Structura Structura

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the Self-Organizing Universe

DRAFT

"As he scribbled his odds and ends, he made a note reaffirming his belief that art always serves beauty, and beauty is delight in form, and form is the key to organic life, since no living thing can exist without it, so that every work of art, including tragedy, expresses the joy of existence. And his own ideas and notes also brought him joy, a tragic joy, a joy full of tears that exhausted him and made his head ache."

—Boris Pasternak[1]

[1] *Doctor Zhivago*, pg. 378.

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Logos

"In the beginning was the ***ratio***," and so begins Genesis, with the first word of the first book of the Pentatuch, the Hebrew Bible adopted by the derivative Christian faith. It is ironic that the first word, when translated into Greek, should be sufficiently ambiguous that those who would rewrite the meaning of the text in English would assert that the beginning was "the word," instead of the intended "ratio."

Contemplation of the universe as it is viewed through the lens of contemporary physics and cosmology inevitably demands simpler explanations. Occam's Razor intuits that the existing models are far too complex, and layering more complexity atop the existing analytical models does not appear to offer the elusive, comprehensive solution. The mathematics establishment derides the Golden Mean/Phi Ratio Geometry as a trivial branch of numerology, especially since the accuracy of the associated arithmetic is less than perfect. These criticisms do not consider that many of the Phi Ratio constructs are actually elements of spherical constructs, and not the flat, planar constructs of Euclidean space. Further, most of the constants used in analytical equations have coefficients, and little epsilons that are used to bring non-linear, and linear computations, respectively, into agreement with observed natural structure.

Analytical mathematics will always have less fidelity than that enjoyed by geometry because it is based on the symbolisms of language, and will always be subject to vagaries of semantics, and syntax. Translation from one universe, or language, to another always results in losses due to syntactic, semantic subtleties, and insertion of noise. When modeling the natural order with analytical mathematics we find a description dominated by irrational, and occasionally transcendental numbers. The mathematics establishment claims that the geometry is inexact, yet the Geometry has a much higher fidelity to nature than conventional analytical math. One feature of conventional math can be seen in the coefficients (a, b) and little epsilons (ε) that tweak expressions to fit; the classic fudge factors lumped as relativistic effects, for one.

$$y = ax^2 + bx + c + \epsilon$$

Equation 1

Science often hides behind a façade of analytical mathematics, assuming an indisputable position on the plain of argumentation. In this sense, science begs the question, and stands on self-righteous grounds that retards the progress that would result from honest discourse.

Grand Unification is another distraction that has been wasting resources for decades. It would be more productive to continue concurrent investigations that, when the models are correct, will see the fusion of forces, particles, and effects into common fundamentals. Forcing these things together prematurely will only postpone the day when gravity, magnetism, weak and strong forces, and other phenomena share the common root.

There is a sense that science lacks the energy, or discipline to pursue the rigor that it often cites in a self-congratulatory manner. It seems that for each instance of a seemingly irreconcilable, or even paradoxical experimental result, the physics community devises yet another confusing distraction in the form of a "new" particle: bosons, hadrons, baryons, mesons, *et alia*. The quest for a unified model of everything seems to be an attempt to close the debate through validation of the *status quo* in a single stroke. Dark Matter, and Dark Energy represent the most recent effort to preserve a standard model that is broken through by complexification. These approaches are consuming a significant fraction of research resources, while falling short of answering fundamental questions.

A pervasive example of the lack of enthusiasm for rigor is probability, and statistics. These often become a substitute for the curiosity, and determination necessary to identify the connections, causes, and consequences of observed effects. Thankfully, much of science that has been attributed to "random effects" in previous research is giving way to intriguing developments in contemporary mathematics, which tweeze structure from apparent randomness, and these exciting developments include Mandlebrot's Fractals, followed by the new fields of Chaos, Complexity, and Emergence. These developments encourage researchers who sometimes operate outside of the mainstream doctrine to develop new insights, tools, and paradigms.

We have discovered value in the Ancient and Sacred Phi Ratio Geometry, at a time that the Geometry has been enjoying resurgence in popular interest, in both classic studies, and new applications. This paper offers a re-examination of prevailing thought in physics, astronomy, cosmology, and such in the context of Phi Ratio Geometry (PRG). The Phi Ratio Geometry has been repeatedly demonstrated to feature a ***very high fidelity to nature***, and physics, and also brings a structural inductive process to bear.

Our research experience has typically expended half of the effort on the principal investigations, while the other half has often been consumed in developing tools to support the primary research. Mathematics is probably the most powerful tool in the grip, often providing insights that would otherwise be scarce. Study of PRG, and concurrent development of PRG tools has resulted in a number of insights into cognitive structures, seismically-transparent structures, and other breakthroughs that have novel applications.

Accepted physics is challenged by visionaries who see the universe differently, and those same visionaries offer many solutions to long-standing problems. However, it should be well-understood that we do not seek to trash all contemporary paradigms. Paradigms serve the purpose of supporting collaboration, and common effort among a number of individuals located all over the world. All scientific and technical advances stand on the shoulders of prior art, yet those advances dispute elements of prior art, and provide breakthroughs that revolutionize a discipline that may be considered to have been settled, or that stubbornly refuses to converge.

The models described here are based upon a neo-classical Phi Ratio Geometry, Masonic and Hermetic, extended by recent discovery, and is expressed as a *Structure of Structures.* The archaic origins, and history of the Phi Ratio Geometry are highlighted throughout the text. The Phi Ratio has a number of features that are exhibited throughout the natural world, and these examples will be cited extensively as we review the subject.

In an effort to bring some order to the resurgence of interest in Phi, we also apply some terminology to PRG constructs, thereby eliminating confusions that exist in the popular names for things. The use of Latin monikers is intended to continue taxonomic traditions that have a long history in scientific disciplines, along with the mystique, and reverence for ancient PRG origins.

In the section titled ***Foundations*** we briefly review the basics, and ontology of Phi Ratio Geometry. One of our surprising discoveries is that ***the Geometry is not entirely about Phi***.

The ***Tools*** section discusses the classic implementations of tools throughout the millennia, along with our notions of the functionality that we might expect of an automated PRG Computer-Assisted Design package.

In ***Explorations*** we diverge from the usual Golden Mean / PRG treatment to drill down beneath the popular views of the subject, to discover the building blocks that lurk in the fine-grained structure of the Geometry. We also explore possible new metrology for measuring the universe.

To obtain an introduction to the ***application*** of Phi Ratio Geometry to the many ambitions of contemporary society, please consult a companion document, *Phenomena and Applications of Phi Ratio Geometry*. Said volume should be considered to be a relief valve, so that this volume remains manageable, and focused.

We hope that you enjoy the fresh perspectives offered in this volume, a ***Structure of Structures***.

Foundations

Phi is nature's very unique constant. Although it has a value as a ratio, it is not an absolute. The Phi Ratio Geometry has been revered as both an Ancient, and Sacred Practice. Stellar minds like Galileo and DaVinci put it to practical, and aesthetic use as the Golden Ratio, or the Golden Mean in art, architecture, and science. The Greeks built the Parthenon; the Western Europeans, the great Cathedrals and Domas; and in antiquarian history, someone built the unrivaled complex at Giza.

Nature exhibits the Phi Ratio in every existential aspect. After centuries of general neglect of geometry[2], and the dominant use of geometry to expose our youth to methods of mathematical proof, there is today a re-emergence of interest in the Phi Ratio Geometry[3] as a heritage, and a practical art that has a high fidelity to nature.

At the nexus of the geometry is a feature that causes the Phi Ratio to be ***unique*** *in all of nature*. The Phi Ratio is at the congruence, the intersection, of the linear universe, and the non-linear universe. In addition to the **exponential progression** of a radix, any power of Phi is also the **sum** of any two consecutive powers of Phi – the linear and the non-linear in perfect alignment. This addition of consecutive terms to obtain the next term is more widely familiar as the Fibonacci Series than is the Phi Ratio, and often Fibonacci is incorrectly credited for the doings of Phi.

Some practitioners have advanced beyond mere echolalia, working through the mechanics, and rediscovering the core of the discipline. Others are intrigued by the Fibonacci number series, yet their enthusiasm often stops short of pursuing the foundational Phi art. After tutelage in the art of PRG under Professor Frederick Oliver Mills, the author spent years applying rigor, and thoroughness to the topic, and has discovered more than a dozen attributes of the Phi Ratio. For example, the Phi Ratio is persistent, and pervasive – and Life exhibits signatures of foundational Phi Ratio Geometry. This volume emphasizes a distinction, one of many that will be exposited that the Phi Ratio is not the fractal index that founds the universe, but is instead, a signature of the universe's structural validity.

A common mistake made by many is to reflexively expand, or contract a something by Phi, or the Phi Reciprocal. This is often used as a magical incantation that somehow brings "goodness" to an otherwise ordinary something. Instead, it is essential that a natural object is normalized to one, followed by a search for features at Phi, and 1/Phi boundaries. Phi is better used for reverse engineering something in order to find imbedded, intrinsic relationships. This will be seen as a new distinction, for while the pervasiveness of the Phi Ratio is an indicator, and not a

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[2] R. Buckminster Fuller, who also lamented the misplaced dominance of orthonormal thinking, in *Synergetics I*.

[3] The basic mechanics of Phi Ratio Geometry are simple, and easily understood. There are many books on the topic, and a general introduction to the subject has been written by Priya Hemenway, *Divine Proportion: PHI In Art, Nature, and Science*. Her book is recommended because of a solid execution, and accuracy. Too many recent authors rush to print, and the results have technical errors that only confuse the novice.

building block, the *corpus* of the Phi Ratio Geometry will likely be found to provide those requisite building blocks, and are attributable to irrationals such as √2, √3, and √5, and at least two others, the √1 and √4 that are not irrational, and are the familiar integers 1 and 2.

The expression of these many irrationals is a clarion that indicates that perhaps the body of contemporary mathematics, of which modern man is rightfully proud, in many ways fails to hold fidelity to the natural phenomena that mathematics is intended, and claims to describe. The resulting design doctrine will also be seen as existential proof of conjectures that are most definitely *not* tricks of light and shadow.

For proper operation the Phi Ratio Geometry requires humility. Those whose egos already hold all of the answers, or whose belief in the certitude of their education, or their cultural biases will be less successful in their application of the Geometry than those who are observant. The Geometry reveals itself if one is receptive to the constancy, and consistency of the universe.

We are among a growing number of afficienados who are studying the Phi Ratio Geometry, and we approach the topic from scientifically rigorous, mystical, historical, and practical perspectives. The Geometry is beginning to solve long-standing problems in Physics, Cosmology, Architecture and Cognition, and doing so with ease.

This volume offers attributes that the author has prized free of the Geometry that are intrinsic to it, and are largely unknown, both to the Ancients, and contemporary practitioners.

We will soon be developing Computer-Based Tools based upon these findings, to support research in Machine Cognition, and Seismic Transparency.

This volume is a fusion of documents that included Foundations, Tools, Scaffolds, Pentads, Self-Organization, and more. They were suffering from common, introductory material, sprinkled with diverging topics in some cases, and a lot of confusion throughout. The result is ***Structura Structura***, which attempts to unify all of the material, reduce redundancy, and make the material self-consistent. If successful, the material will result in a book that will never be complete, and will endure many revisions as others contribute to the craft, the science, the art, and the mathematics of the Phi Ratio Geometry. PRG is already contributing to the science of cognition, earthquake-proof structures, and other disruptive technologies.

Introduction to the Phi Ratio Geometry

Some computer science terminology is pressed into our service in order to provide a linguistic structure to the topic. A powerful concept extracted from the study of Ancient and Sacred Geometry is recursion. Recursion is similar to iteration in that it employs repetition. Iteration repeats a process without coupling the instances of iteration.

In contrast, recursion uses the structure created by one instance of recursion as the basis for the next instance of the process. Essentially, recursion conveys state data between invocations and iteration does not.

Examples which support the distinction

Iteration:

```
For i = 1, 10, i++ do
        {printf(x);};
```

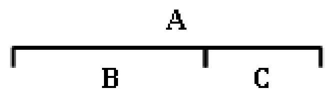
Recursion:

```
Defn: Recurr(x,y)
        While (x =< y) do
                Printf(x)
                x := x + 1
                Recurr(x,y);
. .
Recurr(1,10);
```

It is clear that the loop counter in the first example only determines the termination of the loop and does not influence the state of the data, *ergo*, it is iteration. It is also clear that recursion carries state information from one instance (context) to the next.

Deriving Phi

The fundamental relationship that is Phi is a simple linear ratio, depicted in Figure 1. The ratio of A to B is the same as that of B to C.



$$A = B + C$$

$$A = 1.618033988749$$
$$B = 1.0$$
$$C = 0.618033988749$$

$$A/B = B/C$$
$$1.6180339/1 = 1/0.6180339$$
$$1.6180339 = 1.6180339$$

Figure 1 Phi Ratio Relationships, A is to B as B is to C

To apply Phi it is sufficient to normalize an object (assign an alias of one), and thereafter use this new Phi basis for describing additional dimensions of the object. This step ignores the absolute dimensional metrics that may otherwise be applied to the object. Thereafter, other features of a domain will be discovered to have Phi (1.6180), and Inverse Phi (0.6180) relationships to the object now deemed to have the value of one. The normalization step may be invoked at any progression of Phi Ratio scale, whether increasing the length by 1.618…, or when diminishing the scale by 0.618…

At the nexus of the geometry is a feature to which we originally referred when we asserted Phi's *uniqueness in nature*. The Phi Ratio is to be found anywhere that the linear, and the non-linear must co-exist. At the cornerstone of Phi discourse is that, in addition to the **multiplicative**, or **exponential progression** of a radix, *any power of Phi is also the* **sum** *of any two consecutive powers of Phi* – the linear and the non-linear in perfect alignment.

In terms of Mathematical Induction, Phi is recursively defined:

$\Phi^{(0)}$ *is 1 [base case]*

$\Phi^{(1)}$ *is* Φ *[base case]*

For all integers n > 1: $\Phi^{(n)}$ is $(\Phi^{(n-1)} + \Phi^{(n-2)})$ [recursive case].

Equation 2

The irrational nature of the Phi Ratio should not deter one from embracing it. While Phi does exhibit an infinity of positions to the right of the decimal, this is a feature of our base-10 metrology. This is an artifact of the decimal system that is the basis of our systems of measurement, and not a flaw of PRG. There are an number of indicators of the discordance between nature, and the decimal radix-based, symbolic, analytical mathematics that we use to describe nature. However, we will demonstrate that the square roots and Phi can be rationalized to crisp, clean and succinct counting numbers. Key values of the Phi Ratio Geometry are 1, Phi, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, $\sqrt{5}$, and Pi, and we will make the case that these seemingly indigestible (irrational) numbers have a utility every bit as valid as integers.

Table 1 Exponential and Additive Series of Phi are Identical in Result

Exponential Progression			Arithmetic Progression
Φ^0	0	1.000000	
			+ 2.618034
Φ^1	1	1.618034	
			+ 4.236068
Φ^2	2	2.618034	
			+ 6.854102
Φ^3	3	4.236068	
			+ 11.090170
Φ^4	4	6.854102	
			+ 17.944272
Φ^5	5	11.090170	
			+ 29.034442
Φ^6	6	17.944272	
			+ 46.978714
Φ^7	7	29.034442	
	etc.,	*etc.,*	

Phi Ratio analytical computational complexity is seldom any more difficult than its *genus* quadratic equations, or the Pythagorean Theorem. The remaining rigor is generally found in the mechanics of structured drawing, *e.g*. Figure 2, a new variation of graphic derivation of Phi Ratio that concurrently yields Phi, and its reciprocal with minimal effort.

This depiction has a higher fidelity to the Phi Ratio Quadratic Equation than other, existing procedures, and is easier to understand.

Hypotenuse of a double-square

$$\sqrt{2^2 + 1^2} = \sqrt{5} = 2.23606$$

$$\Phi \text{ (Phi)} \quad \frac{1 + \sqrt{5}}{2} = 1.618033$$

$$1/\Phi \quad \frac{1 - \sqrt{5}}{2} = 0.618033$$

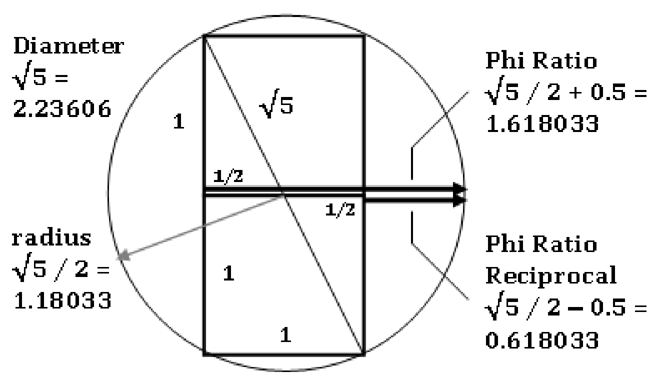


Figure 2 Deriving the Phi Ratio as a Quadratic Equation

Phi Ratio as Integer

The Fibonacci Series is identical in form to the Phi Ratio additive series cited in Equation 2.

> ***F*** *(0) is 0 [base case]*
>
> ***F*** *(1) is 1 [base case]*

For all integers n > 1: ***F*** (n) is (***F*** (n-1) + ***F*** (n-2)) [recursive case]

Equation 3

The first several terms of the Fibonacci Series are 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, . . . The series converges to the Phi Ratio (1.618033989) to three decimal places by the ***F***(12) term, where 89 / 55 yields 1.618 and the last two digits repeat to infinity. As additive series are wont to do, the convergence continues to tighten up, yet is ever-elusive, forever oscillating about Phi.

Distinctions

Any course of learning, whether formally in the classroom, or informally in our daily lives, is based principally on drawing distinctions, and classifying things, and phenomena based upon sometimes very subtle differences. There are also many distinctions to be found among the elements that constitute the Geometry.

Most treatments of Phi Ratio, Platonic Solids Geometry and the Hermetic Sciences are somewhat superficial and deal principally with the mechanics of the subject. The history of the subject is intriguing and the deliberate efforts to occult, or hide, Phi Ratio art have been successful at keeping it from the mainstream of mathematics and therefore, physics, engineering, architecture and life sciences. This has the unfortunate effect of slowing the rate that progress could be obtained in scientific, engineering, and social development.

Phi Ratio Geometry has many attributes, some of which are not so obvious until the investigator has spent many hours poking at it, or interacting with a tutor. Other, second-order features of the geometry are subtle, requiring an extended effort to tweeze them from the rich fabric that is the geometry.

It is common knowledge that the Phi Ratio is irrational, since its enumeration in the decimal number radix results in infinite decimal fractions. Less well known is that the Phi Ratio is NOT a transcendental like Pi (π), since it is the result of a simple quadratic.

Further study results in the awareness that Phi Ratio Geometry is:

Recursive,

Persistent,

Pervasive,

Relative,

Scalable,

Symmetric,

Self-similar,

Self-referential,

Fractal,

Mesoscopic,

Organic, and

Inductive.

Many of these attributes are very similar to one another, and while subtle, these nuances are essential to one's use of the Geometry as design and analysis tools. This fact is key: Phi Ratio Geometry is recursive, persistent and pervasive. All else derives from these.

Recursive

The Phi Ratio Geometry is ***recursive***, in that all expressions may be found to be more than merely iterative, but in fact, depend upon prior states to construct the next larger, or smaller, examples of itself. This property is fundamental to all other properties and once understood, can be anticipated and discovered throughout the universe.

Although the concept of recursion is derived from computer science, nature holds the senior claim – to use the term self-similar to describe the result of recursion seems too modest a descriptor for the powerful, persistent and pervasive process of recursion. Recursion is often confused with iteration, where the principal distinction is that recursion builds upon the prior state, and iteration can be entirely stateless. Iteration is simple repetition at a single scale without regard to the state of the prior iteration instance.

The distinction that offers value to the philosopher, scientist, engineer or architect is that repetitive application of Phi rigors leads the practitioner to anticipate another recursion, an inductive step that further explicates the universe. This effect has already lead to discoveries and re-discoveries.

Persistent

The Phi Ratio Geometry is ***persistent***, because it surfaces again, and again, at new scales of structure, phenomena, and contexts. It can also be said to endure, because geometry outlasts other organizational structures. For example, when other artistic and architectural expressions of ancient societies are eroded by time, political forces and weather, fundamental Phi relationships persist in the stones and ashlars which constitute megalithic structures.

Pervasive

The Phi Ratio Geometry is ***pervasive***, and can be found to be an *imprimatur signature* that confirms ubiquitously that natural processes, and constructs have been properly assembled – or emerged in new contexts, sometimes where they would not be expected. In the search for life beyond Earth, it is initially sufficient to identify the presence of the pentagonal branch of Phi.

As the universe cools, the fine-grained structure continues to express the geometry everywhere and at all scales. A phenomenon is observed and assumed to operate identically across the universe as it does in our vicinity.

Relative

The Phi Ratio Geometry is ***relative***, since as a ratio, it is ***not absolute***. This is the reason that Phi cannot be the Fractal basis of the Universe, in spite of the fact that it is so persistent and pervasive in the animal, vegetable, and mineral domains. Whatever the fractal basis of the universe, it could change overnight, and yet because Phi is a ratio, the universe can expand and contract without breaking itself, or our perceptions of it.

In an effort to fit the Phi Ratio as the Fractal Basis of the Universe, it became clear that Phi could not serve in such a role. The issue was parked on the back burner until the findings of the square root foundations of the PRG emerged.

Scalable

The Phi Ratio Geometry is ***scalable***, and at any scale, a construct may be renormalized to one, yet still retain the available multiples both larger, by Phi, and smaller, by Phi's reciprocal. The Geometry is all about scaling. Phi Ratio Geometry is scalable in that it can be applied to any metric and can be *normalized* to any element of the universe or abstraction that one cares to apply it to. A line may also be *remeasured* by *cutting the line* or in ancient practices, *crossing the lights*, where Phi (1.6180…) or Phi reciprocals (0.6180…) are cast as line segments.

This normalization is possible precisely because Phi is a ratio, and not an absolute. Normalization can be said to occur continuously. A tidy example is the Fibonacci Series. While early ratios of successive members sport a ratio of ½, 3/5, 5/8, *etc*., the sequence very rapidly converges near Phi Ratio, to three decimal places by F11/F10 (55/34).

Ergo, at any time a Fibonacci number can be normalized to 1, and the next Fibonacci Number can be thought of as an integer equivalent of 1 * Phi with a reasonable accuracy[4].

EXPLORE: Planck's constant is a limit to scaling[5].

Symmetric

The Phi Ratio Geometry is ***symmetric***, and exhibits a number of bilateral, and rotational symmetries that are simply demonstrated. It is clear that symmetry drives nature everywhere. The symmetry is in the reciprocal nature of phi, and 1/phi, driving Phi's recursive engine.

Self-similar

The Phi Ratio Geometry is ***self-similar***, and in the manner of the Mandlebröt set, exhibits examples that usually differ slightly from those at other scales, and are seldom exact copies. This has lead to the ancient maxim, "as above, so below," an observation that failed to obtain a name until now, when the term recursion was borrowed from the lexicon of computing to describe the phenomenon.

Self-similarity can also be seen to include the ordinal number of an object. For example, the structures of organisms exhibit five (5) at many scales, which contrasts to the cubic, hexagonal and tetrahedral structures that dominate inorganic chemistry and geology. Examples include granite, which has a brittleness that follows cubic structure at many scales, and in organic structure pentagrams, and stellates that are ubiquitous at numerous scales. DaVinci's Vitruvian Man is a classic example of the pentagonal form.

Self-Referential

The Phi Ratio Geometry is ***self-referential***, in that it points back to itself, sometimes through symmetry, and other times through repetitive instantiation, invoking or echoing itself. The constructs of the geometry can often be seen to express themselves repeatedly as the scales of

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[4] By the way, observe that the value of Phi-squared (Φ^2 = 2.6180337898) is a cozy neighbor of *e* (2.71828).

[5] Fractals, Chaos, Power Laws: Minutes from an Infinite Paradise, Manfred Schroeder, pg. 2.

recursion increase and decrease.[6] It often occurs in complex PRG structures that signature Phi devices are found, much as the fractal Mandelbrot "blot" recurs.

Fractal

The Phi Ratio Geometry is ***fractal***, in that it is expressed at many levels of scale in many nonlinear constructs. Because Phi Ratio Geometry is recursively scalable, it offers the appearance of having a fractal basis. It is a unique number in this universe, a radix where linear intersects with the non-linear. The geometric scaling is a familiar one, where successive increments of the exponent of a radix,

$$\mathbf{r \times r^{n} = r^{n+1}}$$

generates the next order-of-magnitude of scale.

The very powerful distinction unique to Phi is that the very same power sequence may be derived recursively by expansion through successive addition of pairs of orders-of-magnitude to derive the next,

$$\mathbf{r^{n+2} = r^{n} + r^{n+1}}$$

An integer version of this progression is the Fibonacci series,

$$F = 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, \ . \ . \ .$$

which is developed by adding two successive numbers to derive the next. The Fibonacci Series converges to Phi rapidly, to a precision of on decimal place by terms F7 (8) and F6 (5), two decimal places in the ratio of terms F9 (21) and F8 (13), and three by F14 (233) and F13 (144).

Mesoscopic

The Phi Ratio Geometry is ***mesoscopic***, and natural phenomena can be seen to emerge and extinguish as scales of Phi are expanded or contracted, and other phenomena begin to dominate, in conjunction with shifting energy content. In the physical universe phenomena occur at specific scales and energy states (temperature), and mesoscopic behaviors engage and dominate a domain of several adjacent orders of magnitude, or scale.

Phenomena are bounded for plasma, gasses, liquids, solids, and cryogenic solids. Even the celebrated Albert Einstein considered that the Gravitational Constant may not be indefinitely constant. Our view of mesoscalar effects would suggest that a shift of the gravitational coefficient is more responsible for the observed acceleration of the expanding universe than the imagined dark matter, dark energy or dark particle, which dissipate scarce research funding.

There are phenomena which occur in three dimensions where a protein, for example, ***snaps to an invisible grid*** which has alignments resulting from the existing molecular structure – and does so seemingly uphill to the total bonding energies of the attachment.

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[6] While not making any claim that the Mandelbrot set is a Phi Ratio creature, it expresses self-similarity at several scales.

Organic

The Phi Ratio Geometry is ***organic***, in that the tenacity frequently attributed to organic life may actually result from the persistence, pervasiveness, and symmetry of Phi Ratio Geometry, and tuned, amplified, or constrained by several of the previously-cited attributes. These features may actually provide better strategies in the search for evidence of extraterrestrial intelligence than the radio monitoring of the SETI program.

Inductive

The Phi Ratio Geometry is ***inductive***, a "process" that occurs during the course of devising constructs while using PRG methods insights are intuited. These productions are enhanced when PRG scaffolds are available.

Classical Phi Ratio Structures

These definitions will provide needed clarity and succinct taxonomy to the many constructions of the Phi Ratio Geometry, particularly when we apply them to real-world applications.

The Phi Cube

The Vesica Pisces

There are an infinitude of possible *vesica*, since intersecting circles are as divisible and continuous as the number line. However, there is by definition, only one *Vesica Piscis*, the plural form is *Vesica Pisces*, and this cardial form is distinguished from all the rest in that two (identical) circles with the same radius intersect so that each has a point on its perimeter (tangent) that crosses through the center of its companion circle.

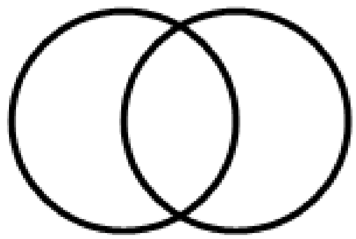


Figure 3 The Vesica Piscis Features Perimeters Tangent to the Centers

The name of this device translates to “bladder of the fish,” where the bladder is seen as the lens-shaped area common to both circles. A symbol emerges that has heritage from early Christian and late Roman eras that is shown below. Most of the heritage and mythology intrinsic to the Phi Ratio Geometry comes to us originally from the Egyptian Freemasons, and was conveyed and adorned by the Graeco-Roman Empires, leading into the Christian, and later, the Knights Templar, and later still, the Masonic Traditions. In these mythologies, and methodologies, the *Vesica Piscis* is the “*mother of creation*,” and in the mythological view, the lens is the birth channel, and imagination fills in the rest.

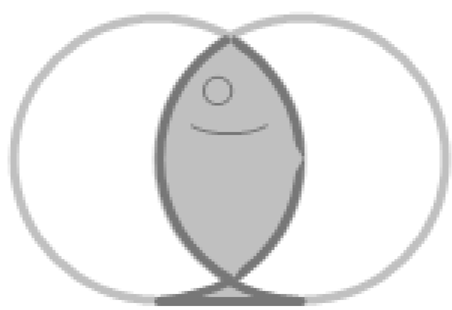


Figure 4 The *Vesica Piscis* has Ancient Heritage

In practical terms, the *Vesica Piscis* is a simple, circular grid that is extensible in a very dramatic way, and can be employed to devise two, three, and four dimensional objects. We know of no efforts to take this any further, but we know from our visit with mathematical induction that this is probably unbounded, and because of the inexorably recursive nature of the Phi Ratio Geometry we can further infer that time is most decidedly not the fourth dimension.

The *Vesica* is the progenitor that derives geometric objects, transcendental and irrational numbers with ease. These values are simply derived from the *Vesica*, where many numbers are irrational, and a few are transcendental, such as Pi. The Phi Ratio results from a quadratic equation, and is therefore, not a transcendental number.

Two dimensional shapes, or objects that may be quickly derived from the foundation of the *Vesica Piscis* include the triangle, square, rectangle, pentagon, hexagon, heptagon, *etc*. The entire family of three dimensional Platonic solids that include the Tetrahedron, Hexahedron (cube), Octahedron, Dodecahedron, and Icosahedron, can be assembled on the framework of the *Vesica Piscis*. Other, more complex polyhedra can similarly be constructed.

Squaring the Circle

Contrary to the opinion of some mainstream mathematicians, the poser as to whether a square of area equivalent to a given circle may be derived geometrically, has straightforward proof that is internally consistent.

The *Vesica Piscis* provides a scaffolding, which is generous with grid points, and the *Cruxis* variant provides a square whose area is identical to a circle in a straightforward manner. The VP Cruxis features four prominent vesica that have √3 chords (major axes), and four minor vesica on the diagonals that feature √2 major axes.

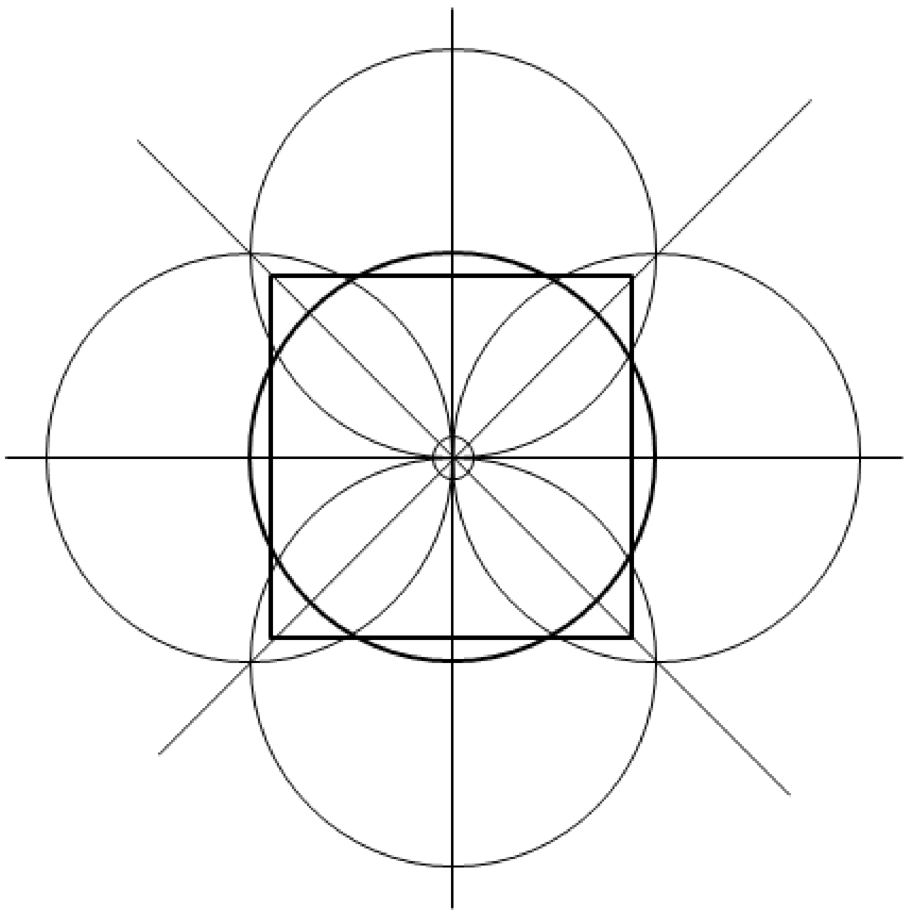


Figure 5 Squaring the Circle

The Area of the reference circle with radius one is πr^2 , as is also the area of the objective square. It follows, therefore, that the length of the side of the objective square is $\sqrt{\pi r^2}$, or 1.772 units.

Imbedded Equilateral Triangles

With the *Vesica Piscis* as a scaffolding, and especially with a supporting grid, isosceles, and equilateral triangles are simple to fabricate. These triangles are also simple to measure, particularly because these triangles often have intrinsic values that are the result of the scaffolding. In Figure 6 an interior (imbedded) triangle is shown to be equilateral, with unit one sides identical to the radius of the reference circle.

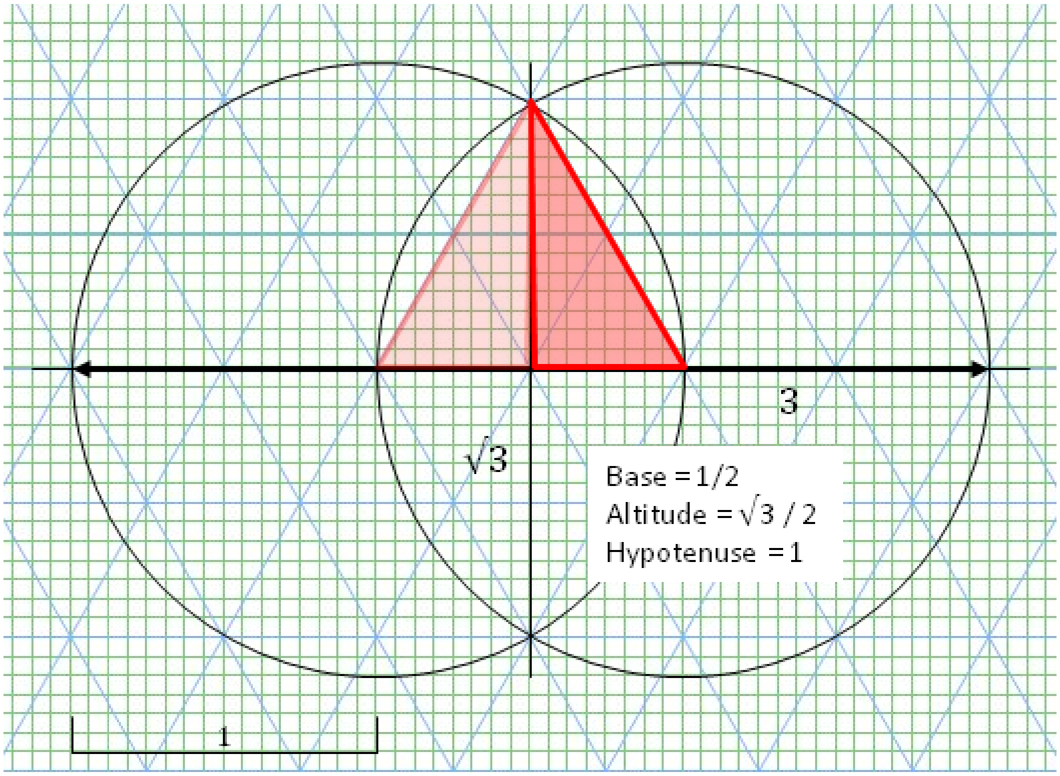


Figure 6 The Isosceles Triangle is Bisected into Symmetric Equilateral Triangles

The intersections of the interior hexagon with the reference circle mark also intersect the four perimeter edges (sides) of the objective square. The sides of the triangles can be confirmed to be of length one by sweeping out the radii with a compass.

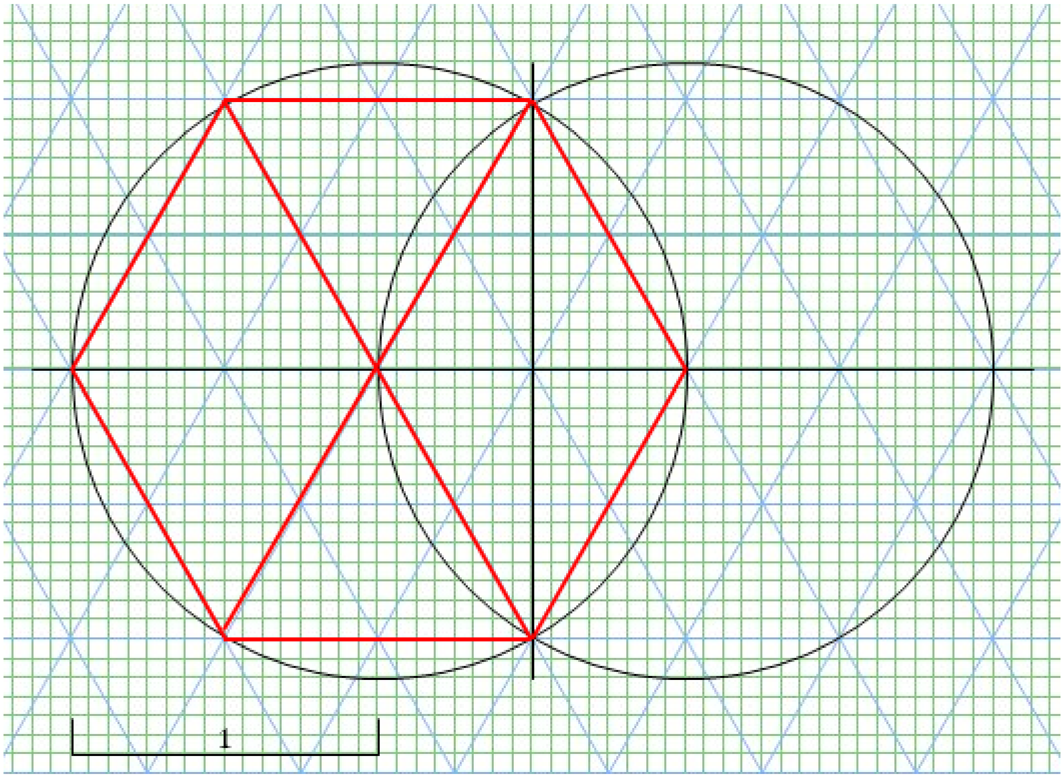


Figure 7 Hexagons are an Extension of Isosceles Triangles Imbedded in the *Vesica Piscis*

The Pythagorean 3-4-5 Triangle

The Pythagorean 3-4-5 Triangle has its origins among the tools of early Freemasonic traditions. An elegant, yet simple geometric proof is depicted in Figure 8, below. The squares of each of the respective sides can be compared so that Pythagoras' equation can be readily confirmed.

Equation 4

The base of 3, and altitude of 4, when squared, and summed are equal to the square of the hypotenuse of 5.

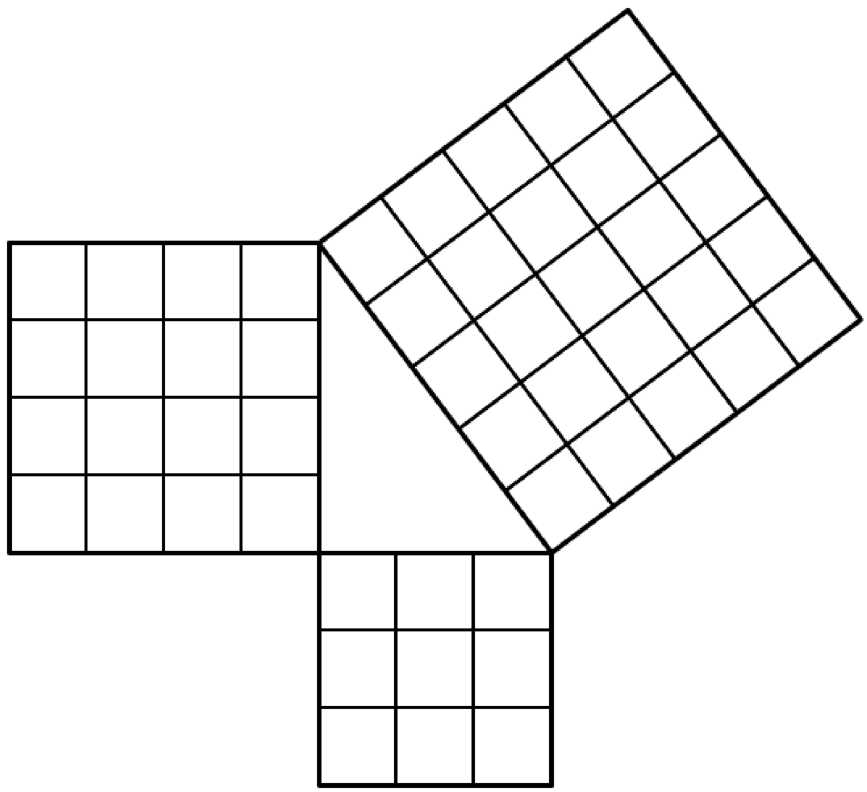


Figure 8 The Classic Pythagorean 3-4-5 Triangle

Spira

There are a number of spira, or spirals, to be found in the PRG, and adjacent popular culture. These include the Spiral of Theodorus, the *Spira Mirabilis*, the Fibonacci Spiral, the Phi Spiral, and more – and we will work our way through this menagerie of species in order to clarify the topic.

The Spiral of Theodorus

The square root three of the *Vesica Piscis'* long axis, and the imbedded square roots of two, and five of the stacked unit squares presaged the square roots that form the Theodorus Spiral. This device hails from ancient Greece, and is resplendent in roots, as can be seen in Figure 9, below.

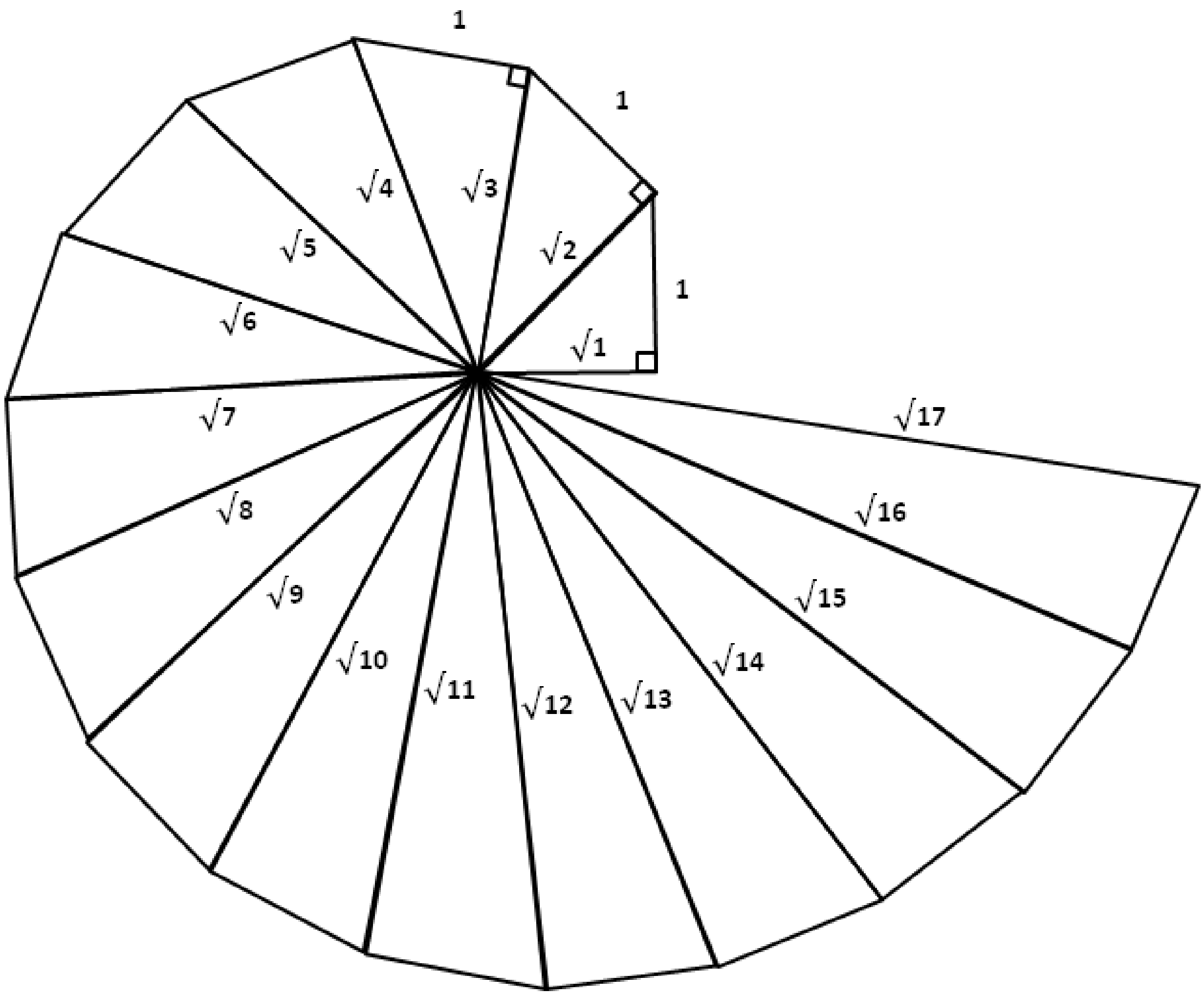


Figure 9 The Theodorus Spira

Fibonacci Spiral

The spiral shown in Figure 10 is built upon an arrangement of Squares of Fibonacci Series numbers, and the observant eye will see the 1, 1, 2, 3, 5, *f(n)* values of the interior sides of squares that are superimposed on the grid. This spiral is not a perfect Phi Ratio spiral, but is sufficiently close for most applications, and features the advantage of being easy to construct.

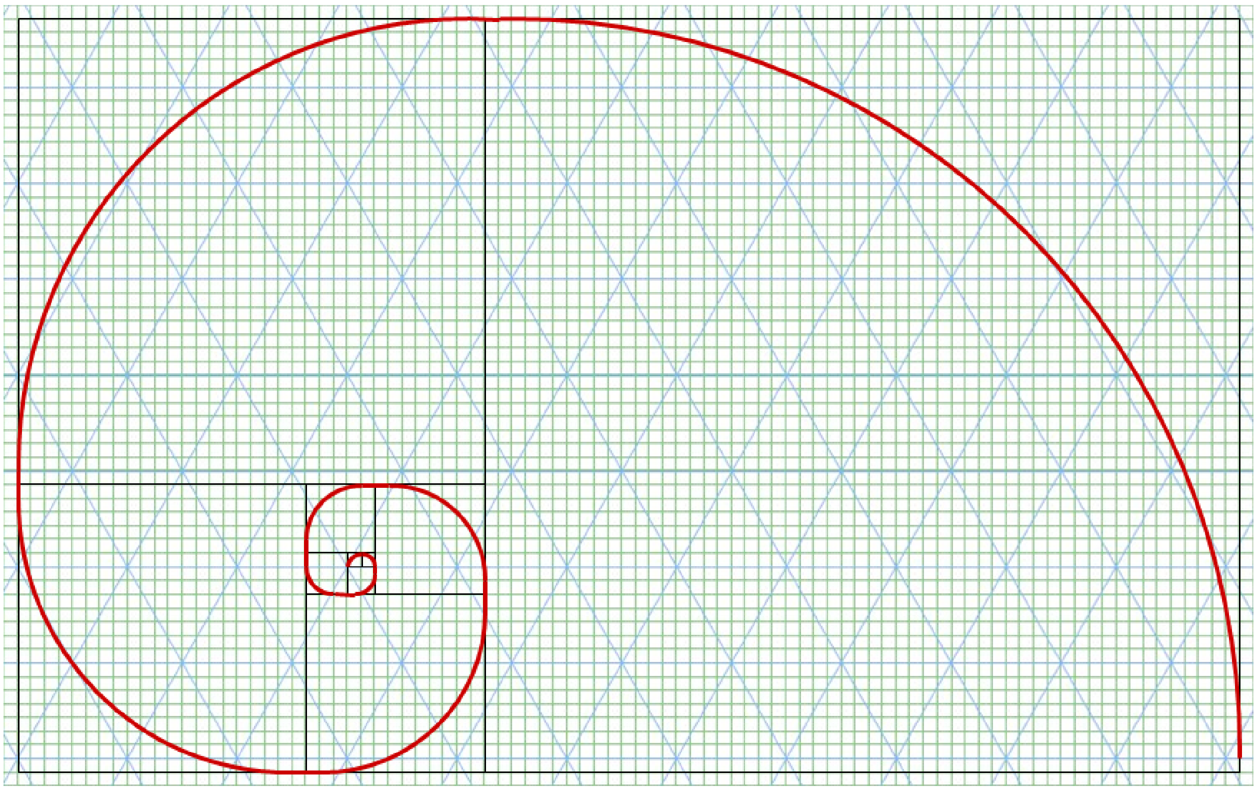
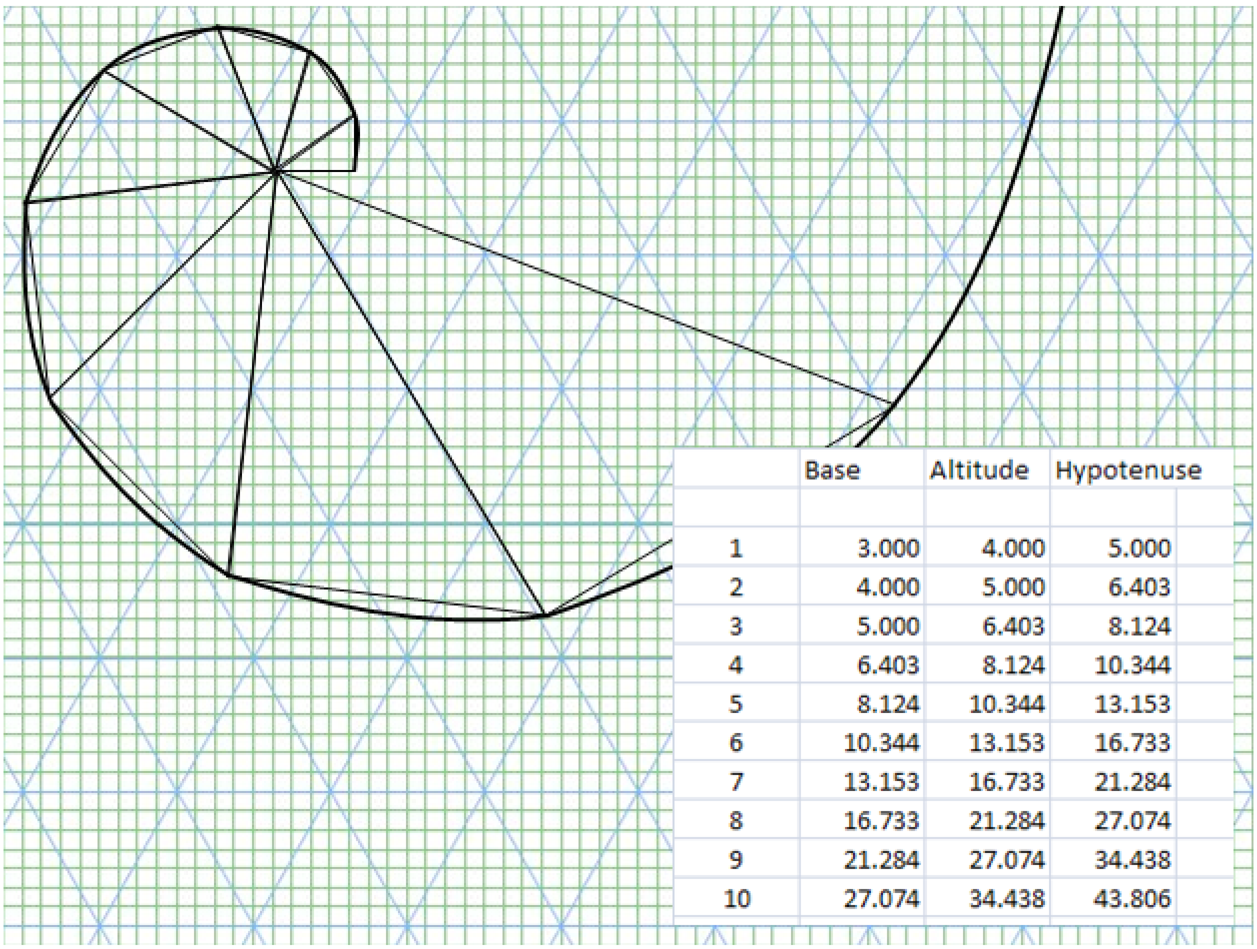


Figure 10 The Fibonacci Spiral is Constructed from Fibonacci Integers

Spira Mirabilis

In the concurrent courses of cognition development and Phi Ratio Geometry rediscovery, the author has found considerable confusion in terminology. The term *Spira Mirabilis* is ambiguously used to describe both the discrete Phi Ratio, or Fibonacci Spiral, and the macro-structure of the contravailing, or entwined spirals. In order to bring order to the domain, the curve depicted in Figure 11, below reserves the original designation as *Spira Mirabilis*.

Soon an original 3-4-5 triangle will be described as a √1√2√3, and the entire sequence of triangles can be seen to form a classic √2√3 *Spira Mirabilis*, through a pivot of a progression of Square Roots of Fibonacci integers.



