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Bing-yuan Cao
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Volume 1

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Preface

This book is the proceedings of the 5th Annual Conference on Fuzzy Information and Engineering (ACFIE2010) from Sep. 23-27, 2010 in Huludao, China. The conference proceedings is published by Springer-Verlag (Advances in Intelligent and Soft Computing, ISSN: 1867-5662).

This year, we have received more than 191 submissions. Each paper has undergone a rigorous review process. Only high-quality papers are included. The 5th Annual Conference on Fuzzy Information and Engineering (ACFIE2010), built on the success of previous conferences, the ACFIE2005 (Guangzhou, China), is a major symposium for scientists, engineers and practitioners in China to present their updated results, ideas, developments and applications in all areas of fuzzy information and engineering. It aims to strengthen relations between industry research laboratories and universities, and to create a primary symposium for world scientists in fuzzy fields as follows:

1) The mathematical theory of fuzzy systems;
2) Fuzzy logic, systems and control;
3) Fuzzy optimization and decision-making;
4) Fuzzy information, identification and clustering;
5) Fuzzy engineering application and soft computing method; etc.

This book contains 89 papers, divided into five main parts:

In Section I, we have 15 papers on "the mathematical theory of fuzzy systems".

In Section II, we have 15 papers on "fuzzy logic, systems and control".

In Section III, we have 24 papers on "fuzzy optimization and decision-making".

In Section IV, we have 17 papers on "fuzzy information, identification and clustering".

In Section V, we have 18 papers on "fuzzy engineering application and soft computing method".

In addition to the large number of submissions, we are blessed with the presence of eight renowned keynote speakers and several distinguished panelists and we shall organize workshops.

On behalf of the Organizing Committee, we appreciate Liaoning Technology University in China, Fuzzy Information and Engineering Branch of China Operation Research Society for sponsorship; Fuzzy Information and Engineering Branch of International Institute of General Systems Studies China Branch (IIGSS-GB) Co-Sponsorships. We are grateful to the supports coming from the two international magazines published by Springer-Verlag GmbH: Fuzzy Optimization & Decision Making (FODM) and Fuzzy Information & Engineering. We are showing gratitude to the members of the Organizing Committee, the Steering Committee, and the Program Committee for their hard work. We wish to express our heart-felt appreciation to the keynote and panel speakers, workshop organizers, session chairs, reviewers, and students. In particular, I am thankful to Wei-wei Fu, Tao Song and Jian Han, who have contributed a lot to the development of this issue. Meanwhile, we thank the publisher, Springer, for publishing the ACFIE 2010 proceedings as J. Advances in Intelligent and Soft Computing (AISC) and J. Advances in Soft Computing (ASC)(ASC 40 in ICFIE'07, ASC 54 in ICFIE'08 and AISC 62 in ICFIE'09 published by the Springer Publisher has been included into ISTP). Finally, we appreciated all the authors and participants for their great contributions that made this conference possible and all the hard work worthwhile.

September 2010 in Huludao,
P.R. China

Bing-yuan Cao
Si-zong Guo

Organization

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Contents

The Mathematical Theory of Fuzzy Systems

Fuzzy Logic, Systems and Control

Fuzzy Information, Identification and Clustering

Fuzzy Engineering Application and Soft Computing Method

On the Completion of L-Fuzzy Topological Vector Spaces

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Abstract. This paper discusses the completion of L-fuzzy topological vector spaces given by Fang and Yan [1] in 1997. We characterize the completable of L-fuzzy topological space and show that the corresponding completion is uniquely determined up to fuzzy order-hemomorphism.

Keywords: L-fuzzy topological vector spaces, λ-cauchy net, linear order-hemomorphism, Completion.

1 Introduction

In [1], Fang and Yan introduced the notion of L-topological vector space, which is a generalization of the usual topological vector space and the fuzzy topological vector space in the sense of Katsaras [8] According to the standardized terminology in [2], it will be called L-topological vector spaces (L-tvs). It is well know that the theory of completion of spaces plays a quite important role in classical function analysis and its application. So it is natural and necessary to find the counterpart of L-completion in research of L-tvs. The purpose of this paper is to extend this theorem to L-topological vector spaces.

2 Basic Concepts and Lemmas

Throughout this paper. Let X be a vector space over the field K (R or C) and θ denote the zero element of X . L denotes a complete and completely distribute lattice equipped with an order-reversing involution $':L\to L$, $0 and 1$ are their smallest and greatest element, respectively. We always assume that they are regular. (i.e. the intersection of each pair of non-zero elements in L is not zero,which is equivalent to $1\in M(L)$). An L-fuzzy set which takes the constant value $\lambda\in L$ on X is denoted by λ^* . An L-fuzzy set on X is called an L-fuzzy point if it takes value 0 for all $y\in X$ except one, says $x\in X$ .If its value at x is $\lambda\in L\setminus\{0\}$, we denote this L-fuzzy point by x_λ . An L topology δ on X is called statified if it

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contains all constant L-fuzzy sets on X. We always assume that the L-topologies refered to in this paper are all stratified. Other symbols which are not mentioned may refer to [2,3,4].

Definition 2.1[5,6]. *Let* (L,δ) *be an* L*-topological space and* $x_\lambda \in M\left(L^X\right)$, $P \in L^X$ *is called a closed R-neighborhood of* x_λ *,if* $P \in \delta'$ *and* $x_\lambda \notin P$. *The set of all closed R-neighborhood of* x_λ *is denoted by* $\eta^-(x_\lambda)$.

$A \in L^X$ *is called an R-neighborhood of* x_λ *,if there exists* $P \in \eta^-(x_\lambda)$ *such that* $A \le P$. *The set of all R-neighborhood of* x_λ *is denoted by* $\eta(x_\lambda)$.

Definition 2.2[1]. *The addition and the scalar multiplication operator of* L*-fuzzy sets on* X *are defined as follows: for* $A,B \in L^X$ *and* $k \in K$.

$$(A+B)(x) = \bigvee_{y+z=x}\left(A(y)\wedge B(z)\right);\ (kA)(x) = A\left(x/k\right),\ k \ne 0$$

$$(0A)(x) = \begin{cases} \bigvee_{y\in X} A(y), x = 0 \\ 0, x \ne 0 \end{cases}$$

In particular, for L*-fuzzy points* x_λ, y_μ *and* $k \in K$ *we have*

$$(x_\lambda + A)(y) = \lambda \wedge A(y-x);\ \ (x+A)(y) = A(y-x)$$
$$x_\lambda + y_\mu = (x+y)_{\lambda\wedge\mu};\qquad kx_\lambda = (kx)_\lambda$$

Definition 2.3[1]. *Let* δ *be an* L*_topology on* X. *The pair* $\left(L^X,\delta\right)$ *is called an* L*-tvs if the following the mapping(the addition and the scalar multiplication on* X *):*

(1) $f: X\times X \to X$, $(x,y)\to x+y$

(2) $g: K\times X \to X$, $(k,x)\to kx$

are both continuous, where $X\times X$ *and* $K\times X$ *are equiped with the corresponding product* L*-topologies* $\delta\times\delta$ *and* $J_K\times\delta$ *,respectively, and* J_K *denote the usual topology on* K.

Definition 2.4[3]. *Let* $\left(L^X,\delta\right)$ *be an* L*-tvs and* $\lambda \in M(L)$. *Let* D *be a directed set. A molecular net* $\left\{x_{\lambda_n}^{(n)}\right\}_{n\in D}$ *in* L^X *is called a* $\lambda-$ *cauchy net, if for each R-neighborhood* W *of* θ_λ*, there exists* $n_0 \in D$ *such that* $x_{\lambda_n}^{(n)} - x_{\lambda_m}^{(m)} \notin W$ *for all* $m,n \succ n_0$.

An L *-tvs* $\left(L^X,\delta\right)$ *is complete in every stratum, if for every* $\lambda \in M(L)$ *and* $\lambda-$ *cauchy net*

$$S = \left\{x_{\lambda_n}^{(n)}\right\}_{n\in D}\ in\ \left(L^X,\delta\right),\ S \to x_\lambda \in M\left(L^X\right).$$

Lemma 2.1 [1]. *Let (L^X,δ) be an L -tvs and β_λ $(\lambda\in M(L))$ be a closed R-neighborhood base of θ_λ. Then β_λ $(\lambda\in M(L))$ has the following properties:*

(1) If $W\in\beta_\lambda$ or $W=\lambda^*$, then for any $x\in X$ and $\alpha\in M(L)$ with $x_\alpha\notin W$, there exists $P\in\beta_\alpha$ suth that $W\le x+P$.
(2) If $P,Q\in\beta_\lambda$, then there exists $W\in\beta_\lambda$ such that $P\vee Q\le W$.
(3) If $W\in\beta_\lambda$ there exists $P\in\beta_\lambda$ such that $P'+P'\le W'$
(4) If $W\in\beta_\lambda$, then exists $P\in\beta_\lambda$ such that $tP'\le W'$ for all $t\in K$ with $|t|\le 1$
(5) If $W\in\beta_\lambda$, then for each $x\in X$ there exists $t>0$ such that $x_\lambda\notin tW$.

Conversely, if for each $\lambda\in M(L)$ there is a family β_λ of L -fuzzy sets on X satisfying the above conditions (1)-(5), then there exists a unique L -topology δ on X such that (L^X,δ) is an L -tvs and β_λ is a closed R-neighborhood base of θ_λ.

Lemma 2.2 [1]. *Let (L^X,δ) be an L-tvs, then for each $\lambda\in M(L)$ there exists a closed R-neighborhood base β_λ of θ_λ and Q' is balanced for each $Q\in\beta_\lambda$.*

Lemma 2.3 [3]. *Let (L_1^X,δ_1) and (L_2^Y,δ_2) be two L -tvs, and let X_0 be a crisp linear subspace of X which is dense in every stratum of (L_1^X,δ_1). Let (L_2^Y,δ_2) be Hausdorff and complete in every stratum, and let*

$G:L_1^{X_0}\to L_2^Y$ be a continuous fuzzy linear order homomorphism. Then there exists a unique continuous fuzzy linear order homomorphism $F:L_1^X\to L_2^Y$ such that $F|_{L_1^{X_0}}=G$.

3 Main Result

Theorem 3.1. *Let (L^X,δ) be an L -tvs, then there exists a complete Hausdorff L -tvs (L^{X_1},δ) over K contain (L^X,δ) as a dense subspace.*

Proof: (a) For each $\lambda\in M(L)$. Let L^{X_1} be the set of all λ -cauchy net in (L^X,δ).

We define scalar multiplication and addition as following, respectively.

For each $\xi=\{x_\lambda^{(p)}:p\in\beta_\lambda\}$, $t\in K$. We define $t\xi=\{tx_\lambda^{(p)}:p\in\beta_\lambda\}$, where β_λ is a R-neighborhood base of θ_λ for each $\lambda\in M(L)$. Note that β_λ is directed by inclusion.

Obviously $t\xi \in L^{X_1}$.

For any $\xi = \left\{ tx_\lambda^{(p)} : p \in \beta_\lambda \right\}$, $\eta = \left\{ ty_\lambda^{(q)} : q \in \beta_\lambda \right\} \in L^{X_1}$.

Put $s = \left\{ x_\lambda^{(p)} + y_\lambda^{(q)} : (p,q) \in \beta_\lambda \times \beta_\lambda \right\}$; $s^{(p,q)} = x^{(p)} + y^{(q)}$ for each $(p,q) \in \beta_\lambda \times \beta_\lambda$

We define a partial order on $\beta_\lambda \times \beta_\lambda$ as follows:

$$(p_1, q_1) \le (p_2, q_2) \text{ if } p_1 \le p_2 \text{ , } q_1 \le q_2$$

It is obvious that the set $\left\{ s_\lambda^{(p,q)} : (p,q) \in \beta_\lambda \times \beta_\lambda \right\}$ is a molecular net in L^{X_1} , we can prove that $s_\lambda^{(p,q)} \to \theta_\lambda$ .In fact for each $W \in \beta_\lambda$ there exists $P \in \beta_\lambda$ such that $P' + P' \le W'$ by Lemma 2.1.Since there exists $p_0, q_0 \in \beta_\lambda$ such that $x_\lambda^{(p)} - x_\lambda^{(p_1)} \notin P$ for all $p, p_1 \ge p_0$ and $y_\lambda^{(q)} - y_\lambda^{(q_1)} \notin P$ For all $q, q_1 \ge q_0$.

Then, $\left(x_\lambda^{(p)} + y_\lambda^{(q)} \right) - \left(x_\lambda^{(p_1)} + y_\lambda^{(q_1)} \right) = \left(x_\lambda^{(p)} - x_\lambda^{(p_1)} \right) + \left(y_\lambda^{(q)} - y_\lambda^{(q_1)} \right) \notin P + P$ for all $(p,q), (p_1, q_1) \ge (p_0, q_0)$.

(b) For each $V \in \beta_\lambda$ we denoted V^* by the set of all $\xi = \left\{ x_\lambda^{(p)} \mid p \in \beta_\lambda \right\}$ satisfying the following conditions:

There exists $p_0 \in \beta_\lambda$ and $U \in \beta_\lambda$ such that $V \subset \left\{ x_\lambda^{(p)} \mid p \ge p_0 \right\} + U$.

There exists an R-neighborhood of θ_λ consisting of closed and co-balanced L-fuzzy sets on X for each $\lambda \in M(L)$ by lemma 2.2. and denoted by η .Put $\eta^* = \left(V^* \mid V \in \beta_\lambda \right)$.

We shall prove that η^* satisfies condition (1)-(5) in Lemma 2.1.(1)-(4)is stratified by Lemma2.1.We need only to prove(5).

For any $V \in \beta_\lambda$ there exists $U \in \beta_\lambda$ such that $U' + U' + U' \subset V'$ by [1.Theorem3.1].

Since $\left\{ x_\lambda^{(p)} \mid p \in \beta_\lambda \right\}$ is λ -cauchy net, there exists $p_0 \in \beta_\lambda$ such that $x_\lambda^{(p)} - x_\lambda^{(p_1)} \notin U$ for all

$p, p_1 \ge p_0$ .For $x_\lambda^{(p_0)}$ there exists $t \in K$ such that $t \cdot x_\lambda^{(p_0)} \notin U$, we have

$t \cdot x_\lambda^{(p)} = t \cdot x_\lambda^{(p_0)} + t \left(x_\lambda^{(p)} - x_\lambda^{(p_0)} \right) \notin U + U$, implies that $t \cdot x_\lambda^{(p)} + U' \subset V'$ .i.e.

$V \subset \left\{ t \cdot x_\lambda^{(p)} \mid p \in \beta_\lambda \right\} + U$. This completes the proof (5).

Thus by Lemma 2.1 there exists a unique L -topology δ on X such that $\left(L^{X_1}, \delta \right)$ is L -tvs and η^* is a closed R-neighborhood base of θ_λ

(c) $\left(L^X, \delta \right)$ is isometric to a dense subspaces M of $\left(L^{X_1}, \delta \right)$.

For each $x \in X$ and $\lambda \in M(L)$, obviously $\{ x_\lambda \cdots x_\lambda \cdots \}$ is a λ -cauchy net in $\left(L^X, \delta \right)$. We denote it by x_λ^* and define fuzzy linear order-homomorphism $F : L^X \to L^{X_1}$, there exist an ordinary linear operator $T:X\to X_1$ and a finite meet-preserving order homomorphism

$\varphi:L\to L$ such that $F=T_\varphi$ .We know that $T_\varphi(x_\lambda)$ is dense in $\left(L^{X_1},\delta\right)$.

In fact, for each $\xi=\left\{x_\lambda^{(p)}:p\in\beta_\lambda\right\}\in L^{X_1}$ and each $V^*\in\eta^*$ where $V\in\eta$ .Then there exist $U\in\eta$ such that $U^{'}+U^{'}\subset V^{'}$ by Lemma2.1,since $\left\{x_\lambda^{(p)}:p\in\beta_\lambda\right\}$ is λ -cauchy net in $\left(L^X,\delta\right)$,there exists $p_0\in\beta_\lambda$ such that $x_\lambda^{(p)}-x_\lambda^{(p_0)}\notin U$ for all $p\ge p_0$ implies

$$\left\{x_\lambda^{(p)}-x_\lambda^{(p_0)}\mid p\ge p_0\right\}+U^{'}\subset V^{'}\text{ i.e, }V\subset\left\{x_\lambda^{(p)}-x_\lambda^{(p_0)}\mid p\ge p_0\right\}+U$$

This completes $T(X)$ is a dense subspace of X_1 .

(d) $\left(L^{X_1},\delta\right)$ is a complete L -tvs.

Let $\xi_\lambda=\left\{x_\lambda^{(p)}:p\in\beta_\lambda\right\}$ be a λ -cauchy net in X_1 , since $T(X)$ is a dense subspace of X_1 .

For each $p^*\in\eta^*$ and $p\in\beta_\lambda$ there exist $x^{\left(\lambda,p^*\right)}\in X$ such that $\xi_\lambda-x^{\left(\lambda,p^*\right)}\notin p^*$. We define the partia order on η^* as follows:

$$V_2^*\subset V_1^*\ \text{ iff }\ V_1^*\le V_2^*$$

Put $\varepsilon=\left\{x_{\left(\lambda,V^*\right)}\right\}$, we shall prove $\varepsilon\in L^{X_1}$. In fact for each $V\in\beta_\lambda$ there exists $W\in\beta_\lambda$, such that

$$W^{'}+W^{'}+W^{'}+W^{'}+W^{'}\subset V^{'}\ \text{ Put }U_1=W^{'}+W^{'}+W^{'}\text{and }U_2=U_1+W^{'}$$

Since $\{\xi_\lambda\}$ is λ -cauchy net for all $\lambda\in M(L)$, there exists $\lambda_0\in M(L)$ such that $\xi_\lambda-\xi_{\lambda_1}\notin W$ for all $\lambda,\lambda_1\succ\lambda_0$ .This implies $x^{\left(\lambda,p^*\right)}-x^{\left(\lambda^{'},p^{*'}\right)}=\left(\xi_\lambda-\xi_{\lambda^{'}}\right)+\left(\xi_{\lambda^{'}}-x^{\left(\lambda^{'},p^{*'}\right)}\right)+\left(x^{\left(\lambda,p^*\right)}-\xi_\lambda\right)$

$$\in\left(W^*\right)^{'}+\left(W^*\right)^{'}+\left(W^*\right)^{'}\subset U_1^*$$

On the other hand $\xi_{\lambda^{'}}-\xi_\lambda=\xi_{\lambda^{'}}-x^{\left(\lambda^{'},W^*\right)}+x^{\left(\lambda^{'},W^*\right)}-\xi_\lambda\in\left(U^*\right)^{'}$ this complete the proof.

(e) The uniqueness of $\left(L^{X_1},\delta\right)$.

This is stratified by Lemma 2.3.

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Strong ω-Compactness in $L\omega$-Spaces

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Abstract. In this paper, the notion of strong ω-compactness is introduced in $L\omega$-spaces by means of open β_a-ω-cover and strong open β_a-ω-cover. It is proved that the intersection of a strongly ω-compact L-set and a ω-closed set is strongly ω-compact, that strong ω-compactness is preserved by continuously generalized Zadeh funtions. Under the condition that $\beta(a \wedge b) = \beta(a) \cap \beta(b)$, for any $a, b \in L$, the Tychonoff Theorem for strong ω-compactness is true.

Keywords: $L\omega$-spaces, open β_a-ω-cover, strong open β_a-ω-cover, strong ω-compactness.

1 Introduction

In L-fuzzy topology, operators play an important role. There are different operators, such as closure operator (Liu [3]), δ-closure operators (Saha [10]), N-closure operator (Chen [4]), and so on. In [1], Chen introduced a kind of generalized fuzzy spaces called $L\omega$-spaces by means of ω-operator. Some topological properties such as separation axioms (Chen [7]) and connectivity (Huang [9]) are discussed in $L\omega$-spaces. Recently Han and Meng introduced ω-compactness in $L\omega$-spaces by means of open β_a-ω-cover [6]. In this paper, we shall introduce a new concept of strong ω-compactness in $L\omega$-spaces. Strong ω-compactness has many good properties, the intersection of a strongly ω- compact L-set and an ω-closed set is strongly ω-compact, the strong ω-compactness is preserved by continuously generalized Zadeh funtions, and it implies ω-compactness. Under the condition that $\beta(a \wedge b) = \beta(a) \cap \beta(b)$, for any $a, b \in L$, the Tychonoff Theorem for strong ω-compactness is true.

2 Preliminaries

Throughout this paper, L is a completely distributive lattice with an order-reserving involution (briefly F-lattice), X is a nonempty set, L^X is the set of all L-fuzzy sets on X. The smallest element and the largest element in L^X are denoted respectively by 0_X *and* 1_X.

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An element a in L is called a prime element if $a \geq b \wedge c$ implies $a \geq b$ or $a \geq c$. a in L is called a coprime element if a' is a prime element. The set of nonunit prime elements in L is denoted by $P(L)$. The set of nonzero coprime elements in L is denoted by $M(L)$. The set of nonzero coprime elements in L^X is denoted by $M(L^X)$.

The binary relation $\prec$ in L is defined as follows: for $a, b \in L$, if and only if every subset $D \subseteq L$, the relation $b \leq supD$ always implies the existence of $d \in D$ with $a \leq d$ [2]. In a F-lattice, each member b is a sup of $\{a \in L \mid a \prec b\}$. In the sense of Wang [8], $\{a \in L \mid a \prec b\}$ is the greatest minimal family of b, in symbol $\beta(b)$. Moreover for an L-set $A \in L^X$, $\beta(A)$ denotes the greatest minimal family of A.

Definition 2.1[1]. Let X be a nonempty crisp set. $\omega : L^X \rightarrow L^X$ is called ω-operator, if it satisfies the following conditions for all $A, B \in L^X$:
(1)$\omega(1_X) = 1_X$;
(2)$A \leq B$, then $\omega(A) \leq \omega(B)$;
(3)$A \leq \omega(A)$.
An L-set $A \in L^X$ is called an ω-set if $\omega(A) = A$. Let $\Omega = \{A \in L^X \mid \omega(A) = A\}$, the pair (L^X, Ω) is called a $L\omega$-spaces(also can be call L-order-preserving operator space).

Definition 2.2[1]. Let (L^X, Ω) be a $L\omega$-spaces, $p \in L^X$, $x_\alpha \in M(L^X)$. If there exists a $Q \in \Omega$ such that $x_\alpha \not\leq Q$ and $p \leq Q$, then we call p an ω-remote neighborhood of x_α. The collection of all ω-remote neighborhoods of x_α is denoted by $\omega\eta(x_\alpha)$.

Definition 2.3[1]. Let (L^X, Ω) be a $L\omega$-space, $A \in L^X$. $x_\alpha \in M(L^X)$ is called an ω-adherence point of A, if for each $p \in \omega\eta(x_\alpha)$,$A \not\leq p$. The union of all ω-adherence points of A is called the ω-closure of A which denoted by $\omega cl(A)$. If $\omega cl(A) = A$, then A can be called as a ω-closed set. A is called an ω-open set if A' is an ω-closed set. The family of all ω-closed sets and ω-open sets in (L^X, Ω) are denoted by $\omega c(L^X)$ and $\omega o(L^X)$.

Definition 2.4[1]. Let $(L_i^{X_i}, \Omega_i)(i = 1, 2)$ be two $L\omega$- spaces, an order homomorphism $f : L_1^{X_1} \rightarrow L_2^{X_2}$ is called (ω_1, ω_2)-continuous if $f^{-1}(B)$ is ω-closed set in $(L_1^{X_1}, \Omega_1)$ for every $B \in \omega c(L_2^{X_2})$.

Definition 2.5[5]. Let L_1, L_2 be two F-lattice and X, Y be two nonempty crisp sets, $p : X \rightarrow Y$ is a crisp mapping, $q : L_1 \rightarrow L_2$ is an order homomorphism, $f^{\rightarrow} : L_1^X \rightarrow L_2^Y$ is a fuzzy mapping induced by p and q, where $f^{\rightarrow}$ is defined as follows:

$$f^{\rightarrow}(A)(y) = \bigvee_{p(x)=y} q(A(x)), \;\; A \in L_1^X, \;\; y \in Y.$$

Then $f^{\rightarrow}$ is called generalized Zadeh function.

Lemma 2.6[5]. Let $f^{\rightarrow}: L_1^X \rightarrow L_2^Y$ be a generalized Zadeh function induced by $p: X \rightarrow Y$ and $q: L_1 \rightarrow L_2$, then $f^{\rightarrow}$ is an order homomorphism and the inverse $f^{\leftarrow}: L_2^Y \rightarrow L_1^X$ of $f^{\rightarrow}$ satisfies $f^{\leftarrow}(B)(x) = q^{-1}(B(p(x))), x \in X, B \in L_2^Y$.

Definition 2.7[8]. Let $X = \prod\limits_{t \in T} X_t$ be the product of a family of nonempty crisp sets $\{X_t\}_{t \in T}$, $\forall t \in T$, $p_t : X \rightarrow X_t$ is an order projective mapping, $p_t^{\rightarrow} : L^X \rightarrow L^{X_t}$ is a Zadeh function induced by p_t, i.e.,

$$p_t^{\rightarrow}(A)(x_t) = \vee\{A(x) \mid p_t(x) = x_t\}, A \in L^X, x = \{x_t\}_{t \in T} \in X, x_t \in X_t.$$

Then $p_t^{\rightarrow}$ is still called a projective mapping.

Definition 2.8[8]. Let L be a F-lattice, $\{X_t\}_{t \in T}$ be a family of nonempty crisp sets, $\forall t \in T$, $A_t : X_t \rightarrow L$ is an L-set on X_t, $A : \prod\limits_{t \in T} X_t \rightarrow L$ is defined as follows:

$$\forall x = \{x_t\}_{t \in T} \in \prod_{t \in T} X_t, \quad A(x) = \bigwedge_{t \in T} A_t(x_t)$$

Then A is called the product of $\{A_t\}_{t \in T}$, denoted by $A = \prod\limits_{t \in T} A_t$. Obviously, we have the following equality:

$$\prod_{t \in T} A_t = \bigwedge_{t \in T} (A_t \circ p_t) = \bigwedge_{t \in T} p_t^{\leftarrow}(A_t), \text{ \textit{where} } p_t^{\leftarrow} \text{ \textit{is the inverse of} } p_t^{\rightarrow}.$$

Definition 2.9[6]. Let (L^X, Ω) be a $L\omega$-space, $a \in M(L)$ and $G \in L^X$. A family $\mu \subset \omega o(L^X)$ is called an open H_a-ω-*cover* of G, if for any $x \in X$ with $x_a \not\leq G'$, it follows that $x_\alpha \prec \bigvee\limits_{A \in \mu} A$.

Definition 2.10[6]. Let (L^X, Ω) be a $L\omega$-*space* and $G \in L^X$. Then G is called ω-compact if for any $a \in M(L)$, each open H_a-ω-cover of G has a finite subfamily which is an open H_a-ω- cover of G. (L^X, Ω) is called ω-compact if 1_X is ω- compact.

Obviously, $\mu \subset \omega o(L^X)$ is an open H_a-ω-cover of G if and only if for any $x \in X$ it follows that $a \in \beta(G'(x) \vee \bigvee\limits_{A \in \mu} A(x))$. This shows that H_a-ω-cover is equivalent to β_a-cover in [11] when $L\omega$-space is fuzzy topological space. Hench, the H_a-ω-cover also can be call β_a-ω-cover.

3 Strong ω-Compactness

Definition 3.1. Let (L^X, Ω) be an $L\omega$-space, $a \in M(L)$ and $G \in L^X$. A family $\mu \subset \omega o(L^X)$ is called a strong open β_a-ω-cover of G if $a \in \beta(\bigwedge_{x\in X}(G'(x) \vee \bigvee_{A\in\mu} A(x)))$.

Definition 3.2. Let (L^X, Ω) be an $L\omega$-space, $a \in M(L)$ and $G \in L^X$. Then G is called strongly ω-compact if each open β_a-ω-cover of G has a finite subfamily which is a strong open β_a-ω-cover of G. (L^X, Ω) is called strongly ω-compact if 1_X is strongly ω-compact.

Obviously, strong ω-compact implies ω-compact.

Theorem 3.3. Let (L^X, Ω) be a $L\omega$-space, G is strongly $\omega-$compact and H is ω-closed set, then $G \wedge H$ is strongly ω- compact.

Proof. Suppose that $\mu \subset \omega o(L^X)$ is an open β_a-ω -cover of $G \wedge H$. Then $\mu \cup H'$ is an open β_a-ω-cover of G. By strong ω-compactness of G, we know that $\mu \cup H'$ has a finite subfamily ν which is a strong open β_a-ω-cover of G. Take $\varphi = \nu/H'$, then φ is a finite strong open β_a-ω-cover of $G \wedge H$. This shows that $G \wedge H$ is strongly ω-compact.

Theorem 3.4. Let (L_1^X, Ω_1), (L_2^Y, Ω_2) be two $L\omega$-spaces, $f^{\rightarrow} : (L_1^X, \Omega_1) \rightarrow (L_2^Y, \Omega_2)$ be a generalized Zadeh function and G be strongly ω_1- compact in (L_1^X, Ω_1). If $f^{\rightarrow}$ is (ω_1, ω_2)- continuous, $q : L_1 \rightarrow L_2$ is a bijection, then $f^{\rightarrow}(G)$ is strongly ω_2-compact in (L_2^Y, Ω_2).

Proof. For the beginning, by the condition of $q : L_1 \rightarrow L_2$ is an order homomorphism and a bijection, we know that $q^{-1} : L_2 \rightarrow L_1$ is an order homomorphism. Hence, q and q^{-1} have the properties of union-preserving, intersection-preserving, order-reserving involution- preserving, minimal set-preserving and co-prime-preserving.

Let $a \in M(L_2)$ and $\mu \subset \omega o(L_2^Y)$ be an open β_a-ω_2-cover of $f^{\rightarrow}(G)$. Then for any $y \in Y$, we have that $a \in \beta(f^{\rightarrow}(G)'(y) \vee \bigvee_{A\in\mu} A(y))$. By the following equation:

$$f^{\rightarrow}(G)'(y) = (\bigvee_{p(x)=y} q(G(x)))' = \bigwedge_{p(x)=y} q(G'(x)) = q(\bigwedge_{p(x)=y} G'(x)),$$

$$\begin{aligned} f^{\rightarrow}(G)'(y) \vee \bigvee_{A\in\mu} A(y) &= q(\bigwedge_{p(x)=y} G'(x)) \vee \bigvee_{A\in\mu} qq^{-1}A(y), \\ &= q[(\bigwedge_{p(x)=y} G'(x)) \vee (\bigvee_{A\in\mu} q^{-1}A(y))], \\ &= q[\bigwedge_{p(x)=y} (G'(x) \vee \bigvee_{A\in\mu} f^{\leftarrow}(A)(X))], \\ &= \bigwedge_{p(x)=y} q(G'(x) \vee \bigvee_{A\in\mu} f^{\leftarrow}(A)(x)) \end{aligned}$$

We obtain that $a \in \bigcap_{p(x)=y} q[\beta(G'(x) \vee \bigvee_{A\in\mu} f^{\leftarrow}(A)(x))]$, moreover for any $x \in X$, $a \in q[\beta(G'(x) \vee \bigvee_{A\in\mu} f^{\leftarrow}(A)(x))]$, therefore $q^{-1}(a) \in \beta(G'(x) \vee \bigvee_{A\in\mu} f^{\leftarrow}(A)(x))$. This shows that $f^{\leftarrow}(\mu) = \{f^{\leftarrow}(A)|A \in \mu\}$ is an open $\beta_{q^{-1}(a)}$-ω_1-cover of G. By strongly ω-compact of G we know that μ has a finite subfamily ν such that $f^{\leftarrow}(\nu)$ is a strong open cover of $\beta_{q^{-1}(a)}$-ω_1-cover of G i.e.

$$q^{-1}(a) \in \beta(\bigwedge_{x\in X} (G'(x) \vee \bigvee_{A\in\mu} f^{\leftarrow}(A)(x))) \quad (*)$$

By the following equality and $(*)$ we can obtain that

$$a \in \beta(\bigwedge_{y\in Y} (f^{\rightarrow}(G)'(y) \vee \bigvee_{A\in\nu} A(y))).$$

$$\begin{aligned} q^{-1}(\bigwedge_{y\in Y} (f^{\rightarrow}(G)'(y) \vee \bigvee_{A\in\nu} A(y))) &= \bigwedge_{y\in Y} (q^{-1}(f^{\rightarrow}(G)'(y) \vee \bigvee_{A\in\nu} q^{-1}A(y)) \\ &= \bigwedge_{y\in Y} [q^{-1}(\bigwedge_{p(x)=y} q(G'(x))) \vee \bigvee_{A\in\nu} f^{\leftarrow}(A)(x)] \\ &= \bigwedge_{y\in Y} [(\bigwedge_{x\in p^{-1}(y)} G'(x) \vee \bigvee_{A\in\nu} f^{\leftarrow}(A)(x))] \\ &= \bigwedge_{y\in Y} \bigwedge_{x\in p^{-1}(y)} [G'(x) \vee \bigvee_{A\in\nu} f^{\leftarrow}(A)(x)] \\ &= \bigwedge_{x\in X} [G'(x) \vee \bigvee_{A\in\nu} f^{\leftarrow}(A)(x)]. \end{aligned}$$

This shows that ν is a strong open β_a-ω_2-cover of $f^{\rightarrow}(G)$. Therefore $f^{\rightarrow}(G)$ is strongly ω_2-compact.

Corollary 3.5. Let (L^X, Ω_1), (L^Y, Ω_2) be two $L\omega$-spaces, $f^{\rightarrow} : (L^X, \Omega_1) \rightarrow (L^Y, \Omega_2)$ be a generalized Zadeh function and G be strongly ω_1-compact in (L^X, Ω_1). If $f^{\rightarrow}$ is (ω_1, ω_2)-continuous, then $f^{\rightarrow}(G)$ is strongly ω_2-compact in (L^Y, Ω_2).

4 The Tychonoff Theorem

Theorem 4.1. Suppose that for any a, $b \in L$, $\beta(a \wedge b) = \beta(a) \cap \beta(b)$. Let R be an ω-subbase of $L\omega$-space (L^X, Ω_1) and $G \in L^X$. If for each $a \in M(L)$, every open β_a-ω-cover μ of G consisting of members of R has finite subfamily which is a strong open β_a-ω-cover of G, then G is strongly ω-compact.

Proof. Suppose that every open β_a-ω-cover of G consisting of members of R has a finite subfamily which is a strong open β_a-ω-cover of G. Now we prove that every open β_a-ω-cover of G consisting of members of $\omega o(L^X)$ also has a finite subfamily which is a strong open β_a-ω-cover of G. Suppose that μ is an open β_a-ω-cover of G consisting of members of $\omega o(L^X)$ and it has no finite subfamily which is a strong open β_a-ω-cover of G, then for any A_1, $A_2, \ldots, A_n \in \mu$, we have that

$$a \notin \beta(\bigwedge_{x \in X} (G'(x) \vee A_1(x) \vee A_2(x) \vee \ldots \vee A_n(x))).$$

Let
$\Gamma = \{\Psi | \mu \subseteq \Psi \subseteq \omega o(L^X)\}$, Ψ has no finite subfamily being a strong open β_a-ω-cover of G.

By $\mu \in \Gamma$ we know that $(\Gamma, \subseteq)$ is a nonempty partially ordered set and each chain has an upper bound, hence by Zorn Lemma, Γ has a maximal element Φ. Now we prove that Φ satisfies the following conditions:

(1)Φ is an open β_a-ω-cover of G;
(2) for every $B \in \omega o(L^X)$, if $C \in \Phi$ and $C \geq B$, then $B \in \Phi$;
(3) if for each B, $C \in \omega o(L^X)$, $B \wedge C \in \Phi$, then $B \in \Phi$ or $C \in \Phi$.

The proof of (1) and (2) is easy, we only verify (3). If $B \notin \Phi$ and $C \notin \Phi$, then $\{B\} \cup \Phi \notin \Gamma$ and $\{C\} \cup \Phi \notin \Gamma$, hence, $\{B\} \cup \Phi$ has a finite subfamily $\{A_1, A_2, \ldots, A_m\}$ which is a strong open β_a-ω-cover of G and $\{C\} \cup \Phi$ has a finite subfamily $A_{m+1}, A_{m+2}, \ldots, A_{m+n}$ which is a strong open β_a-ω-cover of G, so we have that

$$a \in \beta(\bigwedge_{x \in X} (G' \vee A_1 \vee A_2 \vee \ldots \vee A_m \vee B)(x))$$

and

$$a \in \beta(\bigwedge_{x \in X} (G' \vee A_{m+1} \vee A_{m+2} \vee \ldots \vee A_{m+n} \vee C)(x)).$$

Let $A = A_1 \vee A_2 \vee \ldots \vee A_m$, then,

$$a \in \beta(\bigwedge_{x\in X} (G' \vee A \vee B)(x)) \ and \ a \in \beta(\bigwedge_{x\in X} (G' \vee A \vee C)(x)).$$

Hence,

$$\begin{aligned} a &\in \beta(\bigwedge_{x\in X} (G' \vee A \vee B)(x)) \bigcap \beta(\bigwedge_{x\in X} (G' \vee A \vee C)(x)) \\ &= \beta(\bigwedge_{x\in X} (G' \vee A \vee (B \wedge C)(x)). \end{aligned}$$

This implies that $B \wedge C \notin \Phi$, which contradicts $B \wedge C \in \Phi$, (3) is proved. From (2) and (3) it is immediate that if $D \in \Phi$, $P_1, P_2, \ldots, P_n \in \omega o(L^X)$ and $D \geq \bigwedge_{i=1}^{n} P_i$, then there exists $i(1 \leq i \leq n)$ such that $P_i \in \Phi$.

Now consider $R \cap \Phi$. If $R \cap \Phi$ is an open β_a-ω-cover of G, then by the hypothesis, it has a finite subfamily φ which is a strong open β_a-ω-cover of G. Obviously φ is also a finite subfamily of Φ, this contradicts the sense of Φ. There fore $R \cap \Phi$ is not an open β_a-ω-cover of G, this implies that for any $A \in R \cap \Phi$ and there exists an $x \in X$ such that $a \notin \beta(G'(x) \vee A(x))$.

By (1) we know that Φ is an open β_a-ω-cover of G, so for all $x \in X$, there exists $D \in \Phi$ such that $a \in \beta(G'(x) \vee D(x))$. By R is a ω-subbase of (L^X, Ω), we know that

$$D = \bigvee_{i\in I} \bigwedge_{j\in J_i} A_{ij} \ \ (where\ for\ each\ i \in I, j_i\ is\ a\ finite\ set\ and\ A_{ij} \in R).$$

Then there exists $i \in I$ such that

$$a \in \beta(G'(x) \vee \bigvee_{j\in J_i} A_{ij}(x)).$$

Thus for each $j \in J_i$ we have that $a \in \beta(G'(x) \vee A_{ij}(x))$, by $D \geq \bigwedge_{j\in J_i} A_{ij}$ we know that there is $j \in J_i$ such that $A_{ij} \in \Phi$, this contradicts $a \notin \beta(G'(x) \vee A_{ij}(x))$. The proof is obtained.

Theorem 4.2. Suppose that for any $a\,, b \in L$, $\beta(a \wedge b) = \beta(a) \cap \beta(b)$. Let (L^X, Ω) be the product of a family of ω-spaces $\{(L^{X_i}, \Omega_i)\}_{i\in I}$, if for each $i \in I$, G_i is a strongly ω-compact constant L-set in (L^{X_i}, Ω_i), then $G = \prod_{i\in I} G_i$ is strongly ω-compact in (L^X, Ω).

Proof. Let $R = \{p_i^{\leftarrow}(D_i) \mid i \in I,\, D_i \in \omega o(L^{X_i})\}$ be a ω-subbase of $L\omega$-space (L^X, Ω). Suppose that $\mu \subseteq R$ is an open β_a-ω-cover of G, let

$$\mu = \bigcup_{i\in J} \mu_i,\ J \subset I,\ \mu_i = \{p_i^{\leftarrow}(B_i) \mid B_i \in U_i \subset \omega o(L^{X_i})\}.$$

Then for any $x \in X$, $a \in \beta(G'(x) \vee \bigvee_{A\in\mu} A(x)) = \beta(G'(x) \vee \bigvee_{i\in J} (\bigvee_{A\in\mu_i} A(x)))$.

(1) If there exists $i \in I$ such that for any $x_i \in X_i$, $a \in \beta(G'_i(x_i))$, since G_i is constant L-set, we know that $a \in \beta(\bigwedge_{x_i\in X_i} G'_i(x_i))$. By $G(x) = \bigwedge_{i\in I} G_i(x_i)$, we have that

$$\bigwedge_{x\in X} G'(x) = \bigwedge_{x\in X}\bigvee_{i\in I} G'_i(x_i) = \bigvee_{i\in I}\bigwedge_{x\in X} G'_i(x_i) = \bigvee_{i\in I}\bigwedge_{x_i\in X_i} G'_i(x_i)$$

hence, $a \in \beta(\bigwedge_{x\in X} G'(x))$. This shows that any finite subfamily of μ is a strong open β_a-ω-cover of G.

(2) If for any $i \in I$, there exists $x_i \in X_i$ such that $a \notin \beta(G'_i(x_i))$. Now we prove that there exists $k \in J$ such that U_k is an open β_a-ω-cover of G_k. If there is no $k \in J$ such that U_k is an open β_a-ω-cover of G_k, then $\forall i \in J$, U_i is not an open β_a-ω-cover of G_i, hence, there exists $y_i \in X_i$ such that $a \notin \beta(G'_i(y_i) \vee \bigvee_{B\in U_i} B(y_i))$. Let $z = \{z_i\}_{i\in I}$ such that

$$z_i = \begin{cases} y_i, & i \in J, \\ x_i, & i \notin J. \end{cases}$$

By the following equation

$$\begin{aligned} G'(z) &= (\prod_{i\in I} G_i)'(z) = (\bigwedge_{i\in I} p_i^{\leftarrow}(G_i))'(z) \\ &= (\bigvee_{i\in J} p_i^{\leftarrow}(G'_i)(z)) \vee (\bigvee_{i\notin J} p_i^{\leftarrow}(G'_i)(z)) \\ &= (\bigvee_{i\in J} G'_i(p_i(z))) \vee (\bigvee_{i\notin J} G'_i(p_i(z))) \\ &= (\bigvee_{i\in J} G'_i(y_i)) \vee (\bigvee_{i\notin J} G'_i(x_i)). \end{aligned}$$

We obtain that $a \notin \beta(G'(z))$. Moreover for any $i \in J$, by the following fact

$$a \notin \beta(\bigvee_{B\in U_i} B(y_i)) = \beta(\bigvee_{B\in U_i} p_i^{\leftarrow}(B)(z)) = \beta(\bigvee_{A\in\mu_i} A(z)),$$

we have that

$$a \notin \bigcup_{i\in J} \beta(\bigvee_{A\in\mu_i} A(z)) = \beta(\bigvee_{i\in J}(\bigvee_{A\in\mu_i} A(z))) = \beta(\bigvee_{A\in\mu} A(z)).$$

This implies that $a \notin \beta(G'(z) \vee \bigvee_{A\in\mu} A(z))$, which contradicts $a \in \beta(G'(x) \vee \bigvee_{A\in\mu} A(x))$ for any $x \in X$. Thus we obtain the proof that there exists $k \in J$

such that U_k is an open β_a-ω-cover of G_k. By strong ω-compactness of G_k we know that U_k has a finite subfamily D_k which is a strong open β_a-ω- cover of G_k. By

$$a \in \beta(\bigwedge_{x_k \in X_k} (G'_k(x_k) \vee \bigvee_{D \in D_k} D(x_k))) \subseteq \beta(\bigwedge_{x \in X} (G'(x) \vee \bigvee_{D \in D_k} p_k^{\leftarrow}(D)(x))),$$

we know that $\{p_k^{\leftarrow}(D_k)\} = \{p_k^{\leftarrow}(D) \mid D \in D_k\}$ is a strong open β_a-ω-cover of G. The proof is completed.

Corollary 4.3. Let (L^X, Ω) be the product of a family of $L\omega$- spaces $\{(L^{X_i}, \Omega_i)\}_{i \in I}$ and for any a, $b \in L$, $\beta(a \wedge b) = \beta(a) \cap \beta(b)$. Then (L^X, Ω) is strongly ω-compact if and only if for each $i \in I$, (L^{X_i}, Ω_i) is strongly ω-compact.

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(λ, μ)-Fuzzy Sublattices and (λ, μ)-Fuzzy Subhyperlattices

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Abstract. We first introduced the concepts of (λ, μ)-fuzzy sublattices and (λ, μ)-fuzzy ideals of a lattice and listed some properties of them. Then we studied (λ, μ)-fuzzy sublattices and (λ, μ)-fuzzy ideals of complemented lattices. Lastly, we researched (λ, μ)-fuzzy subhyperlattices and (λ, μ)-fuzzy ideals of hyperlattices.

Keywords: (λ, μ)-fuzzy; sublattice; subhyperlattice; ideal.

1 Introduction and Preliminaries

The concept of fuzzy sets was first introduced by Zadeh [14] in 1965 and then the fuzzy sets have been used in the reconsideration of classical mathematics. Recently, Yuan [13] introduced the concept of fuzzy subgroup with thresholds. A fuzzy subgroup with thresholds λ and μ is also called a (λ, μ)-fuzzy subgroup. Yao continued to research (λ, μ)-fuzzy normal subgroups, (λ, μ)-fuzzy quotient subgroups and (λ, μ)-fuzzy subrings in [10, 11, 12].

Hyperstructure theory was first introduced in 1934 by Marty at the VIIIth Congress of Scandinavian Mathematicians (see [8]). After that, algebraic hyperstructures have been developed by many researchers. Feng [5], Ali [1], Koguep [7] had researched hyperlattices.

In this paper, we first introduced the concepts of (λ, μ)-fuzzy sublattices and (λ, μ)-fuzzy ideals of a lattice and listed some properties of them. Then we studied (λ, μ)-fuzzy sublattices and (λ, μ)-fuzzy ideals of complemented lattices. Lastly, we researched (λ, μ)-fuzzy subhyperlattices and (λ, μ)-fuzzy ideals of hyperlattices .

Let us recall some definitions and notions.

By a fuzzy subset of a nonempty set X we mean a mapping from X to the unit interval $[0, 1]$. If A is a fuzzy subset of X, then we denote $A_\alpha = \{x \in X | A(x) \geq \alpha\}$ for all $\alpha \in [0, 1]$.

A partial hypergroupoid $< H; * >$ is a nonempty set H with a function from $H \times H$ to the set of subsets of H. I.e.,

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$$* : H \times H \to \mathbf{P}(H)$$

$$(x, y) \to x * y.$$

A hypergroupoid is a nonempty set H, endowed with a hyperoperation, that is a function from $H \times H$ to the set of nonempty subsets of H.

If $A, B \in \mathbf{P}(H) - \{\emptyset\}$, then we define $A * B = \cup\{a * b | a \in A, b \in B\}$, $x * B = \{x\} * B$ and $A * y = A * \{y\}$.

Definition 1 ([4, 6]). *Let H be a nonempty set , $\sqcup : H \times H \to P^*(H)$ be a hyperoperation, where $P(H)$ is the power set of H and $P^*(H) = P(H) - \{\emptyset\}$ and $\wedge : H \times H \to H$ be an operation. Then $(H, \sqcup, \wedge)$ is called a hyperlattice if for all $a, b, c \in H$:*

(1) $a \in a \sqcup a$, $a = a \wedge a$;
(2) $a \sqcup b = b \sqcup a$,$a \wedge b = b \wedge a$
(3) $(a \sqcup b) \sqcup c = a \sqcup (b \sqcup c)$,$(a \wedge b) \wedge c = a \wedge (b \wedge c)$
(4) $a \in (a \sqcup b) \wedge a$, $a \in (a \wedge b) \sqcup a$
(5) $b \in a \sqcup b \Leftrightarrow a = a \wedge b \Leftrightarrow a \leq b$

The readers can consult [3, 9] to learn more about hyperstructures and fuzzy sets.

Throughout this paper, we will always assume that $0 \leq \lambda < \mu \leq 1$.

2 $(\boldsymbol{\lambda}, \boldsymbol{\mu})$-Fuzzy Sublattices

In this section, we always use L to stand for a lattice.

Definition 2. *A fuzzy subset A of a lattice L is said to be a (λ, μ)-fuzzy sublattice of L if $\forall a, b \in L$,*

$$A(a \wedge b) \vee \lambda \geq (A(a) \wedge A(b)) \wedge \mu$$

and

$$A(a \vee b) \vee \lambda \geq (A(a) \wedge A(b)) \wedge \mu.$$

From the previous definition, we can easily conclude that a fuzzy sublattice is a $(0, 1)$-fuzzy sublattice.

Theorem 1. *Let A be a fuzzy subset of L. Then the following are equivalent:*

(1) A is a (λ, μ)-fuzzy sublattice of L;
(2) A_α is a sublattice of L, for any $\alpha \in (\lambda, \mu]$, where $A_\alpha \neq \emptyset$.

Proof. "(1) $\Rightarrow$ (2)"
Let A be a (λ, μ)-fuzzy sublattice of L. For any $\alpha \in (\lambda, \mu]$, such that $A_\alpha \neq \emptyset$, we need to show that $x \wedge y \in A_\alpha$ and $x \vee y \in A_\alpha$, for all $x, y \in A_\alpha$.

From $x \in A_\alpha$ we know that $A(x) \geq \alpha$. And similarly we obtain that $A(y) \geq \alpha$. Thus $A(x \wedge y) \vee \lambda \geq (A(x) \wedge A(y)) \wedge \mu \geq \alpha$. We also know that $\lambda < \alpha$. Then we conclude that $A(x \wedge y) \geq \alpha$. So $x \wedge y \in A_\alpha$.

$x \vee y \in A_\alpha$ can be proved similarly.

"(2) $\Rightarrow$ (1)"

Conversely, let A_α be a sublattice of L. If there exist $x_0, y_0 \in L$ such that $A(x_0 \wedge y_0) \vee \lambda < \alpha = (A(x_0) \wedge A(y_0)) \wedge \mu$, then $\alpha \in (\lambda, \mu]$, $A(x_0) \wedge A(y_0) \geq \alpha$. So $x_0 \in A_\alpha$ and $y_0 \in A_\alpha$. But $A(x_0 \wedge y_0) < \alpha$, that is $x_0 \wedge y_0 \notin A_\alpha$. This is a contradiction with that A_α is a sublattice of L. Thus $A(x \wedge y) \vee \lambda \geq (A(x) \wedge A(y)) \wedge \mu$ holds for all $x, y \in L$.

$A(x \vee y) \vee \lambda \geq (A(x) \wedge A(y)) \wedge \mu$ can be proved similarly.

Therefore, A is a (λ, μ)-fuzzy sublattice of L .

Definition 3. *Let $(L, \wedge, \vee)$ be a lattice. A nonempty subset I of L is called an ideal of L if for all $a, b \in L$,*

$$a \in L, b \in I \Rightarrow a \wedge b \in I$$

and

$$a, b \in I \Rightarrow a \vee b \in I.$$

Proposition 1. *Suppose I is a subset of a lattice H, then the following are equivalent for all $a, b \in L$,*

(1) $a \in H, b \in I \Rightarrow a \wedge b \in I$;
(2) $a \in I$ and $b \leq a \Rightarrow b \in I$.

Proof. "(1) $\Rightarrow$ (2)"
If $b \leq a$, then $b = a \wedge b$. From (1) we know that $a \wedge b \in I$. And so $b \in I$.

"(2) $\Rightarrow$ (1)"

From $a \wedge b \leq b \in I$ and (2) we know that $a \wedge b \in I$.

Definition 4. *A fuzzy subset A of a lattice L is called a (λ, μ)-fuzzy ideal if for all $a, b \in L$,*

$$A(a \wedge b) \vee \lambda \geq (A(a) \vee A(b)) \wedge \mu$$

and

$$A(a \vee b) \vee \lambda \geq (A(a) \wedge A(b)) \wedge \mu.$$

Proposition 2. *A is a fuzzy subset of a lattice L, then the following are equivalent for all $a, b \in L$,*

(1) $A(a \wedge b) \vee \lambda \geq (A(a) \vee A(b)) \wedge \mu$;
(2) $a \leq b \Rightarrow A(a) \vee \lambda \geq A(b) \wedge \mu$.

Proof. "(1) $\Rightarrow$ (2)"
If $a \leq b$, then $a \wedge b = a$. Thus $A(a) \vee \lambda = A(a \wedge b) \vee \lambda \geq (A(a) \vee A(b)) \wedge \mu \geq A(b) \wedge \mu$.

"(2) $\Rightarrow$ (1)"

From $a \wedge b \leq a$ we know that $A(a \wedge b) \vee \lambda \geq A(a) \wedge \mu$ and from $a \wedge b \leq b$ we conclude that $A(a \wedge b) \vee \lambda \geq A(b) \wedge \mu$. Thus $A(a \wedge b) \vee \lambda \geq (A(a) \wedge \mu) \vee (A(b) \wedge \mu) = (A(a) \vee A(b)) \wedge \mu$.

Hence, we complete the proof.

Theorem 2. *Let A be a (λ, μ)-fuzzy sublattice of L. Then the following are equivalent:*

(1) A is a (λ, μ)-fuzzy ideal of L;

(2) A_α is an ideal of L, for any $\alpha \in (\lambda, \mu]$, where $A_\alpha \neq \emptyset$.

Proof. "(1) $\Rightarrow$ (2)"

Let A be a (λ, μ)-fuzzy ideal of L. For any $\alpha \in (\lambda, \mu]$, such that $A_\alpha \neq \emptyset$, we need to show that $x \wedge y \in A_\alpha$, for all $x \in A_\alpha$ and $y \in L$.

From $A(x) \geq \alpha$ we obtain that $A(x \wedge y) \vee \lambda \geq (A(x) \vee A(y)) \wedge \mu \geq \alpha$. We conclude that $A(x \wedge y) \geq \alpha$ since $\lambda < \alpha$. So $x \wedge y \in A_\alpha$.

"(2) $\Rightarrow$ (1)"

Conversely, let A_α be an ideal of L, for all $\alpha \in (\lambda, \mu]$, where $A_\alpha \neq \emptyset$. If there exist $x_0, y_0 \in L$ such that $A(x_0 \wedge y_0) \vee \lambda < \alpha = (A(x_0) \vee A(y_0)) \wedge \mu$, then $\alpha \in (\lambda, \mu]$, $A(x_0) \vee A(y_0) \geq \alpha$. So $x_0 \in A_\alpha$ or $y_0 \in A_\alpha$. But $A(x_0 \wedge y_0) < \alpha$, that is $x_0 \wedge y_0 \notin A_\alpha$. This is a contradiction with that A_α is an ideal of L. Thus $A(x \wedge y) \vee \lambda \geq (A(x) \vee A(y)) \wedge \mu$ holds for all $x, y \in L$.

The proof is ended.

We use L_1, L_2 to stand for two lattices and define $\sup \emptyset = 0$, where $\emptyset$ is the empty set, in the following of this section.

Theorem 3. *Let $f : L_1 \to L_2$ be a homomorphism and let A be a (λ, μ)-fuzzy sublattice of L_1. Then $f(A)$ is a (λ, μ)-fuzzy sublattice of L_2, where*

$$f(A)(y) = \sup_{x \in L_1} \{A(x) | f(x) = y\}, \quad \forall y \in L_2.$$

Proof. If $f^{-1}(y_1) = \emptyset$ or $f^{-1}(y_2) = \emptyset$ for any $y_1, y_2 \in L_2$, then $(f(A)(y_1 \vee y_2)) \vee \lambda \geq 0 = (f(A)(y_1) \wedge f(A)(y_2)) \wedge \mu$.

Suppose that $f^{-1}(y_1) \neq \emptyset$, $f^{-1}(y_2) \neq \emptyset$ for any $y_1, y_2 \in L_2$. Then

$$\begin{aligned} f(A)(y_1 \vee y_2) \vee \lambda &= \sup_{t \in L_1} \{A(t) | f(t) = y_1 \vee y_2\} \vee \lambda \\ &= \sup_{t \in L_1} \{A(t) \vee \lambda | f(t) = y_1 \vee y_2\} \\ &\geq \sup_{x_1, x_2 \in L_1} \{(A(x_1 \vee x_2)) \vee \lambda | f(x_1) = y_1, f(x_2) = y_2\} \\ &\geq \sup_{x_1, x_2 \in L_1} \{(A(x_1) \wedge A(x_2)) \wedge \mu | f(x_1) = y_1, f(x_2) = y_2\} \\ &= (\sup_{x_1 \in L_1} \{A(x_1) | f(x_1) = y_1\} \wedge \sup_{x_2 \in L_1} \{A(x_2) | f(x_2) = y_2\}) \wedge \mu \\ &= (f(A)(y_1) \wedge f(A)(y_2)) \wedge \mu. \end{aligned}$$

Similarly, we have $f(A)(y_1 \wedge y_2) \vee \lambda \geq (f(A)(y_1) \wedge f(A)(y_2)) \wedge \mu$.

So, $f(A)$ is a (λ, μ)-fuzzy sublattice of L_2.

Theorem 4. *Let $f : L_1 \to L_2$ be a surjective homomorphism and let A be a (λ, μ)-fuzzy ideal of L_1. Then $f(A)$ is a (λ, μ)-fuzzy ideal of L_2.*

Proof. For any $y_1, y_2 \in L_2$, $f^{-1}(y_1)$ and $f^{-1}(y_2)$ are not empty. We have

$$\begin{aligned} f(A)(y_1 \wedge y_2) \vee \lambda &= \sup_{t \in L_1}\{A(t)|f(t) = y_1 \wedge y_2\} \vee \lambda \\ &= \sup_{t \in L_1}\{A(t) \vee \lambda|f(t) = y_1 \vee y_2\} \\ &\geq \sup_{x_1, x_2 \in L_1}\{(A(x_1 \vee x_2)) \vee \lambda|f(x_1) = y_1, f(x_2) = y_2\} \\ &\geq \sup_{x_1, x_2 \in L_1}\{(A(x_1) \vee A(x_2)) \wedge \mu|f(x_1) = y_1, f(x_2) = y_2\} \\ &= (\sup_{x_1 \in L_1}\{A(x_1)|f(x_1) = y_1\} \vee \sup_{x_2 \in L_1}\{A(x_2)|f(x_2) = y_2\}) \wedge \mu \\ &= (f(A)(y_1) \vee f(A)(y_2)) \wedge \mu. \end{aligned}$$

So, $f(A)$ is a (λ, μ)-fuzzy ideal of L_2.

Theorem 5. *Let $f : L_1 \to L_2$ be a homomorphism and let B be a (λ, μ)-fuzzy sublattice of L_2. Then $f^{-1}(B)$ is a (λ, μ)-fuzzy sublattice of L_1, where*

$$f^{-1}(B)(x) = B(f(x)), \quad \forall x \in L_1.$$

Proof. For any $x_1, x_2 \in L_1$,

$$\begin{aligned} f^{-1}(B)(x_1 \vee x_2) \vee \lambda &= B(f(x_1 \vee x_2)) \vee \lambda \\ &= B(f(x_1) \vee f(x_2)) \vee \lambda \\ &\geq (B(f(x_1)) \wedge B(f(x_2))) \wedge \mu \\ &= (f^{-1}(B)(x_1) \wedge f^{-1}(B)(x_2)) \wedge \mu. \end{aligned}$$

Similarly, we have $f^{-1}(B)(x_1 \wedge x_2) \vee \lambda \geq (f^{-1}(B)(x_1) \wedge f^{-1}(B)(x_2)) \wedge \mu$. So, $f^{-1}(B)$ is a (λ, μ)-fuzzy sublattice of L_1.

Theorem 6. *Let $f : L_1 \to L_2$ be a homomorphism and let B be a (λ, μ)-fuzzy ideal of L_2. Then $f^{-1}(B)$ is a (λ, μ)-fuzzy ideal of L_1.*

Proof. For any $x_1, x_2 \in L_1$,

$$\begin{aligned} f^{-1}(B)(x_1 \wedge x_2) \vee \lambda &= B(f(x_1 \wedge x_2)) \vee \lambda \\ &= B(f(x_1) \wedge f(x_2)) \vee \lambda \\ &\geq (B(f(x_1)) \vee B(f(x_2))) \wedge \mu \\ &= f^{-1}(B)(x_1) \vee f^{-1}(B)(x_2)) \wedge \mu. \end{aligned}$$

So, $f^{-1}(B)$ is a (λ, μ)-fuzzy ideal of L_1.

3 (λ, μ)-Fuzzy Sublattices of Complemented Lattices

Let L be a lattice with the greatest element 1 and the least element 0. Let $x \in L$. By a complement of x in L is meant an element y in L such that $x \wedge y = 0$ and $x \vee y = 1$.

Definition 5 ([1, 2]). *A lattice L with the greatest element 1 and the least element 0 is called complemented if all its elements have unique complements.*

Definition 6. *A fuzzy set A of a complemented lattice L is called a (λ, μ)-fuzzy sublattice of L if $\forall a, b, c \in L$.*

$$A(a \wedge b) \vee \lambda \geq (A(a) \wedge A(b)) \wedge \mu$$

$$A(a \vee b) \vee \lambda \geq (A(a) \wedge A(b)) \wedge \mu$$

and

$$A(c') \vee \lambda \geq A(c) \wedge \mu$$

where c' is the complement of c.

Proposition 3. *If A is a (λ, μ)-fuzzy sublattice of a complemented lattice L, then $A(0) \vee \lambda \geq A(x) \wedge \mu$ and $A(1) \vee \lambda \geq A(x) \wedge \mu$ for all $x \in L$.*

Proof. $\forall x \in L$ and let x' be the complement of x. Then $A(1) \vee \lambda = A(x \vee x') \vee \lambda = (A(x \vee x') \vee \lambda) \vee \lambda \geq ((A(x) \wedge A(x')) \wedge \mu) \vee \lambda = (A(x) \vee \lambda) \wedge (A(x') \vee \lambda) \wedge (\mu \vee \lambda) \geq A(x) \wedge (A(x) \wedge \mu) \wedge \mu = A(x) \wedge \mu$.

