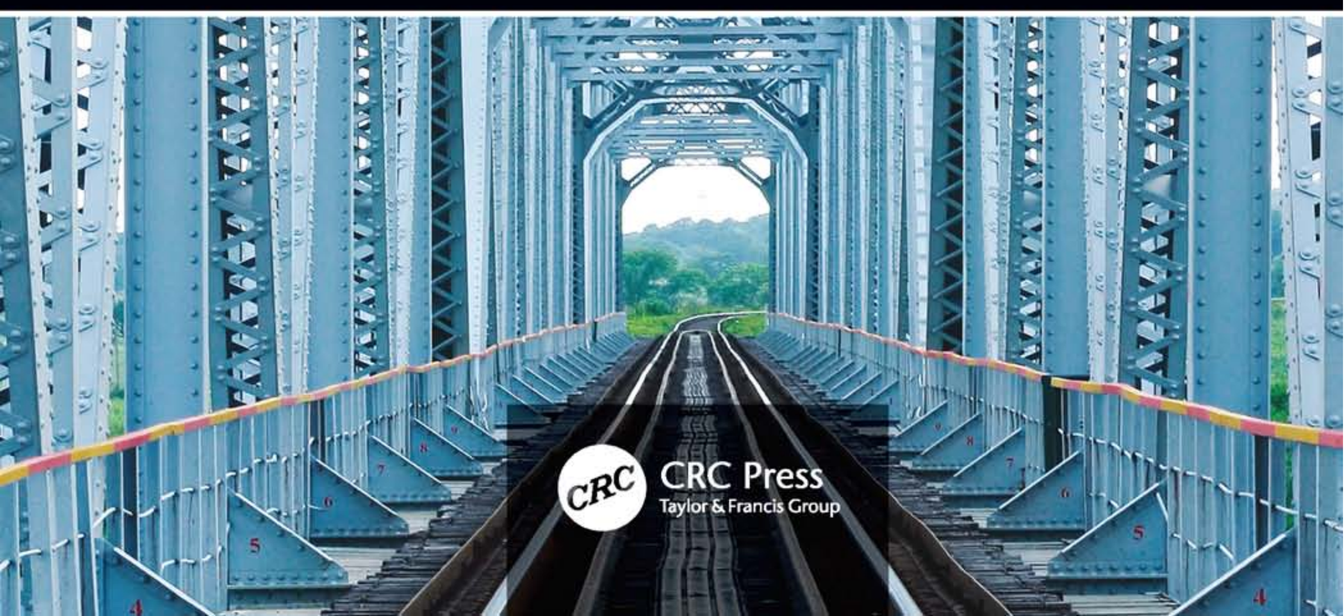
THIRD EDITION

Principles of STRUCTURAL DESIGN

Wood, Steel, and Concrete

RAM S. GUPTA



CRC Press
Taylor & Francis Group

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CRC Press
Taylor & Francis Group
Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an **informa** business

CRC Press
Taylor & Francis Group
6000 Broken Sound Parkway NW, Suite 300
Boca Raton, FL 33487-2742

Printed on acid-free paper

International Standard Book Number-13: 978-1-138-49353-7 (Hardback)

Visit the Taylor & Francis Web site at
http://www.taylorandfrancis.com

and the CRC Press Web site at
http://www.crcpress.com

eResource material is available for this title at
https://www.crcpress.com/9781138493537.

Contents

Preface

Like previous editions, the third edition of *Principles of Structural Design: Wood, Steel, and Concrete* has a code connection focus. All major codes have been revised since the publication of the second edition, so the need for a new edition became imperative.

The guiding principle of this new edition, like previous versions, is to provide a comprehensive design book for wood, steel, and concrete that is suitable for general engineering, civil engineering, construction engineering, and management and architecture, and is also appropriate for undergraduate-level coursework. The book adopts the latest load resistance factor design (LRFD) approach, which now is a standard practice for the design of wood, steel, and concrete structures.

Principles of Structural Design: Wood, Steel, and Concrete is complete book because it has all the necessary design resources: the section dimensions, section properties, and reference strength design values are incorporated in the book, along with the important load tables and other design aids.

The book has four sections. Section I covers the design philosophies and the basics of structural design. It summarizes the American Society of Civil Engineers, ASCE/SEI 7-16 Standards on Minimum Design Loads in relation to dead load, live load, roof live load, snow load, wind load, and earthquake loads for structures; these concepts are universally followed in the United States. In ASCE/SEI 7-16, the snow and wind load data (maps) have been revised, and certain major changes have been made in the provisions on earthquake load Finally, the combination of loads to be considered for adequate design of structures is discussed in this section.

Section II relates to wood structures. American Wood Council, in the latest National Design Specification (NDS; 2015), includes a new generation of engineered wood product called cross-laminated timber (CLT) for the first time. It is a very strong product with superior acoustic, fire, seismic, and thermal performance. This product offers an alternative to concrete, masonry, and steel in terms of moderate high-rise structures. This product along with the sawn lumber, structural glued laminated timber, and structural composite lumber is covered in detail in this book.

Another change made in the NDS 2015 is the revision of the reference design values of Southern Pine Lumber to the lower side; Appendix B.3 has been revised accordingly. First, the conceptual designs of tension, compression, and bending wood members have been considered under common conditions. Then the effects of the column and beam stabilities have been reviewed together with the combined forces.

Section III deals with steel structures, and steel is a widely used construction material. The fifteenth edition of *American Institute of Steel Construction Manual 2017*,which also includes the Specifications for Structural Steel Buildings (AISC) 360-16, has revised the shear lag factor for net tensile area and also has simplified the design of slender column members. A new category of high-strength bolts has been added in the manual. For tensile members, compression members, flexure members, braced and unbraced frames, the chapters in Section III have been updated together with open-web joists and girders.

Connections are weak links in a structure. The designs of wood connections and steel bolted and welded connections are covered in detail in Chapter 8 and Chapter 13, respectively.

The design of concrete beams and columns according to the American Concrete Institute's ACI Building Code Requirements for Structural Concrete, 318-14, is covered in Section IV. A new chapter on pre-stressed concrete structures has been added. A new chapter on the application of simulation in structural design has also been added. It describes the modern-day practice of mathematical modeling of structural elements.

Principles of Structural Design: Wood, Steel, and Concrete uses both English and metric units. It incorporates fully solved examples on all topics covered in the text. The solved examples have been expanded into a similar set of exercise problems at the end of each chapter. A separate solutions manual is available.

Author

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1 Design Criteria

CLASSIFICATION OF BUILDINGS

Buildings and other structures are classified based on the risk associated with unacceptable performance of the structure, according to Table 1.1. The risk categories range from I to IV, where category I represents buildings and other structures that pose no danger to human life in the event of failure, and category IV represents all essential facilities. Each structure is assigned the highest applicable risk category. Assignment of more than one risk category to the same structure based on use and loading conditions is permitted.

BUILDING CODES

To safeguard public safety and welfare, towns and cities across the United States follow certain codes for the design and construction of buildings and other structures. Until recently, towns and cities modeled their codes based on the following three regional codes, which are normally revised at 3-year intervals:

1. The Building Officials and Code Administrators National Building Code
2. The Uniform Building Code
3. The Standard Building Code

The International Codes Council was created in 1994 for the purpose of unifying these codes into a single set of standards. The council included representatives from the three regional code organizations. The end result was the preparation of the *International Building Code* (IBC), which was first published in 2000, with a second revision in 2003 and a third revision in 2006. The latest is the seventh edition of 2018. Now, practically all local and state authorities follow the IBC. For the specifications of loads to which structures should be designed, the IBC refers directly to the American Society of Civil Engineers's publication *Minimum Design Loads for Buildings and Other Structures*, which is commonly referred to as *American Society of Civil Engineers* (ASCE) 7-16.

STANDARD UNIT LOADS

The primary loads on a structure are dead loads due to the weight of structural components and live loads due to structural occupancy and usage. The other common loads are snow loads, wind loads, and seismic loads. Some specific loads to which a structure could also be subjected to comprise soil loads, hydrostatic forces, flood loads, rain loads, and ice loads (atmospheric icing). ASCE 7-16 specifies the standard unit loads that should be adopted for each category of loading. These have been described in Chapters 2 through 5 of the book for the main categories of loads.

TRIBUTARY AREA

Since the standard unit load in ASCE 7-16 is for a unit area, it needs to be multiplied by the effective area of the structural element on which it acts to ascertain the total load. In certain cases, ASCE 7-16 specifies the concentrated load; then its location needs to be considered for maximum effect. In the parallel framing system shown in Figure 1.1, the beam CD receives the load from the floor

TABLE 1.1
Risk Category of Buildings and Other Structures

Nature of Occupancy	Category
Agriculture, temporary structures, storage	I
All buildings and structures except those classified as I, III, and IV	II
Buildings and other structures that can cause a substantial economic impact and/or mass disruption of day-to-day civil lives, including the following:	III
More than 300 people congregation	
Day care with more than 150	
School with more than 250 and college with more than 500	
Resident healthcare with 50 or more	
Jail	
Power generation, water treatment, wastewater treatment, telecommunication centers	
Essential facilities, including the following:	IV
Hospitals	
Fire, police, ambulance	
Emergency shelters	
Facilities needed in emergency	

Source: American Society of Civil Engineers, Reston, VA.

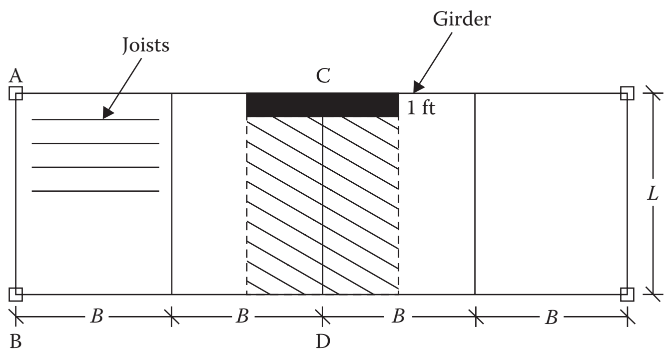


FIGURE 1.1 Parallel framing system.

that extends halfway to the next beam ($B/2$) on each side, as shown by the hatched area. Thus, the tributary area of the beam is $B \times L$ and the load is $W = w \times B \times L$, where w is the standard unit load. The exterior beam AB receives the load from one side only extending halfway to the next beam. Hence, the tributary area is $\frac{1}{2}B \times L$.

Suppose we consider a strip of 1 ft width, as shown in Figure 1.1. The area of the strip is $1 \times B$. The load of the strip is $w \times B$, which represents the uniform load per running foot (or meter) of the beam.

The girder is point loaded at the locations of beams by beam reactions. However, if the beams are closely spaced, the girder could be considered to bear a uniform load from the tributary area of ½$B \times L$.

In Figure 1.2, beam AB supports the rectangular load from area A, B, 2, 1; the area is $BL/2$ and the load is $wBL/2$. It also supports the triangular load from area A, B, 3; this area is (½)$BL/2$ and the load is $wBL/4$. This has a distribution as shown in Figure 1.3.

Beam AC supports the triangular load from area A, C, 3, which is $wBL/4$. However, the loading on the beam is not straightforward because the length of the beam is not L but $L_1 = (\sqrt{L^2 + B^2})$ (Figure 1.4). The triangular loading is as shown in Figure 1.4 to represent the total load (the area under the load diagram) of $wBL/4$.

The framing of a floor system can be arranged in more than one manner. The tributary area and the loading pattern on the framing elements will be different for different framing systems, as shown in Figures 1.5 and 1.6.

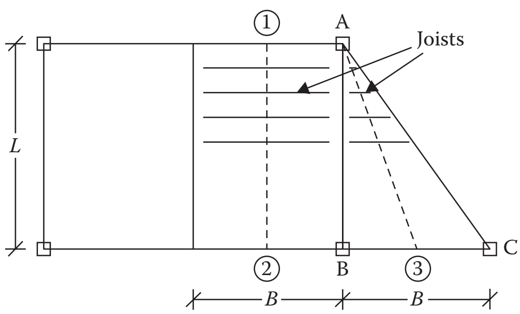


FIGURE 1.2 A triangular framing system.

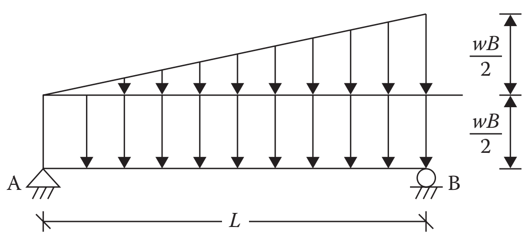


FIGURE 1.3 Load distribution on beam AB of Figure 1.2.

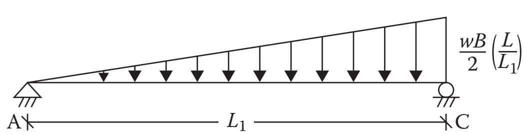


FIGURE 1.4 Load distribution on beam AC of Figure 1.2.

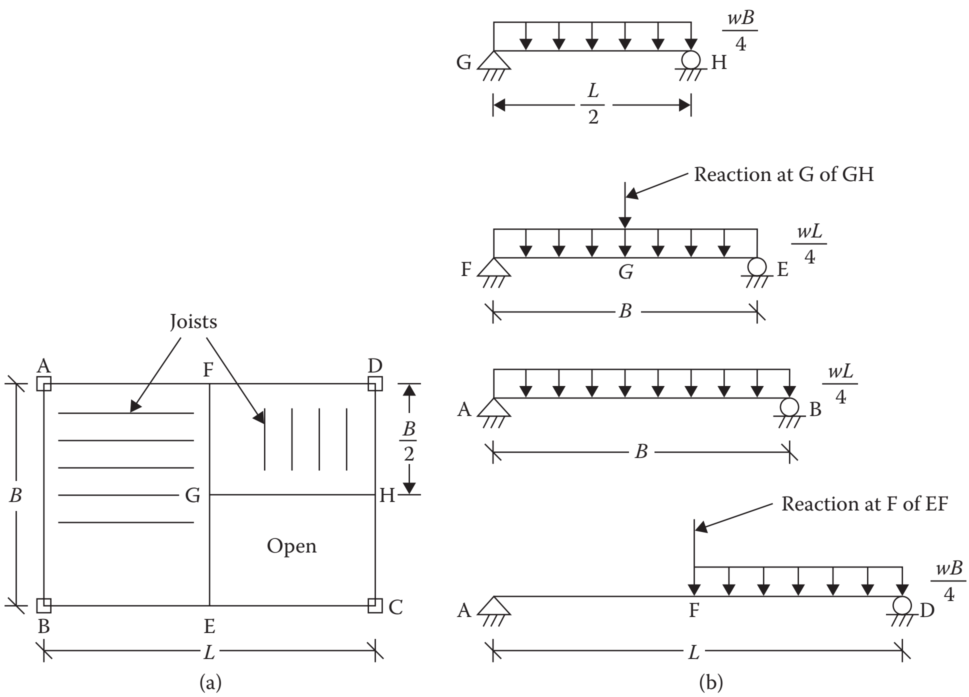


FIGURE 1.5 (a) Framing arrangement. (b) Distribution of loads on elements of frame in Figure 1.5a.

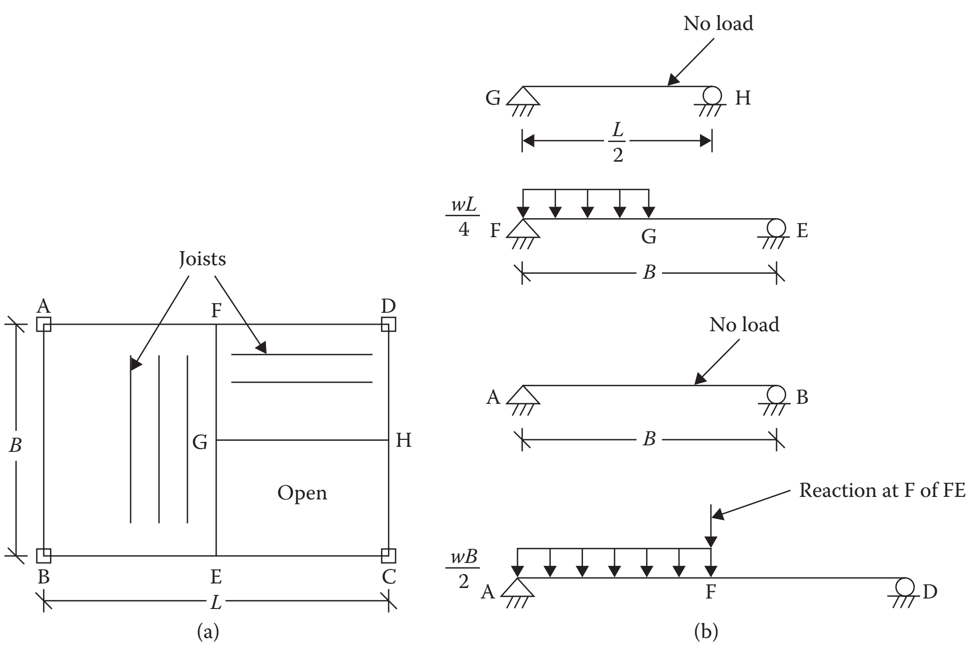


FIGURE 1.6 A different framing arrangement and distribution of load

Example 1.1

In Figure 1.2, the span L is 30 ft and the spacing B is 10 ft. The distributed standard unit load on the floor is 60 lb/ft^2. Determine the tributary area, and show the loading on beams AB and AC.

Solution

Beam AB:

1. Rectangular tributary area per foot of beam length = 1 × 5 = 5 ft^2/ft
2. Uniform load per foot = (standard unit load × tributary area) = (60 lb/ft^2) (5 ft^2/ft) = 300 lb/ft
3. Triangular tributary area (total) = ½(5)(30) = 75 ft^2
4. Total load of triangular area = 60 × 75 = 4500 lb
5. For load at the end of w per foot, area of triangular load diagram = ½wL
6. Equating items (4) and (5), ½wL = 4500 or w = 300 lb/ft
7. The loading is shown in Figure 1.7.

Beam AC:

1. Tributary area = 75 ft^2
2. Total load = 60 × 75 = 4500 lb
3. Length of beam AC, $L = (\sqrt{30^2 + 10^2}) = 31.62\,\text{ft}$
4. Area of triangular load diagram = ½wL =0 ½w(31.62)
5. Equating (2) and (4), ½w(31.62) = 4500 or w = 284.62 lb/ft
6. The loading is shown in Figure 1.8.

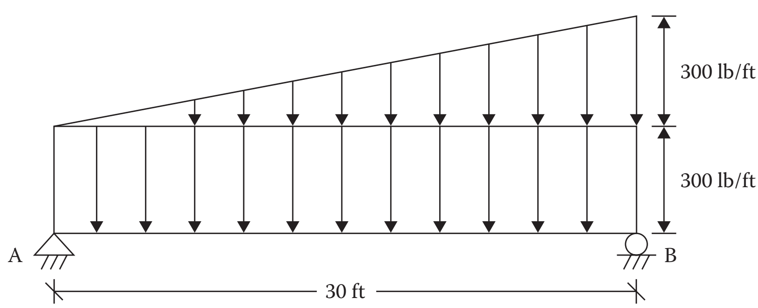


FIGURE 1.7 Distribution of loads on beam AB of Example 1.1.

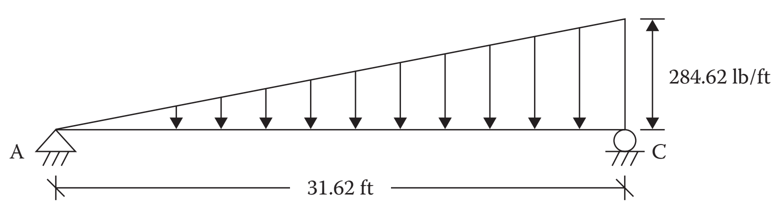


FIGURE 1.8 Distribution of loads on beam AC of Example 1.1.

WORKING STRESS DESIGN, STRENGTH DESIGN, AND UNIFIED DESIGN OF STRUCTURES

There are two approaches to design: (1) the traditional approach and (2) a comparatively newer approach. The distinction between them can be understood from the stress–strain diagram. The stress–strain diagram with labels for a ductile material is shown in Figure 1.9. The diagram for a brittle material is similar except that there is only one hump indicating both the yield and the ultimate strength point, and the graph at the beginning is not really (but close to) a straight line.

Allowable stress is ultimate strength divided by a factor of safety. It falls on the straight-line portion within the elastic range. In the allowable stress design (ASD) or working stress design method, the design is carried out so that when the computed design load, known as the *service load*, is applied on a structure, the actual stress created does not exceed the allowable stress limit. Since the allowable stress is well within the ultimate strength, the structure is safe. This method is also known as the *elastic design approach*.

In the other method, known variously as *strength design*, *limit design*, or *load resistance factor design* (LRFD), the design is carried out at the ultimate strength level. Since we do not want the structure to fail, the design load value is magnified by a certain factor known as the *load factor*. Since the structure at the ultimate level is designed for loads higher than actual loads, it does not fail. In strength design, the strength of the material is taken to be the ultimate strength, and a resistance factor (< 1) is applied to the ultimate strength to account for uncertainties associated with determining the ultimate strength.

The LRFD method is more efficient than the ASD method. In the ASD method, a single factor of safety is applied to arrive at the design stress level. In LRFD, different load factors are applied depending on the reliability to which the different loads can be computed. Moreover, resistance factors are applied to account for the uncertainties associated with the strength values.

The American Concrete Institute was the first regulatory agency to adopt the (ultimate) strength design approach in early 1970 because concrete does not behave as an elastic material and does not display the linear stress–strain relationship at any stage. The American Institute of Steel Construction (AISC) adopted the LRFD specifications in the beginning of 1990. On the other hand, the American Forest and Paper Association (American Wood Council) included the

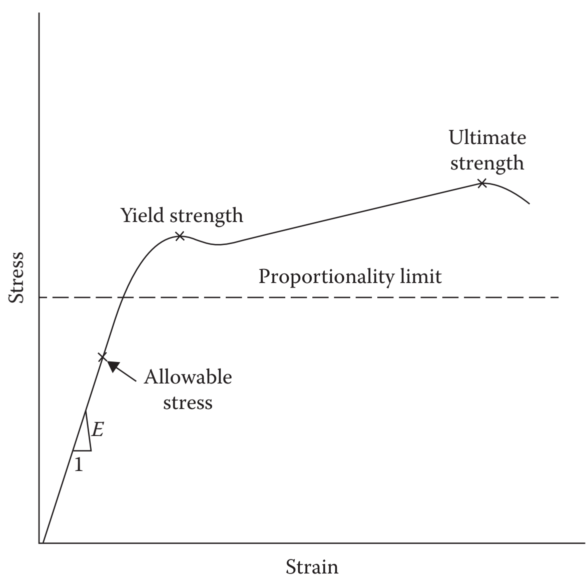


FIGURE 1.9 Stress–strain relation of a ductile material.

LRFD provisions only recently, in the 2005 edition[1] of the *National Design Specification for Wood Construction*.

The *AISC Manual 2005*[2] proposed a unified approach wherein it had combined the ASD and the LRFD methods together in a single documentation. The principle of unification is as follows.

The nominal strength of a material is a basic quantity that corresponds to its ultimate strength. In terms of force, the nominal (force) strength is equal to yield or ultimate strength (stress) times the sectional area of a member. In terms of moment, the nominal (moment) strength is equal to ultimate strength times the section modulus of the member. Thus:

$$P_n = F_y A \tag{1.1}$$

$$M_n = F_y S \tag{1.2}$$

where:

A is area of cross section
S is section modulus

In the ASD approach, the nominal strength of a material is divided by a factor of safety to convert it to the allowable strength. Thus:

$$\text{Allowable (force) strength} = \frac{P_n}{\Omega} \tag{1.3}$$

$$\text{Allowable (moment) strength} = \frac{M_n}{\Omega} \tag{1.4}$$

where Ω is factor of safety.

For a safe design, the load or moment applied on the member should not exceed the allowable strength. Thus, the basis of ASD is as follows:

$$P_a \leq \frac{P_n}{\Omega} \tag{1.5}$$

and

$$M_a \leq \frac{M_n}{\Omega} \tag{1.6}$$

where:

P_a is service design load combination
M_a is moment due to service design load application

Using Equation 1.5 or 1.6, the required cross-sectional area or the section modulus of the member can be determined.

The common ASD procedure works at the stress level. The service (applied) load, P_a, is divided by the sectional area, A, or the service moment, M, is divided by the section modulus, S, to obtain the applied or created stress due to the loading, σ_a. Thus, the cross-sectional area and the section modulus are not used on the strength side but on the load side in the usual procedure. It is the ultimate or yield strength (stress) that is divided by the factor of safety to obtain the permissible stress, σ_p. To safeguard the design, it is ensured that the applied stress, σ_a, does not exceed the permissible stress, σ_p.

[1] The latest is NDS 2015.
[2] The latest is AISC 2016.

For the purpose of unifying the ASD and LRFD approaches, the aforementioned procedure considers strength in terms of the force or the moment. In the LRFD approach, the nominal strengths are the same as those given by Equations 1.1 and 1.2. The design strengths are given by:

$$\text{Design (force) strength} = \phi P_n \tag{1.7}$$

$$\text{Design (moment) strength} = \phi M_n \tag{1.8}$$

where ϕ is resistance factor.

The basis of design is:

$$P_u \le \phi P_n \tag{1.9}$$

$$M_u \le \phi M_n \tag{1.10}$$

where:

P_u is factored design loads

M_u is maximum moment due to factored design loads

From the aforementioned relations, the required area or the section modulus can be determined, which are the parts of P_n and M_n in Equations 1.1 and 1.2.

A link between the ASD and the LRFD approaches can be made as follows. From Equation 1.5 for ASD, at the upper limit:

$$P_n = \Omega P_a \tag{1.11}$$

Considering only the dead load and the live load, $P_a = D + L$. Thus:

$$P_n = \Omega(D + L) \tag{1.12}$$

From Equation 1.9 for LFRD, at the upper limit:

$$P_n = \frac{P_u}{\phi} \tag{1.13}$$

Considering only the factored dead load and live load, $P_u = 1.2D + 1.6L$. Thus:

$$P_n = \frac{(1.2D + 1.6L)}{\phi} \tag{1.14}$$

Equating Equations 1.12 and 1.14:

$$\frac{(1.2D + 1.6L)}{\phi} = \Omega(D + L) \tag{1.15}$$

or

$$\Omega = \frac{(1.2D + 1.6L)}{\phi(D + L)} \tag{1.16}$$

TABLE 1.2
Ω as a Function of ϕ for Various *L/D* Ratios

L/D Ratio (Select)	From Equation 1.16
1	1.4/ϕ
2	1.47/ϕ
3	1.5/ϕ
4	1.52/ϕ

The factor of safety, Ω, has been computed as a function of the resistance factor, ϕ, for various selected live-to-dead load ratios in Table 1.2.

The 2005 AISC specifications used the relation $\Omega = 1.5/\phi$ throughout the manual to connect the ASD and LRFD approaches. Wood and concrete structures are relatively heavier; that is, the *L/D* ratio is less than 3 and the factor of safety, Ω, tends to be lower than 1.5/ϕ, but a value of 1.5 could reasonably be used for these structures as well because the variation of the factor is not significant. This book uses the LRFD basis of design for all structures.

ELASTIC AND PLASTIC DESIGNS

The underlying concept in the preceding section is that a limiting state is reached when the stress level at any point in a member approaches the yield strength value of the material and the corresponding load is the design capacity of the member. Let us revisit the stress–strain diagram for a ductile material like steel. The initial portion of the stress–strain curve of Figure 1.9 has been drawn again in Figure 1.10 to a greatly enlarged horizontal scale. The yield point F_y is a very important property of structural steel. After an initial yield, a steel element elongates in the plastic range without any appreciable change in stress level. This elongation is a measure of ductility and serves a useful purpose in steel design. The strain and stress diagrams for a rectangular beam due to increasing loading are shown in Figures 1.11 and 1.12.

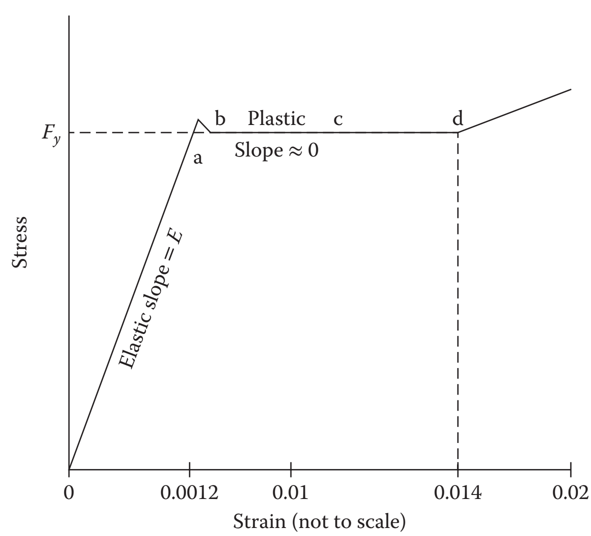


FIGURE 1.10 Initial portion of stress–strain relation of a ductile material.

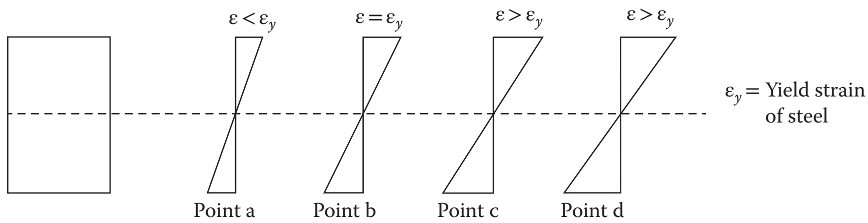


FIGURE 1.11 Strain variation in a rectangular section.

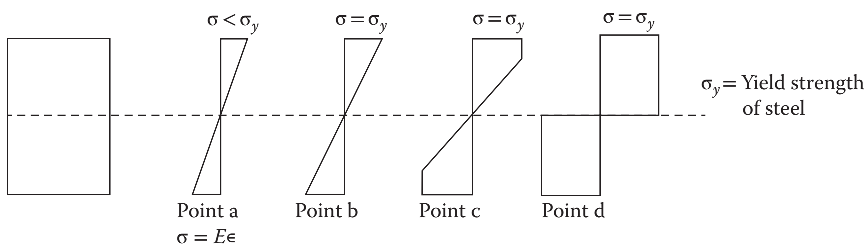


FIGURE 1.12 Stress variation in a rectangular section.

Beyond the yield strain at point b, as a load increases the strain continues to rise in the plastic range and the stress at yield level extends from the outer fibers into the section. At point d, the entire section has achieved the yield stress level, and no more stress capacity is available to develop. This is known as the *fully plastic state*, and the moment capacity at this state is known as the *full plastic moment*. The full moment is the ultimate capacity of a section. Beyond this, a structure will collapse. When full moment capacity is reached, we say that a *plastic hinge* has formed. In a statically determinate structure, the formation of one plastic hinge leads to a collapse mechanism. Two or more plastic hinges are required in a statically indeterminate structure for a collapse mechanism. In general, for a complete collapse mechanism,

$$n = r + 1 \tag{1.17}$$

where:

n is number of plastic hinges
r is degree of indeterminacy

Elastic Moment Capacity

As stated earlier, structures are commonly designed for elastic moment capacity; that is, the failure load is based on the stress reaching a yield level at any point. Consider that, on the rectangular beam of width b and depth d of Figure 1.10 at position b, when the strain has reached the yield level, a full elastic moment, M_E, acts. This is shown in Figure 1.13.

Total compression force is as follows:

$$C = \frac{1}{2}\sigma_y A_c = \frac{1}{2}\sigma_y \frac{ba}{2} \tag{a}$$

Total tensile force is as follows:

$$T = \frac{1}{2}\sigma_y A_t = \frac{1}{2}\sigma_y \frac{bd}{2} \tag{b}$$

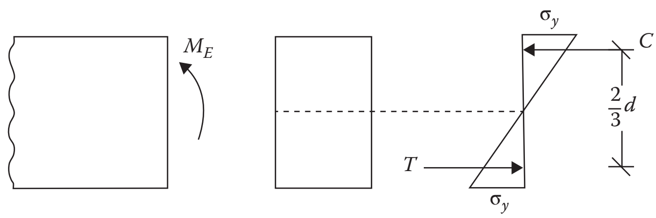


FIGURE 1.13 Full elastic moment acting on a rectangular section.

These act at the centroids of the stress diagram in Figure 1.13.

$$M_E = \text{force} \times \text{moment arm}$$
$$M_E = \left(\frac{1\sigma_y}{2}\frac{bd}{2}\right) \times \left(\frac{2d}{3}\right) \tag{c}$$

$$M_E = \sigma_y \frac{bd^2}{6} \tag{1.18}$$

It should be noted that $bd^2/6 = S$, the section modulus, and the aforementioned relation is given by $M = \sigma_y S$. In terms of moment of inertia, this relation is $M = \sigma_y I/c$. In the case of a nonsymmetrical section, the neutral axis is not in the center, and there are two different values of c and accordingly two different section moduli. The smaller M_E is used for the moment capacity.

Plastic Moment Capacity

Consider a full plastic moment, M_p, acting on the rectangular beam section at the stress level d of Figure 1.10. This is shown in Figure 1.14.

Total compression force is as follows:

$$C = \sigma_y A_c = \sigma_y \frac{bd}{2} \tag{a}$$

Total tensile force is as follows:

$$T = \sigma_y A_t = \sigma_y \frac{bd}{2} \tag{b}$$

$$M_p = \text{force} \times \text{moment arm}$$
$$= \sigma_y \frac{bd}{2} \times \frac{d}{2} \tag{c}$$

or

$$M_p = \sigma_y \frac{bd^2}{4} \tag{1.19}$$

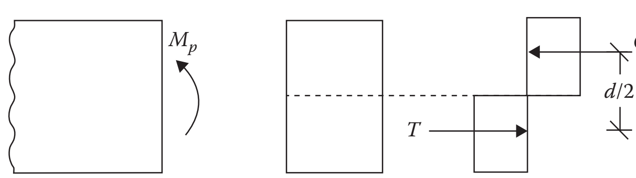


FIGURE 1.14 Full plastic moment acting on a rectangular section.

This is given by:

$$M_p = \sigma_y Z \tag{1.20}$$

where Z is called the *plastic section modulus*.

For a rectangle, the *plastic section modulus* is 1.5 times the elastic section modulus and the plastic moment capacity (M_p) is 1.5 times the elastic moment capacity (M_E). The ratio between the full plastic and the full elastic moment of a section is called the *shape factor*. In other words, for the same design moment value, the section is smaller according to the plastic design.

The plastic analysis is based on the collapse load mechanism and requires knowledge of how a structure behaves when stress exceeds the elastic limit. The plastic principles are used in the design of steel structures.

Example 1.2

For the steel beam section shown in Figure 1.15, determine the (a) elastic moment capacity, (b) plastic moment capacity, and (c) shape factor. The yield strength is 210 MPa.

Solution

a. Elastic moment capacity
   1. Refer to Figure 1.15a.
   2. $C = T = \frac{1}{2}\left(210\times10^6\right)\left(0.05\times0.075\right) = 393.75\times10^3\,\text{N}$
   3. $M_E = (393.75 \times 10^3) \times 0.1 = 39.38 \times 10^3$ N·m
b. Plastic moment capacity
   1. Refer to Figure 1.15b.
   2. $C = T = (210 \times 10^6)(0.05 \times 0.075) = 787.5 \times 10^3$ N
   3. $M_p = (787.5 \times 10^3) \times 0.075 = 59.06 \times 10^3$ N·m
c. Shape factor

$$\text{SF} = \frac{M_p}{M_E} = \frac{59.06\times10^3}{39.38\times10^3} = 1.5$$

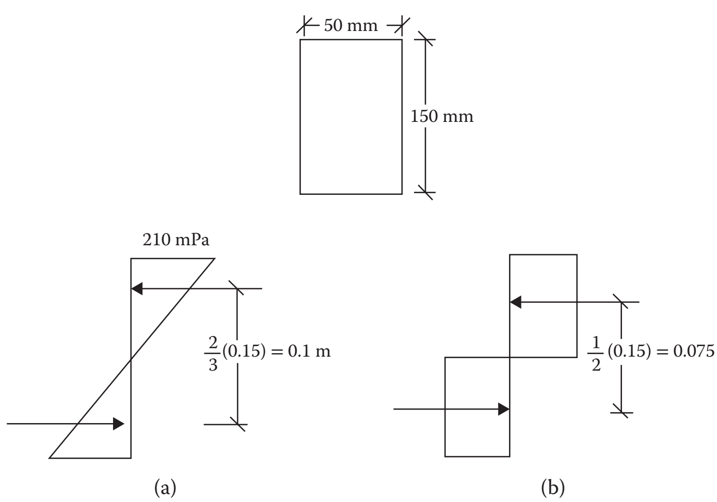


FIGURE 1.15 (a) Elastic moment capacity of beam section. (b) Plastic moment capacity of beam section.

Example 1.3

The design moment for a rectangular beam is 40 kN·m. The yield strength of the material is 200 MPa. Design a section having a width–depth ratio of 0.5 according to the (a) elastic theory and (b) plastic theory.

Solution

a. Elastic theory
   1. $M_E = \sigma_y S$
      or
      $$S = \frac{M_E}{\sigma_y} = \frac{40\times 10^3}{200\times 10^6} = 0.2\times 10^{-3}\,\text{m}^3$$
   2. $\frac{1}{6}bd^2 = 0.2\times 10^{-3}$
      $\frac{1}{6}(0.5d)(d^2) = 0.2\times 10^{-3}$
      or
      $d = 0.134$ m
      and
      $b = 0.076$ m
b. Plastic theory
   1. $M_p = \sigma_y Z$
      or
      $$Z = \frac{M_p}{\sigma_y} = \frac{40\times 10^3}{200\times 10^6} = 0.2\times 10^{-3}\,m^3$$
   2. $\frac{1}{4}bd^2 = 0.2\times 10^{-3}\,m^3$
      $\frac{1}{4}(0.5d)(d^2) = 0.2\times 10^{-3}\,m^3$
      or
      $d = 0.117$ m
      and
      $b = 0.058$ m

COMBINATIONS OF LOADS

Various types of loads that act on a structure are described in the "Standard Unit Loads" section of this chapter. For designing a structure, its elements, or its foundation, loads are considered to act in the following combinations with load factors as indicated in order to produce the most unfavorable effect on the structure or its elements. Dead load, roof live load, floor live load, and snow load are gravity loads that act vertically downward. Wind load and seismic load have vertical as well as lateral components. The vertically acting roof live load, live load, wind load (simplified approach), and snow load are considered to be acting on the horizontal projection of any inclined surface. However, dead load and the vertical component of earthquake load act over the entire inclined length of the member.

For LRFD, ASCE 7-16 recommends the following seven combinations with respect to common types of loads. Since ASCE 7-10, the factor for wind load has been changed to 1 (strength level) from an earlier factor of 1.6. The wind speed maps have been changed accordingly:

1. $1.4D$ (1.21)
2. $1.2D + 1.6L + 0.5(L_r \text{ or } S)$ (1.22)
3. $1.2D + 1.6(L_r \text{ or } S) + fL \text{ or } 0.5W$ (1.23)

4. $1.2D + 1.0W + fL + 0.5(L_r \text{ or } S)$ (1.24)
5. $1.2D + E_v + E_h + fL + 0.2S$ (1.25)
6. $0.9D + 1.0W$ (1.26)
7. $0.9D - E_v + E_h$ (1.27)

where:
D is dead load
D_i is weight of ice
E_h is horizontal earthquake load
E_v is vertical earthquake load
F is fluid load
F_a is flood load
L is live load
L_r is roof live load
S is snow load or rain load
W is wind load
W_i is wind-on-ice load
$f = 0.5$ for all occupancies when the unit live load does not exceed 100 psf except for garage and public assembly and $f = 1$ when unit live load is 100 psf or more and for any load on garage and public places

OTHER LOADS

1. When a fluid load, F, is present, it should be included with the dead load (the same factor) in combinations 1 through 5 and 7 above.
2. When a lateral load, H, due to earth pressure, bulk material, or groundwater pressure is present, then include it with a factor of 1.6 if it adds to the load effect; if it acts against the other loads, use a factor of 0.9 when it is permanent and a factor of 0 when it is temporary.
3. When a structure is located in a flood zone, in V-zones, or coastal A-zones, the wind load in the above load combinations is replaced by $1.0W + 2.0F_a$, where F_a is a flood load; in noncoastal A-zones, $1.0W$ in the above combinations is replaced by $0.5W + 1.0F_a$.
4. When a structure is subjected to atmospheric ice weight, D_i, and wind-on-ice load, W_i, then replace $1.0W$ with $Di + W_i$, and in Equation 1.22, replace $0.5(L_r \text{ or } S)$ with $0.2D_i + 0.5S$.

Example 1.4

A simply supported roof beam receives loads from the following sources, taking into account the respective tributary areas. Determine the loading diagram for the beam according to the ASCE 7-16 combinations.

1. Dead load (1.2 k/ft acting on a roof slope of 10°)
2. Roof live load (0.24 k/ft)
3. Snow load (1 k/ft)
4. Wind load at roof level (15 k)
5. Earthquake load at roof level (25 k)
6. Vertical earthquake load (0.2 k/ft)

Solution

1. The dead load and the vertical earthquake load, which is related to the dead load, act on the entire member length. The other vertical forces act on the horizontal projection.
2. Adjusted dead load on horizontal projection = 1.2/cos 10° = 1.22 k/ft
3. Adjusted vertical earthquake load on horizontal projection = 0.2/cos 10° = 0.20 k/ft
4. Equation 1.21: $W_u = 1.4D = 1.4(1.22) = 1.71$ k/ft
5. Equation 1.22: $W_u = 1.2D + 1.6L + 0.5$ (L_r or S). This combination is shown in Table 1.3.
6. Equation 1.23: $W_u = 1.2D + 1.6$ (L_r or S) + ($0.5L$ or $0.5W$). This combination is shown in Table 1.4.
7. Equation 1.24: $W_u = 1.2D + 1.0W + 0.5L + 0.5(L_r$ or $S)$. This combination is shown in Table 1.5.
8. Equation 1.25: $W_u = 1.2D + E_v + E_h + 0.5L + 0.2S$. This combination is shown in Table 1.6.
9. Equation 1.26: $W_u = 0.9D + 1.0W$. This combination is shown in Table 1.7.
10. Equation 1.27: $W_u = 0.9D + E_h - E_v$. This combination is shown in Table 1.8.

Item 5 can be eliminated as it is less than items 6, 7, and 8, which should be evaluated for the maximum effect, and items 4, 9, and 10 for the least effect.

TABLE 1.3
Dead, Live, and Snow Loads for Item 5 Combination

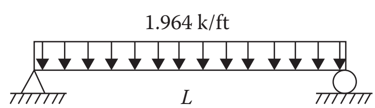
Source	*D* (k/ft)	*L* (k/ft)	L_r or *S* (k/ft)	Combined Value	Diagram
Load	1.22	—	1		
Load factor	1.2	1.6	0.5		
Factored vertical load	1.464	—	0.5	1.964 k/ft	
Factored horizontal load	—	—	—		

TABLE 1.4
Dead, Live, Snow, and Wind Loads for Item 6 Combination

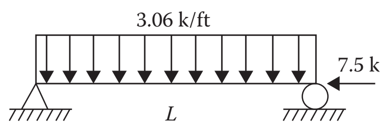
Source	*D* (k/ft)	*L* (k/ft)	*S* (k/ft)	*W* (k)	Combined Value	Diagram
Load	1.22	—	1	15		
Load factor	1.2	0.5	1.6	0.5		
Factored vertical load	1.464		1.6		3.06 k/ft	
Factored horizontal load				7.5	7.5 k	

TABLE 1.5
Dead, Live, Snow, and Wind Loads for Item 7 Combination

Source	*D* (k/ft)	*L* (k/ft)	*S* (k/ft)	*W* (k)	Combined Value
Load	1.22	—	1	15	
Load factor	1.2	0.5	0.5	1	
Factored vertical load	1.464	—	0.5		1.964 k/ft
Factored horizontal load				15	15 k

Diagram

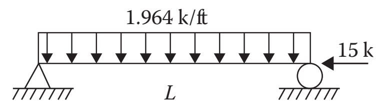


TABLE 1.6
Dead, Live, Snow, and Earthquake Loads for Item 8 Combination

Source	*D* (k/ft)	*L* (k/ft)	*S* (k/ft)	*Ev* (k/ft)	E_h (k)	Combined Value
Load	1.22	—	1	0.2	25	
Load factor	1.2	0.5	0.2	1	1	
Factored vertical load	1.464	—	0.2	0.2		1.864 k/ft
Factored horizontal load					25	25 k

Diagram

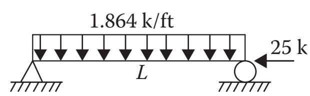


TABLE 1.7
Dead and Wind Loads for Item 9 Combination

Source	*D* (k/ft)	*W* (k)	Combined Value
Load	1.22	15	
Load factor	0.9	1	
Factored vertical load	1.1		1.1 k/ft
Factored horizontal load		15	15 k

Diagram

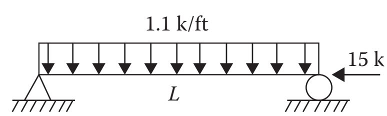
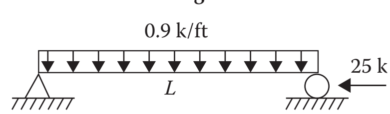


TABLE 1.8
Dead and Earthquake Load for Item 10 Combination

Source	*D* (k/ft)	E_v (k/ft)	E_h (k)	Combined Value
Load	1.22	(−)0.2	25	
Load factor	0.9	1	1	
Factored vertical load	1.1	(−)0.2		0.9 k/ft
Factored horizontal load			25	25 k

Diagram



CONTINUOUS LOAD PATH FOR STRUCTURAL INTEGRITY

Since ASCE 7-10, a new provision[3] has been made that all structures should be provided with a continuous load path and a complete lateral force–resisting system of adequate strength for the integrity of the structure. A concept of *notional load* has been adopted for this purpose. The notional load, N, has been stipulated as follows:

1. All parts of the structure between separation joints shall be interconnected. The connection should be capable of transmitting the lateral force induced by the parts being connected. Any smaller portion of a structure should be tied to the remainder of the structure through elements that have the strength to resist at least 5% of the weight of the portion being connected.
2. Each structure should be analyzed for lateral forces applied independently in two orthogonal directions. In each direction, the lateral forces at all levels should be applied simultaneously. The minimum design lateral force should be:

$$F_x = 0.01W_x \tag{1.28}$$

where:
F_x is design lateral force applied at story x
W_x is dead load of the portion assigned to level x

3. A positive connection to resist the horizontal force acting parallel to the member should be provided for each beam, girder, or truss either directly to its supporting elements or to slabs acting as diaphragms. Where this connection is through a diaphragm, the member's supporting element should be connected to the diaphragm also. The connection should have the strength to resist 5% (unfactored) dead load plus live load reaction imposed by the supported member on the supporting member.
4. A wall that vertically bears the load or provides lateral shear resistance from a portion of a structure should be anchored to the roof, to all floors, and to members that are supported by the wall or provide support to the wall. The anchorage should make a direct connection capable of resisting a horizontal force, perpendicular to the plane of the wall, equal to 0.2 times the weight of the wall tributary to the connection but not less than 5 psf.
While considering load combinations, the notional load, N, specified in items 1 through 4 in this list should be combined with dead and live loads as follows:

$$1.\ 1.2D + 1.0N + fL + 0.2S \tag{1.29}$$

$$2.\ 0.9D + 1.0N \tag{1.30}$$

This is similar to the cases when earthquake loads are considered as in load combination Equations 1.25 and 1.27.

PROBLEMS

Note: In Problems 1.1 through 1.6, the loads given are factored loads.

1.1 A floor framing plan is shown in Figure P1.1. The standard unit load on the floor is 60 lb/ft^2. Determine the design uniform load per foot on the joists and the interior beam.

1.2 In Figure 1.5, length $L = 50$ ft and width $B = 30$ ft. For a floor loading of 100 lb/ft^2, determine the design loads on beams GH, EF, and AD.

[3] It was part of the seismic design criteria of category A.

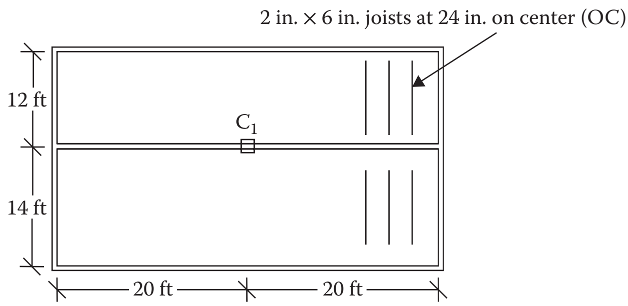


FIGURE P1.1 Floor framing system (Problem 1.1).

1.3 In Figure 1.6, length $L = 50$ ft and width $B = 30$ ft and the loading is 100 lb/ft². Determine the design loads on beams GH, EF, and AD.

1.4 An open well is framed so that beams CE and DE sit on beam AB, as shown in Figure P1.2. Determine the design load for beam CE and girder AB. The combined unit of dead and live loads is 80 lb/ft².

1.5 A roof is framed as shown in Figure P1.3. The load on the roof is 3 kN/m². Determine the design load distribution on the ridge beam.

1.6 Determine the size of the square wood column C_1 from Problem 1.1 and shown in Figure P1.1. Use a resistance factor of 0.8, and assume no slenderness effect. The yield strength of wood in compression is 4000 psi.

1.7 The service dead and live loads acting on a round tensile member of steel are 10 k and 20 k, respectively. The resistance factor is 0.9. Determine the diameter of the member. The yield strength of steel is 36 ksi.

1.8 A steel beam spanning 30 ft is subjected to a service dead load of 400 lb/ft and a service live load of 1000 lb/ft. What is the size of a rectangular beam if the depth is twice the width? The resistance factor is 0.9. The yield strength of steel is 50 ksi.

1.9 Design the interior beam from Problem 1.1 in Figure P1.1. The resistance factor is 0.9. The depth is three times the width. The yield strength of wood is 4000 psi.

1.10 For a steel beam section shown in Figure P1.4, determine the (1) elastic moment capacity, (2) plastic moment capacity, and (3) shape factor. The yield strength is 50 ksi.

1.11 For the steel beam section shown in Figure P1.5, determine the (1) elastic moment capacity, (2) plastic moment capacity, and (3) shape factor. The yield strength is 210 MPa.

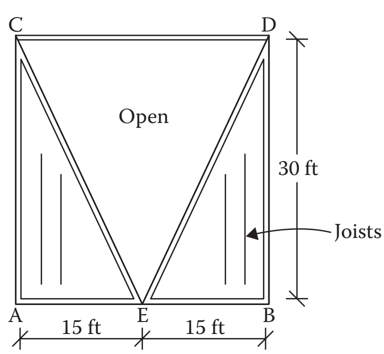


FIGURE P1.2 An open well framing system (Problem 1.4).

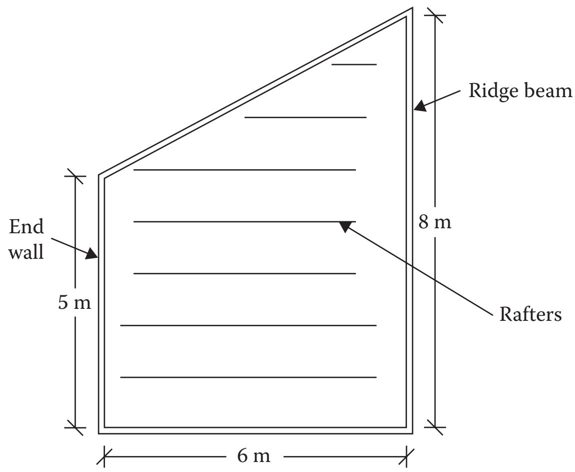


FIGURE P1.3 Roof frame (Problem 1.5).

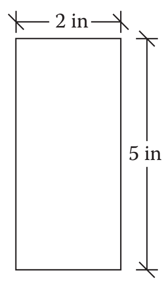


FIGURE P1.4 Rectangular beam section (Problem 1.10).

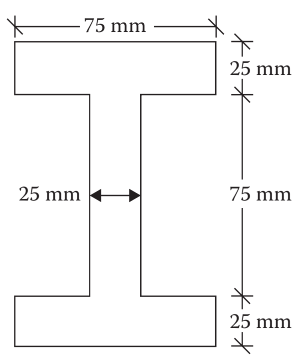


FIGURE P1.5 AN I-beam section (Problem 1.11).

Hint: For elastic moment capacity, use the relation $M_E = \sigma_y I/c$. For plastic capacity, find the compression (or tensile) forces separately for the web and flange of the section and apply these at the centroid of the web and flange, respectively.

1.12 For a circular wood section as shown in Figure P1.6, determine the (1) elastic moment capacity, (2) plastic moment capacity, and (3) shape factor. The yield strength is 2000 psi.

1.13 For the asymmetric section shown in Figure P1.7, determine the plastic moment capacity. The plastic neutral axis (where $C = T$) is at 20 mm above the base. The yield strength is 275 MPa.

1.14 The design moment capacity of a rectangular beam section is 2000 ft·lb. The material's strength is 10,000 psi. Design a section having a width–depth ratio of 0.6 according to the (1) elastic theory and (2) plastic theory.

1.15 For Problem 1.14, design a circular section.

1.16 The following vertical loads are applied on a structural member. Determine the critical vertical load in pounds per square foot for all the ASCE 7-16 combinations.

1. Dead load (on a 15° inclined member): 10 psf
2. Roof live load: 20 psf
3. Wind load (vertical component): 15 psf
4. Snow load: 30 psf
5. Earthquake load (vertical only): 2 psf

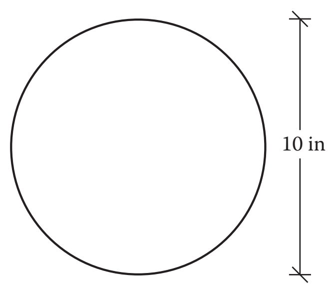


FIGURE P1.6 A circular wood section (Problem 1.12).

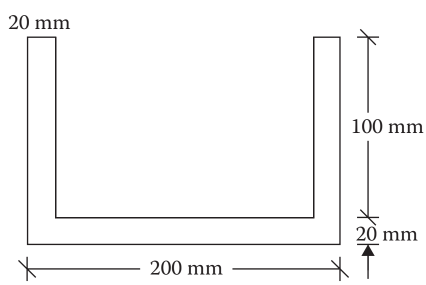


FIGURE P1.7 An asymmetric section (Problem 1.13).

1.17 A floor beam supports the following loads. Determine the load diagrams for the various load combinations.

1. Dead load: 1.15 k/ft
2. Live load: 1.85 k/ft
3. Wind load (horizontal): 15 k
4. Earthquake load (horizontal): 20 k
5. Earthquake load (vertical): 0.3 k/ft

1.18 A simply supported floor beam is subject to the loads shown in Figure P1.8. Determine the loading diagrams for various load combinations.

1.19 A beam supports the loads shown in Figure P1.9. Determine the load diagrams for various load combinations.

1.20 In Problem 1.18, if load case 5 controls the design, determine the maximum axial force, shear force, and bending moment for which the beam should be designed.

1.21 How is the structural integrity of a building ensured?

1.22 A three-story building has a total weight of 1000 k. The heights of the first, second, and third floors are 10 ft, 9 ft, and 8 ft, respectively. Determine the magnitudes of the minimum notional lateral forces that have to be considered for the structural integrity of the building assuming that the weight of the building is distributed according to the height of the floors.

1.23 Two end walls in shorter dimension (width) support the floor slabs of the building in Problem 1.22. Determine the notional forces on the anchorages at each floor level. The wall load is 40 psf.
Hint: The weight of the wall assigned to each floor is according to the effective height of the wall for each floor.

1.24 A girder of 40 ft span is supported at two ends. It has a dead load of 1 k/ft and a live load of 2 k/ft. A positive connection is provided at each end between the girder and the supports. Determine the notional force for which the connection should be designed.

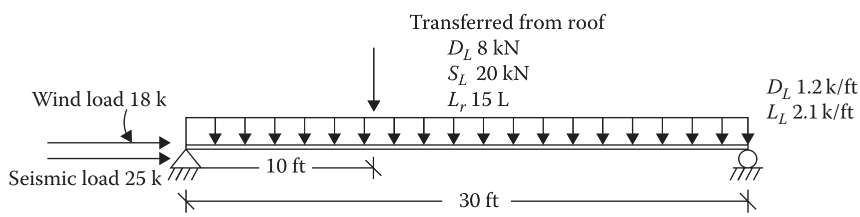


FIGURE P1.8 Loads on a beam for Problem 1.18.

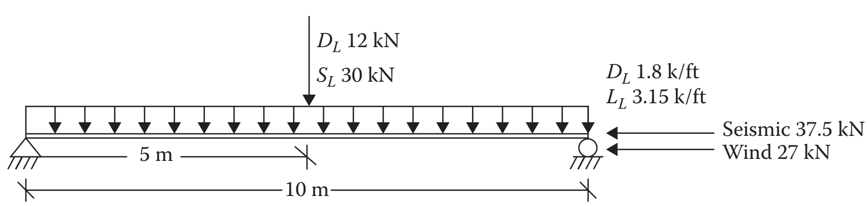


FIGURE P1.9 Loads on a beam for Problem 1.19.

2 Primary Loads
Dead Loads and Live Loads

DEAD LOADS

Dead loads are due to the weight of all materials that constitute a structural member. This also includes the weight of fixed equipments that are built into the structure, such as piping, ducts, air conditioning, and heating equipment. The specific or unit weights of materials are available from different sources. Dead loads are expressed in terms of uniform loads on a unit area (e.g., pounds per square foot). The weights are converted to dead loads, taking into account the tributary area of a member. For example, a beam section weighing 4.5 lb/ft when spaced 16 in. (1.33 ft) on center has a uniform dead load of 4.5/1.33 = 3.38 psf. If the same beam section is spaced 18 in. (1.5 ft) on center, the uniform dead load is 4.5/1.5 = 3.5 psf. The spacing of a beam section may not be known to begin with, as this might be an objective of the design.

The estimation of dead load of a member requires knowledge about what items and materials constitute that member. For example, a wood roof comprises roof covering, sheathing, framing, insulation, and ceiling. A reasonable dead load for a structural member may be assumed, only to be revised when it is found to be grossly out of order.

The dead load of a building of light frame construction is about 10 lb/ft^2 for a flooring or roofing system without plastered ceilings and 20 lb/ft^2 with plastered ceilings. For a concrete flooring system, each 1 in. thick slab has a uniform load of about 12 lb/ft^2 (psf); this is 36 psf for a 3 in. slab. To this, at least 10 psf should be added for the supporting system. Dead loads are gravity forces that act vertically downward. On a sloped roof, the dead load acts over the entire inclined length of the member.

Example 2.1

The framing of a roof consists of the following: asphalt shingles (2 psf), 0.75 in. plywood (2.5 psf), 2 × 8 framing at 12 in. on center (2.5 psf), fiberglass 0.5 in. insulation (1 psf), and plastered ceiling (10 psf). Determine the roof dead load. Make provisions for reroofing (3 psf).

Solution

	Dead Load (psf)
Shingles	2
Plywood	2.5
Framing	2.5
Insulation	1
Ceiling	10
Reroofing	3
Roof Dead Load	**21**

LIVE LOADS

Live loads also act vertically down like dead loads but are distinct from the latter because they are not an integral part of the structural element. Roof live loads, L_r, are associated with maintenance of a roof by workers, equipment, and material. They are treated separately from the other types of live loads, L, that are imposed by the use and occupancy of the structure. However, the occupancy-related load on a roof is considered to be a live load rather than a roof live load. ASCE 7-16 specifies the minimum uniformly distributed load or the concentrated load that should be used as a live load for an intended purpose. Both the roof live load and the floor live load are subjected to a reduction when they act on a large tributary area since it is less likely that the entire large area is loaded to the same magnitude of high unit load. This reduction is not allowed when an added measure of safety is desired for important structures.

FLOOR LIVE LOADS

The floor live load is estimated by the following equation:

$$L = kL_0 \tag{2.1}$$

where:

L_0 is basic design live load (see the section "Basic Design Live Load, L_0")
k is area reduction factor (see the section "Effective Area Reduction Factor")

Basic Design Live Load, L_0

ASCE 7-16 provides a comprehensive table for basic design loads arranged by occupancy and use of a structure. This has been consolidated under important categories in Table 2.1.

TABLE 2.1
Summarized Basic Design Live Loads

	Uniform Load	
Category	**lb/ft²**	**kN/m²**
Residential		
Storage area	20	0.96
Sleeping area (dwelling)	30	1.44
Living area, stairs (dwelling)	40	1.92
Hotel room	40	1.92
Garage	40	1.92
Office	50	2.40
Computer room/facility	100	4.79
School classroom	40	1.92
Hospital		
Patient room	40	1.92
Operation room/lab	60	2.87
Library		
Reading room	60	2.87
Stacking room	150	7.18
Industrial manufacturing/warehouse		
Light	125	6.00
Heavy	250	11.97
Corridor/lobby		
First floor	100	4.79
Above first floor	80	3.83
Public places[a]	100	4.79

[a] Balcony, ballroom, fire escape, gymnasium, public stairs/exits, restaurant, stadium, store, terrace, theater, yard, and so on.

To generalize, the basic design live loads are as follows: above-the-ceiling storage and roof areas: 20 psf; one- or two-family sleeping areas: 30 psf; normal-use rooms, classrooms, and garages: 40 psf; special-use rooms (office, operating, reading, and fixed-seat stadium and arena): 50–60 psf; public assembly places: 100 psf; lobbies, corridors, platforms, ballrooms, gymnasiums, retail stores, grandstands, and bleachers: 100 psf for first floor and 80 psf for other floors (see "Other Provisions for Floor Live Loads" for assembly fixed-seat areas); light industrial uses and light warehouses: 125 psf; and heavy industrial uses and heavy warehouses: 250 psf.

Effective Area Reduction Factor

Members that have more than 400 ft^2 of influence area are subject to a reduction in basic design live loads. The influence area is defined as the tributary area, A_T, multiplied by an element factor, K_{LL}, as listed in Table 2.2.

The following cases are excluded from the live load reduction:

1. Heavy live loads that exceed 100 psf, except for members supporting two or more floors where the live load can be reduced to 80% but not less than k in Equation 2.2.
2. Passenger car garages
3. Public assembly areas

Except for the three items listed above, for all other cases the reduction factor is given by:

$$k = \left(0.25 + \frac{15}{\sqrt{K_{LL}A_T}}\right) \tag{2.2}$$

As long as the following limits are observed, Equation 2.2 can be applied to any area. However, with the limits imposed, the k factor becomes effective when $K_{LL}A_T$ is greater than 400, as stated earlier:

1. The k factor should not be more than 1.
2. The k factor should not be less than 0.5 for members supporting one floor and 0.4 for members supporting more than one floor.

TABLE 2.2
Live Load Element Factor, K_{LL}

Structure Element	K_{LL}
Interior columns	4
Exterior columns without cantilever slabs	4
Edge columns with cantilever slabs	3
Corner columns with cantilever slabs	2
Edge beams without cantilever slabs	2
Interior beams	2
All other members not identified including the following:	1
Edge beams with cantilever slabs	
Cantilever beams	
One-way slabs	
Two-way slabs	

Note: Members without provisions for continuous shear transfer normal to their span.

Example 2.2

The first-floor framing plan of a single-family dwelling is shown in Figure 2.1. Determine the magnitude of the live load on the interior column C.

Solution

1. From Table 2.1, $L_0 = 40$ psf
2. Tributary area $A_T = 20 \times 17.5 = 350$ ft^2
3. From Table 2.2, $K_{LL} = 4$
4. $K_{LL}A_T = 4 \times 350 = 1400$ ft^2
5. From Equation 2.2:

$$K = \left(0.25 + \frac{15}{\sqrt{K_{LL}A_T}}\right) = \left(0.25 + \frac{15}{\sqrt{1400}}\right) = 0.65$$

6. From Equation 2.1, $L = kL_0 = 0.65\ (40) = 26$ psf

Example 2.3

An interior column supports the following unit live loads from three floors on a surface area of 20 ft × 30 ft each: first floor = 40 psf, second floor = 40 psf, and third floor = 35 psf. Determine the design unit live load on the column.

Solution

1. Total load = 40 + 40 + 35 = 115 psf
2. Tributary area $A_T = 20 \times 30 = 600$ ft^2
3. From Table 2.2, $K_{LL} = 4$
4. $K_{LL}A_T = 4 \times 600 = 2400$ ft^2
5. From Equation 2.2:

$$k = \left(0.25 + \frac{15}{\sqrt{2400}}\right) = 0.556 > 0.4 \text{ OK}$$

6. From Equation 2.1, $L = kL_0 = 0.556\ (115) = 63.94$ psf
7. Heavy load reduction permitted for more than one floor:
   $L = 0.8 \times 115 = 92$ psf ← controls

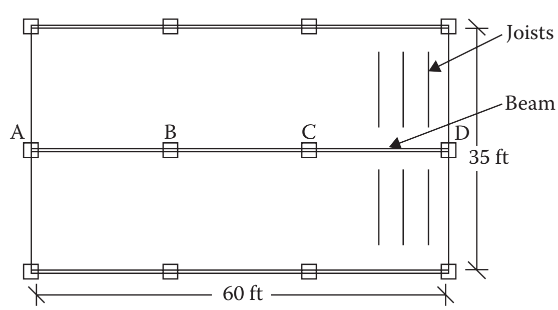


FIGURE 2.1 Floor framing plan.

OTHER PROVISIONS FOR FLOOR LIVE LOADS

Besides uniformly distributed live loads, ASCE 7-16 also indicates the concentrated live loads in certain cases that are assumed to be distributed over an area of 2.5 ft × 2.5 ft. The maximum effect of either the uniformly distributed load or the concentrated load has to be considered. In most cases, the uniformly distributed loads have higher magnitudes.

In buildings where partitions are likely to be erected, a uniform partition live load is provided in addition to the basic design loads, whether or not partitions are shown on plans. The minimum partition load is 15 psf. Partition live loads are not subjected to reduction for large effective areas.

Live loads include an allowance for an ordinary impact. However, where unusual vibrations and impact forces are involved, live loads should be increased. The moving loads shall be increased by an impact factor as follows: (1) elevator, 100%; (2) light shaft or motor-driven machine, 20%; (3) reciprocating machinery, 50%; and (4) hangers for floor or balcony, 33%. After including these effects:

$$\text{Total LL/unit area} = \text{unit LL}(1 + \text{IF}) + \text{PL} \tag{2.3}$$

where:
LL is live load
IF is impact factor, in decimal point
PL is partition load, which is a minimum of 15 psf

For assembly areas like stands, grandstands, and bleachers and for stadiums and arenas with fixed seats, in addition to vertical loads specified under "Basic Design Live Load," horizontal swaying forces are applied to each row of seats as follows:

1. In the direction parallel to each row of seats—24 lb per linear foot
2. In the direction perpendicular to each row of seats—10 lb per linear foot

Both of these need not be applied simultaneously.

ROOF LIVE LOADS, L_r

Roof live loads occur for a short time during the roofing or reroofing process. In load combinations, either the roof live load, L_r, or the snow load, S, is included because both are not likely to occur simultaneously.

The standard roof live load for ordinary flat, sloped, or curved roofs is 20 psf. This can be reduced to a minimum value of 12 psf based on the tributary area being larger than 200 ft^2 and/or the roof slope being more than 18.4°. When less than 20 psf of roof live loads are applied to a continuous beam structure, the reduced roof live load is applied to adjacent spans or alternate spans, whichever produces the greatest unfavorable effect.

The roof live load is estimated by:

$$L_r = R_1 R_2 L_0 \tag{2.4}$$

where:
L_r is reduced roof live load on a horizontally projected surface
L_0 is basic design load for an ordinary roof, which is 20 psf
R_1 is tributary area reduction factor (see the section "Tributary Area Reduction Factor, R_1")
R_2 is slope reduction factor (see the section "Slope Reduction Factor")

Tributary Area Reduction Factor, R_1

The tributary area reduction factor, R_1, is given by:

$$R_1 = 1.2 - 0.001A_T \tag{2.5}$$

where A_T is the horizontal projection of the roof tributary area in square feet.

This is subject to the following limitations:

1. R_1 should not exceed 1.
2. R_1 should not be less than 0.6.

Slope Reduction Factor

The slope reduction factor is given by:

$$R_2 = 1.2 - 0.6\tan\theta \tag{2.6}$$

where θ is the roof slope angle.

This is subject to the following limitations:

1. R_2 should not exceed 1.
2. R_2 should not be less than 0.6.

Example 2.4

The horizontal projection of a roof framing plan of a building is similar to Figure 2.1. The roof pitch is 7 on 12. Determine the roof live load acting on column C.

Solution

1. L_0 = 20 psf
2. A_T = 20 × 17.5 = 350 ft²
3. From Equation 2.5, R_1 = 1.2–0.001 (350) = 0.85
4. Pitch of 7 on 12, tan θ = 7/12 or θ = 30.256°
5. From Equation 2.6, R_2 = 1.2–0.6 tan 30.256° = 0.85
6. From Equation 2.4, L_r = (0.85) (0.85) (20) = 14.45 psf > 12 psf

The aforementioned computations are for an ordinary roof. Special-purpose roofs, such as roof gardens, have loads up to 100 psf. These are permitted to be reduced according to floor live load reduction, as discussed in the "Floor Live Loads" section.

PROBLEMS

2.1 A floor framing consists of the following: hardwood floor (4 psf), 1 in. plywood (3 psf), 2 in. × 12 in. framing at 4 in. on center (2.6 psf), ceiling supports (0.5 psf), and gypsum wallboard ceiling (5 psf). Determine the floor dead load.

2.2 In Problem 2.1, the floor covering is replaced by a 1 in. concrete slab and the framing by 2 in. × 12 in. at 3 in. on center. Determine the floor dead load.
Hint: Weight in pounds of concrete/unit area = 1 ft × 1 ft × 1/12 ft × 150.

2.3 For the floor framing plan of Example 2.2, determine the design live load on the interior beam BC.

2.4 An interior steel column of an office building supports a unit load, as indicated in Table 2.1, from the floor above. The column-to-column distance among all columns in the floor plan is 40 ft. Determine the design live load on the column.

2.5 The framing plan of a gymnasium is shown in Figure P2.1. Determine the live load on column A.

2.6 Determine the live load on the slab resting on column A from Problem 2.5.

2.7 The column in Problem 2.4 supports the same live loads from two floors above. Determine the design live load on the column.

2.8 A corner column with a cantilever slab supports the following live loads over an area of 25 ft × 30 ft: first floor = 30 psf, second floor = 25 psf, and third floor = 20 psf. Determine the design live load.

2.9 The column in Problem 2.8 also supports an elevator and hangers of a balcony. Determine the design load.

2.10 The building in Problem 2.5 includes partitioning of the floor, and it is equipped with a reciprocating machine that induces vibrations on the floor. Determine the design live load on beam AB.

2.11 Determine the roof live load acting on the end column D of the roofing plan shown in Figure P2.2.

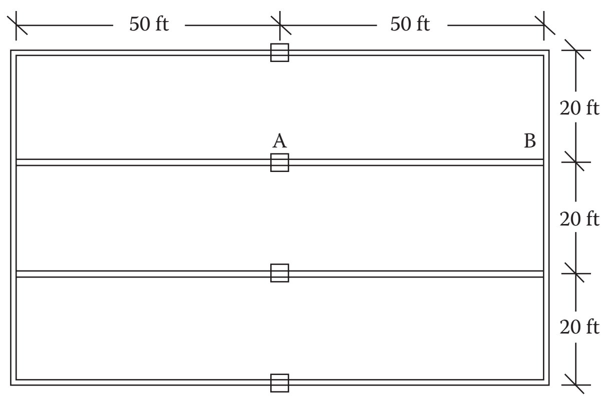


FIGURE P2.1 Framing plan for Problem 2.5.

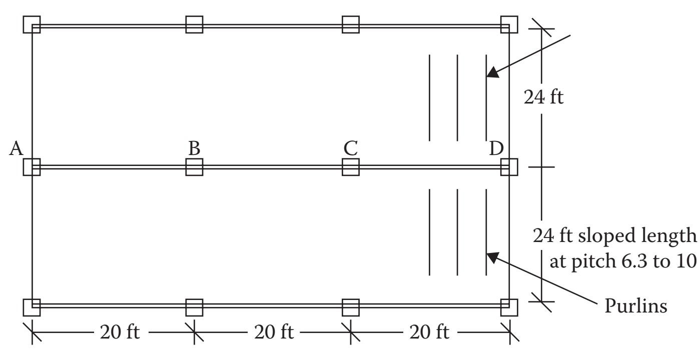


FIGURE P2.2 Roofing plan for Problem 2.11.

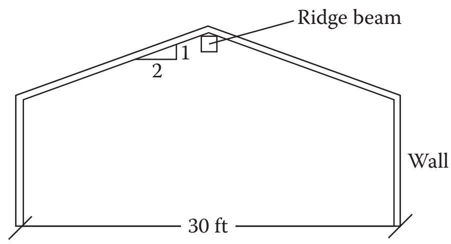


FIGURE P2.3 Side elevation of building for Problem 2.13.

2.12 Determine the roof live load on the purlins of Figure P2.2 if they are 4 ft apart.

2.13 A roof framing section is shown in Figure P2.3. The length of the building is 40 ft. The ridge beam has supports at two ends and at midlength. Determine the roof live load on the ridge beam.

2.14 Determine the load on the walls due to the roof live load from Problem 2.13.

2.15 An interior column supports loads from a roof garden. The tributary area to the column is 250 ft^2. Determine the roof live load. Assume a basic roof garden load of 100 psf.

3 Snow Loads

INTRODUCTION

Snow is a controlling roof load in about half of all the states in the United States. It is a cause of frequent and costly structural problems. Snow loads are assumed to act on the horizontal projection of the roof surface. Snow loads have the following components:

1. Balanced snow load
2. Rain-on-snow surcharge
3. Partial loading of the balanced snow load
4. Unbalanced snow load due to a drift on one roof
5. Unbalanced load due to a drift from an upper roof to a lower roof
6. Sliding snow load

The following snow loading combinations are considered:

1. Balanced snow load *plus* rain-on-snow when applicable, or the minimum snow load
2. Partial loading (of balanced snow load without rain-on-snow)
3. Unbalanced snow load (without rain-on-snow)
4. Balanced snow load (without rain-on-snow) *plus* drift snow load
5. Balanced snow load (without rain-on-snow) *plus* sliding snow load

For low-slope roofs, ASCE 7-16 prescribes a minimum load that acts by itself and is not combined with other snow loads.

MINIMUM SNOW LOAD FOR LOW-SLOPE ROOFS

The slope of a roof is defined as a *low slope* if mono, hip, and gable roofs have a slope of less than 15° and a curved roof has a vertical angle from eave to crown of less than 10°.

The minimum snow load for low-slope roofs should be obtained from the following equations:

1. When the ground snow load, p_g, is 20 lb/ft^2 or less

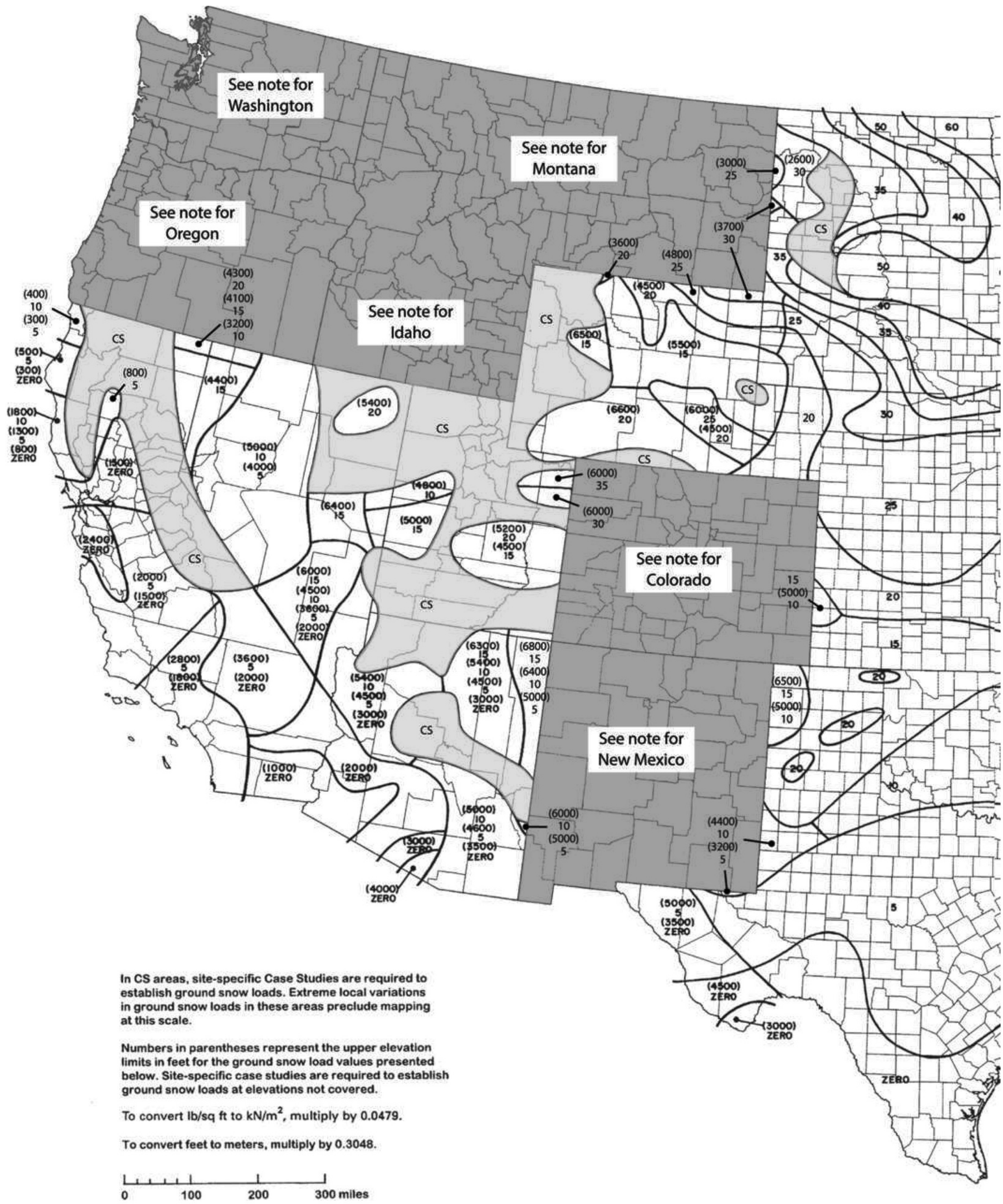
$$p_m = I\ p_g \tag{3.1}$$

2. When the ground snow load is more than 20 lb/ft^2

$$p_m = 20\,I \tag{3.2}$$

where p_g is 50-year ground snow load from Figure 3.1. ASCE 7-16 has identified the states of Colorado, Idaho, Montana, New Hampshire, New Mexico, Oregon, and Washington as areas where extreme variations in ground snow loads preclude showing values on the map in Figure 3.1. For these, ASCE 7-16 has included tables of the regional data generated by state agencies. Also, certain areas as shown on the map in Figure 3.1 are identified as case study (CS) areas where site-specific studies are required to establish ground snow loads. The notes on Figure 3.1 refer to ASCE 7-16.

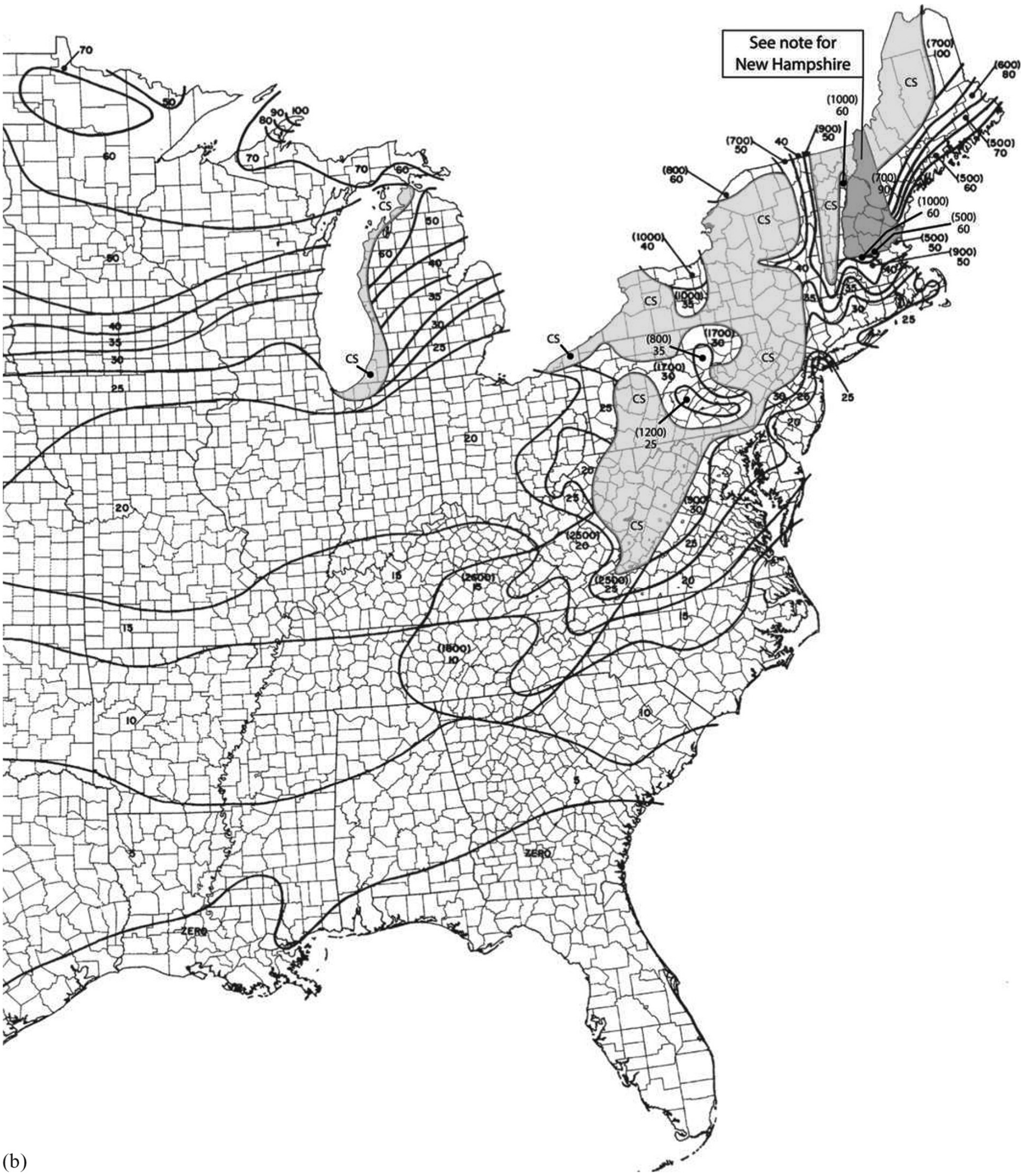
I is importance factor (see the "Importance Factor" section).



Note: See Table 7.2-2 for Colorado; see Table 7.2-3 for Idaho; see Table 7.2-4 for Montana; see Table 7.2-5 for Washington; see Table 7.2-6 for New Mexico; see Table 7.2-7 for Oregon; see Table 7.2-8 for New Hampshire.

(a)

FIGURE 3.1 Ground snow loads, p_g, for the United States (lb/ft^2). *(Continued)*



(b)

FIGURE 3.1 (*Continued*) Ground snow loads, p_g, for the United States (lb/ft2).

As stated, the minimum snow load, p_m, is considered a separate uniform load case. It is not combined with other loads—balanced, rain-on-snow, unbalanced, partial, drift, or sliding loads.

BALANCED SNOW LOAD

This is the basic snow load to which a structure is subjected. The procedure to determine the balanced snow load is:

1. Determine the ground snow load, p_g, from the snow load map in ASCE 7-16, reproduced in Figure 3.1.
2. Convert the ground snow load to flat roof snow load (roof slope ≤ 5°), p_f, with consideration given to the (1) roof exposure, (2) roof thermal condition, and (3) occupancy category of the structure:

$$p_f = 0.7C_eC_tIp_g \tag{3.3}$$

3. Apply a roof slope factor to the flat roof snow load to determine the sloped (balanced) roof snow load.
4. Combining the preceding steps, the sloped roof snow load is calculated from

$$p_s = 0.7C_sC_eC_tIp_g \tag{3.4}$$

where:
p_g is 50-year ground snow load from Figure 3.1
I is importance factor (see the "Importance Factor" section)
C_t is thermal factor (see the "Thermal Factor, C_t" section)
C_e is exposure factor (see the "Exposure Factor, C_e" section)
C_s is roof slope factor (see the "Roof Slope Factor, C_s" section)

It should be noted that when the slope is larger than 70°, the slope factor $C_s = 0$, and the balanced snow load is zero.

Importance Factor

Depending on the risk category identified in Table 1.1 of Chapter 1, the importance factor is determined from Table 3.1.

TABLE 3.1
Importance Factor for Snow Load

Risk Category	Importance Factor
I. Structures of low hazard to human life	0.8
II. Standard structures	1.0
III. High-occupancy structures	1.1
IV. Essential structures	1.2

Source: Courtesy of American Society of Civil Engineers, Reston, VA.

Thermal Factor, C_t

The factors are given in Table 3.2. The intent is to account for the heat loss through the roof and its effect on snow accumulation. For modern, well-insulated, energy-efficient construction with eave and ridge vents, the common C_t value is 1.1.

Exposure Factor, C_e

The factors, as given in Table 3.3, are a function of the surface roughness (terrain type) and the location of the structure within the terrain (sheltered to fully exposed).

It should be noted that exposure A representing centers of large cities, where over half the buildings are taller than 70 ft, is not recognized separately in ASCE 7-16. This type of terrain is included in exposure B.

Sheltered areas correspond to roofs that are surrounded on all sides by obstructions that are within a distance of 10 h_0, where h_0 is the height of the obstruction above roof level. Fully exposed roofs have no obstruction within 10 h_0 on all sides, including no large rooftop equipment or tall parapet walls. Partially exposed roofs represent structures that are not sheltered or fully exposed. Partial exposure is the most common exposure condition.

Roof Slope Factor, C_s

This factor decreases as the roof slope increases. Also, the factor is smaller for slippery roofs and warm roof surfaces.

ASCE 7-16 provides the graphs of C_s versus roof slope for three separate thermal factors, C_t, that is, C_t of $\leq$ 1.0 (warm roofs); C_t of 1.1 (cold, well-insulated, and ventilated roofs); and C_t of 1.2 (cold roofs). On the graph for each value of the thermal factor, two curves are shown. The dashed

TABLE 3.2
Thermal Factor, C_t

Thermal Condition	C_t
All structures except as indicated below	1.0
Structures kept just above freezing and other structures with cold, ventilated, well-insulated roofs (R-value > 25 ft^2 hr·°F/Btu)	1.1
Unheated and open air structures	1.2
Structures intentionally kept below freezing	1.3
Continuously heated greenhouses	0.85

TABLE 3.3
Exposure Factor for Snow Load

Terrain	Fully Exposed	Partially Exposed	Sheltered
B. Urban, suburban, wooded, closely spaced dwellings	0.9	1.0	1.2
C. Open areas of scattered obstructions, flat open country and grasslands	0.9	1.0	1.1
D. Flat unobstructed areas and water surfaces, smooth mud and salt flats	0.8	0.9	1.0
Above the tree line in mountainous region	0.7	0.8	—
Alaska: treeless	0.7	0.8	—

TABLE 3.4
Roof Slope Factor, C_s

Thermal Factor	Unobstructed Slippery Surface	Other Surfaces
	$R \geq 30$ ft^2 hr·°F/Btu for unventilated and $R \geq 20$ ft^2 hr·°F/Btu for ventilated	
Warm roofs ($C_t \leq 1$)	$\theta = 0^\circ - 5^\circ$ $C_s = 1$	$\theta = 0^\circ - 30^\circ$ $C_s = 1$
	$\theta = 5^\circ - 70^\circ C_s = 1 - \frac{\theta - 5^\circ}{65^\circ}$	$\theta = 30^\circ - 70^\circ C_s = 1 - \frac{\theta - 30^\circ}{40^\circ}$
	$\theta > 70^\circ$ $C_s = 0$	$\theta > 70^\circ$ $C_s = 0$
Cold roofs ($C_t = 1.1$)	$\theta = 0^\circ - 10^0$ $C_s = 1$	$\theta = 0^\circ - 37.5^\circ$ $C_s = 1$
	$\theta = 10^\circ - 70^\circ C_s = 1 - \frac{\theta - 10^\circ}{60^\circ}$	$\theta = 37.5^\circ - 70^\circ C_s = 1 - \frac{\theta - 37.5^\circ}{32.5^\circ}$
	$\theta > 70^0$ $C_s = 0$	$\theta > 70^0$ $C_s = 0$
Cold roofs ($C_t = 1.2$)	$\theta = 0^\circ - 15^\circ$ $C_s = 1$	$\theta = 0^\circ - 45^0 C_s = 1$
	$\theta = 15^\circ - 70^\circ C_s = 1 - \frac{\theta - 15^\circ}{55^\circ}$	$\theta = 45^\circ - 70^\circ C_s = 1 - \frac{\theta - 45^\circ}{25^\circ}$
	$\theta > 70^\circ$ $C_s = 0$	$\theta > 70^0$ $C_s = 0$

Note: θ is the slope of the roof.

line is for an unobstructed slippery surface, and the solid line is for other surfaces. The dashed line of unobstructed slippery surfaces has smaller C_s values.

An unobstructed surface has been defined as a roof on which no object exists that prevents snow from sliding and there is a sufficient space available below the eaves where the sliding snow can accumulate. The slippery surface includes metal, slate, glass, and membranes. For the warm roof case ($C_t \leq 1$), to qualify as an unobstructed slippery surface, there is a further requirement with respect to the R (thermal resistance) value.

The values of C_s can be expressed mathematically, as given in Table 3.4. For nonslippery surfaces like asphalt shingles, which is a common case, the C_s factor is relevant only for roofs having a slope larger than 30°; for slopes larger than 70°, $C_s = 0$.

RAIN-ON-SNOW SURCHARGE

An extra load of 5 lb/ft^2 has to be added due to rain-on-snow for locations where the following two conditions apply: (1) the ground snow load, p_g, is ≤ 20 lb/ft^2 and (2) the roof slope is less than $W/50$, W being the horizontal eave-to-ridge roof distance. This extra load is applied only to the balanced snow load case and should not be used in combination with minimum, unbalanced, partial, drift, and sliding load cases.

Example 3.1

Determine the balanced load for an unheated building of ordinary construction, as shown in Figure 3.2, in a suburban area with tree obstruction within a distance of 10 h_0. The ground snow load is 20 psf.

Solution

A. Parameters
  1. $p_g = 20$ psf
  2. Unheated roof, $C_t = 1.20$

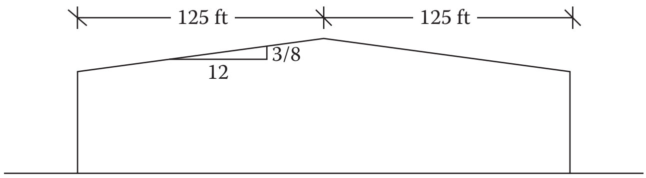


FIGURE 3.2 Low-slope roof.

3. Ordinary building, $I = 1.0$
4. Suburban area (terrain B), sheltered, exposure factor, $C_e = 1.2$
5. Roof angle, $\tan\theta = \dfrac{3/8}{12} = 0.0313;\ \theta = 1.8°$
6. $\theta < 15°$; it is a low slope; the minimum load equation applies.
7. $\dfrac{W}{50} = \dfrac{125}{50} = 2.5$
8. $\theta < 2.5°$ and $p_g = 20$ psf, rain-on-snow surcharge = 5 lb/ft²
9. From Table 3.4, $C_s = 1.0$

B. Snow loads
1. Minimum snow load, from Equation 3.1:

   $p_m = (1)(20) = 20 \text{ lb/ft}^2$
2. From Equation 3.4:

   $$p_s = 0.7C_sC_eC_tIp_g$$
   $$= 0.7(1)(1.2)(1.2)(1)(20) = 20.16 \text{ lb/ft}^2$$
3. Add rain-on-snow surcharge:

   $p_b = 20.16 + 5 = 25.16 \text{ lb/ft}^2 \leftarrow$ controls

Example 3.2

Determine the balanced snow load for an essential facility in Seattle, Washington, having a roof eave to ridge width of 100 ft and a height of 25 ft. It is a warm roof structure.

Solution

A. Parameters
1. Seattle, Washington, $p_g = 20$ psf
2. Warm roof, $C_t = 1.00$
3. Essential facility, $I = 1.2$
4. Category B, urban area, partially exposed (default), exposure factor, $C_e = 1.00$
5. Roof slope, $\tan\theta = \dfrac{25}{100} = 0.25;\ \theta = 14°$
6. $\theta < 15°$; the minimum snow equation is applicable.
7. θ is not less than $W/50$, so there is no rain-on-snow surcharge.
8. For a warm roof, other structures, from Table 3.4 $C_s = 1$

B. Snow loads
1. $p_m = (1.2)(20) = 24$ lb/ft^2 ← controls
2. $p_s = 0.7\, C_s C_e C_t I p_g$

$= 0.7(1)(1)(1.2) = 16.8$ lb/ft^2

PARTIAL LOADING OF THE BALANCED SNOW LOAD

Partial loads are different from unbalanced loads. In unbalanced loads, snow is removed from one portion and is deposited in another portion. In the case of partial loading, snow is removed from one portion through scour or melting but is not added to another portion. The intent is that in a continuous span structure, a reduction in snow loading on one span might induce heavier stresses in some other portion than those that occur with the entire structure being loaded. The provision requires that a selected span or spans should be loaded with one-half of the balanced snow load and the remaining spans with the full balanced snow load. This should be evaluated for various alternatives to assess the greatest effect on the structural member.

Partial load is not applied to the members that span perpendicular to the ridgeline in gable roofs having slopes between 2.38° and 30.3°.

UNBALANCED ACROSS THE RIDGE SNOW LOAD

Balanced and unbalanced loads are analyzed separately.

The unbalanced loading condition results when a blowing wind depletes snow from the upwind direction to pile it up in the downward direction.

Unbalanced snow loading for hip and gable roofs is discussed here. For curved, saw tooth, and dome roofs, a reference is made to Sections 7.6.2 through 7.6.4 of ASCE 7-16.

For an unbalanced load to occur on any roof, it should be neither a very low-slope roof nor a steep roof. Thus, the following two conditions should be satisfied for across the ridge unbalanced snow loading:

1. The roof slope should be equal to or larger than 2.38°.
2. The roof slope should be less than 30.2°.

When the preceding two conditions are satisfied, the unbalanced load distribution is expressed in two different ways:

1. For narrow roofs ($W \leq 20$ ft) of simple structural systems like prismatic wood rafters or light gauge roof rafters spanning from eave to ridge, the windward side is taken as free of snow, and on the leeward side the total snow load is represented by a uniform load from eave to ridge as follows (note this is the total load and is not an addition to the balanced load):

$$p_u = I p_g \tag{3.5}$$

2. For wide roofs ($W > 20$ ft) of any structures as well as narrow roofs other than the simple structures mentioned in the preceding discussion, the load is triangular in shape but is represented by a more user-friendly rectangular surcharge over the balanced load. On the windward side, a uniform load of $0.3p_s$ is applied, where p_s is the balanced snow load mentioned in the "Balanced Snow Load" section. On the leeward side, a rectangular load is placed adjacent to the ridge, on top of the balanced load, p_s, as follows:

$$\text{Uniform load, } p_u = \frac{h_d \gamma}{\sqrt{s}} \tag{3.6}$$

$$\text{Horizontal extent from ridge, } L = \frac{8h_d\sqrt{s}}{3} \tag{3.7}$$

where:

$\frac{1}{s}$ is roof slope

γ is unit weight of snow in lb/ft³, given by:

$$\gamma = 0.13p_g + 14 \leq 30\,\text{lb/ft}^3 \tag{3.8}$$

h_d is height of drift in feet on the leeward roof, given by:

$$\frac{h_d}{\sqrt{I}} = 0.43(W)^{1/3}(p_g + 10)^{1/4} - 1.5 \tag{3.9}$$

W is horizontal distance from eave to ridge for the windward portion of the roof in feet
If $W < 20$ ft, use $W = 20$ ft.

Example 3.3

Determine the unbalanced drift snow load for Example 3.1.

Solution

1. Roof slope, $\theta = 1.8°$
2. Since the roof slope < 2.38°, there is no unbalanced snow load.

Example 3.4

Determine the unbalanced drift snow load for Example 3.2.

Solution

A. On the leeward side
   1. Roof slope, $\theta = 14°$; it is not a very low-slope roof for unbalanced load.
   2. $W > 20$ ft; it is a wide roof.
   3. $p_g = 20$ psf and $p_s = 16.8$ lb/ft² (from Example 3.2)
   4. slope $= \frac{1}{s} = \frac{25}{100}$ or $s = 4$
   5. $\frac{h_d}{\sqrt{1}} = 0.43\,W^{1/3}\left(p_g + 10\right)^{1/4} - 1.5$

$$= 0.43\,\left(100\right)^{1/3}\left(20\ + 10\right)^{1/4} - 1.5$$

$$h_d = 3.16$$

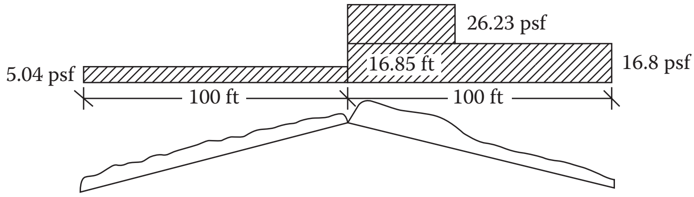


FIGURE 3.3 Unbalanced snow load on a roof.

6. Unit weight of snow
   $\gamma = 0.13p_g + 14 \le 30$

   $= 0.13(20) + 14 = 16.6 \text{ lb/ft}^3$

7. $p_u = \dfrac{h_d \gamma}{\sqrt{s}}$

   $= \dfrac{(3.16)(16.6)}{\sqrt{4}} = 26.23 \text{ lb/ft}^2$

8. Horizontal extent, $L = \dfrac{8h_d\sqrt{s}}{3} = \dfrac{8(3.16)\sqrt{4}}{3} = 16.85 \text{ ft}$

B. On the windward side
   1. $p_u = 0.3\ p_s = 0.3\ (16.8) = 5.04$ psf.
   2. This is sketched in Figure 3.3.

SNOW DRIFT FROM A HIGHER TO A LOWER ROOF

Snow drifts are formed in the wind shadow of a higher structure onto a lower structure. The lower roof can be part of the same structure or it can be an adjacent separated structure.

This drift is a surcharge that is superimposed on the balanced snow roof load of the lower roof. The drift accumulation, when the higher roof is on the windward side, is shown in Figure 3.4. This is known as the *leeward snow drift*.

When the higher roof is on the leeward side, the drift accumulation, known as the *windward snow drift*, is more complex. It starts as a quadrilateral shape because of the wind vortex and ends up in a triangular shape, as shown in Figure 3.5.

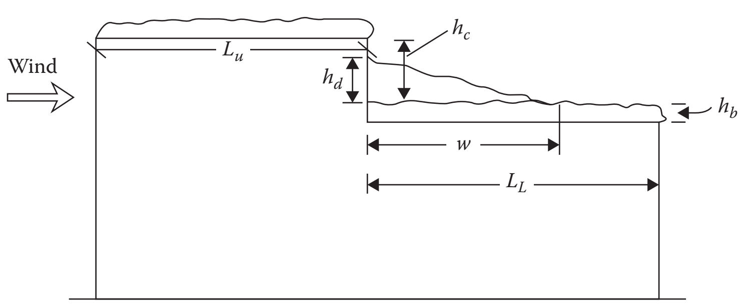


FIGURE 3.4 Leeward snow drift.

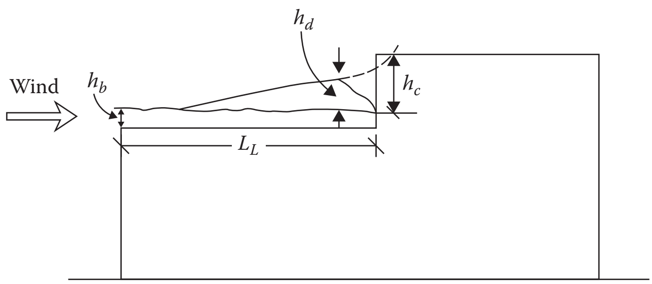


FIGURE 3.5 Windward snow drift.

Leeward Snow Drift on Lower Roof of Attached Structure

In Figure 3.4, if h_c/h_b is less than 0.2, the drift load is not applied, where h_b is the balanced snow depth determined by dividing the balanced snow load, p_s, by a unit load of snow, γ, computed by Equation 3.7. The term h_c represents the difference in elevation between the high and low roofs minus h_b, as shown in Figure 3.4.

The drift is represented by a triangle, as shown in Figure 3.6. Drift height h_d should not be more than 0.6 L_L.

$$\frac{h_d}{\sqrt{I}} = 0.43(L_u)^{1/3}(p_g + 10)^{1/4} - 1.5 \tag{3.10}$$

where:

L_u is the horizontal length of the roof upwind of the drift, as shown in Figure 3.4

L_L is the horizontal length of the roof leeward of the drift

The corresponding maximum snow load is:

$$p_d = \gamma h_d \tag{3.11}$$

The width of the snow load (base of the triangle) has the following values for two different cases:

1. For $h_d \le h_c$

$$w = 4\, h_d \tag{3.12}$$

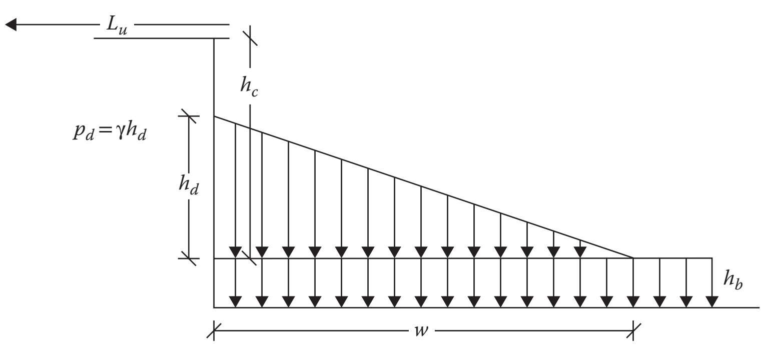


FIGURE 3.6 Configuration of snow drift.

2. For $h_d > h_c$

$$w = \frac{4h_d^2}{h_c} \tag{3.13}$$

but w should not be greater than $8h_c$.

In Equation 3.13, w is computed by the value of h_d from Equation 3.10, which is higher than h_c for the case of Equation 3.13. However, since the drift height cannot exceed the upper roof level, the height of the drift itself is subsequently changed as follows:

$$h_d = h_c \tag{3.14}$$

If width, w, is more than the lower roof length, L_L, then the drift shall be tapered to zero at the end of the roof.

Windward Snow Drift on Lower Roof of Attached Structure

In Figure 3.5, if h_c/h_b is less than 0.2, the drift load is not applied. The drift is given by a triangle similar to the one shown in Figure 3.6. However, the value of h_d is replaced by the following:

$$\frac{h_d}{\sqrt{I}} = 0.75[0.43(L_L)^{1/3}(p_g + 10)^{1/4} - 1.5] \tag{3.15}$$

where L_L is the lower roof length, as shown in Figure 3.5.

Equations 3.12 and 3.13 also apply to the windward width. The larger of the values of the leeward and windward heights, h_d, from the "Leeward Snow Drift on Lower Roof of Attached Structure" and "Windward Snow Drift on Lower Roof of Attached Structure" sections is used in the design.

Leeward Snow Drift on Lower Roof of Separated Structure

If the vertical separation distance between the edge of the higher roof, including any parapet, and the edge of the adjacent lower roof, excluding any parapet, is h, and the horizontal separation between the edges of the two adjacent buildings is s, then the leeward drift to the lower roof is applicable if the following two conditions are satisfied:

1. The horizontal distance, s, is less than 20 ft.
2. The horizontal distance, s, is less than six times the vertical distance, h ($s \leq 6$h).

In such a case, the height of the snow drift is the smaller of the following:

1. h_d as calculated by Equation 3.10 based on the length of the higher structure
2. $\dfrac{(6h - s)}{6}$

The horizontal extent, w, is the smaller of the following:

1. $6h_d$
2. $(6h - s)$

Windward Snow Drift on Lower Roof of Separated Structure

The same equations as for the windward drift on an attached structure, that is, Equation 3.15 for h_d and Equation 3.12 or 3.13 for w, are used. However, the portion of the drift between the edges of the two adjacent roofs is truncated.

At intersecting corners, drift is considered to occur concurrently except that the two drifts are not superimposed.

Example 3.5

A two-story residential building has an attached garage, as shown in Figure 3.7. The residential part is heated and has a well-insulated, ventilated roof, whereas the garage is unheated. Both roofs of 4 on 12 slope have metal surfaces consisting of purlins spanning eave to ridge.

The site is a forested area in a small clearing among huge trees. The ground snow load is 40 psf. Determine the snow load on the lower roof.

Solution

1. The upper roof is subjected to the balanced snow load and the unbalanced across the ridge load due to wind in the transverse direction.
2. The lower roof is subjected to the balanced snow load, the unbalanced across the ridge load due to transverse directional wind, and the drift load from upper to lower roof due to longitudinal direction wind. Only the lower roof is analyzed here.
3. For the lower roof, the balanced load:
   a. Unheated roof, $C_t = 1.2$
   b. Residential facility, $I = 1.0$
   c. Terrain B, sheltered, $C_e = 1.2$
   d. 4 on 12 slope, $\theta = 18.43°$
   e. For slippery unobstructed surface at $C_t = 1.2$, from Table 3.4:

$$C_s = 1 - \frac{(\theta - 15)}{55} = 1 - \frac{(18.43 - 15)}{55} = 0.94$$

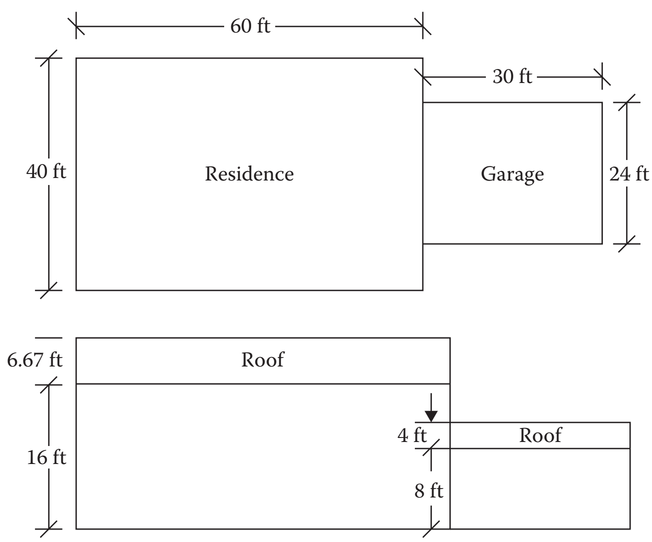


FIGURE 3.7 Higher-lower roof drift.

f. $p_s = 0.7C_sC_eC_tIp_g$

$$= 0.7(0.94)(1.2)(1.2)(1)(40) = 37.90 \text{ lb/ft}^2$$

4. For the lower roof, across the ridge unbalanced load:
   a. $W = 12 < 20$ ft, roof rafter system, the simple case applies
   b. Windward side no snow load
   c. Leeward side

$$p_u = Ip_g = 1(40) = 40 \text{ psf}$$

5. For lower roof, upper–lower roof drift snow load:
   a. From Equation 3.8:

$$\gamma = 0.13p_g + 14 = 0.13(40) + 14 = 19.2 \text{ lb/ft}^3$$

   b. $h_b = \dfrac{p_s}{\gamma} = \dfrac{37.9}{19.2} = 1.97 \text{ ft}$
   c. $h_c = (22.67–12)–1.97 = 8.7$ ft

$$\frac{h_c}{h_b} = \frac{8.7}{1.97} = 4.4 > 0.2 \text{ drift load to be considered}$$

   d. Leeward drift
      From Equation 3.10:

$$\frac{h_d}{\sqrt{1}} = 0.43\ Lu^{1/3}\left(p_g + 10\right)^{1/4} - 1.5$$

$$= 0.43\left(60\right)^{1/3}\left(40 + 10\right)^{1/4} - 1.5 = 2.97 < 0.6\left(30\right) \text{ or } 18 \text{ OK}$$

$$h_d = 2.97$$

      Since $h_c > h_d$, $h_d = 2.97$ ft
   e. $p_d = \gamma\, h_d = (19.2)(2.97) = 57.03 \text{ lb/ft}^2$
   f. From Equation 3.12:

$$w = 4h_d = 4(2.97) = 11.88 \text{ ft}$$

   g. Windward drift

$$\frac{h_d}{\sqrt{1}} = 0.75\left[0.43L_u^{1/3}\left(p_g + 10\right)^{1/4} - 1.5\right]$$

$$= 0.75\left[0.43\ \left(60\right)^{1/3}\left(40 + 10\right)^{1/4} - 1.5\right] = 1.54$$

      $h_d = 1.54$ ft
      Leeward controls, $h_d = 2.97$ ft

6. Figure 3.8 presents the three loading cases for the lower roof.

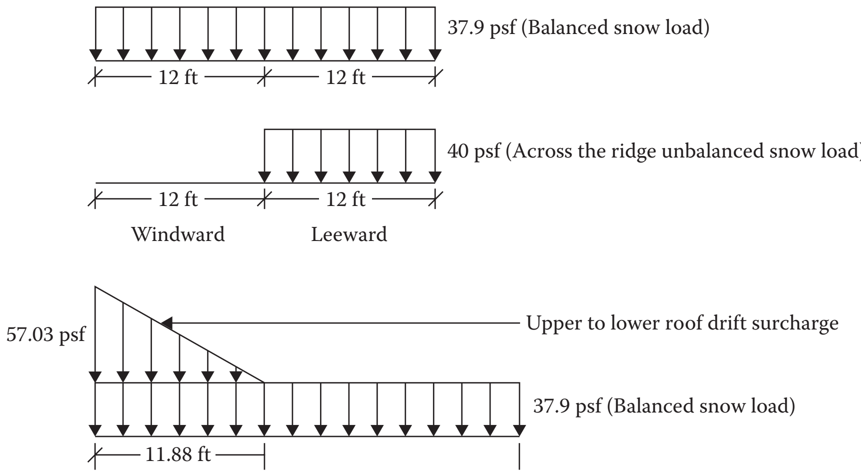


FIGURE 3.8 Loading on a lower roof.

SLIDING SNOW LOAD ON LOWER ROOF

A sliding snow load from an upper to a lower roof is superimposed on the balanced snow load. It is not used in combination with partial, unbalanced, drift, or rain-on-snow loads. The sliding load (plus the balanced load) and the lower roof drift load (plus the balanced load) are considered as two separate cases, and the higher one is used. One basic difference between a slide and a drift is that, in the former case, snow slides off the upper roof along the slope by the action of gravity, and the lower roof should be in front of the sloping surface to capture this load. In the latter case, wind carries the snow downstream, and thus the drift can take place lengthwise perpendicular to the roof slope, as in Example 3.5.

The sliding snow load is applied to the lower roof when the upper slippery roof has a slope of more than $\theta = 2.4°$ (1/4 on 12) or when the nonslippery upper roof has a slope greater than 9.5° (2 on 12).

With reference to Figure 3.9, the total sliding load per unit distance (length) of eave is taken as $0.4\ p_f W$, which is uniformly distributed over a maximum lower roof width of 15 ft. If the width of

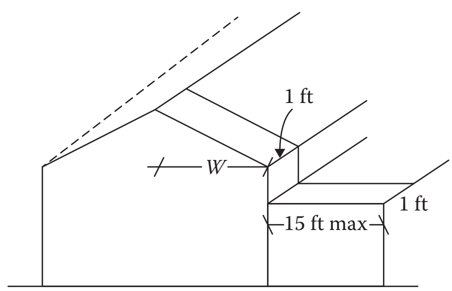


FIGURE 3.9 Sliding snow load.

the lower roof is less than 15 ft, the sliding load is reduced proportionately. The effect is that it is equivalent to distribution over a 15 ft width. Thus:

$$p_{SL} = \frac{0.4 p_f W}{15} \tag{3.16}$$

where:
p_f is flat upper roof snow load (psf) from Equation 3.3
W is horizontal distance from ridge to the eave of the upper roof

Example 3.6

Determine the sliding snow load on an unheated flat roof garage attached to a residence, as shown in Figure 3.10. It is in a suburban area with scattered trees; p_g = 20 psf. Assume that the upper roof flat snow load is 18 psf.

Solution

A. Balanced load on garage
1. Unheated roof, C_t = 1.2
2. Normal usage, I = 1
3. Terrain B, partial exposure, C_e = 1
4. Flat roof, C_s = 1
5. Minimum snow load
   Since p_g = 20 and θ = 0, the minimum load applies, but it is not combined with any other types (balanced, unbalanced, drift, and sliding) of loads.

$$p_m = Ip_g$$
$$= (1)(20) = 20$$

6. Balanced snow load

$$p_s = 0.7 C_s C_e C_t I p_g$$
$$= 0.7(1)(1)(1.2)(1)(20) = 16.8 \text{ lb/ft}^2$$

7. Rain-on-snow surcharge = 5 psf
   Since p_g = 20 and θ < W/50, rain-on-snow surcharge applies, but it is not included in the unbalanced, drift, and sliding load cases.

B. θ < 2.38°; there is no unbalanced across the ridge load.
C. Drift load is not considered in this problem.

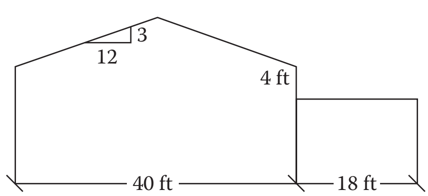


FIGURE 3.10 Sliding snow load on a flat roof.

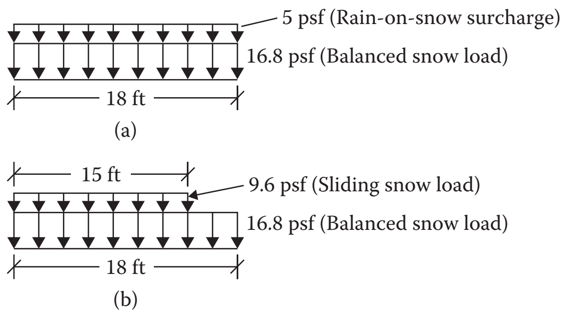


FIGURE 3.11 Loading on lower roof. (a) Balanced snow load. (b) Balanced plus sliding snow load.

D. Sliding snow load
   1. Upper roof slope $\theta = 14° > 9.5°$, sliding applies
   2. $p_f = 18$ lb/ft^2 (given)
   3. $p_{SL} = \dfrac{0.4\,p_f W}{15} = \dfrac{(0.4)(18)(20)}{15} = 9.6$ psf

Figure 3.11 presents the loading cases for the garage.

SLIDING SNOW LOAD ON SEPARATED STRUCTURES

The lower separated roof is subjected to a truncated sliding load if the following two conditions are satisfied:

1. The separation distance between the structures, s, is less than 15 ft
2. The vertical distance between the structures, h, is greater than the horizontal distance, s— that is, ($h > s$)

The sliding load per unit area, p_{SL}, is the same as that given by Equation 3.16, but the horizontal extent on the lower roof is $(15 - s)$. Thus, the load per unit length is:

$$S_L = \frac{0.4\,p_f W(15 - s)}{15} \tag{3.17}$$

PROBLEMS

3.1 Determine the balanced snow load on the residential structure in a suburban area, shown in Figure P3.1. The roof is well insulated and ventilated. There are a few trees behind the building to create obstruction. The ground snow load is 20 lb/ft^2.

3.2 Solve Problem 3.1, but now the eave-to-ridge distance is 30 ft.

3.3 Consider a heated warm roof structure in an urban area surrounded by obstructions from all sides. The eave-to-ridge distance is 25 ft, and the roof height is 7 ft. The ground snow load is 30 psf. Determine the balanced snow load.

3.4 The roof of a high-occupancy structure is insulated and well ventilated in a fully open countryside. The eave-to-ridge distance is 20 ft, and the roof height is 4 ft. The ground snow load is 25 psf. Determine the balanced snow load.

3.5 Determine the unbalanced load for Problem 3.1.

3.6 Determine the unbalanced load for Problem 3.2.

3.7 Determine the unbalanced snow load for Problem 3.3.

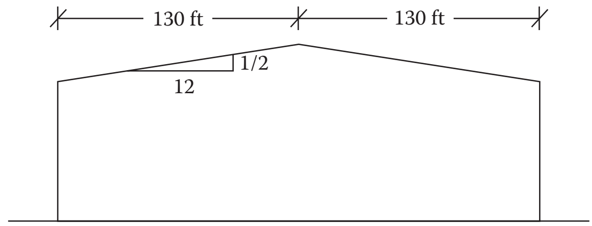


FIGURE P3.1 Suburban residence of Problem 3.1.

3.8 Determine the unbalanced snow load for Problem 3.4.

3.9 Determine snow load on the lower roof of a building where the ground snow load is 30 lb/ft^2. The elevation difference between the roofs is 5 ft. The higher roof is 70 ft wide and 100 ft long. The building is a heated and unventilated office building. The lower roof is 60 ft wide and 100 ft long. It has an unheated storage area. Both roofs have 5 on 12 slope of metallic surfaces without any obstructions. The building is located in an open country with no obstructions. The building is laid out lengthwise, as shown in Figure P3.2.

3.10 Solve Problem 3.9 except that the roofs' elevation difference is 3 ft.

3.11 Solve Problem 3.9 when the building is laid out side by side, as shown in Figure P3.3. The lowest roof is flat.

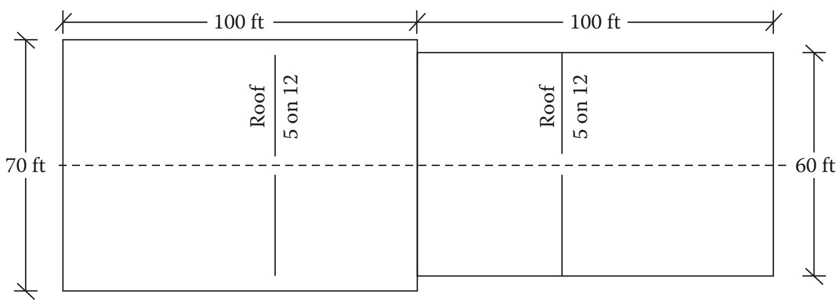


FIGURE P3.2 Different level roofs—lengthwise for Problem 3.9.

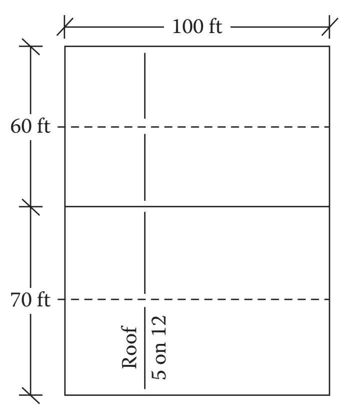


FIGURE P3.3 Different level roofs—side by side for Problem 3.11.

3.12 Solve Problem 3.9 when the two roofs are separated by a horizontal distance of 15 ft.

3.13 Solve Problem 3.11 when the two roofs are separated by a horizontal distance of 10 ft.

3.14 Solve Problem 3.11 for the sliding snow load.

3.15 Determine the snow load due to sliding effect for a heated storage area attached to an office building with a well-ventilated and well-insulated roof in an urban area in Rhode Island having scattered obstructions, as shown in Figure P3.4.

3.16 Determine the sliding load for an unheated garage attached to a cooled roof of the residence shown in Figure P3.5 in a partially exposed suburban area. The ground snow load is 15 lb/ft^2.

3.17 Solve Problem 3.15 when the two roofs are separated by a horizontal distance of 2 ft.

3.18 Solve Problem 3.16 when the two roofs are separated by a horizontal distance of 3 ft.

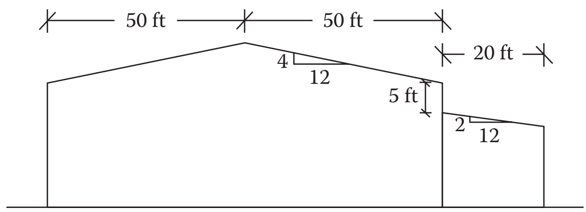


FIGURE P3.4 Sliding snow on urban building for Problem 3.15.

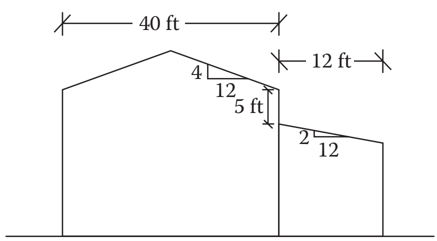


FIGURE P3.5 Sliding snow on suburban building for Problem 3.16.

4 Wind Loads

INTRODUCTION

ASCE 7-10 had made major revisions to wind load provisions: from one single chapter (Chapter 6) in ASCE 7-05, to six chapters (Chapters 26 through 31) that reflected the strength (load resistance factor design) level of design. These features are retained in ASCE 7-16.

Two separate categories have been identified for wind load provisions:

1. Main wind force–resisting system (MWFRS): The MWFRS represents the entire structure comprising an assemblage of the elements constituting the structure that can sustain wind from more than one surface.
2. Components and cladding (C and C): These are the individual elements that face wind directly and pass on the loads to the MWFRS.

The broad distinction is apparent. The entire lateral force–resisting system as a unit that transfers loads to the foundation belongs to the first category. In the second category, the cladding comprises wall and roof coverings like sheathing and finish material, curtain walls, exterior windows, and doors. The components include fasteners, purlins, girts, and roof decking. However, there are certain elements like trusses and studs that are part of the MWFRS but could also be treated as individual components.

The C and C loads are higher than MWFRS loads since they are developed to represent the peak gusts over small areas that result from localized funneling and turbulence of wind.

An interpretation is that, while using MWFRS, the combined interactions of the axial and bending stresses due to the vertical loading together with the lateral loading should be used. But in the application of C and C, either the axial or the bending stress should be considered individually. They are not combined together since the interaction of loads from multiple surfaces is not intended to be used in C and C.

DEFINITION OF TERMS

1. **Low-rise building:** An enclosed or partially enclosed building that has a mean roof height of less than or equal to 60 ft and it (the mean roof height) does not exceed the least horizontal dimension.
2. **Open, partially enclosed, enclosed, and partially open building:** An open building has at least 80% open area in each wall; that is, $A_o/A_g \geq 0.8$, where A_o is the total area of openings in a wall and A_g is the total gross area of that wall.

   A partially enclosed building complies with all three conditions: (1) the total area of openings in a wall that receives the external positive pressure exceeds the sum of the areas of openings in the balance of the building, including the roof, by more than 10%; (2) the total area of openings in a wall that receives the positive external pressure exceeds 4 ft^2 or 1% of the area of that wall, whichever is smaller; and (3) the percentage of openings in the balance of the building envelope does not exceed 20%.

   An enclosed building is one that is not open and that is not partially enclosed, expressed as the following: For each wall $A_o/A_g < 0.01$ or 4 ft^2, whichever is smaller.

   A partially open building does not comply with the requirements of open, partially enclosed, or enclosed buildings

3. **Regular-shaped building:** A building not having any unusual irregularity in spatial form.
4. **Diaphragm building:** Roof, floor, or other membrane or bracing system in a building that transfers lateral forces to the vertical MWFRS.
5. **Hurricane-prone regions:** Areas vulnerable to hurricanes comprising (1) the U.S. Atlantic Ocean and Gulf of Mexico coasts where the basic wind speed is more than 115 miles/h, and (2) Hawaii, Puerto Rico, Guam, the Virgin Islands, and American Samoa, covered as special wind regions in basic wind speed maps.
6. **Special wind regions:** Regions mentioned in item (2) under hurricane-prone regions. These should be examined for higher local winds.
7. **Mean roof height[1]:** The average of the height to the highest point on the roof and the eave height, measured from the ground surface. For a roof angle of 10° or less, it is taken to be the eave height.
8. **Effective wind area:** The span length multiplied by the effective width that need not be less than one-third of the span length.

WIND HAZARD MAPS

For each risk category listed in Table 1.1 in Chapter 1, the basic wind speed, *V*, used to determine design wind loads, is ascertained from one of the maps in Figures 4.1 through 4.4. ASCE 7-16 has reduced the basic wind speeds compared to ASCE 7-10, especially in western parts of the United States. New maps for four different regions of Hawaii have been included in ASCE 7-16 for all four risk categories.

PROCEDURES FOR MWFRS

The following procedures have been stipulated for MWFRS in ASCE 7-16:

1. Wind tunnel procedure: This applies to all types of buildings and structures of all heights, as specified in Chapter 31 of ASCE 7-16.
2. Analytical directional procedure: This applies to regular-shaped enclosed, partially enclosed, and open buildings of all heights, as specified in Part 1 of Chapter 27 in ASCE 7-16.
3. Simplified directional procedure: This applies to regular-shaped enclosed simple diaphragm buildings of 160 ft or less height, as specified in Part 2 of Chapter 27 in ASCE 7-16.
4. Directional procedure: For building appurtenances and structures other than buildings.
5. Analytical envelope procedure: This applies to regular-shaped enclosed or partially enclosed low-rise buildings of 60 ft or less height, as specified in Part 1 of Chapter 28 in ASCE 7-16.
6. Simplified envelope procedure: This applies to enclosed, simple diaphragm low-rise buildings of 60 ft or less height, as specified in Part 2 of Chapter 28 in ASCE 7-16. Since this procedure can be applied to one- and two-story buildings in most locations, it has been adopted in this book.

SIMPLIFIED PROCEDURE FOR MWFRS FOR LOW-RISE BUILDINGS

The following are the steps of the procedure:

1. Determine the basic wind speed, *V*, from one of the maps in Figures 4.1 through 4.4 corresponding to the risk category of the building.
2. Determine the upwind exposure category depending on the surface roughness that prevails in the upwind direction of the structure, as indicated in Table 4.1.

[1] For seismic loads, the height is measured from the base of the structure.

3. Determine the height and exposure adjustment coefficient λ from Table 4.2.
4. The topographic factor, K_{zt}, has to be applied to a structure that is located on an isolated hill of at least 60 ft height for exposure *B* and of at least 15 ft height for exposures *C* and *D*, and it should be unobstructed by any similar hill for at least a distance of 100 times the height of the hill or 2 miles, whichever is less; the hill should also protrude above the height of upwind terrain features within a 2-mile radius by a factor of 2 or more. The factor is assessed by the three multipliers presented in Figure 26.8-1 of ASCE 7-16. For typical cases, $K_{zt} = 1$.
5. Determine p_{s30} from Table 4.3, reproduced from ASCE 7-16. For roof slopes more than 25° and less than or equal to 45°, check for both load cases 1 and 2 in the table.
6. The combined windward and leeward net wind pressure, p_s, is determined by the following simplified equation:

$$p_s = \lambda K_{zt} p_{s30} \tag{4.1}$$

where:
λ is the adjustment factor for structure height and exposure (Tables 4.1 and 4.2)
K_{zt} is the topographic factor; for typical cases, it is equal to 1
p_{s30} is the simplified standard design wind pressure (Table 4.3a–d)

The pressure p_s is the pressure that acts horizontally on the vertical and vertically on the horizontal projection of the structure surface. It represents the net pressure that algebraically sums up the external and internal pressures acting on a building surface. In the case of MWFRS, for the horizontal pressures that act on the building envelope, the p_s combines the windward and leeward pressures.

The plus and minus signs signify the pressures acting toward and away, respectively, from the projected surface.

Horizontal Pressure Zones for MWFRS

The horizontal pressures acting on the vertical plane are separated into the following four pressure zones, as shown in Figure 4.5:

A: End zone of wall
B: End zone of (vertical projection) roof
C: Interior zone of wall
D: Interior zone of (vertical projection) roof

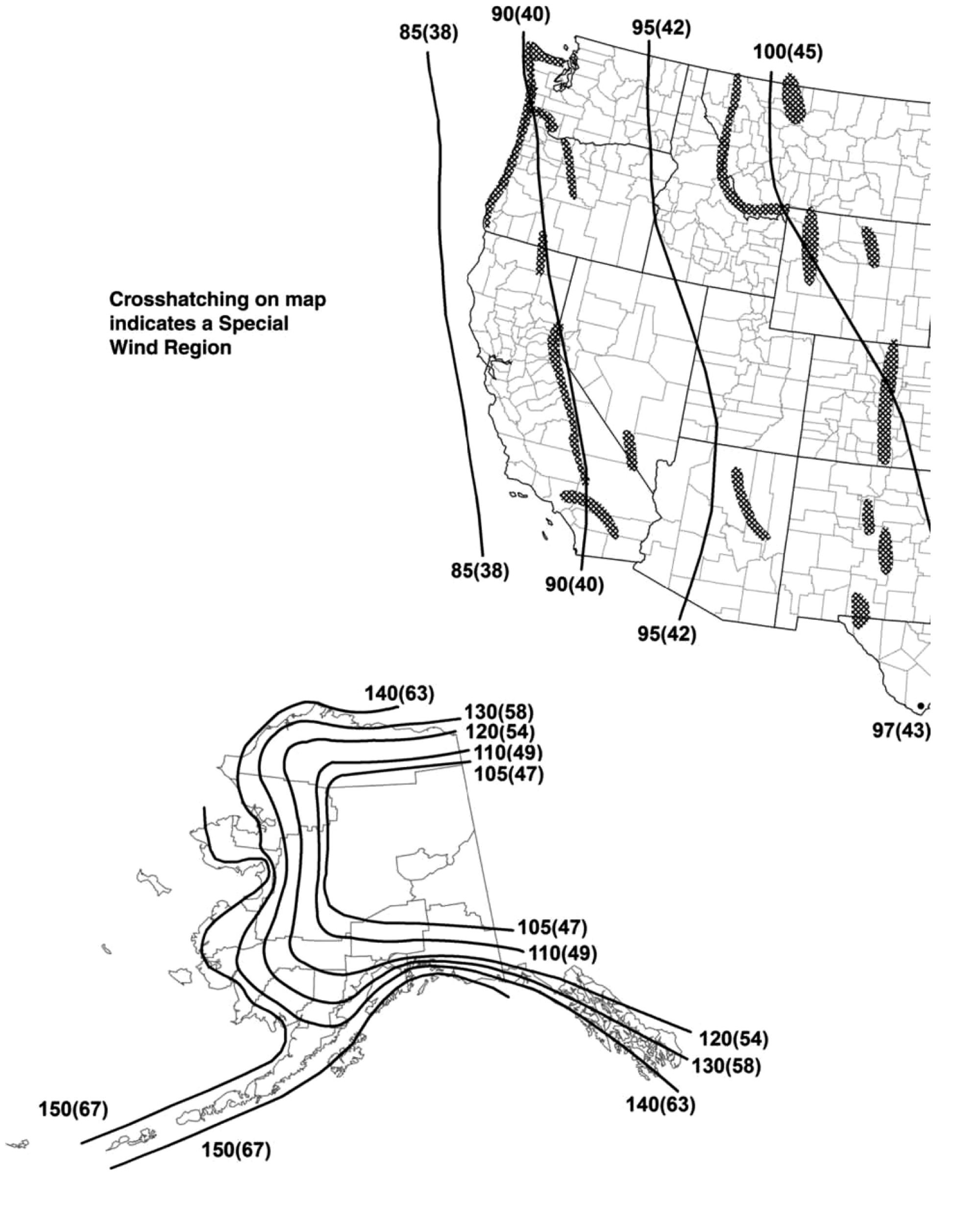
The dimension of end zones A and B are taken to be equal to $2a$, where the value of a is smaller than the following two values:

1. 0.1 times the least horizontal dimension
2. 0.4 times the roof height, h

The height, h, is the mean height of the roof from the ground. For a roof angle < 10°, it is the height to the eave.

If the pressure in zone B or D is negative, treat it as zero in computing the total horizontal force.

For Case B of Figure 4.5, wind acting in the longitudinal direction (wind acting on width), use $\theta = 0$, and zones B and D do not exist.

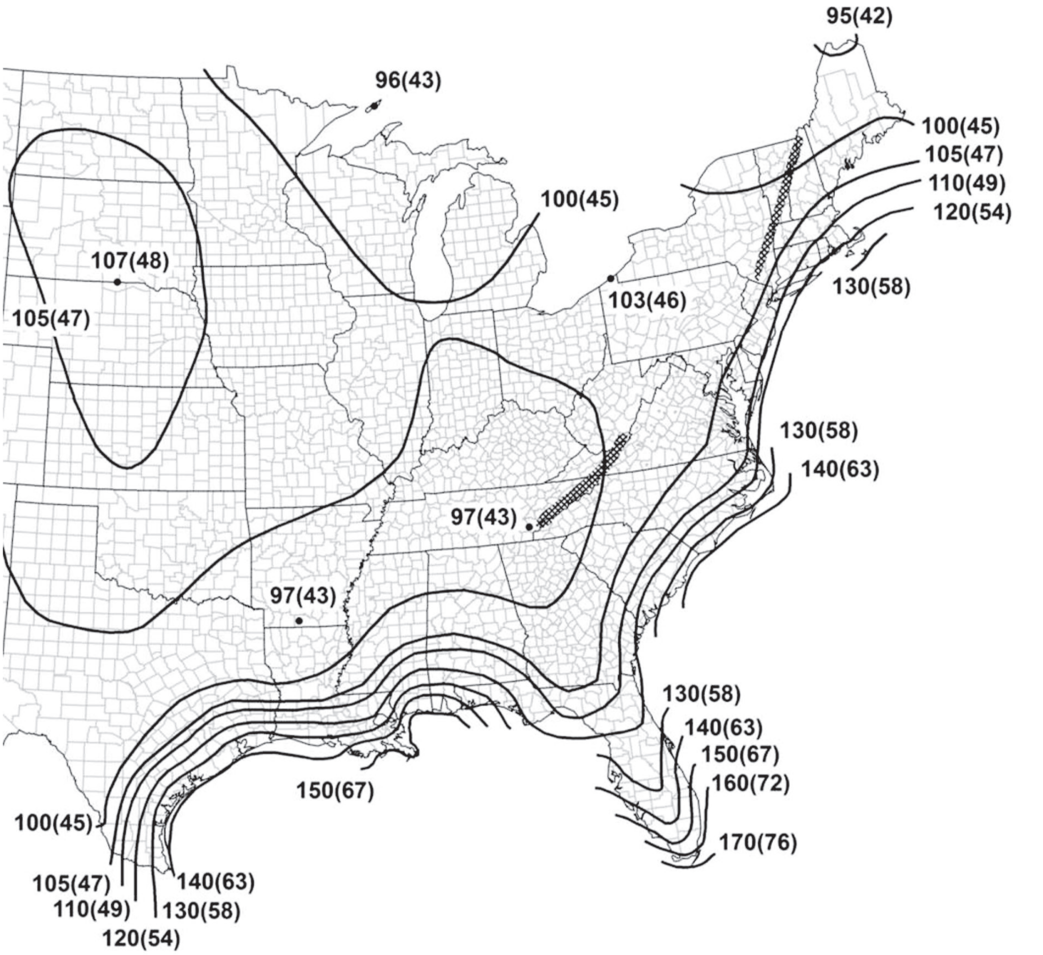


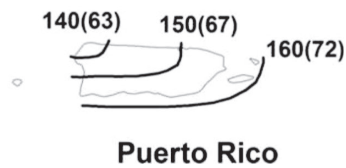
Notes

1. Values are nominal design 3-s gust wind speeds in mi/h (m/s) at 33 ft (10 m) above ground for Exposure Category C.
2. Linear interpolation is permitted between contours. Point values are provided to aid with interpolation.
3. Islands, coastal areas, and land boundaries outside the last contour shall use the last wind speed contour.
4. Mountainous terrain, gorges, ocean promontories, and special wind regions shall be examined for unusual wind conditions.
5. Wind speeds correspond to approximately a 15% probability of exceedance in 50 years (Annual Exceedance Probability = 0.00333, MRI = 300 years).
6. Location-specific basic wind speeds shall be permitted to be determined using www.atcouncil.org/windspeed.

(a)

FIGURE 4.1 Basic wind speeds for risk category I buildings and other structures. (*Continued*)

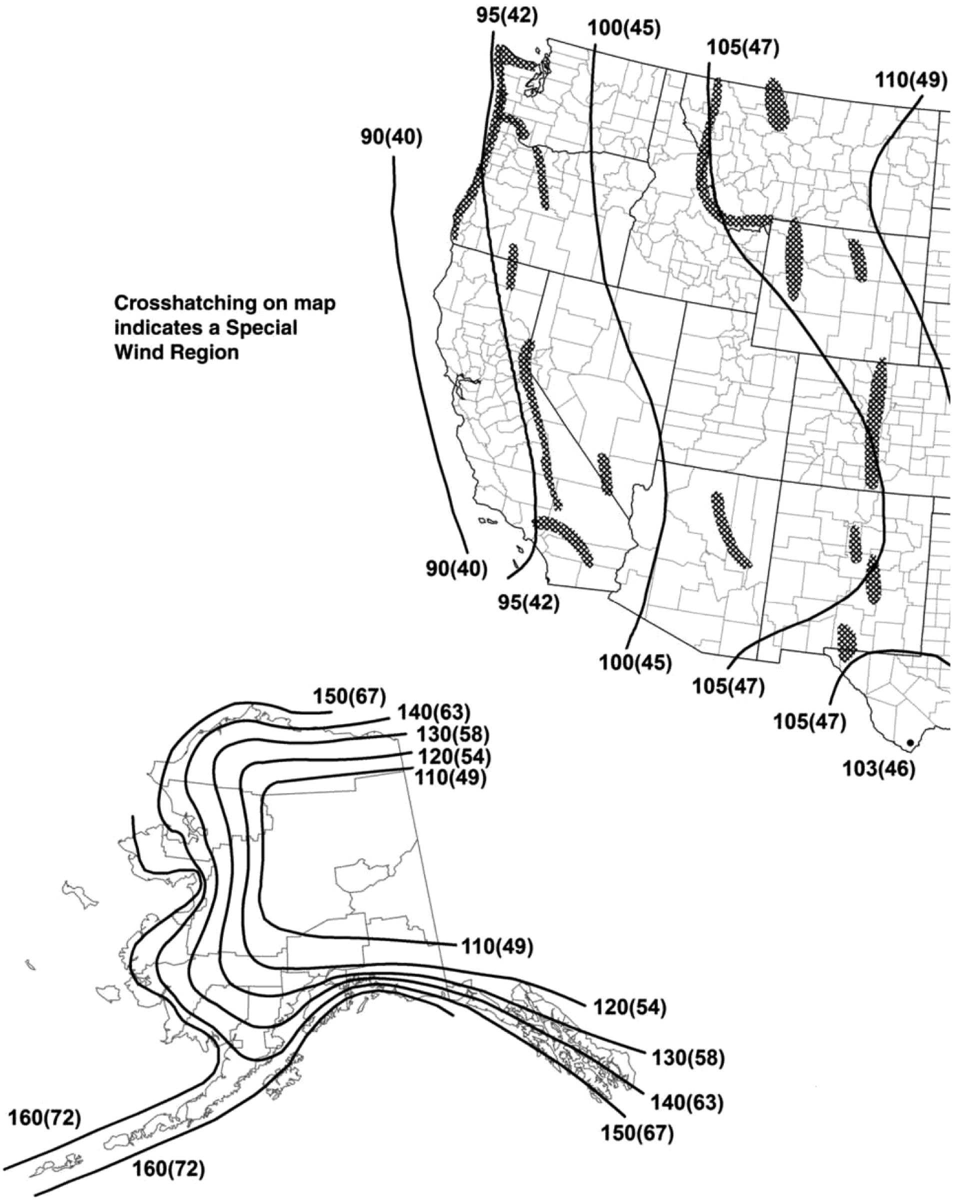




Location	Vmph	(m/s)
Guam	180	(80)
Virgin Islands	150	(67)
American Samoa	150	(67)
Hawaii	(See Figure 26.5-2A)	

(b)

FIGURE 4.1 (*Continued*) Basic wind speeds for risk category I buildings and other structures.

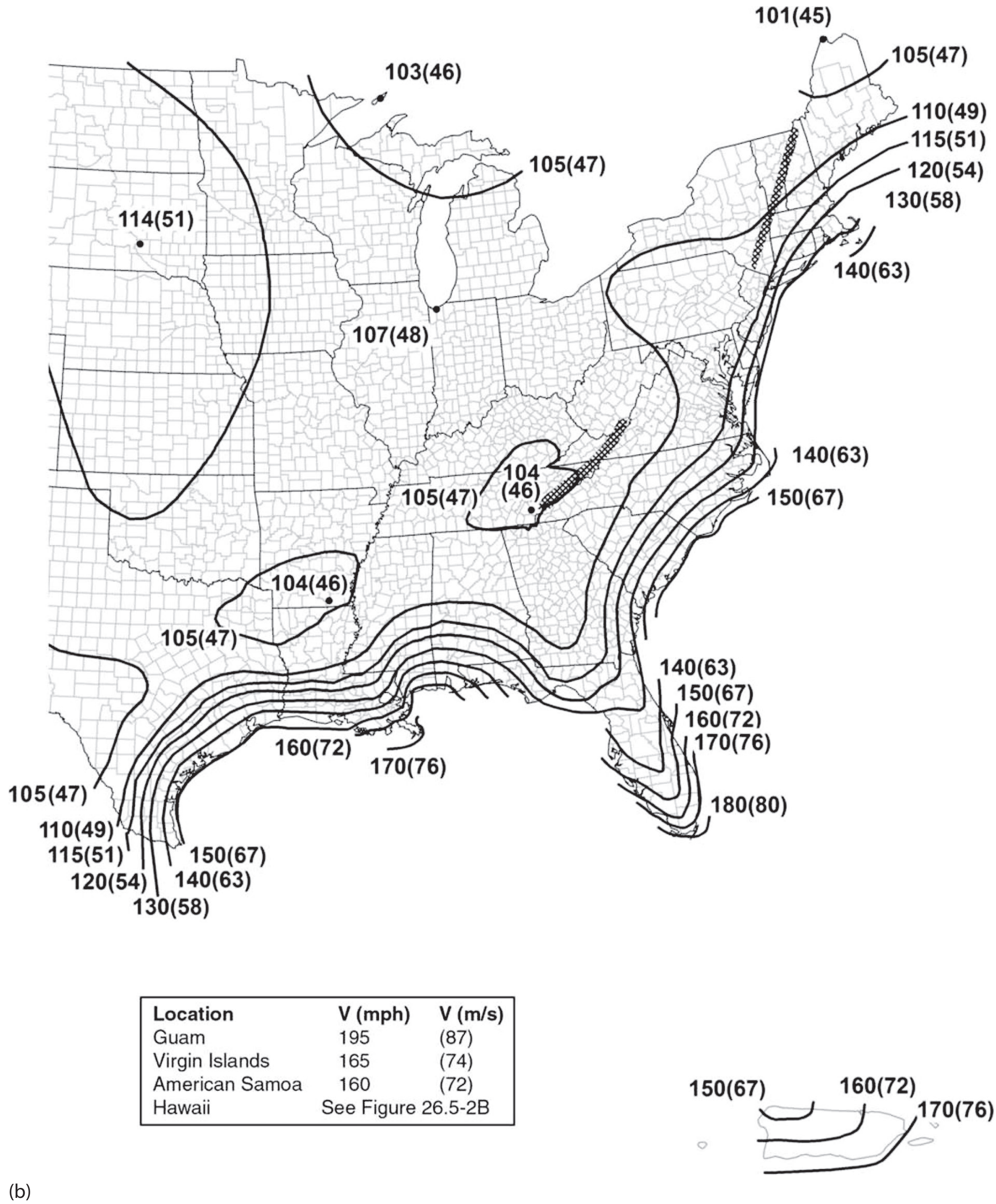


Notes

1. Values are nominal design 3-s gust wind speeds in mi/h (m/s) at 33 ft (10 m) above ground for Exposure Category C.
2. Linear interpolation is permitted between contours. Point values are provided to aid with interpolation.
3. Islands, coastal areas, and land boundaries outside the last contour shall use the last wind speed contour.
4. Mountainous terrain, gorges, ocean promontories, and special wind regions shall be examined for unusual wind conditions.
5. Wind speeds correspond to approximately a 7% probability of exceedance in 50 years (Annual Exceedance Probability = 0.00143, MRI = 700 years).
6. Location-specific basic wind speeds shall be permitted to be determined using www.atcouncil.org/windspeed.

(a)

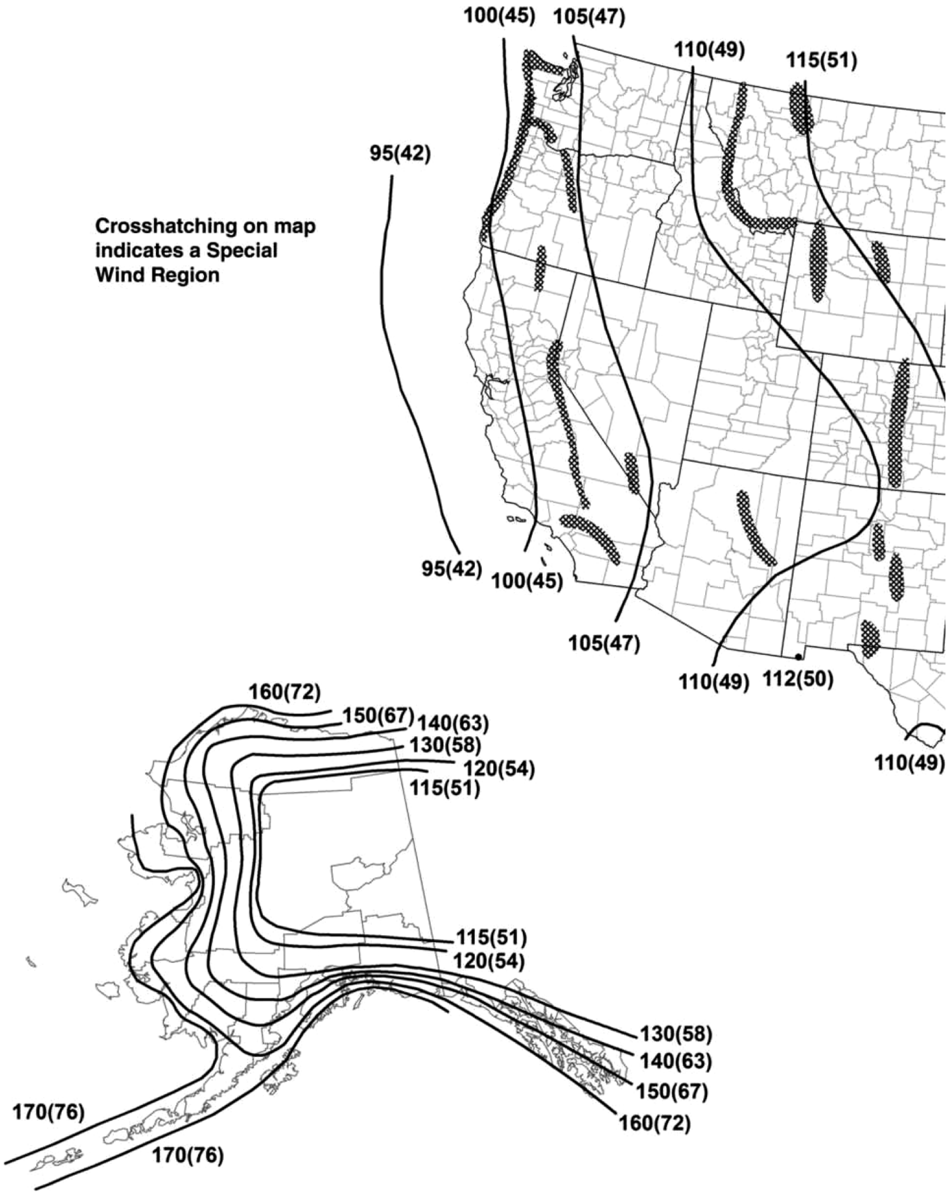
FIGURE 4.2 Basic wind speeds for risk category II buildings and other structures. (*Continued*)



Location	V (mph)	V (m/s)
Guam	195	(87)
Virgin Islands	165	(74)
American Samoa	160	(72)
Hawaii	See Figure 26.5-2B	

(b)

FIGURE 4.2 (*Continued*) Basic wind speeds for risk category II buildings and other structures.

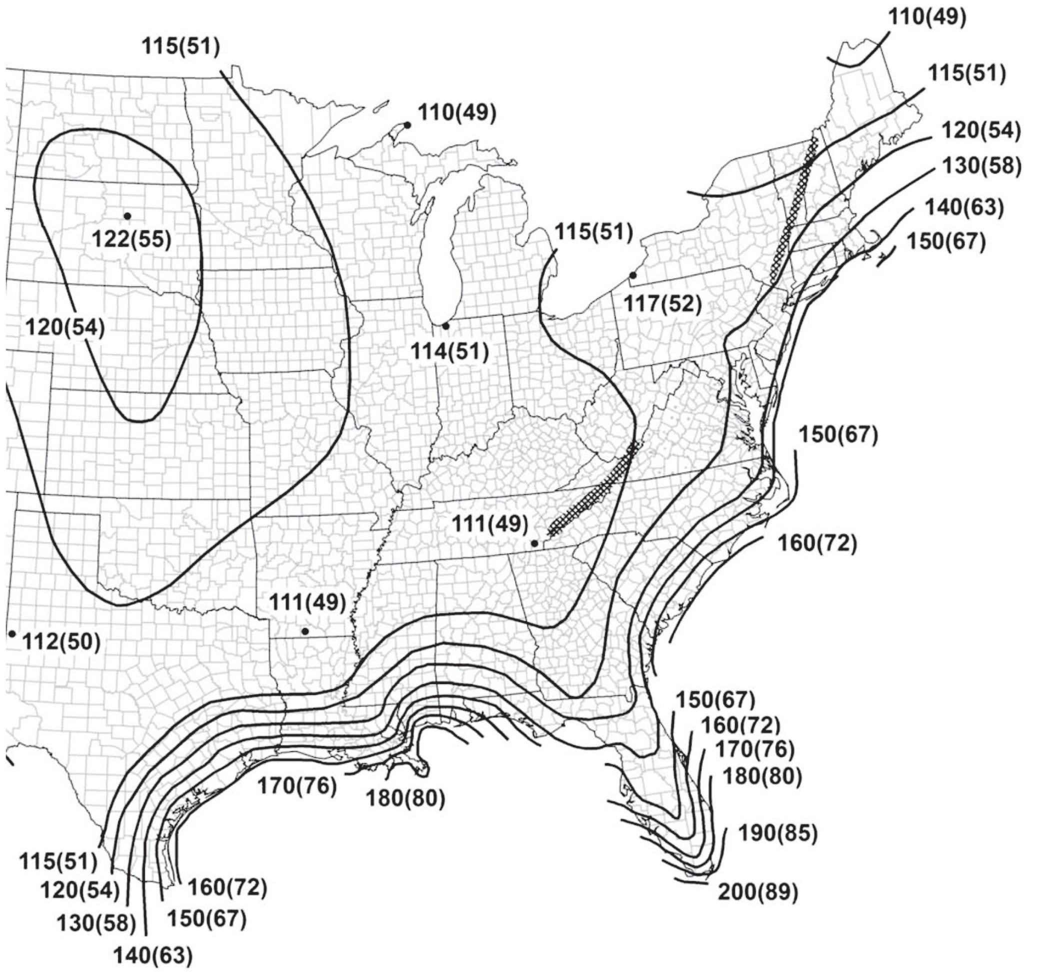


Notes

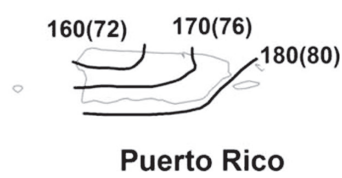
1. Values are nominal design 3-s gust wind speeds in mi/h (m/s) at 33 ft (10 m) above ground for Exposure Category C.
2. Linear interpolation is permitted between contours. Point values are provided to aid with interpolation.
3. Islands, coastal areas, and land boundaries outside the last contour shall use the last wind speed contour.
4. Mountainous terrain, gorges, ocean promontories, and special wind regions shall be examined for unusual wind conditions.
5. Wind speeds correspond to approximately a 3% probability of exceedance in 50 years (Annual Exceedance Probability = 0.000588, MRI = 1,700 years).
6. Location-specific basic wind speeds shall be permitted to be determined using www.atcouncil.org/windspeed.

(a)

FIGURE 4.3 Basic wind speeds for risk category III buildings and other structures. (*Continued*)

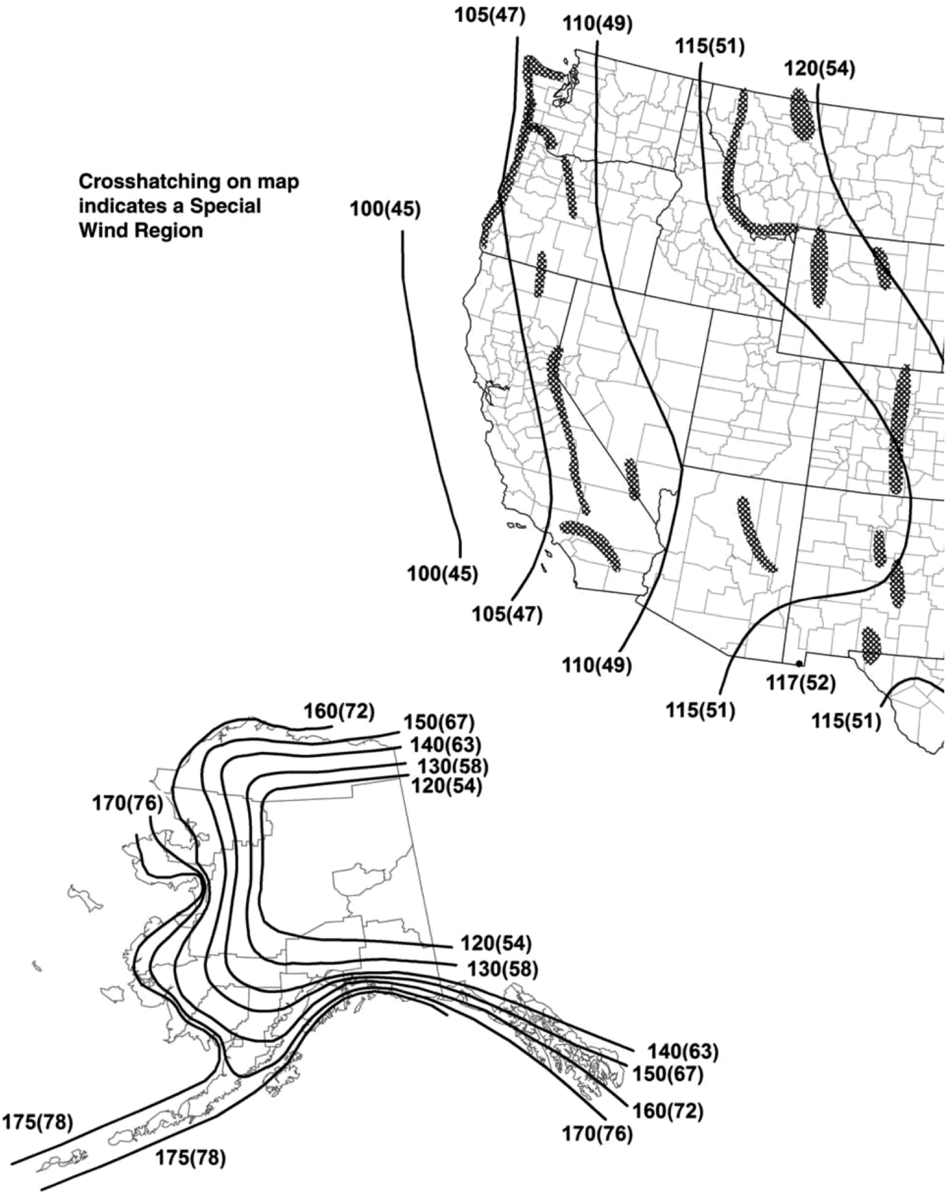


Location	V (mph)	V (m/s)
Guam	210	(94)
Virgin Islands	175	(78)
American Samoa	170	(76)
Hawaii	See Figure 26.5-2C	



(b)

FIGURE 4.3 (*Continued*) Basic wind speeds for risk category III buildings and other structures.

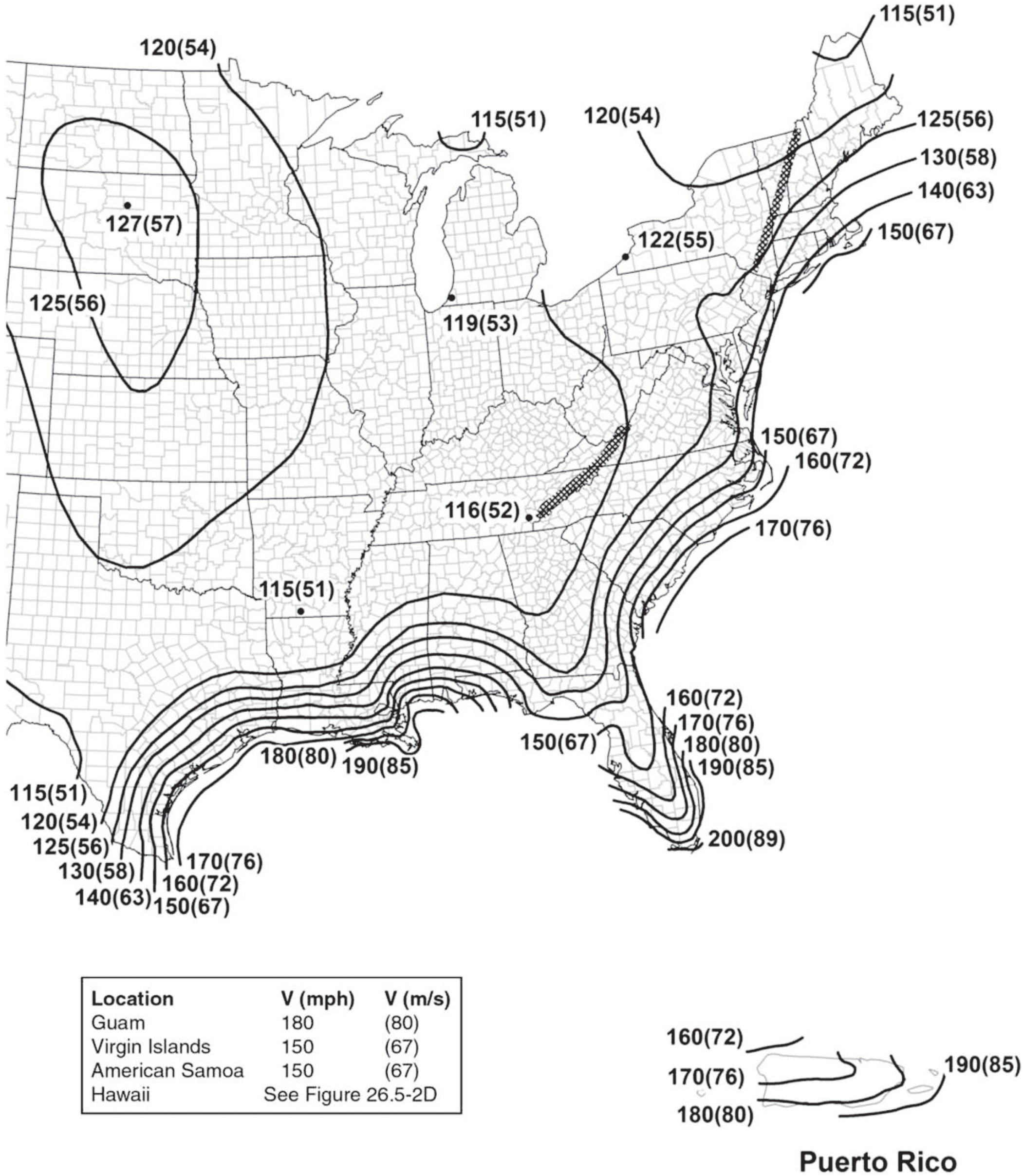


Notes

1. Values are nominal design 3-s gust wind speeds in mi/h (m/s) at 33 ft (10 m) above ground for Exposure Category C.
2. Linear interpolation is permitted between contours. Point values are provided to aid with interpolation.
3. Islands, coastal areas, and land boundaries outside the last contour shall use the last wind speed contour.
4. Mountainous terrain, gorges, ocean promontories, and special wind regions shall be examined for unusual wind conditions.
5. Wind speeds correspond to approximately a 1.6% probability of exceedance in 50 years (Annual Exceedance Probability = 0.00033, MRI = 3,000 years).
6. Location-specific basic wind speeds shall be permitted to be determined using www.atcouncil.org/windspeed.

(a)

FIGURE 4.4 Basic wind speeds for risk category IV buildings and other structures. *(Continued)*



Location	V (mph)	V (m/s)
Guam	180	(80)
Virgin Islands	150	(67)
American Samoa	150	(67)
Hawaii	See Figure 26.5-2D	

(b)

FIGURE 4.4 (*Continued*) Basic wind speeds for risk category IV buildings and other structures.

TABLE 4.1
Exposure Category for Wind Load

Surface Roughness	Exposure Category
Urban and suburban areas, wooded areas, closely spaced dwellings	*B*
Scattered obstructions—flat open country, grasslands	*C*
Flat unobstructed areas, smooth mud and salt flats, water surfaces	*D*

TABLE 4.2
Adjustment Factor for Height and Exposure

	Exposure		
Mean Roof Height (ft)	*B*	*C*	*D*
15	1.00	1.21	1.47
20	1.00	1.29	1.55
25	1.00	1.35	1.61
30	1.00	1.40	1.66
35	1.05	1.45	1.70
40	1.09	1.49	1.74
45	1.12	1.53	1.78
50	1.16	1.56	1.81
55	1.19	1.59	1.84
60	1.22	1.62	1.87

Vertical Pressure Zones for MWFRS

The vertical pressures on the roof are likewise separated into the following four zones, as shown in Figure 4.6.

E: End zone of (horizontal projection) windward roof
F: End zone of (horizontal projection) leeward roof
G: Interior zone of (horizontal projection) windward roof
H: Interior zone of (horizontal projection) leeward roof

When zones E and G fall on a roof overhang, the pressure values under the columns E_{OH} and G_{OH} in Table 4.3 are used for the windward side. For the leeward side, the basic values are used.

The dimension of end zones E and F is taken to be the horizontal distance from edge to ridge and equal to $2a$ in the other direction, as shown in Figure 4.6 for both Case A, transverse direction, and Case B, longitudinal direction. For longitudinal wind direction, roof angle = 0 is used.

Minimum Pressure for MWFRS

The minimum wind load computed for MWFRS is based on pressures of 16 psf for zones A and C and pressures of 8 psf for zones B and D, while assuming the pressures for zones E, F, G, and H are equal to zero.

TABLE 4.3
Simplified Design Wind Pressure, p_{s30} (psf) for Exposure *B* at h = 30 ft (h = 9.1 m)

			Zones									
			Horizontal Pressures				Vertical Pressures				Overhangs	
Basic Wind Speed (mph)	Roof Angle (degrees)	Load Case	A	B	C	D	E	F	G	H	E_{OH}	G_{OH}
85	0–5°	1	11.5	−5.9	7.6	−3.5	−13.8	−7.8	−9.6	−6.1	−19.3	−15.1
	10°	1	12.9	−5.4	8.6	−3.1	−13.8	−8.4	−9.6	−6.5	−19.3	−15.1
	15°	1	14.4	−4.8	9.6	−2.7	−13.8	−9.0	−9.6	−6.9	−19.3	−15.1
	20°	1	15.9	−4.2	10.6	−2.3	−13.8	−9.6	−9.6	−7.3	−19.3	−15.1
	25°	1	14.4	2.3	10.4	2.4	−6.4	−8.7	−4.6	−7.0	−11.9	−10.1
		2	—	—	—	—	−2.4	−4.7	−0.7	−3.0	—	—
	30–45	1	12.9	8.8	10.2	7.0	1.0	−7.8	0.3	−6.7	−4.5	−5.2
		2	12.9	8.8	10.2	7.0	5.0	−3.9	4.3	−2.8	−4.5	−5.2
90	0–5°	1	12.8	−6.7	8.5	−4.0	−15.4	−8.8	−10.7	−6.8	−21.6	−16.9
	10°	1	14.5	−6.0	9.6	−3.5	−15.4	−9.4	−10.7	−7.2	−21.6	−16.9
	15°	1	16.1	−5.4	10.7	−3.0	−15.4	−10.1	−10.7	−7.7	−21.6	−16.9
	20°	1	17.8	−4.7	11.9	−2.6	−15.4	−10.7	−10.7	−8.1	−21.6	−16.9
	25°	1	16.1	2.6	11.7	2.7	−7.2	−9.8	−5.2	−7.8	−13.3	−11.4
		2	—	—	—	—	−2.7	−5.3	−0.7	−3.4	—	—
	30–45	1	14.4	9.9	11.5	7.9	1.1	−8.8	0.4	−7.5	−5.1	−5.8
		2	14.4	9.9	11.5	7.9	5.6	−4.3	4.8	−3.1	−5.1	−5.8
95	0–5°	1	14.3	−7.4	9.5	−4.4	−17.2	−9.8	−12.0	−7.6	−24.1	−18.8
	10°	1	16.1	−6.7	10.7	−3.9	17.2	−10.5	−12.0	−8.1	−24.1	−18.8
	15°	1	18.0	−6.0	12.0	−3.4	−17.2	−11.2	−12.0	−8.6	−24.1	−18.8
	20°	1	19.8	−5.2	13.2	−2.9	−17.2	−12.0	−12.0	−9.1	−24.1	−18.8
	25°	1	18.0	2.9	13.0	3.0	−8.0	−10.9	−5.8	−8.7	−14.9	−12.7
		2	—	—	—	—	−3.0	−5.9	−0.8	−3.8	—	—
	30–45	1	16.1	11.0	12.8	8.8	1.2	−9.8	0.4	−8.4	−5.6	−6.5
		2	16.1	11.0	12.8	8.8	6.2	−4.8	5.4	−3.4	−5.6	−6.5
100	0–5°	1	15.9	−8.2	10.5	−4.9	−19.1	−10.8	−13.3	−8.4	−26.7	−20.9
	10°	1	17.9	−7.4	11.9	−4.3	−19.1	−11 6	−13.3	−8.9	−26.7	−20.9
	15°	1	19.9	−6.6	13.3	−3.8	−19.1	−12.4	−13.3	−9.5	−26.7	−20.9
	20°	1	22.0	−5.8	14.6	−3.2	−19.1	−13.3	−13.3	−10.1	−26.7	−20.9
	25°	1	19.9	3.2	14.4	3.3	−8.8	−12.0	−6.4	−9.7	−16.5	−14.0
		2	—	—	—	—	−3.4	−6.6	−0.9	−4.2	—	—
	30–45	1	17.8	12.2	14.2	9.8	1.4	−10.8	0.5	−9.3	6.3	−7.2
		2	17.8	12.2	14.2	9.8	6.9	−5.3	5.9	−3.8	−6.3	−7.2

(*Continued*)

TABLE 4.3 (*Continued*)
Simplified Design Wind Pressure, p_{s30} (psf) for Exposure *B* at *h* = 30 ft (*h* = 9.1 m)

Basic Wind Speed (mph)	Roof Angle (degrees)	Load Case	Zones									
			Horizontal Pressures				Vertical Pressures				Overhangs	
			A	B	C	D	E	F	G	H	E_{OH}	G_{OH}
105	0–5°	1	17.5	−9.1	11.6	−5.4	−21.0	−11.9	−14.6	−9.2	−29.4	−23.0
	10°	1	19.7	−8.2	13.1	−4.8	−21.0	−12.8	−14.6	−9.9	−29.4	−23.0
	15°	1	22.0	−7.3	14.6	−4.1	−21.0	−13.7	−14.6	−10.5	−29.4	−23.0
	20°	1	24.2	−6.4	16.1	−3.5	−21.0	−14.6	−14.6	−11.1	−29.4	−23.0
	25°	1	21.9	3.5	15.9	3.6	−9.7	−13.3	−7.1	−10.7	−18.2	−15.5
		2	—	—	—	—	−3.7	−7.2	−1.0	−4.6	—	—
	30–45	1	19.7	13.4	15.6	10.8	1.5	−11.9	0.5	−10.3	−6.9	−7.9
		2	19.7	13.4	15.6	10.8	7.6	−5.9	6.6	−4.2	−6.9	−7.9
110	0–5°	1	19.2	−10.0	12.7	−5.9	−23.1	−13.1	−16.0	−10.1	−32.3	−25.3
	10°	1	21.6	−9.0	14.4	−5.2	−23.1	−14.1	−16.0	−10.8	−32.3	−25.3
	15°	1	24.1	−8.0	16.0	−4.6	−23.1	−15.1	−16.0	−11.5	−32.3	−25.3
	20°	1	26.6	−7.0	17.7	−3.9	−23.1	−16.0	−16.0	−12.2	−32.3	−25.3
	25°	1	24.1	3.9	17.4	4.0	−10.7	−14.6	−7.7	−11.7	−19.9	−17.0
		2	—	—	—	—	−4.1	−7.9	−1.1	−5.1	—	—
	30–45	1	21.6	14.8	17.2	11.8	1.7	−13.1	0.6	−11.3	−7.6	−8.7
		2	21.6	14.8	17.2	11.8	8.3	−6.5	7.2	−4.6	−7.6	−8.7
115	0–5°	1	21.0	−10.9	13.9	−6.5	−25.2	−14.3	−17.5	−11.1	−35.3	−27.6
	10°	1	23.7	−9.8	15.7	−5.7	−25.2	−15.4	−17.5	−11.8	−35.3	−27.6
	15°	1	26.3	−8.7	17.5	−5.0	−25.2	−16.5	−17.5	−12.6	−35.3	−27.6
	20°	1	29.0	−7.7	19.4	−4.2	−25.2	−17.5	−17.5	−13.3	−35.3	−27.6
	25°	1	26.3	4.2	19.1	4.3	−11.7	−15.9	−8.5	−12.8	−21.8	−18.5
		2	—	—	—	—	−4.4	−8.7	−1.2	−5.5	—	—
	30–45	1	23.6	16.1	18.8	12.9	1.8	−14.3	0.6	−12.3	−8.3	−9.5
		2	23.6	16.1	18.8	12.9	9.1	−7.1	7.9	−5.0	−8.3	−9.5
120	0–5	1	22.8	−11.9	15.1	−7.0	−27.4	−15.6	−19.1	−12.1	−38.4	−30.1
	10°	1	25.8	−10.7	17.1	−6.2	−27.4	−16.8	−19.1	−12.9	−38.4	−30.1
	15°	1	28.7	−9.5	19.1	−5.4	−27.4	−17.9	−19.1	−13.7	−38.4	−30.1
	20°	1	31.6	−8.3	21.1	−4.6	−27.4	−19.1	−19.1	−14.5	−38.4	−30.1
	25°	1	28.6	4.6	20.7	4.7	−12.7	−17.3	−9.2	−13.9	−23.7	−20.2
		2	—	—	—	—	−4.8	−9.4	−1.3	−6.0	—	—
	30–45	1	25.7	17.6	20.4	14.0	2.0	−15.6	0.7	−13.4	−9.0	−10.3
		2	25.7	17.6	20.4	14.0	9.9	−7.7	8.6	−5.5	−9.0	−10.3
125	0–5°	1	24.8	−12.9	16.4	−7.6	−29.8	−16.9	−20.7	−13.1	−41.7	−32.6
	10°	1	27.9	−11.6	18.6	−6.7	−29.8	−18.2	−20.7	−14.0	−41.7	−32.6
	15°	1	31.1	−10.3	20.7	−5.9	−29.8	−19.5	−20.7	−14.8	−41.7	−32.6
	20°	1	34.3	−9.1	22.9	−5.0	−29.8	−20.7	−20.7	−15.7	−41.7	−32.6
	25°	1	31.1	5.0	22.5	5.1	−13.8	−18.8	−10.0	−15.1	−25.7	−21.9
		2	—	—	—	—	−5.2	−10.2	−1.4	−6.6	—	—
	30–45	1	27.9	19.1	22.2	15.2	2.1	−16.9	0.7	−14.5	−9.8	−11.2
		2	27.9	19.1	22.2	15.2	10.7	−8.3	9.3	−6.0	−9.8	−11.2

(*Continued*)

TABLE 4.3 (*Continued*)
Simplified Design Wind Pressure, p_{s30} (psf) for Exposure *B* at h = 30 ft (h = 9.1 m)

			Zones									
			Horizontal Pressures				Vertical Pressures				Overhangs	
Basic Wind Speed (mph)	Roof Angle (degrees)	Load Case	A	B	C	D	E	F	G	H	E_{OH}	G_{OH}
130	0–5°	1	26.8	−13.9	17.8	−8.2	−32.2	−18.3	−22.4	−14.2	−45.1	−35.3
	10°	1	30.2	−12.5	20.1	−7.3	−32.2	−19.7	−22.4	−15.1	−45.1	−35.3
	15°	1	33.7	−11.2	22.4	−6.4	−32.2	−21.0	−22.4	−16.1	−45.1	−35.3
	20°	1	37.1	−9.8	24.7	−5.4	−32.2	−22.4	−22.4	−17.0	−45.1	−35.3
	25°	1	33.6	5.4	24.3	5.5	−14.9	−20.4	−10.8	−16.4	−27.8	−23.7
		2	—	—	—	—	−5.7	−11.1	−1.5	−7.1	—	—
	30–45	1	30.1	20.6	24.0	16.5	2.3	−18.3	0.8	−15.7	−10.6	−12.1
		2	30.1	20.6	24.0	16.5	11.6	−9.0	10.0	−6.4	−10.6	−12.1
140	0–5°	1	31.1	−16.1	20.6	−9.6	−37.3	−21.2	−26.0	−16.4	−52.3	−40.9
	10°	1	35.1	−14.5	23.3	−8.5	−37.3	−22.8	−26.0	−17.5	−52.3	−40.9
	15°	1	39.0	−12.9	26.0	−7.4	−37.3	−24.4	−26.0	−18.6	−52.3	−40.9
	20°	1	43.0	−11.4	28.7	−6.3	−37.3	−26.0	−26.0	−19.7	−52.3	−40.9
	25°	1	39.0	6.3	28.2	6.4	−17.3	−23.6	−12.5	−19.0	−32.3	−27.5
		2	—	—	—	—	−6.6	−12.8	−1.8	−8.2	—	—
	30–45	1	35.0	23.9	27.8	19.1	2.7	−21.2	0.9	−18.2	−12.3	−14.0
		2	35.0	23.9	27.8	19.1	13.4	−10.5	11.7	−7.5	−12.3	−14.0
150	0–5°	1	35.7	−18.5	23.7	−11.0	−42.9	−24.4	−29.8	−18.9	−60.0	−47.0
	10°	1	40.2	−16.7	26.8	−9.7	−42.9	−26.2	−29.8	−20.1	−60.0	−47.0
	15°	1	44.8	−14.9	29.8	−8.5	−42.9	−28.0	−29.8	−21.4	−60.0	−47.0
	20°	1	49.4	−13.0	32.9	−7.2	−42.9	−29.8	−29.8	−22.6	−60.0	−47.0
	25°	1	44.8	7.2	32.4	7.4	−19.9	−27.1	−14.4	−21.8	−37.0	−31.6
		2	—	—	—	—	−7.5	−14.7	−2.1	−9.4	—	—
	30–45	1	40.1	27.4	31.9	22.0	3.1	−24.4	1.0	−20.9	−14.1	−16.1
		2	40.1	27.4	31.9	22.0	15.4	−12.0	13.4	−8.6	−14.1	−16.1
160	0–5°	1	40.6	−21.1	26.9	−12.5	−48.8	−27.7	−34.0	−21.5	−68.3	−53.5
	10°	1	45.8	−19.0	30.4	−11.1	−48.8	−29.8	−34.0	−22.9	−68.3	−53.5
	15°	1	51.0	−16.9	34.0	−9.6	−48.8	−31.9	−34.0	−24.3	−68.3	−53.5
	20°	1	56.2	−14.8	37.5	−8.2	−48.8	−34.0	−34.0	−25.8	−68.3	−53.5
	25°	1	50.9	8.2	36.9	8.4	−22.6	−30.8	−16.4	−24.8	−42.1	−35.9
		2[a]	—	—	—	—	−8.6	−16.8	−2.3	−10.7	—	—
	30–45	1	45.7	31.2	36.3	25.0	3.5	−27.7	1.2	−23.8	−16.0	−18.3
		2[a]	45.7	31.2	36.3	25.0	17.6	−13.7	15.2	−9.8	−16.0	−18.3
170	0–5°	1	45.8	−23.8	30.4	−14.1	−55.1	−31.3	−38.3	−24.2	−77.1	−60.4
	10°	1	51.7	−21.4	34.4	−12.5	−55.1	−33.6	−38.3	−25.8	−77.1	−60.4
	15°	1	57.6	−19.1	38.3	−10.9	−55.1	−36.0	−38.3	−27.5	−77.1	−60.4
	20°	1	63.4	−16.7	42.3	−9.3	−55.1	−38.3	−38.3	−29.1	−77.1	−60.4
	25°	1	57.5	9.3	41.6	9.5	−25.6	−34.8	−18.5	−28.0	−47.6	−40.5
		2[a]	—	—	—	—	−9.7	−18.9	−2.6	−12.1	—	
	30–45	1	51.5	35.2	41.0	28.2	4.0	−31.3	1.3	−26.9	−18.1	−20.7
		2[a]	51.5	35.2	41.0	28.2	19.8	−15.4	17.2	−11.0	−18.1	−20.7

(*Continued*)

TABLE 4.3 (*Continued*)
Simplified Design Wind Pressure, p_{s30} (psf) for Exposure *B* at h = 30 ft (h = 9.1 m)

			Zones									
			Horizontal Pressures				Vertical Pressures				Overhangs	
Basic Wind Speed (mph)	Roof Angle (degrees)	Load Case	A	B	C	D	E	F	G	H	E_{OH}	G_{OH}
180	0–5°	1	51.4	−26.7	34.1	−15.8	−61.7	−35.1	−43.0	−27.2	−86.4	−67.7
	10°	1	58.0	−24.0	38.5	−14.0	−61.7	−37.7	−43.0	−29.0	−86.4	−67.7
	15°	1	64.5	−21.4	43.0	−12.2	−61.7	−40.3	−43.0	−30.8	−86.4	−67.7
	20°	1	71.1	−18.8	47.4	−10.4	−61.7	−43.0	−43.0	−32.6	−86.4	−67.7
	25°	1	64.5	10.4	46.7	10.6	−28.6	−39.0	−20.7	−31.4	−53.3	−45.4
		2[a]	—		—	—	−10.9	−21.2	−3.0	−13.6	—	—
	30–45	1	57.8	39.5	45.9	31.6	4.4	−35.1	1.5	−30.1	−20.3	−23.2
		2[a]	57.8	39.5	45.9	31.6	22.2	−17.3	19.3	−12.3	−20.3	−23.2
190	0–5°	1	57.2	−29.7	38.0	−17.6	−68.8	−39.1	−47.9	−30.3	−96.3	−75.4
	10°	1	64.6	−26.8	42.9	−15.6	−68.8	−42.0	−47.9	−32.3	−96.3	−75.4
	15°	1	71.9	−23.8	47.9	−13.6	−68.8	−44.9	−47.9	−34.3	−96.3	−75.4
	20°	1	79.2	−20.9	52.8	−11.6	−68.8	−47.9	−47.9	−36.3	−96.3	−75.4
	25°	1	71.8	11.6	52.0	11.8	−31.9	−43.5	−23.1	−34.9	−59.4	−50.6
		2[a]	—	—	—	—	−12.1	−23.7	−3.3	−15.1	—	—
	30–45	1	64.4	44.0	51.2	35.2	5.0	−39.1	1.7	−33.6	−22.6	−25.9
		2[a]	64.4	44.0	51.2	35.2	24.8	−19.3	21.5	−13.8	−22.6	−25.9
200	0–5°	1	63.4	−32.9	42.1	−19.5	−76.2	−43.3	−53.1	−33.5	−106.7	−83.5
	10°	1	71.5	−29.7	47.6	−17.3	−76.2	−46.5	−53.1	−35.8	−106.7	−83.5
	15°	1	79.7	−26.4	53.1	−15.0	−76.2	−49.8	−53.1	−38.0	−106.7	−83.5
	20°	1	87.8	−23.2	58.5	−12.8	−76.2	−53.1	−53.1	−40.2	−106.7	−83.5
	25°	1	79.6	12.8	57.6	13.1	−35.4	−48.2	−25.6	−38.7	−65.9	−56.1
		2[a]	—	—	—	—	−13.4	−26.2	−3.7	−16.8	—	—
	30–45	1	71.3	48.8	56.7	39.0	5.5	−43.3	1.8	−37.2	−25.0	−28.7
		2[a]	71.3	48.8	56.7	39.0	27.4	−21.3	23.8	−15.2	−25.0	−28.7

[a] Load cases 1 and 2 must be checked for roof angle greater than 25 and less or equal to 45 degree. Load case 2 at 25 degree provided only for interpolation between 25 and 30 degrees.

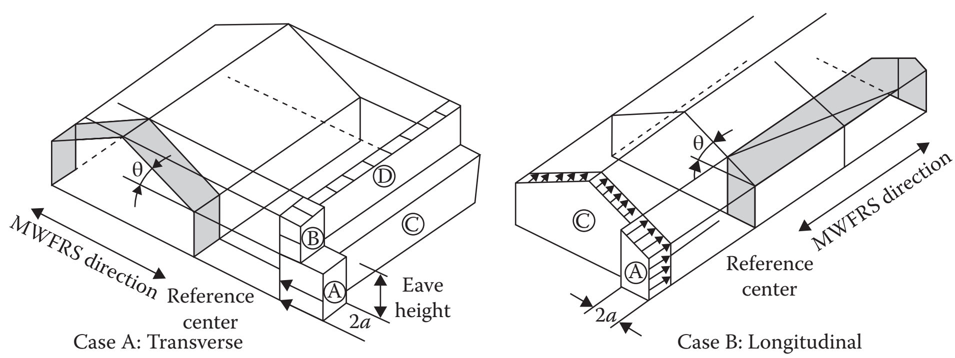


FIGURE 4.5 Horizontal pressure zones.

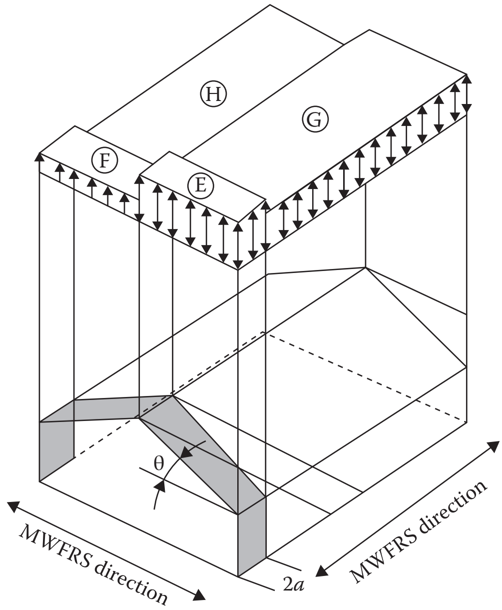


FIGURE 4.6 Vertical pressure zones.

Example 4.1

A two-story essential facility shown in Figure 4.7 is an enclosed wood-frame building located in Colorado. Determine the design wind pressures for the MWFRS in both principal directions of the building and the forces acting on the transverse section of the building. The wall studs and roof rafters are 16 in. on center, and $K_{zt} = 1.0$.

Solution

I. Design parameters
   1. Roof slope, $\theta = 14°$
   2. $h_{mean} = 22 + \dfrac{6.25}{2} = 25.13\,\text{ft}$
   3. End zone dimension, a, smaller than
      a. 0.4 of the $h_{mean} = 0.4(25.13) = 10$ ft
      b. 0.1 of the width = 0.1(50) = 5 ft ← controls
   4. Length of end zone = $2a$ = 10 ft

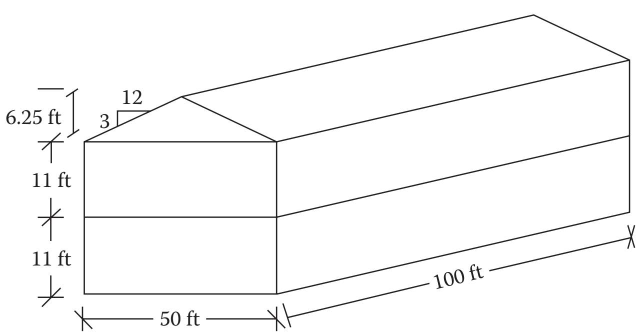


FIGURE 4.7 Two-story framed building.

5. Basic wind speed, V for category IV = 115 mph
6. Exposure category = B
7. λ from Table 4.3 up to 30 ft = 1.0
8. K_{zt} = 1.00 (given)
9. $p_s = \lambda K_{zt}\, p_{s30} = (1)(1)p_{s30} = p_{s30}$

II. **Case A:** For transverse wind direction

A.1 Horizontal Wind Pressure on the Wall and Roof Projection

	Pressure (psf) (Table 4.3)			
Zone	**Roof Angle = 10°**	**Roof Angle = 15°**	**Interpolated for 14°**	**$p_s = p_{s30}$ (psf)**
A: End zone wall	23.7	26.3	25.78	25.78
B: End zone roof	−9.8	−8.7	−8.92	−8.92
C: Interior wall	15.7	17.5	17.14	17.14
D: Interior roof	−5.7	−5.0	−5.14	−5.14

Note: These pressures are shown in the section view in Figure 4.8a.

A.2 Horizontal Force at the Roof Level

		Tributary				
Location	**Zone**	**Height (ft)**	**Width (ft)**	**Area (ft²)**	**Pressure (psf)**	**Load (lb)**
End	A	11[a]	$2a = 10$	110	25.78	2,836
	B	6.25	10	62.5	−8.92 → 0	0
Interior	C	11	$L–2a = 90$	990	17.14	16,969
	D	6.25	90	562.5	−5.14 → 0	0
Total						19,805

Note: Pressures in zones B and D are assumed to be zero.

[a] It is also a practice to take one-half of the floor height for each level. In such a case, the wind force on one-half of the first-floor height from the ground is not applied.

A.3 Horizontal Force at the Second-Floor Level (Top of First Floor)

		Tributary				
Location	**Zone**	**Height (ft)**	**Width (ft)**	**Area (ft²)**	**Pressure (psf)**	**Load (lb)**
End	A	11	10	110	25.78	2,836
Interior	C	11	90	990	17.14	16,969
Total						19,805

Note: Total horizontal force is 39,610. The application of the forces is shown in Figure 4.8b.

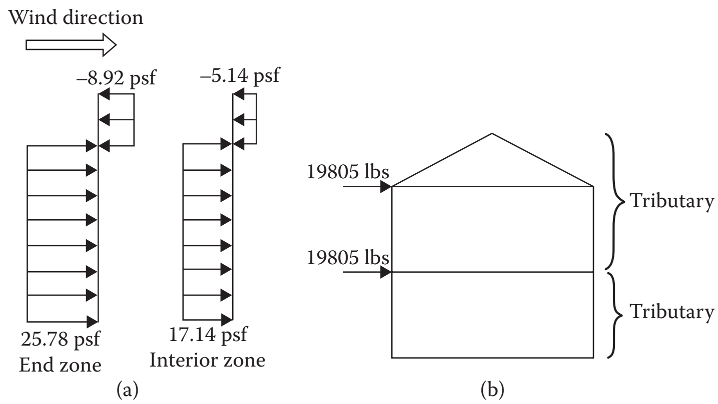


FIGURE 4.8 (a) Horizontal pressure distribution and (b) Horizontal force: transverse wind.

