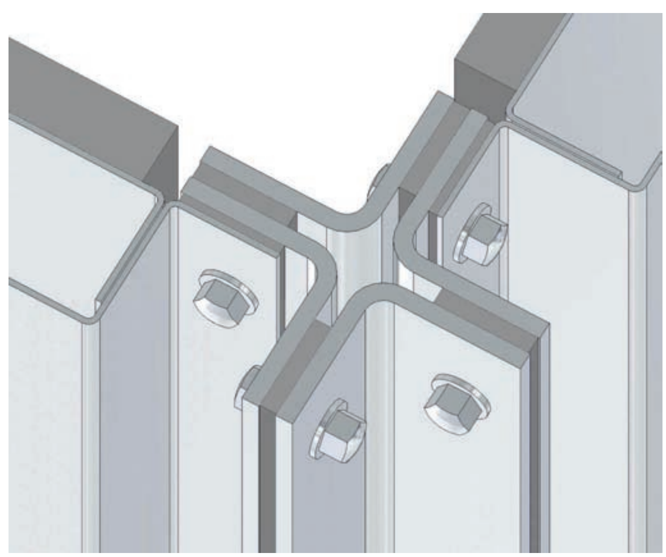


Illustration 8.26
Centre Le Corbusier, Zurich, Switzerland (1967).
Contrary to the free-standing columns of Mies' projects, Le Corbusier's columns here are in line with the pavilion's façade.
Architect: Le Corbusier.

Illustration 8.27
Catalina House, Tucson, AZ, USA (1998).
Load-bearing walls for this house are made of rammed earth. Tapering of exterior wall thickness is evident at large window opening at right. Elsewhere, substantial thickness of wall allows for strategic recesses to be made, and surface texture is provided by means of the formwork used and by its sequential layering construction method.
Architect: Rick Joy Architects.
Structural engineer: Southwest Structural Engineers, Inc.

8.8 The Wall

We may think of the wall as an extrapolation of the column that we have been looking at so far in this chapter; that is, by extending the latter's cross-section along one axis in space, an essentially linear structural element effectively becomes a planar one. Moreover, given their typical vertical orientation and support at the foundation level, structural walls usually serve an essential gravity load-carrying function in a building, with the resulting compression stresses anticipated to be present throughout, increasing in magnitude toward their base as dead and live loads accumulate. Such walls are known as *gravity load-bearing walls* (or simply as *bearing walls* for short) and both material capacity and buckling concerns will typically need to be addressed, just as they were for columns.

Because of their planar aspect, however, it must be pointed out that in addition to carrying gravity loads walls may also be called upon to resist other types of loading. For example, a wall offers great capacity *in the direction of its plane* for resisting lateral loads on a building, and when it functions in this manner it becomes known as a *shear wall*. Shear walls act as stabilizing elements in many buildings of all scales; these were introduced in Chapter 2 and will be discussed more extensively in Chapter 10. Another possibility, especially if a wall is located along the exterior face of a building and/or is situated below ground level and holds back the earth, is that it may be subjected to lateral loads that are *perpendicular to the wall surface*, such as may be caused by wind or earth pressures; in the latter case, these are given the specific name of *retaining walls*. The structural behavior of these particular types of walls will also be discussed, but in the subsections that follow.

Gravity Load-bearing Walls

The wall, when taken as a planar compression element supporting vertical loads, will in many respects behave similarly to the column. Just as for the column, therefore, we will need to distinguish between structural elements whose proportions are short and stocky, in which case material capacity governs, versus those that are tall and thin, for which buckling may also become of significant concern.

The compressive material capacity of walls is an obvious constraint. In principle, for relatively short, thick walls it is a simple matter of dividing the load needing to be carried per linear meter or foot of wall by the material capacity in order to establish the corresponding required wall area and thus its required thickness; i.e.:

$$A_{req'd} = P / \sigma_u$$

and

$$t_{req'd} = A_{req'd} / \text{unit length}$$

Frequently used structural materials historically such as stone, brick, rammed earth, and timber (these are still commonly used today in various contexts) have a relatively limited capacity in compression compared to other higher strength materials and this led/leads to rather thick walls in such buildings being needed simply to carry dead loads – including, typically, quite high self-weights – as well as any roof and occupancy live loads, as the case may be. (e.g., Ill. 8.27.)

A further spatial consequence of using low-capacity materials is that load-bearing wall thicknesses can increase rather quickly as gravity loads accumulate – as is quite dramatically evidenced

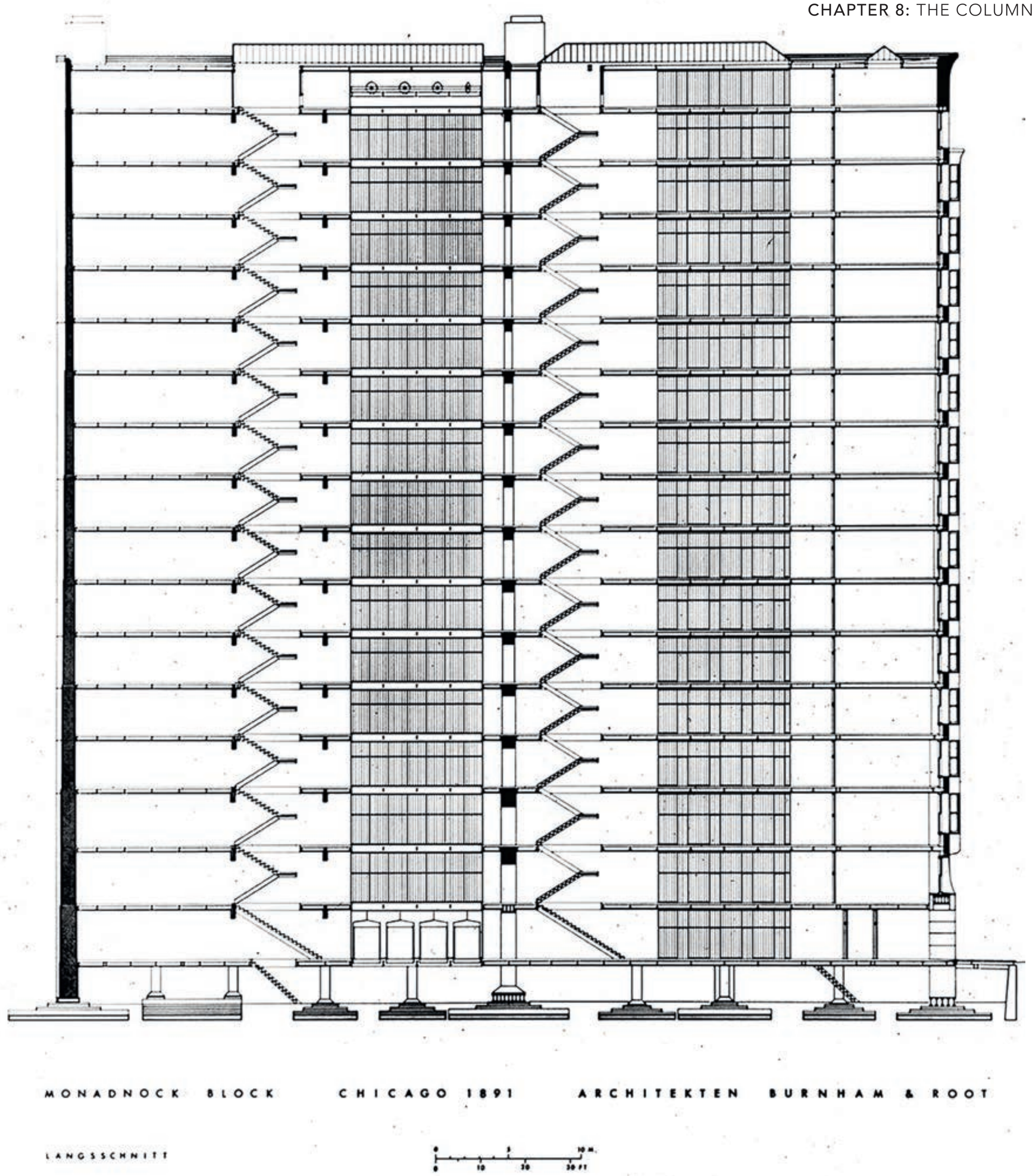


Illustration 8.28
Monadnoch Building, Chicago, IL, USA (1892).
Masonry wall thicknesses increase dramatically over the height of this 16-story building in order to cope with increasing compression loads. Wall thickness at top is 300mm (1ft), at base is 2m (6ft).
Architect: Burnham & Root.

in the late-1800s Monadnoch Building where the full-perimeter brick walls of this 16-story structure start at 300mm (1ft) thick at the top but progressively thicken to 2m (6ft) at the base. (Ill. 8.28.) By contrast, certain contemporary high-strength wall materials such as reinforced concrete (having material compressive strength that is perhaps 10 to 20 times or more that of masonry) allows wall extents and thicknesses to be greatly reduced while still maintaining stresses within acceptable limits. We saw this, for example, in the walls of the Bregenz Kunsthal in Chapter 2 (Ill. 2.5, 2.6) and will discuss this topic once again in Chapter 10, when shear walls are considered more fully.

But aside from a load-bearing wall's material capacity and consequent length and thickness, since it is a compressive structure we also need to be concerned with the potential for failure caused by buckling, just as we saw for the column. In the case of a wall, however, the weak direction in this respect will obviously be in the direction perpendicular to the wall surface, since in its own plane it can be considered to be fully braced against buckling. In fact,

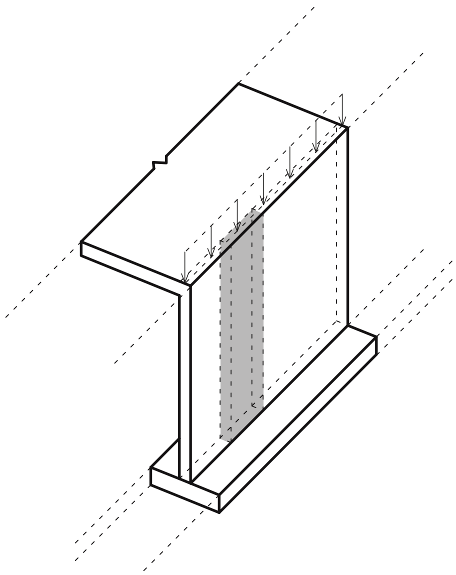


Figure 8.12
"Column-like" unit length of gravity load-bearing wall that can be used to assess wall capacity. This segment can be treated as fully braced in the plane of the wall.

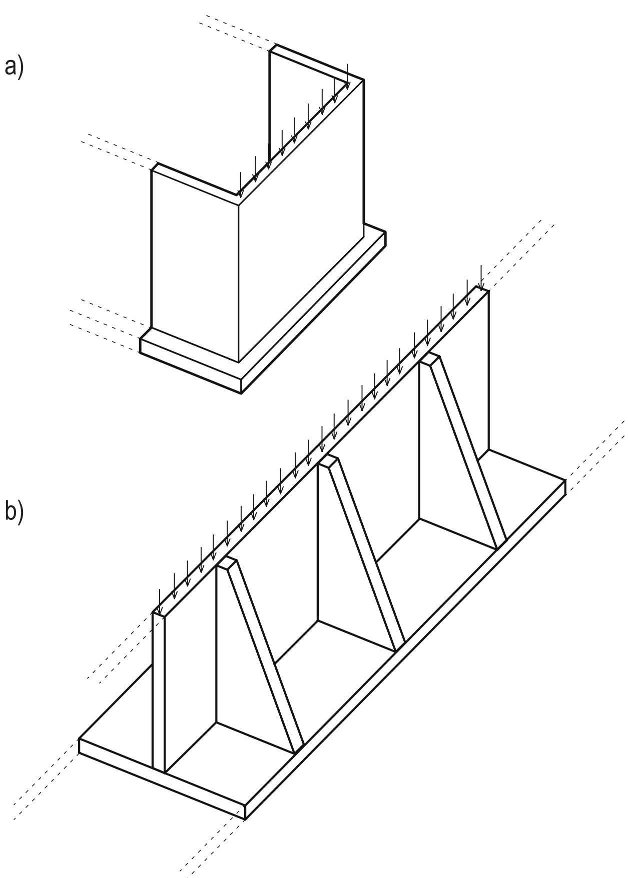


Figure 8.13
Gravity load-bearing wall that is braced against out-of-plane buckling by (a) end transverse walls or (b) intermittent buttresses.

it is quite common to conceptualize the behavior of a long planar wall as that of a series of side-by-side columns having rectangular cross-section of unit length (1m or 1ft) and whose width is the wall thickness. (Fig. 8.12.) Calculating the critical buckling load for such a representative "column" segment, in just the same way as we did for the column, will tell us what the wall's load-bearing capacity is per unit length. Then, just as for the isolated column element, it will be a matter of comparing the wall's capacity as established by material constraints to that derived from buckling considerations in order to determine which one controls.

In all this, we have been assuming that the wall is sufficiently long in plan that its only transverse lateral support is considered to be provided at its top and bottom; i.e., at the foundation and/or floor levels, as the case may be. Other relative proportions and lateral support conditions may arise, however: for example, if we have a situation where the wall is braced by transverse vertical walls and it is supported horizontally at the foundation level (Fig. 8.13, e.g., Ill. 8.29), the deflected shape of the wall will look very different and buckling failure will occur at a higher load level according to the length-to-height ratio of the wall. And yet another approach that has a similar effect of strengthening a thin wall surface against buckling is to fold or curve the surface out of its plane (e.g., Ill. 8.30), which at a conceptual level we can liken to what we did with the column by moving material away from its central axis, thereby increasing its moment of inertia and thus its buckling capacity according to the Euler formula. The detailed theory for the buckling of such wall panels, however, is well beyond the scope of this book.

Finally, if we examine an external load-bearing wall in a multistory building, we may find in certain cases that we are in fact dealing with a wall that supports both vertical loads and wind-induced horizontal wind loads. (Fig. 8.14., e.g. Ill. 8.31.) It is likely that such a wall will be propped sideways by the floor slabs and the wall will then span vertically as a surface panel between them, with the horizontal wind loads producing bending moments M in the wall, and corresponding sets of bending stresses σ_b developing within it in the vertical direction. The theory for this is the same as for beams and slabs, discussed in Chapter 7. If this wall also supports a vertical load P, however, the associated compressive stresses σ_c will be additive to the bending stresses, and there will be further demand on the wall material. One way of considering such a combined loading case is to recognize that the wall needs to be restricted in either carrying vertical loads P, or horizontal

Illustration 8.29
The House of Spiritual Retreat, near Seville, Spain (designed about 1979, built 2005).
Two large, white-stuccoed concrete walls mutually brace one another. Open stairs lead to a "mirador", or lookout, over the Andalucian landscape. Living spaces are underground, opening onto a recessed terrace at the base of the walls.

Architect: Emilio Ambasz and Associates.

Illustration 8.30
Atlántida Church, Atlántida, Uruguay (1952).
Curved walls provide much increased lateral stiffness, as well as increased buckling resistance for vertical loads. (See also Ill. 13.41, 13.42.)

Architect and structural engineer: Eladio Dieste.

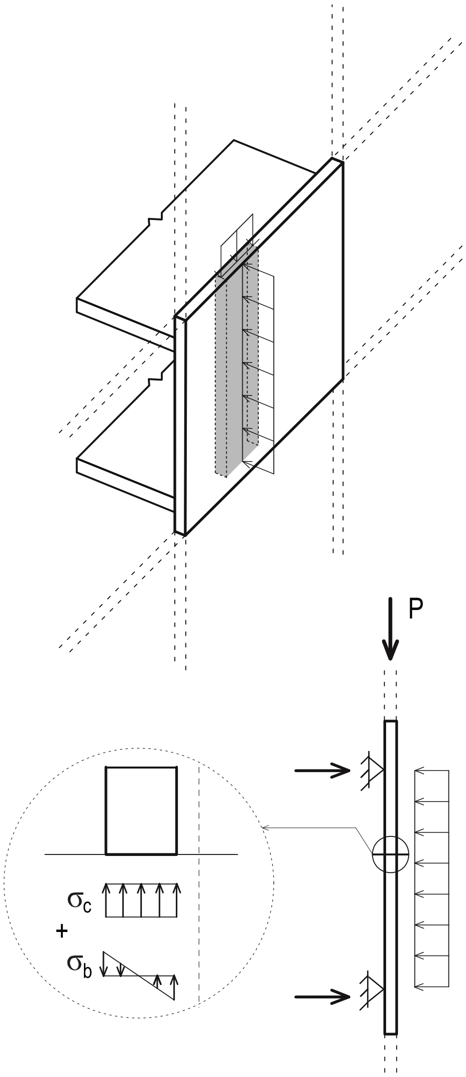


Figure 8.14
Wall with both vertical and horizontal loads.
Resulting stress distributions shown.

Illustration 8.31
Zollverein School of Management and Design, Essen, Germany (2006).
Load-bearing concrete façade walls with openings.
Architect: SANAA. Structural engineer: Bollinger + Grohmann.

loads, or both together. Thus, if a wall of this type is exposed to large horizontal loads, its vertical load-bearing capacity will be greatly diminished, and vice versa. Taken together, the capacity of the wall can be considered as

$$M/M_u + P/P_{cr.} \leq 1$$

where M_u = bending moment capacity of the wall's cross-section and $P_{cr.}$ is the critical load of the wall as a compression element, both taken separately as if not being influenced by the other. Moreover, the sideways deflection caused by the horizontal load is highly detrimental to a slender wall's buckling behavior, with a consequent reduction of its capacity to support vertical loads. (Again, the theory for this is well beyond the scope of this book.)

It should also be mentioned that this type of external gravity load-bearing wall, while certainly not completely relegated to the past (as can be seen with the example of the Zollverein School in Ill. 8.31), was in fact much more common previously than it is today (e.g., Monadnoch Building, Ill. 8.28). Exterior wall construction on many contemporary multistory buildings work essentially only as a "curtain" that resists the lateral wind pressures as an independent vertical panelized system spanning from one floor level to the next, which then transfer these loads to lateral-load resisting subsystems that are often located elsewhere in the building (see Chapters 2 and 10), while gravity loads are typically channeled vertically through an open structural system framework consisting of columns and beams (e.g., see Ill. 3.17).

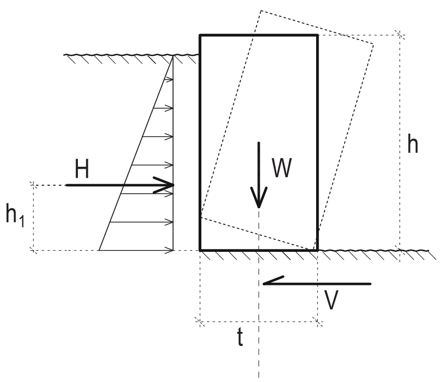


Figure 8.15
Retaining wall as gravity wall. The weight of the wall prevents overturning and sideways sliding.

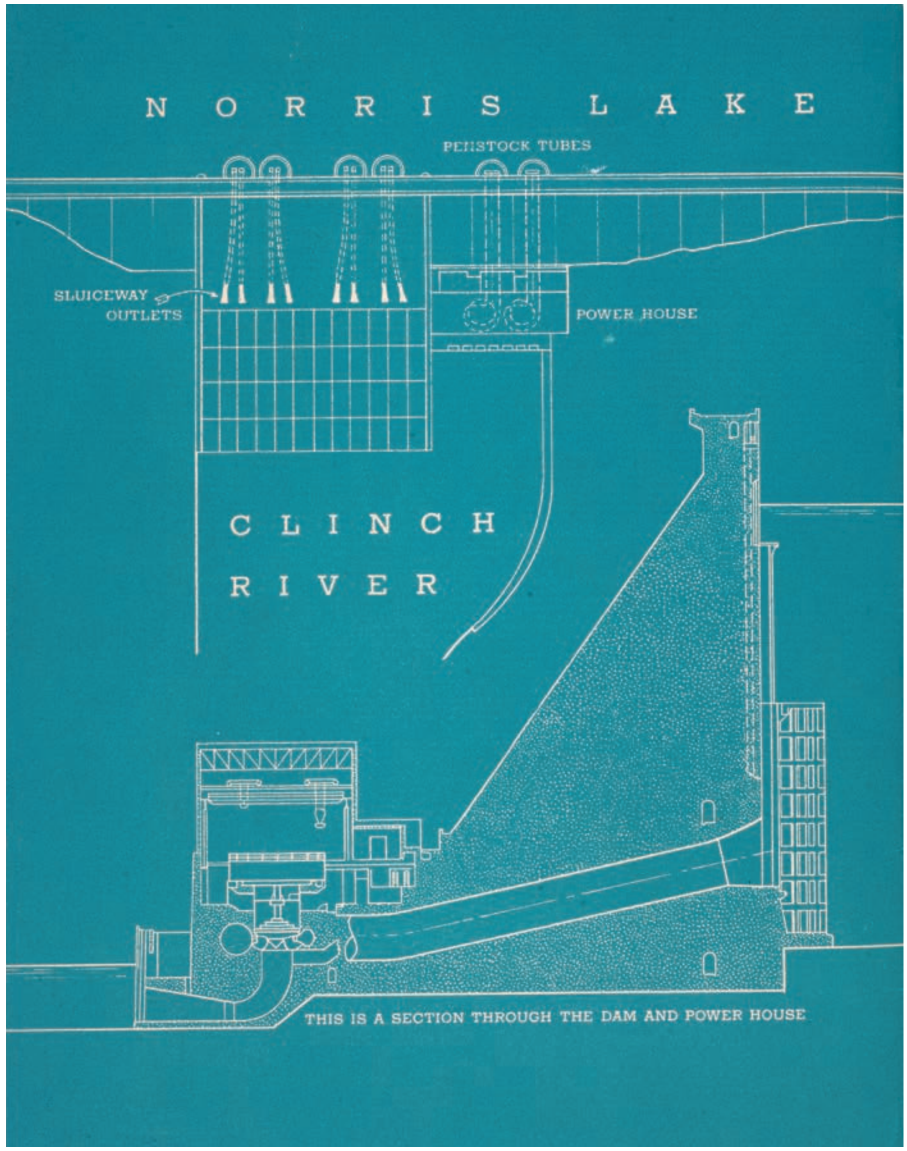


Illustration 8.32
Norris Dam, near Knoxville, TN, USA (1936).
Section drawing indicates some of the remarkable dimensions of this gravity-type concrete dam: 81m (265ft tall), 64m (208ft) thick at the base and 567m (1860ft) across in width – resulting in 765 000m^3 (1 000 000yd^3) of concrete.[10]

Retaining Walls

A special case of walls having lateral loads acting perpendicular to their surface is the condition of *retaining walls*. We may sort such walls into two groups according to the way in which they work to resist horizontal loads; i.e., whether they act as massive *gravity walls* or else thinner *cantilevering flexural walls*. What retaining walls generally have in common is their ability to support loads without being propped against displacement at the top. Such walls are free-standing, and their prime function is to hold back the pressure of earth, rocks, or, perhaps, water. Accumulations of these natural substances will cause a horizontal pressure to be exerted on the retaining wall having increasing intensity from top to bottom, as was discussed in Chapter 3. (Section 3.7.)

The *gravity wall* resists the effect of the horizontal pressure by means of its own self-weight, which needs to be large enough to prevent the wall's overturning or sliding sideways. (Fig. 8.15.) The latter calls for an adequate friction at the base of the wall in order to stop it from being pushed sideways; alternatively, such a mechanism can be assisted by some sort of mechanical anchoring system. If the weight W acts along the centerline of a wall having thickness t, and the resultant of the horizontal load is H, then overturning along an axis at the front of the base of the wall is prevented if

$$Wt/2 \geq \kappa H h_1$$

where h_1 = the distance from the resultant load H to the wall's base, and κ (Greek letter "kappa") is a suitable safety factor, frequently

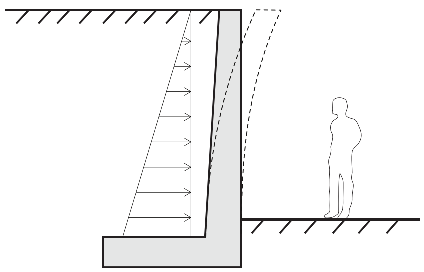


Figure 8.16
Retaining wall deflects as a vertical cantilever due to lateral loading.

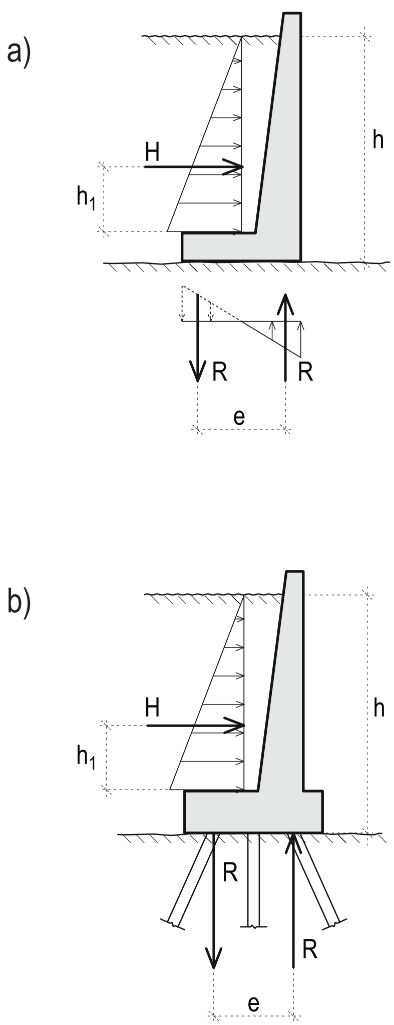


Figure 8.17
Vertically cantilevered retaining walls, and bending moment resolution at their base. Wall foundation on rock (a) and on piles (b).

set at 1.5. The idea here is to make sure that the stabilizing moment produced by the weight of the wall (Wt/2), which prevents the wall from turning over, is significantly larger than the moment which may cause such an overturning (Hh_1). In practice, gravity wall thicknesses t are typically 0.5–0.6h, where h is the total unsupported height of the gravity wall; this obviously results in a wall of quite some thickness if it has a height of any significance.

Instead of relying on self-weight, a retaining wall may be strong enough to cantilever in bending fashion from the foundation (Fig. 8.16), in which case the necessary wall thicknesses will be dramatically reduced from its gravity wall equivalent. Reinforced concrete is a typical material for this situation, as are sheet pile walls made of steel. This wall's behavior is fundamentally one of bending, and accordingly the wall thickness may be proportioned to be thickest at the base, thinning out toward the top. (Fig. 8.17.) The cantilevered retaining wall relies on a foundation capable of establishing a resisting couple (moment) at its base, either by resting on rock and anchored to it, or by being supported by piles having the ability to resist tension and compression as well as horizontal shear forces. If the resultant horizontal load H from the earth pressure acts at a distance h_1 from the base, the requirement for rotational equilibrium will give the resultant of the counteracting compression and tension reaction forces R at the foundation as

$$Re = Hh_1, \text{ or}$$
$$R = Hh_1/e$$

where e = the distance between the reaction forces R. This means that the wider the footing, the less the vertical force R needs to be in order to be accommodated by the foundation.

A project that encapsulates well both the ethos and rationale of various types of retaining walls is the Waterline Museum at Fort Vechten. (Ill. 8.33.) Here, a museum that is dedicated to the way water was used in the defense of the Netherlands between 1588 and 1940 consists of multiple parts, including that of a historic fort that was an integral part of the Waterline defense system, having its own impressive stone wall fortifications and surrounding moat, and then, within that larger context, a recent building addition dedicated to literally putting on display a working miniaturized model of the Waterline system. This display is located below ground, and fully surrounded by its own set of concrete gravity-load-bearing walls and cantilevering retaining walls. (Ill. 8.34.)

Illustration 8.33
Waterline Museum on Fort bij Vechten, Bunnik, Netherlands (2015).
Retaining walls establish both the elevated levels of the terrain for a historic fort along the Dutch Waterline defense infrastructure, and enable a contemporary museum to be "carved out" from this ground.

Architect: Studio Anne Holtrop.

Illustration 8.34
Waterline Museum.
A perimeter retaining wall establishes the museum's sunken courtyard that puts on display a working model of this defense system.

Illustration 8.35
Castelldefels ramp (near Barcelona), Catalonia, Spain (1993). Folded Cor-Ten steel plates form the outside edge of a 1km-long public walkway that zigzags up a hillside, connecting town center to a historic fortified stronghold located high above. These plates are actually the left-in-place outside surface of formwork used to pour the concrete retaining walls that hold the earthwork for the ramp in place.

Architect: José Antonio Martínez Lapeña and Elías Torres Architects.

8.9 Urban Ramps and Retaining Walls

Walkway at Castelldefels

High on a 60m (130ft) hilltop overlooking the Catalonian town of Castelldefels and the Mediterranean Sea beyond is a fortified sixteenth-century stronghold, built around a tenth-century church and upon the ruins of earlier Iberian and Roman villas. A relatively recently built 1km-long (0.6mi) pedestrian access ramp zigzags its way up from the urban center to the castle, following the irregular topography with each bend of the pathway and responding in subtle ways to the natural landscape of the hillside. (Ill. 8.35.)

Contrasting with such delicate site responsiveness is the distinctive main visual feature of this walkway: the upright Cor-Ten steel plates that fold back and forth in origami-like fashion alongside the whole length of the outside edge of the pathway. These plates are actually the left-in-place outside surface of the formwork used to build a network of reinforced concrete retaining walls that hold in place the rocks and earthwork necessary to create the smooth, wide walking surfaces of the ramp. The corrugated steelwork is made visually prominent by extending this "formwork" vertically above the level of the walkway so as to also serve as its safety railing. The back-and-forth folding of the plates, of course, not only provides the steel wall surface with significant flexural bending stiffness such that it can cantilever vertically – despite being quite thin – but it also echoes the visual crenellation patterns of the castle high above. The strength of the retaining wall structures is deliberately emphasized here rather than being hidden away – as it could easily have been.

Olympic Sculpture Park in Seattle

Zigzagging its way down the slope from the center of Seattle to that city's long-neglected urban waterfront is the Olympic Sculpture Park, a major brownfields land reclamation project that is now at once a generous urban recreational park and an outdoor annex for the Seattle Art Museum used to display some of its larger sculpture pieces in a spectacular outdoor environment. The 3.6ha (9 acre) park is located on what had been for many years an oil company's industrial site, until urban renewal efforts in the 1990s caused its soils and anything remaining of the site's natural landscape to

Illustration 8.36
Olympic Sculpture Park, Seattle, WA (2007).
Continuous pathway angles back and forth down the sloping site toward the waterfront. Earth is held in place by means of series of inclined retaining walls that are also a strong visual feature of the sculpted landscape.
Architect: Weiss/Manfredi Architects. Structural engineer: Magnusson Klemencic Associates.

be stripped clean. This soil was then replaced with more than 200 000m^3 (260 000yd^3) of clean landfill in order to create a large-scale park in the city, with native plants used to recreate four distinct and functioning natural ecosystems. This project, however, is about much more than typical land reclamation. All along, there has been the simultaneous objective to create a park that would double as an outdoor venue for the display of large sculpture artwork set in the context of the magnificent views of Puget Sound and of the snow-capped mountains of the Olympic Peninsula beyond. And all of the new earth fill material that was brought to the site provided its own opportunity for being sculpted: architect Weiss/Manfredi's design for the park features a 670m (2200ft) continuous pathway angling back and forth down the richly contoured site with the earth being held in place by an extensive system of retaining walls. (Ill. 8.36.)

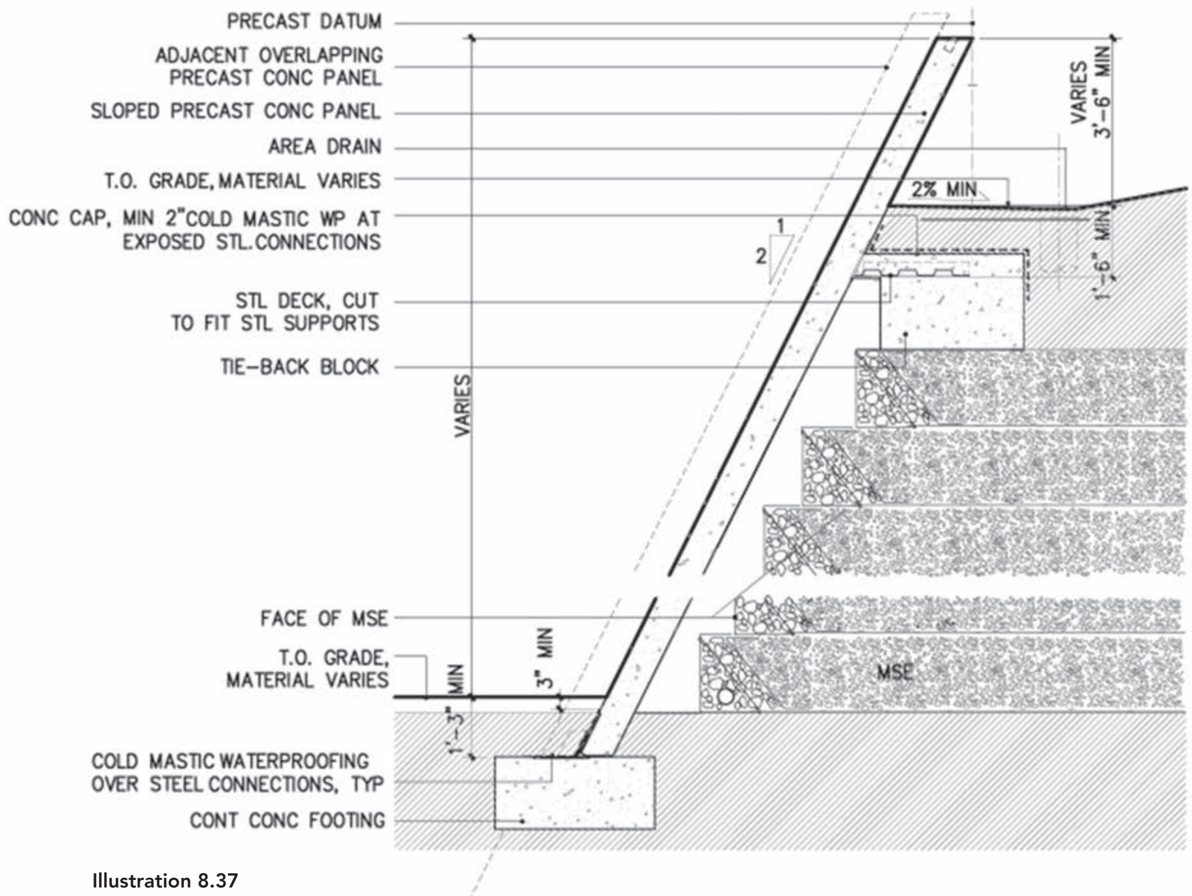


Illustration 8.37
Olympic Sculpture Park.
Drawing of angled retaining wall construction details. Overlapping precast concrete panels are the visible part of these walls, but the layers of mechanically stabilized earth (MSE) hidden behind these are really what are doing the earth-retaining work; this is faced at its outside edge by blocks of wire baskets filled with rocks and gravel.

These walls, that in many ways create the visual identity of the park, are also of considerable interest from the point of view of their construction. Unexpectedly, they are made up of two parallel parts: one that is completely visible, consisting of a series of 3.6m (12ft) long overlapping, back-sloping, precast concrete panels that in places are up to 9.1m (30ft) high, and a second part that is hidden behind the first but that is actually doing all the hard work of holding the tons of soil of the molded landscape in position. (Ill. 8.37.) The workhorse part of the wall is constructed using mechanically stabilized earth (MSE); that is, it is composed of alternating layers of compacted soil 450mm (18in) thick and sheets of geotextile fabric that stretch back to a distance of about 80 percent of the wall's height. This thick band of MSE is faced at the outside edge by a wall made of steel wire baskets filled with rocks and gravel. This outer surface is then in turn protected from the elements by the layer of vertical precast concrete panels, the two being connected only top and bottom by slotted connections but otherwise separated slightly in order to allow for their relative movement in moderate-size earthquakes. Also, the movement allowance in the connections and the overlapping of the exterior precast panels accounts for differential settlement of the ground occurring without causing significant and unsightly cracking of the long walls. Aside from these inherent benefits of built-in flexibility, the dual retaining wall system was also determined to be considerably cheaper to build at the scale of this project than would have been the case of a conventional cast-in-place reinforced concrete wall system with subsequent earth backfilling.

A new topography has been creatively molded at the Olympic Sculpture Park, and its irregular contours have been seen to be held in place by making the earth that is so much a part of this project actually do the work of holding it all together. Moreover, the repetitive aspect of the exterior precast panels can also be seen as a sculptural response in their own right, framing views and providing modulated backdrops for the contemplation of the works of art. Alexander Calder's Red Eagle has certainly found an appropriate place to land and survey the landscape.

The Truss and the Space Frame

CHAPTER

9

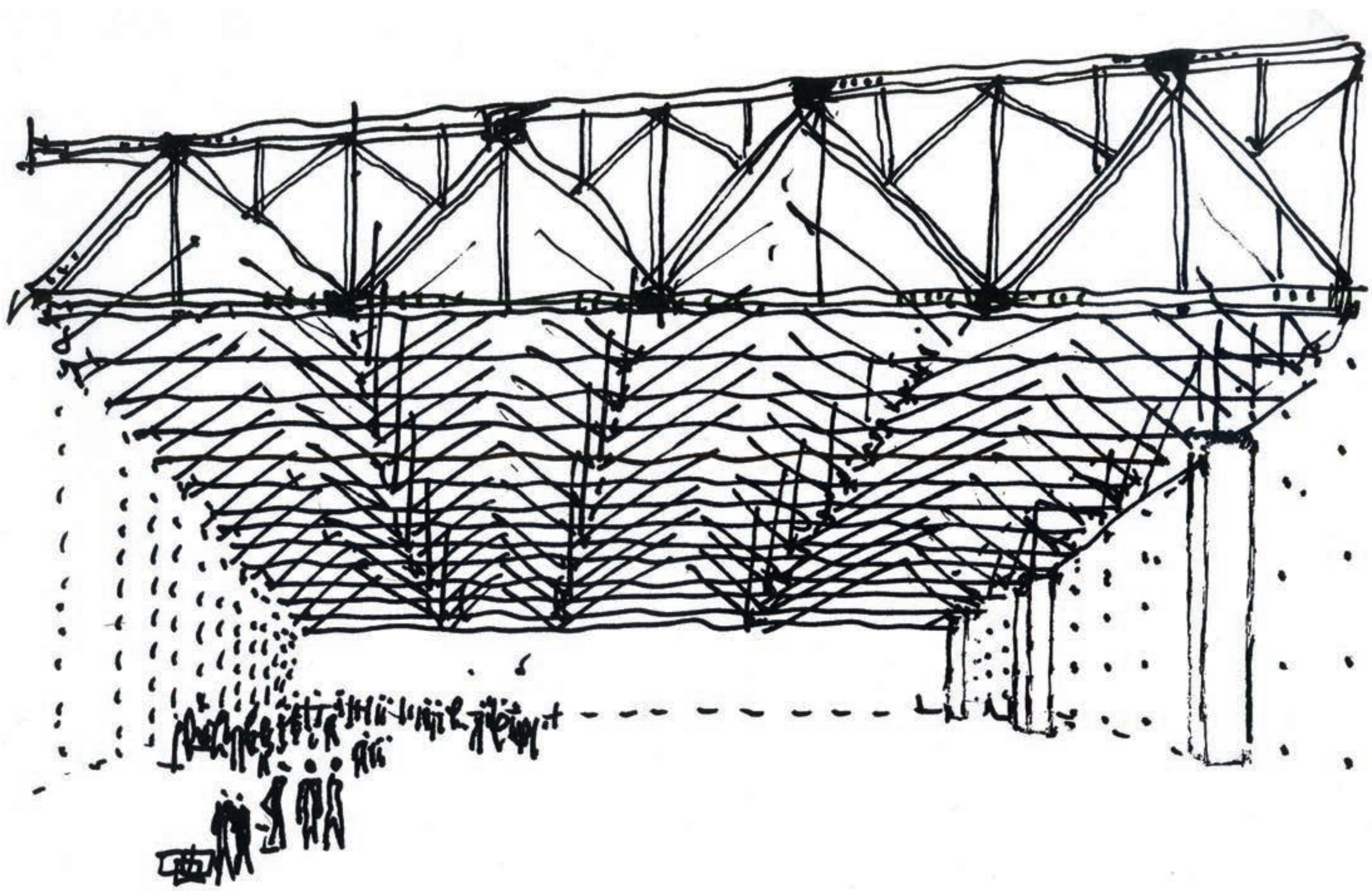


Illustration 9.1
Hangar for Oslo Airport, Fornebu, Norway (built during World War II, 1941). The roof structure was built using flat trusses that span 30m (120ft) that are made of large wood timbers. The truss joints were detailed with steel plates connecting single and double members. Over the hangar doors, the wooden trusses are carried by a transverse steel truss that rests on concrete columns.

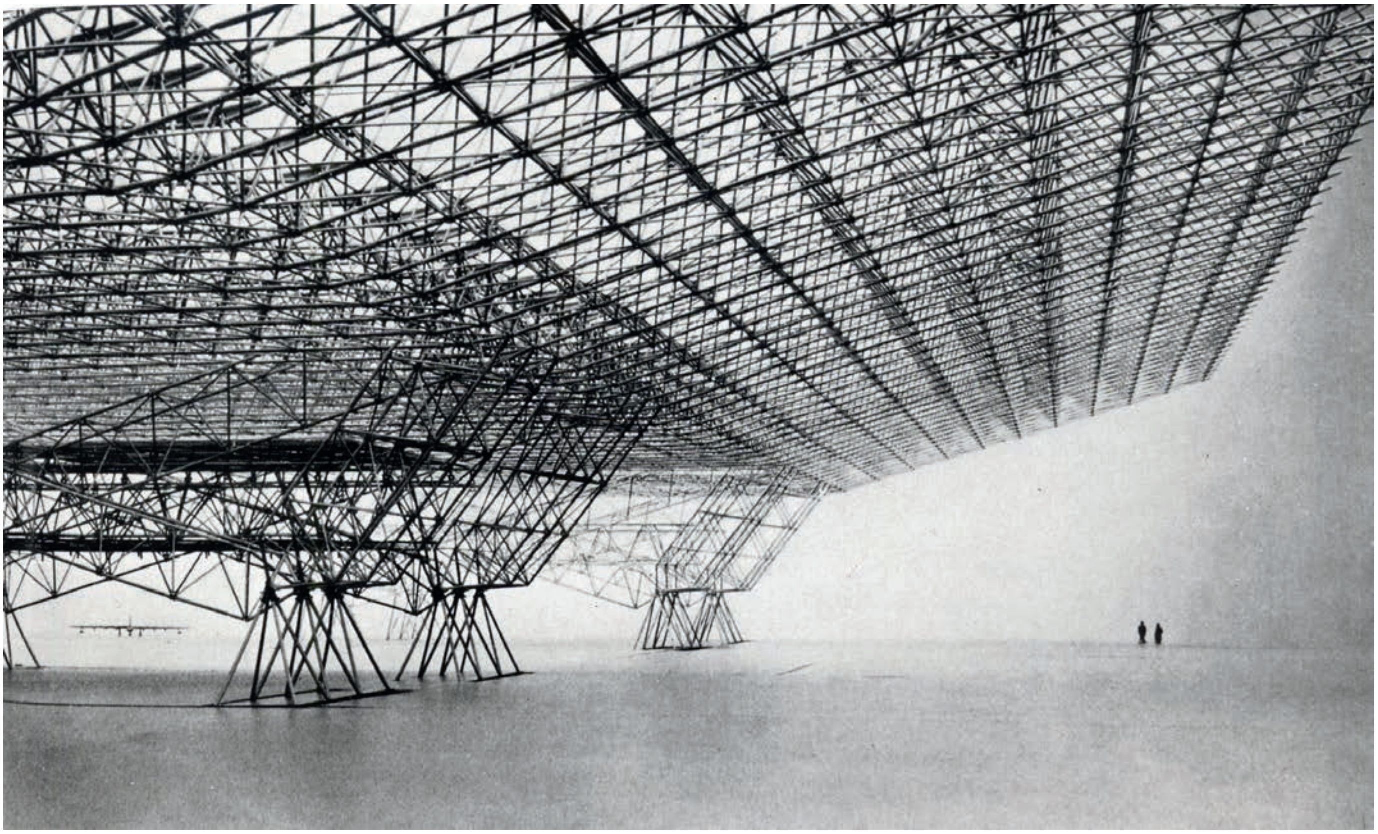


Illustration 9.2
Project for a hangar for the US Air Force (1951).
Model of the space frame structure made of steel.
Architect: Konrad Wachsmann.

9.1 USAF Hanger and BMW World – The Space Frame Evolves

Konrad Wachsmann (1901–1980) contributed greatly to the development of industrial building processes, both as an educator and an instigator of research projects. His career parallels that of other great architectural personalities, such as Walter Gropius and Mies van der Rohe: Wachsmann was born in Germany and migrated to the United States in the late 1930s.

A research project for the US Air Force led to perhaps his most significant work, the development of a structural system that could be used for large hangars. (Ill. 9.2.) In contrast to Pier Luigi Nervi's famous concrete hangar at Orvieto, Wachsmann's choice of material for such structures was steel. The aim of his research work, undertaken at the Chicago Institute of Design in the latter part of the 1950s, was to develop a building system for long spans and cantilevers that was based on the use of standard construction elements whose dimensions varied minimally. His choice of structural form to accomplish this objective was that of a space frame system composed of an accumulation of basic tetrahedron-shaped units. These are open pyramidal forms having four equilateral triangular sides and structural members that connect the three joints of the geometric form.

Because triangles are stable structural forms, there is no need for the joints to be rigid – they can all be of the simpler and cheaper pinned variety. While the potential of the space frame was at that time already becoming known through Alexander Graham Bell's experiments (see Ill. 9.34), it was Wachsmann who first used the system extensively in an architectural setting. The logistical problem that he faced in applying the space frame system to the scale of building structures, though, was in finding an effective connection joint that was relatively simple and cheap – obviously a critical factor given the number of joints in such a system. The solution that he devised was to use a spherical joint made of chromium steel that could connect up to 20 steel tube members coming in at different angles; a simple wedge principle was used to hold the ends of these members in place.

Wachsmann's huge space frame proposal rested on a number of stable polygons constructed using the same basic principles as that of the roof structure. At the time, such projects carried with

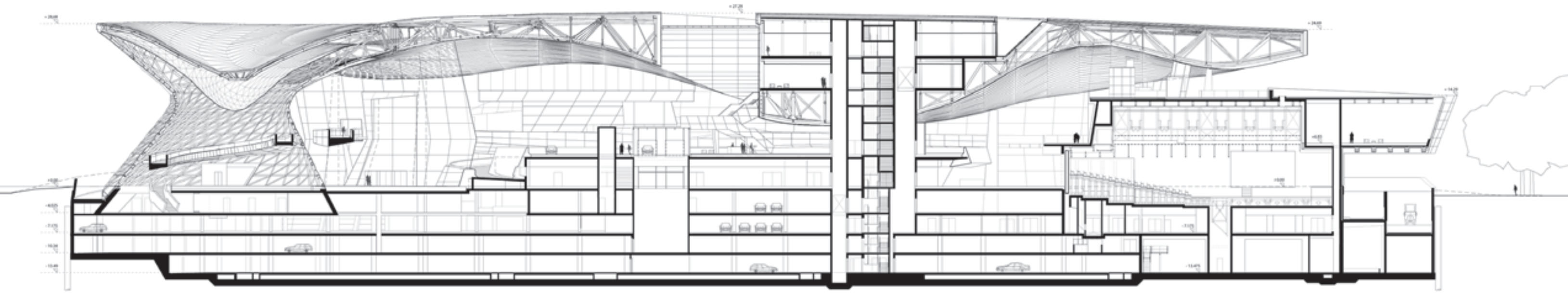


Illustration 9.3
BMW Welt, Munich, Germany (2006).
Section shows undulating truss forms; triangulation can easily adapt to overall structural form that varies.
Architect Coop-Himmelb(l)au. Structural engineer: Bollinger + Grohmann.

Illustration 9.4
BMW Welt, Munich.
Exterior view of undulating roof form.

them the message that this new type of architecture offered the possibility of having a free, dynamic, sheltered space of almost limitless dimensions and lightness. This kind of structure is, of course, particularly well suited to occupancy functions that require big, open spaces such as airport hangers, exhibition halls, etc. Also, in terms of design potential, the nature of the space frame is based on an uncompromising geometric ordering principle, with clear rules for the addition of secondary building elements such as roof covering, façades, and other installations; only the most capable of architects understand and master that game.

Almost 50 years later, different architects and engineers were faced with a similar challenge: that of designing a structure that could cover the exceptionally large roof area of 120 by 200m (394 by 656ft) for an "experience" and distribution center in Munich for the automobile manufacturer BMW. Further challenging the designers – architects Coop-Himmelb(l)au with structural engineers Bollinger + Grohmann – was the fact that the building site is quite close to that city's famously innovative and iconic Olympic Stadium, which has its own celebrated and unique roof form and structural system. (Ill. 11.38). For BMW World, the roof structure is an equally striking, if different, visual feature, one that the designers conceptualized as a "cloud" with a softly curved and undulating shape, especially on its underside. (Ill. 9.3, 9.4.) The top of the roof is slightly bulging upward in the middle, like a cushion, and is fitted with solar panels. Both upper and lower layers of the roof system consist of 5 by 5m (16.7 by 16.7ft) grids of structural members, and these layers are connected by means of numerous diagonal bars that together form an overall space frame structure whose height varies continuously between 2 and 12m (6 and 36ft). The supports for this huge roof system are a large double-cone structure at one end of the building and a number of concrete columns and stair and elevator cores elsewhere.

Part of the original idea for having an undulating roof structure was to make the space between the upper and lower structural grids available for functions related to administration offices and lounge spaces. The story of how the very particular shape was then conceived is interesting: letting those types of functions represent certain imaginary gravitational forces, these were applied to the structure by help of a computer program that produced a virtual deformation and, thereby, suggested the space needed in between the layers. The resulting structure is certainly visually arresting as well

Illustration 9.5
Trajan's Column, Rome, Italy (113 AD).
Relief detail shows braced form of wooden frames used for railings of pontoon bridge having closely spaced boat supports.

as being relatively light and consistently efficient: it is constructed from steel tubes having only 324mm (12in) diameter for both the upper and lower layers' members and 244mm (10in) diameter for the diagonals. While the lineage from Wachsmann's early spatial ambitions and construction strategies to this contemporary built reality is clear, BMW World's designers have also demonstrated that trusses and space frames of the twenty-first century do not necessarily have to follow the regular, straight-line design geometry of the twentieth.

9.2 Spanning Truss History

Trusses have a long and distinguished history; indeed, as a structural type they can be traced all the way back to antiquity. In very simple forms, trusses were quite commonly used for pitched roofs, and the Romans also employed trusses extensively in bridge construction (e.g., Ill. 9.5); indeed, both of these would have been seen at the time as structures that were part of the vernacular tradition. During the Renaissance, a more conscious attitude to trusses' particular way of carrying load and to their potential applications gradually developed, not least of which is represented by the theoretical and practical works of the Italian architect Andrea Palladio (1508–1580). It was, however, not until engineering became an established profession in the first half of the nineteenth century that the potential of trusses was systematically explored (and exposed), although for the most part by bridge builders. Longer spans and heavier loads incited by the rapidly expanding railway transportation system called for efficient and reliable structures. The development of the truss, therefore, primarily took place in civil engineering structures; in architecture, both spans and loads were usually smaller and did not encourage the formal development of trusses to the same degree. Besides, tradition, style, and custom weighed more heavily on architectural practice than on that of engineering, partly explaining why truss forms in general were slow to be admitted into "high-end" architecture. A relatively early example of a systematic use of trusses as both load-bearing structure and expressive architectural element, however, can be found in the Crystal Palace built for the World's Fair of 1851, where both wrought and cast iron trussed beams were used in a large-scale building, albeit a temporary one. (Ill. 9.6.)

But just what, exactly, is this thing we call a truss? We may begin to answer this question by saying that at its most basic a truss is a structure that is made up of linear elements arranged into triangular configurations. (e.g., Fig. 9.1.) Since a triangle is a stable structural form overall – that is, the triangle will not significantly be deformed when external forces are applied to it in any direction within its plane – then such an arrangement provides a highly effective geometric framework to build upon. More specifically, and as we will discuss in considerably more detail shortly, this triangular arrangement of structural elements is very efficient at being able to resist large overall deformations.

More generally, we can say that a truss is made up of an arrangement of many such triangular configurations connected together side by side. These may be found in the form of the

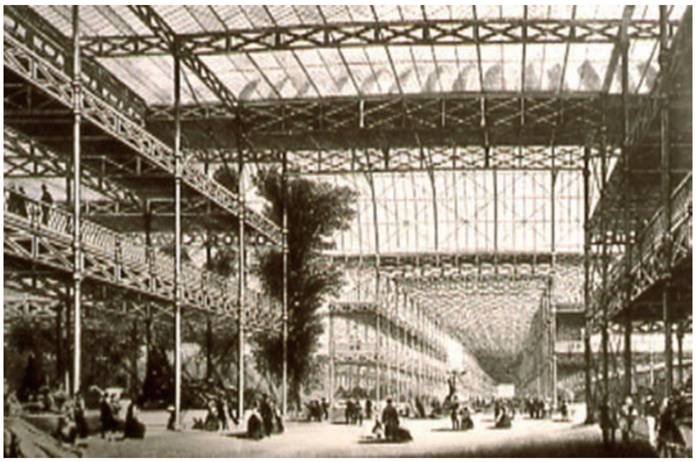


Illustration 9.6
Crystal Palace, London, UK (1851).
The shorter trusses of 7.2m (24ft) span were made of cast iron, while the longest spans (21.6m or 72ft) were executed in the more bending-resistant material of wrought iron.
Designer: Joseph Paxton. Structural engineer: Fox and Henderson.

supporting structure for a simple pitched roof, or else in the place of a long-spanning horizontal beam, or stabilizing a wind-loaded building by means of a diagonally braced frame, or even adopting the overall form of a strategically curved arch, etc. Although these examples are seemingly of very disparate form and function, what they all have in common is that their dimensions and relative proportions are such that it is possible to create a light, open structural subsystem called a truss out of many linear structural elements connected together into stable triangular configurations.

In terms of materials, we can say that before metals such as iron and later steel became so commonly used for trussed structures, wood was the main material used for this structural form, as we just saw in the relief depictions of Roman bridges on Trajan's Column. Given that individual truss members experience either axial tension or compression forces (more on this shortly), and since wood performs quite well in either of these stress conditions (see Chapter 5, Section 5.2), it makes sense that this would have been the material of choice for early trusses; certainly stone would have been out of the question given its fundamental weakness in tension. Historically, then, this would likely have left the designing and assembling of truss connection details as perhaps the greatest challenge to building these types of structures (something that is still of major consequence from both a functional and visual perspective – but more about that later in this chapter in Section 9.6.) During the nineteenth century, advances in material production led to the pronounced development of trusses made of cast iron and, especially, wrought iron, with the latter's more ductile material behavior being favored for the

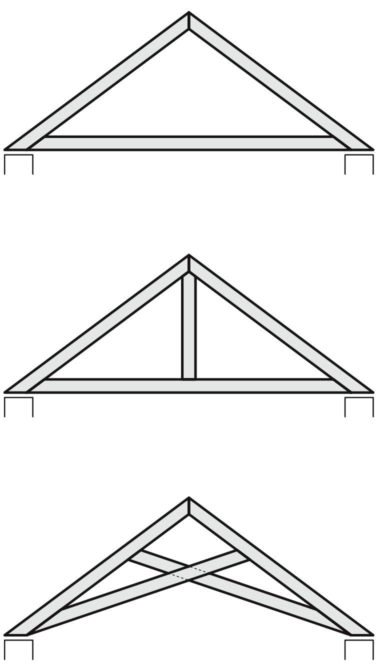


Figure 9.1
Early roof trusses; simple triangle, King Post, and scissors.

Illustration 9.7
"Le Pont de l'Europe," Paris, France (1876).
Painting of historic bridge in which lattice-form iron trusses are evident.
Painting by Gustave Caillebotte.

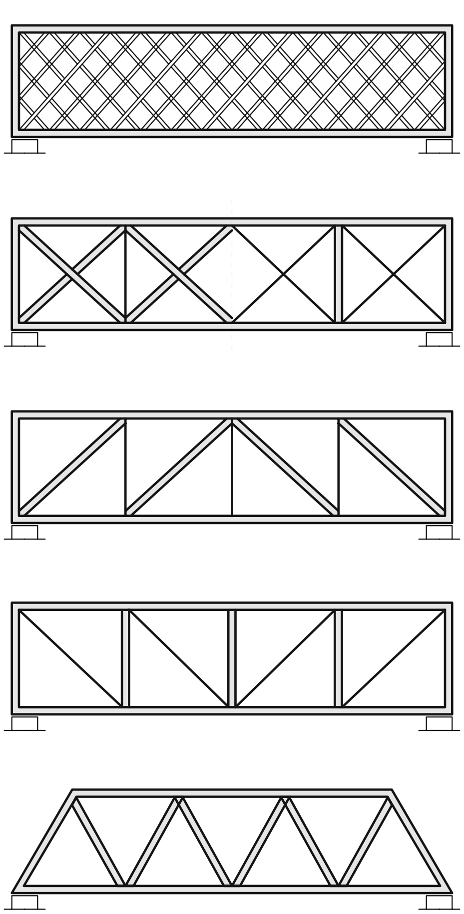


Figure 9.2
From the top: (a) lattice trussed beam, or Town truss; (b) truss as proposed by Long; (c) the Howe truss; (d) the Pratt truss; and (e) the Warren truss.

tension members of a truss, with cast iron eventually mainly restricted to its compression members.

A period of intense truss development brought on largely by the rapid expansion of the railway infrastructure network led to innovative trussed bridge forms that were often named after their inventors. (Fig. 9.2.) An early example of this was the so-called lattice truss (a), also known as the Town truss (1820), in which diagonals of alternating directions form a tight web of members (these are named after Ithiel Town, 1784–1844). "Le Pont de l'Europe", as can be seen in the painting in Illustration 9.7, is a classic example of just this form of truss. In 1830 the so-called Long truss (b) with vertical members was introduced, in which the resulting rectangles were cross-braced by diagonals in both directions (Stephen Long, 1784–1864). A similar truss was proposed by William Howe (1803–1852) in 1838, later modified into the so-called Howe truss (c) in which the rectangles are braced with single diagonals acting in compression. Notably, the truss forms proposed by Long and Howe were both anticipated by Palladio in the sixteenth century, but apparently without a full knowledge of how they performed structurally. In 1844 the Pratt truss (d) was proposed (Thomas Pratt, 1812–1875); this version is an exact reversal of the Howe truss in which the diagonals are oriented in the opposite direction with the result that, as we shall see later, tension forces occur in the diagonals and compression forces are in the vertical members, instead of the reverse in the Howe. Finally, the so-called Warren truss (e) of 1846, named after James Warren (1808–1908), features diagonals that have alternating directions. We shall look more closely into the structural behavior of some of these various truss forms in Section 9.5.

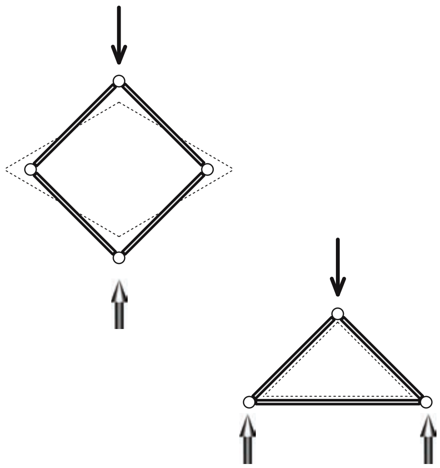


Figure 9.3
Pin-connected rectangle vs. pin-connected triangle. When loads are acting, the rectangle will form a collapse mechanism, whereas the triangle will be stable and essentially retain the same overall shape.

Illustration 9.8
The trussed arrangement of a vulture's wing bone provides large stiffness with little material, reducing weight as much as possible.
After D'Arcy Thompson.

9.3 Triangulation and Internal Stability

Since trusses are constructed from a series of stable triangles, we will first look in more detail at the geometric and structural properties of this shape. Contrary to the case of the quadrilateral, which may easily change shape from a square or rectangle into a parallelogram when the angles between intersecting sides are left free to change, triangles are locked into one shape the moment all three sides are linked together. (Fig. 9.3.) This is a splendid geometric property that has great potential for structural design, since one very basic condition for load-bearing structures is their ability to form stable frameworks.

Ideally (but also typically), the individual component members of a truss are linear and the forces in them are either purely compressive or purely tensile. And since axial force members are able to exploit a material to its full capacity over the whole of a member's cross-section – although admittedly sometimes less so in compression than in tension because of the danger of buckling, as we saw in Chapter 8 – by making structures from an assemblage of linear elements in triangular configurations we will thereby be promoting a strategy of load-bearing efficiency. Generally, we know that members in pure tension and compression will be thinner and more materially efficient than is the case for elements that primarily employ bending behavior to carry load (such as do beams). As we will see, all of this points toward trusses having the qualities of being relatively light and open structures that minimize physical and visual weight. (e.g., Ill. 9.8, 9.9.)

A precondition to having pure axial forces in all truss members is that they must in principle be free to rotate relative to one another about their connection points: this requires the simplest of connections and is a concept that is attempted to be mimicked in reality. Historically, such connections were made by literal hinges

Illustration 9.9
Joso High Bridge over Snake River, between Franklin and Walla Walla counties, WA, USA, (1914).
The similarities and differences between a truss and a beam with no openings in its steel-plate web are evident: both here have to carry the same transverse train loading, but the truss is able to use its strategic triangulated configuration of its component parts in order to span a much greater distance between supports.

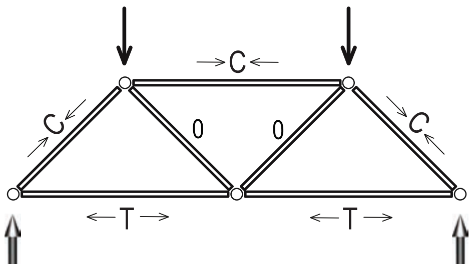


Figure 9.4
Simple truss with pin-joints and loads acting at connection points, resulting in members having pure axial forces, either tension (T) or compression (C). The structural members that establish the height of the truss are called chords; in the present case the lower and upper chords, respectively. The members connecting the chords are termed diagonal and vertical members or, generally, interstitial members.

between members made with the help of a steel pin, hence the name of a *pin-joint* or *pinned connection*, or simply a *hinge*. We should note that axial forces in members that are connected at a pin-joint are all considered to be acting directly through the same point (i.e., the center of the pin), and thus there is no lever arm for moments to be created at the connection (recall the discussion of this topic in Chapter 4, Section 4.4); pinned connections, therefore, are always considered to be free of any such bending moments.

A second precondition for having a truss in which no bending moments are present in the individual members is that all external loads must be applied and all supports must be provided at the connection points, the pin-joints. (e.g., Fig. 9.4.) If they are not, and loads were instead applied transversely along the length of the members, then each of these would have to respond by bending between connection points, thereby resulting in a much less materially efficient structure. Strictly speaking, of course, for a truss structure to be completely free of bending moments we would have to ignore the dead weight of each member (i.e., gravity, in reality, acts all along the lengths of all members of a truss). But, fortunately, the effect of axial forces in truss members produced from loads that are acting at the pin-joints is typically far greater than whatever effect the weights of the individual members may have on them, and so it is usually not too far off the mark to idealize things this way. As a general approach, then, we will ignore the local bending effects due to truss members' self-weight, and this can typically be considered to be sufficiently accurate as long as the dead weight of the total structure is included in the overall applied load calculations.

Having established these fundamental characteristics of the quintessential truss – i.e., triangular configurations of component members that are pin-jointed together and subjected only to axial forces, and with loads that are applied and reactions provided only at joint locations – we are now in a position to look more closely at their behavior. First, however, a few words about nomenclature. For a truss that is oriented horizontally, the structural members that run along the length at the top and bottom are called truss chords, whereas the members connecting these are termed diagonal and vertical members or, more generally, can be called interstitial members.

The first step in the design and analysis of a truss is to determine whether the proposed structure is truly a stable arrangement of members. If it is entirely composed of triangular shapes – as is often and even typically the case – then, as we have discussed, we can rely on this geometry to be completely sufficient to provide the truss' internal stability of form. There are also cases that exist in which there are more members in a truss than are strictly required for its stability, perhaps with "extra" diagonals and overlapping triangular arrangements of members (this is called a statically indeterminate truss structure), but we will not consider those situations at this introductory level.

Generally, a careful visual inspection of a truss is usually adequate to establish the nature of the forces in its various members; i.e., whether compression or tension is likely to occur in these, or whether in fact a member has no force in it at all for a particular loading condition. For example, we can examine the Leonhardt House (1956) on Long Island designed by the architect Philip Johnson (1906–2005). (Ill. 9.10.) Its open glass box hovers over the site and the open landscape below and it is supported by columns located along two axes that are transverse to the orientation of this main living space. In between these lines of support, the box spans freely by means of two simple, full-height steel trusses along the outer glass walls, while beyond the outer column supports these trusses cantilever toward the open view. Vertical loads act on each truss as a result of the roof and floor dead weights as well as the occupancy live load on the floor and potential snow loads on the roof.

Whichever loads are acting on this structure at any one time, these will inevitably lead to the overall truss form being very slightly deformed overall, and it is the anticipation of this response that allows us to be able to predict whether the various members of the truss are acting in tension or compression. To help visualize what is happening, it may be helpful to temporarily think of the truss as though it were a horizontal beam and to imagine how this by-now familiar structure (from Chapter 7) would behave and deflect under transverse loading. First, for example, let us imagine that we have a vertical live load acting at a point midway between the supports and that this load is sufficiently large so that it dwarfs any dead load effects on the structure. (Fig. 9.5a.) The result is obviously going to be a tendency for the imagined analogous beam to sag downward between the supports, and from an overall perspective the truss also will deform and behave in a similar fashion. Since the form of the trussed structure consists of (diagonally braced) rectangles, however, these will themselves have to deform in order to accommodate the truss' overall sagging at mid-span. The vertical member at the center of the truss will obviously be pushed downward with respect

Illustration 9.10
Leonhardt House, Lloyd's Harbor, Long Island, NY, USA (1956).
Architect: Philip Johnson.

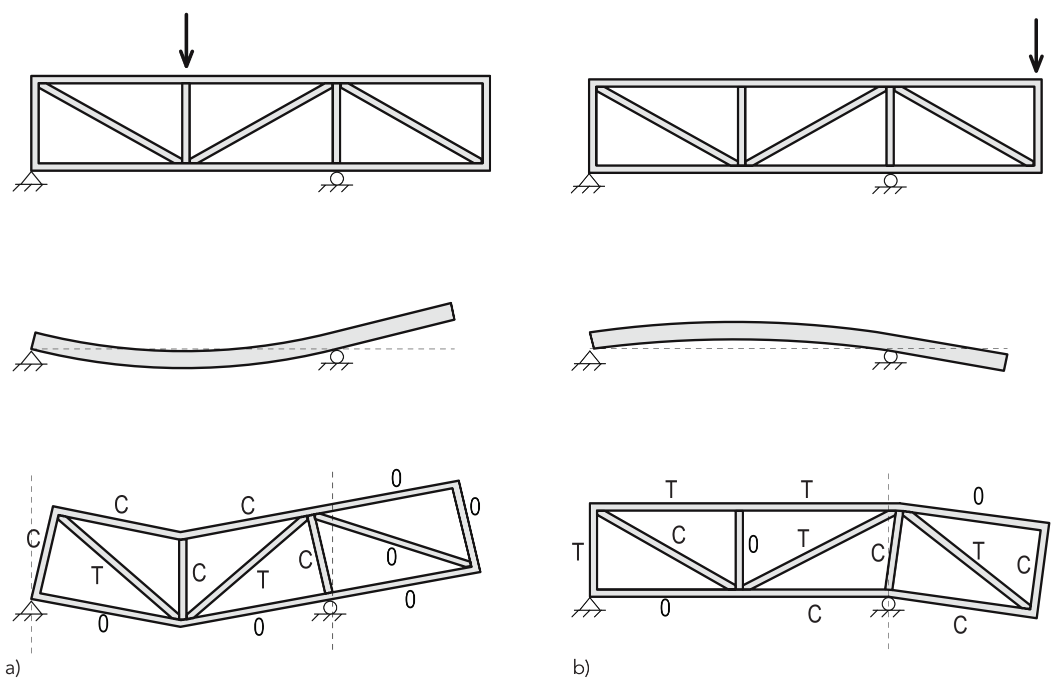


Figure 9.5
(a) The truss of the Leonhardt House with a load acting at mid-span. Mimicking a beam's response, the truss' anticipated deformation leads to being able to predict the types of forces acting in its members. T is tension and C is compression.
(b) The truss of the Leonhardt House with a load acting at the tip of the cantilevered part of the truss, the deformed shape of its analogous beam, and the predicted truss member forces.

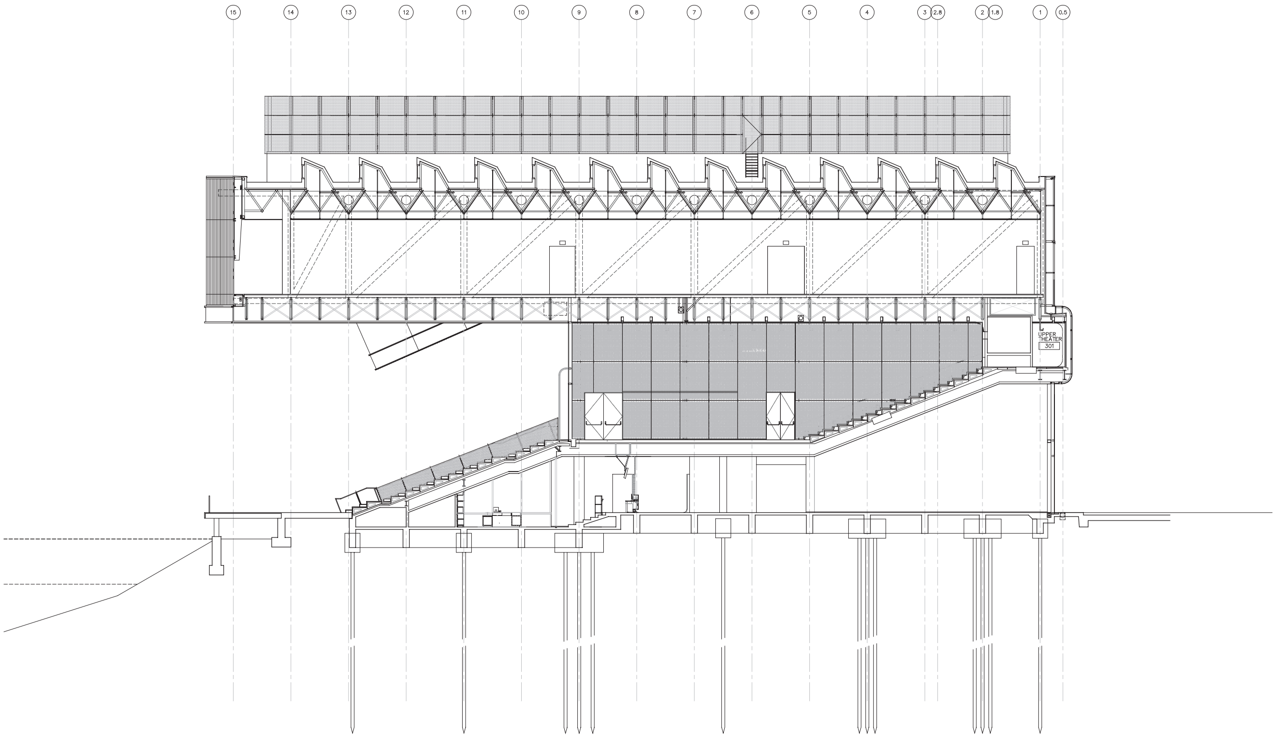


Illustration 9.11
Institute of Contemporary Art, Boston, MA, USA (2006).
Cantilevering trusses support the top-floor volume that projects the main art gallery spaces toward the waters of Boston harbor. A glass wall at the end of the cantilever allows for unimpeded views of the water. Because of their orientation, the truss diagonals can be anticipated to be acting in tension within the cantilever, which helps to minimize these members' cross-sectional dimensions.
Architect: Diller Scofidio + Renfro. Structural engineer: Arup.

to the level of the supports, meaning that both rectangular panels of the truss' middle span will be deformed into parallelograms that effectively make their diagonals stretch. Elongation of these diagonal members tells us that tension forces are at work in them. At the same time, the two vertical members of the truss directly over the column supports will be pulled slightly inward by the tension in the diagonals, resulting in a shortening of the upper chord. Such shortening will, of course, correspond to compression forces being present in these members.

As a second example, a large load is considered to be acting on its own at the tip of the cantilevered end of the truss. (Fig. 9.5b.) Such a load will make that portion of the analogous, imaginary beam deflect downward. Thus the outer rectangular panel of the truss will sag down at its end and it will deform in such a way that the diagonal becomes longer, corresponding to a tension force acting in it. Moreover, by further analogy to the analogous beam's downward bending behavior, the bottom part of the cantilevered truss can be expected to be pushed in toward the support. A compression force can, therefore, be predicted to act in the bottom chord member of the truss at this location. With a little more difficulty, we may also anticipate that for the cantilevered part of the truss to bend downward, the vertical member at the support will have to rotate slightly out in the direction of the cantilever which, in turn, will leave the diagonal to the left of the support with no other option than to stretch according to the imposed displacement at its end.

Obviously, taking dead loads into account will make such a visual analysis process more complicated, but in principle it will follow along the same lines. By always trying to imagine what type of local deformation (shortening or elongation) will take place in the truss members, one can make a fairly reliable prediction of the type of force that can be expected in them (i.e., once again, whether they will be in compression or tension). Such a visual inspection method can be described as a qualitative study of structural behavior, and it can be very useful both for the preliminary design of structures as well as for checking whether a numerical result obtained in some other more sophisticated manner corresponds to what one would expect to happen.

One final thing to note from this brief study of the Leonhardt House trusses: our intuitive analyses for both of these load conditions have suggested that the diagonals of its trusses have been strategically oriented to work as tension elements for key load conditions, which means that they could be (indeed, they were) designed as relatively thin structural members compared to what they would have been had they been oriented differently, working in compression and therefore subject to the possibility of buckling failure. The thin tension diagonal works wonderfully, of course, when the site and the views through the glass walls are so spectacular – something that is surely no design accident. The implications about such strategic orientation of truss diagonals, whatever the design intentions, go far beyond this one example. (e.g., Ill. 9.11.)

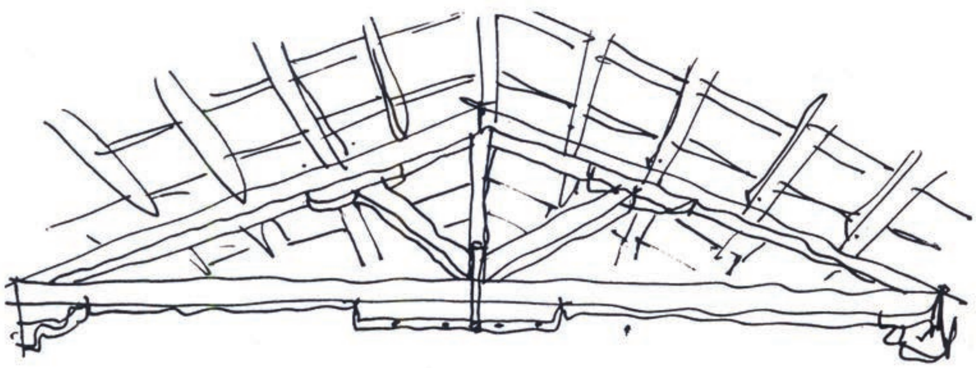


Illustration 9.12
St. Domenico Church, Siena, Italy (1125).
Roof structure as an example of the Western European structural philosophy of using rigid triangles in the form of trusses.

9.4 Roof Systems from East and West

St. Domenico Church

Wooden roof trusses, such as those that span the twelfth-century nave of St. Domenico in Siena, exemplify a structural form that historically was widely used in Europe, and that was incorporated into the simplest of barns and the largest of basilicas alike. One common aspect of these two types of buildings, of course, is that they function much better with a large-span roof, thus minimizing any columnar obstructions of their interior space. Another is their typically linear overall plan configuration, with the resultant doubly pitched roof a virtual requirement given the need to shed rain and snow. This naturally leads to the typical triangular section for their roof structure, which, in turn, conveniently conforms to the basic stable shape so characteristic of the truss. (e.g., Ill. 9.12.) Even so, there were limits to what could be done in the Middle Ages: because of the substantial span of the St. Domenico truss, for example, the made-of-one-piece upper timber chords end up determining the width of the nave while its collar beam, the lower chord, needed to be spliced together at the middle with double fishplates.[1] Given such practical limitations at the time, if the addition of side aisles to a central nave was deemed desirable it would typically be accomplished by means of a second, parallel, trussed shed roof that would have stood independently of the main one.

It should be pointed out that in spite of its distinctively triangular arrangement of components, the St. Domenico roof structure is no "ideal" truss, at least in the sense of the things that we have been talking about in the previous section. Because of the continuous attachment of the roof to the top member of the truss, the weight of the roof surface is transmitted all along the length of these inclined members, thereby violating the ideal of loads being applied only at truss connections. In this case, then, the top chord truss members are subject *not only* to compressive axial forces *but also* to significant bending moments, a situation that results in their needing to have larger dimensions than would otherwise be necessary.

Chongfu Monastery

At about the same time, roof forms in Asia were developed in accordance with other design principles; in early Chinese buildings, it seems, the static potential of wood truss structures was either unknown or not employed very extensively. Instead, steep roof slopes were constructed by having a series of increasingly shorter beams resting one on top of another in an alternating, orthogonal arrangement. (e.g., Ill. 9.13.) The width of the span of the system was thus determined by the load-bearing capacity of the longest beam at the bottom. If the space's width needed to be increased, an extra row of columns (and stack of beams) had to be erected.

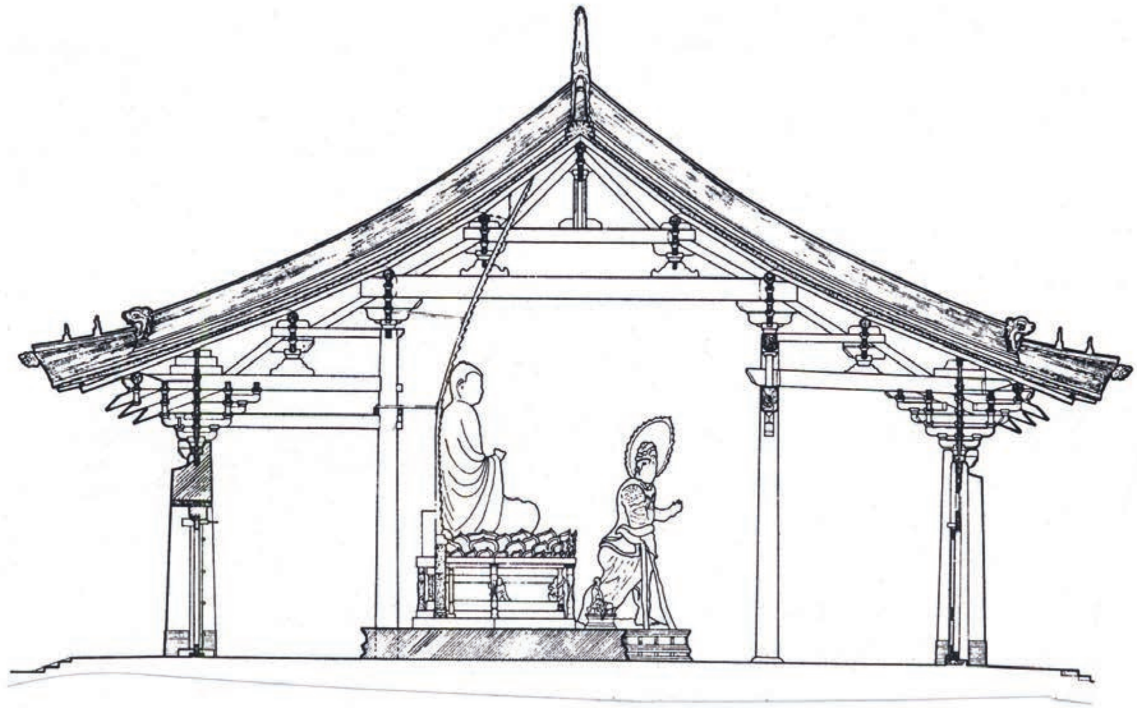


Illustration 9.13
Chongfu Monastery, Shuozhoa, China (from the Jin period 1125–1234). Typical Chinese roof structure of stacked beams.

It is evident that the roof was accorded significant attention in Chinese architecture; to wit, the numerous sloping roof forms characteristic of Chinese pagodas and temple complexes. Here it is the variation and nuances of bracketing (beam-to-beam and beam-to-column connections), of beam stacking patterns, and of the design of the beams themselves that created the basis for the rich traditions of such sloping roof forms.

When we compare the typical Eastern and Western roof systems, we cannot help but be struck by the lack of similarity between their structural approaches in spite of their nearly identical exterior profile and functional requirements; the two approaches can be seen to bear witness to different cultural and building philosophies. In the Eastern building tradition, the roof structures are designed without strict concern for efficiency or quantity of materials, but rather for the beauty of a particular structural order. Beams gradually become shorter and shorter the higher up they are in the system, and they carry correspondingly less and less load; the overall sloped form of the roof takes its shape from this series of gradually shortening spans. In Europe, a different tradition called for stable triangles that were combined into one large triangle. Triangles make for a lightweight and efficient structure, one that may even be seen as a bit boring in its precision. The individual members accept their portion of the load, none of them receiving significantly more or less than the other; in short, they distribute the work evenly among themselves. This European tradition has been documented since the late Roman Empire, and it is possible that it began in Greek antiquity; in either case, we can playfully say that the Western roof truss represents a sort of structural democratic ideal through a fairly even distribution of loads and responsibility. Continuing perhaps more dangerously along these lines, the Eastern architectural tradition can be seen to contrast with this approach by expressing structural layering: the bottom beam must carry all the other beams, while the next one is responsible for one less layer, and so on up to the top; in this way the beams use one another in a structural hierarchy that ignores cooperation. If the European truss system can metaphorically be called democratic, then the historical Eastern roof structure can be described as feudal!

The Nomadic Museum

Perhaps as a fitting coda to this briefly developed theme of Eastern vs. Western pitched roof structures is a the system used for a prefabricated photo gallery building that was designed to be mounted, dismantled, and (partly) transported to other sites around the globe. For the Japanese architect Shigeru Ban, this project also posed the additional challenge of applying his favored theme of using cheap and recyclable materials but here on a monumental scale.

The Nomadic Museum was first located in 2005 in New York City's Meatpacking District, stretching out along Pier 54 located on the Hudson River; its second installation occurred at a Santa Monica pier in California. The New York version's layout consisted of two parallel "building" units running some 200m (656ft), each of which was composed of 152 used shipping containers that were stacked four units high, and all still having their owners' corporate logos and the scratches and bruises from heavy shipping use. The containers were attached to each other at their corners so as to form a strong checkerboard pattern of solids and voids. To create the needed enclosure, the voids between the containers were filled with a sloping tensile PVC membrane, which was also used to create the doubly pitched roof for the temporary building. The exhibition hall had similarities in plan organization to that of a simple basilica: between the massive exterior container walls were two rows of paper-tube columns defining a central nave, the floor of which was covered with reused wooden boards. In the side aisles between the column lines and container walls was the exhibition space for hanging the large-scale photos of the artist Gregory Colbert; these spaces were further distinguished and defined by changing the ground cover to that of a simple layer of gravel.

One of the fascinating aspects of Ban's work over the years has been his use of recycled paper tubes for structure which, perhaps counterintuitively, prove to have the strength necessary for supporting a roof or for performing as members of a truss; and this despite their being cheap, low weight, low-tech structural elements that are easily replaceable. Ban refers to cardboard as "evolved wood," which helps to convey the notion that these materials can have similar qualities, and that one is the source for the other (see Section 5.6). In this project, the cardboard-tube columns were 750mm (30in) in diameter and 10m (34ft) high, and these supported at their top triangular roof trusses made of a strategic combination of smaller paper tubes with post-tensioned steel rods threaded through them (for a similar basic system module, see Section 6.6). The central triangle of tubes over each pair of columns worked wonderfully to give the roof its overall height and sloped profile as well as to provide it with the necessary stability to be able to resist gravity loads and lateral wind forces. (Ill. 9.14.)

Aside from these elements' structural and space-defining functions, the kit-of-parts construction method for the Nomadic Museum's container wall and its tube-and-tension-rod roof truss structural system lent itself extraordinarily well to being quickly disassembled, packed up, and then sent on to the next destination (as could be the project's boardwalk and lighting and other equipment). A few of the containers in the side walls could be used to ship these elements to the next site, while the remainder that were needed to fully reconstruct the walls could be rented at the exhibition's next port of call. Ban's sustainable design approach for the use and reuse of building elements, both found and manufactured from recyclable materials, couldn't be any clearer.

Illustration 9.14
Nomadic Museum, New York City, USA (2005).
View of central space of temporary gallery building with long rows of cardboard tube columns and the roof's central triangular arrangement of smaller tubes – at once giving the pitched roof its overall form but also establishing its stability and load-carrying capability.
Architect: Shigeru Ban. Structural engineer: Buro Happold.

9.5 How Trusses Work

Following our qualitative studies of the truss' inherent geometric stability and of visualization techniques for anticipating which of its members are likely to be subject to compression and tension forces, a more analytical study involving a few calculations will provide a way to develop a more complete understanding of the workings of this structural form. This is important for two reasons: first, it can confirm our initial predictions about the nature (tension or compression) of the axial forces in the various members of a truss and, second, a numerical analysis of forces will be useful for determining the shape and size of individual members as well as establishing a logic for the overall form of the truss itself. For pedagogical reasons, we will show how such calculations can be carried out by traditional hand methods, which have proven to be helpful in conveying a deeper understanding of just how trusses function. The reality, though, is that calculating all truss member forces by hand for the various load conditions that exist can be a rather cumbersome and time-consuming affair, and so in contemporary practice this task is generally left to being handled by computer structural analysis software programs.

Historically, two methods have typically been applied to calculating forces in statically determinate trusses. The first of these conveniently uses the fact that the various members of a truss are aligned to be coincident through its many pin-joints, and considers the equilibrium of each of these connections in turn; for obvious reasons, this strategy is known as the *method of joints*. We have described truss member forces as being either compressive or tensile, and these will necessarily cause reaction forces that act on individual truss joints in the form of pushing or pulling forces, respectively. At a truss joint where these multiple vector forces meet we know that we must have equilibrium or else the joint will translate or rotate, which we obviously know not to be the case. In Chapter 4 Statics we discussed the conditions necessary for having equilibrium of any generalized set of intersecting forces, and we will apply these same principles here in the context of truss joints.

In order to study in detail the requirements for equilibrium of a joint in a truss and, thereby, show how we can find member forces from this, we will illustrate the use of the method of joints by considering the specific example of a portion of one of the roof trusses of the Mont-Cenis Training Academy in Herne, Germany. (Ill. 9.15.) Here, a glass-and-solar-cell-clad big box of an enclosure system is created by long-spanning wooden roof and side-wall trusses, thereby creating a large and spatially open interior volume. Within this overall exterior building envelope, a number of smaller buildings have been inserted – in a "houses-within-a-house" manner – thereby creating an interior, campus-like atmosphere uniting the training academy facilities, associated housing units, commercial spaces, various municipal offices, and public spaces. The section drawing through a portion of this building features two wooden trusses: one that is horizontal and carrying the roof's gravity dead and live loads to its tall column supports and the other one vertical and bearing the glass façade against sideways wind pressures and suctions. (Ill. 9.16.) Our limited focus of attention here will be on the short cantilevering portion of the roof truss, to the left of the column support in the drawing. (The vertical truss is not considered to be providing any support at the outer end of the roof truss which can, therefore, be taken to be a freely deflecting cantilever structure to the left of the column support.)

Let us now imagine that a vertical point load P acts on the outermost joint j_1 of the cantilevered part of the roof structure, as shown in the analytical line diagram of Figure 9.6. This load sets up a force in the diagonal member and a force in the horizontal member intersecting at that joint; anticipating the deformations that will take place under the load allows us to confidently predict that the diagonal member is in compression C_1 and the horizontal member is in tension T_1. Studying the equilibrium of force vectors C_1 and T_1 and load P *graphically* confirms that the forces are indeed compressive and tensile, respectively. The vectors point in toward the joint (pushing) and out from the joint (pulling), corresponding to compression and tension in the members. Since equilibrium is a necessary requirement, the two member forces and the load P together make up a closed triangle of force vectors, ending up with there being no net resultant force acting on the joint. (It will be recalled from Chapter 4, Sections 4.3, 4.5, that from a graphical perspective intersecting forces are in equilibrium when the force vectors form a closed polygon.) This requirement provides graphical information for both the direction and the magnitude of the various forces involved. By repeatedly applying the method of joints in this graphical manner we could, if we wished, consider the equilibrium of all the joints of this truss and find the forces in all of its members, and this was quite a common approach historically.

Illustration 9.15
Mont-Cenis Training Academy, Herne, Germany (1999).
View of exterior, showing roof structure supported by a series of transverse wood trusses.

Architect: Jourda architectes & Perraudin architectes & HHS Planer+Architekten AG. Structural engineer: Arup GmbH; Schlaich Bergermann und Partner.

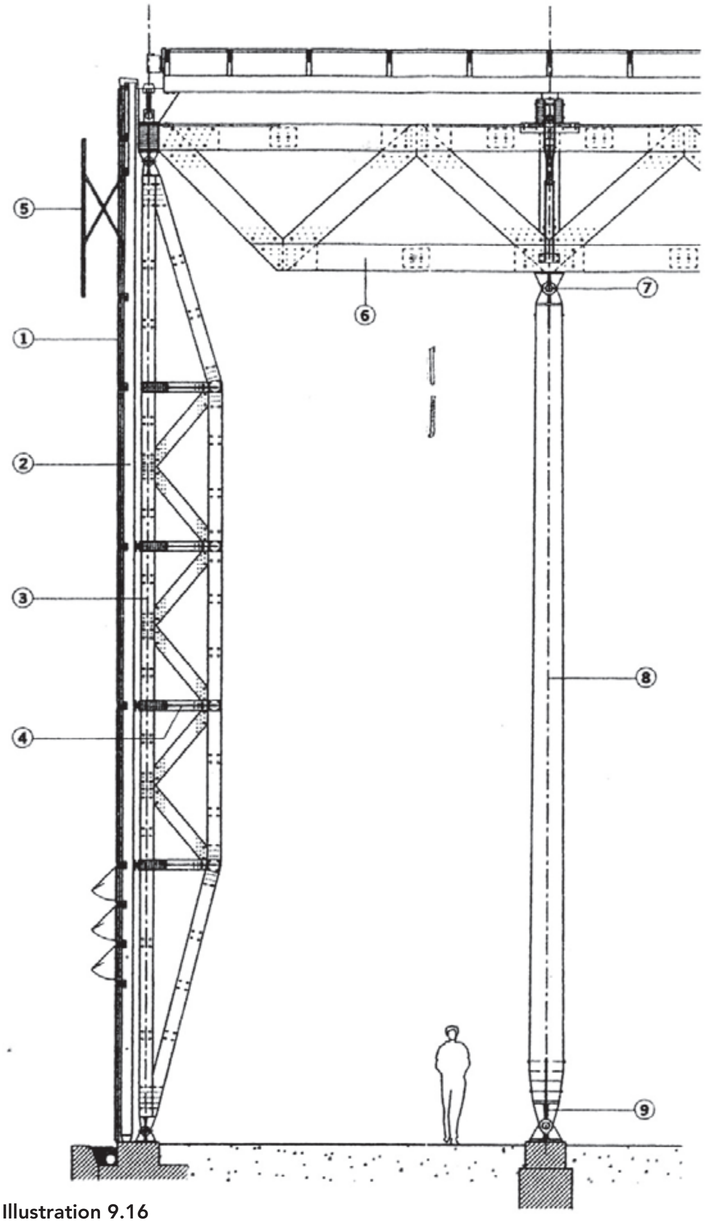


Illustration 9.16
Mont-Cenis Training Academy.
Section drawing showing end of roof truss and one of its column supports, as well as a trussed mullion bracing the façade against wind loads.

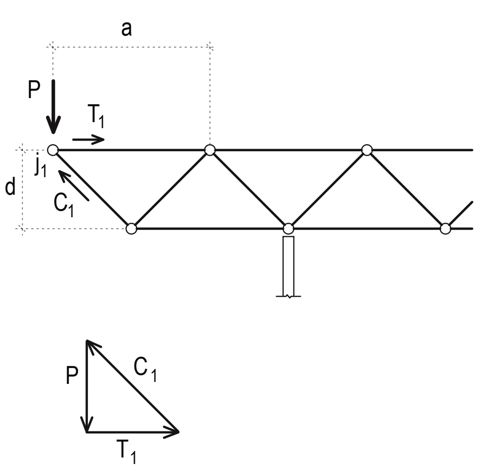


Figure 9.6
The *method of joints* applied to one end of the Mont-Cenis Academy truss: force diagrams can be used to graphically analyze the equilibrium of joints. The forces acting on the joint j_1 are what are being shown.

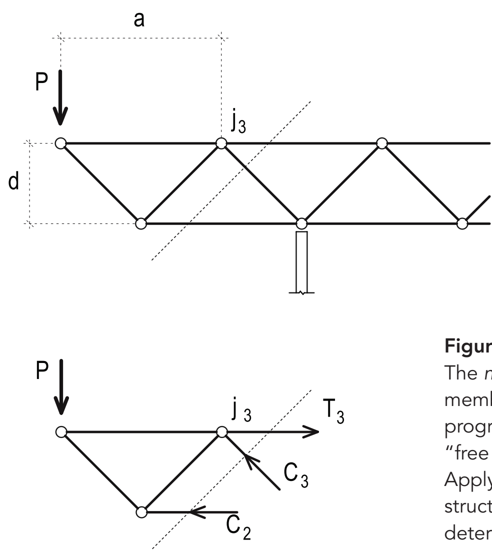


Figure 9.7
The *method of sections* offers a way of finding selected member forces without having to calculate member forces progressively through the truss, one joint at a time. A so-called "free body" diagram is shown of a sub-assembly of the truss. Applying the three equilibrium equations that apply to a planar structure will enable the three unknown member forces to be determined.

If, instead of doing this graphically, we wish to calculate the member forces *mathematically*, it can also be recalled from Chapter 4 that the requirements for equilibrium offer a set of equations that involve, and that will therefore allow for the solution of, the unknown member forces. Since at a joint we are considering the equilibrium of forces that all pass through a common point, rotational equilibrium is not involved and we thus have at our disposal only the two equations that establish translational equilibrium, namely $\Sigma F_x = 0$ and $\Sigma F_y = 0$. These equations imply, of course, that if the sum of all forces acting horizontally on the joint and of all forces acting vertically on it are equal to zero, then we can be assured that the joint is in equilibrium and at rest. Solving with these equations for the unknown member forces will allow us to establish their magnitudes and directions. See Section 6.4 for a more detailed presentation of this process - even though there it was done in the context of a joint that was part of an inclined hanger system, the application of the method here is just the same; i.e., the analytical method of joints does not care whether we are dealing with forces in hangers or in trusses, as abstracted force vectors are all handled the same way when it comes to mathematical calculations.

It should be noted, however, that when using the method of joints for analyzing a truss, because we have only two equations of equilibrium available at each one of its joints, we must begin the whole process at a location where there are no more than two unknown member forces – which often will be at an end support point or at the tip of a cantilever. The rather tedious aspect of the method of joints when it is applied to overall trusses, therefore, is that it is typically necessary to go through the equilibrium calculations for many joints before eventually getting to the one that may be of primary and immediate interest to the designer; i.e., in most cases there is no way to jump into the middle of a truss and directly solve for the member forces at that location without having first gone through the analysis of many other joints beforehand. This prospect can certainly help to explain the reason for the development of an alternative method for determining truss member forces.

This second technique, called the *method of sections*, is in many cases a more effective and efficient approach to the problem of finding the member forces in a truss which are thought to be of critical concern.[2] According to this technique, we need to consider the conditions for equilibrium of a *portion* of the truss (i.e., not just of a joint), identified by imagining a line cutting through the structure's 2-D line-diagram representation and that thus divides the truss into two sub-assemblies. (Fig. 9.7.) Of course, any portion of the truss must be in equilibrium just as much as the whole of it is. According to the conceptual abstraction of this method, the member forces that pass through the imaginary cut line are notionally considered to be "external" forces, provisionally acting on the selected portion of the truss. That sub-assembly must of course be held in equilibrium by the sum of all forces acting on it (including any and all support reactions, external loads, and cut-member forces). Since we must ensure that any portion of the planar truss is *prevented both from translating and rotating* and since we have three equations of equilibrium at our disposal to do so, we will be able to calculate *directly* the magnitudes and directions of up to three unknown member forces *anywhere* in the truss. All of these statements will be made clearer by means of the example that follows.

If we wish to calculate the three forces in members C_2, C_3, and T_3 of the Mont-Cenis Academy truss, we can make an imaginary cut through the truss that passes through these members and then establish a set of three equilibrium equations for the chosen sub-assembly. For one equation, for example, we can see that the member force C_3 has a vertical component that has to balance the vertical load P. Hence, $\Sigma F_y = 0$ yields

$$C_{3y} - P = 0, \text{ or}$$
$$C_{3y} = P$$

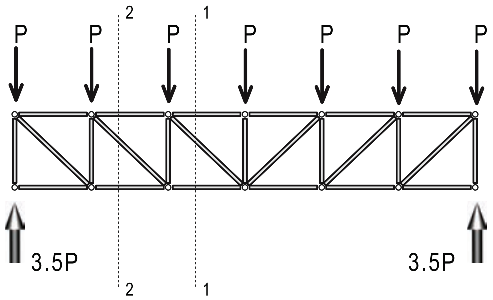


Figure 9.8
A simply supported Pratt truss with a total of 7P vertical point loads. Since this structure's proportions and transverse loading render it beam-like, we anticipate that the top chord will be in compression and the bottom chord in tension. The diagonals change direction at the centerline; all are thus acting in tension while the vertical members act in compression. Sections 1-1 and 2-2 define two sub-assemblies that can be analyzed for equilibrium, seeking out the member forces by the method of sections.

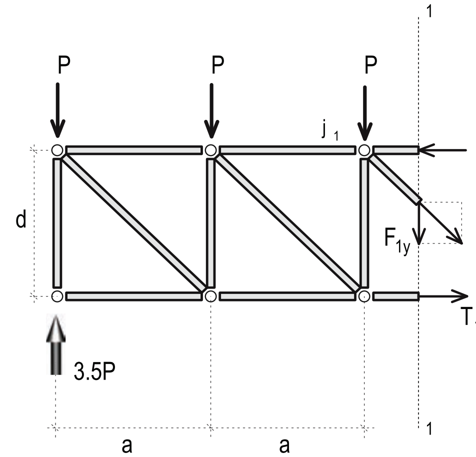


Figure 9.9
Free-body diagram of sub-assembly cut at section 1-1, showing loads and cut-member forces as "external forces."

In writing this equation, we have taken as a sign convention that positive vertical forces are directed upward (+y) and we will do likewise for horizontal forces acting to the right (+x).

From trigonometry we know that

$$C_3 = C_{3y}/\sin\alpha = P/\sin\alpha$$

where α is the angle between the force vector and a horizontal line.

Next, we can make sure that no rotation takes place about joint j_3 (the choice of this point is arbitrary; in principle, it could just as well be any other); the requirement for this to be true is that ΣM = 0 about that point. Since neither force T_3 nor force C_3 have a moment arm with respect to joint j_3, the moment equation will only comprise load P and force C_2:

$$+C_2d - Pa = 0$$
$$C_2 = Pa/d$$

where a = load P's moment arm (equal to the length of the horizontal member between P and joint j_3) and d = the structural depth of the truss.

Finally, a third equation expresses the requirement for horizontal equilibrium, $\Sigma F_x = 0$:

$$+T_3 - C_2 - C_{3x} = 0$$
$$T_3 = C_2 + C_{3x} = C_2 + C_3\cos\alpha$$
$$T_3 = Pa/d + P\cos\alpha/\sin\alpha = Pa/d + P/\tan\alpha$$

We should note that in order to write the above equations, we have had to make assumptions about the directions of the unknown forces that represent the effect of the cut members, and these assumptions have direct implications on whether these members are considered to be in tension or compression. Fortunately, if we make an incorrect assumption our error will be indicated by the force magnitudes coming out as negative in the end; a negative sign accompanying the calculated magnitude of a member force simply means that the direction of the force in the cut member is opposite to the one that we initially assumed.

All three unknown member forces that we set out to find in this example have now been established in terms of the truss' dimensions and of the load P, both of which we would typically be in a position to determine in a specific situation. It is also important to note in passing that these equations are indicating that the magnitudes of the forces in the members of the truss depend not only on the external loads that are being carried but also on the geometry of the truss itself; this is a topic that we will come back to in Section 9.7.

Before that, though, we can conveniently use the method of sections to study various force patterns in common trusses. For example, we can examine the so-called Pratt truss shown in Figure 9.8. Vertical equilibrium of the symmetrical truss will require two vertical support reactions of magnitude 3.5P.

Section 1-1 isolates the left part of the truss shown in Figure 9.9. Rotational equilibrium (ΣM = 0) about joint 1 of this sub-assembly requires that

$$+3.5P(2a) - P(2a) - Pa - T_1d = 0$$

Since neither of the unknown forces in the top chord or in the diagonal member F_1, nor the load P acting on the joint have any moment arms about joint 1, these forces do not need to appear in the writing of this equation. The only unknown force, therefore, is the tension force in the bottom chord, T_1; solving for it yields

$$T_1d = 7Pa - 2Pa - Pa = 4Pa$$
$$T_1 = 4Pa/d$$

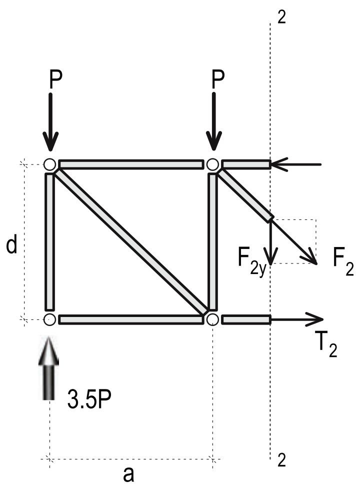


Figure 9.10
Free-body diagram of sub-assembly cut at section 2-2, showing loads and cut-member forces as "external forces."

A similar analysis of the small sub-assembly to the left of section 2-2 (Fig. 9.10) leads to

$$+3.5P(a) - Pa - T_2d = 0$$
$$T_2d = 3.5Pa - Pa = 2.5Pa$$
$$T_2 = 2.5Pa/d$$

Note that if we initially had let T_1 and T_2 point the other way and thereby initially assume that they are compression forces acting on the sub-assembly, those forces would have come out as negative at the end of solving these equations. This result would indicate that such an initial assumption for their direction would have been incorrect, and that they are truly tension forces – as our earlier qualitative methods for predicting which truss members are in tension or compression would lead us to expect.

Now, comparing the relative magnitudes of T_1 and T_2 shows that *the tension in the bottom chord decreases toward the supports for this simply supported truss. The same applies to the compression force in the top chord*. It will be recalled from Chapter 7 that this pattern reflects the one that we have observed for bending moment diagram variations for simply supported beams (and of the directly associated tension and compression stresses in the lower and upper parts of beam cross-sections, respectively); this is a general characteristic for trusses, or what we can see now could logically be called "trussed beams." Furthermore, looking at the algebraic expressions for the tension forces T_1 and T_2 above, we can observe that the larger the truss depth d becomes, the smaller the axial forces in the chords will be. Therefore, *increasing the height of a truss for a given load condition means that the forces in the top and bottom chords will decrease*; this is generally true for trusses of all kinds, and is analogous to what we saw in Chapter 7 that bending stresses are reduced if we move material away from a beam's neutral axis.

Shifting our attention now to the diagonal members, we can look at the requirement for vertical equilibrium ($\Sigma F_y = 0$) of the sub-assembly to the left of section 1-1 in Figure 9.9:

$$3.5P - P - P - P - F_{1y} = 0$$
$$F_{1y} = 0.5P$$

where F_{1y} is the vertical component of force F_1 in the diagonal member. Letting F_1 point out from the free-body diagram defines this as a tension force. Since the calculation resulted in a positive force (+0.5P), our assumption about the nature of the diagonal force was correct. From this simple study of equilibrium we can conclude that under gravity loading the *diagonals in Pratt trusses act in tension*. Conversely, *the vertical members can be shown to be all in compression*.

As a result of the second imaginary cut (see Fig. 9.10), vertical equilibrium of the corresponding sub-assembly to the left of section 2-2 yields

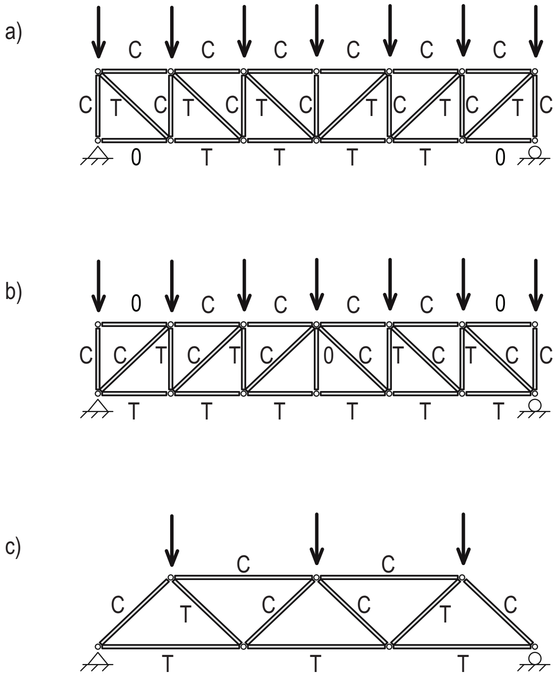


Figure 9.11
Diagrams of (a) Pratt, (b) Howe, and (c) Warren trusses (having parallel top and bottom chords) showing compression C and tension T in the members due to downward loads at each of the top joints. Note that a few members carry no forces in these load conditions.

Illustration 9.17
Hopkins House, Hampstead, UK (1976).
Interior view of the house, with Warren-truss-like configuration of open-web steel joists left in full sight. Diagonals' constantly changing orientation will cause them to alternately be in tension and compression. Relative openness of these trusses allows light to penetrate deep into the living space, and prevents the visual interruption that solid beam webs would cause.