Again, $A(0) \vee \lambda = A(x \wedge x') \vee \lambda \vee \lambda \geq ((A(x) \wedge A(x')) \wedge \mu) \vee \lambda = (A(x) \vee \lambda) \wedge (A(x') \vee \lambda) \wedge (\mu \vee \lambda) \geq A(x) \wedge (A(x) \wedge \mu) \wedge \mu = A(x) \wedge \mu$.

Proposition 4. *If A is a (λ, μ)-fuzzy sublattice of a complemented lattice L. Suppose that $A(0) \neq A(1)$, then either $A(0) \vee A(1) \leq \lambda$ holds or $A(0) \wedge A(1) \geq \mu$ holds.*

Proof. Suppose $A(0) < A(1)$.
If $A(0) \vee A(1) > \lambda$ and $A(0) \wedge A(1) < \mu$. Four cases are possible:

(1) If $A(0) > \lambda$ and $A(0) < \mu$, then $A(0) \vee \lambda = A(0) < A(1)$. Note that $A(0) < \mu$, we obtain that $A(0) < A(1) \wedge \mu$. That is $A(0) \vee \lambda < A(1) \wedge \mu$. This is a contradiction with the previous proposition.

(2) If $A(0) > \lambda$ and $A(1) < \mu$, then $A(0) \vee \lambda = A(0) < A(1) = A(1) \wedge \mu$. This is a contradiction with the previous proposition.

(3) If $A(1) > \lambda$ and $A(0) < \mu$, then from $A(0) < \mu$ and $A(0) < A(1)$ we obtain that $A(0) < A(1) \wedge \mu$. From $\lambda < A(1)$ and $\lambda < \mu$ we conclude that $\lambda < A(1) \wedge \mu$. So $A(0) \vee \lambda < A(1) \wedge \mu$. This is a contradiction with the previous proposition.

(4) If $A(1) > \lambda$ and $A(1) < \mu$, then from $A(0) < A(1)$ and $\lambda < A(1)$ we obtain $A(0) \vee \lambda < A(1) = A(1) \wedge \mu$. This is a contradiction with the previous proposition.

If $A(1) < A(0)$, we can prove the results dually.

Theorem 7. *Let A be a fuzzy subset of a complemented lattice L. Then the following are equivalent:*

(1) A is a (λ, μ)-fuzzy sublattice of L;
(2) A_α is a sublattice of L, for any $\alpha \in (\lambda, \mu]$, where $A_\alpha \neq \emptyset$.

Proof. "(1) $\Rightarrow$ (2)"
Let A be a (λ, μ)-fuzzy sublattice of L. For any $\alpha \in (\lambda, \mu]$, such that $A_\alpha \neq \emptyset$, we need to show that $x' \in A_\alpha$, for all $x \in A_\alpha$.

Since $A(x) \geq \alpha$, Then $A(x') \vee \lambda \geq A(x) \wedge \mu \geq \alpha \wedge \mu = \alpha$. Note that $\lambda < \alpha$, we obtain $A(x') \geq \alpha$. That is $x' \in A_\alpha$.

"(2) $\Rightarrow$ (1)"

Conversely, let A_α is a sublattice of L. We need to prove that $A(x') \vee \lambda \geq A(x) \wedge \mu,\ \ \forall x \in L$. If there exists $x_0 \in L$ such that $A(x_0') \vee \lambda = \alpha < A(x_0) \wedge \mu$, then $A(x_0) \geq \alpha$, $A(x_0') < \alpha$ and $\alpha \in (\lambda, \mu]$. Thus $x_0 \in A_\alpha$ and $x_0' \notin A_\alpha$. This is a contradiction with that A_α is a sublattice of L. Hence $A(x') \vee \lambda \geq A(x) \wedge \mu$ holds for any $x \in L$.

Therefore, A is a (λ, μ)-fuzzy sublattice of L .

Definition 7. *A fuzzy subset A of a complemented lattice L is called a (λ, μ)-fuzzy ideal if for all $a, b, c \in L$,*

$$A(a \wedge b) \vee \lambda \geq (A(a) \vee A(b)) \wedge \mu$$

$$A(a \vee b) \vee \lambda \geq (A(a) \wedge A(b)) \wedge \mu$$

and

$$A(c') \vee \lambda \geq A(c) \wedge \mu$$

where c' is the complement of c.

Theorem 8. *Let A be a (λ, μ)-fuzzy sublattice of a complemented lattice L. Then the following are equivalent:*

(1) A is a (λ, μ)-fuzzy ideal of L;
(2) A_α is an ideal of L, for any $\alpha \in (\lambda, \mu]$, where $A_\alpha \neq \emptyset$.

Proof. The proof is obvious and omitted.

Theorem 9. *Let $f : L_1 \to L_2$ be a homomorphism, where L_1 and L_2 are two complemented lattices and let A be a (λ, μ)-fuzzy sublattice of L_1. Then $f(A)$ is a (λ, μ)-fuzzy sublattice of L_2.*

Proof. Take any $y \in L_2$, suppose y' is the complement of y in L_2.

If $f^{-1}(y) = \emptyset$, then $f(A)(y') \vee \wedge \geq 0 = f(A)(y) \wedge \mu$.
Suppose that $f^{-1}(y) \neq \emptyset$, then we have

$$\begin{aligned} f(A)(y') \vee \lambda &= \sup_{x' \in L_1} \{A(x') | f(x') = y'\} \vee \lambda = \sup_{x' \in L_1} \{A(x') \vee \lambda | f(x') = y'\} \\ &\geq \sup_{x \in L_1} \{A(x) \wedge \mu | f(x') = y'\} \ = \sup_{x \in L_1} \{A(x) \wedge \mu | f(x) = y\} \\ &= \sup_{x \in L_1} \{A(x) | f(x) = y\} \wedge \mu \ \ = f(A)(y) \wedge \mu. \end{aligned}$$

So, $f(A)$ is a (λ, μ)-fuzzy sublattice of L_2.

Theorem 10. *Let $f : L_1 \to L_2$ be a surjective homomorphism, where L_1 and L_2 are two complemented lattices and let A be a (λ, μ)-fuzzy ideal of L_1. Then $f(A)$ is a (λ, μ)-fuzzy ideal of L_2.*

Proof. The proof is obvious and omitted.

Theorem 11. *Let $f : L_1 \to L_2$ be a homomorphism, where L_1 and L_2 are two complemented lattices and let B be a (λ, μ)-fuzzy sublattice of L_2. Then $f^{-1}(B)$ is a (λ, μ)-fuzzy sublattice of L_1.*

Proof. For any $x \in L_1$,

$$\begin{aligned} f^{-1}(B)(x') \vee \lambda &= B(f(x')) \vee \lambda \\ &= B((f(x))') \vee \lambda \\ &\geq B(f(x)) \wedge \mu \\ &= f^{-1}(B)(x) \wedge \mu. \end{aligned}$$

So, $f^{-1}(B)$ is a (λ, μ)-fuzzy sublattice of L_1.

Theorem 12. *Let $f : L_1 \to L_2$ be a homomorphism, where L_1 and L_2 are two complemented lattices and let B be a (λ, μ)-fuzzy ideal of L_2. Then $f^{-1}(B)$ is a (λ, μ)-fuzzy ideal of L_1.*

Proof. The proof is obvious and omitted.

4 (λ, μ)-Fuzzy Subhyperlattices

Throughout this section H always denotes a hyperlattice. The meet, hyperjoin and partial order of H, will be denoted as $\wedge$, $\sqcup$, and $\leq$, respectively.

Definition 8. *A fuzzy subset A of a hyperlattice H is said to be a (λ, μ)-fuzzy subhyperlattice of H if $\forall a, b \in H$,*

$$A(a \wedge b) \vee \lambda \geq (A(a) \wedge A(b)) \wedge \mu$$

and

$$\inf_{t \in a \sqcup b} A(t) \vee \lambda \geq (A(a) \wedge A(b)) \wedge \mu.$$

Theorem 13. *Let A be a fuzzy subset of H. Then the following are equivalent:*

(1) A is a (λ, μ)-fuzzy subhyperlattice of H;
(2) A_α is a subhyperlattice of H, for any $\alpha \in (\lambda, \mu]$, where $A_\alpha \neq \emptyset$.

Proof. "(1) ⇒ (2)"
Let A be a (λ, μ)-fuzzy subhyperlattice of H. For any $\alpha \in (\lambda, \mu]$, such that $A_\alpha \neq \emptyset$, we need to show that $x \wedge y \in A_\alpha$ and $x \sqcup y \subseteq A_\alpha$, for all $x, y \in A_\alpha$.

From $x \in A_\alpha$ we know that $A(x) \geq \alpha$. And similarly we obtain that $A(y) \geq \alpha$. Thus $A(x \wedge y) \vee \lambda \geq (A(x) \wedge A(y)) \wedge \mu \geq \alpha \wedge \mu = \alpha$. Note that $\lambda < \alpha$ and so $x \wedge y \in A_\alpha$.

From $A(x) \geq \alpha$ and $A(y) \geq \alpha$ we know that $\inf_{t \in x \sqcup y} A(t) \vee \lambda \geq (A(x) \wedge A(y)) \wedge \mu \geq \alpha \wedge \mu = \alpha$ and $\lambda < \alpha$, we conclude that $\inf_{t \in x \sqcup y} A(t) \geq \alpha$. So $A(t) \geq \alpha$ for any $t \in x \sqcup y$. Thus $x \sqcup y \subseteq A_\alpha$.

"(2) $\Rightarrow$ (1)"

If there exist $x_0, y_0 \in H$ such that $A(x_0 \wedge y_0) \vee \lambda < \alpha = (A(x_0) \wedge A(y_0)) \wedge \mu$, then $\alpha \in (\lambda, \mu]$, $A(x_0) \wedge A(y_0) \geq \alpha$. So $x_0 \in A_\alpha$ and $y_0 \in A_\alpha$. But $A(x_0 \wedge y_0) < \alpha$, that is $x_0 \wedge y_0 \notin A_\alpha$. This is a contradiction with that A_α is a subhyperlattice of H. Thus $A(x \wedge y) \vee \lambda \geq (A(x) \vee A(y)) \wedge \mu$ holds for all $x, y \in H$.

Again, if there exist $x_0, y_0 \in H$ such that $\inf_{t \in x_0 \sqcup y_0} A(t) \vee \lambda < \alpha = (A(x_0) \wedge A(y_0)) \wedge \mu$, then $\alpha \in (\lambda, \mu]$, $A(x_0) \wedge A(y_0) \geq \alpha$. So $x_0 \in A_\alpha$ and $y_0 \in A_\alpha$. But $\inf_{t \in x_0 \sqcup y_0} A(t) < \alpha$, that is $A(t) < \alpha$ for some $t \in x_0 \sqcup y_0$. So $x_0 \sqcup y_0 \not\subseteq A_\alpha$. This is a contradiction with that A_α is a subhyperlattice of H. Thus $\inf_{t \in x \sqcup y} A(t) \vee \lambda \geq (A(x) \wedge A(y)) \wedge \mu$ holds for all $x, y \in H$.

Definition 9. *Let $(H, \sqcup, \wedge)$ be a hyperlattice. A nonempty subset I of H is called an ideal of H if for all $a, b \in H$,*

$$a, b \in I \Rightarrow a \sqcup b \subseteq I$$

and

$$a \in H, b \in I \Rightarrow a \wedge b \in I.$$

Proposition 5. *Suppose I is a subset of a hyperlattice H, then the following are equivalent for all $a, b \in H$,*

(1) $a \in H, b \in I \Rightarrow a \wedge b \in I$;
(2) $a \in I$ and $b \leq a \Rightarrow b \in I$.

Proof. "(1) $\Rightarrow$ (2)"
If $b \leq a$, then $b = a \wedge b$. From (1) we know that $a \wedge b \in I$. And so $b \in I$.

"(2) $\Rightarrow$ (1)"

From $a \wedge b \leq b \in I$ and (2) we know that $a \wedge b \in I$.

Definition 10. *A fuzzy subset A of a hyperlattice H is a (λ, μ)-fuzzy ideal of H if for all $a, b \in H$,*

$$A(a \wedge b) \vee \lambda \geq (A(a) \vee A(b)) \wedge \mu$$

and

$$\inf_{t \in a \sqcup b} A(t) \vee \lambda \geq (A(a) \wedge A(b)) \wedge \mu.$$

Theorem 14. *Let A be a (λ, μ)-fuzzy subhyperlattice of H. Then the following are equivalent:*

(1) A is a (λ, μ)-fuzzy ideal of H;
(2) A_α is an ideal of H, for any $\alpha \in (\lambda, \mu]$, where $A_\alpha \neq \emptyset$.

Proof. "(1) $\Rightarrow$ (2)"
Let A be a (λ, μ)-fuzzy ideal of H. For any $\alpha \in (\lambda, \mu]$, such that $A_\alpha \neq \emptyset$, we need to show that $x \wedge y \in A_\alpha$, for all $x \in A_\alpha$ and $y \in H$.

From $A(x) \geq \alpha$ we obtain that $A(x \wedge y) \vee \lambda \geq (A(x) \vee A(y)) \wedge \mu \geq \alpha$. Note that $\lambda < \alpha$, we conclude that $A(x \wedge y) \geq \alpha$. So $x \wedge y \in A_\alpha$.

"(2) $\Rightarrow$ (1)"

If there exist $x_0, y_0 \in H$ such that $A(x_0 \wedge y_0) \vee \lambda < \alpha = (A(x_0) \vee A(y_0)) \wedge \mu$, then $\alpha \in (\lambda, \mu]$, $A(x_0) \vee A(y_0) \geq \alpha$. So $x_0 \in A_\alpha$ or $y_0 \in A_\alpha$. But $A(x_0 \wedge y_0) < \alpha$, that is $x_0 \wedge y_0 \notin A_\alpha$. This is a contradiction with that A_α is an ideal of H. Thus $A(x \wedge y) \vee \lambda \geq (A(x) \vee A(y)) \wedge \mu$ holds for all $x, y \in H$.

The proof is ended.

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$(\in, \in \vee q_{(\lambda,\mu)})$-Fuzzy h-Ideals of Hemirings

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Abstract. The definitions of $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left (resp. right) h-ideals of hemirings, generalized fuzzy left (resp. right) h-ideals of hemirings, prime (semiprime) $(\in, \in \vee q_{(\lambda,\mu)})$-left (resp. right) h-ideals of hemirings and prime (semiprime) generalized fuzzy left (resp. right) h-ideals of hemirings are given. Meanwhile, some fundamental properties of them are discussed. Finally, the implication-based fuzzy left (resp. right) h-ideals of hemirings are considered.

Keywords: $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left (resp. right) h-ideals, generalized fuzzy, prime (semiprime), implication-based.

1 Introduction

The concept of semirings was introduced by Vandiver in 1935. Ideals of semirings play a central role in the structure theory. However, their properties do not coincide with the usual ring ideals in general which makes people difficult to obtain similar theorems of the usual ring ideals. Indeed, many results in rings apparently have no analogues in semirings using only ideals. Henriksen [1] defined a more restricted class of ideals in semirings, which is called the class of k-ideals, with the property that if the semiring S is a ring then a complex in S is a k-ideal if and only if it is a ring ideal. Another more restricted class of ideals has been given in hemirings by Iizuka [2]. According to Iizuka's definition, an ideal in any additively commutative semiring S can be given which coincides with a ring ideal provided that S is a hemiring. We now call this ideal an h-ideal of the hemiring S. The properties of h-ideals and k-ideals of hemirings were thoroughly investigated by La Torre [3] and by using the h-ideals and k-ideals. La Torre established some analogous ring theorems for hemirings.

After the introduction of fuzzy sets by Zadeh [4] in 1965, the fuzzy set theory has been widely used in mathematics and many other areas in more than 40 years. Rosenfeld [5] defined fuzzy subgroups in 1971. Since then fuzzy algebra came into being. Zou and Liu [6] systematically summed up some

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aspects of fuzzy algebra. Ghosh [7] characterize fuzzy k-ideal of semirings in 1998. Jun [8] considered fuzzy h-ideals of hemirings in 2004. Moreover, Zhan [9] et al. discussed the h-hemiregular hemirings by using the fuzzy h-ideals. As a continuation of this investigation, Yin [10] et al. introduced the concepts of fuzzy h-bi-ideals and fuzzy h-quasi-ideals of hemirings. By using these fuzzy ideals, some characterization theorems of h-hemiregular and h-intra-hemiregular hemirings are obtained. Other important results related with fuzzy h-ideals of a hemiring were given in [11-14].

Bhakat and Das gave the concept of $(\in, \in q)$-fuzzy subgroups by using the "belong to" relation $(\in)$ and "quasi-coincident with" relation (q) between a fuzzy point x_λ and a fuzzy set A in 1992 and 1996, and do some research on it [15-18]. Bavvaz [19] introduced the concept of $(\in, \in q)$-fuzzy subnear-rings , fuzzy H_v- submodules [20] and fuzzy R-subgroups [21]. Meanwhile, he discussed their properties. Liao [22] et al. extended the common "quasi-coincident" to "generalized quasi-coincident". This "generalized quasi-coincident" is a uniform and generalization of concepts of Rosenfeld's fuzzy algebra, Bhakat and Das's fuzzy algebra and $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy algebra. When $\lambda = 0$ and $\mu = 1$, we get the ordinary results of Rosenfeld's fuzzy algebra. When $\lambda = 0$ and $\mu = 0.5$, we draw the conclusion of Bhakat and Das's fuzzy algebra. When $\lambda = 0.5$ and $\mu = 1$, we can have the results of $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy algebra. Liao et al. have done some research on fuzzy algebra [22-27]. This paper is a continuation of these researches.

In this paper, we recall some basic definitions and results of hemirings in Section 2. In Section 3, the definitions of $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left (resp. right) h-ideals of semirings and generalized fuzzy left (resp. right) h-ideals of semirings are given. The equivalent relationship between them is described and some fundamental properties of them are discussed. In Section 4, the concepts of prime (semiprime) $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left (resp. right) h-ideals of semirings and prime (semiprime) generalized fuzzy left (resp. right) h-ideals of semirings are given and some fundamental properties of them are discussed. Finally, in Section 5, we consider the implication-based fuzzy left (resp. right) h-ideals.

2 Preliminaries

Definition 1. *[9] Recall that a semiring is an algebraic system $(R, +, \cdot)$ consisting of a nonempty set R together with two binary operations " $+$ " and " $\cdot$ " on R which are called addition and multiplication, respectively such that $(R, +)$ and $(R, \cdot)$ are semigroups which are linked by the following distributive laws:*

$$a(b + c) = ac + bc \text{ and } (a + b)c = ac + bc$$

are satisfied for all $a, b, c \in R$.

By zero of a semiring $(R, +, \cdot)$we mean an element $0 \in R$ such that $0 \cdot x = x \cdot 0 = 0$ and $0 + x = x + 0 = x$ for all $x \in R$. A semiring with a zero such that$(R, +)$is a commutative semigroup is called a hemiring.

Now, we recall some basic concepts of a hemiring R. From now on we write R and S for semirings unless otherwise specified.

A left ideal (right ideal and ideal) A of R is called a left h-ideal of (right h-ideal and h-ideal), respectively, if for any $x, z \in R$ and $a, b \in A, x+a+z = b+z$ implies $x \in A$.

The h-closure of A in R is defined by

$$\overline{A} = \{x \in R | x + a_1 + z = a_2 + z, \text{for some}\, a_1, a_2 \in A, z \in R\}.$$

A left h-ideal (right h-ideal and h-ideal) A of R is called prime (resp. semiprime) if $xy \in A \Rightarrow x \in A$ or $y \in A$ (resp. $x^2 \in A \Rightarrow x \in A$), for all $x, y \in R$.

Definition 2. *A map $A : R \to [0, 1]$ is said to be a fuzzy subset of R. A fuzzy subset A of R of the form $A(y) = \begin{cases} \lambda(\neq 0), & y = x \\ 0, & y \neq x \end{cases}$ is said to be a fuzzy point support x and value λ is denote by x_λ.*

Definition 3. *[22] Let $\alpha, \lambda, \mu \in [0, 1]$ and $\lambda < \mu$. Let A be a fuzzy subset of R. If $A(x) \geq \alpha$ then x_α is said to belong to A, denoted by $x_\alpha \in A$. A fuzzy point x_α is said to be generalized quasi-coincident with A if $\alpha > \lambda$ and $A(x) + \alpha > 2\mu$, denoted by $x_\alpha q_{(\lambda,\mu)} A$. If $x_\alpha \in A$ or $x_\alpha q_{(\lambda,\mu)} A$, then denoted by $x_\alpha \in \vee q_{(\lambda,\mu)} A$.*

Definition 4. *[8] Let F and G be fuzzy sets of R. Then the h-product of F and G is defined by*

$$(F \circ_h G)(x) = \sup_{x+a_1b_1+z=a_2b_2+z} (\min\{F(a_1), F(a_2), G(b_1), G(b_2)\})$$

and $(F \circ_h G)(x) = 0$ if x cannot be expressed as $x + a_1b_1 + z = a_2b_2 + z$.

Definition 5. *[10] Let F and G be fuzzy sets of R. Then the h-intrinsic product of F and G is defined by*

$$(F \odot_h G)(x) = \sup_{x+\sum_{i=1}^m a_ib_i+z=\sum_{j=1}^n a'_jb'_j+z} (\min\{F(a_i), F(a'_j), G(b_i), G(b'_j)\})$$

for all $i = 1, 2, \ldots, m; j = 1, 2, \ldots, n;$ and $(F \odot_h G)(x) = 0$ if x cannot be expressed as $x + \sum_{i=1}^m a_ib_i + z = \sum_{j=1}^n a'_jb'_j + z$.

Definition 6. *[28]*

(i) A fuzzy set A of R is called prime if it satisfies:
$A(xy) = A(x)$ or $A(xy) = A(y)$, for all $x, y \in R$;

(ii) A fuzzy set A of R is called semiprime if it satisfies:
$A(x) = A(x^2)$, for all $x \in R$.

Definition 7. *[8] A fuzzy set A of R is said to be a fuzzy left (resp. right) h-ideal of R if for all $x, y, z, a, b \in R$, it satisfies:*

(F1a) $A(x + y) \geq A(x) \wedge A(y)$,
(F1b) $A(xy) \geq A(y)$ *(resp.* $A(xy) \geq A(x)$*),*
(F1c) $x + a + z = b + z$ *implies* $A(x) \geq A(a) \wedge A(b)$.

3 $(\in, \in \vee q_{(\lambda,\mu)})$-Fuzzy h-Ideals

Definition 8. *A fuzzy subset A of R is said to be an$(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left (resp. right) h-ideal if for all $\alpha_1, \alpha_2 \in (\lambda, 1]$ and $x, y, z, a, b \in R$, the following conditions hold:*

(F2a) $x_{\alpha_1}, y_{\alpha_2} \in A$ *imply* $(x + y)_{\alpha_1 \wedge \alpha_2} \in \vee q_{(\lambda,\mu)} A$,
(F2b) $x_{\alpha_1}, y_{\alpha_2} \in A$ *imply* $(xy)_{\alpha_1 \wedge \alpha_2} \in \vee q_{(\lambda,\mu)} A$,
(F2c) $a_{\alpha_1}, b_{\alpha_2} \in A$ *imply* $x_{\alpha_1 \wedge \alpha_2} \in \vee q_{(\lambda,\mu)} A$, *for all* $x, y, z, a, b \in R$ *with* $x + a + z = b + z$.

Definition 9. *Let $\lambda, \mu \in [0, 1]$ and $\lambda < \mu$. A fuzzy subset A of R is said to be a generalized fuzzy left (resp. right) h-ideal, if for all $x, y, z, a, b \in R$, the following conditions hold:*

(F3a) $A(x + y) \vee \lambda \geq A(x) \wedge A(y) \wedge \mu$,
(F3b) $A(xy) \vee \lambda \geq A(y) \wedge \mu$ *(resp.* $A(xy) \vee \lambda \geq A(x) \wedge \mu$*),*
(F3c) $x + a + z = b + z$ *implies* $A(x) \vee \lambda \geq A(a) \wedge A(b) \wedge \mu$.

Theorem 1. *Let A be a fuzzy subset of R, the following statements are equivalent:*

(1) A is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left h-ideal of R.
(2) A is a generalized fuzzy left h-ideal of R.
(3) For any $\alpha \in (\lambda, \mu]$, nonempty set A_α is a left h-ideal of R.

Proof. (1)⇒(2): Based on [23], we only need to prove (F2c)⇒(F3c).
(F2c)⇒(F3c): Assume that there exist $x, z, a, b \in R$ such that $x+a+z = b+z$ and $A(x) \vee \lambda < \alpha = A(a) \wedge A(b) \wedge \mu$, then $\lambda < \alpha \leq \mu$, $A(a) \geq \alpha$ and $A(b) \geq \alpha$, so $a_\alpha \in A$ and $b_\alpha \in A$. Based on the Definition of 8, $x_\alpha \in \vee q_{(\lambda,\mu)} A$. But $A(x) < \alpha \leq \mu$. Therefore $A(x) + \alpha < \alpha + \alpha \leq 2\mu$, which is a contradiction. Thus (F3c) holds.

(2)⇒(1): Based on [23], we only need to prove (F3c)⇒(F2c).

(F3c)⇒(F2c): Let $x, z, a, b \in R$ be such that $x + a + z = b + z$. If for all $\alpha_1, \alpha_2 \in (\lambda, 1]$, for $a_{\alpha_1}, b_{\alpha_2} \in A$, then $A(a) \geq \alpha_1$ and $A(b) \geq \alpha_2$. So choose $\alpha = \alpha_1 \wedge \alpha_2$, since A is a generalized fuzzy left h-ideal of R, we have $A(x) \vee \lambda \geq A(a) \wedge A(b) \wedge \mu \geq \alpha \wedge \mu$. If $\alpha \leq \mu$, by $\lambda < \alpha$, then $A(x) \geq \alpha$, so $x_\alpha \in A$; If $\alpha > \mu$, by $\lambda < \mu$, then $A(x) \geq \mu$, so $A(x) + \alpha \geq \mu + \alpha > 2\mu$. Hence $x_\alpha q_{(\lambda,\mu)} A$. Therefore, $x_\alpha \in \vee q_{(\lambda,\mu)} A$.

(2)⇒(3):$\forall \alpha \in (\lambda, \mu], \forall x, y \in A_\alpha, z \in R$, then $A(x) \geq \alpha$ and $A(y) \geq \alpha$. Since A is a generalized fuzzy left h-ideal of R, we have $A(x + y) \vee \lambda \geq$

$A(x) \wedge A(y) \wedge \mu \geq \alpha \wedge \mu = \alpha$ and $A(zy) \vee \lambda \geq A(y) \wedge \mu \geq \alpha \wedge \mu = \alpha$. By $\lambda < \alpha$, hence $A(x+y) \geq \alpha$ and $A(zy) \geq \alpha$, i.e., $x+y, zy \in A_\alpha$. Let $x, z \in R$ and $a, b \in A_\alpha$ be such that $x+a+z = b+z$. Then $A(x) \vee \lambda \geq A(a) \wedge A(b) \wedge \mu \geq \alpha \wedge \mu$. By $\lambda < \alpha$, then $A(x) \geq \alpha$, so $x \in A_\alpha$. Therefore, A_α is a h-ideal of R.

(3)⇒(2): We only prove (F3c) hold true because the proofs of the others are similar. Assume that there exist $x, z, a, b \in R$ such that $x+a+z = b+z$. But $A(x) \vee \lambda < \alpha = A(a) \wedge A(b) \wedge \mu$, then $\lambda < \alpha \leq \mu$, $A(a) \geq \alpha$ and $A(b) \geq \alpha$, so $a \in A_\alpha$ and $b \in A_\alpha$. Since A_α is a left h-ideal of R, we have $x \in A_\alpha$. But $A(x) < \alpha$, which is a contradiction. Thus (F3c) holds. Therefore, A is a generalized fuzzy left ideal of R.

Theorem 2. *If $\{A\}_{i \in I}$ is a family of $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left h-ideal of R. Then $\cap_{i \in I} A_i$ is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left h-ideal of R.*

Remark 1. If $\{A\}_{i \in I}$ is a family of $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left h-ideal of R. Is $\cup_{i \in I} A_i$ is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left h-ideal of R?

Example 1. The fuzzy subsets

$$A(x) = \begin{cases} 0.8 & \text{if } x \in \langle 4 \rangle, \\ 0.5 & \text{if } x \in \langle 2 \rangle - \langle 4 \rangle, \\ 0 & \text{otherwise}, \end{cases} \quad B(x) = \begin{cases} 0.8 & \text{if } x \in \langle 6 \rangle, \\ 0.5 & \text{if } x \in \langle 3 \rangle - \langle 6 \rangle, \\ 0 & \text{otherwise}, \end{cases}$$

defined on a hemiring $(N_0, +, \cdot)$, where N_0 is the set of all nonnegative integers, are $(\in, \in \vee q_{(0.5,0.8)})$-fuzzy left h-ideal of N_0. Because $(A \cup B)(10) \vee 0.5 = 0.5 \leq 0.8 = (A \cup B)(4) \wedge (A \cup B)(6) \wedge 0.8$, $A \cup B$ is not $(\in, \in \vee q_{(0.5,0.8)})$- fuzzy h-ideal of N_0.

Theorem 3. *Let $\{A\}_{i \in I}$ is a family of $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left h-ideals such that $A_i \subseteq A_j$ or $A_j \subseteq A_i$ for all $i, j \in I$. Then $\cup_{i \in I} A_i$ is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left h-ideal of R.*

Theorem 4. *Let A be a nonempty subset of a hemiring R. Let B be a fuzzy set in R defined by*

$$B(x) = \begin{cases} s & \text{if } x \in A, \\ t & \text{otherwise}, \end{cases}$$

where $t < s, 0 \leq t < \mu$ and $\lambda < s \leq 1$. Then B is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left h-ideal of R if and only if A is a left h-ideal of R.

Proof. When $0 \leq t \leq \lambda, \lambda < s < \mu$, $B_\alpha = \begin{cases} A & \lambda < \alpha \leq s, \\ \emptyset & s < \alpha \leq \mu. \end{cases}$

When $0 \leq t \leq \lambda, \mu \leq s \leq 1, \forall \alpha \in (\lambda, \mu], B_\alpha = A$.

When $\lambda < t < \mu, t < s < \mu$, $B_\alpha = \begin{cases} R & \lambda < \alpha \leq t, \\ A & t < \alpha \leq s, \\ \emptyset & s < \alpha \leq \mu. \end{cases}$

When $\lambda < t < \mu, \mu \leq s \leq 1$, $B_\alpha = \begin{cases} R & \lambda < \alpha \leq t, \\ A & t < \alpha \leq \mu. \end{cases}$

Based on Theorem 1, then B is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left h-ideal of R if and only if A is a left h-ideal of R.

Corollary 1. *Let A be a nonempty subset of a hemiring R. Then A is a left h-ideal of R if and only if χ_A is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left h-ideal of R.*

Theorem 5. *If A and B are $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left h-ideals of R, then so is $A \cap_\mu B$, where $A \cap_\mu B$ is defined by*

$$(A \cap_\mu B)(x) = A(x) \wedge B(x) \wedge \mu \text{ for all } x \in R.$$

Theorem 6. *Let $f : R \to S$ be a homomorphism and let B be an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left h-ideal of S. Then $f^{-1}(B)$ is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left h-ideal of R.*

Theorem 7. *Let A be an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left h-ideal of R and let $f : R \to R$ be an homomorphism. Then the mapping $A^f : R \to [0,1]$, defined by $A^f(x) = A(f(x))$ for all $x \in R$, is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left h-ideal of R.*

Theorem 8. *Let $f : R \to S$ be a automorphism and let A be an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left h-ideal of R. Then $f(A)$ is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left h-ideal of S.*

Theorem 9. *If A and B are an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy right h-ideal and an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left h-ideal of R, respectively, then $(A \cap B) \cup \overline{\lambda} \supseteq (A \circ_h B) \cap \overline{\mu}$, where $\overline{\alpha}$ is defined by $\overline{\alpha}(x) = \alpha$ for all $x \in R$,$\forall \alpha \in [0,1]$.*

Theorem 10. *If A and B are an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy right h-ideal and an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left h-ideal of R, respectively, then $(A \cap_\mu B) \cup \overline{\lambda} \supseteq (A \odot_{(h,\mu)} B) \cup \overline{\lambda}$, where $(A \odot_{(h,\mu)} B)(x) = \sup\limits_{x+\sum_{i=1}^m a_i b_i + z = \sum_{j=1}^n a'_j b'_j + z} (\min\{A(a_i), A(a'_j), G(b_i), G(b'_j), \mu\})$ for all i=1,2,...,m; j=1,2,...,n; and $(A \odot_{(h,\mu)} B)(x) = 0$ if x cannot be expressed as $x + \sum_{i=1}^m a_i b_i + z = \sum_{j=1}^n a'_j b'_j + z$.*

Proof. Let A and B are an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy right h-ideal and an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left h-ideal of R, respectively. Then the proof is obvious if $(A \odot_{(h,\mu)} B)(x) = 0$. Otherwise, for every $a_i, b_i, a'_j, b'_j \in R$, satisfying $x + \sum_{i=1}^m a_i b_i + z = \sum_{j=1}^n a'_j b'_j + z$. As A is an $(\in, \in \vee q_{(\lambda,\mu)})$- fuzzy right ideal, by induction on i, we can easily have $A(\sum_{i=1}^m a_i b_i) \vee \lambda \geq A(a_i b_i) \wedge \mu$.

Therefore, we have $A(x) \vee \lambda = A(x) \vee \lambda \vee \lambda \geq \{A(\sum_{i=1}^m a_i b_i) \wedge A(\sum_{j=1}^n a'_j b'_j) \wedge \mu\} \vee \lambda \vee \lambda = \{(A(\sum_{i=1}^m a_i b_i) \vee \lambda) \wedge (A(\sum_{j=1}^n a'_j b'_j) \vee \lambda) \wedge (\mu \vee \lambda)\} \vee \lambda \geq \{(A(a_i b_i) \wedge \mu) \wedge (A(a'_j b'_j) \wedge \mu) \wedge \mu\} \vee \lambda = (A(a_i b_i) \vee \lambda) \wedge (A(a'_j b'_j) \vee \lambda) \wedge (\mu \vee \lambda) \geq A(a_i) \wedge A(a'_j) \wedge \mu$.

We can prove $B(x) \vee \lambda \geq B(b_i) \wedge B(b'_j) \wedge \mu$ similarly.

Thus, $(A \odot_{(h,\mu)} B)(x) \vee \lambda = \sup_{x+\sum_{i=1}^{m} a_i b_i + z = \sum_{j=1}^{n} a'_j b'_j + z} (\min\{A(a_i), A(a'_j), B(b_i), B(b'_j), \mu\}) \vee \lambda \leq \{(A(x) \vee \lambda) \wedge (B(x) \vee \lambda) \wedge \mu)\} \vee \lambda = \{((A(x) \wedge B(x)) \vee \lambda) \wedge \mu\} \vee \lambda = (A(x) \wedge B(x) \wedge \mu) \vee (\lambda \wedge \mu) \vee \lambda = (A \cap_\mu B)(x) \vee \lambda$.

Consequently, $(A \cap_\mu B) \cup \overline{\lambda} \supseteq (A \odot_{(h,\mu)} B) \cup \overline{\lambda}$.

4 Prime (Semiprime) $(\in, \in \vee q_{(\lambda,\mu)})$-Fuzzy h-Ideals

Definition 10

(i) An $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left (resp. right) h-ideal A of R is called prime if for all $x, y \in R$ and $\alpha \in (\lambda, 1]$, we have

(F4a) $(xy)_\alpha \in A \Rightarrow x_\alpha \in \vee q_{(\lambda,\mu)} A$ or $y_\alpha \in \vee q_{(\lambda,\mu)} A$.

(ii) An $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left (resp. right) h-ideal A of R is called semiprime if for all $x \in R$ and $\alpha \in (\lambda, 1]$, we have

(F4b) $(x^2)_\alpha \in A \Rightarrow x_\alpha \in \vee q_{(\lambda,\mu)} A$.

Definition 11

(i) A generalized fuzzy left (resp. right) h-ideal A is called prime if for all $x, y \in R$, we have

(F5a) $A(x) \vee A(y) \vee \lambda \geq A(xy) \wedge \mu$.

(ii) A generalized fuzzy left (resp. right) h-ideal A is called semiprime if for all $x \in R$, we have

(F5b) $A(x) \vee \lambda \geq A(x^2) \wedge \mu$.

Theorem 11. *Let A be a fuzzy subset of R, the following statements are equivalent:*

(1)A is a prime $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left h-ideal of R.

(2)A is a prime generalized fuzzy left h-ideal of R.

(3)For any $\alpha \in (\lambda, \mu]$, nonempty set A_α is a prime left h-ideal of R.

Proof. (1)$\Rightarrow$(2): By Theorem 1, we only need to prove (F4a)$\Rightarrow$ (F5a).

(F4a) $\Rightarrow$ (F5a): Let A be a prime $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left h-ideal of R. If there exist $x, y \in R$ such that $A(x) \vee A(y) \vee \lambda < \alpha = A(xy) \wedge \mu$, then $\lambda < \alpha \leq \mu$ and $(xy)_\alpha \in A$, but $x_\alpha \notin A$ and $y_\alpha \notin A$. Since $A(x) + \alpha \leq \alpha + \alpha \leq 2\mu$ and $A(y) + \alpha \leq \alpha + \alpha \leq 2\mu$, and hence, $x_\alpha \overline{q_{(\lambda,\mu)}} A$ and $y_\alpha \overline{q_{(\lambda,\mu)}} A$. Consequently, we have $x_\alpha \overline{\in \vee q_{(\lambda,\mu)}} A$ and $y_\alpha \overline{\in \vee q_{(\lambda,\mu)}} A$, which is a contradiction. Thus, (F5a) holds.

(2)$\Rightarrow$(1): By Theorem 1, we only need to prove (F5a)$\Rightarrow$(F4a).

(F5a)$\Rightarrow$(F4a): Let$(xy)_\alpha \in A$, then $A(xy) \geq \alpha$. So we have $A(x) \vee A(y) \vee \lambda \geq A(xy) \wedge \mu \geq \alpha \wedge \mu$. If $\alpha \leq \mu$, by $\lambda < \alpha$, then $A(x) \geq \alpha$ or $A(y) \geq \alpha$, so $x_\alpha \in A$ or $y_\alpha \in A$; If $\alpha > \mu$, by $\lambda < \mu$, then $A(x) \geq \mu$ or $A(y) \geq \mu$, so $A(x) + \alpha \geq \mu + \alpha > 2\mu$ or $A(y) + \alpha \geq \mu + \alpha > 2\mu$. Hence $x_\alpha q_{(\lambda,\mu)} A$ or $y_\alpha q_{(\lambda,\mu)} A$. Therefore, $x_\alpha \in \vee q_{(\lambda,\mu)} A$ or $y_\alpha \in \vee q_{(\lambda,\mu)} A$.

(2)⇒(3): Let A a prime generalized fuzzy left h-ideal of R and $\alpha \in (\lambda, \mu]$. Then, by Theorem 1, nonempty set A_α is a left h-ideal of R for all $\alpha \in (\lambda, \mu]$. Let $xy \in A_\alpha$. Then $A(x) \vee A(y) \vee \lambda \geq A(xy) \wedge \mu \geq \alpha \wedge \mu = \alpha$, by $\lambda < \mu$, and so $A(x) \geq \alpha$ or $A(y) \geq \alpha$. Thus, $x \in A_\alpha$ or $y \in A_\alpha$. This shows that A_α is a prime left h-ideal of R, for all $\alpha \in (\lambda, \mu]$.

(3)⇒(2): Assume that A_α is a prime left h-ideal of R for all $\alpha \in (\lambda, \mu]$. Then by Theorem 1, A is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left ideal of R. Let $(xy)_\alpha \in A$, then $xy \in A_\alpha$. Since A_α is prime, thus $x \in A_\alpha$ or $y \in A_\alpha$, that is, $x_\alpha \in A$ or $y_\alpha \in A$. Thus, $x_\alpha \in \vee q_{(\lambda,\mu)} A$ or $y_\alpha \in \vee q_{(\lambda,\mu)} A$. Therefore, A must be a prime $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left h-ideal of R. Based on (1)⇔(2) in Theorem 12, A must be also a prime generalized fuzzy left h-ideal of R.

By Theorem 12, we have the following corollary.

Corollary 2. *Let A be a fuzzy subset of R, the following statements are equivalent:*

(1) A is a semiprime $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left h-ideal of R.
(2) A is a semiprime generalized fuzzy left h-ideal of R.
(3) For any $\alpha \in (\lambda, \mu]$, nonempty set A_α is a semiprime left h-ideal of R.

For any fuzzy set A of R and $\alpha \in [0, 1]$, we denote $Q_\alpha = \{x \in R | x_\alpha q_{(\lambda,\mu)} A\}$ and $[A]_\alpha = \{x \in R | x_\alpha \in \vee q_{(\lambda,\mu)} A\}$. It is clear that $[A]_\alpha = A_\alpha \cup Q_\alpha$.

Lemma 1. *A fuzzy set A is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left h-ideal of R if and only if nonempty set $[A]_\alpha$ is a left h-ideal of R for all $\alpha \in (\lambda, 2\mu - \lambda]$, where $2\mu - \lambda \leq 1$.*

Theorem 12. *A fuzzy set A is a prime $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left h-ideal of R if and only if nonempty set $[A]_\alpha$ is a prime left h-ideal of R for all $\alpha \in (\lambda, 2\mu - \lambda]$, where $2\mu - \lambda \leq 1$.*

Proof. Let A is a prime $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left h-ideal of R, then by Lemma 1, we have $[A]_\alpha$ is a left h-ideal of R for all $\alpha \in (\lambda, 2\mu - \lambda]$. To prove $[A]_\alpha$ is prime, let $xy \in [A]_\alpha$. Since $[A]_\alpha = A_\alpha \cup Q_\alpha$, we have $xy \in A_\alpha$ or $xy \in Q_\alpha$.

Case 1: $xy \in Q_\alpha - A_\alpha$. Then $A(xy) + \alpha > 2\mu$ and $A(xy) < \alpha$.

(1) If $A(xy) \leq \mu$, then $A(x) \vee A(y) \vee \lambda \geq A(xy) \wedge \mu = A(xy) > 2\mu - \alpha$. Since $\alpha \in (\lambda, 2\mu - \lambda]$, that is, $2\mu - \alpha \geq \lambda$, which implies $A(x) > 2\mu - \alpha$ or $A(y) > 2\mu - \alpha$. Thus, $A(x) + \alpha > 2\mu$ or $A(y) + \alpha > 2\mu$, that is, $x \in Q_\alpha \subseteq [A]_\alpha$ or $y \in Q_\alpha \subseteq [A]_\alpha$.

(2) If $A(xy) > \mu$, then $\mu < A(xy) < \alpha$. Thus, $A(x) \vee A(y) \vee \lambda \geq A(xy) \wedge \mu = \mu$. By $\lambda < \mu$, we have $A(x) > \mu$ or $A(y) > \mu$, which implies $A(x) + \alpha > \mu + \alpha > 2\mu$ or $A(y) + \alpha > \mu + \alpha > 2\mu$. Consequently, $x \in Q_\alpha \subseteq [A]_\alpha$ or $y \in Q_\alpha \subseteq [A]_\alpha$.

Case 2: $xy \in A_\alpha$. Then $A(xy) \geq \alpha$.

(1) If $\alpha \leq \mu$, then $A(x) \vee A(y) \vee \lambda \geq A(xy) \wedge \mu \geq \alpha \wedge \mu = \alpha$. Since $\lambda < \alpha$, we have $A(x) \geq \alpha$ or $A(y) \geq \alpha$, which implies $x \in A_\alpha \subseteq [A]_\alpha$ or $y \in A_\alpha \subseteq [A]_\alpha$.

(2) If $\alpha > \mu$, then $A(x) \vee A(y) \vee \lambda \geq A(xy) \wedge \mu \geq \alpha \wedge \mu = \mu$. Since $\lambda < \mu$, we have $A(x) \vee A(y) \geq \mu$, which implies $A(x) + \alpha \geq \mu + \alpha > 2\mu$ or $A(y) + \alpha \geq \mu + \alpha > 2\mu$. Hence, $x \in Q_\alpha \subseteq [A]_\alpha$ or $y \in Q_\alpha \subseteq [A]_\alpha$.

Therefore, $[A]_\alpha$ is a prime left h-ideal of R.

Conversely, let nonempty set $[A]_\alpha$ be prime left h-ideal of R for all $\alpha \in (\lambda, 2\mu - \lambda]$, where $2\mu - \lambda \leq 1$, then by Lemma 1, we know A is a prime $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left h-ideal of R. Let $(xy)_\alpha \in A$, then $(xy) \in A_\alpha \subseteq [A]_\alpha$. Since $[A]_\alpha$ is prime, we have $x \in [A]_\alpha$ or $y \in [A]_\alpha$. This implies $x_\alpha \in \vee q_{(\lambda,\mu)} A$ or $y_\alpha \in \vee q_{(\lambda,\mu)} A$. Therefore, A is a prime $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left h-ideal of R.

Corollary 3. *A fuzzy set A of R is a semiprime $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left h-ideal of R if and only if nonempty set $[A]_\alpha$ is a semiprime left h-ideal of R of for all $\alpha \in (\lambda, 2\mu - \lambda]$, where $2\mu - \lambda \leq 1$.*

It is prime (semiprime) $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy right h-ideal of R that have the similar results of prime (semiprime) $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left h-ideal of R.

5 Implication-Based Fuzzy h-Ideals

In fuzzy logic, we denote the truth value of fuzzy proposition α by $[\alpha]$. In the following, we display the fuzzy logic and its corresponding set-theoretical notions as follows:

$[x \in A] = A(x)$,
$[x \overline{\in} A] = 1 - A(x)$,
$[P \wedge Q] = \min\{[P], [Q]\}$,
$[P \vee Q] = \max\{[P], [Q]\}$,
$[P \to Q] = \min\{1, 1 - [P] + [Q]\}$,
$[\forall x A(x)] = \inf[A(x)]$,
$\models A$ if and only if $[A] = 1$ for all valuations.

In this paper, we only consider the following implication operator:

(1) Lukasiewicz operator:
$I_a(x, y) = \min\{1, 1 - x + y\}$
(2) Gödel implication operator:
$$I_g(x, y) = \begin{cases} 1 \text{ if } x \leq y, \\ y \text{ if } x > y. \end{cases}$$
(3) The contraposition of Gödel implication operator:
$$I_{cg}(x, y) = \begin{cases} 1 & \text{if } x \leq y, \\ 1 - x & \text{if } x > y. \end{cases}$$
(4) Gaines-Rescher implication operator:
$$I_{gr}(x, y) = \begin{cases} 1 \text{ if } x \leq y, \\ 0 \text{ if } x > y. \end{cases}$$

In the following definition, we consider the implication operators in Lukasiewicz system of continuous-valued logic.

Definition 12. *A fuzzy set A of R is called a fuzzified left (resp. right) h-ideal of R if it satisfies:*

(F6a) For any $x, y \in R, \models [x \in A] \wedge [y \in A] \rightarrow [x + y \in A]$.

(F6b) For any $x, y \in R, \models [y \in A] \rightarrow [xy \in A]$ *(resp.* $\models [x \in A] \rightarrow [xy \in A]$*).*

(F6c) For any $x, z, a, b \in R$ *with* $x + a + z = b + z, \models [a \in A] \wedge [b \in A] \rightarrow [x \in A]$.

Clearly, Definition 12 is equivalent to Definition 7. Therefore a fuzzified left (resp. right) h-ideal is a Rosenfeld's fuzzy left (resp. right) h-ideal.

In[29], the concept of t-tautology is introduced, i.e., $\models_t P$ if and only if $[P] \geq t$.

We next extend the concept of implication-based fuzzy left (resp. right) h-ideal in the following way:

Definition 13. *Let A be a fuzzy set of R and* $t \in (0, 1]$ *is a fixed number. A is called a t-implication-based fuzzy left (resp. right) h-ideal of R if the following conditions hold:*

(F7a) For any $x, y \in R, \models_t [x \in A] \wedge [y \in A] \rightarrow [x + y \in A]$.

(F7b) For any $x, y \in R, \models_t [y \in A] \rightarrow [xy \in A]$ *(resp.* $\models_t [x \in A] \rightarrow [xy \in A]$*).*

(F7c) For any $x, z, a, b \in R$ *with* $x + a + z = b + z, \models_t [a \in A] \wedge [b \in A] \rightarrow [x \in A]$.

Now, if I is an implication operator, then we deduce the following corollary:

Corollary 4. *A fuzzy set A of R is a t-implication-based fuzzy left (resp. right) h-ideal of R if and only if it satisfies the following conditions:*

(F8a) For any $x, y \in R, I(A(x) \wedge A(y), A(x + y)) \geq t$.

(F8b) For any $x, y \in R, I(A(y), A(xy)) \geq t$ *(resp.* $I(A(x), A(xy)) \geq t$*).*

(F8c) For any $x, z, a, b \in R$ *with* $x + a + z = b + z, I(A(a) \wedge A(b), A(x)) \geq t$.

Theorem 13

(i) Let $I = I_{gr}$. *Then A is an* $(\in, \in \vee q_{(\lambda,\mu)})$*-fuzzy left (resp. right) h-ideal of R if and only if for some* $t \in (0, 1]$, *it satisfies the following conditions:*

(F9a) For any $x, y \in R, I(A(x) \wedge A(y) \wedge \mu, A(x + y) \vee \lambda) \geq t$.

(F9b) For any $x, y \in R, I(A(y) \wedge \mu, A(xy) \vee \lambda) \geq t$ *(resp.* $I(A(x) \wedge \mu, A(xy) \vee \lambda) \geq t$*).*

(F9c) For any $x, z, a, b \in R$ *with* $x + a + z = b + z, I(A(a) \wedge A(b) \wedge \mu, A(x) \vee \lambda) \geq t$.

(ii) Let $I = I_g$. *Then A is an* $(\in, \in \vee q_{(\lambda,\mu)})$*-fuzzy left (resp. right) h-ideal of R if and only it satisfies the following conditions:*

(F10a) For any $x, y \in R, I(A(x) \wedge A(y), A(x + y) \vee \lambda) \geq \mu$.

(F10b) For any $x, y \in R, I(A(y), A(xy) \vee \lambda) \geq \mu$ *(resp.* $I(A(x), A(xy) \vee \lambda) \geq \mu$*).*

(F10c) For any $x, z, a, b \in R$ *with* $x + a + z = b + z, I(A(a) \wedge A(b), A(x) \vee \lambda) \geq \mu$*.*

(iii) Let $I = I_{cg}$*. Then* A *is an* $(\in, \in \vee q_{(\lambda,\mu)})$*-fuzzy left (resp. right)* h*-ideal of* R *if and only if it satisfies the following conditions:*

(F11a) For any $x, y \in R, I(A(x) \wedge A(y) \wedge \mu, A(x + y)) \geq 1 - \lambda$*.*

(F11b) For any $x, y \in R, I(A(y) \wedge \mu, A(xy)) \geq 1 - \lambda$ *(resp.* $I(A(x) \wedge \mu, A(xy)) \geq 1 - \lambda$*).*

(F11c) Ffor any $x, z, a, b \in R$ *with* $x + a + z = b + z, I(A(a) \wedge A(b) \wedge \mu, A(x)) \geq 1 - \lambda$*.*

Proof. We only prove $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left h-ideal of R because the proofs of $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy right h-ideal of R is similar.

Next we only prove (ii) because the proofs of (i) and (iii) are similar.

(ii) Let A be an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left h-ideal of R. We have

(F3a) $A(x + y) \vee \lambda \geq A(x) \wedge A(y) \wedge \mu$.

(F3b) $A(xy) \vee \lambda \geq A(y) \wedge \mu$.

(F3c) $x + a + z = b + z$ implies $A(x) \vee \lambda \geq A(a) \wedge A(b) \wedge \mu$.

From (F3a), if $A(x) \wedge A(y) \wedge \mu = A(x) \wedge A(y)$, then $I_g(A(x) \wedge A(y), A(x + y) \vee \lambda) = 1 \geq \mu$. Otherwise, $I_g(A(x) \wedge A(y), A(x + y) \vee \lambda) = 1 \geq \mu$.

(F10b), (F10c) can be proved similarly.

Conversely, if we have

(F10a) $I_g(A(x) \wedge A(y), A(x + y) \vee \lambda) \geq \mu$,

(F10b) $I_g(A(y), A(xy) \vee \lambda) \geq \mu$,

(F10c) $I_g(A(a) \wedge A(b), A(x) \vee \lambda) \geq \mu$ with $x + a + z = b + z$.

From (F10a), we have $A(x + y) \vee \lambda \geq A(x) \wedge A(y)$ or $A(x) \wedge A(y) > A(x + y) \vee \lambda \geq \mu$, which implies $A(x + y) \vee \lambda \geq A(x) \wedge A(y) \wedge \mu$.

Similarly, we can prove that the conditions (F3b) and (F3c) hold.

Therefore, A is $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left h-ideal of R.

6 Conclusion

In study the structure of a fuzzy algebraic system, we notice that fuzzy ideals with special properties always play an important role. In this paper, we consider the the definitions of $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left (resp. right) h-ideals of hemirings and generalized fuzzy left (resp. right) h-ideals of hemirings.The equivalence relationship between them is described. Using level subsets, intersection and union of fuzzy subsets, the equivalent conditions and the properties of $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left (resp. right) h-ideals are investigated. Otherwise, the properties of homomorphic image and homomorphic preimage in a homomorphism of hemirings are shown. Some characterization theorems of prime and semiprime $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left (resp. right) h-ideals of hemirings are obtained. Finally, we also consider the implication-based left (resp. right) h-ideals of hemirings.

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Several Equivalent Conditions of Fuzzy Subgroups of Some Groups*

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Abstract. By considering the generators of groups, we give two equivalent conditions of fuzzy subgroups of groups and we obtain some useful corollaries by them. Based on these corollaries we construct the structures of fuzzy subgroups of some groups.

Keywords: Fuzzy subgroup, generator, cyclic group.

1 Introduction

That the structure of fuzzy subgroups can be given clearly is one of the main goals about the study of fuzzy subgroups. Some authors try to determine the number of fuzzy subgroups of certain finite Abelian groups with respect to a suitable equivalence relation (see [3]-[6]). This is motivated by the realization that in a theoretical study of fuzzy groups, fuzzy subgroups are distinguished by their level sets and not by their images in $[0,1]$. And some authors show that the structure of fuzzy subgroups of a group G can be characterized clearly by the structure of G (see [7]-[9]). In this paper, by considering the generators of groups, we give two equivalent conditions of fuzzy subgroups. The structures of fuzzy subgroups of some groups can be characterized clearly by these equivalent conditions.

2 Several Equivalent Conditions of Fuzzy Subgroups of Some Groups

Throughout this paper, we shall denote a group by X, the identity of X by e, and a fuzzy subgroup of X by A. Let Y be a nonempty subset of a group X.

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The subgroup generated by Y is denoted by $\langle Y\rangle$. We write $\wedge$ for minimum or infimum and $\vee$ for maximum or supremum.

Definition 1. [2] *Let X be a group. A fuzzy subset A of X is called a fuzzy subgroup of X if*

(1) $A(xy) \geq A(x) \wedge A(y), x, y \in X$;
(2) $A(x^{-1}) \geq A(x), x \in X$.

From the perspective of group generators, we shall give two equivalent conditions of fuzzy subgroups of a group.

Theorem 1. *Let X be a group. Then the following assertions are equivalent:*

(1) *A is a fuzzy subgroup of X.*

(2) *Let H, K be subgroups of X. If $H \leq K$ and there exist $k_1, k_2, \cdots, k_s \in K$ such that $K = \langle k_1, k_2, \cdots, k_s\rangle$, then $A(h) \geq A(k_1) \wedge A(k_2) \wedge \cdots \wedge A(k_s)$ for all $h \in H$.*

(3) *Let H be a subgroup of X. If there exist $h_1, h_2, \cdots, h_r \in H$ and $k_1, k_2, \cdots, k_s \in H$ such that $H = \langle h_1, h_2, \cdots, h_r\rangle = \langle k_1, k_2 \cdots k_s\rangle$, then*

$$A(h_1) \wedge A(h_2) \wedge \cdots \wedge A(h_r) = A(k_1) \wedge A(k_2) \wedge \cdots \wedge A(k_s).$$

Proof. (1) $\Rightarrow$ (2) Let H, K be subgroups of X. Suppose that $H \leq K$ and there exist $k_1, k_2, \cdots, k_s \in K$ such that $K = \langle k_1, k_2, \cdots, k_s\rangle$. For every $h \in H$, by $H \leq K$, we have $h \in K$. Since $K = \langle k_1, k_2, \cdots, k_s\rangle$, it follows that $\exists l_1, l_2, \cdots, l_s \in Z$ such that $h = k_{i1}^{l_1} k_{i2}^{l_2} \cdots k_{is}^{l_s}$ where $i1, i2, \cdots, is \in \{1, 2, \cdots, s\}$ and $im \neq in$ when $m \neq n$, where $m, n \in \{1, 2, \cdots, s\}$. Hence, by Definition 1, we have $A(h) = A(k_{i1}^{l_1} k_{i2}^{l_2} \cdots k_{is}^{l_s}) \geq A(k_{i1}^{l_1}) \wedge A(k_{i2}^{l_2}) \wedge \cdots \wedge A(k_{is}^{l_s}) \geq A(k_{i1}) \wedge A(k_{i2}) \wedge \cdots \wedge A(k_{is}) = A(k_1) \wedge A(k_2) \wedge \cdots \wedge A(k_s)$. So (2) holds.

(2) $\Rightarrow$ (3) Let H be a subgroup of X. Assume that there exist $h_1, h_2, \cdots, h_r \in H$ and $k_1, k_2, \cdots, k_s \in H$ such that $H = \langle h_1, h_2, \cdots, h_r\rangle = \langle k_1, k_2 \cdots k_s\rangle$. Then since $\langle h_1, h_2, \cdots, h_r\rangle = \langle k_1, k_2 \cdots k_s\rangle$, we have that $\langle h_1, h_2, \cdots, h_r\rangle \leq \langle k_1, k_2 \cdots k_s\rangle$ and $\langle h_1, h_2, \cdots, h_r\rangle \geq \langle k_1, k_2 \cdots k_s\rangle$. On the one hand, by $\langle h_1, h_2, \cdots, h_r\rangle \leq \langle k_1, k_2 \cdots k_s\rangle$ and assertion (2), we have $A(h_i) \geq A(k_1) \wedge A(k_2) \wedge \cdots \wedge A(k_s)$ for all $h_i \in H, i = 1, 2, \ldots, r$. Thus $A(h_1) \wedge A(h_2) \wedge \cdots \wedge A(h_r) \geq A(k_1) \wedge A(k_2) \wedge \cdots \wedge A(k_s)$. On the other hand, similarly, by $\langle h_1, h_2, \cdots, h_r\rangle \geq \langle k_1, k_2 \cdots k_s\rangle$ and assertion (2), we have that $A(k_1) \wedge A(k_2) \wedge \cdots \wedge A(k_s) \geq A(h_1) \wedge A(h_2) \wedge \cdots \wedge A(h_r)$. Hence $A(h_1) \wedge A(h_2) \wedge \cdots \wedge A(h_r) = A(k_1) \wedge A(k_2) \wedge \cdots \wedge A(k_s)$. So (3) holds.

(3) $\Rightarrow$ (1) Clearly, $\langle x^{-1}\rangle = \langle x\rangle$ for all $x \in X$. Therefore by assertion (3), we conclude that $A(x^{-1}) = A(x)$ for any $x \in X$. Further, clearly, $\langle xy, x\rangle = \langle x, y\rangle$ for all $x, y \in X$. Therefore by assertion (3), we conclude $A(xy) \wedge A(x) = A(x) \wedge A(y)$. Hence $A(xy) \geq A(xy) \wedge A(x) = A(x) \wedge A(y)$. Therefore by Definition 1, we conclude that A is a fuzzy subgroup of X. So (1) holds.

Corollary 1. *Let X be a group. A is a fuzzy subgroup of X if and only if for all $x_1, y_1, x_2, y_2 \in X$ if $\langle x_1, y_1\rangle = \langle x_2, y_2\rangle$, then $A(x_1)\wedge A(y_1) = A(x_2)\wedge A(y_2)$.*

Proof. The necessity follows from Theorem 1. The same to the (3) $\Rightarrow$ (1) of Theorem 1 analysis the sufficiency can be obtained.

Corollary 2. *Let X be an Abelian group and any element of X be finite order. Then the following assertions are equivalent:*

(1) *A is a fuzzy subgroup of X.*
(2) *If $(k - l, o(x)) = 1$ or $(k - l, o(y)) = 1$, then $A(x) \wedge A(y) = A(xy) \wedge A(x^k y^l)$ for all $x, y \in X, k, l \in Z$.*
(3) *If $(l, o(y)) = 1$, then $A(xy) \wedge A(y) = A(x) \wedge A(x^k y^l)$ for all $x, y \in X, k, l \in Z$.*
(4) *If $(l, o(x)) = 1$, then $A(xy) \wedge A(x) = A(y) \wedge A(x^k y^l)$ for all $x, y \in X, k, l \in Z$.*
(Notation: $o(x), o(y)$ for the order of x, y)

Proof. (1) $\Rightarrow$ (2) Let $x, y \in X$, $k, l \in Z$ and $(k-l, o(x)) = 1$ or $(k-l, o(y)) = 1$. Now we show that $A(x) \wedge A(y) = A(xy) \wedge A(x^k y^l)$.

We may assume that $(k-l, o(x)) = 1$. Since $(xy)^{-l}x^k y^l = x^{k-l} \in \langle xy, x^k y\rangle$ and $(k - l, o(x)) = 1$, it follows that $\langle x^{k-l}\rangle = \langle x\rangle$. Hence $x \in \langle xy, x^k y^l\rangle$ and so $x^{-1}(xy) = y \in \langle xy, x^k y^l\rangle$. Consequently, $\langle x, y\rangle \leq \langle xy, x^k y^l\rangle$. It is clear that $\langle xy, x^k y^l\rangle \leq \langle x, y\rangle$. Hence $\langle xy, x^k y^l\rangle = \langle x, y\rangle$. By Theorem 1, we have $A(xy) \wedge A(x^k y^l) = A(x) \wedge A(y)$.