B.1 Vertical Wind Pressure on the Roof

	Pressure (psf) (Table 4.3)			
Zone	**Roof Angle = 10°**	**Roof Angle = 15°**	**Interpolated to 14°**	$p_s = p_{s30}$ **(psf)**
E: End, windward	−25.2	−25.2	−25.2	−25.2
F: End, leeward	−15.4	−16.5	−16.28	−16.28
G: Interior, windward	−17.5	−17.5	−17.5	−17.5
H: Interior, leeward	−11.6	−12.6	−12.4	−12.4

Note: The pressures are shown in the sectional view in Figure 4.9a.

B.2 Vertical Force on the Roof

		Tributary				
Zone		**Length (ft)**	**Width (ft)**	**Area (ft²)**	**Pressure (psf)**	**Load (lb)**
Windward	E: End	25	$2a = 10$	250	−25.2	−6,300
	G: Interior	25	$L–2a = 90$	2250	−17.5	−39,375
	Total					−45,675
Leeward	F: End	25	10	250	−16.28	−4,070
	H: Interior	25	90	2250	−12.4	−27,900
	Total					−31,970

Note: The application of vertical forces is shown in Figure 4.9b.

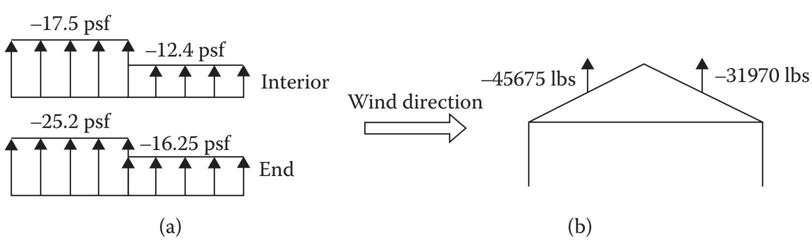


FIGURE 4.9 (a) Vertical pressure distribution on roof and (b) Vertical force on roof: transverse wind.

C. Minimum force on the MWFRS by transverse wind
The minimum pressure is 16 psf acting on the vertical projection of the wall and 8 psf on the vertical projection of the roof. Thus,
Minimum wind force = [16(22) + 8(6.25)] × 100 = 40,200 lb

D. Applicable wind force
The following two cases should be considered for maximum effect:
1. The combined forces of A.2, A.3, and B.2
2. Minimum force C

III. **Case B:** For longitudinal wind direction

A.1 Horizontal wind pressures on the wall
Zones B and D do not exist. Using θ = 0, pressure on zone A = 21.0 psf and pressure on zone C = 13.9 psf from Table 4.3.

A.2 Horizontal force at the roof level
From Figure 4.10:

$$\text{Tributary area for end zone A} = \frac{1}{2}(11 + 13.5)(10) = 122.5\ \text{ft}^2$$

$$\text{Tributary area for interior zone C} = \frac{1}{2}(13.5 + 17.25)(15) + \frac{1}{2}(17.25 + 11)(25)$$
$$= 230.63 + 353.12 = 583.75\ \text{ft}^2$$

Zone	Tributary Area (ft^2)	Pressure (psf)	Load[a] (lb)
A	122.5	21.0	2,573
C	583.75	13.9	8,114
Total			10,687

[a] The centroids of area are different, but the force is assumed to be acting at roof level.

A.3 Horizontal force at the second floor level

$$\text{Tributary area for end zone A} = 11 \times 10 = 110\ \text{ft}^2$$

$$\text{Tributary area for interior zone C} = 11 \times 40 = 440\ \text{ft}^2.$$

Zone	Tributary Area (ft^2)	Pressure (psf)	Load (lb)
A	110	21.0	2310
C	440	13.9	6116
Total			8426

Note: The application of forces is shown in the sectional view in Figure 4.10.

B.1 Vertical wind pressure on the roof (longitudinal case) use θ = 0

Zone	p_{s30} (psf)	$p_s = p_{s30}$
End E	−25.2	−25.2
End F	−14.3	−14.3
Interior G	−17.5	−17.5
Interior H	−11.1	−11.1

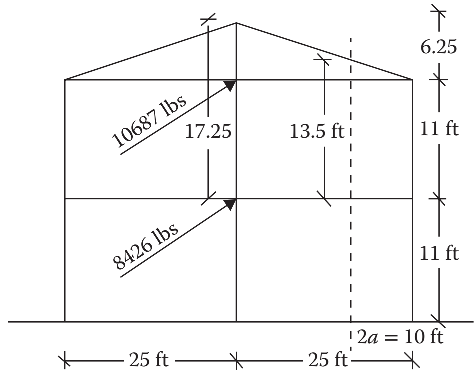


FIGURE 4.10 Horizontal force on wall and roof projection: longitudinal wind.

B.2 Vertical Force on the Roof

		Tributary				
Zone		**Length (ft)**	**Width (ft)**	**Area (ft²)**	**Pressure (psf)**	**Load (lb)**
End E	E	$2a$ = 10	$B/2$ = 25	250	−25.2	−6,300
	F	$2a$ = 10	25	250	−14.3	−9875
	Total					−9,875
Interior	G	L−$2a$ = 90	25	2250	−17.5	−39,375
	H	90	25	2250	−11.1	−24,975
	Total					−64,350

Note: The application of forces is shown in Figure 4.11.

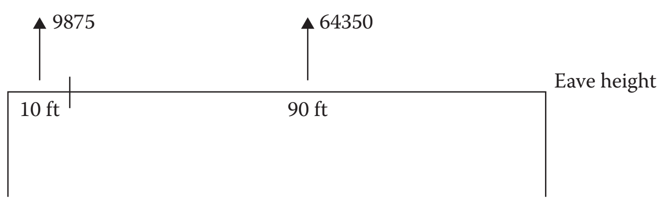


FIGURE 4.11 Vertical force on roof: Longitudinal wind.

PROCEDURES FOR COMPONENTS AND CLADDING

ASCE 7-16 stipulates that when the tributary area is greater than 700 ft², the C and C elements can be designed using the provisions of MWFRS. Chapter 30 of ASCE 7-16 specifies procedures for C and C; these are parallel to the procedures applicable to MWFRS. Two analytical procedures—one for high-rise buildings and one for low-rise buildings—use equations similar to the analytical procedures of MWFRS. Two simplified procedures—one for regular-shaped enclosed buildings of more than 60 ft and up to 160 ft in height and one for regular-shaped enclosed low-rise buildings of 60 ft or less—determine wind pressures directly from tables. ASCE 7-16 also covers C and C for open buildings and appurtenances.

SIMPLIFIED PROCEDURE FOR COMPONENTS AND CLADDING FOR LOW-RISE BUILDINGS

The C and C cover the individual structural elements that directly support a tributary area against the wind force. The conditions and the steps of the procedure are essentially similar to the MWFRS. The pressure acts normal, however, to each surface, that is, horizontal to the wall and perpendicular to the roof. The following similar equation is used to determine the wind pressure. The adjustment factor, λ, and the topographic factor, K_{zt}, are determined from the similar considerations as for MWFRS:

$$p_{net} = \lambda K_{zt}\, p_{net30} \tag{4.2}$$

where:

λ is the adjustment factor for structure height and exposure (Tables 4.1 and 4.2)
K_{zt} is the topographic factor
p_{net30} is the simplified standard design wind pressure (Table 4.4a–g)

However, the pressures p_{net30} are different from p_{s30}. Besides the basic wind speed, the pressures are a function of the roof angle, the effective wind area supported by the element, and the zone of the structure surface. The magnitude of p_{net} represents the net pressures, which are the algebraic summation of the internal and external pressures acting normal to the surface of the C and C.

The effective area is the tributary area of an element but need not be lesser than the span length multiplied by the width equal to one-third of the span length; that is, $A = L^2/3$.

Table 4.4, reproduced from ASCE 7-16, lists p_{net30} values for effective wind areas of 10 ft^2, 20 ft^2, 50 ft^2, and 100 ft^2 for the roof and wall. An element having an effective area in excess of 100 ft^2 should use pressures corresponding to an area of 100 ft^2. A linear interpolation is permitted for intermediate areas.

The zones shown in Figure 4.12 have been identified for the C and C.

The dimension a is smaller than the following two values:

1. 0.4 times the mean height to roof, h_{mean}
2. 0.1 times the smaller horizontal dimension

But the value of a should not be less than the following:

1. 0.04 times the smaller horizontal dimension
2. 3 ft

There are two values of the net pressure that act on each element: a positive pressure acting inward (toward the surface) and a negative pressure acting outward (away from the surface). The two pressures must be considered separately for each element.

Minimum Pressures for C and C

The positive pressure, p_{net}, should not be less than +16 psf and the negative pressure should not be less than −16 psf.

TABLE 4.4a
Components and Claddings, Walls and All Types of Roofs 0–7 Degrees, Wind Speed 95–130 mph, Net Design Wind Pressure, p_{net30}, in lb/ft², for Exposure B at h = 30 ft

	Zone	Effective Wind Area (ft²)	Basic Wind Speed (mph)													
			95		100		105		110		115		120		130	
Walls	4	10	16.2	−17.6	18.0	−19.5	19.8	−21.5	21.8	−23.6	23.8	−25.8	25.9	−28.1	30.4	−33.0
	4	20	15.5	−16.9	17.2	−18.7	18.9	−20.6	20.8	−22.6	22.7	−24.7	24.7	−26.9	29.0	−31.6
	4	50	14.5	−15.9	16.1	−17.6	17.8	−19.4	19.5	−21.3	21.3	−23.3	23.2	−25.4	27.2	−29.8
	4	100	13.8	−15.2	15.3	−16.8	16.9	−18.5	18.5	−20.4	20.2	−22.2	22.0	−24.2	25.9	−28.4
	5	10	16.2	−21.7	18.0	−24.1	19.8	−26.6	21.8	−29.1	23.8	−31.9	25.9	−34.7	30.4	−40.7
	5	20	15.5	−20.3	17.2	−22.5	18.9	−24.8	20.8	−27.2	22.7	−29.7	24.7	−32.4	29.0	−38.0
	5	50	14.5	−18.3	16.1	−20.3	17.8	−22.4	19.5	−24.6	21.3	−26.9	23.2	−29.3	27.2	−34.3
	5	100	13.8	−16.9	15.3	−18.7	16.9	−20.6	18.5	−22.6	20.2	−24.7	22.0	−26.9	25.9	−31.6
Flat/Hip/Gable Roof	1	10	6.6	−25.9	7.3	−28.7	8.1	−31.6	8.9	−34.7	9.7	−37.9	10.5	−41.3	12.4	−48.4
0–7 Degrees	1	20	6.2	−24.2	6.9	−26.8	7.6	−29.5	8.3	−32.4	9.1	−35.4	9.9	−38.5	11.6	−45.2
	1	50	5.6	−21.9	6.3	−24.3	6.9	−26.8	7.6	−29.4	8.3	−32.1	9.0	−34.9	10.6	−41.0
	1	100	5.2	−20.2	5.8	−22.4	6.4	−24.7	7.0	−27.1	7.7	−29.6	8.3	−32.2	9.8	−37.8
	1′	10	6.6	−14.9	7.3	−16.5	8.1	−18.2	8.9	−19.9	9.7	−21.8	10.5	−23.7	12.4	−27.8
	1′	20	6.2	−14.9	6.9	−16.5	7.6	−18.2	8.3	−19.9	9.1	−21.8	9.9	−23.7	11.6	−27.8
	1′	50	5.6	−14.9	6.3	−16.5	6.9	−18.2	7.6	−19.9	8.3	−21.8	9.0	−23.7	10.6	−27.8
	1′	100	5.2	−14.9	5.8	−16.5	6.4	−18.2	7.0	−19.9	7.7	−21.8	8.3	−23.7	9.8	−27.8
	2	10	6.6	−34.1	7.3	−37.8	8.1	−41.7	8.9	−45.7	9.7	−50.0	10.5	−54.4	12.4	−63.9
	2	20	6.2	−31.9	6.9	−35.4	7.6	−39.0	8.3	−42.8	9.1	−46.8	9.9	−50.9	11.6	−59.8
	2	50	5.6	−29.0	6.3	−32.2	6.9	−35.5	7.6	−38.9	8.3	−42.5	9.0	−46.3	10.6	−54.4
	2	100	5.2	−26.8	5.8	−29.7	6.4	−32.8	7.0	−36.0	7.7	−39.3	8.3	−42.8	9.8	−50.2
	3	10	6.6	−46.5	7.3	−51.5	8.1	−56.8	8.9	−62.3	9.7	−68.1	10.5	−74.2	12.4	−87.1
	3	20	6.2	−42.1	6.9	−46.7	7.6	−51.4	8.3	−56.5	9.1	−61.7	9.9	−67.2	11.6	−78.9
	3	50	5.6	−36.3	6.3	−40.2	6.9	−44.4	7.6	−48.7	8.3	−53.2	9.0	−57.9	10.6	−68.0
	3	100	5.2	−31.9	5.8	−35.4	6.4	−39.0	7.0	−42.8	7.7	−46.8	8.3	−50.9	9.8	−59.8

(*Continued*)

TABLE 4.4a (*Continued*)

Components and Claddings, Walls and All Types of Roofs 0–7 Degrees, Wind Speed 95–130 mph, Net Design Wind Pressure, p_{net30}, in lb/ft², for Exposure *B* at *h* = 30 ft

	Zone	Effective Wind Area (ft²)	Basic Wind Speed (mph)													
			95		100		105		110		115		120		130	
Walls	4	10	35.3	−38.2	40.5	−38.2	46.1	−50.0	52.0	−56.4	58.3	−63.2	64.9	−70.4	72.0	−78.1
	4	20	33.7	−36.7	38.7	−36.7	44.0	−47.9	49.6	−54.1	55.7	−60.6	62.0	−67.5	68.7	−74.8
	4	50	31.6	−34.6	36.2	−34.6	41.2	−45.1	46.6	−51.0	52.2	−57.1	58.1	−63.7	64.4	−70.5
	4	100	30.0	−33.0	34.4	−33.0	39.2	−43.1	44.2	−48.6	49.6	−54.5	55.2	−60.7	61.2	−67.3
	5	10	35.3	−47.2	40.5	−47.2	46.1	−61.7	52.0	−69.6	58.3	−78.0	64.9	−87.0	72.0	−96.3
	5	20	33.7	−44.0	38.7	−44.0	44.0	−57.5	49.6	−64.9	55.7	−72.8	62.0	−81.1	68.7	−89.9
	5	50	31.6	−39.8	36.2	−39.8	41.2	−52.0	46.6	−58.7	52.2	−65.8	58.1	−73.4	64.4	−81.3
	5	100	30.0	−36.7	34.4	−36.7	39.2	−47.9	44.2	−54.1	49.6	−60.6	55.2	−67.5	61.2	−74.8
Flat/Hip/Gable Roof	1	10	14.3	−56.2	16.5	−56.2	18.7	−73.4	21.1	−82.8	23.7	−92.9	26.4	−103.5	29.3	−114.6
0–7 Degrees	1	20	13.4	−52.5	154	−52.5	17.6	−68.5	19.8	−77.4	22.2	−86.7	24.8	−96.6	27.4	−107.1
	1	50	12.3	−47.6	14.1	−47.6	16.0	−62.1	18.1	−70.1	20.3	−78.6	22.6	−87.6	25.0	−97.1
	1	100	11.4	−43.9	13.0:	−43.9	14.8	−57.3	16.7	−64.7	18.8	−72.5	20.9	−80.8	23.2	−89.5
	1′	10	14.3	−32.3	16.5	−32.3	18.7	−42.1	21.1	−47.6	23.7	−53.3	26.4	−59.4	29.3	−65.9
	1′	20	13.4.	−32.3	15.4	−32.3	17.6	−42.1	19.8	−47.6	22.2	−53.3	24.8	−59.4	27.4	−65.9
	1′	50	12.3	−32.3	14.1	−32.3	16.0	−42.1	18.1	−47.6	20.3	−53.3	22.6	−59.4	25.0	−65.9
	1′	100	11.4	−32.3	13.0	−32.3	14.8	−42.1	16.7	−47.6	18.8	−53.3	20.9	−59.4	23.2	−65.9
	2	10	14.3	−74.1	16.5	−74.1	18.7	−96.8	21.1	−109.3	23.7	−122.5	26.4	−136.5	29.3	−151.2
	2	20	13.4	−69.3	15.4	−69.3	17.6	−90.6	19.8	−102.2	22.2	−114.6	24.8	−127.7	27.4	−141.5
	2	50	12.3	−63.0	14.1	−63.0	16.0	−82.3	18.1	−92.9	20.3	−104.2	22.6	−116.1	25.0	−128.7
	2	100	11.4	−58.3	13.0;	−58.3	14.8	−76.1	16.7	−85.9	18.8	−96.3	20.9	−107.3	23.2	−118.9
	3	10	14.3	−101.0	16.5	−101.0	18.7	−131.9	21.1	−148.9	23.7	−166.9	26.4	−186.0	29.3	−206.1
	3	20	13.4	−91.5	15.4	−91.5	17.6	−119.5	19.8	−134.9	22.2	−151.2	24.8	−168.5	27.4	−186.7
	3	50	12.3	−78.9	14.1	−78.9	16.0	−103.0	18.1	−116.3	20.3	−130.4	22.6	−145.3	25.0	−161.0
	3	100	11.4	−69.3	13.0	−69.3	14.8	−90.6	16.7	−102.2	18.8	−114.6	20.9	−127.7	23.2	−141.5

TABLE 4.4b
Components and Claddings, Gable Roofs > 7–20 Degrees, Wind Speed 95–130 mph, Net Design Wind Pressure, p_{net30}, in lb/ft², for Exposure *B* at *h* = 30 ft

			Basic Wind Speed (mph)													
	Zone	Effective Wind Area (ft²)	95		100		105		110		115		120		130	
Gable Roof >7–20 Degrees	1	10	9.8	–30.0	10.9	–33.2	12.0	–36.6	13.2	–40.2	14.4	–44.0	15.7	–47.9	18.4	–56.2
	1	20	8.9	–30.0	9.8	–33.2	10.8	–36.6	11.9	–40.2	13.0	–44.0	14.1	–47.9	16.6	–56.2
	1	50	7.6	–18.2	8.4	–20.2	9.3	–22.3	10.2	–24.5	11.1	–26.7	12.1	–29.1	14.2	–34.2
	1	100	6.6	–9.4	7.3	–10.4	8.1	–11.4	8.9	–12.5	9.7	–13.7	10.5	–14.9	12.4	–17.5
	2e	10	9.8	–30.0	10.9	–33.2	12.0	–36.6	13.2	–40.2	14.4	–44.0	15.7	–47.9	18.4	–56.2
	2e	20	8.9	–30.0	9.8	–33.2	10.8	–36.6	11.9	–40.2	13.0	–44.0	14.1	–47.9	16.6	–56.2
	2e	50	7.6	–18.2	8.4	–20.2	9.3	–22.3	10.2	–24.5	11.1	–26.7	12.1	–29.1	14.2	–34.2
	2e	100	6.6	–9.4	7.3	–10.4	8.1	–11.4	8.9	–12.5	9.7	–13.7	10.5	–14.9	12.4	–17.5
	2n	10	9.8	–43.8	10.9	–48.5	12.0	–53.4	13.2	–58.7	14.4	–64.1	15.7	–69.8	18.4	–81.9
	2n	20	8.9	–37.8	9.8	–41.9	10.8	–46.2	11.9	–50.7	13.0	–55.4	14.1	–60.4	16.6	–70.8
	2n	50	7.6	–30.0	8.4	–33.2	9.3	–36.6	10.2	–40.2	11.1	–44.0	12.1	–47.9	14.2	–56.2
	2n	100	6.6	–24.1	7.3	–26.7	8.1	–29.4	8.9	–32.3	9.7	–35.3	10.5	–38.4	12.4	–45.1
	2r	10	9.8	–43.8	10.9	–48.5	12.0	–53.4	13.2	–58.7	14.4	–64.1	15.7	–69.8	18.4	–81.9
	2r	20	8.9	–37.8	9.8	–41.9	10.8	–46.2	11.9	–50.7	13.0	–55.4	14.1	–60.4	16.6	–70.8
	2r	50	7.6	–30.0	8.4	–33.2	9.3	–36.6	10.2	–40.2	11.1	–44.0	12.1	–47.9	14.2	–56.2
	2r	100	6.6	–24.1	7.3	–26.7	8.1	–29.4	8.9	–32.3	9.7	–35.3	10.5	–38.4	12.4	–45.1
	3e	10	9.8	–43.8	10.9	–48.5	12.0	–53.4	13.2	–58.7	14.4	–64.1	15.7	–69.8	18.4	–81.9
	3e	20	8.9	–37.8	9.8	–41.9	10.8	–46.2	11.9	–50.7	13.0	–55.4	14.1	–60.4	16.6	–70.8
	3e	50	7.6	–30.0	8.4	–33.2	9.3	–36.6	10.2	–40.2	11.1	–44.0	12.1	–47.9	14.2	–56.2
	3e	100	6.6	–24.1	7.3	–26.7	8.1	–29.4	8.9	–32.3	9.7	–35.3	10.5	–38.4	12.4	–45.1
	3r	10	9.8	–52.0	10.9	–57.6	12.0	–63.5	13.2	–69.7	14.4	–76.2	15.7	–83.0	18.4	–97.4
	3r	20	8.9	–44.6	9.8	–49.4	10.8	–54.4	11.9	–59.7	13.0	–65.3	14.1	–71.1	16.6	–83.4
	3r	50	7.6	–34.7	8.4	–38.4	9.3	–42.4	10.2	–46.5	11.1	–50.8	12.1	–55.4	14.2	–65.0
	3r	100	6.6	–27.2	7.3	–30.2	8.1	–33.3	8.9	–36.5	9.7	–39.9	10.5	–43.5	12.4	–51.0

(Continued)

TABLE 4.4b (*Continued*)

Components and Claddings, Gable Roofs, 7–20 Degrees, Wind Speed 140–200 mph, Net Design Wind Pressure, p_{net30}, in lb/ft², for Exposure B at h = 30 ft

			Basic Wind Speed (mph)													
	Zone	Effective Wind Area (ft²)	95		100		105		110		115		120		130	
Gable Roof >7–20 Degrees	1	10	21.4	−65.1	24.5	−65.1	27.9	−85.1	31.5	−96.0	35.3	−107.7	39.4	−120.0	43.6	−132.9
	1	20	19.3	−65.1	22.1	−65.1	25.2	−85.1	28.4	−96.0	31.8	−107.7	35.5	−120.0	39.3	−132.9
	I	50	16.5	−39.6	18.9	−39.6	21.5	−51.8	24.3	−58.4	27.2	−65.5	30.3	−73.0	33.6	−80.9
	1	100	14.3	−20.3	16.5	−20.3	18.7	−26.5	21.1	−30.0	23.7	−33.6	26.4	−37.4	29.3	−41.5
	2e	10	21.4	−65.1	24.5	−65.1	27.9	−85.1	31.5	−96.0	35.3	−107.7	39.4	−120.0	43.6	−132.9
	2e	20	19.3	−65.1	22.1	−65.1	25.2	−85.1	28.4	−96.0	31.8	−107.7	35.5	−120.0	39.3	−132.9
	2e	50	16.5	−39.6	18.9	−39.6	21.5	−51.8	24.3	−58.4	27.2	−65.5	30.3	−73.0	33.6	−80.9
	2e	100	14.3	−20.3	16.5	−20.3	18.7	−26.5	21.1	−30.0	23.7	−33.6	26.4	−37.4	29.3	−41.5
	2n	10	21.4	−95.0	24.5	−95.0	27.9	−124.1	31.5	−140.1	35.3	−157.1	39.4	−175.0	43.6	−193.9
	2n	20	19.3	−82.1	22.1	−82.1	25.2	−107.3	28.4	−121.1	31.8	−135.8	35.5	−151.3	39.3	−167.7
	2n	50	16.5	−65.1	18.9	−65.1	21.5	−85.1	24.3	−96.0	27.2	−107.7	30.3	−120.0	33.6	−132.9
	2n	100	14.3	−52.3	16.5	−52.3	18.7	−68.3	21.1	−77.1	23.7	−86.4	26.4	−96.3	29.3	−106.7
	2r	10	21.4	−95.0	24.5	−95.0	27.9	−124.1	31.5	−140.1	35.3	−157.1	39.4	−175.0	43.6	−193.9
	2r	20	19.3	−82.1	22.1	−82.1	25.2	−107.3	28.4	−121.1	31.8	−135.8	35.5	−151.3	39.3	−167.7
	2r	50	16.5	−65.1	18.9	−65.1	21.5	−85.1	24.3	−96.0	27.2	−107.7	30.3	−120.0	33.6	−132.9
	2r	100	14.3	−52.3	16.5	−52.3	18.7	−68.3	21.1	−77.1	23.7	−86.4	26.4	−96.3	29.3	−106.7
	3e	10	21.4	−95.0	24.5	−95.0	27.9	−124.1	31.5	−140.1	35.3	−157.1	394	−175.0	43.6	−193.9
	3e	20	19.3	−82.1	22.1	−82.1	25.2	−107.3	28.4	−121.1	31.8	−135.8	35.5	−151.3	39.3	−167.7
	3e	50	16.5	−65.1	18.9	−65.1	21.5	−85.1	24.3	−96.0	27.2	−107.7	30.3	−120.0	33.6	−132.9
	3e	100	14.3	−52.3	16.5	−52.3	18.7	−68.3	21.1	−77.1	23.7	−86.4	26.4	−96.3	29.3	−106.7
	3r	10	21.4	−112.9	24.5	−112.9	27.9	−147.5	31.5	−166.5	35.3	−186.7	39.4	−208.0	43.6	−230.5
	3r	20	19.3	−96.8	22.1	−96.8	25.2	−126.4	28.4	−142.7	31.8	−159.9	35.5	−178.2	39.3	−197.5
	3r	50	16.5	−75.4	18.9	−75.4	21.5	−98.4	24.3	−111,1	27.2	−124.6	30.3	−138.8	33.6	−153.8
	3r	100	14.3	−59.2	16.5	−59.2	18.7	−77.3	21.1	−87.2	23.7	−97.8	26.4	−109.0	29.3	−120.7

Note: Plus and minus signs signify pressures acting toward and away from the surfaces, respectively. For effective wind areas between those given above, the load may be interpolated; otherwise, use the load associated with the lower effective area.

Metric conversions: 1.0 ft = 0.3048 m; 1.0 ft² = 0.0929 m²; 1.0 lb/ft² = 0.0479 kN/m²

TABLE 4.4c

Components and Claddings, Gable Roofs > 20–27 Degrees, Wind Speed 95–130 mph, Net Design Wind Pressure, p_{net30}, in lb/ft², for Exposure B at h = 30 ft

			Basic Wind Speed (mph)													
	Zone	Effective Wind Area (ft²)	95		100		105		110		115		120		130	
Gable Roof >20–27 Degrees	1	10	9.8	−23.1	10.9	−25.6	12.0	−28.2	I3.2:	−31.0	14.4	−33.9	15.7	−36.9	18.4	−43.3
	1	20	8.9	−23.1	9.8	−25.6	10.8	−28.2	11.9	−31.0	13.0	−33.9	14.1	−36.9	16.6	−43.3
	1	50	7.6	−19.6	8.4	−21.7	9.3	−24.0	10.2	−26.3	11.1	−28.8	12.1	−31.3	14.2	−36.7
	1	100	6.6	−17.0	7.3	−18.8	8.1	−20.7	8.9	−22.8	9.7	−24.9	10.5	−27.1	12.4	−31.8
	2e	10	9.8	−23.1	10.9	−25.6	12.0	−28.2	13.2	−31.0	14.4	−33.9	15.7	−36.9	18.4	−43.3
	2e	20	8.9	−23.1	9.8	−25.6	10.8	−28.2	11.9	−31.0	13.0	−33.9	14.1	−36.9	16.6	−43.3
	2e	50	7.6	−19,6	8.4	−21.7	9.3	−24.0	10.2	−26.3	11.1	−28.8	12.1	−31.3	14.2	−36.7
	2e	100	6.6	−17.0	7.3	−18.8	8.1	−20.7	8.9	−22.8	9.7	−24.9	10.5	−27.1	12.4	−31.8
	2n	10	9.8	−36.9	10.9	−40.9	12.0	−45.0	13.2	−49.4	14.4	−54.0	15.7	−58.8	18.4	−69.0
	2n	20	8.9	−32.3	9.8:	−35.8	10.8	−39.5	11.9	−43.3	13.0	−47.3	14.1	−51.5	16.6	−60.5
	2n	50	7.6	−26.2	8.4	−29.1	9.3	−32.1	10.2	−35.2	11.1	−38.5	12.1	−41.9	14.2	−49.1
	2n	100	6.6	−21.7	7.3	−24.0	8.1	−26.5	8.9;	−29.0	9.7	−31.7	10.5	−34.6	12.4	−40.6
	2r	10	9.8	−36.9	10,9	−40.9	12,0	−45.0	13.2	−49.4	14.4	−54.0	15.7	−58.8	18.4	−69.0
	2r	20	8.9	−32.3	9.8	−35.8	10.8	−39.5	11.9	−43.3	13.0	−47.3	14.1	−51.5	16.6	−60.5
	2r	50	7.6	−26.2	8.4	−29.1	9.3	−32.1	10.2	−35.2	11.1	−38.5	12.1	−41.9	14.2	−49.1
	2r	100	6.6	−21.7	7.3	−24.0	8.1	−26.5	8.9	−29.0	9.7	−31.7	10.5	−34.6	12.4	−40.6
	3e	10	9.8	−36.9	−10.9	−40.9	12.0	−45.0	13.2	−49.4	14.4	−54.0	15.7	−58.8	18.4	−69.0
	3e	20	8.9	−32.3	9.8	−35.8	10.8	−39.5	11.9	−43.3	13.0	−47.3	14.1	−51.5	16.6	−60.5
	3e	50	7.6	−26.2	8.4	−29.1	9.3	−32.1	10.21	−35.2	11.1	−38.5	12.1	−41.9	14.2	−49.1
	3e	100	6.6	−21.7	7.3	−24.0	8.1	−26.5	8.9	−29.0	9.7	−31.7	10.5	−34.6	12.4	−40.6
	3r	10	9.8	−47.5	10.9	−52.6	12.0	−58.0	13.2	−63.7	14.4	−69.6	15.7	−75.8	18.4	−89.0
	3r	20	8.9	−38.8	9.8	−43.0	10,8	−47.4	11.9	−52.0	13.0	−56.8	14.1	−61.9	16.6	−72.6
	3r	50	7.6	−27.2	8.4	−30.2	9.3	−33.3	10.2	−36.5	11.1	−39.9	12.1	−43.5	14.2	−51.0
	3r	100	6.6	−27.2	7.3	−30.2	8.1	−33.3	8.9	−36.5	9.7	−39.9	10.5	−43.5	12,4−	−51.0

(Continued)

TABLE 4.4c (*Continued*)
Components and Claddings, Gable Roofs > 20–27 Degrees, Wind Speed 95–130 mph, Net Design Wind Pressure, p_{net30}, in lb/ft², for Exposure B at h = 30 ft

			Basic Wind Speed (mph)													
	Zone	Effective Wind Area (ft²)	95		100		105		110		115		120		130	
Gable Roof >20 to 27 Degrees	1	10	21.4	−50.2	24.5	−57.6	27.9	−65.6	31.5	−74.0	35.3	−83.0	39.4	−92.5	43.6	−102.4
	1	20	19.3	−50.2	22.1	−57.6	25.2	−65.6	28.4	−74.0	31.8	−83.0	35.5	−92.5	39.3	−102.4
	1	50	16.5	−42.6	18.9	−48.9	21.5	−55.7	24.3	−62.8	27.2	−70.4	30.3	−78.5	33.6	−87.0
	1	100	14.3	−36.9	16.5	−42.3	18.7	−48.2	21.1	−54.4	23.7	−60.9	26.4	−67.9	29.3	−75.2
	2e	10	21.4	−50.2	24.5	−57.6	27.9	−65.6	31.5	−74.0	35.3	−83.0	39.4	−92.5	43.6	−102.4
	2e	20	19.3	−50.2	22.1	−57.6	25.2	−65.6	28.4	−74.0	31.8	−83.0	35.5	−92.5	39.3	−102.4
	2e	50	16.5	−42.6	18.9	−48.9	21.5	−55.7	24.3	−62.8	27.2	−70.4	30.3	−78.5	33.6	−87.0
	2e	100	14.3	−36.9	16.5	−42.3	18.7	−48.2	21.1	−54.4	23.7	−60.9	26.4	−67.9	29.3	−75.2
	2n	10	21.4	−80.1	24.5	−91.9	27.9	−104.6	31.5	−118.1	35.3	−132.4	39.4	−147.5	43.6	−163.4
	2n	20	19.3	−70.1	22.1	−80.5	25.2	−91.6	28.4	−103.4	31.8	−115.9	35.5	−129.2	39.3	−143.1
	2n	50	16.5	−57.0	18.9	−65.4	21.5	−74.4	24.3	−84.0	27.2	−94.2	30.3	−105.0	33.6	−116.3
	2n	100	14.3	−47.1	16.5	−54.0	18.7	−61.5	21.1	−69.4	23.7	−77.8	26.4	−86.7	29.3	−96.0
	2r	10	21.4	−80.1	24.5	−91.9	27.9	−104.6	31.5	−118.1	35.3	−132.4	39.4	−147.5	43.6	−163.4
	2r	20	19.3	−70.1	22.1	−80.5	25.2	−91.6	28.4	−103.4	31.8	−115.9	35.5	−129.2	39.3	−143.1
	2r	50	16.5	−57.0	18.9	−65.4	21.5	−74.4	24.3	−84.0	27.2	−94.2	30.3	−105.0	33.6	−116.3
	2r	100	14.3	−47.1	16.5	−54.0	18.7	−61.5	21.1	−69.4	23.7	−77.8	26.4	−86.7	29.3	−96.0
	3e	10	21.4	−80.1	24.5	−91.9	27.9	−104.6	31.5	−118.1	35.3	−132.4	39.4	−147.5	43.6	−163.4
	3e	20	19.3	−70.1	22.1	−80.5	25.2	−91.6	28.4	−103.4	31.8	−115.9	35.5	−129.2	39.3	−143.1
	3e	50	16.5	−57.0	18.9	−65.4	21.5	−74.4	24.3	−84.0	27.2	−94.2	30.3	−105.0	33.6	−116.3
	3e	100	14.3	−47.1	16.5	−54.0	18.7	−61.5	21.1	−69.4	23.7	−77.8	26.4	−86.7	29.3	−96.0
	3r	10	21.4	−103.2	24.5	−118.5	27.9	−134.8	31.5	−152.2	35.3	−170.6	39.4	−190.1	43.6	−210.6
	3r	20	19.3	−84.2	22.1	−96.7	25.2	−110.0	28.4	−124.2	31.8	−139.2	35.5	−155.1	39.3	−171.9
	3r	50	16.5	−59.2	18.9	−67.9	21.5	−77.3	24.3	−87.2	27.2	−97.8	30.3	−109.0	33.6	−120.7
	3r	100	14.3	−59.2	16.5	−67.9	18.7	−77.3	21.1	−87.2	23.7	−97.8	26.4	−109.0	29.3	−120.7

TABLE 4.4d
Components and Claddings, Gable Roofs > 27–45 Degrees, Wind Speed 95–130 mph, Net Design Wind Pressure, p_{net30}, in lb/ft², for Exposure *B* at *h* = 30 ft

	Zone	Effective Wind Area (ft²)	Basic Wind Speed (mph) 140		150		160		170		180		190		200	
Gable Roof >27–45 Degrees	1	10	14.9	−27.2	16.5	−30.2	18.2	−33.3	19.9	−36.5	21.8	−39.9	23.7	−43.5	27.8	−51.0
	1	20	13.2	−23.1	14.6	−25.6	16.1	−28.2	17.7	−31.0	19.3	−33.9	21.1	−36.9	24,7	−43.3
	1	50	11.0	−17.6	12.2	−19.5	13.5	−21.5	14.8	−23.6	16.1	−25.8	17.6	−28.1	20.6	−33.0
	1	100	9.4	−13.5	10.4	−14.9	11.4	−16.5	12.5	−18.1	13.7	−19.8	14.9	−21.5	17.5	−25.2
	2e	10	14.9	−27.2	16.5	−30.2	18.2	−33.3	19.9	−36.5	21.8	−39.9	23.7	−43.5	27.8	−51.0
	2e	20	13.2	−23.1	14.6	−25.6	16.1	−28.2	17.7	−31.0	19.3	−33.9	21.1	−36.9	24.7	−43.3
	2e	50	11.0	−17.6	12.2	−19.5	13.5	−21.5	14.8	−23.6	16.1	−25.8	17.6	−28.1	20.6	−33.0
	2e	100	9.4	−13.5	10.41	−14.9	11.4	−16.5	12.5	−18.1	13.7	−19.8	14.9	−21.5	17.5	−25.2
	2n	10	14.9	−30.0	16.5	−33.2	18.2	−36,6	19.9	−40.2	21.8	−44.0	23.7	−47.9	27.8	−56.2
	2n	20	13.2	−26.8	14.6	−29.7	16.1	−32.8	17.7	−35.9	19.3	−39.3	21.1	−42.8	24.7	−50.2
	2n	50	11.0	−22.6	12.2	−25.0	13.5	−27.6	14.8	−30.3	16.1	−33.1	17.6	−36.1	20.6	−42.3
	2n	100	9.4	−19.4	10.4	−21.5	11.4	−23.7	12.5	−26.0	13.7	−28.5	14.9	−31.0	17.5	−36.4
	2r	10	14.9	−27.2	16.5	−30.2	18.2	−33.3	19.9	−36.5	21.8	−39.9	23.7	−43.5	27.8	−51.0
	2r	20	13.2	−23.1	14.6	−25.6	16.1	−28.2	17.7	−31.0	19.3	−33.9	21.1	−36.9	24.7	−43.3
	2r	50	11.0	−17.6	12.2	−19.5	13.5	−21.5	14.8	−23.6	16.1	−25.8	17.6	−28.1	20.6	−33.0
	2r	100	9.4	−13.5	10.4	−14.9	11.4	−16.5	12.5	−18.1	13.7	−19.8	14.9	−21.5	17.5	−25.2
	3e	10	14.9	−36.8	16.5	−40.8	18.2	−44.9	19.9	−49.3	21.8	−53.9	23.7	−58.7	27.8	−68.9
	3e	20	13.2	−32.6	14.6	−36.1	16.1	−39.8	17.7	−43.7	19.3	47.8	21.1	−52.0	24.7	−61.0
	3e	50	11.0	−27.1	12.2	−30.0	13.5	−33.1	14.8	−36.3	16.1	−39.7	17.6	−43.2	20.6	−50.7
	3e	100	9.4	−22.9	10.4	−25.3	11.4	−27.9	12.5	−30.7	13.7	−33.5	14.9	−36.5	17.5	−42.8
	3r	10	14.9	−30.0	16.5	−33.2	18.2	−36.6	19.9	−40.2	21.8	−44.0	23.7	−47.9	27.8	−56.2
	3r	20	13.2	−26.8	14.6	−29.7	16.1	−32.8	17.7	−35.9	19.3	−39.3	21.1	−42.8	24.7	−50.2
	3r	50	11.0	−22.6	12.2	−25.0	13.5	−27.6	14.8	−30.3	16.1	−33.1	17.6	−36.1	20.6	−42.3
	3r	100	9.4	−19.4	10.4	−21.5	11.4	−23.7	12.5	−26.0	13.7	−28.5	14.9	−31.0	17.5	−36.4

(*Continued*)

TABLE 4.4d (*Continued*)
Components and Claddings, Gable Roofs > 27–45 Degrees, Wind Speed 95–130 mph, Net Design Wind Pressure, p_{net30}, in lb/ft², for Exposure *B* at *h* = 30 ft

			Basic Wind Speed (mph)													
	Zone	Effective Wind Area (ft²)	140		150		160		170		180		190		200	
Gable Roof >27 to 45 Degrees	1	10	32.3	−59.2	37.0	−67.9	42.1	−77.3	47.6	−87.2	53.3	−97.8	59.4	−109	65.9	−120.7
	1	20	28.7	−50.2	32.9	−57.6	37.4	−65.5	42.3	−74.0	47.4	−82.9	52.8	−92.4	58.5	−102.4
	1	50	23.9	−38.3	27.5	−43.9	31.2	−50.0	35.3	−56.4	39.5	−63.3	44.0	−70.5	48.8	−78.1
	1	100	20.3	−29.3	23.3	−33.6	26.5	−38.2	30.0	−43.2	33.6	−48.4	37.4	−53.9	41.5	−59.8
	2e	10	32.3	−59.2	37.0	−67.9	42.1	−77.3	47.6	−87.2	53.3	−97.8	59.4	−109.0	65.9	−120.7
	2e	20	28.7	−50.2	32.9	−57.6	37.4	−65.5	42.3	−74.0	47.4	−82.9	52.8	−92.4	58.5	−102.4
	2e	50	23.9	−38.3	27.5	−43.9	31.2	−50.0	35.3	−56.4	39.5	−63.3	44.0	−70.5	48.8	−78.1
	2e	100	20.3	−29.3	23.3	−33.6	26.5	−38.2	30.0	−43.2	33.6	−48.4	37.4	−53.9	41.5	−59.8
	2n	10	32.3	−65.1	37.0	−74.8	42.1	85.1	47.6	−96.0	53.3	−107.7	59.4	−120.0	65.9	−132.9
	2n	20	28.7	−58.2	32.9	−66.8	37.4	76.0	42.3	−85.9	47.4	−96.2	52.8	−107.2	58.5	−118.8
	2n	50	23.9	−49.1	27.5	−56.3	31.2	64.1	35.3	−72.4	39.5	−81.1	44.0	−90.4	48.8	−100.2
	2n	100	20.3	−42.2	23.3	−48.4	26.5	55.1	30.0	−62.2	33.6	−69.7	37.4	−77.7	41.5	−86.1
	2r	10	32.3	−59.2	37.0	−67.9	42.1	77.3	47.6	−87.2	53.3	−97.8	59.4	−109.0	65.9	−120.7
	2r	20	28.7	−50.2	32.9	−57.6	37.4	65.5	42.3	−74.0	47.4	−82.9	52.8	−92.4	58.5	−102.4
	2r	50	23.9	−38.3	27.5	−43.9	31.2	50.0	35.3	−56.4	39.5	−63.3	44.0	−70.5	48.8	−78.1
	2r	100	20.3	−29.3	23.3	−33.6	26.5	38.2	30.0	−43.2	33.6	−48.4	37.4	−53.9	41.5	−59.8
	3e	10	32.3	−79.9	37.0	−91.7	42.1	104.3	47.6	−117.8	53.3	−132.0	59.4	−147.1	65.9	−163.0
	3e	20	28.7	−70.8	32.9	−81.3	37.4	92.5	42.3	−104.4	47.4	−117.0	52.8	−130.4	58.5	−144.5
	3e	50	23.9	−58.8	27.5	−67.5	31.2	76.8	35.3	−86.6	39.5	−97.1	44.0	−108.2	48.8	−119.9
	3e	100	20.3	−49.7	23.3	−57.0	26.5	64.9	30.0	−73.2	33.6	−82.1	37.4	−91.5	41.5	−101.4
	3r	10	32.3	−65.1	37.0	−74.8	42.1	85.1	47.6	−96.0	53.3	−107.7	59.4	−120.0	65.9	−132.9
	3r	20	28.7	−58.2	32.9	−66.8	37.4	76.0	42.3	−85.9	47.4	−96.2	52.8	−107.2	58.5	−118.8
	3r	50	23.9	−49.1	27.5	−56.3	31.2	64.1	35.3	−72.4	39.5	−81.1	44.0	−90.4	48.8	−100.2
	3r	100	20.3	−42.2	23.3	−48.4	26.5	55.1	30.0	−62.2	33.6	−69.7	37.4	−77.7	41.5	−86.1

Notes: Plus and minus signs signify pressures acting toward and away from the surfaces, respectively. For effective wind areas between those given above, the load may be interpolated; otherwise, use the load associated with the lower effective area.

Metric conversions: 1.0 ft = 0.3048 m; 1.0 ft² = 0.0929 m²; 1.0 lb/ft² = 0.0479 kN/m².

TABLE 4.4e

Components and Claddings, Hip Roofs > 7–27 Degrees, Wind Speed 95–130 mph, Net Design Wind Pressure, p_{net30}, in lb/ft², for Exposure *B* at *h* = 30 ft

		Zone	Effective Wind Area (ft²)	Basic Wind Speed (mph)													
				95		100		105		110		115		120		130	
Hip Roof > 7–20 Degrees	***h/D* ≤ 0.5**	1	10	12.1	−20.4	13.4	−22.6	14.8	−24.9	16.2	−27.3	17.7	−29.8	19.3	−32.5	22.7	−38.1
		1	20	10.5	−20.4	11.6	−22.6	12.8	−24.9	14.0	−27.3	15.3	−29.8	16.7	−32.5	19.6	−38.1
		1	50	8.3	−18.0	9.2	−20.0	10.1	−22.0	11.1	−24.1	12.1	−26.4	13.2	−28.7	15.5	−33.7
		1	100	6.6	−16.2	7.3	−18.0	8.1	−19.8	8.9	−21.8	9.7	−23.8	10.5	−25.9	12.4	−30.4
		2e	10	12.1	−27.2	13.4	−30.2	14.8	−33.3	16.2	−36.5	17.7	−39.9	19.3	−43.5	22.7	−51.0
		2e	20	10.5	−25.0	11.6	−27.7	12.8	−30.6	14.0	−33.5	15.3	−36.7	16.7	−39.9	19.6	−46.8
		2e	50	8.3	−22.1	9.2	−24.5	10.1	−27.0	11.1	−29.6	12.1	−32.3	13.2	−35.2	15.5	−41.3
		2e	100	6.6	−19.8	7.3	−22.0	8.1	−24.2	8.9	−26.6	9.7	−29.1	10.5	−31.7	12.4	−37.2
		2r	10	12.1	−35.5	13.4	−39.3	14.8	−43.4	16.2	−47.6	17.7	−52.0	19.3	−56.6	22.7	−66.5
		2r	20	10.5	−32.0	11.6	−35.5	12.8	−39.1	14.0	−42.9	15.3	−46.9	16.7	−51.1	19.6	−59.9
		2r	50	8.3	−27.4	9.2	−30.3	10.1	−33.4	11.1	−36.7	12.1	−10.1	13.2	−43.7	15.5	−51.2
		2r	100	6.6	−23.9	7.3	−26.4	8.1	−29.2	8.9	−32.0	9.7	−35.0	10.5	−38.1	12.4	−44.7
		3	10	12.1	−27.2	13.4	−30.2	14.8	−33.3	16.2	−36.5	17.7	−39.9	19.3	−43.5	22.7	−51.0
		3	20	10.5	−25.0	11.6	−27.7	12.8	−30.6	14.0	−33.5	15.3	−36.7	16.7	−39.9	19.6	−46.8
		3	50	8.3	−22.1	9.2	−24.5	10.1	−27.0	11.1	−29.6	12.1	−32.3	13.2	−35.2	15.5	−41.3
		3	100	6.6	−19.8	7.3	−22.0	8.1	−24.2	8.9	−26.6	9.7	−29.1	10.5	−31.7	12.4	−37.2
	***h/D* ≥ 0.8**	1	10	12.1	−27.2	13.4	−30.2	14.8	−33.3	16.2	−36.5	17.7	−39.9	19.3	−43.5	22.7	−51.0
		1	20	10.5	−27.2	11.6	−30.2	12.8	−33.3	14.0	−36.5	15.3	−39.9	16.7	−43.5	19.6	−51,0
		1	50	8.3	−21.0	9.2	−23.2	10.1	−25.6	11.1	−28.1	12.1	−30.7	13.2	−33.5	15.5	−39.3
		1	100	6.6	−16.2	7.3	−18.0	8.1	−19.8	8.9	−21.8	9.7	−23.8	10.5	−25.9	12.4	−30.4
		2e	10	12.1	−38.2	13.4	−42.4	14.8	−46.7	16.2	−51.3	17.7	−56.0	19.3	−61.0	22.7	−71.6
		2e	20	10.5	−34.4	11.6	−38.1	12.8	−12.1	14.0	−46.2	15.3	−50.5	16.7	−54.9	19.6	−64.5
		2e	50	8.3	−29.4	9.2	−32.6	10.1	−35.9	11.1	−39.4	12.1	−43.1	13.2	−46.9	15.5	−55.0
		2e	100	6.6	−25.6	7.3	−28.3	8.1	−31.2	8.9	−34.3	9.7	−37.5	10.5	−40.8	12.4	−47.9

(*Continued*)

TABLE 4.4e (*Continued*)
Components and Claddings, Hip Roofs > 7–27 Degrees, Wind Speed 95–130 mph, Net Design Wind Pressure, p_{net30}, in lb/ft², for Exposure B at h = 30 ft

		Effective Wind Area	Basic Wind Speed (mph)													
	Zone	(ft²)	95		100		105		110		115		120		130	
	2r	10	12.1	−35.5	13.4	−39.3	14.8	−43.4	16.2	−7.6	17.7	−52.0	19.3	−56.6	22.7	−66.5
	2r	20	10.5	−32.0	11.6	−35.5	12.8	−39.1	14.0	−42.9	15.3	−46.9	16.7	−51.1	19.6	−59.9
	2r	50	8.3	−27.4	9.2	−30.3	10.1	−33.4	11.1	−36.7	12.1	−40.1	13.2	−43.7	15.5	−51.2
	2r	100	6.6	−23.9	7.3	−26.4	8.1	−29.2	8.9	−32.0	9.7	−35.0	10.5	−38.1	12.4	−44.7
	3	10	12.1	−38.2	13.4	−42.4	14.8	−46.7	16.2	−51.3	17.7	−56.0	19.3	−61.0	22.7	−71.6
	3	20	10.5	−34.4	11.6	−38.1	12.8	−42.1	14.0	−46.2	15.3	−50.5	16.7	−54.9	19.6	−64.5
	3	50	8.3	−29.4	9.2	−32.6	10.1	−35.9	11.1	−39.4	12.1	−434	13.2	−46.9	15.5	−55.0
	3	100	6.6	−25.6	7.3	−28.3	8.1	−31.2	8.9	−34.3	9.7	−37.5	10.5	−40.8	12.4	−47.9
Hip Roof >	1	10	12.1	−21.7	13.4	−24.1	14.8	−26.6	16.2	−29.1	17.7	−31.9	19.3	−34.7	22.7	−40.7
20–27	1	20	10.5	−19.3	11.6	−21.3	12.8	−23.5	14.0	−25.8	15.3	−28−2	16.7	−30.7	19.6	−36.1
Degrees	1	50	8.3	−16.0	9.2	−17.7	10.1	−19.5	11.1	−21.4	12.1	−23.4	13.2	−25.5	15.5	−29.9
	1	100	6.6	−13.5	7.3	−14.9	8.1	−16.5	8.9	−18.1	9.7	−19.8	10.5	−21.5	12.4	−25.2
	2e	10	12.1	−30.0	13.4	−33.2	14.8	−36.6	16.2	−40.2	17.7	−44.0	19.3	−47.9	22.7	−56.2
	2e	20	10.5	−26.8	11.6	−29.7	12.8	−32.8	14.0	−35.9	15.3	−39.3	16.7	−42.8	19.6	−50.2
	2e	50	8.3	−22.6	9.2	−25.0	10.1	−27.6	11.1	−30.3	12.1	−33.1	13.2	−36.1	15.5	−42.3
	2e	100	6.6	−19.4	7.3	−21.5	8.1	−23.7	8.9	−26.0	9.7	−28.5	10.5	−31.0	12.4	−36.4
	2r	10	12.1	−30.0	13.4	−33.2	14.8	−36.6	16.2	−40.2	17.7	−44.0	19.3	−47.9	22.7	−56.2
	2r	20	10.5	−26.8	11.6	−29.7	12.8	−32.8	14.0	−35.9	15.3	−39.3	16.7	−42.8	19.6	−50.2
	2r	50	8.3	−22.6	9.2	−25.0	10.1	−27.6	11.1	−30.3	12.1	−33.1	13.2	−36.1	15.5	−42.3
	2r	100	6.6	−19.4	7.3	−21.5	8.1	−23.7	8.9	−26.0	9.7	−28.5	10.5	−31.0	12.4	−36.4
	3	10	12.1	−30.0	13.4	−33.2	14.8	−36.6	16.2	−40.2	17.7	−44.0	19.3	−47.9	22.7	−56.2
	3	20	10.5	−26.8	11.6	−29.7	12.8	−32.8	14.0	−35.9	15.3	−39.3	16.7	−42.8	19.6	−50.2
	3	50	8.3	−22.6	9.2	−25.0	10.1	−27.6	11.1	−30.3	12.1	−33.1	13.2	−36.1	15.5	−42.3
	3	100	6.6	−19.4	7.3	−21.5	8.1	−23,7	8.9	−26.0	9.7	−28.5	10.5	−31.0	12.4	−36.4

Notes: Plus and minus signs signify pressures acting toward and away from the surfaces, respectively. For effective wind areas between those given above, the load may be interpolated; otherwise, use the load associated with the lower effective area.

Metric conversions: 1.0 ft = 0.3048 m; 1.0 ft² = 0.0929 m²; 1.0 lb/ft² = 0.0479 kN/m².

TABLE 4.4f

Components and Claddings, Hip Roofs > 7–27 Degrees, Wind Speed 140–200 mph, Net Design Wind Pressure, p_{net30}, in lb/ft², for Exposure *B* at $h = 30$ ft

			Effective Wind	Basic Wind Speed													
		Zone	Area (ft²)	140		150		160		170		180		190		200	
Hip Roof	$h/D \leq 0.5$	1	10	26.3	−44.2	30.2	−50.8	34.3	−57.8	38.8	−65.2	43.5	−73.1	48.4	−81.5	53.7	−90.2
> 7–20		1	20	22.7	−44.2	26.1	−50.8	29.6	−57.8	33.5	−65.2	37.5	−73.1	41.8	−81.5	46.3	−90.2
Degrees		1	50	17.9	−39.1	20.6	−44.9	23.4	−51.1	26.5	−57.7	29.7	−64.7	33.0	−72.1	36.6	−79.8
		1	100	14.3	−35.3	16.5	−40.5	18.7	−46.1	21.1	−52.0	23.7	−58.3	26.4	−64.9	29.3	−72.0
		2e	10	26.3	−59.2	30.2	−67.9	34.3	−77.3	38.8	−87.2	43.5	−97.8	48.4	−109.0	53.7	−120.7
		2e	20	22.7	−54.3	26.1	−62.4	29.6	−71.0	33.5	−80.1	37.5	−89.8	41.8	−100.1	46.3	−110.9
		2e	50	17.9	−47.9	20.6	−55.0	23.4	−62.6	26.5	−70.7	29.7	−79.2	33.0	−88.3	36.6	−97.8
		2e	100	14.3	−43.1	16.5	−49.5	18.7	−56.3	21.1	−63.5	23.7	−71.2	26.4	−79.4	29.3	−87.9
		2r	10	26.3	−77.1	30.2	−88.5	34.3	−100.7	38.8	−113.7	43.5	−127.4	48.4	−142.0	53.7	−157.3
		2r	20	22.7	−69.5	26.1	−79.8	29.6	−90.8	33.5	−102.5	37.5	−114.9	41.8	−128.0	46.3	−141.8
		2r	50	17.9	−59.4	20.6	−68.2	23.4	−77.6	26.5	−87.6	29.7	−98.2	33.0	−109.5	36.6	−121.3
		2r	100	14.3	−51.8	16.5	−59.5	18.7	−67.7	21.1	−76.4	23.7	−85.7	26.4	−95.5	29.3	−105.8
		3e	10	26.3	−59.2	30.2	−67.9	34.3	−77.3	38.8	−87.2	43.5	−97.8	48.4	−109.0	53.7	−120.7
		3e	20	22.7	−54.3	26.1	−62.4	29.6	−71.0	33.5	−80.1	37.5	−89.8	41.8	−100.1	46.3	−110.9
		3e	50	17.9	−47.9	20.6	−55.0	23.4	−62.6	26.5	−70.7	29.7	−79.2	33.0	−88.3	36.6	−97.8
		3e	100	14.3	−43.1	16.5	−49.5	18.7	−56.3	21.1	−63.5	23.7	−71.2	26.4	−79.4	29.3	−87.9
	$h/D \geq 0.8$	1	10	26.3	−59.2	30.2	−67.9	34.3	−77.3	38.8	−87.2	43.5	−97.8	48.4	−109.0	53.7	−120.7
		1	20	22.7	−59.2	26.1	−67.9	29.6	−77.3	33.5	−87.2	37.5	−97.8	41.8	−109.0	46.3	−120.7
		1	50	17.9	−45.6	20.6	−52.3	23.4	−59.5	26.5	−67.2	29.7	−75.3	33.0	−83.9	36.6	−93.0
		1	100	14.3	−35.3	16.5	−40.5	18.7	−46.1	21.1	−52.0	23.7	−58.3	26.4	−64.9	29.3	−72.0
		2e	10	26.3	−83.1	30.2	−95.4	34.3	−108.5	38.8	−122.5	43.5	−137.3	48.4	−153.0	53.7	−169.5
		2e	20	22.7	−74.8	26.1	−85.8	29.6	−97.7	33.5	−110.2	37.5	−123.6	41.8	−137.7	46.3	−152.6
		2e	50	17.9	−63.8	20.6	−73.2	23.4	−83.3	26.5	−94.1	29.7	−105.5	33.0	−117.5	36.6	−130.2
		2e	100	14.3	−55.5	16.5	−63.7	18.7	−72.5	21.1	−81.8	23.7	−91.8	26.4	−102.2	29.3	−113.3
		2r	10	26.3	−77.1	30.2	−88.5	34.3	−100.7	38.8	−113.7	43.5	−127.4	48.4	−142.0	53.7	−157.3
		2r	20	22.7	−69.5	26.1	−79.8	29.6	−90.8	33.5	−102.5	37.5	−114.9	41.8	−128.0	46.3	−141.8
		2r	50	17.9	−59.4	20.6	−68.2	23.4	−77.6	26.5	−87.6	29.7	−98.2	33.0	−109.5	36.6	−121.3
		2r	100	14.3	−51.8	16.5	−59.5	18.7	−67.7	21.1	−76.4	23.7	−85.7	26.4	−95.5	29.3	−105.8
		3e	10	26.3	−83.1	30.2	−95.4	34.3	−108.5	38.8	−122.5	43.5	−137.3	48.4	−153.0	53.7	−169.5
		3e	20	22.7	−74.8	26.1	−85.8	29.6	−97.7	33.5	−110.2	37.5	−123.6	41.8	−137.7	46.3	−152.6
		3e	50	17.9	−63.8	20.6	−73.2	23.4	−83.3	26.5	−94.1	29.7	−105.5	33.0	−117.5	36.6	−130.2
		3e	100	14.3	−55.5	16.5	−63.7	18.7	−72.5	21.1	−81.8	23.7	−91.8	26.4	−102.2	29.3	−113.3

(Continued)

TABLE 4.4f (*Continued*)
Components and Claddings, Hip Roofs > 7–27 Degrees, Wind Speed 140–200 mph, Net Design Wind Pressure, p_{net30}, in lb/ft², for Exposure B at h = 30 ft

			Basic Wind Speed													
	Zone	Effective Wind Area (ft²)	140		150		160		170		180		190		200	
Hip Roof > 20–27 Degrees	1	10	26.3	−47.2	30.2	−54.2	34.3	−61.7	38.8	−69.6	43.5	−78.0	48.4	−87.0	53.7	−96.3
	1	20	22.7	−41.8	26.1	−48.0	29.6	−54.6	33.5	−61.7	37.5	−69.1	41.8	−77.0	46.3	−85.3
	1	50	17.9	−34.7	20.6	−39.8	23.4	−45.3	26.5	−51.1	29.7	−57.3	33.0	−63.9	36.6	−70.8
	1	100	14.3	−29.3	16.5	−33.6	18.7	−38.2	21.1	−43.2	23.7	−48.4	26.4	−53.9	29.3	−59.8
	2e	10	26.3	−65.1	30.2	−74.8	34.3	−85.1	38.8	−96.0	43.5	−107.7	48.4	−120.0	53.7	−132.9
	2e	20	22.7	−58.2	26.1	−66.8	29.6	−76.0	33.5	−85.9	37.5	−96.2	41.8	−107.2	46.3	−118.8
	2e	50	17.9	−49.1	20.6	−56.3	23.4	−64.1	26.5	−72.4	29.7	−81.1	33.0	−90.4	36.6	−100.2
	2e	100	14.3	−42.2	16.5	−48.4	18.7	−55.1	21.1	−62.2	23.7	−69.7	26.4	−77.7	29.3	−86.1
	2r	10	26.3	−65.1	30.2	−74.8	34.3	−85.1	38.8	−96.0	43.5	−107.7	48.4	−120.0	53.7	−132.9
	2r	20	22.7	−58.2	26.1	−66.8	29.6	−76.0	33.5	−85.9	37.5	−96.2	41.8	−107.2	46.3	−118.8
	2r	50	17.9	−49.1	20.6	−56.3	23.4	−64.1	26.5	−72.4	29.7	−81.1	33.0	−90.4	36.6	−100.2
	2r	100	14.3	−42.2	16.5	−48.4	18.7	−55.1	21.1	−62.2	23.7	−69.7	26.4	−77.7	29.3	−86.1
	3e	10	26.3	−65.1	30.2	−74.8	34.3	−85.1	38.8	−96.0	43.5	−107.7	48.4	−120.0	53.7	−132.9
	3e	20	22.7	−58.2	26.1	−66.8	29.6	−76.0	33.5	−85.9	37.5	−96.2	41.8	−107.2	46.3	−118.8
	3e	50	17.9	−49.1	20.6	−56.3	23.4	−64.1	26.5	−72.4	29.7	−81.1	33.0	−90.4	36.6	−100.2
	3e	100	14.3	−42.2	16.5	−48.4	18.7	−55.1	21.1	−62.2	23.7	−69.7	26.4	−77.7	29.3	−86.1

Notes: Plus and minus signs signify pressures acting toward and away from the surfaces, respectively. For effective wind areas between those given above, the load may be interpolated; otherwise, use the load associated with the lower effective area.

Metric conversions: 1.0 ft = 0.3048 m; 1.0 ft² = 0.0929 m²; 1.0 lb/ft² = 0.0479 kN/m².

TABLE 4.4g
Components and Claddings, Hip Roofs > 27–45 Degrees, Wind Speed 95–130 mph, Net Design Wind Pressure, P_{net30}, in lb/ft², for Exposure B at h = 30 ft

	Zone	Effective Wind Area (ft²)	Basic Wind Speed (mph) 95		100		105		110		115		120		130	
Hip Roof > 27–45 Degrees	1	10	11.5	−2.3.1	12.7	−25.6	14.0	−28.2	15.4	−31.0	16.8	−33.9	18.3	−36.9	21.5	−43.3
	1	20	10.0	−20.6	11.1	−22.8	12.2	−25.1	13.4	−27.6	14.7	−30.1	16.0	−32.8	18.7	−38.5
	1	50	8.1	−17.2	8.9	−19.1	9.9	−21.0	10.8	−23.1	11.8	−25.2	12.9	−27.4	15.1	−32.2
	1	100	6.6	−14.7	7.3	−16.2	8.1	−17.9	8.9	−19.6	9.7	−21.5	10.5	−23.4	12.4	−27.4
	2e	10	11.5	−27.6	12.7	−30.6	14.0	−33.8	15.4	−37.1	16.8	−40.5	18.3	−44.1	21.5	−51.8
	2e	20	10.0	−21.9	11.1	−24.2	12.2	−26.7	13.4	−29.3	14.7	−32.1	16.0	−34.9	18.7	−41.0
	2e	50	8.1	−14.3	8.9	−15.8	9.9	−17.4	10.8	−19.1	11.8	−20.9	12.9	−22.8	15.1	−26.7
	2e	100	6.6	−13.5	7.3	−14.9	8.1	−16.5	8.9	−18.1	9.7	−19.8	10.5	−21.5	12.4	−25.2
	2r	10	11.5	−37.6	12.7	−41.6	14.0	−45.9	15.4	−50.4	16.8	−55.0	18.3	−59.9	21.5	−70.3
	2r	20	10.0	−31.1	11.1	−34.5	12.2	−38.0	13.4	−41.7	14.7	−45.6	16.0	−49.7	18.7	−58.3
	2r	50	8.1	−22.7	8.9	−25.1	9.9	−27.7	10.8	−30.4	11.8	−33.2	12.9	−36.1	15.1	−42.4
	2r	100	6.6	−16.2	7.3	−18.0	8.1	−19.8	8.9	−21.8	9.7	−23.8	10.5	−25.9	12.4	−30.4
	3	10	11.5	−36.7	12.7	−40.7	14.0	−44.8	15.4	−49.2	16.8	−53.8	18.3	−58.6	21.5	−68.7
	3	20	10.0	−27.9	11.1	−30.9	12.2	−34.1	13.4	−37.4	14.7	−40.9	16.0	−44.5	18.7	−52.7
	3	50	8.1	−16.2	8.9	−18.0	9.9	−19.8	10.8	−21.8	11.8	−23.8	12.9	−25.9	15.1	−30.4
	3	100	6.6	−16.2	7.3	−18.0	8.1	−19.8	8.9	−21.8	9.7	−23.8	10.5	−25.9	12.4	−30.4

(*Continued*)

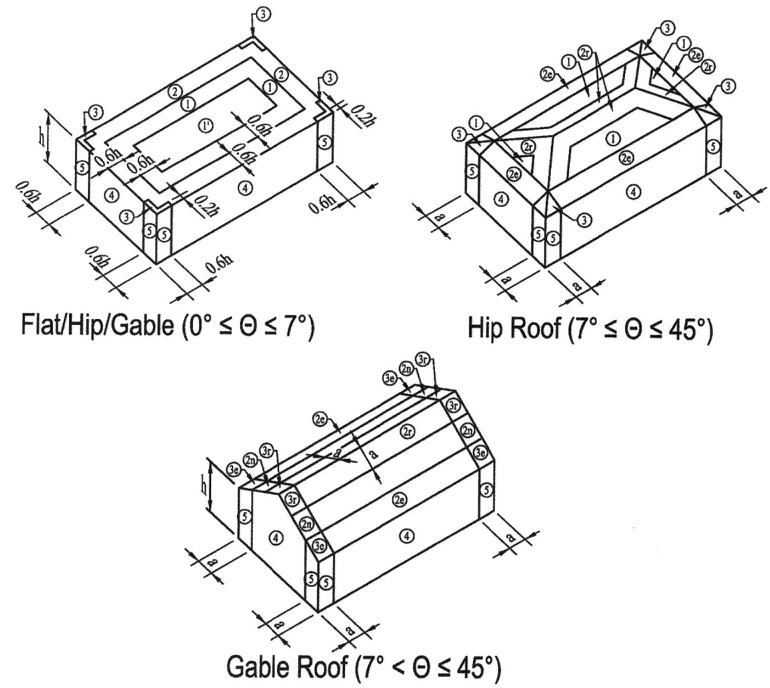
TABLE 4.4g (*Continued*)

Components and Claddings, Hip Roofs > 27–45 Degrees, Wind Speed 140–200 mph, Net Design Wind Pressure, P_{net30}, in lb/ft², for Exposure *B* at $h = 30$ ft

	Zone	Effective Wind Area (ft²)	Basic Wind Speed (mph) 140		150		160		170		180		190		200	
Hip Roof > 27–45 Degrees	1	10	24.9	–50.2	28.6	–57.6	32.5	–65.6	36.7	–74.0	41.2	–83.0	45.9	–92.5	50.8	–102.4
	1	20	21.7	–44.7	24.9	–51.3	28.4	–58.3	32.0	–65.9	35.9	–73.8	40.0	–82.3	44.3	–91.2
	1	50	17.5	–37.4	20.1	–42.9	22.9	–48.8	25.8	–55.1	29.0	–61.8	32.3	–68.8	35.8	–76.2
	1	100	14.3	–31.8	16.5	–36.5	18.7	–41.6	21.1	–46.9	23.7	–52.6	26.4	–58.6	29.3	–64.9
	2e	10	24.9	–60.0	28.6	–68.9	32.5	–78.4	36.7	–88.5	41.2	–99.2	45.9	–110.5	50.8	–122.5
	2e	20	21.7	–47.5	24.9	–54.6	28.4	–62.1	32.0	–70.1	35.9	–78.6	40.0	–87.5	44.3	–97.0
	2e	50	17.5	–31.0	20.1	–35.6	22.9	–40.5	25.8	–45.7	29.0	–51.2	32.3	–57.1	35.8	–63.3
	2e	100	14.3	–29.3	16.5	–33.6	18.7	–38.2	21.1	–43.2	23.7	–48.4	26.4	–53.9	29.3	–59.8
	2r	10	24.9	–81.6	28.6	–93.6	32.5	–106.5	36.7	–120.3	41.2	–134.8	45.9	–150.2	50.8	–166.5
	2r	20	21.7	–67.6	24.9	–77.6	28.4	–88.3	32.0	–99.7	35.9	–111.8	40.0	–124.6	44.3	–138.0
	2r	50	17.5	–49.2	20.1	–56.5	22.9	–64.3	25.8	–72.5	29.0	–81.3	32.3	–90.6	35.8	–100.4
	2r	100	14.3	–35.3	16.5	–40.5	18.7	–46.1	21.1	–52.0	23.7	–58.3	26.4	–64.9	29.3	–72.0
	3	10	24.9	–79.7	28.6	–91.5	32.5	–104.1	36.7	–117.5	41.2	–131.7	45.9	–146.8	50.8	–162.7
	3	20	21.7	–60.6	24.9	–69.5	28.4	–79.1	32.0	–89.3	35.9	–100.1	40.0	–11.5	44.3	–123.6
	3	50	17.5	–35.3	20.1	–40.5	22.9	–46.1	25.8	–52.0	29.0	–58.3	32.3	–64.9	35.8	–72.0
	3	100	14.3	–35.3	16.5	–40.5	18.7	–46.1	21.1	–52.0	23.7	–58.3	26.4	–64.9	29.3	–72.0

Notes: Plus and minus signs signify pressures acting toward and away from the surfaces, respectively. For effective wind areas between those given above, the load may be interpolated; otherwise, use the load associated with the lower effective.area.

Metric conversions: 1.0 ft = 0.3048 m; 1.0 ft² = 0.0929 m²; 1.0 lb/ft² = 0.0479 kN/m².



Notation

a = 10% of least horizontal dimension or $0.4h$, whichever is smaller, but not less than either 4% of least horizontal dimension or 3 ft (0.9 m).

Exception: For buildings with $\theta = 0°$ to $7°$ and a least horizontal dimension greater than 300 ft (90 m), dimension a shall be limited to a maximum of $0.8\,h$.

h = Mean roof height, in ft (m), except that eave height shall be used for roof angles <10°.

θ = Angle of plane of roof from horizontal, in degrees.

Notes

1. Pressures shown are applied normal to the surface, for Exposure B, at $h = 30$ ft (9.1 m). Adjust to other conditions using Eq. (30.4-1).
2. Plus and minus signs signify pressures acting toward and away from the surfaces, respectively.
3. For hip roofs with $\theta \le 25°$, Zone 3 shall be treated as Zone 2e and 2r.
4. For effective wind areas between those given, values may be in terpolated; otherwise use the value associated with the lower effective wind area.
5. If overhangs exist, the lesser horizontal dimension of the building shall not include any overhang dimension, but the edge distance, a, shall be measured from the outside edge of the overhang.

FIGURE 4.12 Zones for components and cladding.

Example 4.2

Determine design wind pressures and forces for the studs and rafters of Example 4.1.

Solution

A. Parameters
   1. $\theta = 14°$
   2. $a = 5$ ft (from Example 4.1), which is more than (1) 0.04 (50) = 2 ft and (2) 3 ft
   3. $p_{net} = p_{net30}$ (from Example 4.1)

B. Wind pressures on studs (wall) at each floor level
   1. Effective area

$$A = L \times W = 11 \times \frac{16}{12} = 14.7\,\text{ft}^2$$

$$A_{min} = \frac{L^2}{3} = \frac{(11)^2}{3} = 40.3\,\text{ft}^2$$

2. Net wall pressures for $V = 115$ mph

Zone	p_{net30} Interpolated for Effective Area 40.3 ft² (psf)		$p_{net} = p_{net30}$ (psf)	
End: 5	21.75	−27.80	21.75	−27.80
Interior: 4	21.75	−23.75	21.75	−23.75

C. Wind forces on studs
C.1 On end studs that have higher pressures
1. Positive $W = p_{net}$ (tributary area[2])
$= 21.75\ (14.7) = 319.73$ lb (inward)
2. Negative $W = p_{net}$ (tributary area)
$= -27.80(14.7) = -408.66$ lb (outward)

These are shown in Figure 4.13.

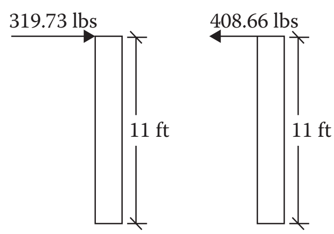


FIGURE 4.13 Wind force on end studs.

D. Wind pressures on rafters (roof)

1. Length of rafter $= \dfrac{25}{\cos 14°} = 25.76\,\text{ft}$

2. $A = (25.76)\left(\dfrac{16}{12}\right) = 34.35\,\text{ft}^2$

3. $A_{min} = \dfrac{L^2}{3} = \dfrac{(25.76)^2}{3} = 221\ \text{ft}^2, \text{use}\,100\ \text{ft}^2$

4. Net roof pressures at θ between 7° and 27°

Zone	p_{net30} at 100 ft² (psf)		$p_{net} = p_{net30}$ (psf)	
Corner 3	9.7	−44.0	9.7[a]	−44.0
End 2	9.7	−27.8	9.7[a]	−27.8
Interior 1	9.7	−19.8	9.7[a]	−19.8

[a] Use a minimum of 16 psf.

E. Wind forces on rafters
E.1 On end rafters
1. Positive $W = p_{net}$(tributary area) = 16 (34.35) = 549.6 lb (inward)
2. Negative $W = p_{net}$(tributary area) = −27.8(34.35) = −954.9 lb (outward)
3. These are shown in Figure 4.14.

[2] Use the tributary area, not the effective area.

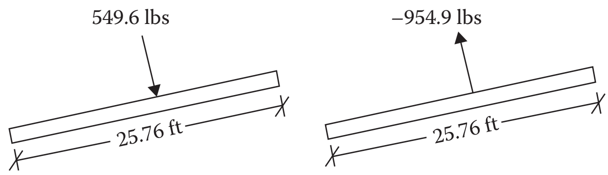


FIGURE 4.14 Wind force on end rafters.

PROBLEMS

4.1 A circular-shaped office building is located in downtown Boston, Massachusetts. It has a height of 160 ft, to which the lateral load is transferred to the MWFRS through the floor and roof system. The front-facing wall that receives the positive external pressure has an area of 1600 ft^2, of which 400 ft^2 is an open area. The other three side walls have a wall area of 1600 ft^2 and openings of 100 ft^2 each. Is this office building an open, partial open, or enclosed building? Which is the most appropriate MWFRS procedure to determine the wind loads?

4.2 A square 100-ft-high office building transfers loads through floors and roof systems to the walls and foundations. All wall sizes are 1000 ft^2, and there are openings of 200 ft^2 each. Is this office building a partial open or enclosed building? What is the most appropriate procedure to determine the wind loads?

4.3 Consider a 100 ft × 50 ft five-story building where the first three stories are 9 ft each and the other two stories are 8 ft each. It is located in a remote open countryside in the state of Maine. The roof slope is 8°. Determine the exposure category and the height adjustment factor.

4.4 Consider a four-story coastal building in Newport, Rhode Island, where the height of each floor is 12.5 ft. The width of the building is 50 ft and the roof slope is 14°. Determine the exposure category and the adjustment factor for height.

4.5 Determine the horizontal wind pressures and forces on the wall and the vertical pressures and forces acting on the roof due to wind acting in the transverse direction on an MWFRS, as shown in Figure P4.1. It is a standard occupancy single-story building located in an urban area in Rhode Island where the basic wind speed is 140 mph. $K_{zt} = 1$.

4.6 In Problem 4.5, determine the horizontal pressures and forces and the vertical pressures and forces in the longitudinal direction.

4.7 An enclosed two-story, heavily occupied building located in an open, flat terrain in Ohio is shown in Figure P4.2. Determine the wind pressures on the walls and roofs of the MWFRS in the transverse direction. Also determine the design wind forces in the transverse direction. $K_{zt} = 1$.

4.8 In Problem 4.7, determine the wind pressures and forces on the walls and roof in the longitudinal direction.

4.9 A three-story industrial steel building, shown in Figure P4.3, located in unobstructed terrain in Cape Cod, Massachusetts, has a plan dimension of 200 ft × 90 ft. The structure consists of nine moment-resisting steel frames spanning 90 ft at 25 ft in the center. It is roofed with steel deck, which is pitched at 1.25° on each side from the center. The building is 36 ft high, with each floor having a height of 12 ft; $K_{zt} = 1$. Determine the MWFRS horizontal and vertical pressures and the forces due to wind in the transverse direction of the building.

4.10 In Problem 4.9, determine the MWFRS horizontal and vertical pressures and the forces in the longitudinal direction.

4.11 The building in Problem 4.5 has wall studs and roof trusses spaced at 12 in. in the center. Determine the elemental wind pressures and forces on the studs and roof trusses.

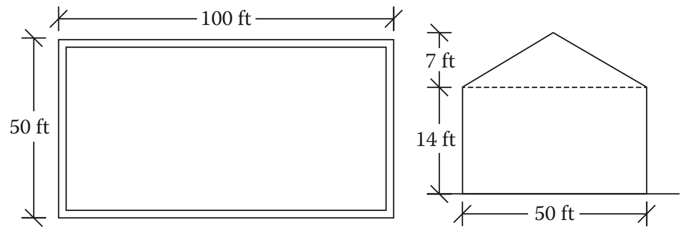


FIGURE P4.1 A single-story building in an urban area for Problem 4.5.

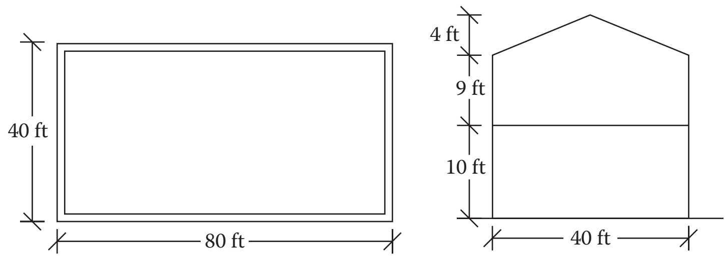


FIGURE P4.2 A two-story building in open terrain for Problem 4.7.

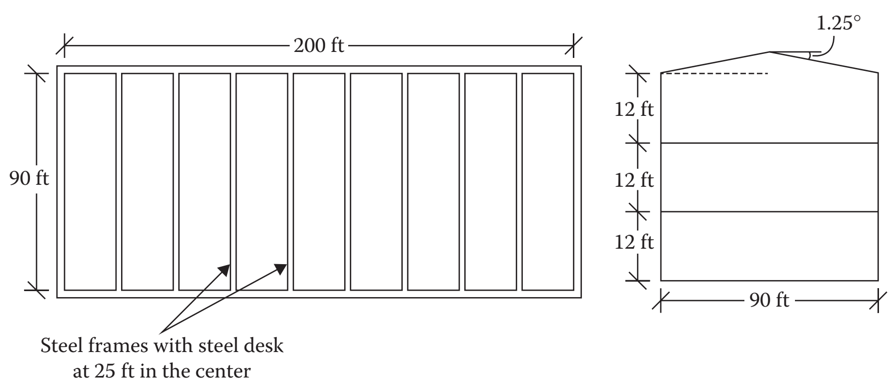


FIGURE P4.3 A three-story industrial building for Problem 4.9.

4.12 The building in Problem 4.7 has wall studs and roof trusses spaced at 16 in. in the center. Determine the elemental wind pressures and forces on the studs and roof trusses.

4.13 Determine the wind pressures and forces on the wall panel and roof decking from Problem 4.9. The decking is supported on joists that are 5 ft in the center, spanning across the steel frames shown in Figure P4.3.

5 Earthquake Loads

SEISMIC FORCES

The earth's outer crust is composed of very big, hard plates as large or larger than a continent. These plates float on the molten rock beneath. When these plates encounter each other, appreciable horizontal and vertical ground motion of the surface occurs, which is known as an *earthquake*. For example, in the western portion of the United States, an earthquake is caused by the two plates comprising the North American continent and the Pacific basin. The ground motion induces a very large inertia force known as the *seismic force* in a structure that often results in the destruction of the structure. The seismic force acts vertically like dead and live loads and laterally like wind load. But unlike the other forces that are proportional to the exposed area of the structure, the seismic force is proportional to the mass of the structure and is distributed in proportion to the structural mass at various levels.

In all other types of loads, including the wind load, the structural response is static wherein the structure is subjected to a pressure applied by the load. However, in a seismic load, there is no such direct applied pressure.

If ground movement could take place slowly, the structure would ride over it smoothly, moving along with it. But the quick movement of ground in an earthquake accelerates the mass of the structure. The product of the mass and acceleration is the internal force created within the structure. Thus, the seismic force is a dynamic entity.

SEISMIC DESIGN PROCEDURES

Seismic analyses have been dealt with in detail in ASCE 7-16 in 13 chapters, from Chapters 11 to 23. There are three approaches to evaluating seismic forces:

1. Modal response spectrum analysis
2. Seismic response history procedure
3. Equivalent lateral force analysis

The first two procedures are permitted to be applied to any type of structure; the third approach is applicable to structures that have no or limited structural irregularities.

In modal response spectrum analysis, an analysis is conducted to determine the natural modes of vibrations of the structure. For each mode, the force-related parameters are determined. The values of these design parameters for various modes are then combined by one of the three methods to determine the modal base shear.

The seismic response history procedure uses either a linear mathematical model of the structure or a model that accounts for the nonlinear hysteretic behavior of the structural elements. The model is analyzed to determine its response to the ground motion acceleration history compatible with the design response spectrum of the site.

In equivalent lateral force analysis, the seismic forces are represented by a set of supposedly equivalent static loads on a structure. It should be understood that no such simplified forces are fully equivalent to the complicated seismic forces, but it is considered that a reasonable design of a structure can be produced by this approach. This approach has been covered in the book in this chapter.

DEFINITIONS

Structural Height

Structural height, h_n, is the vertical distance from the base to the highest level of the seismic force–resisting system of the structure. For sloped roofs, it is from the base to the average height of the roof.[1]

Stories above Base and Grade Plane

Some seismic provisions in ASCE 7-16 refer to the number of stories (floors) *above the grade plane*, whereas some other provisions are based on the number of stories *above the base or including the basement*.

A *grade plane* is a horizontal reference datum that represents the average of the finished ground level adjoining the structure at all exterior walls. If the finished ground surface is 6 ft above the base of the building on one side and is 4 ft above the base on the other side, the grade plane is 5 ft above the base line.

Where the ground level slopes away from the exterior walls, the plane is established by the lowest points between the structure and the property line or, where the property line is more than 6 ft from the structure, between the structure and points 6 ft from the structure.

A *story above the grade plane* is a story in which the floor surface or roof surface at the top of the story and is more than 6 ft above the grade plane or is more than 12 ft above the lowest finished ground level at any point on the perimeter of the structure, as shown in Figure 5.1.

Thus, a building with four stories above the grade plane and a basement below the grade plane is a five-story building above the base.

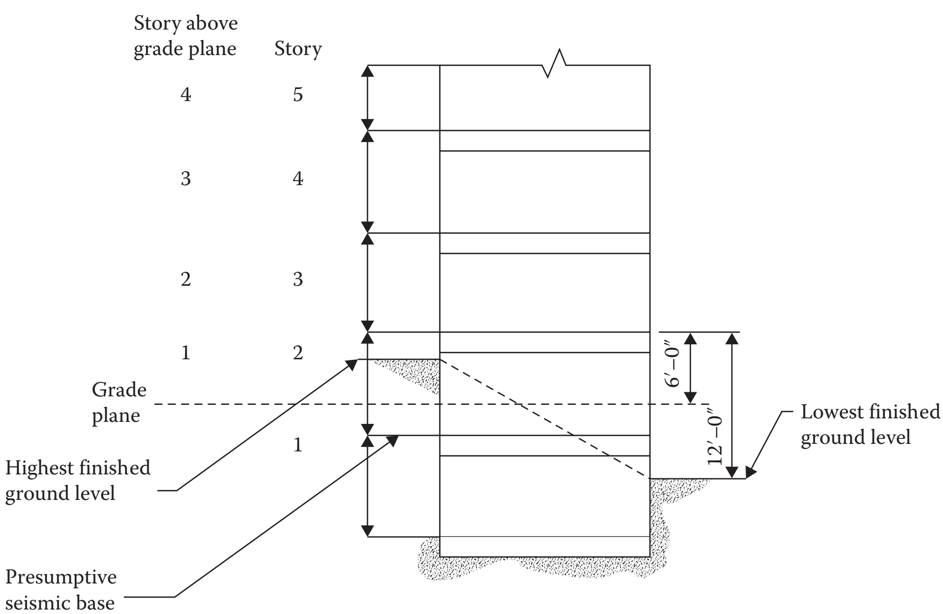


FIGURE 5.1 Story above grade plane and story above base.

[1] For wind loads, mean roof height, h, is measured from the ground surface.

Fundamental Period of Structure

The basic dynamic property of a structure is its fundamental period of vibration. When a mass of body (in this case, a structure) is given a horizontal displacement (in this case, due to an earthquake), the mass oscillates back and forth. This is termed the *free vibration*. The *fundamental period* is defined as the time (in seconds) it takes to go through one cycle of free vibration. The magnitude depends on the mass of the structure and its stiffness. It can be determined by theory. ASCE 7-16 provides the following formula to approximate the fundamental time T_a:

$$T_a = C_t h_n^{\,x} \tag{5.1}$$

where:

T_a is the approximate fundamental period in seconds
h_n is the height of the highest level of the structure above the base in ft. For a sloped roof, it is from the base to the average height of the roof.
C_t is the building period coefficient, as given in Table 5.1
x is the exponential coefficient, as given in Table 5.1

Example 5.1

Determine the approximate fundamental period for a five-story office building above the base, of moment-resisting steel. Each floor has a height of 12 ft.

Solution

1. Height of building from ground = 5 × 12 = 60 ft
2. $T_a = 0.028(60)^{0.8} = 0.74$ seconds

Site Classification

Based on the soil properties at a site, shear wave velocity, penetration resistance, and shear strength, a site is identified as site class A, B, C, D, E, or F in accordance with Table 5.2. Where soil properties are not known in sufficient detail, site class D is used. When site class D is the default class, the value of coefficient F_a, as discussed subsequently, shall not be less than 1.2. When soil conditions are consistent with site class B but the site-specific velocity measurements are not made, F_a and F_v are taken as 1.0.

Many restrictions are placed on the site classification; these are presented in Chapter 20 of ASCE 7-16.

TABLE 5.1
Value of Parameters C_t and x

Structure Type	C_t	x
Moment-resisting frame of steel	0.028	0.8
Moment-resisting frame of concrete	0.016	0.9
Braced steel frame	0.03	0.75
All other structures	0.02	0.75

TABLE 5.2
Soil Classification for Spectral Acceleration

Class	Type
A	Hard rock
B	Rock
C	Soft rock or very dense soil
D	Stiff soil
E	Soft soil
F	Requires site-specific evaluation

SEISMIC GROUND MOTION VALUES

Design values are based on national seismic hazard maps. These maps have been updated in ASCE 7-16, reflecting up to 20% changes in values compared to ASCE 7-10. ASCE 7-16 includes a new site-specific ground motion hazard analysis procedure for certain cases, as discussed later on below.

Mapped Acceleration Parameters

Two terms are applied to consider the most severe earthquake effects:

1. Maximum Considered Earthquake Geometric Mean (MCE_G)
   Peak Ground Acceleration
   The earthquake effects for this standard are determined for geometric mean peak ground acceleration without adjustment for targeted risk. MCE_G, adjusted for site class effects, is used for soil-related issues—liquefaction, lateral spreading, and settlement.
2. Risk-Targeted Maximum Considered Earthquake (MCE_R)
   Ground Motion Response Acceleration
   Earthquake effects for this standard are determined for the orientation that results in the largest maximum response to horizontal ground motions with adjustment for targeted risk. MCE_R, adjusted for site class effects, is used to evaluate seismic induced forces.

Risk-Targeted Maximum Considered Earthquake (MCE_R) Spectral Response Acceleration Parameters

At the onset, the risk-adjusted maximum considered earthquake (MCE_R)[2] ground motion parameters for a place are read from the spectral maps of the United States. There are two types of mapped accelerations: (1) short-period (0.2 seconds) spectral acceleration, S_s, which is used to study the acceleration-controlled portion of the spectra, and (2) 1-second spectral acceleration, S_1, which is used to study the velocity-controlled portion of the spectra. These acceleration parameters represent 5% damped ground motions at 2% probability of exceedance in 50 years. The maps for the conterminous United States, reproduced from Chapter 22 of ASCE 7-16, are given in Figures 5.2 and 5.3. These maps and the maps for Alaska, Hawaii, Puerto Rico, and the Virgin Islands are also available at the United States Geological Survey (USGS) site at http://eathquake.usgs.gov/designmaps. The values given in Figures 5.2 and 5.3 are percentages of the gravitational constant, g; that is, 200 means 2.0 g.

ASCE has developed a hazard tool at the following site: https://asce7hazardtool.online/. This tool can be utilized to ascertain values of seismic design parameters, as well as wind and snow design parameters by entering latitude and longitude or the address of a site.

[2] For practical purposes, it represents the maximum earthquake that can reasonably occur at the site.

Adjustments to Spectral Response Acceleration Parameters for Site Class Effects

The mapped values of Figures 5.2 and 5.3 are adjusted as follows:

$$S_{MS} = F_a S_s \tag{5.2}$$

$$S_{M1} = F_v S_1 \tag{5.3}$$

where:

S_{MS} and S_{M1} are adjusted short-period and 1 s spectral accelerations
F_a and F_v are site coefficients for short and 1 s spectra, as given in Tables 5.3 and 5.4

The factor F_a is 0.8 for soil class A, 0.9 for soil class B, and higher than 1 for soils C onward, up to 2.4 for soil type E.

The factor F_v is 0.8 for soils A and B and higher than 1 for soils C onward, up to 4.2 for soil E.

A reference is made to section 11.4.8 of ASCE 7-16 for soil F.

Design Spectral Acceleration Parameters

Design spectral acceleration parameters are the primary variables to prepare the design spectrum. The design spectral accelerations are two-thirds of the adjusted acceleration, as follows:

$$S_{DS} = \frac{2}{3} S_{MS} \tag{5.4}$$

$$S_{D1} = \frac{2}{3} S_{M1} \tag{5.5}$$

where S_{DS} and S_{D1} are short-period and 1 s design spectral accelerations.

Design Response Spectrum

The design response spectrum is a graph that shows the design value of the spectral acceleration for a structure based on the fundamental period. A generic graph is shown in Figure 5.4; from this graph, a site-specific graph is created based on the mapped values of accelerations and the site soil type.

The controlling time steps at which the shape of the design response spectrum graph changes are as follows:

1. Initial period

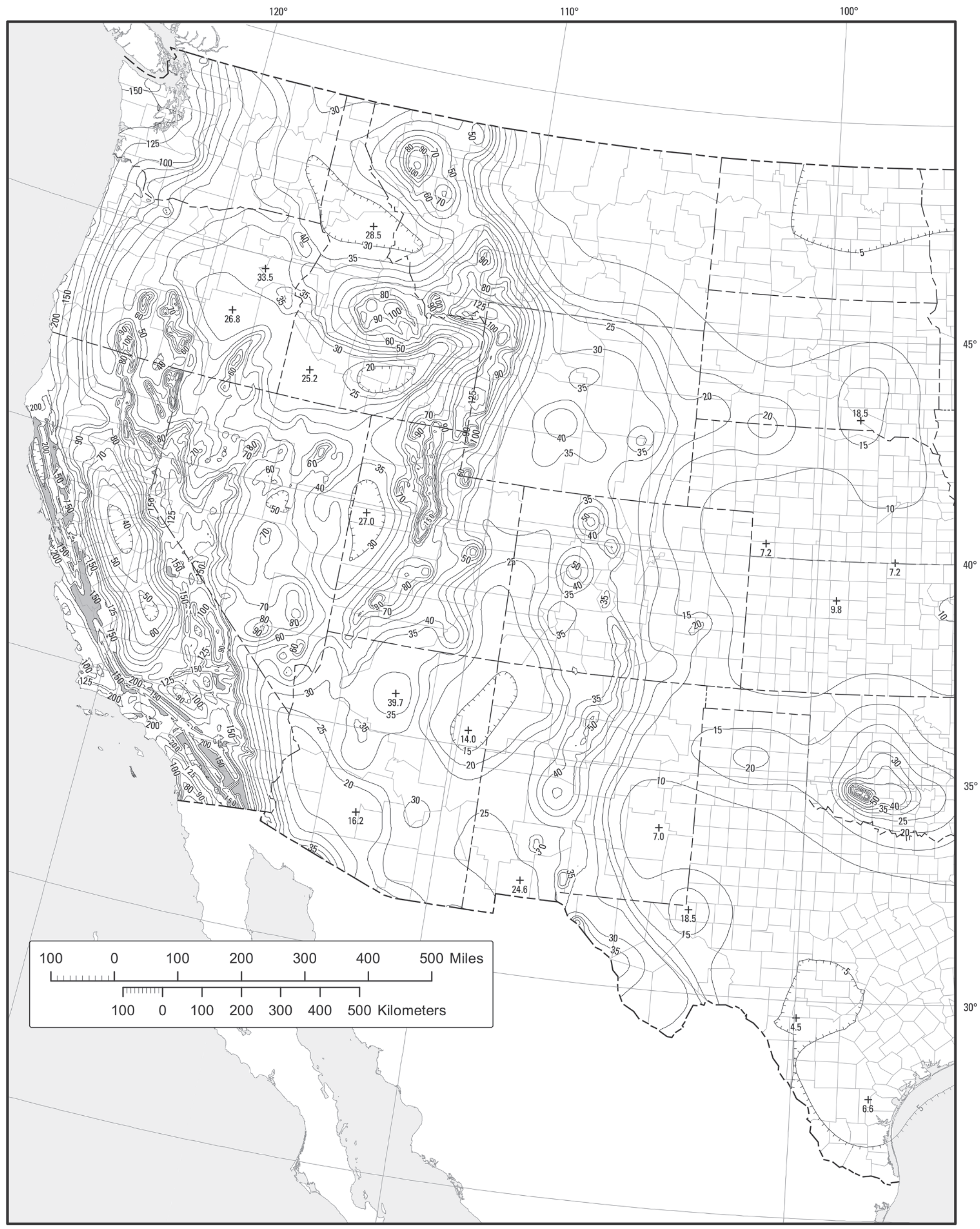
$$T_0 = 0.2 \frac{S_{D1}}{S_{DS}} \tag{5.6}$$

2. Short-period transition for small structures

$$T_s = \frac{S_{D1}}{S_{DS}} \tag{5.7}$$

3. Long-period transition for large structures

T_L is shown in Figure 5.5, which is reproduced from Figures 22.14 through 22.17 of ASCE 7-16.



Notes:

Maps prepared by United States Geological Survey (USGS) in collaboration with the Federal Emergency Management Agency (FEMA)-funded Building Seismic Safety Council (BSSC) and the American Society of Civil Engineers (ASCE). The basis is explained in commentaries prepared by BSSC and ASCE and in the references.

Ground motion values contoured on these maps incorporate:

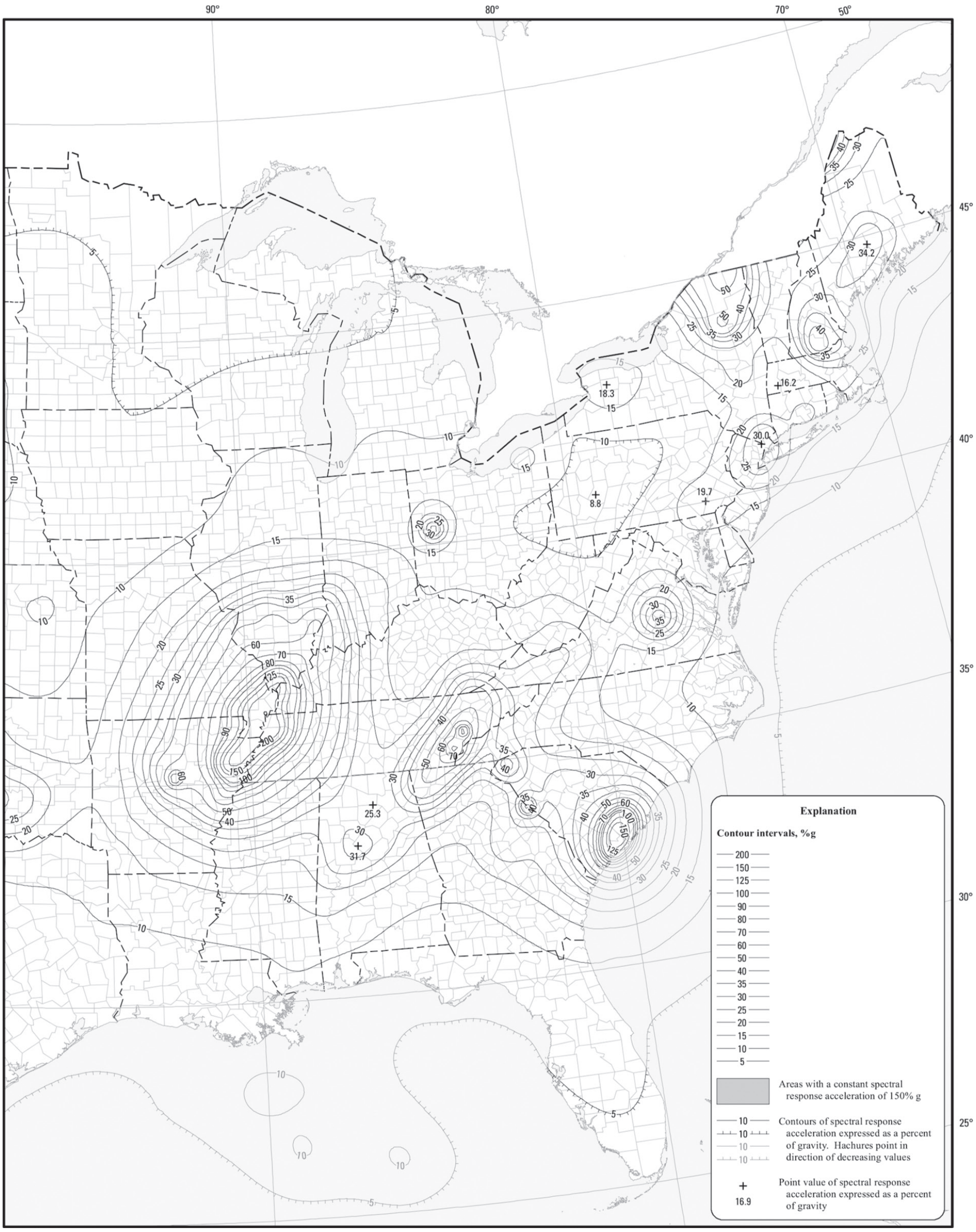
- a target risk of structural collapse equal to 1% in 50 years based upon a generic structural fragility
- a factor of 1.1 to adjust from a geometric mean to the maximum response regardless of direction
- deterministic upper limits imposed near large, active faults, which are taken as 1.8 times the estimated median response to the characteristic earthquake for the governing fault (1.8 is used to represent the 84th percentile response), but not less than 150% g.

As such, the values are different from those on the uniform-hazard 2014 USGS National Seismic Hazard Maps posted at: https://doi.org/10.5066/F7HT2MHG.

Larger, more detailed versions of these maps are not provided because it is recommended that the corresponding USGS web tool (https://doi.org/10.5066/F7NK3C76) be used to determine the mapped value for a specified location.

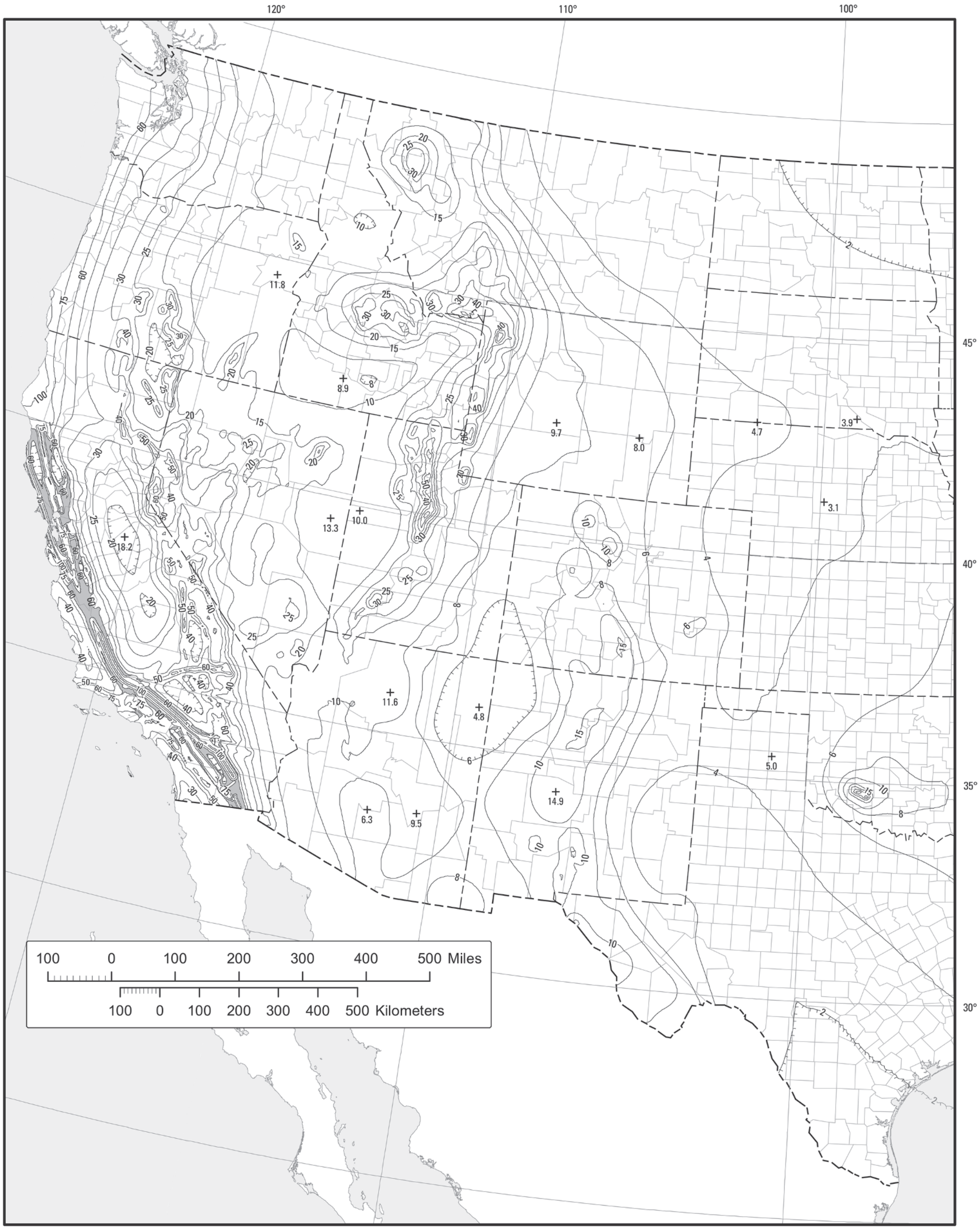
(a)

FIGURE 5.2 Short period (S_s) Risk-targeted maximum considered earthquake (MCE_R) ground motion parameter for the conterminous United States for 0.2 s spectral response acceleration (5% of Critical Damping), site class B. (*Continued*)



(b)

FIGURE 5.2 (*Continued*) Short period (S_s) Risk-targeted maximum considered earthquake (MCE_R) ground motion parameter for the conterminous United States for 0.2 s spectral response acceleration (5% of Critical Damping), site class B.



Notes:

Maps prepared by United States Geological Survey (USGS) in collaboration with the Federal Emergency Management Agency (FEMA)-funded Building Seismic Safety Council (BSSC) and the American Society of Civil Engineers (ASCE). The basis is explained in commentaries prepared by BSSC and ASCE and in the references.

Ground motion values contoured on these maps incorporate:

- a target risk of structural collapse equal to 1% in 50 years based upon a generic structural fragility
- a factor of 1.3 to adjust from a geometric mean to the maximum response regardless of direction
- deterministic upper limits imposed near large, active faults, which are taken as 1.8 times the estimated median response to the characteristic earthquake for the governing fault (1.8 is used to represent the 84th percentile response), but not less than 60% g.

As such, the values are different from those on the uniform-hazard 2014 USGS National Seismic Hazard Maps posted at: https://doi.org/10.5066/F7HT2MHG.

Larger, more detailed versions of these maps are not provided because it is recommended that the corresponding USGS web tool (https://doi.org/10.5066/F7NK3C76) be used to determine the mapped value for a specified location.

(a)

FIGURE 5.3 One second (S_1) Risk-targeted maximum considered earthquake (MCE_R) ground motion parameter for the conterminous United States for 1.0 s spectral response acceleration (5% of Critical Damping), site class B. (*Continued*)

(b)

FIGURE 5.3 (*Continued*) One second (S_1) Risk-targeted maximum considered earthquake (MCE_R) ground motion parameter for the conterminous United States for 1.0 s spectral response acceleration (5% of Critical Damping), site class B.

TABLE 5.3
Site Coefficient, F_a

Site Class	MCE$_R$ at Short Period				
	$S_S \leq 0.25$	$S_S = 0.5$	$S_S = 0.75$	$S_S = 1.0$	$S_S \geq 1.25$
A	0.8	0.8	0.8	0.8	0.8
B	0.9	0.9	0.9	0.9	0.9
C	1.3	1.3	1.2	1.2	1.2
D	1.6	1.4	1.2	1.1	1.0
E	2.4	1.7	1.3	See Section 11.4.8 of ASCE 7-16.	See Section 11.4.8 of ASCE 7-16
F	See Section 11.4.8 of ASCE 7-16.				

Note: Use straight-line interpolation for intermediate values of S_S.

TABLE 5.4
Site Coefficient, F_v

Site Class	MCE$_R$ at 1-Second Period					
	$S_1 \leq 0.1$	$S_1 = 0.2$	$S_1 = 0.3$	$S_1 = 0.4$	$S_1 = 0.5$	$S_1 \geq 0.6$
A	0.8	0.8	0.8	0.8	0.8	0.8
B	0.8	0.8	0.8	0.8	0.8	0.8
C	1.5	1.5	1.5	1.5	1.5	1.4
D	2.4	2.2	2.0	1.9	1.8	1.7
E	4.2	Sec 11.4.8	Sec 11.4.8	Sec 11.4.8	Sec 11.4.8	Sec 11.4.8
F	See Section 11.4.8 of ASCE 7-16.					

Note: Use straight-line interpolation for intermediate values of S_1.

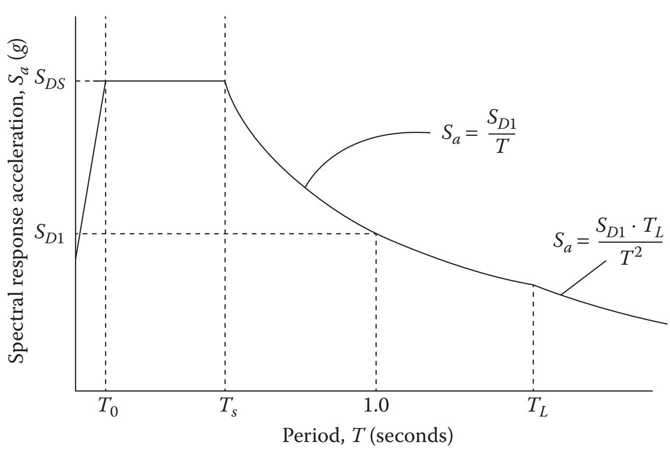


FIGURE 5.4 Design response spectrum.

The characteristics of the design response spectrum are as follows:

1. For the fundamental period, T_a, having a value between 0 and T_0, the design spectral acceleration, S_a, varies as a straight line from a value of 0.4 S_{DS} and S_{DS}, as shown in Figure 5.4, expressed by:

$$S_a = S_{DS}\left(0.4 + 0.6\frac{T}{T_0}\right) \tag{5.8}$$

2. For the fundamental period, T_a, having a value between T_0 and T_s, the design spectral acceleration, S_a, is constant at S_{DS}.
3. For the fundamental period, T_a, having a value between T_s and T_L, the design spectral acceleration, S_a, is given by:

$$S_a = \frac{S_{D1}}{T} \tag{5.9}$$

where T is the time period between T_s and T_L.
4. For the fundamental period, T_a, having a value larger than T_L, the design spectral acceleration, S_a, is given by:

$$S_a = \frac{S_{D1}T_L}{T^2} \tag{5.10}$$

The complete design response spectrum[3] graph is shown in Figure 5.4.

Example 5.2

At a location in California, the mapped values of the MCE$_R$ accelerations S_s and S_1 are 1.5 g and 0.66 g, respectively. The site soil class is D. Prepare the design spectral response curve for this location.

Solution

1. Adjustment factors for soil class D are as follows:

$F_a = 1.0$

$F_v = 1.7$

2. $S_{MS} = F_a S_s$

$= (1.0)(1.5) = 1.5$ g

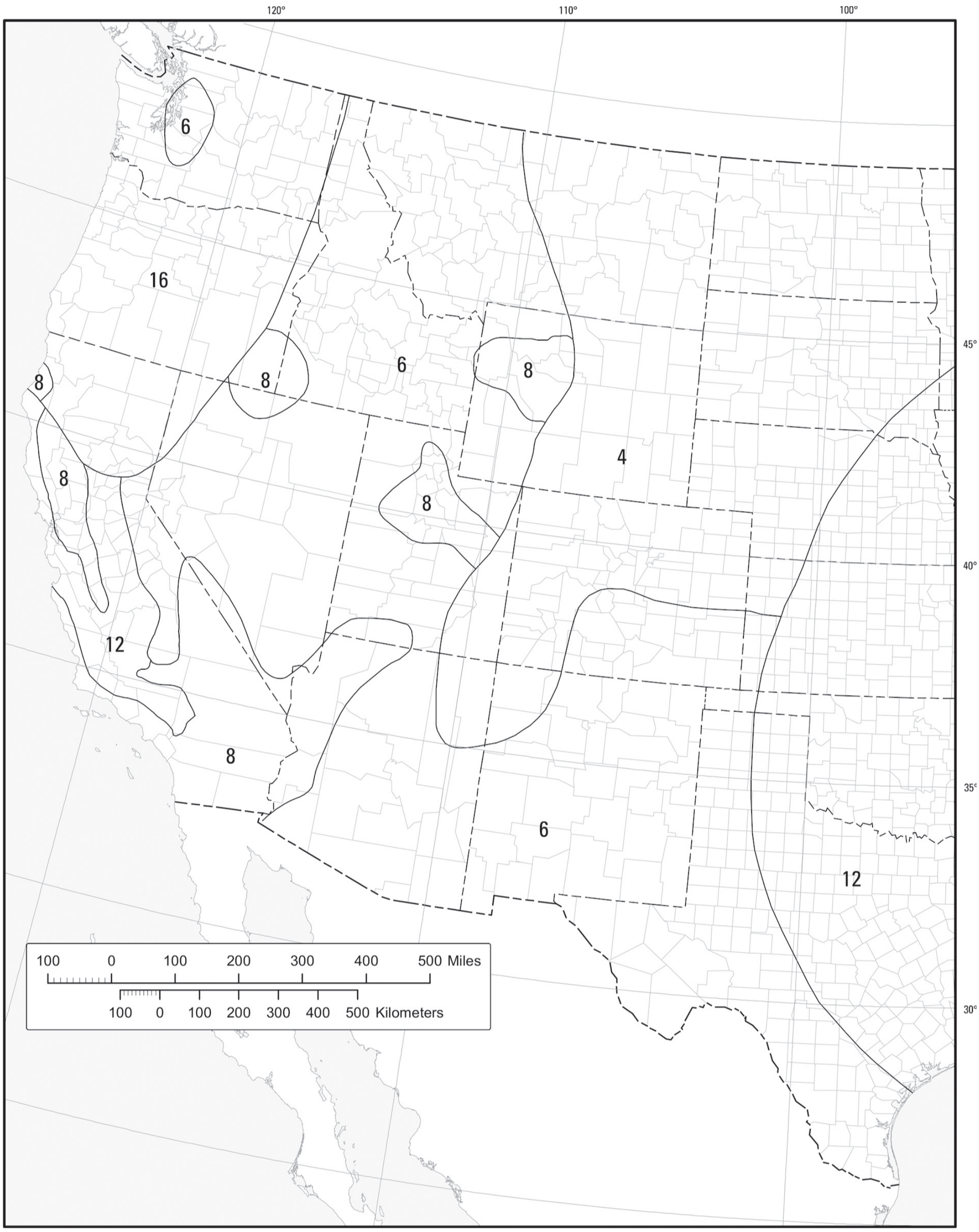
$S_{M1} = F_v S_1$

$= (1.7)(0.66) = 1.12$ g

3. $S_{DS} = \frac{2}{3}S_{MS} = \frac{2}{3}(1.5) = 1$ g

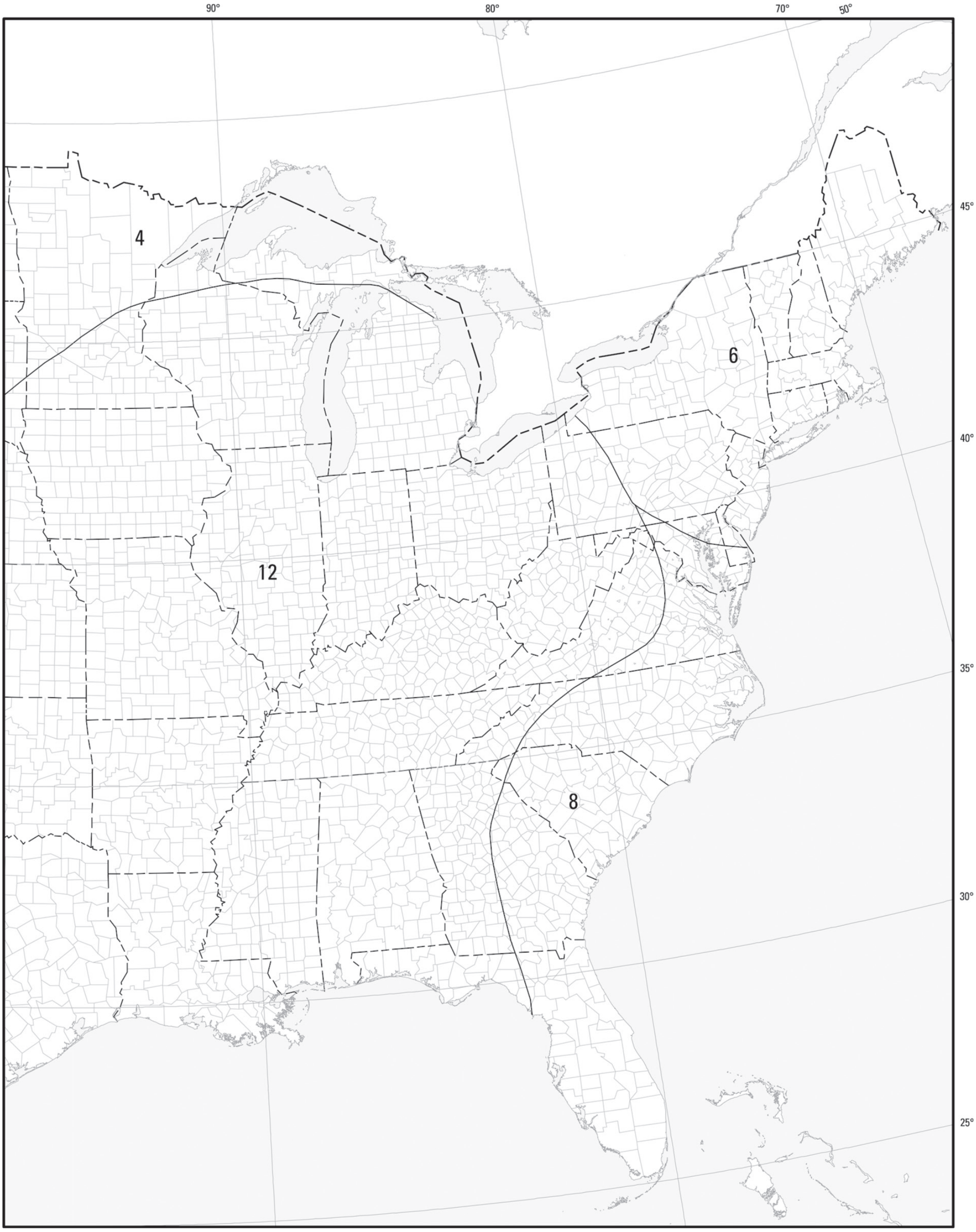
$S_{D1} = \frac{2}{3}S_{M1} = \frac{2}{3}(1.12) = 0.75$ g

[3] Where an MCE$_R$ response spectrum is required, multiply the design response spectrum, S_a, by 1.5.



(a)

FIGURE 5.5 Mapped long-period transition period, T_L (s), for the conterminous United States. *(Continued)*



(b)

FIGURE 5.5 (*Continued*) Mapped long-period transition period, T_L (s), for the conterminous United States.

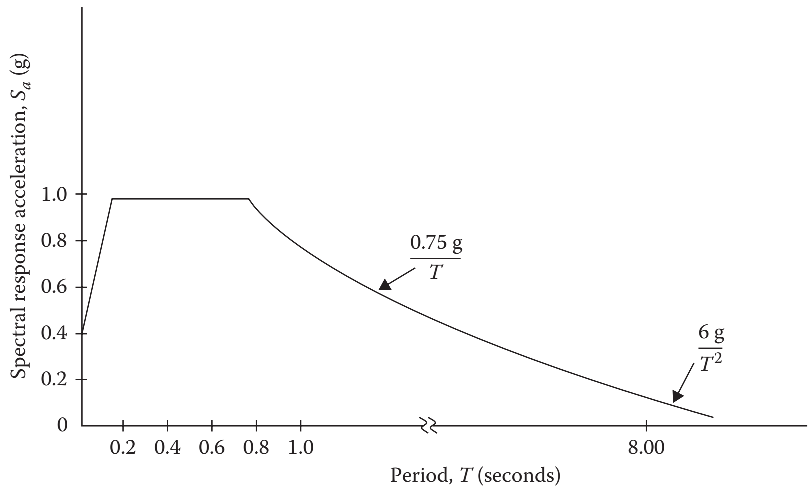


FIGURE 5.6 Design spectral acceleration, Example 5.2.

4. $T_0 = \dfrac{0.2\, S_{D1}}{S_{DS}} = \dfrac{0.2(0.75)}{1} = 0.15$ s

   $T_s = \dfrac{S_{D1}}{S_{DS}} = \dfrac{0.75}{1} = 0.75$ s

   $T_L = 8$ s (from Figure 5.5)

5. The design spectral acceleration at time 0 is 0.4 (1 g) or 0.4 g. It linearly rises to 1 g at time 0.15 seconds. It remains constant at 1 g up to time 0.75 seconds. From time 0.75–8 seconds, it drops at a rate of 0.75 g/*T*. At 0.75 seconds, it is 0.75 g/0.75 = 1 g, and it progresses to a value of 0.75 g/8 = 0.094 g at time 8 seconds. Thereafter, it drops at a rate of $S_{D1}T_L/T^2$ or 6 g/T^2. This is shown in Figure 5.6.

SITE-SPECIFIC GROUND MOTION PROCEDURE

For structures on site class F (liquefiable soils, sensitive clays, and collapsible weak cement soils), site class E with $S_s \geq 1$ and $S_1 \geq 0.2$, and site class D with $S_1 \geq 0.2$, a new procedure of site-specific analysis has to be performed as specified in Section 21.1 of ASCE 7-16.

However, for site class D and site class E, exceptions to the required site-specific procedure have been provided for the following conditions:

1. For class E sites with $S_s \geq 1$, the site coefficient, F_a, is taken as equal to that of class C sites.
2. For class E sites when $S_1 \geq 0.2$, and the fundamental period $T_a \leq T_s$ in equivalent static force procedure.
3. For class D sites with $S_1 \geq 0.2$, if C_s is determined as follows:
   a. For $T_a \leq 1.5\ T_s$, C_s is determined by Equation 5.12.
   b. For $T_a > 1.5\ T_s$ and $\leq T_L$, C_s is taken as 1.5 times the value computed by Equation 5.13.
   c. For $T_a > T_L$, C_s is taken as 1.5 times the value computed by Equation 5.14.

TABLE 5.5
Importance Factor for Seismic Coefficient

Risk Category	Importance Factor
I and II	1.0
III	1.25
IV	1.5

IMPORTANCE FACTOR, *I*

The importance factor, I, for a seismic coefficient, which is based on the risk category of the structure, is indicated in Table 5.5. The risk category is given in Table 1.1 in Chapter 1.

SEISMIC DESIGN CATEGORY

A structure is assigned a seismic design category (SDC) from A through F based on the risk category of the structure and the design spectral response acceleration parameters, S_{DS} and S_{D1}, of the site. The seismic design categories are given in Tables 5.6 and 5.7. A structure is assigned to the severest category determined from the two tables except for the following cases:

1. When $S_1 \leq 0.04$ and $S_s \leq 0.15$, a structure is assigned category A.
2. When $S_1 \geq 0.75$ g, a structure is assigned category E for risk categories I, II, and III and assigned category F for risk category IV.
3. When $S_1 < 0.75$ g and certain conditions of the small structure are met, as specified in 11.6 of ASCE 7-16, only Table 5.6 is applied.

TABLE 5.6
SDC Based on S_{DS}

	Risk Category		
S_{DS} Range	I or II (Low-Risk and Standard Occupancy)	III (High Occupancy)	IV (Essential Occupancy)
0 to <0.167 g	A	A	A
0.167 g to <0.33 g	B	B	C
0.33 g to <0.5 g	C	C	D
≥ 0.5 g	D	D	D
When $S_1 \geq 0.75$ g	E	E	F

TABLE 5.7
SDC Based on S_{D1}

Range S_{D1}	I or II (Low-Risk and Standard Occupancy)	III (High Occupancy)	IV (Essential Occupancy)
0 to <0.067 g	A	A	A
0.067 g to <0.133 g	B	B	C
0.133 g to <0.20 g	C	C	D
≥0.2 g	D	D	D
When S_1 ≥0.75 g	E	E	F

EXEMPTIONS FROM SEISMIC DESIGNS

ASCE 7-16 exempts the following structures from seismic design requirements:

1. The structures belonging to SDC A; these need to comply only to the requirements of the "Continuous Load Path for Structural Integrity" section of Chapter 1.
2. Detached one- and two-family dwellings in SDC A, SDC B, and SDC C or where $S_s < 0.4$.
3. Conventional wood-frame one- and two-family dwellings up to two stories in any seismic design category.
4. Agriculture storage structures used only for incidental human occupancy.

EQUIVALENT LATERAL FORCE (ELF) PROCEDURE TO DETERMINE SEISMIC FORCE

The design base shear, V, due to seismic force is expressed as:

$$V = C_s W \tag{5.11}$$

where:

W is the effective dead weight of structure, discussed in the "Effective Weight of Structure, W" section

C_s is the seismic response coefficient, discussed in the "Seismic Response Coefficient, C_s" section

Effective Weight of Structure, *W*

Generally, the effective weight of structure is taken as the dead load of the structure. However, where a structure carries a large live load, a portion is included in W. For a storage warehouse, 25% of the floor live load is included with the dead load in W. Where provision for partitions is required, the actual partition weight or a minimum of 10 psf of floor area, whichever is greater, is added to W. When the flat roof snow load exceeds 30 psf, 20% of the snow load is included in W.

Seismic Response Coefficient, C_s

Besides depending on the fundamental period and design spectral accelerations, C_s is a function of the importance factor and the response modification factor. The importance factor, I, is given in Table 5.5. The response modification factor, R, is discussed in the "Response Modification Factor or Coefficient, R" section.

The seismic response coefficient, C_s is determined as follows:

1. For $T_a \le T_s$ of Equation 5.7:

$$C_s = \frac{S_{DS}}{(R/I)} \tag{5.12}$$

2. For $T_a > T_s$ and $\le T_L$:

$$C_s = \frac{S_{D1}}{T(R/I)} \tag{5.13}$$

3. For $T_a > T_L$:

$$C_s = \frac{S_{D1}\, T_L}{T^2\,(R/I)} \tag{5.14}$$

Minimum Value of C_s

1. C_s should not be less than the following or 0.01, whichever is larger:

$$\text{Minimum } C_s = 0.044\, S_{DS}\, I \tag{5.15}$$

2. When $S_1 \geq 0.6$ g, C_s should not be less than the following:

$$\text{Minimum } C_s = \frac{0.5 S_1}{(R / I)} \tag{5.16}$$

The values of C_s for different periods are shown in Figure 5.7.

Maximum S_{DS} Value in Determining C_s

The value of C_s is calculated using S_{DS} equal to 1 but not less than 70% of S_{DS} calculated by Equation (5.4), provided all of the following conditions are satisfied:

1. The structure does not have irregularities.
2. The structure does not exceed five stories above the base.
3. The structure is classified as risk category I or II.
4. The site soil properties are not site class E or F.
5. The fundamental period $T \leq 0.5$ s.
6. The redundancy factor ρ, as defined subsequently, is taken as 1.

Response Modification Factor or Coefficient, R

The response modification factor accounts for the following:

1. Ductility, which is the capacity to withstand stresses in the inelastic range
2. Over strength, which is the difference between the design load and the failure load
3. Damping, which is the resistance to vibration by the structure
4. Redundancy, which is an indicator that a component's failure does not lead to failure of the entire system

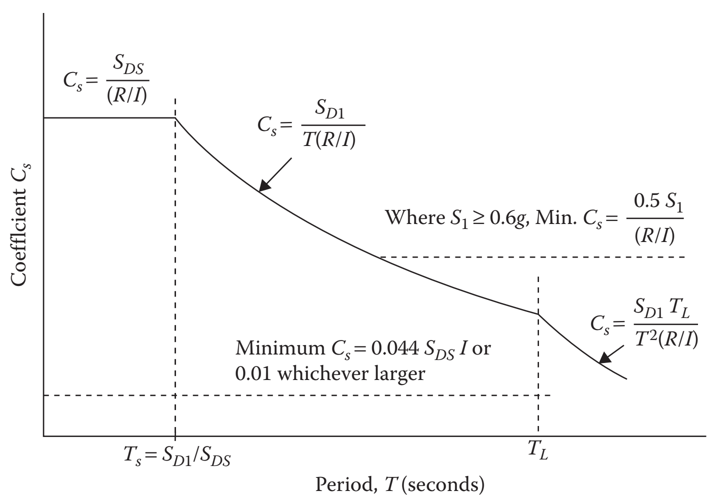


FIGURE 5.7 Seismic response coefficient for base shear.

A large value for the response modification factor reduces the seismic response coefficient and hence the design shear. The factor ranges from 1 to 8. Ductile structures have a higher value, and brittle ones have a lower value. Braced steel frames with moment-resisting connections have the highest value, and concrete and masonry shear walls have the smallest value. For wood-frame construction, the common R factor is 6.5 for wood and light metal shear walls and 5 for special reinforced concrete shear walls. An exhaustive list is provided in Table 12.2-1 in ASCE 7-16.

Example 5.3

The five-story moment-resisting steel building of Example 5.1 is located in California, where S_s and S_1 are 1.5 g and 0.66 g, respectively. The soil class is D. Determine (1) the SDC and (2) the seismic response coefficient, C_s.

Solution

1. From Example 5.1:
   T_a = 0.74 seconds
2. From Example 5.2:
   S_{DS} = 1 g and S_{D1} = 0.75 g
   T_o = 0.15 s and T_s = 0.75 s
3. To compute the SDC:
   a. Risk category II
   b. From Table 5.6, for $S_1 \geq 0.75$ g and category II, SDC is E
4. To compute the seismic coefficient:
   a. Importance factor from Table 5.5, $I = 1$
   b. Response modification factor, $R = 8$
   c. T_a (of 0.74 seconds) < T_s (of 0.75 seconds)
   d. From Figure 5.7, for $T_a < T_s$, $C_s = S_{DS}/(R/I)$

$$C_s = \frac{1}{(8/1)} = 0.125\text{ g}$$

The maximum S_{DS} limit for the C_S value does not apply.

DISTRIBUTION OF SEISMIC FORCES

The seismic forces are distributed throughout the structure in reverse order. The shear force at the base of the structure is computed from the base shear (see Equation 5.10). Then story forces are assigned at the roof and floor levels by distributing the base shear force over the height of the structure.

The primary lateral force–resisting system consists of horizontal and vertical elements. In conventional buildings, the horizontal elements consist of the roof and floors, which act as horizontal diaphragms. The vertical elements consist of studs and end shear walls.

The seismic force distribution for vertical elements (e.g., walls), designated by F_x, is different from the force distribution for horizontal elements, designated by F_{px}, which is applied to the design of the horizontal components. It should be understood that both F_x and F_{px} are horizontal forces that are distributed differently at each story level. The forces acting on horizontal elements at different levels are not additive, whereas all of the story forces on vertical elements are considered to be acting concurrently and are additive from top to bottom.

Distribution of Seismic Forces on Vertical Wall Elements

The distribution of horizontal seismic forces acting on a vertical element (wall) is shown in Figure 5.8. The lateral seismic force induced at any level is determined from the following equations:

$$F_x = C_{vx}V \tag{5.17}$$

and

$$C_{vx} = \frac{W_x h_x^k}{\sum W_i h_i^k} \tag{5.18}$$

Substituting Equation 5.18 in Equation 5.17, we obtain

$$F_x = \frac{(Vh_x^k)W_x}{\sum W_i h_i^k} \tag{5.19}$$

where:

- i is the index for floor level ($i = 1$ for the first level, and so on)
- F_x is the horizontal seismic force on vertical elements at floor level x
- C_{vx} is the vertical distribution factor
- V is shear at the base of the structure (from Equation 5.11)
- W_i or W_x is the effective seismic weight of the structure at index level i or floor level x
- h_i or h_x is the height from the base to index level i or floor x
- k is an exponent related to the fundamental period of structure, T_a, as follows: (1) for $T_a \leq 0.5$ s, $k = 1$ and (2) for $T_a > 0.5$ s, $k = 2$

The total shear force, V_x, in any story is the sum of F_x from the top story up to the x story. The shear force of an x story level, V_x, is distributed among the various vertical elements in that story on the basis of the relative stiffness of the elements.

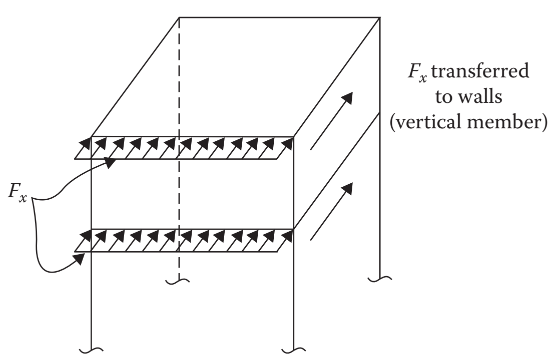


FIGURE 5.8 Distribution of horizontal seismic force to vertical elements.

Distribution of Seismic Forces on Horizontal Elements (Diaphragms)

The horizontal seismic forces transferred to the horizontal components (diaphragms) are shown in Figure 5.9. The floor and roof diaphragms are designed to resist the following minimum seismic force at each level:

$$F_{px} = \frac{\sum_{i=x}^{n} F_i W_{px}}{\sum_{i=x}^{n} W_i} \tag{5.20}$$

where:

F_{px} is the diaphragm design force

F_i is the lateral force applied to level i, which is the summation of F_x from level x (being evaluated) to the top level

W_{px} is the effective weight of the diaphragm at level x (The weight of walls parallel to the direction of F_{px} need not be included in W_{px}.)

W_i is the effective weight at level i, which is the summation of weight from level x (being evaluated) to the top

The force determined by Equation 5.20 is subject to the following two conditions:
The force should not be more than

$$F_{px}(\text{max}) = 0.4 S_{DS} I W_{px} \tag{5.21}$$

The force should not be less than

$$F_{px}(\text{min}) = 0.2 S_{DS} I W_{px} \tag{5.22}$$

DESIGN EARTHQUAKE LOAD IN LOAD COMBINATIONS

An earthquake causes horizontal accelerations as well as vertical accelerations. Accordingly, the earthquake load has two components. In load combinations, it appears in the following two forms:

$$E = E_{horizontal} + E_{vertical} \quad \text{(in Equation 1.25)} \tag{5.23}$$

and

$$E = E_{horizontal} - E_{vertical} \quad \text{(in Equation 1.27)} \tag{5.24}$$

$E_{horizontal}$ is combined with horizontal forces, and $E_{vertical}$ is combined with vertical forces.

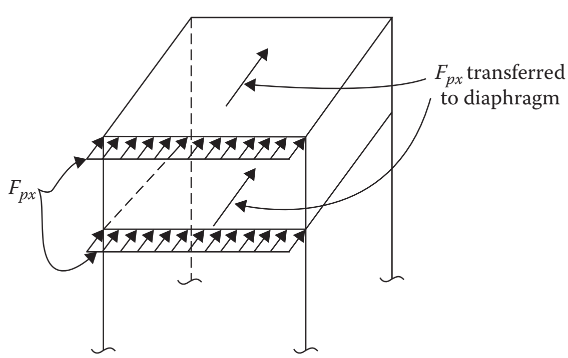


FIGURE 5.9 Distribution of horizontal seismic force to horizontal elements.

The seismic forces are at the load resistance factor design (strength) level and have a load factor of 1. To be combined for the allowable stress design, these should be multiplied by a factor of 0.7.

The horizontal seismic load effect is determined as follows:

$$E_{horizontal} = \rho Q_E \tag{5.25}$$

where:

Q_E is the horizontal seismic forces F_x or F_{px} as determined in the "Distribution of Seismic Forces" section

ρ is the redundancy factor

For F_x forces, the redundancy factor, ρ, is 1.0 for seismic design categories A, B, and C. It is 1.3 for SDC D, SDC E, and SDC F, except for special conditions. The redundancy factor is always 1.0 for F_{px} forces.

Vertical Seismic Load Effect ($E_{vertical}$)

The vertical seismic load effect can be determined as follows:

$$E_{vertical} = 0.2\, S_{DS}\, W \tag{5.26}$$

where:

W is dead load W_x, as determined in the "Distribution of Seismic Forces" section above.

ASCE 7-16, now provides for an optional vertical ground motions procedure in lieu of Equation 5.26 to determine $E_{vertical}$ for seismic design categories C, D, E, and F. This procedure develops MCE$_R$ vertical design response spectrum similar to the design spectrum shown in Figure 5.4.

Maximum S_{DS} Value in Determining $E_{vertical}$

The value of $E_{vertical}$ is calculated using S_{DS} equal to 1, but not less than 70% of S_{DS} calculated by Equation 5.4 provided all of the conditions listed under "Maximum S_{DS} Value in Determining C_S" are met.

Example 5.4

The two-story wood-frame essential facility shown in Figure 5.10 is located near Salem, Oregon. The structure is a bearing wall system with reinforced shear walls. The loads on the structures are as follows:

Roof dead load (DL) = 20 psf (in horizontal plane)
Floor dead load (DL) = 15 psf
Partition live load (PL) = 15 psf
Exterior wall dead load (DL) = 60 psf

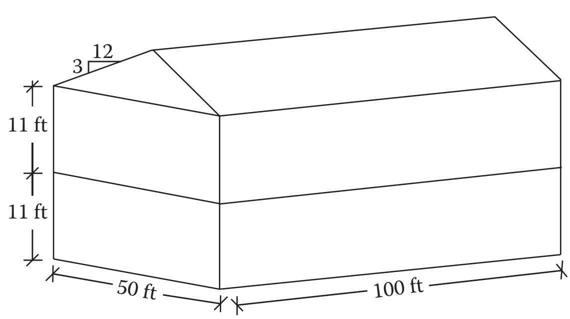


FIGURE 5.10 A two-story wood frame structure.

Determine the earthquake loads acting on the vertical elements of the structure.

Solution

A. Design parameters
   1. Risk category = Essential, IV
   2. Importance factor from Table 5.5 for IV category = 1.5
   3. Mapped MCE_R response accelerations:
      $S_s = 1$ g and $S_1 = 0.4$ g
   4. Site soil class (default) = D
   5. Seismic force–resisting system: Bearing wall with reinforced shear walls
   6. Response modification coefficient = 5

B. Seismic response parameters
   1. Fundamental period (from Equation 5.1) $T_a = C_t h^x$. From Table 5.1, $C_t = 0.02$, $x = 0.75$.

      $T_a = 0.02(25.125)^{0.75} = 0.224$ s

   2. For the default selection of class D, $F_a = 1.2$

      $S_{MS} = F_a S_s = 1.2(1) = 1.2$ g

   3. From Table 5.4, $F_v = 1.9$

      $S_{M1} = F_v S_1 = 1.9(0.4) = 0.76$ g

   4. $S_{DS} = \frac{2}{3} S_{MS} = \frac{2}{3}(1.2) = 0.80$ g

      $S_{D1} = \frac{2}{3} S_{M1} = \frac{2}{3}(0.76) = 0.51$g

   5. Based on risk category and S_{DS}, SDC = D. Based on risk category and S_{D1}, SDC = D.
   6. $T_s = S_{D1}/S_{DS} = 0.51/0.80 = 0.64$ s
      Since $T_a < T_s$, $C_s = S_{DS}/(R/I) = 0.80$ g$/(5/1.5) = 0.24$ g[4]. The maximum S_{DS} limitation for C_s does not apply.

C. Effective seismic weight at each level
   1. W at roof level[5]
      i. Area(roof DL) = $(50 \times 100)(20)/1000 = 100$ k
      ii. 2 Longitudinal walls = 2(wall area)(wall DL)
          $= \frac{2(100 \times 11)(60)}{1000} = 132$ k
      iii. 2 End walls = 2(wall area)(DL)
          $= \frac{2(50 \times 1)(60)}{1000} = 66$ k

          Total = 298 k
   2. W at second floor[6]
      i. Area(floor DL + partition load[7]) = $(50 \times 100)(15 + 15)/1000 = 150$ k
      ii. 2 Longitudinal walls = 132 k
      iii. 2 End walls = 66 k
          Total = 348 k
          Total effective building weight $W = 646$ k

---

[4] This is for the mass of the structure. For weight, the value is 0.24.

[5] It is also a practice to assign at the roof level one-half the second-floor wall height.

[6] It is also a practice to assign at the second-floor level, the wall load from one-half of the second-floor wall and one-half of the first-floor wall. This leaves the weight of one-half of the first-floor wall not included in the effective weight.

[7] ASCE 7-16 prescribes 15 psf for partition load. For earthquake load, a minimum of 10 psf is recommended (Section 12.7.2 of ASCE 7-16).

D. Base shear

$V = C_S W = 0.24(646) = 155$ k

E. Lateral seismic force distribution on the vertical shear walls
1. From Equation 5.19, since $T_a < 0.5$ s, $k = 1$

$$F_x = \frac{(Vh_x)W_x}{\sum W_i h_i}$$

2. The computations are arranged in Table 5.8.

F. Earthquake loads for the vertical members
1. The redundancy factor ρ for SDC D is 1.3.
2. The horizontal and vertical components of the earthquake loads for vertical members (walls) are given in Table 5.9.
3. The earthquake forces are shown in Figure 5.11.

TABLE 5.8
Seismic Force Distribution on Vertical Members

Level, x (1)	W_x, k (2)	h_x, ft (3)	$W_x h_x$, k[a] (4)	Vh_x or $155h_x$, k ft[b] (5)	F_x, k[c] (6)	V_x (Shear at Story), k[d] (7)
Roof	298	22	6,556	3410	97.86	97.86
Second	348	11	3828	1705	57.14	155
Σ	646		10384			

[a] Column 2 × column 3.
[b] 155 × column 3.
[c] Column 2 × column 5/summation of column 4.
[d] Cumulate column 6.

TABLE 5.9
Earthquake Loads on Vertical Elements

Level, x	W_x, k	F_x, k	$E_{horizontal} = \rho F_x$, k	$E_{vertical} = 0.2S_{DS}W_x$, k
Roof	298	97.86	127.22	47.68
Second	348	57.14	74.28	55.68

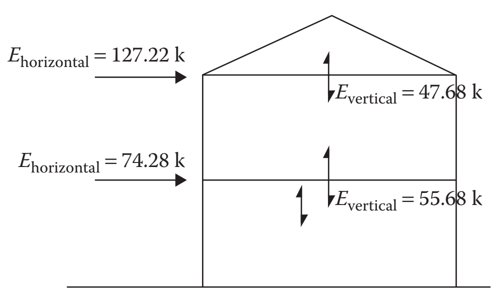


FIGURE 5.11 Earthquake loads on vertical elements.

Example 5.5

For Example 5.4, determine the earthquake loads acting on the horizontal members (diaphragms).

Solution

A. Lateral seismic force distribution on the horizontal members
   1. From Equation 5.20:

$$F_{px} = \frac{\left(\sum_{i=x}^{n} F_i\right) W_{px}}{\sum_{i=x}^{n} W_i}$$

   2. The computations are arranged in Table 5.10.

B. Earthquake loads for vertical members
   1. The redundancy factor ρ for F_{px} is always 1.0.
   2. The horizontal and vertical components of the earthquake loads for horizontal members (diaphragms) are given in Table 5.11.
   3. The earthquake forces on the horizontal members are shown in Figure 5.12.

TABLE 5.10
Seismic Force Distribution on Horizontal Members

							Maximum[e]	Minimum
Level, x	**W_x, k**	**W_{px}, k[a]**	**F_x from Table 5.8, k**	**ΣF_i, k[b]**	**ΣW_i, k[c]**	**F_{px}, k[d]**	**$0.4S_{DS}IW_{px}$, k**	**$0.2S_{DS}IW_{px}$, k**
(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**	**(9)**
Roof	298	232	97.86	97.86	298	76.19	111.36	55.68
Second	348	282	57.14	155	646	67.66[f]	135.36	67.68

[a] W_x—parallel exterior walls weight = 298 − 66 = 232 k.
[b] Summation of column 4.
[c] Summation of column 2
[d] Column 3 × column 5/column 6.
[e] $0.4S_{DS}IW_{px} = 0.4(0.8)(1.5)(232) = 111.36$ k.
[f] F_{px} should be at least 67.68 k (column 9).

TABLE 5.11
Earthquake Loads on Horizontal Elements

Level, x	**W_{px}, k**	**F_{px}, k**	**$E_{horizontal} = F_{px}$, k**	**$E_{vertical} = 0.2S_{DS}W_{px}$, k**
Roof	232	76.19	76.19	37.12
Second	282	67.68	67.68	45.12

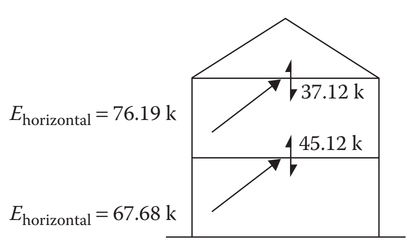


FIGURE 5.12 Earthquake loads on horizontal elements.

SOIL–STRUCTURE INTERACTION

The above combination of forces did not consider the interaction between the structure foundation and the soil, which tends to reduce the base shear force and its distribution thereof. If this option is exercised, the effective shear is determined as:

$$\bar{V} = V - \Delta V \tag{5.27}$$

The shear reduction, ΔV, is computed as follows:

$$\Delta V = [C_s - \bar{C}_s \quad (5.6 - \ln 100\beta)/4]\bar{W} \tag{5.28}$$

But ΔV should not exceed the following:

$\Delta V \leq 0.3\ V$ for $R \leq 3$
$\Delta V \leq (0.5\text{–}R/15)V$ for $R > 3$ and <6
$\Delta V \leq 0.1\ V$ for $R \geq 6$

where:

V is base shear from Equation 5.11
C_s is the seismic response coefficient (Figure 5.7)
$\bar{C}_s$ is the seismic response coefficient from Figure 5.7, using the effective period $\bar{T}$ for a flexibly supported structure.
β is the effective viscous damping ratio of the soil structural system. It is a complicated expression. The building codes assume a minimum value of 0.05 and a maximum value of 0.2.
$\bar{W}$ is the adjusted seismic weight of the structure for effective modal period. Alternatively, it is taken as the weight of structure.
$\bar{T}$ is the effective period computed by a relation in ASCE 7-16 as a function of various stiffness parameters related to the foundation. It is higher than the fundamental period, T_a.

Example 5.6

For Example 5.4, determine the base shear force, accounting for the soil–structure interaction. The effective period is computed to be 0.4 seconds, and the effective viscous damping ratio is 0.1.

Solution

1. From Example 5.4:
   $S_{DS} = 0.80$
   $T_s = 0.64$ s
   $I = 1.5$
   $R = 5$
   $C_s = 0.24$
   $W = 646$ k
   $V = 155$ k
2. Since $\bar{T}$ of 0.4 s is $< T_s$, $\bar{C}_s = \dfrac{S_{DS}}{R/I} = \dfrac{0.8}{5/1.5} = 0.24$
3. $\bar{W} = 646$ k
4. $\bar{C}_s$ (5.6–ln 100 β)/4 = 0.24(5.6–ln 10)/4 = 0.198
5. From Equation 5.28, ΔV = (0.24–0.198)646 = 27.13
   or Since $R > 3$ but <6 ΔV = (0.5–5/15)155 = 25.8 ← controls
6. From Equation 5.27, $\bar{V}$ = 155–25.8 = 129.2 k

PROBLEMS

5.1 Determine the approximate fundamental period for a five-story concrete office building with each floor having a height of 12 ft.

5.2 Determine the approximate fundamental period for a three-story wood-frame structure having a total height of 25 ft.

5.3 At a location in Washington State, the mapped values of MCE_R accelerations S_s and S_1 are 1.4 g and 0.7 g, respectively. The site soil class is C. The long-period transition period is 8 seconds. Prepare the design response acceleration curve for this location.

5.4 At a location in California, the mapped values of S_s and S_1 are 1.8 g and 0.75 g, respectively. The site soil class is B. The long-period transition period is 6 seconds. Prepare the design response acceleration curve.

5.5 The five-story concrete office building from Problem 5.1 is located in Washington State; each floor has a height of 12 ft. where S_s and S_1 are 1.4 g and 0.7 g, respectively, and the site soil class is C. Determine (1) the SDC and (2) the seismic response coefficient. Assume $R = 2.0$.

5.6 The three-story wood-frame commercial building from Problem 5.2 is located in California. It has a total height of 25 ft, where S_s and S_1 are 1.8 g and 0.75 g, respectively, and the soil group is B. Determine (1) the SDC and (2) the seismic response coefficient. Assume $R = 6.5$.

5.7 A two-story office building, as shown in Figure P5.1, is located in Oregon, and $S_s = 1.05$ g and $S_1 = 0.35$ g. The building has a plywood floor system and plywood sheathed shear walls ($R = 6.5$). The soil in the foundation is very dense. The loads on the building are as follows:

Roof dead load (on the horizontal plane) = 20 psf
Floor dead load = 15 psf
Partition load = 15 psf
Exterior wall dead load = 50 psf
Determine the lateral and vertical earthquake loads that will act on the vertical elements of the building.

5.8 For the building from Problem 5.7, determine the earthquake loads that will act on the horizontal elements of the building.

5.9 The building in Problem 5.6 has three stories—the first two stories are 8 ft each and the top story is 9 ft with a flat roof. It has a plan dimension of 120 ft × 60 ft. The roof and floor dead loads are 20 psf, and the wall dead load is 60 psf. Determine the earthquake loads acting on the vertical members of the building.

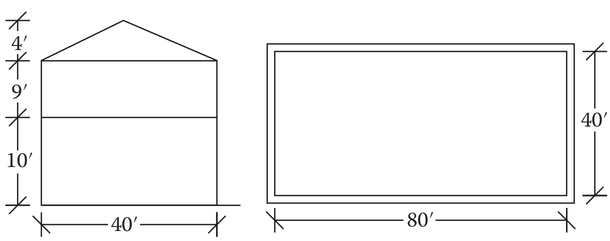


FIGURE P5.1 An office building in Ogegon for Problem 5.1.

5.10 For the building from Problem 5.9, determine the earthquake loads acting on the horizontal elements of the building.

5.11 A three-story industrial steel building (Figure P5.2), located where S_s and S_1 are 0.61 g and 0.18 g, respectively, has a plan dimension of 200 ft × 90 ft. The structure consists of nine gable moment-resisting steel frames spanning 90 ft at 25 ft apart; $R = 4.5$. The building is enclosed by insulated wall panels and is roofed with steel decking. The building is 36 ft high, and each floor height is 12 ft. The building is supported on spread roofing on medium dense sand (soil class D).

The steel roof deck is supported by joists at 5 ft in the center, between the main gable frames. The flooring consists of a concrete slab over steel decking, supported by floor beams at 10 ft apart. The floor beams rest on girders that are attached to the gable frames at each end.

The following loads have been determined in the building:

Roof dead load (horizontal plane) = 15 psf
Third floor storage live load = 120 psf
Slab and deck load on each floor = 40 psf
Weight of each framing = 10 k
Weight of non-shear-resisting wall panels = 10 psf
Include 25% of the storage live load for seismic force. Since the wall panels are non-shear-resisting, these are not to be subtracted for F_{px}.

Determine the lateral and vertical earthquake loads acting on the vertical elements of the building.

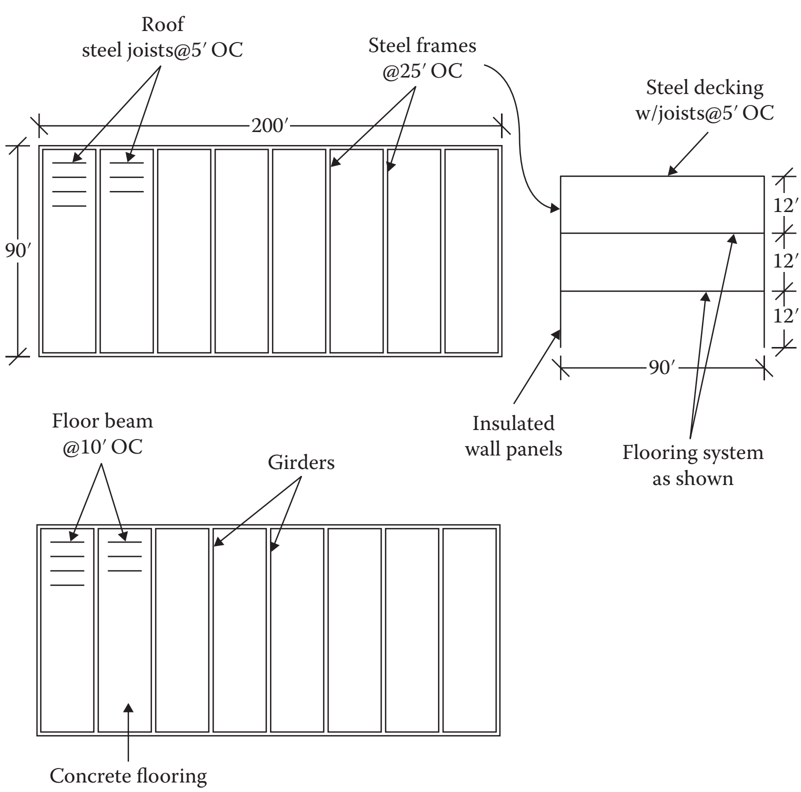


FIGURE P5.2 An industrial steel building for Problem 5.11.

5.12 For the building from Problem 5.11, determine the lateral and vertical earthquake loads acting on the horizontal elements of the building.

5.13 For Problem 5.7, determine the base shear force, accounting for the soil–structure interaction. The effective period is computed to be 0.4 seconds. The damping factor is 0.1.

5.14 For Problem 5.9, determine the base shear force, accounting for the soil–structure interaction. The effective period is computed to be 0.5 seconds. The damping factor is 0.1.

5.15 For Problem 5.11, determine the base shear force, accounting for the soil–structure interaction. The effective period is computed to be 0.8 seconds. The damping factor is 0.05.

6 Wood Specifications

ENGINEERING PROPERTIES AND DESIGN REQUIREMENTS

The National Design Specifications (NDS) for wood construction of the American Forest and Paper Association (AF&PA; 2015 edition) provide the basic standards and specifications for the following wood products:

- Sawn lumber
- Structural glued laminated timber
- Structural composite lumber
- Cross-laminated timber
- Timber piles and poles
- Prefabricated wood I-joists
- Wood structural panels

Cross-laminated timber is a new generation of engineering wood product that was introduced in the 2015 NDS as a code compliant construction material. This text covers the first four products as they pertain to common structural elements. In addition to the above wood products, the NDS covers the practice to be followed in the design of the fasteners.

The numerical values of the permissible level of stresses for design with respect to bending, tension, compression, shear, modulus of elasticity, and modulus of stability of a specific lumber are known as the *reference design values*. The reference design values are the basic values that are multiplied by many factors to obtain the adjusted design values.

The design requirement is that each member or connection shall be of sufficient size and capacity to carry the applied load without exceeding the adjusted design values of the specific wood product. To distinguish an adjusted value from a reference value, a prime notation is added to the symbol of the reference value to indicate that the necessary adjustments have been made. Thus:

$$F'_{(\,)} = F_{(\,)} \times \text{(products of adjustment factors)} \tag{6.1}$$

The parentheses ( ) are replaced by a property like tension, compression, and bending.

A subscript of n is added to recognize that it is a nominal (strength) value for the load resistance factor design (LRFD). Thus, the adjusted nominal design stress is expressed as:

$$F'_{(\,)n} = \phi\, F'_{(\,)} K_F\, \lambda \text{ (other adjustment factors)} \tag{6.2}$$

The adjustment factors common to all wood products are described below. Besides these, other adjustment factors are applied, the applicability of which is discussed with the specific wood product.

Format Conversion Factor, K_F

For wood structures, the allowable stress design (ASD) is a traditional basis of design. The LRFD provisions were introduced in 2005. The reference design values given in the NDS are based on the ASD (i.e., these are permissible stresses). The reference design values for LRFD have to be converted from the ASD values.

To determine the nominal design stresses for LRFD, the reference design values of the NDS tables, as reproduced in the appendixes, have to be multiplied by a format conversion factor, K_F. The format conversion factor serves a purpose reverse of the factor of safety to obtain the nominal strength values for LRFD application.

The format conversion factors for the different types of stresses are reproduced in Table 6.1. They come from Table N1 of the NDS.

Resistance Factor, ϕ

The resistance factor, also referred to as the *strength reduction factor*, is used to account for all uncertainties whether related to the materials manufacturing, structural construction, or design computations that may cause actual values to be less than the theoretical values. The resistance factor, given in Table 6.2, is a function of the mode of failure.

TABLE 6.1
Conversion Factor for Stresses

Application	Property	K_F
Member	Bending F_b	2.54
	Tension F_t	2.7
	Shear F_v, Radial tension, F_{rt}	2.88
	Compression F_c	2.4
	Compression perpendicular to grain $F_{c\perp}$	1.67
	E_{min}	1.76
	E	1.0
Connections	All design values	3.32

TABLE 6.2
Resistance Factor, ϕ

Application	Property	ϕ
Member	F_b	0.85
	F_t	0.80
	F_v, F_{rt}	0.75
	F_c, $F_{c\perp}$,	0.9
	E_{min}	0.85
Connections	All design values	0.65

TABLE 6.3
Time Effect Factor

Load Combination	λ
$1.4D$	0.6
$1.2D + 1.6L + 0.5(L_r \text{ or } S)$	0.7 when L is from storage 0.8 when L is from occupancy 1.25 when L is from impact
$1.2D + 1.6(L_r \text{ or } S) + (fL \text{ or } 0.5W)$	0.8
$1.2D + 1.0W + fL + 0.5(L_r \text{ or } S)$	1.0
$1.2D + 1.0E + fL + 0.2S$	1.0
$0.9D + 1.0W$	1.0
$0.9D + 1.0E$	1.0

Note: $f = 0.5$ for $L \leq 100$ psf; otherwise $f = 1$.

Time Effect Factor,[1] λ

Wood has a unique property because it can support a higher load when applied for a short duration. The nominal reference design values are representative of the short duration loading. For loading of long duration, the reference design value has to be reduced by a time effect factor. The different types of loads represent different load durations. Accordingly, the time effect factor depends on the combination of the loads. For various load combinations, the time effect factor is given in Table 6.3. It should be remembered that the factor is applied to the nominal reference (stress) value and not to the load.

Wet Service Factor, C_M

The specified reference design values are used for dry service conditions. When the moisture content exceeds 19% in the case of sawn lumber or 16% in the case of glued laminated, structural composite, and cross-laminated timber, a wet service factor is applied. This factor is specified by the NDS along with reference design values for sawn lumber and glued laminated timber. For structural composite lumber and cross-laminated timber, the factor should be obtained from the manufacturer.

Temperature Factor, C_t

When structural members experience sustained exposure to elevated temperatures of more than 100°F, an adjustment factor, as shown in Table 6.4, is applied.

TABLE 6.4
Temperature Factor, C_t

Reference Design Value	In-Service Moisture Condition	Temperature >100°F ≤125°F	Temperature >125°F ≤150°F
F_t, E, E_{min}	Wet or Dried	0.9	0.9
F_b, F_v, F_c, $F_{c\perp}$	Dry	0.8	0.7
F_b, F_v, F_c, $F_{c\perp}$	Wet	0.7	0.5

[1] The time effect factor is relevant only to the LRFD. For ASD, this factor, known as the *load duration factor*, C_D, has different values.

Fire Retardant Treatment Factor

The adjustment values for lumber and structural glued laminated timber that is pressure-treated with fire retardant chemicals should be obtained from the company providing the treatment.

DESIGN WITH SAWN LUMBER

The second part of the NDS, referred to as the NDS supplement, contains the reference design values for the strength of different varieties of wood grouped according to the species of trees. The pieces of wood sawn from the same species or even from the same source show a great variation in engineering properties. Accordingly, the lumber is graded to establish the strength values. The pieces of lumber having similar mechanical properties are placed in the same class known as the *grade* of the wood. Most lumber is visually graded. However, a small percentage is graded mechanically. In each grade, the relative size of the wood section and the suitability of that size for a structural application are used as additional guides to establish the strength.

The sawed lumber is classified according to the size into (1) dimension lumber; (2) timber beams and stringers (B & S) and post and timbers (P & T); and (3) the decking, with design values assigned to each grade. "Dimension" lumber has smaller sizes. It has a nominal thickness of 2–4 in. and a width[2] of 2–16 in. Thus, the sizes of the dimension lumber range from 2 in. × 2 in. to 4 in. × 16 in. Timber has a minimum nominal thickness of 5 in. Dimension lumber is further classified, based on the suitability of the specific size for use as a structural member, into structural light framing, light framing, studs, joists, and planks.

"Beams and Stringers" refers to lumber of rectangular cross section at least 5 in. thick, with width (depth) more than 2 in. greater than thickness. It is graded with respect to its strength in bending.

"Post and Timbers" refers to lumber of nearly square cross section 5 in. × 5 in. and larger, with width (depth) not more than 2 in. greater than thickness.

"Decking" refers to lumber from 2 to 4 in. thick, grooved on the narrow face. It is intended to be used as a roof, floor or wall member.

The size and use categorization of the commercial lumber is given in Table 6.5.

Lumber is referred to by the nominal size. However, lumber used in construction is mostly dressed lumber. In other words, lumber is surfaced to a net size, which is taken to be 0.5 in. less than the nominal size for sizes up to 6 in., and 0.75 in. less for nominal sizes over 6 in. and below 16 in., and 1 in. less for sizes 16 in. and above. In the case of large sections, sometimes

[2] In the terminology of lumber grading, the smaller cross-sectional dimension is thickness and the larger dimension is width. In the designation of engineering design, the dimension parallel to the neutral axis of a section, as placed, is width and the dimension perpendicular to the neutral axis is depth. Thus, a member loaded about the strong axis (placed with the smaller dimension parallel to the neutral axis) has the "width," what is referred to as the "thickness" in lumber terminology.

TABLE 6.5
Categories of Lumber

Name	Symbol	Nominal Dimension: Thickness (Smaller Dimension)	Nominal Dimension: Width
A. Dimension Lumber			
1. Light framing	LF	2–4 in.	2–4 in.
2. Structural light framing	SLF	2–4 in.	2–4 in.
3. Structural joist and plank	SJ & P	2–4 in.	5 in. or more
4. Stud		2–4 in.	2 in. or more
5. Decking		2–4 in.	4 in. or more
B. Timber			
1. Beam and stringer	B & S	5 in. or more	At least 2 in. more than thickness
2. Post and timber	P & T	5 in. or more	Not more than 2 in. more than thickness

the lumber is rough sawed. The rough sawed dimensions are approximately ⅛ in. larger than the dressed size.

The sectional properties of the standard dressed sawn lumber are given in Appendix B.1.

For each size and use category, the reference design values are listed for different grades of lumber. Thus, the design value may be different for the same grade name but in a different size category. For example, the select structural grade appears in the B & S, and P & T categories, and the design values for a given species are different for the select structural grade in all of these categories.

The following reference design values are provided in tables:

Appendix B.2: Reference design values for dimension lumber other than Southern Pine
Appendix B.3: Reference design values for Southern Pine dimension lumber
Appendix B.4: Reference design values for timber

Although the reference design values are given according to the size and use combination, the values depend on the size of the member rather than its use. Thus, a section 6×8 listed under Post and Timber (P & T) with its reference design values indicated therein can be used for Beam and Stringer (B & S), but its design values as indicated for P & T will apply.

The reference design values from Appendixes B.2, B.3, and B.4 are multiplied by the adjustment factors specified in Table 6.6. Besides the common adjustment factors discussed previously, the other factors applied to lumber are described below.

In addition, there are some special factors like the column stability factor, C_P, and the beam stability factor, C_L, which are discussed in the context of column and beam designs in Chapter 7.

TABLE 6.6
Applicability of Adjustment Factors for Sawn Lumber

		Wet Service Factor	Temperature Factor	Beam Stability Factor	Size Factor	Flat Use Factor	Incising Factor	Repetitive Member Factor	Column Stability Factor	Buckling Stiffness Factor	Bearing Area Factor	Format Conversion Factor	Resistance Factor	Time Effect Factor
												K_F	ϕ	
$F_b' = F_b$	×	C_M	C_t	C_L	C_F	C_{fu}	C_i	C_r	—	—	—	2.54	0.85	λ
$F_t' = F_t$	×	C_M	C_t	—	C_F	—	C_i	—	—	—	—	2.70	0.80	λ
$F_v' = F_v$	×	C_M	C_t	—	—	—	C_i	—	—	—	—	2.88	0.75	λ
$F_c' = F_c$	×	C_M	C_t	—	C_F	—	C_i	—	C_p	—	—	2.40	0.90	λ
$F_{c\perp}' = F_{c\perp}$	×	C_M	C_t	—	—	—	C_i	—	—	—	C_b	1.67	0.90	—
$E' = E$	×	C_M	C_t	—	—	—	C_i	—	—	—	—	—	—	—
$E_{min}' = E_{min}$	×	C_M	C_t	—	—	—	C_i	—	—	C_T	—	1.76	0.85	—

MORE FACTORS APPLICABLE TO LUMBER

Incising Factor, C_i

Some species of wood do not accept the pressure treatment easily and require incision. In such cases, the following incising factor is applied to dimension lumber:

$$F_b, F_t, F_c, F_v = 0.8$$

$$F_{c\perp} = 1.0$$

$$E, E_{\min} = 0.95$$

Size Factor, C_F

The size of a wood section has an effect on its strength. The factor for size is handled differently for dimension lumber and for timber.

Size Factor, C_F, for Dimension Lumber

For visually graded dimension lumber, the size factors for species other than Southern Pine are presented together with the reference design values in Appendix B.2. For visually graded Southern Pine dimension lumber, the size factors are generally built into the design values except for the bending values for 4 in. thick (breadth) dimension lumber. The size factors for Southern Pine dimension lumber wherever applicable are given together with the reference design values in Appendix B.3. No size factor adjustment is required for the mechanically graded lumber.

Size Factor, C_F, for Timber

For timber sections exceeding 12 in. depth, a reduction factor is applied only to bending, as follows:

$$C_F = \left(\frac{12}{d}\right)^{1/9} \tag{6.3}$$

where d is the dressed depth of the section.

For beams of circular cross section greater than 13.5 in. diameter, the size factor should be determined from Equation 6.3 on the basis of an equivalent square beam of the same cross-sectional area.

Repetitive Member Factor, C_r

This factor is applied only to the dimension lumber and then only to the bending strength value. A repetitive member factor $C_r = 1.15$ is applied when all of the following three conditions are met:

1. The members are used as joists, truss chords, rafters, studs, planks, decking, or similar members that are joined by floor, roof, or other load-distributing elements.
2. The members are in contact or are spaced not more than 24 in. on center.
3. The members are not less than three in number.

The reference design values for decking are already multiplied by C_r.

Flat Use Factor, C_{fu}

The reference design values are for bending about the major axis; that is, the load is applied on the narrow face. The flat use factor refers to members that are loaded about the weak axis; that is, the load is applied on the wider face. The reference value is increased by a factor, C_{fu} in such cases.

This factor is applied only to bending to dimension lumber and to bending and E and E_{min} to timber. The values of C_{fu} are listed along with the reference design values in Appendixes B.2 through B.4.

Buckling Stiffness Factor, C_T

This is a special factor that is applied when all of these conditions are satisfied: (1) compression chord of a truss; (2) a 2 x 4. or smaller sawn lumber; (3) subjected to combined flexure and axial compression, under dry conditions; and (4) has ⅜ in. or thicker plywood sheathing nailed to the narrow face of the chord.

For such a case, the E_{min} value is allowed to be increased by the factor C_T that is more than 1, in column stability C_P calculations. Conservatively, this can be taken as 1.

Bearing Area Factor, C_b

This is a special factor applied only to the compression reference design value perpendicular to the grain, $F_{c\perp}$. This is described in Chapter 7 for support bearing cases.

LRFD BASIS LUMBER DESIGN

As discussed in the "Working Stress, Strength Design, and Unified Design of Structures" section in Chapter 1, LRFDs are performed at the strength level in terms of the force and moment. Accordingly, the adjusted nominal design stress values from the above section are changed to the strength values by multiplying by the cross-section area or the section modulus. Thus, the basis of design in LRFD is as follows:

$$\text{Bending: } Mu = \phi M_n = F'_{bn}\, S = \phi\, F_b\, \lambda\, K_F C_M C_t C_F C_{fu} C_i (C_L) S \tag{6.4}$$

$$\text{Tension: } T_u = \phi T'_n = \phi\, F_t\, \lambda\, K_F C_M C_t C_F C_i A \tag{6.5}$$

$$\text{Compression: } P_u = \phi P'_n = \phi\, F_c\, \lambda\, K_F C_M C_t C_F C_i (C_P) A \tag{6.6}$$

$$P_{u\perp} = \phi P_n'^{\perp} = \phi\, F_{c\perp}\, \lambda\, K_F C_M C_t C_i C_b A \tag{6.7}$$

$$\text{Shear: } V_u = \phi V'_n = \phi F_v\, \lambda\, K_F\, C_M C_t C_i \left(\frac{2}{3} A^*\right) \tag{6.8}$$

$$\text{Stability: } E'_{min(n)} = \phi E_{min}\, K_F C_M C_t C_i C_T \tag{6.9}$$

$$\text{Modulus of elasticity: } E'_n = E C_M C_t C_i \tag{6.10}$$

The left hand side (LHS) of the above equations are the factored loads combination and the factored moments combination.

The design of an element is an iterative procedure since the reference design value and modification factors in many cases are a function of the size of the element that is to be determined. Initially, the nominal design value could be assumed to be one and a half times the basic reference design value for the smallest listed size of the specified species from the table in Appendix B.2 through B.4 corresponding to the type of design lumber.

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* $\tau\max = \frac{3V}{2A}$ or $V = \tau\max = \frac{2A}{3}$

Example 6.1

Determine the adjusted nominal reference design values and the nominal strength capacities of the Douglas Fir-Larch #1 2 in. × 8 in. roof rafters at 18 in. on center that support dead and roof live loads. Consider the dry-service conditions, normal temperature range, and no-incision application.

Solution

1. The reference design values of Douglas Fir-Larch #1 2 in. × 8 in. section obtained from Appendix B.2 and the adjustment factors obtained from Tables 6.1, 6.2, and 6.3 are given in the table below.
2. The computed adjusted nominal reference design values are arranged in the table:

Property	Reference Design Value (psi)	Adjustment Factors					F'_{0n} (psi)
		ϕ	λ for $D+L_r$	K_F	C_F	C_r	
Bending	1000	0.85	0.8	2.54	1.2	1.15	2383.54
Tension	675	0.80	0.8	2.7	1.2		1399.68
Shear	180	0.75	0.8	2.88			311.04
Compression	1500	0.9	0.8	2.40	1.05		2721.6
Compression⊥	625	0.9	0.8	1.67			751.5
E	1.7×10^6						1.7×10^6
E_{min}	0.62×10^6	0.85		1.76			0.93×10^6

3. Strength capacities
   For a 2 in. × 8 in. section, $S = 13.14\text{ in.}^3$, $A = 10.88\text{ in.}^2$:
   $M_u = F'_{bn}\, S = (2383.54)(13.14) = 31{,}319.66$ in.-lb
   $T_u = F'_{tn}\, A = (1399.68)(10.88) = 15{,}228.52$ lb
   $V_u = F'_{vn}\,(2/3A) = (311.04)(2 \times 10.88/3) = 2257.21$ lb
   $P_u = F'_{cn}\, A = (2721.6)(10.88) = 29611$ lb

Example 6.2

Determine the adjusted nominal reference design values and the nominal strength capacities of a Douglas Fir-Larch #1 6 in. × 16 in. floor beam supporting a combination of load comprising dead, live, and snow loads. Consider the dry-service conditions, normal temperature range, and no incision application.

Solution

1. The reference design values of Douglas Fir-Larch #1 6 in. × 16 in. beams and stringers were obtained from Appendix B.4.
2. The adjustment factors and the adjusted nominal reference design values are given in the table below:

Property	Reference Design Value (psi)	Adjustment Factors				F'_{0n} (psi)
		ϕ	λ for D, L, S	K_F	C_F[a]	
Bending	1350	0.85	0.8	2.54	0.976	2275.76
Tension	675	0.80	0.8	2.7		1166.4
Shear	170	0.75	0.8	2.88		293.8
Compression	925	0.9	0.8	2.4		1598.4
E	1.6×10^6					1.6×10^6
E_{min}	0.58×10^6	0.85		1.76		0.87×10^6

[a] $C_F = \left(\frac{12}{d}\right)^{1/9} = \left(\frac{12}{15}\right)^{1/9} = 0.976$

3. Strength capacities
   For section 6 in. × 16 in., $S = 206.3$ in.^{3}, $A = 82.5$ in.^{2}:
   $M_u = F'_{bn}\,S = (2275.76)(206.3) = 469{,}489$ in.-lb
   $T_u = F'_{tn}\,A = (1166.4)(82.5) = 96{,}228$ lb
   $V_u = F'_{vn}\,(2/3A) = (293.8)(2 \times 82.5/3) = 16{,}167$ lb
   $P_u = F'_{cn}\,A = (1598.4)(82.5) = 13{,}1868$ lb

Example 6.3

Determine the unit load (per square foot load) that can be imposed on a floor system consisting of 2 in. × 6 in. Southern Pine select structural joists spaced at 24 in. on center spanning 12 ft. Assume that the dead load is one-half of the live load. Ignore the beam stability factor.

Solution

1. For Southern Pine 2 in. × 6 in. select structural dressed lumber, the reference design value $F_b = 2100$ psi.
2. Size factor is included in the tabular value.
3. Time effect factor for dead and live loads = 0.8
4. Repetitive factor = 1.15
5. Format conversion factor = 2.54
6. Resistance factor = 0.85
7. Nominal reference design value

$$F'_{bn} = \phi F_b\,\lambda K_F C_M C_t C_r C_F C_{fu} C_i$$
$$= 0.85(2100)((0.8)(2.54)(1)(1.15)(1)(1) = 4171 \text{ psi}$$

8. For 2 in. × 6 in. $S = 7.56$ in.^{3}:
   $M_u = F'_{bn}S = (4171)(\ 7.56) = 31{,}533$ in.-lb or 2678 ft-lb
9. $M_u = \dfrac{w_u L^2}{8}$
   or
   $w_u = \dfrac{8M_u}{L^2} = \dfrac{8(2678)}{(12)^2} = 148.78$ lb/ft
10. Tributary area/ft of joists $= \dfrac{24 \times 1}{12} = 2$ ft^2/ft
11. w_u = (design load/ft^2)(tributary area/ft)
    $148.78 = (1.2D + 1.6L)(2)$
    or
    $148.78 = [1.2D + 1.6(2D)](2)$
    or
    $D = 16.91$ lb/ft^2
    and
    $L = 33.82$ lb/ft^2

Example 6.4

For a Southern Pine Dense No. 1 floor system, determine the size of the joists at 18 in. on center, spanning 12 ft; the column receiving loads from an area of 100 ft^2 is acted upon by a dead load of 30 psf and a live load of 40 psf. Assume that the beam and column stability factors are not a concern.

Solution

A. Joist design

1. Factored unit combined load = 1.2(30) + 1.6(40) = 100 psf
2. Tributary area/ft = (18/12) × 1 = 1.5 ft^2/ft
3. Design load/ft w_u = 100(1.5) = 150 lb/ft
4. $Mu = \dfrac{w_u L^2}{8} = \dfrac{(150)(12)^2}{8} =$ 2700 ft-lb or 32,400 in.-lb
5. For a trial section, select the reference design value of 2–4 in. wide section and assume the nominal reference design value to be one-and-half times of the table value.
   From Appendix B.3, for Southern Pine Dense No. 1, F_b = 1650 psi
   Nominal reference design value = 1.5(1650) = 2475 psi
6. Trial size

   $$S = \frac{M_u}{F'_{bn}} = \frac{32{,}400}{2475} = 13.1 \text{ in.}^3$$

   Use 2 in. × 8 in. S = 13.14 in.^{3}
7. From Appendix B.3, F_b = 1350 psi
8. Adjustment factors
   $\phi = 0.85$
   $\lambda = 0.8$
   $C_r = 1.15$
   $K_F = 2.54$
9. Adjusted nominal reference design value
   $F'_{bn} = 0.85(1350)(0.8)(1.15)(2.54) = 2681.5$ psi
10. $M_u = F'_{bn}\, S$
    or

    $$S_{reqd} = \frac{Mu}{F'_{bn}} = \frac{32{,}400}{2681.5} = 12.1 \le 13.14 \text{ in.}^3$$

    Selected size 2 in. × 8 in. is **OK.**

B. Column design

1. Factored unit load (Step A.1 above) = 100 psf
2. Design load = (unit load)(tributary area)
   = (100)(100) = 10,000 lb
3. For a trial section, select the reference design value of 2–4 in. wide section and assume the nominal reference design value to be one-and-a-half times of the table value.
   From Appendix B.3, for Southern Dense Pine No. 1, F_c = 1750 psi
   Nominal reference design value = 1.5(1750) = 2625 psi

4. Trial size

$$A = \frac{P_u}{F'_{cn}} = \frac{10{,}000}{2650} = 3.81 \text{ in.}^2$$

Use 2 in. × 4 in. $A = 5.25$ in.[2]

5. $F_c = 1750$ psi
   $\lambda = 0.8$
   $K_F = 2.40$
   $\phi = 0.90$
6. Adjusted nominal reference design value

$$F'_{cn} = 0.9(1750)(0.8)(2.4) = 3024 \text{ psi}$$

$$A_{reqd} = \frac{P_u}{F'_{cn}} = \frac{10{,}000}{3024} = 3.30 < 5.25 \; in.^2$$

Selected size 2 in. × 4 in. is **OK**.

STRUCTURAL GLUED LAMINATED TIMBER

Glued laminated timber (GLULAM) members are composed of individual pieces of dimension lumber that are bonded together by an adhesive to create the required size. For Western species, the common width[3] (breadth) are 3-⅛, 5-⅛, 6-¾, 8-½, 10-½, and 12-½ in. (there are other interim sections as well). The laminations are typically in 1-½ in. incremental depths. For Southern Pine, the common widths are 3, 5, 6-¾, 8-½ and 10-½ in. and the depth of each lamination is 1-⅜ in. Usually the lamination of GLULAM is horizontal (the wide faces are horizontally oriented). A typical cross section is shown in Figure 6.1.

The sectional properties of Western species structural glued laminated timber are given in Appendix B.5. For Southern Pine structural glued laminated timber, these are given in Appendix B.6.

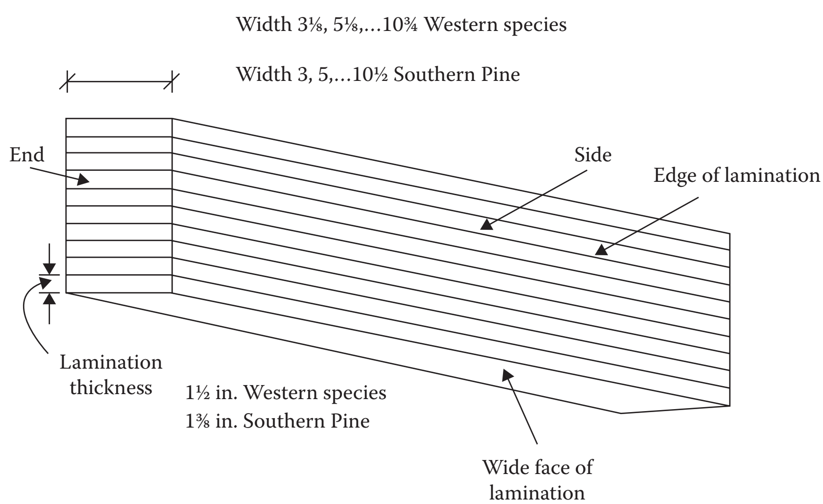


FIGURE 6.1 A structural glued laminated (GLULAM) section.

[3] Not in terms of the lumber grading terminology.

Because of their composition, large GLULAM members can be manufactured from smaller trees from a variety of species such as Douglas Fir, Hem Fir, and Southern Pine. GLULAM has much greater strength and stiffness than sawn lumber.

REFERENCE DESIGN VALUES FOR GLULAM

The reference design values for GLULAM are given in Appendix B.7 for members stressed primarily in bending (beams). The reference design values are given in Appendix B.8 for members stressed primarily in axial tension or compression.

Appendix B.7, relating to bending members, is a summary table based on the stress class. The first part of the stress class symbol refers to the bending stress value for the grade in hundreds of psi followed by the letter F. For example, 24F indicates a bending stress of 2400 psi for normal duration loaded in the normal manner; that is, loads are applied perpendicular to the wide face of lamination. The second part of the symbol is the modulus of elasticity in millions of psi. Thus, 24F-1.8E indicates a class with the bending stress of 2400 psi and the modulus elasticity of 1.8×10^6 psi. For each class, the NDS provides the expanded tables according to the combination symbol and the types of species making up the GLULAM. The first part of the combination symbol is the bending stress level; that is, 24F refers to 2400 psi bending stress. The second part of the symbol refers to the lamination stock; V stands for visually graded and E for mechanically graded or E-rated. Thus, the combination symbol 24F-V5 refers to the grade of 2400 psi bending stress of visually graded lumber stock. Under this, the species are indicated by abbreviations, that is, DF for Douglas Fir, SP for Southern Pine, and HF for Hem Fir.

The values listed in Appendix B.7 are more complex than sawn lumber. The first six columns are the values for bending about the strong (*X–X*) axis when the loads are perpendicular to the wide face of lamination. These are followed by values for bending about the *Y–Y* axis. The axially loaded values are also listed in case the member is picked up for the axial load conditions.

For F_{bx}, two values have been listed in column 1 and 2 of Appendix B.7 (for bending) as F_{bx}^{+} and F_{bx}^{-}. In a rectangular section, the compression and tension stresses are equal in extreme fibers. However, it has been noticed that the outer tension laminations are in a critical state, and therefore high-grade laminations are placed at the bottom of the beam, which is recognized as the tensile zone of the beam. The other side is marked as "top" of the beam in the lamination plant. Placed in this manner, the top marked portion is subjected to compression and the bottom to tension. This is considered to be the condition in which "tension zone is stressed in tension" and the F_{bx}^{+} value of the first column is used for bending stress. This is a common condition.

However, if the beam is installed upside down or, in the case of a continuous beam, when the negative bending moment condition develops (i.e., the top fibers are subject to tension), the reference values in the second column known as the "compression zone stressed in tension" F_{bx}^{-}

Appendix B.8 lists the reference design values for principally axially load-carrying members. Here members are identified by numbers such as 1, 2, 3 followed by the species, such as DF, HF, SP, and by the grade. The values are not complex like those in Appendix B.7 (the bending case).

It is expected that the members with the bending combination in Appendix B.7 will be used as beams as they make efficient beams. However, it does not mean that they cannot be used for axial loading. Similarly, an axial combination member can be used for a beam. The values with respect to all types of loading modes are covered in both tables.

ADJUSTMENT FACTORS FOR GLULAM

The reference design values of Appendixes B.7 and B.8 are applied by the common adjustment factors—resistance, time effect, format conversion, wet service, temperature, and fire-retardant factors as described earlier for all wood products. The special factors like the beam stability factor, C_L, the column stability factor, C_P, and the bearing area factor, C_b are similar to those described for sawn lumber. In addition, other adjustment factors as listed in Table 6.7 are applied to structural GLULAM. The factors that are different in GLULAM are discussed below.

Flat Use Factor for GLULAM, C_{fu}

The flat use factor is applied to the reference design value only (1) for the case of bending that is loaded parallel to the laminations and (2) when the dimension parallel to the wide face of lamination (depth in flat position) is less than 12 in. The factor is

$$C_{fu} = \left(\frac{12}{d}\right)^{1/9} \tag{6.11}$$

where d is the depth of the section.

Equation 6.11 is similar to the size factor (Equation 6.3) of sawn timber lumber.

Volume Factor for GLULAM, C_v

The volume factor is applied to bending only for horizontally laminated timber for loading applied perpendicular to laminations (bending about the X–X axis), it is applied to F_{bx}^{+} and F_{bx}^{-}. The beam stability factor, C_L, and the volume factor, C_v, are not used together; only the smaller of the two is applied to adjust F'_{bn}. The concept of the volume factor for GLULAM is similar to the size factor for sawn lumber because test data indicate the size effects extend to volume in the case of GLULAM. The volume factor is:

$$C_v = \left(\frac{5.125}{b}\right)^{1/x}\left(\frac{12}{d}\right)^{1/x}\left(\frac{21}{L}\right)^{1/x} \leq 1 \tag{6.12}$$

where:

b is the width (in.)
d is the depth (in.)
L is the length of member between points of zero moments (ft)
$x = 20$ for Southern Pine and 10 for other species

TABLE 6.7
Applicability of Adjustment Factors for Structural Glued Laminated Timber

		Wet Service Factor	Temperature Factor	Beam Stability Factor[a]	Volume Factor[a]	Flat Use Factor	Curvature Factor	Stress Interaction Factor	Shear Reduction Factor	Column Stability Factor	Bearing Area Factor	Format Conversion Factor K_F	Resistance Factor ϕ	Time Effect Factor
$F_b'^{*} = F_b$	×	C_M	C_t	C_L	C_V	—	C_c	C_I	—	—	—	2.54	0.85	λ
$F_b'^{**} = F_b$	×	C_M	C_t	C_L	—	C_{fu}	C_c	C_I	—	—	—	2.54	0.85	λ
$F_t' = F_t$	×	C_M	C_t	—	—	C_{fu}	—	—	—	—	—	2.70	0.80	λ
$F_v' = F_v$	×	C_M	C_t	—	—	—	—	—	C_{vr}	—	—	2.88	0.75	λ
$F_{rt}' = F_{rt}$	×	C_M	C_t	—	—	—	—	—	—	—	—	2.88	0.75	λ
$F_c' = F_c$	×	C_M	C_t	—	—	—	—	—	—	C_P	—	2.40	0.90	λ
$F_{c\perp}' = F_{c\perp}$	×	C_M	C_t	—	—	—	—	—	—	—	C_b	1.67	0.90	—
$E' = E$	×	C_M	C_t	—	—	—	—	—	—	—	—	—	—	—
$E_{min}' = E_{min}$	×	C_M	C_t	—	—	—	—	—	—	—	—	1.76	0.85	—

[a] The beam stability factor, C_L, shall not apply simultaneously with the volume factor, C_V, for structural glued laminated timber bending members. Therefore, the lesser of these adjustment factors shall apply.

[*] Perpendicular to lamination.

[**] Parallel to lamination.

Curvature Factor for GLULAM, C_c

The curvature factor is applied to bending stress only to account for the stresses that are introduced in laminations when they are bent into curved shapes during manufacturing. The curvature factor is:

$$C_c = 1 - 2000\left(\frac{t}{R}\right)^2 \tag{6.13}$$

where:

t is the thickness of lamination, 1-½- in. or 1-⅜ in.
R is the radius of curvature of the inside face of lamination

The ratio t/R may not exceed 1/100 for Southern Pine and 1/125 for other species. The curvature factor is not applied to straight portion(s) of a member regardless of curvature in the other portion(s).

Stress Interaction Factor, C_I

The stress interaction factor is applied only to (1) the tapered section of a member and (2) the reference bending stress. For members tapered in compression, either C_I or the volume factor C_V, whichever is smaller, is applied. For members tapered on tension face, either C_I or the beam stability factor C_L, whichever is smaller, is applied.

The factor depends on the angle of taper, bending stress, shear stress, and compression stress perpendicular to grains for compression face taper and radial tensile stress for tension face taper. It is less than 1. Refer to Section 5.3.9 of NDS 2015.

Shear Reduction Factor, C_{vr}

The reference shear design values, F_{vx} and F_{vy}, are multiplied by a factor $C_{vr} = 0.72$ when any of the following conditions apply:

1. Non-prismatic members
2. Members subject to impact or repetitive cyclic loading
3. Design of members at notches
4. Design of members at connections

Example 6.5

Determine the adjusted nominal reference design stresses and the strength capacities of a 6-¾ in. × 18 in. GLULAM from Doulas Fir-Larch of stress class 24F-1.7E, used primarily for bending. The span is 30 ft. The loading consists of the dead and live load combination along the major axis.

Solution

1. The adjusted reference design values are computed in the table below:

		Adjustment Factors				$F'_{()n}$ (psi)
Property	**Reference Design Value (psi)**	**ϕ**	**λ**	**K_F**	**C_V**	
Bending	2400	0.85	0.8	2.54	0.90[a]	3730.75
Tension	775	0.8	0.8	2.7		1339.2
Shear	210	0.75	0.8	2.88		362.88
Compression	1000	0.9	0.8	2.4		1728.0
E	1.7×10^6					1.7×10^6
E_{min}	0.88×10^6	0.85		1.76		1.32×10^6

[a] $C_V = \left(\frac{5.125}{6.75}\right)^{1/10}\left(\frac{12}{18}\right)^{1/10}\left(\frac{21}{30}\right)^{1/10} = 0.9$

2. Strength capacities
   For the 6-3⁄4 in. × 18 in. section, $S_x = 364.5$ in.^{3}, $A = 121.5$ in.^{2}
   Bending: $\phi M_n = F'_{bn}\,S = (3730.75)(364.5) = 1.36 \times 10^6$ in.-lb
   Tension: $\phi T_n = F'_{tn} = (1339.2)(121.5) = 162.71 \times 10^3$ lb
   Shear: $\phi V_n = F'_{vn}\left(\frac{2A}{3}\right) = (362.88)(2/3 \times 121.5) = 29.39 \times 10^3$ lb
   Compression: $\phi P_n = F'_{cn}\,A = (1728)(121.5) = 210 \times 10^3$ lb

Example 6.6

The beam in Example 6.5 is installed upside down. Determine the design strengths.