Architect: Michael Hopkins & Partners. Structural engineer: Anthony Hunt Associates.

$$3.5P - P - P - F_{2y} = 0$$
$$F_{2y} = 1.5P$$

Since F_2 is closer to the support than F_1, and $F_{2y} > F_{1y}$, we observe that *the forces in the diagonal members of the truss increase toward the supports*. This observation is generally applicable for all interstitial (both diagonal and vertical) members, and recalls the pattern that we observed for the variation of shear force in solid beams.

Looking once again at some of the various truss configurations that we saw in Section 9.2, we can now make a number of observations and rationalize some of the visual patterns that we see there. For example, a symmetrical Pratt truss is visually quite distinctive in having its diagonal members change direction at mid-span (Fig. 9.11a); by applying the results of the preceding analysis to both ends of the truss it can be confirmed that such an arrangement results in all of the diagonals being in tension and all of the vertical members being in compression, as long as the loads act vertically downward. As we saw with such trusses at the Leonhardt House (see Ill. 9.10), this implies that the tension diagonal members can be designed to be quite thin, thus minimizing any potential obstruction of view. With the so-called Howe truss (Fig. 9.11b), on the other hand, the diagonals are all sloped in the opposite direction to the way they are in the Pratt, with the consequence that all the diagonals will be in compression (and thicker, therefore) for exactly the same load case, and the vertical members will be tension. Finally, the Warren truss' zigzagging diagonals (Fig. 9.11c, e.g., Ill. 9.17) can be shown to alternate tension and compression forces from one diagonal to the next, reversing the pattern at mid-span.

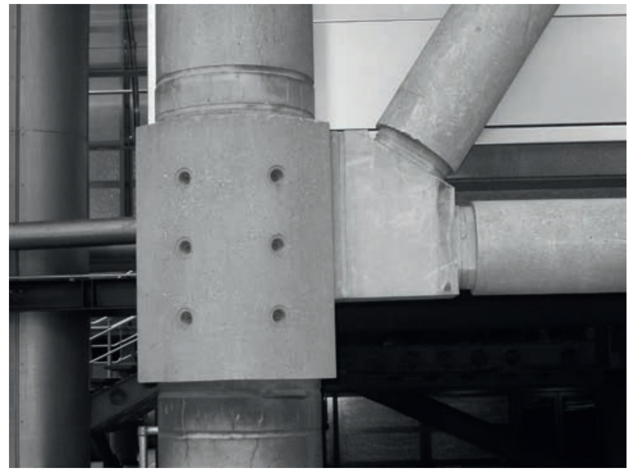
Before we leave these diagrams showing the truss member forces that can be anticipated based on the equilibrium considerations that we have just been discussing, we have a bit of a curious feature to consider: close scrutiny of the Pratt and Howe trusses shown in Figure 9.11 reveals that even for the very common case of downward vertical loads a few of the members of such trusses are, in fact, not supporting any loads whatsoever, i.e., they are neither in tension nor in compression. In general, such so-called *zero force members* meet other truss members at right angles, with no other members connecting in to the joint at any other angle that could balance a push or pull from the perpendicular member. In spite of this, and what would seem to be an opportunity to get rid of "useless" parts of the truss, we should keep in mind that this situation might change completely under a different load condition. For example, if a load were to act up from below and directly on the vertical member at the mid-span of the Howe truss (perhaps from wind uplift, for example), this member would then be subjected to a compression force instead of being a zero force member.[3] Moreover, a load hanging from below the truss at mid-span would result in a tension force acting in that same vertical member.

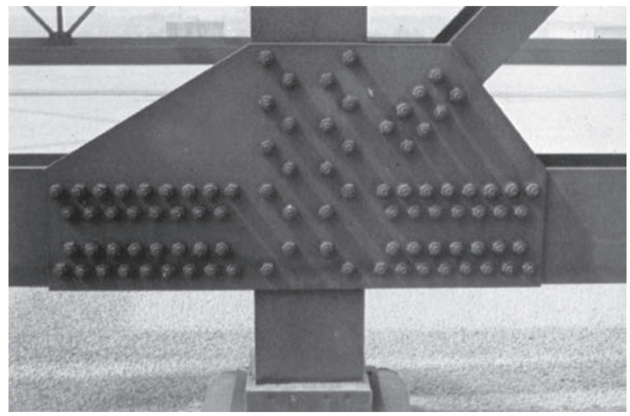
Illustration 9.18
Four joints in plane trusses of different materials.
(a) Concrete joint at Lloyd's building, London, UK (1986).
Some eccentricity at joint since centerlines of all members do not meet at a single point.
Architect: Richard Rogers Partnership. Structural engineer: Arup.

(b) Joint in Cor-Ten steel in Wills Factory, Bristol, UK (1974).
Gusset plate allows for multiple members to be connected together; centerlines all intersect.
Architect: YRM with SOM. Structural engineer: Felix Samuely.

(c) Joint in laminated wood truss, Haakon's Hall, Lillehammer, Norway (1993).
Cuts through wood are visible where three steel plates are hidden that are used to connect the members; member centerlines all intersect.
Architect: Østgaard. Structural engineer: Reinertsen Engineering.

(d) A hybrid joint combining steel and reinforced concrete in the Educatorium, University of Utrecht, the Netherlands (1997).
Architect: OMA. Structural engineer: ABT.


a)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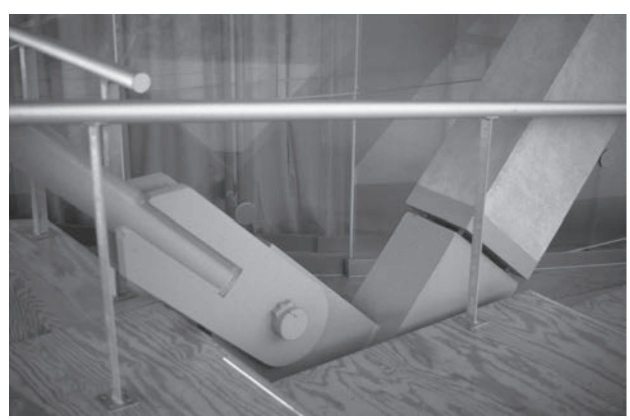

d)

9.6 Joint Adventures

So far in this chapter, we have obviously based quite a lot of our understanding about how a truss works on the so-called *pin-joint* – but what do such connections actually look like? We have taken for granted the assumption that truss members are connected by joints that allow all connected members to rotate independently of each other, and this is a necessary concept to prevent bending moments from being created in the connections and which thus allows all members of a truss to be subjected to pure axial forces, either compressive or tensile.

It is relatively easy to imagine trusses made from a number of members that are all ideally pin-connected, where the joints behave like true hinges. Historical truss bridges are well known for having members connected in this fashion. This is rarely practical or desirable for most trusses today, however, partly for economical and maintenance reasons, and the chords, in particular, are frequently made from one continuous element. The interstitial diagonal and vertical members in trusses are commonly connected to the upper and lower chords by the help of steel plates and bolts or, in the case of metal trusses, by means of direct welding, and the resulting connections may, therefore, be quite far removed from being able to behave like true pin-joints.

In reality, then, joints often have a certain rigidity which may enable some local bending moments to develop in the vicinity of the joint, producing stresses that will be additional to the stresses from the axial forces – and the joints will indeed have to be designed for these additional stresses. However, to prevent having bending moments further out along the length of the members of a truss, and so that these can be as slender and efficient as possible, we typically try and have their extended centerlines intersect, thus identifying a theoretical point in each connection about which the member centerline axes have no eccentricity. And if any external loads are also applied through such points, then these will have no moment arm that could potentially produce bending moments in the members – so this is clearly how truss members should be aligned if possible and also where trusses should be loaded and supported.

Trusses made from different materials like steel, timber, concrete, or aluminum or even combinations of materials may have a certain superficial likeness in overall form, but their connections often offer design problems and opportunities in which each material appears with a uniqueness of form and technical resolution. The detailed design of truss joint connections often expresses just what a material is really capable of. (e.g., Ill. 9.18.)

9.7 How Trusses Look

Overall Shape Variations

Horizontal trusses that span simply between two supports commonly have *overall shapes* that fit into one of four main types: parallel chord trusses, pitched trusses (which can also be inverted), bowstring trusses (convex upward or downward), and lenticular trusses which are lens shaped and also informally called "fish-belly" trusses. (Fig. 9.12a–d.) Likewise, horizontally cantilevering trusses are often shaped either with parallel top and bottom chords or else with diminishing profile depth the further one gets from the support. (Fig. 9.12e–f.)

The general strategy for those simply supported truss forms having larger structural depths at mid-span than toward the supports (or, conversely for the cantilever, of larger depth toward the support) is to provide increased resistance to overall external moment where this is needed most. Here we need to think in terms of maintaining overall equilibrium at each section cut through a truss, while recalling our analogous discussion in Chapter 7 about the variation in the magnitude of bending moments over the length of a beam.

Starting with the parallel chord truss and highlighting its way of resisting external moments will clarify just what we are taking about: such a truss counters external moments by means of couples formed by the forces in their upper and lower chords. (Fig. 9.13.) In order to be able to maintain rotational equilibrium we will need to have that $M_{ext} = M_{int} = Cd = Td$, where C is the compression force in the upper chord, T is the tension force in the lower chord, and d is the truss depth. Since the moment produced by the external loads M_{ext} is largest at the mid-span of a simply supported structure

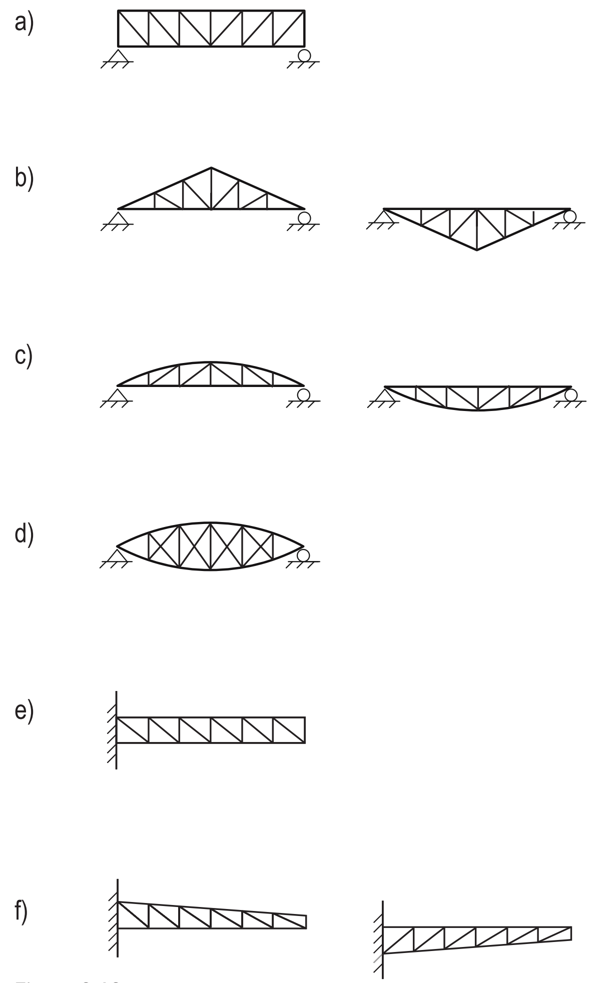


Figure 9.12
Some overall shapes of trusses that act like beams.

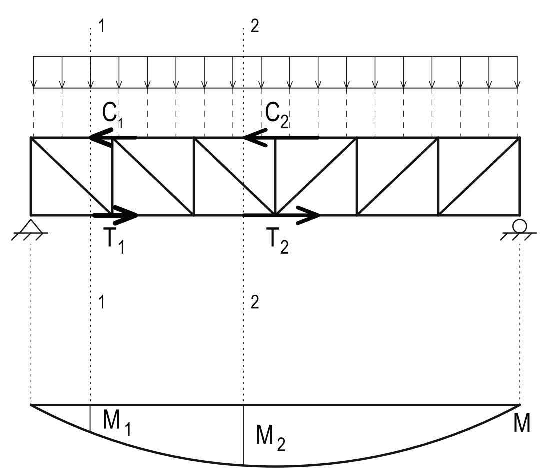


Figure 9.13
Parallel chord truss with external moment resisted by a couple formed by the forces in upper and lower chords. Variation of the moment diagram leads to a similar change of chord forces along the length of the truss. C_2 is larger than C_1, T_2 is larger than T_1, while $C_1 = T_1$ and $C_2 = T_2$.

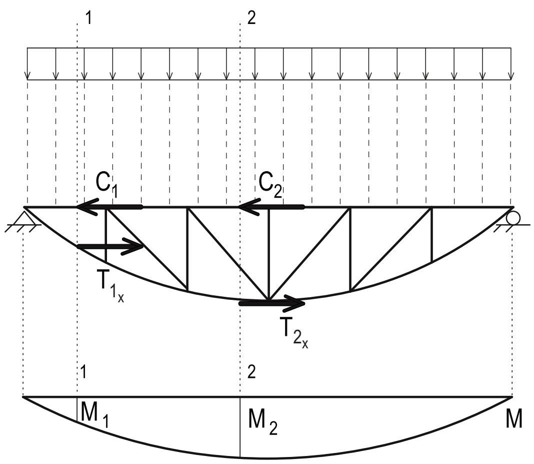


Figure 9.14
Illustration of the horizontal component, constant along the length, of chord forces in a bowstring truss with a parabolic lower chord designed to follow the variation of the moment diagram. $C_1 = C_2 = T_{1x} = T_{2x}$.

to which symmetrically distributed loads are applied (e.g., recall the shape of the bending moment diagram for such beams), the compression and tension forces in the truss' chords will also need to be largest at that location (since C = T = M/d).

If we next consider the effect upon this of increasing the overall depth of the truss at mid-span, it is evident that for the same loading the axial forces in the truss chords will be able to be smaller than they were before, while still producing the required balancing moment. Toward the supports, the external moments become smaller, and so if we correspondingly shape the truss depth to follow this variation we end up with something that we recognize as the familiar bowstring truss configuration, whose structural depth gradually lessens toward the ends. (Fig. 9.14.) Of course, the consequence of such a shaping of the truss is that the chord forces along the entire length of the truss become more uniform in magnitude.[4] Therefore, in shaped trusses of this kind we may be able to use members having the same cross-sectional dimensions for all the chord members, thereby contributing to construction efficiency through a simplification of member specifications and the ensuing manufacturing logistics. Conversely, of course, for trusses with parallel chords the repeated use of the same cross-section necessarily implies a certain loss of material efficiency since it is only used to full capacity near the mid-span.

Illustrating these contrasting truss-shaping strategies, the constant truss depths of, for example, the Joso High Bridge and the Mont-Cenis Academy trusses (see Ill. 9.9 and 9.16, respectively) can be compared to the truss depth variations of the Berlin Olympic Stadium and Bern Tram Depot that we will examine more closely in the next section, which closely follow these structures' overall bending moment diagrams. (See Ill. 9.28 and Ill. 9.31.)

Member Size and Shape Variations

Moving on from this discussion of variations to the overall or global form of the truss, we will next elaborate on the sometimes significant differences between a truss' individual component members. *What can be deduced about the design and necessary size and shape of individual truss members*? It is not difficult to imagine that there may need to be distinction between members that are in tension and those in compression; i.e., since slender compression members are apt to buckle, they will generally be expected to have larger cross-sectional dimensions than the tension members. Such a difference of member sizes and shapes is often quite obvious to recognize in the top and bottom chords of trusses that are used for very large spans. (e.g., Ill. 9.19, 9.20.) This will not always be the case, however, especially for smaller trusses, because the demands for efficient production and manufacturing may argue against the individual design and specification of each member in such a truss. Besides, a designer may wish to maintain a unity of member proportions throughout, notwithstanding any material savings.[5]

But beyond such fairly obvious distinguishing between top and bottom chord compression and tension forces, there is plenty of opportunity in the truss to further vary member sizes and shapes, if perhaps more subtly. As we know, the external bending moment that needs to be balanced varies along the span of a truss and so too, then, will the magnitude of the chord forces – which means that the top and bottom chord members sizes can be varied from one end of a truss to the other. As we have also established, overall shear demands vary in a truss from one end of a span to the other, even if differently from the way external bending moments change – and so this can lead to its own independent logic for the

Illustration 9.19
Royal Albert Bridge, between Plymouth and Saltash, England, UK (1859).
The heavy train loads that this bridge was designed to carry led to its having an overall lenticular shape as well as dramatic differences in the cross-sectional shape of its compression vs. tension chords.

Designer and structural engineer: Isambard Kingdom Brunel.

Illustration 9.20
Centre Georges Pompidou, Paris, France (1976).
The Warren-type trusses that typically clear-span nearly the full width of this multistory museum building demonstrate differences between the dimensions of compression (top) and tension (bottom) chord members, with the former having a larger diameter to prevent buckling. Moreover, the tension members are solid bars while the compression members are circular hollow profiles which more efficiently provide resistance to buckling tendencies.

Architect: Renzo Piano and Richard Rogers. Structural engineer: Arup.

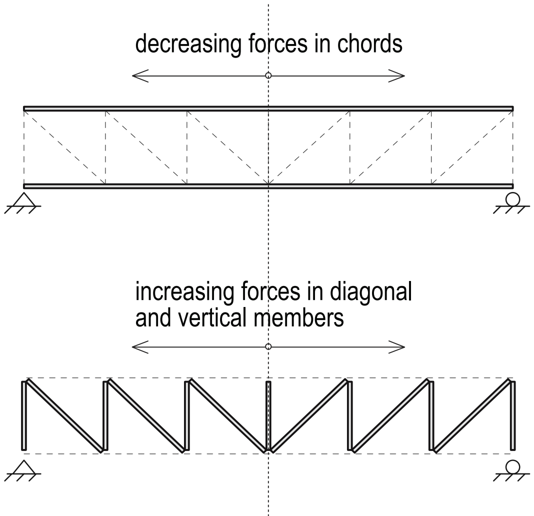


Figure 9.15
Diagram showing how the magnitudes of the forces change along the truss length. Forces in the chords are larger at mid-span than closer to the supports, while the interstitial members experience larger forces the closer they get to the supports. This reflects the analogous situation for bending moments and shear forces in beams that are simply supported.

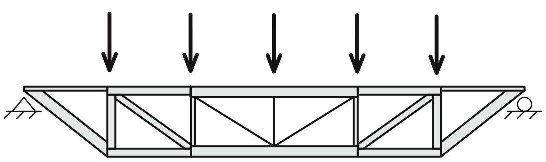


Figure 9.16
Parallel chord truss.
Illustration of possible variation of member sizes according to the variation of member axial forces. Chords may actually become smaller toward the supports as the overall moment demand decreases, while interstitial members may become larger, following the increase of shear force.

sizing and shaping and density of interstitial members. All of these possible variations are summarized diagrammatically for a simply supported truss in Figures 9.15 and 9.16. For example, the Xstrata Treetop Walkway's steel trusses illustrate the need for increasing shear capacity toward the supports by making explicit its gradually steeper and shorter diagonal truss members. (Ill. 9.21.)

There are also situations in which truss member forces can change from one load condition to another, with members in compression to begin with and then in tension, or vice versa. In this case, the member will obviously have to be designed for the condition that puts the most demand on the member, which almost inevitably will be when it is in the compression condition. We will recall from Chapter 8 that it is advantageous to design compression members to have symmetrical cross-sections about both cross-sectional axes so as to prevent their buckling, and that hollow sections fit this bill

Illustration 9.21
Xstrata Treetop Walkway, Kew Gardens, London, UK (2008).
This walkway runs 200m (656ft) among the trees at Kew and is raised 18m (59ft) above the ground, enabling a unique vista of the garden from the treetops. A continuous truss bridge supported on pylons, the structure features a variation of the density of the structural members: as the trusses approach the supports, the density of diagonal members increases in response to the increase of shear forces. The spacing of the diagonals is dictated by the so-called Fibonacci sequence, a mathematical relationship between numbers in a series often associated with nature's forms.[6] This principle is used both for the vertical trusses which form the balustrade, and the walkway deck structure which forms horizontal trusses resisting wind.

Architect: Marks Barfield Architects. Structural engineer: Jane Wernick Associates.

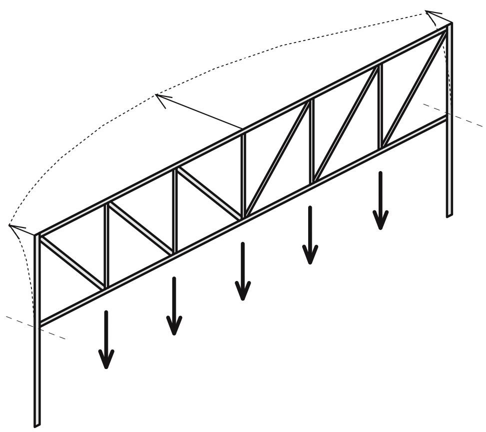


Figure 9.17
Possible buckled shape of a truss failing by local buckling of its compression chord.

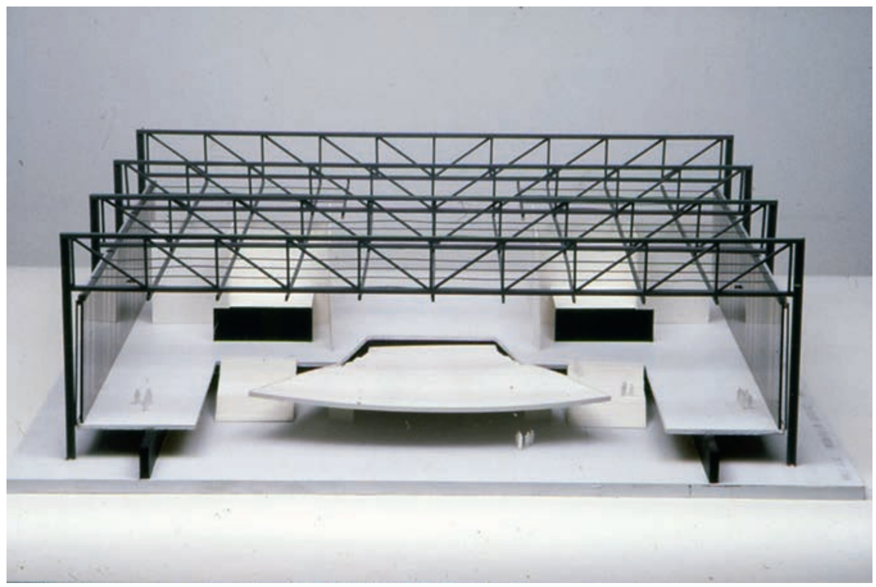


Illustration 9.22
Project for a theater, Mannheim, Germany (1953).
Model showing exposed steel trusses spanning the entire space of the proposed theater. When the truss structure is above the plane of the roof, there is no bracing of its compression chord, which then needs to be designed in size and shape to be adequate to prevent local sideway buckling.
Architect: Ludwig Mies van der Rohe. AHO model by Niels Marius Askim and Lars Lantto.

exceptionally well; this type of member is, therefore, often found in trusses. Moreover, since the slenderness ratio of a compression element also depends on the member's length, it is generally a good strategy to keep the compression members of a truss as short as possible, thereby increasing a truss' structural efficiency by using less material. As for the truss members that do in fact remain in tension under all load conditions, these may be designed as solid bars just as effectively as any other cross-sectional shape, thereby having a much thinner appearance than their compression counterparts.

Lateral Bracing and 3-D Truss Variations

We must also consider the effect on truss member shapes and sizes caused by the typical need to provide lateral bracing to the compression chord of a truss, which will be at the top for a simply supported condition subject to gravity loading.[7] This is analogous to our discussion in Chapter 8 about beams' tendency to be subject to *lateral-torsional buckling* or *warping* for the part of these that are subjected to compression stresses. Considering the typically long lengths of trusses, the chances are good that their compression chord will be likely to buckle laterally if measures are not taken to prevent it from doing so. (Fig. 9.17.) When secondary roof or floor elements rest on and are connected directly to this upper chord, lateral bracing of this member is typically easily provided by means of the large in-plane stiffness of the roof or floor plate. In some cases, however, trusses support roofs that are attached to their bottom edge, effectively leaving the upper compression chord unbraced laterally. (e.g., Ill. 9.22.) One way of accounting for this is to design a trusses' compression chord members to have an asymmetrical cross-section; i.e., having them be wider in the

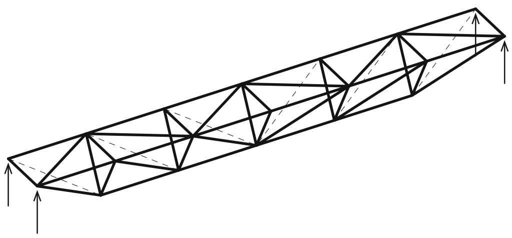


Figure 9.18
Three-dimensional trusses, or space trusses, have good stiffness in the lateral direction, as well as significant torsional stiffness.

Illustration 9.23
Fuhlsbüttel Airport terminal building, Hamburg, Germany (1993).
Three-dimensional space trusses with triangular cross-sections support the long span roof.
Architect: von Gehrkan, Marg und Partner. Structural engineer: ARGE Kockjoy-Schwarz + Dr. Weber.

lateral direction than in their vertical, in-the-truss-plane dimension, thereby reducing their lateral slenderness ratio and increasing their buckling capacity. For example, the side-by-side double-hollow-tube top chord for the Pompidou Centre's gallery-spanning trusses illustrates this approach quite well. (See Ill. 9.20.)

Further along this same line of thinking, a natural development is to use *three-dimensional trusses*, or *space trusses*, that can through their cross-sectional geometry provide far more lateral stiffness than can the individual elements of a planar truss. (Fig. 9.18., e.g., Ill. 9.23.) The 3-D truss commonly has a triangular overall cross-section, with either one tension chord and two compression chords spaced apart but connected intermittently in order to reduce these members' lateral unsupported length or else two tension chords and one compression chord with the latter laterally braced by the slanting of the two sets of interstitial members. Also, a square or rectangular overall cross-sectional configuration can obviously be made to work, since in this case the bracing can be provided to both the top and bottom chords – often by means of transverse horizontal trusses connecting between the main load-carrying trusses – effectively creating a 3-D trussed tube structure. An example of this configuration can be seen with the Høse Bridge in southwestern Norway designed by the architects Sami Rintala and Dagur Eggertsson and the structural engineering office of Dr. Techn. Kristoffer Apeland, whereby top and bottom transverse trusses serve not only to laterally brace the compression chords of the main side trusses but also to create the roof and walkway of this uniquely trussed bridge. (Ill. 9.24, 9.25.)

Illustration 9.24
Høse Bridge, Sand, Suldal, Norway (2013).
Truss diagonals' orientations vary; steel side plates in the middle of the bridge reorient the view, acoustic environment, sense of space downward toward the water.
Architect: Rintala Eggertsson Architects. Structural engineer: Dr. Techn. Kristoffer Apeland AS.

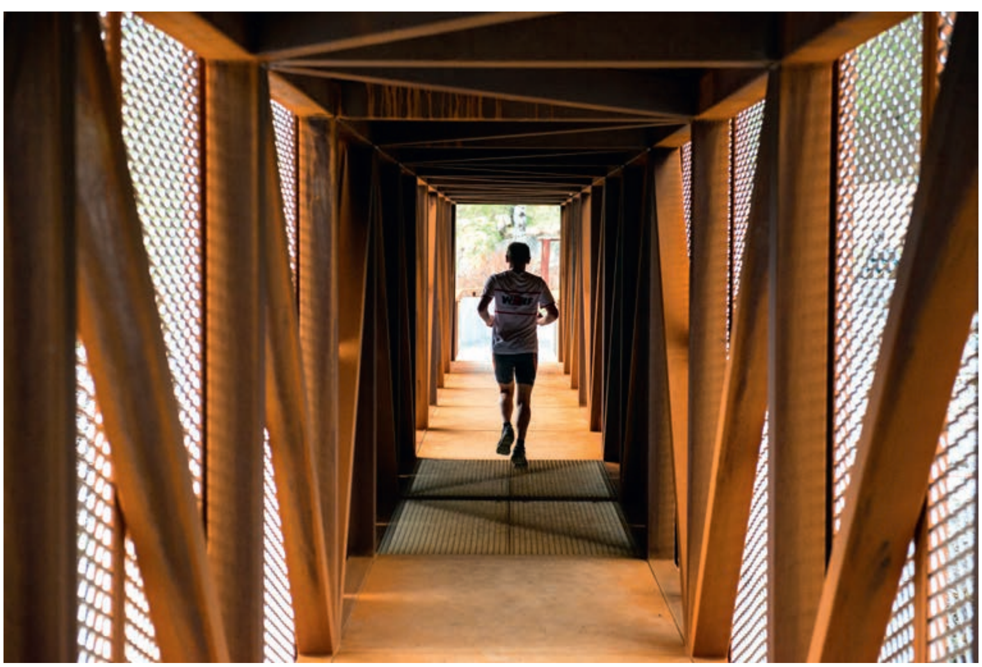


Illustration 9.25
Høse Bridge.
Horizontal, transverse trusses combine with vertical side trusses to form an overall trussed tube, providing bracing to the chords of the vertical side trusses as well as a system for resisting lateral wind loads. At mid-span, floor becomes open steel grating.

a)

b)

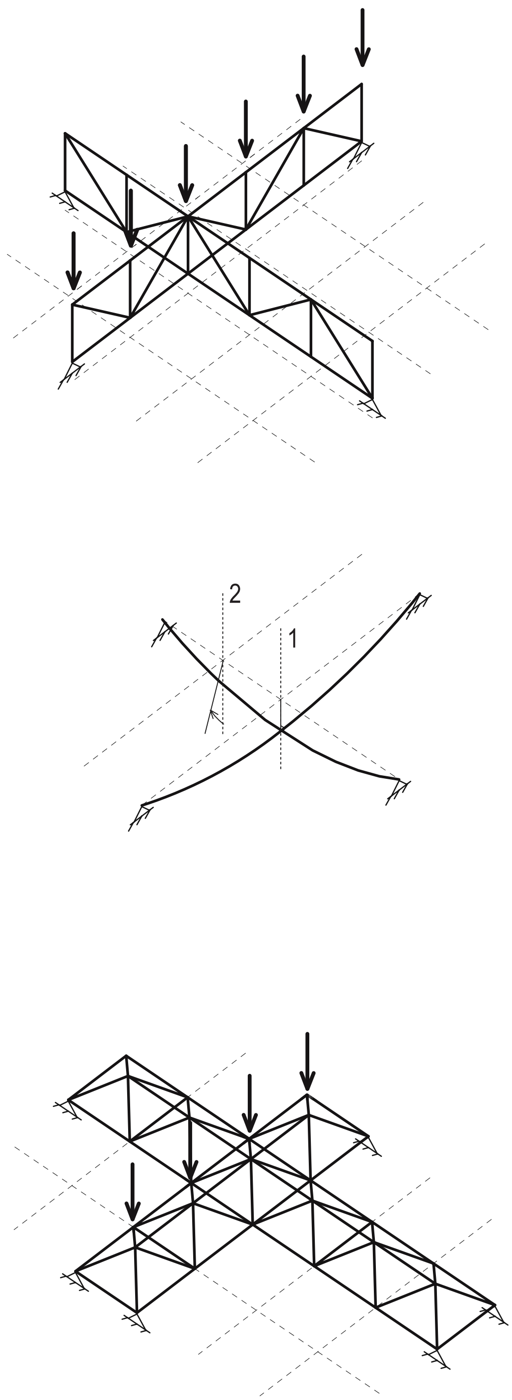


Figure 9.19
Truss grids of (a) plane trusses and
(b) space trusses and their deformations.

Truss Grid Variations

Yet another strategy for avoiding the buckling of compression chords is to combine multiple trusses together into the design of a *planar-truss grid* or even a *3-D truss grid* system. Planar truss grids, in which a set of two-dimensional trusses intersect with each other in a two-way or even a three-way arrangement, will significantly reduce the unsupported length of any of the planar trusses' compression chords, thereby improving their resistance to buckling. Two-way rectangular grids are commonly established parallel to the support boundaries or perhaps diagonal to them. (Fig. 9.19a.) Aside from reducing buckling tendencies, such grid systems present the additional structural advantage of sharing applied transverse loads among a number of trusses, with the same improved load-carrying benefit that we saw with beam grids in Chapter 7 in Section 7.10.[8]

The 3-D or space-truss grid is an improvement on this combined system yet again. (Fig. 9.19b.) Because of their overall hollow, tube-like cross-section, individual space trusses have considerable torsional stiffness. This causes a grid of such trusses, where intersecting 3-D trusses are connected to each other top and bottom, to be become especially stiff. The reason again goes back to our discussion in Chapter 7 about beam grids since, as we saw there, the vertical deflection of a beam (or of a truss, as is the case here) running in one direction results in both the vertical deflection and the *twisting* of the intersecting beams (or trusses); moreover, the 3-D truss is well equipped to resist *both* of these types of deformation, resulting in a very stiff system. Such a grid of 3-D trusses is, therefore, capable of carrying larger forces, or spanning greater distances or, conversely, of deflecting much less than plane truss grids. In a unique vertical extrapolation of such advantages, and especially of the ability of interesting alignments of 3-D trussed tubes to resist bending and twisting behavior simultaneously, the Nanjing Sifang Art Museum's rising, folded, 3-D tube truss configuration stands out as especially noteworthy. (Ill. 9.26.)

Illustration 9.26
Nanjing Sifang Art Museum, Nanjing, China (2010).
Three-dimensional space truss with rectangular cross-section rises and folds into a unique continuous walkway configuration for this art museum. Hollow trussed form is well suited to resisting significant bending and twisting behavior that results from the cantilevering corners of this museum/walkway.
Architect: Stephen Holl Architects. Structural engineer: Guy Nordenson & Associates. Cornell model by Kenneth Chow and Stefan Krawitz.

9.8 Two Trussed Roofs in Berlin and Bern

Because of the relatively efficient use of material in trusses, this structural subsystem has frequently been used to span long distances or else to carry exceptionally heavy loads or both at the same, such as for train or roadway bridges or for athletic stadium or industrial shed roofs, etc. In all of these cases, the very large flexural demands being imposed by the applied transverse loads tend to favor giving the trusses maximum depth where the bending moments are largest, and then reducing this dimension as the moment demand goes down – i.e., such trusses become very clear and distinctive expressions of these structures' overall bending moment diagrams. In this section, we will look at two examples of such expressive truss forms.

Olympic Stadium in Berlin

The Olympic Stadium in Berlin, which was designed by architect Werner March (1894–1976) in 1936, was carefully modernized in 2004 by architects von Gerkan, Marg und Partner; as a result, it combines the facilities and features of a modern arena with the substance of a historical building. The wide oval of the stadium, then as now, is broken at the Marathon Gate at the west end, leaving an unobstructed view toward the distant bell-tower of the city's original Olympic facilities. The exterior appearance of the stadium is almost the same as when it opened in 1936; only the edge of its new slim roof minimally reveals the changes that have taken place inside.

Within the stadium, the new roof's filigree steel construction

Illustration 9.27
The Olympic Stadium, Berlin, Germany (1936/2004).
Overall view of the stadium interior.
Architect: von Gerkan, Marg und Partner. Structural engineer: Krebs und Kiefer; Schlaich Bergermann und Partner.

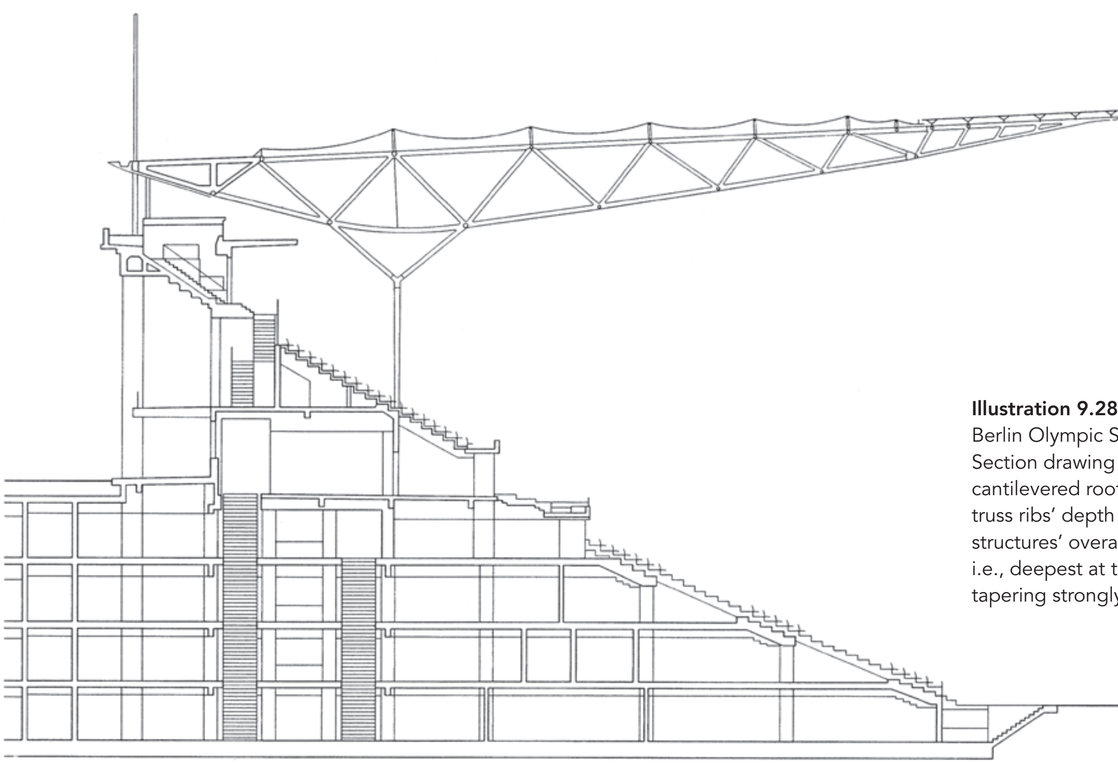


Illustration 9.28
Berlin Olympic Stadium.
Section drawing showing trussed, cantilevered roof over grandstand. The radial truss ribs' depth follows the profile of these structures' overall bending moment diagram; i.e., deepest at the cantilever support and tapering strongly toward the free end.

and translucent cladding intentionally contrasts with the heavy gray sandstone bulk of the historical stadium. Following the original plan of the stadium, the new roof that protects the tiered seating has been left open at the Marathon Gate. This break in continuity precluded the use of an inner tension ring that would have been necessary for the suspension roof systems that are so often found covering many modern stadia (see Chapter 11). Instead, the roof is carried by 76 radial trusses that dramatically cantilever into the stadium interior, the whole arrangement having been likened to the skeleton of an airplane wing. (Ill. 9.27, 9.28.) Each truss projects inward toward the playing surface a remarkable distance of almost 50m (160ft) and has a total length of 68m (220ft). The truss depth clearly follows the profile of the bending moment diagram for such a cantilevering condition: both are maximum directly in line with the column support and taper to almost nothing at the free end above the playing surface and at the "back"/outside end where the truss is anchored down. Of course, this tapering of the truss depth lends itself well to opening up the views and to orienting the overall sense of space toward the playing surface from the vantage point of the seating situated below the roof.

To counterbalance the effects of their tremendously large projections toward the interior of the stadium, the cantilever trusses are supported on a circumferential space truss (having triangular cross-section and whose depth is integrated with that of the cantilevering trusses) and by having their outer ends tied down. The space truss is supported by 20 so-called structural "tree" columns spaced around the building; each of these 250–350mm (10–14in) diameter cylindrical steel "trunks" have four cast steel "branches" that reach out spatially to carry the truss. The vertical tie-downs are anchored to the original stadium columns as well as to a reinforced concrete ring that adds the necessary counterweight for the equilibrium of this daring roof structure.

Most of the roof's upper surface consists of a membrane made of Teflon-coated translucent fiberglass textile with transparent panels underneath that allow a certain amount of light to reach the stadium seating, while still permitting a view from below of the roof's trussed structure. Toward the center of the stadium, the roofing becomes a single layer of glass panels that can clearly be seen to be supported by the ends of the radial trusses. All of the conduits and other infrastructure needed for the sound systems and lighting of modern-day shows and events have also been integrated into the roof, thereby rendering obsolete the need for the typical towering (and design-conflicting) masts found in many other such large stadia.

Tram Depot in Bern

The Neue Tramdepot Bern was designed as a close collaboration between architects and structural engineers for a tram parking and maintenance facility located on the outskirts of that city, both to address current needs but also expressly anticipating a two-phase expansion in the coming years as the urban transportation network expands. The initial building fits into a relatively tightly constrained site, and has two long side-by-side spaces for tram parking and for their maintenance/washing that run along the 200m (650ft) stem of the L-shaped building as well as an interior turn-around set of tracks located within the enlarged foot portion of the L. (Ill. 9.29.)

Twenty-six transverse trusses span across the parking and maintenance spaces of the first phase of this industrial shed, supported on columns situated along the outside edges of the building as well as along a central row of columns. (Ill. 9.30.) The truss depth clearly follows the overall bending moment diagram for such a "continuous-beam-like" condition; i.e., deepest over the central supports and tapering toward the outside edges. (Ill. 9.31a.) For the second phase, the parking space for the trams is dramatically increased by extending each of the transverse trusses outward to a repositioned set of exterior column supports. This has clearly been anticipated, however, and it is not a matter of tearing down the existing structure but rather "clipping" onto the end of each existing truss a significant truss extension segment. (Ill. 9.31b.) For the third phase, only the "foot" portion of the L-shaped building is enlarged, but this time even more dramatically by adding more tram parking within an extension that is perpendicular to the existing lines of parking. The truss extensions for this addition still work off of the existing alignments of the original trusses, resulting in exceptional spans of 90m (300ft). (Ill. 9.31c.) The bending moment diagram for this condition will result in its having a second peak of almost equal value to the first, but this time near mid-span of this extension segment. The at-first seemingly bizarre profile of the roof truss then makes perfect sense, with one peak located over the interior column support point and the other near the middle of the elongated span.

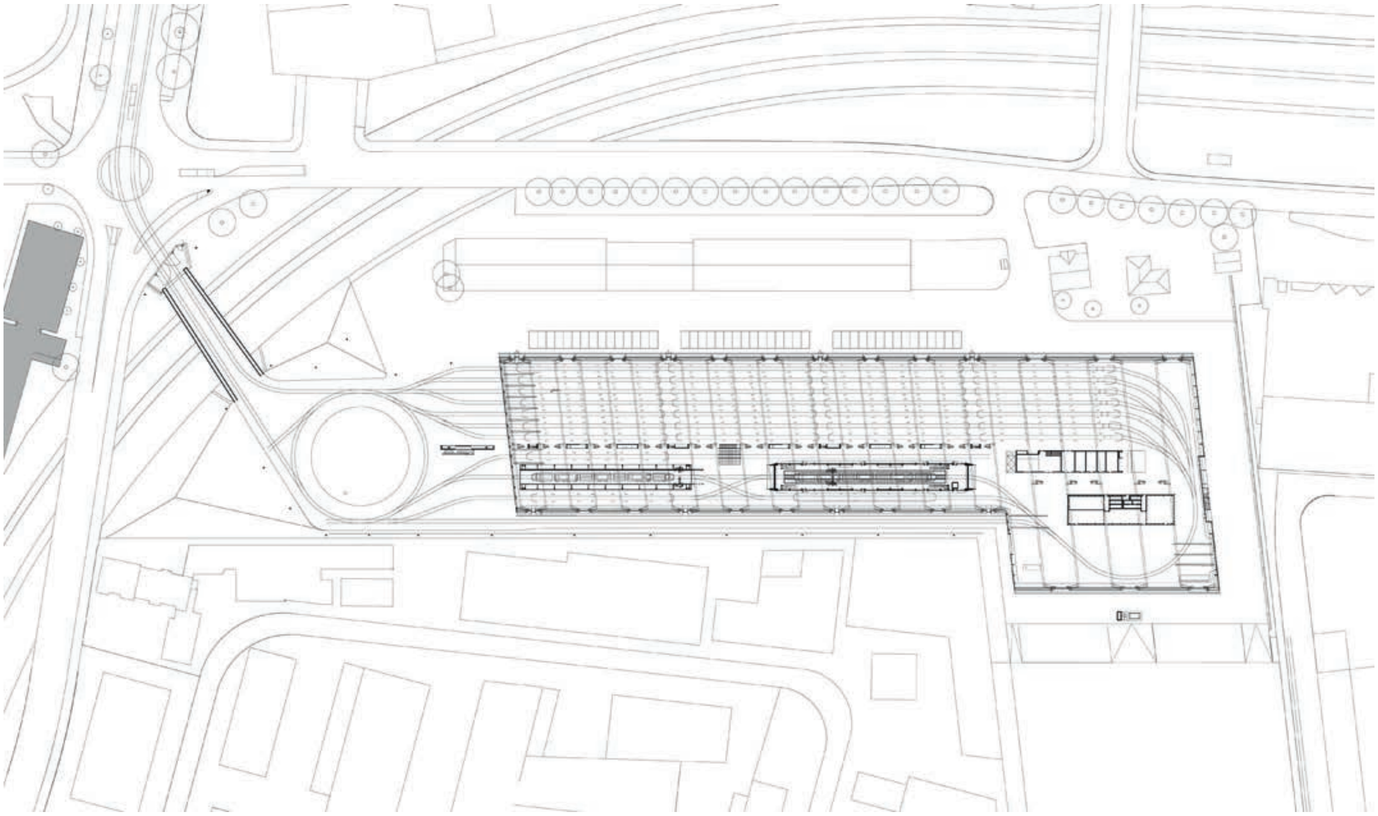


Illustration 9.29
Neue Tramdepot Bern, Switzerland (2011).
Plan of L-shaped building and site in initial configuration, showing long tram parking and maintenance spaces as well as enclosed turn-around at one end.
Architect and structural engineer: Penzel Valier AG.

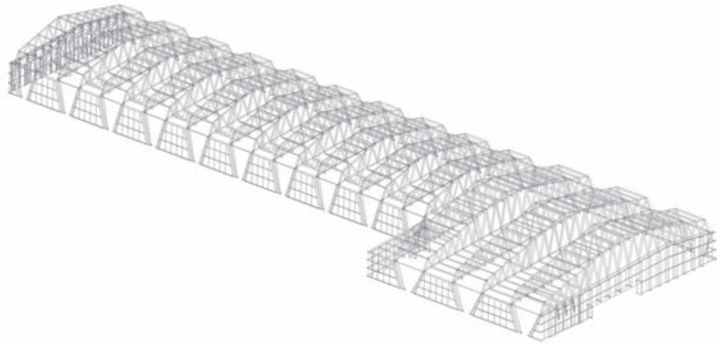


Illustration 9.30
Neue Tramdepot Bern.
Twenty-six transverse trusses span across the parking and maintenance spaces of the first phase of the project.

The perimeter envelope for this unheated, "utilitarian" tram shed largely consists of translucent white glass that allows the movement and activities taking place within to be sensed but not directly seen during the daytime and for the building volume to become a glowing lantern at night. The openness of the roof trusses also allows natural daylight to bathe the interior space, thereby minimizing energy demands. (Ill. 9.32.) Furthering such an environmentally friendly design approach, the roof also is partly covered by bands of solar cells and it collects rainwater that is stored for washing the trams. Finally to be noted here is the angled orientation of the lines of columns supporting the roof – these are inclined both along the main axis of the building but also in the transverse direction as well – so as to provide lateral stability for the building in every direction, a topic that will be discussed much more extensively in the next chapter. The Neue Tramdepot Bern is a building that integrates well not only its trussed structural system into its overall architectural expression and programmatic function, but other design concerns as well – and all this over the course of many years and several planned expansion phases. This is a building, then, that takes to heart the timeless lessons about design forethought.

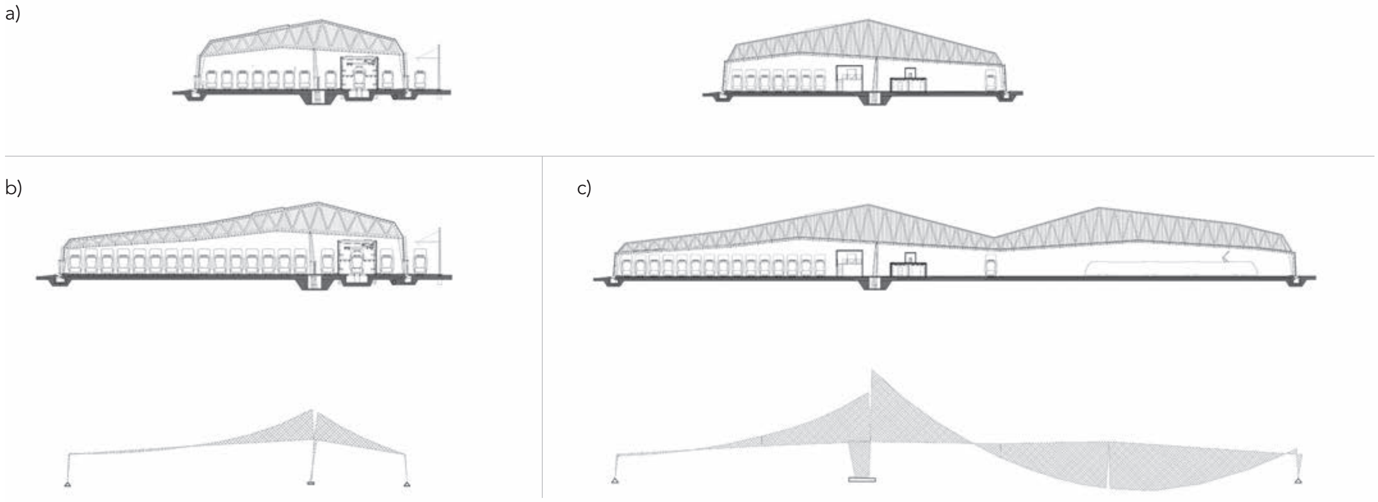


Illustration 9.31
Neue Tramdepot Bern.
a) Section drawings for the original L-shaped building; b) & c) section drawings and corresponding bending moment diagrams for the two anticipated expansion phases.

Illustration 9.32
Neue Tramdepot Bern.
Interior view of space, with natural daylight penetrating through relatively open trusses.

Illustration 9.33
"La Tour Rouge" (1911).
Eiffel Tower exhibits well the three-dimensional spatial form possibilities of a trussed structural system.
Painting by Robert Delaunay.

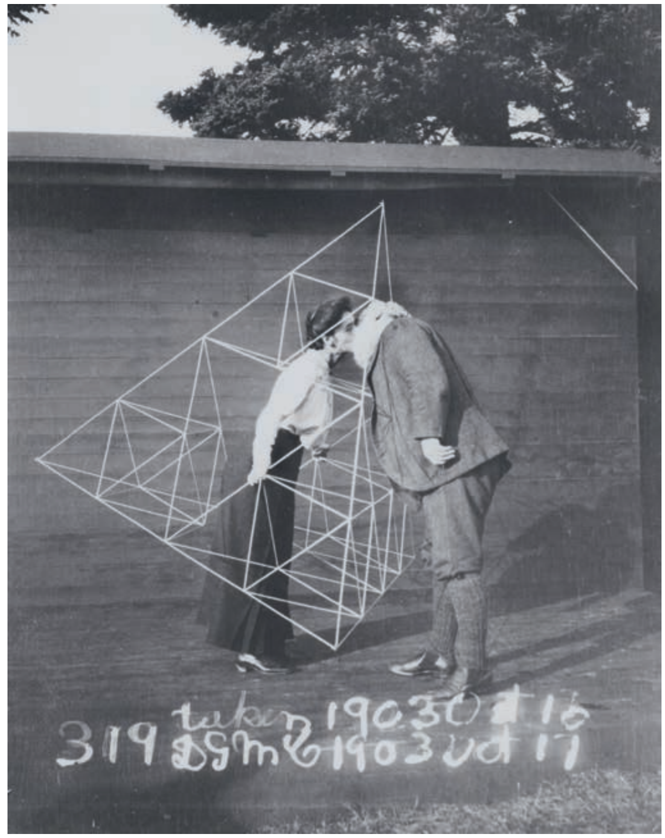


Illustration 9.34
Early space frame "experiment" (about 1900).
Inventor: Alexander Graham Bell.

9.9 Space Frames – 3-D Truss Action

Truss-like surface and spatial structures are called *space frames*. Just like planar (2-D) trusses, such structures are lightweight and constructed from interlocking struts forming stable geometric patterns that have long been recognized as being able to carry transverse loads very efficiently. Indeed, in spite of its name, with its avant-garde "Space Age" connotations, what is effectively space frame construction has been with us for quite a long time in the sense of its *spatial* form of structural framing. Sophisticated church tower timber structures of the Middle Ages (see Ill. 4.25) and the nineteenth-century wrought iron structures of the Eiffel Tower in Paris (1889; Ill. 9.33, 10.32) as well as the Firth of Forth Railway Bridge near Edinburgh (1890; Ill. 7.36) are all iconic examples of different forms of spatial truss construction. But the invention of the "conventional" space frame as we have come to know it today is generally credited to the Scottish-American inventor Alexander Graham Bell (1847–1922).[9] Around 1900, Bell worked with light and efficient spatial structures for passenger-carrying kites and observation towers. (Ill. 9.34.) In the 1920s, the American Richard Buckminster Fuller (1895–1983) made his own entrance on to the scene of space frame development, one in which he would have a long-lasting legacy (e.g., see Section 13.1 for his famous geodesic "Bucky Dome" in Montréal). And eventually, it would indeed turn out that the space frame's more literal association with the Space Age is something not completely without merit, as satellites and space stations and proposed experimental space colony structures have been designed and built over the past half century using this system's fundamental principles, which we will review presently.

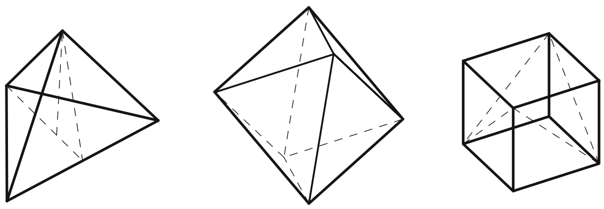


Figure 9.20
Some close-packing polyhedra; the tetrahedron, the octahedron, and the cube. When the cube's squares are braced by diagonals, all three polyhedrons are stable figures.

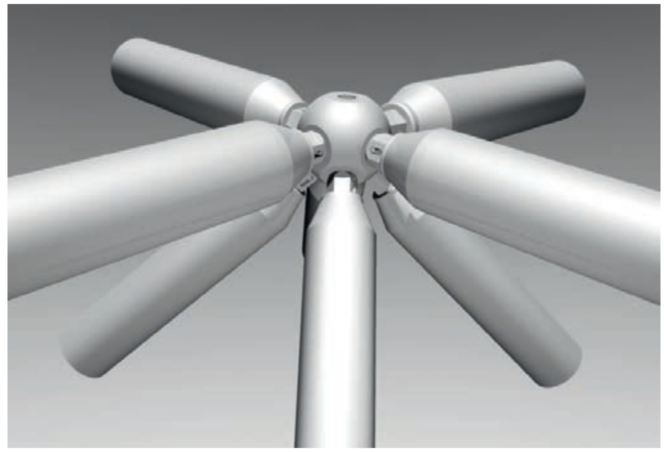


Illustration 9.35
Typical steel joint for space frames by MERO.

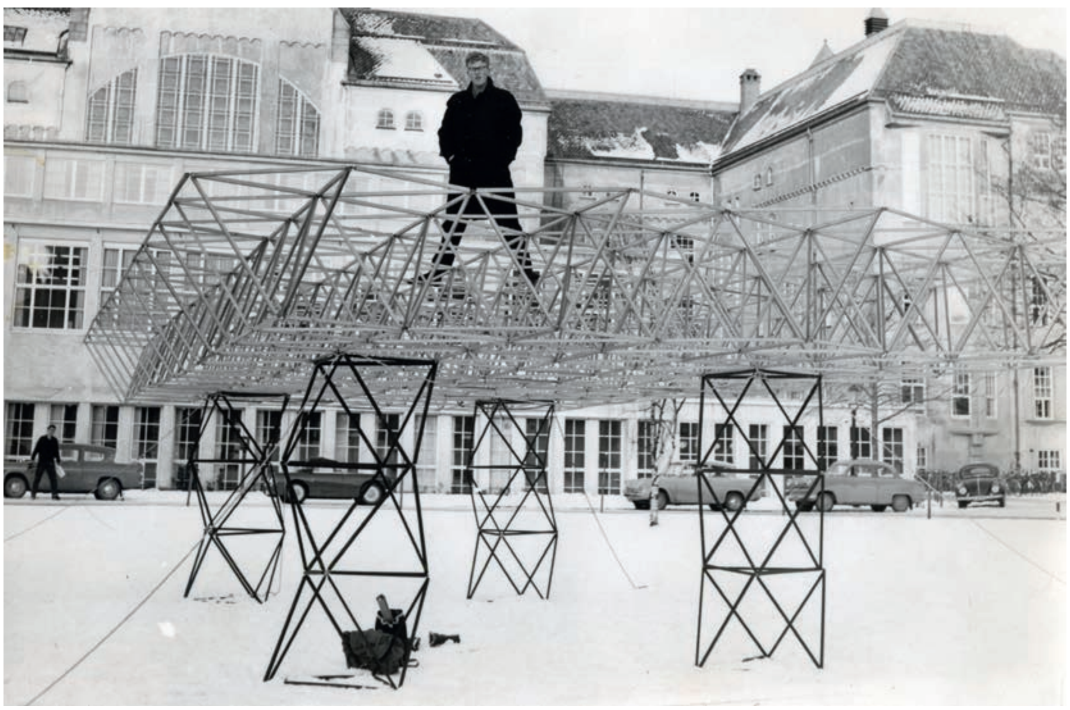


Illustration 9.36
Space frame study by students at the Technical University, NTH, Trondheim, Norway (1963).
The structure is based on a series of tetrahedrons, and the 20 by 20mm (0.75 by 0.75in) members were sawn from wooden drawing boards. Co-author Arne Eggen proudly acts as a point load.

Space frames are made from axial structural members organized in spatial, stable, so-called polyhedral units that can be seen as open building blocks. (Fig. 9.20.) The connections between these axial members or bars are considered to be hinged, often by means of highly specialized and patented joint systems. (Ill. 9.35.) And, similarly to plane trusses, the loads that are applied to space frames and the supports that are provided are expected to act directly at the joints.

Typical space frames are given sufficient structural depth so that they can form an open, multi-member and double-layered "thickened" planar surface that can be considered to have considerable stiffness with respect to transverse loading (e.g., Ill. 9.36) – and thereby not needing to rely on strategic curvature or folding to obtain such stiffness, although these strategies can also be used to provide space frames with even greater structural capacity. But even the most common and basic form of a flat space frame can span freely over considerable distances while maintaining an air of remarkable lightness. In terms of its overall structural behavior, such space frames can be thought of as analogous to a two-way concrete slab, and some of the lessons learned for that structure in Chapter 7, Section 7.12, can be reapplied here: e.g., in order to take advantage of the multi-directional spanning and load-sharing potential of space frames we should provide supports that are distributed fairly uniformly in each direction, and in such a way that the difference between the span lengths is not too great.

A space frame typically consists of top and bottom surface grids that are mutually connected to each other by means of diagonal

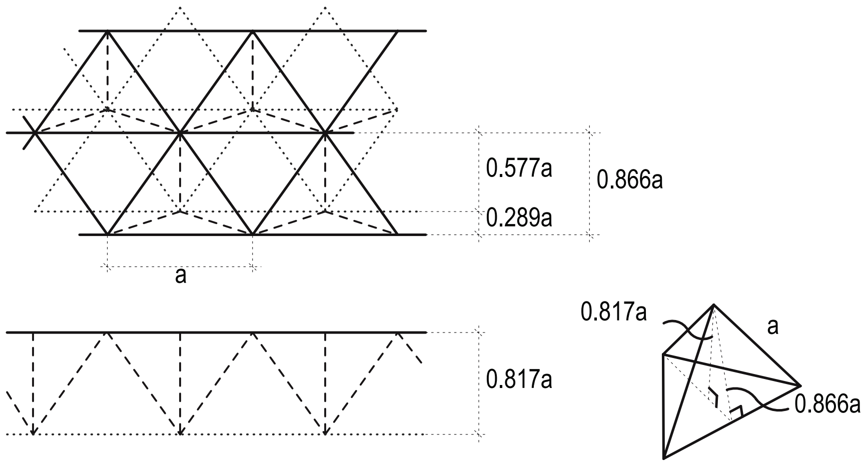


Figure 9.21
Space frame with equilateral triangles on one layer offset from equilateral triangles on the other. It is made from one member length only, and achieves a particularly high stiffness by having three intersecting trusses (that are inclined and continuous) distributing loads in three directions. Plan and section. To ease the spatial reading of the diagram, different line symbols are used; full lines are "level 1," which means they depict members closest to the point of observation; dotted lines are "level 3," meaning they represent structural members farthest from the point of observation, finally dashed lines are "level 2" that connect members of levels 1 and 3.

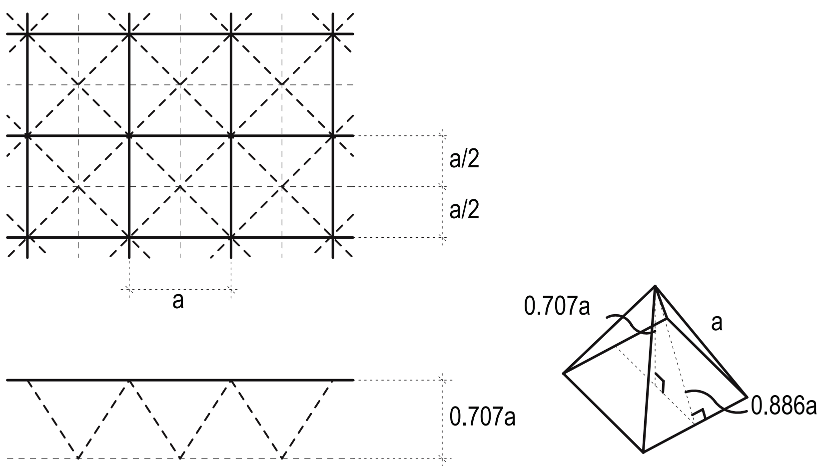


Figure 9.22
Space frame with squares on offset squares. The close-packing polyhedra are the tetrahedron and the half-octahedron, and only one member length is employed. For key to the line symbols, see Figure 9.21.

members, the whole system thereby forming a three-dimensional network of struts or bars. The geometry can also be interpreted as that of a close packing of diverse polyhedra, which are spatial figures composed of at least four faces (called polygons) intersecting along their edges. (Fig. 9.21.) The "edges" in this case are formed by the structural members, and those in turn intersect at the corners (vertices) in structural joints. The so-called regular polyhedra are known as the five Platonic solids.[10] Of those, the *tetrahedron*, which is a pyramid of four equilateral triangles, the *octahedron*, which may be seen as two pyramids with a square base joined along the base with the finished figure having eight faces of equilateral triangles, and the self-evident *cube*, all fill space by themselves or by combining with each other. While both the tetrahedron and the octahedron are stable, the cube needs to be braced by diagonals in order to be stable as a spatial figure. If a space frame is composed entirely of closely packed and stable polyhedral "building blocks," then the space frame as a whole is surely an internally stable structural system.[11]

A form that has been of particular interest is the space frame having equilateral triangular grids in both top and bottom layers, with the upper layer offset with respect to the bottom one in such a way that the corners (vertices) of the triangles in one layer are vertically projected into the center of gravity of the triangles in the other layer (this is called an oblique translation). This is in fact one of the stiffest space frames that can be configured. It consists of closely packed octahedra and tetrahedra, and has the distinct advantage of being able to be constructed from members of only one length. A common variation is also the space frame with square grids in both layers, where the squares are offset half a square in both directions. (Fig. 9.22.) To achieve different visual patterns, a large number of other possible variations exist that nonetheless observe in each case the requirements for stability. (e.g., Ill. 9.37.)

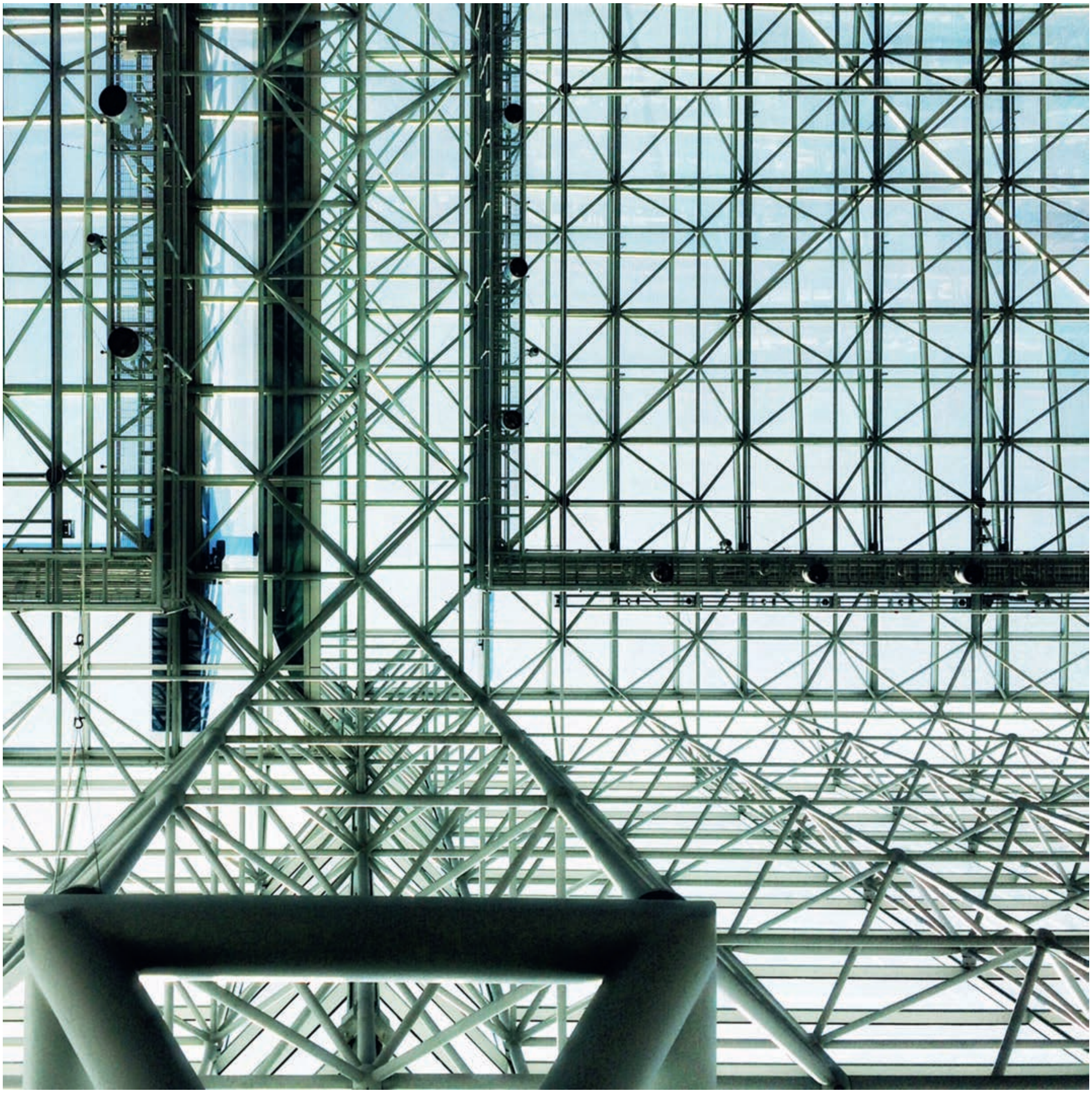


Illustration 9.37
Jacob K. Javits Convention Center, New York City, NY, USA (1986).
Looking up the space frame side wall toward the space frame roof: "conventional" forms and materials – but the results are exceptional in scale and visual patterns.
Architect: Pei Kobb Freed & Partners. Structural engineer: Thornton Tomasetti.

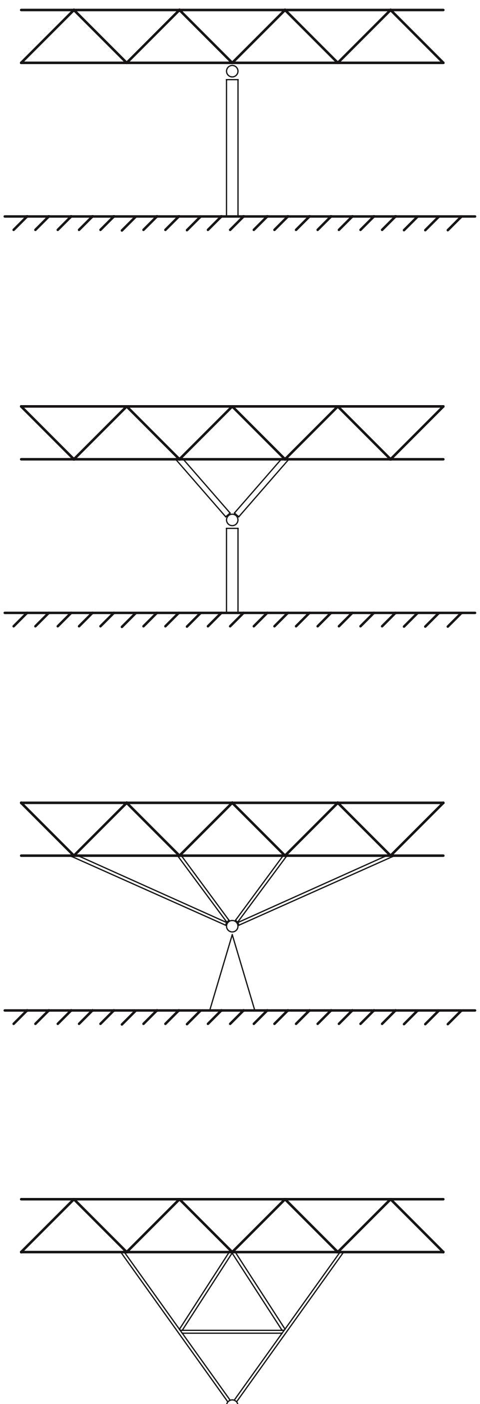


Illustration 9.38
"The Alhambra" (2013).
Sculpture in aluminum and stainless steel by Mark Hagan. Multiple points of support avoid overloading individual elements of the space frame.

Figure 9.23
Supporting space frames on columns attached to only one joint will result in particularly high axial stresses in the few members connecting at this joint, while a distribution of the forces among several support points is more favorable.

Because of the pattern of more or less equally sized members throughout, space frames are particularly well adapted to carrying uniformly distributed loads. Large point loads, on the other hand, will unduly stress a limited number of members. Similarly, then, the quite common situation of supporting space frames on a limited number of pillars or columns is far from ideal from a structural point of view. The image of a floating open structure anchored to the ground on only a few thin columns may look too nice for its own good; however, it is a scheme that has frequently been used in spite of its easily anticipated problems. A support configuration that better addresses the situation has compression struts that splay out from the tops of columns in order to connect to several joints of the space frame, thus allowing the total load to be distributed among a larger number of its members. (Fig. 9.23, Ill. 9.38.)

In space frames, then, regular spatial polyhedral shapes are quite common; these can be seen as advantageous from the point of view of repetition and standardized fabrication of members but they are not necessarily the most desirable or even the most materially efficient configuration in all cases. Providing inspiration for such relative freedom of space frame form is the work of the Venezuelan artist Gego, whose 1976 "Trunk no. 5" wire sculpture, for example, can be considered well ahead of its time. (Ill. 9.39.) As digital design and manufacturing technologies are increasingly handling complex geometries in a more efficient manner, future architecture projects will probably incorporate space frames of more variety and visual interest, as well as of higher structural efficiency, than has mostly been the case to date.[12] (e.g. Ill. 9.40.)

Illustration 9.39
"Trunk no. 5" (1976).
Sculpture in stainless steel by Gego.
Trained as an architect in Germany and spending a lifetime as an artist in Venezuela, Gego (born Gertrud Goldschmidt (1912–1994)) made her re-entry into the European art scene through an important exhibition named "Defying Structures" in Barcelona and Porto in 2006. She moved from the works of the drawn line on paper to real space. In space the line has a physical body, and to relate it to other lines it requires connecting support points, and articulations. Many of Gego's works done in metal wire demonstrate a freedom of form within a structural order based on rigid triangles.

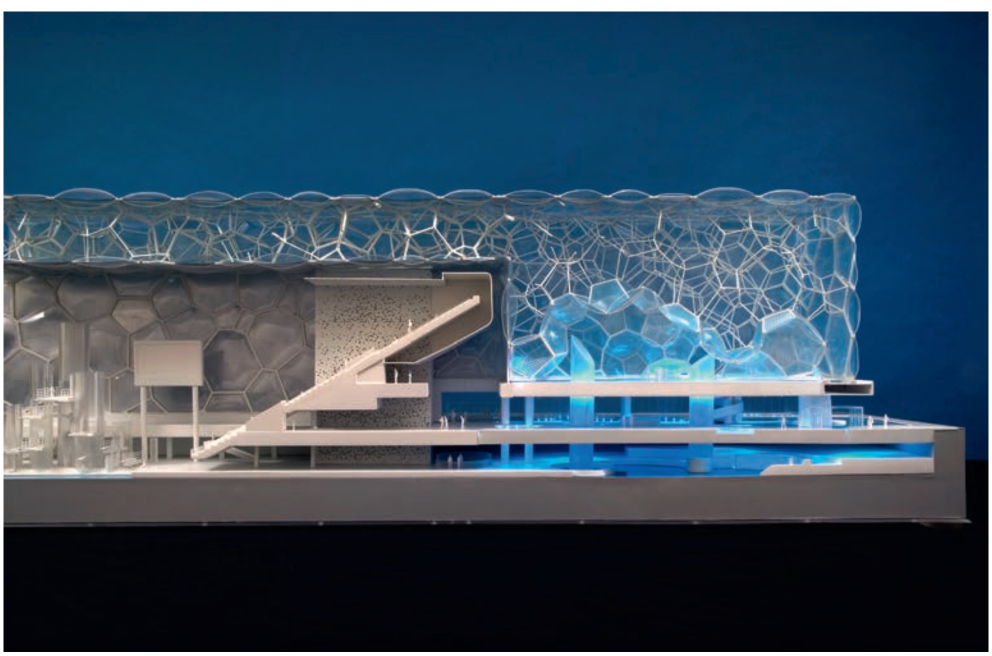


Illustration 9.40
Beijing National Aquatics Center, The Watercube, Beijing, China (2008).
Essentially forming a huge cubic volume measuring 177 x 177 x 31m (580 x 580 x 102ft), this building's light and seemingly irregular structural framing system is inspired by research into the configuration of water bubbles; conceptually packed "bubbles" become geometric 12-sided and 14-sided polyhedra,[13] the flat sides of which are pentagons and hexagons. These shapes are then created from steel tube members that form a unique space frame structure that runs through the thickness of the 3.6m (12ft) walls and the 7.5m (25ft) roof of the building.

Architects: PTW Architects. Structural engineering: Arup.

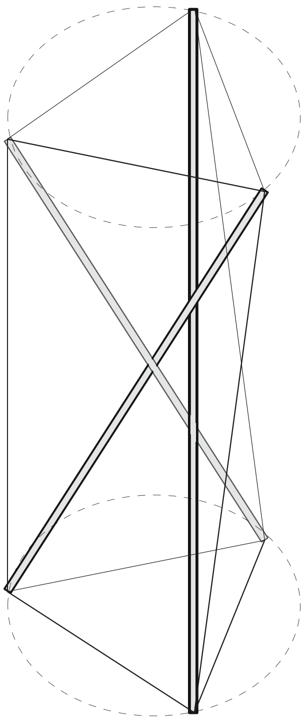


Figure 9.24
The simplest tensegrity structure. Three compression members of equal length are symmetric to one another. Each end is connected to three tension cables and defines in this particular case the corners of equilateral triangles. The triangles at the top and bottom are twisted with respect to each other.

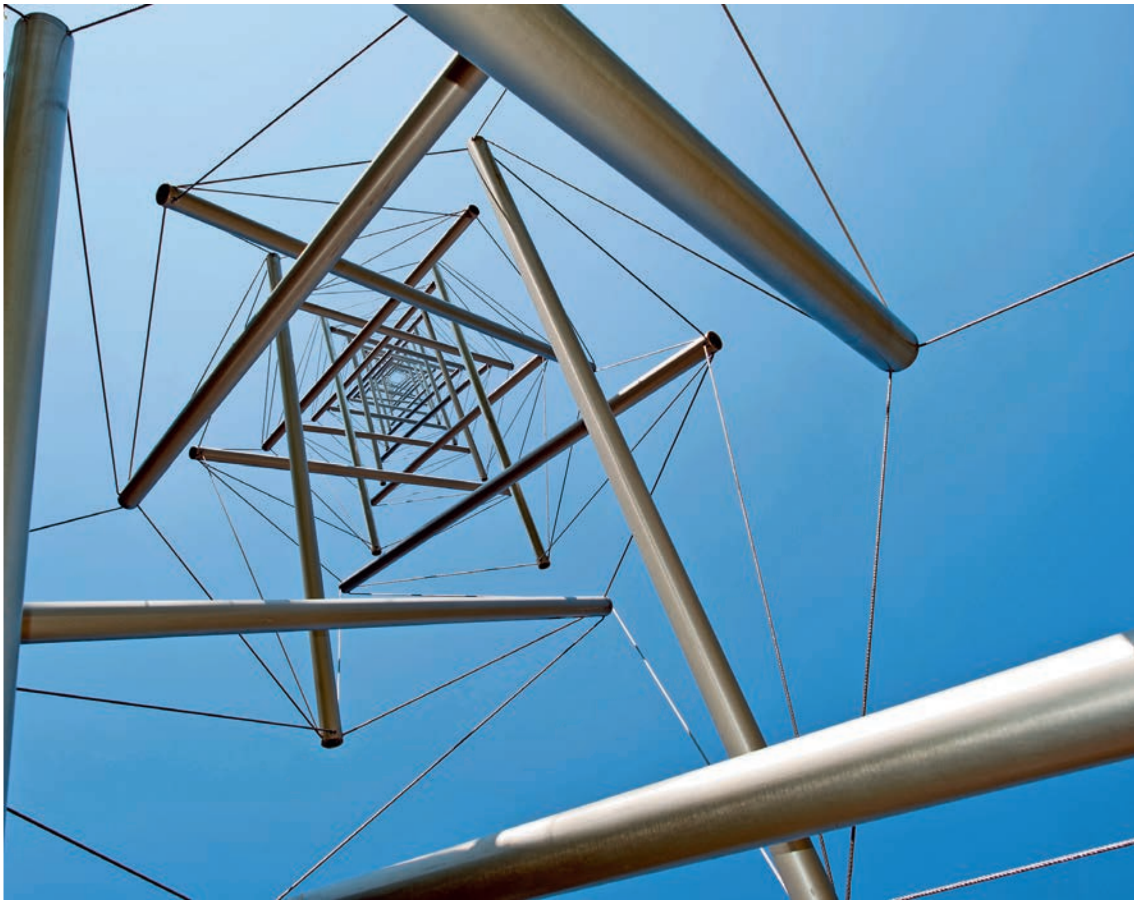


Illustration 9.41
"The Needle Tower" (1968). Kröller-Müller Museum, The Hague, the Netherlands.
Tensegrity sculpture by Kenneth Snelson. Aluminum and stainless steel, 30 x 6 x 6m (98 x 20 x 20ft).

9.10 Tensegrity – When Columns Fly

While not being explicitly a true truss in the sense that we have discussed that structural form so far in this chapter, the so-called *tensegrity* structure nonetheless also consists of a spatial arrangement of multiple axially loaded tension and compression members that, considered as a whole, has the effective structural depth to be able to counter transverse loading. These elements are here configured in a very particular way, however: the lightweight and airy system's tension members are continuous while its compression members are isolated from one another – thereby seeming to "fly." (Fig. 9.24.) The tension elements of the system are usually made of thin steel wires and cables whereas the compression elements are typically thicker rods, perhaps made of metal (preferably aluminum due to that material's relative light weight, but steel can also be used), or else wood struts or even glass tubes have been used on occasion. High precision is needed for the manufacturing and assembly of a tensegrity structure, and their equilibrium relies on the whole system being highly pre-stressed.

The word tensegrity is a contraction of "tensile integrity," as coined by Buckminster Fuller (1895–1983). These structures have three almost simultaneous origins in the works of Fuller, Kenneth Snelson (1927–2016), and Georges Emmerich (1925–1996), all of whom explored different aspects of tensegrity-like phenomena around 1950. Snelson, who was a student of Fuller's, in particular pioneered dramatic sculptural works that applied the use of the tensegrity principle. He considered himself an artist and several

Illustration 9.42
The Snowdon Aviary, London Zoo, London, UK (1963).
Among the most widely recognized of tensegrity structures. Key support for aviary's enclosing cable net is provided by four pyramidal aluminum compression "masts," each of which is perceived to be "floating" in the air, completely supported by angled tension cables.
Architect: Cedric Price (1934–2003) and Lord Snowdon (1930–2017). Structural engineer: Frank Newby (1926–2001).

of his works are displayed in museums and sculpture gardens. His Needle Tower from 1968, for example, is made of aluminum tubes that are amazingly held together in perfect balance by a single continuous stainless steel wire. (Ill. 9.41.) As previously mentioned, one of the fascinating qualities of tensegrity structures is that they seem to float in the air. The reason for this perception is the fact that the compression elements do not touch each other but are, in the words of Buckminster Fuller, like "small islands in a sea of tension." Tensegrity sculptures such as Snelson's not only explore these structural principles but also work visually in relation to them as well as in contrast with the natural landscape.

Despite the visual and conceptual attraction of the form, however, an obvious difficulty arises when it comes to employing tensegrity principles in building projects that have significant weight needing to be carried and that need to provide the typical stable building surfaces of floor slabs, façade cladding, etc. From a conventional building performance point of view, tensegrity structures tend to be quite flexible and finding ways to counter that reality is likely to mean that some of the initial attraction and mysteriousness of "free-floating" structures may end up being substantially diminished. But such challenges notwithstanding, tensegrity has found its place in architecture when the design loads are not too large and/or when relatively significant deflections can be accommodated. For example, the "floating" aluminum pyramidal forms used to support the original cable net enclosures for the Snowdon Aviary at the London Zoo established what is perhaps tensegrity's truest "flying" form. (Ill. 9.42.)

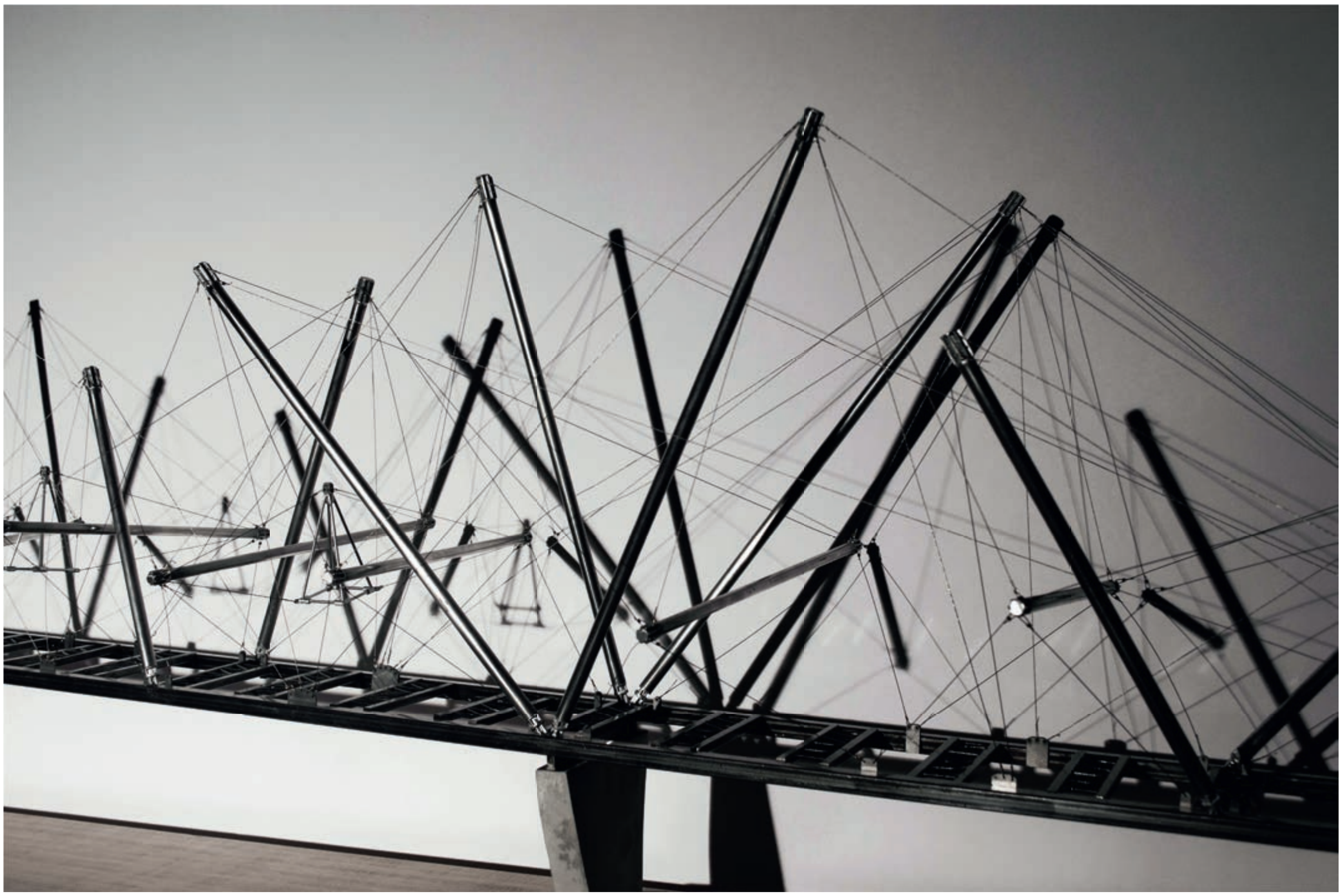


Illustration 9.43
Kurilpa Bridge, Brisbane, Australia (2009).
Hybrid cable-stayed/tensegrity structure – the mast-stabilizing horizontal spars conform to tensegrity principles; i.e., compression struts that at first glance seemingly "float" in the air, but that in reality are supported by angled tension cables at both ends.

Architect: Cox Rayner Architects. Structural engineer: Arup. Cornell model by Bennett Adamson and Thomas Rushton.

More recently, the floating-compression-member system has once again proven to be inspiring and challenging to architects and engineers alike. For example, a small tensegrity pedestrian bridge was designed for the National Building Museum in Washington, DC, that, if built, would have connected high-level galleries on either side of the museum's main hall. This project was developed by Wilkinson Eyre Architects in collaboration with structural engineer Cecil Balmond. The structure, whose underlying geometry was based on a series of tetrahedral elements, was also conceived of as an exhibited object that would have actively demonstrated its structural behavior. The bridge, as designed, was to be constructed from a network of glass tubes acting as the compressive elements, all connected by cables; moreover, a system of LED (light-emitting diode) lights built into the glass tubes would be activated according to variations of their member forces caused by the museum visitors walking along the bridge.

A second contemporary example is a hybrid version of a tensegrity bridge that was designed for and built in Brisbane, Australia. In this case, the structural system consists of a "conventional" cable-stayed mast system for the support of the bridge's pedestrian walkway and cycling path, but these thin, pinned-base masts are given lateral stability by means of transverse, "floating" struts and small pyramidal forms angled this way and that, supported at their ends only by means of tension cables. (Ill. 9.43.) The overall impression of this hybrid stayed/tensegrity system is one of playfulness and yet also one of clear efficiency of material use, in which tension and compression elements are brought together spatially to produce an overall "trussed" structure of rather remarkable qualities.

The Frame and the Shear Wall

CHAPTER

10

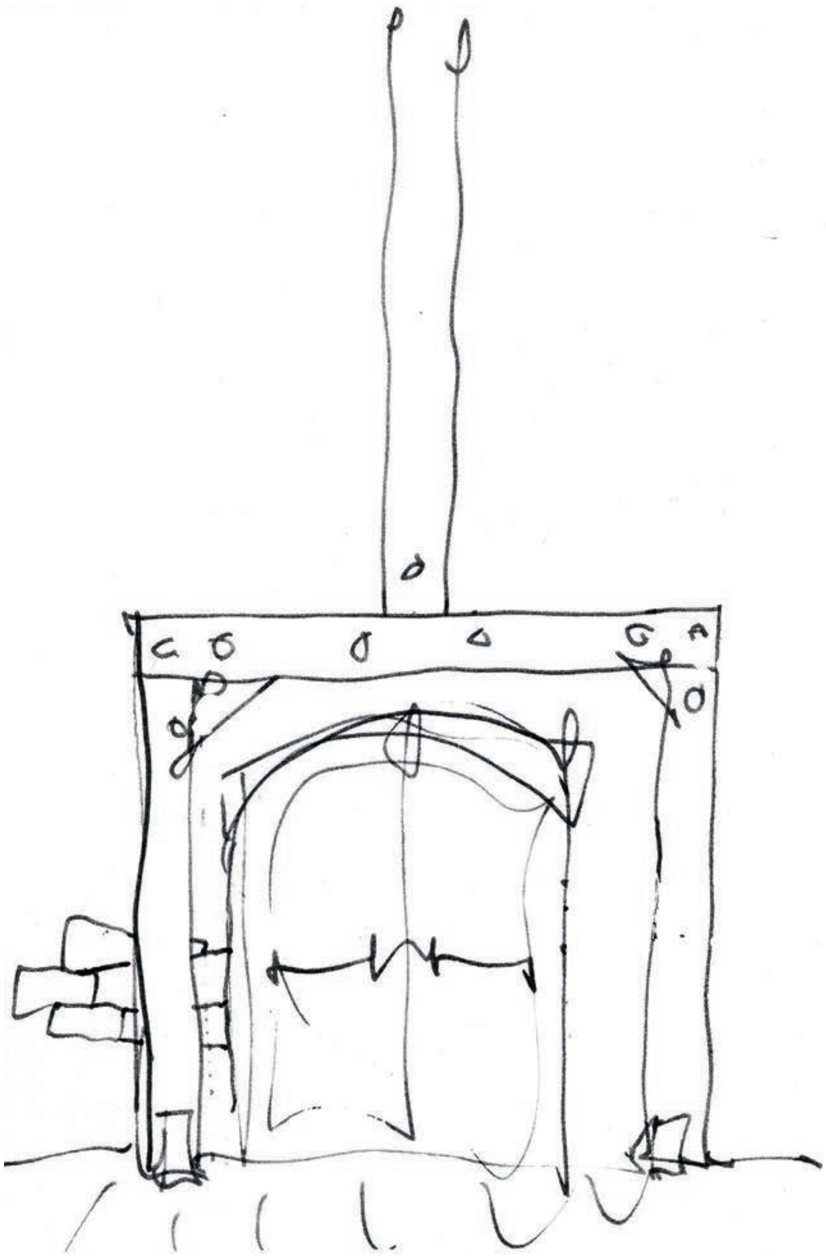


Illustration 10.1
Storhamarlåven, Hedmark Museum, Hamar, Norway (rebuilt 1974).
The simple frame structure supports the column above and distributes its load to both sides of the auditorium door opening, thereby both preventing the passage from being blocked and highlighting the doors' design.
Architect: Sverre Fehn. Structural engineer: Terje Orlien.

Illustration 10.2
Greenwich Academy Upper School, Greenwich, CT, USA (2002).
View of landscaped roof of new building, showing courtyard that cuts into the two-story building mass, as well as four glass-covered pavilions that extend upward above the roof level. Glass roofs of these volumes can be seen to distinctively slope and twist along their length.
Architect: Skidmore, Owings & Merrill (SOM). Structural engineer: DiBlasi Associates.

10.1 Greenwich Academy – Framing Light and Space

In the wooded, hilly setting of the Greenwich Academy in Greenwich, Connecticut, USA is a building with a typical high school program that has been rendered remarkable through the close design collaboration of the architect Richard Duffy of Skidmore Owings and Merrill and the space and light artist James Turrell. Completed in 2002, the 4200m^2 (45 000ft^2) Upper School and Library Building provides a rapidly growing school campus with the usual mix of new classrooms, computer and art facilities, science laboratories, and a library and reading room. But as it is located on a topographically complex site and hard against several previously existing buildings, gardens, and playing fields, perhaps the new structure's most important function is to act as a connector, linking all of these disparate pieces together, and it is on this connective aspect that we will focus our attention.

An understanding of the site is essential for this project: over the building's footprint the terrain slopes rather steeply from an upper level garden and traditional building entrance down to a lower terrace about 7m (23ft) below that has multiple playing fields merging with the natural, wooded landscape. The new building, rather than being built at the top or bottom of this hill is instead built into it, resulting in a two-story structure whose rooftop meets the ground surface of the upper level. The landscape is then made to flow through the large new building in several ways: the entire flat rooftop is landscaped to act as an extension of the gardens, the two-story-high glass walls around the perimeter at the lower level allow the interior and exterior spaces to read as one, several

Illustration 10.3
Greenwich Academy Upper School. Glass pavilions project above the landscaped rooftop – at once creating access points from the upper level of the sloping site and drawing daylight deep into the two-story building, but also creating night-time light chambers for the work of artist James Turrell.

exterior courtyards are integrated within the overall scheme, and, most relevant to our concern here, there are four distinctive glass-covered pavilions that have been called "chambers" or "crystals" that project up through the green roof and give the building its most distinctive feature. (Ill. 10.2.)

The program for the building is organized into four subject areas: math/science, computer-art/ceramics, languages/humanities, and library/reading room, and each of these zones is centered and given identity by one of the glass pavilions. With a height of 9–12m (30–40ft), all the glass volumes project well above the planted roof and thereby draw plenty of light into the two-story-thick building. They distinguish themselves individually, however, by being oriented in plan at right angles to each other and by having roof planes of different heights that are both sloping and warping independently. The overall effect when seen from the upper entrance level, however, is to observe a new type of unified crystalline landscape for the school that is integrated into the natural one.

The structural system that supports the glass enclosures consists of a series of glue-laminated timber frames that have been prominently featured as part of the design. (Ill. 10.3.) The color and warmth of the wood frames provide a striking visual contrast to the enveloping transparent glass and the sky seen through it, and they also suggest a relationship to the trees of the surrounding forests, albeit one that has gone through a manufacturing process: the columns and beams of the frames are made of several 50mm (2in) thick layers of fir that have been pressed and glued together in a factory. The connections between the timber frames' columns and beams are also emphasized in the design: right-angled steel plates cover the joints on both sides, and several bolts hold the pieces together securely. (Ill. 10.4.) We will see in this chapter that this type of connection is one which provides essential stiffness and rigidity to the frame, allowing it to withstand gravity as well as the lateral forces that wind will produce (in this case by acting on the pavilion

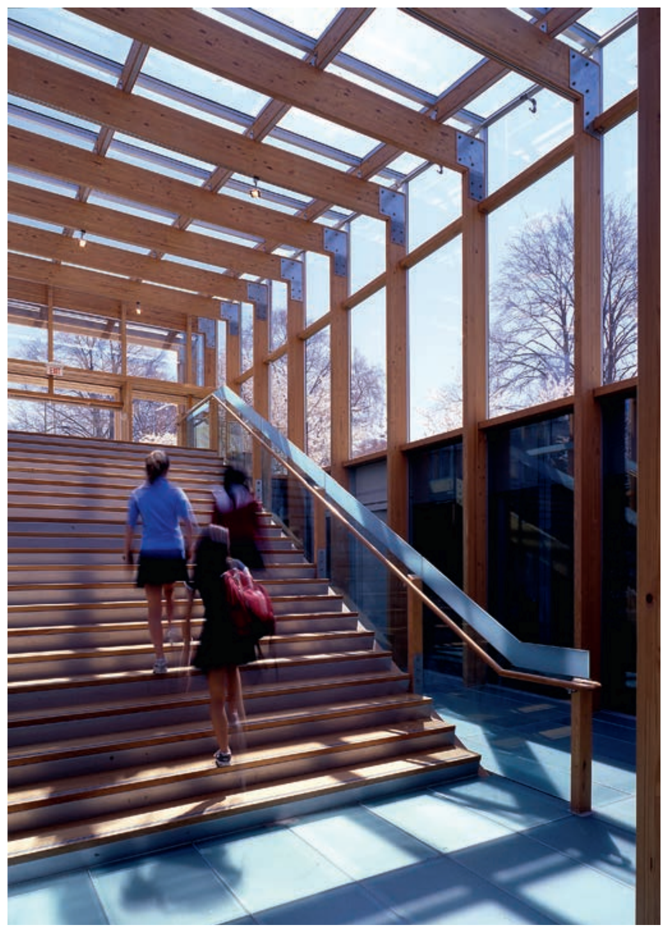


Illustration 10.4
Greenwich Academy Upper School.
Glue-laminated timber frames provide structural support for gravity and lateral loads while allowing maximum light penetration and pedestrian through-circulation. Steel plates establishing rigid connections at the corners of the frames are distinctively featured.

projections above the roof.) But this type of frame, typically called a *rigid frame* specifically because of the special characteristics of its connections, has other design purposes as well: in contrast to other lateral-load-resisting systems that we will encounter in this chapter, it allows for free and easy through-passage – an essential aspect here with the pavilions acting as part of the connective circulation path through the school. Moreover, with only two side columns and a beam at the top, the rigid frame also is recognized as being particularly useful from an architectural design perspective in order to visually orient and frame views and space without any intervening structure, important features in suggesting directions of movement and eventual destination points. But there is yet a further visual aspect of the Greenwich Academy's rigid frames that makes them truly unique.

The artist James Turrell is well known for his space and light installations, including for the yet-to-be-completed Roden Crater in Arizona where a natural volcanic cinder cone is being turned into a massive naked-eye observatory for the sky and heavens. The Greenwich Academy's Upper School Turrell Lighting Installation is much smaller in scale but directly relevant to our discussion of frames: light-emitting diodes (LEDs) have been embedded into each of the timber frames of the glass pavilions, which at this point can perhaps best be called "light chambers" for reasons that will shortly become obvious. The lights give life to the building at night – but certainly not in the usual manner. A programmable electronic control board has been used by Turrell to adjust the mix and intensity of the red, blue, and yellow LEDs of the frames into every hue and color imaginable. Sometimes the lights only trace the outline of these structures, but because the glass of the pavilions has been etched to contain and refract light, at other times the light turns the planar surfaces into glowing volumes. Furthermore, all this can be dynamically choreographed, with the colors merging with each other and seemingly made to jump from one volume to another, figuratively linking one academic discipline area to another. Turrell has turned this into a dazzling light and space show; for us here, though, this project compellingly expresses the potential achieved by the linkage of art and light and architecture and space – all through the medium of the rigid frame.

10.2 A Triad of Stabilizing Subsystems

Structural systems exist to carry loads, which in an architectural context result mainly from the need and desire to inhabit interior spaces of various shapes, sizes, and functions. As we have seen in the preceding chapters and as we will continue to explore in those that follow, these systems' overall forms vary based not only on load-carrying function but also on aesthetic preferences, cultural influences, visual effects, circulation considerations, construction costs, material performances and availability, sustainability objectives, lighting and acoustic effects, thermal comfort, and myriad other concerns or attributes that may be brought to bear. Regardless of these other influences, however, a structure's mechanical role and function is obviously to carry vertical gravity loads, typically stemming from the lifting aloft of a roof or floor surface that covers and spans across an occupiable space. But as we saw in Chapter 3, buildings are also regularly subjected to other types of loading in addition to gravity, such as the lateral forces produced by wind, earthquakes, contained earth or water, etc. Such sideways forces tend to cause a lateral instability in a built structure that must also be resisted if the building is to remain standing and functional. As was briefly discussed earlier in Section 2.5, three broad categories of structural *subsystems* have been devised in order to counter such potential instability caused by lateral loads on buildings – namely, shear walls, braced frames, and rigid frames (Fig. 10.1) – and it is the objective of the remainder of this chapter to describe in more detail their respective fundamental structural behaviors and pros and cons in relation to other architectural design objectives.

The Shear Wall

The first of these lateral load subsystems to be considered is the *shear wall*, which at its simplest is a thickened planar surface to which sideways forces are applied. The key distinction to other lateral-load-resisting wall types that we have discussed earlier in Chapter 8 (e.g., retaining walls) is that in the case of the shear wall it is understood to be primarily used for lateral load resistance *in the direction of its plane*, rather than transversely to it. Although historical as well as contemporary masonry walls made of brick or stone or concrete block quickly come to mind, in fact shear walls

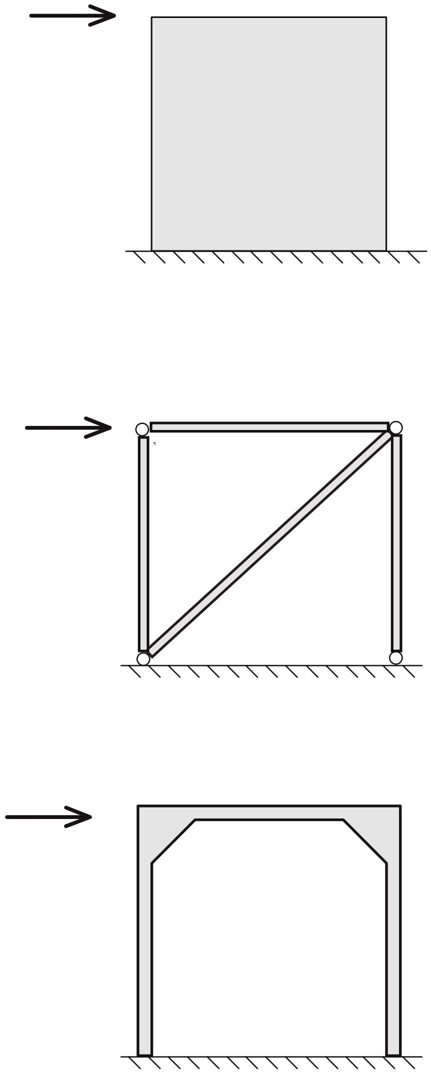


Figure 10.1
Stabilizing subsystems (from top); shear wall, braced frame, and rigid frame.

can also be made of other materials including reinforced concrete, timber, or steel. (e.g., Ill. 10.5, 10.6.)

Irrespective of the material, however, at its most basic as a lateral-load-resisting subsystem one can imagine a planar shear wall to be supported vertically and horizontally at ground level and to have applied to it horizontal, in-plane loads. In response, the wall will tend to displace and flex sideways in its plane in the direction of the loading. We can imagine various ways in which this wall could fail as a result of over-loading: two would certainly

Illustration 10.5
Torre Cube, Guadalajara, Mexico (2005).
Trio of tall, thin, reinforced concrete shear walls resist lateral loads, carry gravity loads from the wood-clad cantilevering volumes of residential space, and act as service/elevator cores.
Architect: Estudio Carme Piños. Structural engineer: Luis Bozzo.

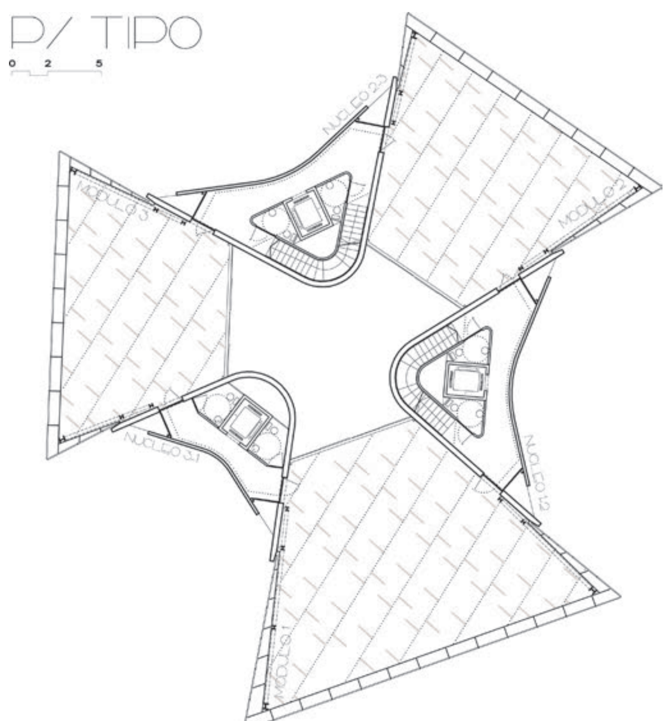


Illustration 10.6
Torre Cube.
Plan drawing shows curved aspect of these load-carrying shear walls that also act to establish and shape space, direct views, and are an essential part of the building's design aesthetic.

Illustration 10.7
Makoko Floating School, Lagos, Nigeria (2013).[1]
Timber braced frame creates spatial and visual openness for multiple communal activities to occur and be visible on different floor levels, while also minimizing dead loads for the floating "foundation raft" of 64 plastic barrels.
Architect: NLÉ / Kunlé Adeyemi with various structural and naval consultants.

be for the wall to slide sideways or else overturn about its base, but if such clearly unacceptable support failures are prevented then the adequacy of the wall ends up being a matter of ensuring that the material of which it is made does not fail due to having its capacity exceeded. *A shear wall, then, can fundamentally be seen to behave like a cantilever beam, albeit a vertical one*. Like all cantilever beams subject to transverse loads (see Chapter 7), shear walls will have both internal shear forces and bending moments that must be dealt with by their associated shear and bending stresses. The relative significance of each of these will depend mostly upon the height-to-length proportions of the wall (to be clear, by length here we mean the plan dimension of the wall parallel to its plane); i.e., the more "squat" the wall the more shear will govern its design, while the taller and slenderer it is the more bending action will be determinant of its capacity. There will be more discussion of these topics later in this chapter in Section 10.4.