(1) $\Leftarrow$ (2) Taking $k = 1, l = 0, y = e$. Since $(k - l, o(y)) = (1, 1) = 1$, by assertion (2) it follows that $A(x) \wedge A(e) = A(xe) \wedge A(x^1 e^0)$ for all $x \in X$. Namely, $A(x) \wedge A(e) = A(x) \wedge A(x) = A(x)$. Thus we have $A(e) \geq A(x) \wedge A(e) = A(x)$ for all $x \in X$. That is, $\forall x \in X$, $A(e) \geq A(x)$.

Taking $k = -1, l = 0, y = e$. Since $(k - l, o(y)) = (-1, 1) = 1$, by assertion (2) it follows that $A(x) \wedge A(e) = A(xe) \wedge A(x^{-1}e^0)$ for all $x \in X$. Namely, $A(x) \wedge A(e) = A(x^{-1}) \wedge A(x)$. By the proof from the previous paragraph, we know $A(e) \geq A(x)$. Hence $A(x) = A(x) \wedge A(e) = A(x^{-1}) \wedge A(x)$ and so $A(x^{-1}) \geq A(x^{-1}) \wedge A(x) = A(x)$. Thus $A(x^{-1}) \geq A(x)$ for all $x \in X$.

Since x and y are finite order for all $x, y \in X$, we denote the order of x by $o(x)$. Taking $k = 2, l = 1$. Clearly $(k - l, o(x)) = (1, o(x)) = 1$. By assertion (2), we have $A(x)\wedge A(y) = A(xy)\wedge A(x^2 y^1)$. Consequently, $A(xy) \geq A(xy) \wedge A(x^2 y^1) = A(x) \wedge A(y)$. We have proved that $A(xy) \geq A(x) \wedge A(y)$ for all $x, y \in X$.

Consequently, by Definition 1, A is a fuzzy subgroup of X. So (1) holds.

The same to the above analysis (1) $\Leftrightarrow$ (3) and (1) $\Leftrightarrow$ (4) can be obtained.

Corollary 3. *Let X be an Abelian group, A be a fuzzy subgroup of X, $x, y \in X$ and they be of finite order. If $A(x) = A(y) < A(xy)$, then we conclude that if $(k - l, o(x)) = 1$ or $(k - l, o(y)) = 1$, then $A(x^k y^l) = A(x) = A(y)$ for all $k, l \in Z$.*

Proof. By Corollary 2 we have $A(xy) \wedge A(x^k y^l) = A(x) \wedge A(y)$. Since $A(x) = A(y)$, it follows that

$$A(xy) \wedge A(x^k y^l) = A(x) \wedge A(y) = A(x) = A(y).$$

If $A(xy) \wedge A(x^k y^l) = A(xy)$, then we have $A(xy) \wedge A(x^k y^l) = A(xy) = A(x) = A(y)$, which contradicts the condition $A(x) = A(y) < A(xy)$. Hence $A(xy) \wedge A(x^k y^l) = A(x^k y^l)$. So $A(x^k y^l) = A(x) = A(y)$.

Corollary 4. *Let X be a group and A be a fuzzy subgroup of X. If $(n_1, n_2, \cdots, n_k) = d$ for all $n_1, n_2, \cdots, n_k \in Z$, then $A(x^d) = A(x^{n_1}) \wedge A(x^{n_2}) \wedge \cdots \wedge A(x^{n_k})$.*

Proof. Since $(n_1, n_2, \cdots, n_k) = d$, it follows that $\langle x^d \rangle = \langle x^{n_1}, x^{n_2}, \cdots, x^{n_k} \rangle$ for all $x \in X$. By Theorem 1, we have $A(x^d) = A(x^{n_1}) \wedge A(x^{n_2}) \wedge \cdots \wedge A(x^{n_k})$.

Corollary 5. $^{[1]lemma2.1.1}$ *Let X be a group and A be a fuzzy subgroup of X. If the element x of X is of n order, then we have $A(x^k) = A(x^d)$ for all $k \in Z$, where $d = (k, n)$.*

Proof. Since $d = (k, n)$, it follows by Corollary 4 that $A(x^d) = A(x^k) \wedge A(x^n) = A(x^k) \wedge A(e) = A(x^k)$ (Since A is a fuzzy subgroup, it follows that $\forall x \in X, A(e) \geq A(x)$). Hence $A(x^k) = A(x^d)$.

It is useful for these corollaries to study fuzzy subgroups of finitely generated groups. Now we construct the structure of fuzzy subgroups of particular groups which is finitely generated in terms of these corollaries.

Theorem 2. *Let X be a group and $X = Z_p \times Z_p$, where p is a prime. Then A is a fuzzy subgroup of X if and only if A is one of the following types:*

(1) $\forall x, y \in X$, $A(x) = A(y) \leq A(e)$, where e is an identity element of X;
(2) $\exists x, y \in X$, such that $X = \langle x, y \rangle$ and

$$A(x^k y^l) = \begin{cases} \lambda, & k \neq l, \\ \mu, & k = l \neq p. \end{cases}$$

for all $k, l \in [1, p]$, where $\lambda, \mu \in [0, 1], \mu > \lambda$ and $A(e) \geq \mu$.

Proof. Since $X = Z_p \times Z_p$ where p is a prime, it follows that any element of X is order p. This conclusion will be applied in the following proof.

We first prove the necessity. Taking $x_1, x_2 \in Z_p \times Z_p = X$, such that $X = \langle x_1, x_2 \rangle$. Clearly, $\langle x_1, x_2 \rangle = \langle x_1 x_2, x_2 \rangle = \langle x_1 x_2, x_1 \rangle = X$. Now we show that there must be two membership degrees equal in $A(x_1), A(x_2)$, and $A(x_1 x_2)$. Assume that $A(x_1) = A(x_2)$, then the result obtains. We may assume that $A(x_1) > A(x_2)$, then $A(x_1 x_2) = A(x_2)$ by Theorem 2.1.6 of [1]. So there must be two numbers equal in in $A(x_1), A(x_2)$, and $A(x_1 x_2)$.

In the preceding segment we obtain that there are two elements(we may assume that x,y) in X such that $X = \langle x, y\rangle$ and $A(x) = A(y)$. We write $A(x) = A(y) = \lambda$. By Definition 1 $A(xy) \geq A(x) \wedge A(y)$. If we write $A(xy) = \mu$, then $\mu \geq \lambda$.

(i) Assume that $\mu > \lambda$. Namely $\lambda = A(x) = A(y) < A(xy) = \mu$. We have $k - l \in [1 - p, p - 1]$ for all $k, l \in [1, p]$. If $k \neq l$, namely $k - l \neq 0$, then $(k - l, p) = 1$. Since $o(x) = p$, it follows that $(k - l, o(x)) = 1$. By Corollary 3 $A(x^k y^l) = A(x) = A(y) = \lambda$. If $k = l \neq p$, namely $k = l \in [1, p - 1]$, then $(k, p) = 1$. And since $o(xy) = p$, it follows from Corollary 5 that $A(x^k y^l) = A((xy)^k) = A(xy) = \mu$.

By definition 1, $A(e) = A((xy)^p) \geq A(xy) = \mu$. Consequently, when $\mu > \lambda$, A is the type (2).

(ii) Assume that $\mu = \lambda$. That is, $A(x) = A(y) = A(xy) = \lambda$. Since x, y and xy are all order p, then, by Corollary 5, we have $A(x^k) = A(y^l) = A((xy)^r) = \lambda$ for all $k, l, r \in [1, p-1]$. Assume that $A(x^k y^l) = \lambda$ for all $k, l \in [1, p-1]$, then A is the type (1). Assume that $\exists k_0, l_0 \in [1, p - 1]$ such that $A(x^{k_0} y^{l_0}) \neq \lambda$. Since $A(x^{k_0} y^{l_0}) \geq A(x^{k_0}) \wedge A(y^{l_0}) = \lambda$, it follows that $A(x^{k_0} y^{l_0}) > \lambda$. We may write $A(x^{k_0} y^{l_0}) = \mu$, namely, $\mu > \lambda$. Clearly $\langle x^{k_0}, y^{l_0}\rangle = \langle x, y\rangle = X$, and we know $A(x^{k_0}) = A(y^{l_0}) = \lambda$ by the preceding proof. We can write $x_0 = x^{k_0}, y_0 = y^{l_0}$, then $X = \langle x_0, y_0\rangle$, $A(x_0) = A(y_0) = \lambda, A(x_0 y_0) = \mu$ and $\mu > \lambda$. So the (i) argument may be applied to the $X = \langle x_0, y_0\rangle$. Hence A is now the type (2).

Consequently, the necessity holds.

Conversely, assume that A is the type (1). it is clear that A is a fuzzy subgroup of X.

Assume that A is the type (2). Since X is a finite group, it follows that we need only prove $A(ab) \geq A(a) \wedge A(b)$ for all $a, b \in X$ by Theorem 2.1.7 of [1]. Since X is an Abelian group and $X = \langle x, y\rangle$, then $\exists k_1, l_1, k_2, l_2 \in [1, p]$ for all $a, b \in X$ such that $a = x^{k_1} y^{l_1}, b = x^{k_2} y^{l_2}$. Hence $A(ab) = A(x^{k_1} y^{l_1} x^{k_2} y^{l_2}) = A(x^{k_1+k_2} y^{l_1+l_2})$. By the type (2) we have $A(ab) = A(x^{k_1+k_2} y^{l_1+l_2}) \geq \lambda$.

Assume that $k_1 \neq l_1$ or $k_2 \neq l_2$; then $A(a) = A(x^{k_1} y^{l_1}) = \lambda$ or $A(b) = A(x^{k_2} y^{l_2}) = \lambda$ by the type (2); namely, $\lambda \geq A(a) \wedge A(b)$. Hence $A(ab) \geq \lambda \geq A(a) \wedge A(b)$.

Assume that $k_1 = l_1$ and $k_2 = l_2$. Discussion of two cases (i) $k_1 = l_1 = p$ or $k_2 = l_2 = p$; We may assume that $k_1 = l_1 = p$, then $a = x^{k_1} y^{l_1} = (xy)^p = e$.Hence $ab = b$. Thus $A(ab) = A(b) \geq A(a) \wedge A(b)$. $(ii) k_1 = l_1 \neq p$ and $k_2 = l_2 \neq p$. By the type (2) we have $A(a) = A(x^{k_1} y^{l_1}) = \mu, A(b) = A(x^{k_2} y^{l_2}) = \mu$. Now since $k_1 + k_2 = l_1 + l_2$, it follows that $A(ab) = A(x^{k_1+k_2} y^{l_1+l_2}) = A((xy)^{k_1+k_2})$. Assume that $p|(k_1 + k_2)$, then $A(ab) = A((xy)^{k_1+k_2}) = A(e)$. By the type (2), $A(e) \geq \mu$, hence $A(ab) \geq \mu = A(a) \wedge A(b)$. Or else $(p, k_1 + k_2) = 1$, then we have $\exists s \in Z, r \in [1.p-1]$ such that $k_1+k_2 = sp+r$ by division with remainder. Hence $A(ab) = A((xy)^{k_1+k_2}) = A((xy)^{sp+r}) = A((xy)^r) = \mu$ by (2). Thus $A(ab) = A(a) \wedge A(b) = \mu$. Consequently, we can always have $A(ab) \geq A(a) \wedge A(b)$ for all $a, b \in X$. This finishes the proof of the sufficiency.

Example. *Let* $K_4 = \{e, a, b, c\}$ *is a Klein 4-group, where* $a^2 = b^2 = c^2 = e, ab = ba = c, ac = ca = b, bc = cb = a$ *and* e *is an identity. If we do not distinguish between a group and groups that are isomorphic to it, then* $K_4 = Z_2 \times Z_2$. *We can completely list the fuzzy subgroups of* K_4*:*

(1) $A(e) \geq A(c) \geq A(a) = A(b)$;
(2) $A(e) \geq A(b) \geq A(a) = A(c)$;
(3) $A(e) \geq A(a) \geq A(b) = A(c)$;

There is only letters difference in these three cases. They are essentially the same. So we only discuss case (1). *Clearly,* A *is one of the following types:*

type 1: $A(e) \geq A(c) = A(a) = A(b)$, *namely:* $A(e) \geq A(ab) = A(a) = A(b)$;
type 2: $A(e) \geq A(c) > A(a) = A(b)$, *namely:* $A(e) \geq A(ab) > A(a) = A(b)$;
This is consistent with our Theorem 2.

Theorem 3. *Let* X *be a cyclic group and* $X = \langle x \rangle$. *Then* A *is a fuzzy subgroup of* X *if and only if for all* $m, n \in Z$ *if* $(m, n) = d$, *then* $A(x^d) = A(x^m) \wedge A(x^n)$.

Proof. By Corollary 4 the necessity holds.
We shall prove the sufficiency.

(i) We shall prove $A((x^k)^{-1}) \geq A(x^k)$ for all $k \in Z$.
Since $(-k, 0) = k$, it follows by Corollary 4 that $A(x^k) = A(x^{-k}) \wedge A(x^0) = A(x^{-k}) \wedge A(e)$. And since $A(x^{-k}) \geq A(x^{-k}) \wedge A(e)$, it follows that $A(x^{-k}) \geq A(x^k)$. So $A((x^k)^{-1}) \geq A(x^k)$.
(ii) We shall prove $A(x^m x^n) \geq A(x^m) \wedge A(x^n)$ for all $m, n \in Z$.
We write $d = (m, n)$. Hence $(m + n, d) = d$. By the condition we have $A(x^d) = A(x^m) \wedge A(x^n)$ and $A(x^d) = A(x^{m+n}) \wedge A(x^d)$. Hence $A(x^{m+n}) \wedge A(x^d) = A(x^m) \wedge A(x^n)$. Since $A(x^m x^n) = A(x^{m+n}) \geq A(x^{m+n}) \wedge A(x^d)$, it follows that $A(x^m x^n) \geq A(x^m) \wedge A(x^n)$.
Since X is cyclic, it follows that we can write $x^k, k \in Z$ for any element of X where $X = \langle x \rangle$. Hence by the above proof and Definition 1, A is a fuzzy subgroup of X. This finishes the proof of the sufficiency.

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The Characters of the Complex Number-Valued Fuzzy Measurable Function

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Abstract. Reference [1] through an example pointed out some defects in reference [2] of fuzzy measurable function,and department a new definition, so many characters of the number-valued measurable function could be inherit, the paper based on this new definition are given complex number-valued fuzzy measurable function and complex fuzzy number-valued fuzzy measurable function, and focus on the transmissibility of basic character by complex number-valued fuzzy measurable function's new definition.

Keywords: Fuzzy, measurable function, number-valued measurable, complex number-valued fuzzy measurable.

1 Preliminary

Definition 1. *A fuzzy -algebra is a class of some fuzzy subsets on* X*, denoted by* $\mathscr{F}(\ \ \mathscr{F}(\mathrm{X}))$*, and satisfy the following nature.*

(i) $\emptyset\ \mathrm{X}\ \ \mathscr{F}$,
(ii) If $E\ \ \mathscr{F}$*, then* $E^c\ \ \mathscr{F}$,
(iii) If $E_n\ \ \mathscr{F}$ *then* $\bigcup_{n\ 1}^{\infty} E_n\ \ \mathscr{F}$.

Call the $(\mathrm{X}\ \mathscr{F})$ *be fuzzy measurable space,where* $\mathscr{F}$ *is a fuzzy -algebra.*
Apparently fuzzy -algebra is closed by $\bigcup$.

Definition 2. $(\mathrm{X}\ \mathscr{F})$ *be a fuzzy measurable space.* $\mathrm{E}\ \ \mathscr{F}$.

(i) We call the mapping $f\ :\ \mathrm{X}\ \ \ [\ \infty\ \ \infty]$ *be the real-number fuzzy measurable function on* E*(by* $(\mathrm{X}\ \mathscr{F})$*),if every* $[\ \infty\ \ \infty]$ $\mathrm{E}\ \ _{\mathrm{F}}\ \ \mathscr{F}$ *and* $\mathrm{E}\ \ ^{c}_{\mathrm{F}}\ \ \mathscr{F}$*.then* $\mathrm{F}\ \ \ x\ \ \mathrm{X}\, f(x)$,$\mathrm{F}-\ \ \ x\ \ \mathrm{X}\, f(x)$.
(ii) We call the mapping $f\ :\ \mathrm{X}\ \ \ \mathrm{F}\ (\mathrm{R})$ *be the fuzzy-number fuzzy measurable function on* E*(by* $(\mathrm{X}\ \mathscr{F})$*),if every* $[0\ \ 1]$*,*$f_{\lambda}(x)$ *and* $f_{\lambda}(x)$ *both are real-number fuzzy measurable function on* E*(by* $(\mathrm{X}\ \mathscr{F})$*).where* $f(x)\ \ \ \bigcup_{\lambda\ [0\ 1]}\ [f(x)_{\lambda}\ \ f(x)_{\lambda}] \triangleq$

$[f_{\lambda}(x)\ \ f_{\lambda}(x)]$.

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Definition 3. *(i) We call the function* $f(x)$ $g(x)$ $ih(x)$ *be the complex number function on* X,*if* $f(x)$ $g(x)$ *are real-number functions,where* F x X $g(x)$ $h(x)$ β.
(ii) We call the function $f(x)$ $g(x)$ $ih(x)$ *be the complex fuzzy number function, if* $g(x)$ $h(x)$ *are fuzzy number functions.* $f(x)$ $[g_\lambda(x)\ g_\lambda(x)]$ i $[h_\lambda(x)\ h_\lambda(x)]$.

Definition 4. *(i) We call the function* $f(x)$ $g(x)$ $ih(x)$ *be the complex-number fuzzy measurable function on* E,*if* $g(x)$ $h(x)$ *both are the real-number fuzzy measurable function on* E.
(ii) We call the function $f(x)$ $g(x)$ $ih(x)$ *be the complex fuzzy number measurable function on* E,*if every* $[0\ 1]$ $g_\lambda(x)$ $g_\lambda(x)$ $h_\lambda(x)$ $h_\lambda(x)$ *both are real-number fuzzy measurable function on* E.

We use the special notation CFM(E) to mean the total of complex-number fuzzy measurable functions,and CFM(E) to mean the total of complex fuzzy number measurable functions.

2 The Nature of Complex-Number Fuzzy Measurable Function

Theorem 1. *The follow sentences are equivalence*

(i) f CFM(E)*;*
(ii) β $[\ \infty\ \ \infty], E_F$ $\mathscr{F}, E^c_F$ $\mathscr{F}$*;*
(iii) β $[\ \infty\ \ \infty], E_{F\_-}$ $\mathscr{F}, E^c_{F\_-}$ $\mathscr{F}$*;*
(iv) $a\ b\ c\ d$ $[\ \infty\ \ \infty], E_{x\,a\ g(x)\ b\,c\ h(x)\ d}$ $\mathscr{F}$.

Proof. (i) (ii)
From the Definition 1.3, we know E_F $\mathscr{F}, E_F$ $\mathscr{F}$, so E_F (E_F) (E_F) $\mathscr{F}$ and because E^c_F $\mathscr{F}, E^c_F$ $\mathscr{F}$, so E^c_F E $(^c_F \bigcup {}^c_F)$ $(E^c_F) \bigcup E^c_F$ $\mathscr{F}$.

(ii) (iii)

$$E_{F\_-}\quad E \bigcup_{n\ 1}^{\infty} F_{\frac{1}{n}\ \frac{1}{n}} \quad \bigcup_{n\ 1}^{\infty} E_{F_{\frac{1}{n}\ \frac{1}{n}}};$$

$$E^c_{F\_-}\quad E \bigcup_{n\ 1}^{\infty} F_{\frac{1}{n}\ \frac{1}{n}} \quad {}^{c}{}_{n\ 1}^{\infty} E \bigcup {}^c_{F_{\frac{1}{n}\ \frac{1}{n}}};$$

n $1\ 2\ 3\ 4$ $E_{F_{\frac{1}{n}\ \frac{1}{n}}}$ F $E^c_{F_{\frac{1}{n}\ \frac{1}{n}}}$ F, so E_{F_-} $\mathscr{F}, E^c_{F\_-}$ $\mathscr{F}$.

(iii) (iv)
If you discover $_{x\,a\ g(x)\ b\,c\ h(x)\ d}$ $F^c_{b\,d}$ $F^c_{a\,c}$,then this proof is very easy,and we omit it.

(iv) (i)

$$E \quad _{F} \quad E \quad \bigcup_{n\ 1}^{\infty} \quad x\,a \quad f(x) \quad a \quad n \quad \bigcup_{n\ 1}^{\infty} E \quad x\,a \quad f(x) \quad a \quad n \quad ;$$

$$E \quad _{F}^{c} \quad E \quad \bigcup_{n\ 1}^{\infty} \quad x\,a \quad f(x) \quad a \quad n \quad ^{c} \quad E \quad _{n\ 1}^{\infty} \quad ^{c}x\,a \quad f(x) \quad a \quad n \quad _{n\ 1}^{\infty} \quad E \quad ^{c}x\,a \quad f(x) \quad a \quad n \quad .$$

So $E \quad _{F} \quad \mathscr{F}, E \quad _{F}^{c} \quad \mathscr{F}$, and $f \quad \text{CFM}(E)$.

Then Theorem 1 prove over.

Consequence 2.1 *The following condition are equivalence*

(i) $f \quad \text{CFM}(E)$;

(ii) $[0\ 1] \quad [0\ 1] \quad \beta \quad (\quad \infty \quad \infty)$, *then*

$E \quad _{G_{\lambda}-} \quad \mathscr{F}, E \quad _{G_{\lambda}-}^{c} \quad \mathscr{F}, E \quad _{G_{\lambda}-} \quad \mathscr{F}, E \quad _{G_{\lambda}-}^{c} \quad \mathscr{F};$

$E \quad _{H\ -} \quad \mathscr{F}, E \quad _{H\ -}^{c} \quad \mathscr{F}, E \quad _{H\ -} \quad \mathscr{F}, E \quad _{H\ -}^{c} \quad \mathscr{F},$

$G_{\lambda}- \quad x\,g_{\lambda}(x) \quad ; G_{\lambda}- \quad x\,g_{\lambda}(x) \quad ; H\ - \quad x\,h\ (x) \quad ; H\ - \quad x\,h\ (x) \quad ;$

(iii) $[0\ 1] \quad [0\ 1] \quad _{1} \quad _{2}\beta_{1}\beta_{2} \quad (\quad \infty \quad \infty)$,

$E \quad _{x \quad 1 \quad g_{\lambda}(x) \quad 2 \quad 1 \quad g_{\lambda}(x) \quad 2 \quad 1 \quad h\ (x) \quad 2 \quad 1 \quad h\ (x) \quad 2} \quad \mathscr{F};$

$E \quad ^{c}_{x \quad 1 \quad g_{\lambda}(x) \quad 2 \quad 1 \quad g_{\lambda}(x) \quad 2 \quad 1 \quad h\ (x) \quad 2 \quad 1 \quad h\ (x) \quad 2} \quad \mathscr{F}.$

Theorem 2. *if* $f \quad \text{CFM}(E)$, *then* $\beta \quad [\quad \infty \quad \infty], E \quad _{x\,g(x) \quad h(x)} \quad \mathscr{F}$ *and* $E \quad ^{c}_{x\,g(x) \quad h(x)} \quad \mathscr{F}$.

Proof. since $\beta \quad (\quad \infty \quad \infty), E \quad _{x\,g(x) \quad h(x)} \quad E \left(\quad _{F} \quad _{F_{--}} \quad \left(E \quad _{F} \quad \left(E \quad ^{c}_{F_{--}} \quad ; \right.\right.\right.$

$E \quad ^{c}_{\{x\,g(x)= \quad h(x)=} \quad E \left(\quad _{F} \quad _{F_{--}} \right)^{c} \quad E \left(\quad ^{c}_{F} \quad _{F_{--}} \quad \left(E \quad ^{c}_{F} \quad \left(E \quad _{F_{--}} \right.\right.\right. ,$

Because $f \quad \text{CFM}(E)$, so $\left(E \quad _{F} \quad \mathscr{F}, \left(E \quad ^{c}_{F_{--}} \quad \mathscr{F}, \left(E \quad ^{c}_{F} \quad \mathscr{F}, \left(E \quad _{F_{--}} \quad \mathscr{F}. \right.\right.\right.\right.$

then $E \quad _{x\,g(x) \quad h(x)} \quad \mathscr{F}, E \quad ^{c}_{x\,g(x) \quad h(x)} \quad \mathscr{F}$.

Since $,\beta \quad \infty$, we can prove easy, because of

$$E \quad _{x\,g(x) \quad \infty\,h(x)} \quad E \quad _{n\ 1}^{\infty} \quad _{F_{n}} \quad _{F_{\overline{n}}-} \Big);$$

$$E \quad ^{c}_{x\,g(x) \quad \infty\,h(x)} \quad E \quad _{n\ 1}^{\infty} \quad _{F_{n}} \quad _{F_{\overline{n}}-} \Big)^{c} \quad \bigcup_{n\ 1}^{\infty} E \quad _{F_{n}} \quad _{F_{\overline{n}}-}^{c} \quad ;$$

and so or $\beta \quad \infty$, then theorem 2 prove over.

Consequence 2.2 *if* $f \quad \text{CFM}(E)$, $[0\ 1] \quad [0\ 1], \quad _{1} \quad _{2}\ \beta_{1}\ \beta_{2} \quad [\quad \infty \quad \infty]$, *then*

$$E \quad x\, g_\lambda(x) \quad {}_1\, g_\lambda(x) \quad {}_2\, h\ (x) \quad {}_1\, h\ (x) \quad {}_2 \quad \mathscr{F};$$

$$E \quad {}^c_x\, g_\lambda(x) \quad {}_1\, g_\lambda(x) \quad {}_2\, h\ (x) \quad {}_1\, h\ (x) \quad {}_2 \quad \mathscr{F}$$

Theorem 3. *If* $f_1, f_2,$ CFM(E)*,then* $E \quad re(f_1)\ \ re(f_2)\ imf(f_1)\ \ imf(f_2)$

Proof. Obvious.

Theorem 4. *If f be the complex-number fuzzy measurable function on E,then f is the real-number fuzzy measurable function on E.*

Proof. Let $f(x) \quad g(x) \quad h(x)$. Because be the complex-number fuzzy measurable function on E, so $g(x)\ \ h(x)$ both are the real-number fuzzy measurable functions on E,

$$f \qquad \overline{g^2(x) \quad h^2(x)} \quad 0$$

f , since 0,in the other mean,

$$g^2(x) \qquad h^2(x)$$

because $h(x)$ is the real-number fuzzy measurable function,so $h^2(x)$ is the real-number fuzzy measurable function [3].

Also f is the real-number fuzzy measurable function on E.

Theorem 5. *If $f_1 \quad f_1$ are the complex-number fuzzy measurable functions on E,then the following functions be the real-number fuzzy measurable functions on E,if these function have means.*

(i) $f_1 \quad f_2;$
(ii) $\overline{f};$
(iii) $f_1 f_2.$

Proof.(i)

$$f_1(x) \quad f_2(x) \quad (g_1(x) \quad g_2(x)) \quad i(h_1(x) \quad h_2(x))$$

we know $g_1(x) \quad g_2(x), h_1(x) \quad h_2(x)$ both are the real-number fuzzy measurable functions.so $f_1 \quad f_2$ is the complex-number fuzzy measurable function.

(ii) Let $f(x) \quad g(x) \quad ih(x)$, then $\overline{f}(x) \quad g(x) \quad ih(x)$.Because $h(x)$is the real-number fuzzy measurable function,so $h(x)$is the real-number fuzzy measurable function too. So $\overline{f}$ is the complex-number fuzzy measurable function.

(iii) Let $f_1(x) \quad g_1(x) \quad h_1(x), f_2(x) \quad g_2(x) \quad h_2(x)$. Then $g_1(x), g_2(x), h_1(x), h_2(x)$ both are the real-number fuzzy measurable functions,

$$f_1 f_2 \quad (g_1 g_2 \quad h_1 h_2) \quad i(g_1 h_2 \quad g_2 h_1)$$

Because $g_1 g_2 \quad h_1 h_2, g_1 h_2 \quad g_2 h_1$both are the real-number fuzzy measurable functions.so $f_1 f_2$ is the complex-number fuzzy measurable function.

Consequence 2.3 *Let* $f_1 \quad g_1 \quad ih_1, f_2 \quad g_2 \quad ih_2$ CFM(E)

(i) If $f_1 \quad f_2$ *has mean,then* $f_1 \quad f_2$ CFM(E)*;*

(ii) If $f_3(x) \quad f_1 f_2\ (x) \triangleq f_1(x) f_2(x)$ *has mean,then* $f_3(x)$ CFM(E).

Theorem 6. *Let* (X $\mathscr{F}$) *be fuzzy measurable space,* $\{f_n(x)\}$ *are a sequence of real-number fuzzy measurable functions on* $E \quad \mathscr{F}$*,then* $\mu(x) \quad \inf_n(f_n(x))$ *and* $(x) \quad \sup_n(f_n(x))$ *both are the fuzzy measurable functions on E.*

Proof. Because $\mu(x) \quad \inf_n(f_n(x))$and $(x) \quad \sup_n(f_n(x))$ are very similar to prove,so we prove $\mu(x) \quad \inf_n(f_n(x))$ only.

$$E \quad x\ \mu(x) \quad E \quad \overset{\infty}{\underset{n\ 1}{}} \quad x\ f_n(x) \quad \overset{\infty}{\underset{n\ 1}{}} E \quad x\ f_n(x)$$

$$E \quad {}^{c}_{x\ \mu(x)} \quad E \quad \overset{\infty}{\underset{n\ 1}{}} \quad {}^{c}_{x\ f_n(x)} \quad \overset{\infty}{\underset{n\ 1}{}} E \quad {}^{c}_{x\ f_n(x)}$$

Because $f_n(x)$ is measurable on E,so $\mu(x) \quad \inf_n(f_n(x))$is measurable on E,and $(x) \quad \sup_n(f_n(x))$ is measurable on E too.

Consequence 2.4 *Let* (X $\mathscr{F}$) *be fuzzy measurable space,* $f_n(x) \quad g_n(x) \quad h_n(x)$ *are a sequence of complex-number fuzzy measurable functions on* $E \quad \mathscr{F}$*,we denote* $\inf_n f_n(x) \quad \inf_n g_n(x) \quad \inf_n h_n(x)$*, and* $\sup_n f_n(x) \quad \sup_n g_n(x) \quad \sup_n h_n(x)$*, then* $\inf_n f_n(x)$*and*$\sup_n f_n(x)$ *both are measurable on* $E \quad \mathscr{F}$

Proof. From the Theorem 6, $\inf_n g_n(x)$,$\inf_n h_n(x)$,$\sup_n g_n(x)$,$\sup_n h_n(x)$ both are the real-number fuzzy measurable functions on $E \quad \mathscr{F}$,so $\inf_n f_n(x)$, $\sup_n f_n(x)$ are the complex -number fuzzy measurable on $E \quad \mathscr{F}$.

Theorem 7. *Let* (X $\mathscr{F}$) *be fuzzy measurable space,*$f_n(x) \quad g_n(x) \quad h_n(x)(n \quad)$ *are a sequence of complex-number fuzzy measurable functions on* $E \quad \mathscr{F}$*, and* $f(x) \quad \lim_{n \quad \infty} f_n(x)$*, is existence, then f(x) is complex-number fuzzy measurable functions on E.*

Proof. Because

$$\underline{\lim}_{n \quad \infty} \quad \sup_n \quad \inf_{m\ n} f_m(x) \quad \overline{\lim}_{n \quad \infty} \quad \inf_n \quad \sup_{m\ n} f_m(x)$$

and then repeat use Consequence 2.1, we can prove $\underline{\lim}_{n \quad \infty}$ $\overline{\lim}_{n \quad \infty}$ both are measurable on E.

$f(x) \quad \lim_{n \quad \infty} f_n(x) \quad \underline{\lim}_{n \quad \infty} f_n(x) \quad \overline{\lim}_{n \quad \infty} f_n(x)$,

so $f(x) \quad \lim_{n \quad \infty} f_n(x)$ is measurable on E. This completes the proof of theorem.

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The Solution Algorithm of Complex Fuzzy-Valued Function Integral by Fuzzy Structured Element

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Abstract. In this paper, we redefined the complex fuzzy integral using the fuzzy structured element theory, and led into the monotone functions[1] of the same sequence in the range between [-1,1], we also obtained some properties and solution algorithm of complex fuzzy-valued function integral by fuzzy structured element.

Keywords: Fuzzy structured element, complex fuzzy-valued function, Riemann integral.

1 Introduction

In 1989, Buckley proposed the concept of fuzzy complex numbers marked the beginning of fuzzy complex analysis; In 1993, Zhang Yue gave the concept of complex fuzzy set-valued function using the interval-valued function and discussed some integral problems. In 1996, Qiu-Ji qing proposed the concept of complex fuzzy measure and complex fuzzy integral; In 1998, Qiu-Ji qing introduced integral concept on the smooth curve and complex ambiguity function in the complex plane combined with strong measurable function. Since 2000, Ma-Sheng quan integrated real analysis and interval analysis methods to discuss fuzzy complex integral, and he has made a series of achievement in this field. He also redefined complex fuzzy integral, and distinguish between real and imaginary parts in order to discuss their properties and some converge theorems of complex fuzzy integral. In 2002, Guo-Si zong proposed the fuzzy structured element, which improved the development of fuzzy analysis.

2 Complex Fuzzy-Valued Function and Fuzzy Structured Element

Define the complex fuzzy-valued function as following

$$\tilde{f}(z) = \tilde{u}(x, y) + i\tilde{v}(x, y)$$

Where $\tilde{u}(x, y)$ is real part and $\tilde{v}(x, y)$ is imaginary part.

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The definition of fuzzy-valued function based on fuzzy structured element is given in reference [2].

Theorem 1.[2] *For a given regular fuzzy structured element* E *in* Y *and arbitrary bounded fuzzy-valued function* $\tilde{u}(x)$ *in* X*, then there exists a binary function* $g(x,y)$ *in* $X \times Y$*, and for arbitrary* $x \in X$*,* $g(x,y)$ *is a bounded monotone function with respect to* y *in the interval [-1,1], such that* $\tilde{u}(x) = g(x,E)$.

Particularly, let $g(x,y) = h_1(x) + h_2(x)y$*, where* $h_1(x)$ *and* $h_2(x)$ *are bounded functions in* X *and* $h_2(x) \geq 0$*, so* $g(x,y)$ *is a bounded monotone function in the interval [-1,1] with respect to y, and for arbitrary* $x \in X$*,* $g(x,y)$ *is a linear function, then we called*

$$\tilde{u}(x) = g(x,E) = h_1(x) + h_2(x)E$$

is a fuzzy-valued function[3] *linear formed by fuzzy structured element* E.

The theorem can be naturally extended to the multi-fuzzy valued functions.

For a given canonical structured element E in W and any bounded fuzzy-valued function $\tilde{u}(x,y)$ in $X \times Y$, there always exists a bounded monotone function $g(x,y,\omega)$ with respect to ω in the interval [-1,1] in $X \times Y \times W$, and for any determinate point $(x,y) \in X \times Y$, it follows that $\tilde{u}(x,y) = g(x,y,E)$.

Hypothesis that the complex fuzzy-valued function $\tilde{f}(x,y) = \tilde{u}(x,y) + i\tilde{v}(x,y)$, in which, $\tilde{u}(x,y)$ and $\tilde{v}(x,y)$ are bounded fuzzy-valued functions. For a given canonical structured element E, according to the fuzzy –valued function represented by fuzzy structured element, then we have

$$\tilde{u}(x,y) = g_1(x,y,E) \quad \tilde{v}(x,y) = g_2(x,y,E)$$

Where $g_1(x,y,\omega)$ and $g_2(x,y,\omega)$ are bounded monotone functions with respect to ω in the interval [-1,1], so the complex fuzzy-valued function can be expressed as

$$\tilde{f}(x,y) = g_1(x,y,E) + ig_2(x,y,E) \tag{1}$$

Particularly, if $\tilde{u}(x,y)$ and $\tilde{v}(x,y)$ are fuzzy-valued functions [3] linear formed by fuzzy structured element E, we have

$$\tilde{u}(x,y) = g_1(x,y,E) = u(x,y) + h_1(x,y)E,$$
$$\tilde{v}(x,y) = g_2(x,y,E) = v(x,y) + h_2(x,y)E.$$

Then the complex fuzzy-valued function $\tilde{f}(x,y)$ can be expressed as following

$$\tilde{f}(x,y) = [u(x,y) + h_1(x,y)E] + i\,[v(x,y) + h_2(x,y)E] \tag{2}$$

Here, $u(x,y)$, $v(x,y)$, $h_1(x,y)$, $h_2(x,y)$ are bounded real-valued functions, the formula (2-2) is called complex fuzzy-valued function linear formed by fuzzy structured element E .

Sometimes, we denote $\tilde{f}(z)$ instead of $\tilde{f}(x,y)$ in order to simplify.

3 Structured Element Representation of Complex Fuzzy-Valued Function Integral

Reference [4] and [5] proposed the curve integral of complex fuzzy-valued functions.

Suppose that $\tilde{f}(z) = \tilde{u}(x,y) + i \cdot \tilde{v}(x,y)$ is a bounded complex fuzzy-valued function defined on the piecewise smooth curve in complex plane C , then we easily establish the following

$$\begin{aligned}\int_C \tilde{f}(z)dz &= \int_C \tilde{u}(x,y)dz + i\int_C \tilde{v}(x,y)dz \\ &= \int_C \tilde{u}(x,y)\,dx - \tilde{v}(x,y)\,dy + i\int_C \tilde{v}(x,y)\,dx + \tilde{u}(x,y)\,dy\end{aligned} \tag{3}$$

Under normal circumstances, the definition of fuzzy-valued function Riemann integral $\int_{\mathrm{D}} \tilde{f}(x)dx$ is the expansion of interval-valued function integral. Suppose that $\tilde{f}(x)$ is a fuzzy-valued function in $D \subseteq X$, the λ-level set of $\tilde{f}(x)$ is a closed interval, we have that $f_\lambda(x) = [f_{1(\lambda)}(x), f_{2(\lambda)}(x)]$. For any $\lambda \in [0,1]$, $f_{1(\lambda)}(x)$ and $f_{2(\lambda)}(x)$ are Riemann integrable on D, then we say $\tilde{f}(x)$ is integral in D, denote the integral value as following

$$\int_{\mathrm{D}} \tilde{f}(x)\,\mathrm{d}\,x = \bigcup_{\lambda\in[0,1]} \lambda \cdot \int_{\mathrm{D}} f_\lambda(x)\,\mathrm{d}\,x \tag{4}$$

Where

$$\int_{\mathrm{D}} f_\lambda(x)\,\mathrm{d}\,x = [\int_{\mathrm{D}} f_{1(\lambda)}(x)\,\mathrm{d}\,x, \quad \int_{\mathrm{D}} f_{2(\lambda)}(x)\,\mathrm{d}\,x]$$

Basing on the fuzzy-valued function represented by the fuzzy structured element E , the integral form of fuzzy –valued function was given in reference[3].

Theorem 2. *Let* $\tilde{f}(x)=g(x,E)$ *be a fuzzy-valued function represented by fuzzy structured element E, if function* $g(x,y)$ *is integral（Riemann）in D* $\subseteq X$ *with respect to x, then the fuzzy-valued function* $\tilde{f}(x)$ *is integral in D, we have*

$$\int_{D}\tilde{f}(x)dx=\int_{D}g(x,y)dx\Big|_{y=E}$$

Let $\tilde{f}(x)$ *be a continuous fuzzy-valued function in D=[a,b], denote*

$$\tilde{F}(x)=\int_a^x\tilde{f}(x)dx=G(x,E),$$

then we have

$$\int_a^b\tilde{f}(x)dx\quad[G(b,y)-G(a,y)]\Big|_{y=E} \tag{5}$$

Using the conclusion of theorem 3.1, it's easy to obtain the complex fuzzy- valued function carve integral that represented by the fuzzy structured element E.

For a given canonical structured element E, the representation of fuzzy-valued function based on fuzzy structured element, it is apparent that

$$\tilde{u}(x,y)=g_1(x,y,E)\quad\tilde{v}(x,y)=g_2(x,y,E)$$

Then we have

$$\begin{aligned}\int_C\tilde{f}(z)dz&=\int_C\tilde{u}(x,y)dx-\tilde{v}(x,y)dy+i\int_C\tilde{v}(x,y)dx+\tilde{u}(x,y)dy\\&=\int_C[g_1(x,y,w)dx-g_2(x,y,w)dy]\Big|_{w=E}\\&\quad+i\int_C[g_1(x,y,w)dy+g_2(x,y,w)dx]\Big|_{w=E}\end{aligned} \tag{6}$$

Particularly, if $\tilde{u}(x,y),\tilde{v}(x,y)$ are fuzzy-valued functions[6] linear formed by fuzzy structured element E, then we denote

$$\tilde{f}(z)=[u(x,y)+h_1(x,y)E]+i\,[v(x,y)+h_2(x,y)E]$$

Where $h_1(x,y)\geq 0, h_2(x,y)\geq 0$, and we have

$$\tilde{f}(z)=[u(x,y)+i\cdot v(x,y)]+[h_1(x,y)+i\cdot h_2(x,y)]E$$

That is to say, the complex fuzzy-valued function $\tilde{f}(z)$ is linear formed by fuzzy structured element E. Denote the complex function

$$\varphi_1(z) = u(x, y) + i \cdot v(x, y)$$
$$\varphi_2(z) = h_1(x, y) + i \cdot h_2(x, y)$$

Thus, we may immediately obtain the complex fuzzy-valued function

$$f(z) = \varphi_1(z) + \varphi_2(z) \cdot E$$

Theorem 3. *Suppose that* $u(x, y), v(x, y)$ *and* $h_1(x, y) \geq 0, h_2(x, y) \geq 0$ *are Riemann integral in complex* C. *Then the complex fuzzy-valued function*

$$\tilde{f}(z) = [u(x, y) + h_1(x, y)E] + i\ [v(x, y) + h_2(x, y)E]$$

is integral in C, *the following integral value is easy to obtain*

$$\int_C \tilde{f}(z)dz = \int_C [u(x, y) + i \cdot v(x, y)]dz + E \cdot \int_C [h_1(x, y) + i \cdot h_2(x, y)]dz$$
$$= \int_C \varphi_1(z)dz + E \cdot \int_C \varphi_2(z)dz$$

Proof.

$$\int_C \tilde{f}(z)dz = \int_C \tilde{u}(x, y)dx - \tilde{v}(x, y)dy + i\int_C \tilde{v}(x, y)dx + \tilde{u}(x, y)dy$$
$$= \int_C [u(x, y) + h_1(x, y)\omega]dx - [v(x, y) + h_2(x, y)\omega]dy \Big|_{\omega=E}$$
$$+ i \cdot \int_C [v(x, y) + h_2(x, y)\omega]dx + [u(x, y) + h_1(x, y)\omega]dy \Big|_{\omega=E}$$

$$= \int_C u(x, y)dx - v(x, y)dy + \omega\int_C h_1(x, y)dx - h_2(x, y)dy \Big|_{\omega=E}$$
$$+ i[\int_C v(x, y)dx + u(x, y)dy + \omega\int_C h_2(x, y)dx + h_1(x, y)dy]\Big|_{\omega=E}$$
$$= \int_C u(x, y)dx - v(x, y)dy + E\int_C h_1(x, y)dx - h_2(x, y)dy$$
$$+ i[\int_C v(x, y)dx + u(x, y)dy + E\int_C h_2(x, y)dx + h_1(x, y)dy]$$
$$= \int_C u(x, y)dx - v(x, y)dy + +i \cdot \int_C v(x, y)dx + u(x, y)dy$$
$$+[\int_C h_1(x, y)dx - h_2(x, y)dy\ +i\int_C h_2(x, y)dx + h_1(x, y)dy] \cdot E$$

$$= \int_C [u(x,y) + i \cdot v(x,y)]dz + E \cdot \int_C [h_1(x,y) + i \cdot h_2(x,y)]dz$$
$$= \int_C \varphi_1(z)dz + E \cdot \int_C \varphi_2(z)dz$$

This completes our proof.

For example, let complex fuzzy-valued function $\tilde{f}(z) = x + xE + i(y + yE)$, L is a straight line segment connected 0 to 1 + i. Try to obtain $\int_L \tilde{f}(z)dz$.

Solution. As is known that

$$\tilde{u}(x,y) = g_1(x,y,w) = x + xw,\ \tilde{v}(x,y) = g_2(x,y,w) = y + yw$$

According to the definition of carve integral based on the structured element. Where

$$\int_L \tilde{f}(z)dz = \int_L \tilde{u}(x,y)dx - \tilde{v}(x,y)dy + i\int_L \tilde{v}(x,y)dx + \tilde{u}(x,y)dy$$

Denote

$$\int_L \tilde{f}(z)dz = \int_L (x + xw)dx - (y + yw)dy + i\int_L (y + yw)dx + (x + xw)dy$$
$$= (1+w)\left[\int_{(0,0)}^{(1,1)} xdx - ydy + i\int_{(0,0)}^{(1,1)} ydx + xdy\right]\Big|_{w=E}$$
$$= (1+w)\, i\,\big|_{w=E}$$
$$= (1+E)\, i$$

Obviously, when $E = 0$, the result is consistent with the classical complex function.

4 Properties of Complex Fuzzy-Valued Function Carve Integral

Property 1. *Let* $\tilde{f}(x,y)$ *and* $\tilde{g}(x,y)$ *are complex-valued fuzzy functions, here,* x *is the real part of* z *, and* y *is the imaginary part. Hence we have the following conclusions*

(i) If $\tilde{f} \pm \tilde{g}$ *is integrable on carve* L*, then*

$$\int_L [\tilde{f}(x,y) \pm \tilde{g}(x,y)]dz = \int_L \tilde{f}(x,y)dz \pm \int_L \tilde{g}(x,y)dz$$

(ii) If k is a complex constant, and $k \cdot \tilde{f}(x, y)$ is integrable on carve L, then

$$\int_L k \cdot \tilde{f}(x, y) dz = k \cdot \int_L \tilde{f}(x, y) dz$$

Proof. Suppose that E is a regular symmetric fuzzy structured element. It must be existed monotone functions g_1, g_2, g_3 and g_4 of the same sequence in the interval [-1,1]. it follows that

$$\tilde{f}(x, y) = [g_1(x, y, w) + ig_2(x, y, w)]\big|_{w=E}$$
$$\tilde{g}(x, y) = [g_3(x, y, w) + ig_4(x, y, w)]\big|_{w=E}$$

Then we have

(i) $$\int_L [\tilde{f}(x, y) \pm \tilde{g}(x, y)]\, dz$$

$$= \int_L [(g_1(x, y, E) + ig_2(x, y, E)) \pm (g_3(x, y, E) + ig_4(x, y, E))] dz$$

$$= \int_L [(g_1(x, y, w) + ig_2(x, y, w)) \pm (g_3(x, y, w) + ig_4(x, y, w))] dz\Big|_{w=E}$$

$$= \int_L [g_1(x, y, w) + ig_2(x, y, w)] dz\Big|_{w=E}$$

$$\pm \int_L [g_3(x, y, w) + ig_4(x, y, w)] dz\Big|_{w=E}$$

$$= \int_L \tilde{f}(x, y) dz \pm \int_L \tilde{g}(x, y) dz$$

(ii) $$\int_L k \cdot \tilde{f}(x, y) dz = \int_L k[g_1(x, y, E) + ig_2(x, y, E)] dz$$

$$= \int_L k[g_1(x, y, w) + ig_2(x, y, w)] dz\Big|_{w=E}$$

$$= k\int_L [g_1(x, y, w) + ig_2(x, y, w)] dz\Big|_{w=E}$$

$$= k\int_L \tilde{f}(x, y) dz$$

From the conclusions above, combined with the properties of complex function, the following result is easy to obtain.

Property 2. *Suppose that* α *,* β *are constants, if complex fuzzy -valued functions* $\tilde{f}(x, y)$, $\tilde{g}(x, y)$ *are integral on carve* L*, then we have*

$$\int_L [\alpha\tilde{f}(x, y) + \beta\tilde{g}(x, y)]\, dz = \alpha\int_L \tilde{f}(x, y)\, ds + \beta\int_L \tilde{g}(x, y)\, dz\, .$$

Property 3. *Suppose that curve* L *is connected by smooth carves* L_1, L_2*, if integral of complex fuzzy-valued function* $\tilde{f}$ *on the carve* L *is existed, then the integral of complex-valued function* $\tilde{f}$ *on the carve* L_1, L_2 *is existed too. Otherwise, if the carve integral of the complex fuzzy-valued function* $\tilde{f}$ *on* L_1, L_2 *is existed, the carve integral of complex fuzzy-valued function* $\tilde{f}$ *on* L *is also existed.*

$$\int_L \tilde{f} dz = \int_{L_1} \tilde{f} dz + \int_{L_2} \tilde{f} dz\, .$$

5 Conclusion

On complex fuzzy integral theory, whether international or domestic research, their work is still relatively small, complex fuzzy integral development of the theory is not yet mature. Based on this, we redefined the complex fuzzy integral using the fuzzy structured element method, and discuss the relevant properties and the solution algorithm. There is an example to analyze at last.

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Discussion on Natural Fuzzy Extension and Joint Fuzzy Extension of the Rational Function

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Abstract. According to the knowledge of interval analysis, relations between natural fuzzy extension and joint fuzzy extension of rational function are discussed in this paper . On the basis of the natural fuzzy extension of rational function , two solutions to the joint fuzzy extension of rational function are put forward. Based on the structured element method, the fuzzy arithmetic with equality constraints is turned into the operation of two monotone functions with the same monotonic form on [0,1]. From this transition we get analytical expression of joint fuzzy extension of the rational function.

Keywords: Interval analysis, Joint fuzzy extension, Natural fuzzy extension, Structured element.

1 Natural Extension and Joint Extension of Interval Valued Functions

Since the interval number is a special case of the fuzzy number in fuzzy analysis , thus we first review the knowledge of interval analysis before researching fuzzy arithmetic [3].

Definition 1. *Let* $X_i \ (i=1,2,\cdots,n)$, Y *be sets of real number, and* f *a bounded function from* $X_1 \times X_2 \times \cdots \times X_n$ *to* Y. *Denote* $\bar{N}(X_i)$ *as the set of all interval numbers in* $X_i \ (i=1,2,\cdots,n)$. *Then we define the joint interval extension of real function as following,*

$$\hat{f}(A_1,A_2,\cdots,A_n)=\left\{f(x_1,x_2,\cdots,x_n)\middle| x_i\in A_i, i=1,2,\cdots,n, A_i\in \bar{N}(X_i)\right\} \quad (1)$$

The function where independent variables are interval variables $A_1, A_2, \cdots, A_n$ is interval function, denoted as $F(A_1, A_2, \cdots, A_n)$.

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Definition 2. *Let* $f(x_1, x_2, \cdots, x_n)$ *be a real function,* $F(A_1, A_2, \cdots, A_n)$ *a interval function . For any real number* $x_i \in A_i$ $(i = 1, 2, \cdots, n)$ *, define* F *as a interval extension of* f *iff* $F(x_1, x_2, \cdots, x_n) = f(x_1, x_2, \cdots, x_n)$ *.*

It is obvious that $\hat{f}$ is a interval extension of f , and we can notice that interval extension of a real-valued function is not unique. For example, if $F(A)$ is a interval extension of $f(x)$, then interval function $F_1(A) = F(A) + A - A$ is also a interval extension of $f(x)$. Another example is that $f(x) = x - x^2 = x(1-x)$. From the above expression we can get two equivalent expressions of $f(x)$, it follows that its two different interval extensions are $F_1(A) = A - A^2 = A - A \cdot A$ and $F_2(A) = A(1-A)$.

Definition 3. *Let* $f(x_1, x_2, \cdots, x_n)$ *be a real rational function of* $x_1, x_2, \cdots, x_n$ *. If instead of real variables with the corresponding interval variables* $A_1, A_2, \cdots, A_n$ *in* $f(x_1, x_2, \cdots, x_n)$ *, thus we get* $f(A_1, A_2, \cdots, A_n)$ *, defined as a rational interval function, and also called as the natural interval extension of the rational function* $f(x_1, x_2, \cdots, x_n)$ *.*

Theorem 1. *If* $f(A_1, A_2, \cdots, A_n)$ *is the natural interval extension of the rational function* $f(x_1, x_2, \cdots, x_n)$ *, then*

$$f(A_1, A_2, \cdots, A_n) \supseteq \hat{f}(A_1, A_2, \cdots, A_n) \tag{2}$$

Proof refers to [3].

The following example shows the differences and similarities between joint interval extension and natural interval extension of rational function.

Example 1. Consider joint extension and natural extension of the polynomial $f(x) = 1 - 5x + x^3/3$, here variable x is any element in $[3, 4]$.

In $[3, 4]$, it is easy to show that $f(4) = 1/3$ is the maximum of $f(x)$ and $f(3) = -5$ the minimum of $f(x)$. Thus $\hat{f}(A) = [-5, 1/3]$ according to definition1 of joint interval extension.

On the other hand, if the interval number $A = [3, 4]$ is substituted into the polynomial $f(x)$ directly, we can get

$$f(A) = 1 - 5A + A^3/3 = 1 - 5[3, 4] + [3, 4]^3/3$$

After that , using four arithmetic operations of interval numbers, we can get the natural extension of $f(x)$ in interval A

$$f(A) = [-10, 22/3]$$

Obviously

$$f(A)=[-10,22/3]\supset[-5,1/3]=\hat{f}(A)$$

The main reason for creating error of natural extension is that interval number A of $5A$ and $A^3/3$ are identical [8] in $f(A)=1-5A+A^3/3$. For any real number $x_0\in A$, $f(x_0)=1-5x_0+x_0{}^3/3$. However, in the arithmetic of natural extension, interval number A of $5A$ and $A^3/3$ are considered as two independent variables. If all variables of interval rational function are independent, then natural extension and joint extension of function must be equal. Hence, we easily establish the following conclusion.

Theorem 2[3]. *If every variable* x_i $(i=1,2,\cdots,n)$ *appears only once in function* $f(x_1,x_2,\cdots,x_n)$, *and* $f(A_1,A_2,\cdots,A_n)$ *is the natural interval extension of* $f(x_1,x_2,\cdots,x_n)$, *then we have*

$$F(A_1,A_2,\cdots,A_n)=\overline{f}(A_1,A_2,\cdots,A_n)$$

2 Fuzzy Extension of Rational Function

We can extend the interval extension to the fuzzy extension of function.

Definition 4. *The function where independent variables are fuzzy variables* $\tilde{A}_1,\tilde{A}_2,\cdots,\tilde{A}_n$ *is defined as fuzzy function, denoted* $F(\tilde{A}_1,\tilde{A}_2,\cdots,\tilde{A}_n)$.

Definition 5. *Let* $f(x_1,x_2,\cdots,x_n)$ *be a bounded function in* $X_1\times X_2\times\cdots\times X_n$, $(A_i)_\lambda$ *the* $\lambda-$*level set of fuzzy number* $\tilde{A}_i$, *and* $\hat{f}((A_1)_\lambda,(A_2)_\lambda,\cdots,(A_n)_\lambda)$ *the joint extension of* $f(x_1,x_2,\cdots,x_n)$. *By the extension principle, we define the joint fuzzy extension of function of several variables as following.*

$$\hat{f}(\tilde{A}_1,\tilde{A}_2,\cdots,\tilde{A}_n)=\bigcup_{\lambda\in[0,1]}\lambda\wedge\hat{f}((A_1)_\lambda,(A_2)_\lambda,\cdots,(A_n)_\lambda)$$

Definition 6. *Let* $f(x_1,x_2,\cdots,x_n)$ *be a real rational function of* $x_1,x_2,\cdots,x_n$. *If instead of real variables with the corresponding fuzzy variables* $A_1,A_2,\cdots,A_n$ *in* $f(x_1,x_2,\cdots,x_n)$, *thus we get* $f(A_1,A_2,\cdots,A_n)$, *defined as a rational fuzzy function, and also called the natural interval extension of the rational function* $f(x_1,x_2,\cdots,x_n)$.

For arbitrary $\lambda \in (0,1]$,if $f[(A_1)_\lambda,(A_2)_\lambda,\cdots,(A_n)_\lambda]$ and $\hat{f}[(A_1)_\lambda,(A_2)_\lambda,\cdots,(A_n)_\lambda]$ are natural extension and joint extension of the rational function $f(x_1,x_2,\cdots,x_n)$ respectively, then by the multivariate extension principle, we have

$$\hat{f}(\tilde{A}_1,\tilde{A}_2,\cdots,\tilde{A}_n)=\bigcup_{\lambda\in[0,1]}\lambda\wedge\hat{f}((A_1)_\lambda,(A_2)_\lambda,\cdots,(A_n)_\lambda)$$

It follows that

$$f(\tilde{A}_1,\tilde{A}_2,\cdots,\tilde{A}_n)\supseteq\hat{f}(\tilde{A}_1,\tilde{A}_2,\cdots,\tilde{A}_n)$$

From Theorem2 , the following result is easy to obtain.

Theorem 3. *If every variable* x_i $(i=1,2,\cdots,n)$ *appears only once in* $f(x_1,x_2,\cdots,x_n)$, $f(\tilde{A}_1,\tilde{A}_2,\cdots,\tilde{A}_n)$ *and* $\hat{f}(\tilde{A}_1,\tilde{A}_2,\cdots,\tilde{A}_n)$ *are the natural fuzzy extension and the joint fuzzy extension of* $f(x_1,x_2,\cdots,x_n)$ *respectively, then*

$$f(\tilde{A}_1,\tilde{A}_2,\cdots,\tilde{A}_n)=\hat{f}(\tilde{A}_1,\tilde{A}_2,\cdots,\tilde{A}_n)$$

3 Fuzzy Structured Element and Transformation with the Same Monotonic Form of Monotone Functions

Let *E* be a fuzzy set on *R*. If its membership function $E(x)$ satisfies the following properties:

1) $E(0)=1$;

2) $E(x)$ is monotonic increasing and continuous from the right on [-1,0), monotonic decreasing and continuous from the left on (0,1];

3) when $x\notin(-1,1)$, $E(x)=0$, then *E* is defined as a fuzzy structured element.

If the fuzzy structured element *E* satisfies: 1 $\forall x\in[-1,1]$, $E(x)>0$; 2 $E(x)$ is continuous and strict monotonic increasing on [-1,0), and continuous and strict monotonic decreasing on (0,1], then we define *E* as a regular fuzzy structured element. If $E(x)=E(-x)$, then *E* is called as a symmetric structured element.

Suppose that *E* is a symmetric structured element , $f(x)$ is a monotonic, bounded function on [-1,1]. If $g(x)=f(-x)$, then $f(E)=g(E)$ $\forall x\in[-1,1]$.

Theorem 4[5]. *Let E be an arbitrary fuzzy structured element ,* $E(x)$ *the membership function of E. For any monotonic,bounded function* $f(x)$ *on [-1,1],* $f(E)$ *is a bounded, closed fuzzy number. Conversely, for arbitrary bounded,* *closed fuzzy number* $\tilde{A}$*, there always exists a monotonic, bounded function* f *on [-1,1], making* $\tilde{A} = f(E)$*. Then we say that fuzzy number* $\tilde{A}$ *is generated by fuzzy structured element.*

Theorem 5[5]. *If the fuzzy number* $\tilde{A} = f(E)$*, then its membership function is* $E(f^{-1}(x))$*. Here* $f^{-1}(x)$ *is denoted as the cyclic symmetric function of* $f(x)$ *with variables x and y (If* $f(x)$ *is continuous,strict monotonic, then* $f^{-1}(x)$ *is the inverse function of* $f(x)$ *).*

For arbitrary $\lambda \in (0,1]$, we assume that E is a fuzzy structured , and its $\lambda-$level set as $E_\lambda = \{ x \mid E(x) \geq \lambda \} = [e_\lambda^-, e_\lambda^+]$. According to the definition of structured element, we have $e_\lambda^- \in [-1,0]$, $e_\lambda^+ \in [0,1]$.

Theorem 6. *Let* $\tilde{A}$ *be a bounded,closed fuzzy number, E a regular fuzzy structured element. For arbitrary* $\tilde{A}$*, there always exists a monotonic function* f *on [-1,1], making* $\tilde{A} = f(E)$*. Furthermore , for arbitrary* $\lambda \in (0,1]$*, if* f *is monotone increasing on [-1,1], then* $A_\lambda = f(E_\lambda) = [f(e_\lambda^-), f(e_\lambda^+)]$ *is the* $\lambda-$ *level set of* $\tilde{A}$ *; if* f *is monotone decreasing on [-1,1], then* $A_\lambda = [f(e_\lambda^+), f(e_\lambda^-)]$.

Proof refers to [5].

Let D[-1,1] denote the set of all monotone functions with the same monotonic form on [-1,1] . The monotone transformation with the same monotonic form has been defined in [6] $\tau_i : D[-1,1] \to D[-1,1]$, $i = 0,1,2,3$, for arbitrary $f \in D[-1,1]$, we denote

$$\tau_0(f) = f \ ; \ \tau_1(f) = f^{\tau_1} ; \tau_2(f) = f^{\tau_2} ; \tau_3(f) = f^{\tau_3}$$

where $f^{\tau_1}(x) = -f(-x)$; $f^{\tau_2}(x) = \dfrac{1}{f(-x)}$ ($f(-x) \neq 0$); $f^{\tau_3}(x) = -\dfrac{1}{f(x)}$, ($f(x) \neq 0$) for arbitrary $x \in [-1,1]$.