	Base	Altitude	Hypotenuse
1	3.000	4.000	5.000
2	4.000	5.000	6.403
3	5.000	6.403	8.124
4	6.403	8.124	10.344
5	8.124	10.344	13.153
6	10.344	13.153	16.733
7	13.153	16.733	21.284
8	16.733	21.284	27.074
9	21.284	27.074	34.438
10	27.074	34.438	43.806

Figure 11 The *Spira Mirabilis* is a Discrete √2√3 Spiral

The Nexa Spira

Clarity is essential in technical discourse, so another distinction in the lexicon will further differentiate one species from another. Therefore, we continue to designate the *spira*, or spiral, defined by the Phi Ratio as the *Spira Mirabilis*, an appropriate appellation. However, the complex structure that is also frequently called *Spira Mirabilis*, and is also called *Bernoulli*'s Spiral, equi-angular Spiral, *et al*., we identify in these works as *Nexa Spira*, or *entwined spiral*, and literally, the *woven basis*. In the original Roman context, this weave was commonly employed in the base of baskets, and the Romans were precise in their use of language.

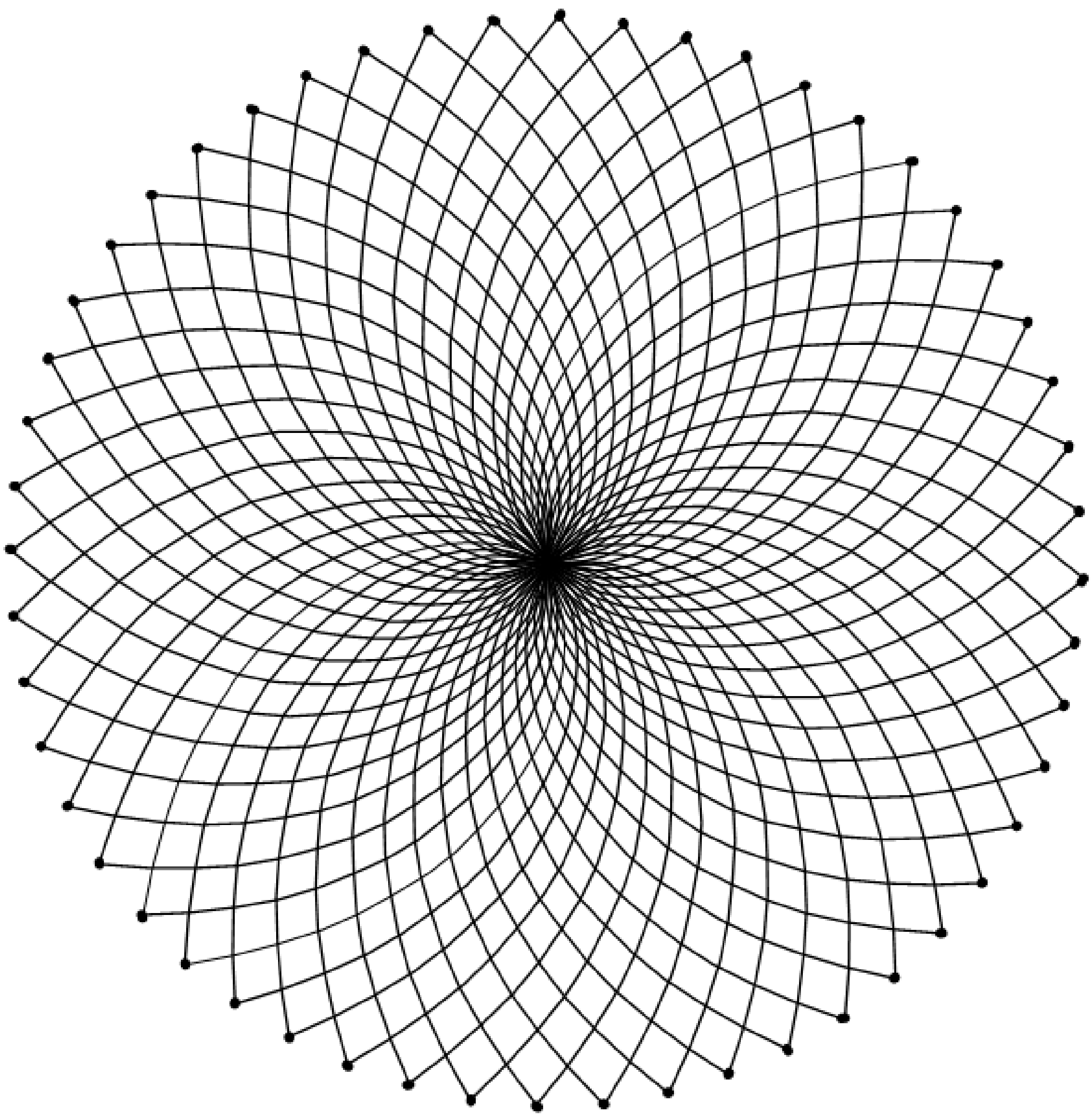


Figure 12 The *Nexa Spira*, or Entwined Spiral is a Non-Linear Grid

SquareRoot(Fibonacci) Spiral

Another discovery is a Spiral that is constructed by pivoting a succession of 3-4-5 triangles, where the base, altitude, and hypotenuse are each specified by Fibonacci integers. This construction is described in more detail in Explorations, page 31.

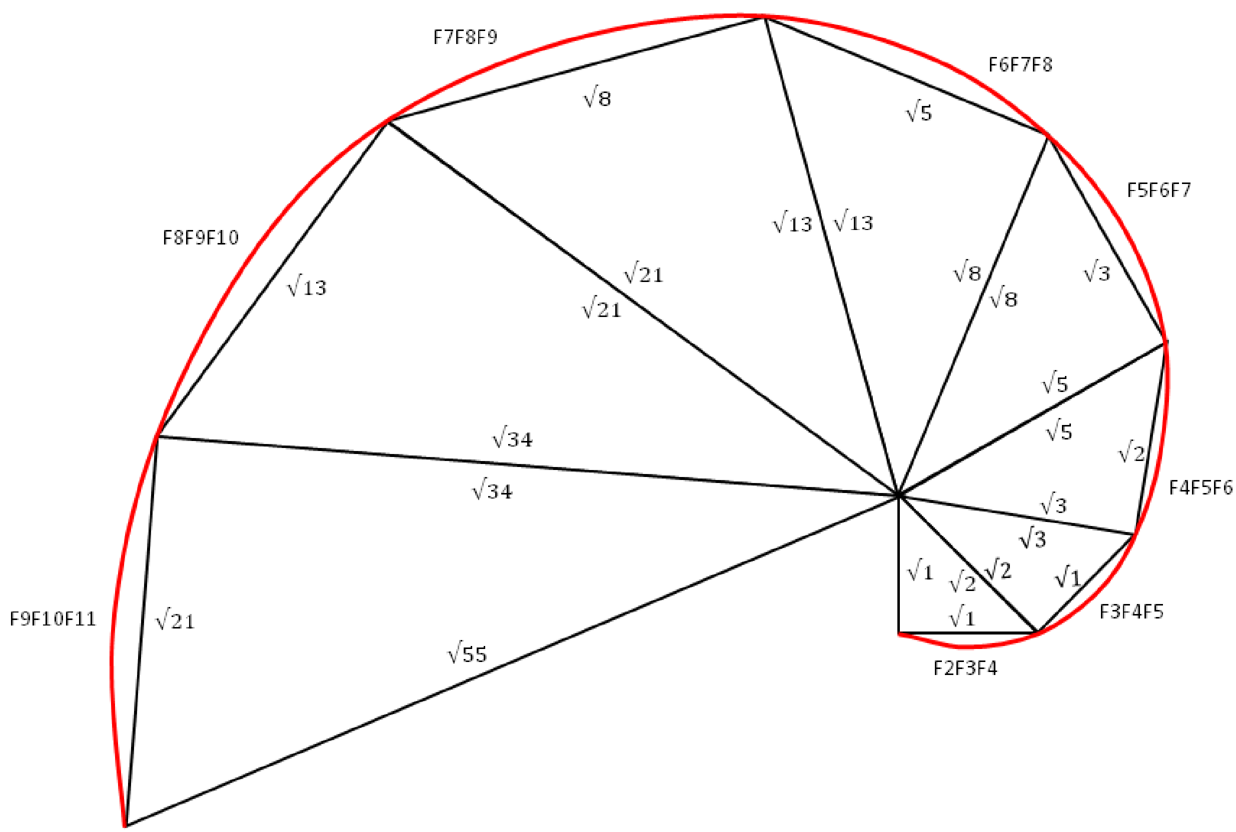


Figure 13 Pivoting a Series of √(Fibonacci) Triangles Generates this Spiral

Circulus Profunditas

The *Circulus Profunditas* is a construct that the author has developed as a two-dimensional yardstick that has proven to have numerous applications, and has also spawned a variety of insights. The *Circulus* is a set of concentric circles that are graduated at Phi Ratio intervals, and as is demonstrated in Figure 14, below, it features an intrinsic vanishing-point perspective. It provides a deep view, hence its name.

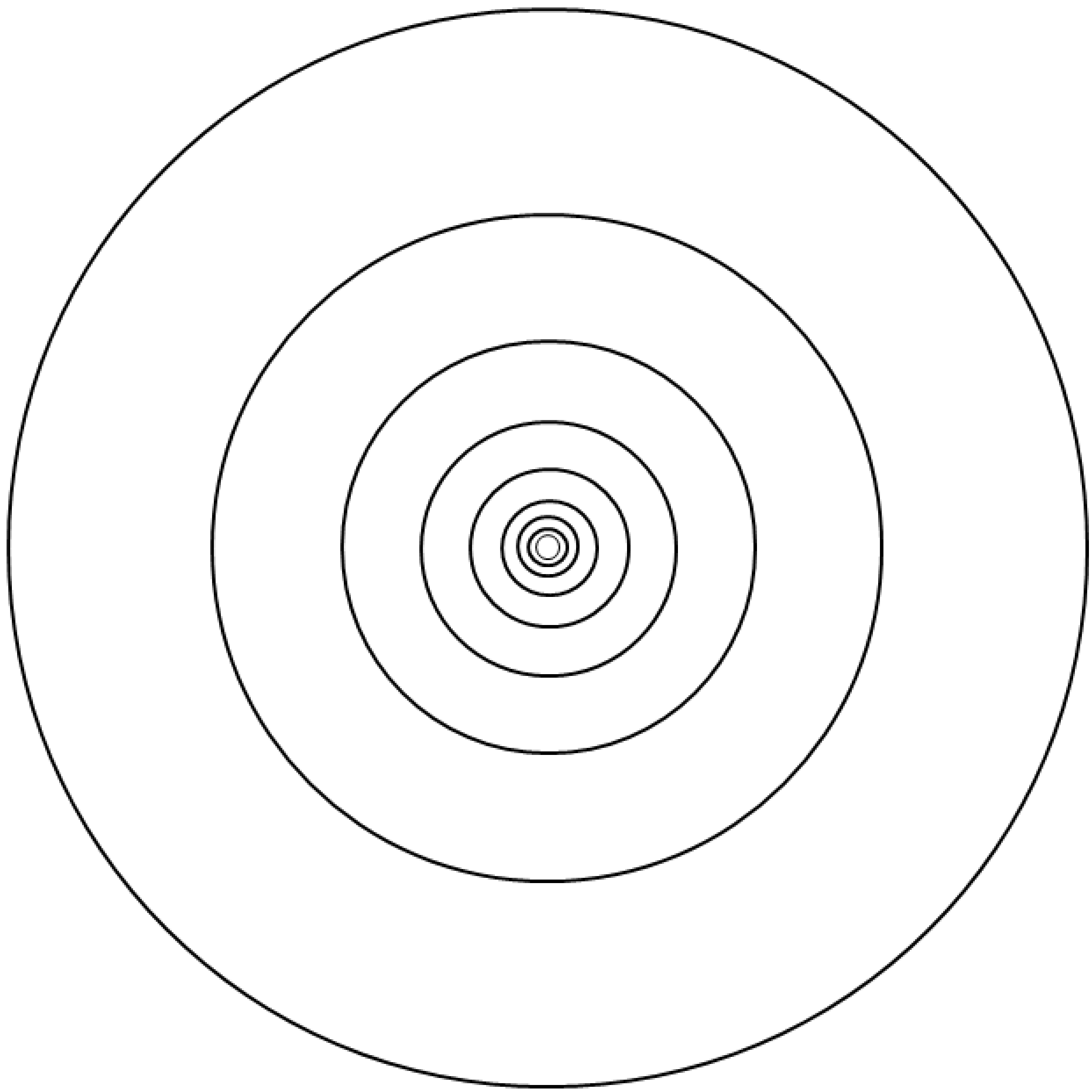


Figure 14 Concentric Circles in Phi Ratio Exhibit Intrinsic Vanishing-Point Perspective

Explorations

A handy tool not taught in the school showed up in routine use in the PRG. In Figure 15 we observe the Pythagorean Theorem applied to the unit square, and then generalized for a square of any size. As shown elsewhere, the PRG assumes that any length can be normalized to one, since Phi is relative, and not absolute.

The last example in the figure extends this notion to the cube, using three sides as terms under the radical.

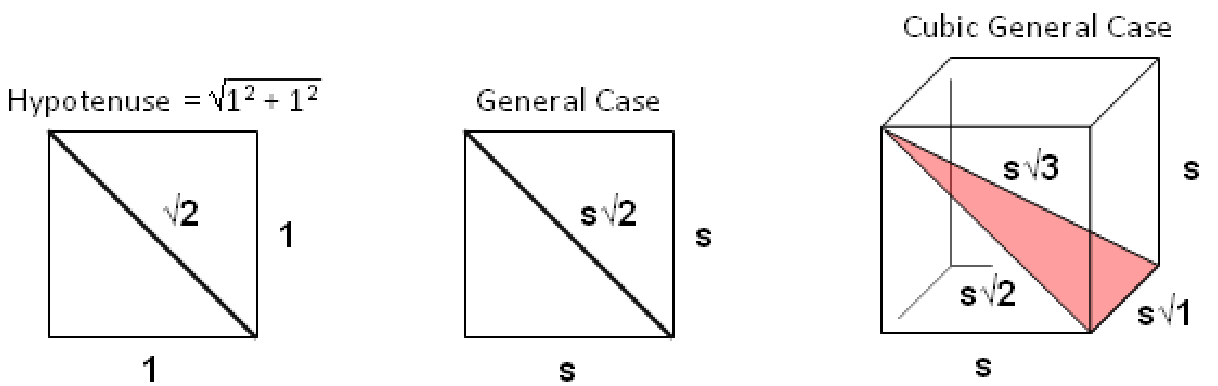


Figure 15 The Reference Hypotenuse for the Square and the Cube

This may seem to be a trivial example, yet even the most trivial situation may become the foundation for a new thread of investigation. We do know from prior work that the interior triangle of the cube that includes the cubic's hypotenuse, and the hypotenuse of one of its facets, is a 3-4-5 Pythagorean Triangle.

The author shared this little curiosity with Professor Mills, and his response was that I was in error, and the triangle was a $\sqrt{2}$, $\sqrt{3}$, and $\sqrt{5}$ triangle. I wasn't wrong, and now armed with two cases, I set forth to determine just how many square roots would satisfy this configuration.

Later application of Mathematical Induction to a now-restricted set of square roots that happened to be roots of the Fibonacci numbers would justify the exercise, per Figure 16.

The Fibonacci Additive Series, like all additive series, converges on Phi, except that it does so faster than others. The Lucas Series are next in this tendency. Another divergent factoid that folds into this discussion is that the 3-4-5 triangle has been frequently observed in nature, particularly in living organisms, and the reason is clarified here.

In Figure 16 the sequence of roots of the triangle's sides are clearly Fibonacci-based, and after cursory examination the altitude, and hypotenuse are seen to follow, with offsets. It may also be observed that the series rotates around the triangle in a counter-clockwise fashion.

The feature suggests a variation on the Theodorus Spiral, Figure 9, page 25, which is depicted in Table 2, below. All of these findings owe their genesis to the simple √2, and √3 "rules," previously cited in Figure 15.

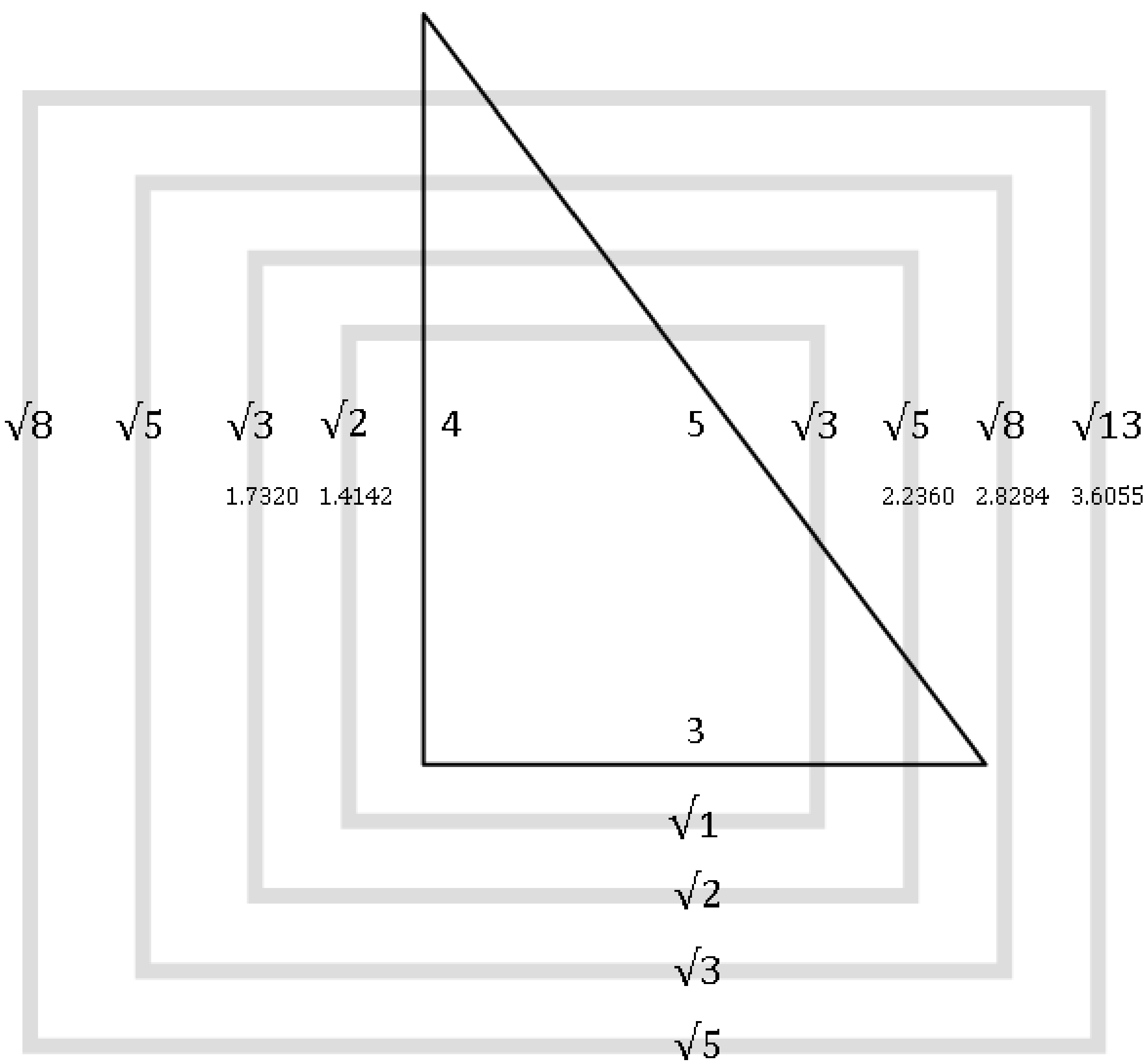
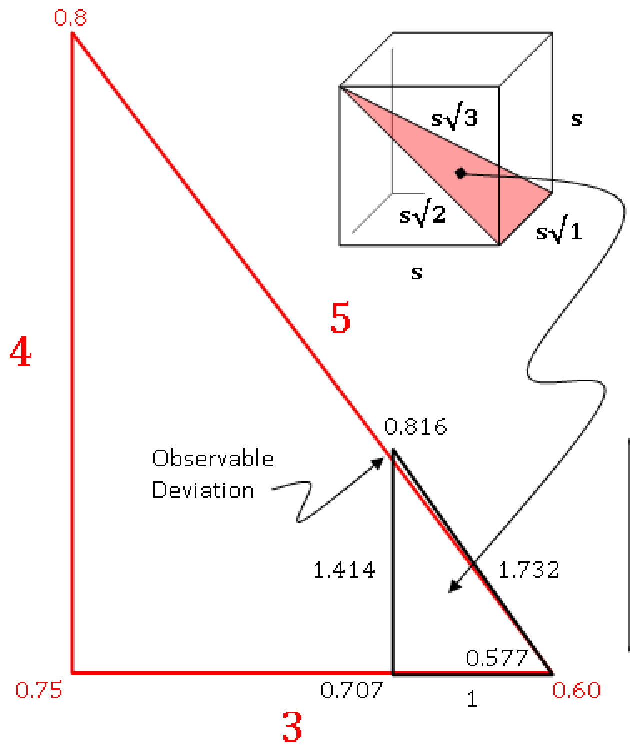
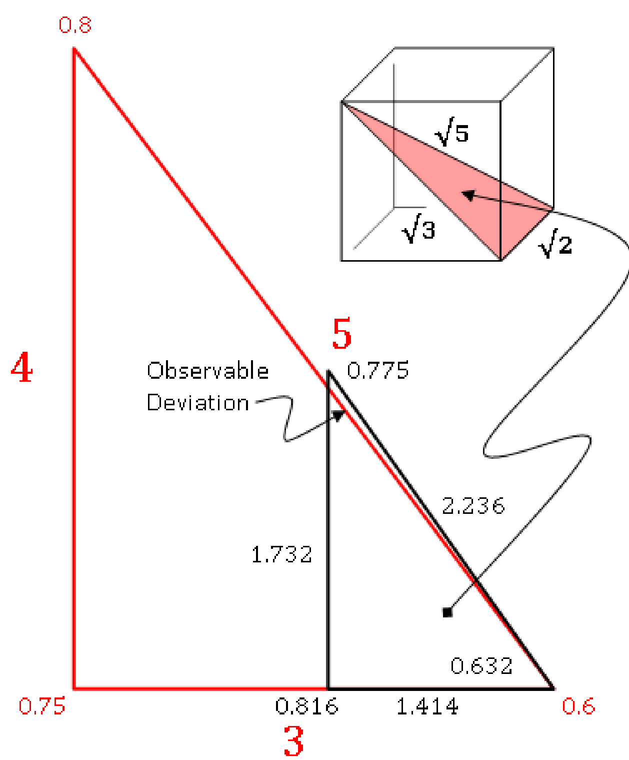


Figure 16 √F3√F4√F5 and √F4√F5√F6 triangles Generalize by Mathematical Induction

Figure 17 An Ideal 3-4-5 Triangle versus √F3-√F4-√F5 Right Triangle



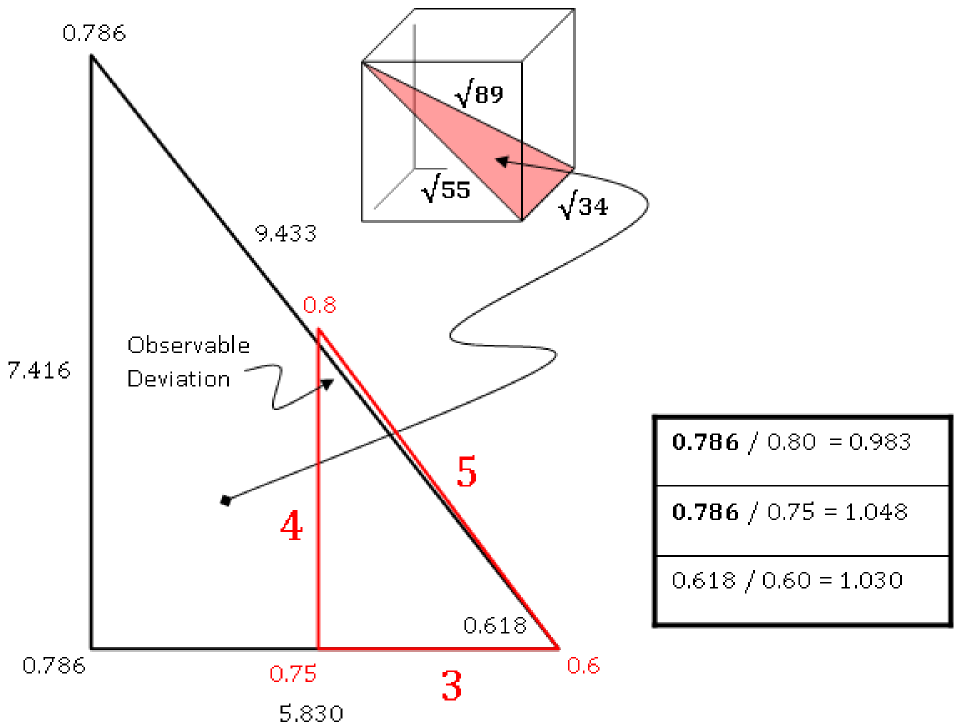
0.816 / 0.80 = 1.020	
0.707 / 0.75 = 0.943	
0.577 / 0.60 = 0.962	



0.775/ 0.80 = 0.9682	
0.816 / 0.75 = 1.089	
0.632 / 0.60 = 1.054	

Figure 18 An Ideal 3-4-5 Triangle versus √F4-√F5-√F6 Right Triangle

Figure 19 An Ideal 3-4-5 Triangle versus √F11-√F12-√F13 Right Triangle



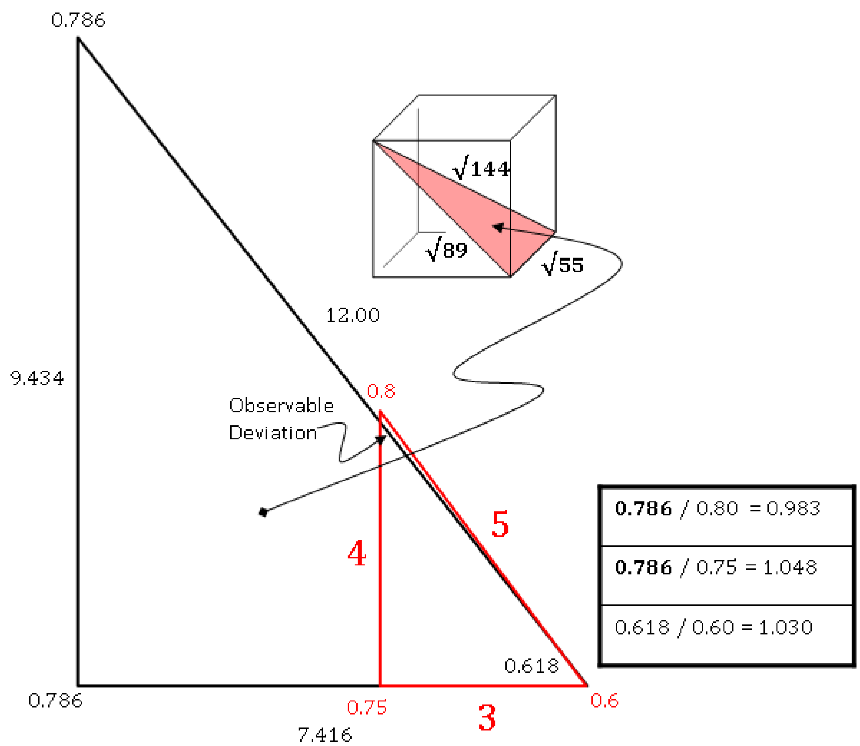


Figure 20 An Ideal 3-4-5 Triangle versus √F12-√F13-√F14 Right Triangle

Table 2 Square Root (Fibonacci) Equivalents of 3-4-5 Triangles

Fibonacci Indices	3-4-5 Triangle Sides Base/Altitude/Hypotenuse
F(n,n+1,n+2)	
1 .. 3	$\sqrt{1}\ \sqrt{1}\ \sqrt{2}$
2 .. 4	$\sqrt{1}\ \sqrt{2}\ \sqrt{3}$
3 .. 5	$\sqrt{2}\ \sqrt{3}\ \sqrt{5}$
4 .. 6	$\sqrt{3}\ \sqrt{5}\ \sqrt{8}$
5 .. 7	$\sqrt{5}\ \sqrt{8}\ \sqrt{13}$
6 .. 8	$\sqrt{8}\ \sqrt{13}\ \sqrt{21}$
7 .. 9	$\sqrt{13}\ \sqrt{21}\ \sqrt{34}$
8 .. 10	$\sqrt{21}\ \sqrt{34}\ \sqrt{55}$
9 .. 11	$\sqrt{34}\ \sqrt{55}\ \sqrt{89}$
10 .. 12	$\sqrt{55}\ \sqrt{89}\ \sqrt{144}$
11 .. 13	$\sqrt{89}\ \sqrt{144}\ \sqrt{233}$

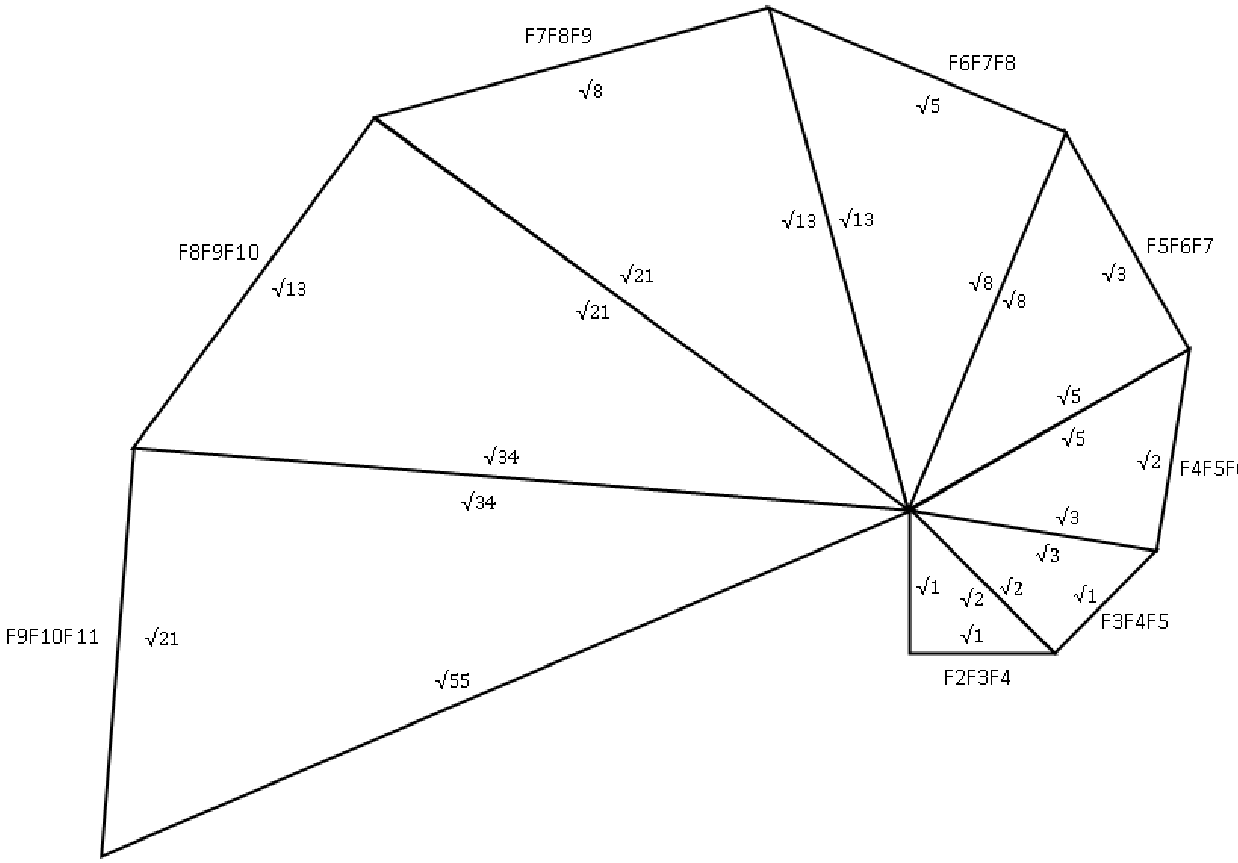
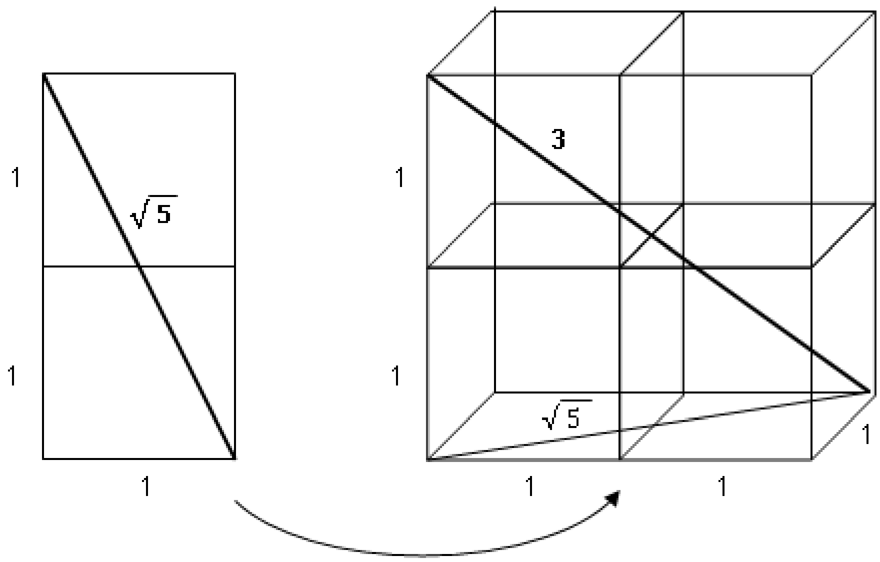


Figure 21 Pivoting Successive Square Root(Fibonacci) Triangles Trace a *Spira Mirabilis*

Once again, application of Mathematical Induction, and simultaneously, PRG recursion, leads us to further explore the Fibonacci Roots. We know that the "long hypotenuse" of stacked unit squares is √5 in length, Figure 2, page 11, and the stacked squares suggest a new structure, per Figure 22, below. Compare the "long hypotenuse" of this construct with the √3 hypotenuse of the individual cubes, and the √5 hypotenuse.



$$h = \sqrt{(1+1)^2 + 1^2} \qquad h = \sqrt{(1+1)^2 + (1+1)^2 + 1^2} \;=\; \sqrt{2^2 + \sqrt{5}^{\,2}}$$

$$h = \sqrt{4 + 1} \qquad h = \sqrt{4 + 4 + 1} \;=\; \sqrt{4 + 5}$$

$$h = \sqrt{5} \qquad h = \sqrt{9} = 3 \;=\; \sqrt{9} = 3$$

Figure 22 Spatial Relationships Among 2, 3 and √5 in Cubic Space

This structure may offer a gateway into expanding the 2-D (planar) Phi Ratio Geometry that we're growing comfortable with, into the 3D universe that is truly the realm of Phi.

The Vesica Piscis is the reference generator for developing all sorts of Phi Ratio-based constructs. The VP depicted in Figure 23 includes Professor Mills' original Square Root Two Rhombus plus other square root edges obtained when the induction of PRG recursion is employed.

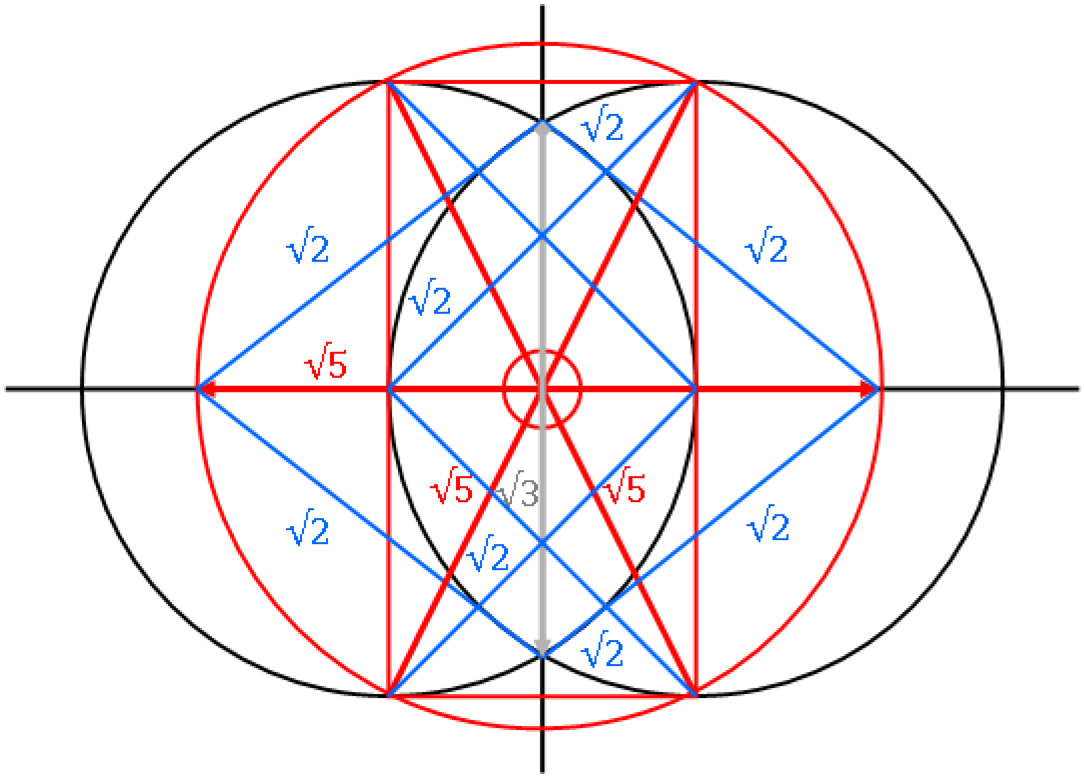


Figure 23 The *Vesica Piscis* Generates Square Roots in Profusion

The square roots can now be seen to be the fundamental building blocks, as important as the integers, and possibly foundational to the integers. *Ergo*, it is essential to complete understanding that one appreciates that Phi Ratio relationships are the signature, the Creator's *imprimatur* that a structure is properly constructed.

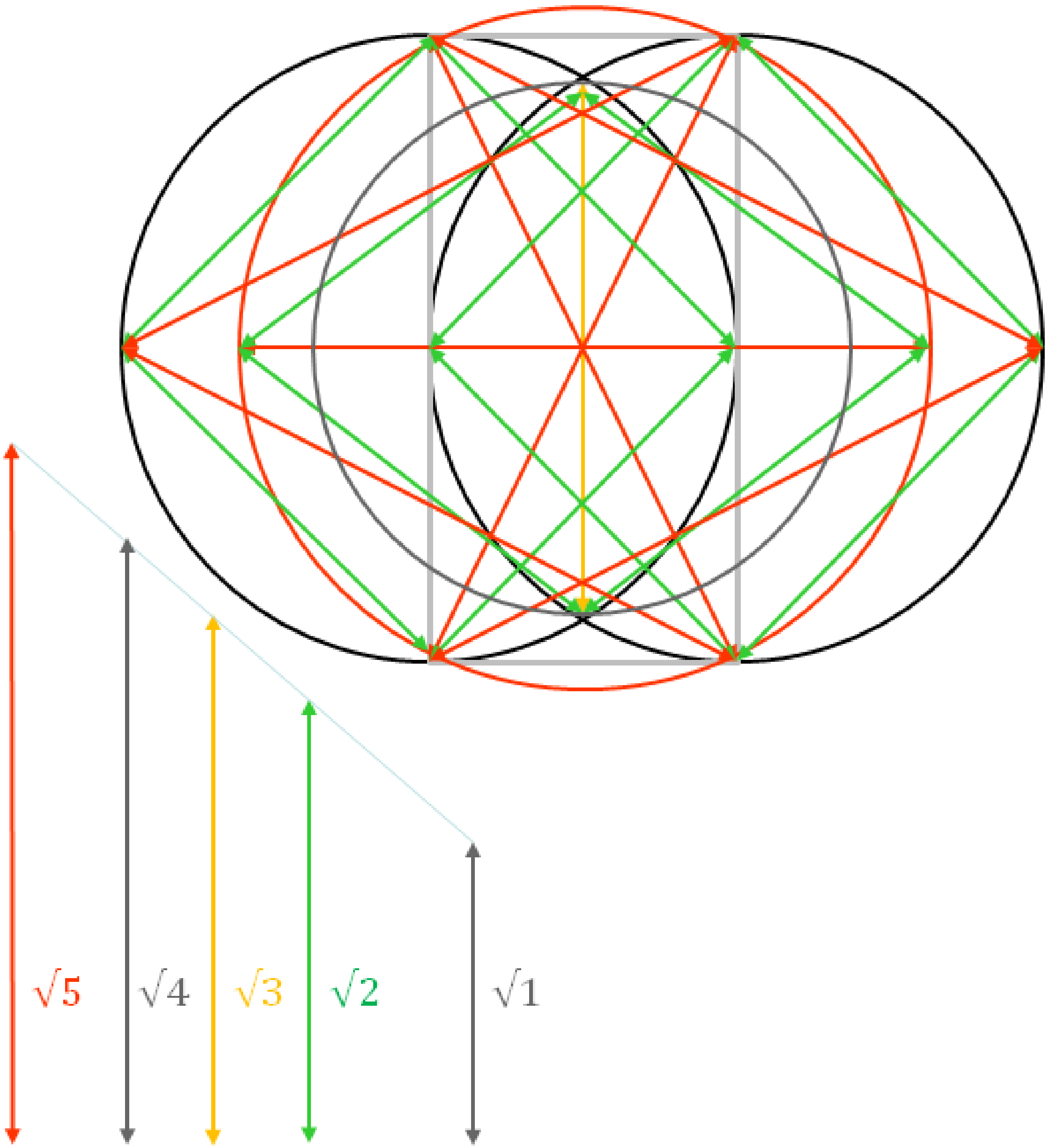


Figure 24 Recursive Induction Generates Additional Square Roots

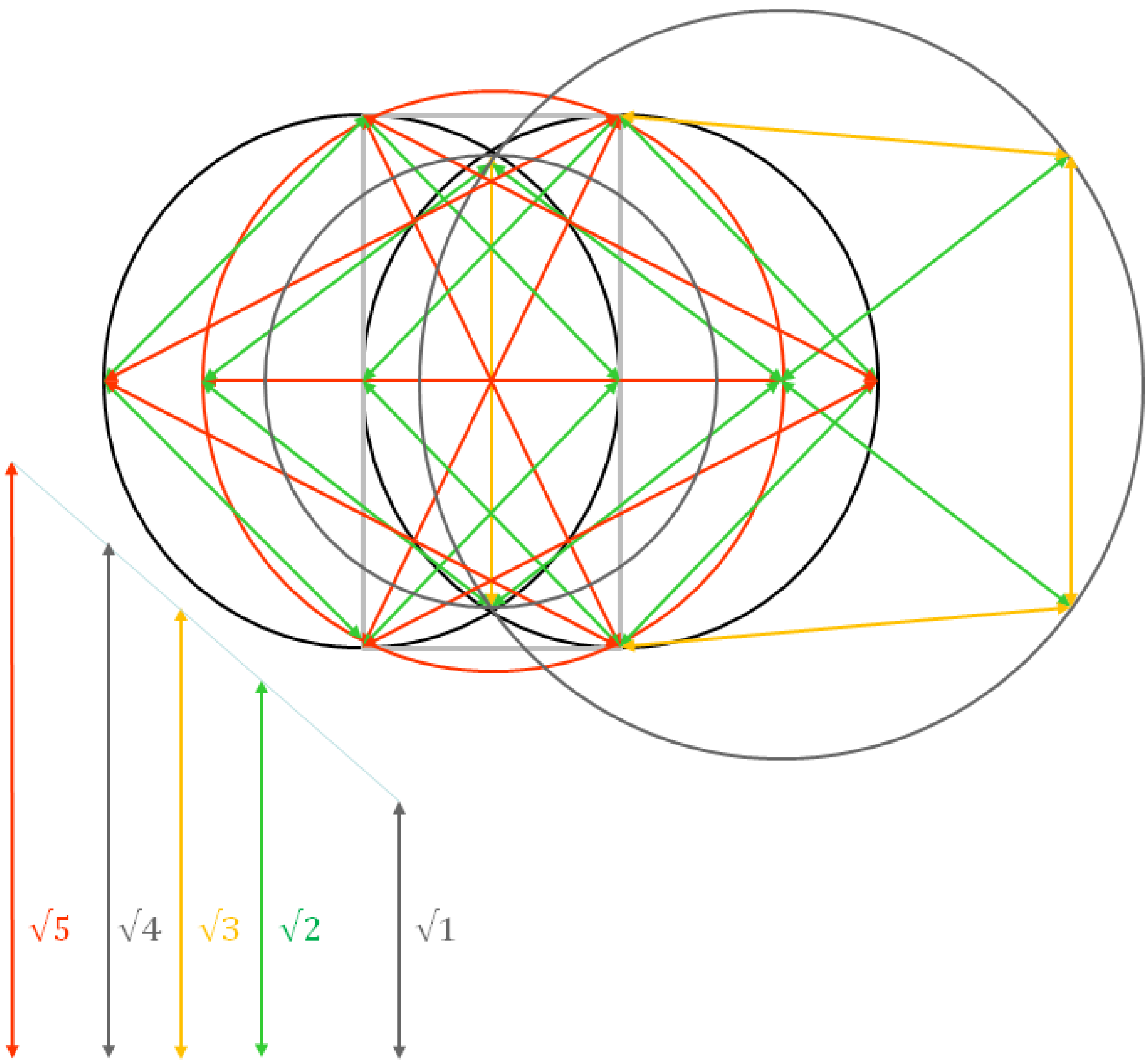


Figure 25 Addition of a SQRT(4) Circle Depicts Further Extensibility of the Concept

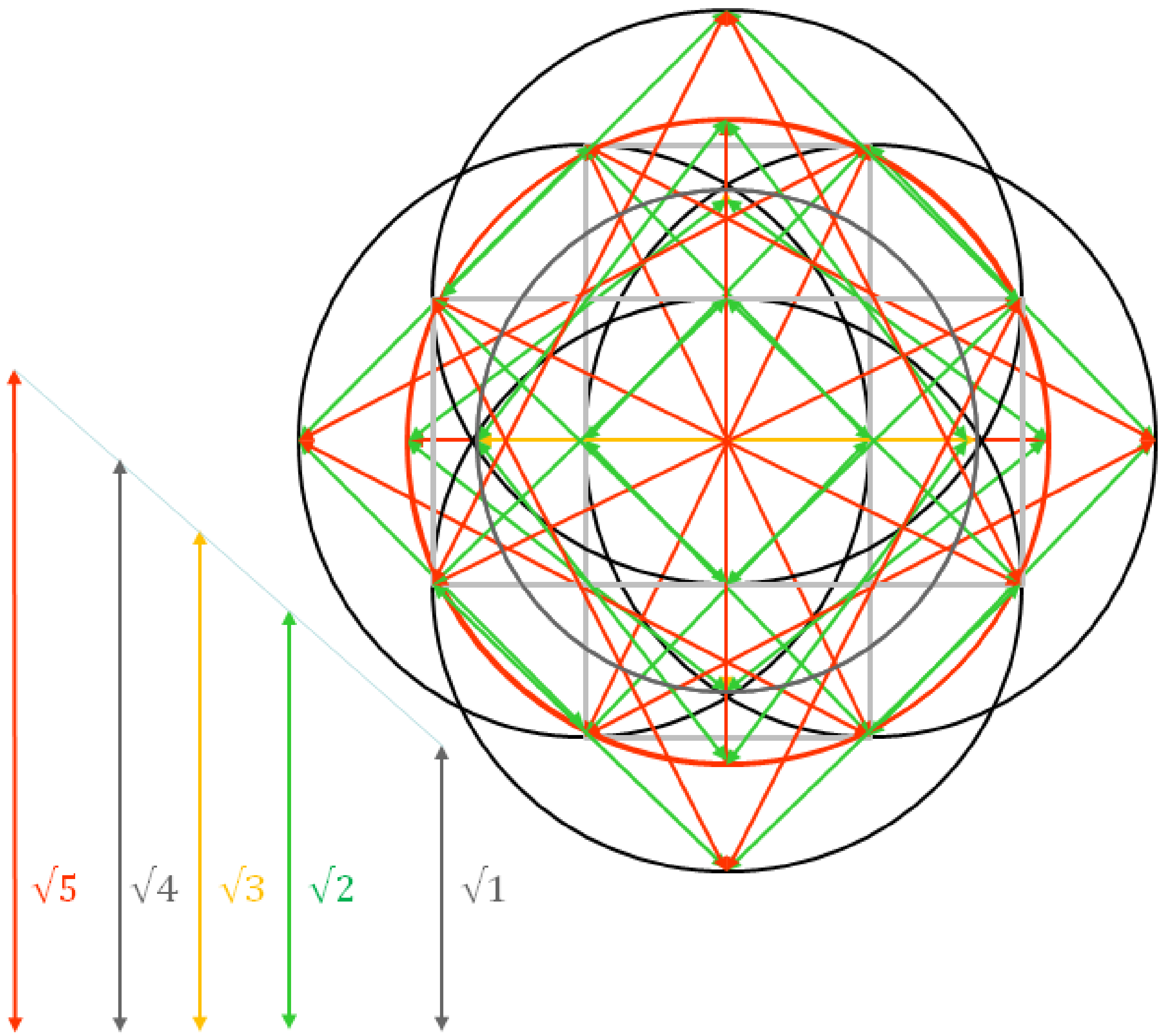


Figure 26 Square Roots Discovered within a *Vesica Cruxis* Scaffolding

A New Basis for the Natural Numbers

At this point in our narrative it may become clear that certain square roots provide the underpinnings of the Phi Ratio Geometry, and that Phi generally serves as a confirmation.

An additional example may "seal the deal." What began in 1621 as Bachet's Conjecture as a translation, was later amended by Adrien-Marie Legendre in 1798, and later still by Carl Friedrich Gauss. The Four-Square Theorem was finally completed as a proof in 1770 by Joseph L. LaGrange[7].

The proof admits fractions that include ½ to the integers that construct the natural numbers,

$$n = i^2 + j^2 + k^2 + m^2$$

and this was seen as a loophole that would permit the use of the Fibonacci-based roots: √2, √3, and √5. The compilation represented by ad only expected integer values – and this will merit further investigation.

Table 3 is one of several that demonstrates that these roots not only work, but that they may also operate as counting numbers with a little more effort.

If this is borne out, a digital computer, and programs based on such arithmetic may also be possible. Such developments would open the gate for wider use of Phi Ratio Geometry tools in science, and engineering.

Physicists have recently discovered that unidentified fractional values are lurking in particle physics, where they had only expected integer values – and this will merit further investigation.

Table 3 Modified LaGrange Sum of Four-Squares

1				√1
2				√2
3				√3
4				√4
5			2	√1
6			2	√2
7			2	√3
8			2	√4
9		2	2	√1
10		2	2	√2
11		2	2	√3
12		2	2	√4
13	2	2	2	√1
14	2	2	2	√2
15	2	2	2	√3
16	2	2	2	√4
17	2	2	2	√5
18		3	3	0
19		3	3	√1
20		3	3	√2
21		3	3	√3
22		3	3	√4
23		3	3	√5
24	2	3	3	√2
25	2	3	3	√3
26	2	3	3	√4
27	2	3	3	√5
28	3	3	3	√1
29	3	3	3	√2
30	3	3	3	√3
31	3	3	3	√4
32	3	3	3	√5
33		4	4	√1
34		4	4	√2
35		4	4	√3
36		4	4	√4
37		4	4	√5
38	2	4	4	√2
39	2	4	4	√3
40	2	4	4	√4
41	2	4	4	√5
42	3	4	4	√1
43	3	4	4	√2
44	3	4	4	√3
45	3	4	4	√4
46	3	4	4	√5
47	4	5	√3	√3
48	4	4	4	0
49	4	4	4	√1
50	4	4	4	√2
51	4	4	4	√3
52	4	4	4	√4
53	4	4	4	√5
54	5	5	√3	√1
55	5	5	√3	√2
56	5	5	√3	√3
57	5	5	√3	√4
58	5	5	√3	√5
59	5	5	3	0
60	5	5	3	√1

[7] Wikipedia, searchkey: Lagrange's Four Square Theorem.

Ontogenic Growth

When building lifeforms, Nature's principal method is self-replicating strands of RNA (Ribose Nucleic Acid), and DNA (Dioxyribose Nucleic Acid). The information imbedded in these strands is used to fold molecules into proteins, processes that are entirely based upon geometry, energy, bonding sites, and ***recursion at several levels of scale***.