The Braced Frame

The second type of lateral-load-resisting subsystem is the *braced frame*, which fundamentally is a two-columns-and-a-beam assembly of elements used to carry gravity loads and create space but that is also provided with one or more additional diagonal members that help to prevent the frame structure from wracking sideways too much when lateral loads are applied to it. We are already somewhat familiar with the braced frame's fundamental behavior from our discussion of the truss in Chapter 9 since, *in essence, a braced frame is nothing other than a vertical cantilevering truss going by a different name*. Of course, using the braced frame as a lateral-load-stabilizing subsystem is an effective way of not having the major visual obstruction of the typical shear wall while maintaining a comparably high degree of lateral stiffness. Aside from the common single- and double-diagonal (or X-) braced frames (e.g., Ill. 10.7, 10.8), several variations of form can be identified that belong to this subsystem category: V-bracing configurations,

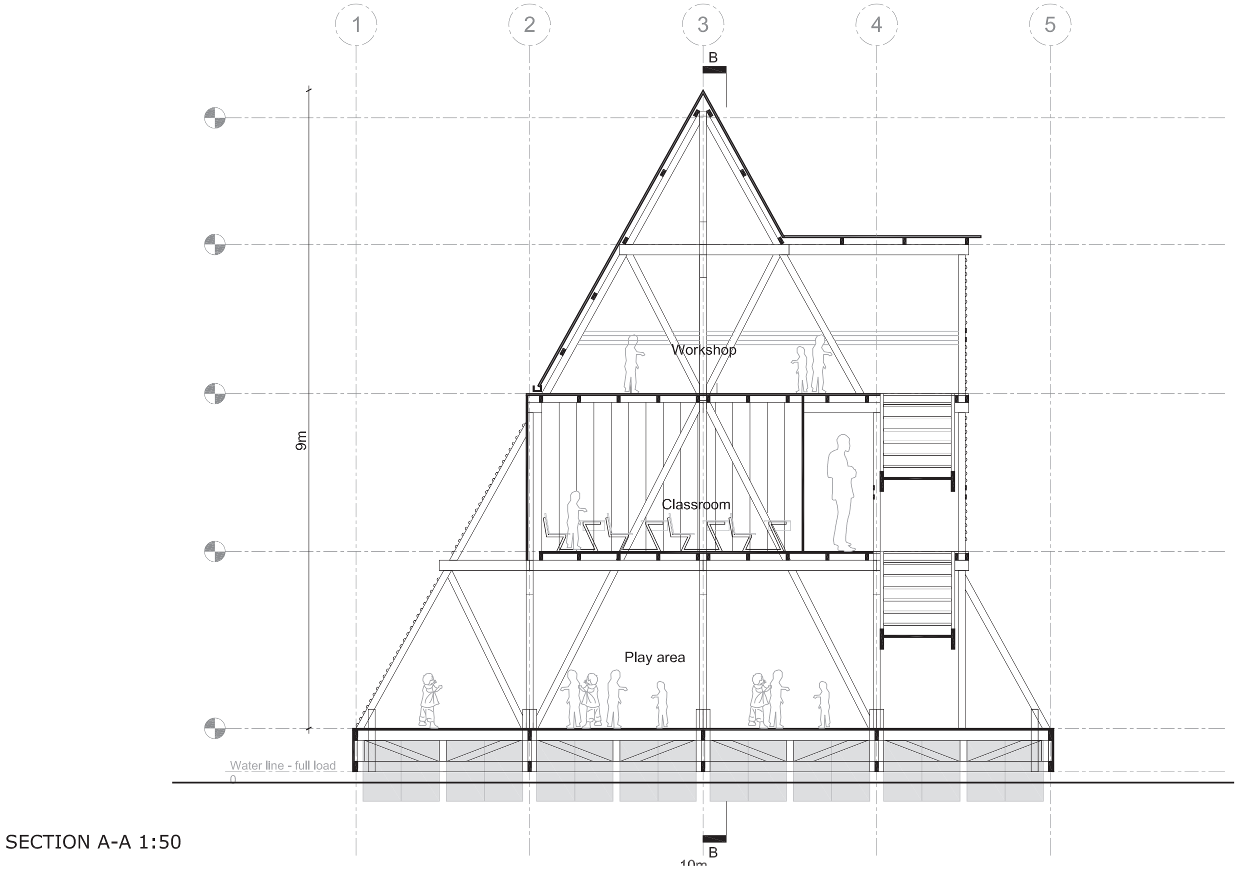


Illustration 10.8
Makoko Floating School.
Drawing clearly highlights bracing used to stabilize overall timber framework.

lattices that are created by diagonal and horizontal members in a triangular grid (also called diagrids), inclined columns that can act as both vertical-load-carrying and lateral-stability-giving bracing elements simultaneously, and even angled buttresses and guy cables placed on the exterior of frames can all be understood to be variants of this type of lateral-load-resisting system. We will return to some of these alternative forms in Section 10.5.

Forces in braced frames may be predicted and calculated according to the same principles and analytical methods that were set forth for trusses in Chapter 9, so we do not need to repeat those again in this chapter. Also, the same general conclusion about braced frames can be reached as it was for trusses: namely, that this is a structural form that resolves what is, from an overall perspective, bending-type behavior into a set of highly efficient axial tension and compression forces in its linear elements that are strategically spaced apart. The implications of this in terms of the visual transparency of the braced frame as a lateral-load-resisting system could not be any clearer compared to the shear wall.

The Rigid Frame

Third among the basic lateral-load-resisting subsystems is the so-called *rigid frame*, whose form and behavior we haven't yet considered in detail in this book and which will be, therefore, the subject of more substantial treatment and analytical focus later in this chapter. For now, however, the rigid frame can be characterized as simply an assembly of columns and beams that are connected together by means of *rigid joints* – and thus, its name. The rigid frame represents a structural assembly that offers not only support for gravity loads but also quite good lateral stability, all the while providing an almost complete visual and circulation openness; i.e., the system is free of the obstructing solid plane of the shear wall or the interfering diagonal member of the braced frame. As we will see in Section 10.6, stability of the rigid frame is achieved instead by providing an adequate number of rigid joints located at the connections between its columns and beams, and/or at the column bases.

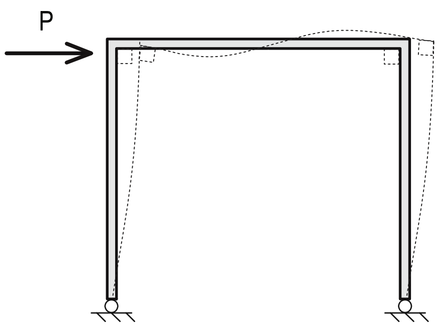


Figure 10.2
Rigid joints prevent relative rotation between the connected members. This means that the elements themselves must bend when the frame deflects sideways. The stiffness of the structural elements thus provides stability, although in a flexible way.

Illustration 10.9
Casa El Mirador, Valle de Bravo, Mexico (2013). Rigid frame structure created by bending-resistant flanged steel columns and beams that are welded together at their connections. (See also Ill. 7.38.)

Architect: CC Arquitectos. Structural engineer: Miguel Campero, Jorge Soto, Pedro de la Fuente.

In contrast to the hinged/pinned connections of braced frames, rigid joints are bending-resistant connections that do not allow relative rotation to occur between the members they are connecting; in other words, the angle between these members does not change even if the connection as a whole rotates. Such rigid joints cause the columns and beams of the frame to bend when loads are applied to it (Fig. 10.2); i.e., when a rigid frame deflects sideways in response to lateral loading, the *bending* stiffness of both the column *and* beam elements contribute significantly to giving the frame its lateral stability – in contrast to being provided only by the *axial* stiffness of these elements and especially of the diagonal in the case of the braced frame. Given what we know from preceding chapters about the relative difference in stiffness and load-carrying efficiency of structural members that are subject to bending versus axial stresses, it should be easy to anticipate that for similarly sized elements the rigid frame is, therefore, quite a bit more flexible than is a braced frame – notwithstanding its name. Or, conversely, the column and beam elements of a rigid frame typically need to be considerably "beefed up" from the dimensions that they would have as part of a corresponding braced frame if the lateral stiffnesses of the two subsystems are to be roughly equal to each other. We will consider more closely the implications of these observations later in this chapter; they are quite far-reaching in terms of establishing rigid frames' overall forms, connection details, and material selections – all in support of various design intentions. (e.g., Ill. 10.9.)

10.3 French Frames

Palais de Bois

Auguste Perret (1873–1954) remained an *architecte-ingénieur* throughout his career and was a pioneer said to be *"sans pareil"* in exploring the architectonic potential of reinforced concrete. He was also a father figure in his famous studio that attracted many young talented designers; both Walter Gropius (1883–1969) and Ludwig Mies van der Rohe (1886–1969) worked for Perret and also Le Corbusier worked there as an apprentice for a short period of time. Perret's mastery, however, was by no means limited to the exploration of reinforced concrete. His aptly named Palais de Bois, a temporary gallery space built in the Bois de Boulogne for the

Illustration 10.10
Le Palais de Bois, Porte-Maillot, Paris, France (1924).
Architect: Auguste Perret.

1924 Salon des Tuileries in Paris, demonstrates that he was equally comfortable in handling wood. (Ill. 10.10.) As it happened at that time, the Western world showed a great deal of interest in Asian art and design, and in Perret's Palais de Bois one can recognize an architecture that drew its inspiration from traditional Japanese wood construction.

This was a building with a 300m (984ft) long space that was created by a regularly repeating timber framed structure along its length; the typical section of the gallery resembled that of a basilica, with a high nave providing clerestory lighting from above. The structure consisted of a very simple, clear and exposed system made of sawn timber of standard dimensions straight from the lumber mill, so that when the building was taken down these elements were able to be reused elsewhere. Organized in a clearly hierarchic fashion, the post-and-beam form of the structure consisted of two single columns carrying double beams; resting on the beams were rafters and purlins covered by boards. The frame-like structural system was important for opening up the interior space of the Palais de Bois, but on its own the building would not have been stable. Judging by photographs, it seems that the stabilizing function was provided instead by solid walls that filled in selected gaps in the side aisles between the frame's columns. So, while the celebrated frame of this building was certainly essential in shaping its overall form and defining the quality of its interior space, it was supplemented in doing so by the lateral in-plane stiffness of a number of walls; i.e., one structural system cannot be seen as complete here without the other. Likely without appreciating such subtleties, however, it was said that while the Parisian public of the 1920s very much enjoyed the atmosphere of the gallery, with its open space and particular material quality, they also found the structure *"un peu primitif."*

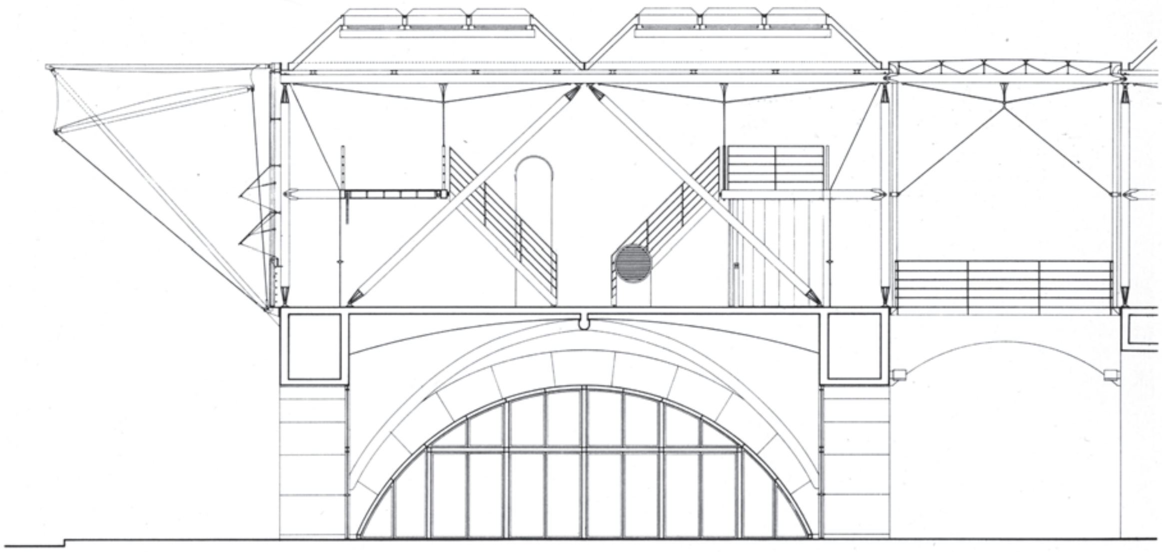


Illustration 10.11
The School of Architecture, Lyon, France (1987).
Section drawing highlights the differences between the upper and lower levels, with massive concrete arches and domes at ground level vs. timber and steel linear components connected together into a structural framework above.
Architect: Jourda & Perraudin. Structural engineer: Rice Francis Ritchie.

Illustration 10.12
The School of Architecture in Lyon.
Studio space features diagonally braced glue-laminated timber frames.

École d'Architecture de Lyon

The building for the Architecture School in Lyon is the result of a design competition won by architects Françoise-Hélène Jourda (1955–2015) and Gilles Perraudin. The building plan follows a symmetrical classical order with an internal two-story-high street which is flanked on both sides by a long line of classrooms and studios and which leads in the direction of the auditoria and faculty offices at the end of the building. The various spaces and functions are regulated by a strict modular layout, but there is also a certain life and vitality to the system. As would befit a building having the pedagogical program and mission of an architecture school, there are many highly distinctive spatial qualities and material uses and details to be found here. The lower, "learning" level of the building, where classrooms and review spaces are located, are cast out of heavy concrete into forms such as domes and solid walls, while the upper, "doing" level of the studio spaces is characterized by an open, light, laminated-wood structural system, with its exterior

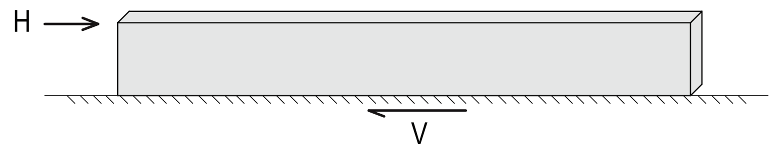


Figure 10.3
Lateral load applied to shear wall having relatively long, low proportions. Shear response of wall dominates over that of bending.

walls completely glazed. (Ill. 10.11.) According to the architects, there is in this dual system a reference to a famous father-and-son duo from Greek mythology: the ground floor represents the artisan, Daidalos, and it is the place where the young students learn the basic crafts; the upstairs world is that of Icarus, where upper level students practice the art of flying.

We shall focus here on the structure of the upper level. As we have already mentioned, it is noticeably thinner and lighter than that below. Looking along the length of the studio space, a regular structural rhythm is clearly established by the transverse wooden frames that are spaced at equal intervals. (Ill. 10.12.) The glue-laminated timber columns and beams that make up these frames each have identical cross-sections of 200 × 200mm (8 × 8in), and they are connected at their corners by means of cast steel hinge joints. Such frames, with hinges at their four corners, would not be stable structures on their own, let alone when dealing with lateral loading acting on the building. The necessary sideways stiffness is provided by the distinctive inverted-V diagonal bracing within the frames of these studio spaces. Triangular geometry results from the presence of such bracing and, as we know from studying trusses in Chapter 9, this provides the necessary stability for a laterally loaded pin-jointed frame not to collapse. The basic image of a braced frame reads clearly throughout the architecture studio space: a beam on two posts with diagonal bracing, forming a remarkable space.

The highly visible and articulated series of metal joints that are used for the wooden members' connections throughout this building (see Ill. 8.16 for close-up detail) were designed in collaboration with the structural engineer Peter Rice (1935–1992), who had used cast steel technology when he worked on the giant *"gerberettes"* of the Centre Georges Pompidou in Paris. (See Ill. 9.20.) Not to be confused with elements that are made from the structurally brittle material of cast iron, cast steel components have become a distinctive part of our contemporary architectural toolbox due to this material's malleability and its high strength. And just as cast iron did in the nineteenth century, cast steel today offers wonderful opportunities for designing and shaping intricate structural components, and especially for resolving complex connection geometries. In the present building, both the hinged joints at the top and bottom of the columns as well as the expressively designed connections to the diagonal members are all convincing proof of this.

10.4 Shear Walls – Basic Behavior and Form Variations

There is a tendency to associate walls in structures to something that belongs to the past, which is not surprising given the predominance of walled structures in historical buildings around the world – whether it be the Inca ruins that still astound with their mortarless and intricately fitted stone walls or else the many examples of Roman and Renaissance builders' ingenuity with their multilayered walls of brick or stone, which they multiplied and organized in plan and opened up in elevation into exceptional structures incorporating arches, vaults, and domes for arcades, aqueducts, arenas, cathedrals, and more. But walls do not just belong to historical structures; in fact, the majority of buildings constructed around the world today continue to be supported by walled systems, whether made of rammed earth, fired mud bricks, stone blocks, concrete masonry units, sheathed timber-stud construction, structurally insulated panels (SIPs), solid wood panels, reinforced concrete, pre-cast and post-tensioned concrete panels, or stiffened steel plates. In short, walls are most definitely not passé.

Low Shear Walls

In the case of low, long walls subject to in-plane loading (Fig. 10.3), it is clear that their plan-view geometry provides a very large section modulus that will correspondingly greatly diminish any bending stresses resulting from bending moments in the wall when it acts as a vertical cantilever (recall from Chapter 7 that $\sigma = M/S$, and the implications of that when S is very large). As a consequence, therefore, in such relatively common situations historically but also that are still the case today in most low-rise masonry and light timber-framed buildings, shear behavior is quite often the critical action in terms of challenging such a wall material's maximum capacity with respect to lateral loading. Hence, it becomes understandable why the name *shear* wall was adopted as the generic name for this lateral load subsystem (instead of, say, "bending moment wall").

Although the detailed methods for correctly accounting for shear behavior in walls are beyond the introductory scope of this book, we can nevertheless approach this topic at a conceptual level here in order to develop a basic appreciation for the relative capacities

Illustration 10.13
Sant Pere de Rodes, Port de la Selva, Catalonia, Spain (originally built c.900, renovation 1994). Orthogonal arrangement of masonry walls for this Benedictine monastery reflect such walls' typical long, low, thick proportions; shear stresses from lateral loading are likely to be limiting rather than bending stresses.
Architect (for renovation): J. Antonio Martínez Lapeña and Elías Torres Tur.

of shear walls that are subjected to horizontal forces in their plane. Obviously, if the stresses in the wall that result from such a shear force reach a level at which they exceed the shear strength of the wall material, then this will fail. As an general indication of when this might happen, the mean shear stress τ in a rectangular wall cross-section subjected to the shear force V = H can be calculated and compared to the material's strength in shear, τ_u; i.e.,

$$\tau = V/A \leq \tau_u$$

where A = the area of the cross-section of the wall as seen in plan view. Without doing so here, this approximation can be used for very rough, back-of-the-envelope type calculations for the thick, long masonry walls that characterize so many historical structures. (Ill. 10.13.) The perhaps somewhat unexpected conclusion from this is that one finds that most of the lengths and thicknesses of such walls were actually quite necessary given the low shear stress capacity of their masonry construction materials and the relatively large magnitude of the lateral wind forces acting on what could sometimes be quite large building façades. Moreover, accounting for the significant self-weight of thick masonry walls, plus some quite reasonable allowances for typical floor and roof dead weights and occupancy live loads, leads to the realization that the vertical compression stresses due to gravity loads alone come close to matching typical maximum material capacities for masonry walls. Seen in light of both of these statements, it can thus be concluded that the length, thickness, and solidity of masonry walls is not particularly wasteful – contrary to what one might expect for historical structures but also not so far from the mark for many modern masonry block and timber shear wall structures. That is, perhaps contrary to a lay person's instinctive sense that such long lengths of walls in buildings are the result of our predecessors' scientific ignorance and/or the wasteful bad habits of current low-tech construction conventions, there is in fact good structural capacity rationale for such dimensions.

The even more generally applicable lesson for the building designer is that material choice/selection/availability can have very significant consequences for shear walls, not "only" for the required dimensions of structural elements but also, and perhaps just as importantly, for their resulting spatial implications as well. In the case of masonry and sheathed stud-wall timber construction, for example, these materials' relatively low shear stress capacities inevitably lead to the structural need for long extents of such walls that, in turn, tend to substantially enclose and subdivide space; one does not come without the other and we should not expect otherwise – at least without changing structural subsystems or the material of which they are made.

The reinforced concrete shear walls of the Bregenz Kunsthall seen in Chapter 2, Section 2.1, suggest in starkly contrasting fashion just what is possible with a higher strength material like concrete compared to masonry construction. The three distinct, orthogonally oriented shear walls of Bregenz have greatly reduced length compared to the preceding examples of the Sant Pere de Rodes monastery (see Ill. 10.13) or of the Palladian Villa Foscari (see Ill. 2.9a). Since there are no columns in the Bregenz Kusthaus, all the gravity loads of the building's multiple floor levels and of its art gallery live load occupancy allowance must be carried by these three walls, just as must all lateral loads on the building also be resisted by them, something that is only able to be accomplished because of the greatly increased material capacity of reinforced concrete compared to masonry. (See Chapter 5, Table 5.1.) Moreover, from a spatial perspective, these reinforced concrete walls' substantially reduced lengths allow, for example, for the sides of each floor plate of the Kunsthaus to remain relatively open so that indirect, diffuse, natural lighting can illuminate the galleries through the distinctive glass-paneled sides of the building as well as its cleverly designed hanging glass ceilings. (See Ill. 2.4, 2.6.)

To carry this material/spatial progression of walls to an extreme, we will now consider the Park Café in Koga, Japan, designed by SANAA. (Ill. 10.14, 10.15.) In this case four very thin steel plate shear walls provide lateral stability for the partially glazed/enclosed pavilion (two walls are aligned in each orthogonal direction; recall from Chapter 2 that a minimum of three such walls are necessary

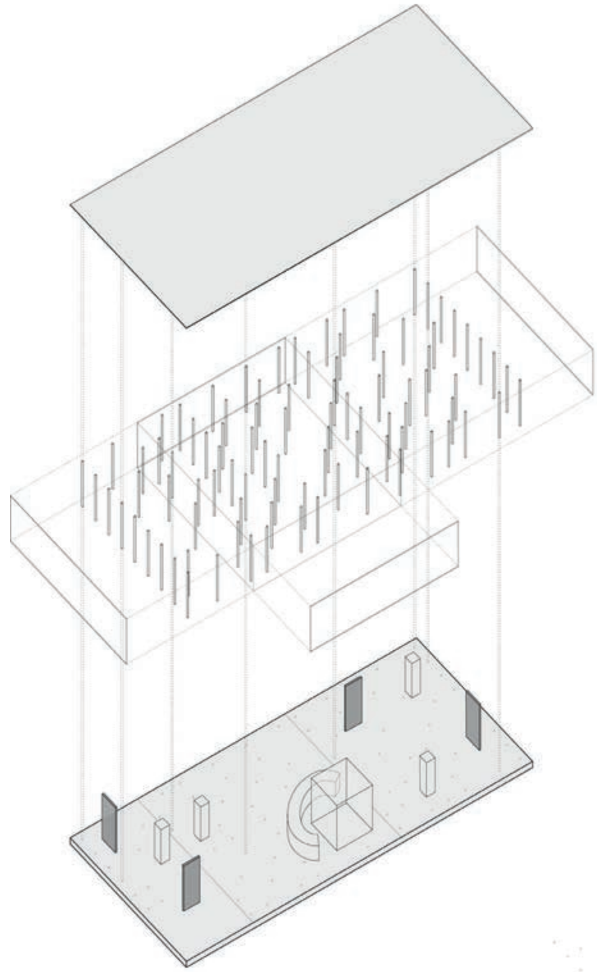


Illustration 10.14
Park Café, Koga, Ibaraki, Japan (1998).
Drawing highlights four thin steel plate shear walls (two each in orthogonal directions) that provide lateral stability for entire café; columns only carry gravity loads.
Architect: SANAA. Structural engineer: Sasaki Structural Consultants.

Illustration 10.15
Koga Park Café.
Very close spacing allows columns to be exceptionally thin, enabling surrounding park landscape to seemingly run right through the building/pavilion. Reinforcing this visual concept, mirror-finished stainless steel cladding covers the structural steel plate shear walls.

for equilibrium, as long as they are not all parallel nor meet at a common point of intersection), whereas the 100 small, irregularly spaced steel columns carry the roof's gravity loads.[2] Very close spacing allows these columns to be exceptionally thin tubes, thus enabling the surrounding park landscape to seemingly run right through the building without significant visual obstruction – despite there being so many columns and that shear walls are in fact also present and doing the work of resisting the lateral loads. Mirror-like polished stainless steel cladding on both faces of the thin steel plate shear walls reinforces this visual concept.

The general lesson of this short progression of examples should be clear: by going from a material such as low-capacity masonry to, first, reinforced concrete and then on to high-capacity steel, one can drastically alter the length and thickness of shear walls that are needed to provide lateral stability to buildings, and in so doing completely upend the sense of their physical and visual presence within the space that these walls support – i.e., one can go from a condition of almost complete heaviness/interiority to one of spectacular lightness/openness. The implications of the choice of different materials in architecture could not be more obvious.

Tall Shear Walls

For shear walls of significant height and/or slenderness, it is likely that bending behavior will limit their lateral-load-resisting capacity rather than that due to shear.[3] (Fig. 10.4a.) For example, the relatively large height-to-length proportions of the reinforced concrete walls of the Torre Cube seen earlier in this chapter (see Ill. 10.5) clearly demonstrate such relative dimensional proportion differences compared to those of lower, longer masonry walls.

In addition to behaving as tall, narrow, flexural cantilevers and thus developing relatively large vertically oriented bending stresses, such shear walls are typically also used to carry gravity loads, which will lead to its own set of axial compression stresses that must be superimposed on the former. (Fig. 10.4b.) The *total* stresses that are directed vertically in the wall resulting from a moment M produced by lateral loading in combination with a gravity load P is thus given by:

$$\sigma = M/S \pm P/A \leq \sigma_u$$

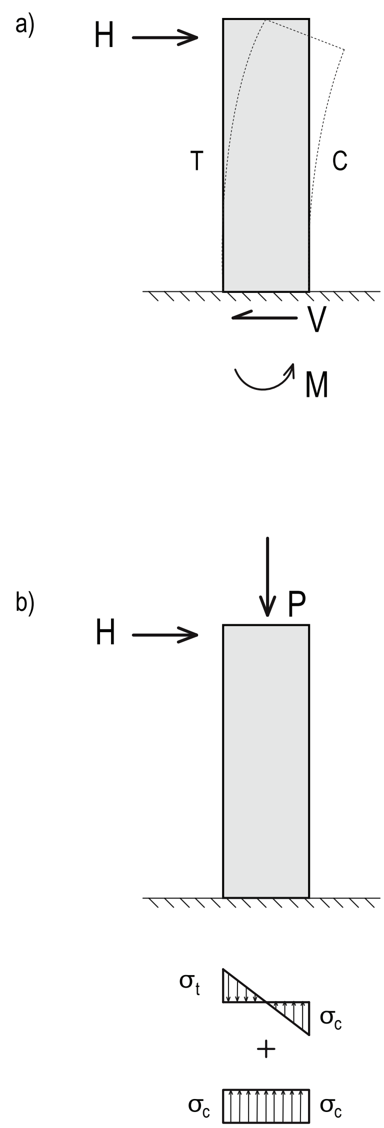


Figure 10.4
(a) Lateral load applied to shear wall having relatively narrow, tall proportions. Bending response of wall dominates. (b) Lateral and gravity loads acting simultaneously on wall lead to linearly varying bending stresses from the former needing to be superimposed on uniform magnitude compression stresses due to the latter.

Illustration 10.16
Svartlamoen Student Housing, Trondheim, Norway (2005).
Building that was one of the earliest to use solid wood panels as vertical shear walls, but also for virtually all other structural components as well.
Architect: Brendeland & Kristoffersen. Structural engineer: Interconsult ASA, Reinertsen Engineering AS.

Illustration 10.17
Svartlamoen Student Housing.
Solid wood panels used throughout for walls and floors create unified, relatively enclosed quality of space.

where S = the section modulus of the cross-section of the wall, A = the area of the same cross-section, and σ_u the material strength, or the lowest value of either its tension or compression strength. The + sign applies to the compression side of the cross-section, where gravity axial stresses are adding to the bending stresses, while the – sign indicates that gravity reduces the resulting stresses on the tension side.

Effectively, then, gravity loads can be considered as a form of pre-stressing of the shear wall before bending stresses act on it, thus effectively reducing the risk of its overturning. If there isn't enough pre-compression from gravity loading, one end of the wall may indeed go into tension, which means that the wall material must be able to withstand tensile stresses and at the base it will have to be anchored down to the ground. (This can be especially true for relatively light shear walls made of solid wood panels that are connected together to act in unison as an overall vertical cantilever (e.g., Ill. 10.16, 10.17), although even heavier concrete shear walls may need to be anchored down when these become exceptionally tall and slender.)

At the other end of the wall, given the large compression stresses likely produced by the combined state of loading, but also given a wall's inherent planar quality and relative thinness transverse to its plane, either the material may fail by exceeding its compressive capacity or else the local buckling failure mechanism that we encountered in Chapter 8 for compression elements may arise. If this is the case, the problem can be addressed by having enlargements or end walls running at 90° to the shear wall plane that effectively can act as stiffeners (Fig. 10.5); at the same time, such projections at the wall ends can also cause the wall to have a huge I-shaped cross-section when seen in plan view – something that will further increase its overall bending capacity because of

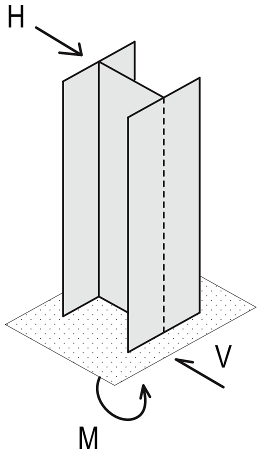


Figure 10.5
Large compression stresses caused by combining effects of lateral loads and gravity loads may cause local buckling at the compression end of the shear wall, a risk that is diminished if transverse end walls are provided, which is effectively also providing the cantilevering shear wall with an advantageously flanged, H-profile section.

Illustration 10.18
Metropolitan Tower and Carnegie Hall Tower, New York City, NY, USA (1987, 1991).
Adjacent tall buildings have exceptional slenderness ratios. (See also Ill. 3.28.)

Architect: Schuman Claman Lichtenstein & Efron (SCLE) Architects and César Pelli Associates, respectively. Structural engineer: Rosenwasser/Grossman Consulting Engineers (for both); one of the present co-authors, Mark Cruvellier, worked extensively on their structural modeling, analysis and design.

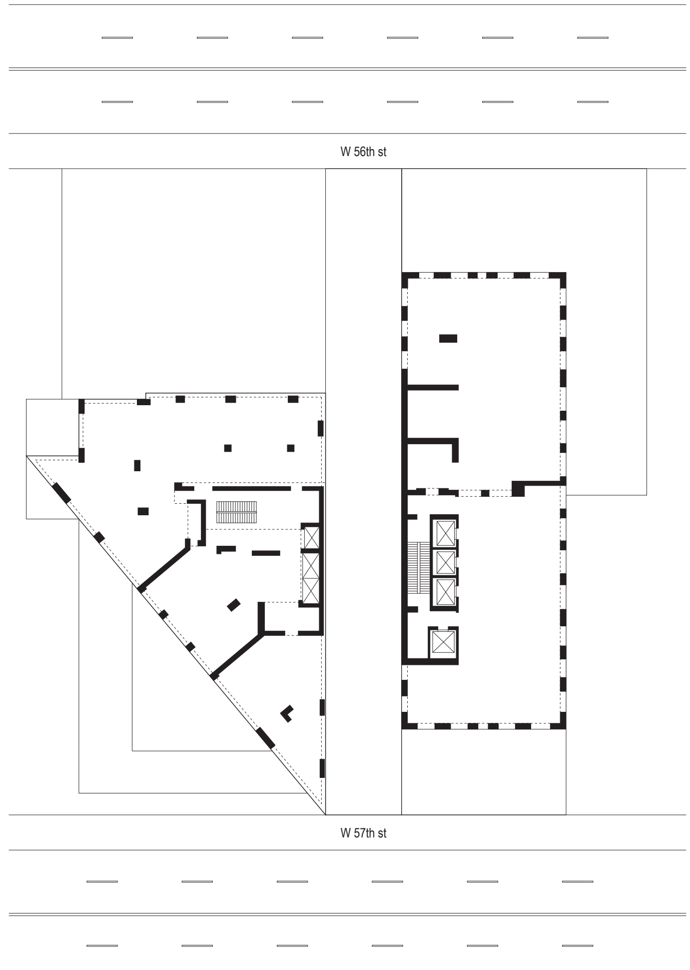


Illustration 10.19
Metropolitan Tower and Carnegie Hall Tower.
Plans for buildings show basic shear wall configurations. For Metropolitan Tower, concrete walls reach out from core toward long diagonal façade, effectively running from one side of the building to the other so as to maximize its moment of inertia; furthering this objective, the wall ends are also somewhat enlarged. For Carnegie Hall Tower, the overall configuration of the shear wall is that of a double perimeter tube, whose thick perimeter walls of high-strength reinforced concrete are perforated to create multiple windows with spectacular views.

an increased section modulus and the beneficial implications of that derived from the flexure formula. The plan configurations for shear walls in buildings of medium to tall height often take advantage of this benefit.

In fact, it is common for such orthogonal assemblies of walls to be gathered together in a building into what is called a *structural core*, an arrangement which not only serves to help stabilize planar wall surfaces against buckling and increases the section moduli of the walls in each direction but also is convenient for surrounding elevator shafts and emergency egress staircases and to run plumbing and other building services up and down the height of a building. (e.g., Ill. 10.18, 10.19) Such cores of intersecting walls often effectively form vertical tubes, which we know from Chapter 2 Statics also makes for a highly efficient way to resist any tendency of the building to twist (in plan), so clearly there can be multiple advantages to such shear wall assemblies.

Left unsaid so far, but implied by our previous discussion of the typically low stress limits of masonry or wood shear walls, is that much taller shear walls such as are presently being considered must be made of materials having significantly larger maximum stress capacity. Reinforced concrete made with various aggregates, water-to-cement ratios, and strengthening additives can be used to meet the requirement for increased compression capacity, and the incorporation of reinforcing bars gives the material tensile capacity as well, which may be necessary to prevent failure due to uplift as buildings get taller and more and more slender.

In the end, whether for low buildings or tall, whether limited by stresses that result from shear or bending behavior, whether made of stone or wood or reinforced concrete or steel, it can be stated that shear walls can be considered to be a remarkably effective and common subsystem for resisting lateral forces in buildings.

Illustration 10.20
Vilharigues Tower, Vouzela, Portugal (original: c.1300; intervention: 2013). Contrasting gravity and lateral-load-resisting structural systems; i.e., thickness and solidity of granite block wall of historical medieval fortification tower ruin vs. glazed, steel-rod braced frame for a contemporary intervention intended to maximize outward views to the surrounding landscape.
Architect for intervention: Renato Rebelo.

10.5 Braced Frames – Basic Behavior and Form Variations

We will now move away from considering walled building systems and their (often) inherently enclosed/enclosing nature to looking at their "opposite": open three-dimensional structural assemblies made up of columns and beams – termed here *frameworks* or simply *frames*. (e.g., Ill. 10.20.) Such constructions are typically not stable unless specific measures are taken to ensure their structural and geometric integrity, and this is what we will look at for the remainder of this chapter, first for frames that are braced by diagonal members and then those made "rigid" without such bracing.

We begin by considering the fundamental problem; that is, the lack of stability of column-and-beam frames that are made up of many discrete structural elements that are assumed to have hinged connections throughout (which will be the case unless very specific counter-measures are taken). One can just imagine trying to build this way and we all intuitively know that the resulting framework, equivalent to the proverbial house of cards (or, perhaps as a better analogy, of matchsticks), would not stand up on its own for very long. And yet, frameworks exist quite commonly – at least once properly stabilized.

Hinged or "pinned" joints either can literally be designed as such or else are to be assumed at connections where no deliberate care has been taken to provide rotational integrity from the end of one structural element to the next, such that effectively free rotation between elements is able to occur. The literal notion of a "pinned connection" dates back to the nineteenth century when true rotational hinges were made by providing element ends with holes through which wrought iron or steel cylindrical rods, otherwise called "pins," were inserted. Today, while much rarer, we may still see examples of such truly hinged joints in frames – sometimes to ensure a particular structural response (or non-response) in the elements being connected, sometimes for expressive design

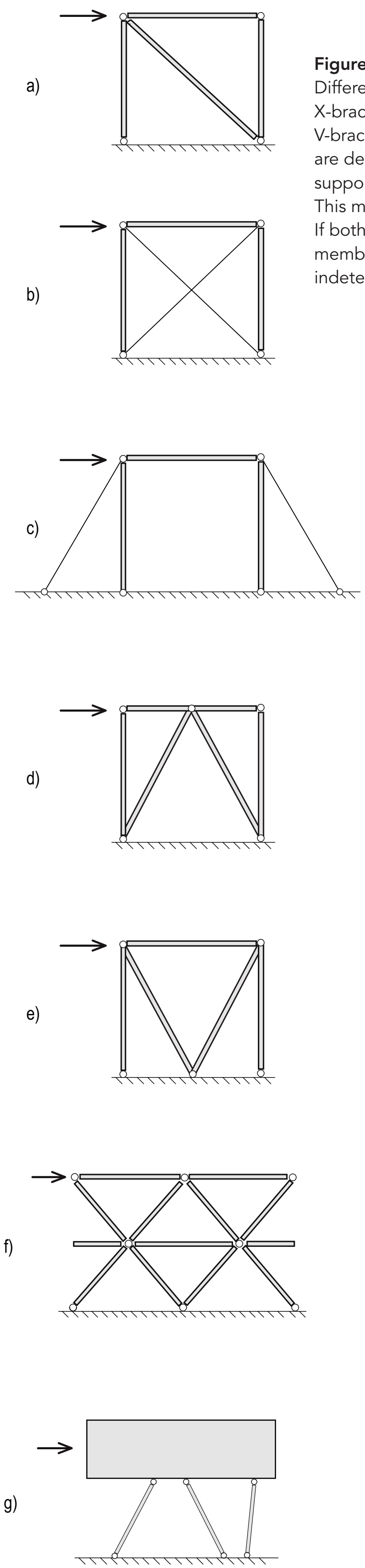


Figure 10.6
Different variants of the braced frame: (a) single-diagonal, (b) X-bracing, (c) external diagonal bracing, (d) inverted-V-bracing, (e) V-bracing, (f) diagrid bracing, and (g) inclined columns. All frames are designed as closed rectangles with hinged connections and supported on one pinned connection and one roller connection. This means that all frames are statically determinate externally. If both supports are pinned connections the lower horizontal member can be omitted, but the braced frame becomes statically indeterminate externally.

aesthetic reasons, and sometimes both at the same time, as we saw earlier at the Architecture School in Lyon (see Ill. 8.16). More commonly, however, hinged joints and support connections are much more mundane-looking and do not visually live up to their name – even while being carefully designed so that significant bending moments are not able to be transmitted between the connecting members. An example of this is the connection commonly used at the ends of steel beams, in which just a few connecting bolts are located close to the beam web's mid-depth while the upper and lower flanges remain physically unattached to the column to which the beam is being connected. Or, as we saw with trusses in Section 9.6, at the scale of an overall truss' dimensions, a hinge connection may effectively be assumed simply when the centerlines of members are deliberately made to align through a common point.

Regardless of the detail resolution, however, such a framework of vertical columns and horizontal beams with hinged connections everywhere does not constitute a stable structure. But even so, assuming for the time being that such stability can and will be provided, a certain amount of structural integrity is nonetheless ensured as hinges do enable the transfer of axial forces and shear forces from one element to the next; the hinged connections of a frame must, therefore, be designed to allow such forces to be transmitted. What is still needing to be established, however, is just how one can prevent such an inherently unstable overall framework from collapsing upon itself.

Which brings us back to describing the second basic strategy for resisting lateral loads on buildings, namely, that of strategically introducing diagonal bracing elements within an assembly of columns and beams. At its simplest, this takes the form of a single member running diagonally across a column-and-beam framed opening, from bottom left corner to upper right, or upper left to lower right, and we will review the workings of this basic configuration shortly. We will also briefly discuss the various forms of the braced frame, including X-bracing, V-bracing, diagrids, inclined columns, etc. (Fig. 10.6), and the structural response of each of these in turn.

Single Diagonal Brace

The structural response to lateral loading of this most basic bracing configuration consisting of a single diagonal element within a four-sided frame (Fig. 10.6a) was described earlier as being analogous

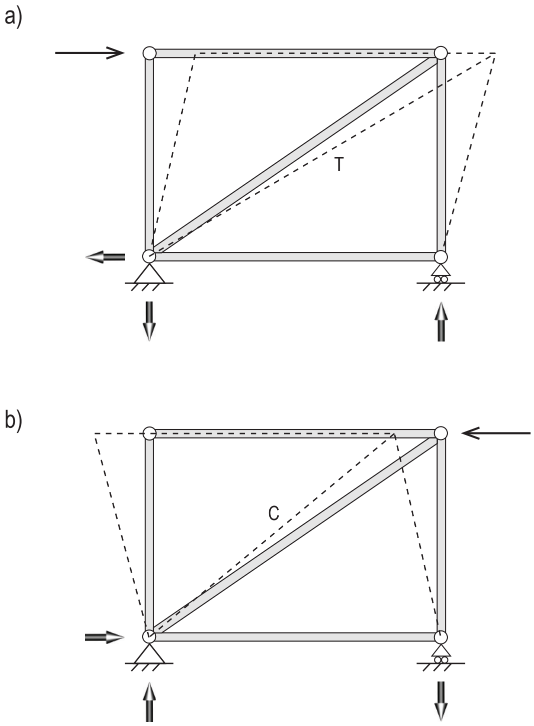


Figure 10.7
Diagrams of deformation and member force response for a single braced panel, with lateral load first applied from left to right and then from right to left.

Illustration 10.21
Casa Kiké, Cahuita, Costa Rica (2007).
Expressive single diagonal wood bracing members stabilize rectangular frame panels along sides of this writer's house/pavilion. Diagonals are integrated into bookshelves, and visual pattern is extended into diagonal bracing beams for the roof.
Architect: Gianni Botsford Architects. Structural engineer: Tall Engineers.

to that of a one-panel segment of a truss (see Section 9.3). A diagonal brace is able to stabilize a pin-jointed frame and a lateral/transverse load on it can be countered by means of an axial tension or compression force within the diagonal – one or the other depending on the loading direction and whether the diagonal tends to be elongated or compressed in response to it. (Fig. 10.7.) Equilibrium considerations for the pin-jointed frame will dictate that its columns and beams will also be subject to axial tension or compression forces whose magnitudes can be calculated (recall the Method of Joints and Method of Sections that were demonstrated for the analysis of forces in trusses in Chapter 9, Section 9.5). Lateral forces on such a frame, then, are able to be resolved into "pure" axial forces in all of its various members, where the word "pure" is used here in the sense that this is the frame members' *only* structural response; i.e., individually, they are not subject to any bending behavior.

Unlike for a horizontally spanning truss subjected to always-downward gravity loading, however, the reversibility of lateral loads produced by wind or earthquakes acting on such a frame means that the dimensions of a single diagonal brace will necessarily always be established by its having to work as a compression element – since buckling considerations then come into play, whereas these don't when the member works in tension. This inevitably leads to having a single diagonal brace which is larger and thus which is more visually "present"/intrusive than the absolute minimum it could be if it were always working in tension. (e.g., Ill. 10.21, 10.22). This observation is among the reasons for us to next consider using an X-bracing form instead.

Illustration 10.22
Highline 23, New York City, NY, USA (2009).
Multistory diagonals are emphasized as part of a braced frame system for resisting lateral loading on narrow building site. System provides a distinguishing feature while also allowing for views of the elevated urban parkway.
Architect: Neil M. Dinari Architects. Structural engineer: Desimone Consulting Engineers.

Illustration 10.23
R128 House, Stuttgart, Germany (2000).
Steel X-bracing minimizes amount of material needed for resisting lateral loads on this exposed site, thus opening up the completely glazed house space to maximum views.
Building designer and structural engineer: Werner Sobek.

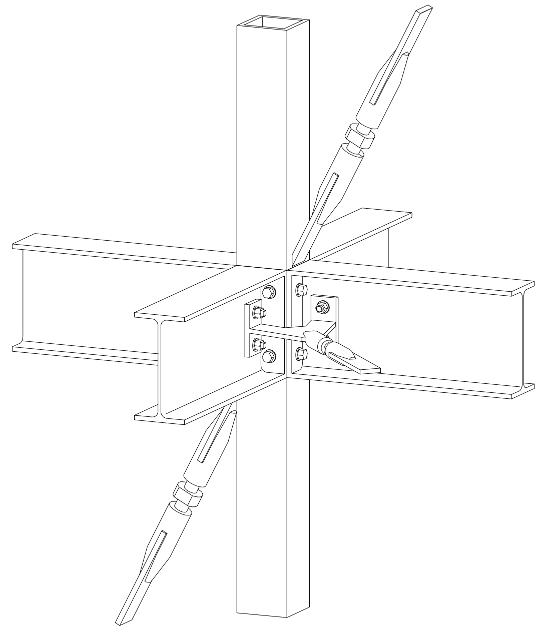


Illustration 10.24
R128 House.
Connection detail that also highlights strategic cross-sectional shapes of different steel elements; i.e., hollow tube for column in compression, flanged section for beams in bending, and thin flat bars for diagonal tension elements. Also of note: all element centerlines are carefully aligned to intersect at a common point.

X-bracing, V-bracing

Certain arrangements of thin tension rods or cables can be used in lieu of the single-diagonal compression member in order to make a braced frame even more visually light and spatially open. The most common form of this is in an X-braced configuration, thus sometimes referred to as *cross-bracing* (see previous Fig. 10.6b). Because of the typically reversible nature of lateral loading, with this type of arrangement one of the thin diagonal bracing rods will always be inactive due to its lack of compression stiffness; i.e., the diagonal member that happens to be leaning toward the direction of short-duration horizontal loading will temporarily buckle and not contribute to either the panel's strength nor to its stiffness. When the load direction is reversed, the opposite happens. The advantage of such a configuration is that it allows each of the two diagonals to be sized for the tension-only condition and since this will be established by material capacity alone, the common use of high-strength steel for cross-bracing will inevitably result in minimal cross-sectional dimensions, such that it may sometimes even barely be visible from a distance. The trade-off, of course, is that this minimum condition needs to be present twice; i.e., each diagonal of the X-brace must be designed so as to be able to resist the full lateral loading acting on the frame. (e.g., Ill. 10.23, 10.24.)

It should be noted that an X-braced system need not necessarily be of the "tension-member-only" type. In the end, minimal dimensions and material efficiency may not be the only design criterion that needs to be paid attention to; for example, there may be aesthetic or conceptual reasons that may favor a heavier but symmetrical X-bracing system that is celebrated and made visible over its minimal tension rod cross-bracing alternative or the inevitably directional and asymmetrical aspect of a single diagonal brace.

Yet a further variation of the X-bracing configuration, although one which works essentially the same way, is to have two oppositely inclined diagonal members but not have them be within the frame being stabilized; e.g., the two bracing elements can be placed on

Illustration 10.25
Steilneset Memorial, Vardø, Norway (2011).
Lateral wind loads from any direction on this very exposed site at the edge of the Barents Sea are withstood by means of inclined wooden struts on both sides of the central frame, thus allowing for uninterrupted circulation down the long, fabric-enclosed interior space of this memorial. Put on display within this are the stories and artifacts associated with 91 people who were accused of witchcraft, tried, and then burned roughly 400 years ago.

Architect: Peter Zumthor Architect and (artist) Louise Bourgeois. Structural engineer: Finn-Erik Nilsen/Oslo Metropolitan University. Cornell model by Jackie Krosnokutskaya and Kevin Alexander.

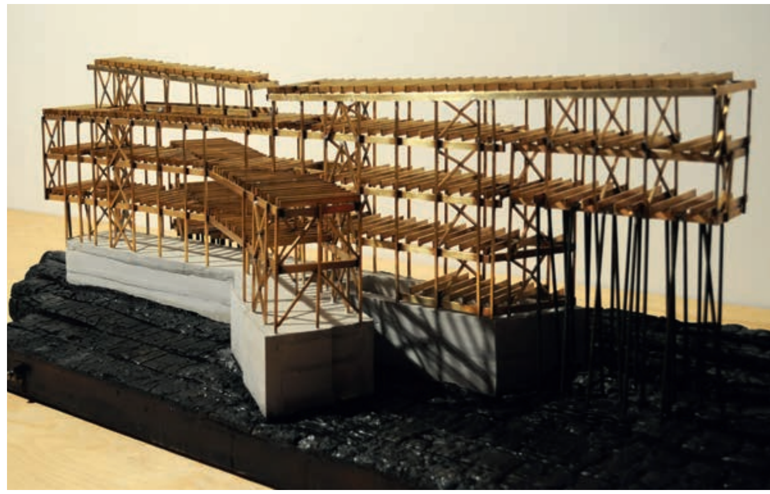


Illustration 10.26
Fogo Island Inn, Newfoundland, Canada (2013).
Organized as an X in plan, building volumes of two and four stories intersect and partly share the same footprint; building must be stabilized by appropriately positioned and oriented cross-bracing within the framework. Notably, this bracing does not occupy every bay of the structure, as the floors effectively transfer the lateral loads to where these can be resisted.

Architect: Saunders Architecture. Structural engineer: DBA Consulting Engineers. Cornell model by Natalie Hemlick and Vinayak Portonovo.

the outside of the frame, one on each side of it. (see Fig. 10.6c). This configuration, of course, has the advantage of liberating the interior frame for through-circulation or completely open views. A potential consequence is that the bracing may now be much more visible on the outside of the building enclosure, which may be an advantage or disadvantage depending on one's point of view. Examples of this approach can be recognized in structures as disparate as guy-cable-stabilized transmission towers and Gothic cathedrals with their flying buttresses. The Steilneset Memorial designed by architect Peter Zumthor together with artist Louise Bourgeois can also be considered as a compelling contemporary example of this particular arrangement. (Ill. 10.25.)

Alternatively, a building designer might wish to have a braced frame provided by a so-called "V-brace" or "inverted-V-brace" (see Fig. 10.6d,e) – also known as a "chevron" configuration – which divides the beam and column rectangular panel opening into three triangles (an example of this was seen in the studio spaces of the École d'Architecture de Lyon (see Ill. 10.11, 10.12)). If such a frame is proportioned appropriately, this arrangement of bracing elements may allow for through-passage circulation in the middle, something that the X-bracing system is unlikely to be able to provide within a typical story height.

Finally, from considering the stabilization of a single two-dimensional frame one can apply the same basic principles and form options to a three-dimensional multi-bay, multistory framework with diagonals strategically placed within the structural system. Given the inherent efficiency of the bracing elements, it is certainly not necessary from a structural perspective to provide diagonal members in each and every bay of the frame on every level, although in some instances this approach will indeed be taken, perhaps for uniformity of aesthetic reasons. More often, however, the diagonals will be much more selectively and strategically distributed throughout the overall system, as we saw at the R128 House (see Ill. 10.23) and as is evident in a model of the structural system for the Fogo Island Inn. (Ill. 10.26.)

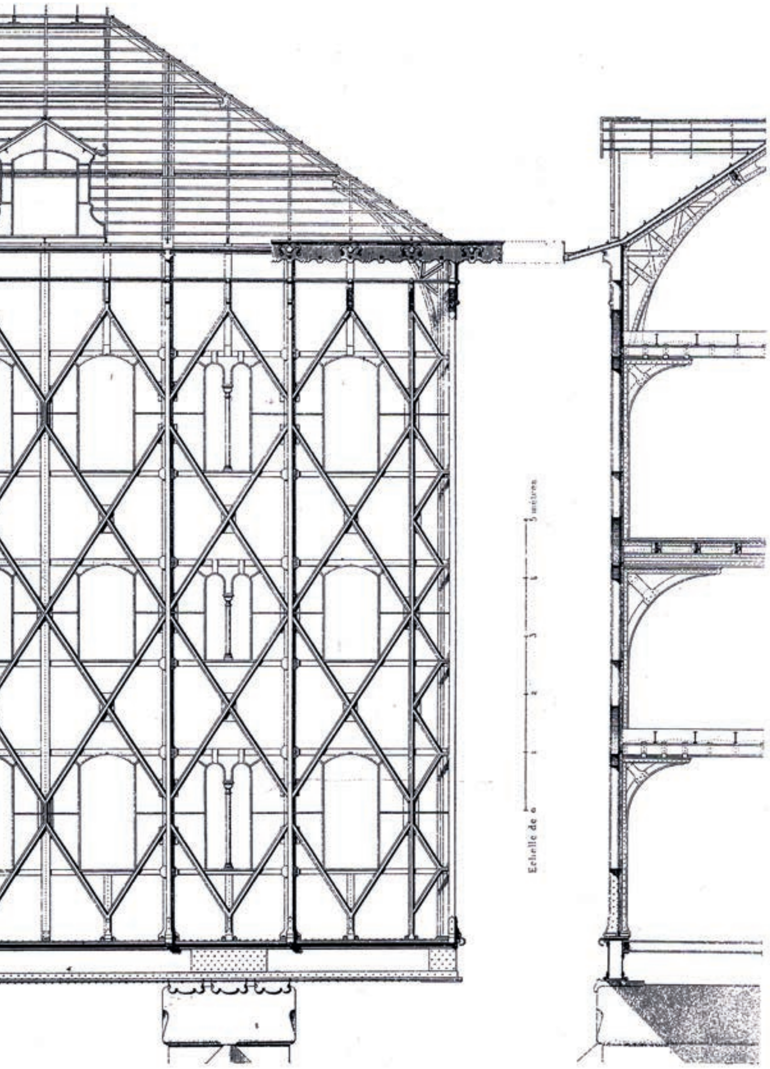


Illustration 10.27
Chocolaterie Menier, Noisiel-sur-Marne, France (original construction 1872, renovated 1996).
Elevation and section of the structural system. A wrought iron diagrid (without being helped by the brick infill) provides the necessary bracing along one direction, while in the building's cross-section curved struts contribute to rigid connections between beams and columns, thus establishing rigid frames for stability (see Section 10.6).

Architect: Jules Saulnier. Structural engineer: Logre and Sèraphin.

Illustration 10.28
Chocolaterie Menier.
Detail of windows and medallion of tiles depicting the cocoa plant as well as highlighting the many intersection points of the diagrid bracing.

Diagrids

Another subgroup of the braced frame, *diagrids* have proven to be an efficient structural form that is quite attractive to architects. (e.g., Ill. 10.27, 10.28.) The name diagrid is, of course, an abbreviation of "diagonal grids," which points to its use of many diagonal structural elements that are arranged in triangular or lozenge-shaped patterns (see Fig. 10.6f) and that together share the load-carrying and stabilizing functions of the X-bracing elements that we have just discussed. Diagrids are often (but not always) made out of steel so as to minimize the dimensions and visual obstruction of the bracing elements; on the other hand, the sharply angled geometry at which the many diagonal members meet will typically call for careful attention to be paid to connection detailing and the construction process. On the other hand, a diagrid has the advantage of being able to simultaneously provide resistance both to gravity loads and lateral loads; because of this, the system can be used to advantage in tall buildings where material savings can become substantial in comparison to a conventional rigid or braced frame system. (e.g., Ill. 10.29.) Deliberately irregular diagrids can also be made to work, such as for the Tod's Omotesando Building (Ill. 10.30), although the members of such a structure will inevitably be heavier than they otherwise might be.

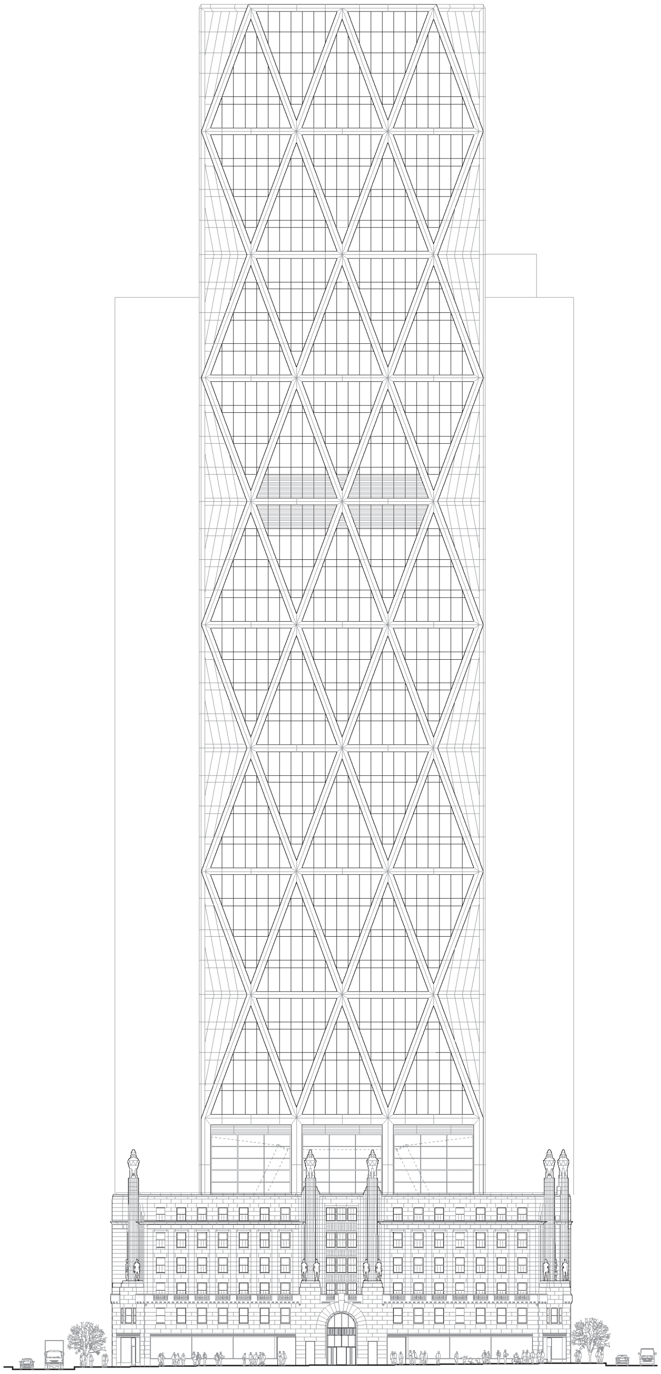


Illustration 10.29
Hearst Tower, New York City, NY, USA (2004).
A perimeter diagrid is used for a new tower structure rising up from its 1928 base.

Architect: Foster + Partners. Structural engineer: Cantor Seinuk Structural Engineers.

Illustration 10.30
Tod's Omotesando Building, Tokyo, Japan (2004).
Criss-crossing structural "branches" of concrete: stability can also be achieved by means of a structural configuration where diagonals are more randomly designed and distributed, perhaps not forming closed triangles at all, in which case both axial forces and bending moments will result.

Architect: Toyo Ito & Associates. Structural engineer: OAK Inc.

Inclined Columns

Still within the broad category of braced frames, we next consider the case of the inclined gravity load-bearing column which, because of its sloped aspect, inherently contributes to the overall lateral load resistance of a building in which it is placed. In order to understand how this is possible, we will imagine the situation of a building that is raised up in the air on several columns (see Fig. 10.6g). This building volume is to be considered a stiff box; i.e., it is a space that is braced within itself according to its own stabilizing structure. This means that the building volume has the ability to transfer forces vertically and horizontally to the columns that are supporting it, and also that as a volume it will not deform significantly no matter how loads are applied to it. While it is clear that the support columns beneath this building volume can be used to carry the gravity loads down to the ground, how can we expect them to provide lateral bracing between the level of the raised building and the ground without introducing any shear walls, conventionally braced frames, or rigid frames?

If we are studying, for the time being, the stability of the building as a planar problem, it is obvious that three parallel columns that are hinged top and bottom will not offer any stability; i.e., the three columns will quite simply rotate over sideways when any lateral load is applied at the top. (Fig. 10.8a). If, as we have seen, we arrange two of the columns to form an inverted-V with its vertex connected to the underside of the building volume, this will in principle provide the necessary stiffness to hinder lateral movement since a stable triangular form has been created. To prevent the building from rotating about the vertex of the inverted-V column pair, however, at least one more column is needed which does not intersect with the other two. (Fig. 10.8b)

Experimenting a bit further with alternate column configurations, we can determine that it is actually not necessary to arrange the columns according to triangular geometry. For example, we can separate the two columns of the V or inverted-V while still letting one column lean to the left and the other lean to the right and keeping the third one in an upright position (Fig. 10.8c). If properly detailed, this arrangement is fully capable of providing the required lateral stability to the columnar support system. In order to explain this conclusion, it is necessary to go back to the fact that the box/building (having its own internal stiffening system) is unable to deform significantly. If the building were to move uncontrollably

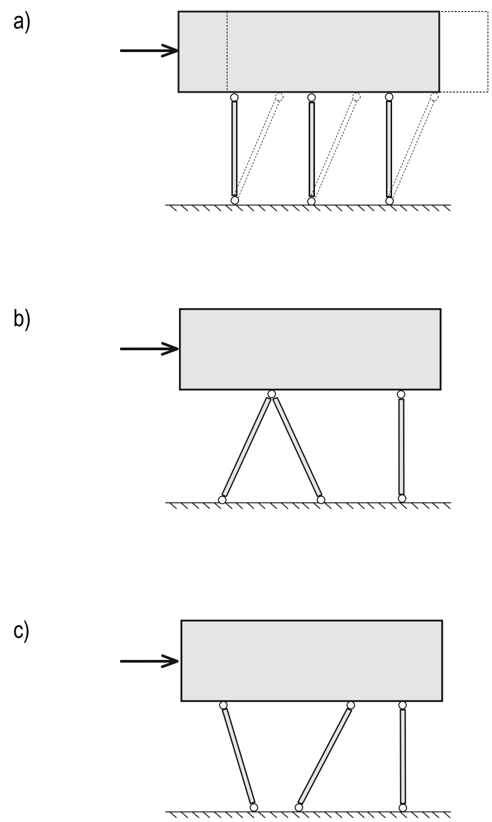


Figure 10.8
Resisting lateral loads by tilted, or inclined, columns. (a) If all columns are parallel (or intersect), no bracing effect is possible. (b) pair of columns in V- or inverted-V-configuration provide lateral stability, but a third column is needed to prevent rotation of elevated building volume. (c) V- or inverted-V-configuration is "opened" up.

sideways, one of the tilted columns would rotate in a way that its top would move downward while the other inclined column top would move upward; i.e., the building volume would also tend to rotate as a whole. But this rotation is not possible because of the presence of the third column (unless the latter were to first fail by buckling or by excessive tension stress depending on the wind direction, but of course that clearly would not be allowed

Illustration 10.31
Sharp Center for Design, Ontario College of Art and Design, Toronto, Canada (2004).
Inclined columns lift a two-story volume of space above a preexisting building; they not only carry gravity loads but also effectively provide bracing for its lateral stability in all directions.
Architect: Will Alsop of Alsop Architects. Structural engineer: Carruthers & Wallace Ltd.

to happen). The only way for the box/building to react to lateral loads, therefore, is for it to try and rotate/displace as a rigid box – but the geometry of the inclined column arrangement will prevent that as well, thereby "locking" the building in its original position. Perhaps somewhat perplexingly, it can be reasoned that stability can also be established for the situation where two columns tilt the same way but have different angles to the horizontal plane, while a third column has a different inclination yet again or else is vertical.

So at least three columns are required to prevent lateral movement in this plane, as well as rotation, while more columns will obviously mean increased stiffness and stability. The basic rules for their alignments is that the columns should not all intersect, nor be parallel. Overall stability of a three-dimensional building may indeed call for inclined columns in yet other directions. (e.g., Ill. 10.31.) Despite this seeming freedom of arrangement, however, we should be aware of the fact that providing lateral stability by means of inclining the columns means introducing more compression forces into certain columns than are necessary to carry only the vertical loads, the result of which will be thicker columns than were originally needed. Lateral stiffness does not come for free.

To conclude this discussion about the range of braced frame configurations and their implications, it can be observed somewhat paradoxically that this lateral-load-resisting subsystem is at once physically minimalistic (in the sense that members are efficiently designed for axial loads alone, and so can be of the smallest possible cross-sectional dimension) and yet it is a subsystem which is also highly visually expressive. Its selection for being included in the design of an overall building structural system, therefore, clearly needs to be carefully considered and thought through. Of course, there are certain well-known examples for which this has successfully been done; e.g., the Eiffel Tower, which from an overall perspective is nothing but a braced frame that has been both elegantly and strategically shaped to be like the bending moment diagram that is needed to resist the lateral loads acting on it. (Ill.10.32.)

Illustration 10.32
Eiffel Tower, Paris, France (1889).
Silhouette against sunrise highlights truss/braced frame qualities of tower structure, with distinct axial elements spaced apart, and overall form mimicking that of bending moment diagram for vertical cantilever.
Designer and structural engineer: Gustave Eiffel.

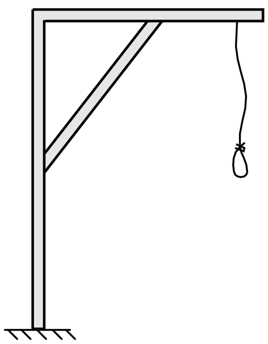


Figure 10.9
Gallows, displaying continuity between post and beam provided by the inclined strut which is attached to both elements.

Illustration 10.33
Branch firmly fixed to the tree trunk, assuring flexural continuity between the two elements. The gradual thickening of the cantilevering branch as it approaches the trunk reflects the bending moment diagram, and this happens "naturally."

10.6 Rigid Frames – Basic Behavior

We now come to the third of the three general types of lateral-load-resisting subsystems: *rigid frames*. For the two alternatives that we have looked at so far in this chapter, namely shear walls and braced frames, we had studied corresponding structural behaviors and their implications in previous chapters – i.e., flexural cantilevers and trusses, respectively – and so we could apply that knowledge directly, even if in a different context, and move on from there. But that is not possible for the rigid frame. In Chapter 2 and Section 10.2 we generally described the defining and essential characteristics of this system, namely, that there are *rigid* connections (otherwise known as *fixed*, or *moment-resistant* or, for short, *moment* connections) between columns and beams, and that the consequence of this condition is that it is primarily the *bending* response of both the columns and beams of the frame that gives this system its stiffness, or certainly that contributes to it in a very significant way. There is no directly corresponding structural behavior that we have studied in detail in this book so far, and so we need to take a closer analytical look at the underpinnings of these statements in order to better understand just how the rigid frame behaves in response to loading and thereby be able to appreciate and apply its design possibilities in various situations.

Rigid Connections

In order to be able to establish flexural continuity between members, what we are calling rigid connections need to be present in this type of frame. But just what is meant by "continuity" in this context of beam-to-column connections?

This condition can, for example, be observed by looking at a macabre gallows structure, in which the simple form consists of a single upright supporting a horizontally projecting element from which a rope is hung. (Fig. 10.9.) Clearly, the horizontal portion of the structure cannot simply rest on the column, for in that case even the smallest vertical load (even its own self-weight) would cause the beam to rotate and fall against the column. To prevent this from happening, a strut is provided that connects the two elements, keeping the beam from rotating about its support point. It is much the same with a large branch jutting out sideways from the trunk of a tree (Ill. 10.33): for the branch to be able to carry load, its connection to the tree trunk must be such that a rotation at the base of the branch will be matched by an equal rotation of the trunk at that location, something that will happen in the case of the tree because of the internal overlapping of a multitude of long wood fiber cells in the natural "construction" of this joint.

In building construction, the analogous moment connections between beams and columns are a matter of careful detailing according to the material that is being used. Some examples are shown in the bottom row of Figure 10.10 for steel, concrete, and wood. Corresponding examples of pinned connections using the

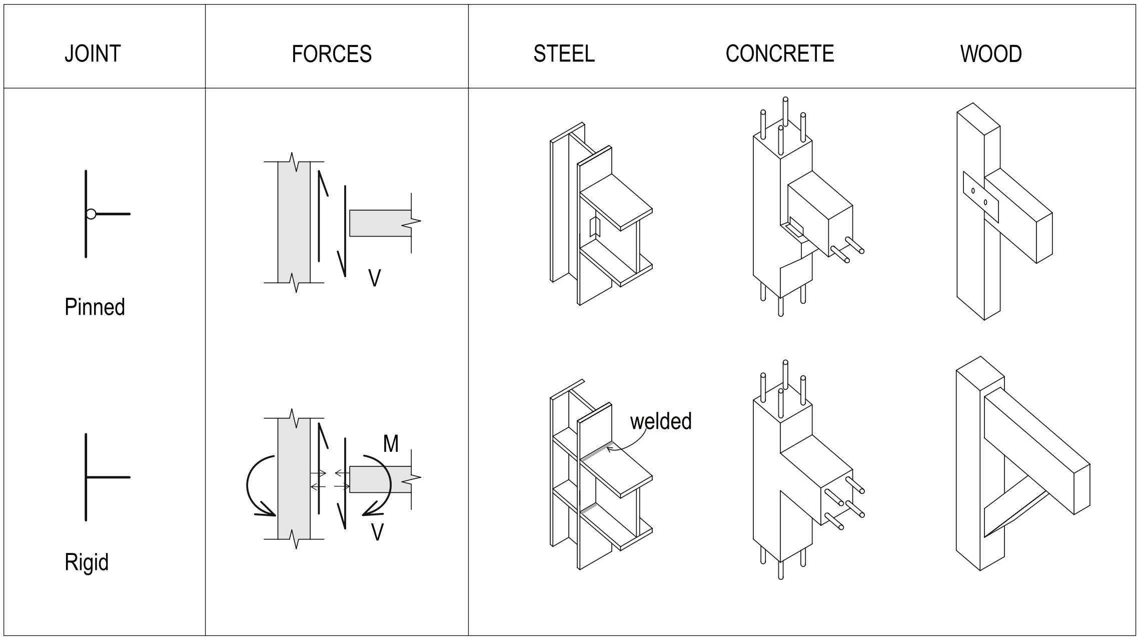


Figure 10.10
Diagram depicting the statical conditions for pinned and rigid beam/column connections, and schematic solutions in steel, RC, and wood.

Derived from Illustration 9.23 of *Structures*, 6th ed.; original permission courtesy of Daniel Schodek.

same materials are shown in the top row of this figure in order to contrast the differences in behavior and the physical resolution of the details, which are in some cases quite subtle and in others highly different. We have already seen several examples of such rigid connections earlier in this book, and we will consider a few more cases now so as to better understand such joints' specific detailing and ways of working to transmit forces and moments.

A way to stabilize a frame in timber construction that was quite common historically was to insert a short diagonal strut between the column and the beam member a small distance out from where they meet, similarly to the gallows structure just discussed. (e.g., see Ill. 5.1, see also Ill. 10.58.) This strut forms a stable triangular shape in the corner of the frame, effectively locking together the alignments of the ends of the two members in such a way that a rotation of one will be transmitted to the other. This strategy of the so-called *knee-brace* in the corners of the timber frame is still employed at times today (e.g., see Ill. 10.51), although a less-visible and more common technique whereby all necessary force and moment transmissions take place roughly within the dimensions of the joint is also possible using specially detailed bolted-through steel connection plates whose inherent in-plane material stiffness effectively "splices" the two wood components together. This can be clearly seen at the Greenwich Academy Upper School (Ill. 10.4) as well as at the corner of the timber frame located on the auditorium stage of the Hedmark Museum. (Ill. 10.34, see also Ill. 10.1.)

Illustration 10.34
Storhamarlåven, Hedmark Museum, Hamar, Norway (1974).
Rigid connections in wood frame by help of inserted triangular steel plates and bolts.

Architect: Sverre Fehn. Structural engineer: Terje Orlien.

Illustration 10.35
Art Gallery, 242 State Street, Los Altos, CA, USA (2014).
Steel rigid frame used for the mechanical opening of the front of a renovated art gallery space; also acts as its iconic "signpost."
Architect: Olson Kundig.

Illustration 10.36
242 State Street Art Gallery.
Flanges and webs of column and beam elements are all welded together, enabling full transfer of bending moments, shear forces, and axial forces.

For steel rigid frames, full rigidity in the connection is typically achieved by specifically detailed bolted or welded connections. Explicit care must be taken to design the joint so that it is able to resist and transmit all axial forces, shear forces, and bending moments from the end of one structural element to the other. The strategy for how to accomplish this is to make sure that both of the outer parts of the cross-sections of the connected elements are firmly attached together, and for steel sections this typically means that the flanges of the columns and beams are directly connected to each other. This type of connection will allow a force couple to develop within the connection, which means that both compression and tension forces in the flanges can be transmitted and that they have a certain distance between them that provides the necessary lever arm to produce bending moment. Connecting steel sections along their web will also provide for the necessary transmission of shear forces. The corners of the frames at the Casa El Mirador seen earlier (Ill. 10.9) and that used for lifting open the glass front wall for the adaptive reuse renovation project at the 242 State Street Art Gallery (Ill. 10.35, 10.36) are clear examples of this type of steel rigid connection.

For reinforced concrete rigid frames, establishing continuity through the joint can relatively easily be done by ensuring that longitudinal reinforcing bars at the top and bottom of beams and vertical bars at the corners of columns are allowed to run through

Illustration 10.37
Nida House, Navidad, Chile (2015).
Reinforced concrete rigid frame is at the essence of this house design. Increasing width of floor plates with height above ground leads to columns being offset in plan from a strict conventional layout.
Architect: Pezo von Ellrichshausen. Structural engineer: Luis Mendieta.

Illustration 10.38
Nida House.
Reinforcing bars embedded within the concrete enable moment continuity between the beam and column elements, but are hidden from view in the end.

the connection, or else are properly anchored into it. (e.g., Ill. 10.37, 10.38.) In that sense, as long as it is properly detailed, reinforced concrete construction is inherently of the rigid frame type; i.e., by building in reinforced concrete one almost "automatically" obtains this type of lateral-force-resisting subsystem without too much additional effort or significant expense or visual consequence. A contrasting situation made of similar material helps to make this point: a frame composed of precast concrete elements, with its kit-of-parts assembly, does not provide for the same rigidity through its discontinuous connections – unless the assembly is ultimately post-tensioned together.

Equilibrium Basics

Moment connections between the columns and beams of a rigid frame inevitably lead to its members being subject to flexural deformations when such a structure is loaded, with the result that bending moments and shear forces typically occur everywhere throughout the frame in both its constituent columns *and* beams (in addition to axial forces, typically). We will see, for example, that a single vertical point load acting on the beam element of a frame will produce bending moments in the beam *as well as* in the columns, and that a lateral load acting at the top of a frame will result in bending moments in the columns *as well as* in the beam.

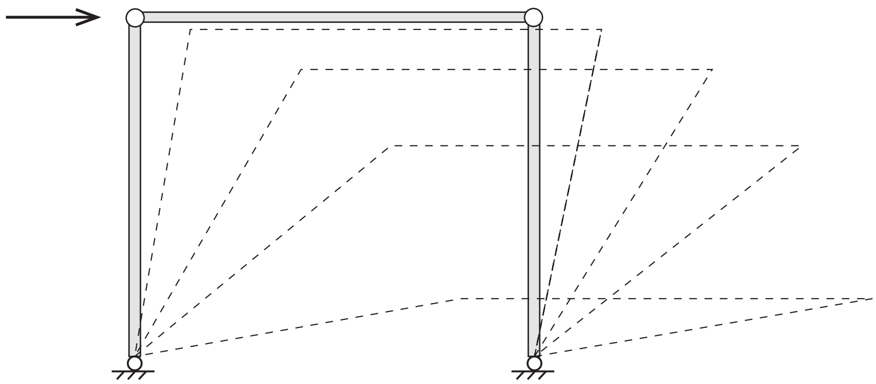


Figure 10.11
A frame with four pinned connections is unstable.

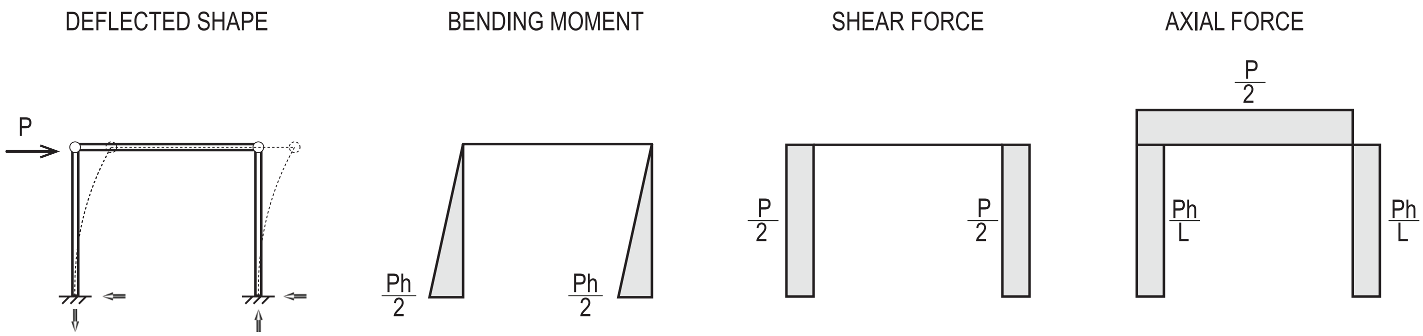


Figure 10.12
Frame with columns having fixed bases but with pinned beam-to-column connections – and corresponding bending moment, shear force, and axial force diagrams. The frame can similarly be stabilized with only one column having a fixed base, but structural demand on this column from the lateral load will then be twice as large.

But this is getting ahead of ourselves; in order to understand how all this happens, we first need to go back to equilibrium basics.

We begin with what we can recognize as a proto-typical rigid frame consisting of two columns supported at their bases at the ground and with a beam connecting them across the top. Clearly a situation that would have hinges at all four corners of the frame results in a structure that is not stable. (Fig. 10.11.)

But the frame can in fact be made to stand on its own if it has a certain minimum number of its connection points that are made rigid/moment-resistant. A relatively simple case to consider is one whereby the columns of the frame are fixed at their bases but both beam-to-column connections are hinged. (Fig. 10.12.) Here the fixed-base columns are the only means of providing lateral stability to the frame and these must act as two independent vertical cantilevers, each resisting half of the total lateral loading. Of course, one can take this a step further and still have stability with only one of the columns being fixed at its base. (We have already studied the details of the structural behavior of cantilevers in Chapter 7 – as well as earlier in this chapter in the context of vertical shear walls – and so we can simply apply here what we have learned before.) In either of these fixed-base-column situations, everything as far as lateral stability is concerned depends on the bending stiffness of the cantilevering column(s), with no benefit whatsoever derived from the configuration of the rest of the structure. Shear and bending moment diagrams for the columns can be established from simple statics and, accordingly,

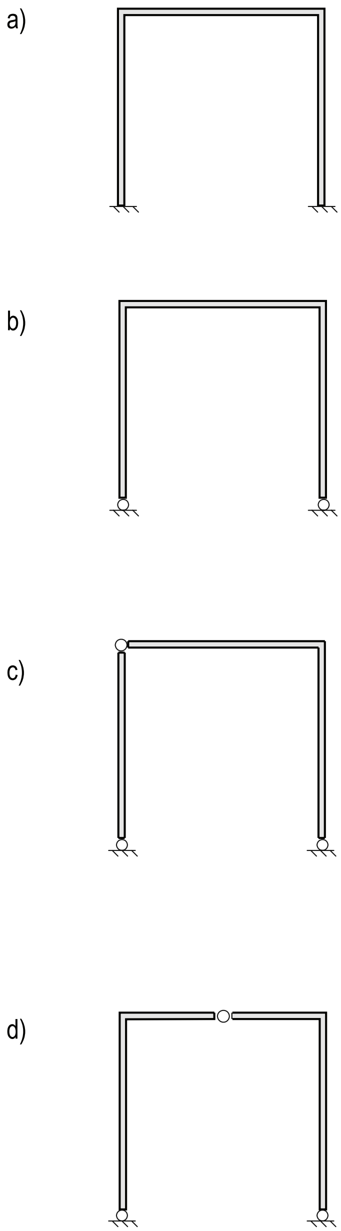


Figure 10.13
Frames with different numbers of hinges and, thus, different degrees of indeterminacy. (a) no hinges, three degrees; (b) two hinges, one degree; (c) and (d) three hinges, zero degree or statically determinate.

a tapering profile for the columns would make sense, i.e., widest at the base and diminishing progressively toward the top. (A tree trunk profile comes to mind, for example, something that is clearly not a natural accident.) But a constant-width column is, of course, also possible, as long as it is sized for the maximum forces and bending moments at the base.

Now let us go to the condition in which the two base supports as well as the two beam-to-column connections are of the fixed/rigid variety. (Fig. 10.13a.) We know from our study of statics and overall equilibrium in Chapter 4 that this structure has three unknown reactions at each column base: horizontal and vertical force reactions and a moment. There are thus six unknown support reactions for the frame structure as a whole but only three equations of equilibrium with which to solve for them, and so we have a structure which is three times indeterminate. There are well-established special methods to determine these unknowns (by hand and also by means of the computer), but we will present here only one method of rigid frame analysis – one that directly uses basic statics and equilibrium considerations – since such an approach helps to visualize fundamental concepts about the interconnected workings of the rigid frame system.

We will proceed by considering yet another special case of the rigid frame, one for which we provide a certain strategic number of "moment releases" since, as we have already seen, a frame can still be stable even if some of its connections are pinned while others are rigid. In fact, the maximum number of hinges in order for this to be the case is three, and since a planar three-hinged frame is statically determinate (the original six unknowns we were just confronting gets reduced to three by means of the moment releases at the hinges), we know that for such a structure we can relatively simply calculate its support reactions and internal forces and moments by typical hand methods using the relevant three equations for equilibrium. The three hinges may be located at the two column bases plus one at mid-span of the beam, for example; the two remaining connections in the frame must be rigid in order for the structure to remain standing up. (Fig. 10.13d.) Such a particular configuration of the frame is named a *portal frame*. If fewer than these three hinges are provided the structure is statically indeterminate – which is still possible, of course, and is quite common in real-life construction – but the method of analyzing such a structure becomes more complex than that which will be presented here.

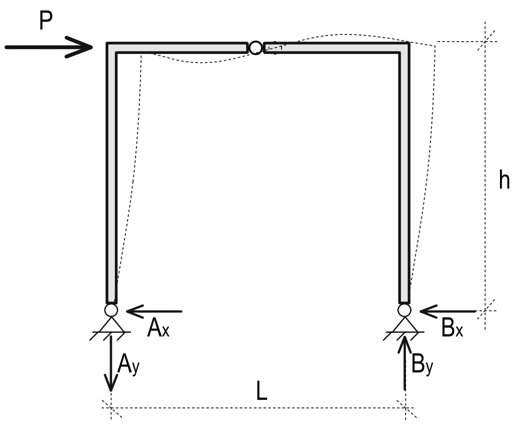


Figure 10.14
Three-hinge frame with lateral load P at the top. Resulting deformations and reaction forces.

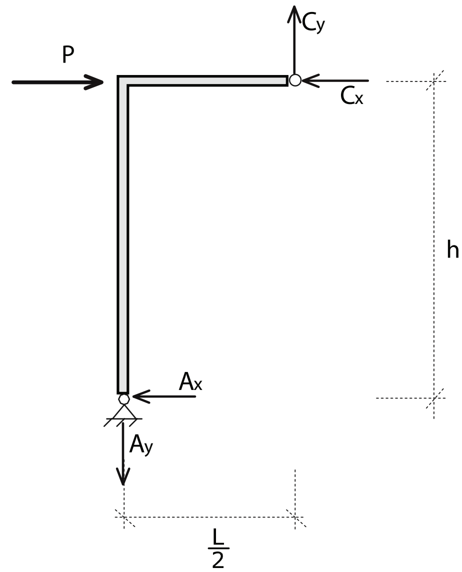


Figure 10.15
Free-body diagram of the left half of the frame.

Analysis of Forces and Moments

We will now analyze closely the three-hinged portal frame having pinned supports at the column bases and a hinge at mid-span of the beam and which is acted upon by a lateral load P acting at the top of the frame from left to right.[4] (Fig. 10.14.) A study of how the frame can be anticipated to deform under load – this can be reassuringly recreated using a flexible physical model – is revealing: the beam/column connection to the left tends to open up, but is restrained from doing so by the continuity of the connection. This results in bending (and, therefore, bending moments) in both the column and the beam, with tension stresses on the inside of both elements in the vicinity of the corner. At the same time, according to the anticipated deformed-shape diagram, the right-hand connection experiences the opposite: here, the joint angle tries to close and the bending moments produce tension stresses on the outside. Also, the sideways deformation of the columns clearly indicates the direction of the necessary horizontal forces at the supports, acting on the columns' lower ends. Those can be seen to also represent the shear forces in the columns.

In order to calculate the magnitude of the moments and the forces within the frame itself, we start by finding the base support reactions. Applying the equilibrium equation which expresses the rotational equilibrium of the frame as a whole about support B, $\Sigma M_B = 0$, yields

$$+ Ph - A_y L = 0$$
$$A_y = Ph/L$$

where h = the height of the frame, L = the span, and a direction is assumed for A_y, which is the vertical support force acting on the column. We guessed that it would be a downward force and we were right: the treatment of the equilibrium equation resulted in a positive value for the axial end force, which means that the direction of A_y is downward as anticipated and the column is subjected to tension. In accordance with Newton's third law the vertical force from the column acting on the base connection at support A has an upward direction, indicating that the foundation must be prevented from uplift.

To maintain vertical equilibrium of the frame, B_y must be of equal magnitude to A_y but acting in the opposite direction:

$$B_y = Ph/L$$

To find the horizontal support reactions it is convenient to demand rotational equilibrium of the left half of the frame about the mid-span hinge C, $\Sigma M_C = 0$. (Fig. 10.15.) A_x is assumed to be directed to the left, and therefore produces a positive moment about point C (clockwise rotation). This yields:

$$+A_x h - (A_y) \times (L/2) = 0$$
$$+A_x h - (Ph/L) \times (L/2) = 0$$
$$+A_x h = Ph/2$$
$$A_x = P/2$$

Requiring horizontal equilibrium of external loads and forces for the frame as a whole, $\Sigma F_x = 0$, means that

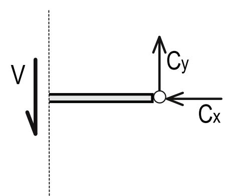


Figure 10.16
Free-body diagram of a portion of the beam. Shear force shown.

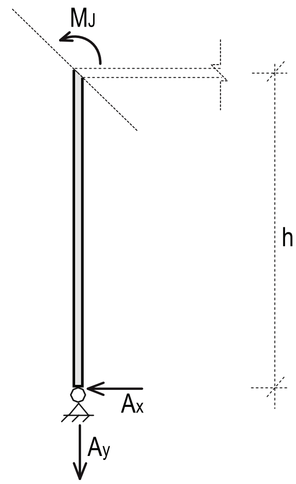


Figure 10.17
Free-body diagram of the left column. Bending moment shown.

$+P - A_x - B_x = 0$
$+P - P/2 - B_x = 0$
$B_x = P/2$

All external reactions are now found. Since no other loads act transversely along the columns, a check of horizontal equilibrium at any cross-section in these will show that the shear force V remains constant at V = P/2 throughout their height. Since the tendency of both horizontal reactions that produce shear forces in the columns is to rotate the elements they act on clockwise, the shear forces in both columns are defined as positive. A check of the vertical equilibrium of the columns will reveal that the left column has a constant axial tension force N which is $N = A_y = Ph/L$, while the right column has an axial compression force of the same magnitude.

What about the forces in the beam? A study of the free-body diagram of the left half of the frame (see Fig. 10.15) makes us anticipate a horizontal force C_x and a vertical force C_y at the hinge C. Requiring horizontal equilibrium of this half leads to

$+P - A_x - C_x = 0$
$+P - P/2 - C_x = 0$
$C_x = P/2$

The direction of the force is such that it produces an axial compression force in the beam. A check of internal horizontal equilibrium of this portion of the beam satisfies us that the compression force is constant along its length. At the same time, the oppositely directed reaction force to C_x produces compression force of the same magnitude in the other half of the beam.

Now for the beam's shear force: vertical equilibrium of one half of the frame yields

$+C_y - A_y = 0$
$C_y = A_y = Ph/L$

with C_y having an upward direction. If we study a cross-section somewhere along the length of the left half of the beam, we will find that equilibrium demands that there is an internal shear force V of constant magnitude throughout the length of this portion. (Fig. 10.16.) It is always directed downward, tending to rotate the element in question counterclockwise; the shear force is, therefore, assigned to be negative. Again, the reaction force to C_y acts in the opposite direction on the other half of the beam and also tends to rotate this part counterclockwise, resulting in a negative sign also for the shear force in this part of the beam.

At this point, only the bending moments remain to be determined. Demanding rotational equilibrium of the left column about the beam/column joint (Fig. 10.17) gives us the equation

$-M_J + A_x \times h = 0$
$-M_J + P/2 \times h = 0$
$M_J = Ph/2$

where M_J = the internal bending moment in the column at the joint. This moment increases linearly from zero at the pinned base support, and it produces tension on the inside of the column. Since rigid frames are characterized by continuity at the joints, we can conclude that a bending moment of the same magnitude is acting

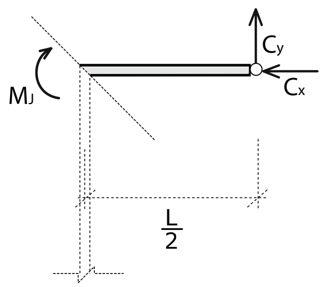


Figure 10.18
Free-body diagram of the left half of the beam.
Bending moment shown.

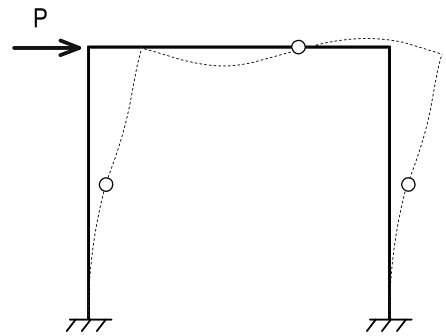


Figure 10.19
The rigid frame with two fixed supports.
Anticipated deformation suggests that this behaves as if having three hinges – located at mid-heights and mid-span of columns and beam, respectively. The associated bending moment, shear, and axial force diagrams for the top portion of the frame are therefore similar to diagrams applying to true three-hinge portal frames with hinges at the same locations.

at the corresponding end of the beam, and that this becomes zero at the mid-beam hinge. (Fig. 10.18.) To check on this assumption we require rotational equilibrium of the relevant portion of the beam about the joint, and find that

$$+M_J - C_y \times L/2 = 0$$
$$+M_J - Ph/L \times L/2 = 0$$
$$M_J = Ph/2$$

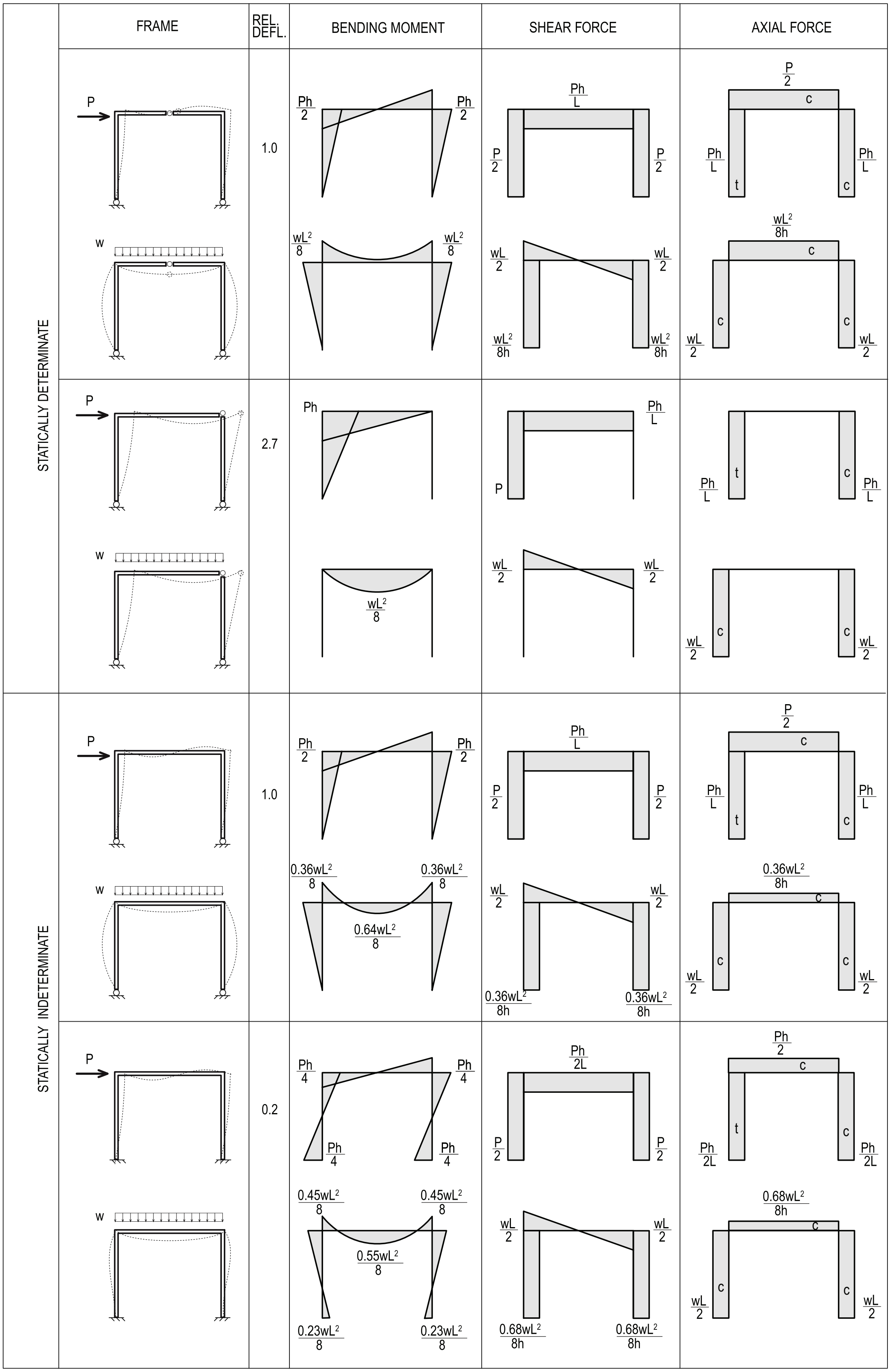
which is what we had previously predicted would be needed to balance the moment in the column. The bending moment in the beam produces tension force on the underside of the beam in this left-hand half of the frame. Finally, a similar check of the right column and the portion of the beam to the right will reveal bending moments of the same magnitude, but in this case producing tension on the outside of the column and on the upper face of the beam.

From the results of these calculations, we are now able to draw the bending moment, shear force, and axial force diagrams for this three-hinged portal frame. These are presented across the top row of the table in Figure 10.20. We could also determine bending moments, shear forces, and axial forces in this frame when supporting a uniformly distributed load (UDL) along the beam; this is done in much the same manner, always applying the three equilibrium equations in different situations. If both the lateral point load and the gravity UDL act at the same time on the frame, the resulting moments and forces will be obtained by simply adding together their respective diagrams according to the principle of superposition.

In the case of frames having two fixed supports (and no hinges) the frame is strictly speaking statically indeterminate with six unknown reactions, as we've previously described. A simplified way of dealing with this situation is to recognize the anticipated deformation resulting from such a frame being subjected to loading.[5] If a lateral load P acts on this frame, the columns will deform in such a way that their curvature will be reversed at mid-height (whereas they were only singly curved from bottom to top in the preceding case), while the beam will also reverse its deformation curvature at mid-span (which is the same as before). (Fig. 10.19.) The points at which the member curvatures are reversed are called points of *inflection*; these are also, therefore, the locations where the direction of the bending moments change. As a result, at these locations tension changes to compression or vice versa on the internal and external faces of the members. At the very place where the change takes place, the bending moment must be zero, and we can thus assume *effective* hinges at these locations. This conveniently means that we can make the assumption that the frame is *effectively* statically determinate, since three additional equilibrium equations are created ($\Sigma M = 0$ about each of the effective hinges) that together with the three basic equilibrium equations ($\Sigma M = 0$, $\Sigma F_x = 0$, $\Sigma F_y = 0$) applied to the frame as a whole makes a total of six equations, which would enable us to solve for the six unknown reactions.

We could obviously carry on in this way as well as by other methods of analysis for rigid frames having various combinations of hinge points as well as for frames without any hinges at all, and the table in Figure 10.20 summarizes the results of this in the form of corresponding bending moment, shear force, and axial force diagrams.

FACING PAGE
Figure 10.20
A number of portal frames and rigid frames with lateral load and gravity load applied, and with their corresponding bending moment, shear force, and axial force diagrams.
Note the comparison of the stiffness of the different frames, given by a relative number for sideways deflection. These numbers apply for frames where height h equals span L, and where all elements have the same cross-sectional properties and material properties.
(c = compression; t = tension)


FRAME
REL. DEFL.
BENDING MOMENT
SHEAR FORCE
AXIAL FORCE
STATICALLY DETERMINATE
STATICALLY INDETERMINATE
P
w
1.0
2.7
1.0
0.2
Ph/2
Ph/L
P/2
Ph/L
t
c
wL²/8
wL/2
wL²/8h
Ph
P
0.36wL²/8
0.64wL²/8
0.36wL²/8h
Ph/4
Ph/2L
Ph/2
0.45wL²/8
0.55wL²/8
0.23wL²/8
0.68wL²/8h

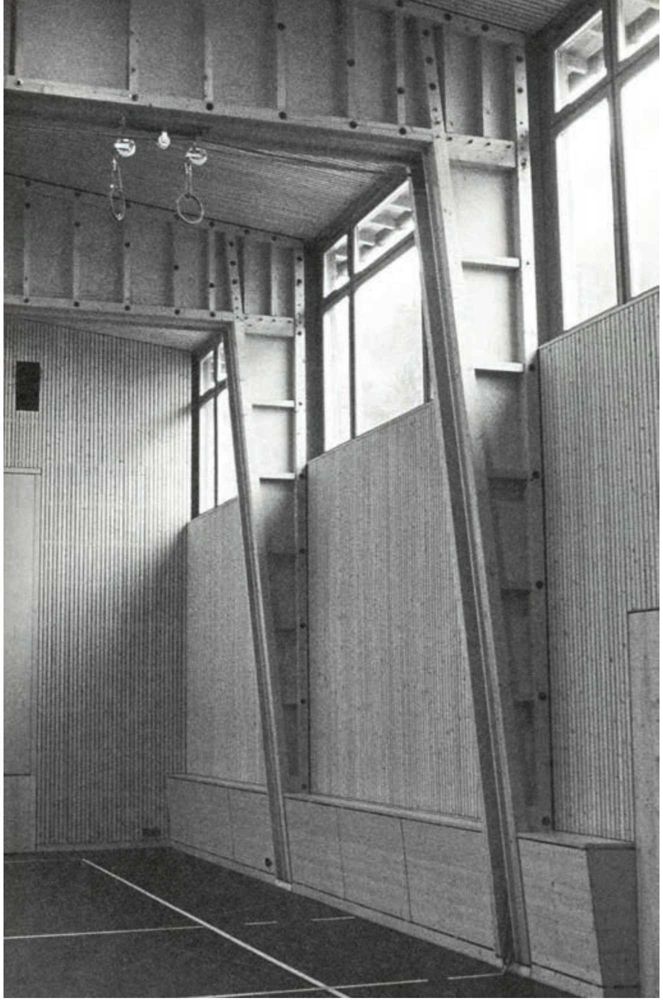


Illustration 10.39
Multipurpose Hall, Alvaschagn, Switzerland (1991).
Series of timber frames resist gravity and lateral loads, but also provide necessary openness for use of multipurpose space. Rigidity of top corner connection between beam and column elements is evident, while tapering of column suggests a pin connection at the base.
Architect: Bearth & Deplazes. Structural engineer: Conzett, Bronzini, Gartmann AG.

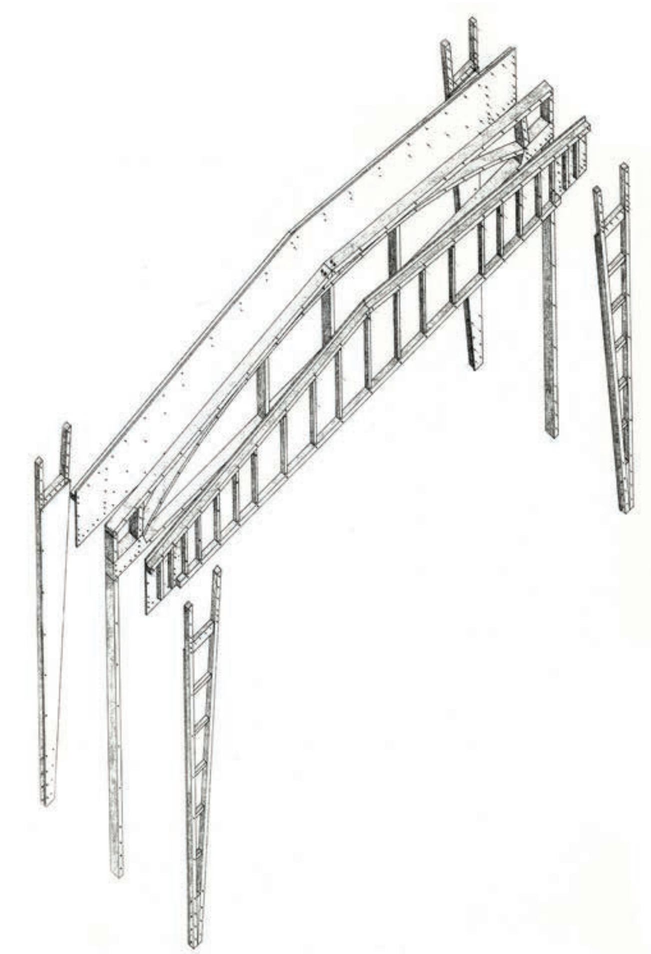


Illustration 10.40
Multipurpose Hall.
Stiffened timber plates overlap at the corners of the frames, ensuring the necessary rigid connection between beam and column elements.

The relative differences in the lateral deflections of the various rigid frames given in Figure 10.20 should also be noted: for example, the frame with only one rigid connection deflects considerably more than do either of the pin-supported frames having two rigid joints; almost three times as much, in fact. Also to be noticed: the amount of deformation is exactly the same for the first and third frame configurations in Figure 10.20, suggesting that despite seeming differences these two frames are effectively the same for this load condition. In the two-hinged frame (the third frame configuration), there is zero bending moment in the beam at its mid-point, reflecting a so-called inflection point where moments shift direction and sign. Again, this means that this inflection point effectively acts as a hinge and that this two-hinged frame behaves in a similar fashion to its three-hinged counterpart (the first frame configuration) – for this load condition, at least. One more observation about the lateral deflections: the rigid frame having two fixed supports deflects considerably less, merely one-fifth, in fact, of the amount of the two previously mentioned frames; i.e., the fixed support frame is obviously a considerably stiffer configuration with respect to lateral sidesway.

10.7 Rigid Frames – Form Variations

As we have seen for all other basic structural elements or subsystems, there are many ways in which to vary the formal expression of rigid frames in order to suit various loading and spanning requirements, but also according to myriad possible design intentions – whether these be conceptual, spatial, visual, etc. In the next few pages we will briefly address some of these possibilities, which can either be exploited or not as the case may be and as suited to design intentions.

Bending Moment Variations

The consequences of the force and moment diagrams in terms of what they can tell us about the potential shaping of rigid frames are quite powerful, especially concerning the latter – i.e., the moment diagrams – just as was the case for beams as we saw in Chapter 7. For example, for an efficient use of materials the dimensions of a rigid frame's columns and beams may, to a certain extent, at least, reflect the variations of the bending moment diagram, while their capacities also need to be checked for the shear and axial forces. (e.g., Ill. 10.39, 10.40, 10.41). The rigid frames' moment

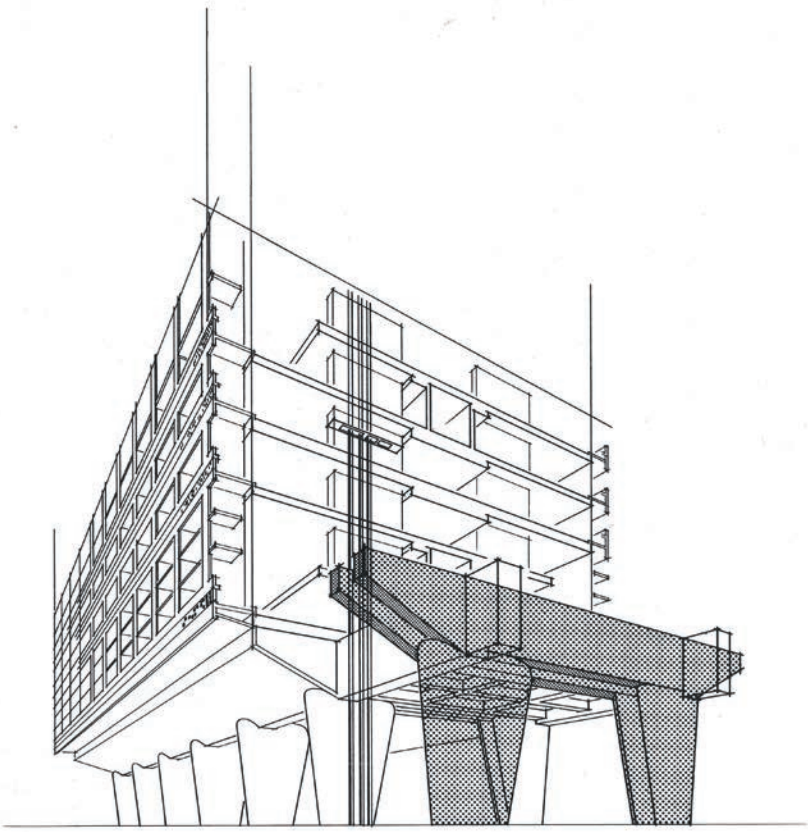


Illustration 10.41
Unité d'Habitation, Marseille, France (1952).
Series of concrete rigid frames literally lift the base of the apartment block up off the ground. Columns of the frame vary in width, maximum at beam-column connection where bending moment is greatest, and minimum at their base, approximating a hinged condition there and also furthering the visual effect of the building's disengagement from the ground. Hidden pairs of concrete beams of large constant depth complete the transverse rigid frames.

Architect: Le Corbusier. Structural engineer: Vladimir Bodiansky.

Illustration 10.42
Buchholtz Sports Hall, Uster, Switzerland (1998).
Single-story, single-bay steel rigid frames are repeated every so often along linear axis to give one large occupiable space for playing/watching sporting events.

Architect: Camenzind Evolution. Structural engineer: Geilinger Stahlbau.

Illustration 10.43
Buchholtz Sports Hall.
Variations in the width of the column and depth of the beam reflect the bending moment diagrams, with maximum at the beam–column connection and minimum at the hinge locations at the column base and part-way along the beam span.