It is apparent that if E is a symmetric fuzzy structured element, and $f(x)$ is a bounded , monotone increasing function on [-1,1], then we have

$$f(e_\lambda^-) = -f^{\tau_1}(e_\lambda^+), f(e_\lambda^+) = -f^{\tau_1}(e_\lambda^-).$$

Lemma 1. *Let* $\tilde{A}$*,* $\tilde{B}$ *be two fuzzy numbers , E a fuzzy structured element. If f , g are two monotone functions with the same monotonic form on [-1,1] , and* $\tilde{A} = f(E)$*,* $\tilde{B} = g(E)$*, then*

(*a*) *If both* $\tilde{A}$ *and* $\tilde{B}$ *are arbitrary bounded fuzzy numbers, then* $\tilde{A}+\tilde{B} =$ *(f + g) (E), and its membership function is* $\mu_{\tilde{A}+\tilde{B}}(x) = E[(f+g)^{-1}(x)]$ *;*

(*b*) *If both* $\tilde{A}$ *and* $\tilde{B}$ *are positive fuzzy numbers, then* $\tilde{A}\cdot\tilde{B} = [f\cdot g](E)$*, and its membership function is* $\mu_{\tilde{A}\cdot\tilde{B}}(x) = E[(f\cdot g)^{-1}(x)]$.

Lemma 2. *Let E be a symmetric fuzzy structured element, f a monotonic,bounded function on [-1,1] , and the fuzzy number* $\tilde{A} = f(E)$*, then*

$$-\tilde{A} = f^{\tau_1}(E)\,; \quad \frac{1}{\tilde{A}} = f^{\tau_2}(E)\,; \quad -\frac{1}{\tilde{A}} = f^{\tau_1\tau_2}(E) = f^{\tau_3}(E)$$

Proofs of both Lemma 1 and Lemma 2 refer to [5].

We assume that $\tilde{A}$ and $\tilde{B}$ are arbitrary bounded fuzzy numbers, and their $\lambda-$ level sets as $A_\lambda = [a_\lambda^-, a_\lambda^+]$ and $B_\lambda = [b_\lambda^-, b_\lambda^+]$ respectively. For every $\lambda \in (0,1]$, if there are both $a_\lambda^- \le b_\lambda^-$ and $a_\lambda^+ \le b_\lambda^+$, then we define $A \le B$; If there exists $\lambda \in (0,1]$ such that either $a_\lambda^- < b_\lambda^-$ or $a_\lambda^+ < b_\lambda^+$, then we define $A < B$.

4 Structured Element of Natural Extension of Fuzzy Rational Function

In many practical applications, forms of natural extension of fuzzy rational functions are not uncommon. Take queuing theory for example, we denote λ as the average arrival rate of a customer , μ as the service rate of a service desk, and $\rho = \lambda/\mu$ as the traffic intensity. In steady state, the average team length in steady state L, and the average waiting time for customers T in the model of single service waiting system, and the average staying time for customers W in the model of single service mixing system are

$$L(\rho) = \frac{\rho}{1-\rho} \qquad T(\mu,\lambda) = \frac{1}{\mu-\lambda} - \frac{1}{\mu} \qquad W(\rho,\lambda) = \frac{\rho}{1+\rho} \div \frac{\lambda}{1+\rho}$$

respectively. If their parameters, $\tilde{\lambda}, \tilde{\mu}, \tilde{\rho}$ respectively , are all fuzzy numbers, then the corresponding natural extensions are

$$L(\tilde{\rho}) = \frac{\tilde{\rho}}{1-\tilde{\rho}} \qquad T(\tilde{\mu},\tilde{\lambda}) = \frac{1}{\tilde{\mu}-\tilde{\lambda}} - \frac{1}{\tilde{\mu}} \qquad W(\tilde{\rho},\tilde{\lambda}) = \frac{\tilde{\rho}}{1+\tilde{\rho}} \div \frac{\tilde{\lambda}}{1+\tilde{\rho}}$$

How to operate on natural fuzzy extension of function in order to make the natural fuzzy extension equal the joint fuzzy extension of rational function is the key issue of this paper.

Definition 7. *In* $f(x_1,x_2,\cdots,x_n)$ *, the occurrence number of a variable* x_i $(i=1,2,\cdots,n)$ *is more than 1, while other variables* x_j $(j \neq i)$ *are all appear only once, then we define* $f(x_1,x_2,\cdots,x_n)$ *as the identity qualified formula of variable* x_i *, and also called single variable identity qualified formula. Furthermore, we define variable* x_i *as qualified variable.*

For example, $f(x_1,x_2,\cdots,x_n)=\sum_{i=1}^{n} x_i + x_1{}^2$ is an identity qualified formula of variable x_1.

Theorem 7. *Let E be a fuzzy structured element,* $g_i(x)$ $(i=1,2,\cdots,n)$ *monotone functions with the same monotonic form on [-1,1], and fuzzy numbers* $\tilde{A}_k = g_k(E)$ $(k=1,2,\cdots,n)$ *. Supposing that* $f(x_1,x_2,\cdots,x_n)$ *is a identity qualified formula of variable* x_i *, for arbitrary* λ *∈(0,1],* $\hat{f}((A_1)_\lambda,(A_2)_\lambda,\cdots,(A_n)_\lambda)$ *is the joint interval extension of* $f(x_1,x_2,\cdots,x_n)$ *. Without loss of generality, assume that* x_1 *is qualified variable.*

(1) *If* $f(x_1,x_2,\cdots,x_n)$ *is monotone increasing on qualified variable,* $x_1 \in (A_1)_\lambda = g_1(E_\lambda) = [g_1(e_\lambda^-), g_1(e_\lambda^+)]$*, and the interval number*

$$f(g_1(e_\lambda^-),(A_2)_\lambda,\cdots,(A_n)_\lambda) = [m_1(f), M_1(f)]$$
$$f(g_1(e_\lambda^+),(A_2)_\lambda,\cdots,(A_n)_\lambda) = [m_2(f), M_2(f)]$$

where $m_i(f)$ *and* $M_i(f)$ *are real numbers,* $i=1,2$ *,then we have*

$$\hat{f}((A_1)_\lambda,(A_2)_\lambda,\cdots,(A_n)_\lambda) = [m_1(f), M_2(f)]$$

(2) *If* $f(x_1,x_2,\cdots,x_n)$ *is monotone increasing on qualified variable,* $x_1 \in (A_1)_\lambda = g_1(E_\lambda) = [g_1(e_\lambda^-), g_1(e_\lambda^+)]$*, and the interval number*

$$f(g_1(e_\lambda^-),(A_2)_\lambda,\cdots,(A_n)_\lambda) = [m_1(f), M_1(f)]$$
$$f(g_1(e_\lambda^+),(A_2)_\lambda,\cdots,(A_n)_\lambda) = [m_2(f), M_2(f)]$$

where $m_i(f)$ *and* $M_i(f)$ *are real numbers,* $i=1,2$ *, then we have*

$$\hat{f}((A_1)_\lambda,(A_2)_\lambda,\cdots,(A_n)_\lambda) = [m_2(f), M_1(f)]$$

(3) *If* $f(x_1,x_2,\cdots,x_n)$ *is not monotone increasing on qualified variable,* $x_1 \in (A_1)_\lambda = g_1(E_\lambda) = [g_1(e_\lambda^-), g_1(e_\lambda^+)]$ *, but it takes the maximum when* $x_1 = a$ *,*

and the minimum when $x_1 = b \quad (a, b \in [g_i(e_\lambda^-), g_i(e_\lambda^+)])$. *Suppose that the interval number*

$$f(a,(A_2)_\lambda,\cdots,(A_n)_\lambda) = [m_1(f), M_1(f)]$$
$$f(b,(A_2)_\lambda,\cdots,(A_n)_\lambda) = [m_2(f), M_2(f)]$$

where $m_i(f)$ *and* $M_i(f)$ *are real numbers,* $i = 1,2$, *then we have*

$$\hat{f}((A_1)_\lambda,(A_2)_\lambda,\cdots,(A_n)_\lambda) = [m_1(f), M_2(f)]$$

Proof. It suffices to show (1), proofs of (2) and (3) are similar.

Since function $f(x_1, x_2, \cdots, x_n)$ is monotone increasing on qualified variable $x_1 \in [g_1(e_\lambda^-), g_2(e_\lambda^+)]$, thus for any $x_i \in (A_i)_\lambda \quad (i = 2,3,\cdots,n)$, we have

$$\begin{aligned} f((A_1)_\lambda, x_2,\cdots,x_n) &= f([g_1(e_\lambda^-), g_2(e_\lambda^+)], x_2,\cdots,x_n) \\ &= [f(g_1(e_\lambda^-), x_2,\cdots,x_n), f(g_2(e_\lambda^+), x_2,\cdots,x_n)] \end{aligned}$$

For a given $x_1 = g_1(e_\lambda^-) \in R$, since other variables $x_2, x_3, \cdots, x_n$ appearing once, by Theorem 2, thus $f(g_1(e_\lambda^-),(A_2)_\lambda,\cdots,(A_n)_\lambda)$ is the joint interval extension of $f(g_1(e_\lambda^-), x_2,\cdots,x_n)$. Furthermore ,

$$f(g_1(e_\lambda^-),(A_2)_\lambda,\cdots,(A_n)_\lambda) = [m_1(f), M_1(f)]$$
$$f(g_1(e_\lambda^+),(A_2)_\lambda,\cdots,(A_n)_\lambda) = [m_2(f), M_2(f)]$$

where $m_i(f)$ and $M_i(f)$ are real numbers, $i = 1,2$, then

$$\hat{f}((A_1)_\lambda,(A_2)_\lambda,\cdots,(A_n)_\lambda) = [m_1(f), M_2(f)]$$

Hence the proof .

Theorem 7 presents a solution of the joint interval extension of a function. By the extension principle , we can get the corresponding joint fuzzy extensions of the function.

Theorem 8. *Let E be a fuzzy structured element. For arbitrary* $\lambda \in (0,1]$, *if* $f(A_\lambda) = h(E_\lambda)$, *then* $f(\tilde{A}) = h(E)$.

Proof. By the extension principle that $f(\tilde{A}) = \bigcup_{\lambda \in (0,1]} \lambda * f(A_\lambda)$, we have $h(E) = \bigcup_{\lambda \in (0,1]} \lambda * h(E_\lambda)$

Since $f(A_\lambda) = h(E_\lambda)$, thus $f(\tilde{A}) = h(E)$.

Example 2. $f(\tilde{A}) = 1 - 5\tilde{A} + \frac{1}{3}\tilde{A}^3$ ($\tilde{A} > 3$).

Suppose that E is a symmetric fuzzy structured element, $g(x)$ a monotone function with the same monotonic form on [-1,1], and $\tilde{A} = g(E)$. The $\lambda-$level set of $\tilde{A}$ is

$$A_\lambda = g(E_\lambda) = [g(e_\lambda^-), g(e_\lambda^+)]$$

Assume that

$$f(x) = 1 - 5x + \frac{1}{3}x^3 \quad (x \in A_\lambda)$$

According to calculation

$$f'(x) = -5 + x^2 > 0$$

Thus $f(x)$ is monotone increasing in interval A_λ, and

$$\hat{f}(g(e_\lambda^-)) = 1 - 5g(e_\lambda^+) + \frac{1}{3}g^3(e_\lambda^-)$$

$$\hat{f}(g(e_\lambda^+)) = 1 - 5g(e_\lambda^-) + \frac{1}{3}g^3(e_\lambda^+)$$

By Theorem 7, we have

$$\hat{f}(A_\lambda) = [1 - 5g(e_\lambda^+) + \frac{1}{3}g^3(e_\lambda^-), 1 - 5g(e_\lambda^-) + \frac{1}{3}g^3(e_\lambda^+)]$$

$$= (1 + 5g^{\tau_1} + \frac{1}{3}g^3)[e_\lambda^-, e_\lambda^+] = (1 + 5g^{\tau_1} + \frac{1}{3}g^3)(E_\lambda)$$

Assume that

$$(1 + 5g^{\tau_1} + \frac{1}{3}g^3)(x) = h(x)$$

Then

$$\hat{f}(A_\lambda) = h(E_\lambda)$$

By Theorem 8, we have

$$\hat{f}(\tilde{A}) = h(E) = (1 + 5g^{\tau_1} + \frac{1}{3}g^3)(E)$$

By Theorem 5, its membership function is

$$\mu_{f(\tilde{A})}(x) = E[(1 + 5g^{\tau_1} + \frac{1}{3}g^3)^{-1}(x)] .$$

5 Conclusion

In this paper, the relation between the joint fuzzy extension and the natural fuzzy extension of rational function is mainly discussed, on the basis of which we find two solutions to the joint fuzzy extension of function. At the end of the paper, we have solved the joint fuzzy extension of the known funtion and proved simple and effective of the two methods. It is also applicable to other problems such as some fuzzy queuing models, but both of the two methods have some limitations. In the future we will continue to look for the more general method to solve the joint fuzzy of rational function.

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$(s,t]$-Intuitionistic Convex Fuzzy Sets

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Abstract. Based on the concept of cut sets on the intuitionistic fuzzy sets [1] and the neighborhood relations between a fuzzy point and an intuitionistic fuzzy set [2], we define 16 kinds of (α, β)-intuitionistic convex fuzzy sets for $\alpha, \beta \in \{\in, q, \in \wedge q, \in \vee q\}$, and the relationships between them are investigated. According to the above discussion, the $(s,t]$-intuitionistic convex fuzzy set is derived. Lastly the results obtained for operations properties of the $(s,t]$-intuitionistic convex fuzzy sets.

Keywords: Fuzzy points, the cut sets of intuitionistic fuzzy sets, intuitionistic convex fuzzy sets.

1 Introduction

The concept of convex fuzzy set over vector space was first introduced by Zadeh in 1965 [3]. Based on this notion, some scholars studied the properties of convex fuzzy set[4-7] and the topology properties[8-10]. It is worthy pointing out that Bhakat and Das[11,12] gave the concepts of the (α, β)-fuzzy subgroups by using the "belong to" relation ($\in$) and "quasi-coincident with" relation (q) between a fuzzy point x_t and a fuzzy set A, and obtained the significant $(\in, \in \vee q)$-fuzzy subgroup. Yuan et al. [13] gave the definition of a fuzzy subgroup with thresholds from the aspects of multi-implication, which generalized the Rosenfeld's fuzzy subgroup and $(\in, \in \vee q)$-fuzzy subgroup to (λ, μ)- fuzzy subgroup. The results in [11-13] were further generalized in papers [14-18]. Yuan et al.[19,20] applied the above ideas and approaches in [11-13] for the research of convex fuzzy subset and fuzzy topology. K.Atanassov[21] introduced the concept of fuzzy set in 1986. Since then, the intuitionistic fuzzy subgroup and the intuitionistic fuzzy topology [3-24] were presented consequently. However, the research of convex intuitionistic fuzzy set is rather scarce, so the area of its application is limited. Therefore, in this paper, we deal with the various equivalent conditions for the topic of intuitionistic convex fuzzy sets, and the operations of intuitionistic convex fuzzy sets are established, which provide the basis of the theory of intuitionistic fuzzy extremum problems. This paper will be organized as follows. In

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section 2, some preparatory expositions on the fuzzy points, the cut sets of intuitionistic fuzzy sets and the neighborhood relations between a fuzzy point and an intuitionistic fuzzy set are given. In section 3, we provide the summary of (α, β)-intuitionistic convex fuzzy set based on the works of the intuitionistic fuzzy subgroups [2]. In section 4, the notion of the $(s, t]$-intuitionistic convex fuzzy set is presented, and the corresponding theorems are shown. In this paper, we suppose that E denote a vector space over real number set $\mathbb{R}$.

2 Preliminaries

Let X be a universe of discourse. A fuzzy point A , denoted by x_λ ($\lambda \in [0,1]$), is a fuzzy set that satisfying $A(y) = \lambda$ if $y = x$; otherwise $A(y) = 0$. For $x, y \in X$, $s, t \in [0,1]$, using the extension principle in the sense of Zadeh, the operation between two fuzzy points and scalar multiplication are defined as follows:

$$x_s + y_t = (x+y)_{s\wedge t}, kx_s = (kx)_s, k \in \mathbb{R}.$$

We call $A = (X, \mu_A, \nu_A)$ an intuitionistic fuzzy subset over X, and denote it by $A(x) = (\mu_A(x), \nu_A(x))$, if $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ are two mappings and for all $x \in X$, $\mu_A(x) + \nu_A(x)) \le 1$. The class of the intuitionistic fuzzy subsets over X is denoted by $\mathcal{IF}(X)$. Let $A, B \in \mathcal{IF}(X)$ be the intuitionistic fuzzy subsets over X. The order and operations are defined as follows:

$$A \subseteq B : \mu_A(x) \le \mu_B(x), \nu_A(x) \ge \nu_B(x);$$
$$A \cap B : (A \cap B)(x) = A(x) \wedge B(x) = (\mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x));$$

$$\begin{aligned} A + B : (A+B)(z) &= \bigvee_{x+y=z} (A(x) \wedge B(y)) \\ &= (\bigvee_{x+y=z} (\mu_A(x) \wedge \mu_B(y)), \bigwedge_{x+y=z} (\nu_A(x) \vee \nu_B(y)). \end{aligned}$$

Let X, Y be two sets, and $f : X \to Y$ be a mapping. If $A \in \mathcal{IF}(X)$ and $C \in \mathcal{IF}(X)$, then $f(A)$ and $f^{-1}(C)$ are defined as follows:

$$\mu_{f(A)}(y) = \bigvee\nolimits_{f(x)=y} \mu_A(x), \nu_{f(A)}(y) = \bigwedge\nolimits_{f(x)=y} \nu_A(x);$$
$$\mu_{f^{-1}(C)}(x) = \mu_C(f(x)), \nu_{f^{-1}(C)}(x) = \nu_C(f(x)).$$

A is said to be a convex fuzzy subset over E if and only if for all $x, y \in E$ and $\lambda \in [0,1]$,

$$A((1-\lambda)x + \lambda y) \ge A(x) \wedge A(y).$$

Definition 2.1. *[10] Let A be a fuzzy subset over X and x_t be a fuzzy point.*

(1) If $A(x) \ge t$, then we say x_t belongs to A, and denote $x_t \in A$;

(2) If $t + A(x) > 1$*, then we say* x_t *is quasi-coincident with* A*, and denote* $x_t qA$*;*
(3) $x_t \in \wedge A \Longleftrightarrow x_t \in A$ *and* $x_t qA$*;*
(4) $x_t \in \vee A \Longleftrightarrow x_t \in A$ *or* $x_t qA$*.*

Definition 2.2. *[1] Let* $A = (X, \mu_A, \nu_A)$ *be an intuitionistic fuzzy subset over* X *and* $a \in [0, 1]$*,*

(1) We call

$$A_a(x) = \begin{cases} 1 & \mu_A(x) \geq a \\ 1/2 & \mu_A(x) < a \leq 1 - \nu_A(x) \\ 0 & a > 1 - \nu_A(x) \end{cases}$$

and

$$A_{\underline{a}}(x) = \begin{cases} 1 & \mu_A(x) > a \\ 1/2 & \mu_A(x) \leq a < 1 - \nu_A(x) \\ 0 & a \geq 1 - \nu_A(x) \end{cases}$$

the a*-the upper cut set and* a*-strong upper cut set of fuzzy set* A*, respectively.*

(2) We call

$$A_{[a]}(x) = \begin{cases} 1 & a + \mu_A(x) \geq 1 \\ 1/2 & \nu_A(x) \leq a < 1 - \mu_A(x) \\ 0 & \nu_A(x) > a \end{cases}$$

and

$$A_{[\underline{a}]}(x) = \begin{cases} 1 & a + \mu_A(x) > 1 \\ 1/2 & \nu_A(x) < a \leq 1 - \mu_A(x) \\ 0 & \nu_A(x) \geq a \end{cases}$$

the a*-the upper* Q*-cut set and* a*-strong upper* Q*-cut set of fuzzy set* A*, respectively.*

Remark 2.1. (1) It is obvious that $A_{[\underline{a}]}(x) = A_{\underline{1-a}}(x)$, $A_{\underline{a}} \subseteq A_a$. If $a < b$, then $A_a \supseteq A_b$, $A_{\underline{a}} \supseteq A_{\underline{b}}$, and $A_{\underline{a}} \supseteq A_b$.
(2) Let X be a set, we call the mapping $A : X \to 0, 1/2, 1$ a 3-valued fuzzy set.

Definition 2.3. *(1) Let* $[x_a \in A]$ *and* $[x_a qA]$ *represent the grades of membership of* $x_a \in A$ *and* $x_a qA$*, respectively, then*

$$[x_a \in A] \triangleq A_a(x);$$

$$[x_a qA] \triangleq A_{[a]}(x).$$

(2) Let $[x_a \in \wedge qA]$ *and* $[x_a \in \vee qA]$ *represent the grade of membership of* $x_a \in A$ *and* $x_a qA$*,* $x_a \in A$ *or* $x_a qA$*, respectively, then*

$$[x_a \in \wedge qA] \triangleq [x_a \in A] \wedge [x_a qA] = A_a(x) \wedge A_{[a]}(x);$$

$$[x_a \in \vee qA] \triangleq [x_a \in A] \vee [x_a qA] = A_a(x) \vee A_{[a]}(x).$$

Definition 2.4. *Let x_a be a fuzzy point, $s \in (0,1)$ and $A = (X, \mu_A, \nu_A)$ be intuitionistic fuzzy set of X. The neighbourhood relations between a fuzzy point x_a and an intuitionistic fuzzy set A are regulated as follows:*

(1) $[x_a q_s A] = \begin{cases} 1 & a + \mu_A(x) > 2s \\ 1/2 & \nu_A(x) < 2s - a \leq 1 - \mu_A(x); \\ 0 & \nu_A(x) + 2s \geq a + 1 \end{cases}$

(2) $[x_a \in \wedge q_s A] \triangleq [x_a \in A] \wedge [x_a q_s A]$;

(3) $[x_a \in \vee q_s A] \triangleq [x_a \in A] \vee [x_a q_s A]$.

Remark 2.2. When $s = 0.5$, $[x_a q_s A] = [x_a q A]$.

3 (α, β)-Intuitionistic Convex Fuzzy Set

Definition 3.1. *An intuitionistic fuzzy set A of E is said to be a (α, β)-intuitionistic convex fuzzy set if*

$$[((1-\lambda)x_s + \lambda y_t)\beta A] \geq [x_s \alpha A] \wedge [y_t \alpha A]$$

whenever $x, y \in E$, $s, t \in (0,1]$ and $\lambda \in [0,1]$, where $\alpha, \beta \in \{\in, q, \in \wedge q, \in \vee q\}$.

In Definition 3.1, α and β can be chosen from 4 kinds of relations, respectively, thus there are 16 kinds of (α, β)-intuitionistic convex fuzzy sets.

Theorem 3.1. *(1) A is a $(\in, \in)$-intuitionistic convex fuzzy set of E if and only if*

$$\mu_A((1-\lambda)x + \lambda y) \geq \mu_A(x) \wedge \mu_A(y),$$

$$\nu_A((1-\lambda)x + \lambda y) \leq \nu_A(x) \vee \nu_A(y),$$

whenever $x, y \in E, \lambda \in [0,1]$.

(2) A is a $(\in, \in \vee q)$-intuitionistic convex fuzzy set of E if and only if

$$\mu_A((1-\lambda)x + \lambda y) \geq \mu_A(x) \wedge \mu_A(y) \wedge 0.5,$$

$$\nu_A((1-\lambda)x + \lambda y) \leq \nu_A(x) \vee \nu_A(y) \vee 0.5,$$

whenever $x, y \in E, \lambda \in [0,1]$.

(3) A is a $(\in \wedge q, \in)$-intuitionistic convex fuzzy set of E if and only if

$$\mu_A((1-\lambda)x + \lambda y) \vee 0.5 \geq \mu_A(x) \wedge \mu_A(y),$$

$$\nu_A((1-\lambda)x + \lambda y) \wedge 0.5 \leq \nu_A(x) \vee \nu_A(y),$$

whenever $x, y \in E, \lambda \in [0,1]$.

Proof. (1) "$\Rightarrow$" Put $t = \mu_A(x) \wedge \mu_A(x)$, then $[((1-\lambda)x_t + \lambda y_t) \in A] \geq [x_t \in A] \wedge [y_t \in A] = 1$. It is clear that $\mu_A((1-\lambda)x + \lambda y) \geq t = \mu_A(x) \wedge \mu_A(y)$. Let $s = \nu_A((1-\lambda)x + \lambda y)$, for $t > 1 - s(t \in (0,1)$, we have that $0 = [((1-\lambda)x_t + \lambda y_t) \in A] \geq [x_t \in A] \wedge [y_t \in A]$. Subsequently, $[x_t \in A] = 0$ or $[y_t \in A] = 0$, i.e. $\nu_A(x) > 1-t$ or $\nu_A(y) > 1-t$. Thus $\nu_A(x) \vee \nu_A(y) > 1-t$. Consequently $\nu_A(x) \vee \nu_A(y) \geq \vee\{1-t | 1-t < s\} = s = \nu_A((1-\lambda)x + \lambda y)$.
"$\Leftarrow$" Let $a = [x_s \in A] \wedge [y_t \in A]$. In the case $a = 1$, that is to say $[x_s \in A] = [y_t \in A] = 1$, which implies that $\mu_A((1-\lambda)x + \lambda y) \geq \mu_A(x) \wedge \mu_A(y)$. Hence $[((1-\lambda)x_s + \lambda y_t) \in A] = 1$. In the case $a = 1/2$, that is to say $[x_s \in A] \geq 1/2$, $[y_t \in A] \geq 1/2$, thus $1 - \nu_A(x) \geq s$, $1 - \nu_A(y) \geq t$. Furthermore, $1 - \nu_A((1-\lambda)x + \lambda y) \geq 1 - \nu_A(x) \vee \nu_A(y) = (1 - \nu_A(x)) \wedge (1 - \nu_A(y)) \geq s \wedge t$. So $[((1-\lambda)x_s + \lambda y_t) \in A] \geq 1/2$. Therefore, $[((1-\lambda)x_s + \lambda y_t) \in A] \geq [x_s \in A] \wedge [y_t \in A]$, which implies that A is a $(\in, \in)$-intuitionistic convex fuzzy set of E.

(2) "$\Rightarrow$" Put $t = \mu_A(x) \wedge \mu_A(x) \wedge 0.5$, then $[((1-\lambda)x_t + \lambda y_t) \in \vee q A] \geq [x_t \in A] \wedge [y_t \in A] = 1$, which implies that $\mu_A((1-\lambda)x + \lambda y) \geq t$ or $\mu_A((1-\lambda)x + \lambda y) > 1 - t \geq 0.5 \geq t$, thus $\mu_A((1-\lambda)x + \lambda y) \geq \mu_A(x) \wedge \mu_A(y) \wedge 0.5$. Let $1 - s = \nu_A(x) \vee \nu_A(y) \vee 0.5$, then $[((1-\lambda)x_s + \lambda y_s) \in \vee q A] \geq [x_s \in A] \wedge [y_s \in A] \geq 1/2$, which implies that $s \leq 1 - \nu_A((1-\lambda)x + \lambda y)$ or $\nu_A((1-\lambda)x + \lambda y) < s \leq 1 - s$. Furthermore, $\nu_A((1-\lambda)x + \lambda y) \leq 1 - s = \nu_A(x) \vee \nu_A(y) \vee 0.5$.
"$\Leftarrow$" Put $a = [x_s \in A] \wedge [y_t \in A]$. In the case $a = 1$, suppose that $[((1-\lambda)x_s + \lambda y_t) \in \vee q A] \leq 1/2$, then $\mu_A(x) \geq s$, $\mu_A(y) \geq t$, $\mu_A((1-\lambda)x + \lambda y) < s \wedge t$, and $\mu_A((1-\lambda)x + \lambda y) \leq 1 - s \wedge t$, thus $0.5 > \mu_A((1-\lambda)x + \lambda y) \geq \mu_A(x) \wedge \mu_A(y) \wedge 0.5$. So $\mu_A((1-\lambda)x + \lambda y) \geq \mu_A(x) \wedge \mu_A(y) \geq s \wedge t$. This is a contradiction to $\mu_A((1-\lambda)x + \lambda y) < s \wedge t$. Therefore, we have $[((1-\lambda)x_t + \lambda y_t) \in \vee q A] = 1$. In the case $a = 1/2$, we have $1 - \nu_A(x) \geq s$, $1 - \nu_A(y) \geq t$, and $1 - \nu_A(x) \vee \nu_A(y) \geq s \wedge t$. Suppose that $[((1-\lambda)x_s + \lambda y_t) \in \vee q A] = 0$, then $s \wedge t > 1 - \nu_A((1-\lambda)x + \lambda y)$ and $\nu_A((1-\lambda)x + \lambda y) \geq s \wedge t$, thus $\nu_A((1-\lambda)x + \lambda y) > 0.5$, consequently $\nu_A((1-\lambda)x + \lambda y) \leq \nu_A(x) \vee \nu_A(y)$ and $1 - \nu_A((1-\lambda)x + \lambda y) \geq 1 - \nu_A(x) \vee \nu_A(y) \geq s \wedge t$. This is a contradiction to $1 - \nu_A((1-\lambda)x + \lambda y) < s \wedge t$. Thus, we have $[((1-\lambda)x_s + \lambda y_t) \in \vee q A] \geq 1/2$. Ultimately, $[((1-\lambda)x_s + \lambda y_t) \in \vee q A] \geq [x_s \in A] \wedge [y_t \in A]$. This shows that A is a $(\in, \in \vee q)$-intuitionistic convex fuzzy set of E.

(3) Similar to (2), so we can obtain the conclusion.

Theorem 3.2. *(1) A is a $(\in, \in)$-intuitionistic convex fuzzy set of $E \iff \forall t \in [0,1]$, A_t is a 3-valued convex fuzzy set of E.*
(2) A is a $(\in, \in \vee q)$-intuitionistic convex fuzzy set of $E \iff \forall t \in (0, 0.5]$, A_t is a 3-valued convex fuzzy set of E.
(3) A is a $(\in \wedge q, \in)$-intuitionistic convex fuzzy set of $E \iff \forall t \in (0.5, 1]$, A_t is a 3-valued convex fuzzy set of E.

The proof of Theorem 3.2 is similar to Theorem 3.1, so the detail is omitted.

Theorem 3.3. *A is a $(\in \wedge q, \in \vee q)$-intuitionistic convex fuzzy set of E if and only if for any $x, y \in E$ and $\lambda \in [0,1]$,*

(1) $\mu_A((1-\lambda)x+\lambda y) \geq \mu_A(x) \wedge \mu_A(y) \wedge 0.5$ or $\mu_A((1-\lambda)x+\lambda y) \vee 0.5 \geq \mu_A(x) \wedge \mu_A(y)$. *(3.1)*

(2) $\nu_A((1-\lambda)x+\lambda y) \leq \nu_A(x) \vee \nu_A(y) \vee 0.5$ or $\nu_A((1-\lambda)x+\lambda y) \wedge 0.5 \leq \nu_A(x) \vee \nu_A(y)$. *(3.2)*

Proof. "$\Rightarrow$" (1)Suppose that $\mu_A((1-\lambda)x+\lambda y) \vee 0.5 < t = \mu_A(x) \wedge \mu_A(y)$, then $\mu_A(x) \geq t > 0.5$,$\mu_A(y) \geq t > 0.5$. Thus $[((1-\lambda)x_{0.5} + \lambda y_{0.5}) \in \vee qA] \geq [x_{0.5} \in \wedge qA] \wedge [y_{0.5} \in \wedge qA] = 1$, which implies that $\mu_A((1-\lambda)x+\lambda y) \geq 0.5$ or $\mu_A((1-\lambda)x+\lambda y) + 0.5 > 1$. Consequently, $\mu_A((1-\lambda)x+\lambda y) \geq 0.5 \geq \mu_A(x) \wedge \mu_A(y) \wedge 0.5$. (2)Suppose that $\nu_A((1-\lambda)x+\lambda y) \wedge 0.5 > t = 1 - s = \nu_A(x) \vee \nu_A(y)$, then $s \leq 1 - \nu_A(x), s \leq 1 - \nu_A(y)$ and $s > 0.5$. Thus $[((1-\lambda)x_{0.5} + \lambda y_{0.5}) \in \vee qA] \geq [x_{0.5} \in \wedge qA] \wedge [y_{0.5} \in \wedge qA] \geq 1/2$, which implies that $0.5 \leq 1 - \nu_A((1-\lambda)x+\lambda y)$ or $\nu_A((1-\lambda)x+\lambda y) < 0.5$. Consequently,$\nu_A((1-\lambda)x+\lambda y) \leq 0.5 \leq \nu_A(x) \vee \nu_A(y) \vee 0.5$.

"$\Leftarrow$" For any $x, y \in E$ and $s, t \in (0,1]$, put $a = [x_s \in \wedge qA] \wedge [y_t \in \wedge qA]$. In the case $a = 1$, then $\mu_A(x) \geq s, \mu_A(x) > 1 - s, \mu_A(y) \geq t$ and $\mu_A(y) > 1 - t$, thus $\mu_A(x) \wedge \mu_A(y) > 0.5$. Suppose that $[((1-\lambda)x_s + \lambda y_t) \in \vee qA] \leq 1/2$, then $\mu_A((1-\lambda)x+\lambda y) < s \wedge t$ and $\mu_A((1-\lambda)x+\lambda y) \leq 1 - s \wedge t$, thus $\mu_A((1-\lambda)x+\lambda y) < 0.5 < \mu_A(x) \wedge \mu_A(y)$. Furthermore, $\mu_A((1-\lambda)x+\lambda y) < \mu_A(x) \wedge \mu_A(y) \wedge 0.5$ and $\mu_A((1-\lambda)x+\lambda y) \vee 0.5 < \mu_A(x) \wedge \mu_A(y)$, which is a contradiction to (3.1). Consequently,$[((1-\lambda)x_s + \lambda y_t) \in \vee qA] = 1$. In the case $a = 1/2$, then $1 - \nu_A(x) \geq s > \nu_A(x)$ or $1 - \nu_A(y) \geq t > \nu_A(y)$, thus $\nu_A(x) \vee \nu_A(y) < 0.5$. Suppose that $[((1-\lambda)x_s + \lambda y_t) \in \vee qA] = 0$, then $\nu_A((1-\lambda)x+\lambda y) \geq s \wedge t > 1 - \nu_A((1-\lambda)x+\lambda y)$, thus $\nu_A((1-\lambda)x + \lambda y) > 0.5$, furthermore $\nu_A((1-\lambda)x+\lambda y) \wedge 0.5 = 0.5 > \nu_A(x) \vee \nu_A(y)$ and $\nu_A((1-\lambda)x+\lambda y) > \nu_A(x) \vee \nu_A(y) \vee 0.5$, which is a contradiction to (3.2). Consequently, $[((1-\lambda)x_s + \lambda y_t) \in \vee qA] \geq 1/2$. From the above we have $[((1-\lambda)x_s + \lambda y_t) \in \vee qA] \geq [x_s \in \wedge qA] \wedge [y_t \in \wedge qA]$. This shows that A is a $(\in \wedge q, \in \vee q)$-intuitionistic convex fuzzy set of E.

By analogy with the results of the intuitionistic fuzzy subgroups in the paper[2], we can have the following theorems.

Theorem 3.4. *Let A be a (α, β)-intuitionistic convex fuzzy set of E, if $\alpha \neq \in \wedge q$, then $A_{\underline{0}}$ is a 3-valued convex fuzzy set, i.e.$\forall x, y \in A$ and $0 \leq \lambda \leq 1$,*

$$A_{\underline{0}}((1-\lambda)x+\lambda y) \geq A_{\underline{0}}(x) \wedge A_{\underline{0}}(y).$$

Theorem 3.5. *Let A be a (α, β)-intuitionistic convex fuzzy set of E. For $x \in E$, let $A(x) = (\mu_A(x), \nu_A(x))$. If $\mu_A(x) > 0$, then we have $A(x) = (1,0)$ for $(\alpha, \beta) \in \{(\in, q), (\in, \in \wedge q), (\in \vee q, q), (\in \vee q, \in \wedge q), (q, \in), (q, \in \wedge q), (\in \vee q, \in)\}$.*

Theorem 3.6. *If A is a (q,q)-intuitionistic convex fuzzy set of E,then*

(1) $\mu_A(x) > 0 \Rightarrow A(x) = (\mu_A(0), \nu_A(0))$;
(2) $\mu_A(x) = 0, \nu_A(x) < 1 \Rightarrow A(x) = (0, \nu_A(0))$.

Theorem 3.7. *Let A be a $(q, \in \vee q)$-intuitionistic convex fuzzy set of E, and $H = \{x|x \in E, \mu_A(x) > 0\}$,$K = \{x|x \in E, \mu_A(x) = 0, \nu_A(x) < 1\}$, then*

(1) If $\mu_A(x)$ is not a constant on H, then for any $x \in H, A(x) \geq (0.5, 0.5)$, i.e.$\mu_A(x) \geq 0.5, \nu_A(x) \leq 0.5$;
(2) If $\nu_A(x)$ is not a constant on K, then for any $x \in K, \nu_A(x) \leq 0.5$.

Theorem 3.8. *If A is a $(\in \vee q, \in \vee q)$-intuitionistic convex fuzzy set of E, then A is a $(\in, \in \vee q)$-intuitionistic convex fuzzy set of E.*

Theorem 3.9. *Let A be a $(\in \wedge q, \beta)$-intuitionistic convex fuzzy set of E, and $N = \{x|x \in E, \nu_A(x) > 0.5\}$, where $\beta \in \{q, \in \wedge q\}$. Then for any $x \in N, A(x) = (\mu_A(0), \nu_A(0))$, i.e. A is a constant on N.*

4 $(s,t]$-Intuitionistic Convex Fuzzy Set and Their Operations

Definition 4.1. *Let $s,t \in [0,1]$ and $s < t$. A is called a $(s,t]$-intuitionistic convex fuzzy sets of E, if*

(1) $\mu_A((1-\lambda)x + \lambda y) \vee s \geq \mu_A(x) \wedge \mu_A(y) \wedge t$,
(2) $\nu_A((1-\lambda)x + \lambda y) \wedge (1-s) \leq \nu_A(x) \vee \nu_A(y) \vee (1-t)$,

whenever $0 \leq \lambda \leq 1$.We denote the class of all $(s,t]$-intuitionistic convex fuzzy sets of E by $(s,t] - \mathcal{IF}(E)$.

Theorem 4.1. *A is a $(s,t]$-intuitionistic convex fuzzy set of E if and only if A_a is a 3-valued convex fuzzy set of E for any $a \in (s,t]$.*

Proof. "$\Rightarrow$" Let $a \in (s,t]$.(1)If $A_a(x) \wedge A_a(y) = 1$, then $\mu_A(x) \geq a > s, \mu_A(y) \geq a > s$. By $\mu_A((1-\lambda)x + \lambda y) \vee s \geq \mu_A(x) \wedge \mu_A(y) \wedge t \geq a$, we have that $\mu_A((1-\lambda)x + \lambda y) \geq a$, which implies that $A_a((1-\lambda)x + \lambda y) = 1$.(2)If $A_a(x) \wedge A_a(y) = 1/2$, then $1 - \nu_A(x) \geq a, 1 - \nu_A(y) \geq a$, thus $\nu_A(x) \vee \nu_A(y) \leq 1 - a < 1 - s$. By $\nu_A((1-\lambda)x + \lambda y) \wedge (1-s) \leq \nu_A(x) \vee \nu_A(y) \vee (1-t) \leq 1 - a$, we have that $\nu_A((1-\lambda)x + \lambda y) \leq 1 - a$, which implies that $A_a((1-\lambda)x + \lambda y) \geq 1/2$. From the above we have $A_a((1-\lambda)x + \lambda y) \geq A_a(x) \wedge A_a(y)$ for any $x, y \in E$. This leads to a conclusion that A_t is a 3-valued convex fuzzy set of E for any $t \in [0,1]$.

"$\Leftarrow$" (1)If $\mu_A((1-\lambda)x + \lambda y) \vee s < a = \mu_A(x) \wedge \mu_A(y) \wedge t$, then $\mu_A(x) \geq a, \mu_A(y) \geq a$, and $a \in (s,t]$, we have$A_a((1-\lambda)x + \lambda y) \geq A_a(x) \wedge A_a(y) = 1$ and hence one has $\mu_A((1-\lambda)x + \lambda y) \geq a$, but this is a contradiction to $\mu_A((1-\lambda)x + \lambda y) < a$. Consequently, $\mu_A((1-\lambda)x + \lambda y) \vee s \geq \mu_A(x) \wedge \mu_A(y) \wedge t$.

(2)If $\nu_A((1-\lambda)x+\lambda y)\wedge(1-s)>1-a=\nu_A(x)\vee\nu_A(y)\vee(1-t)$, then $a\in(s,t], 1-\nu_A(x)\geq a, 1-\nu_A(y)\geq a$. Thus $A_a((1-\lambda)x+\lambda y)\geq A_a(x)\wedge A_a(y)\geq 1/2$. It follows that $\nu_A((1-\lambda)x+\lambda y)\leq 1-a$. But this is a contradiction to $\nu_A((1-\lambda)x+\lambda y)>1-a$. Consequently, $\nu_A((1-\lambda)x+\lambda y)\wedge(1-s)\leq\nu_A(x)\vee\nu_A(y)\vee(1-t)$. Therefore, A is a $(s,t]$- intuitionistic convex fuzzy set of E.

Similar to Theorem 3.2, we can have the following results.

Theorem 4.2. *Let $s,t\in[0,1]$ and $0<s<t$, then A is a $(s,t]$-intuitionistic convex fuzzy set of E if only if for any $a,b\in(0,t], x,y\in E$ and $\lambda\in[0,1]$*

$$[((1-\lambda)x_a+\lambda y_b)\in A]\geq[x_a\in\wedge q_s A]\wedge[y_b\in\wedge q_s A].$$

Theorem 4.3. *Let $s,t\in[0,1]$ and $s<t<1$, then A is a $(s,t]$-intuitionistic convex fuzzy set of E if only if for any $a,b\in(s,1], x,y\in E$ and $\lambda\in[0,1]$*

$$[((1-\lambda)x_a+\lambda y_b)\in\vee q_t A]\geq[x_a\in A]\wedge[y_b\in A].$$

We can generalize the operation properties of the convex sets to the $(s,t]$-intuitionistic convex fuzzy set. We have the following results.

Theorem 4.4. *Let A,B be the $(s,t]$-intuitionistic convex fuzzy sets of E, then $A\cap B$ is also a $(s,t]$-intuitionistic convex fuzzy set of E.*

Proof. On the one hand, $\mu_{A\cap B}((1-\lambda)x+\lambda y)\vee s=(\mu_A((1-\lambda)x+\lambda y)\wedge\mu_B((1-\lambda)x+\lambda y))\vee s=(\mu_A((1-\lambda)x+\lambda y)\vee s)\wedge(\mu_B((1-\lambda)x+\lambda y)\vee s)\geq(\mu_A(x)\wedge\mu_A(y)\wedge t)\wedge(\mu_B(x)\wedge\mu_B(y)\wedge t)=(\mu_A(x)\wedge\mu_B(x))\wedge(\mu_A(y)\wedge\mu_B(y))\wedge t=\mu_{A\cap B}(x)\wedge\mu_{A\cap B}(y)\wedge t$. On the other hand, we can similarly prove that $\nu_{A\cap B}((1-\lambda)x+\lambda y)\wedge(1-s)\leq\nu_{A\cap B}(x)\vee\nu_{A\cap B}(y)\vee(1-t)$. By the Definition 4.1, $A\cap B$ is a $(s,t]$-intuitionistic convex fuzzy sets of E.

Theorem 4.5. *Let E,F be two vector spaces, and f is a linear transformation from E to F. If $A\in(s,t]-\mathcal{IF}(E)$ and $B\in(s,t]-\mathcal{IF}(F)$, then we have that $f(A), f^{-1}(B)$ are $(s,t]$-intuitionistic convex fuzzy sets of F and E, respectively.*

Proof. By f is a linear transformation from E to F, there exist $x_1,x_2\in E$ such that $f(x_1)=y_1, f(x_2)=y_2$, whenever $y_1,y_2\in F$. For any $\lambda\in[0,1]$, we have $\mu_{f(A)}((1-\lambda)y_1+\lambda y_2)\vee s=\bigvee_{f(x)=((1-\lambda)y_1+\lambda y_2)}\mu_A(x)\vee s\geq\bigvee_{f(x_1)=y_1,f(x_2)=y_2}\mu_A((1-\lambda)x_1+\lambda x_2)\vee s=\bigvee_{f(x_1)=y_1,f(x_2)=y_2}(\mu_A((1-\lambda)x_1+\lambda x_2)\vee s)\geq\bigvee_{f(x_1)=y_1,f(x_2)=y_2}(\mu_A(x_1)\wedge\mu_A(x_2)\wedge t)=(\bigvee_{f(x_1)=y_1}\mu_A(x_1))\wedge(\bigvee_{f(x_2)=y_2}\mu_A(x_2))\wedge t=\mu_{f(A)}(y_1)\wedge\mu_{f(A)}(y_2)\wedge t$,Similarly, we can prove that $\nu_{f(A)}((1-\lambda)y_1+\lambda y_2)\wedge(1-s)\leq\nu_{f(A)}(y_1)\vee\nu_{f(A)}(y_2)\vee(1-t)$. Therefore, $f(A)$ is a $(s,t]$-intuitionistic convex fuzzy set of F. Analogously, we can have that $f^{-1}(B)$ is a $(s,t]$-intuitionistic convex fuzzy sets of E.

Theorem 4.6. *Let A, B be two $(s,t]$-intuitionistic convex fuzzy sets of E, then $A+B$ is also a $(s,t]$-intuitionistic convex fuzzy set of E.*

The proof for this is analogous to the one of Theorem 4.5.

Remark 4.1. By Theorem 4.5, we have that $-B$ is a $(s,t]$-intuitionistic convex fuzzy set of E. Obviously, $A-B$ is also a $(s,t]$-intuitionistic convex fuzzy set of E.

5 Discussions

Motivated by earlier research works [1,2,12,18,20], the definitions of (α,β)-intuitionistic convex fuzzy sets are formulated based on the concept of cut sets of intuitionistic fuzzy sets and the Lukasiewicz implication in 3-valued logic, and the $(s,t]$-intuitionistic convex fuzzy sets of E are proposed in this paper. Owing to excellent works in the paper [2], the convexity theory for an intuitionistic fuzzy set has the similar structures to the intuitionistic fuzzy subgroups is formed. Furthermore, the operations of $(s,t]$-intuitionistic convex fuzzy sets are considered in this paper. We will try to explore intuitionistic fuzzy convex analysis, intuitionistic fuzzy optimization and relationships between them in the future.

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Fuzzy-valued Ordinary Differential Equation

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Abstract. In this paper we put forward concepts of ordinary differential equation of a fuzzy-valued function for the first time, discuss the existence and uniqueness of the solution at common points by applying decomposition theorem in fuzzy sets as well, so that a solution is obtained for the equation.

Keywords: Fuzzy-valued functions; ordinary differential equation; decomposition theorem; existence and uniqueness of solution.

1 Introduction

We extend the ordinary differential equation to an interval and a fuzzy-valued case, put forward: 1) interval ordinary differential equation; 2) ordinary differential equation of a fuzzy-valued function; and obtained useful results.

2 Concepts Interval and Its Ordinary Differential Equation

Definition 1. *If we use $\mathscr{R}$ to denote a real number set, then we call the real number close interval $\bar{x} = [x^-, x^+] = \{x|x^- \leqslant x \leqslant x^+, x^-; x^+ \in \mathscr{R}\}$ an interval number, while the degenerated close interval $[x, x]$ is seen as a real number x itself ($x = 0$ is especially example).*

Definition 2. *Suppose $\bar{x}_1 = [x_1^-, x_1^+], \bar{x}_2 = [x_2^-, x_2^+]$, below " $*$ " means carrying on arithmetic "$+, -, \times, \div$" to the real number, by use of a classical expansion principle, we have*

$$\bar{x}_1 * \bar{x}_2 = \{z|\exists(x_1, x_2) \in [x_1^-, x_1^+] \times [x_2^-, x_2^+], z = x_1 * x_2\}.$$

If the income as a result is still a close interval number, we say an operation of $\mathscr{R}$ given by the formula above. When " $*$ " denotes division, $0 \in \bar{x}_2$ is an exception.

The relevant definition of continuum and differentiable in $y = f(x)$ at [a,b] can be seen in [1].

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Definition 3. *Suppose*

$$\bar{F}:[a,b]\to I_{\mathscr{R}}, I_{\mathscr{R}}=\{[x_1',x_2']|x_1'\leqslant x_2', x_1',x_2'\in\mathscr{R}\},$$
$$x\to[x_1^-,x_2^+]$$

then $x\to\bar{F}(x)=[F^-(\cdot),F^+(\bigstar)]$ *is an interval function, here* $x_1'=F^-(\cdot), x_2'=F^+(\bigstar)$, *and "*$(\cdot)$*" denotes* $(x, f^-(x), \dfrac{df^-(x)}{dx}),\cdots,\dfrac{d^n f^-(x)}{dx^n})$, *"*$(\bigstar)$*" denotes* $(x,f^+(x),\dfrac{df^+(x)}{dx}),\cdots,\dfrac{d^n f^+(x)}{dx^n})$, *and* F^-, F^+ *with* f^-, f^+ *are all the ordinary functions on* $[a,b]$, *hence* $\forall\in[a,b], f^-(x)\leqslant f^+(x), F^-(\cdot)\leqslant F^+(\bigstar), f^-(x)$, $f^+(x)$, *such that* $F^-(\cdot), F^+(\bigstar)$ *continues at [a,b], then call* $\bar{F}(x)$ *continuous at [a,b].*

Definition 4. *Suppose* $\bar{f}(x)$ *to be a function defined at* $[a,b]$, *and at* $x_0\in[a,b]$, *if there exists ordinary derivatives* $\dfrac{d^n f^-(x_0)}{dx^n}$ *and* $\dfrac{d^n f^+(x_0)}{dx^n}$, *we call that interval function* $\bar{f}(x)$ *there exists* n*-th derivative*

$$[\min\{\frac{d^n f^-(x_0)}{dx^n},\frac{d^n f^+(x_0)}{dx^n}\},\max\{\frac{d^n f^-(x_0)}{dx^n},\frac{d^n f^+(x_0)}{dx^n}\}]$$

at x_0. *While* $[\min\{\dfrac{d^n f^-(x)}{dx^n},\dfrac{d^n f^+(x)}{dx^n}\},\max\{\dfrac{d^n f^-(x)}{dx^n},\dfrac{d^n f^+(x)}{dx^n}\}]$ *is an* n*-th derivative function of* $f(x)$ *at* x.

Definition 5. *The equation with unknown interval derivative is called an interval differential equation, the differential equation containing an unknown is called an interval ordinary differential equation, i.e.,*

$$\frac{d\bar{f}(x)}{dx}=\bar{f}(x)\triangleq[\frac{df^-(x)}{dx},\frac{df^+(x)}{dx})]=[f^-(x),f^+(x)], \tag{1}$$

calling it the 1*-st interval ordinary differential equation, but*

$$\begin{aligned}&\frac{d^n\bar{f}(x)}{dx^n}+\cdots+a_{n-1}(x)\frac{d\bar{f}(x)}{dx}+a_n(x)\bar{f}(x)=0\\ &\triangleq[\frac{d^n f^-(x)}{dx^n},\frac{d^n f^+(x)}{dx^n})]+\cdots+a_{n-1}(x)[\frac{df^-(x)}{dx},\frac{df^+(x)}{dx})]\\ &\quad+a_n(x)[f^-(x),f^+(x)]=[0,0]\end{aligned} \tag{2}$$

is called the n*-th interval ordinary differential equation, where* $a_1(x),\cdots,a_{n-1}(x)$, $a_n(x)$ *are known functions.*

The following functions to discuss are all supposed to be the same order derivable [1] (the antitone derivable can be similarly discussed).

Definition 6. *If a function $\bar{\varphi}(x)$ is placed into (1) or (2), such that it is identical, then $\bar{\varphi}(x)$ is called an interval solution to them. Seeking the interval solutions to (1) or (2) is called solving interval differential equation. Here $\bar{\varphi}(x) = [\varphi^-(x), \varphi^+(x)]$, where φ^-, φ^+ are ordinary functions.*

Definition 7. *The fixed solutions problem is*

$$\begin{cases} \bar{F}(x, \bar{f}(x), \frac{d\bar{f}(x)}{dx}, \frac{d^2\bar{f}(x)}{dx^2}, \cdots, \frac{d^n\bar{f}(x)}{dx^n}) = 0 & (3) \\ \bar{f}(x_0) = \bar{f}_0, \frac{d\bar{f}(x_0)}{dx} = \bar{f}_0^{(1)}, \cdots, \frac{d^{n-1}\bar{f}(x_0)}{dx^{n-1}} = \bar{f}_0^{(n-1)} & (4) \end{cases}$$

$$\triangleq \begin{cases} \bar{F}([x,x], [f^-(x), f^+(x)], [\frac{df^-(x)}{dx}, \frac{df^+(x)}{dx}], \cdots, [\frac{d^nf^-(x)}{dx^n}, \frac{d^nf^+(x)}{dx^n}]) \\ \qquad = [0,0], \\ [f^-(x_0), f^+(x_0)] = [f_0^-, f_0^+], [\frac{df^-(x_0)}{dx}, \frac{df^+(x_0)}{dx}] = [f_0^{(1)-}, f_0^{(1)+}], \cdots, \\ [\frac{d^{n-1}f^-(x_0)}{dx^{n-1}}, \frac{d^{n-1}f^+(x_0)}{dx^{n-1}}] = [f_0^{-(n-1)}, f_0^{+(n-1)}], \end{cases}$$

a solution satisfying (3) and (4), called an interval special one to the fixed solutions problem.

3 Ordinary Differential Equation of A Fuzzy-Valued Function

Definition 8. *Let $\tilde{A} \in \mathscr{F}(\mathscr{R})$ be a fuzzy subset on $\mathscr{R}$. If, for $\forall\alpha \in [0,1], A_\alpha = [A_\alpha^-, A_\alpha^+]$ with $A_1 \neq \phi$, then $\tilde{A}$ is called a fuzzy number, whole sets of which are written as $\mathscr{F}(\mathscr{R})$.*

Definition 9. *If*

1) *There exists a unique $y_0 \in \mathscr{R}$, such that $\mu_{\tilde{A}}(y_0) = 1$;*
2) *$\mu_{\tilde{A}}(y)$ continues with respect to y;*
3) *$\exists[y_1, y_2]$, such that $S(\tilde{A}) \subset [y_1, y_2]$;*
4) *$\forall s, \forall t > s, \forall y \in (s,t)$, there is $\mu_{\tilde{A}}(y) > \min(\mu_{\tilde{A}}(s), \mu_{\tilde{A}}(t))$.*

Then $\tilde{A} \in \mathscr{F}(\mathscr{R})$ is called a convex normal fuzzy number.[3,4]

Definition 10. *Let $\tilde{f} : [a,b] \to \mathscr{F}(\mathscr{R}), x \mapsto \tilde{f}(x)$. Then $\tilde{f}$ is called a fuzzy-valued function defined at $[a,b]$, and when $\tilde{f}(x)$ is a convex normal fuzzy number, $\tilde{f}$ is called a convex normal fuzzy function.*

Definition 11. *Let $\bar{f}_\alpha : [a,b] \to I_{\mathscr{R}}, x \mapsto \bar{f}_\alpha(x) \triangleq [\tilde{f}(x)]_\alpha$. Then $\bar{f}_\alpha$ is called an $\alpha-$cut function of $\tilde{f}$, if and only if $\forall\alpha \in (0,1], \bar{f}_\alpha$ continues, so as $\tilde{f}$.*

The operation of fuzzy numbers is defined by fuzzy extension principle [2].

If $f(\tilde{A}^{(1)}, \tilde{A}^{(2)}, \cdots, \tilde{A}^{(m)}) \triangleq \bigcup_{\alpha\in(0,1]} \alpha f(A_\alpha^{(1)}, A_\alpha^{(2)}, \cdots, A_\alpha^{(m)})$, we have $\tilde{A}, \tilde{B} \in \mathscr{F}(\mathscr{R})$, then

1) $(\tilde{A} \pm \tilde{B})_\alpha = A_\alpha \pm B_\alpha$;
2) $(k\tilde{A})_\alpha = kA_\alpha$.

Definition 12. *Let $\tilde{f}(x)$ be defined at $[a, b]$ and $\bar{f}_\alpha(x)$ be differentiable $\forall\alpha \in (0, 1]$. Then $\frac{d\tilde{f}(x)}{dx} = \bigcup_{\alpha\in(0,1]} \alpha \frac{d\bar{f}_\alpha(x)}{dx}$ is called a fuzzy-valued derivative at ordinary point x.*

In the following, it is supposed that $\tilde{f}(x)$ is the same order derivable at $[a, b]$ (similar discussion by antitone derivable), then, the fuzzy-valued derivative can be simply expressed as $\frac{d\tilde{f}(x)}{dx} = \bigcup\limits_{\alpha\in(0,1]} \alpha\{\frac{df_\alpha^-(x)}{dx}, \frac{df_\alpha^+(x)}{dx}\}$.

Definition 13. *Let $\tilde{f} : [a, b] \times [c, d] \to \mathscr{F}(\mathscr{R}), (x, y) \mapsto \tilde{f}(x, y)$ be defined as a binary fuzzy-valued function at $[a, b] \times [c, d]$; its $\alpha-$cut function is $\bar{f}_\alpha$:*

$$x \mapsto \bar{f}_\alpha(x, y) \triangleq [\tilde{f}(x, y)]_\alpha = [f_\alpha^-(x, y), f_\alpha^+(x, y)].$$

If for $\forall\alpha \in (0, 1]$, f_α^- and f_α^+ differentiable at (x, y), then the partial derivative of $\tilde{f}$ at (x, y) is defined as:

$$\frac{\partial\tilde{f}(x, y)}{\partial x} = \bigcup_{\alpha\in(0,1]} \alpha\{\frac{\partial\tilde{f}_\alpha^-(x, y)}{\partial x}, \frac{\partial\tilde{f}_\alpha^+(x, y)}{\partial x}\},$$
$$\frac{\partial\tilde{f}(x, y)}{\partial y} = \bigcup_{\alpha\in(0,1]} \alpha\{\frac{\partial\tilde{f}_\alpha^-(x, y)}{\partial y}, \frac{\partial\tilde{f}_\alpha^+(x, y)}{\partial y}\}.$$

Theorem 1. *If $\tilde{f}_1(x), \tilde{f}_2(x)$ is the same order derivable, $\frac{d\tilde{f}_1(x)}{dx}, \frac{d\tilde{f}_2(x)}{dx}$ is a normal convex, then*

1) $\frac{d}{dx}(\tilde{f}_1(x) \pm \tilde{f}_2(x)) = \frac{d\tilde{f}_1(x)}{dx} \pm \frac{d\tilde{f}_2(x)}{dx}$;
2) $\frac{d}{dx}(k\tilde{f}(x)) = k\frac{d\tilde{f}(x)}{dx}$.

Definition 14. *The equation with unknown fuzzy-valued derivative is called a fuzzy-valued differential equation,*

$$\frac{d\tilde{f}(x)}{dx} = \tilde{f}(x) \triangleq \bigcup_{\alpha\in(0,1]} \alpha\frac{d\bar{f}_\alpha(x)}{dx} = \bigcup_{\alpha\in(0,1]} \alpha\bar{f}_\alpha(x) \tag{5}$$

is called the 1-st fuzzy-valued differential one, and

$$\begin{aligned} &\frac{d^n\tilde{f}(x)}{dx^n} + \cdots + a_{n-1}(x)\frac{d\tilde{f}(x)}{dx} + a_n(x)\tilde{f}(x) = 0 \triangleq \\ &\bigcup_{\alpha\in(0,1]} \alpha\frac{d^n\bar{f}_\alpha(x)}{dx^n} + \cdots + a_{n-1}(x) \bigcup_{\alpha\in(0,1]} \alpha\frac{d\bar{f}_\alpha(x)}{dx} + \\ &a_n(x) \bigcup_{\alpha\in(0,1]} \alpha\bar{f}_\alpha(x) = 0 \end{aligned} \tag{6}$$

is called the nth fuzzy-valued differential one, where $a_i(x)(1 \leqslant i \leqslant n)$ is a known ordinary function (also a fuzzy-valued function), with $a_n(x) \neq 0$.

Definition 15. *If a fuzzy-valued function $\tilde{f}(x)$ is substituted to (5) or (6), such that it is identical, then the function $\tilde{f}(x)$ is a solution to it. The process of finding solution to (5) or (6) is called finding a solution to fuzzy-valued differential equation.*

Definition 16. *Let a problem of fuzzy-valued fixed solution be*

$$\begin{cases} \tilde{F}(x, \tilde{f}(x), \dfrac{d\tilde{f}(x)}{dx}, \dfrac{d^2\tilde{f}(x)}{dx^2}, \cdots, \dfrac{d^n\tilde{f}(x)}{dx^n}) = 0, \\ \tilde{f}(x_0) = \tilde{f}_0, \dfrac{d\tilde{f}(x_0)}{dx}) = \tilde{f}_0', \cdots, \dfrac{d^{n-1}\tilde{f}(x_0)}{dx^{n-1}} = \tilde{f}_0^{(n-1)}. \qquad (a) \end{cases} \tag{7}$$

Then a solution satisfying (7) is called a special solution, while an arbitrary expression of the solution to (7) $\tilde{f}(x) = \tilde{\varphi}(x, \tilde{C}_0, \tilde{C}_1, \cdots, \tilde{C}_{n-1})$ is called a fuzzy-valued general solution to problem. Here

$$(7) \triangleq$$

$$\begin{cases} \bar{F}(x, \bigcup\limits_{\alpha\in(0,1]} \alpha\bar{f}_\alpha(x), \bigcup\limits_{\alpha\in(0,1]} \alpha\dfrac{d\bar{f}_\alpha(x)}{dx}, \cdots, \bigcup\limits_{\alpha\in(0,1]} \alpha\dfrac{d^n\bar{f}_\alpha(x)}{dx^n} = 0, \\ \bigcup\limits_{\alpha\in(0,1]} \alpha\bar{f}_\alpha(x_0) = \bigcup\limits_{\alpha\in(0,1]} \alpha\bar{f}_{0\alpha}, \\ \bigcup\limits_{\alpha\in(0,1]} \alpha\dfrac{d\bar{f}_{0\alpha}(x_0)}{dx} = \bigcup\limits_{\alpha\in(0,1]} \alpha\bar{f}_{0\alpha}', \\ \cdots\cdots\cdots\cdots\cdots \\ \bigcup\limits_{\alpha\in(0,1]} \alpha\dfrac{d^{(n-1)}\bar{f}_{0\alpha}(x_0)}{dx^{(n-1)}} = \bigcup\limits_{\alpha\in(0,1]} \alpha\bar{f}_{0\alpha}^{(n-1)}, \end{cases}$$

$$\tilde{f}(x) \triangleq \bigcup_{\alpha\in(0,1]} \alpha\bar{\varphi}_\alpha(x, \bar{C}_{0\alpha}, \bar{C}_{1\alpha}, \cdots, \bar{C}_{(n-1)\alpha}).$$

If at least to a certainty range arbitrary given by initial value condition (7)(a), a specifically fixed value of arbitrary fuzzy number $\tilde{C}_0, \tilde{C}_1, \cdots, \tilde{C}_{n-1}$ can all be found, such that the corresponding solution satisfies this condition.