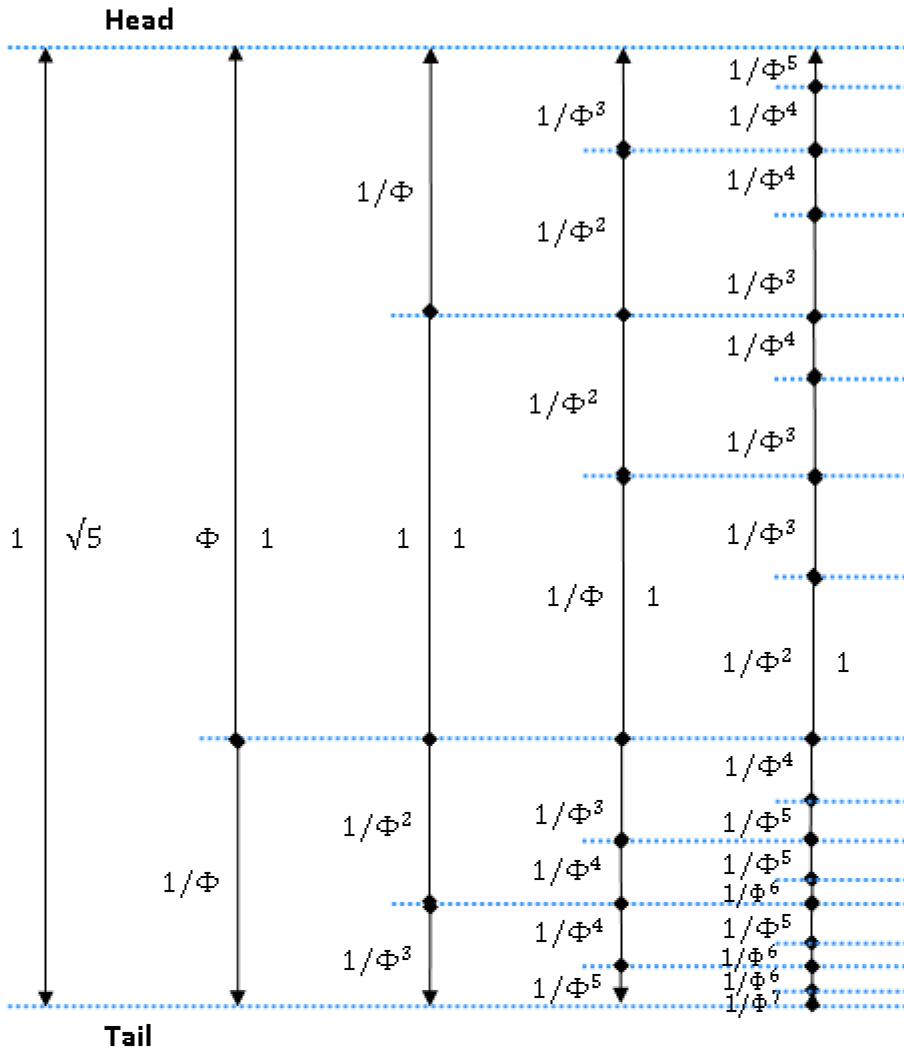


Figure 27 Differentiation Occurs During Ontogenic Growth at Phi Boundaries

While Phi Ratio relationships have been used to reverse-engineer our local fringe of the universe, it is the square roots, and likely cube roots, and others that are actually the fundamental building blocks. One example of the role of square roots is that the Square Root of Five is itself recursive in nature, and is the basis for Phi recursion, as depicted in Figure 27.

The Spherical Universe

The Universe is not an orthogonal, or rectilinear domain. The universe is expressed as spheres, circles, and angular relationships in a domain where a straight line is a rare occurrence.

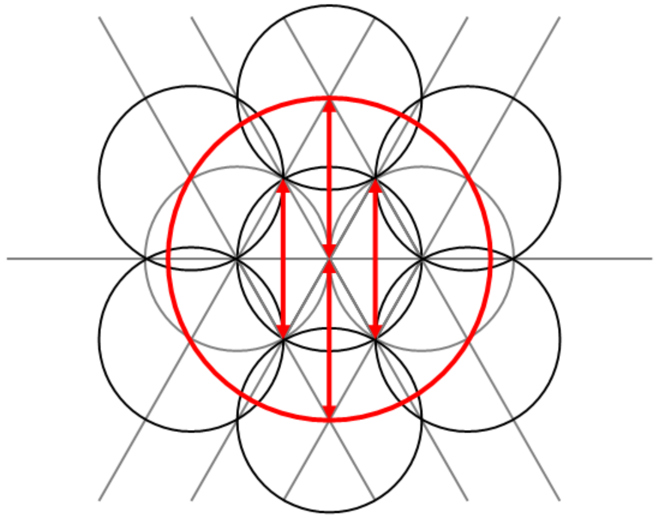


Figure 28 The Centers of the FOL Six-Around-One are Radius √3

The Extent of Space

Mathematical Induction is the cornerstone for the *corpus* of proofs that constitute contemporary mathematics, and also stands as the core of the Scientific Method. Guiseppe Peano, (1858-1932), offered a number of axioms that are key to proofs, and defining the role, and the process of, mathematical induction:

> *"Let P be a predicate on ω. If*
>
> *(i) P(0) = TRUE, and*
>
> *(ii) for every k, If P(k) = TRUE then P(k + 1) = TRUE*
>
> *Then P(n) = TRUE for all n."*

—Mitchell Wand[8]

The dimensionality of a universe, **D**, is expressed as a ***space*** described by a number of orthonormal dimensions, ***n***, passing through the reference origin of the objective universe. The dimension of a universe is described by $\mathbf{D}^{n}$.

The ***basis step***, (i), where:

> *the predicate n = 0, is tested, and found to be True because the resulting universe, D^{0}, is a singularity, a point of no dimension, or resolution.*
>
> *We then move on to the **inductive step**, (ii), where a series of linked predicates are tested, (k+1):*
>
> *when n = 1, the universe that results, D1, is a line that has an extent in one dimension, and none in any other.*
>
> *the two dimensions that result in the case of n = 2, describe a universe, D2, which is a plane that features no thickness.*
>
> *a familiar three dimensioned universe, D3, is a cubic space.*

The conclusion is important to our deliberations, and is strongly connected to each of the precursory steps. We must be careful, therefore, that we do not alter the language, or the intent of the inductive process. When Peano asserts that P(*n*) is True for all *n*, he is not describing everything in the universe, but rather, he is insisting on consistency, that P(n) is True for all *n* that are described by the predicate. The predicate does not describe time, however, it does describe additional ***spatial dimensions***, where n = 4 is descriptive of a hypercube, and rotated spaces can be projected onto other dimensions.

Our experience with the universe should have already disabused us of any notion that the universe would be as crisp, and as neat as squares, cubes, and tesseracts. Any universe resembling ours could not be expected to have orthonormal dimensions, where $x_{max} = y_{max} = z_{max}$. The universe more likely resembles a roughly spherical "blob," imperfect because of

[8] Induction, Recursion and Programming, Mitchell Wand, pg 32.

chaotic, interior processes that occur as the volume unfolds, and hyperplasmas form to alter local densities, magnetic and gravitational effects.

Projection of Dimensional Space

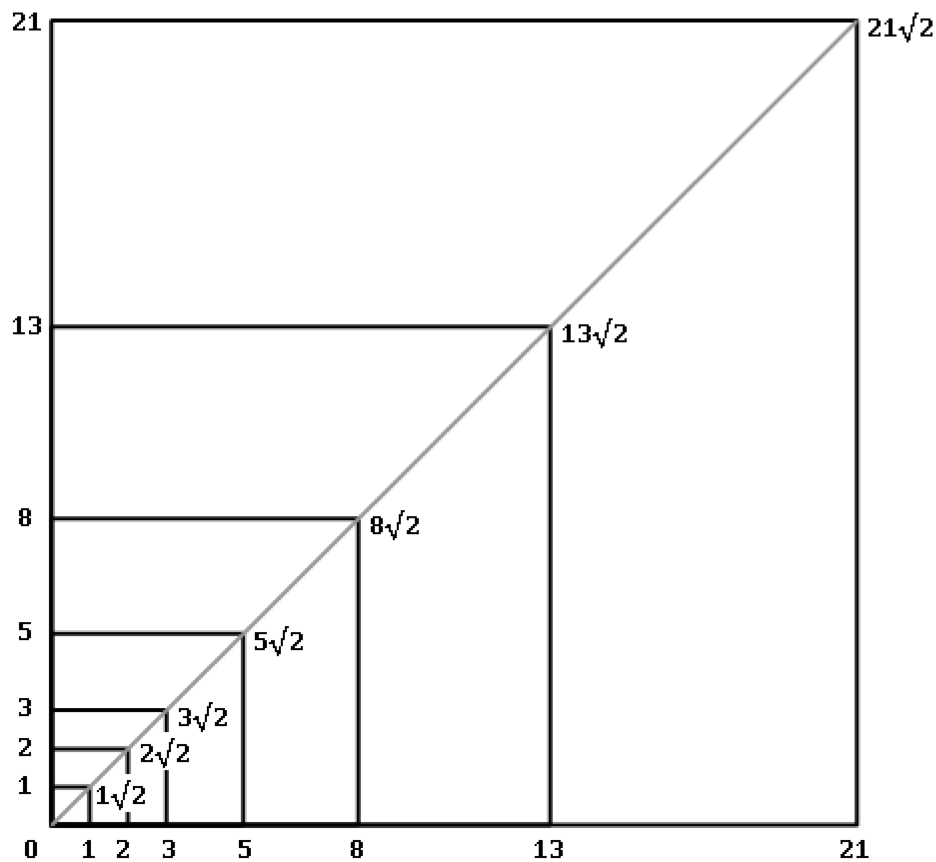


Figure 29 Phi Ratio Relationships Persist

The recursive Phi Ratio universe show us three of its dimensions. The current paradigms insist that time is a fourth dimension that we can access. It is possible that structural recursion would suggest that time isn't a dimension at all, and that we have direct access to three dimensions in a continuum of dimensions.

Time is not a Dimension

People are very much committed to their concepts of time, especially since they are overwhelmed by time subjectively. Motion occurs within the domain of at least three dimensions, driven by the dominant effects that are in effect at that instant, such as temperature (energy density).

Apparent time has direction due to entropy, the general cooling of the universe since the "Big Bang," or similar event. Entropy is evidenced as irreversible chaos, where structure breaks down. [does this imply that black holes are counter-entropic?]

Time is a *degree-of-freedom*, a displacement that is enabled by the existence of containing dimensions. The intuitions that prevent the recognition of this insight are based upon experiential views, reinforced by the consequent motion within a universe of the pendulum of a clock, a tuning fork, a 32,768 Hertz Quartz crystal, and the atoms of Cesium oscillating in synchrony with LASER coherence. Each of these reference metrics are based not upon time, but time based upon motion in space.

Projections to/from 3D

Projection of hyper constructs

Compartmentalizations of parallel 3D universes as 4D and 5D symmetry rotations

Redefine dimensions as spherical partitions, not measurement dimensions

Recompartmentalization parallel universes

Temperature and local state

The confusion about time, and other dimensions, and our inability to find anything above dimension four, is basically due to our confusion of metrics with our perceptions.

Dimensions Above Three

The three dimensions familiar to humanity's day-to-day existence are projected from the fourth, fifth, and higher dimensions. This assertion is based upon the other attributes of the PRG, where Phi relationships are recursive, pervasive, and persistent. Mathematical Induction alone would suggest that familiar three-dimensional Phi structures have analogs in higher-dimensional space.

Phi Ratio Foam

In a fashion similar to the construction of the Serpinski Gasket, multiple-scaled, or fractal structures can be devised that exhibit Phi Ratio relationships. This is an example of recursion, aggressively applied to a brachiated, or tree construction.

This idea is presented in a more gentle way by the packaging of a Dutch Cocao powder, shown in Figure 30, below. The image of the nun dominates the initial, large frame, and is repeated at two additional scales on the smaller package, and the mug, and yet again at levels below our ability to perceive on the third level of the package, and the fourth level package evident on the mug.

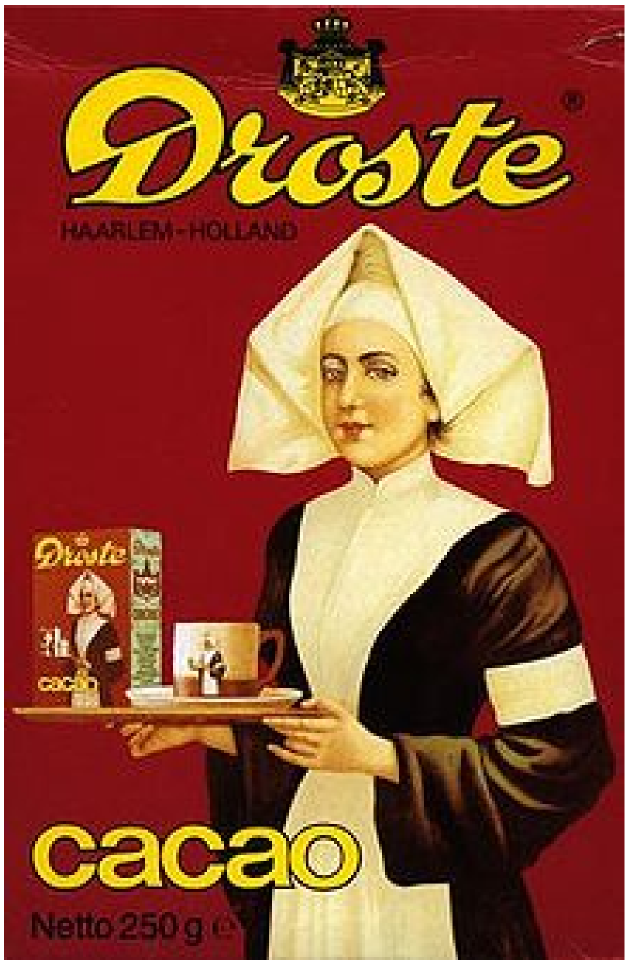


Figure 30 The Droste Labelling Employs Concurrent Recursive Branches

Sierpinski Examples

One of the early examples of fractal constructions were the Sierpinski Snowflake, and the Sierpinski Gasket. These are each accomplished through a recursive substitution rule specific to the intended structure. In Figure 31, below, a line segment is trisected, and the middle third is replaced with the sides of an equilateral triangle, less the base. This simple rule is repetitively applied in Figure 32, and Figure 33, continuing at steadily reducing scales of 1/3.

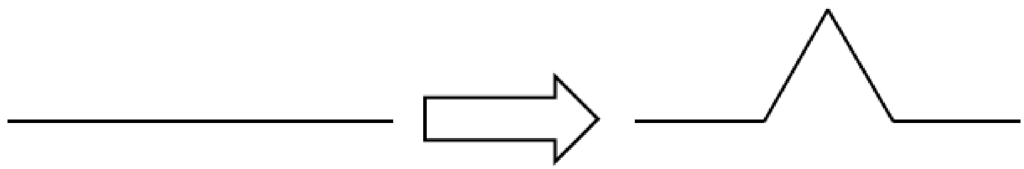


Figure 31 The Snowflake Substitution Rule

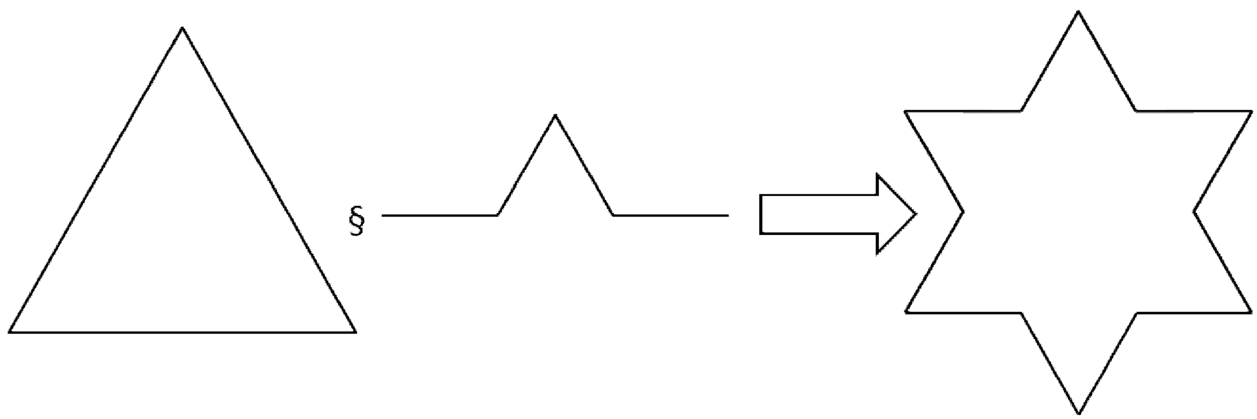


Figure 32 The First Recursion of the Substitution Rule Applied to Each Edge

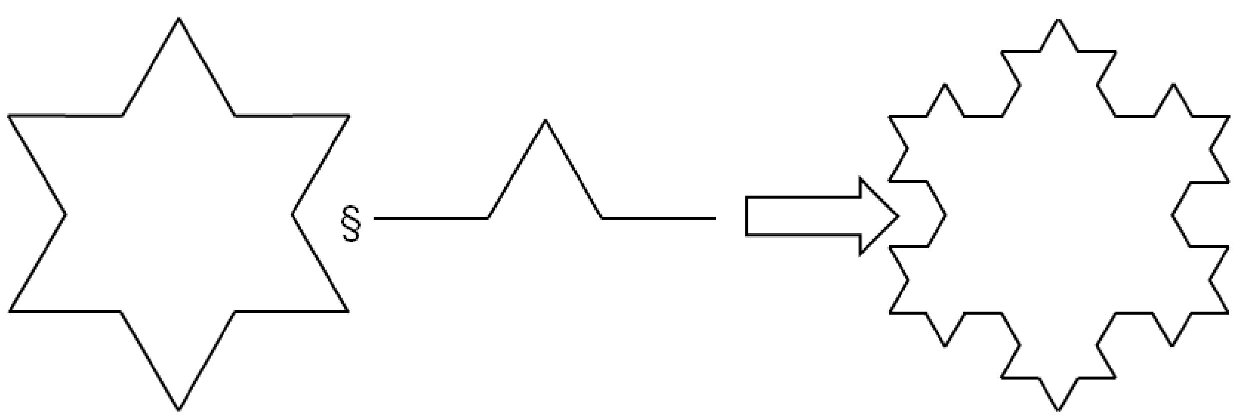


Figure 33 The Second Recursion of the Substitution Rule Applied to Each Edge

Another simple Sierpinski example is the Sierpinski Gasket, which recursively applies a void within a triangle. The first application of the rule is the first step in the gasket's construction, per Figure 34.

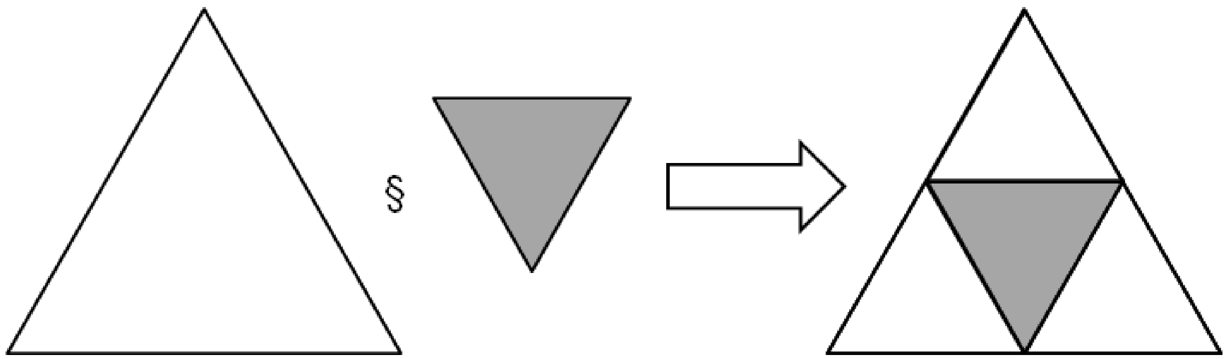


Figure 34 The First Application of a Void to the Sierpinski Gasket

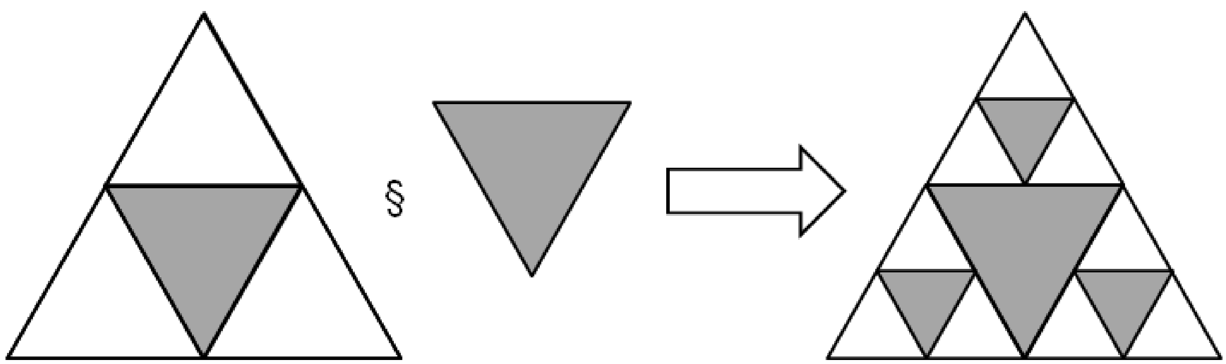


Figure 35 A Second Recursion of the Sierpinski Gasket Substitution

One foundational feature of the Phi Ratio Geometry is the pervasiveness of recursion as one of the fundamental construction tools, and while the Sierpinski devices are not a PRG construction, they quite vividly, and clearly depict the power of a simple rule applied recursively. While the previous half dozen illustrations tell this in a modest way, Figure 36, and Figure 37 make the point impressively.

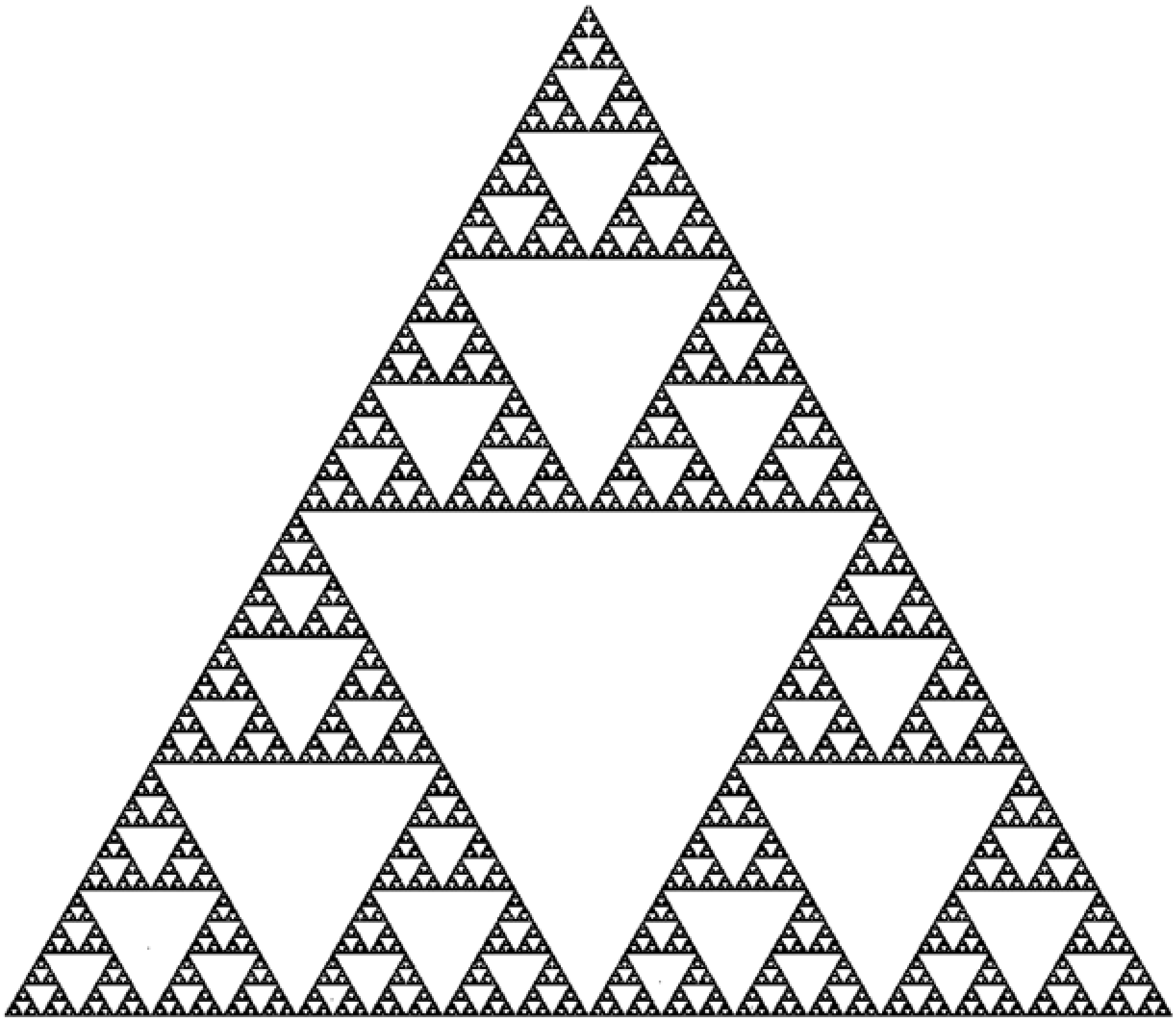


Figure 36 The Sierpinski Gasket has a Complex Structure by the Sixth Recursion

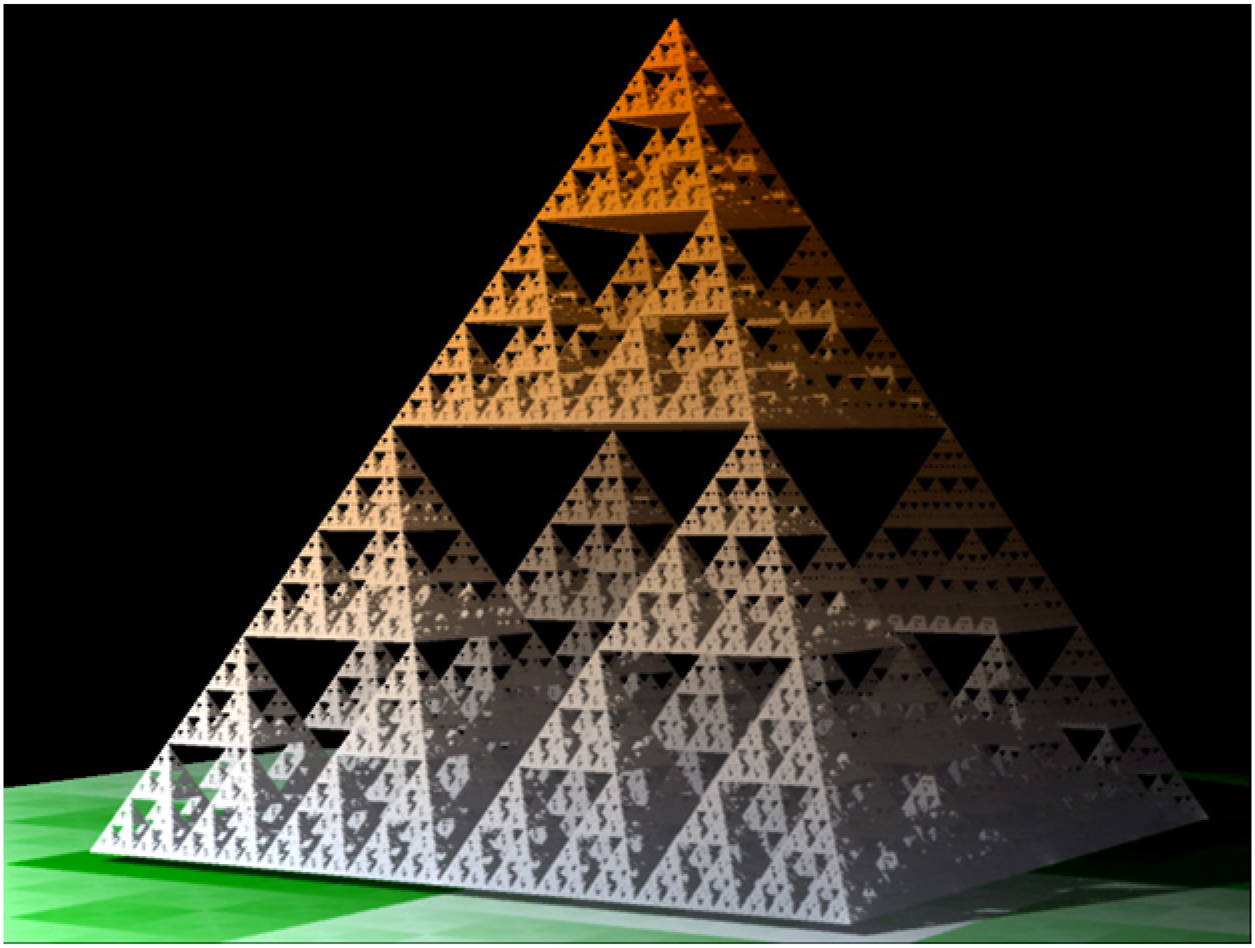


Figure 37 The Sierpinski Pyramid Takes Fractals to a Third Dimension

Now that we have introduced the basic concepts of substitution, and fractals, we move on to Phi Ratio Foam.

The Circles and Spheres of PRG

A circle is filled with smaller circles, all based upon successive Phi Ratio values that reduce by 1/Φ, Φ-1, or 0.6180339… Each reduction represents a smaller scale of Phi-proportioned circles that bifurcate.

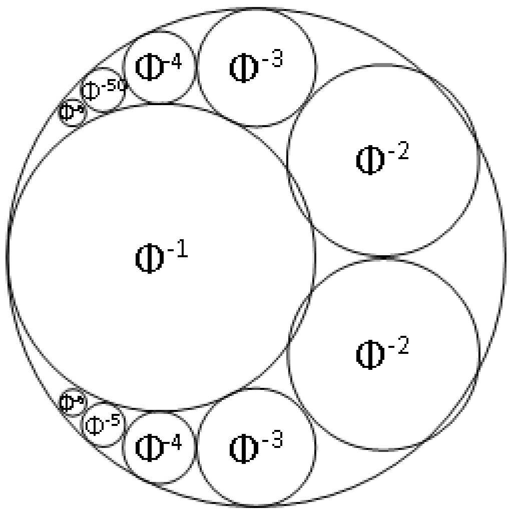


Figure 38 Dense-packing of a Circle with Phi Ratio Fractional Circles

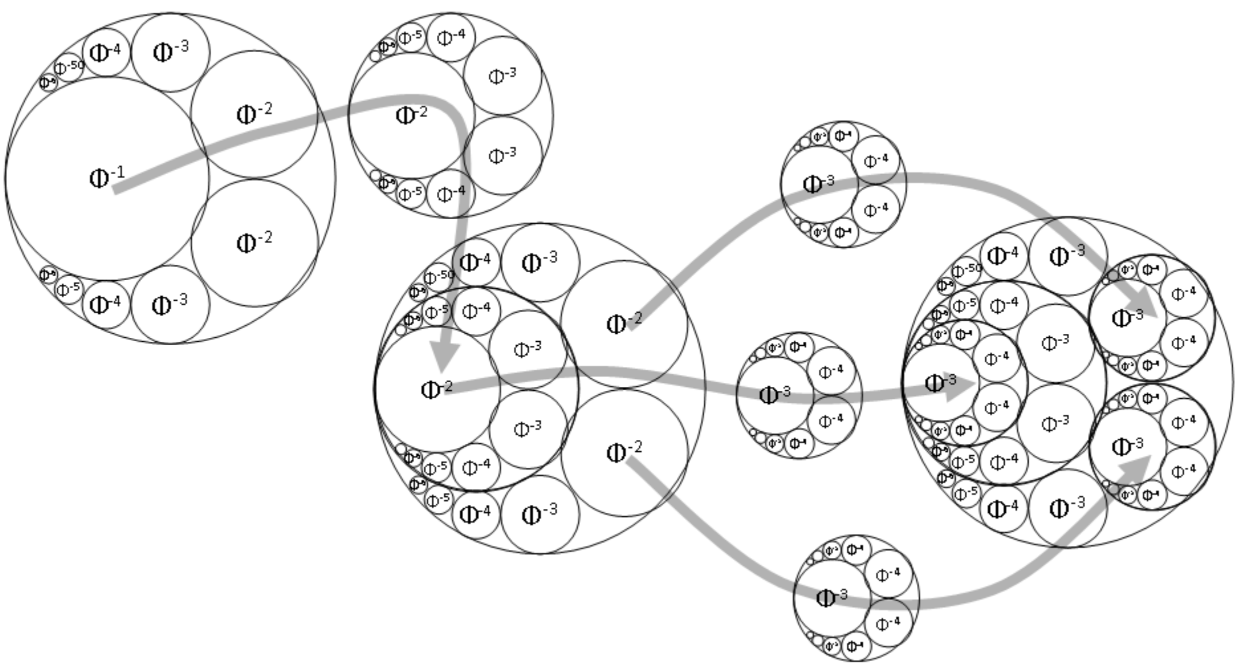


Figure 39 Recursion Generates a Phi Foam that is a Fractal

A spatial, three-dimensioned sphere fills voids with successive bifurcations, reiterating that the Phi Ratio is highly recursive. In a manner similar to that of the Droste Chocolate package, the brachiation of the circles explodes exponentially into a forest of branches, and each recursion can be considered to be a cross-section of a tree.

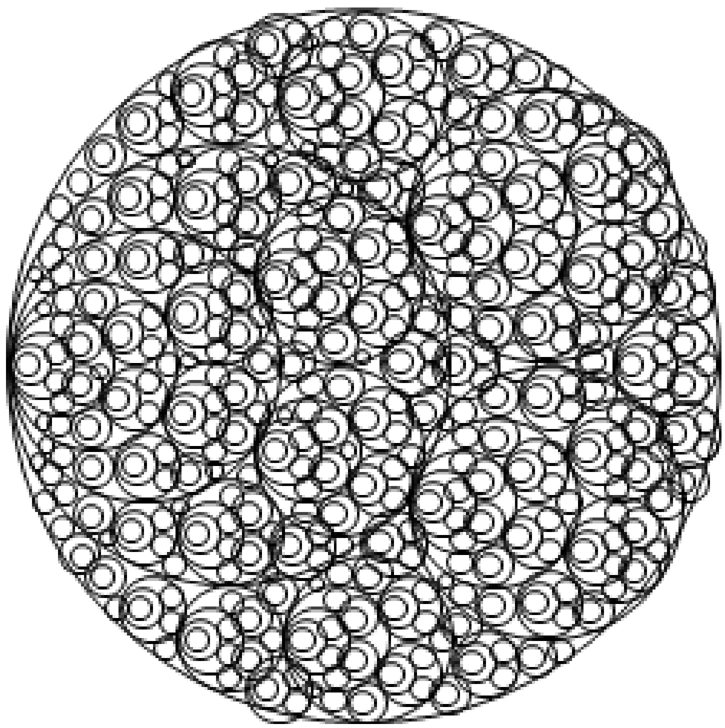


Figure 40 The Universe may Feature an Internal Structure of Densely-Packed Phi Ratio Foam

First Order Universe

The universe can be regarded as a continuum of scales that become visible at the subatomic, and progress to scales that contain the universe where the ultimate scale is conjecture. It is apparent that matter exists in ***bands of effect***, where groups of scale are contiguous, and these are periodically interrupted by ***bands of void***. Imagine that the universe is centered on a single atom, and that scales are drawn at Phi intervals to the borders of the universe, if there is such a thing, as suggested by Figure 41, below.

The void provides the volume within which the operant phenomena of one or more bands of effect alternate with bands of void. The voids are filled with aggregations of effects that are occurring at lower levels of scale.

Phi structures are evident on scales from Φ_c .. Φ_j, followed by voids which span an extent from Φ_k .. Φ_r which is followed by matter spanning several new scales of phi ratio structure, Φ_s .. Φ_z. This is consistent with the fundamental recursive nature of the Phi Ratio Universe.

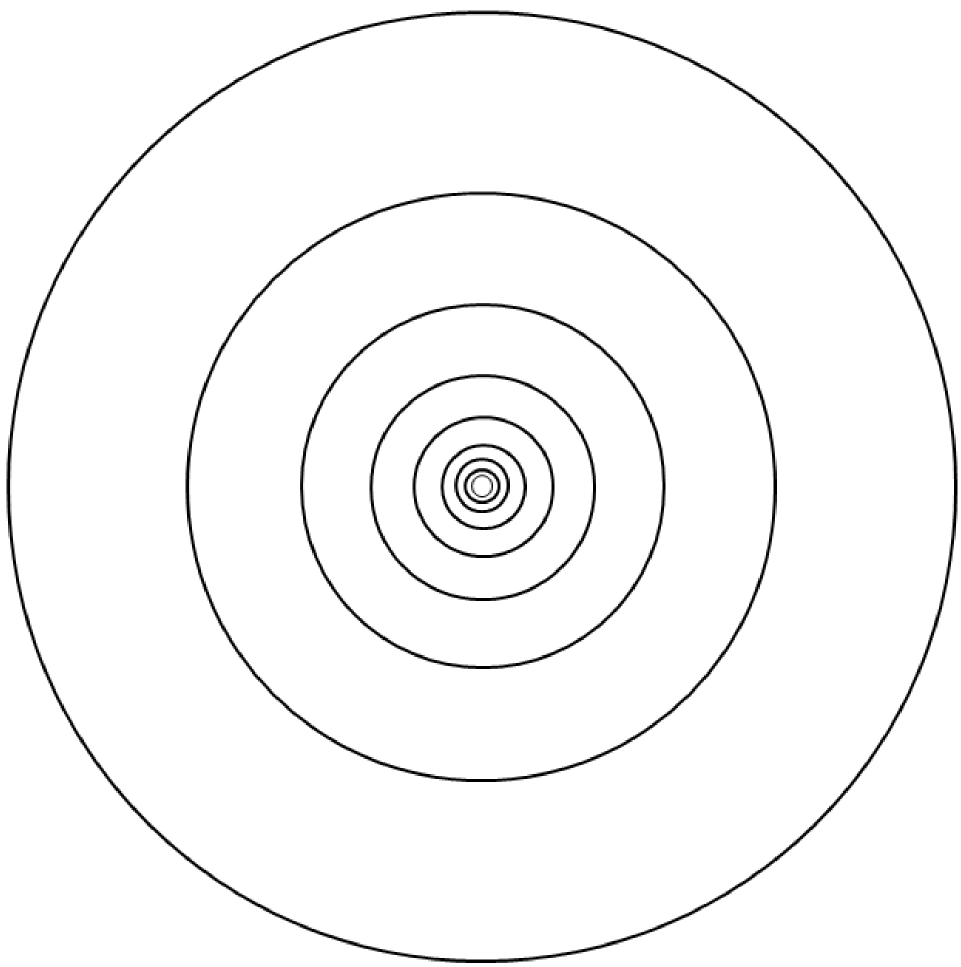


Figure 41 *Circulus Profunditas*, Bands of Scale under Phi Ratio Expansion

Phi is not in the driver's seat. It is essential that we recall that Phi is not the cause, and is the signature that the system is properly constructed.

Voids occur within the recursively unfolding Phi Ratio expression, and we can speculate that the voids are a consistent group of exponent intervals within an ***octave***, alternating with another octave, a consistent group of exponent intervals that describe material structure, or phenomena.

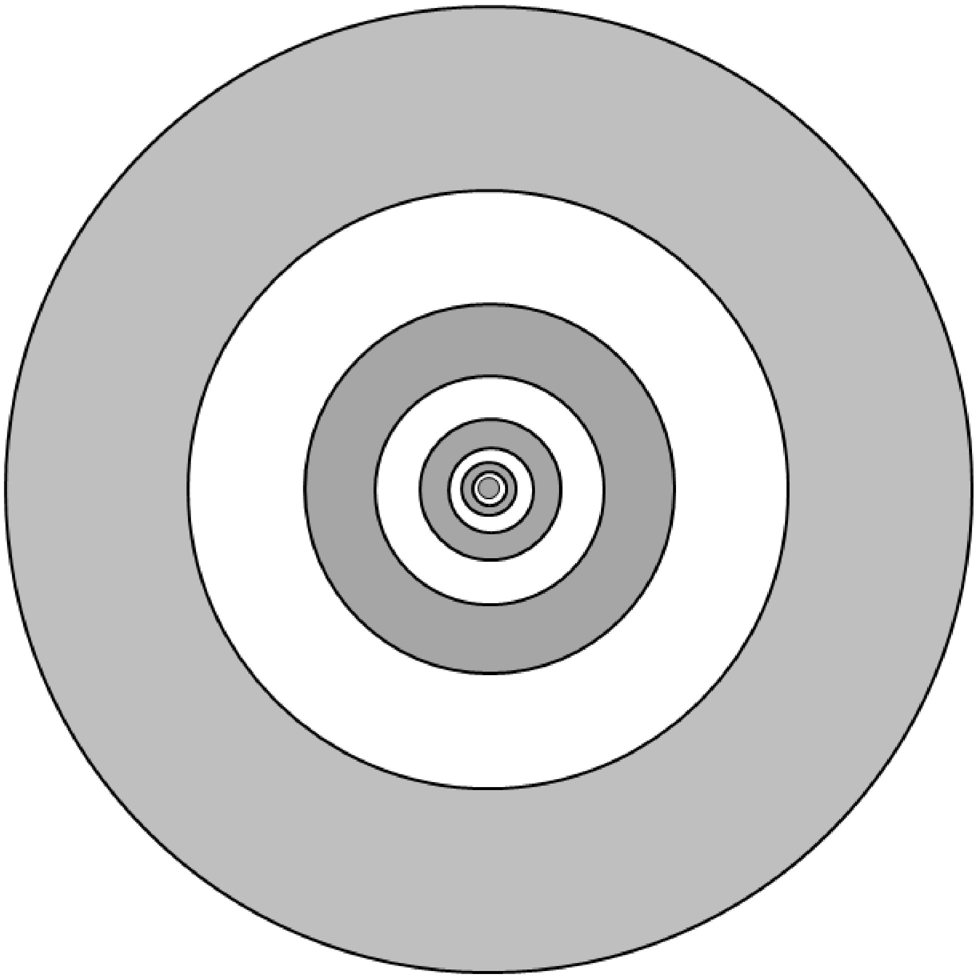


Figure 42 Each Band Contains an Octave of Phi Scales, Φ^n .. Φ^{n+7}

We expand the meaning of the term ***mesoscalar*** to describe effects that dominate a band of effect, and ***macroscalar*** to define the next larger void within which the mesoscalar effects operate. Each phenomenon has a range of bands of effect, and void within which it dominates other effects, and is co-resident with others. These effects include gravity, electrostatics, magnetics, weak force, strong force, charge, and so on. Mesoscale describes phenomenological extent.

For example, the weak and strong forces, in conjunction with charge, are dominant at the atomic level, and more pronounced than gravity, which is weaker than the other effects at the atomic scale.

In an effort to provide metrics across all scales, we now demonstrate yet another application of *Vesica Pisces* as a scaffolding. Construction of complex *Vesica* may yield new and interesting properties that have not been previously apparent. For example, since the Phi Ratio in nature is persistent and pervasive, it may be possible to probe the boundaries of mesoscalar effects with diagrams such as that in Figure 43.

Here we have a *Vesica Piscis* that is grown by one level as a Phi expansion (x 1.6180…), and diminished by six orders of magnitude by successive application of Phi's reciprocal (x 0.6180…). Certain mesoscalar properties of various phenomena may be identified by such constructions. For example, it should be noted that at no scale do we see an intersection of two circles of the same scale at a perimeter tangent, and the circles can be observed to diverge even though the centers remain constant. Certain design methods that we have extracted from the geometry would cause us to ask questions such as "what is the relationship of this image to Planck's constant at atomic scales?"

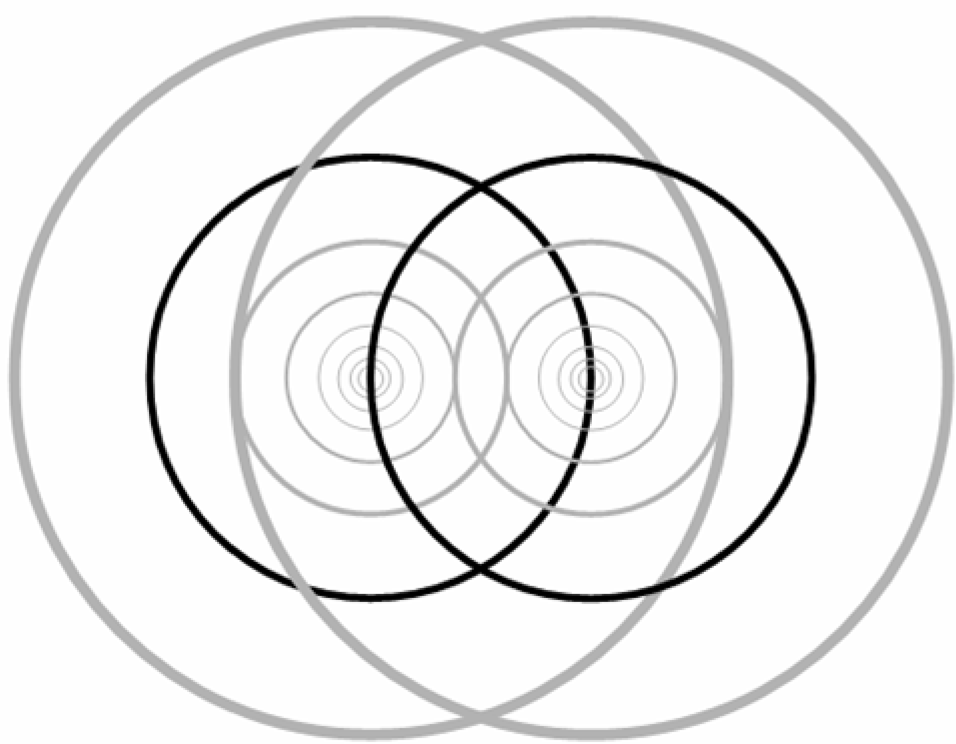


Figure 43 A Vesica Piscis (Bold) Expanded and Diminished by Phi Ratio

Recall that in Carl Sagan's popular science fiction novel, *Contact*, that he imagined that after computing Pi to a gozillion places, the resulting bit pattern at the end of the now-terminating fraction was a circle. If we continue the Phi Ratio expansion of a *Vesica Piscis*, the result is a construct with a familiar limit. Ladies and Gentlemen, in Figure 44, I give you a non-fictional version of Carl's signature circle.

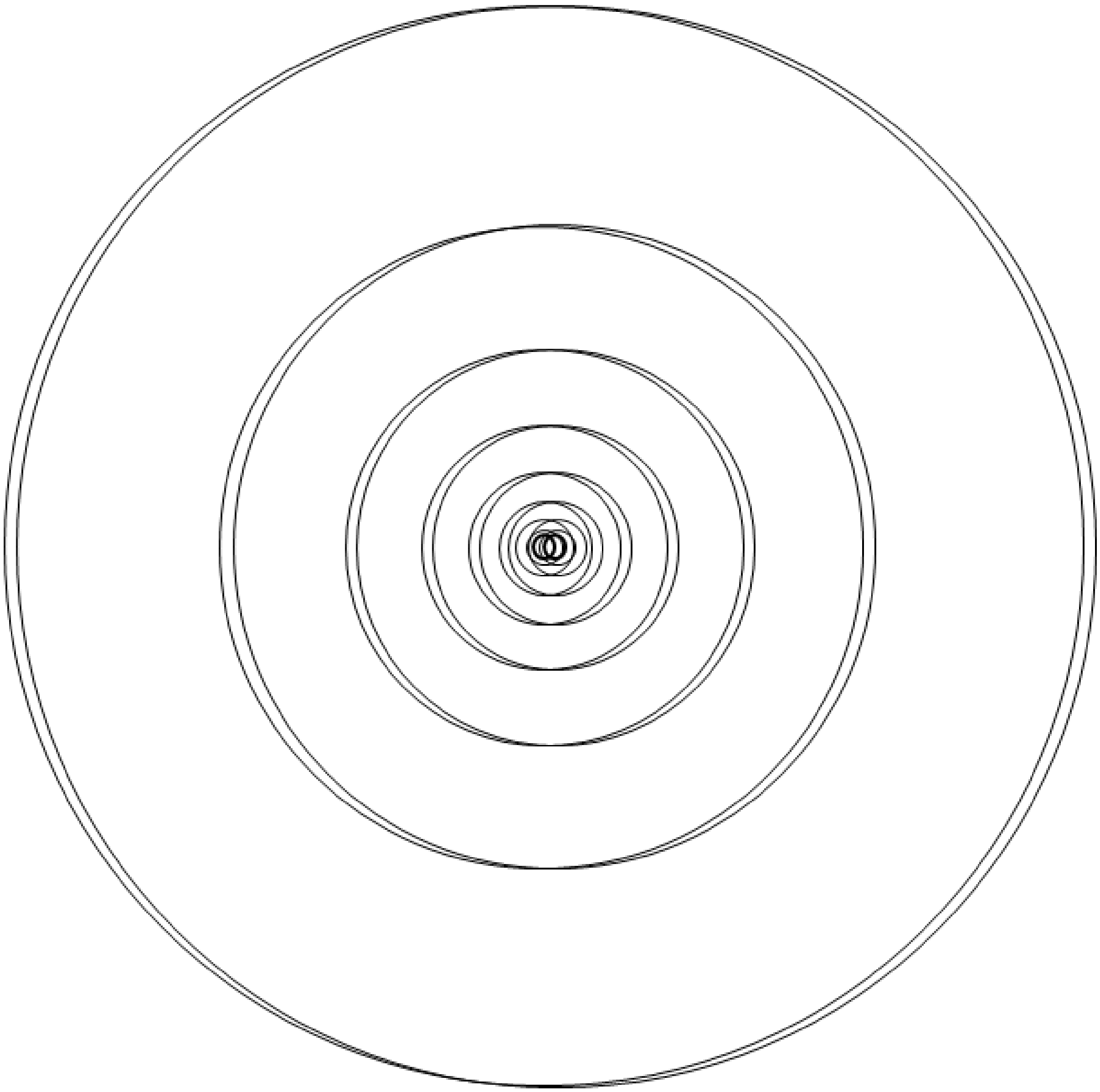


Figure 44 The Limit of the *Vesica Piscis* Phi Power Expansion is a Signature Circle

The practical limit of a band is depicted in Figure 44, above, as the outer *Vesica* pair less one, where the difference appears to be nearly equal to the radius of the original *Vesica Piscis* that can be seen deep in the origin of this sequence. This suggests that the depth of each band of effect, and each band of void is an octave of scales, spanning Φ^0 .. Φ^7. The depth of an octave was intuited by Tygh Simpson in September of 2014, and may soon be verified by calibration of the bands.

As becomes evident in Figure 45, below, the *Vesica Piscis* that forms the foundation of the ***Vesica Profunditas***, is quickly lost in the numerical precision of the exponential expansion of the structure. In Figure 45, the original radius of one has already been reduced to nine *per cent* of the Φ^5 exterior radii, and will be reduced to less than one *per cent* before achieving Φ^{10}, at which point the two circles composing the outer *vesica* can barely be discerned.

At no scale in the *V. Profundis* are the two circles tangent to one another, and the apparent tangency in the Φ^{-1} and Φ^{-2} scales are actually the visual intersection of one of each of those respective scaled circles.

Figure 45 The Limit of the *Vesica Profunditas* Phi Power Expansion is a Signature Circle

Bands of Effect

The prevailing assumption that phenomena operate in a manner that is consistent across the universe is probably correct at identical levels of scale. For example, the distance of an electron from the nucleus of Hydrogen at zero degrees Celcius is likely identical in each and every galaxy that we can view through a telescope.

However, scale changes everything.

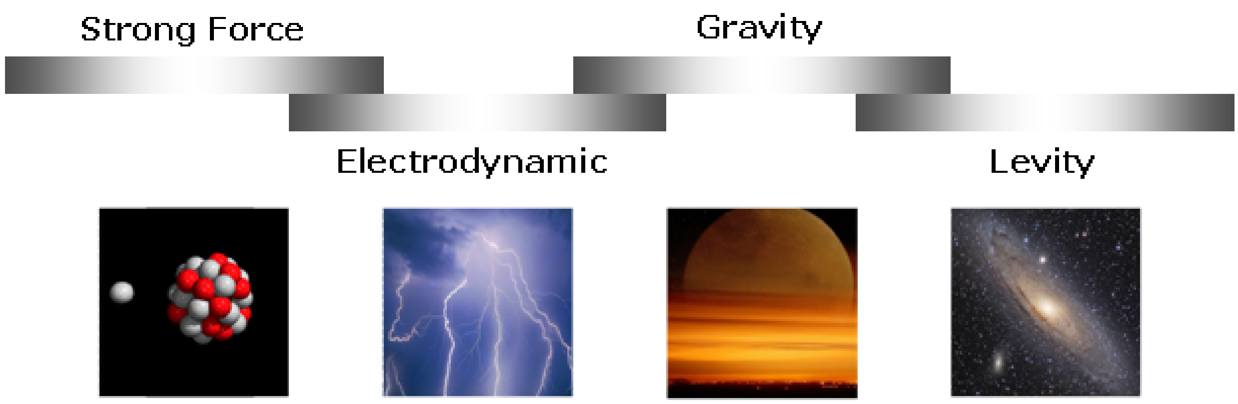


Figure 46 Bands of Effect Overlap or Represent Gradients and Polarities of an Effect

No less a genius than Albert Einstein suggested that many constants probably aren't constant, especially gravity. We suggest that the reason that the dark particle, and the dark force have yet to be discovered is that physics is off on a "wild goose chase." If the universe contained as much dark matter as has been proposed, then distant, and not-so-distant, stars and galaxies would be occluded from time to time.

We would like to suggest that at the scale of galaxies gravity's coefficient changes sign, and magnitude, and gravity becomes levity – transitioning to a repulsive force.

Bands of Void

If it were not for space, we would be packed elbow to elbow with objects of every possible scale, collapsing into a monstrous blackhole which would quickly explode from the energy of its mass.

Bands of void provide freedom, isolation into local domains

The void of space is a necessary condition for separating bodies of mass from excessively coalescing and impacting each other.

Mesoscalar Effects

Mesoscalar phenomena evidence themselves in groups of scale, Φ_j .. Φ_k, in bands of effect that are bounded and isolated from one another by bands of void, Φ_m .. Φ_n. These bands may employ one or more octaves of scale, where an octave would be defined as Φ_j .. Φ_{j+7}.

As bands change, mesoscalar effects are subject to variations in observed “constants,” or are dominated by other effects in the new domain, *e.g.*, the strong force giving way to gravity, gravity changing to levity – repelling galaxies.

This may be a better explanation than the elusive particles proposed to instantiate “dark energy.”