Solution

1. Bending reference design value for compression zone stressed in tension = 1450 psi from Appendix B.7
2. Adjustment factors from Example 6.5:
   $\phi = 0.85$
   $\lambda = 0.80$
   $C_V = 0.90$
   $K_F = 2.54$
3. Adjusted nominal design value
   $F'_{bn} = 0.85(1450)(0.8)(0.9)(2.54) = 2254$ psi
4. Strength capacity
   $F'_{bn}\,S = (2254)(364.5) = 0.882 \times 10^6$ in.-lb
5. The other values are the same as in Example 6.5.

Example 6.7

The beam used in Example 6.5 is flat with loading along the minor axis; determine the design strengths.

Solution

1. The adjusted reference design values are computed in the table below:

Property	Reference Design Value (psi)	Adjustment Factors				F'_{0n} (psi)
		ϕ	λ	K_F	C_{fu}	
Bending	1050	0.85	0.8	2.54	1.066[a]	1933.26
Tension	775	0.8	0.8	2.7		1339.2
Shear	185	0.75	0.8	2.88		319.68
Compression	1000	0.9	0.8	2.4		1728.0
E	1.3×10^6					1.3×10^6
E_{min}	0.67×10^6	0.85		1.76		1.00×10^6

[a] $C_{fu} = \left(\frac{12}{6.75}\right)^{1/9} = 1.066$

2. Strength capacities
   For section 6-3⁄4 in. × 18 in. section, $S_y = 136.7$ in.^{3}, $A = 121.5$ in.^{2}
   Bending: $\phi M_n = F'_{bn}\,S = (1933.26)(136.7$[4]$) = 0.26 \times 10^6$ in.-lb
   Tension: $\phi T_n = F'_{tn}\,A = (1339.2)(121.5) = 16{,}2713$ lb
   Shear: $\phi\,V_n = F'_{vn}\left(\frac{2A}{3}\right) = (319.68)(2/3 \times 121.5) = 25.89 \times 10^3$ lb
   Compression: $\phi P_n = F'_{cn}\,A = (1728)(121.5) = 209 \times 10^3$ lb

[4] S_y value.

Example 6.8

What are the unit dead and live loads (per ft^2) that are resisted by the beam in Example 6.5? The beam is spaced 10 ft on center. Assume that the unit dead load is one-half of the live load.

Solution

1. From Example 6.5:
   $M_u = \phi\, M_n = 1.36 \times 10^6$ in.-lb or 113,333.33 ft·lb
2. $M_u = 113333.33 = \dfrac{w_u L^2}{8}$
   $$w_u = \frac{113{,}333.33\ (8)}{(30)^2} = 1007.41\ \text{lb/ft}$$
3. Tributary area/ft of beam = 10 × 1 = 10 ft^2/ft
   w_u = (design load/ft^2)(tributary area, ft^2/ft)
   $10{,}071.41 = (1.2D + 1.6L)10$
   or
   $10{,}071.41 = [1.2D + 1.6(2D)]10$
   or
   $D = 22.9$ lb/ft^2
   and
   $L = 45.8$ lb/ft^2

STRUCTURAL COMPOSITE LUMBER

Structural composite lumber (SCL) is an engineered product manufactured from smaller logs. The manufacturing process involves sorting and aligning the strands or veneer, applying adhesive, and bonding under heat and pressure. The stranding is the process of making 3–12 in. slices of a log similar to grating a block of cheese. The veneering is the process of rotary peeling by a knife placed parallel to the outer edge of a spinning log. The log is peeled from outside toward the center similar to removing paper towels from a roll. The slices cut into sheets are called veneer.

The following are four common types of SCL products:

1. Laminated strand lumber (LSL)
2. Oriented strand lumber (OSL)
3. Laminated veneer lumber (LVL)
4. Parallel strand lumber (PSL)

The lamination of SCL is vertical (wide faces of laminations are oriented vertically) compared to the horizontal lamination of GLULAM (wide faces oriented horizontally). The strength and stiffness of SCL are generally higher than GLULAM. The first two of the items in the list above are strand products; the other two are veneer products.

LVL is commonly fabricated in 1-¾ in. width, and multiple plies can be nailed together for wider width. The common depths are 9-½, 11-⅞, 14, 16, and 18-¾ in. LVL is commonly used for beams and flanges in composite I-joists. Microlam, which has a proprietary name, is an LVL product.

PSL has common widths of 3-½, 5-¼, and 7 in. and depths of 9-¼, 9-½, 11-¼, 11-⅞, 14, 16, and 19 in. It is generally more expensive than the other three kinds of composite wood. It is used for large beams, headers, and columns. Parallam, which has a proprietary name, is a PSL product.

LSL is commonly fabricated in 1-¼, 1-½, 1-¾, and 3-¼ widths and 9-¼ to 16 in. depth. It is less expensive than other composite wood. It has a higher allowable shear strength to allow larger penetration. It is ideal for short spans.

OSL is similar to LSL, but the strands are shorter than those in LSL. It Is typically produced in 4 ft × 8 ft sheets. It is useful for joists, studs, and rafters.

The typical reference design values for SCL are listed in Appendix B.9. SCL is equally strong in bending flatwise or edgewise. Many brands of SCL are commercially available. The reference values and technical specifications for the specific brand may be obtained from the manufacturer's literature.

ADJUSTMENT FACTORS FOR STRUCTURAL COMPOSITE LUMBER

The adjustment factors listed in Table 6.8 are applied to SCL. The common adjustment factors—resistance, time effect, format conversion, wet service, temperature, and fire retardant—described for lumber are applicable to the structural composite lumber.

Also, special factors like the beam stability factor, C_L; column stability factor, C_P; and bearing area factor, C_b, described for sawn lumber and structural glued laminated timber are similar for SCL. The other factors distinct for structural composite lumber are discussed below.

REPETITIVE MEMBER FACTOR, C_r

To the members used in repetitive assembly, as defined in the "Repetitive Member Factor, C_r" section earlier, a repetitive factor C_r of 1.04 is applied.

VOLUME FACTOR, C_v

The value of the size (volume) factor, C_V, is obtained from the SCL manufacturer's literature. When $C_V \le 1$, only the lesser of the volume factor, C_V, and the beam stability factor, C_L, is applied. However, when $C_V > 1$, both the volume factor and the beam stability factor are used together.[5]

Example 6.9

Design a Parallam beam for a span of 30 ft. The dead and live loads are 229 lb/ft and 458 lb/ft, respectively (the loading of Example 6.5). The volume factor given by the manufacturer is 1.05. Assume that the beam stability factor is not a concern.

Solution

1. Design load, $w_u = 1.2(229) + 1.6(458) = 1007.6$ lb/ft
2. Design bending moment, $M_u = \dfrac{1000.6\ (30)^2}{8} = 113.36 \times 10^3$ ft-lbs or 1360×10^3 in.-lb
3. From Appendix B.9, for Parallam, $F_b = 5360$ psi
4. Adjusted $F'_{bn} = \phi\lambda\, K_F\, C_v\, F_b = (0.85)(0.8)(2.54)(1.05)(5360) = 9720$ psi
5. Required $S = \dfrac{1360 \times 103}{9720} = 147$ in.^{3}
6. Choose a 3-½ in. wide section.

   $S = \dfrac{1}{6}(3.5)d^2 = 147$

   $d = 15.9$ in. ≈ 16 in.
   Select a 3-½ in. × 16 in. section.

[5] When the volume factor C_v >1, it is used in the calculation of the beam stability factor, as discussed in Chapter 7.

TABLE 6.8
Applicability of Adjustment Factors for Structural Composite Lumber

		Wet Service Factor	Temperature Factor	Beam Stability Factor	Volume Factor	Repetitive Member Factor	Column Stability Factor	Bearing Area Factor	Format Conversion Factor	Resistance Factor	Time Effect Factor
									K_F	ϕ	
$F_b' = F_b$	×	C_M	C_t	C_L	C_V	C_r	—	—	2.54	0.85	λ
$F_t' = F_t$	×	C_M	C_t	—	—	—	—	—	2.70	0.80	λ
$F_v' = F_v$	×	C_M	C_t	—	—	—	—	—	2.88	0.75	λ
$F_c' = F_c$	×	C_M	C_t	—	—	—	C_P	—	2.40	0.90	λ
$F_{c\perp}' = F_{c\perp}$	×	C_M	C_t	—	—	—	—	C_b	1.67	0.90	—
$E' = E$	×	C_M	C_t	—	—	—	—	—	—	—	—
$E_{min}' = E_{min}$	×	C_M	C_t	—	—	—	—	—	1.76	0.85	—

CROSS-LAMINATED TIMBER (CLT)

Cross-laminated timber (CLT) is a new generation of engineered wood product developed in Germany and Austria in the 1900s. By the 2000s, it saw a much wider use in Europe. It was introduced in the 2015 NDS for the first time as a code-compliant material.

CLT consists of at least three ⅝ in. to 2 in. thick layers of solid-sawn lumber where the adjacent layers are cross-oriented and bonded with structural adhesive to form a solid element. It has better rigidity in both directions. A CLT panel is shown in Figure 6.2.

CLT is very strong, with superior acoustic, fire, seismic, and thermal performance. It is proving to be an alternative to concrete, masonry, and steel. Up to 18 stories of buildings have been designed using CLT.

The American National Standards Institute (ANSI) and the Engineered Wood Association (formerly the American Plywood Association [APA]) have specified the design properties (reference design values) under PRG320 that represent the specifications intended for use by CLT manufacturers in the United States and Canada, as given in Appendix B.10. The types of lumber used in various CLT grades are explained in Appendix B.10. In configurations of three, five, and seven layers, the thickness of each layer is 1-⅜ in.

These reference design values are multiplied by the adjustment factors shown in Table 6.9.

The wet service factor, C_M, is obtained from the manufacturer. The temperature factor, C_t; beam stability factor, C_L; column stability factor, C_p; and bearing area factor, C_b are as defined previously. The volume factor, C_v, is not applicable to CLT.

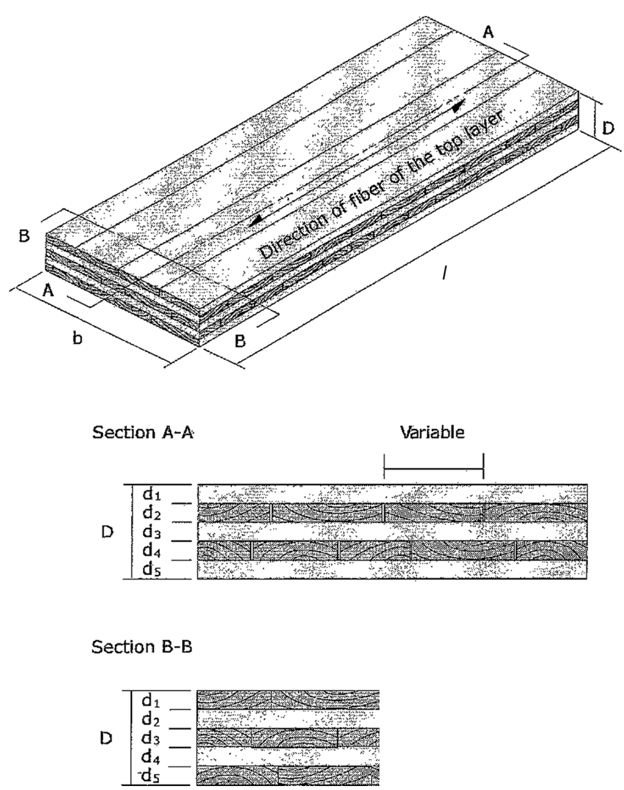


FIGURE 6.2 Example of CLT panel cross-sections and direction of fibers of the top layers.

TABLE 6.9
Applicability of Adjustment Factors for Cross-Laminated Timber

		Wet Service Factor	Temperature Factor	Beam Stability Factor	Column Stability Factor	Bearing Area Factor	Format Conversion Factor	Resistance Factor	Time Effect Factor
$F_b(S_{eff})' = F_b(S_{eff})$	×	C_M	C_t	C_L	—	—	2.54	0.85	λ
$F_t(A_{parallel})' = F_t(A_{parallel})$	×	C_M	C_t	—	—	—	2.70	0.80	λ
$F_v(t_v)' = F_v(t_v)$	×	C_M	C_t	—	—	—	2.88	0.75	λ
$F_s(Ib/Q)_{eff}' = F_s(Ib/Q)_{eff}$	×	C_M	C_t	—	—	—	2.88	0.75	—
$F_c(A_{parallel})' = F_c(A_{parallel})$	×	C_M	C_t	—	C_P	—	2.40	0.90	λ
$F_{c\perp}(A)' = F_{c\perp}(A)$	×	C_M	C_t	—	—	C_b	1.67	0.90	—
$(EI)_{app}' = (EI)_{app}$	×	C_M	C_t	—	—	—	—	—	—
$(EI)_{app\text{-}min}' = (EI)_{app\text{-}min}$	×	C_M	C_t	—	—	—	1.76	0.85	—

Due to the cross orientation of layers (fibers), CLT is a non-homogeneous, anisotropic material with stiffness, E, much smaller in cross direction. Accordingly, the properties of CLT sections are represented by the effective values as discussed below.

EFFECTIVE FLEXURE STIFFNESS AND FLEXURAL STRENGTH

Based on the moment of inertia along the centroid of each layer and the transfer of axis to the neutral axis of the section, the following expression emerges:

$$EI_{eff} = \textstyle\sum_{i=1}^{n} E_i b_i \dfrac{h_i^3}{12} + \displaystyle\sum_i^n E_i A_i z_i^2 \tag{6.14}$$

where:
E_i = the modulus of elasticity of each individual layer
E_0 along the major axis, E_{90} along the minor axis
b_i = the width of each layer of cross section, usually 12 in.
h_i = the thickness of each individual layer
A_i = the area of cross section of each individual layer
z_i = the distance from the center of each individual layer to the neutral axis [NA] of the section

For the outermost fiber, the section modulus is given by:

$$S_{eff} = \frac{2EI_{eff}}{E_1 h} \tag{6.15}$$

where:
E_1 = the modulus of elasticity of the outermost layer
h = the total thickness of the section

ANSI and APA also provide the bending capacity $F_b\, S_{eff}$ table by multiplying S_{eff} from Equation 6.15 by the F_b value from Appendix B.10. A further conservation factor of 0.85 is used in the standard PRG320.

Example 6.10

For a five-layered E2 panel, determine EI_{eff}, S_{eff}, and F_bS_{eff}. Assume the stiffness in the transverse direction to be 1/30[6] of along the fiber stiffness.

Solution

See Figure 6.3, which is a cross section of a five-layer CLT.

1. From Appendix B.10 for E2:

   $E_0 = 1.5 \times 10^6$ psi

   $E_{90} = 1.4 \times 10^6$ psi

   $F_b = 1650$ psi

[6] Sometimes the transverse stiffness is assumed to be negligible also.

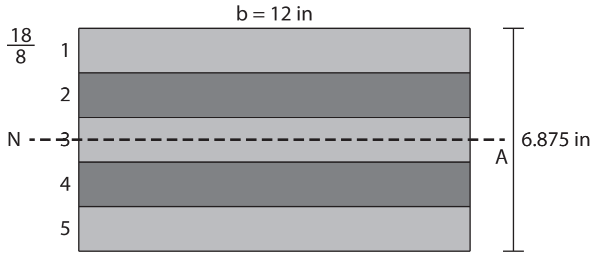


FIGURE 6.3 A 5-layer cross-laminated timber (CLT) section.

2.

Layer	A in.^{2}	$I_0 = bh^3$ 12 in.^{4}	$E \times 10^6$ psi	$EI_0 \times 10^6$ in.^{2}.lb	z in.	$EAz^2 \times 10^6$ in.^{2}.lb	$\Sigma \times 10^6$ in.^{2}.lb
1	1.375 × 12 = 16.5	2.60	1.5	3.9	2.75	187.2	191.1
2	16.5	2.6	1.4/30 = 0.05	0.13	1.375	1.56	1.69
3	16.5	2.6	1.5	3.9	0	0	3.90
4	16.5	2.6	0.05	0.13	1.375	1.56	1.69
5	16.5	2.6	1.5	3.9	2.75	187.2	191.1
							389.5

3. $EI_{eff} = 389.5 \times 10^6$ in.^{2}-lb/ft

4. $S_{eff} = \dfrac{2\,(389.5 \times 10^6)}{(1.5 \times 10^6)(6.875)} = 75.5$ in.^{3}

5. $F_b S_{eff} = (0.85)(1650)(75.5) = 105900$ in.-lb/ft or 8824 ft-lb/ft

EFFECTIVE SHEAR STRENGTH FACTOR

In shear stress distribution, $\tau = VQ/Ib$, where Q is the moment with respect to NA of the area above or below the level where the shear stress is to be determined. Thus, for a constant width, b (canceling b from numerator and denominator):

$$\left(\frac{Ib}{Q}\right)_{eff} = \frac{EI_{eff}}{\sum_{i=1}^{n} E_i h_i z_i} \tag{6.16}$$

where:

h_i = (only the top half section) the thickness of each individual layer except the middle layer, which is one-half of the thickness

z_i = (only the top half section) the distance from the centroid of the layer to NA except the middle layer, where it is the centroid of one-half layer to NA

The shear strength capacity is given by:

$$V_s = F_s(Ib/Q)_{eff} \tag{6.17}$$

where:

F_s = the rolling shear strength of CLT

Example 6.11

For a five-layered E2 CLT panel, determine (1) $(Ib/Q)_{eff}$ and (2) the shear strength capacity.

Solution

1. From Appendix B.10, F_s = 60 psi
2. From Example 6.10, EI_{eff} = 389.5 × 10^6 in.^{2}- lb/ft
3.

Layer	$E \times 10^6$ psi	h in.	z in.	$Ehz \times 10^6$ lb
1	1.5	1.375	2.75	5.672
2	1.4/30 = 0.05	1.375	1.375	0.095
3	1.5	0.688	0.344	0.355
				6.122

4. $(Ib/Q)_{eff} = \dfrac{389.5\times10^6}{6.122\times10^6} = 63.62 \text{ in.}^2$
5. V_s = 60 (63.62) = 3817 lb

EFFECTIVE SHEAR STIFFNESS

For shear modulus G, PRG320 assumes that $G = E/16$ and that the G of the minor axis, that is, $E_{90}/16$, is divided by 10 for the rolling shear modulus. The effective shear stiffness is given by:

$$GA_{eff} = \frac{a^2}{\left(\dfrac{h_1}{2G_1 b}\right) + \left(\sum_{i=2}^{n-1} \dfrac{h_i}{G_i b_i}\right) + \left(\dfrac{h_n}{2G_n b}\right)} \tag{6.18}$$

where:
i = the index for each layer, i.e., top layer i = 1
a = the center to center distance between the top and bottom layer

Example 6.12

For a five-layered E2 CLT panel, determine the effective shear stiffness.

Solution

1. h = 1.375 in.

   b = 12 in.

   a = 5.5 in.

2.

Layer	$G \times 10^6$ psi	$\dfrac{h}{G\,b} \times 10^{-6}$
1	1.5/16 = 0.09375	1.222
2	1.4/(16 × 10) = 0.0085	13.095
3	0.09375	1.222
4	0.0085	13.095
5	0.09375	1.222

3. $GA_{eff} = \dfrac{(5.5)^2}{\left(\frac{1.222}{2}\right) + \left(13.095 + 1.222 + 13.095\right) + \left(\frac{1.222}{2}\right)} = 1.06 \times 10^6$ lb

For various configurations of CLT, ANSI and APA have also computed the effective flexural capacity, effective shear capacity, and effective shear stiffness, as given in Table 6.10. This table is a very helpful design tool.

Example 6.13

Select a CLT section for a 20 ft span floor. The dead and live loads are 30 psf and 50 psf, respectively.

Solution

1. Consider a 1 ft wide section:

   $w_u = 1.2\text{ DL} + 1.6\text{ LL} = 1.2(30) + 1.6(50) = 116$ lb/ft

2. $M_u = \dfrac{w_u L^2}{8} = \dfrac{116\,(20)^2}{8} = 5800$ ft-lb/ft
3. Try a three-layer E2 Grade CLT. From Table 6.10, $F_b S_{eff} = 3825$ ft-lb/ft
4. Adjustment factors
   $K_F = 2.54$
   $\phi = 0.85$
   $\lambda = 0.8$
5. $(F_b S_{eff})'_n = (3825)(2.54)(0.85)(0.8) = 6607 > 5800$ **OK**

SUMMARY OF ADJUSTMENT FACTORS

A: Common to sawn lumber, GLULAM, SCL, CLT
   1. Time effect, $\lambda \le 1$[7]
   2. Temperature, $C_t \le 1$
   3. Wet service, $C_M \le 1$
   4. Format conversion, $K_F > 1$
   5. Resistance factor, $\phi < 1$
   6. Beam stability factor, C_L (applied to F_b only) ≤ 1
   7. Column stability factor, C_P (applied only to $F_{c\parallel}$) ≤ 1
   8. Bearing area factor, C_b (applied only to $F_{c\perp}$) ≥ 1

B: Sawn lumber
   1. Incision factor, $C_i \le 1$
   2. Size factor, $C_F \le 1$
   3. Repetitive, $C_r = 1.15$
   4. Flat use factor, C_{fu} (applied only to F_b), for dimension lumber ≥ 1, for timber ≤ 1

C: GLULAM
   1. Volume factor, $C_V \le 1$
   2. Curvature factor, $C_c \le 1$
   3. Flat use factor, C_{fu} (applied only to F_b) for GLULAM ≥ 1

D: SCL
   1. Volume Factor, $C_V \le 1$ or ≥ 1
   2. Repetitive Factor, $C_r = 1.04$

[7] The time effect factor λ is not applied with $F_{c\perp}$.

TABLE 6.10
Design Capacities for CLT (For Use in the U.S.) a, b, c

CLT Layup	CLT t_p (in.)	Lamination Thickness (in.) in CLT Layup =	⊥	=	⊥	=	⊥	=	Major Strength Direction $(F_bS)_{eff,f,0}$ (lbf-ft/ft of width)	$(EI)_{eff,f,0}$ (10^6 lbf-in.²/ft of width)	$(GA)_{eff,f,0}$ (10^6 lbf/ft of width)	$V_{s,0}$ (lbf/ft of width)	Minor Strength Directions $(F_bS)_{eff,f,90}$ (lbf-ft/ft of width)	$(EI)_{eff,f,90}$ (10^6 lbf-in.²/ft of width)	$(GA)_{eff,f,90}$ (10^6 lbf/ft of width)	$V_{s,90}$ (lbf/ft of width)
E1	4 1/8	1 3/8	1 3/8	1 3/8					4,525	115	0.46	1,430	160	3.1	0.61	495
	6 7/8	1 3/8	1 3/8	1 3/8	1 3/8	1 3/8			10,400	440	0.92	1.970	1,370	81	1.2	1,430
	9 5/8	1 3/8	1 3/8	1 3/8	1 3/8	1 3/8	1 3/8	1 3/8	18,375	1,089	1.4	2,490	3,125	309	1.8	1,960
E2	4 1/8	1 3/8	1 3/8	1 3/8					3,825	102	0.53	1,910	165	3.6	0.56	660
	6 7/8	1 3/8	1 3/8	1 3/8	1 3/8	1 3/8			8,825	389	1.1	2,625	1,430	95	1.1	1,910
	9 5/8	1 3/8	1 3/8	1 3/8	1 3/8	1 3/8	1 3/8	1 3/8	15,600	963	1.6	3,325	3,275	360	1.7	2,625
E3	4 1/8	1 3/8	1 3/8	1 3/8					2,800	81	0.35	1,110	110	2.3	0.44	385
	6 7/8	1 3/8	1 3/8	1 3/8	1 3/8	1 3/8			6,400	311	0.69	1,530	955	61	0.87	1,110
	9 5/8	1 3/8	1 3/8	1 3/8	1 3/8	1 3/8	1 3/8	1 3/8	11,325	769	1.0	1,940	2,180	232	1.3	1,520
E4	4 1/8	1 3/8	1 3/8	1 3/8					4,525	115	0.50	1,750	140	3.4	0.62	605
	6 7/8	1 3/8	1 3/8	1 3/8	1 3/8	1 3/8			10,400	440	1.0	2,410	1,230	88	1.2	1,750
	9 5/8	1 3/8	1 3/8	1 3/8	1 3/8	1 3/8	1 3/8	1 3/8	18,400	1,089	1.5	3,050	2,800	335	1.9	2,400
V1	4 1/8	1 3/8	1 3/8	1 3/8					2,090	108	0.53	1,910	165	3.6	0.59	660
	6 7/8	1 3/8	1 3/8	1 3/8	1 3/8	1 3/8			4,800	415	1.1	2,625	1,430	95	1.2	1,910
	9 5/8	1 3/8	1 3/8	1 3/8	1 3/8	1 3/8	1 3/8	1 3/8	8,500	1,027	1.6	3,325	3,275	360	1.8	2,625
V2	4 1/8	1 3/8	1 3/8	1 3/8					2,030	95	0.46	1,430	160	3.1	0.52	495
	6 7/8	1 3/8	1 3/8	1 3/8	1 3/8	1 3/8			4,675	363	0.91	1,970	1,370	81	1.0	1,430
	9 5/8	1 3/8	1 3/8	1 3/8	1 3/8	1 3/8	1 3/8	1 3/8	8,275	898	1.4	2,490	3,125	309	1.6	1,960
V3	4 1/8	1 3/8	1 3/8	1 3/8					1,740	95	0.49	1,750	140	3.4	0.52	605
	6 7/8	1 3/8	1 3/8	1 3/8	1 3/8	1 3/8			4,000	363	0.98	2,420	1,230	88	1.0	1,750
	9 5/8	1 3/8	1 3/8	1 3/8	1 3/8	1 3/8	1 3/8	1 3/8	7,100	899	1.5	3,050	2,800	335	1.6	2,400

For SI: 1 in. = 25.4 mm; 1 ft = 304.8 mm; 1 lbf = 4.448 N

[a] The second subscript 0 is for along grains values and 90 for perpendicular to grains values.

[b] This table represents one of many possibilities that the CLT could be manufactured by varying lamination grades, thicknesses, orientations, and layer arrangements in the layup.

[c] Custom CLT layups that are not listed in this table shall be permitted in accordance with 7.2.1 of ANSI Standard for Performance Rated CLT.

F_bS (lbf-ft/ft) and Vs (lbf/ft)—Nearest 25 for values greater than 2,500, nearest 10 for values between 1,000 and 2,500, or nearest 5 otherwise.

EI (lbf-in.²/ft) and GA (lbf/ft)—Nearest 10^6 for values greater than 10^7, nearest 10^5 for values between 10^6 and 10^7, or nearest 10^4 otherwise

E: Special Factors
1. Buckling stiffness factor, C_T (applied only to sawn lumber and to E_{min}) ≥ 1
2. Stress interaction factor, C_I (applied only to GLULAM and tapered section) ≤ 1
3. Shear reduction factor, C_{vr} (applied only to F_v for GLULAM in some cases) = 0.72

PROBLEMS

Note: In Problems 6.1 through 6.5, Determine the adjusted reference design values and the strength capacities for the following members. In all cases, consider the dry-service conditions, the normal temperature range, and the no-incision application. In practice, all loading combinations must be checked. However, in these problems only a single load condition should be considered for each member, as indicated in the problem.

6.1 Floor joists are 2 in. × 6 in. at 18 in. on center (OC) of Douglas Fir-Larch #2. They support the dead and live loads.

6.2 Roof rafters are 2 in. × 8 in. at 24 in. OC of Southern Pine #2. The loads are dead load and roof live load.

6.3 Five floor beams are of 4 in. × 8 in. dimension lumber Hem Fir #1, spaced 5 ft apart. The loads are dead and live loads.

6.4 Studs are 2 in. × 8 in. at 20 in. OC of Hem Fir #2. The loads are dead load, live load, and wind load.

6.5 Interior column is 5 in. × 5 in. of Douglas Fir-Larch #2 to support the dead and live loads.

6.6 Determine the unit dead and live loads (on per sq ft area) that can be resisted by a floor system consisting of 2 in. × 4 in. joists at 18 in. OC of Douglas Fir-Larch #1. The span is 12 ft. The dead and live loads are equal.

6.7 Determine the unit dead load on the roof. The roof beams are 4 in. × 10 in. of Hem Fir #1. The beams are located at 5 ft on center and the span is 20 ft apart. They support the dead load and a snow load of 15 psf.

6.8 A 6 in. × 6 in. column of Douglas Fir-Larch #1 supports the dead load and live load on an area of 100 ft^2. Determine the per square ft load if the unit dead load is one-half of the unit live load.

6.9 A floor system is acted on by a dead load of 20 psf and a live load of 40 psf. Determine the size of the floor joists of Douglas Fir-Larch Structural. They are located 18 in. OC and span 12 ft. Assume that the beam stability factor is not a concern.

6.10 In Problem 6.9, determine the size of the floor joists when used in a flat position.

6.11 Determine the size of a column of Southern Pine #2 of dimension lumber that receives loads from an area of 20 ft × 25 ft. The unit service loads are 20 psf dead load and 30 psf live load. Assume that the column stability factor is not a concern.

6.12 For Problem 6.11, design a column of Southern Pine Select Structural timber. Use wet service factor since the reference design values are for wet condition.

6.13 A GLULAM beam section is 6-¾ in. × 37.5 in. from Douglas Fir 24F-1.7E Class. The load combination comprises the dead load, snow load, and wind load. The bending is about the *X*-axis. Determine the adjusted nominal reference design stresses and the strength capacities for bending, tension, shear, compression, modulus of elasticity, and modulus of stability (E_{min}). The span is 30 ft.

6.14 Determine the wind load for Problem 6.13 if the unit dead load is 50 psf and the unit snow and wind loads are equal. The beams are 10 ft apart.

6.15 The beam in Problem 6.13 is installed upside down. Determine the strength capacities.

6.16 The beam in Problem 6.13 is used flat with bending about the minor axis. Determine the design capacities.

6.17 A 5-⅛ in. × 28.5 in., 26F-1.9E Southern Pine GLULAM is used to span 35 ft. The beam has a radius of curvature of 10 ft and lamination thickness of 1-⅜ in. The load combination is the dead load and the snow load. Determine the adjusted nominal reference design stresses and the strength capacities for loading perpendicular to the laminations for the beam installed according to the specifications.

6.18 The beam in Problem 6.17 is installed upside down. Determine the percentage reduction in the strength capacities.

6.19 The beam in Problem 6.17 is loaded along the laminations, about the minor axis. Determine the percentage change in the strength capacities.

6.20 A 1-¾ in. × 7-¼ in. size LVL of 1.9E class is used for the roof rafters spanning 20 ft, located 24 in. on center. Determine the strength design capacities for the dead and snow load combination. The volume factor is given by $(12/d)^{1/7.5}$.

6.21 Two 1-¾ in. × 16 in. (two sections side by side) of Parallam of 2.0E class are used for a floor beam spanning 32 ft, spaced 8 ft on center. The loading consists of the dead and live loads. Determine the strength capacities for bending, tension, compression, and shear. The volume factor is given by $(12/d)^{1/7.5}$.

6.22 Determine the unit loads (per ft^2) on the beam in Problem 6.21 if the live load is one-and-a-half times the dead load.

6.23 Design a Microllam LVL beam for a span of 8 m. The dead and live loads are 3.3 kN/m and 6.6 kN/m, respectively. The volume factor given by the manufacturer is 1.08 and the beam stability factor is 0.85 (1 psi = 6.9×10^{-3} N/mm^2).

6.24 For a seven-layered E1 CLT panel, determine EI_{eff}, S_{eff}, and $F_b\, S_{eff}$. Assume the stiffness in the transverse direction to be 1/30 of the fiber stiffness.

6.25 For a seven-layered E1 CLT panel, determine $(Ib/Q)_{eff}$ and the shear strength capacity.

6.26 For a seven-layered E1 CLT panel, determine the effective shear stiffness.

6.27 Select a V2 grade CLT section for an 18 ft span floor system carrying a dead load of 40 lb/ft^2 and a live load of 50 lb/ft^2. Do not consider the beam stability factor.

6.28 A beam of 30 ft span carries a dead load of 50 lb/ft and a live load of 100 lb/ft. Select an E1 grade CLT section. Ignore the beam stability factor.

7 Flexure and Axially Loaded Wood Structures

INTRODUCTION

The conceptual design of wood members was presented in Chapter 6. The underlying assumption of design in that chapter was that an axial member was subjected to axial tensile stress or axial compression stress only, and a flexure member was subjected to normal bending stress only. However, the compression force acting on a member tends to buckle a member out of the plane of loading, as shown in Figure 7.1. This buckling occurs in the columns and in the compression flange of the beams unless the compression flange is adequately braced. The beam and column stability factors C_L and C_P, respectively, are applied to account for the effect of this lateral buckling.

This chapter presents the detailed designs of flexure members, axially loaded tensile and compression members, and the members subjected to the combined flexure and axial force made of sawn lumber, glued laminated timber (GLULAM), structural composite lumber (SCL), and cross-laminated timber (CLT).

DESIGN OF BEAMS

In most cases, for the design of a flexure member or beam, the bending capacity of the material is a critical factor. Accordingly, the basic criterion for the design of a wood beam is developed from a bending consideration.

In a member subjected to flexure, compression develops on one side of the section; under compression, lateral stability is an important factor. It could induce a buckling effect that could undermine the moment capacity of the member. An adjustment factor is applied in wood design when the buckling effect could prevail, as discussed later on below.

A beam is initially designed for bending capacity. It is checked for shear capacity. It is also checked from the serviceability consideration of the limiting state of deflection. If the size is not found adequate for the shear capacity or the deflection limits, the design is revised.

The bearing strength of a wood member is considered at the beam supports or where loads from other members frame onto the beam. The bearing length (width) is designed on this basis.

BENDING CRITERIA OF DESIGN

For the bending capacity of a member, as discussed in Chapter 6:

$$M_u = F'_{bn} S \tag{7.1}$$

In the case of CLT, Equation 7.1 takes the following form:

$$M_u = F'_{bn}\, S_{eff} \tag{7.2}$$

M_u represents the design moment due to the factored combination of loads. The design moment for a uniformly distributed load, w_u, is given by $M_u = w_u L^2 / 8$ and for a concentrated load, P_u, centered at

FIGURE 7.1 Buckling due to compression.

mid-span, $M_u = P_u L/4$. For other cases, M_u is ascertained from the analysis of structure. For standard loading cases, M_u is listed in Appendix A.3.

The span length, L, is taken as the distance from the center of one support to the center of the other support. However, when the provided (furnished) width of a support is more than what is required from the bearing consideration, it is permitted to take the span length as the clear distance between the supports plus one-half of the required bearing width at each end.

F'_{bn} is the adjusted load resistance factor design (LRFD) reference value for bending. To start, the reference bending design value, F_b, for the appropriate species and grade is obtained. These values are listed in Appendixes B.2 through B.4 for sawn lumber and Appendixes B.7 through B.10 for GLULAM, SCL, and CLT. Then the value is adjusted by multiplying the reference value by a string of factors. The applicable adjustment factors were given in Chapter 6: in Table 6.6 for sawn lumber, Table 6.7 for GLULAM, Table 6.8 for SCL, and Table 6.9 for CLT.

For sawn lumber, the adjusted reference bending design value is restated as:

$$F'_{bn} = \phi F_b \lambda K_F C_M C_t C_F C_r C_{fu} C_i C_L \tag{7.3}$$

For GLULAM, the adjusted reference bending design value is restated for two cases.

For loads perpendicular to lamination (load on narrow face) on GLULAM:

$$F'_{bn} = \phi F_b \lambda\, K_F C_M C_t C_c C_I (C_v \text{ or } C_L) \tag{7.4}$$

For loads parallel to lamination (load on wide face) on GLULAM:

$$F'_{bn} = \phi F_b \lambda K_F C_M C_t C_c C_{fu} C_I\, C_L \tag{7.5}$$

For SCL, the adjusted reference bending design value is:

$$F'_{bn} = \phi F_b \lambda K_F C_M C_t C_r (C_v \text{ or/and } C_L) \tag{7.6}$$

For CLT, the adjusted reference design value is:

$$F'_{bn} = \phi\, F_b\, \lambda\, K_F\, C_M\, C_t\, C_L \tag{7.7}$$

where:

F_b is the tabular reference bending design value
ϕ is the resistance factor for bending = 0.85
λ is the time factor (Table 6.2)

C_M is the wet-service factor
C_t is the temperature factor
C_F is the size factor
C_r is the repetitive member factor
C_{fu} is the flat use factor
C_i is the incision factor
C_L is the beam stability factor
C_c is the curvature factor
C_v is the volume factor
C_I is the stress interaction factor
K_F is the format conversion factor = 2.54

Using the assessed value of F'_{bn}, from Equations 7.3 through 7.7, based on the adjustment factors known initially, the required section modulus, S, is determined from Equation 7.1 or S_{eff} from Equation 7.2,[1] and a trial section is selected having the section modulus higher than the computed value. In the beginning, some section-dependent factors such as C_F, C_v, and C_L will not be known while others such as λ, K_F, and ϕ will be known. The design is performed considering all possible load combinations along with the relevant time factor. If loads are of one type only, that is, all vertical or all horizontal, the highest value of the combined load divided by the relevant time factor determines which combination is critical for design.

Based on the trial section, all adjustment factors including C_L are then computed and the magnitude of F'_{bn} is reassessed. A revised S is obtained from Equation 7.1 or 7.2, and the trial section is modified, if necessary. For CLT, a revised $F'_b\, S_{eff}$ with all adjustment factors or a revised F'_b and S_{eff} therefrom is computed, and the section is modified, if necessary.

BEAM STABILITY FACTOR, C_L

As stated earlier, the compression stress, besides causing an axial deformation, can cause a lateral deformation if the compression zone of the beam is not braced against the lateral movement. In the presence of the stable one-half tensile portion, the buckling in the plane of loading is prevented. However, the movement could take place sideways (laterally), as shown in Figure 7.2.

The bending design described in Chapter 6 assumed that no buckling was present and adjustments were made for other factors only. The condition of no buckling is satisfied when the bracing requirements, as listed in Table 7.1, are met. In general, when the depth-to-breadth ratio is 2 or less, no lateral bracings are required. When the depth-to-breadth ratio is more than 2 but does not exceed 4, the ends of the beam should be held in position by one of the these methods: full-depth solid blocking, bridging, hangers, nailing, or bolting to other framing members. The stricter requirements are stipulated as stated in Table 7.1 to hold the compression edge in line for a depth-to-breadth ratio of higher than 4.

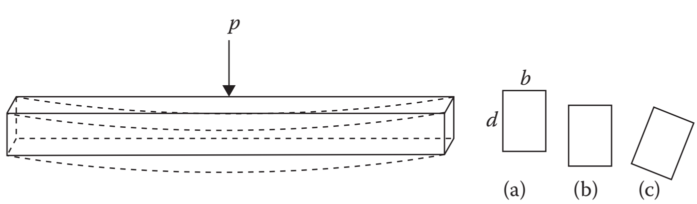


FIGURE 7.2 Buckling of a bending member: (a) original position of beam, (b) deflected position without lateral instability, (c) compression edge buckled laterally.

[1] For CLT, Table 6.10 provides a direct value of $F_b\, S_{eff}$.

TABLE 7.1
Bracing Requirements for Lateral Stability

Depth/Breadth Ratio[a]	Bracing Requirements
	Sawn Lumber
≤2	No lateral bracing required.
>2 but ≤4	The ends are to be held in position, as by full-depth solid blocking, bridging, hangers, nailing, or bolting to other framing members, or by other acceptable means.
>4 but ≤5	The compression edge is to be held in line for its entire length to prevent lateral displacement, as by sheathing or subflooring, and the ends at points of bearing are to be held in position to prevent rotation and/or lateral displacement.
>5 but ≤6	Bridging, full-depth solid blocking, or diagonal cross bracing is to be installed at intervals not exceeding 8 ft.; the compression edge is to be held in line for its entire length to prevent lateral displacement, as by sheathing or subflooring; and the ends at points of bearing are to be held in position to prevent rotation and/or lateral displacement.
> 6 but ≤7	Both edges of a member are to be held in line for their entire length, and the ends at points of bearing are to be held in position to prevent rotation and/or lateral displacement.
Combined bending and compression	The depth/breadth ratio may be as much as 5 if one edge is held firmly in line. If, under all load conditions, the unbraced edge is in tension, the depth/breadth ratio may be as much as 6.
	Glued Laminated Timber
≤1	No lateral bracing required.
>1	The compression edge is supported throughout its length to prevent lateral displacement, and the ends at points of bearing are laterally supported to prevent rotation.

[a] Nominal dimensions.

When the requirements of Table 7.1 are not met, the following beam stability factor has to be applied to account for the buckling effect:

$$C_L = \left(\frac{1+\alpha}{1.9}\right) - \sqrt{\left(\frac{1+\alpha}{1.9}\right)^2 - \left(\frac{\alpha}{0.95}\right)} \tag{7.8}$$

where:

$$\alpha = \frac{F_{bEn}}{F_{bn}'^{*}} \tag{7.9}$$

where $F_{bn}'^{*}$ is the reference bending design value adjusted for all factors except C_v, C_{fu}, and C_L.
For SCL, when $C_v > 1$, C_v is also included in calculating $F_{bn}'^{*}$.
F_{bEn} is the Euler-based LRFD critical buckling stress for bending:

$$F_{bEn} = \frac{1.2E'_{y\ min(n)^{*}}}{R_B^2} \tag{7.10}$$

where:
$E'_{y\ min(n)}$ is the adjusted nominal stability modulus of elasticity along the minor axis
R_B is the slenderness ratio for bending

[*] Use y axis

$$R_B = \sqrt{\frac{L_e d}{b^2}} \le 50 \tag{7.11}$$

where L_e is the effective unbraced length, as discussed in the "Effective Unbraced Length" section.

When R_B exceeds 50 in Equation 7.10, the beam dimensions should be revised to limit the slenderness ratio R_B to 50.

Effective Unbraced Length

The effective unbraced length is a function of several factors, such as the type of span (simple, cantilever, continuous); the type of loading (uniform, variable, concentrated loads); the unbraced length, L_u, which is the distance between the points of lateral supports; and the size of the beam.

For a simple one span or cantilever beam, the following values can be conservatively used for the effective length:

$$\text{For} \quad \frac{L_u}{d} < 7, \quad L_e = 2.06 L_u \tag{7.12}$$

$$\text{For} \quad 7 \le \frac{L_u}{d} \le 14.3, \quad L_e = 1.63 L_u + 3d \tag{7.13}$$

$$\text{For} \quad \frac{L_u}{d} > 14.3, \quad L_e = 1.84 L_u \tag{7.14}$$

Example 7.1

A 5½ in. × 24 in. GLULAM beam is used for a roof system having a span of 32 ft, which is braced only at the ends. GLULAM consists of the Douglas Fir 24F-1.8E. Determine the beam stability factor. Use the dead and live conditions only.

Solution

1. Reference design values

$F_b = 2400$ psi

$E = 1.8 \times 10^6$ psi

$E_{y(min)} = 0.83 \times 10^6$ psi

2. Adjusted design values

$F'^*_{bn} = \phi F_b \lambda K_F$

$= (0.85)(2400)(0.8)(2.54) = 4147$ psi or 4.15 ksi

$E'_{y\,min(n)} = \phi E_{y(min)} K_F$

$= (0.85)(0.83 \times 10^6)(1.76)$

$= 1.24 \times 10^6$ psi or 1.24×10^3 ksi

3. Effective unbraced length

$$\frac{L_u}{d} = \frac{32\times 12}{24} = 16 > 14.3$$

From Equation 7.14:

$$L_e = 1.84L_u = 1.84(32) = 58.88 \text{ ft or } 701.28 \text{ in.}$$

4. From Equation 7.11:

$$R_B = \sqrt{L_e d/b^2}$$

$$= \sqrt{(701.28)(24)/(5.5)^2}$$

$$= 23.59 < 50 \textbf{ OK}$$

5. $F_{bEn} = \dfrac{1.2E'_{y\ min(n)}}{RB^2}$

$$= \frac{12.2\ (1.24\times 103)}{(23.59)^2} = 2.7$$

6. $\alpha = \dfrac{F_{bEn}}{F'^{*}_{bn}} = \dfrac{2.7}{4.15} = 0.65$

7. From Equation (7.8):

$$C_L = \frac{1.65}{1.9} - \sqrt{\left(\frac{1.65}{1.9}\right)^2 - \left(\frac{0.65}{0.95}\right)} = 0.6$$

SHEAR CRITERIA

A transverse loading applied to a beam results in vertical shear stresses in any transverse (vertical) section of a beam. Because of the complementary property of shear, an associated longitudinal shear stress acts along the longitudinal plane (horizontal face) of a beam element. In any mechanics of materials text, it can be seen that the longitudinal shear stress distribution across the cross section is given by:

$$f_v = \frac{VQ}{Ib} \tag{7.15}$$

where:

f_v is the shear stress at any plane across the cross section
V is the shear force along the beam at the location of the cross section
Q is the moment of the area above the plane or below the plane where stress is desired to the top or to the bottom edge of the section. Moment is taken about the neutral axis
I is the moment of inertia along the neutral axis
b is the width of the section

Equation (7.15) also applies for the transverse shear stress at any plane of the cross section as well, because the transverse and the longitudinal shear stresses are complementary, numerically equal, and opposite in sign.

In the case of CLT, though the basic equation is Equation (7.15), the formulation is different due to heterogeneous sections, as discussed in Chapter 6.

SHEAR STRENGTH OF SAWN LUMBER, GLULAM, AND SCL

For a rectangular cross section, which is usually the case with wood beams, the shear stress distribution by Equation (7.15) is parabolic with the following maximum value at the center:

$$f_{v(max)} = \frac{3V_u}{2A} \tag{7.16}$$

In terms of V_u, the basic equation for shear design of the beam is:

$$V_u = \frac{2}{3} F'_{vn} A \tag{7.17}$$

where:
V_u is the maximum shear force due to factored load on the beam
F'_{vn} is the adjusted reference shear design value
A is the area of the beam

The National Design Specification (NDS) permits that the maximum shear force, V_u, to be taken as the shear force at a distance equal to the depth of the beam from the support. However, V_u is usually taken to be the maximum shear force from the diagram, which is at the support for a simple span.

For sawn lumber, the adjusted reference shear design value is:

$$F'_{vn} = \phi F_v \lambda\, K_F C_M C_t C_i \tag{7.18}$$

For GLULAM, the adjusted reference shear design value is:

$$F'_{vn} = \phi F_v \lambda\, K_F C_M C_t C_{vr} \tag{7.19}$$

For SCL, the adjusted reference shear design value is:

$$F'_{vn} = \phi F_v \lambda\, K_F\, C_M C_t \tag{7.20}$$

Shear Strength of CLT

$$F_S(Ib/Q)_{eff'} = \phi\, F_S\, (Ib/Q)_{eff}\, \lambda\, K_F\, C_M\, C_t \tag{7.21}$$

where:
F_v is the tabular reference shear design value
F_S is the reference value of rolling shear
ϕ is the resistance factor for shear = 0.75
λ is the time factor (see the "Time Effect Factor, λ" section in Chapter 6)
C_M is the wet-service factor
C_t is the temperature factor
C_i is the incision factor
C_{vr} is the shear reduction factor
K_F is the format conversion factor, which equals 2.88
$(Ib/Q)_{eff}$ as given by Equation (6.16)

DEFLECTION CRITERIA

Note that deflection is a service requirement. Thus, it is computed using the service loads (not the factored loads).

The deflection in a beam comprises flexural deflection and shear deflection; the latter is normally a very small quantity. The reference design values for modulus of elasticity, E, as given in NDS 2015, with adjustments as shown in Equation (7.23), include a shear deflection component, which means that only the flexural deflection is to be considered in beam design.

However, where the shear deflection could be appreciable, as on a short heavily loaded beam, it should be accounted for separately in addition to the flexural deflection. The shear deflection is computed by integrating the shear strain term $V_{(x)}Q/GIb$ and by expressing the shear force in terms of x. The form of the shear deflection is $\delta = kWL/GA'$, where k is a constant that depends on the loading condition, G is modulus of rigidity, and A' is the modified beam area. When the shear deflection is considered separately, a shear free value of modulus of elasticity should be used. For sawn lumber and GLULAM, it is approximately 1.03 and 1.05 times, respectively, of the listed NDS reference design value.

DEFLECTION OF SAWN LUMBER, GLULAM, AND SCL

The flexural deflection is a function of the type of loading, type of beam span, moment of inertia of the section, and modulus of elasticity. For a uniformly loaded simple span member, the maximum deflection at mid-span is:

$$\delta = \frac{5wL^4}{384E'I} \tag{7.22}$$

where:
- w is the uniform combined service load per unit length
- L is the span of the beam
- E' is the adjusted modulus of elasticity:

$$E' = EC_M C_t C_i \tag{7.23}$$

- E, in Equation (7.23), is the reference modulus of elasticity
- I is the moment of inertia along the neutral axis

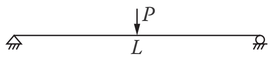
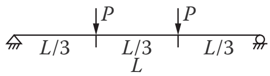
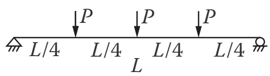
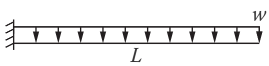
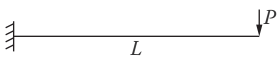
Depending on the loading condition, however, the theoretical derivation of the expression for deflection might be quite involved. For some commonly encountered load conditions, when the expression of the bending moment is substituted in the deflection expression, a generalized form of deflection can be expressed as follows:

$$\delta = \frac{ML^2}{CE'I} \tag{7.24}$$

where:
- w is the service loads combination
- M is the moment due to the service loads

The values of constant C are indicated in Table 7.2 for different load cases.

TABLE 7.2
Deflection Loading Constants

Diagram of Load Condition	Constant C for Equation (7.20)
	9.6
	12
	9.39
	10.13
	4
	3

In simplified form, the designed factored moment, M_u, can be converted to the service moment by dividing by a factor of 1.5 (i.e., $M = M_u/1.5$). The service live load moment, M_L, is approximately two-thirds of the total moment M (i.e., $M_L = 2M_u/4.5$). The factor C from Table 7.2 can be used in Equation (7.24) to compute the expected deflection.

DEFLECTION OF CLT

For CLT, $E'I$ in Equations (7.22) and (7.24) is represented by $E'I_{eff}$. To account for shear deformation as well, the effective flexural stiffness $E'I_{eff}$ is reduced to $E'I_{app}$. Thus, for CLT, the deflection equations take the following forms.

For a uniformly loaded simple span:

$$\delta = \frac{5}{384} \frac{wL4}{E'I_{app}} \tag{7.25}$$

For other loading conditions:

$$\delta = \frac{ML^2}{CE'I_{app}} \tag{7.26}$$

where:

$$EI_{app} = \frac{EI_{eff}}{1 + \dfrac{K_S EI_{eff}}{GA_{eff} L^2}} \tag{7.27}$$

$$E'I_{app} = EI_{app}\, C_M\, C_t \tag{7.28}$$

EI_{eff} is the effective flexural stiffness given by Equation (6.14)
GA_{eff} is the effective shear stiffness given by Equation (6.18)
K_S is a constant for loading conditions given in Table 7.3

The actual (expected) maximum deflection δ should be less than or equal to the allowable deflections, Δ, as given in Table 7.4. Often a check is made for two cases: the live load alone as well as for the total load. Thus:

$$\delta_L \leq \Delta_L \tag{7.29}$$

$$\delta_{TL} \leq \Delta_{TL} \tag{7.30}$$

When the above criteria are not satisfied, a new beam size is determined using the allowable deflection as a guide and then computing the desired moment of inertia on that basis.

TABLE 7.3
K_s Values for Various Loading Conditions

Loading	End Fixity	K_s
Uniformly distributed	Pinned	11.5
	Fixed	57.6
Concentrated at midspan	Pinned	14.4
	Fixed	57.6
Concentrated at quarter points	Pinned	10.5
Constant moment	Pinned	11.8
Uniformly distributed	Cantilevered	4.8
Concentrated at free-end	Cantilevered	3.6

TABLE 7.4
Recommended Deflection Criteria

Classification	Live or Applied Load Only	Dead Load Plus Applied Load
Roof beams		
No ceiling	Span/180	Span/120
Without plaster ceiling	Span/240	Span/180
With plaster ceiling	Span/360	Span/240
Floor beams[a]	Span/360	Span/240
Highway bridge stringers	Span/300	
Railway bridge stringers	Span/300–Span/400	

Source: American Institute of Timber Association, *Timber Construction Manual*, 5th ed., John Wiley & Sons, New York, 2005.
[a] Additional limitations are used where increased floor stiffness or reduction of vibration is desired.

CREEP DEFLECTION

In addition to the elastic deflection discussed above, beams deflect more with time. This is known as the *creep* or the time-dependent deflection. When this is foreseen as a problem, the member size designed on the basis of elastic or short-term deflection is increased to provide for extra stiffness. The total long-term deflection is computed as:

$$\delta_t = K_{cr}\delta_{LT} + \delta_{ST} \tag{7.31}$$

where:

δ_t is the total deflection

K_{cr} is a creep factor; it equals 1.5 for lumber, GLULAM, and SCL, and 2.0 for CLT under dry conditions

δ_{LT} is the elastic deflection due to dead load and a portion (if any) of the live load representing the long-term design load

δ_{ST} is the elastic deflection due to the remaining design load representing the short-term design load

Example 7.2

Design roof rafters spanning 16 ft and spaced 16 in. on center (OC). The plywood roof sheathing prevents local buckling. The dead load is 12 psf, and the roof live load is 20 psf. Use Douglas Fir-Larch #1 wood.

Solution

A. Loads
   1. Tributary area/ft $= \frac{16}{12} \times 1 = 1.333\,\text{ft}^2/\text{ft}$
   2. Loads per feet

      $$w_D = 12 \times 1.333 = 16 \text{ lb/ft}$$

      $$w_L = 20 \times 1.333 = 26.66 \text{ lb/ft}$$

   3. Loads combination

      $$w_u = 1.2w_D + 1.6w_L$$

      $$= 1.2(16) + 1.6(26.66) = 61.86\,\text{lb/ft}$$

   4. Maximum bending moment

      $$M_u = \frac{w_u L^2}{8} = \frac{(61.86)(16)^2}{8} = 1974.52\,\text{ft lb or } 23.75 \text{ in.-k}$$

   5. Maximum shear

      $$V_u = \frac{w_u L}{2} = \frac{(61.86)(16)}{2} = 494.9 \text{ lb}$$

B. Reference design values (Douglas Fir-Larch #1, 2 in. and wider)
   1. F_b = 1000 psi
   2. F_v = 180 psi
   3. $E = 1.7 \times 10^6$ psi
   4. $E_{min} = 0.62 \times 10^6$ psi

C. Preliminary design

1. Initially adjusted bending design value

$$F'_{bn}(\text{estimated}) = \phi F_b \lambda K_F C_r$$

$$= (0.85)(1000)(0.8)(2.54)(1.15) = 1986$$

2. $S_{reqd} = \dfrac{M_u}{F'_{bn}(\text{estimated})} = \dfrac{(23.75 \times 1000)}{1986} = 11.96$

3. Try 2 in. × 8 in. $S = 13.14\ \text{in.}^3$

$A = 10.88\ \text{in.}^2$

$I = 47.63\ \text{in.}^4$

D. Revised design

1. Adjusted reference design values

	Reference Design Values (psi)	ϕ	λ	K_F	C_F	C_r	$F'_{(\,)n}$ (psi)
F'_{bn} [a]	1000	0.85	0.8	2.54	1.2	1.15	2384
F'_{vn}	180	0.75	0.8	2.88	—	—	311
E'	1.7×10^6	—	—	—	—	—	1.7×10^6
$E'_{min(n)}$	0.62×10^6	0.85	—	1.76	—	—	0.93×10^6

[a] Without C_L factor.

E. Check for bending strength.

Bending capacity = $F'_{bn} S$

$$= \frac{(2384)(14.14)}{1000} = 31.33 > 23.75 \text{ in.-K } \textbf{OK}$$

F. Check for shear strength.

$$\text{Shear capacity} = F'_{vn}\left(\frac{2}{3}A\right)$$

$$= 311\left(\frac{2}{3} \times 10.88\right) = 2255 > 494.5 \text{ lb } \textbf{OK}$$

G. Check for deflection.

1. Deflection is checked for service load:

$w = 16 + 26.66 = 42.66$ lb/ft.

2. $\delta = \dfrac{5}{384}\dfrac{wL^4}{E'I} = \dfrac{5}{384}\dfrac{(42.66)(16)^4(12)^3}{(1.7 \times 10^6)(47.63)} = 0.78\ \text{in.}$

3. Allowable deflection (without plastered ceiling)

$$\Delta = \frac{L}{180} = \frac{16 \times 12}{180} = 1.07\ \text{in.} > 0.78\ \text{in.} \quad \textbf{OK}$$

Example 7.3

A structural GLULAM is used as a beam to support a roof system. The tributary width of the beam is 16 ft. The beam span is 32 ft. The floor dead load is 15 psf and the live load is 40 psf. Use Douglas Fir GLULAM 24F-1.8E. The beam is braced only at the supports.

Solution

A. Loads
   1. Tributary area/ft. = 16 × 1 = 16 ft^2/ft
   2. Loads per feet

$$w_D = 15 \times 16 = 240 \text{ lb/ft}$$

$$w_L = 40 \times 16 = 640 \text{ lb/ft}$$

   3. Design load, $w_u = 1.2w_D + 1.6w_L$

$$= 1.2(240) + 1.6(640) = 1312 \text{ lb/ft or } 1.31 \text{ k/ft}$$

   4. Design bending moment

$$M_u = \frac{w_u L^2}{8} = \frac{(1.31)(32)^2}{8} = 167.68 \text{ ft.-k or } 2012.16 \text{ in.-k}$$

   5. Design shear

$$V_u = \frac{w_u L}{2} = \frac{1.31(32)}{2} = 20.96 \text{ k}$$

B. Reference design values

$F_b = 2400$ psi

$F_v = 265$ psi

$E = 1.8 \times 10^6$ psi

$E_{y(min)} = 0.83 \times 10^6$ psi

C. Preliminary design
   1. Initially adjusted bending reference design value:

$$F'_{bn}(\text{estimated}) = \phi F_b \lambda K_F$$

$$= (0.85)(2400)(0.8)(2.54) = 4145 \text{psi or } 4.15 \text{ ksi}$$

   2. $S_{reqd} = \dfrac{2012.16}{4.15} = 484.86 \text{ in.}^3$

Try $5\frac{1}{2}$ in. × 24 in. $S = 528 \text{ in.}^3$

$A = 132 \text{ in.}^2$

$I = 6336 \text{ in.}^4$

D. Revised adjusted design values

Type	Reference Design Values (psi)	ϕ	λ	K_F	F'_{0n} (psi)
F'^*_{bn}	2400	0.85	0.8	2.54	4145
F'_{vn}	265	0.75	0.8	2.88	457.9
E'	1.8×10^6	—	—	—	1.8×10^6
$E'_{min(n)}$	0.83×10^6	0.85	—	1.76	1.24×10^6

Note: $F'_{bn}{}^*$ is the reference bending design value adjusted for all factors except C_V, C_{fu}, and C_L.

E. Volume factor, C_v,

$$C_v = \left(\frac{5.125}{b}\right)^{1/10}\left(\frac{12}{d}\right)^{1/10}\left(\frac{21}{L}\right)^{1/10}$$

$$= \left(\frac{5.125}{5.5}\right)^{1/10}\left(\frac{12}{24}\right)^{1/10}\left(\frac{21}{32}\right)^{1/10} = 0.89$$

F. Beam stability factor, C_L:

From Example 7.1, $C_L = 0.60$.

Since $C_L < C_v$, use the C_L factor.

G. Bending capacity
1. $F'_{bn} = (4145)(0.6) = 2487$ psi or 2.49 ksi
2. Moment capacity $= F'_{bn}S$
   $= 2.49\ (528)$
   $= 1315$ in.-k $< 2112.16\ (M_u)$

A revised section should be selected and steps E, F, and G should be repeated.

H. Check for shear strength[2].

$$\text{Shear capacity} = F'_{vn}\left(\frac{2A}{3}\right) = 457.9\left(\frac{2}{3}\times 132\right) = 40{,}295 \text{ lb or } 40.3 \text{ k} > 20.29 \text{ k} \quad \textbf{OK}$$

I. Check for deflection.
1. Deflection checked for service load

   $w = 240 + 640 = 880$ lb/ft

2. $$\delta = \frac{5}{384}\frac{wL^4}{EI} = \frac{5}{384}\frac{(880)(32)^4(12)^3}{(1.8\times10^6)(6336)} = 1.82 \text{ in.}$$

3. Permissible deflection (without plastered ceiling)

$$\Delta = \frac{L}{180} = \frac{32\times12}{180} = 2.13 \text{ in.} > 1.82 \text{ in.} \quad \textbf{OK}$$

[2] Based on the original section.

Example 7.4

Design a CLT floor system spanning 16 ft. The dead and live loads are 40 psf each.

Solution

A. Moment capacity

1. $w_u = 1.2(40) + 1.6(40) = 112$ lb/ft^2

2. $M_u = \dfrac{112(16)^2}{8} = 3584$ ft-lb/ft width

3. Select a three-layer E2 section. From Table 6.10:

   $F_b S_{eff} = 3825$ ft-lb/ft

4. Adjustment factors

   $\phi = 0.85$

   $\lambda = 0.80$

   $K_F = 2.54$

   $C_L = 1$ (stability criteria satisfied)

5. $F_b S_{eff}' = 3825(0.85)(0.8)(1)(2.54) = 6607 > 3585$ ft-lb/ft **OK**

B. Check for shear.

6. $V_u = \dfrac{w_u L}{2} = \dfrac{112(16)}{2} = 896$ lb

7. From Table 6.10, $F_S\,(I\,b/Q)_{eff} = 1910$ lb
8. Adjusted $F_S\,(I\,b/Q)_{eff}' = 1910(0.75)(2.88) = 4126$ lb > 896 **OK**

C. Deflection check

9. From Table 6.10, for three-layer E2:

   $(EI)_{eff} = 102 \times 10^6$ in.^{2}-lb/ft

   $(GA)_{eff} = 0.53 \times 10^6$ lb/ft

10. From Table 7.3, $K_S = 11.5$
11. From Equation (7.27):

$$EI_{app} = \frac{102 \times 10^6}{1 + \dfrac{11.5\left(102 \times 10^6\right)}{\left(0.53 \times 10^6\right)\left(16 \times 12\right)^2}}$$

$$= 96.2 \times 10^6 \text{ in.}^2\text{-lb/ft}$$

12. $EI_{app'} = 96.2 \times 10^6\,(1)(1)$
13. Service load, $w = 80$ psf. From Equation (7.25):

$$\delta = \frac{5\,(80)(16)^4(12)^3}{384\left(96.2 \times 10^6\right)} = 1.23 \text{ in.}$$

$$\text{Allowable } \Delta \text{ (from Table 7.4)} = \frac{L}{240} = \frac{16 \times 12}{240} = 0.8 \text{ in.} \quad \textbf{NG}$$

Select a five-layer E2 section.

BEARING AT SUPPORTS

The bearing perpendicular to the grains occurs at the supports or wherever a load-bearing member rests onto the beam, as shown in Figure 7.3. The relation for bearing design is:

$$P_u = F'_{C\perp n} A \tag{7.32}$$

The adjusted compressive design value perpendicular to the grain is obtained by multiplying the reference design value by the adjustment factors. Including these factors, Equation (7.32) becomes:

For sawn lumber:

$$P_u = \phi F_{C\perp} K_F C_M C_t C_i C_b A \tag{7.33}$$

For GLULAM, SCL, and CLT:

$$P_u = \phi F_{C\perp} K_F C_M C_t C_b A \tag{7.34}$$

where:

P_u is the reaction at the bearing surface due to the factored load on the beam
$F_{C\perp}$ is the reference compressive design value perpendicular to the grain
$F'_{C\perp n}$ is the adjusted compressive design value perpendicular to the grain
ϕ is the resistance factor for compression = 0.9
C_M is the wet-service factor
C_t is the temperature factor
C_i is the incision factor
C_b is the bearing area factor, as discussed below
K_F is the format conversion factor for bearing = 1.875/ϕ
A is the area of bearing surface (perpendicular to the grain)

Bearing Area Factor, C_b

The bearing area factor is applied only to a specific case when the bearing length l_b is less than 6 in. and also the distance from the end of the beam to the start of the contact area is larger than 3 in., as shown in Figure 7.4. The factor is not applied to the bearing surface at the end of a beam, which

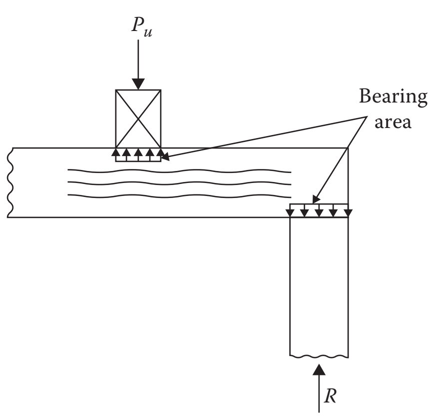


FIGURE 7.3 Bearing perpendicular to grain.

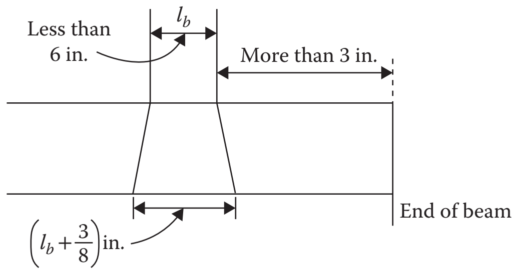


FIGURE 7.4 Bearing area factor.

may be of any length, or where the bearing length is 6 in. or more at any other location than the end. This factor accounts for the additional wood fibers that could resist the bearing load. It increases the bearing length by ⅜ in. Thus:

$$C_b = \frac{l_b + 3/8}{l_b} \tag{7.35}$$

where l_b, the bearing length, is the contact length parallel to the grain.

Example 7.5

For Example 7.3, determine the bearing surface area at the beam supports.

Solution

1. Reaction at the supports

$$R_u = \frac{w_u L}{2} = \frac{1.31(32)}{2} = 20.96 \text{ k}$$

2. Reference design value for compression perpendicular to grains

$$F_{C\perp n} = 650 \text{psi}$$

3. Initially adjusted perpendicular compression reference design value

$$F'_{c\perp n} = \phi F_{C\perp}\ K_F C_M C_t C_i$$

$$= 0.9(650)(1.67)(1)(1) = 977 \text{psi or } 0.977 \text{ ksi}$$

4. $A_{reqd} = \dfrac{R_u}{F'_{c\perp n}} = \dfrac{20.96}{0.977} = 21.45 \text{ in.}^2$

5. Initial bearing length

$$l_b = \frac{A}{b} = \frac{21.45}{5.5} = 3.9 \text{ in.}$$

6. Bearing area factor

$$C_b = \frac{l_b + 3/8}{l_b}$$

$$= \frac{3.9 + 0.375}{3.9} = 1.1$$

7. Adjusted perpendicular compression design value

$$F'_{C\perp n} = 0.977(1.1) = 1.07 \text{ ksi}$$

8. $A = \dfrac{R_u}{F'_{C\perp n}} = \dfrac{20.96}{1.07} = 19.6 \text{ in.}^2$

9. Bearing length, $l_b = \dfrac{19.6}{5.5} = 3.6$ in.

DESIGN OF AXIAL TENSION MEMBERS

Axially loaded wood members generally comprise studs, ties, diaphragms, shear walls, and trusses where loads directly frame into joints to pass through the member's longitudinal axis or with a very low eccentricity. These loads exert either tension or compression without any appreciable bending in members. For example, a truss has some members in compression and some in tension. The treatment of a tensile member is relatively straightforward because only the direct axial stress is exerted on the section. However, the design is typically governed by the net section at the connection because the hole opening separates out from the fastener in a stretched condition.

The tensile capacity of a member is given by:

$$T_u = F'_{tn} A_n \tag{7.36}$$[3]

Axial tension members in wood generally involve a relatively small force, for which a dimensional lumber section is used that requires inclusion of a size factor.

Including the adjustment factors, the tensile capacity is represented as follows:

For sawn lumber:

$$T_u = \phi F_t \lambda K_F C_M C_t C_F C_i A_n \tag{7.37}$$

For GLULAM, SCL, and CLT:

$$T_u = \phi F_t \lambda K_F C_M C_t A_n \tag{7.38}$$

where:

T_u is the factored tensile load on the member
F_t is the reference tension design value parallel to the grain
F'_{tn} is the adjusted tension design value parallel to the grain
ϕ is the resistance factor for tension = 0.8
λ is the time effect factor (see the "Time Effect Factor, λ" section in Chapter 6)
C_M is the wet-service factor
C_t is the temperature factor
C_i is the incision factor
C_F is the size factor for sawn dimension lumber only
K_F is the format conversion factor for tension = 2.70
A_n is the net cross-sectional area as follows:

$$A_n = A_g - \Sigma A_h \tag{7.39}$$

[3] In axial members, fibers run parallel to the direction of the load. A is also referred to as $A_{parallel}$.

where:

A_g is the gross cross-sectional area

ΣA_h is the sum of the projected area of the holes

For CLT, the area of those cross sections that run parallel to the load is included.

In determining the net area of a nail or a screw connection, the projected area of the nail or screw is neglected. For a bolted connection, the projected area consists of rectangles given by:

$$\Sigma A_h = nbh \tag{7.40}$$

where:

n is the number of bolts in a row

b is the width (thickness) of the section

h is the diameter of the hole, usually $d + \frac{1}{16}$ in.

d is the diameter of the bolt

Example 7.6

Determine the size of the bottom (tension) chord of the truss shown in Figure 7.5. The service loads acting on the horizontal projection of the roof are dead load = 20 psf and snow load = 30 psf. The trusses are 5 ft on center. The connection is made by one bolt of ¾ in. diameter in each row. Lumber is Douglas Fir-Larch #1.

Solution

A. Design loads
  1. Factored unit loads = $1.2D + 1.6S = 1.2(20) + 1.6(30) = 72$ psf
  2. Tributary area, ft²/ft = $5 \times 1 = 5$ ft²/ft
  3. Load/ft, $w_u = 72(5) = 360$ lb/ft
  4. Load at joints

$$\text{Exterior} = 360\left(\frac{7.5}{2}\right) = 1350 \text{ lb or } 1.35 \text{ k}$$

$$\text{Interior} = 360(7.5) = 2700 \text{ lb or } 2.7 \text{ k}$$

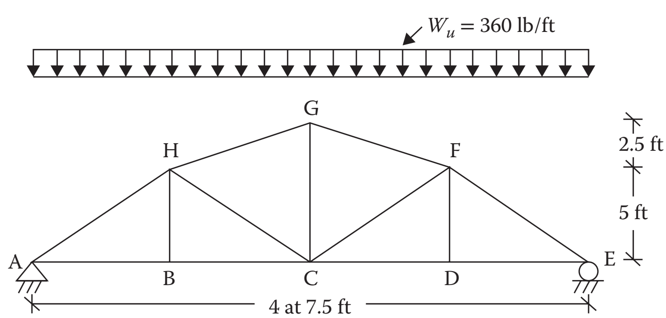


FIGURE 7.5 Roof truss of Example 7.5.

B. Analysis of truss
  1. Reactions at A and E: $A_y = 1.35 + 3\left(\frac{2.7}{2}\right) = 5.4$ k
  2. For members at joint A, taking moment at H:

$$(5.4 - 1.35)7.5 - F_{AB}(5) = 0$$

$$F_{AB} = 6.075\text{k}$$

$$F_{BC} = F_{AB} = 6.075\text{k}$$

C. Reference design value and the adjustment factors
  1. $F_t = 675$ psi
  2. $\lambda = 0.8$
  3. $\phi = 0.8$
  4. Assume a size factor $C_F = 1.5$, which will be checked later.
  5. $K_F = 2.70$
  6. $F'_{tn} = (0.8)(675)(0.8)(1.5)(2.7) = 1750$ psi or 1.75 ksi

D. Design
  1. $A_{n\,\text{reqd}} = \dfrac{P_u}{F'_{tn}} = \dfrac{6.075}{1.75} = 3.47 \text{ in.}^2$
  2. For one bolt in a row and an assumed 2-in.-wide section:

$$h = \frac{3}{4} + \frac{1}{16} = 0.813 \text{ in.}$$

$$\sum nbh = (1)(1.5)(0.813) = 1.22 \text{ in.}^2$$

  3. $A_g = A_n + A_h = 3.47 + 1.22 = 4.69$ in.^{2}

     Select a 2 in.×4 in. section, A=5.25 in.^{2}

  4. Verify the size factor and revise the adjusted value if required.

DESIGN OF COLUMNS

The axial compression capacity of a member in terms of the nominal strength is:

$$P_u = F'_{cn}A \tag{7.41}$$

In Equation (7.41), F'_{cn} is the adjusted LRFD reference design value for compression. To start, the reference design compression value, F_c, for the appropriate species and grade is ascertained. These values are listed in Appendixes B.2 through B.4 for sawn lumber and Appendixes B.7 through B.10 for GLULAM, SCL, and CLT. Then the adjusted value is obtained, multiplying the reference value by a string of factors. The applicable adjustment factors for sawn lumber, GLULAM, SCL, and CLT are given in Tables 6.6 through 6.9 of Chapter 6, respectively.

For sawn lumber, the adjusted reference compression design value is:

$$F'_{cn} = \phi F_c \lambda K_F C_M C_t C_F C_i C_P \tag{7.42}$$

For GLULAM, SCL, and CLT, the adjusted reference compression design value is:

$$F'_{cn} = \phi F_c \lambda K_F C_M C_t C_P \tag{7.43}$$

where:

F_c is the tabular reference compression design value parallel to the grain
ϕ is the resistance factor for compression = 0.90
λ is the time factor (see the "Time Effect Factor, λ" section in Chapter 6)
C_M is the wet-service factor
C_t is the temperature factor
C_F is the size factor for dimension lumber only
C_i is the incision factor
C_P is the column stability factor, discussed below
K_F is the format conversion factor = 2.40

In the case of CLT, the area of those cross sections that run parallel to the load is included.

Depending on the relative size of a column, it might act as a *short column* when only the direct axial stress is borne by the section, or it might behave as a *long column* with a possibility of buckling and a corresponding reduction of the strength. This latter effect is considered by a column stability factor, C_P. Because this factor can be ascertained only when the column size is known, the column design is a trial procedure.

The initial size of a column is decided using an estimated value of F'_{cn} by adjusting the reference design value, F_c, for whatever factors are initially known in Equation (7.42) or (7.43).

On the basis of the trial section, F'_{cn} is adjusted again from Equation (7.42) or (7.43) using all relevant modification factors and the revised section is determined from Equation (7.41).

COLUMN STABILITY FACTOR, C_P

As stated, the column stability factor accounts for buckling. The slenderness ratio, expressed as *KL/r*, is a limiting criterion of buckling. For wood, the slenderness ratio is adopted in a simplified form as *KL/d*, where *d* is the least dimension of the column section. The factor, *K*, known as the *effective length factor*, depends on the end support conditions of the column. The column end conditions are identified in Figure 7.6, and the values of the effective length factors for these conditions are also indicated therein.

When a column is supported differently along the two axes, the slenderness ratio *K* is determined with respect to each axis and the highest ratio is used in design.

The slenderness ratio should not be greater than 50.

The expression for a column stability factor is similar to that of the beam stability factor, as follows:

$$C_P = \left(\frac{1+\beta}{2c}\right) - \sqrt{\left(\frac{1+\beta}{2c}\right)^2 - \left(\frac{\beta}{c}\right)} \tag{7.44}$$

where:

c is the buckling–crushing interaction factor (0.8 for sawn lumber; 0.85 for round timber poles; 0.9 for GLULAM, SCL, and CLT).

$$\beta = \frac{F_{cEn}}{F'^{*}_{cn}} \tag{7.45}$$

F'^{*}_{cn} is the reference design value for compression parallel to the grain adjusted by all factors except C_P
F_{cEn} is the Euler critical buckling stress

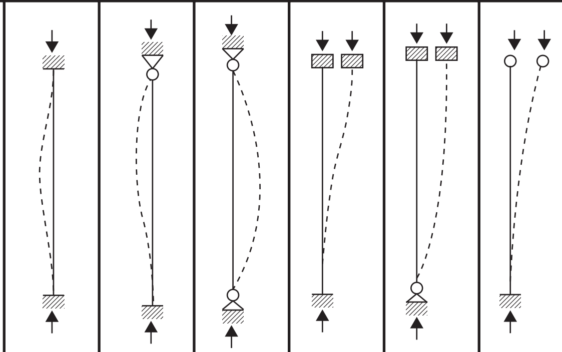
Buckling mode	Translation fixed No sway Braced frame case			Translation free Sway Unbraced frame case		
End conditions	Both ends fixed	One fixed one hinged	Both ends hinged	Both ends fixed	One fixed one hinged	One fixed one free
						
Theoretical value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended value	0.65	0.80	1.0	1.2	2.0	2.10
End condition code		Rotation fixed, translation fixed Rotation free, translation fixed Rotation fixed, translation free Rotation free, translation free				

FIGURE 7.6 Buckling length coefficients, K.

CRITICAL BUCKLING FOR SAWN LUMBER, GLULAM, AND SCL

$$F_{cEn} = \frac{0.822E'_{min(n)}}{(KL/d)^2} \tag{7.46}$$

Determine F_{cEn} for both axes and use the smaller value.

$$\frac{KL}{d} \leq 50 \tag{7.47}$$

Use the $E'_{min(n)}$ value for width or depth corresponding to d dimension in the equation.

$E'_{min(n)}$ is the adjusted modulus of elasticity for buckling.

For sawn lumber:

$$E'_{min(n)} = \phi E_{min} K_F C_M C_t C_i C_T \tag{7.48}$$

For GLULAM and SCL:

$$E'_{min(n)} = \phi E_{min} K_F C_M C_t \tag{7.49}$$

where:

ϕ (= 0.85) is the resistance factor for the stability modulus of elasticity
C_T is the buckling stiffness factor applicable to limited cases, as explained in Chapter 6
K_F (= 1.76) is the format conversion factor for the stability modulus of elasticity

CRITICAL BUCKLING FOR CLT

$$F_{cEn} = \frac{\pi^2 EI_{app-min'}}{A_{parallel}\left(KL\right)^2} \tag{7.50}$$

where:

$$EI_{app\text{-}min} = 0.518\ EI_{app} \tag{7.51}$$

EI_{app} is given by Equation (7.27)

$$EI_{app\text{-}min'} = EI_{app\text{-}min}\ \phi\ C_M C_t\ K_F \tag{7.52}$$

The column behavior is dictated by the interaction of the crushing and buckling modes of failure. When C_P is 1, the strength of a column is F'^*_{cn} (the adjusted reference compressive design value without C_P), and the mode of failure is by crushing. As the C_P reduces (that is, the slenderness ratio is effective), the column fails by the buckling mode.

Example 7.7

Design a 12-ft-long, simply supported column. The axial loads are dead load = 1500 lb, live load = 1700 lb, and snow load = 2200 lb. Use Southern Pine #1.

Solution

A. Loads
The controlling combination is the highest ratio of the factored loads to the time effect factor.

1. $\frac{1.4D}{\lambda} = \frac{1.4(1500)}{0.6} = 3500 \text{ lb}$
2. $\frac{1.2D + 1.6L + 0.5S}{\lambda} = \frac{1.2(1500) + 1.6(1700) + 0.5(2200)}{0.8} = 7025 \text{ lb}$
3. $\frac{1.2D + 1.6S + 0.5L}{\lambda} = \frac{1.2(1500) + 1.6(2200) + 0.5(1700)}{0.8} = 7713 \text{ lb} \leftarrow \text{Controls1}$

So, $P_u = 1.2D + 1.6S + 0.5L = 6170 \text{ lb}$

B. Reference design values
For 2- to 4-in.-wide section:

$F_c = 1850$ psi

$E = 1.7 \times 10^6$ psi

$E_{y\,min} = 0.62 \times 10^6$ psi

C. Preliminary design

$F'_{cn} = \phi F_c \lambda K_F = (0.9)(1850)(0.8)(2.40) = 3196.8 \text{ psi}$

$A_{reqd} = \frac{6170}{3196.8} = 1.93 \text{ in.}^2$

Try $2 \text{in.} \times 4 \text{in.}$ section, $A = 5.25$ in.^{2}

D. Adjusted design values

Type	Reference Design Values (psi)	ϕ	λ	K_F	C_F	$F'_{()n}$ (psi)
Compression	1850	0.9	0.8	2.4	1.0	3196.8 (F'^{*}_{cn})
E	1.7×10^6	—	—	—	—	1.7×10^6
E_{min}	0.62×10^6	0.85	—	1.76	—	0.937×10^6

E. Column stability factor
1. Both ends hinged, $K = 1.0$
2. $\frac{KL}{d} = \frac{1(12\times 12)}{1.5} = 96 > 50$ **NG**
3. Revise the section to 4 in. × 4 in., $A = 12.25$ in.^{2}
4. $\frac{KL}{d} = \frac{1(12\times 12)}{3.5} = 41.14 < 50$ **OK**
5. $F_{cEn} = \frac{0.822(0.93\times 10^6)}{(41.14)^2} = 451.68\,\text{psi}$
6. $\beta = \frac{F_{cEn}}{F'^{*}_{cn}} = \frac{451.68}{3196.8} = 0.14$
7. $C_P = \left(\frac{1+\beta}{2c}\right) - \sqrt{\left(\frac{1+\beta}{2c}\right)^2 - \left(\frac{\beta}{c}\right)}$

$$= \frac{1.14}{1.6} - \sqrt{\left(\frac{1.14}{1.6}\right)^2 - \left(\frac{0.14}{0.8}\right)}$$

$$= 0.713 - \sqrt{(0.508) - (0.175)} = 0.136$$

F. Compression capacity
1. $P_u = F'^{*}_{cn} C_p A$

$$= (3196.8)(0.136)(12.25) = 5325 \text{ lb} < 6170 \text{lb}\ \textbf{NG}$$

Use section $4\,\text{in.}\times 6\,\text{in.}, A = 19.25$ in.^{2}

2. $KL/d = 41.14$
3. $F_{cEn} = 451.68$ psi for the smaller dimension
4. $\beta = 0.14$
5. $C_P = 0.136$
6. Capacity = (3196.8)(0.136)(19.25) = 8369 > 6170 lb **OK**

DESIGN FOR COMBINED BENDING AND COMPRESSION

The members stressed simultaneously in bending and compression are known as *beam-columns*. The effect of combined stresses is considered through an interaction equation. When bending occurs simultaneously with axial compression, a *second order effect* known as the P–Δ moment takes place. For an explanation of this effect, first consider only the transverse loading that causes a deflection, Δ. Now, when an axial load P is applied, it causes an additional bending moment equal to $P\cdot\Delta$. In a simplified approach, this additional bending stress is not computed directly. Instead, it is accounted for indirectly by amplifying the bending stress component in the interaction equation. This approach is similar to the design of steel structures.

The amplification is defined as follows:

$$\text{Amplification factor} = \frac{1}{\left(1 - \dfrac{P_u}{F_{cEx(n)}A}\right)} \tag{7.53}$$

where $F_{cEx(n)}$ is the Euler-based stress with respect to the x axis slenderness as follows:

$$F_{cEx(n)} = \frac{0.822E'_{x\,min(n)}}{(KL/d)_x^2} \tag{7.54}$$

where:

$E'_{xmin(n)}$ is given by Equations (7.47) and (7.48)

$E_{x\,min}$ is the stability modulus of elasticity along the x axis

$(KL/d)_x$ is the slenderness ratio along the x axis

As P–Δ increases, the amplification factor or the secondary bending stresses increase.

In Equation (7.53), the amplification factor increases with a larger value of P_u. The increase of Δ is built into the reduction of the term $F_{cEx(n)}$.

In terms of the load and bending moment, the interaction formula is expressed as follows:

$$\left(\frac{P_u}{F'_{cn}A}\right)^2 + \frac{1}{\left(1 - \dfrac{P_u}{F_{cEx(n)}A}\right)}\left(\frac{M_u}{F'_{bn}S}\right) \leq 1 \tag{7.55}$$

where:

F'_{cn} is the reference design value for compression parallel to the grain and adjusted for all factors (see Equations 7.42 and 7.43)

$F_{cEx(n)}$ (see Equation 7.54)

F'_{bn} is the reference bending design value adjusted for all factors (see Equations 7.3 through 7.7)

P_u is the factored axial load

M_u is the factored bending moment

A is the area of cross section

S is the section modulus along the major axis

For CLT, in Equation (7.55), area A refers to $A_{parallel}$ and section modulus S refers to S_{eff}. In addition, a term appears along with M_u to represent the eccentricity of the applied axial load. However, the total moment can be included in the term M_u.

Note that, while determining the column adjustment factor C_P, F_{cEn} in Equation 7.46 is based on the maximum slenderness ratio (generally with respect to the y axis), whereas the $F_{cEx(n)}$ (in Equation 7.54) is based on the x axis slenderness ratio.

Equation (7.55) should be evaluated for all the load combinations.

The design proceeds with a trial section that in the first iteration is checked by the interaction formula with the initial adjusted design values (without the column and beam stability factors) and without the amplification factor. This value should be only a fraction of 1, preferably not exceeding 0.5.

Then the final check is made with the fully adjusted design values, including the column and beam stability factors together with the amplification factor.

Example 7.8

A 16-ft-long column in a building is subjected to a total vertical dead load of 4 k, and a roof live load of 5 k. In addition, a wind force of 200 lb/ft acts laterally on the column. Design the column of 2DF GLULAM.

Solution

A. Load combinations

Vertical loads

1. $1.4D = 1.4(4) = 5.6$ k
2. $1.2D + 1.6L + 0.5L_r = 1.2(4) + 1.6(0) + 0.5(5) = 7.3$ k
3. $1.2D + 1.6L_r + 0.5L = 1.2(4) + 1.6(5) + 0.5(0) = 12.8$ k

Vertical and lateral loads

4. $1.2D + 1.6L_r + 0.5W$ broken down into 4a and 4b as follows:
   - 4a. $1.2D + 1.6L_r = 1.2(4) + 1.6(5) = 12.8$ k (vertical)
   - 4b. $0.5W = 0.5(200) = 100$ lb/ft (lateral)
5. $1.2D + 1.0W + 0.5L + 0.5L_r$ broken down into 5a and 5b as follows:
   - 5a. $1.2D + 0.5L_r + 0.5L = 1.2(4) + 0.5(5) = 7.3$ k (vertical)
   - 5b. $1.0W = 1(200) = 200$ lb/ft (lateral)

Either 4 (4a + 4b) or 5 (5a + 5b) could be critical. Both will be evaluated.

B. Initially adjusted reference design values

					$F'_{()n}$	
Property	**Reference Design Values (psi)**	**ϕ**	**λ**	**KF**	**(psi)**	**(ksi)**
Bending	1,700	0.85	0.8	2.54	2,936	2.94
Compression	1,950	0.90	0.8	2.40	3,369.6	3.37
E	1.6×10^6		—	—	1.6×10^6	1.6×10^6
$E_{x\,min}$	830,000	0.85	—	1.76	12,420,000	1.242×10^6
$E_{y\,min}$	830,000	0.85	—	1.76	12,420,000	1.242×10^6

I. Design Load case 4

C. Design loads

$$P_u = 12.8k$$

$$M_u = \frac{w_u L^2}{8} = \frac{100(16)^2}{8} = 3200\,\text{ft-lb}\,38.4\,\text{in.-k}$$

D. Preliminary design

1. Try a $5\frac{1}{8}$ in. $\times 7\frac{1}{2}$ in. section, $S_x = 48.05$ in.^{3} $A = 38.44$ in.^{2}
2. Equation (7.55) with the initial design values but without the amplification factor:

$$\left[\frac{12.8}{3.37(38.44)}\right]^2 + \left[\frac{38.4}{2.94(48.05)}\right] = 0.27$$

a small fraction of 1 **OK**

E. Column stability factor, C_P

1. Hinged ends, $K = 1$
2. $(KL/d)_y = \dfrac{(1)(16 \times 12)}{5.125} = 37.46 < 50$ **OK**

3. $F_{cEn} = \dfrac{0.822(1.242\times10^3)}{(37.46)^2} = 0.728$

4. $\beta = \dfrac{F_{cEn}}{F'_{cn}} = \dfrac{0.728}{3.37} = 0.216$

5. c = 0.9 for GLULAM

6. $C_P = \left[\dfrac{1+0.216}{(2)(0.9)}\right] - \sqrt{\left(\dfrac{1+0.216}{2(0.9)}\right)^2 - \left(\dfrac{0.216}{0.9}\right)} = 0.21$

7. F'_{cn} = 3.37(0.21) = 0.71 ksi

F. Volume factor, C_v

$$C_v = \left(\frac{5.125}{b}\right)^{1/10}\left(\frac{12}{d}\right)^{1/10}\left(\frac{21}{L}\right)^{1/10} = \left(\frac{5.125}{5.125}\right)^{1/10}\left(\frac{12}{7.5}\right)^{1/10}\left(\frac{21}{16}\right)^{1/10} = 1.07, \quad \text{use} \quad 1.0.$$

G. Beam stability factor

1. $\dfrac{L_u}{d} = \dfrac{16(12)}{7.5} = 25.6 > 14.3$

$L_e = 1.84L_u = 1.84(16\times12) = 353.28$ in.

2. $R_B = \sqrt{\dfrac{L_e d}{b^2}} = \sqrt{\dfrac{(353.28)(7.5)}{(5.125)^2}} = 10.04$

3. $F_{bEn} = \dfrac{1.2(1.242\times10^3)}{(10.04)^2} = 14.82$

4. $\alpha = \dfrac{F_{bEn}}{F_{bn}*} = \dfrac{14.82}{2.94} = 5.04$

5. $C_L = \left(\dfrac{1+5.04}{1.9}\right) - \sqrt{\left(\dfrac{1+5.04}{1.9}\right)^2 - \left(\dfrac{5.04}{0.95}\right)} = 0.99$

6. F'_{bn} = (2.94)(0.99) = 2.91 ksi

H. Amplification factor

1. Based on the x axis, $(KL/d)_x = \dfrac{1(16\times12)}{7.5} = 25.6$

2. $F_{cEx(n)} = \dfrac{0.8222E_{x\,min(n)}}{(KL/d)^2}$

$$= \frac{0.822\left(1.242\times10^3\right)}{(25.6)^2} = 1.56$$

3. Amplification factor $= \dfrac{1}{(1-(P_u / F_{cEx(n)}A))}$

$$= \frac{1}{1-(12.8/(1.56)(38.44))} = \frac{1}{0.787} = 1.27$$

I. Interaction equation, Equation (7.55)

$$\left[\frac{12.8}{(0.71)(38.44)}\right]^2 + \left[\frac{1.27(38.4)}{(2.91)(48.05)}\right] = 0.22 + 0.35 = 0.57 < 1 \quad \textbf{OK}$$

II. Design load case 5

J. Design loads

$$P_u = 7.3\,k$$

$$M_u = \frac{w_u L^2}{8} = \frac{200(16)^2}{8} = 6400\,\text{ft-lb or } 76.8\,\text{in.-k}$$

K. Column stability factor, $C_P = 0.21$ and $F'_{cn} = 0.71$ ksi, from step E

L. Beam stability factor, $C_L = 0.99$ and $F'_{bn} = 2.91$ ksi, from step G

M. Amplification factor

$$= \frac{1}{(1-(P_u / F_{cEx(n)}A))}$$

$$= \frac{1}{[1-(7.3/(1.56)(38.44))]} = \frac{1}{0.878} = 1.14$$

N. Interaction equation, Equation (7.55)

$$\left[\frac{7.3}{(0.71)(38.44)}\right]^2 + \left[\frac{1.14(76.8)}{(2.91)(48.05)}\right] = 0.07 + 0.626 = 0.7 < 1 \quad \textbf{OK}$$

Example 7.9

For a 10-ft-high wall resisting a dead load of 30 k/ft and a live load of 45 k/ft, a five-layer E1 CLT panel is used. Is this section adequate?

Solution

1. $P_u = 1.2(30) + 1.6(45) = 108$ k/ft
2. From Appendix B.10, $F_c = 1800$ psi.
   From Table 6.10 $EI_{eff} = 440 \times 10^6$ in.^{2}-lb/ft.

   $GA_{eff} = 0.92 \times 10^6$ lb

   From Table 7.3 for constant moment, $K_S = 11.8$.
3. From Equation (7.27):

$$EI_{app} = \frac{440 \times 10^6}{1 + \dfrac{11.8\,(440 \times 10^6)}{\left(0.92 \times 10^6\right)\left(10 \times 12\right)^2}}$$

$$= 316 \times 10^6 \text{ in.}^2\text{-lb/ft}$$

4. $EI_{app\text{-}min} = 0.518\,(316 \times 10^6) = 164 \times 10^6$ in.^{2}-lb/ft
5. $EI_{app\text{-}min'} = 164 \times 10^6\ \phi\ K_F$

$$= 164 \times 10^6\,(0.85)(1.76) = 245 \times 10^6 \text{ in.}^2\text{-lb/ft}$$

6. $A_{parallel}$ = (3 parallel layers) $(1.375 \times 12) = 49.5$ in.^{2}
7. From Equation (7.50) for pinned end, $K = 1$.

$$F_{cEn}' = \frac{\pi^2\left(245 \times 10^6\right)}{(49.5)\ \left(1 \times 10 \times 12\right)^2}$$

$$= 3390 \text{ psi}$$

8. Adjusted $F^*_{cn} = 1800(2.4)(0.9)(0.8) = 3110$ psi
9. From Equation (7.45):

$$\beta = \frac{3390}{3110} = 1.1$$

10. From Equation (7.44):

$$C_P = \frac{1+1.1}{2\,(0.9)} - \sqrt{\frac{[1+1.1]^2}{2\,(0.9)} - \frac{1.1}{0.9}} = 0.79$$

11. Column capacity

$$F'_{cn}\,A_{parallel} = 1800\,(2.4)(0.9)(0.8)(0.79)(49.5)$$

$$= 121{,}632 \text{ lb or } 121.6 \text{ k} > 108 \text{ k } \textbf{OK}$$

PROBLEMS

7.1 Design the roof rafters with the following information; check for shear and deflection.
Span: 10 ft
Spacing: 16 in. OC
Species: Southern Pine #1 Dense
Dead load = 15 psf
Roof live load = 20 psf
Roof sheathing provides the full lateral support.

7.2 Design the beam in Problem 7.1 if the beam is supported only at the ends.

7.3 Design the roof rafters in Figure P7.1 with the following information:
Spacing 24 in. on center
Species: Douglas Fir-Larch #1
Dead load: 15 psf
Snow load: 40 psf
Wind load (vertical): 18 psf
Unbraced length: support at ends only

7.4 Design the floor beam in Figure P7.2 for the following conditions:
Span, $L = 12$ ft
$P_D = 500$ lb (service)
$P_L = 1000$ lb (service)
Unbraced length: one-half of the span
Species: Hem Fir #1

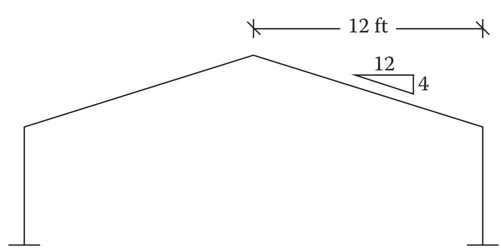


FIGURE P7.1 Roof rafters for Problem 7.3.

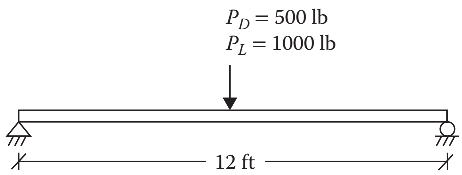


FIGURE P7.2 Floor beam for Problem 7.4.

7.5 Design the beam in Problem 7.4 for the unbraced length equal to the span.

7.6 Design the floor beam in Figure P7.3 with the following information:
$w_D = 100$ lb/ft (service)
$P_L = 400$ lb (service)
Species: Douglas Fir-Larch Select Structural
Unbraced length: at the supports
The beam section should not be more than 10 in. deep.

7.7 The floor framing plan of a building is shown in Figure P7.4. Dead loads are as follows:
Floor = 12 psf
Joists = 7 psf
Beams = 9 psf
Girders = 10 psf
Live load = 40 psf
Design the beams of Southern Pine select structural timber. The beam is supported only at the ends. The beam should not be more than 12 in. in depth.

7.8 Design girders for Problem 7.7 of 24F-1.8E Southern Pine GLULAM with a width of 6¾ in. and having a lateral bracing at the supports only.

7.9 A Douglas Fir structural GLULAM of 24F-1.8E is used to support a floor system. The tributary width of the beam is 12 ft and the span is 40 ft. The dead and live loads are 15 psf and 40 psf, respectively. Design a beam of 10 ¾ width, braced only at the supports.

7.10 For the beam shown in Figure P7.5, the loads are applied by purlins spaced at 10 ft. on center. The beam has lateral supports at the ends and at the locations where the purlins frame onto the beam. Design the beam of 24F-1.8E Douglas Fir GLULAM. Use an 8¾-wide section.

7.11 Design Problem 7.10 for a beam that is used flat with bending along the minor axis. Use a 10¾-wide section.

7.12 Select a CLT V2 Grade section for an 18 ft span floor system carrying a dead load of 40 psf and a live load of 50 psf. Check for shear and deflection.

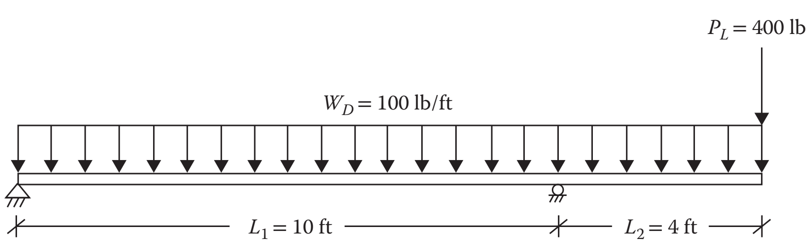


FIGURE P7.3 Floor beam for Problem 7.6.

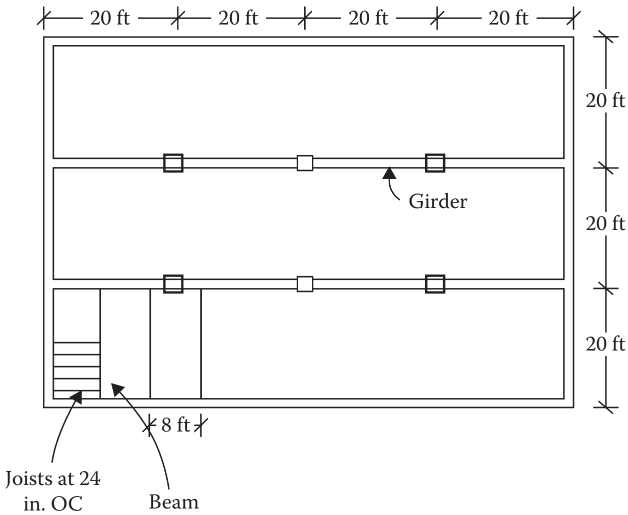


FIGURE P7.4 Floor framing plan for Problem 7.7.

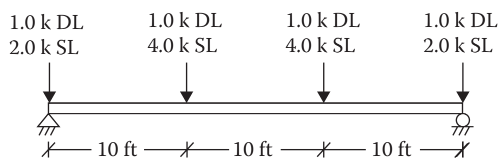


FIGURE P7.5 Load on beam by purlins for Problem 7.10.

7.13 Design the bearing plate for the supports from Problem 7.4.

7.14 Design the bearing plate for the supports from Problem 7.9.

7.15 Determine the length of the bearing plate placed under the interior loads of the beam from Problem 7.10.

7.16 Roof trusses, spanning 24 ft at 4 ft on center, support a dead load of 16 psf and a snow load of 50 psf only. The lumber is Hem Fir #1. The truss members are connected by a single row of ¾-in. bolts. Design the bottom chord. By truss analysis, the tensile force due to the service loads in the bottom chord members is 5.8 k. Assume that dry wood is used under normal temperature conditions.
Hint: Divide the force in the chord between dead and snow loads in the above ratio between dead and snow loads for factored load determination.

7.17 A Warren-type truss supports only dead load. The lumber is Douglas Fir-Larch #2. The end connection consists of two rows of ½-in. bolts. Determine the size of the tensile member. By truss analysis, the maximum force due to the service load in the bottom chord is 5.56 k tension. Assume dry wood and normal temperature conditions.

7.18 Design a simply supported 10-ft-long column using Douglas Fir-Larch #1. The loads comprise 10 k of dead load and 10 k of roof live load.

7.19 Design a 12-ft-long, simply supported column of Southern Pine #1 Dense. The axial loads are dead load = 1000 lb, live load = 2000 lb, and snow load = 2200 lb.

7.20 Design the column from Problem 7.19 if a full support is provided by the sheeting about the smaller dimension.

7.21 What is the largest axial load that can be applied to a 4 in. × 6 in. #1 Hem Fir column? The column is 15 ft long, fixed at both ends.

7.22 A 6 in. × 8 in. column carries dead and snow loads of equal magnitude. The lumber is Douglas Fir-Larch #1. If the unbraced length of the column, which is fixed at one end and hinged at the other end, is 9 ft, what is the load capacity of the column?

7.23 Determine the axial compression capacity of a 20-ft-long GLULAM 6¾ in. × 11 in. column, hinged at both ends, of SPN1D14 Southern Pine of more than four laminations.

7.24 Determine the capacity column from Problem 7.23 if it is braced at the center in the weaker direction.

7.25 A GLULAM column of 24F-1.8E Douglas Fir carries a dead load of 20 k and a roof live load of 40 k. The column has a simply supported length of 20 ft. Design an 8¾-in.-wide column.

7.26 The column in Problem 7.25 is braced along the weaker axis at 8 ft. from the top. Design a 6¾-in.-wide column.

7.27 A 2 in. × 6 in. exterior stud wall is 12 ft tall. The studs are 16 in. on center. The studs carry the following vertical loads per foot horizontal distance of the wall:
Dead = 400 lb/ft
Live = 1000 lb/ft
Snow = 1500 lb/ft
The sheathing provides the lateral support in the weaker direction. The lumber is Douglas Fir-Larch #1. Check the studs. Assume a simple end support condition and that the loads on studs act axially.

7.28 The first-floor (10-ft-high) bearing wall of a building consists of 2 in. × 6 in. studs at 16 in. on center. The following roof loads are applied: roof dead load = 10 psf, roof live load = 20 psf, wall dead load = 5 psf, floor dead load = 7 psf, live load = 40 psf, lateral wind load = 25 psf. The tributary width of the roof framing to the bearing wall is 8 ft. The sheathing provides a lateral support to studs in the weaker direction. Check whether the wall studs made of Douglas Fir-Larch #2 are adequate.

7.29 A beam column is subjected to an axial dead load of 1 k, a snow load of 0.8 k, and a lateral wind load of 160 lb/ft. The column height is 10 ft. Design a beam-column of section 4 × ____ of Southern Pine #1 Dense.

7.30 A tall 20-ft.-long building column supports a dead load of 4 k and a live load of 5 k along with a lateral wind load of 240 lb/ft. Design a beam-column of 5⅛ × ____section made of 2DFL2 GLULAM, more than four laminations.

7.31 A vertical 4 in. × 12 in. Southern Pine dense #1, 12-ft-long member is embedded at the base to provide fixity. The other end is free to sway without rotation along the weaker axis and is hinged along the strong axis. The bracing about the weak axis is provided every 4 ft by wall girts and only at the ends about the strong axis. The dead load of 1000 lb and the roof live load of 4000 lb act axially. A uniform wind load of 240 lb/ft acts along the strong axis. The sheathing provides a continuous lateral support to the compression side. Check the member for adequacy.
Hint: Consider that the member is fixed at one end and has a spring support at the other end. For this case, take the design end bending moment to be 70% of the maximum bending moment on the column acting like a cantilever.

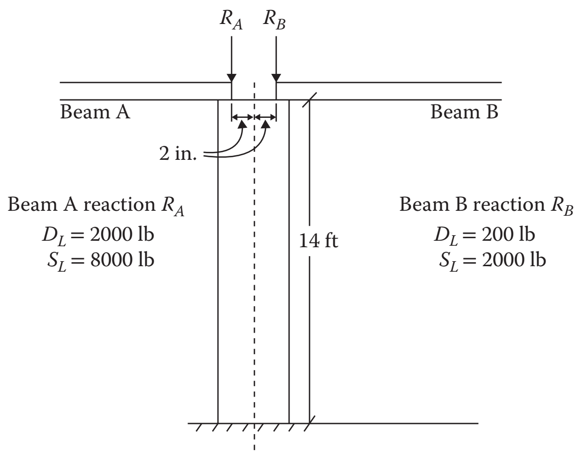


FIGURE P7.6 Column supporting two beams for Problem 7.33.

7.32 Solve Problem 7.31 when no lateral support to the compression side is provided. If a 4 in. × 12 in. section in not adequate, select a new section of a maximum 12 in. depth.

7.33 Choose a 5-in.-wide Southern Pine SPN1D14 GLULAM column supporting two beams, as shown in Figure P7.6. The beam reactions cause bending about the major axis only. The bottom is fixed and the top is hinged.

7.34 For a 10-ft-high wall resisting a dead load of 20 k/ft and a live load of 40 k/ft, check whether a five-layer V2 CLT panel is adequate. Assume the pined end condition.

7.35 Design a 9 ft long column of E2 grade CLT to sustain a dead load of 20 k and a live load of 30 k. Assume both ends are hinged.

8 Wood Connections

TYPES OF CONNECTIONS AND FASTENERS

There are two broad types of wood connections: (1) the mechanical connections that attach members with some kind of fasteners and (2) the adhesive connections that bind members chemically under controlled environmental conditions such as that seen in glued laminated timber (GLULAM). The mechanical connections, with the exception of moment splices, are not expected to transfer any moment from one element to another. The mechanical connections are classified according to the direction of load on the connector. Shear connections or lateral load connections have the load or the load component applied perpendicular to the length of the fastener. The withdrawal connections have the tensile load applied along (parallel to) the length of the fastener. When the load along the fastener length is in compression, a washer or a plate of sufficient size is provided so that the compressive strength of the wood perpendicular to the grain is not exceeded. The mechanical type of connectors can be grouped as follows:

1. Dowel-type connectors
2. Split ring and shear plate connectors
3. Timber rivets
4. Pre-engineered metal connectors

Dowel-type connectors, comprising nails, staples and spikes, bolts, lag bolts, and lag screws, are the common type of fasteners that are discussed in this chapter. The split ring and shear plate connectors fit into precut grooves and are used in shear-type connections to provide additional bearing area for added load capacity. Timber rivets or GLULAM rivets are nail-like fasteners of hardened steel (minimum strength of 145 ksi) with a countersunk head and rectangular-shaped cross section; they have no similarity to steel rivets. These are used primarily in GLULAM members for large loads.

Pre-engineered metal connectors include joist hangers, straps, ties, and anchors; they are used as accessories along with dowel-type fasteners. They make connections simpler and easier to design. In certain cases, such as earthquakes and high winds, they are an essential requirement. The design strength values for specific connectors are available from the manufacturers.

DOWEL-TYPE FASTENERS (NAILS, SCREWS, BOLTS, PINS)

The basic design equation for dowel-type fasteners is:

$$R_Z \text{ or } R_W \leq NZ'_n \tag{8.1}$$

where:

R_Z is the factored lateral design force on a shear-type connector
R_W is the factored axial design force on a withdrawal-type connector
N is the number of fasteners
Z'_n is the adjusted reference design value of a fastener, given as:

$$Z'_n = \text{reference design value } (Z) \times \text{adjustment factors} \tag{8.2}$$

The reference design value, Z, refers to the basic load capacity of a fastener. The shear-type connections rely on the bearing strength of wood against the metal fastener or the bending yield strength of the fastener (not the shear rupture of the fastener, as in steel design). The withdrawal-type connections rely on the frictional or interfacial resistance to the transfer of loads. Until the 1980s, the capacities of fasteners were obtained from the empirical formulas based on field and laboratory tests. In the newer approach, however, the yield mechanism is considered from the principles of engineering mechanics. The yield-related approach is limited to the shear-type or laterally loaded connections. The withdrawal-type connections are still designed from the empirical formulas.

YIELD LIMIT THEORY FOR LATERALLY LOADED FASTENERS

Yield limit theory considers the various modes (limits) by which a connection can yield under a lateral load. The capacity is computed for each mode of yielding. Then the reference value is taken as the smallest of these capacities.

In yield limit theory, the primary factors that contribute to the reference design value are:

1. Fastener diameter, D
2. Bearing length, l
3. Dowel-bearing strength of wood, F_{ew}, controlled by the (1) specific gravity of wood; (2) angle of application of load to the wood grain, θ; and (3) relative size of the fastener
4. Bearing strength of metal side plates, F_{ep}
5. Bending yield strength, F_{yb}

A subscript m or s is added to the above factors to indicate whether they apply to the main member or the side member. For example, l_m and l_s refer to bearing lengths of the main member and side member, respectively. For bolted connections, the bearing length, l, and member thickness are identical, as shown in Figure 8.1.

For nail, screw, or lag bolt connections, the bearing length of the main member, l_m, is less than the main member thickness, as shown in Figure 8.2.

Depending on the mode of yielding, one of the strength terms corresponding to items 3, 4, or 5 above or their combinations are the controlling factor(s) for the capacity of the fastener. For example, in the bearing-dominated yield of the wood fibers in contact with the fastener, the term F_{ew} for wood

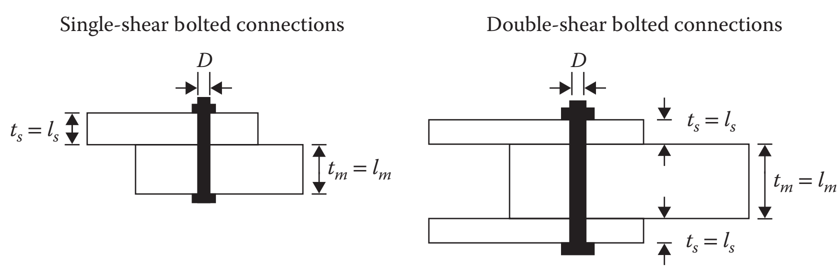


FIGURE 8.1 Bearing length of bolted connection.

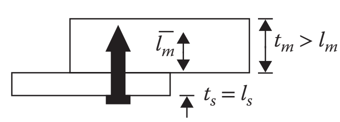


FIGURE 8.2 Bearing length of nail or screw connection.

is a controlling factor; for a metal side member used in a connection, the bearing strength of metal plate F_{ep} controls.

For a fastener yielding in bending with the localized crushing of the wood fibers, both F_{yb} and F_{ew} are the relevant factors. The various yield modes are described in the "Yield Mechanisms and Yield Limit Equations" section.

1. The *dowel-bearing strength of wood*, also known as the *embedded strength, F_{ew}* (item 3 above), is the crushing strength of the wood member. Its value depends on the specific gravity of wood. For large-diameter fasteners (≥¼ in.), the bearing strength also depends on the angle of load to grains of wood. The National Design Specifications (NDS) provides the values of specific gravity, *G*, for various species and their combinations, and also includes the formulas and tables for the dowel-bearing strength, F_{ew}, for the two cases of loading: the load acting parallel to the grains and the load applied perpendicular to the grains.
2. The *bearing strength of steel members* (item 4 above) is based on the ultimate tensile strength of steel. For hot-rolled steel members (usually of thickness ≥¼ in.), $F_{ep} = 1.5\ F_u$, and for cold-formed steel members (usually <¼ in.), $F_{ep} = 1.375\ F_u$.
3. The *fastener bending yield strength, F_{yb}* (item 5 above), has been listed by the NDS for various types and diameters of fasteners. These values can be used in the absence of the manufacturer's data.

YIELD MECHANISMS AND YIELD LIMIT EQUATIONS

Dowel-type fasteners have the following four possible modes of yielding:

Mode I: Bearing yield of wood fibers when stress distribution is uniform over the entire thickness of the member. In this case, due to the high lateral loading, the dowel-bearing stress of a wood member uniformly exceeds the strength of the wood. This mode is classified as I_m if the bearing strength is exceeded in the main member and as I_s if the side member is overstressed, as shown in Figure 8.3.

Mode II: Bearing yield of wood by crushing due to maximum stress near the outer fibers. The bearing strength of wood is exceeded in this case also. However, the bearing stress is not uniform. In this mode, the fastener remains straight but undergoes a twist that causes flexure-like nonuniform distribution of stress, with the maximum stress at the outer fibers. The wood fibers are accordingly crushed at the outside face of both members, as shown in Figure 8.4. Mode II yield occurs simultaneously in the main and side members. It is not applicable to a double-shear connection because of symmetry by the two side plates.

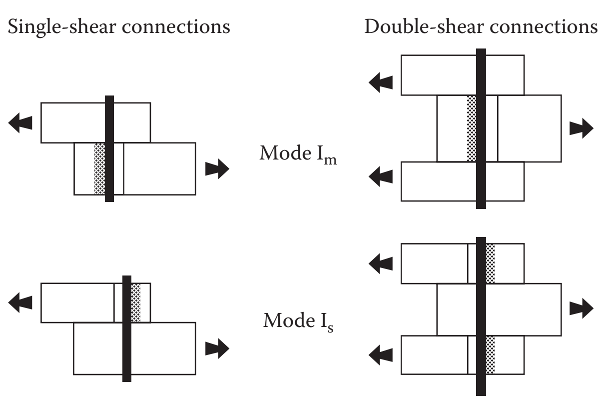


FIGURE 8.3 Mode I yielding. (Courtesy of American Forest & Paper Association, Washington, DC.)

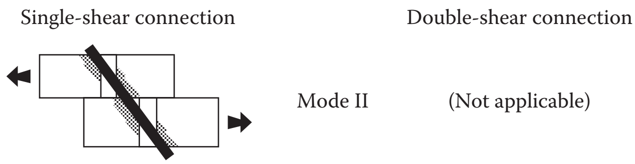


FIGURE 8.4 Mode II yielding. (Courtesy of American Forest & Paper Association, Washington, DC.)

Mode III: Fastener bends at one point within a member and wood fibers in contact with the fastener yield in bearing. This is classified as III_m when fastener bending occurs and the wood bearing strength is exceeded in the main member. III_s indicates the bending and crushing of wood fibers in the side member, as shown in Figure 8.5. Mode III_m is not applicable to a double-shear connection because of symmetry by the two side plates.

Mode IV: Fastener bends at two points in each shear plane and wood fibers yield in bearing near the shear plane(s). Mode IV occurs simultaneously in the main and side members in a single shear, as shown in Figure 8.6. However, in a double shear, this can occur in each plane; hence yielding can occur separately in the main member and the side member.

To summarize, in a single-shear connection, there are six modes of failures: I_m, I_s, II, III_m, III_s, and IV. Correspondingly, there are six yield limit equations derived for the single-shear connections. For a double-shear connection, there are four modes of failures: I_m, I_s, IV_m, and IV_s. There are four corresponding yield limit equations for the double-shear connections.

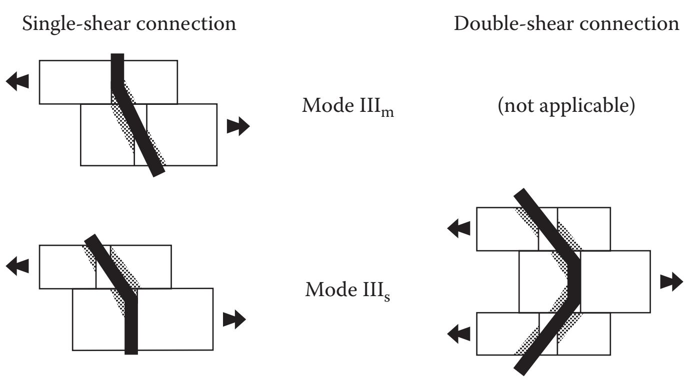


FIGURE 8.5 Mode III yielding. (Courtesy of American Forest & Paper Association, Washington, DC.)

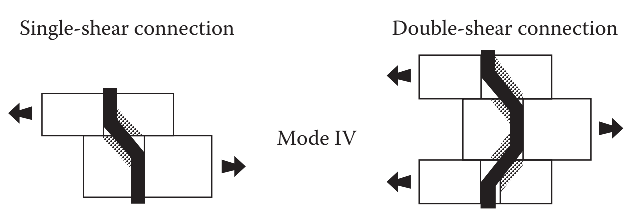


FIGURE 8.6 Mode IV yielding. (Courtesy of American Forest & Paper Association, Washington, DC.)

REFERENCE DESIGN VALUES FOR LATERAL LOADS (SHEAR CONNECTIONS)

For a given joint configuration, depending on the single- or the double-shear connection, six or four yield limit equations are evaluated, and the smallest value obtained from these equations is used as a reference design value, Z.

Instead of using the yield limit equations, the NDS provides the tables for the reference design values that evaluate all relevant equations and adopts the smallest values for various fastener properties and specific gravity of species. The selected reference design values for the lateral loading are included in Appendix B.11, Appendix B.13, Appendix B.15, Appendix B.17, and Appendix B.18 for different types of fasteners.

As stated above under the dowel-bearing strength of wood for fasteners of ¼ in. or larger, the angle of loading with respect to the wood grain also affects the reference design values. The NDS tables include two cases: one for the loads parallel to the grain and one for the loads perpendicular to the grain. The loads that act at other angles involve the application of the Hankinson formula, which has not been considered in this text.

A reference design value, Z, obtained by the yield limit equations or from the NDS tables, is then subjected to the adjustment factors to get the adjusted reference design value, Z_n', to be used in Equation 8.1. The adjustment factors are discussed in the "Adjustments of the Reference Design Values" section.

REFERENCE DESIGN VALUES FOR WITHDRAWAL LOADS

Dowel-type fasteners are much less strong in withdrawal capacity. The reference design values for different types of fasteners in lb/in. of penetration are given by the empirical formulas, which are functions of the specific gravity of species and the diameter of the fasteners. The NDS provides the tables based on these formulas. The selected reference design values for withdrawal loading are included in Appendix B.12, Appendix B.14, Appendix B.16, and Appendix B.19 for different types of fasteners.

ADJUSTMENTS OF THE REFERENCE DESIGN VALUES

Table 8.1 specifies the adjustment factors that apply to the lateral loads and withdrawal loads for dowel-type fasteners. The last three factors, ϕ_z, λ, and K_F, are relevant to load resistance factor design (LRFD) only. For connections, their values are:

$\phi_z = 0.65$

λ = as given in the "Time Effect Factor, λ" section in Chapter 6

$K_F = 3.32$

The other factors are discussed below.

WET SERVICE FACTOR, C_M

For connections, the listed reference design values are for seasoned wood having a moisture content of 19% or less. For wet woods or those exposed to wet conditions, the multiplying factors of less than 1 are specified in the NDS Table 11.3.3 of the *National Design Specification for Wood Construction* 2015 cited in the bibliography.

TEMPERATURE FACTOR, C_t

For connections that experience sustained exposure to higher than 100°F temperature, a factor of less than 1 shall be applied, as specified in the NDS Table 11.3.4 of the *National Design Specification for Wood Construction* 2015 cited in the bibliography.

TABLE 8.1
Adjustment Factors for Dowel-Type Fasteners

	LRFD Only			Factors						
Loads	Time Effect	Resistance Factor	Format Conversion	Wet Service	Temperature	Group Action	Geometry	End Grain	Diaphragm	Toenail
Lateral loads	λ	ϕ_z	K_F	C_M	C_t	C_g	C_Δ	C_{eg}	C_{di}[a]	C_{tn}[a]
Withdrawal	λ	ϕ_z	K_F	C_M	C_t	—	—	C_{eg}	—	C_{tn}[a]

[a] This factor applies to nails and spikes only.

Group Action Factor, C_g

A row of fasteners consists of a number of fasteners in a line parallel to the direction of loading. The load carried by fasteners in a row is not equally divided among the fasteners; the end fasteners in a row carry a larger portion of the load compared to the interior fasteners. The unequal sharing of loads is accounted for by the group action factor, C_g.

For dowel-type fasteners of diameter less than ¼ in. (i.e., nails and wood screws), $C_g = 1$. For ¼ in. or larger diameter fasteners, C_g is given by a formula, which is quite involved. The NDS provides tabulated values for simplified connections. The number of fasteners in a single row is the primary consideration. For bolts and lag screws, conservatively, C_g has the values indicated in Table 8.2 (nails and screws have $C_g = 1$).

Geometry Factor, C_Δ

When the diameter of a fastener is less than ¼ in. (nails and screws), $C_\Delta = 1$. For ≥¼ in. diameter fasteners the geometry factor accounts for the end distance, edge distance, and spacing of fasteners, as defined in Figure 8.7.

1. The edge distance requirements, according to the NDS, are given in Table 8.3, where *l/D* is the lesser of the following:

   a. $$\frac{l_m}{D} = \frac{\text{bearing length of bolt in main member}}{\text{bolt diameter}}$$

   b. $$\frac{l_s}{D} = \frac{\text{combined bearing length of bolts in all side members}}{\text{bolt diameter}}$$

2. The spacing requirements between rows, according to the NDS, are given in Table 8.4, where *l/D* is defined above.
3. The end distance requirements, according to the NDS, are given in Table 8.5.
4. The spacing requirements for fasteners along a row, according to the NDS, are given in Table 8.6.

TABLE 8.2
Conservative Value of the Group Action Factor

Number of Fasteners in One Row	C_g
2	0.97
3	0.89
4	0.80

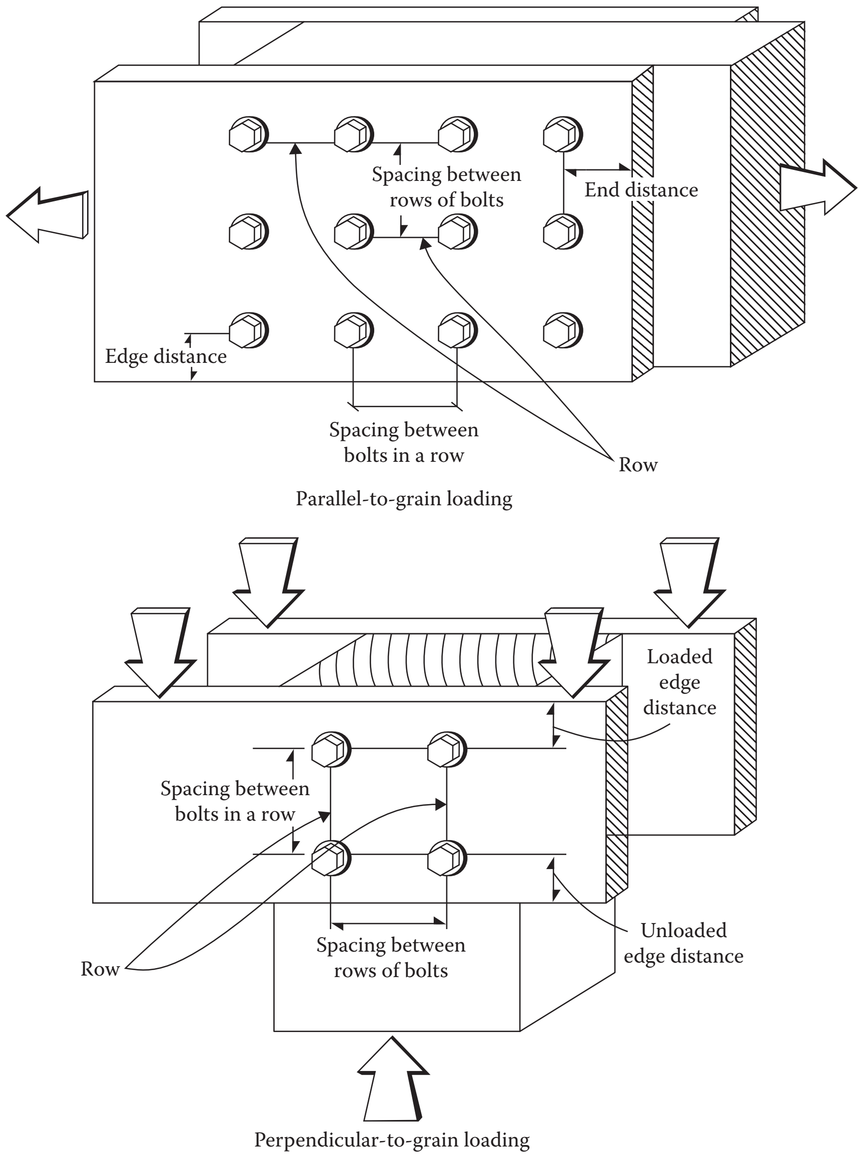


FIGURE 8.7 Connection geometry. (Courtesy of American Forest & Paper Association, Washington, DC.)

TABLE 8.3
Minimum Edge Distance

Direction of Loading	Minimum Edge Distance
1. Parallel to grains	
When $l/D \leq 6$	$1.5D$
When $l/D > 6$	$1.5D$ or half spacing between rows, whichever is greater
2. Perpendicular to grains	
Loaded edge	$4D$
Unloaded edge	$1.5D$

TABLE 8.4
Minimum Spacing between Rows

Direction of Loading	Minimum Spacing
1. Parallel to grains	$1.5D$
2. Perpendicular to grains	
When $l/D \leq 2$	$2.5D$
When $l/D > 2$ but <6	$(5l + 10D)/8$
When $l/D \geq 6$	$5D$

TABLE 8.5
Minimum End Distance

Direction of Loading	End Distance for $C_\Delta = 1$	Minimum End Distance for $C_\Delta = 0.5$
1. Parallel to grains		
Compression	$4D$	$2D$
Tension—softwood	$7D$	$3.5D$
Tension—hardwood	$5D$	$2.5D$
2. Perpendicular to grains	$4D$	$2D$

TABLE 8.6
Minimum Spacing in a Row

Direction of Loading	Spacing for $C_\Delta = 1$	Minimum Spacing
1. Parallel to grains	$4D$	$3D$
2. Perpendicular to grains	On side plates (attached member) spacing should be $4D$	$3D$

The provisions for C_Δ are based on the assumption that the edge distance and the spacing between rows are met in accordance with Tables 8.3 and 8.4, respectively. In addition, the perpendicular to grain distance between the outermost fastener rows should not exceed 5 in. for sawn lumber and GLULAM, with $C_M = 1$.

The requirements for the end distance and the spacing along a row for $C_\Delta = 1$ are given in the second column of Tables 8.5 and 8.6. The tables also indicate the (absolute) minimum requirements that must be met. When the actual end distance and the actual spacing along a row are less than those indicated for $C_\Delta = 1$, the value of C_Δ should be computed by the following ratio:

$$C_\Delta = \frac{\text{actual end distance or actual spacing along a row}}{\text{end distance for } C_\Delta = 1 \text{ from Table 8.5 or spacing for } C_\Delta = 1 \text{ from Table 8.6}}$$

For fasteners located at an angle, the geometry factor, C_Δ, also depends on the shear area. For C_Δ to be 1, the minimum shear area of an angled member, as shown in Figure 8.8, should be equal to the shear area of a parallel member connection having the minimum end distance as required for $C_\Delta = 1$ from Table 8.5, as shown in Figure 8.9. If the angled shear area is less, the geometry factor C_Δ is determined by the ratio of the actual shear area to that required for $C_\Delta = 1$ from Figure 8.9.

The geometry factor is the smallest value determined from the consideration of the end distance, spacing along the row, and the angled shear area.

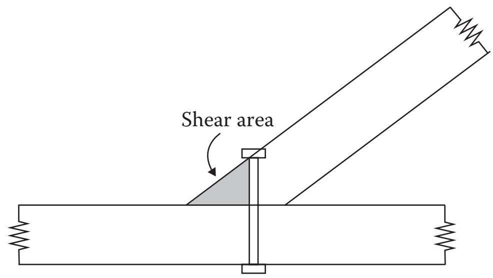


FIGURE 8.8 Shear area for fastener loaded at angle.

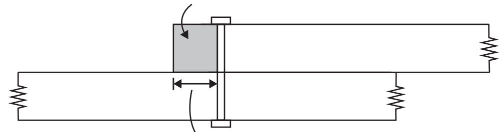


FIGURE 8.9 Shear area of parallel member connection.

End Grain Factor, C_{eg}

In a shear connection, load is perpendicular to the length (axis) of the fastener, and in a withdrawal connection, load is parallel to the length of the fastener. But in both cases, the length (axis) of the fastener is perpendicular to the wood fibers (fastener is installed in the side grains). However, when a fastener penetrates an end grain so that the fastener axis is parallel to the wood fibers, as shown in Figure 8.10, it is a weaker connection.

For a withdrawal-type loading, $C_{eg} = 0.75$. For a lateral (shear)-type loading, $C_{eg} = 0.67$.

Diaphragm Factor, C_{di}

This applies to nails and spikes only. When nails or spikes are used in diaphragm construction, $C_{di} = 1.1$.

Toenail Factor, C_{tn}

This applies to nails and spikes only. In many situations, it is not possible to directly nail a side member to a holding member. Toenails are used in the side member at an angle of about 30° and start at about one-third of the nail length from the intersection of the two members, as shown in Figure 8.11.

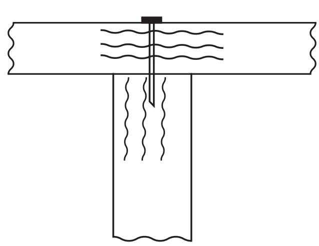


FIGURE 8.10 End grain factor.

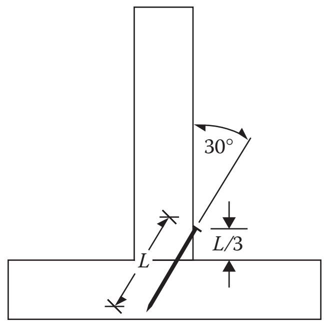


FIGURE 8.11 Toenail factor.

For lateral loads, C_{tn} = 0.83. For withdrawal loads, C_{tn} = 0.67. For withdrawal loads, the wet-service factor is not applied together with C_{tn}.

Example 8.1

The reference lateral design value for the parallel-to-grain loaded lag screw connection shown in Figure 8.12 is 1110 lb. Determine the adjusted reference design value. The diameter of the screws is ⅞ in. The connection is subjected to dead and live tensile loads in dry softwood at normal temperatures.

Solution

1. Adjusted reference design value, $Z'_n = Z \times (\phi_z \lambda K_F\, C_g C_\Delta)$, since C_M and $C_t = 1$.
2. $\phi_z = 0.65$
3. $\lambda = 0.8$
4. $K_F = 3.32$
5. Group action factor, C_g
   For three fasteners in a row, $C_g = 0.89$ (from Table 8.2).
6. Geometry factor, C_Δ
   a. End distance = 4 in.
   b. End distance for $C_\Delta = 1$, $7D = 7\left(\dfrac{7}{8}\right) = 6.125\,\text{in.}$
   c. End factor $= \dfrac{4.0}{6.125} = 0.65 \leftarrow$ controls
   d. Spacing along a row = 3 in.

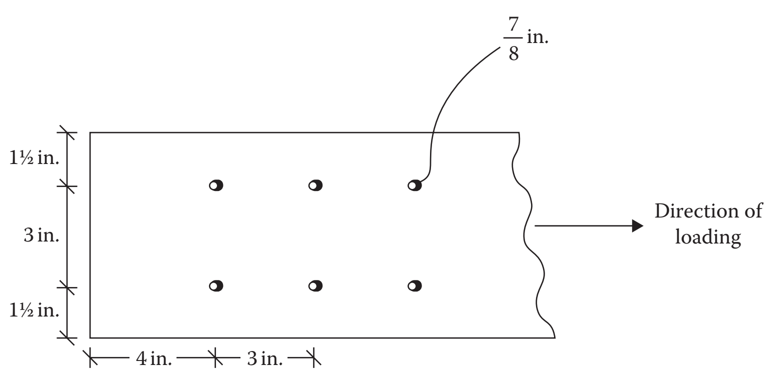


FIGURE 8.12 Parallel-to-grain loaded connection.

e. Spacing for $C_\Delta = 1$, $4D = 3.5$ in.

f. Spacing factor $= \dfrac{3.0}{3.5} = 0.857$

7. $Z'_n = 1110 \sim (0.65)(0.8)(3.32)(0.89)(0.65) = 1108.6\ \text{lb}$

Example 8.2

The reference lateral design value for the perpendicular-to-grain loaded bolted connection shown in Figure 8.13 is 740 lb. Determine the adjusted reference design value. The bolt diameter is ⅞ in. Use soft dry wood and normal temperature conditions. The connection is subjected to dead and live loads.

Solution

1. Adjusted reference design value, $Z'_n = Z \times (\phi_z \lambda K_F C_g C_\Delta)$, since C_M and $C_t = 1$.
2. $\phi_z = 0.65$
3. $\lambda = 0.8$
4. $K_F = 3.32$
5. Group action factor, C_g
   For two fasteners in a row, $C_g = 0.97$ (from Table 8.2).
6. Geometry factor, C_Δ
   a. End distance = 2 in.
   b. End distance for $C_\Delta = 1$, $4D = 4\left(\dfrac{7}{8}\right) = 3.5$ in.
   c. End factor $= \dfrac{2.0}{3.5} = 0.57 \leftarrow$ controls
   d. Spacing along a row = 3 in.
   e. Spacing for $C_\Delta = 1$, $4D = 4\left(\dfrac{7}{8}\right) = 3.5$ in.
   f. Spacing factor $= \dfrac{3.0}{3.5} = 0.857$
7. $Z'_n = 740(0.65)(0.80)(3.32)(0.97)(0.57) = 706.3\ \text{lb}$

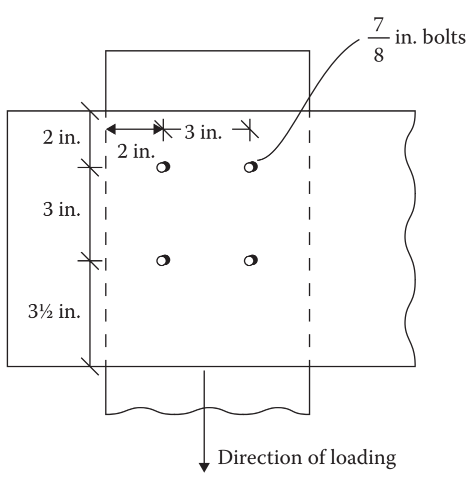


FIGURE 8.13 Perpendicular-to-grain loaded connection.

Example 8.3

The connection of Example 8.1 when loaded in withdrawal mode has a reference design value of 500 lb. Determine the adjusted reference withdrawal design value.

Solution

1. Adjusted reference design value, $Z'_n = Z \times (\phi_z \lambda K_F)$.
2. $\phi_z = 0.65$
3. $\lambda = 0.8$
4. $K_F = 3.32$
5. $Z'_n = 500(0.65)(0.80)(3.32) = 863\,\text{lb}$

NAIL AND SCREW CONNECTIONS

Once the adjusted reference design value is determined, Equation 8.1 can be used with the factored load to design a connection for any dowel-type fasteners. Nails and wood screws generally fall into small-size fasteners having a diameter of less than ¼ in. For small-size fasteners, the angle of load with respect to the grain of wood is not important. The group action factor, C_g, and the geometry factor, C_Δ, are not applicable. The end grain factor, C_{eg}; the diaphragm factor, C_{di}; and the toenail factor, C_{tn}, apply only to specific cases. Thus, for a common type of dry wood under normal temperature conditions, no adjustment factors are required except for the special LRFD factors of ϕ_z, λ, and K_F.

The basic properties of nails and wood screws are described below.

Common, Box, and Sinker Nails

Nails are specified by the pennyweight, abbreviated as *d*. A nail of a specific pennyweight has a fixed length, *L*; shank diameter, *D*; and head size, *H*. There are three kinds of nails: common, box, and sinker. Common and box nails have a flat head, and sinker nails have a countersunk head, as shown in Figure 8.14. For the same pennyweight, box and sinker nails have a smaller diameter and hence a lower capacity compared to common nails.

The reference lateral design values for the simple nail connector are given in Appendix B.11. The values for the other cases are included in the NDS specifications. The reference withdrawal design values for nails of different sizes for various wood species are given in Appendix B.12.

Post-Frame Ring Shank Nails

Post-frame ring shank nails are threaded nails. There are two types of threads. In annular nails, the threads are perpendicular to the nail axis. The threads of helical nails are aligned at an angle between 30° and 70° to the nail axis. Annular nails, shown in Figure 8.15, are called the post-frame ring shank nails. The threaded nails have higher withdrawal strength because of wood fibers lodged between the threads.

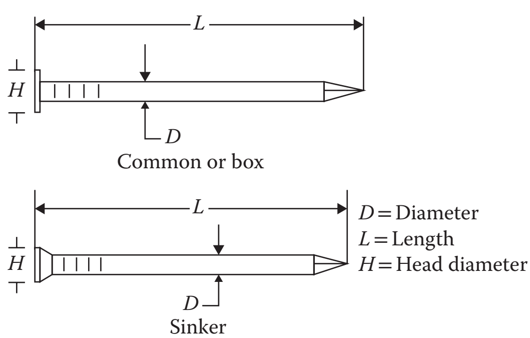


FIGURE 8.14 Typical specifications of nails.

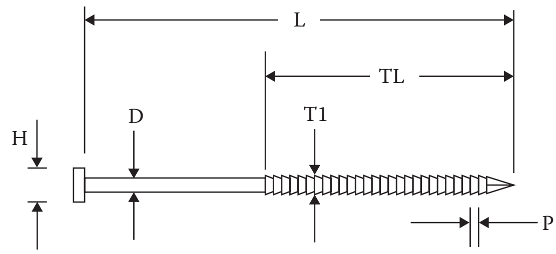


FIGURE 8.15 Typical specifications of post-frame ring shank nails.

The typical dimensions of post-frame ring shank nails are given in Table 8.7. The reference design values for post-frame ring shank nails using a single-shear connection are given in Appendix B.13. The reference withdrawal design values per inch penetration are given in Appendix B.14.

Wood Screws

Wood screws are identified by a number. A screw of a specific number has a fixed diameter (outside to outside of threads) and a fixed root diameter, as shown in Figure 8.16. Screws of each specific number are available in different lengths. There are two types of screws: *cut thread screws* and *rolled thread screws*. The thread length, T, of a cut thread screw is approximately two-thirds the screw length, L. In a rolled thread screw, the thread length, T, is at least four times the screw diameter, D, or two-thirds the screw length, L, whichever is greater. The screws that are too short to accommodate the minimum thread length have threads extended as close to the underside of the head as practical.

TABLE 8.7
Typical Dimensions of Post-Frame Ring Shank Nails

D (in.)	L (in.)	H (in.)	Root Diameter, D_r (in.)
0.135	3, 3.5	5/16	0.128
0.148	3, 3.5, 4, 4.5	5/16	0.140
0.177	3, 3.5, 4, 4.5, 5, 6, 8	3/8	0.169
0.20	3.5, 4, 4.5, 5, 6, 8	15/32	0.193
0.207	4, 4.5, 5, 6, 8	15/32	0.199

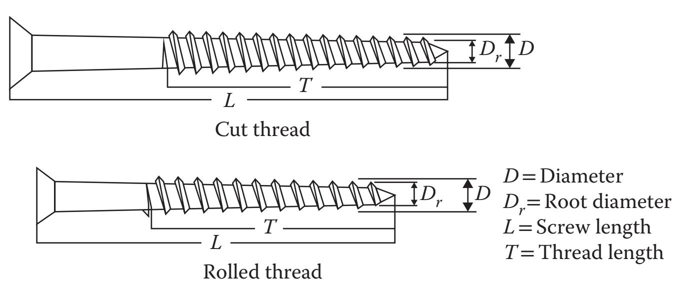


FIGURE 8.16 Typical specifications of wood screws.

The screws are inserted in their lead hole by turning with a screwdriver; they are not driven by a hammer. The minimum penetration of the wood screw into the main member for single shear or into the side member for double shear should be six times the diameter of the screw. Wood screws are not permitted to be used in a withdrawal-type connection in end grain.

The reference lateral design values for simple wood screw connections are given in Appendix B.15. The values for other cases are included in the NDS specifications. The reference withdrawal design values for wood screws are given in Appendix B.16.

Example 8.4

A 2 in. × 6 in. diagonal member of Southern Pine is connected to a 4 in. × 6 in. column, as shown in Figure 8.17. It is acted upon by a service wind load component of 2 k. Design the nailed connection. Neglect the dead load.

Solution

1. Factored design load, $R_Z = 1(2) = 2$ k or 2000 lb
2. Use 30*d* nails, three in a row.
3. Reference design value for a side thickness of 1.5 in.
   From Appendix B.11, $Z = 203$ lb.
4. For nails, the adjusted reference design value:

   $$Z'_n = Z \times (\phi_z \lambda K_F)$$

   where:
   $\phi_z = 0.65$
   $\lambda = 1.00$
   $K_F = 3.32$
   $Z'_n = 203(0.65)(1)(3.32) = 438$ lb
5. From Equation 8.1:

   $$N = \frac{R_z}{Z'_n} = \frac{2000}{438} = 4.57 \text{ nails}$$

6. For number of nails per row, $n = 3$

   $$\text{Number of rows} = \frac{4.57}{3} = 1.52 \text{ (use 2)}$$

   Provide two rows of three nails, each of 30*d* size.

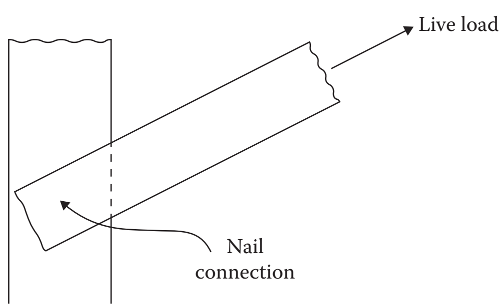


FIGURE 8.17 Diagonal member nail connection.

BOLT AND LAG SCREW CONNECTIONS

Bolts and lag screws are used for larger loads. The angle of load to grains is an important consideration in large-diameter (≥¼ in.) connections comprising bolts and lag screws. However, this text uses the reference design tables, in lieu of the yield limit equations, which include only the two cases of parallel-to-grain and perpendicular-to-grain conditions. The group action factor, C_g, and the geometry factor, C_Δ, apply to bolts and lag screws. Although the end grain factor, C_{eg}, is applicable, it is typical to a nail connection. The other two factors, the diaphragm factor, C_{di}, and the toenail factor, C_{tn}, also apply to nails. An important consideration in bolt and lag screw connection design is to accommodate the number of bolts and rows within the size of the connecting member satisfying the requirements of the end, edge, and in-between bolt spacing.

The larger diameter fasteners often involve the use of prefabricated steel accessories or hardware. The NDS provides details of the typical connections involving various kinds of hardware.

Bolts

In steel structures, the trend is to use high-strength bolts. However, this is not the case in wood structures, where low-strength A307 bolts are commonly used. Bolt sizes used in wood construction range from ½ in. through 1 in. diameter, in increments of ⅛ in. The NDS restricts the use of bolts to a largest size of 1 in. The bolts are installed in the predrilled holes. The NDS specifies that the hole size should be a minimum of 1/32 in. to a maximum of 1/16 in. larger than the bolt diameter for uniform development of the bearing stress.

Most bolts are used in the lateral-type connections. They are distinguished by the single-shear (two-member) and double-shear (three-member) connections. For more than double shear, the single-shear capacity at each shear plane is determined and the value of the weakest shear plane is multiplied by the number of shear planes.

The connections are further recognized by the types of main and side members, such as wood-to-wood, wood-to-metal, wood-to-concrete, and wood-to-masonry connections. The last two are simply termed *anchored connections*.

Washers of adequate size are provided between the wood member and the bolt head, and between the wood member and the nut. The size of the washer is not of significance in shear. For bolts in tension and compression, the size should be adequate so that the bearing stress is within the compression strength perpendicular to the wood grain.

The reference lateral design values for a simple bolted connection are given in Appendix B.17.

Lag Screws

Lag screws are relatively larger than wood screws. They have wood screw threads and a square or hexagonal bolt head. The dimensions for lag screws include the nominal length, L; diameter, D; root diameter, D_r; unthreaded shank length, S; minimum thread length, T; length of tapered tip, E; number of threads per in., N; height of head, H; and width of head across flats, F, as shown in Figure 8.18.

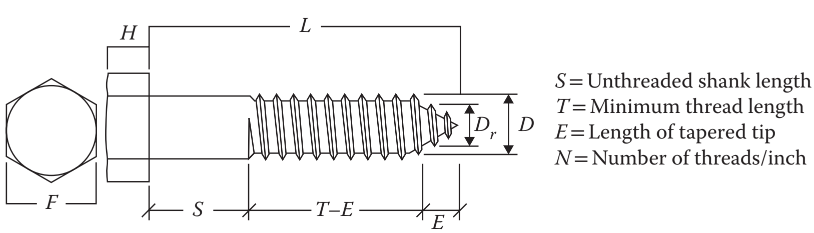


FIGURE 8.18 Typical specifications of lag screws.

Lag screws are used when an excessive length of bolt is required to access the other side or when the other side of a through-bolted connection is not accessible. Lag screws are used in shear as well as in withdrawal applications.

Lag screws are installed with a wrench, whereas wood screws are installed with a screwdriver. Lag screws involve pre-bored holes with two different diameter bits. The larger diameter hole has the same diameter and length as the unthreaded shank of the lag screw, and the lead hole for the threaded portion is similar to that for a wood screw, the size of which depends on the specific gravity of the wood. The minimum penetration (excluding the length of the tapered tip) into the main member for single shear and into the side member for double shear should be four times the lag screw diameter, *D*.

The reference lateral design values for simple lag screw connections are given in Appendix B.18. The other cases are included in the NDS specifications. The reference withdrawal design values for lag screws are given in Appendix B.19.

Example 8.5

The diagonal member of Example 8.4 is subjected to a wind load component of 4 k. Design the bolted connection. Use ⅝-in. bolts.

Solution

1. Factored design load, R_Z = 1(4) = 4 k or 4000 lb.
2. Use ⅝-in. bolts, two in a row.
3. Reference design value
   a. For a side thickness of 1.5 in.
   b. Main member thickness of 3.5 in.
   c. From Appendix B.17, Z = 940 lb
4. Adjusted reference design value, $Z'_n = Z \times (\phi_Z \lambda K_F C_g C_\Delta)$
5. $\phi_z = 0.65$
   $\lambda = 1.0$
   $K_F = 3.22$
6. Group action factor, C_g
   For two fasteners in a row, $C_g = 0.97$ (from Table 8.2).
7. Geometry factor, C_Δ
   a. End distance to accommodate within 6 in. column size = 2.5 in.
   b. Spacing within 6 in. column = 2 in.
   c. End distance for $C_\Delta = 1$, $7D = 4.375$ in.
   d. End factor $= \dfrac{2.5}{4.375} = 0.57 \leftarrow$ controls
   e. Spacing $C_\Delta = 1$, $4D = 2.5$ in.
   f. Spacing factor $= \dfrac{2}{2.5} = 0.8$
8. $Z'_n = 940(0.65)(1)(3.22)(0.97)(0.57) = 1087.8\text{ lb}$
9. From Equation 8.1:

$$N = \frac{R_z}{Z'_n} = \frac{4000}{1087.8} = 3.7$$

10. Number of bolts per row, $n = 2$

$$\text{Number of rows} = \frac{3.7}{2} = 1.85 \text{ (use 2)}$$

Provide 2 rows of two ⅝-in. bolts.

PROBLEMS

8.1 The reference lateral design value of the parallel-to-grain loaded lag screw connection shown in Figure P8.1 is 740 lb. The screw diameter is ⅝ in. The loads comprise dead and live loads. Determine the adjusted reference design value for soft dry wood at normal temperature.

8.2 The reference lateral design value of the perpendicular-to-grain loaded lag screw connection shown in Figure P8.2 is 500 lb. The screw diameter is ⅝ in. The loads comprise dead and live loads. Determine the adjusted reference design value for soft dry wood at normal temperature.

8.3 The connection in Problem 8.1 has a reference withdrawal design value of 400 lb. Determine the adjusted reference design value.

8.4 Problem 8.2 is a nailed connection by 0.225-in.-diameter nails. The holding member has fibers parallel to the nail axis. The reference design value is 230 lb. Determine the adjusted reference design value.

8.5 A spliced parallel-to-grain-loaded connection uses two rows of ⅞-in. lag screws with three fasteners in each row, as shown in Figure P8.3. The load carried is $1.2D + 1.6L$. The reference design value is 1500 lb. The connection is in hard dry wood at normal temperature. Determine the adjusted reference design value.

8.6 The connection in Problem 8.5 is subjected to a perpendicular-to-grain load from the top only. The reference design value is 1000 lb. Determine the adjusted reference design value.

8.7 The connection in Problem 8.5 is subjected to withdrawal loading. The reference design value is 500 lb. Determine the adjusted reference design value.

8.8 The connection shown in Figure P8.4 uses ¾-in.-diameter bolts in a single shear. There are two bolts in each row. The reference design value is 2000 lb. It is subjected to lateral wind load only (no live load). Determine the adjusted reference design value for soft dry wood at normal temperature.

8.9 For the connection shown in Figure P8.5, the reference design value is 1000 lb. Determine the adjusted reference design value for dry wood under normal temperature conditions.

8.10 Toenails of 50*d* pennyweight (0.244 in. diameter, 5½ in. length) are used to connect a beam to the top plate of a stud wall, as shown in Figure P8.6. It is subjected to dead and live loads. The lateral reference design value is 250 lb. Determine the adjusted reference design value for soft wood under normal temperature and dry conditions. Show the connection.

8.11 Design a nail connection to transfer tensile service dead and live loads of 400 lb and 600 lb, respectively, acting along the axis of a 2 in. × 6 in. diagonal member connected to a 4 in. × 4 in. vertical member. Use Southern Pine soft dry wood. Assume two rows of 30*d* common nails.

8.12 A 2 in. × 8 in. diagonal member is connected by 20*d* common nails to a 4 in. × 6 in. vertical member. It is acted on by a combined factored dead and snow load of 1.5 k. Design the connection. Use Douglas Fir-Larch dry wood ($G = 0.5$).

8.13 Determine the tensile capacity of a spliced connection acted on by the dead and snow loads. The joint connects two 2 in. × 6 in. Southern Pine members together by 10*d* common nails via one side plate of 1 in. thickness, as shown in Figure P8.7.

8.14 Two 2 in. × 8 in. members of Douglas Fir-Larch ($G = 0.5$) are to be spliced connected via a single 1½-in.-thick plate on top with two rows of #9 size screws. The service loads comprise 200 lb of dead load and 500 lb of live load that act normal to the fibers. Design the connection.

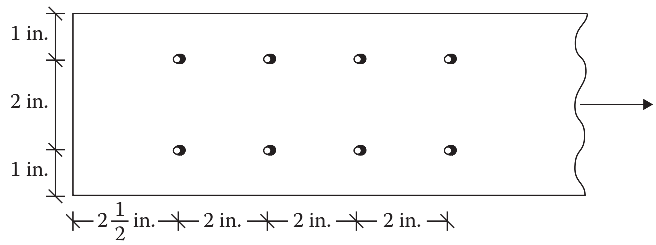


FIGURE P8.1 Parallel-to-grain screw connection for Problem 8.1.

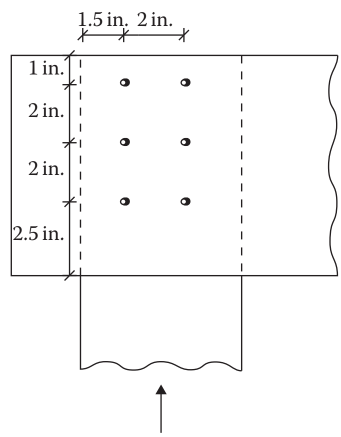


FIGURE P8.2 Perpendicular-to-grain screw connection for Problem 8.2.

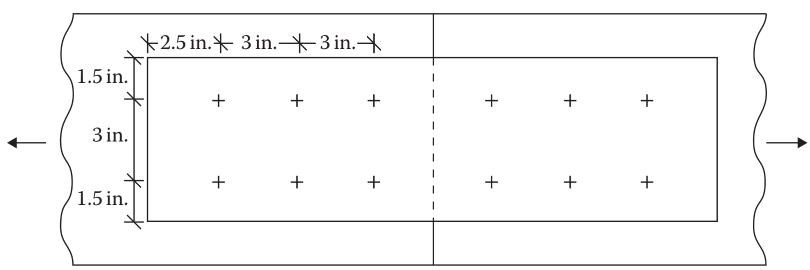


FIGURE P8.3 Spliced parallel-to-grain connection.

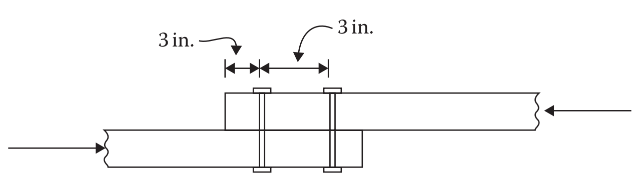


FIGURE P8.4 A single-shear connection.

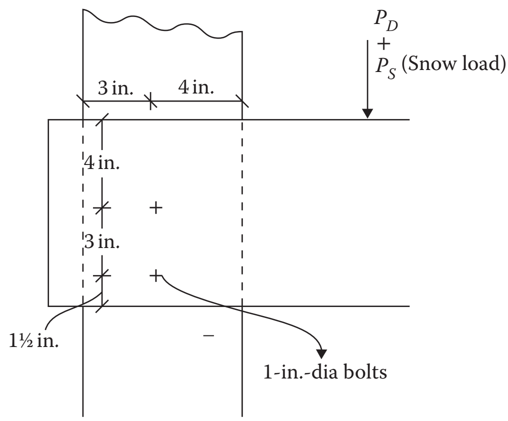


FIGURE P8.5 Perpendicular-to-grain bolted connection for Problem 8.9.

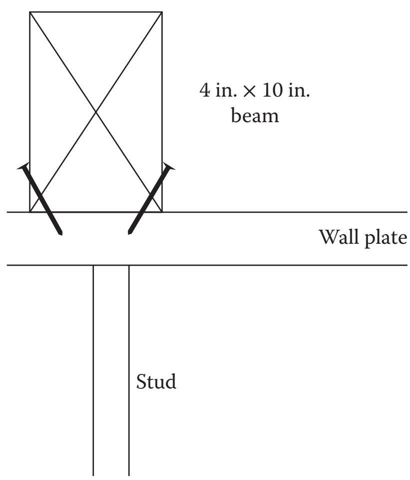


FIGURE P8.6 Toenail connection to a top plate.

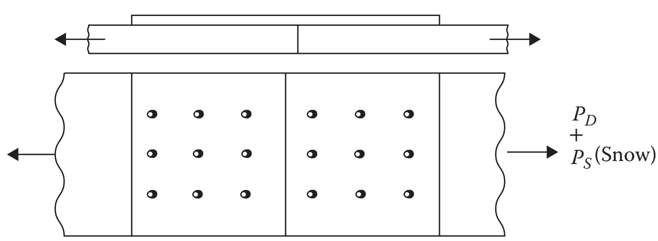


FIGURE P8.7 A spliced nail connection.

8.15 Southern Pine, 10-ft-long 2 in. × 4 in. wall studs, spaced at 16 in. on center (OC) are toenailed on to Southern Pine top and bottom plates with two 10*d* nails at each end. The horizontal service wind load of 30 psf acts on the studs. Is the connection adequate?

8.16 The service dead load and live load in Problem 8.11 are doubled. Design a lag screw connection using ½-in. lag screws. Assume the edge distance, end distance, and bolt spacing along the diagonal of 2 in. each.
Hint: Only two bolts per row can be arranged along the diagonal within a 4 × 4 column size.

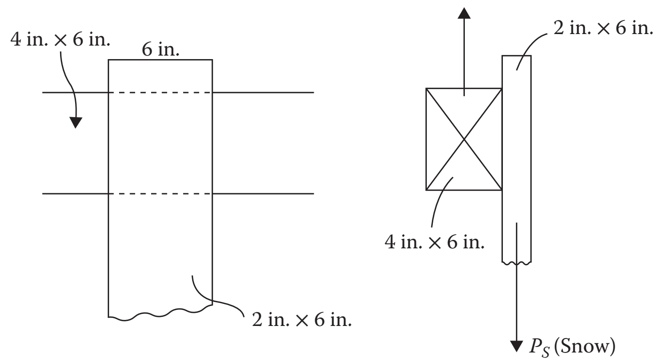


FIGURE P8.8 A beam–column shear connection.

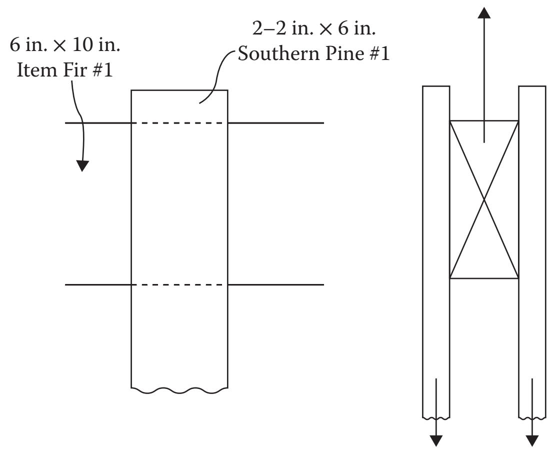


FIGURE P8.9 A beam–column double-shear connection.

8.17 A 2 in. × 6 in. is connected to a 4 in. × 6 in. member, as shown in Figure P8.8. Design a ½ in. lag screw connection to transfer the dead and snow (service) loads of 0.4 k and 1.2 k, respectively. The wood is soft Hem Fir-Larch in dry conditions at normal temperature.
Hint: For a beam size of 6 in., only three bolts can be arranged per row of the vertical member.

8.18 Determine the number and placement of ⅝-in. bolts to transfer the service dead and snow loads of 0.2 k and 2.85 k, respectively, through a joint, as shown in Figure P8.9. The single shear reference design value is 830 lb, which should be doubled for two shear planes.

8.19 The controlling load on the structural member in Problem 8.17 is an unfactored wind load of 3.2 k that acts horizontally. Design the ½-in. bolted connection.
Hint: Load acts normal to the grain, and three rows can be arranged within the column size for the horizontally acting load.

8.20 To elongate a beam span, two short lengths of a beam of 3 in. × 10 in. are spliced connected by one 2 in. × 10 in. side member of Southern Pine soft dry wood. The connection consists of six 1-in. bolts in two rows in each splice. Determine the joint capacity for dead and live loads. The end distance and bolt spacing are 3.5 in. each. If the dead load is one-half of the live load, what is the magnitude of each load?

9 Tension Steel Members

PROPERTIES OF STEEL

Steel structures commonly consist of frames, cables and trusses, and plated structures. The bracing in the form of diagonal members provides the lateral stiffness. For steel elements, the standard shapes, which are specified according to the American Society of Testing Materials (ASTM), are generally used. The properties of these elements are listed in the beginning of the manual of the American Institute of Steel Construction (AISC; 2017) under the "Dimensions and Properties" section. A common element is an I-shaped section having horizontal flanges that are connected at the top and bottom of a vertical web. This type of section is classified into W, M, S, and HP shapes, the difference in these shapes essentially being in the width and thickness of flanges. A typical designation of "W14 × 68" means a wide flange section having a nominal depth of 14 in. and a weight of 68 lb/ft of length. The other standard shapes are channels (C and MC), angles (∟), and tees (WT, MT, and ST).

Tubular shapes are common for compression members. The rectangular and square sections are designated by the letters HSS along with the outer dimensions and the wall thickness. The round tubing is designated as HSS round (for Grade 42) and pipes (for Grade 35) along with the outer diameter and the wall thickness. The geometric properties of the frequently used wide flange sections are given in Appendixes C.1a and b, with those for channel sections in Appendixes C.2a and b; angle sections in Appendixes C.3a through c; rectangular tubing in Appendixes C.4a and b; square tubing in Appendix C.5; round tubing in Appendix C.6; and pipes in Appendix C.7. The enlarged tables in the 2017 AISC Manual include heavier sections.

The structural shapes are available in many grades of steel classified according to the ASTM specifications. The commonly used grades of steel for various structural shapes are listed in Table 9.1.

The yield strength is a very important property of steel because so many design procedures are based on this value. For all grades of steel, the modulus of elasticity is practically the same at a level of 29×10^3 ksi, which means the stress–strain relation of all grades of steel is similar.

A distinguished property that makes steel a very desirable structural material is its ductility, a property that indicates that a structure will withstand an extensive amount of deformation under very high levels of stress without failure.

PROVISIONS FOR DESIGN STEEL STRUCTURES

The AISC *Specification for Structural Steel Buildings* (AISC 360)-2016 is intended to cover common design criteria. This document forms part of the AISC *Steel Construction Manual*. However, it is not feasible to cover within such a document all special and unique problems that are encountered within the full range of structural design. Accordingly, AISC 360 covers the common structures of low seismicity, and a separate AISC document, *Seismic Provisions for Structural Steel Buildings* (AISC 341)-2016, addresses the high-seismic applications. The latter document is incorporated within the *Seismic Design Manual*.

The seismic provisions are not required for the following structures, which are designed according to AISC 360:

1. Structures in seismic design category A
2. Structures in seismic design categories B and C where the response modification factor (coefficient), R, is 3

TABLE 9.1
Common Steel Grades

ASTM Classification	Yield Strength, F_y (ksi)	Ultimate Strength, F_u (ksi)	Applicable Shapes
A36, A709[a]	36	58	W, M, S, HP, ∟, C, MC, WT
A529, A572 Grade 50[a]	50	65	Same
A913, A992 Grade 50[a]	50	65	Same
A501, A618, A1085[b]	36 and 50	58, 65, and 70	HSS—rectangular, square and round
A500 Grade B[a]	46	58	HSS—rectangular and square
A1065 Grade 50[b]	50	60	
A500 Grade B[a]	42	58	HSS—round
A53 Grade B	35	60	Pipe—round

[a] Grades with higher yield strength and ultimate strength are available.

[b] Added in the fifteenth edition of the AISC Manual (2017).

UNIFIED DESIGN SPECIFICATIONS

A major unification of the codes and specifications for structural steel buildings has been accomplished by the AISC. Formerly, the AISC provided four design publications, one separately for the allowable stress design (ASD) method, the load resistance factor design (LRFD) method, the single-angle members, and the hollow tubular structural sections. However, the thirteenth edition of the *Steel Construction Manual* of the AISC (2005) combined all these provisions in a single volume. In addition, the 2005 AISC specifications established common sets of requirements for both the ASD and LRFD methods for analyses and designs of structural elements.

The current fifteenth edition of the *Steel Construction Manual* of the AISC (2017) updated the tables of element shapes to conform to ASTM A6. This update included adding and deleting some shapes and slightly changing areas in some cases.

The factors unifying the two methods are as follows:

1. The nominal strength is the limiting state of failure of a steel member under different modes like compression, tension, or bending. It is the capacity of the member. The same nominal strength applies to both the ASD and LRFD methods of design.
2. For ASD, the available strength is the allowable strength, which is the nominal strength divided by a factor of safety. The available strength for LRFD is the design strength, which is the nominal strength multiplied by a resistance (uncertainty) factor.
3. The required strength for a member is given by the total of the service loads that act on the structure for the ASD method. The required strength for the LRFD method is given by the total of the factored (magnified) loads.
4. The required strength for loads should be within the available strength of the material.

Since the allowable strength of ASD and the design strength of LRFD are both connected with the nominal strength as indicated in item 2, there can be a direct relationship between the factor of safety of ASD and the resistance factor of LRFD. This was discussed in the “Working Stress Design, Strength Design, and Unified Design of Structures” section in Chapter 1.

Limit States of Design

All designs are based on checking that the limit states are not exceeded. For each member type (tensile, column, beam), the AISC specifications identify the limit states that should be checked.

The limit states consider all possible modes of failures like yielding, rupture, and buckling, and also consider the serviceability limit states like deflection and slenderness.

The limit states design process consists of the following:

1. Determine all applicable limit states (modes of failures) for the type of member to be designed.
2. Determine the expression for the nominal strength (and the available strength) with respect to each limit state.
3. Determine the required strength from consideration of the loads applied on the member.
4. Configure the member size by equating items 2 and 3 above.

In ASD, safety is established through a safety factor, which is independent of the types of loading. In LRFD, safety is established through a resistance factor and a load factor that varies with load types and load combinations.

DESIGN OF TENSION MEMBERS

In the 2017 AISC Manual, Chapter D of Part 16 applies to members that are subject to axial tension, and Section J4 of Chapter J applies to connections and connecting elements like gusset plates that are in tension.

The limiting states for the tensile members and the connecting elements are controlled by the following modes:

1. Tensile strength
2. Shear strength of connection
3. Block shear strength of connection along the shear/tension failure path

The shear strength of connection (item 2) will be discussed in Chapter 13 on steel connections.

TENSILE STRENGTH OF ELEMENTS

The serviceability limit state of the slenderness ratio *L*/*r*[1] being less than 300 for members in tension is not mandatory in the new specifications, although Section D1 recommends this value of 300 except for rods and hangers.

The design tensile strength of a member shall be the lower of the values obtained for the limit states of (1) the *tensile yielding* at the gross area and (2) the *tensile rupture* at the net area.

Thus, the strength is the lower of the following two values:

Based on the limit state of yielding of the gross section

$$P_u = 0.9F_y A_g \tag{9.1}$$

Based on the limit state of rupture of the net section

$$P_u = 0.75F_u A_e \tag{9.2}$$

[1] L is the length of the member, and r is the radius of gyration $= \sqrt{I/A}$.

where:

P_u is the factored design tensile load
F_y is the yield strength of steel
F_u is the ultimate strength of steel
A_g is the gross area of the member
A_e is the effective net area

In connecting members, if a portion of a member is not fully connected like a leg of an angle section, the unconnected part is not subjected to the full stress. This is referred to as a *shear leg*. A factor is used to account for the shear lag. Thus:

$$A_e = A_n U \tag{9.3}$$

where:

A_n is the net area
U is the shear lag factor

Net Area, A_n

The net area is the product of the thickness and the net width of a member. To compute net width, the sum of the widths of the holes for bolts is subtracted from the gross width. The hole width is taken as $\frac{1}{16}$ in. greater than the bolt diameter. For a bolt size 1 in. or larger, the standard hole size is $\frac{1}{8}$ in. larger.

For a chain of holes in a zigzag line shown as a–b in Figure 9.1, a quantity $s^2/4g$ is added to the net width for each zigzag of the gage space, g, in the chain. Thus:

$$A_n = bt - \sum ht + \sum \left(\frac{s^2}{4g}\right)t \tag{9.4}$$

where:

s is the longitudinal (in the direction of loading) spacing between two consecutive holes (pitch)
g is the transverse (perpendicular to force) spacing between the same two holes (gage)
b is the width of the member
t is the thickness of the member
h is the size of the hole

For angles, the gage for holes in the opposite legs, as shown in Figure 9.2, is $g = g_1 + g_2 - t$.

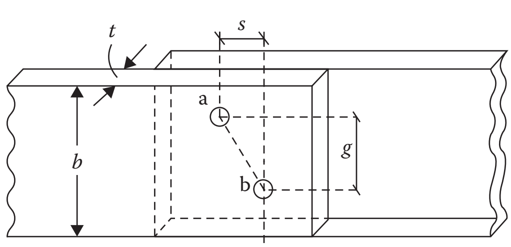


FIGURE 9.1 Zigzag pattern of holes.

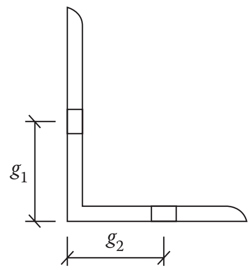


FIGURE 9.2 Gage for holes in angle section.

Example 9.1

An angle ∟ 5 × 5 × ½[2] has a staggered bolt pattern, as shown in Figure 9.3. The holes are for bolts of ⅞ in. diameter. Determine the net area.

Solution

1. $A_g = 4.79$ in.[2], $t = 0.5$ in.
2. $h = d + \left(\frac{1}{16}\right) = \left(\frac{7}{8}\right) + \left(\frac{1}{16}\right) = 0.94$ in.
3. $g = g_1 + g_2 - t = 3 + 2 - 0.5 = 4.5$ in.
4. Section through line a–b–d–e: deducting for two holes
   $A_n = A_g - \Sigma ht$
   $= 4.79 - 2(0.94) = 3.85$ in.[2]
5. Section through line a-b-c-d-e: deducting for three holes and adding $s^2/4g$ for b–c and c–d

$$A_n = A_g - 3ht + \left(\frac{s^2}{4g}\right)_{bc} t + \left(\frac{s^2}{4g}\right)_{cd} t$$

$$= 4.79 - 3(0.94)(0.5) + \left[\frac{2^2}{4(4.5)}\right]0.5 + \left[\frac{2^2}{4(1.5)}\right]0.5$$

$$= 3.82 \text{ in.}^2 \leftarrow \text{ controls}$$

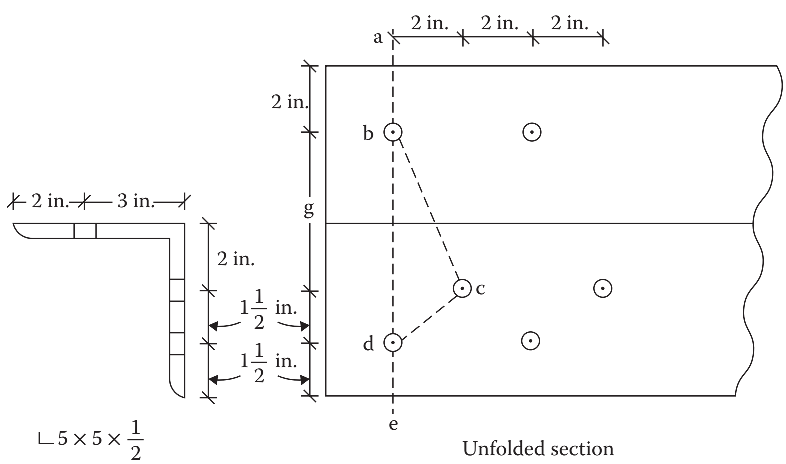


FIGURE 9.3 Bolt pattern for Example 9.1.

[2] The sectional properties relating to area, gages etc. of angle 5 × 5 × 1/2 are not included in the Appendix C.3a.

SHEAR LAG FACTOR, *U*

For members that are fully in contact and the entire area participates in transmitting the load, $U = 1$ for bolted and welded connections.

For members where load is transmitted to some but not all of the cross-sectional elements, the following provision applies. For an open cross section such as W, M, S, C, HP, WT, ST, single angles, and double angles, the shear lag factor, U, with not be less than the ratio of the gross area of the connected element to gross area of the member. This provision does not apply to plates and HSS sections.

BOLTED CONNECTION

1. For all tensile members except HSS where load is transferred to some but not all of the cross section:

$$U = 1 - \frac{\bar{x}}{L} \tag{9.5}$$

where:

$\bar{x}$ is eccentricity, that is, the distance from the connection plane to the centroid of the resisting member, as shown in Figure 9.4

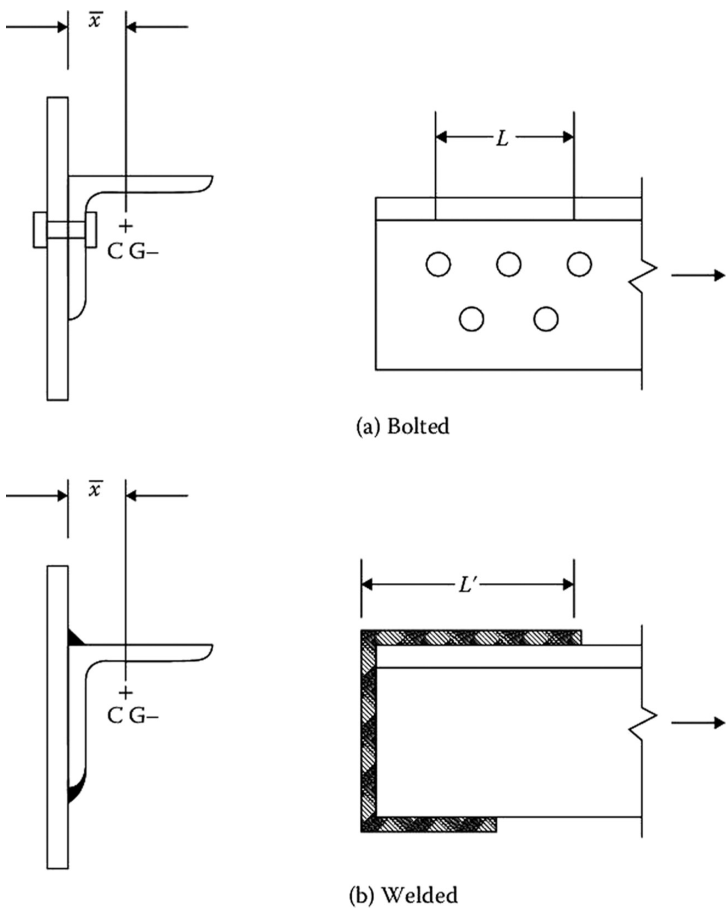


FIGURE 9.4 Eccentricity of the resisting member.

L is the length of connection

In lieu of Equation 9.5, the following values can be applied:

a. For W, M, S, HP, and T shapes
   Flange connected with three or more bolts per line in the direction of loading:

   $$b_f \geq 2/3\ d\ U = 0.9$$

   $$b_f < 2/3\ d\ U = 0.85$$

   Web connected with four or more bolts per line in the direction of loading:

   $$U = 0.70.$$

   For other cases not listed above, use Equation 9.5.

b. For angle shapes
   For single or double angles with four or more bolts in the direction of loading, $U = 0.8$.
   For single or double angles with three bolts in the direction of loading, $U = 0.60$.
   For single or double angles with less than three bolts in the direction of loading, use Equation 9.5.

c. For plates
   As the flat plates are fully in contact, the shear lag factor is $U = 1$. For bolted slice plates, the effective net area should not be more than 85% of the gross area ($A_e = A_n \leq A_g$).

WELDED CONNECTION

1. For a longitudinal and transverse weld combination for all tensile members except HSS, use Equation 9.5.
2. For a transverse weld only (Figure 9.5a) for all tensile members, $U = 1$. (A_e is the area of the directly connected element.)
3. For a longitudinal weld only (Figure 9.5b) for plates, angles, channels, tees, and W-shapes:

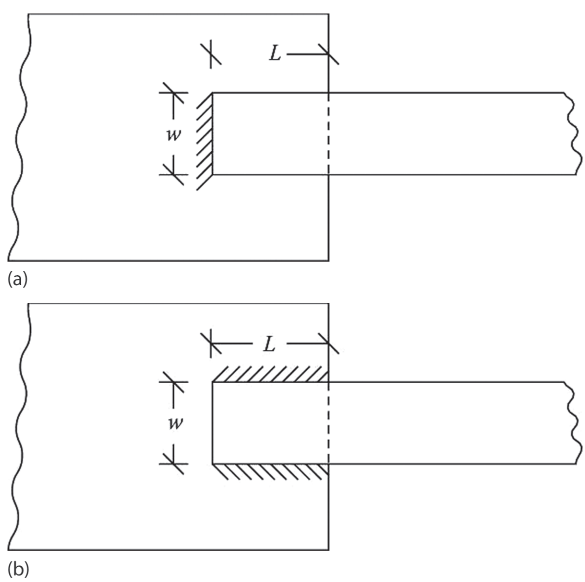


FIGURE 9.5 Longitudinal weld.

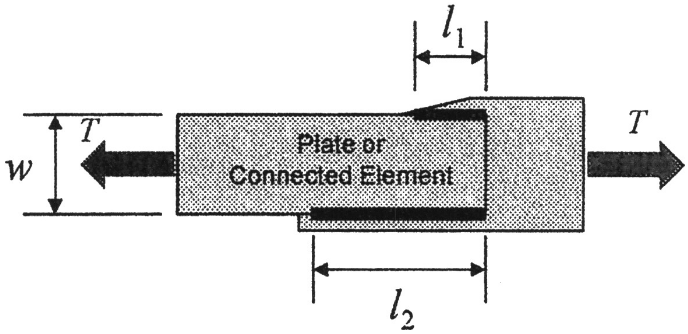


FIGURE 9.6 Eccentricity of the resisting member.

$$U = \frac{3l^2}{3l^2 + w^2}\left(1 - \frac{\bar{x}}{L}\right) \tag{9.6}$$

where:

$$l = \frac{l_1 + l_2}{2}$$

l_1, l_2, and w are shown in Figure 9.6.

FOR HSS SHAPES

Refer to Table D3.1 of the 2017 AISC Manual.

Example 9.2

Determine the effective net area for the single-angle member in Example 9.1.

Solution

1. Since the number of bolts in the direction of loading is 3, $U = 0.6$.
2. From Example 9.1, $A_n = 3.82$ in.^{2}.
3. $A_e = A_nU = (3.82)(0.6) = 2.29$ in.^{2}

Example 9.3

What is the design strength of the element of Example 9.1 for A36 steel?

Solution

1. $A_g = 4.75$ in.^{2}
2. $A_e = 2.29$ in.^{2} (from Example 9.2)
3. From Equation 9.1:

$$P_u = 0.9F_y A_g = 0.9(36)(4.75) = 153.9 \text{ k}$$

4. From Equation 9.2:

$$P_u = 0.75F_u A_e = 0.75(58)(2.29) = 99.6 \text{ k} \leftarrow \text{controls}$$

BLOCK SHEAR STRENGTH

In certain connections, a *block* of material at the end of the member may tear out. In the single-angle member shown in Figure 9.7, the block shear failure may occur along plane abc. The shaded block fails by shear along plane ab and tension in section bc.

Figure 9.8 shows a tensile plate connected to a gusset plate. In this case, the block shear failure could occur in both the gusset plate and the main tensile member. The tensile failure occurs along section bc; the shear failure occurs along planes ab and cd.

The welded member shown in Figure 9.9 experiences block shear failure along welded planes abcd. It has a tensile area along bc and a shear area along ab and cd.

Both the tensile area and shear area contribute to the strength. The resistance to shear block is the sum of the strengths of the two surfaces.

The resistance (strength) to shear block is given by a single two-part equation:

$$R_u = \phi R_n = \phi(0.6F_u A_{nv} + U_{bs} F_u A_{nt}) \leq \phi(0.6F_y A_{gv} + U_{bs} F_u A_{nt}) \tag{9.7}$$

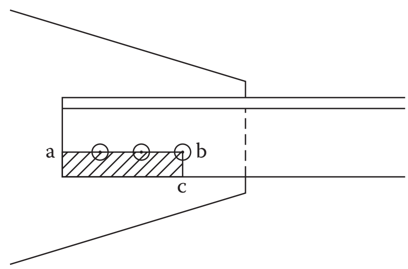


FIGURE 9.7 Block shear in a single-angle member.

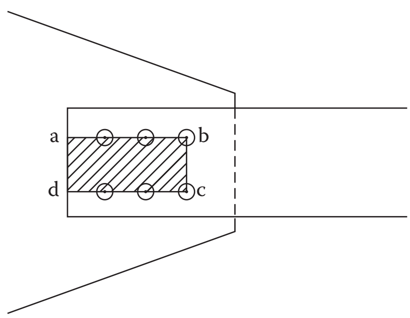


FIGURE 9.8 Block shear in a plate member.

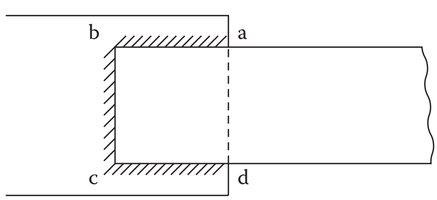


FIGURE 9.9 Block shear in a welded member.

where:

ϕ is the resistance factor, 0.75
A_{nv} is the net area subjected to shear
A_{nt} is the net area subjected to tension
A_{gv} is the gross area along the shear surface
U_{bs} is 1.0 when the tensile stress is uniform (most cases)
U_{bs} is 0.5 when the tensile is nonuniform

Example 9.4

An ∟ 6 × 4 × ½[3] tensile member of A36 steel is connected by three $^{15}\!/\!_{16}$ in. bolts, as shown in Figure 9.10. Determine the strength of the member.

Solution

I. Tensile strength of member
  A. Yielding in gross area
    1. $A_g = 4.75$ in.^{2}
    2. $h = \left(^{15}\!/\!_{16}\right) + \left(^{1}\!/\!_{16}\right) = 1$ in.
    3. From Equation 9.1:

$$P_u = 0.9(36)(4.75) = 153.9 \text{ k}$$

  B. Rupture in net area
    1. $A_n = A_g -$ one hole area
       $= 4.75 - (1)(1)\left(\frac{1}{2}\right) = 2.25$ in.^{2}
    2. $U = 0.6$ for three bolts in a line
    3. $A_e = UA_n = 0.6\ (4.25) = 2.55$ in.^{2}
    4. From Equation 9.2:

$$P_u = 0.75(58)(2.55) = 110.9 \text{ k} \leftarrow \text{controls}$$

II. Block shear strength
  A. Gross shear area along ab

$$A_{gv} = 10\left(\tfrac{1}{2}\right) = 5 \text{ in.}^2$$

  B. Net shear area along ab
    $A_{nv} = A_{gv} -$ 1½ hole area
    $= 5 - 2.5(1)(½) = 3.75$ in.^{2}
  C. Net tensile area along bc
    $A_{nt} = 2.5\ t -$ (½) hole
    $= 2.5(½) - ½(1)(½) = 1.0$ in.^{2}
  D. $U_{bs} = 1.0$
  E. From Equation 9.7:

$$\phi\left(0.6F_uA_{nv} + U_{bs}F_uA_{nt}\right) = 0.75\left[0.6(58)(3.75) + (1)(58)(1.0)\right] = 1421.4 \text{ k}$$

$$\phi\left(0.6F_yA_{gv} + U_{bs}F_uA_{nt}\right) = 0.75\left[0.6(36)(5) + (1)(58)(1.0)\right] = 124.5 \text{ k}$$

The strength is 110.9 k controlled by rupture of the net section.

[3] The sectional properties relating to area, gages, etc. are not included in the Appendix C.3a.

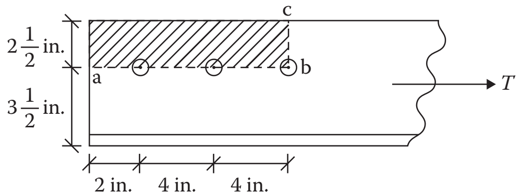


FIGURE 9.10 The three-bolt connection of Example 9.4.

DESIGN PROCEDURE FOR TENSION MEMBERS

The type of connection used for a structure affects the choice of the tensile member. The bolt-type connections are convenient for members consisting of angles, channels, and W and S shapes. The welded connection suits plates, channels, and structural tees.

The procedure to design a tensile member is:

1. Determine the critical combination(s) of factored loads.
2. For each critical load combination, determine the gross area required by Equation 9.1 and select a section.
3. Account for holes or welds based on the connection requirements, and determine the effective net area.
4. Use Equation 9.2 to compute the loading capacity of the effective net area of the selected section. This capacity should be more than the design load(s) of step 1. If it is not, revise the selection.
5. Check the block shear strength using Equation 9.7. If it is not adequate, revise either the connection or the member size.
6. The limitation of the maximum slenderness ratio of 300 is not mandatory in the 2017 AISC Manual. However, it is still a preferred practice except for rods and hangers.

Although rigid frames are common in steel structures, roof trusses having nonrigid connections are used for industrial or mill buildings. The members in the bottom chord of a truss are commonly in tension. Some of the web members are in tension, and the others are in compression. With changing of the wind direction, the forces in the web members alternate between tension and compression. Accordingly, the web members have to be designed to function both as tensile as well as compression elements.

Example 9.5

A roof system consists of a Warren-type roof truss, as shown in Figure 9.11. The trusses are spaced 25 ft apart. The following loads are passed on to the truss through the purlins. Design the bottom chord members consisting of the two angles section separated by a ⅜ in. gusset plate. Assume one line of two ¾ in. diameter bolts spaced 3 in. at each joint. Use A572 steel.

Dead load (deck, roofing, insulation) = 10 psf
Snow = 29 psf
Roof live load = 20 psf
Wind (vertical) = 16 psf

Solution

A. Computation of loads
   1. Adding 20% to dead load for the truss weight, $D = 12$ psf
   2. Consider the following load combinations:
      a. $1.2D + 1.6(L_r \text{ or } S) + 0.5W = 1.2(12) + 1.6(29) + 0.5(16) = 68.8$ psf ← controls
      b. $1.2D + W + 0.5(L_r \text{ or } S) = 1.2(12) + 16 + 0.5(29) = 44.9$ psf

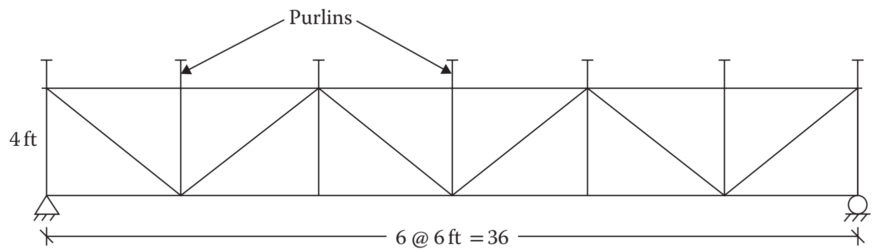


FIGURE 9.11 A Warren roof truss.

3. Tributary area of an entire truss = 36 × 25 = 900 ft^2
4. Total factored load on the truss = 68.8 × 900 = 61920 lb or 61.92 k
5. This load is distributed through purlins in six parts, on to five interior joints and one-half on each end joint since the exterior joint tributary is one-half that of the interior joints. Thus, the joint loads are:

$$\text{Interior joints} = \frac{61.92}{6} = 10.32 \text{ k}$$

$$\text{Exterior joints} = \frac{10.32}{2} = 5.16 \text{ k}$$

B. Analysis of the truss
1. The loaded truss is shown in Figure 9.12.
2. Reaction at L_0 and L_6 = 61.62/2 = 30.96 k
3. The bottom chord members L_2L_3 and L_3L_4 are subjected to the maximum force. A free-body diagram of the left of section a-a is shown in Figure 9.13.
4. M at $U_2 = 0$

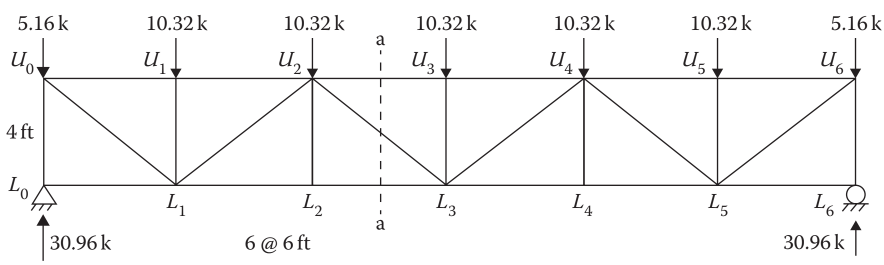


FIGURE 9.12 Truss analysis for Example 9.5.

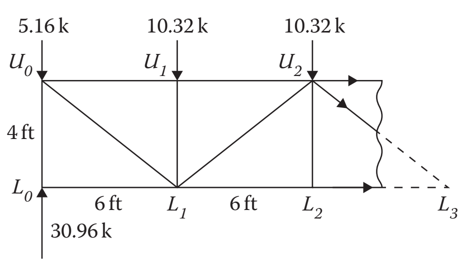


FIGURE 9.13 Free-body diagram of truss.

$$-30.96(12)+5.16(12)+10.32(6)+F_{L_2L_3}(4)=0$$

$$F_{L_2L_3}=61.92 \text{ k} \leftarrow P_u$$

C. Design of member
   1. From Equation 9.1:

$$A_g=\frac{P_u}{0.9F_y}=\frac{61.92}{0.9(50)}=1.38 \text{ in.}^2$$

   Try 2 ∟ 3 × 2 × ¼ A_g = 2.4 in.^{2}, centroid $\bar{x}$ = 0.487 (from Appendix C.3b).
   2. $h=\left(\frac{3}{4}\right)+\left(\frac{1}{16}\right)=0.81$ in.

$$A_n = A_g - \text{one hole area}$$

$$= 2.40 - (1)(0.81)\left(\tfrac{1}{4}\right) = 2.2 \text{ in.}^2$$

   3. From Equation 9.6:

$$U=1-\frac{0.487}{3}=0.84$$

$$A_e = 0.84\ (2.2) = 1.85 \text{ in.}^2$$

   4. From Equation 9.2:

$$P_u = 0.75F_uA_e$$

$$= 0.75(65)(1.85) = 90.18 \text{ k } \textbf{OK}$$

D. Check for block shear strength (similar to Example 9.4).

PROBLEMS

9.1 A ½ in. × 10 in. plate is attached to another plate by means of six ¾-in. diameter bolts, as shown in Figure P9.1. Determine the net area of the plate.