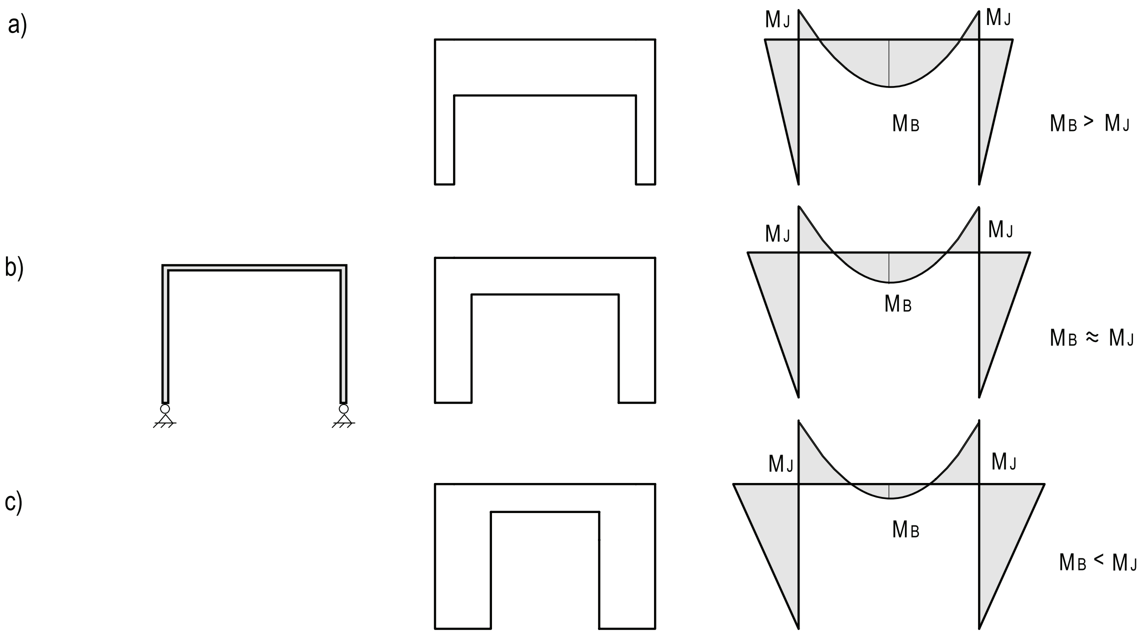


Figure 10.21
The importance of relative beam-to-column flexural stiffnesses for statically indeterminate rigid frames: i.e., stiffer members "attract" larger bending moments. If the relative stiffnesses change between beam and columns, then so does the bending moment distribution in the frame.
(a) represents the case of strong beam + weak column, (b) balanced beam-to-column stiffnesses, (c) weak beam + strong columns.

connections are in most cases quite heavily stressed as a result of large bending moments in the corners of the frames. This suggests the possibility of having larger structural dimensions for the frame at these locations, a configuration that we can readily observe in some rigid frames. (e.g., Ill. 10.42, 10.43.) That being said, it should also be pointed out that it is not usually possible for the form of a frame to truly follow a single bending moment diagram, as there will certainly be multiple load conditions to which a frame will be subjected over time, and these will result in ever-changing combinations of diagrams and stress conditions. Clearly, member dimensions cannot constantly be changing from one minute to the next, so the shape of the frame may instead reflect the bending moment diagram for the *dominant* load condition, presuming that other load conditions are also checked and found not to produce even larger bending moments at any cross-section along the frame.

There may, on the other hand, be aesthetic design or conceptual or even practical reasons for the member dimensions of a frame to be uniform, in which case the cross-section that is needed wherever there is maximum combined-force-and-moment demand is kept throughout, even though this is conceptually "wasteful" in terms of purely efficient usage of material. (e.g., see Ill. 10.9, Ill. 10.37, and Ill. 10.38, although in the latter case there will in fact be variations in the number and size of the hidden reinforcing bars within the constant dimensions of the concrete frames' columns and beams.)

Such differing approaches to the shaping (or non-shaping) of rigid frame members in relation to bending moment diagrams brings to mind a similar discussion that we had in Chapter 7 when we considered the design possibilities for the profiles of beams, which serves here to further emphasize the relative importance of flexure for this particular type of lateral-load-resisting subsystem.

Beam/Column Stiffness Variations

Rigid frames depend, as we have seen, on *both* column and beam elements contributing to the overall load-resisting mechanism, and it therefore becomes relevant to discuss the importance of the relative magnitudes of beam-to-column stiffnesses. In the most general cases, we have seen that these frames are statically indeterminate and the magnitude of the internal forces and moments is then dependent on the relative proportions of their members; i.e., stiffer members will do "more work," which in this case effectively means that they will "attract" larger bending moments. For example, in Figure 10.21 we examine the case of a pin-supported rigid frame: in case (a) the frame has a very large and stiff beam interacting with thin, flexible columns. When a uniformly distributed gravity load acts on the beam it will deflect substantially, as the flexible columns will not be able to offer significant restraint to the ends of the beam. The beam will experience bending moments which will be virtually the same as those of a simply supported beam, with large moments at mid-span. (e.g., Ill. 10.44, 10.45.) In case (b) the proportions of beam-to-column relative stiffness are more balanced. The columns' bending resistance will provide partial restraint to the ends of the beam, and the elements will share

Illustration 10.44
Crown Hall, Illinois Institute of Technology, Chicago, IL, USA (1952).
Erecting of the main structure. In spite of the great difference between the depth of the beam and column, the bending moments in the two elements at this rigid connection must always be the same for equilibrium of the joint. Clearly the aesthetics of having a constant depth beam across the full width of the building came into play in this design, as well as the desire to have these beams' presence noticed from ground level.

Architect: Ludwig Mies van der Rohe. Structural engineer: Frank J. Kornacker.

Illustration 10.45
IIT's Crown Hall.
Series of parallel frames' deep plate-girder beams are left exposed above the roof and span clear over the building's architecture studio space. These beams have also served as a distinguishing aspect for this iconic building.

Illustration 10.46
Picture Window House, Shizuoka, Japan (2001).
Overall rigid frame is created by the combination of trussed vertical end supports and the long-span truss contained within the enclosed living space of this house. Together these create a framed "picture window" terrace from which to enjoy the views and breezes of the Pacific Ocean.
Architect: Shigeru Ban Architects.

between them the moments needed to carry the load. In the third case (c), the beam is quite flexible whereas the columns are disproportionately stiff, offering almost full restraint to the ends of the beam. The result is that the bending moment in the beam is significantly reduced at mid-span while the moments at the beam ends are quite large, resembling the condition of a beam that is fully fixed at both ends.

Trussed Frame Variations

Within this general discussion about potential variations of the rigid frame, we can also consider different kinds of structural elements and how these can be used for the column and beam components. Because both of these elements need to be designed to be able to resist bending moments (as well as shear forces and axial forces) an effective member type to consider aside from that having a strategically oriented rectangular or I-shaped sectional profile is the truss, since these even more efficiently distribute materials so as to maximize the moment of inertia of the member and, therefore, increase its bending capacity, as we saw in Chapter 9. (e.g., Ill. 10.46, Ill. 10.47, 10.48.) *Trussed frames* are, therefore, quite advantageous, although the problem of designing the corresponding rigid "beam-to-column" connection while maintaining an ordered and pleasing look, given the many lines of intersecting truss members, has challenged and fascinated many architects over the years.

Aesthetic complications aside, providing effectively fixed conditions at the corner connections of trussed rigid frames can be achieved relatively simply. In terms of overall dimensions, trusses have significant depth, and so connections that are effectively rigid can be achieved by connecting the outer chords of the intersecting "column" and "beam" trussed elements. It is sufficient in such cases that each local connection is simply able to transmit axial forces; as with the truss itself, the spatial quality of the overall connection will allow couples of axial forces that are at some distance from each other to produce the required bending moment that needs to be transmitted, while shear will typically be carried by diagonal members within the connection. Given this rationale, then, what may at first seem to be a somewhat surprising use of trusses for both the beam and column elements of a rigid frame becomes quite logical – indeed, highly strategic.

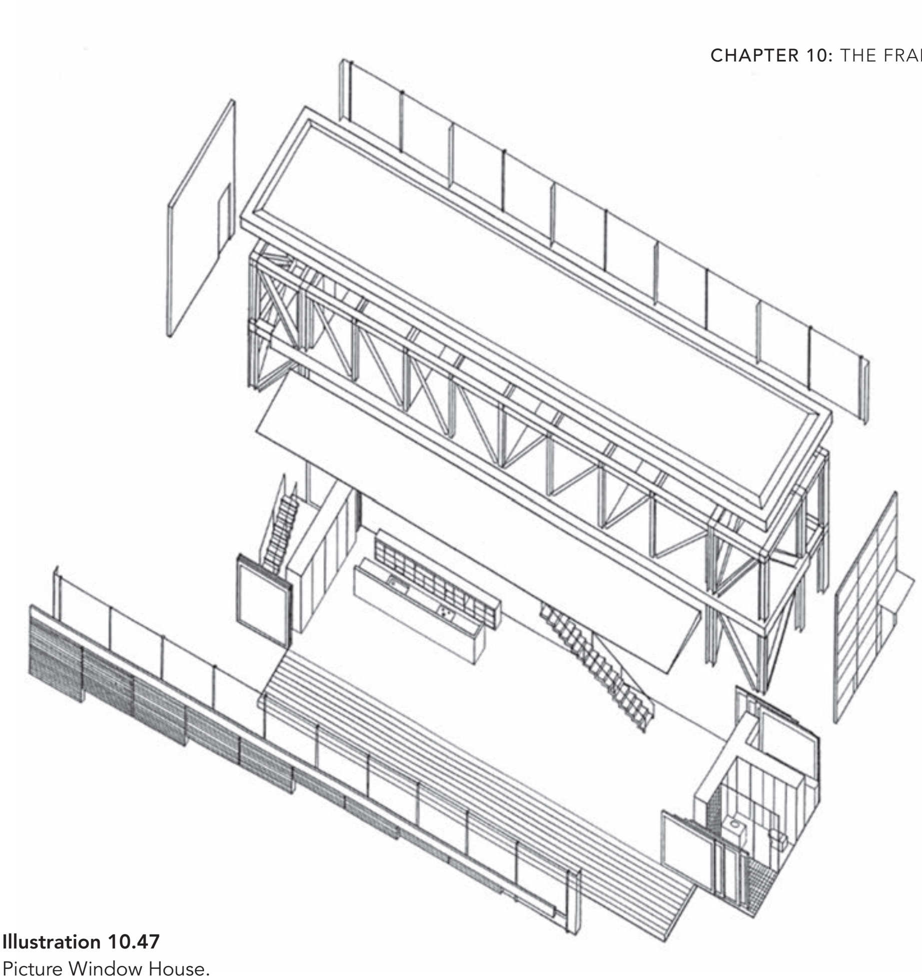


Illustration 10.47
Picture Window House.
Axon drawing highlights house's trussed rigid frame structural system; diagonal bracing changes orientation at mid-span in order to be in tension for predominant gravity load condition.

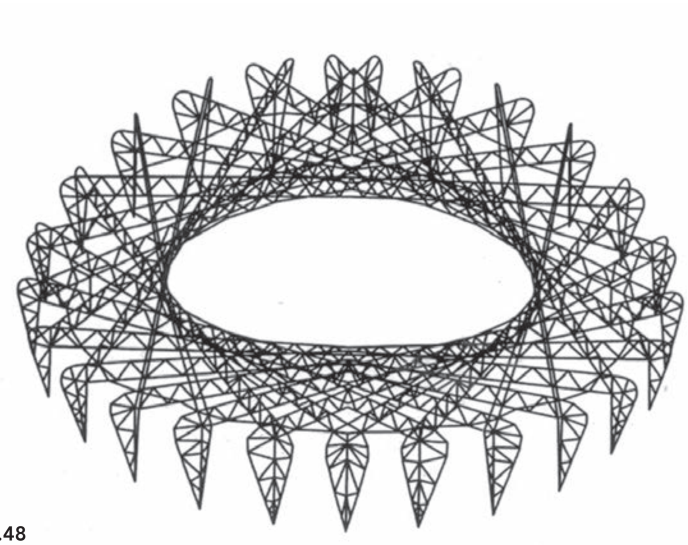


Illustration 10.48
Beijing National Stadium (The Bird's Nest), Beijing, China (2008).
Intersecting ribs of trussed steel frames regularly structure a three-dimensional ring around the perimeter of this stadium. Note the variation of the in-plane depth of each individual trussed frame, reflecting changing bending moment demands; e.g., maximum at the top corner and tapering to a minimum at the base. Alignments of secondary supporting elements between these ribs is much more haphazard, giving the whole its " bird's nest" character; not shown here, but obvious in the finished stadium.
Architect: Herzog & de Meuron. Structural engineer: Arup.

Illustration 10.49
Maryhill Overlook, Goldendale, WA, USA (1998).
Post-tensioned concrete slab is repeatedly folded into concrete walls, thereby creating multiple rigid frames and unexpectedly unifying its "column" and "beam" component elements.
Architect: Allied Works Architecture. Structural engineer: Architectural Concrete Associates.

Illustration 10.50
Maryhill Overlook.
Secondary alignments of cuts and folds create viewing axes and benches in this structure/sculpture located on a high bluff overlooking the Columbia River; its overall alignment points to a flat-topped stone promontory across the gorge.

Slab Frame Variations

One can also go to the other end of the spectrum of such visual expressiveness and structural efficiency, however. Indeed, it may seem completely unexpected that rigid frames are possible – moreover, quite common – in reinforced concrete buildings in which no beams are even seen to be present; i.e., where only slabs span the space between columns in what is called the *flat plate* condition that we discussed previously in Section 7.12. But as we saw there, significant widths (or "strips") of these slabs – much wider than the columns – can effectively act as "beam" elements by having relatively heavy sets of reinforcing bars distributed across them in order to deal with the bending moments and so compensate for the lack of depth of this critical element in the frame. These sets of reinforcing bars are strategically placed toward the top and bottom of the slab thickness according to the anticipated flexural deformations of a rigid frame that are produced in response to gravity and lateral loading. What is more, this is typically done in both orthogonal directions within a slab depth in order to have not only gravity but also lateral-load-resistance capability in any direction. Much strategic forethought is thus hidden away within the depth of a seemingly "banal" reinforced concrete flat slab. (e.g., Ill. 10.49, 10.50.)

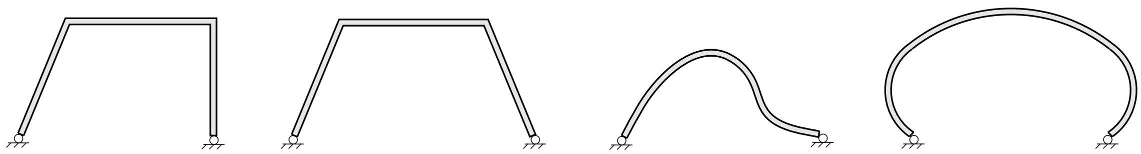


Figure 10.22
Variants of rigid frame forms, which are not necessarily rectangular and symmetrical.

Trapezoidal Frame Variations

Aside from changing the configuration of the beam and column components that make up a rigid frame – whether, as we have seen, according to bending moment diagram variations or deciding whether to use beams or trusses or slabs – there are yet further possibilities for changing the "look" of a rigid frame. For example, we may be used to thinking about rigid frames as orthogonal arrangements of straight, vertical columns and flat, horizontal beams,

Illustration 10.51
FLAMME-Iga Complex, Iga, Japan (2006).
Rigid frames may also be given a trapezoidal form. One consequence of designing tilted columns to achieve a trapezoidally shaped frame is that its lateral stiffness increases. Compared to a similar frame with vertical columns, the trapezoidal frame deflects less laterally.
Architect: Ryuichi Sasaki + Junpei Kiz + Tetsuo Kobori/ Phiframe. Structural engineer: Rhythm Design.

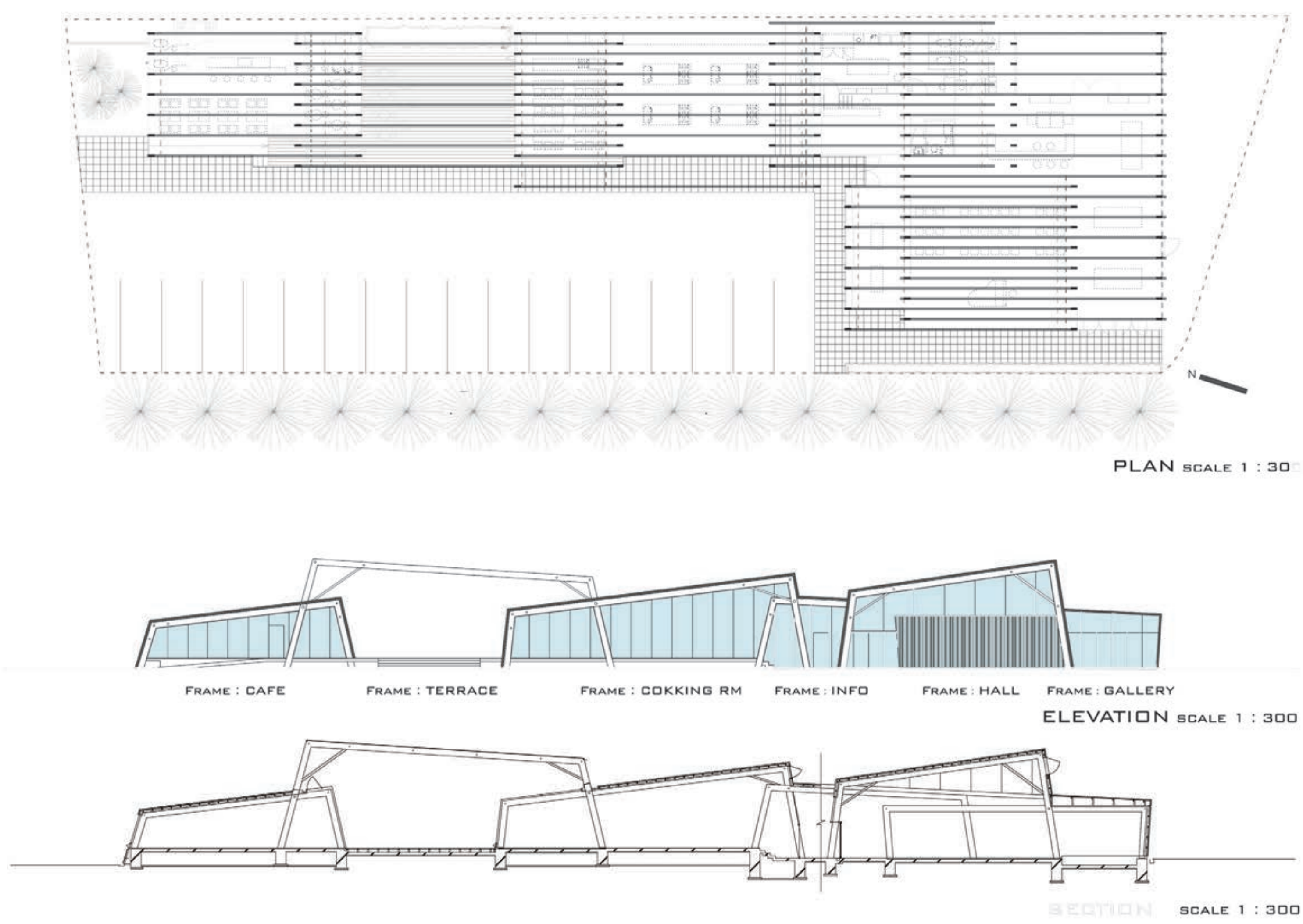


Illustration 10.52
Equestrian Center, Valle de Bravo, Mexico (2012).
Two-hinged, trussed timber rigid frame with pitched roof.
Architect: CC Arquitectos.

but this need not be the case. Beam and column elements may indeed meet at various angles, but as long as the resulting trapezoidal configuration makes up a rigid entity as we have discussed it so far, we can consider such a structure to be a rigid frame and treat it as such. (Fig. 10.22, e.g., Ill. 10.51 and Ill. 10.52.)

Curved Frame Variations

Indeed, the geometry of a frame may even follow a curved line without the typical kinks that we commonly associate with the connections between columns and beams and this can still be described as behaving overall in a rigid frame fashion. (e.g., Ill. 10.53, 10.54.) At some point, it may seem that the threshold between a curved structural element such as an arch (Chapter 12) and a curved frame becomes ambiguous, but in the end the distinction is a question of how the structure supports the loads. For example,

Illustration 10.53
"AURA-S" (2008) in Villa Foscari ("La Malcontenta"), near Venice, Italy.
This installation takes the "harmonic proportions" developed by Palladio – and employed in the design of Villa Foscari – and manifests them as wave forms representing musical intervals.
Sculptural installation by Zaha Hadid and Patrik Schumacher.

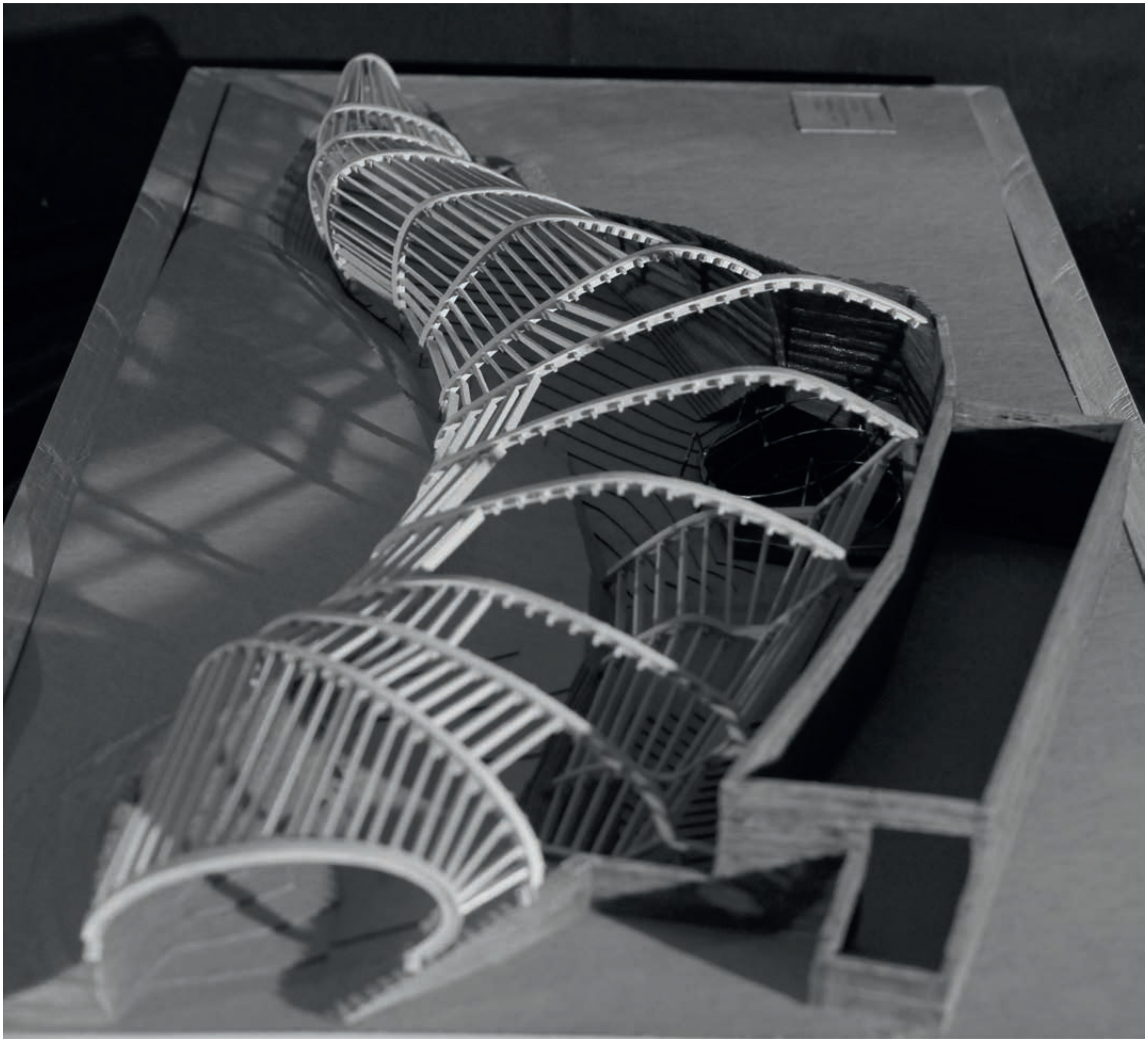


Illustration 10.54
Water Pavilion, Neeltje Jans Island, Netherlands (1997).
Set of curved steel moment-resistant frames define constantly changing external form and interior space of this pavilion, which puts on display various water technologies.
Architect: NOX/Lars Spuybruk. Structural engineer: Ingenieursbureau Zonneveld. Model: Nathan C. Friedman.

contrary to what we will find with the arch, structural frames carry symmetrical gravity loads *primarily* by bending and shear action of their members, whereas this is accomplished largely by means of axial compression in arches, at least for those that can comfortably answer to that name. Yet, asymmetrical gravity loads and lateral loads will still produce significant bending moments and shear forces in both frames and arches, so that this load condition will not necessarily help us make the distinction between these structural types. Nonetheless, a distinction remains; the question of naming things correctly here is not so much about geometric form as it is about primary structural behavior.

Illustration 10.55
Willis Tower (formerly Sears Tower), Chicago, IL, USA (1973). Multi-bay, multistory steel rigid frame configuration is evident in photo taken during construction.

Architect: Bruce Graham of Skidmore, Owings and Merrill (SOM). Structural engineer: Fazlur Khan of SOM.

Multi-bay, Multistory, Mega-scale Frame Variations

Finally, although we have mostly focused here on single-bay, single-story frames, it should be borne in mind that the rigid frame can in fact be and often is multiplied and configured to work in many different ways – including as multistory, multi-bay three-dimensional systems. (e.g., Ill. 10.55, 10.56.) In all such cases, each rigid frame sub-unit fundamentally deforms and behaves according to the same basic principles that we have just established, and the corresponding element-shaping and rigid-connection-detailing lessons can be applied.

Moreover, the same structural behavior principles and possible form variations that we have seen so far for a single rigid frame unit of typical story-height and column-spacing dimensions can be extended into a corresponding mega-scale subsystem unit, i.e., that overall is many stories tall and that spans an exceptionally long distance. (e.g., Ill. 10.57; also, see Ill. 1.6.[6])

Illustration 10.57
Hotel Arts, Barcelona, Catalonia, Spain (1994).
Trussed mega-scale rigid frame configuration is clear.
Architect: Bruce Graham of Skidmore, Owings and Merrill (SOM). Structural engineer: SOM.

FACING PAGE
Illustration 10.56
Jian Wai SOHO, Beijing, China (2004).
Consistent expression of rigid frames visually unifies this complex of 20 housing towers with lower levels of commercial facilities, public spaces, pedestrian bridges, etc. Overall design strategy is one of a unifying 3-D rigid frame matrix grid that is both strategically extruded in places and carved out in others.
Architect: Riken Yamamoto & Field Shop. Structural engineer: Plus One Structural Des. & Eng.

Illustration 10.58
Trestle framed building.
Model by architecture students at AHO.

10.8 Nordic Moments, Nordic Spaces

Historical Trestle Frames

In rural western Norway, houses have long been constructed using a technique called trestle framing. (Ill. 10.58.) Traditional and well known in other countries as well, a trestle frame consists of two posts connected by a transverse beam with short diagonal struts in the corners. This rigid frame configuration has been seen earlier in this chapter and the way it works was discussed. (See Fig. 10.10.)

Such a structural system is spatially flexible in the sense that a building can be extended in a modular fashion by adding trestles in the longitudinal direction or by adding aisles on either side of the original frames. Another advantage is that the system can easily and clearly fit onto sloping and uneven terrain, since the stability of the structure for lateral loading is largely being addressed at the constant level of the beam (vs. a braced frame in the same condition, where each diagonal will have its own angle of inclination). The posts can simply vary in length to the level of the constant beam datum, while their bases rest on solid flat stones sticking up from the ground in order to prevent moisture damage.

The trestle frame structure was frequently used in Norway for simple and airy buildings having no need for insulation, such as storehouses, barns, or boat shelters. Since the walls in trestle buildings have no load-bearing function, their sole mission is to provide some protection from the elements; indeed, in some cases it can even be an advantage to have the wind blowing freely through the building in order to dry corn or hay, all the while keeping the rain out. Materials for this type of structure are all found in the local surroundings, representing a historical lesson in the use of natural resources. Pinewood is used for the structure and cladding and sometimes naturally bent timber is used for diagonal struts; either slate or peat is selected for the roof, depending on the local climate and traditions. The anonymous character of the trestle frame building hides what is in reality a well-conceived and well-developed structure, both from the point of view of an overall building system as well as its careful and thoughtful detailing.

Illustration 10.59
Three variations of laminated leg for chairs designed in the 1930s.
Designer: Alvar Aalto.

Illustration 10.60
Otaniemi Technical University, Otaniemi, Finland (1966).
Series of concrete rigid frames of non-conventional form envelops the space of the auditorium.
Architect: Alvar Aalto. Structural engineer: Magnus Malmberg.

Aalto's Curving Frames

In the 1930s Alvar Aalto (1898–1976) experimented with the sculptural possibilities of wood. Wood is, of course, a material that is universally used and has a long history; it is also ideal for small structures such as furniture, in which joints were traditionally either glued or doweled. Aalto's experiments with the material led, over time, to novel practical solutions and applications, such as form-pressed veneers and curving laminated wood structures. These experiments laid the groundwork for Aalto's classic series of curving wood furniture.

As has been established, the rigid frame in its simplest form is a beam connected to two columns where the joint between the beam and the column forms a rigid connection. With this in mind, at a much smaller scale the base of a chair can also be considered for using the same structural configuration with the connection between seat and chair leg effectively forming the corner of a rigid frame. (Ill. 10.59.) If the connections between the seat and legs of the chair are not rigid enough, when someone sits down on the chair the legs will have a tendency to slide apart. The basis for most of Aalto's furniture is this stable frame form; in the design of the actual frame corners lies evidence of Aalto's innovative genius, from simple bent-wood chair legs to the fan-shaped versions where the leg is composed of glued wedge-formed laminates. In one clean blow, the traditional corner connection solutions were replaced with a new unifying concept.

Over time, Aalto's work with furniture began to influence his larger-scale building designs. Two projects, in particular, can be cited in this connection: the auditorium of the Technical University in Otaniemi outside Helsinki, completed in 1966 (Ill. 10.60), and the Riola Church in Italy in 1968. The curving concrete frames incorporated in both of these projects constitute a synthesis of form and structure, very much in keeping with his earlier furniture experiments in bent-wood frames.

Illustration 10.61
Stegastein Overlook, Aurland, Norway (2006).
Platform provides a viewpoint over the Aurland Fjord.
Architect: Todd Saunders and Tommie Wilhelmsen.
Structural engineer: Node Engineers.

Stegastein Overlook

In recent years, small but nonetheless significant architectural installations built along Norwegian tourist routes have gained wide attention. These installations all have similar functions as lookout points, rest areas, and sometimes simply benches. Consistently embodying a strong design concept and having a high quality of construction, they enhance the experience of the place.

"Nature first and architecture second" was the guiding principle for architects Todd Saunders and Tommie Wilhelmsen when they began the design for a lookout point at Aurland, which is a place having a magnificent view over the fjord landscape on the west coast of Norway. In order not to interfere with the overwhelming landscape by inserting too many elements, a simple yet strong form in the shape of a tilted wooden V has been chosen that conserves and complements the natural context. (Ill. 10.61, 10.62.) Shooting out from the shoulder of the road, the structure is an elevated platform that emphasizes a strong horizontal datum, bringing people out into the open air of the vast space above the fjord. At the end of the platform, the dizzying feeling of being in the middle of open space is especially strongly felt; this feeling is heightened by the clear glass railing at the end of the walkway.

The distinctive frame form of the lookout, however, with its deep laminated wood beams that turn a sharp corner at the end,

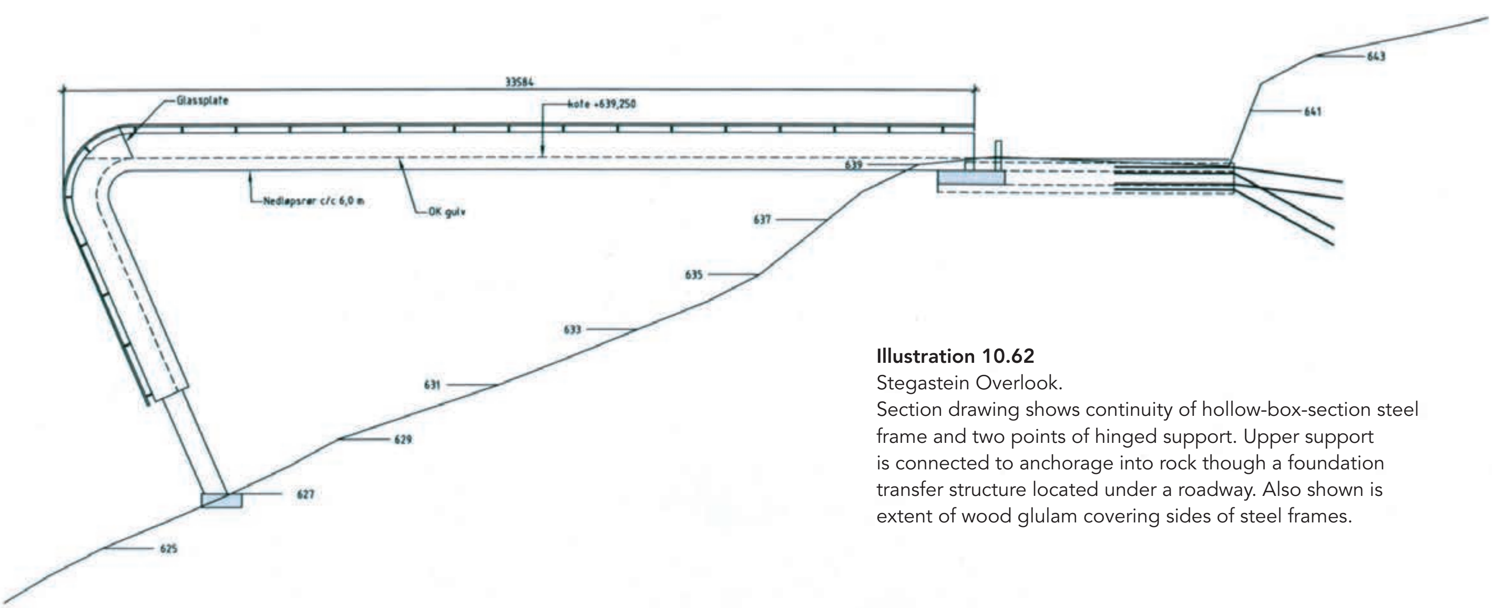


Illustration 10.62
Stegastein Overlook.
Section drawing shows continuity of hollow-box-section steel frame and two points of hinged support. Upper support is connected to anchorage into rock though a foundation transfer structure located under a roadway. Also shown is extent of wood glulam covering sides of steel frames.

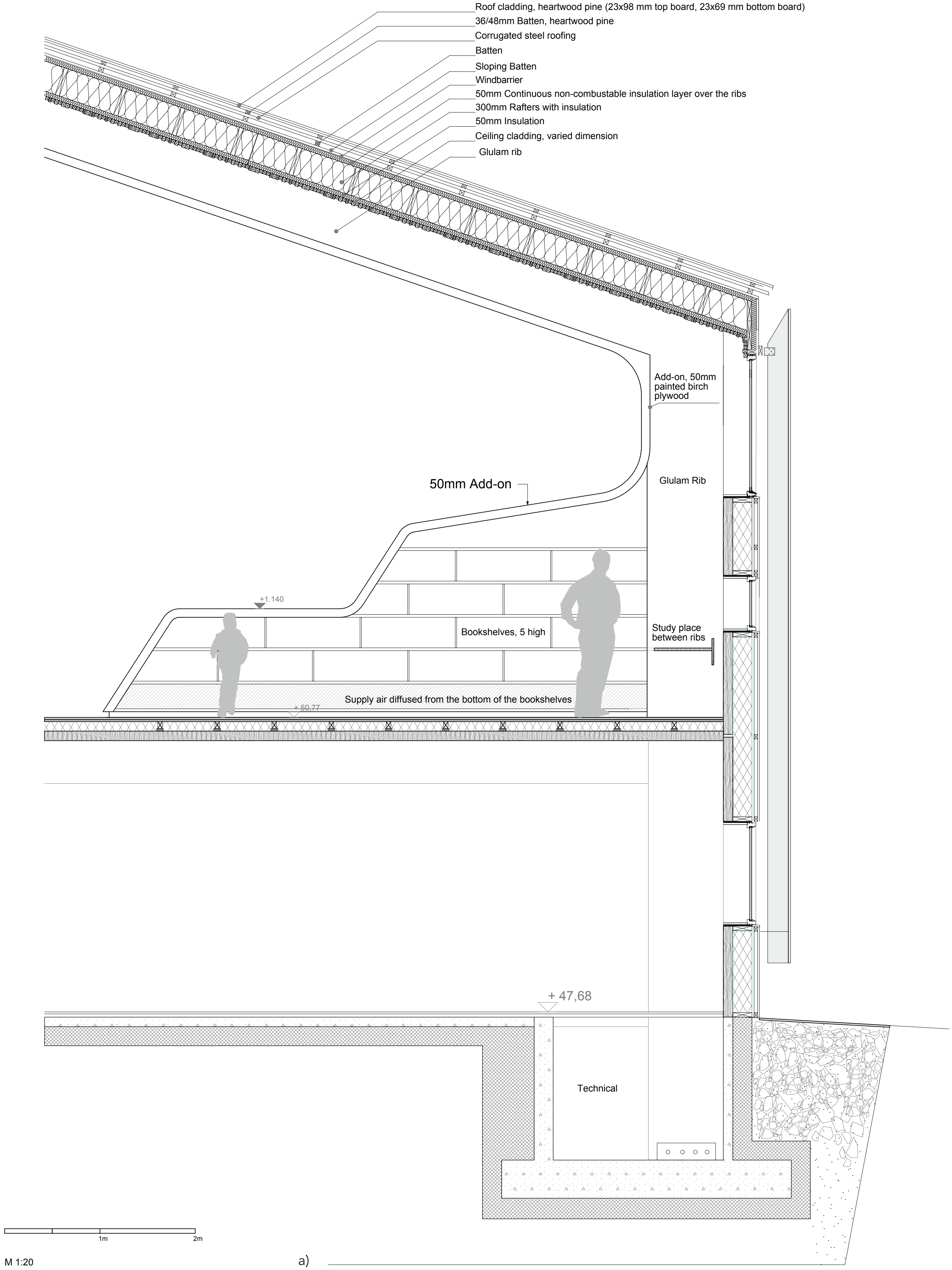
Illustration 10.63
Vennesla Library and Cultural Center, Vennesla, Norway (2011).
Multiple glue-laminated wooden rib frames, each of slightly different profile, give the interior of this building its distinctive visual identity. CNC-milled plywood board coverings give the ribs a unified visual appearance.
Architect: Helen & Hard Architects. Structural engineer: Rambøll.

is not quite what it appears to be. For this project, Node Engineers developed a structure based on two parallel two-hinged steel frames that are manufactured by welding together segments having hollow-box cross-sections. The bent corner is indeed rigid, as its form suggests, while the two ends of each frame where it meets the ground are hinged supports that can rotate relative to the concrete foundations. Under the walkway, a transverse steel truss connects the two frames in order to assist them in resisting the lateral wind loads as a cantilevered braced frame structure. This steel truss is hidden by wooden crossbeams that support a deck made of solid wood; the two-hinged steel frames are also hidden from view, clad as they are by the deep laminated-wood side coverings. These wooden facing elements also become the parapets for the edges of the walkway, which continue together with top handrails right over the bend at the end, notionally leading one on down into the void.

At the Stegastein Overlook, the visually strong yet also open quality of a rigid frame is featured; this choice of structural system allowed its designers to create the desired interaction and relationship between structure and nature. At this stop along the road, one can walk out into the thin air among the treetops, experiencing nature and the space of the larger landscape.

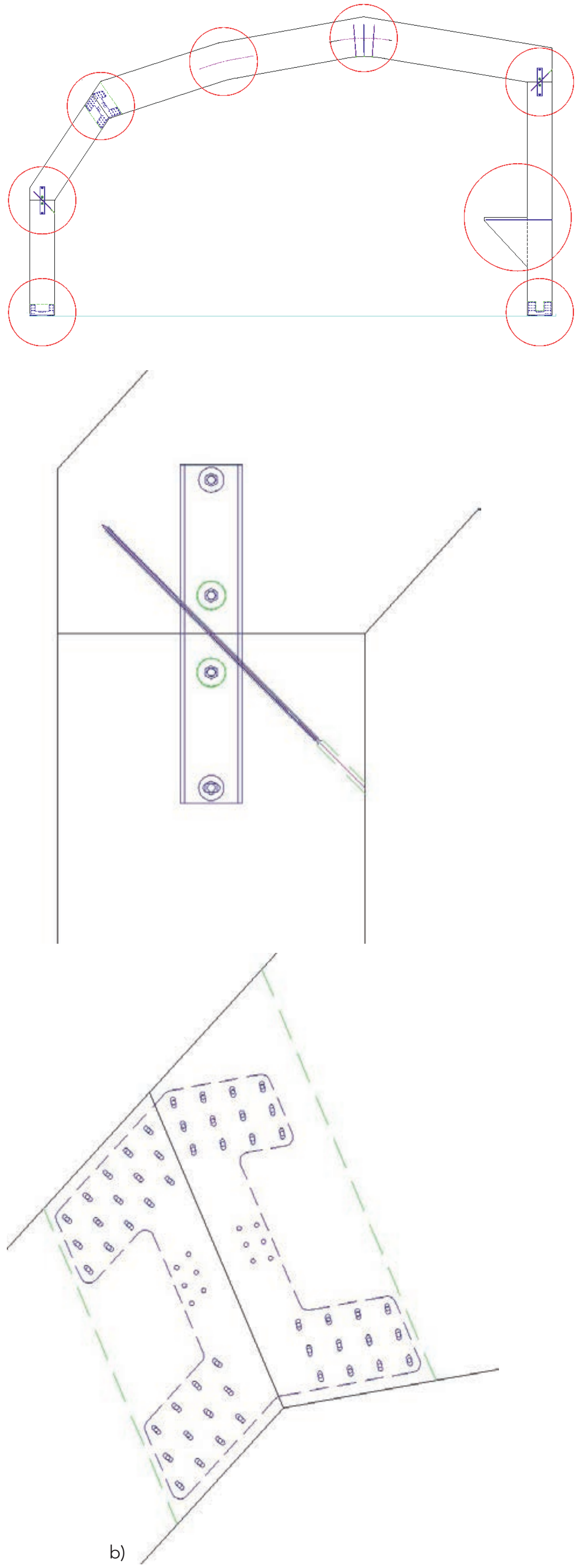
Vennesla Library and Cultural Centre

Finally, a striking multiple-frame-rib building can be found in the southern Norwegian community of Vennesla, where architects Helen & Hard designed a municipal library that also serves as an inviting public gathering space and as a circulation connector between the town's main square and a community learning center. The building bends in plan in response to being wedged between irregularly placed adjacent buildings, and its folded roof geometry takes cues from the surroundings as well. Any such sharp angles on the outside, however, are smoothed out in the interior by the curves of its 27 closely spaced wooden glulam ribs that give this building its distinctive identity. (Ill. 10.63). Each rib serves multiple functions, being at once the structure that carries gravity and lateral loads as well as defines the opening of the building's extruded space, but beyond that these are uniquely shaped to integrate bookcases and informal seating as well as wide bands of lighting and hidden air-handling ductwork. CNC-milled plywood boards give the ribs a unified visual appearance (although in fact for each apparent paired-set of ribs only one is load-carrying and structural) and the varying geometry of their profiles can easily be accommodated


Roof cladding, heartwood pine (23x98 mm top board, 23x69 mm bottom board)
36/48mm Batten, heartwood pine
Corrugated steel roofing
Batten
Sloping Batten
Windbarrier
50mm Continuous non-combustable insulation layer over the ribs
300mm Rafters with insulation
50mm Insulation
Ceiling cladding, varied dimension
Glulam rib
Add-on, 50mm painted birch plywood
50mm Add-on
Glulam Rib
+1.140
Bookshelves, 5 high
Study place between ribs
Supply air diffused from the bottom of the bookshelves
+ 50,77
+ 47,68
Technical
1m
2m
M 1:20

a)

Illustration 10.64
Vennesla Library and Cultural Center.
FACING PAGE – (a) Architect's section detail drawing, includes showing the extension of the rib to a lower level, where a fixed connection is shown at its base.
THIS PAGE – (b) Structural engineer's drawing showing details of rib construction, including connection details for its multiple parts.



b)

by using digitally controlled pre-fabrication technology, such that the interior effectively reads as one smoothly transitioning volume of space and enveloping form, morphing from a larger opening at the more public end of the building to a lower, narrower one at the other.

Looking a little more closely at the structural details of each of these ribs is instructive. Although the finishing is made to look as though these are made of one piece, the ribs are in fact composed of multiple glue-laminated segments of about 220mm x 1200mm (8.5in x 47in) cross-sectional dimensions (these dimensions vary slightly) that are connected together in various strategic ways as shown in Illustration 10.64a and b. The vertical column segments at the sides run down through the floor of the main space to the basement level and have a fixed connection at the foundation achieved by means of a bolted steel connector plate that runs across the width of the section. (We should note, in passing, that the orientation of this column segment is such that its moment of inertia is maximized in the plane of the frame/rib.) In contrast, the top of the column segment is effectively pin-connected to the end of the beam portion of the rib by means of a narrow, vertical, channel-shaped steel connector that is located at the neutral axis of the column (this is hidden between the structural rib and its non-loadbearing "twin" profile), thus ensuring the transmission of shear and axial forces between the two structural members but not of bending moment. Elsewhere across the span, the glulam beam segments are moment-connected to one another, with full-depth steel connector plates ensuring the flexural continuity of the member. So, effectively, what we have in each of these seemingly continuous wooden ribs is a hidden post-and-beam frame that has been detailed to mask any visual disruptions to the desired smoothness of its final aesthetic. On the one hand, the frame is everything in this building, creating its extruded space and being highlighted in its repeated form, yet on the other the idiosyncratic construction details of the rigid frames that make this possible are hidden from view.

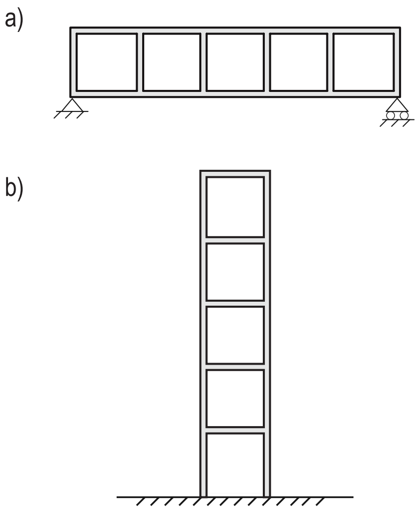


Figure 10.23
Drawing of basic configurations of Vierendeel structure: (a) spanning between supports and (b) cantilevering vertically from the ground.

Illustration 10.65
A "classic" rigid joint connection between steel beam and column elements used for the Schooten Bridge, proudly displayed here by Belgian Professor Henri Dustin (1882–1935), Director of the Laboratory for Testing of Materials at the Université Libre de Bruxelles. (Photo taken in 1934.)

10.9 The Vierendeel – Adapting the Rigid Frame

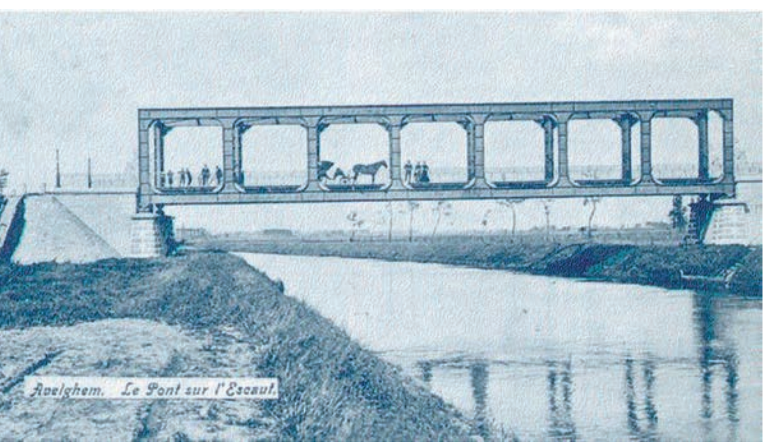


Illustration 10.66
Waterhoek Bridge, Avelgem, Belgium (1902). The first Vierendeel-type bridge built from steel.

To conclude this chapter, we will briefly look at a very particular arrangement of multiple rigid frame panels that are connected together side by side into an overall structural form that is sometimes used to carry gravity loads and span over an expanse of space (or to cantilever from the ground and resist lateral forces). (Fig. 10.23.) This combination of structural elements should remind us of the open, composite-member structures that we discussed in the previous chapter, i.e., trusses, albeit with a key difference: in this instance, there are no diagonal members across the individual panels of the system, which should suggest that rigid frame behavior must somehow be coming into play in order to stabilize this structural configuration.

Planar structures constructed from a number of (usually, but not necessarily) orthogonally connected members that interact with each other according to rigid frame principles are called *Vierendeel frames/beams* or *Vierendeel trusses* (although misleadingly so with regard to the latter in terms of their primary structural behavior and detailing, as we shall see shortly) or else they can simply be called *Vierendeels*, thus avoiding all the naming confusion. They are given this nomenclature because of the Belgian engineer and professor Arthur Vierendeel (1852–1940), who invented the visually distinctive system and used it in the design of a series of bridges. (e.g., Ill. 10.65, 10.66.)

Illustration 10.67
Bridge Pavilion, Valle de Calamuchita, Cordoba, Argentina (2014).
Vierendeel structure means that there are no diagonals to obstruct the views from this small bridge/pavilion.
Architect: Alarciaferrer Arquitectos. Structural engineer: German Serboraria.

A Vierendeel structure depends in a fundamental manner on the bending and shear stiffnesses of its individual members, and on the rigid joints that connect them together; i.e., it is explicitly rigid frame behavior that is providing this system's stability. Contrary to conventional trusses that have diagonal members and pinned connections and which, therefore, carry loads by means of (only) axial member forces, structures that are connected in a Vierendeel configuration rely heavily on bending moments and shear forces in its members (as well as axial forces.) Applying what we know from previous chapters about relative load-carrying-mechanism efficiencies, a Vierendeel is thus much less efficient than a correspondingly sized conventional truss, but it can on the other hand have distinct advantages in certain situations where diagonal structural members would interfere with the desired circulation, through-views or, perhaps, with favored design aesthetics. (e.g., Ill. 10.67, 10.68.)

We will now take a closer look at how this system works. (Fig. 10.24). Just as in any beam-like structure spanning between supports, transverse loads acting on a Vierendeel produce overall shear forces and bending moments that need to be balanced at every cross-section. The overall bending moments in the structure are able to be resisted by force couples consisting of axial compression and tension forces in the top and bottom chords, just as in a typical truss. For resisting the overall shear force, however, things must clearly be different: whereas in a

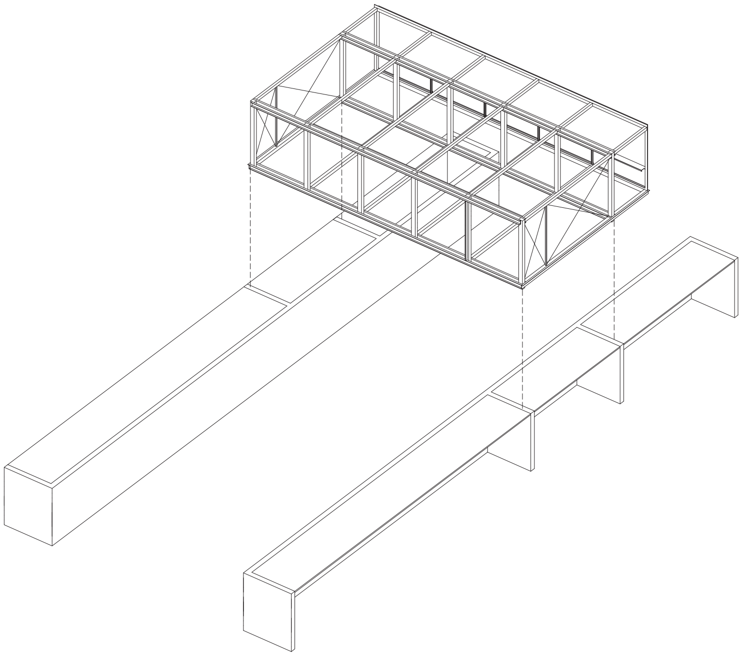


Illustration 10.68
Bridge Pavilion.
Drawing highlights two built-up steel Vierendeels used to support living space above water channel, spanning across from one concrete retaining wall to another. X-bracing at the two ends stabilizes frame structure in short direction.

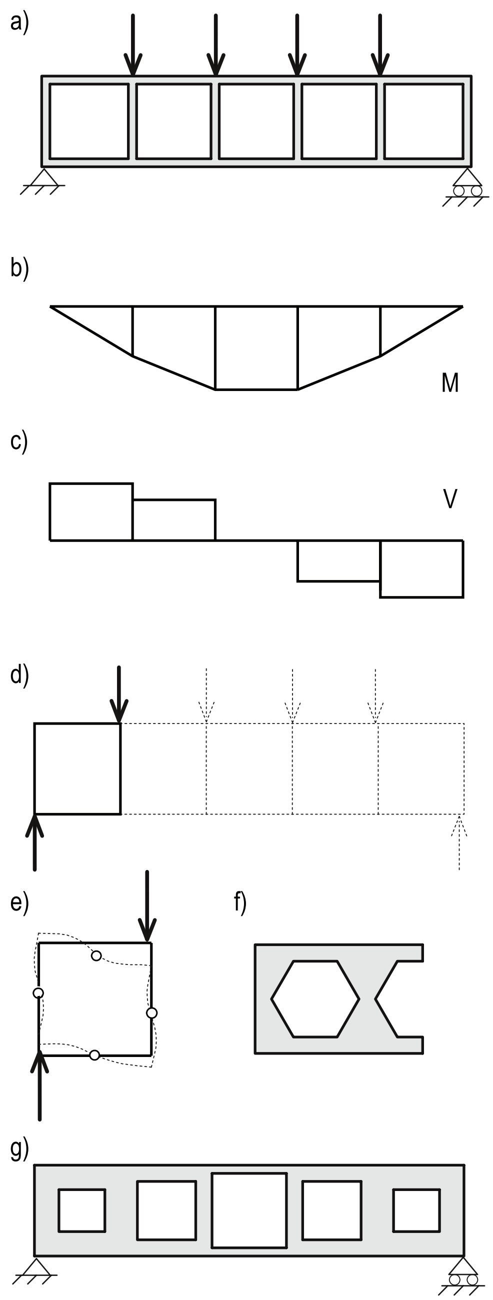


Figure 10.24
(a) A Vierendeel beam with rigid joints between horizontal and vertical structural elements. (b) Bending moment diagram for this load configuration. (c) Shear force diagram for this load configuration. (d) Identifying the rectangle experiencing the largest shear force, close to the support. (e) Deformation of the rectangle, with inflection points. (f) Panel shape that would correspond to "local" bending moments in each element caused by deformed shape. (g) Vierendeel configuration showing (in exaggerated fashion) relative structural element dimensions corresponding to maximum "local" bending moments and shear forces in each member.

conventional truss this was done by means of an axial force in the diagonal member, in the Vierendeel there is no such member present. Instead, the overall shear in the structure must be carried instead by "local" shear forces in the upper and lower chord members being summed up together.

Beyond this, we can learn more about the workings of the Vierendeel structure by examining its deflected shape under load. (Fig. 10.24e). It can be seen that because of the rigid connections each individual element of the structure has to deform into reverse curvature, creating inflection points halfway along them. Since such points effectively act as hinges, we can observe that shear forces and bending moments tend to be produced in each of the members of a Vierendeel in a similar fashion to those we described for a rigid frame with fixed supports: i.e., overall shear forces through the structure cause "local" shear forces and bending moments in all of the individual members of the structure as it deforms.

Such forces and moments in the members can be estimated from an analytical model in which two three-hinged portal frames, mirrored toe-to-toe against each other, represent the structural system for each rectangle. (Again, Fig. 10.24e). The shape and magnitude of the bending moment and shear force diagrams for each structural unit of the Vierendeel can thus be found using the logic that we have studied earlier. For a simply supported Vierendeel, the tendency is that *local* bending moments and shear forces in the individual members are highest toward the overall structure's ends (since overall shear forces are largest there), and axial forces in the upper and lower chords are highest toward its middle (since overall bending moments have maximum magnitude there).

Logical and efficient shapes for Vierendeel structures can be derived from the observation of force and moment distributions, as are suggested in Figures 10.24f and 10.24g and as can be seen in the Vierendeel structures spanning over the machine showroom of the Trumpf Smart Factory Chicago (Ill. 10.69, 10.70). But it is also not at all unusual that for visual consistency's sake the maximum member dimensions that are required anywhere in such a structure are used throughout, even though such an approach can obviously be somewhat more wasteful of material for large loads and spans. One way of maintaining visual consistency while also still having a relatively efficient overall structure, however, is to keep the more visually obvious outer member dimensions constant while

Illustration 10.69
Trumpf Smart Factory Chicago, Chicago, IL, USA (2017).
Shaping of component steel plates that are used to build up the Vierendeels reflects the vertical members' bending moment diagram; i.e., large bending moments at the top and bottom connections, with minimal bending moment at mid-height.
Architect: Barkow Leibinger. Structural engineer: Knippers Helbig Advanced Engineering.

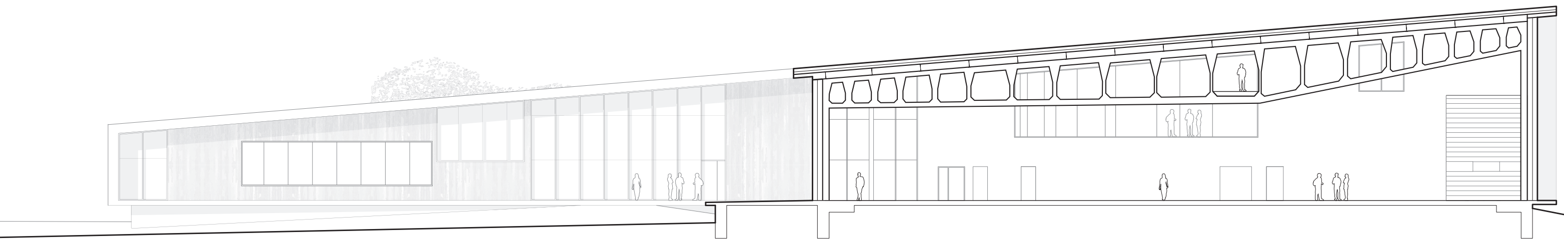


Illustration 10.70
Trumpf Smart Factory Chicago.
Vierendeel structure spans over factory machine/display room, leaving ground floor clear of any obstructions.

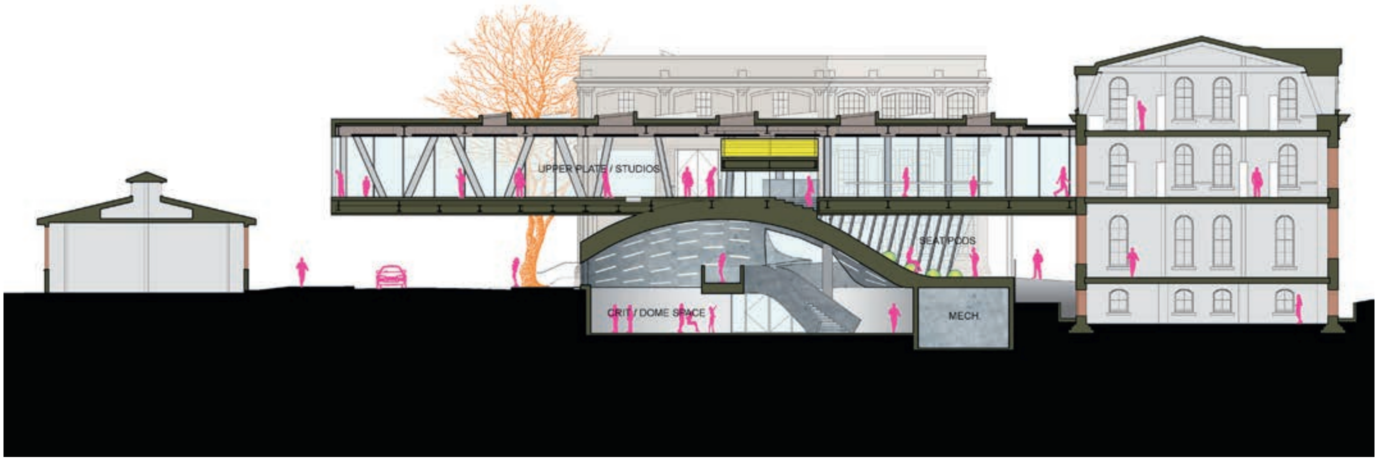


Illustration 10.70
Milstein Hall, College of Architecture, Art, and Planning, Cornell University, Ithaca, NY, USA (2011). Cantilevered studio space enabled by modified version of floor-to-ceiling Vierendeel system. To right of the dome, vertical column elements connect rigidly to beams at top and bottom of the open studio space, creating a "classic" Vierendeel configuration; to its left, verticals gradually become more and more inclined, approaching the configuration and stiffness (and obstruction-to-through-passage) of a conventional truss.
Architect: The Office for Metropolitan Architecture (OMA). Structural engineer: Robert Silman Associates.

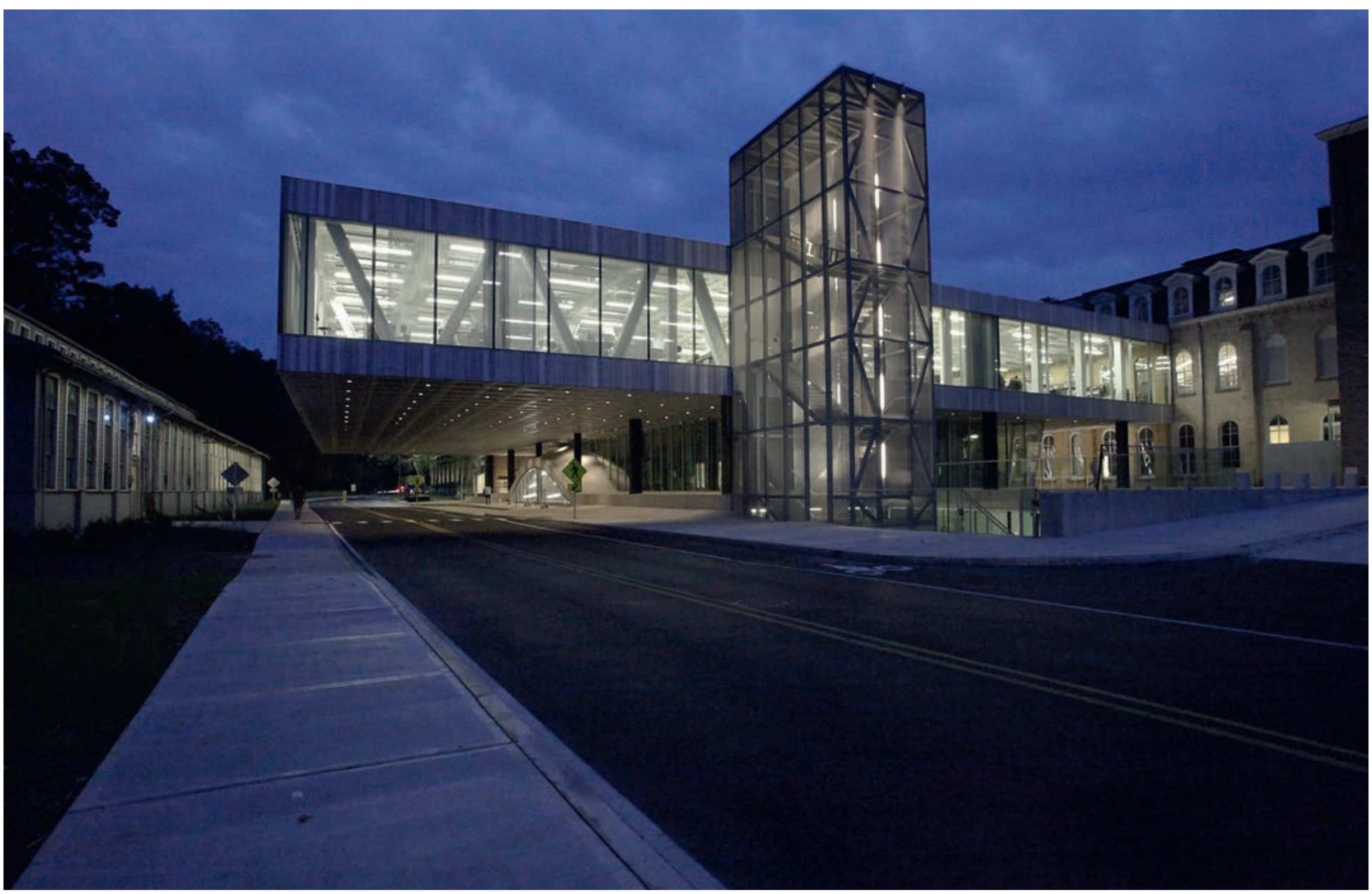


Illustration 10.71
Cornell University's Milstein Hall.

varying their sectional thicknesses; for example, a Vierendeel can be constructed of square hollow steel tubes of consistent outside dimensions but in which the tube wall thicknesses vary. Of course, such an approach can also be effectively hidden within a concrete Vierendeel by strategically varying the number and size of reinforcing bars according to the anticipated response.

To summarize this discussion, then: Vierendeel structures do indeed have some overall resemblance to conventional trusses in the sense that both can be considered to be large beams with holes cut out of them, in one case square or rectangular holes and in the other triangular ones. But that distinction actually makes all the difference in the way that they behave to carry load and thus are detailed and their members shaped. As a final observation, and as has been explored recently at the architecture school at Cornell University, structures need not necessarily be "purely" one thing or the other. Here, a full-story-height rectangular Vierendeel system that allows through circulation in a certain portion of the studio area "morphs" into something that is more of a conventional truss configuration with diagonal members toward the end of the cantilevered part, where additional stiffness is required against deflection. (Ill. 10.71, 10.72.)

The Cable and the Membrane

CHAPTER

11

Illustration 11.1
Lord's Cricket Ground, London, UK (1991). Fabric roof protecting the spectators' upper deck.

Architect: Hopkins Architects. Structural engineer: Arup.

Illustration 11.2
Portuguese National Pavilion, Expo '98, Lisbon, Portugal (1998).
Suspended roof structure is made of highly tensioned steel cables encased in a thin concrete surface. End walls provide vertical and horizontal support reactions.
Architect: Alvaro Siza Vieira. Structural engineer: Arup.

11.1 Portuguese Tension

The Expo in Lisbon in 1998 included a very significant work of architecture that relied in an essential fashion on the unique shape and load-carrying capabilities of the suspension cable: Alvaro Siza Vieira's Portuguese National Pavilion. (Ill. 11.2.) Largely defined by a hanging roof that is shallowly draped between two porticos, this covered space served as a public plaza for official Expo ceremonies and as a covered entrance to the interior of the pavilion proper. Seen from the vantage point of this plaza, with the hanging roof above and its curved shadow traced out on the ground, the framed view of the adjacent river estuary and its endless maritime activity is indeed an extraordinary sight to see. And by spanning 70m (230ft) with such minimal thickness, this suspended concrete structure is, of course, spectacular in its own right. With the concrete seemingly in tension, but in fact not – rather, it is the cables that pass through the concrete surface that are the tension elements here – this concrete "flying carpet" reveals new and unexpected ideas for how to resolve the design essentials of such a "simple" structural system.

Working in collaboration with the structural engineer Cecil Balmond of Arup, Siza ruled out a fabric membrane for the suspended roof at least partly because of such a system's need

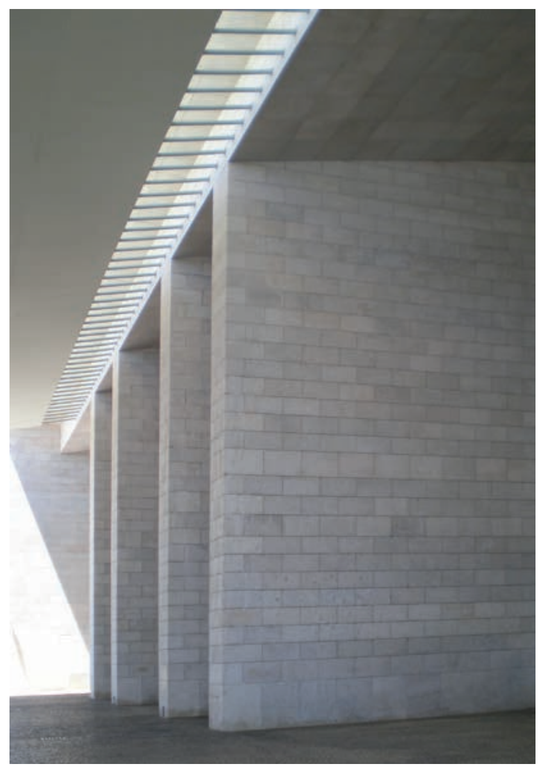


Illustration 11.3
Portuguese National Pavilion.
Detail showing exposed steel cables near support. Also, cables being anchored into horizontal beam running across tops of walls as well as alignment of series of shear walls that counter the tension forces in the cables.
Architect: Alvaro Siza Vieira. Structural engineer: Arup.

Illustration 11.4
Braga Stadium, Braga, Portugal (2004).
Suspension structure spanning between and covering the two grandstands.
(See also Ill. 11.12, 11.13.)
Architect: Eduardo Souto de Moura. Structural engineer: AFA Associados.

for a secondary stiffening structure (there will be more on this topic in Section 11.4) – a feature that would have added unwanted visual depth to the roof's thin profile. Instead, a curved concrete slab was selected due to its inherent self-weight, which makes it fully capable of stabilizing the roof against any uplift tendencies caused by the wind. The 200mm (8in) thick concrete roof surface is carried by a series of suspension cables that are tightly stretched above the plaza and anchored into two deep concrete-walled porticos that bracket the open space. In order to resist the considerable inward pull of the tension forces of the roof's cables, the support porticos incorporate a number of parallel shear walls that are aligned with the direction of the cables. It is to be noted that the roof's concrete surface stops short of the supporting structures at both ends, thereby creating gaps of blue sky right where one expects to find the roof connected to its supports. These gaps in the surface, however, do allow short lengths of the stainless steel-covered suspension cables to be exposed for all to see, thereby revealing and celebrating how the roof system is primarily working. (Ill. 11.3.) But this detail, together with that of hidden oiled sheathings within the concrete surface through which the tensioned cables pass, actually allows the concrete roof to move independently of its supports, something that is quite necessary because of the significant effects of temperature fluctuations and seismic activity in this region.

Moving next to the northern Portuguese city of Braga, we can find two famous outdoor venues that attract very different crowds of worshipers. One of these, situated on a green hillside and thus able to be seen from afar, is the famous Baroque eighteenth-century Escadaria de Bom Jesus; during religious festivities the faithful can be seen meandering up a monumental processional staircase to the pilgrimage church.

A little north of the city, in a former granite quarry, is a very different kind of attraction: Braga Stadium, a modern gathering venue for this era's equally fervent worshipers – football fans. (Ill. 11.4.) The 30 000 seat arena was designed by architect Eduardo Souto de Moura and structural engineers AFA Associados for the 2004 European soccer championship. Two grandstands rise steeply on the long sides of the playing field while the ends (typically curved and therefore often referred to as "curvas") remain open – one end facing a rocky hillside, the other offering a view of the distant landscape. Souto de Moura opposed the traditional arena arrangement with seating at the "curvas" as he considered that watching the game from behind the goals was a rather poor experience. By avoiding such construction, the architect also created an uncommonly open football arena, one in which the surrounding landscape is visible from every seat.

To protect the spectators from the sun and the rain, a unique two-part roof structure has been built that is very clearly supported by a series of suspension cables stretched between the tops of the two grandstands. As in Lisbon, concrete slabs guarantee that a certain weight will always be acting on the suspension roof structure, thus preventing it from fluttering with the wind (as well as partially shielding the spectators from the elements, of course). In this case, the slabs are cast on corrugated metal decking that sits on top of the suspension cables, but this is done only over the seating areas, thereby leaving the cables completely exposed to view over the playing field. Souto de Moura at first proposed that this roof would also be a continuous concrete slab like the one that Siza had designed in Lisbon, but after a visit to Peru to visit the Incas' rope bridges he opted instead for a structural form inspired by an associated cultural artifact with its own set of cables/fibers: a large loom with two unfinished pieces of cloth on each side.

Along the east side of the stadium a series of outwardly angled concrete walls rise up to a staggering height of 47m (154ft). (See section drawing through stadium, Ill. 11.12.) Of course, the inclination of these walls reacts strategically against the heavy inward pull of the roof's tension cables while also conveniently suiting the necessary slope of the tiered seating. Dramatically exposed between these vertical planar walls are cantilevered sets of stairs for the processions of spectators to climb up to their seats – echoing the sight of worshipers on the monumental staircase at the Escadaria de Bom Jesus. On the west side of Braga Stadium, a very different spatial condition exists: the grandstand structure is set against the excavated rock of the hillside, thus creating impressive internal spaces between the underside of the concrete seating structure and the dynamited granite rock face.

Finally, it is to be noted that the drainage of a suspended roof surface such as this one offers its own set of interesting design challenges and opportunities. Because of the two-part configuration of the roof at the Braga Stadium, Souto de Moura faced the prospect of rainwater draining straight onto the playing field, which would clearly have been unacceptable. Instead, gutters are located along the inside edges of the roof surfaces, which are slightly tilted toward one end of the stadium. The rainwater thus drains off the roof at two points into spectacularly projecting concrete troughs, from which it is then led down the hillside in a sinuous open canal.

11.2 Hanging by a Rope

Much can be learned from relaxing in a hammock (Ill. 11.5); in fact, perhaps Donald Duck's favorite resting spot is not all about having fun after all, at least for those of us who are interested in structures. First of all, we must acknowledge that in order to support a hammock we need to have two trees or similarly stiff vertical structures at its ends. The trees need to be fairly large so that our weight in the hammock does not make these supports bend inward too much, which could make our hanging bed sag excessively and possibly touch the ground. Second, we know from experience that a hammock is a suspension structure made of canvas or rope, and that its consequent lack of stiffness is a challenge when we try to climb into it: the hammock quite simply changes form when we try to wriggle or ease our way into our favorite relaxing position. When we initially sit in it, the hammock stretches out to form two fairly straight lines from our body up toward the supports. Having

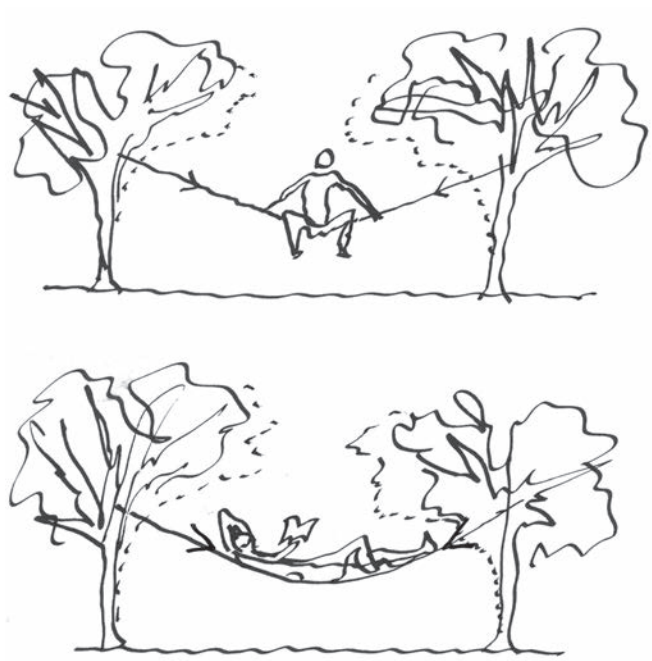


Illustration 11.5
The hammock; a simple suspension structure. Different load configurations influence the shape of the suspension structure.

Illustration 11.6
Traditional nomadic tent of the Berbers, Morocco.

Illustration 11.7
Incan rope bridge, Andes Mountains (nineteenth century).
Suspension cable profile is evident, and its "lively" nature can be sensed. Dead weight anchorage seen at left.
Engraving by Rodolfo Cronau.

mastered the climbing-in operation, however, we notice that the hammock gradually changes form from that of a V to a shallow U when we lie down. Indeed, when our body mass is fairly evenly distributed along the length of the hammock, it changes into a softly curved upwardly concave shape that is fairly comfortable for resting and even for sleeping in. But alas, if we roll over in our sleep and pull our knees up (assuming that we do not fall out), we will once again change the hammock's shape and have to accommodate to a different resting position. Also, if someone comes along and gives us a push, the hammock will swing freely. This endless adaptability of the hammock might be seen as an advantage for a temporary sleeping structure (ask any mariner!), but the lack of stability needs to be taken seriously if the same structural principle is to be employed in the context of buildings, which we are primarily concerned with here. We will come back to this issue of the stability of hanging structures in Section 11.4. But before completely leaving this descriptive study of a hammock's characteristics and behavior, we should duly note and emphasize one of its undisputed advantages: its weight or, rather, its lack thereof. A hammock is able to support persons of all shapes and body weights, and it can do so while being extremely light itself. The fundamental reason for this is that a hammock works *all* by means of tension forces and, as we have already seen in Chapter 6, tension is a highly efficient way of carrying load. Note that the basic difference between the tension structures being considered here and those of Chapter 6 is that in the latter case loads were being applied *axially*, i.e., along the axis of the tension element, whereas this chapter is all about cables and other hanging structural elements that work in tension when loads are applied *transversely* to their longitudinal axis.

Applying the observations of this informal discussion about our experiences in a hammock to an analogous suspension cable system, we can make the following general statements that will apply to such structures:

- A hanging rope, cable, or chain needs supports at its ends that are fairly stiff; i.e., that do not displace too much into the span.
- A cable structure on its own has no bending stiffness in and of itself and it will consequently be shaped in accordance with the location and magnitude of the loads that it supports.
- A suspension cable will change its profile if the loads that are applied to it change location.
- The cable's lack of rigidity will in most cases have to be dealt with so as to ensure that the structure does not move excessively.
- A hanging cable structure is also fundamentally unstable in the sideways direction.
- Hanging cable structures work primarily in tension, and so they are quite light.

Historically, these basic characteristics of suspension cable structures can be found to have been recognized and applied in such (nearly) primordial structures as tents and vernacular rope bridges. (e.g., Ill. 11.6, 11.7.) For example, the Coliseum in Rome (finished ad 80) is thought to have offered spectators shading from the intense sun by means of a retractable rope and fabric roof. And in sailing ships a highly efficient array of ropes and fabrics is formed with a single purpose in mind: to efficiently catch the wind and create forward movement. As efficient as such a system is for harnessing energy, however, it must be recognized that its geometry is completely unstable; i.e., when wind forces blow against the

Illustration 11.8
Fall Creek Suspension Bridge, Cornell University campus, Ithaca, NY, USA (1961).
Suspension cables drape across a steep-sided gorge, with intermittent vertical hangers supporting a spectacular pedestrian walkway. Stability of the cable profile is provided by deck-level stiffening trusses forming the railings; the structure is nonetheless still quite "active" and footfall vibrations are distinctly felt.
Structural engineers: Professors S.C. Hollister and William McGuire.

sails they bulge out into taut, curved shapes but when the wind stops the sails go slack. This is rather obviously not a situation that we want to duplicate in building structures, but we shall see eventually that by pre-tensioning such sail-like fabric surfaces into very particular curved geometries we can make such spectacular forms stiff and stable enough that they can, in fact, be incorporated into the realm of architecture (Sections 11.6 and 11.8). Trusses and other strategies have also been used to stabilize suspension cable structures for bridges and buildings alike. (Ill. 11.8.) But now we are getting far ahead of ourselves; we first need to go back and look in detail at how suspension cables work, and what it takes to make them applicable in the context of architecture.

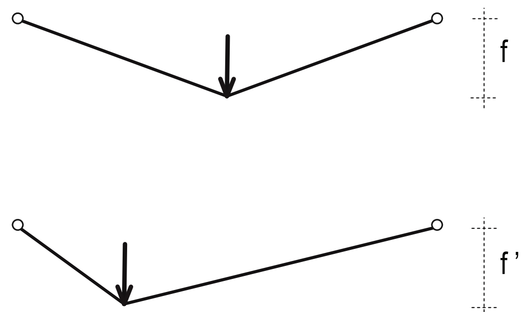


Figure 11.1
Cable with point load. Straight cable lines are formed.

11.3 Cable Shapes and Cable Forces

As was discussed in relation to the familiar example of a hammock in the preceding section, a hanging suspension cable is by nature flexible and it will adjust its shape when the distribution of loads applied to it changes. When supporting a single point load, for example, a cable takes on a V-shape where the bottom of the "V" is the point where the load is located and the "V" is obviously symmetrical if the load acts at mid-span. (Fig. 11.1.) The vertical distance between the lowest point of the cable and the level of the supports is called the sag, f. If this point load is moved toward one of the supports, the cable will adjust its profile by forming a skewed V, and the sag will be reduced.[1] We will later see that this change in profile also entails increased cable forces. If we put a few more point loads on the cable, its shape becomes polygonal; i.e., the cable forms straight lines between the loads, and there is a change of slope wherever a point load acts. (Fig. 11.2a.) In all of this, we assume that the cable weight is quite small compared to the magnitude of the loads; if it is not, the cable will also tend to curve slightly between the locations of the point loads.

Imagine that we now increase the number of point loads even further. The number of direction changes along the cable will keep increasing and the straight lines between the point loads will become shorter and shorter. (e.g., Ill. 11.9.) As the load condition approaches what we have previously defined as a distributed load the cable tends to become continuously curved rather than polygonal. For

Illustration 11.9
Buckingham Palace Ticket Office, London, UK (1995).
Vertical hangers supporting a fabric membrane roof canopy apply four point loads to the primary suspension cable between its mast supports, causing it to adopt a polygonal profile. The membrane structure that is stretched between the hangers and a system of struts and ties protects the wood-clad ticket booth.
Architect: Hopkins Architects. Structural engineer: Architen Landrell. Cornell model by Tiffany Lin.

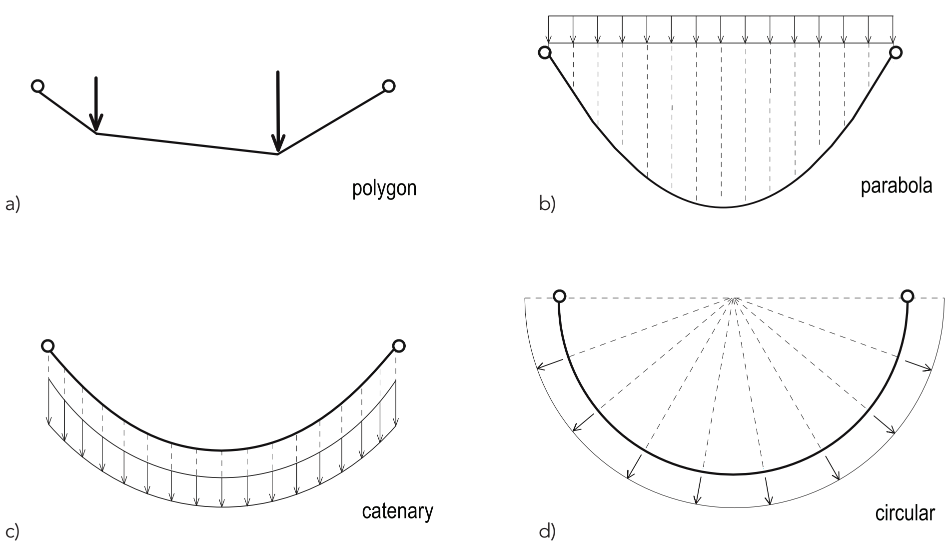


Figure 11.2
Funicular lines for a cable subjected to (a) two point loads, (b) load uniformly distributed along the span, shaping the cable in the form of a parabola, (c) load uniformly distributed along the curve, shaping the cable in the form of a catenary, and (d) radial load, shaping the cable in the form of a circle. Funicular lines suggest a structural geometry by which a particular loading distribution will cause pure axial forces in the structure.

Illustration 11.10
Clifton Suspension Bridge, Bristol, UK (1864).
Within the main span, the "cables" made of chain-like segments connected together have a mostly parabolic profile due to the uniformly distributed load imposed by many closely spaced vertical hangers supporting the deck, although the substantial self-weight of the chains can also cause a certain tendency toward a catenary profile as well.
Structural engineer: William Henry Barlow and John Hawkshaw, based on an earlier design by Isambard Kingdom Brunel.

a *uniformly distributed load* (UDL) the shape of the cable in fact becomes that of a parabola. (Fig 11.2b., e.g., Ill. 11.10.) If the load is distributed in another fashion, i.e., not evenly distributed across the span but, instead, evenly *along the length* of the cable, then the shape turns into what we call a *catenary*.[2] (Fig. 11.2c.) This particular load configuration is typically associated with the cable's self-weight, and a catenary profile is formed when a cable or a chain hangs freely. A radially applied set of loads will cause the cable to assume a semicircular shape. (Fig. 11.2d.)

What is common in each of these cases is that despite the different shapes the cables carry the loads by *tension forces alone*, and that they do so out of necessity. Cables, ropes, and chains have no other way of behaving structurally because they all lack the geometric properties that would enable them to act as either bending or compression elements. Structures such as these that respond to a particular load situation by setting up pure axial forces within its elements (whether in tension or compression) are referred to as *funicular structures*.[3,4]

Since the internal forces in a cable structure are always purely tensile, and these forces are necessarily always directed along the line of the cable, we can as a result relatively simply observe the obligatory relationship between a cable's sag and the magnitude of its tension force. Consider, for example, the force polygon of the tension force at the very end of a cable next to its support. (Fig. 11.3.) Since vertical equilibrium of the overall system demands that the vertical component of the support reaction is a constant (no matter what the sag of the cable is), the total tension force in the cable must vary according to the angle at which the cable meets the horizontal at the support. The larger the sag, therefore, the smaller is the horizontal force component necessary to close the

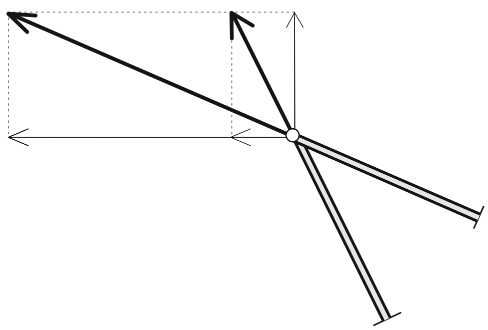


Figure 11.3
The influence of sag/inclination of cable on support reactions: when cables carry vertical loads, a decreased sag means larger horizontal force reaction, and hence the total tension force increases. The vertical force reaction stays the same.

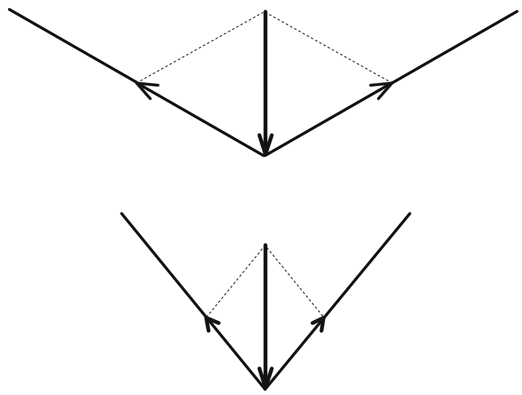


Figure 11.4
Diagram of force polygons depicting the variation of the magnitude of the tension force with respect to the variation of cable sag/cable inclination: i.e., the less the sag, the larger the force.

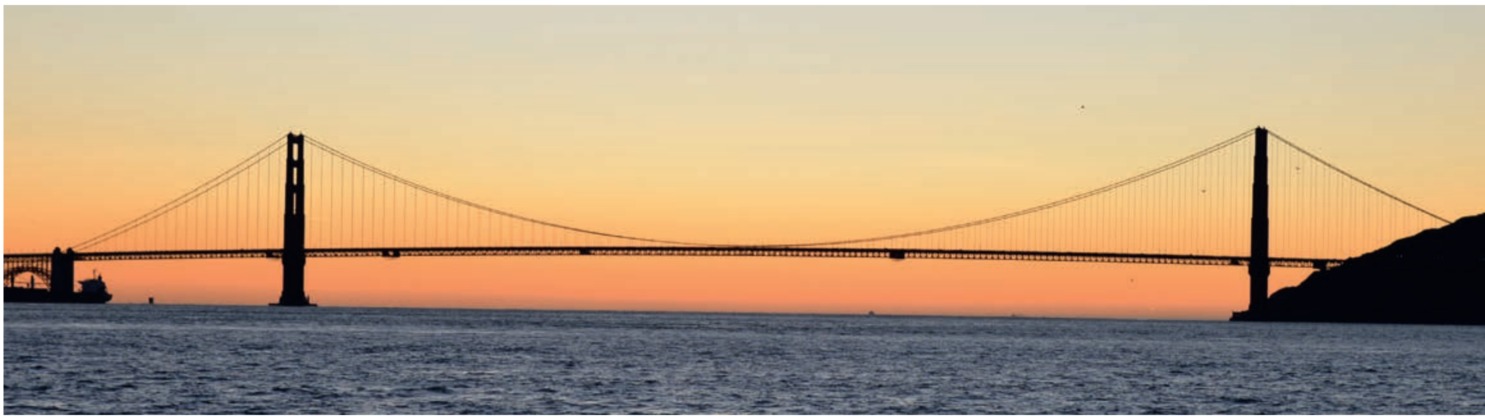


Illustration 11.11
Golden Gate Bridge, San Francisco, CA, USA (1937).
Relative span-to-sag proportions are evident, as is the parabolic cable profile caused by evenly spaced hangers.
Architect: Irving Morrow. Structural engineer: Joseph Strauss and Charles Ellis.

force polygon and the smaller will be the total tension force in the cable. Conversely, less sag corresponds to the need for a larger horizontal support reaction and to a larger tension force in the cable.

Since a cable with less sag means that it will be subject to larger forces (Fig. 11.4), there is a consequent need for this to be a thicker cable, which, in turn, means that it will weigh more for that reason. On the other hand, a cable with more sag and, therefore, smaller forces is by definition a longer cable and thus will also weigh more for a different reason. Hence, for efficiency of form, we seek a compromise between the relative proportions of sag to span; for cable roofs the ratio between them is typically in the range of

$$1/15 < f/L < 1/10$$

where f = sag and L = span. For bridges we might expect a relatively larger sag with ratios in the range of 1/12 to 1/8. If f/L is significantly larger than 1/8 the situation may require unreasonably high support structures. On the other hand, a very small ratio between sag and span may lead to a condition where the cable forces may become very large and perhaps enough to result in significant elastic deformations of the cable. An optimum relationship between these geometrical dimensions is typically sought, therefore, within the range that has just been suggested. An example can bring this to life: if we consider the Golden Gate Bridge in San Francisco (Ill. 11.11), the main span is 1280m (4200ft) and the cable sag is about 152m (500ft), which translates into a sag-to-span ratio of 1/8.4.

Beyond these general observations about how the relative magnitudes of suspension cable forces vary according to the amount

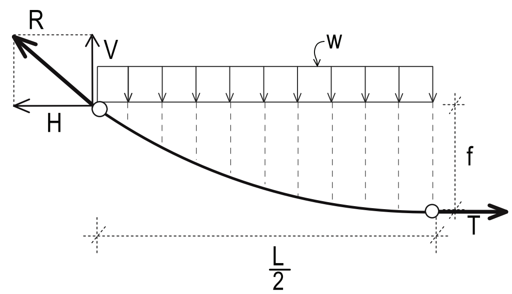


Figure 11.5
Free-body diagram of one-half of cable subjected to a uniformly distributed load w.

of sag, we are also in a position of being able to determine precisely the magnitudes and distributions of tension forces in suspension cable structures. For example, a cable supporting a point load P at mid-span may be analyzed according to the same equilibrium principles described in Chapter 4 and applied in Chapters 7 and 9 in the context of beams and trusses, respectively. Demanding that we have equilibrium of vertical forces at the lowest point where the load P is located will allow the magnitude of the tension force in the cable to be established, and we can thus verify the observation made above that a cable forming a smaller angle with the load (corresponding to a larger sag) will be subject to a smaller tension force. (See Fig. 11.4.) In the case of a single point load the tension force will obviously be constant along the entire length of the cable.

If we wish to consider a cable supporting a UDL, however, the picture is a bit more complicated. We will be content here with looking at a cable where the two supports are positioned at the same elevation, with the cable spanning the length L and having a sag f when subjected to the distributed load w. (Fig. 11.5.) The tension force in the cable at the point of maximum sag (at mid-span) is called T. For there to be equilibrium, the support reactions must have exactly the same direction as the cable tangents at the two points of support, and these are seen to be composed of horizontal and vertical force components, H and V, respectively.

Horizontal equilibrium of a free-body diagram depicting one-half of the cable shows that:

$$H = T$$

Demanding vertical equilibrium of this part of the cable structure results in:

$$V = wL/2$$

Since the tension force T at the lowest point in the cable acts horizontally, it will not be part of the equation for vertical equilibrium given above. Furthermore, equilibrium of moments about the support gives:

$$(wL/2)(L/4) - (T)(f) = 0$$
$$T = wL^2/8f$$

Since H = T, the magnitude of the horizontal component at the support is also given by this equation. One way of interpreting this result is to observe that the moment $wL^2/8$ produced by the distributed load w across the span L is resisted by the couple comprising forces T and H having a moment arm f (i.e., $T \times f = wL^2/8$). This is analogous to a simply supported straight beam, of course, with the difference being that the moment arm in that case is established within the limited beam depth and thus results in the need for considerably larger material expenditure.

Having calculated both V and H, the support reaction R, which is equal (but oppositely directed) to the cable tension force at that point, is given by:

$$R^2 = H^2 + V^2$$
$$R^2 = (wL^2/8f)^2 + (wL/2)^2$$

Since no horizontal loads act on the cable, the horizontal force component H is constant throughout the length of the cable, and is always in equilibrium with the horizontal force reaction at the supports. At the lowest point of the cable (which is at mid-span for a symmetrical case), this horizontal force is the sole cable force component and thus is the total tension force in the cable at that point. In other words, this is the minimum value of the cable's tension force anywhere along its length. Elsewhere, there is always a vertical force component which varies along the length and has its largest value at the supports where the cable slope is largest. We may therefore conclude that the total tension force in the cable is also largest at the supports, and is given by the equation above.[5]

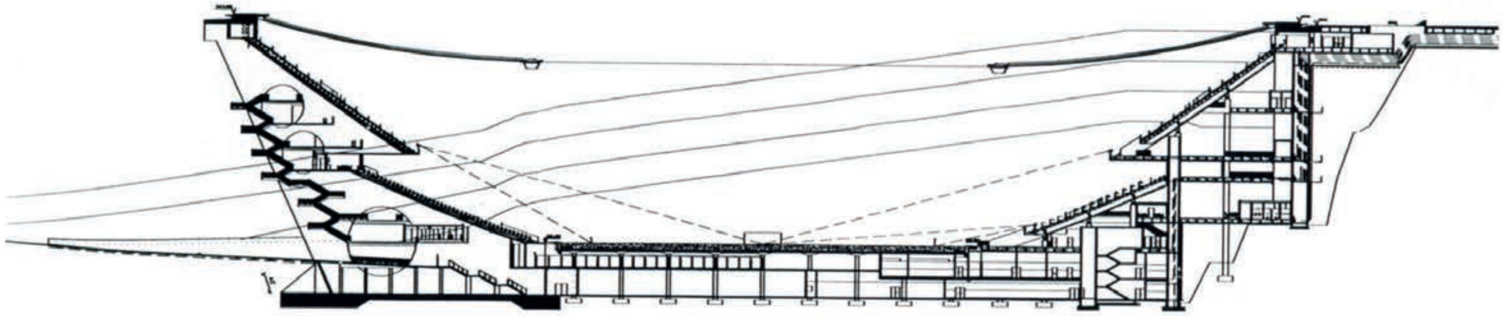


Illustration 11.12
Braga Stadium, Braga, Portugal (2004).
Section drawing.
Architect: Eduardo Souto de Moura. Structural engineer: AFA Associados.

Recalling the situation of Braga Stadium that we discussed in Section 11.1 and whose section drawing is shown in Illustration 11.12, we are now in a position to do an approximate calculation of some of the main cable forces in that structure in order to see what kind of stress levels these must cope with. (While doing so, however, it must be understood that the control of the dynamic effects of wind is also vital for the stability and safety of such a roof, but that the theoretical foundations for being able to incorporate such an analysis of structural behavior are beyond the scope of this book.)

The suspension cables at the Braga Stadium are arranged in pairs that are spaced 3.75m apart (12ft) and that are anchored in slabs at each end that act as horizontal beams between the tops of the vertical wall/pier supports. As was previously mentioned, a unique aspect of this stadium's cable structure is that it only supports loads in the areas covering the stands. So the suspension cables in this case are in fact not subjected to a distributed load along their whole length, but instead have loads acting only along the parts of the span closest to the two supports. In addition to the weight of the concrete slabs, there are also point loads acting on the cables that are produced by the transverse steel trusses at the outer end of the covered roof areas. These trusses both act as stiffening beams for the roof system and provide a convenient place to mount floodlights and loudspeakers to animate the events below as well as to carry a drainage trough to divert the rainfall run-off water.

For our purposes here, it will be enough to consider only the situation where dead loads from the roof are acting on the suspension system, and we will ignore the weight of the cables themselves. Given these simplifying assumptions, the cables in the open portion between the covered roofs can be considered to carry essentially no transverse load at all, resulting in their shape being able to be approximated as a straight, horizontal line. This part of the span measures 88.4m (290ft), leaving the length "a" of the two covered roof parts as 57.3m (188ft) each, for a total span of 203m (666ft).

The thickness of the concrete slab elements is estimated to be 0.245m (9.66in) and the weight density of reinforced concrete is taken as 25kN/m^3 (157lb/ft^3).

The weight w per unit of length for a roof strip width corresponding to the cable-pair spacing of 3.75m (12ft) is thus:

$$w = 25\text{kN/m}^3 \times 0.245\text{m} \times 3.75\text{m} = 9.88\text{kN/m}$$

The weight of the supported part of the truss per cable pair is estimated to be:

$$P = 7.2\text{kN}$$

And the cable sag at the outer end of the covered roof is approximated as:

$$f = 10.1\text{m}$$

We can now find the cable force H at that outer end point (which, again, is considered to be horizontal because no loads are assumed to be acting on the cable outside the covered roof area) by requiring equilibrium of moments about the left support.

$$(w \times a \times a/2) + (P \times a) - (H \times f) = 0$$
$$(w \times 57.3\text{m} \times 57.3\text{m}/2) + (P \times 57.3\text{m}) - (H \times 10.1\text{m}) = 0$$
$$(H \times 10.1\text{m}) = (9.88\text{kN/m} \times 57.3\text{m} \times 57.3\text{m}/2) + (7.2\text{kN} \times 57.3\text{m})$$
$$(H \times 10.1\text{m}) = 16632.1\text{kNm}$$
$$H = 1646.7\text{kN}$$

At the supports, the horizontal component H will join forces with the vertical component V, which is

$$V = (w \times a) + P$$
$$V = (w \times 57.3\text{m}) + P = (9.88\text{kN/m} \times 57.3\text{m}) + 7.2\text{kN} = 573.3\text{kN}$$

Illustration 11.13
Braga Stadium.
"Inside" the space of the stadium; drama of cable system made evident.

The total tension force in the cable at the supports, therefore, will be

$$T^2 = V^2 + H^2 = 573.3^2 + 1646.7^2$$
$$T = 1743.7\text{kN}$$

This tension force is shared between two cables, however, and if we assume that the net area of the cables corresponds to cable diameters of d = 84mm (3.33in), this condition will result in a tension stress acting within each cable of:

$$\sigma = T/A = T/(2 \times 3.14\ d^2/4) = 1\ 743\ 700\ \text{N}/(2 \times 5539\text{mm}^2) = 157\text{N/mm}^2\ (22\ 771\text{lb/sq.in})$$

This stress that we have calculated is in fact quite low for a steel cable, about one-tenth of the strength that we might expect for steel wires, reflecting the fact that we have included in our calculations only a limited part of the total loads that may be acting on this roof. For a more accurate calculation of the cable stresses, the effects of wind would need to be taken into account, as would the dead weight of the cables. We should also note that we have not considered any safety factors against failure, which we would be expected to do according to structural design practices in all countries. Nevertheless, even from the limited investigation that we have just carried out we can safely conclude that suspension cable systems are remarkably efficient and effective structures for spanning long distances with a minimum amount of material; moreover, they can be quite spectacular in doing so. (e.g., Ill. 11.13.)

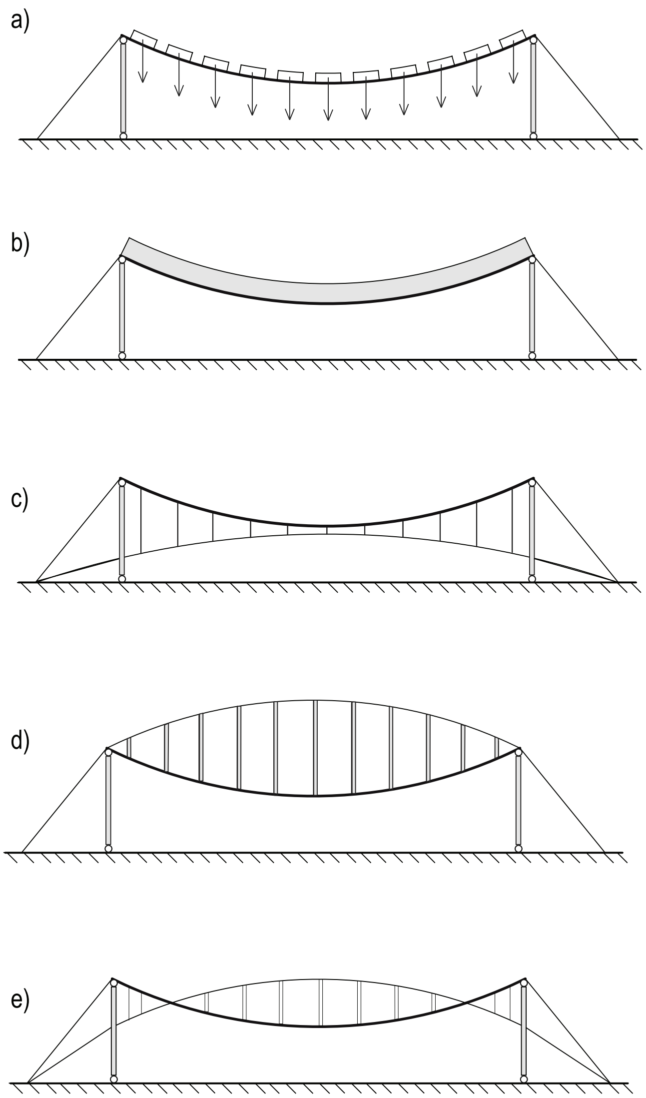


Figure 11.6
Principles for stabilizing suspended cable systems. In (a) by adding weight, (b) by connecting to or forming rigid structural elements, or by having a dual-cable system with counter-curvature cables, (c) below the main cable, (d) above it, or (e) overlapping with the main cable (i.e., partly above and partly below).

11.4 Stabilizing and Supporting Suspension Cables

Cable Stabilization Techniques

As we have described with reference to the hammock, suspension structures have no bending stiffness per se and they are, therefore, potentially vulnerable to loads which change magnitude and distribution over shorter or longer periods of time; e.g., snow loads on a roof exemplify a type of gravity load which does just that by being moved about by the wind. In addition to being aware of the effect of asymmetrical and changing live loads, it is important to note that the *dynamic* response of such a structure to wind may also be critical in establishing what is structurally necessary to stabilize a cable roof structure. Wind loads change continuously and may easily set such a hanging roof in motion by creating a suction force on the roof's top side whose magnitude may exceed the system's typically low dead weight. This condition will cause the roof to bulge upward, thus changing its shape dramatically. This new roof profile will, in turn, respond to the wind load differently and the shape is likely to change yet again, establishing a cyclical process that sets the roof into a large-scale fluttering behavior. And beyond needing to stabilize such obviously undesirable large-scale geometric changes, it is crucial that the natural vibration frequencies of a hanging structure be designed to be quite different from the frequencies of any likely wind gusts, as this condition can cause a so-called resonance behavior that can rather quickly result in excessive and violent vibrations of its own.[6] The collapse in 1940 of the Tacoma Narrows Bridge in Washington State in the USA is a well-known structural accident resulting from just this very effect, and its widely available video footage is quite convincing in portraying the ever-increasing torsional oscillations of its bridge deck before eventually failing catastrophically. A more recent example of the problems that a structure's dynamic response may cause was observed on opening day for the Millennium Bridge in London (Ill. 5.3), which led to its short-term closure and retrofitting; since that time the bridge has operated without further problems.

While the Tacoma Narrows failure may be an extreme example that is convenient to illustrate a point, it remains that its lesson is clear: measures need to be taken to prevent suspension structures from moving and vibrating excessively. Fortunately, there are some relatively simple ways of accomplishing this objective. The options

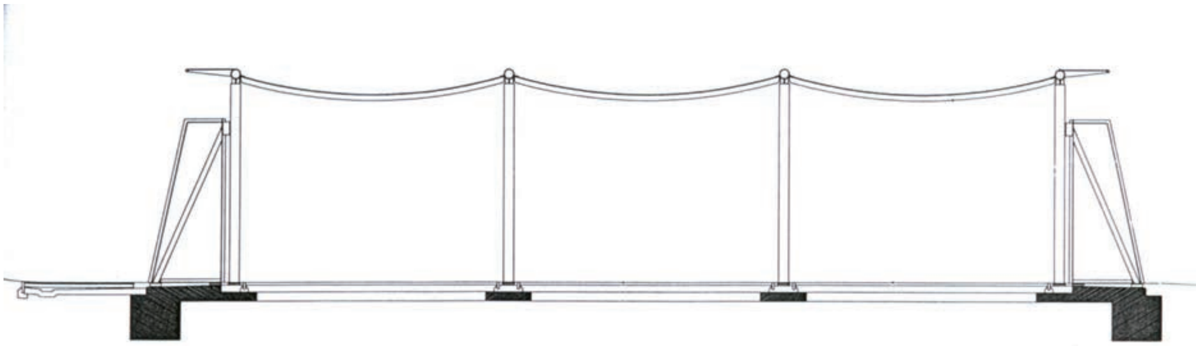


Illustration 11.14
The Royal Albert Dock Regatta Centre, London, UK (1999).
Suspended steel sheet roof with stiffness provided by bending-stiff T-profiles, with the stem of the T facing upward.
Architect: Ian Ritchie. Structural engineer: Arup.