The Phenomena of Energy

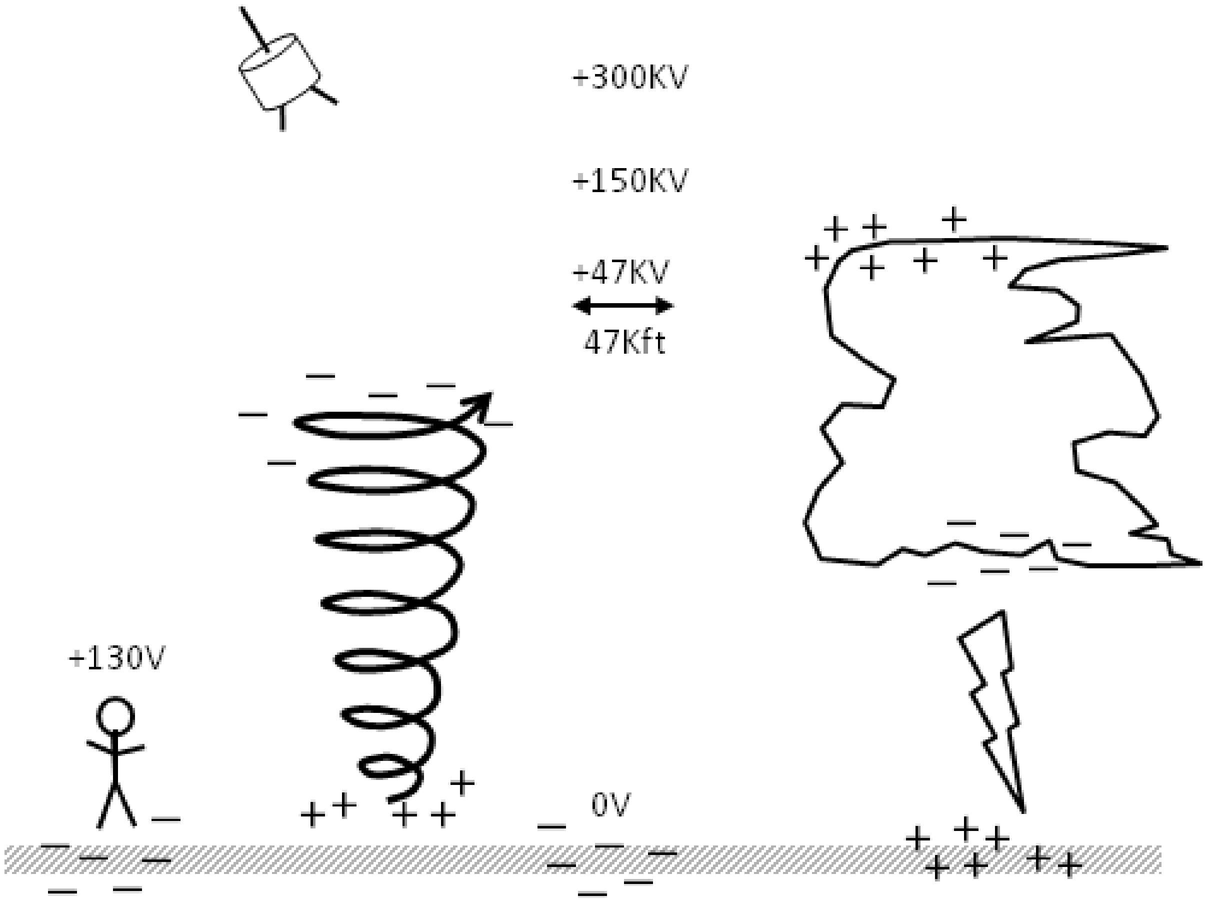


Figure 47 The Earth is Surrounded by an ElectroDynamic Gradient

The wave versus particle is yet another physics conundrum traceable to metrics. Recalling Heisenberg's Uncertainty Principle, a photon cannot be evaluated simultaneously for particle, and wave characteristics. The solution is simple, and it isn't a strawman of wave vs. particle, and instead, it is a particle spinning at a frequency directly proportional to its energy.

There are probably other attributes of a photon that can be attributed to the alignment of the photon's rotational axis with respect to the direction of travel.

The Cooling Universe

Entropy Reconsidered

Entropy

Ectopy / Negentropy

Etheric Energy

Thermodynamics

Conventional wisdom would have us believe that clouds of asteroids are the condensed gas leftovers from planet formation. How many laws of physics would such a concept violate? We are surrounded by examples of natural process, and it isn't necessary to invent, or imply novel processes that don't work.

Asteroids are the leftovers from prior generations of planetary systems, when their suns that went nova, and destroyed planets. The radioactive and not-so-unstable atoms formed by the death of stars aggregate, then decay into smaller daughter products, interact with their neighbor atoms to form new minerals that repeat the decay process, trading partners in mineralogy. Later, these minerals accrete, and form the rocky prototypes that form planets with molten cores, which morphogenesis develops into more complex structures when they cool. All the while, radioactive decay continues to restructure existing minerals from the inside, assuring that there is no stability over long stretches of time.

While the Big Bang Theory is currently in vogue, the ancients used an oruburus to depict the creation of the universe. Their continuous arc in the heavens is more likely a helix than a circle, yet it describes a continuous cosmology, of decay, and renewal.

However, should the Big Bang endure, we might consider that perhaps the universe develops in complexity with each expansion / contraction cycle.

As the universe cools from superhot, post-Big Bang states of matter to colder states of matter, new interactions and physics rules assert themselves. These phenomena are not visible when matter is excessively energetic. Successive unmasking of these already known and identified behaviors can develop patterns which are readily and recursively extrapolated from known physics.

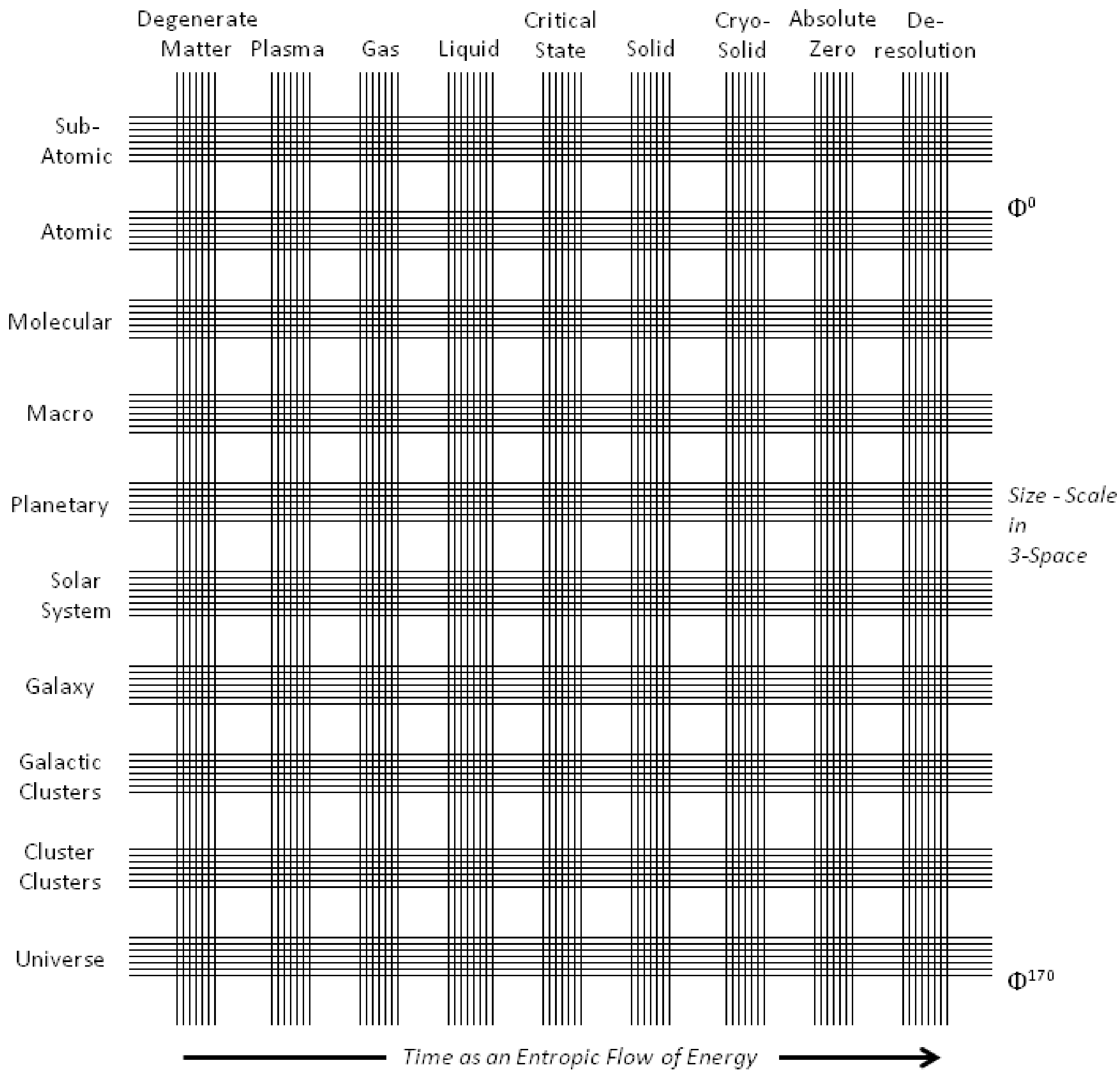


Figure 48 Phi Ratio Scaled Bands of Effect and Void *versus* Energy States

Matter as Condensed Energy

Hyperplasma, plasma, gas, liquid, supercritical liquid, solid, cryogenic solid, absolute zero

The Conch shell expresses the Phi Ratio signature that results from constructing with roots.

Time and Rates of Change

From Kenneth Swartz we have fundamental insight into particle physics. If the flow of time is a continuum, and may not be linear after all, allowing the rate of change of time to be the first derivative of the effect. Rate of Change (RoC) is likely to be somehow proportional to the mass of each particle, and the presence of electrons reduces the headlong rush of time experienced by the proton, and neutron.

Table 4 Mass Figures for Atomic Particles

Particle	***Absolute Mass***	***Relative Mass***
proton	1.6727E-24 gram	1.007316 amu
neutron	1.6750E-24 gram	1.008701 amu
electron	9.110E-28 gram	0.000549 amu

One conjecture would posit that were the relative rates of change to be dependent upon their masses, the RoC of the electron may be normalized to one, for our consideration, while the Proton's RoC is 1835 times faster.

Benjamin Franklin has been accused of wrongly assigning the respective polarities of the proton and the electron. When viewed from the time perspective, it is clear that Franklin was wrongfully charged by his detractors.

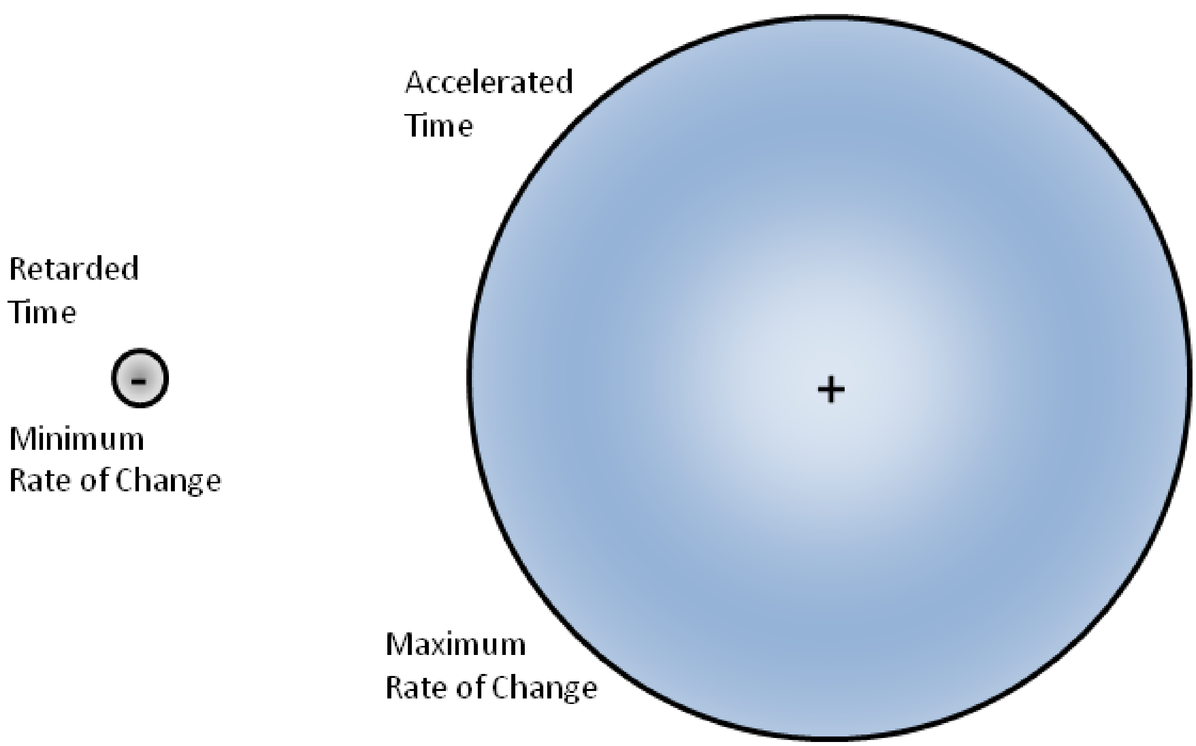


Figure 49 The Rate of Change (Time) may be Proportional to Mass

Atoms that are enclosed by fields of negative, or positive charge may endure a retarded, or an accelerated rate of change, respectively.

Richard Westfall was aware of the charge phenomenon, and considered application of charge to the frame and container of isotopic generators (RTGs, *etc*.) in order to facilitate the storage, integration, and deployment of such aboard mobility platforms, and to deliver such to their final destinations. However, the insight into the RoC effect is entirely Ken's.

Emerging Patterns of Thought

The personal discoveries of elegant features of the Phi Geometry offer up '*Eureka*' moments. In the literature of Sacred Geometry can be found many references to the beauty and esthetic appreciation of the subject.

The Divine Proportion Huntley

R. Buckminster Fuller, in *Synergetics I*, admitted to being puzzled about the relationship between his tetrahedral geometry, the pervasiveness of the sixty-degree angle and the foundational structure of the universe. Had he the tools available from Sacred Geometry and the Phi Ratio, this self-taught genius would have contributed so much more our heritage than he did.

such recursion is found elsewhere in nature, and particularly in Darwin's evolution of species, where the competition between prey and predatory species, and species attempting to occupy the same grazing, or habitat niches press competitors for adaptative mutations. These adaptations are instantiated at a profoundly low level of the DNA genotype and are realized as adaptive attributes in the phenotype.

The Spontaneous Emergence of Phi

it might seem that there is a common school of architecture

while we're not in a position to dispute or support such a notion, it is clear that the Phi Ratio-based Geometry is extensible from simple observation of nature in her most basic form.

Self-Organizing Systems

Nuts and Bolts Applications

Phi Ratio Applied to Architecture

Phi Ratio Applied to Chemistry

Monumental Applications

Rennes les Chateau

Giza

Cydonia

Communicating through Space

What are crop circles and why do they consistently exhibit so many Phi Geometry and fractal features?

one of the expensive *lessons-learned* by modern science in the last few decades is that optimism and millions of lines of program code will never result in true artificial intelligence. While computer scientists and everyday programmers have been able to develop very clever programs, they have yet to develop a program which is innately clever.

and with all of the competing schools of modern psychology, we still don't comprehend how the complexity of the mind is imprinted on the human brain, or any brain, for that matter.

Shaping Mankind's Future

Scaling the Universe

"re-measuring" is the same as "scaling process"

everything spins

bands of mesoscopic effect

remeasuring, renormalization probably occurs at void boundaries

Snap-to-grid may overcome other energy issues to force atoms together which may not otherwise have energy sufficient to connect.

A worthwhile course of study is to identify math constructs which generate phi ratio'd structures. Any additive series will converge on Phi. Phi Ratio may not be the fractal basis (radix) for scaling the universe but it is the measuring index of the result.

The electric charge radius of a proton: 0.84087 Femtometers

The proton magnetic radius: 0.87 Femtometers

Structure

For a while this research effort operated on the assumption that phi was the fractal basis of the universe.

A suspicion that Phi is not the fractal basis, but rather, a signature index to its formative success, began to take hold.

Rachel Fletcher describes a rational geometry that is based upon √3, yet exhibits a familiar Phi Ratio-based geometry. Her exposition makes use of the Vesica Pisces and other Phi Ratio rigors, yet recursively employs √3 rationale for progressive expansion of her constructs.

The √3 Geometry exhibits a feature that evades Phi Ratio rationale, in that it synthesizes right angle constructs that appear to be extensible into at least three dimensions, and plausibly beyond. While these structures can be properly described by Phi Ratio, Phi is not their genesis.

Tools

In the movie 2001: A Space Odyssey, Stanley Kubrick and Arthur C. Clarke compressed a couple million years of human evolution into the memorable scene where Moon-Watcher tosses his new-found tool into the sky, where the Tapir femur is gracefully transformed into a contemporary spacecraft orbiting Earth. The process of the *tool-shaping-the-brain-which-shapes-the-tool* is posited with stunning clarity. Tools take many forms in our intensively technological culture, and mathematics is the toolbox, which shapes our view of everything.

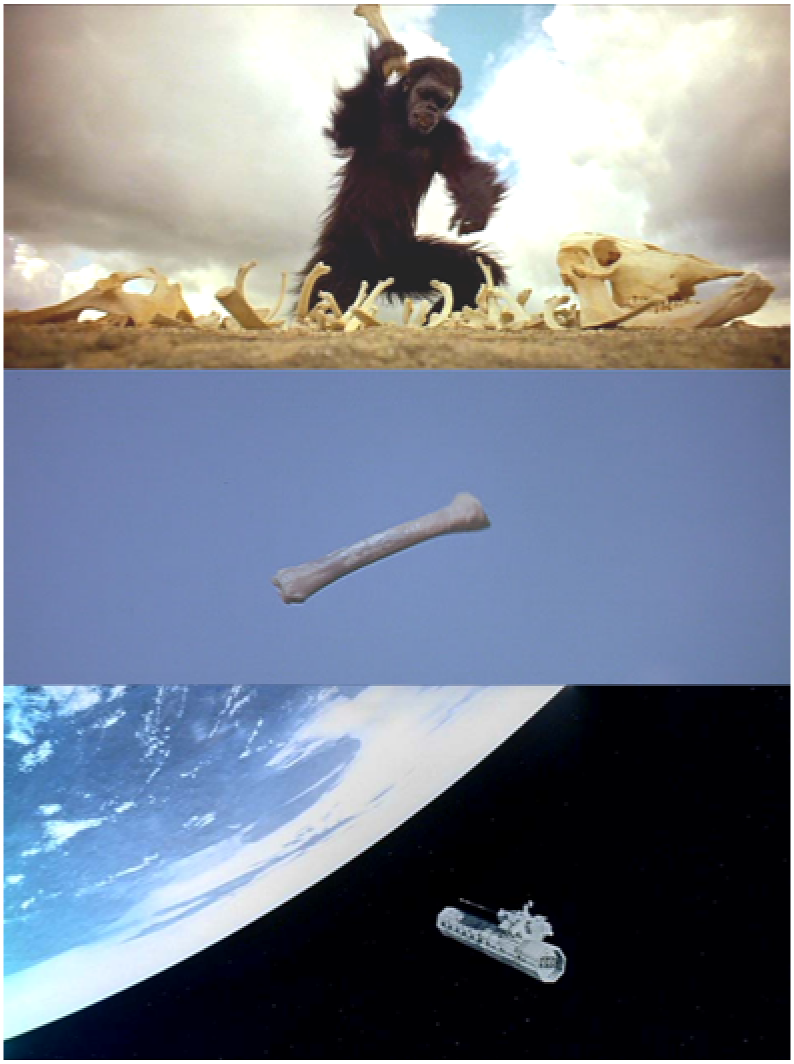


Figure 50 An Opening Sequence from *2001: A Space Odyssey*

The heritage of Phi Ratio Geometry is as old as the oldest of human civilizations and is the foundation of Freemasonry in ancient Egypt, the empires of Sumeria and Assyria, Pharoanic Egypt, Greece, the Indus Valley of India, lost civilizations of Burma and SouthEast Asia, along with the pre-Columbian empires of the Olmecs, Incas, Mayans and more recently, the Aztecs and finally, the Hermites, Knights Templars and Masons of the Mediterranean and Europe. It appears around the globe contemporaneously and not, as civilizations have emerged and declined, and challenges time as megalithic structures.

Those contemporary subject matter experts who would regard Phi Ratio Geometry as odd, esoteric, pedantic, and otherwise unworthy of modern thought, should consider the value and contexts of tools. More specifically, Phi Ratio Geometry is often better, more correct tool for many applications. Conventional mathematics views the universe through a reductionist, analytical perspective and toolset. The extent of the contortions which mathematics will exercise in its efforts to characterize natural phenomena is sometimes astounding to those on the periphery of math and physics. There is a disconnect between the mathematical system of the Western World, and natural phenomena and structure that it seeks to model.

To those who would decry the slight deviations of the Geometry from perfect measure, deviations that are typically accurate to three and four decimal places – know this, that many conventional mathematical models are replete with coefficients, and small epsilon error terms that are used to absorb those irregularities that are found in the analytical model, errors that are often more significant than those exhibited by the Geometry.

That irrational and transcendental numbers are central to nature, and yet are an apparent afterthought, or tack-on, in the many domains of analytical mathematics should alert us to the discord that exists between that which we would measure and model, and the tools that we employ to do so.

R. Buckminster Fuller, who has been recently redeemed by revisionists as being worthy, perhaps more narrowly for his views on regenerative nature and ecosystems than his broader contributions to math and science, struggled to define his seminal[9] work on spherical, tetrahedral and self-described 60° geometry. Fuller was incredibly prolific in his time and he could have made so much more progress, and actually succeeded in the university, if the Phi Ratio Geometries were available as tools for his nimble and creative genius.

A fusion of Phi Ratio Geometry with a computer graphics system representation of tools will begin to offer clarity to many disciplines and accelerate the design and development of technologies more appropriate to today's cybernetic society – a hybrid of organic and machine.

The mechanics and history of Phi Ratio Geometry are available from many books on the topic which have been published in the last twenty years. After a review of powerful key elements, we explore many of the attributes of the geometry as a prelude to defining a computer-assisted design toolset (CAD) which is intended to provide access for researchers in many disciplines to natural self-organizational mechanisms.

Practical application of Phi Ratio Geometry may accelerate design and development by removing the years of discontinuities caused by typical *ad hoc*, trial-and-error methods. Development is enhanced by allowing a single school of practitioners to advance a technology in its entirety through pervasive application of a consistent methodology. A fully-elaborated

[9] Certainly we chose to diminish the Platonic contributions in the modern era.

Phi Ratio Geometry offers an aggressive approach to design which elegantly conforms to natural phenomena with high fidelity.

The ***Hermetic Universe*** is a Computer-Assisted Design platform consisting of a number of interactive and interconnected programs that perform specialized elements of computation, projection and data management. The tools are partitioned so that esoteric Phi Ratio purists may use them without compromising their work with Newtonian analytical mathematics, such as the calculus and differential equations.

The current version of the Hermetic Universe Tool offers two principal classes of tools, tools which are intrinsic to the classic treatments of the geometry, and those which are not, which are typically system-enabled functions which are in a class of their own.

A fusion of Phi Ratio Geometry with a computer graphics system representation of tools will begin to offer clarity to many disciplines and accelerate the design and development of technologies more appropriate to today's cybernetic society – a hybrid of organic and machine.

The mechanics and history of Phi Ratio Geometry are available from many books on the topic, which have been published in the last twenty years. After a review of powerful key elements, we explore many of the attributes of the geometry as a prelude to defining a computer-assisted design toolset (CAD) that is intended to provide access for researchers in many disciplines to natural self-organizational mechanisms.

The ***Hermetic Universe*** is a Computer-Assisted Design platform consisting of a number of interactive and interconnected programs that perform specialized elements of computation, projection and data management. The tools are partitioned so that esoteric Phi Ratio purists may use them without compromising their work with Newtonian analytical mathematics, such as the calculus and differential equations.

The current version of the ***Hermetic Universe*** offers two principal classes of tools, those that are intrinsic to the classic treatments of the geometry, and those which are not, which are typically system-enabled functions which are in a class of their own.

The Phi Ratio Geometry features a high fidelity to natural structures, that is achieved by contemporary mathematics through extremely complex processes that employ highly idealized or abstracted models that diverge from faithfulness to the original natural object or phenomenon.

The CAD software attempts to use the features of contemporary computers to best advantage, all while attempting to facilitate the porting of the CAD system to future systems that may be radically divergent from those same computers. This has implications for the design of the CAD software, and the most significant of these is an effort to avoid advanced analytical mathematics in the system, in favor of employing Phi Ratio Geometry to best advantage. It is expected that such a methodology will encounter difficulties, yet novel solutions are anticipated as the result of such an approach.

Renormalization (re-measuring)

One example is normalization, and its cousin, renormalization.

Setting a dimension, whether measured or not, to be "one" is "normalization."

While a line, measured in centimeters, or the hypotenuse of a right triangle, measured in royal cubits, retains these arbitrary parametrics, they can also be re-asserted as "one" in order to conduct Phi Ratio operations.

Re-normalizing is a re-scaling which is structurally evident in the Fibonacci Series expansion, for example. When expanding by Phi or contracting by its reciprocal, the effect is identical to increasing or decreasing the exponent of Phi, *ie.*, Phi^{n+1} or Phi^{n-1}. In the universe of numbers, this behavior is unique to Phi, and is expressed as an equality:

$$F_n = F_{n-1} + F_{n-2}$$

which is arithmetically equivalent to

$$F_n = F_{n-1} \times F_1$$

or more directly,

$$F_n = F_{n-1} + F_{n-2} = F_{n-1} \times F_1$$

This feature probably explains the many "strange" behaviors and lore associated with Phi, and it is clear that ***the Phi Ratio is where the linear world meets the non-linear***.

The additive series is quickly recognizable in its integer form as the Fibonacci Numbers, where each term of the series is the sum of the previous two terms:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, . . .

While the ratio of two consecutive numbers begins as 1, then 0.5, it rapidly converges to Phi, to three decimal places of accuracy, when evaluating the 34/55 or 89/55 ratios. The convergence continues to tighten as one traverses the series into larger number pairs.

With this series in hand, it is worth revisiting the spiral of Square Root (Fibonacci) triangles of Figure 21, on page 36.

Phi cut

Scaffolding

We first describe a rational universe that appeals to the purely abstract, and orthonormal views of Euclid upon which Western civilization has long been constructed.

We will then disregard the discontinuity introduced into our observed universe of dimensions in the modern era, in favor of a continuum of dimensions that surely have an upper bound of implementation, as most Phi Ratio constructs do.

The geometric structures that serve as scaffolds are themselves constructed as Phi Ratio forms, and the classic examples include the *Vesica Piscis*, and the Mouth of Ra constructions.

In this course, we will remove time as a dimension, and replace it with a notional view that time is a discernable degree of freedom in existing space.

Because the universe is composed of spheres at all levels of scale, and the Phi Ratio has a high fidelity to the universe, universe then eschews orthonormality.

Further, the Phi Ratio is not the fractal basis of the Universe. It is, however, the signature that indicates that things are properly assembled, at the many levels of scale that the Universe operates. The Phi Ratio is a relationship among other numbers, and does not have an absolute numerical value, by definition. If those other numbers change, then the ratio does, as well.

The Universe can be shown to be based, instead, on the Square Roots of the Fibonacci Series: $\sqrt{1}$, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{8}$, $\sqrt{13}$, and so on.

Admittedly, these square roots are not particularly pretty when described by analytical processes using radix ten fractional representation (decimals), but then, what branch of contemporary mathematics is cleanly descriptive of nature? The only branch of mathematics that is succinctly, and precisely descriptive of nature are the counting numbers, which returns us to the origins of mathematics – slashes on a bone, or charcoal stripes on a cavewall that count numbers of ungulates, or days in a Lunar cycle.

The square roots of prime numbers, on the other hand, enjoy an elegance in their simplicity, and in their pervasiveness, and persistence in the universe that rivals the most advanced math to be found in our universities.

A very fundamental aspect of the Phi Ratio, for which this geometry is named, when it isn't being called the Sacred, Hermetic, or Ancient Geometry, is that the ***Phi Ratio is the signature result of synthesizing geometry*** through the use of square roots of prime numbers – and not the cause of the synthesis. When structures and phenomena, whether natural or artificial, are properly constructed, the Phi Ratio appears as the signature of the Creator, affirming that the construct is correctly rendered.

The *Vesica Pisces*, a Phi Ratio creature, can be used to recalibrate the values for each of these formative numbers.

For the last many centuries, and a few millenia the Phi Ratio has been observed in nature, revered in art and ritual, and applied to architecture, engineering, and municipal planning. The Phi Ratio

Geometry has also been recently misapplied in a significant fraction of those venues. The Phi Ratio is a relationship, a ratio[10] of two numbers, and is therefore not an absolute. And while the Phi Ratio is an irrational number, with a never-ending decimal fraction, it is not a transcendental because it is the algebraic result of a quadratic equation.

Companions to the integers, roots are the building blocks that, when viewed from a different perspective, are not exclusively the messy irrationals that the decimal number system, and modern math supposes them to be. In **Error! Reference source not found.** the classic Pythagorean 3-4-5 Triangle is shown to be a creature of the Phi Ratio Geometry, and the many observed instances of the pervasive 3-4-5 Triangle are actually expressions of the pervasive Phi Ratio that are usually indistinguishable when viewed near the limits of our measurement systems methodologies.

Purists among the ranks of mathematicians attempt to debunk the Phi Ratio Geometry as numerology and yet the evidence exists, and difficult to dispute. The math and physics community often uses coefficients (non-linear) and little-epsilon (linear) corrections to “fudge” their working equations to more closely fit the observed “real world.” Those factors and terms of correction are a result of constraints imposed by contemporary measurement, compounded by the granularity of the decimal radix. The latter is a wondrous invention, and its limitations are consistently overlooked, or ignored, especially in the metrology of science.

While the analytical expressions of the geometry sometimes exhibit slight variation from the ideal, the geometry itself has a very high fidelity to natural expression, Q.E.D.

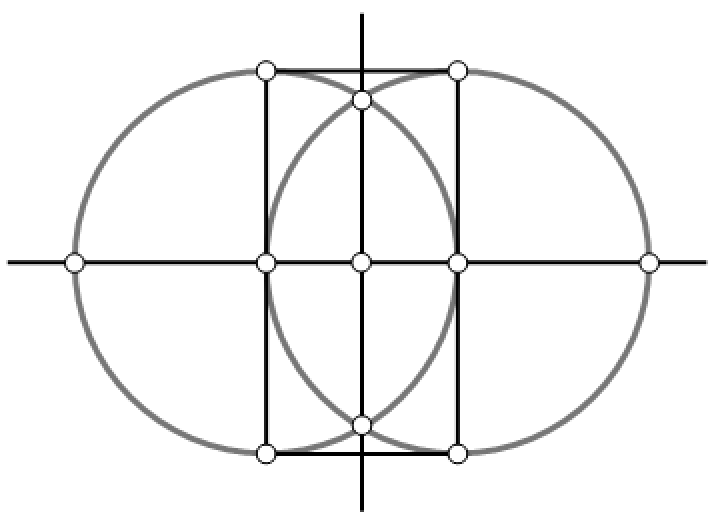


Figure 51 The *Vesica Piscis* with Primary Gridpoints Highlighted

[10] In Greek: logos.

When the *Vesica Pisces* is elaborated with meridians, and a double square, a number of key intersections emerge that form the grid upon which new geometric devices can be constructed. This is a natural mathematics, a math of space that is consistent with nature because it is derived from nature. When modeling the natural order we find that the natural world is dominated by transcendental and irrational numbers, and treatment by conventional mathematics is crude, awkward, and incongruous.

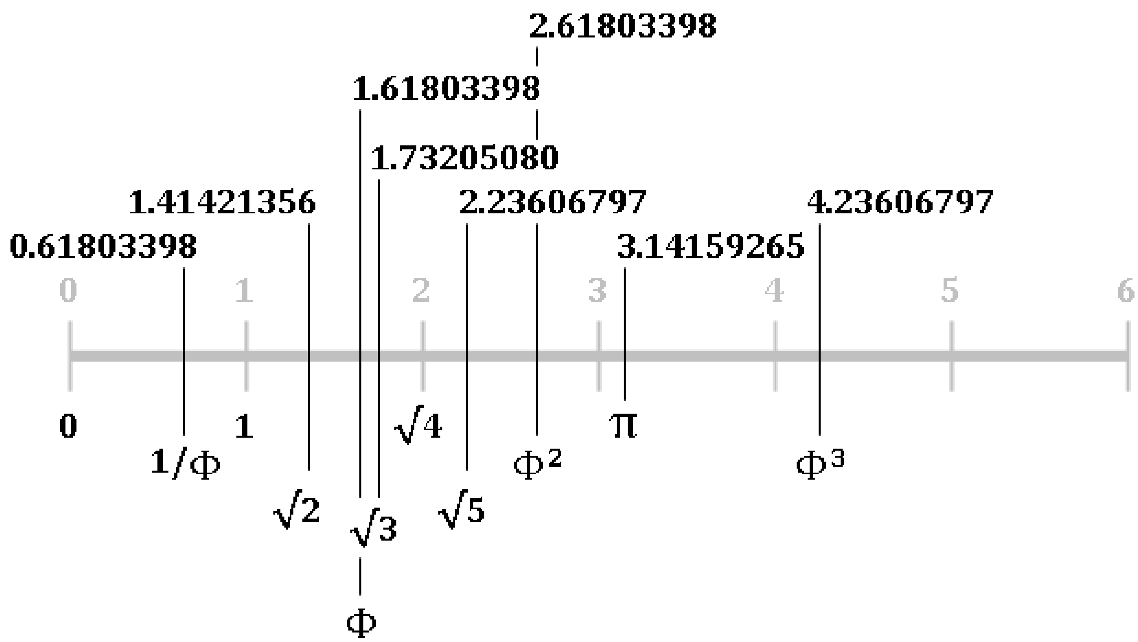


Figure 52 A Number Line Contains Irrational and Transcendental Numbers

The Phi Ratio Geometry has two recurring pillars, Phi and Pi, and it becomes clear to the PRG practitioner that the Geometry is "all about" circular planes, and spherical space.

Circulus

A simple circle can serve as a scaffolding, upon which complex constructions can be rationally developed. Nearly all other Phi Ratio Geometry scaffoldings begin with a circle, and most rely on two, three, four, or more circles for realizing complex, or merely accurate, constructions.

The *Vesica Piscis*, Mouth of Ra, and all of the other scaffoldings that are featured here begin with at least one circle, providing a grid work upon which the architecture labors.

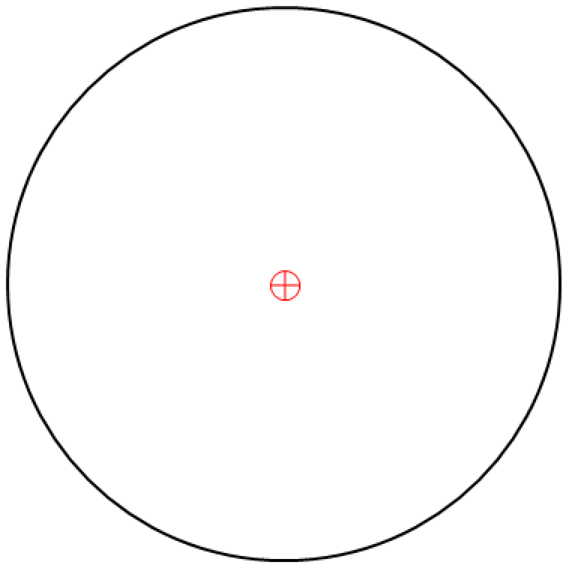


Figure 53 As with the Universe, a Circle is the origin

Vesica Pisces

There are an infinitude of possible *Vesica*, since intersecting circles are as divisible and continuous as the number line. However, there is by definition, only one *Vesica Piscis*, and it is distinguished from all the rest in that two [identical] circles with the same radius intersect so that each has a point on its perimeter [tangent] that crosses through the center of the companion circle. The name of this device translates to "bladder of the fish," where the bladder is seen as the lens-shaped area common to both circles.

Most of the heritage and mythology associated with the Phi Ratio Geometry comes to us originally from the Egyptian Freemasons, and was conveyed and adorned by the Graeco-Roman Empires, leading into the Christian, and later, the Knights Templar, and later still, the Masonic Traditions. In these mythologies the *Vesica Piscis* is the "*mother of creation*," and in methodology it is the foundational grid for constructing geometry.

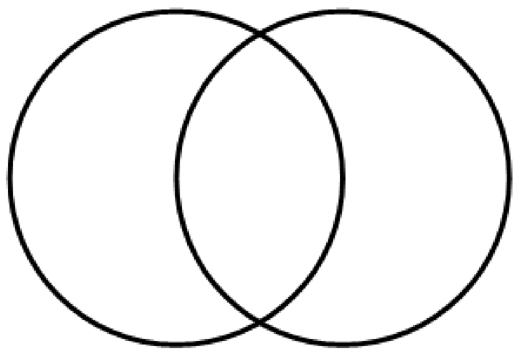


Figure 54 The *Vesica Piscis* Features Perimeters Tangent to the Centers of Unit Circles

In practical terms, the *Vesica Piscis* is a simple, circular grid that is extensible in a very dramatic way, and can be used to devise two, three, and four dimensional objects. When the *Vesica* is elaborated with meridians, and a double square, a number of key intersections emerge that form the grid upon which geometric devices can be constructed. This is a natural mathematics, a math of space that is consistent with nature from which it is derived. We find that the natural world is crisp and clear when viewed through the Geometry, yet when we employ mathematics based upon the positional, decimal number system, we find nature to be dominated by transcendental and irrational numbers. This is an artifact of translation.

The *Vesica* is the progenitor that derives geometric objects with ease. The Phi Ratio results from a quadratic equation, and is therefore, not a transcendental number. In practical terms, the *Vesica Piscis* is a simple, **circular grid** that is extensible in a very dramatic way, and can be employed to devise a multitude of two, three, and four-dimensional objects.

When the *Vesica* is elaborated with meridians, and a double square, a number of key intersections emerge that form the grid upon which new geometric devices can be constructed, per Figure 51 on page 79. This is a natural mathematics, a math of space that is consistent with nature because it is derived from nature.

Two-dimensional shapes, or objects that may be quickly derived from the foundation of the *Vesica Piscis* include the triangle, square, rectangle, pentagon, hexagon, heptagon, *etc*. The entire family of three dimensional Platonic solids of Tetrahedra, Hexahedra, Octahedra, Dodecahedra, and the

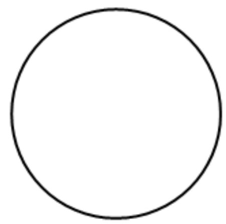


Figure 55 Draw the Beginning and End

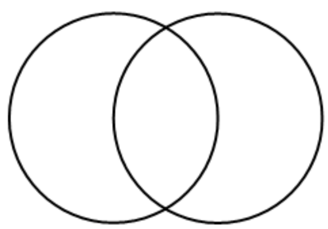


Figure 56 Draw the Dyadic Companion, the *Vesica Piscis*

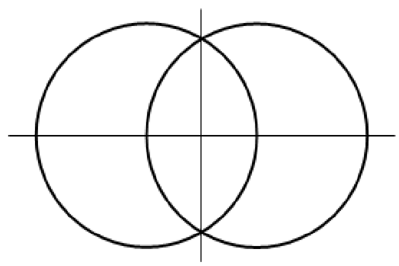


Figure 57 Establish Registration Axes

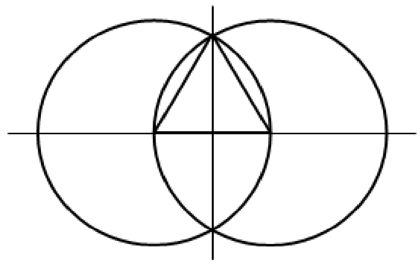


Figure 58 Inscribe an Equilateral Triangle for Reference

Icosahedra can all be assembled on the framework of the *Vesica Piscis*. Other, more complex polyhedra can similarly be constructed.

Vesica Profunditas

When the component circles of *Vesica Pisces* are expanded, or contracted about the centers of their respective circles, as in Figure 14, the outer circles merge to form an ellipse that is initially dimpled at the bottom and top of the *Vesica* lens, and this soon smooths out as the expansion reaches its limit of a circle.

Observe that at no scale do the circles of any *Vesica Piscis* touch at a tangent point, in spite of the apparent contact among differing levels of scale.

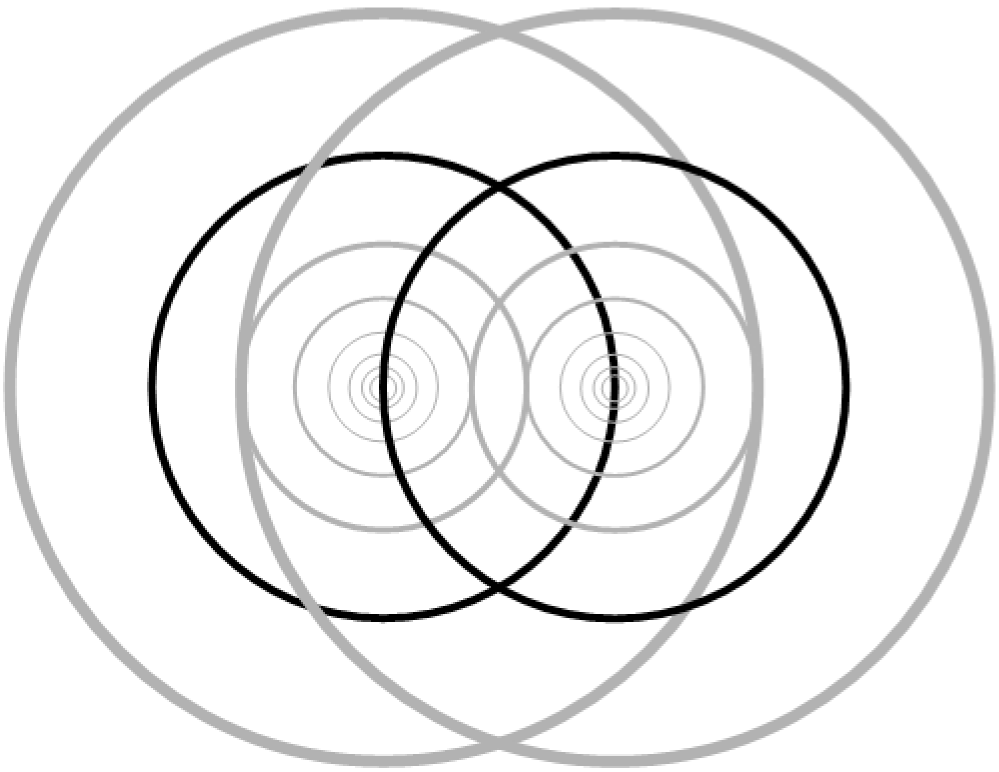


Figure 59 The *Vesica Profunditas* is a *Vesica Piscis* Enhanced or Diminished by Phi Ratio

The name of this structure attempts to capture the fact that a *Vesica Piscis* is deeply imbedded within the structure of the *Vesica Profunditas*, and the distinction between the component circles vanishes at the scale where the original separation of the centers by a *radius of one* is irrelevant, immeasurable, or indistinct. This leads to the octave boundaries that are posited as the depth of bands of effect alternating with bands of void.

A *Vesica* is any lens-shaped intersection of two circles. A formal *Vesica Piscis* has a distinctive relationship, by definition: the two circles have identical radii (diameters), and each center is tangent to the perimeter of the other circle, per Figure 60, below[11].

Any plurality of the *Vesica Piscis* is termed *Vesica Pisces*. The width of the construct is three, and the chord of the lens that defines its major axis is √3. These numbers can be of any, arbitrary measure, but are normalized to their values of 1, 2, √3, Φ, 1/Φ and others.

The value of a *Vesica Piscis* or any other scaffolding is that they provide a grid that is is a radial grid, and not rectilinear, providing points of reference that are usually integer, square root, or Phi Ratio relationships.

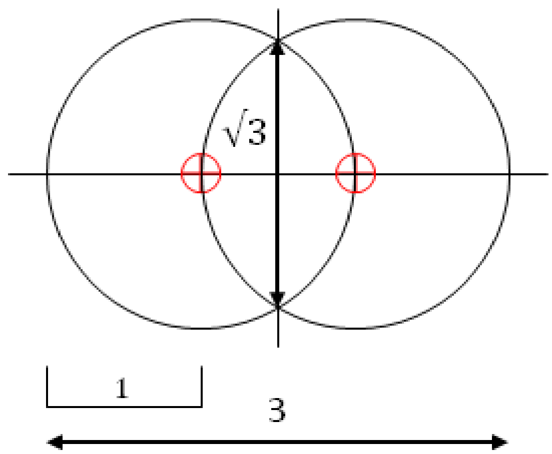


Figure 60 *Vesica Piscis* with Meridian and Equator

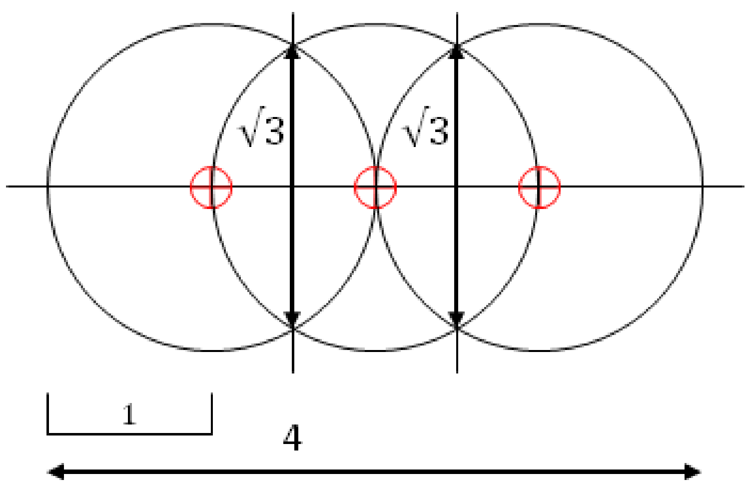


Figure 61 *Vesica Piscis Duplex* Features Two *Vesica* Lenses

[11] The crosshair indicates the center of a circle when drawn with a compass

A *Vesica Piscis Triplex* is obtained through the addition of a second, and third *Vesica* lens to the ensemble, where the new circle is centered at the apex of the original *Vesica* lens.

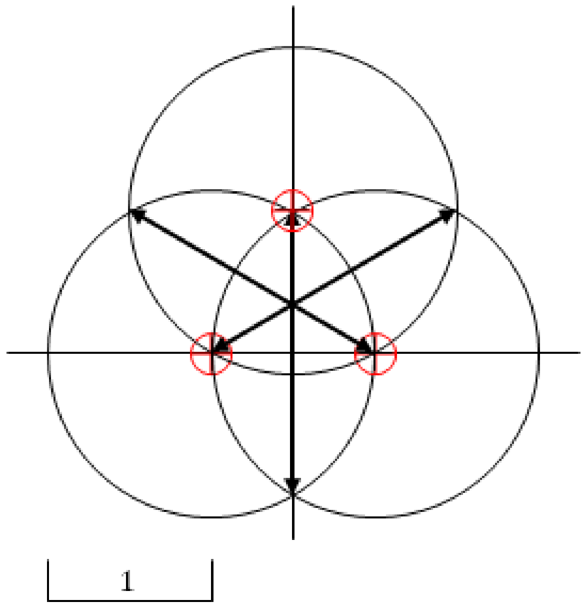


Figure 62 *Vesica Pisces Triplex*

A *Vesica Cruxis* is obtained when another *Vesica Piscis* is overlain at an aspect perpendicular to the original. An intermediate circle with a diameter of one is used to register the centers of the new circles on the vertical axis.

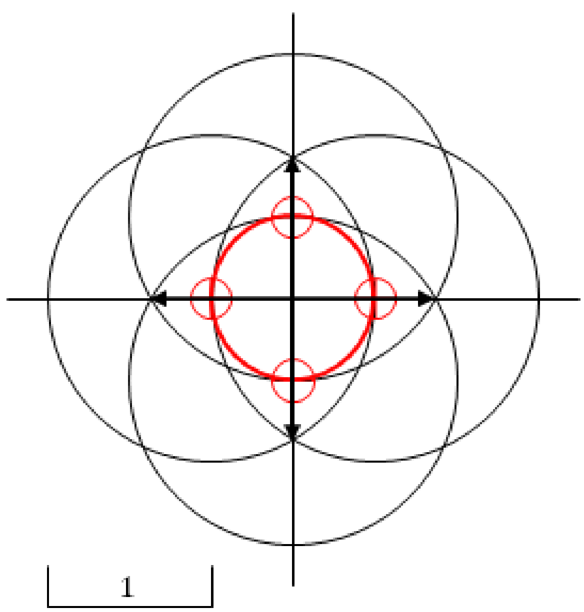


Figure 63 *Vesica Pisces Cruxis*

A pair of unit squares are stacked vertically, yielding √2 and √5 diagonals, which augment the √3 of the *Vesica*'s major axis (vertical).

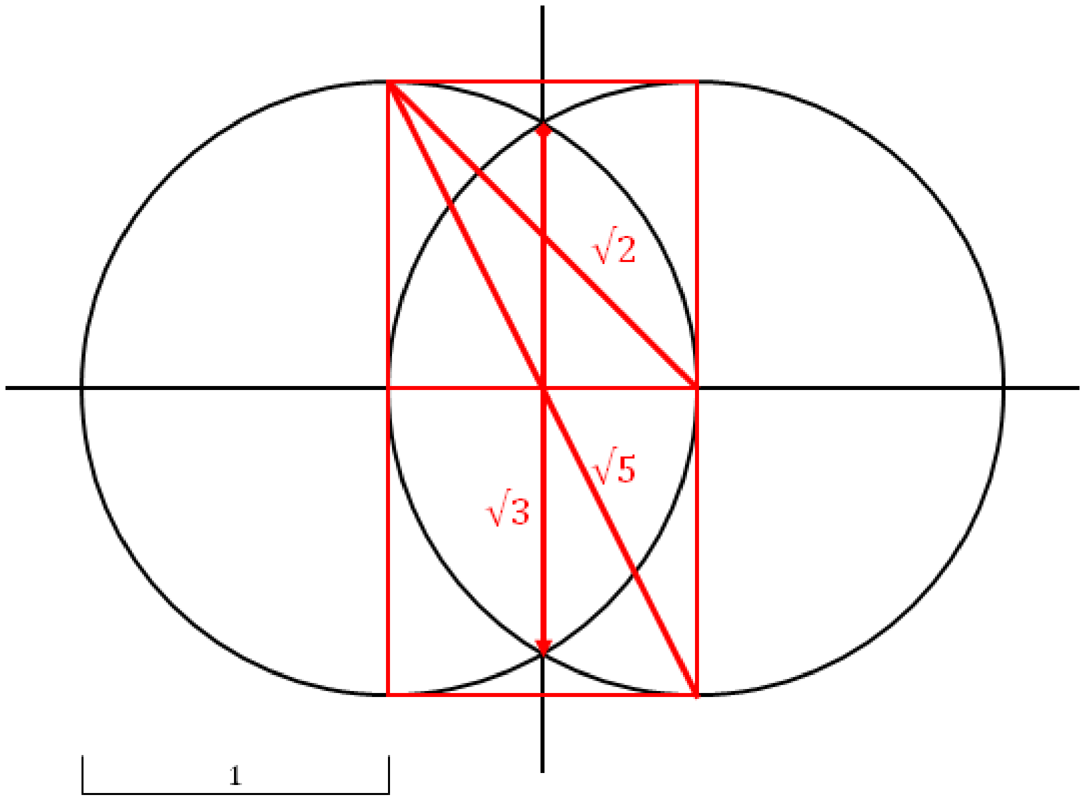


Figure 64 Deriving Square Roots of 2, 3 and 5

The Vesica Piscis is, perhaps, the most versatile of all of the scaffolding tools, and for that reason it is depicted here with many derivatives, and applications.

A circle centered on the origin, per Figure 65, sweeps the √5 hypotenuse, crossing the other √5 hypotenuse, and subtending the horizontal axis. Professor Frederick O. Mills has discovered a rhombus with vertices on the √5 arc at the major horizontal axis, the endpoints of the √3 *Vesica* lens minor axis, exhibiting √2 sides, and the appearance of 3::4::5 Right Triangles.

This structure exhibits the square roots of three prime numbers as a simple extension of the *Vesica Piscis*. As described previously, connecting 3::4::5 Right Triangles with square roots was simple, especially given the pervasiveness of both in the geometry. This also reiterates the value of the Vesica Pisces family of scaffoldings, which is considerable.