9.2 A ¾ in. × 10 in. plate is connected to a gusset plate by ⅞-in. diameter bolts, as shown in Figure P9.2. Determine the net area of the plate.

9.3 An ∟ 5 × 5 × ½ has staggered holes for ¾-in. diameter bolts, as shown in Figure P9.3. Determine the net area for the angle (A_g = 4.79 in.^{2}, centroid $\bar{x}$ = 1.42 in.).

9.4 An ∟ 8 × 4 × ½ has staggered holes for ⅞-in. diameter bolts, as shown in Figure P9.4. Determine the net area (A_g = 5.80 in.^{2}, centroid $\bar{x}$ = 0.854 in.).

9.5 A channel section C 9 × 20 has the bolt pattern shown in Figure P9.5. Determine the net area for ¾-in. bolts.

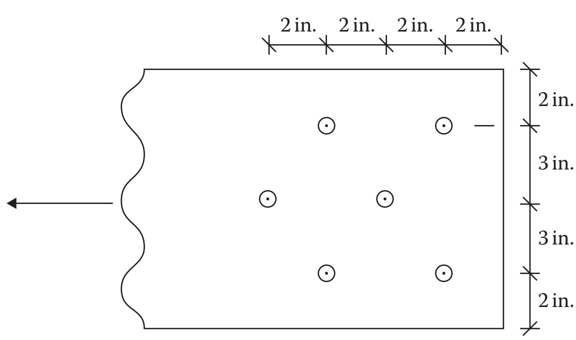


FIGURE P9.1 Plate to plate connection.

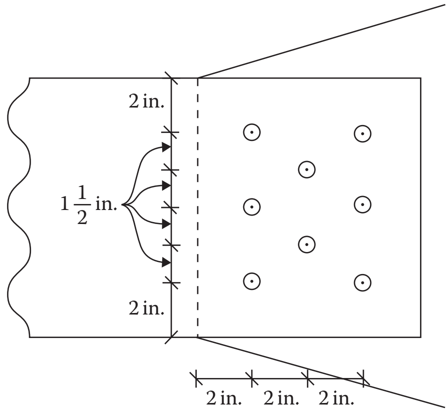


FIGURE P9.2 Plate to gusset plate connection.

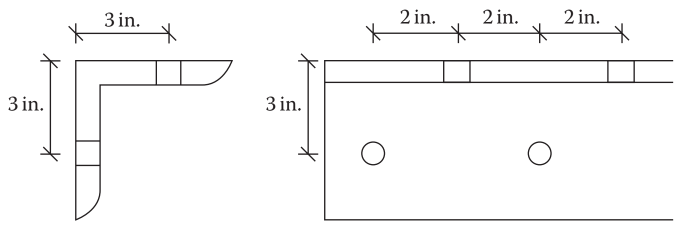


FIGURE P9.3 Staggered angle connection.

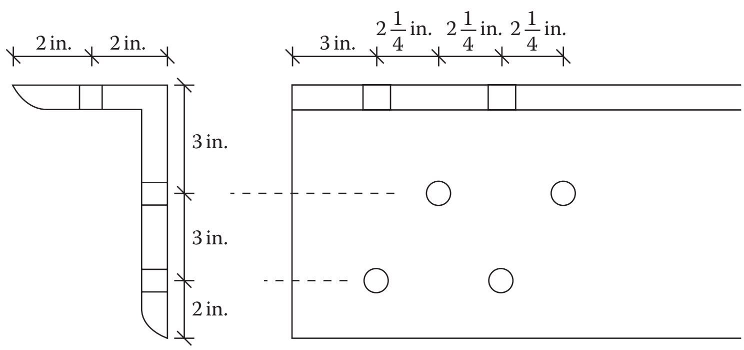


FIGURE P9.4 Staggered long leg angle connection.

9.6 Determine the effective net area for Problem 9.2.

9.7 Determine the effective net area for Problem 9.3.

9.8 Determine the effective net area for Problem 9.4.

9.9 Determine the effective net area for the connection shown in Figure P9.6 for an ∟ 5 × 5 × ½.

9.10 For Problem 9.9, determine the effective net area with welding in the transverse direction only.

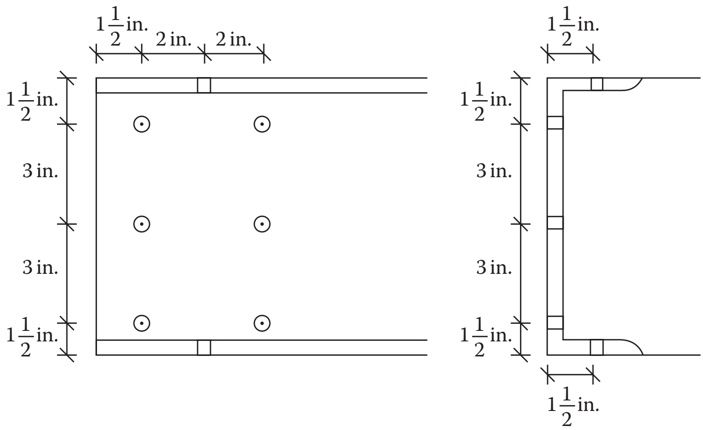


FIGURE P9.5 Staggered channel connection.

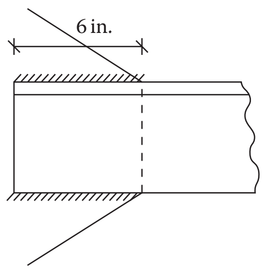


FIGURE P9.6 Welded connection.

9.11 Determine the tensile strength of the plate in Problem 9.1 for A36 steel.

9.12 The tensile member in Problem 9.4 is subjected to a dead load of 30 k and a live load of 60 k. Is the member adequate? Use A572 steel.

9.13 Is the member in Problem 9.9 adequate to support the following loads all acting in tension? Use A992 steel.
Dead load = 25 k
Live load = 50 k
Snow load = 40 k
Wind load = 35 k

9.14 An angle of A36 steel is connected to a gusset plate with six ¾-in. bolts, as shown in Figure P9.7. The member is subjected to a dead load of 25 k and a live load of 40 k. Design a 3½ in. size (3½ × ?) member.

9.15 An angle of A36 steel is connected by ⅞-in. bolts, as shown in Figure P9.8. It is exposed to a dead load of 20 k, a live load of 45 k, and a wind load of 36 k. Design a 4 in. size (4 × ?) member. Use A992 steel.

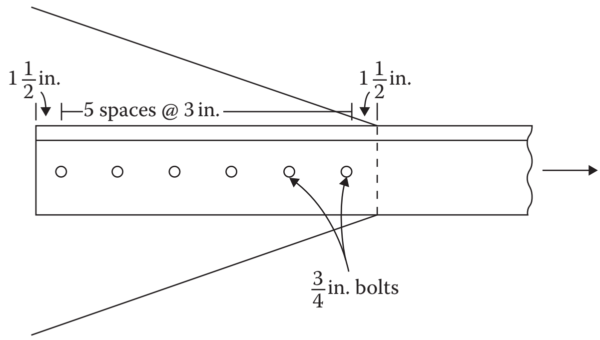


FIGURE P9.7 Connection for Problem 9.14.

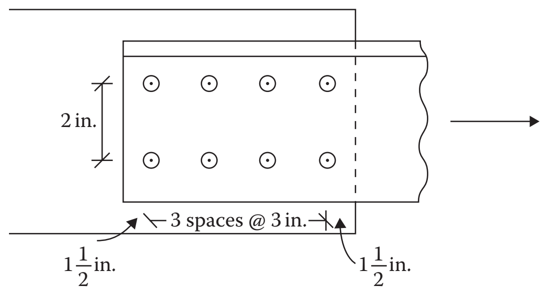


FIGURE P9.8 Two-row connection for Problem 9.15.

9.16 Compute the strength including the block shear capacity of a member comprising ∟ 3½ × 3½ × ½, as shown in Figure P9.9. The bolts are ¾-in. The steel is A36.

9.17 A tensile member comprises a W 12 × 30 section of A36 steel, as shown in Figure P9.10. Each side of the flanges has three holes for ⅞-in. bolts. Determine the strength of the member, including the block shear strength.

9.18 Determine the strength of the welded member shown in Figure P9.11, including the block shear capacity. The steel is A572.

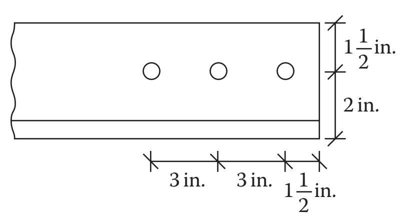


FIGURE P9.9 Tensile member for Problem 9.16.

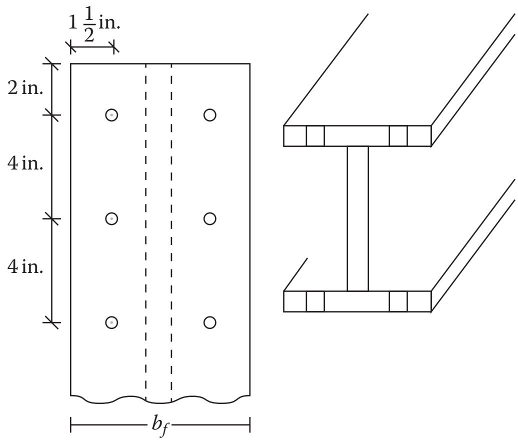


FIGURE P9.10 Wide flange tensile member for Problem 9.17.

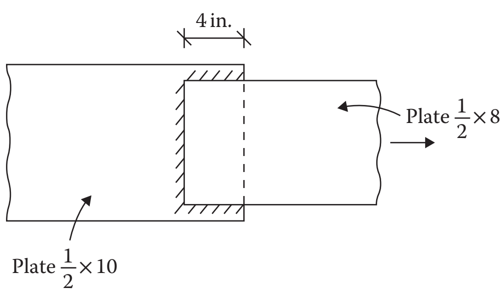


FIGURE P9.11 Welded member for Problem 9.18.

10 Compression Steel Members

STRENGTH OF COMPRESSION MEMBERS OR COLUMNS

The basic strength requirement of compression in the load resistance factor design is:

$$P_u \le \phi P_n \tag{10.1}$$

where:
- P_u is the factored axial load
- $\phi = 0.9$, the resistance factor for compression
- P_n is the nominal compressive strength of the column

For a compression member that fails by yielding, $P_n = F_y A_g$, similar to a tensile member. However, the steel columns are leaner; that is, the length dimension is much larger than the cross-sectional dimension. Accordingly, the compression capacity is more often controlled by the rigidity of the column against buckling instead of yielding. There are two common modes of failure in this respect:

1. *Local instability:* If the parts (elements) comprising a column are relatively very thin, a localized buckling or wrinkling of one or more of these elements may occur prior to the instability of the entire column. Based on the ratio of width to thickness of the element, a section is classified as a *slender* or a *nonslender* for the purpose of local instability.
2. *Overall instability:* Instead of an individual element getting wrinkled, the entire column may bend or buckle lengthwise under the action of the axial compression force. This can occur in three different ways:
   a. *Flexural buckling:* A deflection occurs by bending about the weak axis, as shown in Figure 10.1. The slenderness ratio is a measure of the flexural buckling of a member. When the buckling occurs at a stress level within the proportionality limit of steel, it is called *elastic buckling*. When the stress at buckling is beyond the proportionality limit, it is called *inelastic buckling*. Columns of any shape can fail in this mode by either elastic or inelastic buckling.
   b. *Torsional buckling:* This type of failure is caused by the twisting of the member longitudinally, as shown in Figure 10.2. The doubly symmetric hot-rolled shapes like W, H, or round are normally not susceptible to this mode of buckling. The torsional buckling of doubly symmetric sections can occur only when the torsional unbraced length exceeds the lateral flexural unbraced length. The thinly built-up sections might be exposed to torsional buckling.
   c. *Flexural-torsional buckling:* This failure occurs by the combination of flexural and torsional buckling when a member twists while bending, as shown in Figure 10.3. Only the sections with a single axis of symmetry or the nonsymmetric sections such as a channel, tee, and angle are subjected to this mode of buckling.

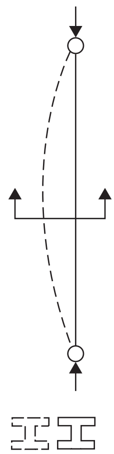


FIGURE 10.1 Flexural buckling.

The nominal compressive strength, P_n, in Equation 10.1 is the lowest value obtained according to the limit states of flexural buckling, torsional buckling, and flexural-torsional buckling. The flexural buckling limit state is applicable to all sections.

In addition, the doubly symmetric sections having torsional unbraced length larger than the weak-axis flexural unbraced length, the doubly symmetric sections built from thin plates, singly symmetric sections, and nonsymmetric sections are subjected to torsional buckling or flexural-torsional buckling that requires substantive evaluations. It is desirable to prevent it when feasible. This can be done by bracing the member to prevent twisting. The overall limit states are considered separately for the nonslender and the slender sections established by the local instability criteria.

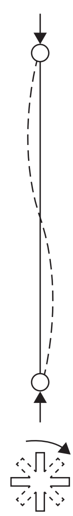


FIGURE 10.2 Torsional buckling.

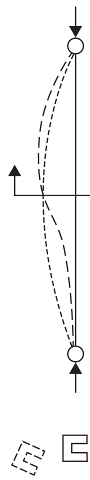


FIGURE 10.3 Flexural–torsional buckling.

LOCAL BUCKLING CRITERIA

In the context of local buckling, the elements of a structural section are classified into the following two categories:

1. *Unstiffened element:* This has an unsupported edge (end) parallel to (along) the direction of the load, like an angle section.
2. *Stiffened element:* This is supported along both of its edges, like the web of a wide flange section.

The two types of elements are illustrated in Figure 10.4.

When the ratio of width to thickness of an element of a section is greater than the specified limit λ_r, as shown in Table 10.1, it is classified as a slender shape. The cross section of a slender

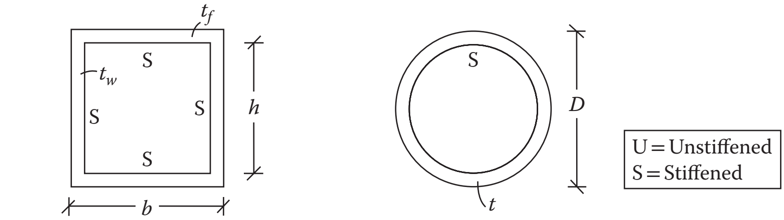


FIGURE 10.4 Stiffened and unstiffened elements.

TABLE 10.1
Slenderness Limit for Compression Member

Element	Width: Thickness Ratio	λ_r	Magnitude for 36 ksi	Magnitude for 50 ksi
W, S, M, H	$b_f/2t_f$	$0.56\sqrt{E/F_y}$	15.89	13.49
	h/t_w	$1.49\sqrt{E/F_y}$	42.29	35.88
C	b_f/t_f	$0.56\sqrt{E/F_y}$	15.89	13.49
	h/t_w	$1.49\sqrt{E/F_y}$	42.29	35.88
T	$b_f/2t_f$	$0.56\sqrt{E/F_y}$	15.89	13.49
	d/t_w	$0.75\sqrt{E/F_y}$	21.29	18.16
Single ∟ or double ∟ with separation	b/t	$0.45\sqrt{E/F_y}$	12.77	10.84
Double ∟ with continuous contact	b/t	$0.56\sqrt{E/F_y}$	15.89	13.49
Other unstiffened element	b/t	$0.45\sqrt{E/F_y}$	12.77	10.84
Other stiffened element	b/t	$1.49\sqrt{E/F_y}$	42.29	35.88
Box, tubing	b/t	$1.4\sqrt{E/F_y}$	Box (46 ksi steel) 35.15	Tubing (42 ksi steel) 36.79
Circular	D/t	$0.11(E/F_y)$	Pipe (35 ksi steel) 91.14	

element is not fully effective in resisting a compressive force. Such elements should be avoided or else their strength should be reduced, as discussed in the "Slender Compression Members" section. The terms are explained in Figure 10.4.

FLEXURAL BUCKLING CRITERIA

The term (KL/r), known as the *slenderness ratio*, is important in column design. Not only does the compression capacity of a column depend on the slenderness ratio, but the ratio also sets a limit between the elastic and nonelastic buckling of the column. When the slenderness ratio exceeds a value of $4.71\sqrt{E/F_y}$, the column acts as an elastic column and the limiting (failure) stress level is within the elastic range.

According to the classic Euler formula, the critical load is inversely proportional to $(KL/r)^2$, where K is the effective length factor (coefficient), discussed in the "Effective Length Factor for Slenderness Ratio" section; L is the length of the column; and r is the radius of gyration given by $\sqrt{I/A}$.

Although it is not a mandatory requirement in the 2017 AISC Manual, the AISC recommends that the slenderness ratio for a column not exceed a value of 200.

EFFECTIVE LENGTH FACTOR FOR SLENDERNESS RATIO

The original flexural buckling or Euler formulation considered the column pinned at both ends. The term K was introduced to account for the other end conditions because the end condition makes a column buckle differently. For example, if a column is fixed at both ends, it will buckle at the points of inflection about $L/4$ distance away from the ends, with an effective length of one-half of the column length. Thus, the effective length of a column is the distance at which the column is assumed to buckle in the shape of an elastic curve. The length between the supports, L, is multiplied by a factor to calculate the effective length.

When columns are part of a frame, they are constrained at the ends by their connection to beams and to other columns. The effective length factor for such columns is evaluated by the use of the alignment charts or nomographs given in Figures 10.5 and 10.6; the former is for the braced frames where sidesway is prevented, and the latter is for the moment frames where sidesway is permitted.

In the nomographs, the subscripts A and B refer to two ends of a column for which K is desired. The term G is the ratio of the column stiffness to the girder stiffness, expressed as:

$$G = \frac{\sum I_c / L_c}{\sum I_g / L_g} \tag{10.2}$$

where:

I_c is the moment of inertia of the column section
L_c is the length of the column
I_g is the moment of inertia of the girder beam meeting the column
L_g is the length of the girder
$\sum$ is the summation of all members meeting at joint A for G_A and at joint B for G_B

The values of I_c and I_g are taken about the axis of bending of the frame. For a column base connected to the footing by a hinge, G is taken as 10; when the column is connected rigidly (fixed) to the base, G is taken as 1.

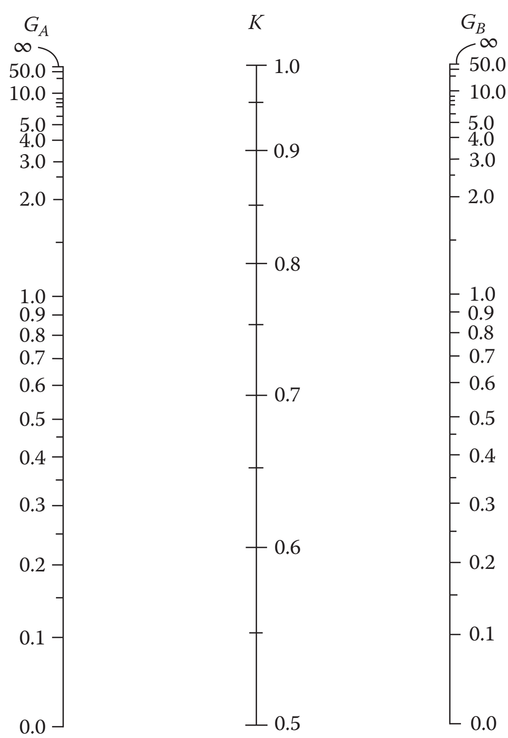


FIGURE 10.5 Alignment chart, sidesway prevented. (Courtesy of American Institute of Steel Construction, Chicago, IL.)

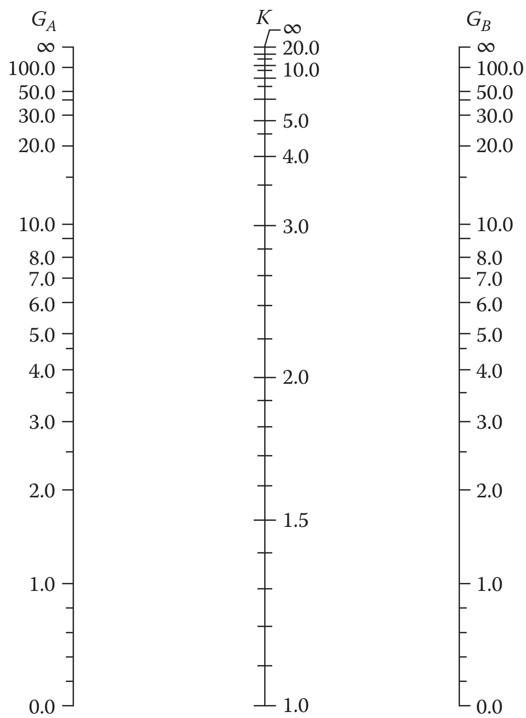


FIGURE 10.6 Alignment chart, sidesway not prevented. (Courtesy of American Institute of Steel Construction, Chicago, IL.)

After determining G_A and G_B for a column, K is obtained by connecting a straight line between points G_A and G_B on the nomograph. Since the values of I (moment of inertia) of the columns and beams at the joint are required to determine G, the factor K cannot be determined unless the size of the columns and the beams are known. On the other hand, the factor K is required to determine the column size. Thus, these nomographs need some preliminary assessments of the value of K and the dimensions of the columns and girders.

One of the conditions for the use of the nomographs or the alignment charts is that all columns should buckle elastically; that is, $KL > 4.71\sqrt{E/F_y}$. If a column buckles inelastically, a stiffness reduction factor, τ_a, has to be applied. The factor τ_a is the ratio of the tangent modulus of elasticity to the modulus of elasticity of steel. The value has been tabulated in the AISC Manual as a function of P_u/A_g. Without τ_a, the value of K is on the conservative side.

However, in lieu of applying the monographs, in a simplified method the factors (coefficients) as listed in Figure 7.6 are used to ascertain the effective length. This Figure 7.6 is used for isolated columns also. When Figure 7.6 is used for the unbraced frame columns, the lowest story (base) columns could be approximated by the condition with $K = 2$ for the hinged base and $K = 1.2$ for the fixed base, and the upper story columns are approximated by the condition with $K = 1.2$. For braced frames, the condition with $K = 0.65$ is a good approximation.

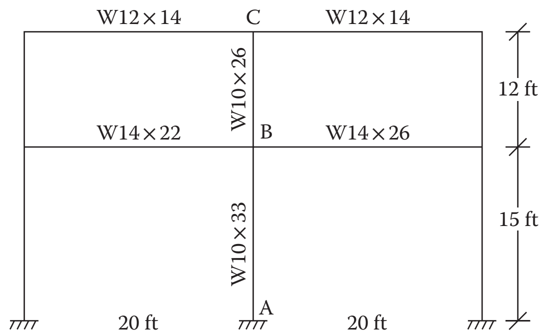


FIGURE 10.7 An unbraced frame.

Example 10.1

A rigid unbraced moment frame is shown in Figure 10.7. Determine the effective length factors with respect to weak axis for members AB and BC.

Solution

1. The section properties and G ratios are arranged in the table below:

	Column				Girder				
Joint	**Section**	***I*** **(in.^{4})**	***L*** **(ft)**	***I/L***	**Section**	***I*** **(in.^{4})**	***L*** **(ft)**	***I/L***	***G***
A	Fixed								1
B	W10 × 33	171	15	11.40[a]	W14 × 22	199	20	9.95	
	W10 × 26	144	12	12.00	W14 × 26	245	20	12.25	
	Σ			23.40				22.20	23.4/22.20 = 1.05
C	W10 × 26	144	12	12.00	W12 × 14	88.6	20	4.43	
					W12 × 14	88.6	20	4.43	
	Σ			12.00				8.86	12.00/8.86 = 1.35

[a] Mixed units (I in in.^{4} and L in ft) can be used since the ratio is being considered.

2. Column AB
   From Figure 10.6, the alignment chart for an unbraced frame (sidesway permitted) connecting a line from $G_A = 1$ to $G_B = 1.05$, $K = 1.3$.
3. Column BC
   From the alignment chart, with $G_A = 1.05$ (point B) and $G_B = 1.35$ (point C), $K = 1.38$.

LIMIT STATES FOR COMPRESSION DESIGN

The limit states of design of a compression member depend on the category to which the compression member belongs, as described in the "Strength of Compression Members or Columns" section. The limit states applicable to different categories of columns are summarized in Table 10.2.

TABLE 10.2
Applicable Limit States for Compressive Strength

	Local Buckling (Local Instability)	
Type of Column	**Nonslender Column, $\lambda \leq \lambda_r$**	**Slender Column, $\lambda > \lambda_r$**
Overall Instability		
1. Doubly symmetric members	Flexural buckling in elastic or inelastic region	Nonslender relations apply by replacing gross area A_g with the effective area A_e, which is determined by using reduced width b_e given by formulas in Section E7 of AISC 360-16. The critical stress F_{cr} remains the same, regardless of element slenderness.
2. Doubly symmetric thin plate built-up members or large unbraced torsional length members	Lowest of the following two limits: 1. Flexural buckling in elastic or inelastic region 2. Torsional buckling	
3. Singly symmetric or nonsymmetric	Lowest of the following three limits: 1. Flexural buckling in elastic or inelastic region 2. Torsional buckling 3. Flexural-torsional buckling members	

AISC 360-16[1] has organized the provisions for compression members as follows:

1. Flexural buckling of nonslender members
2. Torsional buckling and flexural-torsional buckling of nonslender members
3. Single-angle members
4. Built-up members by combining two shapes
5. Slender members

The discussion below follows the same order.

NONSLENDER MEMBERS

Flexural Buckling of Nonslender Members in Elastic and Inelastic Regions

Based on the limit state for flexural buckling, the nominal compressive strength P_n is given by:

$$P_n = F_{cr} A_g \tag{10.3}$$

where:
F_{cr} is the flexural buckling state (stress)
A_g is the gross cross-sectional area

Including the nominal strength in Equation 10.1, the strength requirement of a column can be expressed as:

$$P_u = \phi F_{cr} A_g \tag{10.4}$$

The flexural buckling stress, F_{cr}, is determined as follows.

[1] Part of the 2017 AISC Manual.

Inelastic Buckling

When $Lc \le 4.71\sqrt{E/F_y}$, or $F_y/F_e \le 2.25$,[2] we have inelastic buckling, for which:

$$F_{cr} = (0.658^{F_y/F_e})F_y \tag{10.5}$$

where F_e is the elastic critical buckling or Euler stress calculated according to Equation 10.6:

$$F_e = \frac{\pi^2 E}{(L_C/r)^2} \tag{10.6}$$

where:
- L_C is the effective length of the member = KL
- K is the effective length factor discussed earlier
- L is the laterally unbraced length
- r is the radius of gyration = $\sqrt{I/A}$
- E is the modulus of elasticity of steel = 29,000 ks

Elastic Buckling

When $Lc > 4.71\sqrt{E/F_y}$, or $F_y/F_e > 2.25$, we have elastic buckling, for which:

$$F_{cr} = 0.877F_e \tag{10.7}$$

The value of $4.71\sqrt{E/F_y}$, at the threshold of inelastic and elastic buckling, is given in Table 10.3 for various types of steel.

The available critical stress ϕF_{cr} in Equation 10.4 for both the inelastic and elastic regions is given in Table 10.4 in terms of L_C/r for F_y = 50 ksi, as adapted from the 2017 AISC Manual. The manual contains similar tables for other F_y values.

Torsional and Flexural-Torsional Buckling of Nonslender Members

According to the commentary in Section E of AISC 360-16, in the design with hot-rolled column sections, the torsional buckling of symmetric shapes and the flexural-torsional buckling of non-symmetric shapes are the failure modes that are not usually considered in design. They usually do not govern or the critical load differs very little from the flexural buckling mode. Hence, this section usually applies to double-angle, tee-shaped, and other built-up members.

TABLE 10.3
Numerical Limits of Inelastic-Elastic Buckling

Type of Steel	$4.71\sqrt{E/F_y}$
A36	133.7
A992	113.43
A572	113.43

[2] Both expressions provide the same value.

TABLE 10.4
Available Critical Stress ϕF_{cr} for Compression Members (F_y = 50 ksi and ϕ = 0.90)

L_c/r	ϕF_{cr}, ksi	L_c/r	ϕF_{cr}, ksi	L_c/r	ϕF_{cr}, ksi	L_c/r	ϕF_{cr}, ksi	L_c/r	ϕF_{cr}, ksi
1	45.0	41	39.8	81	27.9	121	15.4	161	8.72
2	45.0	42	39.5	82	27.5	122	15.2	162	8.61
3	45.0	43	39.3	83	27.2	123	14.9	163	8.50
4	44.9	44	39.1	84	26.9	124	14.7	164	8.40
5	44.9	45	38.8	85	26.5	125	14.5	165	8.30
6	44.9	46	38.5	86	26.2	126	14.2	166	8.20
7	44.8	47	38.3	87	25.9	127	14.0	167	8.10
8	44.8	48	38.0	88	25.5	128	13.8	168	8.00
9	44.7	49	37.7	89	25.2	129	13.6	169	7.89
10	44.7	50	37.5	90	24.9	130	13.4	170	7.82
11	44.6	51	37.2	91	24.6	131	13.2	171	7.73
12	44.5	52	36.9	92	24.2	132	13.0	172	7.64
13	44.4	53	36.7	93	23.9	133	12.8	173	7.55
14	44.4	54	36.4	94	23.6	134	12.6	174	7.46
15	44.3	55	36.1	95	23.3	135	12.4	175	7.38
16	44.2	56	35.8	96	22.9	136	12.2	176	7.29
17	44.1	57	35.5	97	22.6	137	12.0	177	7.21
18	43.9	58	35.2	98	22.3	138	11.9	178	7.13
19	43.8	59	34.9	99	22.0	139	11.7	179	7.05
20	43.7	60	34.6	100	21.7	140	11.5	180	6.97
21	43.6	61	34.3	101	21.3	141	11.4	181	6.90
22	43.4	62	34.0	102	21.0	142	11.2	182	6.82
23	43.3	63	33.7	103	20.7	143	11.0	183	6.75
24	43.1	64	33.4	104	20.4	144	10.9	184	6.67
25	43.0	65	33.0	105	20.1	145	10.7	185	6.60
26	42.8	66	32.7	106	19.8	146	10.6	186	6.53
27	42.7	67	32.4	107	19.5	147	10.5	187	6.46
28	42.5	68	32.1	108	19.2	148	10.3	188	6.39
29	42.3	69	31.8	109	18.9	149	10.2	189	6.32
30	42.1	70	31.4	110	18.6	150	10.0	190	6.26
31	41.9	71	31.1	111	18.3	151	9.91	191	6.19
32	41.8	72	30.8	112	18.0	152	9.78	192	6.13
33	41.6	73	30.5	113	17.7	153	9.65	193	6.06
34	41.4	74	30.2	114	17.4	154	9.53	194	6.00
35	41.2	75	29.8	115	17.1	155	9.40	195	5.94
36	40.9	76	29.5	116	16.8	156	9.28	196	5.88
37	40.7	77	29.2	117	16.5	157	9.17	197	5.82
38	40.5	78	28.8	118	16.2	158	9.05	198	5.76
39	40.3	79	28.5	119	16.0	159	8.94	199	5.70
40	40.0	80	28.2	120	15.7	160	8.82	200	5.65

Source: American Institute of Steel Construction, Chicago, IL.

The nominal strength is governed by Equation 10.3. Also, the F_{cr} value is determined according to Equation 10.5 or Equation 10.7 except for two-angle and tee-shaped members. For two-angle and tee-shaped sections, F_{cr} is determined directly by a different type of equation. For simplicity, Equation 10.5 or Equation 10.7 can be used with y axis strength.

However, to determine the Euler stress, F_e, for this category, instead of Equation 10.6, a different set of formulas is used. These include the warping and torsional constants for the section.

SINGLE-ANGLE MEMBERS

For single angles with $b/t \le 0.71\sqrt{E/F_y}$, only the flexural limit state is to be considered. This applies to all currently produced hot-rolled angles. Thus, the flexural-torsional limit state applies only to fabricated angles with $b/t > 0.71\sqrt{E/F_y}$, for which the provisions of the "Torsional and Flexural-Torsional Buckling of Nonslender Members" section apply.

The AISC provides a simplified approach in which load is applied through one connected leg. The slenderness ratio is computed by a specified equation. Then, Equations 10.4 through 10.7 are used to determine the capacity.

BUILT-UP MEMBERS

The members are made by interconnecting elements by bolts or welding. The empirical relations for the effective slenderness ratio for the composite section is used to consider the built-up member acting as a single unit. Depending on the shape of the section, it is designed according to the flexural buckling or flexural-torsional buckling.

SLENDER COMPRESSION MEMBERS

In the 2010 AISC Manual and prior standards, the approach to design slender members having $\lambda > \lambda_r$ was similar to the nonslender members with a reduction factor, Q, included in the expression $4.7\sqrt{E/F_y}$ that distinguishes the inelastic and elastic regions, and Q was also included for F_{cr} in Equations 10.5 and 10.6. The slenderness reduction factor Q has two components: Q_s for the slender unstiffened elements, and Q_a for the slender stiffened elements. The values of Q_s and Q_a were given by a set of formulas for different shapes of columns.

In AISC 360-16, the slender elements design has been simplified by the effective area approach similar to the American Iron and Steel Institute (AISI; 2007). The stiffened and unstiffened elements are treated similarly.

The slender effect is accommodated through an effective area. Thus:

$$P_n = F_{cr} A_e \tag{10.8}$$

where:

F_{cr} is the critical stress as given for nonslender members by Equations 10.5 or 10.7, or as discussed in section "Torsional and Flexural-Torsional Buckling of Nonslender Members"

$A_e = \Sigma$ of the effective areas of cross section elements based on reduced effective widths, defined below.

EFFECTIVE WIDTH OF SLENDER ELEMENTS, b_e

When $\lambda \le \lambda_r \le \sqrt{F_y/F_{cr}}$, the effective width is assumed to be equal to the full section width, b. However, when $\lambda > \lambda_r > \sqrt{F_y/F_{cr}}$, the effective width is reduced by a relation containing local elastic buckling stress, F_{el}, given by a formula and width adjustment factors, c_1, c_2, and c_3, provided in the tables for different shapes. Refer to E 7-1 of AISC 360-16.

In a higher effective length (KL) range, the old Q-factor approach and new effective width approach provide similar results. However, for lower effective lengths, the Q-factor approach is more conservative, providing a lesser strength capacity for the same section.

All W shapes have nonslender flanges for A992 steel. All W shapes listed for the columns in the AISC Manual have nonslender webs (except for W14 × 43). However, many W shapes meant to be used as beams have slender webs in the compression.

This chapter considers only the doubly symmetric nonslender members covered in the "Nonslender Members" section. By proper selection of a section, this condition, that is, $\lambda \le \lambda_r$, could be satisfied.

USE OF THE COMPRESSION TABLES

Section 4 of the 2017 AISC Manual contains tables concerning "available strength in axial compression, in kips" for various shapes and sizes. These tables directly give the capacity as a function of effective length L_C (= KL) with respect to least radius of gyration for various sections. The design of columns is a direct procedure from these tables. An abridged table for F_y = 50 ksi is given in Appendix C.8. The AISC Manual contains tables for F_y = 65 ksi and F_y = 70 ksi as well.

When the values of K and/or L are different in the two directions, both K_xL_x and K_yL_y are computed. If K_xL_x is bigger, it is adjusted as $K_xL_x/(r_x/r_y)$. The higher of the adjusted $K_xL_x/(r_x/r_y)$ and K_yL_y value is entered in the table to pick a section that matches the factored design load, P_u.

When designing for a case when K_xL_x is bigger, the adjustment of $K_xL_x/(r_x/r_y)$ is not straightforward because the values of r_x and r_y are not known. The initial selection could be made based on the K_yL_y values, and then the adjusted value of $K_xL_x/(r_x/r_y)$ is determined based on the initially selected section.

Example 10.2

A 25-ft-long column has one end rigidly fixed to the foundation. The other end is braced (fixed) in the weak axis and free to translate in the strong axis. It is subjected to a dead load of 120 k and a live load of 220 k. Design the column using A992 steel.

Solution

A. Analytical solution
  1. Assuming a column weight of 100 lbs/ft, total weight of column = 2500 lb or 2.5 k
  2. Factored design load
     P_u = 1.2(120 + 2.5) + 1.6(220) = 499 k
  3. For yield limit state:
     $$A_g = \frac{P_u}{\phi F_y} = \frac{499}{0.9(50)} = 11.1 \text{ in.}^2$$
  4. The size will be much larger than step 3 to allow for the buckling mode of failure.
     Select a section W14×61
     $$A = 17.9$$
     $$r_x = 5.98$$
     $$r_y = 2.45$$
     $$\frac{b_f}{2t_f} = 7.75$$
     $$\frac{h}{t_w} = 30.4$$
     $$0.56\sqrt{\frac{E}{F_y}} = 0.56\sqrt{\frac{29{,}000}{50}} = 13.49$$
     $$1.49\sqrt{\frac{E}{F_y}} = 1.49\sqrt{\frac{29{,}000}{50}} = 35.88$$
  5. Since $b_f/2t_f < 0.56\sqrt{E/F_y}$ and $h/t_w < 1.49\sqrt{E/F_y}$, it is a nonslender section.

6. $K_x = 1.2$ from Figure 7.6

   $K_y = 0.65$

   $$\frac{K_x L_x}{r_x} = \frac{1.2\,(25 \times 12)}{5.98} = 60.2$$

   $$\frac{K_y L_x}{r_y} = \frac{0.65\,(25 \times 12)}{2.45} = 79.59 \leftarrow \text{controls}$$

   Since 79.59 < 200 **OK**

7. From Table 10.3, $4.71\sqrt{E/F_y} = 113.43$. Since 79.59 < 113.43, inelastic buckling

8. $$F_e = \frac{\pi^2 E}{(KL/r)^2}$$

   $$= \frac{\pi^2(29{,}000)}{(79.59)^2} = 45.14\,\text{ksi}$$

9. $F_{cr} = (0.658^{50/45.14})50 = 31.45$ ksi

10. $\phi P_n = (0.9)(31.45)(17.9) = 507$ k **OK**

B. Use of Appendix C.8

1. $K_x L_x = 1.2(25) = 30\,\text{ft}$

   $K_y L_y = 0.65(25) = 16.25\,\text{ft}$

2. Select preliminary section based on $K_y L_y$ = 16.25 ft, Section W14 × 61, Capacity = 507 k (interpolated), from Appendix C.8.

3. For W14 × 61, r_x = 5.98 in., r_y = 2.45 in.

   $$\text{Adjusted}\,\frac{K_x L_x}{r_x / r_y} = \frac{1.2(25)}{5.98/2.45} = 12.29$$

   Use the larger value of $K_y L_y$ of 16.25 ft.

4. Section from Appendix C.8, W14 × 61 with capacity = 507 k

Example 10.3

An unbraced column hinged at base, shown in Figure 10.8, is fabricated from Grade 50 steel. Determine the limit state that will control the design of the column.

Solution

1. The doubly symmetric built-up section will be subjected to flexural-torsional buckling.

2. $$\frac{b}{t} = \frac{10}{0.25} = 40$$

   $$1.4\sqrt{\frac{E}{F_y}} = 1.4\sqrt{\frac{29{,}000}{50}} = 33.72$$

   Since 40 > 33.72, it is a slender column; the effective area has to be applied.

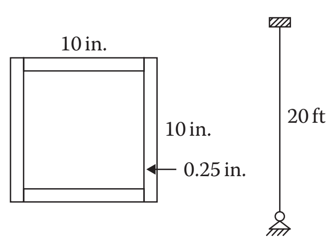


FIGURE 10.8 Built-up column.

3. $I = I_{out} - I_{in}$

$$= \frac{1}{12}(10)(10)^3 - \frac{1}{12}(9.5)(9.5)^3$$

$$= 154.58 \text{ in.}^2$$

$$A = (10)(10) - (9.5)(9.5)$$

$$= 9.75 \text{in.}^2$$

$$r = \sqrt{I/A} = \sqrt{154.58/9.75} = 3.98 \text{ in.}$$

$$K = 2.0$$

$$\frac{KL}{r} = \frac{2.0(20 \times 12)}{3.98} = 120.6$$

4. From Table 10.2, $4.71\sqrt{\frac{E}{F_y}} = 113.43$

$$\frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}}, \text{elastic flexural buckling}$$

5. The lowest of the following two limit states will control:
   a. Elastic flexural buckling with the effective area applied
   b. Torsional buckling with the effective area applied

PROBLEMS

10.1 A W8 × 31 column of A36 steel is 20 ft long. Along the *y* axis, it is hinged at both ends. Along the *x* axis, it is hinged at one end and free to translate at the other end. In which direction is it likely to buckle ($r_x = 3.47$ in., $r_y = 2.02$ in.)?

10.2 An HSS 5 × 2½ × ¼ braced column is supported, as shown in Figure P10.1. Determine the controlling (higher) slenderness ratio.

10.3 A single-story, single-bay frame has the relative *I* values shown in Figure P10.2. Determine the effective length of the columns along the *x* axis. Sway is permitted in *x* direction.

10.4 The frame of Figure P10.3 is braced and bends about the *x* axis. All beams are W18 × 35, and all columns are W10 × 54. Determine the effective length factors for AB and BC.

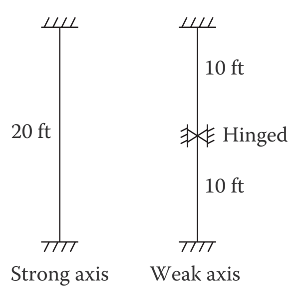


FIGURE P10.1 An HSS column.

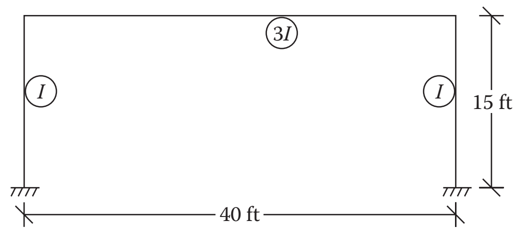


FIGURE P10.2 Frame for Problem 10.3.

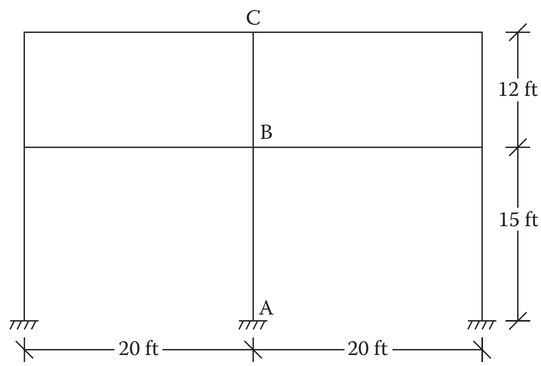


FIGURE P10.3 Frame for Problem 10.4.

10.5 An unbraced frame of Figure P10.4 bends along the x axis. Determine the effective length factors for AB and BC.

10.6 Determine the effective length factors for AB and BC of the frame of Figure P10.4 for bending along the y axis. Will the factors determined in Problem 10.5 or the factors determined in Problem 10.6 control the design?

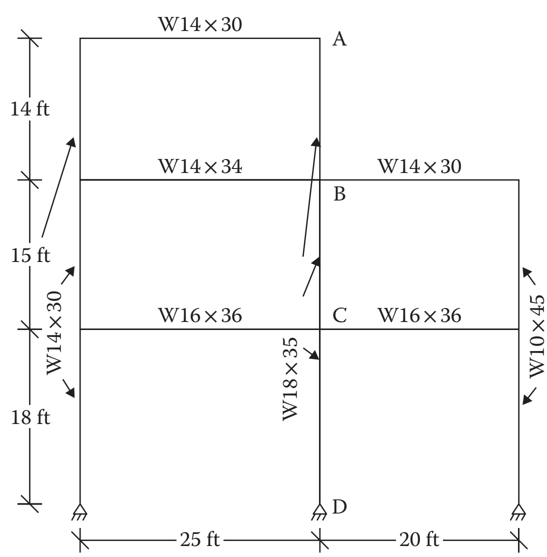


FIGURE P10.4 Frame for Problem 10.5.

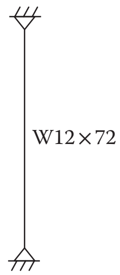


FIGURE P10.5 Column for Problem 10.7.

10.7 Determine the strength of the column of A992 steel in Figure P10.5 when the length is (a) 15 ft and (b) 30 ft.

10.8 Compute the strength of the member of A36 steel shown in Figure P10.6.

10.9 Compute the strength of the member (translation permitted) shown in Figure P10.7. It is made of A500 Grade B steel.

10.10 A W18 × 130 section is used as a column with one end pinned and the other end fixed against rotation but free to translate. The length is 12 ft. Determine the strength of the A992 steel column.

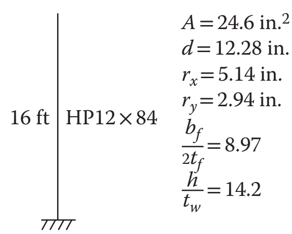


FIGURE P10.6 Column for Problem 10.8.

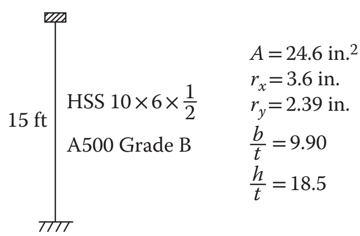


FIGURE P10.7 Column for Problem 10.9.

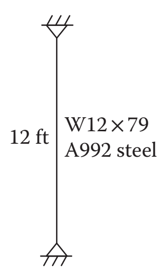


FIGURE P10.8 Column for Problem 10.11.

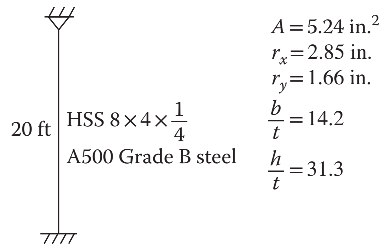


FIGURE P10.9 Column for Problem 10.12.

10.11 Determine the maximum dead and live loads that can be supported by the compression member shown in Figure P10.8. The live load is twice the dead load.

10.12 Determine the maximum dead and live loads supported by the braced column of Figure P10.9. The live load is one-and-a-half times the dead load.

10.13 Determine whether the braced member of A992 steel in Figure P10.10 is adequate to support the loads as indicated.

10.14 Check whether the A36 steel member of Figure P10.11 unbraced at the top is adequate for the indicated loads.

10.15 An HSS 6 × 4 × 5/16 braced section (46 ksi steel) shown in Figure P10.12 is applied by a dead load of 40 k and a live load of 50 k. Check the column adequacy.

10.16 Select an HSS section for the braced column shown in Figure P10.13.

10.17 Design a standard pipe section of A53 Grade B steel for the braced column shown in Figure P10.14.

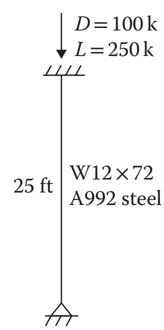


FIGURE P10.10 Column for Problem 10.13.

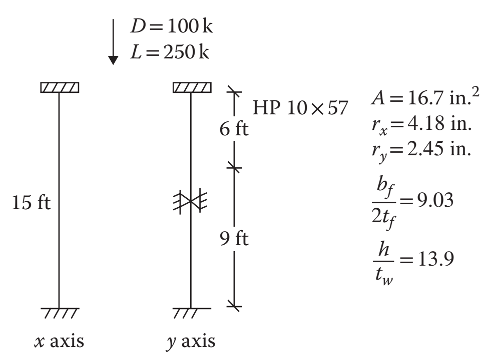


FIGURE P10.11 Column for Problem 10.14.

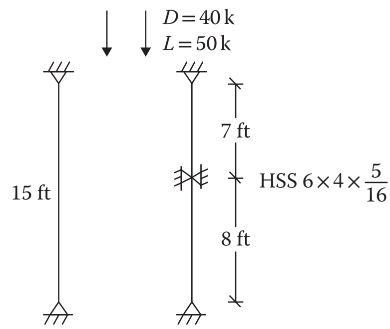


FIGURE P10.12 Column for Problem 10.15.

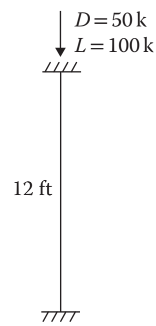


FIGURE P10.13 Column for Problem 10.16.

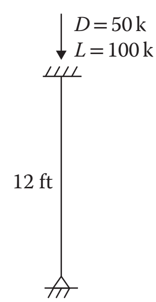


FIGURE P10.14 Column for Problem 10.17.

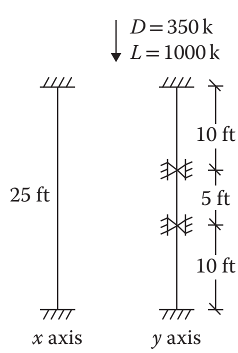


FIGURE P10.15 Column for Problem 10.18.

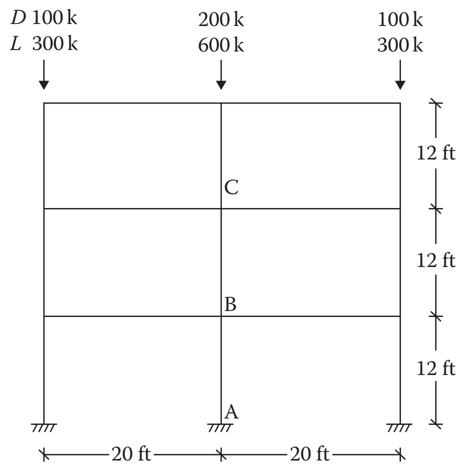


FIGURE P10.16 Frame for Problem 10.19.

10.18 Select a W14 shape of A992 steel for the braced column of 25 ft length shown in Figure P10.15. Both ends are fixed. There are bracings at 10 ft from top and bottom in the weaker direction.

10.19 Design a W14 section column AB of the frame shown in Figure P10.16. It is unbraced along the x axis and braced in the weak direction. The loads on the column are dead load = 200 k and live load = 600 k. First determine the effective length factor using Figure 7.6. After selecting the preliminary section for column AB, use the alignment chart (nomograph) with the same size for column BC as for column AB to revise the selection. Use W16 × 100 for the beam sections meeting at B.

10.20 Design the column AB in Problem 10.19 for the frame braced in both directions.

10.21 A WT12 × 34 column of 18 ft length is pinned at both ends. Show what limiting states will determine the strength of the column. Use A992 steel (A = 10 in.^{2}, r_y = 1.87 in., $b_f/2t_f$ = 7.66, d/t_w = 28.7).

10.22 The A572 braced steel column in Figure P10.17 is fixed at one end and hinged at the other end. Indicate the limit states that will control the strength of the column.

10.23 A double-angle braced section with a separation of ⅜ in. is subjected to the loads shown in Figure P10.18. Determine the limit states that will govern the design of the column. Use Grade 50 steel (A = 3.86 in.^{2}, r_y = 1.78 in., b/t = 16).

10.24 A cruciform column is fabricated from Grade 50 steel, as shown in Figure P10.19. Determine the limit states that will control the design. Use the properties of a single angle to determine the values of the composite section.

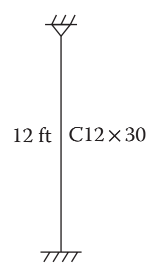


FIGURE P10.17 Column for Problem 10.22.

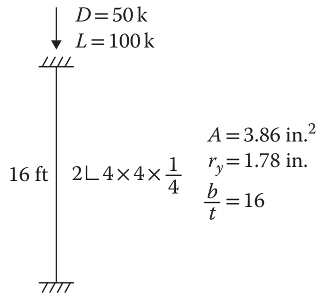


FIGURE P10.18 Column for Problem 10.23.

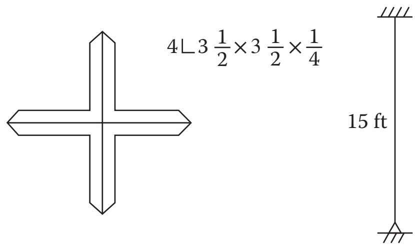


FIGURE P10.19 Cruciform column for Problem 10.24.

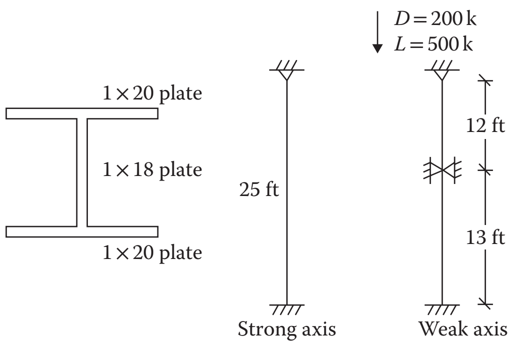


FIGURE P10.20 Built-up column for Problem 10.25.

10.25 For the braced column section and the loading shown in Figure P10.20, determine the limit states for which the column should be designed. Use A992 steel.

11 Flexural Steel Members

BASIS OF DESIGN

Beams are the structural members that support transverse loads on them and are subjected to flexure and shear. An I shape is a common cross section for a steel beam where the material in the flanges at the top and bottom is most effective in resisting bending moment and the web provides for most of the shear resistance. As discussed in the "Design of Beams" section of Chapter 7—in the context of wood beams—the design process involves selection of a beam section on the basis of the maximum bending moment to be resisted. The selection is then checked for shear capacity. In addition, the serviceability requirement imposes the deflection criteria for which the selected section should be checked.

The basis of design for bending or flexure is:

$$M_u \leq \phi M_n \tag{11.1}$$

where:
- M_u is the factored design (imposed) moment
- ϕ is the resistance factor for bending = 0.9
- M_n is the nominal moment strength of steel

NOMINAL STRENGTH OF STEEL IN FLEXURE

Steel is a ductile material. As discussed in the "Elastic and Plastic Designs" section in Chapter 1, steel can acquire the plastic moment capacity, M_p, wherein the stress distribution above and below the neutral axis is represented by the rectangular blocks corresponding to the yield strength of steel; that is, $M_p = F_y Z$, with Z being the plastic section modulus of the section.

However, certain other factors undermine the plastic moment capacity. One such factor relates to the unsupported (unbraced) length of the beam, and another relates to the slender dimensions of the beam section. The design capacity is determined considering both factors. The effect of the unsupported length on strength is discussed first in the "Lateral Unsupported Length" section. The beam's slender dimensions affect the strength similar to the local instability of compression members. This is described in the "Noncompact and Slender Beam Sections for Flexure" section.

LATERAL UNSUPPORTED LENGTH

As a beam bends, it develops compression stress in one part and tensile stress in the other part of its cross section. The compression region acts analogous to a column. If the entire member is slender, it will buckle outward similar to a column. However, in this case, the compression portion is restrained by the tensile portion. As a result, a twist occurs in the section. This form of instability, as shown in Figure 11.1, is called *lateral torsional buckling.*

Lateral torsional buckling can be prevented in two ways:

1. Lateral bracings can be applied to the compression flange at close intervals, which prevents the lateral translation (buckling) of the beam, as shown in Figure 11.2. This support can be provided by a floor member securely attached to the beam.
2. Cross bracings or a diaphragm can be provided between adjacent beams, as shown in Figure 11.3, which directly prevent the twisting of the sections.

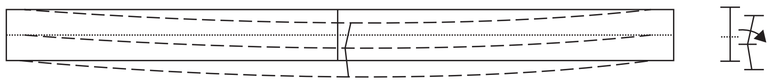


FIGURE 11.1 Buckling and twisting effect in a beam.

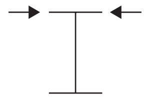


FIGURE 11.2 Lateral bracing of compression flange.

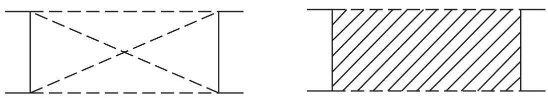


FIGURE 11.3 Cross bracing or diaphragm.

Depending on the lateral support condition on the compression side, the strength of the limit state of a beam is due to either the plastic yielding of the section or the lateral torsional buckling of the section. The latter condition has two further divisions: inelastic lateral torsional buckling and elastic lateral torsional buckling. These three zones of the limit states are shown in Figure 11.4 and are described here.

In Figure 11.4, the first threshold value for the unsupported or the unbraced length is L_p, given by the following relation:

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}} \tag{11.2}$$

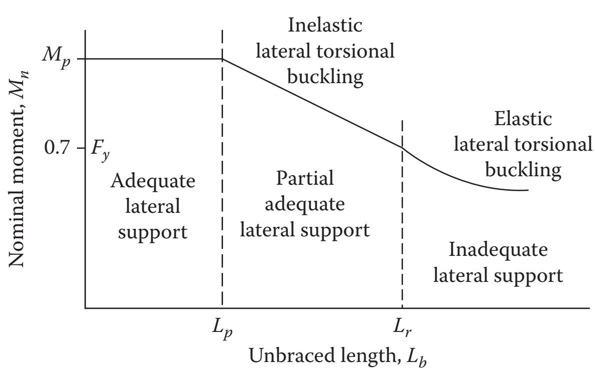


FIGURE 11.4 Nominal moment strength as a function of unbraced length.

where:

L_p is the first threshold limit for the unsupported length (in inches)

r_y is the radius of gyration about the y axis, listed in Appendixes C.1 through C.7

The second threshold value is L_r, which is conservatively given by the following relation:

$$L_r = \pi r_{ts} \sqrt{\frac{E}{0.7F_y}} \tag{11.3}$$ [1]

where:

L_r is the second threshold of the unsupported length (in inches)

r_{ts} is the special radius of gyration for L_r, listed in Appendixes C.1 through C.7.

FULLY PLASTIC ZONE WITH ADEQUATE LATERAL SUPPORT

When the lateral support is continuous or closely spaced so that the unbraced (unsupported) length of a beam, L_b, is less than or equal to L_p from Equation 11.2, the beam can be loaded to reach the plastic moment capacity throughout the section. The limit state in this case is the yield strength, given as follows:

$$M_u = \phi F_y Z, \quad \text{with } \phi = 0.9 \tag{11.4}$$

where Z is the plastic section modulus. Generally it is Z_x along the strong axis in which position a beam is placed.

The lateral torsional buckling does not apply in this zone.

INELASTIC LATERAL TORSIONAL BUCKLING ZONE

When the lateral unsupported (unbraced) length, L_b, is more than L_p but less than or equal to L_r, the section does not have sufficient capacity to develop the plastic moment capacity, that is, the full yield stress, F_y, in the entire section. Before all fibers are stressed to F_y, buckling occurs. This leads to inelastic lateral torsional buckling.

At $L_b = L_p$, the moment capacity is the plastic capacity, M_p. As the length, L_b, increases beyond the L_p value, the moment capacity becomes less. At the L_r value of the unbraced length, the section buckles elastically, attaining the yield stress only at the top or the bottom fiber. Accounting for the residual stress in the section during manufacturing, the effective yield stress is $F_y - F_r$, where F_r is residual stress. The residual stress is taken as 30% of the yield stress. Thus, at $L_b = L_r$, the moment capacity is $(F_y - F_r)S$ or $0.7F_yS$.

When the unbraced length, L_b, is between the L_p and L_r values, the moment capacity is linearly interpolated between the magnitudes of M_p and $0.7F_yS_x$, as follows:

$$M_u = \phi\left[M_p - (M_p - 0.7F_yS)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right]C_b \tag{11.5}$$

where $M_p = F_yZ$.

[1] This approximation can be very conservative. AISC 360-16 uses a very complex relation.

MODIFICATION FACTOR C_b

The factor C_b is introduced in Equation 11.5 to account for a situation when the moment within the unbraced length is not uniform (constant). A higher moment between the supports increases the resistance to torsional buckling, thus resulting in an increased value of C_b. This factor has the following values:

	C_b
1. No transverse loading between brace points	1
2. Uniformly loaded simple supported beam	1.14
3. Centrally loaded simple supported beam	1.32
4. Cantilever beam	1
5. Equal end moments of opposite signs	1
6. Equal end moments of the same sign (reverse curvature)	2.27
7. One end moment is 0	1.67

A value of 1 is conservatively taken.

ELASTIC LATERAL TORSIONAL BUCKLING ZONE

When the unbraced length, L_b, exceeds the threshold value of L_r, the beam buckles before the effective yield stress, $0.7F_y$, is reached anywhere in the cross section. This is elastic buckling. The moment capacity is made up of the torsional resistance and the warping resistance of the section:

$$M_u < 0.7\phi F_y S \tag{11.6}$$

At $L_b = L_r$, the capacity M_u is exactly $0.7\phi F_y S$.

NONCOMPACT AND SLENDER BEAM SECTIONS FOR FLEXURE

The aforementioned discussion on beam strength did not account for the shape of a beam; that is, it assumes that the beam section is robust enough to not experience any localized problem. However, if the flange and the web of a section are relatively thin, they might buckle, as shown in Figure 11.5, even before lateral torsional buckling occurs. This mode of failure is called *flange local buckling* or *web local buckling*.

Sections are divided into three classes based on the width to thickness ratios of the flange and the web. (The threshold values of classification are given in Table 11.1.):

When $\lambda \le \lambda_p$, the shape is compact.
When $\lambda > \lambda_p$ but $\lambda \le \lambda_r$, the shape is noncompact.
When $\lambda > \lambda_r$, the shape is slender.

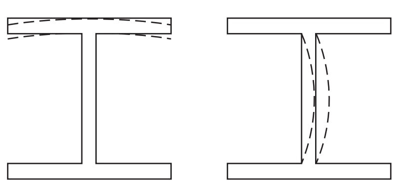


FIGURE 11.5 Local buckling of section.

TABLE 11.1
Shape Classification Limits

Element	λ	λ_p	λ_r
Flange	$b_f/2t_f$ [a]	$0.38\sqrt{\dfrac{E}{F_y}}$	$1.0\sqrt{\dfrac{E}{F_y}}$
Web	h/t_w	$3.76\sqrt{\dfrac{E}{F_y}}$	$5.70\sqrt{\dfrac{E}{F_y}}$

[a] For channel shape, this is b_f/t_f.

Both the flange and the web are evaluated by the aforementioned criteria. Based on the aforementioned limits, the flange of a section might fall into one category, whereas the web of the same section might fall into the other category.

The values of λ_p and λ_r for various types of steel are listed in Table 11.2.

In addition to the unsupported length, the bending moment capacity of a beam also depends on the compactness or width–thickness ratio, as shown in Figure 11.6.

This localized buckling effect could be the flange local buckling or the web local buckling, depending on which one falls into the noncompact or slender category. All W, S, M, HP, C, and MC shapes listed in the 2017 AISC Manual have compact webs at $F_y \leq 70$ ksi. Thus, only the flange criteria need to be applied. Fortunately, most of the shapes satisfy the flange compactness requirements as well, as discussed below.

Without accounting for the lateral unsupported length effect, that is, assuming a fully laterally supported beam, the strength limits described in the following sections are applicable based on the compactness (width-thickness) criteria.

TABLE 11.2
Magnitude of the Classification Limits

Element	Limits	A36 $F_y = 36$ ksi	A572 A992 $F_y = 50$ ksi
Flange	$0.38\sqrt{\dfrac{E}{F_y}}$	10.79	9.15
	$1.0\sqrt{\dfrac{E}{F_y}}$	28.38	24.08
Web	$3.76\sqrt{\dfrac{E}{F_y}}$	106.72	90.55
	$5.70\sqrt{\dfrac{E}{F_y}}$	161.78	137.27

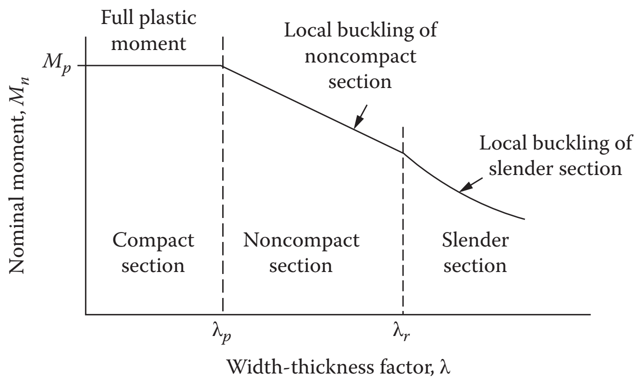


FIGURE 11.6 Nominal moment strength as a function of compactness.

COMPACT FULL PLASTIC LIMIT

As long as $\lambda \le \lambda_p$, the beam moment capacity is equal to M_p and the limit state of the moment is given by the yield strength expressed by Equation 11.4.

NONCOMPACT FLANGE LOCAL BUCKLING[2]

For sections having a value of λ between the λ_p and λ_r limits shown in Table 11.1, the moment capacity is interpolated between M_p and $0.7F_yS_x$ as a gradient of the λ values, on the same line like that in Equation 11.5. The moment capacity is expressed as follows:

$$M_u = \phi\left[M_p - (M_p - 0.7F_yS)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)\right] \tag{11.7}$$

SLENDER FLANGE LOCAL BUCKLING

For sections with $\lambda > \lambda_r$, the moment-resisting capacity is inversely proportional to the square of slenderness ratio, as follows:

$$M_u = \frac{0.9\phi E k_c S}{\lambda^2} \tag{11.8}$$

where $k_c = \dfrac{4}{\sqrt{h/t_w}}$, and $k_c \ge 0.35$ and ≤ 0.76.

SUMMARY OF BEAM RELATIONS

Considering both the lateral support and the compactness criteria, the flexural strength (the moment capacity) is taken to be the lowest value obtained according to the limit states of the lateral torsional buckling and the compression flange local buckling. The applicable limits and corresponding equations are shown in Table 11.3.

The 2017 AISC Manual also covers cases of noncompact and slender web buckling, although all current W, S, M, HP, C, and MC shapes have compact webs up to F_y of 70 ksi. The equations are similar to Equations 11.4, 11.7, and 11.8 of the flange buckling criteria with application of a web plastification factor, R_{pc}, for a noncompact web, and a bending strength reduction factor, R_{pg}, for a slender web.

[2] All webs are compact for $F_y \le 70$ ksi.

TABLE 11.3
Applicable Limiting States of Beam Design

Zone	Unbraced Length, L_b	Flange Local Buckling[a]: Compact $\lambda \leq \lambda_p$	Flange Local Buckling[a]: Noncompact (inelastic) $\lambda > \lambda_p$ and $\leq \lambda_r$	Flange Local Buckling[a]: Slender (Elastic) $\lambda > \lambda_r$
Fully plastic zone	Adequate lateral support $L_b \leq L_p$	Limit state: Yield strength: Equation 11.4.[b] Lateral torsional buckling does not apply	Limit state: Inelastic flange local buckling: Equation 11.7. Lateral torsional buckling does not apply	Limit state: Elastic flange local buckling: Equation 11.8. Lateral torsional buckling does not apply
Lateral torsional buckling	Partial inadequate support $L_b > L_p$ and $L_b \leq L_r$	Limit state: Inelastic lateral torsional buckling: Equation 11.5.	Limit states: Lower of the following two: 1. Inelastic lateral torsional buckling: Equation 11.5. 2. Noncompact flange local buckling: Equation 11.7.	Limit states: Lower of the following two: 1. Inelastic lateral torsional buckling: Equation 11.5. 2. Slender flange local buckling: Equation 11.8.
Lateral torsional buckling	Inadequate support, $L_b > L_r$	Limit state: Elastic lateral torsional buckling: Equation 11.6.	Limit states: Lower of the following two: 1. Elastic lateral torsional buckling: Equation 11.6. 2. Noncompact flange local buckling: Equation 11.7.	Limit states: Lower of the following two: 1. Elastic lateral torsional buckling: Equation 11.6. 2. Slender flange local buckling: Equation 11.8.

[a] Web local buckling is not included since all I-shaped and C-shaped sections have compact webs up to $F_y = 70$ ksi. In the case of a web local buckling member, formulas are similar to the flange local buckling. Equations 11.7 and 11.8 are modified for (1) the web plastification factor (R_{pc}) and (2) the bending strength reduction factor (R_{pg}).

[b] Most beams fall into the adequate laterally supported compact category. This chapter considers only this state of design.

TABLE 11.4
List of Noncompact Flange Sections

W21 × 48
W14 × 99
W14 × 90
W12 × 65
W10 × 12
W8 × 31
W8 × 10
W6 × 15
W6 × 9
W6 × 8.5
M4 × 6

All W, S, M, C, and MC shapes have compact flanges for F_y of 36 and 50 ksi, except for the sections listed in Table 11.4. Thus, a beam is compact if the sections listed in Table 11.4 are avoided.

Most beam sections fall in the full plastic zone where Equation 11.4 can be applied. In this chapter, it is assumed that the condition of adequate lateral support is satisfied, if necessary, by providing bracings at intervals less than the distance L_p and also that the condition of flange and web compactness is fulfilled.

DESIGN AIDS

The 2017 AISC Manual provides the design tables. A beam can be selected by entering the table either with the required section modulus or with the design bending moment. These tables are applicable to adequately supported compact beams for which the yield limit state is applicable. For simply supported beams with uniform load over the entire span, tables are provided in the AISC Manual that show the allowable uniform loads corresponding to various spans. These tables are applicable for adequately supported beams but also extend to noncompact members.

Also included in the AISC Manual are more comprehensive charts that plot the total moment capacity against the unbraced length, starting at spans less than L_p and continuing to spans greater than L_r, and covering compact as well as noncompact members. These charts are applicable to the condition $C_b = 1$. The charts can be directly used to select a beam section.

A typical chart is given in Appendix C.9. Enter the chart with the given unbraced length on the bottom scale, and proceed upward to meet the horizontal line corresponding to the design moment on the left-hand scale. Any beam listed above and to the right of the intersection point satisfies the design requirement. The section listed at the first solid line after the intersection represents the most economical section.

Example 11.1

A floor system is supported by steel beams, as shown in Figure 11.7. The live load is 100 psf. Design the beam. Determine the maximum unbraced length of the beam to satisfy the requirement of adequate lateral support; $F_y = 50$ ksi.

Solution

A. Analytical
1. Tributary area of beam per foot = 10 × 1 = 10 ft²/ft
2. Weight of slab per foot $= 1 \times 10 \times \frac{6}{12} \times 150 = 750$ lb/ft
3. Estimated weight of beam per foot = 30 lb/ft
4. Dead load per foot = 780 lb/ft
5. Live load per foot = 100 × 10 = 1000 lb/ft
6. Design load per foot

$$w_u = 1.2(780) + 1.6(1000) = 2536 \text{ lb/ft or } 2.54 \text{ k/ft}$$

7. Design moment

$$M_u = \frac{w_u L^2}{8} = \frac{2.54\,(25)^2}{8} = 198.44 \text{ ft-k}$$

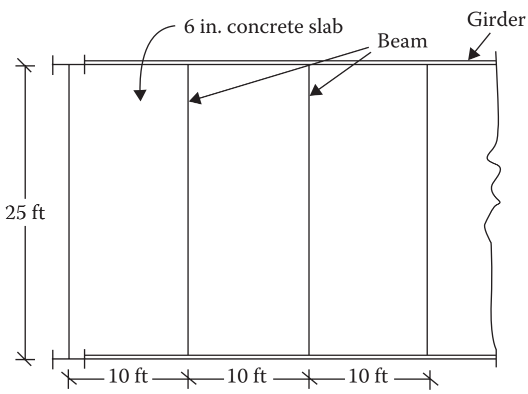


FIGURE 11.7 A floor system supported by beams.

8. From Equation 11.4:

$$Z_x = \frac{198.44(12)}{(0.9)(50)} = 52.91\,\text{in.}^3$$

9. Select W14 × 34
   Z_x = 54.6 in.^{3}
   r_x = 5.83 in.
   r_y = 1.53 in.

$$\frac{b_f}{2t_f} = 7.41$$

$$\frac{h}{t_w} = 43.1$$

10. Since $\frac{b_f}{2t_f} = 7.41 < 9.15$ from Table 11.2, it is a compact flange.
    Since $\frac{h}{t_w} = 43.1 < 90.55$ from Table 11.2, it is compact web.
    Equation 11.4 applies; selection is **OK**.
11. Unbraced length from Equation 11.2

$$L_p = 1.76\,r_y\sqrt{\frac{E}{F_y}}$$

$$= 1.76\,(1.53)\sqrt{\frac{29{,}000}{50}}$$

$$= 64.85\text{in. or } 5.4 \text{ ft}$$

B. Use of chart
   From Appendix C.9, for an unbraced length of 45.4 ft and a design moment of 198 ft- k, the suitable sections are W16 × 31 and W14 × 34.

Example 11.2

The compression flange of the beam in Example 11.1 is braced at 10-ft intervals. Design the beam when the full plastic limit state applies (adequate lateral support exists).

Solution

1. At upper limit, $L_b = L_p$

$$\text{or } 10 \times 12 = 1.76\,r_y\sqrt{\frac{29{,}000}{50}}$$

   or r_y = 2.83 in. minimum
2. Select W14 × 109
   Z_x = 192 in.^{3}
   r_y = 3.73 in.

$$\frac{b_f}{2t_f} = 8.49$$

$$\frac{h}{t_w} = 21.7$$

3. $M_u = \phi F_y Z_x$ = (0.9)(50)(192) = 8640 in.· k or 720 ft· k > 198.44 **OK**

4. Since $\frac{b_f}{2t_f} = 8.49 < 9.15$, compact

   Since $\frac{h}{t_w} = 21.7 < 90.55$, compact

Example 11.3

The compression flange of the beam in Example 11.1 is braced at 10-ft intervals. Design the beam when the inelastic lateral torsional limit state applies.

Solution

A. Analytical
   1. At upper limit, $L_b = L_r$

      $$\text{or } 10 \times 12 = \pi r_{ts} \sqrt{\frac{29{,}000}{(0.7)50}}$$

      or $r_{ts} = 1.33$ in. minimum
   2. Minimum Z_x required for the plastic limit state:

      $$Z_x = \frac{M_u}{\phi F_y} = \frac{198.44 \times 12}{(0.9)(50)} = 52.2\,\text{in.}^3$$

   3. Select W14 × 43
      $Z_x = 69.6$ in.^{3}
      $S_x = 62.6$ in.^{3}
      $r_y = 1.89$ in.
      $r_{ts} = 2.18$ in. > minimum r_{ts} of 1.33

      $$\frac{b_f}{2t_f} = 7.54$$

      $$\frac{h}{t_w} = 37.4$$

   4. $$L_p = 1.76(1.89)\sqrt{\frac{29{,}000}{50}} = 80.11\text{ in.} \quad \text{or} \quad 6.68\text{ ft}$$

      $$L_r = \pi(2.18)\sqrt{\frac{29{,}000}{50}} = 197.04\text{ in.} \quad \text{or} \quad 16.42\text{ ft}$$

   5. $M_p = F_y Z_x = 50(69.6) = 3480$ in.· k
      $0.7F_y S_x = 0.7(500)(62.6) = 2190$ in. · k

   6. $$M_u = \phi\left[M_p - (M_p - 0.7F_y S_x)\frac{(L_b - L_p)}{(L_r - L_b)}\right]C_b$$

      $$= 0.9\left[3480 - (3480 - 2190)\frac{(10 - 6.68)}{(16.42 - 6.68)}\right](1)$$

      $= 2736.3$ in.-k or 228 ft-k > 198.44 **OK**

7. Since $\frac{b_f}{2t_f} = 7.54 < 7.54 < 9.15$ compact

   Since $\frac{h}{t_w} = 37.4 < 90.55$ compact

B. Use of the chart
   From Appendix C.9, for an unbraced length of 10 ft and a design moment of 198 ft·k, W14 × 43 is a suitable section.

SHEAR STRENGTH OF STEEL

The section of beam selected for the moment capacity is checked for its shear strength capacity. The design relationship for shear strength is:

$$V_u = 0.6\, F_y\, \phi\, A_w\, C_{v1} \tag{11.9}$$

where:
V_u is the factored shear force applied
ϕ is the resistance factor for shear
A_w is the web area $= dt_w$
C_{v1} is the shear strength coefficient

1. For all I-shaped members with $h/t_w \leq 2.24\sqrt{E/F_y}$, $\phi = 1$ and $C_{v1} = 1$ in Equation 11.9.
   All W, S, and HP sections except W44 × 230, W40 × 149, W36 × 135, W33 × 118, W30 × 90, W24 × 55, W16 × 26, and W12 × 14 meet the above criterion of item 1 for $F_y = 50$ ksi.
2. For all other I-shaped and channel sections, when $h/t_w \leq 1.1\sqrt{K_v\, E/F_y}$, $\phi = 0.9$ and $C_{v1} = 1$.
   All M shapes except M12.5 × 12.4, M12.5 × 11.6, M12 × 11.8, M12 × 10.8, M12 × 10, M10 × 8, and M10 × 7.5 meet the criterion of item 2 for $F_y = 50$ ksi.
3. For all other I-shaped and channel sections, when $h/t_w > 1.1\sqrt{K_v\, E/F_y}$, $\phi = 0.9$ and

$$C_{v1} = \frac{1.10\sqrt{K_v\, E/F_y}}{h/t_w} \tag{11.10}$$

   where plate shear buckling coefficient K_v is as follows:
   1. For webs without transverse stiffeners, $K_v = 5.34$.
   2. For webs with transverse stiffeners:

$$K_v = 5 + \frac{5}{(a/h)^2} \tag{11.11}$$

   a being the clear distance between transverse stiffeners.

For members that utilize the post buckling strength (tension field action)—single angles and tees, rectangular HSS, and other singly and doubly symmetric members—the shear strength relation is similar to Equation 11.9, with the shear buckling coefficient, C_{v1}, replaced by another form of the coefficient, C_{v2}.

As stated, however, most of the sections of $F_y < 50$ ksi steel have compact shapes that satisfy Equation 11.9.

Example 11.4

Check the beam of Example 11.1 for shear strength.

Solution

1. $V_u = \frac{w_u L}{2}$

   $= \frac{2.54\,(25)}{2} = 31.75$ k
2. For W14 × 34, $h/t_w = 43.1$
   $A_w = dt_w = 14(0.285) = 3.99$
   $2.24\sqrt{E/F_y} = 53.95$
3. Since $h/t_w \leq 2.24\sqrt{E/F_y}$
   $V_u = 0.6(1)(50)(3.99)(1) = 119.7$ k > 31.75 k **OK**

BEAM DEFLECTION LIMITATIONS

Deflection is a service requirement. A limit on deflection is imposed so that the serviceability of a floor or a roof is not impaired due to the cracking of plastic or of concrete slab, or the distortion of partitions or any other kind of undesirable occurrence. There are no standard limits because such values depend on the function of a structure. For cracking of plaster, usually a live load deflection limit of span/360 and a total load limit of span/240 are observed. It is imperative to note that, being a serviceability consideration, the deflections are always computed with service (unfactored) loads and moments.

For a common case of a uniformly distributed load on a simple beam, the deflection is given by the following formula:

$$\delta = \frac{5}{384}\frac{wL^4}{EI} \tag{11.12}$$ [3]

Depending on the loading condition, however, the theoretical derivation of the expression for deflection might be quite involved. For various load conditions on a simply supported beam, and cantilever and fixed beams, the expressions for maximum deflection are given in Appendix A.3.3. For commonly encountered load conditions on simply supported and cantilever beams, when the expression of the bending moment is substituted in the deflection expression, a generalized form of deflection can be expressed as follows:

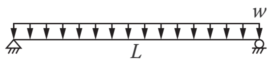
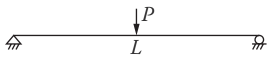
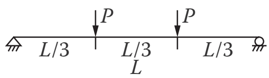
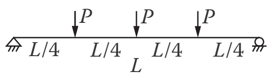
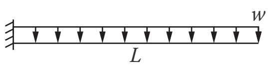
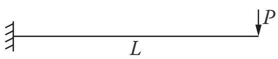
$$\delta = \frac{ML^2}{CEI} \tag{11.13}$$

where:
w is the combination of the service loads
M is the moment due to the service loads

The values of constant *C* are indicated in Table 11.5 for different load cases.

[3] In foot-pound-second units, the numerator is multiplied by $(12)^3$ to convert δ in inch unit when *w* is in kips per ft, *L* is in feet, *E* is in kips per square inch, and *I* is in inch^4. Equation 11.3 is also multiplied by $(12)^3$ when *M* is in ft-k.

TABLE 11.5
Deflection Loading Constants

Diagram of Load Condition	Constant C for Equation 11.13
	9.6
	12
	9.39
	10.13
	4
	3

In simplified form, the designed factored moment, M_u, can be converted to the service moment by dividing by a factor of 1.5 (i.e., $M = M_u/1.5$). The service live load moment, M_L, is approximately two-thirds of the total moment M (i.e., $M_L = 2M_u/4.5$). The factor C from Table 11.5 can be used in Equation 11.13 to compute the expected deflection, which should be checked against the permissible deflection, Δ, to satisfy the deflection limitation.

Example 11.5

Check the beam in Example 11.1 for deflection limitation. The maximum permissible live load deflection is $L/360$. Use (A) the conventional method and (B) the simplified procedure.

Solution

A. Conventional method
  1. Service live load = 1000 lb/ft or 1 k/ft
  2. For W14 × 34, $I = 340$ in.^{4}
  3. From Equation 11.12:

$$\delta = \frac{5}{384}\,\frac{(1.0)(25)^4(12)^3}{(29{,}000)(340)} = 0.89 \text{ in.}$$

  4. $$\Delta = \frac{L \times 12}{360} = \frac{25 \times 12}{360} = 0.83 \text{ in.}$$

Since 0.89 in. > 0.83 in., **NG** (border case).

B. Simplified procedure

1. $M_L = \frac{2M_u}{4.5} = \frac{2(198.44)}{4.5} = 88.2\,\text{ft-k}$

2. From Equation 11.13:

$$\delta = \frac{ML^2 \times (12)^3}{CEI}$$

$$= \frac{(88.20)(25)^2(12)^3}{(9.6)(290{,}000)(340)}$$

$$= 0.99 \text{ in.}$$

3. $\Delta < \delta$ **NG** (border case)

PROBLEMS

11.1 Design a beam of A36 steel for the loads in Figure P11.1. Determine the maximum unbraced length of beam to satisfy the requirement of adequate lateral support.

11.2 Design a simply supported 20 ft span beam of A992 steel having the following concentrated loads at the midspan.
Service dead load = 10 k
Service live load = 25 k
Determine the maximum unbraced length of beam to satisfy the requirement of adequate lateral support.

11.3 Design a beam of A992 steel for the loading shown in Figure P11.2. The compression flange bracing is provided at each concentrated load. The selected section should be such that the full lateral support condition is satisfied. Determine the maximum unbraced length of beam to satisfy the requirement of adequate lateral support.

11.4 Design a cantilever beam of A992 steel for the loading shown in Figure P11.3. The compression flange bracing is provided at each concentrated load. The selected section should be such that the full lateral support condition is satisfied. Determine the maximum unbraced length of beam to satisfy the requirement of adequate lateral support.

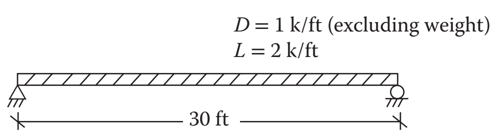


FIGURE P11.1 Beam for Problem 11.1.

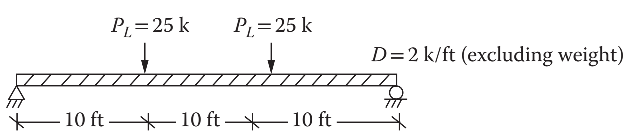


FIGURE P11.2 Beam for Problem 11.3.

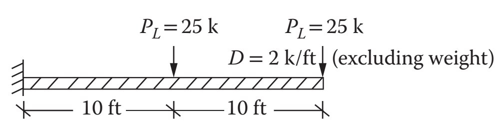


FIGURE P11.3 Beam for Problem 11.4.

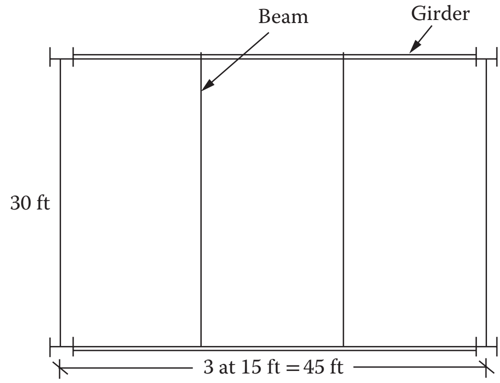


FIGURE P11.4 Floor system for Problem 11.5.

11.5 A floor system supporting a 6 in. concrete slab is shown in Figure P11.4. The live load is 100 psf. Design a beam of section W14 × ?? of A36 steel. Recommend the compression flange bracing so that the beam has the full lateral support.

11.6 Design a W18 × ? section of A992 steel girder for Problem 11.5. Recommend the compression flange bracing so that the beam has the full lateral support.

11.7 The beam in Problem 11.6 is braced at 15-ft intervals. Design a W14 × ? section of A992 steel for the full plastic limit state (for the adequate lateral support case).

11.8 The beam in Problem 11.6 is braced at 15-ft intervals. Design a W14 × ? section of A992 steel for the inelastic lateral torsional buckling limit state.

11.9 From the sections listed below, which sections of A992 steel are compact, noncompact, and slender?

1. W21 × 93	2. W18 × 97
3. W14 × 99	4. W12 × 65
5. W10 × 68	6. W8 × 31
7. W6 × 15	

11.10 A grade 50 W21 × 62 section is used for a simple span of 20 ft. The only dead load is the weight of the beam. The beam is fully laterally braced. What is the largest service concentrated load that can be placed at the center of the beam? What is the maximum unbraced length?

11.11 A W18 × 97 beam of A992 steel is selected to span 20 ft. If the compression flange is supported at the end and at the midpoint, which formula do you recommend to solve for the moment capacity? Determine the maximum unbraced length of beam to satisfy the requirement of adequate lateral support.

11.12 A W18 × 97 beam of A992 steel is selected to span 20 ft. It is supported at the ends only. Which formula do you recommend to solve for the moment capacity?

11.13 A W21 × 48 section is used to span 20 ft and is supported at the ends only. Which formula do you recommend to solve for the moment capacity?

11.14 A W21 × 48 section is used to span 20 ft and is supported at the ends and at the center. Which formula do you recommend to solve for the moment capacity?

11.15 Check the selected beam section in Problem 11.1 for shear strength capacity.

11.16 Check the selected beam section in Problem 11.2 for shear strength capacity.

11.17 Check the selected beam section in Problem 11.3 for shear strength capacity.

11.18 What is the shear strength of a W16 × 26 A992 beam with transverse stiffeners 2 ft apart?

11.19 What is the shear strength of a W12 × 14 A992 beam without transverse stiffeners?

11.20 Compute the total load and the live load deflections for the beam in Problem 11.1 by the (1) conventional method and (2) simplified procedure. The permissible deflection for the total load is *L*/240 and for the live load is *L*/360.

11.21 Compute the total load and the live load deflections for the beam in Problem 11.2 by the (1) conventional method and (2) simplified procedure. The permissible deflection for the total load is *L*/240 and for the live load is *L*/360.

11.22 Compute the total load and the live load deflections for the beam in Problem 11.3 by the (1) conventional method and (2) simplified procedure. The permissible deflection for the total load is *L*/240 and for the live load is *L*/360. Redesign the beam if necessary.

11.23 Check the total load and the live load deflections for the beam in Problem 11.5 by the (1) conventional method and (2) simplified procedure. The permissible deflection for the total load is *L*/240 and for the live load is *L*/360. Redesign the beam if necessary.

11.24 Check the total load and the live load deflections for the beam in Problem 11.6 by the (1) conventional method and (2) simplified procedure. The permissible deflection for the total load is *L*/240 and for the live load is *L*/360. Redesign the beam if necessary.

12 Combined Forces on Steel Members

DESIGN APPROACH TO COMBINED FORCES

The design of tensile, compression, and bending members was separately treated in Chapters 9 through 11, respectively. In actual structures, the axial and the bending forces, specifically the compression due to gravity loads and the bending due to lateral loads, generally act together. An interaction formula is the simplest way for such cases. In such a formula, the sum of the ratios of factored design load to limiting axial strength and factored design moment to limiting moment strength should not exceed 1.

Test results show that assigning an equal weight to the axial force ratio and the moment ratio in the interaction equation provides sections that are too large. Accordingly, the American Institute of Steel Construction (AISC) suggested the following modifications to the interaction equations in which the moment ratio is reduced when the axial force is high, and the axial force ratio is reduced when the bending moment is high:

1. For $\frac{P_u}{\phi P_n} \geq 0.2$

$$\frac{P_u}{\phi P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \leq 1 \tag{12.1}$$

2. For $\frac{P_u}{\phi P_n} < 0.2$

$$\frac{1}{2}\frac{P_u}{\phi P_n} + \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \leq 1 \tag{12.2}$$

where:

ϕ is the resistance factor for axial force (0.9 or 0.75 for a tensile member and 0.9 for a compression member)
ϕ_b is the resistance factor for bending (0.9)
P_u is the factored design load, determined by structural analysis (required force)
P_n is the nominal axial capacity, determined according to Chapters 9 and 10
M_{ux} and M_{uy} are factored design moments about x and y axes, as determined by structural analysis, including second-order effects (required moments)
M_{nx} and M_{ny} are nominal bending capacities along the x and y axes if only bending moments are present, which are determined by different methods as mentioned in Chapter 11

COMBINATION OF TENSILE AND FLEXURE FORCES

Some members of a structural system are subject to axial tension as well as bending. An example is the bottom chord of a trussed bridge. The hanger type of structures acted on by transverse loads is another example.

The analysis in which a member size is known and the adequacy of the member to handle a certain magnitude of force is to be checked is a direct procedure with Equation 12.1 or Equation 12.2. However, the design of a member that involves the selection of a suitable size for a known magnitude of load is a trial-and-error procedure by the interaction equation, Equation 12.1 or Equation 12.2.

The 2017 AISC Manual presents a simplified procedure to make an initial selection of a member size. This procedure necessitates, however, the application of factors that are available from specific tables in the manual. Since the AISC Manual is not a precondition for this chapter, that procedure is not used here.

Example 12.1

Design a member to support the load shown in Figure 12.1. It has one line of four holes for a ⅞ in. bolt in the web for the connection. The beam has adequate lateral support. Use grade 50 steel.

Solution

A. Analysis of structure
   1. Assume a beam weight of 50 lb/ft
   2. $W_u = 1.2(2.05) = 2.46$ k/ft
   3. $M_u = \dfrac{W_u L^2}{8} = \dfrac{(2.46)(12)^2}{8} = 44.28$ ft-k or 531.4 in.-k
   4. $P_u = 1.6(100) = 160$ k
B. Design
   1. Try a W10 × 26 section.[1]
   2. $A_g = 7.61$ in.^{2}
   3. $I_x = 144$ in.^{4}
   4. $Z_x = 31.3$ in.^{3}
   5. $t_w = 0.26$ in.
   6. $b_f/2t_f = 6.56$
   7. $h/t_w = 34.0$
C. Axial (tensile) strength
   1. $U = 0.7$ from the "Shear Lag" section of Chapter 9 for W shapes; $h = \frac{7}{8} + \frac{1}{16} = 0.938$, $A_h = 0.938(0.26) = 0.24$ in.^{2}
   2. $A_n = A_g - A_h = 7.61 - 0.24 = 7.37$ in.^{2}
   3. $A_e = 0.7(7.37) = 5.16$ in.^{2}

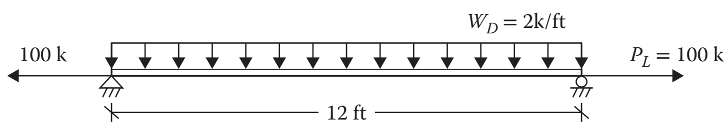


FIGURE 12.1 A tensile and flexure forces member.

[1] As a guess, the minimum area for axial load alone should be Unit weight $= \frac{17}{20} = 0.85$ psf < assumed 1 psf **OK**. The selected section is twice this size because a moment, M_u, is also acting.

4. Tensile strength

$\phi F_y A_g = 0.9(50)(7.62) = 342.9$ k

$\phi F_u A_e = 0.75(65)(5.16) = 251.55$ k ← Controls

D. Moment strength

1. $0.38\sqrt{\frac{E}{F_y}} = 9.15 > 6.56$; it is a compact flange

$3.76\sqrt{\frac{E}{R_y}} = 90.55 > 34.0$; it is a compact web

2. Adequate lateral support (given)
3. Moment strength

$\phi_b F_y Z = 0.9(50)(31.3)) = 1408.5$ in.-k

E. Interaction equation

1. Since $\frac{P_u}{\phi P_n} = \frac{160}{251.06} = 0.64 > 2$, use Equation 12.1.

2. $\frac{P_u}{\phi P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi M_{nx}}\right)$

$(0.64) + \frac{8}{9}\left(\frac{531.4}{1408.5}\right) = 0.97 < 1$ **OK**

COMBINATION OF COMPRESSION AND FLEXURE FORCES: THE BEAM-COLUMN MEMBERS

Instead of axial tension, when an axial compression acts together with a bending moment, which is a more frequent case, a secondary effect sets in. The member bends due to the moment. This causes the axial compression force to act off center, resulting in an additional moment equal to axial force times lateral displacement. This additional moment causes further deflection, which in turn produces more moment, and so on, until an equilibrium is reached. This additional moment, known as the P–Δ effect, or the *second-order moment*, is not as much of a problem with axial tension, which tends to reduce the deflection.

There are two kinds of second-order moments, as discussed in the following sections.

Members without Sidesway

Consider the isolated beam-column member AB of a frame with no sway in Figure 12.2. Due to load w_u on the member itself, a moment M_{u1} results, assuming that the top joint B does not deflect with respect to the bottom joint A (i.e., there is no sway). This causes the member to bend, as shown in Figure 12.3. The total moment consists of the primary (first-order) moment, M_{u1}, and the second-order moment, $P_u\delta$. Thus:

$$M_{nosway} = M_{u1} + P_u\delta \tag{12.3}$$

where M_{u1} is the first-order moment in a member, assuming no lateral movement (no translation).

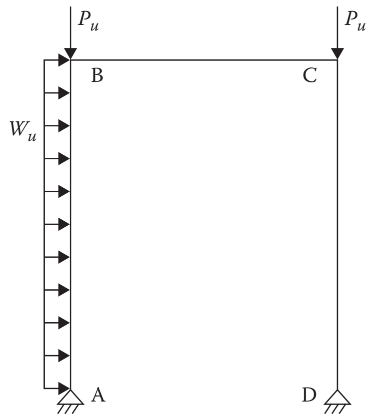


FIGURE 12.2 Second-order effect on a frame.

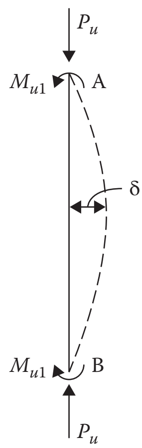


FIGURE 12.3 Second-order moment within a member.

Members with Sidesway

Now consider that the frame is subject to a sidesway where the ends of the column can move with respect to each other, as shown in Figure 12.4. M_{u2} is the primary (first-order) moment caused by the lateral translation only of the frame. Since the end B is moved by Δ with respect to A, the second-order moment is $P_u\Delta$.

Therefore, the total moment is:

$$M_{sway} = M_{u2} + P_u\Delta \tag{12.4}$$

where M_{u2} is the first-order moment caused by the lateral translation.

It should be understood that the moment M_{nosway} (Equation 12.3) is the property of the member and the moment M_{sway} (Equation 12.4) is a characteristic of a frame. When a frame is braced against sidesway, M_{sway} does not exist. For an unbraced frame, the total moment is the sum of M_{nosway} and M_{sway}. Thus:

$$M_u = (M_{u1} + P_u\delta) + (M_{u2} + P_u\Delta) \tag{12.5}$$

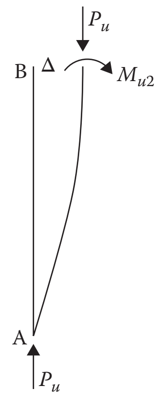


FIGURE 12.4 Second-order moment due to sidesway.

The second-order moments are evaluated directly or through the factors that magnify the primary moments. In the second case:

$$\begin{aligned} M_u &= B_1 M_{u1} + B_2 M_{u2} \\ &\quad (no\,sway) \quad (sway) \end{aligned} \tag{12.6}$$

where B_1 and B_2 are magnification factors when first-order moment analysis is used.

For braced frames, only the factor B_1 is applied. For unbraced frames, both factors B_1 and B_2 are applied.

Magnification Factor B_1

This factor is determined assuming the braced (no sway) condition. It can be demonstrated that for a sine curve the magnified moment directly depends on the ratio of the applied axial load to the elastic (Euler) load of the column. The factor is expressed as follows:

$$B_1 = \frac{C_m}{1-(P_u / P_{e1})} \geq 1 \tag{12.7}$$

where:

C_m is the moment modification factor, which is discussed below
P_u is the applied factored axial compression load
P_{e1} is the Euler buckling strength, which is given as follows:

$$P_{e1} = \frac{\pi^2 EA}{(L_C/r)^2} \tag{12.8}$$

The slenderness ratio L_C/r or KL/r is along the axis on which the bending occurs. Equation 12.7 suggests that B_1 should be greater than or equal to 1; it is a magnification factor.

Moment Modification Factor, C_m

The modification factor C_m is an expression that accounts for the nonuniform distribution of the bending moment within a member. Without this factor, B_1 may be overmagnified. When a column is bent in a single curvature with equal end moments, deflection occurs, as shown in Figure 12.5a.

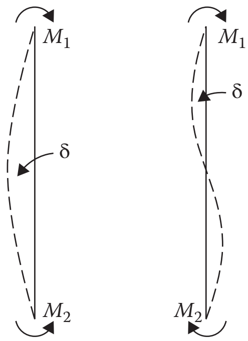


FIGURE 12.5 Deflection of a column under different end moment conditions.

In this case, $C_m = 1$. When the end moments bend a member in a reverse curvature, as shown in Figure 12.5b, the maximum deflection that occurs at some distance away from the center is smaller than the first case; using $C_m = 1$ overdoes the magnification. The purpose of the modifier C_m is to reduce the magnified moment when the variation of the moment within a member requires that B_1 should be reduced. The modification factor depends on the rotational restraint placed at the member's ends. There are two types of loadings for C_m:

1. When there is no transverse loading between the two ends of a member, the modification factor is given by:

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) \le 1 \tag{12.9}$$

   where:
   M_1 is the smaller end moment
   M_2 is the larger of the end moments

   The ratio (M_1/M_2) is negative when the end moments have opposite directions, causing the member to bend in a single curvature. (This is opposite to the sign convention for concrete columns in the "Short Columns with Combined Loads" section in Chapter 16.) The ratio is taken to be positive when the end moments have the same direction, causing the member to bend in a reverse curvature.
2. When there is a transverse loading between the two ends of a member:

   a. $C_m = 0.85$ for a member with the restrained (fixed) ends.
   b. $C_m = 1.0$ for a member with unrestrained ends.

Example 12.2

The service loads[2] on a W12 × 72 braced frame member of A572 steel are shown in Figure 12.6. The bending is about the strong axis. Determine the magnification factor B_1. Assume the pinned-end condition.

Solution

A. Design loads
   1. Weight = 72(14) = 1008 lb or 1 k
   2. P_u = 1.2(101) + 1.6(200) = 441 k

[2] Axial load on a frame represents the loads from all the floors above up to the frame level in question.

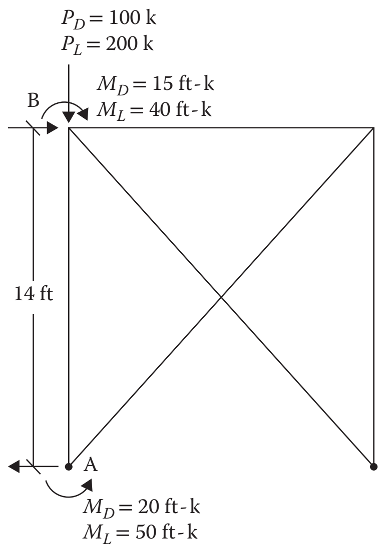


FIGURE 12.6 Braced frame for Example 12.2.

3. $(M_{u1})_B = 1.2(15) + 1.6(40) = 82$ ft-k
4. $(M_{u1})_A = 1.2(20) + 1.6(50) = 104$ ft-k

B. Modification factor
1. $\dfrac{M_1}{M_2} = \dfrac{-82}{104} = -0.788$
2. $C_m = 0.6 - 0.4(-0.788) = 0.915$

C. Euler buckling strength
1. For a braced frame, $K = 1$
2. For W12 × 72, $A = 21.1$ in.^{2}
   $r_x = 5.31$ in., bending in the x direction

3. $\dfrac{KL}{r_x} = \dfrac{(1)(14 \times 12)}{5.31} = 31.64$

4. $P_{e1} = \dfrac{\pi^2 EA}{(KL/r)^2}$

$$= \frac{\pi^2(29{,}000)(21.2)}{(31.64)^2} = 6055 \text{ k}$$

5. $B_1 = \dfrac{C_m}{1-(P_u/P_{e1})}$

$$= \frac{0.915}{1-\left(\dfrac{440}{6{,}055}\right)} = 0.99 < 1$$

Use $B_1 = 1$

K Values for Braced Frames

Figure 7.6 and the monographs in Figures 10.5 and 10.6 are used to determine the effective length factor, *K*. According to the commentary in Appendix 7 of AISC 360-16, braced frames are commonly idealized as vertical, cantilevered, pin-connected truss systems. The effective length factor of the components of a braced frame is normally taken as 1.

BRACED FRAME DESIGN

For braced frames, only the magnification factor B_1 is applied. As stated earlier, the use of an interaction equation, Equation 12.1 or Equation 12.2, is direct in analysis when the member size is known. However, it is a trial-and-error procedure for designing a member.

Instead of making a blind guess, design aids are available to make a feasible selection prior to the application of the interaction equation. The procedure presented in the 2017 AISC Manual for initial selection needs an intensive input of data from special tables included in the manual. In a previous version of the AISC Manual, a different approach that was less data intensive was suggested. This less data-intensive approach is described here.

The interaction equations can be expressed in terms of an equivalent axial load. With respect to Equation 12.1, this modification is demonstrated as follows:

$$\frac{P_u}{\phi P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 1$$

Multiplying both sides by ϕP_n:

$$P_u + \frac{8}{9}\frac{\phi P_n}{\phi_b}\left(\frac{M_{ux}}{M_{nx}} + \frac{M_{uy}}{M_{ny}}\right) = \phi P_n \tag{12.10}$$

By treating ϕP_n as P_{eff}, Equation 12.10 can be expressed as:

$$P_{eff} = P_u + mM_{ux} + mUM_{uy} \tag{12.11}$$

where:

P_u is the factored axial load
M_{ux} is the magnified factored moment about the x axis
M_{uy} is the magnified factored moment about the y axis

The values of the coefficient m, reproduced from a previous AISC Manual, are given in Table 12.1.

The AISC Manual uses an iterative application of Equation 12.11 to determine the equivalent axial compressive load, P_{eff}, for which a member could be picked up as an axially loaded column only. However, this also requires the use of an additional table to select the value of U.

This chapter suggests an application of Equation 12.11 just to make an educated guess for a preliminary section. The initially selected section will then be checked by the interaction equations.

The procedure is as follows:

1. For the known value of effective length, L_c or KL, pick up the value of m from Table 12.1 for a selected column shape category. For example, for a column of W12 shape and the computed KL of 16, the magnitude of m is 2 from Table 12.1.
2. Assume $U = 3$ in all cases.
3. From Equation 12.11, solve for P_{eff}.

TABLE 12.1
Values of Factor *m*

F_y	36 ksi							50 ksi						
KL (ft)	10	12	14	16	18	20	22 and over	10	12	14	16	18	20	22 and over
First Approximation														
All shapes	2.4	2.3	2.2	2.2	2.1	2.0	1.9	2.4	2.3	2.2	2.0	1.9	1.8	1.7
Subsequent Approximations														
W, S 4	3.6	2.6	1.9	1.6	—	—	—	2.7	1.9	1.6	1.6	—	—	—
W, S 5	3.9	3.2	2.4	1.9	1.5	1.4	—	3.3	2.4	1.8	1.6	1.4	1.4	—
W, S 6	3.2	2.7	2.3	2.0	1.9	1.6	1.5	3.0	2.5	2.2	1.9	1.8	1.5	1.5
W 8	3.0	2.9	2.8	2.6	2.3	2.0	2.0	3.0	2.8	2.5	2.2	1.9	1.6	1.6
W 10	2.6	2.5	2.5	2.4	2.3	2.1	2.0	2.5	2.5	2.4	2.3	2.1	1.9	1.7
W 12	2.1	2.1	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	1.9	1.9	1.8	1.7
W 14	1.8	1.7	1.7	1.7	1.7	1.7	1.7	1.8	1.7	1.7	1.7	1.7	1.7	1.7

Note: Values of m are for $C_m = 0.85$. When C_m is any value other than 0.85, multiply the tabular value of m by $C_m/0.85$.

4. Pick up a section having cross-sectional area larger than the following:

$$A_g = \frac{P_{eff}}{\phi F_y}$$

5. Confirm the selection using the appropriate interaction equation, Equation 12.1 or Equation 12.2.

Example 12.3

For a braced frame, the axial load and the end moments obtained from structural analysis are shown in Figure 12.7. Design a W14 member of A992 steel. Use $K = 1$ for the braced frame.

Solution

A. Critical load combinations
   1. $1.2D + 1.6L$
      a. Assume a member weight of 100 lb/ft; total weight = 100(14) = 1400 lb or 1.4 k
      b. $P_u = 1.2(81.4) + 1.6(200) = 417.7$ k
      c. $(M_{u1})_x$ at A = 1.2(15) + 1.6(45) = 90 ft-k
      d. $(M_{u1})_x$ at B = 1.2(20) + 1.6(50) = 104 ft-k
   2. $1.2D + L + W$
      a. $P_u = 1.2(81.4) + 200 = 297.7$ k
      b. $(M_{u1})_x$ at A = 1.2(15) + 45 = 63 ft-k
      c. $(M_{u1})_x$ at B = 1.2(20) + 50 = 74 ft-k
      d. $(M_{u1})_y = 192$ ft-k

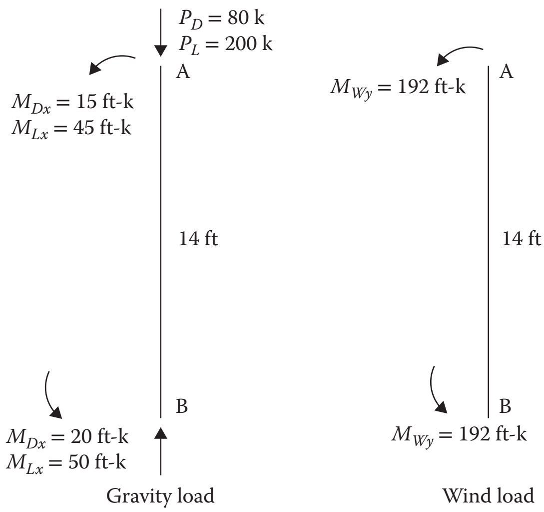


FIGURE 12.7 Column member of a braced frame.

B. Trial selection
 1. For load combination (1)
    From Table 12.1 for KL = 14 ft, m = 1.7
    $P_{eff} = 417.7 + 1.7(104) = 594.5$ k
 2. For load combination (2), let U = 3.
    $P_{eff} = 297.7 + 1.7(74) + 1.7(3)(192) = 1402.7$ k ← controls
 3. $A_g = \dfrac{P_{eff}}{\phi F_y} = \dfrac{1402.7}{(0.9)(50)} = 31.17\,\text{in.}^2$
 4. Select W14 × 109 A = 32.0 in.^{2}
    $Z_x = 192$ in.^{3}
    $Z_y = 92.7$ in.^{3}
    $r_x = 6.22$ in.
    $r_y = 3.73$ in.
    $b_f/2t_f = 8.49$
    $h/t_w = 21.7$

 Checking of the trial selection for load combination (b)

C. Along the strong axis
 1. Moment strength
    $\phi M_{nx} = \phi F_y Z_x = 0.9(50)(192) = 8640$ in.-k or 720 ft-k
 2. Modification factor for magnification factor B_1: reverse curvature

$$\frac{(M_{nt})_x \text{ at } A}{(M_{nt})_x \text{ at } B} = \frac{63}{74} = 0.85$$

$$C_{mx} = 0.6 - 0.4(0.85) = 0.26$$

3. Magnification factor, B_1

   $K = 1$

   $$\frac{KL}{r_x} = \frac{(1)(14\times 12)}{6.22} = 27.0$$

   $$(P_{e1})_x = \frac{\pi^2 EA}{(KL/r_x)^2} = \frac{\pi^2(29,000)(32)}{(27.0)^2} = 12,551$$

4. $$(B_1)_x = \frac{C_m}{1-(P_u/P_{e1})}$$

   $$= \frac{0.26}{1-(297.7/12551)} = 0.27 < 1; \text{use } 1$$

5. $(M_u)_x = B_1(M_{u1})_x$

   $= 1(74) = 74$ ft-k

D. Along the minor axis

1. Moment strength

   $\phi M_{ny} = \phi F_y Z_y = 0.9(50)(92.7) = 4171.5$ in.-k or 347.63 ft-k

2. Modification factor for magnification factor B_1: reverse curvature

   $$\frac{(M_{u1})_x \text{ at A}}{(M_{u1})_x \text{ at B}} = \frac{192}{192} = 1$$

   $C_{mx} = 0.6 - 0.4(1) = 0.2$

3. Magnification factor, B_1

   $K = 1$

   $$\frac{KL}{r_y} = \frac{(1)(14\times 12)}{3.73} = 45.0$$

   $$(P_{e1})_y = \frac{\pi^2 EA}{(KL/r_y)^2} = \frac{\pi^2(29,000)(32)}{(45.0)^2} = 4,518.4$$

4. $$(B_1)_y = \frac{C_m}{1-(P_u/P_{e1})}$$

   $$= \frac{0.2}{1-(297.7/4518.4)} = 0.21 < 1; \text{ use } 1$$

5. $(M_u)_y = (B_1)_y (M_{nt})_y$

   $= 1(192) = 192$ ft-k

E. Compression strength

1. $$\frac{KL}{r_x} = \frac{(1)(14\times 12)}{6.22} = 27.0$$

2. $$\frac{KL}{r_y} = \frac{(1)(14\times 12)}{3.73} = 45.0 \leftarrow \text{controls}$$

3. Since $4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29,000}{50}} = 113.43 > 45$; inelastic buckling

4. $$F_e = \frac{\pi^2 E}{(KL/r_y)^2} = \frac{\pi^2(29,000)}{(45.0)^2} = 141.2$$

5. $F_{cr} = (0.658^{50/141.2})50 = 43.11$

6. $$\phi P_n = 0.9 F_{cr} A_g$$
$$= 0.9(43.11)(32) = 1241.6\,\text{k}$$

F. Interaction equation

$$\frac{P_u}{\phi P_n} = \frac{297.7}{1241.6} = 0.24 > 0.2$$

$$\frac{P_u}{\phi P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right)$$

$$0.24 + \frac{8}{9}\left(\frac{74}{720} + \frac{192}{347.63}\right)$$

$= 0.82 < 1$ **OK**

MAGNIFICATION FACTOR FOR SWAY, B_2

The term B_2 is used to magnify column moments under the sidesway condition. For sidesway to occur in a column on a floor, it is necessary that all the columns on that floor should sway simultaneously. Hence, the total load acting on all columns on a floor appears in the expression for B_2. The 2017 AISC Manual presents the following two relations for B_2:

$$B_2 = \frac{1}{1 - \dfrac{\Sigma P_u}{\Sigma H}\left(\dfrac{\Delta H}{L}\right)} \tag{12.12}$$

or

$$B_2 = \frac{1}{1 - \dfrac{\Sigma P_u}{\Sigma P_{e2}}} \tag{12.13}$$

where:

ΔH is the lateral deflection of the floor (story) in question
L is the story height
ΣH is the sum of horizontal forces on the floor in question
ΣP_u is the total design axial force on all the columns on the floor in question
ΣP_{e2} is the summation of the elastic (Euler) capacity of all columns on the floor in question, given by:

$$\Sigma P_{e2} = \Sigma \frac{\pi^2 EA}{(KL/r)^2} \tag{12.14}$$

The term P_{e2} is similar to the term P_{e1}, except that the factor K used in Equation 12.14 is with respect to the plane of bending under the unbraced condition, whereas K in P_{e1} is in the plane of bending for the braced condition.

A designer can use Equation 12.12 or Equation 12.13; the choice is a matter of convenience. In Equation 12.12, the initial size of the members is not necessary since A and r are not required as a part of P_{e2}, unlike in Equation 12.13. Further, a limit on $\Delta H/L$, known as the *drift index*, can be set by the designer to control the sway. This is limited to 0.004 with factored loads.

K VALUES FOR UNBRACED FRAMES

According to the commentary in Appendix 7 of AISC 360-16, the lateral moment resisting frames generally have an effective length factor, K, greater than 1. However, when the sidesway amplification factor, B_2, is less than or equal to 1, the effective length factor $K = 1$ can be used.

As stated in Chapter 10, for the unbraced frame, the lower-story columns can be designed using $K = 2$ for pin-supported bases and 1.2 for fixed bases. For upper-story columns, $K = 1.2$.

Example 12.4

An unbraced frame of A992 steel at the base floor level is shown in Figure 12.8. The loads are factored. Determine the magnification factor for sway for the column bending in the y axis.

Solution

A. Exterior columns
   1. Factored weight of column = 1.2(0.096 × 15) = 1.7 k
   2. $P_u = 240 + 1.7 = 241.7$ k
   3. $K = 2$
   4. For W12 × 96, $A = 28.2$ in.², $r_y = 3.09$ in.
   5. $\dfrac{KL}{r_y} = \dfrac{2(15\times 12)}{3.09} = 116.50$
   6. $P_{e2} = \dfrac{\pi^2 EA}{(KL/r_y)^2} = \dfrac{\pi^2(29{,}000)(28.2)}{(116.5)^2} = 594.1\text{k}$

B. Interior columns
   1. Factored weight of column = 1.2(0.12 × 15) = 2.2 k
   2. $P_u = 360 + 2.2 = 362.2$ k
   3. $K = 2$
   4. For W12 × 120, $A = 35.2$ in.², $r_y = 3.13$ in.
   5. $\dfrac{KL}{r_y} = \dfrac{2(15\times 12)}{3.13} = 115.0$
   6. $P_{e2} = \dfrac{\pi^2 EA}{(KL/r_y)^2} = \dfrac{\pi^2(290{,}000)(35.2)}{(115)^2} = 76\ 1\text{k}$

C. For the entire story

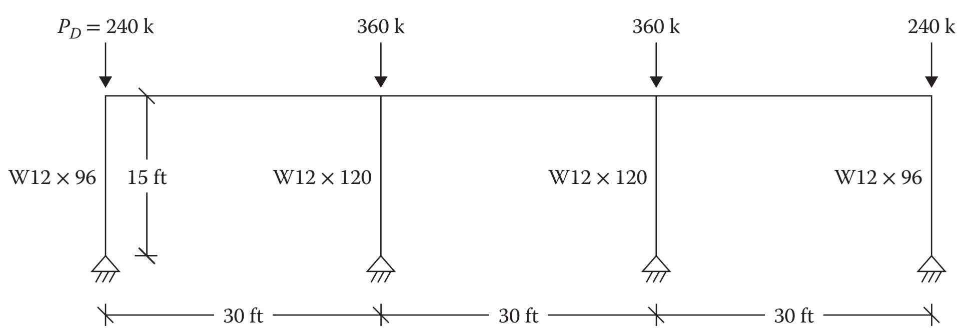


FIGURE 12.8 Unbraced frame for Example 12.4.

1. $\Sigma P_u = 2(241.7) + 2(362.2) = 1208$ k
2. $\Sigma P_{e2} = 2(594.1) + 2(761) = 2710$ k
3. From Equation 12.13:

$$B_2 = \frac{1}{1 - \left(\frac{\sum P_u}{\sum P_{e2}}\right)}$$

$$= \frac{1}{1 - \left(\frac{1208}{2710}\right)} = 1.80$$

Example 12.5

In Example 12.4, the total factored horizontal force on the floor is 200 k and the allowable drift index is 0.002. Determine the magnification factor for sway.

Solution

From Equation 12.12:

$$B_2 = \frac{1}{1 - \frac{\Sigma P_u}{\Sigma H}\left(\frac{\Delta H}{L}\right)}$$

$$= \frac{1}{1 - \left(\frac{1208}{200}\right)(0.002)} = 1.01$$

UNBRACED FRAME DESIGN

The interaction Equations 12.1 and 12.2 are used for unbraced frame design also. M_{ux} and M_{uy} in the equations are computed by Equation 12.6 magnified for both B_1 and B_2.

The trial size can be determined from Equation 12.11 following the procedure stated in the "Braced Frame Design" section. When an unbraced frame is subjected to symmetrical vertical (gravity) loads along with a lateral load, as shown in Figure 12.9, the moment M_{u1} in member AB is computed for the gravity loads. This moment is amplified by the factor B_1 to account for the P–δ effect. The moment M_{u2} is computed due to the horizontal load H. It is then magnified by the factor B_2 for the P–Δ effect.

When an unbraced frame supports an asymmetric loading, as shown in Figure 12.10, the eccentric loading causes it to deflect sideways. First, the frame is considered to be braced by a fictitious support called an *artificial joint restraint (AJR)*. The moment M_{u1} and the deflection δ are computed, and they are amplified by the factor B_1.

To compute M_{u2}, a force equal to AJR but opposite in direction is then applied. This moment is magnified by the factor B_2 for the P–Δ effect.

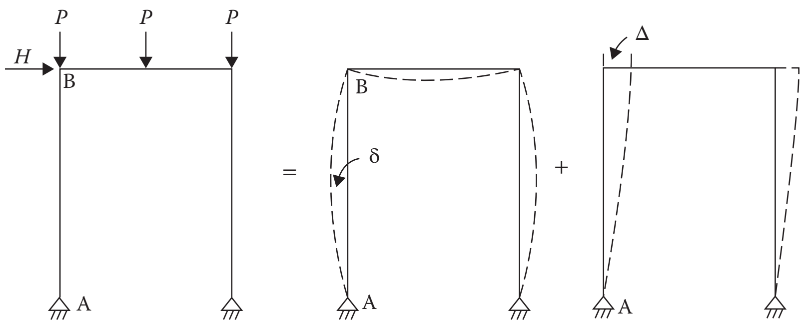


FIGURE 12.9 Symmetrical vertical loads and a lateral load on an unbraced frame.

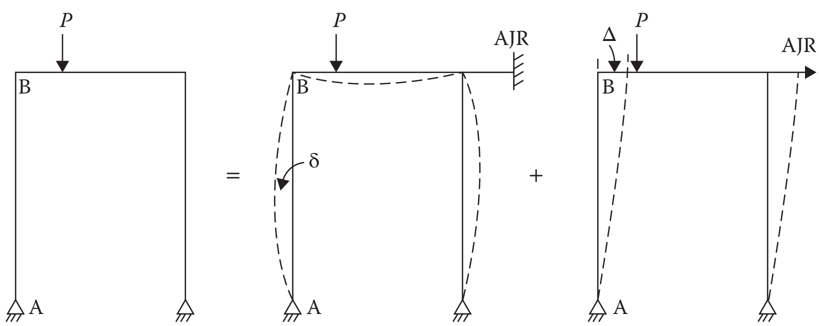


FIGURE 12.10 Asymmetric loading on an unbraced frame: AJR, artificial joint restraint.

When both asymmetric gravity loads and lateral loads are present, the aforementioned two cases are combined; that is, AJR force is added to the lateral loads to compute M_{u2} for the P–Δ effect.

Alternatively, two structural analyses are performed. The first analysis is performed as a braced frame; the resulting moment is M_{u1}. The second analysis is done as an unbraced frame. The results of the first analysis are subtracted from those of the second analysis to obtain M_{u2}.

Example 12.6

An unbraced frame of A992 steel is subjected to dead load, live load, and wind load. The structural analysis provides the axial forces and the moments on the column along the x axis, as shown in Figure 12.11. Design for a maximum drift of 0.5 in.

Solution

A. Critical load combinations
   1. $1.2D + 1.6L$
      a. Assume a member weight of 100 lb/ft, total weight = 100(15) = 1500 lb or 1.5 k
      b. $P_u = 1.2(81.5) + 1.6(210) = 433.8$ k
      c. $(M_{u1})_x$ at A = 1.2(15) + 1.6(45) = 90 ft-k
      d. $(M_{u1})_x$ at B = 1.2(20) + 1.6(50) = 104 ft-k
      e. $(M_{u2}) = 0$ since the wind load is not in this combination

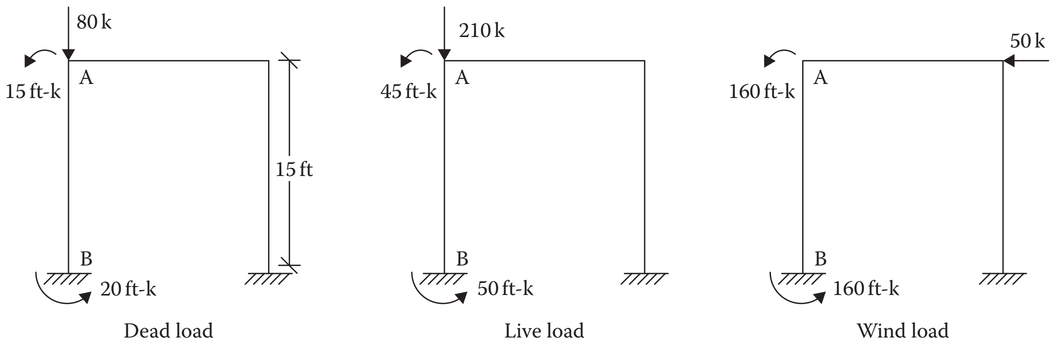


FIGURE 12.11 Loads on an unbraced frame.

2. $1.2D + L + W$
   a. $P_u = 1.2(81.5) + 210 = 307.8$ k
   b. $(M_{u1})_x$ at A = 1.2(15) + 45 = 63 ft-k
   c. $(M_{u1})_x$ at B = 1.2(20) + 50 = 74 ft-k
   d. $(M_{u2})_x$ at A = 160 ft-k
   e. $(M_{u2})_x$ at B = 160 ft-k

B. Trial selection
1. For load combination (1)
   Fixed base, $K = 1.2, KL = 1.2(15) = 18$ ft
   From Table 12.1 for W12 section, $m = 1.9$
   $P_{eff} = 433.8 + 1.9(104) = 631.4$ k
2. For load combination (2)
   $P_{eff} = 307.8 + 1.9(74) + 1.9(160) = 752.4$ k← controls
3. $A_g = \dfrac{P_{eff}}{\phi F_y} = \dfrac{751.4}{(0.9)(50)} = 16.7$
4. Select W12 × 72 (W12 × 65 has the noncompact flange)
   $A = 21.1$ in.^{2}
   $Z_x = 108$ in.^{3}
   $r_x = 5.31$ in.
   $r_y = 3.04$ in.
   $b_f/2t_f = 8.99$
   $h/t_w = 22.6$

Checking of the trial selection for critical load combination (b)

C. Moment strength
1. $0.38\sqrt{\dfrac{E}{F_y}} = 9.15 > \dfrac{b_f}{2t_f}$, compact
2. $3.76\sqrt{\dfrac{E}{F_y}} = 90.55 > \dfrac{h}{t_w}$, compact
3. $\phi M_{nx} = \phi F_y Z_x = 0.9(50)(108) = 4860$ in.-k or 405 ft-k

D. Modification factor for magnification factor B_1: reverse curvature
1. $\dfrac{(M_{u1})_x \text{ at A}}{(M_{u1})_x \text{ at B}} = \dfrac{63}{74} = 0.85$
2. $C_{mx} = 0.6 - 0.4(0..85) = 0.26$

E. Magnification factor, B_1
1. $K = 1$ for braced condition
2. $\dfrac{KL}{r_x} = \dfrac{(1)(15 \times 12)}{5.31} = 33.9$

3. $`(P_{e1})_x = \dfrac{\pi^2 EA}{(KL/r_x)^2} = \dfrac{\pi^2(29{,}000)(21.1)}{(33.9)^2} = 5250$

4. $(B_1)_x = \dfrac{C_m}{1-(P_u/P_{e1})}$

$$= \frac{0.26}{1-(307.8/5250)} = 0.28 < 1;\text{use}\,1$$

F. Magnification factor for sway, B_2

1. $K = 1.2$ for unbraced condition
2. $\dfrac{KL}{r_x} = \dfrac{(12)(15\times 12)}{5.31} = 40.68$
3. $(P_{e2})_x = \dfrac{\pi^2 EA}{(KL/r_x)^2} = \dfrac{\pi^2(29{,}000)(21.1)}{(40.68)^2} = 3{,}645.7$
4. $\Sigma P_u = 2(307.8) = 615.6$ k, since there are two columns in the frame
5. $\Sigma(P_{e2})_x = 2(3645.7) = 7291.4$ k
6. $\dfrac{\Delta H}{L} = \dfrac{0.5}{15\times 12} = 0.00278$
7. From Equation 12.12:

$$B_2 = \frac{1}{1-\dfrac{\Sigma P_u}{\Sigma H}\left(\dfrac{\Delta H}{L}\right)}$$

$$= \frac{1}{1-\left(\dfrac{615.6}{50}\right)(0.00278)} = 1.035$$

8. From Equation 12.13:

$$B_2 = \frac{1}{1-\dfrac{\Sigma P_u}{\Sigma P_{e2}}}$$

$$= \frac{1}{1-\left(\dfrac{615.6}{7291.4}\right)} = 1.09 \leftarrow \text{Controls}$$

G. Design moment

$$(M_u)_x = B_1(M_{u1})_x + B_2(M_{u2})_x$$
$$= 1(74) + 1.09(160) = 248.4\,\text{ft-k}$$

H. Compression strength

1. $\dfrac{KL}{r_x} = \dfrac{(1.2)(15\times 12)}{5.31} = 40.7$
2. $\dfrac{KL}{r_y} = \dfrac{(1.2)(15\times 12)}{3.04} = 71.05 \leftarrow \text{Controls}$
3. $4.71\sqrt{\dfrac{E}{F_y}} = 4.71\sqrt{\dfrac{29{,}000}{50}} = 113.43 > 71.05,$ inelastic buckling
4. $F_e = \dfrac{\pi^2 E}{(KL/r_y)^2} = \dfrac{\pi^2(29{,}000)}{(71.05)^2} = 56.64$

5. $F_{cr} = (0.658^{50/56.64})50 = 34.55$ ksi
6. $\phi P_n = 0.9 F_{cr} A_g$

$$= 0.9(34.55)(21.1) = 656.2 \text{ k}$$

I. Interaction equation
1. $\dfrac{P_u}{\phi P_n} = \dfrac{307.8}{656.2} = 0.47 > 0.2, \text{apply Equation 12.1}$
2. $\dfrac{P_u}{\phi P_n} + \dfrac{8}{9}\left(\dfrac{M_{ux}}{\phi_b M_{nx}}\right) = 0.47 + \dfrac{8}{9}\left(\dfrac{248.4}{405}\right)$

$$= 1.0 \ \textbf{OK} \text{ (border case)}$$

Select a W12 × 72 section.

OPEN-WEB STEEL JOISTS

A common type of floor system for small- to medium-sized steel frame buildings consists of open-web steel joists with or without joist girders. Joist girders, when used, are designed to support open-web steel joists. Floor and roof slabs are supported by open-web joists. A typical plan is shown in Figure 12.12.

Open-web joists are parallel chord trusses where web members are made from steel bars or small angles. A section is shown in Figure 12.13. Open-web joists are pre-engineered systems that can be quickly erected. The open spaces in the web can accommodate ducts and piping.

The AISC specifications do not cover open-web joists. A separate organization, the Steel Joist Institute (SJI), is responsible for the specifications related to open-web steel joists and joist girders. The SJI's publication titled *Standard Specifications* deals with all aspects of open-web joists, including their design, manufacture, application, erection, stability, and handling. Three categories of joists are presented in the standard specifications:

1. Open-web joists, K-series: For span range 8–60 ft, depth 8–30 in., chords $F_y = 50$ ksi, and web $F_y = 36$ or 50 ksi
2. Long span steel joists, LH-series: For span range 21–96 ft, depth 18–48 in., chords $F_y = 36$ or 50 ksi, and web $F_y = 36$ or 50 ksi
3. Deep long span joists, DLH-series: For span range 61–144 ft, depth 52–72 in., chords $F_y = 36$ or 50 ksi, and web $F_y = 36$ or 50 ksi

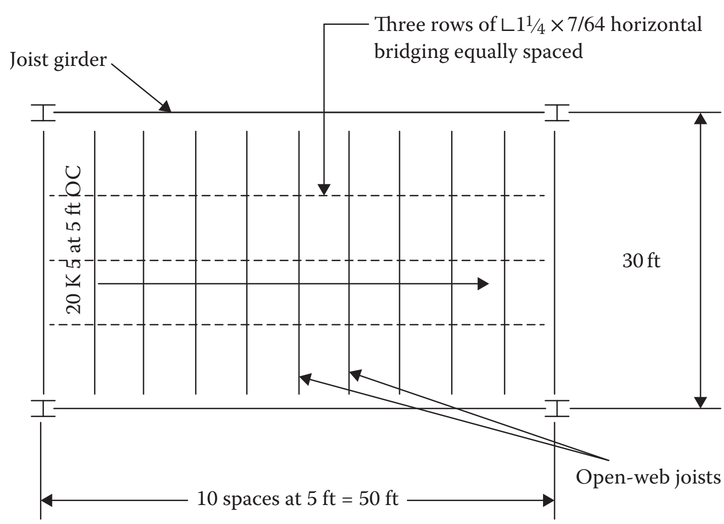


FIGURE 12.12 An open-web joist floor system.

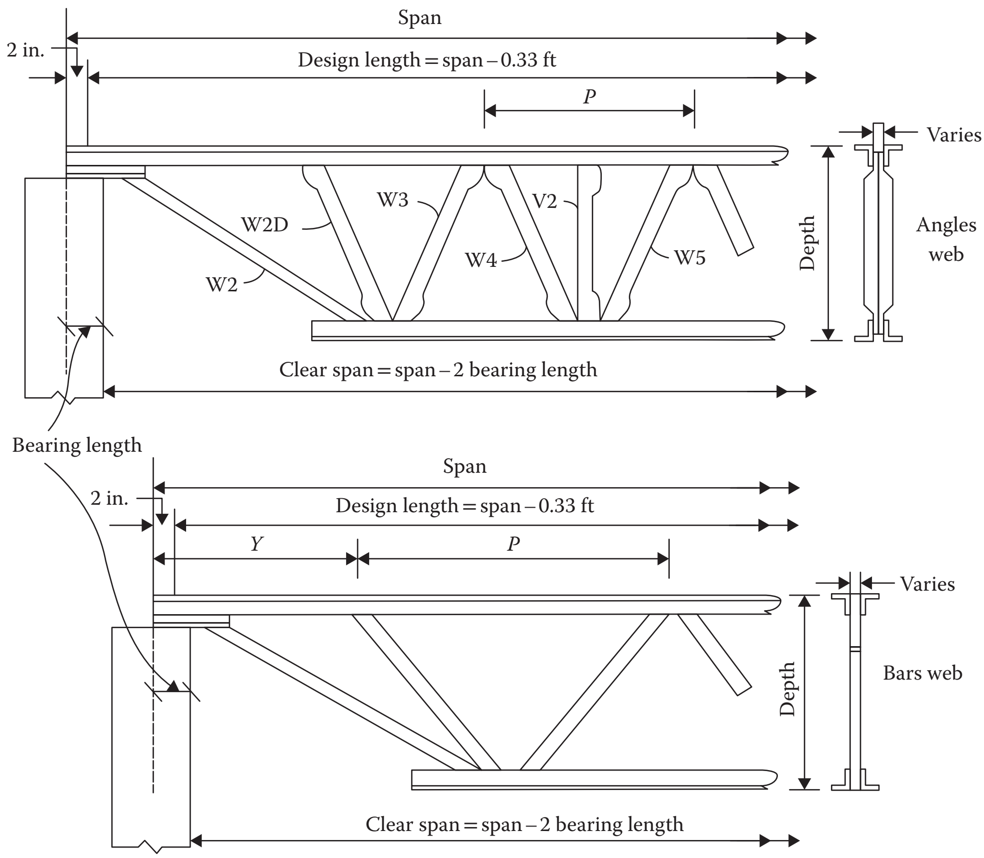


FIGURE 12.13 Open-web steel joist.

Open-web joists use a standardized designation, for example, "18 K 6" means that the depth of the joist is 18 in. and that it is a K-series joist that has a relative strength of 6. The higher the strength number, the stronger the joist. Different manufacturers of 18 K 6 joists can provide different member cross sections, but they all must have a depth of 18 in. and a load capacity as tabulated by the SJI.

The joists are designed as simply supported, uniformly loaded trusses supporting a floor or a roof deck. They are constructed so that the top chord of a joist is braced against lateral buckling.

The SJI specifications stipulate the following basis of design:

1. The bottom chord is designed as an axially loaded tensile member. The design standards and limiting states of Chapter 9 for tensile members are applied.
2. The top chord is designed for axial compression forces only when the panel length, l, does not exceed 24 in., which is taken as the spacing between lines of bridging. The design is done according to the standards of Chapter 10 on columns. When the panel length exceeds 24 in., the top chord is designed as a continuous member subject to the combined axial compression and bending, as discussed in this chapter.
3. The web is designed for the vertical shear force determined from a full uniform loading, but it should not be less than one-fourth of the end reaction. The combined axial compression and bending are investigated for the compression web members.
4. Bridging comprising a cross-connection between adjoining joists is required for the top and bottom chords. This consists of one or both of the following types:

a. Horizontal bridging by a continuous horizontal steel member: the ratio of the length of bracing between the adjoining joists to the least radius of gyration, l/r, should not exceed 300.
b. Diagonal bridging by cross bracing between the joists with the l/r ratio determined on the basis of the length of the bracing and its radius of gyration not exceeding 200.

The number of rows of top chord and bottom chord bridging should not be less than that prescribed in the bridging tables of SJI standards. The spacing should be such that the radius of gyration of the top chord about its vertical axis should not be less than $l/145$, where l is the spacing in inches between the lines of bridging.

For design convenience, the SJI has included in its standard specifications the standard load tables that can be directly used to determine joist size. Tables for K-series joists are included in Appendix C.10a and C.10b. The loads in the tables represent the uniformly distributed loads. The joists are designed for a simple span uniform loading, which produces a parabolic moment diagram for the chord members and a linearly sloped (triangular-shaped) shear diagram for the web members, as shown in Figure 12.14a.

To address the problem of supporting the uniform loads together with the concentrated loads, special K-series joists, known as KCS joists, are designed. KCS joists are designed for flat moments and rectangular shear envelopes, as shown in Figure 12.14b.

As an example, in Appendix C.10a and C.10b, under the column "18 K 6," across a row corresponding to the joist span, the first figure is the total pounds per foot of load that an 18 K 6 joist can support and the second light-faced figure is the unfactored live load from the consideration of $L/360$ deflection. For a live load deflection of $L/240$, multiply the load figure by the ratio 360/240, that is, 1.5.

Example 12.7 demonstrates the use of the joist table.

Example 12.7

Select an open-web steel joist for a span of 30 ft to support a dead load of 35 psf and a live load of 40 psf. The joist spacing is 4 ft. The maximum live load deflection is $L/240$.

Solution

A. Design loads
   1. Tributary area per foot = 4 ft^2/ft
   2. Dead load per foot = $35 \times 4 = 140$ lb/ft

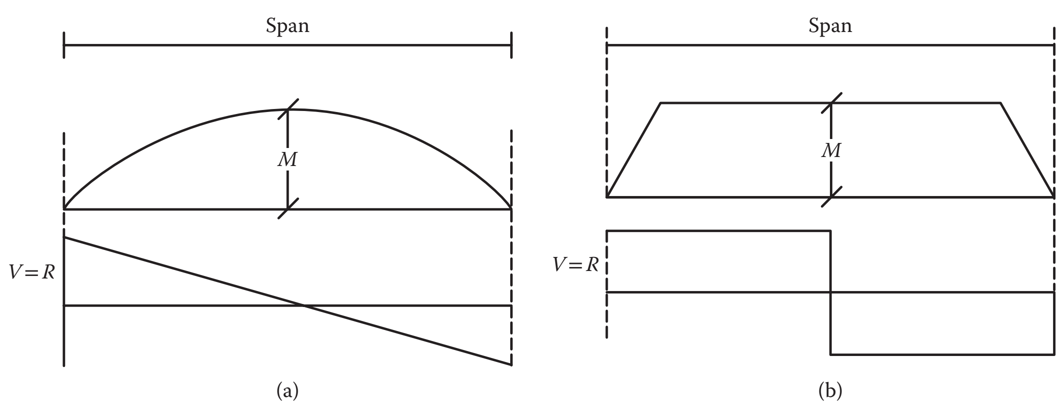


FIGURE 12.14 Shear and moment envelopes: (a) standard joist shear and bending moment diagrams and (b) KCS joist shear and bending moment diagrams.

3. Weight of joist per foot = 10 lb/ft
4. Total dead load = 150 lb/ft
5. Factored dead load = 1.2(150) = 180 lb/ft
6. Live load per foot = $40 \times 4 = 160$ lb/ft
7. Factored live load = 1.6(160) = 256 lb/ft
8. Total factored load = 436 lb/ft

B. Standard load table at Appendix C.10a and C > 10b (from the table for joists starting at size 18K 3)
1. Check the row corresponding to span 30. The section suitable for a total factored load of 436 lb/ft is 18 K × 6, which has a capacity of 451 lb/ft.
2. Live load capacity for $L/240$ deflection

$$= \frac{360}{240}(175) = 262.5 \text{ lb/ft} > 256 \text{ lb/ft} \quad \textbf{OK}$$

3. The joists of a different depth might be designed by selecting a joist of another size from the standard load table of SJI (from the table starting at size 8K 1). In fact, SJI includes an economy table for the lightest joist selection.

JOIST GIRDERS

The loads on a joist girder are applied through open-web joists that the girder supports. This load is equal in magnitude and evenly spaced along the top chord of the girder applied through the panel points.

The bottom chord is designed as an axially loaded tension member. The radius of gyration of the bottom chord about its vertical axis should not be less than $l/240$, where l is the distance between the lines of bracing.

The top chord is designed as an axially loaded compression member. The radius of gyration of the top chord about the vertical axis should not be less than span/575.

The web is designed for vertical shear for full loading but should not be less than one-fourth of the end reaction. The tensile web members are designed to resist at least 25% of the axial force in compression.

The SJI, in its standard specifications, has included the girder tables that are used to design girders. Selected tables have been included in Appendix C.11.

The following are the design parameters of a joist girder:

1. Span of the girder.
2. Number of spacings or size (distance) of spacings of the open-web joists on the girder: when the spacing size is known, the number equals the span/size of spacing; for the known number of spacings, size equals the span/number.
3. The point load on the panel points in kips: total factored unit load in pounds per square foot is multiplied by the spacing size and the length of the joist (joist span or bay length) converted to kips.
4. Depth of girder.

For any of the first three known parameters, the fourth one can be determined from the girder tables. In addition, the table gives the weight of the girder in pounds per foot to confirm that it has been adequately included in the design loads.

Usually, the first three parameters are known and the depth of the girder is determined. A rule of thumb is about an inch of depth for each foot of span for an economic section. Each joist girder uses a standardized designation; for example, "36G 8N 15F" means that the depth of the girder is 36 in., it provides for eight equal joist spaces, and it supports a factored load of 15 k at each panel location (a symbol K at the end, in place of F, is used for the service load capacity at each location).

Example 12.8

Specify the size of the joist girder for the floor system shown in Figure 12.15.

Solution

A. Design loads
1. Including 1 psf for the weight of the girder, total factored load
   $= 1.2(15 + 1) + 1.6(30) = 67.2$ psf
2. Panel area $= 6 \times 20 = 120$ ft^2
3. Factored concentrated load/panel point
   $67.2 \times 120 = 8064$ lb or 8.1 k, use 9 k

B. Joist details
1. Space size = 6 ft
2. Number spaces $= \dfrac{30}{6} = 5$

C. Girder depth selection
1. Refer to Appendix C.11. For 30 ft span, 5 N, and 9 k load, the range of depth is 24–36 in. Select 28G 5N 9F.
2. From Appendix C.11, weight per foot of girder = 17 lb/ft.
   Unit weight $= \dfrac{17}{20} = 0.85\text{psf} < \text{assumed}1\text{ psf}$ **OK**
3. The information shown in Figure 12.16 will be specified to the manufacturer.

PROBLEMS

Note: In all problems, assume the full lateral support conditions.

12.1 A W12 × 35 section of A992 steel with a single line (along the tensile force) of four ¾-in. bolts in the web is subjected to a tensile live load of 65 k and a bending moment only due to the dead load, including the weight of the member along the weak axis of 20 ft-k. Is this member satisfactory?

12.2 A W10 × 33 member is to support a factored tensile force of 100 k and a factored moment along the x axis of 100 ft-k, including the weight of the member. It is a fully welded member of grade 50 steel. Is the member adequate for the loads?

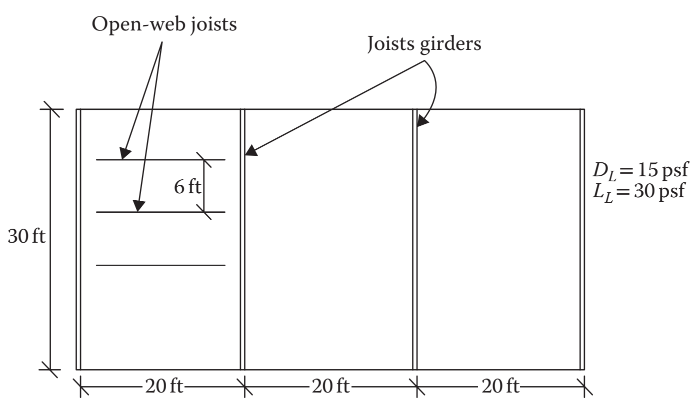


FIGURE 12.15 Floor system for Example 12.8.

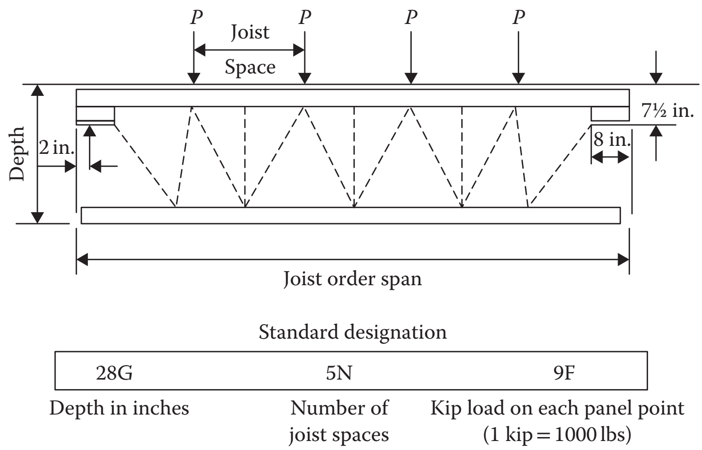


FIGURE 12.16 Selection of joist girder.

12.3 A 12-ft long hanger supports a tensile dead load of 50 k and a live load of 100 k at an eccentricity of 4 in. with respect to the x axis. Design a W10 section of A992 steel. There is one line of three bolts of ¾ in. diameter on one side of the top flange and one line of three bolts of the same size on the other side of the top flange. The bottom flange has a bolt pattern similar to the top flange.

12.4 Design a W8 or W10 member to support the loads shown in Figure P12.1. It has a single line of four holes for ⅞-in. bolts in the web. The member consists of A992 steel.

12.5 The member in Problem 12.4, in addition to the loading along the x axis, has a factored bending moment of 40 ft-k along the y axis. Design the member.

Hint: Since a sizable bending along the y axis is involved, initially select a section at least four times of that required for axial load alone.

12.6 A horizontal beam section W10 × 26 of A992 steel is subjected to the service live loads shown in Figure P12.2. The member is bent about the x axis. Determine the magnitude of the magnification factor B_1.

12.7 A braced frame member W12 × 58 of A992 steel is subjected to the loads shown in Figure P12.3. The member is bent about the x axis. Determine the magnitude of the magnification factor B_1. Assume pin-end conditions.

12.8 In Problem 12.7, the moments at the ends A and B are both clockwise. The ends are restrained (fixed). Determine the magnification factor B_1.

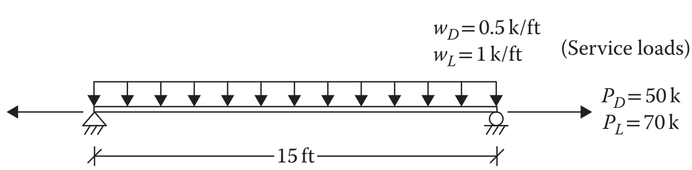


FIGURE P12.1 Tensile and flexure member for Problem 12.4.

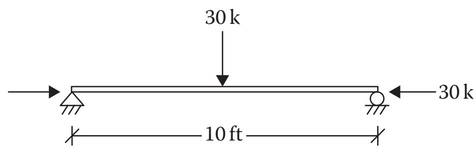


FIGURE P12.2 Compression flexure member for Problem 12.6.

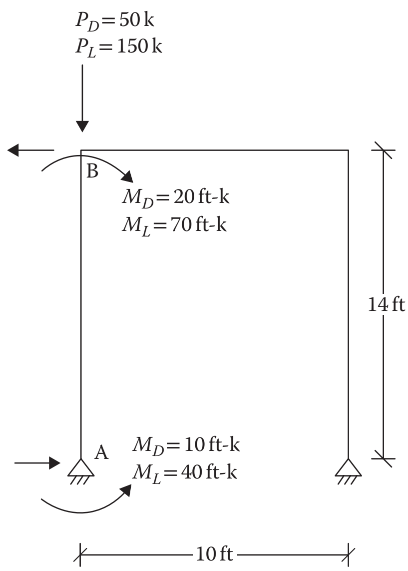


FIGURE P12.3 Braced frame member for Problem 12.7.

12.9 In Problem 12.8, in addition to the loads shown, a uniformly distributed wind load of 1 k/ft acts laterally between A and B. Determine the magnification factor B_1.

12.10 In Problem 12.7, in addition to the shown *x*-axis moments, the moments in the *y* axis at A and B are as follows:

- At B $(M_D)_y$ =10 ft-k, $(M_L)_y$ = 20 ft-k, both clockwise
- At A $(M_D)_y$ = 8 ft-k, $(M_L)_y$ = 15 ft-k, both counterclockwise

Determine the magnification factor B_1.

12.11 The member of the A572 steel section shown in Figure P12.4 is used as a beam column in a braced frame. It is bent about the strong axis. Is the member adequate?

12.12 A horizontal component of a braced frame is shown in Figure P12.5. It is bent about the strong axis. Is the member adequate? Use A992 steel.

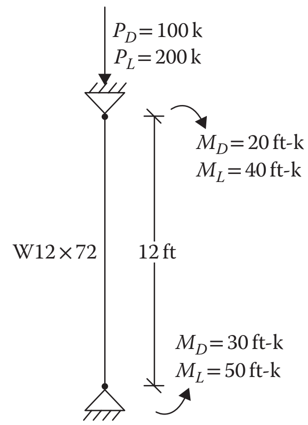


FIGURE P12.4 Beam-column member for Problem 12.11.

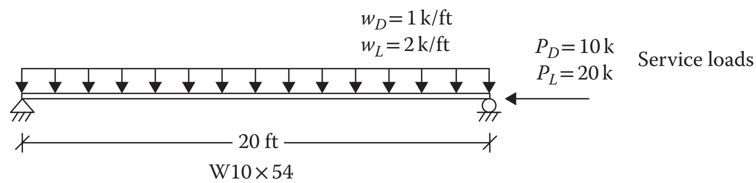


FIGURE P12.5 Horizontal component of a braced frame for Problem 12.12.

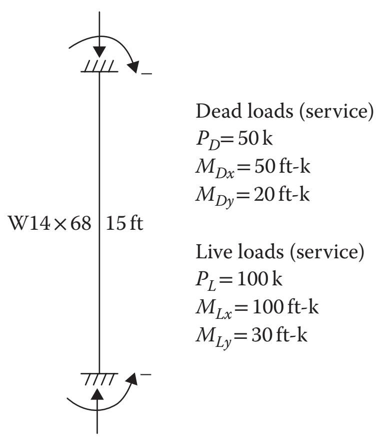


FIGURE P12.6 Restrained braced frame member for Problem 12.13.

12.13 The member of the A572 steel section shown in Figure P12.6 is used as a beam column in a braced frame. It has restrained ends. Is the member adequate?

12.14 A W12 × 74 section of A572 steel is part of a braced frame. It is subjected to service, dead, live, and seismic loads, as shown in Figure P12.7. The bending is along the strong axis. It has pinned ends. Is the section satisfactory?

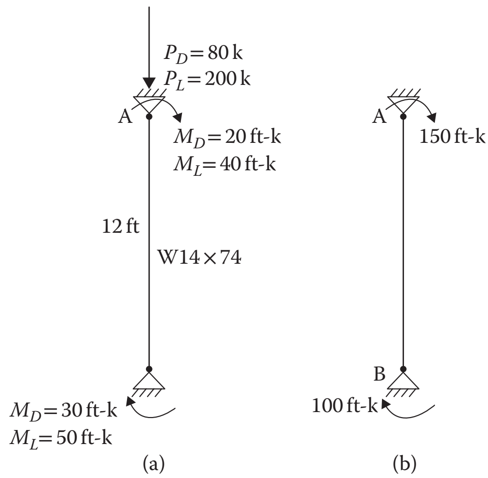


FIGURE P12.7 (a) Gravity and (b) seismic loads on a braced frame.

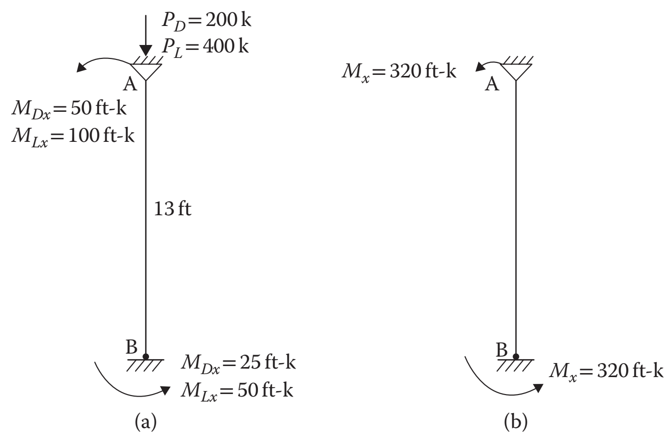


FIGURE P12.8 (a) Gravity and (b) wind loads on a braced frame.

12.15 For a braced frame, the service axial load and the moments obtained by structural analysis are shown in Figure P12.8. Design a W14 section of A992 steel. One end is fixed, and the other is hinged.

12.16 In Problem 12.15, the gravity dead and live loads and moments act along the x axis, and the wind load moments act along the y axis (instead of the x axis). Design the member.

12.17 For a 12-ft -high beam column in an unbraced A36 steel frame, a section W10 × 88 is selected for P_u = 500 k. There are five columns of the same size bearing the same load and having the same buckling strength. Assume that the members are fixed at the support in the x direction, hinged at the support in the y direction, and free to sway (rotation is fixed) at the other end in both directions. Determine the magnification factors in both directions.

12.18 In Problem 12.17, the drift along the x axis is 0.3 in. as a result of a factored lateral load of 300 k. Determine the magnification factor B_2.

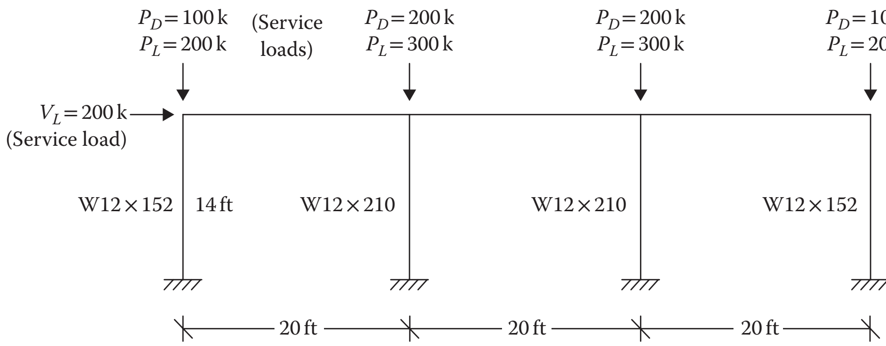


FIGURE P12.9 Unbraced frame for Problem 12.19.

12.19 An unbraced frame of A992 steel is shown in Figure P12.9. Determine the magnification factors along both axes.

12.20 The allowable story drift in Problem 12.19 is 0.5 in. in the x direction. Determine the magnification factor B_2 along the x axis for exterior columns.

12.21 A 10-ft -long W12 × 96 column of A992 steel in an unbraced frame is subjected to the following factored loads:

1. $P_u = 240$ k; $(M_{u1})_x = 50$ ft-k; $(M_{u1})_y = 30$ ft-k; $(M_{u2})_x = 100$ ft-k; $(M_{u2})_y = 70$ ft-k
2. It is bent in reverse curvature with equal and opposite end moments.
3. There are five similar columns in a story.
4. The column is fixed at the base and is free to translate without rotation at the other end.

Is the section satisfactory?

12.22 Select a W12 column member of A992 steel of an unbraced frame for the following conditions; all loads are factored:

1. $K = 1.2$ for the sway case and $K = 1$ for the unsway case
2. $L = 12$ ft
3. $P_u = 350$ k
4. $(M_{u1})_x = 75$ ft-k
5. $(M_{u1})_y = 40$ ft-k
6. $(M_{u2})_x = 150$ ft-k
7. $(M_{u2})_y = 80$ ft-k
8. Allowable drift = 0.3 in.
9. It has intermediate transverse loading between the ends.
10. Total factored horizontal force = 100 k
11. There are four similar columns in a story.

12.23 An unbraced frame of A992 steel is subjected to dead, live, and wind loads in the x axis; the wind load causes the sway. Structural analysis provided the loads as shown in Figure P12.10. Design a W14 section for a maximum drift of 0.5 in. Each column is subjected to the same axial force and moment.

12.24 A one-story unbraced frame of A992 steel is subjected to dead, roof, live, and wind loads. The bending is in the x axis. Structural analysis provided the loads as shown in Figure P12.11. The moments at the base are 0. Design a W12 section for a maximum drift of 0.5 in. The lateral wind load causes the sway.

12.25 Select a K-series, open-web, steel joist spanning 25 ft to support a dead load of 30 psf and a live load of 50 psf. The joist spacing is 3.5 ft. The maximum live load deflection is $L/360$.

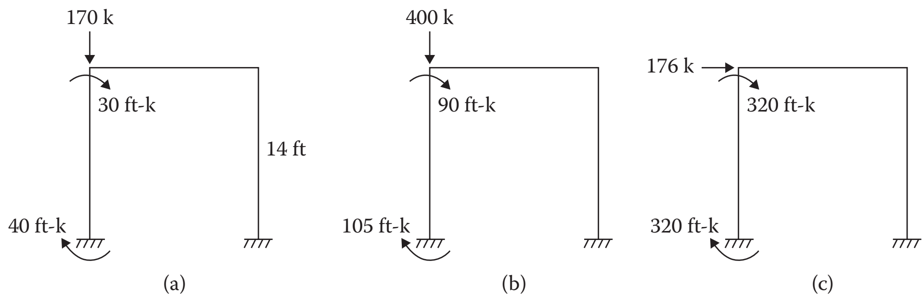


FIGURE P12.10 (a) Dead, (b) live, and (c) wind loads on the unbraced frame for Problem 12.23.

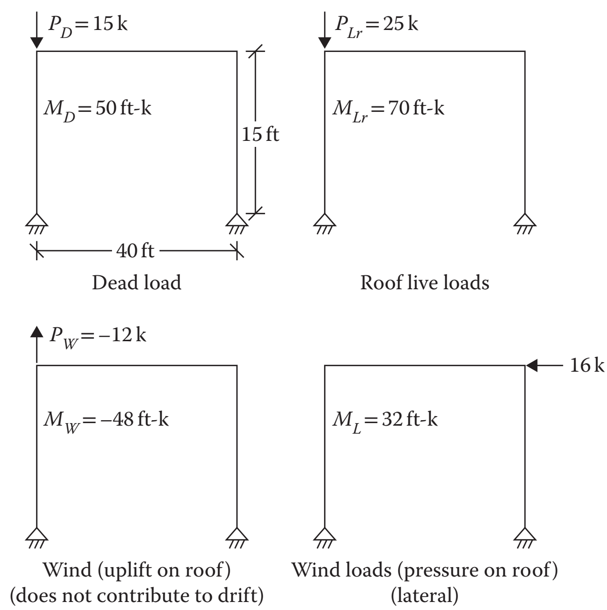


FIGURE P12.11 Dead, roof live, and wind loads on the unbraced frame for Problem 12.24.

12.26 Select an open-web steel joist for the following flooring system:
1. Joist spacing: 3 ft
2. Span length: 20 ft
3. Floor slab: 3 in. concrete
4. Other dead load: 30 psf
5. Live load: 60 psf
6. Maximum live load deflection: $L/240$

12.27 On an 18 K 10 joist spanning 30 ft, how much total unit load and unfactored live load in pounds per square foot can be imposed? The joist spacing is 4 ft. The maximum live load deflection is $L/300$.

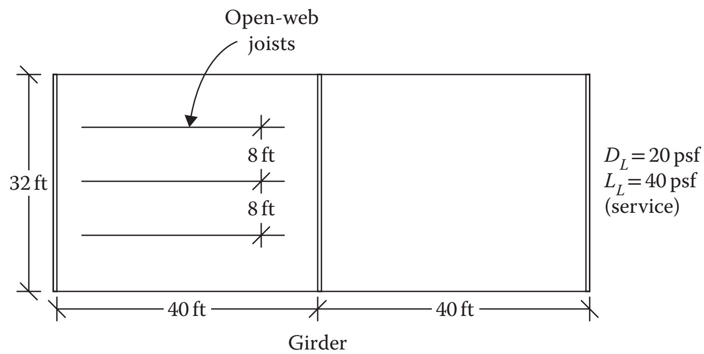


FIGURE P12.12 Open-web joist and joist girder floor system for Problem 12.29.

12.28 The service dead load in pounds per square foot on an 18 K 6 joist is one-half of the live load. What are the magnitudes of these loads on the joist loaded to the capacity at a span of 20 ft, spaced 4 ft on center?

12.29 Indicate the joist girder designation for the flooring system shown in Figure P12.12.

12.30 For a 30 ft × 50 ft bay with joists spaced 3.75 ft on center, indicate the designation of the joist girders to be used for a dead load of 20 psf and a live load of 30 psf.

13 Steel Connections

TYPES OF CONNECTIONS AND JOINTS

The failure of most structure occurs at a connection. Thus, the American Institute of Steel Construction (AISC) placed a lot of emphasis on connections and has brought out separate detailed design specifications related to connections in the 2005 and subsequent *Steel Design Manual*.

Steel connections are made by bolting and welding; riveting is obsolete now. Bolting of steel structures is rapid and requires less skilled labor. On the other hand, welding is simple, and many complex connections with bolts become very simple when welds are used. But the requirements of skilled workers and inspections make welding difficult and costly, which can be partially overcome by shop welding instead of field welding. When a combination is used, welding can be done in the shop and bolting can be done in the field. The AISC's *Steel Design Manual* 2017 (Specifications 360-16) introduces a new category of high-strength bolts of 200 ksi capacity, and all materials have been updated to current ASTM Standards.

Based on the mode of load transfer, the connections are categorized as follows:

1. Simple or axially loaded connection when the resultant of the applied forces passes through the center of gravity of the connection
2. Eccentrically loaded connection when the line of action of the resultant of the forces does not pass through the center of gravity of the connection

The following types of joints are formed by the two connecting members:

1. *Lap joint:* As shown in Figure 13.1, the line of action of the force in one member and the line of action of the force in the other connecting member have a gap between them. This causes a bending within the connection, as shown by the dashed lines. For this reason, the lap joint is used for minor connections only.
2. *Butt joint:* The butt joint provides a more symmetrical loading, as shown in Figure 13.2, which eliminates the bending condition.

The connectors (bolts or welds) are subjected to the following types of forces (and stresses):

1. *Shear:* The forces acting on the splices shown in Figure 13.3 can shear the shank of the bolt. The weld in Figure 13.4 resists the shear.
2. *Tension:* The hanger-type connection shown in Figures 13.5 and 13.6 imposes tension in bolts and welds.
3. *Shear and tension combination:* The column-to-beam connections shown in Figures 13.7 and 13.8 cause both shear and tension in bolts and welds. The welds are weak in shear and are usually assumed to fail in shear regardless of the direction of the loading.

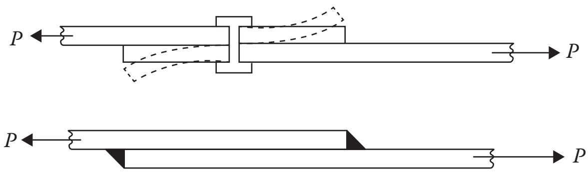


FIGURE 13.1 Lap joint.

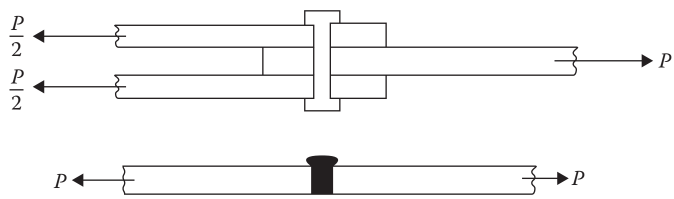


FIGURE 13.2 Butt joint.

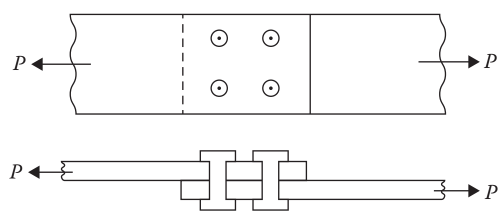


FIGURE 13.3 Bolts in shear.

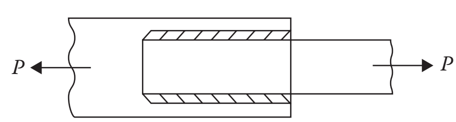


FIGURE 13.4 Welds in shear.

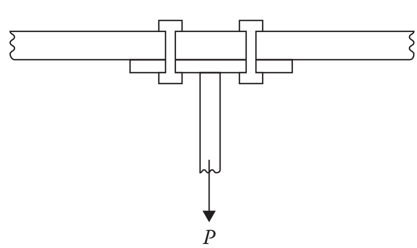


FIGURE 13.5 Bolts in tension.

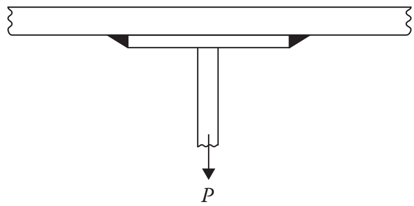


FIGURE 13.6 Welds in tension.

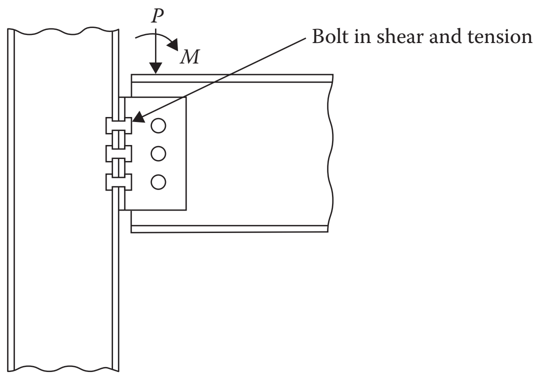


FIGURE 13.7 Bolts in shear and tension.

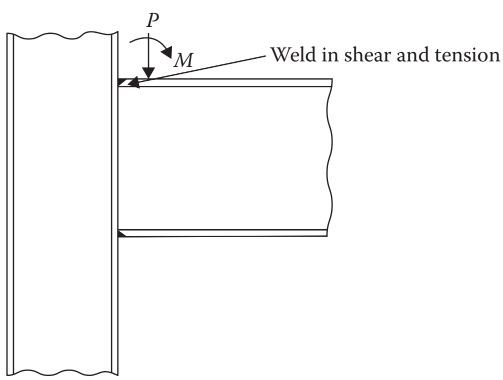


FIGURE 13.8 Welds in shear and tension.

BOLTED CONNECTIONS

Ordinary or common bolts, also known as *unfinished bolts*, are classified as A307 bolts. The characteristics of A307 steel are very similar to A36 steel. The strength of ordinary bolts is considerably less than those of high-strength bolts. Their use is recommended for structures subjected to static loads and for secondary members like purlins, girts, and bracings. With the advent of high-strength bolts, the use of ordinary bolts has been neglected, although for ordinary construction, common bolts are quite satisfactory. High-strength bolts are twice, or more, as strong as ordinary bolts.

High-Strength Bolts

There are three groups of high-strength bolts:

Group A: ASTM F3125[1] Grades A325, F1852 and ASTM A354 Grade BC
Group B: ASTM F3125[1] Grades A490, F2280 and ASTM A354 Grade BD
Group C: ASTM F3043 and F3111

Group C is a new addition in AISC 360-16. Its use is limited to specific building locations in noncorrosive environmental conditions.

Types of Connections

High-strength bolts are used in two types of connections: bearing-type and slip-critical or friction type.

In bearing-type connections, in which the common bolts can also be used, no frictional resistance in the faying (contact) surfaces is assumed, and a slip between the connecting members occurs as the load is applied. This brings the bolt in contact with the connecting member, and the bolt bears the load. Thus, the load transfer takes place through the bolt.

In bearing-type connections, the bolts can be tightened to the snug-tight condition, which means the tightness that can be obtained by the full effort of a person using a spud wrench or pneumatic wrench.

In slip-critical connections, the bolts are torqued to a high tensile stress in the shank. This develops a clamping force on the connected parts. The shear resistance to the applied load is provided by the clamping force, as shown in Figure 13.9.

Thus, in a slip-critical connection, the bolts themselves are not stressed since the entire force is resisted by the friction developed on the contact surfaces. For this purpose, the high-strength bolts are tightened to a very high degree. The minimum pretension applied to bolts is 0.7 times the tensile strength of steel. The pretension for bolts of different diameters is given in Table 13.1.

The methods available to tighten the bolts comprise the (1) turn of the nut method, (2) calibrated wrench method, (3) direct tension indicator method, and (4) twist-off type tension control method in which bolts are used whose tips are sheared off at a predetermined tension level.

The slip-critical connection is a costly process subject to inspections. It is used for structures subjected to dynamic loading, such as bridges, where stress reversals and fatigued loading take place.

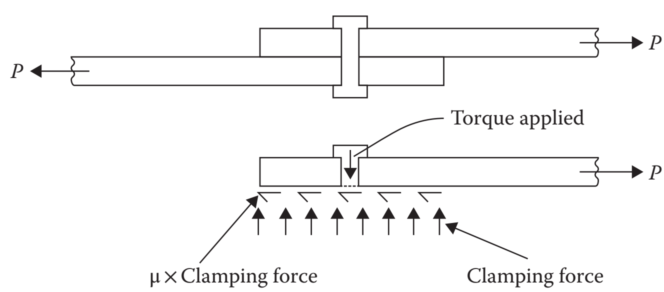


FIGURE 13.9 Frictional resistance in a slip-critical connection.

[1] These bolts also have metric specifications.

TABLE 13.1
Minimum Pretension on Bolts, k

Bolt Diameter (in.)	Area (in.^{2})	Group A (e.g., A325 Bolts)	Group B (e.g., A490 Bolts)	Group C (e.g., F3043 Grade 2)
½	0.0196	12	15	—
⅝	0.307	19	24	—
¾	0.442	28	35	—
⅞	0.601	39	49	—
1	0.785	51	64	90
1¼	1.227	71	102	143
1½	1.766	103	148	

SPECIFICATIONS FOR SPACING OF BOLTS AND EDGE DISTANCE

1. *Definitions:* The following definitions are given with respect to Figure 13.10.
   Gage, g: This is the center-to-center distance between two successive lines of bolts, perpendicular to the axis of a member (perpendicular to the load).
   Pitch, p: This is the center-to-center distance between two successive bolts along the axis of a member (in line with the force).
   Edge distance, L_e: This is the distance from the center of the outermost bolt to the edge of a member.
2. *Minimum spacing:* The minimum center-to-center distance for standard, oversized, and slotted holes should not be less than 2⅔d, but a distance of 3d is preferred (d is the bolt diameter). However, the clear distance between holes shall not be less than d.
3. *Maximum spacing:* The maximum spacing of bolts of the painted members or the unpainted members not subject to corrosion should not exceed 24 times the thickness of the thinner member or 12 in., whichever is less. The maximum spacing for members subject to corrosion should not exceed 14 times the thickness or 7 in., whichever is less.
4. *Minimum edge distance:* The minimum edge distance in any direction is tabulated by the AISC. It is generally 1¾ times the bolt diameter for the sheared edges and 1¼ times the bolt diameter for the rolled or gas cut edges.
5. *Maximum edge distance:* The maximum edge distance should not exceed 12 times the thickness of the thinner member or 6 in., whichever is less.

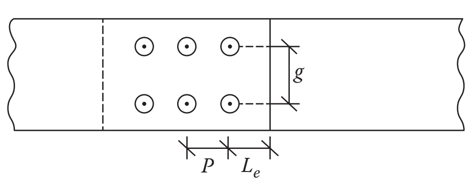


FIGURE 13.10 Definition sketch.

BEARING-TYPE CONNECTIONS

The design basis of a connection is:

$$P_u \le \phi R_n \tag{13.1}$$

where:

P_u is the applied factored load on a connection
ϕ is the resistance factor = 0.75 for a connection
R_n is the nominal strength of a connection

In terms of the nominal unit strength (stress), Equation 13.1 can be expressed as:

$$P_u \le \phi F_n A \tag{13.2}$$

For bearing-type connections, F_n refers to the nominal unit strength (stress) for the various limit states or modes of failure, and A refers to the relevant area of failure.

The failure of a bolted joint in a bearing-type connection can occur by the following modes:

1. Shearing of the bolt across the plane between the members, in single shear in the lap joint and in double shear in the butt joint, as shown in Figure 13.11.
   For a single shear:

$$A = \frac{\pi}{4} d^2$$

   For a double shear:

$$A = \frac{\pi}{2} d^2$$

2. Bearing failure on the contact area between the bolt and the plate, as shown in Figure 13.12:

$$A = d \cdot t$$

3. Tearing out of the plate from the bolt, as shown in Figure 13.13:

$$A = \text{tearing area} = 2L_c\, t$$

4. Tensile failure of the plate, is shown in Figure 13.14. This condition was discussed in Chapter 9 for tension members. It is not part of the connection.

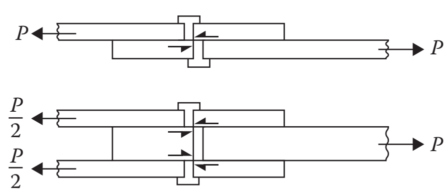


FIGURE 13.11 Shear failure.

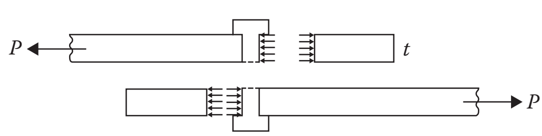


FIGURE 13.12 Bearing failure.

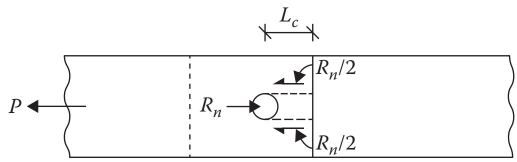


FIGURE 13.13 Tearing out of plate.

FIGURE 13.14 Tensile failure of plate.

LIMIT STATE OF SHEAR RUPTURE

For the shearing type of the limiting state, F_n in Equation 13.2 is the nominal unit shear strength of bolts, F_{nv}, which is taken as 50% of the ultimate strength of bolts. The cross-sectional area, A_b, is taken as the area of the unthreaded part or the body area of bolt. If the threads are in the plane of shear or are not excluded from shear plane, a factor of 0.8 is applied to reduce the area. This factor is incorporated in the strength, F_{nv}.

Thus, for the shear limit state, the design strength is given by:

$$P_u \leq 0.75 F_{nv} A_b n_b \tag{13.3}$$

where:

F_{nv} is the nominal unit shear strength of bolt
$A_b = (\pi/4)d^2$ for single shear and $A = (\pi/2)d^2$ for double shear
n_b is the number of bolts in the connection

The nominal shear strength for different types of bolts is given in Table 13.2. The table identifies whether the threads are not excluded from the shear plane, commonly referred to as an N-type connection, like 32-N, or the threads are excluded from the shear plane, commonly referred to as an X-type connection, like 325-X.

In the case of double shear, if the combined thickness of two outside elements is more than the thickness of the middle element, the middle element is considered in design, using twice the bolt area as the area for shear strength and using the thickness of the middle element for bearing strength. However, if the combined thickness of the outer elements is less than the middle element, then the outer element is considered in design, using a single bolt for shear strength and outer element thickness for bearing strength, with one-half of the total load, which each of the outside element shares.

TABLE 13.2
Nominal Unit Shear Strength, F_{nv}

Bolt Type	F_{nv} (ksi)
A307	27
Group A (e.g., A325-N) threads not excluded from shear plane	54
Group A (e.g., A325-X) threads excluded from shear plane	68
Group B (e.g., A490-N) threads not excluded from shear plane	68
Group B (e.g., A490-X) threads excluded from shear plane	84

Bearing and Tearout Limit State

The other two modes of failure, that is, the bearing and the tearing out of a member, are based not on the strength of bolts but on the parts being connected. The areas for bearing and tearing are described in the preceding discussion. The nominal unit strength in the bearing and the (shear) tearout depend on the deformation around the holes that can be tolerated and on the types of holes. The bearing strength is very high because tests have shown that the bolts and the connected member actually do not fail in bearing but the strength of the connected parts is impaired.

1. For standard, oversized, short-slotted holes, independent of the direction of loading, and long-slotted holes with slots parallel to the force where deformation of the hole is ≤0.25 in. (i.e., deformation is a consideration):
   a. For tearout

$$P_u = 1.2\phi L_c t F_u n_b \tag{13.4a}$$

   b. For bearing

$$P_u = 2.4\phi d t F_u n_b \tag{13.4b}$$

   where F_u is the ultimate strength of the connected member
2. For standard, oversized, short-slotted holes independent of the direction of loading and long-slotted holes parallel to the force where deformations is >0.25 in. (i.e., deformation is not a consideration):
   a. For tearout

$$P_u = 1.5\phi L_c t F_u n_b \tag{13.5a}$$

   b. For bearing

$$P_u = 3\phi d t F_u n_b \tag{13.5b}$$

3. For long-slotted holes, slots being perpendicular to the force:
   a. For tearout

$$P_u = 1.0\phi L_c t F_u n_b \tag{13.6a}$$

   b. For bearing

$$P_u = 2.0\phi d t F_u n_b \tag{13.6b}$$

   where:
   ϕ is 0.75
   L_c is illustrated in Figure 13.15.

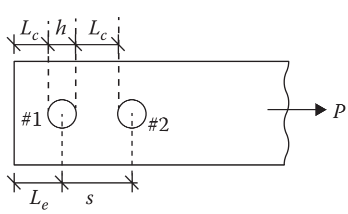


FIGURE 13.15 Definition of *Lc*.

For edge bolt #1:

$$L_c = L_e - \frac{h}{2} \tag{13.7}$$

For interior bolt #2:

$$L_c = s - h \tag{13.8}$$

where h = the hole diameter $(d + 1/16)$ in.

Example 13.1

A channel section C9 × 15 of A36 steel is connected to a ⅜-in. steel gusset plate, with ⅞-in. diameter, Group A: A325 bolts. A service dead load of 20 k and live load of 50 k is applied to the connection. Design the connection. The slip of the connection is permissible. The threads are excluded from the shear plane. Deformation of the hole is a consideration.

Solution

A. The factored load

$$P_u = 1.2(20) + 1.6(50) = 104 \text{ k}$$

B. Shear limit state
1. $A_b = (\pi/4)(7/8)^2 = 0.601$ in.^{2}
2. For Group A: A325-X, $F_{nv} = 68$ ksi
3. From Equation 13.3:

$$\text{No. of bolts} = \frac{P_u}{0.75F_{nv}A_b}$$

$$= \frac{104}{0.75(68)(0.601)} = 3.39 \text{ or 4 bolts}$$

C. Bearing limit state
1. Minimum edge distance

$$L_e = 1\frac{3}{4}\left(\frac{7}{8}\right) = 1.53 \text{ in., use 2 in.}$$

2. Minimum spacing

$$s = 3\left(\frac{7}{8}\right) = 2.63 \text{ in., use 3 in.}$$

3. $h = d + \dfrac{1}{16} = 0.938$ in.
4. For holes near edge:

$$L_e = L_e - \frac{h}{2}$$

$$= 2 - \frac{0.938}{2} = 1.5 \text{ in.}$$

$t = 5/16$ in. for the web of the channel section
For a standard size hole of deformation <0.25 in. (deformation is a consideration):

$$\text{Tearout strength/bolt} = 1.2\phi L_c t F_u$$

$$= 1.2(0.75)(1.5)\left(\frac{5}{16}\right)(58) = 24.5 \leftarrow \text{Controls}$$

$$\text{Bearing strength/bolt} = 2.4\phi d t F_u$$

$$= 2.4(0.75)\left(\frac{7}{8}\right)\left(\frac{5}{16}\right)(58) = 28.55 \text{ k}$$

5. For interior holes:

$$L_c = s - h = 3 - 0.938 = 2.06$$

6. $$\text{Tearout strength/bolt} = 1.2(0.75)(2.06)\left(\frac{5}{16}\right)(58) = 33.6 \text{ k}$$

$$\text{Bearing strength/bolt} = 2.4(0.75)\left(\frac{7}{8}\right)\left(\frac{5}{16}\right)(58) = 28.55 \text{ k} \leftarrow \text{Controls}$$

7. Suppose there are n lines of holes with two bolts in each, then:

$$P_u = 104 = n(24.5) + \mathrm{n}(28.55)$$

$$n = 1.96$$

$$\text{Total no. of bolts} = 2(1.96) = 3.92 \text{ or } 4 \text{ bolts}$$

Select four bolts either by shear or bearing.

8. The section has to be checked for the tensile strength and the block shear by the procedure given in Chapter 9 under the "Tensile Strength of Elements" and "Block Shear Strength" sections, respectively.

SLIP-CRITICAL CONNECTIONS

In a slip-critical connection, the bolts are not subjected to any stress. The resistance to slip is equal to the product of the tensile force between the connected parts and the static coefficient of friction. This is given by:

$$P_u = \phi D_u \mu h_f T_b N_s n_b \tag{13.9}$$

where:

ϕ is a resistance factor with different values, as follows:

1. Standard holes and short-slotted holes perpendicular to the direction of the load, $\phi = 1$
2. Short-slotted holes parallel to the direction of the load, $\phi = 0.85$
3. Long-slotted holes, $\phi = 0.70$

D_u is the ratio of installed pretension to minimum pretension; use $D_u = 1.13$. Other values are permitted.

μ is the slip (friction) coefficient, as given in Table 13.3.

TABLE 13.3
Slip (Friction) Coefficient

Class	Surface	μ
Class A	Unpainted clean mill scale, or Class A coating on blast cleaned steel, or Hot dipped galvanized and roughened surface	0.30
Class B	Unpainted blast cleaned surface, or Class B coating on blast cleaned steel	0.5

h_f is the factor for fillers, as follows:

1. No filler or where the bolts have been added to distribute loads in fillers, $h_f = 1$
2. One filler between connected parts, $h_f = 1$
3. Two or more fillers, $h_f = 0.85$

T_b is the minimum bolt pretension, as given in Table 13.1
N_s is the number of slip (shear) planes
n_b is the number of bolts in the connection

Although there is no bearing on bolts in a slip-critical connection, the AISC requires that it should also be checked as a bearing-type connection by Equation 13.3 and the relevant equation from Equations 13.4 through 13.6.

Example 13.2

A double-angle tensile member consisting of 2∟ 3 × 2½ × ¼ is connected by a gusset plate ¾ in. thick. It is designed for a service load of 15 k and live load of 30 k. No slip is permitted. Use ⅝-in. Group A: A325 bolts and A572 steel. Holes are standard size, and bolts are excluded from the shear plane. There are no fillers, and the surface is Coat A on blast cleaned steel. Deformation of the hole is a consideration.

Solution

A. Factored design load

$$P_u = 1.2(15) + 1.6(30) = 66 \text{ k}$$

B. For the slip-critical limit state:

1. $D_u = 1.13$
2. Standard holes, $\phi = 1$
3. No fillers, $h_f = 1$
4. Class A surface, $\mu = 0.3$
5. From Table 13.1, $T_b = 19$ ksi for ⅝-in. bolts
6. For double shear (double angle), $N_s = 2$

From Equation 13.9:

$$n_b = \frac{Pu}{\phi D_u \mu h_f T_b N_s}$$

$$= \frac{66}{1(1.13)(0.3)(1)(19)(2)} = 5.12$$

C. Check for the shear limit state as a bearing-type connection.
   1. $A_b = 2(\pi/4)(5/8)^2 = 0.613$ in.^{2}
   2. From Table 13.2, for Group A: A325-X, $F_{nv} = 68$ ksi
   3. From Equation 13.3:

$$\text{No. of bolts} = \frac{P_u}{0.75 F_{nv} A_b}$$
$$= \frac{66}{0.75(68)(0.613)} = 2.11$$

D. Check for the bearing limit state as a bearing-type connection.
   1. Minimum edge distance

$$L_e = 1\frac{3}{4}\left(\frac{5}{8}\right) = 1.09 \text{ in., use } 1.5 \text{ in.}$$

   2. Minimum spacing

$$s = 3\left(\frac{5}{8}\right) = 1.88 \text{ in., use } 2 \text{ in.}$$

   3. $h = d + \dfrac{1}{16} = 0.688$ in.

   4. For holes near the edge:

$$L_c = L_e - \frac{h}{2}$$
$$= 1.5 - \frac{0.688}{2} = 1.156$$

   $t = 2\ (1/4) = 0.5$ in. ← Thinner than the gusset plate
   For a standard size hole of deformation <0.25 in. (deformation is a consideration):

$$\text{Tearout strength / bolt} = 1.2\phi L_c t F_u$$
$$= 1.2(0.75)(1.156)(0.5)(65) = 33.81 \text{ k} \leftarrow \text{Controls}$$

$$\text{Bearing strength/bolt} = 2.4\phi d t F_u$$
$$= 2.4(0.75)\left(\frac{5}{8}\right)(0.5)(65) = 36.56\text{k}$$

   5. For interior holes:

$$L_c = s - h = 2 - 0.688 = 1.31 \text{ in.}$$

   6. Tearout strength/bolt = 1.2(0.75)(1.31)(0.5)(65) = 38.31 k

$$\text{Bearing strength/bolt} = 2.4(0.75)\left(\frac{5}{8}\right)(0.5)(65) = 36.56 \text{ k} \leftarrow \text{Controls}$$

   7. Suppose there are n lines of holes with two bolts in each, then:

$$P_u = 66 = n(33.81) + n(36.56)$$
$$n = 0.94$$

   Total number of bolts = 2

E. The slip-critical limit controls the design.
Number of bolts selected = 6 for symmetry.
F. Check for the tensile strength of bolt, per the "Tensile Strength of Elements" section in Chapter 9.
G. Check for the block shear, per the "Block Shear Strength" section in Chapter 9.

TENSILE LOAD ON BOLTS

This section applies to tensile loads on bolts, in both the bearing type of connections and the slip-critical connections. The connections subjected to pure tensile loads (without shear) are limited. These connections exist in hanger-type connections for bridges, flange connections for piping systems, and wind-bracing systems in tall buildings. A hanger-type connection is shown in Figure 13.16.

Tension by the external loads acts to relieve the clamping force between the connected parts that causes a reduction in the slip resistance. This topic has been considered in the "Combined Shear and Tensile Forces on Bolts" section. As far as the tensile strength of the bolt is concerned, however, it is computed without giving any consideration to the initial tightening force or pretension.

The tensile limit state of rupture follows the standard form of Equation 13.2:

$$T_u \leq 0.75 F_{nt} A_b n_b \tag{13.10}$$

where:

T_u is the factored design tensile load
F_{nt} is the nominal unit tensile strength, as given in Table 13.4

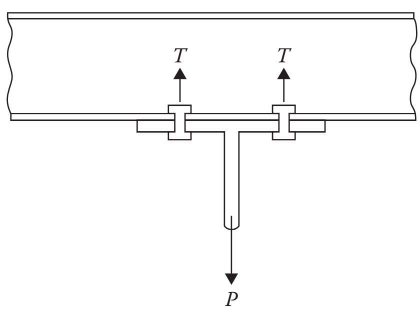


FIGURE 13.16 T-type hanger connection.

TABLE 13.4
Nominal Unit Tensile Strength, F_{nt}

Bolt Type	F_{nt} (ksi)
A307	45
Group A: A325	90
Group B: A490	113
Group C: F3043	150

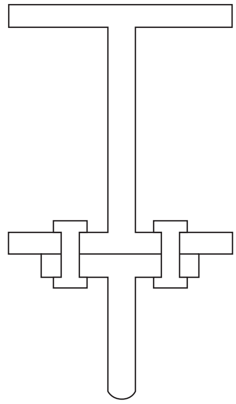


FIGURE 13.17 A tensile connection for Example 13.3.

Example 13.3

Design the hanger connection shown in Figure 13.17 for the service dead and live loads of 30 k and 50 k, respectively. Use Group A: A325 bolts.

Solution

1. Factored design load

$$P_u = 1.2(30) + 1.6(50) = 116 \text{ k}$$

2. Use ⅞-in. bolt.

$$A_b = \frac{\pi(7/8)^2}{4} = 0.601 \text{ in.}^2$$

3. From Equation 3.10:

$$n_b = \frac{P_u}{0.75F_{nt}A_b}$$

$$= \frac{116}{0.75(90)(0.601)} = 2.86$$

Use four bolts, two on each side.

COMBINED SHEAR AND TENSILE FORCES ON BOLTS

Combined Shear and Tension on Bearing-Type Connections

Many connections are subjected to a combination of shear and tension. A common case is a diagonal bracing attached to a column.

When both tension and shear are imposed, the interaction of these two forces in terms of the combined stress must be considered to determine the capacity of the bolt. A simplified approach to deal with this interaction is to reduce the unit tensile strength of a bolt to F'_{nt} (from the original F_{nt}). Thus, the limiting state equation is:

$$T_u \le 0.75F'_{nt}A_b n_b \tag{13.11}$$

where the adjusted (reduced) nominal unit tensile strength is given as follows[2]:

$$F'_{nt} = 1.3F_{nt} - \left(\frac{F_{nt}}{0.75F_{nv}}\right) f_v \le F_{nt} \tag{13.12}$$

where f_v is actual shear stress given by the design shear force divided by the area of the number bolts in the connection.

To summarize, for combined shear and tension in a bearing-type connection, the procedure comprises the following steps:

1. Use the unmodified shear limiting state equation (Equation 13.3).
2. Use the tension limiting state equation (Equation 13.11) as a check.
3. Use the relevant bearing limiting state equation from Equations 13.4 through 13.6 as a check.

Example 13.4

A WT12 × 27.5 bracket of A36 steel is connected to a W14 × 61 column, as shown in Figure 13.18, to transmit the service dead and live loads of 15 and 45 k, respectively. Design the bearing-type 0connection between the column and the bracket using ⅞-in. Group A: 325-X bolts. Deformation is a consideration.