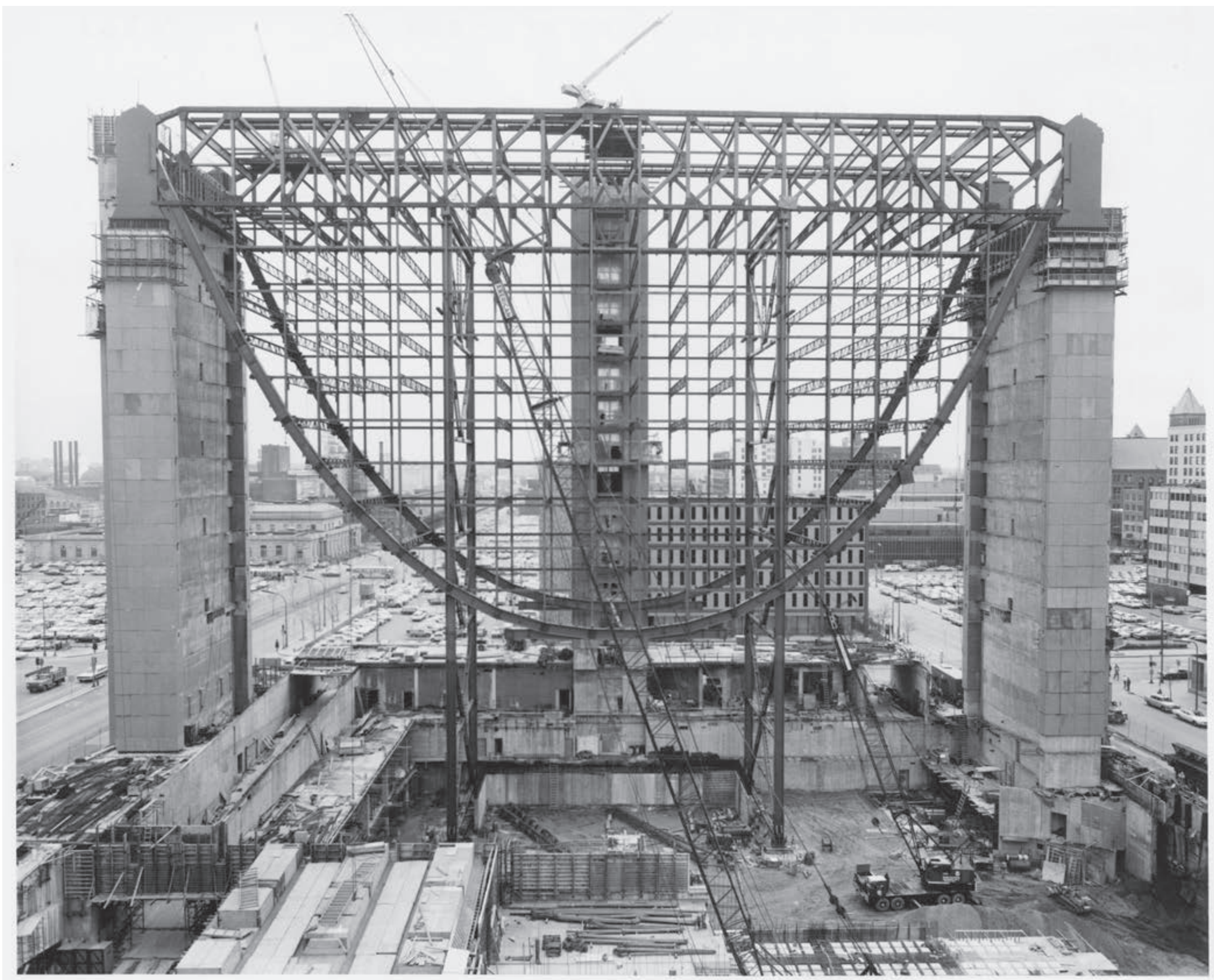


Illustration 11.15
Marquette Plaza; formerly, Federal Reserve Bank Building, Minneapolis, MN, USA (1973).
Upper level stiffening truss is used to stabilize the profile of the suspension cables that support the floors of this building. Vertical support for the cables is provided by the two end concrete core walls; stiffening truss also acts as horizontal compression strut, keeping the tops of the walls from deflecting inward.
Architect: Gunnar Birkerts. Structural engineer: Leslie E. Robertson & Associates.

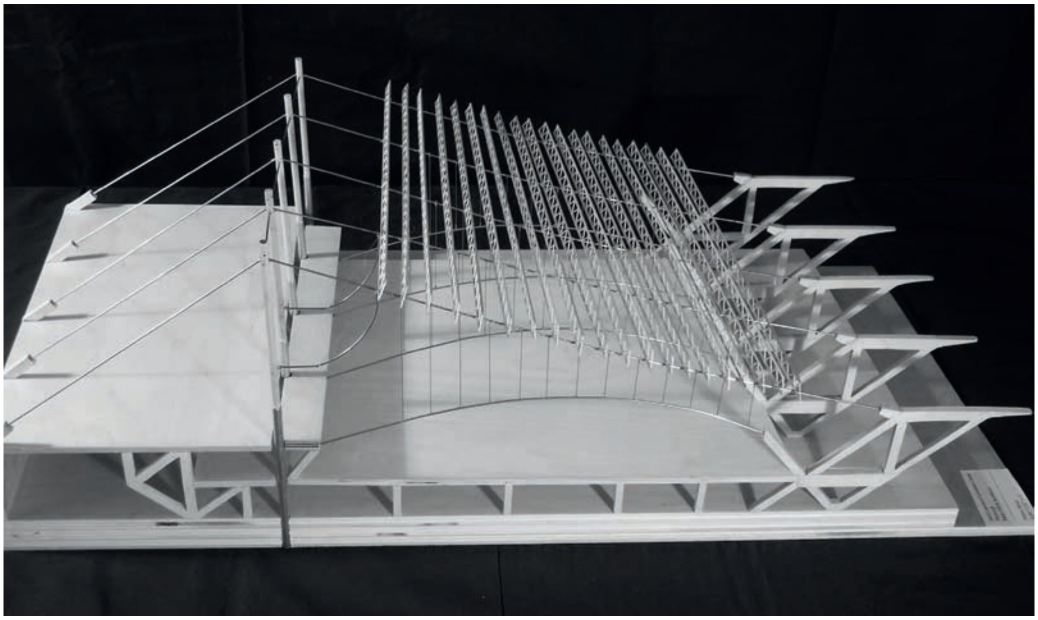


Illustration 11.16
David L. Lawrence Convention Center, Pittsburgh, PA, USA (renovated 2003).
Curved profile of suspension cables supporting the light roof trusses is stabilized by counter-curvature cables below; the two sets of cables have noticeably different curvatures and are connected to each other by means of vertical tension ties. Cable anchorages are also evident and produce overall self-balancing system. Pittsburgh's numerous adjacent suspension bridges provided inspiration for this system.
Architect: Rafael Viñoly Architects. Structural engineer: Dewhurst Macfarlane and Partners.

that are available each affect overall structural form and visual expression in its own way, however, and so these will have significant design implications when considered in the context of architecture.

One fundamental way to stabilize and stiffen suspension structures that we are already familiar with from the preceding examples in this chapter is simply to add weight to the system, thus countering any uplift tendencies produced by the wind or asymmetrical live loading. (Fig. 11.6a.) The added dead load must be supported directly by, or suspended from, the cable system; as we have seen, such supplemental weight can come from secondary structural elements that bridge the gaps between parallel suspension cables (e.g., the slabs in the case of both of the Portuguese structures discussed in Section 11.1) and/or it can come from additional ballast that is placed on top of the roof surface (as will be seen to be the case for the Hohenems garage example discussed in Section 11.5). The dead weight of the roof also acts to pre-tension the cables, thereby improving their response in terms of the natural periods of vibration of the roof. This additional benefit aside, however, the primary approach here is to simply add dead weight to the system, which may seem to be a counterintuitive strategy to use with a structural system that is meant to exploit material efficiency.

A second important stabilizing strategy for a suspended roof is to connect it to rigid structural elements that can act as curved beams or slabs, providing both bending stiffness and sometimes also weight to the cable structure. (Fig. 11.6b.) In the case of the Lisbon pavilion, the stiffness of the continuous concrete "flying carpet" will resist the surface's flexing and hence it is able to make a certain contribution toward the suspension system's overall stability. For the roof of the Royal Albert Dock Regatta Centre, a suspended sheet steel roof is given transverse stiffness by bending-stiff T-shaped steel rib projections. (Ill. 11.14.) In suspension bridges, the necessary stiffness for the cable system is commonly provided by trussed edge beams or by the bridge deck structure (e.g., see the Cornell University pedestrian bridge and the Golden Gate Bridge, shown previously in Ill. 11.8 and Ill. 11.11, respectively), and this can also be found in buildings on occasion, such as in the former Federal Reserve Bank Building in Minneapolis, seen under construction in Illustration 11.15. What is happening in such cases is that any potential cable movements are restricted by the cables being tied to bending-stiff elements, forcing the cables to follow these elements' much more limited flexibility. A secondary benefit of the bending stiffness of such elements is that it enables a distribution of point loads to a longer segment of the cable structure, the result of which will be reduced local cable deformations.

A third general alternative for stabilizing suspension cable systems involves the use of two cables and is thus referred to as a *dual-cable system*; one of these cables is the primary load-carrying suspension cable, while the second is an inverted stabilizing cable which is intermittently connected along its length to the primary one. (Fig. 11.6c; e.g., such opposite curvature cables are clearly made evident in the model of the suspended roof system for the Lawrence Convention Center in Pittsburgh (Ill. 11.16), and in the glass wall stabilizing system for Fingal County Municipal Hall in Dublin that will be discussed in the next section – see Ill. 11.26.) Any tendency to displace under loading by of one of the cables in such a system will immediately be countered by the curvature of its opposing counterpart. The two cables are often arranged in the same vertical plane (although other arrangements are certainly possible, as will be discussed shortly) but they are typically made to have different curvatures so as to have different natural frequencies one from the other. This difference in curvature also means that the tension forces in the two cables will be different, as per our earlier discussion about the effects of different cable sags.

Illustration 11.17
American Museum of Natural History/Rose Center for Earth and Space, New York City, NY, USA (2000).
Multiple "cable beams" brace large glass surface against wind loads. The glass is of a type with greatly reduced iron content for increased clarity (in this context, iron is a "polluting" impurity which commonly gives glass its greenish color).
Architect: Polshek Partnership Architects. Structural engineer: Weidlinger Associates.

Illustration 11.18
Madison Square Garden, New York City, NY, USA (1968).
Aerial view of radial suspension system under construction. Outer compression ring is evident for anchoring suspension cables, as is inner tension ring. Central mast was temporary and used for construction.
Architect: Charles Luckman Associates. Structural engineer: Severud Associates.

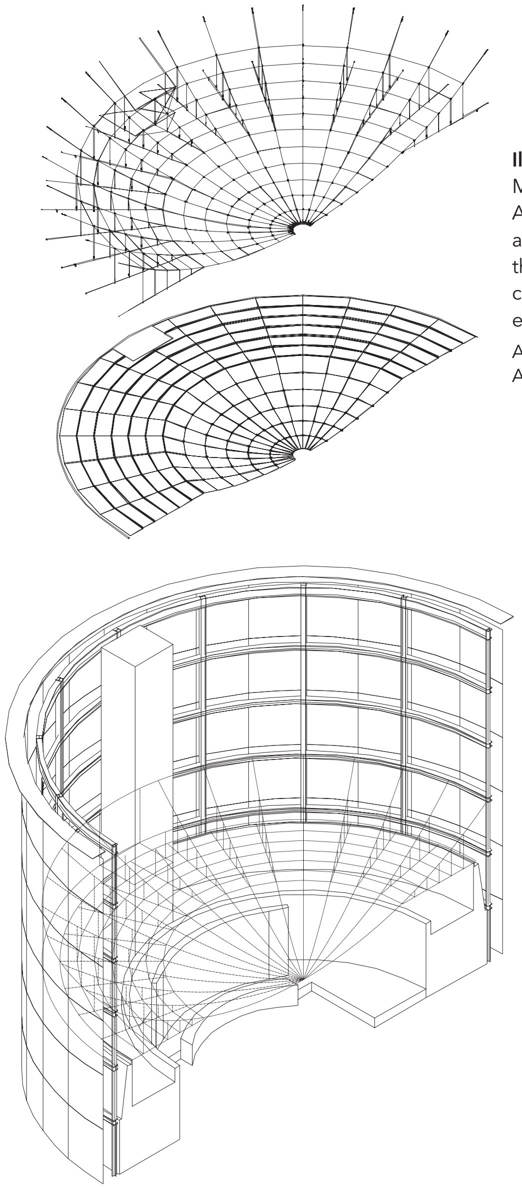


Illustration 11.19
Main courtroom, Sandra Day O'Connor Courthouse, Phoenix, AZ, USA (2000).
At bottom, axon drawing of half of courtroom volume; ceiling's radial cables are anchored around the outer perimeter on a cylindrical steel rigid frame. At top, the cable structure and the glass ceiling profile drawn independently of each other; glass panels above the central courtroom space are directly attached to the draped suspension cables, but around the enveloping circulation zone they are horizontal and hung by short vertical hangers.
Architect: Richard Meier & Partners (for the courthouse building), James Carpenter Design Associates Inc. (for the glass ceiling). Structural engineer: Arup.

Illustration 11.20
Sandra Day O'Connor Courthouse main courtroom.
Lens-shaped glass ceiling over main courtroom displays radial arrangement of suspension cables, anchored near the center by a small tension ring. Weight of glass panes helps to stabilize this structure, which is completely housed within a much larger building atrium.

There are two obvious arrangement options for this dual-cable system: one is to have the suspension cable placed directly above the inverted secondary cable, with the two being connected by a number of tension ties – the overall system then has opposite curvature top and bottom. (Fig. 11.6c.) We can also arrange the two opposing cables in such a way that the system becomes lens-shaped; the lower cable in this convex system is the primary suspension one while the upper cable provides stiffness to the system. (Fig. 11.6d.) This second arrangement results in the need for the cable-connecting elements to be compression struts in order to keep the cables apart. A third variation is to have the opposing curvature cables intersect with each other, resulting in some of the connecting members needing to be in tension while the others are in compression. (Fig. 11.6e.) However the cables are arranged, the system as a whole is typically highly pre-stressed, and as such the complete dual-cable system can start taking on the somewhat unexpected flexural load-carrying characteristics of a beam or a truss (i.e., the notional "compression" chord of such a structural type is never allowed to actually go into compression because the system is so highly pre-stressed in tension) and at that point these dual-cable systems are sometimes referred to as *cable trusses* or *cable beams*. (e.g., Ill. 11.17.) Because of their filigree quality, such strategies are often employed as the stabilizing structural elements for structural glass walls.

More three-dimensional spatial variations of these stabilizing options also exist whereby inverted cables are offset at halfway intervals between a series of parallel suspension cables (i.e., they are not in the same vertical plane); down the length of such a system there will hence be cables having alternating upward and downward curvatures, creating a series of ridges and valleys in the resulting surface. And the suspension cable system also can be configured to be arranged radially if the building plan is circular, as in the system used for the famous roof of Madison Square Garden in New York City and, at a much smaller scale, for the showcase glass ceiling in the Sandra Day O'Connor Courthouse in Phoenix, Arizona. (Ill. 11.18 and Ill. 11.19, 11.20, respectively.)

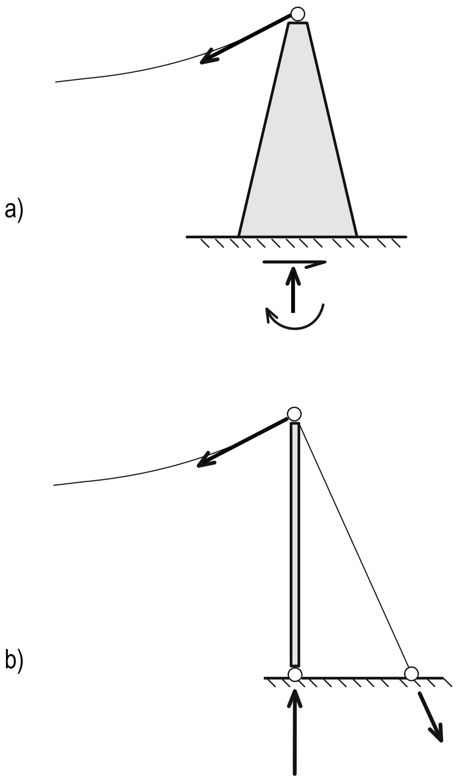


Figure 11.7
End support of cable structures.
(a) Cantilevered piers acting in bending, and (b) guyed, or back-stayed, masts. The latter principle results in axial forces only, with compression and tension in the mast and guy, respectively.

Cable End Supports and Anchorages

As for the requirements at the end supports of suspension cables, it is clear that the rather substantial inward pull of the cable(s) must be countered, and that this will be an additional imposition on the support beyond the usual vertical reaction forces needed to resist gravity loads; both of these force reactions must, therefore, be met by the support structures. We can differentiate between two typical ways in which suspension cable support structures accomplish this task: one involves resisting the inward pull by significant bending action, with the support effectively being made to function as a vertical shear wall, while the other works by an effective combination of axial compression and tension forces in guyed, or back-stayed, masts as discrete and spaced-apart structural elements – all of which recalls the basic functioning of shear walls and braced frames that we have previously considered in Chapter 10. (Fig. 11.7.)

In the first case, steel or reinforced concrete piers are typically aligned with the spanning direction, and these receive the pulling forces directly from the cables or, perhaps more commonly, indirectly via a transverse distribution beam or truss that anchors a number of cables and spans horizontally between the tops of the piers. As we have seen in the case of both the Lisbon Pavilion and the Braga Stadium (Section 11.1), and as can also be found in the Dulles Airport Terminal structure (Ill. 11.21, 11.22), such piers cantilever vertically from the foundations and necessarily have to be of substantial structural dimension. At all levels of the support pier bending moments, lateral (shear) forces and axial forces are all present, and the bending moments are especially large, typically increase from top to bottom of the wall, putting their greatest demand on it at the foundation level. If possible, two opposing outwardly leaning piers of this type at opposite ends of the structure can not only help to reduce the imposed bending moments by having some of the cables' inward pull resisted by the walls also acting as an inclined compression strut, but also the bases of these may be connected to each other underground by compression struts or slabs, thereby letting the inwardly pushing lateral forces be balanced and freeing the foundation from having to distribute these forces to the ground. The equivalent to this latter self-balancing system in the case of a radial arrangement is to anchor the cable tension forces by means of a compression ring(s) around the outer perimeter and an inner tension ring (e.g., these can be seen in the photo of the construction of Madison Square Garden roof (see Ill. 11.18) as well as with the cylindrical steel frame and central tension ring used to anchor the Sandra Day O'Connor courtroom ceiling system (see Ill. 11.19)).

It should be recognized, however, that the bending mechanism of the preceding piers is inherently inefficient as a load-resisting mechanism, and by trying to anchor large tension forces high up in the air at the tops of the piers this "problem" is only heightened. A much more materially efficient support structure is one that uses the guyed, or back-stayed mast. (See Fig. 11.7b.) By this system a mast or column receives the cables' vertical reaction forces in compression as any vertical support must, but in addition the inward pull of the suspension cable is countered by an opposing tension in tie-back cables or guys. A consequence of this anchoring system

Illustration 11.21
Washington Dulles International Airport, terminal building, VA, USA (1962/extended 2009). Suspension cables support a concrete roof having catenary curvature due to its primary loading from the roof's self-weight. Supports at each end are outwardly inclined to work in compression to help resist the inward pull of the cables. Their tapering profile indicates that they also work as vertical cantilevers in bending.

Architect: Eero Saarinen/SOM. Structural engineer: Ammann and Whitney/SOM.

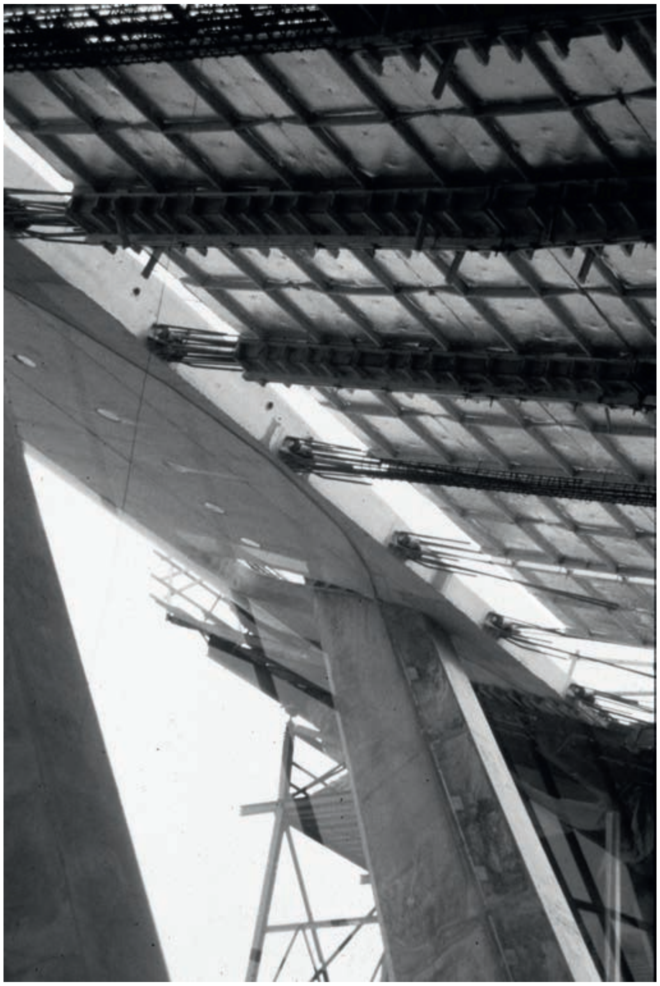


Illustration 11.22
Washington Dulles International Airport, terminal building. Cable-to-support connection detail as seen during construction of terminal building expansion. Closely spaced steel cables connected to inclined longitudinal concrete beam, which is in turn supported by the series of inclined masts.

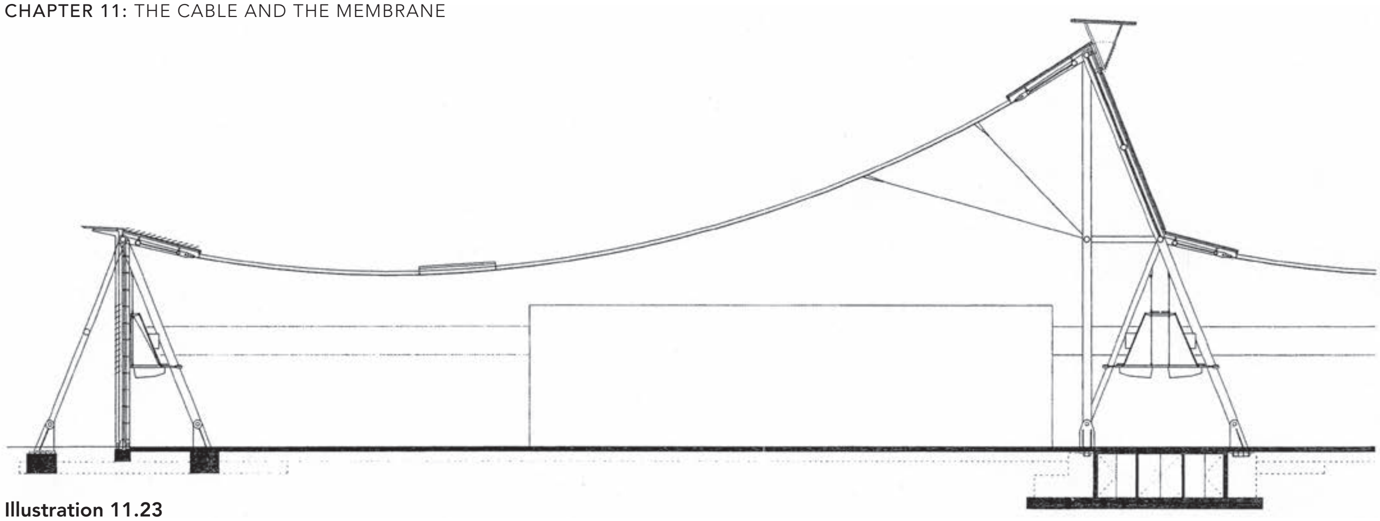
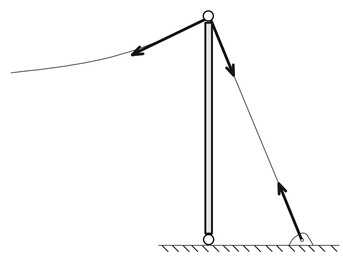


Illustration 11.23
Messehalle 26, Hanover, Germany (1996).
Section drawing for three-bay suspended roof structure. Braced frame structures provide lateral support for unbalanced horizontal forces from cables at mast-top anchorages. Inclined ties help to prevent vibration in cables.
Architect: Herzog and Partners. Structural engineer: Schlaich Bergermann und Partners.



is that the vertical force in the mast will be increased since it must also bear the vertical force component of the tie-back cable, but dealing with this is still likely to leave this end support system a much lighter and more open affair than is the (comparatively inefficient) bending-reliant behavior of the pier/shear wall system. (e.g., this condition can be observed in the Buckingham Palace Ticket Office (see Ill. 11.9), the Clifton and Golden Gate Bridges (see Ill. 11.10 and Ill. 11.11), the Lawrence Convention Center (see Ill. 11.16), as well as in the analogous mast-and-backstay(s) condition of the cable-stayed Ypsilon Footbridge (see Ill. 6.23.)) As a slight variation on this approach, the supports for the suspension cable roof of the Messehalle 26 in Hanover employs diagonal bracing subsystems to balance the resultant horizontal forces at the cable supports. (Ill. 11.23.)

The "lighter" guyed-mast system does not come without its own structural "price," however: the tie-back cables' foundations have to be able to withstand both lateral forces as well as uplift, which makes the foundation work for the project significantly more complex than it otherwise might be. (Fig. 11.8.) Typical solutions for anchoring tension forces in the ground include: (a) mobilizing gravity to counter uplift by following the ballast principle; i.e., by placing a large weight in the ground sufficient to counteract any uplift force. The lateral inward pull of the anchoring cable may then be resisted by passive earth compressive forces against the side of foundation. (b) resisting uplift and horizontal forces by the use of friction piles or some kind of wedge principle in softer ground; and (c) using steel anchor bolts that can be fastened in holes drilled into solid rock and that are then filled with cement, in which case the bedrock is expected to resist both uplift and lateral forces.

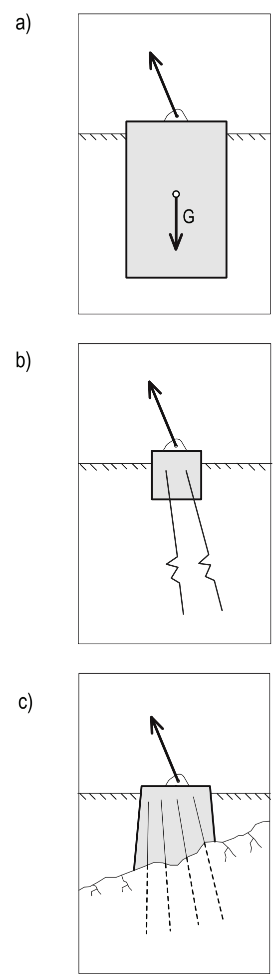


Figure 11.8
Tensile force anchoring systems: (a) gravity counterweight, (b) friction piles, and (c) rock anchors.

Illustration 11.24
Municipal garage, Hohenems, Austria (2000).
Distinctive suspended wood panel roof above exterior municipal garage's work space.
Architect: Reinhard Drexel. Structural engineer: Merz, Kaufmann und Partner.

11.5 Distinctive Small-Scale Systems

We have already seen several examples of the various techniques that can be used to stabilize suspended structures; here we will look at two more buildings that are both distinguished by their comparatively small scale and unusual applications of common materials. These two cases are strongly distinguished from one another, however, by their different stabilizing systems' main axis of orientation/operation: i.e., vertical in the first instance but horizontal in the second. Then again, the two are united by both addressing the various program needs of local governments, thus helping to make the case that suspension structures can have their place in more familiar and modest settings, and that they need not be limited to the conventionally "heroic" situations of long span bridges and stadia roofs.

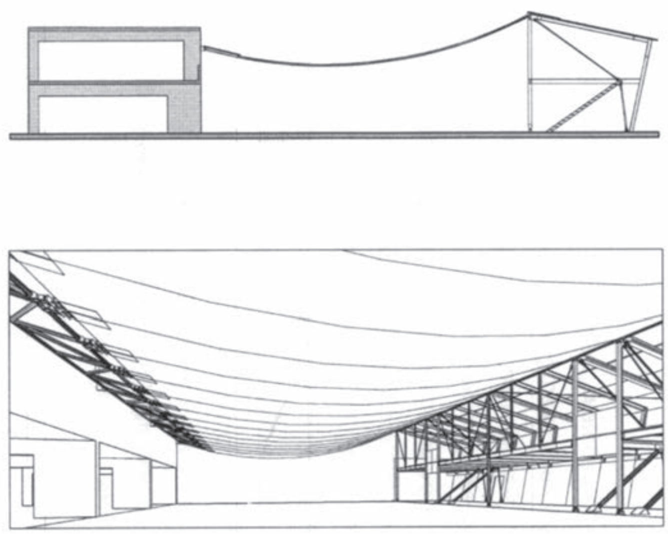


Illustration 11.25
Hohenems municipal garage.
Section and perspective drawings. Anchorage for suspension system is concrete rigid frame on one side vs. braced frame at the other.

Hohenems' Weighted-Down Wood Panel Suspension Canopy

The western Austrian province of Vorarlberg is an important locus for research on the sustainability of building practices, including the possibilities of modern wood construction; in this vein, teams of architects, engineers, and manufacturers have been collaborating in recent years to produce innovative design work that is built using prefabricated timber components. Indeed, while it may seem that a municipal storage facility would not be the first place that one thinks of going in search of such architectural innovation, the hanging timber roof over the exterior public works yard in Hohenems by architect Reinhard Drexel and structural engineers Merz, Kaufmann und Partner makes this particular building a noteworthy exception to the rule. (Ill. 11.24, 11.25.)

At this facility, a dramatically curved, suspended roof drapes over the open and column-free central space, allowing the various municipal vehicles their necessary freedom to move about. The roof is supported by buildings along two edges of this very wide hanging span: along one side it is connected by means of a steel anchoring truss to the rigid-framed structure of an administration building, while along the opposite edge the roof is anchored by the facility's long, open storage house and its exposed series of braced frames. (Ill. 11.25.) These two adjacent building structures thus not only have to support themselves but also must act as the necessary abutments for both the vertical and the horizontal forces that are produced by the suspension roof system.

In the case of such a thin structural system, the roof must necessarily carry the inevitable gravity loads primarily by means of tension, and the overall hanging geometry of the roof 's profile consequently closely follows that traced by a hanging chain or, as it is often referred to by its Latin equivalent, a catenary line. But it is the details of this particular suspended roof system that make it truly unique: rather than the more typical steel suspension cables that are often used in this situation, in this case the main tension-carrying elements are a series of 18m x 1.8m (59ft x 6ft) curved timber panels that are built from a layer of 39mm (1.5in) thick plywood sheets. To counteract the strong possibility that such a lightweight structure would tend to be lifted up by the wind, the top sides of these panels are supplied with a built-up grid of two 24mm (1in) thick timber ribs in order to hold in place a layer of gravel ballast. The roof 's top surface is made using OSB plates (oriented strand boards) in which the rectangular wood strands are aligned to provide maximum tensile strength in the hanging direction; these plates also serve as an underlay for two layers of waterproof bituminous sheeting.

Beyond its unique and unexpected material composition, however, this suspended roof and the open space below it have some other remarkable architectural qualities: rather impressively, in spite of a span of close to 20m (66ft), the structure connecting the two sides of the public works yard is only a total of 110mm (4.33in) thick, far exceeding the typical span-to-depth ratios of more conventional roofing systems. Also, an unexpected natural light washes over the underside of the roof surface, greatly reducing the need for artificial lighting in the work area; this effect is made possible by the roof surface curving upward toward its supports but then stopping just short of these. For example, a skylight-covered open steel truss anchors the roof next to the administration building, thus allowing natural light to bounce off the underside of the roof and light the workspace. Finally, with the two other sides of the public works yard completely open, the distinctive profile and surprising material composition of this hanging roof canopy are highlighted and appropriately celebrated; this is indeed a fine space for a city's municipal trucks and tractors to operate in.

Swords' Cable-Stabilized Hanging Glass Façade

In the case of Fingal County Hall, located in the town of Swords just north of Dublin, Ireland, the local government sought to have a building built that would be an open expression of administrative transparency, provide services to the community, and be a pleasant and energy-conscious working environment for a staff of about 450 people. A design competition was held in the late 1990s that was won by Bucholz McEvoy Architects with a building design that also manages to integrate itself within a challenging urban site: it is located at the intersection of multiple street axes, across from the historic Swords Castle, and a small crescent-shaped park with an existing grove of tall Holm oak trees as well as a 150-year-old Himalayan cedar that had to be respected.

In the end, the centerpiece of the design is a curved 5-story-high public foyer/atrium, whose 32m (105ft) long, 18m (59ft) high exterior façade wall is all made of suspended, transparent glass panels. (Ill. 11.26.) Because of its location this strategic gesture at once connects the interior space with the trees just outside, with the Norman castle across the street, and with the views along extended street axes. And from the outside, of course, this huge glass wall makes the functions of the local government directly visible, whether by day or night. Reinforcing the point about local government transparency and accessibility, the atrium space contains service counters, meeting rooms, and open circulation stairways and balconies; i.e., the atrium itself defines and becomes the zone of overlap between public and private functions, the latter of which are largely contained in three narrow, parallel Office Wings that project back and away from this curved band of common atrium space.

We will focus here on the structural support for this huge glass "window" wall. In order to minimize the dimensions of any and all supporting structure, this is almost completely done by means of

Illustration 11.26
Fingal County Hall, Swords (near Dublin), Ireland (2000).
Interior of foyer space showing counter-curvature cable system stabilizing hanging glass wall.
Architect: Bucholz McEvoy Architects. Structural engineer: Arup (building), RFR (glass wall).

Illustration 11.27
Fingal County Hall.
Plan drawing highlights counter-curvature of cable system used to stabilize entry's hanging glass wall.

tension. To begin with, we can consider gravity loads: the thin glass panels of this wall are all hanging vertically, each panel physically connected to the one immediately above it by specialized multi-pronged bolted connectors, with each of these vertical bands of connected glass panels then hung at the top from an outwardly projecting, vertically curved flanged steel beam located at the roof level. There is a long series of such cantilevering beams, as can be seen in Illustration 11.26. The glass of each panel, then, is working in tension, carrying the glass's self-weight from bottom to top, with the top-most panel therefore having to carry the most tension load. This requires, of course, that the tensile material capacity of the glass is not exceeded, but that is certainly possible as glass is quite strong in tension. Such glass panels need to be specially produced for this purpose, however, given glass's normally brittle failure tendencies; the connections/connectors also require specialized technology to ensure the proper and smooth transfer of forces from one panel to the next. But all this is able to be done today with the right specialized expertise, and this hanging system is well able to account for the glass wall's gravity loads, which are carried up and away from the base of the glass wall – contrary to "normal" gravity load paths – but then, that is a good thing as thin glass panels do not lend themselves very well to carrying significant compressive loads.

But there remains an inherent lateral instability with such a long "hanging curtain" glass wall, with inward wind pressures (or outward wind suctions, depending on the wind direction) tending to cause the "curtain" to displace rather significantly. As can be seen in Illustrations 11.26 and 11.27, such displacements are being prevented over the height of this wall by means of a series of intermittently spaced, horizontal pairs of intersecting counter-curvature stabilization cables, which take just the form that we saw diagrammed in Figure 11.6e. The glass wall surface can be seen to be laterally connected to these stabilizing cable-pairs by means of a multitude of short timber-tipped spars/struts. Of course, the opposing curvature of these sets of cables allows the system to work equally well whether the wind pressure is tending to cause the wall to move inward or suction is pulling it outward, or even both at the same time over such a large expanse of façade. Moreover, because these pre-stressed cables are all made to be working in tension, the dimensions of the hanging glass wall's stabilizing system have been minimized, which maximizes the visual transparency that is such a key aspect of the design intensions for this project. At the same time, their sweeping gestures within the atrium space can be considered to be putting on their own performance for all to see.

Illustration 11.28
Spider's web, a natural cable net.

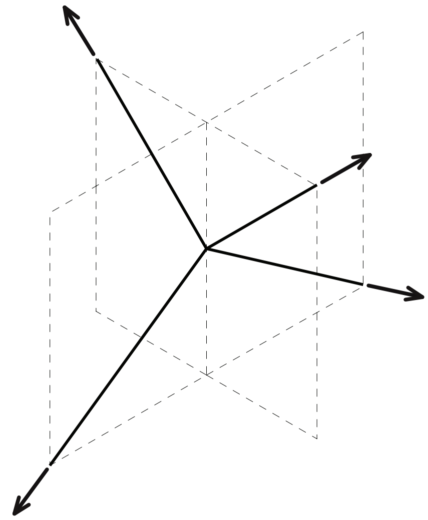


Figure 11.9
Two orthogonally directed cables provide mutual stability.

11.6 Cable Nets – A Grid of Cables

While the suspension structures and the stability principles discussed in this chapter so far have essentially been for two-dimensional in-plane systems that can, where necessary or desirable, be "extruded" or "spun" in space to form overall three-dimensional structural systems for buildings, there are other ways of designing and constructing tensile structures. Evidence for this can easily be found in the woods and in unused corners of attics or basements, where a wonderful cable-like structure can be observed in the form of the spider's web. (e.g., Ill. 11.28.) This ingenious product of nature provides an interesting inspiration for human-made structures, one in which the efficiency of tension is fully exploited in a *spatial* manner. Here is the enticing suggestion that there is a way for cables alone to form an integrated, three-dimensional structural system; i.e., not only are thin tension "cables" carrying all loads and stabilizing the system, but these are also creating the suggestion of a surface as well – and in architecture surfaces are typically needed for roofing, for enclosure, and for walking on. In the spider's web, then, we find a three-dimensional tensile structure in which support and surface are integrated as one in a remarkably elegant manner, and that seems full of possibility.

Alas, a spider's web is not a stiff structure, but instead moves in the smallest of winds. To see how we might accomplish something like a natural web in the context of real building structures where major deflections are not permitted, we begin by briefly going back to a single suspended cable held up at both ends, but one which is now connected to a second cable oriented at 90^0 to the first. (Fig. 11.9.) Pulling downward at both ends of the transverse cable puts the suspended cable into the shape of a V; moreover, the tensioning of the downward cable will ensure that the system of two cables will be stabilized, one locked into the other. The same can be done with a series of suspended cables spanning in one direction. If we connect these to a second set of perpendicular cables having opposite downward curvature, the first set will form into a concave shape while the second set of cables will form into a convex shape (Fig. 11.10.) As with the single cable pair, these two cable sets will mutually provide each other with stiffness and stability. In order to do so, however, the cables need to be pre-stressed (i.e., slack, loose cables will do nothing for this system) and the system must have double curvature of the *anticlastic* type just described. In other words, the curvatures in the two main directions must be opposite to each other (the seat of a saddle is perhaps the most familiar analogous form that comes to mind to help visualize this shape).[7] For reasons that are self-evident from the pattern of the grid of interconnected tensioned cables thus established, we call such a system a *cable net* structure.

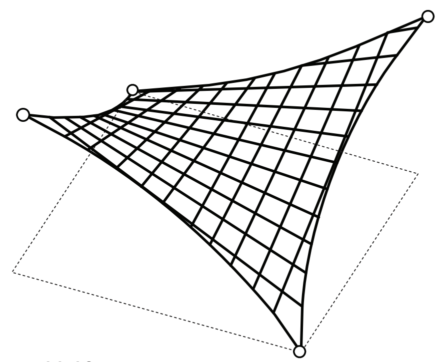


Figure 11.10
The 3-D cable structure: the cable net. Since tensile structures basically respond to external loads by trying to adjust their geometry, it is necessary that 3-D shapes are double-curved and pre-stressed so that the surface geometry becomes stiff. Two sets of cables form an anticlastic, or saddle-shaped, surface.

Illustration 11.29
German Pavilion, Expo '67, Montreal, Canada (1967).
Multiple high points at tops of masts and low point anchorages enabled the unique double-curvature shape of the cable net surface that covered the full extent of the terraced landscape of the pavilion.
Architect: Atelier Frei Otto and Rolf Gutbrod. Structural engineer: Leonhardt und Andrä.

Generally speaking, the way in which a cable net structure is given the very specific three-dimensional geometry that it requires to both carry loads and be stable is as follows: its anticlastic surface geometry is created by providing alternating high points (e.g., at mast tops) and low points (e.g., at ground supports) for the net to be anchored to. Many variations of the overall anticlastic surface geometry are possible with cable nets, including shapes having very prominent high points, or those having more subtly rounded forms. Cable nets may be differently shaped in plan to accommodate various program layouts; in some cases perhaps a single cable net can be used to cover an entire space or in others several nets can be strategically "cut and pasted" together to form a composite surface geometry. There is in reality no limit to the shape variations that are possible as long as the basic requirement of anticlastic surface geometry is observed everywhere. (e.g. Ill. 11.29.)

Along the edges of the cable net between the various mast tops and ground anchorages we usually find another, larger set of cables, called *edge cables*, on to which the orthogonal cables of the net are fastened. The edge cables are typically curved to resist the series of tension forces coming from the cable net; much less frequently, very particularly curved bending-type elements may be used instead as the edge supports. (e.g., Ill. 11.30, 11.31 and Ill. 11.32, 11.33.)

Beyond the overall surface geometry, the detailed resolution of the individual components of a cable net structure is also an important aspect of their design. Given their visual prominence

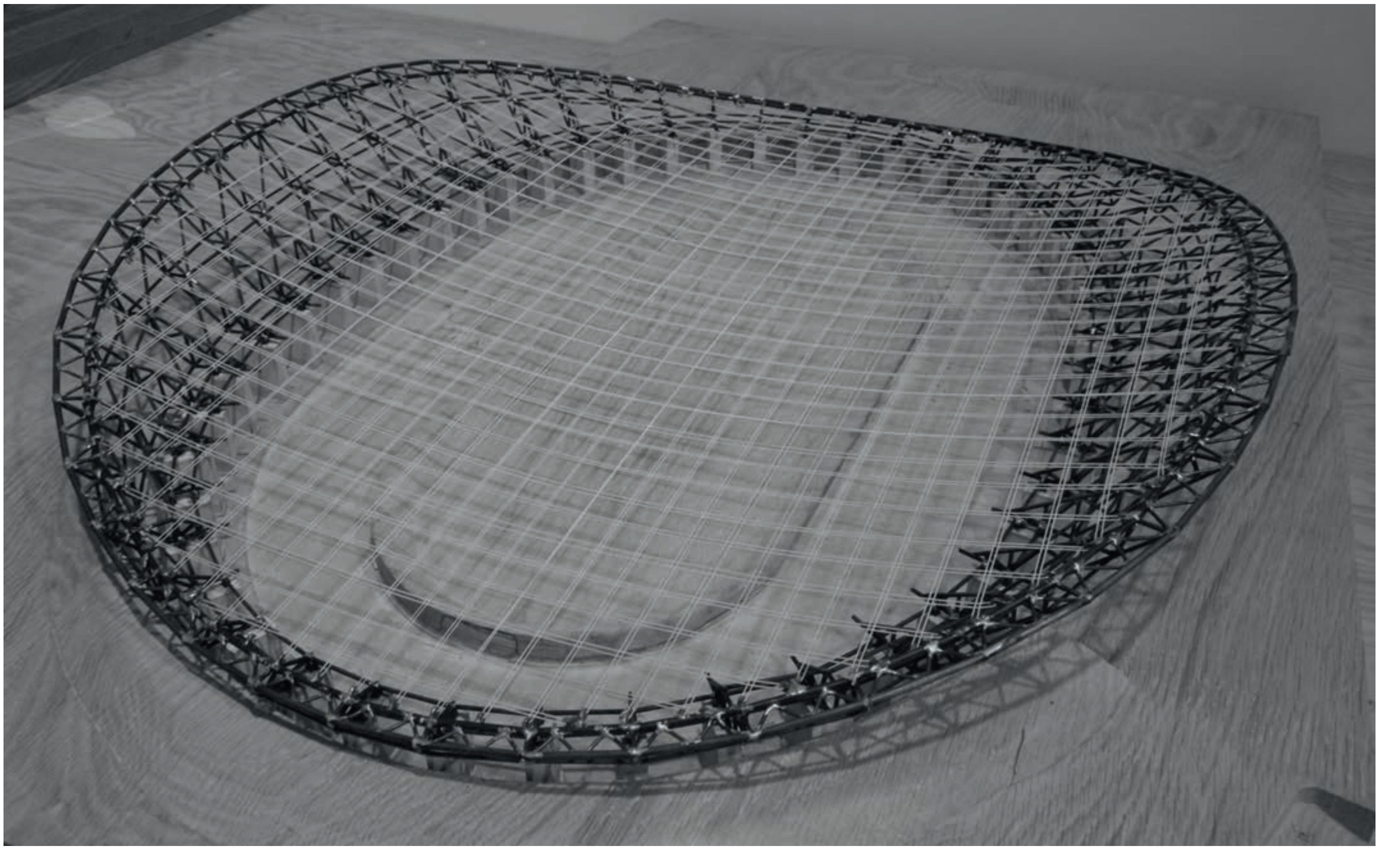


Illustration 11.30
Lee Valley VeloPark (Olympic Velodrome), London, UK (2011).
Curved truss provides cable net anchorage all around the roof perimeter; elevation of truss undulates so as to generate the double curvature of the cable net needed for stability.

Architect: Hopkins Architects. Structural engineer (for cable net): Schlaich Bergermann und Partner. Cornell model by Karl Pops and Alex Stojkovic.

Illustration 11.31
Lee Valley VeloPark.
Constantly varying height needed for support of large doubly curved cable net roof gives building its distinctive elevational profile.

Illustration 11.32
Yoyogi Olympic Stadium, Tokyo, Japan (1964).
Two halves of the suspension roof system of main swimming/diving stadium are vertically supported along the main centerline by two large 333mm (13in) suspension cables. At their lower, outer edges, the inward pull of the suspension roof halves is anchored by concrete arches lying down *almost* flat to the ground – but not quite flat, in order to still allow light to be admitted into the stadium from the sides. At right, an analogous but different, spiraling form is created for the smaller gymnasium facility; here the main suspension cable curves back upon itself around a central mast.
Architect: Kenzo Tange. Structural engineer: Kawaguchi & Engineers.

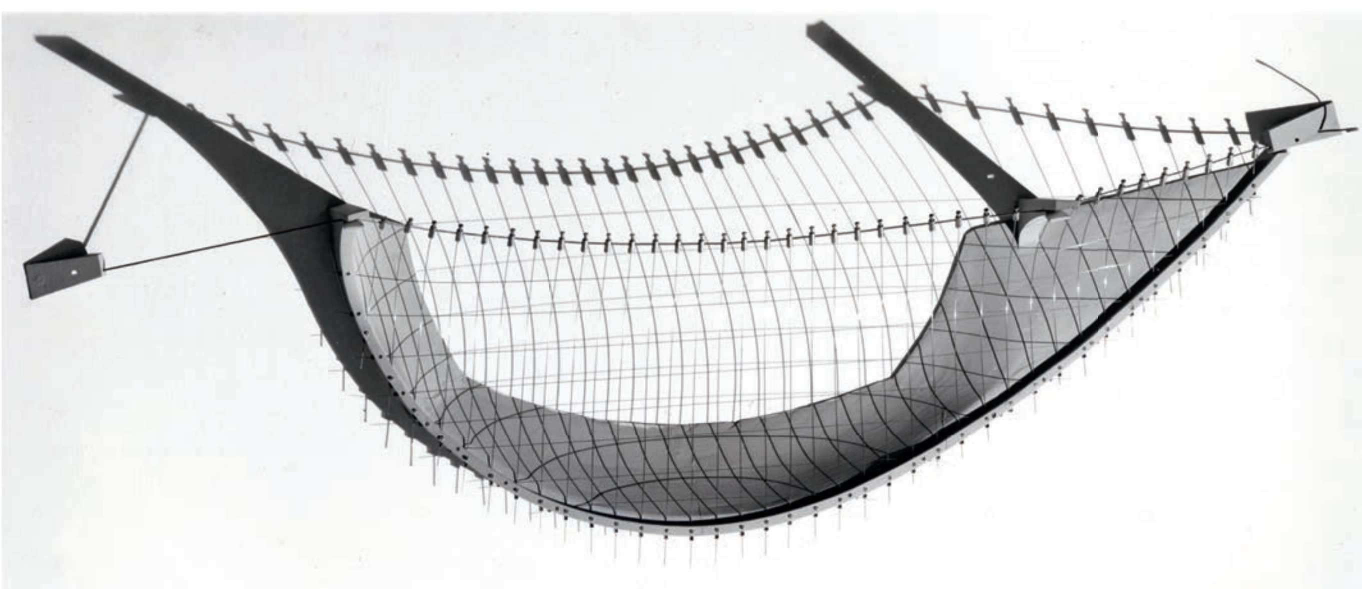


Illustration 11.33
Yoyogi Olympic Stadium.
AHO model by Elise Christie and Pål Biørnstad.

Illustration 11.34
Kempinski Hotel Airport Munich, Munich, Germany (1994).
Detail of hanging glass façade supported by steel wire cable net. Glass panes are clamped with plates of cast, stainless steel through which wire cables are threaded.
Architect: Murphy/Jahn Architectural Group. Structural engineer: Schlaich Bergermann und Partner.

Illustration 11.35
Cable net canopy, Expo '92, Seville, Spain (1992).
Doubly curved cable net, with sun-shading pieces of fabric inserted into the grid openings. Conical fabric covered structures are passive wind towers that draw hot air upward by applying the Venturi principle for wind blowing across the opening at the top.

and importance in establishing the surface geometry, the masts themselves often become distinctive elements in the design of such structures. Also, the anchorages are often at ground level and so must be thoughtfully considered. The connections between intersecting cables are repeated perhaps hundreds of times in a single net and these can become distinctive design components in their own right. (e.g., Ill. 11.34.) And, finally, the openings in the grid of a cable net are usually "filled" in some way in order to provide the occupants below with some form of shelter – whether this is by means of a fabric mesh as for the shading canopies of the 1992 World's Fair in Seville (Ill. 11.35, 11.36) or, as will be seen in Section 11.7 to follow, by means of acrylic glass panels used in Günther Behnisch and Frei Otto's Munich Olympic Stadium, which remains after almost 40 years one of the most compelling examples of what is possible both structurally and spatially using the cable net.

Illustration 11.36
Cable net canopy, Expo '92.
Detail of cable net with sun-shading pieces of fabric inserted into the grid openings.

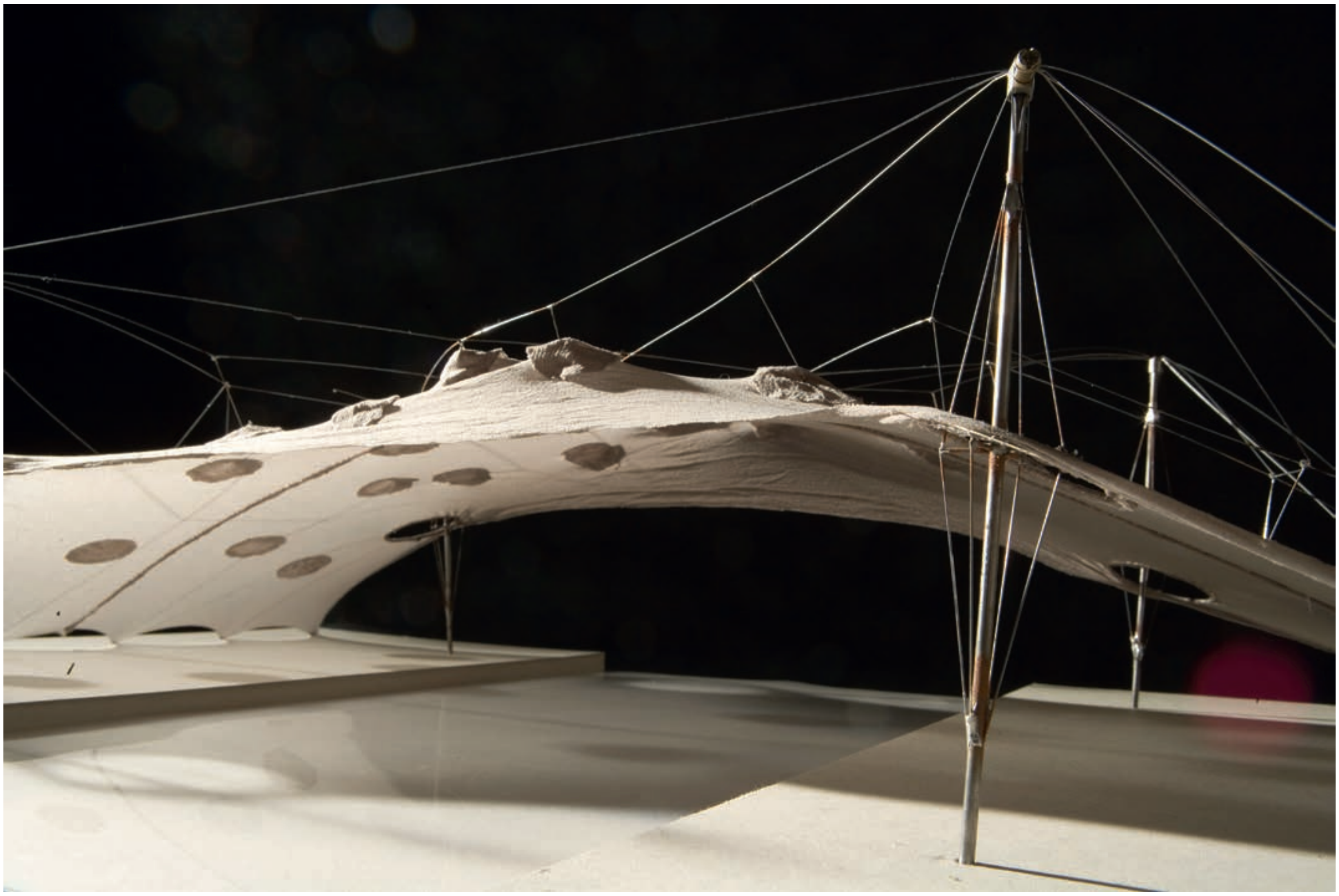


Illustration 11.37
Project for a tensile membrane structure covering the port of Bremen, Germany (1963).
Architect: Frei Otto. Model study by students at AHO.

11.7 Frei Otto – The Master of Cable Nets

During a period of time when most architects were occupied with building with such solid materials as concrete and brick, a man in Berlin spoke of membranes and designed transparent structures; he was trying to understand the structural laws of nature and make use of them for his constructions. As early as 1954, Frei Otto (1925–2015) caused a stir in the architecture world with his theoretical work on suspended roofs.[8] He produced innovative ideas throughout his life and is well established as one of the great architects, engineers, and visionaries of the last century. As head of the Institute of Lightweight Structures at the University of Stuttgart, he spent a lifetime researching and initiating lightweight structures, thereby minimizing the use of energy and materials and building in harmony with nature.

In the early 1960s the Bremen Port Authority asked Frei Otto to study the possibility of roofing over the vast Neustadt harbor basin, one of the busiest ports in Germany. It was hoped that this would allow the loading and unloading of ships to be carried out more quickly and make the work independent of the weather. The roof design that Frei Otto presented measured some 1500m by 380m (4920ft by 1250ft) and it was supported by 19 masts. (Ill. 11.37.) Its structure consisted of an upper, irregular, and very wide suspended cable net, which, in turn, served as a support for a lower, finer net with a regular mesh of 400mm (16in) spacing. The cable net formed a huge vaulted space over not only the storage area but also the navigable channel, and the quays and railway sidings. The roof covering proper was to consist of transparent PVC panels with bituminous weatherproof coating. At the time the project was presented, however, waterproof containers became more commonly used, and this was at least part of the reason that the Bremen plans went unrealized.

Certainly Frei Otto's best known project is the unique roof for the main stadium of the 1972 Summer Olympics in Munich. The winning design, done in collaboration with architects Behnisch & Partner, was based on an ambitious concept to continue and strengthen the landscape. (Ill. 11.38.) The project called upon some of the most notable designers of the period: aside from the

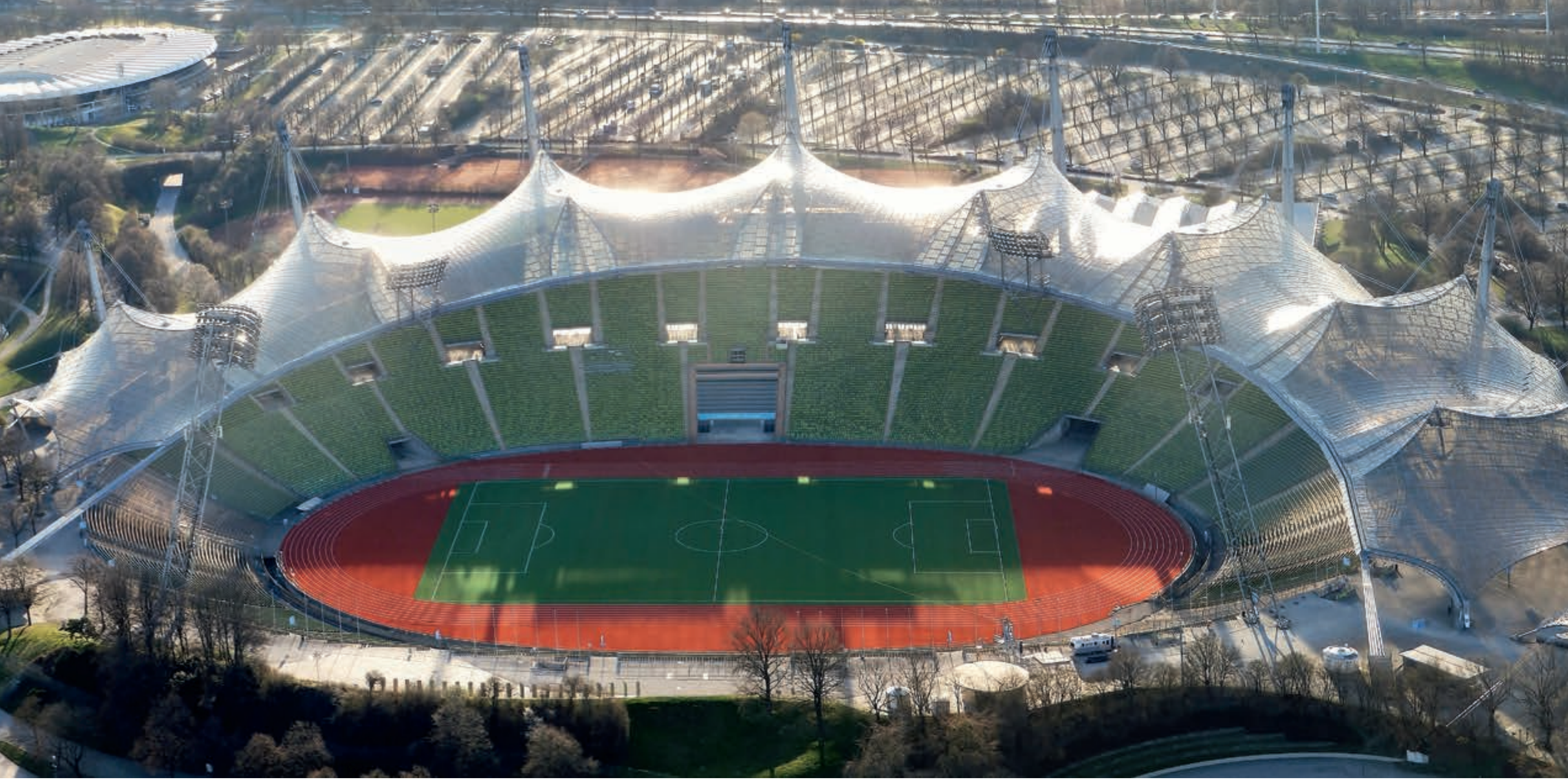


Illustration 11.38
Olympic Stadium, Munich, Germany (1972).
Overall form of an exceptional cable net canopy for stadium seating with acrylic glass covering. Multiple high and low point anchorages to the masts and the ground in alternating fashion produce the multiple doubly curved geometries needed for cable net stability throughout. Across the front edge, a set of larger cables anchors the cable net without interfering with spectator views.

Architect: Frei Otto with Günther Behnisch. Structural engineer: Leonhardt & Andrä.

Illustration 11.39
Munich Olympic Stadium.
Cable net with acrylic glass covering as seen from underneath. Fine grid is the cable net structure; darker, larger squares are the sealant between adjacent acrylic panels.

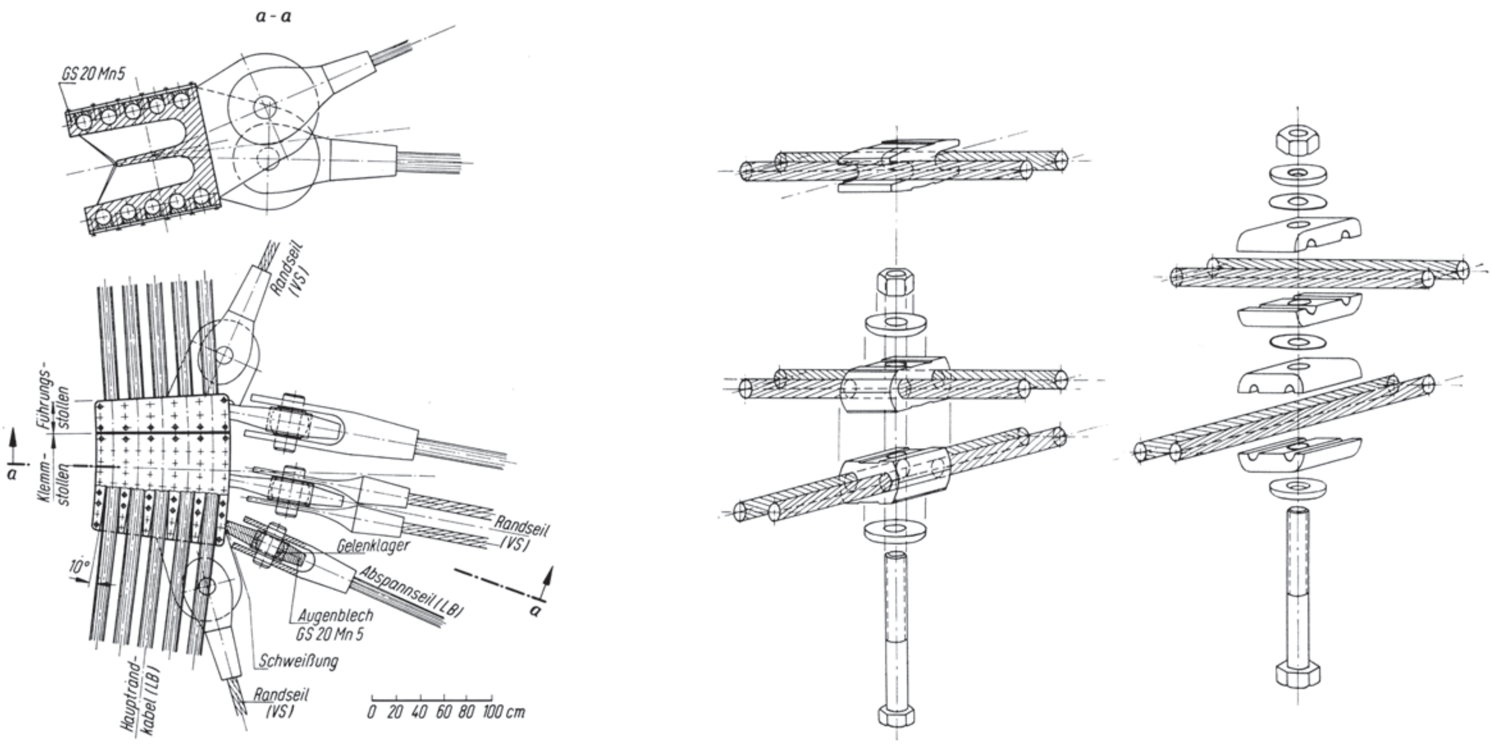


Illustration 11.40
Munich Olympic Stadium.
Detail of cable fittings.

architects Frei Otto and Günther Behnisch, the structural engineers Jörg Schlaich, Rudolf Bergermann, Fritz Leonhardt, and Heinz Isler were all involved.

A cable net structure was chosen to realize the architects' vision. Tulle, a thin, netted fabric usually found in women's stockings, was used for model shape-finding studies; however, here for the first time computer-aided calculations were applied to determine the exact shape of the tensile membrane. A form of structure consisting of nearly regular saddle-shaped surfaces framed by edge cables came close to the preferred design. The cables needed to hold the cable net in position are connected low to the ground at many anchorage points as well as to mid-air anchorages hung from the tops of a series of tall masts; in order not to disturb the open sightlines of the public, the masts are situated behind the grandstands. (Ill. 11.39.) The cable net's regular 750mm (29.5in) square mesh is covered with acrylic glass plates on flexible rubber supports. Light, transparent roofs such as this that are both open and yet still give sufficient protection for the spectators thus cover and connect various arenas of Munich's Olympic Park, making the sports facilities at once an extension of the natural landscape and into a collective meeting place for all nations.

Illustration 11.41
Temporary exhibition fabric tent (2014) in front of Institut du Monde Arabe, Paris, France.
As for the cable net forms, a stable fabric membrane surface must be doubly curved everywhere – here by mast-created high points and low point anchorages to the ground. In keeping with traditional Bedouin tents as were seen in Ill. 11.6, for this temporary exhibition installation natural camel and goat wool fibers were used to make the tensioned membrane, as opposed to contemporary structural fabric materials.
Architect: Oulalou + Choi Architectures (formerly KILO).

11.8 Fabric Membranes – A Tight Weave of Fibers

The cables of cable nets are spaced apart which, as we've seen, often results in the need to fill the "voids" with other surface materials. But very similar structural forms can also be made from fabrics, thereby creating a continuous surface that works simultaneously to carry load, provide stability, as well as to create enclosure. (e.g., Ill. 11.41.) In fact, we may think of a fabric membrane as a cable net in which the net has gradually become denser and denser while the cables have become thinner and thinner, resulting in a very tight weave of thin fibers running in orthogonal directions. Fabric membranes can then be understood to need to follow exactly the same double-curvature shape requirements that we have just described for cable nets. (Fig. 11.11.) In fact, structural fabrics (textiles and foils, see Section 5.8) along with cable nets together form what are typically called structural *tensile membranes*, which can be characterized as thin surface structures that are primarily carrying loads and being stabilized through tension stresses developed within an anticlastically curved surface. The overall structural behavior of cable nets and fabric membranes is indeed quite similar, and we can discuss forms and forces relating to tensile membranes in ways that apply to both variants.[9]

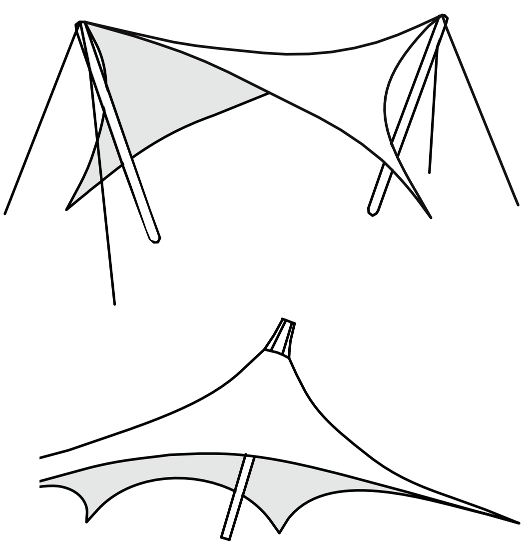


Figure 11.11
Two common examples of fabric membranes having anticlastic shapes.

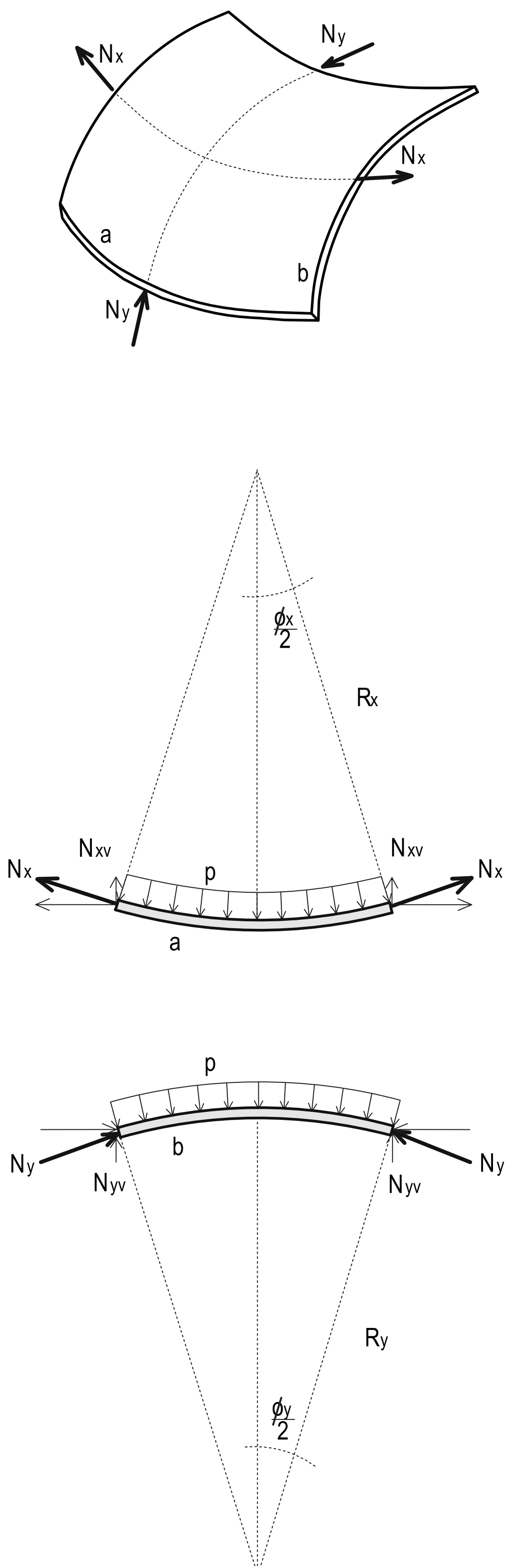


Figure 11.12
The geometry of the doubly curved membrane element. Free-body diagrams showing transverse load p and internal axial force reactions N_x and N_y.

In order to develop a better understanding of just what takes place within a fabric membrane and how such a structure works to carry load, we need to discuss the theoretical basis for an equation that reliably is able to predict its behavior. To begin this process, consider the equilibrium of a small elemental piece of a membrane surface – since it is part of that surface, the elemental piece will itself be of *anticlastic* curved shape. (Fig. 11.12.) When this element is acted upon by a uniformly distributed load p (kN/m^2, psi) over its surface, reactive forces are set up within the membrane. The free-body diagram of the element shown is cut along the principal curvatures of the surface so that only axial membrane forces N_x and N_y will be set up to resist the load (i.e., there will be no tangential shear forces along the edges of the element). Because the element is considered to be very small, we can think of its curvatures as being essentially constant along the element edge; i.e., we can consider the surface element to be effectively spherical in each of the two opposing principal directions. Furthermore, both of the curvatures are assumed to be quite small, although they are permitted to be different in magnitude one from the other; i.e., $1/R_x$ and $1/R_y$, where R is the radius of curvature.

We are now in a position to consider the equilibrium of the forces acting on this element in the vertical direction, thus expressing the essential relationship between the axial membrane forces N_x and N_y acting along the element edges of unit length, and the externally applied load p; i.e., the vertical components of the axial forces need to balance the external resultant load in order for the element not to move up or down and in order to have equilibrium, just as we have considered to be the case all along in this book. Without going through it here, it can be derived without too much difficulty[10] that the following algebraic interrelationship necessarily must exist between the transverse load and the membrane forces and its surface geometry:

$$p = N_x/R_x + N_y/R_y \tag{11.1}$$

This formulation of the equilibrium equation for a tensile membrane is an important result that will be seen to have general application to these types of surfaces, and for that reason it is called the *membrane equation*; as such, it bears some discussion of its implications. For one thing, the equation states mathematically that the axial forces per unit length in the two principal directions of a membrane are each proportional to their respective curvature radii.

Illustration 11.42
COP 22 Village, Marrakesh, Morrocco (2016).
Shading fabric canopy exhibits strong membrane curvatures everywhere, giving the surface stability against excessive deformations.
Architect: Oulalou + Choi Architectures.

Illustration 11.43
Our Dynamic Earth, Edinburgh, Scotland, UK (1999).
Steel mast-supported membrane stretched between curved ribs. Membrane can be seen to be doubly curved everywhere, but curvatures are modest, thereby increasing the magnitude of membrane forces and of the fabric strength required.
Architect: Michael Hopkins and Partners. Structural engineer: Arup.

For a given applied load p, therefore, an increase of radius (i.e., less surface curvature) will result in a proportional increase of the axial forces in the membrane. Taking this to an extreme suggests that large flat areas must particularly be avoided in a membrane since as R approaches infinity so must N, which will clearly cause material strength capacities to be exceeded. On the other hand, if the curvatures of a membrane increase, the forces decrease in the same ratio. From the material behavior studies presented in Chapter 5 (Section 5.2) we generally know that a decrease of internal forces means reduced deformations; hence, we can conclude that an increase of curvatures leads to stiffer tensile membranes. (e.g., Ill. 11.42, Ill. 11.43.)

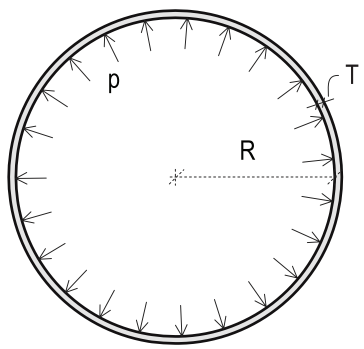


Figure 11.13
Section through an inflated spherical balloon.
The internal air pressure sets the membrane in tension.

In order to begin to develop a sense for just what this membrane equation implies in practical terms with regard to material capacities, we will begin by considering the very particular case in which the surface curvatures are made equal to each other ($R_x = - R_y = R$). The membrane equation states that the axial forces ($N_x = N_y = N$) will then be the same in the two directions, or:

$$p = 2N/R$$
$$N = pR/2 \qquad (11.2)$$

We can consider the implications of this simple equation in terms of a very familiar example: a spherical rubber balloon (Fig. 11.13) having a radius R = 100mm (4in) and an internal air pressure p which is only 10 percent higher than a typical air pressure of about 1000hPa (which is 0.1MPa or 0.1N/mm^2) (14.5psi). The resulting tension forces in both directions from equation 11.2 will be, therefore, N = pR/2 = (0.11N/mm^2 × 100mm)/2 = 5.5N/mm (31.4lbs/in). The tension stress in a rubber balloon whose thickness is t = 0.2mm (8/1000in) will thus amount to σ = (5.5N/mm)/0.2mm = 27.5N/mm^2 (3990psi), which is about the same magnitude as the tensile strength of rubber – and so this means that in this case the balloon is about to burst! It obviously doesn't take much extra internal pressure, therefore, to cause a balloon material's capacity to be exceeded. We will return once again to such so-called pneumatic membranes next in this chapter in Section 11.9, but first we need to spend a bit more time considering the implications of the general membrane equation (i.e., 11.1) to see what this can tell us about the much more common situation in an architectural context of having a thin membrane that has anticlastic surface geometry.

In fact, the general membrane equation

$$p = N_x/R_x + N_y/R_y \qquad (11.1)$$

actually has quite far-reaching consequences, for, as we will see eventually in Chapter 13, this very same formula can be applied to more than just tensile membrane surfaces. This is because nothing that was done in developing this equation from basic equilibrium principles was specific to the condition of a tension membrane; i.e., for its development (which, once again, can be found in this chapter's endnote 10) we are simply dealing with the equilibrium of forces and the doubly curved geometry of a surface element, but not with whether the forces in the membrane are in tension or compression. As a result, equation 11.1 can be applied to any membrane surface, whether it has compressive or tensile forces acting in it, or both.

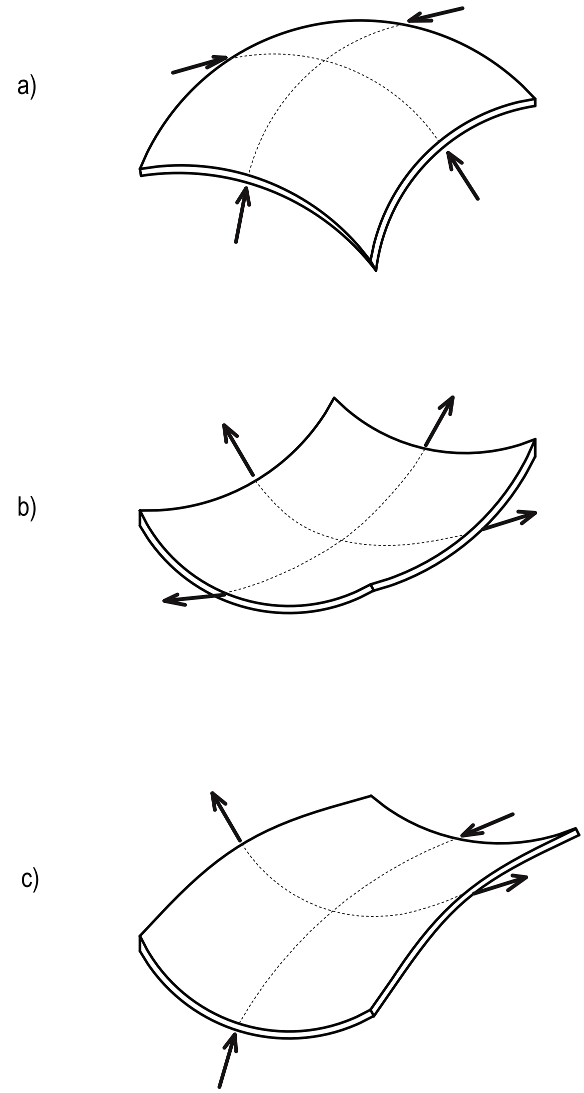


Figure 11.14
Variants of a double-curved surface. Surfaces (a) and (b) are synclastic with the two main axes curved in the same direction, while (c) is an anticlastic surface where the axes are oppositely curved.

Now, we can establish in the most general terms that doubly curved membrane shapes belong to one of three categories: in two of these cases the two main axes of the surface may both be curved in the same direction – resulting in either convex/convex or concave/concave surfaces (both are termed as having synclastic curvature, Fig. 11.14a,b) – and in the third case the two axes may be oppositely curved rendering the surface concave/convex (which is called an anticlastic surface, Fig. 11.14c). Thinking in terms of an applied gravity loading p acting on the surfaces of these various

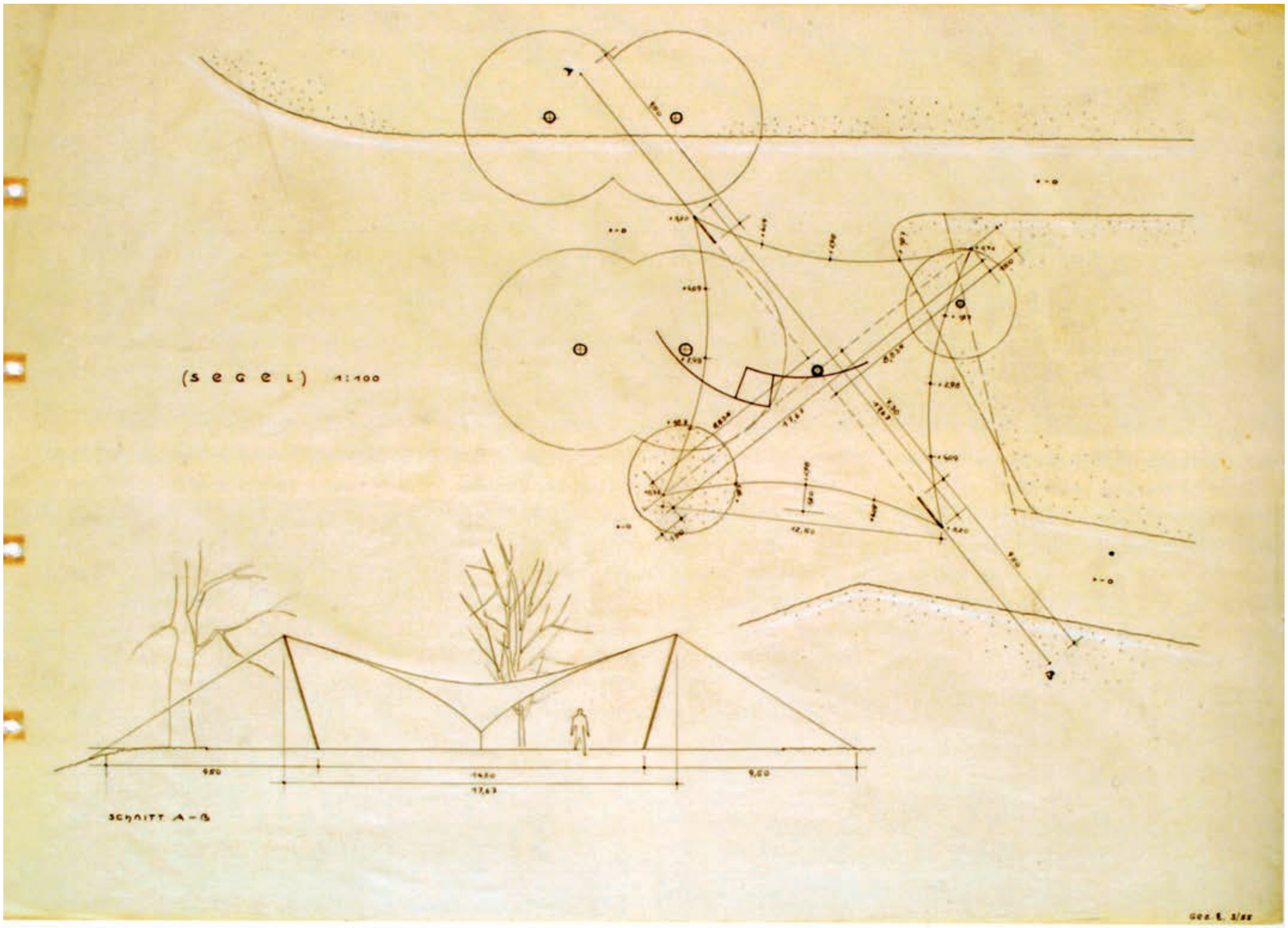


Illustration 11.44
Music Pavilion at the Federal Garden Exhibition, Kassel, Germany (1955).
Original drawing for this early four-point tensile membrane structure of 1mm (0.04in) thickness cotton fabric spanning 18m (59ft).
Architect and structural engineer: Frei Otto with Peter Strohmeyer.

categories of membrane, this would tend to cause compression/compression, tension/tension, or tension/compression membrane forces in the three different cases, respectively.

Since we have established earlier in this section that in order for a tensile membrane to have surface stability it must have anticlastic geometry (unless it is pressurized, as was the balloon that we just considered and as we will look at more extensively in the next section), then tensile membrane surfaces must be associated with the latter of these three member force categories; i.e., tension/compression. But this conclusion is still somewhat perplexing since it implies that transverse gravity loading will require that there is compression in one of the two principal directions of the tensile membrane, which we know to be an impossible condition (i.e., we know very well from experience that this thin flexible surface has zero capacity against buckling). The essential realization at this point, therefore, is that in order for a tensile membrane to work by means of tension in both directions there is clearly a need for it to be *pre-stressed*. Moreover, the pre-tensioning of a tensile membrane must always be large enough to prevent it from ever going into compression under loading and thus becoming slack; i.e., the pre-tensioning forces in the convex direction (the direction of "compressive" forces), which will inevitably be reduced under the external loading of the surface, must never be allowed to reach zero.

In light of these conclusions, we will next study the configuration and behavior of a relatively simple saddle-shaped fabric membrane of shallow profile covering a square plan. The specific example that we will look at is the Music Pavilion that was designed by Frei Otto and built for the Federal Garden Exhibition in Kassel, Germany, in 1955 – which, to put things into context, was quite early in terms of the history of development of structural fabric membranes. (Ill. 11.44.) This tensile membrane had a so-called four-point configuration with two high points at the mast-tops and two low-point supports at ground level anchorages, and with its geometry describing an anticlastic surface of hyperbolic-paraboloid shape (this specific geometry will be discussed in Section 13.7 in the context of rigid shells, but for now it will be enough to say that sections cut through this surface in the principal directions will both be parabolas).

Before proceeding with this example, we should note that a precise description and calculation of the forces in a tensile membrane is actually a more complicated matter than we have let on so far, however: i.e., deformations are substantial, and the membrane forces are actually dependent on the final shape of the surface *after* deformation and on its orientation in space. In other words, the structure is non-linear (i.e., the more the structure deflects, the larger are the forces, which further increases the deflections, and so on) and its precise and detailed analysis is well beyond the scope of this book. For approximation purposes, however, we can nevertheless define the surface geometry from that corresponding to an initial state of uniform surface tension and study the resulting behavior of the membrane, all this with the sole intention of developing a fundamental understanding of a tensile membrane's main design constraints.

The radii of the two curvatures that should be used for the calculations with the membrane equation 11.1 are, strictly speaking, the radii right at the apex points (i.e., at the top of the "arching" direction, and the bottom of the "suspended" direction), but for an approximate calculation for a shallow membrane such as the one being considered here it is enough to consider these radii as being constant throughout the two principal curvatures (i.e., even though the profile is really parabolic). In an initial state where there is no transverse load and only pre-stressing acts (i.e., p = 0), the membrane equation 11.1 tells us that

$$T_x/R_x + T_y/R_y = 0$$

where the axial forces are tensile and given the symbol T. We further will restrict this analysis to the special case with the two curvatures being identical, but opposite to each another, i.e., $R_x = -R_y = R$. This yields

$$T_x/R + T_y/{-R} = 0$$
$$T_x - T_y = 0, \text{ or}$$
$$T_x = T_y = T_0$$

Therefore, when no external transverse loads act on the surface, the membrane must be in a state of uniform tension T_0. Such a pre-stressing may be relatively easily applied to a membrane of the Kassel pavilion-type by means of a simultaneous and uniform tightening of all four edge cables.

When an external load w acts on the membrane, however, this will induce additional tension force T_s in the "suspended" membrane direction and a reduction T_a of the tension force in the "arched" direction. The magnitude of these tension forces can be found by help of the membrane equation, but it is more convenient to directly consider the load to be shared equally between narrow strips of the membrane running in the two directions. Recalling that the horizontal force reaction at the support for a cable to be $H = wL^2/8f$ (as was found in Section 11.3) and considering the difference between the magnitude of the horizontal force reaction H and the tension membrane force T that we are looking for here to be quite small since this is a very shallow membrane, we can approximate that the tension force in the membrane per unit length is

$$T_s = T_a = (w/2)(L^2/8f) = wL^2/16f$$

since, as previously mentioned, we consider half of the external load (w/2) to be supported in each direction.

When both the external loading *and* the necessary pre-stressing act at the same time, then, we find the total tension forces in the two principal directions in the membrane to be

$$T_{s\ Total} = T_0 + wL^2/16f \leq \text{material capacity}$$
$$T_{a\ Total} = T_0 - wL^2/16f \geq 0$$

For the tension force in the arched direction to be positive at all times, which is the same as saying that the membrane must be prevented from going slack, an approximate minimum magnitude of the pre-stress can be considered to be:

$$T_{0\ min} = wL^2/16f$$

which means that the pre-stressing forces in a membrane of this type (again, for approximation purposes here only) must be at least as large as the tensile forces (T_s and T_a) generated in each direction by the external loads. If they are exactly equal (not considering

Illustration 11.45
Kassel Music Pavilion.
Photo of fabric membrane pavilion.

safety factors) we find, moreover, that the maximum membrane force per unit width T_T (in the suspended direction) to be

$$T_T = T_0 + wL^2/16f = wL^2/16f + wL^2/16f$$
$$T_T = wL^2/8f$$

whereas the membrane forces in the other direction come close to zero, but should never really reach that point.

So what does this analytical development and its conclusions really mean physically? It is evident that in order to make these structures work we need to pre-stress the tension membrane to a considerable degree. Moreover, what we have seen and is generally true is that at least one-half of the potential load-carrying capacity of a tensile membrane is typically "spent" on dealing with the pre-stressing that is required to make it function. Also, the design of the arched direction of the membrane will need to consider the forces produced by the pre-stressing (determined here to be of the same magnitude as the forces produced by one-half of the full gravity loading, $wL^2/16f$) plus that due to maximum wind suction which causes uplift and thus increases the tension in that direction.

In the case of a membrane having the same geometry as that of the Federal Garden Exhibition's Music Pavilion, we can use this analysis in order to establish what the approximate maximum membrane force per unit width would be under typical loading. If the (wind and gravity) transverse load is taken to be $w = 0.3kN/m^2$ ($6.3lbs/ft^2$), the span L = 18m (59ft) and the sag f = 3.0m (10ft), we will have

$$T_T = wL^2/8f = [0.3kN/m^2\ (18m)^2]/[8\ (3.0m)] = 4.1kN/m = 4.1N/mm\ (23.4lbs/in)$$

We should put this result into context. Today, structural fabrics are made of glass fibers (and polyester fibers); a strip of typical glass fiber fabric, for example, has a tensile strength somewhere in the order of 16N/mm (91lbs/in), which is well above the maximum membrane force per unit length that we just calculated. It should be acknowledged that the range of tensile strengths for different fabric materials is great and that there is commonly a difference between the strengths of the fabric in the warp and weft directions (see Section 5.8); nonetheless, these preliminary calculations are good enough to give the general sense that such a tensile membrane system is more than capable of carrying the anticipated loads, which is really all that we were interested in demonstrating here. The actual Federal Garden Exhibition's Music Pavilion membrane was made of cotton, a fabric which today is considered too weak to be of interest for membrane structures. But the relatively small magnitude of the maximum tension stress that we just calculated suggests that even such a cotton membrane would have been adequate to carry the applied loads reasonably safely; of course, the pavilion's successful operation during the 1955 exhibition provides irrefutable confirmation of this. (Ill. 11.45.)

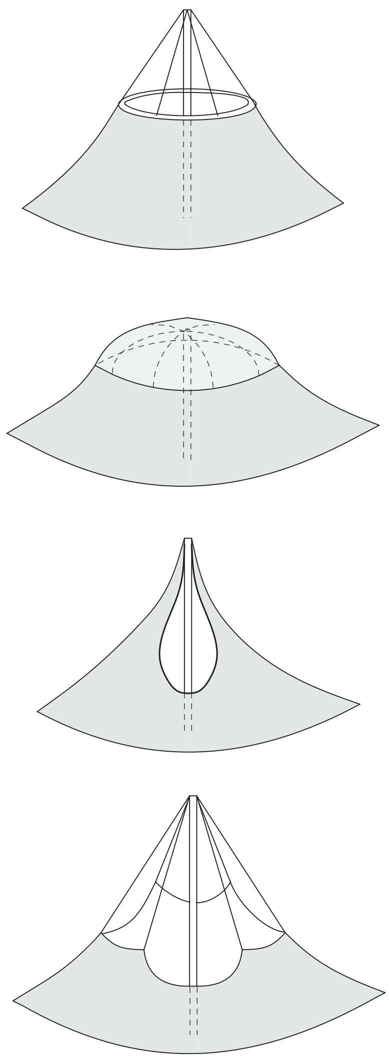


Figure 11.15
Commonly used detailing of fabric membrane supports observes the need for distributing the concentrated force to a larger membrane length, thus reducing stresses in the fabric at this point.

To summarize this discussion of the structural behavior of shallow, anticlastic tensile membranes, then, the following general principles can be stated:

- The membrane must be pre-stressed, commonly by tightening its edge cables.
- Under gravity loading, the suspended and the arched directions of the membrane are considered to share the load equally.
- When gravity loads act, the tension in the suspended portion of the membrane increases from the initial pre-stress, while the tension in the arched portion decreases. The latter must never be allowed to go slack.
- The maximum membrane forces are found in the suspended direction of the membrane (assuming that wind suction is less than gravity loading), and are a sum of the pre-stressing forces and the forces produced by gravity loads.

Finally, we need to consider what it takes to create the necessary doubly curved shape of fabric structures and how some of their particular design detailing can be attended to. As we have seen in the preceding example and as has been previously discussed, fabric membranes are typically supported and given their shape by their connection to the tops of masts and to low point anchorages as well as to cables at the edges of the membrane. Care should be taken to avoid a direct connection between the fabric and a narrow point of support, however, because of the concentration of membrane forces that would result; i.e., fabrics may easily be punctured unless the transition of forces to them takes place over a large enough portion of the membrane. As a result, it is typically necessary to provide rings, cable hoops or loops, or other elements at mast tops that have the capacity to distribute the forces over an adequate membrane length. (Fig. 11.15, e.g., Ill. 11.46.)

As for the edge cables needed to anchor the sides of a tensile membrane, we are in a position of being able to predict their magnitude and, just as importantly for our purposes here, see what shape these elements must have. If we consider an initial situation with pre-tensioning as the sole load on the system, both primary directions of the membrane are subject to tension forces T_0 per unit length. This typical pre-stressing of the membrane will be supported by edge cables which will adjust their shape into circular segments because of the imposed loading pattern; i.e., the loads acting on the edge cables are the force reactions from the

Illustration 11.46
Rocca di Frassinello vineyard, Tuscany, Italy (2007).
Connection of fabric membrane to high point cable support is achieved by specialized steel "cap" that distributes the concentrated tension force into a tension stress level that the membrane is able to accommodate.
Architect: Renzo Piano Building Workshop. Structural engineer: Favero & Milan.

membrane in the two perpendicular directions and the resultant distributed load will be radial to the circular edge cables with a magnitude of T_0 per unit length.[11] (Fig. 11.16.) Since circular cables are funicular structures for radial loads, it can be established that the equilibrium between the loads and the tension forces S_0 in the edge cables can be expressed as

$$S_0 = T_0 \times r$$

where r = the curvature radius of the edge cables.

If the radius r of the edge cable decreases, thereby introducing more curvature, the tension forces S_0 in the edge cable will also decrease. Once again, in tension structures such as the ones we are considering, the curvatures should be made to be significant in order to reduce forces and thus the required structural dimensions; the result will be an efficiency of material and a visually more delicate structure. It should be noted that when external loads act on the membrane in addition to the pre-stressing that we have just considered, the shape of the edge cables will tend to approach parabolas rather than circles. Regardless, the edges of a tensile membrane are typically distinctively curved inward; this is yet another of the many curved surface geometry characteristics that distinguish such structures from the traditional rule of the straight line.

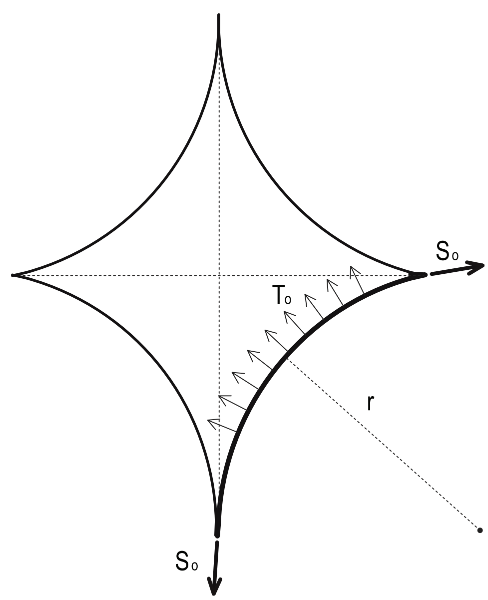


Figure 11.16
Geometry and load on membrane edge cables.

Illustration 11.47
BC Place Stadium, Vancouver, Canada (1983).
Air-supported membrane roof bulges upward; sheltered over 60 000 spectators for the opening and closing ceremonies of World's Fair (1986) and Winter Olympics (2010).
Architect: PBK Architects. Structural engineer: Geiger Berger Associates (roof), PBK Engineering (building).

11.9 Pneumatic Structures

The word "pneumatic" refers to devices or structures that in some way operate by help of air pressure. The common balloon is perhaps the quintessential example of such a "structure" that takes its form and volume from internal air pressure, and one that we investigated briefly in the previous section in order to get somewhat familiar with the membrane equation and to see by just how much the rubber membrane is stressed in normal conditions of inflation. For the balloon, an internal air pressure which is higher than the external atmospheric pressure pre-stresses and stabilizes the extremely light rubber membrane so that its roughly spherical shape is maintained while being able to be subjected to moderate external loads. We are well aware, however, that there are certain problems associated with the stability of balloons: they typically leak their internal air pressure and deflate over time, and concentrated loads (point loads) tend to deflect the rubber membrane excessively and may rather easily puncture it.

Air pressure likewise offers unique and innovative shape-making potential for structures in the context of architecture, and certainly one of their most striking qualities is their remarkable lightness and transportability. Just after World War II a breakthrough was made in applying these advantages to inhabitable shelters in the form of several dome-shaped inflated membranes housing radar antennae; i.e., the so-called radomes. Built in various climates for military purposes, these structures paved the way for the application of air-supported membranes to the broader world of architecture in the decades that followed; e.g., the 1970 Osaka World's Fair is perhaps the best known for some of its innovative and highly distinctive inflated structures, such as the United States and Fuji Film pavilions. It must be acknowledged, however, that since then the forms of pneumatic structures have been slow to be accepted into mainstream architecture for buildings of major and lasting significance, with the roofs for large sports stadia and temporary exhibition structures largely seeming to be their accepted place. Nevertheless, as we shall see shortly, interesting innovations and experiments continue to take place with pneumatic membranes.

It is common to classify pneumatic structures according to two main types; these are:

- *Air-supported structures* that consist of a single membrane enclosing a building space. The stability of the membrane depends on a (slight) air pressure differential between the outdoor (atmospheric) pressure and the indoor pressure, the latter being higher than the former. The most common building types that use this principle are sports stadia and small to medium-sized temporary buildings for various purposes. Because of their lightweight nature, and the fact that air is effectively providing the support for the surface, quite large spans can be achieved. (e.g., Ill. 11.47.)
- *Air-inflated structures* that make use of inflated structural elements that are individually created by pressurized air. With this approach the air pressure of the occupied space remains atmospheric. Air-inflated structures form structural elements like beams, slabs, arches, and vaults and carry external loads in a more traditional manner than do air-supported structures. However, this type of system calls for much higher air pressures in the structural elements than that which is necessary for air-supported structures. (e.g., Ill. 11.48.)

  Hybrid structures also exist that apply and combine the techniques and advantages of both principles.

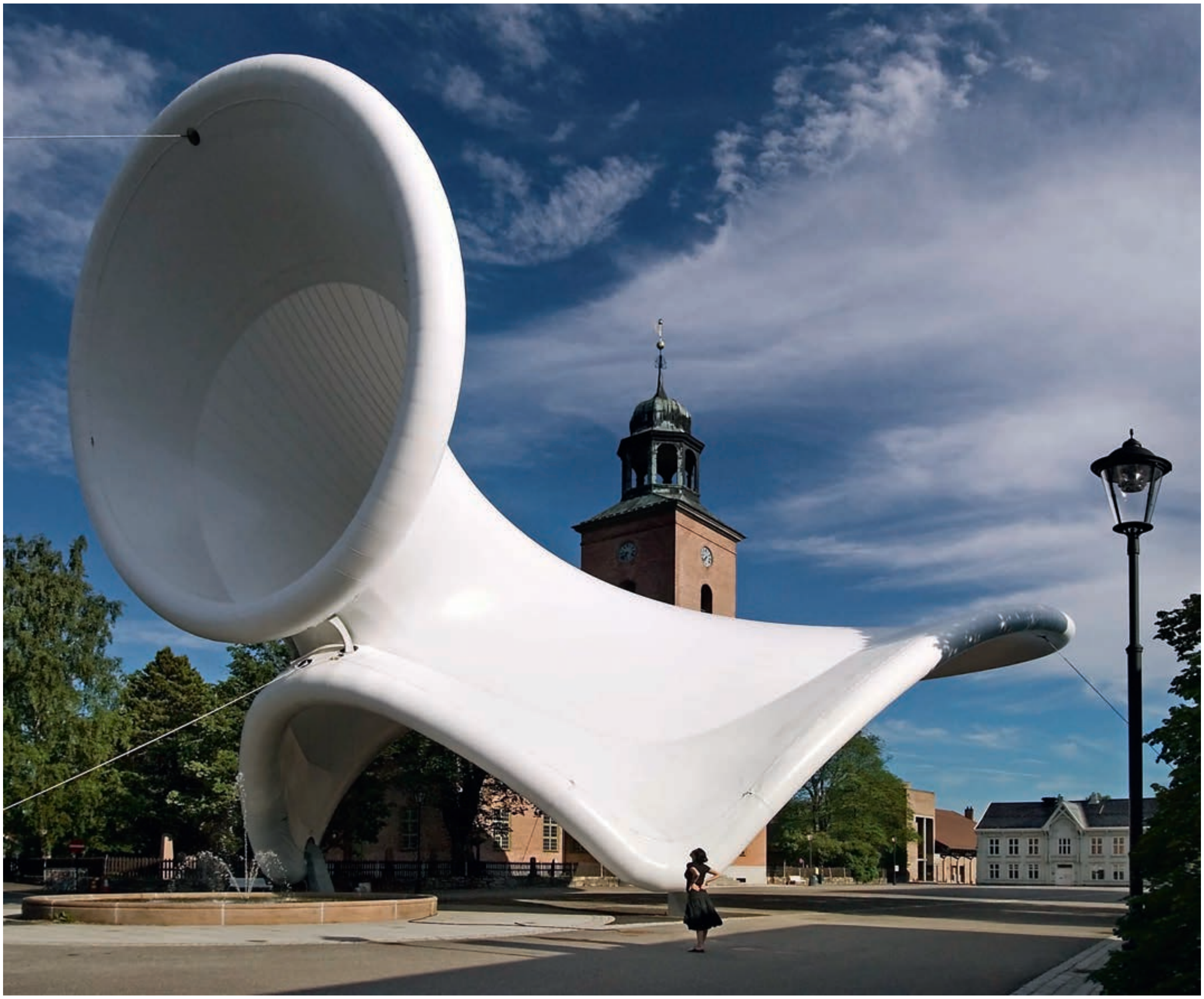


Illustration 11.48
"Tubaloon," Kongsberg, Norway (2006).
Main stage for Kongsberg Jazz Festival. A doubly curved tension membrane is stretched between the edges of this hybrid structure. Hidden, curved galvanized steel tubes strengthen air-inflated PVC membrane tubes along the edges. These elements are mutually supportive: while the steel tubes brace the inflated membrane tubes, they are simultaneously being braced against buckling by the membrane.
Architect: Snøhetta. Structural engineer: Airlight SA.

Obviously, a common aspect of both these types of inflated structures is that their internal air pressures must be large enough to keep the membrane in tension throughout when external and dead loads act; i.e., if the membrane were to try and enter into a state of compression, this would inevitably result in wrinkles and folds forming in the very thin fabric and a loss of load-carrying capability. To predict whether a membrane experiences net external pressure over its entire surface or both pressure and suction forces in various places due to wind blowing over it is not an easy and straightforward issue, and is something typically best left to sophisticated wind tunnel studies. What we can do here, however, is to look at and compare the necessary air pressures for the uniform loading of representative examples of each of the two pneumatic structure types, with the objective of developing an understanding of their fundamental similarities and differences.

We will first look at an *air-supported*, cylindrical membrane. (Fig. 11.17.) In accordance with the membrane equation (11.1), the air-pressure difference between the internal and external conditions p_{net}

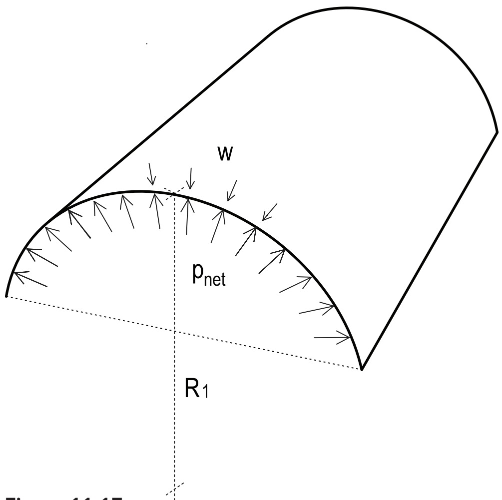


Figure 11.17
Air-supported cylindrical membrane.

is related to membrane tension per unit length in both directions, T_1 and T_2, and their respective radii according to

$$p_{net} = T_1/R_1 + T_2/R_2$$

This cylindrical geometrical shape has orthogonal radii R_1 and $R_2 = \infty$, and so the last part of the equation approaches zero. This means that

$$p_{net} = T_1/R_1, \text{ or}$$
$$T_1 = p_{net}R_1$$

When the membrane is also subjected to an external load w per unit area of the surface, additional surface forces are obviously produced. Such additional loads push down and tend to compress the pre-tensioned membrane. Assuming that we control these additional loads in such a way that the inflated membrane always retains its original, convex shape produced by the internal air pressure, we can determine the *effective* compressive forces produced in the membrane by once again using the membrane equation. Their magnitude per unit length will be

$$C = wR_1$$

For stability of form it is obviously critical that the membrane always retains its tension, which means that the unit tension force T_1 must always be larger than, or equal to, the force C tending to create compression. This means that

$$p_{net}R_1 \geq wR_1 \text{ , or}$$
$$p_{net} \geq w$$

Thus, according to this calculation (that, once again, greatly simplifies reality), the necessary pressure for an air-supported membrane must be at least of equal magnitude to that of the external distributed load per unit area. If, as an example, the membrane is to support a significant external snow load w of $2.5kN/m^2$ ($52.5lbs/ft^2$), the necessary pre-tensioning air-pressure differential (ignoring typically small membrane dead weight) is of equal magnitude, namely $0.0025N/mm^2$ (0.36psi), a mere fraction of normal atmospheric pressure at sea level of $0.1N/mm^2$ (14.7psi).[12] The pressure necessary to keep an *air-supported* membrane stable is, thus, surprisingly low. For these membranes a differential pressure between inside and outside of about $0.001–0.0025N/mm^2$ (0.15–0.36psi) is considered normal. Air-supported structures are thus mostly of the dome or vault type with a convex shape resulting from a larger than atmospheric air pressure in the interior. (e.g., Ill. 11.49.) Maintaining this pressure differential, however, is critical and can present its own challenges: air-supported structures typically need to be furnished with two sets of doors in order to create an airlock; even so, some air leakage is to be expected and a continuous operation of fans in order to maintain pressure is necessary. In regions that are prone to the occasional very heavy snowfall, this will typically have to manually be removed from the roof, a not-so-insignificant amount of exercise.