Theorem 2. *(existence theorem on implicit function) Suppose neighborhood at point $(x_1^0, x_2^0, \cdots, x_n^0, \tilde{u}_0)$, if*

1) $\tilde{F}(x_1, x_2, \cdots, x_n, \tilde{u})$ is continuously convex normal fuzzy-valued function, with $\tilde{F}(x_1^0, x_2^0, \cdots, x_n^0, \tilde{u}_0) = 0$;

2) $\dfrac{\partial\tilde{F}}{\partial u}(x_1^0, x_2^0, \cdots, x_n^0, \tilde{u}_0)$ is the same order fuzzy-valued partial derivative and continuous, with $\dfrac{\partial\tilde{F}}{\partial u}(x_1^0, x_2^0, \cdots, x_n^0, \tilde{u}_0) \neq 0$.

Then neighborhood at this point, $\tilde{F}(x_1, x_2, \cdots, x_n, \tilde{u}) = 0$ contains a unique fuzzy-valued solution $\tilde{u} = \tilde{\varphi}(x_1, x_2, \cdots, x_n)$.

Proof. Because

$$\tilde{F}(x_1, x_2, \cdots, x_n, \tilde{u}) = \bigcup_{\alpha\in(0,1]} \alpha\bar{F}_\alpha(x_1, x_2, \cdots, x_n, \bar{u}_\alpha),$$

$$\frac{\partial\tilde{F}}{\partial u}(x_1, x_2, \cdots, x_n, \tilde{u}) = \bigcup_{\alpha\in(0,1]} \alpha\frac{\partial\bar{F}_\alpha}{\partial u}(x_1, x_2, \cdots, x_n, \bar{u}_\alpha),$$

again $\frac{\partial\tilde{F}}{\partial u}(x_1^0, x_2^0, \cdots, x_n^0, \tilde{u}_0) \neq 0$, we know the following according to the assumption

$$\bigcup_{\alpha\in(0,1]} \alpha[\frac{\partial\bar{F}_\alpha^-}{\partial u}(x_1^0, x_2^0, \cdots, x_n^0, \bar{u}_{0\alpha}^-), \frac{\partial\bar{F}_\alpha^+}{\partial u}(x_1^0, x_2^0, \cdots, x_n^0, \bar{u}_{0\alpha}^+)] \neq 0,$$

i.e.,

$$\bigcup_{\alpha\in(0,1]} \alpha\frac{\partial F_\alpha^-}{\partial u}(x_1^0, x_2^0, \cdots, x_n^0, u_{0\alpha}^-) \neq 0,$$

$$\bigcup_{\alpha\in(0,1]} \alpha\frac{\partial F_\alpha^+}{\partial u}(x_1^0, x_2^0, \cdots, x_n^0, u_{0\alpha}^+) \neq 0,$$

with $\forall\alpha \in (0,1]$, all containing

$$\bar{F}_\alpha(x_1^0, x_2^0, \cdots, x_n^0, \bar{u}_{0\alpha}) \neq 0,\ \frac{\partial\bar{F}_\alpha}{\partial u}(\bar{F}'_u)_\alpha(x_1^0, x_2^0, \cdots, x_n^0, \bar{u}_{0\alpha}) \neq 0,$$

and continuing near at $(x_1^0, x_2^0, \cdots, x_n^0, \bar{u}_{0\alpha})$, such that a unique interval solution $\bar{u}_\alpha = \bar{\varphi}_\alpha(x_1^0, x_2^0, \cdots, x_n^0)$ exists in $\bar{F}_\alpha(x_1^0, x_2^0, \cdots, x_n^0, \bar{u}_{0\alpha}) = 0$ at this point.

Therefore at this point, $\tilde{F}(x_1^0, x_2^0, \cdots, x_n^0, \tilde{u}_0) = 0$ contains a unique fuzzy-valued solution $\tilde{u} = \bigcup_{\alpha\in(0,1]} \alpha\bar{\varphi}_\alpha(x_1, x_2, \cdots, x_n)$.

If we solved $\frac{d^n\tilde{f}(x)}{dx^n}$ from relation form (7), then we obtain an equation like

$$\frac{d^n\tilde{y}(x)}{dx^n} = \tilde{f}(x, \tilde{y}(x), \frac{d\tilde{y}(x)}{dx}, \cdots, \frac{d^{n-1}\tilde{y}(x)}{dx^{n-1}}), \tag{8}$$

where $\tilde{f}$ is a known fuzzy-valued function dependent upon $n+1$ variation x, called a normal type fuzzy-valued differential equation.

Theorem 3. *If by considering the region,* $\frac{\partial\tilde{F}}{\partial\tilde{f}^{(n)}(x)} \neq 0$*, then (7) contains a normal type fuzzy-valued differential equation (8).*

Proof. Because at the considered region $\frac{\partial \tilde{F}}{\partial \tilde{f}^{(n)}(x)} \neq 0$, while

$$\frac{\partial \tilde{F}}{\partial \tilde{f}^{(n)}(x)} \neq 0 \Longleftrightarrow \bigcup_{\alpha\in(0,1]} \alpha \frac{\partial}{\partial x}(\bar{F}_\alpha^{(n-1)}(x)) \neq 0,$$

and according to existence theorem in the fuzzy-valued implicit function, the theorem holds.

Theorem 4. *Any of the nth normal type fuzzy-valued differential equation (8) is equivalent to a 1-order one*

$$\begin{cases} \frac{d\tilde{y}(x)}{dx} = \tilde{y}_1(x), \\ \frac{d\tilde{y}_1(x)}{dx} = \tilde{y}_2(x), \\ \quad\vdots \\ \frac{d\tilde{y}_{n+1}}{dx} = \tilde{f}(x; \tilde{y}(x), \tilde{y}_1(x), \cdots, \tilde{y}_{n-1}(x)). \end{cases} \tag{9}$$

Proof. Because (8) $\Longleftrightarrow$

$$\bigcup_{\alpha\in(0,1]} \alpha \frac{d^n \bar{y}_\alpha(x)}{dx^n} = \bigcup_{\alpha\in(0,1]} \alpha \bar{f}_\alpha(x, \bar{y}_\alpha(x), \frac{d\bar{y}_\alpha(x)}{dx}, \cdots, \frac{d^{n-1}\bar{y}_\alpha(x)}{dx^{n-1}}).$$

Suppose $\tilde{y}(x) = \tilde{\varphi}_(x)$ to be a solution to (8) in interval $I = [a, b]$, let $\tilde{\varphi}_1(x) = \frac{d\tilde{\varphi}(x)}{dx}, \cdots, \tilde{\varphi}_{n-1}(x) = \frac{d^{n-1}\tilde{\varphi}(x)}{dx^{n-1}}$. Then it is equivalent to

$$\begin{cases} \bigcup_{\alpha\in(0,1]} \alpha \frac{d\bar{\varphi}_\alpha(x)}{dx} = \bigcup_{\alpha\in(0,1]} \alpha \bar{\varphi}_{1\alpha}(x), \\ \bigcup_{\alpha\in(0,1]} \alpha \frac{d\bar{\varphi}_{1\alpha}(x)}{dx} = \bigcup_{\alpha\in(0,1]} \alpha \bar{\varphi}_{2\alpha}(x), \\ \quad\vdots \\ \bigcup_{\alpha\in(0,1]} \alpha \frac{d\bar{\varphi}_{(n-1),\alpha}(x)}{dx} = \bigcup_{\alpha\in(0,1]} \alpha \bar{f}_\alpha(x, \varphi_\alpha(x), \bar{\varphi}_{1\alpha}(x), \cdots, \bar{\varphi}_{(n-1)\alpha}(x)), \end{cases}$$

i.e.,

$$\begin{cases} \frac{d\tilde{\varphi}(x)}{dx} = \tilde{\varphi}_1(x), \\ \frac{d\tilde{\varphi}_1(x)}{dx} = \tilde{\varphi}_2(x), \\ \quad\vdots \\ \frac{d\tilde{\varphi}_{n-1}(x)}{dx} = \tilde{f}(x, \tilde{\varphi}(x), \tilde{\varphi}_1(x), \cdots, \tilde{\varphi}_{n-1}(x)). \end{cases}$$

Because $\tilde{\varphi}_i(x) = \bigcup_{\alpha\in(0,1]} \alpha\bar{\varphi}_{i\alpha}(x), (1 \leqslant i \leqslant n-1)$ is an unknown fuzzy-valued function, it shows that,

$$\forall\alpha, \bar{y}_\alpha(x) = \bar{\varphi}_\alpha(x), \bar{y}_{1\alpha}(x) = \frac{d\bar{\varphi}_\alpha(x)}{dx}, \cdots, \bar{y}_{(n-1)\alpha}(x) = \frac{d^{n-1}\bar{\varphi}_\alpha(x)}{dx^{n-1}}$$

is equivalent to a solution corresponding to a classical problem in interval $I = [a,b]$. Thereby, $\tilde{y}(x) = \tilde{\varphi}(x), \tilde{y}_1(x) = \frac{d\tilde{\varphi}(x)}{dx}, \cdots, \tilde{y}_{n-1}(x) = \frac{d\tilde{\varphi}^{n-1}(x)}{dx^{n-1}}$ is equivalent to a solution to (9) in interval $I = [a,b]$. Therefore, the theorem is certificated.

Corollary 1. *The arbitrary initial value problem of (8) is equivalent to the interval value problem of 1-st normal fuzzy-valued differential equations.*

Only the 1st case is discussed below, because the similar conclusion can be got to the others.

Definition 17. $d(\tilde{x},\tilde{y}) = d_H(\tilde{x},\tilde{y})$ *is Hausdorff measure induced by the measure d, defined as*

$$d_H(\tilde{x},\tilde{y}) = \begin{cases} \max(\sup\{d(x,\tilde{x})|\ x\in\tilde{x}\}, \sup\{d(y,\tilde{y})|y\in\tilde{y}\}), \textit{if}\ \tilde{x},\tilde{y}\neq\phi, \\ 0, \qquad \textit{if}\ \tilde{x},\tilde{y}=\phi, \\ \infty, \qquad \tilde{x}=\phi,\tilde{y}\neq\phi\ \textit{or}\ \tilde{x}\neq\phi,\tilde{y}=\phi. \end{cases}$$

When $\tilde{x},\tilde{y}$ *is a non-empty closed set of a closed region æ, we have*

$$d_H(\tilde{x},\tilde{y}) = \max\{\sup[d(x,\tilde{x})|x\in\tilde{x}], \sup[d(y,\tilde{y})|y\in\tilde{y}]\}.$$

Theorem 5. *(existence theorem of the solution) Given* $\frac{d\tilde{y}}{dx} = \tilde{f}(x,\tilde{y})$ *and the initial value* $(x_0,\tilde{y}_0)$*, with* $\tilde{f}(x,\tilde{y})$*, a convex normal fuzzy-valued function, continuing on closed region* æ : $|x-x_0| \leqslant a, d(\tilde{y},\tilde{y}_0) \subseteq [b^-,b^+](a>0, b^->0, b^+>0,$ *and* $b^-<b^+)$*, then in* $\frac{d\tilde{y}}{dx} = \tilde{f}(x,\tilde{y})$ *exists at least a fuzzy-valued solution with a value* $\tilde{y}_0$ *taken at* $x=x_0$*, at the same time, confirmed and continuing at a certain interval containing* x_0.

Proof. From existence theorem of the solution in the interval, so $\bigcup_{\alpha\in(0,1]} \alpha\frac{d\bar{y}_\alpha}{dx} = \bigcup_{\alpha\in(0,1]} \alpha\bar{f}_\alpha(x,\bar{y}_\alpha)$, contains at least a solution $\bar{y}_0$ with confirmed and continuing at certain interval of x_0. Again, $\bigcup_{\alpha\in(0,1]} \alpha\frac{d\bar{y}_\alpha}{dx} = \bigcup_{\alpha\in(0,1]} \alpha\bar{f}_\alpha(x,\bar{y}_\alpha) \Longleftrightarrow \frac{d\tilde{y}}{dx} = \tilde{f}(x,\tilde{y})$, hence the conclusion of the theorem holds.

Theorem 6. *(unique theorem of the solution) Under the condition of Theorem 5, if fuzzy variation* $\tilde{y}$ *still satisfies Lipschitz condition in non-empty closed region æ, i.e.,* $\exists N>0$*, such that any of the two values* $\tilde{y}_1,\tilde{y}_2$ *in æ always contains*

$$d_H[\tilde{f}(x,\tilde{y}_1),\tilde{f}(x,\tilde{y}_2)] \subseteq N d_H(\tilde{y}_1,\tilde{y}_2), \tag{10}$$

then a unique confirmed and continuous fuzzy-valued solution exists in

$$\begin{cases} \dfrac{d\tilde{y}}{dx} = \tilde{f}(x,\tilde{y}), \\ \tilde{f}(x,\tilde{y})|_{x=x_0,\tilde{y}=\tilde{y}_0} = \tilde{f}_0. \end{cases} \tag{11}$$

Proof. We know from the prove of Theorem 5, for arbitrary $\alpha \in (0,1]$, $\dfrac{d\bar{y}_\alpha}{dx} = \bar{f}_\alpha(x,\bar{y}_\alpha)$ continues at æ : $|x-x_0| \leqslant a, |\bar{y}_\alpha - \bar{y}_{0\alpha}| \subseteq [b^-,b^+]$. Again

$$(10) \Longleftrightarrow \bigcup_{\alpha\in(0,1]} \alpha d_H[\bar{f}_\alpha(x,\bar{y}_{1\alpha}),\bar{f}_\alpha(x,\bar{y}_{2\alpha})] \subseteq N \bigcup_{\alpha\in(0,1]} \alpha d_H(\bar{y}_{1\alpha},\bar{y}_{2\alpha})$$

for arbitrary α above, then

$$d_H[\bar{f}_\alpha(x,\bar{y}_{1\alpha}),\bar{f}_\alpha(x,\bar{y}_{2\alpha})] \subseteq N d_H(\bar{y}_{1\alpha},\bar{y}_{2\alpha}).$$

From Theorem 5, we know

$$\begin{cases} \dfrac{d\bar{y}_\alpha}{dx} = \bar{f}_\alpha(x,\bar{y}_\alpha), \\ \bar{f}_\alpha(x,\bar{y}_\alpha)|_{x=x_0,\bar{y}=\bar{y}_{0\alpha}} = \bar{f}_{0\alpha} \end{cases}$$

contains a unique confirmed and continuous interval solution $\bar{\varphi}_\alpha(x,\bar{y}_\alpha)$.

Because of arbitrariness of α at $(0,1]$, then $\tilde{f} = \bigcup\limits_{\alpha\in(0,1]} \alpha\bar{\varphi}_\alpha(x,\bar{y}_\alpha)$, that is a unique confirmed and continuous fuzzy-valued solution to (11) at æ.

Theorem 7. *Let $\tilde{f}(x,\tilde{y})$ be a convex normal fuzzy-valued function of the same order integrable. Then in $\dfrac{d\tilde{y}}{dx} = \tilde{f}(x,\tilde{y})$ exists fuzzy-valued solution $\tilde{y} = \tilde{\varphi}(x) + \tilde{c}$, where $\tilde{c}$ is a fuzzy constant.*

Proof. Since

$$\begin{aligned} &\int\Big(\frac{d\tilde{y}}{dx}\Big)dx = \int \tilde{f}(x,\tilde{y})dx \Longleftrightarrow \frac{d}{dx}\int \tilde{y}dx = \int \tilde{f}(x,\tilde{y})dx \\ &\triangleq \frac{d}{dx}\Big(\bigcup_{\alpha\in(0,1]} \alpha \int \bar{y}_\alpha dx\Big) = \bigcup_{\alpha\in(0,1]} \alpha \int \bar{f}_\alpha(x,\bar{y}_\alpha)dx, \end{aligned}$$

for $\forall\alpha \in (0,1]$, there exists $\bar{y}_\alpha = \int \bar{f}_\alpha(x,\bar{y}_\alpha)dx$. When $\bar{f}_\alpha(x,\bar{y}_\alpha)$ is the same order integrable and with same order primal function $\bar{\varphi}_\alpha(x)$, there exists

$$\bar{y}_\alpha = \bar{\varphi}_\alpha(x) + \bar{c}_\alpha \Longleftrightarrow \bigcup_{\alpha\in(0,1]} \alpha\bar{y}_\alpha = \bigcup_{\alpha\in(0,1]} \alpha(\bar{\varphi}_\alpha(x) + \bar{c}_\alpha).$$

Therefore,

$$\tilde{y}(x) = \tilde{\varphi}(x) + \tilde{c}.$$

4 Numerical Example

Consider the second order fuzzy ordinary differential equation

$$\frac{d^2\tilde{y}}{dx^2} + 2\frac{d\tilde{y}}{dx} + \tilde{y} = \tilde{0},$$

(high order can be solved similar). In accordance with above method: change it into

$$\bigcup_{\alpha\in(0,1]} \alpha\frac{d^2\bar{y}_\alpha}{dx^2} + 2\bigcup_{\alpha\in(0,1]} \alpha\frac{d\bar{y}_\alpha}{dx} + \bigcup_{\alpha\in(0,1]} \alpha\bar{y}_\alpha = \bigcup_{\alpha\in(0,1]} \alpha\bar{f}_\alpha(x,\bar{0}_\alpha) = 0. \quad (12)$$

Since (0,1] is the infinite set of very difficult to solve, we can according to experience or needs, select some of the $\alpha \in (0,1]$ (finite) to solve [5]. For the Equation (12), taking an α, the corresponding interval ordinary differential equation is

$$\alpha\frac{d^2\bar{y}_\alpha}{dx^2} + 2\alpha\frac{d\bar{y}_\alpha}{dx} + \alpha\bar{y}_\alpha = 0,$$

by which we gain access to its interval solution $\bar{y}_\alpha$. In the actual decision-making process, if a finite number of intervals selected in interval ordinary differential equation solution is not ideal, we get solutions by a weighted method until the best solution is obtained.

5 Conclusion

In summary, the interval and fuzzy-valued differential equation are promotion of classical differential equation, containing information more than the classical differential. Therefore, a series of properties of ordinary differential equation and effective solution would be easily extended to the above equation in the second category.

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Comparation between Some Approaches to Solve First Order Linear Fuzzy Differential Equation

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Abstract. This paper studies first order fuzzy differential equation with fuzzy initial value problems and discusses the relationship between the present four methods for solving fuzzy initial value problem. It proves that under certain conditions the results from these four methods are identical, This will be proved by the example given in the paper.

Keywords: Fuzzy initial value; Hukuhara differentiability; Differential inclusions; Extension principle; Depict equation.

1 Introduction

The importance of the study of differential equations from both theoretical point of view and practical application is well known. However, in some cases, such as Biological Sciences, Quality Control, Electronics Technology etc, many model parameters are uncertain, which are given by fuzzy subsets, so the study of fuzzy differential equation has become important. The concept of fuzzy derivative was first introduced by Chang and Zadeh [1], and followed by Dubois and Prade [2]. The first order linear fuzzy differential equation and fuzzy initial value problems were regularly treated by Nieto and Rodriguez-Lopez [3] and Seikkala [4]. Current research focuses on first order fuzzy differential equation with fuzzy initial value, there are mainly four methods: the first involves the Hukuhara derivative [5]; the second, suggested by Hullermeier [6],is based on a family of differential inclusions and the third solution through Zadeh extension principle[7] applied to the deterministic solution. The last one finds the solutions of fuzzy differential equation via depict equation[8,9]. For some other recent and novel approaches, see, for example [10,11,12]. This paper mainly studies the relationship between the four methods.

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Take the following first order linear fuzzy differential equation as an example to study the solution of the equation.

$$\begin{cases} x'(t) = f(t, x(t)) \\ x(t_0) = \tilde{x}_0 \end{cases} \tag{1}$$

where $\tilde{x}_0 \in R_f$,We denote by R_f the family of all the fuzzy numbers of R.

2 Basic Concepts

We place a $\sim$ over a symbol to denote a fuzzy set, so $\tilde{A}, \tilde{B}, \ldots,$ represent all fuzzy sets of R. Let us denote by R_f the class of fuzzy numbers, i.e., normal, convex, upper semi-continuous and compactly supported fuzzy subsets of the real numbers.For $\forall\lambda \in (0, 1]$,denote $[\tilde{A}]^\lambda = \{x \mid \tilde{A}(x) \geq \lambda, x \in R\}$, the $\lambda-$ level of $\tilde{A}$. For $\lambda = 0$, we have $[\tilde{A}]^0 = \{x \mid \tilde{A}(x) > 0, x \in R\}$, the support of $\tilde{A}$, denote $[\tilde{A}]^\lambda = [\underline{A}_\lambda, \bar{A}_\lambda]$, it's bounded compact interval.

Definition 2.1. *Let $\tilde{A}, \tilde{B} \in R_f$ denote $[\tilde{A}]^\lambda = [\underline{A}_\lambda, \bar{A}_\lambda], [\tilde{B}]^\lambda = [\underline{B}_\lambda, \bar{B}_\lambda]$,then*

$$D(\tilde{A}, \tilde{B}) = \sup_{\lambda \in [0,1]} max\{|\underline{A}_\lambda - \underline{B}_\lambda|, |\bar{A}_\lambda - \bar{B}_\lambda|\}$$

is called Hausdorf distance, denote by D.

3 Solution of Fuzzy Differential Equation

3.1 Solution via Hukuhara Derivative

Definition 3.1[5]. *Let $\tilde{A}, \tilde{B} \in R_f$, if there exist $\tilde{C} \in R_f$, such that $\tilde{A} = \tilde{B} + \tilde{C}$, then $\tilde{C}$ is called the Hukuhara difference of $\tilde{A}$ and $\tilde{B}$, denoted by $\tilde{C} = \tilde{A} -_H \tilde{B}$.*

Definition 3.2[5]. *Let fuzzy value function $f : [a, b] \to R_f$, and $t \in [a, b]$, we say that f is Hukuhara differentiable at t, if there exists an element $f'(t) \in R_f$,that the limits $\lim\limits_{h\to 0} \frac{f(t+h) -_H f(t)}{h}$ and $\lim\limits_{h\to 0} \frac{f(t) -_H f(t-h)}{h}$ exist and are equal to $f'(t)$. Here the limits are taken in distance D.*

Theorem 3.1[13]. *Let fuzzy value function $f : [a, b] \to R_f$ be Hukuhara differentiable and denote $[f(t)]^\lambda = [f_-^\lambda, f_+^\lambda]$. Then the boundary functions f_-^λ and f_+^λ are differentiable and $[f'(t)]^\lambda = [{f'}_-^\lambda, {f'}_+^\lambda]$.*

Theorem 3.1 gives us a procedure to solve the fuzzy differential Equation (1).

Denote

$$[x(t)]^\lambda = x_\lambda(t) = [u_\lambda(t), v_\lambda(t)], [\tilde{x}_0]^\lambda = [u_\lambda^0, v_\lambda^0]$$

and

$$[f(t, x(t))]^\lambda = [f_\lambda(t, u_\lambda(t), v_\lambda(t)), g_\lambda(t, u_\lambda(t), v_\lambda(t))]$$

and proceed as follows:

(i) Solve the differential system

$$\begin{cases} u'_\lambda(t) = f_\lambda(t, u_\lambda(t), v_\lambda(t)), & u_\lambda(0) = u_\lambda^0 \\ v'_\lambda(t) = g_\lambda(t, u_\lambda(t), v_\lambda(t)), & v_\lambda(0) = v_\lambda^0 \end{cases}$$

(ii) Ensure that $[u_\lambda(t), v_\lambda(t)]$ and $[u'_\lambda(t), v'_\lambda(t)]$ are valid level sets.
Using the Stacking Theorem[14], pile up the levels $[u_\lambda(t), v_\lambda(t)]$ to a fuzzy solution $x(t)$.

3.2 Solution via Differential Inclusions

Let us assume that fuzzy value function $f : [a, b] \times R_f \to R_f$ is obtained via Zadeh extension principle from a continuous function $h : [a, b] \times R \to R$. Now Equation (1) can be computed levelwise, i.e.,

$$[f(t, x)]^\lambda = h(t, [x]^\lambda), \quad \forall \lambda \in [0, 1]$$

for all $t \in [a, b]$, and $\lambda \in [0, 1]$. Following Diamonds [6], we interpret the fuzzy initial value problem (1) as a set of differential inclusions

$$y'_\lambda(t) = h(t, y_\lambda(t)),\ y_\lambda(0) \in [\tilde{x}_0]^\lambda \tag{2}$$

Under suitable assumptions, the attainable sets $\vartheta_\lambda(t) = \{y_\lambda(t) | y_\lambda$ is a solution of Equation (2)$\}$, which is called a solution of Equation (1).

3.3 Solution via Extension Principle

Let U be an open subset in R, consider the following equation

$$\begin{cases} x'(t) = f(t, x(t)) \\ x(t_0) = x_0 \end{cases} \tag{3}$$

where $x_0 \in U$, If there exists a solution $x(t, x_0)$ of Equation (3), and $x(t, x_0)$ is continuous on U. Then, we can define the operator: $L_t : U \to R$ by $L_t(x_0) = x(t, x_0)$, which is the unique solution of Equation (3) and it is continuous relative to x_0.

Then application of the extension principle to L_t, leads to the extension

$$\tilde{L}_t : F(U) \to F(R)$$

.
$\tilde{L}_t$ is the solution[7] of Problem (1).

3.4 Solution via Depict Equation

Definition 3.3.[13] *If the fuzzy initial value $\tilde{x}_0$ of Equation (1) is replaced by x_0 , then the equation is called depict equation.*

From Definition 3.3, the depict equation of Equation (1) is equation (3).

Theorem 3.2.[9] *Provided the solution of depict Equation (2) is $x(t, x_0)$, if $x(t, x_0)$ is monotonical increasing function with respect to argument x_0, then $x(t, x_0)$ is the solution of Equation (1).*

Theorem 3.2 gives us a procedure to solve the fuzzy differential equation, finding the solution of depict equation with crisp parameter, then crisp parameter is replaced with fuzzy parameter.

4 Relationship between These Methods

O. Kaleva and M.T. Mizukoshi had studied some relationships between these methods.

Theorem 4.1.[13] *If h of Equation (2) is nondecreasing with respect to the second argument, then fuzzy solution of Equation (1) via Hukuhara derivative and solution via differential inclusions are identical.*

Example 4.1.[13]: Consider the following fuzzy initial value problem

$$\begin{cases} x'(t) = x^2(t) \\ x(0) = \tilde{x}_0 \end{cases} \tag{4}$$

where $\tilde{x}_0$ is a triangular fuzzy numberthe membership function is

$$u_{\tilde{x}_0}(y) = \begin{cases} 3 - y, \ 2 \leqslant y \leqslant 3 \\ y - 1, \ 1 \leqslant y \leqslant 2 \\ 0, \ elsewhere \end{cases}$$

Since $h(x) = x^2$ is continuous, we can solve the equation levelwise. Since $h(x)$ is increasing when $x > 0$, we have to solve a differential system

$$\begin{cases} u'_\lambda(t) = u^2_\lambda(t), \quad u_\lambda(0) = 1 + \lambda \\ v'_\lambda(t) = v^2_\lambda(t), \quad v_\lambda(0) = 3 - \lambda \end{cases}$$

The solutions are

$$[x(t)]^\lambda = [u_\lambda(t), v_\lambda(t)] = \left[\frac{1+\lambda}{1-t-\lambda t}, \frac{3-\lambda}{1-3t-\lambda t}\right]$$

Which agrees with Theorem 4.1.

Theorem 4.2.[15] *Let U be an subset in R and $x_0 \in U$. Suppose that f is continuous, if there exists one unique solution $x(t, x_0)$ of the Equation (3) and it is continuous in U. Then, there exists $\tilde{L}_t(\tilde{x}_0)$ and $\tilde{L}_t(\tilde{x}_0) = \vartheta(\tilde{x}_0)$.*

Theorem 4.2 tells us the solution via Zadeh extension of Equation (1) and solution via differential inclusions are identical.

Consider the Example 4.1, in this case there exists an attainable set $\tilde{L}_t(\tilde{x}_0)$, the function $L_t(x_0)$ is nondecreasing with respect to x_0 Then, we have

$$[\tilde{L}_t(\tilde{x}_0)]^\lambda = L_t([\tilde{x}_0]^\lambda) = [L_t(1+\lambda), L_t(3-\lambda)] = \left[\frac{1+\lambda}{1-t-\lambda t}, \frac{3-\lambda}{1-3t-\lambda t}\right]$$

Which agrees with Theorem 4.2

Now let us discuss the relationship between the solution via Hukuhara derivative and via depict equation.

Theorem 4.3. *If $f(t, x)$ of Equation (1) is nondecreasing with respect to the second argument x, then fuzzy solution of Equation (1) via Hukuhara derivative and solution via depict equation are identical.*

Proof: Let $[x(t)]^\lambda = [\underline{x}_\lambda, \bar{x}_\lambda]$, then $[x'(t)]^\lambda = [\underline{x}'_\lambda, \bar{x}'_\lambda]$, since

$$[f(t, x(t))]^\lambda = f(t, [x(t)]^\lambda) = f(t, [\underline{x}_\lambda, \bar{x}_\lambda])$$

and $f(t, x)$ is nondecreasing with respect to the second argument x, and $\underline{x}_\lambda \leqslant \bar{x}_\lambda$, it follows that $f(t, [\underline{x}_\lambda, \bar{x}_\lambda]) = [f(t, \underline{x}_\lambda), f(t, \bar{x}_\lambda)]$, suppose $[\tilde{x}_0]^\lambda = [\underline{x}^0_\lambda, \bar{x}^0_\lambda]$, then we have

$$\begin{cases} \underline{x}'_\lambda = f(t, \underline{x}_\lambda), & \underline{x}_\lambda(0) = \underline{x}^0_\lambda \\ \bar{x}'_\lambda = f(t, \bar{x}_\lambda), & \bar{x}_\lambda(0) = \bar{x}^0_\lambda \end{cases} \tag{5}$$

Differential system (5) is consist of two ordinary differential equations, in fact (5) is one ordinary differential equation with different initial values. Because $x(t, x_0)$ is the solution of depict equation,so we have $[x(t, \tilde{x}_0)]^\lambda = x(t, [\tilde{x}_0]^\lambda) = x(t, [\underline{x}^0_\lambda, \bar{x}^0_\lambda])$, from Theorem 3.2, since $x(t, x_0)$ is nondecreasing with respect to the second argument x_0, then $x(t, [\underline{x}^0_\lambda, \bar{x}^0_\lambda]) = [x(t, \underline{x}^0_\lambda), x(t, \bar{x}^0_\lambda)]$. $x(t, \underline{x}^0_\lambda)$ is the solution of the first equation of (5), $x(t, \bar{x}^0_\lambda)$ is the solution of the second equation of (5), so the conclusion is obtained.

Theorem 4.4. *If $f(t,x)$ of Equation (1) is nondecreasing with respect to the second argument x, then above four methods are identical.*

Proof: From Theorem 4.1, Theorem 4.2, Theorem 4.3, the result follows. Let us again consider Example 4.1, now solving the Equation (4) via depict equation. Since the depict equation of Equation (4) is

$$\begin{cases} x'(t) = x^2(t) \\ x(0) = x_0 \end{cases}$$

there exists unique solution$x(t) = \frac{1}{\frac{1}{x_0} - t}$, since it is nondecreasing with respect to the argument x_0, from theorem 3.2, the solution of Equation (4) exists. Since

$$[\tilde{x}_0]^\lambda = [1 + \lambda, 3 - \lambda]$$

we have

$$[x(t)]^\lambda = \frac{1}{\frac{1}{[\tilde{x}_0]^\lambda} - t}$$

and

$$[x(t)]^\lambda = \frac{1}{\frac{1}{[1+\lambda, 3-\lambda]} - t}$$

so

$$[x(t)]^\lambda = \left[\frac{1+\lambda}{1 - t - \lambda t}, \frac{3-\lambda}{1 - 3t - \lambda t}\right]$$

Which agrees with Theorem 4.3, therefore, we may conclude that if $f(t,x)$ of Equation (1) is an increasing function, then the four methods produce the same solution.

5 Conclusion

This article briefly introduces some present methods for solving the first-order fuzzy differential equations with fuzzy initial value problems. Each method has its own characteristics. These methods via Zadeh extension and via depict equation are relatively easy, because they don't relate to the definition of derivative of fuzzy value function. The article studies the relationship between the four methods, proving that under certain conditions the results from them are identical, the study of fuzzy differential equations are our next work.

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Separability Extension of Right Twisted Weak Smash Product

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Abstract. In this paper, we study the concept of right twisted weak smash product over weak Hopf algebras and give a criterion for A*H to be separable over A.

Keywords: Right twisted weak smash product, Separability extension.

1 Introduction

Weak Hopf algebras have been proposed by G. Bohm and F. Nill as a generalization of ordinary Hopf algebras in the following sense: the defining axioms are the same, but the multiplicativity of the counit and the comultiplicativity of the unit are replaced by weaker axions. The initial motivation to study weak Hopf algebras was their connection with the theory of algebra extension, and another important application of weak Hopf algebras is that they provide a natural framework for the study of dynamical twsits in Hopf algebras. It turns out that many important properties of ordinary Hopf algebras have "weak" analogues. In this paper, we mainly study the concept of the right twisted smash products over weak Hopf algebrasand investigate their properties.

2 Right Twisted Weak Smsh Product

We work over a fixed field k, we follow Sweedler's book for terminology on coalgebras, bialgebras and Hopf algebras. Let C be a coallgebra, the sigma notation

$$\vartriangle(c) = c_1 \otimes c_2$$

for all $c \in C$ will be used frequently later.

Let H be a weak Hopf algebra with antipode S, and A be an algebra. A is called an H-bimodule angebra if the following conditions hold:

(1) A is an H-bimodule with the left H-module structure map $"\rightarrow"$ and with the right H-module structure map $"\leftarrow"$;

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(2) A is not only left H-module algebra with the left module action "$\rightarrow$" but also right H-module algebra with the right module action "$\leftarrow$".

Let A be an H-bimodule algebra. A right twisted weak smash product A*H is defined on the vector space $A\otimes H$. Define a multiplication

$$(a\otimes h)(b\otimes h)=a(h_1\rightarrow b\leftarrow S(h_3))\otimes h_2 g$$

on tensor space $A\otimes H$, for all $a,b\in A,\ \ h,l\in H$. Let a*x denote the class of $a\otimes x$ in $A\otimes H$,the multiplication in A*H is given by the familiar formula

$$\hat{1}_1\rightarrow a\otimes \hat{1}_2\, h=a\otimes h$$
$$a\leftarrow S(\hat{1}_2)\otimes \hat{1}_1\, h=a\otimes h$$

3 Separability Extension

In this section we mainly give a criterion for A*H to be separable over A.

Definition 1. *Let $R\subseteq T$ be any ring extension.T is called separable over R if there exists an idempotent*

$$e=\sum_i x_i\otimes y_i\in T\otimes_R T$$

such that $\sum_i x_i y_i=\mathbf{1}, te=et$,hold for all $t\in T$ .We call e a separability idempotent of T over R.

Here we consider $T\otimes_R T$ as a T-T bimodule via $t(x\otimes y)=tx\otimes y$

$$(x\otimes y)t=x\otimes yt$$

for all $t\in T, x\otimes y\in T\otimes_R T$.

For a weak Hopf algebra H, G. Bohm and F. Nill ([2],Proposition 2.11) proved that H^L, H^R are separable over the basic field k with separability idempotents $S(\mathbf{1}_1)\otimes\mathbf{1}_2, \mathbf{1}_1\otimes S(\mathbf{1}_2)$ and that H is separable over its basic field k if and only if H is semisimple if and only if there exists a normalized left integral l in H.

Lemma 1. *Let H be a weak Hopf algebra and r a non-zero element in H.The following conditions are equivalent:*

1) *r is a right integral,*
2) $r_1 x\otimes r_2=r_1\otimes r_2 S(x)$, *for all* $x\in H$.

Proof: Suppose r is a right integral. Then for all $x \in H$ we have

$$
\begin{aligned}
r_1 x \otimes r_2 &= \Delta(r)(x \otimes \mathbf{1}) \\
&= \Delta(r\mathbf{1})(x \otimes \mathbf{1}) \\
&= \Delta(r)(\mathbf{1}_1 x \otimes \mathbf{1}_2) \\
&= \Delta(r)(x_1 \otimes x_2 S(x_3)) \\
&= \Delta(r x_1)(\mathbf{1} \otimes S(x_2)) \\
&= \Delta(r \textstyle\prod^R (x_1))(\mathbf{1} \otimes S(x_2)) \\
&= \Delta(r)\Delta(\mathbf{1}_1 \varepsilon(x_1 \mathbf{1}_2))(\mathbf{1} \otimes S(x_2)) \\
&= \Delta(r)(\mathbf{1}_1 \otimes \mathbf{1}_2 \varepsilon(x_1 \mathbf{1}_3))(\mathbf{1} \otimes S(x_2)) \\
&= \Delta(r)(\mathbf{1}_1 \otimes \mathbf{1}_2 \mathbf{1}_{1'} \varepsilon(x_1 \mathbf{1}_{2'}))(\mathbf{1} \otimes S(x_2)) \\
&= \Delta(r)(\mathbf{1}_1 \otimes \mathbf{1}_2 \textstyle\prod^R (x_1))(\mathbf{1} \otimes S(x_2)) \\
&= \Delta(r)(\mathbf{1}_1 \otimes \mathbf{1}_2 \textstyle\prod^R (x_1) S(x_2)) \\
&= \Delta(r)(\mathbf{1}_1 x \otimes \mathbf{1}_2 S(x)) \\
&= \Delta(r)\Delta(\mathbf{1})(\mathbf{1} \otimes S(x)) \\
&= \Delta(r)(\mathbf{1} \otimes S(x)) \\
&= r_1 \otimes r_2 S(x)
\end{aligned}
$$

Conversely, for all $x \in H$, we have

$$r(x) = \varepsilon(r_1 x_1) r_2 x_2 = \varepsilon(r_1) r_2 S(x_1) x_2 = r \prod^R (x)$$

So, r is a right integral in H.

Theorem 1. *Let H be a weak Hopf algebra with a bijective weak antipode S, and let A be a left H-module algebra. Assume there exists a normalized left (or right) integral in H and* $a \leftarrow S(h_1) * h_2 = a \leftarrow S(h_2) * h_1$ *. Then the right twisterd weak smash product algebra A*H is separable over A.*

Proof: Let r be a normalized right integral in H, and set

$e = (1 * S(r_1)) \otimes (1 \otimes r_2)(a * 1)$ .We will show that e is a separability idempotent of A*H over A.

Let $\mu : A * H \otimes_A A * H \to A * H$ denotes the multiplication map. We have

$$
\begin{aligned}
u(e) &= (1 * S(r_1))(1 * r_2) \\
&= 1 * S(r_1) r_2 = 1 * \prod^R (r) = 1 * 1
\end{aligned}
$$

For every $a \in A$, the following holds:

$$\begin{aligned}
ea &= (1*S(r_1)) \otimes (1*r_2)(a*1) \\
&= (1*S(r_1)) \otimes (r_2 \to a \leftarrow S(r_4)*r_3) \\
&= (1*S(r_1)) \otimes (r_2 \to a \leftarrow S(r_4)*\hat{1}) \otimes (1*r_3) \\
&= (\hat{1}_1 \to a \leftarrow S(r_4)S^2(r_1)*S(r_2)) \otimes (1*r_3\hat{1}_2) \\
&= (a \leftarrow S(r_2)*S^2(r_1)S(r_3)) \otimes (1*r_4) \\
&= (a*S(\prod{}^R(r_1))S(r_2)) \otimes (1*r_3) \\
&= ae
\end{aligned}$$

Hence ea=ae shows that e is A-centralizing. For every $x \in H$,by Lemma 1 the following holds:

$$\begin{aligned}
xe &= (1*x)(1*S(r_1)) \otimes (1*r_2) \\
&= (1*S(r_1S^{-1}(x))) \otimes (1*r_2) \\
&= (1*S(r_1)) \otimes (1*r_2S(S^{-1}(x))) \\
&= (1*S(r_1)) \otimes (1*r_2x) \\
&= ex
\end{aligned}$$

Hence ex=xe shows that e is H-centralizing and then, e is a separability idempotent of A*H over A.

Recall a ring extension $R \subseteq T$ is semisimple if every exact sequence of left T-modules, which splits as a sequence of left R-modules, splits.K.Hirata and K.Sugano[3] proved that separable extensions are semisimple extensions. Hence any separable extension of s semisimple artinian ring is itself semsiimple artinian. In particular, from Theorem .3 we get the following important corollary which generalizes a result of M.Cohen and D.Fishman given in [4]. That is, for a weak Hopf algevra H and an H-module algebra A we have A*H is semisimple artinian whenever A and H are semisimple artinian.

Corollary 1**.** *Ler H be a weak Hopf algebra with a bijective weak antipode S ,and let A be a left H-module algebra. Assume there exists a normalized left (or right) integral r in H(i.e. H is semisimple artinian)and*

$$a \leftarrow S(h_1)*h_2 = a \leftarrow S(h_2)*h_1$$

*Then if A is semisimple artinian, so is A*H.*

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Quantity Change Theorem and Quality Change Theorem

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Abstract. Based on the theory of variable fuzzy sets, quantity change theorem and quality change theorem are first given according to opposite fuzzy sets and the definitions of a relative difference function and a relative proportion function. The research could make the variable fuzzy sets more systematic, complete and scientific, furthermore, which is the extending of classical fuzzy sets established by Zadeh (Zadeh 1965).

Keywords: variable fuzzy sets, opposite fuzzy sets, relative difference function, relative proportion function, quantity change theorem and quality change theorem.

1 Introduction

In the nature every material system changes continually, and the revolutionary progress of eternal generation and extinction is everywhere. Revolution is a general phenomenon and a general law of movement in material systems. According to abundant data of modern science, there must be an interim period between the generation and the extinction, where interim system forms generate. Owing to the existing of interim system forms, a successive and external development progress is generated in the nature. The concept of the fuzzy set was proposed by Zadeh.(Zadeh 1965), which can give media and fuzziness a scientific describe, and have great significance in the academic. The fuzzy set is static without considering the relativity and variability, therefore, the theory conflicts with variability of interim form. It is a defect of fuzzy sets because of approaching to the media, variable fuzzy phenomenon, variable fuzz objects and fuzzy concepts by static concepts, theory and method of fuzzy sets.

The theory and method of engineering fuzzy sets was established according to relative relationship member by author(Chen 1998) in 1998, afterward the theory of variable fuzzy sets was proposed by author(Chen 2005; Chen 2005) in 2005. The achievements above are the innovation and extending of static fuzzy sets theory, which are very important in theory and applications. Based on the theory of engineering fuzzy sets and variable fuzzy sets, quantity change theorem and quality change theorem are given according to opposite fuzzy sets and the definitions of a relative difference function and a relative proportion function, and

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establish quantity change theorem and quality change theorem, it is significant in theory and applications of variable fuzzy sets.

2 Quantity Change Theorem and Quality Change Theorem

Definition 1. *Suppose that there is opposite fuzzy concept (object or phenomenon) in the universe* U , $\underset{\sim}{A}$ *expresses characteristic of attractability and* $\underset{\sim}{A}^c$ *states repellency. Hence, to any element* u ($u \in U$), $\mu_{\underset{\sim}{A}}(u)$ *and* $\mu_{\underset{\sim}{A}^c}(u)$ *are the relative membership degrees (RMD) of attractability and repellency in a range of the continuous interval [0, 1] (for* $\underset{\sim}{A}$ *) and [1, 0] (for* $\underset{\sim}{A}^c$ *), besides* $\mu_{\underset{\sim}{A}}(u) + \mu_{\underset{\sim}{A}^c}(u) = 1$.

Let

$$\underset{\approx}{A} = \left\{ u, \mu_{\underset{\sim}{A}}(u), \mu_{\underset{\sim}{A}^c}(u) \middle| u \in U \right\} \tag{1}$$

If $\mu_{\underset{\sim}{A}}(u) + \mu_{\underset{\sim}{A}^c}(u) = 1 \quad 0 \le \mu_{\underset{\sim}{A}}(u) \le 1$ and $0 \le \mu_{\underset{\sim}{A}^c}(u) \le 1$

Then $\underset{\approx}{A}$ could be named the relative fuzzy set of *u*, moreover, there are $\mu_{\underset{\sim}{A}}(u) = 1$ and $\mu_{\underset{\sim}{A}^c}(u) = 0$ at the left top p_l, in a similar way, $\mu_{\underset{\sim}{A}}(u) = 0$ and $\mu_{\underset{\sim}{A}^c}(u) = 1$ at the right top p_r.

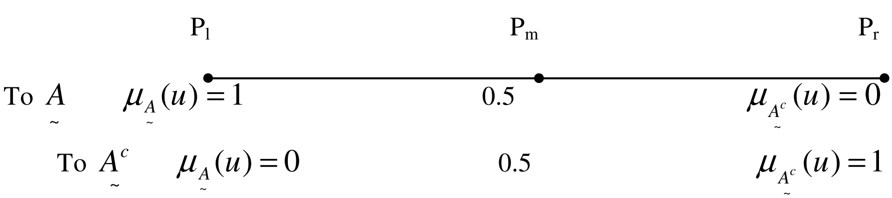

Fig. 1. Sketch map of opposite fuzzy sets $\underset{\approx}{A}$

Point P_m is the quality change point of gradient mode for $\underset{\sim}{A}$ within [1, 0] and for $\underset{\sim}{A}^c$ within [0, 1], where $\mu_{\underset{\sim}{A}}(u) = \mu_{\underset{\sim}{A}^c}(u) = 0.5$.

If there are n elements in the universe U:

$$U = \{u_1, u_2, \cdots, u_n\}$$

Then, the relative fuzzy set $\underset{\approx}{A}$ can be given by

$$\underset{\approx}{A} = \frac{(\mu_{\underset{\sim}{A}}(u_1), \mu_{\underset{\sim}{A}^c}(u_1))}{u_1} + \frac{(\mu_{\underset{\sim}{A}}(u_2), \mu_{\underset{\sim}{A}^c}(u_2))}{u_2} + \ldots + \frac{(\mu_{\underset{\sim}{A}}(u_n), \mu_{\underset{\sim}{A}^c}(u_n))}{u_n} \quad (2)$$

Definition 2. *Let*

$$\underset{\sim}{V} = \left\{(u, D) \middle| u \in U, D(u) = \mu_{\underset{\sim}{A}}(u) - \mu_{\underset{\sim}{A}^c}(u), D \in [-1, 1]\right\} \quad (3)$$

$$A_+ = \{u | u \in U, 0 < D(u) \leq 1\} \quad (4)$$

$$A_- = \{u | u \in U, -1 \leq D(u) < 0\} \quad (5)$$

$$A_0 = \{u | u \in U, D(u) = 0\} \quad (6)$$

Here $\underset{\sim}{V}$ *is defined as the variable fuzzy set of U, Respectively,* A_+, A_- *and* A_0 *are defined as the attractive region, the repellent region and the quality change boundary of gradient mode.*

Definition 3. *Suppose C is variable factors set of* $\underset{\sim}{V}$ *,*

$$C = \{C_A, C_B, C_C\} \quad (7)$$

Where C_A *is the variable models set,* C_B *is the variable parameters set,* C_C *the other variable factors exclude model and its parameter. Let*

$$A^+ = C(A_-) = \{u | u \in U, -1 \leq D(u) < 0, 0 < D(C(u)) \leq 1\} \quad (8)$$

$$A^- = C(A_+) = \{u | u \in U, 0 < D(u) \leq 1, -1 \leq D(C(u)) < 0\} \quad (9)$$

The two subsets are defined as variable change regions of $\underset{\sim}{V}$ *about* C *, and* $D_{\underset{\sim}{A}}(C(u))$ *is the relative difference function from element u to C. Let*

$$A^{(+)} = C(A_{(+)}) = \{u | u \in U, 0 < D(u) \leq 1, 0 < D(C(u)) \leq 1\} \quad (10)$$

$$A^{(-)} = C\left(A_{(-)}\right) = \left\{u \middle| u \in U, -1 \le D(u) < 0, -1 \le D\left(C(u)\right) < 0\right\} \tag{11}$$

The two subsets are defined as quantity change regions of $\underset{\sim}{V}$ *about* C.

Theorem 1. *Let*

$$D(u) = \mu_{\underset{\sim}{A}}(u) - \mu_{\underset{\sim}{A^c}}(u) \tag{12}$$

if $\mu_{\underset{\sim}{A}}(u) > \mu_{\underset{\sim}{A^c}}(u)$, *then* $0 < D(u) < 1$,

and if $\mu_{\underset{\sim}{A}}(u) = \mu_{\underset{\sim}{A^c}}(u)$, *then* $D(u) = 0$,

and if $\mu_{\underset{\sim}{A}}(u) < \mu_{\underset{\sim}{A^c}}(u)$, *then* $-1 < D(u) < 0$.

Where $D(u)$ *is defined as the relative difference degree of element u to* $\underset{\sim}{A}$. *Mapping is defined as the relative difference function of u to* $\underset{\sim}{A}$ *as follows:*

$$\begin{aligned} &D : U \to [-1,1] \\ &u \mapsto D(u) \in [-1,1] \end{aligned} \tag{13}$$

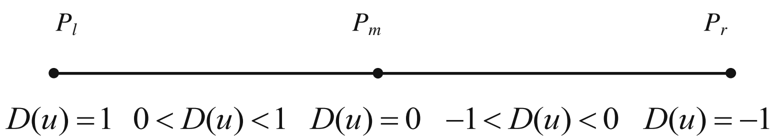

Fig. 2. Sketch map of relative difference function

According to Definition 1 to 3 and theorem 1, the quantity change theorem and quality change theorem (gradient mode and mutational mode) can be established.

(1) If $D(u) > 0$ and $D(C(u)) > 0$, then there is $D(u) \cdot D(C(u)) > 0$ (14)
This situation is quantity change.
(2) If $D(u) > 0$ and $D(C(u)) < 0$, then $D(u) \cdot D(C(u)) < 0$ (15)
This is quality change of gradient mode and throughout the point $D_{\underset{\sim}{A}}(u) = 0$.
(3) If $D(u) < 0$ and $D(C(u)) < 0$, then $D(u) \cdot D(C(u)) > 0$ (16)
It belongs to quantity change.
(4) If $D(u) < 0$ and $D(C(u)) > 0$, then $D(u) \cdot D(C(u)) < 0$ (17)

This situation is quality change of gradient mode and throughout the point $D(u)=0$.

According to the formula (14) to (17), the quantity change theorem and quality change theorem are only related to the sign of $D(u)\cdot D(C(u))$, whatever $D(u)$ is. There are $D(u)=1$ at the left top P_l and $D(u)=-1$ at the right top P_r, and two points both are quality change of mutational mode.

From formula (14) and (16), a conclusion can be draw.

(1) If $D(u)>0$, and $D(u)$ is changing from one plus figure to1, then there is

$$D(u)\cdot D(C(u))=D(u)=\left|D(u)\right| \tag{18}$$

Quality change of mutational mode is happened without passing point P_m, namely $D(u)=0$.

(2) If $D(u)<0$, and $D(u)$ is changing from one negative number to -1, then there is

$$-\left|D(u)\right|\cdot D(C(u))=-\left|D(u)\right|(-1)=\left|D(u)\right| \tag{19}$$

Quality change of mutational mode is happened without passing point t P_m.

From formula (18) and (19), while without overpassing point P_m, the quality change of mutational mode can be obtained as follows:

$$D(u)\cdot D(C(u))=\left|D(u)\right| \tag{20}$$

Similarly, from formula (15) and (17), it can be concluded

(1) If $D(u)>0$, and passing point P_m, besides, $D(u)$ approaches to "-1" finally, then

$$D(u)\cdot D(C(u))=D(u)\times(-1)=-\left|D(u)\right| \tag{21}$$

Quality change of mutational will be occurred.

(2) If $D(u)<0$, and passing point P_m, furthermore, $D(u)$ approaches to"-1" finally, then

$$-\left|D(u)\right|\times(1)=-\left|D(u)\right| \tag{22}$$

This is a quality change of mutational.

According to formula (21) and (22), when point P_m is overpassed, the quality change of mutational mode is only depends on formula (23).

$$D(u)\cdot D(C(u))=-\left|D(u)\right| \tag{23}$$

Based on the above proof, the quantity change theorem and quality change theorem are described as follows:

(1) The quantity change

$$D(u)\cdot D(C(u))>0 \tag{24}$$

(2) The quality change of gradient mode

$$D(u)\cdot D(C(u))<0 \tag{25}$$

(3) The critical point

$$D(u)\cdot D(C(u))=0 \tag{26}$$

(4)The quality change of mutational mode
(a) Without passing through the gradient quality change boundary

$$D(u)\cdot D(C(u))=\left|D(u)\right| \tag{27}$$

(b)Throughout the gradient quality change boundary

$$D(u)\cdot D(C(u))=-\left|D(u)\right| \tag{28}$$

Definition 4. *Suppose*

$$E(u)=\mu_{\underset{\sim}{A}}(u)/\mu_{\underset{\sim}{A}^c}(u) \tag{29}$$

When $\mu_{\underset{\sim}{A}}(u)>\mu_{\underset{\sim}{A}^c}(u)$, *1<* $E(u)$ *<∞; When* $\mu_{\underset{\sim}{A}}(u)=\mu_{\underset{\sim}{A}^c}(u)$, $E(u)$ *=1*
When $\mu_{\underset{\sim}{A}}(u)<\mu_{\underset{\sim}{A}^c}(u)$, *1>* $E(u)$ *≥0.*

E *is called the relative proportion of the value of* u *to* $\underset{\approx}{A}$. *Map*

$$\begin{aligned} E \quad U &\to [0,\infty \\ u &\mapsto E(u)\in[0,\infty \end{aligned} \tag{30}$$

is called the relative proportion function of $\underset{\approx}{A}$ *for* u.

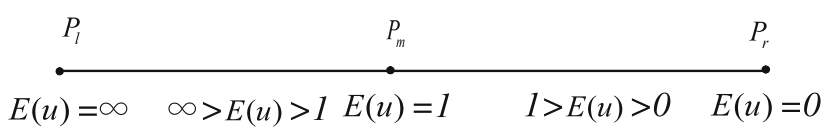

Fig. 3. Sketch map of relative proportion function

The relative proportion function expresses the relative value of any point $\mu_{\underset{\sim}{A}}(u)$ and $\mu_{\underset{\sim}{A^c}}(u)$ on the continuum axis, the relative ratio of the two opposing sides or the proportion of opposing basic fuzzy attribute level, P_m point of $E(u)$ =1 describes the dynamic equilibrium between two opposing basic fuzzy of the property to qualitative point. P_l point and P_r point of $E(u)$ =∞ and 0 describes two basic fuzzy attribute points to abrupt qualitative change. Therefore, the relative proportion of function integrity, vividly described the dialectical materialism on the qualitative change in two forms:gradation and mutation.

Theorem 2. *Suppose* $E(u)$ *is the relative proportion function of any element u on the domain U, as follows:* $E(u)=\dfrac{\mu_{\underset{\sim}{A}}(u)}{\mu_{\underset{\sim}{A^c}}(u)}$, *to transform C for u, the transformed relative proportion function is* $E(C(u))$.

(1) For example inequality:

$$E(u)>1, E(C(u))<1, E(C(u))\neq 0、1、\infty \tag{31}$$

or

$$E(u)<1, E(C(u))>1, E(C(u))\neq 0、1、\infty \tag{32}$$

qualitative change was tapered.

(2) For example equation:

$$E(u)\bullet E(C(u))=0 \tag{33}$$

or

$$E(u)\bullet E(C(u))=\infty \tag{34}$$

qualitative change was abrupt. Formula (32) is non-explosive mutations, Formula (33) is the explosive mutation. According to dialectical materialism philosophy, there are non-explosive mutation and explosive mutation, where 0 indicates non-explosive mutation, ∞ indicates explosive mutation.

(3) For example equation:

$$E(u)=1 \tag{35}$$

Qualitative change is the critical point of a gradual change, shortly tapered the point of qualitative change.

Suppose $E(u)$ is the relative proportion function of any element u on the domain U, to transform C for u , the transformed relative proportion function is $E(C(u))$.

(1) For example inequality:

$$E(u)>1, E(C(u))>1, E(C(u))\neq 0、1、\infty \tag{36}$$

or $$E(u) < 1, E(C(u)) < 1, E(C(u)) \neq 0、1、\infty \qquad (37)$$

is quantitative.

Formula (31) to (35), formula (36), (37) were expressed as a function of the relative proportions of qualitative and quantitative theorem.

3 Conclusion

Based on variable fuzzy set theory, the first expression of mathematical theorems ——quantity change theorem and quality change theorem with strict laws of materialist dialectics, one of the three "quality of exchange law," the quality of each variable given that the form of two theorems in the theory and application, especially in dialectical philosophy of mathematics and mathematical research, the theorems are very important.

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Fuzzy Value Function's Curvilinear and Surface Integral Base on Fuzzy Structured Element Method （Ⅰ）—— Fuzzy-valued Function's Curvilinear Integral

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Abstract. According to the fuzzy structured element representation theorem of fuzzy-valued function, under the background of engineering, this paper gives the definition of fuzzy-valued function's first form curvilinear integral, presents the representation theorem of two-dimensional fuzzy number and fuzzy vector based on fuzzy structured element, and gives the definition of the fuzzy-valued function's second form curvilinear integral. In fact, both calculation method and property of fuzzy-valued function's curvilinear integral have been presented in this paper.

Keywords: Fuzzy structured element, Fuzzy vector, Fuzzy-valued function, Curvilinear integral.

1 Preliminaries

The concept of fuzzy structured element is given in [1] where element method of fuzzy-valued function analysis including the structured element representation of fuzzy-valued function, differential of fuzzy-valued function, Riemann integral is also structured.

Suppose that E is a regular fuzzy structured element whose membership function is $E(x)$, then for any given bounded fuzzy number $\tilde{A}$, there must exist a monotonic bounded function $f(x)$ in $[-1,1]$ such that $\tilde{A} = f(E)$ and the membership function of $\tilde{A}$ is $\mu_{\tilde{A}}(x) = E\left(f^{-1}(x)\right)$. Specially, if $f(x) = a + bx$, $b > 0$ then we say that $\tilde{A} = f(E) = a + bE$ is the fuzzy number formed with fuzzy structured element E, and it is easy to get the membership function is $\mu_{\tilde{A}}(x) = E\left(\frac{x-a}{b}\right)$.

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For any given bounded fuzzy-valued function $\tilde{f}(x)$ in X, there must be a monotonic bounded binary function $g(x,y)$ on the variable y in $[-1,1]$ to make $\tilde{f}(x)=g(x,E)$, and we say that $\tilde{f}(x)$ is the fuzzy-valued function formed with structured element E.

Lemma 1. *Suppose that $\tilde{f}(x) = g(x,E)$ is the fuzzy-valued function formed of structured element E. If $g(x,y)$ is integrable (in the sense of Riemann) on the variable x in D $\subseteq$ R, so the fuzzy-valued function $\tilde{f}(x)$ is integrable in D, and*

$$\int_{\mathrm{D}} \tilde{f}(x)dx = \int_{\mathrm{D}} g(x,y)dx\Big|_{y=E} .$$

The above conclusions can be seen in [1~3].

2 The First Fuzzy-Valued Function Curvilinear Integral

As we all know, the physical meaning of the first curvilinear integral $\int_L f(x,y)ds$ is the quality of curve L, where $f(x,y)$ is the density function in an arbitrary point (x,y) of plane curve L. However, the density function in an arbitrary point (x,y) of plane curve L is difficult to accurately represent because of various factors in many application problems, so we can regard it as a fuzzy-valued function $\tilde{f}(x,y)$, then we put forward the first fuzzy-valued function curvilinear integral, which is defined as

$$\int_L \tilde{f}(x,y)ds = \lim_{\|T\|\to 0}\sum_{i=1}^{n}\tilde{f}(\xi_i,\eta_i)\Delta s_i,\quad \|T\| = \max_{i=1,2,\cdots,n}\Delta s_i .$$

Here, $\tilde{f}(x,y)$ is a fuzzy-valued function defined on the curve L, Δs_i is the arc length area of the i arc when the curve L is divided into n small arcs, $i=1,2,\cdots,n$ (ξ_i,η_i) is the point on the i arc.

Suppose that E is a regular fuzzy structured element, and $\tilde{f}(x,y)$ is a fuzzy-valued function defined on the plane curve L whose structured element is $\tilde{f}(x,y)=g(x,y,E)$, where $g(x,y,z)$ is monotonous about the variable z in $[-1,1]$. So the line integral of fuzzy-valued function $\tilde{f}(x,y)$ on the curve L can be expressed as $\int_L \tilde{f}(x,y)ds = \int_L g(x,y,z)ds\big|_{z=E}$. If $g(x,y,z)$ is integrable with respect to the variables x, y on L, then we say that the fuzzy-valued function $\tilde{f}(x,y)$ is integrable on L.

The first fuzzy-valued function curvilinear integral can be converted to line integral with respect to arc length of common function $\int_L g_z(x,y)ds$ according to Lemma 1.

By the property of the first curvilinear integral of the normal function can naturally introduce the same property of the first fuzzy-valued function curvilinear integral.