Solution

A. Design data
  1. Thickness of the bracket = 0.505 in.
  2. Thickness of the column = 0.645 in.
  3. P_u = 1.2 (15) + 1.6 (45) = 90 k
  4. Design shear, $V_u = P_u\,(3/5)$ = 54 k
  5. Design tension, $T_u = P_u\,(4/5)$ = 72 k

B. For the shear limiting state:
  1. $A_b = (\pi/4)(7/8)^2 = 0.601$ in.²
  2. For Group A, A325-X, F_{nv} = 68 ksi

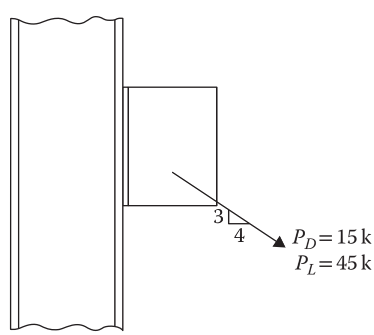


FIGURE 13.18 Column-bracket connection for Example 13.4.

[2] When the actual shear stress $f_v \le 0.3\phi F_{nv}$ or the actual tensile stress $f_t \le 0.3\phi F_{nt}$, the adjustment of F_{nt} is not required.

3. From Equation 13.3:

$$n_b = \frac{V_n}{0.75F_{nv}A_b}$$

$$= \frac{54}{0.75(68)(0.601)} = 1.762 \text{ bolts}$$

Use four bolts, two on each side (minimum two bolts on each side).

C. For the tensile limiting state:

1. $F_{nt} = 90$ ksi

2. Actual shear stress

$$f_v = \frac{V_u}{A_b n_b} = \frac{54}{(0.601)(4)} = 22.46 \text{ ksi}$$

3. Adjusted unit tensile strength from Equation 13.12

$$F'_{nt} = 1.3F_{nt} - \left(\frac{F_{nt}}{0.75F_{nv}}\right) f_v \le F_{nt}$$

$$= 1.3(90) - \frac{90}{0.75(68)}(22.46) = 77.36 \text{ ksi } \textbf{OK}$$

4. From Equation 13.11:

$$n_b = \frac{T_u}{0.75F'_{nt}A_b}$$

$$= \frac{72}{0.75(77.36)(0.601)} = 2.06 < 4 \text{ bolts } \textbf{OK}$$

D. Check for the bearing limit state.

1. Minimum edge distance

$$L_e = 1\frac{3}{4}\left(\frac{7}{8}\right) = 1.53 \text{ in., use } 2 \text{ in.}$$

2. Minimum spacing

$$s = 3\left(\frac{7}{8}\right) = 2.63 \text{ in.,use } 3 \text{ in.}$$

3. $h = d + \frac{1}{16} = 0.938$ in.

4. For holes near edge:

$$L_e = L_e - \left(\frac{h}{2}\right)$$

$$= 2 - \left(\frac{0.938}{2}\right) = 1.53 \text{ in.}$$

$t = 0.505$ in. ← Thickness of WT flange

For a standard size hole of deformation <0.25 in. (deformation is a consideration):

$$\text{Tearout strength} = 1.2\phi L_c t F_u n_b$$

$$= 1.2(0.75)(1.53)(0.505)(58)(4) = 161\ \text{k} \leftarrow \text{Controls} > 54\ \text{k}\ \textbf{OK}$$

$$\text{Bearing strength} = 2.4\phi d t F_u n_b$$

$$= 2.4(0.75)\left(\frac{7}{8}\right)(0.505)(58)(4) = 184.5\ \text{k}$$

Combined Shear and Tension on Slip-Critical Connections

As discussed in the "Tensile Load on Bolts" section, the externally applied tension tends to reduce the clamping force and the slip-resisting capacity. A reduction factor, k_s, is applied to the previously described slip-critical strength. Thus, for the combined shear and tension, the slip-critical limit state is:

$$V_u = \phi D_u \mu h_f T_b N_s n_b k_s \tag{13.13}$$

where:

$$k_s = 1 - \frac{Tu}{D_u T_b n_b} \tag{13.14}$$

V_u is the factored shear load on the connection
T_u is the factored tension load on the connection
T_b is the minimum bolt pretension, as given in Table 13.1
N_s is the number of slip (shear) planes
n_b is the number of bolts in the connection
h_f is a factor for fillers, defined in Equation 13.9
μ is the slip (friction) coefficient, as given is Table 13.3

Combining Equations 13.13 and 13.14, the relation for the number of bolts is:

$$n_b = \frac{1}{D_u T_b}\left(\frac{V_u}{\phi \mu N s h_f} + T_u\right) \tag{13.15}$$

To summarize, for the combined shear and tension in a slip-critical connection, the procedure is:

1. Use the shear limiting state equation (Equation 13.3).[3]
2. Use the (original) tensile limit state equation (Equation 13.10).
3. Use the relevant bearing limiting state equation from Equations 13.4 through 13.6 as a check.
4. Use the (modified) slip-critical limit state equation (Equation 13.13).

[3] The slip-critical connections must also be checked for bearing capacity and shear strength.

Example 13.5

Design Example 13.4 as a slip-critical connection. The holes are standard size. There is no filler. The surface is unpainted clean mill scale.

Solution

A. Design loads from Example 13.4
   1. $V_u = 54$ k
   2. $T_u = 72$ k

B. For the shear limiting state, $n_b = 1.762$ from Example 13.4 (use four bolts, min. two on each side).

C. For the tensile limiting state:

$$n_b = \frac{T_u}{0.75F_{nt}A_b}$$

$$= \frac{72}{0.75(90)(0.601)} = 1.77 < 4 \text{ bolts } \textbf{OK}$$

D. For the bearing limit state:
   Strength = 161 k (from Example 13.4) > 54 k **OK**

E. For the slip-critical limit state:
   1. Standard holes, $\phi = 1$
   2. No filler, $h_f = 1$
   3. Class A surface, $\mu = 0.3$
   4. From Table 13.1, $T_b = 39$ ksi
   5. For single shear, $N_s = 1$
   6. From Equation 13.15:

$$n_b = \frac{1}{D_u T_b}\left(\frac{V_u}{\phi\mu N s h_f} + T_u\right)$$

$$= \frac{1}{1.13(39)}\left[\frac{54}{(1)(0.3)(1)(1)} + 72\right]$$

$$= 5.72 \text{ bolts (select 6 bolts, 3 on each side of web)}$$

WELDED CONNECTIONS

Welding is a process in which the heat of an electric arc melts the welding electrode and the adjacent material of the part being connected simultaneously. The electrode is deposited as a filler metal into the steel, which is referred to as the *base metal*. There are two types of welding processes. The *shielded metal arc welding (SMAW)*, usually done manually, is the process used for field welding. The *submerged arc welding (SAW)* is an automatic or semiautomatic process used in shop welding. The strength of a weld depends on the weld metal used, which is the strength of the electrode used. An electrode is specified by the letter E followed by the tensile strength in ksi, and the last two digits specify the type of coating. Since strength is a main concern, the last two digits are specified by XX, a typical designation being E 70 XX. The electrode should be selected to have a larger tensile strength than the base metal (steel). For steel of 58 ksi strength, the electrode E 70 XX is used, and for 65 ksi steel, the electrode E 80 XX is used. Electrodes of high-strength E 120 XX are available.

The two common types of welds are fillet welds and groove or butt welds, as shown in Figure 13.19. Groove welds are stronger and more expensive than fillet welds. Most of the welded connections are made by fillet welds because of a larger allowed tolerance.

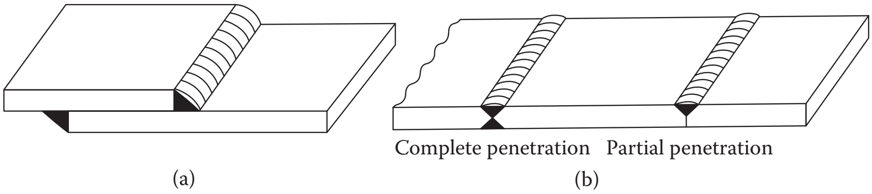


FIGURE 13.19 Types of welds: (a) fillet and (b) groove or butt welds.

The codes and standards for welds are prepared by the American Welding Society. With a few exceptions, these have been adopted in the 2017 AISC Manual.

GROOVE WELDS

Effective Area of Groove Weld

The effective area of a complete-joint-penetration (CJP) groove weld is the length times the thickness of the thinner part joined. The effective area of a partial-joint-penetration (PJP) groove weld is the length times the depth (effective throat) of the groove.[4] The minimum effective throat for PJP welds has been listed in the *AISC* 360-16. It is ⅛ in. for ¼ in. material thickness to ⅝ in. for over 6 in. thick material joined.

FILLET WELDS

Effective Area of Fillet Weld

The effective area is the effective length multiplied by the effective throat size. The cross section of a fillet weld is assumed to be a 45° right angle triangle, as shown in Figure 13.20. Any additional build-up of weld is neglected. The size of the fillet weld is denoted by the sides of the triangle, *w*, and the throat dimension, given by the hypotenuse, *t*, which is equal to 0.707*w*. When the SAW process is used, the greater heat input produces a deeper penetration.

The effective throat size is:

$$T_e = t = 0.707w \tag{13.16}$$

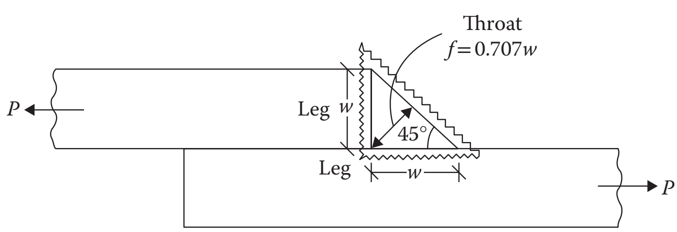


FIGURE 13.20 Fillet weld dimensions.

[4] For gas metal arc welds (GMAWs) and flux cored arc welds (FCAWs), the groove depth is subtracted by ⅛ in.

TABLE 13.5
Minimum Size of Weld (in.)

Base Material Thickness of Thinner Part (in.)	*w* (in.)
≤¼	⅛
>¼ to ≤½	3/16
≤½ to ≤¾	¼
>¾	5/16

Minimum Size of Fillet Weld

The minimum size of a fillet weld should not be less than the dimension shown in Table 13.5.

Maximum Size of Fillet Weld

1. Along the edges of material less than ¼ in. thick, the weld size should not be greater than the thickness of the material.
2. Along the edges of material ¼ in. or more, the weld size should not be greater than the thickness of the material less 1/16 in.

Length of Fillet Weld

1. The effective length of an end-loaded fillet weld, L_E, is equal to the actual length for the length up to 100 times the weld size. When the length exceeds 100 times the weld size, the actual length is multiplied by a reduction factor $\beta = 1.2 - 0.002\ (l/w)$, where l is the actual length and w is the weld size. When the length exceeds 300 times the weld size, the effective length is 180 w.
2. The minimum length should not be less than four times the weld size.
3. If only longitudinal welds are used, the length of each side should not be less than the perpendicular distance between the welds.

STRENGTH OF WELD

CJP Groove Welds

Since the weld metal is always stronger than the base metal (steel), the strength of a CJP groove weld is taken as the strength of the base metal. The design of the connection is not done in the normal sense.

For the combined shear and tension acting on a CJP groove weld, there is no explicit approach. The generalized approach is to reduce the tensile strength by a factor of $(f_v/F_t)^2$ subject to a maximum reduction of 36% of the tensile strength.

PJP Welds and Fillet Welds

A weld is weakest in shear and is always assumed to fail in the shear mode. Although a length of weld can be loaded in shear, compression, or tension, the failure of a weld is assumed to occur in the shear rupture through the throat of the weld. Thus:

$$P_u = \phi F_w A_w \tag{13.17}$$

where:

ϕ is the resistance factor = 0.75
F_w is the strength of the weld = $0.6F_{EXX}$ (F_{EXX} is the strength of the electrode)
A_w is the effective area of weld = T_eL

However, there is a requirement that the weld shear strength cannot be larger than the base metal shear strength. For the base metal, the shear yield and shear rupture strengths are taken to be 0.6 times the tensile yield of steel and 0.6 times the ultimate strength of steel, respectively. The yield strength is applied to the gross area, and the rupture strength is applied to the net area of shear surface; in the case of a weld, however, both areas are the same. The resistance factor is 1 for shear yield and 0.75 for shear rupture.

Thus, the PJP groove and the fillet welds should be designed to meet the strengths of the weld and the base metal, whichever is smaller, as follows:

1. Weld shear rupture limit state
   By the substitution of the terms in Equation 13.17:

$$P_u = 0.45F_{EXX}T_eL_E \tag{13.18}$$

   where:
   F_{EXX} is the strength of the electrode, ksi
   L_E is the effective length of the weld
   T_e is the effective throat dimension from Equation 13.16
2. Base metal shear limit state
   a. Shear yield strength:

$$P_u = 0.60F_ytL_E \tag{13.19}$$

   where t is the thickness of the thinner connected member
   b. Shear rupture strength:

$$P_u = 0.45F_utL_E \tag{13.20}$$

In addition, the block shear strength should also be considered using Equation 9.7.

Example 13.6

A tensile member consisting of one ∟ 3½ × 3½ × ½ section carries a service dead load of 30 k and live load of 50 k, as shown in Figure 13.21. A single ¾-in. plate is directly welded to the column flange using a CJP groove. Fillet welds attach the angles to the plate. Design the welded connection. The longitudinal length of the weld cannot exceed 5 in. Use the return (transverse) weld, if necessary. Use E70 electrodes. The steel is A36.

Solution

A. Angle-plate (bracket) connection
   1. Factored load

$$P_u = 1.2(30) + 1.6(50) = 116 \text{ k}$$

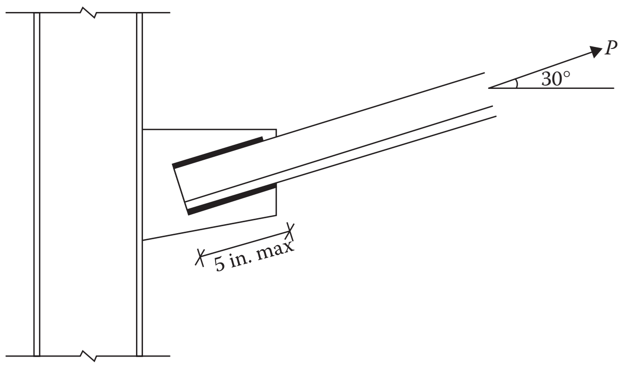


FIGURE 13.21 Column-bracket welded connection for Example 13.6.

2. Maximum weld size: For thinner member, thickness of angle, $t = ½$ in.

$$w = t - \left(\frac{1}{16}\right) = \frac{7}{16} \text{ in.}$$

3. Throat dimension, SMAW process

$$T_e = 0.707\left(\frac{7}{16}\right) = 0.309 \text{ in.}$$

4. For the weld shear limit state, from Equation 13.18:

$$L_E = \frac{P_u}{0.45 F_{EXX} T_e}$$

$$= \frac{116}{0.45(70)(0.309)} = 11.92 \text{ in.} \sim 12 \text{ in.} \leftarrow \text{Controls}$$

5. For the steel shear yield limit state, from Equation 13.19:

$$L_E = \frac{P_u}{0.6 F_y t}$$

$$= \frac{116}{0.6(36)\left(\frac{1}{2}\right)} = 10.74 \text{ in.}$$

6. For the steel rupture limit state, from Equation 13.20:

$$L_E = \frac{P_u}{0.45 F_u t}$$

$$= \frac{116}{0.45(58)\left(\frac{1}{2}\right)} = 8.9 \text{ in.}$$

7. Provide a 5-in.-long weld on each side[5] (maximum in this problem), with 1 in. return on each side.

[5] Theoretically, the lengths on two sides should be unequally distributed so that the centroid of the weld will pass through the center of gravity of the angle member.

8. The longitudinal length of welds (5 in.) should be at least equal to the transverse distance between the longitudinal weld on two sides (3½). **OK**
9. Length of 12 in. is greater than $4w$ or $4 \times \frac{7}{16}$ that is 1.75 in. **OK**

B. Column-plate connection

1. The connection is subjected to tension and shear as follows:

$$T_u = P_u \cos 30° = 116 \cos 30° = 100.5 \text{ k}$$

$$V_u = P_u \sin 30° = 116 \sin 30° = 58 \text{ k}$$

2. For the CJP groove, the design strengths are the same as for the base metal.
3. This is the case of the combined shear and tension in a groove weld. Using a maximum reduction of 36%,[6] the tensile strength = $0.76F_t$.
4. For the base material tensile limit state:

$$T_u = \phi(0.76F_t)tL$$

where t is gusset plate thickness

$$100.5 = 0.9(0.76)(36)\left(\frac{3}{4}\right)L$$

or

$$L = 5.44\text{in.} \leftarrow \text{Controls}$$

Use a 6 in. weld length.

5. For the base metal shear yield limit state:

$$V_u = 0.6F_y tL$$

$$58 = 0.6(36)\left(\frac{3}{4}\right)L$$

or

$$L = 3.6 \text{ in.}$$

6. For the base metal shear rupture limit state:

$$V_u = 0.45F_u tL$$

$$58 = 0.45(58)\left(\frac{3}{4}\right)L$$

or

$$L = 3.0 \text{ in.}$$

[6] See the "CJP Groove Welds" section.

FRAME CONNECTIONS

There are three types of beam-to-column frame connections:

1. Fully restrained (FR) or rigid frame or moment frame connection
   - It transfers the full joint moment and shear force.
   - It retains the original angle between the members or rotation is not permitted.
2. Simple or pinned frame or shear frame connection
   - It transfers shear force only.
   - It permits rotation between the members.
3. Partially restrained (PR) frame connection
   - It transfers some moment and the entire shear force.
   - It permits a specified amount of rotation.

The relationship between the applied moment and the rotation (variation of angle) of members for rigid, semirigid, and simple framing is shown in Figure 13.22.

A fully rigid joint has a small change in angle with the application of moment. A simple joint can support some moment (although theoretically the moment capacity should be zero). A semirigid joint is where the actual moment and rotation are accounted for.

SHEAR OR SIMPLE CONNECTION FOR FRAMES

A variety of beam-to-column or beam-to-girder connections is purposely made flexible for rotation at the ends of the beam. These connections are designed for the end reaction (shear force). They are used for structures where the lateral forces due to wind or earthquake are resisted by the other systems, like truss framing or shear walls. Following are the main categories of simple connections.

Single-Plate Shear Connection or Shear Tab

This simple and economical approach is becoming very popular. The holes are pre-punched in a plate and in the web of the beam to be supported. The plate is welded (usually shop welded) to the supporting column or beam. The pre-punched beam is bolted to the plate at the site. An example is shown in Figure 13.23.

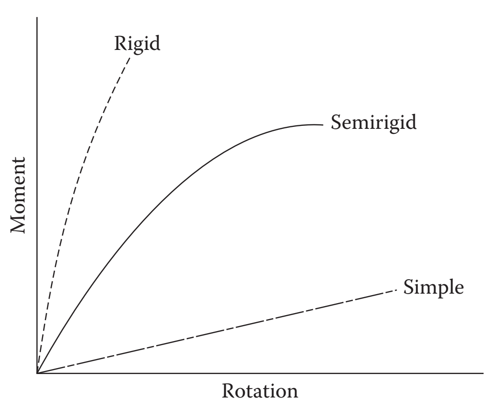


FIGURE 13.22 Moment-rotation characteristics.

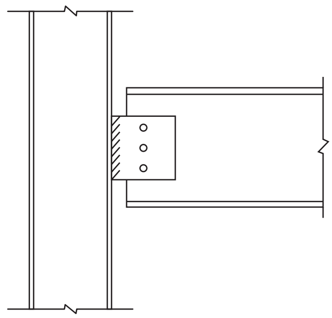


FIGURE 13.23 Single-plate or shear tab connection.

Framed-Beam Connection

The web of the beam to be supported is connected to the supporting column through a pair of angles, as shown in Figure 13.24.

Seated-Beam Connection

The beam to be supported sits on an angle attached to the supporting column flange, as shown in Figure 13.25.

End-Plate Connection

A plate is welded against the end of the beam to be supported. This plate is then bolted to the supporting column or beam at the site. An end-plate connection is shown in Figure 13.26. These connections are becoming popular but not as much as the single-plate connection.

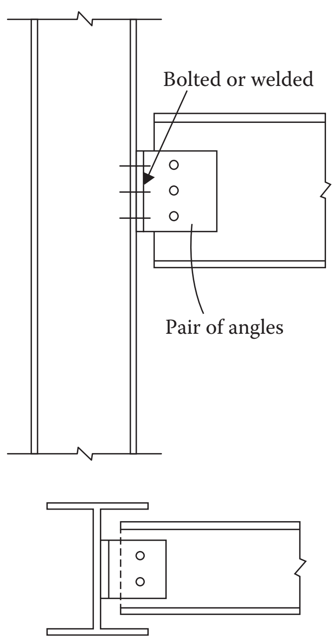


FIGURE 13.24 Framed-beam connection.

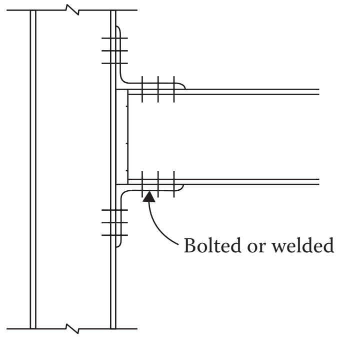


FIGURE 13.25 Seated-beam connection.

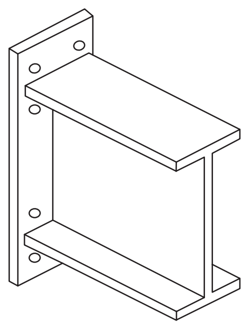


FIGURE 13.26 End-plate connection.

The design of the simple connections proceeds along the lines of the bearing-type connections described in the "Bearing-Type Connections" section. The limiting states considered are: (1) shear on bolts; (2) bearing yield strength; (3) shear rupture strength between the bolt and the connected part, as discussed in the "Bearing-Type Connections" section; and (4) block shear strength of the connected part.

The 2017 AISC Manual includes a series of tables to design the different types of bolted and welded connections. The design of only a single-plate shear connection for frames is presented here.

SINGLE-PLATE SHEAR CONNECTION FOR FRAMES

The following are the conventional configurations for a single-plate shear connection:

1. A single row of bolts that includes two to twelve bolts.
2. The distance between the bolt line and weld line should not exceed 3.5 in.
3. Provision of the standard or short-slotted holes.
4. The horizontal distance to edge $L_e \geq 2d_b$ (bolt diameter).
5. The plate and beam must satisfy $t \leq (d_b/2) + (1/16)$.
6. For welded connections, the weld shear rupture and the base metal shear limits should be satisfied.
7. For bolted connections, the bolt shear, the plate shear, and the bearing limit states should be satisfied.
8. The block shear of the plate should be satisfactory.

Example 13.7

Design a single-plate shear connection for a W14 × 82 beam joining a W12 × 96 column by a ⅜-in. plate, as shown in Figure 13.27. The factored reaction at the support of the beam is 50 k. Use ¾-in.-diameter Group A: A325-X bolts, A36 steel, and E70 electrodes.

Solution

A. Design load

$P_u = R_u = 50$ k

B. For W14 × 82:

$d = 14{,}3$ in., $tw = 0.51$ in.
$t_f = 0.855$ in., $b_f = 14.7$ in.
$F_y = 36$ ksi, $F_u = 58$ ksi

C. For W12 × 96:

$d = 12.7$ in., $t_w = 0.55$ in.
$t_f = 0.9$ in., $b_f = 12.2$ in.
$F_y = 36$ ksi, $F_y = 58$ ksi

D. Column-plate welded connection

1. For ⅜-in. plate:

$$\text{Weld max size} = t - \left(\frac{1}{16}\right) = \left(\frac{3}{8}\right) - \left(\frac{1}{16}\right) = \left(\frac{5}{16}\right) \text{ in.}$$

2. $T_e = 0.707\,(5/16) = 0.22$ in.
3. For the weld shear limit state, from Equation 13.18:

$$L_E = \frac{P_u}{0.45 F_{EXX} T_e}$$

$$= \frac{50}{0.45(70)(0.22)} = 7.21 \text{ in.} \sim 8 \text{ in.} \leftarrow \text{Controls}$$

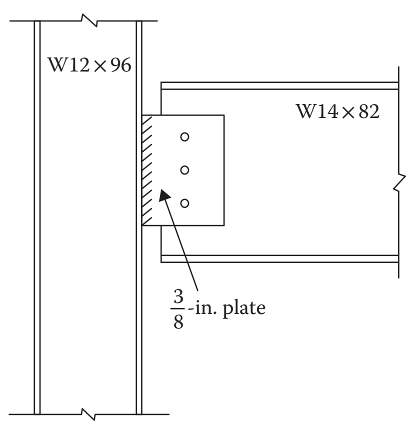


FIGURE 13.27 Single-plate connection for Example 13.7.

4. For the steel shear yield limit state, from Equation 13.19:

$$L_E = \frac{P_u}{0.6F_y t}$$

$$= \frac{50}{0.6(36)\left(\frac{3}{8}\right)} = 6.17 \text{ in.}$$

5. For the steel rupture limit state, from Equation 13.20:

$$L_E = \frac{P_u}{0.45F_u t}$$

$$= \frac{50}{0.45(58)\left(\frac{3}{8}\right)} = 5.1 \text{ in.} < 8 \text{ in.}$$

6. Up to 100 times of the weld size (in this case 100 ($\frac{5}{16}$) = 31.25 in.), effective length is equal to actual length; hence $L = L_E = 8$ in.
7. L of 8 in. > $4w$ or $4 \times \frac{5}{8}$, that is, 1.25 in. **OK**

E. Beam-plate bolted connection

E.1 The single shear limit state

1. $A_b = (\pi/4)(\frac{3}{4})^2 = 0.441$ in.^{2}
2. For A325-X, from Table 13.2 $F_{nv} = 68$ ksi
3. From Equation 13.3:

$$\text{No. of bolts} = \frac{P_u}{0.75F_{nv}A_b}$$

$$= \frac{50}{0.75(68)(0.441)} = 2.22 \text{ or 3 bolts}$$

E.2 The bearing limit state

1. Minimum edge distance

$$L_e = 1\frac{3}{4}d_b = 1\frac{3}{4}\left(\frac{3}{4}\right) = 1.31 \text{in., use } 1.5 \text{ in.}$$

2. Minimum spacing

$$s = 3d_b = 3\left(\frac{3}{4}\right) = 2.25 \text{in., use 2.5 in.}$$

3. $h = d + \dfrac{1}{16} = 0.813$ in.

4. For holes near the edge:

$$L_c = L_e - \left(\frac{h}{2}\right)$$

$$= 1.5 - \left(\frac{0.813}{2}\right) = 1.093 \text{ in.}$$

$t = \frac{3}{8}$ in. thinner member
For a standard size hole of deformation <0.25 in.:

$$\text{Tearout strength per bolt} = 1.2\varphi L_c\, tF_u$$
$$= 1.2(0.75)(1.093)(3/8)(58) = 21.4 \leftarrow \text{Controls}$$

$$\text{Bearing strength per bolt} = 2.4\phi dtF_u$$

$$= 2.4(0.75)\left(\frac{3}{4}\right)\left(\frac{3}{8}\right)(58) = 29.36\text{ k}$$

5. For other holes:

   $$L_c = s - h = 2.5 - 0.813 = 1.687\text{ in.}$$

6. $\text{Tearout strength per bolt} = 1.2(0.75)(1.687)\left(\frac{3}{8}\right)(58) = 33.02\text{ k}$

   $$\text{Bearing strength per bolt} = 2.4(0.75)\left(\frac{3}{4}\right)\left(\frac{3}{8}\right)(58) = 29.36\text{ k} \leftarrow \text{Controls}$$

7. Total strength for three bolts/two near edges

   $P_u = 2(21.4) + 29.36 = 72.16 > 50$ k **OK**

8. The section has to be checked for block shear by the procedure given in Chapter 9.

MOMENT-RESISTING CONNECTION FOR FRAMES

Fully restrained (rigid) and partially restrained (semirigid) are two types of moment-resisting connections. It is customary to design a semirigid connection for some specific moment capacity, which is less than the full moment capacity.

Figure 13.28 shows a moment-resisting connection that has to resist a moment, *M*, and a shear force (reaction), *V*. The two components of the connection are designed separately. The moment is transmitted to the column flange as a couple by the two tees attached at the top and bottom flanges

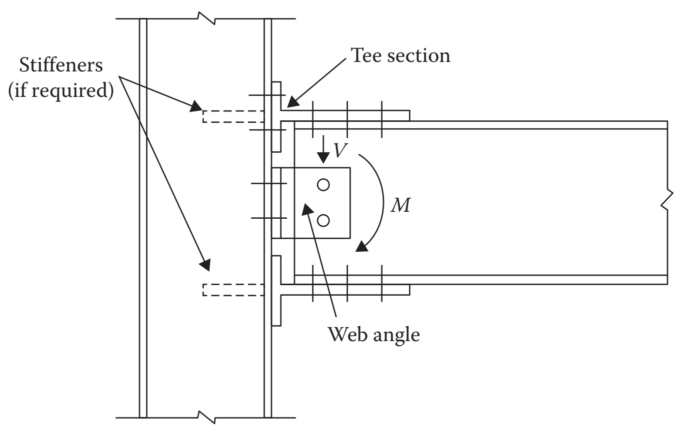


FIGURE 13.28 Moment-resisting connection.

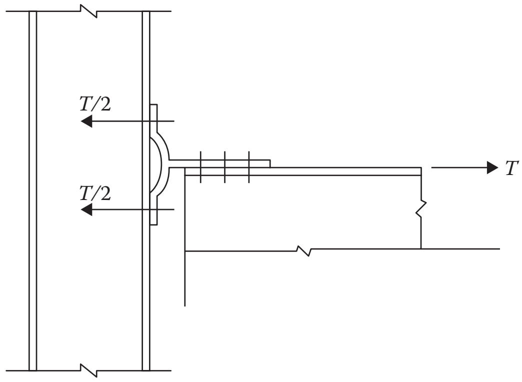


FIGURE 13.29 Prying action in connection.

of the beam. This results in tension, T, on the top flange and compression, C, on the bottom flange. From the couple expression, the two forces are given by:

$$C = T = \frac{M}{d} \tag{13.21}$$

where d is taken as the depth of the beam.

The moment is taken care of by designing the tee connection for the tension T. Note that the magnitude of the force T can be decreased by increasing the distance between the tees (by a deeper beam).

The shear load is transmitted to the column by the beam–web connection. This is designed as a simple connection of the type discussed in the "Shear or Simple Connection for Frames" section. This can be a single plate, two angles (framed), or seat angle connection.

The connecting tee element is subjected to prying action, as shown in Figure 13.29. This prying action could be eliminated by connecting the beam section directly to the column through a CJP groove weld, as shown in Figure 13.30.

The welded length should not exceed the beam flange width, b_f, of both the beam and the column; otherwise, a thicker plate has to be welded at the top and bottom of the beam.

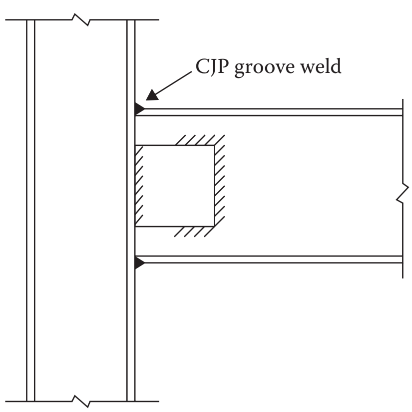


FIGURE 13.30 Welded moment-resisting connection.

Example 13.8

Design the connection of Example 13.7 as a moment-resisting connection subjected to a factored moment of 200 ft-k and a factored end shear force (reaction) of 50 k. The beam flanges are groove welded to the column.

Solution

A. Design for the shear force has been done in Example 13.7.
B. Flanges welded to the column:

1. $C = T = \dfrac{M}{d}$

$$= \frac{200(12)}{14.3} = 167.83 \text{ k}$$

2. The base material limit state

$$T_u = \phi FytL, \text{where } t = t_f$$

or

$$L = \frac{T_u}{\phi Fyt}$$

$$= \frac{167.83}{(0.9)(36)(0.855)} = 6.06 \text{ in.} < b_f$$

Provide a 6-in.-long CJP weld.

PROBLEMS

13.1 Determine the strength of the bearing-type connection shown in Figure P13.1. Use A36 steel, Group A: A325, ⅞-in. bolts. The threads are not excluded from the shear plane. Deformation of the hole is a consideration.

13.2 Determine the strength of the bearing-type connection shown in Figure P13.2. Use A36 steel, Group A: A325, ⅞-in. bolts. The threads are excluded from the shear plane. Deformation of holes is not a consideration.

13.3 Design the bearing-type connection for the bolt joint shown in Figure P13.3. The steel is A572 and the bolts are Group A: A325, ¾-in. diameter. The threads are excluded from the shear plane. Deformation of holes is a consideration.

13.4 A chord of a truss shown in Figure P13.4 consists of 2 C9 × 20 of A36 steel connected by a 1-in. gusset plate. Check the bearing-type connection by Group B: A490 bolts and assume that threads are excluded from the shear plane. Deformation of holes is not a consideration.

13.5 Design the bearing-type connection shown in Figure P13.5 (threads excluded from the shear plane) made with ⅞-in. Group B: A490 bolts. Use A572 steel. Deformation of holes is a consideration.

13.6 Solve Problem 13.1 for the slip-critical connection of unpainted clean mill scale surface. The holes are standard size, and there are no fillers.

13.7 Solve Problem 13.2 for the slip-critical connection of unpainted blast cleaned surface. The holes are standard size. Two fillers are used between connected members.

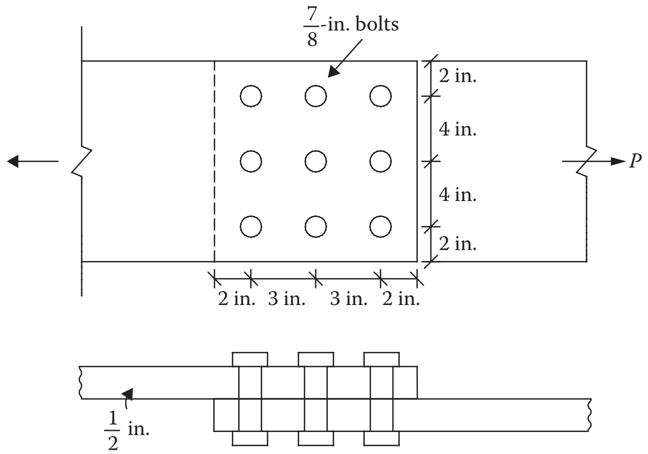


FIGURE P13.1 Connection for Problem 13.1.

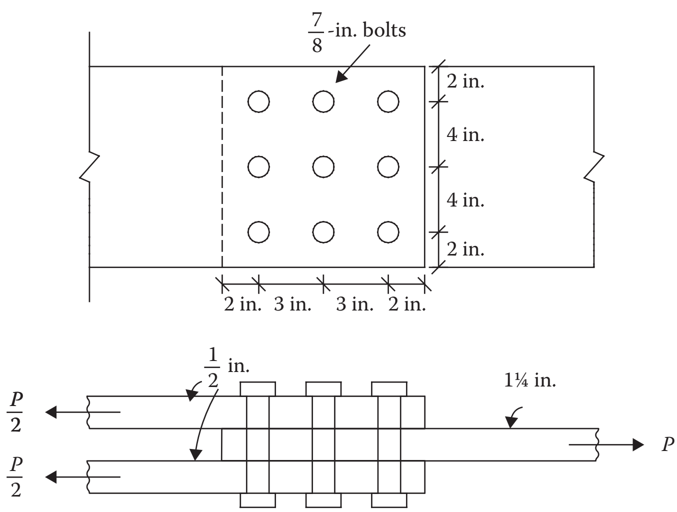


FIGURE P13.2 Connection for Problem 13.2.

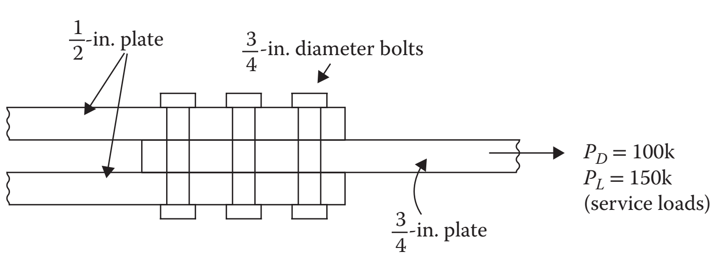


FIGURE P13.3 Connection for Problem 13.3.

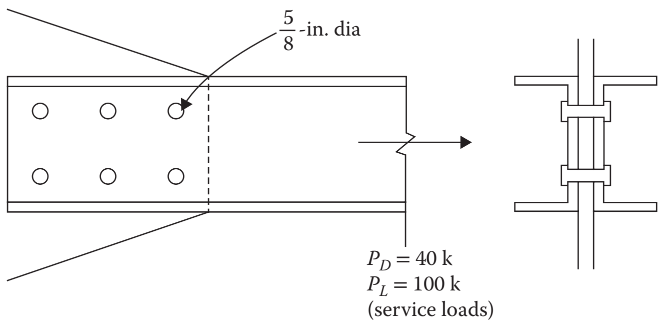


FIGURE P13.4 Truss chord connection for Problem 13.4.

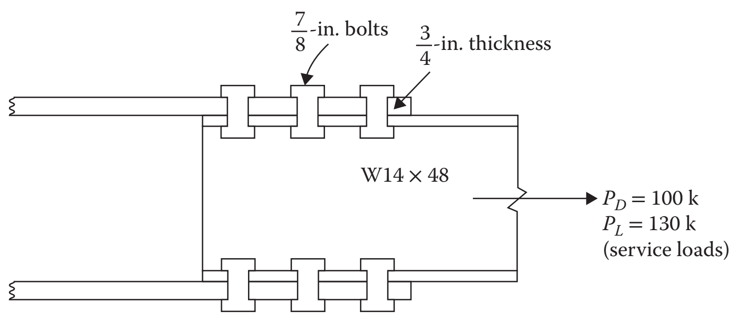


FIGURE P13.5 Connection for Problem 13.5.

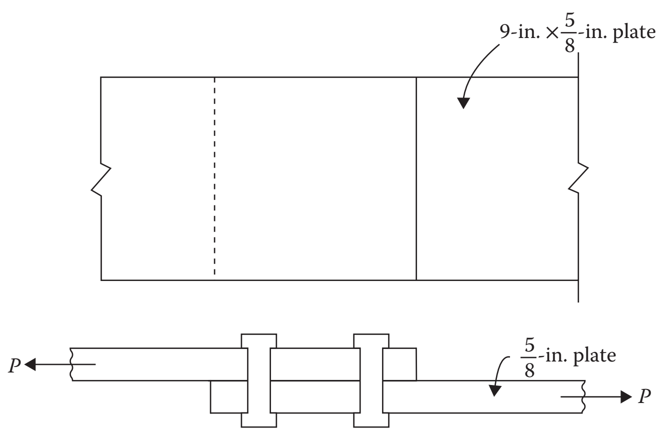


FIGURE P13.6 Connection for Problem 13.8.

13.8 Design a slip-critical connection for the plates shown in Figure P13.6 to resist a service dead load of 30 k and a live load of 50 k. Use 1-in. Group A: A325 bolts and A572 steel. Assume a painted class A surface. The holes are standard size. There are no fillers. The threads are excluded from the shear plane and hole deformation is a consideration.

13.9 A single angle 3½ × 3 × ¼ tensile member is connected by a ⅜-in.-thick gusset plate. Design a no-slip (slip-critical) connection for the service dead and live loads of 8 and 24 k, respectively. Use ⅞-in. Group A: A325 bolts and A36 steel. Assume an unpainted blast cleaned surface. The holes are standard size. There is one filler. The threads are not excluded from the shear plane, and the hole deformation is not a consideration.

13.10 The tensile member shown in Figure P13.7 consists of two ∟ 4 × 3½ × ½ and carries a wind load of 176 k acting at 30°. A bracket consisting of a tee section connects this tensile member to a column flange. The connection is slip-critical. Design the bolts for the tensile member only. Use ⅞-in. Group B: A490-X bolts and A572 steel. Assume an unpainted blast cleaned surface. The holes are short-slotted parallel to the direction of loading. There are no fillers, and hole deformation is not a consideration.

13.11 Determine the strength of the bolts in the hanger connection shown in Figure P13.8. Neglect the prying action.

13.12 Are the bolts in the hanger connection adequate in Figure P13.9?

13.13 A WT12 × 31 is attached to a ¾-in. plate as a hanger connection to support service dead and live loads of 25 k and 55 k, respectively. Design the connection for ⅞-in. Group A: A325 bolts and A572 steel. Neglect the prying action.

In Problems 13.14 through 13.16, the threads are excluded from shear planes, and deformation is a consideration.

13.14 Design the column-to-bracket connection from Problem 13.10. Slip is permitted.

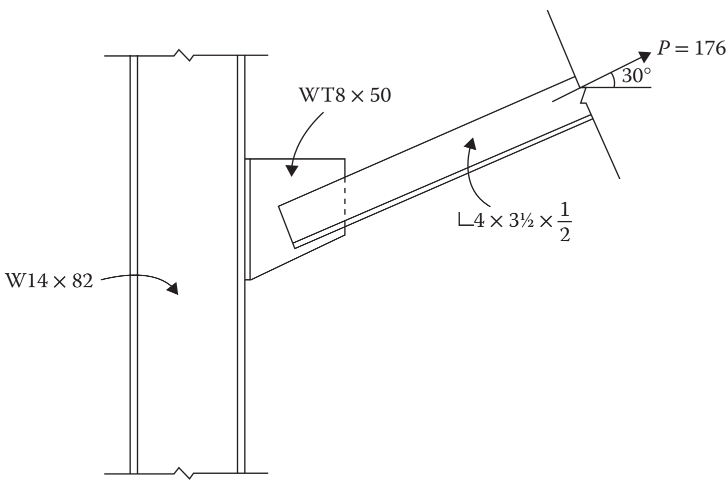


FIGURE P13.7 Connection for Problem 13.10.

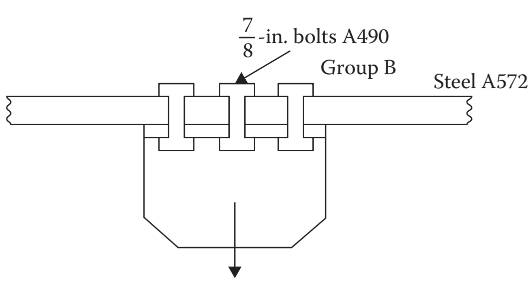


FIGURE P13.8 Hanger-type connection for Problem 13.11.

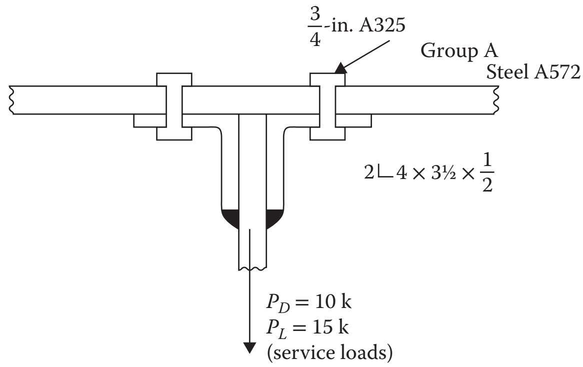


FIGURE P13.9 Hanger-type connection for Problem 13.12.

13.15 In the bearing-type connection shown in Figure P13.10, determine the load capacity, P_u.

13.16 A tensile member is subjected to service dead and live loads of 30 k and 50 k, respectively, through a ⅞-in. plate, as shown in Figure P13.11. Design the bearing-type connection. The steel is A572 and the bolts are ¾-in., Group B: A490-X.

In Problems 13.17 through 13.19, the connecting surface is unpainted clean mill scale. The holes are standard size and there are no fillers.

13.17 Design the connection from Problem 13.14 as a slip-critical connection.

13.18 Solve Problem 13.15 as a slip-critical connection.

13.19 Design the connection in Problem 13.16 as a slip-critical connection. Bolts are pre-tensioned to 40 k.

13.20 Determine the design strength of the connection shown in Figure P13.12. The steel is A572, and the electrodes are E 70.

13.21 In Problem 13.20, the applied loads are a dead load of 50 k and a live load of 150 k. For the longitudinal welding shown in Figure P13.12, determine the thickness of the plates. The weld size is based on plate thickness.

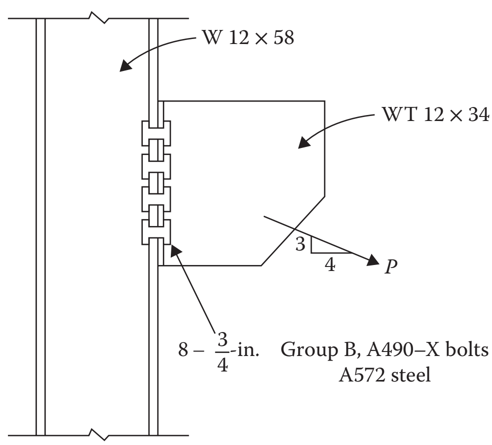


FIGURE P13.10 Combined shear-tension connection for Problem 13.15.

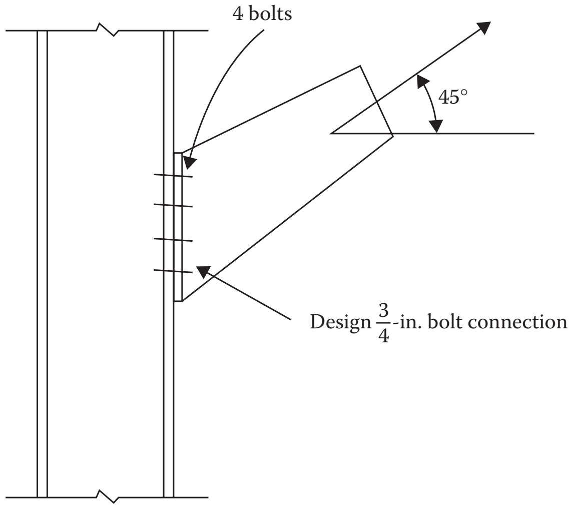


FIGURE P13.11 Combined shear-tension connection for Problem 13.16.

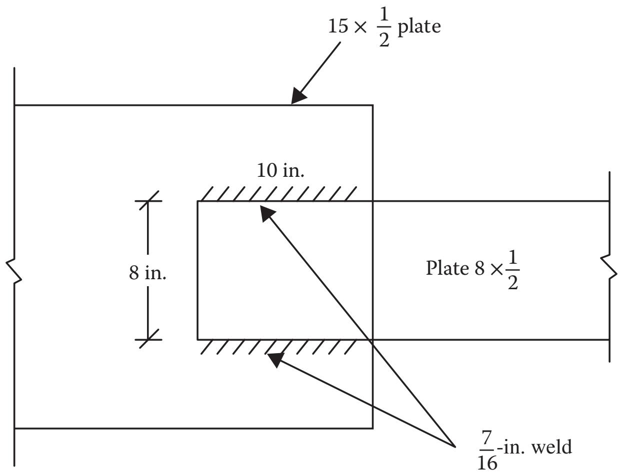


FIGURE P13.12 Welded connection for Problem 13.20.

13.22 A ¼-in.-thick flat plate is connected to a gusset plate of $\frac{5}{16}$-in. thickness by a $\frac{3}{16}$-in. weld, as shown in Figure P13.13. The maximum longitudinal length is 4 in. Use the return (transverse) weld, if necessary. The connection has to resist a dead load of 10 k and a live load of 20 k. What is the length of the weld? Use E 70 electrodes. The steel is A36.

13.23 Two ½ in. × 10 in. A36 plates are to be connected by a lap joint for a factored load of 80 k. Use E 80 electrodes. The steel is A36. Determine the weld size for the entire width (transverse) welding of the plate.

13.24 The plates in Problem 13.23 are welded as a PJP butt connection. The minimum effective throat width according to AISC specifications is $\frac{3}{16}$ in. Is the width adequate or does it need to be increased?

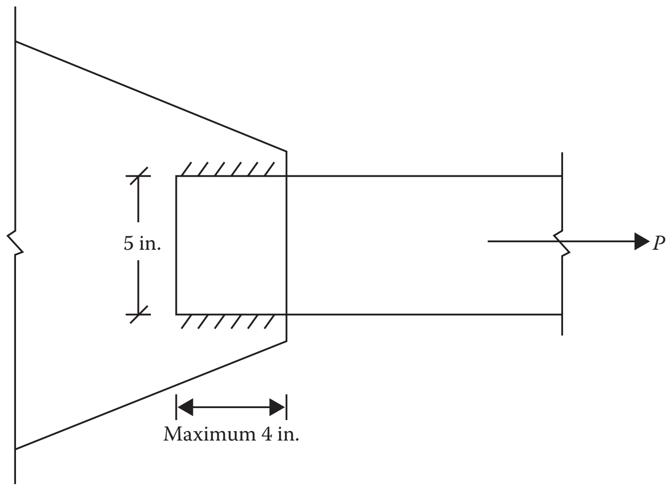


FIGURE P13.13 Welded connection for Problem 13.22.

13.25 Design the longitudinal fillet welds to connect the ∟ 4 × 3 × ½ angle tensile member shown in Figure P13.14 to resist a service dead load of 50 k and live load of 80 k. Use E 70 electrodes. The steel is A572.

13.26 A tensile member consists of 2 ∟ 4 × 3 × ½ carries a service dead load of 50 k and live load of 100 k, as shown in Figure P13.15. The angles are welded to a ¾-in. gusset plate, which is welded to a column flange. Design the connection of the angles to the gusset plate and the gusset plate to the column. The gusset plate is connected to the column by a CJP groove, and the angles are connected by a fillet weld. Use E 70 electrodes. The steel is A572.

13.27 Design a single-plate shear connection for a W14 × 53 beam joining a W14 × 99 column by a ¼-in. plate. The factored reaction is 60 k. Use A36 steel. Use ⅝-in. Group A: A325-X bolts and E70 welds. The deformation of the hole is a consideration.

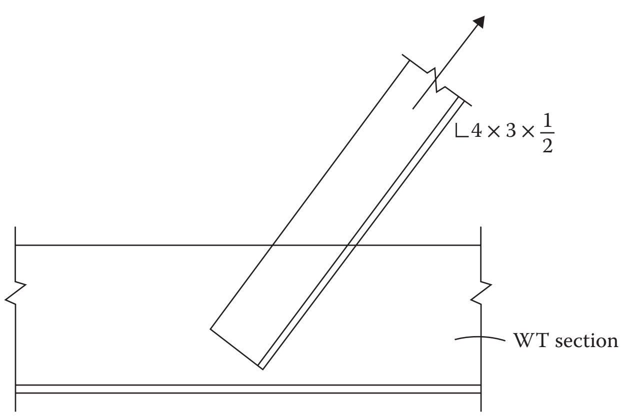


FIGURE P13.14 Welded connection for Problem 13.25.

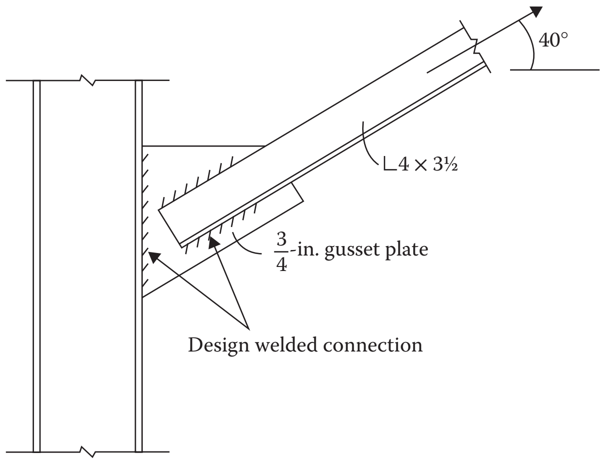


FIGURE P13.15 Welded connection for Problem 13.26.

13.28 Design a single-plate shear connection for a W16 × 67 beam joining a W18 × 71 column by a ½-in. plate to support a factored beam reaction of 70 k. Use ¾-in. Group B: A490-X bolts and E80 welds. The beam, columns, and plate have A992 steel. The deformation of hole is a consideration.

13.29 Design the connection for Problem 13.27 as a moment connection to resist a factored moment of 200 ft-k in addition to the factored reaction of 60 k.

13.30 Design the connection for Problem 13.28 as a moment-resisting connection to resist a factored moment of 300 ft-k and a factored shear force of 70 k.

14 Flexural Reinforced Concrete Members

PROPERTIES OF REINFORCED CONCRETE

Concrete is a mixture of cement, sand, gravel, crushed rock, and water. Water reacts with cement in a chemical reaction known as *hydration* that sets the cement with other ingredients into a solid mass, high in compression strength. The strength of concrete depends on the proportion of the ingredients. The most important factor for concrete strength is the water–cement ratio. More water results in a weaker concrete. However, an adequate amount is needed for concrete to be workable and easy to mix. An adequate ratio is about 0.25 by weight. The process of selecting the relative amounts of ingredients for concrete to achieve a required strength at the hardened state and to be workable in the plastic (mixed) state is known as *concrete mix design*. The specification of concrete in terms of the proportions of cement, fine (sand) aggregate, and coarse (gravel and rocks) aggregate is called the *nominal mix*. For example, a 1:2:4 nominal mix has one part cement, two parts sand, and four parts gravel and rocks by volume. Nominal mixes having the same proportions could vary in strength. For this reason, another expression for specification known as the *standard mix* uses the minimum compression strength of concrete as a basis.

The procedure for designing a concrete mix is a trial-and-error method. The first step is to fix the water–cement ratio for the desired concrete strength using an empirical relationship between the compressive strength and the water–cement ratio. Then, based on the characteristics of the aggregates and the proportioning desired, the quantities of the other materials, comprising cement, fine aggregate, and coarse aggregate, are determined.

There are some other substances that are not regularly used in the proportioning of the mix. These substances, known as *mixtures*, are usually chemicals that are added to change certain characteristics of concrete such as accelerating or slowing the setting time, improving the workability of concrete, and decreasing the water–cement ratio.

Concrete is quite strong in compression, but it is very weak in tension. In a structural system, steel bars are placed in the tension zone to compensate for this weakness. Such concrete is known as *reinforced concrete*. At times, steel bars are also used in the compression zone to give extra strength with a leaner concrete size, as in reinforced concrete columns and doubly reinforced beams.

COMPRESSION STRENGTH OF CONCRETE

The strength of concrete varies with time. The specified compression strength, denoted as f_c', is the value that concrete attains 28 days after the placement. Beyond that stage, the increase in strength is very small. The strength, f_c', ranges from 2500 to 9000 psi, with a common value between 3000 and 5000 psi.

The stress–strain diagram of concrete is not linear to any appreciable extent; thus, concrete does not behave elastically over a major range. Moreover, concrete of different strengths have stress–strain curves that have different slopes. Therefore, in concrete, the modulus of elasticity cannot be ascertained directly from a stress–strain diagram.

The American Concrete Institute (ACI) is a primary agency in the United States that prepares national standards for structural concrete, namely, the Building Code Requirements for Structural Concrete, 318-14. It provides the empirical relations for the modulus of elasticity based on the compression strength, f_c'.

Although the stress–strain curves have different slopes for concrete of different strengths, the following two characteristics are common to all types of concrete:

1. The maximum compression strength, f_c', in all concrete is attained at a strain level of approximately 0.002 in./in.
2. The point of rupture of all curves lies in the strain range of 0.003–0.004 in./in. Thus, it is assumed that concrete fails at a strain level of 0.003 in./in.

DESIGN STRENGTH OF CONCRETE

To understand the development and distribution of stress in concrete, let us consider a simple rectangular beam section with steel bars at the bottom (in the tensile zone), which is loaded by an increasing transverse load.

The tensile strength of concrete is small, and thus concrete soon cracks at the bottom at a low transverse load. The stress at this level is known as the *modulus of rupture*, and the bending moment is referred to as the *cracking moment*. Beyond this level, the tensile stress is handled by the steel bars and the compression stress by the concrete section above the neutral axis. Concrete is a brittle (not a ductile) material, and the distribution of stress within the compression zone could be considered linear only up to a moderate load level, when the stress attained by concrete is less than ½ f_c', as shown in Figure 14.1. In this case, the stress and strain bear a direct proportional relationship.

As the transverse load increases further, the strain distribution remains linear (Figure 14.2b), but the stress distribution acquires a curvilinear shape similar to the shape of the stress–strain curve. As the steel bars reach the yield level, the distribution of strain and stress at this load is as shown in Figure 14.2b and c.

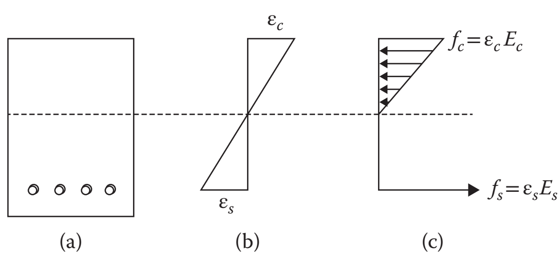


FIGURE 14.1 Stress–strain distribution at moderate loads: (a) section, (b) strain, and (c) stress.

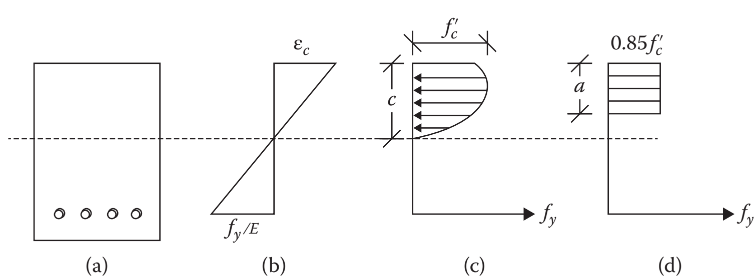


FIGURE 14.2 Stress–strain distribution at ultimate load: (a) section, (b) strain, (c) stress, and (d) equivalent stress.

For simplification, Whitney (1942) proposed a fictitious but equivalent rectangular stress distribution of intensity 0.85 f_c', as shown in Figure 14.2d. This has since been adopted by the ACI. The property of this rectangular block of depth *a* is such that the centroid of this rectangular block is the same as the centroid of the actual curved shape and that the area under the two diagrams in Figure 14.2c and d are the same. Thus, for design purposes, the ultimate compression of concrete is taken to be 0.85 f_c', uniformly distributed over the depth, *a*.

STRENGTH OF REINFORCING STEEL

The steel bars used for reinforcing are round, deformed bars with some form of patterned ribbed projections onto their surfaces. The bar sizes are designated from #3 through #18. For #3 to #8 sizes, the designation represents the bar diameter in one-eighths of an inch; that is, the #5 bar has a diameter of ⅝ in. The #9, #10, and #11 sizes have diameters that provide areas equal to the areas of the 1 in. × 1 in. square bar, 1⅛ in. × 1⅛ in. square bar, and 1¼ in. × 1¼ in. square bar, respectively. Sizes #14 and #18 are available only by special order. They have diameters equal to the areas of a 1½ in. × 1½ in. square bar and 2 in. × 2 in. square bar, respectively. The diameter, area, and unit weight per foot for various sizes of bars are given in Appendix D.1.

The most useful properties of reinforcing steel are the yield stress, f_y, and the modulus of elasticity, *E*. A large percentage of reinforcing steel bars are not made from new steel but are rolled from melted, reclaimed steel. These are available in different grades. Grade 40, Grade 50, and Grade 60 are common. Grade 40 means that the steel has a yield stress of 40 ksi, and so on. A new provision in ACI 318-14 for high-strength reinforcement that does not have a sharply defined yield point, is that the yield strength f_y should be taken as 0.2% proof stress (the value of stress where the line at 0.002 strain drawn parallel to the initial stress–strain slope meets the stress–strain curve).

The modulus of elasticity of reinforcing steel of different grades varies over a very small range. It is adopted as 29,000 ksi for all grades of steel.

Concrete structures are composed of the beams, columns, or column–beam types of structures where they are subjected to flexure, compression, or the combination of flexure and compression. The theory and design of simple beams and columns have been presented in the book.

LOAD RESISTANCE FACTOR DESIGN BASIS OF CONCRETE

Until mid-1950, concrete structures were designed by the elastic or working stress design (WSD) method. The structures were proportioned so that the stresses in concrete and steel did not exceed a fraction of the ultimate strength, known as the *allowable* or *permissible stresses*. It was assumed that the stress within the compression portion of concrete was linearly distributed. However, beyond a moderate load, when the stress level is only about one-half the compressive strength of concrete, the stress distribution in a concrete section is not linear.

In 1956, the ACI introduced a more rational method wherein the members were designed for a nonlinear distribution of stress and the full strength level was to be explored. This method was called the ultimate strength design (USD) method. Since then, the name has been changed to the *strength design method*.

The same approach is known as the load resistance factor design (LRFD) method in steel and wood structures. Thus, concrete structures were the first ones to adopt the LRFD method of design in the United States.

ACI Publication No. 318, revised numerous times, contains the codes and standards for concrete buildings. ACI 318-56 of 1956 for the first time included the codes and standards for USD in an appendix to the code. ACI 318-63 provided equal status to WSD and USD methods, bringing both of them within the main body of the code. ACI 318-02 code made USD, with the name changed to the strength design method, the mandatory method of design. ACI 318-14 provides the latest design provisions.

In the strength design method, the service loads are amplified using the load factors. The member's strength at failure, known as the theoretical or the nominal capacity, is somewhat reduced by a strength reduction factor to represent the usable strength of the member. The amplified loads must not exceed the usable strength of the member, namely:

$$\text{Amplified loads on member} \le \text{usable strength of member} \tag{14.1}$$

Depending on the type of structure, the loads are the compression forces, shear forces, or bending moments.

REINFORCED CONCRETE BEAMS

A concrete beam is a composite structure where a group of steel bars are embedded into the tension zone of the section to support the tensile component of the flexural stress. The areas of the group of bars are given in Appendix D.2. The minimum widths of beam that can accommodate a specified number of bars in a single layer are indicated in Appendix D.3. These tables are very helpful in designs.

In the case of beams, Equation 14.1 takes the following form, similar to wood and steel structures:

$$M_u \le \phi M_n \tag{14.2}$$

where:

M_u is the maximum moment due to the application of the factored loads
M_n is the nominal or theoretical capacity of the member
ϕ is the strength reduction (resistance) factor for flexure

According to the flexure theory, $M_n = F_b S$, where F_b is the ultimate bending stress and S is the section modulus of the section. The application of this formula is straightforward for a homogeneous section for which the section modulus or the moment of inertia could be directly found. However, for a composite concrete–steel section and a nonlinear stress distribution, the flexure formula presents a problem. A different approach, termed the *internal couple method*, is followed for concrete beams.

In the internal couple method, two forces act on the beam cross section represented by a compressive force, C, acting on one side of the neutral axis (above the neutral axis in a simply supported beam) and a tensile force, T, acting on the other side. Since the forces acting on any cross section of the beam must be in equilibrium, C must be equal and opposite of T, thus representing a couple. The magnitude of this internal couple is the force (C or T) times the distance, Z, between the two forces, called the *moment arm*. This internal couple must be equal and opposite to the bending moment acting at the section due to the external loads.

This is a common approach for determining the nominal moment, M_n, in concrete structures.

DERIVATION OF THE BEAM RELATIONS

The stress distribution across a beam cross section at the ultimate load is shown in Figure 14.3, representing the concrete stress by a rectangular block as stated in the "Design Strength of Concrete" section.

The ratio of stress block and depth to the neutral axis is defined by a factor β_1, as follows:

$$\beta_1 = \frac{a}{c} \tag{14.3}$$

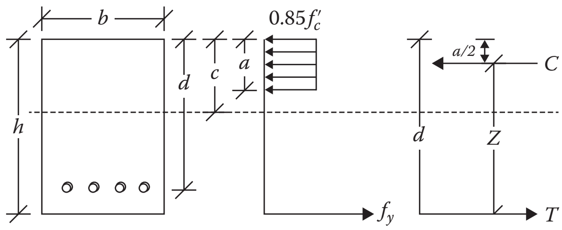


FIGURE 14.3 Internal forces and couple acting on a section.

Sufficient test data are available to evaluate β_1. According to the ACI:

1. For $f_c' \le 4000$ psi, $\beta_1 = 0.85$. (14.4a)
2. For $f_c' > 4000$ psi but ≤ 8000 psi, (14.4b)

$$\beta_1 = 0.85 - \left(\frac{f_c' - 4000}{1000}\right)(0.05).$$

3. For $f' > 8000$ psi, $\beta_1 = 0.65$. (14.4c)

With reference to Figure 14.3, since force = (stress)(area):

$$C = (0.85 f_c')(ab) \tag{a}$$

$$T = f_y A_s \tag{b}$$

Since $C = T$:

$$(0.85 f_c')(ab) = f_y A_s \tag{c}$$

or

$$a = \frac{A_s f_y}{0.85 f_c' \, b} \tag{d}$$

or

$$a = \frac{\rho f_y d}{0.85 f_c'} \tag{14.5}$$

where:

$$\rho = \text{steel ratio} = \frac{A_s}{bd} \tag{14.6}$$

Since moment = (force)(moment arm):

$$M_n = T\left(d - \frac{a}{2}\right) = f_y A_s \left(d - \frac{a}{2}\right) \tag{e}$$

Substituting a from Equation 14.5 and A_s from Equation 14.6 into (e), we obtain:

$$M_n = \rho f_y bd^2\left(1 - \frac{\rho f_y}{1.7 f_c'}\right) \tag{f}$$

Substituting (f) into Equation 14.2 at equality, we obtain:

$$\frac{M_u}{\phi bd^2} = \rho f_y\left(1 - \frac{\rho f_y}{1.7 f_c'}\right) \tag{14.7}$$

Equation 14.7 is a very useful relation for analysis and design of a beam.

If we arbitrarily define the expression on the right side of Equation 14.7 as $\bar{K}$, called the *coefficient of resistance*, then Equation 14.7 becomes:

$$M_u = \phi bd^2 \bar{K} \tag{14.8}$$

where:

$$\bar{K} = \rho f_y\left(1 - \frac{\rho f_y}{1.7 f_c'}\right) \tag{14.9}$$

The coefficient $\bar{K}$ depends on (1) ρ, (2) f_y, and (3) f_c'. The values of $\bar{K}$ for different combinations of ρ, f_y, and f_c' are listed in Appendix D.4 through D.10.

In place of Equation 14.7, these tables can be directly used in beam analyses and designs.

STRAIN DIAGRAM AND MODES OF FAILURE

The strain diagrams in Figures 14.1 and 14.2 show a straight-line variation of the concrete compression strain, ε_c, to the steel tensile strain, ε_s; the line passes through the neutral axis. Concrete can have a maximum strain of 0.003, and the strain at which steel yields is $\varepsilon_y = f_y/E$. When the strain diagram is such that the maximum concrete strain of 0.003 and the steel yield strain of ε_y are attained at the same time, it is said to be a balanced section, as shown by the solid line labeled I in Figure 14.4.

In this case, the amount of steel and the amount of concrete balance each other out, and both of these reach the failing level (attain the maximum strains) simultaneously. If a beam has more steel than the balanced condition, then the concrete reaches a strain level of 0.003 before the steel attains the yield strain of ε_y. This is shown by condition II in Figure 14.4. The neutral axis moves down in this case.

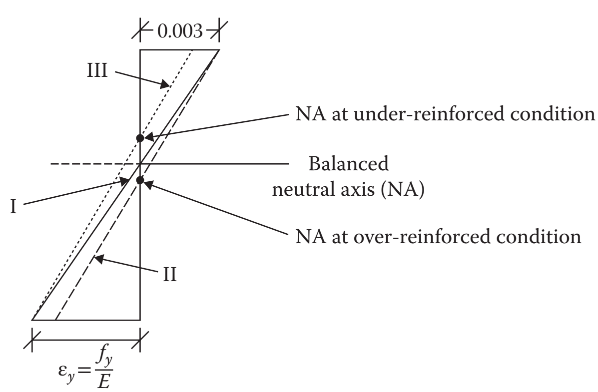


FIGURE 14.4 Strain stages in a beam.

The failure is initiated by crushing of concrete, which is sudden since concrete is brittle. This mode of failure in compression is undesirable because a structure fails suddenly without any warning.

If a beam has lesser steel than the balanced condition, then the steel attains its yield strain before the concrete can reach the maximum strain level of 0.003. This is shown by condition III in Figure 14.4. The neutral axis moves up in this case. The failure is initiated by the yielding of the steel, which is gradual because of the ductility of steel. This is a tensile mode of failure, which is more desirable because at least there is an adequate warning of an impending failure. The ACI recommends the tensile mode of failure or the under-reinforcement design for a concrete structure.

BALANCED AND RECOMMENDED STEEL PERCENTAGES

To ensure the under-reinforcement conditions, the percentage of steel should be less than the balanced steel percentage, ρ_b, which is the percentage of steel required for the balanced condition.

From Figure 14.4, for the balanced condition:

$$\frac{0.003}{c} = \frac{f_y/E}{d-c} \tag{a}$$

By substituting $c = a/\beta_1$ from Equation 14.3, $a = \rho f_y d\,/\,0.85 f_c'$ from Equation 14.5, and $E = 29 \times 10^6$ psi in Equation (a), the following expression for the balanced steel is obtained:

$$\rho_b = \left(\frac{0.85\beta_1 f_c'}{f_y}\right)\left(\frac{870{,}000}{87{,}000 + f_y}\right) \tag{14.10}$$

The values for the balanced steel ratio, ρ_b, calculated for different values of f_c' and f_y, are tabulated in Appendix D.11. Although a tensile mode of failure ensues when the percentage of steel is less than the balanced steel, the ACI code defines a section as tension controlled only when the tensile strain in steel, ε_t, is equal to or greater than 0.005 as the concrete reaches its strain limit of 0.003. The strain range between $\varepsilon_y = (f_y/E)$ and 0.005 is regarded as the transition zone.

The values of the percentage of steel for which ε_t is equal to 0.005 are also listed in Appendix D.11 for different grades of steel and concrete. It is recommended to design beams with a percentage of steel that is not larger than these listed values for ε_t of 0.005.

If a larger percentage of steel is used than $\varepsilon_t = 0.005$, to be in the transition region, the strength reduction factor ϕ should be adjusted, as discussed in the "Strength Reduction Factor for Concrete" section.

MINIMUM PERCENTAGE OF STEEL

Just as the maximum amount of steel is prescribed to ensure the tensile mode of failure, a minimum limit is also set to ensure that the steel is not too small that it causes failure by rupture (cracking) of the concrete in the tension zone. The ACI recommends the higher of the following two values for the minimum steel in flexure members:

$$(A_s)_{min} = \frac{3\sqrt{f_c'}}{f_y} bd \tag{14.11}$$

or

$$(A_s)_{min} = \frac{200}{f_y} bd \tag{14.12}$$

where:

b is the width of the beam
d is the effective depth of the beam

The values of ρ_{min}, which is $(A_s)_{min}/bd$, are also listed in Appendix D.11, where the higher of the values from Equations 14.10 and 14.11 have been tabulated.

The minimum amount of steel for slabs is controlled by shrinkage and temperature requirements, as discussed in the "Specifications for Slabs" section.

STRENGTH REDUCTION FACTOR FOR CONCRETE

In Equations 14.2 and 14.7, a strength reduction factor, ϕ, is applied to account for all kinds of uncertainties involved in strength of materials, design and analysis, and workmanship. The values of the factor recommended by the ACI are listed in Table 14.1.

For the transition region between the compression-controlled and the tension-controlled stages when ε_t is between ε_y (assumed to be 0.002) and 0.005, as discussed above, the value of ϕ is interpolated between 0.65 and 0.9 by the following relation:

$$\phi = 0.65 + (\varepsilon_t - 0.002)\left(\frac{250}{3}\right) \tag{14.13}$$[1]

The values[2] of ε_t for different percentages of steel are also indicated in Appendixes D.4 through D.10. When it is not listed in these tables, ε_t is larger than 0.005.

SPECIFICATIONS FOR BEAMS

The ACI specifications for beams are as follows:

1. *Width-to-depth ratio:* There is no code requirement for the *b*/*d* ratio. From experience, the desirable *b*/*d* ratio lies between ½ and ⅔.
2. *Selection of steel:* After a required reinforcement area is computed, Appendix D.2 is used to select the number of bars that provide the necessary area.
3. The minimum beam widths required to accommodate multiples of various size bars are given in Appendix D.3. This is a useful design aid, as demonstrated in the example.

TABLE 14.1
Strength Reduction Factors

Structural System	ϕ
1. Tension-controlled beams and slabs	0.9
2. Compression-controlled columns	
Spiral	0.70
Tied	0.65
3. Shear and torsion	0.75
4. Bearing on concrete	0.65

[1] For spiral reinforcement this is $\phi = 0.7 + (\varepsilon_t - 0.002)(250/3)$

[2] ε_t is calculated by the formula $\varepsilon_t = (0.00255 f_c' \beta_1 / \rho f_y) - 0.003$.

4. The reinforcement is located at a certain distance from the surface of the concrete called the *cover*. The cover requirements in the ACI code are extensive. For beams, girders, and columns that are not exposed to weather or are not in contact with the ground, the minimum clear distance from the bottom of the steel to the concrete surface is 1½ in. There is a minimum cover requirement of 1½ in. from the outermost longitudinal bars to the edge toward the width of the beam, as shown in Figure 14.5.
5. *Bar spacing:* The clear spacing between the bars in a single layer should not be less than any of the following three conditions:
   1. 1 in.
   2. The diameter of the bar
   3. 1⅓ × maximum aggregate size
6. *Bar placement:* If the bars are placed in more than one layer, those in the upper layers are required to be placed directly over the bars in the lower layers, and the clear distance between the layers must not be less than 1 in.
7. *Concrete weight:* Concrete is a heavy material. The weight of the beam is significant. An estimated weight should be included in design . If it is found to be appreciably less than the weight of the section designed, then the design should be revised. Table 14.2 could be used as a guide for estimation of concrete weight.

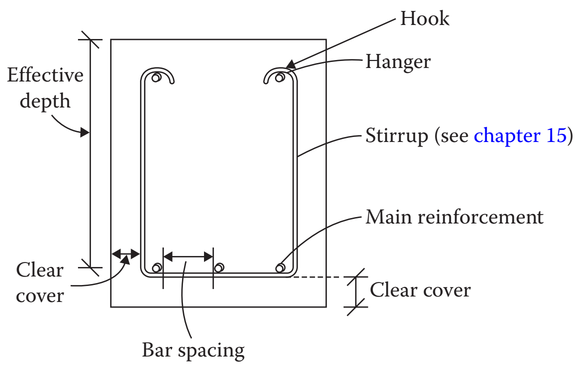


FIGURE 14.5 Sketch of beam specifications.

TABLE 14.2
First Estimate of Beam Weight

Design Moment, M_u (ft-k)	Estimated Weight (lb/ft)
≤200	300
> 200 but ≤300	350
> 300 but ≤400	400
> 400 but ≤500	450
> 500	500

ANALYSIS OF BEAMS

Analysis relates to determining the factored or service moment or the load capacity of a beam of known dimensions and known reinforcement. The analysis procedure is:

1. Calculate the steel ratio from Equation 14.6:

$$\rho = \frac{A_s}{bd}$$

2. Calculate $(A_s)_{min}$ from Equations 14.11 and 14.12 or use Appendix D.11. Compare this to the A_s of the beam to ensure that it is more than the minimum.
3. For known value of the steel ratio ρ, read ε_t from Appendixes D.4 through D.10, or by the formula in footnote 2 accompanying Equation 14.13. If no value is given, then $\varepsilon_t = 0.005$. If $\varepsilon_t < 0.005$, determine ϕ from Equation 14.13.
4. For known ρ, compute $\bar{K}$ from Equation 14.9 or read the value from Appendixes D.4 through D.10.
5. Calculate M_u from Equation 14.7:

$$M_u = \phi b d^2 \bar{K}$$

6. Break down into the loads if required.

Example 14.1

The loads on a beam section are shown in Figure 14.6. Determine whether the beam is adequate to support the loads; $f'_c = 4{,}000$ psi and $f_y = 60{,}000$ psi.

Solution

A. Design loads and moments
   1. Weight of beam/ft = (12/12) × (20/12) × 1 × 150 = 250 lb/ft or 0.25 k/ft
   2. Factored dead load, w_u = 1.2 (1.25) = 1.5 k/ft
   3. Factored live load, P_u = 1.6 (15) = 24 k
   4. Design moment due to dead load = $w_u L^2/8 = 1.5(20)^2/8 = 75$ ft-k
   5. Design moment due to live load = $P_u L/4 = 24(20)/4 = 120$ ft-k
   6. Total design moment, M_u = 195 ft-k
   7. A_s = 3.16 in.^{2} (from Appendix D.2, for 4 bars of size #8)

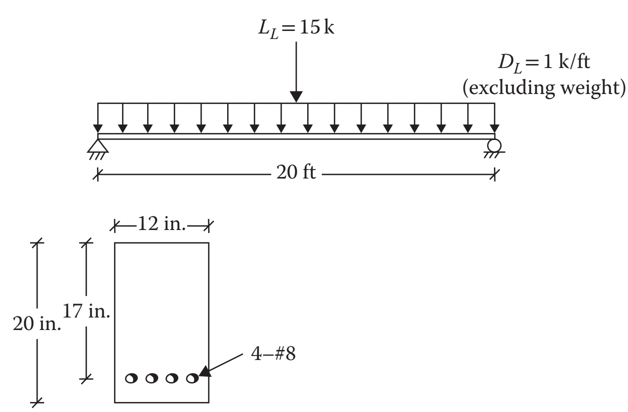


FIGURE 14.6 Beam for Example 14.1.

8. $\rho = A_s/bd = 3.16/12 \times 17 = 0.0155$
9. $(A_s)_{min} = 0.0033$ (from Appendix D.11) < 0.0155 **OK**
10. $\varepsilon_t \geq 0.005$ (since ε_t value is not listed in Appendix D.9), $\phi = 0.9$
11. $\bar{K} = 0.8029$ ksi (for $\rho = 0.0155$ from Appendix D.9)
12. $M_u = \phi bd^2\bar{K} = (0.9)(12)(17)^2(0.8029) = 2506$ in.-k or 209 ft-k > 195 ft-k **OK**

DESIGN OF BEAMS

In wood beam design in Chapter 7 and steel beam design in Chapter 11, beams were designed for bending moment capacity and checked for shear and deflection. For concrete beams, shear is handled independently, as discussed in Chapter 16. For deflection, the ACI stipulates that when certain depth requirements are met, deflection does not interfere with the use of or cause damage to the structure. These limiting values are given in Table 14.3 for normal weight (120–150 lb/ft^3) concrete and Grade 60 steel. For other grade concrete and steel, adjustments are made as indicated in the footnotes to Table 14.3.

When the minimum depth requirement is met, deflection does not need to be computed. For members of lesser thickness than those listed in Table 14.3, the deflections should be computed to check for safe limits. This book assumes that the minimum depth requirement is satisfied.

Beam design falls into the two categories, which are discussed below.

Design for Reinforcement Only

When a beam section has been fixed from architectural or any other consideration, only the amount of steel has to be selected. The procedure follows:

1. Determine the design moment, M_u, including the beam weight, for various critical load combinations.
2. Using $d = h{-}3$ and $\phi = 0.9$, calculate the required $\bar{K}$ from Equation 14.8, expressed as:

$$\bar{K} = \frac{M_u}{\phi bd^2}$$

3. From Appendixes D.4 through D.10, find the value of ρ corresponding to $\bar{K}$ of step 2. From the same tables, confirm that $\varepsilon_t \geq 0.005$. If $\varepsilon_t < 0.005$, reduce ϕ by Equation 14.13, recompute $\bar{K}$, and find the corresponding ρ.
4. Compute the required steel area, A_s, from Equation 14.6:

$$A_s = \rho bd$$

TABLE 14.3
Minimum Thickness of Beams and Slabs for Normal Weight Concrete and Grade 60 Steel

	Minimum Thickness, *h* (in.)			
Member	**Simply Supported**	**Cantilever**	**One End Continuous**	**Both Ends Continuous**
Beam	$L/16$	$L/18.5$	$L/21$	$L/8$
Slab (one-way)	$L/20$	$L/24$	$L/28$	$L/10$

Note: L is the span in inches.

For lightweight concrete of unit weight 90 < 120 lb/ft^3, the table values should be multiplied by $(1.65 < 0.005Wc)$ but not less than 1.09, where Wc is the unit weight in lb/ft.^{3}

For other than Grade 60 steel, the table value should be multiplied by $(0.4 + fy/100)$, where fy is in ksi.

5. Check for the minimum steel $A_{s(min)}$ from Appendix D.11.
6. Select the bar size and the number of bars from Appendix D.2. From Appendix D.3, check whether the selected steel (size and number) can fit into width of the beam, preferably in a single layer. They can be arranged in two layers. Check to confirm that the actual depth is at least equal to $h - 3$.
7. Sketch the design.

Example 14.2

Design a rectangular reinforced beam to carry a service dead load of 1.6 k/ft and a live load of 1.5 k/ft on a span of 20 ft. The architectural consideration requires the width to be 10 in. and depth to be 24 in. Use $f_c' = 3{,}000$ psi and $f_y = 60{,}000$ psi.

Solution

1. Weight of beam/ft = (10/12)(24/12) × 1 × 150 = 250 lb/ft or 0.25 k/ft
2. $w_u = 1.2\ (1.6 + 0.25) + 1.6\ (1.5) = 4.62$ k/ft
3. $M_u = w_u L^2/8 = 4.62(20)^2/8 = 231$ ft-k or 2772 in.-k
4. $d = 24 - 3 = 21$ in.
5. $\bar{K} = 2772/(0.9)(10)(21)^2 = 0.698$ ksi
6. $\rho = 0.0139$ $\varepsilon_t = 0.0048$ (from Appendix D, Table D.6)
7. From Equation 14.13:
   $\phi = 0.65 + (0.0048 - 0.002)(250/3) = 0.88$
8. Revised $\bar{K} = 2772/(0.88)(10)(21)^2 = 0.714$ ksi
9. Revised $\rho = 0.0143$ (from Appendix D.6)[3]
10. $A_s = \rho bd = (0.0143)(10)(21) = 3$ in.^{2}
11. $A_{s(min)} = 0.0033$ (from Appendix D.11) < 0.0143 **OK**
12. Selection of steel

Bar Size	No. of Bars	A_s from Appendix D.2	Minimum Width in One Layer from Appendix D.3
#6	7	3.08	15 **NG**
#7	5	3.0	12.5 **NG**
#9	3	3.0	9.5 **NG**

Note: Select three bars of #9

13. The beam section is shown in Figure 14.7.

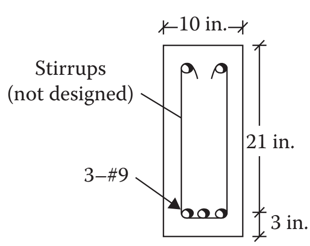


FIGURE 14.7 Beam section for Example 14.2.

[3] For $\bar{K} = 0.714$ ksi, $\varepsilon_t = 0.0046$, ϕ requires a minor adjustment again by Equation 14.13.

Design of Beam Section and Reinforcement

Designing the beam section and reinforcement entails determining the beam dimensions and selecting the amount of steel. The procedure follows:

1. Determine the design moment, M_u, including the beam weight for various critical load combinations.
2. Select the steel ratio, ρ, corresponding to $\varepsilon_t = 0.005$ from Appendix D.11.
3. From Appendixes D.4 through D.10, find $\bar{K}$ for the steel ratio of step 2.
4. For b/d ratio of $\frac{1}{2}$ and $\frac{2}{3}$, find two values of d from the following expression:

$$d = \left[\frac{M_u}{\phi(b/d)\bar{K}}\right]^{1/3} \tag{14.14}$$ [4]

5. Select the effective depth to be between the two values of step 4.
6. If the depth from Table 14.3 is larger, use that value.
7. Determine the corresponding width, b, from

$$b = \frac{M_u}{\phi d^2 \bar{K}} \tag{14.15}$$

8. Estimate h and compute the weight of the beam. If this is excessive compared to the assumed value of step 1, repeat steps 1 through 7.
9. From now on, follow steps 4 through 7 of the design procedure in the "Design for Reinforcement Only" section for the selection of steel.

Example 14.3

Design a rectangular reinforced beam for the service loads shown in Figure 14.8. Use $f'_c = 3{,}000$ psi and $f_y = 60{,}000$ psi.

Solution

1. Factored dead load excluding beam weight, $w_u = 1.2(1.5) = 1.8$ k/ft
2. Factored live load, $P_u = 1.6(20) = 32$ k
3. Design moment due to dead load $= w_u L^2/8 = 1.8(30)^2/8 = 202.5$ ft-k
4. Design moment due to live load $= P_u L/3 = 32(30)/3 = 320$ ft-k
5. Total moment, $M_u = 522.5$ ft-k
6. Weight of beam from Table 14.2: 0.5 k/ft
7. Revised Factored dead load, including weight: $1.2(1.5 + 0.5) = 2.4$ k/ft
8. Moment due to dead load $= 2.4(30)^2/8 = 270$ ft-k
9. Total design moment = 320 + 270 = 590 ft-k or 7080 in.-k
10. $\rho = 0.0136$ (from Appendix D.11, for $\varepsilon_t = 0.005$)
11. $\bar{K} = 0.684$ ksi (from Appendix D.6)

[4] This relation is the same as $M_u = \phi\, bd^2 \bar{K}$.

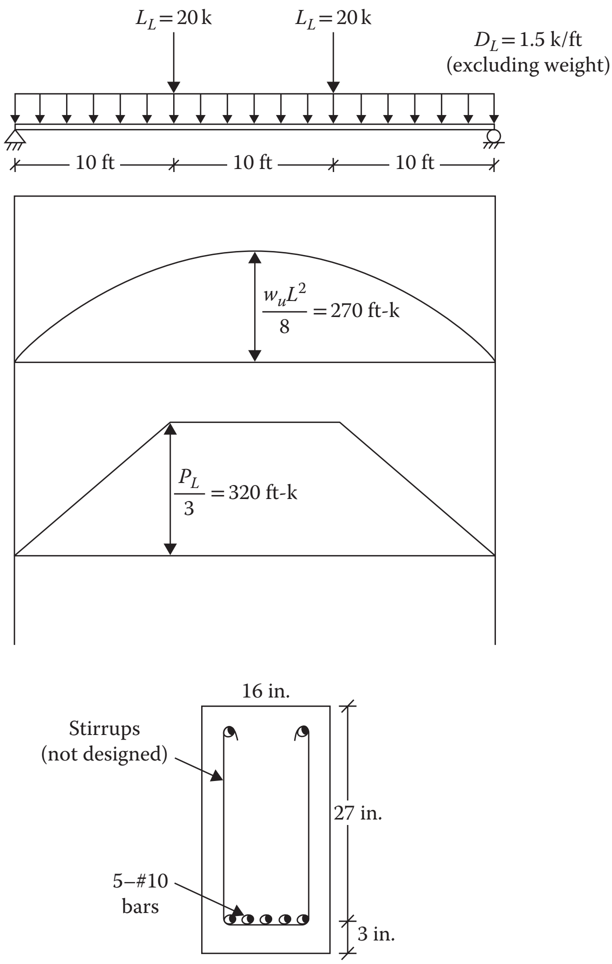


FIGURE 14.8 Loads, bending moments, and beam section for Example 14.3.

12. Values of d

Select b/d Ratio	Calculate d from Equation 14.14
1/2	28.3[a]
2/3	25.8

[a] $[7080/(0.9 \times \frac{1}{2} \times 0.684)^{1/3}$

13. Depth for deflection (from Table 14.3)

$$h = \frac{L}{16} = \frac{30 \times 12}{16} = 22.5 \text{ in.}$$

or $d = h{-}3 = 22.5{-}3 = 19.5$ in.

Use $d = 27$ in.

14. From Equation 14.15:

$$b = \frac{7080}{(0.9)(27)^2(0.684)} = 15.75 \text{ in., use 16 in.}$$

15. $h = d + 3 = 30$ in.
    Weight of beam/ft = (16/12)(30/12) × 1 × 150 = 500 lb/ft or 0.50 k/ft **OK**
16. $A_s = \rho bd = (0.0136)(16)(27) = 5.88 \text{ in.}^2$
17. Selection of steel

Bar Size	No. of Bars	As From Appendix D.2	Minimum Width in One Layer from Appendix D.3
#9	6	6.00	16.5 NG
#10	5	6.35	15.5

18. Select five bars of #10.

ONE-WAY SLAB

Slabs are the concrete floor systems supported by reinforced concrete beams, steel beams, concrete columns, steel columns, concrete walls, or masonry walls. If they are supported on two opposite sides only, they are referred to as *one-way slabs* because the bending is in one direction only, perpendicular to the supported edge. When slabs are supported on all four edges, they are called *two-way slabs* because the bending is in both directions. A rectangular floor plan has slab supported on all four sides. However, if the long side is two or more times longer than the short side, the slab could be considered a one-way slab spanning the short direction.

A one-way slab is analyzed and designed as 12-in.-wide beam segments placed side by side having a total depth equal to the slab thickness, as shown in Figure 14.9.

The amount of steel computed is considered to exist in 12-in. width on average. Appendix D.12 is used for this purpose; for the different bar sizes, it indicates the center-to-center spacing of the bars for a specified area of steel. The relationship is:

$$K \text{ Bar spacing center to center} = \frac{\text{required steel area}}{\text{area of 1 bar}} \times 12 \tag{14.16}$$

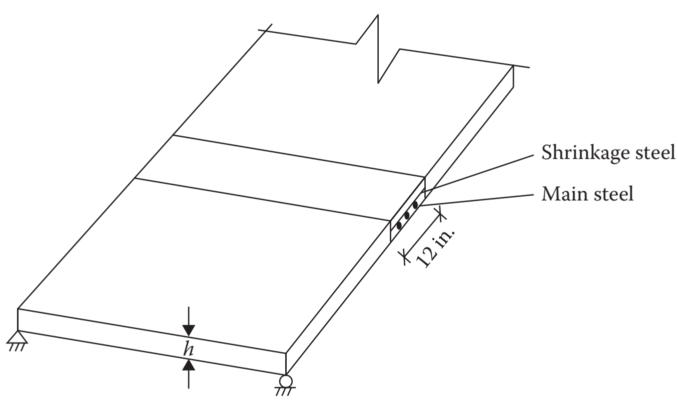


FIGURE 14.9 Simply supported one-way slab.

SPECIFICATIONS FOR SLABS

The ACI specifications for one-way slab follow:

1. *Thickness:* Table 14.3 indicates the minimum thickness for one-way slabs where deflections are not to be calculated. The slab thickness is rounded off to the nearest ¼ in. on the higher side for slabs up to 6 in. and to the nearest ½ in. for slabs thicker than 6 in.
2. *Cover:* (1) For slabs that are not exposed to the weather or are not in contact with the ground, the minimum cover is ¾ in. for #11 and smaller bars and (2) for slabs exposed to the weather or in contact with the ground, the minimum cover is 3 in.
3. *Spacing of bars:* The main reinforcement should not be spaced on center to center more than (1) three times the slab thickness or (2) 18 in., whichever is smaller.
4. *Shrinkage steel:* Some steel is placed in the direction perpendicular to the main steel to resist shrinkage and temperature stresses. The minimum area of such steel is:
   a. For Grade 40 or 50 steel, shrinkage $A_s = 0.002bh$.
   b. For Grade 60 steel, shrinkage $A_s = 0.0018bh$, where $b = 12$ in.
   The shrinkage and temperature steel should not be spaced farther apart than (1) five times the slab thickness or (2) 18 in., whichever is smaller.
5. *Minimum main reinforcement:* The minimum amount of main steel should not be less than the shrinkage and temperature steel.

ANALYSIS OF ONE-WAY SLAB

The analysis procedure follows:

1. For the given bar size and spacing, read A_s from Appendix D.12.
2. Find the steel ratio:

$$\rho = \frac{A_s}{bd}$$

   where $b = 12\,\text{in.}$, $d = h - 0.75\,\text{in.} - 1/2(\text{bar diameter})$[5]

3. Check for the minimum shrinkage steel and also that the main reinforcement A_s is more than $A_{s(min)}$:

$$A_{s(min)} = 0.002bh$$

4. For ρ of step 2, read $\bar{K}$ and ε_t (if given in the same appendixes) from Appendix D.4 through D.10.
5. Correct ϕ from Equation 14.13 if $\varepsilon_t < 0.005$.
6. Find M_u as follows and convert to loads if necessary:

$$M_u = \phi bd^2\bar{K}$$

Example 14.4

The slab of an interior floor system has the cross section shown in Figure 14.10. Determine the service live load that the slab can support in addition to its own weight on a span of 10 ft; $f'_c = 3{,}000$ psi, $f_y = 40{,}000$ psi.

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[5] For slabs laid on the ground, $d = h - 3 - ½$ (bar diameter).

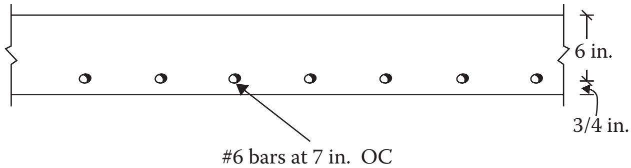

FIGURE 14.10 Cross section of slab of a floor system (perpendicular to span) for Example 14.4.

Solution

1. $A_s = 0.75$ in.^{2} (from Appendix D.12)
2. $d = 6 - ½ \times 0.75 = 5.625$ in.
   and $\rho = A_s/bd = 0.75/(12)(5.625) = 0.011$
3. $A_{s(min)} = 0.002bh = (0.002)(12)(6.75) = 0.162 < 0.75$ in.^{2} **OK**
4. $\bar{K} = 0.402$ (from Appendix D.4), $\varepsilon_t > 0.005$ for $\rho = 0.011$
5. $M_u = \phi bd^2 \bar{K} = (0.9)(12)(5.625)^2(0.402) = 137.37$ in.-k or 11·45 ft-k
6. $w_u = 8M_u/L^2 = 8(11.45)/10^2 = 0.916$ k/ft
7. Weight of a slab/ft = (12/12)(6.75/12)(1)(150/1000) = 0.084 k/ft
8. $w_u = 1.2(w_D) + 1.6(w_L)$
   or $0.916 = 1.2(0.084) + 1.6\ w_L$ or $w_L = 0.51$ k/ft
   Since the slab width is 12 in., the live load is 0.51 k/ft^2

DESIGN OF ONE-WAY SLAB

1. Determine the minimum h from Table 14.3. Compute the slab weight/ft for $b = 12$ in.
2. Compute the design moment, M_u. The unit load per square foot automatically becomes load/ft since the slab width = 12 in.
3. Calculate an effective depth, d, from:

$$d = h - \text{cover} - 1/2 \times \text{assumed bar diameter}$$

4. Compute $\bar{K}$ assuming $\phi = 0.90$:

$$\bar{K} = \frac{M_u}{\phi bd^2}$$

5. From Appendix D.4 through D.10, find the steel ratio ρ and note the value of ε_t (if ε_t is not listed then $\varepsilon_t > 0.005$).
6. If $\varepsilon_t < 0.005$, correct ϕ from Equation 14.13 and repeat steps 4 and 5.
7. Compute the required A_s:

$$A_s = \rho bd$$

8. From the table in Appendix D.12, select the main steel satisfying the condition that the bar spacing is $\leq 3h$ or 18 in.
9. Select shrinkage and temperature of steel:

$$\text{Shrinkage } A_s = 0.002bh(\text{Grade 40 or 50 steel})$$

or

$$0.0018bh(\text{Grade 60 steel})$$

10. From Appendix D.12, select size and spacing of shrinkage steel with a maximum spacing of $5h$ or 18 in., whichever is smaller.
11. Check that the main steel area of step 7 is not less than the shrinkage steel area of step 9.
12. Sketch the design.

Example 14.5

Design an exterior one-way slab exposed to the weather to span 12 ft and to carry a service dead load of 100 pounds per square foot (psf) and live load of 300 psf in addition to the slab weight. Use $f'_c = 3{,}000$ psi and $f_y = 40{,}000$ psi.

Solution

1. Minimum thickness for deflection from Table 14.3

$$h = \frac{L}{20} = \frac{12(12)}{20} = 7.2\,\text{in.}^2$$

For exterior slab, use h = 10 in.

2. Weight of slab = (12/12)(10/12)(1)(150/1000) = 0.125 k/ft
3. w_u = 1.2(0.1 + 0.125) + 1.6(0.3) = 0.75 k/ft
4. $M_u = w_u L^2/8$ = 13.5 ft-k or 162 in.-k
5. Assuming #8 size bar (diameter = 1 in.):

$$d = h - \text{cover} - \tfrac{1}{2}(\text{bar diameter})$$

$$= 10 - 3 - \tfrac{1}{2}(1) = 6.5 \text{ in.}$$

6. $\bar{K} = (M_u)/(\phi b d^2) = (162)/(0.9)(12)(6.5)^2 = 0.355$
7. $\rho = 0.014$, ε_t 0.005 (from Appendix D.4)
8. $A_s = \rho bd$ = (0.014)(12)(6.5) = 1.09 in.^{2}/ft
   Provide #8 size bars @ 8 in. on center (from Appendix D.12); A_s = 1.18 in.^{2}
9. Check for maximum spacing.
   a. $3h$ = 3(10) = 30 in.
   b. 18 in. > 8 in. **OK**
10. Shrinkage and temperature steel

$$A_s = 0.002bh$$

$$= 0.002(12)(7.5) = 0.18 \text{ in.}^2/\text{ft}$$

Provide #3 size bars @ 5½ in. on center (from Appendix D.12); A_s = 0.24 in.^{2}

11. Check for maximum spacing of shrinkage steel.

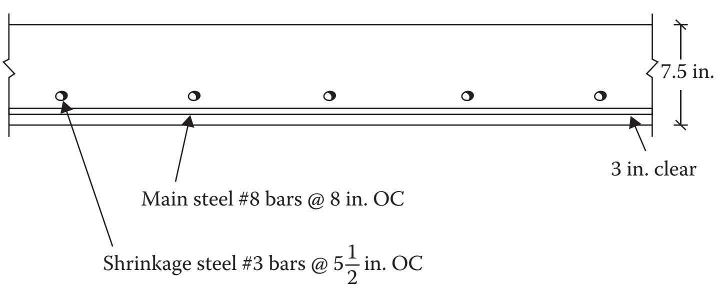

FIGURE 14.11 Design section for Example 14.5.

a. $5h = 5(10) = 50$ in.
b. 18 in. > 5½ in. **OK**
12. Main steel > shrinkage steel **OK**
13. A designed section is shown in Figure 14.11.

PROBLEMS

14.1 A beam cross section is shown in Figure P14.1. Determine the service dead load and live load/ft for a span of 20 ft. The service dead load is one-half of the live load; $f_c' = 4,000$ psi, $f_y = 60,000$ psi.

14.2 Calculate the design moment for a rectangular reinforced concrete beam having a width of 16 in. and an effective depth of 24 in. The tensile reinforcement is five bars of size #8; $f_c' = 4,000$ psi, $f_y = 40,000$ psi.

14.3 A reinforced concrete beam has the cross section shown in Figure P14.2 for a simple span of 25 ft. It supports a dead load of 2 k/ft (excluding beam weight) and live load of 3 k/ft; $f_c' = 4,000$ psi, $f_y = 60,000$ psi. Is the beam adequate?

14.4 Determine the dead load (excluding the beam weight) for the beam section shown in Figure P14.3 that has a span of 30 ft. The service dead load and live load are equal; $f_c' = 5,000$ psi, $f_y = 60,000$ psi.

14.5 The loads on a beam and its cross section are shown in Figure P14.4; $f_c' = 4,000$ psi, $f_y = 50,000$ psi. Is this beam adequate?

14.6 Design a reinforced concrete beam to resist a factored design moment of 150 ft-k. It is required that the beam width be 12 in. and the overall depth be 24 in.; $f_c' = 3,000$ psi, $f_y = 60,000$ psi.

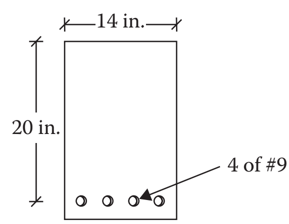

FIGURE P14.1 Beam section for Problem 14.1.

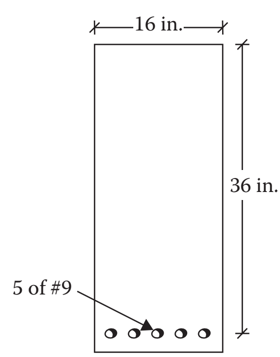

FIGURE P14.2 Beam section for Problem 14.3.

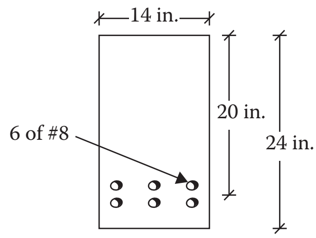

FIGURE P14.3 Beam section for Problem 14.4.

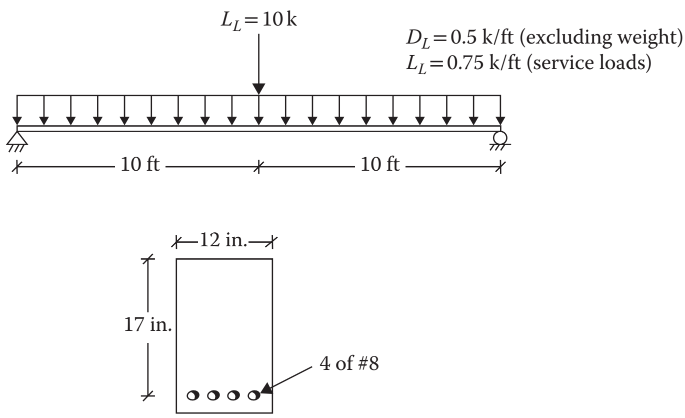

FIGURE P14.4 Loads and section for Problem 14.5.

14.7 Design a reinforced concrete beam of a span of 30 ft. The service dead load is 0.85 k/ft (excluding weight) and the live load is 1 k/ft. The beam has to be 12 in. wide and 26 in. deep; $f_c' = 4{,}000$ psi, $f_y = 60{,}000$ psi.

14.8 Design a reinforced beam for a simple span of 30 ft. There is no dead load except the weight of the beam and the service live load is 1.5 k/ft. The beam can be 12 in. wide and 28 in. overall depth; $f_c' = 5{,}000$ psi, $f_y = 60{,}000$ psi.

14.9 A beam carries the service loads shown in Figure P14.5. From architectural consideration, the beam width is 12 in. and the overall depth is 20 in.; $f_c' = 4{,}000$ psi, $f_y = 60{,}000$ psi. Design the beam reinforcement.

14.10 In Problem 14.9, the point dead load has a magnitude of 6.5 k (instead of 4 k). Design the reinforcement for the beam of the same size as in Problem 14.9; $f_c' = 4{,}000$ psi, $f_y = 60{,}000$ psi.

14.11 Design a rectangular reinforced beam for a simple span of 30 ft. The uniform service loads are dead load of 1.5 k/ft (excluding beam weight) and live load of 2 k/ft; $f_c' = 4{,}000$ psi, $f_y = 60{,}000$ psi.

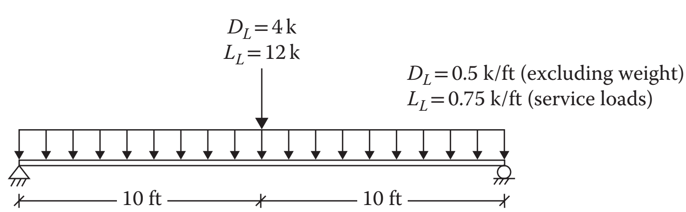

FIGURE P14.5 Loads on beam for Problem 14.9.

14.12 Design a simply supported rectangular reinforced beam for the service loads shown in Figure P14.6. Provide the reinforcement in a single layer; $f_c' = 4{,}000\,\text{psi}$, $f_y = 60{,}000\,\text{psi}$. Sketch the design.

14.13 Design a simply supported rectangular reinforced beam for the service loads shown in Figure P14.7. Provide the reinforcement in a single layer; $f_c' = 3{,}000\,\text{psi}$, $f_y = 40{,}000\,\text{psi}$. Sketch the design.

14.14 Design the cantilever rectangular reinforced beam shown in Figure P14.8. Provide a maximum of #8 size bars, in two rows if necessary; $f_c' = 3{,}000\,\text{psi}$, $f_y = 50{,}000\,\text{psi}$. Sketch the design.
Hint: Reinforcement will be at the top. Design as usual.

14.15 Design the beam for the floor shown in Figure P14.9. The service dead load (excluding beam weight) is 100 psf and live load is 300 psf; $f_c' = 3{,}000\,\text{psi}$, $f_y = 40{,}000\,\text{psi}$.

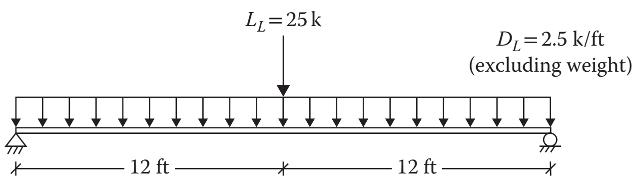

FIGURE P14.6 Loads on beam for Problem 14.12.

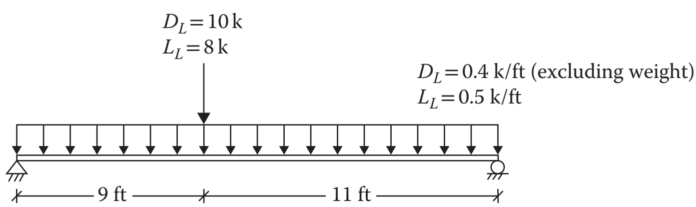

FIGURE P14.7 Loads on beam for Problem 14.13.

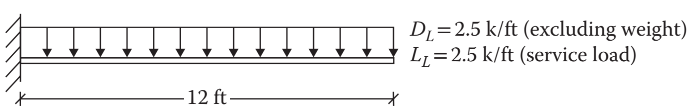

FIGURE P14.8 Cantilevered beam for Problem 14.14.

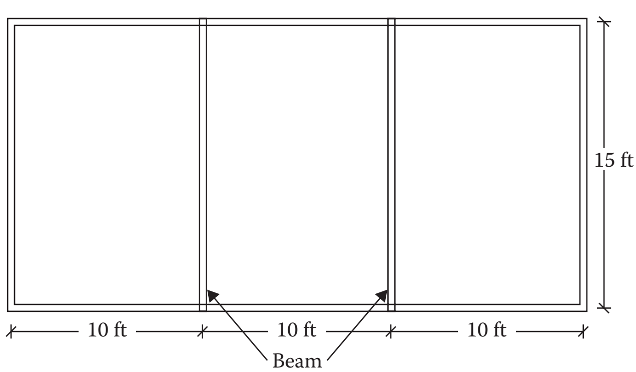

FIGURE P14.9 Floor system for Problem 14.15.

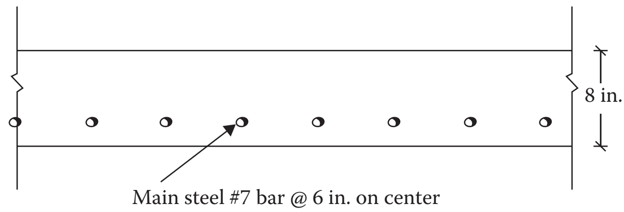

FIGURE P14.10 Cross section of slab for Problem 14.17.

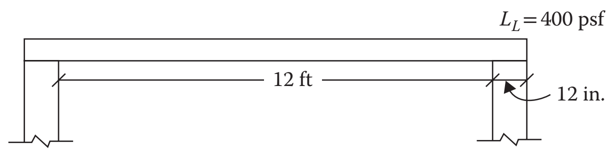

FIGURE P14.11 One-way slab for Problem 14.21.

14.16 A 9-in.-thick one-way interior slab supports a service live load of 500 psf on a simple span of 15 ft. The main reinforcement consists of #7 size bars at 7 in. on center. Check whether the slab can support the load in addition to its own weight. Use $f_c' = 3{,}000$ psi, $f_y = 60{,}000$ psi.

14.17 The one-way interior slab shown in Figure P14.10 spans 12 ft; $f_c' = 3{,}000$ psi, $f_y = 40{,}000$ psi. Determine the service load that the slab can carry in addition to its own weight.

14.18 A one-way slab, exposed to the weather, has a thickness of 9 in. The main reinforcement consists of #8 size bars at 7 in. on center. The slab carries a dead load of 500 psf in addition to its own weight on a span of 10 ft; $f_c' = 4{,}000$ psi, $f_y = 60{,}000$ psi. What is the service live load that the slab can carry?

14.19 A 8½-in.-thick one-way slab interior spans 10 ft. It was designed with the reinforcement of #6 size bars at 6.5 in. on center, to be placed with a cover of 0.75 in. However, the same steel was misplaced at a clear distance of 2 in. from the bottom; $f_c' = 4{,}000$ psi, $f_y = 60{,}000$ psi. How much is the capacity of the slab reduced to carry the superimposed service live load in addition to its own weight due to misplacement of the steel?

14.20 Design a simply supported, one-way interior slab to span 15 ft and to support the service dead and live loads of 150 psf and 250 psf, respectively, in addition to its own weight; $f_c' = 4{,}000$ psi, $f_y = 50{,}000$ psi. Sketch the design.

14.21 Design the concrete floor slab shown in Figure P14.11; $f_c' = 3{,}000$ psi, $f_y = 40{,}000$ psi. Sketch the design.

14.22 Design the slab of the floor system in Problem 14.15. $f_c' = 3{,}000$ psi, $f_y = 40{,}000$ psi.
Hint: The slab weight is included in the service dead load.

14.23 For Problem 14.15, design the thinnest slab so that the strain in steel is not less than 0.005; $f_c' = 3{,}000$ psi, $f_y = 40{,}000$ psi.

14.24 Design a balcony slab exposed to the weather. The cantilevered span is 8 ft and the service live load is 100 psf. Use the reinforcement of #5 size bars; $f_c' = 4{,}000$ psi, $f_y = 60{,}000$ psi. Sketch the design.
Hint: Reinforcement is placed on top. For the thickness of slab, in addition to the provision of main steel and shrinkage steel, at least 3 in. of depth (cover) should exist over and below the steel.

15 Doubly and T-Shaped Reinforced Concrete Beams

DOUBLY REINFORCED CONCRETE BEAMS

The aesthetics or architectural considerations sometimes necessitate a small beam section that is not adequate to resist the moment imposed on the beam. In such cases, the additional moment capacity could be achieved by adding more steel on both the compression and tensile sides of the beam. Such sections are known as *doubly reinforced* beams. The compression steel also makes beams more ductile and more effective in reducing deflections.

The moment capacity of doubly reinforced beams is assumed to comprise two parts, as shown in Figure 15.1. One part is due to the compression concrete and tensile steel, shown in Figure 15.1b (as described in Chapter 14). The other part is due to the compression steel and the additional tensile steel, as shown in Figure 15.1c.

Thus:

$$A_s = A_{s1} + A_{s2}$$

$$M_u = M_{u1} + M_{u2}$$

$$M_{u1} = \phi A_{s1} f_y \left(d - \frac{a}{2} \right)$$

and

$$M_{u2} = \phi A_{s2} f_y (d - d')$$

The combined capacity is given by:

$$M_u = \phi A_{s1} f_y \left(d - \frac{a}{2} \right) + \phi A_{s2} f_y (d - d') \tag{15.1}$$

where:

ϕ is the resistance factor
d is the effective depth
A_s is the area of steel on the tensile side of the beam; $A_s = A_{s1} + A_{s2}$
A'_s is the area of steel on the compression side of the beam
The compression steel area A'_s depends on the compression stress level f_s', which can be the yield stress f_y or less. The value of f_s' is decided by the strain in concrete at compression steel level, which in turn depends on the location of the neutral axis.

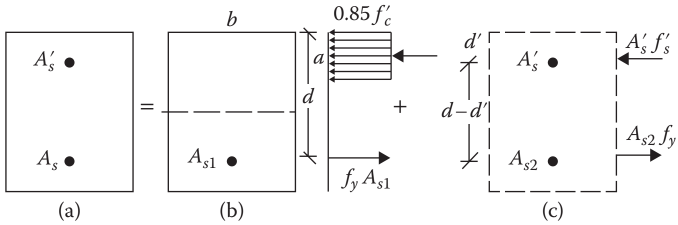

FIGURE 15.1 Moment capacity of doubly reinforced beam.

From the strain diagram, when concrete attains the optimal strain level at the top, as shown in Figure 15.2:

$$\varepsilon'_s = \frac{0.003(c - d')}{c} \tag{15.2}$$

$$\varepsilon_t = \frac{0.003(d - c)}{c} \tag{15.3}$$

1. When $\varepsilon'_s \geq f_y/E$, the compression steel has yielded, $f'_s = f_y$, and from the forces shown in Figure 15.1c:

$$A_{s2} = A'_s \tag{15.4}$$

2. When $\varepsilon'_s < f_y/E$, the compression steel has not yielded, $f'_s = \varepsilon'_s E$, and again from the forces shown in Figure 15.1c:

$$A_{s2} = \frac{A'_s f'_s}{f_y} \tag{15.5}$$

3. When $\varepsilon_t \geq 0.005$, $\phi = 0.9$.
4. When $\varepsilon_t < 0.005$, compute ϕ from Equation 14.13.

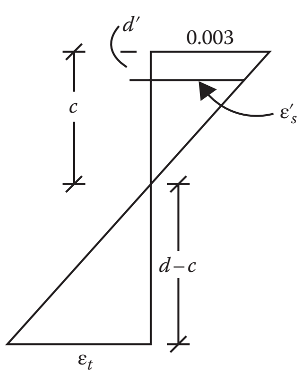

FIGURE 15.2 Strain diagram of concrete.

To ascertain the value of neutral axis, c, the tensile strength of the beam is equated with the compression strength. Thus, from Figure 15.1:

$$\text{Tensile force} = \text{compression force}$$

$$A_{s1} f_y + A_{s2} f_y = 0.85 f_c{}' ab + A'_s f_s{}'$$

$$(A_{s1} + A_{s2}) f_y = 0.85 f_c{}' ab + A'_s \varepsilon'_s E$$

Substituting $a = \beta_1 c$ from Equation 14.3, ε'_s from Equation 15.2 and $E = 29{,}000$ ksi:

$$A_s f_y = 0.85 f_c{}' \beta_1 cb + A'_s \frac{(c-d')}{c}(0.003)(29{,}000) \tag{15.6}$$

where β_1 is given in Equation 14.4, and the others terms are explained in Figure 15.1.

ANALYSIS OF DOUBLY REINFORCED BEAMS

A summary of the steps for analysis of a doubly reinforced beam follows:

1. From Equation 15.6, determine c; from Equation 14.3, compute a.
2. From Equation 15.2 compute ε'_s. If $\varepsilon'_s < f_y/E$ (that is, the compression steel has not yielded), use Equation 15.5 to determine A_{s2}. Otherwise, use Equation 15.4 to determine A_{s2}.
3. From Equation 15.3, compute ε_t; from that determine ϕ as stated in steps 3 and 4 of the previous section.
4. Compute the moment capacity from Equation 15.1.

Example 15.1

Determine the moment capacity of the beam shown in Figure 15.3. Use $f'_c = 3{,}000$ psi, $f_y = 60{,}000$ psi.

Solution

1. From Equation 15.6:

$$A_s f_y = 0.85 f_c{}' \beta_1 cb + A'_s \frac{(c-d')}{c}(0.003)(29{,}000)$$

$$6.24(60) = 0.85(3)(0.85)c(14) + 2\frac{(c-2.5)}{c}(0.003)(29{,}000)$$

$$374.4 = 30.345c + \frac{174(c-2.5)}{c}$$

$$374.4c = 30.345c^2 + 174c - 435$$

$$30.345c^2 - 200.4c - 435 = 0$$

$$c^2 - 6.60c - 14.335 = 0$$

$$c = \frac{+6.60 \pm \sqrt{(6.60)^2 + 4(14.335)}}{2} = 8.32 \text{ in. (positive value)}$$

$$a = \beta_1 c = 0.85(8.32) = 7.07 \text{ in.}$$

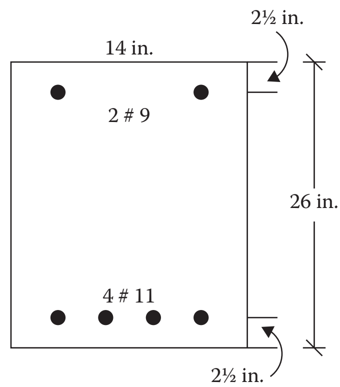

FIGURE 15.3 Beam section of Example 15.1.

2. From Equation 15.2:

$$\varepsilon'_s = \frac{0.003(c - d')}{c}$$

$$= \frac{0.003(8.32 - 2.5)}{8.32} = 0.0021$$

$$\frac{f_y}{E} = \frac{60}{29{,}000} = 0.0021$$

Since $\varepsilon'_s = f_y/E$, the compression steel has yielded.

$$f'_s = f_y = 60 \text{ ksi}$$

$$A_{s2} = A'_s = 2 \text{ in.}^2$$

$$A_{s1} = 6.24 - 2 = 4.24 \text{ in.}^2$$

3. From Equation 15.3:

$$\varepsilon_t = \frac{0.003(d - c)}{c}$$

$$= \frac{0.003(23.5 - 8.32)}{8.32} = 0.0055$$

Since $\varepsilon_t > 0.005$, $\phi = 0.9$.

4. Moment capacity from Equation 15.1

$$M_u = \phi A_{s1} f_y \left(d - \frac{a}{2}\right) + \phi A_{s2} f_y (d - d')$$

$$= 0.9(4.24)(60)\left(23.5 - \frac{7.07}{2}\right) + 0.9(2)(60)(23.5 - 2.5)$$

$$= 6839.19 \text{ in.-k or } 569.9 \text{ ft-k}$$

Example 15.2

Determine the moment capacity of the beam shown in Figure 15.4. Use $f_c{}'$=4,000 psi, f_y = 60,000 psi.

Solution

1. From Equation 15.6:

$$A_s f_y = 0.85 f_c{}' \beta_1 cb + A'_s \frac{(c-d')}{c}(0.003)(29{,}000)$$

$$6.24(60) = 0.85(4)(0.85)c(14) + 1.58\frac{(c-3)}{c}(0.003)(29{,}000)$$

$$374.4 = 40.46c + 137.46\frac{(c-3)}{c}$$

$$374.4c = 40.46c^2 + 137.46c - 412.38$$

$$40.46c^2 \quad 236.94c - 412.38 = 0$$

$$c^2 \quad 5.856c \quad 10.19 = 0$$

$$c = \frac{+5.856 \pm \sqrt{(5.856)^2} + 4(10.19)}{2} = 7.26 \text{ in. (positive value)}$$

$$a = \beta_1 c = 0.85(7.26) = 6.17 \text{ in.}$$

2. From Equation 15.2:

$$\varepsilon'_s = 0.003\,\frac{(c-d')}{c}$$

$$= \frac{0.003(7.26-3)}{7.26} = 0.0018$$

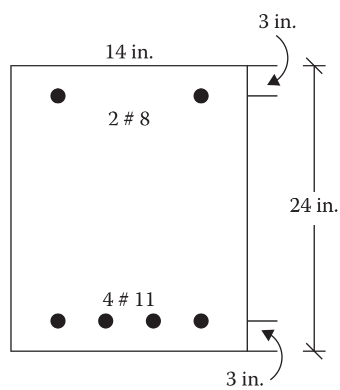

FIGURE 15.4 Beam section of Example 15.2.

$$\frac{f_y}{E} = \frac{60}{29{,}000} = 0.0021$$

Since $\varepsilon'_s < f_y/E$, the compression steel has not yielded.

$$f_s{}' = \varepsilon'_s E = 0.0018(29{,}000) = 52.20 \text{ ksi}$$

From Equation 15.5:

$$A_{s2} = \frac{A'_s f'_s}{f_y} = \frac{1.58(52.20)}{60} = 1.375 \text{ in.}^2$$

$$A_{s1} = 6.24 - 1.375 = 4.865 \text{ in.}^2$$

3. From Equation 15.3:

$$\varepsilon_t = \frac{0.003(d-c)}{c}$$

$$= \frac{0.003(24-7.26)}{7.26} = 0.0069$$

Since $\varepsilon_t > 0.005$, $\phi = 0.9$.

4. Moment capacity from Equation 15.1

$$M_u = \phi A_{s1} f_y \left(d - \frac{a}{2}\right) + \phi A_{s2} f_y (d - d')$$

$$= 0.9(4.865)(60)\left(24 - \frac{6.17}{2}\right) + 0.9(1.375)(60)(24-3)$$

$$= 7053.9 \text{ in.-k or } 587.8 \text{ ft-k}$$

DESIGN OF DOUBLY REINFORCED BEAMS

A summary of the steps to design a doubly reinforced beam follows:

1. Determine the factored moment, M_u, due to applied loads.
2. Ascertain ρ corresponding to $\varepsilon_t = 0.005$ from Appendix D.11, and $\bar{K}$ from Appendixes D.4 through D.10. Also determine $A_{s1} = \rho bd$.
3. Compute $M_{u1} = \phi bd^2\bar{K}$, assuming $\phi = 0.9$.
4. Compute $M_{u2} = M_u - M_{u1}$.
5. Compute (1) $a = \dfrac{A_{s1} f_y}{0.85 f'_c b}$ and (2) $c = a/\beta_1$.
6. Compute ε'_s from Equation 15.2.
   When $\varepsilon'_s \geq f_y/E$, the compression steel has yielded; $f_s{}' = f_y$.
   When $\varepsilon'_s < f_y/E$, the compression steel has not yielded; $f_s{}' = \varepsilon'_s E$.
7. Compute:

$$A'_s = \frac{M_{u2}}{\phi f_s{}'(d-d')}$$

8. Compute:

$$A_{s2} = \frac{A'_s f_s'}{f_y}$$

If the amount of compression steel A'_s and tensile steel A_s are selected exactly as computed, ε_t equals 0.005; that is, the tension-controlled condition prevails. However, selecting different amounts of steel may change this condition, resulting in a reduced value of ϕ (a value less than 0.9). Technically, after the amounts of steel are selected, it converts to a problem of analysis, as described in the previous section, to confirm that the resisting moment capacity is adequate for the applied bending moment.

Example 15.3

A simply supported beam of span 30 ft is subjected to a dead load of 2.4 k/ft and a live load of 3.55 k/ft. From architectural consideration, the beam dimensions are fixed, as shown in Figure 15.5. Design the beam. Use $f_c' = 4{,}000$ psi and $f_y = 60{,}000$ psi.

Solution

1. $\text{Weight of beam/ft} = \frac{31}{12} \times \frac{15}{12} \times 1 \times 150 = 484 \text{ lb/ft or } 0.484 \text{ k/ft}$

   $$w_u = 1.2(2.4 + 0.484) + 1.6(3.55) = 9.141 \text{ k/ft}$$

   $$M_u = \frac{wL^2}{8} = \frac{9.41(30)^2}{8} = 1028.4 \text{ ft-k}$$

2. From Appendix D.11, $\rho = 0.0181$.
   From Appendix D.9, $\bar{K} = 0.911$ ksi.

   $$A_{s1} = \rho bd = (0.0181)(15)(28) = 7.6 \text{ in.}^2$$

3. $M_{u1} = \phi bd^2\bar{K} = (0.9)(15)(28)^2(0.911) = 9642 \text{ in.-k or } 803.5 \text{ ft-k}$
4. $M_{u2} = M_u - M_{u1} = 1028.4 - 803.5 = 224.9 \text{ ft-k or } 2698.8 \text{ in.-k}$

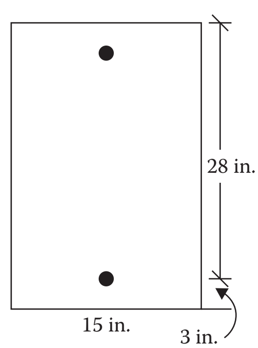

FIGURE 15.5 Size of beam for Example 15.3.

5. $a = \frac{A_{s1} f_y}{0.85 f'_c b} = \frac{760(60)}{0.85(4)(15)} = 8.94$ in.

$c = \frac{a}{\beta_1} = \frac{8.94}{0.85} = 10.52$ in.

6. $\varepsilon'_s = 0.003 \frac{(c - d')}{c}$

$= \frac{0.003(10.52 - 3)}{10.52} = 0.0021$

$\frac{f_y}{E} = \frac{60}{29{,}000} = 0.0021$

Since $\varepsilon'_s = f_y / E$, the compression steel has yielded.

$f_s' = f_y = 60$ ksi

7. $A'_s = \frac{M_{u2}}{\phi f'_s (d - d')} = \frac{2698.8}{0.9(60)(28 - 3)} = 2.0$ in.^{2}; use 2 bars of #9, $A'_s = 2$ in.^{2}

8. $A_{s2} = \frac{A'_s f_s'}{f_y} = \frac{2.0(60)}{60} = 2$ in.^{2}

$A_s = 7.6 + 2.0 = 9.6$ in.^{2}; use 8 bars of #10, $A_s = 10.2$ in.^{2} (two layers)

MONOLITHIC SLAB AND BEAM (T-BEAMS)

Concrete floor systems generally consist of slabs and beams that are monolithically cast together. In such cases, the slab acts as part of the beam, resulting in a T-shaped beam section as shown in Figure 15.6. The slab portion is called a flange, and the portion below the slab is called a web. The slab spans from beam to beam. But the American Concrete Institute (ACI) code defines a limited width that can be considered part of the beam. According to the ACI, this effective flange width should be the smallest of the following three values:[1]

1. b_f = one-fourth of the span (15.7a)
2. $b_f = b_w + 16\, h_f$ (15.7b)
3. b_f = center-to-center spacing of beams (15.7c)

A T-beam has five relevant dimensions: (1) flange width, b_f; (2) flange thickness, h_f; (3) width of web or stem, b_w; (4) effective depth of beam, d; and (5) tensile steel area, A_s. In analysis problems, all five of these parameters are known and the objective is to determine the design capacity of the beam. In the design of T-beams, the flange is designed separately as a slab spanning between the beams (webs) according to the procedure described for one-way slabs in Chapter 14. The effective width of the flange is ascertained according to Equation 15.7. The size of the web is fixed to satisfy the shear capacity or other architectural requirements. Thus, the values of b_f, h_f, b_w, and d are preselected, and the design consists of computing the area of tensile steel.

Under a positive bending moment, the concrete on the flange side resists compression, and the steel in the web resists tension. Depending on the thickness of the flange, the compression stress block might fully confine within the flange or it might fully cover the flange thickness and further extend into the web. The former condition exists most often.

[1] For an L-shaped beam, the overhang portion of the flange should be the smallest of (1) one-twelfth of the span length; (2) six times the slab thickness, h_f; and (3) one-half of the clear distance between beams.

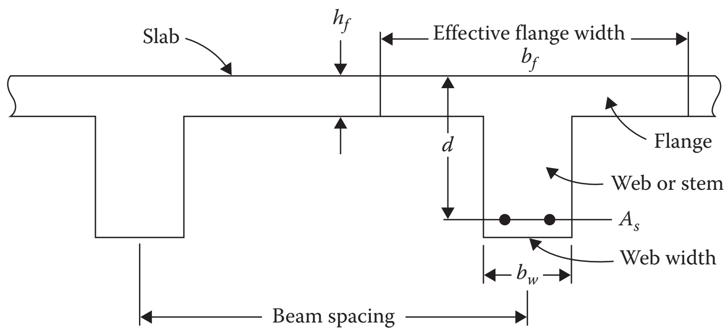

FIGURE 15.6 T beam comprising slab and supporting beam of a floor system.

In the first case, a T-beam acts like a rectangular beam of width b_f because all the concrete area below the compression stress block is considered to be cracked, and thus any shape of concrete below this compression stress block does not matter.

The minimum steel requirements as specified by Equations 14.11 and 14.12 also apply to T-beams.

ANALYSIS OF T-BEAMS

1. Determine the effective flange width, b_f, from Equation 15.7.
2. Check for minimum steel using Equations 14.11 and 14.12, using web width, b_w, for beam width.
3. Determine the area of the compression block, A_c:

$$A_c = \frac{A_s f_y}{0.85 f_c'} \tag{15.8}$$

4. In most cases, $A_c \le b_f h_f$; that is, the compression stress block lies within the flange. In such cases, the depth of the stress block is given by:

$$a = \frac{A_c}{b_f} \tag{15.9}$$

   and the centroid of the compression block from the top is given by:

$$\bar{y} = \frac{a}{2} \tag{15.10}$$

5. When $A_c > b_f h_f$, the compression stress block extends into the web to an extent, A_c, exceeding the flange area, $b_f h_f$. The centroid is determined for the area of the flange and the area extending into the web, as demonstrated in Example 15.4.
6. Determine (1) $c = a/\beta_1$, where β_1 is given in Equation 14.4; (2) $\varepsilon_t = 0.0003(d-c)/c$; and (3) $Z = d - \bar{y}$.
7. If $\varepsilon_t < 0.005$, adjust ϕ by Equation 14.13.
8. Calculate the moment capacity:

$$\phi M_n = \phi A_s f_y Z$$

Example 15.4

Determine the moment capacity of the T-beam spanning 20 ft, as shown in Figure 15.7. Use $f_c' = 3{,}000$ psi and $f_y = 60{,}000$ psi.

Solution

1. Effective flange width, b_f

   a. $\dfrac{\text{span}}{4} = \dfrac{20 \times 12}{4} = 60\,\text{in.}$
   b. $b_w + 16h_f = 11 + 16(3) = 59$ in.
   c. Beam spacing = 3 × 12 = 36 in. ← Controls
2. Minimum steel

   a. $\dfrac{3\sqrt{f_c'}\,b_w d}{f_y} = \dfrac{3\sqrt{3{,}000}(11)(24)}{60{,}000} = 0.723\ \text{in.}^2$
   b. $\dfrac{200 b_w d}{f_y} = \dfrac{200(11)(24)}{60{,}000} = 0.88\ \text{in.}^2 < 6.35\ \text{in.}^2$ **OK**
3. Area of compression block

   $$A_c = \frac{A_s f_y}{0.85 f_c'} = \frac{(6.35)(60{,}000)}{(0.85)(3{,}000)} = 149.41\,\text{in.}^2$$

   $$b_f h_f = (36)(3) = 108\ \text{in.}^2$$

   Since 149.41 > 108, the stress block extends into web by a distance a_1 below the flange.

4. $a_1 = \dfrac{A_c - b_f h_f}{b_w} = \dfrac{149.41 - 108}{11} = 3.76\ \text{in.}^2$

   $$a = 3 + 3.76 = 6.76\ \text{in.}$$

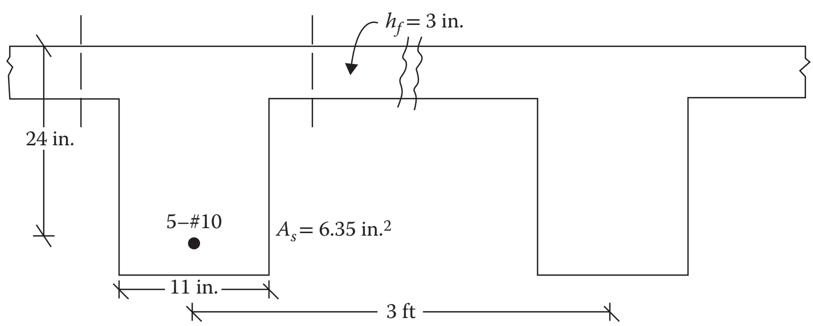

FIGURE 15.7 T beam dimensions for Example 15.4.

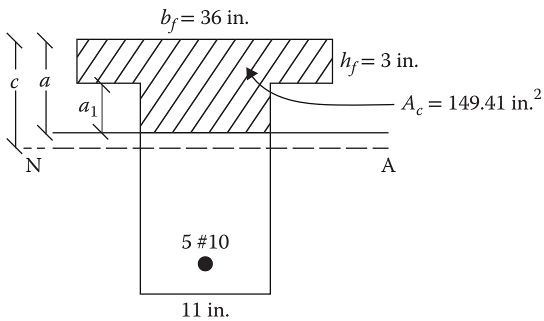

FIGURE 15.8 Compression stress block for Example 15.4.

5. In Figure 15.8, the centroid of the compression block from the top:

$$\bar{y} = \frac{[36 \times 3 \times 1.5] + [11 \times 3.76 \times (3 + 3.76/2)]}{149.41} = 2.435 \text{ in.}$$

6. $c = \dfrac{6.76}{0.85} = 7.95 \text{in.}$

$$\varepsilon_t = \frac{0.003(24 - 7.95)}{7.95} = 0.0061 > 0.005; \text{ hence } \phi = 0.9$$

$Z = d - \bar{y} = 24 - 2.435 = 21.565$ in.

7. Moment capacity
$\phi M_n = \phi A_s f_y Z = 0.9(6.35)(60)(21.565) = 7394.64$ in.-k or 616.2 ft-k

DESIGN OF T-BEAMS

As stated earlier, design consists of determining only the tensile steel area of a T-beam. This process is the reverse of the analysis. The steps follow:

1. Compute the factored design moment, including the dead load.
2. Determine the effective flange width, b_f, from Equation 15.7.
3. Adopt the effective depth $d = h - 3$ when the overall depth, h, is given. Assume the moment arm, Z, to be the larger of the following: (1) $0.9d$ or (2) $(d - h_f/2)$.
4. Calculate the steel area:

$$A_s = \frac{M_u}{\phi f_y Z}, \text{ for initial value of } \phi = 0.9$$

5. Calculate the area of the compression block, A_c:

$$A_c = \frac{A_s f_y}{0.85 f_c'} \tag{15.8}$$

6. Determine the depth of the stress block, a. In most cases, $A_c \le b_f h_f$; that is, the compression stress block lies within the flange. In such cases, the depth of the stress block is given by:

$$a = \frac{A_c}{b_f} \tag{15.9}$$

and the centroid of the compression block from the top is given by:

$$\bar{y} = \frac{a}{2} \tag{15.10}$$

When $A_c > b_f h_f$, the compression stress block extends into the web to the extent that A_c exceeds the flange area $b_f h_f$. The centroid is determined for the areas in the flange and the web, as shown in Example 15.4.

7. Determine (1) $c = a/\beta_1$, where β_1 is given in Equation 14.4, and (2) $\varepsilon_t = \frac{0.003(d-c)}{c}$.
8. If $\varepsilon_t < 0.05$, adjust ϕ by Equation 14.13 and recalculate the steel area from step 4.
9. Compute the revised moment arm:

$$Z = d - a$$

If the computed Z is appreciably different than the assumed Z of step 3, repeat steps 4 through 9 until the value of Z stabilizes.

10. Select steel for the final value of A_s computed.
11. Check the minimum steel by using Equations 14.11 and 14.12 or referring to Appendix D.11.

Example 15.5

Design a T-beam for the floor system spanning 20 ft, as shown in Figure 15.9. The moments due to the dead load (including beam weight) and the live load are 200 ft-k and 400 ft-k, respectively. Use $f_c' = 3{,}000$ psi and $f_y = 60{,}000$ psi.

Solution

1. Factored design moment = 1.2(200) = 1.6(400) = 960 ft-k or 11,520 in.-k.
2. Effective flange width, b_f

   a. $\frac{\text{span}}{4} = \frac{20 \times 12}{4} = 60\,\text{in.} \leftarrow \text{Controls}$

   b. $B_w + 16h_f = 15 + 16(3) = 63\,\text{in.}$

   c. $\text{Beam spacing} = 6 \times 12 = 72\,\text{in.}$

3. Moment arm

   $Z = 0.9d = 0.9(24) = 21.6\,\text{in.}$

   $Z = d - \frac{h_f}{2} = 24 - \frac{3}{2} = 22.5\,\text{in.} \leftarrow \text{Controls}$

4. Steel area

   $A_s = \frac{M_u}{\phi f_y Z} = \frac{11{,}520}{(0.9)(60)(22.5)} = 9.48\ \text{in.}^2$

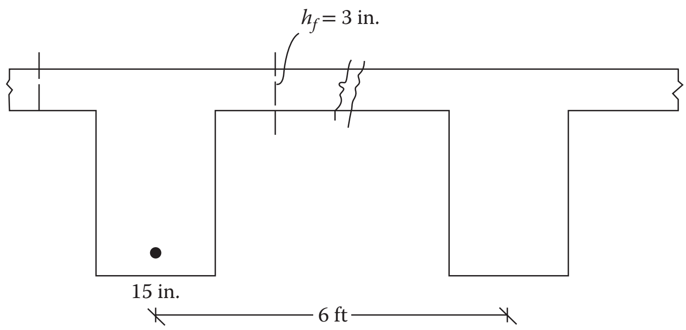

FIGURE 15.9 T beam section for Example 15.5.

5. Area of compression block

$$A_c = \frac{A_s f_y}{0.85 f'_c} = \frac{(9.48)(60{,}000)}{(0.85)(3{,}000)} = 223.06\,\text{in.}^2$$

$$b_f h_f = (60)(3) = 180 \text{ in.}^2$$

Since 223.06 > 180, the stress block extends into the web by a distance a_1 below the flange.

$$a_1 = \frac{A_c - b_f h_f}{b_w} = \frac{223.06 - 80}{15} = 2.87\,\text{in.}^2$$

6. $a = 3 + 2.87 = 5.87$ in.
7. In Figure 15.10, the centroid of the compression block from the top is:

$$\bar{y} = \frac{[60 \times 3 \times 1.5] + [15 \times 2.87 \times (3 + 2.87/2)]}{223.06} = 2.066\,\text{in.}$$

8. $c = \dfrac{5.87}{0.85} = 6.91\,\text{in.}$

$$\varepsilon_t = \frac{0.003(24 - 6.91)}{6.91} = 0.0074 > 0.005;\ \text{hence, } \phi = 0.9$$

$$Z = d - \bar{y} = 24 - 2.066 = 21.93 \text{ in.}^2$$

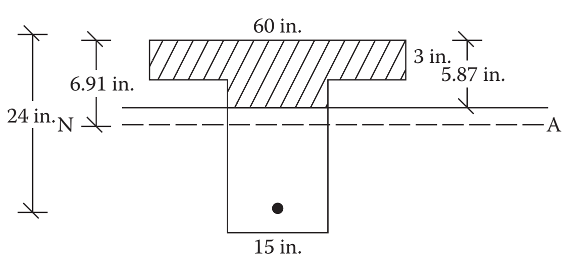

FIGURE 15.10 Compression stress block for Example 15.5.

[2] This is close to the assumed Z of step 3. Technically, steps 4 through 8 should be repeated with $Z = 21.93$ in.

9. Revised steel area

$$A_s = \frac{M_u}{\phi f_y Z} = \frac{11{,}520}{(0.9)(60)(21.93)} = 9.73\,\text{in.}^2$$

Select 10 bars of #9, A_s = 10 in.^{2} in two layers.
The steel area could be refined further by a small margin by repeating steps 4 through 9.

10. Minimum steel

a. $$\frac{3\sqrt{A'_c}\,b_w d}{f_y} = \frac{3\sqrt{3{,}000}(15)(24)}{60{,}000} = 0.99\,\text{in.}^2$$

b. $$\frac{200 b_w d}{f_y} = \frac{200(15)(24)}{60{,}000} = 1.20\text{ in.}^2 < 9.73\text{ in.}^2\ \textbf{OK}$$

PROBLEMS

15.1 Determine the design strength of the beam shown in Figure P15.1. Use $f_s' = 4{,}000\,\text{psi}$ and $f_y = 60{,}000\,\text{psi}$.

15.2 Determine the design strength of the beam shown in Figure P15.2. Use $f_c' = 3{,}000\,\text{psi}$ and $f_y = 60{,}000\,\text{psi}$.

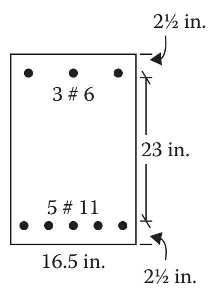

FIGURE P15.1 Beam section for Problem 15.1.

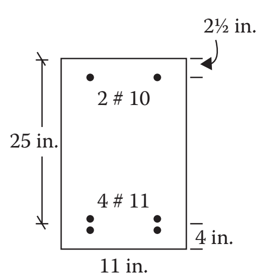

FIGURE P15.2 Beam section for Problem 15.2.

15.3 Determine the design strength of the beam shown in Figure P15.3. Use $f_c' = 4{,}000\,\text{psi}$ and $f_y = 60{,}000\,\text{psi}$.

15.4 Determine the design strength of the beam shown in Figure P15.4. Use $f_c' = 5{,}000\,\text{psi}$ and $f_y = 60{,}000\,\text{psi}$.

15.5 A beam of the dimensions shown in Figure P15.5 is subjected to a dead load of 690 lb/ft and a live load of 1500 lb/ft. It has a simple span of 35 ft. Design the beam. Use $f_c' = 4{,}000\,\text{psi}$ and $f_y = 60{,}000\,\text{psi}$.

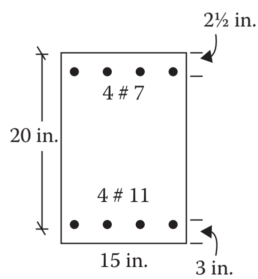

FIGURE P15.3 Beam section for Problem 15.3.

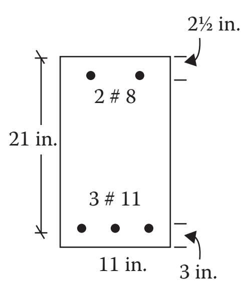

FIGURE P15.4 Beam section for Problem 15.4.

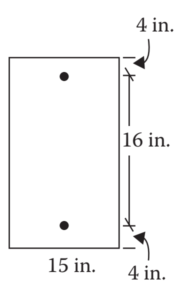

FIGURE P15.5 Beam dimensions for Problem 15.5.

15.6 Design a beam to resist the moment due to a service dead load of 150 ft-k (including weight) and the moment due to a service live load of 160 ft-k. The beam width is limited to 11 in., and the effective depth is limited to 20 in. The compression steel is 3 in. from the top. Use $f_c' = 3{,}000\,\text{psi}$ and $f_y = 60{,}000\,\text{psi}$.

15.7 Design a beam to resist the total factored moment (including weight) of 1,000 ft-k. The dimensions are as shown in Figure P15.6. Use $f_c' = 4{,}000\,\text{psi}$ and f_y = 50,000 psi.

15.8 Determine the design moment capacity of the T-beam shown in Figure P15.7. The T-beam spans 25 ft. Use $f_c' = 4{,}000\,\text{psi}$ and $f_y = 60{,}000\,\text{psi}$.

15.9 Determine the design capacity of the beam in Problem 15.8. The slab thickness is 3 in. and the center-to-center spacing of beams is 3 ft. Use $f_c' = 3{,}000\,\text{psi}$ and $f_y = 60{,}000\,\text{psi}$.

15.10 Design a T-beam for the floor system shown in Figure P15.8. The live load is 200 psf; the dead load is 60 psf excluding the weight of the beam. The slab thickness is 4 in., the effective depth is 25 in., and the width of the web is 15 in. Use $f_c' = 3{,}000\,\text{psi}$ and $f_y = 60{,}000\,\text{psi}$.

15.11 Design the T-beam shown in Figure P15.9 that spans 25 ft. The moment due to a service dead load is 200 ft-k (including beam weight) and due to a service live load is 400 ft-k. Use $f_c' = 4{,}000\,\text{psi}$ and $f_y = 60{,}000\,\text{psi}$.

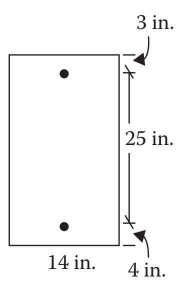

FIGURE P15.6 Beam dimensions for Problem 15.7.

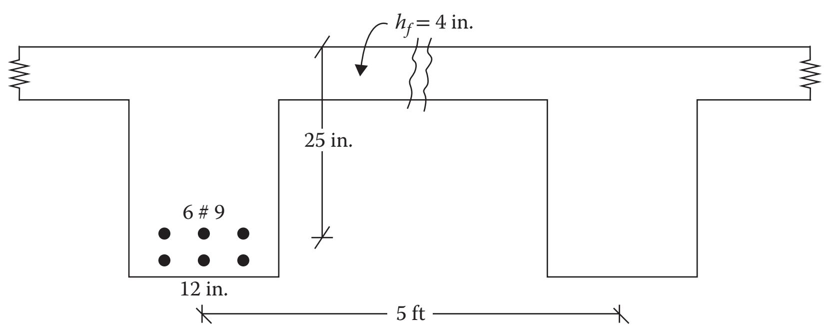

FIGURE P15.7 T beam section for Problem 15.8.

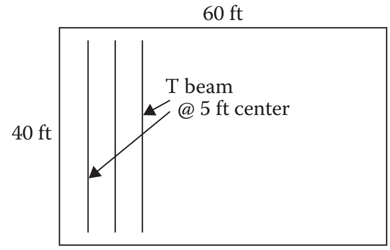

FIGURE P15.8 Floor system for Problem 15.10.

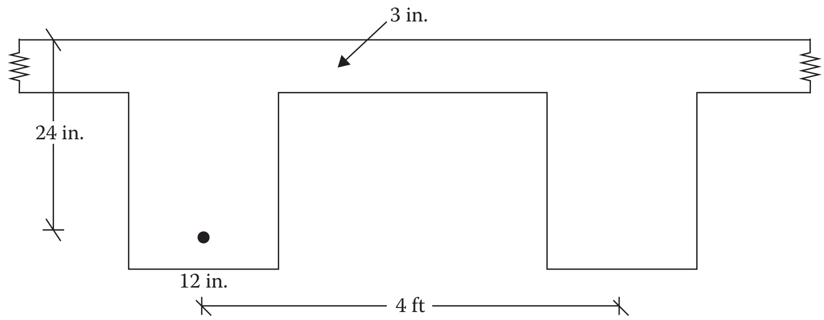

FIGURE P15.9 T beam section for Problem 15.11.

16 Shear and Torsion in Reinforced Concrete

STRESS DISTRIBUTION IN BEAM

The transverse loads on a beam segment cause a bending moment and a shear force that vary across the beam cross section and along the beam length. At point (1) in the beam shown in Figure 16.1, these contribute to the bending (flexure) stress and the shear stress, respectively, expressed as follows:

$$f_b = \frac{My}{I} \tag{16.1a}$$

and

$$f_v = \frac{VQ}{Ib} \tag{16.1b}$$

where:

M is the bending moment at a horizontal distance x from the support
y is the vertical distance of point (1) from the neutral axis
I is the moment of inertia of the section
V is the shear force at x
Q is the moment taken at the neutral axis of the cross-sectional area of the beam above point (1)
b is the width of section at (1)

The distribution of these stresses is shown in Figure 16.1. At any point (2) on the neutral axis, the bending stress is zero and the shear stress is maximum (for a rectangular section). On a small element at point (2), the vertical shear stresses act on the two faces, balancing each other, as shown in Figure 16.2. According to the laws of mechanics, the complementary shear stresses of equal magnitude and opposite sign act on the horizontal faces, as shown, to avoid any rotation of the element.

If we consider a free-body diagram along the diagonal a–b, as shown in Figure 16.3, and resolve the forces (shear stress times area) parallel and perpendicular to the plane a–b, the parallel force cancels and the total perpendicular force acting in tension is $1.414 f_v A$. Dividing by the area $1.414\,A$ of plane a–b, the tensile stress that acts on a–b is f_v. Similarly, if we consider a free-body diagram along the diagonal c–d, as shown in Figure 16.4, the total compression stress on the plane c–d is f_v. Thus, the planes a–b and c–d are subjected to tensile stress and compression stress, respectively, which have a magnitude equal to the shear stress on the horizontal and vertical faces. These stresses

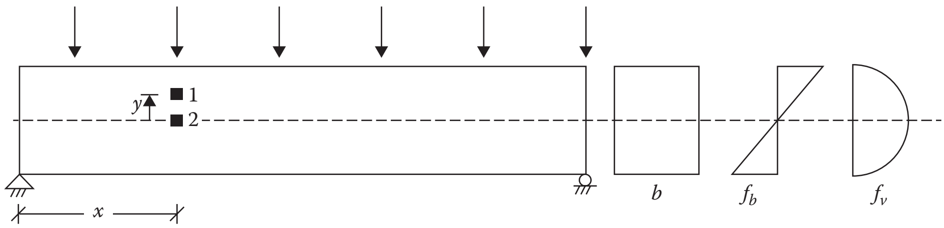

FIGURE 16.1 Flexure and shear stresses on transverse loaded beam.

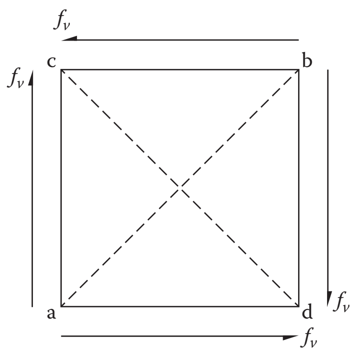

FIGURE 16.2 Shear stresses at neutral axis.

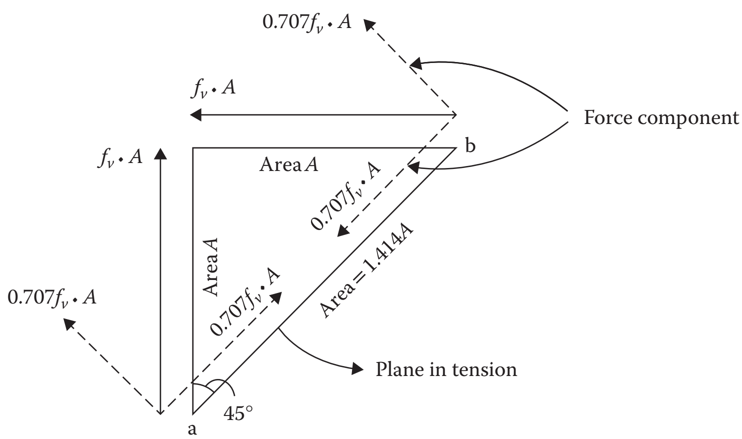

FIGURE 16.3 Free-body diagram along plane a–b of element of Figure 16.2.

on the planes a–b and c–d are the principal stresses (since they are not accompanied by any shear stress). The concrete is strong in compression but weak in tension. Thus, the stress on plane a–b, known as the *diagonal tension*, is of great significance. It is not the direct shear strength of concrete but the shear-induced diagonal tension that is considered in the analysis and design of concrete beams.

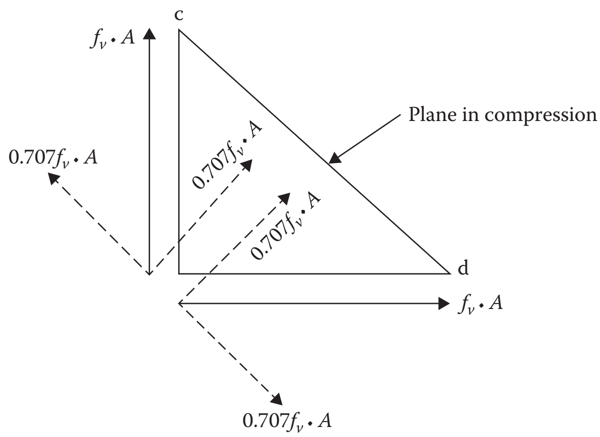

FIGURE 16.4 Free-body diagram along plane c–d of element of Figure 16.2.

DIAGONAL CRACKING OF CONCRETE

Concrete has a tendency to crack along the plane subjected to tension when the level of stress exceeds a certain value. The cracks form near the mid-depth where the shear stress (including the diagonal tension) is maximum and move in a path diagonal to the tensile surface, as shown in Figure 16.5. These are known as the *web-shear cracks*. They appear nearer to the support where shear is high. In a region where the moment is higher than the cracking moment capacity, the vertical flexure cracks appear first and the diagonal shear cracks develop as an extension to the flexure cracks. Such cracks are known as the *flexure-shear cracks*. They are more frequent in beams. The longitudinal (tensile) reinforcement does not prevent shear cracks, but it prevents the cracks from widening.

After a crack develops, the shear resistance along the cracked plane is provided by the following factors:

1. Shear resistance provided by the uncracked section above the crack, V_{cz}. This is about 20%–40% of the total shear resistance of the cracked section.
2. Friction developed due to interlocking of the aggregates on opposite sides of the crack, V_a. This is about 30%–50% of the total.
3. Frictional resistance between concrete and longitudinal (main) reinforcement called the *dowel action*, V_d. This is about 15%–25% of the total.

In a deep beam, some tie–arch action is achieved by the longitudinal bars acting as a tie and the uncracked concrete above and to the sides of the crack acting as an arch.

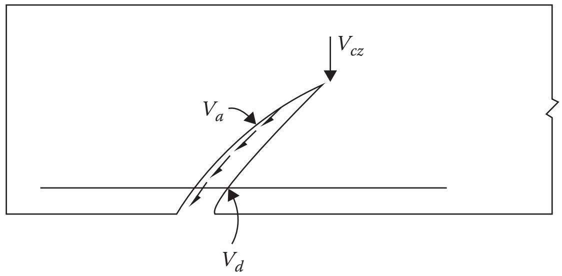

FIGURE 16.5 Shear resistance of cracked concrete.

Once the applied shear force exceeds the shear resistance offered by the above three factors in a cracked section, the beam fails suddenly unless a reinforcement known as the *web* or *shear reinforcement* is provided to prevent the further opening of the crack. It should be understood that the web reinforcement does not prevent the diagonal cracks that happen at almost the same loads with or without a web reinforcement. It is only after a crack develops that the tension that was previously held by the concrete is transferred to the web reinforcement.

STRENGTH OF WEB (SHEAR) REINFORCED BEAM

As stated above, the web reinforcement handles the tension that cannot be sustained by a diagonally cracked section. The actual behavior of web reinforcement is not clearly understood, in spite of the many theories presented. The truss analogy is the classic theory, which is very simple and widely used. The theory assumes that a reinforced concrete beam with web reinforcement behaves like a truss. A concrete beam with vertical web reinforcement in a diagonally cracked section is shown in Figure 16.6. The truss members shown by dotted lines are superimposed in Figure 16.6. The analogy between the beam and the truss members is shown in Table 16.1.

According to the above concept, the web reinforcement represents the vertical tensile member. According to the truss analogy theory, the entire applied shear force that induces the diagonal tension is resisted only by the web reinforcement. But observations have shown that the tensile stress in the web reinforcement is much smaller than the tension produced by the entire shear force. Accordingly, the truss analogy theory was modified to consider that the applied shear force is resisted by two components: the web reinforcement and the cracked concrete section. Thus:

$$V_n = V_c + V_s \tag{16.2}$$

Including a capacity reduction factor, ϕ, we have:

$$V_u \le \phi V_n \tag{16.3}$$

For the limiting condition:

$$V_u = \phi V_c + \phi V_s \tag{16.4}$$

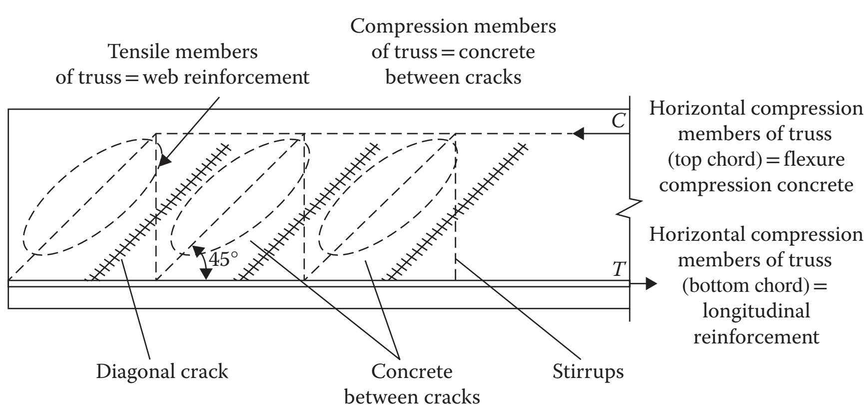

FIGURE 16.6 Truss analogy of beam.

TABLE 16.1
Beam–Truss Analogy

Truss	Beam
Horizontal tensile member (bottom chord)	Longitudinal reinforcement
Horizontal compression member (top chord)	Flexure compression concrete
Vertical tensile members	Web reinforcement
Diagonal compression members	Web concrete between the cracks in the compression zone

where:
- V_n is the nominal shear strength
- V_u is the factored design shear force
- V_c is the shear contribution of concrete
- V_s is the shear contribution of the web reinforcement
- ϕ is the capacity reduction factor for shear = 0.75 (Table 14.1)

Equation 16.4 serves as a design basis for web (shear) reinforcement.

SHEAR CONTRIBUTION OF CONCRETE

With flexure reinforcement but without web reinforcement, concrete does not contribute in resisting the diagonal tension once the diagonal crack is formed. Therefore, the shear stress in concrete at the time of diagonal cracking can be assumed to be the ultimate strength of concrete in shear. Many empirical relations have been suggested for the shear strength. The American Concrete Institute (ACI) has suggested the following relation:

$$V_c = 2\lambda\sqrt{f'_c}\,bd \tag{16.5}$$

The expression λ was introduced in the ACI 2008 code to account for lightweight concrete; for normal weight concrete, $\lambda = 1$. An alternative, much more complicated expression has been proposed by the ACI for V_c. This alternative is a function of the longitudinal reinforcement, bending moment, and shear force at various points of the beam.

SHEAR CONTRIBUTION OF WEB REINFORCEMENT

The web reinforcement takes the form of stirrups that run along the face of a beam. The stirrups enclose the longitudinal reinforcement. The common types of stirrups are U -shaped or UU-shaped as shown in Figures 16.7a and b and are arranged vertically or diagonally. When a significant amount of torsion is present, closed stirrups are used, as shown in Figure 16.7c.

The strength of a stirrup of area A_v is $f_y A_v$. If n number of stirrups cross a diagonal crack, then the shear strength by stirrups across a diagonal is:

$$V_s = f_y A_v n \tag{16.6}$$

In a 45° diagonal crack, the horizontal length of the crack equals the effective depth, d, as shown in Figure 16.8. For stirrups spaced s on center, $n = d/s$. Substituting this in Equation 16.6, we have:

$$V_s = f_y A_v \frac{d}{s} \tag{16.7}$$

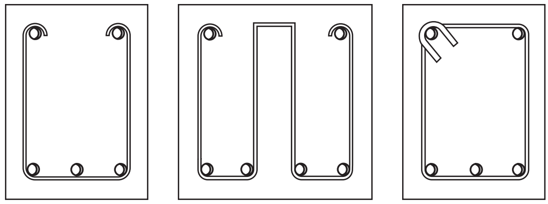

FIGURE 16.7 Types of stirrups: (a) open stirrup, (b) double stirrup, and (c) closed stirrup.

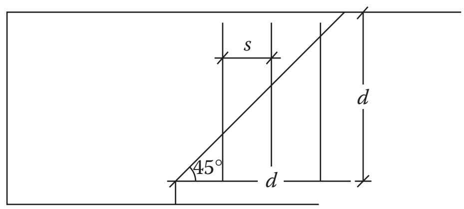

FIGURE 16.8 Vertical stirrup in a diagonal crack.

where:

A_v is the area of the stirrups
s is the spacing of the stirrups

For a U-shaped stirrup, A_v is twice the area of the bar; for a UU-shaped stirrup, A_v is four times the bar area.

When the stirrups are inclined at 45°, the shear force component along the diagonal matches the stirrups' (web reinforcements) strength, or:

$$V_s = 1.414 f_y A_v \frac{d}{s} \tag{16.8}$$

Equations 16.7 and 16.8 can be expressed as a single relation:

$$V_s = \alpha f_y A_v \frac{d}{s} \tag{16.9}$$

where $\alpha = 1$ for the vertical stirrups, and 1.414 for the inclined stirrups.

SPECIFICATIONS FOR WEB (SHEAR) REINFORCEMENT

The requirements of ACI 318-14 for web reinforcement are summarized below:

1. According to Equation 16.4, when $V_u \leq \phi V_c$, no web reinforcement is necessary. However, the ACI code requires that a minimum web reinforcement should be provided when V_u exceeds $1/2\phi V_c$, except for slabs, shallow beams (≤10 in.), and footing.
2. *Minimum steel:* When web reinforcement is provided, the amount of it should fall between the specified lower and upper limits. The reinforcing should not be so low that the web reinforcement steel yields as soon as a diagonal crack develops. The minimum web reinforcement area should be the *higher* of the following two values:

$$(A_v)_{min} = \frac{0.75\sqrt{f_c'}bs}{f_y} \tag{16.10}$$

or

$$(A_v)_{min} = \frac{50bs}{f_y} \tag{16.11}$$

3. *Maximum steel:* The maximum limit of web reinforcement is set because concrete eventually disintegrates, no matter how much steel is added. The upper limit is:

$$(A_v)_{max} = \frac{8\sqrt{f_c'}bs}{f_y}a \tag{16.12}$$

4. *Stirrup size:* The most common stirrup size is #3 bar. When the value of shear force is large, #4 bar might be used. The use of larger than #4 size is unusual. For a beam width of ≤24 in., a single loop stirrup U is satisfactory. Up to a width of 48 in., a double loop UU is satisfactory.
5. *Stirrup spacing*
   a. *Minimum spacing:* The vertical stirrups are generally not closer than 4 in. on center.
   b. *Maximum spacing* when $V_s \le 4\sqrt{f_c'}bd$
      The maximum spacing is the smaller of the following:
      i. $s_{max} = \dfrac{d}{2}$
      ii. $s_{max} = 24$ in.
      iii. $s_{max} = \dfrac{A_v f_y}{0.75\sqrt{f_c'}b}$ (based on Equation 16.10)
      iv. $s_{max} = \dfrac{A_v f_y}{50b}$ (based on Equation 16.11)
   c. *Maximum spacing* when $V_s > 4\sqrt{f_c'}bd$.
      The maximum spacing is the smaller of the following:
      i. $s_{max} = \dfrac{d}{4}$
      ii. $s_{max} = 12$ in.
      iii. $s_{max} = \dfrac{A_v f_y}{0.75\sqrt{f_c'}b}$ (based on Equation 16.10)
      iv. $s_{max} = \dfrac{A_v f_y}{50b}$ (based on Equation 16.11)
6. *Stirrups pattern:* The size of the stirrups is held constant while the spacing of the stirrups is varied. The shear force generally decreases from the support toward the middle of the span, indicating that the stirrups' spacing can continually increase from the end toward the center. From a practical point of view, the stirrups are placed in groups; each group has the same spacing. Only two to three such groups of incremental spacing are used within a pattern. The increment of spacing should be a multiple of whole inches, perhaps a multiple of 3 in. or 4 in.
7. *Critical section:* For a normal kind of loading where a beam is loaded at the top and there is no concentrated load applied within a distance *d* (effective depth) from the support, the section located at a distance *d* from the face of the support is called the *critical section*. The shear force at the critical section is taken as the design shear value, V_u, and the shear force from the face of the support to the critical section is assumed to be the same as at the critical section. When the support reaction is in tension at the end region of a beam or the loads are applied at the bottom (to the tension flange), or it is a bracket (cantilevered) section, no design shear force reduction is permitted, and the critical section is taken at the face of the support itself. Some designers place their first stirrup at a distance *d* from the face of the support, while others place the first stirrup at one-half of the spacing calculated at the end.

ANALYSIS FOR SHEAR CAPACITY

The process involves the following steps to check for the shear strength of an existing member and to verify the other code requirements:

1. Compute the concrete shear capacity using Equation 16.5.
2. Compute the web reinforcement shear capacity using Equation 16.9.
3. Determine the total shear capacity using Equation 16.4. This should be more than the applied factored shear force on the beam.
4. Check for the spacing of the stirrups from step 5 in the "Specifications for Web (Shear) Reinforcement" section.

Example 16.1

Determine the factored shear force permitted on the reinforced concrete beam shown in Figure 16.9. Use f'_c = 4,000 psi, f_y = 60,000 psi. Check for the web reinforcement spacing.

Solution

A. Concrete shear capacity from Equation 16.5

$$V_c = 2\lambda\sqrt{f'_c}\,bd$$

$$= 2(1)\sqrt{4000}(16)(27) = 54{,}644 \text{ lb or } 54.64 \text{ k}$$

B. Web shear capacity from Equation 16.9

$$A_v = 2(0.11) = 0.22 \text{ in.}^2$$

$$V_s = \alpha f_y A_v \frac{d}{s}$$

$$= 1(60{,}000)(0.22)\left(\frac{27}{12}\right) = 29{,}700 \text{ lb or } 29.7 \text{ k}$$

C. Design shear force from Equation 16.4

$$V_u = \phi V_c + \phi V_s$$

$$= 0.75(54.64) + 0.75(29.7) = 63.26 \text{ k}$$

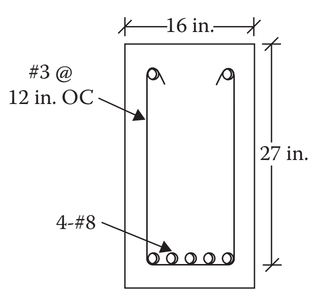

FIGURE 16.9 Section for Example 16.1.

D. Maximum spacing
1. $4\sqrt{f'_c}bd/1000 = 4\sqrt{4000}(16)(27)/1000 = 109.3$ k
2. Since V_s of 29.7 k < 109.3 k, maximum spacing is smaller of:
   a. $d/2 = 27/2 = 13.5$ in. > 12 in. ← Controls **OK**
   b. 24 in.
   c. $$s_{max} = \frac{A_v f_y}{0.75\sqrt{f'_c}b}$$
   $$= \frac{(0.22)(60{,}000)}{(0.75)\sqrt{4{,}000}(16)} = 17.4$$
   d. $$s_{max} = \frac{A_v f_y}{50b}$$
   $$= \frac{(0.22)(60{,}000)}{50(16)} = 16.5 \text{ in.}$$

DESIGN FOR SHEAR CAPACITY

A summary of the steps to design for web reinforcement is presented below:

1. Based on the factored loads and clear span, draw a shear force, V_u, diagram.
2. Calculate the critical V_u at a distance d from the support and show this on the V_u diagram as the critical section. When the support reaction is in tension, the shear force at the end is the critical V_u.
3. Calculate $\phi V_c = (0.75)2\sqrt{f'_c}bd$ and draw a horizontal line at the ϕV_c level on the V_u diagram. The portion of the V_u diagram above this line represents ϕV_s, the portion of the shear force that has to be provided by the web reinforcement or stirrups.
4. Calculate ½ϕV_c and show it by a point on the V_u diagram. The stirrups are needed from the support to this point. Below the $1/2\phi V_c$ point on the diagram toward the center, no stirrups are needed.
5. Make the tabular computations indicated in steps 5, 6, and 7 for the theoretical stirrups spacing. Starting at the critical section, divide the span into a number of segments. Determine V_u at the beginning of each segment from the slope of the V_u diagram. At each segment, calculate V_s from the following rearranged Equation 16.4:
$$V_s = \frac{(V_u - \phi V_c)}{\phi}$$
6. Calculate the stirrup spacing for a selected stirrup size at each segment from the following rearranged Equation 16.9:
$$s = \alpha f_y A_v \frac{d}{V_s} \quad (\alpha \text{ being 1 for vertical stirrup})$$
7. Compute the maximum stirrup spacing from the equations in step 5 in the "Specifications for Web (Shear) Reinforcement" section.
8. Draw a spacing versus distance diagram from step 6. On this diagram, draw a horizontal line at the maximum spacing of step 7 and a vertical line from step 4 for the cut-off limit stirrup.
9. From the diagram, select a few groups of different spacing and sketch the design.

Example 16.2

The service loads on a reinforced beam are shown in Figure 16.10, along with the designed beam section. Use $f_c' = 4{,}000$ psi and $f_y = 60{,}000$ psi. Design the web reinforcement.

Solution

A. V_u diagram
1. Weight of beam = (15/12) × (21/12) × 1 × (150/1000) = 0.33 k/ft
2. $w_u = 1.2(3 + 0.33) = 4$ k/ft
3. $P_u = 1.6(15) = 24$ k
4. M at B = 0

$$R_A\,(24) - 4(24)(12) - 24(18) - 24(6) = 0$$

$$R_A = 72 \text{ k}$$

5. The shear force diagram is shown in Figure 16.11.
6. The enlarged V_u diagram for one-half span is shown in Figure 16.12.

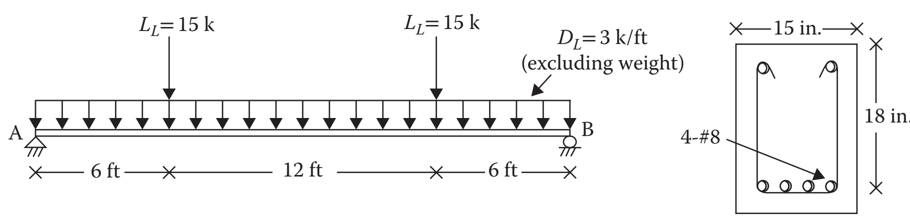

FIGURE 16.10 Load on beam and section for Example 16.2.

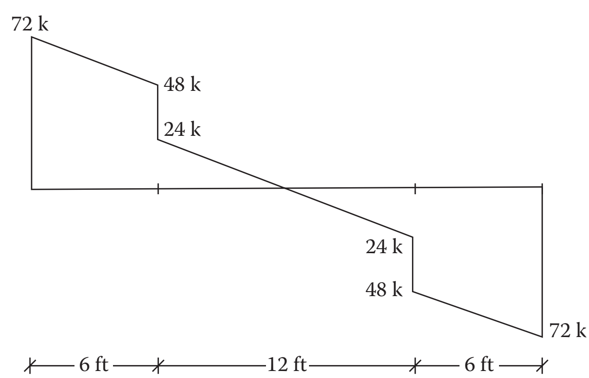

FIGURE 16.11 Shear force diagram for Example 16.2.

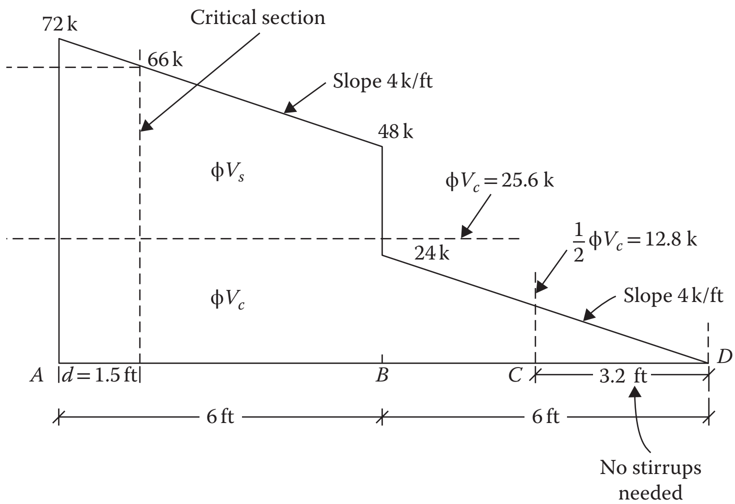

FIGURE 16.12 V_u diagram for Example 16.2.

B. Concrete and steel strengths
1. Critical V_u at a distance, $d = 72 - (18/12)(4) = 66$ k
2. $\phi V_c = 0.75(2)\sqrt{f'_c}bd = 0.75(2)\sqrt{4000}(15)(18)/1000 = 25.61$ k
3. $\frac{1}{2}\phi V_c = 12.8$ k
4. Distance from the beam center line to $(\frac{1}{2})(\phi V_c/\text{slope}) = 12.8/4 = 3.2$ ft

C. Stirrups design: Use #3 stirrups.

Distance from Support, *x*, ft	V_u**, k**	$V_s = \dfrac{V_u - \varphi V_c, k}{\varphi}$	$s = f_y A_v \dfrac{d}{V_s}$**, in.**
1	2	3[c]	4[d]
1.5 (=*D*)	66[a]	53.87	4.41
2	64	51.20	4.64
4	56	40.53	5.86
6^-	48	29.87	7.96
6^+	24[b]	0	∞

[a] $V_u = V_u$ @end − (slope)(distance) = 72 − 4(Col. 1)

[b] $V_u = V_u$ @ B in Figure 16.12 − (slope)(distance − 6) = 24 − 24(Col.1 − 6)

[c] (Col.2 − item B.2)/ϕ

[d] (600,000/1000)(0.22)(18)/Col.3

Distance versus spacing from the above table are plotted in Figure 16.13.

D. Maximum spacing
1. $4\sqrt{f'_c}bd/1000 = 4\sqrt{4000}(15)(18)/1000 = 68.3$ k
2. $V_{s\ critical}$ of 53.87 k < 68.3 k
3. Maximum spacing is the smaller of:
   a. $\dfrac{d}{2} = \dfrac{18}{2} = 9$ in. ← controls
   b. 24 in.

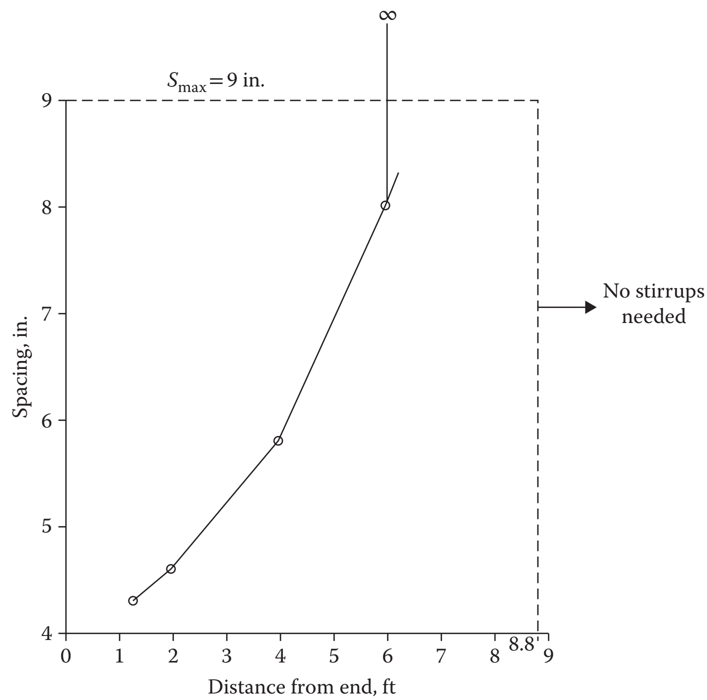

FIGURE 16.13 Distance-spacing graph for Example 16.2.

c. $s_{max} = \dfrac{A_v f_y}{0.75\sqrt{f'_c}\,b}$

$$= \frac{(0.22)(60{,}000)}{(0.75)\sqrt{4{,}000}(15)} = 18.55$$

d. $s_{max} = \dfrac{A_v f_y}{50b}$

$$= \frac{(0.22)(60{,}000)}{50(15)} = 17.6 \text{ in.}$$

The s_{max} line is shown in Figure 16.13.

E. Selected spacings

Distance Covered, ft	Spacing, in.	No. of Stirrups
0–5	4	15
5–6	6	2
6–8.8	9	4

TORSION IN CONCRETE

Torsion occurs when a member is subjected to a twist about its longitudinal axis due to a load acting off center of the longitudinal axis. Such a situation can be seen in the spandrel girder shown in Figure 16.14.

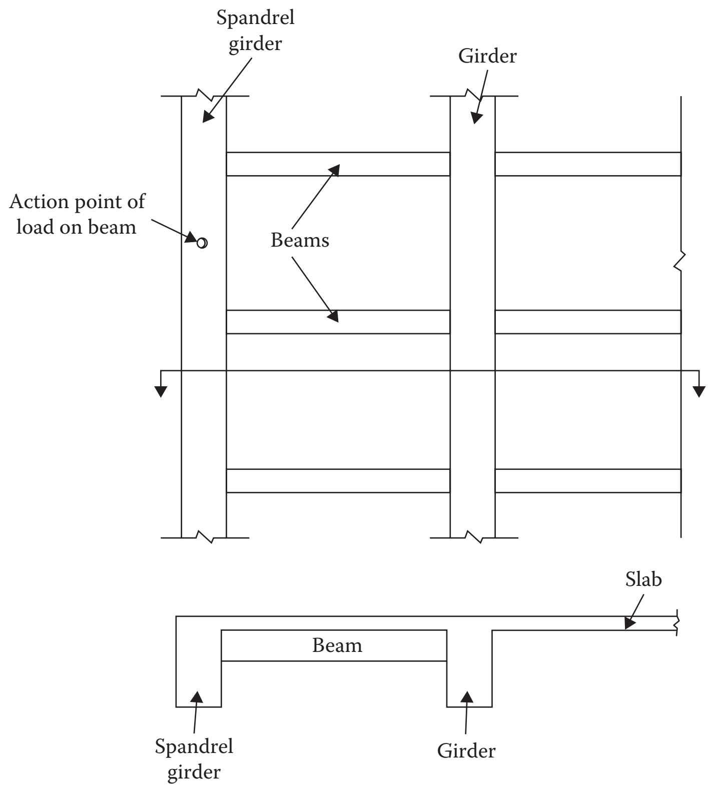

FIGURE 16.14 Beam subjected to torsion.

The moment developed at the end of the beam produces a torsion in the spandrel girder. A similar situation develops when a beam supports a member that overhangs across the beam. An earthquake can cause substantial torsion to the members. The magnitude of torsion can be given by:

$$T = Fr \tag{16.13}$$

where:
- F is the force or reaction
- r is the perpendicular distance of the force from the longitudinal axis

A load factor is applied to the torsion to convert T to T_u, similar to the moment. A torsion produces torsional shear on all faces of a member. The torsional shear leads to diagonal tensile stress very similar to that caused by the flexure shear. The concrete cracks along the spiral lines that run at 45° from the faces of a member when this diagonal tension exceeds the strength of concrete. After the cracks develop, any additional torsion makes the concrete fail suddenly unless torsional reinforcement is provided. Similar to shear reinforcement, providing torsional reinforcement does not change the magnitude of the torsion at which the cracks form. However, once the cracks are formed, the torsional tension is taken over by the torsional reinforcement to provide additional strength against the torsional tension.

PROVISION FOR TORSIONAL REINFORCEMENT

ACI 318-14 provides that as long as the factored applied torsion, T_u, is less than one-fourth of the cracking torque, T_r, torsional reinforcement is not required. Equating T_u to one-fourth of cracking torque T_r, the threshold limit is expressed as:

$$[T_u]_{limit} = \phi\sqrt{f_C'}\,\frac{A_{cp}^2}{P_{cp}} \tag{16.14}$$

where:

T_u is the factored design torsion
A_{cp} is the area enclosed by the outside parameter of the concrete section = width × height
P_{cp} is the outside parameter of the concrete = 2 (b + h)
ϕ = 0.75, for torsion

When T_u exceeds the threshold limit given by Equation 16.14, torsional reinforcement has to be designed. The process consists of performing the following computations:

1. Verify from Equation 16.14 that the cross-sectional dimensions of the member are sufficiently large to support the torsion acting on the beam.
2. If required, design the closed loop stirrups to support the torsional tension ($T_u = \phi T_n$) as well as the shear-induced tension ($V_u = \phi V_n$).
3. Compute the additional longitudinal reinforcement to resist the horizontal component of the torsional tension. There must be a longitudinal bar in each corner of the stirrups.

When an appreciable torsion is present that exceeds the threshold value, it might be more expedient and economical to select a larger section than would normally be chosen to satisfy Equation 16.14 so that torsional reinforcement does not have to be provided. The book uses this approach.

Example 16.3

The concentrated service loads shown in Figure 16.15 are located at the end of a balcony cantilever section, 6 in. to one side of the centerline. Is the section adequate without any torsional reinforcement? If not, redesign the section so that no torsional reinforcement has to be provided. Use $f_c' = 4{,}000$ psi and $f_y = 60{,}000$ psi.

Solution

The beam is subjected to moment, shear force, and torsion. It is being analyzed for torsion only.

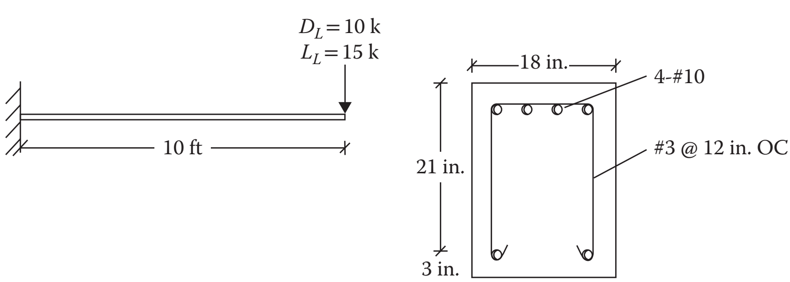

FIGURE 16.15 Cantilever beam and section for Example 16.3.

A. Checking the existing section
1. Design load contributing to torsion

$$P_u = 1.2(10)+1.6(15) = 36\text{ k}$$

2. Design torsion

$$T_u = 36\left(\frac{6}{12}\right) = 18\,\text{ft-k}$$

3. Area enclosed by the outside parameter

$$A_{cp} = bh = 18\times 24 = 432\text{ in.}^2$$

4. Outside parameter

$$P_{cp} = 2(b+h) = 2(18+24) = 84\text{ in.}$$

5. Torsional capacity of concrete

$$= \phi\sqrt{f'_C}\,\frac{A_{cp}^2}{P_{cp}}$$

$$= (0.75)\sqrt{4000}\,\frac{(432)^2}{84}$$

$$= 105{,}385.2\text{ in.-lb or } 8.78\text{ ft-k} < 18\text{k} \quad \textbf{NG}$$

B. Redesign the section
1. Assume a width of 24 in.
2. Area enclosed by the outside parameter, $A_{cp} = (24h)$
3. Parameter enclosed, $P_{cp} = 2(24 + h)$
4. Torsional capacity $= \phi\sqrt{f'_c}\,\dfrac{A_{cp}^2}{P_{cp}}$

$$= (0.75)\sqrt{4000}\,\frac{(24h)^2}{2(24+h)}$$

$$= 13{,}661\frac{h^2}{(24+h)}\text{ in.-lb or } 1.138\frac{h^2}{(24+h)}\text{ ft-k}$$

5. For no torsional reinforcement:

$$T_u = \phi\sqrt{f'_c}\,\frac{A_{cp}^2}{P_{cp}}$$

or

$$18 = 1.138\frac{h^2}{(24+h)}$$

or

$$h = 29\text{ in.}$$

A section 24 × 29 will be adequate.

PROBLEMS

16.1–16.3 Determine the concrete shear capacity, web reinforcement shear capacity, and design shear force permitted on the beam sections shown in Figures P16.1 through P16.3. Check for the spacing of web reinforcement. Use $f_c' = 3{,}000$ psi and $f_y = 40{,}000$ psi.

16.4 The reinforced beam with a 20-ft span shown in Figure P16.4 is subjected to a dead load of 1 k/ft (excluding beam weight) and a live load of 2 k/ft. Is the beam satisfactory to resist the maximum shear force? Use $f_c' = 3{,}000$ psi and $f_y = 60{,}000$ psi.

16.5 The service dead load (excluding the beam) is one-half of the service live load on the beam of span 25 ft shown in Figure P16.5. What is the magnitude of these loads from shear consideration? Use $f'_c = 4{,}000$ psi and $f_y = 60{,}000$ psi.

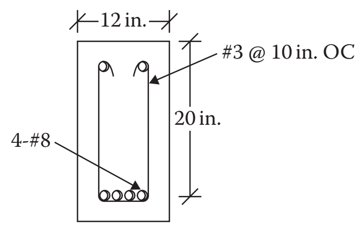

FIGURE P16.1 Beam section for Problem 16.1.

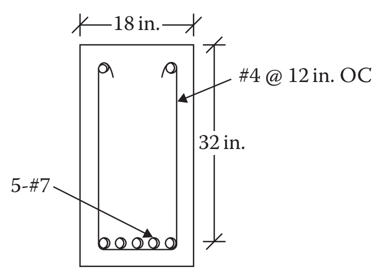

FIGURE P16.2 Beam section for Problem 16.2.

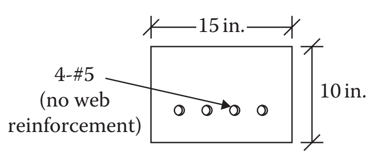

FIGURE P16.3 Beam section for Problem 16.3.

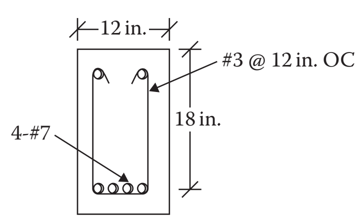

FIGURE P16.4 Beam section for Problem 16.4.

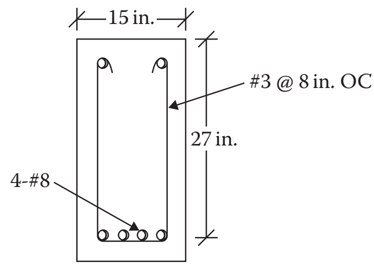

FIGURE P16.5 Beam section for Problem 16.5.

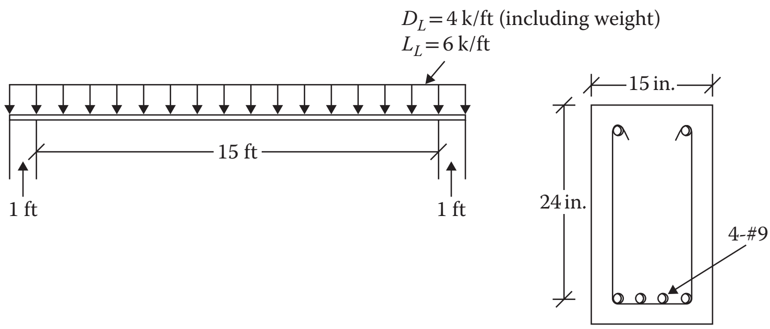

FIGURE P16.6 Loads on beam and section for Problem 16.7.

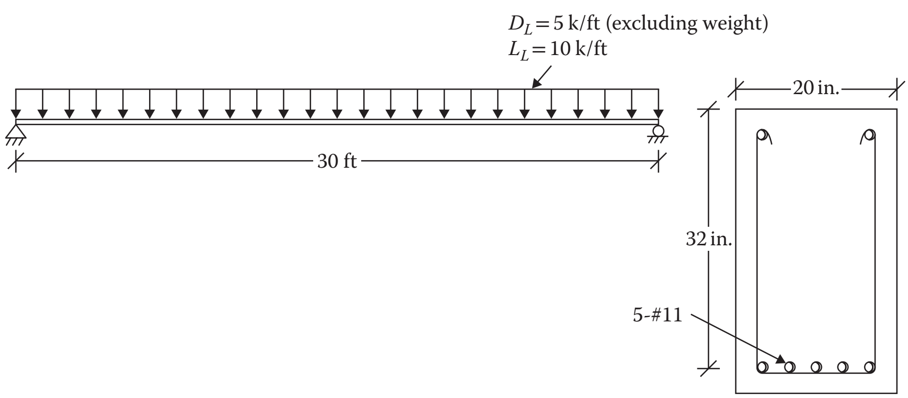

FIGURE P16.7 Loads on beam and section for Problem 16.8.

16.6 A simply supported beam is 15 in. wide and has an effective depth of 24 in. It supports a total factored load of 10 k/ft (including the beam weight) on a clear span of 22 ft. Design the web reinforcement. Use $f'_c = 4{,}000$ psi and $f_y = 60{,}000$ psi.

16.7 Design the web reinforcement for the service loads shown in Figure P16.6. Use $f'_c = 4{,}000$ psi and $f_y = 60{,}000$ psi.

16.8 For the beam and service loads shown in Figure P16.7, design the web reinforcement using #4 stirrups. Use $f'_c = 5{,}000$ psi and $f_y = 60{,}000$ psi.

16.9 For the service loads on a beam (excluding beam weight) shown in Figure P16.8, design the web reinforcement. Use $f'_c = 4{,}000$ psi and $f_y = 50{,}000$ psi.

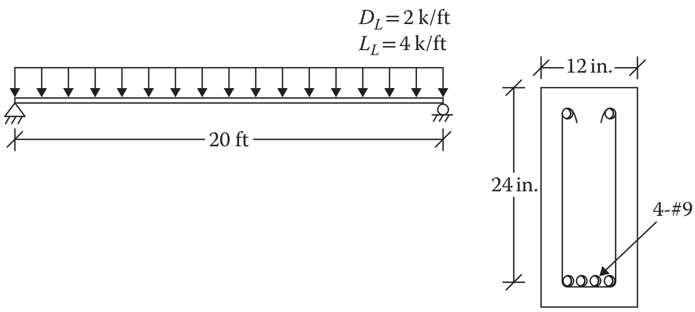

FIGURE P16.8 Loads on beam and section for Problem 16.9.