We will next consider the basic requirements of an *air-inflated* structure. Let us imagine that we are spanning a distance L using a series of tightly packed, side-by-side inflated cylindrical beams of radius r. (Fig. 11.18.) These "beams" are necessarily closed at both ends in order to maintain their internal air pressure. When air pressure p_i is applied, a resultant force acts on the circular end of the beam of magnitude

$$N = p_i \pi r^2$$

This force causes tension stresses σ_t in the membrane parallel to the length of the inflated structural element. Since the force is distributed over the entire circumference, and by designating the membrane thickness as t, we find that

$$\sigma_t = p_i \pi r^2/2\pi rt = p_i r/2t$$

Illustration 11.49
United States Pavilion, Expo '70, Osaka, Japan (1970).
Air-supported membrane can result in a remarkable span of column-free space.
Architect: Davis-Brody. Structural engineer: David Geiger.

If the inflated cylindrical beam is assumed to support its proportionate share "q" of an external load w per unit area, we can say that q = w × 2r since 2r equals the beam's width. Applying what we learned in Chapter 7 about the bending behavior of beams, the maximum bending moment along the length of this simple-span inflated beam is, therefore

$M = qL^2/8 = wrL^2/4$

Since for a thin-walled tube the section modulus $S = \pi r^2 t$, the maximum compressive (and tensile) bending stresses caused by the external load are thus given by

$\sigma_c = M/S$
$\sigma_c = (wrL^2/4)/(\pi r^2 t) = (wL^2)/(4\pi rt)$

Now, if the external load is not to cause the membrane to wrinkle in the upper parts of the beam, the pre-tensioning stresses σ_t must always remain larger than the compression stresses caused by the external load. This means that

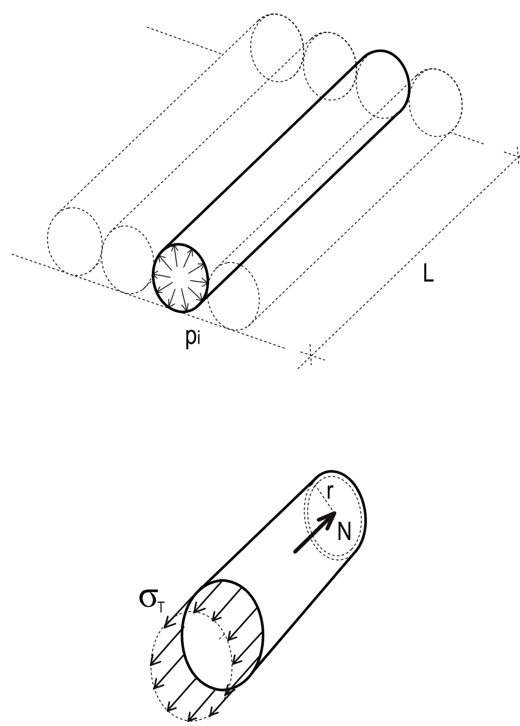


Figure 11.18
Air-inflated tubes for a roof structure.

Illustration 11.50
Fuji Group Pavilion, Expo '70, Osaka, Japan (1970).
Series of horseshoe-shaped inflated membrane tubes tethered together stood out for its original form and structural system.
Designer and structural engineer: Kawaguchi & Engineers.

$\sigma_t \geq \sigma_c$, or
$p_i\ r/2t \geq (wL^2)/(4\pi rt)$
$p_i \geq (wL^2)/(2\pi r^2)$

As a specific example, let us also assume that the beam height to span ratio is h/L = 2r/L = 1/10. This yields L = 20r. If we substitute this ratio into the equation above, the necessary air pressure in this particular case yields

$p_i \geq 64\ w$

The simplifications that we implicitly made in arriving at this result prevent us from directly comparing it with that for the air-supported membrane (where p_{net} simply needed to be ≥ w), but we can certainly observe that in the present case of an air-inflated structure the pressure differential of 64 times the distributed load is substantially larger than that for an air-supported structure. If, as we considered before, the gravity snow load acting on the inflated membrane is w = 2.5kN/m^2 (52.5lbs/ft^2), the necessary internal air pressure becomes $p_i \geq 64 \times 0.0025$N/mm^2 = 0.16N/mm^2 (23.2psi) or a pressure differential of 1.6 times atmospheric pressure – certainly quite a difference from the small fraction of standard atmospheric pressure that was needed for the air-supported membrane. Moreover, above and beyond the much higher required internal air pressures that must be provided, the resulting membrane tension forces will be substantially larger in an air-inflated membrane, requiring a thicker/stronger membrane material. (e.g., Ill. 11.50.)

Despite such clear differences between the air pressures necessary to stabilize the two basic types of pneumatic structures, as a whole these structural forms do share many common attributes when it comes to their being used in the context of buildings. The typical advantages of air-supported structures are a lower initial cost than for conventional buildings, and a potential lowering of operating costs due to their relative simplicity of design. They are easy and quick to assemble or to dismantle and relocate, and offer unobstructed

Illustration 11.51
Montreux Parking Garage, Montreux, Switzerland (2005).
There are ways to exploit the benefits of air-inflated structures, increasing their versatility while dramatically reducing the necessary air pressure. Here, a hybrid beam structure is designed in which steel profiles acting as chords interact with an inflated membrane to create highly effective long-span beams. So-called Tensairity® beams of glass-fiber membranes span 28m (90ft) across a parking space.
Architect: Lüscher Architekten. Structural engineer: Mauro Pedretti.

open interior space since there is no need for columns. Also among the design decisions to be made is the particular challenge of how or whether to make the inherent lightness of a pneumatic structure visible, which partly may depend upon the chosen membrane material; these are commonly vinyl-coated (PVC) polyester or PTFE-coated glass-fiber fabrics (see Chapter 5), each having their own particular mechanical and light-transmission properties.

Among pneumatic membranes' disadvantages is the risk of collapse if air pressure is lost or if the fabric is compromised in some way; although a collapse is fortunately a typically slow process, with ample time available to evacuate the space and/or to supply emergency power to maintain the original pressure differential. Pneumatic membrane structures' shape and remarkable lightness typically turn wind loads into a major challenge; in combination with the permanent uplift forces resulting from the internal pressurization, membranes must as a result be carefully secured to the ground by heavy weights or by means of tension anchors. Furthermore, with regard to indoor climate control it needs to be acknowledged that, even with a second, inner membrane lining, pneumatic structures cannot attain the insulation values of hard-walled structures. Hence, increased heating/cooling costs must be expected in such structures and this can also be considered to be a disadvantage from an overall environmental sustainability perspective. Finally, tensile membranes are known to deteriorate more quickly than conventional building materials do, and they need to be replaced at regular multi-year intervals. Accounting for all these short- and long-term pros and cons of the pneumatic structure is certainly not a straightforward matter; how much any one of these issues factors into a decision to use this structural strategy will vary from case to case.

It should be mentioned before closing this chapter that new and innovative applications for air-inflated membranes continue to be developed for building components. For example, the Tensairity® system that was illustrated at the Montreux Parking Garage uses an inflated membrane in combination with steel top and bottom chords for highly effective and distinctive long-span beams. (Ill. 11.51, 11.52, 11.53.) Also, translucent foils (ETFE), sometimes

Illustration 11.52
Montreux Parking Garage.
View from below canopy; series of air-inflated membrane beams span across space; doubly curved fabric membrane spans between them.

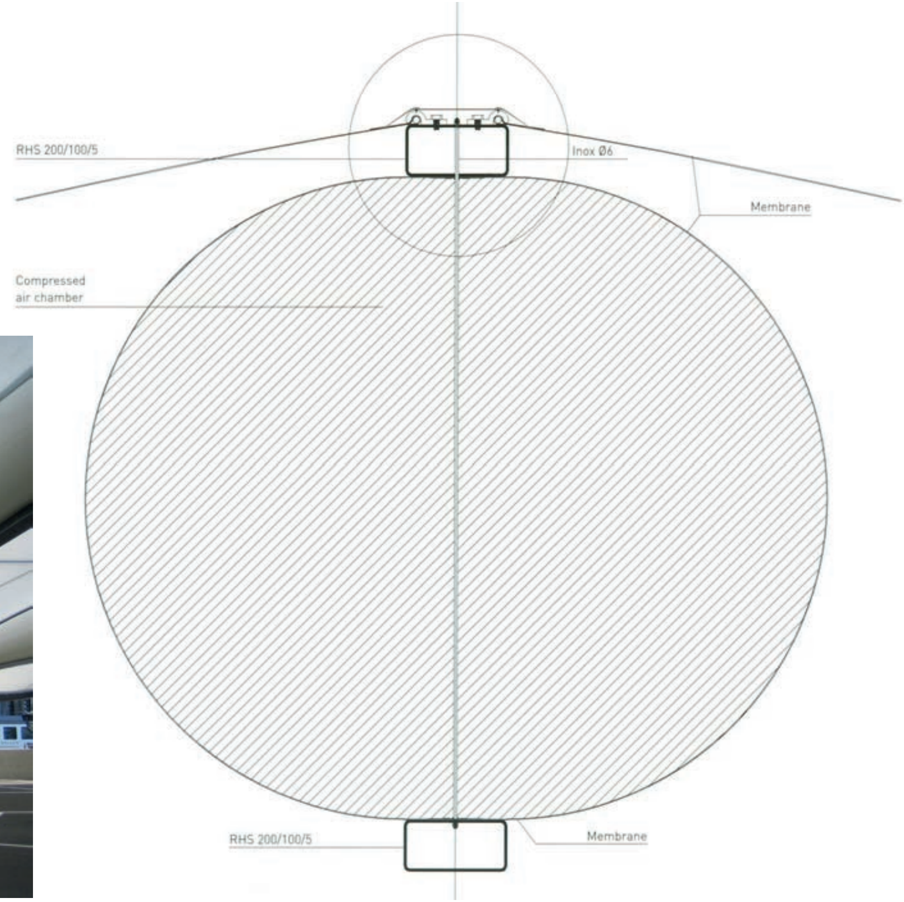


Illustration 11.53
Montreux Parking Garage.
Section drawing of air-inflated beam with top and bottom steel chords in the form of two RHS steel sections.

combined with artificial lighting, offer new and intriguing possibilities for architectural expression as smaller scale "bubble" or "pillow" cladding elements in such buildings as the Allianz Arena (Ill. 5.47), the Beijing Water Cube (Ill. 9.40), and the Eden Project (Section 13.1). All things considered, the unique qualities offered by pneumatic structures for exploiting stiffness, lightness, material efficiency, translucency, and color make it reasonable to believe that these structures will become more common in the architecture of the near and sustainable future.

11.10 Ephemeral Interventions

Fehn in Osaka

While pneumatic structures remain on the relative margins of engaging with architecture, there are a few tantalizing exceptions to this statement that suggest that it need not necessarily be so. The World's Fair in Osaka in 1970 provided an early hint of some of the experiments with this relationship, and we have already seen an example of that with the very long span air-supported roof of the United States Pavilion, which was a first for its kind. (See Ill. 11.49.) Another inflated pavilion from that Expo also has often been remarked upon, this one built for the Fuji Group, and it consisted of several upstanding horseshoe-shaped air-inflated tubes lashed together side by side and that were strikingly colored in several shades of Pop-Culture orange. (See Ill. 11.50.) Both of these designs clearly belong to an early experimental period for inflated structures technology, whether in terms of spanning potential or formal resolution. But there was yet another pneumatic structure designed for this same Osaka Expo that, even though it was never built and so has remained relatively obscure over the years, in hindsight can clearly be seen to be far ahead of its time in terms of the type's architectural formal development and conceptual underpinning. This was the 1969 design proposal for an intervention that would have been located *within* the Scandinavian Pavilion that had already been designed at that point and was eventually built for the 1970 Expo; this proposed pneumatic structure's design, however, came from a most unexpected source: Sverre Fehn (1924–2009), the renowned master architect of Nordic Modernism whose palette of materials typically consisted of concrete, wood, and masonry – hardly materials that could be used for an inflated structure. And yet, Fehn's design concept for this project drove him to adapt from his familiar construction materials and methods.

For Osaka, he proposed a delicate pneumatic structure that would be completely contained within the steel and glass of the single-story horizontal space (2000m^2 (21 500ft^2), and 8.5m (28ft) high} of the Scandinavian Pavilion designed by the Danish architect Bent Severin (1925–2012); indeed this was clearly intended to be a "house-within-a-house" scenario, with the interior container designed as a formal counterpoint to the outer one. (Ill. 11.54, 11.55.)

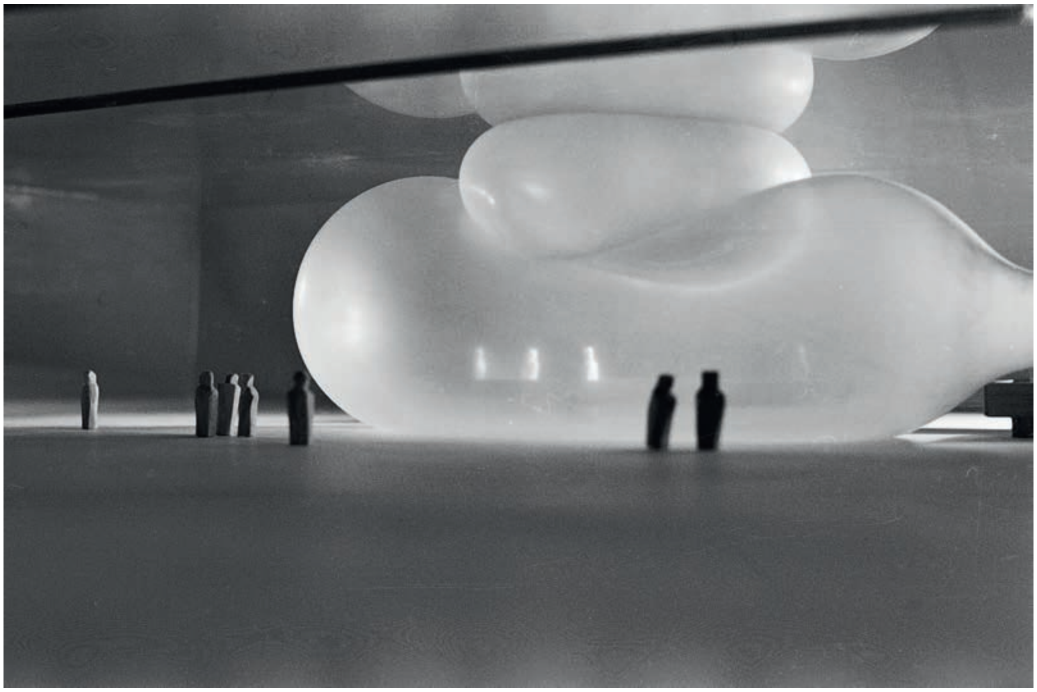


Illustration 11.54
Scandinavian Pavilion proposal for Expo '70 in Osaka, Japan (unbuilt; designed 1969).
Model of double inflated membrane intervention that was proposed to be contained within and constrained by floor and ceiling of the pavilion itself that was designed by Bent Severin.
Architect: Sverre Fehn.

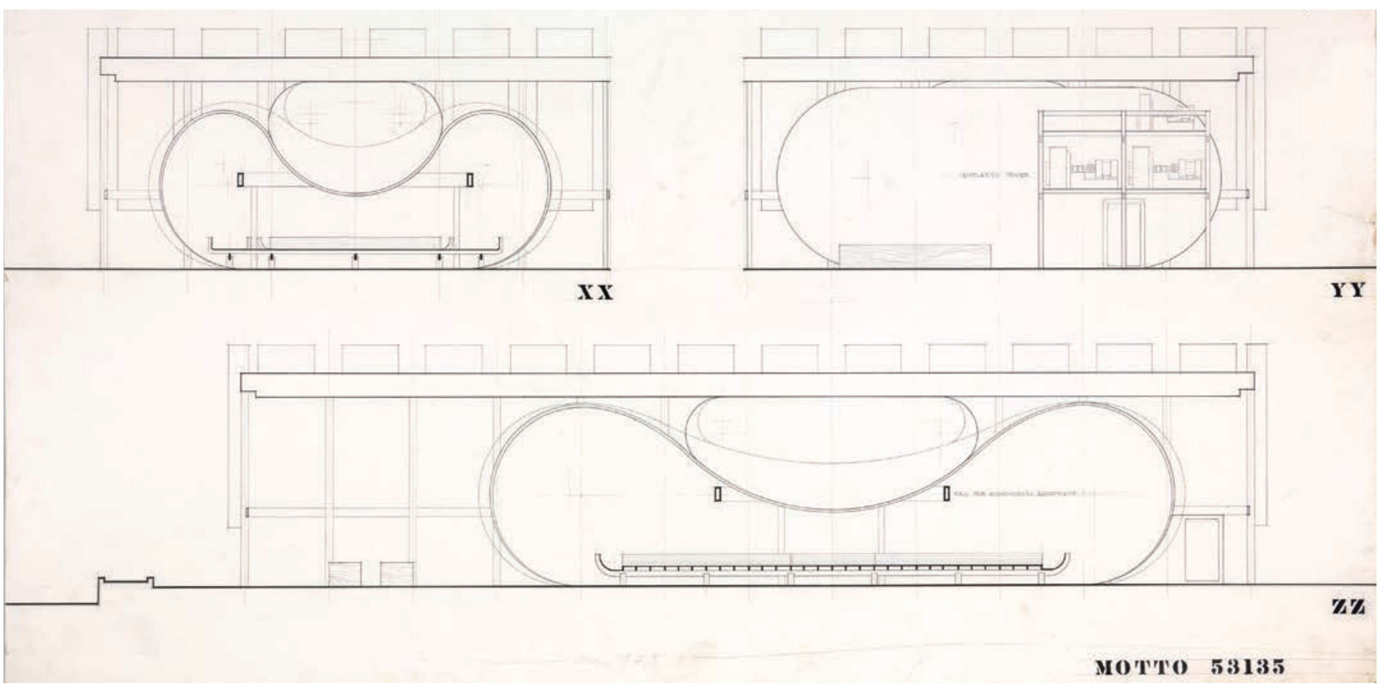


Illustration 11.55
Scandinavian Pavilion proposal, Expo '70.
Section drawings relate scale of intervention to that of pavilion proper. Drawings also convey the changing shape of inflated membranes, as the top chamber inflates, it presses down on the lower one, causing it to bulge upward and outward at the sides, and vice versa.

Illustration 11.56
Ode to Osaka Pavilion, Norwegian Architecture Museum, Oslo, Norway (2015).
Interior space of lower inflated membrane chamber.
Architect: Manthay Kula Architects.

Fehn's design called for an interacting two-balloon system, with one air chamber laying on top of the other and the two squeezed together between the hard, flat surfaces of the floor and ceiling of Severin's pavilion. The lower inflated volume was designed to be occupied by up to 100 people at a time. The upper air chamber's internal pressure and, therefore, its volume could be modulated, thereby affecting the shape of the lower one, since these were co-dependent in their "boxed-in" situation.

Ahead of his time, Fehn was concerned with people being able to breathe clean air within an ever-more-polluted environment that was being brought on by industrial development; the air inside these two bubbles was thus conceived of as a sanctuary of clean air within the ever-more-polluted condition of industrial Osaka. Moreover, the intervention was designed as a responsive form of an artificial breathing lung; i.e., when the pressure in the upper air chamber was mechanically varied this made the pressure and shape of the lower chamber bulge out and contract accordingly. Reinforcing the point none-too-subtly, Fehn proposed projecting images of Scandinavian nature onto the inside surfaces of the "breathing," inflated, occupiable structure, which would have made for a decidedly powerful statement relative to the overtly optimistic and futuristic "Progress and Harmony for Mankind" theme that had been set forth for Expo '70.

Ode to Osaka

As a coda to Fehn's 1969 design proposal for the Scandinavian Pavilion, in 2015 the Norwegian practice of Manthay Kula Architects designed and built a temporary installation called the Ode to Osaka Pavilion at the Norwegian National Museum of Architecture in Oslo. (Ill. 11.56, 11.57.) It is important to point out that in spite of obvious visual similarities, this more recent intervention was conceived of, designed, and built independently as an *interpretation* (not as a

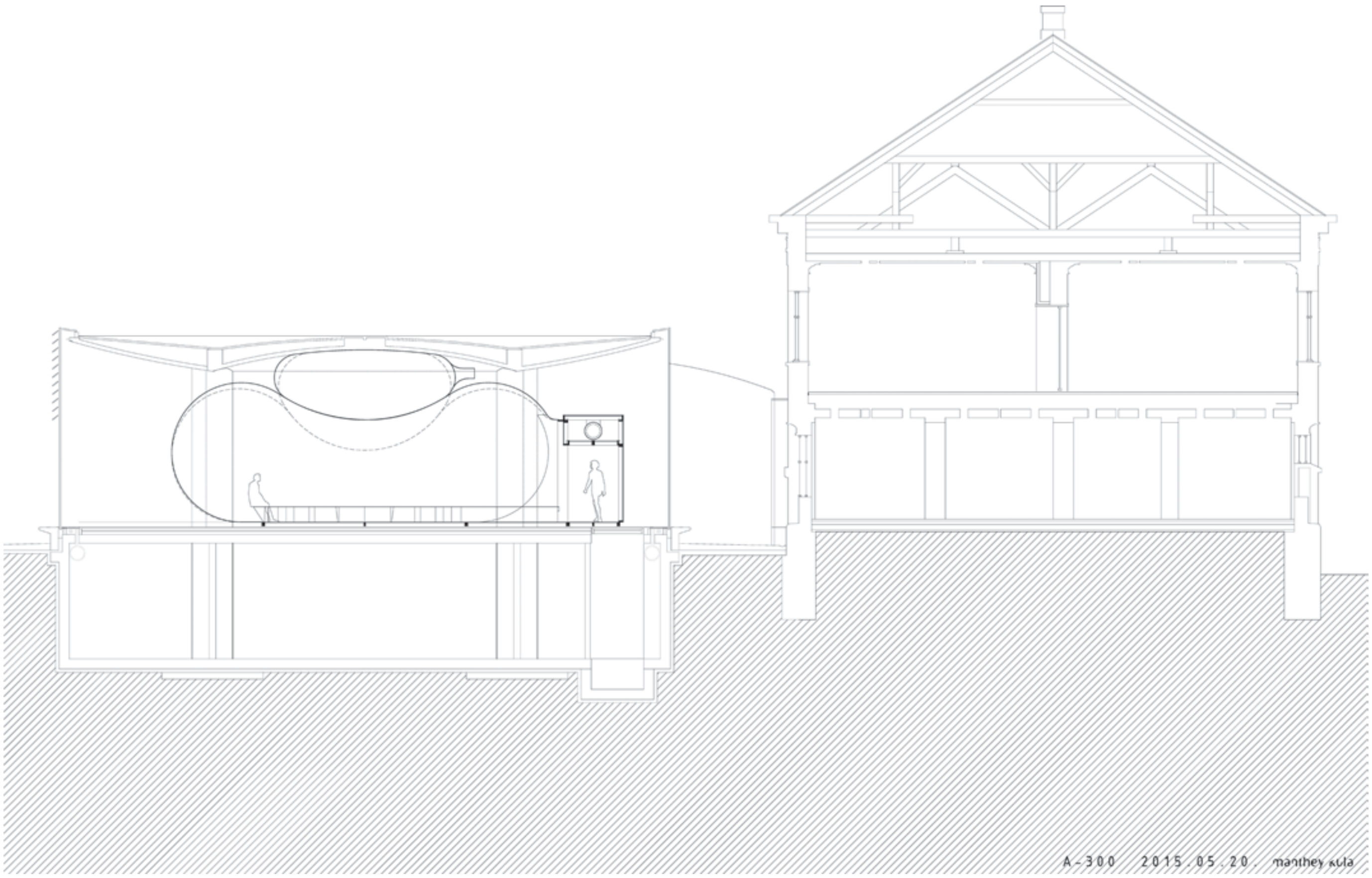


Illustration 11.57
Ode to Osaka Pavilion.
Section drawing through inflated double-air-chamber installation, set within the context and confines of Sverre Fehn's concrete and glass pavilion addition to the museum. Historical museum building seen at right.

reproduction) of Sverre Fehn's conceptual design for Expo '70, even as it was clearly and purposefully derivative from that earlier project. At the same time, however, the Ode to Osaka intervention had the unique and powerful additional resonance of being situated within the confines of a 2008 pavilion at the museum that had been designed by none other than Fehn himself.

So in this installation we had a Sverre Fehn-inspired inflatable form and space that had been especially designed by Manthay Kula to be in active dialogue with the hard architecture of the concrete and wood and glass of a Fehn-designed form and space. The delicate and finely crafted translucent white fabric form of the inflated structure was designed in relation to the four concrete columns that structure and strongly define this preexisting space, and the pair of inflated balloon chambers, one located on top of the other as intended at Osaka, here were pressed against the floor and ceiling space-defining boundaries of Fehn's pavilion. Moreover, the air pressure in the upper chamber was modulated mechanically as it was to have been at Osaka, with the lower one bulging and contracting accordingly; i.e., the structure audibly and visually "breathed." Unlike at Osaka, however, where images and information were to have been projected on to the inside surface of the fabric, in Oslo these were left white. The homage in this case was to the *space* itself, to the experience of that empty space – and to the master architect that was Sverre Fehn.

Hirshhorn Bubble

The other unbuilt pneumatic project that we will examine here is the 2009 design proposal for an extension to and/or insertion into the Hirshhorn Museum in Washington, DC, by architects Diller Scofidio + Renfro (DS+R). In this case, an inflatable translucent fabric "bubble" was to have been inserted within the central courtyard of the 4-story-tall cylindrical-ring-shaped 1974 Modernist building

Illustration 11.58
Proposal for Hirshhorn Museum expansion, Washington, DC, USA (designed 2009). Rendered overview, with proposed inflated dome at lower right seen in context of other domes on the Mall.
Architect: Diller Scofidio + Renfro (DS+R).

designed by Gordon Bunshaft of SOM that is located directly on the Mall. (Ill. 11.58, 11.59, 11.60.) The "Bubble" was conceived of as being a seasonal structure that would have been deployed on an annual basis for a month or two at a time (which would have taken about a week of staging time and half an hour to inflate), and it was to contain space at the ground level for a temporary stage and semicircular seating for up to 800 people for various special events like symposia, lectures, and debates as well as for performances and informal exhibitions, including film and image projections onto the fabric membrane.

As can be seen, the inflated air chamber would have run right up the height of the cylindrical void, although not quite touching the encircling concrete ring of gallery spaces, and it then bulged out at the top in a skewed form that would have been in interesting dialogue with the many historical domes of the Mall. At the base, a small off-shoot from the central chamber squeezed between the ground and the underside of the pilotis-elevated concrete ring of the existing building and then pushed outward from the bottom side of the Hirshhorn, giving the impression that the substantial weight of the drum was pressing down on the easily deformable balloon structure.

Unlike for Sverre Fehn's completely interior proposal for Osaka, in this case the wind would have very much been able to act on the inflated form, especially on that part protruding out from the top, and caused it to substantially move and deform, and so some movement of the inflated structure had to be permitted and yet not so much that it would touch the preexisting structure; steel cables were therefore to tether the balloon to attachments on a roof level truss. Also, the bubble had to be anchored down so as not to "float away" in the wind, and this was to be done by means of other steel cables anchoring it to a large tube of water around the base of the structure, acting as a very necessary (yet temporary) counterweight/bench. Given the evident multiple design criteria, extensive trial testing needed to be carried out to find a suitable material that would satisfy the need for this membrane to be sufficiently flexible, durable, foldable, and translucent and luminous enough, and these trials concluded with the selection of a modified (PVC-) polyvinyl-chloride-coated polyester fabric.

Design similarities and differences notwithstanding, in all three of the cases examined in this section there is a compelling and tantalizing contemporary sustainability theme being addressed: i.e., demonstrating the possibilities of inflatable structures to be thoughtfully and creatively designed as insertions into and in relation to preexisting buildings. Sizable floor area and volume expansions to/into preexisting buildings become possible at a fraction of the "normal" cost and material expenditures for corresponding space, simply by using existing construction and building frameworks to support extremely lightweight additions/insertions. Moreover, these examples also demonstrate just how such inflatable projects need not be, as is commonly assumed, mindless technology-driven forms but that in the right hands they can have their place as part of a thoughtful, conceptual design process. The design of inhabitable spaces "in context" thereby takes on a whole new set of possibilities.

Illustration 11.59
Proposal for Hirshhorn Museum expansion.
Rendering of proposal, with inflated chamber emerging from the top and side of the elevated concrete cylinder of the existing 1974 Bunshaft/SOM-designed museum building.

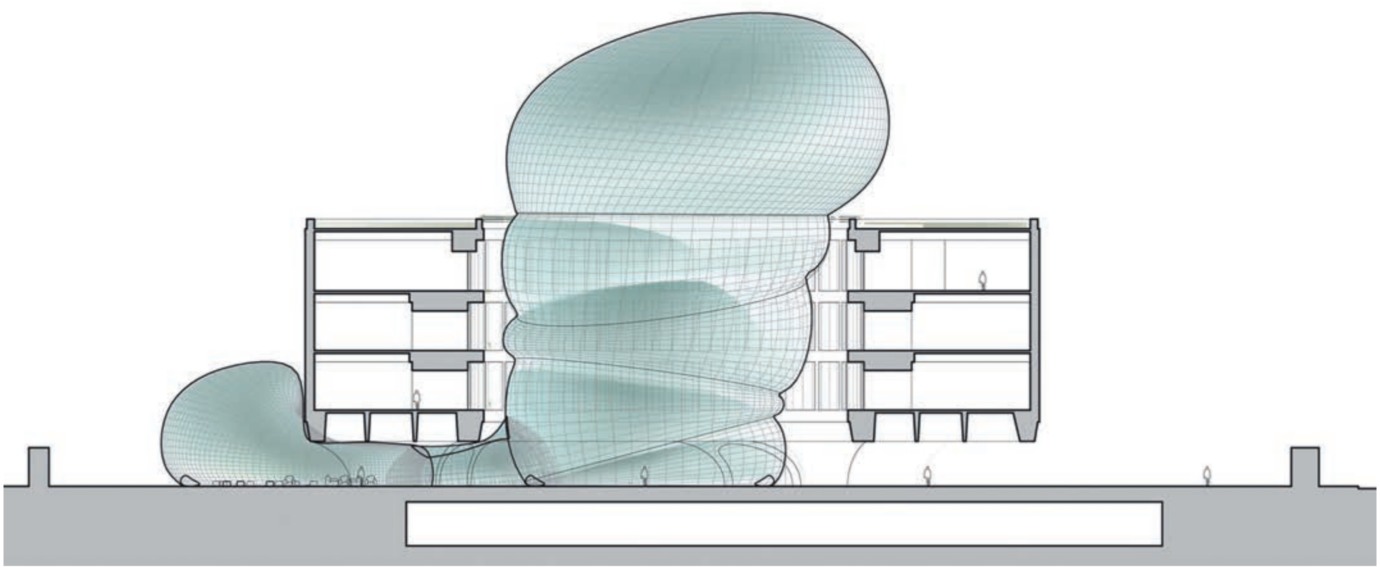


Illustration 11.60
Proposal for Hirshhorn Museum expansion.
Section drawing of proposed intervention, showing relationship to levels of preexisting cylindrical building.

The Arch and the Vault

CHAPTER

12

Illustration 12.1
Chartres Cathedral, Chartres, France (1260).
Gothic arches of the nave's side wall and cross-vaulted ceiling stretch vertical space, enhancing lightness of structure and dwarfing human congregants.

Illustration 12.2
Padre Pio Church, San Giovanni Rotondo, Italy (2004).
Overlapping arches made of several large stone segments form distinctive aspect of church; segmented roof is propped off these by steel struts; stone paving integrates interior and exterior spaces.
Architect: Renzo Piano Building Workshop. Structural engineer: Arup and Favero and Milan Ingegneria S.r.l.

12.1 Padre Pio Church – The Stone Arch Revisited

Padre Pio (1887–1968) was the mystical Capuchin monk who became so famous for his bleeding stigmata and healing powers that he achieved sainthood in relatively short order in 2002. The southern Italian town of San Giovanni Rotondo was his home and is his last resting place. As sainthood in modern Italy is a serious affair and can attract hundreds of thousands of faithful, the architect Renzo Piano was commissioned to design the Padre Pio Pilgrimage Church in order to meet the spiritual and practical demands of such modern mass veneration. (Ill. 12.2.)

Pilgrims following the processional route walk along the wall of a narrow city street and suddenly arrive at a large open space. To the left are views overlooking the rolling green scenery of the Puglian landscape and the far-off coast of the Adriatic Sea. Straight ahead is a stone-paved piazza that slopes gently down toward the front of the church; the stone paving then continues on into the church, integrating the exterior and interior spaces and giving the whole a feeling of an open house. Overall the plan arrangement follows that of a spiral, subtly drawing people deeper toward the focus of the interior space. (Ill. 12.3.) But it is really the structural form of this church which is the most distinctive and the reason for our own pilgrimage to it, consisting of an overlapping array of *stone* arches radiating out from the central altar where the remains of Padre Pio now lie.

Even though mastering the art of designing arches made of stone has very long traditions in Europe, the use of this material for this purpose is somewhat unexpected for us today. One relevant point of comparison for the structure of the church in San Rotondo might be to the flying buttressed Gothic cathedrals: as did the master builders for those structures in their time, Padre Pio Church's designers have also used some of the most advanced building techniques available to them. Each arch at Padre Pio is composed of up to 50 different segments of stone that were quarried from the local pale Apricena marble, but in this case each stone was precisely designed and cut using digital technology instead of by age-old methods. In this way, the stones of the arches could be made to bear in compression exceptionally evenly against one another and they could be independently sized and shaped with little additional cost. Moreover, the subtle variations of the digital stone-cutting process allowed the shapes of the arches to follow those of tilted parabolas, and a great variety of spans was also made possible for this most traditional of building materials and structural forms – with some reaching a staggering 50m (160ft). (Ill. 12.4.) Once in place, the stone segments of each arch are pre-stressed

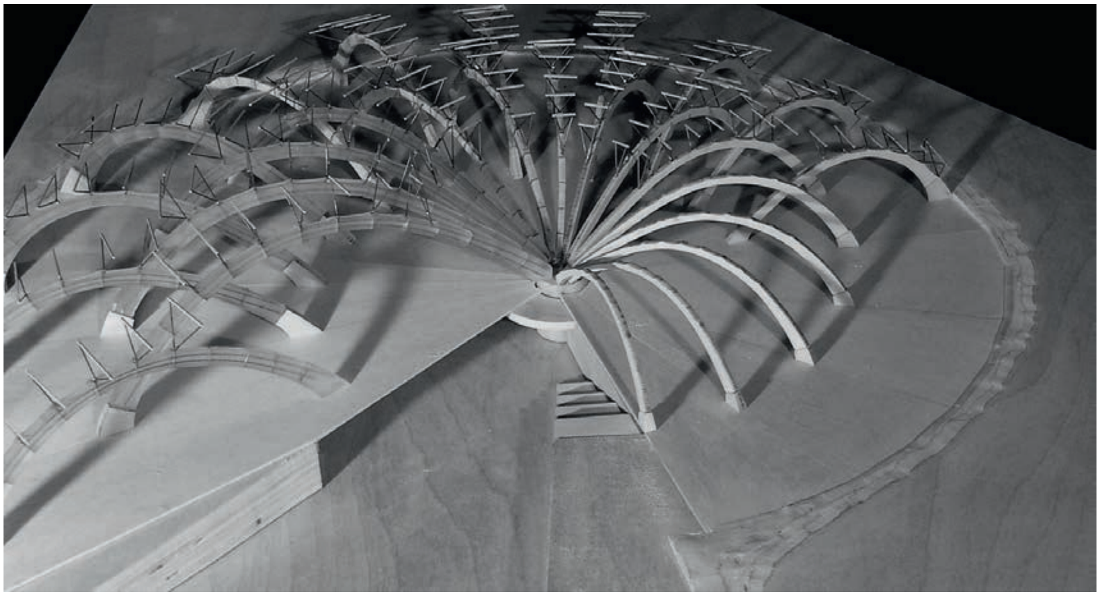


Illustration 12.3
Padre Pio Church.
Overall spiral arrangement of arches focuses space and funnels circulation toward central altar.
Cornell model by Eric Vollmer.

Illustration 12.4
Padre Pio Church.
Arched stone ribs vary in height and span. Thin V-configured steel struts serve to raise the timber-lined roof above arches and side wall.

Illustration 12.5
Granaries of Ramesseum, Luxor, Egypt (approximately 1400 bc).
Rubble fill above sides of clay brick vaults helps stabilize form; presages the favored Roman technique.

together and stabilized against the effects of earthquake motions by means of internal stainless steel cables.

By using stone for both the arches and the floors, the church is given a consistent spatial unity and there is direct reference made to local building traditions; other natural and long-lasting materials used in the project are laminated larch timber for the roof beams and pre-oxidized copper for the roof itself, whose overlapping segments are "floated" on pairs of steel struts that project upward from the arches, allowing rays of daylight to enter deep into the interior space. Early work on the Padre Pio Church was one of the last of several uniquely creative collaborations between the architect Renzo Piano and the structural engineer Peter Rice (1935–1992) and it reflects Rice's previous experience and contemporary experimentation with that most traditional of building materials and structural types: the stone arch.[1] (We will briefly encounter one of these earlier "experiments" a bit later in this chapter (see Ill. 12.19).)

12.2 Arch Form as Historical Indicator

The arch represents one of the most widely known and basic forms of structure and it has a long and distinguished history. The particular details of the arch shape, on the other hand, have developed differently according to cultural context and as an understanding of its behavior under loading evolved over the ages.

The origins of the arch are lost in the ancient cultures along the Nile, Tigris, and Euphrates rivers, centuries before recorded time. However, in the granaries of Luxor in Egypt one can find long arch-vaulted storehouses built of clay brick that have endured some 3400 years. (Ill. 12.5.) The Egyptian architect Hassan Fathy (1899–1989) studied such ancient building techniques in order to apply them to contemporary social housing; after visiting the granaries he wrote in understated fashion: "It seemed like a fairly durable substance."[2]

Illustration 12.6
Taq-I Kisra, Ctesiphon, Iraq (600–300 bc).
Near-parabolic profile of huge clay-brick arch is precursor of things to come hundreds of years later.

A remarkable historical landmark of the arch form, the Taq-I Kisra, stands in the forgotten city of Ctesiphon north of Baghdad. (Ill. 12.6.) Viewed from a distance the outstanding curved shape of the massive brick structure looms over the desert; it consists of a huge arch rising some 40m (131ft) above the ground, 27m (89ft) wide at its base, reputedly the widest single span of unreinforced clay brick in the world. Besides its overwhelming scale and structural accomplishment, however, this structure possesses another distinctive visual quality: the almost-parabolic curve gives it an air of uplift and lightness which is very different from the more "grounded" semicircular arch profile later adopted by and forever associated with the Romans.

Examples of this latter arch form abound, of course, when walking around the Italian capital. For example, along the Via Sacra from the Campidoglio to the Forum, one passes through the Triumphal Arch of Emperor Septimius Severus. (Ill. 12.7.) This monument was erected in the second century by the soldier-emperor to celebrate one of his greatest victories and the typical semicircular arch form frames the view of some of the most important historical sites in Rome. Not far from this are many other examples of the period, including the arch-dominated forms of the Foro di Augusto, the Coliseum, and the Tiber-spanning arches of the Ponte Fabricio and Ponte Cestio. Further afield, the Pont du Gard aqueduct near Nîmes ensured the provision of water to that city in (now) southern France and is but one of innumerable examples of how the Romans took advantage of the intrinsic potential of the arch to achieve truly remarkable structures throughout their expansive empire. (Ill. 12.8.)

The pointed arch, on the other hand, was both introduced and extensively developed by the Gothic master builders during the Middle Ages. In a series of French cathedrals beginning with

Illustration 12.7
The Triumphal Arch of Emperor Septimius Severus, Rome, Italy (146–211).
Semicircular arch form was much favored by Roman designers, here decorated for celebratory purposes in the Roman Forum.

Illustration 12.8
Pont du Gard, near Nîmes, France (first century).
Multi-layered series of stone arches sitting atop of one another combined to help supply Roman cities with water; lead-plate-lined channel ran along the top for this purpose.

Chartres (see Ill. 12.1), and including shortly thereafter Reims, Amiens, Beauvais, and others, the Gothic arch evolved and was taken to perceived perfection of form. The characteristic elongated profile of this arch, and the accompanying development of the flying buttress system to support tall cathedral walls was, of course, particularly well suited to their designers' desire for verticality and lightness of space. This topic of the evolution of the profile of the arch over time and within a single cathedral is the focus of the following section.

The distinctive form of the arch in Islamic architecture is also clear in our minds: it curves outward at first from the base but then curls back in on itself with its closure and we can find offspring of this arch form in the Moorish palaces in Spain and Venice. (Ill. 12.9.)

Illustration 12.9
Pena Palace, Sintra, Portugal (1847).
Openings in garden wall display classic profile of Islamic arches, with lower part curving back inward from the vertical alignment of the base of a semicircular arch form.

Illustration 12.10
Florence, Italy.
Varying bridge profiles across the Arno River including, at center, the segmental arches of Ponte Vecchio (1345).

Illustration 12.11
Gateway Arch, St. Louis, MO, USA (1965).
Overall profile is approximately that of inverted catenary (shape of hanging chain suspended at two ends). Resistance to transverse wind is provided by vertical cantilever action of the wide and hollow steel tube form of the arch legs; this also allows for an internal elevator up to an observation platform at apex.

Architect: Eero Saarinen. Structural engineer: Severud, Elstad, Krueger, and Associates.

Beginning during the Middle Ages and continuing through the Renaissance and on to this day, arches composed of circle segments and later of parabolas and ellipses or combinations of several of these basic geometrical curves have been built. (Ill. 12.10.) As we will see shortly, the parabola-shaped arch in particular can be taken as a manifestation of the advancement in scientific understanding of how arches work (though it is perhaps ironic, if not accidental, that an arch very close to this shape was built at Taq-I Kisra roughly two millennia earlier (see Ill. 12.6)).

Bracketing the ages, certainly the most striking contemporary example of a similar form is to be found in St. Louis' Gateway Arch, monument to the opening up of the western American continent. (Ill. 12.11.) In its incredible scale and construction out of metal plates, this structure also serves as a convenient reminder of the important developments in the production of iron and steel that took place during the Industrial Revolution and that have so significantly affected arches as well as all other types of structural forms for the past 150 years. When paralleled by the need for the large open spaces of train sheds and industrial buildings in the mid- to late-nineteenth century, this innovation in material led to the building of remarkable spaces created by long series of arched ribs all covered by glass – a system and combination representing both physically and metaphorically the deep transformations then taking place in society.

12.3 La Cathédrale du Mans – An Arch Form Evolves

While the exterior of an early Christian church was typically a continuous, enclosing wall envelope and that of the Romanesque church often a stronghold fortification, the wall of the Gothic church became relatively thin and transparent. As admirer and engineer Pier Luigi Nervi (1891–1979) would put it hundreds of years later: "The Gothic builders were the first real forerunners of modern technology, eliminating the heavy masses of masonry used by the Romans and replacing them with equilibrium of forces created by the interplay of thrust and counterthrust of slender ribs."[3]

Like many cathedrals, that at Le Mans (begun in 1056) was developed and built over a period of a couple of hundred years; nonetheless, there is evident in its finished form a never-ending desire to achieve the original ambition and conceptual idea of the project. The church is also telling in its as-built details of the story of evolving architectural styles during that two-century period: the cathedral has a distinctive longitudinal Romanesque quality, with five groined vaults along the nave, each with slightly pointed arches covering a square. (Ill. 12.12.) This nave, with its simplified harmony and fine proportions, is considered to be one of the most beautiful Roman interiors in transition: despite the subtle peak in their form, the arch profile here is still strongly based on the tradition of rounded arches. While covering square floor areas, what can perhaps be considered to be the most important architectural innovation that the pointed arch and the groined vault introduce is not yet fully exploited; namely, the possibility that this form opens up for covering not merely square bays but also rectangular ones. Since the height of a vault made with pointed arches is no longer directly bound by the size of the arch span (which is the case for a semicircular arch where the height is half the span), plans and vertical spaces can be made more flexible and irregular. The thirteenth century then saw the construction of the apse of the cathedral with its 13 side chapels fanning out in plan, and here the vaults are built with a distinctly pointed Gothic profile (Ill. 12.13.) On the outside of the cathedral, one can observe the drama of three-story-high arched flying buttresses counterbalancing the outward thrusts of the interior vaults around the apse, and the plan drawing of the cathedral makes these differences quite evident. (Ill. 12.14.)

Illustration 12.12
Cathédrale St.-Julien du Mans, Le Mans, France (1254).
Rounded, Romanesque-style arches of nave.

Illustration 12.13
Cathédrale St.-Julien du Mans.
Pointed, Gothic-style arches of apse. The difference in the arch profiles is especially apparent along the sides of the nave in this and the preceding image, as well as in the outline of the stained-glass windows at the far ends of these views.

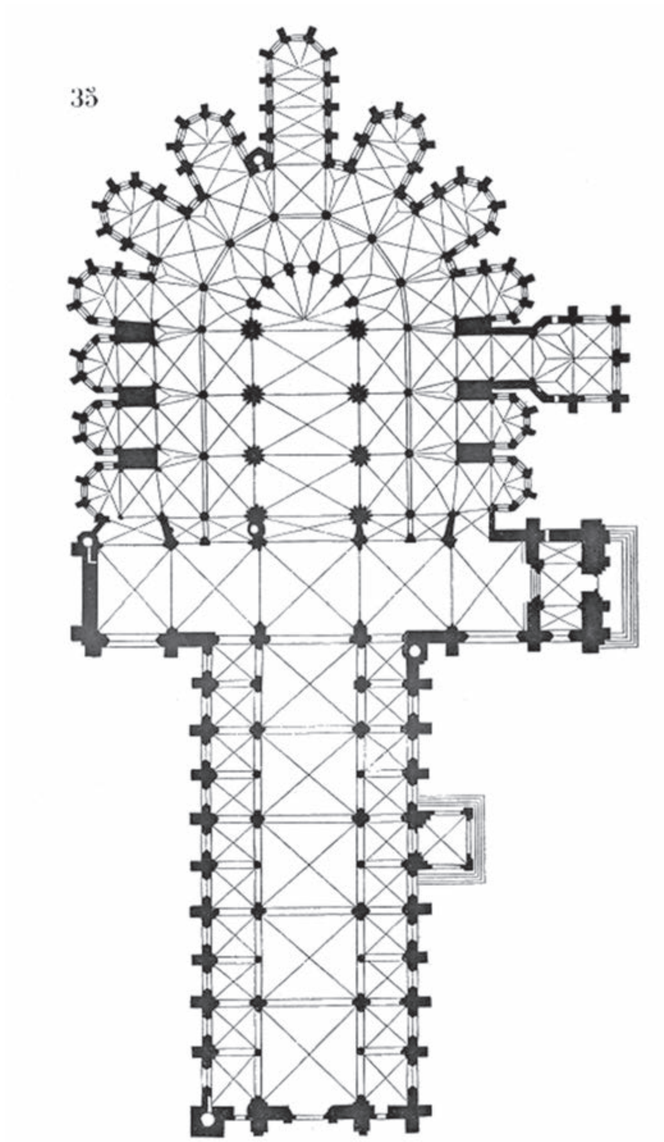


Illustration 12.14
Cathédrale St.-Julien du Mans.
Plan drawing, where difference between the two parts of the nave and apse portions of the cathedral is clearly evident; the structural bays of the nave with rounded arches are squares while the bays of the apse with pointed arches are rectangles.

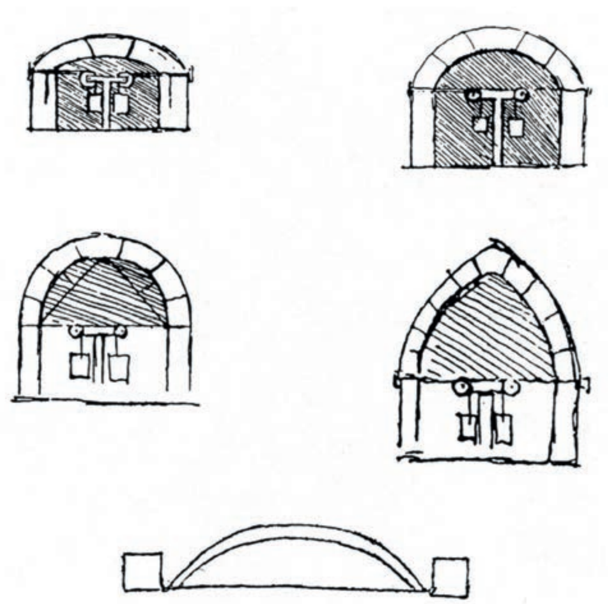


Illustration 12.15
Leonardo da Vinci's sketches of arch support conditions and horizontal reactions.

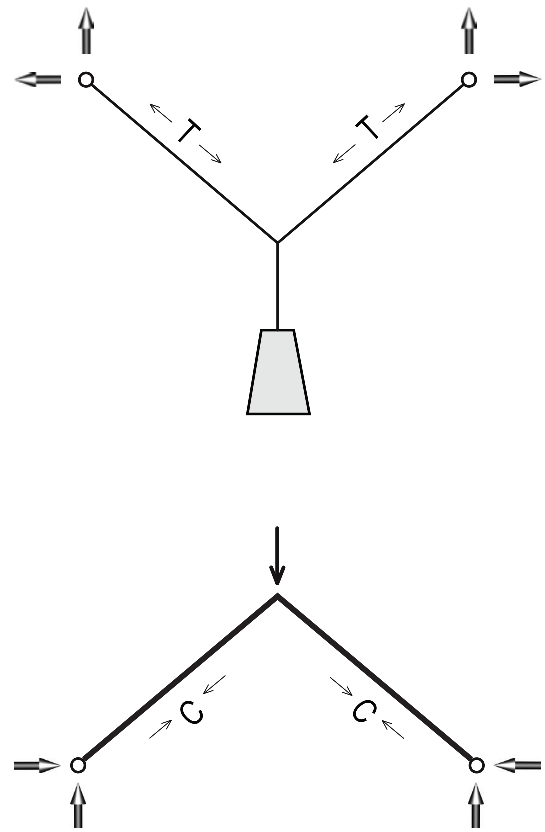


Figure 12.1
Straight lines of point-loaded cable structure and corresponding inverted arch form.

12.4 Understanding Arch Behavior

Common for all types of arches is the *elevated position of the center of the structural form* compared to the level of the supports on either side. Also, as we will see shortly, these support conditions are very particular such as to ensure that the *load is primarily borne by compression forces* in the arch, which depends on the supports being held in place both vertically and horizontally, with no movements allowed. These two physical characteristics of overall profile and of load-carrying mechanism have helped to give the arch a very long and distinguished history in architecture; i.e., the compressive stress strength of traditional materials such as stones and bricks is extremely well suited to primary compressive arch action and the vaulted form is particularly appropriate to enlarging and heightening interior space while allowing for the development of unique and expressive architectural styles.

In order to better understand the fundamental logic of the arch and how it carries load, it is helpful to briefly revisit once again the behavior of the simple cable that we discussed in the preceding chapter. As we suggested then, if we hold a rope with two hands and hang a weight at the rope's midpoint the rope will take on an overall "V" configuration so as to be in pure axial tension. (Fig. 12.1a.) Moreover, we have established that in addition to the downward pull that we feel in our hands as we hold up the ends of the rope, we will also experience forces tending to pull our hands inward toward each other. In order to maintain equilibrium, therefore, we know that we must react against these tendencies by pulling both upward and outward with our hands. If we were to now conceptually "freeze" the rope in order make it stiff (or substitute it with a material such as wood or stone, for example) and then invert the resulting form, the load at the apex would now be borne by pure compression in the "legs" of the inverted-V arched structure. (Fig. 12.1b.) As before, our hands must still hold up the ends since gravity does not change direction, but now the compression within the structure will press outward at the supports and we must react by pushing inward in order to have equilibrium. As an aside, it is worth confirming what we instinctively already know: that it is not possible for an arch to be made of thin rope; in compression, a

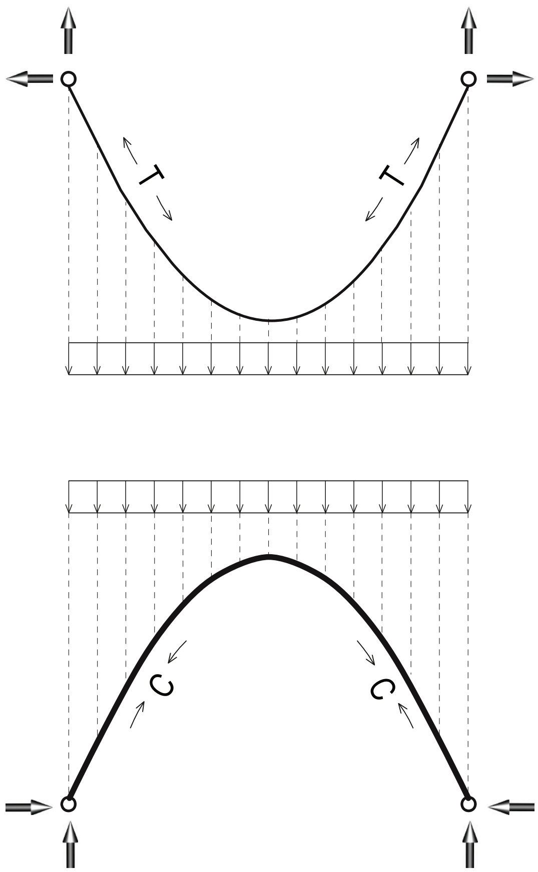


Figure 12.2
Parabolic cable profile for load that is uniform along horizontal span and corresponding inverted arch form.

Illustration 12.16
L'Umbracle, City of the Arts and Sciences, Valencia, Spain (2002). Catenary (nearly parabolic) profile of light steel arch ribs corresponds to uniform-along-the-arch loading condition produced by self-weight of ribs and cross bars, resulting in "pure" compression forces along the arch.
Architect and structural engineer: Santiago Calatrava.

rope will buckle out of alignment before any load can be carried. Therefore, we understand intuitively that an arch profile needs to be thicker than that of an equivalent cable structure, and our experience of arches in the form of stone blocks and large, curved timber glue-laminated members indeed confirms this.

We can go further with this reversal-of-form analogy. If we again take the rope and this time subject it to vertical load that is uniformly distributed along the horizontal span between the supports, we will recall from the previous chapter that it will take the shape of a parabola so as to be able to bear the load in tension throughout. (Fig. 12.2a.) When this configuration is inverted into a parabolically shaped arch, the structure can analogously be understood to bear the uniformly distributed loading by means of "pure" compression force along the axis of the arch.[4] (Fig. 12.2b.) Close approximations to such loading conditions and to the associated overall parabolic form can indeed be found in the work of a wide range of architects; notably in some of the structural-action-expressive projects of Santiago Calatrava. (e.g., Ill. 12.16.)

Beyond the preceding two relatively simple rope-to-arch inversions, it can correctly be extrapolated that for every loading condition, no matter how simple or complex, there is a very particular arch geometry for which there will be "pure" compression stresses throughout the structure. Such a shape, whatever it may be, whether an inverted V or a parabola or something else entirely – is labeled as the *ideal or funicular* configuration of the arch for the associated load condition. It is of historical interest to note in this context that in the era before the advent of computer structural modeling some very complex three-dimensional arch forms were created using just this cable-to-arch-reversal process, of which surely the best known are the phenomenal hanging models used by the Catalan architect Antoni Gaudí (1852–1926) to design the Colonia Güell chapel near Barcelona. (Ill. 12.17, 12.18.) Other, more recent examples of essentially funicular forms can also be recognized for the very particular and predominant loading conditions of the arches of the Pavilion of the Future (Ill. 12.19) and the Campo Volantín Footbridge. (Ill. 12.20.)

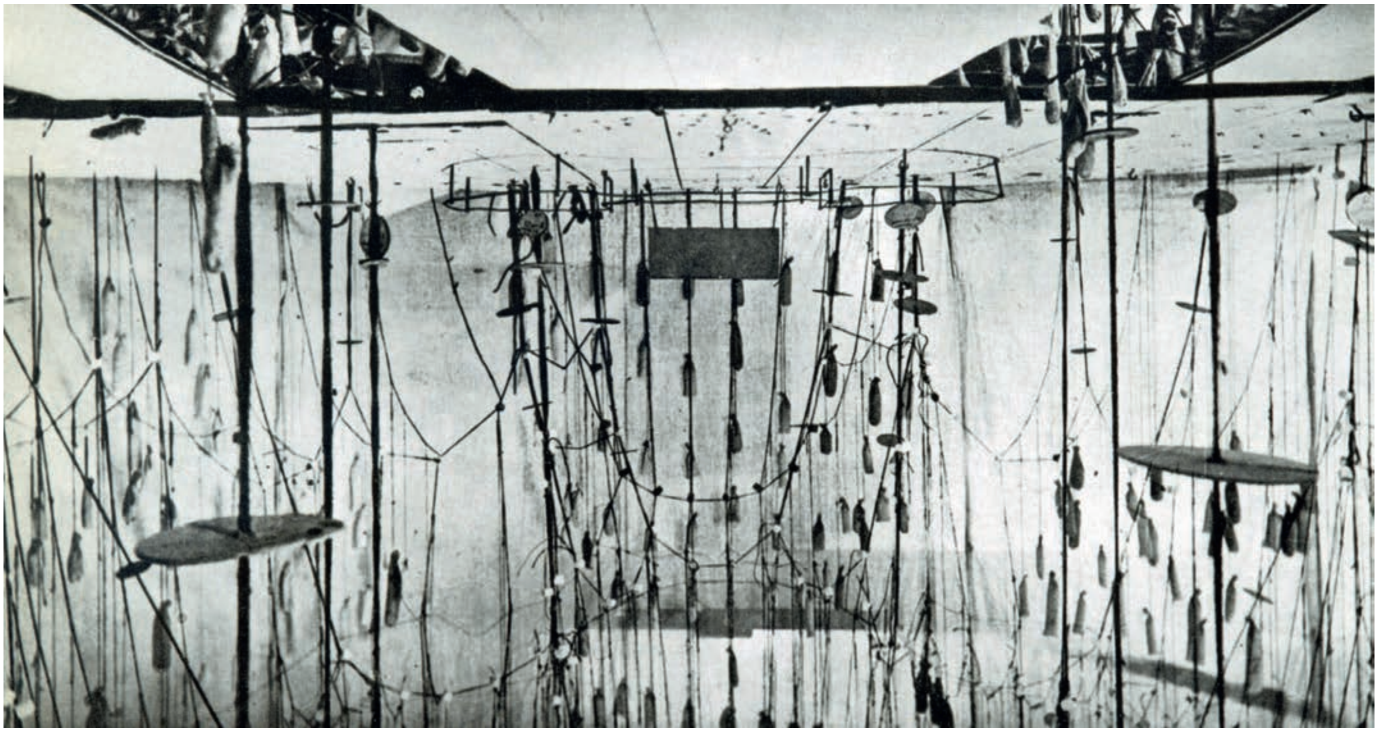


Illustration 12.17
Model for Colonia Güell Chapel, near Barcelona, Spain (1915).
Antoni Gaudí's hanging chain model.

Illustration 12.18
Colonia Güell Chapel.
Unusually angled arch forms established from hanging chain model so as to favor their being in "pure" compression and minimizing bending as much as possible.

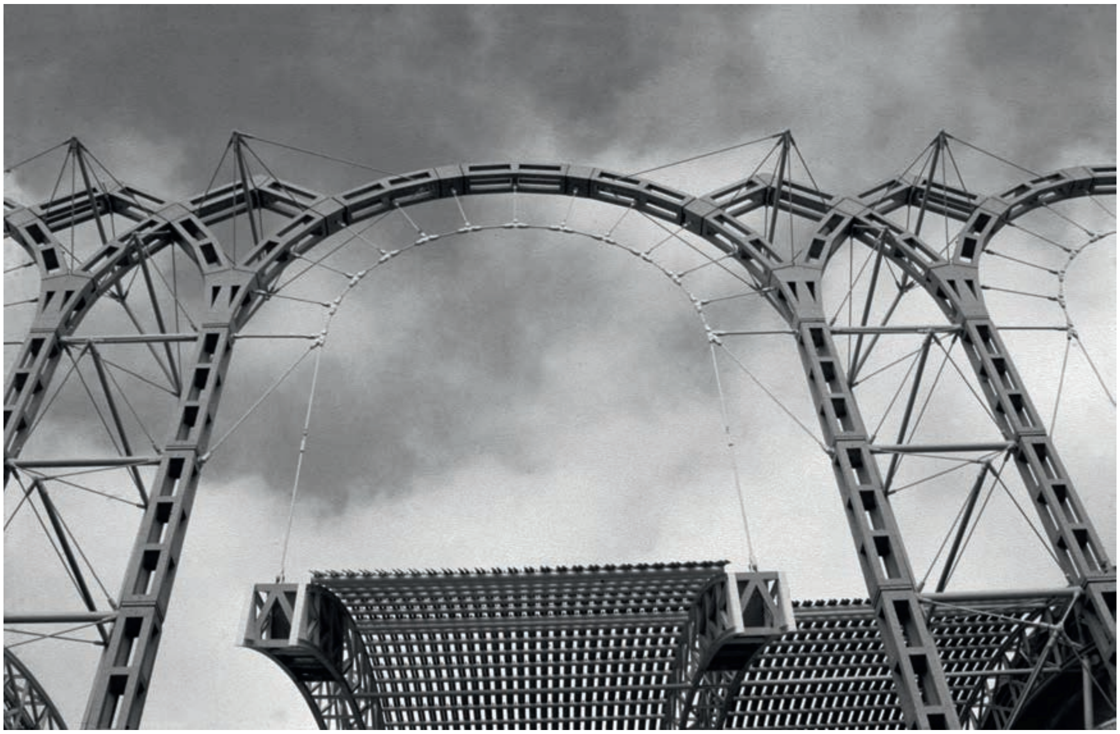


Illustration 12.19
Pavilion of the Future, Seville, Spain (1992).
Vertical hanger loads are converted into radial set of tension rods; semicircular arch form corresponds to the funicular profile for this load distribution. Example of one of Peter Rice's "experiments" in resurrecting stone arch technology, and a precursor to the stone arches of the Padre Pio Church discussed in Section 12.1.
Architect: Martorell Bohigas Mackay. Structural engineer: Peter Rice.

Illustration 12.20
Campo Volantín Footbridge, Bilbao, Spain (1997).
Inclined cable hangers supporting pedestrian walkway connect mostly toward the middle of the arch, resulting in near-parabolic funicular form; the sides of the arch are straight, however, where no load-carrying cables connect to it.
Architect and structural engineer: Santiago Calatrava.

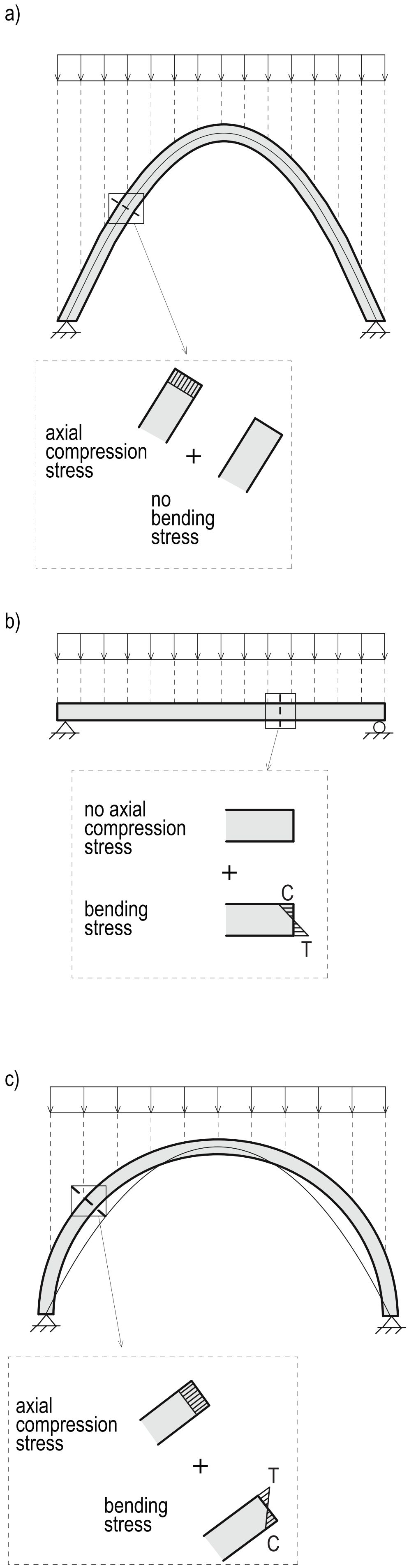


Figure 12.3
Stress condition variations according to structural form: (a) axial but no bending stresses for parabolic arch supporting uniformly distributed load, (b) bending but no axial stresses for simply supported horizontal beam, (c) both axial and bending stresses for semicircular arch subject to uniform load over horizontal span.

But as compelling as the structural logic and "purity" of such forms may be, there is a problem with limiting our understanding of the arch to such unique conditions. If an arch could only work in the load-specific funicular configurations that we have so far been discussing, the infinite variability of loading caused by live loads (whether due to occupancy or wind, for example) would imply that the structure would forever need to be readjusting its shape in order to always ensure compression stresses throughout – clearly something that could not happen, without causing severe physical and emotional distress in an inhabited building! Evidently, our understanding of arch behavior cannot yet be considered complete.

As previously noted, an arch requires substantial cross-sectional dimension because of its buckling tendency as a compression structure; the consequence of this physical thickness is that the arch also inherently has a structural capacity to offer some resistance to the tendency to change shape as loading conditions vary. Such resistance to deformations transverse to the curved centerline of the arch (i.e., against bulging outward or bowing inward from the original profile) is provided by the arch's *flexural* stiffness and thus by *bending stresses being established throughout the arch* – something that may at first seem quite unexpected, for now we are talking about an arch that is not only carrying loads in compression but that is also working in beam-like fashion where bending moments are produced. (Fig. 12.3.) Where the arch tends to bulge outward, bending action's tension stresses will develop on the outside face of the arch and compression on the inside, with a linear variation of magnitudes from one side to the other across the arch thickness, and vice versa where the arch tends to bow inward.

From this discussion, it can correctly be inferred that the general condition for arches to carry load is by means of some combination of both axial compression stresses *and* bending stresses. As was implied earlier as an extrapolation from cable behavior, the funicular profile is the (no-bending) *exceptional* case rather than the general rule governing arch behavior. There is, as a result of this typically *combined* load-carrying mechanism, the potential for significant variation in the shaping of the arch, and rather than seeing a certain arch profile as being the only one capable of working for a particular loading distribution, we can instead consider that a wide range of arch shapes are in fact possible for that purpose depending on which materials are being used. And coming back to our original concern, the converse must also be true: i.e., any particular arch profile that is subjected to variation of load over time will be able

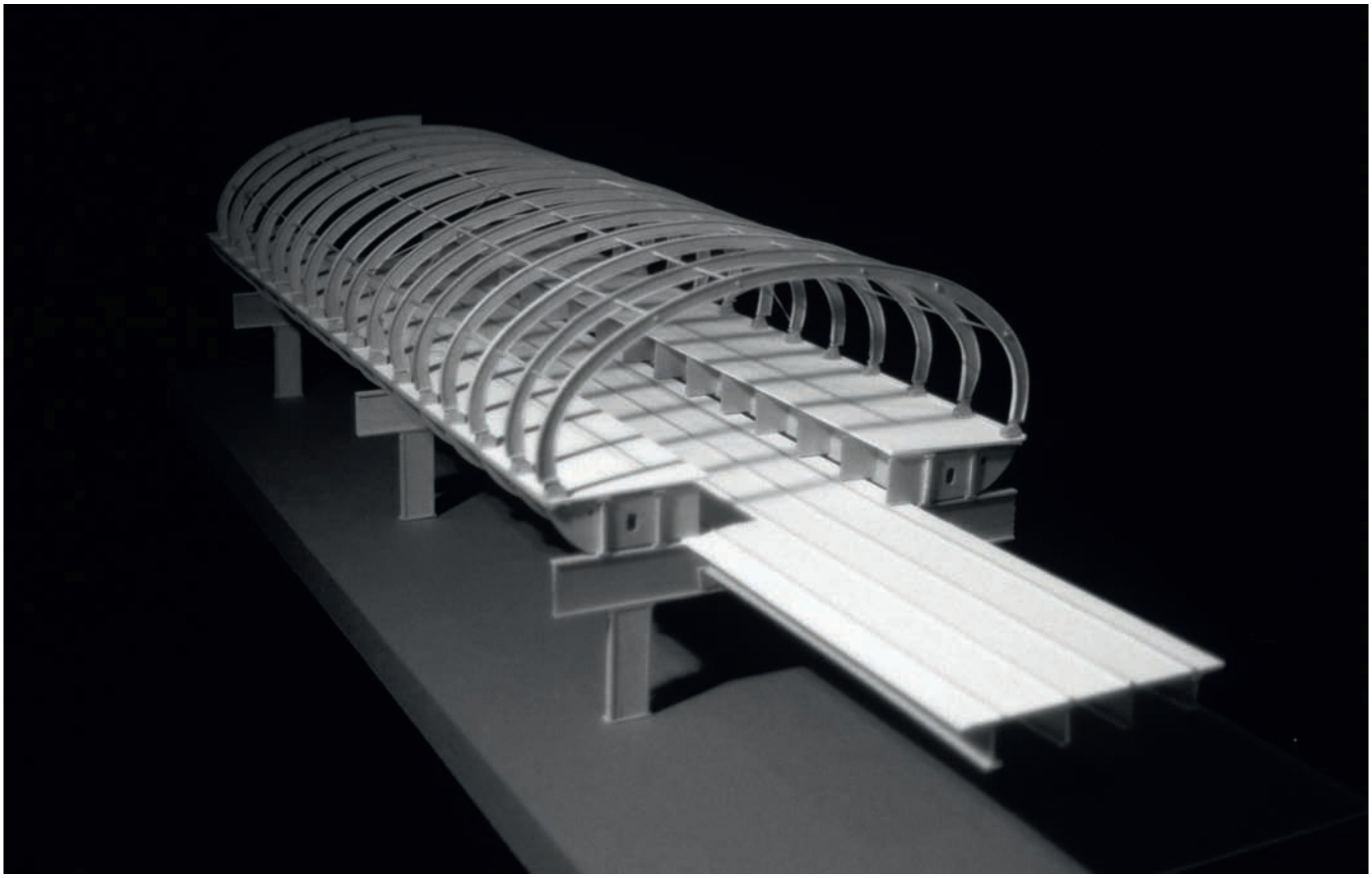


Illustration 12.21
Brin Station, Genoa, Italy (1994).
Elliptical arch ribs curve back upon themselves, requiring material that can withstand significant bending without failing.
Architect: Renzo Piano Building Workshop. Structural engineer: D. and L. Mascia. Cornell model by Jacob Werner.

to handle these changes by adjusting not its shape, but rather its load-carrying mechanism to new combinations of compression and bending stresses so as to always maintain equilibrium, at least as long as material capacities are not exceeded.

These statements lead us to an interesting reconsideration of the most famous of arch profiles: i.e., that of the Roman masonry semicircle. Given our preceding discussion, such a profile is now known to clearly not be “ideally” shaped for the common condition of loads that are uniformly distributed along a horizontal span, nor would its masonry material seemingly be well suited to the tension stresses that would result from any significant bending action in the arch. Yet the Romans adopted this shape almost to the exclusion of all others and did so with such structural success that many examples remain standing 2000 years later. Clearly, they had to have developed an effective means of dealing with the inner workings of their preferred arch form. Indeed, upon closer examination, we find that the Romans typically took to filling in the areas over the curved ends of their semicircular arches with rubble and concrete, thereby changing the loading pattern and stabilizing the arch so as reduce potential deformations and be better able to deal with bending behavior. (e.g., Ill. 12.7 and Ill. 12.8.)[5] Indeed, the loose stone fill material weighing down the vaulted structures of Egyptian master builders is evidence that this basic arch-stabilizing principle had been understood well before the Romans adopted it with such success. (Ill. 12.5.)

Such techniques would obviously have been much more critical historically when masonry building construction prevailed, partly because the dead load of those structures was so dominant over live load magnitudes but mostly because of the weakness of masonry materials to withstand any net tension stresses that could result from significant bending action in an arch. Today, of course, material capacities are much greater and often include considerable tensile capability, thereby giving the arch designer much more freedom in choosing geometric profiles, as is evident in the elliptical thin steel ribs of the Brin metro station in Genoa that curve back inward upon themselves near their base or with the multiple undulations of the Zentrum Paul Klee. (Ill. 12.21, 12.22.) Nonetheless, it is telling that the cross-section of each arch rib in these two examples is shaped

Illustration 12.22
Zentrum Paul Klee, Bern, Switzerland (2005).
(top) Multiple, continuous, undulating arch ribs are subject to much bending; (bottom) each arch rib is in cross-section a deep steel plate box-girder, a form well suited to this purpose.

Architect: Renzo Piano Building Workshop. Structural engineer: Arup.

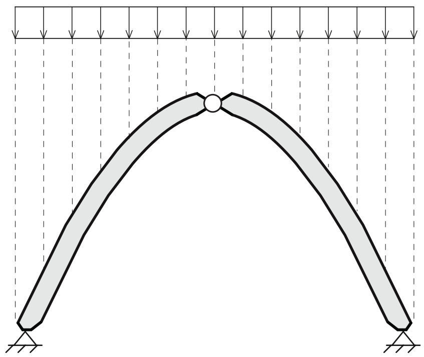


Figure 12.4
Symmetrical three-hinged arch configuration, with hinges at top and at two base supports.

in the form of a flanged "I"-shape, which we know from Chapter 7 is strategic for resisting bending action; obviously, some of the previous lessons learned about how structures work and can be strategically shaped can be reapplied in different circumstances.

It nevertheless remains an inescapable truth that the closer an arch's profile is to the funicular shape for a particular load condition the more compression stresses will do the work of carrying the load and the less will bending behavior have to be called upon. And since from our discussion about beams we know just how inefficient bending action is as a means of carrying load, we can conclude that despite the freedom of form that is technically possible today there is distinct material efficiency advantage for an arch to follow as closely as possible the funicular curve for the predominant load condition. This will be an especially important strategy, obviously, for arches that carry very heavy loads or that we otherwise seek to make as slim and slender as possible.

We will examine more closely the topic of combined compression and bending action in the arch in the context of material stresses in Section 12.6; for now, however, it is enough to observe that great freedom of arch shape is made possible by it.

12.5 To Hinge or Not To Hinge?

As a means of further developing an understanding of basic arch behavior, it is convenient to next consider another very particular type of arch: the widely known and historically popular *three-hinged arch*. As its name implies, this type of arch has three points along its profile where the structure can rotate without restraint; i.e., the arch is in some way articulated so as to be purposefully free from any bending action at those locations. Often, although not necessarily, these hinge locations are placed at the two base support points as well as at the apex of the arch (Fig. 12.4); aside from the unique detailing that is needed in order to allow for free rotations to occur, these strategic locations may also be part of the reason that hinge points have often been richly decorated or otherwise emphasized in arch structures. (e.g., Ill.12.23–12.26.)

Illustration 12.23
Abattoirs de la Mouche, Lyon, France (1913).
Hinge detail at base of arch that expresses ability of structure to rotate freely at that point; dimension of trussed arch rib tapers to minimum possible.
Designer and structural engineer: Tony Garnier.

Illustration 12.24
Abattoirs de la Mouche.
Hinge detail at apex of the arch that expresses ability of structure to rotate freely; trussed form of arch elsewhere expresses its ability to resist bending as well as compression.

Illustration 12.25
Brentwood Skytrain Station, Vancouver, BC, Canada (2002).
Hinge detail at base of arch expresses ability of structure to rotate freely; flanged steel profile and rectangular glulam segments of arch provide both bending and compression capacity elsewhere in the arch.

Architect: Busby + Associates Architects (now part of Perkins+Will). Structural engineer: Fast & Epp Partners.

Illustration 12.26
Nature Boardwalk Pavilion, Lincoln Zoo South Pond, Chicago, IL, USA (2010).
Hinge detail that expresses ability of structure to rotate freely; orientation of rectangular section of the arch corresponds to arch's being able to withstand bending and compression in its plane.

Architect: Jeanne Gang Architects. Structural Engineer: Magnusson Klemencic Associates.

Illustration 12.27
Canary Wharf Underground Station, London, UK (1999).
Example of fixed support at base of steel arch; rotation is prevented, and arch rib thickens to match large moment reaction from support. Arch rib dimension diminishes up to apex, where a cylindrical rod provides a hinge connection between the left and right halves of the structure. (See also Ill. 12.49.)
Architect: Foster + Partners. Structural engineer: Arup.

Our choice of examining this particular arch form is not coincidental. In fact, its popularity over the past 150 years has had much to do with the fact that by introducing three hinges the arch "magically" becomes statically determinate. Of course, there is a rational explanation behind this strategy. If the two base support points of a continuous planar arch are at first considered to be fixed, then there will be three possible reactions at each support (i.e., vertical and horizontal forces and a moment reaction), which would lead to a total of six unknown reactions needing to be determined – not something that is easily done by hand methods when there are only three equations of equilibrium available to solve for these. If both of these supports are hinged, however, then we are down to only four unknown reactions. And if we provide a further physical release by introducing a hinge at the top of the arch, then we have another equation of equilibrium available to us at that point ($\Sigma M = 0$), which means that we now have a total of four equations that are available to solve the four unknown force reactions at the supports. Once that is done, all of its forces and stresses can be completely and precisely determined by hand calculation methods, certainly an important consideration before the advent of computers, but also an attribute that we will use to advantage here in the pages that follow. Today, more complex forms and more sophisticated methods of structural analysis permit structurally indeterminate two-hinged or even continuous (hingeless) arches to routinely be used to aesthetic and structural benefit (i.e., greater stiffness and, therefore, less distortion under load). One very clear example of just such a fixed support condition can be seen at the base of the arched entrance for the Canary Wharf Underground Station in London. (Ill. 12.27.)

This is not to say, however, that statically determinate three-hinged arches are for the past: when Sir Nicholas Grimshaw designed the Waterloo Train Station in 1993, he clearly chose

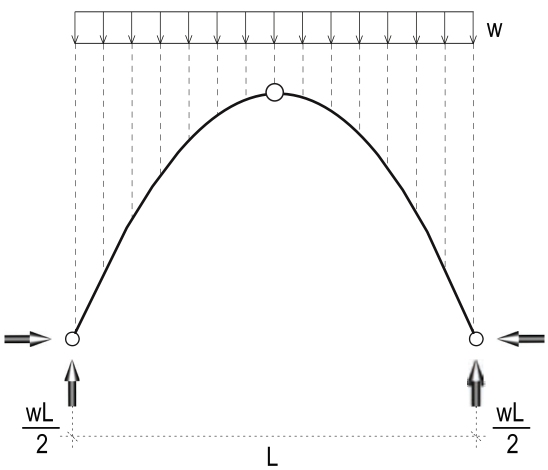


Figure 12.5
Pinned arch supports provide vertical and horizontal force reactions.

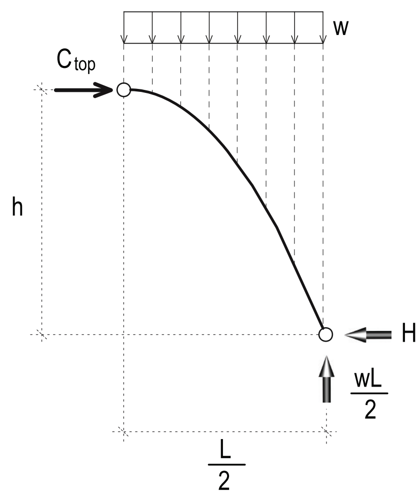


Figure 12.6
Free-body diagram for right-hand half of arch when "cut" at apex.

to reference the typical arch forms of the past century's train sheds with a clearly articulated three-hinged arch profile that greets passengers upon arrival in London. (See Ill. 12.31.) The three-hinged arch form also has the attribute of being relatively structurally forgiving in the sense that, for example, the vertical settlement of one support will not occasion supplemental stresses in the arch segments, an especially important consideration for long-span roof structures or bridges where the ground conditions might very well vary from one arch support to the other. In the end, therefore, the decision today on whether or not to include hinges in an arch ends up being some carefully considered combination of technical necessity/advantage and aesthetic/conceptual design objectives. For our purpose at this point, however, there are some important lessons that can be gleaned from analyzing in detail the workings of the three-hinged arch that also can be applied more generally to other arch structures, and it is to this task that we now turn our attention.

We begin by considering a symmetrical three-hinged arch of height h and total span L that carries a uniform load w. (Fig. 12.5.) At each of the two hinged base supports, there is the possibility of both vertical and horizontal reaction forces. The symmetry of the situation leads to the obvious fact that the vertical support reaction at each end of the arch will be half the total applied load; i.e.,

$$V_R = V_L = (wL)/2$$

It is to be noted in passing that the arch's vertical reactions do not depend on the height of the arch (except for the contribution of some extra self-weight resulting from a higher arch).

The horizontal reactions are somewhat less easily determined, but this can be done by dividing the symmetrical arch into two identical half structures. At the location of the hinges we know that there cannot be any bending moment since we allow the adjacent arch elements to rotate freely at that point, and the symmetry of the loading situation dictates that there will be no relative vertical displacement or vertical shearing of the left half of the arch relative to the right, or vice versa. The resulting half-arch free-body diagram (Fig 12.6) must necessarily satisfy all the conditions of statical equilibrium, including that

$\Sigma M = 0$ (taken about any point in the structure)

If we choose to take $\Sigma M = 0$ about the top hinge point for the half structure since, again, there cannot be any bending moment in the hinge and also remembering that this must be true if the part of the structure that we are considering is not to rotate in space, the result is the following equation:

$$(H)\,(h) - (wL/2)\,(L/2) + (wL/2)\,(L/4) = 0$$

which when simplified and rearranged leads to

$$H = (wL/8)\,(L/h)$$

Contemplating this result, it is clear that the magnitude of the horizontal reaction needed to support an arch is dependent not only on the magnitude of the vertical load, but it is also inversely proportional to the height-to-span ratio, h/L; i.e., for the same loading, a relatively "flat" arch will thrust outward much more strongly and its base supports will need to be correspondingly bigger to resist this force than will be the case for an arch which is relatively "tall." The physical options and architectural implications of this observation with regard to support conditions will be discussed in Section 12.7.

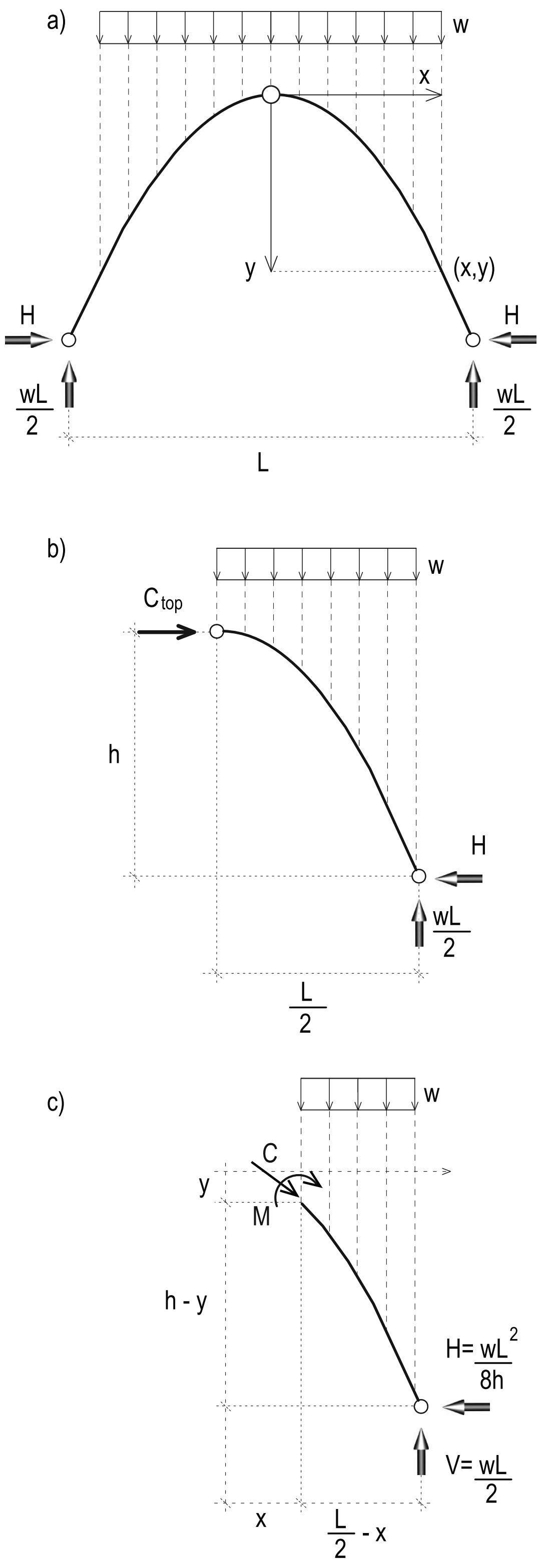


Figure 12.7
(a) Three-hinged arch subject to uniform loading, base support reactions, (b) free-body diagram for right-hand half of structure when cut at apex hinge, where there is compression force but no bending moment, (c) free-body diagram for lower segment of arch when cut at arbitrary location, where generally there is bending moment M as well as compression force C.

12.6 Compression Forces and Bending Moments in Arches

The basic principle of arch load-carrying behavior has already been described but it is worth repeating for emphasis: the arch carries load by means of both internal axial compression forces *and* bending moments. Methods of computing the varying magnitudes of these two distinct structural actions can quickly get quite involved, but a limited investigation here will prove beneficial to gaining a better understanding of arch behavior in general.

In the previous section, it was shown that the base support reactions for a three-hinged arch of height h and span L that is subject to uniformly distributed vertical loading w, as shown in Fig 12.7a, are:

$$V = (wL)/2 \text{ and } H = (wL^2)/(8h)$$

Applying Pythagoras' most famous theorem, the effective reaction R whose magnitude and direction must necessarily be equal and opposite to the net axial compression force C in the arch at the support is, therefore,

$$R = C_{supp.} = \sqrt{[(wL/2)^2 + (wL^2/8h)^2]}$$

Furthermore, reexamination of the horizontal equilibrium equation for the half arch that we looked at previously leads to the observation that the compressive force at the top of the arch must be equal in magnitude to the horizontal support reaction (Fig. 12.7b); i.e.,

$$\Sigma F_x = 0$$
$$C_{top} - H = 0$$
$$C_{top} = (wL^2)/(8h)$$

When the load on the arch is symmetrical, as it is here, C_{top} is a horizontal force; i.e., there is no vertical force component at that point since the (vertical) external load on the half arch exactly balances the vertical reaction force at the support. From these results, it can be extrapolated that the magnitude of the axial compressive force C in the arch is a minimum at its apex and increases to a maximum at its base. Moreover, as do the horizontal reactions, the magnitudes of the compression forces in arches also depend on the relative height-to-width proportions of the arch; i.e., the flatter the arch, the larger the compressive axial forces that must be dealt with throughout the structure. This axial force will everywhere be directed along the arch profile centerline.

Let us now consider what is happening with regard to bending action in the arch. With a three-hinged arch configuration, we ensure that at the hinge points there will be zero bending moment. But this is not necessarily so elsewhere along the arch; in fact, generally speaking it will not be the case. In order to determine the bending moment M in the arch at any generalized point having coordinates (x,y) we must consider the equilibrium of the free-body diagram of the structure on either side of an imaginary cut through the arch at that location. (Fig. 12.7c.) For there to be rotational equilibrium of this free body, the sum of the internal and external bending moments taken about the cut must be equal to zero; i.e.,

$$\Sigma M = 0$$
$$M + w [L/2 - x] [(L/2 - x)/2] - [(wL/2) (L/2 - x)] + [(wL^2/8h)(h - y)] = 0$$

which, upon simplification and solving for M yields:

$$M = (wL^2/8h) (y) - (w/2) (x)^2 \qquad (12.1)$$

This equation defines the bending moment in an arch according to the particular geometric relationship between the x and y coordinates for the arch profile being considered; i.e., whether it is parabolic, semicircular, etc.

We will first consider what we will soon see to be the special case of a parabolic arch. (Fig. 12.8.) The general form of the mathematical equation that defines a parabola, with respect to an x–y coordinate system that originates at the top of the arch is:

$$y = ax^2$$

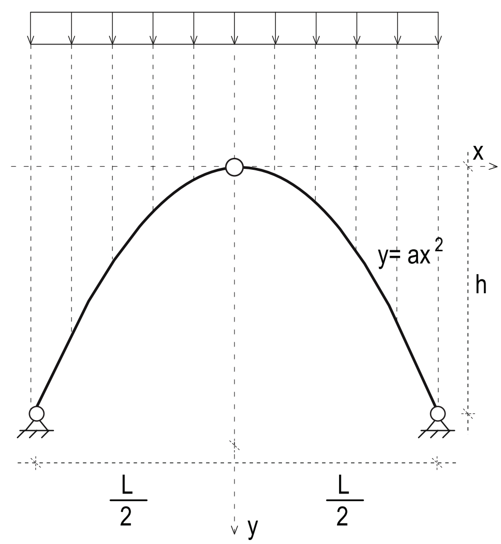


Figure 12.8
Parabolic arch form and equation.

Knowing that the arch passes through the point having coordinates (L/2, h) means, therefore, that

$$h = a (L/2)^2$$
$$a = 4h/L^2$$

So that the equation which defines the arch's profile is

$$y = (4h/L^2)\, x^2$$

For the parabolic arch form, therefore, equation (12.1) becomes:

$$M = [wL^2/8h] [(4h/L^2)(x^2)] - [w/2] [x^2]$$

which can be simplified to

$$M = [w/2] [x^2] - [w/2] [x^2]$$

Illustration 12.28
Santa Justa Train Station, Seville, Spain (1991).
Parabolic arch profile subject to uniform load: funicular form minimizes bending action and arch rib thickness.
Architect: Cruz y Ortiz Arquitectos. Structural engineer: INECO.

Since the two terms on the right are found to be equal to each other, we can now see that for a parabolic arch subject to uniform vertical loading there will be no bending moment anywhere in the arch – i.e., we have a funicular form. This is irrespective of the arch height and span. Of course, things can get somewhat more complicated than this with the real-life variability of loading, but it is an important lesson nonetheless for the efficient shaping of arched structures, particularly those in which gravity dead loads predominate. (e.g., Ill. 12.28.)

If, on the other hand, an arch carrying a uniformly distributed vertical load is anything other than parabola-shaped, it can be inferred and shown that it will experience bending. This will be demonstrated for a semicircular arch (Fig. 12.9), whose profile's mathematical equation relative to a coordinate system that originates at the apex of the arch shape is

$$y = h - \sqrt{(h^2 - x^2)}$$

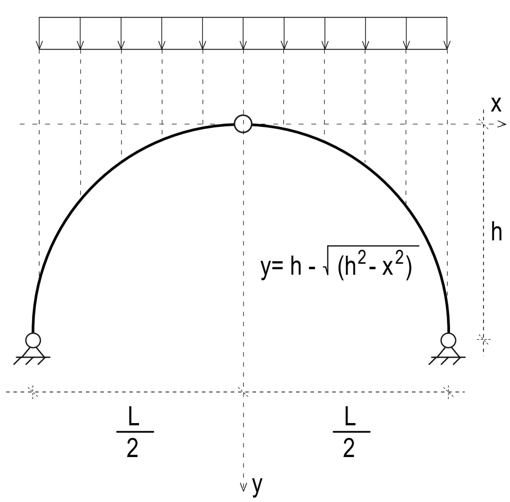


Figure 12.9
Semicircular arch form and equation.

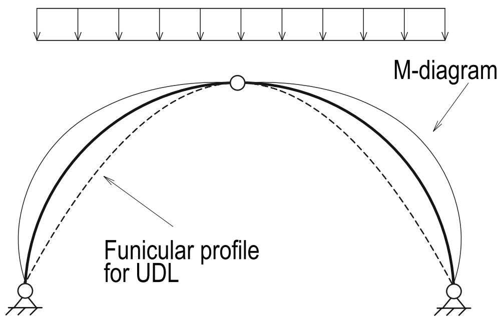


Figure 12.10
Relative magnitude of bending moment in semicircular arch corresponds to deviation of arch centerline from parabolic funicular profile for UDL condition.

Illustration 12.29
Sports Hall, École d'Ingénieurs ESIEE, Marne-la-Vallée, France (1987).
Semicircular arch profile subject to uniform load: rib thickness needs to account for bending as well as compression.
Architect: Dominique Perrault Architecte (DPA). Structural engineer: B.E.F.S. S.A.

Substituting for y into equation (12.1), we find that

$$M = [wL^2/8h]\,[h - \sqrt{(h^2 - x^2)}] - [w/2]\,[x^2]$$

In this case, the two terms on the right-hand side are generally not equal to each other, and the bending moment is therefore not equal to zero everywhere as it was for the parabola. Using the arch profile as a baseline, the variation of bending moments calculated in this manner at various points along the arch can be presented graphically. In fact, it can be demonstrated that the magnitude of the bending moment at any point is proportional to the magnitude of the deviation of the arch profile from the parabolic funicular compression line for this load condition. (Fig. 12.10., e.g. Ill. 12.29.) Other arch shapes besides the semicircle can also be analyzed in an analogous fashion, and we will find that the bending moment will again vary in magnitude along the arch profile. In fact, such will be the case for any non-parabolic arch form to which uniform loading is applied, or conversely, for any parabolic arch form to which non-uniform loading is applied.

To conclude this section, let us reexamine all this from a slightly different point of view: i.e., in terms of the combined states of stress in the arch. For example, for a parabola-shaped arch that has uniformly distributed load applied along the length of the span the arch's funicular line of compression runs along the arch centerline; i.e., at each cross-section of the arch, the compression forces will be centered at mid-thickness of the arch material and everywhere the magnitude of the stresses will be uniform across the thickness. If we now change the profile of the arch but we do not modify the loading distribution the position of the funicular compression line will no longer be at the centerline of the structural form. The stresses in the arch will then generally involve a superposition of tensile-to-compressive bending stresses in addition to the uniform compressive stresses typically associated with arch action. (Fig. 12.11.) Whether we in fact still have net compression stresses all over a cross-section will obviously depend upon the relative magnitudes of these superimposed sets of stresses.[6] If the bending stresses become large enough, net tension over a portion of a cross-section may result, with the potential for consequent problems if the arch material is not up to the task.

As a general summary, then, we have seen that arches will experience *combined* axial compression and bending moment sets of stresses. Two examples that reflect well this fundamental

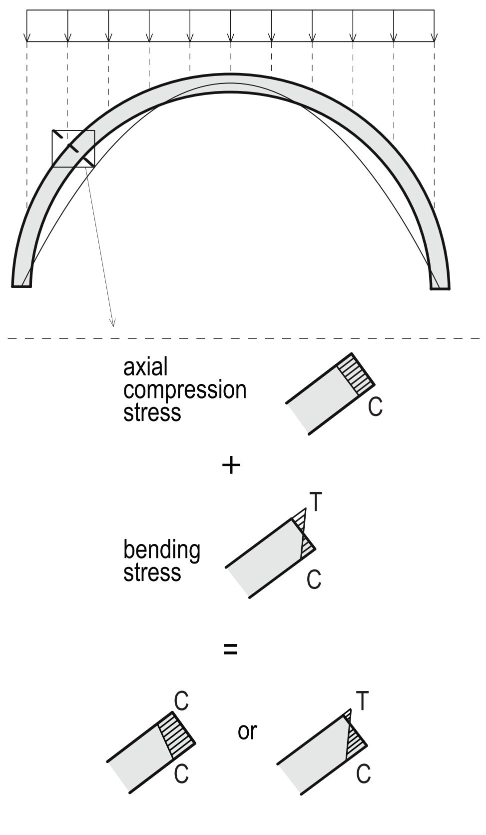
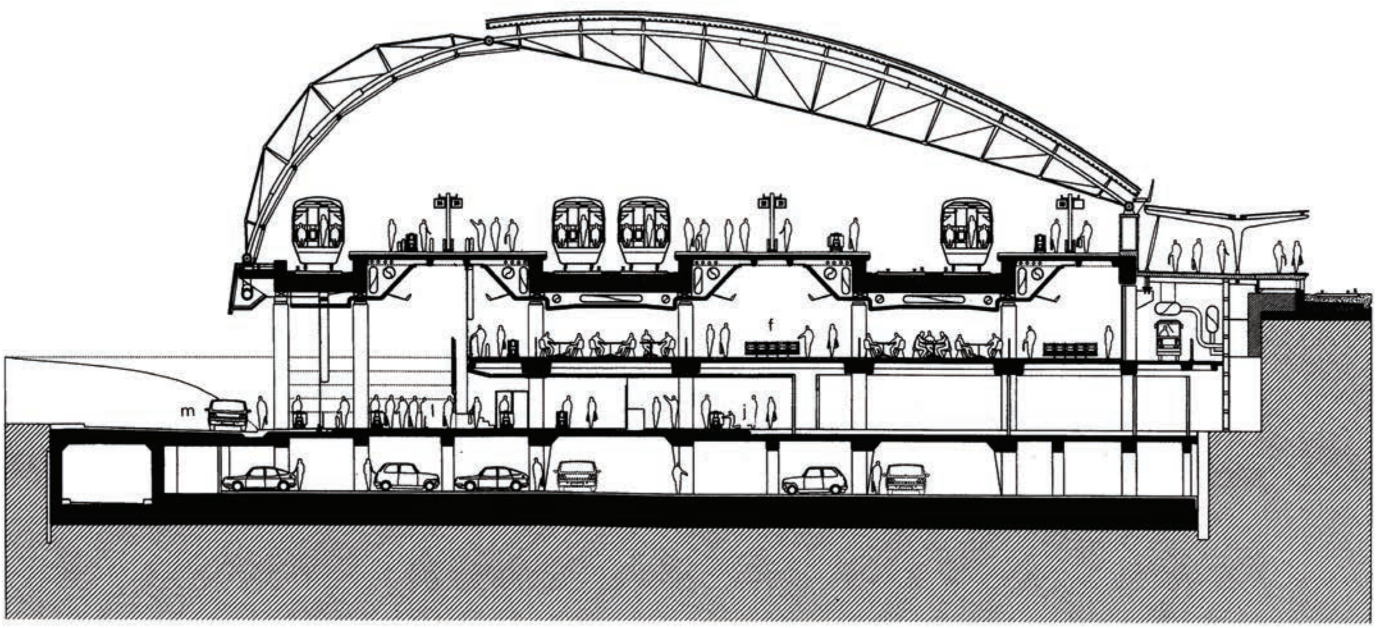


Figure 12.11
Superposition of axial compressive stress and bending stresses; different relative magnitudes of these may or may not result in net tension stresses at various locations along the arch and across its depth/thickness.

Illustration 12.30
Salginatobel Bridge, near Schiers, Switzerland (1930).
Vertical walls of three-hinged concrete arch vary in dimension according to bending moment demand.
Designer and structural engineer: Robert Maillart.



cross section through departure and arrival halls

Illustration 12.31
Waterloo Train Station. London, UK (1993).
Trussed arch ribs of roof vary in depth/thickness, from a minimum at three hinge points to maximum values halfway between. The truss ribs are triangulated in the third dimension so as to be able to resist buckling.
Architect: Nicholas Grimshaw and Partners. Structural engineer: Anthony Hunt Associates.

dual behavior by emphasizing the strong form-making influence of variations in flexural demand are Robert Maillart's (1872–1940) reinforced concrete Salginatobel Bridge (Ill. 12.30), and Nicholas Grimshaw's steel-trussed Waterloo Train Station. (Ill. 12.31.) In both cases, the arches are true three-hinged arches, and they can clearly be seen to thicken quite substantially where bending moments are largest and to have impressively minimal dimensions at the hinge points, bringing to mind the opportunity for variations in beam and truss depths that we saw in Chapters 7 and 9, respectively. For the case of the Waterloo Station it is also to be noted that the hinge is not at the top of the arch, nor is the arch symmetrical, suggesting the possibility of shaping arch profiles according to other design concerns – in this case the desire to emphasize the height of the left glazed side of the arched space that opens up toward the adjacent urban context as opposed to that of the right lower, almost solidly clad side that is next to a much less attractive expanse of commuter railway tracks.

A very different strategy for dealing with the same characteristic bending response of arches is also clearly illustrated on the façades of the Broadgate Exchange House (Ill. 12.32, 12.33), where diagonal members are used to stabilize relatively thin steel arches of constant dimension instead of having them be substantially thickened as in the two preceding examples. Beyond this, however, with the bases of the Broadgate's outwardly thrusting arches lifted so clearly up off the ground on to pedestals that are visibly linked to each other by means of horizontal tension ties, the next section's primary topic of discussion – that of arch supports and foundations – is also conveniently introduced.

Illustration 12.32
Broadgate Exchange House, London, UK (1990). Relatively thin arches support multistory building; arch profile is stabilized by diagonal members. Outward thrusts at base supports negated through horizontal tension tie. (See Ill. 6.29.)

Architect: Skidmore, Owings & Merrill (SOM). Structural engineer: SOM.

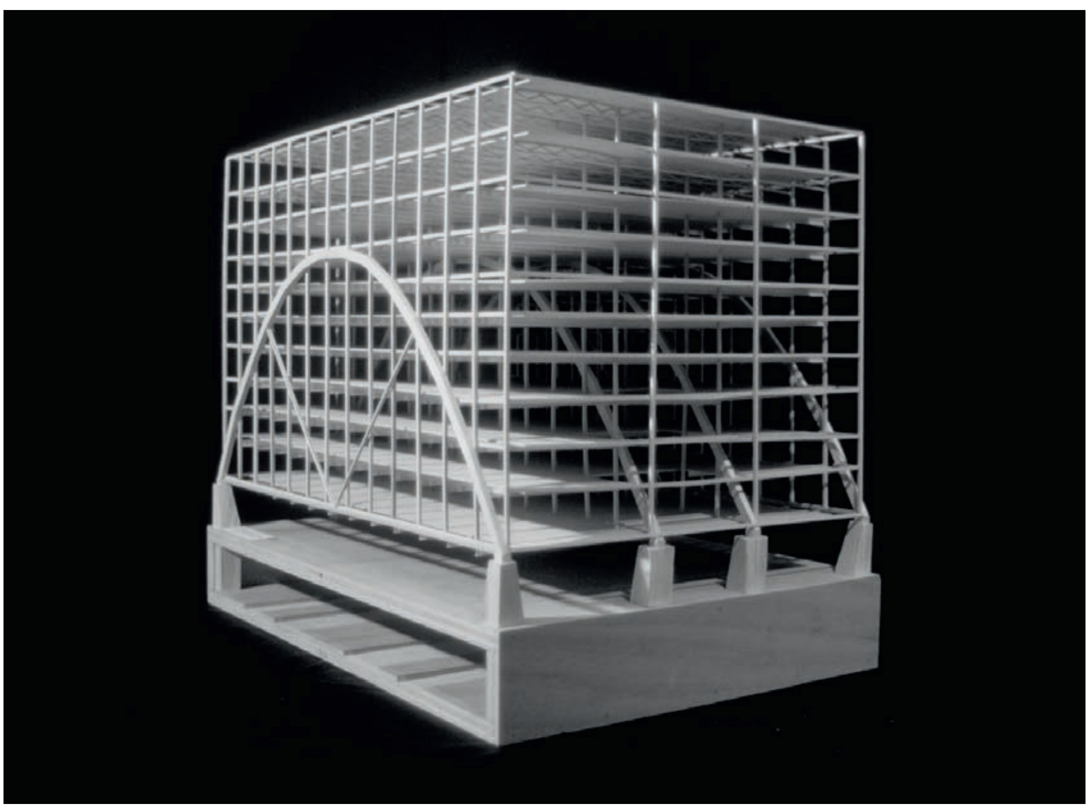


Illustration 12.33
Broadgate Exchange House. Four arches support the multiple floors of the building, two on the outside faces, two internally.

Cornell model by Jennifer Miller.

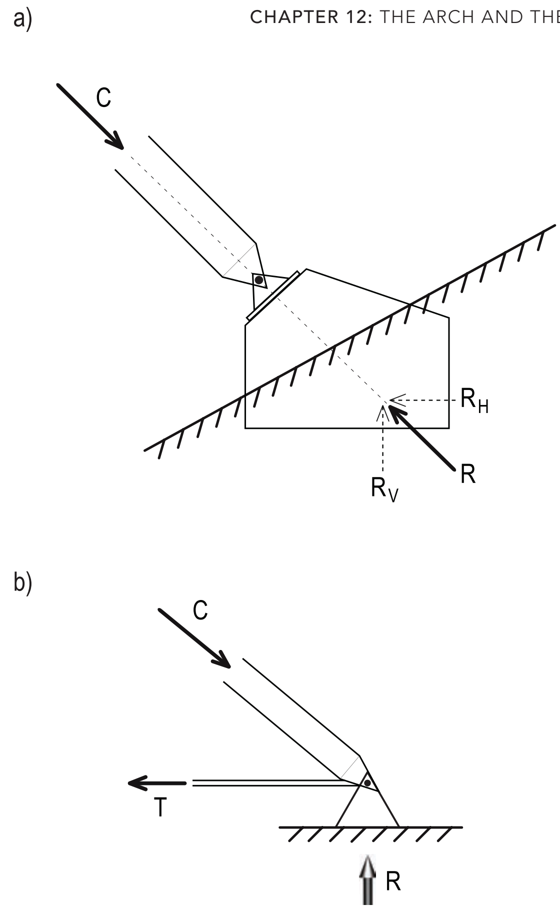


Figure 12.12
Alternate end support conditions to resolve outward thrust of arch: (a) foundation/ground able to counter directly by balancing inward compression force, (b) tie rod counters by means of tension needing to be anchored at opposite support.