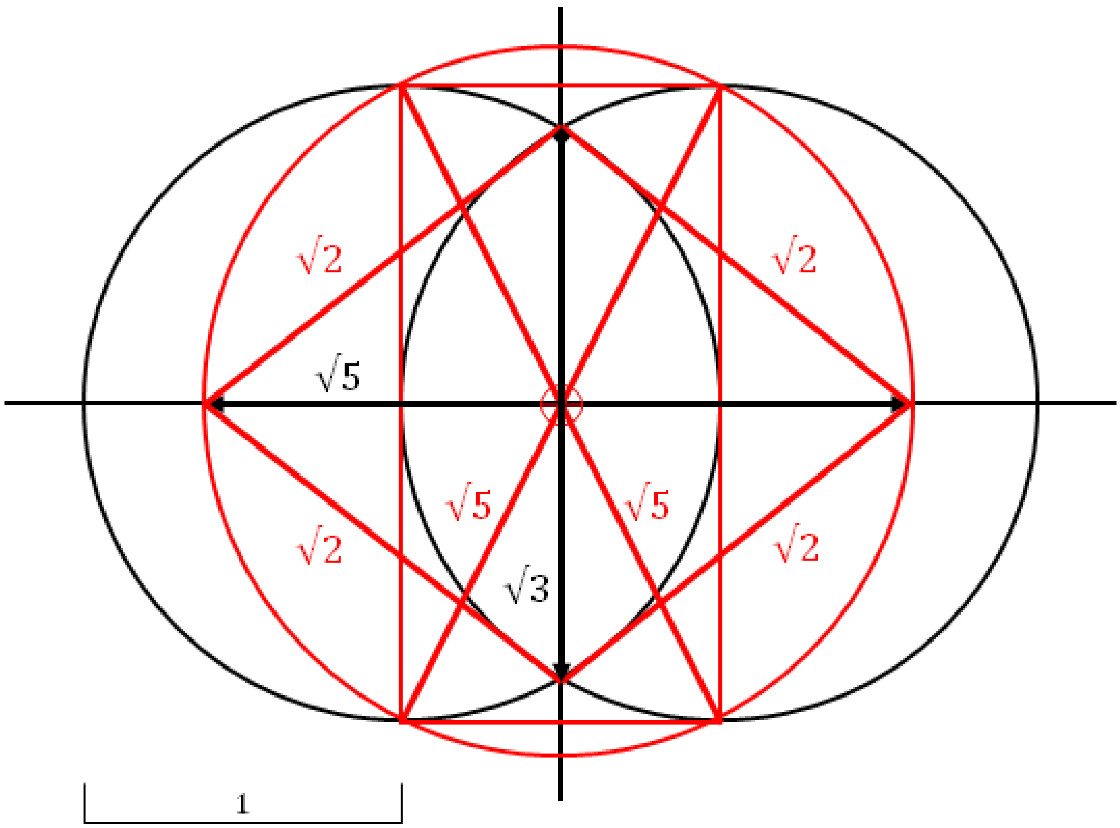
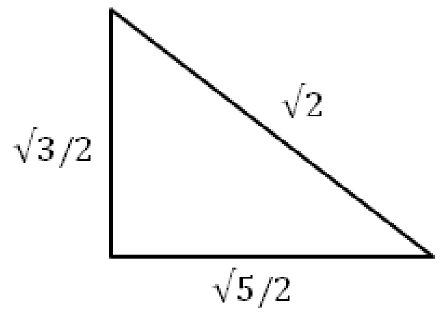


Figure 65 *Vesica Piscis* with Imbedded Rhombus and √5 Circle

In addition to Professor Mills' *Three Root Rhombus*, the *Vesica Pisces* and derivative constructs can be seen to feature many other square roots when one is alert to them – such as those highlighted here.

This is a key feature of the Phi Ratio Geometry. Square roots are the building blocks of the universe, and Phi Ratio relationships are the ***signature imprimatur*** that a construct, or phenomenon, has been correctly synthesized. Phi Ratio is a relationship, not an absolute, and ***is not the Fractal basis of the Universe***. This distinction is important. We do not build with the Phi Ratio, we reverse-engineer with it.



$$(\sqrt{2})^2 = (\sqrt{3/2})^2 + (\sqrt{5/2})^2$$

$$2 = 3/4 + 5/4$$

$$2 = 8/4$$

Figure 66 *Vesica Piscis* Imbedded √2 Rhombus Quartered for Symmetry

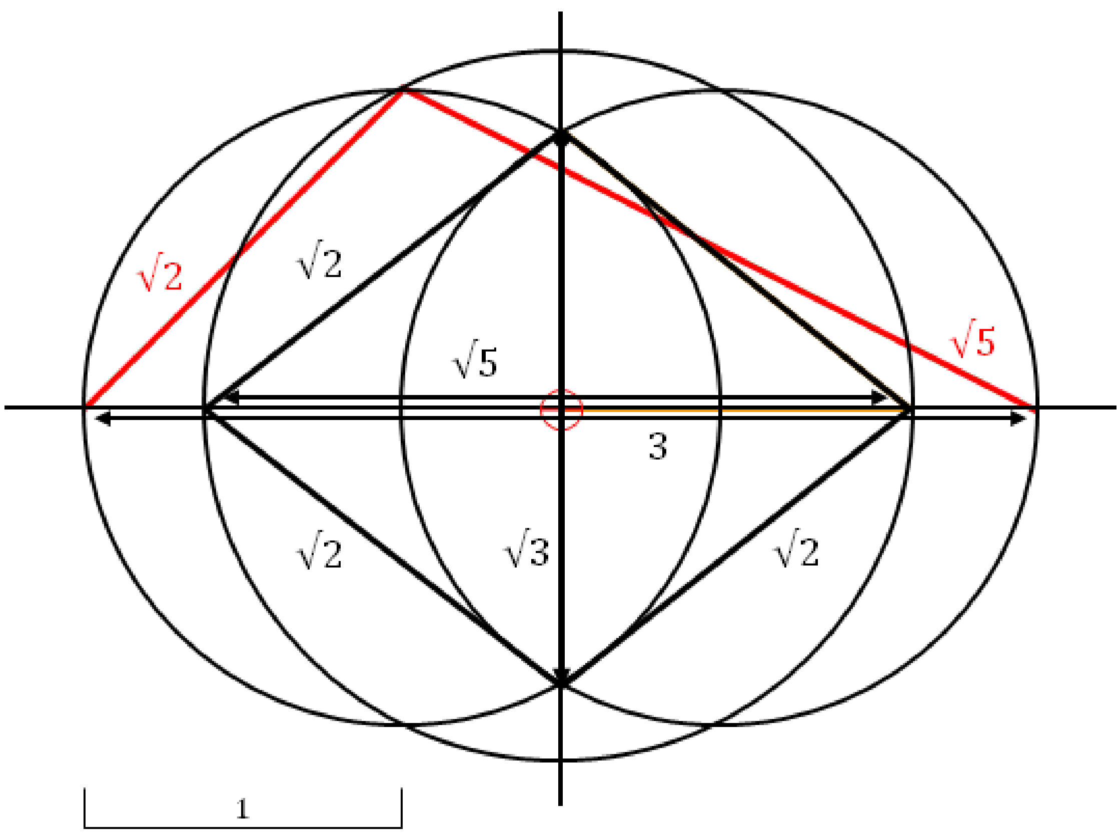
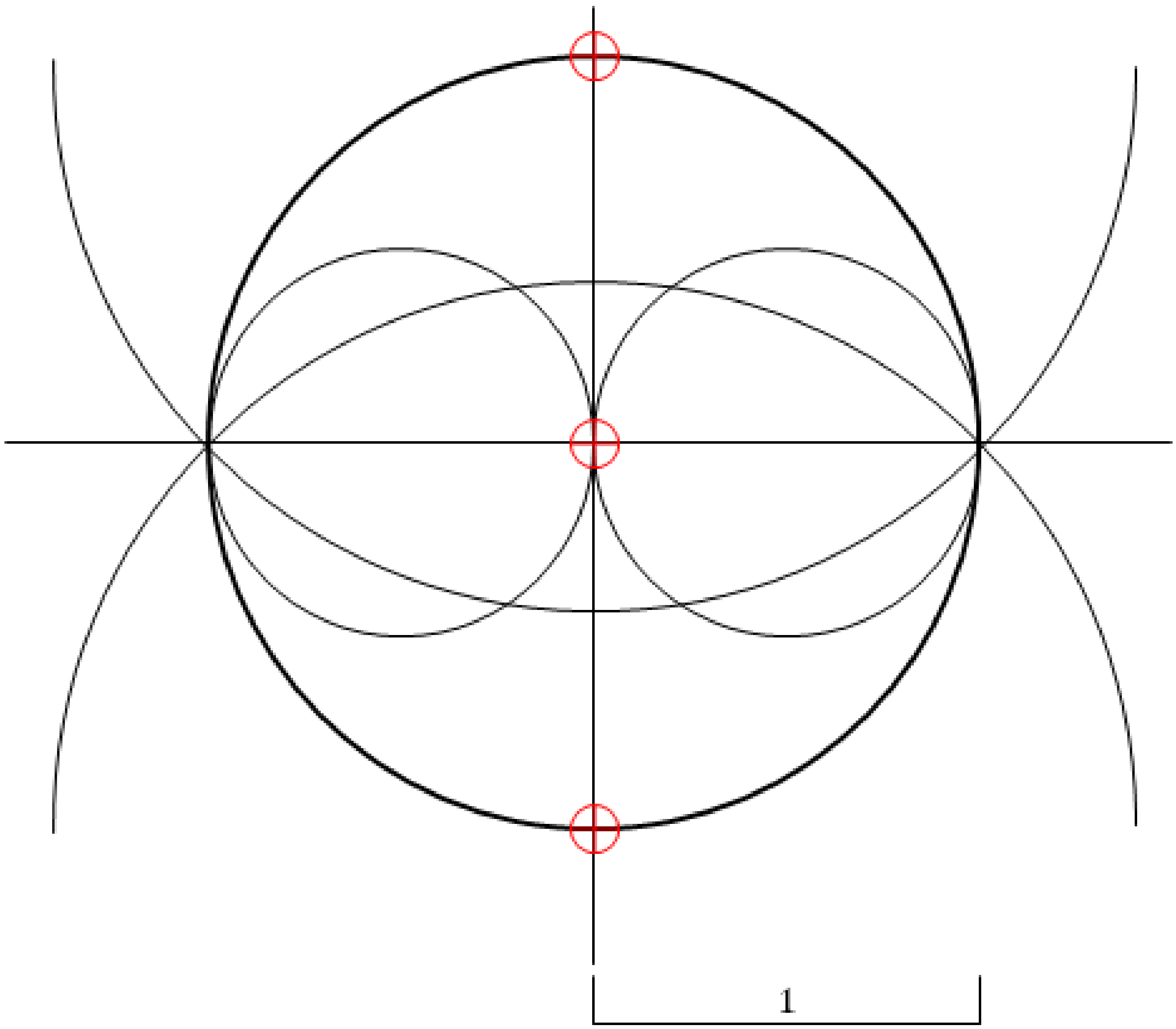


Figure 67 Other Roots are Exhibited in Vesica Piscis Scaffolding

Mouth of Ra

Another PRG construct that occasionally surfaces is the Mouth of Ra, which serves as a scaffolding or grid.



Star Recursion

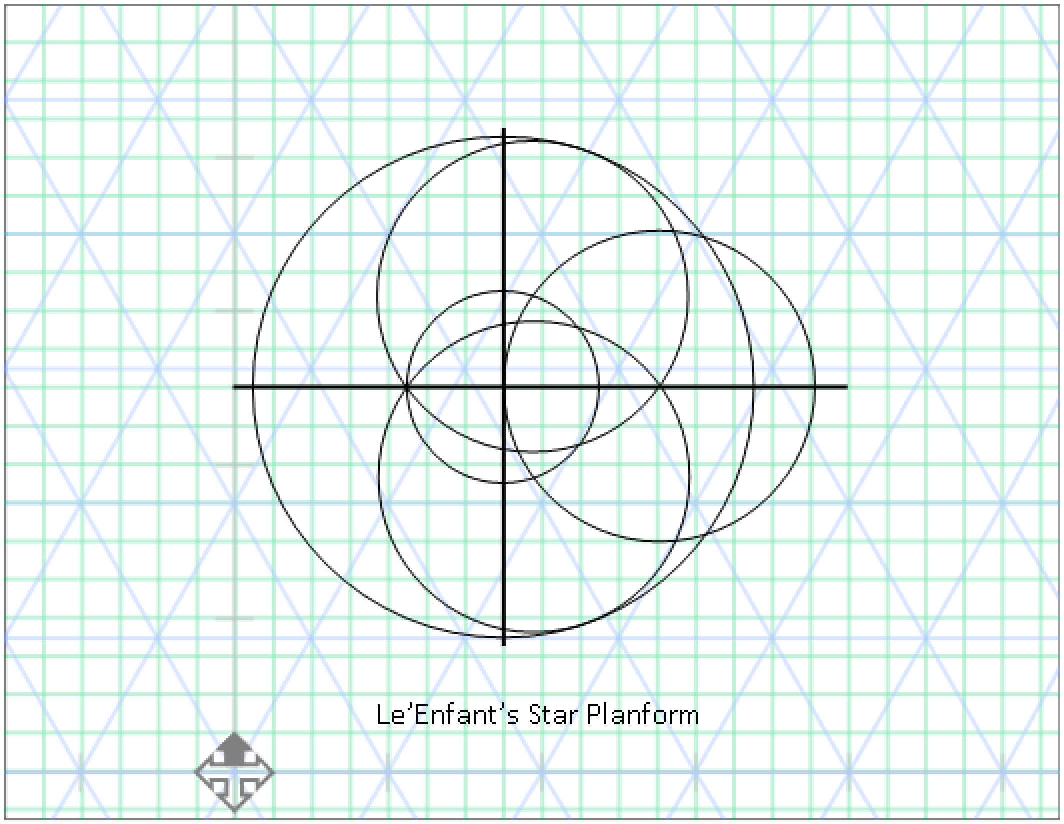


Figure 68 Le'Enfant's Star Basis Vesica Bench

Nicholas R. Mann, *The Sacred Geometry of Washington, D.C.: The Integrity and Power of the Original Design*

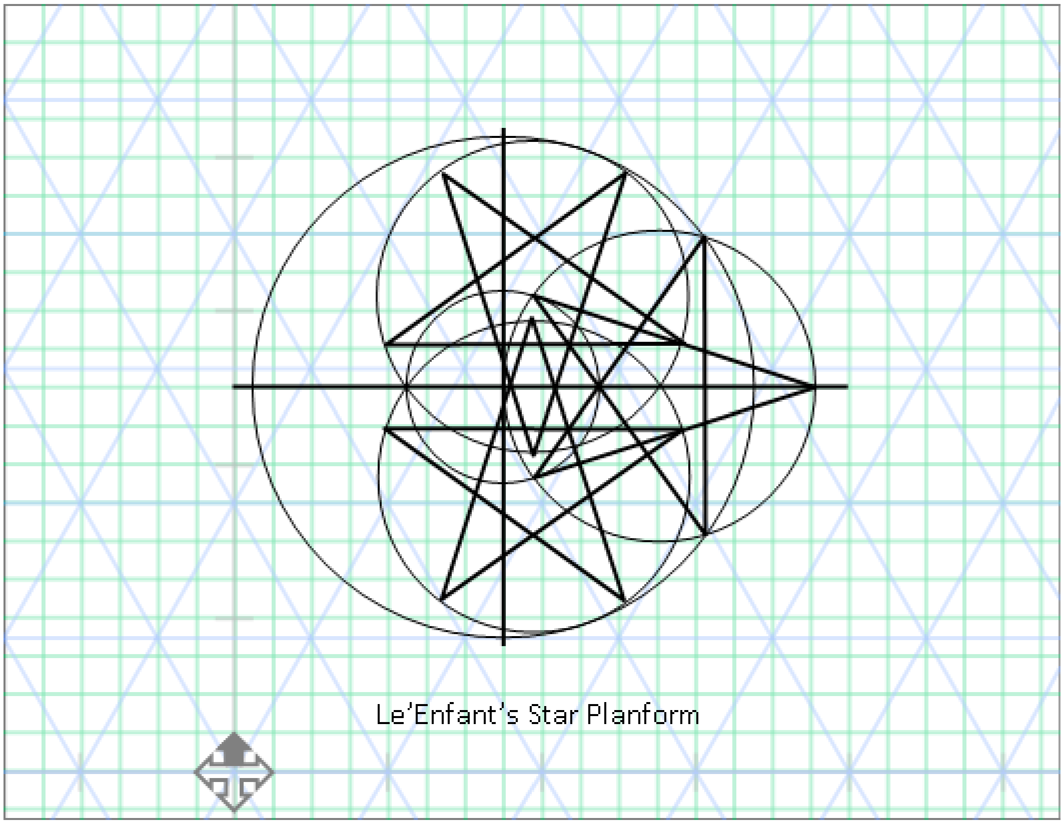


Figure 69 Le'Enfant's Star Planform for the District of Columbia

Masonic Tools

Compass, square, plumb, straightedge, and an apron in which to stow them. The occluded tool was the ortholinear, or rectilinear grid, and cannot easily be carried about in an apron. The grid, however, was to be discovered on the floor of each Masonic Temple, only to be confused by the unknowing with a chess board.

Grids

Grids are an essential device, or tool, for translating analytical mathematics, such as Algebraic Equations, to a spatial, or planar representation such as the Cartesian plane.

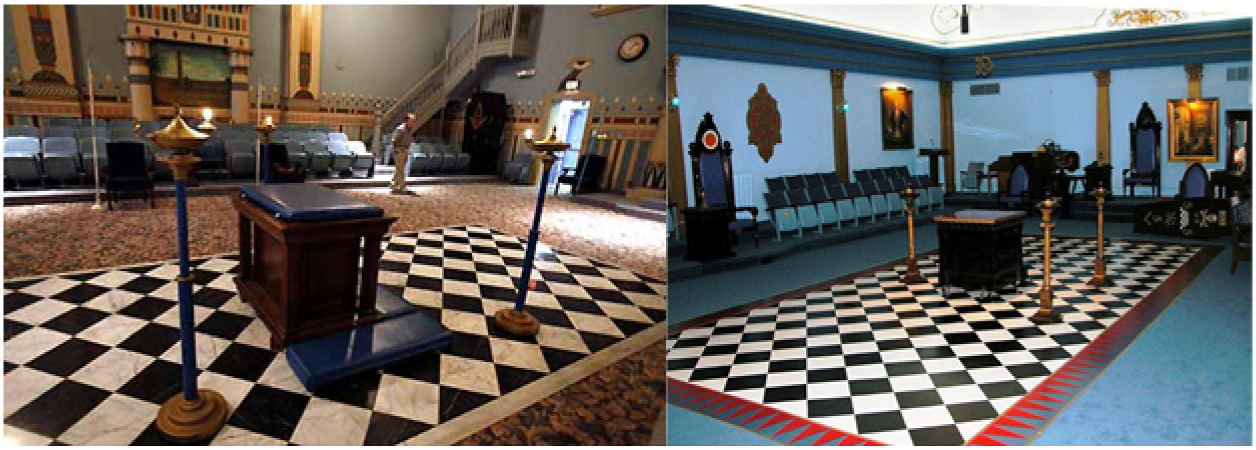


Figure   Each Masonic Temple has a Gridded Floor

Figure   Masonic Banners Depict the Gridded Floor as a Central Feature

Professor Frederick Oliver Mills and I are acquainted with Masonic lore, and it is common knowledge that the Freemasons have been around for millenia, and that they shepherded the Platonic, Archimedean, and Ancient (Golden Mean) Geometry through the turbulent Middle Ages, and the Crusades. Professor Mills relentlessly pursued the grid topic for three or four years, and made much progress on prizing out the utility, purpose, and mythology that support the unstated role of grids to the Geometry.

The grids that resulted from Professor Mill's efforts, however, were confined to ortholinear representations, and were often tied to optimizing the depiction of a given PRG construction. The figures also, and always featured heavy black borders that could not be discarded no matter the argument. The author's thinking is that each of these constructions, no matter the size, or scale, was a finite device in an infinite ocean, and to place a heavy boundary around them was to irrationally constrain one's thinking. Such borders are deliberately absent in the accompanying illustrations because the perceivable universe is unbounded, in any practical sense.

Grids as an Occluded tool

The Masonic Tradition makes use of a tool that is probably not recognized as such, yet has applicability that is profound. The Masons themselves may not be aware of its significance. Professor Frederick Oliver Mills made the discovery of this tool during the course of his extensive investigations of the Phi Ratio Geometry.

Grids as Tools

Grids are synthesized through recursive application of intervals, which can be a constant or an expression, perhaps as simple as a statement in a Forth syntax (postfix).

The engineer's pad with its familiar blue or green grid in a quadrille or a tenth-inch pitch, is a useful tool when shaping the universe around us. Its principal utility is that it establishes a scale and metric to what would otherwise be just another crude sketch of an idea in its seminal stage, replacing the right angle rule of the stonemason of an earlier age. The engineer's pad lends credibility and a bit of technical authenticity to the sketch.

Frequently a drawing is scribbled without regard to the limits and order enforced by the light blue

graticules, yet the grid often serves as a guide to at least the first few linear traces of pencil or ink, as the concept takes form and mind attempts to crystallize a fleeting thought.

Two dimensional objects cast on the grid can take advantage of the ortholinearity of its parallel lines on two axes, the intersection of any of which may be an arbitrary origin. Three-dimensional objects may be successfully depicted, using the gossamer lines and imprecise intersections as references for a higher order.

The engineer, artist or student soon takes advantage of this tool in his everyday thinking and his worldview is shaped to be overlapping orthonormal grids of myriad scales, which define the billions of objects and their locations within his experiential universe.

The grid is a very valuable tool when employing Sacred Geometry, for the above-cited conveniences. The grid can be employed, whether at the major graticule lines or the minor, to reliably draw a 3-4-5

Pythagorean triangle – or to sketch a near-circle by filling in the arcs between roughed-in hashmarks. The grid is alluded to in the black and white chessboard pattern found on the floors of Masonic Temples.

With the re-normalization (or re-measuring) facility of the geometry, the foundational grid may be exchanged for another, so that when a 55 x 55 grid no longer offers the requisite precision, a 144 x 144 grid may cheerfully take its place.

Professor Mills has determined the grid to be a powerful, though forgotten, tool of the geometry. It has often indicated or implied a missing piece, or identified the location for the next polygonal vertex of a construction. Having made this discovery, it is reinforced by tradition where the alternating black and white squares of the Masonic temple join the compass, rule and plumb already ensconced in the Mason's tool apron.

However powerful this tool paradigm, the grid cannot be viewed as a firmament or ether upon which all things reside.

The Orthonormal Grid

The typical grid may be represented as an 8 x 8 pixel area bounded by major or minor graduations. A 2 x 2 may be considered to be an atomic grid, not further reduceable, and an application of such a grid with such a fine, binary granularity may be rare. The projection or graphic display is considered to be a first quadrant construct, (+x,+y), in the standard Cartesian Plane employed by conventional mathematics.

The pixels which edge the left and the bottom of the subgrid are colored to be graduations. These are the essential component of the representation for it is these edges and intersections of rectilinear, orthogonal edges which provide the cardinality of the geometry's vertices, pivots, lengths and diameters. Very few points of interest will ever lay within the colored interior of a subgrid, *ergo*, most points of interest will be at the (0,0) coordinates of the subgrid.

Since the digital computer display is a Cartesian plane of pixels, the display is addressed in terms of X and Y coordinates. When the Hermetic Universe grows to three dimensions, then the addressing will include the Z axis. In conventional systems the addition of additional axes also represents an expansion into new dimensions of storage, and non-linear growth of the requisite address fields, as well. This will not be the case in the Hermetic Universe, and this assertion will be elaborated elsewhere since it is a direct result of our *machine cognition* research.

In CAD and drawing systems a canvas is a bounded plane upon which images are drawn, drafted or otherwise constructed. A canvas is usually a plain white backdrop that is bounded as a dimensioned plane that are often specified by sizes of printer paper stock. Since the canvas is a uniform white, its color value for the entire plane can be held in a single cell, instead of wasting a plane of memory for a uniform backdrop. Therefore, plane zero can be specified to be the canvas plane that conveys the dimension for all subsequent layers, and whose color is held as a single value. This also permits the storage-saving feature of now allowing for a backdrop color that doesn't cost an entire plane of memory.

When a drawing image is painted on the display, each pixel is computed by first of all copying the background canvas cell, Layer 0, to the next active image buffer. Each layer of the display is then written to the image buffer, starting with Layer 1, and is followed by the next layer in sequence, until the top layer is copied.

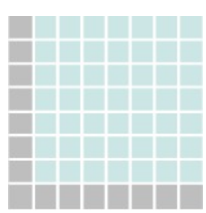


Figure 70 An Atomic 8 x 8 Gridblock

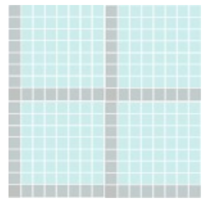


Figure 71 A Small Cluster of Gridblocks

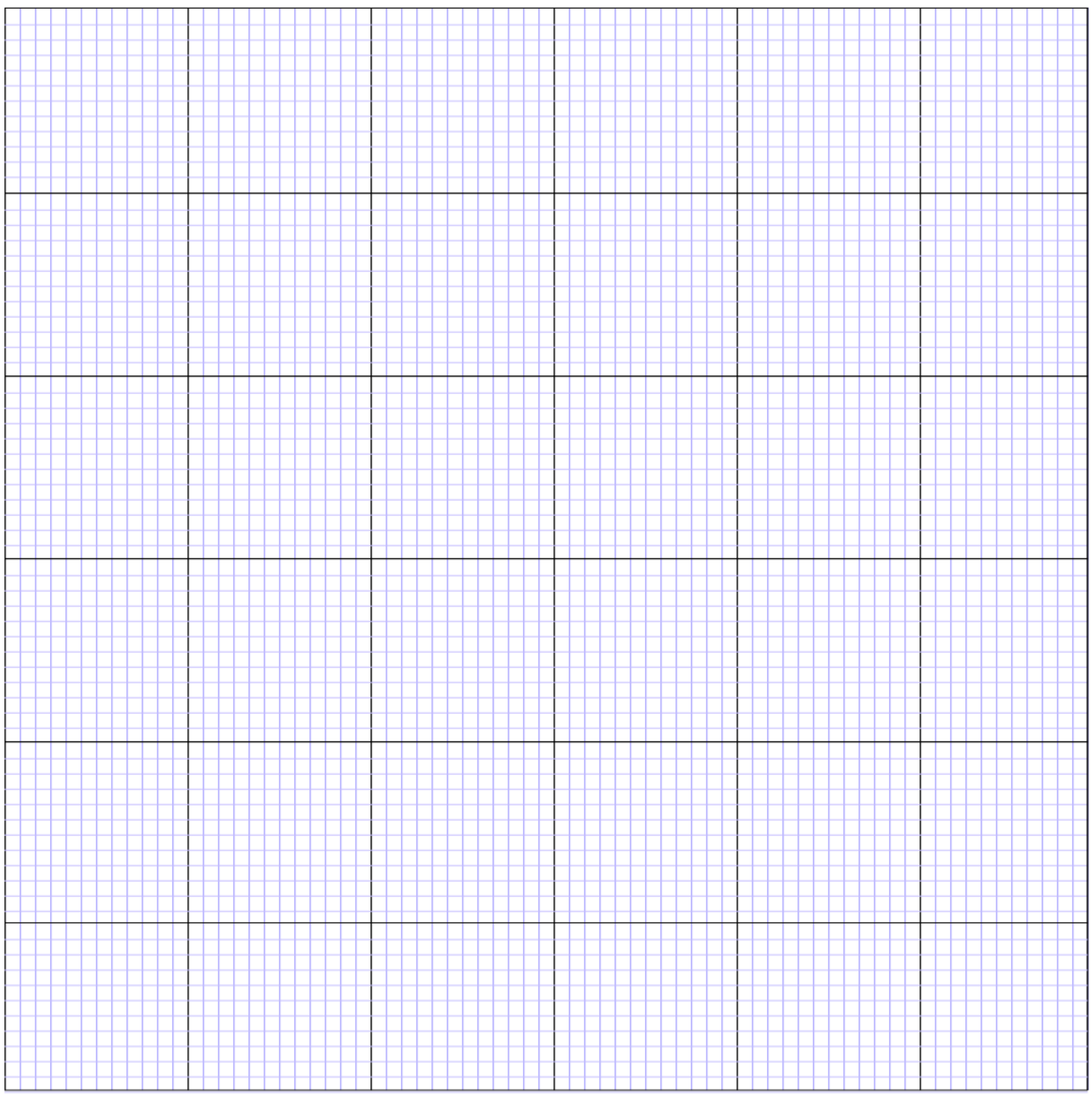


Figure 72 A 6 x 6 Grid of 12 x 12 Graticules

The Equilateral Triangle Grid

The base-to-altitude ratio of eight to seven (7 : 8) is sufficiently accurate that is can be used as an equilateral triangle grid. The triangle is the smallest of all Platonic polygons that can be tiled, and the hexagon the largest. The octagon, for example, does not tile without the use of a second polygon to fill the interstices in a Penrose bipartite tiling.

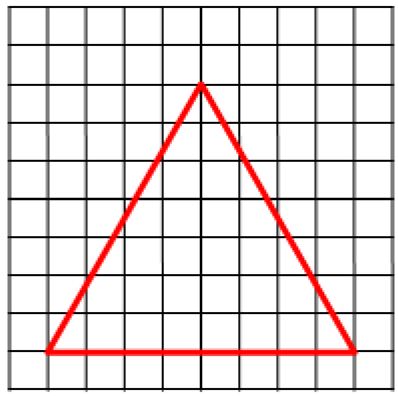


Figure 73 The 7 :: 8 Equilateral Triangle on a Grid

A tiling of equilateral triangles perfectly aligns itself upon an orthonormal grid when each triangle has a base of 8, and an altitude of 7. The base conveniently divides into symmetric lengths of 4, permitting a perfect tiling, devoid of any irregularities. The accumulated error across a plane of these triangles is 0.772% of the span. The sides (hypotenuse) of a 7 : 8 equilateral triangle are simply computed by Pythagoras to be the square root of 42 + 72, or 8.062.

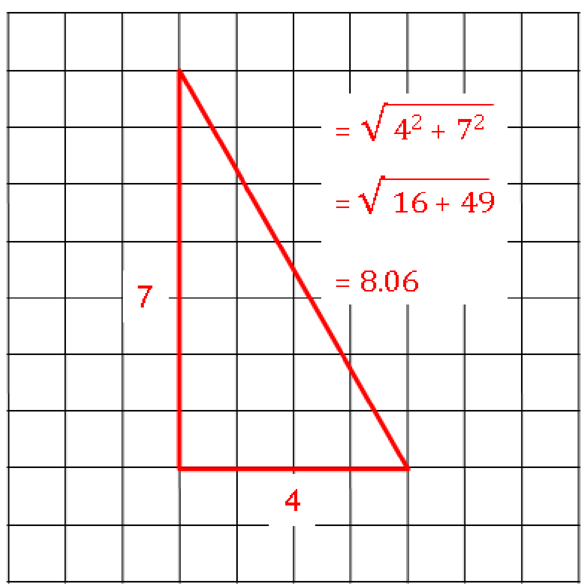


Figure 74 Evidence that 7 :: 8 is an Equilateral Triangle

The equilateral triangle tiling provides a coarser grid where Phi Ratio is pervasive. A secondary, and coarser tiling of hexagons can be seen against the triangular tiling.

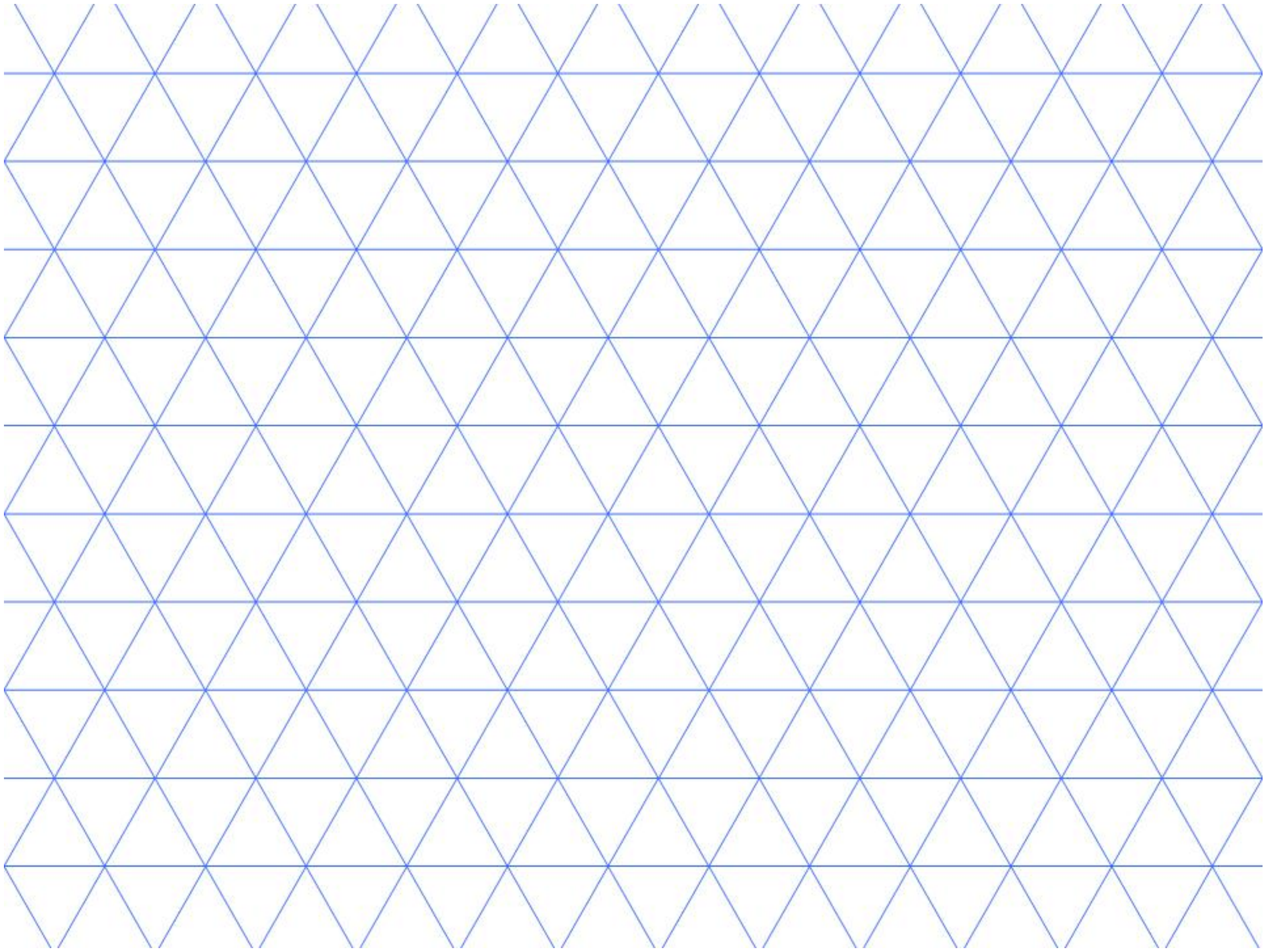


Figure 75 A Tiling of Equilateral Triangles also Exhibits Hexagonal Tiling

Mixed Grids

Color combinations, in conjunction with the pitch of the grid, the apparent density, and the point size of the gridlines, will synthesize a hybrid, or complex grid that has demonstrated utility when constructing objects in the Phi Ratio domain. The complex grid of Figure 76 is a crude prototype that proves the point, and has been used with MS PowerPoint as a background perfectly aligned to the allow *snap-to-grid* behavior at 1/10th inch intervals. This very same complex grid may be generated algorithmically, on one or more grid layers, with or without the snap-to-grid feature.

When inserted as a slide background in MicroSoft PowerPoint, and aligned with a 0.1" or 0.05" snap-to grid the image serves as a very useful tool. Recall that the floors of Masonic Temples feature a chessboard of square tilings, thereby evoking the one Masonic tool that does not conveniently fit into the work apron of the Practical Mason: the grid.

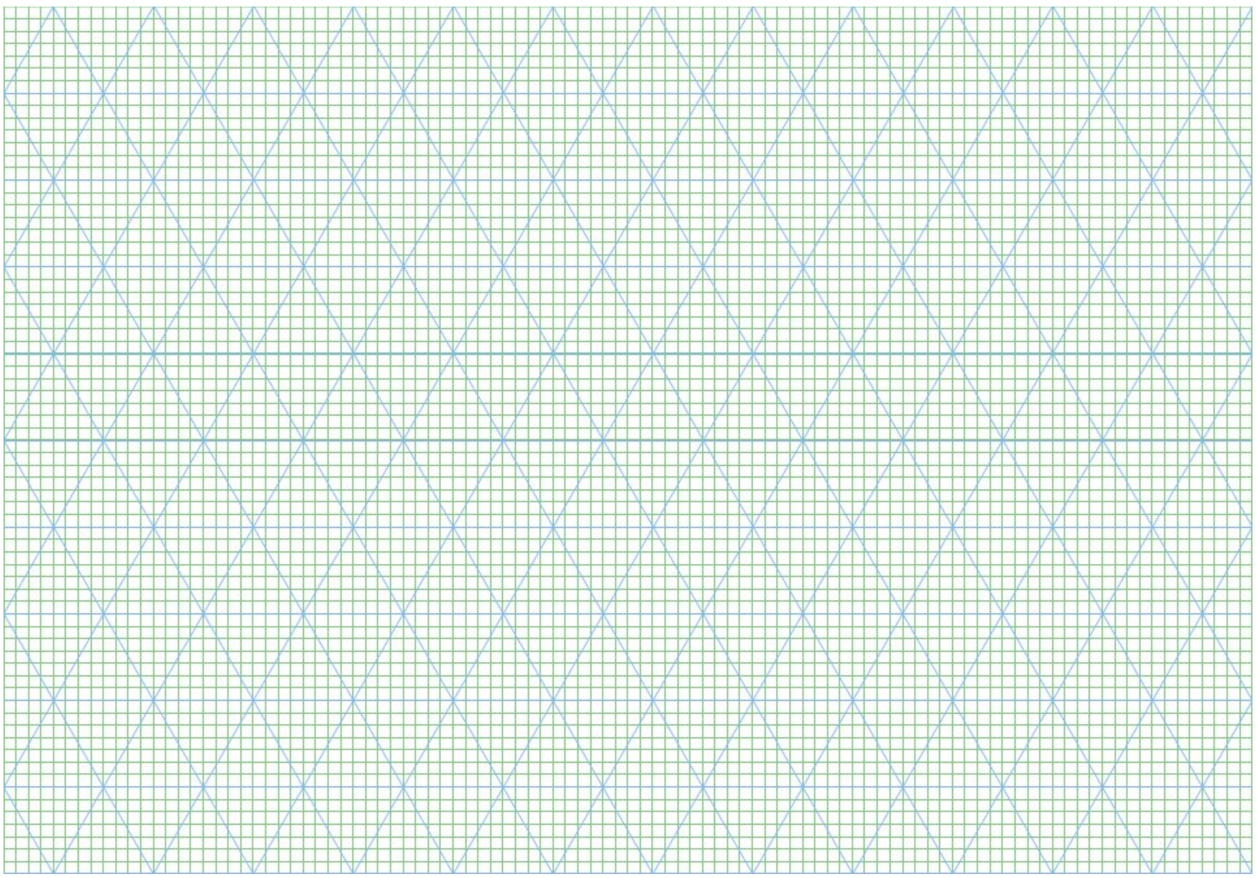


Figure 76 An Equilateral Triangle Tiling Superimposed on a Standard Ortholinear Grid

The green squares provide an ortholinear, or rectilinear grid, while the blue triangles form a concurrent tiling of triangles, and hexagons. Some displays tend to lose the bases of the triangles, instead creating the appearance of diamonds.

Scales

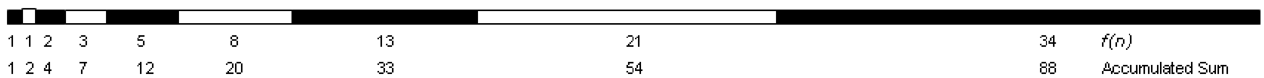


Figure 77 A Fibonacci-based Scale for Drawing Sheets

Fibonacci Grid

Custom Grids

Recursive Indexing (exponent of Phi, Fibonacci, etc.)

Radial Grid

The Vesica Pisces is a radial grid

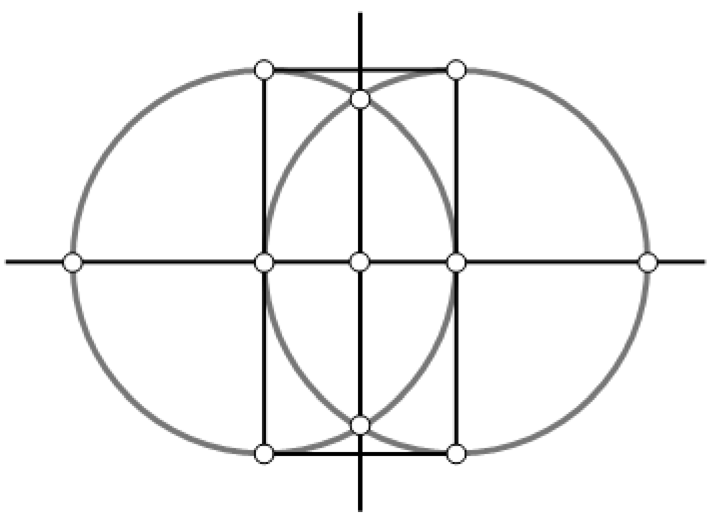


Figure 78 A Vesica Pisces is a Radial Grid that can be Extensively Elaborated

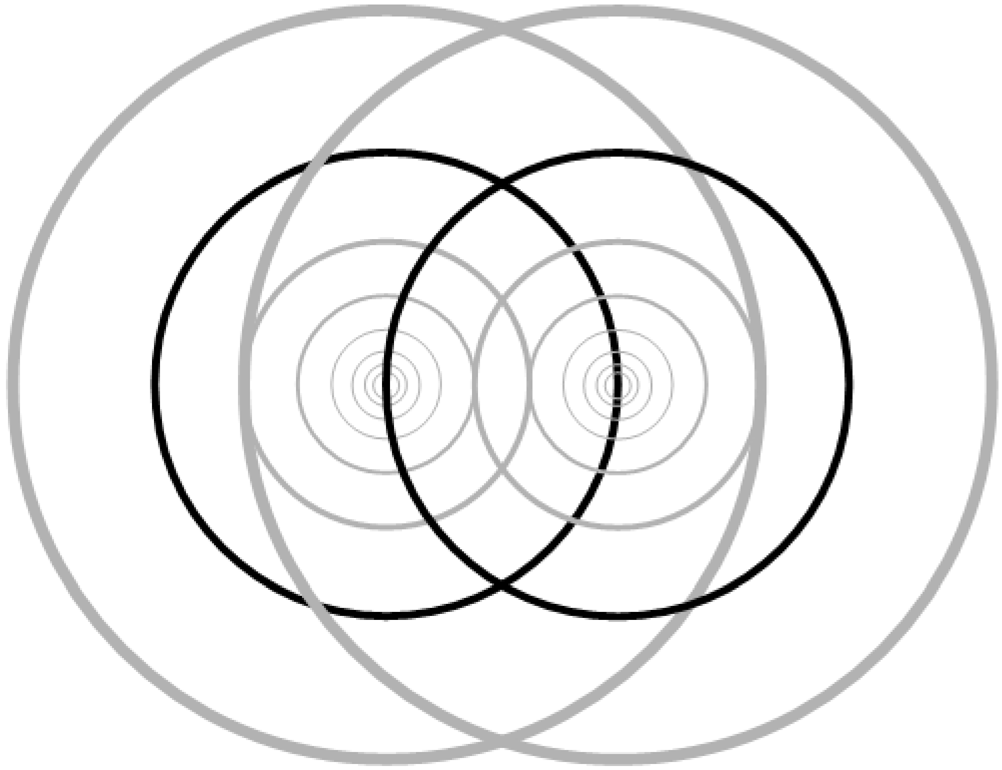


Figure 79 A *Vesica Profunditas* is Replete with Recursive Phi Gridpoints

A Form (Pragma) as a Scaffolding

An object may be specified as an ordered list of grid coordinates, and the suggested minimum grid for its rendering to a precision that is also specified in the descriptor list. This is a methodology first explored for Phi Ratio constructions by Professor Mills, and is instantiated as an extended tool capability.

Tilings

Tilings are repeated use of a polygon as a continuum on a plane. The use of ortholinear, rectilinear, triangular, and hexagonal grids are particular instances of a tiling. In this system, a function has been defined which allows the user to rapidly apply substance to a tiling of polygons by creating a field of objects which are defined by the graticules of the grid. These objects, once defined, can be assigned attributes and arbitrarily moved about the drawing board.

Mosaics

Additional functions permit mixed use of polygons in the creation of images and these include coloration, dissolution, return to origin, and other operators appropriate for developing mosaics.

Scaling

Esoteric

The esoteric tools are also known as the Masonic tools and were used by the Operating Mason to build

the stone edifices of the Middle Ages. These same tools were employed by the architects of the time,

who may have also been among the ranks of the Speculative Masons.

Straight-edge

Right Angle

Compass

Plumb (protractor)

Often overlooked is the Apron necessary to carry these tools at the worksite.

Symmetry

Radial

Bilateral

Rotation

Mirroring

Recursion

Confusing Tools with Nature

linear, planar, decimal radix

analytical vs. geometric
PRG represents a form of rigor that encourages us to drill down into the confusion of poorly-understood phenomena, and obtain the clarity of understanding.
probability is just another example of hand-waving when we're unsure, or uncertain, about what's truly going on

Defining the System

The Hermetic System will be developed as a succession of functions that mimic the way that nature uses simple rules to develop ever more complex structures and behavior.

Limitations of the 4::3 display pixel aspect ratio, in conjunction with an angular resolution limited to a degree do introduce slight distortions to these diagrams. The *Hermetic Universe* ™ CAD system under development will correct for these problems, particularly when a 1::1 display pixel is used, such as on a Macintosh computer.

In the course of developing Phi Ratio Tools, and Design Methodology based upon second-order Phi Ratio effects, we have developed novel design doctrine. Some of the treatments found herein are prototypes, or proof-of-concept investigations and may diverge from methods that are more common. Until we have polished and completed our studies, if that is possible, we reserve the right to release this material on our terms.

The first phase of system development seeks to devise the drawing domain, the canvas, that provides the medium, or universe, upon which all else is rendered. The canvas only exists as an abstraction in the computer's memory, and is actually a specification for a layer of memory, zero, a specification whose dimensional data is transferred to all subsequent layers. The mechanism that constructs the working layers in memory extends the canvas, and also integrates the many layers whenever a new instance of the image buffer is compiled.

The second phase of the system definition is the coordinate frame declaration. This can include a quartering X and Y axis, with the origin in the center. Other possibilities include a frame of any of the first, second, third or fourth quadrants, and more. All coordinate origins are established as a displacement from the conventional computer display origin, which is in the upper lefthand corner, or the fourth quadrant aspect, as in Figure 80.

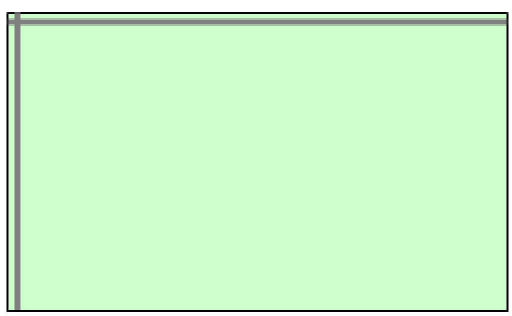


Figure 80 A Display's Origin is in the Upper Lefthand Corner

The third phase of the system is to construct a special type of layer that hosts a grid. This may be as simple as setting an attribute, and sequencing it in the layer stackup towards the background. This is extended by the development of code that allows a linear grid to be defined parametrically. A foundational interval is established for each of the horizontal and vertical axes, or both in tandem to support square cells.

The next step of this phase is to instantiate graticles at one or more intervals. Each of the graticles is essentially a MODulus interval that highlights a vertical or horizontal line at each zero interval by a divergent color, or heavier point line, or both. It is possible that a multiple grid interval may be employed, such as 8, 64, and 256, or 10, and 80. Non-linear derivatives of these orthonormal grids can also be extended to include log-log, log-linear, Fibonacci-linear, and Fibonacci-Fibonacci, where the Fibonacci Series is the instantiation of Phi in integer form with an accuracy exceeding three decimal places, by term F(10), or better.

The Canvas as a Layer

First, a two-dimensional Cartesian space is defined, which is ortholinear, and rectilinear by definition. For many conventional applications, the Cartesian grid is sufficient, and one version is depicted in

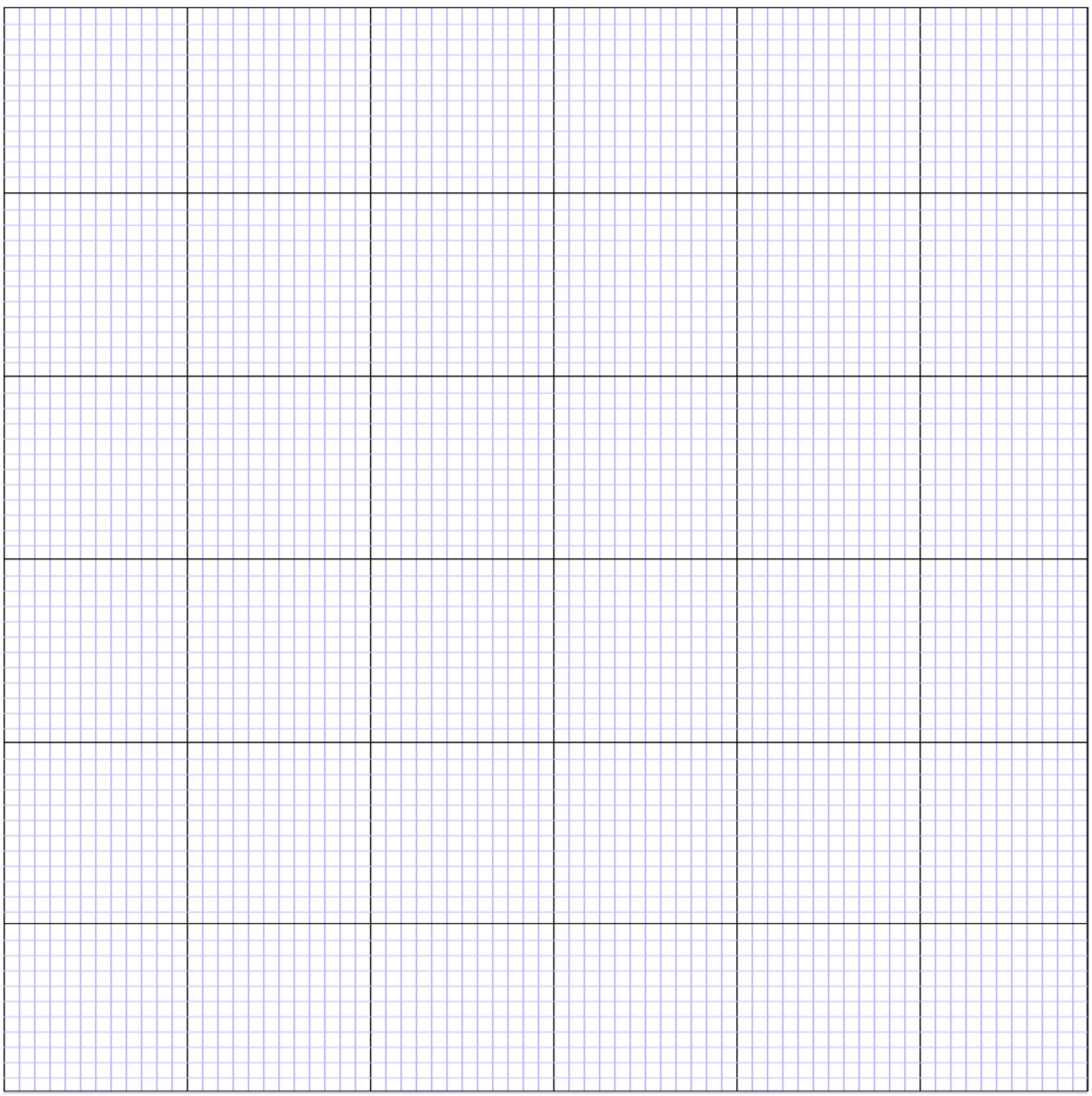


Figure 72, below. The grid has a defined extent that is sufficient to render any of the user's intended drawings, and this expanse is the **canvas**. The canvas is defined by the maximum pixel to be represented on each of the X and Y axis, and the total area of the canvas is specified as the product of X and Y, where X = 0, and Y = 0 defines the origin. The canvas is assumed to be in the fourth quadrant, using current math, and computer display conventions. The canvas conserves considerable memory by being reduced to its descriptor, where the background color, transparency, and other attributes are in compact form as Layer Zero, per Figure 81.

The user has an additional option at this point, and that is a choice of color for the canvas that includes a palette of opaque colors that can be rendered translucent, and could be made entirely transparent in the aggregate.

The canvas may be specified to be larger than the display area at a 1::1 scale. This may be later modified to be larger, extending the available “blank” area for an existing drawing, or may be reduced, a move that truncates the existing image crossing each edge into the proverbial “bit bucket”.

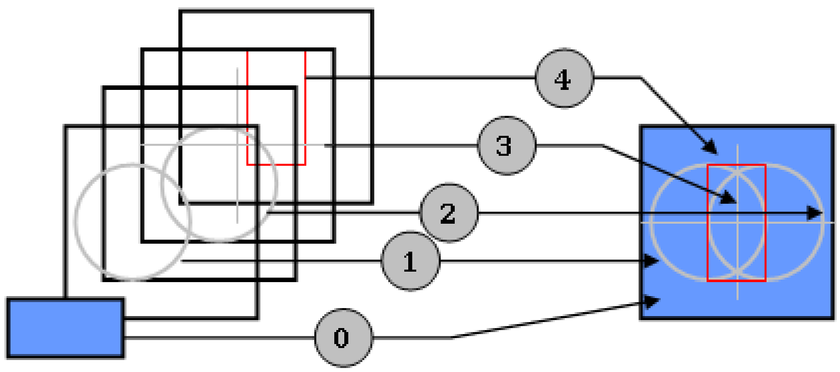


Figure 81 Copying a Layer Stackup to an Image Buffer

The Coordinate Layer

There are a number of possible coordinate aspects that may offer utility to various technical applications. They can be specified by label, or possibly by the location of the origin, roughly based on the face of a clock, per Figure 82. The selected coordinate system may be operant, yet may be transparent, or may be depicted, with optional numerical indexing.