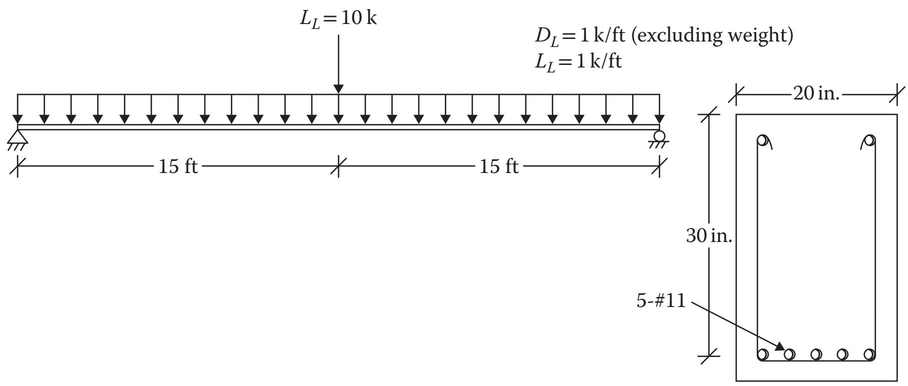

FIGURE P16.9 Loads on beam and section for Problem 16.10.

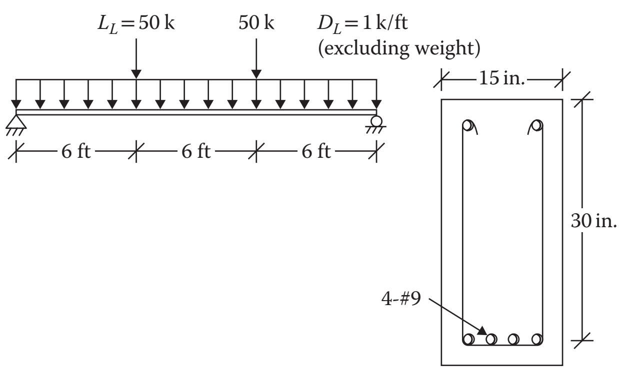

FIGURE P16.10 Loads on beam and section for Problem 16.11.

16.10 Design the web reinforcement for the service loads on the beam shown in Figure P16.9. Use $f'_c = 3{,}000$ psi and $f_y = 40{,}000$ psi.

16.11 A simply supported beam carries the service loads (excluding the beam weight) shown in Figure P16.10. Design the web reinforcement. Use $f'_c = 4{,}000$ psi and $f_y = 60{,}000$ psi.

16.12 A simply supported beam carries the service loads (excluding the beam weight) shown in Figure P16.11. Design the web reinforcement. Use #4 size stirrups. Use $f'_c = 4{,}000$ psi and $f_y = 60{,}000$ psi.

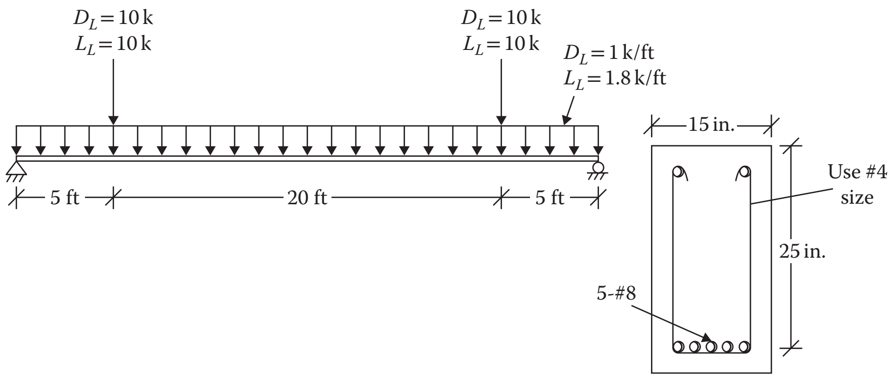

FIGURE P16.11 Loads on beam and section for Problem 16.12.

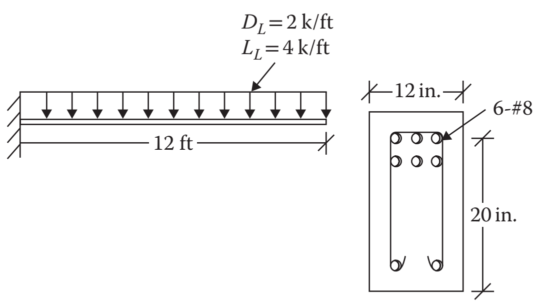

FIGURE P16.12 Loads on cantilever beam and section for Problem 16.13.

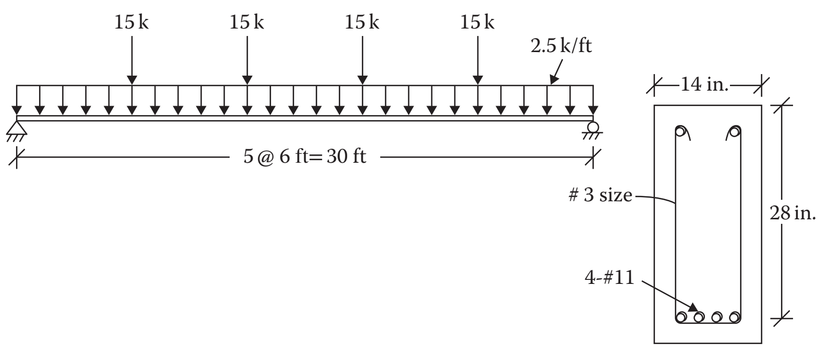

FIGURE P16.13 Loads on beam and section for Problem 16.14.

16.13 A cantilever beam carries the service loads, including the beam weight, shown in Figure P16.12. Design the web reinforcement. Use $f'_c = 4{,}000$ psi and $f_y = 60{,}000$ psi. *Hint:* $V_{critical}$ is at the support.

16.14 A beam carries the factored loads (including beam weight) shown in Figure P16.13. Design the #3 size web reinforcement. Use $f'_c = 3{,}000$ psi and $f_y = 40{,}000$ psi.

16.15 A beam supported on the walls carries the uniform distributed loads and the concentrated loads from the upper floor shown in Figure P16.14. The loads are service loads, including the weight of the beam. Design the #3 size web reinforcement. Use $f'_c = 4{,}000$ psi and $f_y = 50{,}000$ psi.

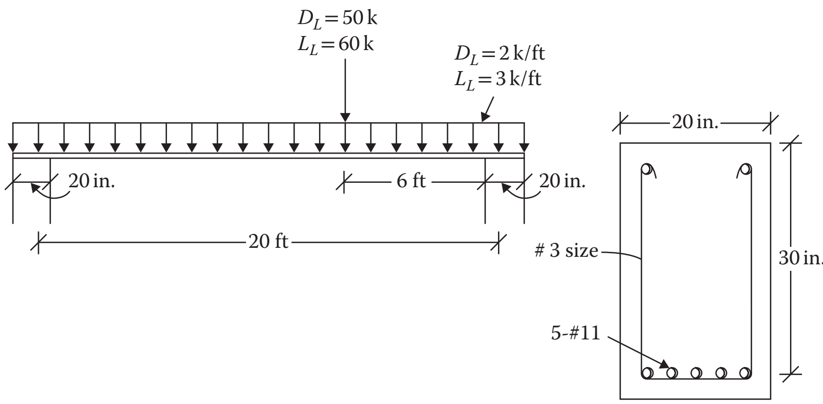

FIGURE P16.14 Loads and section of beam.

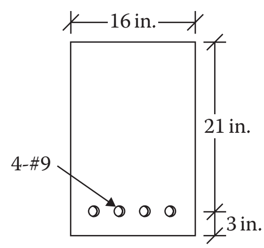

FIGURE P16.15 Beam section under torsion for Problem 16.16.

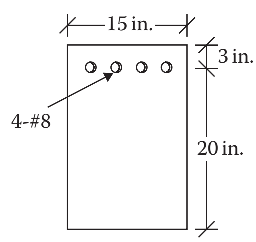

FIGURE P16.16 Cantilever section under torsion for Problem 16.17.

16.16 Determine the torsional capacity of the beam section in Figure P16.15 without torsional reinforcement. Use $f'_c = 4{,}000$ psi and $f_y = 60{,}000$ psi.

16.17 Determine the torsional capacity of the cantilever beam section shown in Figure P16.16 without torsional reinforcement. Use $f'_c = 3{,}000$ psi and $f_y = 40{,}000$ psi.

16.18 The spandrel beam shown in Figure P16.17 is subjected to a factored torsion of 8 ft-k. Is this beam adequate if no torsional reinforcement is used? If not, redesign the section. The width cannot exceed 16 in. Use $f'_c = 4{,}000$ psi and $f_y = 50{,}000$ psi.

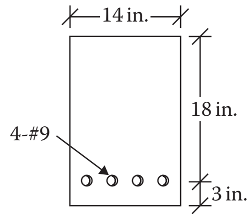

FIGURE P16.17 Beam section under torsion for Problem 16.18.

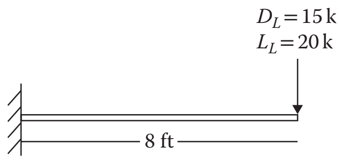

FIGURE P16.18 Torsional loads on cantilever for Problem 16.19.

16.19 Determine the total depth of a 24-in.-wide beam if no torsional reinforcement is used. The service loads, shown in Figure P16.18, act 5 in. to one side of the centerline. Use $f'_c = 4{,}000$ psi and $f_y = 60{,}000$ psi.

16.20 A spandrel beam is exposed to a service dead load of 8 k and live load of 14 k acting 8 in. off center of the beam. The beam section is 20 in. wide and 25 in. deep. Is the section adequate without torsional reinforcement? If not, redesign the section using the same width. Use f'_c $f'_c = 5{,}000$ psi and $f_y = 60{,}000$ psi.

17 Compression and Combined Forces Reinforced Concrete Members

TYPES OF COLUMNS

Concrete columns are divided into four categories, as described in the following sections.

PEDESTALS

The column height is less than three times the least lateral dimension. A pedestal is designed with plain concrete (without reinforcement) for a maximum compression strength of $0.85\phi\ f_c'A_g$, where ϕ is 0.65 and A_g is the cross-sectional area of the column.

COLUMNS WITH AXIAL LOADS

The compressive load acts coinciding with the longitudinal axis of the column or at a small eccentricity so that there is no induced moment or there is a moment of little significance. This is a basic case, although not quite common in practice.

SHORT COLUMNS WITH COMBINED LOADS

The columns are subjected to an axial force and a bending moment. However, the buckling effect is not present and the failure is initiated by crushing of the material.

LARGE OR SLENDER COLUMNS WITH COMBINED LOADS

In this case, the buckling effect is present. Due to an axial load, P, the column axis buckles by an amount, Δ. Thus, the column is subjected to the secondary moment or the P–Δ moment.

As concrete and steel both can share compression loads, steel bars directly add to the strength of a concrete column. The compression strain is equally distributed between concrete and steel that are bonded together. It causes a lengthwise shortening and a lateral expansion of the column due to Poisson's effect. The column capacity can be enhanced by providing a lateral restraint. The column is known as a *tied* or a *spiral* column depending on whether the lateral restraint is in the form of the closely spaced ties or the helical spirals wrapped around the longitudinal bars, as shown in Figure 17.1a and b.

Tied columns are ordinarily square or rectangular, and spiral columns are round but they could be otherwise too. The spiral columns are more effective in terms of the column strength because of their hoop stress capacity. But they are more expensive. Thus, tied columns are more common, and spiral columns are used only for heavy loads.

Composite columns are reinforced by steel shapes that are contained within the concrete sections or by concrete being filled in within the steel section or tubing, as shown in Figure 17.1c and d. The latter is commonly called a *lally* column.

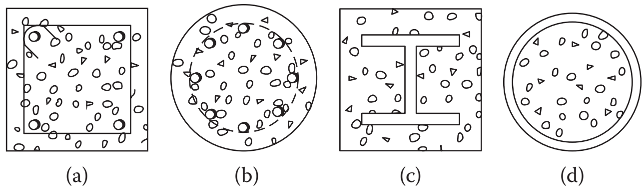

FIGURE 17.1 Types of columns: (a) tied column, (b) spiral column, and (c) and (d) composite columns.

AXIALLY LOADED COLUMNS

Axially loaded columns include columns with a small eccentricity. The small eccentricity is defined when the compression load acts at a distance, e, from the longitudinal axis controlled by the following conditions:

$$\text{For spiral columns:}\quad e \le 0.05h \tag{17.1}$$

$$\text{For tied columns:}\quad e \le 0.1h \tag{17.2}$$

where h is the column dimension along distance, e.

In the case of columns, unlike beams, it does not matter whether the concrete or steel reaches ultimate strength first because both of them deform/strain together, which distributes the matching stresses between them. Also, high strength is more effective in columns because the entire concrete area contributes to the strength, unlike the contribution from concrete in the compression zone only in beams, which is about 30%–40% of the total area.

The basis of the design is the same as for wood or steel columns; that is:

$$P_u \le \phi P_n \tag{17.3}$$

where:

P_u is the factored axial load on the column
P_n is the nominal axial strength
ϕ = the strength reduction factor, which is 0.70 for spiral columns, 0.65 for tied columns

The nominal strength is the sum of the strength of concrete and the strength of steel. The concrete strength is the ultimate (uniform) stress 0.85 f_c' times the concrete area (A_g–A_{st}), and the steel strength is the yield stress, f_y, times the steel area, A_{st}. However, to account for the small eccentricity, a factor (0.85 for spiral and 0.8 for tied) is applied. Thus:

$$P_n = 0.85[0.85 f_c'(A_g - A_{st}) + f_y A_{st}]\,\text{for spiral columns} \tag{17.4}$$

$$P_n = 0.80[0.85 f_c'(A_g - A_{st}) + f_y A_{st}]\,\text{for tied columns} \tag{17.5}$$

Including a strength reduction factor of 0.7 for spiral and 0.65 for tied columns in the previous equations, Equation 17.3 for column design is as follows:

For spiral columns with $e \le 0.05h$:

$$P_u = 0.60[0.85 f_c'(A_g - A_{st}) + f_y A_{st}] \tag{17.6}$$

For tied columns with $e \le 0.1\ h$:

$$P_u = 0.52[0.85 f_c'(A_g - A_{st}) + f_y A_{st}] \tag{17.7}$$

STRENGTH OF SPIRALS

Note that a higher factor is used for spiral columns than tied columns. The reason is that in a tied column, as soon as the shell of a column spalls off, the longitudinal bars buckle immediately with the lateral support gone. But a spiral column continues to stand and resist more load, with the spiral and longitudinal bars forming a cage to confine the concrete.

Because the utility of a column is lost once its shell spalls off, the American Concrete Institute (ACI) assigns only slightly more strength to the spiral compared to the strength of the shell that gets spalled off.

With reference to Figure 17.2:

$$\text{Strength of shell} = 0.85 f_c'(A_g - A_c) \tag{a}$$

$$\text{Hoop tension in spiral} = 2 f_y A_{sp} = 2 f_y \rho_s A_c \tag{b}$$

where ρ_s is spiral steel ratio = A_{sp}/A_c. Equating the two expressions (a) and (b) and solving for ρ_s:

$$\rho_s = 0.425 \frac{f_c'}{f_y}\left(\frac{A_g}{A_c} - 1\right) \tag{c}$$

Making the spiral a little stronger:

$$\rho_s = 0.45 \frac{f_c'}{f_y}\left(\frac{A_g}{A_c} - 1\right) \tag{17.8}$$

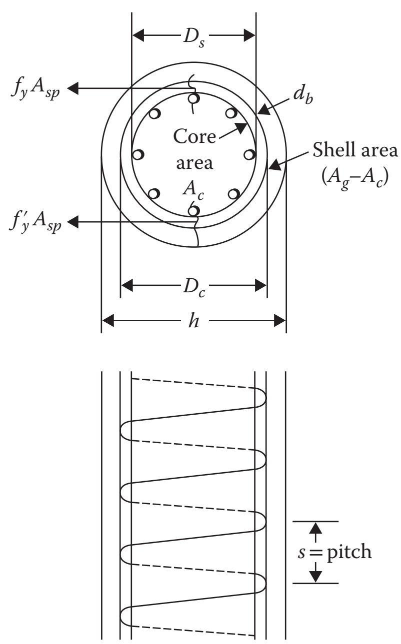

FIGURE 17.2 Spiral column section and profile.

Once the spiral steel is determined, the following expression derived from the definition of ρ_s is used to set the spacing or pitch of the spiral:

By definition, from Figure 17.2:

$$\rho_s = \frac{\text{volume of spiral in one loop}}{\text{volume of concrete in pitch, } s} \tag{d}$$

$$= \frac{\pi(D_c - d_b)A_{sp}}{(\pi D_c^2/4)s} \tag{e}$$

where the various terms are explained in Figure 17.2.

If the diameter difference, that is, d_b, is neglected:

$$\rho_s = \frac{4A_{sp}}{D_c s} \tag{f}$$

or

$$s = \frac{4A_{sp}}{D_c \rho_s} \tag{17.9}$$

Appendix D.13, based on Equations 17.8 and 17.9, can be used to select the size and pitch of spirals for a given diameter of a column.

SPECIFICATIONS FOR COLUMNS

1. *Main steel ratio:* The steel ratio, ρ_g, which is the ratio area of steel to area of concrete; *Ast/Ag*, should not be less than 0.01 (1%) and not more than 0.08 (8%). Usually a ratio of 0.03 is adopted.
2. *Minimum number of bars:* A minimum of four bars is used within the rectangular or circular ties and six within the spirals.
3. *Cover:* A minimum cover over the ties or spiral shall be 1½ in.
4. *Spacing:* The clear distance between the longitudinal bars should neither be less than 1.5 times the bar diameter nor 1½ in. To meet these requirements, Appendix D.14, can be used to determine the maximum number of bars that can be accommodated in one row within a given size of a column.
5. *Tie requirements:*
   a. The minimum size of the tie bars is #3 when the size of longitudinal bars is #10 or smaller or when the column diameter is 18 in. or less. The minimum size is #4 for longitudinal bars larger than #10 or a column larger than 18 in. Usually, #5 is the maximum size.
   b. The center-to-center spacing of ties should be the smaller of the following:
      i. 16 times the diameter of longitudinal bars
      ii. 48 times the diameter of ties
      iii. Least column dimension
   c. The ties shall be so arranged that every corner and alternate longitudinal bar has the lateral support provided by the corner of a tie having an included angle of not more than 135°. Figure 17.3 shows the tie arrangements for several columns.
   d. Longitudinal bar shall not have more than 6 in. clear distance on either side of a tie. If it is more than 6 in., a tie is provided, as shown in Figure 17.3c and e.

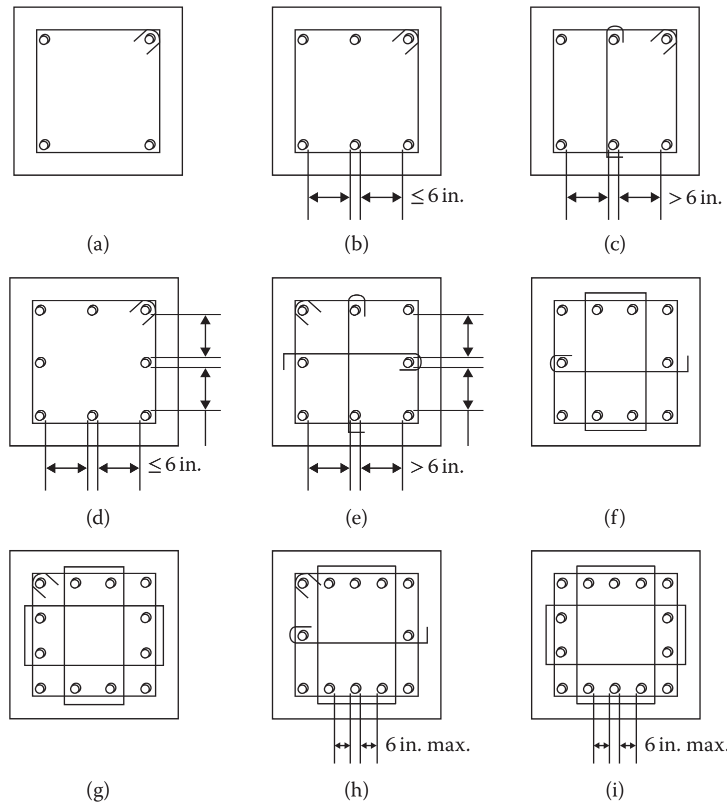

FIGURE 17.3 Tie arrangements for several columns (a) through (i).

6. *Spiral requirements*:
   a. The minimum spiral size is ⅜ in. (#3). Usually the maximum size is ⅝ in. (#5).
   b. The clear space between spirals should not be less than 1 in. or more than 3 in.

ANALYSIS OF AXIALLY LOADED COLUMNS

The analysis of columns of small eccentricity involves determining the maximum design load capacity and verifying the amount and details of the reinforcement according to the code. The procedure is summarized below:

1. Check that the column meets the eccentricity requirement ($\leq 0.05\ h$ for spiral and $\leq 0.1\ h$ for tied column).
2. Check that the steel ratio, ρ_g, is within 0.01–0.08.
3. Check that there are at least four bars for a tied column and six bars for a spiral column and that the clear spacing between bars is determined according to the "Specifications for Columns" section.
4. Calculate the design column capacity using Equation 17.6 or Equation 17.7.
5. For ties, check the size, spacing, and arrangement using the information in the "Specifications for Columns" section. For spirals, check the size and spacing using the information in the "Specifications for Columns" section.

Example 17.1

Determine the design axial load on a 16-in. square axially loaded column reinforced with eight #8 size bars. Ties are #3 at 12 in. on center. Use $f_c' = 4{,}000$ psi and $f_y = 60{,}000$ psi.

Solution

1. $A_{st} = 6.32$ in.^{2} (from Appendix D.2)
2. $A_g = 16 \times 16 = 256$ in.^{2}
3. $\rho_g = \dfrac{A_{st}}{A_g} = \dfrac{6.32}{256} = 0.0247$

   This is >0.01 and <0.08. **OK**
4. h = 2(cover) + 2(tie diameter) + 3(bar diameter) + 2(spacing)
   or 16 = 2(1.5) + 2(0.375) + 3(1) + 2(s)
   or s = 4.625 in.
   $s_{min} = 1.5(1) = 1.5$ in.; spacing s is more than s_{min}. **OK**
   $s_{max} = 6$ in.; spacing s is less than s_{max}. **OK**
5. From Equation 17.7:
   $$P_u = \frac{0.52[0.85(4000)(256-6.32)+(60000)(6.32)]}{1000}$$
   $$= 638.6\text{ k}$$
6. Check the ties.
   a. #3 size **OK**
   b. The spacing should be the smaller of the following:
      i. $16 \times 1 = 16$ in. ← Controls, more than given 12 in. **OK**
      ii. $48 \times 0.375 = 18$ in.
      iii. 16 in.
   c. Clear distance from the tie = 4.625 in. (step 4) < 6 in. **OK**

Example 17.2

A service dead load of 150 k and live load of 220 k is axially applied on a 15-in. diameter circular spiral column reinforced with six #9 bars. The lateral reinforcement consists of ⅜ in. spiral at 2 in. on center. Use $f_c' = 4{,}000$ psi and $f_y = 60{,}000$ psi. Is the column adequate?

Solution

1. $A_{st} = 6$ in.^{2} (from Appendix D.2)
2. $A_g = \dfrac{\pi}{4}(15)^2 = 176.63\text{ in.}^2$
3. $\rho_g = \dfrac{A_{st}}{A_g} = \dfrac{6}{176.63} = 0.034$

   This is >0.01 and <0.08. **OK**
4. $(D_c - d_b) = h - 2(\text{cover}) - 2(\text{spiral diameter})$
   $= 15 - 2(1.5) - 2(0.375) = 11.25$ in.
5. Perimeter, $p = \pi(D_c - d_b) = \pi(11.25) = 35.33$ in.
   p = 6(bar diameter) + 6(spacing)
   or 35.33 = 6(1.128) + 6(s)
   or s = 4.76 in.
   $s_{min} = 1.5(1) = 1.5$ in.; spacing s is more than s_{min}. **OK**
   $s_{max} = 6$ in.; spacing s is less than s_{max}. **OK**

6. $\phi P_n = \dfrac{0.60[0.85(4{,}000)(176.63-6)+(60{,}000)(6)]}{1{,}000} = 564 \text{ k}$
7. $P_u = 1.2(150)+1.6(220) = 532 \text{ k} < 564 \text{ k}$ **OK**
8. Check for spiral.
   a. ⅜ in. diameter **OK**
      D_c = 15–3 = 12 in.
   b. $A_c = \dfrac{\pi(12)^2}{4} = 113.04 \text{ in.}^2$

      $A_{sp} = 0.11 \text{ in.}^2$

      From Equation 17.8:

      $$\rho_s = 0.45\frac{(4)}{(60)}\left(\frac{176.63}{113.04}-1\right) = 0.017$$

      From Equation 17.9:

      $$s = \frac{4(0.11)}{(12)(0.017)} = 2.16 \text{ in.} > 2 \text{ in.(given)} \textbf{ OK}$$

   c. Clear distance = 2 – ⅜ = 1.625 in. > 1 in. **OK**

DESIGN OF AXIALLY LOADED COLUMNS

Design involves fixing the column dimensions, selecting reinforcement, and deciding the size and spacing of ties and spirals. For a direct application, Equations 17.6 and 17.7 are rearranged as follows by substituting $A_{st} = \rho_g A_g$:

For spiral columns:

$$P_u = 0.60A_g[0.85f_c'(1-\rho_g)+f_y\rho_g] \tag{17.10}$$

For tied columns:

$$P_u = 0.52A_g[0.85f_c'(1-\rho_g)+f_y\rho_g] \tag{17.11}$$

The design procedure involves the following:

1. Determine the factored design load for various load combinations.
2. Assume $\rho_g = 0.03$. A lower or higher value could be taken depending on a bigger or smaller size of column being acceptable.
3. Determine the gross area, A_g, from Equation 17.10 or Equation 17.11. Select the column dimensions to a full-inch increment.
4. For the actual gross area, calculate the adjusted steel area from Equation 17.6 or Equation 17.7. Make the selection of steel using Appendix D.2, and check, from Appendix D.14, that the number of bars can fit in a single row of the column.
5. For spirals, select the spiral size and pitch from Appendix D.13. For ties, select the size of tie, decide the spacing, and arrange ties as per item 5 of, "Specifications for Columns" section.
6. Sketch the design.

Example 17.3

Design a tied column for an axial service dead load of 200 k and service live load of 280 k. Use $f_c' = 4{,}000$ psi and $f_y = 60{,}000$ psi.

Solution

1. $P_u = 1.2(200) + 1.6(280) = 688$ k
2. For $\rho_g = 0.03$ from Equation 17.11:

$$A_g = \frac{P_u}{0.52[0.85f_c'(1-\rho_g) + f_y\rho_g]}$$

$$= \frac{688}{0.52[0.85(4)(1-0.03) + 60(0.03)]}$$

$$= 259.5 \text{ in.}^2$$

   For a square column, $h = \sqrt{259.5} = 16.1$ in.; use 16 in. × 16 in.; $A_g = 256$ in.^{2}
3. From Equation 17.7:
   $688 = 0.52\ [0.85(4)(256 - A_{st}) + 60(A_{st})]$
   $688 = 0.52(870.4 + 56.6A_{st})$
   $A_{st} = 8$ in.^{2}
   Select 8 bars of #9 size. A_{st} (provided) = 8 in.^{2}
   From Appendix D.14, for a core size of 16 − 3 = 13 in., 8 bars of #9 size can be arranged in a row.
4. Design of ties
   a. Select #3 size.
   b. Spacing should be the smaller of the following:
      i. 16(1.128) = 18 in.
      ii. 48(0.375) = 18 in.
      iii. 16 in. ← Controls
   c. Clear distance
      16 = 2(cover) + 2(tie diameter) + 3(bar diameter) + 2(spacing)
      16 = 2(1.5) + 2(0.375) + 3(1.128) + 2s
      or s = 4.43 in. < 6 in. **OK**
5. The sketch is shown in Figure 17.4.

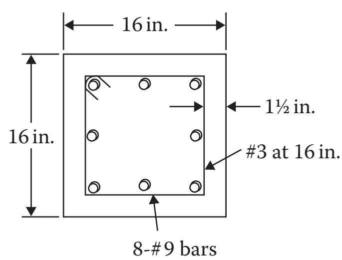

FIGURE 17.4 Tied column section of Example 17.3.

Example 17.4

For Example 17.3, design a circular spiral column.

Solution

1. $P_u = 1.2(200) + 1.6(280) = 688$ k
2. For $\rho_g = 0.03$ from Equation 17.10:

$$A_g = \frac{P_u}{0.60[0.85f_c'(1\ \ \rho_g) + f_y\rho_g]}$$

$$= \frac{688}{0.60[0.85(1-0.03)+60(0.03)]} = 225 \text{ in.}^2$$

For a circular column, $\dfrac{\pi h^2}{4} = 225$, and $h = 16.93$ in.; use 17 in.; $A_g = 227$ in.^{2}

3. From Equation 17.6:

$$688 = 0.60[0.85(4)(227 - A_{st}) + 60(A_{st})]$$

$$688 = 0.60(771.8 + 56.6\,A_{st})$$

or $A_{st} = 6.62$ in.^{2}

Select 7 bars of #9 size. A_{st} (provided) = 7 in.^{2}
From Appendix D.14, for a core size of 17 − 3 = 14 in., 9 bars of #9 can be arranged in a single row. **OK**

4. Design of spiral
   a. From Appendix D.13, for 17-in. diameter column, spiral size = ⅜ in., pitch = 2 in.
   b. Clear distance
      2 − ⅜ = 1.625 in. > 1 in. **OK**
5. The sketch is shown in Figure 17.5.

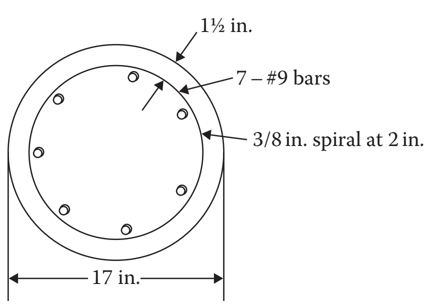

FIGURE 17.5 Spiral column of Example 17.4.

SHORT COLUMNS WITH COMBINED LOADS

Most of the reinforced concrete columns belong to this category. The condition of an axial loading or a small eccentricity is rare. The rigidity of the connection between beam and column makes the column rotate with the beam, resulting in a moment at the end. Even an interior column of equally spanned beams receives unequal loads due to variations in the applied loads, producing a moment on the column.

Consider that a load, P_u, acts at an eccentricity, e, as shown in Figure 17.6a. Apply a pair of loads P_u, one acting up and one acting down through the column axis, as shown in Figure 17.6b. The applied loads cancel each other and, as such, have no technical significance. When we combine the load P_u acting down at an eccentricity e with the load P_u acting upward through the axis, a couple, $M_u = P_u e$, is produced. In addition, the downward load P_u acts through the axis. Thus, a system of force acting at an eccentricity is equivalent to a force and a moment acting through the axis, as shown in Figure 17.6c. Inverse to this, a force and a moment, when acting together, are equivalent to a force acting with an eccentricity.

As discussed with wood and steel structures, buckling is a common phenomenon associated with columns. However, concrete columns are stocky, and a great number of columns are not affected by buckling. These are classified as the *short columns*. It is the slenderness ratio that determines whether a column could be considered a short or a slender (long) column. The ACI sets the following limits when it is a short column and the slenderness effects can be ignored:

a. For members not braced against sidesway:

$$\frac{Kl}{r} \le 22 \tag{17.12}$$

b. For members braced against sidesway:

$$\frac{Kl}{r} \le 34 - 12\left(\frac{M_1}{M_2}\right) \tag{17.13a}$$

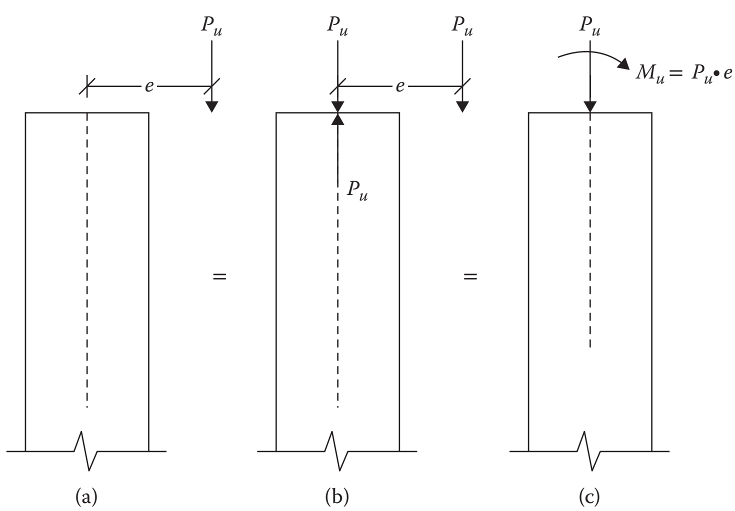

FIGURE 17.6 Equivalent force system on a column: (a) eccentric load on a column, (b) equivalent loaded column with axial and eccentric loads, and (c) equivalent column with axial load and moment.

or

$$\frac{Kl}{r} \le 40 \tag{17.13b}$$

where:

M_1 and M_2 are small and large end moments. The ratio M_1/M_2 is positive if a column bends in a single curvature, that is, the end moments have opposite signs. It is negative for a double curvature when the end moments have the same sign. (This is opposite of the sign convention for steel in the "Magnification Factor, B_1" section in Chapter 12.)

l is the length of the column

K is the effective length factor given in Figure 7.6 and the alignment charts in Figures 10.5 and 10.6

r = the radius of gyration = $\sqrt{I/A}$

= $0.3h$ for a rectangular column

= $0.25h$ for a circular column

If a clear bracing system in not visible, the ACI provides certain rules to decide whether a frame is braced or unbraced. However, conservatively, it can be assumed to be unbraced.

The effective length factor has been discussed in detail in the "Column Stability Factor" section in Chapter 7, and the "Effective Length Factor for Slenderness Ratio" section in Chapter 10. For columns braced against sidesway, the effective length factor is 1 or less; conservatively, it can be set as 1. For members subjected to sidesway, the effective length factor is greater than 1. It is 1.2 for a column fixed at one end and the other end has the rotation fixed but is free to translate (sway).

EFFECTS OF MOMENT ON SHORT COLUMNS

To consider the effect of an increasing moment (eccentricity) together with an axial force on a column, the following successive cases have been presented accompanied with respective stress/strain diagrams.

Case 1: Only Axial Load Acting

The entire section is subjected to a uniform compression stress, $\sigma_c = P_u/A_g$, and a uniform strain of $\varepsilon = \sigma_c/E_c$, as shown in Figure 17.7. The column fails by the crushing of concrete. By another measure, the column fails when the compressive concrete strain reaches 0.003. In the following other cases, the strain measure will be considered because the strain diagrams are linear. The stress variations in concrete are nonlinear.

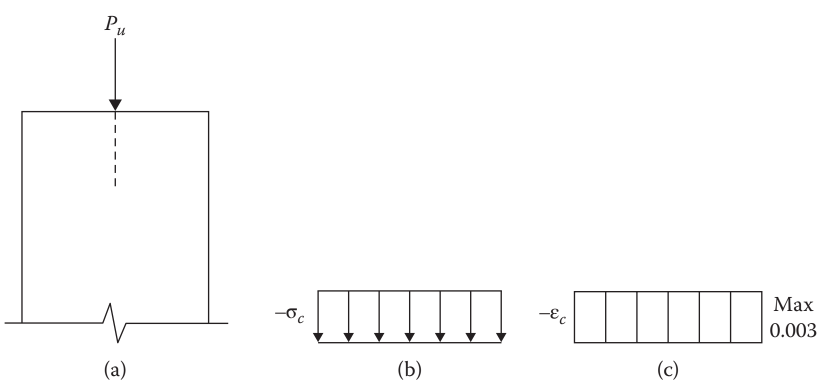

FIGURE 17.7 Axial load only on column: (a) load or column, (b) stress, (c) strain (Case 1).

Case 2: Large Axial Load and Small Moment (Small Eccentricity)

Due to axial load, there is a uniform strain, $-\varepsilon_c$, and due to moment, there is a bending strain of compression on one side and tension on the other side. The sum of these strains is shown in the last diagram of Figure 17.8. As the maximum strain due to the axial load and moment together cannot exceed 0.003, the strain due to the load is smaller than 0.003 because part of the contribution is made by the moment. Hence, the axial load, P_u, is smaller than that in the previous case.

Case 3: Large Axial Load and Moment Larger than Case 2 Section

This is a case when the strain is zero at one face. To attain the maximum crushing strain of 0.003 on the compression side, the strain contribution from both the axial load and moment is 0.0015.

Case 4: Large Axial Load and Moment Larger than Case 3 Section

When the moment (eccentricity) increases somewhat from the previous case, tension develops on one side of the column as the bending strain exceeds the axial strain. The entire tensile strain contribution comes from steel.[1] The concrete on the compression side contributes to compression strain. The strain diagram is shown in Figure 17.10d. The neutral axis (the point of zero strain) is at a distance, c, from the compression face. As the strain in steel is less than yielding, the failure occurs by crushing of concrete on the compression side.

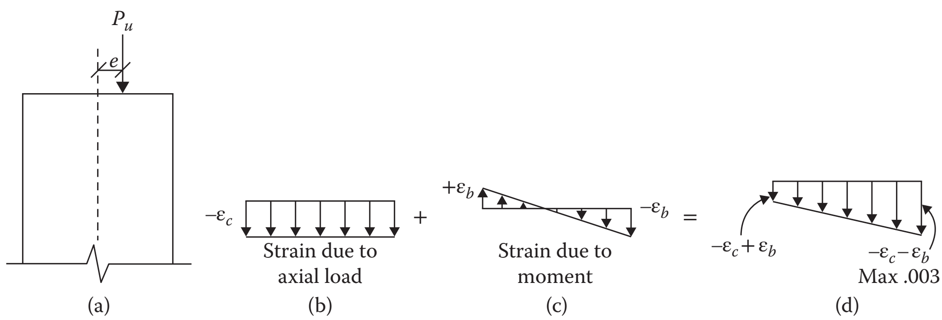

FIGURE 17.8 Axial load with small moment on column: (a) load on column, (b) axial strain, (c) bending strain, and (d) combined strain (Case 2).

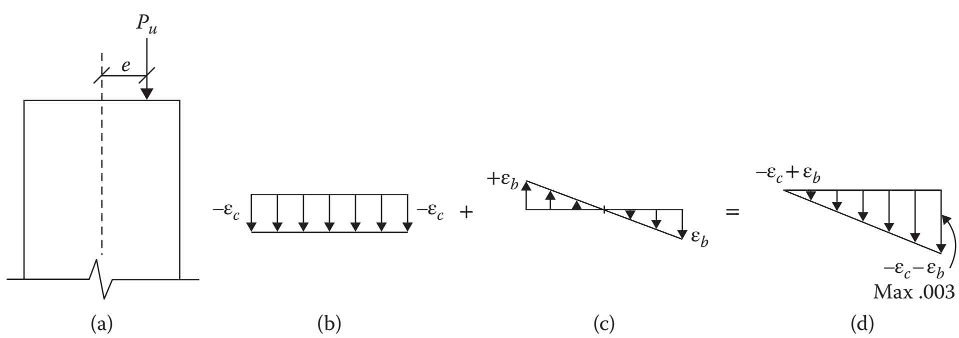

FIGURE 17.9 Axial load and moment on column: (a) load on column, (b) axial strain, (c) bending strain, and (d) combined strain (Case 3).

[1] The concrete is weak in tension, so its contribution is neglected.

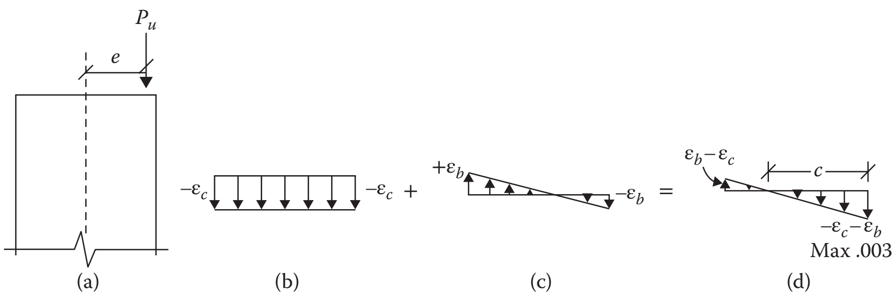

FIGURE 17.10 Axial load and moment on column: (a) load on column, (b) axial strain, (c) bending strain, and (d) combined strain (Case 4).

Case 5: Balanced Axial Load and Moment

As the moment (eccentricity) continues to increase, the tensile strain steadily rises. A condition is reached when the steel on the tension side attains the yield strain, $\varepsilon_y = f_y/E$ (for Grade 60 steel, this strain is 0.002), simultaneously as the compression strain in concrete reaches the crushing strain of 0.003. The failure of concrete occurs at the same time as steel yields. This is known as the *balanced condition*. The strain diagrams in this case are shown in Figure 17.11. The value of c in Figure 17.11d is less compared to the previous case; that is, the neutral axis moves up toward the compression side.

Case 6: Small Axial Load and Large Moment

As the moment (eccentricity) is further increased, steel reaches the yield strain, $\varepsilon_y = f_y/E$, before concrete attains the crushing strain of 0.003. In other words, when compared to the concrete strain of 0.003, the steel strain has already exceeded its yield limit, ε_y, as shown in Figure 17.12d. The failure occurs by the yielding of steel. This is called the *tension-controlled condition*.

Case 7: No Appreciable Axial Load and Large Moment

This is the case when the column acts as a beam. The eccentricity is assumed to be at infinity. The steel has long before yielded prior to concrete reaching a level of 0.003. In other words, when compared to a concrete strain of 0.003, the steel strain is 0.005 or more. This is shown in Figure 17.13b.

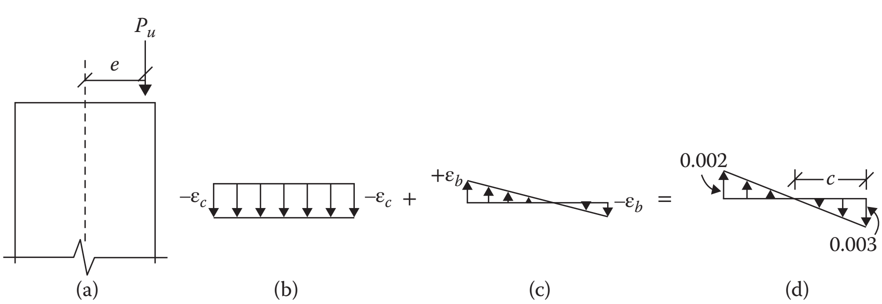

FIGURE 17.11 Balanced axial load and moment on column: (a) load on column, (b) axial strain, (c) bending strain, and (d) combined strain (Case 5).

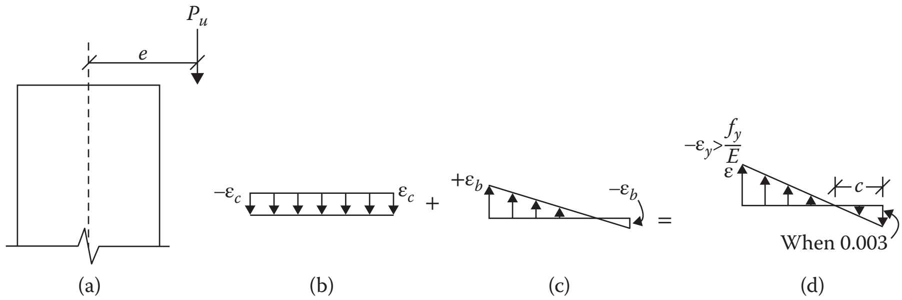

FIGURE 17.12 Small axial load and large moment on column: (a) load on column, (b) axial strain (c) bending strain, and (d) combined strain (Case 6).

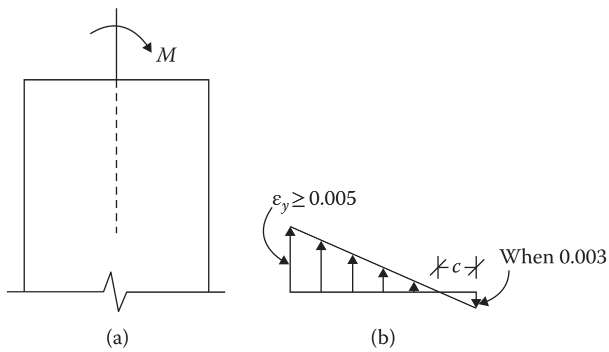

FIGURE 17.13 Moment only column: (a) load on column, (b) combined strain (Case 7).

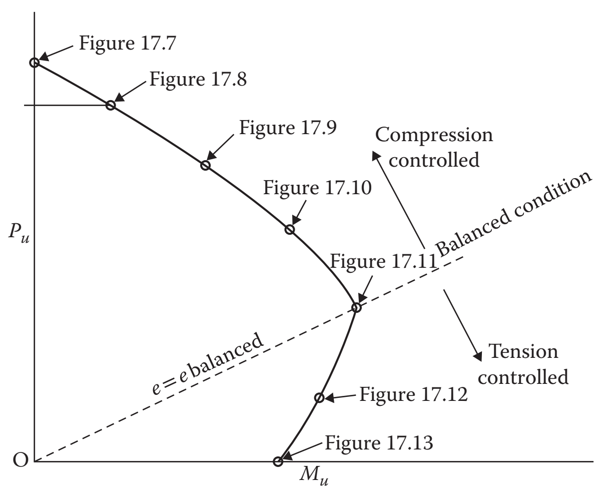

FIGURE 17.14 Column interaction diagram.

As discussed in the "Axially Loaded Columns" section, when a member acts as a column, the strength (capacity) reduction factor, ϕ, is 0.7 for spiral columns and 0.65 for tied columns. This is the situation for Cases 1 through 5. For beams, as in Case 7, the factor is 0.9. For Case 6, between the column and the beam condition, the magnitude of ϕ is adjusted by Equation 14.13, based on the value of strain in steel, ε_t.

If the magnitudes of the axial loads and the moments for all seven cases are plotted, it will appear like the shape shown in Figure 17.14. This is known as the *interaction diagram*.

CHARACTERISTICS OF THE INTERACTION DIAGRAM

The interaction diagram presents the capacity of a column for various proportions of the loads and moments. Any combination of loading that falls inside the diagram is satisfactory, whereas any combination falling outside represents a failure condition.

From Cases 1 through 5, where compression control exists, as the axial load decreases, the moment capacity increases. Below this balanced stage, the position is different. First, for the same moment, the axial capacity is higher in the compression control zone than in the tensile control zone. In the tensile control zone, as the axial load increases, the moment capacity also increases because any axial compression load tends to reduce the tensile strain (and stress), which results in raising of the moment-resisting capacity.

Any radial line drawn from origin O to any point on the diagram represents a constant eccentricity, that is, a constant ratio of the moment to the axial load. A line from point O to a point on the diagram for the condition in “Case 5: Balanced Axial Load and Moment” represents the $e_{balanced}$ eccentricity.

As the amount of steel varies within the same column, although the shape of the diagram (curve) remains similar to Figure 17.4, the location of the curve shifts to represent the appropriate magnitudes of the axial force and the moment; that is, the shapes of the curves are parallel.

The interaction diagram serves as a very useful tool in the analysis and design of columns for the combined loads.

APPLICATION OF THE INTERACTION DIAGRAM

The ACI has prepared the interaction diagrams in dimensionless units for rectangular and circular columns with different arrangements of bars for various grades of steel and various strengths of concrete. The abscissa has been represented as $R_n = M_u/\phi f_c{}' A_g h$ and the ordinate as $K_n = P_u/\phi f_c{}' A_g$. Several of these diagrams for concrete strength of 4,000 psi and steel strength of 60,000 psi are included in Appendix D.15 through D.22.

On these ACI diagrams, the radial strain line of value = 1 represents the balanced condition. Any point on or above this line represents compression control and $\phi = 0.7$ (spiral) or 0.65 (tied). The line of $\varepsilon_t = 0.005$ represents that the steel has yielded or beam behavior. Any point on or below this line has $\phi = 0.9$. Between these two lines is the transition zone for which ϕ has to be corrected by Equation 14.13.

The line labeled K_{max} indicates the maximum axial load with the limiting small eccentricity of $0.05h$ for spiral and $0.1h$ for tied columns.

The other terms in these diagrams are:

1. $\rho_g = \dfrac{A_{st}}{A_g}$
2. h = column dimension in line with eccentricity (perpendicular to the plane of bending)
3. $$\gamma = \frac{\text{center-to-center distance of outer row of steel}}{h} \tag{17.14}$$
4. Slope of radial line from origin = h/e

ANALYSIS OF SHORT COLUMNS FOR COMBINED LOADING

This analysis involves determining the axial load strength and the moment capacity of a known column. The steps in the analysis are:

1. From Equation 17.12 or Equation 17.13, confirm that it is a short column (there is no slenderness effect).
2. Calculate the steel ratio, $\rho_g = A_{st}/A_g$, and check for the value to be between 0.01 and 0.08.
3. Calculate γ from Equation 17.14.
4. Select the right interaction diagram to be used based on γ, type of cross section, f_c', and f_y.
5. Calculate the slope of the radial line = h/e.
6. Locate a point for coordinates $K_n = 1$ and $R_n = 1/\text{slope}$, or $R_n = e/h$ (or for any value of K_n, $R_n = K_n e/h$). Draw a radial line connecting the coordinate point to the origin. Extend the line to intersect with ρ_g of step 2. If necessary, interpolate the interaction curve.
7. At the intersection point, read K_n and R_n.
8. If the intersection point is on or above the strain line = 1, $\phi = 0.7$ or 0.65. If it is on or below the line, $\varepsilon_t = 0.005$ and $\phi = 0.9$. If it is between, correct ϕ by using Equation 14.13. (This correction is rarely applied.)
9. Compute $P_u = K_n \phi f_c' A_g$ and $M_u = R_n \phi f_c' A_g h$.

Example 17.5

A 10-ft-long braced column with a cross section is shown in Figure 17.15. The end moments are equal and have the same sign. Use $f_c' = 4{,}000$ psi and $f_y = 60{,}000$ psi. Find the axial design load and the moment capacity for an eccentricity of 6 in.

Solution

1. For same sign (double curvature), $\dfrac{M_1}{M_2} = -1$.
2. $K = 1$ (braced), $l = 10 \times 12 = 120$ in., $r = 0.3h = 0.3(16) = 4.8$ in.
3. $\dfrac{Kl}{r} = \dfrac{1(120)}{4.8} = 25$
4. Limiting value from Equation 17.13

$$\frac{Kl}{r} = 34 - 12\left(\frac{M_1}{M_2}\right)$$

$$= 34 - 12(-1) = 46 > 40$$

Limit value of 40 used. Since step (3) < step (4), short column.

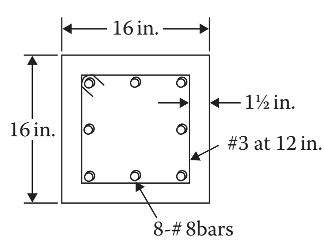

FIGURE 17.15 Eccentrically loaded column for Example 17.5.

5. $A_g = 16 \times 16 = 256 \text{ in.}^2$

   $A_{st} = 6.32 \text{ in.}^2$

   $$\rho_g = \frac{6.32}{256} = 0.025$$

6. Center to center of steel = 16 − 2(cover) − 2(tie diameter) − 1(bar diameter) = 16 − 2(1.5) − 2(0.375) − 1(1) = 11.25 in.

   $$\gamma = \frac{11.25}{16} = 0.70$$

7. Use the interaction diagram in Appendix D.17.

8. $\text{slope} = \dfrac{h}{e} = \dfrac{16}{6} = 2.67$

9. $K_n = 1,\ R_n = \dfrac{1}{\text{slope}} = \dfrac{1}{2.67} = 0.375$

   Draw a radial line connecting the aforementioned coordinates to the origin.

10. At $\rho_g = 0.025$, $K_n = 0.48$ and $R_n = 0.18$.
11. The point is above the line where strain = 1; hence, $\phi = 0.65$.
12. $P_u = K_n \phi f_c' A_g = 0.48(0.65)(4)(256) = 319.5$ k

    $M_u = R_n \phi f_c' A_g h = 0.18(0.65)(4)(256)(16) = 1917$ in.-k or 159.74 ft-k.

DESIGN OF SHORT COLUMNS FOR COMBINED LOADING

This involves determining the size, selecting steel, and fixing ties or spirals for a column. The steps follow:

1. Determine the design-factored axial load and moment.
2. Based on ρ_g = 1% and axial load only, estimate the column size by Equation 17.10 or Equation 17.11, rounding on the lower side.
3. For a selected size (diameter) of bars, estimate γ for the column size of step 2.
4. Select the right interaction diagram based on f_c', f_y, the type of cross section, and γ of step 3.
5. Calculate $K_n = P_u / \phi f_c' A_g$ and $R_n = M_u / \phi f_c' A_g h$, assuming $\phi = 0.7$ (spiral) or 0.65 (ties).
6. Use the appropriate diagram in Appendixes D.15 through D.22. Read ρ_g at the intersection point of K_n and R_n. This should be less than 0.05. If not, change the dimension and repeat steps 3–6.
7. Check that the interaction point of step 6 is above the line where strain = 1. If it is not, adjust ϕ and repeat steps 5 and 6.
8. Calculate the required steel area, $A_{st} = \rho_g A_g$ and select reinforcement from Appendix D.2; check that it fits in one row from Appendix D.14.
9. Design ties or spirals from steps 5 and 6 of the "Specifications for Columns" section.
10. Confirm from Equation 17.12 or Equation 17.13 that the column is short (no slenderness effect).

Example 17.6

Design a 10-ft-long circular spiral column for a braced system to support service dead and live loads of 300 k and 460 k, respectively, and service dead and live moments of 100 ft-k each. The moment at one end is zero. Use f_c' = 4,000 psi and f_y = 60,000 psi.

Solution

1. $P_u = 1.2(300)+1.6(460) = 1096$ k

   $M_u = 1.2(100)+1.6(100) = 280$ ft-k or 3360 in.-k

2. Assume $\rho_g = 0.01$ from Equation 17.10.

   $$A_g = \frac{P_u}{0.60[0.85f_c'(1-\rho_g)+f_y\rho_g]}$$

   $$= \frac{1096}{0.60[0.85(4)(1-0.01)+60(0.01)]}$$

   $$= 460.58 \text{ in.}^2$$

   $$\frac{\pi h^2}{4} = 460.58$$

   or $h = 24.22$ in.
   Use $h = 24$ in., $A_g = 452$ in.^{2}

3. Assume #9 size of bar and ⅜ in. spiral center-to-center distance.
   Center-to-center distance = 24–2(cover)–2(spiral diameter)–1(bar diameter)
   = 24–2(1.5)–2(3/8)–1.128 = 19.12 in.

   $$\gamma = \frac{19.12}{24} = 0.8$$

   Use the interaction diagram in Appendix D.21.

4. $$K_n = \frac{P_u}{\phi f_c' A_g} = \frac{1096}{(0.7)(4)(452)} = 0.866$$

   $$R_n = \frac{M_u}{\phi f_c' A_g h} = \frac{3360}{(0.7)(4)(452)(24)} = 0.11$$

5. At the intersection point of K_n and R_n, $\rho_g = 0.025$.
6. The point is above the strain line = 1; hence, $\phi = 0.7$. **OK**
7. $A_{st} = (0.025)(452) = 11.3$ in.^{2}
   From Appendix D.2, select 12 bars of #9; $A_{st} = 12$ in.^{2}
   From Appendix D.14, for a core diameter of 24–3 = 21 in., 15 bars of #9 can be arranged in a row.
8. Selection of spirals: From Appendix D.13, size = ⅜ in., pitch = 2¼ in.
   Clear distance = 2.25–3/8 = 1.875 > 1 in. **OK**
9. $K = 1$, $l = 10 \times 12 = 120$ in., $r = 0.25(24) = 6$ in.

   $$\frac{Kl}{r} = \frac{1(120)}{6} = 20$$

   $$\left(\frac{M_1}{M_2}\right) = 0$$

   $$34-12\left(\frac{M_1}{M_2}\right) = 34$$

   Because $(Kl/r) \leq 34$, short column.

LONG OR SLENDER COLUMNS

When the slenderness ratio of a column exceeds the limits given by Equation 17.12 or Equation 17.13, it is classified as a *long* or *slender* column. In a physical sense, when a column bends laterally by an amount, Δ, the axial load, P, introduces an additional moment equal to $P\,\Delta$. When this $P\,\Delta$ moment cannot be ignored, the column is a long or slender column.

There are two approaches to deal with this additional or secondary moment. The nonlinear second-order analysis is based on a theoretical analysis of the structure under application of an axial load, a moment, and the deflection. As an alternative approach, the ACI provides a first-order method that magnifies the moment acting on the column to account for the $P - \Delta$ effect. The magnification expressions for the braced (nonsway) and unbraced (sway) frames are similar to the steel magnification factors discussed in the "Magnification Factor, B_1" and the "Magnification Factor for Sway, B_2" sections in Chapter 12. After the moments are magnified, the procedure for short columns from the "Analysis of Short Columns for Combined Loading" and "Design of Short Columns for Combined Loading" sections in this chapter can be applied for analysis and design of the column using the interaction diagrams.

The computation of the magnification factors is appreciably complicated for concrete because of the involvement of the modulus of elasticity of concrete and the moment of inertia with creep and cracks in concrete.

A large percentage of columns do not belong to the slender category. It is advisable to avoid the slender columns whenever possible by increasing the column dimensions, if necessary. As a rule of thumb, a column dimension of one-tenth of the column length in braced frames meets the short column requirement. For a 10-ft length, a column of 1 ft (or 12 in.) or more is a short braced column. For unbraced frames, a column dimension one-fifth of the length satisfies the short column requirement. A 10-ft-long unbraced column of 2 ft (or 24 in.) dimension avoids the slenderness effect.

PROBLEMS

17.1 Determine the design axial load capacity and check whether the reinforcements meet the specifications for the column shown in Figure P17.1. Use $f_c' = 4{,}000$ psi and $f_y = 60{,}000$ psi.

17.2 Determine the design axial load capacity and check whether the reinforcements meet the specifications for the column shown in Figure P17.2. Use $f_c' = 4{,}000$ psi and $f_y = 60{,}000$ psi.

17.3 Determine the design axial load capacity of the column in Figure P17.3 and check whether the reinforcement is adequate. Use $f_c' = 5{,}000$ psi and $f_y = 60{,}000$ psi.

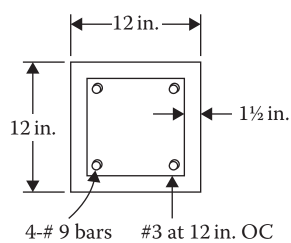

FIGURE P17.1 Column section for Problem 17.1.

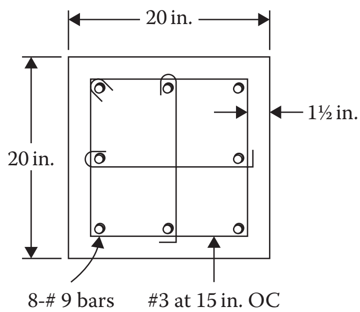

FIGURE P17.2 Column section for Problem 17.2.

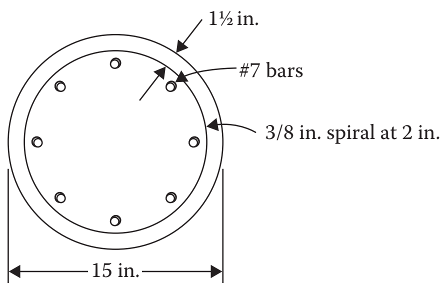

FIGURE P17.3 Column section for Problem 17.3.

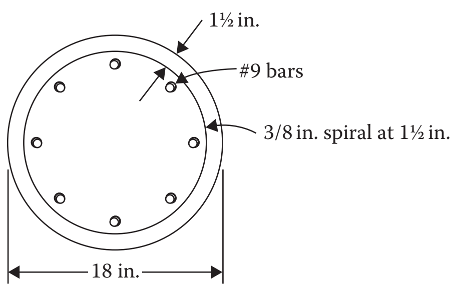

FIGURE P17.4 Column section for Problem 17.4.

17.4 Determine whether the maximum service dead load and live load carried by the column shown in Figure P17.4 are equal. Check for spiral steel. Use $f_c' = 3{,}000$ psi and $f_y = 40{,}000$ psi.

17.5 Compute the maximum service live load that may be axially placed on the column shown in Figure P17.5. The service dead load is 150 k. Check for ties specifications. Use $f_c' = 3{,}000$ psi and $f_y = 40{,}000$ psi.

17.6 A service dead load of 100 k and service live load of 450 k are axially applied on a 20-in.-diameter circular column reinforced with six #8 bars. The cover is 1½ in. and the spiral size is ½ in. at a 2 in. pitch. Use $f_c' = 4{,}000$ psi and $f_y = 60{,}000$ psi. Is the column adequate?

17.7 Design a tied column to carry a factored axial design load of 900 k. Use $f_c' = 5{,}000$ psi and $f_y = 60{,}000$ psi.

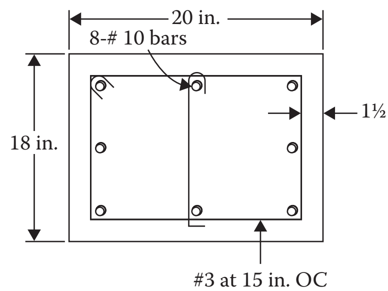

FIGURE P17.5 Column section for Problem 17.5.

17.8 For Problem 17.7, design a circular spiral column.

17.9 Design a tied column to support a service dead axial load of 300 k and live load of 480 k. Use $f_c' = 4{,}000$ psi and $f_y = 60{,}000$ psi.

17.10 Redesign a circular spiral column for Problem 17.9.

17.11 Design a rectangular tied column to support an axial service dead load of 400 k and live load of 590 k. The larger dimension of the column is approximately twice the shorter dimension. Use $f_c' = 5{,}000$ psi and $f_y = 60{,}000$ psi.

17.12 Design the smallest circular spiral column to carry an axial service dead load of 200 k and live load of 300 k. Use $f_c' = 3{,}000$ psi and $f_y = 60{,}000$ psi.
Hint: For the smallest dimension, use 8% steel. It is desirable to use #11 steel to reduce the number of bars to be accommodated in a single row.

17.13 For the 8-ft-long braced column shown in Figure P17.6, determine the axial load strength and the moment capacity at an eccentricity of 5 in. in the larger dimension. Use $f_c' = 4{,}000$ psi and $f_y = 60{,}000$ psi.

17.14 The unbraced column shown in Figure P17.7 has a length of 8 ft, and a cross section as shown. The factored moment-to-load ratio on the column is 0.5 ft, $K = 1.2$. Use $f_c' = 4{,}000$ psi and $f_y = 60{,}000$ psi. Determine the strength of the column.

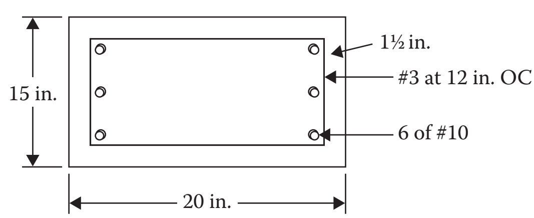

FIGURE P17.6 Column section for Problem 17.13.

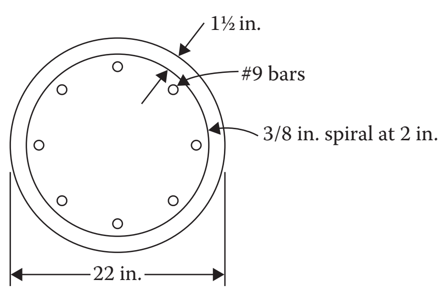

FIGURE P17.7 Column section for Problem 17.14.

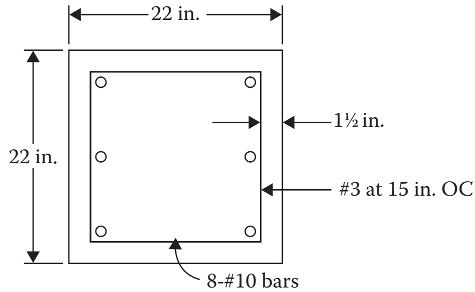

FIGURE P17.8 Column section for Problem 17.15.

17.15 On a 10-ft-long column of an unbraced frame system, the load acts at an eccentricity of 5 in. The column section is shown in Figure P17.8. Use $f_c' = 4{,}000$ psi and $f_y = 60{,}000$ psi. What are the axial load capacity and moment strength of the column?

17.16 Design an 8-ft-long circular spiral column of a braced system to support a factored axial load of 1200 k and a factored moment of 300 ft-k. The end moments are equal and have the same signs. Use $f_c' = 4{,}000$ psi and $f_y = 60{,}000$ psi.

17.17 Design a tied column for Problem 17.16. Arrange the reinforcement on all faces.

17.18 For an unbraced frame, design a circular column of 10-ft length that supports service dead and live loads of 400 k and 600 k, respectively, and service dead and live moments of 120 ft-k and 150 ft-k, respectively. The end moments are equal and have opposite signs; $K = 1.2$. Use $f_c' = 4{,}000$ psi and $f_y = 60{,}000$ psi.

17.19 Design a tied column for Problem 17.18 having reinforcement on all faces.

17.20 A braced frame has a 10-ft-long column. Design a tied column with reinforcing bars on two end faces only to support the following service loads and moments:

$$P_D = 150 \text{ k}, P_L = 200 \text{ k}$$

$$M_D = 50 \text{ ft-k}, M_L = 70 \text{ ft-k}$$

If necessary, adjust the column dimensions to qualify it as a short column. The column has equal end moments and a single curvature. Use $f_c' = 4{,}000$ psi and $f_y = 60{,}000$ psi.

18 Pre-Stressed Concrete Structures

PRE-STRESSING OF CONCRETE

Concrete is very strong in compression and very weak in tension. It is thus reinforced by standard steel on the tensile side, as discussed in previous chapters. To compensate for tensile weakness, another concept developed in the 1920s and 1930s, and expanded in more recent time, is that if the tensile region of concrete is pre-stressed in compression prior to imposition of the tensile-causing external forces, the induced tensile stresses will be compensated by the built-in compression stresses within the member.

With this pre-stressing, the entire concrete section will be in compression, and thus the moment of inertia contributed by the section will be much larger. The resultant advantages are: (1) the smaller size of section required, (2) the reduced deflection, (3) the increased span, (4) no cracking of the concrete section, and (5) improved long-term durability of the structure.

Disadvantages include (1) the more complicated designs, and (2) increased costs for materials, fabrication, and delivery.

There are two methods to pre-stress the concrete. Both are discussed below.

Pre-Tensioning

In this method, steel wires, or steel strands consisting of many wires or steel bars, known as tendons, are stressed prior to casting of concrete. The pre-stress is imparted to concrete by the bonding between steel and concrete. The process consists of the following stages:

Stage 1: The tendons are stretched between bulkheads of a casting bed and anchored at live and dead ends, as shown in Figure 18.1.
Stage 2: The concrete is cast around the stretched tendons.
Stage 3: After concrete hardens to about 75% of the specified strength, f'_c, tendons are released to transfer the stress to concrete.

At each end of the member, there is a transmission length of approximately 50 times the tendon diameter over which the tendon force (stress) is transferred to the concrete. Sometimes, the tendon is debonded at the ends for this length.

Post-Tensioning

In this process, the concrete is already set with a duct or sleeve cast into the concrete. The process consists of the following stages:

Stage 1: The tendons are fed through the duct, as shown in Figure 18.2. At each end of the tendons, an anchorage assembly is fixed to the surrounding concrete.
Stage 2: The tendons are then stretched (tensioned) through the anchorages.
Stage 3: The tendons are then locked off at the anchorages. The large force applied to tension the tendons result in a significant permanent compression being applied to concrete.

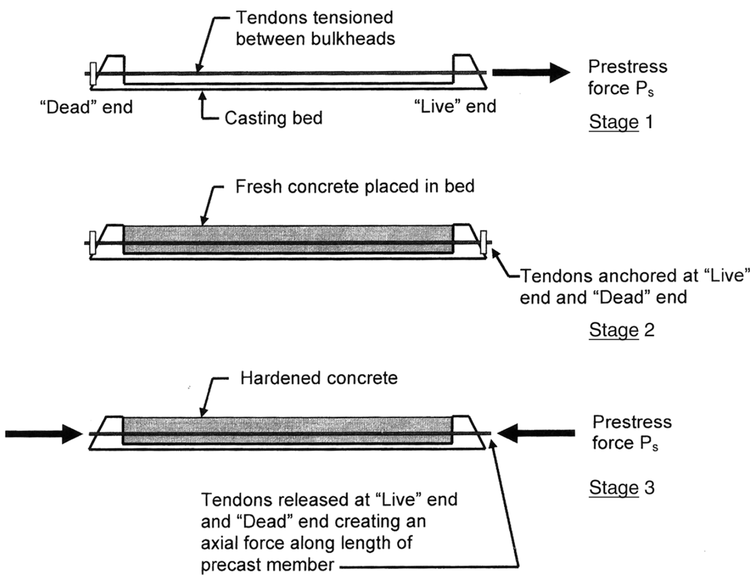

FIGURE 18.1 Pre-tensioning process.

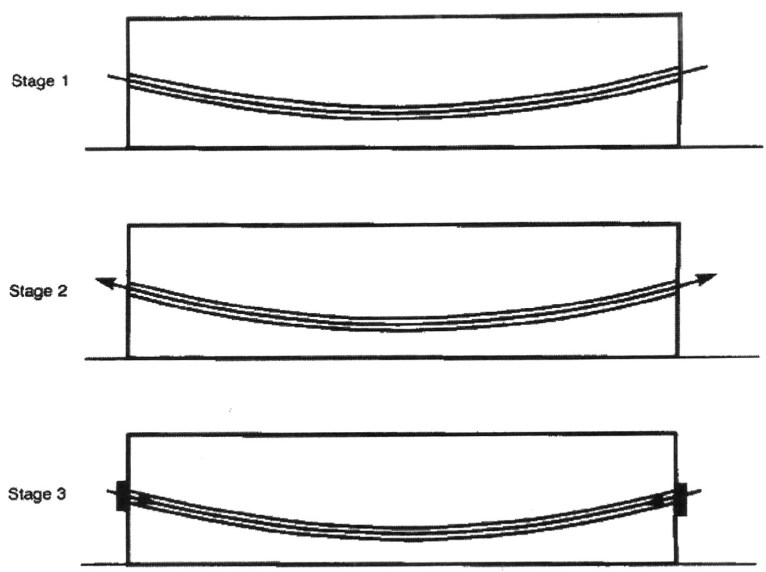

FIGURE 18.2 Post-tensioning process.

There are two types of post-tensioning:

Bonded post-tensioning, in which the tendons are subsequently bonded to the surrounding concrete by internal grouting of the duct after stretching.

Unbonded post-tensioning, in which the tendons are permanently debonded from the surrounding concrete by means of a greased sheath over the tendons. The anchorages to post-tensioned members distribute the lad to the concrete.

Both bonded and unbonded techniques are widely used. The benefits of bonded post-tensioning are: (1) reduced reliance on end anchorages, (2) increased strength in flexure, (3) improved crack control, and (4) improved fire performance.

The advantages of unbonded post-tensioning are: (1) faster fabrication and installation, (2) improved site-labor productivity, (3) reduced concrete cover, (4) simpler replacement, and (5) superior ductility.

Stressing and Anchorage Devices

In the pre-stressing process, a tendon is anchored at one end and stretched. After stretching, it is anchored at the other end too. In the pre-tension system, the tendon is released after casting of the concrete, but in the post-tension system, the end anchorages are integral components of the system in order to maintain stress in the concrete.

A large number of anchorage systems are available, which are based on the following principles:

1. Wedge action that provides the frictional grip on tendons
2. Direct bearing by the heads at the ends of tendons
3. Looping of the tendons around concrete

The devices used for tensioning or stretching of steel are grouped as follows:

1. Hydraulic jacking: Force is applied through a hydraulic jack with calibrated pressure gages that indicate the magnitude of the force developed during tensioning of the tendons.
2. Electrical pre-stressing: The tendons are coated with thermoplastic material such as sulfur or low melting alloy and buried in concrete. After the concrete is set, electric current of low voltage and high amperage is passed, which heats and elongates the tendons.
3. Chemical pre-stressing: The tendons are embedded in concrete made of expanding cement. By expansion of the concrete, steel is elongated and thus gets pre-stressed.

Pre-Tensioning versus Post-Tensioning

The advantage of a pre-tensioning system is that elaborate end-anchorages and rubber core or metal sheathing are not required. There is a greater certainty of the pre-stressing force, which depends on end-anchorages in a post-tensioned system. The disadvantage of a pre-tensioned system is that large casting beds with strong end abutments are needed, which can be provided only in pre-casting factories. Hence, pre-tensioned members are limited in size because of the difficulty of transportation from the factory to the construction site. Also, the loss of stress is greater in pre-tensioned members.

Post-tensioning is suitable in construction work involving stage pre-stressing. It is widely used for concrete dams, tanks, and long-span bridges.

MATERIALS FOR PRE-STRESSED CONCRETE

Pre-stressed concrete utilizes high-strength concrete and high-strength steel. In addition, ordinary reinforcing steel is also used as shear reinforcement or supplemental reinforcement. The properties of ordinary reinforcing steel are summarized in Chapter 14. Tables in Appendix D provide important design characteristics of ordinary reinforcing bars. Grade 40 (yield strength of 40 ksi), Grade 50, and Grade 60 are common reinforcing steel. The modulus of elasticity of 29,000 ksi is commonly adopted.

High-Strength Steel

Normal reinforcing steel has ultimate strength of 58–60 ksi (400–415 MPa). The losses in pre-stressed steel could approach this level. For pre-stressed concrete, high-strength steel is used that could possess three to four times the strength of the reinforcing steel. This high strength is achieved by increasing the carbon content to 0.6%–0.8%. Besides the high tensile strength, the other desirable properties of pre-stressed steel are that it (1) remains elastic to high stress levels; (2) possesses high ductility; (3) has good bonding properties; (4) has low levels of *relaxation*, which is an indicator of the loss of tension over a period of time; and (5) does not corrode much.

The types of tendons consist of wires, strands that are made of several wires, and bars. There are two types of strands: the stress-relieved or normal relaxation strands and low relaxation strands. The latter has lower loss of tension over time.

The typical characteristics of steel used for tendons follow:

1. Wires: Ultimate Strength 235–250 ksi (1620–1720 MPa)

Plain round wires	0.06–0.36 in. diameter
Indented wires	0.2–0.28 in. diameter
Deformed or twisted wires	0.28–0.5 in. diameter

2. Strands: Ultimate Strength 250–270 ksi (1720–1860 MPa)

Two wire strands	0.2 in. diameter
Seven wire strands	0.25–0.6 in. diameter
Nineteen wire strands	0.7–0.9 in. diameter

3. Bars: Ultimate Strength 145–160 ksi (1000–1100 MPa)

Round bars	0.36–1.26 in. diameter
Threaded bars	0.9–1.26 in. diameter

Wires are suitable for smaller structures. Between strands and bars, the strands are much easier to handle than bars because of their flexibility and they have superior properties due to better quality control.

The yield strength of pre-stressing steel is not well defined; it is considered to be a magnitude corresponding to a strain of 1% for wires and strands, and 0.7% for bars. In another method, the stress corresponding to an offset (line drawn parallel to stress–strain diagram) at a strain level of 0.2% or 0.1% of strain (depending on the standard used) is taken as the yield strength. The modulus of elasticity of steel is generally 28×10^3 ksi.

Allowable Stress in Pre-Stressed Steel

The maximum tensile stress during pre-stressing, that is, at the time of tensioning behind the anchorages, should not exceed 80% of the ultimate strength. There is no upper limit indicated for the stress at the transfer stage and after the long-term losses.

High-Strength Concrete

The compression strength of common concrete in reinforcing concrete structures is 3–5 ksi (21–31.4 MPa). In pre-stressed concrete structures, the compression strength is in the range of 5 and 10 ksi (34.4 and 68.9 MPa). It is even possible to produce ultra-high-strength concrete in the range of 10 and 15 ksi (70 and 105 MPa). A higher early strength is desirable to enable sooner application of the pre-stressing force.

In high-strength concrete, cement used is common Portland cement, but to achieve a higher strength, high cement content, a low water-to-cement ratio, and good quality of aggregates are required.

For pre-stressed concrete structures, several other properties besides the compression strength are important, such as shrinkage, creep, elastic modulus. A large elastic modulus reduces the shortening of the member. The knowledge of the time-dependent properties like shrinkage of will allow prediction of losses over the time and long term deformation of the structure.

Shrinkage of Concrete

A concrete mix contains more water than required for chemical hydration reaction of the cement. The loss of this excess water over time through evaporation leads to gradual shortening of a member, a condition known as *shrinkage*. As a member shortens, some of the pre-stress is lost. The shrinkage is affected by many factors like amount of excess water, relative humidity, ambient temperature, the ratio of the surface area to the volume of the member and aggregate type; rocks being less susceptible to shrinkage. Shrinkage is independent of loading.

The amount of shrinkage depicted by shrinkage-strain is time dependent and tends asymptotically to an ultimate value. Almost 50% of maximum strain occurs within a month and 90% within a year. The American Concrete Institute (ACI) has recommended an empirical time-strain relation.

Several equations are found in the literature to evaluate ultimate shrinkage strain. The average value of ultimate strain is in the range of 4×10^{-4} and 8×10^{-4}.

Creep of Concrete

Loading on a structure creates stress that results in elastic strain on a material. However, in concrete, there is a strain that is induced in excess of the elastic strain. This is known as *creep*. Unlike shrinkage, creep is caused by loading. If a load is maintained, the creep strain tends asymptotically with time toward a maximum value called *ultimate creep strain*. This is a substantial amount, as shown in Figure 18.3.

Creep causes significant stress losses in pre-stressed steel and long-term deflection of the member. The factors that affect shrinkage also contribute to creep because both are of similar origin.

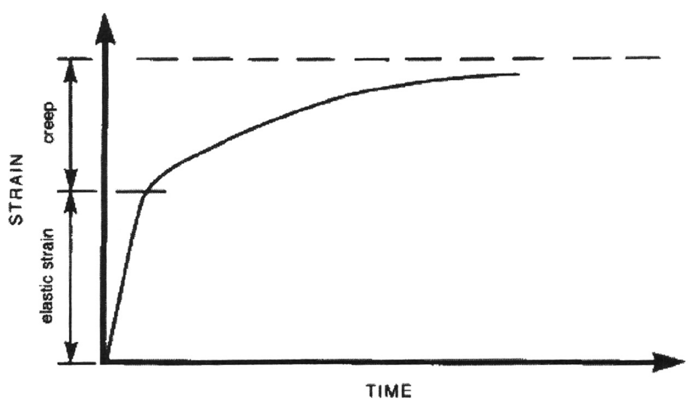

FIGURE 18.3 Creep strain of concrete.

The creep of concrete continues for a long time, tending toward a limiting value in infinite time but about 50% of 20-year creep occurs in 3 months and about 75% in one year. If creep of one year is taken as unity, the creep after 30 years is 1.36.

A *creep coefficient* is a measure of creep. It is defined as the ratio of ultimate creep strain to elastic strain. It is 1.6 at 28 days of loading and 1.1 at 1-year loading. The ACI has recommended an empirical time-strain relation for creep as well.

Allowable Stress in Concrete

Concrete subjected to pre-stressing is checked at two stages:

At Transfer: This is immediately after pre-stress transfer from tendons to concrete. The concrete at this stage is less strong. This situation is temporary. The additional stress is due to the weight of the member.

At Service Loads: This is when the external load is also imposed. At this stage, concrete attains full strength. However, losses to pre-stress occur. Since we deal with only service (unfactored) loading, only the allowable stresses are considered.

The allowable tensile and compression stresses in concrete at the stage of transfer and at service loads are defined in terms of the concrete (28-day) strength, f_c'. The values of allowable stresses recommended by ACI Code 318 are given in Table 18.1.

TABLE 18.1
Allowable Stresses in Concrete on Pre-Stressed Members

	psi	MPa
1. Stresses immediately after pre-stress transfer (before pre-stress losses) shall not exceed the following:		
(a) Extreme fiber stress in compression	$0.60 f'_{ci}$	$0.60 f'_{ci}$*
(b) Extreme fiber stress in tension except as permitted in (c)	$-3\sqrt{f'_{ci}}$	$-0.25\sqrt{f'_{ci}}$
(c) Extreme fiber stress in tension at ends of simply supported members	$-6\sqrt{f'_{ci}}$	$-0.50\sqrt{f'_{ci}}$
Where computed tensile stresses exceed these values, bonded auxiliary reinforcement (non-pre-stressed or pre-stressed) shall be provided in the tensile zone to resist the total tensile force in the concrete computed with the assumption of an uncracked section.		
2. Stresses at service loads (after allowance for all pre-stress losses) shall not exceed the following:		
(a) Extreme fiber stress in compression due to pre-stress plus sustained load	$0.45 f'_c$	$0.45 f'_c$
(b) Extreme fiber stress in compression due to pre-stress and total load	$0.60 f_c'$	$0.60 f_c'$
(c) Extreme fiber stress in tension in pre-compressed tensile zone	$-6\sqrt{f_c'}$	$-0.50\sqrt{f_c'}$
(d) Extreme fiber stress in tension in pre-compressed tensile zone of members (except two-way slab systems) where analysis based on transformed cracked sections and on bilinear moment-deflection relationships shows that immediate and long-time deflections comply with requirements stated elsewhere in the code	$-12\sqrt{f_c'}$	$-\sqrt{f_c'}$
3. The permissible stresses of sections 1 and 2 may be exceeded if shown by test or analysis that performance will not be impaired.		

Source: Adapted from ACI 318 Code.

* f'_{ci} is compression strength of concrete at time of initial stress; around $0.80 f'_c$.

In addition to the above listed checks, the capacity of the pre-stressed section is also considered for ultimate (factored) strength loading. If required, the section is suitably strengthened.

PRE-STRESS LOSSES

The tendons stretched during pre-stressing are subjected to contraction due to causes discussed below. This results in reduction of the pre-stress of tendons. This loss of stress is an important factor in the performance of the member. However, determination of these losses is a complex problem because of time dependent characteristics of concrete and steel, and the determination is only as an estimate.

Loss Due to Elastic Shortening (ES)

The loss by elastic shortening shortening of concrete is related to the modular ratio and stress in concrete as follows:

$$\mathrm{ES} = K_{CS}\frac{E_S}{E_c} f_c \tag{18.1}$$

where:

K_{CS} = 1.0 for a pre-tensioned member
= 0.5 for a post-tensioned member
E_S = the modulus of elasticity of steel = 28×10^3 ksi
E_c = the modulus of elasticity of concrete = $33\, w_c^{1.5}\sqrt{f_c'}$
w_c = the unit weight of concrete ≈ 145 lbs/ft^3
f_c' = the compression strength of concrete, in psi
f_c = the concrete stress at the level of steel at transfer due to pre-stressing force

1. For pre-tensioned steel with eccentricity, e (Figure 18.4):

$$f_c = \frac{F}{A} + \frac{Fe^2}{I} \tag{18.2}$$

where:
A is the cross-sectional area of the section
I is the moment of inertia of the section

2. For post-tensioned steel (Figure 18.5):

$$f_c = \frac{F}{A} + \frac{F(e_1^{\,2} + e_2^{\,2})}{I} \tag{18.3}$$

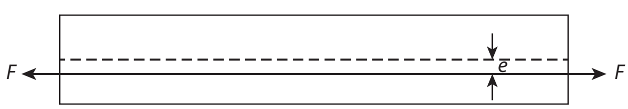

FIGURE 18.4 Initial stress in pre-tensioned concrete.

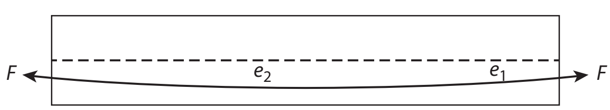

FIGURE 18.5 Initial stress in post-tensioned concrete.

Example 18.1

A pre-stressed concrete beam, 12 in. wide and 18 in. deep, is pre-tensioned by straight wires of 0.4 in.^{2} cross sectional area carrying an initial force of 100 kip at an eccentricity of 2 in. Estimate the percent loss of stress in steel due to elastic shortening. The modulus of elasticity of steel is 28×10^3 ksi and the compression strength of concrete is 5 ksi.

Solution

1. Area of concrete section = 12 × 18 = 216 in.^{2}
2. Moment of inertia of concrete section = 1/12 (12)(18)^3 = 5832 in.^{4}
3. Modulus of elasticity of concrete = $33 \times 145^{1.5} \times 5000^{0.5}$ = 4,074,000 psi or 4074 ksi
4. Modular ratio = $\dfrac{E_s}{E_c} = \dfrac{28 \times 10^3}{4074} = 6.87$
5. Initial stress in steel = $\dfrac{100}{0.4} = 250$ ksi
6. $f_c = \dfrac{F}{A} + \dfrac{Fe^2}{I}$

   $f_c = \dfrac{100}{216} + \dfrac{100(2)^2}{5832} = 0.5$ ksi
7. From Equation 18.2:
   ES = 1(6.87)(0.5) = 3.44 ksi
8. Percentage loss due to elastic shortening = $\dfrac{3.44 \times 100}{250} = 1.4\%$

Loss Due to Shrinkage (SH) of Concrete

As discussed above, several empirical relations have been suggested for shrinkage strain. In simplified form:

$$\text{SH} = e_{sh}\, E_S \tag{18.4}$$

where:

e_{sh} = shrinkage strain
= 0.0003 for pre-tensioned member
= $\dfrac{0.0002}{\log(t+2)}$ for post-tensioned member

t = age of concrete in days

Example 18.2

Determine the loss of stress due to shrinkage if the age of concrete transfer is 15 days. Assume the beam first to be (1) pre-tensioned, then (2) post-tensioned.

Solution

A. Pre-tensioned beam
   1. Initial stress in steel = $\dfrac{100}{0.4} = 250$ ksi
   2. Shrinkage strain = 0.0003
   3. Loss due to shrinkage = $0.0003(28 \times 10^3)$ = 8.4 ksi
   4. Percentage loss = $\dfrac{8.4 \times 100}{250} = 3.36\%$

B. Post-tensioned beam

1. Shrinkage strain = $\dfrac{0.0002}{\log(15+2)} = 0.00016$
2. Loss due to shrinkage = $0.00016(28 \times 10^3) = 4.5$ ksi
3. Percent loss = $\dfrac{4.5 \times 100}{250} = 1.8\%$

Loss Due to Creep (CR) of Concrete

Creep strain is related to elastic strain as follows:

$$\text{CR} = \frac{C_c E_S f_c}{E_c} \tag{18.5}$$

where C_c = the creep coefficient = 1.6 at 28 days and 1.1 at 1 year.

Example 18.3

For Example 18.1, determine the loss due to creep.

Solution

From Example 18.1:

1. Modular ratio = $\dfrac{E_s}{E_c} = \dfrac{28 \times 10^3}{4074} = 6.87$
2. Initial stress in steel = $\dfrac{100}{0.4} = 250$ ksi
3. $f_c = \dfrac{F}{A} + \dfrac{Fe^2}{I}$

   $f_c = \dfrac{100}{216} + \dfrac{100(2)^2}{5832} = 0.5\,\text{ksi}$
4. Loss due to creep (from Equation 18.5) = 1.6(6.87)(0.5) = 5.5 ksi
5. Percentage loss = $\dfrac{5.5 \times 100}{250} = 2.2\%$

Loss Due to Relaxation (RE) of Steel

Creep in steel occurs when the stress in steel is more than 50% of its yield strength. This is also termed *relaxation*, and it contributes to loss of stress. The relaxation loss depends on the type and grade of steel. The following equation, with adjustment for all other losses, estimates the relaxation:

$$\text{RE} = K_{RE} - f(\text{ES} + \text{SH} + \text{CR}) \tag{18.6}$$

The values of K_{RE} and f are given in Table 18.2.

TABLE 18.2
Factors for Relaxation Loss

Grade and Type of Steel	K_{RE}, psi	K_{RE}, N/mm²	f
Grade 250–270 ksi, (1720–1860 MPa): normal strands or wires	20,000	137.7	0.15
Grade 250–270 ksi, (1720–1860 MPa): low relaxation strands or wires	5000	34.4	0.04
Grade 145–160 ksi, (1000–1100 MPa): bars	6000	41.3	0.05

Example 18.4

Determine the loss due to relaxation of steel of normal relaxation for Example 18.1.

Solution

1. Loss due to elastic shortening (ES) from Example 18.1: 3.44 ksi
2. Loss due to shrinkage (SH) from Example 18.2: 8.4 ksi
3. Loss due to creep (CR) from Example 18.3: 5.5 ksi
4. From Table 17.2: $K_{RE} = 20$ ksi, $f = 0.15$
5. From Equation 17.6:

   $$RE = 20 - 0.15(3.44 + 8.4 + 5.5) = 17.4 \text{ ksi}$$

6. Percentage loss due to relaxation $= \dfrac{17.4 \times 100}{250} = 7\%$

Loss Due to Friction (FL)

Friction between the sides of the duct and steel contributes to loss of stress. This is due to the length effect, described as *wobbling effect*, and the curvature effect, as shown in Figure 18.6. The differential wobbling effect, $KFdx$ and the differential frictional effect, $\mu Fd\theta$, when integrated for the entire length, L, and chord angle, α, provides the following relation:

$$FL = F\left[1 - e^{-(KL + \mu\alpha)}\right] \tag{18.7}$$

where:

K = the coefficient of wobbling effect, ≈15×10^{-4} to 50×10^{-4} per m (5×10^{-4} to 16×10^{-4} per ft)
μ = the coefficient of friction, ≈0.15 – 0.25.

Example 18.5

For Example 18.1, determine the loss due to friction for a tendon of length 25 ft. The tendon is curved 1^0 from horizontal. The coefficient of friction is 0.2 and coefficient is 5×10^{-4}.

Solution

1. Initial pre-stress = 250 ksi
2. The tendon is curved on each side from the apex. Hence, the angle subtended at the center of the arc is twice the angle from horizontal from each side.

   $\alpha = 2^0 = 0.035$ radians

3. $FL = 250 \times [1 - \text{Exp} - (5 \times 10^{-4} \times 25 + 0.20 \times 0.035)] = 4.83$ ksi
4. Percentage loss due to friction $= \dfrac{4.83 \times 100}{250} = 1.93\%$

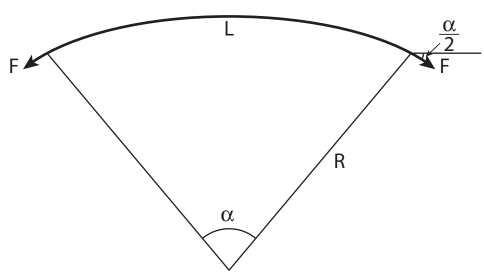

FIGURE 18.6 Length and curvature effects.

TABLE 18.3
Estimate of Pre-Stress Losses

Type of Steel	Total Loss, ksi	Percentage Loss of Stress, %
Pre-tensioned steel		
Normal relaxation	45	18–25
Low relaxation	35	
Post-tensioned steel		
Normal relaxation	32	15–20
Low relaxation	20	
Bars	22	

TOTAL LOSSES OF STRESS

Total losses equal summation of the above losses. According to the California Department of Transportation, the estimates of the total losses given in Table 18.3 may be used in lieu of the preceding methods for normal weight concrete, normal pre-stress, and average exposure condition. According to various sources approximate percentage losses are also indicated in the table.

ANALYSIS OF STRESSES DURING PRE-STRESSING

Designing pre-stressed concrete involves ensuring that the stresses in the concrete are within the allowable limits according to the codes of practice. The stresses are checked at two stages: (1) at transfer, when the concrete first feels the stresses of the pre-stressed tendon, and (2) at the service stage, when external loads are imposed on the structure. Then the ultimate (factored) strength designs and shear strength designs are also considered.

TENDON WITH ECCENTRICITY

Figure 18.7 shows a concrete beam subjected to a constant eccentric force of magnitude F. This is equivalent to a concentric force of F and a moment of $F.e$ as shown in Figure 18.8. The stresses developed at the top and bottom of the beam due to these loads are as shown in Figure 18.9 where, in compression, stress is shown as positive. In the case of concentric tendon (there is no eccentricity), the second stress term due to bending moment is not present.

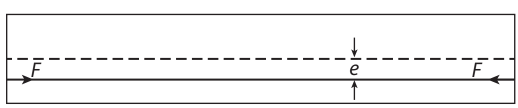

FIGURE 18.7 Eccentric loaded beam.

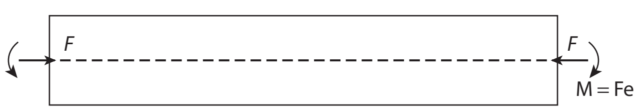

FIGURE 18.8 Equivalent loading.

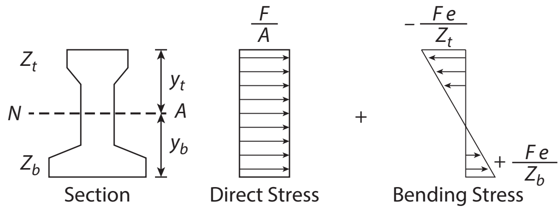

FIGURE 18.9 Stresses due to an eccentric force.

where:

Z_t = the section modulus at the top = I/y_t

Z_b = the section modulus at the bottom = I/y_b

Consider that the beam carries a uniform dead load of w_D and live load of w_L. The moments due to these loads at the center of span L are:

$$M_D = \frac{w_D L^2}{8} \quad \text{and} \quad M_L = \frac{w_L L^2}{8}$$

The stresses due to these moments are shown in Figure 18.10.

Stresses at Transfer

At the transfer stage, the loading includes the pre-stressing force and the moment due to dead load alone. The stresses at the top and the bottom are as shown in Figure 18.11; tensile stress is taken as negative.

$$\left(f_{\text{top}}\right)_{\text{transfer}} = \frac{F}{A} - \frac{Fe}{Z_t} + \frac{M_D}{Z_t} \tag{18.8}$$

$$\left(f_{\text{bottom}}\right)_{\text{transfer}} = \frac{F}{A} + \frac{Fe}{Z_b} - \frac{M_D}{Z_b} \tag{18.9}$$

Stresses at Service Load

At service load, the stresses are induced by the live load in addition to pre-stress and self-weight. But due to pre-stress losses, both F/A and Fe/Z components are reduced. The combined stresses at service loads are shown in Figure 18.12.

$$\left(f_{\text{top}}\right)_{\text{transfer}} = \alpha\left(\frac{F}{A} - \frac{Fe}{Z_t}\right) + \frac{M_D}{Z_t} + \frac{M_L}{Z_t} \tag{18.10}$$

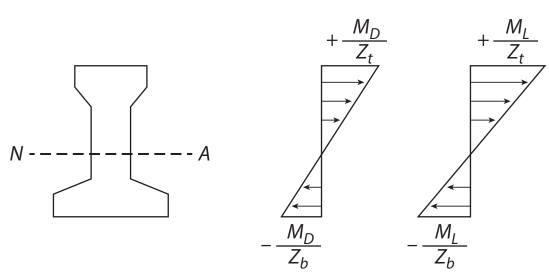

FIGURE 18.10 Stresses due to dead load and live load.

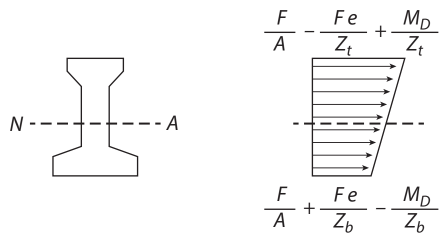

FIGURE 18.11 Stresses at transfer.

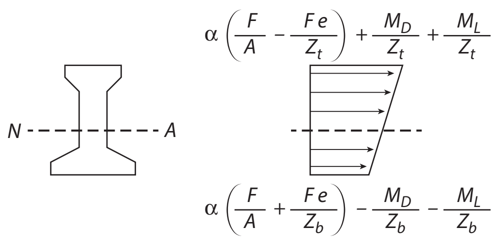

FIGURE 18.12 Stresses at service loads.

$$\left(f_{\text{bottom}}\right)_{\text{transfer}} = \alpha\left(\frac{F}{A} - \frac{Fe}{Z_b}\right) - \frac{M_D}{Z_b} - \frac{M_L}{Z_b} \tag{18.11}$$

where, α = the reduced pre-stress factor = (1–pre-stress losses expressed in fraction)

Since the area of steel is very small compared to concrete, the values of A and I are generally based on the concrete cross sectional properties alone.

When the tendon has a curved shape, as shown in Figure 18.13 (which is generally the case with post-tensioned member), the stress varies at different sections as e varies. In the above relations, the stress is computed with the maximum eccentricity e_2.

The above stresses at transfer and at service loads should be within the allowable stresses according to the codes as indicated in Table 18.1. Thus, the following criteria are used in pre-stressed concrete design:

1. At Transfer
   Top Fiber

$$\frac{F}{A} - \frac{Fe}{Z_t} + \frac{M_D}{Z_t} \geq f_{tt} \tag{18.12}$$ [1]

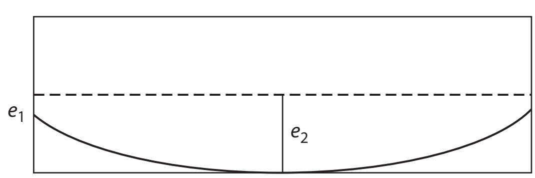

FIGURE 18.13 Parabolic tendon.

[1] The computed values should be higher than the lowest (minimum) allowable value for tension of concrete and lesser than the maximum allowable value for compression of concrete. In these relations, tension is a negative quantity. Hence, the computed value should be a lesser negative number, preferably a positive value.

Bottom Fiber

$$\frac{F}{A} + \frac{Fe}{Z_b} - \frac{M_D}{Z_b} \geq f_{ct} \tag{18.13}$$[1]

where:
f_{tt} = allowable tensile stress in concrete at initial transfer of pre-stress
f_{ct} = allowable compression stress in concrete at initial transfer of pre-stress

2. At Service Loads
Top Fiber

$$\alpha\left(\frac{F}{A} - \frac{Fe}{Z_t}\right) + \frac{M_D}{Z_t} + \frac{M_L}{Z_t} \leq f_{cs} \tag{18.14}$$

Bottom Fiber

$$\alpha\left(\frac{F}{A} + \frac{Fe}{Z_b}\right) - \frac{M_D}{Z_b} - \frac{M_L}{Z_b} \leq f_{ts} \tag{18.15}$$

where:
f_{cs} = allowable compression stress in concrete under service loads
f_{ts} = allowable tensile stress in concrete under service loads

By combining Equations 18.12 and 18.14, the following relation is derived for section modulus, Z_t:

$$Z_t = \frac{M_D(1-\alpha) + M_L}{f_{cs} - \alpha f_{tt}} \tag{18.16}$$

Similarly, combining Equations 18.13 and 18.15 yields:

$$Z_b = \frac{-M_D(1-\alpha) - M_L}{f_{ts} - \alpha f_{ct}} \tag{18.17}$$

The above values of Z_t ($=I/y_t$) and Z_b ($=I/y_b$) are expressed in terms of b and d to select the size of a member. The procedure is iterative because of the self-weight of a beam; hence, M_D depends on the beam section.

For a selected section, the minimum pre-stressing force, F, and the eccentricity, e, are determined from Equations 18.12 and 18.15. In a graphic procedure, the four relationships expressed by Equations 18.12 through 18.15 are plotted with $1/F$ as ordinate and e as abscissa. The feasibility (common) region formed by the four lines covers the values of the pre-stressing force and eccentricity.

Example 18.6

An asymmetrical I-section beam as shown is pre-stressed by a force of 100 kN applied through a tendon of wires of 50 mm diameter located 60 mm from the bottom. The beam carries a load of 4 kN/m over a span of 8 m. The pre-stress losses are 20%. Compute the stresses in concrete at the (1) transfer stage and (2) service load stage. The density of concrete is 24 kN/m^3 (Figure 18.14).

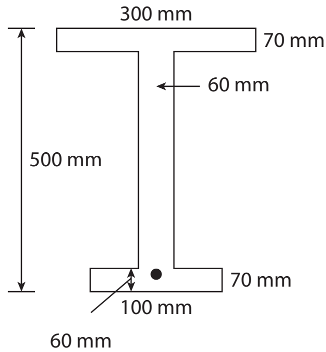

FIGURE 18.14 Beam section from Example 18.6.

Solution

1. Area of cross section, $A = (300 \times 70) + (100 \times 70) + (360 \times 60) = 49.6 \times 10^3$ mm²
2. Centroid from bottom

$$y_b = \frac{(300\times70)465 + (360\times60)250 + (100\times70)35}{49.6\times10^3} = 310.7 \text{ mm}$$

3. $y_t = 500 - 310.7 = 189.3$ mm
4. Eccentricity, $e = 310.7 - 60 = 250.7$ mm
5. Moment of inertia

Section	I_{CG} mm⁴ 10^6	A mm² 10^3	d mm	$A\,d^2$ mm⁴ 10^6	I_{NA} mm⁴ 10^6
Top Flange	$\frac{1}{12}(300)70^3 = 8.575$	$300 \times 70 = 21.0$	$189.3 - 35 = 154.3$	500	508.58
Web	$\frac{1}{12}(60)360^3 = 233.28$	$360 \times 60 = 21.6$	$310.7 - 250 = 60.7$	79.58	312.86
Bottom Flange	$\frac{1}{12}(100)70^3 = 2.858$	$100 \times 70 = 7.0$	$310.7 - 35 = 275.7$	532.1	534.96
				Total	1356.4

6. Section modulus

$$Z_t = \frac{1356.4\times10^6}{189.3} = 7.17\times10^6 \text{ mm}^3$$

$$Z_b = \frac{1356.4\times10^6}{310.7\ 6} = 4.37\times10^6 \text{ mm}^3$$

7. Load at transfer = self-weight = section area × density

$$= 49.6 \times 10^{-3} \times 24 = 1.19 \text{ kN/m}$$

8. Maximum bending moment due to this loading

$$M_D = \frac{(1.19)(8)^2}{8} = 9.52 \text{ kNm or } 9.52\times10^6 \text{ Nmm}$$

9. Maximum bending moment due to imposed load

$$M_L = \frac{(4)(8)^2}{8} = 32.0 \text{ kNm or } 32 \times 10^6 \text{ Nmm}$$

10. Stresses at transfer
    At top from Equation 18.8:

$$f_{\text{top}} = \frac{100 \times 10^3}{49.6 \times 10^3} - \frac{\left(100 \times 10^3\right) 250.7}{7.17 \times 10^6} + \frac{\left(9.52 \times 10^6\right)}{7.17 \times 10^6} = -0.15 \, \frac{\text{N}}{\text{mm}^2} \text{ or } 0.15 \text{ MPa}$$

At bottom from Equation 18.9:

$$f_{\text{bottom}} = \frac{100 \times 10^3}{49.6 \times 10^3} + \frac{\left(100 \times 10^3\right) 250.7}{4.37 \times 10^6} - \frac{\left(9.52 \times 10^6\right)}{4.37 \times 10^6} = 5.58 \, \frac{\text{N}}{\text{mm}^2} \text{ or } 5.58 \text{ MPa}$$

11. Stresses at service load
    At top from Equation 18.10:

$$f_{\text{top}} = 0.8 \left[\frac{100 \times 10^3}{49.6 \times 10^3} - \frac{\left(100 \times 10^3\right) 250.7}{7.17 \times 10^6} \right] + \frac{9.52 \times 10^6}{7.17 \times 10^6} + \frac{32 \times 10^6}{7.17 \times 10^6}$$

$$= 4.29 \, \frac{\text{N}}{\text{mm}^2} \text{ or } 4.29 \text{ MPa}$$

At bottom from Equation 18.11:

$$f_{\text{top}} = 0.8 \left[\frac{100 \times 10^3}{49.6 \times 10^3} + \frac{\left(100 \times 10^3\right) 250.7}{4.37 \times 10^6} \right] - \frac{9.52 \times 10^6}{4.37 \times 10^6} - \frac{32 \times 10^6}{4.37 \times 10^6}$$

$$= -3.3 \, \frac{\text{N}}{\text{mm}^2} \text{ or } -3.3 \text{ MPa}$$

Example 18.7

The allowable stresses are the following:

f_{tt} = allowable tensile stress in concrete at initial transfer of pre-stress = –1.58 MPa
f_{ct} = allowable compression stress in concrete at initial transfer of pre-stress = 24 MPa
f_{cs} = allowable compression stress in concrete under service loads = 30 MPa
f_{ts} = allowable tensile stress in concrete under service loads = –3.54 MPa

Check whether the beam of Example 18.6 is adequate.

Solution

Since –0.15 > –1.58 MPa **OK**
5.58 < 24 MPa **OK**
4.29 < 30 MPa **OK**
–3.3 > 3.54 MPa **OK**

Example 18.8

Design a rectangular pre-stressed beam section that supports a live load of 2 k/ft on a span of 25 ft. Pre-stress losses = 20%. $f_c' = 5000$ psi, $f_{tt} = -190$ psi, $f_{ct} = 2400$ psi, $f_{cs} = 3000$ psi, $f_{ts} = -424$ psi.

Solution

1. Assume the self-weight of beam = 300 lb/ft
2. Maximum bending moment due to self-weight, $M_D = \frac{300\,(25)^2}{8(1000)} = 23.44$ ft-k or 281.28 in.-k
3. Maximum bending moment due to live load, $M_L = \frac{2(25)^2}{8} = 156.25$ ft-k or 1875 in.-k
4. From Equation 18.16:

   $$Z_t = \frac{281.28(1-0.8)+1875}{3\text{–}0.8\,(-0.19)} = 612.7 \text{ in.}^3$$

5. From Equation 18.17:

   $$Z_b = \frac{-281.28\,(1-0.8)-1875}{-0.424-0.8\,(2.4)} = 829.92 \text{ in.}^3$$

6. Since the section is rectangular, Z_t and Z_b are equal. Hence, Z_b (higher value) governs.
7. Since $Z = \frac{bd^2}{6}$, for a width $b = 15$ in. (selected), depth required is:

   $d = \sqrt{6}\,Z/b = \sqrt{6}\,(829.92)/15 = 18.2$ in. Select 20 in.

   Self-weight of beam $= \frac{(15\times 20)}{12\times 12} \times 145 = 302$ lbs/ft ≈ assumed weight **OK**

Example 18.9

For the beam in Example 18.8, determine the minimum initial pre-stressing force and optimum eccentricity.

Solution

1. Area of cross section, $A = 15 \times 20 = 300$ in.^{2}
2. Moment of inertia, $I = 1(15)(20^3)/12 = 10 \times 10^3$ in.^{4}
3. Section modulus, $Z_t = Z_b = 1 \times 10^3$ in.^{3}
4. From Equation 18.12:

   $$\frac{F}{300} - \frac{Fe}{1\times 10^3} + \frac{281.28}{1\times 10^3} \geq -0.19$$

5. From Equation 18.15:

   $$\frac{0.8F}{300} + \frac{0.8Fe}{1\times 10^3} - \frac{281.28}{1\times 10^3} - \frac{1875}{1\times 10^3} \geq -0.424$$

6. Solving with the equality sign

   $F \geq 254$ k (minimum value)

   $e \leq 5.3$ in. (maximum value)

ULTIMATE LIMIT STATE DESIGN

Under service loads condition including self-weight, pre-stressed concrete members act as uncracked members. The concrete section and pre-stressing force are determined based on allowable stress for loads at service level, as discussed above.

However, the moment capacity and shear design of pre-stressed members are based on an ultimate limit state design. Under factored loads condition, the concrete in the tensile region develops cracks. After cracking, the section behaves more or less like an ordinary reinforced member. Thus, the approach of ordinary reinforced concrete applies, with the difference that the stress in the tendon steel is the sum of the strain caused by the pre-stressing force and the change in strain in the concrete adjacent to the steel.

If the capacity is insufficient, ordinary reinforcing steel is added at the tensile side and might be added at the compression side as well.

CRACKING MOMENT

If the fracture cracking capacity of concrete is f_r, then:

$$f_r = \frac{F}{A} + \frac{Fe}{Z_b} - \frac{M_{cr}}{Z_b} \tag{18.18}$$

where, M_{cr} = the cracking moment.

Rearranging Equation 18.18, we have:

$$M_{cr} = F\left(e + \frac{Z_b}{A}\right) - f_r Z_b \tag{18.19}$$

The fracture capacity of concrete is given by the modulus of rupture of concrete as follows:

$$f_r = -7.5\sqrt{f_c'} \quad \text{psi} \tag{18.20}$$

$$f_r = -0.6\sqrt{f_c'} \quad \text{MPa} \tag{18.21}$$

Note the negative sign for f_r.

Example 18.10

What is the cracking moment for a pre-stressed rectangular beam of 15 in. width and 20 in. depth? The pre-stressing force is 300 k acting at an eccentricity of 5 in. The compression strength of concrete is 5000 psi.

Solution

1. $f_r = -7.5\sqrt{5000} = -530.33$ psi or -0.53 ksi
2. Area of cross section, $A = 15 \times 20 = 300$ in.^{2}
3. Section modulus, $Z_b = (15)(20)^2/6 = 1000$ in.^{3}
4. From Equation 18.19:

   $M_{cr} = 300(5 + 1000/300) - (-0.53)(1000) = 3030$ in.-k or 252.5 ft-k.

STRAINS AT DIFFERENT STAGES OF LOADING

Stage 1: At Transfer

The strain diagram is shown in Figure 18.15. The strain in concrete adjacent to the tendon is ε_{ct}. The stress in the tendon and the corresponding strain are:

$$f_{st} = \frac{F}{A} \quad \text{and} \quad \varepsilon_{st} = \frac{f_{st}}{E_s} \tag{18.22}$$

Stage 2: After Application of External Load

First, the concrete adjacent to the tendon is decompressed to make the strain in the tendon (steel) equal to $\varepsilon_{ct} + \varepsilon_{st}$. Then at full load, additional strain similar to what happens in a normal reinforced section is imposed, as shown in Figure 18.16:

From the similarity of the triangles:

$$\frac{\varepsilon_0}{c} = \frac{\varepsilon_{cp}}{d_p - c}$$
$$\text{or } \varepsilon_{cp} = \frac{\varepsilon_0 (d_p - c)}{c} \tag{18.23}$$

The strain in concrete adjacent to the tendon is:

$$\varepsilon_{\text{concrete}} = \varepsilon_{cp} + \varepsilon_{ct}$$
$$= \frac{\varepsilon_0 (d_p - c)}{c} + \varepsilon_{ct} \tag{18.24}$$

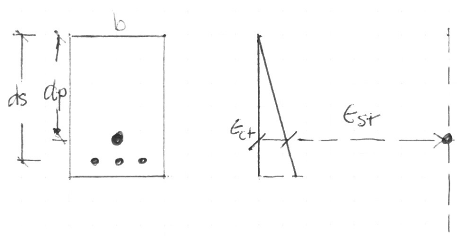

FIGURE 18.15 Strain diagram at transfer.

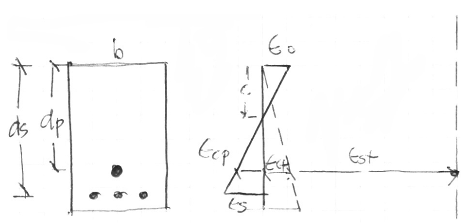

FIGURE 18.16 Strain diagram after application of load.

The strain in the tendon (steel) is:

$$\varepsilon_{\text{tendon}} = \varepsilon_{cp} + \varepsilon_{ct} + \varepsilon_{st}$$

$$= \frac{\varepsilon_0\left(d_p - c\right)}{c} + \varepsilon_{ct} + \varepsilon_{st} \tag{18.25}$$

where:

ε_{ct} = the strain in the concrete adjacent to the tendon at transfer
ε_{st} = the initial strain in the tendon due to the pre-stress at transfer
ε_0 = the compression strain in the concrete at the top fiber after application of the load
ε_{cp} = the tensile strain in the concrete adjacent to the tendon after application of the load
ε_s = the tensile strain in normal reinforcing steel after application of the load (if present)

STRESSES AND FORCES AFTER APPLICATION OF THE LOAD

These stresses and forces are similar to normal reinforced concrete members, as shown below:

1. As for normal reinforced concrete, the compression force is:

$$C = 0.85 f_c' \, a \, b \tag{18.26}$$

From Chapter 14, Equations 14.3 and 14.4:

$$a = \beta_1 c \tag{14.3}$$

$$\beta_1 = 0.85 \text{ for } f_c' \le 4000 \text{ psi} \tag{14.4a}$$

$$\beta_1 = 0.85 - \left(f_c' - \frac{4000}{1000}\right)(0.05) \text{ for } f_c' > 4000 \text{ psi but} \le 8000 \text{ psi} \tag{14.4b}$$

$$\beta_1 = 0.65 \text{ for } f_c' > 8000 \text{ psi} \tag{14.4c}$$

2. Assuming that the reinforcing steel has yielded, the tensile force in reinforcing steel is:

$$T_s = A_s f_y \tag{18.27}$$

where:

A_s = the area of the reinforcing steel
f_y = the yield strength of the reinforcing steel

3. The tensile force in the tendon is:

$$T_p = f_{ps} A \tag{18.28}$$

where:

A = the area of tendon steel
f_{ps} = the stress in the tendon (pre-stressed steel)

The stress in pre-stressed steel, f_{ps}, is unknown. There are three approaches to determine f_{ps}:

1. Approximate analysis wherein f_{ps} is assumed to be equal to the yield strength of pre-stressed steel, f_{py}.
2. Semi-empirical approach according to Codes.
3. Rigorous approach using strain compatibility.

In the semi-empirical approach, the stress in the tendon lies somewhere between the yield strength of pre-stressed steel, f_{py}, and ultimate strength, f_{pu}. Numerous empirical formulas have been proposed to assess the stress in tendon.

In the rigorous method, effort is directed to determine the depth of neutral axis from forces equilibrium. Then, using Equations 18.22 through 18.24, the strains in concrete, reinforcing steel, and pre-stressed steel are determined and therefrom the corresponding stresses. The true value of f_{ps} is difficult to calculate; hence, it is estimated using the other two methods.

Assuming $f_{ps} = f_{py}$ by balancing of forces, $C = T_p + T_s$:

$$a = \frac{f_{py}A + f_y A_s}{0.85 f_c' b} \tag{18.29}$$

ULTIMATE MOMENT CAPACITY

Refer again to Figure 18.17. Taking the moment at the location of the compression force, C:

$$M_n = T_p Z_1 + T_s Z_2 \tag{a}$$

$$= f_{ps} A Z_1 + f_y A_s Z_2 \tag{b}$$

Assuming the depth of the line of action of the compression force, C, at 0.15 d_s from the top and using an approximate procedure wherein $f_{ps} = f_{py}$:

$$M_u = \phi f_{py} A(d_p - 0.5a) + \phi f_y A_s (d_s - 0.5a) \tag{18.30}$$

where, ϕ = the resistance factor = 0.9 for steel yielding case.

The moment capacity with pre-stressed steel only is often more than the factored moment due to loads. If it is inadequate marginally, then tensile reinforcement steel is added, which is presented by the second term of Equation 18.30. However, if the capacity is substantially inadequate, then it is more expedient to increase the tendon size since it is at least four times more effective.

Maximum and Minimum Reinforcement

The maximum reinforcement (both pre-stressed and non-pre-stressed) should be:

$$\frac{c}{d_p} \le 0.42 \tag{18.31}$$

The minimum reinforcement should be so that:

$$M_u > 1.2 M_{cr} \tag{18.32}$$

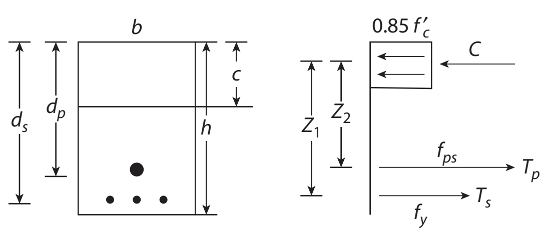

FIGURE 18.17 Stress and force diagram.

Example 18.11

Determine the ultimate moment capacity of a rectangular, pre-stressed, concrete 14 in. wide × 30 in. deep section. The steel tendon of 2 in.^{2} area is located 5 in. from the bottom. The tendon is pre-stressed to 300-kip force. The compression strength of concrete is 5 ksi, and the yield strength of pre-stressed steel is 230 ksi.

Solution

1. From Equation 18.29:

$$a = \frac{230(2)}{0.85(5)(14)} = 7.73\text{ in.}$$

2. Since there is no normal reinforcement:

$$M_u = \phi\, f_{py}\, A(d_p - 0.5a)$$

$$= 0.9(230)(2)(25 - 0.5 \times 7.73) = 8750\text{ in.-k or }729.2\text{ ft-k.}$$

Example 18.12

The beam in Example 18.11 is subjected to a dead load of 1 k/ft (in addition to self-weight) and a live load of 4 k/ft over a span of 30 ft. Is the beam adequate? If not, design the beam using reinforcing steel of 50 ksi.

Solution

1. Self-weight of beam = $\frac{14}{12} \times \frac{30}{12} \times 1 \times 145$ = 422.9 lbs/ft or 0.42 k/ft
2. Bending moment due to dead load: $M_D = 1.42(30)^2/8 = 159.75$ ft-k
3. Bending moment due to live load: $M_L = 4(30)^2/8 = 450.0$ ft-k
4. Factored moment on beam = 1.2 (159.75) + 1.6 (450.0) = 911.7 ft-k or 10940.8 in.-k
5. Since 911.7 > 729.2, **NG** reinforced steel to be provided
6. Initial design, from Equation 18.30, assuming 2 in. cover and a without normal reinforcing steel:
   $10{,}940.8 = 0.9(230)(2)(25 - 0.5 \times 7.73) + 0.9(50)A_s\,(27 - 0.5 \times 7.73)$
   or $A_s = 1.92$ in.^{2}
   Provide 3 bars of #9.$A_s = 3$ in.^{2}
7. Revised $a = \dfrac{230(2) + 50(3.0)}{0.85 \times 5 \times 14} = 10.25\text{ in.}$
8. Design capacity

$$M_u = 0.9 \times 230 \times 2 \times (25 - 0.5 \times 10.25) + 0.9 \times 50 \times 3 \times (28 - 0.5 \times 10.25)$$

$$= 11{,}316.4 > 10{,}940.8\ \textbf{OK}$$

ULTIMATE SHEAR STRENGTH DESIGN

As discussed in Chapter 15 on shear in reinforced concrete, the applied shear force is resisted by the concrete section and additionally by the web reinforcement if the former is not adequate. As stated in Equation 15.4 of Chapter 15:

$$V_u = \phi\, V_c + \phi\, V_s \tag{15.4}$$

where:

V_u = the factored applied (design) shear force
V_c = the shear strength contributed by concrete
V_s = the shear contribution of the web reinforcement
ϕ = the capacity reduction factor for shear = 0.75

In pre-stressed concrete, the shear contribution by concrete is different and more complex. The pre-stressed concrete offers two major advantages:

1. The shear force at any section is less because of the vertical component of the pre-stressing force that generally acts opposite to the external load.[2]
2. The diagonal tension is smaller due to the compression induced by the pre-stressing force.

SHEAR STRENGTH PROVIDED BY CONCRETE

Two types of shear-related cracks can develop in pre-stressed concrete. Both are evaluated, and the lower value is adopted.

Shear Capacity of Cracked Section (Flexure Induced Shearing)

The cracks start as flexure cracks at the tensile side normal to the beam axis and then, due to the effect of the diagonal tension, they propagate in the inclined direction. Usually these cracks govern near midspan where the flexure stress is large.

Since concrete is cracked and the inclination of the tendon near the center is small, the vertical component of pre-stressing force is not included in this case.

Empirically, the cracked shear capacity of concrete is given by:

$$V_{ci} = \left(1 - 0.55\frac{\alpha F}{A_p f_{pu}}\right) v_c b d_p + \frac{M_0}{M} V \tag{18.33}$$

V_{ci} should not be less than the following:

$$V_{ci} \geq 1.7\sqrt{f_c'}\, b d_p \qquad \text{(FPS units)} \tag{18.34}$$

$$V_{ci} \geq 0.1\sqrt{f_c'}\, b d_p \qquad \text{(SI units)} \tag{18.35}$$

where:

α = the reduced pre-stress factor = (1 – pre-stress losses in fraction)
F = the pre-stressing force on the tendon
A_p = the area of the cross section of the tendon
f_{pu} = the ultimate tensile strength of the tendon
b = width of the beam
d_p = the depth from the top to the center of the tendon
M_0 = the moment required to remove 80% of the compressive stress of pre-stressed steel

$$= 0.8\left(\frac{F}{A} + \frac{Fe^2}{I}\right)\frac{I}{e}$$

v_c = the ultimate shear strength of concrete, as shown in Figure 18.18

[2] The contribution of the vertical component is considered only in web-shearing cracking (uncracked in flexure) as discussed subsequently in the section "Shear Capacity of Uncracked Section (Web-Shear Cracking)."

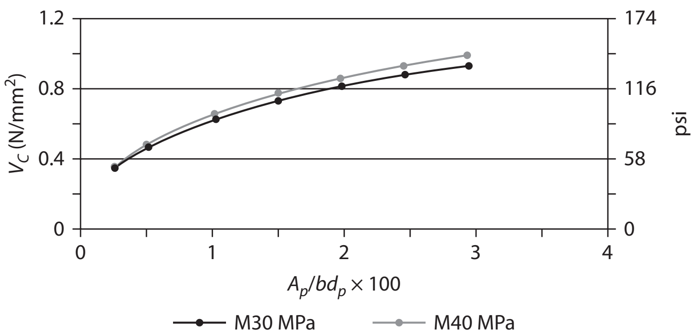

FIGURE 18.18 Shear strength of concrete.

Shear Capacity of Uncracked Section (Web-Shear Cracking)

When the magnitude of the diagonal tension is relatively high compared to the flexural stress, cracking of the web develops before flexural cracking. This type of failure is more prevalent near supports where the shear due to the support reaction is high. A simplified relation based on the principal tensile stress in the web is as follows:

$$V_{cw} = 0.67b\,h\sqrt{f_t^2 + 0.8 f_t f_{cp}} + V_p \tag{18.36}$$

where:

h = the depth of the beam

f_t = the tensile stress in the web

= $3.5\sqrt{f_c{}'}$ (FPS units)

= $0.24\sqrt{f_c{}'}$ (SI units)

f_{cp} = the compressive stress due to pre-stressing

= F/A

V_p = the vertical component of the pre-stressing force in the tendon

Example 18.13

A T-beam section has a web width of 200 mm and a depth of 1000 mm; the area of the cross section is 300×10^3 mm^2, the moment of inertia is 40×10^9 mm^4, and the centroid is 300 mm from the top. The tendons of area 1800 mm^2 are stretched by a force of 3060 kN at an eccentricity of 350 mm. Check the section for shear at a location where the shear force is 500 kN and the bending moment is 800 kN. At this location, the tendon is inclined 3^0 from horizontal. The concrete and steel strengths are 50 MPa and 1600 MPa, respectively. Assume pre-stress losses of 20%.

Solution:

1. Known:
   $\alpha = 0.8$
   $F = 3060 \times 10^3$ N
   $A = 300 \times 10^3$ mm^2
   $I = 40 \times 10^9$ mm^4

$e = 350$ mm
$A_p = 1800$ mm^2
$b = 200$ mm
d_p = centroid + eccentricity = 300 + 350 = 650 mm
$f_{pu} = 1600$ N/mm^2
$V = 500 \times 10^3$ N
$M = 800 \times 10^6$ Nmm

2. $M_0 = 0.8\left(\frac{F}{A}+\frac{Fe^2}{I}\right)\frac{I}{e} = 0.8\left(\frac{3060\times10^3}{300\times10^3}+\frac{3060\times10^3\times350^2}{40\times10^9}\right)\left(\frac{40\times10^9}{350}\right)$

$= 1.79\times10^9$ Nmm

3. $A_p/bd_p = 1800/(200 \times 650) = 0.014$
   From Figure 18.18, $v_c = 0.7$ N/mm^2
4. From Equation 18.33:

$$V_{ci} = (1-\frac{0.55(0.8)(3060\times10^3)}{1800\times1600}\times0.7\times200\times650+\frac{(1.79\times10^9)}{800\times10^6}500\times10^3$$

$$=1167.2\times10^3\,\text{N}$$

5. From Equation 18.35:

$0.1\sqrt{fc'}\,bd_p = 0.1\sqrt{50}\times 200\times 650 = 91.92\times 10^3$ N

$1167.2 \times 10^3 > 91{,}92 \times 10^3$ **OK**

6. $f_t = 0.24\sqrt{fc'} = 0.24\sqrt{50} = 1.7$

$f_{cp} = F/A = 3060 \times 10^3/300 \times 10^3 = 10.2$

7. $V_p = (3060 \times 10^3) \sin 3^0 = 160 \times 10^3$ N
8. From Equation 18.36:

$V_{cw} = 0.67 \times 200 \times 1000 \times (1.7^2 + 0.8 \times 1.7 \times 10.2)^{0.5} + 160 \times 10^3$

$= 708.58 \times 10^3$ N

9. Shear strength of section = 708.58 kN (smaller of V_{ci} and V_{cw})

   Since 708.58 kN is larger than the applied shear at section of 500 kN, no web reinforcement is required.

SHEAR STRENGTH PROVIDED BY WEB REINFORCEMENT

In the region where the factored design shear force V_u exceeds V_c, V_c is the equal to smaller of the values of V_{ci} from Equation 18.33 and V_{cw}, from Equation 18.36 the magnitude of V_s is positive from Equation 15.4. In this region the web reinforcement in the form of stirrups is provided.

All specifications and relations, including maximum and minimum steel, and spacing of stirrups as discussed in Chapter 15 in the context of conventional reinforced members, are applicable to pre-stressed members. The notations of f_y and d in these equations refer to the non-pre-stressed steel in tension.

PROBLEMS

18.1 A pre-stressed concrete beam, 300 mm wide and 500 mm deep, is pre-tensioned by straight wires of 260 mm^2 cross-sectional area, carrying an initial force of 450 kN at an eccentricity of 50 mm. Estimate the percentage loss of stress in steel due to elastic shortening. The modulus of elasticity of steel is 195 kN/mm^2 (195 GPa). The modulus of elasticity and compression strength of concrete are 28 kN/mm^2 (28 GPa) and 35 MPa (35 N/mm^2), respectively.

18.2 A pre-stressed concrete beam, 14 in. wide and 20 in. deep, is pre-tensioned by straight wires of 0.5 in.^{2} cross-sectional area, carrying an initial force of 200 kip at an eccentricity of 3 in. Estimate the percentage loss of stress in steel due to elastic shortening. The modulus of elasticity of steel is 28×10^3 ksi, and the compression strength of concrete is 4 ksi.

18.3 A pre-stressed concrete beam, 14 in. wide and 18 in. deep, is post-tensioned by wires of 0.6 in.^{2} cross-sectional area, carrying an initial force of 150 kip at an eccentricity of 3 in. at center and 2 in. at the ends. Estimate the percentage loss of stress in steel due to elastic shortening. The modulus of elasticity of steel is 28×10^3 ksi, and the compression strength of concrete is 5 ksi.

18.4 A pre-stressed concrete beam, 200 mm wide and 400 mm deep, is post-tensioned by a tendon of 200 mm^2 cross-sectional area, carrying an initial force of 400 kN at an eccentricity of 50 mm at the center and 30 mm at the ends. Estimate the percentage loss of stress in steel due to elastic shortening. The modulus of elasticity of steel is 195 kN/mm^2 (195 GPa). The modulus of elasticity and compression strength of concrete are 40 kN/mm^2 (28 GPa) and 35 MPa (35 N/mm^2), respectively.

18.5 For Problem 18.1, determine the loss of stress due to shrinkage if the age of concrete at transfer is 15 days. Assume the beam to be (1) pre-tensioned, (2) post-tensioned.

18.6 For Problem 18.2, determine the loss of stress due to shrinkage if the age of concrete at transfer is 10 days. Assume the beam to be (1) pre-tensioned, (2) post-tensioned.

18.7 In Problem 18.1, determine the loss of stress due to creep at 28 days.

18.8 For Problem 18.2, determine the loss of stress due to creep at 1 year.

18.9 For Problem 18.1, determine the loss of stress due to the relaxation of steel.

18.10 In Problem 18.2, determine the loss of stress due to the relaxation of steel.

18.11 For Problem 18.1, determine the loss of stress due to friction for a tendon of length 10 m. The tendon is curved 2^0 from horizontal. The coefficient of friction is 0.3, and the coefficient of wave effect is 20×10^{-4} per m.

18.12 For Problem 18.2, determine the loss of stress due to friction for a tendon of length 30 ft. The tendon is curved 1.5^0 from horizontal. The coefficient of friction is 0.25, and the coefficient of wave effect is 5×10^{-4} per ft.

18.13 An asymmetrical I-section beam is pre-stressed by a force of 150 kN applied through a tendon of wires of 60 mm diameter located 50 mm from the bottom. The beam carries a load of 3 kN/m over a span of 10 m. The pre-stress losses are 20%. Compute the stresses in concrete at the (1) transfer stage, and (2) service load stage. The density of concrete is 25 kN/m^3.

18.14 The allowable stresses are the following:
f_{tt} = the allowable tensile stress in concrete at initial transfer of pre-stress = −1.58 MPa
f_{ct} = the allowable compression stress in concrete at initial transfer of pre-stress = 24 MPa

f_{cs} = the allowable compression stress in concrete under service loads = 30 MPa
f_{ts} = the allowable tensile stress in concrete under service loads = –3.54 MPa
Check whether the beam of Problem 18.13 is adequate.

18.15 A rectangular beam of 12 in. width and 18 in. depth is pre-stressed by a force of 20 kip applied through a tendon of wires of 2 in. diameter located 3 in. from the bottom. The beam carries a load of 2 k/ft over a span of 20 ft. The pre-stress losses are 25%. Compute the stresses in concrete at the (1) transfer stage, and (2) service load stage. The density of concrete is 145 lbs/ft^3.

18.16 The compressive strength of concrete is 5000 psi. Accordingly, the allowable stresses are the following:
f_{tt} = the allowable tensile stress in concrete at initial transfer of pre-stress = −190 psi
f_{ct} = the allowable compression stress in concrete at initial transfer of pre-stress = 2400 psi
f_{cs} = the allowable compression stress in concrete under service loads = 30,000 psi
fts = the allowable tensile stress in concrete under service loads = −424 psi
Check whether the beam of Problem 18.15 is adequate.

18.17 Design a rectangular pre-stressed beam section (on allowable stress basis) that supports a live load of 2.5 k/ft on a span of 30 ft. Pre-stress losses = 22%. $f_c' = 6000$ psi, $f_{tt} = -208$ psi, $f_{ct} = 2880$ psi, $f_{cs} = 3600$ psi, $f_{ts} = -465$ psi.

18.18 Design a rectangular pre-stressed beam section (on allowable stress basis) that supports a live load of 30 kN/m on a span of 10 m. Pre-stress losses = 22%. $f_c' = 50$ MPa, $f_{tt} = -1.58$ MPa, $f_{ct} = 24$ MPa, $f_{cs} = 30$ MPa, $f_{ts} = -3.54$ MPa.

18.19 For the beam in Problem 18.17, determine the minimum initial pre-stressing force and optimum eccentricity.

18.20 For the beam in Problem 18.18, determine the minimum initial pre-stressing force and optimum eccentricity.

18.21 What is a cracking moment for a pre-stressed rectangular beam of 10 in. width and 14 in. depth? The pre-stressing force is 100 k acting at an eccentricity of 3 in. The compression strength of concrete is 4000 psi.

18.22 What is a cracking moment for a pre-stressed rectangular beam of 200 mm width and 400 mm depth? The pre-stressing force is 1000 kN acting at an eccentricity of 100 mm. The compression strength of concrete is 40 MPa.

18.23 Determine the ultimate moment capacity of a rectangular pre-stressed concrete 350 mm wide × 750 mm deep section. The steel tendon has transfer pre-stress of 1200 kN of 1000 mm^2 area and located 100 mm from the bottom. The compression strength of concrete is 50 MPa, and the yield strength of pre-stressed steel is 1600 MPa. The density of concrete is 24 kN/m^3.

18.24 The beam in Problem 18.23 is subjected to a dead load of 4 kN/m (in addition to self-weight) and a live load of 15 kN/m over a span of 8 m. Is the beam adequate? If not, design the beam using reinforcing steel of 350 MPa.

18.25 Determine the ultimate moment capacity of a rectangular, pre-stressed, concrete 14 in. wide × 24.6 in. deep section. The steel tendon of 2.6 in.^{2} area is located 5 in. from the bottom. The tendon is pre-stressed to 300-kip force. The compression strength of concrete is 7 ksi, and the yield strength of pre-stressed steel is 230 ksi.

18.26 The beam in Problem 18.25 is subjected to a dead load of 1.5 k/ft (in addition to self-weight) and a live load of 5 k/ft over a span of 25 ft. Is the beam adequate? If not, design the beam using reinforcing steel of 50 ksi.

18.27 A T-beam section has a web width of 10 in. and a depth of 30 in; the area of cross section is 460 in.^{2}; the moment of inertia is 96×10^3 in.^{4}; and the centroid is 12 in. from the top. The tendons of area 2.8 in.^{2} are stretched by a force of 700 kip at an eccentricity of 14 in. Check the section for shear at a location where the shear force is 115 kip and the bending moment is 600 ft-k. At this location, the tendon is inclined 3^0 from horizontal. The concrete and steel strengths are 7 ksi and 230 ksi, respectively. Assume pre-stress losses of 20%.

18.28 A rectangular beam section has a width of 250 mm and a depth of 600 mm. The tendons of area 1500 mm^2 are stretched by a force of 3000 kN at an eccentricity of 200 mm. Check the section for shear at a location where the shear force is 400 kN and the bending moment is 600 kNm. At this location, the tendon is inclined 4^0 from horizontal. The concrete and steel strengths are 50 MPa and 1600 MPa, respectively. Assume prestress losses of 18%.

19 Application of Simulations in Structural Design

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INTRODUCTION

The structural integrity and stability of any building or structure is only as good as its individual parts. Much of engineering relies on the use of standards and codes to provide uniformity in design, as well as to ensure the safety of all structural components and the structure as a whole. As discussed in previous chapters, the standard design practice utilizes traditional analytical tools in making design decisions. However, these standards do not answer all "what if" questions:

- What if this joint is fixed instead of hinged?
- What if concrete is used instead of steel?
- What if there is an imperfection in the structure, such as a crack?
- What if a prefabrication technique is used to build this bridge instead of an on-site method?

Moreover, newer architecture may involve uncommon elements that are not suited to traditional analysis. The design decisions could be made utilizing other techniques such as physical prototyping. With physical prototyping, you can build a simple representative prototype of the structure using large blocks and then load the structure in different ways to study the various effects. If you want your structure to look more realistic, you could use smaller blocks, but the smaller the blocks, the longer it will take to build your structure.

However, with the advent of computing, mathematical modeling, instead of physical modeling, has become a convenient tool of analysis. More physics-based methods such as finite element methods (FEMs), or finite element analysis (FEA), give engineers the ability to assess the influence of relevant variables in a virtual environment and solve complex real-life designs. Through visualizing the effect of a wide range of variables in a virtual design environment, civil engineers can narrow the scope of field investigations, save considerable time and cost on projects, and move more quickly to the ground breaking stage.

Several educational, open-source commercial packages (including the one from ANSYS, the affiliation of the authors of this chapter) are available today that facilitate the learning and deployment of FEM. Currently, civil engineers use these tools for projects such as high-rise buildings, bridges, dams, and stadiums, among others. By experimenting with innovative design in a virtual environment, engineers and designers can enforce safety, strength, comfort, and environmental considerations.

This chapter introduces modeling techniques using simple examples to better equip the next generation of civil engineers. We start with a simple example and solve it first analytically, then use the simulation technique. A brief discussion about real-life equivalents follows, and the chapter finishes by explaining how students can access the tools discussed.

ANALYZING A SIMPLE BEAM USING ANALYTICAL METHOD

Suppose you opened an office in a building and want to calculate the size of a beam that should be used to attach a sign board to the building. Based on space and aesthetic considerations, the sign must be cantilevered. The beam should not deflect more than one millimeter under the load of the company logo. You designed a beam comprised of a hollow rectangular tube, as shown in Figure 19.1. Now your objective is to use as little material as possible to save money. Also, due to time constraints, the beam must be ordered in the next few hours to meet the deadline.

Fortunately, you have gone through this textbook and you know that there is an equation that can predict the deflection of a cantilevered beam. You also know that a large sign hanging off the end of the beam can be replaced with a simple load as it doesn't add any significant stiffness to the beam, as shown in Figure 19.2.

From Appendix A.3, you know that the end deflection of a cantilevered beam can be calculated by the following equation:

$$\delta = \frac{PL^3}{3EI} \tag{19.1}$$

where:
P = Load
L = Length of beam
E = Modulus of elasticity
I = Moment of inertia
δ = Deflection

The sign will be made out of steel with E of 210 GPa, and it will weigh 1000 N and have a length of 2 m. With these variables specified, the required moment of inertia can be calculated for the cross section of the beam. Although the edges of the beam are rounded, you can assume the cross section with the sharp edges, as shown in Figure 19.3, to calculate the moment of inertia more easily.

From Appendix A.2, the moment of inertia of this cross section is:

$$I = \frac{1}{12} b_1 (h_1)^3 - \frac{1}{12} b_2 (h_2)^3 \tag{19.2}$$

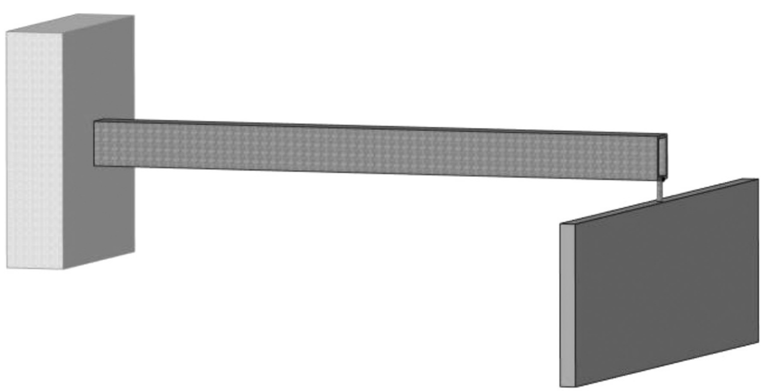


FIGURE 19.1 The sign modeled as a cantilevered beam.

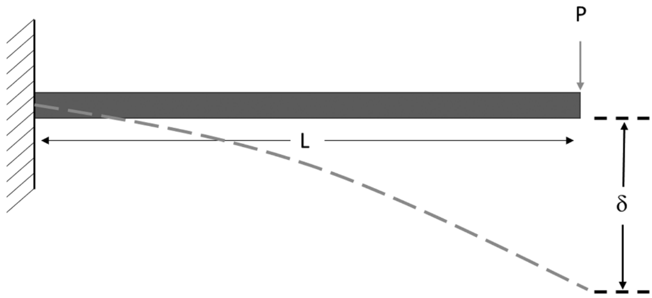


FIGURE 19.2 Deflection of sign modeled as a cantilevered beam.

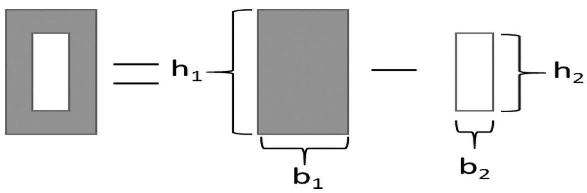


FIGURE 19.3 Moment of inertia of elements.

With the following measurements:

$h_1 = 0.15\,\text{m}$
$b_1 = 0.10\,\text{m}$
$h_2 = 0.13\,\text{m}$
$b_2 = 0.08\,\text{m}$

you can compute $I = 13 \times 10^{-6}\ \text{m}^4$ from Equation 19.2 and then the vertical deflection of 0.99 mm from Equation 19.1. This satisfies the requirement.

In the process of doing the hand calculation, you made some basic core engineering assumptions and simplifications:

1. The critical response was the deflection at the end of the beam.
2. The weight of the beam itself is ignored in calculating the load.
3. The load was exactly at the end of the beam.
4. The cross section of the beam was constant.
5. The rounded edges of the beam were ignored.
6. The building itself was perfectly rigid and acted as fixed support.

If any of these requirements change, you have to repeat all the steps in the computation. Next, we will do the same problem by the simulation technique using mathematical modeling.

MATHEMATICAL MODELING TECHNIQUE

Even though the simple equations (such as for the beam) have exact solutions, it is rarely so in the real world. In order to solve the more realistic and complex equations, a numerical approximation method must be used. The most popular method for solving complex models is called the *finite element method* (FEM).

Instead of creating a model from smaller physical blocks, each block can be represented with a mathematical equation. For instance, you can think of each block as a spring that deflects according to Hooke's Law:

$$f = kx \tag{19.3}$$

Instead of building a physical model, linear algebra is used to combine, or "assemble," these mathematical equations. The spring response of the entire model would be the following matrix equation:

$$\begin{bmatrix} f_1 \\ \vdots \\ f_n \end{bmatrix} = \begin{bmatrix} k_{11} & \cdots & k_{1n} \\ \vdots & \ddots & \vdots \\ k_{n1} & \cdots & k_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \tag{19.4}$$

where:
$f_1, f_2, \ldots f_n$ are elemental force
$x_1, x_2, \ldots x_n$ represent deflection of corresponding elements

As with the hand calculation of the beam example, creating the mathematical model requires some simplification of assumptions, including:

- How much detail to include?
- What loads will the model include?
- What material properties and physical effects should be modeled?

Answering these and other questions will guide the creation of the model.

The simplified model using Hooke's Law is just one example of a finite element model. Much more complex physical phenomena can be solved using FEM. FEMs, as well as similar numerical solutions techniques, have a rich history. Several textbooks on these methods are available for those who are interested in learning more about them.

MATHEMATICAL MODELING OF BEAM WITH SIGN BOARD

Model Setup and Input

This requires input that includes geometry, material properties, loads, and constraints. The geometry is created by a CAD tool and imported to a solver to apply loads (force, moments, etc.) and impose constraints (fixed support). The advent of computing and user-interface facilities means that several commercially available programs, including ANSYS, are available to solve these problems with minimal effort because most of the solver methods are already coded. As with hand calculations, there are trade-offs when using finite elements because we make assumptions to solve these problems. A CAD-generated model of beam is shown in Figure 19.4.

- Simplification
  Even though the program can handle all the details, let's use the similar model that we used for the analytical solution and suppress all other geometry.
- Material properties
  - Structural steel
  - Young modulus of elasticity = 210 GPa
  - Poisson ratio = 0.3

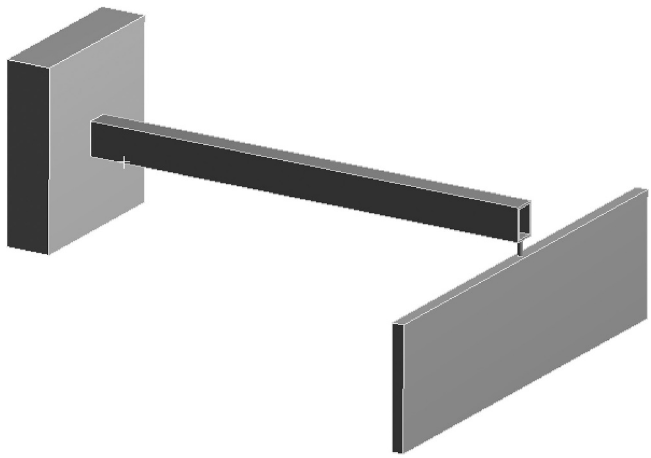


FIGURE 19.4 CAD geometry for analysis.

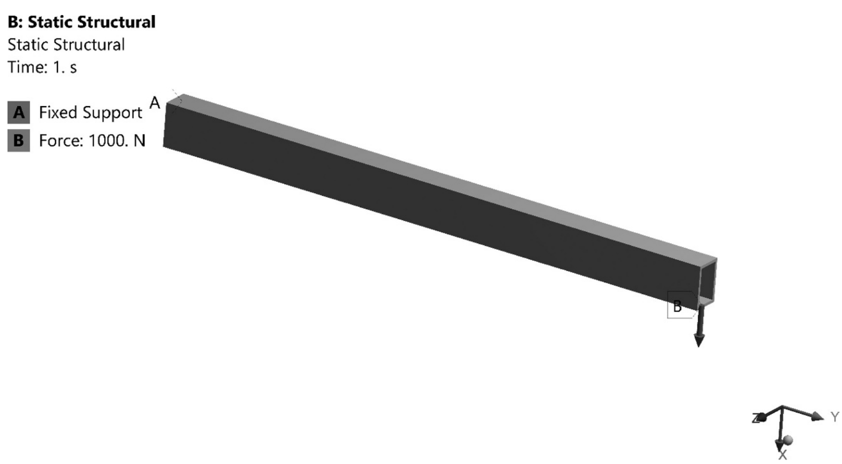


FIGURE 19.5 Simulation domain.

- Application of loads (distributed or point) and constraints

  The loads can be point load or distributed load, as discussed in previous chapters. For this example, assume a force of 1000 N at the face where the sign is hanging. The fixed support is located where the beam is attached to the building structure. Figure 19.5 shows the simulated domain with load and end constraint.
- Finite element mesh

  FEMs require converting the geometry into finite element mesh. Depending on the complexity, this could be thousands to millions of nodes and elements to get a properly converged and accurate solution. For this case, let's use a coarse mesh, as shown in Figure 19.6.

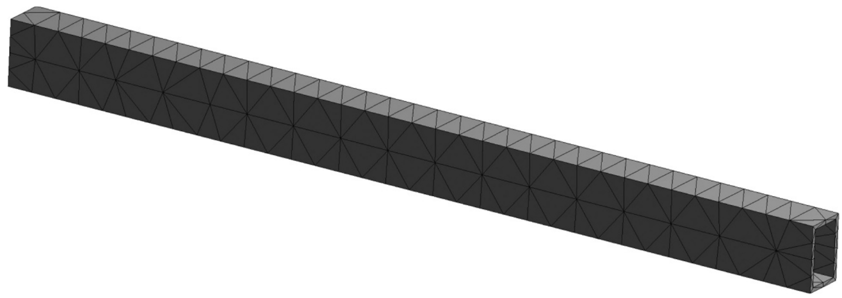


FIGURE 19.6 Finite element mesh.

MODEL OUTPUT

Solution and Post-Processing

ANSYS uses proprietary solvers to solve a set of equations that allow you to visualize simulation results, such as deformation and the stresses on the step and frame. ANSYS includes very detailed post-processing of the computed results. You can think of this as having hundreds of strain gages and other sensors.

Deformation

The maximum deformation is about 0.98487 mm, which matches our analytical solution. We can also plot the deformation along the length of the beam, as shown in Figures 19.7 and 19.8.

Equivalent Stresses and Strain

In structural engineering and strength of materials, a part or beam may be subjected to different types of forces and/or moments, or a complex combination of these. All these forces and moments, or their combinations, result in different types of stresses at different points in the beam. Depending on the material and the stress generated, the beam may fail.

Equivalent stress (also known as von-Mises stress) is used in design work because it allows any arbitrary three-dimensional stress state to be represented as a single positive stress value. The von-Mises stress is often computed as a value to determine if a given material will yield or fracture (Figure 19.9). It is mostly used for ductile materials, such as metals. The value of the maximum stress should be less than material's yield stress. The equivalent strain is calculated from the component strain (Figure 19.10).

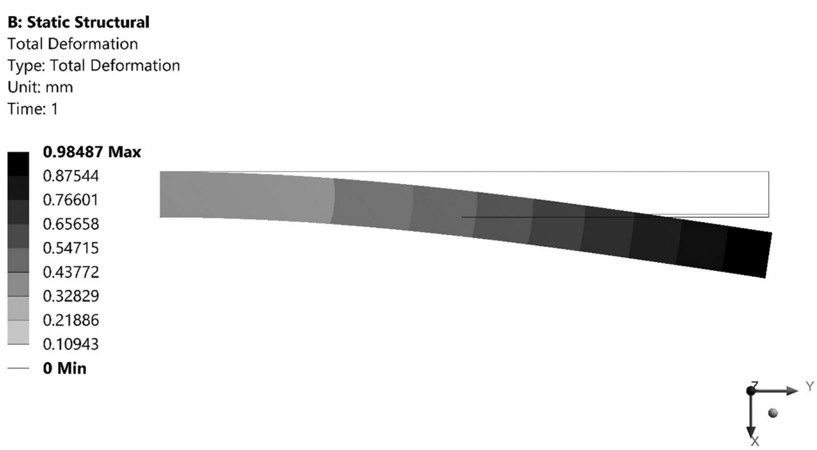


FIGURE 19.7 Total deformation (exaggerated for display).

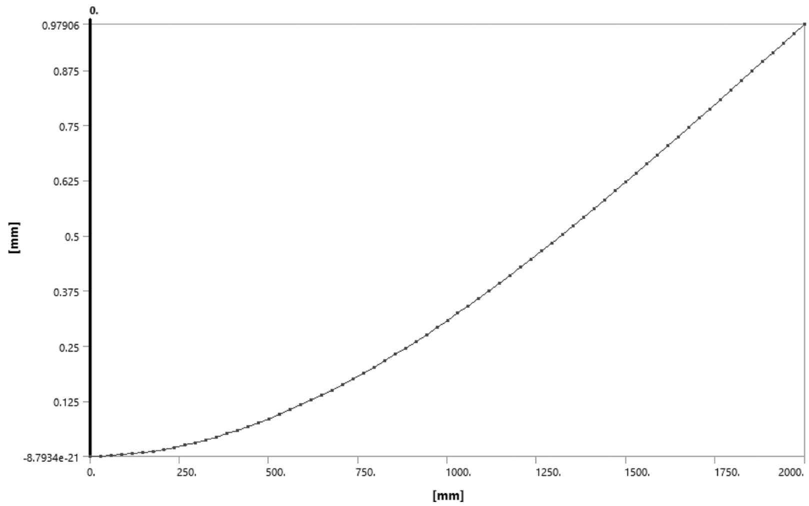


FIGURE 19.8 Deformation along the length of the beam.

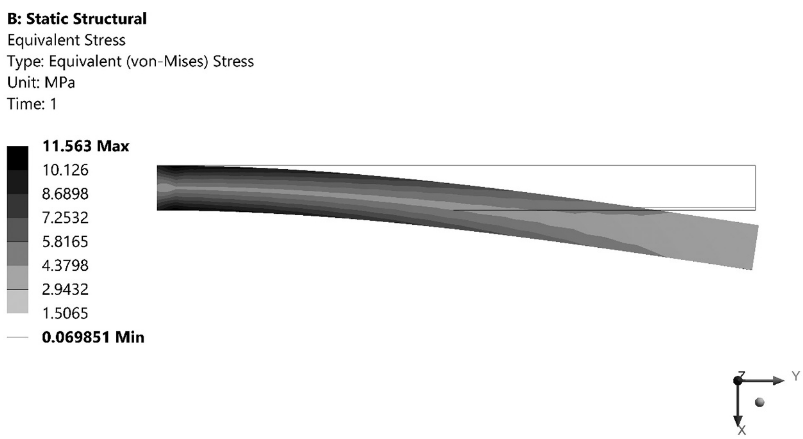


FIGURE 19.9 Equivalent stress.

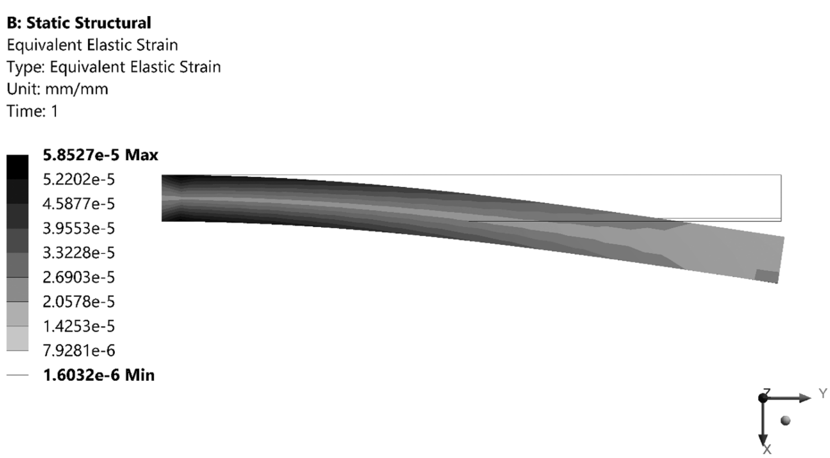


FIGURE 19.10 Equivalent strain.

EXPLORING MODEL OUTPUT FOR "WHAT IF?"

Now that you have answered the question of how much deflection the cantilever beam would have under a specific load, more questions could arise. Some examples follow:

- How much load can it take before the beam breaks?
- What other materials or shapes can this beam be made of?
- How would the sign respond to high winds?

Once the model is created, you can easily parameterize different variables, set up a design point (DP) table, and analyze different designs. For example, in Table 19.1, only the force is parameterized, and the deflection, stress, and strain are computed with one click. Behind the scenes, ANSYS runs each design point and displays these results in tabular form, which can be exported for further analysis.

MATHEMATICAL MODELING OF A STAIRCASE

With the success of the cantilever project, you have been asked to design a staircase for the building. You could simply build a physical model of the staircase, but that would take time and could be expensive. The process of designing and analyzing it using FEM is very similar to the beam simulation.

- Clean geometry model with relevant parts
- Material: structural steel with Young modulus = 210 GPa
- Specify forces and supports:
    - Adult standing on the steps exerting 800 N (force)
    - Ends of the rails are rigidly attached to the rest structure (fixed support)
- Finite element mesh
- Solution and post-processing

Figure 19.11 shows the steps: geometry with specified constraints and load, finite element mesh, deformation (slightly exaggerated for display) caused by loads and von-Mises stresses.

TABLE 19.1
Parameterization of a Variable Force

No.	Name	P4- Force X Component	P1-Total Deformation Maximum	P2-Equivalent Stress Maximum	P3-Equivalent Elastic Strain Maximum
	Units	N	mm	MPa	mm/mm × 10^{-5}
1	DP 0-current	1000	0.98487	11.563	5.8527
2	DP 1	500	0.49243	5.7816	2.9264
3	DP 2	750	0.73865	8.6724	4.3895
4	DP 3	1500	1.4773	17.345	8.7791
5	DP 4	2000	1.9697	23.126	11.7050

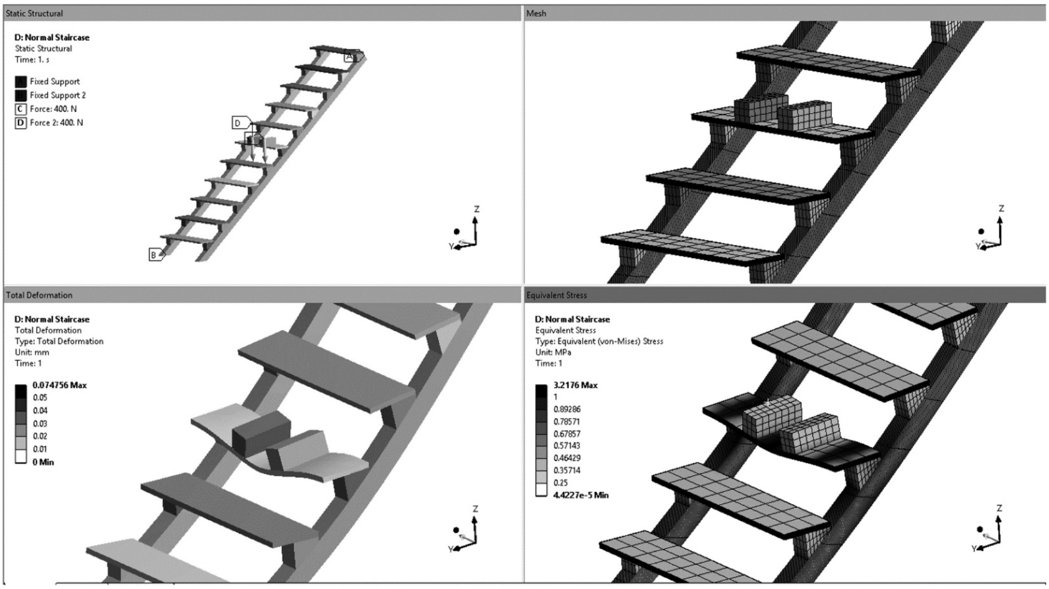


FIGURE 19.11 Staircase analysis.

What If?

As you did for the simple beam, you can ask more design-related questions, including:

- How many people could stand on the staircase before it broke.
- What if someone wanted to add internal supports under each step?
  - What size should they be?
  - Where should they be placed?
  - How many should be used?
- What other materials can the staircase be made of? Wood?
- How would the staircase respond to an earthquake?

Once the model is set up, most of these questions are like modifying geometry and/or changing inputs and computing their impacts.

REAL-LIFE STRUCTURAL ENGINEERING PROBLEMS

FEA allows engineers to tackle some of the most complex and important design challenges facing society today, such as:

- Ensuring that a structure can meet seismic and other dynamic requirements
- Determining if the structure can meet requirements for blast and explosion
- Evaluating the structural integrity of a structure under high winds, even tsunamis
- Using cutting-edge technologies such 3D printing to create beams, trusses, and even buildings

Some of these are purely structural analysis, while others require a more multi-physics approach. All these scenarios are being analyzed virtually using simulation tools.

Think about a world-class stadium that will require structural analysis, ventilation, and airflow (as it results in wind load). Virtual simulation is the key to completing these products on time and on budget. For more detailed information, visit *http://www.ansys.com*.

To address the complex FEA needs of the civil engineering field, Ingeciber S.A., developed CivilFEM® (*www.civilfem.com/*). This advanced software provides unique concrete, seismic, and geotechnical solutions for all kinds of bridges, foundations, tunnels, dams, and even nuclear power plants. Behind its custom interface, it uses industry standard ANSYS solvers. CivilFEM® APPs and wizards allow engineers to check and design ANSYS Workbench™ models using the major international construction standards. It includes check and design tools for beams and shells, and both steel and reinforced concrete for European (EC2 and EC3), American (AISC, LRFD, AISC ASD and ACI318), and American nuclear ACI 349 standards. This allows engineers to work in the construction and civil engineering fields by analyzing their models from ideation, without the need for additional training, in a user-friendly and agile way.

ACCESSING ANSYS FOR STUDENTS

ANSYS provides free student software products perfect for work done outside the classroom, such as homework, capstone projects, student competitions, and more. These products can be downloaded by students around the world from *www.ansys.com/student*. Technical support is provided via Student Communities.

There are several initiatives to teach FEA and these methods. A good example is Cornell University's SimCafe, found at *www.simcafe.org*. Using SimCafe, students engage in a systematic process for checking results, including comparing to their hand calculations. Many of the SimCafe modules juxtapose traditional textbook content with the numerical approach that ANSYS software uses. SimCafe includes a template that contributors can use to create new tutorials for the site. The template ensures that tutorials follow a standard format and use best practices in instruction, even though they are developed by different authors.

Similar to ANSYS, CivilFEM also has an academic version for instructors and students. Please visit their website.

SUMMARY

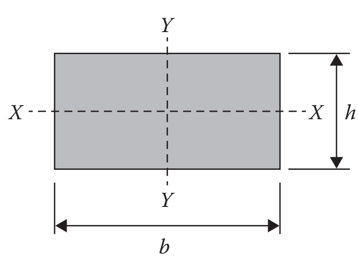
This chapter complements and extends the theory and practices of structural design for wood, steel, and concrete. While simulation is a very powerful tool, it does not replace the judgment of the engineer. The designs of most new structures, dams, and buildings use simulation tools to reduce the cost of design and development. It is recommended that the students reading this chapter visit the links provided in the text above to become a versatile civil engineer.

Appendix A: General

APPENDIX A.1
Useful Conversion Factors

Multiply	By	To Obtain
Pounds (m)	0.4356	Kilogram
Kilogram	2.205	Pounds (m)
Mass in slug	32.2	Weight in pound
Mass in kilogram	9.81	Weight in Newton (N)
Pound (f)	4.448	Newton
Newton	0.225	Pounds
U.S. or short ton	2000	Pounds
Metric ton	1000	Kilogram
U.S. ton	0.907	Metric ton
Foot	0.3048	Meter
Meter	3.281	Feet
Mile	5280	Feet
Mile	1609	Meter
	1.609	Kilometer
Square feet	0.0929	Square meter
Square mile	2.59	Square kilometer
Square kilometer	100	Hectare (ha)
Liter	1000	Cubic centimeter
Pounds per ft.^{2}	47.88	N/m^2 or pascal
Standard atmosphere	101.325	Kilopascal (kPa)
Horsepower	550	Foot-pound/second
	745.7	Newton-meter/second or Watt
°F	$\frac{5}{9}$°F − 32	°C
°C	$\frac{9}{5}$°C + 32	°F
Log to base *e* (i.e., $\log_e$, where $e = 2.718$)	0.434	Log to base 10 (i.e., $\log_{10}$)

APPENDIX A.2
Geometric Properties of Common Shapes



Rectangle:

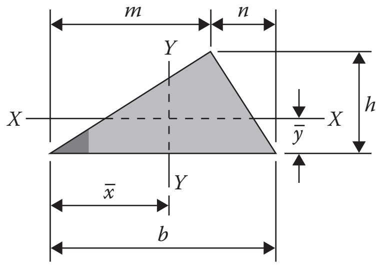
$A = bh,$

$I_x = \frac{1}{12} bh^3,$

$r_x = \sqrt{\frac{I_x}{A}} = 0.288h.$

(*Continued*)

APPENDIX A.2 (*Continued*)
Geometric Properties of Common Shapes



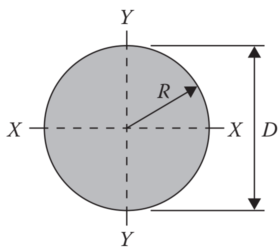
Triangle:

$A = \frac{1}{2} bh,$

$\bar{y} = \frac{h}{3},$

$\bar{x} = \frac{b+m}{3},$

$I_x = \frac{1}{36} bh^3.$



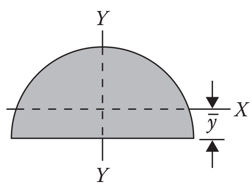
Circle:

$A = \frac{1}{4}\pi D^2 = \pi R^2,$

$I_x = \frac{\pi D^4}{64} = \frac{\pi R^4}{4},$

$r_x = \sqrt{\frac{I_x}{A}} = \frac{D}{4} = \frac{R}{2},$

$J = I_x + I_y = \frac{\pi D^4}{32} = \frac{\pi R^4}{2}.$



Semicircle:

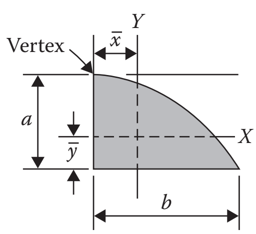
$A = \frac{1}{8}\pi D^2 = \frac{1}{2}\pi R^2,$

$\bar{y} = \frac{4r}{3\pi},$

$I_x = 0.00682 D^4 = 0.11 R^4,$

$I_y = \frac{\pi D^4}{128} = \frac{\pi R^4}{8},$

$r_x = 0.264 R.$

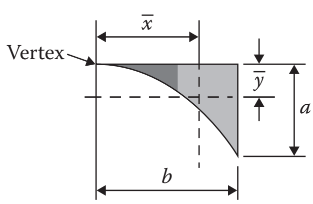


Parabola:

$A = \frac{2}{3} ab,$

$\bar{x} = \frac{3}{8} b,$

$\bar{y} = \frac{2}{5} a.$



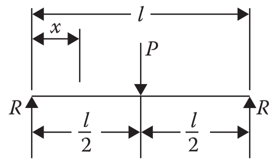
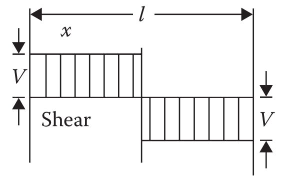
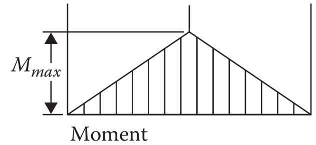
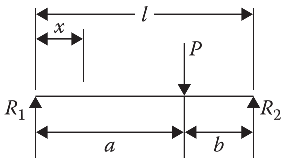
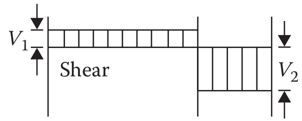
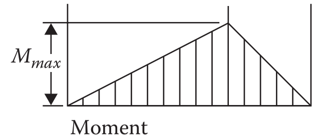
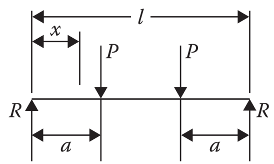
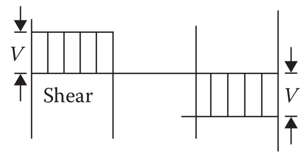
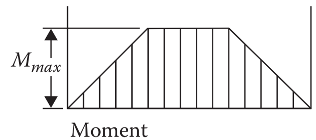
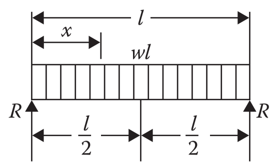
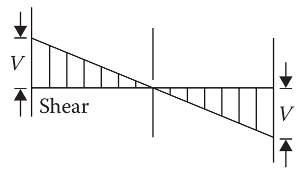
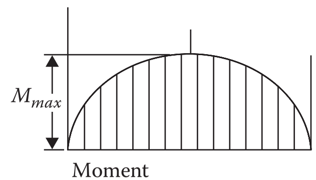
Spandrel of parabola:

$A = \frac{1}{3} ab,$

$\bar{x} = \frac{3}{4} b,$

$\bar{y} = \frac{3}{10} a.$

APPENDIX A.3
Shears, Moments, and Deflections

Loading	Shear Force Diagram	Moment Diagram	Maximum Moment	Slope at End	Maximum Deflection
			$\frac{Pl}{4}$	$\frac{Pl^2}{16EI}$	$\frac{Pl^3}{48EI}$
			$\frac{Pab}{l}$ at $x = a$	$\frac{Pa(l^2 - a^2)}{6EI}$ at R_2	$\frac{Pb(l^2 - b^2)^{3/2}}{9\sqrt{3}EIl}$ at $x = \sqrt{\frac{l^2 - b^2}{3}}$
			Pa	$\frac{Pa(l - a)}{2EI}$	$\frac{Pa(3l^2 - 4a^2)}{24EI}$
			$\frac{wl^2}{8}$	$\frac{wl^3}{24EI}$	$\frac{5}{384}\frac{wl^4}{EI}$

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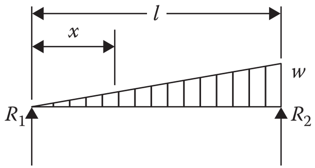
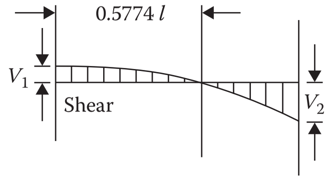
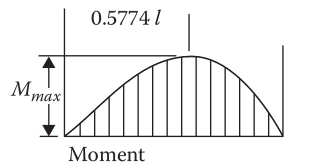
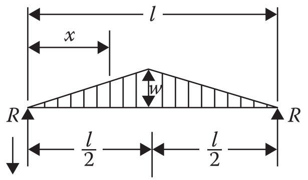
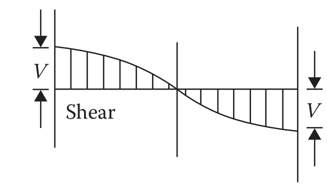
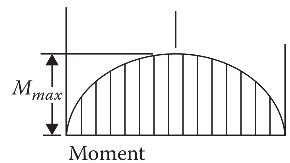
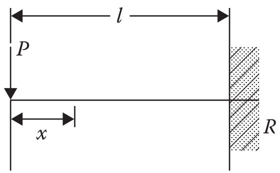
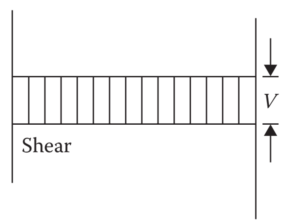
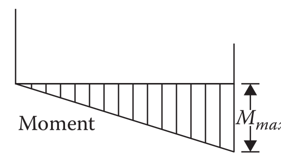
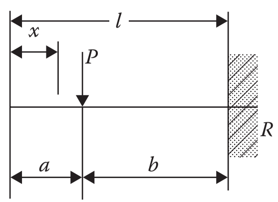
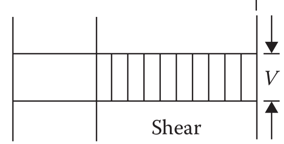
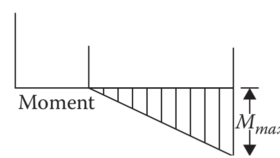
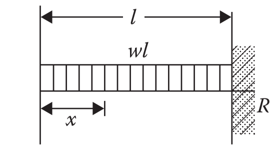
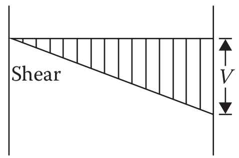
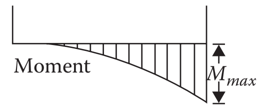
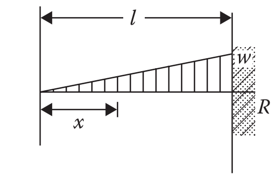
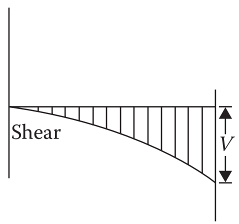
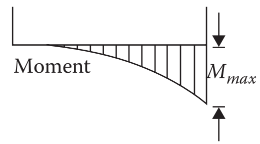
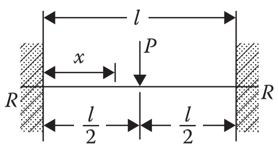
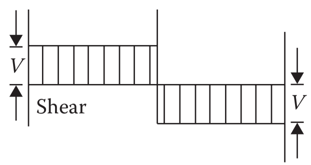
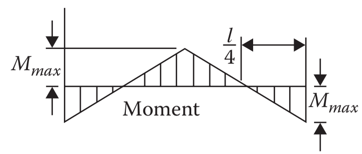
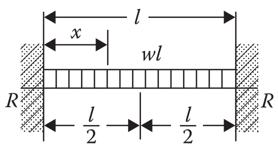
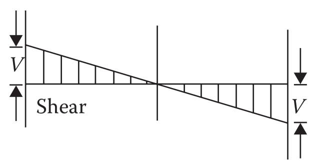
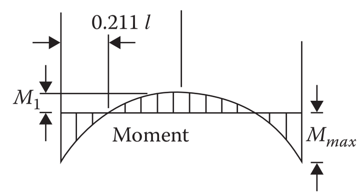
APPENDIX A.3 (*Continued*)
Shears, Moments, and Deflections

Loading	Shear Force Diagram	Moment Diagram	Maximum Moment	Slope at End	Maximum Deflection
			$\frac{wl^2}{9\sqrt{3}}$	$\frac{8wl^3}{360EI}$ at R_2	$\frac{2.5wl^4}{384EI}$ at $x = 0.519l$
			$\frac{wl^2}{12}$	$\frac{5wl^3}{192EI}$	$\frac{wl^4}{120EI}$
			$-Pl$	$\frac{Pl^2}{2EI}$	$\frac{Pl^3}{3EI}$
			$-Pb$	$\frac{Pb^2}{2EI}$	$\frac{Pb^2(3l-b)}{6EI}$ at free end

(*Continued*)

APPENDIX A.3 (*Continued*)
Shears, Moments, and Deflections

Loading	Shear Force Diagram	Moment Diagram	Maximum Moment	Slope at End	Maximum Deflection
			$-\frac{wl^2}{2}$	$\frac{wl^3}{6EI}$	$\frac{wl^4}{8EI}$
			$-\frac{wl^2}{6}$	$\frac{wl^3}{24EI}$	$\frac{wl^4}{8EI}$
			$-\frac{Pl}{8}$	0	$\frac{Pl^3}{192EI}$
			$-\frac{wl^2}{12}$	0	$\frac{wl^4}{384EI}$

Note: w, load per unit length; W, total load.

APPENDIX A.4
Typical Properties of Engineering Materials

Material	Strength (psi) (Yield Values Except Where Noted)			Modulus of Elasticity (E) (ksi)	Coefficient of Thermal Expansion (F^{-1}) (10^{-6})
	Tension	Compression	Shear		
Wood (dry)					
Douglas fir	6,000	3,500[a]	500	1,500	2
Redwood	6,500	4,500[a]	450	1,300	2
Southern Pine	8,500	5,000[a]	600	1,500	3
Steel	50,000	50,000	30,000	29,000	6.5
Concrete					
Structural, lightweight	150[b]	3,500[b]	130[b]	2,100	5.5
Brick masonry	300[b]	4,500[b]	300[b]	4,500	3.4
Aluminum, structural	30,000	30,000	18,000	10,000	12.8
Iron, cast	20,000[b]	85,000[b]	25,000[b]	25,000	6
Glass, plate	10,000[b]	36,000[b]	—	10,000	4.5
Polyester, glass-reinforced	10,000[b]	25,000[b]	25,000[b]	1,000	35

[a] For the parallel-to-grain direction.
[b] Denotes ultimate strength for brittle materials.

Appendix B: Wood

APPENDIX B.1
Section Properties of Standard Dressed (S4S) Sawn Lumber

			x–x Axis		y–y Axis		Approximate Weight in Pounds per Linear Foot (lb/ft) of Piece When Density of Wood Equals					
Nominal Size ($b \times d$)	Standard Dressed Size (S4S) ($b \times d$) (in. × in.)	Area of Section A (in.^{2})	Section Modulus S_{xx} (in.^{3})	Moment of Inertia I_{xx} (in.^{4})	Section Modulus S_{yy} (in.^{3})	Moment of Inertia I_{yy} (in.^{4})	25 lbs/ft^3	30 lbs/ft^3	35 lbs/ft^3	40 lbs/ft^3	45 lbs/ft^3	50 lbs/ft^3
Boards												
1 × 3	¾ × 2 ½	1.875	0.781	0.977	0.234	0.088	0.326	0.391	0.456	0.521	0.586	0.651
1 × 4	¾ × 3 ½	2.625	1.531	2.680	0.328	0.123	0.456	0.547	0.638	0.729	0.820	0.911
1 × 6	¾ × 5 ½	4.125	3.781	10.40	0.516	0.193	0.716	0.859	1.003	1.146	1.289	1.432
1 × 8	¾ × 7 ¼	5.438	6.570	23.82	0.680	0.255	0.944	1.133	1.322	1.510	1.699	1.888
1 × 10	¾ × 9 ¼	6.938	10.70	49.47	0.867	0.325	1.204	1.445	1.686	1.927	2.168	2.409
1 × 12	¾ × 11 ¼	8.438	15.82	88.99	1.055	0.396	1.465	1.758	2.051	2.344	2.637	2.930
Dimension Lumber (see NDS 4.1.3.2) and Decking (see NDS 4.1.3.5)												
2 × 3	1 ½ × 2 ½	3.750	1.56	1.953	0.938	0.703	0.651	0.781	0.911	1.042	1.172	1.302
2 × 4	1 ½ × 3 ½	5.250	3.06	5.359	1.313	0.984	0.911	1.094	1.276	1.458	1.641	1.823
2 × 5	1 ½ × 4 ½	6.750	5.06	11.39	1.688	1.266	1.172	1.406	1.641	1.875	2.109	2.344
2 × 6	1 ½ × 5 ½	8.250	7.56	20.80	2.063	1.547	1.432	1.719	2.005	2.292	2.578	2.865
2 × 8	1 ½ × 7 ¼	10.88	13.14	47.63	2.719	2.039	1.888	2.266	2.643	3.021	3.398	3.776
2 × 10	1 ½ × 9 ¼	13.88	21.39	98.93	3.469	2.602	2.409	2.891	3.372	3.854	4.336	4.818
2 × 12	1 ½ × 11 ¼	16.88	31.64	178.0	4.219	3.164	2.930	3.516	4.102	4.688	5.273	5.859
2 × 14	1 ½ × 13 ¼	19.88	43.89	290.8	4.969	3.727	3.451	4.141	4.831	5.521	6.211	6.901
3 × 4	2 ½ × 3 ½	8.75	5.10	8.932	3.646	4.557	1.519	1.823	2.127	2.431	2.734	3.038
3 × 5	2 ½ × 4 ½	11.25	8.44	18.98	4.688	5.859	1.953	2.344	2.734	3.125	3.516	3.906
3 × 6	2 ½ × 5 ½	13.75	12.60	34.66	5.729	7.161	2.387	2.865	3.342	3.819	4.297	4.774
3 × 8	2 ½ × 7 ¼	18.13	21.90	79.39	7.552	9.440	3.147	3.776	4.405	5.035	5.664	6.293
3 × 10	2 ½ × 9 ¼	23.13	35.65	164.9	9.635	12.04	4.015	4.818	5.621	6.424	7.227	8.030
3 × 12	2 ½ × 11 ¼	28.13	52.73	296.6	11.72	14.65	4.883	5.859	6.836	7.813	8.789	9.766
3 × 14	2 ½ × 13 ¼	33.13	73.15	484.6	13.80	17.25	5.751	6.901	8.051	9.201	10.35	11.50

(*Continued*)

APPENDIX B.1 (*Continued*)
Section Properties of Standard Dressed (S4S) Sawn Lumber

			x–x Axis		y–y Axis		Approximate Weight in Pounds per Linear Foot (lb/ft) of Piece When Density of Wood Equals					
Nominal Size ($b \times d$)	Standard Dressed Size (S4S) ($b \times d$) (in. × in.)	Area of Section A (in.^{2})	Section Modulus S_{xx} (in.^{3})	Moment of Inertia I_{xx} (in.^{4})	Section Modulus S_{yy} (in.^{3})	Moment of Inertia I_{yy} (in.^{4})	25 lbs/ft^3	30 lbs/ft^3	35 lbs/ft^3	40 lbs/ft^3	45 lbs/ft^3	50 lbs/ft^3
3 × 16	2 ½ × 15 ¼	38.13	96.90	738.9	15.89	19.86	6.619	7.943	9.266	10.59	11.91	13.24
4 × 4	3 ½ × 3 ½	12.25	7.15	12.51	7.146	12.51	2.127	2.552	2.977	3.403	3.828	4.253
4 × 5	3 ½ × 4 ½	15.75	11.81	26.58	9.188	16.08	2.734	3.281	3.828	4.375	4.922	5.469
4 × 6	3 ½ × 5 ½	19.25	17.65	48.53	11.23	19.65	3.342	4.010	4.679	5.347	6.016	6.684
4 × 8	3 ½ × 7 ¼	25.38	30.66	111.1	14.80	25.90	4.405	5.286	6.168	7.049	7.930	8.811
4 × 10	3 ½ × 9 ¼	32.38	49.91	230.8	18.89	33.05	5.621	6.745	7.869	8.993	10.12	11.24
4 × 12	3 ½ × 11 ¼	39.38	73.83	415.3	22.97	40.20	6.836	8.203	9.570	10.94	12.30	13.67
4 × 14	3 ½ × 13 ¼	46.38	102.41	678.5	27.05	47.34	8.051	9.661	11.27	12.88	14.49	16.10
4 × 16	3 ½ × 15 ¼	53.38	135.66	1,034	31.14	54.49	9.266	11.12	12.97	14.83	16.68	18.53
Timbers (5 × 5 in. and Larger)												
Post and Timber (see NDS 4.1.3.4)												
5 × 5	4 ½ × 4 ½	20.25	15.19	34.17	15.19	34.17	3.516	4.219	4.922	5.625	6.328	7.031
6 × 6	5 ½ × 5 ½	30.25	27.73	76.26	27.73	76.26	5.252	6.302	7.352	8.403	9.453	10.50
6 × 8	5 ½ × 7 ¼	39.88	48.18	174.7	36.55	100.5	6.923	8.307	9.69	11.08	12.46	13.85
8 × 8	7 ¼ × 7 ¼	52.56	63.51	230.2	63.51	230.2	9.125	10.95	12.78	14.60	16.43	18.25
8 × 10	7 ¼ × 9 ¼	67.06	103.4	478.2	81.03	293.7	11.64	13.97	16.30	18.63	20.96	23.29
10 × 10	9 ¼ × 9 ¼	85.56	131.9	610.1	131.9	610.1	14.85	17.83	20.80	23.77	26.74	29.71
10 × 12	9 ¼ × 11 ¼	104.1	195.1	1,098	160.4	742.0	18.07	21.68	25.29	28.91	32.52	36.13
12 × 12	11 ¼ × 11 ¼	126.6	237.3	1,335	237.3	1,335	21.97	26.37	30.76	35.16	39.55	43.95
12 × 14	11 ¼ × 13 ¼	149.1	329.2	2,181	279.5	1,572	25.68	31.05	36.23	41.41	46.58	51.76
14 × 14	13 ¼ × 13 ¼	175.6	387.7	2,569	387.7	2,569	30.48	36.58	42.67	48.77	54.86	60.96
14 × 16	13 ¼ × 15	198.8	496.9	3,727	438.9	2,908	34.51	41.41	48.31	55.21	62.11	69.01
16 × 16	15 × 15	225.0	562.5	4,219	562.5	4,219	39.06	46.88	54.69	62.50	70.31	78.13
16 × 18	15 × 17	255.0	722.5	6,141	637.5	4,781	44.27	53.13	61.98	70.83	79.69	88.54

(*Continued*)

APPENDIX B.1 (*Continued*)
Section Properties of Standard Dressed (S4S) Sawn Lumber

Nominal Size ($b \times d$)	Standard Dressed Size (S4S) ($b \times d$) (in. × in.)	Area of Section A (in.^{2})	*x–x* Axis: Section Modulus S_{xx} (in.^{3})	*x–x* Axis: Moment of Inertia I_{xx} (in.^{4})	*y–y* Axis: Section Modulus S_{yy} (in.^{3})	*y–y* Axis: Moment of Inertia I_{yy} (in.^{4})	Approximate Weight in Pounds per Linear Foot (lb/ft) of Piece When Density of Wood Equals: 25 lbs/ft^3	30 lbs/ft^3	35 lbs/ft^3	40 lbs/ft^3	45 lbs/ft^3	50 lbs/ft^3
18 × 18	17 × 17	289.0	818.8	6,960	818.8	6,960	50.17	60.21	70.24	80.28	90.31	100.3
18 × 20	17 × 19	323.0	1,023	9,717	915.2	7,779	56.08	67.29	78.51	89.72	100.9	112.2
20 × 20	19 × 19	361.0	1,143	10,860	1,143	10,860	62.67	75.21	87.7	100.3	112.8	125.3
20 × 22	19 × 21	399.0	1,397	14,663	1,264	12,003	69.27	83.13	97.0	110.8	124.7	138.5
22 × 22	21 × 21	441.0	1,544	16,207	1,544	16,207	76.56	91.88	107.2	122.5	137.8	153.1
22 × 24	21 × 23	483.0	1,852	21,292	1,691	17,750	83.85	100.6	117.4	134.2	150.9	167.7
24 × 24	23 × 23	529.0	2,028	23,320	2,028	23,320	91.84	110.2	128.6	146.9	165.3	183.7
Beams and Stringers (see NDS 4.1.3.3)												
6 × 10	5 ½ × 9 ¼	50.88	78	363	47	128	8.83	10.6	12.4	14.1	15.9	17.7
6 × 12	5 ½ × 11 ¼	61.88	116.0	653	56.72	156.0	10.74	12.89	15.04	17.19	19.34	21.48
6 × 14	5 ½ × 13 ¼	72.88	160.9	1,066	66.80	183.7	12.65	15.18	17.71	20.24	22.77	25.30
6 × 16	5 ½ × 15	82.50	206.3	1,547	75.63	208.0	14.32	17.19	20.05	22.92	25.78	28.65
6 × 18	5 ½ × 17	93.50	264.9	2,252	85.71	235.7	16.23	19.48	22.73	25.97	29.22	32.47
6 × 20	5 ½ × 19	104.5	330.9	3,144	95.79	263.4	18.14	21.77	25.40	29.03	32.66	36.28
6 × 22	5 ½ × 21	115.5	404.3	4,245	105.9	291.2	20.05	24.06	28.07	32.08	36.09	40.10
6 × 24	5 ½ × 23	126.5	484.9	5,577	116.0	318.9	21.96	26.35	30.75	35.14	39.53	43.92
8 × 12	7 ½ × 11 ¼	81.6	152.9	860.2	98.6	357.3	14.16	16.99	19.82	22.66	25.49	28.32
8 × 14	7 ¼ × 13 ¼	96.1	212.1	1,405	116.1	420.8	16.68	20.01	23.35	26.68	30.02	33.36
8 × 16	7 ¼ × 15	108.8	271.9	2,039	131.4	476.3	18.88	22.66	26.43	30.21	33.98	37.76
8 × 18	7 ¼ × 17	123.3	349.2	2,968	148.9	539.9	21.40	25.68	29.96	34.24	38.52	42.80
8 × 20	7 ¼ × 19	137.8	436.2	4,144	166.4	603.4	23.91	28.70	33.48	38.26	43.05	47.83
8 × 22	7 ¼ × 21	152.3	532.9	5,595	184.0	666.9	26.43	31.72	37.01	42.29	47.58	52.86
8 × 24	7 ¼ × 23	166.8	639.2	7,351	201.5	730.4	28.95	34.74	40.53	46.32	52.11	57.90
10 × 14	9 ¼ × 13 ¼	122.6	270.7	1,793	189.0	873.9	21.28	25.53	29.79	34.05	38.30	42.56

(*Continued*)

APPENDIX B.1 (*Continued*)
Section Properties of Standard Dressed (S4S) Sawn Lumber

			x–x Axis		y–y Axis		Approximate Weight in Pounds per Linear Foot (lb/ft) of Piece When Density of Wood Equals					
Nominal Size ($b \times d$)	Standard Dressed Size (S4S) ($b \times d$) (in. × in.)	Area of Section A (in.^{2})	Section Modulus S_{xx} (in.^{3})	Moment of Inertia I_{xx} (in.^{4})	Section Modulus S_{yy} (in.^{3})	Moment of Inertia I_{yy} (in.^{4})	25 lbs/ft^3	30 lbs/ft^3	35 lbs/ft^3	40 lbs/ft^3	45 lbs/ft^3	50 lbs/ft^3
10 × 16	9 ¼ × 15	138.8	346.9	2,602	213.9	989	24.09	28.91	33.72	38.54	43.36	48.18
10 × 18	9 ¼ × 17	157.3	445.5	3,787	242.4	1,121	27.30	32.76	38.22	43.68	49.14	54.60
10 × 20	9 ¼ × 19	175.8	556.5	5,287	270.9	1,253	30.51	36.61	42.72	48.82	54.92	61.02
10 × 22	9 ¼ × 21	194.3	679.9	7,139	299.5	1,385	33.72	40.47	47.21	53.96	60.70	67.45
10 × 24	9 ¼ × 23	212.8	815.5	9,379	328.0	1,517	36.94	44.32	51.71	59.10	66.48	73.87
12 × 16	11 ¼ × 15	168.8	421.9	3,164	316.4	1,780	29.30	35.16	41.02	46.88	52.73	58.59
12 × 18	11 ¼ × 17	191.3	541.9	4,606	358.6	2,017	33.20	39.84	46.48	53.13	59.77	66.41
12 × 20	11 ¼ × 19	213.8	676.9	6,430	400.8	2,254	37.11	44.53	51.95	59.38	66.80	74.22
12 × 22	11 ¼ × 21	236.3	826.9	8,682	443.0	2,492	41.02	49.22	57.42	65.63	73.83	82.03
12 × 24	11 ¼ × 23	258.8	992	11,407	485.2	2,729	44.92	53.91	62.89	71.88	80.86	89.84
14 × 18	13 ¼ × 17	225.3	638.2	5,425	497.4	3,295	39.11	46.93	54.75	62.57	70.39	78.21
14 × 20	13 ¼ × 19	251.8	797.2	7,573	555.9	3,683	43.71	52.45	61.19	69.93	78.67	87.41
14 × 22	13 ¼ × 21	278.3	974	10,226	614.5	4,071	48.31	57.97	67.63	77.29	86.95	96.6
14 × 24	13 ¼ × 23	304.8	1,168	13,434	673.0	4,459	52.91	63.49	74.07	84.65	95.23	105.8
16 × 20	15 × 19	285.0	902.5	8,574	712.5	5,344	49.48	59.38	69.27	79.17	89.06	99.0
16 × 22	15 × 21	315.0	1,103	11,576	787.5	5,906	54.69	65.63	76.56	87.50	98.4	109.4
16 × 24	15 × 23	345.0	1,323	15,209	862.5	6,469	59.90	71.88	83.85	95.8	107.8	119.8
18 × 22	17 × 21	357.0	1,250	13,120	1,012	8,598	61.98	74.38	86.77	99.2	111.6	124.0
18 × 24	17 × 23	391.0	1,499	17,237	1,108	9,417	67.88	81.46	95.03	108.6	122.2	135.8
20 × 24	19 × 23	437.0	1,675	19,264	1,384	13,146	75.87	91.04	106.2	121.4	136.6	151.7

Source: Courtesy of the American Wood Council.
Note: NDS, National Design Specification.