12.7 The Foundations of the Arch

The way in which a building meets the ground can lead to its architectonic and structural clarification. In contrast to more conventional vertical load-carrying systems such as columns and walls, arches with their sweeping curving profiles and outwardly thrusting compression forces have a much more dynamic quality about them, with the consequent potential for distinctive visual expression at the supports. Resolving the play of forces at the transition between the arch and its foundation is therefore an important aspect in the design of such structures: we shall now look at a few ways in which the arch "lands."

Where competent ground conditions can be found, such as rock, the arch's outward and downward push is allowed to bear almost directly against it (Fig. 12.12a); this has frequently been the way in which bridges across steep valleys have been built, such as that just seen with the Salginatobel Bridge. (See Ill. 12.30.) An alternative "grounded" strategy can be found in the abutments for the bridge at East 45th Street in New York City, not far from Grand Central Station. (Ill. 12.34.) Here the load-bearing arches are built in steel while the foundations use the rather incredible massiveness of granite-clad concrete blocks to transfer the thrust of the arches into the ground. The load transfer point is given special design attention with an angled surface that meets the compression of the arch. In this design there is a clear articulation and transition in terms of material, mass, and geometry of what is being supported and what is doing the supporting, with the angle of inclination of the receiving support accentuating the arch's outward thrust into the massive block. Such a thin-metal-arch-to-massive-foundation resolution has many variations, and its basic configuration was sketched by da Vinci hundreds of years ago. (See Ill. 12.15, lower drawing.) A more recent example of this approach can be seen in the low slung arches of Marc Mimram's Passerelle Léopold Sédar-Senghor in Paris where the required resistance to very strong lateral thrusts is resolved by massive concrete abutments measuring up

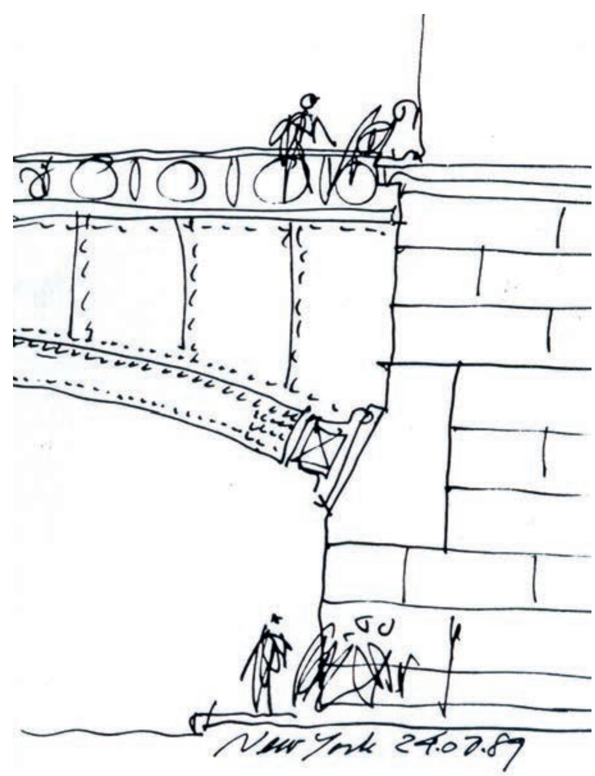


Illustration 12.34
East 45th St. overpass, New York City, NY, USA. Massive stone-clad block meets and counters outward-thrusting steel arch.

Illustration 12.35
Passerelle Léopold Sédar-Senghor (formerly the Passerelle Solférino), Paris, France (1999).
Open Vierendeel-form arch rib of footbridge's lower pathway pushes hard into massive concrete foundation blocks embedded in the walled banks of the Seine.
Architect and structural engineer: Marc Mimram.

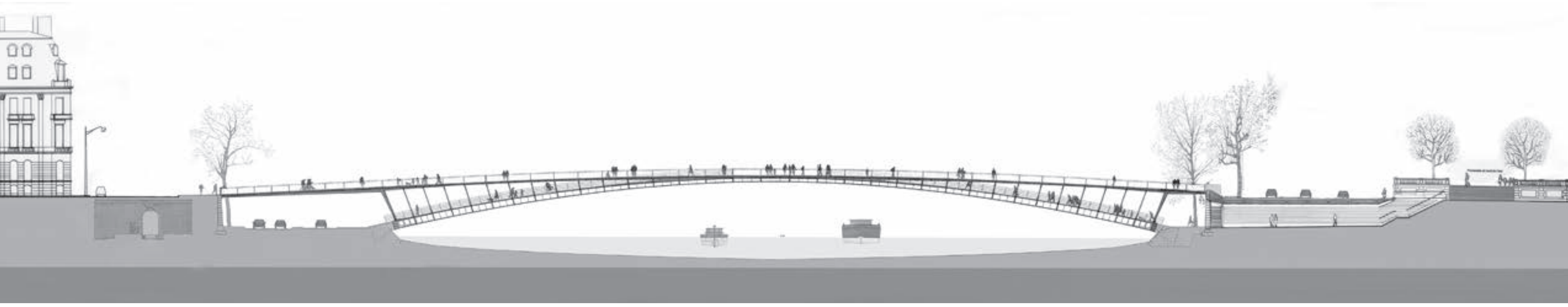


Illustration 12.36
Passerelle Léopold Sédar-Senghor.
Section highlights the low profile of the arch, multiple pathways along and over the arch, and integration of bridge into urban context.

to 15m x 15m x 18m (50ft x 50ft x 59ft) that weigh 9731 tons (21 450kips) and that are embedded into the banks of the Seine River.[7] (Ill. 12.35, 12.36.)

Alternative strategies such as friction or end-bearing piles may be needed for the foundations of arch supports where ground conditions are very poor and unable to resist the arch's outward thrust or the weight of such massive abutments. And when it is desired that the arch be elevated up off the ground and made to "float" on vertical piers, as was just seen at the Broadgate Exchange House (Ill. 12.32), a different method of addressing the arch's horizontal support can involve the use of a tension tie so that the outward push at one end is visually and physically balanced against that at the other. (Fig. 12.12b.) In fact, this has been an approach that has been frequently used over time, whether to lighten the potential massiveness of supports on the open side of a *transversely* vaulted arcade, such as at the Ospedale degli Innocenti (Ill. 12.37) designed by Filippo Brunelleschi (1377–1446) or, as we shall see shortly, to seemingly "float" the undulating ceramic tile roof over the Barcelona food market structure designed by Enric Miralles/Benedetta Tagliabue (Section 12.8).

Along the *length* of an arcade, the series of arches that typically abut one another have the advantage of base thrusts that conveniently negate or balance one another, resulting in the need to carry only vertical forces at the intermediate supports and allowing the possibility for these to therefore be much more slender than might otherwise be expected to support an arch. (Fig. 12.13.) The same will obviously be true for the mid-river piers of the myriad multiple-arch bridges of the world, representing a

Illustration 12.37
Ospedale degli Innocenti arcade, Florence, Italy (1424). Into-the-courtyard thrust of arcade's transverse vaults countered by tension tie rods.
Architect: Filippo Brunelleschi.

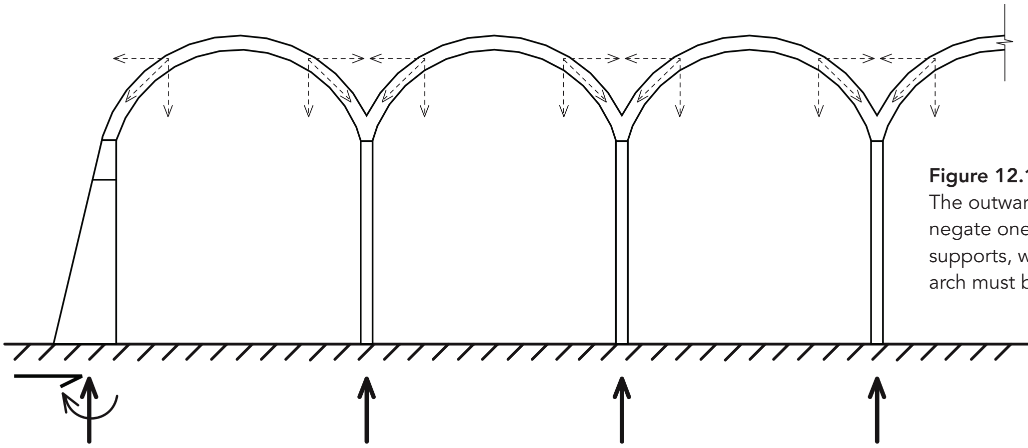


Figure 12.13
The outward thrusts of adjacent arches along arcade negate one another, allowing for slender interior supports, whereas the unbalanced thrust of the exterior arch must be countered by the outermost support.

Illustration 12.38
Santa Caterina Market, Barcelona, Spain (2005).
Urban context of market; undulating roof covered by multi-colored ceramic tiles patterned on pixelated image of fruits and vegetables. At center, piercing through roof, are three great transverse steel-trussed arch ribs from which the middle part of roof is suspended.
Architect: Enric Miralles/Benedetta Tagliabue of EMBT. Structural engineer: Robert Brufau and José Maria Velasco (roof).

distinct advantage in terms of reducing any obstruction to water flow (e.g., see Ill. 12.10). Only at the ends of such an arcade or series of arches will the last, unbalanced outward arch thrust need to be dealt with expressly by one means or another.

However an arch's support is provided, we should once again emphasize that the reactions' essential function is to establish the arch's primary load-bearing mechanism of compression. If the tendency of the arch to push outward is *not* resisted in some way then the arch mechanism will not work, whether the structure has an arch-like form or not. Indeed, if it is not resisted, what we are essentially left with is a structure that has no choice other than to work as a beam (albeit a beam having a curved elevational profile) that carries load *only* by means of what we know to be the much-less-efficient bending stress mechanism and, therefore, results in a structure having much larger cross-sectional dimensions. If the support conditions prevent outward movement, however, the converse becomes true: compressive arch action is developed that allows the possibility for carrying greater loads or spanning much longer distances – an attribute of the arch that has long been recognized and applied over the course of architectural history and that continues to be strategically used to this day.

12.8 Santa Caterina Market – A Roof Takes Flight

Like other European cities, Barcelona has several covered public food markets that each form an essential part of its neighborhoods' quality of life, both in an economic and a social sense. The site of the Santa Caterina Market in the Gothic quarter of the city near the medieval cathedral has a rich architectural history dating back at least to the Roman necropolis unearthed during excavations for the new building. Framed on three sides by the preexisting arched walls of the previous market from the nineteenth century, the structure by architects Enric Miralles (1955–2000) and Benedetta Tagliabue was opened in 2005. It essentially consists of a "flying carpet" roof of brightly colored ceramic tiles that provides inspiring and lofty spaces for the varied daily activities of its users: 60 vendors' stalls share the open floor space with cafés, a supermarket, community services, and an area that is reserved for exhibiting the historical artifacts that have been discovered on the site.

Architects sometimes refer to the roof as a building's fifth elevation, and this is undoubtedly the case here where the ridges and valleys of the irregularly folded surface can be prominently seen from many elevated vantage points. (Ill. 12.38.) Drawing upon the Catalan tradition of glazed tile ornamentation used by such Barcelona architects as Antoni Gaudí (1852–1926) and Lluís Montaner (1850–1923), the undulating roof over the market of Santa Caterina

Illustration 12.39
Santa Caterina Market.
Varying profiles of three-hinged glulam timber arches create undulations of roof. Bottom end of arches carried by steel trusses that run length of market, supported at the ends by columns and at center by being hung from three large transverse arches. Ends of the latter are tied together by pairs of tie rods, seen in foreground of image.

is clad in 300 000 hexagonal tiles that form an abstract, pixelated pattern of market fruits and vegetables in 67 different colors; its inventive graphic and material design involved a combination of computer simulations and practical testing.

Under the tiles, three layers of pinewood planking help shape and stiffen the curved roof surfaces that are supported on a series of three-hinged glue-laminated timber arches. (Ill. 12.39.) These arches' profiles change as a reflection of the varying height and width of the roof undulations, while their bases are supported on a set of horizontal tubular steel trusses that run the length of the building at the bottom of the roof's "valleys." The trusses are carried at the ends of the market on tree-like columns, but in the middle of the open market space they are suspended from above on vertical hangers coming down from three great transverse tied arches that dramatically pierce through the roof surface in places. (Ill. 12.40.) The structural system for the roof thus can be seen to have a richly interwoven and hierarchical complexity as each layer is laid perpendicular to the one supporting it but these also overlap and intersect spatially with one another. Adding further to the visual richness of this busy place, it is at times hard to distinguish between the rehabilitated parts of the preexisting market structure and the new construction; historical walls and timber roof trusses are cleverly incorporated into the perimeter service areas of the new design.

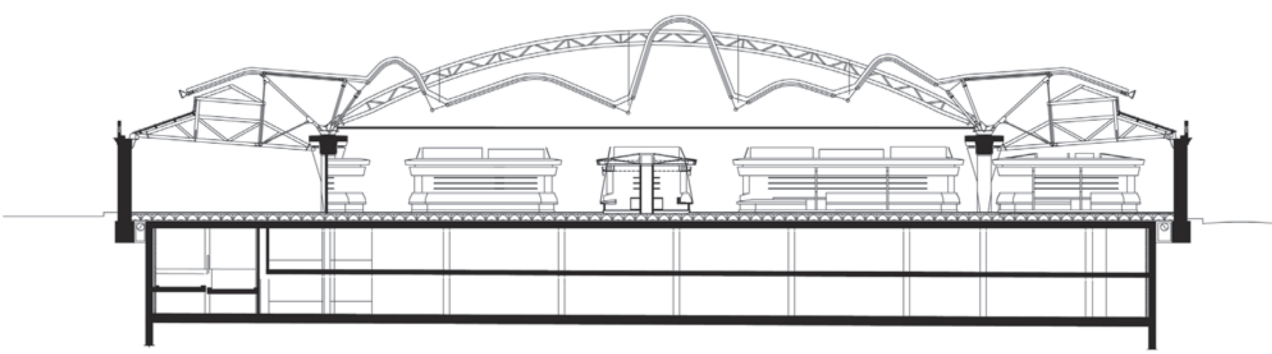


Illustration 12.40
Santa Caterina Market.
Section through structure, highlighting the series of three-hinged glulam arches that create the varied, undulating roof form, as well as the trussed "great" arches that span from side to side of the market and whose base supports are connected by horizontal tension ties.

Illustration 12.41
Art Warehouse, Boeotia, Greece (2013).
Smooth, continuous reinforced concrete arched form establishes most basic vaulted structure and space for this art studio/warehouse/gallery.
Architect: A31 Architecture.

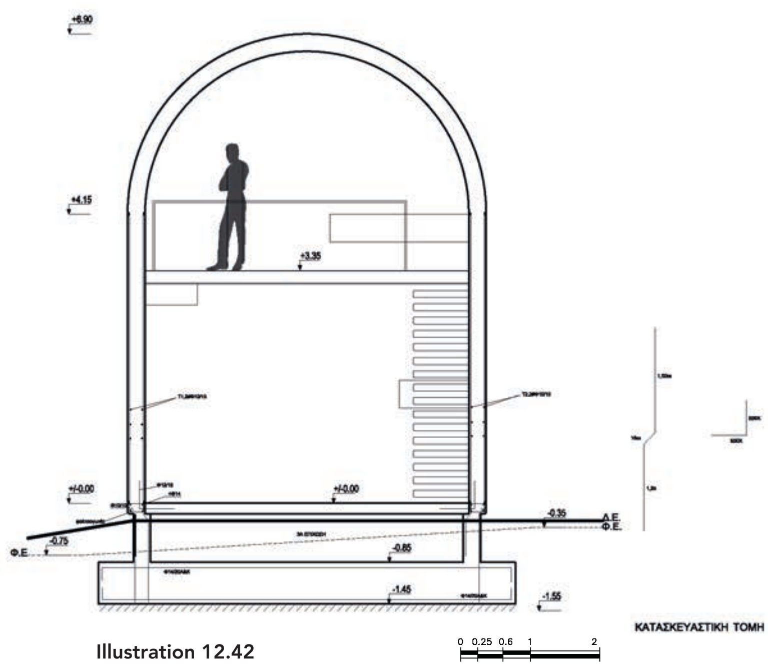


Illustration 12.42
Boeotia Art Warehouse.
Section drawing reveals relative simplicity of reinforced concrete vaulted form. Also indicated is that there are two layers of steel reinforcing bars within the thickness of the concrete – one each close to the outer and inner surfaces – so as to be able to resist any transverse bending that may occur in addition to the more commonly expected compression stresses.

12.9 The Vault and Light

Until this point we have mostly discussed the shapes and structural behavior of arches in the two dimensions of their elevational profile, but we have not expressly considered what is needed for these in the third dimension of space. A single arch standing on its own will of course need to be able to resist all lateral forces that are applied transversely to it, essentially by means of two curved, vertically cantilevered structures that join together at the top of the arch. It is typical, therefore, to find that a single arch will need to have significant width/thickness and structural capacity perpendicular to its plane, as can readily be observed in the case of the Gateway Arch. (See Ill. 12.11.)

But the more common situation in an architectural context is to have either a continuously curved surface with an arched cross-section or a series of arched ribs placed one after another along a longitudinal axis, thus creating what is commonly referred to as a vault. The extension of the arch into the three-dimensional vault form has been tremendously important in the history of building. At its simplest, the space created runs continuously in an arched cross-sectional form from one end of a building to the other, thereby creating a room with a raised central ceiling and a strong directional orientation. (e.g., Ill. 12.41, 12.42.) This room is typically used for building projects that serve special

Illustration 12.43
Kimmel Center for the Performing Arts, Philadelphia, PA, USA (2001).
Glazed vaulted form envelopes and unifies multiple smaller volumes and program elements; it is also a distinctive profile on skyline.

Architect: Rafael Viñoly Architects. Structural engineer: Dewhurst Macfarlane and Partners in association with Goldreich Engineering, PC.

functions; e.g., churches, train stations, markets, athletic facilities, and museums are only some of the building programs that are frequently associated with the vaulted form. The cross-sectional profile can put special emphasis on the central axis, while the vault's length, with or without repetitive arched ribs, encourages linear programmatic functions and progressions; i.e., the basic part of cathedrals' central naves and altars or of train stations' platforms leading to head terminal buildings are spatial organizations that readily lend themselves to this form. Because of its embracing shape the vault can be considered to have an intrinsic communal quality and an ability to gather and shelter many people in an inwardly oriented, womb-like interior space. And the distinctively curved and bulging shape of the arched vault on the outside can also certainly offer the opportunity to highlight the presence of such structures and its contained programmatic functions. (e.g., Ill. 12.43.)

While in its cross-sectional form a vault can be understood to behave structurally as a set of independent arches that just happen to be located next to one another, it remains for us to consider how such a structure is stabilized in the direction of the

Illustration 12.44
Sea Folk Museum, Toba, Japan (1992).
Glue-laminated arches mimic construction of ribs of the inverted hull of a wooden ship; stabilizing and partially enclosing solid surface made of wood planking is like that of its hull. Daylighting is carefully controlled, entering museum along ridgeline at top and alongside water's edge at bottom.
Architect: Naito Architect and Associates. Structural engineer: Structural Design Group Co., Ltd.

Illustration 12.45
Sea Folk Museum.
Bracing between adjacent parallel arch ribs stabilizes the system that might otherwise tend to fall over against each other like a series of dominoes.

vault's longitudinal axis. Clearly there is structural advantage to be gained for this purpose by linking one transverse arch rib to another rather than having them all acting independently of one another. Frequently this interconnection is made by means of a relatively solid and continuous enclosing roof surface, as seen in the appropriately boat-like wood planking at the Sea Folk Museum (Ill. 12.44, 12.45), but it can also be provided by means of angled cross-bracing members running in the vault surface between the series of arch ribs. As we have discussed previously in other contexts, such a bracing system provides its own opportunity for additional visual expression, detailing, and for the admission of light into the space. At the Berlin Central Rail Station, for example, the series of transverse trussed arch ribs are stabilized by many thin diagonal bracing rods that not only transfer any longitudinal loading to the ground but also provide the desired high degree of visual transparency for the glazed vault surface. (Ill. 12.46.) As yet another example of the infinite number of possibilities, the stiffening of the glass vault of the Kimmel Center in downtown Philadelphia is accomplished by means of a folded Vierendeel steel framework (Ill. 12.43) that serves the double structural function of (a) giving

Illustration 12.46
Berlin Central Rail Station, Berlin, Germany (2006).
Thin, criss-crossing, diagonal rod system stiffens gridded shell structure (see Section 13.8) between intermittent arch ribs, allowing light to flood the station interior.
Architect: von Gerkan, Marg, und Partner. Structural engineer: Schlaich Bergermann und Partner.

the vault's transverse section its required bending stiffness (we will discuss the advantage of folding surfaces more fully in Section 13.5) and (b) providing what is essentially a stiffening rigid frame grid for the vault's longitudinal direction stability. All the while, this open structural framework also allows light to pass through relatively freely and makes the interior space an integral part of the surrounding urban environment.

It is by paying such simultaneous attention to structural necessity and detailing as well as to daylight or reflected artificial light that life is given to a vault. The variations in light tell us the time of day, which season it is, what the weather is like, and so forth. Whereas the light that enters a building through side openings in a façade quickly dissipates as it reaches deeper into the room, a long skylight along the ridge of a vault gives evenly dispersed light throughout the entire space. Moreover, such a visual opening gives us a direct view of the sky and a completely different interior light quality than that from lateral lighting, which is normally reflected from the surrounding buildings and landscape. From an architectural design perspective, therefore, it is important to understand that in order to have the same light intensity in a vaulted space we need

Illustration 12.47
Isfahan market, Iran.
Part of extensive network of covered street markets; small, intermittent openings in masonry vaulting allow only a limited amount of light to set the atmosphere of the space.

far less glass area when using skylights than when depending on lateral lighting. The direction that the light is coming from is also decisive in determining how we experience people and objects and spaces, a theme that has many variations.

The souk is a traditional street market in the Middle East and North Africa. Many of these streets are covered with various forms of masonry groined vaults, such as at Baza-e Bozorg in Isfahan, Iran, where interior covered streets run continuously for kilometers from the famous Maidğn-e Naqsh-e Jahğn to the Friday mosque. (Ill. 12.47.) Light from the sky seeps gently in through rectangular openings at the top of the vaults, contributing to the staging of daily life in the souk well away from the intense heat and beating sun of the world outside. It is natural in such climatic conditions to have only a few openings at the top of the vault for the controlled admission of light, but also to be able to evacuate the hot air that builds up and rises within such a space of intense human activity. Provision must obviously be made for the main arching compression forces to find a way around these openings that break the continuity of the vault surface, but since these are typically quite short and small this is relatively easily accomplished.

Illustration 12.48
Paddington Station, London, UK (1854).
Lighting is provided by continuous glazing in the upper portions of the ribbed vaults, following the direction of train movement and of passengers along the platforms, and accenting the innovations in structural forms and materials of the Industrial Revolution.
Designer and structural engineer: Isambard Kingdom Brunel.

London's great railway stations of the nineteenth century were, according to John Betjeman, "cathedrals of industrial architecture."[8] In a period that saw the huge expansion of the railway network in England, these stations represented the state of the art in technical innovation. Perhaps nowhere is this better demonstrated than at Isambard Kingdom Brunel's (1806–1859) Paddington Station that opened in 1854. (Ill. 12.48.) The column and trussed arch structure is wonderfully developed with one grand central arched span and two lesser vaults on the sides; moreover, two transepts give an additional spatial feeling to the station. Longitudinal stability of the vaults is achieved by cross-bracing the cast iron trussed arches. The glazed skylights are constructed as continuous ridges with the natural light washing over the many small members of the trussed arches, the train tracks, and the platforms, all contributing in an essential way to give the great hall its elegant yet industrious character. Today Paddington Station is as much alive as it ever was, and the relatively recent addition of a connection line to Heathrow Airport has given the station renewed relevance – something that would have no doubt delighted Brunel.

Illustration 12.49
Canary Wharf Underground Station, London, UK (1999).
Glass panels between arched ribs of entrance canopy at once draws daylight down into Underground station and people up to the ground surface; artificial lighting at night allows the distinctive form to announce the buried station's presence. (See also Ill. 12.27.)
Architect: Foster + Partners. Structural engineer: Arup.

A century and a half later, but not so far away geographically, the Jubilee Line extension to the London Underground system comprises 11 stations by just as many architects, of which the Canary Wharf Station by Norman Foster + Partners is the largest. By using cut-and-cover techniques, the 310m (1017ft) long station is laid out underneath a landscaped park where only its glazed canopies are visible above ground so as to provide access to but also to suggest the presence of the hidden world below. (Ill. 12.49.) The entrance canopies, one at each end of the station, are constructed by means of tapering box-section arches made of steel. Tubular members tie the arches together and also act as purlins supporting stainless steel "spider" connectors for affixing the glass panels between the arch ribs. While glowing with light at night-time, by day these structures allow the natural light to be carried deep down into the station to the platform level; orientation, always a problem in underground stations, is thereby dramatically enhanced.

Finally, and further demonstrating the many possibilities for interplay between arched vaults of various forms and light, is the example provided by St. Henry's Ecumenical Art Chapel in Turku, Finland. (Ill. 12.50.) Here, a series of tall, narrow, and sharply peaked glulam arches are stabilized in the longitudinal direction by solid timber planking on both sides, from top to bottom and along the full length of the vault – or, rather, almost so. For at the far end, at the altar, the siding stops just short, admitting a bright light that powerfully orients the space.

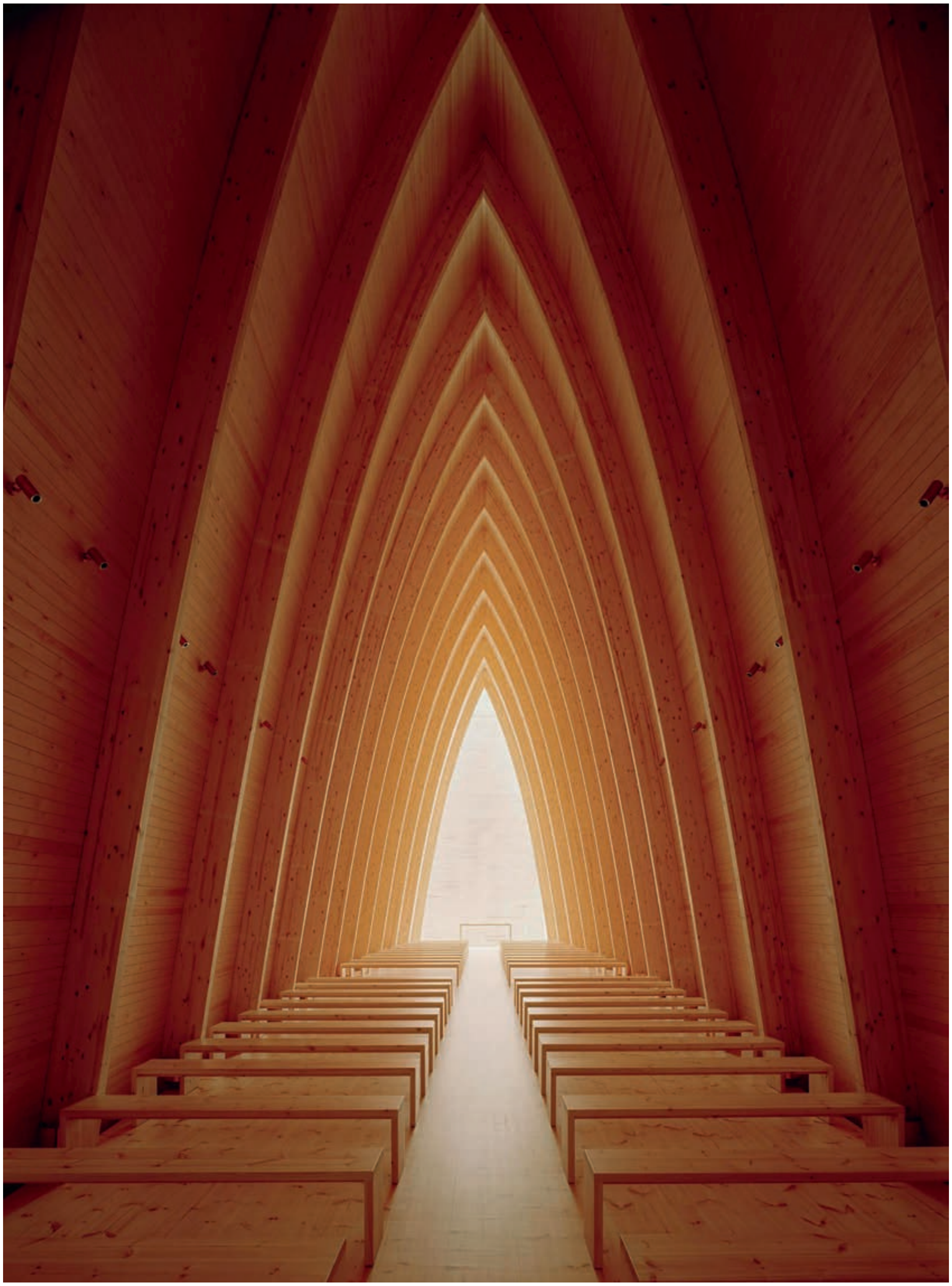


Illustration 12.50
St. Henry's Ecumenical Art Chapel, Turku, Finland (2005).
Glulam arch ribs carry gravity loads and have sufficient depth to stabilize the cross-section of the vault against deformation caused by wind loading on the sides of the chapel; in the longitudinal direction, wood planking effectively turns the two sides into shear walls. But these stop just short of far end of the space, allowing light to be admitted.
Architect: Sanaksenaho Architects. Structural engineer: Kalevi Narmala.

The Dome and the Shell

CHAPTER

13

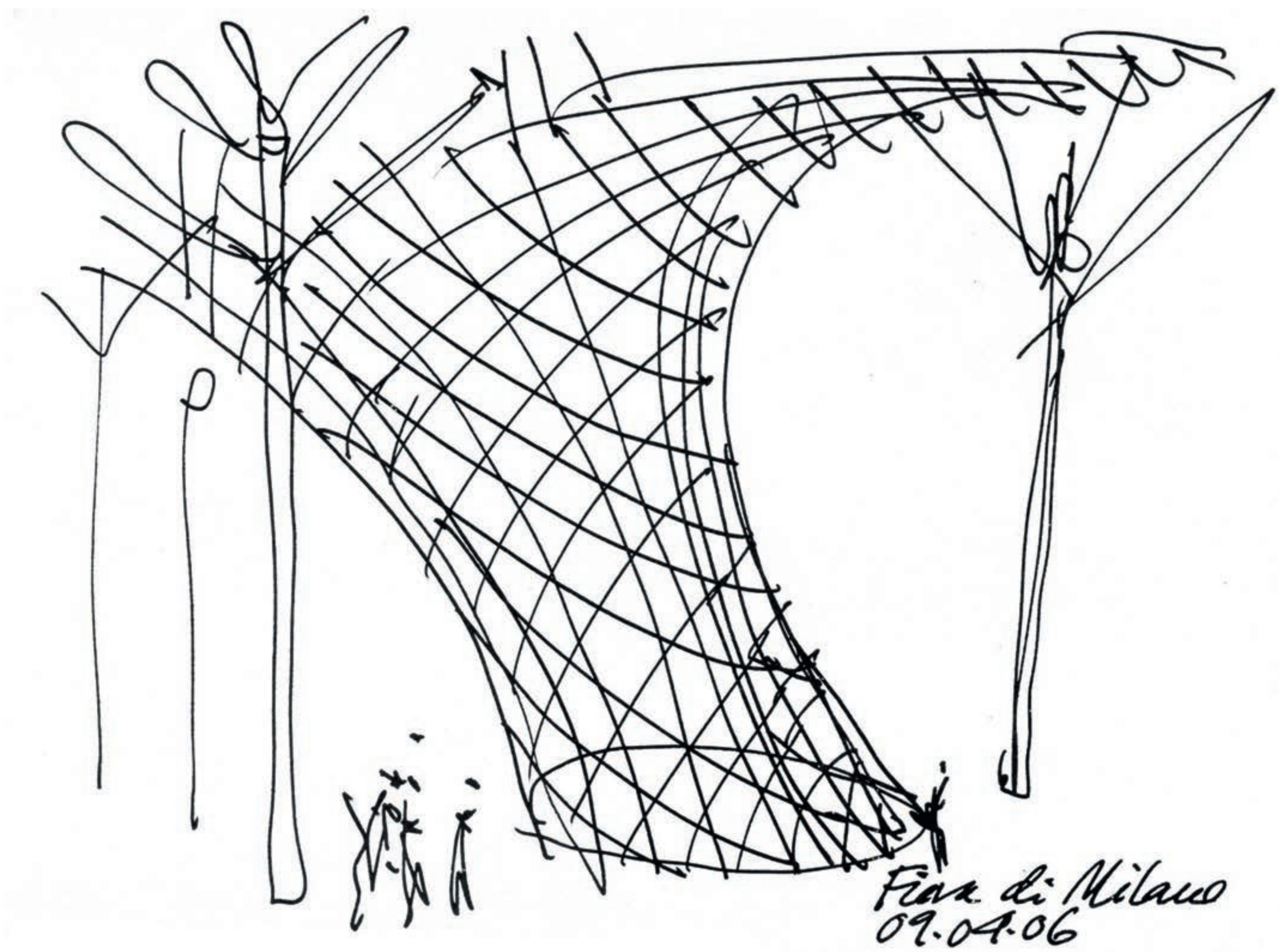


Illustration 13.1
Fiera Milano, Milan, Italy (2005).
Grid shell "funnel," part of the articulation of a kilometer-length glass canopy.
Architect: Massimiliano Fuksas Architetto. Structural engineer: Schlaich Bergermann und Partner together with Mero-TSK International.

Illustration 13.2
The Eden Project, St. Bazey, Cornwall, England, UK (2001).
Bubble-like domes nestled into landscape. Exterior surface covered by inflated EFTE pillows, whose relative transparency to UV rays promotes the growth of plants in one of the world's largest greenhouses.
Architect: Nicholas Grimshaw and Partners. Structural engineer: Anthony Hunt and Associates with, for the domes, MERO (UK).

13.1 Geodesic Domes in the Landscape

In an abandoned china-clay pit in the midst of the green hills and valleys of Cornwall is the Eden Project, a complex of greenhouse structures designed by Nicholas Grimshaw and Partners that are used to showcase global biodiversity and humans' dependence on plants. A winding path leads down from the visitor center on the edge of the site to reveal an amazing scene as one walks toward a series of overlapping, soap-bubble-like domes of various sizes that are elegantly nestled into the landscape. (Ill. 13.2.) The large, transparent domes each encapsulate a different climatic region of the earth and, in keeping with the project's programmatic objectives, these were designed with ecological and sustainability issues in mind: i.e., the overall structural form of the steel geodesic domes is exceedingly light and efficient in terms of material usage, and the innovative foil bubble enclosure system carries that design objective to an even higher level.

The project easily reveals its fundamental geometry: the eight intersecting spherical domes are actually the top segments of several imaginary globes whose centers are located deep underground. The domes have diameters ranging from 18m (59ft) to 65m (213ft) in accordance with the various heights of the plants that are native to the different biomes being sheltered. (Ill. 13.3.) Structurally, each dome is of geodesic form, which generally can be defined as an overall curved surface that is made up of many small flat panels of triangular, pentagonal, hexagonal, or other polygonal shapes that are connected together, with each panel having a slightly different orientation than its neighbors so as to produce an overall rounded surface. The geodesic system is typically achieved by means of lightweight steel tubing defining the various polygonal shapes and that are connected together at their intersection points by means of patented cast steel spherical nodes. Because the number of nodes has a significant impact upon the cost of building such a structure, it is usual to try and use the largest possible polygon modules.

In order to give stability to such a lightweight and seemingly flimsy assembly, larger geodesic structures need to have an effective thickness to them, something typically accomplished by interconnecting inner and outer layers of elements. At the Eden Project an outer geodesic layer is formed by 625 hexagons and 16 pentagons at the top of the spheres, whereas an inner layer is created by 190 large triangles. (Ill. 13.4.) The interconnection of these two layers is achieved by yet more steel tubing elements that join the many nodes of the two surfaces, thus creating multifaceted three-dimensional polygonal shapes within the dome thickness; in the present case, for example, icosahedrons of 20 plane faces can be identified. Such complex geometry can be readily modeled by computer software programs, with the resulting information also able to be communicated to automated production lines that can cut the myriad pieces to exacting specifications. The assembly of the structure is then "merely" done as a kit-of-parts on site, although the size of the temporary scaffolding needed to build the Eden Project domes set world records.

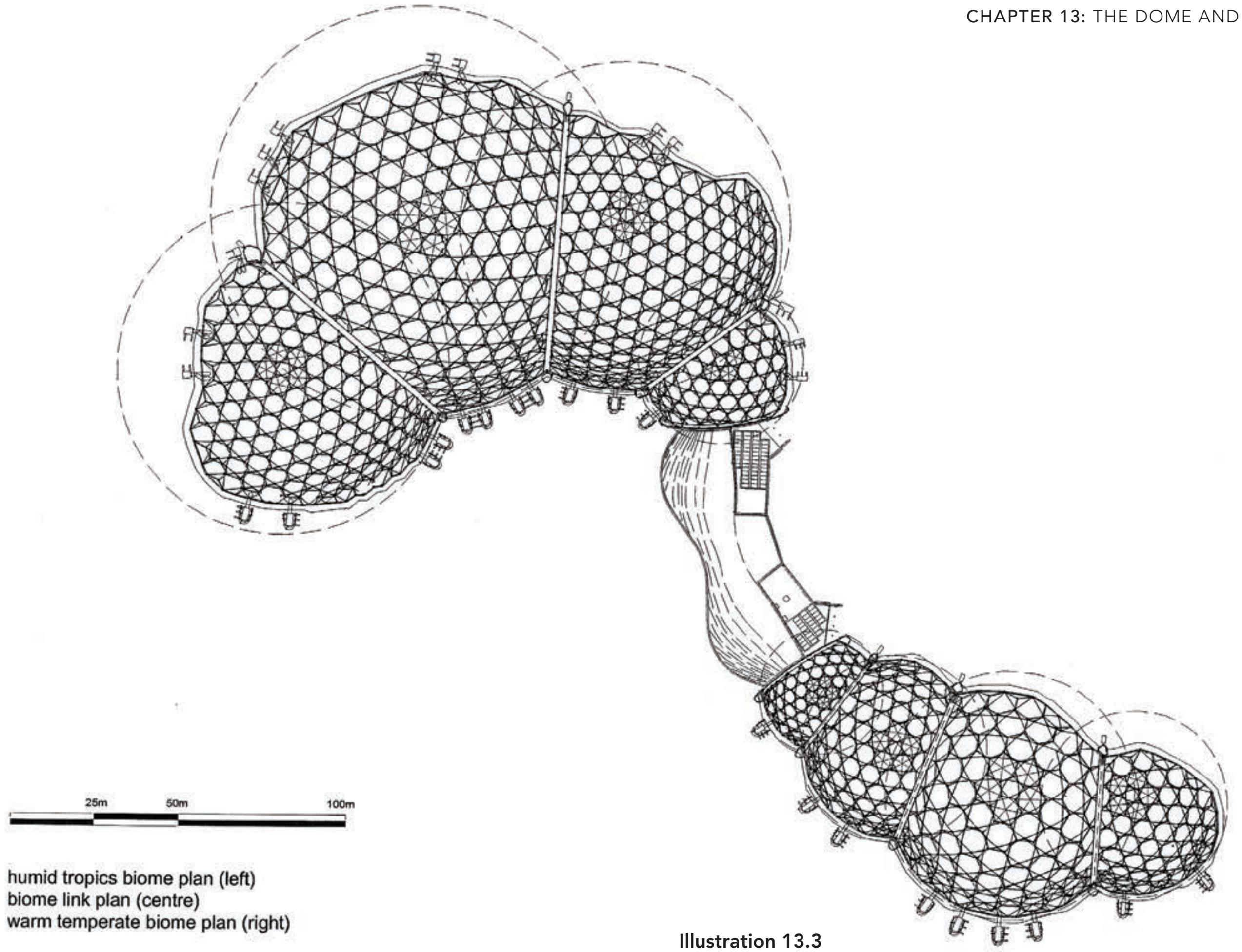


Illustration 13.3
The Eden Project.
Plan arrangement of the project's many domes; various sizes accommodate the plants of different bio-climatic regions of the world.

Illustration 13.4
The Eden Project.
Domes are composed of double-layer surfaces for stability; the geometry of the outer layer is hexagonal, that of the inner layer is triangular.

Illustration 13.5
United States Pavilion at Expo ’67, Montréal, QC, Canada (1967).
Perhaps the most famous of geodesic domes. Today the open steel framework lives on as the Montréal Biosphère, a museum dedicated to the environment.
Architect: Buckminster Fuller. Structural engineer: Simpson Gumpertz and Heger Inc.

A further distinguishing feature of the Eden Project domes is that their cladding system is made of inflated foil pillows that fit into the hexagons of the outer geodesic layer. Each pillow consists of three layers of ETFE foil (ethylene-tetra-fluoro-ethylene, described in Section 5.8) that are placed on top of each other and then sealed together along the sides; the resulting two chambers are inflated with air. From a structural point of view, the opposite curvatures of the two faces of the pillows means that the inner layer can deal with the tendency to deform inward due to wind pressure or snow loads, while the outer layer is able to handle any tendency of the pillow to bulge outward due to wind suction. The layered air cushions also provide considerable thermal insulation to the interior environment, and this insulation can in fact be regulated by adjusting the air pressure in the pillows. The polymer ETFE material also has exceptional properties in the context of this project: i.e., it is more transparent than glass with regard to ultraviolet light penetration, which is obviously a necessary characteristic for proper plant growth. Finally, and perhaps most amazingly, the foil pillows weigh less than 1 percent of the dead weight of an equivalent area of insulating double-layer glass – thus contributing greatly to reducing of the overall amount of material needed for these domes’ steel-tube structural elements.

In a sense, then, a geodesic dome can be thought of as a sort of space-frame-like or latticed structure applied to the enclosure of a spherical space. It is an extremely lightweight system that is able to cover a very large space by means of the interplay of many small, short elements. Moreover, as we will see later in this chapter, the lightness and efficiency of domes in general as an overall system for carrying load means that very large spans and enclosed volumes of space can be achieved with support only needing to be provided around the base perimeter of the structure – i.e., without any interior and space-intervening columns.

The development of the Eden Project can be directly linked to the pioneering work of the American engineer, designer, and philosopher Richard Buckminster (“Bucky”) Fuller and his vision of maximum enclosed volume within a minimum surface area. Fuller’s early geodesic dome designs of the 1940s aroused great interest at the time in schools of architecture around the world. The United States Pavilion built for the World’s Fair in Montréal in 1967 remains today as perhaps the most compelling built example of the

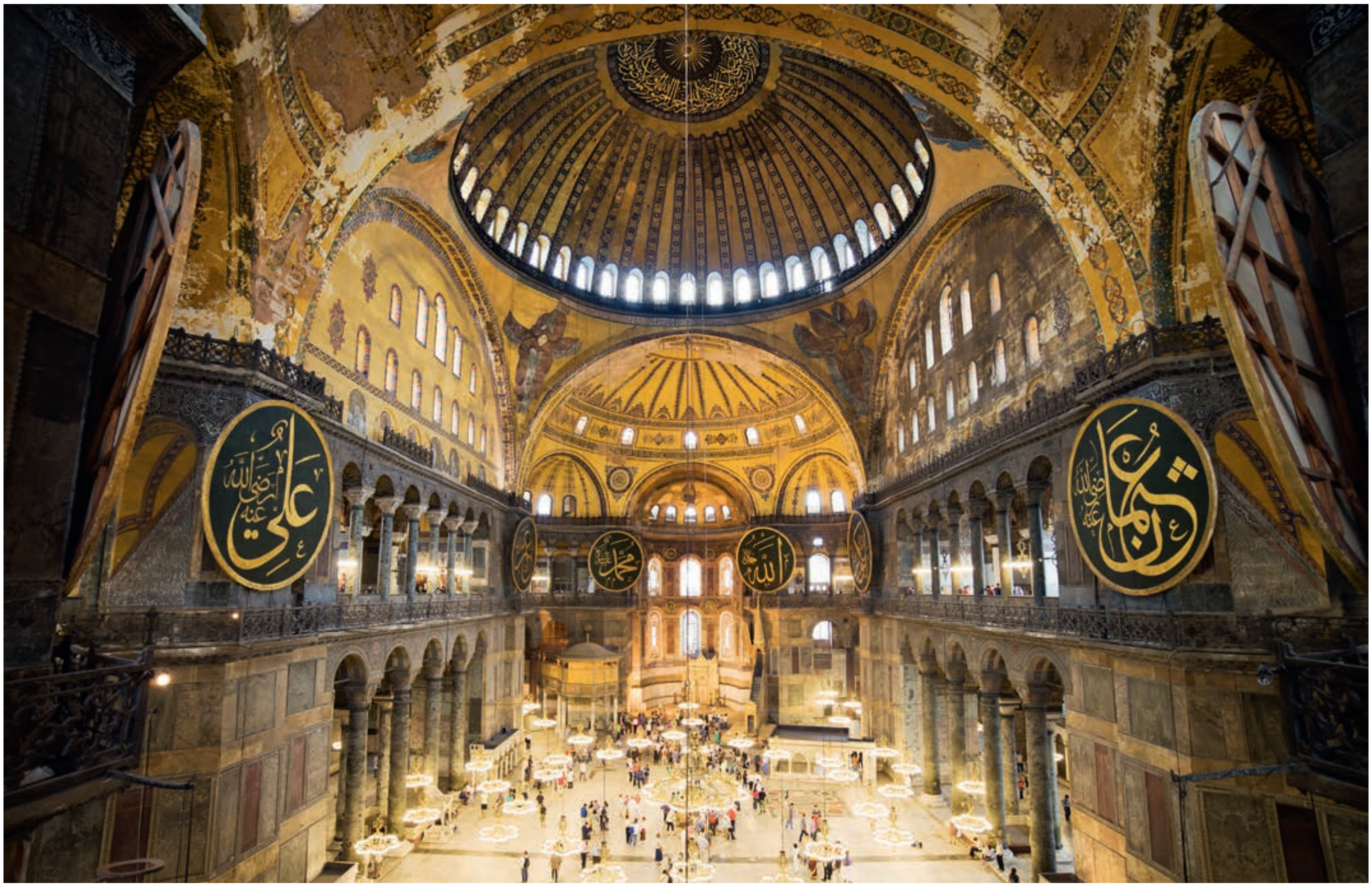


Illustration 13.6
Hagia Sophia, Istanbul, Turkey (537).
Interior of domed and semi-domed space dwarfs human scale.
Architects: Isidorus and Anthemios.

tremendous spatial possibilities of this form of construction. (Ill. 13.5.) It is also interesting to note in the context of the cladding discussion above that this dome was originally covered with lightweight acrylic panels, but that these burned in a fire in 1976. Fortunately, the fire consumed the acrylic panels so quickly that it left intact the open steel latticework as an ongoing and compelling reminder of Fuller's futuristic ambitions; today, it encapsulates a museum dedicated to the environment, and the so-called "Bucky-dome" continues to stand out on the Montréal skyline, with nightly lightshows highlighting and playing off of its iconic form.

In self-serving fashion, one of Fuller's typical leading questions was to ask of other projects: "How much does the building weigh?" and the answer in the case of the Eden Project is a remarkably minimal 667 metric tonnes (735 US tons). To give a measure of comparison, this is the equivalent of the weight of about seven of the Valley Temple stones at Chefren that we saw earlier in Section 7.2; i.e., the number of pieces of granite needed to form just three of the closely spaced portals, each comprising two columns topped by a beam. The contrast in terms of the volume of space contained by equal weights of construction materials could hardly be greater.

13.2 Traditional Dome – Arch Action Revisited

In the preceding chapter we mentioned that a vault can be understood both spatially and in terms of its structural behavior as a series of separate arches placed side-by-side along a linear axis. In an analogous manner, a dome can be thought of as a spatial form that is created by an arch that is spun about a central vertical axis. This will lead to the characteristic domed shape that has long dominated urban skylines and whose lofty spaces we experience with awe and craned necks from within. (e.g., Ill. 13.6.)

A particular type of dome can in fact be built in a similar manner to the form we have just described, with a set of radial arch ribs connecting the top of the dome to its base. (Fig. 13.1.) In such a situation, loads are carried in a similar manner to the arch action that we became familiar with in the last chapter: i.e., the dome ribs carry loads in compression just as arches do, and these will resist any potential changes in shape by means of their flexural stiffness. As will be recalled, it is this combined compression-plus-bending

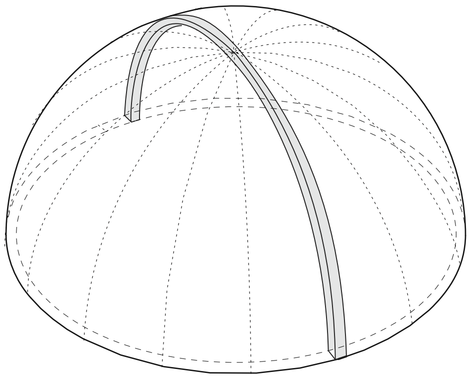


Figure 13.1
Dome created by "thick" arch ribs having flexural (bending) stiffness, all intersecting at central vertical axis.

behavior that is fundamental to understanding how arches work and that, therefore, also describes how *ribbed* domes function.

Two examples of such ribbed dome structures, linked through time both programmatically and symbolically, are those sitting atop the historic United States Capitol Building in Washington, DC (Ill. 13.7) and the relatively recent addition to the roof of the Reichstag parliament building in Berlin.[1] (Ill. 13.8, 13.9) In the case of the Capitol Building dome, the rib structural depth and flexural stiffness is provided by means of cast iron trusswork, whereas for the Reichstag dome the bending capacity comes from triangularly shaped hollow tubes made of welded steel plates.

While still considering the workings of this relatively basic type of dome created by intersecting arched ribs, we can use the opportunity to make other observations about this structural form that will remain relevant for the other types of dome structures that we will encounter shortly. Especially noteworthy, in this regard, is the typical need for two horizontal rings linking all of the ribs, one at the top of the dome and another around its base. (Fig. 13.2.) At the top, the need to resolve what would otherwise be a hopeless congestion of structural materials as the arch ribs all try to intersect at a single, common point is typically resolved by a circular opening, termed an oculus (or "eye" to/from the sky, for obvious reasons). Compression in each of the arch ribs means that the ring forming the opening will be subject to symmetrical radial inward forces, causing it to be in uniform circumferential compression. Of course, aside from structural necessity the oculus also often serves other architectural functions, such as for the admission of light and

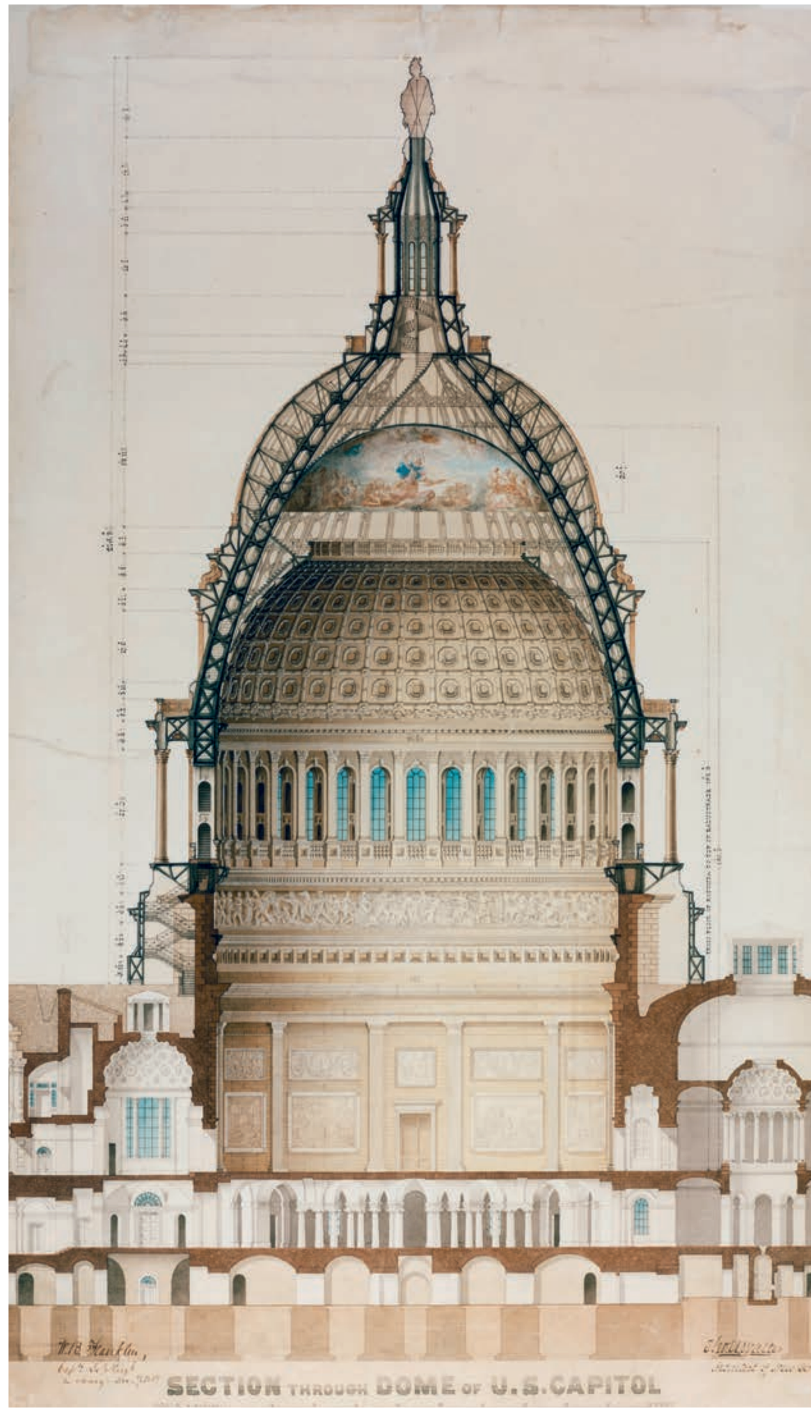


Illustration 13.7
Capitol Building, Washington, DC, USA (1866).
Section highlights single elongated, curving, cast iron structural rib, of which there are 36 around the perimeter; the famous outer dome profile and two inner domes are created by working off of this ribbed structure.
Architect: Thomas U. Walter. Engineer in charge of construction: Montgomery C. Meigs.

Illustration 13.8
Reichstag, Berlin, Germany (1999).
Dome was built on top of historic building as a symbol for the reunification of Germany and open parliamentary government.
Architect: Norman Foster + Partners. Structural engineer: Leonhardt Andrä und Partner.

Illustration 13.9
Reichstag dome.
Glazing between the steel ribs of the dome allows light to bounce off central mirror core to bring natural light into Bundestag debating chamber below. Spiral walkway brackets off ribs, also allowing for views of unified city.

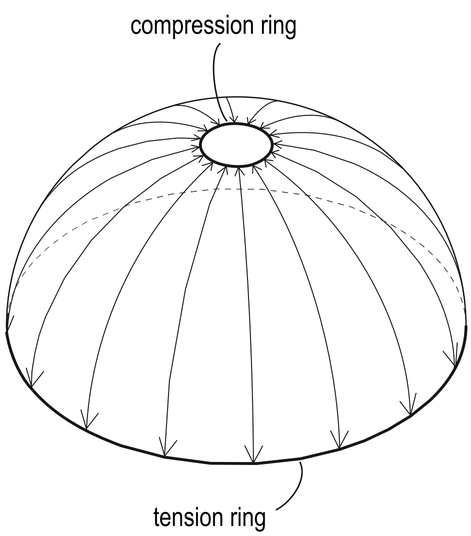


Figure 13.2
Compression and tension rings balance radial inward push and outward thrust at top and bottom of dome, respectively.

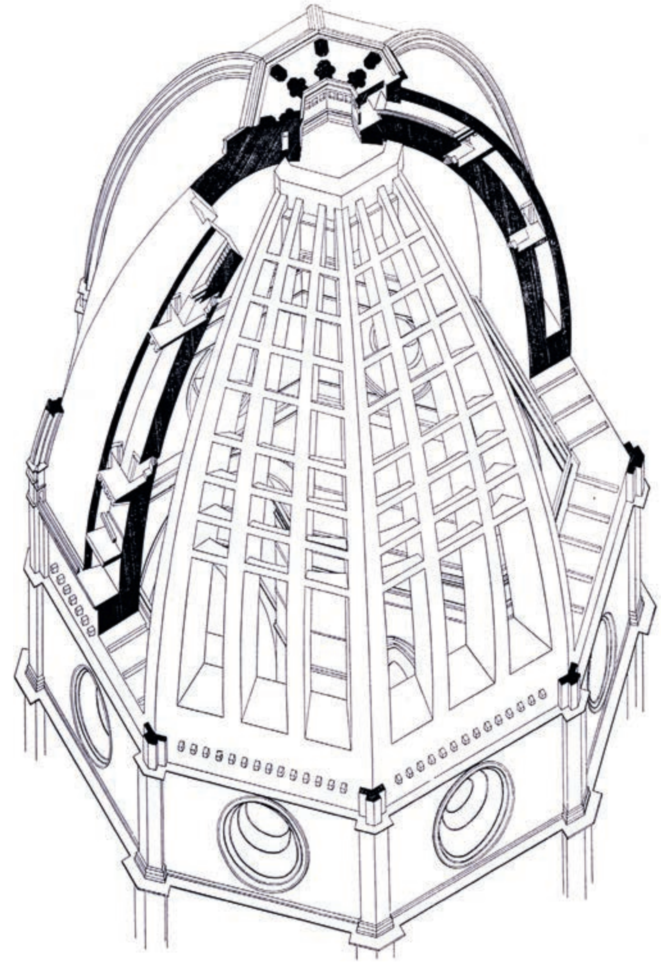


Illustration 13.10
Basilica di Santa Maria del Fiore, Florence, Italy (1436). Axon cut-away reveals the arch-like ribs of the dome that are joined at the top in an ocular compression ring, as well as the dome's exterior and interior surfaces. The radial outward thrusts at the bottom of the dome are resisted by a tension ring – see detail, Ill. 13.11.
Architect: Filippo Brunelleschi. From Sanpaolesi, pl. VII.

the evacuation of heated air that naturally rises into a domed space. For structural and visual and symbolic reasons, therefore, the oculus has been a feature of domes that has long attracted much design attention – the most famous example undoubtedly being the completely-open-to-the-sky aperture in the dome of the Pantheon in Rome. (see Ill. 13.19, 13.20; more about this structure follows in Section 13.4).

At the base of the dome, on the other hand, the outward thrusts resulting from the compression in all of the arched ribs also needs to be resolved. To do so, one option is to provide a base ring that will be subject to a set of outward radial horizontal forces that will as a result put it into a state of circumferential tension. (Fig. 13.2.) Such a tension ring can be thought of as the three-dimensional equivalent of the straight tie rod that we used to anchor the support points of a two-dimensional arch in Section 12.7. Examples of tension rings that have been used over the course of time have included iron chains for St. Peter's Basilica in Rome, and sawn timber members linked together by means of carefully detailed metal dowels in Florence's Duomo (Ill. 13.10, 13.11); more recently, at the bases of contemporary domes,

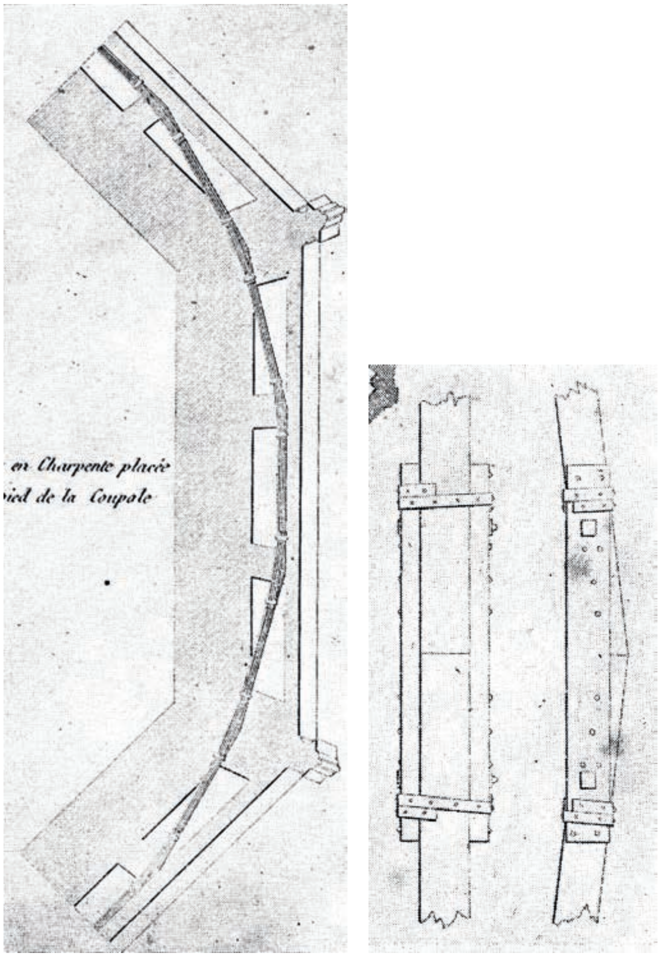


Illustration 13.11
Basilica di Santa Maria del Fiore.
(left) Partial plan view of timber tension ring at base of dome.
(right) Detail of tension ring "chain," composed of timber segments linked by sets of iron plates.
From Rondelet, pl. 189.

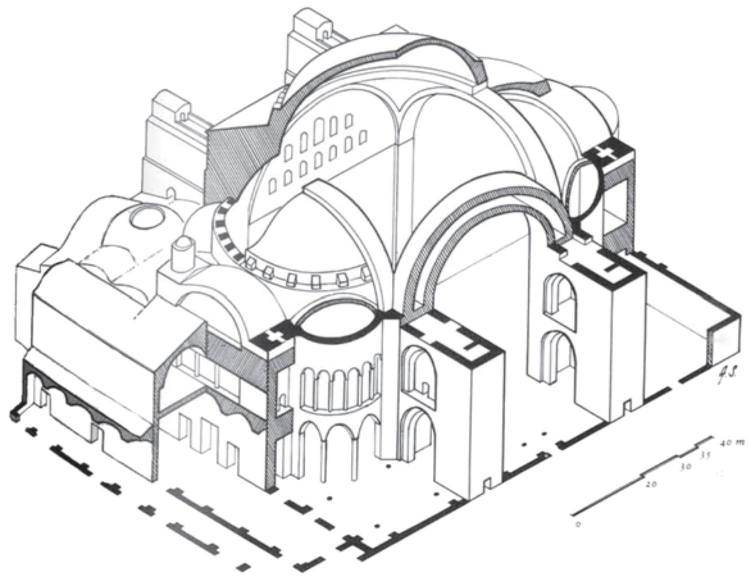


Illustration 13.12
Hagia Sophia, Istanbul, Turkey (537).
Heavy masonry walls buttress outward lateral thrusts of central dome in one direction, while a succession of relatively shallow dome segments and supporting columns and walls do the same in the other. Gradually increasing height of "piled-up" structures along sides allows for relatively shallow-angled transmission of thrusts from base of dome down to ground. (Also, Ill. 13.6.)
Architects: Isidorus and Anthemios.

continuous steel hoops or post-tensioned wire strands encased within concrete base rings have achieved the same essential load-balancing objectives.

But just as for the arch, alternative methods exist to balance the outward thrusts at the base of a dome besides anchoring it by means of a tension ring. For example, the large, relatively flat dome of Hagia Sophia, whose base is lifted high up into the air in order to make possible the incredible interior space, can be seen to be anchored by thick buttressing walls all around that act as orthogonal sets of shear walls (Chapter 10) that resist the outward thrust at the base of the dome as well as carry the dome's heavy gravity loads (Ill. 13.12), and a remarkably similar strategy is employed with a series of radial cross walls present through the thickness of the cylindrical drum at Rome's Pantheon.[2] Yet another alternative to resolving the radial outward thrusts is the provision of a series of inclined compressive buttresses angling inward to resist the outward push occurring at the base of a dome, as can be seen in another famous Roman dome: the Palazzetto dello Sport. (Ill. 13.13.)

Illustration 13.13
Palazzetto dello Sport, Rome, Italy (1957).
Exterior of concrete dome, with Y-shaped buttresses around perimeter to counter outward radial thrusts.
Architect: Annibale Vitellozzi. Structural engineer: Pier Luigi Nervi.

But at this point we need to go back to where we started with the dome, for while it was convenient to begin with familiar arch-like radial ribs in terms of gaining a basic understanding of dome behavior, it is a fact that not all domes are built this way; indeed, many of the world's most famous and structurally daring domes quite clearly are not. Such domes can be observed to have a more uniform thickness of material throughout their form, with no evidence whatsoever of thickened arching ribs. Just how this seemingly more monolithic form of construction functions needs some explanation.

To begin to visualize what is happening, we must examine the deformation tendencies of a hemispheric dome under uniform vertical loading (caused by its self-weight, for example). The top part of the dome, being relatively horizontal, will tend to be pushed inward relative to its original profile; the lower, more vertical parts of the dome, on the other hand, will bulge outward from the original shape (Fig. 13.3); analogously, such deformation tendencies can easily be observed by pushing downward on a tennis ball. The outward "ballooning" of the lower part implies an expansion of the material surface, and will necessarily result in tension stresses immediately being established in a circumferential hooping fashion around the lower part of the dome, while in the upper part the hoop forces will be compressive.[3] The forces that follow the arched sections of the dome, called meridional forces, will always be compressive.

Since we know that the masonry of such historical domes is a material that is notoriously weak in tension, "failure" of a sort would have ensued immediately with a series of cracks running up from the base of the dome. (Fig. 13.4.) Such cracking can be observed in everyday life by trying to flatten out the spherically shaped peel

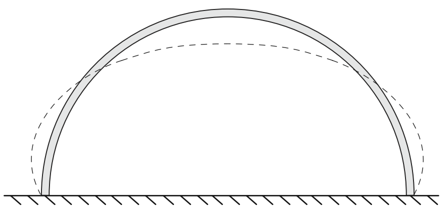


Figure 13.3
Deformation tendencies of dome due to vertical gravitational loads: top sinks inward, sides bulge outward.

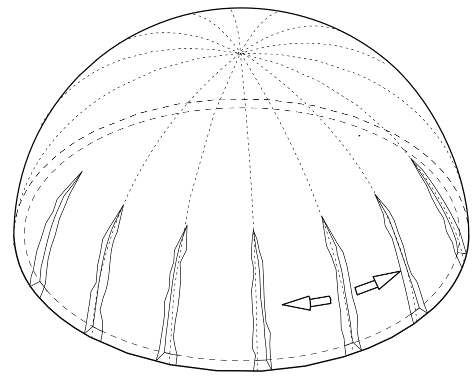


Figure 13.4
Radial cracks tend to develop around lower half of hemispherical masonry dome due to circumferential tension stresses (also called hoop stresses) caused by tendency to bulge outward.

of an orange, and in architecture this phenomenon has long been observed in masonry domes that have survived hundreds and even thousands of years.[4] It is interesting to speculate that perhaps it was the observation of such a cracking pattern around the base of domes that might have led ancient builders to realize that they could without distress open up the base of domes with regularly spaced window openings, thereby phenomenally "lifting" domes up into the sky. (Ill. 13.14.)

In historic masonry domes of hemispheric shape, then, the hoop tension forces in the lower part of the dome were often resisted by iron chains. Given that, the reaction forces needed along their circular support only needed to be vertical, and the dome could be lifted upward on a vertical, cylindrical "drum." If, on the other hand, a *shallow* dome of this type was used, where only the segmental upper part of the spherical dome remained (such as at Hagia Sophia, see Ill. 13.12) in which both meridional forces and hoop forces are compressive, this worked very well for domes made of masonry since this material has negligible tension strength. Anything comes at a cost, however: the support reaction forces that are needed for such shallow domes are steeply inclined, which means that there will be considerable horizontal force components still needing to be dealt with. In traditional masonry domes those forces typically have been resisted by buttresses or semi-domes around the perimeter. (In more modern shallow domes made of reinforced concrete this horizontal thrust at the support may be dealt with by a tie ring, perhaps made of post-tensioned concrete.)

In terms of the load-carrying behavior of the masonry dome surface itself, the cracking pattern that we have described effectively means that these can be considered to be, in structural essence, nothing but a series of wedge-shaped arch ribs rotated about a central vertical axis. The load-carrying strategy for such wedge-shaped ribs is no different from that for the more obvious ribs discussed earlier: i.e., they both carry load primarily in compression

Illustration 13.14
St. Peter's Basilica, Vatican City (1590).
Window openings around base of dome not only phenomenally "lift" it up into the air, but also may be a reflection of observations of radial cracking around base of such masonry structures.
Architect: Michelangelo (with the involvement of others before and after).

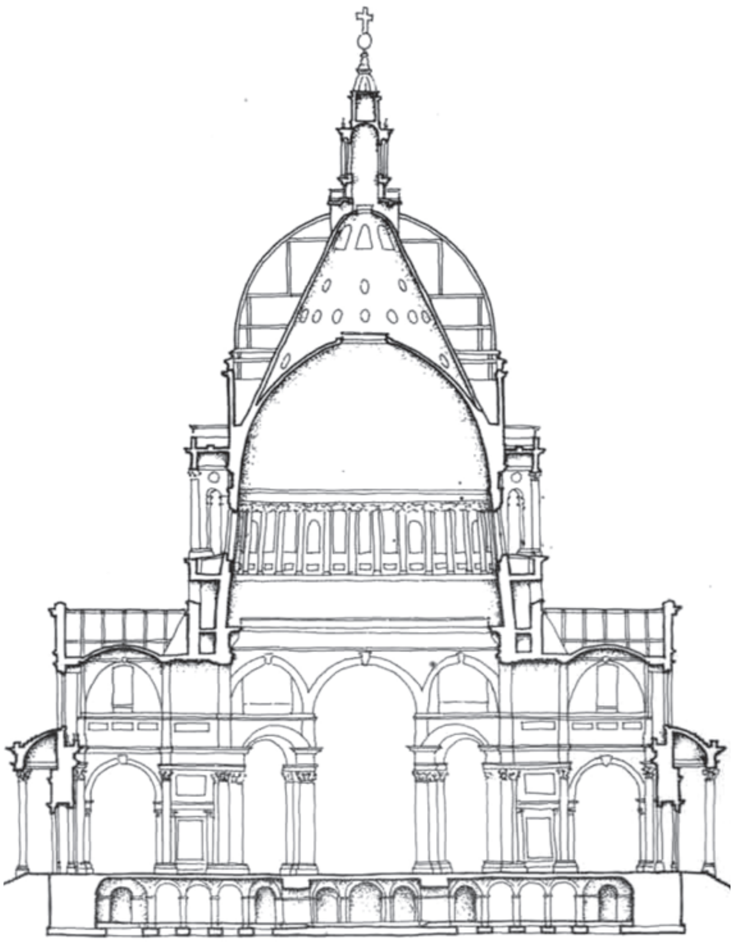


Illustration 13.15
St. Paul's Cathedral, London, UK (1711).
Section suggests funicular conical shape of structural masonry dome that carries large concentrated load of lantern at top as well as distributed load from timber framing that creates more "proper" outer dome shape to suit profile-on-the-skyline expectations. Third, internal, hemispherical dome also added inside to meet conventional aesthetic expectations for its painted surface. (See also Section 5.1.)
Architect: Christopher Wren.

but resist any changes in shape by arch-like bending action taken care of by their significant thickness. Indeed, the relative thickness of masonry domes (typical thickness-to-span ratios are roughly equivalent to those of conventional ribbed arches) can thus be understood and justified, and contrasted with the remarkable slenderness of modern shell domes that we will discuss next.

Before moving on, however, it is noteworthy that, just as with the arch, this problematic reliance on bending behavior in historical masonry domes (problematic in the sense that it tends to lead to heavy, thick structures) has long been recognized, and clever means have been taken to reduce bending action as much as possible. For example, domes have sometimes been shaped to best suit the primary structural load condition, such as at St. Paul's Cathedral in London where the large lantern load at the top is carried on an elongated masonry dome of conical shape (the three-dimensional equivalent of the funicular, compression-only profile of a point-loaded two-dimensional arch). (Ill. 13.15.) As we discussed in Chapter 5, Christopher Wren then "clipped" on to this primary load-bearing cone structure two "decorative" dome forms that were of a more visually familiar and acceptable shape: one on the outside that is supported off the cone by timber framing and another on the inside independently built up of masonry – the whole arrangement resulting in a visually misleading yet nevertheless ingenious and elegant three-layered system.

13.3 Shell Dome – Revolution in Structural Behavior

The thickness-to-span ratios of the domes that we have just discussed typically lie somewhere in the range of 1:50 to 1:100, whether these are of the explicitly radial-rib variety or else are domes that effectively become ribbed because of the vertical cracking of the surface that takes place as the base splays apart. Many modern and large-scale domes have been built, however, that are much thinner than this in proportion to the span, achieving remarkable ratios of up to 1:400 or 1:500; in other words, they are five to eight times as "slender" as the domes that we have considered until this point. Such proportions mimic and sometimes even go well beyond what can be found in nature in the smooth dome-like surfaces of

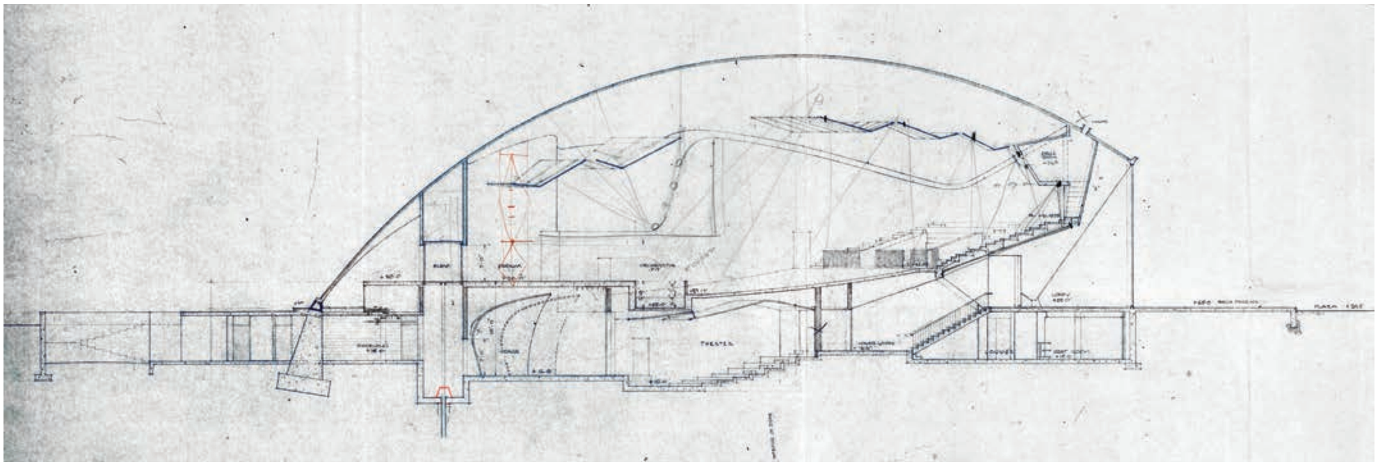


Illustration 13.16
Kresge Auditorium, MIT, Boston, MA, USA (1954).
Relative thinness and remarkable spanning capabilities of modern concrete shell domes is evident in the section drawing.
Architect: Eero Saarinen. Structural engineer: Ammann and Whitney.

Illustration 13.17
Kresge Auditorium, MIT.
Thickened free edges of this thin shell provide local stability to the "cut" surface.

eggshells and seashells, although the chicken egg nonetheless presents some quite impressive proportions: it is only 20 percent as thick as would be expected based on our description in the last section about domes that work as arch segments and that must rely on through-the-thickness bending action for stability. How can this be? And what is going on to make such a difference in spanning proportions possible? There is certainly the strong suggestion that quite a different load-carrying mechanism must be at work in the eggshell and in modern shell-like domes as well.

Part of the answer lies in the material of which such shells are made: i.e., one that is able not only to withstand compressive stresses but that also has significant capacity to resist the hooping tensile stresses that tend to pull the surface apart and that in case of the masonry dome led to its vertical cracking. In shell dome structures such as at MIT's Kresge Auditorium (Ill. 13.16, 13.17), for example, the concrete shells are reinforced with steel bars, giving the shell surfaces simultaneous compression and tension capability throughout and thereby preventing any possibility of having the surface split apart into the discrete arch-like segments we discussed previously. And in the American Air Museum in Cambridge, England (Ill. 13.18), the concrete shell is actually made up of almost 1000 separate precast panels that are post-tensioned together, not only

Illustration 13.18
American Air Museum, Cambridge, England, UK (1997).
Roof shape is that of a small surface segment of a great torus centered deep underground. Span of 90m (295ft) – enough to house a B-52 Stratofortress bomber and display and suspend several other smaller planes as well – achieved by double-layer concrete shell made of 924 precast concrete panels post-tensioned together.
Architect: Foster + Partners. Structural engineer: Arup.

to hold together such an assembly of component parts into a single unified shell structure but also to ensure that no tension stresses ever develop anywhere within the shell surface.

Material differences and advances aside, however, quite a different basic load-carrying mechanism must also be described in order to fully appreciate just how these relatively thin shell domes work. To do so, we will consider here the behavior of a spherical shell dome, which has overall geometrical similarity to the hemispherical masonry domes that we have just considered.

Vertical gravity loads acting on such a structure will, as before, be primarily carried by compressive forces N_ϕ acting along meridional lines within the dome surface (i.e., oriented like the lines of longitude by which we position ourselves on the earth's surface), but now any tendency for the dome to change shape by bulging outward or sagging inward under loading is able to be countered everywhere by hooping forces N_θ acting around the circumference of the dome along the equivalent of lines of latitude (to carry on with the same terrestrial analogy). These are shown in Fig 13.5. Where the dome tends to sag down in the upper region under gravity load, compressive hooping stresses will help prevent its collapsing inward by means of the surface's curvature, while tensile hooping rings of stress in the lower, more vertical parts of the dome will similarly resist the form's tendency to bulge outward. Stability of the domical form and resistance to changes of shape are therefore ensured by efficient *uniform axial stresses* acting through the thickness of the shell rather than by the much less efficient linearly-varying-through-the-thickness bending action that was characteristic of arch rib behavior, with the consequence that much thinner domed surfaces than we had previously considered possible can now be understood and their load-carrying behavior explained.

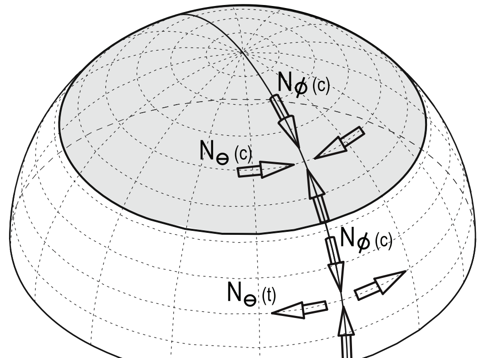


Figure 13.5
Meridional (N_Φ) and circumferential (N_θ) stresses (or hoop stresses) in shell dome; N_Φ is compressive throughout, N_θ is compressive in upper (shaded) "cap," tensile in lower circumferential band.

If carrying load on a curved surface by means of sets of axial forces acting within that surface sounds familiar, it should, for we have encountered this mechanism before when we considered purely tensile membranes in Chapter 11. Recall from there that the membrane equilibrium equation was defined as:

$$p = N_x/R_x + N_y/R_y \tag{13.1}$$

where the magnitude of the transverse radial load p on the surface at a particular location is balanced by the sum of the ratios in the two orthogonal directions of the in-the-plane-of-the-surface axial force N to the radius of curvature R of the surface in that direction.

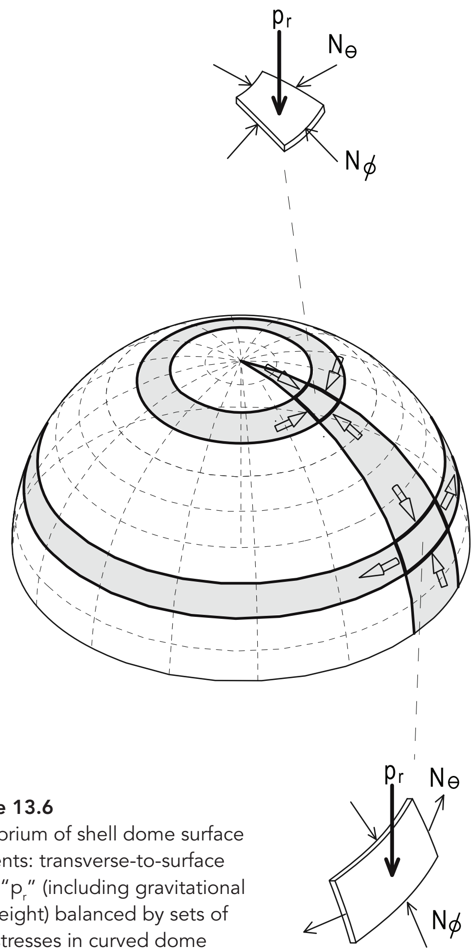


Figure 13.6
Equilibrium of shell dome surface elements: transverse-to-surface loads "p_r" (including gravitational self-weight) balanced by sets of axial stresses in curved dome surface; equilibrium condition different in upper and side regions.

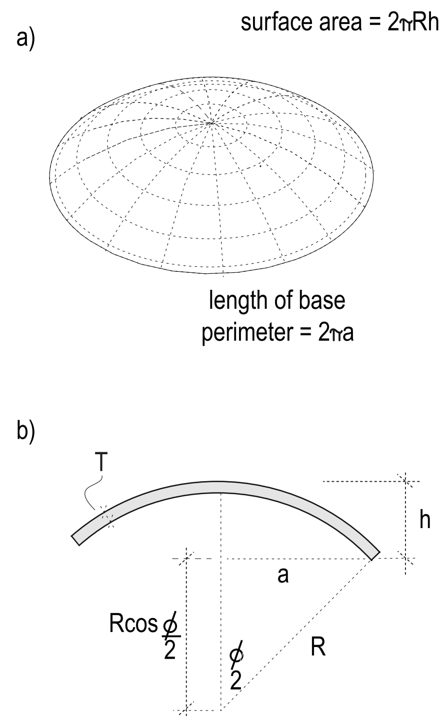


Figure 13.7
Surface and section geometry for dome segment.

We will come back to this equation shortly and apply it to further describe and understand the behavior of shell surfaces. Before doing so, however, let us consider the case of a segment of a spherical shell-dome surface of uniform thickness t, leading to its having a weight per unit surface area w. (Fig. 13.6.) If the radius of the dome is labeled R and its height h, the total surface area A of the segment of the sphere is defined by basic geometry to be equal to 2πRh. The domed segment's total weight W can, therefore, be calculated in terms of its spherical geometry, where the angle $\Phi/2$ is measured between radial lines drawn to the azimuth and to the base of the dome:

$$W = w \times A$$
$$W = w \times (2\pi Rh)$$
$$W = w \times (2\pi R\ [R - R\cos(\Phi/2)]$$
$$W = w \times (2\pi R\ [R\{1 - \cos(\Phi/2)\}])$$
$$W = 2w\pi R^2\ (1 - \cos(\Phi/2)) \qquad (13.2)$$

In order to have vertical equilibrium, the sum of the vertical components of the meridional forces N_Φ acting all around the base of the dome, which has a circumference of 2πa, where $a = R\sin(\Phi/2)$, must necessarily balance this total weight; i.e.,

$$W = \Sigma N_{\Phi\ (\text{vertical})}$$
$$W = N_\Phi \sin(\Phi/2)\ (2\pi a)$$
$$W = N_\Phi \sin(\Phi/2)\ [2\pi R \sin(\Phi/2)]$$

Rearranging this in terms of the meridional force leads to

$$N_\Phi = W/[2\pi R \sin^2(\Phi/2)] \qquad (13.3)$$

which becomes, upon substitution for W from equation 13.2, and then using in succession the standard trigonometric identities $\cos^2(\Phi/2) + \sin^2(\Phi/2) = 1$ and $[1 - \cos^2(\Phi/2)] = [1 + \cos(\Phi/2)]\ [(1 - \cos(\Phi/2)]$:

$$N_\Phi = \{w2\pi R^2\ [1 - \cos(\Phi/2)]\}/[2\pi R \sin^2(\Phi/2)]$$
$$N_\Phi = wR\ [1 - \cos\ (\Phi/2)]/\{[1 - \cos^2(\Phi/2)]\}$$
$$N_\Phi = wR\ [1 - \cos(\Phi/2)]/\{[1 + \cos(\Phi/2)]\ [1 - \cos(\Phi/2)]\}$$
$$N_\Phi = wR/[1 + \cos(\Phi/2)] \qquad (13.4)$$

It is self-evident that the meridional forces N_Φ defined by this equation must be compressive in the shell in order to balance the downward pull of gravity on the shell surface.

Now, in order to establish what are the circumferential forces N_θ in the dome, let us go back to the membrane equation (13.1) and rewrite it in terms of both meridional and circumferential forces in the shell surface, N_Φ and N_θ, respectively. The equation becomes

$$p = N_\Phi/R_1 + N_\theta/R_2$$

For the case of a spherical shell that supports a uniform surface gravity load, $R_1 = R_2 = R$ since the radius is the same in all directions and $p = w \times \cos(\Phi/2)$ because gravity loads act vertically ($w \times \cos(\Phi/2)$ is the radial component of the surface's gravity load at a level defined by the azimuth angle $\Phi/2$).

Therefore,

$$w \cos(\Phi/2) = N_\Phi/R + N_\theta/R$$

and rearranging to solve for N_θ this becomes

$$N_\theta = R\,[w \cos(\Phi/2)] - N_\Phi$$

Substituting from equation 13.4 above for N_Φ results in the desired expression for the circumferential hoop stresses:

$$N_\theta = R\,[w \cos(\Phi/2)] - w\,R/[1 + \cos(\Phi/2)]$$
$$N_\theta = w\,R\,\{\cos(\Phi/2) - \{1/[1 + \cos(\Phi/2)]\}\} \qquad (13.5)$$

Finally, we can consider the implications of equations 13.4 and 13.5 for the meridional and hoop stresses, respectively, within the range from $\Phi/2 = 0$ (i.e., at the top of the hemisphere) to $\Phi/2 = 90°$ (at the base of a full hemisphere) and come to the following observations, as represented in Fig. 13.8:

- N_Φ varies from wR/2 at the top of the dome and increases to wR at the base (i.e., it is never equal to zero).
- N_θ varies from +wR/2 at the top of the dome and changes to –wR at the base. Not only is there significant variation in the hooping force magnitude, therefore, but also change in direction: i.e., maximum compressive hoop forces occur at the top of the dome but diminish as one moves away from this level, eventually becoming tensile hoop forces that increase the further one goes down the hemisphere.

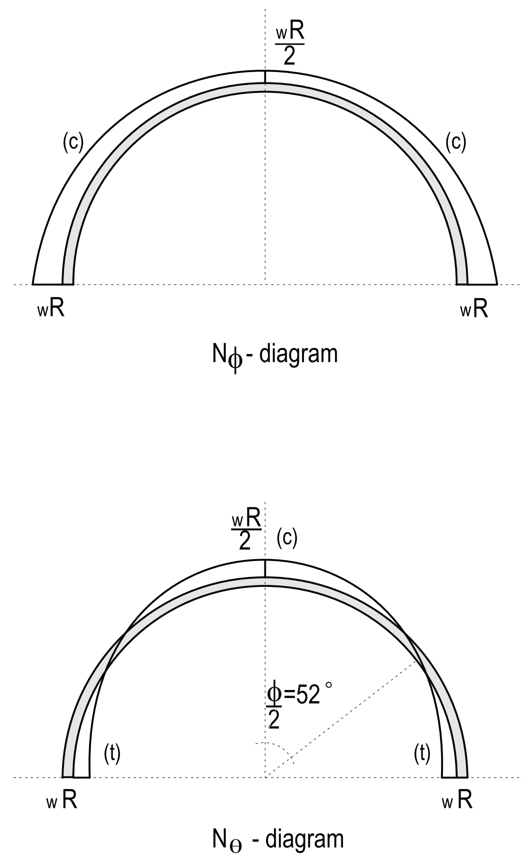


Figure 13.8
Variation of membrane forces, N_Φ and N_θ, within dome; compressive meridional N_Φ increases from top to bottom, hoop N_θ goes from maximum compression at top to maximum tension at base of dome, with changeover from compression to tension at azimuth angle of 52^0.

Furthermore, by setting $N_\theta = 0$, we can establish that it is at an angle of $\Phi/2 = 52°$ that the hoop stresses go from being compressive to tensile, which leads to the perhaps somewhat unexpected result that for a relatively flat hemispherical dome there will be no tensile hoop forces within the shell membrane to have to be contended with. Such a shell is, therefore, able to carry all loads by compression forces only. (In hindsight, we can now remark that the relative flatness of the Hagia Sophia dome that we saw in the previous section actually works remarkably well to minimize any potential tension stresses developing within its masonry structure.)

Beyond such generally applicable observations, it is further enlightening to consider the actual numerical values that these expressions yield for typical concrete dome construction dimensions. For example, let us consider a spherical dome that has a radius of 30m (96ft) and that is only 150mm (5.9in) thick. The weight of the concrete for this thickness is approximately 3.5kN/m^2 (77lbs/ft^2) plus an allowance for various finishes so that we can take w = 3.75kN/m^2 (82.7lbs/ft^2). Let us also assume that a snow load of 0.15kN/m^2 (30lbs/ft^2) must also be accounted for.

Based on the equations and observations made above, the maximum force in the shell can then be calculated to be

$N_{(max)} = wR$
$N_{(max)} = (5.25\text{kN/m}^2)\ (30\text{m})$
$N_{(max)} = 157.5\text{kN}$ per meter of the shell surface

which, when converted into an axial stress σ_Φ for the given shell thickness 150mm, is

$\sigma = N/A_{(\text{unit sectional area})}$
$\sigma = 157\,500\ \text{N}/(1000\text{mm} \times 150\text{mm})$
$\sigma = 1.05\text{N/mm}^2$

It is to be noted that this result is a remarkably small compressive stress that is well within any standard concrete's ability to handle (i.e., the ultimate compression stress for concrete is something in the realm of 30–50N/mm^2 or more, which is at least 30 times the demand in this example). Moreover, at all other points in the shell the magnitude of the stress will be even less than this. It is only by working through these equations and seeing such a result that the amazing potential of shell surfaces can be truly appreciated; i.e., that such impressive spans can be achieved using relatively thin structural surfaces while still keeping stresses well within ultimate limits is certainly not intuitively obvious.

Of course, having realized this point we may desire more: the remarkably small stresses that we found in this example lead to the inevitable question of why not reduce shell thicknesses even farther? Part of the answer to this question is rooted in the need to address compressive structures' old nemesis, buckling, which we have so far been conveniently neglecting here but which in reality cannot be ignored. We will recall from Chapter 8 that the standard way of dealing with buckling is to make a compressive element thicker. Also, there are practical limits to just how thin a concrete shell can be built and still have sufficient protective cover for the steel reinforcing bars or pre-stressing strands within the surface. Notwithstanding such practical limitations and even taking them into account, however, the fundamental lesson of this section is to recognize just how incredibly thin a shell surface can be when it uses surface curvature and membrane stresses to establish equilibrium compared to relying on the through-the-thickness bending action that was discussed in the previous section. We will explore where else this all leads very shortly but, before doing so, let us first make a pilgrimage to two famous domes that span the ages – and while doing this observe their similarities and differences of form.

13.4 Due Duomi a Roma

When in Rome, one can have the good fortune of being able to sense the span of 2000 years of dome construction.

The Pantheon, built in the Campus Martius area in AD 126 for the Emperor Hadrian, remains today as likely the best preserved of all the buildings of antiquity; perhaps not coincidental in this regard is its original fortuitous dedication as a Roman temple to all gods. Once entered, leaving the surrounding noisy commercial atmosphere behind, the Pantheon has always impressed its visitors by suggesting that one is entering another world; "It resembles heaven," Dio Cassius said.[5] (Ill. 13.19, 13.20.)

As a building, the Pantheon can be broken down into two distinct parts: an entrance porch with 16 impressive Corinthian columns and its central feature, a huge domed rotunda. From the outside the impression of the building is one of great compactness; inside,

Illustration 13.19
Pantheon, Rome, Italy (AD 126).
Open oculus at dome apex connects interior to the heavens/natural elements while also serving essential structural function at the point of major stress concentration.
Architect: Apollodorus of Damascus.

Illustration 13.20
"Interior of the Pantheon" (c.1734).
Eighteenth-century foreign tourists mingle with Romans in awe-inspiring space.
Artist: Giovanni Paolo Panini (1692–1765). Samuel N. Kres Collection, National Gallery of Art, Washington, DC.

however, the space opens up dramatically with a dome that spans 43m (141ft). The dome's surface is part of perhaps the world's most well-known imaginary sphere – one which can be completely inscribed inside the space, just touching the floor. The apparently massive walls around the perimeter of the dome (just hinted at in the previous section in terms of their dome-stabilizing structural function) are actually subdivided into vaults and niches that serve the religious and ceremonial program of the building – providing an interesting example of a structural system simultaneously meeting programmatic and statics-based equilibrium objectives. But other than the entrance doorway there are no openings in these side walls; the only other aperture to the outside world is up to the sky through the open oculus at the top of the dome that measures a full 9m (29.5ft) across and from which daylight is distributed into the interior space. In this oculus we have once again an inspired combination of an essential structural feature – in this case the compression ring at the top of a dome – also serving the not-so-subtle dual purpose of an opening for divine contemplation.

The Pantheon's dome is made of Roman concrete whose radially cracked, wedge-shaped-arch functioning has previously been described. And in keeping with the bending behavior that is an essential part of such a dome's method of resisting deformation, its thickness is significant; moreover, the dome grows thicker around its base, extrapolating into three-dimensional form the Romans' typical strategy for the stabilization of the sides of two-dimensional arches (Section 12.2). Roman master builders had learned to utilize the very weight and mass of building materials to maximum effectiveness, likely derived from their observations of the cantilevered (also called corbeled) vaults that had been built in many places in Italy before the Roman era in which the slimmest part of the vault is

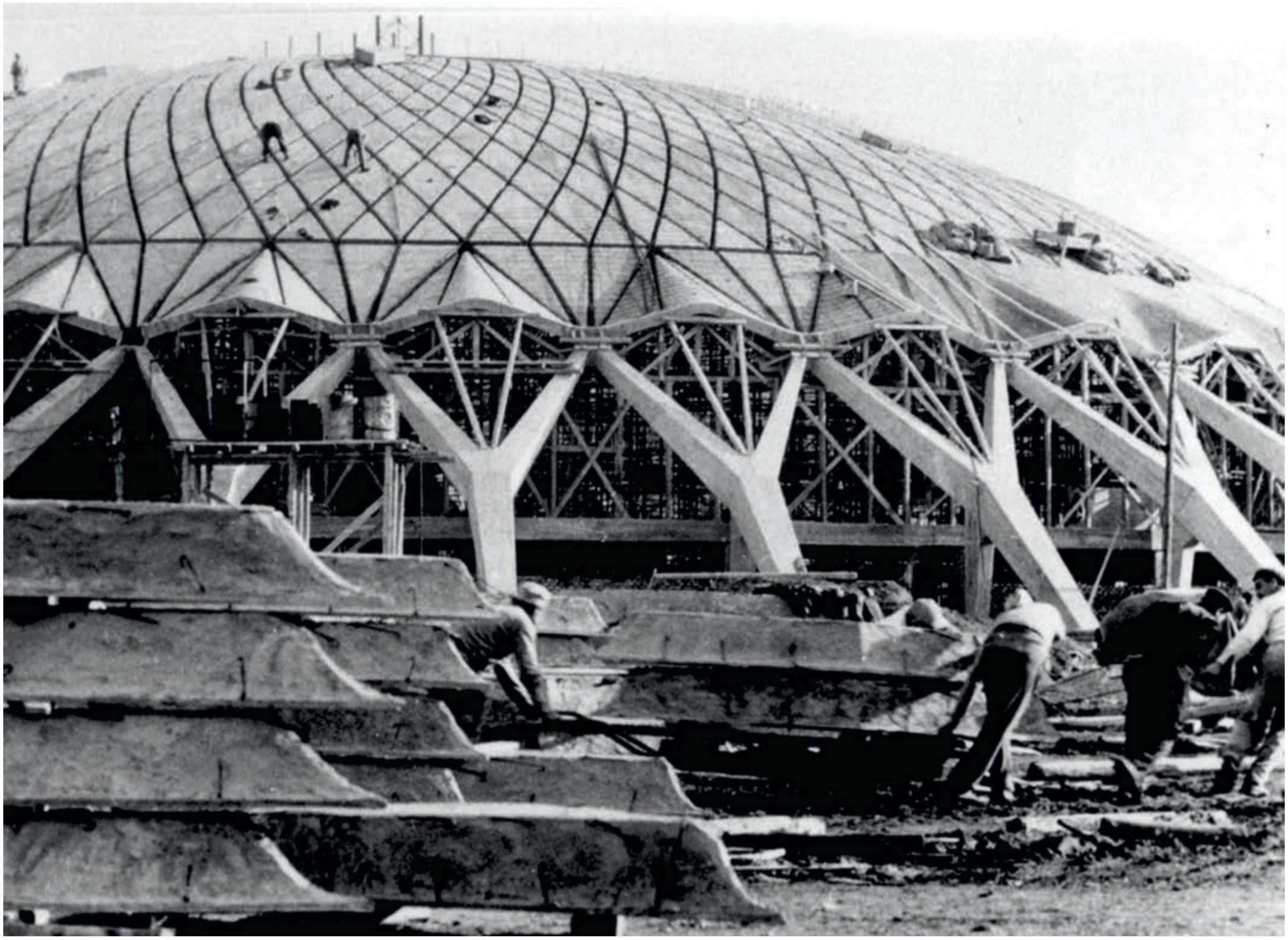


Illustration 13.21
Palazzetto dello Sport, Rome, Italy (1957).
Angled props around the perimeter are yet another way to resist a dome's outward thrusts, in a manner reminiscent of Gothic "flying buttresses."
Architect: Annibale Vitellozzi. Structural engineer: Pier Luigi Nervi.

at the top and the thickness grows toward the foundation.[6] It is also reasonable to hypothesize that the ocular opening at the top perhaps had its origins in earlier domed spaces in which a series of radial cantilevers reached inward toward a common center that was purposefully left open in order to admit light and release rising hot air and smoke.

On the subject of how the Romans actually built the Pantheon's incredible dome, there is unfortunately no documentation that remains. Viollet-le-Duc (1814–1879), among many others who have studied the problem, proposed that an elaborate timber centering scheme may have been used that consisted of a central tower supporting the inner ends of 28 radially arranged-in-plan and bow-shaped-in-section trussed ribs that spanned out to the surrounding masonry drum wall.[7] These ribs would have in turn supported the formwork that was needed to carry the unhardened concrete as it was built up in circumferential bands and to create the dome's coffered underside. And since hoisting the 28 trusses into position would likely have demanded an array of 28 cranes sitting on the top of the wall activated by ropes radiating out on to the surrounding terrain, we can postulate that the dome was built *before* the porch – just the beginning of a speculative excavation, so to speak, into the very rich realm of structural forensics associated with this structure.

Almost two millennia later, and somewhat to the north of the former Campus Martius area, preparations for the Summer Olympics of 1960 gave the brilliant building engineer and contractor Pier Luigi Nervi (1891–1979) the opportunity to try his own hand at another spectacular Roman dome, in his case by exploiting the structural efficiency and plastic richness of precast concrete for the Palazzetto dello Sport.[8] (Ill. 13.21.)

Illustration 13.22
Palazzetto dello Sport.
Ribbed interior of dome's surface is in marked contrast to its smooth outer layer; it also traces innovative construction methodology. Base of dome surface clearly disengaged from ground, allowing light to phenomenally "float" the dome structure.

Nervi's design scheme for the Palazzetto demanded a clear span of some 70m (230ft) and, moreover, a roof that would read as visually independent from the top of the spectator seating that surrounds the athletic playing surface. Toward these objectives, he designed a relatively shallow segment-of-a-sphere dome that covers the entire building in one fell swoop and whose perimeter support is seemingly lifted up off the ground. In contrast with the Pantheon, where the perimeter walls effectively form a thick cylindrical drum up to the base of the dome, the Palazzetto's sides are almost completely glazed. Receiving and countering the downward and outward thrust of the dome are 36 radially arranged, Y-shaped, vertically propped buttressing columns whose angle of inclination is based on radiating tangent lines coming off the dome surface. The openness of this flying-buttress-like supporting structure allows for the striking visual effect of a free-floating dome, with bright light entering from all around the structure and seemingly lifting the roof up into the air.

The Palazzetto dome itself was built using an innovative technique: in order to simplify construction and reduce the amount of temporary timber formwork necessary (acknowledging that, not coincidentally, Nervi was also the contractor for the project) prefabricated and permanent formwork panels were created by casting a relatively thin layer of fine aggregate concrete over the top and sides of triangular plastic molds placed with their open-side downward. These coffered precast concrete panels were then put into position as formwork in preparation for the casting of the dome surface, with their careful placement adjacent to each other creating open channels between the panel sides into which steel reinforcing bars could be placed. Finally, cast-in-place concrete was poured into these channels and over the top of all the coffers in order to make, when it was hardened, a highly efficient thin-ribbed dome made of a composite of both the precast and cast-in-place concrete.

The visual effect of this construction technique results in a fortuitous echoing of the Pantheon's famously coffered inside surface. (Ill. 13.22.) And also like the Pantheon, the Palazzetto's dome has a central oculus and skylight, here in the form of a glazed lantern. In yet a further mimicking of the past, a construction crane was placed in this central opening during construction in order to lift and place the precast concrete panels into position. Unfortunately this lantern has since been closed to light, presumably to avoid glare for some of the spectators as the sun shone through it, but the consequence has significantly reduced daylight in the interior of the space and it thereby perhaps lost some of its magic. (Surely it would verge on the sacrilegious to think about closing the Pantheon's oculus …)

As we have noted, Pier Luigi Nervi, besides designing and doing the calculations for his structures, also owned the contracting firm that built his domes and other remarkable structures. Nervi was thus able to control their building process as well as detect and correct any irregularities during construction. And even while we admire the ingenuity and daring aspect of his œuvre, we become even more impressed by it when it is recognized that these projects were obtained in competitions that were governed by very strict constraints of cost and time. In the context of construction efficiencies, Nervi had some interesting reflections on the reason for the geometrical shape of his domes: he noted that an elliptical dome would result in very complex calculations and construction difficulties compared to a circular one; moreover, a circular dome could be built by the repetition of only one precast form, whereas for an elliptical dome the geometry of an individual forming panel could only be used twice.[9] As we will see shortly in Section 13.8, although some of these truths remain, much has also changed since these reflections were made in the 1950s and 1960s. Today digital techniques for the manufacturing of building elements can allow for very cost-effective ways of incorporating even thousands of subtle differences into the many individual components that may go into making up a single surface for a project. And while on the one hand this newfound freedom is unquestionably exhilarating, the risk is that design and production can turn into anarchy of form; it still takes a steady hand, and perhaps an eye to the overall design sensitivity so evident in the domes of the Pantheon and the Palazzetto dello Sport, to maneuver in this environment.

13.5 Folded Plates and Cylindrical Shells – Beam Action Revisited

Having established the key principles that domed shells are able to carry significant loads by means of "in-plane" axial forces in tension and compression and by using to advantage their overall curved geometry, we can extend these same strategies to other rigid surfaces of non-spherical form. We begin by considering a special category of thin, stiff surfaces that are deliberately shaped so as to be able to carry load in an overall beam-like manner even though they are employing these same fundamental shell-behavior principles in order to do so.

One such structural form is the *folded plate*, in which a thin flat surface having relatively little inherent flexural stiffness of its own (think of a sheet of paper trying to span between two table tops, for example) can be significantly stiffened against large-scale transverse deformation by folding it back and forth into ridges and valleys aligned with the direction of the span. (Fig. 13.9a,b.) Simple experiments can easily be conducted with thin card stock to

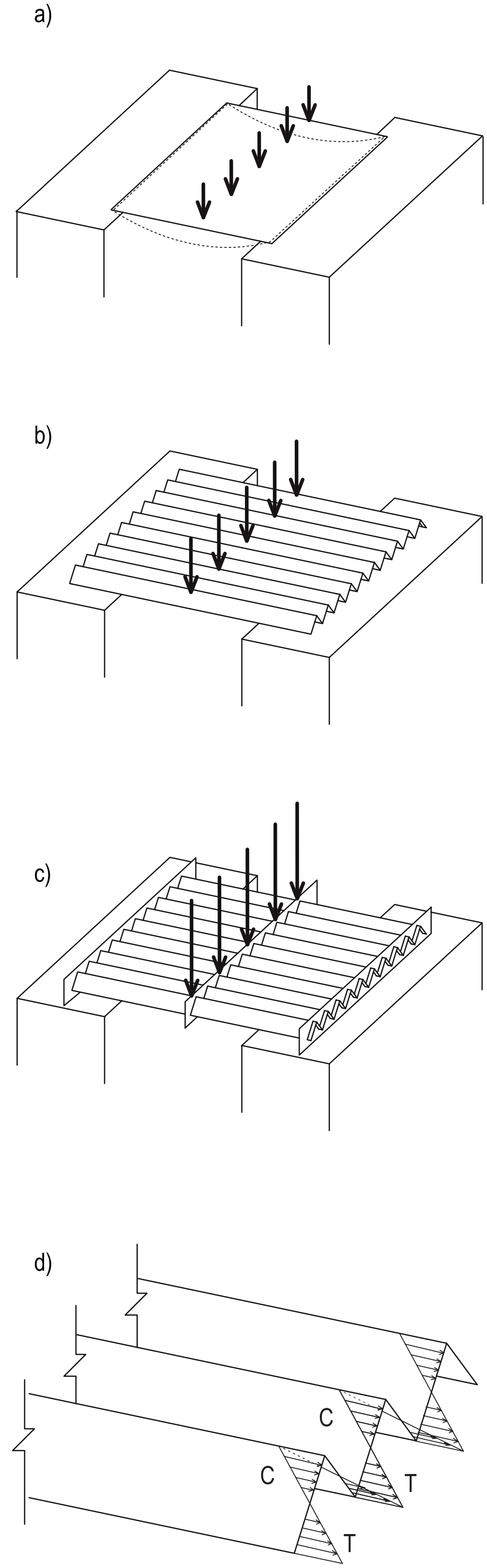


Figure 13.9
Representation of stiffness and load-carrying variations produced by spanning with (a) flat sheet of card stock, (b) folded such surface, and (c) folded surface supplemented by transverse diaphragms; (d) depicts beam-like distribution of stresses acting in planes of folded surface.