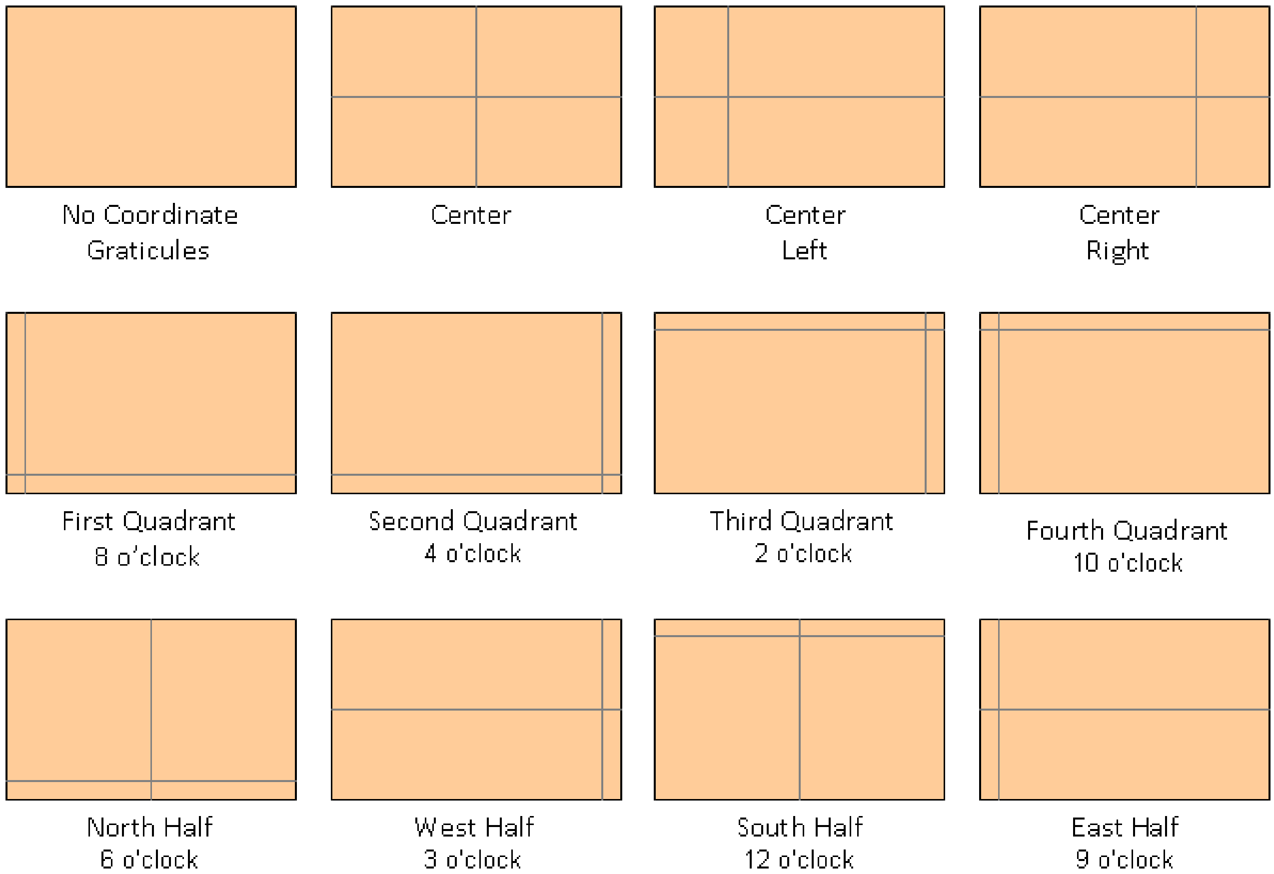


Figure 82 Ortholinear Coordinate Aspects

At some appropriate point in the process, any layer that has been found to be empty during the last buffer copy is handed off to the garbage collector.

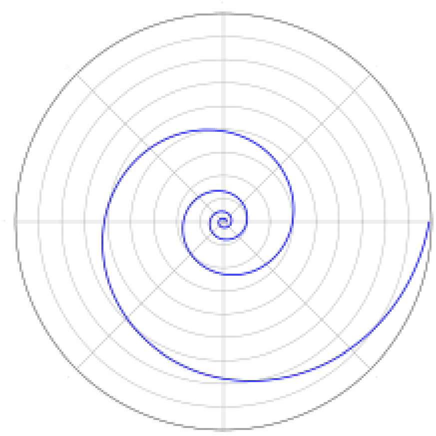
Objects may be selected individually, or as a group, and then copied, or moved from any layer to any other layer.

Developing a reference grid for the CAD user is a matter of selecting a feature, and then adjusting the parameters of each feature.

Geometry software under development offers gridded backgrounds upon which a *Vesica Pisces* or other devices may be constructed. The programs will also allow an object to be drawn and an entirely new grid inserted which establishes a better "fit" of the grid's precision and orientation.

Extended Tools

Spirals



Recursive Replication scaled to the 8x8 square

A gridded, two-dimensional universe of four 12x12 matrices is organized as a 24x24 grid. Super-imposed

on this grid, in the fourth quadrant, is a drawing of the Fibonacci spiral, 1, 1, 2, 3, 5, 8, . . . terminating at a

5x8 box, yielding an 8x8 square on the 24x24 grid.

The inset illustration drawn below in the ungridded fourth quadrant is to clarify the object of interest

imbedded on the grid. When the same drawing is replicated and expanded to a point where the interior

1x1 "box" of the copy (red) precisely fits the 8x8 "box" of the original interior drawing (black), the new,

expanded overlay is the precise continuation of the original Fibonacci sequence. Voila! Fractals.

The approach is repeated by dropping the expanded duplicate (yellow) over the interior 3x3 square of the original interior drawing (red). The resulting structure does not generate the spiral, and instead sends a

larger square off to the upper left, much like a glider gun of the computer game of 'LIFE.'

Figure 10 Replication on the 3x3 square

A normal (90°) rotation of the long aspect occurs with each member of the Fibonacci series, and

replicating on the 3 x 3 square causes these rotations to be skipped. This doesn't appear to have any

value at this point.

Projection

2D > 3D

3D > 2D

3D > 4D

4D > 3D

Pragmas

Since they serve a multitude of roles beyond conventional software, the macro, or script language of the system is known as a pragma. A pragma is a procedure for drawing objects, defining computational scripts, presenting interactive tutorials, supporting production of constructs, and cataloging the myriad objects, schemas, and reference drawings from antiquity through recent times.

System Tools

Metrology

dimensional measures

convert units

reference to metrics

templates

Functions

Snap Object to Grid

Align Object to Grid   float an object until it aligns – ΔxΔy, then try rotate w/ ΔxΔy

Swap Grids

Sequential and Recursive Procedures

Macro Script

Pragmas to capture geometric constructs, *ie., Vesica Pisces*

Macros to capture repetitious steps: rotations, expansions, etc.

Intrinsics to formalize elements that are imbedded and inseperable from the Phi Ratio Geometry, such as constants and conventions

Building Structures through Recursive Application of rules

The spatial computer is a three dimensional Fibonacci Computer

The bits (binary digits – or is it fits?) of a Fibonacci computer represent Fibonacci Values, all else is a void.

The Fibonacci Series is Phi represented in the integer domain.

System Libraries

Pragmas

Pythagoras Plus

The Pythagorean Theorem is often depicted as a squaring of the sides of a right triangle, shown in Figure 12, below. Also observe the participation of a Phi ratio grid on the interior unit square.

Figure 83 A Pragma Constructed from the Pythagorean Theorem

The grids which are normal to each other on the base and the altitude, when extended will intersect the hypotenuse grid at the gridpoints on the triangle's edge. Most of those same gridlines extended from the hypotenuse's grid do not similarly intersect the other two gridpoints on the edge.

Richard Buckminster Fuller, his 60° and Tetrahedral Geometries

Synergetics

Series of Vesica Pisces

Equilateral Triangle

Square

Pentagon

Hexagon

Heptagon

the history and cultures of the world are replete with many examples of the Phi Ratio and Phi Ratio Geometry

However, there exist only a few, isolated examples of the device which are clear instantiations of Phi Geometry and are so stated to the public.

As an occult, or hidden, *corpus* of knowledge, the Phi Ratio and related geometries are ubiquitous and pervasive, and yet the public remains ignorant and indifferent to the subject.

The gods have given mankind many gifts, wine, beer, civilization – and the Phi Geometry. Nearly every civilization has some expression of the presents which were conveyed to mankind through the kings and priests annointed by the gods, yet as each generation unfolds, our memory fades.

There is a resurgence of interest in the subject as evidenced by numerous publications on the subject in recent times. Some of this is due to scattershot New Age interests, self-promoting *psuedo*-scientists, survivalists, Y2K'ers and also from the adherents of the pending *doom-and-gloom* of the inexorable, yet still non-specific 2012 cataclysms.

With the state of science in America's classrooms, anything which will spur the citizenry to explore and understand science is enthusiastically welcomed, no matter how far afield.

The Extraordinary Nature of Phi

Sacred Geometries and the Integer

There's so much more to Geometry than proofs based upon parallel lines

When so many students in the Western world have positted that there has to be more to geometry than what we were issued in high school and college

The fraction is an artificial construct that provides a very powerful tool with which to study the universe. However, it must be understood that in Nature there are no fractions. It cannot be said that one has "half of a grasshopper, yet one still possesses those features embued to a whole grasshopper." Fractional grasshoppers are reduced to decaying structure and form that represents proteins, fibers and fluids, molecules and carapace on scales below that which previously defined a functional, living grasshopper.

The ancients in Egypt used a device called the *seket* in order to describe the ratio of the *sides* of a triangle. The notion of an angle is a modern convenience, and Trigonometry is actually based upon the ratios of the sides of triangles – so the Egyptians weren't wrong, they also didn't have decimals or fractions, yet they managed all of their accomplishments with integers.

Simple Math makes for Simple Tools

Integers

Tool Development Develops the Mind

Examples

Stellates in Phi Ratio Geometry

This section is a compilation that captures a number of stellates, that is stars and pentads, that have been discovered, or synthesized, in the domains of a Geometry that is variously known as the Ancient, Sacred, Golden, Arcane, , Fibonacci, Hermetic, or Alchemic Arts. There are distinctions and chasms among these schools, yet they all seek to describe natural process in terms that are simple, and have high fidelity to nature. This exposition is a broad collection of diverse scaffolding techniques that span centuries of development, and include one from Professor Mills, and a few devised by the author.

Stellates take two canonical forms that are complementary, but are not duals. The five-pointed stellate is a pentad, and either exhibits ***sequential vertex connections*** of a pentagon, or the binal intervals of a star. The stellates can be said to have a number of possible configurations, and are chiefly characterized by the number of points, nodes, sides, or facets that they exhibit in their idealized forms.

Stellates do not garner much attention within the Phi Ratio Geometry community, yet they have proven to be a subtopic that has a wealth of content and context. These facts will certainly assure a substantial support structure in the Computer-Based Tools.

This examination is cross-cutting introduction to the topic, seeking to view Stellates through the many, and various forms of Phi Ratio synthesis methods that are historic, legacy, heritage, and in some cases, novel. These methods are also presented as a cross-section of a variety of scaffolding techniques that are employed to erect a structure, and discarded once the enduring objective has been built.

One scaffolding technique, profuse with stellates exhibiting a signature aesthetic would be that used by the architect Major Pierre Charles L'Enfant. L'Enfant was commissioned by President George Washington to design the District of Columbia for the new republic of the United States of America. Examples of his method are included here.

Constructing Stellates Using a Vesica Piscis Scaffolding

The *Vesica Piscis* has been used as the scaffolding that supports the construction of nearly every PRG device that has been encountered. The author has compiled a number of methods, and devised novel methods for constructing Stellates – stars and pentads; all while finding that the *Vesica Piscis* is a consistently reliable tool.

This *Vesica Piscis*-based method is useful for the construction of pentads:

Step 1: To a Vesica Piscis inscribe a pair of unit squares, as in Figure 64.

Step 2: Attach an identical pair of unit squares, and again identify the $\sqrt{5}$ diagonal.

Step 3: Sweep out a new circle with a diameter of $\sqrt{5}$

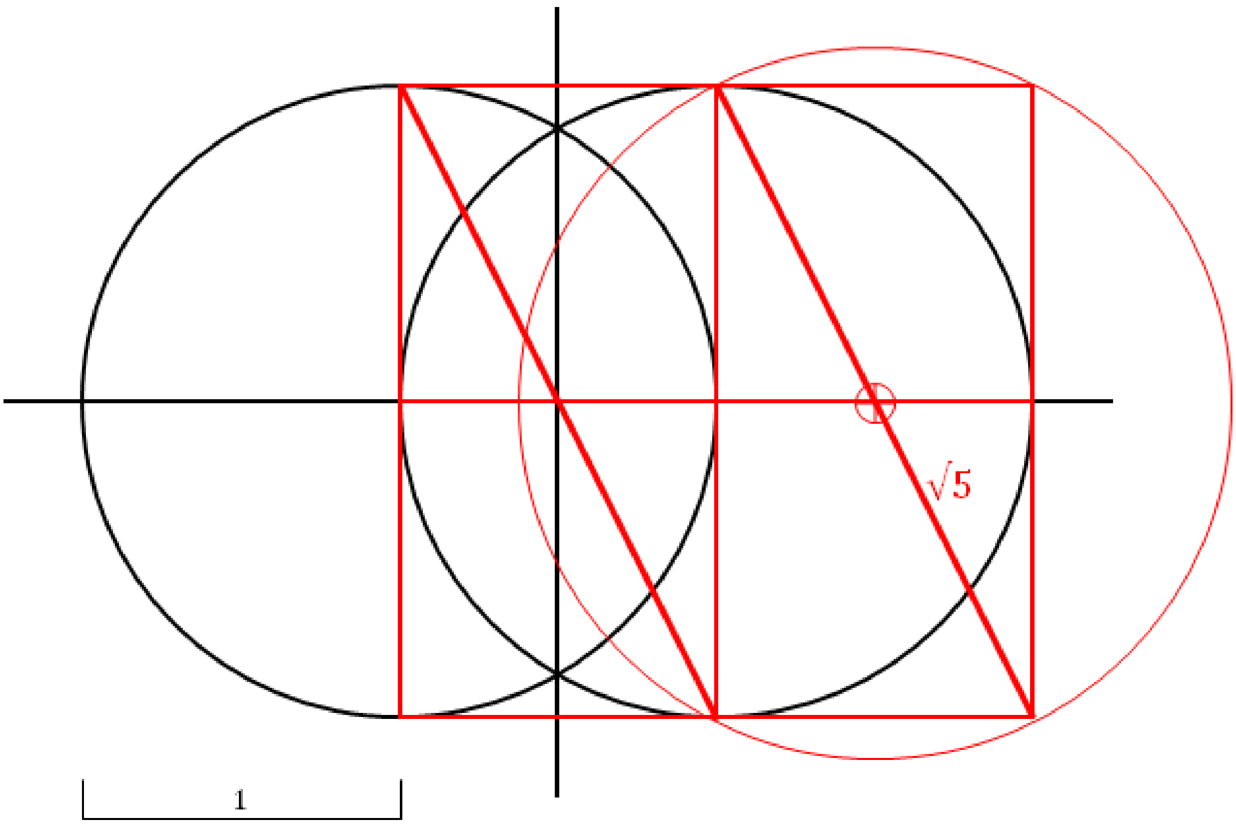


Figure 84 Extending the Vesica Piscis with a $\sqrt{5}$ Circle

Clear away the scaffolding squares, then:

> *Step 4: Sweep out an arc with a radius from the apex of the righthand Vesica circle to its intersection with the √5 circle and the horizontal axis.*
>
> *Step 5: With the same radius, sweep out two new arcs.*

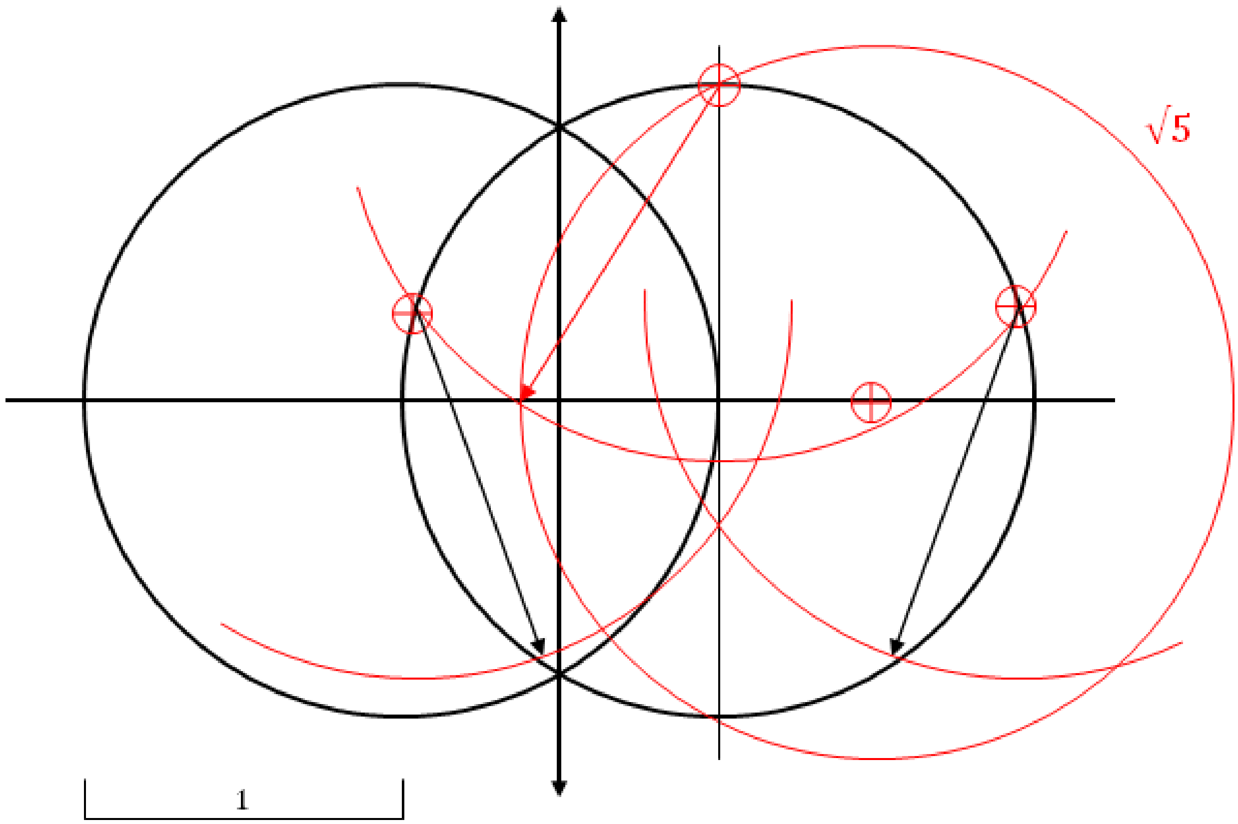


Figure 85 Locate Intersections for Stellate Vertex Placements

Using the scaffolding intersections:

Step 6: Connect the indicated vertices to obtain a pentad.

Step 7: Connect the indicated alternate vertices to obtain a star.

Observe that steps 4 and 5 are recursive, and that other stars may be inscribed at a smaller scale within the star within the pentad.

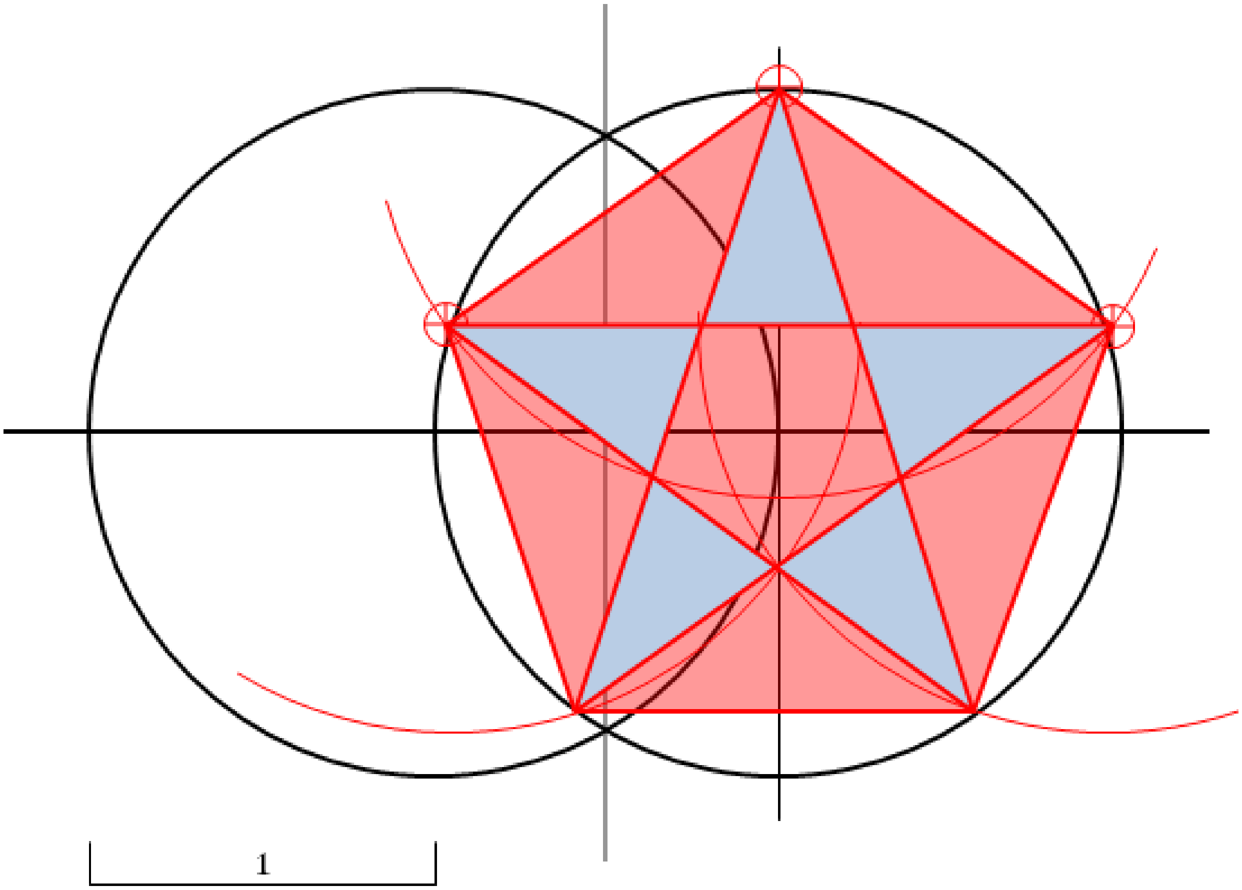


Figure 86 Complete the Pentad or Star

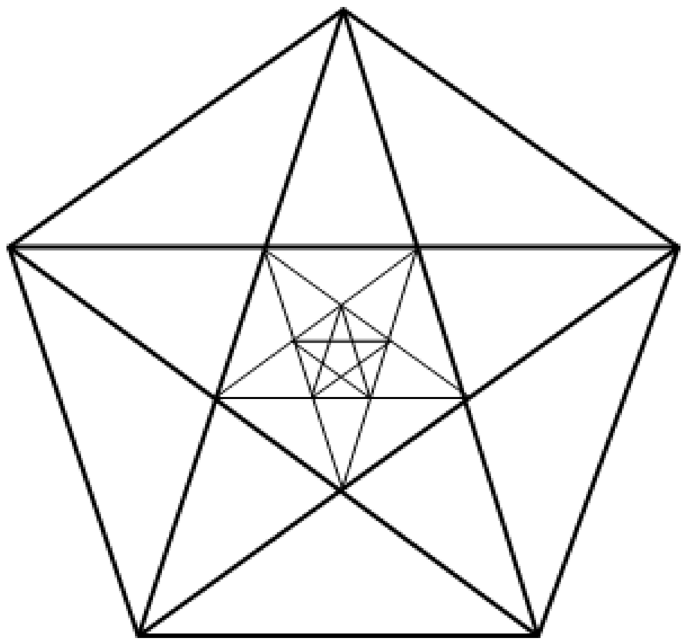


Figure 87 Clear away Scaffolding and Optionally Apply Pentads/Stars Recursively

A regular pentad is composed of five isosceles triangles that feature central vertex angles, which total 360°, and the three interior angles of each triangle total 180°.

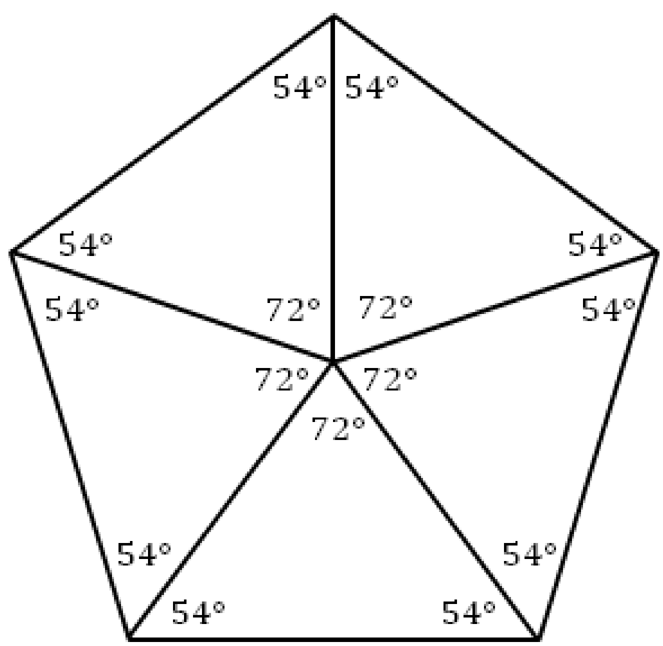


Figure 88 The Interior Angles of a Regular Pentad

The 5::6 relationship between the organic and inorganic domains, and the foundational *Vesica Piscis,* become evident in this diagram, where the scaffolding is removed for clarity.

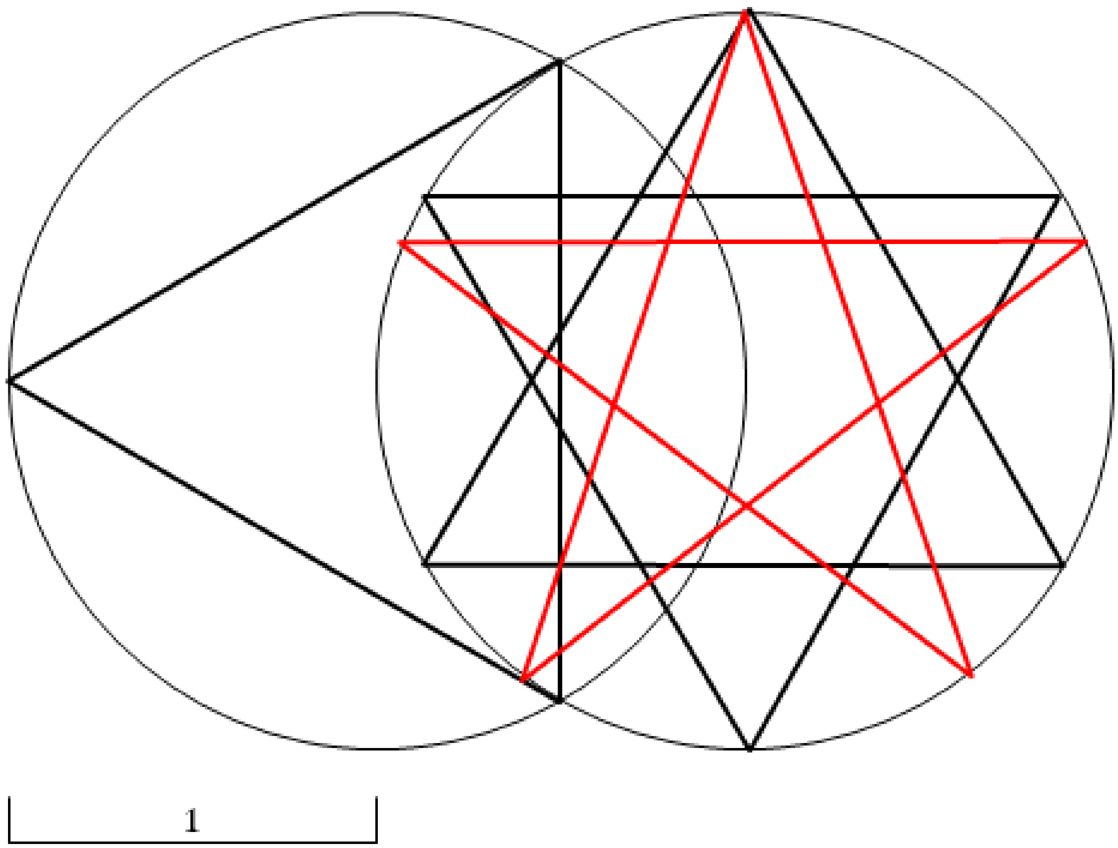


Figure 89 Hexagonal and Pentagonal Stars without Scaffolding

Constructing Stellates Using a Vesica Piscis Triplex Scaffolding

The *Vesica Piscis Triplex* offers a unique approach to the construction of some PRG devices.

> *Step 1: The three interior $\sqrt{3}$ chords are inscribed, describing the center-of-gravity (CG) of the Vesica Pisces Triplex, confirming that the scaffolding is correct.*
>
> *Step 2: A horizontal line tangent to the top of each of the primary Vesica Piscis circles is drawn.*
>
> *Step 3: A second horizontal line perpendicular to the vertical axis, and tangent to the base of the upright Vesica is drawn.*
>
> *Step 4: A new circle, a Phi Ratio multiple (x 1.618033989) of the three Vesica circles, and centered on the construct's CG, intersects the horizontal lines and the vertical axis.*
>
> *Step 5: Connecting these five points of intersection yields a star and a pentad.*

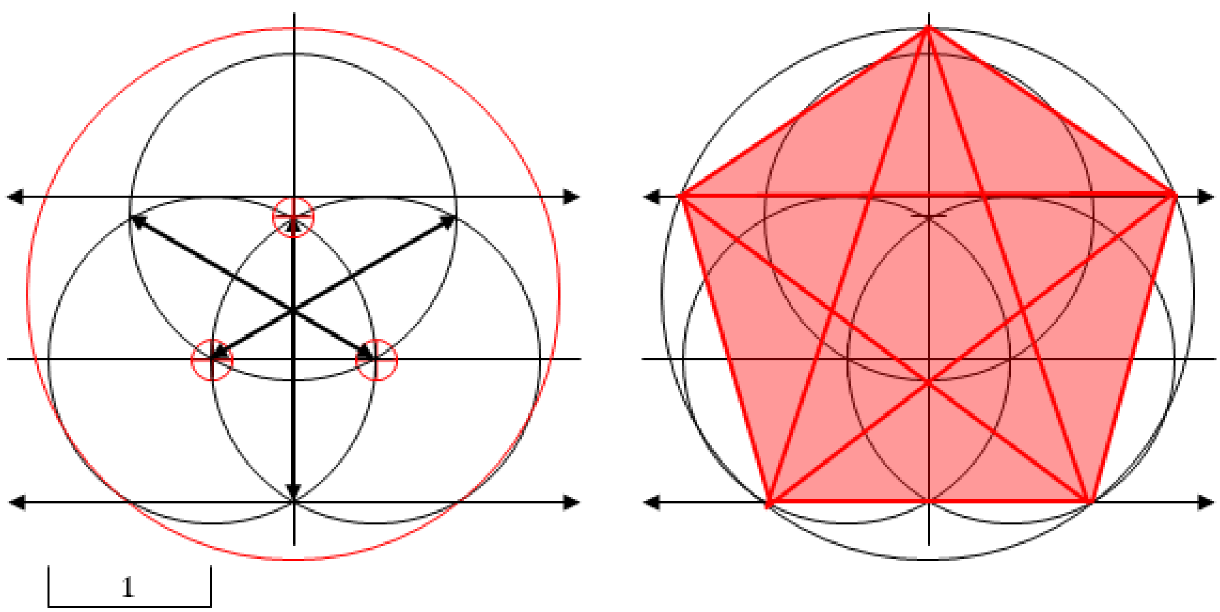


Figure 90 Both Stellates Drawn on the *Vesica Pisces Triplex* Scaffolding

Constructing Stellates Using a √3 Pentad Scaffolding

Stellates may also be fabricated with a double vesica, the Vesica Piscis Duplex.

Step 1: Construct a regular Vesica Piscis

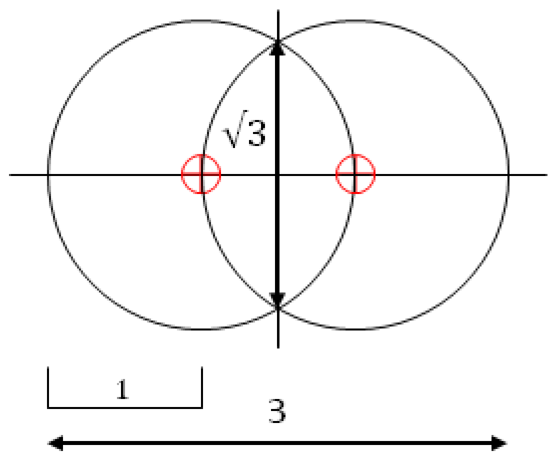


Figure 91 Construct a Reference *Vesica Piscis*

Step 2: Construct a Vesica Piscis Duplex by Appending another radius one circle.

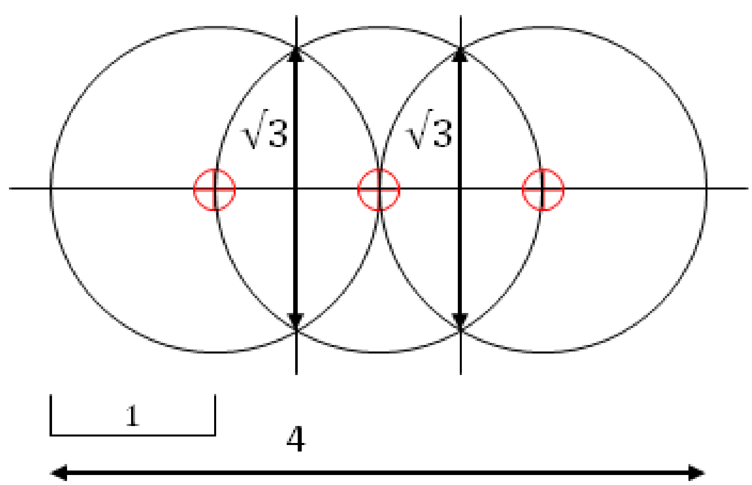


Figure 92 Expand to a *Vesica Piscis Duplex*

Step 3: Construct a Vesica Piscis through the addition of another radius one circle at the apex of the central circle.

The new construct differs from the V. P. Triplex, and a side-by-side comparison is worth the examination. The new *Vesica* confirms the √3 length and the equilateral's side (relative) length of one, by definition.

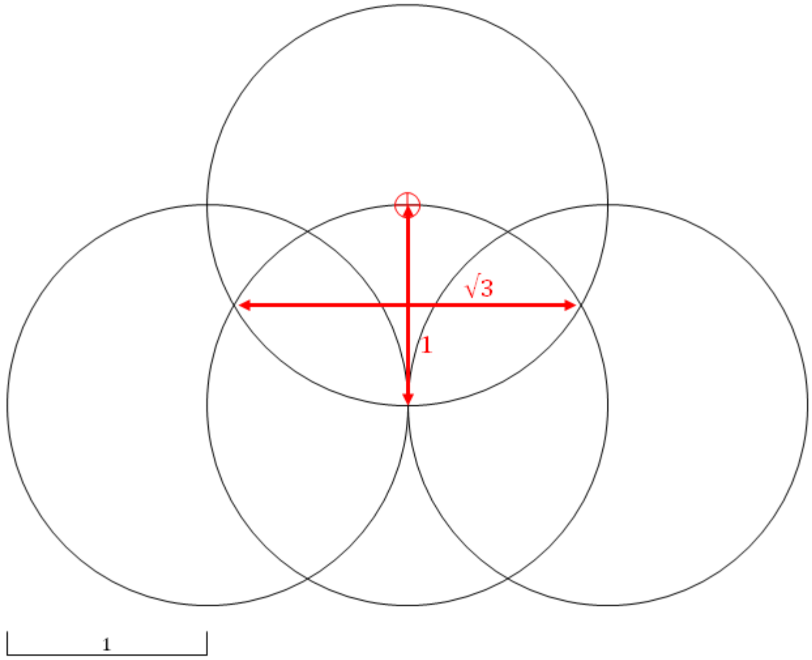


Figure 93

Step 4: Insert an equilateral triangle, connecting the indicated intersections.

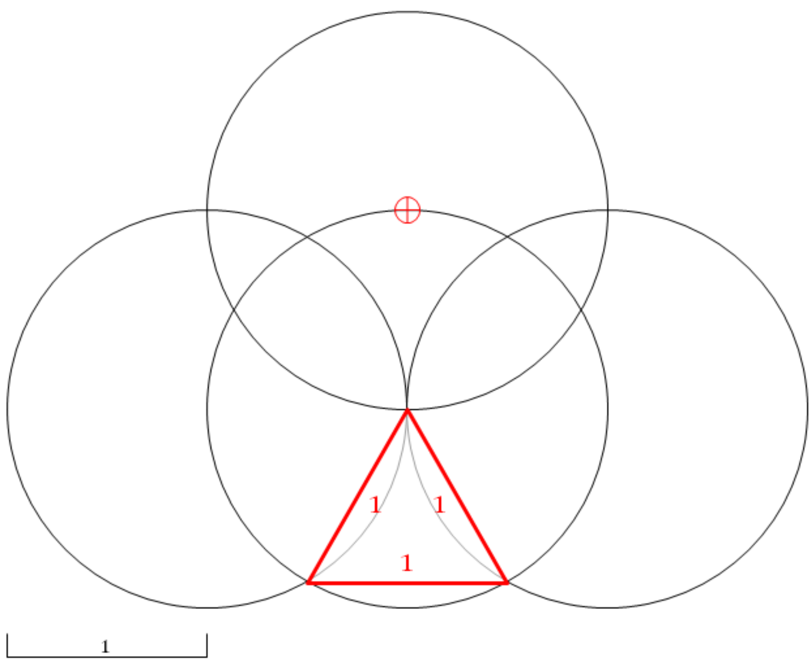


Figure 94

Step 5: Inscribe equilateral triangles in the superior positions of the star, forming a rhombus.

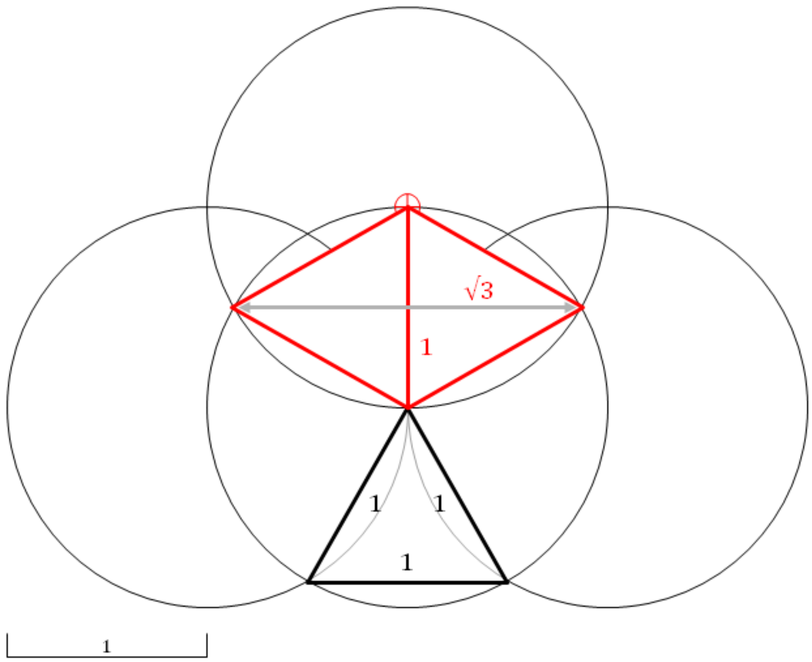


Figure 95

Step 6: Insert an isosceles triangle in each of the flanking gaps.

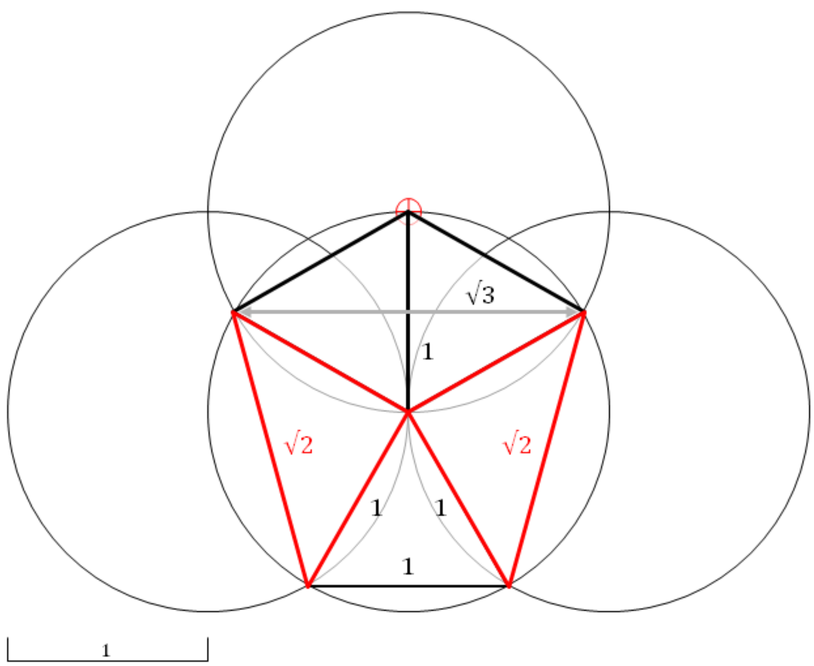


Figure 96

When the edges of the equilateral triangle are renormalized to ONE, the long edge (hypotenuse) of each isosceles triangle is calibrated as √2 and the long axis of the "diamond" formed by the two superior equilateral triangles is the √3 chord of the vesica.

The result is an Extended Pentad.

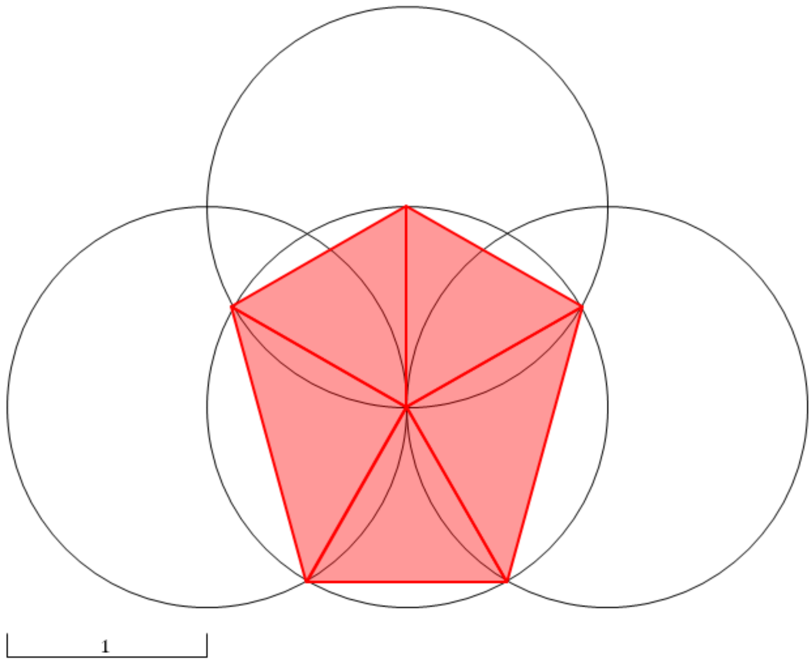


Figure 97

The interior angles of the component triangles are cardinal values.

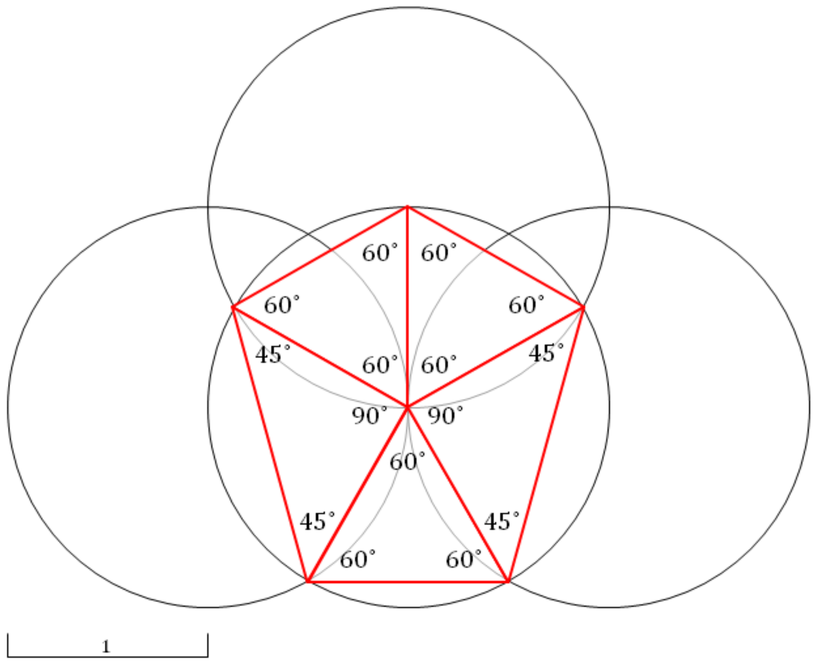


Figure 98

Mills' Whirling Polygon Scaffolding

Professor Mills had earlier developed a method for developing the "Square Root Three Pentad" that he termed the "Whirling Polygon." Nearly any polygon can be constructed on this device, or may be used to establish the points of the device, assuming that one has the patience to iterate the rotated polygon. The simplest polygons are triangles, and squares, although more complex polygons such as pentagons, and hexagons can be used. It does have the benefit of providing a scaffolding for rotating a polygon, as long as the resulting cardinal points are useful.

> *Step 1: Draw an equilateral triangle with vertices at the endpoints of the Vesica's major axis ($\sqrt{3}$ line) to the crossing of the horizontal axis and the circle at the Vesica's 3-line endpoints*
>
> *Step 2: Mirror the equilateral triangle in the other circle, reflecting an opposing triangle, bilaterally symmetric across the $\sqrt{3}$ line*

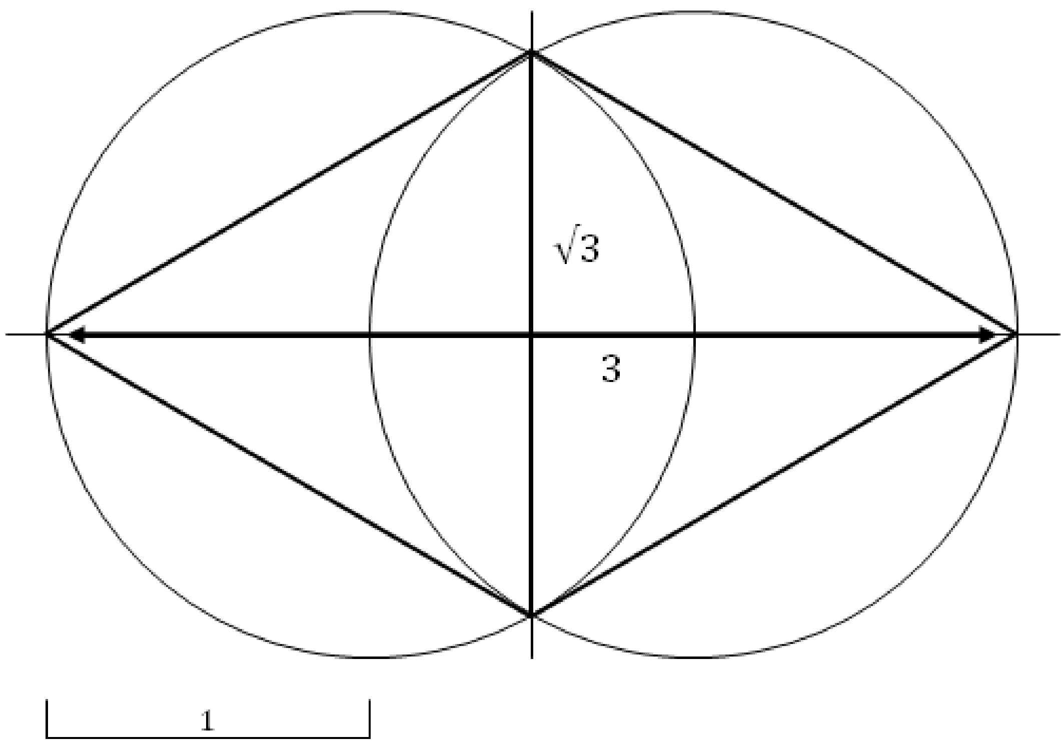


Figure 99

Step 3: Construct a rotated copy of the equilateral triangle from a Vesica circle center

Step 4: Insert alignment edges to the interior intersections of the resulting Star of David, another canonical form

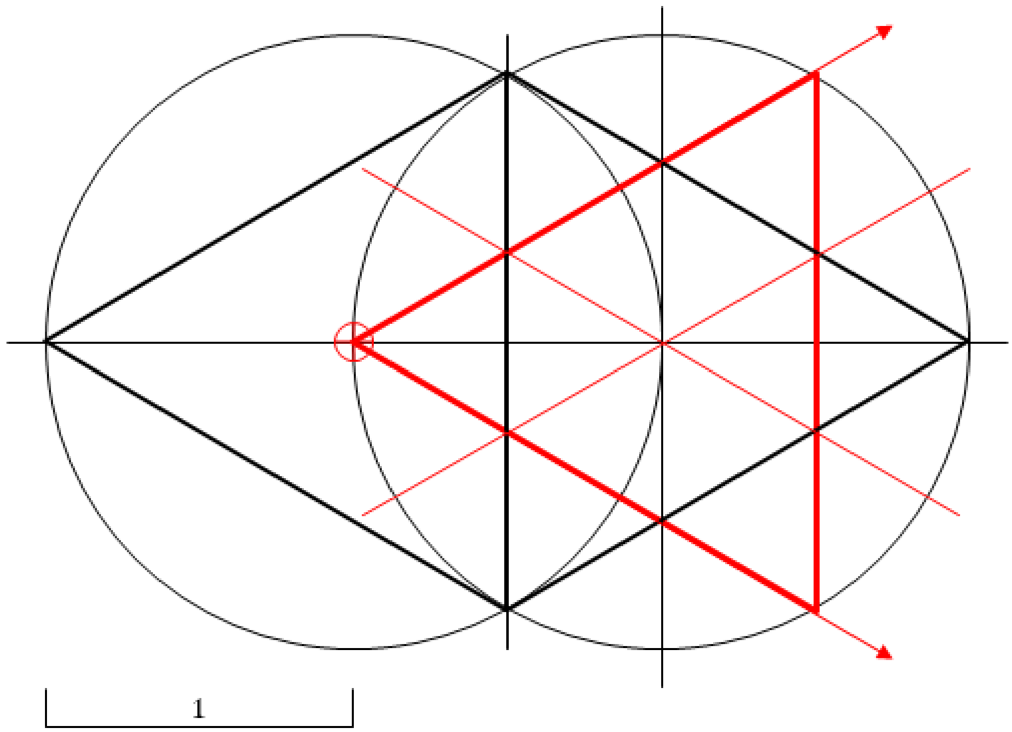


Figure 100

Step 5: Complete a twelve-pointed star on the circumference of the Vesica circle at the indicated intersections, forming a template

Step 6: Inscribe a third circle with a unit radius, creating a second upright Vesica lens, and another canonical device, the Vesica Piscis Duplex

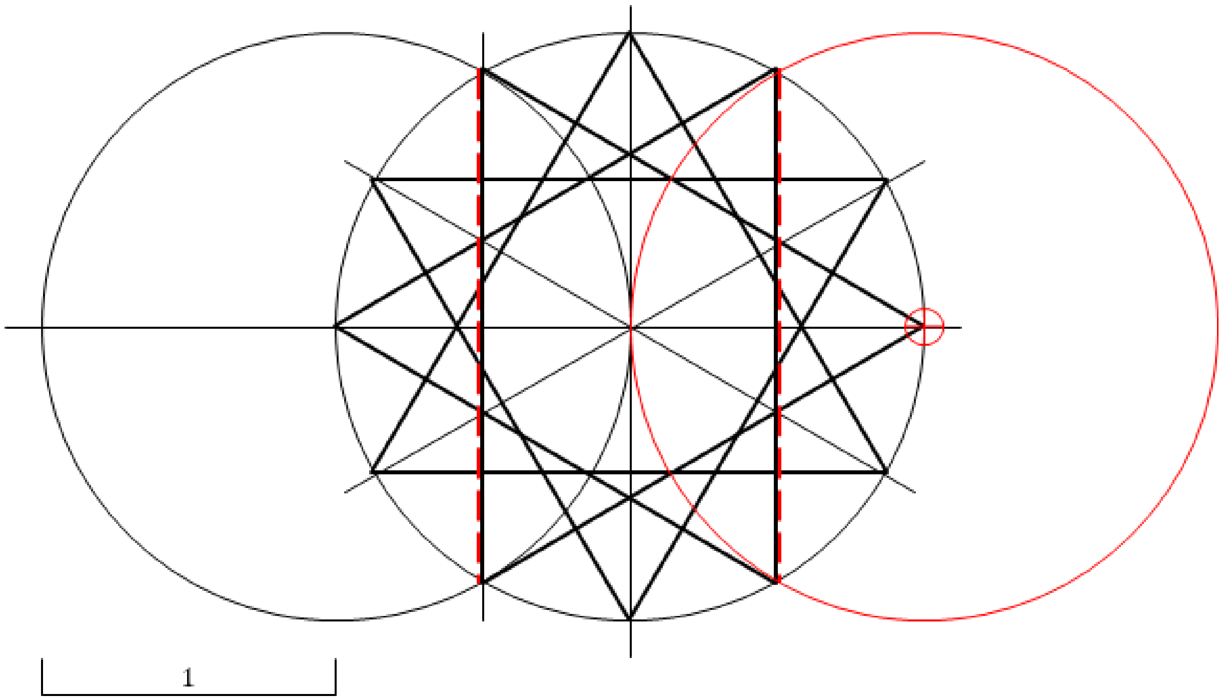


Figure 101

Step 7: Inscribe an equilateral triangle from the bases of the Vesica lenses (√3 lines) to the origin.