APPENDIX B.2
Size Factor, Wet Service Factor, and Flat Use Factor (All Species Except Southern Pine)

Flat Use Factor, C_{fu}

Bending design values adjusted by size factors are based on edgewise use (load applied to narrow face). When dimension lumber is used flatwise (load applied to wide face), the bending design value, F_b, shall also be multiplied by the following flat use factors:

	Thickness (Breadth)	
Width (Depth)	**2 in. and 3 in.**	**4 in.**
2 in. and 3 in.	1.0	—
4 in.	1.1	1.0
5 in.	1.1	1.05
6 in.	1.15	1.05
8 in.	1.15	1.05
10 in. and wider	1.2	1.1

Size Factor, C_F

Tabulated bending, tension, and compression parallel to grain design values for dimension lumber 2–4 in. thick shall be multiplied by the following size factors:

		F_b Thickness (Breadth)			
Grades	**Width (Depth)**	**2 in. and 3 in.**	**4 in.**	F_t	F_c
Select structural, No. 1 and Btr, No. 1, No. 2, No. 3	2 in., 3 in., and 4 in.	1.5	1.5	1.5	1.15
	5 in.	1.4	1.4	1.4	1.1
	6 in.	1.3	1.3	1.3	1.1
	8 in.	1.2	1.3	1.2	1.05
	10 in.	1.1	1.2	1.1	1.0
	12 in.	1.0	1.1	1.0	1.0
	14 in. and wider	0.9	1.0	0.9	0.9
Stud	2 in., 3 in., and 4 in.	1.1	1.1	1.1	1.05
	5 in. and 6 in.	1.0	1.0	1.0	1.0
	8 in. and wider	Use No. 3 Grade tabulated design values and size factors.			
Construction, standard	2 in., 3 in., and 4 in.	1.0	1.0	1.0	1.0
Utility	4 in.	1.0	1.0	1.0	1.0
	2 in. and 3 in.	0.4	—	0.4	0.6

Wet Service Factor, C_M

F_b	F_t	F_v	$F_{c\perp}$	F_c	E and E_{min}
0.85[a]	1.0	0.97	0.67	0.8[b]	0.9

[a] When $(F_b)(C_F) \le 1150$ psi, $C_M = 1$.
[b] When $(F_c)(C_F) \le 750$ psi, $C_M = 1$.

APPENDIX B.2

Reference Design Values for Visually Graded Dimension Lumber (2–4 in. Breadth) (All Species Except Southern Pine)

Species and Commercial Grade	Size Classification	Design Values in Pounds per Square Inch (psi): Bending F_b	Tension Parallel to Grain F_t	Shear Parallel to Grain F_v	Compression Perpendicular to Grain $F_{c\perp}$	Compression Parallel to Grain F_c	Modulus of Elasticity: E	Modulus of Elasticity: E_{min}	Grading Rules Agency
Beech-Birch-Hickory									
Select structural	2 in. and wider	1,450	850	195	715	1,200	1,700,000	620,000	NELMA
No. 1		1,050	600	195	715	950	1,600,000	580,000	
No. 2		1,000	600	195	715	750	1,500,000	550,000	
No. 3		575	350	195	715	425	1,300,000	470,000	
Stud	2 in. and wider	775	450	195	715	475	1,300,000	470,000	
Construction	2–4 in. wide	1,150	675	195	715	1,000	1,400,000	510,000	
Standard		650	375	195	715	775	1,300,000	470,000	
Utility		300	175	195	715	500	1,200,000	440,000	
Cottonwood									
Select structural	2 in. and wider	875	525	125	320	775	1,200,000	440,000	NSLB
No. 1		625	375	125	320	625	1,200,000	440,000	
No. 2		625	350	125	320	475	1,100,000	400,000	
No. 3		350	200	125	320	275	1,000,000	370,000	
Stud	2 in. and wider	475	275	125	320	300	1,000,000	370,000	
Construction	2–4 in. wide	700	400	125	320	650	1,000,000	370,000	
Standard		400	225	125	320	500	900,000	330,000	
Utility		175	100	125	320	325	900,000	330,000	
Douglas Fir-Larch									
Select structural	2 in. and wider	1,500	1,000	180	625	1,700	1,900,000	690,000	WCLIB
No. 1 and Btr		1,200	800	180	625	1,550	1,800,000	660,000	WWPA
No. 1		1,000	675	180	625	1,500	1,700,000	620,000	
No. 2		900	575	180	625	1,350	1,600,000	580,000	
No. 3		525	325	180	625	775	1,400,000	510,000	
Stud	2 in. and wider	700	450	180	625	850	1,400,000	510,000	

(*Continued*)

APPENDIX B.2 (*Continued*)

Reference Design Values for Visually Graded Dimension Lumber (2–4 in. Breadth) (All Species Except Southern Pine)

Species and Commercial Grade	Size Classification	Design Values in Pounds per Square Inch (psi): Bending F_b	Tension Parallel to Grain F_t	Shear Parallel to Grain F_v	Compression Perpendicular to Grain $F_{c\perp}$	Compression Parallel to Grain F_c	Modulus of Elasticity: E	E_{min}	Grading Rules Agency
Construction	2–4 in. wide	1,000	650	180	625	1,650	1,500,000	550,000	
Standard		575	375	180	625	1,400	1,400,000	510,000	
Utility		275	175	180	625	900	1,300,000	470,000	
Douglas Fir-Larch (North)									
Select structural	2 in. and wider	1,350	825	180	625	1,900	1,900,000	690,000	NLGA
No. 1 and Btr		1,150	750	180	625	1,800	1,800,000	660,000	
No. 1/No. 2		850	500	180	625	1,400	1,600,000	580,000	
No. 3		475	300	180	625	825	1,400,000	510,000	
Stud	2 in. and wider	650	400	180	625	900	1,400,000	510,000	
Construction	2–4 in. wide	950	575	180	625	1,800	1,500,000	550,000	
Standard		525	325	180	625	1,450	1,400,000	510,000	
Utility		250	150	180	625	950	1,300,000	470,000	
Douglas Fir (South)									
Select structural	2 in. and wider	1,350	900	180	520	1,600	1,400,000	510,000	WWPA
No. 1		925	600	180	520	1,450	1,300,000	470,000	
No. 2		850	525	180	520	1,350	1,200,000	440,000	
No. 3		500	300	180	520	775	1,100,000	400,000	
Stud	2 in. and wider	675	425	180	520	850	1,100,000	400,000	
Construction	2–4 in. wide	975	600	180	520	1,650	1,200,000	440,000	
Standard		550	350	180	520	1,400	1,100,000	400,000	
Utility		250	150	180	520	900	1,000,000	370,000	
Eastern Hemlock-Balsam Fir									
Select structural	2 in. and wider	1,250	575	140	335	1,200	1,200,000	440,000	NELMA
No. 1		775	350	140	335	1,000	1,100,000	400,000	NSLB

(*Continued*)

APPENDIX B.2 (*Continued*)

Reference Design Values for Visually Graded Dimension Lumber (2–4 in. Breadth) (All Species Except Southern Pine)

Species and Commercial Grade	Size Classification	Design Values in Pounds per Square Inch (psi): Bending F_b	Tension Parallel to Grain F_t	Shear Parallel to Grain F_v	Compression Perpendicular to Grain $F_{c\perp}$	Compression Parallel to Grain F_c	Modulus of Elasticity: E	E_{min}	Grading Rules Agency
No. 2		575	275	140	335	825	1,100,000	400,000	
No. 3		350	150	140	335	475	900,000	330,000	
Stud	2 in. and wider	450	200	140	335	525	900,000	330,000	
Construction	2–4 in. wide	675	300	140	335	1,050	1,000,000	370,000	
Standard		375	175	140	335	850	900,000	330,000	
Utility		175	75	140	335	550	800,000	290,000	
Eastern Hemlock-Tamarack									
Select structural	2 in. and wider	1,250	575	170	555	1,200	1,200,000	440,000	NELMA
No. 1		775	350	170	555	1,000	1,100,000	400,000	NSLB
No. 2		575	275	170	555	825	1,100,000	400,000	
No. 3		350	150	170	555	475	900,000	330,000	
Stud	2 in. and wider	450	200	170	555	525	900,000	330,000	
Construction	2–4 in. wide	675	300	170	555	1,050	1,000,000	370,000	
Standard		375	175	170	555	850	900,000	330,000	
Utility		175	75	170	555	550	800,000	290,000	
Eastern Softwoods									
Select structural	2 in. and wider	1,250	575	140	335	1,200	1,200,000	440,000	NELMA
No. 1		775	350	140	335	1,000	1,100,000	400,000	NSLB
No. 2		575	275	140	335	825	1,100,000	400,000	
No. 3		350	150	140	335	475	900,000	330,000	
Stud	2 in. and wider	450	200	140	335	525	900,000	330,000	
Construction	2–4 in. wide	675	300	140	335	1,050	1,000,000	370,000	
Standard		375	175	140	335	850	900,000	330,000	
Utility		175	75	140	335	550	800,000	290,000	

(*Continued*)

APPENDIX B.2 (*Continued*)

Reference Design Values for Visually Graded Dimension Lumber (2–4 in. Breadth) (All Species Except Southern Pine)

Species and Commercial Grade	Size Classification	Design Values in Pounds per Square Inch (psi): Bending F_b	Tension Parallel to Grain F_t	Shear Parallel to Grain F_v	Compression Perpendicular to Grain $F_{c\perp}$	Compression Parallel to Grain F_c	Modulus of Elasticity: E	E_{min}	Grading Rules Agency
Eastern White Pine									
Select structural	2 in. and wider	1,250	575	135	350	1,200	1,200,000	440,000	NELMA
No. 1		775	350	135	350	1,000	1,100,000	400,000	NSLB
No. 2		575	275	135	350	825	1,100,000	400,000	
No. 3		350	150	135	350	475	900,000	330,000	
Stud	2 in. and wider	450	200	135	350	525	900,000	330,000	
Construction	2–4 in. wide	675	300	135	350	1,050	1,000,000	370,000	
Standard		375	175	135	350	850	900,000	330,000	
Utility		175	75	135	350	550	800,000	290,000	
Hem-Fir									
Select structural	2 in. and wider	1,400	925	150	405	1,500	1,600,000	580,000	WCLIB
No. 1 and Btr		1,100	725	150	405	1,350	1,500,000	550,000	WWPA
No. 1		975	625	150	405	1,350	1,500,000	550,000	
No. 2		850	525	150	405	1,300	1,300,000	470,000	
No. 3		500	300	150	405	725	1,200,000	440,000	
Stud	2 in. and wider	675	400	150	405	800	1,200,000	440,000	
Construction	2–4 in. wide	975	600	150	405	1,550	1,300,000	470,000	
Standard		550	325	150	405	1,300	1,200,000	440,000	
Utility		250	150	150	405	850	1,100,000	400,000	
Hem-Fir (North)									
Select structural	2 in. and wider	1,300	775	145	405	1,700	1,700,000	620,000	NLGA
No. 1 and Btr		1,200	725	145	405	1,550	1,700,000	620,000	
No. 1/No. 2		1,000	575	145	405	1,450	1,600,000	580,000	
No. 3		575	325	145	405	850	1,400,000	510,000	

(*Continued*)

APPENDIX B.2 (*Continued*)
Reference Design Values for Visually Graded Dimension Lumber (2–4 in. Breadth) (All Species Except Southern Pine)

Species and Commercial Grade	Size Classification	Design Values in Pounds per Square Inch (psi)							Grading Rules Agency
		Bending F_b	Tension Parallel to Grain F_t	Shear Parallel to Grain F_v	Compression Perpendicular to Grain $F_{c\perp}$	Compression Parallel to Grain F_c	Modulus of Elasticity E	Modulus of Elasticity E_{min}	
Stud	2 in. and wider	775	450	145	405	925	1,400,000	510,000	
Construction	2–4 in. wide	1,150	650	145	405	1,750	1,500,000	550,000	
Standard		650	350	145	405	1,500	1,400,000	510,000	
Utility		300	175	145	405	975	1,300,000	470,000	
Mixed Maple									
Select structural	2 in. and wider	1,000	600	195	620	875	1,300,000	470,000	NELMA
No. 1		725	425	195	620	700	1,200,000	440,000	
No. 2		700	425	195	620	550	1,100,000	400,000	
No. 3		400	250	195	620	325	1,000,000	370,000	
Stud	2 in. and wider	550	325	195	620	350	1,000,000	370,000	
Construction	2–4 in. wide	800	475	195	620	725	1,100,000	400,000	
Standard		450	275	195	620	575	1,000,000	370,000	
Utility		225	125	195	620	375	900,000	330,000	

Source: Courtesy of the American Forest & Paper Association, Washington, DC.

Notes: Tabulated design values are for normal load duration and dry service conditions. See NDS 4.3 for a comprehensive description of design value adjustment factors.

APPENDIX B.3
Size Factor, Wet Service Factor, and Flat Use Factor for Southern Pine

Flat Use Factor, C_{fu}

Bending design values adjusted by size factors are based on edgewise use (load applied to narrow face). When dimension lumber is used flatwise (load applied to wide face), the bending design value, F_b, shall also be multiplied by the following flat use factors:

	Thickness (Breadth)	
Width (Depth)	**2 in. and 3 in.**	**4 in.**
2 in. and 3 in.	1.0	—
4 in.	1.1	1.0
5 in.	1.1	1.05
6 in.	1.15	1.05
8 in.	1.15	1.05
10 in. and wider	1.2	1.1

Size Factor, C_F

Appropriate size adjustment factors have already been included in the tabular design values of Southern Pine and mixed Southern Pine dimension lumber, except the following cases:

Grade	Size	F_b	F_c	F_t	F_v, $F_{c\perp}$, E, E_{min}
All grades (except Dense Structural 86, Dense Structural 72, Dense Structural 65)	(1) For 4-in. breadth × 8 in. or more depth	1.1			
	(2) For all dimension lumber >12-in. depth, the table values of 12-in. depth multiplied as shown across	0.9	0.9	0.9	1.00
Dense Structural 86, Dense Structural 72, Dense Structural 65	For dimension lumber >12-in. depth, F_b table value of 12 in. multiplied as shown across	$(12/d)^{1/9}$			

Wet Service Factor, C_M

F_b	F_t	F_v	$F_{c\perp}$	F_c	E and E_{min}
0.85[a]	1.0	0.97	0.67	0.8[b]	0.9

[a] When $(F_b)(C_F) \leq 1150$ psi, $C_M = 1$.

[b] When $(F_c)(C_F) \leq 750$ psi, $C_M = 1$.

APPENDIX B.3
Reference Design Values for Visually Graded Southern Pine Dimension Lumber (2″—4″ Thick)

Species and Commercial Grade	Size Classification	Design Values in Pounds per Square Inch (psi)							Specific Gravity[6] G	Grading Rules Agency
		Bending F_b	Tension Parallel to Grain F_t	Shear Parallel to Grain F_v	Compression Perpendicular to Grain $F_{c\perp}$	Compression Parallel to Grain F_c	Modulus of Elasticity E	E_{min}		
Southern Pine										
Dense select structural		2,700	1,900	175	660	2,050	1,900,000	690,000		
Select structural		2,350	1,650	175	565	1,900	1,800,000	660,000		
Non-dense select structural		2,050	1,450	175	480	1,800	1,600,000	580,000		
No. 1 Dense		1,650	1,100	175	660	1,750	1,800,000	660,000		
No. 1	2"–4" wide	1,500	1,000	175	565	1,650	1,600,000	580,000	0.55	
No. 1 Non-dense		1,300	875	175	480	1,550	1,400,000	510,000		
No. 2 Dense		1,200	750	175	660	1,500	1,600,000	580,000		
No. 2		1,100	675	175	565	1,450	1,400,000	510,000		
No. 2 Non-dense		1,050	600	175	480	1,450	1,300,000	470,000		
No. 3 and stud		650	400	175	565	850	1,300,000	470,000		
Construction		875	500	175	565	1,600	1,400,000	510,000		
Standard	4" wide	475	275	175	565	1,300	1,200,000	440,000	0.55	
Utility		225	125	175l	565	850	1,200,000	440,000		
Dense select structural		2,400	1,650	175	660	1,900	1,900,000	690,000		
Select structural		2,100	1,450	175	565	1,800	1,800,000	660,000		
Non-dense select structural		1,850	1,300	175	480	1,700	1,600,000	580,000		
No. 1 Dense		1,500	1,000	175	660	1,650	1,800,000	660,000		
No. 1	5"–6" wide	1,350	875	175	565	1,550	1,600,000	580,000	0.55	
No. 1 Non-dense		1,200	775	175	480	1,450	1,400,000	510,000		
No. 2 Dense		1,050	650	175	660	1,450	1,600,000	580,000		
No. 2		1,000	600	175	565	1,400	1,400,000	510,000		
No. 2 Non-dense		950	525	175	480	1,350	1,300,000	470,000		
No. 3 and stud		575	350	175	565	800	1,300,000	470,000		

(*Continued*)

APPENDIX B.3 (*Continued*)

Reference Design Values for Visually Graded Southern Pine Dimension Lumber (2″—4″ Thick)

Species and Commercial Grade	Size Classification	Design Values in Pounds per Square Inch (psi) Bending F_b	Tension Parallel to Grain F_t	Shear Parallel to Grain F_v	Compression Perpendicular to Grain $F_{c\perp}$	Compression Parallel to Grain F_c	Modulus of Elasticity E	E_{min}	Specific Gravity[6] G	Grading Rules Agency
Dense select structural		2,200	1,550	175	660	1,850	1,900,000	690,000		
Select structural		1,950	1,350	175	565	1,700	1,800,000	660,000		
Non-dense select structural		1,700	1,200	175	480	1,650	1600,000	580,000		
No. 1 Dense		1,350	900	175	660	1,600	1,800,000	660,000		SPIB
No. 1	8" wide	1,250	S00	175	565	1,500	1,600,000	580,000	0.55	
No. 1 Non-dense		1,100	700	175	480	1,400	1,400,000	510,000		
No. 2 Dense		975	600	175	660	1,400	1,600,000	580,000		
No. 2		925	550	175	565	1,350	1,400,000	510,000		
No. 2 Non-dense		875	500	175	480	1,300	1,300,000	470,000		
No. 3 and stud		525	325	175	565	775	1,300,000	470,000		
Dense select structural		1,950	1,300	175	660	1,800	1,900,000	690,000		
Select structural		1,700	1,150	175	565	1,650	1,800,000	660,000		
Non-dense select structural		1,500	1,050	175	480	1,600	1,600,000	580,000		
No. 1 Dense		1,200	800	175	660	1,550	1,800,000	660,000		
No. 1	10" wide	1,050	700	175	565	1,450	1,600,000	580,000	0.55	
No. 1 Non-dense		950	625	175	480	1,400	1,400,000	510,000		
No. 2 Dense		850	525	175	660	1,350	1,600,000	580,000		
No. 2		800	475	175	565	1,300	1,400,000	510,000		
No. 2 Non-dense		750	425	175	480	1,250	1,300,000	470,000		
No. 3 and stud		475	275	175	565	750	1,300,000	470,000		
Dense select structural		1,600	1,250	175	660	1,750	1,900,000	690,000		
Select structural		1,600	1,100	175	565	1,650	1,800,000	660,000		
Non-dense select structural		1,400	975	175	480	1,550	1,600,000	580,000		
No. 1 Dense		1,100	750	175	660	1,500	1,800,000	660,000		

(*Continued*)

APPENDIX B.3 (*Continued*)

Reference Design Values for Visually Graded Southern Pine Dimension Lumber (2″—4″ Thick)

Species and Commercial Grade	Size Classification	Design Values in Pounds per Square Inch (psi): Bending F_b	Tension Parallel to Grain F_t	Shear Parallel to Grain F_v	Compression Perpendicular to Grain $F_{c\perp}$	Compression Parallel to Grain F_c	Modulus of Elasticity: E	Modulus of Elasticity: E_{min}	Specific Gravity[6] G	Grading Rules Agency
No. 1	12" wide	1,000	650	175	565	1,400	1,600,000	580,000	0.55	
No. 1 Non-dense		900	575	175	480	1,350	1,400,000	510,000		
No. 2 Dense		800	500	175	660	1,300	1,600,000	580,000		
No. 2		750	450	175	565	1,250	1,400,000	510,000		
No. 2 Non-dense		700	400	175	480	1,250	1,300,000	470.000		
No. 3 and stud		450	250	175	565	725	1,300,000	470,000		
Southern Pine (Surfaced Dry—Used in Dry Service Condtions—19% or Less Moisture Content)										
Dense structural 86	2" & wider	2,600	1.750	175	660	2,000	1,800,000	660,000		
Dense structural 72		2,200	1,450	175	660	1,650	1,800,000	660,000	0.55	SPIB
Dense structural 65		2,000	1,300	175	660	1,500	1,800,000	660,000		
Southern Pine (Surfaced Green—Used in Any Service Condtion)										
Dense structural 86		2,100	1,400	165	440	1,300	1,600,000	580,000		
Dense structural 72	2-1/2" & wider	1,750	1,200	165	440	1,100	1,600,000	580,000	0.55	SPIB
Dense structural 65	2-1/2n-4" thick	1,600	1,050	165	440	1,000	1,600,000	580,000		
Mixed Southern Pine										
Select structural		2,050	1,200	175	565	1,800	1,600,000	580.000		
No. 1	2" –4" wide	1,450	875	175	565	1,650	1,500,000	550,000	0.51	
No. 2		1,100	675	175	565	1,450	1,400,000	510.000		
No. 3 and stud		650	400	175	565	850	1,200,000	440,000		
Construction		850	500	175	565	1,600	1,300,000	470,000		
Standard	4" wide	475	275	175	565	1.300	1,200,000	440,000	0.51	
Utility		225	125	175	565	850	1,100,000	400,000		

(*Continued*)

APPENDIX B.3 (*Continued*)

Reference Design Values for Visually Graded Southern Pine Dimension Lumber (2″—4″ Thick)

Species and Commercial Grade	Size Classification	Design Values in Pounds per Square Inch (psi): Bending F_b	Tension Parallel to Grain F_t	Shear Parallel to Grain F_v	Compression Perpendicular to Grain F_c	Compression Parallel to Grain F_c	Modulus of Elasticity: E	Modulus of Elasticity: E_{min}	Specific Gravity[6] G	Grading Rules Agency
Select structural		1,850	1,100	175	565	1,700	1,600,000	580,000		
No. 1	5" –6" wide	1,300	750	175	565	1,550	1.500,000	550,000	0.51	
No. 2		1,000	600	175	565	1,400	1,400,000	510,000		
No. 3 and stud		575	350	175	565	775	1,200,000	440,000		
Select structural		1,750	1,000	175	565	1,600	1,600,000	580,000		SPIB
No. 1	8" wide	1,200	700	175	565	1,450	1,500,000	550,000	0.51	
No. 2		925	550	175	565	1,350	1,400,000	510,000		
No. 3 and stud		525	325	175	565	800	1,200,000	440,000		
Select structural		1,500	875	175	565	1,600	1,600,000	580.000		
No. 1	10" wide	1,050	600	175	565	1,450	1,500,000	550,000	0.51	
No. 2		800	475	175	565	1,300	1,400,000	510,000		
No. 3 and stud		475	275	175	565	750	1,200,000	440,000		
Select structural	12" wide	1,400	825	175	565	1,550	1,600,000	580,000	0.51	
No. 1		975	575	175	565	1,400	1,500,000	550,000		
No. 2		750	450	175	565	1,250	1,400,000	510,000		
No. 3 and stud		450	250	175	565	725	1,200,000	440,000		

Spruce Pine.

To obtain recommended design values for Spruce Pine graded to SPIB rules, multiply the appropriate design values for Mixed Southern Pine by the corresponding conversion factor shown below and round to the nearest 100,000 psi for E; to the nearest 10,000 psi for E; to the next lower multiple of 5 psi for F_v and $F_{c\perp}$; to the next lower multiple of 50 psi for F_b, F_t, and F_c if 1,000 psi or greater, 25 psi otherwise.

Conversion Factors for Determining Design Values for Spruce Pine

	Bending	Tension Parallel to Grain	Shear Parallel to Grain	Compression Perpendicular to Grain	Compression Parallel to Grain	Modulus of Elasticity
	F_b	F_t	F_v	F	F_c	E and E_{min}
Conversion factor	0.78	0.78	0.98	0.73	0.78	0.82

APPENDIX B.4
Size Factor, Wet Service Factor, and Flat Use Factor for Timbers

Size Factor, C_F

When visually graded timbers are subjected to loads applied to the narrow face, tabulated design values shall be multiplied by the following size factors:

Depth	F_b	F_t	F_c
$d > 12$ in.	$(12/d)^{1/9}$	1.0	1.0
$d \leq 12$ in.	1.0	1.0	1.0

Flat Use Factor, C_{fu}

When members designated as Beams and Stringers in Appendix B.4 are subjected to loads applied to the wide face, tabulated design values shall be multiplied by the following flat use factors:

Grade	F_b	E and E_{min}	Other Properties
Select structural	0.86	1.00	1.00
No. 1	0.74	0.90	1.00
No. 2	1.00	1.00	1.00

Wet Service Factor, C_M

F_b	F_t	F_v	$F_{c\perp}$	F_c	E and E_{min}
1.0	1.0	1.0	0.67	0.91	1.0

APPENDIX B.4
Reference Design Values for Visually Graded Timbers (5 × 5 in. and Larger)

Species and Commercial Grade	Size Classification	Design Values in Pounds per Square Inch (psi): Bending F_b	Tension Parallel to Grain F_t	Shear Parallel to Grain F_v	Compression Perpendicular to Grain $F_{c\perp}$	Compression Parallel to Grain F_c	Modulus of Elasticity: E	E_{min}	Specific Gravity G	Grading Rules Agency
Beech-Birch-Hickory										
Select structural	Beams and stringers	1,650	975	180	715	975	1,500,000	550,000	0.71	NELMA
No. 1		1,400	700	180	715	825	1,500,000	550,000		NSLB
No. 2		900	450	180	715	525	1,200,000	440,000		
Select structural	Posts and timbers	1,550	1,050	180	715	1,050	1,500,000	550,000		
No. 1		1,250	850	180	715	900	1,500,000	550,000		
No. 2		725	475	180	715	425	1,200,000	440,000		
Coast Sitka Spruce										
Select structural	Beams and stringers	1,150	675	115	455	775	1,500,000	550,000	0.43	NLGA
No. 1		950	475	115	455	650	1,500,000	550,000		
No. 2		625	325	115	455	425	1,200,000	440,000		
Select structural	Posts and timbers	1,100	725	115	455	825	1,500,000	550,000		
No. 1		875	575	115	455	725	1,500,000	550,000		
No. 2		525	350	115	455	500	1,200,000	440,000		
Douglas Fir-Larch										
Dense select structural	Beams and stringers	1,900	1,100	170	730	1,300	1,700,000	620,000	0.50	WWPA
Select structural		1,600	950	170	625	1,100	1,600,000	580,000		
No. 1 Dense		1,550	775	170	730	1,100	1,700,000	620,000		
No. 1		1,350	675	170	625	925	1,600,000	580,000		
No. 2 Dense		1,000	500	170	730	700	1,400,000	510,000		
No. 2		875	425	170	625	600	1,300,000	470,000		

(*Continued*)

APPENDIX B.4 (*Continued*)

Reference Design Values for Visually Graded Timbers (5 × 5 in. and Larger)

Species and Commercial Grade	Size Classification	Design Values in Pounds per Square Inch (psi): Bending F_b	Tension Parallel to Grain F_t	Shear Parallel to Grain F_v	Compression Perpendicular to Grain $F_{c\perp}$	Compression Parallel to Grain F_c	Modulus of Elasticity E	Modulus of Elasticity E_{min}	Specific Gravity G	Grading Rules Agency
Dense select structural	Posts and timbers	1,750	1,150	170	730	1,350	1,700,000	620,000		
Select structural		1,500	1,000	170	625	1,150	1,600,000	580,000		
Dense No. 1		1,400	950	170	730	1,200	1,700,000	620,000		
No. 1		1,200	825	170	625	1,000	1,600,000	580,000		
No. 2 Dense		850	550	170	730	825	1,400,000	510,000		
No. 2		750	475	170	625	700	1,300,000	470,000		
Douglas Fir-Larch (North)										
Select structural	Beams and stringers	1,600	950	170	625	1,100	1,600,000	580,000	0.49	NLGA
No. 1		1,300	675	170	625	925	1,600,000	580,000		
No. 2		875	425	170	625	600	1,300,000	470,000		
Select structural	Posts and timbers	1,500	1,000	170	625	1,150	1,600,000	580,000		
No. 1		1,200	825	170	625	1,000	1,600,000	580,000		
No. 2		725	475	170	625	700	1,300,000	470,000		
Douglas Fir-South										
Select structural	Beams and stringers	1,550	900	165	520	1,000	1,200,000	440,000	0.46	WWPA
No. 1		1,300	625	165	520	850	1,200,000	440,000		
No. 2		825	425	165	520	550	1,000,000	370,000		
Select structural	Posts and timbers	1,450	950	165	520	1,050	1,200,000	440,000		
No. 1		1,150	775	165	520	925	1,200,000	440,000		
No. 2		675	450	165	520	650	1,000,000	370,000		

(*Continued*)

APPENDIX B.4 (*Continued*)
Reference Design Values for Visually Graded Timbers (5 × 5 in. and Larger)

Species and Commercial Grade	Size Classification	Design Values in Pounds per Square Inch (psi): Bending F_b	Tension Parallel to Grain F_t	Shear Parallel to Grain F_v	Compression Perpendicular to Grain $F_{c\perp}$	Compression Parallel to Grain F_c	Modulus of Elasticity E	Modulus of Elasticity E_{min}	Specific Gravity G	Grading Rules Agency
Eastern Hemlock										
Select structural	Beams and stringers	1,350	925	155	550	950	1,200,000	440,000	0.41	NELMA
No. 1		1,150	775	155	550	800	1,200,000	440,000		NSLB
No. 2		750	375	155	550	550	900,000	330,000		
Select structural	Posts and timbers	1,250	850	155	550	1,000	1,200,000	440,000		
No. 1		1,050	700	155	500	875	1,200,000	440,000		
No. 2		600	400	155	550	400	900,000	330,000		
Eastern Hemlock-Tamarack										
Select structural	Beams and stringers	1,400	925	155	555	950	1,200,000	440,000	0.41	NELMA
No. 1		1,150	775	155	555	800	1,200,000	440,000		NSLB
No. 2		750	375	155	555	500	900,000	330,000		
Select structural	Posts and timbers	1,300	875	155	555	1,000	1,200,000	440,000		
No. 1		1,050	700	155	555	875	1,200,000	440,000		
No. 2		600	400	155	555	400	900,000	330,000		
Eastern Spruce										
Select structural	Beams and stringers	1,050	725	135	390	750	1,400,000	510,000	0.41	NELMA
No. 1		900	600	135	390	625	1,400,000	510,000		NSLB
No. 2		575	275	135	390	375	1,000,000	370,000		
Select structural	Posts and timbers	1,000	675	135	390	775	1,400,000	510,000		
No. 1		800	550	135	390	675	1,400,000	510,000		
No. 2		450	300	135	390	300	1,000,000	370,000		

(*Continued*)

APPENDIX B.4 (*Continued*)
Reference Design Values for Visually Graded Timbers (5 × 5 in. and Larger)

Species and Commercial Grade	Size Classification	Design Values in Pounds per Square Inch (psi): Bending F_b	Tension Parallel to Grain F_t	Shear Parallel to Grain F_v	Compression Perpendicular to Grain $F_{c\perp}$	Compression Parallel to Grain F_c	Modulus of Elasticity: E	E_{min}	Specific Gravity G	Grading Rules Agency
Eastern White Pine										
Select structural	Beams and stringers	1,050	700	125	350	675	1,100,000	400,000	0.36	NELMA
No. 1		875	600	125	350	575	1,100,000	400,000		NSLB
No. 2		575	275	125	350	400	900,000	330,000		
Select structural	Posts and timbers	975	650	125	350	725	1,100,000	400,000		
No. 1		800	525	125	350	625	1,100,000	400,000		
No. 2		450	300	125	350	325	900,000	330,000		
Hem-Fir										
Select structural	Beams and stringers	1,300	750	140	405	925	1,300,000	470,000	0.43	WCLIB WWPA
No. 1		1,050	525	140	405	750	1,300,000	470,000		
No. 2		675	350	140	405	500	1,100,000	400,000		
Select structural	Posts and timbers	1,200	800	140	405	975	1,300,000	470,000		
No. 1		975	650	140	405	850	1,300,000	470,000		
No. 2		575	375	140	405	575	1,100,000	400,000		
Hem-Fir (North)										
Select structural	Beams and stringers	1,250	725	135	405	900	1,300,000	470,000	0.46	NLGA
No. 1		1,000	500	135	405	750	1,300,000	470,000		
No. 2		675	325	135	405	475	1,100,000	400,000		
Select structural	Posts and timbers	1,150	775	135	405	950	1,300,000	470,000		
No. 1		925	625	135	405	850	1,300,000	470,000		
No. 2		550	375	135	405	575	1,100,000	400,000		

(*Continued*)

APPENDIX B.4 (*Continued*)

Reference Design Values for Visually Graded Timbers (5 × 5 in. and Larger)

Species and Commercial Grade	Size Classification	Design Values in Pounds per Square Inch (psi): Bending F_b	Tension Parallel to Grain F_t	Shear Parallel to Grain F_v	Compression Perpendicular to Grain $F_{c\perp}$	Compression Parallel to Grain F_c	Modulus of Elasticity E	Modulus of Elasticity E_{min}	Specific Gravity G	Grading Rules Agency
Mixed Maple										
Select structural	Beams and stringers	1,150	700	180	620	725	1,100,000	400,000	0.55	NELMA
No. 1		975	500	180	620	600	1,100,000	400,000		
No. 2		625	325	180	620	375	900,000	330,000		
Select structural	Posts and timbers	1,100	725	180	620	750	1,100,000	400,000		
No. 1		875	600	180	620	650	1,100,000	400,000		
No. 2		500	350	180	620	300	900,000	330,000		
Mixed Oak										
Select structural	Beams and stringers	1,350	800	155	800	825	1,000,000	370,000	0.68	NELMA
No. 1		1,150	550	155	800	700	1,000,000	370,000		
No. 2		725	375	155	800	450	800,000	290,000		
Select structural	Posts and timbers	1,250	850	155	800	875	1,000,000	370,000		
No. 1		1,000	675	155	800	775	1,000,000	370,000		
No. 2		575	400	155	800	350	800,000	290,000		
Mixed Southern Pine (Wet Service Conditions)										
Select structural	5 in. × 5 in. and larger	1,500	1,000	165	375	900	1,300,000	470,000	0.51	SPIB
No. 1		1,350	900	165	375	800	1,300,000	470,000		
No. 2		850	550	165	375	525	1,000,000	370,000		

(*Continued*)

APPENDIX B.4 (*Continued*)

Reference Design Values for Visually Graded Timbers (5 × 5 in. and Larger)

Species and Commercial Grade	Size Classification	Design Values in Pounds per Square Inch (psi): Bending F_b	Tension Parallel to Grain F_t	Shear Parallel to Grain F_v	Compression Perpendicular to Grain $F_{c\perp}$	Compression Parallel to Grain F_c	Modulus of Elasticity: E	Modulus of Elasticity: E_{min}	Specific Gravity G	Grading Rules Agency
Southern Pine (Wet Service Conditions)										
Dense select structural	5 in. × 5 in. and larger	1,750	1,200	165	440	1,100	1,600,000	580,000	0.55	SPIB
Select structural		1,500	1,000	165	375	950	1,500,000	550,000		
No. 1 Dense		1,550	1,050	165	440	975	1,600,000	580,000		
No. 1		1,350	900	165	375	825	1,500,000	550,000		
No. 2 Dense		975	650	165	440	625	1,300,000	470,000		
No. 2		850	550	165	375	525	1,200,000	440,000		
Dense select structural 86		2,100	1,400	165	440	1,300	1,600,000	580,000		
Dense select structural 72		1,750	1,200	165	440	1,100	1,600,000	580,000		
Dense select structural 65		1,600	1,050	165	440	1,000	1,600,000	580,000		

Spruce Pine

To obtain recommended design values for Spruce Pine graded to Southern Pine Inspection Bureau (SPIB) rules, multiply the appropriate design values for Mixed Southern Pine by the corresponding conversion factor shown below. Round off the values to the nearest 100,000 psi for E; to the nearest 10,000 psi for E_{min}; to the next lower multiple of 5 psi for F_v and $F_c\perp$; and to the next lower multiple of 50 psi for F_b, F_t, and F_c if the value 1,000 psi or greater, 25 psi otherwise.

Conversion Factors for Determining Design Values for Spruce Pine

	Bending F_b	Tension Parallel to Grain F_t	Shear Parallel to Grain F_v	Compression Perpendicular to Grain $F_{c\perp}$	Compression Parallel to Grain F_c	Modulus of Elasticity E and E_{min}
Conversion factor	0.78	0.78	0.98	0.73	0.78	0.82

APPENDIX B.5
Section Properties of *Western Species* Structural Glued Laminated Timber (GLULAM)

		x–x Axis			*y–y* Axis	
Depth *d* (in.)	Area *A* (in.^{2})	I_x (in.^{4})	S_x (in.^{3})	r_x (in.)	I_y (in.^{4})	S_y (in.^{3})
			3⅛ in. width		**(r_y = 0.902 in.)**	
6	18.75	56.25	18.75	1.732	15.26	9.766
7½	23.44	109.9	29.30	2.165	19.07	12.21
9	28.13	189.8	42.19	2.598	22.89	14.65
10½	32.81	301.5	57.42	3.031	26.70	17.09
12	37.50	450.0	75.00	3.464	30.52	19.53
13½	42.19	640.7	94.92	3.897	34.33	21.97
15	46.88	878.9	117.2	4.330	38.15	24.41
16½	51.56	1,170	141.8	4.763	41.96	26.86
18	56.25	1,519	168.8	5.196	45.78	29.30
19½	60.94	1,931	198.0	5.629	49.59	31.74
21	65.63	2,412	229.7	6.062	53.41	34.18
22½	70.31	2,966	263.7	6.495	57.22	36.62
24	75.00	3,600	300.0	6.928	61.04	39.06
			5⅛ in. width		**(r_y = 1.479 in.)**	
6	30.75	92.25	30.75	1.732	67.31	26.27
7½	38.44	180.2	48.05	2.165	84.13	32.83
9	46.13	311.3	69.19	2.598	101.0	39.40
10½	53.81	494.4	94.17	3.031	117.8	45.96
12	61.50	738.0	123.0	3.464	134.6	52.53
13½	69.19	1,051	155.7	3.897	151.4	59.10
15	76.88	1,441	192.2	4.330	168.3	65.66
16½	84.56	1,919	232.5	4.763	185.1	72.23
18	92.25	2,491	276.8	5.196	201.9	78.80
19½	99.94	3,167	324.8	5.629	218.7	85.36
21	107.6	3,955	376.7	6.062	235.6	91.93
22½	115.3	4,865	432.4	6.495	252.4	98.50
24	123.0	5,904	492.0	6.928	269.2	105.1
25½	130.7	7,082	555.4	7.361	286.0	111.6
27	138.4	8,406	622.7	7.794	302.9	118.2
28½	146.1	9,887	693.8	8.227	319.7	124.8
30	153.8	11,530	768.8	8.660	336.5	131.3
31½	161.4	13,350	847.5	9.093	353.4	137.9
33	169.1	15,350	930.2	9.526	370.2	144.5
34½	176.8	17,540	1,017	9.959	387.0	151.0
36	184.5	19,930	1,107	10.39	403.8	157.6
			6¾ in. width		**(r_y = 1.949 in.)**	
7½	50.63	237.3	63.28	2.165	192.2	56.95
9	60.75	410.1	91.13	2.598	230.7	68.34
10½	70.88	651.2	124.0	3.031	269.1	79.73
12	81.00	972.0	162.0	3.464	307.5	91.13
13½	91.13	1,384	205.0	3.897	346.0	102.5
15	101.3	1,898	253.1	4.330	384.4	113.9
16½	111.4	2,527	306.3	4.763	422.9	125.3

(*Continued*)

APPENDIX B.5 (*Continued*)
Section Properties of *Western Species* Structural Glued Laminated Timber (GLULAM)

		x–x Axis			y–y Axis	
Depth d (in.)	**Area A (in.^{2})**	**I_x (in.^{4})**	**S_x (in.^{3})**	**r_x (in.)**	**I_y (in.^{4})**	**S_y (in.^{3})**
			6¾ in. width		**(r_y = 1.949 in.)**	
18	121.5	3,281	364.5	5.196	461.3	136.7
19½	131.6	4,171	427.8	5.629	499.8	148.1
21	141.8	5,209	496.1	6.062	538.2	159.5
22½	151.9	6,407	569.5	6.495	576.7	170.9
24	162.0	7,776	648.0	6.928	615.1	182.3
25½	172.1	9,327	731.5	7.361	653.5	193.6
27	182.3	11,070	820.1	7.794	692.0	205.0
28½	192.4	13,020	913.8	8.227	730.4	216.4
30	202.5	15,190	1,013	8.660	768.9	227.8
31½	212.6	17,580	1,116	9.093	807.3	239.2
33	222.8	20,210	1,225	9.526	845.8	250.6
34½	232.9	23,100	1,339	9.959	884.2	262.0
36	243.0	26,240	1,458	10.39	922.6	273.4
37½	253.1	29,660	1,582	10.83	961.1	284.8
39	263.3	33,370	1,711	11.26	999.5	296.2
40½	273.4	37,370	1,845	11.69	1,038	307.5
42	283.5	41,670	1,985	12.12	1,076	318.9
43½	293.6	46,300	2,129	12.56	1,115	330.3
45	303.8	51,260	2,278	12.99	1,153	341.7
46½	313.9	56,560	2,433	13.42	1,192	353.1
48	324.0	62,210	2,592	13.86	1,230	364.5
49½	334.1	68,220	2,757	14.29	1,269	375.9
51	344.3	74,620	2,926	14.72	1,307	387.3
52½	354.4	81,400	3,101	15.16	1,346	398.7
54	364.5	88,570	3,281	15.59	1,384	410.1
55½	374.6	96,160	3,465	16.02	1,422	421.5
57	384.8	104,200	3,655	16.45	1,461	432.8
58½	394.9	112,600	3,850	16.89	1,499	444.2
60	405.0	121,500	4,050	17.32	1,538	455.6
			8¾ in. width		**(r_y = 2.526 in.)**	
9	78.75	531.6	118.1	2.598	502.4	114.8
10½	91.88	844.1	160.8	3.031	586.2	134.0
12	105.0	1,260	210.0	3.464	669.9	153.1
13½	118.1	1,794	265.8	3.897	753.7	172.3
15	131.3	2,461	328.1	4.330	837.4	191.4
16½	144.4	3,276	397.0	4.763	921.1	210.5
18	157.5	4,253	472.5	5.196	1,005	229.7
19½	170.6	5,407	554.5	5.629	1,089	248.8
21	183.8	6,753	643.1	6.062	1,172	268.0
22½	196.9	8,306	738.3	6.495	1,256	287.1
24	210.0	10,080	840.0	6.928	1,340	306.3
25½	223.1	12,090	948.3	7.361	1,424	325.4
27	236.3	14,350	1,063	7.794	1,507	344.5

(*Continued*)

APPENDIX B.5 (*Continued*)
Section Properties of *Western Species* Structural Glued Laminated Timber (GLULAM)

		x–x Axis			*y–y* Axis	
Depth *d* (in.)	Area *A* (in.^{2})	I_x (in.^{4})	S_x (in.^{3})	r_x (in.)	I_y (in.^{4})	S_y (in.^{3})
			8¾ in. width		**(r_y = 2.526 in.)**	
28½	249.4	16,880	1,185	8.227	1,591	363.7
30	262.5	19,690	1,313	8.660	1,675	382.8
31½	275.6	22,790	1,447	9.093	1,759	402.0
33	288.8	26,200	1,588	9.526	1,842	421.1
34½	301.9	29,940	1,736	9.959	1,926	440.2
36	315.0	34,020	1,890	10.39	2,010	459.4
37½	328.1	38,450	2,051	10.83	2,094	478.5
39	341.3	43,250	2,218	11.26	2,177	497.7
40½	354.4	48,440	2,392	11.69	2,261	516.8
42	367.5	54,020	2,573	12.12	2,345	535.9
43½	380.6	60,020	2,760	12.56	2,428	555.1
45	393.8	66,450	2,953	12.99	2,512	574.2
46½	406.9	73,310	3,153	13.42	2,596	593.4
48	420.0	80,640	3,360	13.86	2,680	612.5
49½	433.1	88,440	3,573	14.29	2,763	631.6
51	446.3	96,720	3,793	14.72	2,847	650.8
52½	459.4	105,500	4,020	15.16	2,931	669.9
54	472.5	114,800	4,253	15.59	3,015	689.1
55½	485.6	124,700	4,492	16.02	3,098	708.2
57	498.8	135,000	4,738	16.45	3,182	727.3
58½	511.9	146,000	4,991	16.89	3,266	746.5
60	525.0	157,500	5,250	17.32	3,350	765.6
			10¾ in. width		**(r_y = 3.103 in.)**	
12	129.0	1,548	258.0	3.464	1,242	231.1
13½	145.1	2,204	326.5	3.897	1,398	260.0
15	161.3	3,023	403.1	4.330	1,553	288.9
16½	177.4	4,024	487.8	4.763	1,708	317.8
18	193.5	5,225	580.5	5.196	1,863	346.7
19½	209.6	6,642	681.3	5.629	2,019	375.6
21	225.8	8,296	790.1	6.062	2,174	404.5
22½	241.9	10,200	907.0	6.495	2,329	433.4
24	258.0	12,380	1,032	6.928	2,485	462.3
25½	274.1	14,850	1,165	7.361	2,640	491.1
27	290.3	17,630	1,306	7.794	2,795	520.0
28½	306.4	20,740	1,455	8.227	2,950	548.9
30	322.5	24,190	1,613	8.660	3,106	577.8
31½	338.6	28,000	1,778	9.093	3,261	606.7
33	354.8	32,190	1,951	9.526	3,416	635.6
34½	370.9	36,790	2,133	9.959	3,572	664.5
36	387.0	41,800	2,322	10.39	3,727	693.4

(*Continued*)

APPENDIX B.5 (*Continued*)
Section Properties of *Western Species* Structural Glued Laminated Timber (GLULAM)

		x–x Axis			y–y Axis	
Depth *d* (in.)	**Area *A* (in.^{2})**	**I_x (in.^{4})**	**S_x (in.^{3})**	**r_x (in.)**	**I_y (in.^{4})**	**S_y (in.^{3})**
			10¾ in. width		**(r_y = 3.103 in.)**	
37½	403.1	47,240	2,520	10.83	3,882	722.3
39	419.3	53,140	2,725	11.26	4,037	751.2
40½	435.4	59,510	2,939	11.69	4,193	780.0
42	451.5	66,370	3,161	12.12	4,348	808.9
43½	467.6	73,740	3,390	12.56	4,503	837.8
45	483.8	81,630	3,628	12.99	4,659	866.7
46½	499.9	90,070	3,874	13.42	4,814	895.6
48	516.0	99,070	4,128	13.86	4,969	924.5
49½	532.1	108,700	4,390	14.29	5,124	953.4
51	548.3	118,800	4,660	14.72	5,280	982.3
52½	564.4	129,600	4,938	15.16	5,435	1,011
54	580.5	141,100	5,225	15.59	5,590	1,040
55½	596.6	153,100	5,519	16.02	5,746	1,069
57	612.8	165,900	5,821	16.45	5,901	1,098
58½	628.9	179,300	6,132	16.89	6,056	1,127
60	645.0	193,500	6,450	17.32	6,211	1,156

Source: Courtesy of the American Forest & Paper Association, Washington, DC.

APPENDIX B.6
Section Properties of *Southern Pine* Structural Glued Laminated Timber (GLULAM)

		x–x Axis			*y–y* Axis	
Depth *d* (in.)	**Area *A* (in.^{2})**	**I_x (in.^{4})**	**S_x (in.^{3})**	**r_x (in.)**	**I_y (in.^{4})**	**S_y (in.^{3})**
			3 in. width		**(r_y = 0.866 in.)**	
5½	16.50	41.59	15.13	1.588	12.38	8.250
6⅞	20.63	81.24	23.63	1.985	15.47	10.31
8¼	24.75	140.4	34.03	2.382	18.56	12.38
9⅝	28.88	222.9	46.32	2.778	21.66	14.44
11	33.00	332.8	60.50	3.175	24.75	16.50
12⅜	37.13	473.8	76.57	3.572	27.84	18.56
13¾	41.25	649.9	94.53	3.969	30.94	20.63
15⅛	45.38	865.0	114.4	4.366	34.03	22.69
16½	49.50	1,123	136.1	4.763	37.13	24.75
17⅞	53.63	1,428	159.8	5.160	40.22	26.81
19¼	57.75	1,783	185.3	5.557	43.31	28.88
20⅝	61.88	2,193	212.7	5.954	46.41	30.94
22	66.00	2,662	242.0	6.351	49.50	33.00
23⅜	70.13	3,193	273.2	6.748	52.59	35.06
			5 in. width		**(r_y = 1.443 in.)**	
6⅞	34.38	135.4	39.39	1.985	71.61	28.65
8¼	41.25	234.0	56.72	2.382	85.94	34.38
9⅝	48.13	371.5	77.20	2.778	100.3	40.10
11	55.00	554.6	100.8	3.175	114.6	45.83
12⅜	61.88	789.6	127.6	3.572	128.9	51.56
13¾	68.75	1,083	157.6	3.969	143.2	57.29
15⅛	75.63	1,442	190.6	4.366	157.6	63.02
16½	82.50	1,872	226.9	4.763	171.9	68.75
17⅞	89.38	2,380	266.3	5.160	186.2	74.48
19¼	96.25	2,972	308.8	5.557	200.5	80.21
20⅝	103.1	3,656	354.5	5.954	214.8	85.94
22	110.0	4,437	403.3	6.351	229.2	91.67
23⅜	116.9	5,322	455.3	6.748	243.5	97.40
24¾	123.8	6,317	510.5	7.145	257.8	103.1
26⅛	130.6	7,429	568.8	7.542	272.1	108.9
27½	137.5	8,665	630.2	7.939	286.5	114.6
28⅞	144.4	10,030	694.8	8.335	300.8	120.3
30¼	151.3	11,530	762.6	8.732	315.1	126.0
31⅝	158.1	13,180	833.5	9.129	329.4	131.8
33	165.0	14,970	907.5	9.526	343.8	137.5
34⅜	171.9	16,920	984.7	9.923	358.1	143.2
35¾	178.8	19,040	1,065	10.32	372.4	149.0
			6¾ in. width		**(r_y = 1.949 in.)**	
6⅞	46.41	182.8	53.17	1.985	176.2	52.21
8¼	55.69	315.9	76.57	2.382	211.4	62.65
9⅝	64.97	501.6	104.2	2.778	246.7	73.09
11	74.25	748.7	136.1	3.175	281.9	83.53
12⅜	83.53	1,066	172.3	3.572	317.2	93.97

(*Continued*)

APPENDIX B.6 (*Continued*)
Section Properties of *Southern Pine* Structural Glued Laminated Timber (GLULAM)

		x–x Axis			y–y Axis	
Depth d (in.)	Area A (in.^{2})	I_x (in.^{4})	S_x (in.^{3})	r_x (in.)	I_y (in.^{4})	S_y (in.^{3})
			6¾ in. width		**(r_y = 1.949 in.)**	
13¾	92.81	1,462	212.7	3.969	352.4	104.4
15⅛	102.1	1,946	257.4	4.366	387.6	114.9
16½	111.4	2,527	306.3	4.763	422.9	125.3
17⅞	120.7	3,213	359.5	5.160	458.1	135.7
19¼	129.9	4,012	416.9	5.557	493.4	146.2
20⅝	139.2	4,935	478.6	5.954	528.6	156.6
22	148.5	5,990	544.5	6.351	563.8	167.1
23⅜	157.8	7,184	614.7	6.748	599.1	177.5
24¾	167.1	8,528	689.1	7.145	634.3	187.9
26⅛	176.3	10,030	767.8	7.542	669.6	198.4
27½	185.6	11,700	850.8	7.939	704.8	208.8
28⅞	194.9	13,540	938.0	8.335	740.0	219.3
30¼	204.2	15,570	1,029	8.732	775.3	229.7
31⅝	213.5	17,790	1,125	9.129	810.5	240.2
33	222.8	20,210	1,225	9.526	845.8	250.6
34⅜	232.0	22,850	1,329	9.923	881.0	261.0
35¾	241.3	25,700	1,438	10.32	916.2	271.5
37⅛	250.6	28,780	1,551	10.72	951.5	281.9
38½	259.9	32,100	1,668	11.11	986.7	292.4
39⅞	269.2	35,660	1,789	11.51	1,022	302.8
41¼	278.4	39,480	1,914	11.91	1,057	313.2
42⅝	287.7	43,560	2,044	12.30	1,092	323.7
44	297.0	47,920	2,178	12.70	1,128	334.1
45⅜	306.3	52,550	2,316	13.10	1,163	344.6
46¾	315.6	57,470	2,459	13.50	1,198	355.0
48⅛	324.8	62,700	2,606	13.89	1,233	365.4
49½	334.1	68,220	2,757	14.29	1,269	375.9
50⅞	343.4	74,070	2,912	14.69	1,304	386.3
52¼	352.7	80,240	3,071	15.08	1,339	396.8
53⅝	362.0	86,740	3,235	15.48	1,374	407.2
55	371.3	93,590	3,403	15.88	1,410	417.7
56⅜	380.5	100,800	3,575	16.27	1,445	428.1
57¾	389.8	108,300	3,752	16.67	1,480	438.5
59⅛	399.1	116,300	3,933	17.07	1,515	449.0
60½	408.4	124,600	4,118	17.46	1,551	459.4
			8½ in. width		**(r_y = 2.454 in.)**	
9⅝	81.81	631.6	131.2	2.778	492.6	115.9
11	93.50	942.8	171.4	3.175	562.9	132.5
12⅜	105.2	1,342	216.9	3.572	633.3	149.0
13¾	116.9	1,841	267.8	3.969	703.7	165.6
15⅛	128.6	2,451	324.1	4.366	774.1	182.1
16½	140.3	3,182	385.7	4.763	844.4	198.7
17⅞	151.9	4,046	452.6	5.160	914.8	215.2

(*Continued*)

APPENDIX B.6 (*Continued*)
Section Properties of *Southern Pine* Structural Glued Laminated Timber (GLULAM)

		x–x Axis			*y–y* Axis	
Depth *d* (in.)	Area *A* (in.^{2})	I_x (in.^{4})	S_x (in.^{3})	r_x (in.)	I_y (in.^{4})	S_y (in.^{3})
			8½ in. width		**(r_y = 2.454 in.)**	
19¼	163.6	5,053	525.0	5.557	985.2	231.8
20⅝	175.3	6,215	602.6	5.954	1,056	248.4
22	187.0	7,542	685.7	6.351	1,126	264.9
23⅜	198.7	9,047	774.1	6.748	1,196	281.5
24¾	210.4	10,740	867.8	7.145	1,267	298.0
26⅛	222.1	12,630	966.9	7.542	1,337	314.6
27½	233.8	14,730	1,071	7.939	1,407	331.1
28⅞	245.4	17,050	1,181	8.335	1,478	347.7
30¼	257.1	19,610	1,296	8.732	1,548	364.3
31⅝	268.8	22,400	1,417	9.129	1,618	380.8
33	280.5	25,460	1,543	9.526	1,689	397.4
34⅜	292.2	28,770	1,674	9.923	1,759	413.9
35¾	303.9	32,360	1,811	10.32	1,830	430.5
37⅛	315.6	36,240	1,953	10.72	1,900	447.0
38½	327.3	40,420	2,100	11.11	1,970	463.6
39⅞	338.9	44,910	2,253	11.51	2,041	480.2
41¼	350.6	49,720	2,411	11.91	2,111	496.7
42⅝	362.3	54,860	2,574	12.30	2,181	513.3
44	374.0	60,340	2,743	12.70	2,252	529.8
45⅜	385.7	66,170	2,917	13.10	2,322	546.4
46¾	397.4	72,370	3,096	13.50	2,393	562.9
48⅛	409.1	78,950	3,281	13.89	2,463	579.5
49½	420.8	85,910	3,471	14.29	2,533	596.1
50⅞	432.4	93,270	3,667	14.69	2,604	612.6
52¼	444.1	101,000	3,868	15.08	2,674	629.2
53⅝	455.8	109,200	4,074	15.48	2,744	645.7
55	467.5	117,800	4,285	15.88	2,815	662.3
56⅜	479.2	126,900	4,502	16.27	2,885	678.8
57¾	490.9	136,400	4,725	16.67	2,955	695.4
59⅛	502.6	146,400	4,952	17.07	3,026	712.0
60½	514.3	156,900	5,185	17.46	3,096	728.5
			10½ in. width		**(r_y = 3.031 in.)**	
11	115.5	1,165	211.8	3.175	1,061	202.1
12⅜	129.9	1,658	268.0	3.572	1,194	227.4
13¾	144.4	2,275	330.9	3.969	1,326	252.7
15⅛	158.8	3,028	400.3	4.366	1,459	277.9
16½	173.3	3,931	476.4	4.763	1,592	303.2
17⅞	187.7	4,997	559.2	5.160	1,724	328.5
19¼	202.1	6,242	648.5	5.557	1,857	353.7
20⅝	216.6	7,677	744.4	5.954	1,990	379.0
22	231.0	9,317	847.0	6.351	2,122	404.3
23⅜	245.4	11,180	956.2	6.748	2,255	429.5
24¾	259.9	13,270	1,072	7.145	2,388	454.8
26⅛	274.3	15,600	1,194	7.542	2,520	480.0
27½	288.8	18,200	1,323	7.939	2,653	505.3

(*Continued*)

APPENDIX B.6 (*Continued*)
Section Properties of *Southern Pine* Structural Glued Laminated Timber (GLULAM)

Depth d (in.)	Area A (in.^{2})	I_x (in.^{4})	S_x (in.^{3})	r_x (in.)	I_y (in.^{4})	S_y (in.^{3})
		x–x Axis			y–y Axis	
			10½ in. width		**(r_y = 3.031 in.)**	
28⅞	303.2	21,070	1,459	8.335	2,786	530.6
30¼	317.6	24,220	1,601	8.732	2,918	555.8
31⅝	332.1	27,680	1,750	9.129	3,051	581.1
33	346.5	31,440	1,906	9.526	3,183	606.4
34⅜	360.9	35,540	2,068	9.923	3,316	631.6
35¾	375.4	39,980	2,237	10.32	3,449	656.9
37⅛	389.8	44,770	2,412	10.72	3,581	682.2
38½	404.3	49,930	2,594	11.11	3,714	707.4
39⅞	418.7	55,480	2,783	11.51	3,847	732.7
41¼	433.1	61,420	2,978	11.91	3,979	758.0
42⅝	447.6	67,760	3,180	12.30	4,112	783.2
44	462.0	74,540	3,388	12.70	4,245	808.5
45⅜	476.4	81,740	3,603	13.10	4,377	833.8
46¾	490.9	89,400	3,825	13.50	4,510	859.0
48⅛	505.3	97,530	4,053	13.89	4,643	884.3
49½	519.8	106,100	4,288	14.29	4,775	909.6
50⅞	534.2	115,200	4,529	14.69	4,908	934.8
52¼	548.6	124,800	4,778	15.08	5,040	960.1
53⅝	563.1	134,900	5,032	15.48	5,173	985.4
55	577.5	145,600	5,294	15.88	5,306	1,011
56⅜	591.9	156,800	5,562	16.27	5,438	1,036
57¾	606.4	168,500	5,836	16.67	5,571	1,061
59⅛	620.8	180,900	6,118	17.07	5,704	1,086
60½	635.3	193,800	6,405	17.46	5,836	1,112

Source: Courtesy of the American Forest & Paper Association, Washington, DC.

APPENDIX B.7

Reference Design Values for Structural Glued Laminated Softwood Timber Combinations[a] (Members Stressed Primarily in Bending)

		Bending About x–x Axis (Loaded Perpendicular to Wide Faces of Laminations)							Bending About y–y Axis (Loaded Parallel to Wide Faces of Laminations)					Axially Loaded		Fasteners	
		Bending		Compression Perpendicular to Grain		Shear Parallel to Grain	Modulus of Elasticity		Bending	Compression Perpendicular to Grain	Shear Parallel to Grain	Modulus of Elasticity		Tension Parallel to Grain	Compression Parallel to Grain	Specific Gravity for Fastener Design	
Combination Symbol	Species Outer/ Core	Bottom of Beam Stressed in Tension (Positive Bending) F_{bx}^{+} (psi)	Top of Beam Stressed in Tension (Negative Bending) F_{bx}^{-} (psi)	Tension Face $F_{c\perp x}$ (psi)	Compression Face $F_{c\perp x}$ (psi)	$F_{vx}{}^{b}$ (psi)	For Deflection Calculations E_x (10^6 psi)	For Stability Calculations $E_{x\min}$ (10^6 psi)	F_{by} (psi)	$F_{c\perp y}$ (psi)	$F_{vy}{}^{b,c}$ (psi)	For Deflection Calculations E_y (10^6 psi)	For Stability Calculations $E_{y\min}$ (10^6 psi)	F_t (psi)	F_c (psi)	Top or Bottom Face G	Side Face G
16F–1.3E		**1600**	**925**	**315**		**195**	**1.3**	**0.69**	**800**	**315**	**170**	**1.1**	**0.58**	**675**	**925**	**0.41**	
16F–V3	DF/DF	1600	1250	560	560	265	1.5	0.79	1450	560	230	1.5	0.79	975	1500	0.50	0.50
16F–V6	DF/DF	1600	1600	560	560	265	1.6	0.85	1450	560	230	1.5	0.79	1000	1600	0.50	0.50
16F–E2	HF/HF	1600	1050	375	375	215	1.4	0.74	1200	375	190	1.3	0.69	825	1150	0.43	0.43
16F–E3	DF/DF	1600	1200	560	560	265	1.6	0.85	1400	560	230	1.5	0.79	975	1600	0.50	0.50
16F–E6	DF/DF	1600	1600	560	560	265	1.6	0.85	1550	560	230	1.5	0.79	1000	1600	0.50	0.50
16F–E7	HF/HF	1600	1600	375	375	215	1.4	0.74	1350	375	190	1.3	0.74	875	1250	0.43	0.43
16F–V2	SP/SP	1600	1400	740	650	300	1.5	0.79	1450	650	260	1.4	0.74	1000	1300	0.55	0.55
16F–V3	SP/SP	1600	1450	740	740	300	1.4	0.74	1450	650	260	1.4	0.74	975	1400	0.55	0.55
16F–V5	SP/SP	1600	1600	650	650	300	1.6	0.85	1600	650	260	1.5	0.79	1000	1550	0.55	0.55
16F–E1	SP/SP	1600	1250	650	650	300	1.6	0.85	1400	650	260	1.6	0.85	1050	1550	0.55	0.55
16F–E3	SP/SP	1600	1600	650	650	300	1.7	0.90	1650	650	260	1.6	0.85	1100	1550	0.55	0.55
20F–1.5E		**2000**	**1100**	**425**		**195**	**1.5**	**0.79**	**800**	**315**	**170**	**1.2**	**0.63**	**725**	**925**	**0.41**	
20F–V3	DF/DF	2000	1450	650	560	265	1.6	0.85	1450	560	230	1.5	0.79	1000	1550	0.50	0.50
20F–V7	DF/DF	2000	2000	650	650	265	1.6	0.85	1450	560	230	1.6	0.85	1050	1600	0.50	0.50
20F–V12	AC/AC	2000	1400	560	560	265	1.5	0.79	1250	470	230	1.4	0.74	925	1500	0.46	0.46
20F–V13	AC/AC	2000	2000	560	560	265	1.5	0.79	1250	470	230	1.4	0.74	950	1550	0.46	0.46
20F–V14	POC/POC	2000	1450	560	560	265	1.5	0.79	1300	470	230	1.4	0.74	900	1600	0.46	0.46
20F–V15	POC/POC	2000	2000	560	560	265	1.5	0.79	1300	470	230	1.4	0.74	900	1600	0.46	0.46

(Continued)

APPENDIX B.7 (*Continued*)

Reference Design Values for Structural Glued Laminated Softwood Timber Combinations[a] (Members Stressed Primarily in Bending)

Combination Symbol	Species Outer/ Core	Bending About x–x Axis (Loaded Perpendicular to Wide Faces of Laminations): Bending: Bottom of Beam Stressed in Tension (Positive Bending) F_{bx}^{+} (psi)	Bending: Top of Beam Stressed in Tension (Negative Bending) F_{bx}^{-} (psi)	Compression Perpendicular to Grain: Tension Face $F_{c\perp x}$ (psi)	Compression Perpendicular to Grain: Compression Face $F_{c\perp x}$ (psi)	Shear Parallel to Grain F_{vx}[b] (psi)	Modulus of Elasticity: For Deflection Calculations E_x (10^6 psi)	Modulus of Elasticity: For Stability Calculations E_{xmin} (10^6 psi)	Bending About y–y Axis (Loaded Parallel to Wide Faces of Laminations): Bending F_{by} (psi)	Compression Perpendicular to Grain $F_{c\perp y}$ (psi)	Shear Parallel to Grain F_{vy}[b,c] (psi)	Modulus of Elasticity: For Deflection Calculations E_y (10^6 psi)	Modulus of Elasticity: For Stability Calculations E_{ymin} (10^6 psi)	Axially Loaded: Tension Parallel to Grain F_t (psi)	Axially Loaded: Compression Parallel to Grain F_c (psi)	Fasteners: Specific Gravity for Fastener Design G: Top or Bottom Face	Side Face
20F–E2	HF/HF	2000	1400	500	500	215	1.6	0.85	1200	375	190	1.4	0.74	925	1350	0.43	0.43
20F–E3	DF/DF	2000	1200	560	560	265	1.7	0.90	1400	560	230	1.6	0.85	1050	1600	0.50	0.50
20F–E6	DF/DF	2000	2000	560	560	265	1.7	0.90	1550	560	230	1.6	0.85	1150	1650	0.50	0.50
20F–E7	HF/HF	2000	2000	500	500	215	1.6	0.85	1450	375	190	1.4	0.74	1050	1450	0.43	0.43
20F–E8	ES/ES	2000	1300	450	450	200	1.5	0.79	1000	315	175	1.4	0.74	825	1100	0.41	0.41
24F–E/SPF1	SPF/SPF	2400	2400	560	560	215	1.6	0.85	1150	470	190	1.6	0.85	1150	2000	0.42	0.42
24F–E/SPF3	SPF/SPF	2400	1550	560	650	215	1.6	0.85	1200	470	195	1.5	0.79	900	1750	0.42	0.42
20F–V2	SP/SP	2000	1550	740	650	300	1.5	0.79	1450	650	260	1.4	0.74	1000	1400	0.55	0.55
20F–V3	SP/SP	2000	1450	650	650	300	1.5	0.79	1600	650	260	1.5	0.79	1000	1400	0.55	0.55
20F–V5	SP/SP	2000	2000	740	740	300	1.6	0.85	1450	650	260	1.4	0.74	1050	1500	0.55	0.55
20F–E1	SP/SP	2000	1300	650	650	300	1.7	0.90	1400	650	260	1.6	0.85	1050	1550	0.55	0.55
20F–E3	SP/SP	2000	2000	650	650	300	1.7	0.90	1700	650	260	1.6	0.85	1150	1600	0.55	0 55
24F–1.7E		**2400**	**1450**	**500**		**210**	**1.7**	**0.90**	**1050**	**315**	**185**	**1.3**	**0.69**	**775**	**1000**	**0.42**	
24F–V5	DF/HF	2400	1600	650	650	215	1.7	0.90	1350	375	200	1.5	0.79	1100	1450	0.50	0.43
24F–V10	DF/HF	2400	2400	650	650	215	1.8	0.95	1450	375	200	1.5	0.79	1150	1550	0.50	0.43
24F–E11	HF/HF	2400	2400	500	500	215	1.8	0.95	1550	375	190	1.5	0.79	1150	1550	0.43	0.43
24F–E15	HF/HF	2400	1600	500	500	215	1.7	0.95	1200	375	190	1.5	0.79	975	1500	0.43	0.43
24F–V1	SP/SP	2400	1750	740	650	300	1.7	0.90	1450	650	260	1.5	0.79	1100	1500	0.55	0.55
24F–V4[d]	SP/SP	2400	1650	740	650	210	1.7	0.90	1350	470	230	1.5	0.79	975	1350	0.55	0.43
24F–V5	SP/5P	2400	2400	740	740	300	1.7	0.90	1700	650	260	1.6	0.85	1150	1600	0.55	0.55

(*Continued*)

APPENDIX B.7 (*Continued*)

Reference Design Values for Structural Glued Laminated Softwood Timber Combinations[a] (Members Stressed Primarily in Bending)

		Bending About x–x Axis (Loaded Perpendicular to Wide Faces of Laminations)							Bending About y–y Axis (Loaded Parallel to Wide Faces of Laminations)					Axially Loaded		Fasteners	
		Bending		Compression Perpendicular to Grain		Shear Parallel to Grain	Modulus of Elasticity		Bending	Compression Perpendicular to Grain	Shear Parallel to Grain	Modulus of Elasticity		Tension Parallel to Grain	Compression Parallel to Grain	Specific Gravity for Fastener Design	
		Bottom of Beam Stressed in Tension (Positive Bending)	Top of Beam Stressed in Tension (Negative Bending)	Tension Face	Compression Face		For Deflection Calculations	For Stability Calculations				For Deflection Calculations	For Stability Calculations			Top or Bottom Face	Side Face
Combination Symbol	Species Outer/ Core	F_{bx}^{+} (psi)	F_{bx}^{-} (psi)	$F_{c\perp x}$ (psi)		F_{vx}[b] (psi)	E_x (10^6 psi)	$E_{x\min}$ (10^6 psi)	F_{by} (psi)	$F_{c\perp y}$ (psi)	F_{vy}[b,c] (psi)	E_y (10^6 psi)	$E_{y\min}$ (10^6 psi)	F_t (psi)	F_c (psi)	G	
24F–1.8E		**2400**	**1450**	**650**		**265**	**1.8**	**0.95**	**1450**	**560**	**230**	**1.6**	**0.85**	**1100**	**1600**	**0.50**	
24F–V4	DF/DF	2400	1850	650	650	265	1.8	0.95	1450	560	230	1.6	0.85	1100	1650	0.50	0.50
24F–V8	DF/DF	2400	2400	650	650	265	1.8	0.95	1550	560	230	1.6	0.85	1100	1650	0.50	0.50
24F–E4	DF/DF	2400	1450	650	650	265	1.8	0.95	1400	560	230	1.7	0.90	1100	1700	0.50	0.50
24F–E13	DF/DF	2400	2400	650	650	265	1.8	0.95	1750	560	230	1.7	0.90	1250	1700	0.50	0.50
24F–E18	DF/DF	2400	2400	650	650	265	1.8	0.95	1550	560	230	1.7	0.90	975	1700	0.50	0.50
24F–V3	SP/SP	2400	2000	740	740	300	1.8	0.95	1700	650	260	1.6	0.85	1150	1650	0.55	0.55
24F–V8	SP/SP	2400	2400	740	740	300	1.8	0.95	1700	650	260	1.6	0.85	1150	1650	0.55	0.55
24F–E1	SP/SP	2400	1450	805	650	300	1.8	0.95	1550	650	260	1.7	0.90	1150	1600	0.55	0.55
24F–E4	SP/SP	2400	2400	805	805	300	1.9	1.00	1850	650	260	1.8	0.95	1450	1750	0.55	0.55
26F–1.9E		**2600**	**1950**	**650**		**265**	**1.9**	**1.00**	**1600**	**560**	**230**	**1.6**	**0.85**	**1150**	**1600**	**0.50**	
26F–V1	DF/DF	2600	1950	650	650	265	2.0	1.06	1850	560	230	1.8	0.95	1350	1850	0.50	0.50
26F–V2	DF/DF	2600	2600	650	650	265	2.0	1.06	1850	560	230	1.8	0.95	1350	1850	0.50	0.50
26F–V1	SP/SP	2600	2000	740	740	300	1.8	0.95	1700	650	260	1.6	0.85	1150	1600	0.55	0.55
26F–V2	SP/SP	2600	2100	740	740	300	1.9	1.00	1950	740	260	1.8	0.95	1300	1850	0.55	0.55
26F–V3	SP/SP	2600	2100	740	740	300	1.9	1.00	1950	650	260	1.8	0.95	1250	1800	0.55	0.55
26F–V4	SP/SP	2600	2600	740	740	300	1.9	1.00	1700	650	260	1.8	0.95	1200	1600	0.55	0.55
26F–V5	SP/SP	2600	2600	740	740	300	1.9	1.00	1950	650	260	1.8	0.95	1300	1850	0.55	0.55

(*Continued*)

APPENDIX B.7 (*Continued*)

Reference Design Values for Structural Glued Laminated Softwood Timber Combinations[a] (Members Stressed Primarily in Bending)

		Bending About x–x Axis (Loaded Perpendicular to Wide Faces of Laminations)							Bending About y–y Axis (Loaded Parallel to Wide Faces of Laminations)					Axially Loaded		Fasteners	
		Bending		Compression Perpendicular to Grain		Shear Parallel to Grain	Modulus of Elasticity		Bending	Compression Perpendicular to Grain	Shear Parallel to Grain	Modulus of Elasticity		Tension Parallel to Grain	Compression Parallel to Grain	Specific Gravity for Fastener Design	
	Species	Bottom of Beam Stressed in Tension (Positive Bending)	Top of Beam Stressed in Tension (Negative Bending)	Tension Face	Compression Face		For Deflection Calculations	For Stability Calculations				For Deflection Calculations	For Stability Calculations			Top or Bottom Face	Side Face
Combination Symbol	Outer/ Core	F_{bx}^{+} (psi)	F_{bx}^{-} (psi)	$F_{c\perp x}$ (psi)		F_{vx}[b] (psi)	E_x (10^6 psi)	$E_{x\min}$ (10^6 psi)	F_{by} (psi)	$F_{c\perp y}$ (psi)	F_{vy}[b,c] (psi)	E_y (10^6 psi)	$E_{y\min}$ (10^6 psi)	F_t (psi)	F_c (psi)	G	
28F–2.1E SP		**2800**	**2300**	**805**		**300**	**2.1**	**1.11**	**1600**	**650**	**260**	**1.7**	**0.90**	**1250**	**1750**	**0.55**	
28F–E1	SP/SP	2800	2300	805	805	300	2.1	1.11	1600	650	260	1.7	0.90	1300	1850	0.55	0.55
28F–E2	SP/SP	2600	2800	805	805	300	2.1	1.11	2000	650	260	1.7	0.90	1300	1300	0.55	0.55
30F–2.1E SP		**3000**	**2400**	**805**		**300**	**2.1**	**1.11**	**1750**	**650**	**260**	**1.7**	**0.90**	**1250**	**1750**	**0.55**	
30F–E1	SP/SP	3000	2400	805	805	300	2.1	1.11	1750	650	260	1.7	0.90	1250	1750	0.55	0.55
30F–E2	SP/SP	3000	3000	805	805	300	2.1	1.11	1750	650	260	1.7	0.90	1350	1750	0.55	0.55

[a] The combinations in this table are applicable to members consisting of four or more laminations and are intended primarily for members stressed in bending due to loads applied perpendicular to the wide faces of the laminations. However, reference design values are tabulated for loading both perpendicular and parallel to the wide faces of the laminations. For combinations and reference design values applicable to members loaded primarily axially or parallel to the wide faces of the laminations, see Appendix B.8. For members of two or three laminations, see Appendix B.8.

[b] The reference design values for shear, F_{vx} and F_{vy}, shall be multiplied by the shear reduction factor, C_{vr}, for the conditions defined in NDS 5.3.10.

[c] Reference design values are for structural glued laminated timbers with laminations made from a single piece of lumber across the width or multiple pieces that have been edge-bonded. For structural glued laminated timber manufactured from multiple piece laminations (across width) that are not edge-bonded, value shall be multiplied by 0.4 for members with five, seven, or nine laminations or by 0.5 for all other members. This reduction shall be cumulative with the adjustment in the preceding footnote.

[d] This combination may contain lumber with wane. If lumber with wane is used, the reference design value for shear parallel to grain, F_{vx}, shall be multiplied by 0.67 if wane is allowed on both sides. If wane is limited to one side, F_{vx} shall be multiplied by 0.83. This reduction shall be cumulative with the adjustment in footnote b.

APPENDIX B.8

Reference Design Values for Structural Glued Laminated Softwood Timber (Members Stressed Primarily in Axial Tension or Compression)

			All Loading			Axially Loaded			Bending About y–y Axis (Loaded Parallel to Wide Faces of Laminations)				Bending About x–x Axis (Loaded Perpendicular to Wide Faces of Laminations)		Fasteners
			Modulus of Elasticity			Tension Parallel to Grain	Compression Parallel to Grain		Bending			Shear Parallel to Grain[a,b,c]	Bending	Shear Parallel to Grain[c]	
Combination Symbol	Species	Grade	For Deflection Calculations E (10^6 psi)	For Stability Calculations E_{min} (10^6 psi)	Compression Perpendicular to Grain $F_{c\perp}$ (psi)	Two or More Laminations F_t (psi)	Four or More Laminations F_c (psi)	Two or Three or More Laminations F_c (psi)	Four or More Laminations F_{by} (psi)	Three Laminations F_{by} (psi)	Two Laminations F_{by} (psi)	F_{vy} (psi)	Two Laminations to 15 in. Deep[d] F_{bx} (psi)	F_{vx} (psi)	Specific Gravity for Fastener Design G
							Visually Graded Western Species								
1	DF	L3	1.5	0.79	560	950	1550	1250	1450	1250	1000	230	1250	265	0.50
2	DF	L2	1.6	0.85	560	1250	1950	1600	1800	1600	1300	230	1700	265	0.50
3	DF	L2D	1.9	1.00	650	1450	2300	1900	2100	1850	1550	230	2000	265	0.50
4	DF	L1CL	1.9	1.00	590	1400	2100	1950	2200	2000	1650	230	2100	265	0.50
5	DF	L1	2.0	1.06	650	1650	2400	2100	2400	2100	1800	230	2200	265	0.50
14	HF	L3	1.3	0.69	375	800	1100	1050	1200	1050	850	190	1100	215	0.43
15	HF	L2	1.4	0.74	375	1050	1350	1350	1500	1350	1100	190	1450	215	0.43
16	HF	L1	1.6	0.85	375	1200	1500	1500	1750	1550	1300	190	1600	215	0.43
17	HF	L1D	1.7	0.90	500	1400	1750	1750	2000	1850	1550	190	1900	215	0.43
22	SW	L3	1.0	0.53	315	525	850	725	800	700	575	170	725	195	0.35
69	AC	L3	1.2	0.63	470	725	1150	1100	1100	975	775	230	1000	265	0.46
70	AC	L2	1.3	0.69	470	975	1450	1450	1400	1250	1000	230	1350	265	0.46
71	AC	L1D	1.6	0.85	560	1250	1900	1900	1850	1650	1400	230	1750	265	0.46
72	AC	L1S	1.6	0.85	560	1250	1900	1900	1850	1650	1400	230	1900	265	0.46
73	POC	L3	1.3	0.69	470	775	1500	1200	1200	1050	825	230	1050	265	0.46
74	POC	L2	1.4	0.74	470	1050	1900	1550	1450	1300	1100	230	1400	265	0.46
75	POC	L1D	1.7	0.90	560	1350	2300	2050	1950	1750	1500	230	1850	265	0.46

(*Continued*)

APPENDIX B.8 (*Continued*)

Reference Design Values for Structural Glued Laminated Softwood Timber (Members Stressed Primarily in Axial Tension or Compression)

			All Loading			Axially Loaded			Bending About y–y Axis (Loaded Parallel to Wide Faces of Laminations)				Bending About x–x Axis (Loaded Perpendicular to Wide Faces of Laminations)		Fasteners
			Modulus of Elasticity			Tension Parallel to Grain	Compression Parallel to Grain		Bending			Shear Parallel to Grain[a,b,c]	Bending	Shear Parallel to Grain[c]	
Combination Symbol	Species	Grade	For Deflection Calculations E (10^6 psi)	For Stability Calculations E_{min} (10^6 psi)	Compression Perpendicular to Grain $F_{c\perp}$ (psi)	Two or More Laminations F_t (psi)	Four or More Laminations F_c (psi)	Two or Three or More Laminations F_c (psi)	Four or More Laminations F_{by} (psi)	Three Laminations F_{by} (psi)	Two Laminations F_{by} (psi)	F_{vy} (psi)	Two Laminations to 15 in. Deep[d] F_{bx} (psi)	F_{vx} (psi)	Specific Gravity for Fastener Design G
							Visually Graded Southern Pine								
47	SP	N2M12	1.4	0.74	650	1200	1900	1150	1750	1550	1300	260	1400	300	0.55
47 1:10	SP	N2M10	1.4	0.74	650	1150	1700	1150	1750	1550	1300	260	1400	300	0.55
47 1:8	SP	N2M	1.4	0.74	650	1000	1500	1150	1600	1550	1300	260	1400	300	0.55
48	SP	N2D12	1.7	0.90	740	1400	2200	1350	2000	1800	1500	260	1600	300	0.55
48 1:10	SP	N2D10	1.7	0.90	740	1350	2000	1350	2000	1800	1500	260	1600	300	0.55
48 1:8	SP	N2D	1.7	0.90	740	1150	1750	1350	1850	1800	1500	260	1600	300	0.55
49	SP	N1M16	1.7	0.90	650	1350	2100	1450	1950	1750	1500	260	1800	300	0.55
49 1:14	SP	N1M14	1.7	0.90	650	1350	2000	1450	1950	1750	1500	260	1800	300	0.55
49 1:12	SP	N1M12	1.7	0.90	650	1300	1900	1450	1950	1750	1500	260	1800	300	0.55
49 1:10	SP	N1M	1.7	0.90	650	1150	1700	1450	1850	1750	1500	260	1800	300	0.55
50	SP	N1D14	1.9	1.00	740	1550	2300	1700	2300	2100	1750	260	2100	300	0.55
50 1:12	SP	N1D12	1.9	1.00	740	1500	2200	1700	2300	2100	1750	260	2100	300	0.55
50 1:10	SP	N1D	1.9	1.00	740	1350	2000	1700	2100	2100	1750	260	2100	300	0.55

[a] For members with two or three laminations, the reference shear design value for transverse loads parallel to the wide faces of the laminations, F_{vy}, shall be reduced by multiplying by a factor of 0.84 for two laminations, or 0.95 for three laminations.

[b] The reference shear design value for transverse loads applied parallel to the wide faces of the laminations, F_{vy}, shall be multiplied by 0.4 for members with five, seven, or nine laminations manufactured from multiple piece laminations (across width) that are not edge-bonded. The reference shear design value, F_{vy}, shall be multiplied by 0.5 for all other members manufactured from multiple-piece laminations with unbonded edge joints. This reduction shall be cumulative with the adjustments in footnotes a and c.

[c] The reference design values for shear, F_{vx} and F_{vy}, shall be multiplied by the shear reduction factor, C_{VT}, for the conditions defined in NDS 5.3.10.

[d] For members greater than 15 in. deep, the reference bending design value, F_{bx}, shall be reduced by multiplying by a factor of 0.88.

APPENDIX B.9
Reference Design Values for Structural Composite Lumber

Grade	Orientation	Shear of Elasticity G (psi)	Modulus of Elasticity E (psi)	Flexural Stress F_b[a] (psi)	Tension Stress F_t[b] (psi)	Compression Perpendicular to Grain $F^{c}_{c\perp}$ (psi)	Compression Parallel to Grain $F_c\|$ (psi)	Horizontal Shear Parallel to Grain F_v (psi)
				TimberStrand LSL				
1.3E	Beam/ Column	81,250	1.3×10^6	3,140	1,985	1,240	2,235	745
	Plank	81,250	1.3×10^6	3,510		790	2,235	280
1.55E	Beam	96,875	1.55×10^6	4,295	1,975	1,455	3,270	575
				Microllam LVL				
1.9E	Beam	118,750	1.9×10^6	4,805	2,870	1,365	4,005	530
				Parallam PSL				
1.8E and 2.0E	Column	112,500	1.8×10^6 2.0×10^6	4,435	3,245	775	3,990	355
2.0E	Beam	125,000	2.0×10^6	5,360	3,750	1,365	4,630	540

[a] For 12-in. depth and for other depths, multiply, F_b, by the factors as follows: For TimberStrand LSL, multiply by $(12/d)^{0.092}$; for Microllam LVL, multiply by $(12/d)^{0.136}$; for Parallam, PSL, multiply by $(12/d)^{0.111}$.

[b] F_t has been adjusted to reflect the volume effects for most standard applications.

[c] $F_{c\perp}$ shall not be increased for duration of load.

APPENDIX B.10
Reference Design Values for Cross-Laminated Timber (CLT)

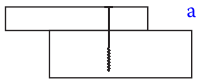
	Major Strength Direction						Minor Strength Direction					
CLT Grades	$F_{b,0}$ (psi)	E_0 (10^6 psi)	$F_{t,0}$ (psi)	$F_{e,0}$ (psi)	$F_{v,0}$ (Psi)	$F_{s,0}$ (psi)	$F_{b,90}$ (psi)	E_{90} (10^6 psi)	$F_{t,90}$ (psi)	$F_{c,90}$ (psi)	$F_{v,90}$ (psi)	$F_{s,90}$ (psi)
E1	1,950	1.7	1,375	1,800	135	45	500	1.2	250	650	135	45
E2	1,650	1.5	1,020	1,700	180	60	525	1.4	325	775	180	60
E3	1,200	1.2	600	1,400	110	35	350	0.9	150	475	110	35
E4	1,950	1.7	1,375	1,800	175	55	575	1.4	325	825	175	55
V1	900	1.6	575	1,350	180	60	525	1.4	325	775	180	60
V2	875	1.4	450	1,150	135	45	500	1.2	250	650	135	45
V3	975	1.6	550	1,450	175	55	575	1.4	325	825	175	55

For SI; 1 psi = 0.006895 MPa
Tabulated values are allowable design values and not permitted to be increased for the lumbar size adjustment factor in accordance with the NDS. The design values shall be used in conjunction with the section properties provided by the CLT manufacturer based on the actual layup used in manufacturing the CLT panel.
Custom CLT grades that are not listed in this table shaft be permitted in.

Stress Grade	Major Strength Direction	Minor Strength Direction
El	1950f-1.7E MSR SPF	#3 Spruce Pine Fir
E2	1650f-1.5E MSR DFL	#3 Dougias Fir Larch
E3	1200f-1.2E MSR Misc	#3 Misc
E4	1950f-1.7E MSR SP	#3 Southern Pine
VI	#2 Doulas Fir Larch	#3 Dourtas Fir Larch
V2	#1/ #2 Spruce Pine Fir	#3 Spruce Pine Fir
V3	#2 Southern Pine	# Southern Pine

APPENDIX B.11

Common Wire, Box, or Sinker Nails: Reference Lateral Design Values (Z) for Single-Shear (Two-Member) Connections

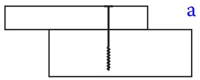

a

Side Member Thickness, t_s (in.)	Nail Diameter, D (in.)	Common Wire Nail (Pennyweight)	Box Nail (Pennyweight)	Sinker Nail (Pennyweight)	G = 0.67, Red Oak (lb)	G = 0.55, Mixed Maple Southern Pine (lb)	G = 0.5, Douglas Fir-Larch (lb)	G = 0.49, Douglas Fir-Larch (North) (lb)	G = 0.46, Douglas Fir (South) Hem-Fir (North) (lb)	G = 0.43, Hem-Fir (lb)	G = 0.42, Spruce-Pine-Fir (lb)	G = 0.37, Redwood (Open Grain) (lb)	G = 0.36, Eastern Softwoods Spruce-Pine-Fir (South) Western Cedars Western Woods (lb)	G = 0.35, Northern Species (lb)
¾	0.099		6d	7d	73	61	55	54	51	48	47	39	38	36
	0.113	6d	8d	8d	94	79	72	71	65	58	57	47	46	44
	0.120			10d	107	89	80	77	71	64	62	52	50	48
	0.128		10d		121	101	87	84	78	70	68	57	56	54
	0.131	8d			127	104	90	87	80	73	70	60	58	56
	0.135		16d	12d	135	108	94	91	84	76	74	63	61	58
	0.148	10d	20d	16d	154	121	105	102	94	85	83	70	69	66
	0.162	16d	40d		183	138	121	117	108	99	96	82	80	77
	0.177			20d	200	153	134	130	121	111	107	92	90	87
	0.192	20d		30d	206	157	138	134	125	114	111	96	93	90
	0.207	30d		40d	216	166	147	143	133	122	119	103	101	97
	0.225	40d			229	178	158	154	144	132	129	112	110	106
	0.244	50d		60d	234	182	162	158	147	136	132	115	113	109
1	0.099		6d	7d	73	61	55	54	51	48	47	42	41	40
	0.113	*6d*[b]	8d	8d	94	79	72	71	67	63	61	55	54	51
	0.120			10d	107	89	81	80	76	71	69	60	59	56
	0.128		10d		121	101	93	91	86	80	79	66	64	61
	0.131	8d			127	106	97	95	90	84	82	68	66	63
	0.135		16d	12d	135	113	103	101	96	89	86	71	69	66
	0.148	10d	20d	16d	154	128	118	115	109	99	96	80	77	74
	0.162	16d	40d		184	154	141	137	125	113	109	91	89	85

(Continued)

APPENDIX B.11 (*Continued*)

Common Wire, Box, or Sinker Nails: Reference Lateral Design Values (Z) for Single-Shear (Two-Member) Connections



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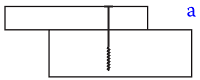
Side Member Thickness, t_s (in.)	Nail Diameter, D (in.)	Common Wire Nail (Pennyweight)	Box Nail (Pennyweight)	Sinker Nail (Pennyweight)	G = 0.67, Red Oak (lb)	G = 0.55, Mixed Maple Southern Pine (lb)	G = 0.5, Douglas Fir-Larch (lb)	G = 0.49, Douglas Fir-Larch (North) (lb)	G = 0.46, Douglas Fir (South) Hem-Fir (North) (lb)	G = 0.43, Hem-Fir (lb)	G = 0.42, Spruce-Pine-Fir (lb)	G = 0.37, Redwood (Open Grain) (lb)	G = 0.36, Eastern Softwoods Spruce-Pine-Fir (South) Western Cedars Western Woods (lb)	G = 0.35, Northern Species (lb)
	0.177			20d	213	178	155	150	138	125	121	102	99	95
	0.192	20d		30d	222	183	159	154	142	128	124	105	102	98
	0.207	30d		40d	243	192	167	162	149	135	131	111	109	104
	0.225	40d			268	202	177	171	159	144	140	120	117	112
	0.244	50d		60d	274	207	181	175	162	148	143	123	120	115
1¼	0.099		*6d*	*7d*	73	61	55	54	51	48	47	42	41	40
	0.113	*6d*	8d	8d	94	79	72	71	67	63	61	55	54	52
	0.120			10d	107	89	81	80	76	71	69	62	60	59
	0.128		10d		121	101	93	91	86	80	79	70	69	67
	0.131	*8d*			127	106	97	95	90	84	82	73	72	70
	0.135		16d	12d	135	113	103	101	96	89	88	78	76	74
	0.148	10d	20d	16d	154	128	118	115	109	102	100	89	87	84
	0.162	16d	40d		184	154	141	138	131	122	120	103	100	95
	0.177			20d	213	178	163	159	151	141	136	113	110	105
	0.192	20d		30d	222	185	170	166	157	145	140	116	113	108
	0.207	30d		40d	243	203	186	182	169	152	147	123	119	114
	0.225	40d			268	224	200	193	177	160	155	130	127	121
	0.244	50d		60d	276	230	204	197	181	163	158	133	129	124

(*Continued*)

APPENDIX B.11 (*Continued*)

Common Wire, Box, or Sinker Nails: Reference Lateral Design Values (*Z*) for Single-Shear (Two-Member) Connections

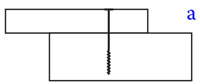


Side Member Thickness, t_s (in.)	Nail Diameter, *D* (in.)	Common Wire Nail (Pennyweight)	Box Nail (Pennyweight)	Sinker Nail (Pennyweight)	*G* = 0.67, Red Oak (lb)	*G* = 0.55, Mixed Maple Southern Pine (lb)	*G* = 0.5, Douglas Fir-Larch (lb)	*G* = 0.49, Douglas Fir-Larch (North) (lb)	*G* = 0.46, Douglas Fir (South) Hem-Fir (North) (lb)	*G* = 0.43, Hem-Fir (lb)	*G* = 0.42, Spruce-Pine-Fir (lb)	*G* = 0.37, Redwood (Open Grain) (lb)	*G* = 0.36, Eastern Softwoods Spruce-Pine-Fir (South) Western Cedars Western Woods (lb)	*G* = 0.35, Northern Species (lb)
1½	0.099			*7d*	73	61	55	54	51	48	47	42	41	40
	0.113		*8d*	*8d*	94	79	72	71	67	63	61	56	54	52
	0.120			10d	107	89	81	80	76	71	69	62	60	59
	0.128		10d		121	101	93	91	86	80	79	70	69	67
	0.131	*8d*			127	106	97	95	90	84	82	73	72	70
	0.135		16d	12d	135	113	103	101	96	89	88	78	76	74
	0.148	10d	20d	16d	154	128	118	115	109	102	100	89	87	84
	0.162	16d	40d		184	154	141	138	131	122	120	106	104	101
	0.177			20d	213	178	163	159	151	141	138	123	121	117
	0.192	20d		30d	222	185	170	166	157	147	144	128	126	120
	0.207	30d		40d	243	203	186	182	172	161	158	135	131	125
	0.225	40d			268	224	205	201	190	178	172	143	138	132
	0.244	50d		60d	276	230	211	206	196	181	175	146	141	135
1¾	0.113		*8d*		94	79	72	71	67	63	61	55	54	52
	0.120			10*d*	107	89	81	80	76	71	69	62	60	59
	0.128		10d		121	101	93	91	86	80	79	70	69	67
	0.135		16d	12d	135	113	103	101	96	89	88	78	76	74
	0.148	10d	20d	16d	154	128	118	115	109	102	100	89	87	84
	0.162	16d	40d		184	154	141	138	131	122	120	106	104	101

(*Continued*)

APPENDIX B.11 (*Continued*)

Common Wire, Box, or Sinker Nails: Reference Lateral Design Values (Z) for Single-Shear (Two-Member) Connections


[a]

Side Member Thickness, t_s (in.)	Nail Diameter, D (in.)	Common Wire Nail (Pennyweight)	Box Nail (Pennyweight)	Sinker Nail (Pennyweight)	$G = 0.67$, Red Oak (lb)	$G = 0.55$, Mixed Maple Southern Pine (lb)	$G = 0.5$, Douglas Fir-Larch (lb)	$G = 0.49$, Douglas Fir-Larch (North) (lb)	$G = 0.46$, Douglas Fir (South) Hem-Fir (North) (lb)	$G = 0.43$, Hem-Fir (lb)	$G = 0.42$, Spruce-Pine-Fir (lb)	$G = 0.37$, Redwood (Open Grain) (lb)	$G = 0.36$, Eastern Softwoods Spruce-Pine-Fir (South) Western Cedars Western Woods (lb)	$G = 0.35$, Northern Species (lb)
	0.177			20d	213	178	163	159	151	141	138	123	121	117
	0.192	20d		30d	222	185	170	166	157	147	144	128	126	122
	0.207	30d		40d	243	203	186	182	172	161	158	140	137	133
	0.225	40d			268	224	205	201	190	178	174	155	151	144
	0.244	50d		60d	276	230	211	206	196	183	179	159	154	147

Source: Courtesy of the American Forest & Paper Association, Washington, DC.

[a] Single-shear connection.

[b] Italic *d* indicates that the nail length is insufficient to provide 10 times nail diameter penetration. Multiply the tabulated values by the ratio (penetration/10 × nail diameter).

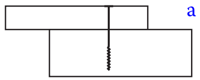
APPENDIX B.12
Nail and Spike Reference Withdrawal Design Values (W)

	Pounds per Inch of Nail Penetration																			
	Common Wire Nails, Box Nails, and Common Wire Spikes Diameter, *D*										Common Nails/Spikes, Box Nails					Threaded Nails Wire Diameter, *D*				
Specific gravity, *G*	0.099 in.	0.113 in.	0.128 in.	0.131 in.	0.135 in.	0.148 in.	0.162 in.	0.192 in.	0.207 in.	0.225 in.	0.244 in.	0.263 in.	0.283 in.	0.312 in.	0.375 in.	0.120 in.	0.135 in.	0.148 in.	0.177 in.	0.207 in.
0.73	62	71	80	82	85	93	102	121	130	141	153	165	178	196	236	82	93	102	121	141
0.71	58	66	75	77	79	87	95	113	121	132	143	154	166	183	220	77	87	95	113	132
0.68	52	59	67	69	71	78	85	101	109	118	128	138	149	164	197	69	78	85	101	118
0.67	50	57	65	66	68	75	82	97	105	114	124	133	144	158	190	66	75	82	97	114
0.58	35	40	45	46	48	52	57	68	73	80	86	93	100	110	133	46	52	57	68	80
0.55	31	35	40	41	42	46	50	59	64	70	76	81	88	97	116	41	46	50	59	70
0.51	25	29	33	34	35	38	42	49	53	58	63	67	73	80	96	34	38	42	49	58
0.50	24	28	31	32	33	36	40	47	50	55	60	64	69	76	91	32	36	40	47	55
0.49	23	26	30	30	31	34	38	45	48	52	57	61	66	72	87	30	34	38	45	52
0.47	21	24	27	27	28	31	34	40	43	47	51	55	59	65	78	27	31	34	40	47
0.46	20	22	25	26	27	29	32	38	41	45	48	52	56	62	74	26	29	32	38	45
0.44	18	20	23	23	24	26	29	34	37	40	43	47	50	55	66	23	26	29	34	40
0.43	17	19	21	22	23	25	27	32	35	38	41	44	47	52	63	22	25	27	32	38
0.42	16	18	20	21	21	23	26	30	33	35	38	41	45	49	59	21	23	26	30	35
0.41	15	17	19	19	20	22	24	29	31	33	36	39	42	46	56	19	22	24	29	33
0.40	14	16	18	18	19	21	23	27	29	31	34	37	40	44	52	18	21	23	27	31
0.39	13	15	17	17	18	19	21	25	27	29	32	34	37	41	49	17	19	21	25	29
0.38	12	14	16	16	17	18	20	24	25	28	30	32	35	38	46	16	18	20	24	28
0.37	11	13	15	15	16	17	19	22	24	26	28	30	33	36	43	15	17	19	22	26
0.36	11	12	14	14	14	16	17	21	22	24	26	28	30	33	40	14	16	17	21	24
0.35	10	11	13	13	14	15	16	19	21	23	24	26	28	31	38	13	15	16	19	23
0.31	7	8	9	10	10	11	12	14	15	17	18	19	21	23	28	10	11	12	14	17

Source: Courtesy of the American Forest & Paper Association, Washington, DC.

APPENDIX B.13

Post-Frame Ring Shank Nails: Reference Lateral Design Values, *Z*, for Single-Shear (Two-Member) Connections

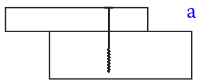


Side Member Thickness t_s (in.)	Nail Diameter *D* (in.)	Nail Length *L* (in.)	*G* = 0.67, Red Oak lb	*G* = 0.55, Mixed Maple, Southern Pine lb	*G* = 0.5, Douglas Fir-Larch lb	*G* = 0.49, Douglas Fir-Larch (North) lb	*G* = 0.46, Douglas Fir (South), Hem-Fir (North) lb	*G* = 0.43, Hem-Fir lb	*G* = 0.42, Spruce-Pine-Fir lb	*G* = 0.37, Redwood (Open Grain) lb	*G* = 0.36, Eastern Softwoods, Spruce-Pine-Fir (South), Western Cedars, Western Woods lb	*G* = 0.35, Northern Species lb
½	0.135	3, 3.5	114	89	80	78	73	67	65	57	56	54
	0.148	3–4.5	127	100	89	87	81	75	73	64	63	61
	0.177	3–8	173	139	125	122	115	107	105	93	91	88
	0.200	3.5–8	188	151	137	134	126	118	115	102	100	95
	0.207	4–8	193	156	142	138	131	122	119	106	102	96
¾	0.135	3, 3.5	138	106	93	90	83	75	73	62	61	58
	0.148	3–4.5	156	118	103	100	92	84	81	70	68	65
	0.177	3–8	204	157	139	134	125	115	112	97	94	91
	0.200	3.5–8	218	168	149	145	135	124	121	105	103	99
	0.207	4–8	223	173	153	149	139	128	125	109	106	103
1	0.135	3, 3.5	138	115	106	103	97	87	84	70	68	65
	0.148	3–4.5	156	130	119	116	107	96	93	78	76	73
	0.177	3–8	227	181	158	153	141	128	124	105	102	98
	0.200	3.5–8	250	193	168	163	151	137	133	113	110	106
	0.207	4–8	259	197	172	166	154	140	136	116	113	109
1¼	0.135	3, 3.5	138	115	106	103	98	92	90	80	77	74
	0.148	3–4.5	156	130	119	116	110	103	101	88	86	82
	0.177	3–8	227	189	173	170	160	143	139	116	112	107
	0.200	3.5–8	250	208	191	184	169	152	147	123	120	115
	0.207	4–8	259	216	195	188	172	155	150	126	123	118

(Continued)

APPENDIX B.13 (*Continued*)

Post-Frame Ring Shank Nails: Reference Lateral Design Values, *Z*, for Single-Shear (Two-Member) Connections

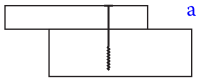

[a]

Side Member Thickness t_s (in.)	Nail Diameter D (in.)	Nail Length L (in.)	$G = 0.67$, Red Oak lb	$G = 0.55$, Mixed Maple, Southern Pine lb	$G = 0.5$, Douglas Fir-Larch lb	$G = 0.49$, Douglas Fir-Larch (North) lb	$G = 0.46$, Douglas Fir (South), Hem-Fir (North) lb	$G = 0.43$, Hem-Fir lb	$G = 0.42$, Spruce-Pine-Fir lb	$G = 0.37$, Redwood (Open Grain) lb	$G = 0.36$, Eastern Softwoods, Spruce-Pine-Fir (South), Western Cedars, Western Woods lb	$G = 0.35$, Northern Species lb
1½	0.135	3, 3.5	138	115	106	103	98	92	90	80	78	76
	0.148	3–4.5	156	130	119	116	110	103	101	90	88	85
	0.177	3–8	227	189	173	170	161	150	147	128	124	118
	0.200	3.5–8	250	208	191	187	177	166	162	136	132	126
	0.207	4–8	259	216	198	194	184	172	167	139	134	128
1¾	0.135	3, 3.5	138	115	106	103	98	92	90	80	78	76
	0.148	3–4.5	156	130	119	116	110	103	101	90	88	85
	0.177	3[b], 3.5[b], 4–8	227	189	173	170	161	150	147	131	128	125
	0.200	3.5[b], 4–8	250	208	191	187	177	166	162	144	141	137
	0.207	4–8	259	216	198	194	184	172	168	149	147	140
2½	0.135	3.5[b]	138	115	106	103	98	92	90	80	78	76
	0.148	3.5[b], 4, 4.5	156	130	119	116	110	103	101	90	88	85
	0.177	4[b], 4.5, 5, 6, 8	227	189	173	170	161	150	147	131	128	125
	0.200	4[b], 4.5, 5, 6, 8	250	208	191	187	177	166	162	144	141	137

(*Continued*)

APPENDIX B.13 (*Continued*)

Post-Frame Ring Shank Nails: Reference Lateral Design Values, *Z*, for Single-Shear (Two-Member) Connections



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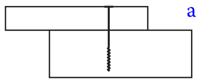
Side Member Thickness t_s (in.)	Nail Diameter *D* (in.)	Nail Length *L* (in.)	*G* = 0.67, Red Oak lb	*G* = 0.55, Mixed Maple, Southern Pine lb	*G* = 0.5, Douglas Fir-Larch lb	*G* = 0.49, Douglas Fir-Larch (North) lb	*G* = 0.46, Douglas Fir (South), Hem-Fir (North) lb	*G* = 0.43, Hem-Fir lb	*G* = 0.42, Spruce-Pine-Fir lb	*G* = 0.37, Redwood (Open Grain) lb	*G* = 0.36, Eastern Softwoods, Spruce-Pine-Fir (South), Western Cedars, Western Woods lb	*G* = 0.35, Northern Species lb
	0.207	4[b], 4.5[b], 5, 6, 8	259	216	198	194	184	172	168	149	147	142
3½	0.148	4.5[b]	156	130	119	116	110	103	101	90	88	85
	0.177	5[b], 6, 8	227	189	173	170	161	150	147	131	128	125
	0.200	5[b], 6, 8	250	208	191	187	177	166	162	144	141	137
	0.207	5[b], 6, 8	259	216	198	194	184	172	168	149	147	142
0.036 (20 gage)	0.135	3, 3.5	130	111	102	100	95	89	88	78	77	75
	0.148	3–4.5	142	125	115	113	107	101	99	88	87	84
	0.177	3–8	171	171	167	164	156	146	143	128	126	122
	0.200	3.5–8	177	177	177	177	172	161	158	141	139	135
	0.207	4–8	178	178	178	178	178	167	164	146	144	140
0.048 (18 gage)	0.135	3, 3.5	131	111	103	101	96	90	88	79	78	76
	0.148	3–4.5	147	125	116	113	108	101	99	89	87	85
	0.177	3–8	213	182	168	164	156	147	144	129	127	123
	0.200	3.5–8	235	200	184	181	172	162	158	142	139	135
	0.207	4–8	237	207	191	187	178	168	164	147	144	140
0.060 (16 gage)	0.135	3, 3.5	132	113	104	102	97	92	90	81	79	77

(*Continued*)

APPENDIX B.13 (*Continued*)

Post-Frame Ring Shank Nails: Reference Lateral Design Values, *Z*, for Single-Shear (Two-Member) Connections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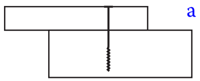
Side Member Thickness t_s (in.)	Nail Diameter D (in.)	Nail Length L (in.)	$G = 0.67$, Red Oak lb	$G = 0.55$, Mixed Maple, Southern Pine lb	$G = 0.5$, Douglas Fir-Larch lb	$G = 0.49$, Douglas Fir-Larch (North) lb	$G = 0.46$, Douglas Fir (South), Hem-Fir (North) lb	$G = 0.43$, Hem-Fir lb	$G = 0.42$, Spruce-Pine-Fir lb	$G = 0.37$, Redwood (Open Grain) lb	$G = 0.36$, Eastern Softwoods, Spruce-Pine-Fir (South), Western Cedars, Western Woods lb	$G = 0.35$, Northern Species lb
	0.148	3–4.5	148	126	117	115	109	103	101	90	89	86
	0.177	3–8	214	183	169	165	157	148	145	130	128	124
	0.200	3.5–8	235	201	185	182	173	163	159	143	140	136
	0.207	4–8	244	208	192	188	179	168	165	148	145	141
0.075 (14 gage)	0.135	3, 3.5	134	115	106	104	100	94	92	83	81	79
	0.148	3–4.5	150	129	119	117	112	105	103	93	91	88
	0.177	3–8	216	185	171	167	160	150	147	132	130	126
	0.200	3.5–8	237	203	187	183	175	164	161	145	142	138
	0.207	4–8	246	210	194	190	181	170	167	150	147	143
0.105 (12 gage)	0.135	3, 3.5	142	122	113	111	106	100	98	88	87	83
	0.148	3–4.5	159	137	127	124	119	112	110	99	97	94
	0.177	3–8	223	192	178	174	166	157	154	138	136	132
	0.200	3.5–8	244	209	194	190	181	171	167	150	148	144
	0.207	4–8	252	216	200	196	187	176	173	155	153	148
0.120 (11 gage)	0.135	3, 3.5	147	127	118	115	110	104	102	92	90	86
	0.148	3–4.5	164	141	131	129	123	116	114	103	101	98
	0.177	3–8	228	197	182	179	171	161	158	142	140	136

(*Continued*)

APPENDIX B.13 (*Continued*)

Post-Frame Ring Shank Nails: Reference Lateral Design Values, *Z*, for Single-Shear (Two-Member) Connections


a

Side Member Thickness t_s (in.)	Nail Diameter *D* (in.)	Nail Length *L* (in.)	*G* = 0.67, Red Oak lb	*G* = 0.55, Mixed Maple, Southern Pine lb	*G* = 0.5, Douglas Fir-Larch lb	*G* = 0.49, Douglas Fir-Larch (North) lb	*G* = 0.46, Douglas Fir (South), Hem-Fir (North) lb	*G* = 0.43, Hem-Fir lb	*G* = 0.42, Spruce-Pine-Fir lb	*G* = 0.37, Redwood (Open Grain) lb	*G* = 0.36, Eastern Softwoods, Spruce-Pine-Fir (South), Western Cedars, Western Woods lb	*G* = 0.35, Northern Species lb
	0.200	3.5–8	249	214	198	194	185	175	171	154	152	147
	0.207	4–8	257	221	204	200	191	180	177	159	156	152
0.134 (10 gage)	0.135	3, 3.5	152	132	122	120	115	108	106	96	93	88
	0.148	3–4.5	169	147	136	134	128	120	118	107	105	102
	0.177	3–8	234	202	187	184	175	165	162	146	144	140
	0.200	3.5–8	254	219	203	199	190	179	176	158	156	151
	0.207	4–8	262	225	209	205	196	185	181	163	160	156
0.179 (7 gage)	0.135	3, 3.5	172	149	139	136	131	123	121	105	102	98
	0.148	3–4.5	191	166	154	151	145	137	134	121	118	113
	0.177	3–8	256	222	206	202	193	183	179	162	159	153
	0.200	3.5–8	276	238	221	217	208	196	192	174	171	166
	0.207	4–8	283	245	227	223	213	201	197	178	175	170
0.239 (3 gage)	0.135	3, 3.5	184	156	144	141	134	126	124	106	102	98
	0.148	3–4.5	207	176	162	159	151	142	139	124	120	114
	0.177	3–8	293	255	236	232	220	207	203	179	174	165
	0.200	3.5–8	312	271	252	248	237	224	220	199[a]	195	189
	0.207	4–8	319	277	258	253	242	229	224	203	199	194

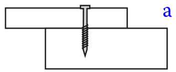
[a] Single shear connection.

[b] Nail length is insufficient to provide 10 times nail diameter penetration. Tabulated lateral design values, *Z*, shall be multiplied by (penetration/10 × nail diameter).

APPENDIX B.14
Post-Frame Ring Shank Nail Reference Withdrawal Design Values, *W*, Pounds per Inch of Ring Shank Penetration into Side Grain of Wood Member

	Diameter, *D* (in.)				
Specific Gravity, *G*	**0.135**	**0.148**	**0.177**	**0.200**	**0.207**
0.73	129	142	170	192	199
0.71	122	134	161	181	188
0.68	112	123	147	166	172
0.67	109	120	143	162	167
0.58	82	90	107	121	125
0.55	74	81	96	109	113
0.51	63	69	83	94	97
0.50	61	67	80	90	93
0.49	58	64	76	86	89
0.47	54	59	70	80	82
0.46	51	56	67	76	79
0.44	47	52	62	70	72
0.43	45	49	59	67	69
0.42	43	47	56	64	66
0.41	41	45	54	61	63
0.40	39	43	51	58	60
0.39	37	41	48	55	57
0.38	35	38	46	52	54
0.37	33	36	44	49	51
0.36	31	35	41	47	48
0.35	30	33	39	44	46
0.31	23	26	31	35	36

APPENDIX B.15
Wood Screws: Reference Lateral Design Values (Z) for Single-Shear (Two-Member) Connections

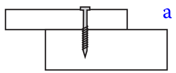

a

Side Member Thickness, t_s (in.)	Wood Screw Diameter, D (in.)	Wood Screw Number	$G = 0.67$, Red Oak (lb)	$G = 0.55$, Mixed Maple Southern Pine (lb)	$G = 0.5$, Douglas Fir-Larch (lb)	$G = 0.49$, Douglas Fir-Larch (North) (lb)	$G = 0.46$, Douglas Fir (South) Hem-Fir (North) (lb)	$G = 0.43$, Hem-Fir (lb)	$G = 0.42$, Spruce-Pine-Fir (lb)	$G = 0.37$, Redwood (Open Grain) (lb)	$G = 0.36$, Eastern Softwoods Spruce-Pine-Fir (South) Western Cedars Western Woods (lb)	$G = 0.35$, Northern Species (lb)
½	0.138	6	88	67	59	57	53	49	47	41	40	38
	0.151	7	96	74	65	63	59	54	52	45	44	42
	0.164	8	107	82	73	71	66	61	59	51	50	48
	0.177	9	121	94	83	81	76	70	68	59	58	56
	0.190	10	130	101	90	87	82	75	73	64	63	60
	0.216	12	156	123	110	107	100	93	91	79	78	75
	0.242	14	168	133	120	117	110	102	99	87	86	83
⅝	0.138	6	94	76	66	64	59	53	52	44	43	41
	0.151	7	104	83	72	70	64	58	56	48	47	45
	0.164	8	120	92	80	77	72	65	63	54	53	51
	0.177	9	136	103	91	88	81	74	72	62	61	58
	0.190	10	146	111	97	94	88	80	78	67	65	63
	0.216	12	173	133	117	114	106	97	95	82	80	77
	0.242	14	184	142	126	123	115	106	103	89	87	84
¾	0.138	6	94	79	72	71	65	58	57	47	46	44
	0.151	7	104	87	80	77	71	64	62	52	50	48
	0.164	8	120	101	88	85	78	71	69	58	56	54
	0.177	9	142	114	99	96	88	80	78	66	64	61
	0.190	10	153	122	107	103	95	86	83	71	69	66
	0.216	12	192	144	126	122	113	103	100	86	84	80

(Continued)

APPENDIX B.15 (*Continued*)

Wood Screws: Reference Lateral Design Values (*Z*) for Single-Shear (Two-Member) Connections

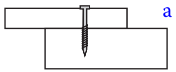


Side Member Thickness, t_s (in.)	Wood Screw Diameter, *D* (in.)	Wood Screw Number	$G = 0.67$, Red Oak (lb)	$G = 0.55$, Mixed Maple Southern Pine (lb)	$G = 0.5$, Douglas Fir-Larch (lb)	$G = 0.49$, Douglas Fir-Larch (North) (lb)	$G = 0.46$, Douglas Fir (South) Hem-Fir (North) (lb)	$G = 0.43$, Hem-Fir (lb)	$G = 0.42$, Spruce-Pine-Fir (lb)	$G = 0.37$, Redwood (Open Grain) (lb)	$G = 0.36$, Eastern Softwoods Spruce-Pine-Fir (South) Western Cedars Western Woods (lb)	$G = 0.35$, Northern Species (lb)
	0.242	14	203	154	135	131	122	111	108	93	91	87
1	0.138	6	94	79	72	71	67	63	61	55	54	51
	0.151	7	104	87	80	78	74	69	68	60	59	56
	0.164	8	120	101	92	90	85	80	78	67	65	62
	0.177	9	142	118	108	106	100	94	90	75	73	70
	0.190	10	153	128	117	114	108	101	97	81	78	75
	0.216	12	193	161	147	143	131	118	114	96	93	89
	0.242	14	213	178	157	152	139	126	122	102	100	95
1¼	0.138	6	94	79	72	71	67	63	61	55	54	52
	0.151	7	104	87	80	78	74	69	68	60	59	57
	0.164	8	120	101	92	90	85	80	78	70	68	66
	0.177	9	142	118	108	106	100	94	92	82	80	78
	0.190	10	153	128	117	114	108	101	99	88	87	84
	0.216	12	193	161	147	144	137	128	125	108	105	100
	0.242	14	213	178	163	159	151	141	138	115	111	106
1½	0.138	6	94	79	72	71	67	63	61	55	54	52
	0.151	7	104	87	80	78	74	69	68	60	59	57
	0.164	8	120	101	92	90	85	80	78	70	68	66
	0.177	9	142	118	108	106	100	94	92	82	80	78
	0.190	10	153	128	117	114	108	101	99	88	87	84
	0.216	12	193	161	147	144	137	128	125	111	109	106

(*Continued*)

APPENDIX B.15 (*Continued*)

Wood Screws: Reference Lateral Design Values (Z) for Single-Shear (Two-Member) Connections


[a]

Side Member Thickness, t_s (in.)	Wood Screw Diameter, D (in.)	Wood Screw Number	$G = 0.67$, Red Oak (lb)	$G = 0.55$, Mixed Maple Southern Pine (lb)	$G = 0.5$, Douglas Fir-Larch (lb)	$G = 0.49$, Douglas Fir-Larch (North) (lb)	$G = 0.46$, Douglas Fir (South) Hem-Fir (North) (lb)	$G = 0.43$, Hem-Fir (lb)	$G = 0.42$, Spruce-Pine-Fir (lb)	$G = 0.37$, Redwood (Open Grain) (lb)	$G = 0.36$, Eastern Softwoods Spruce-Pine-Fir (South) Western Cedars Western Woods (lb)	$G = 0.35$, Northern Species (lb)
	0.242	14	213	178	163	159	151	141	138	123	120	117
1¾	0.138	6	94	79	72	71	67	63	61	55	54	52
	0.151	7	104	87	80	78	74	69	68	60	59	57
	0.164	8	120	101	92	90	85	80	78	70	68	66
	0.177	9	142	118	108	106	100	94	92	82	80	78
	0.190	10	153	128	117	114	108	101	99	88	87	84
	0.216	12	193	161	147	144	137	128	125	111	109	106
	0.242	14	213	178	163	159	151	141	138	123	120	117

Source: Courtesy of the American Forest & Paper Association, Washington, DC.

[a] Single-shear connection.

APPENDIX B.16
Cut Thread or Rolled Thread Wood Screw Reference Withdrawal Design Values (*W*)

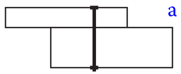
Pounds per Inch of Thread Penetration

Specific Gravity, *G*	Wood Screw Number										
	6	**7**	**8**	**9**	**10**	**12**	**14**	**16**	**18**	**20**	**24**
0.73	209	229	249	268	288	327	367	406	446	485	564
0.71	198	216	235	254	272	310	347	384	421	459	533
0.68	181	199	216	233	250	284	318	352	387	421	489
0.67	176	193	209	226	243	276	309	342	375	409	475
0.58	132	144	157	169	182	207	232	256	281	306	356
0.55	119	130	141	152	163	186	208	231	253	275	320
0.51	102	112	121	131	141	160	179	198	217	237	275
0.50	98	107	117	126	135	154	172	191	209	228	264
0.49	94	103	112	121	130	147	165	183	201	219	254
0.47	87	95	103	111	119	136	152	168	185	201	234
0.46	83	91	99	107	114	130	146	161	177	193	224
0.44	76	83	90	97	105	119	133	148	162	176	205
0.43	73	79	86	93	100	114	127	141	155	168	196
0.42	69	76	82	89	95	108	121	134	147	161	187
0.41	66	72	78	85	91	103	116	128	141	153	178
0.40	63	69	75	81	86	98	110	122	134	146	169
0.39	60	65	71	77	82	93	105	116	127	138	161
0.38	57	62	67	73	78	89	99	110	121	131	153
0.37	54	59	64	69	74	84	94	104	114	125	145
0.36	51	56	60	65	70	80	89	99	108	118	137
0.35	48	53	57	62	66	75	84	93	102	111	130
0.31	38	41	45	48	52	59	66	73	80	87	102

Source: Courtesy of the American Forest & Paper Association, Washington, DC.
Note: Tabulated withdrawal design values (*W*) are in pounds per inch of thread penetration into side grain of main member. Thread length is approximately two-third of the total wood screw length.

APPENDIX B.17

Bolts: Reference Lateral Design Values (*Z*) for Single-Shear (Two-Member) Connections



Thickness			G = 0.67, Red Oak				G = 0.55, Mixed Maple Southern Pine				G = 0.50, Douglas Fir-Larch				G = 0.49, Douglas Fir-Larch (North)				G = 0.46, Douglas Fir (South) Hem-Fir (North)			
Main member, t_m (in.)	Side member, t_s (in.)	Bolt diameter, D (in.)	$Z_{\parallel}$ (lb)	$Z_{s\perp}$ (lb)	$Z_{m\perp}$ (lb)	$Z_{\perp}$ (lb)	$Z_{\parallel}$ (lb)	$Z_{s\perp}$ (lb)	$Z_{m\perp}$ (lb)	$Z_{\perp}$ (lb)	$Z_{\parallel}$ (lb)	$Z_{s\perp}$ (lb)	$Z_{m\perp}$ (lb)	$Z_{\perp}$ (lb)	$Z_{\parallel}$ (lb)	$Z_{s\perp}$ (lb)	$Z_{m\perp}$ (lb)	$Z_{\perp}$ (lb)	$Z_{\parallel}$ (lb)	$Z_{s\perp}$ (lb)	$Z_{m\perp}$ (lb)	$Z_{\perp}$ (lb)
1½	1½	½	650	420	420	330	530	330	330	250	480	300	300	220	470	290	290	210	440	270	270	190
		⅝	810	500	500	370	660	400	400	280	600	360	360	240	590	350	350	240	560	320	320	220
		¾	970	580	580	410	800	460	460	310	720	420	420	270	710	400	400	260	670	380	380	240
		⅞	1130	660	660	440	930	520	520	330	850	470	470	290	830	460	460	280	780	420	420	250
		1	1290	740	740	470	1060	580	580	350	970	530	530	310	950	510	510	300	890	480	480	280
1¾	1¾	½	760	490	490	390	620	390	390	290	560	350	350	250	550	340	340	250	520	320	320	230
		⅝	940	590	590	430	770	470	470	330	700	420	420	280	690	410	410	280	650	380	380	250
		¾	1130	680	680	480	930	540	540	360	850	480	480	310	830	470	470	300	780	440	440	280
		⅞	1320	770	770	510	1080	610	610	390	990	550	550	340	970	530	530	320	910	500	500	300
		1	1510	860	860	550	1240	680	680	410	1130	610	610	360	1110	600	600	350	1040	560	560	320
2½	1½	½	770	480	540	440	660	400	420	350	610	370	370	310	610	360	360	300	580	340	330	270
		⅝	1070	660	630	520	930	560	490	390	850	520	430	340	830	520	420	330	780	470	390	300
		¾	1360	890	720	570	1120	660	560	430	1020	590	500	380	1000	560	480	360	940	520	450	330
		⅞	1590	960	800	620	1300	720	620	470	1190	630	550	410	1170	600	540	390	1090	550	500	360
		1	1820	1020	870	660	1490	770	680	490	1360	680	610	440	1330	650	590	420	1250	600	550	390
3½	1½	½	770	480	560	440	660	400	470	360	610	370	430	330	610	360	420	320	580	340	400	310
		⅝	1070	660	760	590	940	560	620	500	880	520	540	460	870	520	530	450	830	470	490	410
		¾	1450	890	900	770	1270	660	690	580	1200	590	610	510	1190	560	590	490	1140	520	550	450
		⅞	1890	960	990	830	1680	720	770	630	1590	630	680	550	1570	600	650	530	1470	550	600	480
		1	2410	1020	1080	890	2010	770	830	670	1830	680	740	590	1790	650	710	560	1680	600	660	520

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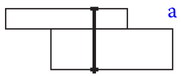
APPENDIX B.17 (*Continued*)

Bolts: Reference Lateral Design Values (Z) for Single-Shear (Two-Member) Connections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Thickness			G = 0.67, Red Oak				G = 0.55, Mixed Maple Southern Pine				G = 0.50, Douglas Fir-Larch				G = 0.49, Douglas Fir-Larch (North)				G = 0.46, Douglas Fir (South) Hem-Fir (North)			
Main member, t_m (in.)	Side member, t_s (in.)	Bolt diameter, D (in.)	$Z_\parallel$ (lb)	$Z_{s\perp}$ (lb)	$Z_{m\perp}$ (lb)	$Z_\perp$ (lb)	$Z_\parallel$ (lb)	$Z_{s\perp}$ (lb)	$Z_{m\perp}$ (lb)	$Z_\perp$ (lb)	$Z_\parallel$ (lb)	$Z_{s\perp}$ (lb)	$Z_{m\perp}$ (lb)	$Z_\perp$ (lb)	$Z_\parallel$ (lb)	$Z_{s\perp}$ (lb)	$Z_{m\perp}$ (lb)	$Z_\perp$ (lb)	$Z_\parallel$ (lb)	$Z_{s\perp}$ (lb)	$Z_{m\perp}$ (lb)	$Z_\perp$ (lb)
3½	1¾	½	830	510	590	480	720	420	510	390	670	380	470	350	660	380	460	340	620	360	440	320
		⅝	1160	680	820	620	1000	580	640	520	930	530	560	460	920	530	550	450	880	500	510	410
		¾	1530	900	940	780	1330	770	720	580	1250	680	640	520	1240	660	620	500	1190	600	580	460
		⅞	1970	1120	1040	840	1730	840	810	640	1620	740	710	550	1590	700	690	530	1490	640	640	490
		1	2480	1190	1130	900	2030	890	880	670	1850	790	780	590	1820	750	760	570	1700	700	700	530
3½	3½	½	830	590	590	530	750	520	520	460	720	490	490	430	710	480	480	420	690	460	460	410
		⅝	1290	880	880	780	1170	780	780	650	1120	700	700	560	1110	690	690	550	1070	650	650	500
		¾	1860	1190	1190	950	1690	960	960	710	1610	870	870	630	1600	850	850	600	1540	800	800	560
		⅞	2540	1410	1410	1030	2170	1160	1160	780	1970	1060	1060	680	1940	1040	1040	650	1810	980	980	590
		1	3020	1670	1670	1100	2480	1360	1360	820	2260	1230	1230	720	2210	1190	1190	690	2070	1110	1110	640
5¼	1½	⅝	1070	660	760	590	940	560	640	500	880	520	590	460	870	520	590	450	830	470	560	430
		¾	1450	890	990	780	1270	660	850	660	1200	590	790	590	1190	560	780	560	1140	520	740	520
		⅞	1890	960	1260	960	1680	720	1060	720	1590	630	940	630	1570	600	900	600	1520	550	830	550
		1	2410	1020	1500	1020	2150	770	1140	770	2050	680	1010	680	2030	650	970	650	1930	600	910	600
5¼	1¾	⅝	1160	680	820	620	1000	580	690	520	930	530	630	470	920	530	630	470	880	500	590	440
		¾	1530	900	1050	800	1330	770	890	680	1250	680	830	630	1240	660	810	620	1190	600	780	590
		⅞	1970	1120	1320	1020	1730	840	1090	840	1640	740	960	740	1620	700	920	700	1550	640	850	640
		1	2480	1190	1530	1190	2200	890	1170	890	2080	790	1040	790	2060	750	1000	750	1990	700	930	700
5¼	3½	⅝	1290	880	880	780	1170	780	780	680	1120	700	730	630	1110	690	720	620	1070	650	690	580
		¾	1860	1190	1240	1080	1690	960	1090	850	1610	870	1030	780	1600	850	1010	750	1540	800	970	710

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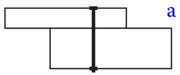
APPENDIX B.17 (*Continued*)

Bolts: Reference Lateral Design Values (Z) for Single-Shear (Two-Member) Connections



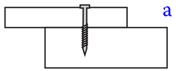
Thickness			G = 0.67, Red Oak				G = 0.55, Mixed Maple Southern Pine				G = 0.50, Douglas Fir-Larch				G = 0.49, Douglas Fir-Larch (North)				G = 0.46, Douglas Fir (South) Hem-Fir (North)			
Main member, t_m (in.)	Side member, t_s (in.)	Bolt diameter, D (in.)	$Z_{\parallel}$ (lb)	$Z_{s\perp}$ (lb)	$Z_{m\perp}$ (lb)	$Z_{\perp}$ (lb)	$Z_{\parallel}$ (lb)	$Z_{s\perp}$ (lb)	$Z_{m\perp}$ (lb)	$Z_{\perp}$ (lb)	$Z_{\parallel}$ (lb)	$Z_{s\perp}$ (lb)	$Z_{m\perp}$ (lb)	$Z_{\perp}$ (lb)	$Z_{\parallel}$ (lb)	$Z_{s\perp}$ (lb)	$Z_{m\perp}$ (lb)	$Z_{\perp}$ (lb)	$Z_{\parallel}$ (lb)	$Z_{s\perp}$ (lb)	$Z_{m\perp}$ (lb)	$Z_{\perp}$ (lb)
		⅞	2540	1410	1640	1260	2300	1160	1380	1000	2190	1060	1230	870	2170	1040	1190	840	2060	980	1100	770
		1	3310	1670	1940	1420	2870	1390	1520	1060	2660	1290	1360	940	2630	1260	1320	900	2500	1210	1230	830
5½	1½	⅝	1070	660	760	590	940	560	640	500	880	520	590	460	870	520	590	450	830	470	560	430
		¾	1450	890	990	780	1270	660	850	660	1200	590	790	590	1190	560	780	560	1140	520	740	520
		⅞	1890	960	1260	960	1680	720	1090	720	1590	630	980	630	1570	600	940	600	1520	550	860	550
		1	2410	1020	1560	1020	2150	770	1190	770	2050	680	1060	680	2030	650	1010	650	1930	600	940	600
5½	3½	⅝	1290	880	880	780	1170	780	780	680	1120	700	730	630	1110	690	720	620	1070	650	690	580
		¾	1860	1190	1240	1080	1690	960	1090	850	1610	870	1030	780	1600	850	1010	750	1540	800	970	710
		⅞	2540	1410	1640	1260	2300	1160	1410	1020	2190	1060	1260	910	2170	1040	1220	870	2060	980	1130	790
		1	3310	1670	1980	1470	2870	1390	1550	1100	2660	1290	1390	970	2630	1260	1340	930	2500	1210	1250	860
7½	1½	⅝	1070	660	760	590	940	560	640	500	880	520	590	460	870	520	590	450	830	470	560	430
		¾	1450	890	990	780	1270	660	850	660	1200	590	790	590	1190	560	780	560	1140	520	740	520
		⅞	1890	960	1260	960	1680	720	1090	720	1590	630	1010	630	1570	600	990	600	1520	550	950	550
		1	2410	1020	1560	1020	2150	770	1350	770	2050	680	1270	680	2030	650	1240	650	1930	600	1190	600
7½	3½	⅝	1290	880	880	780	1170	780	780	680	1120	700	730	630	1110	690	720	620	1070	650	690	580
		¾	1860	1190	1240	1080	1690	960	1090	850	1610	870	1030	780	1600	850	1010	750	1540	800	970	710
		⅞	2540	1410	1640	1260	2300	1160	1450	1020	2190	1060	1360	930	2170	1040	1340	900	2060	980	1280	850
		1	3310	1670	2090	1470	2870	1390	1830	1210	2660	1290	1630	1110	2630	1260	1570	1080	2500	1210	1470	1030

Source: Courtesy of the American Forest & Paper Association, Washington, DC.

[a] Single-shear connection.

APPENDIX B.18

Lag Screws: Reference Lateral Design Values (*Z*) for Single-Shear (Two-Member) Connections

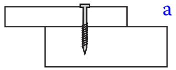


Side Member Thickness, t_s (in.)	Lag Screw Diameter, *D* (in.)	*G* = 0.67, Red Oak				*G* = 0.55, Mixed Maple Southern Pine				*G* = 0.50, Douglas Fir-Larch				*G* = 0.49, Douglas Fir-Larch (North)				*G* = 0.46, Douglas Fir (South) Hem-Fir (North)			
		$Z_{\parallel}$ (lb)	$Z_{s\perp}$ (lb)	$Z_{m\perp}$ (lb)	$Z_{\perp}$ (lb)	$Z_{\parallel}$ (lb)	$Z_{s\perp}$ (lb)	$Z_{m\perp}$ (lb)	$Z_{\perp}$ (lb)	$Z_{\parallel}$ (lb)	$Z_{s\perp}$ (lb)	$Z_{m\perp}$ (lb)	$Z_{\perp}$ (lb)	$Z_{\parallel}$ (lb)	$Z_{s\perp}$ (lb)	$Z_{m\perp}$ (lb)	$Z_{\perp}$ (lb)	$Z_{\parallel}$ (lb)	$Z_{s\perp}$ (lb)	$Z_{m\perp}$ (lb)	$Z_{\perp}$ (lb)
½	¼	150	110	110	110	130	90	100	90	120	90	90	80	120	90	90	80	110	80	90	80
	5⁄16	170	130	130	120	150	110	120	100	150	100	110	100	140	100	110	90	140	100	100	90
	⅜	180	130	130	120	160	110	110	100	150	100	110	90	150	90	110	90	140	90	100	90
⅝	¼	160	120	130	120	140	100	110	100	130	90	100	90	130	90	100	90	120	90	90	80
	5⁄16	190	140	140	130	160	110	120	110	150	110	110	100	150	100	110	100	150	100	110	90
	⅜	190	130	140	120	170	110	120	100	160	100	110	100	160	100	110	90	150	100	110	90
¾	¼	180	140	140	130	150	110	120	110	140	100	110	100	140	100	110	90	130	90	100	90
	5⁄16	210	150	160	140	180	120	130	120	170	110	120	100	160	110	120	100	160	100	110	100
	⅜	210	140	160	130	180	120	130	110	170	110	120	100	170	110	120	100	160	100	110	90
1	¼	180	140	140	140	160	120	120	120	150	120	120	110	150	110	110	110	150	110	110	100
	5⁄16	230	170	170	160	210	140	150	130	190	130	140	120	190	120	140	120	180	120	130	110
	⅜	230	160	170	160	210	130	150	120	200	120	140	110	190	120	140	110	180	110	130	100
1¼	¼	180	140	140	140	160	120	120	120	150	120	120	110	150	110	110	110	150	110	110	100
	5⁄16	230	170	170	160	210	150	150	140	200	140	140	130	200	140	140	130	190	130	140	120
	⅜	230	170	170	160	210	150	150	140	200	140	140	130	200	130	140	120	190	120	140	120
1½	¼	180	140	140	140	160	120	120	120	150	120	120	110	150	110	110	110	150	110	110	100
	5⁄16	230	170	170	160	210	150	150	140	200	140	140	130	200	140	140	130	190	140	140	130
	⅜	230	170	170	160	210	150	150	140	200	140	140	130	200	140	140	130	190	140	140	120
	7⁄16	360	260	260	240	320	220	230	200	310	200	210	180	310	190	210	180	300	180	200	160
	½	460	310	320	280	410	250	290	230	390	220	270	200	390	220	260	200	370	210	250	190
	⅝	700	410	500	370	600	340	420	310	560	310	380	280	550	310	380	270	530	290	360	260

(*Continued*)

APPENDIX B.18 (*Continued*)

Lag Screws: Reference Lateral Design Values (Z) for Single-Shear (Two-Member) Connections



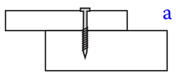
a

Side Member Thickness, t_s (in.)	Lag Screw Diameter, D (in.)	$G = 0.67$, Red Oak				$G = 0.55$, Mixed Maple Southern Pine				$G = 0.50$, Douglas Fir-Larch				$G = 0.49$, Douglas Fir-Larch (North)				$G = 0.46$, Douglas Fir (South) Hem-Fir (North)			
		$Z_\parallel$ (lb)	$Z_{s\perp}$ (lb)	$Z_{m\perp}$ (lb)	$Z_\perp$ (lb)	$Z_\parallel$ (lb)	$Z_{s\perp}$ (lb)	$Z_{m\perp}$ (lb)	$Z_\perp$ (lb)	$Z_\parallel$ (lb)	$Z_{s\perp}$ (lb)	$Z_{m\perp}$ (lb)	$Z_\perp$ (lb)	$Z_\parallel$ (lb)	$Z_{s\perp}$ (lb)	$Z_{m\perp}$ (lb)	$Z_\perp$ (lb)	$Z_\parallel$ (lb)	$Z_{s\perp}$ (lb)	$Z_{m\perp}$ (lb)	$Z_\perp$ (lb)
	¾	950	550	660	490	830	470	560	410	770	440	510	380	760	430	510	370	730	400	480	360
	⅞	1240	720	830	630	1080	560	710	540	1020	490	660	490	1010	470	650	470	970	430	610	430
	1	1550	800	1010	780	1360	600	870	600	1290	530	810	530	1280	500	790	500	1230	470	760	470
1¾	¼	180	140	140	140	160	120	120	120	150	120	120	110	150	110	110	110	150	110	110	100
	5/16	230	170	170	160	210	150	150	140	200	140	140	130	200	140	140	130	190	140	140	130
	⅜	230	170	170	160	210	150	150	140	200	140	140	130	200	140	140	130	190	140	140	120
	7/16	360	260	260	240	320	230	230	210	310	210	210	190	310	210	210	190	300	200	200	180
	½	460	320	320	290	410	270	290	250	390	240	270	220	390	240	260	220	380	220	250	200
	⅝	740	440	500	400	660	360	440	320	610	330	420	290	600	320	410	290	570	300	390	270
	¾	1030	580	720	520	890	480	600	430	830	450	550	390	820	440	540	380	780	420	510	360
	⅞	1320	740	890	650	1150	630	750	550	1070	570	700	510	1060	550	680	490	1010	500	650	470
	1	1630	910	1070	790	1420	700	910	670	1340	610	850	610	1320	590	830	590	1270	550	790	550
2½	¼	180	140	140	140	160	120	120	120	150	120	120	110	150	110	110	110	150	110	110	100
	5/16	230	170	170	160	210	150	150	140	200	140	140	130	200	140	140	130	190	140	140	130
	⅜	230	170	170	160	210	150	150	140	200	140	140	130	200	140	140	130	190	140	140	120
	7/16	360	260	260	240	320	230	230	210	310	210	210	190	310	210	210	190	300	200	200	180
	½	460	320	320	290	410	290	290	250	390	270	270	240	390	260	260	230	380	250	250	220
	⅝	740	500	500	450	670	430	440	390	640	390	420	350	630	380	410	340	610	360	390	320
	¾	1110	680	740	610	1010	550	650	490	960	500	610	450	950	490	600	430	920	460	580	410
	⅞	1550	830	1000	740	1370	690	880	600	1280	630	830	550	1260	620	810	530	1190	580	770	500
	1	1940	980	1270	860	1660	830	1080	720	1550	770	990	660	1520	750	970	640	1450	720	920	620
3½	¼	180	140	140	140	160	120	120	120	150	120	120	110	150	110	110	110	150	110	110	100

(*Continued*)

APPENDIX B.18 (*Continued*)

Lag Screws: Reference Lateral Design Values (*Z*) for Single-Shear (Two-Member) Connections


[a]

Side Member Thickness, t_s (in.)	Lag Screw Diameter, D (in.)	G = 0.67, Red Oak				G = 0.55, Mixed Maple Southern Pine				G = 0.50, Douglas Fir-Larch				G = 0.49, Douglas Fir-Larch (North)				G = 0.46, Douglas Fir (South) Hem-Fir (North)			
		$Z_{\parallel}$ (lb)	$Z_{s\perp}$ (lb)	$Z_{m\perp}$ (lb)	$Z_{\perp}$ (lb)	$Z_{\parallel}$ (lb)	$Z_{s\perp}$ (lb)	$Z_{m\perp}$ (lb)	$Z_{\perp}$ (lb)	$Z_{\parallel}$ (lb)	$Z_{s\perp}$ (lb)	$Z_{m\perp}$ (lb)	$Z_{\perp}$ (lb)	$Z_{\parallel}$ (lb)	$Z_{s\perp}$ (lb)	$Z_{m\perp}$ (lb)	$Z_{\perp}$ (lb)	$Z_{\parallel}$ (lb)	$Z_{s\perp}$ (lb)	$Z_{m\perp}$ (lb)	$Z_{\perp}$ (lb)
	5/16	230	170	170	160	210	150	150	140	200	140	140	130	200	140	140	130	190	140	140	130
	3/8	230	170	170	160	210	150	150	140	200	140	140	130	200	140	140	130	190	140	140	120
	7/16	360	260	260	240	320	230	230	210	310	210	210	190	310	210	210	190	300	200	200	180
	1/2	460	320	320	290	410	290	290	250	390	270	270	240	390	260	260	230	380	250	250	220
	5/8	740	500	500	450	670	440	440	390	640	420	420	360	630	410	410	360	610	390	390	340
	3/4	1110	740	740	650	1010	650	650	560	960	600	610	520	950	580	600	510	920	550	580	490
	7/8	1550	990	1000	860	1400	800	880	710	1340	720	830	640	1320	700	810	620	1280	660	780	570
	1	2020	1140	1270	1010	1830	930	1120	810	1740	850	1060	740	1730	830	1040	720	1670	790	1000	680

Source: Courtesy of the American Forest & Paper Association, Washington, DC.

[a] Single-shear connection.

APPENDIX B.19
Lag Screw Reference Withdrawal Design Values (*W*)

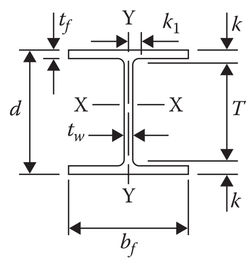
Pounds per Inch of Thread Penetration

	Lag Screw Unthreaded Shank Diameter, *D*										
Specific Gravity, *G*	**¼ in.**	**5⁄16 in.**	**⅜ in.**	**7⁄16 in.**	**½ in.**	**⅝ in.**	**¾ in.**	**⅞ in.**	**1 in.**	**1⅛ in.**	**1¼ in.**
0.73	397	469	538	604	668	789	905	1016	1123	1226	1327
0.71	381	450	516	579	640	757	868	974	1077	1176	1273
0.68	357	422	484	543	600	709	813	913	1009	1103	1193
0.67	349	413	473	531	587	694	796	893	987	1078	1167
0.58	281	332	381	428	473	559	641	719	795	869	940
0.55	260	307	352	395	437	516	592	664	734	802	868
0.51	232	274	314	353	390	461	528	593	656	716	775
0.50	225	266	305	342	378	447	513	576	636	695	752
0.49	218	258	296	332	367	434	498	559	617	674	730
0.47	205	242	278	312	345	408	467	525	580	634	686
0.46	199	235	269	302	334	395	453	508	562	613	664
0.44	186	220	252	283	312	369	423	475	525	574	621
0.43	179	212	243	273	302	357	409	459	508	554	600
0.42	173	205	235	264	291	344	395	443	490	535	579
0.41	167	198	226	254	281	332	381	428	473	516	559
0.40	161	190	218	245	271	320	367	412	455	497	538
0.39	155	183	210	236	261	308	353	397	438	479	518
0.38	149	176	202	227	251	296	340	381	422	461	498
0.37	143	169	194	218	241	285	326	367	405	443	479
0.36	137	163	186	209	231	273	313	352	389	425	460
0.35	132	156	179	200	222	262	300	337	373	407	441
0.31	110	130	149	167	185	218	250	281	311	339	367

Source: Courtesy of the American Forest & Paper Association, Washington, DC.
Notes: Tabulated withdrawal design values (*W*) are in pounds per inch of thread penetration into side grain of main member. Length of thread penetration in main member shall not include the length of the tapered tip.

Appendix C: Steel

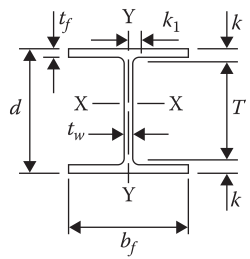
APPENDIX C.1a
W Shapes: Dimensions



				Web			Flange				Distance				
											k				
Shape	Area, A (in.^{2})	Depth, d (in.)		Thickness, t_w (in.)		$\frac{t_w}{2}$ (in.)	Width, b_f (in.)		Thickness, t_f (in.)		k_{des} (in.)	k_{det} (in.)	k_1 (in.)	T (in.)	Workable Gage (in.)
W21 × 93	27.3	21.6	21 5/8	0.580	9/16	5/16	8.42	8 3/8	0.930	15/16	1.43	1 5/8	15/16	18 3/8	5 1/2
× 83[a]	24.4	21.4	21 3/8	0.515	1/2	1/4	8.36	8 3/6	0.835	13/16	1.34	1 1/2	7/8		
× 73[a]	21.5	21.2	21 1/4	0.455	7/16	1/4	8.30	8 1/4	0.740	3/4	1.24	1 7/16	7/8		
× 68[a]	20.0	21.1	21 1/8	0.430	7/16	1/4	8.27	8 1/4	0.685	11/16	1.19	1 3/8	7/8		
× 62[a]	18.3	21.0	21	0.400	3/8	3/16	8.24	8 1/4	0.615	5/8	1.12	1 5/16	13/16		
× 55[a]	16.2	20.8	20 3/4	0.375	3/8	3/16	8.22	8 1/4	0.522	1/2	1.02	1 3/16	13/16		
× 48[a,b]	14.1	20.6	20 5/8	0.350	3/8	3/16	8.14	8 1/8	0.430	7/16	0.930	1 1/8	13/16		
W21 × 57[a]	16.7	21.1	21	0.405	3/8	3/16	6.56	6 1/2	0.650	5/8	1.15	1 5/16	13/16	18 3/8	3 1/2
× 50[a]	14.1	20.8	20	0.380	3/8	3/16	6.53	6 1/2	0.535	9/16	1.04	1 1/4	13/16		
× 44[a]	13.0	20.7	20 5/8	0.350	3/8	3/16	6.50	6 1/2	0.450	7/16	0.950	1 1/8	13/16		
W18 × 311[d]	91.6	22.3	22 3/8	1.52	1 1/2	3/4	12.0	12	2.74	2 3/4	3.24	3 7/16	1 3/8	15 1/2	5 1/2
× 283[d]	83.3	21.9	21 7/8	1.40	1 3/8	11/16	11.8	11 7/8	2.50	2 1/2	3.00	3 3/16	1 5/16		
× 258[d]	76.0	21.5	21 1/2	1.28	1 1/4	5/8	11.8	11 3/4	2.30	2 5/16	2.70	3	1 1/4		
× 234[d]	68.8	21.1	21	1.16	1 3/16	5/8	11.7	11 5/8	2.11	2 1/8	2.51	2 3/4	1 3/16		
× 211	62.3	20.7	20 5/8	1.06	1 1/16	9/16	11.6	11 1/2	1.91	1 15/16	2.31	2 9/16	1 3/16		
× 192	56.2	20.4	20 3/8	0.960	15/16	1/2	11.5	11 1/2	1.75	1 3/4	2.15	2 7/16	1 1/8		
× 175	51.4	20.0	20	0.890	7/8	7/16	11.4	11 3/8	1.59	1 9/16	1.99	2 7/16	1 1/4	15 1/8	

(Continued)

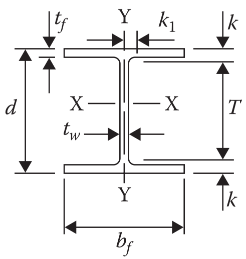
APPENDIX C.1a (*Continued*)
W Shapes: Dimensions



Shape	Area, A (in.^{2})	Depth, d (in.)	Web: Thickness, t_w (in.)			$\frac{t_w}{2}$ (in.)	Flange: Width, b_f (in.)		Flange: Thickness, t_f (in.)		Distance: k: k_{des} (in.)	Distance: k: k_{det} (in.)	Distance: k_1 (in.)	Distance: T (in.)	Distance: Workable Gage (in.)
× 158	46.3	19.7	19 3/4	0.810	13/16	7/16	11.3	11 1/4	1.44	1 7/16	1.84	2 3/8	1 1/4		
× 143	42.0	19.5	19 1/2	0.730	3/4	3/8	11.2	11 1/4	1.32	1 5/6	1.72	2 3/16	1 3/6		
× 130	38.3	19.3	19 1/4	0.670	11/16	3/8	11.2	11 1/8	1.20	1 3/16	1.60	2 1/16	1 3/16		
× 119	35.1	19.0	19	0.655	5/8	5/16	11.3	11 1/4	1.06	1 1/16	1.46	11 3/16	1 3/16		
× 106	31.1	18.7	18 3/4	0.590	9/16	5/16	11.2	11 1/4	0.940	15/16	1.34	11 3/16	1 1/8		
× 97	28.5	18.6	18 5/8	0.535	9/16	5/16	11.1	11 1/8	0.870	7/8	1.27	1 3/4	1 1/8		
× 86	25.3	18.4	18 3/8	0.480	1/2	1/4	11.1	11 1/8	0.770	3/4	1.17	1 5/8	1 1/16		
× 76[a]	22.3	18.2	18 1/4	0.425	7/16	1/4	11.0	11	0.680	11/16	1.08	1 9/16	1 1/16	↓	↓
W18 × 71	20.9	18.5	18 1/2	0.495	1/2	1/4	7.64	7 5/8	0.810	13/16	1.21	1 1/2	7/8	15 1/2	3 1/2[c]
× 65	19.1	18.4	18 3/8	0.450	7/16	1/4	7.59	7 5/8	0.750	3/4	1.15	1 7/16	7/8		
× 60[a]	17.6	18.2	18 1/4	0.415	7/16	1/4	7.56	7 1/2	0.695	11/16	1.10	1 3/8	13/16		
× 55[a]	16.2	18.1	18 1/8	0.390	3/8	3/16	7.53	7 1/2	0.630	5/8	1.03	1 5/16	13/16		
× 50[a]	14.7	18.0	18	0.355	3/8	3/16	7.50	7 1/2	0.570	9/16	0.972	1 1/4	13/16	↓	↓
W18 × 46[a]	13.5	18.1	18	0.360	3/8	3/16	6.06	6	0.605	5/8	1.01	1 1/4	13/16	15 1/2	3 1/2[c]
× 40[a]	11.8	17.9	17 7/8	0.315	5/16	3/16	6.02	6	0.525	1/2	0.927	1 3/16	13/16		
× 35[a]	10.3	17.7	17 3/4	0.300	5/16	3/16	6.00	6	0.425	7/16	0.827	1 1/8	3/4	↓	↓

(*Continued*)

APPENDIX C.1a (*Continued*)
W Shapes: Dimensions



Shape	Area, A (in.^{2})	Depth, d (in.)		Web Thickness, t_w (in.)		Web $\frac{t_w}{2}$ (in.)	Flange Width, b_f (in.)		Flange Thickness, t_f (in.)		Distance k: k_{des} (in.)	Distance k: k_{det} (in.)	Distance k_1 (in.)	Distance T (in.)	Distance Workable Gage (in.)
W16 × 100	29.4	17.0	17	0.585	9/16	5/16	10.4	$10\frac{3}{8}$	0.985	1	1.39	$1\frac{7}{8}$	$1\frac{1}{8}$	$13\frac{1}{4}$	$5\frac{1}{2}$
× 89	26.2	16.8	$16\frac{3}{4}$	0.525	1/2	1/4	10.4	$10\frac{3}{8}$	0.875	7/8	1.28	$1\frac{3}{4}$	$1\frac{1}{16}$		
× 77	22.6	16.5	$16\frac{1}{2}$	0.455	7/16	1/4	10.3	$10\frac{1}{4}$	0.760	3/4	1.16	$1\frac{5}{8}$	$1\frac{1}{16}$		
× 67[a]	19.6	16.3	$16\frac{3}{8}$	0.395	3/8	3/16	10.2	$10\frac{1}{4}$	0.665	11/16	1.07	$1\frac{9}{16}$	1		
W16 × 57	16.8	16.4	$16\frac{3}{8}$	0.430	7/16	1/4	7.12	$7\frac{1}{8}$	0.715	11/16	1.12	$1\frac{3}{8}$	7/8	$13\frac{5}{8}$	$3\frac{1}{2}$[c]
× 50[a]	14.7	16.3	$16\frac{1}{4}$	0.380	3/8	3/16	7.07	$7\frac{1}{8}$	0.630	5/8	1.03	$1\frac{5}{16}$	13/16		
× 45[a]	13.3	16.1	$16\frac{1}{8}$	0.345	3/8	3/16	7.04	7	0.565	9/16	0.967	$1\frac{1}{4}$	13/16		
× 40[a]	11.8	16.0	16	0.305	5/15	3/16	7.00	7	0.505	1/2	0.907	$1\frac{3}{16}$	13/16		
× 36[a]	10.6	15.9	$15\frac{7}{8}$	0.295	5/16	3/16	6.99	7	0.430	7/16	0.832	$1\frac{1}{8}$	3/4		
W16 × 31[a]	9.13	15.9	$15\frac{7}{8}$	0.275	1/4	1/8	5.53	$5\frac{1}{2}$	0.440	7/16	0.842	$1\frac{1}{8}$	3/4	$13\frac{5}{8}$	$3\frac{1}{2}$
× 26[a,e]	7.68	15.7	$15\frac{3}{4}$	0.250	1/4	1/8	5.50	$5\frac{1}{2}$	0.345	3/8	0.747	$1\frac{1}{16}$	3/4	$13\frac{5}{8}$	$3\frac{1}{2}$
W14 × 730[d]	215	22.4	$22\frac{3}{8}$	3.07	$3\frac{1}{16}$	$1\frac{9}{16}$	17.9	$17\frac{7}{8}$	4.91	$4\frac{15}{16}$	5.51	$6\frac{3}{16}$	$2\frac{3}{4}$	10	$3\text{–}7\frac{1}{2}_{2\text{–}3}$[e]
× 665[d]	196	21.6	$21\frac{5}{8}$	2.83	$2\frac{13}{16}$	$1\frac{7}{16}$	17.7	$17\frac{5}{8}$	4.52	$4\frac{1}{2}$	5.12	$5\frac{13}{16}$	$2\frac{5}{8}$		$3\text{–}7\frac{1}{2}_{2\text{–}3}$[e]
× 605[d]	178	20.9	$20\frac{7}{8}$	2.60	$2\frac{5}{8}$	$1\frac{5}{16}$	17.4	$17\frac{3}{8}$	4.16	$4\frac{3}{16}$	4.76	$5\frac{7}{16}$	$2\frac{1}{2}$		$3\text{–}7\frac{1}{2}_{2\text{–}3}$
× 550[d]	162	20.2	$20\frac{1}{4}$	2.38	$2\frac{3}{8}$	$1\frac{3}{16}$	17.2	$17\frac{1}{4}$	3.82	$3\frac{13}{16}$	4.42	$5\frac{1}{8}$	$2\frac{3}{8}$		
× 500[d]	147	19.6	$19\frac{5}{8}$	2.19	$2\frac{3}{16}$	$1\frac{1}{8}$	17.0	17	3.50	$3\frac{1}{2}$	4.10	$4\frac{13}{16}$	$2\frac{5}{8}$		
× 455[d]	134	19.0	19	2.02	2	1	16.8	$16\frac{7}{8}$	3.21	$3\frac{3}{16}$	3.81	$4\frac{1}{2}$	$2\frac{1}{4}$		
× 426[d]	125	18.7	$18\frac{5}{8}$	1.88	$1\frac{7}{8}$	15/16	16.7	$16\frac{3}{4}$	3.04	$3\frac{1}{16}$	3.63	$4\frac{5}{16}$	$2\frac{1}{8}$		

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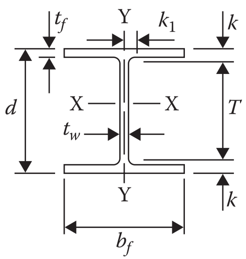
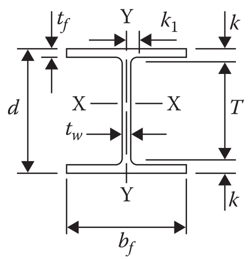
APPENDIX C.1a (*Continued*)
W Shapes: Dimensions



			Web				Flange				Distance				
											k				
Shape	Area, A (in.^{2})	Depth, d (in.)	Thickness, t_w (in.)			$\frac{t_w}{2}$ (in.)	Width, b_f (in.)		Thickness, t_f (in.)		k_{des} (in.)	k_{det} (in.)	k_1 (in.)	T (in.)	Workable Gage (in.)
× 398[d]	117	18.3	$18\frac{1}{4}$	1.77	$1\frac{3}{4}$	7/8	16.6	$16\frac{5}{8}$	2.85	$2\frac{7}{8}$	3.44	$4\frac{1}{8}$	$2\frac{1}{8}$	↓	↓
× 370[d]	109	17.9	$17\frac{7}{8}$	1.66	$1\frac{5}{8}$	13/16	16.5	$16\frac{1}{2}$	2.66	$2\frac{11}{16}$	3.26	$3\frac{15}{16}$	$2\frac{1}{18}$		
× 342[d]	101	17.5	$17\frac{1}{2}$	1.54	$1\frac{9}{16}$	13/16	16.4	$16\frac{3}{8}$	2.47	$2\frac{1}{2}$	3.07	$3\frac{3}{4}$	2		
× 311[d]	91.4	17.1	$17\frac{1}{8}$	1.41	$1\frac{7}{16}$	3/4	16.2	$16\frac{1}{4}$	2.26	$2\frac{1}{4}$	2.86	$3\frac{9}{16}$	$1\frac{15}{16}$		
× 283[d]	83.3	16.7	$16\frac{3}{4}$	1.29	$1\frac{5}{16}$	11/16	16.1	$16\frac{1}{8}$	2.07	$2\frac{1}{16}$	2.67	$3\frac{3}{8}$	$1\frac{7}{8}$		
× 257	75.6	16.4	$16\frac{3}{8}$	1.18	$1\frac{3}{16}$	5/8	16.0	16	1.89	$1\frac{7}{8}$	2.49	$3\frac{3}{16}$	$1\frac{13}{16}$		
× 233	68.5	16.0	16	1.07	$1\frac{1}{16}$	9/16	15.9	$15\frac{7}{8}$	1.72	$1\frac{3}{4}$	2.32	3	$1\frac{3}{4}$		
× 211	62.0	15.7	$15\frac{3}{4}$	0.980	1	1/2	15.8	$15\frac{3}{4}$	1.56	$1\frac{9}{16}$	2.16	$2\frac{7}{8}$	$1\frac{11}{16}$		
× 193	56.8	15.5	$15\frac{1}{2}$	0.890	7/8	7/16	15.7	$15\frac{3}{4}$	1.44	$1\frac{7}{16}$	2.04	$2\frac{3}{4}$	$1\frac{11}{16}$		
× 176	51.8	15.2	$15\frac{1}{4}$	0.830	13/16	7/16	15.7	$15\frac{5}{8}$	1.31	$1\frac{5}{16}$	1.91	$2\frac{5}{8}$	$1\frac{5}{8}$		
× 159	46.7	15.0	15	0.745	3/4	3/8	15.6	$15\frac{5}{8}$	1.19	$1\frac{3}{16}$	1.79	$2\frac{1}{2}$	$1\frac{9}{16}$		
× 145	42.7	14.8	$14\frac{3}{4}$	0.680	11/16	3/8	15.5	$15\frac{1}{2}$	1.09	$1\frac{1}{16}$	1.69	$2\frac{3}{8}$	$1\frac{9}{16}$	↓	↓
W14 × 132	38.8	14.7	$14\frac{5}{8}$	0.645	5/8	5/16	14.7	$14\frac{3}{4}$	1.03	1	1.63	$2\frac{5}{16}$	$1\frac{9}{16}$	10	$5\frac{1}{2}$
× 120	35.3	14.5	$14\frac{1}{2}$	0.590	9/16	5/16	14.7	$14\frac{5}{8}$	0.940	15/16	1.54	$2\frac{1}{4}$	$1\frac{1}{2}$	↓	↓
× 109	32.0	14.3	$14\frac{3}{8}$	0.525	1/2	1/4	14.6	$14\frac{5}{8}$	0.860	7/8	1.46	$2\frac{3}{16}$	$1\frac{1}{2}$		
× 99[b]	29.1	14.2	$14\frac{1}{8}$	0.485	1/2	1/4	14.6	$14\frac{5}{8}$	0.780	3/4	1.38	$2\frac{1}{16}$	$1\frac{7}{16}$		
× 90[b]	26.5	14.0	14	0.440	7/16	1/4	14.5	$14\frac{1}{2}$	0.710	11/16	1.31	2	$1\frac{7}{16}$	↓	↓

(*Continued*)

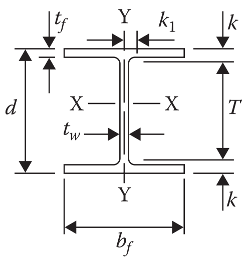
APPENDIX C.1a (*Continued*)
W Shapes: Dimensions



			Web			Flange					Distance				
											k				
Shape	Area, A (in.^{2})	Depth, d (in.)	Thickness, t_w (in.)			$\frac{t_w}{2}$ (in.)	Width, b_f (in.)		Thickness, t_f (in.)		k_{des} (in.)	k_{det} (in.)	k_1 (in.)	T (in.)	Workable Gage (in.)
W14 × 82	24.0	14.3	14¼	0.510	1/2	1/4	10.1	10⅛	0.855	7/8	1.45	1 1/16	1 1/16	10⅞	5½
× 74	21.8	14.2	14⅛	0.450	7/16	1/4	10.1	10⅛	0.785	13/16	1.38	1⅝	1 1/16		
× 68	20.0	14.0	14	0.415	7/16	1/4	10.0	10	0.720	3/4	1.31	1 9/16	1 1/16		
× 61	17.9	13.9	13⅞	0.375	3/8	3/16	10.0	10	0.645	5/8	1.24	1½	1		
W14 × 53	15.6	13.9	13⅞	0.370	3/8	3/16	8.06	8	0.660	11/16	1.25	1½	1	10⅞	5½
× 48	14.1	13.8	13¾	0.340	5/16	3/16	8.03	8	0.595	5/8	1.19	1 7/16	1		
× 43[a]	12.6	13.7	13⅝	0.305	5/16	3/16	8.00	8	0.530	1/2	1.12	1⅜	1		
W14 × 38[a]	11.2	14.1	14⅛	0.310	5/16	3/16	6.77	6¾	0.515	1/2	0.915	1¼	13/16	11⅝	3½[c]
× 34[a]	10.0	14.0	14	0.285	5/16	3/16	6.75	6¾	0.455	7/16	0.855	1 3/16	3/4		3½
× 30[a]	8.85	13.8	13⅞	0.270	1/4	1/8	6.73	6¾	0.385	3/8	0.785	1⅛	3/4		3½
W14 × 26[a]	7.69	13.9	13⅞	0.255	1/4	1/8	5.03	5	0.420	7/16	0.820	1⅛	3/4	11⅝	2¾[c]
× 22[a]	6.49	13.7	13¾	0.230	1/4	1/8	5.00	5	0.335	5/16	0.735	1 1/16	3/4	11⅝	2¾[c]
W12 × 336[d]	98.9	16.8	16⅞	1.78	1¾	7/8	13.4	13⅜	2.96	2 15/16	3.55	3⅞	1 11/16	9⅛	5½
× 305[d]	89.5	16.3	16⅜	1.63	1⅝	13/16	13.2	13¼	2.71	2 11/16	3.30	3⅝	1⅝		
× 279[d]	81.9	15.9	15⅞	1.53	1½	3/4	13.1	13⅛	2.47	2½	3.07	3⅜	1⅝		
× 252[d]	74.1	15.4	15⅜	1.40	1⅜	11/16	13.0	13	2.25	2¼	2.85	3⅛	1½		
× 230[d]	67.7	15.1	15	1.29	1 5/16	11/16	12.9	12⅞	2.07	2 1/16	2.67	2 15/16	1½		

(*Continued*)

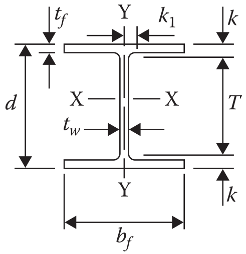
APPENDIX C.1a (*Continued*)
W Shapes: Dimensions



Shape	Area, A (in.^{2})	Depth, d (in.)	Web Thickness, t_w (in.)			Web $\frac{t_w}{2}$ (in.)	Flange Width, b_f (in.)		Flange Thickness, t_f (in.)		Distance k: k_{des} (in.)	Distance k: k_{det} (in.)	Distance k_1 (in.)	Distance T (in.)	Distance Workable Gage (in.)
× 210	61.8	14.7	$14\frac{3}{4}$	1.18	$1\frac{3}{16}$	5/8	12.8	$12\frac{3}{4}$	1.90	$1\frac{7}{8}$	2.50	$21\frac{3}{16}$	$1\frac{7}{16}$		
× 190	56.0	14.4	$14\frac{3}{8}$	1.06	$1\frac{1}{16}$	9/16	12.7	$12\frac{5}{8}$	1.74	$1\frac{3}{4}$	2.33	$2\frac{5}{8}$	$1\frac{3}{8}$		
× 170	50.0	14.0	14	0.960	15/16	1/2	12.6	$12\frac{5}{8}$	1.56	$1\frac{9}{16}$	2.16	$2\frac{7}{16}$	$1\frac{5}{16}$		
× 152	44.7	13.7	$13\frac{3}{4}$	0.870	7/8	7/16	12.5	$12\frac{1}{2}$	1.40	$1\frac{3}{8}$	2.00	$2\frac{5}{16}$	$1\frac{1}{4}$		
× 136	39.9	13.4	$13\frac{3}{8}$	0.790	13/16	7/16	12.4	$12\frac{3}{8}$	1.25	$1\frac{1}{4}$	1.85	$2\frac{1}{8}$	$1\frac{1}{4}$		
× 120	35.2	13.1	$13\frac{1}{8}$	0.710	11/16	3/8	12.3	$12\frac{3}{8}$	1.11	$1\frac{1}{8}$	1.70	2	$1\frac{3}{16}$		
× 106	31.2	12.9	$12\frac{7}{8}$	0.610	5/8	5/16	12.2	$12\frac{1}{4}$	0.990	1	1.59	$1\frac{7}{8}$	$1\frac{1}{8}$		
× 96	28.2	12.7	$12\frac{5}{4}$	0.550	9/16	5/16	12.2	$12\frac{1}{8}$	0.900	7/8	1.50	$11\frac{3}{16}$	$1\frac{1}{8}$		
× 87	25.6	12.5	$12\frac{1}{2}$	0.515	1/2	1/4	12.1	$12\frac{1}{8}$	0.810	13/15	1.41	$11\frac{1}{16}$	$1\frac{1}{16}$		
× 79	23.2	12.4	$12\frac{3}{8}$	0.470	1/2	1/4	12.1	$12\frac{1}{8}$	0.735	3/4	1.33	$1\frac{5}{8}$	$1\frac{1}{16}$		
× 72	21.1	12.3	$12\frac{1}{4}$	0.430	7/16	1/4	12.0	12	0.670	11/16	1.27	$1\frac{9}{16}$	$1\frac{1}{16}$		
× 65[b]	19.1	12.1	$12\frac{1}{8}$	0.390	3/8	3/16	12.0	12	0.605	5/8	1.20	$1\frac{1}{2}$	1		
W12 × 58	17.0	12.2	$12\frac{1}{4}$	0.360	3/8	3/16	10.0	10	0.640	5/8	1.24	$1\frac{1}{2}$	15/16	$9\frac{1}{4}$	$5\frac{1}{2}$
× 53	15.6	12.1	12	0.345	3/8	3/16	10.0	10	0.575	9/16	1.18	$1\frac{3}{8}$	15/16	$9\frac{1}{4}$	$5\frac{1}{2}$
W12 × 50	14.6	12.2	$12\frac{1}{4}$	0.370	3/8	3/16	8.08	$8\frac{1}{8}$	0.640	5/8	1.14	$1\frac{1}{2}$	15/16	91⁄4	$5\frac{1}{2}$
× 45	13.1	12.1	12	0.335	5/16	3/16	8.05	8	0.575	9/16	1.08	$1\frac{3}{8}$	15/16		
× 40	11.7	11.9	12	0.295	5/16	3/16	8.01	8	0.515	1/2	1.02	$1\frac{3}{8}$	7/8		

(*Continued*)

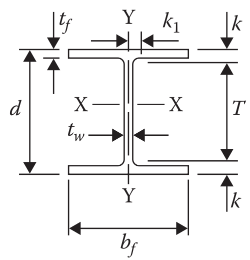
APPENDIX C.1a (*Continued*)
W Shapes: Dimensions



Shape	Area, A (in.²)	Depth, d (in.)	Web: Thickness, t_w (in.)		$\frac{t_w}{2}$ (in.)	Flange: Width, b_f (in.)		Flange: Thickness, t_f (in.)		Distance: k: k_{des} (in.)	k_{det} (in.)	k_1 (in.)	T (in.)	Workable Gage (in.)
W12 × 35[a]	10.3	12.5	$12\frac{1}{2}$	0.300 5/16	3/16	6.56	$6\frac{1}{2}$	0.520	1/2	0.820	$1\frac{3}{16}$	3/4	$10\frac{1}{8}$	$3\frac{1}{2}$
× 30[a]	8.79	12.3	$12\frac{3}{8}$	0.260 1/4	1/8	6.52	$6\frac{1}{2}$	0.440	7/16	0.740	$1\frac{1}{8}$	3/4		
× 26[a]	7.65	12.2	$12\frac{1}{4}$	0.230 1/4	1/8	6.49	$6\frac{1}{2}$	0.380	3/8	0.680	$1\frac{1}{18}$	3/4		
W12 × 22[a]	6.48	12.3	$12\frac{1}{4}$	0.260 1/4	1/8	4.03	4	0.425	7/16	0.725	15/16	5/8	$10\frac{3}{8}$	$2\frac{1}{4}$[e]
× 19[a]	5.57	12.2	$12\frac{1}{8}$	0.235 1/4	1/8	4.01	4	0.350	3/8	0.650	7/8	9/16		
× 16[a]	4.71	12.0	12	0.220 1/4	1/8	3.99	4	0.265	1/4	0.565	13/16	9/16		
× 14[a,e]	4.16	11.9	$11\frac{7}{8}$	0.200 3/16	1/8	3.97	4	0.225	1/4	0.525	3/4	9/16		
W10 × 112	32.9	11.4	$11\frac{3}{8}$	0.755 3/4	3/8	10.4	$10\frac{3}{8}$	1.25	$1\frac{1}{4}$	1.75	$1\frac{15}{16}$	1	$7\frac{1}{2}$	$5\frac{1}{2}$
× 100	29.3	11.1	$11\frac{1}{8}$	0.680 11/16	3/8	10.3	$10\frac{3}{8}$	1.12	$1\frac{1}{8}$	1.62	$1\frac{13}{16}$	1		
× 88	26.0	10.8	$10\frac{7}{8}$	0.605 5/8	5/16	10.3	$10\frac{1}{4}$	0.990	1	1.49	$1\frac{11}{16}$	15/16		
× 77	22.7	10.6	$10\frac{5}{8}$	0.530 1/2	1/4	10.2	$10\frac{1}{4}$	0.870	7/8	1.37	$1\frac{9}{16}$	7/8		
× 68	19.9	10.4	$10\frac{3}{8}$	0.470 1/2	1/4	10.1	$10\frac{1}{8}$	0.770	3/4	1.27	$1\frac{7}{16}$	7/8		
× 60	17.7	10.2	$10\frac{1}{4}$	0.420 7/16	1/4	10.1	$10\frac{1}{8}$	0.680	11/16	1.18	$1\frac{3}{8}$	13/16		
× 54	15.8	10.1	$10\frac{1}{8}$	0.370 3/8	3/16	10.0	10	0.615	5/8	1.12	$1\frac{5}{16}$	13/16		
× 49	14.4	10.0	10	0.340 5/16	3/16	10.0	10	0.560	9/16	1.06	$1\frac{1}{4}$	13/16		
W10 × 45	13.3	10.1	$10\frac{1}{8}$	0.350 3/8	3/16	8.02	8	0.620	5/8	1.12	$1\frac{5}{16}$	13/16	$7\frac{1}{2}$	$5\frac{1}{2}$
× 39	11.5	9.92	$9\frac{7}{8}$	0.315 5/16	3/16	7.99	8	0.530	1/2	1.03	$1\frac{3}{16}$	13/16		
× 33	9.71	9.73	$9\frac{3}{4}$	0.290 5/16	3/16	7.96	8	0.435	7/16	0.935	$1\frac{1}{8}$	3/4		

(*Continued*)

APPENDIX C.1a (*Continued*)
W Shapes: Dimensions



			Web				Flange				Distance				
											k				
Shape	**Area, *A* (in.²)**	**Depth, *d* (in.)**	**Thickness, t_w (in.)**			$\frac{t_w}{2}$ **(in.)**	**Width, b_f (in.)**		**Thickness, t_f (in.)**		k_{des} **(in.)**	k_{det} **(in.)**	k_1 **(in.)**	***T* (in.)**	**Workable Gage (in.)**
W10 × 30	8.84	10.5	$10\frac{1}{2}$	0.300	5/16	3/16	5.81	$5\frac{3}{4}$	0.510	1/2	0.810	$1\frac{1}{8}$	11/16	$8\frac{1}{4}$	$2\frac{3}{4}$[e]
× 26	7.61	10.3	$10\frac{3}{8}$	0.260	1/4	1/8	5.77	$5\frac{3}{4}$	0.440	7/16	0.740	$1\frac{1}{16}$	11/16	↓	↓
× 22[a]	6.49	10.2	$10\frac{1}{8}$	0.240	1/4	1/8	5.75	$5\frac{3}{4}$	0.360	3/8	0.660	15/16	5/8	↓	↓
W10 × 19	5.62	10.2	$10\frac{1}{4}$	0.250	1/4	1/8	4.02	4	0.395	3/8	0.695	15/16	5/8	$8\frac{3}{8}$	$2\frac{1}{4}$[e]
× 17[a]	4.99	10.1	$10\frac{1}{8}$	0.240	1/4	1/8	4.01	4	0.330	5/16	0.630	7/8	9/16	↓	↓
× 15[a]	4.41	10.0	10	0.230	1/4	1/8	4.00	4	0.270	1/4	0.570	13/16	9/16	↓	↓
× 12[a,b]	3.54	9.87	$9\frac{7}{8}$	0.190	3/16	1/8	3.96	4	0.210	3/16	0.510	3/4	9/16	↓	↓

Source: Courtesy of the American Institute of Steel Construction, Chicago, IL.

[a] Shape is slender for compression with $F_y = 50$ ksi.

[b] Shape exceeds compact limit for flexure with $F_y = 50$ ksi.

[c] The actual size, combination, and orientation of fastener components should be compared with the geometry of the cross section to ensure compatibility.

[d] Flange thickness is greater than 2 in. Special requirements may apply per AISC Specification Section A3.1c.

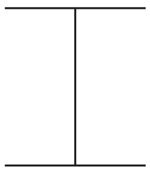
[e] Shape does not meet the h/t_w limit for shear in Specification Section G2.1a with $F_y = 50$ ksi.

APPENDIX C.1b
W Shapes Properties

Shape	Compact Section Criteria		Axis x–x				Axis y–y						Torsional Properties	
	$\frac{b_f}{2t_f}$	$\frac{h}{t_w}$	I (in.^{4})	S (in.^{3})	r (in.)	Z (in.^{3})	I (in.^{4})	S (in.^{3})	r (in.)	Z (in.^{3})	r_{ts} (in.)	h_0 (in.)	J (in.^{4})	C_w (in.^{6})
W21 × 93	4.53	32.3	2,070	192	8.70	221	92.9	22.1	1.84	34.7	2.24	20.7	6.03	9,940
× 83	5.00	36.4	1,830	171	8.67	196	81.4	19.5	1.83	30.5	2.21	20.6	4.34	8,630
× 73	5.60	41.2	1,600	151	8.64	172	70.6	17.0	1.81	26.6	2.19	20.5	3.02	7,410
× 68	6.04	43.6	1,480	140	8.60	160	64.7	15.7	1.80	24.4	2.17	20.4	2.45	6,760
× 62	6.70	46.9	1,330	127	8.54	144	57.5	14.0	1.77	21.7	2.15	20.4	1.83	5,960
× 55	7.87	50.0	1,140	110	8.40	126	48.4	11.8	1.73	18.4	2.11	20.3	1.24	4,980
× 48	9.47	53.6	959	93.0	8.24	107	38.7	9.52	1.66	14.9	2.05	20.2	0.803	3,950
W21 × 57	5.04	46.3	1,170	111	8.36	129	30.6	9.35	1.35	14.8	1.68	20.5	1.77	3,190
× 50	6.10	49.4	984	94.5	8.18	110	24.9	7.64	1.30	12.2	1.64	20.3	1.14	2.570
× 44	7.22	53.6	843	81.6	8.06	95.4	20.7	6.37	1.26	10.2	1.60	20.3	0.770	2,110
W18 × 311	2.19	10.4	6,970	624	8.72	754	795	132	2.95	207	3.53	19.6	176	76,200
× 283	2.38	11.3	6,170	565	8.61	676	704	118	2.91	185	3.47	19.4	134	65,900
× 258	2.56	12.5	5,510	514	8.53	611	628	107	2.88	166	3.42	19.2	103	57,600
× 234	2.76	13.8	4,900	466	8.44	549	558	95.8	2.85	149	3.37	19.0	78.7	50,100
× 211	3.02	15.1	4,330	419	8.35	490	493	85.3	2.82	132	3.32	18.8	58.6	43,400
× 192	3.27	16.7	3,870	380	8.28	442	440	76.8	2.79	119	3.28	18.6	44.7	38,000
× 175	3.58	18.0	3,450	344	8.20	398	391	68.8	2.76	106	3.24	18.4	33.8	33,300
× 158	3.92	19.8	3,060	310	8.12	356	347	61.4	2.74	94.8	3.20	18.3	25.2	29,000

(*Continued*)

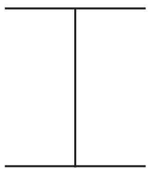
APPENDIX C.1b (*Continued*)
W Shapes Properties



Shape	Compact Section Criteria		Axis x–x				Axis y–y						Torsional Properties	
	$\frac{b_f}{2t_f}$	$\frac{h}{t_w}$	I (in.^{4})	S (in.^{3})	r (in.)	Z (in.^{3})	I (in.^{4})	S (in.^{3})	r (in.)	Z (in.^{3})	r_{ts} (in.)	h_0 (in.)	J (in.^{4})	C_w (in.^{6})
× 143	4.25	22.0	2,750	282	8.09	322	311	55.5	2.72	85.4	3.17	18.2	19.2	25,700
× 130	4.65	23.9	2,460	256	8.03	290	278	49.9	2.70	76.7	3.13	18.1	14.5	22,700
× 119	5.31	24.5	2,190	231	7.90	262	253	44.9	2.69	69.1	3.13	17.9	10.6	20,300
× 106	5.96	27.2	1,910	204	7.84	230	220	39.4	2.66	60.5	3.10	17.8	7.48	17,400
× 97	6.41	30.0	1,750	188	7.82	211	201	36.1	2.65	55.3	3.08	17.7	5.86	15,800
× 86	7.20	33.4	1,530	166	7.77	186	175	31.6	2.63	48.4	3.05	17.6	4.10	13,600
× 76	8.11	37.8	1,330	146	7.73	163	152	27.6	2.61	42.2	3.02	17.5	2.83	11,700
W18 × 71	4.71	32.4	1,170	127	7.50	146	60.3	15.8	1.70	24.7	2.05	17.7	3.49	4,700
× 65	5.06	35.7	1,070	117	7.49	133	54.8	14.4	1.69	22.5	2.03	17.6	2.73	4,240
× 60	5.44	38.7	984	108	7.47	123	50.1	13.3	1.68	20.6	2.02	17.5	2.17	3,850
× 55	5.98	41.1	890	98.3	7.41	112	44.9	11.9	1.67	18.5	2.00	17.5	1.66	3,430
× 50	6.57	45.2	800	88.9	7.38	101	40.1	10.7	1.65	16.6	1.98	17.4	1.24	3,040
W18 × 46	5.01	44.6	712	78.8	7.25	90.7	22.5	7.43	1.29	11.7	1.58	17.5	1.22	1,720
× 40	5.73	50.9	612	68.4	7.21	78.4	19.1	6.35	1.27	10.0	1.56	17.4	0.810	1,440
× 35	7.06	53.5	510	57.6	7.04	66.5	15.3	5.12	1.22	8.06	1.51	17.3	0.506	1,140
× 100	5.29	24.3	1,490	175	7.10	198	186	35.7	2.51	54.9	2.92	16.0	7.73	11,900
× 89	5.92	27.0	1,300	155	7.05	175	163	31.4	2.49	48.1	2.88	15.9	5.45	10,200
× 77	6.77	31.2	1,110	134	7.00	150	138	26.9	2.47	41.1	2.85	15.7	3.57	8,590
× 67	7.70	35.9	954	117	6.96	130	119	23.2	2.46	35.5	2.82	15.6	2.39	7,300

(*Continued*)

APPENDIX C.1b (*Continued*)
W Shapes Properties



	Compact Section Criteria		Axis *x–x*				Axis *y–y*						Torsional Properties	
Shape	$\frac{b_f}{2t_f}$	$\frac{h}{t_w}$	I (in.^{4})	S (in.^{3})	r (in.)	Z (in.^{3})	I (in.^{4})	S (in.^{3})	r (in.)	Z (in.^{3})	r_{ts} (in.)	h_0 (in.)	J (in.^{4})	C_w (in.^{6})
W16 × 57	4.98	33.0	758	92.2	6.72	105	43.1	12.1	1.60	18.9	1.92	15.7	2.22	2,660
× 50	5.61	37.4	659	81.0	6.68	92.0	37.2	10.5	1.59	16.3	1.89	15.7	1.52	2,270
× 45	6.23	41.1	586	72.7	6.65	82.3	32.8	9.34	1.57	14.5	1.87	15.5	1.11	1,990
× 40	6.93	46.5	518	64.7	6.63	73.0	28.9	8.25	1.57	12.7	1.86	15.5	0.794	1,730
× 36	8.12	48.1	448	56.5	6.51	64.0	24.5	7.00	1.52	10.8	1.83	15.5	0.545	1,460
W16 × 31	6.28	51.6	375	47.2	6.41	54.0	12.4	4.49	1.17	7.03	1.42	15.5	0.461	739
× 26	7.97	56.8	301	38.4	6.26	44.2	9.59	3.49	1.12	5.48	1.38	15.4	0.262	565
W14 × 730	1.82	3.71	14,300	1280	8.17	1660	4720	527	4.69	816	5.68	17.5	1450	362,000
× 665	1.95	4.03	12,400	1150	7.98	1480	4170	472	4.62	730	5.57	17.1	1120	305,000
× 605	2.09	4.39	10,800	1040	7.80	1320	3680	423	4.55	652	5.44	16.7	869	258,000
× 550	2.25	4.79	9,430	931	7.63	1180	3250	378	4.49	583	5.35	16.4	669	219,000
× 500	2.43	5.21	8,210	838	7.48	1050	2880	339	4.43	522	5.26	16.1	514	187,000
× 455	2.62	5.66	7,190	756	7.33	936	2560	304	4.38	468	5.17	15.8	395	160,000
× 426	2.75	6.08	6,600	706	7.26	869	2360	283	4.34	434	5.11	15.7	331	144.000
× 398	2.92	6.44	6,000	656	7.16	801	2170	262	4.31	402	5.05	15.5	273	129,000
× 370	3.10	6.89	5,440	607	7.07	736	1990	241	4.27	370	5.00	15.2	222	116,000
× 342	3.31	7.41	4,900	558	6.98	672	1810	221	4.24	338	4.95	15.0	178	103,000

(*Continued*)

APPENDIX C.1b (*Continued*)
W Shapes Properties

Shape	Compact Section Criteria		Axis x–x				Axis y–y						Torsional Properties	
	$\frac{b_f}{2t_f}$	$\frac{h}{t_w}$	I (in.^{4})	S (in.^{3})	r (in.)	Z (in.^{3})	I (in.^{4})	S (in.^{3})	r (in.)	Z (in.^{3})	r_{ts} (in.)	h_0 (in.)	J (in.^{4})	C_w (in.^{6})
× 311	3.59	8.09	4,330	506	6.88	603	1610	199	4.20	304	4.87	14.8	136	89,100
× 283	3.89	8.84	3,840	459	6.79	542	1440	179	4.17	274	4.80	14.6	104	77,700
× 257	4.23	9.71	3,400	415	6.71	487	1290	161	4.13	246	4.75	14.5	79.1	67,800
× 233	4.62	10.7	3,010	375	6.63	436	1150	145	4.10	221	4.69	14.3	59.5	59,000
× 211	5.06	11.6	2,660	338	6.55	390	1030	130	4.07	198	4.64	14.1	44.6	51,500
× 193	5.45	12.8	2,400	310	6.50	355	931	119	4.05	180	4.59	14.1	34.8	45,900
× 176	5.97	13.7	2,140	281	6.43	320	838	107	4.02	163	4.55	13.9	26.5	40,500
× 159	6.54	15.3	1,900	254	6.38	287	748	96.2	4.00	146	4.51	13.8	19.7	35,600
× 145	7.11	16.8	1,710	232	6.33	260	677	87.3	3.98	133	4.47	13.7	15.2	31,700
W14 × 132	7.15	17.7	1,530	209	6.28	234	548	74.5	3.76	113	4.23	13.7	12.3	25,500
× 120	7.80	19.3	1,380	190	6.24	212	495	67.5	3.74	102	4.20	13.6	9.37	22,700
× 109	8.49	21.7	1,240	173	6.22	192	447	61.2	3.73	92.7	4.17	13.4	7.12	20,200
× 99	9.34	23.5	1,110	157	6.17	173	402	55.2	3.71	83.6	4.14	13.4	5.37	18,000
× 90	10.2	25.9	999	143	6.14	157	362	49.9	3.70	75.6	4.11	13.3	4.06	16,000
W14 × 82	5.92	22.4	881	123	6.05	139	148	29.3	2.48	44.8	2.85	13.4	5.07	6,710
× 74	6.41	25.4	795	112	6.04	126	134	26.6	2.48	40.5	2.82	13.4	3.87	5,990
× 68	6.97	27.5	722	103	6.01	115	121	24.2	2.46	36.9	2.80	13.3	3.01	5,380
× 61	7.75	30.4	640	92.1	5.98	102	107	21.5	2.45	32.8	2.78	13.3	2.19	4,710

(*Continued*)

APPENDIX C.1b (*Continued*)
W Shapes Properties

	Compact Section Criteria		Axis *x–x*				Axis *y–y*						Torsional Properties	
Shape	$\frac{b_f}{2t_f}$	$\frac{h}{t_w}$	I (in.^{4})	S (in.^{3})	r (in.)	Z (in.^{3})	I (in.^{4})	S (in.^{3})	r (in.)	Z (in.^{3})	r_{ts} (in.)	h_0 (in.)	J (in.^{4})	C_w (in.^{6})
W14 × 53	6.11	30.9	541	77.8	5.89	87.1	57.7	14.3	1.92	22.0	2.22	13.2	1.94	2,540
× 48	6.75	33.6	484	70.2	5.85	78.4	51.4	12.8	1.91	19.6	2.20	13.2	1.45	2,240
× 43	7.54	37.4	428	62.6	5.82	69.6	45.2	11.3	1.89	17.3	2.18	13.2	1.05	1,950
W14 × 38	6.57	39.6	385	54.6	5.87	61.5	26.7	7.88	1.55	12.1	1.82	13.6	0.798	1,230
× 34	7.41	43.1	340	48.6	5.83	54.6	23.3	6.91	1.53	10.6	1.80	13.5	0.569	1,070
× 30	8.74	45.4	291	42.0	5.73	47.3	19.6	5.82	1.49	8.99	1.77	13.4	0.380	887
W14 × 26	5.98	48.1	245	35.3	5.65	40.2	8.91	3.55	1.08	5.54	1.31	13.5	0.358	405
22	7.46	53.3	199	29.0	5.54	33.2	7.00	2.80	1.04	4.39	1.27	134	0.208	314
W12 × 336	2.26	5.47	4,060	483	6.41	603	1190	177	3.47	274	4.13	13.8	243	57,000
× 305	2.45	5.98	3,550	435	6.29	537	1050	159	3.42	244	4.05	13.6	185	48,600
× 279	2.66	6.35	3,110	393	6.16	481	937	143	3.38	220	4.00	13.4	143	42,000
× 252	2.89	6.96	2,720	353	6.06	428	828	127	3.34	196	3.93	13.2	108	35,800
× 230	3.11	7.56	2,420	321	5.97	386	742	115	3.31	177	3.87	13.0	83.8	31,200
× 210	3.37	8.23	2,140	29.2	5.89	348	664	104	3.28	159	3.82	12.8	64.7	27,200
× 190	3.65	9.16	1,890	263	5.82	311	589	93.0	3.25	143	3.76	12.7	48.8	23,600
× 170	4.03	10.1	1,650	235	5.74	275	517	82.3	3.22	126	3.71	12.4	35.6	20,100
× 152	4.46	11.2	1,430	209	5.66	243	454	72.8	3.19	111	3.66	12.3	25.8	17,200
× 136	4.96	12.3	1,240	186	5.58	214	398	64.2	3.16	98.0	3.61	12.2	18.5	14,700

(*Continued*)

APPENDIX C.1b (*Continued*)
W Shapes Properties

	Compact Section Criteria		Axis *x–x*				Axis *y–y*						Torsional Properties	
Shape	$\frac{b_f}{2t_f}$	$\frac{h}{t_w}$	I (in.^{4})	S (in.^{3})	r (in.)	Z (in.^{3})	I (in.^{4})	S (in.^{3})	r (in.)	Z (in.^{3})	r_{ts} (in.)	h_0 (in.)	J (in.^{4})	C_w (in.^{6})
× 120	5.57	13.7	1,070	163	5.51	186	345	56.0	3.13	85.4	3.56	12.0	12.9	12,400
× 106	6.17	15.9	933	145	5.47	164	301	49.3	3.11	75.1	3.52	11.9	9.13	10,700
× 96	6.76	17.7	833	131	5.44	147	270	44.4	3.09	67.5	3.49	11.8	6.85	9,410
× 87	7.48	18.9	740	118	5.38	132	241	39.7	3.07	60.4	3.46	11.7	5.10	8,270
× 79	8.22	20.7	662	107	5.34	119	216	35.8	3.05	54.3	3.43	11.7	3.84	7,330
× 72	8.99	22.6	597	97.4	5.31	108	195	32.4	3.04	49.2	3.40	11.6	2.93	6,540
× 65	9.92	24.9	533	87.9	5.28	96.8	174	29.1	3.02	44.1	3.38	11.5	2.18	5,780
W12 × 58	7.82	27.0	475	78.0	5.28	86.4	107	21.4	2.51	32.5	2.82	11.6	2.10	3,570
× 53	8.69	28.1	425	70.6	5.23	77.9	95.8	19.2	2.48	29.1	2.79	11.5	1.58	3,160
W12 × 50	6.31	26.8	391	64.2	5.18	71.9	56.3	13.9	1.96	21.3	2.25	11.6	1.71	1,880
× 45	7.00	29.6	348	57.7	5.15	64.2	50.0	12.4	1.95	19.0	2.23	11.5	1.26	1,650
× 40	7.77	33.6	307	51.5	5.13	37.0	44.1	11.0	1.94	16.8	2.21	11.4	0.906	1,440
W12 × 35	6.31	36.2	285	45.6	5.25	51.2	24.5	7.47	1.54	11.5	1.79	12.0	0.741	879
× 30	7.41	41.8	238	38.6	5.21	43.1	20.3	6.24	1.52	9.56	1.77	11.9	0.457	720
× 26	8.54	47.2	204	33.4	5.17	37.2	17.3	5.34	1.51	8.17	1.75	11.8	0.300	607
W12 × 22	4.74	41.8	156	25.4	4.91	29.3	4.66	2.31	0.848	3.66	1.04	11.9	0.293	164
× 19	5.72	46.2	130	21.3	4.82	24.7	3.76	1.88	0.822	2.98	1.02	11.9	0.180	131
× 16	7.53	49.4	103	17.1	4.67	20.1	2.82	1.41	0.773	2.26	0.983	11.7	0.103	96.9
× 14	8.82	54.3	88.6	14.9	4.62	17.4	2.36	1.19	0.753	1.90	0.961	11.7	0.0704	80.4

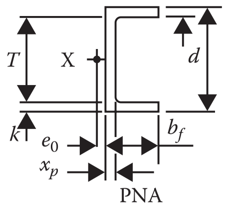
(*Continued*)

APPENDIX C.1b (*Continued*)
W Shapes Properties

	Compact Section Criteria		Axis x–x				Axis y–y						Torsional Properties	
Shape	$\frac{b_f}{2t_f}$	$\frac{h}{t_w}$	I (in.^{4})	S (in.^{3})	r (in.)	Z (in.^{3})	I (in.^{4})	S (in.^{3})	r (in.)	Z (in.^{3})	r_{ts} (in.)	h_0 (in.)	J (in.^{4})	C_w (in.^{6})
W10 × 112	4.17	10.4	716	126	4.66	147	236	45.3	2.68	69.2	3.08	10.2	15.1	6,020
× 100	4.62	11.6	623	112	4.60	130	207	40.0	2.65	61.0	3.04	10.0	10.9	5,150
× 88	5.18	13.0	534	98.5	4.54	113	179	34.8	2.63	53.1	2.99	9.81	7.53	4,330
× 77	5.86	14.8	455	85.9	4.49	97.6	154	30.1	2.60	45.9	2.95	9.73	5.11	3,630
× 68	6.58	16.7	394	75.7	4.44	85.3	134	26.4	2.59	40.1	2.92	9.63	3.56	3,100
× 60	7.41	18.7	341	66.7	4.39	74.6	116	23.0	2.57	35.0	2.88	9.52	2.48	2,640
× 54	8.15	21.2	303	60.0	4.37	66.6	103	20.6	2.56	31.3	2.85	9.49	1.82	2,320
× 49	8.93	23.1	272	54.6	4.35	60.4	93.4	18.7	2.54	28.3	2.84	9.44	1.39	2,070
W10 × 45	6.47	22.5	248	49.1	4.32	54.9	53.4	13.3	2.01	20.3	2.27	9.48	1.51	1,200
× 39	7.53	25.0	209	42.1	4.27	46.8	45.0	11.3	1.98	17.2	2.24	9.39	0.976	992
× 33	9.15	27.1	171	35.0	4.19	38.8	36.6	9.20	1.94	14.0	2.20	9.30	0.583	791
W10 × 30	5.70	29.5	170	32.4	4.38	36.6	16.7	5.75	1.37	8.84	1.60	10.0	0.622	414
× 26	6.56	34.0	144	27.9	4.35	31.3	14.1	4.89	1.36	7.50	1.58	9.86	0.402	345
× 22	7.99	36.9	118	23.2	4.27	26.0	11.4	3.97	1.33	6.10	1.55	9.84	0.239	275
W30 × 19	5.09	35.4	96.3	18.8	4.14	21.6	4.29	2.14	0.874	3.35	1.06	9.81	0.233	104
× 17	6.08	36.9	81.9	16.2	4.05	18.7	3.56	1.78	0.845	2.80	1.04	9.77	0.156	85.1
× 15	7.41	38.5	68.9	13.8	3.95	16.0	2.89	1.45	0.810	2.30	1.01	9.72	0.104	68.3
× 12	9.43	46.6	53.8	10.9	3.90	12.6	2.18	1.10	0.785	1.74	0.983	9.66	0.0547	50.9

Source: Courtesy of the American Institute of Steel Construction, Chicago, IL.