Illustration 13.23
Finnish Embassy, New Delhi, India (1986).
Irregularly folded plate roof unifies the separate volumes of the ambassador's residence, the chancery, staff apartments and a Finnish sauna. The linear arrangement of the folded plates recalls the Nordic landscape of glacier-carved parallel furrows and hills, and the eaves of the angled white roof resemble the snow sculptures formed in the winter ice around the Gulf of Finland. The 1963 competition jury, in recognizing the outstanding design qualities of this project, complimented its qualities of "aesthetic uniqueness against mere rational excellence."
Architect: Raili and Raima Pietilä. Structural engineer: Heimo Kakko & Co.

demonstrate the very significant increase in load-carrying capability produced by such folding of surfaces, and in everyday life we apply this understanding to everything from corrugated cardboard to the instinctive street-wise strategy used for eating a slice of hand-held pizza.

The basic mechanism for carrying loads by means of a folded plate surface is reminiscent of that which is used in conventional beams and slabs (Chapter 7), but now the tension and compression stresses developed in the direction of the span and that vary in characteristic beam-like linear fashion do so from bottom to top of the *entire folded plate section* (Ill. 13.9d), rather than simply through the thickness of the surface, as they did in a flat slab structure. This means that a small element of the surface of a folded plate can essentially be considered as subject to uniform axial stress through its thickness, and that the structure as a whole uses its geometry to advantage by developing a large moment of inertia based on its overall cross-section. We have seen previously with beams just how much was gained by maximizing the distance of structural material from a beam section's overall neutral axis (Section 7.8) and the same principles apply here in the context of the folded plate.

In describing this behavior, we are obviously assuming that the material of this folded surface is capable of carrying the tension and compression stresses equally well. Also being taken as a given is that the folds of the structure are made to hold their shape, typically something ensured by means of cross-walls or stiffening ribs or bulkheads across the folded plate section that are spaced intermittently along the length of the span. (Fig. 13.9c.) Form-stabilizing stiffness is also often provided by having the free outside edges of a folded surface strengthened by means of an edge beam of some sort, or perhaps by including a partial half-fold of the surface.

Whatever the specific details, the generally faceted texture and angularity of form of the folded plate surface typically gives it a unified yet also highly distinctive appearance that may be particularly well suited to certain programmatic objectives and aesthetic preferences. And while the folded plate drawn in Figure 13.9 depicts a regular folding of the surface, in fact the basic principles can be applied to intentionally more irregularly folded surface geometries, such as that for the overriding concrete roof of the Finnish Embassy in New Delhi (Ill. 13.23), as well as for the origami-like folded surfaces made of solid wood panels for the temporary St-Loup Chapel. (Ill. 13.24, 13.25.)

Having arrived at an understanding of how folded plates develop stiffness and how these work to carry load, it also becomes possible

Illustration 13.24
Temporary St-Loup Chapel, Deaconess Community of St-Loup, Pompaples VD, Switzerland (2008).
Built for short-term use during the renovation of this religious community's main buildings, a small trapezoidal folded plate structure was constructed from many cross-laminated timber panels, 40mm (1.5in) thick for the walls and 60mm (2.25in) for the roof.

Architect: localarchitecture and Danilo Mondada Architects. Structural engineer: IBOIS – Hani Buri / Yves Weinand.

Illustration 13.25
Temporary St-Loup Chapel.
The chapel profile rises from the entrance to the chancel, and the highly irregular folding patterns are used to address acoustic and lighting design objectives;
the panels are all of different shape and were fabricated and milled in a factory, then precisely connected together by folded steel plates along the creases and simply cap screwed, producing quite an economical and quick method of construction that was nonetheless visually distinctive.

Illustration 13.26
Cadet Chapel, US Air Force Academy Chapel, Colorado Springs, CO, USA (1962).
Sharply folded aspect of chapel's roof surface suggests airplane wing geometries of the era; while representative of the overall form of a folded plate surface, this structure is not, strictly speaking, that of a "pure," solid-surface-version of this type but instead consists of a series of steel space frame tetrahedrons covered by aluminum panels.
Architect: Walter Netsch of Skidmore, Owings & Merrill (SOM). Structural engineer: SOM.

to understand how such a folded structure can equally well be trussed in each of the planes of the folds rather than be made of a solid concrete slab or wood panel or steel plate. Just as a beam can be rendered more efficient for carrying load by strategically concentrating material into the chord and web members of a truss, so too can each panel of a folded plate also be trussed, with chord members along the lines of the ridges and valleys of the folded surface and diagonal and/or other linear elements connecting between these. Two well-known examples of such trussed "folded plate" structures are the delta-shaped roof of the US Air Force Cadet Chapel (Ill. 13.26) and the supporting structure for the landscaped walking surface/sculpted roof of the Yokohama Terminal cruise ship pier (Ill. 13.27), although in both cases the trussing is in fact not at all evident at first glance because of non-structural metal sheets that the folded surfaces hide for aesthetic reasons; i.e., efficiency of material use and lightness of structure are two things to consider in the design of structures, but visual clarity vs. cluttered appearance is quite another.

Other rigid surface shapes exhibiting a very similar beam-like response to carrying loads are the half- or partial-cylinder, the cycloid (whose particular profile is described in the context of the Kimbell Art Museum in the next section), or other curved shapes lying on their sides and spanning lengthwise some significant distance compared to the surface's cross-sectional thickness. If such a curved surface's material is assumed to have the ability to carry both compressive and tensile stresses, then it is possible to envision this type of structure functioning just as the folded plate does, i.e., with bending stresses developing in the surface in the direction of the span but that vary linearly from top to bottom of the overall, effective cross-section. (Fig. 13.10a.) To repeat ourselves once more for emphasis: for the folded plate or the cylindrical shell what we are really doing is using these forms' overall sectional

Illustration 13.27
Yokohama Terminal, Yokohama, Japan (2002).
Origami-like folded surface made of perforated steel plates that mostly cover hidden trusses. Folds provide strength and stiffness needed to span the large internal space without columns while supporting heavy loads from the occupiable, landscaped rooftop terrace for which this project is perhaps most widely recognized.
Architect: Foreign Office Architects. Structural engineer: Arup.

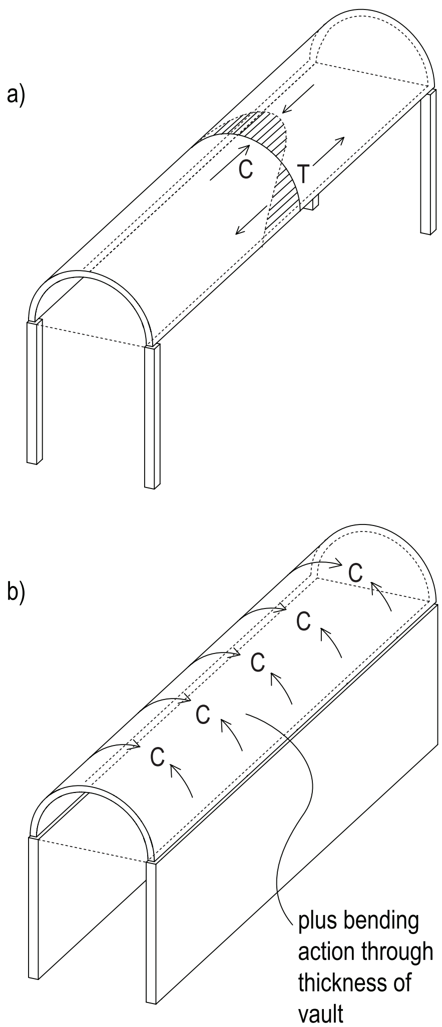


Figure 13.10
Similar forms but different structural behaviors for (a) cylindrical shell and (b) arched vault in terms of locations of supports, spanning directions, and load-carrying mechanisms.

geometry to effectively create beams having a very large moment of inertia, thus enabling large spans to be accomplished, large loads to be carried, and/or for the structure to be much thinner than would typically seem possible.

As an aside at this point, we should be careful to distinguish between the structural behavior of a cylindrical shell and that of the semicircular vault seen in Chapter 12. Even though these two structures have similar overall geometry, in fact they are *supported* fundamentally differently, with the shell carried at its two ends compared to the vault that is supported all along its two longitudinal edges. (Fig. 13.10a,b.) In contrast to the in-the-direction-of-the-span, beam-like load-carrying mechanism that we have just described for the cylindrical shell, it will be recalled that the classical side-supported vault works to carry load in transverse-section arch-like fashion, with compression stresses following the section's arched profile and bending action, while also typically being present, in this case acting only *across the thickness* of the vault surface. As we have seen, this can lead to a significant need to thicken or otherwise strengthen such a vault surface against potentially large bending deformations and stresses.

Finally, getting back to the folded plates, cylindrical shells, and other such shaped, rigid surfaces that work in an overall beam-like fashion while utilizing basic shell structure principles, we can apply to these other lessons that we have previously learned about beams and other bending-action-dependent structures in earlier chapters. For example, just as cantilevered beams can be tapered in elevational profile according to bending moment demands, so too can this be done with the overall depth of folded plates and cylindrical shells, something that can easily be observed in the profile of the renowned canopy structure for the Madrid Hippodrome. (Ill. 13.28, 13.29.) Also, the significant structural benefit that can be derived by continuing a beam some distance past its end supports, thereby reducing maximum bending moments and corresponding bending stresses that the beam needs to be designed for is a strategy often implemented, as can be seen at the Finnish Embassy roof in Ill. 13.23. Likewise, the lateral-load-resistance strategy of the rigid frame, which involves rigidly connecting the ends of its beam element to the top of its supporting columns, can also be exploited in interestingly spatial ways with folded plate and surface structures, as we have just seen with the St-Loup Chapel (see Ill. 13.24, 13.25) and as will see in the next section for the UNESCO Headquarters Assembly Hall (see Ill. 13.30).

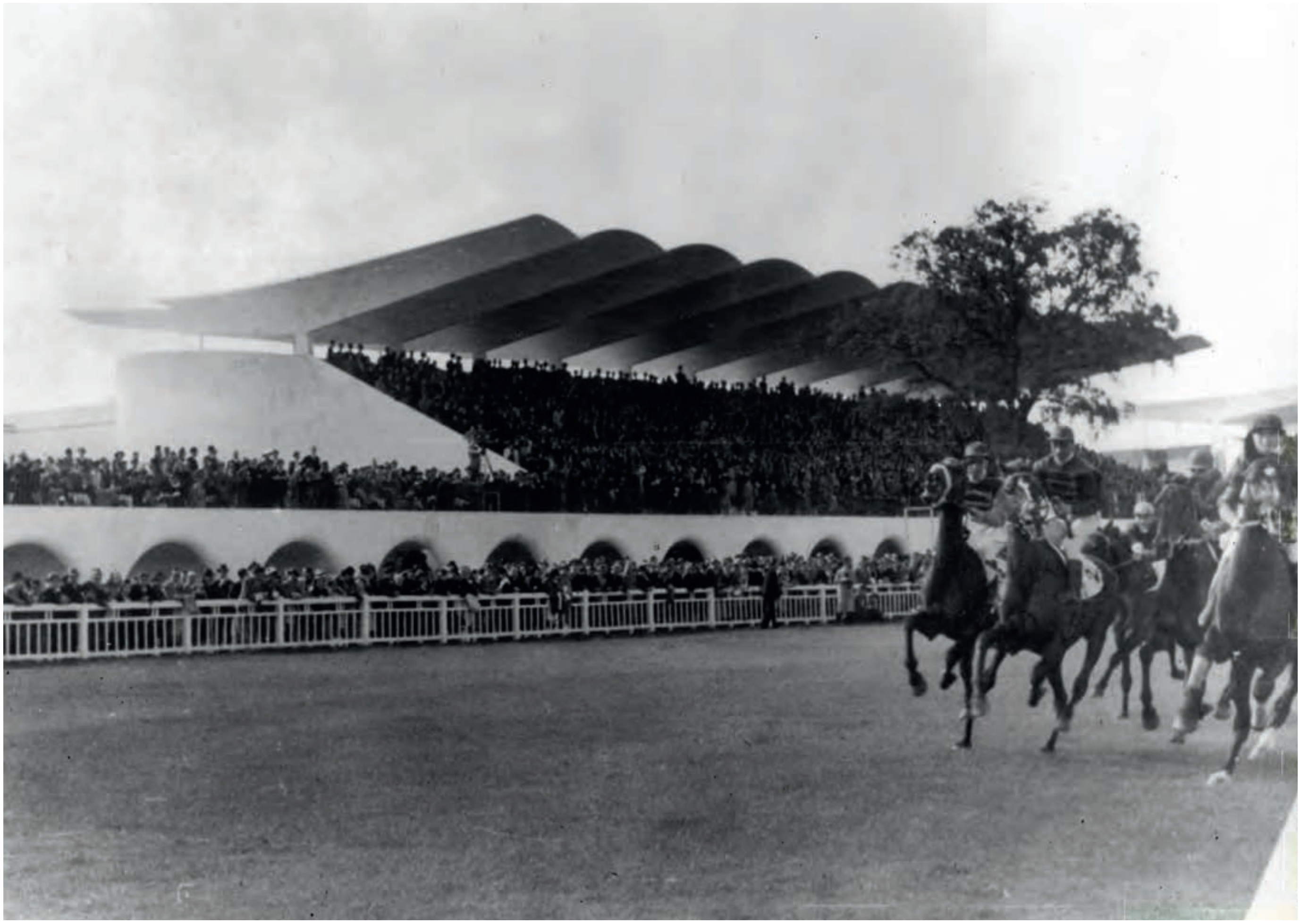


Illustration 13.28
Hipódromo de la Zarzuela, Madrid, Spain (1936).
Multiply-curved concrete surface segments together create a shell form that gives the grandstand canopy the ability to cantilever out 12.8m (42ft); each half-segment of the shells has hyperbolic paraboloid geometry (see Section 13.7).
Architect: Carlos Amiches and Martín Domínguez. Structural engineer: Eduardo Torroja y Miret.

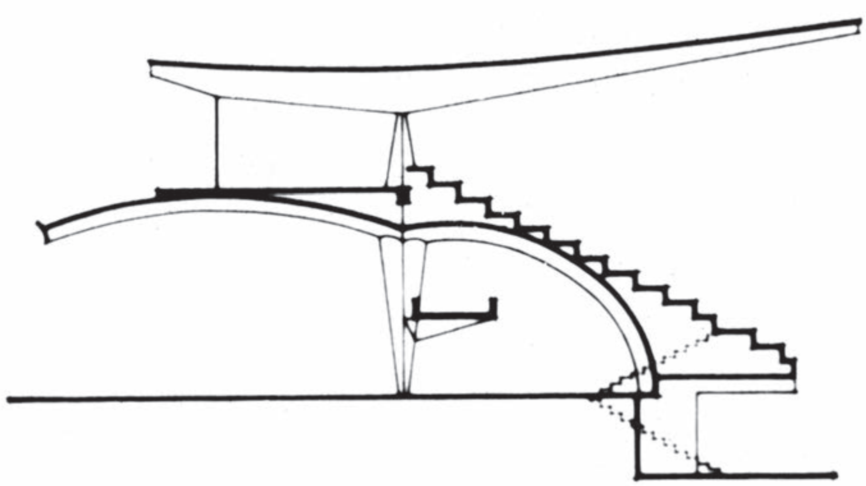


Illustration 13.29
Hipódromo de la Zarzuela.
The overall depth of the effective section from top to bottom of the curved surface can be seen to vary according to bending moment demand.

Illustration 13.30
UNESCO Headquarters Auditorium, Paris, France (1956).
Auditorium space is established and given character by a folded plate ceiling that turns into the backstage wall.
Architect: Marcel Breuer and Bernard Zehrfuss. Structural engineer: Pier Luigi Nervi.

13.6 Modern Classics Spanning Space

To achieve large roof spans, folded plates made from post-tensioned reinforced concrete offer a variety of possibilities, and especially so when the folded surface's cross-section is changed along the length of the span according to structural demand. An early example of this strategy can be found in the UNESCO Headquarters Assembly Hall in Paris, built in 1956 by the engineer Pier Luigi Nervi in collaboration with architects Marcel Breuer (1902–1981) and Bernard Zehrfuss (1911–1996).

The folded roof slab at UNESCO spans over the main auditorium in the long direction as well as over a second smaller space behind it. (Ill. 13.30–13.32) The roof is supported at its ends by correspondingly folded walls as well as on a line of vertical support near the middle of the building. From an overall structural-behavior point of view, the folded slab can be described as running continuously over the middle support and thus tending to be bent concave upward at that location while sagging downward over most of the distance between vertical supports. At the end of the span, the folds of the roof surface meet with the folds of the back-stage wall to create an effectively rigid connection between the two and thereby produce an overall rigid frame configuration (whose characteristics and attributes were discussed in Chapter 10); such a rigid connection also tends to produce concave-upward curvature for the folded roof surface near the end walls. Overall, then, based on an understanding of the deflected shape that we can expect of a rigid frame, the beam-like tension-to-compression bending stress distribution over the roof's full folded section can be understood to produce compression stresses in the lower parts of the folds in the vicinity of the central vertical support as well as near the end walls, whereas compression stresses will be in the top parts of the folds in the middle of the spans where the roof tends to sag downward.

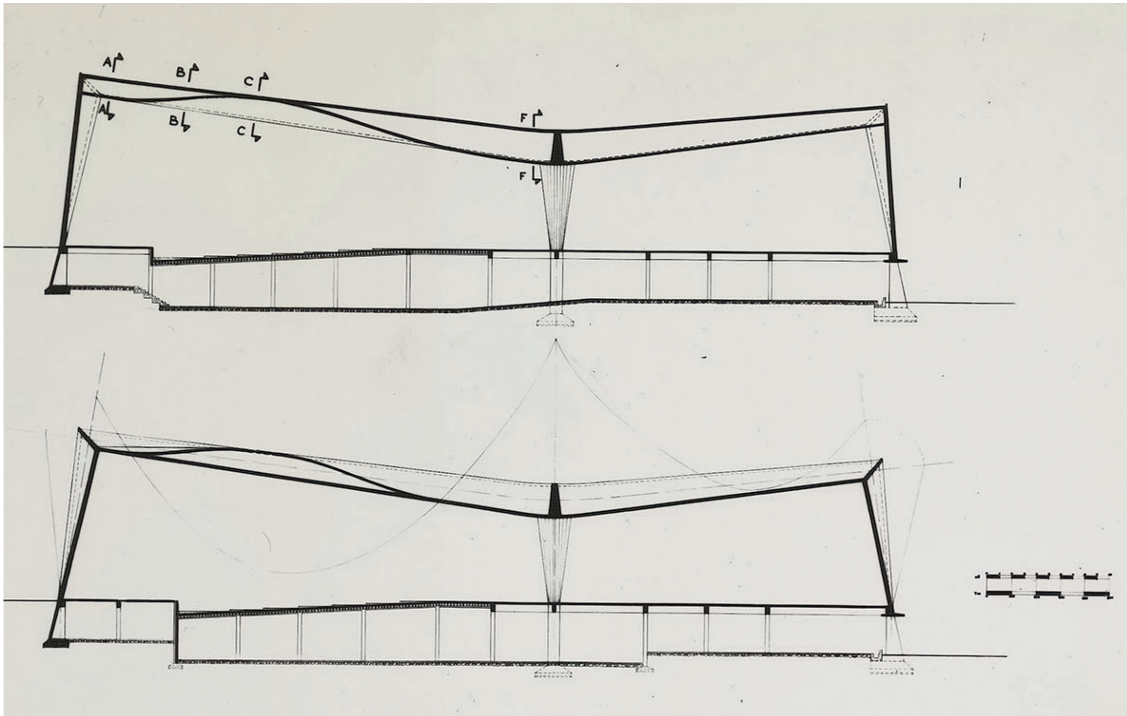


Illustration 13.31
UNESCO Headquarters Auditorium.
Section drawing highlights undulating profile of stiffening slab between buckling-prone compression zones of folded plates; also evident is overall rigid frame structure produced by effectively rigid connection between folded surfaces where roof meets wall.

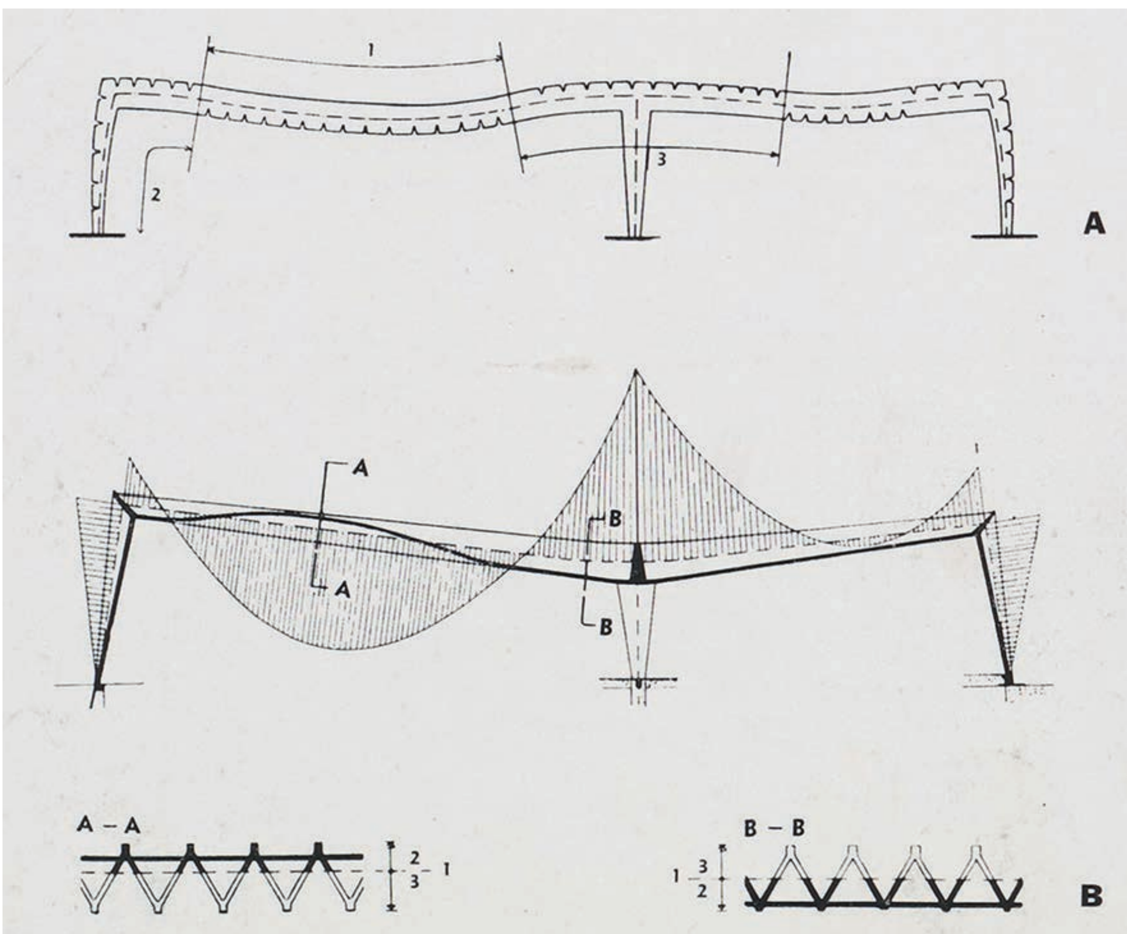


Illustration 13.32
UNESCO Headquarters Auditorium.
Drawings of anticipated deflected shape, bending moment diagram, steel reinforcement bar placement, and section cuts showing varying level of transverse stiffening slab.

Illustration 13.33
Kimbell Art Museum, Fort Worth, TX, USA (1972).
End supports necessitate length-wise, beam-like spanning of distinctively shaped shell roof.
Architect: Louis I. Kahn. Structural engineer: August Komendant.

Now, we know from Chapter 8 that compression stresses can lead to buckling problems and that lateral bracing is sometimes used to counter this phenomenon; this problem is likely to be especially acute in a thin surface structure such as this one that will be all the more prone to buckling failure. Adapting the standard folded plate structural form to these stress variations and buckling concerns in the context of the exceptionally long span over the auditorium, Nervi added to the typical folded surface at UNESCO a secondary horizontal reinforced concrete slab element. Near the middle support, this is positioned in the lower part of the folds to stiffen the section against buckling tendencies there (see Ill. 13.32, detail B-B). Heading out into the middle of the span where the overall beam-like stresses in the folds gradually reverse direction, Nervi makes the initially flat slab element rise up continuously within the folded section profile in order to follow the compression stresses toward the top (detail A-A). Things reverse yet again in the vicinity of the end wall support, with the bracing slab dipping down to the bottom of the folds. In this way the underside of the anti-buckling stiffening element becomes a smoothly curved surface seeming to elegantly undulate through the depth of the folds of the roof surface. Viewed from above one can observe the "negative" form of this variation, with the folded plate's valleys seeming to widen and narrow according to where the points of vertical support are provided. This is a subtle game that only a master can play.

Perhaps one of the clearest examples of the remarkable structural and architectural possibilities of the cylindrical-shell-as-beam strategy can be found at Louis I. Kahn's (1901–1974) Kimbell Art Museum in Fort Worth, Texas. (Ill. 13.33.) Here the roof structure consists entirely of a series of parallel concrete shells, each having a cross-sectional profile that is a so-called cycloid, a curve that is generated by a point on the circumference of a circle as it rolls along a straight line. The shells must make use of their full sectional depth in order to be able to act as a beam in spanning the 30m (100ft) distance between the rows of columns. Moreover, the relatively low level of the shells relative to the ground not only helps to accentuate the span when seen from the outside but they also clearly define the internal spaces of the galleries and the circulation within the museum – one is always conscious of the oriented space that the

Illustration 13.34
Kimbell Art Museum.
Shell form defines interior space, acts as light reflector.

shells create; indeed, one can't help but feel that one is walking *within* the structure, rather than under it.

But there is a further subtlety of detail to these shells that is not only telling of how they work structurally but that also plays a key role in the quality of the internal space: daylight seeps in through a long slit cut in the top of the concrete shells all along their length. (Ill. 13.34.) Clearly with such a cut at the "keystone" location, this structure cannot be working as a series of arches that span transversely to bottom-edge beams that would in turn carry the loads to the supports at the ends; rather, the shells work as overall beam structures that span lengthwise from one column line to another, with overall beam-type compression and tension stresses acting within the concrete surface aligned in the long spanning direction. A few cross ribs of concrete do exist across the opening at the top of the curved form to ensure that there is connection between the two halves of the shell and to help prevent distortion, but not more than that. A couple of other discrete features are also used to help counter the flattening-out tendency of the curved shell surface: edge beams (which are simultaneously exploited for practical purposes such as utility services and water drainage) and the curved, thickened arch-like rib ends of the shells that work in bending to resist the opening up of the shell beam.

Light at the Kimbell Art Museum is carefully controlled with the help of a perforated, polished aluminum reflector running under the slit at the top of the shells. The reflector lets most of the light down into the room, while the remainder is reflected on the interior surface of the vault; in this way its surface is softly washed in daylight rather than being dark and uninviting. Kahn himself describes the choice of the vaulted shape of his roof in this way:

> *My mind is full of Roman greatness and the vault so etched itself in my mind that, though I cannot employ it, it's there always ready. And the vault seems to be the best. And I realize that the light must come from a high point where the light is best in zenith. The vault, rising but not high, not in an august manner, but somehow appropriate to the size of the individual. And a feeling of being home and safe came to mind.*[10]

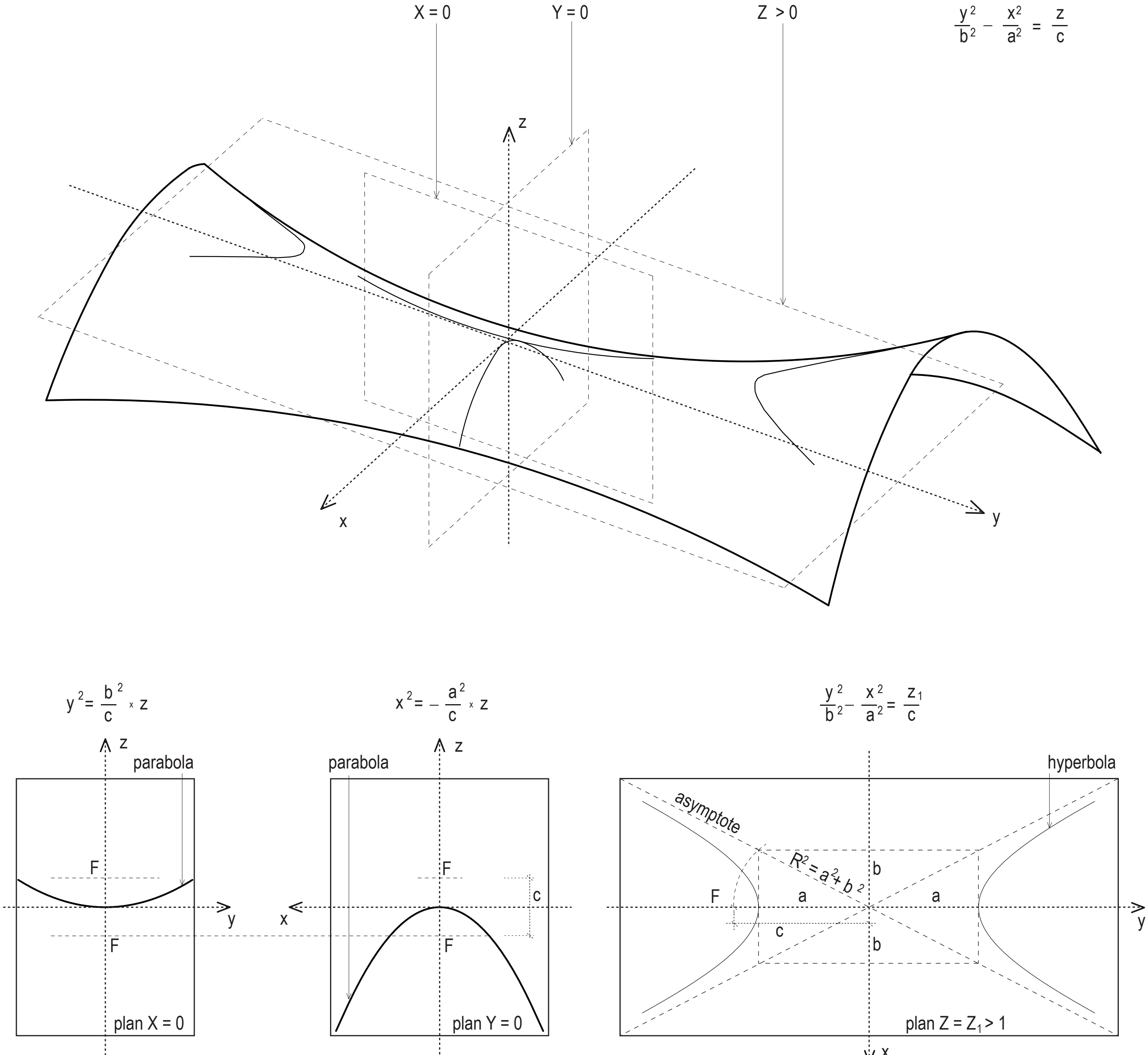


Figure 13.11
Mathematically precise geometry of doubly curved hyperbolic paraboloid surface; e.g., planes at X = 0 and Y = 0 cut surface along parabolas, plane at Z > 0 cuts surface along two hyperbolas.

13.7 The Hypar Shell

Another particular structural shell shape that has enjoyed remarkable popularity over the past half century is that of the *hyperbolic paraboloid*, which describes a pure mathematically defined surface whose technical nomenclature is often abbreviated to the more informal and familiar abbreviation *hypar*. As was seen in Section 11.8 in the context of tensile membranes, this shape can generally be described as one which has an anticlastic doubly curved surface; i.e., it has concave curvature in one direction while being convex at right angles to that. The hypar can perhaps most readily be recognized and brought to mind by thinking of the shape of the surface of a horse-riding saddle.[11]

From a mathematical perspective, the shape of the hypar can be described by starting with one planar parabola, for example one which is opening upward, and then translating along this curve an orthogonal parabola of opposite downward curvature. (Fig. 13.11.) Vertical section cuts taken in one orthogonal direction or another through the resulting surface will therefore always consist of parabolic curves. Horizontal sections, on the other hand, will always cut the doubly curved surface in two places along lines whose geometric functions are defined as matching hyperbolas. Given these very particular mathematical properties the naming of this surface as a hyperbolic paraboloid becomes evident, although initially there

Illustration 13.35
First Nations House of Learning, University of British Columbia, Vancouver, BC, Canada (1993).
Double curvature of hypar-shaped roof shape created using straight tree trunks.
Architect: McFarland Marceau Architects Ltd. Structural engineer: Thorson and Thorson Ltd.

may not seem to be anything especially remarkable about being able to generate and define a surface in this way, and one may wonder about the reason for the form's history of popularity in the world of architecture and building structures.

Another way of describing exactly this same shape, however, begins to hint at its remarkable qualities. Rather unexpectedly, one particularity of the doubly curved anticlastic hypar surface is that *straight* lines can be drawn on it. In fact, perhaps the simplest way of generating a hypar form experimentally is to take two parallel straight rods that are connected by a closely spaced set of strings and then tilt the rods in opposite directions; e.g., lowering the far end of the left rod while lifting the matching end of the right one. The strings tied to the two rods obviously have no capacity for bending, and so they must necessarily stay straight between the rotated rods. The overall surface suggested by the closely spaced strings, however, will quickly be recognized as one having anticlastic double curvature – in fact, they produce just the hypar surface that we have previously described mathematically.

But more importantly than noting this "curiosity" of geometry (at least from a building designer's point of view), it will also quickly be recognized that this means that one can in fact create a complexly curved hypar surface using discrete straight-line elements. The First Nations House of Learning on the University of British Columbia campus, for example, has a roof of this distinctive doubly curved shape simply made using straight tree trunks. (Ill. 13.35.) Extending this potential further, if a compressive membrane shell of this shape is to be built of concrete, it can be completely formed by means of flat boards of wood and also reinforced with straight reinforcing bars rather than by trying to bend or twist either of these materials in some sort of awkward manner. Furthermore, this straight-line particularity of the hypar shell can also be advantageous in a shell surface that needs to be pre-stressed (we will discuss why momentarily), since the post-tensioning tendons running through the shell can be positioned so as to be perfectly straight and then tightened without having them cause supplemental and problematic bending stresses across the thin surface of the concrete shell.[12]

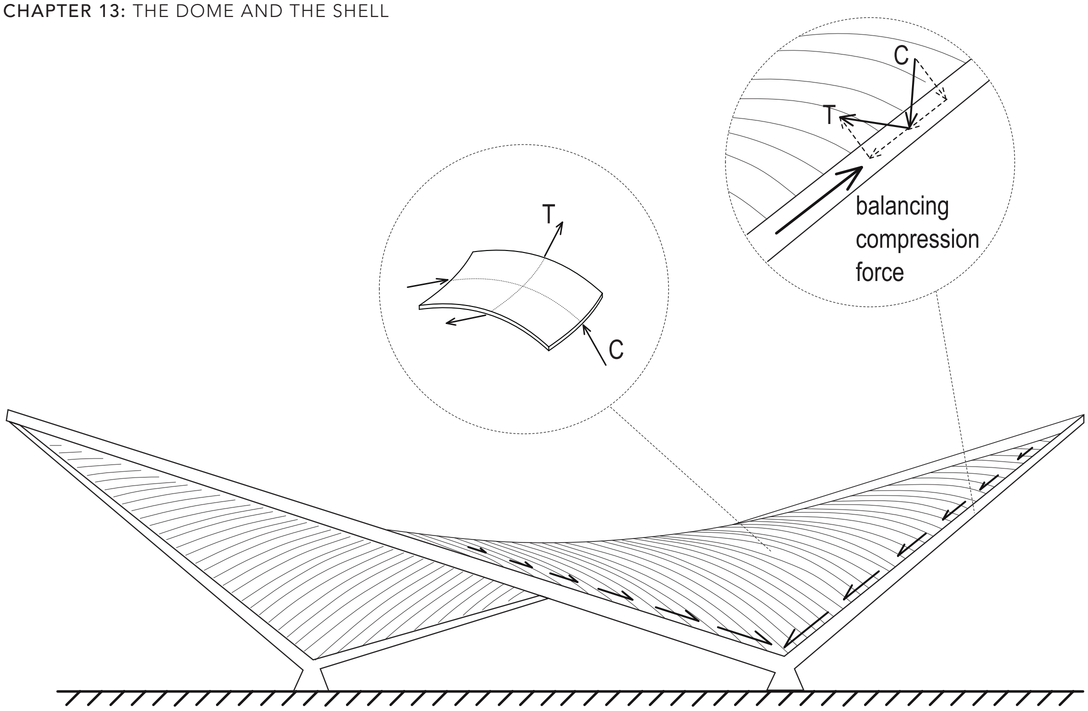


Figure 13.12
Detail depicting membrane stresses acting on small element of hypar surface, with "hanging" tensile stresses in upwardly curved direction and "arching" compressive stresses in downwardly curved direction. Anchoring of both sets of stresses at edge of shell surface indicates need for additional balancing compression force along the edge in order to ensure equilibrium.

Illustration 13.36
Hypar Shell, Cornell University Arboretum, Ithaca, NY, USA (1975).
"Classic" form of doubly curved hyperbolic paraboloid concrete shell.
Cornell student project. Faculty advisor: Professor Donald Greenberg.

Transverse gravity or lateral loads are carried in a thin-shell hypar by means of the in-plane sets of axial membrane stresses that we have encountered previously in our discussion of spherical shell domes – except that here, because of the opposite curvatures of the surface, we fundamentally establish equilibrium everywhere by having tension sets of membrane stresses in the "hanging" direction of the curved surface and compressive membrane stresses in the orthogonal "arching" direction. (Fig. 13.12.) This suggests an equal sharing of load-carrying responsibility between the two directions with the resultant structural efficiency benefits, but it also presumes that one is using a material for the shell surface that is as equally capable of carrying tensile stresses as it is compressive. If this is not the case, then the noted straight-line particularity of the hypar can be used to advantage by pre-stressing the shell surface into a sufficient state of pre-compression so that the tensile membrane stresses that result from loading will not be enough to overcome them, which is conceptually just the reverse of the strategy discussed in Section 11.8 of the need to pre-tension fabric membranes because they completely lack any compression capabilities. Calculating the magnitudes of the membrane stresses in a hypar shell surface in a precise manner is considered beyond the scope of the introductory treatment in this book but, as with the shell dome, one will generally find these to be remarkably small relative to a shell material's capacity – leading to the similar conclusion that we came to before about the incredible advantage that is to be derived from equilibrium being established through surface curvature and the overall shaping of structure in three-dimensional space. This, as we are seeing, is developing into the common theme of the second part of this chapter.

A further design aspect of note with regard to the use of the hypar shell is that the surface that is used does not necessarily need to be of the whole geometric saddle shape; one can "cut" out parts of this surface at will and still have the special geometric and behavioral characteristics of the hypar, although this possibility does have to be compensated for. Consider, for example, what might be identified as the most basic and prototypical of hypar shell surfaces: one which is supported at two low points in the "arching" direction and which reaches skyward with two high points in the orthogonal direction. (e.g., Ill. 13.36.) This surface will carry load by means of membrane stresses throughout as we have described, but in order to generate this load-carrying mechanism there will have to be an effective "anchoring" of the tension and compression

Illustration 13.37
Shells, Oslo School of Architecture and Design, Oslo, Norway (2006). Combination of three hyperbolic paraboloid shells, each measuring 1.5m × 1.5m (5ft × 5ft) and 20mm (0.75in) thick, are suggestive of spatial possibilities at larger scale. Straight lines of formwork lining boards remain evident in finished form made of lightweight fiber-reinforced concrete.

AHO contributers: Eli Malene Haugen, Ragnhild Gødø, Tone Sandøy, Merethe Skjelvik, Silje Hustad Widing, Line Mari Haugland, Sara Brubæk Bua. Faculty advisor: Arne Eggen, one of the present co-authors.

Illustration 13.38
Restaurante Los Manantiales, Xochimilco, Mexico (1957). Radial repetitions of hypar geometry unite into unique restaurant roof. Exceptional thinness of concrete surface stands out, even from a distance.

Architect: Fernando and Joaquin Alvarez Ordóñez. Structural engineer: Félix Candela.

stresses along all of the free edges of the shell. Closer examination of the equilibrium of the edge-anchoring condition will establish that a balancing axial compression force will be necessary along the cut edge, increasing in magnitude toward the base support, and this is usually accommodated by a progressive thickening of the shell. It should be recognized that this provision of anchorage in order for membrane action to be developed in the hypar shell can be considered to be completely analogous to the function and need for anchoring cables along the edges of the tensile membranes that we discussed in Chapter 11. Finally, it is also of design interest to point out that one can take several such discrete pieces of hypar surfaces and effectively "cut and paste" them together, generating all kinds of unexpected surface geometries and potential occupiable spaces that are not so obviously a part of the hypar family. (e.g., Ill. 13.37, Ill. 13.38.)

Illustration 13.39
Indoor Tennis Centre, Heimberg, Switzerland (1980).
Form developed using Isler's experimental process for shell shape finding: hanging membranes solidified, inverted, and adapted to various program and site conditions.
Architect: J.A. Copeland. Structural engineer: Heinz Isler.

13.8 Beyond Surface and Geometric Purity

In hindsight, it can be recognized that concrete shells and compressive membranes had a certain period of popularity for use as large span roof structures in the 1950s and 1960s, perhaps because of their efficiency and the development of theories and methods for predicting their structural behavior, but also maybe because their curvilinear forms departed so radically from the orthogonal architectural regimen of the preceding years and designers found this release appealing and symbolic of a new era. These early shell forms represented a foray into the uncharted territory of the future, but then their primary *raison d'être* seems to have quickly been supplanted by the parallel advances taking place in the similarly shaped but much more easily fabricated tensile membranes of cable nets and fabric structures, so that there were not very many rigid shells built toward the end of the last century. Ironically, in a sense, and until very recently, their use perhaps came to be considered somewhat passé and the form of a bygone era. All the while, though, experimentation was quietly going on and the shell was biding its time. Exploration of shell forms was being conducted in the 1960s to 1980s by relatively few but nonetheless very talented designers. Switzerland's Heinz Isler (1926–2009) was almost alone in his playful but altogether logical experiments of hanging fabrics into geometrically complex but structurally efficient tensile funicular surfaces, spraying or freezing these into rigid forms, and then inverting them into correspondingly thin concrete compressive shell forms, thus building up over the years an impressive repertoire of theaters, warehouses, service stations, etc.[13] (e.g., Ill. 13.39, 13.40.) And in Uruguay the engineer Eladio Dieste (1917–2000) experimented not only with the geometry of shell forms but also with an innovative and quite unexpected use of materials for such a structural type: traditional and standard-sized hollow clay bricks

Illustration 13.40
Indoor Tennis Centre.
Shell geometry varies continuously; shell is remarkably thin at top, can be seen to thicken toward supports. Heinz Isler acts as scale figure.

Illustration 13.41
Church of Christ the Worker, Atlántida, Uruguay (1960).
The beginnings of Dieste's œuvre of curving, long-span, thin shells made of clay bricks (post-tensioned together).
Designer and structural engineer: Eladio Dieste.

Illustration 13.42
Atlántida Church.
Interior of church.

were pre-stressed together into remarkable and quite unexpected curvilinear shapes that seemingly soar miraculously above one's head.[14] (e.g., Ill. 13.41, 13.42; see also Ill. 5.38 and Ill. 8.30.)

As we discuss where shells are today, perhaps it should be pointed out in no uncertain terms that not all shells need to be made from a smooth, uniformly thick, and continuous concrete surface. We have already seen earlier in this chapter the beginnings of variations from this in the ribbed structural form of the dome of the Palazzetto dello Sport developed by Nervi, although in that case the continuous shell surface remained despite having been considerably thinned out. Embedded in that shell dome and its rib pattern, however, is the compelling suggestion of being able to go even further and open up the dome surface to the sky. As well, the relatively low magnitudes of stresses that have generally been shown to exist in a compressive membrane by means of extrapolation from the numerical example of Section 13.3 also begins to suggest that tremendous possibilities exist for the literal opening up of the compressive membrane shell by concentrating membrane stresses into forces in relatively short, discrete, intersecting linear elements, whose overall arrangement forms a tight "mesh" or "grid," thus leading us to what is today's quite popular *grid shell*. We can liken this approach of opening up the solid shell surface while still having it retain its original overall structural behavioral characteristics to what we have described with other types of structures in previous chapters; e.g., trusses that can be thought of as beams with holes cut into them, or space frames' two-way load action being likened to that of the efficient opening up of a solid plate or slab of structural material.[15] The possibilities of form variation that today are achievable by the grid shell are extraordinary (e.g., Ill. 13.43), even if there is a careful rigor inherently involved in creating these so as to ensure that the forces acting on the structure are mainly carried by in-plane membrane action.

If dealing with in-plane membrane stresses by the means of a gridded mesh has loosely been justified above, it may still seem that the stability of such a surface is somewhat miraculous. As a means of briefly addressing this aspect, we will here consider the configuration of a basic grid shell: often it is composed of a rectangular mesh of thin bars of some sort, typically made of steel but alternatively, perhaps, of wood or another material. Depending upon the type of grid shell, one possibility for creating the desired curved shape would be to construct extensive formwork over which

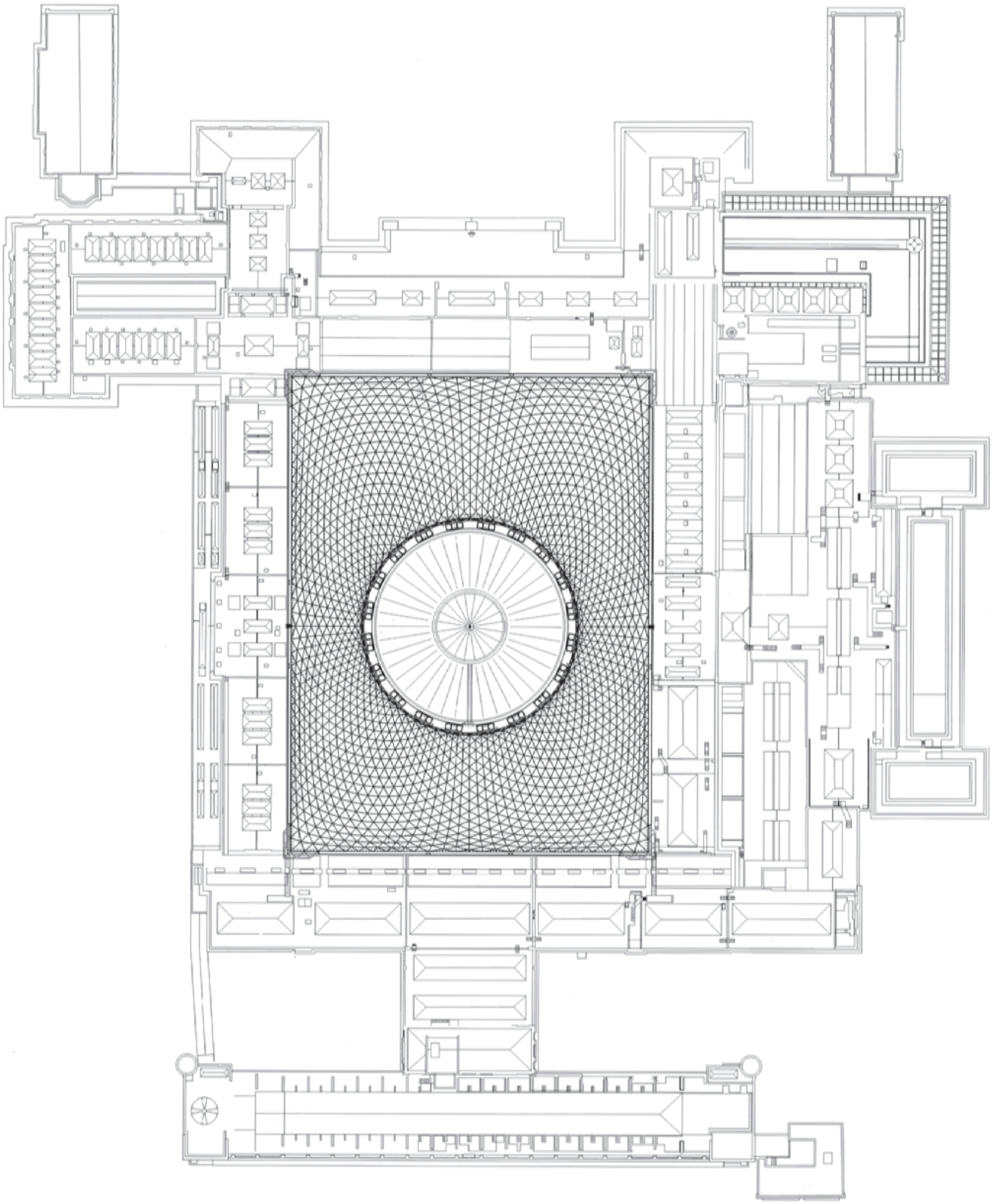
Illustration 13.43
Weald and Downland Gridshell, Singleton, West Sussex, England, UK (2002).
Layered oak laths used to create grid of doubly curved shell; rectangular panels stiffened in plane of surface by secondary lath layer that triangulates system. Material choice inspired by workshop space use for conservation and repair of historical timber-framed structures.
Architect: Edward Cullinan Architects. Structural engineer: Buro Happold.

Illustration 13.44
DZ Bank, Berlin, Germany (2001).
Grid shell curvature varies greatly within confines of courtyard; intermittent sets of fanning tension rods attached to arched ribs help to maintain form.
Architect: Frank Gehry and Partners. Structural engineer: Schlaich Bergermann und Partner.

Illustration 13.45
'Meiso no Mori' Municipal Funeral Hall, Kakamigahara, Japan (2007).
Irregular undulating shape of concrete shell roof informed by analytical studies of structural behavior; it also clearly alludes to the topography of the surrounding hills, while contrasting with orthogonal geometry of "grounded" walls below.
Architect: Toyo Ito and Associates. Structural engineer: Sasaki Structural Consultants.

this gridded surface is built. But an alternative and much more economical method of creating it can also be achieved by building the gridded mesh flat on the ground with loose-fitting connections and then lifting the whole assembly up into the air, or draping it over scaffolding that is later removed. The loose joints of the mesh allow the grid elements to rotate and twist freely in order to assume the desired geometry and then the whole surface is stiffened up, often by means of a series of tightened diagonal wires that create stable triangles within the rectangular mesh but also by other means, such as running longitudinal members over the gridded surface or by tightening all of the joints so as to produce frame-like rigidity in the plane of the shell. Depending on the situation, a thin grid shell surface may in addition need to be further stabilized, perhaps by intermittent sets of "fanning" tension rods that give the form stability every so often across the overall section of the shell surface. Such distinctive stiffening systems can be likened to being appropriately lighter and more transparent versions of the transverse plates or bulkhead ribs that we encountered in Section 13.5 to help folded plates and cylindrical shells keep their shape. Frank Gehry and Jörg Schlaich's collaboration on the incredibly fluid profile variations of the glass-covered grid shell for the courtyard of the DZ Bank in Berlin illustrates well the need for the use of such a supplemental stiffening system. (Ill. 13.44.)

But grid shells, while currently popular, are not the end of the story in terms of the evolution of compressive membranes. Simultaneously there is a limited yet remarkable resurgence of interest in the concrete shell itself – although today it is less likely to be of the pure geometry variety of previous years but instead of a noticeably different shape, such as is exemplified in the striking profiles of the white shell roof of Toyo Ito's 'Meiso no Mori' Municipal Funeral Hall. (Ill. 13.45.) Relatively recent trends in architectural design stemming from the use of digital technology and parametric generation of "free-form" surfaces – with the ability of the designer to "loft" and manipulate and deform virtual surface geometries in ways that were not even close to being feasible before – mean that surfaces are once again coming into vogue, even if these are no longer describable using pure mathematical equations.[16] Undoubtedly this second life of the shell is also being sustained by parallel advances in computer structural analysis techniques and, perhaps even more so, by the application of computer technology to the construction industry, with the possibility of being able to use digital information and manufacturing technologies for either the shaping of molds for non-geometrically definable formwork or for the cutting and precision milling of the thousands of different individual pieces and segments needed for contemporary grid shell surfaces. New shell forms can thereby be designed, produced, and studied, signaling the beginning of a yet-to-be-written chapter on the ever-evolving relationship between structural behavior and architectural design.



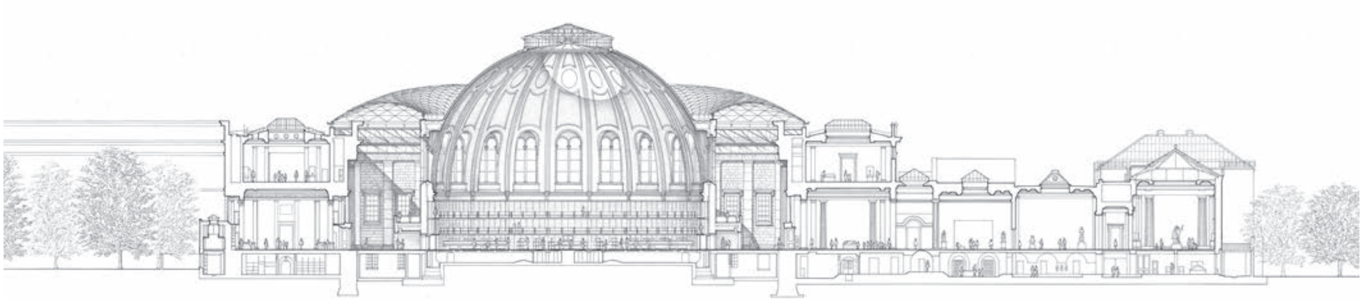


Illustration 13.46
Great Court, British Museum, London, UK (2000).
Section and plan drawings.
Architect: Norman Foster + Partners. Structural engineer: Buro Happold.

Illustration 13.47
Great Court, British Museum.
Geometry of grid shell roof mediates between circle of the central Reading Room drum and rectangle of surrounding classical façades.

13.9 Four Exceptional Shell Forms

Great Court

One of the most remarked-upon buildings of the past quarter century is really not a complete building structure at all – or, at least, the part of it that is new is only a relatively small portion of the whole building. We refer here to the Queen Elizabeth II Great Court of the British Museum in London, and even more specifically to this institution's courtyard roof, which was completed in the year 2000 as a harbinger of things to come in the twenty-first century in terms of structure and architecture. (Ill. 13.46.) The objectives and successes that are associated with this canopy, designed by architect Norman Foster + Partners and structural engineer Buro Happold, have been discussed extensively elsewhere; briefly, however, these include resolving major congestion-of-circulation issues for the museum, creating Europe's largest covered public square, and bringing to life a remarkable and remarkably "lost" and forgotten urban space right in the heart of the city.

We will spend but a few moments here to describe some of the specifics of the great gridded shell that makes all of these things possible. First, of course, one must appreciate the basic geometrical issue of trying to cover, in as minimal a surface membrane fashion as possible, a square interior courtyard that has a domed cylindrical drum located right in the middle of it. (Ill. 13.47.) The old rules of geometry and mathematical formulas just do not apply any longer – instead, in a Brave New World fashion, a form-generating computer program defined the surface while taking both structural and architectural design imperatives into account. The resulting surface shape has been likened to the top of a donut, except that in this case the donut has a square outside edge. The fact that this surface is bulging upward is plain to see and explain: obviously a surface as thin as this one would have no hope of working if it were simply horizontal because of the huge bending moments and stresses that would result. Instead, and in order to carry the roof loads in as much of a compressive-membrane manner as possible, the profile of the shell is generally curved about a rather tight radius in order to give the form sufficient steepness and curvature; the height of the now-encircled Reading Room dome provides ample reminder of the logic of this basic strategy for carrying load primarily in compression.[17] This having being said, the sectional drawing of Ill. 13.46 demonstrates that we have, nonetheless, made significant progress over the past 150 years; i.e., the structure for the new shell roof is noticeably flatter and thinner than that of the historical dome.

The grid of the shell is triangulated by skewed sets of thin intersecting steel members that define both the structural elements and the attachment frames for the glass panels that create the enclosure for the light-filled courtyard: 1656 double-glazed panels are needed to cover the surface, and 1800 nodal connectors join the six steel members at every intersection point (there are 6000 members in all). But mere numbers are by no means the end of the story here: the geometry necessary to create this shell surface is one which is flatter around the perimeter than it is toward the center near the Reading Room dome and it is flattest of all in the four outside corners of the surface. This means that each and every joint has to be precisely shaped to a slightly different geometry – something that only recent developments in digital manufacturing technology have made possible. Going even further, this ever-so-subtle variation of the shell's surface curvature is also relevant in reminding ourselves of where this discussion and this chapter began. The flattest part of the Great Court roof shell around the perimeter requires loads to be carried and surface stability to be established by more bending action in the gridded shell surface than in the center, where it is more curved and where compressive membrane action is therefore more effective; the result is that the structural members of the grid shell are not uniform in dimension everywhere but rather they taper gradually from a minimum near the middle of the Great Court and increase in dimension toward the outside edges. And if this concern with bending sounds vaguely familiar, it should: recall that in historical ribbed and masonry domes we fundamentally relied on combined axial compression as well as bending action in order to have stability; here, in making "progress" toward shell surfaces of "freer form" than those of the geometric purity that we have been looking at in the latter part of the chapter, we have reverted back to relying on the combined actions of axial compression plus bending. There is nothing inherently wrong with that, of course, but if necessary we can console ourselves that there has indeed been considerable progress over time: the Great Court grid shell is considerably lighter – in every sense of the word – than is that of the Reading Room dome it encircles.

Fiera Milano

If the Great Court is said to be Europe's largest covered courtyard, then the Fiera Milano is its largest trade fair complex, which is entirely appropriate as Milan has a long tradition of putting on display the state of the art in furniture and fashion design among many other things. A constructed exhibition fairground on the outskirts of the city of Milan, completed in 2005, covers more than two million square meters of space (21.5 million square feet) and is composed of more than a dozen different buildings of various shapes and sizes so as to be able to accommodate several major exhibitions and conventions simultaneously. This facility also served for the extended site of the World's Fair held in Milan in 2015. In order to unify this immense and disparate complex, a long linear open-air circulation spine connects all the various exhibition halls, and this axis is distinctively covered by an undulating glass and steel canopy structure that gives the project its iconic identity. Extending over a length of no less than 1300m (4250ft) this pedestrian walkway's glass and steel roof canopy is flat in certain places but in others undulates very distinctively to form curved surfaces. (Ill. 13.48.) This roof has been called an artificial/natural landscape (that mimics and frames the view of the Alps on the horizon), and its curving forms likened to dunes, hills, funnels, and even volcanoes that figuratively reach down to the ground and rise up to touch the sky; perhaps the unifying name of vela (sail) best describes its overall fluctuations of form.[18]

The initial design sketches for the roof by the architect Massimiliano Fuksas were made buildable through collaboration with the structural engineers Schlaich Bergermann und Partner and Mero-TSK International, the manufacturer of space frames

Illustration 13.48
Fiera Milano, Milan, Italy (2005). Well over 1km long (approx. 0.75 mile) roof canopy for exhibition and convention center's outdoor circulation spine sweeps along in continuous fashion, in places wrapping over program volumes, at others dipping down to touch the ground. Flat, orthogonally gridded surface structure becomes triangular and faceted at locations of significant curvature.

Architect: Massimiliano Fuksas Architetto. Structural engineer: Schlaich Bergermann und Partner together with Mero-TSK International.

Illustration 13.49
Fiera Milano.
"Funnel" reaching down to ground; curvature allows structure to be self-supporting by means of in-plane-of-surface shell-like behavior.

that decades ago developed the famous patented spherical node connector for such structures (see Ill. 9.35). The Fiera Milano canopy is clearly no regularly defined geometrical surface: it is neither monotonously repetitive nor can it be described wholly by following pure mathematical equations; rather, it can generally be described as being of composite geometry, with some parts perfectly flat and horizontal and other portions that are free-form surfaces of either single or double curvature (and the latter of either synclastic or anticlastic form). The main task of the consultancy was to develop a lightweight structure for the surfaces, which effectively became about finding the necessary geometry for a gridded steel mesh that could create the desired forms and yet also support the dead weight of the glass panels and the live loads on this surface caused by such variables as snow, wind, seismic activity, and (especially because it is an extremely long and open-air structure) temperature changes. In the flat regions of the canopy, things are relatively simple: the grid is quadrilateral when seen in plan view, 2.7 x 2.25m (8.9 x 7.4ft), and it carries the vertical loads in the good, old-fashioned two-way beam-grid manner (recall Section 7.10) with steel members having a bending-efficient T-shaped section. The covering glass panels match that of the steel grid and are attached by means of aluminum frames to the tops of the steel members.

When this flat grid is significantly distorted out of plane into the funnel shapes, however, things become more complicated. (Ill. 13.49.) In order to adopt the desired shape, the sides of four-sided panels would have to twist in space relative to each other, something the overriding flat glass sheets would simply not be able to do without becoming overstressed. The solution that was developed for these regions of the canopy was to change the rectangular grid to a triangular one, which allows each glass panel to be planar; i.e., the surface becomes a faceted one, although the panels are small enough that at the scale of this project it appears overall to be a smoothly curving surface. The strong curvature of the surface in these regions provides the perfect opportunity for the in-plane compressive membrane action that we have been discussing to take over the load-carrying responsibilities. And in conformance with the strategic shaping of what are now primarily axial compression elements, the steel mesh changes in these regions to become made of square hollow members (Section 8.6). Of course, there will inevitably be transition zones between the flat and curved parts of

Illustration 13.50
Rolex Learning Center, École Polytechnique Fédérale de Lausanne, Lausanne, Switzerland (2009).
Double layer of undulating surfaces establish the form and occupiable spaces as well as the structure of an unconventional academic building.
Architect: SANAA. Structural engineer: Bollinger + Grohmann.

the canopy, both in terms of geometry and primary load-carrying methods, and in these areas the triangular and rectangular grids can be observed to overlap each other.

One final aspect of this project will be briefly discussed here: that of being able to manufacture the incredible kit of parts that is necessary to build such a free-form surface. There are roughly 42 000 steel bar elements needed for this more than 1km (almost 1 mile) long gridded mesh, and 18 000 nodes are necessary to connect them all together. The frequent and irregular variation of geometry that we have described for this surface and the precision of form that is evident in the images suggest that the manufacturing of the elements and the connections cannot be done for such a project by conventional methods. Indeed, all of the bar elements and the nodes were defined by variable geometric data that was linked to state-of-the-art digital manufacturing techniques, including using milling machines with computer numerical control (CNC). The Fiera Milano can thus be seen to be a compelling example of the new possibilities that exist for compressive membranes in particular and, more generally, for a new basis of development for structure in terms of architecture.

Rolex Learning Center

The Rolex Learning Center is a large, single-story building that provides spaces for multiple university functions and student activities at the École Polytechnique Fédérale de Lausanne. In plan view, it is a rectangular building that measures 160m x 75m (525ft x 245ft), but this is where typical descriptors about this building end, since both its external form and internal spaces are highly irregular. In distinctive fashion, two seemingly identical undulating white surfaces serve as the building's floor and roof, spaced only 3.3m (11ft) apart, with glass walls all around the perimeter and several large, rounded openings admitting light deep into this "sandwiched" layer of space. (Ill. 13.50.)

One's typical perception and experience of occupied space is distorted here since there are almost no flat, horizontal areas within the mat building's vast expanse. The undulations of the floor and roof surfaces produce more limited views and intimacies of space than one is used to encountering in such a building – especially one having no formal walls – and these aspects are constantly changing with the varying slopes of the surfaces and shapes of the

Illustration 13.51
Rolex Learning Centre.
Thick concrete shell creates walkable surface "landscape."

openings. A few, glass enclosed "pods" do have flat floors and these exist for more formal study/discussion/café or other service spaces, but generally the open space between the sloped surfaces produces a free-flowing space that encourages informal meetings and discussions and other social activities – in a similar way that a classic academic campus' quadrangle does with its intersecting circulation footpaths and grass lawns for lounging, reading, talking, playing, etc. Moreover, in certain areas, the relatively steep slopes are playfully/practically negotiated by means of switchback pathways – a feature which is certainly in keeping with the surrounding alpine landscape. (e.g., Ill. 13.51.)

But looks are actually deceiving here, for in spite of their apparent visual similarity the floor and roof surfaces could not be more different in terms of structural configuration and behavior. The lower surface is a doubly curved 60cm (2ft) thick reinforced concrete shell, occasionally arching up into the air so as to draw people into/under the building. Such a free-form shell surface derives a good part of its stiffness and strength from its curved shape, and thus develops membrane-type axial stresses within it to carry the gravity loads. But universal access and other requirements had to be reconciled with structural optimization, and the shell form could not be shaped to be structurally "ideal/funicular" in profile, meaning

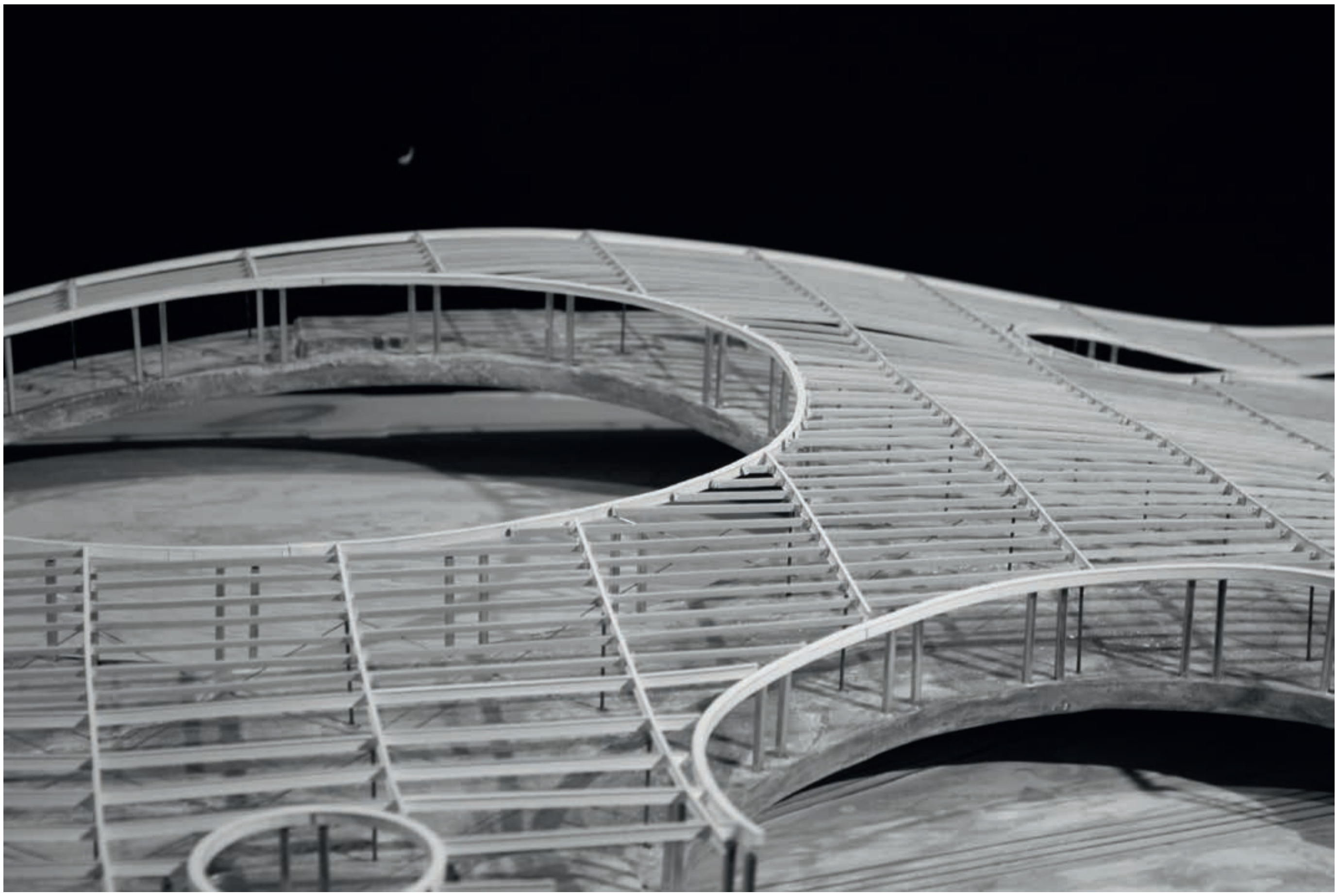


Illustration 13.52
Rolex Learning Centre.
The two surfaces are not identical, despite finished appearances. The lower surface is indeed a thick, free-spanning reinforced concrete shell, but the roof "shell" is actually timber-and-steel-framed construction supported on many, relatively closely-spaced, slender steel columns.
Cornell model by Kao Onishi and Janice Rim.

that bending stresses are *also* present across the thickness of the shell surface in order to maintain equilibrium, leading to the need for a shell which is quite thick. Also, the lifting-up of the surface that gives it its wave-like undulations is accomplished by means of significant arching action within the shell surface, and because of the relative flatness of the arched profile the magnitude of the compression forces is quite large and the thickness of the shell is therefore further strengthened at these locations with reinforcing ribs that are 20cm (8in) thicker than the rest of the shell; the opposing outward thrusts are resolved where the shell meets the ground by means of horizontal pre-stressed cable tension ties.

The roof, while appearing to be an identical shell, is instead – and surprisingly – built in "conventional" post-and-beam fashion, with the columns on a 9m x 9m (30ft x 30ft) grid, and the beams having typical one-way timber-and-steel beam sequential hierarchy. (Ill. 13.52.) It may seem ironic that such a spatially innovative building makes extensive use of such a traditional structural system for its upper "shell" surface, but among the advantages is that the resulting roof has relatively little mass compared to a thick concrete slab, and so any seismic forces that would have to be countered will thus be greatly reduced.

Illustration 13.53
Teshima Art Museum, Teshima Island, Kagawa Prefecture, Japan (2010).
View of the museum's site and context, on a terrace overlooking the Seto Inland Sea. Exterior form of the shell structure fits "naturally" into the landscape but also stands out from it with its stark white concrete material.
Architect: Ryue Nishizawa with Rei Naito (artist collaborator). Structural engineer: Sasaki Structural Consultants.

Teshima Art Museum

Finally, to close things out, we will consider the "free-form" concrete shell of the Teshima Art Museum, an architecture and art and structural engineering design collaboration between the architect Ryue Nishizawa, the artist Rai Naito and the engineering office of Sasaki Structural Consultants.

The building is on a spectacular site on Teshima Island overlooking the Seto Inland Sea, up a steep hillside from the water far below, although the more immediate surrounding context is at the edge of a gently sloping natural terrain of fields and woods and human-made rice terraces. (Ill. 13.53.) The museum was built as part of a plan to revitalize the island's economy, which was devastated in the 1980s by an industrial waste scandal involving the illegal dumping on the island of toxic byproducts from manufacturing processes located elsewhere in Japan. To support the aging inhabitants that still remain on the island, but also to give the community new life, the rehabilitation plan includes a revival of traditional agricultural production as well as new initiatives to make this place a cultural travel destination – to wit: the building of the Teshima Art Museum.

The approach to the structure is deliberately controlled along a narrow, winding, circumferential path leading through a small forest and having occasional glimpses of the spectacular views of the Inland Sea, before seemingly accidentally coming upon the low-slung "natural" form of the shell structure that is – quite literally as we shall see – the Teshima Art Museum. Here the curvature of the shell is most obviously not mathematically "pure"; instead, it variously but subtly swells or constricts here and there; Nishizawa described its overall form as having been inspired by that of a droplet of water lying on a flat surface.[19] The combined yet conflicting effects on the shape of such a droplet when acted upon by gravity as well as other, more elusive natural forces – such as water surface tension, for example – can thus be brought to mind. But knowing of this design inspiration is not necessary to appreciate the final result: a powerful external form that is nonetheless perceived as being in harmony with the natural landforms of the site, effortlessly fitting into the landscape like just another hill or slope, even as it simultaneously stands out from nature with its starkly "alien" white concrete material composition.

On the inside, the space is unexpectedly low and horizontally extended; i.e., the maximum height of the surface is only 4.5m (15ft) for a building whose plan dimensions are roughly 40m x 60m (130ft x 200ft). A concrete floor completely covers the ground, and from around its perimeter a 25cm (1ft) thick concrete shell rises up to cover the space with a smooth, continuous ceiling; i.e., the sense of flow of the interior space is uninterrupted by any columns or beam protrusions from under the shell surface. Two large elliptical openings are made in the shell to admit light and air and rain into the space. And that is all there is to it, or so it would seem – for as we have grown accustomed to by now, there is much more involved in producing such a "simple" building structure than at first meets the eye.

First, we shall consider the thickness of this shell, which just on its own reveals quite a lot about the structural behavior taking place within it. By point of comparison, and as we have seen in Section 13.7, mathematically pure hyperbolic paraboloid shell structures were, as a result of their geometry, principally carrying the predominant self-weight dead loads by means of axial in-plane membrane stresses, and thus needed to be only about 4cm (1.5in) thick – as in the case of some of Félix Candela's exceptional structures (e.g., Ill. 13.38). Clearly, at Teshima, a significant price is being paid for the "impure" form of the shell, since it has a thickness that is about eight times more than Candela's norm for structures that span even greater distances; i.e., in order to ensure the equilibrium of the museum's "free-form" shell, membrane stresses must in general be supplemented by significant bending stresses acting across the shell surface thickness, which necessitates its being significantly thicker.[20]

Of course, creating formwork for pouring a "free-form" concrete shell is also not a simple matter. (Again, a comparison to the straight, wood board forms that could be used for Candela's hypar shells is to be noted by point of comparison – see Section 13.7.) In the case of Teshima, after an overall shape had been agreed upon by the architect, artist, and structural engineer after multiple form iterations and corresponding computer structural analyses had been conducted, an artificial mound of earth was very precisely shaped using 3500 points of elevation in order to ensure that the stresses in the eventual real shell structure would match as closely as possible the structural response predicted during the design phase. A layer of grout was poured over the earth mound and a complex, dense network of reinforcing bars was then carefully put into place, after which a special white concrete with lime additive was poured in a single day so as to avoid any unsightly construction joints, with a final layer of plastic coating then applied to the surface to counter

Illustration 13.54
Teshima Art Museum.
Contemplative interior space.

water infiltration and the potential rusting of the reinforcing bars. The concrete was left to cure for five weeks, after which it had gained sufficient strength so that the earth formwork could be carefully excavated from within, eventually leaving a singular structural shell form to span the open museum space.

The artist Rai Naito's work typically focuses on the interactions of light, water, and air, and in support of this agenda Ryue Nishizawa designed a building which is about "just" that; indeed, nothing else is on display in this museum other than the interaction of "building" with nature. For example, for Naito's inaugural installation entitled *Matrix*, small amounts of water trickle out here and there from 182 tiny ducts in the floor slab at different times throughout the day – to then coalesce with other water droplets, growing slowly into small puddles and rivulets that run along the floor only to disappear down similarly unseen drain holes. Visitors are left alone to contemplate and experience this playful water dance.

Another installation consists of a single string suspended from two points at the edge of one of the openings in the shell roof. The string is so light, of course, that it is caught by the breezes, and renders visible the currents of air movement within the space that otherwise cannot be seen. (Ill. 13.54.) A much more subtle play or reference is also coyly being made by this string when there is no breeze at all and it is only subject to gravity loads. As we saw in Chapter 11, such a string, having uniform load along its length due to its self-weight, will necessarily adopt the pure mathematical shape of a catenary. We then saw in Chapter 12 that an arch could also be so configured in order to be in pure compression, and in this chapter that shell surfaces can be similarly shaped by extension (for example, in the work of Heinz Isler (Section 13.8) with his highly efficient shell structures derived from the hanging shapes of fabric membranes). Of course, as we have just seen, such funicular form is *not* the case with the Teshima Museum shell roof; to be able to appreciate such clever wit, however, one has to be able to understand and appreciate structural behavior and resultant form!

Installations aside, this building on its own highlights the interplay of light, water, and air together and as one with the birds, insects, sun, wind, snow, rain, and other elements of nature. Or one can focus only on the movement of the tree foliage by looking through one of the roof openings. At other times, the experience is all about observing the sky as a colored disk – one that transforms from blue to gray to white to black and other colors as well at sunrise and sunset. This museum at Teshima very deliberately fuses together the environment, art, and architecture – with all three working together as an entity. This building is about a singular structural-shell-created architectural space that envelopes/swallows one whole – and about how it enables the *re*-presentation of the natural elements and forces of nature that surround us all the time, but that we may not otherwise take the time to notice. Such can be the subtle-yet-forceful power of the structural basis of architecture.

Notes

Chapter 1
STRUCTURING SPACE

1 Neuhart, M. and Neuhart, J., *Eames House*, Ernst & Sohn Verlag, 1994, p. 37.
2 www.e-architect.co.uk/bordeaux/maison.
3 From a conversation with one of the present authors (B.N.S.). in Vienna in September 1996.
4 By "sub-optimization" we mean that the structure is not optimal with regard to the building's overall shape, but given the particular architectural context, measures have been taken to make the structure more materially efficient by changing the structural pattern according to the magnitude and distribution of forces.
5 CNC stands for computer numerically controlled processes.
6 As an extension of this approach, one can find that structures in architecture have an *aesthetic* interest; i.e., one which is focused on our visual experiences. While every attempt is made throughout this work to articulate what we see when we view a structure, it is considered to be outside of the scope of this book to elaborate more specifically on this broad topic; for that purpose, the reader is referred to Sandaker, B.N., *On Span and Space. Exploring Structures in Architecture*, Routledge, 2008.

Chapter 2
INTRODUCING STRUCTURAL SYSTEMS

1 While not being uncommon to speak of a *transport* of loads and forces, no transportation is actually taking place. A more precise, but elaborate, description of a horizontal span structure would be to state that the loads act at certain points while being prevented by the structure from reaching the ground at the vertical projection of those points. The structure in this case supports the loads while spanning a certain horizontal distance, resulting in new loads acting on the vertical structural elements and on the foundations well to the side of where the initial loads act.
2 By the notion of "surface elements" used here, we mean to identify a different concept than what in structural mechanics is commonly referred to as "surface structures." The latter notion is used in connection with thin, shell structures that carry loads by means of surface stresses acting in-the-plane-of-the-surface. (See Chapters 11 and 13). By "surface elements" in this context we mean to point out certain visual and spatial properties of a group of structural elements that form "planes" rather than "lines," and where the load-carrying mechanism may vary from one case to the next.
3 In Section 4.4 of Chapter 4 Statics we will define as *moments* those force actions which result in either rotation, flexing, or twisting of an element.
4 Or else when one end is fixed while the other is twisted. It is the relative rotation which counts. In Section 4.4 we will realize that torsion is a particular type of moment.
5 The downward deformation of the mezzanine at this end is actually a sum of the elongation of the hanger and the deformation of the roof beam at that point. The latter effect is not made explicit in the drawings.

Chapter 3
LOADS

1 One of the most compelling accounts of a building designer's responsibilities and fears of catastrophic failure surely has to be the article written about structural engineer Bill LeMessurier and the Citicorp Center in New York City: Morgenstern, Joe, "The Fifty-Nine-Story Crisis," *New Yorker* (May 29, 1995), pp. 45–53.
2 This nomenclature is an obvious allusion to nature's streams and rivers that gather the rainfall from a certain geographic area.
3 Recall at the same time, however, that common examples of *movable* "point" loads, such as those caused by people and furniture on floors, have their unpredictability accounted for by the occupancy live load allowances for floor areas.
4 These loads are "live" in the sense that they clearly can vary significantly over time.
5 For example, a huge snowfall on March 18–19, 2003, in the mountains of Colorado, USA dumped almost 2.5m (7.25ft) of wet heavy snow in certain places, leading to the collapse of hundreds of roof structures.
6 Why times the story height? Because typical building cladding and vertical window attachment systems usually span vertically from the edge of one floor to another, effectively converting lateral pressures into point loads applied at each floor level.
7 This is the same fundamental mechanism, of course, that is used to make airplanes fly; i.e., by means of air flowing over a carefully shaped obstruction – namely the airplane wing.
8 To carry the previous analogy of water flow a bit further, one will encounter a similar phenomenon with the variation of the speed of water in a river channel, with the water in the middle of the channel flowing fastest while that along the banks and bottom moves considerably more slowly.

9 The relatively rare matching of the period of vibration of a structure with the interval of wind gusting can have rather disastrous consequences – as is evidenced by the infamous video footage of the positive reinforcing of oscillations of the Tacoma Narrows Suspension Bridge in Washington State in 1940. The subject of suspension cable stability is further discussed in Chapter 11.

10 Horizontal floor slabs, beam-framing systems, floor and ceiling finishes, and mechanical and service distribution systems are all located at the floor level, whereas between floors there is mostly empty (occupiable) space except for the vertical columns and walls.

11 Only dead loads that are rigidly attached to a building structure need to be considered, as they are the only ones that are "dragged" along and become inertial mass. Live loads are unattached and should not be included in this total as they can remain stationary while the building moves relative to them – a phenomenon perhaps most clearly illustrated in video clips of bars during an earthquake when bottles and glasses are seemingly tossed about, whereas in reality it is the building that is moving back and forth with respect to what are, at least to begin with, stationary objects on tables and shelves.

12 We can observe the effect of this reality in the devastating aftermath of earthquakes where traditional long, low, stiff, and massive masonry walls predominate – such as in the cases of Amarice and Port-au-Prince previously mentioned.

13 This is much the same as the string on a musical instrument which will vibrate differently depending on its thickness and tightness and thereby produce different sounds.

Chapter 4
STATICS

1 Some have seen a certain symbolism in the fact that Newton was born in the same year that Galileo died. This is stretching history a bit, however. At the time, England still used the Julian calendar, which fixed Newton's birth to December 25, 1642. According to the Gregorian calendar of the Venetians, applying to the date of Galileo's death, Newton was born on January 4, 1643!

2 Newton was primarily interested in heavenly bodies, while here we think of bodies as representing beams, columns, struts, or whatever structural elements we are studying.

3 Slug is sometimes called pound mass.

4 The abbreviation lb comes from the Latin word *libra*, meaning "scales, balances," which also described a Roman unit similar to the pound.

5 If the body was able to fall freely, the acceleration would be 9.81m/s^2 for all bodies, independent of their mass. This is the reality of the experiments Galileo is supposed to have conducted from the Leaning Tower of Pisa.

6 Velocity is a vector, for example, while the accompanying scalar is quite simply called speed.

7 Cartesian is named after the French philosopher and mathematician René Descartes (1596–1650) who invented the very convenient system of orienting points and lines in a mathematical space.

8 Archimedes of Syracuse is credited as the first to use the lever principle when in about 250 bc he built stone-throwing catapults as weapons of defense against Roman invaders.

9 André Gide (1869–1951), French author, in *Journal 1889–1939, Vol. 1–3*, Paris, 1948.

10 Jean Prouvé (1901–1984) was a metals craftsman and much celebrated designer. Neither being an engineer nor an architect himself, he nonetheless played an important role in the development of certain aspects of modernism in architecture and design, working throughout his career with a number of well-known architects.

11 In reality, the forces acting at the tip of the cantilevering beam to the right of the column are the sum of the (smaller) roof load and the tension force in the rod.

12 A spatial system demands six independent support reactions to be in equilibrium, corresponding to three translatory equations ($\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$) and three rotational equations ($\Sigma M_x = 0$, $\Sigma M_y = 0$, $\Sigma M_z = 0$).

Chapter 5
MATERIALS

1 The modulus derives its name from the British polymath Thomas Young (1773–1829).

2 The element will also become thicker, but for a thin linear element like the one being discussed here, this will be of little significance.

3 See B. Berge, *The Ecology of Building Materials*, Architectural Press, 2000, p. 190.

4 Ibid., p. 191. Numerical values for primary energy consumption can be found in B. Berge, op. cit. (2000). It is important to recognize that such figures are not like material properties that can be taken as material constants. Also, as technologies for production develop, the values may easily change.

5 For more extensive reading on this subject, the reader is referred to Bill Addis' *Building: 3000 Years of Design Engineering and Construction*, Phaidon, 2007.

6 Among the many books on the structural development of Gothic cathedrals, Robert Mark's *Experiments in Gothic Structure*, MIT Press, 1982, is particularly recommended.

7 Erik Lundberg, *The Visual Language of Architecture*, translated from the Swedish *Arkitekturens Formspråk*, Nordisk Rotogravyr, 1949.

8 Andrea Palladio, *I Quattro Libri dell'Architettura*, Appresso Dominico de' Franceschi, 1570 (Modern English trans. R. Tavernor and R. Schofield, *The Four Books of Architecture*, MIT Press, 2002).

9 Galileo Galilei, *Due Nuovo Scienze*, 1638 (English translation: *Dialogues Concerning Two New Sciences*, Dover Publications, 1954).

10 Erik Reitzel, *Fra Brud til Form*, Polyteknisk forlag, 1979.

11 The reader is referred to the following books for histories of the development of structural theories and behavior: Karl-Eugen Kurrer, *The History of the Theory of Structures: From Arch Analysis to Computational Mechanics*, Ernst & Son, 2008, and the classic Stephen P. Timoshenko, *History of Strength of Materials*, Dover Publications, 1983.

12 An alloy is a homogeneous hybrid of two or more elements, at least one of which is a metal, and where the resulting material has metallic properties. The resulting metallic substance usually has different properties from those of its constituent components.

13 The poor tension strength resulted in cast iron being used almost exclusively in structural elements subjected to compression, such as columns, arches, and arched vaults.

14 "Coke" is a solid fuel made by heating coal without any air present – so that any volatile components are driven off.

15 "Ferrous" comes from *ferrum*, the Latin word for iron. Ferrous metals are iron based, like steel, wrought iron, and cast iron.

16 Galvanic corrosion is an electrochemical process in which one metal corrodes (the least noble, with reference to its position in a galvanic series) when in direct contact with a different type of metal and both metals are moistened by an electrolyte.

17 See also V.B. Bell and P. Rand, *Materials for Architectural Design*, Laurence King Publishing, 2006; C. Lefteri, *Wood: Materials for Inspirational Design*, RotoVision, 2003.

18 See Buro Happold, "Constructing a Prototype Cardboard Building: Design Guide," September 2001 (comm.: design guide published by the company Buro Happold). A. Cripps, "Building Real Buildings with Cardboard," Paper from the International Conference on Sustainable Building 2000, Maastricht, the Netherlands, 2000.

19 Resulting in the so-called soda-lime glass, accounting for about 90 percent of all glass that is made.

20 Glass in which these impurities are removed can be produced. This type was used for the Glass Pyramid at the Louvre in Paris, by architect I.M. Pei, for example.

21 The float glass method was invented in the 1950s by Sir Alastair Pilkington, a British engineer employed by the glass manufacturer Pilkington.

22 In very small samples of glass that are without flaws and cracks (such as in glass fibers) a very high tensile strength can indeed be reached. Tiny surface cracks in glass sheets were identified and studied by the British scientist A.A.Griffith (1893–1963) around 1920.

23 Somewhat confusingly, this same name is also used to refer to carbon filament thread as well as to felt or woven cloth made from carbon filaments.

24 Polyester may also be of the thermoplastic type.

25 For detailed information, see Frank Kaltenbach (ed.), *Translucent Materials: Detail Praxis*, Birkhäuser Edition Detail, 2004.

Chapter 6
THE HANGER AND THE TIE

1 The glass is of the low iron type because of its high degree of transparency, as was illustrated in Section 5.7.

2 In making this statement, we are again ignoring the self-weight of the tension members themselves. For most situations this will be sufficiently accurate, and certainly so for preliminary design. If angled tension members are long enough, however, they will significantly sag under the transverse load of their own weight and then also deflect in a suspension-cable-like fashion as is described later in Chapter 11.

3 The name "guy" is derived from the Old French verb "*guier*," meaning "to guide," and is defined as a rope or cable that is used to guide, stabilize, or secure something that would otherwise tend to change position or configuration.

4 The Skylon was, however, not the first time this peculiar form of structure appeared. One of the competition entries for a Socialist Settlement in the new town of Magnitogorsk in Ural in the Soviet Union was submitted by a team of students led by Ivan Leonidov (1902–1959). In one of the fine white-line-on-black illustrations from the competition material, we notice a similar structure, although the needle here seems to be resting directly on the ground. Stalin's Magnitogorsk was indeed realized during the 1930s, although without the innovation of Leonidov's work.

Chapter 7
THE BEAM AND THE SLAB

1 This is in contrast to a simple hanger or column, which will react to a load applied *along* its axis by lengthening or shortening, respectively, but whose main axis will keep its initial straight alignment.
2 Hoffman, Donald, *Frank Lloyd Wright's Fallingwater and its History*, Dover Publications, 1978, p. 18.
3 Silman, Robert, "The Plan to Save Fallingwater," *Scientific American*, Vol. 283, No. 3, 2000, pp. 88–95.
4 For symmetrical sections such as the one we are looking at here the neutral axis lies at mid-depth of the beam; if the section geometry is asymmetrical the neutral axis will be at the level of the beam cross-section's centroid, or center of area.
5 Galileo Galilei, *Due Nuovo Scienze*, 1638, op. cit.
6 These individuals included Edme Marriotte (1620–1684) who corrected Galileo's mistaken assumption of rotation of the cantilever beam about its bottom edge, and Antoine Parent (1666–1716) and Charles Augustin Coulomb (1736–1806), who are both acknowledged to have independently "solved" the bending behavior problem before Navier, even though the latter today receives most of the credit.
7 Given that symmetry of form is not limited to the design of building façades, it is surely no accident that the form of this bending stress equation is so completely analogous to that of the axial stress formula $\sigma = P/A$.
8 It is clear from this formula why the moment of inertia is also sometimes referred to as the second moment of area.
9 The Swiss engineer Wilhelm Ritter in 1899 and the German engineer Emil Mörsch in 1902 are independently credited with developing this truss analogy. The interested reader is referred to any of several editions of MacGregor, James G., and Wight, James K., *Reinforced Concrete: Mechanics and Design*, 5th ed., Prentice Hall, 2008, for a detailed consideration of this topic.
10 The exterior span will obviously not be quite as symmetrically restrained.
11 It should be noted that in the case of reinforced concrete beams care will have to be taken to place the reinforcing bars at the appropriate top and bottom levels along the length of the continuous beam if it is to work as intended.
12 A "nearly square" panel here is defined as having side lengths with a ratio of no more than 1.5:1.0.
13 The idea of drawing isostatic lines of stress on a structure – that is, lines that represent conditions of uniform structural demand (in a similar sense that *isobars* represent lines of uniform barometric pressure on a weather map) – seems to be credited to the Italian engineer Aldo Arcangeli, who worked in Nervi's office. In the case of a load-carrying structural slab, these lines represent directions of maximum and minimum bending action, and zero torsional shear demand (in contrast to the combined bending/twisting behavior of the orthogonal beam grid discussed in Section 7.10). For an accessible introduction to what can be a rather obscure subject, the reader is referred to Salvadori, Mario, and Heller, Robert, *Structure in Architecture: The Building of Buildings,* 2nd ed., Prentice Hall, 1975, pp. 222–229, 256–261, 272–273.
14 This echoes our discussion in Section 5.3 on the dangers of simply scaling up member dimensions to suit a spanning distance.
15 It should be noted that the benefits of composite action will typically not exist where the floor system reverses curvature as it runs continuously over supporting elements, since at these locations the concrete slab that is at the top of the section will be in tension, cracked, and ineffective in carrying load.
16 Unlike with a reinforced concrete beam where this overload can be dealt with by means of the introduction of shear reinforcing in the form of stirrups, a slab does not typically have enough depth for a similar type of steel reinforcement bar to be effective.

Chapter 8
THE COLUMN AND THE WALL

1 In practical structural design, a limitation is put on the stress level, reducing the stress by a so-called safety factor. Moreover, for some ductile materials, like steel, the yield stress rather than the ultimate stress is more relevant here.
2 At the very point where things start to happen differently, that is, at the precise buckling load, we will find that we might push the column sideways and the column will come to rest in this new position and establish a state of equilibrium there. We call this neutral equilibrium.
3 H.P. Lorange, "Sentrum og periferi," Dreyers Forlag, B.A. Butensschøn A.s & Co., Oslo, 1973. Translated from Norwegian.
4 The moment of inertia is also called the second moment of area.
5 By an "ideal" elastic column we mean a column which is absolutely straight and loaded only up to the point where the material stops acting elastically; i.e., before entering the plastic state.
6 The radius of gyration may be considered to be the distance between

the center of area (centroid) of a cross-section and the point at which all of the area may be conceived as being concentrated while still having the same moment of inertia as the original cross-section.

7 For more precise information about this, one needs to consult European, American, or the respective national codes of practice for the design of structures of different materials.

8 For columns influenced by both failing modes, we may for practical purposes write the load-bearing capacity as $P_{cr} = \beta\sigma_u A$, where β is a reduction factor dependent upon the slenderness ratio.

9 A game better known in North America as Pick-Up Sticks.

10 This was a Depression-era pilot project of the Tennessee Valley Authority (TVA) to control flooding and generate electricity and economic development – a dozen others were built throughout the Tennessee Valley following this model. These are also said to have influenced modern architecture: the Hungarian-born architect Ronald Wank gave the dam design a progressive, modern look – to the extent that Le Corbusier took an interest and visited the Norris Dam and other TVA projects in 1946.

Chapter 9
THE TRUSS AND THE SPACE FRAME

1 The term fishplate is defined as a short metal plate that is used to splice two members together, usually attached by means of bolts. The term is commonly used, for example, in the context of railways for connecting together the ends of metal rail segments. The unusual name stems from the word "fish" that describes a wooden bar with a curved profile that is used to strengthen a ship's mast.

2 The method of sections is associated with the German engineer August Ritter (1826–1908), and is also called the Ritter method.

3 Also, if a truss should undergo large deformations, the angles between members might change substantially and members that were initially zero-force members could be activated, contributing to added "final stage" stiffness and strength before potential collapse.

4 In reality, for trusses shaped to follow the external moment variation, we will find that the *horizontal* component of the force in a chord with parabolic shape is a constant and of equal magnitude to the force at mid-span in the straight chord. The vertical component increases as the slope of the curved chord increases toward the supports, resulting in a somewhat increased total force in that chord toward the supports.

5 There may be the opportunity to subtly have things both ways: with hollow tube members, for example, the outside dimensions of members can be kept uniform while the wall thicknesses vary.

6 The Italian mathematician Leonardo Fibonacci (about 1170–1250) came up with a sequence of numbers in which each number is the sum of the previous two numbers, starting with 0 and 1. Thus the sequence begins 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, etc. The higher up in the sequence, the closer two following numbers divided by each other will approach the so-called "golden ratio" (approximately 1:1.618 or 0.618:1).

7 One must keep in mind that for a cantilevering truss the compression chord will be at the bottom instead.

8 Of the two grid patterns, that with the diagonal grid arrangement provides more structural stiffness than does the rectangular one, especially at the corners where the span of the closest truss is reduced.

9 Alexander Graham Bell is more famous for his patent on the telephone (1876). He was until recently considered its inventor, but, in 2001, the American Congress took that honor away from him and credited instead the Italian Antonio Meucci as the true inventor of the phone. However, the unit of decibel we commonly associate with acoustics, which in reality expresses the magnitude of a physical quantity relative to a certain reference level, is still bearing his name. The decibel is one-tenth of a Bel, which is an abbreviation for Bell.

10 The Platonic solids are the tetrahedron, octahedron, and icosahedron, which are all composed of equilateral triangles (4, 8, and 20 respectively), the cube (composed of squares) and the dodecahedron (composed of pentagons).

11 We should note that when the system of intersecting member forces is not planar but spatial, we will have forces acting in all three dimensions of space. The requirements for equilibrium of each of the structural joints then becomes $\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\Sigma F_z = 0$, where z represents the third axis. A graphical solution will now require a 3-D vector drawing, but otherwise we may approach the mathematical solution in the same way that we did in the planar examples.

12 Of particular interest is the CNC technology that unites digital design and manufacturing. CNC is an abbreviation for computer numerically controlled processes. These refer to computer-driven machine tools used to fabricate components by, for example, selective removal of material.

13 Twelve-sided polyhedral composed of only pentagons are called dodecahedra. Fourteen-sided polyhedral composed of pentagons and hexagons are called tetrakaidecahedra.

Chapter 10
THE FRAME AND THE SHEAR WALL

1 This structure collapsed in 2016. For a discussion of the causes, see NLÉ website for responses to FAQ: www.nleworks.com/case/makoko-floating-school/
2 The layout grid is actually regular, but certain columns are omitted in an irregular fashion.
3 Nonetheless, such a wall in a tall building continues to be called a shear wall, even though it is clearly somewhat of a misnomer at that point.
4 Note that frames must be either pin-supported or fixed at their bases and not supported on rollers as this would compromise their stability and prevent the columns from resisting overall shear.
5 Today, of course, it is more than likely in practice that the computer will solve these problems for us! This doesn't diminish, however, the instructive value of our considering the hand methods here.
6 La Grande Arche perhaps should have more appropriately, if less poetically, been called "La Grande Charpente Rigide."

Chapter 11
THE CABLE AND THE MEMBRANE

1 If the two support structures are located at different heights, the sag refers to the maximum vertical distance between the cable and an imaginary straight line between the two supports.
2 The equation defining the shape of a catenary is more complex than that of the parabola. However, if the sag to span ratio of the catenary is small (≤ 1/10), the geometries of the two are practically the same.
3 The word "funicular" comes from the Latin word for "rope," namely *funis*; "catenary" is derived from *catena*, meaning "chain."
4 We can also mention in this context that all suspended cable shapes carrying loads by pure tension forces have their compressive counterparts in arches: if suspension structures are turned upside-down while substituting the cable with stiff structural elements, these will form arches acting in pure compression. We will discuss this further in the next chapter.
5 Calculating cable forces within an inclined cable (where the supports are at different levels) is much more difficult since the location of the lowest cable point (where the internal tension force is horizontal) is not at mid-span.
6 All structures have a natural vibration tendency when acted on by external loads, characterized by an oscillating motion that repeats itself after an interval of time, called a period. The number of occurrences of a repeating motion per unit time is called the frequency.
7 A doubly curved surface where both are curving in the same way is called synclastic, obvious examples of this being the surface of a ball or a globe.
8 His doctoral dissertation was submitted that year, entitled "The Suspended Roof. Form and Structure."
9 Tensile membranes are sometimes called soft shells because of the similarity of their geometry to the more solid shell forms that we will encounter in Chapter 13.
10 Referring to Fig. 11.12, the total applied load P on the surface of the element is, therefore, given by

$$P = (p \times A) = (p \times a \times b)$$

where A = surface area of the element with edge lengths a and b. Since both edges of the element are curved, their lengths depend on the radius of curvatures R_x and R_y as well as on their respective central angles Φ_x and Φ_y; i.e., since an arc length is the product of the angle (when measured in radians) times the radius, this means that we can express the lengths of the element sides as

$$a = \Phi_x \times R_x \text{ and } b = \Phi_y \times R_y$$

The total resultant force acting on the element surface, therefore, becomes

$$P = (p \times a \times b) = p\,(\Phi_x \times R_x)\,(\Phi_y \times R_y)$$

To ensure vertical equilibrium, however, this applied load is counteracted by the sum of the vertical components of the axial membrane forces that are acting along the four element edges. Individually, these components are

$$N_{xV} = N_x \sin(\Phi_x/2) \text{ and } N_{yV} = N_y \sin(\Phi_y/2)$$

Since there are two of both force components, however, and they act along the perimeters having lengths $a = \Phi_x \times R_x$ and $b = \Phi_y \times R_y$, the equilibrium equation yields

$$p\,(\Phi_x R_x)\,(\Phi_y R_y) = 2\,(N_x \sin(\Phi_x/2))\,(\Phi_y R_y) + 2\,(N_y \sin(\Phi_y/2))\,(\Phi_x R_x)$$

Now, remembering that the curvatures are assumed to be quite small enables us to simplify things somewhat with good conscience by saying that $\sin(\Phi/2) = \Phi/2$, where the angles are still measured in radians.* The equation now can be simplified to

$$p\,(\Phi_x R_x)\,(\Phi_y R_y) = 2\,(N_x (\Phi_x/2))\,(\Phi_y R_y) + 2\,(N_y (\Phi_y/2))\,(\Phi_x R_x)$$

$$p\,(\Phi_x R_x)\,(\Phi_y R_y) = (N_x \Phi_x)\,(\Phi_y R_y) + (N_y \Phi_y)\,(\Phi_x R_x)$$

Finally, dividing both sides of this equation first by $\Phi_y \Phi_x$, and then by $R_x R_y$ yields, in turn,

$$p R_x R_y = N_x R_y + N_y R_x$$

$$p = N_x/R_x + N_y/R_y \qquad \text{(Eq. 11.1)}$$

*An angle of 1 radian results in an arc with an equal length to the radius of the circle. An angle given in radians, then, expresses the ratio of the arc length to the radius. As an example, an angle of 90° will have an arc length of a quarter of the total length of the circle circumference which is $2\pi R$, yielding $\pi R/2$. Hence, this angle written in radians becomes $\pi/2$.

11 To justify that this is actually the case, remember that the perpendicular tension forces T_0 per unit length supported by the circular edge cables will be projected on to the cables along smaller or larger lengths, resulting in force components of different magnitudes (less than T_0) depending upon the position along the circle. Their resultant in the radial direction is T_0, as proved by the help of a geometric analysis.

12 If in the present case the membrane radius is fairly large, we may safely consider a gravity load here as acting radially to the surface. We are content with doing an approximate study, and the error will be very small.

Chapter 12
THE ARCH AND THE VAULT

1 See, for example, Rice, Peter, *An Engineer Imagines*, Ellipsis, 1994, pp. 118–126, for a description of the development of the tall, free-standing granite arcade of the Pavilion of the Future that was built for Expo '92 in Seville, Spain, and which is illustrated in this book in Section 12.4.

2 Fathy, Hassan, *Architecture for the Poor*, American University in Cairo Press, 1989, p. 8.

3 Nervi, Pier Luigi, *Aesthetics and Technology in Building*, Harvard University Press, 1965, p. 5.

4 The word "pure" here is used in the sense that these stresses will be uniform over any cross-section of the arch, even though the magnitude of these stresses will be different from one section to another.

5 Besides, a funicular (no-bending) arch shape for this load condition could probably also be established within the total mass of the arch and in-filled area above.

6 An interesting result of such an analysis was long used for predicting the stability of masonry arches: as long as the funicular line of compression falls within the inner third of the cross-sectional dimension, there will be net compressive stress everywhere over the cross-section. However, if the funicular line is outside this middle third, also called the kern, the arch will experience tension stresses over a portion of the section at that location and the arch will tend to open up and be liable to fail.

7 This bridge is of other interest as well from a design point of view: not only is the arch supporting the principal cross-river walkway at street level, but also it affords multiple pathways and access to the lower banks of the Seine.

8 Betjeman, John, *London's Historic Railway Stations*, John Murray Publisher, 1972, p. 2.

Chapter 13
THE DOME AND THE SHELL

1 The dome built atop the Reichstag in 1999 is actually a long-delayed replacement structure for a dome that was part of the original building but that was destroyed by a suspicious fire in 1933.

2 See Lancaster, Lynne, *Concrete Vaulted Construction in Imperial Rome*, Cambridge University Press, 2005, p. 100.

3 The hoop forces will change from compression to tension at an angle of 52^0 from a vertical through its top.

4 A 1764 drawing by Giambattista Piranesi of the rotunda of the third-century Tempio della Tosse, for example, clearly shows vertical cracking developing intermittently around the lower portion of the dome.

5 Norberg-Schulz, Christian, *Meaning in Western Architecture*, Studio Vista, Cassell Ltd., 1975, p. 50.

6 Mainstone, Rowland, *Developments in Structural Form*, Architectural Press, 2001, pp. 116–117. Melaragno, Michele, *An Introduction to Shell Structures: The Art and Science of Vaulting*, Van Nostrand Reinhold, 1991, p. 26.

7 Taylor, Rabun, *Roman Builders*, Cambridge University Press, 2003, pp. 195–197.

8 This structure's larger twin, the Palazzo de Sport, was also built by Nervi in the EUR sector of southern Rome.

9 Nervi, Pier Luigi, *Aesthetics and Technology in Building*, Harvard University Press, 1965, p. 105.

10 Extract from a speech made by Louis I. Kahn at the Kimbell Art Museum in 1973.

11 The city of Calgary, Canada, celebrates its cowboy heritage by naming its hockey arena, built for the 1988 Olympics, the Saddledome because of its hyperbolic paraboloid concrete shell roof.

12 If all of these reasons help to explain why hypars have been a popular shell form with designers, and if all of these advantages seem somewhat mystical and too good to be true, it is interesting to note that no less a figure than Antoni Gaudí is said to have likened the three straight lines

that can be used to describe the hypar surface (those of the two straight rods and of the string connecting them) as being akin to the Holy Trinity (Melaragno, Michele, *An Introduction to Shell Structures: The Art and Science of Vaulting*, Van Nostrand Reinhold, 1991, p. 10).

13 Isler, Heinz, "Concrete Shells Derived from Experimental Shapes," *Shell and Spatial Structures, Structural Engineering International*, Vol. 4, No. 3, 1994, pp. 142–147.

14 Anderson, Stan (ed.). *Eladio Dieste: Innovation in Structural Art*, Princeton Architectural Press, 2004.

15 Precursors to today's opening up of compressive shell surfaces also includes, of course, the geodesic domes developed by Buckminster Fuller that are still in use and being designed today, as was discussed in Section 13.1.

16 See, for example, the range of examples presented on the topic of "The New Structuralism" in the July/August 2010 issue of *Architectural Design*.

17 The radial compression resulting from the shell pushing inward all around the base of the Reading Room dome is self-equilibrated by a compression ring around that structure. This ring, and thus the whole interior part of the roof shell, is held up in the air by means of 20 concrete-filled steel tube columns that surround the cylindrical drum of the Reading Room; these have been masked by a stone cladding that encases the historical and unable-to-carry-more-load central structure. Around its square outside perimeter, the shell is carried on sliding bearings on top of a new concrete parapet; horizontal movement is thus permitted at the edge of the shell to insure that the perimeter walls are not distressed or overloaded – nor are their classic Georgian façades then needing to be strengthened and perhaps covered over.

18 It is also interesting to note that aside from providing visual interest, the ground-touching funnels also serve the practical function of helping to limit the expansions and contractions along the length of the canopy caused by temperature variations.

19 Benesse Art Site Naoshima; www.benesse-artsite.jp/en/art/teshima-artmuseum

20 Contemporary code regulations to ensure adequate reinforcement cover also partly account for this difference, but the basic lesson of form's influence on shell structural behavior remains nonetheless.

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Chapter 6

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