Step 8: Inscribe equilateral triangles in the superior positions of the star, forming a diamond

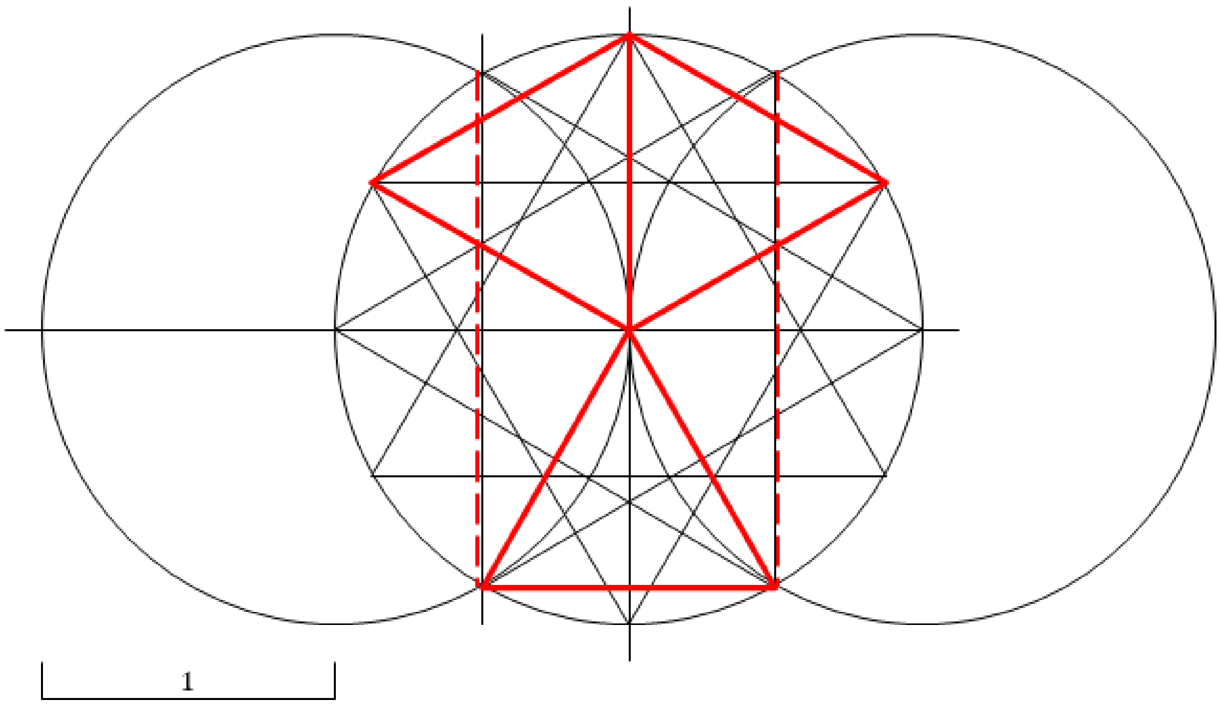


Figure 102

Step 9: Insert an isosceles triangle in each of the flanking gaps.

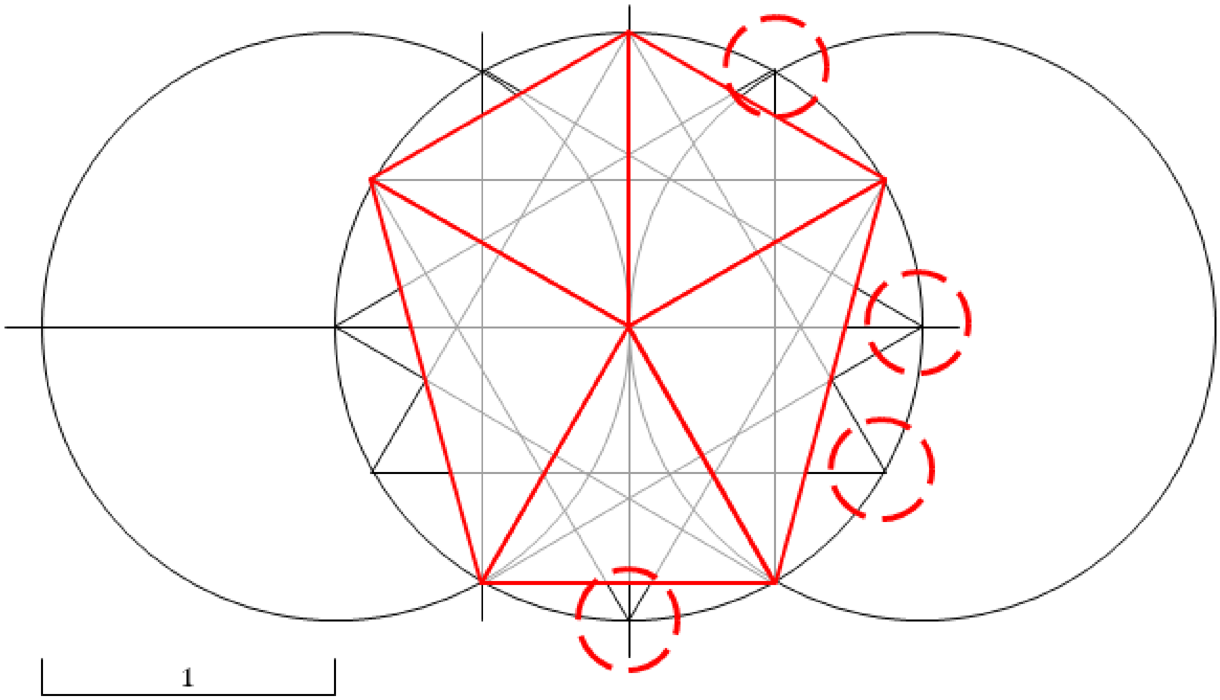


Figure 103

In some respects this method is similar to the previous Square Root Three Pentad scaffolding method, and is based on the *Vesica Piscis Duplex*.

Mouth of Ra Scaffolding

Observe that while this method also employs a pair of circles, they are tangent, and do not form a *Vesica*.

Step 1: Adjoin two tangent circles of ***Diameter*** *1*

Step 2: Insert vertical and horizontal axes

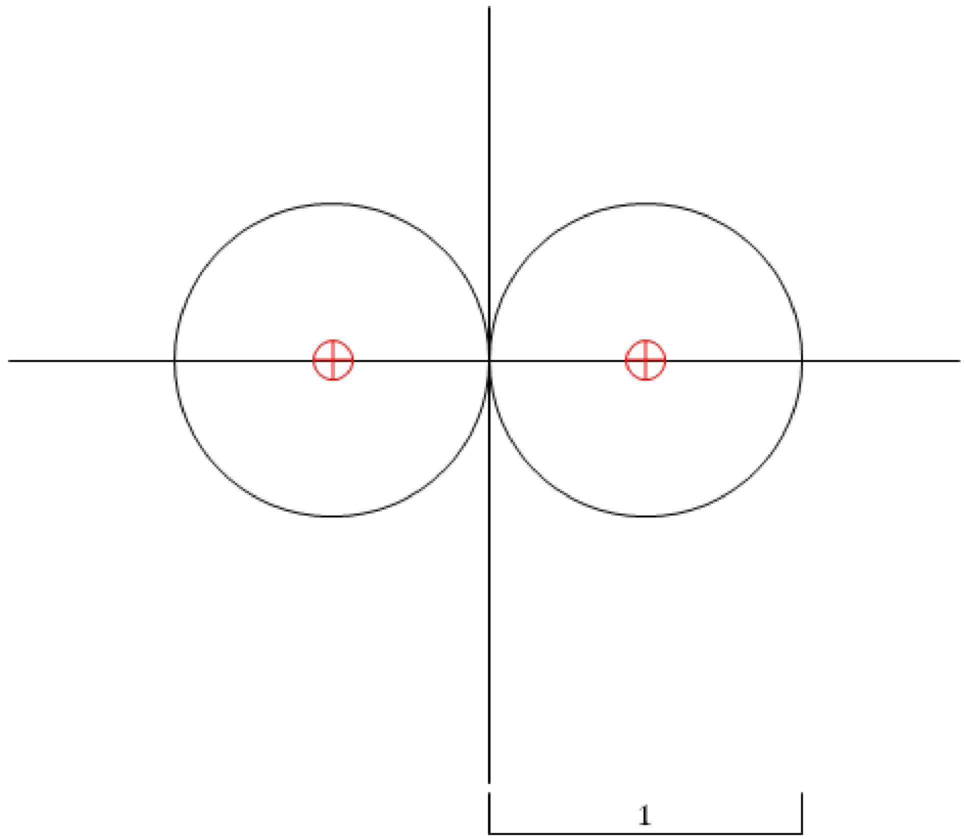


Figure 104

*Step 3: Draw an alignment Circle with **Radius** 1 centered at the origin*

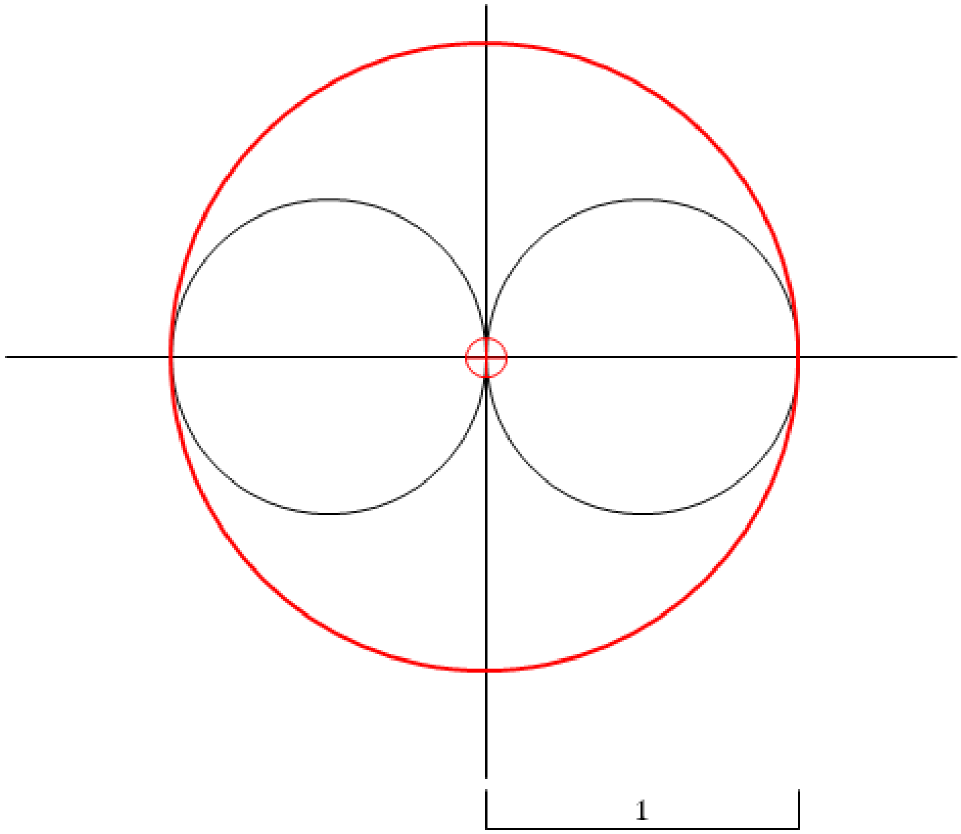


Figure 105

Step 4: Draw an arc with ***Radius*** *1 displaced below the origin by a radius*

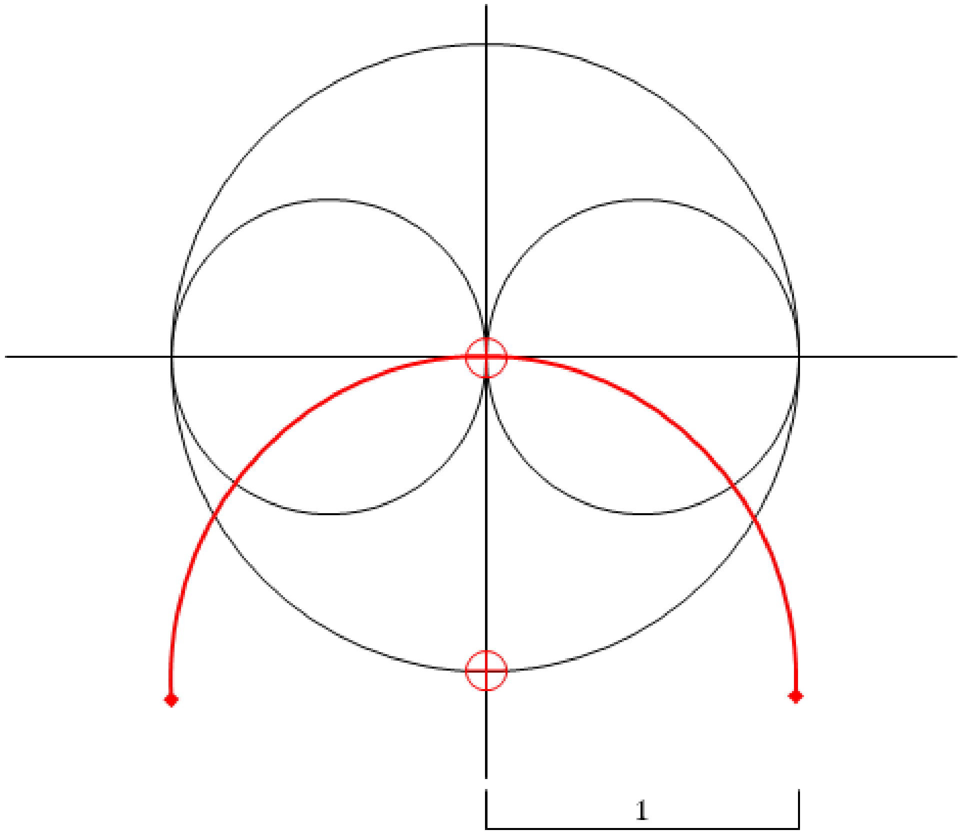


Figure 106

*Step 5: Trace out an arc with **Radius** of 1/Φ from the displaced center. The tangent point with the unit one circles establish this length*

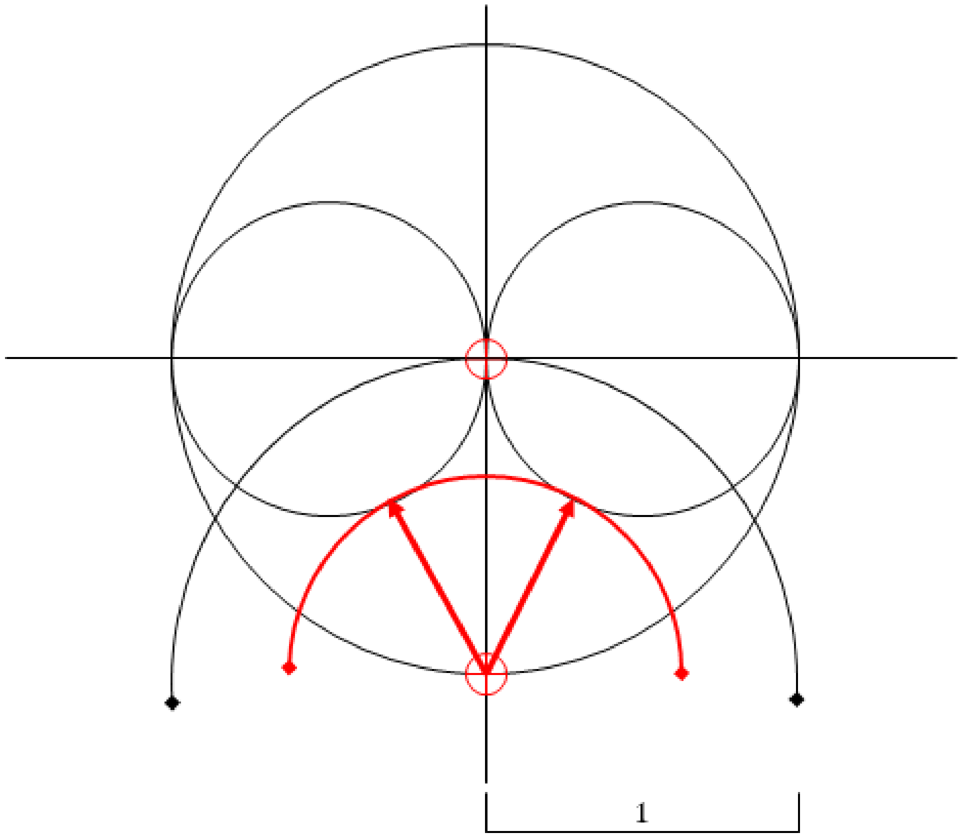


Figure 107

*Step 6: Trace out a Circle with **Radius** of Φ from the displaced center. The opposing tangent point with the unit one circles establishes this length.*

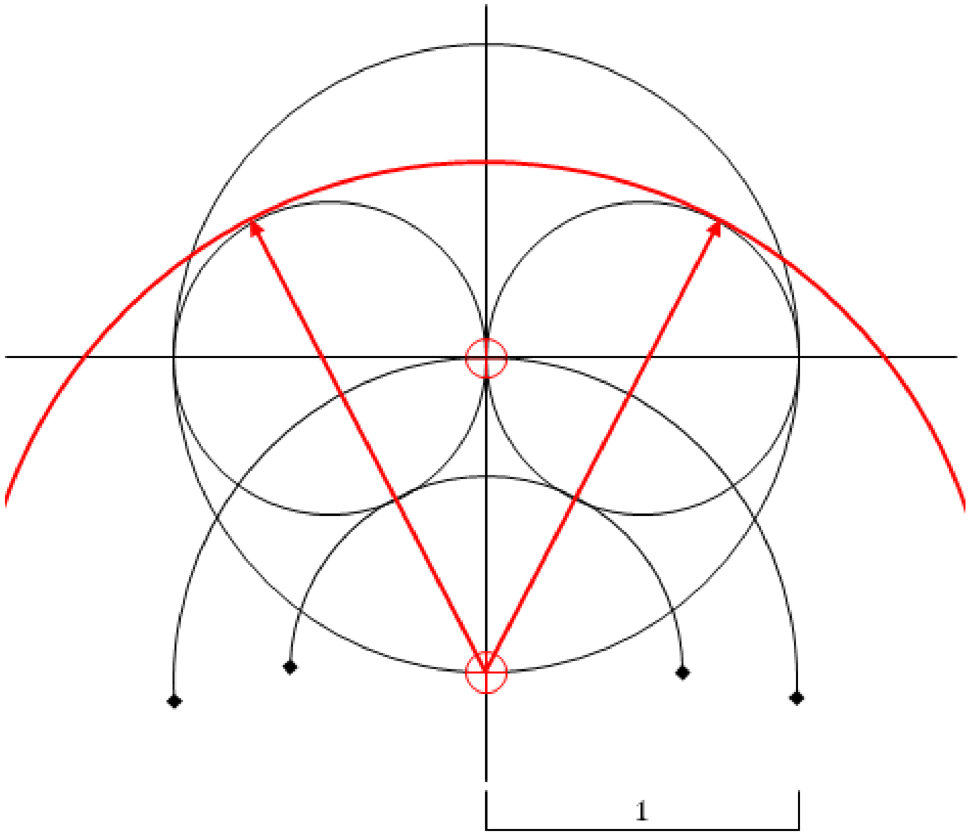


Figure 108

Step 7: Connect the indicated intersections on the Radius 1 Circle with a pentad

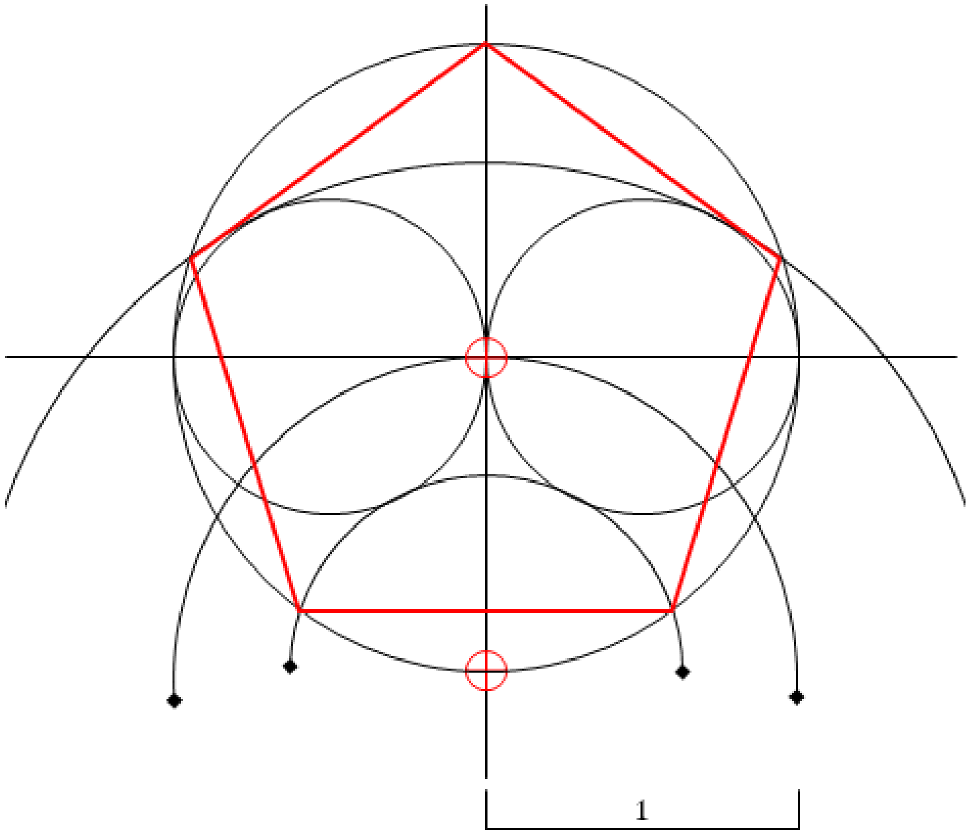


Figure 109

Step 8: Connect the indicated vertices to obtain a star

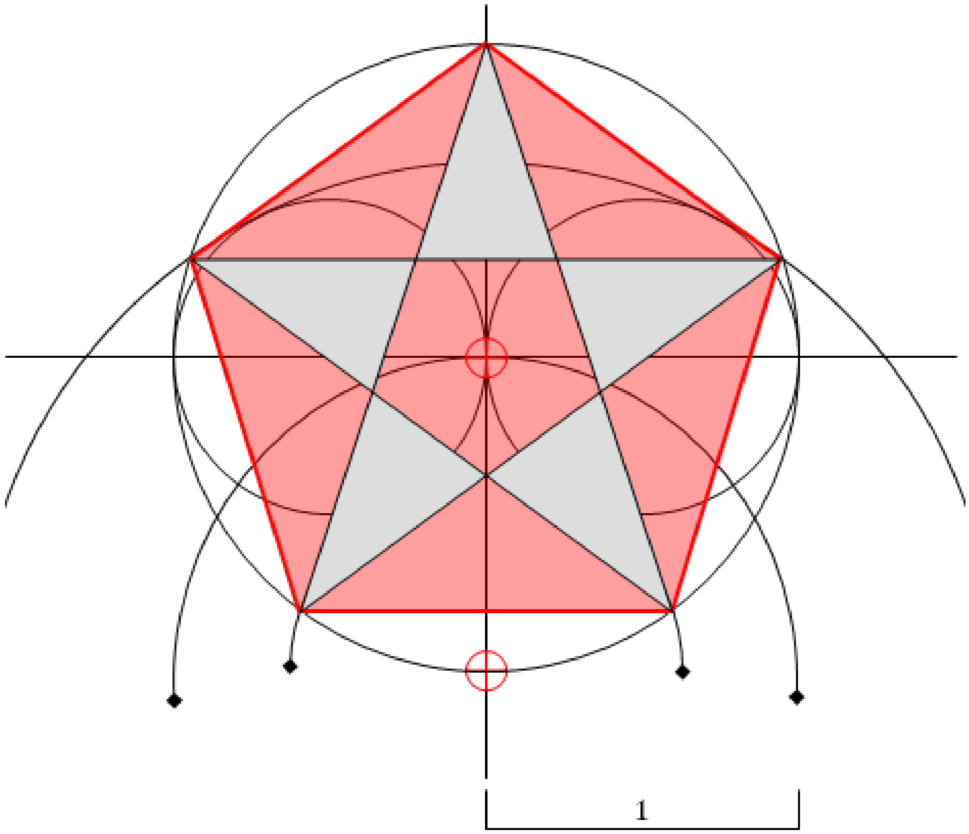


Figure 110

Step 9: Renormalize the resulting structure.

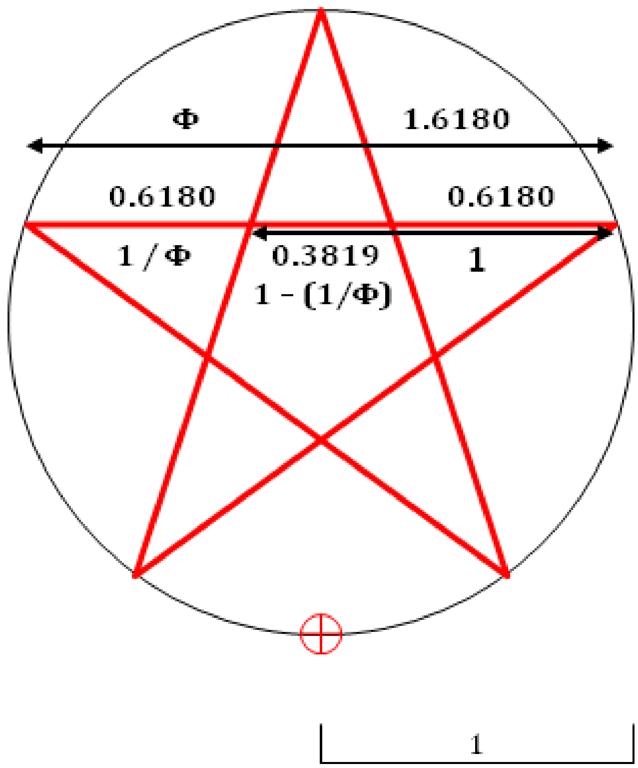


Figure 111 Stellate Arms Exhibit Phi Ratio Proportions

L'Enfant Scaffolding

French Major Pierre Charles L'Enfant was commissioned by George Washington to devise a plan for the District of Columbia, and the City now known as Washington. The work begins with a diamond-shaped district that is oriented to the rising Sun, and drills down into the placement of monuments, key government buildings, commerce and populace. L'Enfant used an original scaffolding that is not a classical Masonic method, yet works well for the city's governance function, and is Phi Ratio Based[12].

> *Step 1: Following the identical procedure for constructing the Mouth of Ra Scaffolding, Figure 104 through Figure 108, trace out circles of 1/Φ, 1, and Φ diameters.*

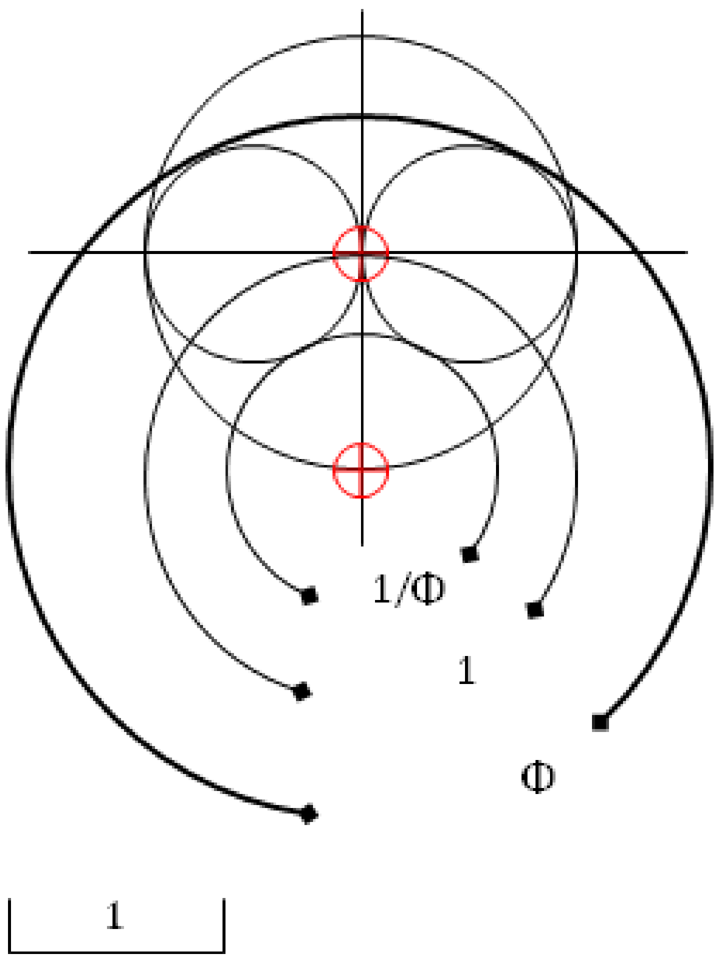


Figure 112

[12] Proceduralized from examples, *The Sacred Geometry of Washington, D.C.: The Integrity and Power of the Original Design*, Nicholas R. Mann, pg. 93.

Step 2: The circles of 1/Φ, and Φ diameters remain in place.

Step 3: The arc of the upper Vesica remains in place.

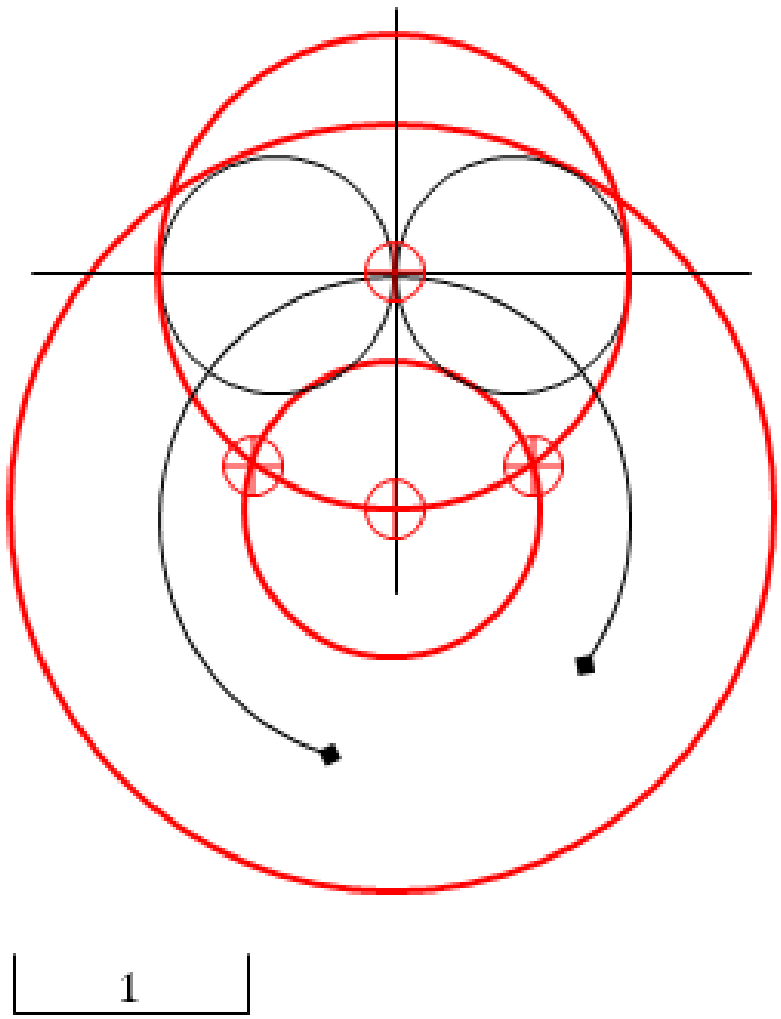


Figure 113

Step 4: The circle of radius 1 is used to calibrate the compass

Step 5: At the indicated intersections inscribe circles with radius 1

Step 6: Confirm that the minor Vesica circles form a major intersection at the 1/Φ circle's base

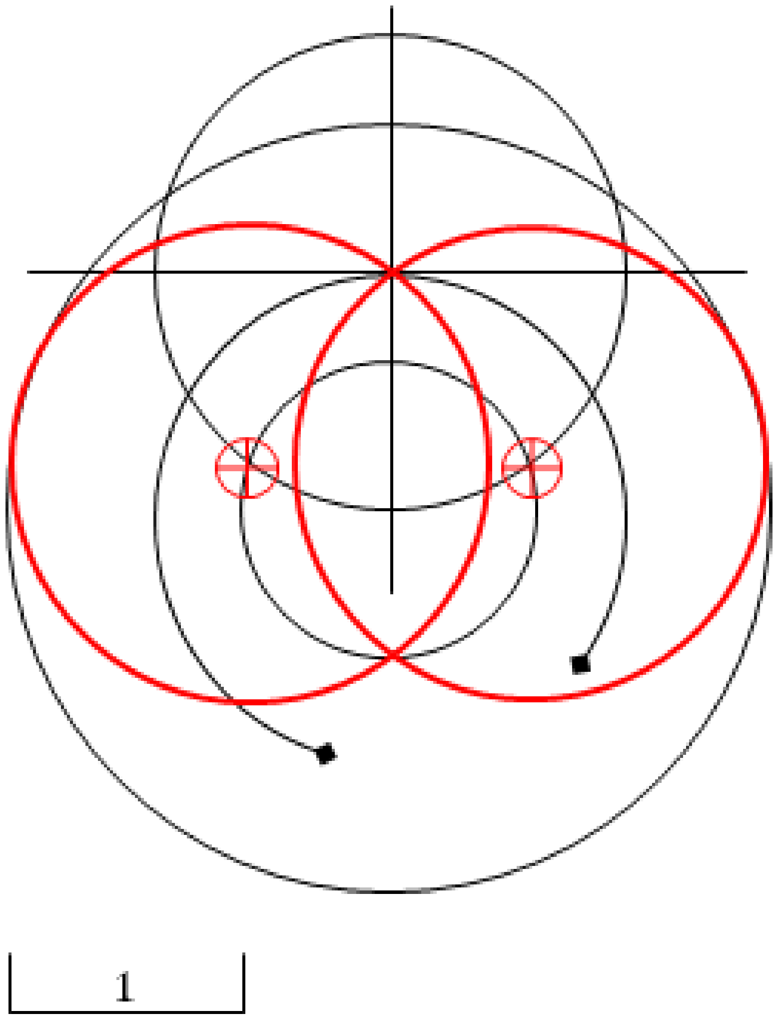


Figure 114

Step 7: Inscribe a pentad at the indicated intersections

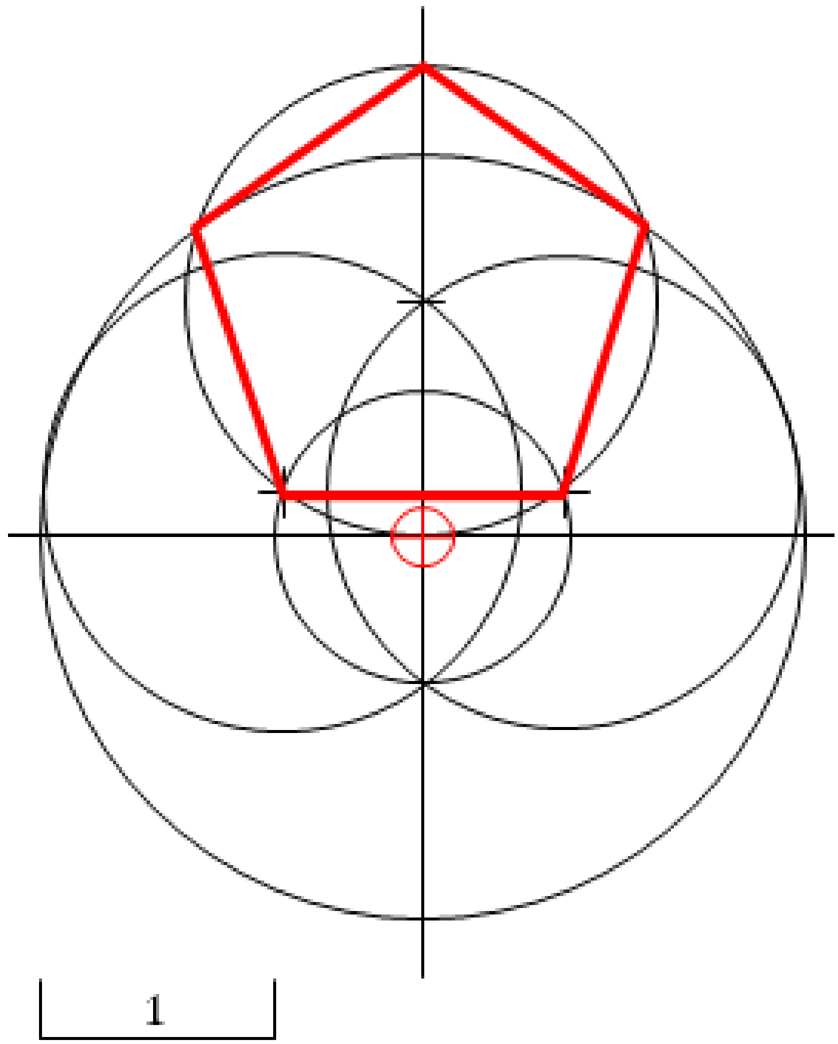


Figure 115

Step 8: Insert a star at the vertices of the pentad

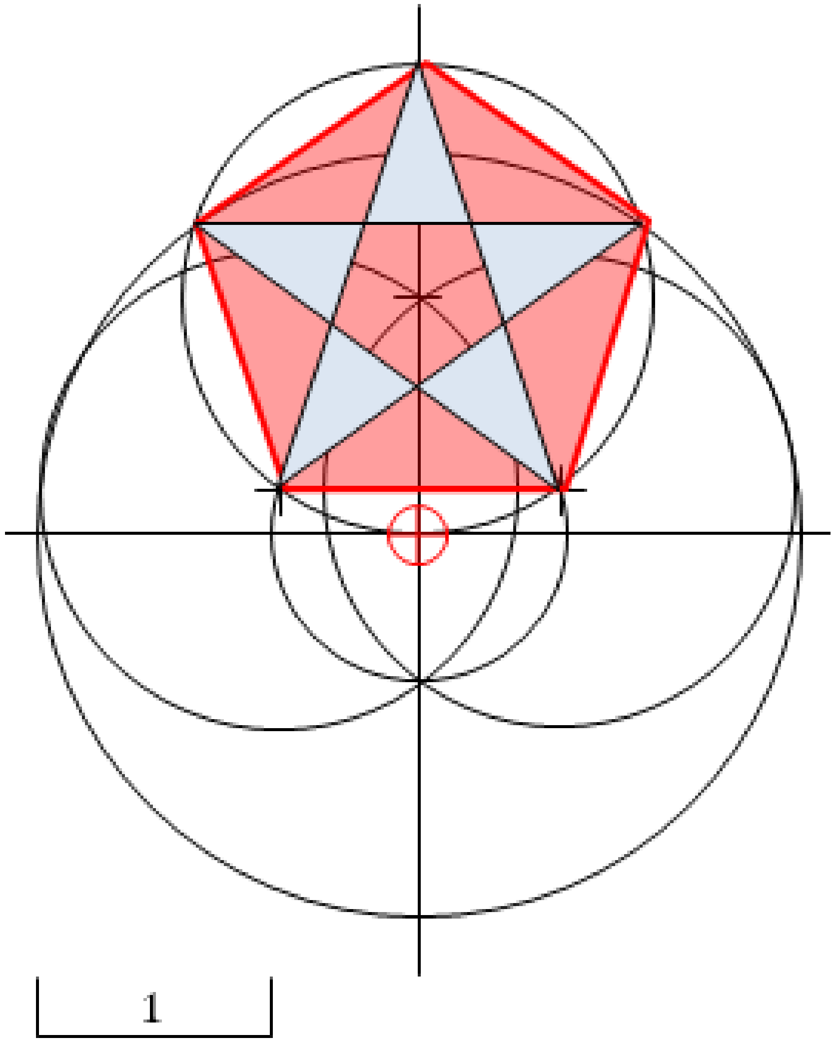


Figure 116

Oblique Stars form the foundation for Washington DC's diagonal streets that cut across the conventional rectilinear street layouts.

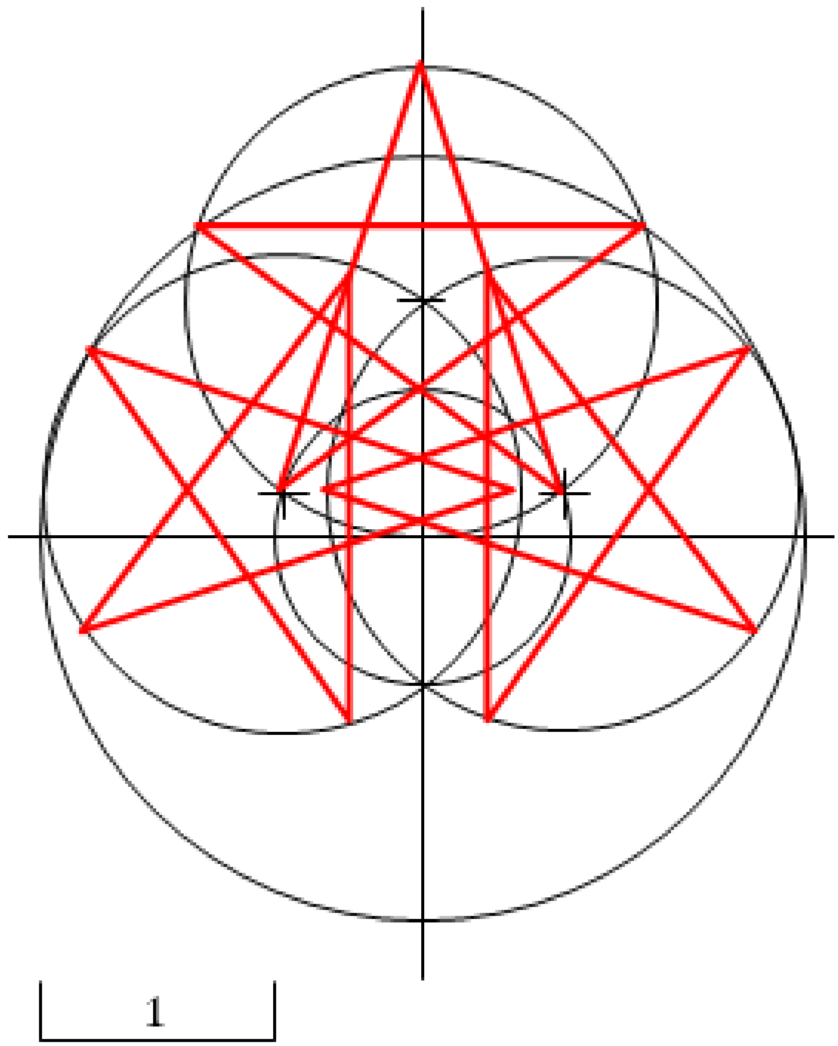


Figure 117

Stars and pentads are generated at Phi Ratio Intervals. The value of this structure is that it suggests that a high-fidelity metrology should be based on Phi Ratio, instead of conventional decimal metrics. Growth rates and cell maturity may be more evident by using "natural" metrics that can be normalized for convenience.

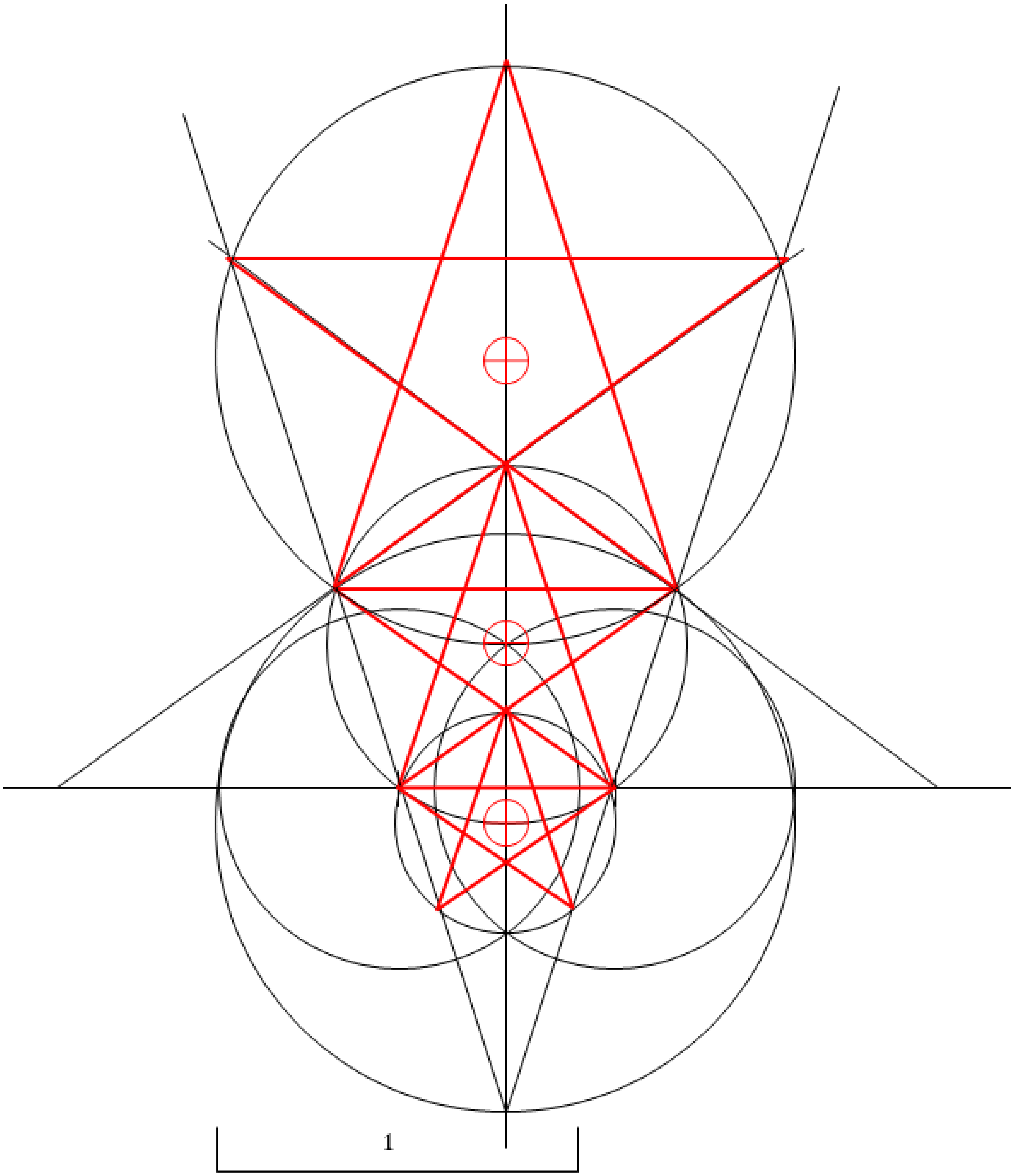


Figure 118 A Stellate Version of Jacob's Ladder

Phyllotaxic Scaffolding

A Phyllotaxic distribution of points about the circle makes use of a cardinal angle of 137.5˚. This angle is known in popular literature, and is the equivalent of 360° x Φ^{-2}, or 360° x (1–1/Φ).

Among other applications, the natural world uses the phyllotaxic distribution of leaves around branches, and branches around stems, which has the result of generally avoiding the casting of shadows on lower leaves, and a large capture cross-section for rain and dew.

Angles are otherwise not widely used in the Ancient and Sacred Geometries, for a number of reasons, some of which are not well understood today. Angles were described as ratios of the lengths of sides, a precursor to today's Trigonometry, but without the decimal fractions. The Egyptians, for example, employed the ratios of sides, or *sekets*, to define an angle – a precursor to modern Trigonometry without the decimal fractions.

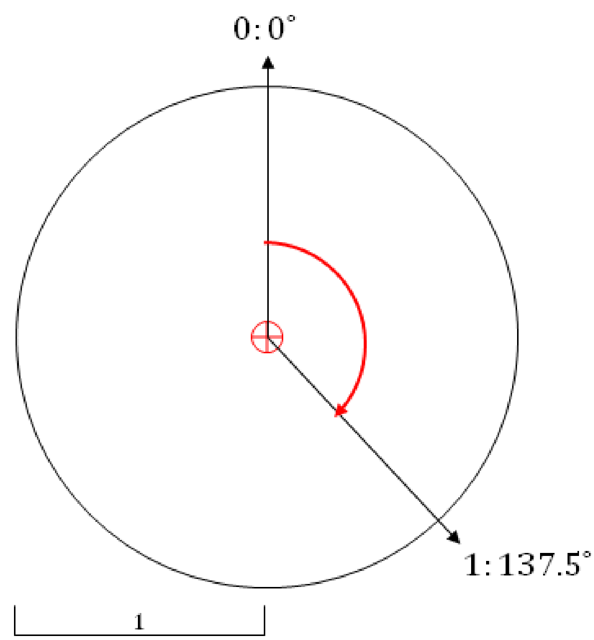


Figure 119 The Phi Cut of 360° is 137.5°

The radial version of the Phi Ratio Cut is depicted in Figure 119, above, and differs in application from all other PRG devices demonstrated heretofore. The angle is already unusual in PRG domains, and is again unusual in that it is applied repeatedly, and additively, as can be seen in Figure 120, Figure 121, and beyond. In Figure 120 there are enough points on the circle to construct a polygon in the form of a circle, in Figure 121 a trapezoid, and in Figure 122 a pentad or a star.

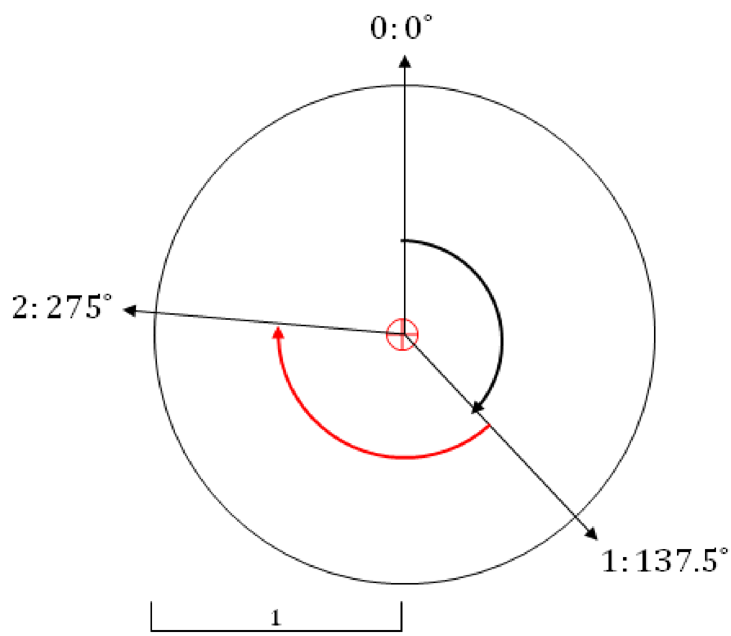


Figure 120

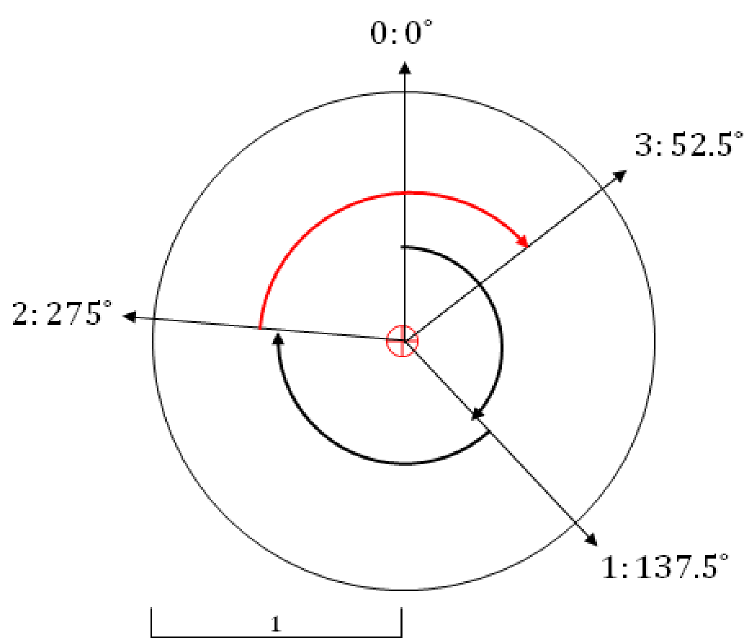


Figure 121

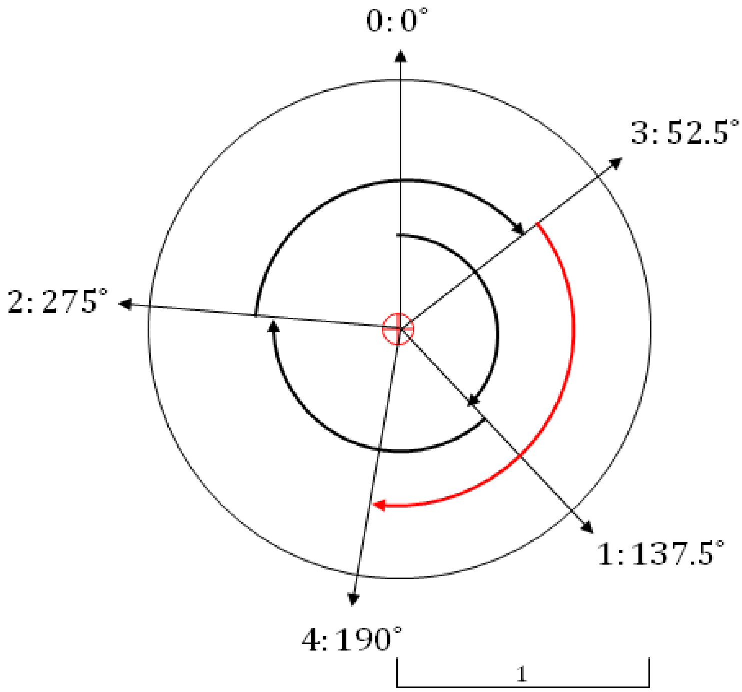


Figure 122

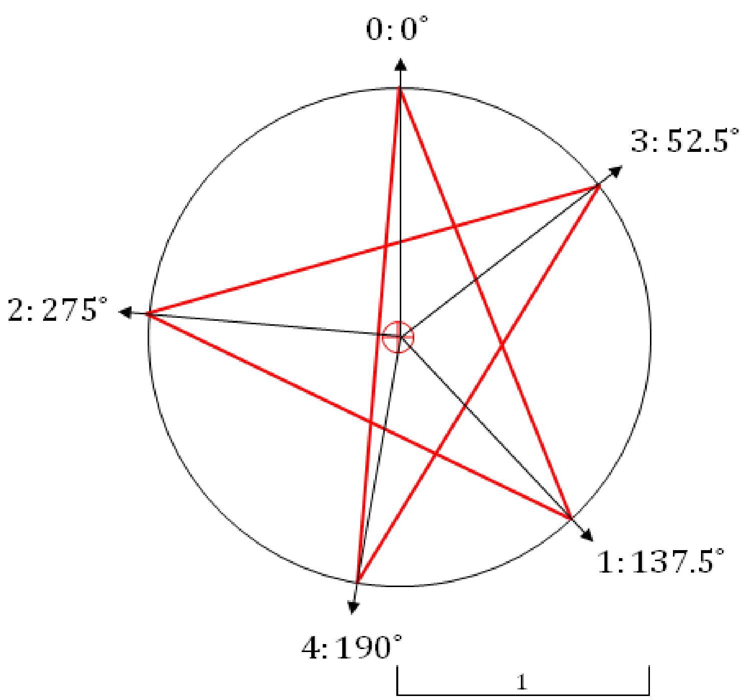


Figure 123

The Phyllotaxic‡[13] scaffolding generates a *pentad stellate* with its first five arms, and is validated through retention of a bilateral symmetry (across the 275° edge), although it diverges from the appearance of a regular star.