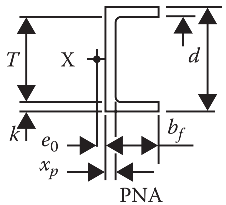
APPENDIX C.2a
C Shapes Dimensions



Shape	Area, A (in.²)	Depth, d (in.)		Web: Thickness, t_w (in.)		Web: $\frac{t_w}{2}$ (in.)	Flange: Width, b_f (in.)		Flange: Thickness, t_f (in.)		Distance: k (in.)	Distance: T (in.)	Distance: Workable Gage (in.)	r_{ts} (in.)	h_0 (in.)
C15 × 50	14.7	15.0	15	0.716	11/16	3/8	3.72	3 3/4	0.650	5/8	1 7/16	12 1/8	2 1/4	1.17	14.4
× 40	11.8	15.0	15	0.520	1/2	1/4	3.52	3 1/4	0.650	5/8	1 7/16	12 1/8	2	1.15	14.4
× 33.9	10.0	15.0	15	0.400	3/8	3/16	3.40	3 3/8	0.650	5/8	1 7/16	12 1/8	2	1.13	14.4
C12 × 30	8.81	12.0	12	0.510	1/2	1/4	3.17	3 1/8	0.501	1/2	1 1/8	9 3/4	1 3/4[a]	1.01	11.5
× 25	7.34	12.0	12	0.387	3/8	3/16	3.05	3	0.501	1/2	1 1/8	9 3/4	1 3/4[a]	1.00	11.5
× 20.7	6.08	12.0	12	0.282	5/16	3/16	2.94	3	0.501	1/2	1 1/8	9 3/4	1 3/4[a]	0.983	11.5
C10 × 30	8.81	10.0	10	0.673	11/16	3/8	3.03	3	0.436	7/16	1	8	1 3/4[a]	0.924	9.56
× 25	7.35	10.0	10	0.526	1/2	1/4	2.89	2 7/8	0.436	7/16	1	8	1 3/4[a]	0.911	9.56
× 20	5.87	10.0	10	0.379	3/8	3/16	2.74	2 3/4	0.436	7/16	1	8	1 1/2[a]	0.894	9.56
× 15.3	4.48	10.0	10	0.240	1/4	1/8	2.60	2 5/8	0.436	7/16	1	8	1 1/2[a]	0.868	9.56
C9 × 20	5.87	9.00	9	0.448	7/16	1/4	2.65	2 5/8	0.413	7/16	1	7	1 1/2[a]	0.850	8.59
× 15	4.40	9.00	9	0.285	5/16	3/16	2.49	2 1/2	0.413	7/16	1	7	1 3/8[a]	0.825	8.59
× 13.4	3.94	9.00	9	0.233	1/4	1/8	2.43	2 3/8	0.413	7/16	1	7	1 3/8[a]	0.814	8.59
C8 × 18.7	5.51	8.00	8	0.487	1/2	1/4	2.53	2 1/2	0.390	3/8	15/16	6 1/8	1 1/2[a]	0.800	7.61
× 13.7	4.03	8.00	8	0.303	5/16	3/16	2.34	2 3/8	0.390	3/8	15/16	6 1/8	1 3/8[a]	0.774	7.61
× 11.5	3.37	8.00	8	0.220	1/4	1/8	2.26	2 1/4	0.390	3/8	15/16	6 1/8	1 3/8[a]	0.756	7.61

(Continued)

APPENDIX C.2a (*Continued*)
C Shapes Dimensions



			Web				Flange				Distance				
Shape	Area, A (in.^{2})	Depth, d (in.)		Thickness, t_w (in.)		$\frac{t_w}{2}$ (in.)	Width, b_f (in.)		Thickness, t_f (in.)		k (in.)	T (in.)	Workable Gage (in.)	r_{ts} (in.)	h_0 (in.)
C7 × 14.7	4.33	7.00	7	0.419	7/16	1/4	2.30	$2\frac{1}{4}$	0.366	3/8	7/8	$5\frac{1}{4}$	$1\frac{1}{4}$[a]	0.738	6.63
× 12.2	3.59	7.00	7	0.314	5/16	3/16	2.19	$2\frac{1}{4}$	0.366	3/8	7/8	$5\frac{1}{4}$	$1\frac{1}{4}$[a]	0.722	6.63
× 9.8	2.87	7.00	7	0.210	3/16	1/8	2.09	$2\frac{1}{8}$	0.366	3/8	7/8	$5\frac{1}{4}$	$1\frac{1}{4}$[a]	0.698	6.63
C6 × 13	3.82	6.00	6	0.437	7/16	1/4	2.16	$2\frac{1}{8}$	0.343	5/16	13/16	4/8	$1\frac{3}{8}$[a]	0.689	5.66
× 10.5	3.07	6.00	6	0.314	5/16	3/16	2.03	2	0.343	5/16	13/16	$4\frac{3}{8}$	$1\frac{1}{8}$[a]	0.669	5.66
× 8.2	2.39	6.00	6	0.200	3/16	1/8	1.92	$1\frac{7}{8}$	0.343	5/16	13/16	$4\frac{3}{8}$	$1\frac{1}{8}$[a]	0.643	5.66
C5 × 9	2.64	5.00	5	0.325	5/16	3/16	1.89	$1\frac{7}{4}$	0.320	5/16	3/4	$3\frac{1}{2}$	$1\frac{1}{8}$[a]	0.616	4.68
× 6.7	1.97	5.00	5	0.190	3/16	1/8	1.75	$1\frac{3}{4}$	0.320	5/16	3/4	$3\frac{1}{2}$	—[b]	0.584	4.68
C4 × 7.2	2.13	4.00	4	0.321	5/16	3/16	1.72	$1\frac{3}{4}$	0.296	5/16	3/4	$2\frac{1}{2}$	1[a]	0.563	3.70
C4 × 6.25	1.77	4.00	4	0.247	1/4	1/8	1.65	$1\frac{3}{4}$	0.272	5/16	3/4	$2\frac{1}{2}$	—[b]	0.546	3.73
× 5.4	1.58	4.00	4	0.184	3/16	1/8	1.58	$1\frac{5}{8}$	0.296	5/16	3/4	$2\frac{1}{2}$	—[b]	0.528	3.70
× 4.5	1.38	4.00	4	0.125	1/8	1/16	1.58	$1\frac{5}{8}$	0.296	5/16	3/4	$2\frac{1}{2}$	—[b]	0.524	3.70
C3 × 6	1.76	3.00	3	0.356	3/8	3/16	1.60	$1\frac{5}{8}$	0.273	1/4	11/16	$1\frac{5}{8}$	—[b]	0.519	2.73
× 5	1.47	3.00	3	0.258	1/4	1/8	1.50	$1\frac{1}{2}$	0.273	1/4	11/16	$1\frac{5}{8}$	—[b]	0.496	2.73
× 4.1	1.20	3.00	3	0.170	3/16	1/8	1.41	$1\frac{3}{8}$	0.273	1/4	11/16	$1\frac{5}{8}$	—[b]	0.469	2.73
× 3.5	1.09	3.00	3	0.132	1/8	1/16	1.37	$1\frac{3}{8}$	0.273	1/4	11/16	$1\frac{5}{8}$	—[b]	0.456	2.73

Source: Courtesy of the American Institute of Steel Constructions, Chicago, IL.

[a] The actual size, combination, and orientation of fastener components should be compared with the geometry of the cross section to ensure compatibility.

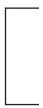
[b] Flange is too narrow to establish a workable gage.

APPENDIX C.2b
C Shapes Properties

Shape	Shear Center, e_0 (in.)	Axis x–x				Axis y–y						Torsional Properties			
		I (in.^{4})	S (in.^{3})	r (in.)	Z (in.^{3})	I (in.^{4})	S (in.^{3})	r (in.)	$\bar{x}$ (in.)	Z (in.^{3})	x_p (in.)	J (in.^{4})	C_w (in.^{6})	$\bar{r}_0$ (in.)	H
C15 × 50	0.583	404	53.8	5.24	68.5	11.0	3.77	0.865	0.799	8.14	0.490	2.65	492	5.49	0.937
× 40	0.767	348	46.5	5.43	57.5	9.17	3.34	0.883	0.778	6.84	0.392	1.45	410	5.71	0.927
× 33.9	0.896	315	42.0	5.61	50.8	8.07	3.09	0.901	0.788	6.19	0.332	1.01	358	5.94	0.920
C12 × 30	0.618	162	27.0	4.29	33.8	5.12	2.05	0.762	0.674	4.32	0.367	0.861	151	4.54	0.919
× 25	0.746	144	24.0	4.43	29.4	4.45	1.87	0.779	0.674	3.82	0.306	0.538	130	4.72	0.909
× 20.7	0.870	129	21.5	4.61	25.6	3.86	1.72	0.797	0.698	3.47	0.253	0.369	112	4.93	0.899
C10 × 30	0.368	103	20.7	3.43	26.7	3.93	1.65	0.668	0.649	3.78	0.441	1.22	79.5	3.63	0.921
× 25	0.494	91.1	18.2	3.52	23.1	3.34	1.47	0.675	0.617	3.18	0.367	0.687	68.3	3.76	0.912
× 20	0.636	78.9	15.8	3.67	19.4	2.80	1.31	0.690	0.606	2.70	0.294	0.368	56.9	3.93	0.900
× 15.3	0.796	67.3	13.5	3.88	15.9	2.27	1.15	0.711	0.634	2.34	0.224	0.209	45.5	4.19	0.884
C9 × 20	0.515	60.9	13.5	3.22	16.9	2.41	1.17	0.640	0.583	2.46	0.326	0.427	39.4	3.46	0.899
× 15	0.681	51.0	11.3	3.40	13.6	1.91	1.01	0.659	0.586	2.04	0.245	0.208	31.0	3.69	0.882
× 13.4	0.742	47.8	10.6	3.48	12.6	1.75	0.954	0.666	0.601	1.94	0.219	0.168	28.2	3.79	0.875
C8 × 18.7	0.431	43.9	11.0	2.82	13.9	1.97	1.01	0.598	0.565	2.17	0.344	0.434	25.1	3.05	0.894
× 13.7	0.604	36.1	9.02	2.99	11.0	1.52	0.848	0.613	0.554	1.73	0.252	0.186	19.2	3.26	0.874
× 11.5	0.697	32.5	8.14	3.11	9.63	1.31.	0.775	0.623	0.572	1.57	0.211	0.130	16.5	3.41	0.862
C7 × 14.7	0.441	27.2	7.78	2.51	9.75	1.37	0.772	0.561	0.532	1.63	0.309	0.267	13.1	2.75	0.875
× 12.2	0.538	24.2	6.92	2.59	8.46	1.16	0.696	0.568	0.525	1.42	0.257	0.161	11.2	2.86	0.862
× 9.8	0.647	21.2	6.07	2.72	7.19	0.957	0.617	0.578	0.541	1.26	0.205	0.0996	9.15	3.02	0.846

(Continued)

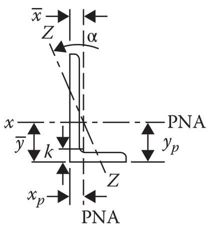
APPENDIX C.2b (*Continued*)
C Shapes Properties



Shape	Shear Center, e_0 (in.)	Axis x–x: I (in.^{4})	Axis x–x: S (in.^{3})	Axis x–x: r (in.)	Axis x–x: Z (in.^{3})	Axis y–y: I (in.^{4})	Axis y–y: S (in.^{3})	Axis y–y: r (in.)	Axis y–y: $\bar{x}$ (in.)	Axis y–y: Z (in.^{3})	Axis y–y: x_p (in.)	Torsional Properties: J (in.^{4})	Torsional Properties: C_w (in.^{6})	Torsional Properties: $\bar{r}_0$ (in.)	Torsional Properties: H
C6 × 13	0.380	17.3	5.78	2.13	7.29	1.05	0.638	0.524	0.514	1.35	0.318	0.237	7.19	2.37	0.858
× 10.5	0.486	15.1	5.04	2.22	6.18	0.860	0.561	0.529	0.500	1.14	0.256	0.128	5.91	2.48	0.842
× 8.2	0.599	13.1	4.35	2.34	5.16	0.687	0.488	0.536	0.512	0.987	0.199	0.0736	4.70	2.65	0.823
C5 × 9	0.427	8.89	3.56	1.84	4.39	0.624	0.444	0.486	0.478	0.913	0.264	0.109	2.93	2.10	0.815
× 6.7	0.552	7.48	2.99	1.95	3.55	0.470	0.372	0.489	0.484	0.757	0.215	0.0549	2.22	2.26	0.790
C4 × 7.2	0.386	4.58	2.29	1.47	2.84	0.425	0.337	0.447	0.459	0.695	0.266	0.0817	1.24	1.75	0.767
C4 × 6.25	0.434	4.00	2.00	1.50	2.43	0.345	0.284	0.441	0.435	0.569	0.221	0.0487	1.03	1.79	0.764
× 5.4	0.501	3.85	1.92	1.56	2.29	0.312	0.277	0.444	0.457	0.565	0.231	0.0399	0.921	1.88	0.742
× 4.5	0.587	3.65	1.83	1.63	2.12	0.289	0.265	0.457	0.493	0.531	0.321	0.0322	0.871	2.01	0.710
C3 × 6	0.322	2.07	1.38	1.09	1.74	0.300	0.263	0.413	0.455	0.543	0.294	0.0725	0.462	1.40	0.690
× 5	0.392	1.85	1.23	1.12	1.52	0.241	0.228	0.405	0.439	0.464	0.245	0.0425	0.379	1.45	0.673
× 4.1	0.461	1.65	1.10	1.17	1.32	0.191	0.196	0.398	0.437	0.399	0.262	0.0269	0.307	1.53	0.655
× 3.5	0.493	1.57	1.04	1.20	1.24	0.169	0.182	0.394	0.443	0.364	0.296	0.0226	0.276	1.57	0.646

Source: Courtesy of the American Institute of Steel Construction, Chicago, IL.

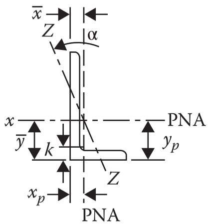
APPENDIX C.3a
Angles Properties



				Axis x–x						Flexural–Torsional Properties		
Shape	**k (in.)**	**Wt. (lb/ft.)**	**Area, A (in.²)**	**J (in.⁴)**	**S (in.³)**	**r (in.)**	**$\bar{y}$ (in.)**	**Z (in.³)**	**y_p (in.⁴)**	**J (in.⁴)**	**C_w (in.⁴)**	**$\bar{r}_0$ (in.)**
L4 × 31⁄2 × 1/2	7/8	11.9	3.50	5.30	1.92	1.23	1.24	3.46	0.500	0.301	0.302	2.03
× 3/8	3/4	9.10	2.68	4.15	1.48	1.25	1.20	2.66	0.427	0.132	0.134	2.06
× 5/16	1$\frac{1}{16}$	7.70	2.25	3.53	1.25	1.25	1.17	2.24	0.400	0.0782	0.0798	2.08
× 1/4	5/8	6.20	1.82	2.89	1.01	1.26	1.14	1.81	0.360	0.0412	0.0419	2.09
L4 × 3 × 5/8	1	13.6	3.99	6.01	2.28	1.23	1.37	4.08	0.808	0.529	0.472	1.91
× 1/2	7/8	11.1	3.25	5.02	1.87	1.24	1.32	3.36	0.750	0.281	0.255	1.94
× 3/8	3/4	8.50	2.49	3.94	1.44	1.26	1.27	2.60	0.680	0.123	0.114	1.97
× 5/16	1$\frac{1}{16}$	7.20	2.09	3.36	1.22	1.27	1.25	2.19	0.656	0.0731	0.0676	1.98
× 1/4	5/8	5.80	1.69	2.75	0.988	1.27	1.22	1.77	0.620	0.0386	0.0356	1.99
L31⁄2 × 31⁄2 × 1/2	7/8	11.1	3.25	3.63	1.48	1.05	1.05	2.66	0.464	0.281	0.238	1.87
× 7/16	1$\frac{3}{16}$	9.80	2.89	3.25	1.32	1.06	1.03	2.36	0.413	0.192	0.164	1.89
× 3/8	3/4	8.50	2.50	2.86	1.15	1.07	1.00	2.06	0.357	0.123	0.106	1.90
× 5/16	1$\frac{1}{16}$	7.20	2.10	2.44	0.969	1.08	0.979	1.74	0.300	0.0731	0.0634	1.92
× 1/4	5/8	5.80	1.70	2.00	0.787	1.09	0.954	1.41	0.243	0.0386	0.0334	1.93
L31⁄2 × 3 × 1/2	7/8	10.2	3.02	3.45	1.45	1.07	1.12	2.61	0.480	0.260	0.191	1.75
× 7/16	1$\frac{3}{16}$	9.10	2.67	3.10	1.29	1.08	1.09	2.32	0.449	0.178	0.132	1.76
× 3/8	3/4	7.90	2.32	2.73	1.12	1.09	1.07	2.03	0.407	0.114	0.0858	1.78
× 5/16	1$\frac{1}{16}$	6.60	1.95	2.33	0.951	1.09	1.05	1.72	0.380	0.0680	0.0512	1.79
× 1/4	5/8	5.40	1.58	1.92	0.773	1.10	1.02	1.39	0.340	0.0360	0.0270	1.80

(*Continued*)

APPENDIX C.3a (*Continued*)
Angles Properties

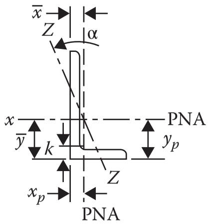


				Axis x–x						Flexural–Torsional Properties		
Shape	***k* (in.)**	**Wt. (lb/ft.)**	**Area, *A* (in.²)**	***J* (in.⁴)**	***S* (in.³)**	***r* (in.)**	**$\bar{y}$ (in.)**	***Z* (in.³)**	**y_p (in.⁴)**	***J* (in.⁴)**	**C_w (in.⁴)**	**$\bar{r}_0$ (in.)**
L31/2 × 21/2 × 1/2	7/8	9.40	2.77	3.24	1.41	1.08	1.20	2.52	0.730	0.234	0.159	1.66
× 3/8	3/4	7.20	2.12	2.56	1.09	1.10	1.15	1.96	0.673	0.103	0.0714	1.69
× 5/16	1 1/16	6.10	1.79	2.20	0.925	1.11	1.13	1.67	0.636	0.0611	0.0426	1.71
× 1/4	5/8	4.90	1.45	1.81	0.753	1.12	1.10	1.36	0.600	0.0322	0.0225	1.72
L3 × 3 × 1/2	7/8	9.40	2.76	2.20	1.06	0.895	0.929	1.91	0.460	0.230	0.144	1.59
× 7/16	1 3/16	8.30	2.43	1.98	0.946	0.903	0.907	1.70	0.405	0.157	0.100	1.60
× 3/8	3/4	7.20	2.11	1.75	0.825	0.910	0.884	1.48	0.352	0.101	0.0652	1.62
× 5/16	1 1/16	6.10	1.78	1.50	0.699	0.918	0.860	1.26	0.297	0.0597	0.0390	1.64
× 1/4	5/8	4.90	1.44	1.23	0.569	0.926	0.836	1.02	0.240	0.0313	0.0206	1.65
× 3/16	9/16	3.71	1.09	0.948	0.433	0.933	0.812	0.774	0.182	0.0136	0.00899	1.67
L3 × 21/2 × 1/2	7/8	8.50	2.50	2.07	1.03	0.910	0.995	1.86	0.500	0.213	0.112	1.46
× 7/16	1 3/16	7.60	2.22	1.87	0.921	0.917	0.972	1.66	0.463	0.146	0.0777	1.48
× 3/8	3/4	6.60	1.93	1.65	0.803	0.924	0.949	1.45	0.427	0.0943	0.0507	1.49
× 5/16	1 1/16	5.60	1.63	1.41	0.681	0.932	0.925	1.23	0.392	0.0560	0.0304	1.51
× 1/4	5/8	4.50	1.32	1.16	0.555	0.940	0.900	1.000	0.360	0.0296	0.0161	1.52
× 3/16	9/16	3.39	1.00	0.899	0.423	0.947	0.874	0.761	0.333	0.0130	0.00705	1.54
L3 × 2 × 1/2	1 3/16	7.70	2.26	1.92	1.00	0.922	1.08	1.78	0.740	0.192	0.0908	1.39
× 3/8	1 1/16	5.90	1.75	1.54	0.779	0.937	1.03	1.39	0.667	0.0855	0.0413	1.42

(*Continued*)

APPENDIX C.3a (*Continued*)

Angles Properties

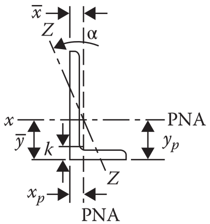


Shape	k (in.)	Wt. (lb/ft.)	Area, A (in.²)	Axis x–x: J (in.⁴)	S (in.³)	r (in.)	$\bar{y}$ (in.)	Z (in.³)	y_p (in.⁴)	Flexural–Torsional Properties: J (in.⁴)	C_w (in.⁴)	$\bar{r}_0$ (in.)
× 5/16	5/8	5.00	1.48	1.32	0.662	0.945	1.01	1.19	0.632	0.0510	0.0248	1.43
× 1/4	9/16	4.10	1.20	1.09	0.541	0.953	0.980	0.969	0.600	0.0270	0.0132	1.45
× 3/16	1/2	3.07	0.917	0.847	0.414	0.961	0.952	0.743	0.555	0.0119	0.00576	1.46
L21⁄2 × 21⁄2 × 1/2	3/4	7.70	2.26	1.22	0.716	0.735	0.803	1.29	0.452	0.188	0.0791	1.30
× 3/8	5/8	5.90	1.73	0.972	0.558	0.749	0.758	1.01	0.346	0.0833	0.0362	1.33
× 5/16	9/16	5.00	1.46	0.837	0.474	0.756	0.735	0.853	0.292	0.0495	0.0218	1.35
× 1/4	1/2	4.10	1.19	0.692	0.387	0.764	0.711	0.695	0.238	0.0261	0.0116	1.36
× 3/16	7/16	3.07	0.901	0.535	0.295	0.771	0.687	0.529	0.180	0.0114	0.00510	1.38
L21⁄2 × 2 × 3/8	5/8	5.30	1.55	0.914	0.546	0.766	0.826	0.982	0.433	0.0746	0.0268	1.22
× 5/16	9/16	4.50	1.32	0.790	0.465	0.774	0.803	0.839	0.388	0.0444	0.0162	1.23
× 1/4	1/2	3.62	1.07	0.656	0.381	0.782	0.779	0.688	0.360	0.0235	0.00868	1.25
× 3/16	7/16	2.75	0.818	0.511	0.293	0.790	0.754	0.529	0.319	0.0103	0.00382	1.26
L21⁄2 × 11⁄2 × 1/4	1/2	3.19	0.947	0.594	0.364	0.792	0.866	0.644	0.606	0.0209	0.00694	1.19
× 3/16	7/16	2.44	0.724	0.464	0.280	0.801	0.839	0.497	0.569	0.00921	0.00306	1.20
L2 × 2 × 3/8	5/8	4.70	1.37	0.476	0.348	0.591	0.632	0.629	0.343	0.0658	0.0174	1.05
× 5/16	9/16	3.92	1.16	0.414	0.298	0.598	0.609	0.537	0.290	0.0393	0.0106	1.06
× 1/4	1/2	3.19	0.944	0.346	0.244	0.605	0.586	0.440	0.236	0.0209	0.00572	1.08

(*Continued*)

APPENDIX C.3a (*Continued*)

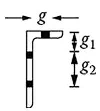
Angles Properties



Shape	k (in.)	Wt. (lb/ft.)	Area, A (in.^{2})	Axis x–x: J (in.^{4})	S (in.^{3})	r (in.)	$\bar{y}$ (in.)	Z (in.^{3})	y_p (in.^{4})	Flexural–Torsional Properties: J (in.^{4})	C_w (in.^{4})	$\bar{r}_0$ (in.)
× 3/16	7/16	2.44	0.722	0.271	0.188	0.612	0.561	0.338	0.181	0.00921	0.00254	1.09
× 1/8	3/8	1.65	0.491	0.189	0.129	0.620	0.534	0.230	0.123	0.00293	0.000789	1.10

Source: Courtesy of the American Institute of Steel Construction, Chicago, IL.
Note: For compactness criteria, refer to the end of Appendix C.3b.

Workable Gages in Angles Legs (in.)



Leg	8	7	6	5	4	3½	3	2½	2	1¾	1½	1⅜	1¼	1
g	4½	4	3½	3	2½	2	1¾	1⅜	1⅛	1	7/8	7/8	3/4	5/8
g_1	3	2½	2¼	2										
g_2	3	3	2½	1¾										

Note: Other gages are permitted to suit specific requirements subject to clearances and edge distance limitations.

APPENDIX C.3b
Angles Other Properties

	Axis y–y						Axis z–z				Q_s
Shape	I (in.^{4})	S (in.^{3})	r (in.)	$\bar{x}$ (in.)	Z (in.^{3})	x_p (in.)	I (in.^{4})	S (in.^{3})	r (in.)	Tan α	F_y = 36 ksi
L4 × 31⁄2 × 1/2	3.76	1.50	1.04	0.994	2.69	0.438	1.80	1.17	0.716	0.750	1.00
× 3/8	2.96	1.16	1.05	0.947	2.06	0.335	1.38	0.938	0.719	0.755	1.00
× 5/16	2.52	0.980	1.06	0.923	1.74	0.281	1.17	0.811	0.721	0.757	0.997
× 1/4	2.07	0.794	1.07	0.897	1.40	0.228	0.950	0.653	0.723	0.759	0.912
L4 × 3 × 5/8	2.85	1.34	0.845	0.867	2.45	0.499	1.59	1.13	0.631	0.534	1.00
× 1/2	2.40	1.10	0.858	0.822	1.99	0.406	1.30	0.927	0.633	0.542	1.00
× 3/8	1.89	0.851	0.873	0.775	1.52	0.311	1.01	0.705	0.636	0.551	1.00
× 5/16	1.62	0.721	0.880	0.750	1.28	0.261	0.851	0.591	0.638	0.554	0.997
× 1/4	1.33	0.585	0.887	0.725	1.03	0.211	0.691	0.476	0.639	0.558	0.912
L31⁄2 × 31⁄2 × 1/2	3.63	1.48	1.05	1.05	2.66	0.464	1.51	1.01	0.679	1.00	1.00
× 7/16	3.25	1.32	1.06	1.03	2.36	0.413	1.34	0.920	0.681	1.00	1.00
× 3/8	2.86	1.15	1.07	1.00	2.06	0.357	1.17	0.821	0.683	1.00	1.00
× 5/16	2.44	0.969	1.08	0.979	1.74	0.300	0.989	0.714	0.685	1.00	1.00
× 1/4	2.00	0.787	1.09	0.954	1.41	0.243	0.807	0.598	0.688	1.00	0.965
L31⁄2 × 3 × 1/2	2.32	1.09	0.877	0.869	1.97	0.431	1.15	0.851	0.618	0.713	1.00
× 7/16	2.09	0.971	0.885	0.846	1.75	0.381	1.03	0.774	0.620	0.717	1.00
× 3/8	1.84	0.847	0.892	0.823	1.52	0.331	0.895	0.692	0.622	0.720	1.00
× 5/16	1.58	0.718	0.900	0.798	1.28	0.279	0.761	0.602	0.624	0.722	1.00
× 1/4	1.30	0.585	0.908	0.773	1.04	0.226	0.623	0.487	0.628	0.725	0.965
L31⁄2 × 21⁄2 × 1/2	1.36	0.756	0.701	0.701	1.39	0.396	0.782	0.649	0.532	0.485	1.00
× 3/8	1.09	0.589	0.716	0.655	1.07	0.303	0.608	0.496	0.535	0.495	1.00
× 5/16	0.937	0.501	0.723	0.632	0.900	0.256	0.518	0.419	0.538	0.500	1.00
× 1/4	0.775	0.410	0.731	0.607	0.728	0.207	0.425	0.340	0.541	0.504	0.965
L3 × 3 × 1/2	2.20	1.06	0.895	0.929	1.91	0.460	0.924	0.703	0.580	1.00	1.00
× 7/16	1.98	0.946	0.903	0.907	1.70	0.405	0.819	0.639	0.580	1.00	1.00
× 3/8	1.75	0.825	0.910	0.884	1.48	0.352	0.712	0.570	0.581	1.00	1.00
× 5/16	1.50	0.699	0.918	0.860	1.26	0.297	0.603	0.496	0.583	1.00	1.00
× 1/4	1.23	0.569	0.926	0.836	1.02	0.240	0.491	0.415	0.585	1.00	1.00
× 3/16	0.948	0.433	0.933	0.812	0.774	0.182	0.374	0.326	0.586	1.00	0.912
L3 × 21⁄2 × 1/2	1.29	0.736	0.718	0.746	1.34	0.417	0.666	0.568	0.516	0.666	1.00
× 7/16	1.17	0.656	0.724	0.724	1.19	0.370	0.591	0.517	0.516	0.671	1.00
× 3/8	1.03	0.573	0.731	0.701	1.03	0.322	0.514	0.463	0.517	0.675	1.00
× 5/16	0.888	0.487	0.739	0.677	0.873	0.272	0.437	0.404	0.518	0.679	1.00
× 1/4	0.734	0.397	0.746	0.653	0.707	0.220	0.356	0.327	0.520	0.683	1.00
× 3/16	0.568	0.303	0.753	0.627	0.536	0.167	0.272	0.247	0.521	0.687	0.912
L3 × 2 × 1/2	0.667	0.470	0.543	0.580	0.887	0.377	0.409	0.411	0.425	0.413	1.00
× 3/8	0.539	0.368	0.555	0.535	0.679	0.292	0.318	0.313	0.426	0.426	1.00
× 5/16	0.467	0.314	0.562	0.511	0.572	0.247	0.271	0.264	0.428	0.432	1.00
× 1/4	0.390	0.258	0.569	0.487	0.463	0.200	0.223	0.214	0.431	0.437	1.00
× 3/16	0.305	0.198	0.577	0.462	0.351	0.153	0.173	0.163	0.435	0.442	0.912
L21⁄2 × 21⁄2 × 1/2	1.22	0.716	0.735	0.803	1.29	0.452	0.521	0.459	0.481	1.00	1.00
× 3/8	0.972	0.558	0.749	0.758	1.01	0.346	0.400	0.373	0.481	1.00	1.00
× 5/16	0.837	0.474	0.756	0.735	0.853	0.292	0.339	0.326	0.481	1.00	1.00
× 1/4	0.692	0.387	0.764	0.711	0.695	0.238	0.275	0.274	0.482	1.00	1.00
× 3/16	0.535	0.295	0.771	0.687	0.529	0.180	0.210	0.216	0.482	1.00	0.983

(*Continued*)

APPENDIX C.3b (*Continued*)
Angles Other Properties

	Axis y–y						Axis z–z				Q_s
Shape	***I*** **(in.^{4})**	***S*** **(in.^{3})**	***r*** **(in.)**	$\bar{x}$ **(in.)**	***Z*** **(in.^{3})**	x_p **(in.)**	***I*** **(in.^{4})**	***S*** **(in.^{3})**	***r*** **(in.)**	**Tan α**	F_y **= 36 ksi**
L21⁄2 × 2 × 3/8	0.513	0.361	0.574	0.578	0.657	0.310	0.273	0.295	0.419	0.612	1.00
× 5/16	0.446	0.309	0.581	0.555	0.557	0.264	0.233	0.260	0.420	0.618	1.00
× 1/4	0.372	0.253	0.589	0.532	0.454	0.214	0.191	0.213	0.423	0.624	1.00
× 3/16	0.292	0.195	0.597	0.508	0.347	0.164	0.149	0.163	0.426	0.628	0.983
L21⁄2 × 11⁄2 × 1/4	0.160	0.142	0.411	0.372	0.261	0.189	0.0975	0.119	0.321	0.354	1.00
× 3/16	0.126	0.110	0.418	0.347	0.198	0.145	0.0760	0.091	0.324	0.360	0.983
L2 × 2 × 3/8	0.476	0.348	0.591	0.632	0.629	0.343	0.203	0.227	0.386	1.00	1.00
× 5/16	0.414	0.298	0.598	0.609	0.537	0.290	0.173	0.20	0.386	1.00	1.00
× 1/4	0.346	0.244	0.605	0.586	0.440	0.236	0.141	0.171	0.387	1.00	1.00
× 3/16	0.271	0.188	0.612	0.561	0.338	0.181	0.109	0.137	0.389	1.00	1.00
× 1/8	0.189	0.129	0.620	0.534	0.230	0.123	0.0751	0.0994	0.391	1.00	0.912

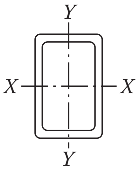
Source: Courtesy of the American Institute of Steel Constructions, Chicago, IL.
Note: For compactness criteria, refer to the end of this table.

APPENDIX C.3c
Compactness Criteria for Angles

	Compression	Flexure	
	Nonslender up to	**Compact up to**	**Noncompact up to**
t	**Width of Angle Leg (in.)**		
$1\frac{1}{8}$	8	8	—
1			—
7/8			—
3/4			—
5/8			—
9/16	7		8
1/2	6	7	
7/16	5	6	
3/8	4	5	
5/16	4	4	
1/4	3	3 1/2	6
3/16	2	2 1/2	4
1/8	1 1/2	1 1/2	3

Note: Compactness criteria given for F_y = 36 ksi and C_v = 1.0 for all angles.

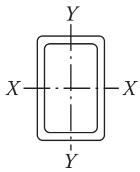
APPENDIX C.4a
Rectangular HSS Dimensions and Properties



Shape	Design Wall Thickness, t (in.)	Nominal Wt. (lb/t.)	Area, A (in.^{2})	Axis x–x					
				b/t	h/t	I (in.^{4})	S (in.^{3})	r (in.)	Z (in.^{3})
HSS6 × 4 × 1/2	0.465	28.43	7.88	5.60	9.90	34.0	11.3	2.08	14.6
× 3/8	0.349	22.37	6.18	8.46	14.2	28.3	9.43	2.14	11.9
× 5/16	0.291	19.08	5.26	10.7	17.6	24.8	8.27	2.17	10.3
× 1/4	0.233	15.62	4.30	14.2	22.8	20.9	6.96	2.20	8.53
× 3/16	0.174	11.97	3.28	20.0	31.5	16.4	5.46	2.23	6.60
× 1/8	0.116	8.16	2.23	31.5	48.7	11.4	3.81	2.26	4.56
HSS6 × 3 × 1/2	0.465	25.03	6.95	3.45	9.90	26.8	8.95	1.97	12.1
× 3/8	0.349	19.82	5.48	5.60	14.2	22.7	7.57	2.04	9.90
× 5/16	0.291	16.96	4.68	7.31	17.6	20.1	6.69	2.07	8.61
× 1/4	0.233	13.91	3.84	9.88	22.8	17.0	5.66	2.10	7.19
× 3/16	0.174	10.70	2.93	14.2	31.5	13.4	4.47	2.14	5.59
× 1/8	0.116	7.31	2.00	22.9	48.7	9.43	3.14	2.17	3.87
HSS6 × 2 × 3/8	0.349	17.27	4.78	2.73	14.2	17.1	5.71	1.89	7.93
× 5/16	0.291	14.83	4.10	3.87	17.6	15.3	5.11	1.93	6.95
× 1/4	0.233	12.21	3.37	5.58	22.8	13.1	4.37	1.97	5.84
× 3/16	0.174	9.42	2.58	8.49	31.5	10.5	3.49	2.01	4.58
× 1/8	0.116	6.46	1.77	14.2	48.7	7.42	2.47	2.05	3.19
HSS5 × 4 × 1/2	0.465	25.03	6.95	5.60	7.75	21.2	8.49	1.75	10.9
× 3/8	0.349	19.82	5.48	8.46	11.3	17.9	7.17	1.81	8.96
× 5/16	0.291	16.96	4.68	10.7	14.2	15.8	6.32	1.84	7.79
× 1/4	0.233	13.91	3.84	14.2	18.5	13.4	5.35	1.87	6.49
× 3/16	0.174	10.70	2.93	20.0	25.7	10.6	4.22	1.90	5.05
× 1/8	0.116	7.31	2.00	31.5	40.1	7.42	2.97	1.93	3.50
HSS5 × 3 × 1/2	0.465	21.63	6.02	3.45	7.75	16.4	6.57	1.65	8.83
× 3/8	0.349	17.27	4.78	5.60	11.3	14.1	5.65	1.72	7.34
× 5/16	0.291	14.83	4.10	7.31	14.2	12.6	5.03	1.75	6.42
× 1/4	0.233	12.21	3.37	9.88	18.5	10.7	4.29	1.78	5.38
× 3/16	0.174	9.42	2.58	14.2	25.7	8.53	3.41	1.82	4.21
× 1/8	0.116	6.46	1.77	22.9	40.1	6.03	2.41	1.85	2.93
HSS5 × 2½ × 1/4	0.233	11.36	3.14	7.73	18.5	9.40	3.76	1.73	4.83
× 3/16	0.174	8.78	2.41	11.4	25.7	7.51	3.01	1.77	3.79
× 1/8	0.116	6.03	1.65	18.6	40.1	5.34	2.14	1.80	2.65

(*Continued*)

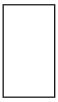
APPENDIX C.4a (*Continued*)
Rectangular HSS Dimensions and Properties



Shape	Design Wall Thickness, *t* (in.)	Nominal Wt. (lb/t.)	Area, *A* (in.^{2})	Axis *x–x* *b/t*	*h/t*	*I* (in.^{4})	*S* (in.^{3})	*r* (in.)	*Z* (in.^{3})
HSS5 × 2 × 3/8	0.349	14.72	4.09	2.73	11.3	10.4	4.14	1.59	5.71
× 5/16	0.291	12.70	3.52	3.87	14.2	9.35	3.74	1.63	5.05
× 1/4	0.233	10.51	2.91	5.58	18.5	8.08	3.23	1.67	4.27
× 3/16	0.174	8.15	2.24	8.49	25.7	6.50	2.60	1.70	3.37
× 1/8	0.116	5.61	1.54	14.2	40.1	4.65	1.86	1.74	2.37
HSS4 × 3 × 3/8	0.349	14.72	4.09	5.60	8.46	7.93	3.97	1.39	5.12
× 5/16	0.291	12.70	3.52	7.31	10.7	7.14	3.57	1.42	4.51
× 1/4	0.233	10.51	2.91	9.88	14.2	6.15	3.07	1.45	3.81
× 3/16	0.174	8.15	2.24	14.2	20.0	4.93	2.47	1.49	3.00
× 1/8	0.116	5.61	1.54	22.9	31.5	3.52	1.76	1.52	2.11
HSS4 × 2½ × 3/8	0.349	13.44	3.74	4.16	8.46	6.77	3.38	1.35	4.48
× 5/16	0.291	11.64	3.23	5.59	10.7	6.13	3.07	1.38	3.97
× 1/4	0.233	9.66	2.67	7.73	14.2	5.32	2.66	1.41	3.38
× 3/16	0.174	7.51	2.06	11.4	20.0	4.30	2.15	1.44	2.67
× 1/8	0.116	5.18	1.42	18.6	31.5	3.09	1.54	1.47	1.88
HSS4 × 2 × 3/8	0.349	12.17	3.39	2.73	8.46	5.60	2.80	1.29	3.84
× 5/16	0.291	10.58	2.94	3.87	10.7	5.13	2.56	1.32	3.43
× 1/4	0.233	8.81	2.44	5.58	14.2	4.49	2.25	1.36	2.94
× 3/16	0.174	6.87	1.89	8.49	20.0	3.66	1.83	1.39	2.34
× 1/8	0.116	4.75	1.30	14.2	31.5	2.65	1.32	1.43	1.66
HSS31/2 × 2½ × 3/8	0.349	12.17	3.39	4.16	7.03	4.75	2.72	1.18	3.59
× 5/16	0.291	10.58	2.94	5.59	9.03	4.34	2.48	1.22	3.20
× 1/4	0.233	8.81	2.44	7.73	12.0	3.79	2.17	1.25	2.74
× 3/16	0.174	6.87	1.89	11.4	17.1	3.09	1.76	1.28	2.18
× 1/8	0.116	4.75	1.30	18.6	27.2	2.23	1.28	1.31	1.54
HSS3½ × 2 × 1/4	0.233	7.96	2.21	5.58	12.0	3.17	1.81	1.20	2.36
× 3/16	0.174	6.23	1.71	8.49	17.1	2.61	1.49	1.23	1.89
× 1/8	0.116	4.33	1.19	14.2	27.2	1.90	1.09	1.27	1.34

Source: Courtesy of the American Institute of Steel Construction, Chicago, IL.

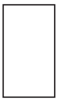
APPENDIX C.4b
Rectangular HSS Other Properties



Shape	Axis y–y				Workable Flat		Torsion		Surface Area
	I (in.^{4})	S (in.^{3})	r (in.)	Z (in.^{3})	Depth (in.)	Width (in.)	J (in.^{4})	C (in.^{3})	(ft.^{2}/ft.)
HSS6 × 4 × 1/2	17.8	8.89	1.50	11.0	$3\frac{3}{4}$	—[a]	40.3	17.8	1.53
× 3/8	14.9	7.47	1.55	8.94	$4\frac{5}{16}$	$2\frac{5}{16}$	32.8	14.2	1.57
× 5/16	13.2	6.58	1.58	7.75	$4\frac{5}{8}$	$2\frac{5}{8}$	28.4	12.2	1.58
× 1/4	11.1	5.56	1.61	6.45	$4\frac{7}{8}$	$2\frac{7}{8}$	23.6	10.1	1.60
× 3/16	8.76	4.38	1.63	5.00	$5\frac{3}{16}$	$3\frac{3}{16}$	18.2	7.74	1.62
× 1/8	6.15	3.08	1.66	3.46	$5\frac{7}{16}$	$3\frac{7}{16}$	12.6	5.30	1.63
HSS6 × 3 × 1/2	8.69	5.79	1.12	7.28	$3\frac{3}{4}$	—[a]	23.1	12.7	1.37
× 3/8	7.48	4.99	1.17	6.03	$4\frac{5}{16}$	—[a]	19.3	10.3	1.40
× 5/16	6.67	4.45	1.19	5.27	$4\frac{5}{8}$	—[a]	16.9	8.91	1.42
× 1/4	5.70	3.80	1.22	4.41	$4\frac{7}{8}$	—[a]	14.2	7.39	1.43
× 3/16	4.55	3.03	1.25	3.45	$5\frac{3}{16}$	$2\frac{3}{16}$	11.1	5.71	1.45
× 1/8	3.23	2.15	1.27	2.40	$5\frac{7}{16}$	$2\frac{7}{16}$	7.73	3.93	1.47
HSS6 × 2 × 3/8	2.77	2.77	0.760	3.46	$4\frac{5}{16}$	—[a]	8.42	6.35	1.23
× 5/16	2.52	2.52	0.785	3.07	$4\frac{5}{8}$	—[a]	7.60	5.58	1.25
× 1/4	2.21	2.21	0.810	2.61	$4\frac{7}{8}$	—[a]	6.55	4.70	1.27
× 3/16	1.80	1.80	0.836	2.07	$5\frac{3}{16}$	—[a]	5.24	3.68	1.28
× 1/8	1.31	1.31	0.861	1.46	$5\frac{7}{16}$	—[a]	3.72	2.57	1.30
HSS5 × 4 × 1/2	14.9	7.43	1.46	9.35	$2\frac{3}{4}$	—[a]	30.3	14.5	1.37
× 3/8	12.6	6.30	1.52	7.67	$3\frac{5}{16}$	$2\frac{5}{16}$	24.9	11.7	1.40
× 5/16	11.1	5.57	1.54	6.67	$3\frac{5}{8}$	$2\frac{5}{8}$	21.7	10.1	1.42
× 1/4	9.46	4.73.	1.57	5.57	$3\frac{7}{8}$	$2\frac{7}{8}$	18.0	8.32	1.43
× 3/16	7.48	3.74	1.60	4.34	$4\frac{3}{16}$	$3\frac{3}{16}$	14.0	6.41	1.45
× 1/8	5.27	2.64	1.62	3.01	$4\frac{7}{16}$	$3\frac{7}{16}$	9.66	4.39	1.47
HSS5 × 3 × 1/2	7.18	4.78	1.09	6.10	$2\frac{3}{4}$	—[a]	17.6	10.3	1.20
× 3/8	6.25	4.16	1.14	5.10	$3\frac{5}{16}$	—[a]	14.9	8.44	1.23
× 5/16	5.60	3.73	1.17	4.48	$3\frac{5}{8}$	—[a]	13.1	7.33	1.25
× 1/4	4.81	3.21	1.19	3.77	$3\frac{7}{8}$	—[a]	11.0	6.10	1.27
× 3/16	3.85	2.57	1.22	2.96	$4\frac{3}{16}$	$2\frac{3}{16}$	8.64	4.73	1.28
× 1/8	2.75	1.83	1.25	2.07	$4\frac{7}{16}$	$2\frac{7}{16}$	6.02	3.26	1.30
HSS5 × $2\frac{1}{2}$ × 1/4	3.13	2.50	0.999	2.95	$3\frac{7}{8}$	—[a]	7.93	4.99	1.18
× 3/16	2.53	2.03	1.02	2.33	$4\frac{3}{16}$	—[a]	6.26	3.89	1.20
× 1/8	1.82	1.46	1.05	1.64	$4\frac{7}{16}$	—[a]	4.40	2.70	1.22
HSS5 × 2 × 3/8	2.28	2.28	0.748	2.88	$3\frac{5}{16}$	—[a]	6.61	5.20	1.07
× 5/16	2.10	2.10	0.772	2.57	$3\frac{5}{8}$	—[a]	5.99	4.59	1.08
× 1/4	1.84	1.84	0.797	2.20	$3\frac{7}{8}$	—[a]	5.17	3.88	1.10
× 3/16	1.51	1.51	0.823	1.75	$4\frac{3}{16}$	—[a]	4.15	3.05	1.12
× 1/8	1.10	1.10	0.848	1.24	$4\frac{7}{16}$	—[a]	2.95	2.13	1.13

(Continued)

APPENDIX C.4b (*Continued*)
Rectangular HSS Other Properties

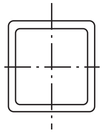


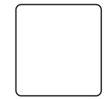
Shape	Axis *y–y* I (in.⁴)	S (in.³)	r (in.)	Z (in.³)	Workable Flat Depth (in.)	Width (in.)	Torsion J (in.⁴)	C (in.³)	Surface Area (ft.²/ft.)
HSS4 × 3 × 3/8	5.01	3.34	1.11	4.18	$2\frac{5}{16}$	—[a]	10.6	6.59	1.07
× 5/16	4.52	3.02	1.13	3.69	$2\frac{5}{8}$	—[a]	9.41	5.75	1.08
× 1/4	3.91	2.61	1.16	3.12	$2\frac{7}{8}$	—[a]	7.96	4.81	1.10
× 3/16	3.16	2.10	1.19	2.46	$3\frac{3}{16}$	—[a]	6.26	3.74	1.12
× 1/8	2.27	1.51	1.21	1.73	$3\frac{7}{16}$	—[a]	4.38	2.59	1.13
HSS4 × $2\frac{1}{2}$ × 3/8	3.17	2.54	0.922	3.20	$2\frac{5}{16}$	—[a]	7.57	5.32	0.983
× 5/16	2.89	2.32	0.947	2.85	$2\frac{5}{8}$	—[a]	6.77	4.67	1.00
× 1/4	2.53	2.02	0.973	2.43	$2\frac{7}{8}$	—[a]	5.78	3.93	1.02
× 3/16	2.06	1.65	0.999	1.93	$3\frac{7}{8}$	—[a]	4.59	3.08	1.03
× 1/8	1.49	1.19	1.03	1.36	$3\frac{7}{16}$	—[a]	3.23	2.14	1.05
HSS4 × 2 × 3/8	1.80	1.80	0.729	2.31	$2\frac{5}{16}$	—[a]	4.83	4.04	0.900
× 5/16	1.67	1.67	0.754	2.08	$2\frac{5}{8}$	—[a]	4.40	3.59	0.917
× 1/4	1.48	1.48	0.779	1.79	$2\frac{7}{2}$	—[a]	3.82	3.05	0.933
× 3/16	1.22	1.22	0.804	1.43	$3\frac{3}{16}$	—[a]	3.08	2.41	0.950
× 1/8	0.898	0.898	0.830	1.02	$3\frac{7}{16}$	—[a]	2.20	1.69	0.967
HSS3 1/2 × $2\frac{1}{2}$ × 3/8	2.77	2.21	0.904	2.82	—[a]	—[a]	6.16	4.57	0.900
× 5/16	2.54	2.03	0.930	2.52	$2\frac{1}{8}$	—[a]	5.53	4.03	0.917
× 1/4	2.23	1.78	0.956	2.16	$2\frac{3}{8}$	—[a]	4.75	3.40	0.933
× 3/16	1.82	1.46	0.983	1.72	$2\frac{11}{16}$	—[a]	3.78	2.67	0.950
× 1/8	1.33	1.06	1.01	1.22	$2\frac{15}{16}$	—[a]	2.67	1.87	0.967
HSS$3\frac{1}{2}$ × 2 × 1/4	1.30	1.30	0.766	1.58	$2\frac{3}{8}$	—[a]	3.16	2.64	0.850
× 3/16	1.08	1.08	0.792	1.27	$2\frac{11}{16}$	—[a]	2.55	2.09	0.867
× 1/8	0.795	0.795	0.818	0.912	$2\frac{15}{16}$	—[a]	1.83	1.47	0.883

Source: Courtesy of the American Institute of Steel Construction, Chicago, IL.

[a] Flat depth or width is too small to establish a workable flat.

APPENDIX C.5
Square HSS: Dimensions and Properties



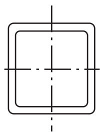


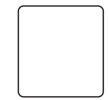
HSS7–HSS4 1/2

Shape	Design Wall Thickness, t (in.)	Nominal Wt. (lb/ft.)	Area, A (in.^{2})	b/t	h/t	I (in.^{4})	S (in.^{3})	r (in.)	Z (in.^{3})	Workable Flat (in.)	Torsion J (in.^{4})	Torsion C (in.^{3})	Surface Area (ft.^{2}/ft.)
HSS7 × 7 × 5/8	0.581	50.81	14.0	9.05	9.05	93.4	26.7	2.58	33.1	$4\frac{3}{16}$	158	47.1	2.17
× 1/2	0.465	42.05	11.6	12.1	12.1	80.5	23.0	2.63	27.9	$4\frac{3}{4}$	133	39.3	2.20
× 3/8	0.349	32.58	8.97	17.1	17.1	65.0	18.6	2.69	22.1	$5\frac{5}{16}$	105	30.7	2.23
× 5/16	0.291	27.59	7.59	21.1	21.1	56.1	16.0	2.72	18.9	$5\frac{5}{8}$	89.7	26.1	2.25
× 1/4	0.233	22.42	6.17	27.0	27.0	46.5	13.3	2.75	15.5	$5\frac{7}{8}$	73.5	21.3	2.27
× 3/16	0.174	17.08	4.67	37.2	37.2	36.0	10.3	2.77	11.9	$6\frac{3}{16}$	56.1	16.2	2.28
× 1/8	0.116	11.56	3.16	57.3	57.3	24.8	7.09	2.80	8.13	$6\frac{7}{16}$	38.2	11.0	2.30
HSS6 × 6 × 5/8	0.581	42.30	11.7	7.33	7.33	55.2	18.4	2.17	23.2	$3\frac{3}{16}$	94.9	33.4	1.83
× 1/2	0.465	35.24	9.74	9.90	9.90	48.3	16.1	2.23	19.8	$3\frac{3}{4}$	81.1	28.1	1.87
× 3/8	0.349	27.48	7.58	14.2	14.2	39.5	13.2	2.28	15.8	$4\frac{5}{16}$	64.6	22.1	1.90
× 5/16	0.291	23.34	6.43	17.6	17.6	34.3	11.4	2.31	13.6	$4\frac{5}{8}$	55.4	18.9	1.92
× 1/4	0.233	19.02	5.24	22.8	22.8	28.6	9.54	2.34	11.2	$4\frac{7}{8}$	45.6	15.4	1.93
× 3/16	0.174	14.53	3.98	31.5	31.5	22.3	7.42	2.37	8.63	$5\frac{3}{16}$	35.0	11.8	1.95
× 1/8	0.116	9.86	2.70	48.7	48.7	15.5	5.15	2.39	5.92	$5\frac{7}{16}$	23.9	8.03	1.97
HSS$5\frac{1}{2}$ × $5\frac{1}{2}$ × 3/8	0.349	24.93	6.88	12.8	12.8	29.7	10.8	2.08	13.1	$3\frac{13}{16}$	49.0	18.4	1.73
× 5/16	0.291	21.21	5.85	15.9	15.9	25.9	9.43	2.11	11.3	$4\frac{1}{8}$	42.2	15.7	1.75
× 1/4	0.233	17.32	4.77	20.6	20.6	21.7	7.90	2.13	9.32	$4\frac{3}{8}$	34.8	12.9	1.77
× 3/16	0.174	13.25	3.63	28.6	28.6	17.0	6.17	2.16	7.19	$4\frac{11}{16}$	26.7	9.85	1.78
× 1/8	0.116	9.01	2.46	44.4	44.4	11.8	4.30	2.19	4.95	$4\frac{15}{16}$	18.3	6.72	1.80
HSS5 × 5 × 1/2	0.465	28.43	7.88	7.75	7.75	26.0	10.4	1.82	13.1	$2\frac{3}{4}$	44.6	18.7	1.53
× 3/8	0.349	22.37	6.18	11.3	11.3	21.7	8.68	1.87	10.6	$3\frac{5}{16}$	36.1	14.9	1.57

(*Continued*)

APPENDIX C.5 (*Continued*)
Square HSS: Dimensions and Properties



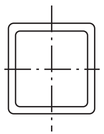


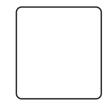
HSS7–HSS4 1/2

Shape	Design Wall Thickness, t (in.)	Nominal Wt. (lb/ft.)	Area, A (in.^{2})	b/t	h/t	I (in.^{4})	S (in.^{3})	r (in.)	Z (in.^{3})	Workable Flat (in.)	Torsion J (in.^{4})	Torsion C (in.^{3})	Surface Area (ft.^{2}/ft.)
× 5/16	0.291	19.08	5.26	14.2	14.2	19.0	7.62	1.90	9.16	$3\frac{5}{8}$	31.2	12.8	1.58
× 1/4	0.233	15.62	4.30	18.5	18.5	16.0	6.41	1.93	7.61	$3\frac{7}{8}$	25.8	10.5	1.60
× 3/16	0.174	11.97	3.28	25.7	25.7	12.6	5.03	1.96	5.89	$4\frac{3}{16}$	19.9	8.08	1.62
× 1/8	0.116	8.16	2.23	40.1	40.1	8.80	3.52	1.99	4.07	$4\frac{7}{16}$	13.7	5.53	1.63
HSS$4\frac{1}{2}$ × $4\frac{1}{2}$ × 1/2	0.465	25.03	6.95	6.68	6.68	18.1	8.03	1.61	10.2	$2\frac{1}{4}$	31.3	14.8	1.37
× 3/8	0.349	19.82	5.48	9.89	9.89	15.3	6.79	1.67	8.36	$2\frac{13}{16}$	25.7	11.9	1.40
× 5/16	0.291	16.96	4.68	12.5	12.5	13.5	6.00	1.70	7.27	$3\frac{1}{8}$	22.3	10.2	1.42
× 1/4	0.233	13.91	3.84	16.3	16.3	11.4	5.08	1.73	6.06	$3\frac{3}{8}$	18.5	8.44	1.43
× 3/16	0.174	10.70	2.93	22.9	22.9	9.02	4.01	1.75	4.71	$3\frac{11}{16}$	14.4	6.49	1.45
× 1/8	0.116	7.31	2.00	35.8	35.8	6.35	2.82	1.78	3.27	$3\frac{15}{16}$	9.92	4.45	1.47
HSS4 × 4 × 1/2	0.465	21.63	6.02	5.60	5.60	11.9	5.97	1.41	7.70	—[a]	21.0	11.2	1.20
× 3/8	0.349	17.27	4.78	8.46	8.46	10.3	5.13	1.47	6.39	$2\frac{5}{16}$	17.5	9.14	1.23
× 5/16	0.291	14.83	4.10	10.7	10.7	9.14	4.57	1.49	5.59	$2\frac{5}{8}$	15.3	7.91	1.25
× 1/4	0.233	12.21	3.37	14.2	14.2	7.80	3.90	1.52	4.69	$2\frac{7}{8}$	12.8	6.56	1.27
× 3/16	0.174	9.42	2.58	20.0	20.0	6.21	3.10	1.55	3.67	$3\frac{3}{16}$	10.0	5.07	1.28
× 1/8	0.116	6.46	1.77	31.5	31.5	4.40	2.20	1.58	2.56	$3\frac{7}{16}$	6.91	3.49	1.30
HSS$3\frac{1}{2}$ × $3\frac{1}{2}$ × 3/8	0.349	14.72	4.09	7.03	7.03	6.49	3.71	1.26	4.69	—[a]	11.2	6.77	1.07
× 5/16	0.291	12.70	3.52	9.03	9.03	5.84	3.34	1.29	4.14	$2\frac{1}{8}$	9.89	5.90	1.08
× 1/4	0.233	10.51	2.91	12.0	12.0	5.04	2.88	1.32	3.50	$2\frac{3}{8}$	8.35	4.92	1.10
× 3/16	0.174	8.15	2.24	17.1	17.1	4.05	2.31	1.35	2.76	$2\frac{11}{16}$	6.56	3.83	1.12
× 1/8	0.116	5.61	1.54	27.2	27.2	2.90	1.66	1.37	1.93	$2\frac{15}{16}$	4.58	2.65	1.13

(*Continued*)

APPENDIX C.5 (*Continued*)
Square HSS: Dimensions and Properties





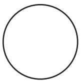
HSS7–HSS4 1/2

Shape	Design Wall Thickness, t (in.)	Nominal Wt. (lb/ft.)	Area, A (in.^{2})	b/t	h/t	I (in.^{4})	S (in.^{3})	r (in.)	Z (in.^{3})	Workable Flat (in.)	Torsion J (in.^{4})	Torsion C (in.^{3})	Surface Area (ft.^{2}/ft.)
HSS3 × 3 × 3/8	0.349	12.17	3.39	5.60	5.60	3.78	2.52	1.06	3.25	—[a]	6.64	4.74	0.900
× 5/16	0.291	10.58	2.94	7.31	7.31	3.45	2.30	1.08	2.90	—[a]	5.94	4.18	0.917
× 1/4	0.233	8.81	2.44	9.88	9.88	3.02	2.01	1.11	2.48	—[a]	5.08	3.52	0.933
× 3/16	0.174	6.87	1.89	14.2	14.2	2.46	1.64	1.14	1.97	23/16	4.03	2.76	0.950
× 1/8	0.116	4.75	1.30	22.9	22.9	1.78	1.19	1.17	1.40	27/16	2.84	1.92	0.967
HSS2½ × 2½ × 5/16	0.291	8.45	2.35	5.59	5.59	1.82	1.46	0.880	1.88	—[a]	3.20	2.74	0.750
× 1/4	0.233	7.11	1.97	7.73	7.73	1.63	1.30	0.908	1.63	—[a]	2.79	2.35	0.767
× 3/16	0.174	5.59	1.54	11.4	11.4	1.35	1.08	0.937	1.32	—[a]	2.25	1.86	0.784
× 1/8	0.116	3.90	1.07	18.6	18.6	0.998	0.799	0.965	0.947	—[a]	1.61	1.31	0.800
HSS2¼ × 2¼ × 1/4	0.233	6.26	1.74	6.66	6.66	1.13	1.01	0.806	1.28	—[a]	1.96	1.85	0.683
× 3/16	0.174	4.96	1.37	9.93	9.93	0.953	0.847	0.835	1.04	—[a]	1.60	1.48	0.700
× 1/8	0.116	3.48	0.956	16.4	16.4	0.712	0.633	0.863	0.755	—[a]	1.15	1.05	0.717
HSS2 × 2 × 1/4	0.233	5.41	1.51	5.58	5.58	0.747	0.747	0.704	0.964	—[a]	1.31	1.41	0.600
× 3/16	0.174	4.32	1.19	8.49	8.49	0.641	0.641	0.733	0.797	—[a]	1.09	1.14	0.617
× 1/8	0.116	3.05	0.840	14.2	14.2	0.486	0.486	0.761	0.584	—[a]	0.796	0.817	0.633

Source: Courtesy of the American Institute of Steel Construction, Chicago, IL.

[a] Flat depth or width is too small to establish a workable flat.

APPENDIX C.6
Round HSS: Dimensions and Properties

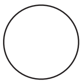


HSS6.625–HSS2.500

Shape	Design Wall Thickness, *t* (in.)	Nominal Wt. (lb/ft.)	Area, *A* (in.²)	*D/t*	*I* (in.⁴)	*S* (in.³)	*r* (in.)	*Z* (in.³)	Torsion *J* (in.⁴)	Torsion *C* (in.³)
HSS6.625 × 0.500	0.465	32.74	9.00	14.2	42.9	13.0	2.18	17.7	85.9	25.9
× 0.432	0.402	28.60	7.86	16.5	38.2	11.5	2.20	15.6	76.4	23.1
× 0.375	0.349	25.06	6.88	19.0	34.0	10.3	2.22	13.8	68.0	20.5
× 0.312	0.291	21.06	5.79	22.8	29.1	8.79	2.24	11.7	58.2	17.6
× 0.280	0.260	18.99	5.20	25.5	26.4	7.96	2.25	10.5	52.7	15.9
× 0.250	0.233	17.04	4.68	28.4	23.9	7.22	2.26	9.52	47.9	14.4
× 0.188	0.174	12.94	3.53	38.1	18.4	5.54	2.28	7.24	36.7	11.1
× 0.125[a]	0.116	8.69	2.37	57.1	12.6	3.79	2.30	4.92	25.1	7.59
HSS6.000 × 0.500	0.465	29.40	8.09	12.9	31.2	10.4	1.96	14.3	62.4	20.8
× 0.375	0.349	22.55	6.20	17.2	24.8	8.28	2.00	11.2	49.7	16.6
× 0.312	0.291	18.97	5.22	20.6	21.3	7.11	2.02	9.49	42.6	14.2
× 0.280	0.260	17.12	4.69	23.1	19.3	6.45	2.03	8.57	38.7	12.9
× 0.250	0.233	15.37	4.22	25.8	17.6	5.86	2.04	7.75	35.2	11.7
× 0.188	0.174	11.68	3.18	34.5	13.5	4.51	2.06	5.91	27.0	9.02
× 0.125[a]	0.116	7.85	2.14	51.7	9.28	3.09	2.08	4.02	18.6	6.19
HSS5.563 × 0.500	0.465	27.06	7.45	12.0	24.4	8.77	1.81	12.1	48.8	17.5
× 0.375	0.349	20.80	5.72	15.9	19.5	7.02	1.85	9.50	39.0	14.0
× 0.258	0.240	14.63	4.01	23.2	14.2	5.12	1.88	6.80	28.5	10.2
× 0.188	0.174	10.80	2.95	32.0	10.7	3.85	1.91	5.05	21.4	7.70
× 0.134	0.124	7.78	2.12	44.9	7.84	2.82	1.92	3.67	15.7	5.64
HSS5.500 × 0.500	0.465	26.73	7.36	11.8	23.5	8.55	1.79	11.8	47.0	17.1
× 0.375	0.349	20.55	5.65	15.8	18.8	6.84	1.83	9.27	37.6	13.7
× 0.258	0.240	14.46	3.97	22.9	13.7	5.00	1.86	6.64	27.5	10.0
HSS5.000 × 0.500	0.465	24.05	6.62	10.8	17.2	6.88	1.61	9.60	34.4	13.8
× 0.375	0.349	18.54	5.10	14.3	13.9	5.55	1.65	7.56	27.7	11.1
× 0.312	0.291	15.64	4.30	17.2	12.0	4.79	1.67	6.46	24.0	9.58
× 0.258	0.240	13.08	3.59	20.8	10.2	4.08	1.69	5.44	20.4	8.15
× 0.250	0.233	12.69	3.49	21.5	9.94	3.97	1.69	5.30	19.9	7.95
× 0.188	0.174	9.67	2.64	28.7	7.69	3.08	1.71	4.05	15.4	6.15
× 0.125	0.116	6.51	1.78	43.1	5.31	2.12	1.73	2.77	10.6	4.25
HSS4.500 × 0.375	0.349	16.54	4.55	12.9	9.87	4.39	1.47	6.03	19.7	8.78
× 0.337	0.313	15.00	4.12	14.4	9.07	4.03	1.48	5.50	18.1	8.06
× 0.237	0.220	10.80	2.96	20.5	6.79	3.02	1.52	4.03	13.6	6.04
× 0.188	0.174	8.67	2.36	25.9	5.54	2.46	1.53	3.26	11.1	4.93
× 0.125	0.116	5.85	1.60	38.8	3.84	1.71	1.55	2.23	7.68	3.41

(*Continued*)

APPENDIX C.6 (*Continued*)
Round HSS: Dimensions and Properties



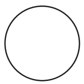
HSS6.625–HSS2.500

Shape	Design Wall Thickness, *t* (in.)	Nominal Wt. (lb/ft.)	Area, *A* (in.^{2})	*D/t*	*I* (in.^{4})	*S* (in.^{3})	*r* (in.)	*Z* (in.^{3})	Torsion *J* (in.^{4})	Torsion *C* (in.^{3})
HSS4.000 × 0.313	0.291	12.34	3.39	13.7	5.87	2.93	1.32	4.01	11.7	5.87
× 0.250	0.233	10.00	2.76	17.2	4.91	2.45	1.33	3.31	9.82	4.91
× 0.237	0.220	9.53	2.61	18.2	4.68	2.34	1.34	3.15	9.36	4.68
× 0.226	0.210	9.12	2.50	19.0	4.50	2.25	1.34	3.02	9.01	4.50
× 0.220	0.205	8.89	2.44	19.5	4.41	2.21	1.34	2.96	8.83	4.41
× 0.188	0.174	7.66	2.09	23.0	3.83	1.92	1.35	2.55	7.67	3.83
× 0.125	0.116	5.18	1.42	34.5	2.67	1.34	1.37	1.75	5.34	2.67
HSS3.500 × 0.313	0.291	10.66	2.93	12.0	3.81	2.18	1.14	3.00	7.61	4.35
× 0.300	0.279	10.26	2.82	12.5	3.69	2.11	1.14	2.90	7.38	4.22
× 0.250	0.233	8.69	2.39	15.0	3.21	1.83	1.16	2.49	6.41	3.66
× 0.216	0.201	7.58	2.08	17.4	2.84	1.63	1.17	2.19	5.69	3.25
× 0.203	0.189	7.15	1.97	18.5	2.70	1.54	1.17	2.07	5.41	3.09
× 0.188	0.174	6.66	1.82	20.1	2.52	1.44	1.18	1.93	5.04	2.88
× 0.125	0.116	4.51	1.23	30.2	1.77	1.01	1.20	1.33	3.53	2.02
HSS3.000 × 0.250	0.233	7.35	2.03	12.9	1.95	1.30	0.982	1.79	3.90	2.60
× 0.216	0.201	6.43	1.77	14.9	1.74	1.16	0.992	1.58	3.48	2.32
× 0.203	0.189	6.07	1.67	15.9	1.66	1.10	0.996	1.50	3.31	2.21
× 0.188	0.174	5.65	1.54	17.2	1.55	1.03	1.00	1.39	3.10	2.06
× 0.152	0.141	4.63	1.27	21.3	1.30	0.865	1.01	1.15	2.59	1.73
× 0.134	0.124	4.11	1.12	24.2	1.16	0.774	1.02	1.03	2.32	1.55
× 0.125	0.116	3.84	1.05	25.9	1.09	0.730	1.02	0.965	2.19	1.46
HSS2.875 × 0.250	0.233	7.02	1.93	12.3	1.70	1.18	0.938	1.63	3.40	2.37
× 0.203	0.189	5.80	1.59	15.2	1.45	1.01	0.952	1.37	2.89	2.01
× 0.188	0.174	5.40	1.48	16.5	1.35	0.941	0.957	1.27	2.70	1.88
× 0.125	0.116	3.67	1.01	24.8	0.958	0.667	0.976	0.884	1.92	1.33
HSS2.500 × 0.250	0.233	6.01	1.66	10.7	1.08	0.862	0.806	1.20	2.15	1.72
× 0.188	0.174	4.65	1.27	14.4	0.865	0.692	0.825	0.943	1.73	1.38
× 0.125	0.116	3.17	0.869	21.6	0.619	0.495	0.844	0.660	1.24	0.990

Source: Courtesy of the American Institute of Steel Construction, Chicago, IL.

[a] Shape exceeds compact limit for flexure with $F_y = 42$ ksi.

APPENDIX C.7
Pipe: Dimensions and Properties

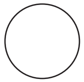


Pipe

Shape	Nominal Wt. (lb/ft.)	Dimensions: Outside Diameter (in.)	Dimensions: Inside Diameter (in.)	Nominal Wall Thickness (in.)	Design Wall Thickness (in.)	Area (in.^{2})	D/t	I (in.^{4})	S (in.^{3})	r (in.)	J (in.^{4})	Z (in.^{3})
Standard Weight (Std.)												
Pipe 12 Std.	49.6	12.8	12.0	0.375	0.349	13.7	36.5	262	41.0	4.39	523	53.7
Pipe 10 Std.	40.5	10.8	10.0	0.365	0.340	11.5	31.6	151	28.1	3.68	302	36.9
Pipe 8 Std.	28.6	8.63	7.98	0.322	0.300	7.85	28.8	68.1	15.8	2.95	136	20.8
Pipe 6 Std.	19.0	6.63	6.07	0.280	0.261	5.20	25.4	26.5	7.99	2.25	52.9	10.6
Pipe 5 Std.	14.6	5.56	5.05	0.258	0.241	4.01	23.1	14.3	5.14	1.88	28.6	6.83
Pipe 4 Std.	10.8	4.50	4.03	0.237	0.221	2.96	20.4	6.82	3.03	1.51	13.6	4.05
Pipe 3½ Std.	9.12	4.00	3.55	0.226	0.211	2.50	19.0	4.52	2.26	1.34	9.04	3.03
Pipe 3 Std.	7.58	3.50	3.07	0.216	0.201	2.07	17.4	2.85	1.63	1.17	5.69	2.19
Pipe 2½ Std.	5.80	2.88	2.47	0.203	0.189	1.61	15.2	1.45	1.01	0.952	2.89	1.37
Pipe 2 Std.	3.66	2.38	2.07	0.154	0.143	1.02	16.6	0.627	0.528	0.791	1.25	0.713
Pipe 1½ Std.	2.72	1.90	1.61	0.145	0.135	0.749	14.1	0.293	0.309	0.626	0.586	0.421
Pipe 1¼ Std.	2.27	1.66	1.38	0.140	0.130	0.625	12.8	0.184	0.222	0.543	0.368	0.305
Pipe 1 Std.	1.68	1.32	1.05	0.133	0.124	0.469	10.6	0.0830	0.126	0.423	0.166	0.177
Pipe 3/4 Std.	1.13	1.05	0.824	0.113	0.105	0.312	10.0	0.0350	0.0671	0.336	0.0700	0.0942
Pipe 1/2 Std.	0.850	0.840	0.622	0.109	0.101	0.234	8.32	0.0160	0.0388	0.264	0.0320	0.0555
Extra Strong (x-strong)												
Pipe 12 x-strong	65.5	12.8	11.8	0.500	0.465	17.5	27.4	339	53.2	4.35	678	70.2
Pipe 10 x-strong	54.8	10.8	9.75	0.500	0.465	15.1	23.1	199	37.0	3.64	398	49.2

(*Continued*)

APPENDIX C.7 (*Continued*)
Pipe: Dimensions and Properties



Pipe

Shape	Nominal Wt. (lb/ft.)	Dimensions: Outside Diameter (in.)	Dimensions: Inside Diameter (in.)	Nominal Wall Thickness (in.)	Design Wall Thickness (in.)	Area (in.^{2})	D/t	I (in.^{4})	S (in.^{3})	r (in.)	J (in.^{4})	Z (in.^{3})
Pipe 8 x-strong	43.4	8.63	7.63	0.500	0.465	11.9	18.5	100	23.1	2.89	199	31.0
Pipe 6 x-strong	28.6	6.63	5.76	0.432	0.403	7.83	16.4	38.3	11.6	2.20	76.6	15.6
Pipe 5 x-strong	20.8	5.56	4.81	0.375	0.349	5.73	15.9	19.5	7.02	1.85	39.0	9.50
Pipe 4 x-strong	15.0	4.50	3.83	0.337	0.315	4.14	14.3	9.12	4.05	1.48	18.2	5.53
Pipe 3½ x-strong	12.5	4.00	3.36	0.318	0.296	3.43	13.5	5.94	2.97	1.31	11.9	4.07
Pipe 3 x-strong	10.3	3.50	2.90	0.300	0.280	2.83	12.5	3.70	2.11	1.14	7.40	2.91
Pipe 2½ x-strong	7.67	2.88	2.32	0.276	0.257	2.10	11.2	1.83	1.27	0.930	3.66	1.77
Pipe 2 x-strong	5.03	2.38	1.94	0.218	0.204	1.40	11.6	0.827	0.696	0.771	1.65	0.964
Pipe 1½ x-strong	3.63	1.90	1.50	0.200	0.186	1.00	10.2	0.372	0.392	0.610	0.744	0.549
Pipe 1¼ x-strong	3.00	1.66	1.28	0.191	0.178	0.837	9.33	0.231	0.278	0.528	0.462	0.393
Pipe 1 x-strong	2.17	1.32	0.957	0.179	0.166	0.602	7.92	0.101	0.154	0.410	0.202	0.221
Pipe 3/4 x-strong	1.48	1.05	0.742	0.154	0.143	0.407	7.34	0.0430	0.0818	0.325	0.0860	0.119
Pipe 1/2 x-strong	1.09	0.840	0.546	0.147	0.137	0.303	6.13	0.0190	0.0462	0.253	0.0380	0.0686
Double-Extra Strong (xx-strong)												
Pipe 8 xx-strong	72.5	8.63	6.88	0.875	0.816	20.0	10.6	154	35.8	2.78	308	49.9
Pipe 6 xx-strong	53.2	6.63	4.90	0.864	0.805	14.7	8.23	63.5	19.2	2.08	127	27.4
Pipe 5 xx-strong	38.6	5.56	4.06	0.750	0.699	10.7	7.96	32.2	11.6	1.74	64.4	16.7
Pipe 4 xx-strong	27.6	4.50	3.15	0.674	0.628	7.66	7.17	14.7	6.53	1.39	29.4	9.50
Pipe 3 xx-strong	18.6	3.50	2.30	0.600	0.559	5.17	6.26	5.79	3.31	1.06	11.6	4.89
Pipe 2½ xx-strong	13.7	2.88	1.77	0.552	0.514	3.83	5.59	2.78	1.94	0.854	5.56	2.91
Pipe 2 xx-strong	9.04	2.38	1.50	0.436	0.406	2.51	5.85	1.27	1.07	0.711	2.54	1.60

Source: Courtesy of the American Institute of Steel Construction, Chicago, IL.

APPENDIX C.8
Available Strength in Axial Compression, Kips: W Shapes, $F_y = 50$ ksi

Shape		W14×						W12×					W10×				
Weight per Foot		82	74	68	61	53	48	58	53	50	45	40	54	49	45	39	33
		$\phi_c P_c$	$\phi_c P_c$	$\phi_c P_c$	$\phi_c P_c$	$\phi_c P_c$	$\phi_c P_c$	$\phi_c P_c$	$\phi_c P_c$	$\phi_c P_c$	$\phi_c P_c$	$\phi_c P_c$	$\phi_c P_c$	$\phi_c P_c$	$\phi_c P_c$	$\phi_c P_c$	$\phi_c P_c$
Design		LRFD	LRFD	LRFD	LRFD	LRFD	LRFD	LRFD	LRFD	LRFD	LRFD	LRFD	LRFD	LRFD	LRFD	LRFD	LRFD
	0	1080	981	900	805	702	634	765	702	657	589	526	711	648	598	517	437
	6	1020	922	845	756	633	572	720	660	595	534	476	671	611	545	470	395
	7	993	902	826	739	610	551	705	646	574	515	459	651	598	527	454	381
	8	968	879	805	720	585	527	687	629	551	494	440	642	584	507	436	365
	9	940	854	782	699	557	502	668	611	526	471	420	624	568	485	416	348
	10	910	827	756	676	528	475	647	592	500	447	398	605	550	461	396	330
	11	878	797	729	651	497	447	625	571	472	422	375	585	532	437	374	311
	12	844	767	701	626	465	419	601	549	443	396	352	564	512	411	352	292
	13	809	735	671	599	433	390	577	526	413	369	328	542	492	385	329	272

(Continued)

APPENDIX C.8 (*Continued*)

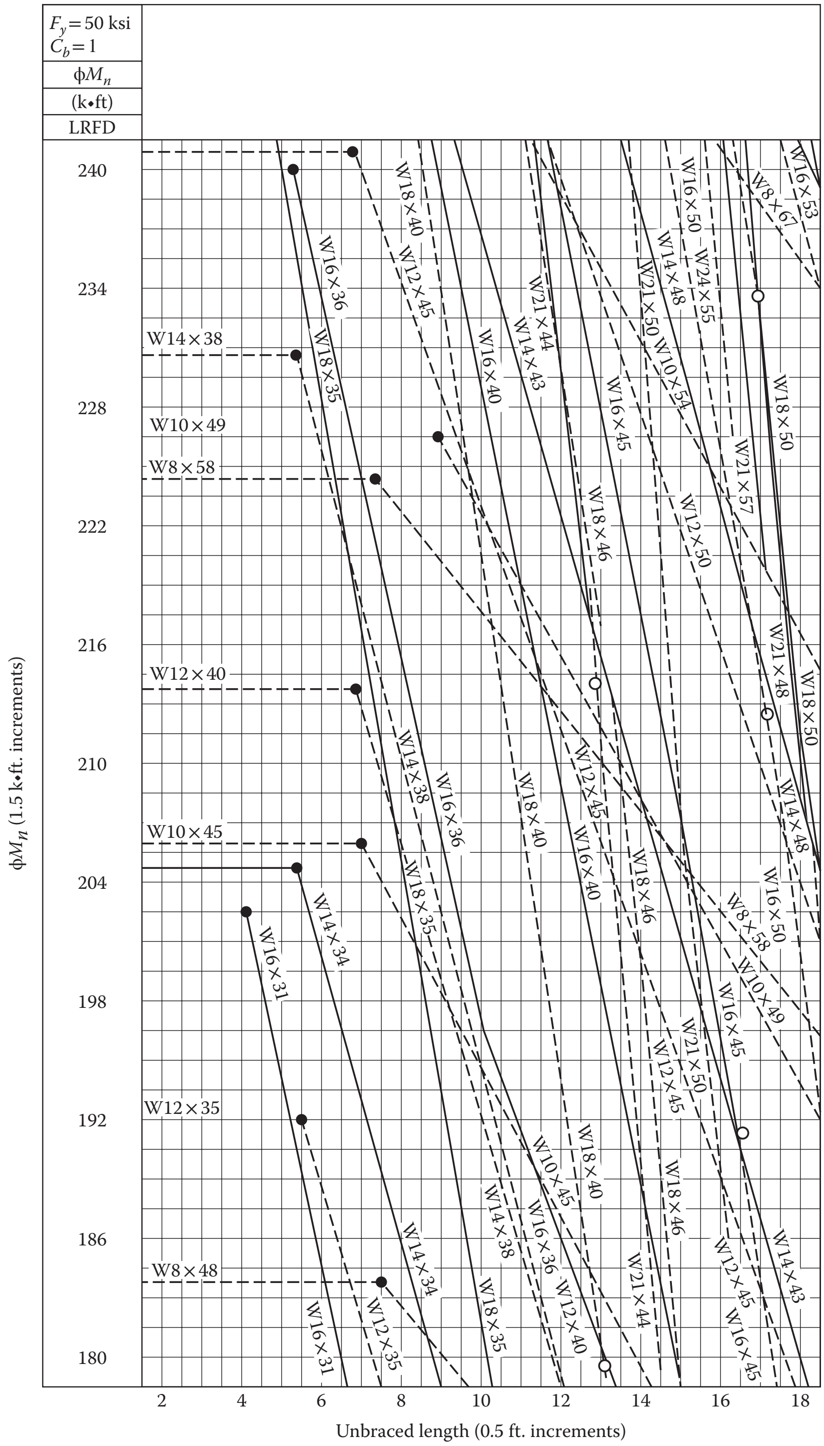
Available Strength in Axial Compression, Kips: W Shapes, $Fy = 50$ ksi

Shape		**W14×**						**W12×**					**W10×**				
Weight per Foot		**82**	**74**	**68**	**61**	**53**	**48**	**58**	**53**	**50**	**45**	**40**	**54**	**49**	**45**	**39**	**33**
		$\phi_c P_c$	$\phi_c P_c$	$\phi_c P_c$	$\phi_c P_c$	$\phi_c P_c$	$\phi_c P_c$	$\phi_c P_c$	$\phi_c P_c$	$\phi_c P_c$	$\phi_c P_c$	$\phi_c P_c$	$\phi_c P_c$	$\phi_c P_c$	$\phi_c P_c$	$\phi_c P_c$	$\phi_c P_c$
Effective Length,	14	772	701	640	571	401	360	551	502	384	343	304	519	471	359	306	253
L_C (ft.), with	15	735	667	608	543	369	331	525	478	355	316	281	495	449	333	283	233
Respect to	16	697	633	577	514	338	303	499	453	326	290	257	471	427	307	260	214
Least Radius of	17	659	598	544	485	308	276	472	428	298	265	235	447	404	282	238	195
Gyration, r_y	18	620	563	512	456	278	249	445	403	270	240	213	422	382	257	217	177
	19	582	529	480	428	250	224	418	378	244	216	191	398	360	234	196	159
	20	545	495	449	399	226	202	392	354	220	195	173	374	337	211	177	143
	22	472	428	388	345	186	167	341	307	182	161	143	327	294	174	146	118
	24	402	365	330	293	157	140	292	261	153	136	120	282	253	146	123	99.5
	26	343	311	281	249	133	119	249	223	130	116	102	240	216	125	105	84.8
	28	295	268	242	215	115	103	214	192	112	100	88	207	186	108	90	73.1
	30	257	234	211	187	100	90	187	167	98	87	77	180	162	94	79	63.7
	32	226	205	185	165	88.1		164	147	86	76	67	159	142	82	69	56.0
	34	200	182	164	146			145	130				141	126			
	36	179	162	147	130			130	116				125	112			
	38	160	146	131	117			116	104				112	101			
	40	145	131	119	105			105	94				102	91			

Source: Courtesy of the American Institute of Steel Construction, Chicago, IL.

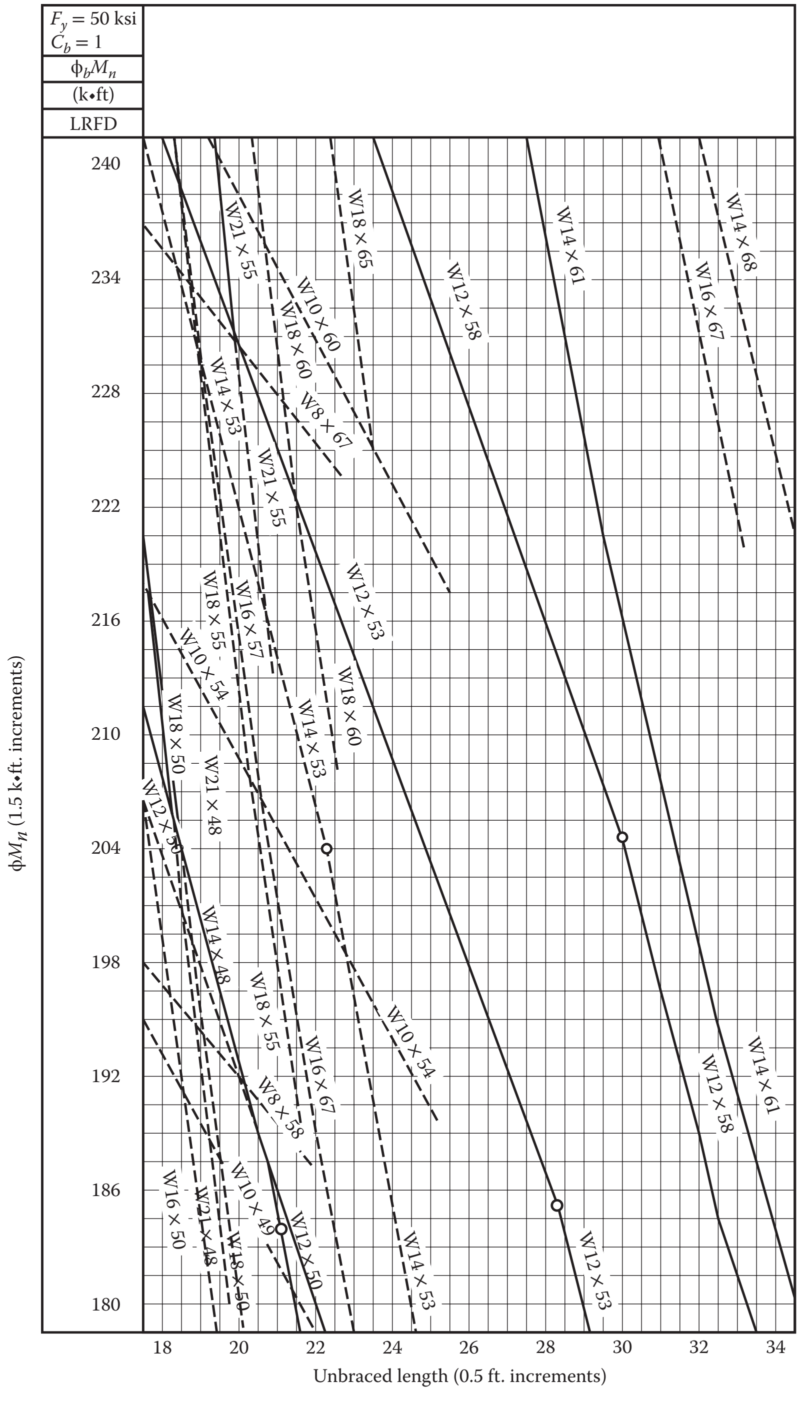
Note: LRFD, load resistance factor design.

APPENDIX C.9
W Shapes: Available Moment versus Unbraced Length Load Resistance Factor Design



(*Continued*)

APPENDIX C.9 (*Continued*)
W Shapes: Available Moment versus Unbraced Length Load Resistance Factor Design



Source: Courtesy of the American Institute of Steel Construction, Chicago, IL.

APPENDIX C.10a
Standard Load Table for Open-Web Steel Joists, K-Series (8K–16K)

Based on 50 ksi Maximum Yield Strength—Loads Shown in Pounds per Linear Foot

Joist Designation	**8K1**	**10K1**	**12K1**	**12K3**	**12K5**	**14K1**	**14K3**	**14K4**	**14K6**	**16K2**	**16K3**	**16K4**	**16K5**	**16K6**	**16K7**	**16K9**
Depth (in.)	**8**	**10**	**12**	**12**	**12**	**14**	**14**	**14**	**14**	**16**	**16**	**16**	**16**	**16**	**16**	**16**
Approx. Wt. (lb/ft.)	**5.1**	**5.0**	**5.0**	**5.7**	**7.1**	**5.2**	**6.0**	**6.7**	**7.7**	**5.5**	**6.3**	**7.0**	**7.5**	**8.1**	**8.6**	**10.0**
Span (ft.)																
↓	**825**															
8	550															
9	**825**															
	550															
10	**825**	**825**														
	480	550														
11	**798**	**825**														
	377	542														
12	**666**	**825**	**825**	**825**	**825**											
	288	455	550	550	550											
13	**565**	**718**	**825**	**825**	**825**											
	225	363	510	510	510											
14	**486**	**618**	**750**	**825**	**825**	**825**	**825**	**825**	**825**							
	179	289	425	463	463	550	550	550	550							
15	**421**	**537**	**651**	**814**	**825**	**766**	**825**	**825**	**825**							
	145	234	344	428	434	475	507	507	507							
16	**369**	**469**	**570**	**714**	**825**	**672**	**825**	**825**	**825**	**825**	**825**	**825**	**825**	**825**	**825**	**825**
	119	192	282	351	396	390	467	467	467	550	550	550	550	550	550	550
17		**415**	**504**	**630**	**825**	**592**	**742**	**825**	**825**	**768**	**825**	**825**	**825**	**825**	**825**	**825**
		159	234	291	366	324	404	443	443	488	526	526	526	526	526	526
18		**369**	**448**	**561**	**760**	**528**	**661**	**795**	**825**	**684**	**762**	**825**	**825**	**825**	**825**	**825**
		134	197	245	317	272	339	397	408	409	456	490	490	490	490	490
19		**331**	**402**	**502**	**681**	**472**	**592**	**712**	**825**	**612**	**682**	**820**	**825**	**825**	**825**	**825**
		113	167	207	269	230	287	336	383	347	386	452	455	455	455	455

(Continued)

APPENDIX C.10a (*Continued*)
Standard Load Table for Open-Web Steel Joists, K-Series (8K–16K)

Based on 50 ksi Maximum Yield Strength—Loads Shown in Pounds per Linear Foot

Joist Designation	8K1	10K1	12K1	12K3	12K5	14K1	14K3	14K4	14K6	16K2	16K3	16K4	16K5	16K6	16K7	16K9
Depth (in.)	8	10	12	12	12	14	14	14	14	16	16	16	16	16	16	16
Approx. Wt. (lb/ft.)	5.1	5.0	5.0	5.7	7.1	5.2	6.0	6.7	7.7	5.5	6.3	7.0	7.5	8.1	8.6	10.0
20		**298**	**361**	**453**	**613**	**426**	**534**	**642**	**787**	**552**	**615**	**739**	**825**	**825**	**825**	**825**
		97	142	177	230	197	246	267	347	297	330	386	426	426	426	426
21			**327**	**409**	**555**	**385**	**483**	**582**	**712**	**499**	**556**	**670**	**754**	**822**	**825**	**825**
			123	153	198	170	212	248	299	255	285	333	373	405	406	406
22			**298**	**373**	**505**	**351**	**439**	**529**	**648**	**454**	**505**	**609**	**687**	**747**	**825**	**825**
			106	132	172	147	184	215	259	222	247	289	323	361	385	385
23			**271**	**340**	**462**	**321**	**402**	**483**	**592**	**415**	**462**	**556**	**627**	**682**	**760**	**825**
			93	116	150	128	160	188	226	194	216	252	282	307	339	363
24			**249**	**312**	**423**	**294**	**367**	**442**	**543**	**381**	**424**	**510**	**576**	**627**	**697**	**825**
			81	101	132	113	141	165	199	170	189	221	248	269	298	346
25						**270**	**339**	**408**	**501**	**351**	**390**	**469**	**529**	**576**	**642**	**771**
						100	124	145	175	150	167	195	219	238	263	311
26						**249**	**313**	**376**	**462**	**324**	**360**	**433**	**489**	**532**	**592**	**711**
						88	110	129	156	133	148	173	194	211	233	276
27						**231**	**289**	**349**	**427**	**300**	**334**	**402**	**453**	**493**	**549**	**658**
						79	98	115	139	119	132	155	173	188	208	246
28						**214**	**270**	**324**	**397**	**279**	**310**	**373**	**421**	**459**	**510**	**612**
						70	88	103	124	106	118	138	155	168	186	220
29										**259**	**289**	**348**	**391**	**427**	**475**	**570**
										95	106	124	139	151	167	198
30										**241**	**270**	**324**	**366**	**399**	**444**	**532**
										86	96	112	126	137	151	178
31										**226**	**252**	**304**	**342**	**373**	**415**	**498**
										78	87	101	114	124	137	161
32										**213**	**237**	**285**	**321**	**349**	**388**	**466**
										71	79	92	103	112	124	147

Source: Courtesy of the Steel Joist Institute, Forest, VA.

APPENDIX C.10b

Standard Load Table for Open-Web Steel Joists, K-Series (18K–22K)

Based on 50 ksi Maximum Yield Strength—Loads Shown in Pounds per Linear Foot

Joist Designation	**18K3**	**18K4**	**18K5**	**18K6**	**18K7**	**18K9**	**18K10**	**20K3**	**20K4**	**20K5**	**20K6**	**20K7**	**20K9**	**20K10**	**22K4**	**22K5**	**22K6**	**22K7**	**22K9**	**22K10**	**22K11**
Depth (in.)	**18**	**18**	**18**	**18**	**18**	**18**	**18**	**18**	**18**	**18**	**20**	**20**	**20**	**20**	**22**	**22**	**22**	**22**	**22**	**22**	**22**
Approx. Wt. (lb/ft.)	**6.6**	**7.2**	**7.7**	**8.5**	**9**	**10.2**	**11.7**	**6.7**	**7.6**	**8.2**	**8.9**	**9.3**	**10.8**	**12.2**	**8**	**8.8**	**9.2**	**9.7**	**11.3**	**12.6**	**13.8**
Span (ft.)																					
↓	**825**	**825**	**825**	**825**	**825**	**825**	**825**														
18	550	550	550	550	550	550	550														
19	**771**	**825**	**825**	**825**	**825**	**825**	**825**														
	494	523	523	523	523	523	523														
20	**694**	**825**	**825**	**825**	**825**	**825**	**825**	**775**	**825**	**825**	**825**	**825**	**825**	**825**							
	423	490	490	490	490	490	490	517	550	550	550	550	550	550							
21	**630**	**759**	**825**	**825**	**825**	**825**	**825**	**702**	**825**	**825**	**825**	**825**	**825**	**825**							
	364	426	460	460	460	460	460	463	520	520	520	520	520	520							
22	**573**	**690**	**777**	**825**	**825**	**825**	**825**	**639**	**771**	**825**	**825**	**825**	**825**	**825**	**825**	**825**	**825**	**825**	**825**	**825**	**825**
	316	370	414	438	438	438	438	393	461	490	490	490	490	490	548	548	548	548	548	548	548
23	**523**	**630**	**709**	**774**	**825**	**825**	**825**	**583**	**703**	**793**	**825**	**825**	**825**	**825**	**777**	**825**	**825**	**825**	**825**	**825**	**825**
	276	323	362	393	418	418	418	344	402	451	468	468	468	468	491	518	518	518	518	518	518
24	**480**	**577**	**651**	**709**	**789**	**825**	**825**	**535**	**645**	**727**	**792**	**825**	**825**	**825**	**712**	**804**	**825**	**825**	**825**	**825**	**825**
	242	284	318	345	382	396	396	302	353	396	430	448	448	448	431	483	495	495	495	495	495
25	**441**	**532**	**600**	**652**	**727**	**825**	**825**	**493**	**594**	**669**	**729**	**811**	**825**	**825**	**657**	**739**	**805**	**825**	**825**	**825**	**825**
	214	250	281	305	337	377	377	266	312	350	380	421	426	426	381	427	464	474	474	474	474
26	**408**	**492**	**553**	**603**	**672**	**807**	**825**	**456**	**549**	**618**	**673**	**750**	**825**	**825**	**606**	**682**	**744**	**825**	**825**	**825**	**825**
	190	222	249	271	299	354	361	236	277	310	337	373	405	405	338	379	411	454	454	454	454
27	**378**	**454**	**513**	**558**	**622**	**747**	**825**	**421**	**508**	**573**	**624**	**694**	**825**	**825**	**561**	**633**	**688**	**768**	**825**	**825**	**825**
	169	198	222	241	267	315	347	211	247	277	301	333	389	389	301	337	367	406	432	432	432
28	**351**	**423**	**477**	**519**	**577**	**694**	**822**	**391**	**472**	**532**	**579**	**645**	**775**	**825**	**522**	**588**	**640**	**712**	**825**	**825**	**825**
	151	177	199	216	239	282	331	189	221	248	269	298	353	375	270	302	328	364	413	413	413

(Continued)

APPENDIX C.10b (*Continued*)
Standard Load Table for Open-Web Steel Joists, K-Series (18K–22K)

Based on 50 ksi Maximum Yield Strength—Loads Shown in Pounds per Linear Foot

Joist Designation	**18K3**	**18K4**	**18K5**	**18K6**	**18K7**	**18K9**	**18K10**	**20K3**	**20K4**	**20K5**	**20K6**	**20K7**	**20K9**	**20K10**	**22K4**	**22K5**	**22K6**	**22K7**	**22K9**	**22K10**	**22K11**
Depth (in.)	**18**	**18**	**18**	**18**	**18**	**18**	**18**	**18**	**18**	**18**	**20**	**20**	**20**	**20**	**22**	**22**	**22**	**22**	**22**	**22**	**22**
Approx. Wt. (lb/ft.)	**6.6**	**7.2**	**7.7**	**8.5**	**9**	**10.2**	**11.7**	**6.7**	**7.6**	**8.2**	**8.9**	**9.3**	**10.8**	**12.2**	**8**	**8.8**	**9.2**	**9.7**	**11.3**	**12.6**	**13.8**
29	**327**	**394**	**444**	**483**	**538**	**646**	**766**	**364**	**439**	**495**	**540**	**601**	**723**	**825**	**486**	**547**	**597**	**664**	**798**	**825**	**825**
	136	159	179	194	215	254	298	170	199	223	242	268	317	359	242	272	295	327	387	399	399
30	**304**	**367**	**414**	**451**	**502**	**603**	**715**	**340**	**411**	**462**	**504**	**561**	**675**	**799**	**453**	**511**	**556**	**619**	**745**	**825**	**825**
-	123	144	161	175	194	229	269	153	179	201	218	242	286	336	219	245	266	295	349	385	385
31	**285**	**343**	**387**	**421**	**469**	**564**	**669**	**318**	**384**	**433**	**471**	**525**	**631**	**748**	**424**	**478**	**520**	**580**	**697**	**825**	**825**
	111	130	146	158	175	207	243	138	162	182	198	219	259	304	198	222	241	267	316	369	369
32	**267**	**322**	**363**	**396**	**441**	**529**	**627**	**298**	**360**	**406**	**442**	**492**	**592**	**702**	**397**	**448**	**489**	**544**	**654**	**775**	**823**
	101	118	132	144	159	188	221	126	147	165	179	199	235	276	180	201	219	242	287	337	355
33	**252**	**303**	**342**	**372**	**414**	**498**	**589**	**280**	**339**	**381**	**415**	**463**	**566**	**660**	**373**	**421**	**459**	**511**	**615**	**729**	**798**
	92	108	121	131	145	171	201	114	134	150	163	181	214	251	164	183	199	221	261	307	334
34	**237**	**285**	**321**	**349**	**390**	**468**	**555**	**264**	**318**	**358**	**391**	**435**	**523**	**621**	**352**	**397**	**432**	**481**	**579**	**687**	**774**
	84	98	110	120	132	156	184	105	122	137	149	165	195	229	149	167	182	202	239	280	314
35	**223**	**268**	**303**	**330**	**367**	**441**	**523**	**249**	**300**	**339**	**369**	**411**	**493**	**585**	**331**	**373**	**408**	**454**	**546**	**648**	**741**
	77	90	101	110	121	143	168	96	112	126	137	151	179	210	137	153	167	185	219	257	292
36	**211**	**253**	**286**	**312**	**348**	**417**	**495**	**235**	**283**	**319**	**348**	**388**	**466**	**553**	**313**	**354**	**385**	**429**	**516**	**612**	**700**
	70	82	92	101	111	132	154	88	103	115	125	139	164	193	126	141	153	169	201	236	269
37								**222**	**268**	**303**	**330**	**367**	**441**	**523**	**297**	**334**	**364**	**406**	**487**	**579**	**663**
								81	95	106	115	128	151	178	116	130	141	156	186	217	247
38								**211**	**255**	**286**	**312**	**348**	**418**	**496**	**280**	**316**	**345**	**384**	**462**	**549**	**628**
								74	87	98	106	118	139	164	107	119	130	144	170	200	228
39								**199**	**241**	**271**	**297**	**330**	**397**	**471**	**267**	**300**	**327**	**364**	**438**	**520**	**595**
								69	81	90	98	109	129	151	98	110	120	133	157	185	211
40								**190**	**229**	**258**	**282**	**313**	**376**	**447**	**253**	**285**	**310**	**346**	**417**	**495**	**565**
								64	75	84	91	101	119	140	91	102	111	123	146	171	195

(*Continued*)

APPENDIX C.10b (*Continued*)

Standard Load Table for Open-Web Steel Joists, K-Series (18K–22K)

Based on 50 ksi Maximum Yield Strength—Loads Shown in Pounds per Linear Foot

Joist Designation	18K3	18K4	18K5	18K6	18K7	18K9	18K10	20K3	20K4	20K5	20K6	20K7	20K9	20K10	22K4	22K5	22K6	22K7	22K9	22K10	22K11
Depth (in.)	**18**	**18**	**18**	**18**	**18**	**18**	**18**	**18**	**18**	**18**	**20**	**20**	**20**	**20**	**22**	**22**	**22**	**22**	**22**	**22**	**22**
Approx. Wt. (lb/ft.)	**6.6**	**7.2**	**7.7**	**8.5**	**9**	**10.2**	**11.7**	**6.7**	**7.6**	**8.2**	**8.9**	**9.3**	**10.8**	**12.2**	**8**	**8.8**	**9.2**	**9.7**	**11.3**	**12.6**	**13.8**
41															**241**	**271**	**295**	**330**	**396**	**471**	**538**
															85	95	103	114	135	159	181
42															**229**	**259**	**282**	**313**	**378**	**448**	**513**
															79	88	96	106	126	148	168
43															**219**	**247**	**268**	**300**	**360**	**427**	**489**
															73	82	89	99	117	138	157
44															**208**	**235**	**256**	**286**	**343**	**408**	**466**
															68	76	83	92	109	128	146

Source: Courtesy of the Steel Joist Institute, Forest, VA.

APPENDIX C.11
Design Guide LRFD Weight Table for Joist Girders Based on 50 ksi Yield Strength

			Joist Girder Weight—Pounds per Linear Foot																	
			Factored Load on Each Panel Point (k)																	
Girder Span (ft.)	Joist Spaces (ft.)	Girder Depth (in.)	6.0	9.0	12.0	15.0	18.0	21.0	24.0	27.0	30.0	36.0	42.0	48.0	54.0	60.0	66.0	72.0	78.0	84.0
20	2N @ 10.00	20	16	19	19	19	19	20	24	24	25	30	37	41	46	50	56	62	70	75
		24	16	19	19	19	19	20	21	21	25	28	32	36	41	42	49	52	53	66
		28	16	19	19	19	19	20	20	21	23	26	28	32	39	40	42	46	48	49
	3N @ 6.67	20	15	15	19	19	20	23	24	27	31	36	44	48	54	74	75	81	84	89
		24	15	16	16	16	19	20	23	26	27	33	36	45	47	53	56	68	79	82
		28	15	16	16	16	17	20	24	24	26	31	36	44	46	49	53	57	68	80
	4N @ 5.00	20	15	15	19	21	25	29	33	38	41	50	57	65	71	88	97	100	107	120
		24	15	16	17	20	23	26	29	32	35	44	50	55	62	71	85	90	100	102
		28	16	16	17	19	22	25	28	30	34	39	49	50	59	63	72	86	91	91
	5N @ 4.00	20	15	17	21	26	31	36	39	48	51	62	71	82	99	99	109	120	141	142
		24	16	16	20	23	26	30	35	39	43	53	60	68	80	91	101	103	110	120
		28	16	16	18	22	27	28	33	37	39	48	55	64	68	77	93	95	107	111
	6N @ 3.33	20	16	19	25	29	36	41	50	57	58	72	82	99	107	118	138	141		
		24	16	18	22	28	31	37	43	46	53	61	70	85	102	102	111	123	144	147
		28	17	18	22	26	30	33	40	42	47	58	68	76	83	96	109	112	119	130
	8N @ 2.50	20	19	25	32	41	51	58	65	72	82	99	118	139	142					
		24	17	22	29	36	42	50	54	61	69	86	103	107	128	149	153			
		28	18	22	29	34	40	47	54	61	67	76	88	107	112	124	135	155	166	
22	2N @ 11.00	20	21	21	21	22	22	23	24	24	25	34	39	43	49	55	62	69	76	78
		24	18	21	21	22	22	22	23	24	24	30	33	41	41	45	51	55	61	73
		28	18	21	21	21	22	22	22	23	24	37	30	33	41	42	46	48	51	58
	3N @ 7.33	20	15	18	18	19	22	24	26	29	33	42	45	53	68	70	76	84	88	94
		24	15	15	19	19	20	23	24	26	30	35	40	45	48	55	61	74	81	84

(*Continued*)

APPENDIX C.11 (*Continued*)
Design Guide LRFD Weight Table for Joist Girders
Based on 50 ksi Yield Strength

			Joist Girder Weight—Pounds per Linear Foot																	
			Factored Load on Each Panel Point (k)																	
Girder Span (ft.)	Joist Spaces (ft.)	Girder Depth (in.)	6.0	9.0	12.0	15.0	18.0	21.0	24.0	27.0	30.0	36.0	42.0	48.0	54.0	60.0	66.0	72.0	78.0	84.0
		28	15	16	16	16	19	20	23	24	27	32	36	45	47	52	54	59	74	82
	4N @ 5.50	20	15	16	19	23	26	30	36	39	44	55	62	71	82	95	96	106	119	134
		24	15	15	17	20	25	27	29	34	38	48	52	58	71	79	89	98	101	107
		28	16	16	16	19	22	25	28	32	35	40	49	54	60	72	79	87	90	97
	5N @ 4.40	20	15	17	24	27	34	38	42	49	55	65	75	96	98	111	126	137		
		24	16	16	20	24	28	33	38	40	48	56	62	73	85	100	101	110	116	133
		28	16	16	18	22	26	30	32	38	41	51	57	65	73	86	92	102	105	111
	6N @ 3.67	20	16	21	27	33	39	49	56	57	65	79	97	106	118	137				
		24	16	19	23	28	32	39	45	51	58	66	82	98	101	109	120	142	144	
		28	16	18	22	26	30	34	39	44	50	61	70	76	89	102	104	113	127	148
	8N @ 2.75	20	19	27	36	43	56	64	71	80	96	106	135	138						
		24	18	24	31	38	46	53	60	68	75	101	105	125	145	149				
		28	18	22	28	34	40	47	54	62	69	79	87	106	118	131	152	164		
25	3N @ 8.33	20	18	18	19	22	26	27	30	37	41	49	59	66	70	76	86	89	97	102
		24	15	18	19	20	22	25	26	28	32	39	43	51	59	67	71	81	84	89
		28	15	15	19	19	20	23	24	27	29	34	39	45	47	55	59	67	81	82
		32	15	16	16	16	20	21	23	24	27	32	36	44	46	52	54	58	74	81
		36	16	16	16	17	17	20	24	24	26	32	36	40	45	48	53	54	68	79
	4N @ 6.25	20	15	18	20	25	29	35	39	42	49	55	70	78	93	99	109	119	134	135
		24	15	16	19	21	26	29	33	37	40	50	57	64	72	88	97	100	106	120
		28	15	15	17	20	24	25	29	34	37	43	51	58	66	72	89	90	101	102
		32	16	16	17	19	21	25	28	32	35	40	49	54	60	69	79	86	91	96

(*Continued*)

APPENDIX C.11 (*Continued*)
Design Guide LRFD Weight Table for Joist Girders
Based on 50 ksi Yield Strength

			Joist Girder Weight—Pounds per Linear Foot																	
			Factored Load on Each Panel Point (k)																	
Girder Span (ft.)	**Joist Spaces (ft.)**	**Girder Depth (in.)**	**6.0**	**9.0**	**12.0**	**15.0**	**18.0**	**21.0**	**24.0**	**27.0**	**30.0**	**36.0**	**42.0**	**48.0**	**54.0**	**60.0**	**66.0**	**72.0**	**78.0**	**84.0**
		36	16	16	17	19	21	26	26	29	34	38	49	50	56	63	73	85	88	92
	5N @ 5.00	20	15	18	25	31	38	43	51	55	58	73	93	100	109	125	134			
		24	15	17	23	26	32	36	42	47	53	61	75	81	98	102	112	129	140	
		28	16	16	20	24	28	31	37	41	47	56	62	72	79	93	101	106	117	125
		32	16	16	19	23	26	30	33	38	41	51	57	65	73	83	93	102	105	111
		36	16	17	18	22	26	28	31	36	39	48	54	64	69	75	88	96	101	108
	6N @ 4.17	20	16	24	29	38	45	55	58	69	78	94	104	116	134					
		24	16	20	25	31	37	44	50	56	64	75	97	99	107	118	138			
		28	16	18	23	28	32	38	44	51	55	67	73	87	101	104	120	134	143	145
		32	16	18	22	26	30	34	39	44	50	61	69	77	89	102	105	113	127	148
		36	16	18	24	25	30	36	39	43	49	58	67	74	84	98	108	116	117	129
	8N @ 3.12	20	21	29	39	48	58	70	78	94	99	115	134							
		24	19	26	33	41	50	57	65	75	81	99	118	138						
		28	18	23	30	38	44	53	60	67	75	86	103	116	127	147				
		32	18	24	28	34	39	47	54	65	71	78	87	105	117	129	152	154		
		36	18	22	29	34	40	46	52	61	63	76	87	101	114	121	136	148	166	167
	10N @ 2.50	20	26	38	49	63	78	94	100	115	134									
		24	23	33	42	54	65	75	89	99	104	130								
		28	21	30	38	48	56	64	74	84	101	109	134	147						
		32	21	28	36	43	52	62	69	76	87	107	118	130	153					
		36	22	28	37	44	52	64	71	77	85	100	116	130	151	157				
28	3N @ 9.33	24	18	18	19	22	24	27	29	36	39	43	53	62	70	71	78	85	89	98
		28	18	18	19	20	22	25	26	28	31	39	43	46	55	61	66	76	83	86
		32	15	18	19	19	21	23	24	27	28	34	39	45	48	53	58	66	80	81

(*Continued*)

APPENDIX C.11 (*Continued*)
Design Guide LRFD Weight Table for Joist Girders
Based on 50 ksi Yield Strength

Girder Span (ft.)	Joist Spaces (ft.)	Girder Depth (in.)	Joist Girder Weight—Pounds per Linear Foot — Factored Load on Each Panel Point (k) 6.0	9.0	12.0	15.0	18.0	21.0	24.0	27.0	30.0	36.0	42.0	48.0	54.0	60.0	66.0	72.0	78.0	84.0
	4N @ 7.00	24	15	16	20	24	27	32	38	40	48	55	62	71	82	95	104	106	120	135
		28	15	15	18	21	25	28	32	36	39	49	56	64	71	79	96	97	106	107
		32	15	15	17	20	23	25	29	33	37	43	50	58	62	70	85	90	99	102
	5N @ 5.60	24	15	18	24	29	34	39	46	52	58	66	78	96	102	111	126	136		
		28	15	17	21	26	30	35	39	46	50	61	68	77	90	99	107	114	130	142
		32	16	17	20	24	27	32	37	41	44	56	62	70	80	93	102	107	112	119
	6N @ 4.67	24	16	21	28	35	41	49	55	63	70	79	96	106	134	137				
		28	15	20	24	30	36	42	50	54	58	71	82	99	107	118	138	142		
		32	16	19	23	28	32	37	43	49	53	64	74	84	101	102	111	123	144	146
	7N @ 4.00	24	18	24	32	41	49	56	64	74	79	96	110	135						
		28	17	22	27	35	43	51	57	62	69	82	99	108	129	140				
		32	16	21	27	31	38	44	52	55	63	74	85	102	108	123	143	146		
	8N @ 3.50	24	20	28	37	48	55	64	74	79	95	105	134							
		28	18	25	32	39	50	58	65	72	81	99	108	129	141					
		32	17	24	29	38	43	53	60	64	70	86	103	113	127	147	149			
	10N @ 2.80	24	24	36	46	57	70	79	96	102	117	137								
		28	23	30	41	50	60	69	82	99	100	120	141							
		32	21	30	38	46	55	66	71	80	93	109	126	147						
30	3N @ 10.00	24	18	18	21	24	27	31	35	38	40	48	58	66	71	80	92	98	117	119
		28	18	18	19	22	25	27	30	35	37	42	49	56	63	70	79	82	93	99
		32	18	18	19	20	22	26	28	31	32	39	46	51	57	64	71	73	83	84
		36	16	19	19	19	21	23	26	28	31	35	39	46	52	57	64	65	73	75
	4N @ 7.50	24	16	18	23	29	33	37	42	49	53	64	76	85	101	104	126	127	149	150
		28	15	16	21	25	30	33	37	42	45	53	61	73	81	86	103	104	126	128
		32	15	16	18	22	26	30	34	37	43	51	55	62	70	77	87	103	105	116
		36	16	16	17	22	24	27	31	34	36	46	52	59	64	74	78	88	91	105

(*Continued*)

APPENDIX C.11 (*Continued*)
Design Guide LRFD Weight Table for Joist Girders
Based on 50 ksi Yield Strength

			Joist Girder Weight—Pounds per Linear Foot																	
		Girder	Factored Load on Each Panel Point (k)																	
Girder Span (ft.)	Joist Spaces (ft.)	Depth (in.)	6.0	9.0	12.0	15.0	18.0	21.0	24.0	27.0	30.0	36.0	42.0	48.0	54.0	60.0	66.0	72.0	78.0	84.0
	5N @ 6.00	24	15	19	25	30	37	43	51	55	58	73	86	96	109	125	134			
		28	15	17	23	27	32	37	44	47	53	61	75	88	97	102	112	128	138	
		32	16	17	21	24	29	35	39	43	48	56	63	77	90	100	101	107	117	133
		36	16	17	20	24	27	31	36	40	43	51	60	70	80	86	94	103	110	118
	6N @ 5.00	24	16	24	29	37	45	52	58	66	73	94	104	116	134					
		28	16	20	27	32	38	44	50	57	65	75	97	99	107	137	140			
		32	16	19	24	29	34	40	45	51	58	65	82	98	100	109	121	142	144	
		36	16	18	23	26	31	37	41	46	52	61	70	84	101	102	111	123	126	148
	8N @ 3.75	24	21	32	40	51	63	73	83	99	111	124	146							
		28	20	30	37	44	53	61	73	80	86	114	126	149						
		32	18	26	34	42	49	55	63	71	79	104	117	130	154	161				
		36	17	23	32	39	46	54	61	69	76	89	108	121	134	154	169			
	10N @ 3.00	24	25	38	51	66	78	99	111	123	134									
		28	24	36	47	57	69	80	94	113	116	138								
		32	22	31	39	52	58	74	82	95	105	129	142							
		36	22	30	39	48	54	68	79	84	91	119	132	151						

Source: Courtesy of the Steel Joist Institute, Forest, VA.

Appendix D: Concrete

APPENDIX D.1
Diameter, Area, and Unit Weight of Steel Bars

Bar Number	3	4	5	6	7	8	9	10	11	14	18
Diameter (in.)	0.375	0.500	0.625	0.750	0.875	1.000	1.128	1.270	1.410	1.693	2.257
Area (in.^{2})	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56	2.25	4.00
Unit weight per foot (lb)	0.376	0.668	1.043	1.502	2.044	2.670	3.400	4.303	5.313	7.65	13.60

APPENDIX D.2
Areas of Group of Steel Bars (in.^{2})

	Bar Size								
Number of Bars	#3	#4	#5	#6	#7	#8	#9	#10	#11
1	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56
2	0.22	0.40	0.62	0.88	1.20	1.58	2.00	2.54	3.12
3	0.33	0.60	0.93	1.32	1.80	2.37	3.00	3.81	4.68
4	0.44	0.80	1.24	1.76	2.40	3.16	4.00	5.08	6.24
5	0.55	1.00	1.55	2.20	3.00	3.93	5.00	6.35	7.80
6	0.66	1.20	1.86	2.64	3.60	4.74	6.00	7.62	9.36
7	0.77	1.40	2.17	3.08	4.20	5.53	7.00	8.89	10.9
8	0.88	1.60	2.48	3.52	4.80	6.32	8.00	10.2	12.5
9	0.99	1.80	2.79	3.96	5.40	7.11	9.00	11.4	14.0
10	1.10	2.00	3.10	4.40	6.00	7.90	10.0	12.7	15.6
11	1.21	2.20	3.41	4.84	6.60	8.69	11.0	14.0	17.2
12	1.32	2.40	3.72	5.28	7.20	9.48	12.0	15.2	18.7
13	1.43	2.60	4.03	5.72	7.80	10.3	13.0	16.5	20.3
14	1.54	2.80	4.34	6.16	8.40	11.1	14.0	17.8	21.8
15	1.65	3.00	4.65	6.60	9.00	11.8	15.0	19.0	23.4
16	1.76	3.20	4.96	7.04	9.60	12.6	16.0	20.3	25.0
17	1.87	3.40	5.27	7.48	10.2	13.4	17.0	21.6	26.5
18	1.98	3.60	5.58	7.92	10.8	14.2	18.0	22.9	28.1
19	2.09	3.80	5.89	8.36	11.4	15.0	19.0	24.1	29.6
20	2.20	4.00	6.20	8.80	12.0	15.8	20.0	25.4	31.2

APPENDIX D.3
Minimum Required Beam Widths (in.)

Number of Bars in One Layer	Bar Size							
	#3 and #4	#5	#6	#7	#8	#9	#10	#11
2	6.0	6.0	6.5	6.5	7.0	7.5	8.0	8.0
3	7.5	8.0	8.0	8.5	9.0	9.5	10.5	11.0
4	9.0	9.5	10.0	10.5	11.0	12.0	13.0	14.0
5	10.5	11.0	11.5	12.5	13.0	14.0	15.5	16.5
6	12.0	12.5	13.5	14.0	15.0	16.5	18.0	19.5
7	13.5	14.5	15.0	16.0	17.0	18.5	20.5	22.5
8	15.0	16.0	17.0	18.0	19.0	21.0	23.0	25.0
9	16.5	17.5	18.5	20.0	21.0	23.0	25.5	28.0
10	18.0	19.0	20.5	21.5	23.0	25.5	28.0	31.0

Note: Tabulated values based on No. 3 stirrups, minimum clear distance of 1 in., and a 1½ in. cover.

APPENDIX D.4
Coefficient of Resistance ($\bar{K}$) versus Reinforcement Ratio (ρ) (f'_c = 3,000 psi; f_y = 40,000 psi)

ρ	$\bar{K}$ (ksi)	ρ	$\bar{K}$ (ksi)	ρ	$\bar{K}$ (ksi)	ρ	$\bar{K}$ (ksi)	ρ	$\bar{K}$ (ksi)	ρ	$\bar{K}$ (ksi)	ε_t[a]
0.0010	0.0397	0.0054	0.2069	0.0098	0.3619	0.0142	0.5047	0.0173	0.5981	0.02033	0.6836	0.00500
0.0011	0.0436	0.0055	0.2105	0.0099	0.3653	0.0143	0.5078	0.0174	0.6011	0.0204	0.6855	0.00497
0.0012	0.0476	0.0056	0.2142	0.0100	0.3686	0.0144	0.5109	0.0175	0.6040	0.0205	0.6882	0.00493
0.0013	0.0515	0.0057	0.2178	0.0101	0.3720	0.0145	0.5140	0.0176	0.6069	0.0206	0.6909	0.00489
0.0014	0.0554	0.0058	0.2214	0.0102	0.3754	0.0146	0.5171	0.0177	0.6098	0.0207	0.6936	0.00485
0.0015	0.0593	0.0059	0.2251	0.0103	0.3787	0.0147	0.5202	0.0178	0.6126	0.0208	0.6963	0.00482
0.0016	0.0632	0.0060	0.2287	0.0104	0.3821	0.0148	0.5233	0.0179	0.6155	0.0209	0.6990	0.00478
0.0017	0.0671	0.0061	0.2323	0.0105	0.3854	0.0149	0.5264	0.0180	0.6184	0.0210	0.7017	0.00474
0.0018	0.0710	0.0062	0.2359	0.0106	0.3887	0.0150	0.5294	0.0181	0.6213	0.0211	0.7044	0.00470
0.0019	0.0749	0.0063	0.2395	0.0107	0.3921	0.0151	0.5325	0.0182	0.6241	0.0212	0.7071	0.00467
0.0020	0.0788	0.0064	0.2431	0.0108	0.3954	0.0152	0.5355	0.0183	0.6270	0.0213	0.7097	0.00463
0.0021	0.0826	0.0065	0.2467	0.0109	0.3987	0.0153	0.5386	0.0184	0.6298	0.0214	0.7124	0.00460
0.0022	0.0865	0.0066	0.2503	0.0110	0.4020	0.0154	0.5416	0.0185	0.6327	0.0215	0.7150	0.00456
0.0023	0.0903	0.0067	0.2539	0.0111	0.4053	0.0155	0.5447	0.0186	0.6355	0.0216	0.7177	0.00453
0.0024	0.0942	0.0068	0.2575	0.0112	0.4086	0.0156	0.5477	0.0187	0.6383	0.0217	0.7203	0.00449
0.0025	0.0980	0.0069	0.2611	0.0113	0.4119	0.0157	0.5507	0.0188	0.6412	0.0218	0.7230	0.00446
0.0026	0.1019	0.0070	0.2646	0.0114	0.4152	0.0158	0.5537	0.0189	0.6440	0.0219	0.7256	0.00442
0.0027	0.1057	0.0071	0.2682	0.0115	0.4185	0.0159	0.5567	0.0190	0.6468	0.0220	0.7282	0.00439
0.0028	0.1095	0.0072	0.2717	0.0116	0.4218	0.0160	0.5597	0.0191	0.6496	0.0221	0.7308	0.00436
0.0029	0.1134	0.0073	0.2753	0.0117	0.4251	0.0161	0.5627	0.0192	0.6524	0.0222	0.7334	0.00432
0.0030	0.1172	0.0074	0.2788	0.0118	0.4283	0.0162	0.5657	0.0193	0.6552	0.0223	0.7360	0.00429
0.0031	0.1210	0.0075	0.2824	0.0119	0.4316	0.0163	0.5687	0.0194	0.6580	0.0224	0.7386	0.00426
0.0032	0.1248	0.0076	0.2859	0.0120	0.4348	0.0164	0.5717	0.0195	0.6608	0.0225	0.7412	0.00423
0.0033	0.1286	0.0077	0.2894	0.0121	0.4381	0.0165	0.5746	0.0196	0.6635	0.0226	0.7438	0.00419

(Continued)

APPENDIX D.4 (*Continued*)

Coefficient of Resistance ($\overline{K}$) versus Reinforcement Ratio (ρ) ($f'_c = 3{,}000$ psi; $f_y = 40{,}000$ psi)

ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ε_t[a]
0.0034	0.1324	0.0078	0.2929	0.0122	0.4413	0.0166	0.5776	0.0197	0.6663	0.0227	0.7464	0.00416
0.0035	0.1362	0.0079	0.2964	0.0123	0.4445	0.0167	0.5805	0.0198	0.6691	0.0228	0.7490	0.00413
0.0036	0.1399	0.0080	0.2999	0.0124	0.4478	0.0168	0.5835	0.0199	0.6718	0.0229	0.7515	0.00410
0.0037	0.1437	0.0081	0.3034	0.0125	0.4510	0.0169	0.5864	0.0200	0.6746	0.0230	0.7541	0.00407
0.0038	0.1475	0.0082	0.3069	0.0126	0.4542	0.0170	0.5894	0.0201	0.6773	0.0231	0.7567	0.00404
0.0039	0.1512	0.0083	0.3104	0.0127	0.4574	0.0171	0.5923	0.0202	0.6800	0.0232	0.7592	0.00401
0.0040	0.1550	0.0084	0.3139	0.0128	0.4606	0.0172	0.5952	0.0203	0.6828	0.02323	0.7600	0.00400
0.0041	0.1587	0.0085	0.3173	0.0129	0.4638							
0.0042	0.1625	0.0086	0.3208	0.0130	0.4670							
0.0043	0.1662	0.0087	0.3243	0.0131	0.4702							
0.0044	0.1699	0.0088	0.3277	0.0132	0.4733							
0.0045	0.1736	0.0089	0.3311	0.0133	0.4765							
0.0046	0.1774	0.0090	0.3346	0.0134	0.4797							
0.0047	0.1811	0.0091	0.3380	0.0135	0.4828							
0.0048	0.1848	0.0092	0.3414	0.0136	0.4860							
0.0049	0.1885	0.0093	0.3449	0.0137	0.4891							
0.0050	0.1922	0.0094	0.3483	0.0138	0.4923							
0.0051	0.1958	0.0095	0.3517	0.0139	0.4954							
0.0052	0.1995	0.0096	0.3551	0.0140	0.4985							
0.0053	0.2032	0.0097	0.3585	0.0141	0.5016							

[a] $d = d_t$, where d_t is distance from extreme compression fiber to the center of outermost steel layer. For single layer steel, $d_t = d$.

APPENDIX D.5
Coefficient of Resistance ($\overline{K}$) (f'_c = 3,000 psi, f_y = 50,000 psi)

ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ε_t[a]
0.0020	0.098	0.0056	0.265	0.0092	0.418	0.0128	0.559	0.0163	0.685	0.0050
0.0021	0.103	0.0057	0.269	0.0093	0.422	0.0129	0.563	0.0164	0.688	0.0049
0.0022	0.108	0.0058	0.273	0.0094	0.427	0.0130	0.567	0.0165	0.692	0.0049
0.0023	0.112	0.0059	0.278	0.0095	0.431	0.0131	0.571	0.0166	0.695	0.0048
0.0024	0.117	0.0060	0.282	0.0096	0.435	0.0132	0.574	0.0167	0.698	0.0048
0.0025	0.122	0.0061	0.287	0.0097	0.439	0.0133	0.578	0.0168	0.702	0.0047
0.0026	0.127	0.0062	0.291	0.0098	0.443	0.0134	0.582	0.0169	0.705	0.0047
0.0027	0.131	0.0063	0.295	0.0099	0.447	0.0135	0.585	0.017	0.708	0.0047
0.0028	0.136	0.0064	0.300	0.0100	0.451	0.0136	0.589	0.0171	0.712	0.0046
0.0029	0.141	0.0065	0.304	0.0101	0.455	0.0137	0.593	0.0172	0.715	0.0046
0.0030	0.146	0.0066	0.309	0.0102	0.459	0.0138	0.596	0.0173	0.718	0.0045
0.0031	0.150	0.0067	0.313	0.0103	0.463	0.0139	0.600	0.0174	0.722	0.0045
0.0032	0.155	0.0068	0.317	0.0104	0.467	0.0140	0.604	0.0175	0.725	0.0044
0.0033	0.159	0.0069	0.322	0.0105	0.471	0.0141	0.607	0.0176	0.728	0.0044
0.0034	0.164	0.0070	0.326	0.0106	0.475	0.0142	0.611	0.0177	0.731	0.0043
0.0035	0.169	0.0071	0.330	0.0107	0.479	0.0143	0.614	0.0178	0.735	0.0043
0.0036	0.174	0.0072	0.334	0.0108	0.483	0.0144	0.618	0.0179	0.738	0.0043
0.0037	0.178	0.0073	0.339	0.0109	0.487	0.0145	0.622	0.018	0.741	0.0042
0.0038	0.183	0.0074	0.343	0.0110	0.491	0.0146	0.625	0.0181	0.744	0.0042
0.0039	0.187	0.0075	0.347	0.0111	0.494	0.0147	0.629	0.0182	0.748	0.0041
0.0040	0.192	0.0076	0.352	0.0112	0.498	0.0148	0.632	0.0183	0.751	0.0041
0.0041	0.197	0.0077	0.356	0.0113	0.502	0.0149	0.636	0.0184	0.754	0.0041
0.0042	0.201	0.0078	0.360	0.0114	0.506	0.0150	0.639	0.0185	0.757	0.0040
0.0043	0.206	0.0079	0.364	0.0115	0.510	0.0151	0.643	0.0186	0.760	0.0040
0.0044	0.210	0.0080	0.368	0.0116	0.514	0.0152	0.646			
0.0045	0.215	0.0081	0.373	0.0117	0.518	0.0153	0.650			
0.0046	0.219	0.0082	0.377	0.0118	0.521	0.0154	0.653			
0.0047	0.224	0.0083	0.381	0.0119	0.525	0.0155	0.657			
0.0048	0.229	0.0084	0.385	0.0120	0.529	0.0156	0.660			
0.0049	0.233	0.0085	0.389	0.0121	0.533	0.0157	0.664			
0.0050	0.238	0.0086	0.394	0.0122	0.537	0.0158	0.667			
0.0051	0.242	0.0087	0.398	0.0123	0.541	0.0159	0.671			
0.0052	0.247	0.0088	0.402	0.0124	0.544	0.0160	0.674			
0.0053	0.251	0.0089	0.406	0.0125	0.548	0.0161	0.677			
0.0054	0.256	0.0090	0.410	0.0126	0.552	0.0162	0.681			
0.0055	0.260	0.0091	0.414	0.0127	0.556					

[a] $d = d_t$, where d_t is distance from extreme compression fiber to the center of outermost steel layer. For single layer steel, $d_t = d$.

APPENDIX D.6
Coefficient of Resistance ($\overline{K}$) versus Reinforcement Ratio (ρ) (f'_c = 3,000 psi; f_y = 60,000 psi)

ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ε_t[a]
0.0010	0.0593	0.0059	0.3294	0.0108	0.5657	
0.0011	0.0651	0.0060	0.3346	0.0109	0.5702	
0.0012	0.0710	0.0061	0.3397	0.0110	0.5746	
0.0013	0.0768	0.0062	0.3449	0.0111	0.5791	
0.0014	0.0826	0.0063	0.3500	0.0112	0.5835	
0.0015	0.0884	0.0064	0.3551	0.0113	0.5879	
0.0016	0.0942	0.0065	0.3602	0.0114	0.5923	
0.0017	0.1000	0.0066	0.3653	0.0115	0.5967	
0.0018	0.1057	0.0067	0.3703	0.0116	0.6011	
0.0019	0.1115	0.0068	0.3754	0.0117	0.6054	
0.0020	0.1172	0.0069	0.3804	0.0118	0.6098	
0.0021	0.1229	0.0070	0.3854	0.0119	0.6141	
0.0022	0.1286	0.0071	0.3904	0.0120	0.6184	
0.0023	0.1343	0.0072	0.3954	0.0121	0.6227	
0.0024	0.1399	0.0073	0.4004	0.0122	0.6270	
0.0025	0.1456	0.0074	0.4054	0.0123	0.6312	
0.0026	0.1512	0.0075	0.4103	0.0124	0.6355	
0.0027	0.1569	0.0076	0.4152	0.0125	0.6398	
0.0028	0.1625	0.0077	0.4202	0.0126	0.6440	
0.0029	0.1681	0.0078	0.4251	0.0127	0.6482	
0.0030	0.1736	0.0079	0.4300	0.0128	0.6524	
0.0031	0.1792	0.0080	0.4348	0.0129	0.6566	
0.0032	0.1848	0.0081	0.4397	0.0130	0.6608	
0.0033	0.1903	0.0082	0.4446	0.0131	0.6649	
0.0034	0.1958	0.0083	0.4494	0.0132	0.6691	
0.0035	0.2014	0.0084	0.4542	0.0133	0.6732	
0.0036	0.2069	0.0085	0.4590	0.0134	0.6773	
0.0037	0.2123	0.0086	0.4638	0.0135	0.6814	
0.0038	0.2178	0.0087	0.4686	0.01355	0.6835	0.00500
0.0039	0.2233	0.0088	0.4734	0.0136	0.6855	0.00497
0.0040	0.2287	0.0089	0.4781	0.0137	0.6896	0.00491
0.0041	0.2341	0.0090	0.4828	0.0138	0.6936	0.00485
0.0042	0.2396	0.0091	0.4876	0.0139	0.6977	0.00480
0.0043	0.2450	0.0092	0.4923	0.0140	0.7017	0.00474
0.0044	0.2503	0.0093	0.4970	0.0141	0.7057	0.00469
0.0045	0.2557	0.0094	0.5017	0.0142	0.7097	0.00463
0.0046	0.2611	0.0095	0.5063	0.0143	0.7137	0.00458
0.0047	0.2664	0.0096	0.5110	0.0144	0.7177	0.00453
0.0048	0.2717	0.0097	0.5156	0.0145	0.7216	0.00447
0.0049	0.2771	0.0098	0.5202	0.0146	0.7256	0.00442
0.0050	0.2824	0.0099	0.5248	0.0147	0.7295	0.00437
0.0051	0.2876	0.0100	0.5294	0.0148	0.7334	0.00432
0.0052	0.2929	0.0101	0.5340	0.0149	0.7373	0.00427

(*Continued*)

APPENDIX D.6 (*Continued*)

Coefficient of Resistance ($\bar{K}$) versus Reinforcement Ratio (ρ) (f_c' = 3,000 psi; f_y = 60,000 psi)

ρ	$\bar{K}$ (ksi)	ρ	$\bar{K}$ (ksi)	ρ	$\bar{K}$ (ksi)	ε_t[a]
0.0053	0.2982	0.0102	0.5386	0.0150	0.7412	0.00423
0.0054	0.3034	0.0103	0.5431	0.0151	0.7451	0.00418
0.0055	0.3087	0.0104	0.5477	0.0152	0.7490	0.00413
0.0056	0.3139	0.0105	0.5522	0.0153	0.7528	0.00408
0.0057	0.3191	0.0106	0.5567	0.0154	0.7567	0.00404
0.0058	0.3243	0.0107	0.5612	0.01548	0.7597	0.00400

[a] $d = d_t$, where d_t is distance from extreme compression fiber to the center of outermost steel layer. For single layer steel, $d_t = d$.

APPENDIX D.7

Coefficient of Resistance ($\overline{K}$) versus Reinforcement Ratio (ρ) ($f'_c = 4{,}000$ psi; $f_y = 40{,}000$ psi)

ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ε_t[a]
0.0010	0.0398	0.0054	0.2091	0.0098	0.3694	0.0142	0.5206	0.0186	0.6626	0.0229	0.7927	0.0271	0.9113	0.00500
0.0011	0.0437	0.0055	0.2129	0.0099	0.3729	0.0143	0.5239	0.0187	0.6657	0.0230	0.7956	0.0272	0.9140	0.00497
0.0012	0.0477	0.0056	0.2166	0.0100	0.3765	0.0144	0.5272	0.0188	0.6688	0.0231	0.7985	0.0273	0.9167	0.00494
0.0013	0.0516	0.0057	0.2204	0.0101	0.3800	0.0145	0.5305	0.0189	0.6720	0.0232	0.8014	0.0274	0.9194	0.00491
0.0014	0.0555	0.0058	0.2241	0.0102	0.3835	0.0146	0.5338	0.0190	0.6751	0.0233	0.8043	0.0275	0.9221	0.00488
0.0015	0.0595	0.0059	0.2278	0.0103	0.3870	0.0147	0.5372	0.0191	0.6782	0.0234	0.8072	0.0276	0.9248	0.00485
0.0016	0.0634	0.0060	0.2315	0.0104	0.3906	0.0148	0.5405	0.0192	0.6813	0.0235	0.8101	0.0277	0.9275	0.00482
0.0017	0.0673	0.0061	0.2352	0.0105	0.3941	0.0149	0.5438	0.0193	0.6844	0.0236	0.8130	0.0278	0.9302	0.00480
0.0018	0.0712	0.0062	0.2390	0.0106	0.3976	0.0150	0.5471	0.0194	0.6875	0.0237	0.8159	0.0279	0.9329	0.00477
0.0019	0.0752	0.0063	0.2427	0.0107	0.4011	0.0151	0.5504	0.0195	0.6905	0.0238	0.8188	0.0280	0.9356	0.00474
0.0020	0.0791	0.0064	0.2464	0.0108	0.4046	0.0152	0.5536	0.0196	0.6936	0.0239	0.8217	0.0281	0.9383	0.00471
0.0021	0.0830	0.0065	0.2501	0.0109	0.4080	0.0153	0.5569	0.0197	0.6967	0.0240	0.8245	0.0282	0.9410	0.00469
0.0022	0.0869	0.0066	0.2538	0.0110	0.4115	0.0154	0.5602	0.0198	0.6998	0.0241	0.8274	0.0283	0.9436	0.00466
0.0023	0.0908	0.0067	0.2574	0.0111	0.4150	0.0155	0.5635	0.0199	0.7029	0.0242	0.8303	0.0284	0.9463	0.00463
0.0024	0.0946	0.0068	0.2611	0.0112	0.4185	0.0156	0.5667	0.0200	0.7059	0.0243	0.8331	0.0285	0.9490	0.00461
0.0025	0.0985	0.0069	0.2648	0.0113	0.4220	0.0157	0.5700	0.0201	0.7090	0.0244	0.8360	0.0286	0.9516	0.00458
0.0026	0.1024	0.0070	0.2685	0.0114	0.4254	0.0158	0.5733	0.0202	0.7120	0.0245	0.8388	0.0287	0.9543	0.00455
0.0027	0.1063	0.0071	0.2721	0.0115	0.4289	0.0159	0.5765	0.0203	0.7151	0.0246	0.8417	0.0288	0.9569	0.00453
0.0028	0.1102	0.0072	0.2758	0.0116	0.4323	0.0160	0.5798	0.0204	0.7181	0.0247	0.8445	0.0289	0.9596	0.00450
0.0029	0.1140	0.0073	0.2795	0.0117	0.4358	0.0161	0.5830	0.0205	0.7212	0.0248	0.8473	0.0290	0.9622	0.00447
0.0030	0.1179	0.0074	0.2831	0.0118	0.4392	0.0162	0.5863	0.0206	0.7242	0.0249	0.8502	0.0291	0.9648	0.00445
0.0031	0.1217	0.0075	0.2868	0.0119	0.4427	0.0163	0.5895	0.0207	0.7272	0.0250	0.8530	0.0292	0.9675	0.00442
0.0032	0.1256	0.0076	0.2904	0.0120	0.4461	0.0164	0.5927	0.0208	0.7302	0.0251	0.8558	0.0293	0.9701	0.00440
0.0033	0.1294	0.0077	0.2941	0.0121	0.4495	0.0165	0.5959	0.0209	0.7333	0.0252	0.8586	0.0294	0.9727	0.00437
0.0034	0.1333	0.0078	0.2977	0.0122	0.4530	0.0166	0.5992	0.0210	0.7363	0.0253	0.8615	0.0295	0.9753	0.00435
0.0035	0.1371	0.0079	0.3013	0.0123	0.4564	0.0167	0.6024	0.0211	0.7393	0.0254	0.8643	0.0296	0.9779	0.00432
0.0036	0.1410	0.0080	0.3049	0.0124	0.4598	0.0168	0.6056	0.0212	0.7423	0.0255	0.8671	0.0297	0.9805	0.00430

(Continued)

APPENDIX D.7 (*Continued*)

Coefficient of Resistance ($\overline{K}$) versus Reinforcement Ratio (ρ) (f_c' = 4,000 psi; f_y = 40,000 psi)

ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ε_t[a]
0.0037	0.1448	0.0081	0.3086	0.0125	0.4632	0.0169	0.6088	0.0213	0.7453	0.0256	0.8699	0.0298	0.9831	0.00427
0.0038	0.1486	0.0082	0.3122	0.0126	0.4666	0.0170	0.6120	0.0214	0.7483	0.0257	0.8727	0.0299	0.9857	0.00425
0.0039	0.1524	0.0083	0.3158	0.0127	0.4701	0.0171	0.6152	0.0215	0.7513	0.0258	0.8754	0.0300	0.9883	0.00423
0.0040	0.1562	0.0084	0.3194	0.0128	0.4735	0.0172	0.6184	0.0216	0.7543	0.0259	0.8782	0.0301	0.9909	0.00420
0.0041	0.1600	0.0085	0.3230	0.0129	0.4768	0.0173	0.6216	0.0217	0.7572	0.0260	0.8810	0.0302	0.9935	0.00418
0.0042	0.1638	0.0086	0.3266	0.0130	0.4802	0.0174	0.6248	0.0218	0.7602	0.0261	0.8838	0.0303	0.9961	0.00415
0.0043	0.1676	0.0087	0.3302	0.0131	0.4836	0.0175	0.6279	0.0219	0.7632	0.0262	0.8865	0.0304	0.9986	0.00413
0.0044	0.1714	0.0088	0.3338	0.0132	0.4870	0.0176	0.6311	0.0220	0.7662	0.0263	0.8893	0.0305	1.0012	0.00411
0.0045	0.1752	0.0089	0.3374	0.0133	0.4904	0.0177	0.6343	0.0221	0.7691	0.0264	0.8921	0.0306	1.0038	0.00408
0.0046	0.1790	0.0090	0.3409	0.0134	0.4938	0.0178	0.6375	0.0222	0.7721	0.0265	0.8948	0.0307	1.0063	0.00406
0.0047	0.1828	0.0091	0.3445	0.0135	0.4971	0.0179	0.6406	0.0223	0.7750	0.0266	0.8976	0.0308	1.0089	0.00404
0.0048	0.1866	0.0092	0.3481	0.0136	0.5005	0.0180	0.6438	0.0224	0.7780	0.0267	0.9003	0.0309	1.0114	0.00401
0.0049	0.1904	0.0093	0.3517	0.0137	0.5038	0.0181	0.6469	0.0225	0.7809	0.0268	0.9031	0.03096	1.0130	0.00400
0.0050	0.1941	0.0094	0.3552	0.0138	0.5072	0.0182	0.6501	0.0226	0.7839	0.0269	0.9058			
0.0051	0.1979	0.0095	0.3588	0.0139	0.5105	0.0183	0.6532	0.0227	0.7868	0.0270	0.9085			
0.0052	0.2016	0.0096	0.3623	0.0140	0.5139	0.0184	0.6563	0.0228	0.7897					
0.0053	0.2054	0.0097	0.3659	0.0141	0.5172	0.0185	0.6595							

[a] $d = d_t$, where d_t is distance from extreme compression fiber to the center of outermost steel layer. For single layer steel, $d_t = d$.

APPENDIX D.8

Coefficient of Resistance ($\overline{K}$) ($f_c' = 4{,}000$ psi; $f_y = 50{,}000$ psi)

ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ε_t[a]
0.0030	0.147	0.0061	0.291	0.0102	0.472	0.0143	0.640	0.0184	0.795	0.0216	0.908	0.0050
0.0031	0.151	0.0062	0.296	0.0103	0.476	0.0144	0.643	0.0185	0.799	0.0217	0.912	0.0050
0.0032	0.156	0.0063	0.300	0.0104	0.480	0.0145	0.648	0.0186	0.802	0.0218	0.915	0.0050
0.0033	0.161	0.0064	0.305	0.0105	0.484	0.0146	0.651	0.0187	0.806	0.0219	0.919	0.0049
0.0034	0.166	0.0065	0.309	0.0106	0.489	0.0147	0.655	0.0188	0.810	0.022	0.922	0.0049
0.0035	0.170	0.0066	0.314	0.0107	0.493	0.0148	0.659	0.0189	0.813	0.0221	0.925	0.0048
0.0036	0.175	0.0067	0.318	0.0108	0.497	0.0149	0.663	0.0190	0.817	0.0222	0.929	0.0048
0.0037	0.180	0.0068	0.323	0.0109	0.501	0.0150	0.667	0.0191	0.820	0.0223	0.932	0.0048
0.0038	0.185	0.0069	0.327	0.0110	0.505	0.0151	0.671	0.0192	0.824	0.0224	0.936	0.0047
0.0039	0.189	0.0070	0.332	0.0111	0.510	0.0152	0.675	0.0193	0.828	0.0225	0.939	0.0047
0.0040	0.194	0.0071	0.336	0.0112	0.514	0.0153	0.679	0.0194	0.831	0.0226	0.942	0.0047
0.0041	0.199	0.0072	0.341	0.0113	0.518	0.0154	0.682	0.0195	0.835	0.0227	0.946	0.0046
0.0042	0.203	0.0073	0.345	0.0114	0.522	0.0155	0.686	0.0196	0.838	0.0228	0.949	0.0046
0.0043	0.208	0.0074	0.350	0.0115	0.526	0.0156	0.690	0.0197	0.842	0.0229	0.952	0.0046
0.0044	0.213	0.0075	0.354	0.0116	0.530	0.0157	0.694	0.0198	0.845	0.023	0.956	0.0045
0.0045	0.217	0.0076	0.359	0.0117	0.534	0.0158	0.698	0.0199	0.849	0.0231	0.959	0.0045
0.0046	0.222	0.0077	0.363	0.0118	0.539	0.0159	0.702	0.0200	0.852	0.0232	0.962	0.0045
0.0047	0.227	0.0078	0.368	0.0119	0.543	0.0160	0.706	0.0201	0.856	0.0234	0.969	0.0044
0.0048	0.231	0.0079	0.372	0.0120	0.547	0.0161	0.709	0.0202	0.859	0.0235	0.972	0.0044
0.0049	0.236	0.0080	0.376	0.0121	0.551	0.0162	0.713	0.0203	0.863	0.0236	0.975	0.0043
0.0050	0.241	0.0081	0.381	0.0122	0.555	0.0163	0.717	0.0204	0.866	0.0237	0.978	0.0043
0.0051	0.245	0.0082	0.385	0.0123	0.559	0.0164	0.721	0.0205	0.870	0.0238	0.982	0.0043
0.0052	0.250	0.0083	0.389	0.0124	0.563	0.0165	0.725	0.0206	0.873	0.0239	0.985	0.0043
0.0053	0.255	0.0084	0.394	0.0125	0.567	0.0166	0.728	0.0207	0.877	0.024	0.988	0.0042
0.0054	0.259	0.0085	0.398	0.0126	0.571	0.0167	0.732	0.0208	0.880	0.0241	0.991	0.0042
0.0055	0.264	0.0086	0.403	0.0127	0.575	0.0168	0.736	0.0209	0.884	0.0242	0.995	0.0042

(Continued)

APPENDIX D.8 (*Continued*)

Coefficient of Resistance ($\overline{K}$) ($f_c' = 4{,}000$ psi; $f_y = 50{,}000$ psi)

ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ε_t[a]
0.0056	0.268	0.0087	0.407	0.0128	0.580	0.0169	0.327	0.0210	0.887	0.0243	0.998	0.0041
0.0057	0.273	0.0088	0.411	0.0129	0.584	0.0170	0.743	0.0211	0.891	0.0244	1.001	0.0041
0.0058	0.278	0.0089	0.416	0.0130	0.588	0.0171	0.747	0.0212	0.894	0.0245	1.004	0.0041
0.0059	0.282	0.0090	0.420	0.0131	0.592	0.0172	0.751	0.0213	0.898	0.0246	1.008	0.0040
0.0060	0.287	0.0091	0.424	0.0132	0.596	0.0173	0.755	0.0214	0.901	0.0247	1.011	0.0040
		0.0092	0.429	0.0133	0.600	0.0174	0.758	0.0215	0.904	0.0248	1.014	0.0040
		0.0093	0.433	0.0134	0.604	0.0175	0.762					
		0.0094	0.437	0.0135	0.608	0.0176	0.766					
		0.0095	0.442	0.0136	0.612	0.0177	0.769					
		0.0096	0.446	0.0137	0.616	0.0178	0.773					
		0.0097	0.450	0.0138	0.620	0.0179	0.777					
		0.0098	0.455	0.0139	0.624	0.0180	0.780					
		0.0099	0.459	0.0140	0.628	0.0181	0.784					
		0.0100	0.463	0.0141	0.632	0.0182	0.788					
		0.0101	0.467	0.0142	0.636	0.0183	0.791					

[a] $d = d_t$, where d_t is distance from extreme compression fiber to the center of outermost steel layer. For single layer steel, $d_t = d$.

APPENDIX D.9

Coefficient of Resistance ($\overline{K}$) versus Reinforcement Ratio (ρ) ($f_c{}' = 4{,}000$ psi; $f_y = 60{,}000$ psi)

ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ε_t[a]
0.0010	0.0595	0.0039	0.2259	0.0068	0.3835	0.0097	0.5322	0.0126	0.6720	0.0154	0.7985	0.01806	0.9110	0.00500
0.0011	0.0654	0.0040	0.2315	0.0069	0.3888	0.0098	0.5372	0.0127	0.6766	0.0155	0.8029	0.0181	0.9126	0.00498
0.0012	0.0712	0.0041	0.2371	0.0070	0.3941	0.0099	0.5421	0.0128	0.6813	0.0156	0.8072	0.0182	0.9167	0.00494
0.0013	0.0771	0.0042	0.2427	0.0071	0.3993	0.0100	0.5471	0.0129	0.6859	0.0157	0.8116	0.0183	0.9208	0.00490
0.0014	0.0830	0.0043	0.2482	0.0072	0.4046	0.0101	0.5520	0.0130	0.6906	0.0158	0.8159	0.0184	0.9248	0.00485
0.0015	0.0889	0.0044	0.2538	0.0073	0.4098	0.0102	0.5569	0.0131	0.6952	0.0159	0.8202	0.0185	0.9289	0.00481
0.0016	0.0946	0.0045	0.2593	0.0074	0.4150	0.0103	0.5618	0.0132	0.6998	0.0160	0.8245	0.0186	0.9329	0.00477
0.0017	0.1005	0.0046	0.2648	0.0075	0.4202	0.0104	0.5667	0.0133	0.7044	0.0161	0.8288	0.0187	0.9369	0.00473
0.0018	0.1063	0.0047	0.2703	0.0076	0.4254	0.0105	0.5716	0.0134	0.7090	0.0162	0.8331	0.0188	0.9410	0.00469
0.0019	0.1121	0.0048	0.2758	0.0077	0.4306	0.0106	0.5765	0.0135	0.7136	0.0163	0.8374	0.0189	0.9450	0.00465
0.0020	0.1179	0.0049	0.2813	0.0078	0.4358	0.0107	0.5814	0.0136	0.7181	0.0164	0.8417	0.0190	0.9490	0.00461
0.0021	0.1237	0.0050	0.2868	0.0079	0.4410	0.0108	0.5862	0.0137	0.7227	0.0165	0.8459	0.0191	0.9529	0.00457
0.0022	0.1294	0.0051	0.2922	0.0080	0.4461	0.0109	0.5911	0.0138	0.7272	0.0166	0.8502	0.0192	0.9569	0.00453
0.0023	0.1352	0.0052	0.2977	0.0081	0.4513	0.0110	0.5959	0.0139	0.7318	0.0167	0.8544	0.0193	0.9609	0.00449
0.0024	0.1410	0.0053	0.3031	0.0082	0.4564	0.0111	0.6008	0.0140	0.7363	0.0168	0.8586	0.0194	0.9648	0.00445
0.0025	0.1467	0.0054	0.3086	0.0083	0.4615	0.0112	0.6056	0.0141	0.7408	0.0169	0.8629	0.0195	0.9688	0.00441
0.0026	0.1524	0.0055	0.3140	0.0084	0.4666	0.0113	0.6104	0.0142	0.7453	0.0170	0.8671	0.0196	0.9727	0.00437
0.0027	0.1581	0.0056	0.3194	0.0085	0.4718	0.0114	0.6152	0.0143	0.7498	0.0171	0.8713	0.0197	0.9766	0.00434
0.0028	0.1638	0.0057	0.3248	0.0086	0.4768	0.0115	0.6200	0.0144	0.7543	0.0172	0.8754	0.0198	0.9805	0.00430
0.0029	0.1695	0.0058	0.3302	0.0087	0.4819	0.0116	0.6248	0.0145	0.7587	0.0173	0.8796	0.0199	0.9844	0.00426
0.0030	0.1752	0.0059	0.3356	0.0088	0.4870	0.0117	0.6296	0.0146	0.7632	0.0174	0.8838	0.0200	0.9883	0.00422
0.0031	0.1809	0.0060	0.3409	0.0089	0.4921	0.0118	0.6343	0.0147	0.7676	0.0175	0.8879	0.0201	0.9922	0.00419
0.0032	0.1866	0.0061	0.3463	0.0090	0.4971	0.0119	0.6391	0.0148	0.7721	0.0176	0.8921	0.0202	0.9961	0.00415
0.0033	0.1922	0.0062	0.3516	0.0091	0.5022	0.0120	0.6438	0.0149	0.7765	0.0177	0.8962	0.0203	0.9999	0.00412
0.0034	0.1979	0.0063	0.3570	0.0092	0.5072	0.0121	0.6485	0.0150	0.7809	0.0178	0.9003	0.0204	1.0038	0.00408
0.0035	0.2035	0.0064	0.3623	0.0093	0.5122	0.0122	0.6532	0.0151	0.7853	0.0179	0.9044	0.0205	1.0076	0.00405
0.0036	0.2091	0.0065	0.3676	0.0094	0.5172	0.0123	0.6579	0.0152	0.7897	0.0180	0.9085	0.0206	1.0114	0.00401
0.0037	0.2148	0.0066	0.3729	0.0095	0.5222	0.0124	0.6626	0.0153	0.7941			0.02063	1.0126	0.00400
0.0038	0.2204	0.0067	0.3782	0.0096	0.5272	0.0125	0.6673							

[a] $d = d_t$, where d_t is distance from extreme compression fiber to the center of outermost steel layer. For single layer steel, $d_t = d$.

APPENDIX D.10

Coefficient of Resistance ($\overline{K}$) versus Reinforcement Ratio (ρ) ($f_c' = 5{,}000$ psi; $f_y = 60{,}000$ psi)

ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ε_t[a]
0.0010	0.0596	0.0048	0.2782	0.0086	0.4847	0.0124	0.6789	0.0162	0.8609	0.0194	1.0047	0.02257	1.1385	0.00500
0.0011	0.0655	0.0049	0.2838	0.0087	0.4899	0.0125	0.6838	0.0163	0.8655	0.0195	1.0090	0.0226	1.1398	0.00499
0.0012	0.0714	0.0050	0.2894	0.0088	0.4952	0.0126	0.6888	0.0164	0.8701	0.0196	1.0134	0.0227	1.1438	0.00496
0.0013	0.0773	0.0051	0.2950	0.0089	0.5005	0.0127	0.6937	0.0165	0.8747	0.0197	1.0177	0.0228	1.1479	0.00492
0.0014	0.0832	0.0052	0.3005	0.0090	0.5057	0.0128	0.6986	0.0166	0.8793	0.0198	1.0220	0.0229	1.1520	0.00489
0.0015	0.0890	0.0053	0.3061	0.0091	0.5109	0.0129	0.7035	0.0167	0.8839	0.0199	1.0263	0.0230	1.1560	0.00485
0.0016	0.0949	0.0054	0.3117	0.0092	0.5162	0.0130	0.7084	0.0168	0.8885	0.0200	1.0307	0.0231	1.1601	0.00482
0.0017	0.1008	0.0055	0.3172	0.0093	0.5214	0.0131	0.7133	0.0169	0.8930	0.0201	1.0350	0.0232	1.1641	0.00479
0.0018	0.1066	0.0056	0.3227	0.0094	0.5266	0.0132	0.7182	0.0170	0.8976	0.0202	1.0393	0.0233	1.1682	0.00475
0.0019	0.1125	0.0057	0.3282	0.0095	0.5318	0.0133	0.7231	0.0171	0.9022	0.0203	1.0435	0.0234	1.1722	0.00472
0.0020	0.1183	0.0058	0.3338	0.0096	0.5370	0.0134	0.7280	0.0172	0.9067	0.0204	1.0478	0.0235	1.1762	0.00469
0.0021	0.1241	0.0059	0.3393	0.0097	0.5422	0.0135	0.7328	0.0173	0.9112	0.0205	1.0521	0.0236	1.1802	0.00465
0.0022	0.1300	0.0060	0.3448	0.0098	0.5473	0.0136	0.7377	0.0174	0.9158	0.0206	1.0563	0.0237	1.1842	0.00462
0.0023	0.1358	0.0061	0.3502	0.0099	0.5525	0.0137	0.7425	0.0175	0.9203	0.0207	1.0606	0.0238	1.1882	0.00459
0.0024	0.1416	0.0062	0.3557	0.0100	0.5576	0.0138	0.7473	0.0176	0.9248	0.0208	1.0648	0.0239	1.1922	0.00456
0.0025	0.1474	0.0063	0.3612	0.0101	0.5628	0.0139	0.7522	0.0177	0.9293	0.0209	1.0691	0.0240	1.1961	0.00453
0.0026	0.1531	0.0064	0.3667	0.0102	0.5679	0.0140	0.7570	0.0178	0.9338	0.0210	1.0733	0.0241	1.2001	0.00449
0.0027	0.1589	0.0065	0.3721	0.0103	0.5731	0.0141	0.7618	0.0179	0.9383	0.0211	1.0775	0.0242	1.2041	0.00446
0.0028	0.1647	0.0066	0.3776	0.0104	0.5782	0.0142	0.7666	0.0180	0.9428	0.0212	1.0817	0.0243	1.2080	0.00443
0.0029	0.1704	0.0067	0.3830	0.0105	0.5833	0.0143	0.7714	0.0181	0.9473	0.0213	1.0859	0.0244	1.2119	0.00440
0.0030	0.1762	0.0068	0.3884	0.0106	0.5884	0.0144	0.7762	0.0182	0.9517	0.0214	1.0901	0.0245	1.2159	0.00437
0.0031	0.1819	0.0069	0.3938	0.0107	0.5935	0.0145	0.7810	0.0183	0.9562	0.0215	1.0943	0.0246	1.2198	0.00434
0.0032	0.1877	0.0070	0.3992	0.0108	0.5986	0.0146	0.7857	0.0184	0.9606	0.0216	1.0985	0.0247	1.2237	0.00431
0.0033	0.1934	0.0071	0.4047	0.0109	0.6037	0.0147	0.7905	0.0185	0.9651	0.0217	1.1026	0.0248	1.2276	0.00428
0.0034	0.1991	0.0072	0.4100	0.0110	0.6088	0.0148	0.7952	0.0186	0.9695	0.0218	1.1068	0.0249	1.2315	0.00425
0.0035	0.2048	0.0073	0.4154	0.0111	0.6138	0.0149	0.8000	0.0187	0.9739	0.0219	1.1110	0.0250	1.2354	0.00423
0.0036	0.2105	0.0074	0.4208	0.0112	0.6189	0.0150	0.8047	0.0188	0.9783	0.0220	1.1151	0.0251	1.2393	0.00420

(Continued)

APPENDIX D.10 (*Continued*)

Coefficient of Resistance ($\overline{K}$) versus Reinforcement Ratio (ρ) (f_c' = 5,000 psi; f_y = 60,000 psi)

ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ρ	$\overline{K}$ (ksi)	ε_t[a]
0.0037	0.2162	0.0075	0.4262	0.0113	0.6239	0.0151	0.8094	0.0189	0.9827	0.0221	1.1192	0.0252	1.2431	0.00417
0.0038	0.2219	0.0076	0.4315	0.0114	0.6290	0.0152	0.8142	0.0190	0.9872	0.0222	1.1234	0.0253	1.2470	0.00414
0.0039	0.2276	0.0077	0.4369	0.0115	0.6340	0.0153	0.8189	0.0191	0.9916	0.0223	1.1275	0.0254	1.2509	0.00411
0.0040	0.2332	0.0078	0.4422	0.0116	0.6390	0.0154	0.8236	0.0192	0.9959	0.0224	1.1316	0.0255	1.2547	0.00408
0.0041	0.2389	0.0079	0.4476	0.0117	0.6440	0.0155	0.8283	0.0193	1.0003	0.0225	1.1357	0.0256	1.2585	0.00406
0.0042	0.2445	0.0080	0.4529	0.0118	0.6490	0.0156	0.8329					0.0257	1.2624	0.00403
0.0043	0.2502	0.0081	0.4582	0.0119	0.6540	0.0157	0.8376					0.0258	1.2662	0.00400
0.0044	0.2558	0.0082	0.4635	0.0120	0.6590	0.0158	0.8423							
0.0045	0.2614	0.0083	0.4688	0.0121	0.6640	0.0159	0.8469							
0.0046	0.2670	0.0084	0.4741	0.0122	0.6690	0.0160	0.8516							
0.0047	0.2726	0.0085	0.4794	0.0123	0.6739	0.0161	0.8562							

[a] $d = d_t$, where d_t is distance from extreme compression fiber to the center of outermost steel layer. For single layer steel, $d_t = d$.

APPENDIX D.11
Values of ρ Balanced, ρ for $\varepsilon_t = 0.005$, and ρ Minimum for Flexure

f_y	f'_c	3,000 psi $\beta_1 = 0.85$	4,000 psi $\beta_1 = 0.85$	5,000 psi $\beta_1 = 0.80$	6,000 psi $\beta_1 = 0.75$
Grade 40	ρ balanced	0.0371	0.0495	0.0582	0.0655
40,000 psi	ρ when $\varepsilon_t = 0.005$	0.0203	0.0271	0.0319	0.0359
	ρ min for flexure	0.0050	0.0050	0.0053	0.0058
Grade 50	ρ balanced	0.0275	0.0367	0.0432	0.0486
50,000 psi	ρ when $\varepsilon_t = 0.005$	0.0163	0.0217	0.0255	0.0287
	ρ min for flexure	0.0040	0.0040	0.0042	0.0046
Grade 60	ρ balanced	0.0214	0.0285	0.0335	0.0377
60,000 psi	ρ when $\varepsilon_t = 0.005$	0.0136	0.0181	0.0212	0.0239
	ρ min for flexure	0.0033	0.0033	0.0035	0.0039
Grade 75	ρ balanced	0.0155	0.0207	0.0243	0.0274
75,000 psi	ρ when $\varepsilon_t = 0.005$	0.0108	0.0144	0.0170	0.0191
	ρ min for flexure	0.0027	0.0027	0.0028	0.0031

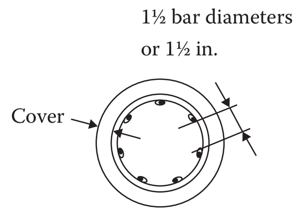
APPENDIX D.12
Areas of Steel Bars per Foot of Slab (in.²)

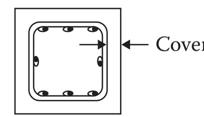
	Bar Size								
Bar Spacing (in.)	**#3**	**#4**	**#5**	**#6**	**#7**	**#8**	**#9**	**#10**	**#11**
2	0.66	1.20	1.86						
2½	0.53	0.96	1.49	2.11					
3	0.44	0.80	1.24	1.76	2.40	3.16	4.00		
3½	0.38	0.69	1.06	1.51	2.06	2.71	3.43	4.35	
4	0.33	0.60	0.93	1.32	1.80	2.37	3.00	3.81	4.68
4½	0.29	0.53	0.83	1.17	1.60	2.11	2.67	3.39	4.16
5	0.26	0.48	0.74	1.06	1.44	1.90	2.40	3.05	3.74
5½	0.24	0.44	0.68	0.96	1.31	1.72	2.18	2.77	3.40
6	0.22	0.40	0.62	0.88	1.20	1.58	2.00	2.54	3.12
6½	0.20	0.37	0.57	0.81	1.11	1.46	1.85	2.34	2.88
7	0.19	0.34	0.53	0.75	1.03	1.35	1.71	2.18	2.67
7½	0.18	0.32	0.50	0.70	0.96	1.26	1.60	2.03	2.50
8	0.16	0.30	0.46	0.66	0.90	1.18	1.50	1.90	2.34
9	0.15	0.27	0.41	0.59	0.80	1.05	1.33	1.69	2.08
10	0.13	0.24	0.37	0.53	0.72	0.95	1.20	1.52	1.87
11	0.12	0.22	0.34	0.48	0.65	0.86	1.09	1.39	1.70
12	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56
13	0.10	0.18	0.29	0.41	0.55	0.73	0.92	1.17	1.44
14	0.09	0.17	0.27	0.38	0.51	0.68	0.86	1.09	1.34
15	0.09	0.16	0.25	0.35	0.48	0.64	0.80	1.02	1.25
16	0.08	0.15	0.23	0.33	0.45	0.59	0.75	0.95	1.17
17	0.08	0.14	0.22	0.31	0.42	0.56	0.71	0.90	1.10
18	0.07	0.13	0.21	0.29	0.40	0.53	0.67	0.85	1.04

APPENDIX D.13
Size and Pitch of Spirals

		f'_c			
Diameter of Column (in.)	**Out to Out of Spiral (in.)**	**2,500**	**3,000**	**4,000**	**5,000**
f_y **= 40,000**					
14, 15	11, 12	$\frac{3}{8}-2$	$\frac{3}{8}-1\frac{3}{4}$	$\frac{1}{2}-2\frac{1}{2}$	$\frac{1}{2}-1\frac{3}{4}$
16	13	$\frac{3}{8}-2$	$\frac{3}{8}-1\frac{3}{4}$	$\frac{1}{2}-2\frac{1}{2}$	$\frac{1}{2}-2$
17–19	14–16	$\frac{3}{8}-2\frac{1}{4}$	$\frac{3}{8}-1\frac{3}{4}$	$\frac{1}{2}-2\frac{1}{2}$	$\frac{1}{2}-2$
20–23	17–20	$\frac{3}{8}-2\frac{1}{4}$	$\frac{3}{8}-1\frac{3}{4}$	$\frac{1}{2}-2\frac{1}{2}$	$\frac{1}{2}-2$
24–30	21–27	$\frac{3}{8}-2\frac{1}{4}$	$\frac{3}{8}-2$	$\frac{1}{2}-2\frac{1}{2}$	$\frac{1}{2}-2$
f_y **= 60,000**					
14, 15	11, 12	$\frac{1}{4}-1\frac{3}{4}$	$\frac{3}{8}-2\frac{3}{4}$	$\frac{3}{8}-2$	$\frac{1}{2}-2\frac{3}{4}$
16–23	13–20	$\frac{1}{4}-1\frac{3}{4}$	$\frac{3}{8}-2\frac{3}{4}$	$\frac{3}{8}-2$	$\frac{1}{2}-3$
24–29	21–26	$\frac{1}{4}-1\frac{3}{4}$	$\frac{3}{8}-3$	$\frac{3}{8}-2\frac{1}{4}$	$\frac{1}{2}-3$
30	17	$\frac{1}{4}-1\frac{3}{4}$	$\frac{3}{8}-3$	$\frac{3}{8}-2\frac{1}{4}$	$\frac{1}{2}-3\frac{1}{4}$

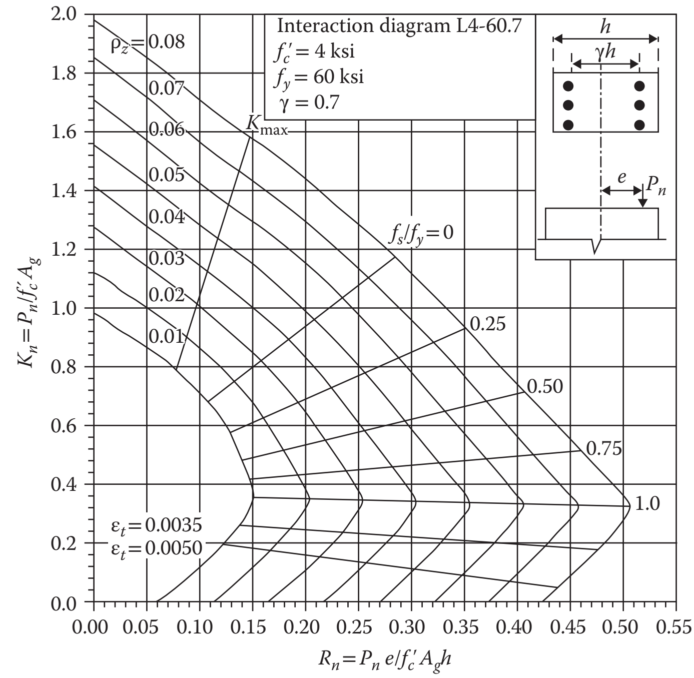
APPENDIX D.14
Maximum Number of Bars in One Row



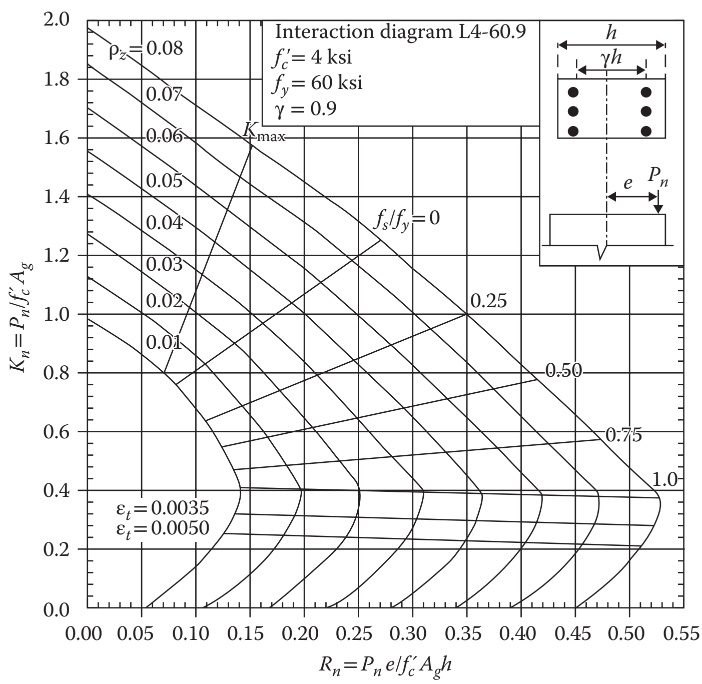


Recommended Spiral or Tie Bar Number	Core Size (in.) = Column Size − 2 × Cover	Circular Area (in.^{2})	Bar Size							Square Area (in.^{2})	Bar Size						
			#5	#6	#7	#8	#9	#10	#11[a]		#5	#6	#7	#8	#9	#10	#11[a]
3	9	63.6	8	7	7	6	—	—	—	81	8	8	8	8	4	4	4
	10	78.5	10	9	8	7	6	—	—	100	12	8	8	8	8	4	4
	11	95.0	11	10	9	8	7	6	—	121	12	12	8	8	8	8	4
	12	113.1	12	11	10	9	8	7	6	144	12	12	12	8	8	8	8
	13	132.7	13	12	11	10	8	7	6	169	16	12	12	12	8	8	8
	14	153.9	14	13	12	11	9	8	7	196	16	16	12	12	12	8	8
	15	176.7	15	14	13	12	10	9	8	225	16	16	16	12	12	12	8
4	16	201.1	16	15	14	12	11	9	8	256	20	16	16	16	12	12	8
	17	227.0	18	16	15	13	12	10	9	289	20	20	16	16	12	12	8
	18	254.5	19	17	15	14	12	11	10	324	20	20	16	16	16	12	12
	19	283.5	20	18	16	15	13	11	10	361	24	20	20	16	16	12	12
	20	314.2	21	19	17	16	14	12	11	400	24	24	20	20	16	12	12
	21	346.4	22	20	18	17	15	13	11	441	28	24	20	20	16	16	12
	22	380.1	23	21	19	18	15	14	12	484	28	24	24	20	20	16	12
5	23	415.5	24	22	21	19	16	14	13	529	28	28	24	24	20	16	16
	24	452.4	25	23	21	20	17	15	13	576	32	28	24	24	20	16	16
	25	490.9	26	24	22	20	18	16	14	625	32	28	28	24	20	20	16
	26	530.9	28	25	23	21	19	16	14	676	32	32	28	24	24	20	16
	27	572.6	29	26	24	22	19	17	15	729	36	32	28	28	24	20	16

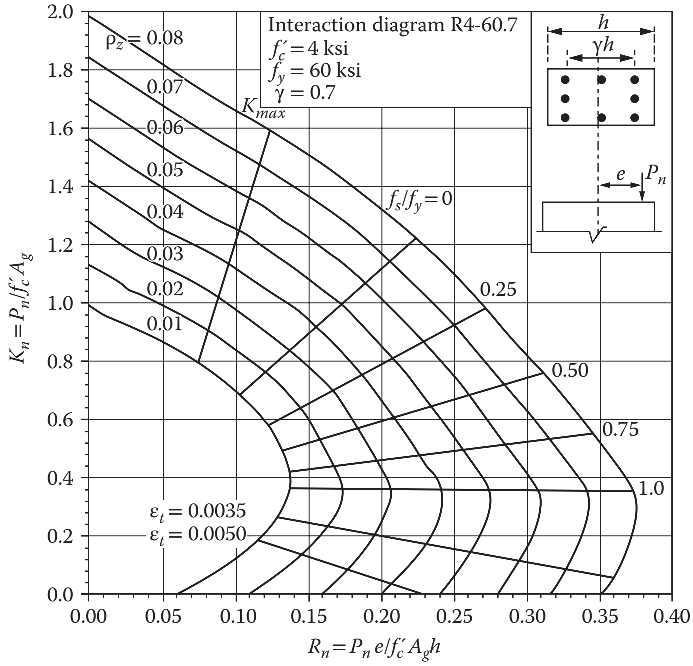
[a] Use No. 4 tie for No. 11 or larger longitudinal reinforcement.



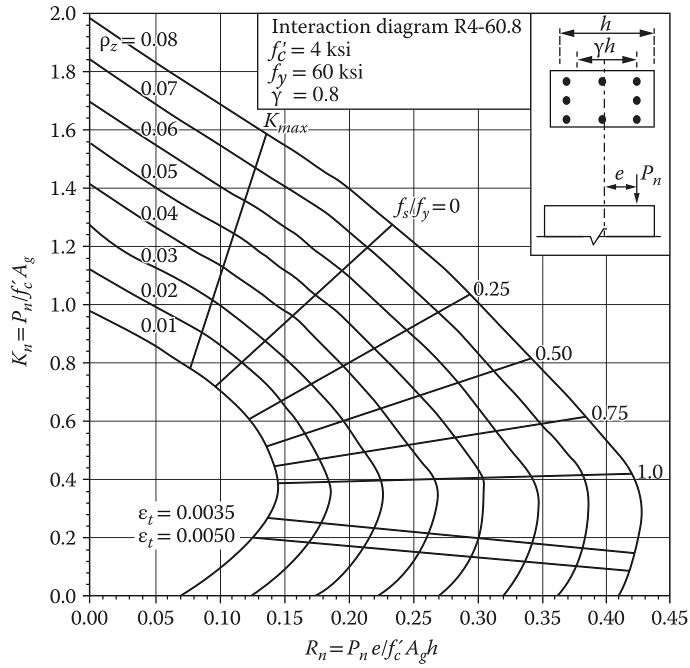
APPENDIX D.15 Column interaction diagram for tied column with bars on end faces only. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)



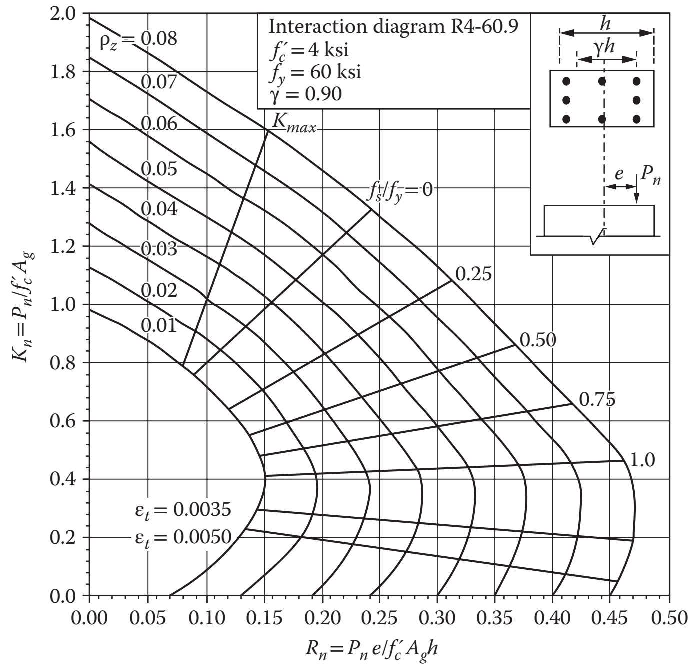
APPENDIX D.16 Column interaction diagram for tied column with bars on end faces only. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)



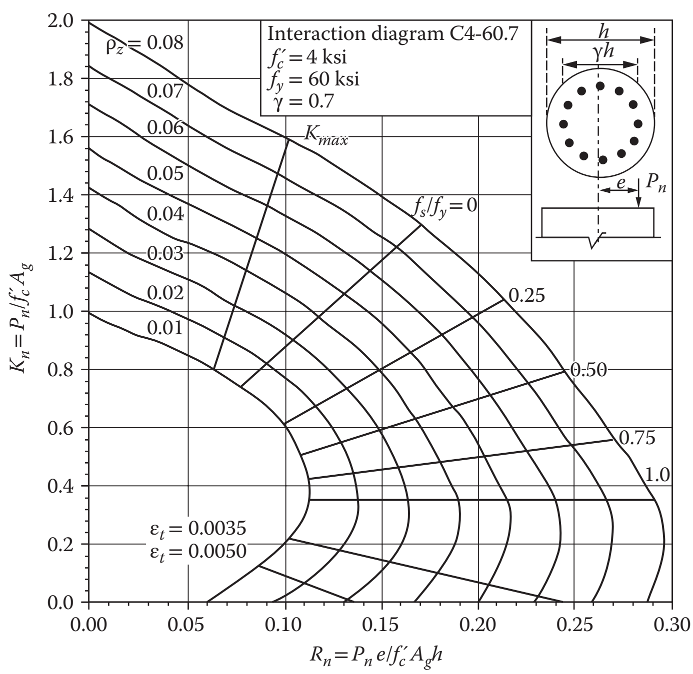
APPENDIX D.17 Column interaction diagram for tied column with bars on all faces. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)



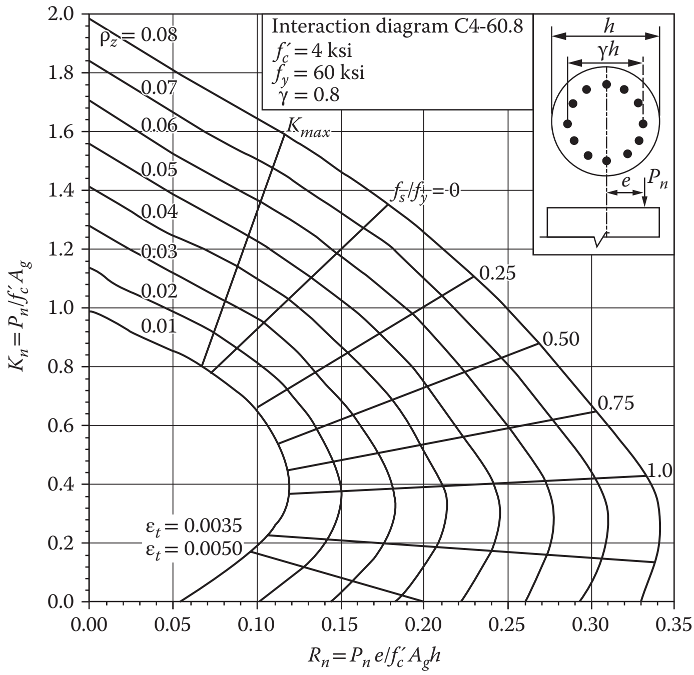
APPENDIX D.18 Column interaction diagram for tied column with bars on all faces. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)



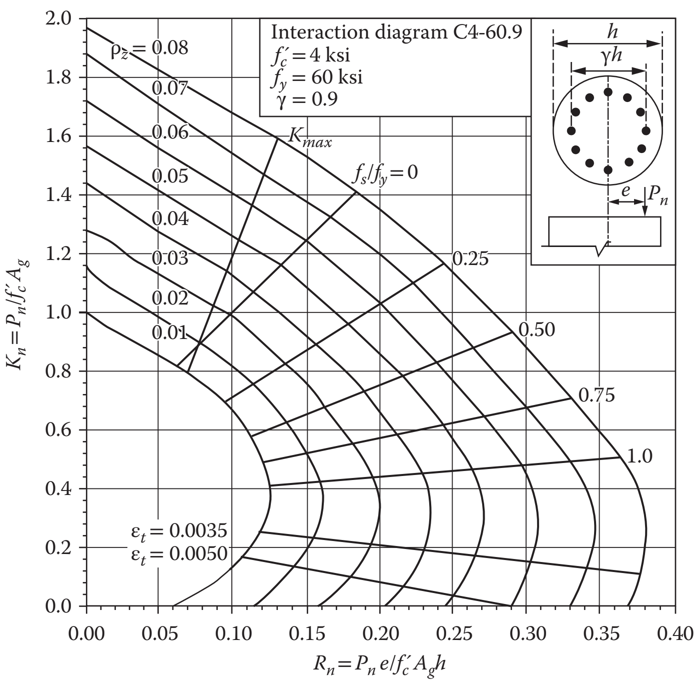
APPENDIX D.19 Column interaction diagram for tied column with bars on all faces. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)



APPENDIX D.20 Column interaction diagram for circular spiral column. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)



APPENDIX D.21 Column interaction diagram for circular spiral column. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)



APPENDIX D.22 Column interaction diagram for circular spiral column. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)

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