Property 1. *α, β are constants, if the first curvilinear integral of fuzzy-valued functions $\tilde{f}_1(x,y)$, $\tilde{f}_2(x,y)$ exist on L, then*

$$\int_L\left[\alpha\tilde{f}_1+\beta\tilde{f}_2\right]ds=\alpha\int_L\tilde{f}_1 ds+\beta\int_L\tilde{f}_2 ds$$

Property 2. *Suppose that the curve L is divided into two arcs L_1, L_2, then*

$$\int_L\tilde{f}ds=\int_{L_1}\tilde{f}ds+\int_{L_2}\tilde{f}ds$$

Property 3. *If the endpoints of curve L are A,B, then $\int_{AB}\tilde{f}ds=\int_{BA}\tilde{f}ds$. That is to say integral value has nothing to do with the direction.*

Property 4. *Suppose that $\tilde{f}(x,y),\tilde{g}(x,y)$ are two fuzzy-valued functions, then*

(1) *If we have $\tilde{f}(x,y)\le\tilde{g}(x,y)$ on L, then $\int_L\tilde{f}(x,y)ds\le\int_L\tilde{g}(x,y)ds$;*

(2) *If we have $\tilde{f}(x,y)\subseteq\tilde{g}(x,y)$ on L, then $\int_L\tilde{f}(x,y)ds\subseteq\int_L\tilde{g}(x,y)ds$.*

3 Plane Fuzzy Vector and the Projection

3.1 Fuzzy Point and Fuzzy Vector in R^2

Definition 1. *The fuzzy subset $\tilde{C}^{(2)}$ in R^2 is called a two-dimensional fuzzy point, if its membership function $\mu_{\tilde{C}^{(2)}}(x,y)$ satisfies:*

(1) *$\mu_{\tilde{C}^{(2)}}(x,y)$ is upper semi-continuous;*

(2) *Exist finite real numbers x, y to make $\mu_{\tilde{C}^{(2)}}(x,y)=1$;*

(3) *For $\forall\lambda\in(0,1]$, $C_\lambda^{(2)}=\left\{(x,y)\middle|\mu_{\tilde{C}^{(2)}}(x,y)\ge\lambda,(x,y)\in R^2\right\}$ is a compact convex set in R^2.*

In the above definition, if exists a unique point (a,b) to make $\mu_{\tilde{C}^{(2)}}(a,b)=1$, then we get the definition of fuzzy point of Buckley [4~5].

Definition 2[6]. *Suppose that* $\tilde{C}^{(2)}$ *is a two-dimensional fuzzy point, if for* $\forall \lambda \in (0,1]$, *the* λ *cut set of* $C_\lambda^{(2)}$ *is circular area in* R^2, *then we say that* $\tilde{C}^{(2)}$ *is a circular fuzzy point.*

The structured element representation method of fuzzy number is introduced in [1], and this method can also be applied to the formulation of two-dimensional fuzzy points.

Definition 3. *Suppose that* $E^{(2)}$ *is a fuzzy set in* R^2, *and its membership function is* $E^{(2)}(x,y)$. *We say* $E^{(2)}$ *is a circular two-dimensional fuzzy structured element, if it satisfies:*

(1) $E^{(2)}(x,y)$ *is upper semi-continuous;*

(2) $E^{(2)}(0,0)=1$ *If* $x^2+y^2 \ge 1$, *then* $E^{(2)}(x,y)=0$

(3) *For* $\forall\ \lambda \in (0,1]$ *the* λ *cut set of* $E^{(2)}$ *is* $E_\lambda^{(2)} = \left\{(x,y)\middle| x^2+y^2 \le r_\lambda^2, r_\lambda^2 \in [0,1]\right\}$, *and for* $0 \le \lambda_1 \le \lambda_2 \le 1$, *we have* $r_{\lambda_1} \ge r_{\lambda_2}$.

In fact, the circular two-dimensional fuzzy structured element $E^{(2)}$ is a special circular fuzzy point.

The structure of the membership function of circular fuzzy structured element is very simple. In the three-dimensional space $X \times Y \times \mu$, Suppose that $\mu = f(x)$ is a monotone decreasing function from $[0,1]$ to $[0,1]$, and $f(0)=1, f(1)=0$. As long as the function $f(x)$ rotate around the μ -axis , the graph we get is the membership function of circular two-dimensional fuzzy structured element, then we have

$$E^{(2)}(x,y) = f(\sqrt{x^2+y^2}).$$

Theorem 1[6]. *Suppose that* $E^{(2)}$ *is a circular two-dimensional fuzzy structured element with membership function* $E^{(2)}(x,y)$. *For an arbitrary linear transformation* $T(X) = AX + B$, *where* $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ *and* $a_{11} > 0, a_{22} > 0$, b_1, b_2 *are finite real numbers,* $A = (a_{ij})$ *is reversible, then* $T(E^{(2)})$ *is a two-dimensional fuzzy point, and the membership function of* $T(E^{(2)})$ *is* $E^{(2)}[T^{-1}(x,y)]$. *Here,* T^{-1} *is the inverse transformation of* T.

Particularly, if $E^{(2)}$ is a circular structured element, $A = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$ $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ where $a > 0$ b_1, b_2 are arbitrary finite real numbers, then $\tilde{C}^{(2)} = B + AE^{(2)}$ is a circular fuzzy point with the membership function

$$\mu_{\tilde{C}^{(2)}}(x,y) = E^{(2)}\left(\frac{x-b_1}{a}, \frac{y-b_2}{a}\right).$$

Theorem 2. *Suppose that* $\tilde{C}^{(2)}$, $\tilde{D}^{(2)}$ *are two fuzzy points formed with the same circular fuzzy structured element, which can express to be two fuzzy vectors. Denote* $\tilde{C}^{(2)} = A_1E^{(2)} + B_1$ $\tilde{D}^{(2)} = A_2E^{(2)} + B_2$ *where* A_1, A_2 *are 2-order squares whose main diagonal elements are real numbers, and others zero, and* B_1, B_2 *are* 1×2 *real matrixes, then we have* $\tilde{C}^{(2)} \pm \tilde{D}^{(2)} = (A_1 + A_2)E^{(2)} + (B_1 \pm B_2)$, *with the membership function* $\mu_{\tilde{C}^{(2)}\pm\tilde{D}^{(2)}}(x,y) = E^{(2)}\left[(A_1 + A_2)^{-1}\cdot(X-(B_1 \pm B_2))\right]$.

Proof. Denote $\tilde{C}^{(2)} = A_1E^{(2)} + B_1 = f(E^{(2)})$ $\tilde{D}^{(2)} = A_2E^{(2)} + B_2 = g(E^{(2)})$. By the extension principle [7], we have

$$(\tilde{C}^{(2)} \pm \tilde{D}^{(2)}) = \bigcup_{\lambda\in[0,1]} \lambda * (\tilde{C}^{(2)} \pm \tilde{D}^{(2)})_\lambda = \bigcup_{\lambda\in[0,1]} \lambda * [f_\lambda(E^{(2)}) \pm g_\lambda(E^{(2)})].$$

Denote $h = f + g$, then $h(E^{(2)}) = f(E^{(2)}) + g(E^{(2)}) = (A_1 + A_2)E^{(2)} + (B_1 + B_2)$. For $\forall\lambda \in (0,1]$, we have

$$f_\lambda(E^{(2)}) = f\ (E_\lambda^{(2)}) = A_1E_\lambda^{(2)} + B_1,\ g_\lambda(E^{(2)}) = g\ (E_\lambda^{(2)}) = A_2E_\lambda^{(2)} + B_2$$
$$h(E^{(2)}) = (A_1 + A_2)E_\lambda^{(2)} + (B_1 + B_2),$$

so

$$f_\lambda(E^{(2)}) + g_\lambda(E^{(2)}) = (A_1 + A_2)E_\lambda^{(2)} + (B_1 + B_2) = h_\lambda(E^{(2)}).$$

Therefore

$$h_\lambda(E^{(2)}) = \bigcup_{\lambda\in[0,1]} \lambda * [f_\lambda(E^{(2)}) + g_\lambda(E^{(2)})] = \bigcup_{\lambda\in[0,1]} \lambda * [f_\lambda(E^{(2)})] + \bigcup_{\lambda\in[0,1]} \lambda * [g_\lambda(E^{(2)})].$$

That is

$$\tilde{C}^{(2)} + \tilde{D}^{(2)} = (A_1 + A_2)E^{(2)} + (B_1 + B_2).$$

As the circular fuzzy structured element satisfies the symmetry, that is to say $E^{(2)} = -E^{(2)}$, for the same reason

$$\tilde{C}^{(2)} - \tilde{D}^{(2)} = A_1E^{(2)} + A_2(-E^{(2)}) + (B_1 - B_2) = (A_1 + A_2)E^{(2)} + (B_1 - B_2).$$

Then it is easy to get its membership function.

Suppose that $E^{(2)}$ is a circular fuzzy structured element, and the fuzzy point $\tilde{C}^{(2)} = AE^{(2)} + B$ k is a real number, then we have $k\tilde{C}^{(2)} = |k|AE^{(2)} + kB$.

Because $E^{(2)} = -E^{(2)}$, $k\tilde{C}^{(2)} = |k| AE^{(2)} + kB$. Then for any real number k_1, k_2, we have

$$k_1\tilde{C}^{(2)} + k_2\tilde{D}^{(2)} = (|k_1| A_1 + |k_2| A_2)E^{(2)} + (k_1B_1 + k_2B_2)$$

3.2 The Direction Fuzzy Numbers of Oval Fuzzy Point

Definition 4. *Suppose that L is a direction vector in plane, $E^{(2)}$ is a circular fuzzy structured element with the membership function $E^{(2)}(x, y)$, and the projection (value) $p_L E^{(2)}$ of $E^{(2)}$ in the L direction is a fuzzy number nothing to do with the direction, which is denoted by $p_L E^{(2)} \triangleq {}^R E$, then we define it as free structured element derived from $E^{(2)}$ with the membership function ${}^R E(u)$.*

Obviously, the projection of circular fuzzy structured element $E^{(2)}$ in any direction has nothing to do with the direction.

As the circular structure element can be get by rotation of single-drop function. Therefore, the two variables of the membership function are symmetrical. The projection in any direction is the same fuzzy number. Its membership function is eliminating a variable of function $E^{(2)}(x, y)$ and instead of the retained variables with u (Nothing to do with the direction). If $E^{(2)}(x, y) = f(\sqrt{x^2 + y^2})$, then ${}^R E(u) = f(u) = E^{(2)}(u)$.

Example. Suppose that $E^{(2)}$ is a conical fuzzy structured element with the membership function

$$E^{(2)}(x, y) = \begin{cases} 1 \quad \sqrt{x^2 + y^2}, & x^2 + y^2 \le 1; \\ \quad 0, & x^2 + y^2 > 1. \end{cases}.$$

Then the membership function of free structured element ${}^R E$ derived from $E^{(2)}$ is

$${}^L E(u) = E^{(2)}(u) = \begin{cases} 1 \quad \sqrt{u^2}, & |u| \le 1 \\ \quad 0, & |u| > 1 \end{cases} = \begin{cases} 1 \quad |u|, & |u| \le 1 \\ \quad 0, & |u| > 1 \end{cases}$$

So ${}^R E$ is a triangular fuzzy number.

Definition 5. *The projection (value) of fuzzy vector $\tilde{C}^{(2)}$ in the L direction is called flat fuzzy number $p_L\tilde{C}^{(2)}$ in the L direction.*

In general, the fuzzy vector expressed by the plane oval fuzzy point is called oval fuzzy vector, while the circular fuzzy vector is a special case of oval fuzzy vector.

Oval fuzzy point $\tilde{C}^{(2)} = B + AE^{(2)}$ can be considered to be obtained by a circular structured element $E^{(2)}$ with expansion, rotation, and then translation, where $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is a non-singular matrix, $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ is the translation coefficient, which is the center of the fuzzy point $\tilde{C}^{(2)}$. In particular, if $a_{12} = a_{21} = 0 \quad a_{11} = a_{22} > 0$ then $\tilde{C}^{(2)}$ is a round fuzzy point.

Suppose that $C^{(2)} = AE^{(2)} + B$ is an oval fuzzy vector. Obviously, B decides the center of the fuzzy vector, while A decides the form of its membership function. For an arbitrary direction vector L suppose that the position of projection point of the center (b_1, b_2) of oval fuzzy point in L is $p_L B = (b_{1L}, b_{2L})$, then the projection of oval fuzzy vector $\tilde{C}^{(2)} = B + AE^{(2)}$ in the L direction is $p_L \tilde{C}^{(2)} = p_L B + p_L (AE^{(2)})$.

Theorem 3. *Suppose that $E^{(2)}$ is a cone structured element, and oval fuzzy vector $C^{(2)} = AE^{(2)} + B$, where $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$, then the projection of oval fuzzy vector $C^{(2)}$ in the L direction is $p_L \tilde{C}^{(2)} = p_L B + a^R E$, where the L direction is $(\cos\theta, \sin\theta)$ $A^{-1} = \begin{pmatrix} a'_{11} & a'_{12} \\ a'_{21} & a'_{22} \end{pmatrix}$*

$$a = \frac{\sqrt{(a'^2_{11} + a'^2_{21})\sin^2\theta - 2\sin\theta\cos\theta(a'_{11}a'_{12} + a'_{21}a'_{22}) + (a'^2_{12} + a'^2_{22})\cos^2\theta}}{\left| a'_{11}a'_{22} - a'_{12}a'_{21} \right|}.$$

Proof. As the fuzzy vector is derived from the cone oval round structured element, according to the center symmetry of structured element, it is easy to know that $p_L(AE^{(2)})$ is a symmetric triangular fuzzy number, that is to say $p_L(AE^{(2)})$ is obtained by a free structured element which is derived from $E^{(2)}$ with stretching transformation.

Obviously, the support set of $AE^{(2)}$ is an ellipse. Because matrix A is nonsingular, and $A^{-1} = \begin{pmatrix} a'_{11} & a'_{12} \\ a'_{21} & a'_{22} \end{pmatrix}$ then the support set equation of $AE^{(2)}$ is: $px^2 + qy^2 + 2rxy - 1 = 0$,

where $p = a'^2_{11} + a'^2_{21} \quad q = a'^2_{12} + a'^2_{22} \quad r = a'_{11}a'_{12} + a'_{21}a'_{22}$.

Suppose that $k = \dfrac{\sin\theta}{\cos\theta}$, then $y = -\dfrac{1}{k}x + b$ are a family of straight lines which are vertical to L , where b is a parameter. Joint it with elliptic equation and simplify it, then we get $(k^2 p + q - 2kr)x^2 + 2(k^2 br - kbq)x + k^2 b^2 q - k^2 = 0$.

Denote $\Delta = 0$ and simplify to get $b^2 = \dfrac{k^2 p + q - 2kr}{k^2(pq - r)}$.

Substitute k, p, q, r respectively, then we have

$$b^2 = \frac{(a_{11}'^2 + a_{21}'^2)\sin^2\theta - 2\sin\theta\cos\theta(a_{11}'a_{12}' + a_{21}'a_{22}') + (a_{12}'^2 + a_{22}'^2)\cos^2\theta}{\sin^2\theta(a_{11}'a_{22}' - a_{12}'a_{21}')^2}$$

Thus the length of the projection of ellipse in the L direction is

$$|b_1 - b_2|\sin\theta =$$

$$\frac{2\sqrt{(a_{11}'^2 + a_{21}'^2)\sin^2\theta - 2\sin\theta\cos\theta(a_{11}'a_{12}' + a_{21}'a_{22}') + (a_{12}'^2 + a_{22}'^2)\cos^2\theta}}{|a_{11}'a_{22}' - a_{12}'a_{21}'|}$$

So $p_L(AE^{(2)}) = a\,^R E$ where $a = \dfrac{|b_1 - b_2|\sin\theta}{2}$ $^R E$ is a free structured element.

Thus $p_L\tilde{C}^{(2)} = p_L B + a\,^R E$.

From the theorem 3, if $C^{(2)}$ is a round fuzzy vector, that is to say $C^{(2)} = AE^{(2)} + B$ $A = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$ $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ then the projection of $C^{(2)}$ in any direction is $p_L\tilde{C}^{(2)} = p_L B + aE$, which has nothing to do with the direction of L.

3.3 Fuzzy Vector Function

All of the plane fuzzy points are defined as $\tilde{N}(R \times R)$ and the fuzzy vector function $\tilde{F}^{(2)}(x, y)$ is the mapping from $R \times R$ to $\tilde{N}(R \times R)$.

Suppose that $E^{(2)}$ is a two-dimensional round fuzzy structured element. For a given point (x, y) there is always an unique fuzzy point $\tilde{F}^{(2)}(x, y)$ which is derived from $E^{(2)}$ corresponding to it. Note

$$\tilde{F}^{(2)}(x, y) = B(x, y) + A(x, y)E^{(2)} \tag{1}$$

where

$B(x, y) = \begin{bmatrix} b_1(x, y) \\ b_2(x, y) \end{bmatrix}$, $A(x, y) = \begin{bmatrix} a_{11}(x, y) & a_{12}(x, y) \\ a_{21}(x, y) & a_{22}(x, y) \end{bmatrix}$, $a_{ii}(x, y) \ge 0, i = 1, 2$ and for any x, y, we have $|A(x, y)| > 0$. Define $\tilde{F}^{(2)}(x, y)$ as the oval fuzzy vector function which is derived from fuzzy structured element $E^{(2)}$.

In particular, if $a_{12}(x, y) = a_{21}(x, y) = 0$, $a_{ii}(x, y) \ge 0, i = 1, 2$ then define $\tilde{F}^{(2)}(x, y)$ as the round fuzzy vector function which is derived from fuzzy structured element $E^{(2)}$.

4 The Second Curve Integral of Fuzzy-Valued Function

Suppose that L is a smooth directing curve in the plane, and fuzzy vector function $\tilde{F}^{(2)}(x,y)$ is that shown in (1), where $a_{ij}(x,y)$, $b_1(x,y), b_2(x,y)$ are all consecutive in L , $i, j = 1,2$.

Suppose that (x,y) is an arbitrary point in L , whose differential arc can be replaced with differential vector ds along the positive tangent direction, and $|ds| = \sqrt{(dx)^2 + (dy)^2}$. Then the direction vector of ds is $\tau = \left(\frac{dx}{\sqrt{(dx)^2 + (dy)^2}}, \frac{dy}{\sqrt{(dx)^2 + (dy)^2}} \right)$. So according to Theorem 3 the projection of $\tilde{F}^{(2)}(x,y)$ in the ds direction is

$$p_L \tilde{F}^{(2)}(x,y) = p_L B(x,y) + a(x,y) \cdot {}^R E$$

where $a(x,y) = \frac{\sqrt{p(x,y)\sin^2\theta - 2r(x,y)\sin\theta\cos\theta + q(x,y)\cos^2\theta}}{|a'_{11}(x,y)a'_{22}(x,y) - a'_{12}(x,y)a'_{21}(x,y)|}$

$p(x,y) = a'^2_{11}(x,y) + a'^2_{21}(x,y)$, $r(x,y) = a'_{11}(x,y)a'_{12}(x,y) + a'_{21}(x,y)a'_{22}(x,y)$, $q(x,y) = a'^2_{12}(x,y) + a'^2_{22}(x,y)$.

Obviously, $\tan\theta = \frac{dy}{dx} = y'(x)$ that is, $\sin^2\theta = \frac{y'^2(x)}{y'^2(x)+1}$

$\cos^2\theta = \frac{1}{y'^2(x)+1}$ $\sin\theta\cos\theta = \frac{y'(x)}{y'^2(x)+1}$

and $p_L B(x,y) = \frac{b_1(x,y)dx}{\sqrt{(dx)^2+(dy)^2}} + \frac{b_2(x,y)dy}{\sqrt{(dx)^2+(dy)^2}}$, then the work done by $\tilde{F}^{(2)}(x,y)$ in the directed differential segment ds is

$$\begin{aligned} p_L \tilde{F}^{(2)}(x,y)ds &= \left[\frac{b_1(x,y)dx}{\sqrt{(dx)^2+(dy)^2}} + \frac{b_2(x,y)dy}{\sqrt{(dx)^2+(dy)^2}} + a(x,y) \cdot {}^R E\right] \cdot \sqrt{(dx)^2+(dy)^2} \\ &= b_1(x,y)dx + b_2(x,y)dy + a(x,y)ds \cdot {}^R E . \end{aligned}$$

If $\int_L \tilde{F}^{(2)}(x,y) \cdot \tau ds$ denotes the work done by variable force $\tilde{F}^{(2)}(x,y)$ along the plane directed curve L , then we have

$$\int_L \tilde{F}^{(2)}(x,y) \cdot \tau ds = \int_L b_1(x,y)dx + \int_L b_2(x,y)dy + \int_L a(x,y)ds \cdot {}^L E ,$$

which can be short for

$$\int_L b_1 dx + b_2 dy + \int_L a ds \cdot {}^R E \tag{2}$$

In (2), $\int_L b_1 dx + b_2 dy$ is the second curve integral in the classic sense, and $\int_L a(x,y)ds$ is the first curve integral in the classic sense.

Thus we can see that the condition of the second curve integral having nothing to do with path is not applicable here. It requires the integral $\int_L b_1(x,y)dx + b_2(x,y)dy$ to satisfy the condition of having nothing to do with path, if $a(x,y)$ is a constant, then the integral also needs to satisfy that the length of two paths are equal. If $a(x,y)$ is a variable then the integral must satisfy

$$\int_{L_1} a(x,y)ds = \int_{L_2} a(x,y)ds .$$

If the above integrals are all exist, denote that $\int_L b_1(x,y)dx + b_2(x,y)dy = \alpha$ $\int_L a(x,y)ds = \beta$ then (2) can be changed into

$$\int_L \tilde{F}^{(2)}(x,y)\cdot \tau ds = \alpha + \beta \cdot {}^R E$$

We can see that the integral result is the fuzzy number generated by the free linear structured element with the membership function ${}^R E\left(\frac{u-\alpha}{\beta}\right)$.

According to the definition of the second curve integral of fuzzy-valued function, it is easy to get the following properties.

Property 5. *If the curve L can be divided into two curves L_1 and L_2, then*

$$\int_L \tilde{F}^{(2)}(x,y)\cdot \tau ds = \int_{L_1} \tilde{F}^{(2)}(x,y)\cdot \tau ds + \int_{L_2} \tilde{F}^{(2)}(x,y)\cdot \tau ds .$$

Property 6. *Suppose that L is a directed arc, and $-L$ is the reverse curves arc of L, then*

$$\int_{-L} \tilde{F}^{(2)}(x,y)\cdot \tau ds = -\int_L \tilde{F}^{(2)}(x,y)\cdot \tau ds .$$

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Fuzzy Value Function's Curvilinear and Surface Integral Base on Fuzzy Structured Element Method (Ⅱ) ——Fuzzy-Valued Function's Surface Integral

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Abstract. This article proposes the concept of the first fuzzy value function surface integral, gives the concept of fuzzy point or fuzzy vector in three-dimension space and the express method based on structured element by imitating the definition of two-dimensional fuzzy point, and defines the concept of the second fuzzy value function surface integral. Besides, the calculation method and relative properties of the two types of fuzzy value function surface integral are also given in this article.

Keywords: Spherical fuzzy structured element, Fuzzy vector, Fuzzy value function, Surface integral.

1 The First Fuzzy Value Function Surface Integral

The application background of the classical first surface integral (with respect to area) is to calculate the quality of surface. If the surface density is fuzzy value function in practical problems, we can similarly define the first fuzzy value function surface integral.

Suppose that Σ is a smooth surface, the surface density is fuzzy value function $\tilde{f}(x,y,z)$. If $\tilde{f}(x,y,z)$ is continuous and bounded on Σ then the first surface integral of $\tilde{f}(x,y,z)$ on Σ is defined as:

$$\iint_{\Sigma} \tilde{f}(x,y,z)d\sigma = \lim_{\|T\|\to 0}\sum_{i=1}^{n}\tilde{f}(\xi_i,\eta_i,\zeta_i)\Delta\sigma_i \qquad \|T\| = \max_{i=1,2,\cdots,n}\Delta\sigma_i$$

where $\Delta\sigma_i$ is the area of the i piece when the surface L is divided into n pieces, $i=1,2,\cdots,n$ $\quad(\xi_i,\eta_i,\zeta_i)$ is an arbitrary point on the i piece.

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If $\tilde{f}(x,y,z)\geq 0$, the physical meaning of $\iint_{\Sigma}\tilde{f}(x,y,z)d\sigma$ is fuzzy quality of surface Σ whose density function is $\tilde{f}(x,y,z)$.

We still use the fuzzy structured element method to discuss the calculation of fuzzy value function surface integral.

Suppose that E is a regular fuzzy structured element, $\tilde{f}(x,y,z)$ is a fuzzy value function defined on surface Σ whose structured element is $\tilde{f}(x,y,z)=g(x,y,z,E)$, where $g(x,y,z,u)$ is monotony with respect to variable u on $[-1,1]$, so the surface integral of fuzzy value function $\tilde{f}(x,y,z)$ on Σ is

$$\iint_{\Sigma}\tilde{f}(x,y,z)d\sigma=\iint_{\Sigma}g(x,y,z,u)\,d\sigma\Big|_{u=E} \tag{1}$$

In the integral expression (1) x,y,z are integral variables in integrand $g(x,y,z,u)$, and u is a parameter which is used to decide distribution of fuzzy membership function, so the calculation can be transformed into the calculation of normal function surface integrals with respect to area $\iint_{\Sigma}g_u(x,y,z)\,d\sigma$.

By the property of the first surface integral of the normal function, we can naturally introduce the same property of the first surface integral of fuzzy value function. So we will not list them here.

Example. Calculate $\iint_{\Sigma}\left(x^2+y^2+2e^E\right)d\sigma$ Σ is the surface of the stereo which is bounded by surface $z=\sqrt{x^2+y^2}$ and plane $z=1$.

Solution. Surface Σ consists of two parts. One part is $\Sigma_1: z=\sqrt{x^2+y^2}$, the other part is $\Sigma_2: z=1$. The projection domain of their common line on plane xoy is D_{xy} $x^2+y^2\leq 1$. Then

$$\begin{aligned}
&\iint_{\Sigma}\left(x^2+y^2+2e^E\right)d\sigma\\
&=\iint_{\Sigma_1}\left(x^2+y^2+2e^E\right)d\sigma+\iint_{\Sigma_2}\left(x^2+y^2+2e^E\right)d\sigma\\
&=\iint_{\Sigma_1}\left(x^2+y^2+2e^u\right)d\sigma\Big|_{u=E}+\iint_{\Sigma_2}\left(x^2+y^2+2e^u\right)d\sigma\Big|_{u=E}\\
&=\sqrt{2}\int_0^{2\pi}d\varphi\int_0^1\left(r^3+2e^u r\right)dr\Big|_{u=E}+\int_0^{2\pi}d\varphi\int_0^1\left(r^3+2e^u r\right)dr\Big|_{u=E}\\
&=\frac{\left(\sqrt{2}+1\right)}{2}\pi+e^E
\end{aligned}$$

Suppose that the membership function of fuzzy structured element E is $E(u)$, then the calculation is fuzzy number, and its membership function is

$$E\left[\ln\left(u-\frac{(\sqrt{2}+1)\pi}{2}\right)\right].$$

2 Space Spherical Fuzzy Point, Fuzzy Vector and Spherical Structured Element

The definition of space fuzzy point can be given according to the definition of two-dimensional fuzzy point [6]. It is the natural extension of plane fuzzy point, and can be understood as a special type of space fuzzy vector.

Definition 1. *The fuzzy subset* $\tilde{C}^{(3)}$ *is called the three-dimensional fuzzy point in real space* R^3, *if the membership function* $\mu_{\tilde{C}^{(3)}}(x,y,z)$ *satisfies:*

(1) $\mu_{\tilde{C}^{(3)}}(x,y,z)$ *in semi-continuous demicontinuous*

(2) *Exist finite real numbers* x,y,z *to make* $\mu_{\tilde{C}^{(3)}}(x,y,z)=1$

(3) *For* $\forall\lambda\in(0,1]$ $C_\lambda^{(3)}=\left\{(x,y,z)\middle|\ \mu_{\tilde{C}^{(3)}}(x,y,z)\geq\lambda,(x,y,z)\in R^2\right\}$ *is a convex compact set in* R^3.

If for $\forall\lambda\in(0,1]$, the λ cut set $C_\lambda^{(3)}$ of fuzzy point $\tilde{C}^{(3)}$ are spheres in R^3, then we say that $\tilde{C}^{(3)}$ is spherical fuzzy point.

Similarly, we can also give the concept of spherical fuzzy structured element.

Definition 2. *Suppose that* $E^{(3)}$ *is a fuzzy set in* R^3 *whose membership function is* $E^{(3)}(x,y,z)$, *then we say that* $E^{(3)}$ *is three-dimensional spherical fuzzy structured element, if it satisfies:*

(1) $E^{(3)}(x,y,z)$ *is upper semicontinuous;*

(2) $E^{(3)}(0,0,0)=1$ *If* $x^2+y^2+z^2\geq 1$ *then* $E^{(3)}(x,y,z)=0$

(3) *For* $\forall$ $\lambda\in(0,1]$, *the* λ *cut set of* $E^{(3)}$ *is* $E_\lambda^{(3)}=\left\{(x,y,z)\middle|x^2+y^2+z^2\leq r_\lambda^2, r_\lambda^2\in[0,1]\right\}$, *and when* $0\leq\lambda_1\leq\lambda_2\leq 1$, *we have* $r_{\lambda_1}\leq r_{\lambda_2}$.

The membership function of spherical fuzzy structured element can not give its graph directly, so we can imagine that it is a ball whose center is the origin $(0,0,0)$, imagine the membership of a point as the density of the ball at the point. And the density is 0 in the outside of the ball whose radius is more than 1, while in the area of the ball whose radius is less than 1, the density increases as the radius reduces until it gets the maximum 1 in the centre of the ball $(0,0,0)$.

We can easily extend it to space spherical fuzzy point according to Theorem 1 in [7].

Suppose that $E^{(3)}$ is a spherical fuzzy structured element, and its membership function is $E^{(3)}(x,y,z)$, then for any linear transformation $T(X)=AX+B$, where $A=\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}, X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}, B=\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$, $a>0$ b_1,b_2,b_3 are finite real numbers, so $T(E^{(3)})=AE^{(3)}+B$ is space spherical fuzzy point, and its membership function is $E^{(3)}[T^{-1}(x,y,z)]$, T^{-1} is the inverse transformation of T here.

Because $T=B+AX$, we have

$$X=A^{-1}(T-B)=\begin{bmatrix} 1/a & 0 & 0 \\ 0 & 1/a & 0 \\ 0 & 0 & 1/a \end{bmatrix}\cdot\begin{bmatrix} x'-b_1 \\ y'-b_2 \\ z'-b_3 \end{bmatrix}=\begin{bmatrix} \frac{x'-b_1}{a} \\ \frac{y'-b_2}{a} \\ \frac{z'-b_3}{a} \end{bmatrix},$$

$$T^{-1}=\left(\frac{x-b_1}{a},\frac{y-b_2}{a},\frac{z-b_3}{a}\right)$$

So the membership function of $AE^{(3)}+B$ is

$$\mu_{AE^{(3)}+B}(x,y,z)=E^{(3)}\left(\frac{x-b_1}{a},\frac{y-b_2}{a},\frac{z-b_3}{a}\right).$$

Besides, similar to the definition 5 in [7], we have the concept of free structured element which is exported by spherical fuzzy structured element in any direction. Suppose that L is a direction vector in space, and $E^{(3)}$ is a spherical fuzzy structured element whose membership function is $E^{(3)}(x,y,z)$. The projection (value) of $E^{(3)}$ in L direction is a fuzzy number which is irrelevant to direction, $p_L E^{(3)} \triangleq {}^R E$ for short so we call it free structured element which is exported by $E^{(3)}$ with the membership function ${}^R E(u)$.

Similar to the two-dimensional circular structured element, the variables in the three-dimensional spherical fuzzy structured element membership function are also symmetrical, so the projection in any direction is the same fuzzy number. The determination of the membership function is to eliminate two variables of the function $E^{(3)}(x,y,z)$, and instead of the variables retained with u.

Example. Suppose that the membership function of spherical fuzzy structured element $E^{(3)}$ is:

$$E^{(3)}(x,y)=\begin{cases}\sqrt{1-x^2-y^2-z^2}, & x^2+y^2+z^2\le 1;\\ 0, & x^2+y^2+z^2>1.\end{cases}$$

Then we have the membership function of free structured element ${}^{R}E$ which is exported by $E^{(3)}$

$${}^{R}E(u)=E^{(3)}(u)=\begin{cases}\sqrt{1-u^2}, & u^2\le 1\\ 0, & u^2>1\end{cases}.$$

Space spherical fuzzy point can be regarded as space fuzzy vector. In the following discussion, projection of space spherical fuzzy vector in a direction (direction fuzzy number) will be given.

There is a space spherical fuzzy point $\tilde{C}^{(3)}=B+AE^{(3)}$, where $E^{(3)}$ is a spherical fuzzy structured element, and $A=\begin{bmatrix}a&0&0\\0&a&0\\0&0&a\end{bmatrix}, B=\begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix}\quad a>0$

b_1,b_2,b_3 are finite real numbers. L is a arbitrary direction vector, and projection point position of the center of spherical fuzzy point (b_1,b_2,b_3) in L is $p_LB=(b_{1L},b_{2L},b_{3L})$. Because the shape of the projection domain which is get by spherical fuzzy point in any direction has nothing to do with direction, the projection (direction fuzzy number) of fuzzy vector $\tilde{C}^{(3)}=B+AE^{(3)}$ in L is

$$p_L\tilde{C}^{(3)}=p_LB+a\cdot{}^{R}E\ . \tag{2}$$

3 The Second Fuzzy Value Function Surface Integral

The application of the classic second surface integral is: steady incompressible flow whose flow velocity in point (x,y,z) is

$$V(x,y,z)=P(x,y,z)i+Q(x,y,z)j+R(x,y,z)k$$

flow from the positive vector to the negative vector along with the surface, and the flow per unit time is

$$q = \lim_{\lambda \to 0} \sum_{i=1}^{n} \left(n_i \cdot V_i \right) \Delta\sigma_i = \iint_{\Sigma} V(x, y, z) \cdot n d\sigma$$

Here $\Delta\sigma_i (i = 1, 2, \cdots, n)$ is the area of the i piece when the surface Σ is divided to n small oriented surface pieces, and λ is the maximum in $\Delta\sigma_i$. Each piece can be approximated as a plane, and n_i is the unit normal vector (it has the same direction with the surface).

The unit normal vector of the oriented surface Σ in arbitrary point is $n = (\cos\alpha, \cos\beta, \cos\gamma)$. Take differential area $d\sigma$ in that point, then we get

$$\cos\alpha d\sigma = dydz \qquad \cos\beta d\sigma = dzdx \qquad \cos\gamma d\sigma = dxdy \tag{3}$$

So the second surface integral can be expressed as

$$\iint_{S} \left(P\cos\alpha + Q\cos\beta + R\cos\upsilon \right) d\sigma = \iint_{S} Pdydz + Qdzdx + Rdxdy \, .$$

We will discuss the second fuzzy value function surface integral now. Calculate the flow that the steady incompressible flow passes through oriented surface Σ along the positive vector in unit time in the event that the flow rate is the fuzzy vector function

$$\tilde{F}^{(3)}(x, y, z) = B(x, y, z) + A(x, y, z) E^{(3)}$$

For the convenience of calculation, we may assume that $E^{(3)}$ is a spherical fuzzy structured element, and the matrix

$$A = \begin{bmatrix} a(x, y, z) & 0 & 0 \\ 0 & a(x, y, z) & 0 \\ 0 & 0 & a(x, y, z) \end{bmatrix} \qquad a(x, y, z) \geq 0 \, .$$

Suppose that the unit normal vector in arbitrary point on surface Σ is $n = (\cos\alpha, \cos\beta, \cos\gamma)$, and take differential area $d\sigma$ at that point. By the expression (2), the projection of the given point $\tilde{F}^{(3)}(x, y, z)$ in method direction is

$$p_n \tilde{F}^{(3)}(x, y, z) = p_n B(x, y, z) + a(x, y, z) \cdot {}^{R}E \, .$$

While $p_n B(x, y, z) = b_1(x, y, z)\cos\alpha + b_2(x, y, z)\cos\beta + b_3(x, y, z)\cos\gamma$,
So the flow passes through oriented surface Σ along the positive vector in unit time is

$$q = \iint_{\Sigma} p_n \tilde{F}^{(3)}(x, y, z) d\sigma = \iint_{\Sigma} [p_n B(x, y, z) + a(x, y, z) \cdot {}^{L}E] d\sigma$$
$$= \iint_{\Sigma} [b_1(x, y, z)\cos\alpha + b_2(x, y, z)\cos\beta + b_3(x, y, z)\cos\gamma + a(x, y, z)\, {}^{R}E] d\sigma$$

$$= \iint_\Sigma b_1(x,y,z)dydz + b_2(x,y,z)dxdz + b_3(x,y,z)dxdy + {}^R E \cdot \iint_\Sigma a(x,y,z)d\sigma ,$$

which can be short for

$$\iint_\Sigma b_1 dydz + b_2 dxdz + b_3 dxdy + \iint_\Sigma a(x,y,z)d\sigma \cdot {}^R E \qquad (4)$$

In the integral equation (4) $\iint_\Sigma b_1 dydz + b_2 dxdz + b_3 dxdy$ is the second surface integral in the classical sense, while $\iint_\Sigma a(x,y,z)d\sigma$ is the first surface integral in the classical sense, and $\int_L a(x,y,z)ds \geq 0$.

If the above integrals are all exist, suppose that $\iint_\Sigma b_1 dydz + b_2 dxdz + b_3 dxdy = \alpha$, and $\iint_\Sigma a(x,y,z)d\sigma = \beta \geq 0$ then(4)can be written as

$$\iint_\Sigma [\tilde{F}^{(3)}(x,y,z)\cdot n]d\sigma = \alpha + \beta \cdot {}^R E .$$

It is easy to know that the result of the integral is a fuzzy number, and its membership function is ${}^L E\left(\frac{u-\alpha}{\beta}\right)$.

According to the definition of the second surface integral about fuzzy value function, we can get the following properties:

Property 1. *Suppose that* Σ *is a oriented surface,* $-\Sigma$ *represents a oriented surface that takes the opposite side with* Σ*, then we have*

$$\iint_{-\Sigma} \tilde{f} \cdot n d\sigma = -\iint_\Sigma \tilde{f} \cdot n d\sigma$$

Property 2. *If we divide the surface* Σ *into* Σ_1 *and* Σ_2*, then we have*

$$\iint_\Sigma \tilde{f} \cdot nds = \iint_{\Sigma_1} \tilde{f} \cdot nds + \iint_{\Sigma_2} \tilde{f} \cdot nds$$

The proof is omitted.

Example. Suppose that $E^{(3)}$ is a spherical structured element, and the space fuzzy vector is

$$\tilde{F}^{(3)}(x,y) = \begin{bmatrix} xy \\ yz \\ zx \end{bmatrix} + \begin{bmatrix} a(x,y,z) & 0 & 0 \\ 0 & a(x,y,z) & 0 \\ 0 & 0 & a(x,y,z) \end{bmatrix} E^{(3)}$$

where $a(x,y,z) = \frac{1}{(1+x+y)^2}$ Σ is the outside of the surface of a tetrahedron which is made by the surface $x+y+z=1$ and the three coordinate planes. Now calculate $\iint_\Sigma [\tilde{F}^{(3)}(x,y,z)\cdot n]d\sigma$.

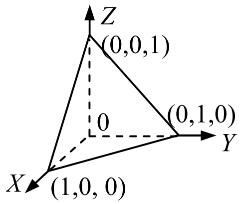

Fig. 1.

Solution. Denote the parts Σ on surface $x=0, y=0, z=0$ and $x+y+z=1$ by $\Sigma_1, \Sigma_2, \Sigma_3, \Sigma_4$ respectively, then according to the integral formula

$$\iint_\Sigma b_1 dydz + b_2 dxdz + b_3 dxdy + \iint_\Sigma a(x,y,z)d\sigma \cdot {}^R E$$

we have

$$\iint_\Sigma b_1 dydz + b_2 dxdz + b_3 dxdy = \iint_\Sigma xydydz + yzdxdz + xzdxdy$$
$$= \iint_{\Sigma_1} + \iint_{\Sigma_2} + \iint_{\Sigma_3} + \iint_{\Sigma_4}$$

Where $\iint_{\Sigma_i} xydydz + yzdxdz + xzdxdy = 0$, $i = 1,2,3$. Denote the projection area of Σ_4 on the coordinate plane $x=0, y=0, z=0$ by D_{yz}, D_{xz}, D_{xy} respectively, then we have

$$\iint_{\Sigma_4} xydydz + yzdxdz + xzdxdy$$
$$= \iint_{D_{xy}} x(1-x-y)dxdy + \iint_{D_{yz}} y(1-y-z)dydz + \iint_{D_{xz}} z(1-z-x)dxdz$$
$$= 3 \times \frac{1}{24} = \frac{1}{8}$$

And $$\iint_\Sigma a(x,y,z)d\sigma = \iint_\Sigma \frac{d\sigma}{(1+x+y)^2} = \iint_{\Sigma_1} + \iint_{\Sigma_2} + \iint_{\Sigma_3} + \iint_{\Sigma_4}$$

$x=0$ on Σ_1, so $\iint_{\Sigma_1} \frac{d\sigma}{(1+x+y)^2} = \int_0^1 dz \int_0^{1-z} \frac{dy}{(1+y)^2} = 1 - \ln 2$

$y=0$ on Σ_2, so $\iint_{\Sigma_2} \frac{d\sigma}{(1+x+y)^2} = \int_0^1 dz \int_0^{1-z} \frac{dx}{(1+x)^2} = 1 - \ln 2$

$z=0$ on Σ_3, so $\iint_{\Sigma_3} \frac{d\sigma}{(1+x+y)^2} = \int_0^1 dx \int_0^{1-x} \frac{dy}{(1+x+y)^2} = \ln 2 - \frac{1}{2}$

$z = 1-x-y$ on Σ_4, so $\sqrt{1+z_x^2+z_y^2} = \sqrt{1+(-1)^2+(-1)^2} = \sqrt{3}$.

Denote the projection area of Σ_4 on the surface xoy by D_{xy}, so

$$\iint_{\Sigma_4} \frac{d\sigma}{(1+x+y)^2} = \iint_{D_{xy}} \frac{\sqrt{3}dxdy}{(1+x+y)^2} = \sqrt{3}\int_0^1 dx \int_0^{1-x} \frac{dy}{(1+x+y)^2} = \sqrt{3}\left(\ln 2 - \frac{1}{2}\right)$$

And we can establish

$$\iint_{\Sigma} \frac{d\sigma}{(1+x+y)^2} = 2(1-\ln 2) + \left(\ln 2 - \frac{1}{2}\right) + \sqrt{3}\left(\ln 2 - \frac{1}{2}\right) = \frac{3-\sqrt{3}}{2} + \left(\sqrt{3}-1\right)\ln 2$$
$$> 0$$

In short, we finally get

$$\iint_{\Sigma} [\tilde{F}^{(3)}(x,y,z)\cdot n] d\sigma = \frac{1}{8} + \left[\frac{3-\sqrt{3}}{2} + \left(\sqrt{3}-1\right)\ln 2\right] \cdot {}^{R}E$$

and its membership function is ${}^{R}E\left(\frac{u-1/8}{(3-\sqrt{3})/2+(\sqrt{3}-1)\ln 2}\right)$.

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Robust Absolute Stability Analysis for Uncertain Fuzzy Neutral Systems

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Abstract. This paper considers the robust absolute stability issues for a class of T-S fuzzy uncertain neutral systems with delays. The uncertainties considered in this paper are norm bounded, and possible time-varying. Based on Lyapunov-Krasovskii functional and linear matrix inequalities approach, the absolute robust stabilization of the fuzzy uncertain neutral systems can be achieved. Two examples are given to demonstrate the effectiveness of the proposed method.

Keywords: Takagi-Sugeno (T-S) fuzzy systems, neutral system, absolute stability.

1 Introduction

T-S fuzzy systems [1] have been shown to be a powerful tool for modeling complex nonlinear system. As is well-known that, by means of the T-S fuzzy model, a nonlinear system can be represented by a weighted sum of some simple linear subsystems and then can be stabilized by a model-based fuzzy control. Therefore, it provides a good opportunity to employ the well-established theory of linear system to investigate the complex nonlinear system. Over the past two decades, many issues related to stability analysis and control synthesis of T-S fuzzy systems have been reported (see, e.g. [2]-[17]).

On the other hand, time delays are frequently encountered in various engineering systems such as aircraft, long transmission lines in pneumatic systems, and chemical or process control systems. It has been shown that the existence of time delays is often one of the main causes of instability and poor performance of a control system [2]. Recently, the T-S fuzzy system with time delay was introduced in [3]. During the past two decades, the study of the time-delay T-S fuzzy system has received much attention(see, e.g. [10, 11, 12, 13]).

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Moreover, it is also well-known that many practical delayed processes can be modeled as general neutral systems, which contain delays both in their states and in the derivatives of their states, such as circuit analysis, computer aided design, realtime simulation of mechanical systems, power systems, chemical process simulation, optimal control [4]; in particular, some practical delayed nonlinear processes have been modeled as general nonlinear neutral systems, such as vacuum-tube oscillation [5], the car-following problem [6], distributed networks–long lines with tunnel diodes [7], the dynamics of the growth of capital stock [8], and nonlinear fluid dynamics [9]. So the stability and stabilization analysis of (nonlinear) neutral systems have recently been extensively investigated (see, e.g. [10, 11, 12]).

As mentioned above, with the T-S fuzzy model, a nonlinear neutral system can be represented as a weighted sum of some simple linear neutral subsystems; then it provides a good chance to make use of the well-established theory of linear neutral systems to investigate the complex nonlinear neutral systems. So the TCS fuzzy neutral system was recently introduced in [14]. The stability analysis of such system has received much attention (see [15, 16, 17]).

However, to the best of the authors knowledge, few results on absolute stability for uncertain fuzzy neutral systems with time-varying delays are available in the literature. In this paper, we are concerned with the problems of robust stabilization for uncertain T-S fuzzy neutral systems with time-varying delays. This paper is organized as follows. Section 2 describes the systems and introduces some notations and lemmas that will be used in the rest of this paper. Section 3 gives our main results in this paper. Examples are given to demonstrate the effectiveness of our theoretical results in section 4. Some conclusions are drawn in Section 5.

2 Problem Statement and Preliminaries

Consider the following T-S uncertain fuzzy neutral system with time-varying delays:

Plant rule i: **If** $s_1(t)$ is u_{i1}, $s_2(t)$ is u_{i2}, $\cdots$, $s_g(t)$ is u_{ig}. **Then**

$$\begin{array}{l} \dot{x}\left(t\right) = A_i(t)x\left(t\right) + B_i(t)x\left(t-h\right) + C_i(t)\dot{x}\left(t-\tau\right) \\ x\left(t_0+\theta\right) = \varphi\left(\theta\right), \forall\theta \in \left[-\rho, 0\right], i = 1, \cdots, r, \end{array} \tag{1}$$

where $x\left(t\right) \in R^n$ is the state vector, h, τ are constant time delays, $\rho = \max\left\{h, \tau\right\}$ and $\varphi\left(\theta\right)$ is the initial condition function. The system matrices are assumed to be uncertain and satisfy

$$[A_i(t), B_i(t), C_i(t)] = [A_i, B_i, C_i] + D_i F_i(t)[E_{ai}, E_{bi}, E_{ci}], i = 1, \cdots, r \tag{2}$$

where $A_i, B_i, C_i, D, E_{ai}, E_{bi}, E_{ci}$ are constant matrices with appropriate dimensions, and and $F_i(t)$ are unknown, real, and possibly time-varying matrix with Lebesgue measurable elements, satisfying

$$F_i^T(t)F_i(t) \leq I, i = 1, \cdots, r. \tag{3}$$

Applying a center-average defuzzier, product inference and singleton fuzzifier, the dynamic fuzzy model (1) can be represented by

$$\begin{array}{l} \dot{x}\left(t\right) = \sum_{i=1}^{r} h_i(s(t))\{A_i(t)x\left(t\right) + B_i(t)x\left(t-h\right) + C_i(t)\dot{x}\left(t-\tau\right)\} \\ x\left(t_0+\theta\right) = \varphi\left(\theta\right), \forall\theta \in \left[-\rho, 0\right], i = 1, \cdots, r, \end{array} \tag{4}$$

where

$$h_i(s(t)) = \frac{\bar{w}_i(s(t))}{\sum_{i=1}^{r} \bar{w}_i(s(t))}, \bar{w}_i(s(t)) = \prod_{j=1}^{g} u_{ij}(s_j(t)), s(t) = [s_1(t), \cdots, s_g(t)]$$

and $u_{ij}(s_j(t))$ is the grade of membership of $s_j(t)$ in u_{ij}. Then, it can be seen that

$$\bar{w}_i(s(t)) > 0, i = 1, 2, \cdots, r, \sum_{i=1}^{r} \bar{w}_i(s(t)) > 0, \forall t > 0,$$

$$h_i(s(t)) > 0, i = 1, 2, \cdots, r, \sum_{i=1}^{r} h_i(s(t)) = 1, \forall t > 0,$$

The following definitions are necessary.

Definition 1. *The equilibrium of* (4) *is said to be absolute stable, if there exist scalars* $\alpha > 0$ *and* $\gamma \geq 1$ *such that for all* $x\left(t\right)$ *the following inequality holds:*

$$\left\| x\left(t\right) \right\| \leq \gamma e^{-\alpha(t-t_0)} \left\| \phi \right\|_{1\rho},$$

where $\|\cdot\|$*denote the Euclidean norm and*

$$\left\| \varphi \right\|_{1\rho} = \max\left\{ \max_{-\rho\leq s\leq 0} \left\| \varphi\left(s\right) \right\|, \max_{-\rho\leq s\leq 0} \left\| \varphi'\left(s\right) \right\| \right\}.$$

Before presenting the main result, we first state the following lemmas which will be used in the proof of our main result.

Lemma 1. [18] *Given matrices* $Q = Q^T$*,* H *and* E *of appropriate dimensions, then*

$$Q + HFE + E^T F^T H^T < 0$$

for all F *satisfying* $F^T(t)F(t) \leq I$*, if and only if exists an* $\varepsilon > 0$ *such that*

$$Q + \varepsilon H H^T + \varepsilon^{-1} E^T E < 0$$

Lemma 2. *For any constant matrices* Q_{11}*,* Q_{12}*,* $Q_{22} \in R^{n\times n}$*,* $\begin{pmatrix} Q_{11} & Q_{12} \\ & Q_{22} \end{pmatrix} >$ 0*, scalar* $h > 0$*, and vector function* $\dot{x} : [-h, 0] \to R^n$ *such that the following integration is well defined, then*

$$-h\int_{t-h}^{t}\begin{bmatrix}x(t)\\ \dot{x}(t)\end{bmatrix}^T\begin{bmatrix}Q_{11} & Q_{12}\\ & Q_{22}\end{bmatrix}\begin{bmatrix}x(t)\\ \dot{x}(t)\end{bmatrix}dt$$
$$\leq -\begin{bmatrix}\int_{t-h}^{t}x(s)ds\\ x(t)\\ x(t-h)\end{bmatrix}^T\begin{bmatrix}Q_{11} & Q_{12} & -Q_{12}\\ & Q_{22} & -Q_{22}\\ & * & Q_{22}\end{bmatrix}\begin{bmatrix}\int_{t-h}^{t}x(s)ds\\ x(t)\\ x(t-h)\end{bmatrix}. \tag{5}$$

Proof. This result is easy to be derived by Jensen's integral inequality. This completes the proof.

Lemma 3. [19] *Suppose that matrices $\{Mi\}_{i=1}^{r} \in R^{nm}$ and a semi-positive-definite matrix $P \in R_{nn}$ are given, then*

$$\left(\sum_{i=1}^{r}h_i(s(t))M_i\right)^T P\left(\sum_{i=1}^{r}h_i(s(t))M_i\right) \leq \sum_{i=1}^{r}h_i(s(t))M_i^T PM_i$$

where the fuzzy basis functions $h_i(s(t))(i = 1, 2, \cdots, r)$ are defined by (4).

From (4), the aim of this paper is to find a new strategy for the exponential stabilization of T-S uncertain fuzzy neutral systems.

3 Stability Analysis

In this section, we are dedicate to establish absolute stability of the T-S uncertaiin fuzzy neutral system (4).

Firstly consider the nominal system, i.e., $F_i(t) = 0$. To this end, choose a new class of Lyapunov functionals candidate for systems (4) as following

$$V(t) = V_1(t) + V_2(t) + V_3(t), \tag{6}$$

where

$$V_1(t) = x^T(t)Px(t),$$

$$V_2(t) = r\int_{-r}^{0}\int_{t+\theta}^{t}\begin{bmatrix}x(s)\\ \dot{x}(s)\end{bmatrix}^T e^{-\alpha(t-s)} \times \begin{bmatrix}Q_{11} & Q_{12}\\ & Q_{22}\end{bmatrix}\begin{bmatrix}x(s)\\ \dot{x}(s)\end{bmatrix}dsd\theta,$$

$$V_3(t) = \int_{t-\tau}^{t}\dot{x}^T(s)e^{-\alpha(t-s)}Rx(s)ds,$$

scalars $\alpha > 0$, matrices $P > 0, R > 0, Q_{11} > 0, Q_{22} > 0$, Q_{12} with appropriate dimensions to be determined. The following lemma gives a decay estimation of $V(t)$ along the state trajectory of systems (4).

Lemma 4. *Given scalars $\alpha > 0$, if there exist matrices $P > 0, Q_{11} > 0, Q_{22} > 0$, Q_{12} and $N_{il}^T(l = 1, \cdots, 5)$ are any matrices with appropriate dimensions, such that the following LMIs hold for all $i = 1, 2, \cdots, r$:*

$$Q = \begin{pmatrix}Q_{11} & Q_{12}\\ & Q_{22}\end{pmatrix} > 0, \tag{7}$$

$$\phi_i = \begin{pmatrix} \phi_{i11} & \phi_{i12} & \phi_{i13} & \phi_{i14} & \phi_{i15} \\ & \phi_{i22} & \phi_{i23} & \phi_{i24} & \phi_{i25} \\ & * & \phi_{i33} & \phi_{i34} & \phi_{i35} \\ & * & * & \phi_{i44} & \phi_{i45} \\ & * & * & * & \phi_{i55} \end{pmatrix} < 0, \tag{8}$$

where

$$\phi_{i11} = \alpha P + PA_i + A_i^T P + h^2 Q_{11} - e^{-\alpha h} Q_{22} + N_{i1}^T A_i + A_i^T N_{i1},$$

$$\phi_{i12} = h^2 Q_{12} - N_{i1}^T + A_i^T N_{i2}, \qquad \phi_{i13} = PC_i + N_{i1}^T C_i + A_i^T N_{i3},$$

$$\phi_{i14} = PB_i + N_{i1}^T B_i + A_i^T N_{i4} + e^{-\alpha h} Q_{22}, \quad \phi_{i15} = -e^{-\alpha h} Q_{12}^T + A_i^T N_{i5},$$

$$\phi_{i22} = h^2 Q_{22} - N_{i2}^T - N_{i2}, \qquad \phi_{i23} = N_{i2}^T C_i - N_{i3},$$

$$\phi_{i24} = N_{i2}^T B_i - N_{i4}, \qquad \phi_{i25} = -N_{i5},$$

$$\phi_{i33} = -e^{-\alpha\tau} R + N_{i3}^T C_i + C_i^T N_{i3}, \qquad \phi_{i34} = N_{i3}^T B_i + C_i^T N_{i4},$$

$$\phi_{i44} = -e^{-\alpha h} Q_{22} + N_{i4}^T B_i + B_i^T N_{i4}, \qquad \phi_{i35} = C_i^T N_{i5},$$

$$\phi_{i45} = B_i^T N_{i5} + e^{-\alpha h} Q_{12}^T, \qquad \phi_{i55} = -e^{-\alpha h} Q_{11}$$

then along the trajectory of the systems (4), it follows that

$$V(t) \le e^{-\alpha(t-t_0)} V(t_0). \tag{9}$$

Proof. Along the trajectories of systems (4), with lemma 2, it holds that

$$\dot{V}_1(t) = 2x^T(t) P \dot{x}(t), \tag{10}$$

$$\begin{aligned} \dot{V}_2(t) = {} & h^2 \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ & Q_{22} \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} - \alpha V_2(t) \\ & -h \int_{t-h}^{t} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix}^T e^{-\alpha(t-s)} \times \begin{bmatrix} Q_{11} & Q_{12} \\ & Q_{22} \end{bmatrix} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix} ds \\ \le {} & h^2 \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ & Q_{22} \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} - \alpha V_2(t) \\ & -e^{-\alpha h} \begin{bmatrix} \int_{t-h}^{t} x(s) ds \\ x(t) \\ x(t-h) \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} & -Q_{12} \\ & Q_{22} & -Q_{22} \\ & * & Q_{22} \end{bmatrix} \begin{bmatrix} \int_{t-h}^{t} x(s) ds \\ x(t) \\ x(t-h) \end{bmatrix}, \end{aligned} \tag{11}$$

$$\dot{V}_3(t) = \dot{x}^T(t) R \dot{x}(t) - \dot{x}^T(t-\tau) e^{-\alpha\tau} R \dot{x}(t-\tau) - \alpha_i V_3(t). \tag{12}$$

For any matrices $N_{il} (l = 1, \cdots, 5)$, it follows from the systems (4) that

$$\begin{aligned} & \textstyle\sum_{i=1}^{r} h_i(s(t)) [x^T(t) N_{i1}^T + \dot{x}^T(t) N_{i2}^T + \dot{x}^T(t-\tau) N_{i3}^T + x^T(t-h) N_{i4}^T \\ & + (\int_{t-h}^{t} x(s) ds)^T N_{i5}^T] [A_i x(t) - \dot{x}(t) + B_i x(t-h) + C_i \dot{x}(t-\tau_i)] = 0. \end{aligned} \tag{13}$$

Then, from (10)-(13), taking Lemma 1-3 into account, one can obtain that

$$\begin{aligned}
&\dot{V}(t)+\alpha V(t)\le\ \alpha x^T(t)Px(t)+2\textstyle\sum_{i=1}^{r}h_i(s(t))x^T(t)P[A_ix(t)+B_ix(t-h)\\
&+C_i\dot{x}(t-\tau)]+\dot{x}^T(t)R\dot{x}(t)+\ h^2\begin{bmatrix}x(t)\\ \dot{x}(t)\end{bmatrix}^T\begin{bmatrix}Q_{11} & Q_{12}\\ & Q_{22}\end{bmatrix}\begin{bmatrix}x(t)\\ \dot{x}(t)\end{bmatrix}\\
&-e^{-\alpha h}\begin{bmatrix}\int_{t-h}^{t}x(s)ds\\ x(t)\\ x(t-h)\end{bmatrix}^T\begin{bmatrix}Q_{11} & Q_{12} & -Q_{12}\\ & Q_{22} & -Q_{22}\\ & * & Q_{22}\end{bmatrix}\begin{bmatrix}\int_{t-h}^{t}x(s)ds\\ x(t)\\ x(t-h)\end{bmatrix}\\
&-\dot{x}^T(t-\tau)e^{-\alpha\tau}R\dot{x}(t-\tau)+\textstyle\sum_{i=1}^{r}h_i(s(t))\sum_{j=1}^{r}h_j(s(t))[x^T(t)N_{i1}^T+\dot{x}^T(t)N_{i2}^T\\
&+\dot{x}^T(t-\tau)N_{i3}^T+\ x^T(t-h)N_{i4}^T+(\textstyle\int_{t-h}^{t}x(s)ds)^TN_{i5}^T]\\
&\times[A_jx(t)-\dot{x}(t)+B_jx(t-h)+C_j\dot{x}(t-\tau)]\\
&=\textstyle\sum_{i=1}^{r}h_i^2(s(t))\xi^T(t)\phi_i\xi(t)<0,
\end{aligned}\tag{14}$$

where

$$\xi^T(t)=[x^T(t)\ \dot{x}^T(t)\ \dot{x}^T(t-\tau)\ x^T(t-h)\ (\int_{t-h}^{t}x(s)ds)^T]$$

Thus, $\dot{V}(t)+\alpha V(t)\le 0$. Integrating these inequalities gives the inequalities (9). This completes the proof.

With the uncertainty described by (2) and (3), the following corollary is straightforward from Lemma 4.

Corollary 1. *Given scalars $\alpha>0$, if there exist scalars $\varepsilon_i>0$ and matrices $P>0,Q_{11}>0,Q_{22}>0$, Q_{12}, $N_{il}^T\{l\in(1,\cdots,5)\}$ with appropriate dimensions, such that the following LMIs hold*

$$Q=\begin{pmatrix}Q_{11} & Q_{12}\\ & Q_{22}\end{pmatrix}>0,\tag{15}$$

$$\varphi_i=\begin{pmatrix}\varphi_{i11} & \phi_{i12} & \varphi_{i13} & \varphi_{i14} & \phi_{i15} & N_{i1}^TD & PD\\ & \phi_{i22} & \phi_{i23} & \phi_{i24} & \phi_{i25} & N_{i2}^TD & 0\\ & * & \varphi_{i33} & \varphi_{i34} & 0 & N_{i3}^TD & 0\\ & * & * & \varphi_{i44} & 0 & N_{i4}^TD & 0\\ & * & * & * & \phi_{i55} & N_{i5}^TD & 0\\ & * & * & * & * & -\varepsilon_iI & 0\\ & * & * & * & * & * & -\varepsilon_iI\end{pmatrix}<0\tag{16}$$

where

$$\varphi_{i11}=\phi_{i11}+2\varepsilon_iE_{ai}^TE_{ai},\quad \varphi_{i13}=\phi_{i13}+2\varepsilon_iE_{ai}^TE_{ci},$$

$$\varphi_{i14}=\phi_{i14}+\varepsilon_iE_{ai}^TE_{bi},\quad \varphi_{i33}=\phi_{i33}+2\varepsilon_iE_{ci}^TE_{ci},$$

$$\varphi_{i34}=\phi_{i34}+\varepsilon_iE_{ci}^TE_{bi},\quad \varphi_{i44}=\phi_{i44}+\varepsilon_iE_{bi}^TE_{bi},$$

with $\phi_{ikl}(i\in S_s,k,l=(1,\cdots,5))$ are defined by lemma 4, then along the trajectory of the system (4), it follows that

$$V(t)\le e^{-\alpha(t-t_0)}V(t_0).\tag{17}$$

Proof. Replaced A_i, B_i, C_i in LMIs (8) respectively with $A_i(t), B_i(t), C_i(t)$ which described in (2) and (3). ϕ_i are changed into $\widetilde{\phi}_i$ as following

$$\widetilde{\phi}_i = \phi_i + H \begin{pmatrix} F(t) & 0 \\ & F(t) \end{pmatrix} E + E^T \begin{pmatrix} F(t) & 0 \\ & F(t) \end{pmatrix}^T H^T < 0. \tag{18}$$

where

$$E = \begin{pmatrix} E_{ai} & 0 & E_{ci} & E_{bi} & 0 \\ E_{ai} & 0 & E_{ci} & 0 & 0 \end{pmatrix}$$

$$H^T = \begin{pmatrix} D^T N_{i1} & D^T N_{i2} & D^T N_{i3} & D^T N_{i4} & D^T N_{i5} \\ D^T P & 0 & 0 & 0 & 0 \end{pmatrix}$$

From lemma 1, the above inequality (18) holds if and only if there exist some scalars $\varepsilon_i > 0$ such that

$$\phi_i + \varepsilon_i E^T E + \varepsilon_i^{-1} H H^T < 0. \tag{19}$$

Applying the Schur complement, the aforementioned inequality (19) is equivalent to LMIs (16). This completes the proof.