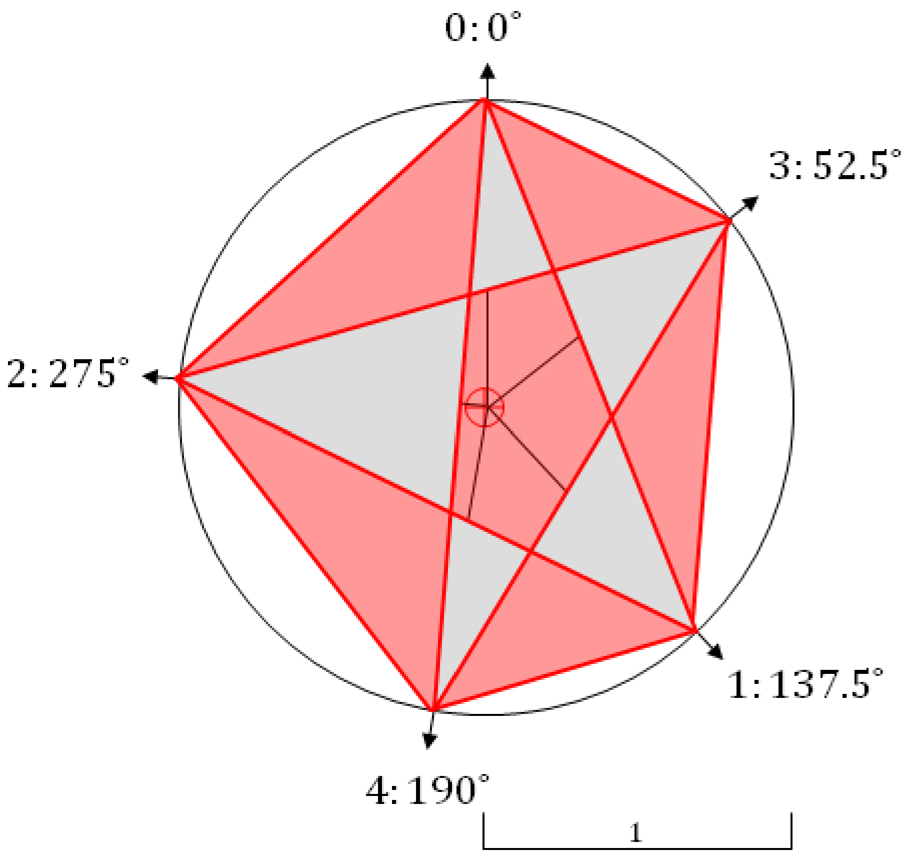


Figure 124

[13] Crisscrossing spirals found in nature. In botany, ***phyllotaxis*** or ***phyllotaxy*** is the arrangement of leaves on a plant stem (from Ancient Greek: (phyllon "leaf" and táxis "arrangement"). A repeating spiral can be represented by the brachiation of leaves, or branches about a stem, per revolution. The numerator and denominator normally consist of a Fibonacci integer and its immediate successor. -- Wikipedia, amended

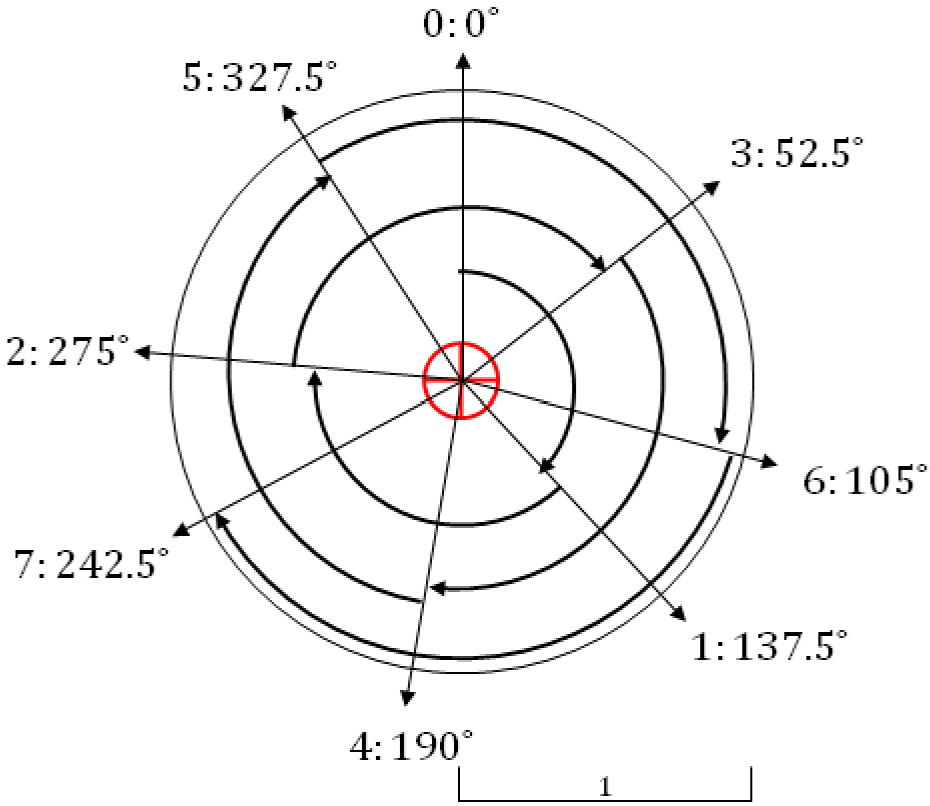


Figure 125

Table 5

	137.5	MOD 360
0	0.0	0.0
1	137.5	137.5
2	275.0	275.0
3	412.5	52.5
4	550.0	190.0
5	687.5	327.5
6	825.0	105.0
7	962.5	242.5

A *Septad Stellate* results in a closed object featuring a single bilateral symmetry (across the 52.5° edge). This construct may be perceived as sufficiently irregular that it is overlooked when it is present, yet it is mirrored across a line of bilateral symmetry.

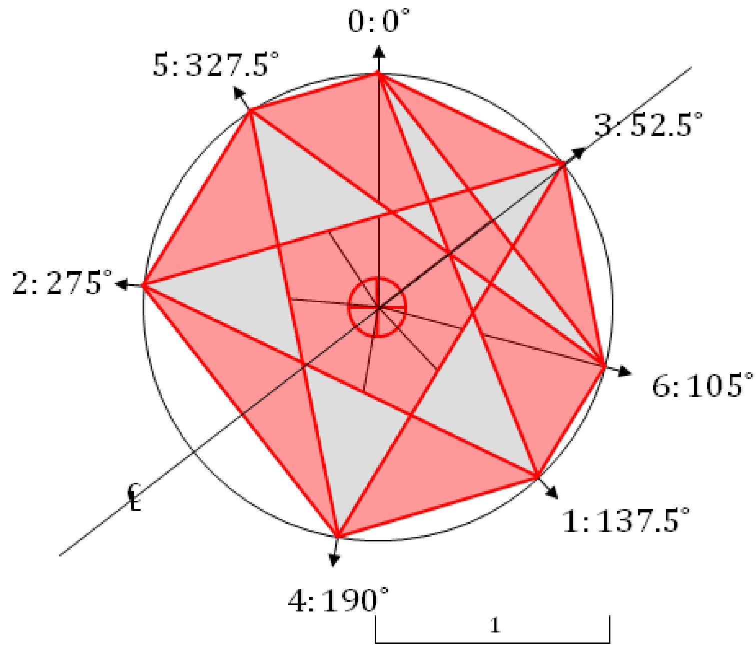


Figure 126

An *Octad Stellate* is also a closed object, mirrored across a line of bilateral symmetry.

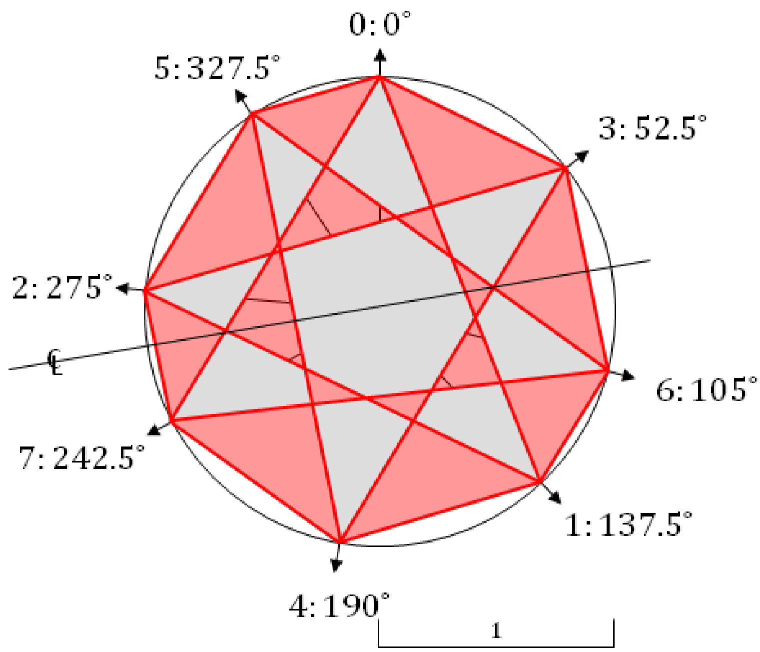


Figure 127

The Phyllotaxic scaffolding is troublesome because it is the only construct in this series that is built on a web of angles. This is not consistent with "standard" Hermetic Practices, where angles are usually the consequence of standard form in geometry, and not the driver.

An interested challenge is to develop a geometric proof that justifies the 137.5˚ interval, derived by 360˚ * (1–1/Φ). The *Vesica Piscis Duplex* superimposed with phyllotaxic radials in Figure 128 provides a generalized conceptual view of such a notion. Then again, there may be another approach that is evocative of the Chambered Nautilus.

Such an accomplishment will pave the way to comprehend Phi Ratio Geometry in spherical space.

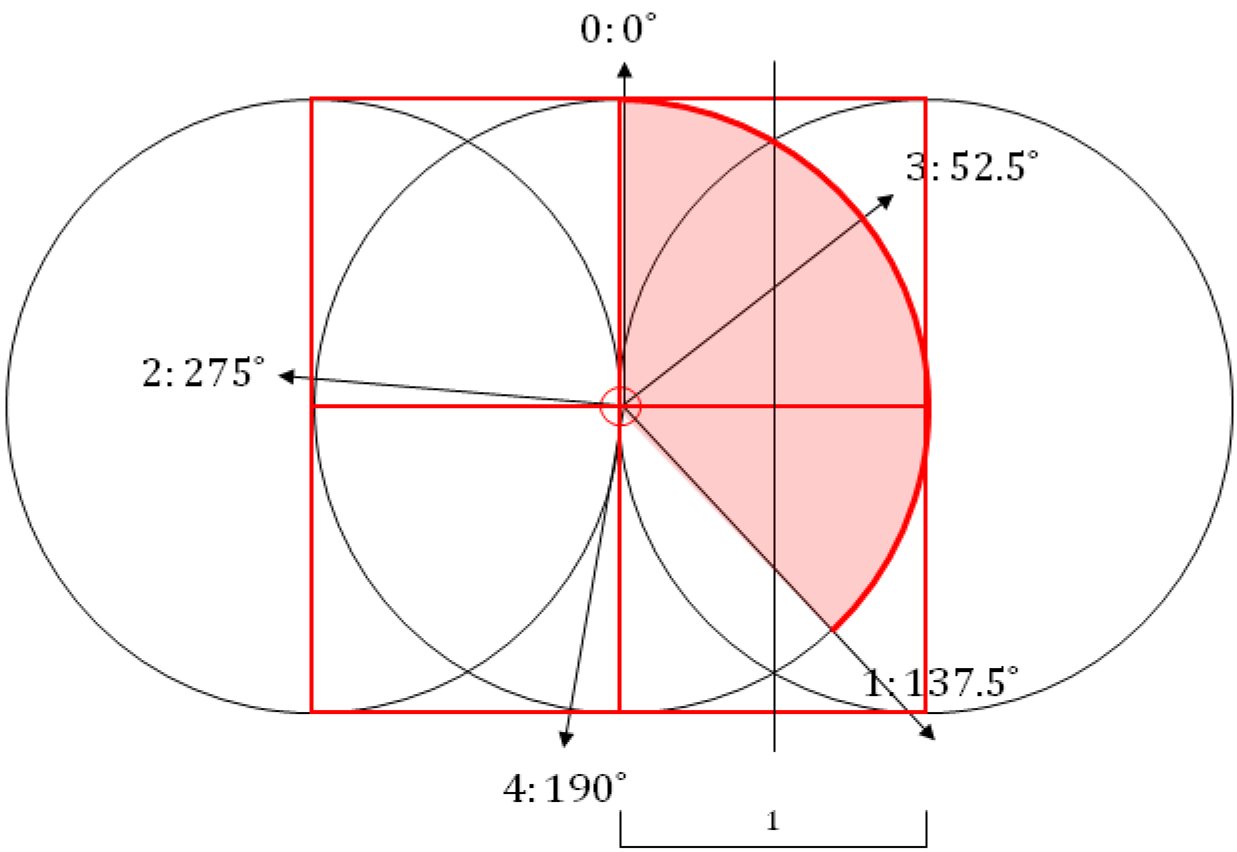


Figure 128

A Heptad

We were told in grade school that it was not possible to draw a Heptagon with a compass, and straightedge. After forty-plus years, we demonstrate how to slay that dragon.

Step 1: Draw an arbitrary circle, preferably in registration with the hexagonal grid:

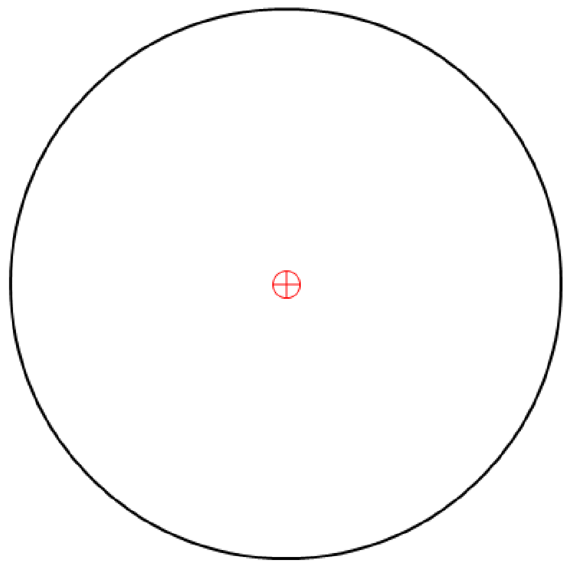


Figure 129

Step 2: Draw a square to frame the circle:

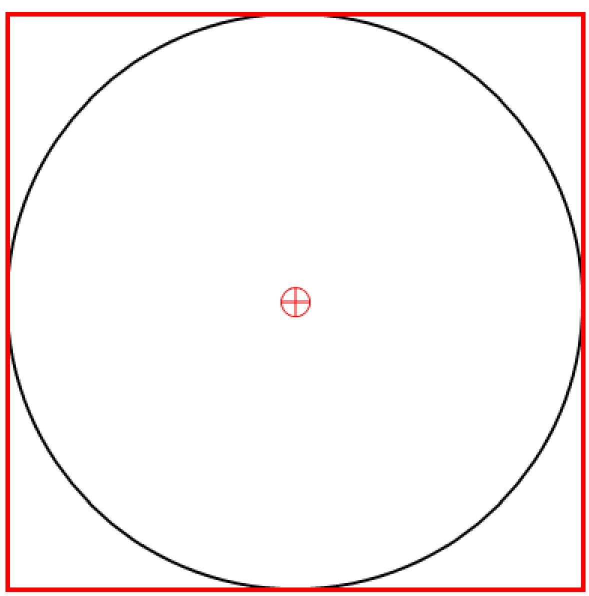


Figure 130

Step 3: Cross diagonals from corners:

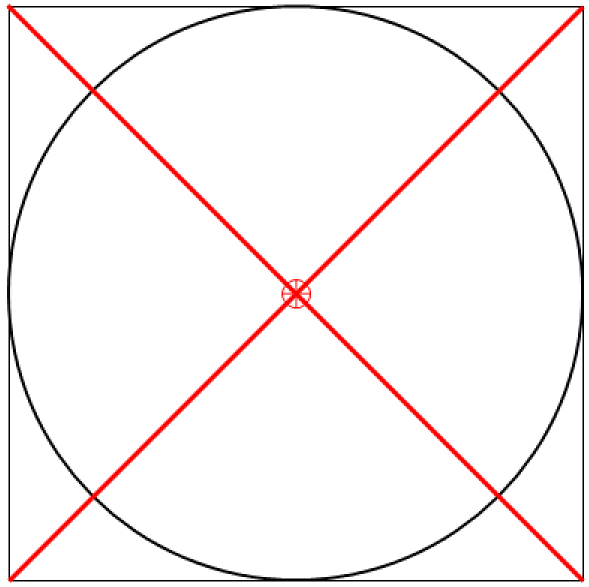


Figure 131

Step 4: Draw a square inside the circle. This provides an esthetic, visual reference, and not a structural reference within the drawing:

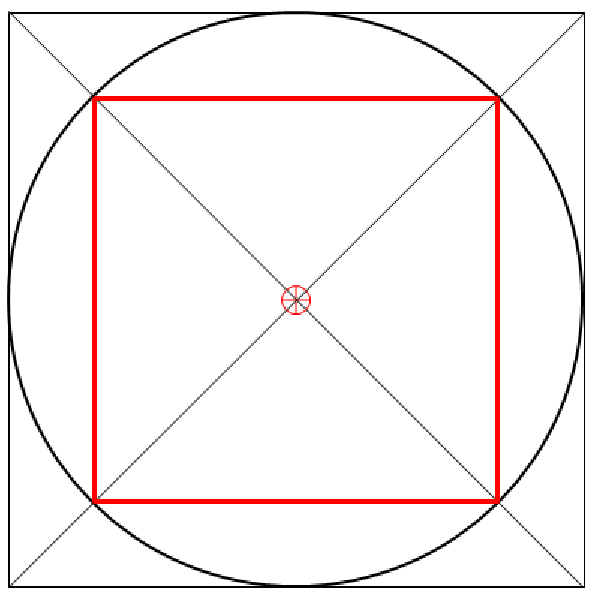


Figure 132

Step 5: Rotate a copy of the interior square:

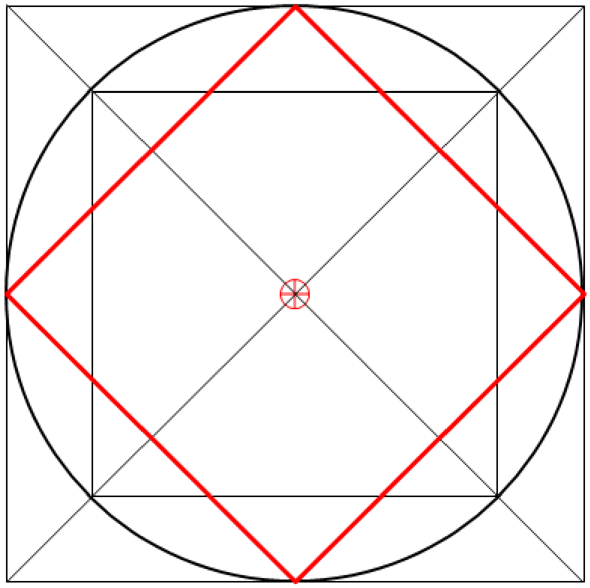


Figure 133

Step 6: Draw a $\sqrt{5}$ hypotenuse based on a 2 x 1 rectangle:

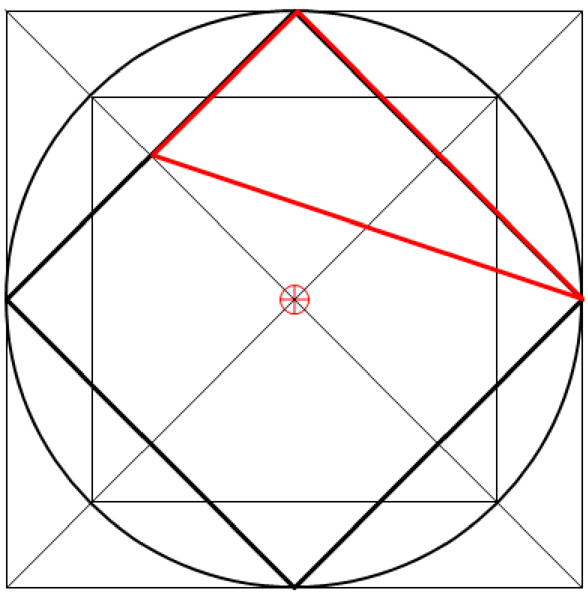


Figure 134

Step 7: Draw a circle with √5 radius referenced to the long hypotenuse within the stacked, and rotated unit squares:

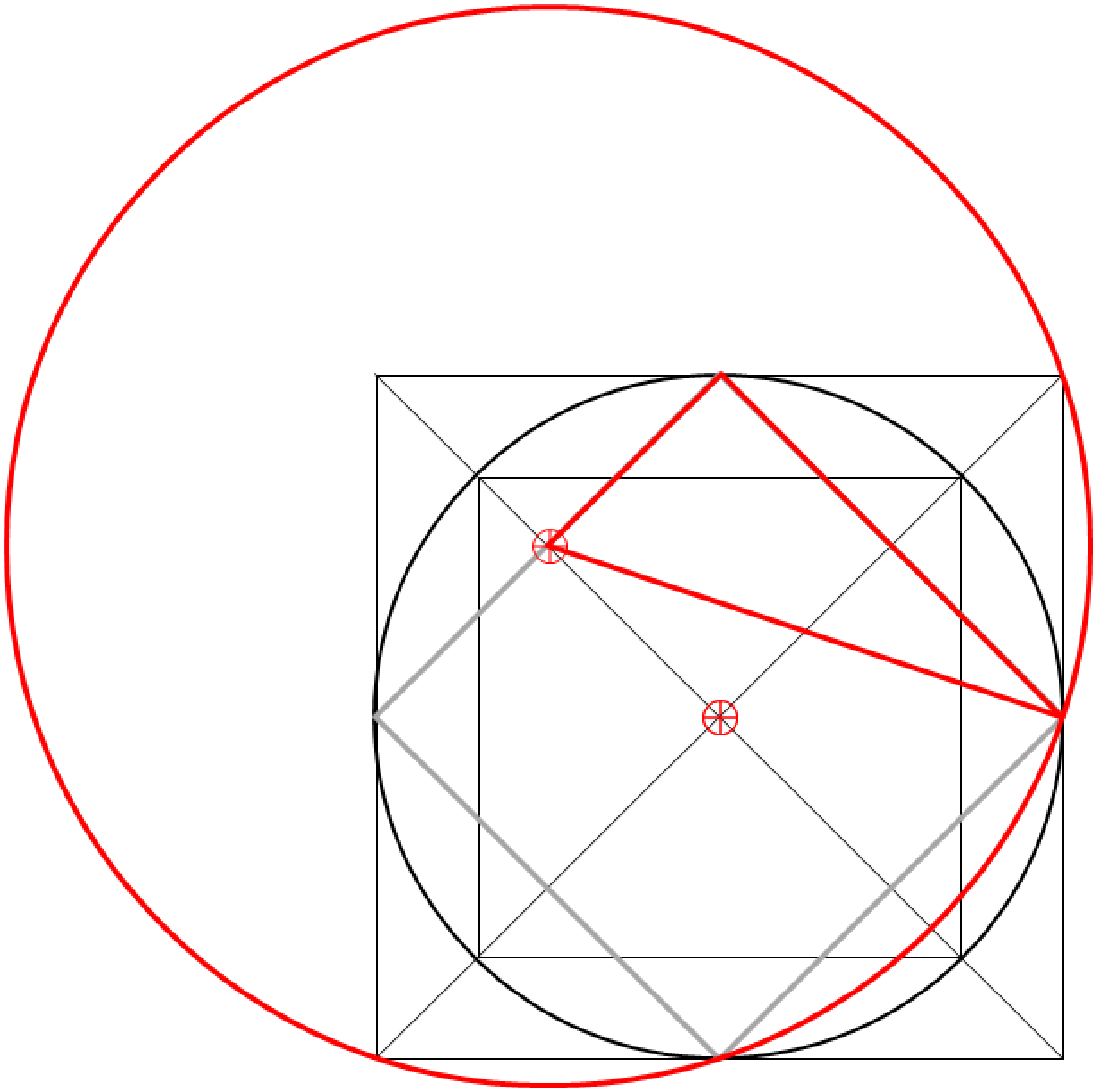


Figure 135

Step 8: Extend the base of the same triangle that was used for the hypotenuse to intersect the √5 circle. This provides a defined length that extends the scaffolding:

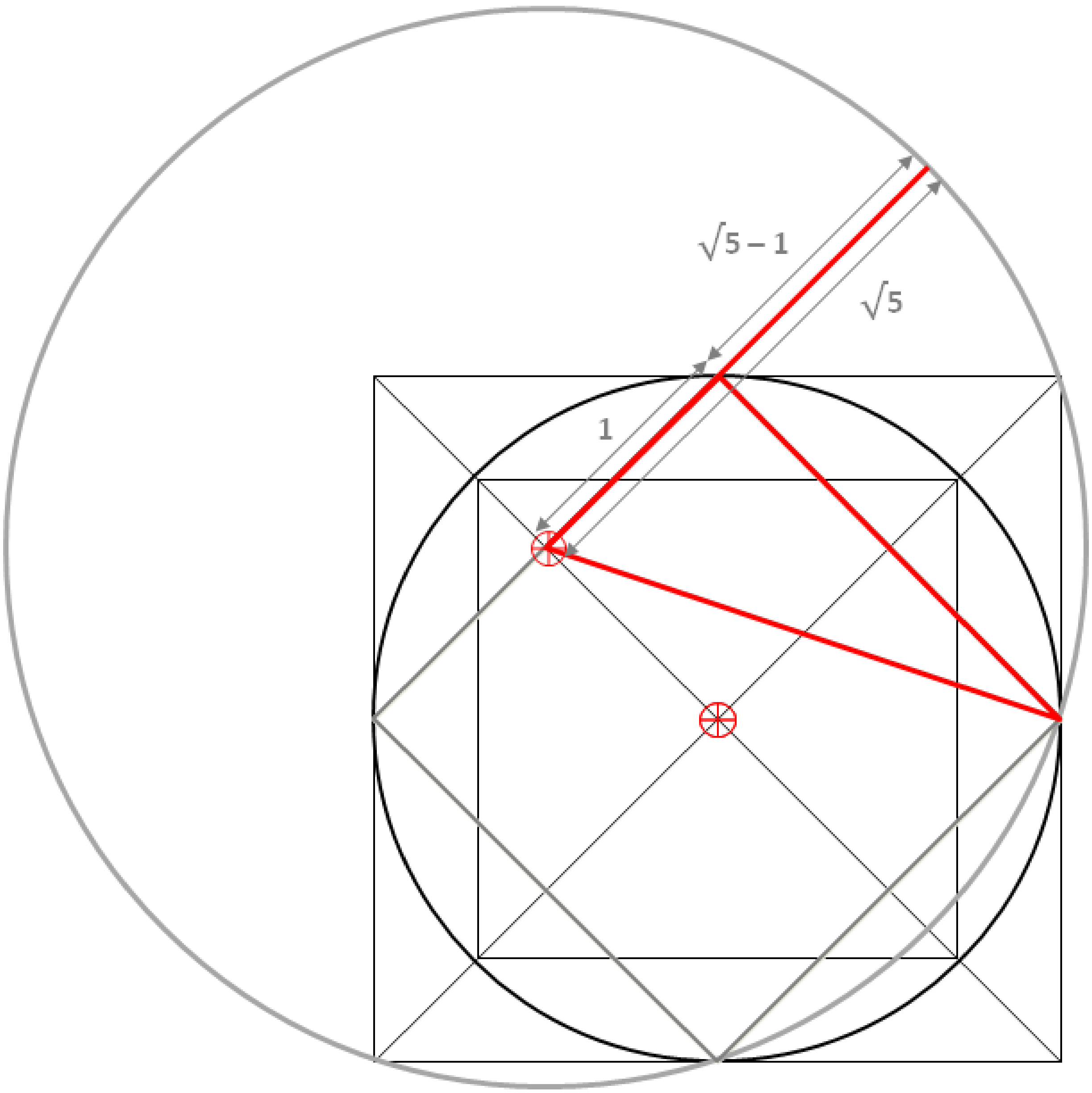


Figure 136

Step 9: At the apex of the square structure, inscribe a circle that intersects the outer $\sqrt{5}$ circle. Observe that the radius of the new circle is diminished by the unit length (1) of the square:

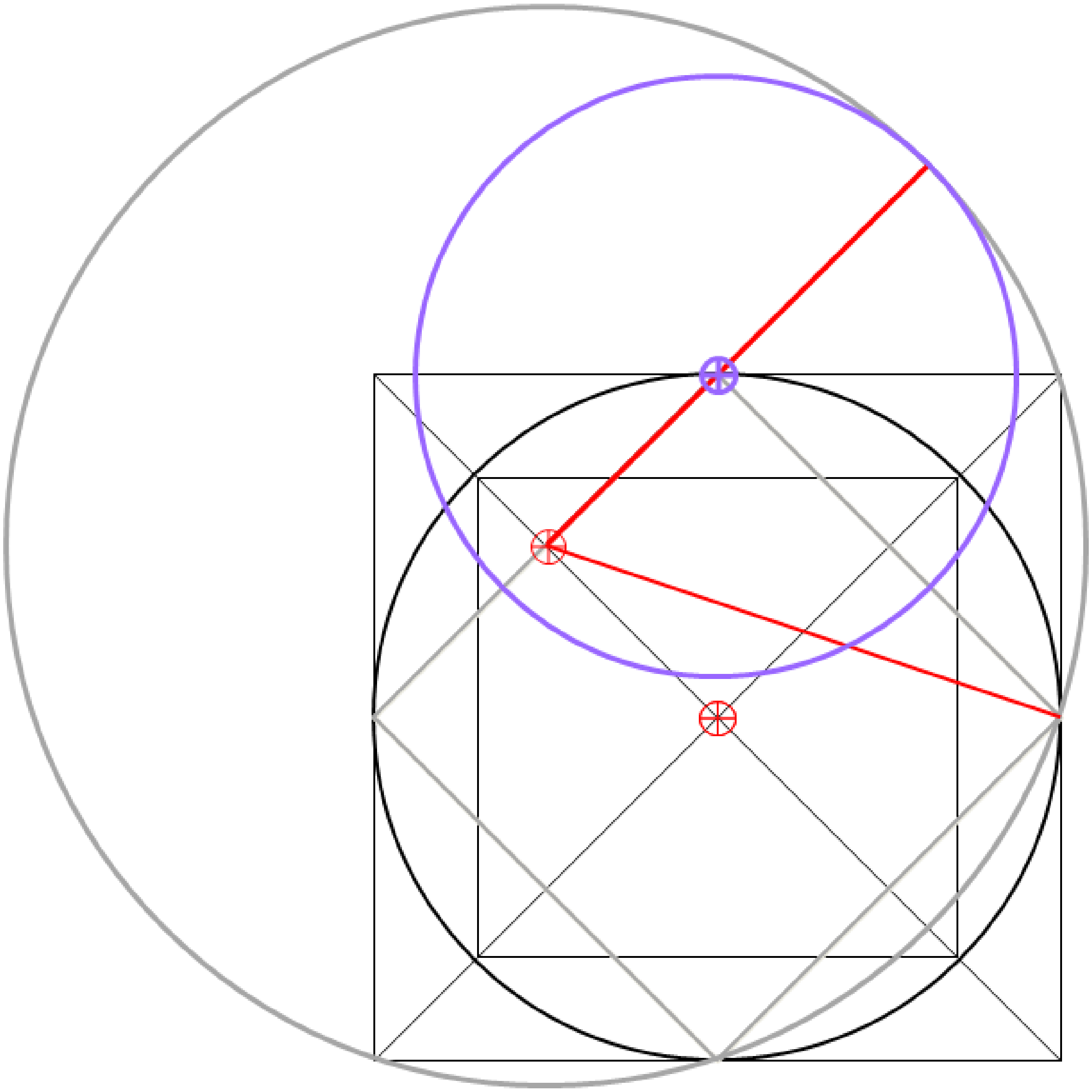


Figure 137

Step 10: Repeat $\sqrt{5} - 1$ radius circles at conjunctions with the foundational circle, forming Vesica Pisces:

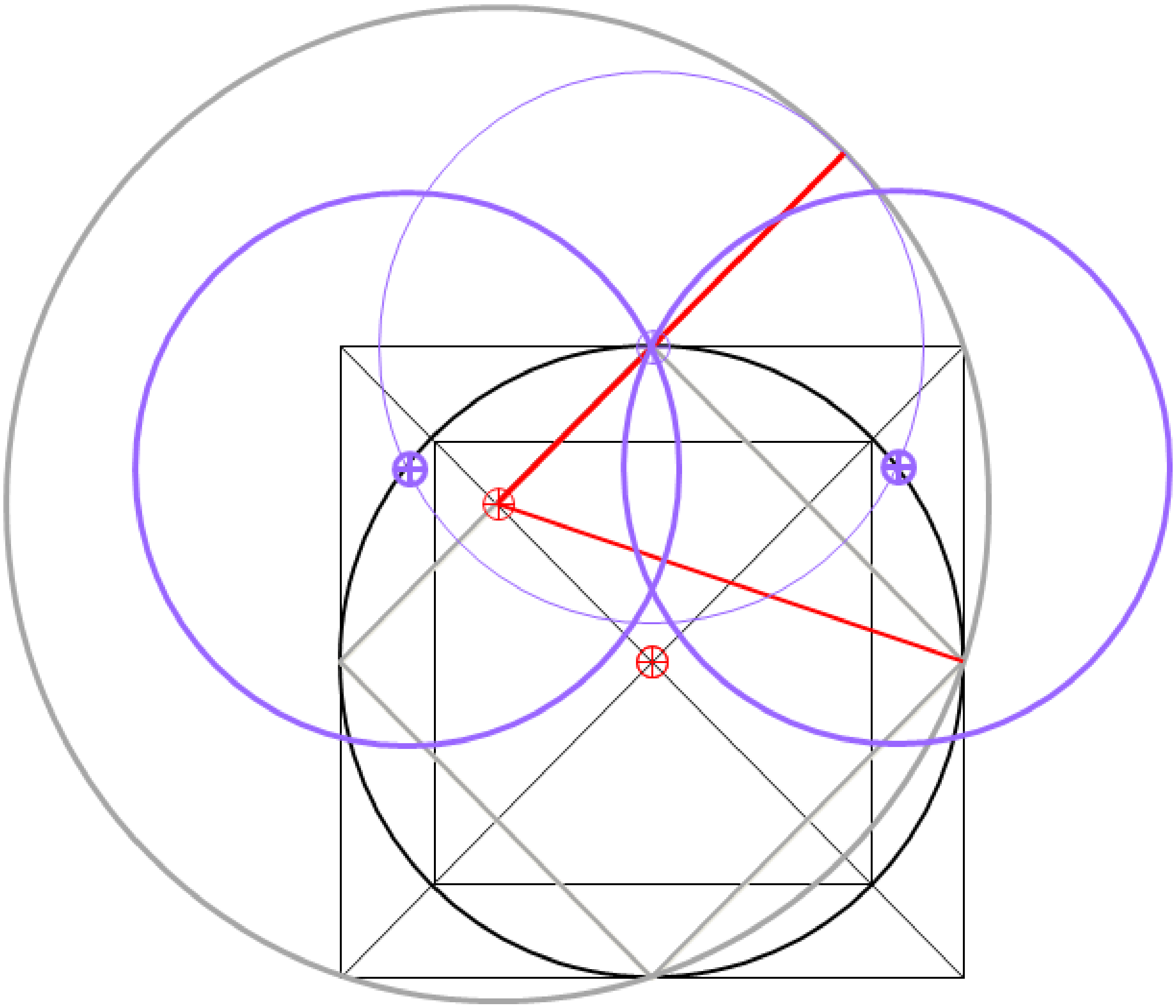


Figure 138

Step 11: Continue to repeat √5 – 1 radius circles at conjunctions with the foundational circle, forming a series of Vesica Pisces in an arc:

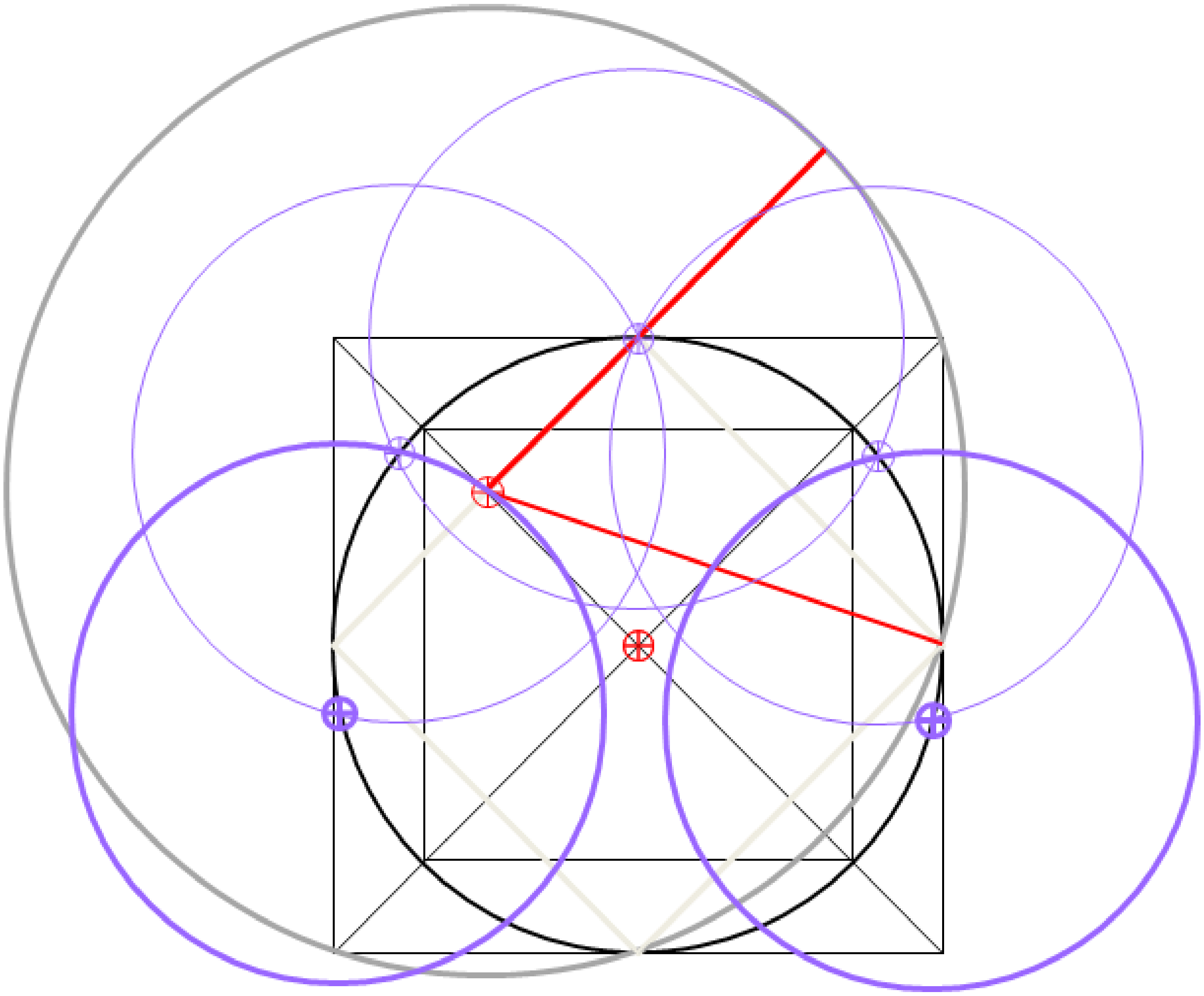


Figure 139

Step 12: Continue to repeat √5 – 1 radius circles at conjunctions with the foundational circle, forming a final pair of Vesica Pisces at the base.

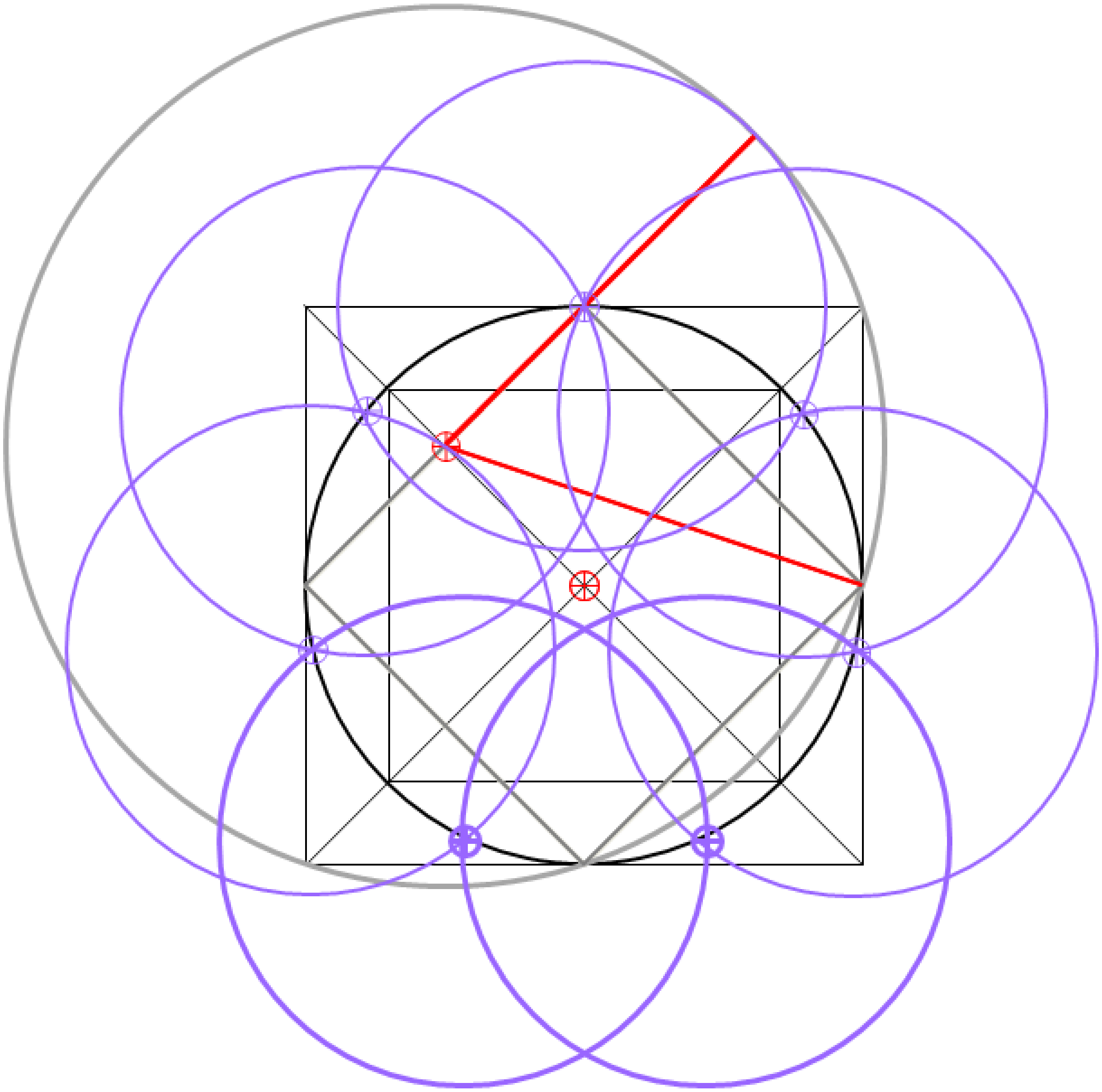


Figure 140

Step 13: Observe the Septad vertices that remain after removing the scaffold:

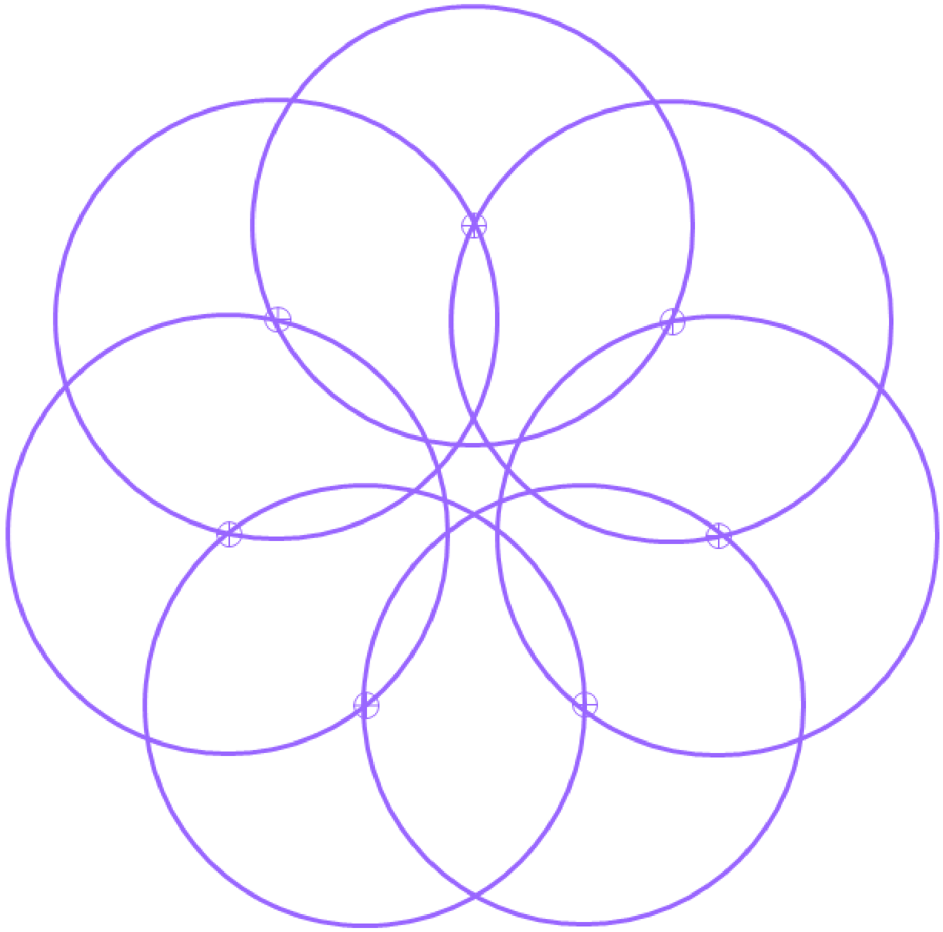


Figure 141

Step 14: Draw chords among the centers of each of the ring of circles, forming the sides of a Septad:

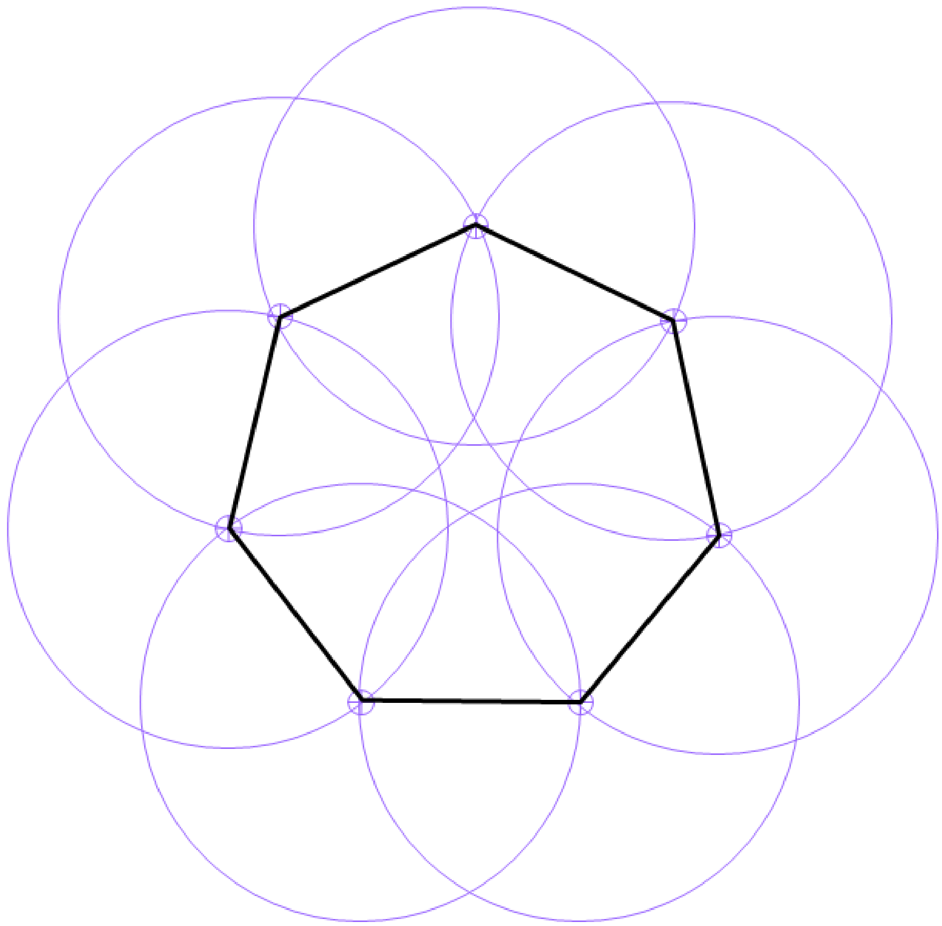


Figure 142

Step 15: Lift the completed Septad from the drawing board.

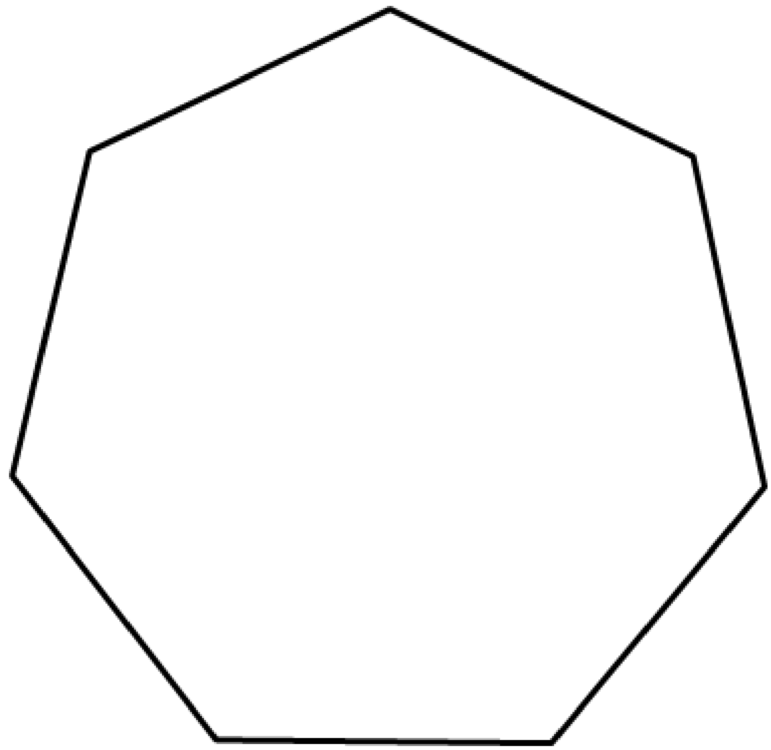


Figure 143

Web-based Features

www.phimatrix.com

www.phipoint.com

www.goldennumber.net

www.naturebynumbers.com