Now, we are in the position to give absolute stability for the system (4).

Theorem 1. *The trivial solution of systems (1) is globally absolutly stable and the state decay estimation is given as*

$$\| x(t) \| \leq \sqrt{\frac{b}{a}} e^{-\frac{1}{2}\alpha(t-t_0)} \| x_0 \|_{c1}, \tag{20}$$

with

$$a = \lambda_{\min}(P), \quad b = \lambda_{\max}(P) + \tau_i \lambda_{\max}(R) + \frac{r^2}{2}\lambda_{\max}(Q) \tag{21}$$

Proof. Notice that

$$V(t_0) \leq b \| x_0 \|_1^2, \quad a \| x(t) \|^2 \leq V(t) \tag{22}$$

Taking (17) into account yields

$$a \| x(t) \|^2 \leq b e^{-\alpha(t-t_0)} \| x_0 \|_1^2$$

which implies (20). This completes the proof.

4 Example

In order to show the effectiveness of the conditions presented in Section 3, in this section, two examples are provided.

Example 1. Consider a neutral T-S fuzzy system (1) with parameters as follows:

$$A_1 = \begin{pmatrix} -0.9 & 0.2 \\ 0.1 & -0.9 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 1 & 1 \\ 1.5 & -2 \end{pmatrix}, B_1 = \begin{pmatrix} -1.1 & -0.2 \\ 0.1 & -1.1 \end{pmatrix},$$

$$B_2 = \begin{pmatrix} -1 & -0.6 \\ 0.5 & -1.2 \end{pmatrix}, C_1 = \begin{pmatrix} -0.2 & 0 \\ 0.2 & -0.1 \end{pmatrix}, \quad C_2 = \begin{pmatrix} 0.2 & 0.1 \\ -0.4 & 0.8 \end{pmatrix},$$

and $h = 0.245, \tau = 0.2455$. However, for $\alpha = 2.3$, we have feasible solution of LMIs in Lemma 4. From (21), we have $a = 16.6045, b = 1356.9$. Using (20), one can obtain

$$\| x(t) \| \leq 1.7986 e^{-2.3(t-t_0)} \| x_0 \|_1,$$

which means that the fuzzy neutral system is absolutly stable by Theorem 3.1. See Fig. 1, where the initial point is $(4, -5)$. Fig. 1 shows the state variables of the fuzzy neutral system.

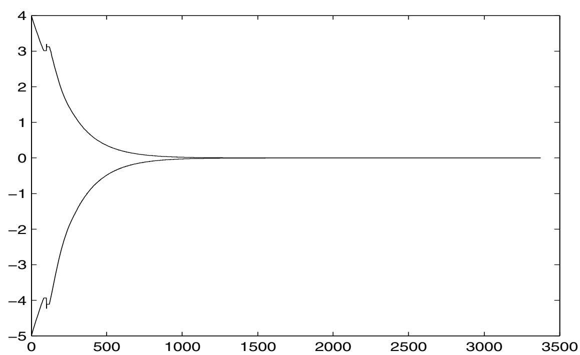

Fig. 1. Behavior of the state component of the fuzzy neutral system

Example 2. Consider the system discussed in Example 1 with uncertain structure as follows

$$D = 0.1459I, E_{ai} = E_{bi} = E_{ci} = I(i = 1, 2), F(t) = \begin{bmatrix} \cos t & 0 \\ 0 & \sin t \end{bmatrix}.$$

For $\alpha_1 = 2.3, \beta_2 = 1.9$, $\lambda^* = 1.6$, μ_1 and μ_2 are as same as it in example 1. Choose $\varepsilon_1 = 0.1, \varepsilon_2 = 0.05$, we have feasible solution of the LMIs in Corollary 1 as following

$$P_1 = \begin{pmatrix} 1.1409 & -0.6198 \\ -0.6198 & 1.8927 \end{pmatrix}, \qquad P_2 = \begin{pmatrix} 0.8973 & -0.5463 \\ -0.5463 & 1.5553 \end{pmatrix},$$

$$R_1 = \begin{pmatrix} 420.5677 & 5.9623 \\ 5.9623 & 410.9918 \end{pmatrix}, \quad R_2 = \begin{pmatrix} 265.0990 & 18.9656 \\ 18.9656 & 268.3814 \end{pmatrix},$$

$$Q_1 = \begin{pmatrix} 1.4344 & -22.2792 & 0.0574 & -0.2406 \\ 18.0982 & 3.6692 & -0.4080 & 0.8578 \\ 0.0574 & -0.4080 & 2.9957 & -0.2636 \\ -0.2406 & 0.8578 & -1.3419 & 3.4756 \end{pmatrix},$$

$$Q_2 = \begin{pmatrix} 1.3255 & 1.5421 & -0.1424 & -0.2460 \\ -5.4866 & 3.5090 & -0.2001 & 0.7516 \\ -0.1424 & -0.2001 & 3.0241 & 0.0234 \\ -0.2460 & 0.7516 & -1.1701 & 2.6254 \end{pmatrix}.$$

Using (20), one can obtain

$$\| x(t) \| \leq 17.9430 e^{-2.3(t-t_0)} \| x_0 \|_1,$$

which means that the T-S uncertain fuzzy neutral system can be robust absolutly stable.

5 Conclusions

By employing Lyapunov functional approach, the robust absolute stability criterion has been presented as a set of linear matrix inequalities problems for the T-S uncertain fuzzy neutral systems. Such conditions are time-dependent and more flexible. Simulation examples are given to demonstrate our theoretical results.

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Solving Fully Fuzzy Systems by Fuzzy Structured Element Method

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Abstract. Using fuzzy structured element method we investigate a fully fuzzy linear system whose coefficient matrix is a fuzzy matrix and right–hand side column is an arbitrary fuzzy vector. According to homeomorphic property between the bounded fuzzy number and the standard monotonic function family with same monotonic form on [-1,1], replace the fully fuzzy linear system by a parametric linear system. Based on the constraint arithmetic operation of fuzzy numbers, the exact fuzzy solutions are obtained by an elimination method. The applicability and superiority of this method are illustrated by two examples.

Keywords: Fully fuzzy linear system, fuzzy structured element method, parametric linear system.

1 Introduction

Many problems in mathematics, engineering, physics, and economics can come down to solving linear systems. The parameters are uncertain in many applications, then establishing a fuzzy system is more applicable than establishing a crisp system. It is necessary to develop an appropriate method for investigating the fuzzy linear systems.

Friedman et al. [1] introduce an $n\times n$ fuzzy linear system (FLS) whose coefficient matrix is crisp and right–hand side column is an arbitrary fuzzy number vector. They solve it by an embedding method and replace the original $n\times n$ fuzzy linear system by a $(2n)\times(2n)$ crisp linear system, then some authors [2-3] study this system by different methods. Asady et al. [4] and Zheng et al. [5] extend the fuzzy linear system to a general $m\times n$ case.

Recently, some authors focus on a fuzzy linear system whose parameters are all fuzzy numbers, which is called fully fuzzy linear system (FFLS). We find FFLS is as important as FLS in the practical applications, therefore finding an effective method is very necessary. Dehahan et al. [6-8] have done a lot of work on solving the FFLS, they study a positive FFLS and find a positive fuzzy solution by using LU decomposition, programming, Jacobi iteration, Gauss–Seidel iteration, SOR, AOR EMA and so on. [9] puts forward a symmetric times triangular

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decomposition method for solving FFLS. [10] discusses the solutions of the $m \times n$ FFLS by a unified iterative scheme which includes Richardson, Jacobi, Gradient and so on. In [11], the block homotopy perturbation method is adopted for finding an approximate solution of FFLS. In [12] the author studies a FFLS whose parameters and unknowns can be any form of fuzzy numbers, and obtain the non-zero solutions by transforming the original $n \times n$ FFLS into a $(2n) \times (4n)$ parametric system, but the solution is approximate all the same.

Since most of the learners study FFLS on the basis of an approximate operation which are proposed by Dubois, these methods are only suitable for the systems whose parameters and unknowns are triangular fuzzy numbers and calculate by three points of the support set of fuzzy numbers, obviously, it makes a lot of information lost, and the errors of solutions will be magnified with the increase of the operation frequency. Based on the concept of fuzzy structured element and the special arithmetic operation of fuzzy numbers [13], we propose a new method for solving the FFLS. The superiority of this method is showed by the comparison with [6] and [12] for the same example.

2 Fuzzy Numbers and Arithmetic Based on Fuzzy Structured Element

Definition 1 [13]. *Let E be a fuzzy set in R and $E(x)$ be the membership function of E. E is cled fuzzy structured element, if it satisfies the following properties: (i) $E(0)=1$, $E(1+0)=E(-1-0)=0$; (ii) $E(x)$ is a function monotonic increasing right continuous on $[-1,0]$, monotonic decreasing left continuous on $(0,1]$; (iii) $E(x)=0$ ($-\infty < x < 1$ or $1 < x < \infty$).*

The class of all fuzzy structured elements is called structured element space, and denoted by S^1.

Definition 2 [13]. *Let E be a fuzzy structured element. If it satisfies the following conditons: (a) $\forall x \in (-1,1)$, $E(x) > 0$; (b) $E(x)$ is a strictly monotonic increasing continuous on $[-1,0]$, strictly monotonic decreasing continuous on $(0,1]$, then E is said to be a regular fuzzy structured element.*

Theorem 1 [13]. *Let E be a fuzzy structured element and $E(x)$ be the membership function of E. Assume the function $f(x)$ be bounded monotonic on $[-1,1]$ and $\hat{f}(E)$ be the prolongation of function f, then $\hat{f}(E)$ is a fuzzy number, and the membership function of $\hat{f}(E)$ is $E(f^{-1}(x))$ where $f^{-1}(x)$ is the prolongation inverse function of $f(x)$.*

In the following, we denote $\hat{f}(x)$ as $f(x)$, and denote $\hat{f}(E)$ as $f(E)$.

Theorem 2 [13]**.** *For a given regular fuzzy structured element E and an arbitrary bounded fuzzy number A, there exists a bounded monotonic function $f(x)$ on $[-1,1]$, which satisfies $A = f(E)$. Conversely, Let E be an arbitrary fuzzy structured element in R and $E(x)$ be the membership function of E, suppose $f(x)$ be a bounded monotonic function on $[-1,1]$, then $f(E)$ is a bounded fuzzy number in R. We call fuzzy number A is generated by E.*

Definition 3. *Suppose f, g be two bounded monotonic functions on $[a,b]$, f, are called two same ordinal functions, if f and g are monotonic increasing or monotonic decreasing on $[a,b]$.*

Definition 4. *Let $D[-1,1]$ be a class of same ordinal bounded monotonic functions on $[-1,1]$. For $f \in D[-1,1]$, we define several transforms as following*

$$\tau_0(f) = f \ , \tau_1(f) = f^{\tau_1}, \tau_2(f) = f^{\tau_2}, \tau_3(f) = f^{\tau_3}$$

where $f^{\tau_1}(x) = -f(-x)$, $f^{\tau_2}(x) = \dfrac{1}{f(-x)}$ ($f(-x) \neq 0$), $f^{\tau_3}(x) = -\dfrac{1}{f(x)}$ ($f(x) \neq 0$), $\forall x \in [-1,1]$.

Property 1. *Let fuzzy numbers $A = f(E)$ and $B = g(E)$ be both generated by the same fuzzy structured element E, where f, g are two same ordinal bounded functions on $[-1,1]$, then*

$$A + B = f(E) + g(E) = (f + g)(E)$$
$$A - B = f(E) - g(E) = f(E) + g^{\tau_1}(E) = (f + g^{\tau_1})(E)$$
$$\forall \lambda \in [0,\infty) \subset R, \lambda A = \lambda f(E) = (\lambda f)(E)$$
$$\forall \lambda \in (-\infty,0) \subset R, \lambda A = -\lambda f^{\tau_1}(E)$$

If $A > 0, B > 0$, *then*

$$A \cdot B = f(E) \cdot g(E) = [f \cdot g](E)$$
$$A \div B = A \cdot \frac{1}{B} = f(E) \cdot g^{\tau_2}(E) = [f \cdot g^{\tau_2}](E)$$

Other cases of fuzzy number multiplication and division see reference [13].

If above f, g are piecewise functions as follows:

$$f(x)=\begin{cases}\underline{f}(x), & x\in[-1,0)\\ \bar{f}(x), & x\in[0,1]\end{cases},\qquad g(x)=\begin{cases}\underline{g}(x), & x\in[-1,0)\\ \bar{g}(x), & x\in[0,1]\end{cases}$$

then A, B are denoted as

$$A=(\underline{f}(E),\bar{f}(E)),\qquad B=(\underline{g}(E),\bar{g}(E))$$

and the arithmetic operation $*$ of A and B are defined by

$$A*B=(\underline{f}(E),\bar{f}(E))*(\underline{g}(E),\bar{g}(E))=(\underline{f}(E)*\underline{g}(E),\bar{f}(E)*\bar{g}(E))$$

Definition 5. *A fuzzy number* $\tilde{A}$ *is called positive(negative) number, denoted by* $\tilde{A}>0$ $(\tilde{A}<0)$, *if its member function* $\mu_{\tilde{A}}(x)$ *satisfies* $\mu_{\tilde{A}}(x)=0$ *for*

$$\forall x\le 0$$

$(\forall x\ge 0)$. *Similarly, a fuzzy number* $\tilde{A}$ *is called nonnegative (nonpositive) numbers, denoted by* $\tilde{A}>0$ $(\tilde{A}<0)$, *if its member function* $\mu_{\tilde{A}}(x)$ *satisfies* $\mu_{\tilde{A}}(x)=0$ *for* $\forall x<0(\forall x>0)$.

Definition 6. *A matrix* $\tilde{A}=(\tilde{a}_{ij})$ *is called a fuzzy matrix, if each element in* $\tilde{A}$ *is a fuzzy number. A vector* $\tilde{b}=(\tilde{b}_j)$ *is called a fuzzy vector, if each element in* $\tilde{b}$ *is a fuzzy number.*

Definition 7. *Let* $\tilde{A}_{ij}=(\tilde{a}_{ij})$ *and* $\tilde{B}=(\tilde{b}_{ij})$ *be two* $m\times n$ *and* $n\times p$ *fuzzy matrices. We define*

$$\tilde{A}\times\tilde{B}=\tilde{C}=(\tilde{c}_{ij})$$

where $\tilde{C}$ *is an* $m\times p$ *matrix and* $\tilde{c}_{ij}=\sum_{k=1}^{n}\tilde{a}_{ik}\times\tilde{b}_{kj}$.

3 Solving Fully Fuzzy Linear System

Definition 8. *The* $n\times n$ *fuzzy linear system*

$$\begin{cases}\tilde{a}_{11}\tilde{x}_1 + \tilde{a}_{12}\tilde{x}_2 + \cdots + \tilde{a}_{1n}\tilde{x}_n = \tilde{b}_1\\ \tilde{a}_{21}\tilde{x}_1 + \tilde{a}_{22}\tilde{x}_2 + \cdots + \tilde{a}_{2n}\tilde{x}_n = \tilde{b}_2\\ \quad\vdots\\ \tilde{a}_{n1}\tilde{x}_1 + \tilde{a}_{n2}\tilde{x}_2 + \cdots + \tilde{a}_{nn}\tilde{x}_n = \tilde{b}_n\end{cases} \tag{1}$$

Above system can be written as

$$\tilde{A}\tilde{x} = \tilde{b}$$

where the coefficient matrix $\tilde{A} = (\tilde{a}_{ij})$, $1 \le i, j \le n$ *is a fuzzy matrix and* $\tilde{b} = (\tilde{b}_i)$, $1 \le i \le n$ *is a fuzzy vector. This system is called a fully fuzzy linear system (FFLS).*

In this paper, we consider the fully fuzzy linear system whose parameters and unknowns are all fuzzy numbers generated by the same fuzzy structured element, then the coefficient matrix $\tilde{A} = (\tilde{a}_{ij})$, right-hand side column $\tilde{b} = (\tilde{b}_i)$ and $\tilde{x} = (\tilde{x}_j)$, $1 \le i, j \le n$ can be respectively denoted by $\tilde{A} = (a_{ij}(E))$, $\tilde{b} = (b_i(E))$ and unknowns $\tilde{x}_j = (x_j(E))$, where $a_{ij}(x)$, $b_i(x)$ and $x_j(x)$ are the same ordinal functions on $[-1,1]$.

In the rest of this paper we investigate a nonnegative FFLS and want to find the nonnegative solution, and Eq.(1) can be transformed into the following form:

$$\begin{cases} a_{11}(E)\cdot x_1(E) + a_{12}(E)\cdot x_2(E) + \cdots + a_{1n}(E)\cdot x_n(E) = b_1(E) \\ a_{21}(E)\cdot x_1(E) + a_{22}(E)\cdot x_2(E) + \cdots + a_{2n}(E)\cdot x_n(E) = b_2(E) \\ \vdots \\ a_{n1}(E)\cdot x_1(E) + a_{n2}(E)\cdot x_2(E) + \cdots + a_{nn}(E)\cdot x_n(E) = b_n(E) \end{cases}$$

By virtue of the definition of nonnegative fuzzy numbers, we get $a_{11}(E) \ne 0$, then let $m_{i1} = a_{11}(E) \times a_{i1}(E)$, and we utilize an elimination process for the coefficients

$$\begin{cases} s_{1j}(E) = a_{1j}(E), & 1 \le j \le n \\ a_{ij}^{(2)}(E) = \dfrac{m_{i1}(E)}{a_{11}(E)} a_{1j}(E) - \dfrac{m_{i1}(E)}{a_{i1}(E)} a_{ij}(E), & a_{i1} \ne 0, 2 \le i \le n, 1 \le j \le n \\ a_{ij}^{(2)}(E) = a_{ij}(E), & a_{i1} = 0, 2 \le i, j \le n \end{cases}$$

and the right-hand sides are calculated as follows:

$$\begin{cases} d_1(E) = b_1(E) \\ b_i^{(2)}(E) = \dfrac{m_{i1}(E)}{a_{11}(E)} b_1(E) - \dfrac{m_{i1}(E)}{a_{i1}(E)} b_i(E), & a_{i1} \ne 0, 2 \le i \le n \\ b_i^{(2)}(E) = b_i(E), & a_{i1} = 0, 2 \le i \le n \end{cases}$$

Use the same procedure for the remaining $(n-1)(n-1)$ parametric system. Assume $k-1$ steps have been done, then we can get

$$\begin{bmatrix} s_{11}(E) & s_{12}(E) & \cdots & s_{1k}(E) & \cdots & s_{1n}(E) \\ 0 & s_{22}(E) & & s_{2k}(E) & & s_{2n}(E) \\ & & \ddots & \vdots & & \vdots \\ \vdots & & & a_{kk}^{(k)}(E) & \cdots & a_{kn}^{(k)}(E) \\ & & & \vdots & & \vdots \\ 0 & \cdots & & a_{nk}^{(k)}(E) & \cdots & a_{nn}^{(k)}(E) \end{bmatrix} \begin{bmatrix} x_1(E) \\ x_2(E) \\ \vdots \\ x_k(E) \\ \vdots \\ x_n(E) \end{bmatrix} = \begin{bmatrix} d_1(E) \\ d_2(E) \\ \vdots \\ b_k^{(k)}(E) \\ \vdots \\ b_n^{(k)}(E) \end{bmatrix}.$$

If $a_{kk}^{(k)}(E) \neq 0$, then let $m_{ik} = a_{kk}(E) \times a_{ik}(E)$, $k+1 \leq i \leq n$ and we do the same operations on the remaining $k \times k$ parametric system in the following

$$\begin{cases} s_{kj}(E) = a_{kj}^{(k)}(E), & k \leq j \leq n \\ a_{ij}^{(k+1)}(E) = \dfrac{m_{ik}(E)}{a_{kk}^{(k)}(E)} a_{kj}^{(k)}(E) - \dfrac{m_{ik}(E)}{a_{ik}^{(k)}(E)} a_{ij}^{(k)}(E), & a_{ik}^{(k)}(E) \neq 0, k+1 \leq i, j \leq n \\ a_{ij}^{(k+1)}(E) = a_{ij}^{(k)}(E), & a_{ik}^{(k)}(E) = 0, k+1 \leq i, j \leq n \end{cases}$$

and

$$\begin{cases} d_k(E) = b_k^{(k)}(E), & \\ b_i^{(k+1)}(E) = \dfrac{m_{ik}(E)}{a_{kk}(E)} b_k^{(k)}(E) - \dfrac{m_{ik}(E)}{a_{ik}(E)} b_i^{(k)}(E), & a_{ik}^{(k)}(E) \neq 0, k+1 \leq i \leq n \\ b_i^{(k+1)}(E) = b_i^{(k)}(E), & a_{ik}^{(k)}(E) = 0, k+1 \leq i \leq n \end{cases}$$

Repeat above elimination process and at last the original parametric system is equivalently transformed into the following system

$$\begin{bmatrix} s_{11}(E) & s_{12}(E) & \cdots & s_{1n}(E) \\ 0 & s_{22}(E) & \cdots & s_{2n}(E) \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & s_{nn}(E) \end{bmatrix} \begin{bmatrix} x_1(E) \\ x_2(E) \\ \vdots \\ x_n(E) \end{bmatrix} = \begin{bmatrix} d_1(E) \\ d_2(E) \\ \vdots \\ d_n(E) \end{bmatrix}$$

If $s_{nn}(E) \neq 0$, then we get the unique solution from above system

$$\begin{cases} x_n(E) = d_n(E)/s_{nn}(E) \\ x_k(E) = (d_k(E) - \sum_{j=k+1}^{n} s_{kj}(E)x_j(E))/s_{kk}(E), \quad k = n-1, n-2, \cdots, 1 \end{cases}$$

Note that above arithmetic operation of parametric numbers follows the constraint operation properties [14].

4 Numerical Example

Example 1. Consider the 2×2 FFLS (The system have been solved in [6] and [12] with different methods with different methods)

$$\begin{bmatrix} (5+E,5+E) & (6+E,6+2E) \\ (7+E,7) & (4,4+E) \end{bmatrix} \begin{bmatrix} (\underline{x}_1(E),\overline{x}_1(E)) \\ (\underline{x}_2(E),\overline{x}_2(E)) \end{bmatrix} = \begin{bmatrix} (50+10E,50+17E) \\ (48+5E,48+7E) \end{bmatrix}$$

We can get two parametric systems from above fuzzy system

$$\begin{cases} (5+E)\cdot \underline{x}_1(E) + (6+E)\cdot \underline{x}_2(E) = 50+10E \\ (7+E)\cdot \underline{x}_1(E) + 4\cdot \underline{x}_2(E) = 48+5E \end{cases}$$

$$\begin{cases} (5+E)\cdot \overline{x}_1(E) + (6+2E)\cdot \overline{x}_2(E) = 50+17E \\ 7\cdot \overline{x}_1(E) + (4+E)\overline{x}_2(E) = 48+7E \end{cases}$$

where E is a fuzzy structured element with the following membership function

$$E(x) = \begin{cases} 1+x, & x\in[-1,0] \\ 1-x, & x\in(0,1] \\ 0, & otherwise \end{cases}$$

Applying an elimination method for solving these two parametric systems, we get the solutions of the FFLS as follows:

$$\tilde{x}_1 = (\underline{x}_1(E),\overline{x}_1(E)) = (\frac{88+38E+5E^2}{22-9E+E^2}, \frac{88+20E-3E^2}{22-5E-E^2})$$

$$\tilde{x}_2 = (\underline{x}_2(E),\overline{x}_2(E)) = (\frac{110+47E+5E^2}{22-9E+E^2}, \frac{110+46E-7E^2}{22-5E-E^2})$$

Dehghan [6] and Tofigh [12] have solved this system and their solutions are respectively showed as

$$\tilde{x}_1 = (4 + \frac{1}{11}E, 0), \quad \tilde{x}_2 = (5 + \frac{1}{11}E, 5 + \frac{1}{2}E)$$

and

$$\tilde{x}_1 = (\frac{88 + 38E + 5E^2}{22 + 9E + E^2}, \frac{88 + 20E - 3E^2}{22 + 5E - E^2}),$$

$$\tilde{x}_2 = (\frac{110 + 47E + 5E^2}{22 + 9E + E^2}, \frac{110 + 46E - 7E^2}{22 + 5E - E^2})$$

The solutions in this paper and solutions in [6] and [12] are plotted and compared (see Fig. 1-3).

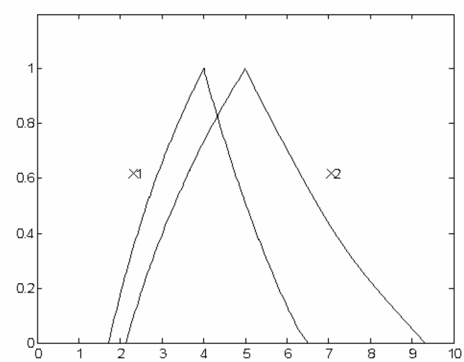

Fig. 1. The exact fuzzy solutions in this paper

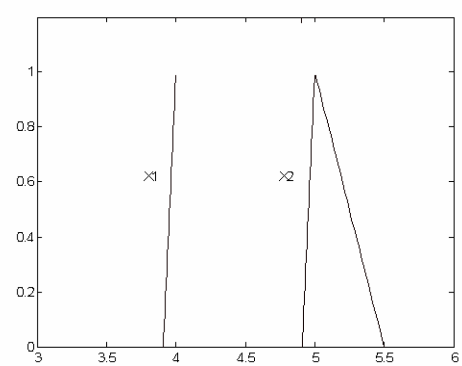

Fig. 2. The approximate fuzzy solutions in [6]

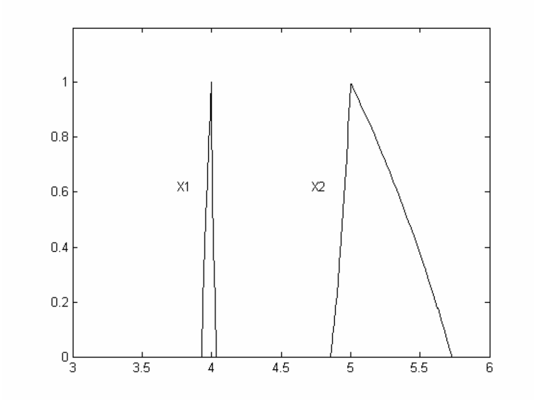

Fig. 3. The approximate fuzzy solutions in [12]

From above comparison of three pictures, clearly, [6] adopt the approximate fuzzy number multiplication Dubois proposed, it results in existing a big error, and the errors of fuzzy solutions will be enlarged with the increase of operation frequency. The multiplication in [12] meets the λ-cut method, but the division does not, and their method cause big errors too. Compared with the solutions in [6] and [12], the superiority of the method in this paper is showed.

Example 2. Consider the following 2×2 FFLS

$$\begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} \\ \tilde{a}_{21} & \tilde{a}_{22} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} = \begin{bmatrix} \tilde{b}_1 \\ \tilde{b}_2 \end{bmatrix}$$

where the membership functions of $\tilde{a}_{ij}, \tilde{b}_i, 0 \le i, j \le 2$ are

$$\mu_{\tilde{a}_{11}}(x) = \begin{cases} 1+\dfrac{(\ln x - 7)}{2}, & x\in [e^5, e^7] \\ 1-\dfrac{(\ln x - 7)}{2}, & x\in (e^7, e^9] \\ 0, & \text{otherwise} \end{cases} \quad \mu_{\tilde{a}_{12}}(x) = \begin{cases} 1+\ln x - 4, & x\in [e^3, e^4] \\ 1-\ln x - 4, & x\in (e^4, e^5] \\ 0, & \text{otherwise} \end{cases}$$

$$\mu_{\tilde{a}_{21}}(x) = \begin{cases} 1+\ln x - 3, & x\in [e^2, e^3] \\ 1-\ln x - 3, & x\in (e^3, e^4] \\ 0, & \text{otherwise} \end{cases} \quad \mu_{\tilde{a}_{22}}(x) = \begin{cases} 1+\ln x - 8, & x\in [e^7, e^8] \\ 1-\ln x - 8, & x\in (e^8, e^9] \\ 0, & \text{otherwise} \end{cases}$$

$$\mu_{\tilde{b}_1}(x)=\begin{cases}1+(\ln x-18)/3, & x\in[e^{15},e^{18}]\\ 1-(\ln x-18)/3, & x\in(e^{18},e^{21}]\\ 0, & \text{otherwise}\end{cases}$$

$$\mu_{\tilde{b}_2}(x)=\begin{cases}1+(\ln x-30)/7, & x\in[e^{23},e^{30}]\\ 1-(\ln x-30)/7, & x\in(e^{30},e^{37}]\\ 0, & \text{otherwise}\end{cases}$$

Transform the fuzzy numbers into the parametric form for fuzzy structured element, then

$$\begin{bmatrix}e^{7+2E} & e^{4+E}\\ e^{3+2E} & e^{8+E}\end{bmatrix}\begin{bmatrix}x_1(E)\\ x_2(E)\end{bmatrix}=\begin{bmatrix}e^{18+3E}\\ e^{30+7E}\end{bmatrix}$$

Solve the parametric system and the exact fuzzy solutions are

$$\tilde{x}_1=x_1(E)=\frac{e^{30+11E}-e^{14+6E}}{e^{8-E}-1}$$

$$\tilde{x}_2=x_2(E)=\frac{e^{27-10E}-e^{19-6E}}{e^{8+E}-1}$$

5 Conclusion

According to homeomorphic property between the bounded fuzzy number and the standard monotonic function family with the same monotonic form on $[-1,1]$, the arithmetic operation of fuzzy numbers are transformed into the operation of the same ordinal monotonic functions on $[-1,1]$, and bring off the solution of FFLS by an elimination method. Compared with the existing methods, the superiorities of this method are as follows: (i) it is applicable to the solution of fully fuzzy linear systems which have any type of fuzzy numbers. (ii) The exact fuzzy numbers can be obtained.

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Solving the System of Fuzzy Polynomial Equations

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Abstract. In this paper, fuzzy polynomial equation and system of fuzzy polynomial equations are defined. The system of fuzzy polynomial equations is investigated based on the fuzzy structured element. The method transforming the system of fuzzy polynomial equations into the parametric equations is proposed. We prove the rationality of this theory and give the necessary and sufficient condition for a fuzzy solution existence, in addition, a numerical example is showed.

Keywords: System of fuzzy polynomial equations, fuzzy structured element method, system of parametric equations.

1 Introduction

The systems of fuzzy polynomial equations are widely applied in fuzzy mathematics, fuzzy engineering and so on, thus it is significant to solve the systems of fuzzy polynomial equations. The operations of fuzzy numbers do not satisfy associative law, that is $\tilde{A}\tilde{B}+\tilde{A}\tilde{C}\neq\tilde{A}(\tilde{B}+\tilde{C})$. It generates that the method cannot be directly applied for solving the systems of fuzzy polynomial equations. Therefore, it is important to find a feasible and effective method for solving.

Since the operations of fuzzy numbers are very complex, there are few research results. Buckely and Qu [1] study the solution of fuzzy equation for two unknowns, and give the necessary and sufficient condition for a fuzzy solution existence, but this method is only suitable for the case where the fuzzy numbers are trapezoidal fuzzy numbers. Two new solution procedures based on the unified extension and possibility theory are introduced [2], but the actual operation is complex. Abbsabandy et al. [3,4] investigate the solution of nonlinear equations, but solving any system of fuzzy nonlinear equations is worth discussing.

The concept of fuzzy structured element and the structured element representations of fuzzy numbers are introduced in [5], [6] proposes homeomorphic property between the bounded fuzzy number and the standard monotonic function family with same monotonic form on $[-1,1]$, on the basis of above results, this paper defines an opposite ordinal transform and develops a new method for solving the system of fuzzy polynomial equations.

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2 Preliminary

Definition 2.1 [5]. *Let* E *be a fuzzy set in R and* $E(x)$ *be the membership function of* E . E *is called fuzzy structured element, if satisfies the following properties: (i)* $E(0)=1$, $E(1+0)=E(-1-0)=0$ *; (ii)* $E(x)$ *is a function monotonic increasing right continuous on* $[-1,0]$ *, monotonic decreasing left continuous on* $(0,1]$ *; (iii)* $E(x)=0$ *(* $-\infty<x<1$ *or* $1<x<\infty$ *).*

Theorem 2.1 [5]. *For a given regular fuzzy structured element* E *and an arbitrary bounded fuzzy number* A *, there exists a bounded monotonic function* $f(x)$ *on* $[-1,1]$ *, which satisfies* $A=f(E)$ *. Conversely, Let* E *be an arbitrary fuzzy structured element in* R *and* $E(x)$ *be the membership function of* E *, assume* $f(x)$ *be a bounded monotonic function on* $[-1,1]$ *, then* $f(E)$ *is a bounded fuzzy number in* R *. We call fuzzy number* A *is generated by* E *.*

Definition 2.2. *Suppose* f , g *two bounded monotonic functions on* $[a,b]$ *,* f , g *are called two same ordinal functions, if* f *and* g *have the same monotonicity on* $[a,b]$ *;* f , g *are called two same ordinal functions, If* f *and* g *have the opposite monotonicity on* $[a,b]$.

Definition 2.3. *Let* $D[-1,1]$ *be a class of monotone functions on* $[-1,1]$ *. For* $f\in D[-1,1]$ *, we define an opposite ordinal transform* $\tau : D[-1,1]\to D[-1,1]$ *on* $[-1,1]$ *in the following*

$$\tau(f)=f^{\tau}$$

where $f^{\tau}(x)=f(-x)$.

Definition 2.4. *A fuzzy number* A *is called positive (negative) number, denoted b y* $A>0$ $(A<0)$ *, if its member function* $\mu_A(x)$ *satisfies* $\mu_A(x)=0$ *for* $\forall x<0$ $(\forall x>0)$.

Property 2.1 [5]. *Let fuzzy numbers* $A=f(E)$ *and* $B=g(E)$ *, where* E *is given fuzzy structured element* f , g *are two same ordinal bounded functions on* $[-1,1]$ *, then*

$$A + B = f(E) + g(E) = (f + g)(E)$$

$$A - B = f(E) - g(E) = f(E) - g^{\tau}(E) = (f - g^{\tau})(E)$$

$\forall \lambda \in [0,\infty) \subset R$, $\lambda A = \lambda f(E) = (\lambda f)(E)$

$\forall \lambda \in (-\infty,0) \subset R$, $\lambda A = -\lambda f^{\tau}(E)$

If $A > 0, B > 0$ *, then* $A \cdot B = [f \cdot g](E)$ *with the membership function* $[f \cdot g]^{-1}(x)$

If $A < 0, B < 0$ *, then* $A \cdot B = [f^{\tau} \cdot g^{\tau}](E)$ *with the membership function* $[f^{\tau} \cdot g^{\tau}]^{-1}(x)$

If $A < 0, B > 0$ *, then* $A \cdot B = [f \cdot g^{\tau}](E)$ *with the membership function* $[f \cdot g^{\tau}]^{-1}(x)$

If $A > 0, B < 0$ *, then* $A \cdot B = [f^{\tau} \cdot g](E)$ *with the membership function* $[f^{\tau} \cdot g]^{-1}(x)$

Theorem 2.2. *Let* E *be a symmetrical fuzzy structured element,* $f(x)$ *monotone function on* $[-1,1]$*, there exists fuzzy number* $A = f(E)$*, then* $\forall k \in N$*, we have*

$$A^k = f^k(E)$$

the membership function of A^k *is*

$$\mu_{A^k}(x) = E[(f^k)^{-1}(x)]$$

Theorem 2.3 [5]**.** *Suppose that* f *is a bounded monotone function on* $[-1,1]$*,* E *is given fuzzy structured element in* R*, fuzzy number* $A = f(E)$*. For any* $\lambda \in [0,1]$*, the* $\lambda-$ *level set* E *of is denoted as* $A_\lambda = E[f(e_\lambda^+), f(e_\lambda^-)]$*, where* $e_\lambda^- \in [-1,0]$*,* $e_\lambda^+ \in [0,1]$*. If* f *is monotone increasing function on* $[-1,1]$ *,then* $A_\lambda = [f(E)]_\lambda = f[e_\lambda^-, e_\lambda^+] = [f(e_\lambda^-), f(e_\lambda^+))]$ *; If* f *is monotone decreasing function on* $[-1,1]$*, then* $A_\lambda = E[f(e_\lambda^+), f(e_\lambda^-)]$.

Theorem 2.4. *Let* E *be a given symmetrical fuzzy structured element in* R*,* $f(x)$*,* $g(x)$ *are bounded monotone functions on* $[-1,1]$ *, if* $f(x) = g(-x)$ *,* $x \in [-1,1]$ *, then* $A = B$ *.*

Proof. Assume f is monotone increasing function on $[-1,1]$, then g is a monotone decreasing function on $[-1,1]$. By Theorem 2.3., $\forall\lambda \in [0,1]$

$$A_\lambda = [f(E)]_\lambda = F(E_\lambda) = [f(e_\lambda^-), f(e_\lambda^+)]$$
$$B_\lambda = [g(E)]_\lambda = g(E_\lambda) = [g(e_\lambda^+), g(e_\lambda^-)]$$

Since E is a symmetrical fuzzy structured element, we have $-e_\lambda^+ = e_\lambda^-$, then we have

$$f(E) = [f(e_\lambda^-), f(e_\lambda^+)] = [f(-e_\lambda^+), f(-e_\lambda^-)] = [g(e_\lambda^+), g(e_\lambda^-)] = g(E)$$

Therefore, $A = B$.

3 Solving the System of Fuzzy Polynomial Equations

Definition 3.1. $\sum_{i=1}^{n} \tilde{a}_i \tilde{x}^i = \tilde{b}$ *is called a fuzzy polynomial equation, where* $\tilde{x}$, $\tilde{b}$ *are two fuzzy numbers,* $\tilde{a}_i$ *is a fuzzy number or real number.*

Theorem 3.1. *Suppose* $\sum_{i=1}^{n} \tilde{a}_i \tilde{x}^i = \tilde{b}$ *is a fuzzy polynomial equation, then there must exists a parametric equation for* $x(u)$, $x^\tau(u)$, $u \in [-1,1]$, *where* $x(u)$, $x^\tau(u)$ *are two opposite ordinal monotone functions.*

Proof. By theorem 2.1, all fuzzy numbers of fuzzy polynomial equations can be represented by the same structured element E, and let $\tilde{a}_k = a_k(E)$, $\tilde{x} = x_k(E)$, where $a_k(u)$, $x_k(u)$ are two same ordinal monotone functions.

If $\tilde{a}_k > 0$, $\tilde{x} > 0$, then $\tilde{a}_k \tilde{x}^k = a_k(E)[x^k(E)]$; If $\tilde{a}_k > 0$, $\tilde{x} < 0$ and k is an even number, then $\tilde{a}_k \tilde{x}^k = a_k(E) x^k(E)$; If $\tilde{a}_k > 0$, $\tilde{x} < 0$ and k is an odd number, then $\tilde{a}_k \tilde{x}^k = a_k^\tau(E)[\tilde{x}^k(E)] = a_k^\tau(E)[x^k(E)]$. Therefore, if $\tilde{a}_k > 0$, then the k th term can be transformed into a system of parametric equations for $x(u)$ and $x^\tau(u)$, $u \in [-1,1]$.

If $\tilde{a}_k < 0$, $\tilde{x} > 0$, then $\tilde{a}_k \tilde{x}^k = a_k(E)[x^k(E)]^\tau = a_k(E)[x^\tau(E)]^k$; If $\tilde{a}_k < 0$, $\tilde{x} < 0$ and k is ,n even number, then $\tilde{a}_k \tilde{x}^k = a_k(E)[x^k(E)]^\tau = a_k(E)[x^\tau(E)]^k$; If $\tilde{a}_k < 0$, $\tilde{x} < 0$ and k is an odd number, then $\tilde{a}_k \tilde{x}^k = a_k^\tau(E)[x^k(E)]^\tau = a_k^\tau(E)[x^\tau(E)]^k$; Therefore, if $\tilde{a}_k < 0$, then the k th term can be transformed into a system of fuzzy parametric equations for $x(u)$ and $x^\tau(u)$, where $u \in [-1,1]$.

If $\tilde{a}_k$ is a real number, we also discuss the cases $\tilde{a}_k > 0$ and $\tilde{a}_k < 0$, it is easy to prove the k th term can be transformed into a system of fuzzy parametric equations for $x(u)$ and $x^\tau(u)$, $u \in [-1,1]$. By the operational properties of structured element fuzzy numbers, for a given fuzzy polynomial equation $\sum_{i=1}^{n} \tilde{a}_i \tilde{x}^i = \tilde{b}$, then there exists a system of polynomial parametric equations for $x(u)$, $x^\tau(u)$, $u \in [-1,1]$.

Definition 3.2. *The system of polynomial equations*

$$F(\tilde{x}) = \begin{bmatrix} F_1(\tilde{x}_1, \tilde{x}_2 \cdots, \tilde{x}_n) \\ F_2(\tilde{x}_1, \tilde{x}_2 \cdots, \tilde{x}_n) \\ \vdots \\ F_n(\tilde{x}_1, \tilde{x}_2 \cdots, \tilde{x}_n) \end{bmatrix} = \begin{bmatrix} \tilde{b}_1 \\ \tilde{b}_2 \\ \vdots \\ \tilde{b}_n \end{bmatrix} \tag{1}$$

is called a system of fuzzy polynomial equations, if $F_i(\tilde{x}_1, \tilde{x}_2 \cdots, \tilde{x}_n) = P_{i1}(\tilde{x}_1) + P_{i2}(\tilde{x}_2) + \cdots + P_{in}(\tilde{x}_n)$, $P_{ij}(\tilde{x}_j), 1 \le i, j \le n$, *is a fuzzy polynomial equation.*

By Theorem 3.1., we have

$$\begin{cases} F_1(x_1(u), x_1^\tau(u), \cdots, x_n(u), x_n^\tau(u)) = b_1(u) \\ \vdots \\ F_n(x_1(u), x_1^\tau(u), \cdots, x_n(u), x_n^\tau(u)) = b_n(u) \\ F_{n+1}(x_1(u), x_1^\tau(u), \cdots, x_n(u), x_n^\tau(u)) = b_1^\tau(u) \\ \vdots \\ F_{2n}(x_1(u), x_1^\tau(u), \cdots, x_n(u), x_n^\tau(u)) = b_n^\tau(u) \end{cases}$$

where $x_i(u)$, $b_i(u)$, $1 \le i \le n$ are two ordinal monotone functions on $[-1,1]$.

Theorem 3.2. *The necessary and sufficient condition for a system of fuzzy polynomial equations which has a positive or negative fuzzy solution* $x_i(E), 1 \le i \le n$ *is*

(a) *The system of parametric equations has solutions* $x_i(u)$, $x_i^\tau(u)$;

(b) $x_i(u)$, $x_i^\tau(u)$ *are symmetrical for* y *axis;*

(c) $x_i(u) > 0$ *or* $x_i(u) < 0$;

(d) $x_i(u)$ *and* $b_i(u)$ *are ordinal monotone functions.*

Proof. By the operational properties and the method of establishing the parametric equations, the sufficient condition is obvious. We prove the necessary condition as follows. If $x_i(E)$ is the solution of Eq.(1), clearly, $x_i(E)$ is a fuzzy number, by virtue of the definition 2.3., $x_i(u)$, $x_i^\tau(u)$ are symmetrical for the y axis. According the definitions of positive and negative fuzzy numbers, obviously, if $x_i(E) > 0$, then $x_i(u) > 0$, $u \in [-1,1]$; if $x_i(E) < 0$, then $x_i(u) < 0$, $u \in [-1,1]$; Owing to the theorem 2.5., a system of parametric equations are established by the monotone functions $x_i(u)$ and $x_i^\tau(u)$, then $x_i(u)$, $x_i^\tau(u)$ must be the solutions of the system of parametric equations.

4 Numerical Example

Example 4.1. Consider the system of fuzzy polynomial equations

$$\begin{cases} 5\tilde{x}_1^2 + 3\tilde{x}_1 + 2\tilde{x}_2 = \tilde{b}_1 \\ \tilde{x}_1^2 - 2\tilde{x}_1 + \tilde{x}_2^2 + 2\tilde{x}_2 = \tilde{b}_2 \end{cases}$$

where

$$\mu_{\tilde{b}_1} = \begin{cases} \sqrt{\frac{1}{5}y - \frac{7}{20}} - \frac{1}{2}, & 3 \le x \le 13 \\ \frac{5}{2} - \sqrt{\frac{1}{5}y - \frac{7}{20}}, & 13 < x \le 33 \\ 0, & otherwise \end{cases}$$

$$\mu_{\tilde{b}_2} = \begin{cases} \sqrt{\frac{1}{5}y + \frac{14}{125}} - \frac{3}{5}, & \frac{31}{25} \le x \le \frac{306}{25} \\ \frac{13}{5} - \sqrt{\frac{1}{5}y + \frac{14}{125}}, & \frac{306}{25} < x \le \frac{831}{25} \\ 0, & otherwise \end{cases}$$

Suppose this fuzzy system has a positive or negative solution, by the 1-level set of all the fuzzy numbers we can easily get $\tilde{x}_1 > 0$, $\tilde{x}_2 < 0$ and obtain the following parametric system of

$$\begin{cases} 5x_1^2(u) + 3x_1(u) + 2x_2(u) = 5u^2 + 15u + 4 \\ 5[x_1^\tau(u)]^2 + 3x_1^\tau(u) + x_2^\tau(u) = 5u^2 - 15u + 4 \\ x_1^2(u) - 2x_1^\tau(u) + [x_2^\tau(u)]^2 + 2x_2(u) = 5u^2 + 16u + 2 \\ [x_1^\tau(u)]^2 - 2x_1(u) + x_2^2(u) + 2x_2^\tau(u) = 5u^2 - 16u + 2 \end{cases}$$

By Matlab the solution of Eq.(1) is found

$$\begin{cases} x_1(u) = 1 + u \\ x_2(u) = -2 + u \end{cases}$$

and

$$\begin{cases} \tilde{x}_1 = 1 + E \\ \tilde{x}_2 = -2 + E \end{cases}$$

with the membership functions

$$\mu_{\tilde{x}_1}(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2 - x, & 1 < x \le 2 \\ 0, & otherwise \end{cases}$$

$$\mu_{\tilde{x}_2}(x) = \begin{cases} 3 + x, & -3 \le x \le -2 \\ -1 - x, & -2 < x \le -1 \\ 0, & otherwise \end{cases}$$

5 Conclusions

In this paper, the system of fuzzy polynomial equations where the unknowns are fuzzy numbers and the parameters are fuzzy numbers or real numbers are investigated. We define an opposite ordinal transform of the monotone functions, and give a method for solving the system of fuzzy polynomial equations, then an example is given.

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Design of Fuzzy Control System with Optimal Guaranteed Cost for Planar Inverted Pendulum

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Abstract. For the problem of control a planar inverted pendulum system (PIP) with non-uniform density pendulum, a design method of optimal guaranteed cost fuzzy control is developed via the parallel distributed compensation(PDC) approach. Firstly, by using the Lagrange equation, the mathematical model of the planar inverted pendulum is derived. Then, a sufficient condition for the existence of guaranteed cost fuzzy controller is presented with taking into account a given quadratic performance index, and a deviation amplitude of the center-of-mass of the rod. Finally, simulation results show the effectiveness of the proposed design method.

Keywords: Guaranteed cost control, T-S fuzzy model, planar inverted pendulum, linear matrix inequalities.

1 Introduction

The inverted pendulum is one type of non-linear, multivariable and highly non-stable system. As a classical control object, it can be applied to verify new theories and methods. The research of inverted pendulum has strong engineering application background, such as the widespread use in rocket control and robot control. In recent years, various modern control theories have been used for controlling the inverted pendulum, for instance, the non-linear control theory ([1]), fuzzy control theory ([2, 3]), especially the variable universe adaptive control theory proposed by LIHX, which has been successfully used in the hardware implement of the quadruple inverted pendulum ([4]). Moreover, compared with the single inverted pendulum system, the spherical

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pendulum is more complex in both the establishment of model and experiment design. Although there have been some literatures [5, 6, 7, 8] about the control of spherical and multi-level inverted pendulum, almost all the previous researches assume that the pendulum is uniform in density. Nevertheless, the mass center will not be the center of pendulum after the pendulum bar has been attached by some peripherals such as decoders, which produces great deviation between the design and real model thus exerting negative impact on the quality of control. To overcome such problem, we firstly establish the model for the case when the pendulum has inhomogeneous density, thereby obtaining a more general result. Considering the fact that in practical setting it is hard to locate the mass center of pendulum exactly, we will present the optimal guaranteed cost robust control method associated with planar inverted pendulum. Currently, the optimal guaranteed cost robust control theory has obtained wide concern and achieved great progress ([9, 10]). In the field of linear system, the robust control has produced many useful results. On the other hand, it also gives some meaningful results about non-linear system theory ([11, 13]). In the aspect of guaranteed cost control, [12, 13] has investigated the control problem of the single and the triple inverted pendulums. In general, the guaranteed cost control adopts quadratic form as the performance index. However, the optimization of such quadratic index tends to reduce the robustness of system. Therefore, it is natural to expect a control rule which keeps balance between guaranteed cost and robustness in order to optimize the overall performance index. This article will develop a comprehensive performance index and discuss the impact of different choices of mass center by numerical experiments.

2 Modeling of Planar Inverted Pendulum

In this section, we establish the mathematical model of planar single-stage inverted pendulum. Planar inverted pendulum system is mainly formed by the rod and the cart. The main parameters in it are shown as follows:

- m_x: the masses of the moving parts and pendulum bearings on X table
- m_y: the masses of the moving parts and pendulum bearings on Y table
- m: the mass of the rod
- l: the actual length from the center of mass of the rod to linked point on the cart (named as the center-of-mass position)
- l_0: the design value of l

This paper is discussed in the following assumptions: (1) parameter m_x, m_y, m can be the exact value; (2) the friction between the components is zero; (3) the exact value of the center-of-mass position l can not be obtained, which can only substitute with a design value l_0.

First of all, we establish the space Cartesian coordinate system on XY table such that X-axis and Y-axis parallel X and Y table, respectively, and Z-axis is perpendicular to XY table. Use u_x and u_y to denote the acting forces that Y table got from X table, and pendulum bearings got from Y table, respectively (Suppose X table is under Y table.). Let α be the angle between Z-axis and the line OA (the projection of the rod in XOZ plane) at a time, and β the angle between Z-axis and the line OB (the projection of the rod in YOZ plane) at a time. The details are shown as Figure 1.

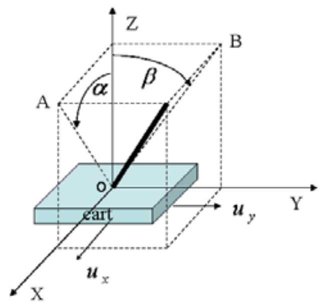

Fig. 1. The planar inverted pendulum and the coordinate system

Let $(x, y, 0)$ denote the position of the cart, then the center-of-mass of the rod is

$$x_c = x + l\frac{\sin\alpha cos\beta}{\sqrt{1-\sin^2\alpha\sin^2\beta}};$$
$$y_c = y + l\frac{\cos\alpha sin\beta}{\sqrt{1-\sin^2\alpha\sin^2\beta}};$$
$$z_c = l\frac{\cos\alpha\cos\beta}{\sqrt{1-\sin^2\alpha\sin^2\beta}}. \quad (1)$$

and the Kinetic energy of the planar inverted pendulum

$$T = T_{cart} + T_{pendulum},$$

where T_{cart} and $T_{pendulum}$ represent the kinetic energy of the cart and the rod respectively. Furthermore, the kinetic energy of of the cart can be expressed as

$$T_{cart} = \frac{1}{2}(m_x\dot{x}^2 + m_y\dot{y}^2).$$

Based on the Konig's theorem([15]), the kinetic energy of the rod

$$T_{pendulum} = \frac{1}{2}mv_c^2 + \frac{1}{2}w^{(0)^T}J_c^{(0)}w^{(0)},$$

where $J_c^{(0)}$ is the inertia matrix of the rod relative to the center-of-mass. For the simple calculation of $J_c^{(0)}$, we assume the linear density of the rod is uniform on the both sides of the the center-of-mass. We denote them as μ_1 and μ_2. So the moment of inertia relative to the axis, which passes through the center-of-mass and is perpendicular to the rod, will be

$$\int_0^l \mu_1 x^2 dx + \int_0^{L-l} \mu_2 x^2 dx = \frac{1}{6} m(L^2 - 2Ll + 2l^2).$$

The expression $\mu_1 l = \mu_2(L-l) = \frac{m}{2}$ is used in the derivation of the above equation.

On the other hand, the gravitational potential energy of the PIP System

$$V = V_{cart} + V_{pendulum},$$

where

$$V_{cart} = 0,\ V_{pendulum} = mgz_c = mgl \frac{\cos\alpha\cos\beta}{\sqrt{1-\sin^2\alpha\sin^2\beta}}.$$

Then we have the Lagrangian function

$$L = T - V, \tag{2}$$

and the Lagrangian equations

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = Q_j,\ (j = 1, 2, 3, 4) \tag{3}$$

where $q_1 = x, q_2 = y, q_3 = \alpha, q_4 = \beta$. After some simple derivations, we get

$$M(q,\dot{q})\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = F, \tag{4}$$

where

$$M(q,\dot{q}) = \begin{bmatrix} m+m_x & 0 & \frac{ml\cos\alpha\cos\beta}{h_0^{3/2}} & -\frac{ml\sin\alpha\sin\beta\cos^2\alpha}{h_0^{3/2}} \\ 0 & m+m_y & -\frac{ml\sin\alpha\sin\beta\cos^2\beta}{h_0^{3/2}} & \frac{ml\cos\alpha\cos\beta}{h_0^{3/2}} \\ \frac{ml\cos\alpha\cos\beta}{h_0^{3/2}} & -\frac{ml\sin\alpha\sin\beta\cos^2\beta}{h_0^{3/2}} & \frac{m\mathcal{L}\cos^2\beta}{2h_0^2} & -\frac{m\mathcal{L}\sin 2\alpha\sin 2\beta}{8h_0^2} \\ -\frac{ml\sin\alpha\sin\beta\cos^2\alpha}{h_0^{3/2}} & \frac{ml\cos\alpha\cos\beta}{h_0^{3/2}} & -\frac{m\mathcal{L}\sin 2\alpha\sin 2\beta}{8h_0^2} & \frac{m\mathcal{L}\cos^2\alpha}{2h_0^2} \end{bmatrix}$$

$$C(q,\dot{q}) = \begin{bmatrix} 0\ 0 & \frac{ml(h_1\dot{\alpha}\sin\alpha\cos\beta + h_2\dot{\beta}\cos\alpha\sin\beta)}{h_0^{5/2}} & \frac{ml(h_2\dot{\alpha}\cos\alpha\sin\beta - h_5\dot{\beta}\sin\alpha\cos\beta\cos^2\alpha)}{h_0^{5/2}} \\ 0\ 0 & \frac{ml(-h_5\dot{\alpha}\cos\alpha\sin\beta\cos^2\beta + h_1\dot{\beta}\sin\alpha\cos\beta)}{h_0^{5/2}} & \frac{ml(h_1\dot{\alpha}\sin\alpha\cos\beta + h_2\dot{\beta}\sin\beta\cos\alpha)}{h_0^{5/2}} \\ 0\ 0 & \frac{m\mathcal{L}(\dot{\alpha}\sin 2\alpha\sin^2 2\beta + 2h_3\dot{\beta}\sin 2\beta)}{8h_0^3} & \frac{m\mathcal{L}(2h_3\dot{\alpha}\sin 2\beta - \dot{\beta}\sin 2\alpha\sin^2 2\beta\sin^2\alpha)}{8h_0^3}. \\ 0\ 0 & \frac{m\mathcal{L}(-\dot{\alpha}\sin 2\beta\sin^2 2\alpha\sin^2\beta + 2h_4\dot{\beta}\sin 2\alpha)}{8h_0^3} & \frac{m\mathcal{L}(2h_4\dot{\alpha}\sin 2\alpha + \dot{\beta}\sin^2 2\alpha\sin 2\beta)}{8h_0^3} \end{bmatrix}$$

$$G(q) = \left[0,\, 0,\, -\frac{mgl}{h_0^{3/2}} \sin\alpha \cos^3\beta,\, -\frac{mgl}{h_0^{3/2}} \sin\beta \cos^3\alpha\right]^T$$
$$F = \left[u_x \; u_y \; 0 \; 0\right]^T,$$

where

$$h_0 = 1 - \sin^2\alpha \sin^2\beta, h_1 = 2\cos^2\alpha \sin^2\beta - \cos^2\beta,$$
$$h_2 = 2\cos^2\beta \sin^2\alpha - \cos^2\alpha, h_3 = \cos^2\beta \sin^2\alpha - \cos^2\alpha,$$
$$h_4 = \cos^2\alpha \sin^2\beta - \cos^2\beta, h_5 = 1 + 2\sin^2\alpha \sin^2\beta,$$
$$\mathcal{L} = (L^2 - 2Ll + 8l^2)/3.$$

Let the state vector $\mathbf{x} = (x_1, x_2, \cdots, x_8)^T = (x, \dot{x}, y, \dot{y}, \alpha, \dot{\alpha}, \beta, \dot{\beta})^T$. Based on the linear method [16], the dynamic system can be linearization between the different operating points. Then the T-S fuzzy model of the PIP System is obtained. The fuzzy rules of the T-S fuzzy model are listed:

$$\begin{aligned}&\text{IF } \; x_1(t) \text{ is } F_1^i \text{ and} \cdots \text{ and } x_n(t) \text{ is } F_n^i\\ &\text{Then } \dot{\mathbf{x}}(t) = A_i(l)\mathbf{x}(t) + B_i(l)\mathbf{u}(t), \mathbf{x}(0) = \mathbf{x}_0, (i = 1, 2, \cdots, r).\end{aligned}$$

where F_j^i, $j = 1, 2, \cdots, n$ is the j-th fuzzy set of the i-th fuzzy rule, $\mathbf{u}(t) = (u_x, u_y)^T$ is the control input and $\mu_j^i(x_j)$ is the membership function of fuzzy set F_j^i.

Now we replace the l in fuzzy sets A_i, B_i with $l_0 + \Delta l$. By the Taylor formula, we have

$$A_i(l) \approx A_i(l_0) + \frac{dA_i}{dl}|_{l_0}\Delta l, B_i(l) \approx B_i(l_0) + \frac{dB_i}{dl} \;|_{l_0}\; \Delta l. \tag{5}$$

Let $\Delta A_i = \frac{dA_i}{dl}|_{l_0}\Delta l, \Delta B_i = \frac{dB_i}{dl} \;|_{l_0}\; \Delta l$. The model (5) can be simplified as

$$\begin{aligned}&\text{IF } x_1(t) \text{ is } F_1^i \text{ and} \cdots \text{ and } x_n(t) \text{ is } F_n^i \text{ Then}\\ &\dot{\mathbf{x}}(t) = (A_i(l_0) + \Delta A_i)\mathbf{x}(t) + (B_i(l_0) + \Delta B_i)\mathbf{u}(t),\\ &\mathbf{x}(0) = \mathbf{x}_0, (i = 1, 2, \cdots, r).\end{aligned} \tag{6}$$

In the following, $A_i(l_0), B_i(l_0)$ can be represented as A_i, B_i respectively.

By using weighted average method, we have the state equation of the PIP System

$$\dot{\mathbf{x}} = \left(\textstyle\sum_{i=1}^r \alpha_i(A_i + \Delta A_i)\right)\mathbf{x} + \left(\textstyle\sum_{i=1}^r \alpha_i(B_i + \Delta B_i)\right)\mathbf{u} \tag{7}$$

where $\alpha_i = \frac{\omega^i}{\sum_{i=1}^r \omega^i}$, $\omega^i = \Pi_{j=1}^n \mu_j^i(x_j)$.

3 Problem Description

In this paper, the control law is designed according to the model (7) for the planar inverted pendulum, which makes the rods maintaining in the vertical direction, the car returning to its original position and the closed-loop system maintaining a certain performance index unchanged when distance to the center-of-mass of the rod l takes values in $[l_0 - \Delta l, l_0 + \Delta l]$. Of course, we hope the deviation Δl are larger in order to improve the robustness of closed-loop control system.

So we first set the deviation of distance to the center-of-mass of rod

$$|\Delta l| \leq \frac{l_0}{\lambda}, \tag{8}$$

where $\lambda > 0$ is a design parameter and λ is expected as small as possible.

Let $D = l_0 I \in R^{8\times 8}$, where I is the unit matrix, $E_{1_i} = \frac{dA_i}{dl} \in R^{8\times 8}, E_{2_i} = \frac{dB_i}{dl} \in R^{8\times 2}$, and $F(t) = \frac{\lambda \Delta l}{l_0}$. Then we get

$$[\Delta A_i, \Delta B_i] = \lambda^{-1} DF(t)[E_{1_i}, E_{2_i}]. \tag{9}$$

By (8) we have

$$F^T(t)F(t) \leq I. \tag{10}$$

We design a fuzzy controller via the parallel distributed compensation approach:

$$\mathbf{u}(t) = \sum_{i=1}^{r} \alpha_i K_i \mathbf{x}(t). \tag{11}$$

By Equations (7) and (9), we obtain the closed-loop system

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \alpha_i \alpha_j [A_i + \lambda^{-1} DFE_{1_i} + (B_i + \lambda^{-1} DFE_{2_i})K_j]\mathbf{x}(t). \tag{12}$$

The quadratic performance index is defined as the following

$$J = \int_0^{\infty} \left[x^T(t)Qx(t) + u^T(t)Ru(t)\right] dt, \tag{13}$$

where Q and R are the given positive definite matrices.

Definition 1. *For uncertain systems (7) and the performance index J, if there exists a control law $u^\star(t)$ having the form (11) and a positive number $J^\star$ such that closed-loop system is asymptotically stable for all admissible uncertainty and $J \leq J^\star$, then $J^\star$ and $u^\star(t)$ are called upper bound of the performance index J and the fuzzy control law keeping the performance for the uncertain system (7), respectively.*

In practical applications, it is hoped that the control law makes the upper bound $J^(star)$ of quadratic performance index J of the uncertain systems to reach the minimum, on the other hand, it is also wished to design the control law which makes λ to reach minimal value in order to improve the robustness of closed-loop system with respect to parameter l. In this paper, the control law u having the form 11 is called the optimal fuzzy control law keeping performance if u makes one performance consisting of two aspects above to reach optimization value.

Next, we give a new performance as follows:

$$\Omega = \xi J^{\star} + \eta \lambda, \tag{14}$$

where ξ and η are weights.

In this paper, the main goal is to design the appropriate state feedback gain matrix K_i for uncertain systems (7) under the performance (11) and parameter λ such that the closed-loop system is asymptotically stable and the performance Ω reaches the optimal value for the allowed l.

4 Designing an Optimal Guaranteed Cost Fuzzy Controller

First of all, in the design of the guaranteed cost fuzzy controller, let the Lyapunov function $V(\mathbf{x}) = \mathbf{x}^T P \mathbf{x}$, where P is a positive definite matrix. Then the following sufficient condition for the existence of the guaranteed cost fuzzy control law can be easily obtained.

Theorem 1. *For the uncertain system (7) and the performance index J, if there are positive definite matrices P and K_i, and a group of matrices X_{ij} (where X_{ii} is symmetrical and $X_{ij} = X_{ji}^T$), such that the following matrix inequalities are satisfied for all the possible uncertainty*

(1) $P[(A_i + B_i K_i) + \lambda^{-1} DF(E_{1_i} + E_{2_i} K_i)] +$
$[(A_i + B_i K_i) + \lambda^{-1} DF(E_{1_i} + E_{2_i} K_i)]^T P + Q + K_i^T R K_i < X_{ii},$
$(i = 1, 2, \cdots, r)$

(2) $P[(A_i + A_j + B_i K_j + B_j K_i) + \lambda^{-1} DF(E_{1_i} + E_{1_j} + E_{2_i} K_j + E_{2_j} K_i)]$
$+[(A_i + A_j + B_i K_j + B_j K_i) + \lambda^{-1} DF(E_{1_i} + E_{1_j} + E_{2_i} K_j + E_{2_j} K_i)]^T P$
$+2Q + K_i^T R K_i + K_j^T R K_j < X_{ij} + X_{ji}, (i < j)$

(3) $$\begin{pmatrix} X_{11} & \cdots & X_{1r} \\ \vdots & \ddots & \vdots \\ X_{r1} & \cdots & X_{rr} \end{pmatrix} < 0,$$

then $\mathbf{u}(t) = \sum_{i=1}^{r} \alpha_i K_i \mathbf{x}(t)$ is a guaranteed cost fuzzy control law and $J^{\star} = \mathbf{x}_0^T P \mathbf{x}_0$ is an upper limit to the performance.

For the conditions of Theorem 1 are not linear matrix inequalities of the P and K_i, It's necessary to transform into linear matrix inequalities.

By using Schur Complement ([18]), we may change conditions of the Theorem 1 to LMIs.

Theorem 2. *For the uncertain system (7) and the performance index J, if there are positive definite matrices Z and W_i, a group of matrices Y_{ij} (where Y_{ii} is symmetrical and $Y_{ij} = Y_{ji}^T$) and constants $\epsilon > 0$, $\mu > 0$, such that the following matrix inequalities are satisfied for all the possible uncertainty*

$$(1) \quad \begin{bmatrix} \Omega_{ii} - Y_{ii} & (E_{1_i}X + E_{2_i}W_i)^T & Z & W_i^T & D \\ E_{1_i}X + E_{2_i}W_i & -\epsilon I & 0 & 0 & 0 \\ Z & 0 & -Q^{-1} & 0 & 0 \\ W_i & 0 & 0 & -R^{-1} & 0 \\ D^T & 0 & 0 & 0 & -\mu I \end{bmatrix} < 0,$$

$$(i = 1, 2 \cdots, r)$$

$$(2) \quad \begin{bmatrix} \Omega_{ij} - Y_{ij} - Y_{ji} & \Gamma_{ij}^T & Z & W_i^T & W_j^T & D \\ \Gamma_{ij} & -\epsilon I & 0 & 0 & 0 & 0 \\ Z & 0 & -\frac{1}{2}Q^{-1} & 0 & 0 & 0 \\ W_i & 0 & 0 & -R^{-1} & 0 & 0 \\ W_j & 0 & 0 & 0 & -R^{-1} & 0 \\ D^T & 0 & 0 & 0 & 0 & -\mu I \end{bmatrix} < 0, \ (i < j)$$

$$(3) \quad \begin{pmatrix} Y_{11} & \cdots & Y_{1r} \\ \vdots & \ddots & \vdots \\ Y_{r1} & \cdots & Y_{rr} \end{pmatrix} < 0.$$

where $\Omega_{ij} = A_iZ + ZA_i^T + B_iW_j + W_j^TB_i^T$,$\Gamma_{ij} = (E_{1_i} + E_{1_j})Z + E_{2_i}W_j + E_{2_j}W_i$, $\lambda^2 = \mu\epsilon$, then $\mathbf{u}(t) = \sum_{i=1}^r \alpha_i K_i \mathbf{x}(t)$ is a guaranteed cost fuzzy control law and $J^\star = \mathbf{x}_0^T P \mathbf{x}_0$ is an upper limit to the performance.

Based on the optimal guaranteed cost fuzzy control method ([9]), the following theorem about the optimal guaranteed cost fuzzy control law can be derived for the uncertain system (7).

Theorem 3. *For the uncertain system (7) and the comprehensive performance index Ω, if the optimal problem*

$$min_{Z,\mu,\epsilon,W_i} \ \ \xi \, Trace(M) + \eta(\mu + \epsilon) \tag{15}$$

with the constraint conditions

$$(1) \ \textit{Conditions (1) (2) (3) of the Theorem 2} \tag{16}$$

$$(2) \ \begin{bmatrix} M & I \\ I & Z \end{bmatrix} > 0 \tag{17}$$

has the solution $(Z^\star, \mu^\star, \epsilon^\star, W_i^\star)$ then $\mathbf{u}(t) = \sum_{i=1}^r \alpha_i K_i \mathbf{x}(t)$ is a optimal guaranteed cost fuzzy control law of the uncertain system, where $K_i = W_i^\star {Z^\star}^{-1}$, ξ,η are the specified weight parameters.

Note 1 In Theorem 3, the minimizing λ is transformed into the minimizing $\mu + \epsilon$, for it is convenient to use the function "mincx" in Matlab.

By using the optimal guaranteed cost fuzzy control law, which is the solution of the optimal problem in Theorem 3, the closed-loop system will be asymptotically stable and the quadratic performance index $J \leq J^{\star}$, as long as the center-of-mass of the rod is limited in the interval $[l_0 - \frac{l_0}{\lambda}, l_0 + \frac{l_0}{\lambda}]$.

5 Simulation

The parameter of PIP are shown as the following:

$$m_x = 0.5(kg), m_y = 0.2(kg),\ m = 0.1(kg), L = 0.50(m),\ g = 9.8(m\dot{s}^{-2}),$$
$$Q = \mathrm{diag}(1,1,1,1,1,1,1,1), R = 1,$$

and the fuzzy IF-THEN rules are listed:

Rule 1: IF α is about 0 and β is about 0 Then
$$\dot{\mathbf{x}}(t) = (A_1 + \Delta A_1)\mathbf{x}(t) + (B_1 + \Delta B_1)\mathbf{u}(t);$$
Rule 2: IF α is about 0 and β is about $\pm\frac{\pi}{6}$ Then
$$\dot{\mathbf{x}}(t) = (A_2 + \Delta A_2)\mathbf{x}(t) + (B_2 + \Delta B_2)\mathbf{u}(t);$$
Rule 3: IF α is about $\pm\frac{\pi}{6}$ and β is about 0 Then
$$\dot{\mathbf{x}}(t) = (A_3 + \Delta A_3)\mathbf{x}(t) + (B_3 + \Delta B_3)\mathbf{u}(t).$$
Rule 4: IF α is about $\pm\frac{\pi}{6}$ and β is about $\pm\frac{\pi}{6}$ and $\alpha\beta > 0$ Then
$$\dot{\mathbf{x}}(t) = (A_4 + \Delta A_4)\mathbf{x}(t) + (B_4 + \Delta B_4)\mathbf{u}(t).$$
Rule 5: IF α is about $\pm\frac{\pi}{6}$ and β is about $\pm\frac{\pi}{6}$ and $\alpha\beta < 0$ Then
$$\dot{\mathbf{x}}(t) = (A_5 + \Delta A_5)\mathbf{x}(t) + (B_5 + \Delta B_5)\mathbf{u}(t).$$

Here, two groups of weight parameters for the objective function in Theorem 3 are used. The solutions are given for the different the center-of-mass positions. The results are shown in Table 1($\xi = 0.5, \eta = 10$) and Table 2($\xi = 1, \eta = 1$).

Table 1.

l_0 (m)	0.18	0.20	0.23	0.25	0.27	0.30	0.35	0.40	0.45
$2\Delta l$ (m)	0.0768	0.1330	0.0875	0.0772	0.0711	0.0670	0.0710	0.0728	0.0761
$\bar{J}$	216.225	136.696	222.442	265.026	301.564	343.525	369.750	399.183	422.739

Table 2.

l_0 (m)	0.18	0.20	0.23	0.25	0.27	0.30	0.35	0.40	0.45
$2\Delta l$ (m)	0.0628	0.0392	0.0608	0.0588	0.0478	0.0494	0.0428	0.0370	0.0280
$\bar{J}$	258.238	161.853	245.495	286.487	298.884	354.332	354.516	370.653	365.613

Where $2\Delta l$ is the length of the theoretic interval and $\bar{J}$ is the optimal solution.

From the results, it's easy to see that, for a given performance index, it has different optimal value for different l_0. Also, the theoretic intervals are different for different l_0 under the given performance. When l_0 is chosen approximately, the corresponding optimal value can reach minimal value (that is, table 1 and table 2 when $l_0 = 0.20$) and theoretic interval have the biggest length. In other words, for the same performance index, the center of mass of the rod can affect quality of the control when l_0 and l are same. On the other hand, the quality of control is worse when l_0 and l are different.

For the fuzzy rules above, the membership functions of "about $\frac{\pi}{6}$", "about $-\frac{\pi}{6}$" and "about 0" are $\mu_1(x) = 1/(1 + exp(12(x + 0.5)))$, $\mu_3(x) = 1/(1 + exp(-12(x - 0.5)))$ and $\mu_2(x) = 1 - \mu_1(x) - \mu_3(x)$, respectively. When $\xi = 0.5, \eta = 10$, figure 2 and figure 3 show the responding effect for $l_0 = 0.18$, $l_0 = 0.20$, $l_0 = 0.40$ under the initial condition $\mathbf{x}(0) = (0, 0, 0, 0, 55°, 0, 50°, 0)^T$ $l_0 = 0.18$, $l_0 = 0.20$, $l_0 = 0.40$. It is easy to know from figures that the quality of control is better when $l_0 = 0.20$ under the same performance index, which is consistent with calculation of table 1.

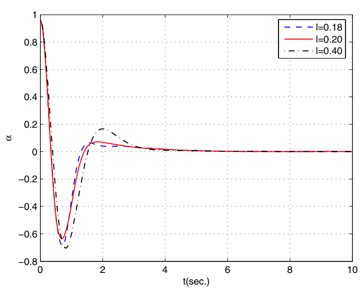

Fig. 2. The response curves of α ($l_0 = 0.18$, $l_0 = 0.20$, $l_0 = 0.40$)

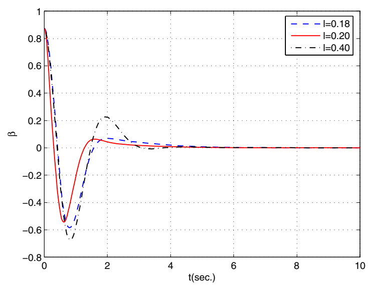

Fig. 3. The response curves of β ($l_0 = 0.18$, $l_0 = 0.20$, $l_0 = 0.40$)

6 Conclusion

In this paper, we make researches on the planar inverted pendulum system modeling and the optimal fuzzy guaranteed cost control problem for the non-uniform pendulum. Firstly, by using Lagrange Equation we set up the planar inverted pendulum system modeling. Secondly, in accordance with the parallel distributed compensation (PDC) structure, we design the state feedback control laws of the fuzzy model based on T-S fuzzy model. Thirdly, we give the existence conditions of the fuzzy state feedback guaranteed cost control laws by use of appropriate linear matrix inequalities. And then consider the quadratic performance index and the deviations between the estimated value of the center of mass position and the actual value. In addition, we put forward the existence conditions of the optimal guaranteed cost fuzzy control laws and describe it as a convex optimization problem subject to linear matrix inequalities. Finally, in the planar inverted pendulum experiments, through choosing the different rods (different centroid) and calculating the optimization problems given by Theorem 3 with the same performance index, we compare the control qualities of the different rods.

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An Intelligent VCR Addition Computing without Both of Overflowing and Perturbation Motion

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Abstract. In mathematics science, addition is rather simple computation of arithmetic. However, no matter what advanced computer in the world, and no matter what backward calculator (such as abacus in China etc.) in people's hand, to do sum of addition is always a NP problem in all the time. Such as capacities of numbers, precise computing (PC), computing precision (CP), perturbation motion (PM) in computing of numbers, etc. This paper makes an algorithm of novel addition with AI-VCR computation in computer, and gives solutions of avoiding the sum's overflowing, insufficient valid figures, PM in arithmetic computation, etc.

Keywords: Artificial intelligence, variable carrying rules, addition, variable capacities of overflowing, perturbation motion.

1 Introduction

In international Mathematics and Computer Science, all researches on Numbers are mechanized [1] and only confined to Same varying rules of FCN (Fixed Carrying Numbers). For example, 10-carrying numbers (D, Decimal numbers), it was used early in SHANG dynasty of China(From B.C. century 17 to 11), until A.D. century 6 it had been used in unity in all over the world[2], today it has been mainly used in unity for 15 centuries(1500 years) in the world; And 2-carrying numbers(B,Binary numbers) was used in main and unity in an inner Computer since 1946. Besides, 5-carrying numbers, 6-carrying numbers, 8-carrying numbers (Q, Octonal numbers), 16-carrying numbers (H, Hexadecimal numbers), etc. All FCN varying rules are called FCR(Fixed Carrying Rules), namely, in a FCN, neighbor Figures' computation rule to be a Same: Binary numbers' Same rules are "Plus 2 as 1, Borrow/Lend 1 as 2", 5-carrying numbers' Same rules are "Plus 5 as 1, Borrow/Lend 1 as 5", 6-carrying numbers' Same rules are "Plus 6 as 1, Borrow/Lend 1 as 6", Octonal numbers' Same rules are "Plus 8 as 1, Borrow/Lend 1 as 8", Decimal numbers' Same rules are "Plus 10 as 1, Borrow/Lend 1 as 10", Hexadecimal numbers' Same rules are "Plus 16 as 1, Borrow/Lend 1 as 16", and the like.

However, in Role of Engineering in Human Society [3], some researches on Numbers must be AI (Artificial Intelligence) and Different varying rules of VCN

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(Variable Carrying Numbers) which was promoted by *Qiusun Ye* in 1995. Namely, in a VCN, neighbor Figures' computation rule to be a Variable. For example, the Numbers of time: 2006 years 8 months 5 days, 1 year=12 or 13(the lunar calendar of Chinese leap year) months, 1 month= 30 or 31 days (Chinese calendar may be 29 or 30 days, specially, in a normal year without including leap month, the February=28 days), 1 day=24 hours, 1 hour=60 minutes, 1 minute=60 seconds. The different Figures (year, month, day, hour, minute and second) computation rules are variable, it is also called VCR (Variable Carrying Rules). *Qiusun Ye* thought that, running & changing of matter in the world was absolute, and the matter's stopping & fixing was relative; so the Numerical Rule, a certain conversion regularity of computation we ought to abide by in a process of describing quantity of matters would be changed in the movement. In an astronomical yearbook, if scientists (who worked in China and in the other countries) didn't inlay AI-Properties of VCR among the Figures such as year, month, day, hour, minute and second; then we couldn't accurately describe periodic changing of weather in one year of which including 4 quarters with 24 climates, peasants wouldn't know planting time for a variety of plants in nature. Of course, today under a big shed with plastic, peasants could also plant a variety of vegetables in different quarter's climates, but these vegetables' taste wouldn't be so good for people in the course of nature. Numbers [4], the objects of its study, is mainly a quantity describing how much/many of matters. The matters ought to be an identical with broad matters in concept of Philosophy. It may be an objective reality matters in general sense such as fire, water, fish, and so on. And it also may be an abstract matters or an appearance of matters like that: sound, city, flood, and the like. To describe how much/many of the above matters mentioned, we can describe them as follows: 1 fish, 2 tons of water, 3 fire shows, 4 sounds, 5 cities, 6 times flood, and so forth. Of course, all of these numbers are integer type here. Sometimes, in need of a practice computation, numbers are also fraction type such as 0.5(or 1/2, decimal number with limited figures 0 and 5); 0.666…(or 2/3, recurring decimal number with limitless figures 6); 0.75(or 3/4, decimal number with limited figures 0, 7 and 5); $(25.513)_6 \div (5)_6 = (3.324111\ldots)_6$, recurring decimal number with limitless figures 1;π=3.1415926…(not recurring decimal number, $\pi$is the Ratio of circumference of a circle to its diameter), etc.

After the 10th Conference of Chinese Association for Artificial Intelligence (CAAI) in 2003, the World Famous Mathematician, the first Top-Prize (¥5,000,000) Winner of National Science & Technology of China in 2000, the 24th Conference Chairman of World Mathematician in 2002, the third International *Yifu Shao* Mathematics Science Prize ($1,000,000) Winner in 2006, the Academician of Chinese Academy of Sciences (CAS), the Academician of the third World Academy of Sciences, the Pre-President of Chinese Mathematics Society(CMS), the Honor President of CMS, the Advisory Committee Honor Chairman of CAAI, Mr. *Wentsun Wu* was sure that, the VCN created by Mr. *Qiusun Ye* was a novel broad concept of numbers, there would be indeed too much potential science value of researches & applications on VCN. The Famous Expert of AI, the Advisory Committee Chairman of CAAI, the Pre-President of CAAI, the Honor President of CAAI, professor *Xuyan Tu* thought that, to research the VCN & its applications, and create Mathematics Theory & Methods of the VCN, it owns very important meanings of academy and very wide value of applications, it may be used not only in Cyphering Science, Communication and Safety of Information, but also in all kinds of variable constructions, variable parameters, constructing models of complex systems and analysis & synthesis, researching & development of new theory or methods & new technology in AI. In this paper, it will introduce a novel practical AI-VCR addition computing without overflowing.

2 Computation Formulae of VCN and FCN

Suppose that, $F_{n-1}F_{n-2}\ldots F_0.F_{-1}F_{-2}\ldots F_{-m}$ is a FCN of including n-Figures integer and m-Figures fraction, and then Real number value of the FCN may be computed in computation formula as follows:

$$R_{FCN} = F_{n-1}F_{n-2}\cdots F_0 \bullet F_{-1}F_{-2}\cdots F_{-m} = \sum_{i=-m}^{n-1} F_i (r+1)^i \tag{2.1}$$

$$Max(I_{FCN}) = R_{n-1}R_{n-2}\cdots R_0 = (r+1)^n - 1 = r\sum_{i=0}^{n-1}(r+1)^i \tag{2.2}$$

$i \in N$, $m \in N$, $N = \{1,2,3,\cdots\}$, $I_{FCN} = F_{n-1}F_{n-2}\cdots F_0$, $r = Max(F_i)$, $R_{n-1} = Max(F_{n-1}), R_{n-2} = Max(F_{n-2}), \cdots, R_0 = Max(F_0)$. In a FCN, all the biggest Figures are equal to each other, namely, $R_{n-1} = R_{n-2} = \cdots = R_0 \equiv R = r$. The Figures' Module(*FM*) are equal to r+1. In a FCN integer of n-Figures, its Numbers' Module(*NM*) is the biggest number plus one, namely,

$$NM = Max(I_{FCN}) + 1 = (r+1)^n \tag{2.3}$$

Suppose that, $F_{n-1}F_{n-2}\ldots F_0.F_{-1}F_{-2}\ldots F_{-m}$ is a VCN of including n-Figures integer and m-Figures fraction, and then Real number value of the VCN may be computed in computation formula as follows:

$$R_{VCN} = F_{n-1}F_{n-2}\cdots F_0 \bullet F_{-1}F_{-2}\cdots F_{-m} = \sum_{k=1}^{n-1} F_k \prod_{i=0}^{k-1}(r_i+1) + F_0 + \sum_{j=-2}^{-m} F_j \prod_{\ell=-1}^{-j+1}(r_\ell+1)^{-1} + F_{-1} \tag{2.4}$$

$k \in N$, $i \in N$, $m \in N$, $j \in I$, $\ell \in I$, $I = \{IntegerNumbers\}$. In a VCN integer of n-Figures, not all the biggest Figures are equal to each other, the Figures' Module (FM_i) are separately equal to R_i+1, its Numbers' Module (*NM*) is the biggest number plus one, or Multiplication of Rolling with all FMs, namely,

$$FM_i = R_i + 1, i \in \{0,1,2,\cdots,n-1\},\ R_i = Max(F_i) \tag{2.5}$$

$$NM = Max(I_{VCN}) + 1 = \underbrace{100\cdots00}_{n-zeros} = \prod_{i=0}^{n-1} FM_i \tag{2.6}$$

3 Concept of AI-VCR, VCO, CN, PM, CP and PC

In a VCN, we know that different figures' computation rules are called VCR, the VCR is relatively fixed in the VCN, but the VCN length of figures after making a certain computation (such as addition, etc.) with another number would be variable, so the VCR is always dynamic in changing value of VCN. When the VCR will be designed in Artificial Intelligence (AI), this computation rule is called AI-VCR [5].

In a computer, we know that capacity of number value is always limited for data word size of CPU (Central Processing Unit), the word size of CPU to be expressed with Bits of binary numbers (binary codes such as 0 and 1), such as 1 Byte=8Bits, 2 Bytes=16Bits, 4 Bytes=32Bits, 8 Bytes=64Bits, etc. When a number value is over more than the biggest number (≥NM), the number will be overflowing in computer. However, if AI-VCR is used of designing a dynamic VCN, then NM of the VCN may be getting larger so that it is limitless, it results in without overflowing forever. This is also called Variable Capacities of Overflowing (VCO).

In Number Theory of Mathematics Science, Numbers are processed in many kinds of arithmetic signs such as ▯ , ▯ , ×, ÷, etc. We called it Computation of Numbers (CN). In CN of traditional FCN and its extensive VCN, Perturbation Motion (PM) is the main origins of errors resulted in computing of numbers. Computing Precision (CP) is concerned with an algorithm of Mathematics, and Precise Computing(PC) is concerned with Valid number of Figures in computer.

4 Realization of AI-VCR Addition Computing with VCO

An optional addition computation SUM with i decimal numbers [R, Real decimal numbers including Integer (Whole number) and Fraction] may be described as follows:

$$S = A_1 + A_2 + \cdots + A_i = \sum_{j=1}^{i} A_j \,,\ j \in N \,,\ \ N = \{1, 2, 3, \cdots\} \tag{4.1}$$

$$A_1 = F_{n1-1}F_{n1-2}\cdots F_0 \bullet F_{-1}F_{-2}\cdots F_{-m1} = D_{n1} \bullet D_{m1} \,,\ n1 \in N \,,\ m1 \in N \,, A_1 \in R$$
$$A_2 = F_{n2-1}F_{n2-2}\cdots F_0 \bullet F_{-1}F_{-2}\cdots F_{-m2} = D_{n2} \bullet D_{m2} \,,\ n2 \in N \,,\ m2 \in N \,, A_2 \in R$$
$$\cdots\cdots$$
$$A_i = F_{ni-1}F_{ni-2}\cdots F_0 \bullet F_{-1}F_{-2}\cdots F_{-mi} = D_{ni} \bullet D_{mi} \,,\ ni \in N \,,\ mi \in N \,, A_i \in R$$

V_{D1}=n1+m1, V_{D2}=n2+m2, …, V_{Di}=ni+mi, V_D is Valid number for Figures which is limited on the Data word size of CPU(Bits of binary numbers) in computer.

According to the Properties of AI-VCR in IFN [6-10] (AI-Fuzzy VCN), we can make an algorithm of AI-VCR addition computing without overflowing as Fig. 1. We definite some concepts from amongst the Initial Works: S_O=0 to be showed the SUM value S is not overflowing after addition computing, S_O=1 to be showed the SUM value S is overflowing after addition computing; SM is temporary variable for the SUM value.

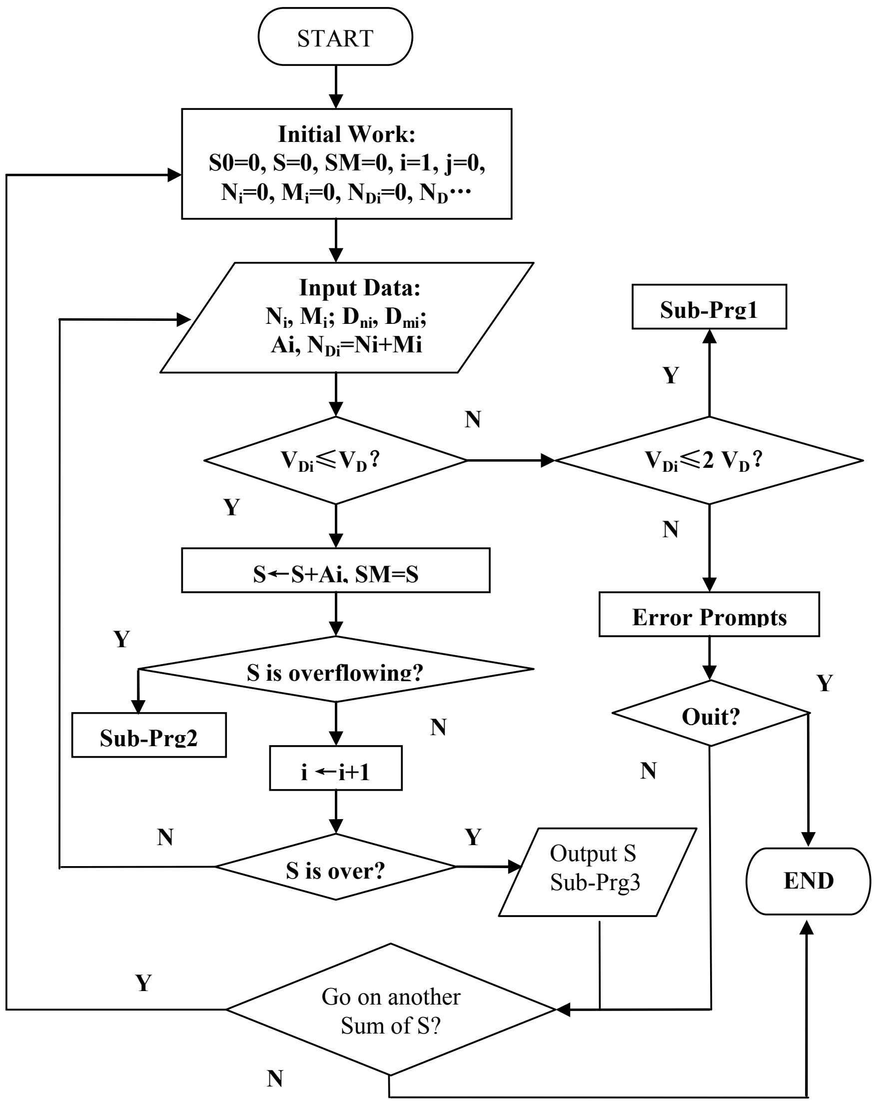

Fig. 1. Algorithm of AI-VCR addition computing without overflowing

Sub-Prg1: When Valid number length of V_{Di} is longer than that length of V_D, namely, V_D $V_{Di} \leq 2$ V_D, this service program to be used in problem-solving of which expressing accumulate number A_i that as follows: $A_i = D_{ni} + D_{mi}$, $AM_i = A_{i.}$

Sub-Prg2: When SUM number S is overflowing, this service program to be used in problem-solving of which inlaying AI-VCR computation regularities of VCO, the typical FCN(Decimal numbers) would be changed into HCN(High Carrying Numbers), it is a kind of VCN which is input in RAM(Read Access Memory) at one-dimension variables of array.

Sub-Prg3: When SUM number S is over, this service program to be used in problem-solving of which outputting the final SUM number S from the RAM, the HCN would be changed back into the typical FCN(Decimal numbers).

5 Analysis of AI-VCR Addition Computing with VCO

In the above mentioned algorithm of addition computing, an addend and summand are the most typical FCN Decimal numbers, owing to its universal research & application in the whole world for 15 centuries, it is in need of changing the HCN sum for addition computing back into the universal Decimal numbers; owing to the VCO of AI-VCR, the Number Value (NV) of an optional VCN wouldn't be overflowing from the sum data in RAM of computer, it looks like that, the flagon in hand of JIGONG monk in Chinese myth wouldn't be canned up with full of wine forever; owing to the Real VCN may be separately processed with both of Integer and Fraction, the Valid Figures to be used would be extended to $2V_D$, and the sum Valid Figures of CP would be limitless (it may surpass $2V_D$), accurate and without PM in computing of numbers, but the complexity of time for sum of PC would be added up in a great deal.

For example, there is an Integer VCN of 3-Figures such as $(516)_{FM2,FM1,FM0}$, supposed that, $V_D=8$, $FM_2=6$, $FM_1=1234567891$, $FM_0=9876543212$, then we can compute the VCN accurately into decimal number **[3]**as follows:

$$
\begin{aligned}
I_{VCN} &= (516)_{6,1234567891,9876543212} \\
&= 5\times 1234567891\times 987654321 + 1\times 9876543212 + 6\times 9876543212^0 \\
&= 60,966,315,627,922,572,678D(DecimalNumber) \\
&= 60966315627922572678(Total:20Figures = 2V_D + 4)
\end{aligned}
$$

••••••

$$I_{VCN} = (516)_{6,10,9} = 5\times 10\times 9 + 1\times 9 + 6\times 9^0$$ ($FM_2=6$, $FM_1=10$, $FM_0=9$)

$$= 465D(DecimalNumber)$$

$$= 465(Total:3Figures = V_D - 5)$$

6 Conclusions

As the word size of an optional CPU of computer in the whole world is limited, it is in need of removing PM in all mathematics computation of numbers such as addition, subtraction, multiplication, division, etc. In this paper, there are 4 properties of realizing computation of AI-VCR addition as follows: Sum of this addition isn't overflowing forever; Sum valid figures of CP are limitless though valid figures of an addend & summand are limited; (3) Sums of this addition are accurate and without PM, namely the errors of this addition are always equal to zero; The complexity of time for sum of PC is higher than that computation ones of universal FCN.

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A Solving Method of Fuzzy Linear System's Equilibrium Point

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Abstract. In this paper, we propose the existence criterion of equilibrium point of fuzzy linear system $\vec{\tilde{x}}\left(k+1\right)=\tilde{A}\vec{\tilde{x}}\left(k\right)$ based on fuzzy arithmetic with identity constraint and fuzzy number expression based on structured element. Then we give a solving method of equilibrium point, propose the concept of generalized equilibrium point of fuzzy linear system, and finally, we use the example in macroeconomics to illustrate the necessity of using fuzzy arithmetic with identity constraint and the feasibility of our method.

Keywords: Fuzzy linear system, equilibrium point, fuzzy arithmetic with identity constraint.

1 Introduction

The existence and solving problem of equilibrium point in fuzzy linear system is a basic issue in the study of the fuzzy system's stability. System equilibrium point was first put forward by economists for economic systems. Walras, Pareto proved the existence of equilibrium point by giving a set of additional conditions. Von Neumann found that equilibrium points exist in game problems when he researched economic phenomena in the view of Game Theory. r [1] proved equilibrium point existential theorem based on utility function for economic system. In recent years, academics achieved a lot in the application of equilibrium point [2]-[10]. However the research on existence and solving problem of equilibrium point of fuzzy system is still scarce and the theoretical analysis always coupled with a complex reasoning process. r [11] researched economic system with fuzzy number map, gave a definition of fuzzy economic system and elaborated the rationality to introduce fuzzy system into economic cybernetics. r [12] researched the solving problem of maximum equilibrium point in discrete fuzzy control system.

The problems mentioned in [11] and [12] can be attributed to the equilibrium solving problem of time-discrete fuzzy system. There are always mutual relevancies between parameters in some time-discrete fuzzy systems, especially

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the economic systems. These mutual relevancies can't be clearly explained when solve the equilibrium point of fuzzy system in common fuzzy arithmetic (such as Max-min arithmetic).Thus, it's necessary to set constraint on fuzzy arithmetic. Fuzzy arithmetic with identity constraint [13] is introduced into this paper. Equilibrium solving problem of linear fuzzy system is changed into the solving problem of fuzzy equations. At last, some meaningful results are obtained. The existential decision condition of equilibrium point and the general solving steps are shown in chapter 3.

2 Preliminaries

Definition 1. *Let* E *be a fuzzy set in the real number field* R *with membership function* $E(x)$, $x \in R$. *If the following formulas are hold*

i) $E(0)=1$, $E(1+0)=E(-1-0)=0$;

ii) $E(x)$ *is a continuous function defined in interval* $[-1,0)\cup(0,1]$. $E(x)$ *is monotone increasing in the right side and monotone decreasing in the left side;*

iii) $E(x)=0$ *if* x *is in* $(1,+\infty)$ *or* $(-\infty,-1)$.

Then fuzzy set E *is a fuzzy structuring element in* R.

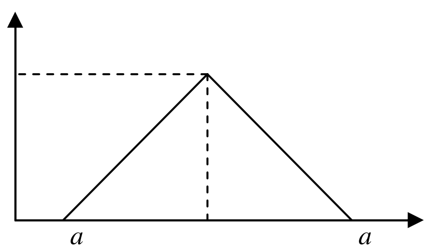

Fig. 1. Membership function of fuzzy number

Definition 2. *Let* $\tilde{A}$ *be a finite fuzzy number. If* $\tilde{A}$ *can be expressed as the following formula*

$$\tilde{A} = a + rE$$

Where' E *is a fuzzy structuring element,* a *and* r *are finite real numbers,* $r > 0$ *,then* $\tilde{A}$ *is a fuzzy number linear spanned by fuzzy structuring element* E.

Once fuzzy structuring element E is identified, all the fuzzy number linear spanned by E will constitute a set, we use $\varepsilon(E)$ to represent this set.

$$\varepsilon(E)=\{\tilde{A} \mid \tilde{A}=a+rE, a\in R, r\in R^{+}\}$$

Obviously, arbitrary element in set $\varepsilon(E)$ is identified by a and r.

Definition 3. $\tilde{x}(k+1)=f[\tilde{x}(k)]$ *is a fuzzy linear iterative system. If the following formula holds*

$$\tilde{x}(k+1)=f[\tilde{x}(k)]=\tilde{x}(k)$$

Then $\tilde{x}(k)$ *is the equilibrium point of fuzzy linear system* $\tilde{x}(k+1)=f[\tilde{x}(k)]$.

See the dynamic multiplier-accelerate model in macroeconomics. Let $Y(k), C(k)$, $I(k)$, $U(k)$ $(k\in N_{+})$ be the National income, National consumption, National investment and Government spending of a country in the k -th Quarter. They have relations described by the following equations

$$Y(k)=C(k)+I(k)+U(k)$$
$$C(k)=bY(k-1)$$
$$I(k)=a[C(k)-C(k-1)]$$

Where $a>0$ is the acceleration system, b is the marginal consumption tendency ($0<b<1$). a and b are known constant quantities.

According to these equations, we may establish the following time discrete economic system model

$$\begin{cases}\vec{x}(k+1)=A\vec{x}(k)+B\vec{u}(k)\\ \vec{y}(k+1)=C\vec{x}(k)+D\vec{u}(k)\end{cases} \tag{1}$$

where $\vec{x}(k)$, $\vec{y}(k)$, $\vec{u}(k)$ are the state vector, output vector, input vector of system(1), the elements in matrixes A, B, C, D are time independent. Let the state vector be

$$\begin{pmatrix}x_1(k)\\ x_2(k)\end{pmatrix}=\begin{pmatrix}C(k)\\ I(k)\end{pmatrix}$$

Then we can rewrite the state equation and output equation as following

$$\begin{cases}\begin{pmatrix} x_1(k+1) \\ x_2(k+1)\end{pmatrix} = \begin{pmatrix} b & b \\ a(b-1) & ab \end{pmatrix}\begin{pmatrix} x_1(k) \\ x_2(k)\end{pmatrix} + \begin{pmatrix} b \\ ab \end{pmatrix}\vec{u}(k) \\ \vec{y}(k) = \begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} x_1(k) \\ x_2(k)\end{pmatrix} + \vec{u}(k) \end{cases} \tag{2}$$

Since marginal consumption tendency is affected by subjective factors, it's reasonable to set parameter b as fuzzy number in system (2).Then we can rewrite system (2) as following

$$\begin{cases}\begin{pmatrix} \tilde{x}_1(k+1) \\ \tilde{x}_2(k+1)\end{pmatrix} = \begin{pmatrix} \tilde{b} & \tilde{b} \\ a(\tilde{b}-1) & a\tilde{b} \end{pmatrix}\begin{pmatrix} \tilde{x}_1(k) \\ \tilde{x}_2(k)\end{pmatrix} + \begin{pmatrix} \tilde{b} \\ a\tilde{b} \end{pmatrix}\tilde{\vec{u}}(k) \\ \tilde{\vec{y}}(k) = \begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} \tilde{x}_1(k) \\ \tilde{x}_2(k)\end{pmatrix} + \tilde{\vec{u}}(k) \end{cases} \tag{3}$$

Internal couples exist in system (3), i.e. the parameters containing b are interrelated with each other. All the parameters that include b will get a perturbation when b is perturbed. This is neglected when solving system (3) in fuzzy number arithmetic without any constraint. It is impossible to avoid such a couple in some cases. Thus, when we solving equilibrium point of a fuzzy system similar to (3) or analysing its behavior, it's necessary to set identity constraint [13] on those arithmetics which involved fuzzy numbers that cause couple. Here introduced the concept of fuzzy arithmetic with identity constraint. It's necessary to use fuzzy arithmetic with identity constraint when the fuzzy numbers involved in a fuzzy arithmetic are always the same (identity).

Definition 4[13]**.** *"identity"means that the fuzzy numbers involved in a fuzzy arithmetic must keeping consistency all the time.*

Take $\begin{pmatrix} \tilde{b} & \tilde{b} \\ a(\tilde{b}-1) & a\tilde{b} \end{pmatrix}$ *for example. If the value of* $\tilde{b}$ *is 3.5,then the value of* $\tilde{b}$ *is also 3.5 in* $a(\tilde{b}-1)$*.If the value of* $\tilde{b}$ *is 2.1,then the value of* $\tilde{b}$ *is also 2.1 in* $a(\tilde{b}-1)$. *i.e., when the value of* $\tilde{X}$ *is fixed, the value of* $\tilde{X}$ *in* $f(\tilde{X})$ *must access the same value. Only in this way can the parameters including* $\tilde{b}$ *get a synchronous response when the value of* $\tilde{b}$ *perturbed. This means fuzzy* *number* $\tilde{X}$ *minus itself the result is 0,* $\tilde{X}$ *divided by itself the result is 1.It should be noted that the result is a real number rather than a fuzzy number. See(4)*

$$\tilde{X} -_R \tilde{X} = 0, \tilde{X} \div_R \tilde{X} = 1 \tag{4}$$

This kind of fuzzy arithmetic is named as "fuzzy arithmetic with identity constraint" in [13].

In the case of fuzzy arithmetic with identity constraint, the internal couples can be accurately reflected when the value of $\tilde{b}$ perturbed. Our research in this paper is based on fuzzy arithmetic with identity constrain. For the convenience of discussion, fuzzy arithmetic with identity constraint is marked as'$-_R$','$\div_R$' and the common arithmetics'$-$','$\div$'.

3 Solving Problem of Equilibrium Point in a Kind of Fuzzy Linear System

The existence of equilibrium points of fuzzy linear system is a basic issue in the research of fuzzy system's stability. Equilibrium point is widespread in control problems of fuzzy economic systems This problem can be attributed to equilibrium solving of the following fuzzy system

$$\vec{\tilde{x}}(k+1) = \tilde{A}\vec{\tilde{x}}(k) \tag{5}$$

Where $\tilde{A}$ is a n-order matrix that represents a linear economic system without external intervention. And its elements $\tilde{a}_{ij}$ ($i, j = 1, 2, \cdots n$) are structuring element linear spanned fuzzy numbers, i.e. $\tilde{a}_{ij} = \alpha_{ij} + \beta_{ij}E$. Thus, $\tilde{A}$ can be expressed as

$$\tilde{A} = A + BE$$

Where A , B are n -order matrixs;the element of A is α_{ij} ,the element of B is β_{ij} .

$\vec{\tilde{x}}$ is the state vector of system(5).The components of $\vec{\tilde{x}}$ is also structuring element linear spanned fuzzy numbers as following

$$\tilde{x}_i = \xi_i + \eta_i E \quad (i = 1, 2, \cdots n).$$

Theorem 1

Proposition I: System (5) has equilibrium point.
The necessary and sufficient conditions of Proposition I is :

In $\tilde{A} = A + BE$ *, A has at least one characteristic root holds* $\lambda_A = 1$ *and* B *has at least one characteristic root holds* $\lambda_B = 1$.

Show that $\tilde{a}_{ij}$ (The elements of $\tilde{A}$) is linear spanned by structuring element as $\tilde{a}_{ij} = \alpha_{ij} + \beta_{ij} E$ .According to the characteristic of structuring element arithmetic, fuzzy matrix $\tilde{A}$ can be expressed by two matrixes A , B as $\tilde{A} = A + BE$. It's the same for vector $\vec{\tilde{x}} = \vec{\xi} + \vec{\eta} E$, where $\vec{\xi}, \vec{\eta}$ are n -dimension vectors. If iterative system $\vec{\tilde{x}}(k+1) = \tilde{A}\vec{\tilde{x}}(k)$ has equilibrium point (i.e. $\vec{\tilde{x}}(k+1) = \tilde{A}\vec{\tilde{x}}(k) = \vec{\tilde{x}}(k)$)then the solving problem of equilibrium point is changed into the solving problem of $\vec{\tilde{x}} = \tilde{A}\vec{\tilde{x}}$. Fuzzy equations $\vec{\tilde{x}} = \tilde{A}\vec{\tilde{x}}$ can be rewired as $\vec{\xi} + \vec{\eta} E = A\vec{\xi} + B\vec{\eta} E$. This is equivalent to $\begin{cases} \vec{\xi} = A\vec{\xi} \\ \vec{\eta} = B\vec{\eta} \end{cases}$.

In the condition of fuzzy arithmetic with identity constraint, we only need to solve the following two linear equations

$$\begin{cases} (A -_R I)\vec{\xi} = 0 \\ (B -_R I)\vec{\eta} = 0 \end{cases} \tag{6}$$

The necessary and sufficient condition of (6) has untrivial solutions is that A has at least one characteristic root holds $\lambda_A = 1$ and B has at least one characteristic root holds $\lambda_B = 1$. Hence the proof.

Example 1. $\tilde{A}$ is the matrix in system (5). Its elements are fuzzy number linear spanned by structuring element as following

$$\tilde{A} = \begin{pmatrix} 3+2E & 1+0.5E \\ -4+3E & -1+2.5E \end{pmatrix} \tag{7}$$

Where, E is the structuring element with expression $E(x) = \begin{cases} 1+x, & [-1,0] \\ 1-x, & (0,1] \\ 0 & \text{el se} \end{cases}$

The following example shows the equilibrium point solving method of system (5). $\tilde{A}$ can be decomposed into two matrixes

$$\tilde{A} = A + BE = \begin{pmatrix} 3 & 1 \\ -4 & -1 \end{pmatrix} + \begin{pmatrix} 2 & 0.5 \\ 3 & 2.5 \end{pmatrix} E$$

Fuzzy state vector $\vec{\tilde{x}}$ can be decomposed into $\vec{\tilde{x}} = \vec{\xi} + \vec{\eta} E$. The equilibrium point of $\vec{\tilde{x}}(k+1) = \tilde{A}\vec{\tilde{x}}(k)$ is equivalent to the solution of $\vec{\tilde{x}} = \tilde{A}\vec{\tilde{x}}$. In the sense of fuzzy arithmetic with identity constraint, the equilibrium point solving problem turns into solving problem of two sets of equations as (8)

$$\begin{cases} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ -4 & -1 \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} \\ \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} 2 & 0.5 \\ 3 & 2.5 \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} \end{cases} \tag{8}$$

Just like (6), we can rewrite (8) as following

$$\begin{cases} \begin{pmatrix} 2 & 1 \\ -4 & -2 \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 1 & 0.5 \\ 3 & 1.5 \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \end{cases} \tag{9}$$

The solution of (9) is $\begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{pmatrix} = \begin{pmatrix} 1 + (-0.5)E \\ -2 + E \end{pmatrix}$, where $(-0.5)E$ is equivalent to $0.5E$. The equilibrium point of fuzzy system (7) is combined by the solutions of (9), i.e. $\begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{pmatrix} = \begin{pmatrix} 1 + 0.5E \\ -2 + E \end{pmatrix}$.

From the background of fuzzy number we can see that in $\tilde{x} = \xi + \eta E$, η represent the width of fuzzy number's value interval. It's gained from experience. In fuzzy system $\tilde{A} = A + BE$, β_{ij} (the element of B) represent the widths of value intervals of $\tilde{a}_{ij}$ (the element of $\tilde{A}$). The values of β_{ij} are determined by experience. Therefore it's difficult to satisfy the condition $\lambda_B = 1$. In order to obtain acceptable results, it seems reasonable to weaken this restricted condition on B. However, even when a weaken condition is given, there may be no numerically accurate equilibrium point in (5) anymore and we can only get generalized equilibrium point. During the iterative process, we don't hope that the value interval of fuzzy state vectors become wider and wider, i.e., the value interval of $\vec{\tilde{x}}(k+1)$ should be included by the value interval of $\vec{\tilde{x}}(k)$. This is described in detail as the following definition of generalized equilibrium point.

Definition 5. *Let the state vector in time* k *and* $k+1$ *be*

$$\vec{\tilde{x}}(k) = \vec{\xi}(k) + \vec{\eta}(k)E$$
$$\vec{\tilde{x}}(k+1) = \vec{\xi}(k+1) + \vec{\eta}(k+1)E$$

If $\exists N, \forall k > N$, *the following formula holds*

$$\left[\xi_i(k+1) - \eta_i(k+1), \xi_i(k+1) + \eta_i(k+1)\right] \subseteq \left[\xi_i(k) - \eta_i(k), \xi_i(k) + \eta_i(k)\right]$$
$$\forall i = 1, 2, \cdots n$$

Then $\vec{\tilde{x}}(k)$ $(k > N)$ *is a generalized equilibrium point.*

Theorem 2. *In the matrixes of fuzzy system* $\tilde{A} = A + BE$, *mark* $\lambda_B^{(i)}$ *as the* i*-th characteristic root of matrix* B. *If it holds that* $\lambda_A = 1$ *(* λ_A *is one of the characteristic roots of* A *) and* $\left|\lambda_B^{(i)}\right| \le 1$ *(* $\forall i = 1, 2, \cdots n$ *), then* $\exists N$, $\forall k > N$ $\vec{\tilde{x}}(k+1) = \tilde{A}\vec{\tilde{x}}(k) \subseteq \vec{\tilde{x}}(k)$ *is the generalized equilibrium point of system(5)*.

Show that the width of value interval of $\tilde{a}_{ij}$ is controlled by β_{ij}.According to contraction mapping principle we can say that, if the absolute value of all the characteristic roots of B are less than or equal to 1, i.e. $\left|\lambda_B^{(i)}\right| < 1 (i = 1, 2, \cdots n)$ then $\vec{\eta}(k)$ will converge to the fixed point of $\vec{\eta}(k+1) = B\vec{\eta}(k)$ in the iterative process. This convergence is a 2-normal linear convergence. Widths of value intervals of $\vec{\tilde{x}}$ are controlled by $\vec{\eta}(k)$. So $\exists N$, $\forall k > N$, $\vec{\tilde{x}}(k+1) = \tilde{A}\vec{\tilde{x}}(k) \subseteq \vec{\tilde{x}}(k)$.

In the condition that $\left|\lambda_B^{(i)}\right| < 1 (i = 1, 2, \cdots n)$ and A has at least one characteristic root holds $\lambda_A = 1$, fuzzy linear system's state vector will access values that approach a limit. And $\vec{\tilde{x}}(k+1) = \tilde{A}\vec{\tilde{x}}(k)$ will have equilibrium points. Hence the proof.

5 Conclusion

Through the discuss on system (5)'s equilibrium point, we found that it's difficult for B to have $\lambda_B \leq 1$ even when the restrict condition is more weaken. Generally speaking, fuzzy linear system (5) is less likely to have equilibrium point. Generally, system (5) has equilibrium point only when we put a feedback on it. Theorem 3.2 shows the sufficient condition of generalized equilibrium point existence in system (5), further research on the necessary condition is needed.

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An Immune Based Multi-parameter Optimization for Intelligent Controller

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Abstract. To solve the puzzle of multi-parameter optimization problem of intelligent controller, the paper proposed an immune-genetic principle based optimization method of multi-parameter controller. Inspired by biology immune principle, firstly it was defined to some concepts such as antibody, antigen and affinity of the parameter optimization problem. Secondly the immune based optimization process of parameter tuning was depicted in detail. Then it took the parameter tuning of human simulated intelligent controller as an example to make the digital simulation for complex system. The simulation result showed that it is better in control quality than other PID tuning method to the intelligent controller, and more suitable for solving the control puzzle of complicated object.

Keywords: Multi-parameter optimization, HSIC, immune algorithm, parameter tuning.

1 Introduction

The control quality of system lies on controller's parameters. Although the human simulated intelligent controller[1][2] (HSIC) has been successfully applied in many fields, but how to choose the controller parameter is still a puzzle. With the increasing of complexity of systems in industrial engineering, also the structure of HSIC becomes more and more complex, there are more parameters needed to be tuned in the control system. But it is very hard to select a set of optimization parameters by handwork tuning, even if it is an expert of control field there must be much difficulty in parameter optimization selection because of being too much parameters needed to be tuned, and those parameters are always contrary or incompatible each other for controlled system performance in static and dynamic quality. Therefore

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it is necessary to research new method of tuning parameter. The paper tried another tack to discuss the parameter tuning method and to resolve multi-parameter optimization problem based on immune-genetic principle.

2 Control System with HSIC

2.1 Generalized Control Model

There are lots of intelligent controller, such as fuzzy logic controller, ANN controller, expert system controller in real time and human simulated intelligent controller etc. Here we only interested in HSIC, because it revealed stronger life power in the praxis applications. It afforded a new thought and a sort of universal method for solving complex system control problem. The HSIC simulates the control behavior of control expert that can perfectly harmonize the various demand of control performance. The basic character of structure and function is that it is hierarchical in information processing and decision organization, on line in identification and memory of the characteristic. And it owns the property of multi-mode control combined open loop with closed loop and qualitative analysis with quantitative control, and all-around application in heuristic and intuition reasoning logic. Essentially HSIC is the multi-mode control method that can alternately use any control algorithm in the HSIC.

The HSIC adopts the generalized model. It is similar to the general control model in form in which the difference is to use generalized controller instead of traditional general controller in the control system, shown in Fig.1. In the Fig.1, r(t) and y(t) is respectively the input and output of the system, and e(t) = r(t) - y (t) shows the error of control system. The u(t) is the control input of controlled object. The advantage of this kind of generalized model is strong in adaptability, and it is not only used to the various linearity system but also the nonlinearity system. The algorithm and its structure of generalized control model depend on the characteristic of controlled object as well as knowledge and experience owned by the control system designer. In the paper, the HSIC is used for substituting generalized controller.

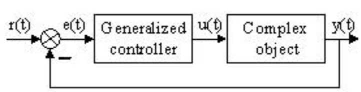

Fig. 1. Generalized control model

2.2 Structure of HSIC

The structure of HSIC can be described as a high order production system, shown as in Fig.2. From top to bottom, it hierarchically includes three levels that are Centrum Chief Level (CC), Organizing and Coordinating Level

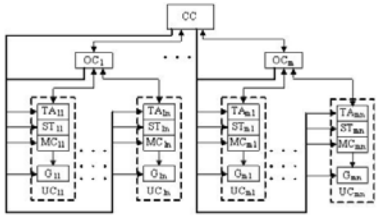

Fig. 2. Generalized control model

(OC), and Unit Control Level (UC). Task programmed of each UC and the harmonization among all the UC realized by OC. The task of each UC and the harmonious demand for other OC will be confirmed by current OC according to the command from CC, harmonious request of other OC, and the feedback characteristic information of each UC. An OC and its all UCs construct a three order production system. And an integrated intelligent control system of a four order production system is formed while some OCs are connected harmoniously each other through a network and submit to the command of CC in higher level. Similarly, with more such integrated intelligent control system united hierarchically, higher order production system can be constructed for the purpose of handling more complicated and difficult control problem.

3 Performance Evaluation Criterion and Unit Control Structure

3.1 Evaluation Criterion for Performance of HSIC

There are three aspects needed to be considered for weighing the performance index of control system, they are the stability, precision and speediness respectively. Obviously it is absolutely necessary to keep stable work of control system, and the higher in stability, the better in control effect. The rising time reflects the speediness of system response, and the shorter in rising time, the faster in control implementing, and also the better in system quality. In order to prevent control energy being too large so as to result in system being overshoot, there must be a conditional constraint term in the objective function that when the error e is large than zero it should be canceled, and if the error e is less than zero then it should be joined in. Therefore the constraint condition should take not only the control input quantity u(t), but also system error e(t) and rising time t_u so as to get better control effect and quality.

In order to obtain satisfactory dynamic characteristic of transition process, it is adopted to the least objective function by IAE. The square term of control input is joined to the objective function so as to prevent control energy to be too large, and the rising time t_u is also introduced into the objective function. The objective performance function can be expressed as formula(1).

$$J = \int_0^{\infty} (w_1|e(t)| + w_2 u^2(t))dt + w_3 \times t_u \quad (1)$$

In which, e(t) is system error, u(t) is controller output that is the input of controlled object, t_u is rising time, w_1, w_2 and w_3 is respectively the weighting value. To avoid overshoot in the control process, there joins a punishment term, once the overshoot is brought the overshoot will be introduced into the objective function. The objective performance function is expressed as in formula (2).

$$\text{if } e(t) < 0 \quad J = \int_0^{\infty} (w_1|e(t)| + w_2 u^2(t) + w_4|e(t)|)dt + w_3 \times t_u \quad (2)$$

In which, w_1, w_2, w_3, and w_4 is respectively the weighting value, In formula(2), when e(t) < 0, it represents that the overshoot of y(t) is produced, therefore it must be constricted. where the w_4 should be greatly bigger than w_1, that is $w_4 >> w_1$, overshoot e(t) is constricted more than the tracking error e(t) to improve the dynamic performance. Therefore it is unnecessary to worry about generating the oscillation phenomenon in the control system.

3.2 Structure of Unit Controller

The concrete control task is completed by control unit, its structure of unit control shown as in Fig.3.

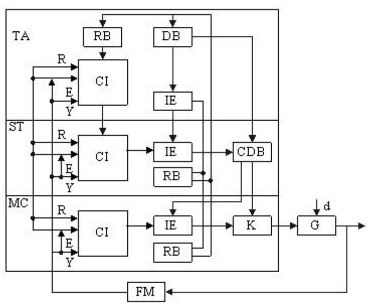

Fig. 3. Structure of unit control

Each unit control has the structure of two order production system, which directly deals with control object by itself through self-organization, self adaptation, and self-adjustment. Each Unit Control Level contains three sublevels that are Motion Control Level (MC), Self- Turning Level (ST) and Task Adaptation Level (TA). Motion Control Level is also called objective level production system, which directly deals with the real-time control problem, and produces quantitative output, constructed a zero order production system by itself. The task of Self-Turning level is to realize the task of self-adjustment of control parameters in Motion Control Level (MC). Self-Turning Level (ST), together with Motion Control Level, constructed one order production system. When a relative big change occurred on the control environment or controlled object, the problem solving strategy of current control task can be reorganized adaptively by Task Adaptation Level. It accomplishes the problem of selection, modification, and even production of characteristic model, reasoning rule, decision-making and control mode in MC or ST, and eventually it fulfills the function of monitor stability of the system. The organization of MC, ST, and TA is a two-order production system. Each MC, ST, and TA has its own database (DB), rule-base (RB), characteristic identifier (CI), and irrational enginery (IE). The information communication among these three levels is accomplished through directly reading and writing of the Common Database (CDB). For a simple controlled object, a single control unit would be enough to handle it, and also because of introduction of characteristic memory, its control quality can be enhanced gradually through self-learning.

4 Algorithm Design of HSIC

For different controlled object, the control algorithm of control unit is also different according the complexity of controlled object. The general control rule set can be derived from the prototype of engineering control algorithm of HSIC. It is expressed as the following.

$$u = K_P e + kK_P \sum_{i=1}^{n-1} e_{m,i} \qquad (e \bullet \dot{e} > 0 \cup e = 0 \cap \dot{e} \neq 0)$$

$$u = kK_P \sum_{i=1}^{n} e_{m,i} \qquad (e \bullet \dot{e} > 0 \cup \dot{e} = 0)$$

where u, K_p, k, e, $\dot{e}$ and $e_{m,i}$ respectively express the control output, proportion coefficient, inhibit coefficient, system error, change rate of system error and the value of i th error peak.

Summarized hominine control experience based on prototype algorithm, the set of parameter control rule in complex system is a suit of control rule that shows different characteristic status. Here we can take that the algorithm would be expressed as the following for some specified object as an example,

if e $>\beta$R then $u_n = u_m$;
if e $<$-βR then u_n = -u_m
if $|e|<\delta_1$ and $|\dot{e}|<\delta_2$ then $u_n = u_{n-1}$
if $|e|>$m$\bullet$R and e$\bullet|\dot{e}|>0$ then $u_n = u_p$
if e$\bullet|\dot{e}|<0$ and $|e/\dot{e}|>$a then $u_n = \alpha p_1$
if e$\bullet|\dot{e}|<0$ and $|e/\dot{e}|>$b then $u_n = p_1 + K_d \dot{e}$
if e$\bullet|\dot{e}|<0$ and b $\leq |e/\dot{e}| \leq$ a then $u_n = u_{p1}$
if e $\dot{e} \geq 0$ and $|e| \in (\delta_1, \theta_1)$, $|\dot{e}| \in (\delta_2, \theta_2)$

then $u_n = p_1 + K_{p2}e + K_{i2}\sum e_j$
if e $\dot{e}$ <0 and $|e| \in (\delta_1, \theta_1)$, $|\dot{e}| \in (\delta_2, \theta_2)$
then $u_n = p_1 + K_{p2}e$
if e $\dot{e} \geq 0$ and $|e| > \theta_2$
then $u_n = p_1 + K_{p1}e + K_{i1}\sum e_j - K_d\dot{y}$.

Where u_n, u_m, and u_p is respectively the n th output value of controller, output hold value related input change ΔR and forced hold value. The e,$\dot{e}$ is respectively expressed as the system error and its change rate. The $\dot{y}$ shows the change rate of object output. P_1 = r$\sum e_{m,i}$(i = 1,2,$\cdots$, l) is the latest hold value, in which $e_{m,i}$ is the j th extremum, r is the weight factor of extremum that can be modified on line. K_{p1}, K_{p2}, K_{i1}, K_{i2}, K_d is respectively proportion, integral and differential coefficient. $\sum e_j$ is accumulative total during e $\dot{e} \geq 0$. β is the switch factor. α, a and b are the coefficient determined by experience rule in the knowledge set. R is setting value. δ_1, δ_2, θ_1, θ_2 expresses respectively the allowable error and range of error rate.

From the above, we can see that the tuning parameters needed are much more than the parameters of general PID controller, therefore it is very hard to the parameter tuning for getting the best control performance. Specially finding the optimal tuning parameters will get more complex.

4.1 Description of Parameter Optimization Process

Based on the immune-genetic principle, the corresponding relation between immune genetic based parameter optimization model and human immune system (HIS) is shown in Table 1[3]. In the parameter optimization, the optimal solution can be regarded as an antigen, and a feasible solution of the problem can be treated as an antibody. Therefore some concepts should be defined that an antigen is the target function of the optimization problem, and is the feasible solution of the target function. In this paper, the real coding is adopted and the antibody is usually defined as a multi-dimension vector $X = \{X_1, X_2, \cdots, X_n\}$each antibody can be represented by a point in the n-dimension space. And the affinity is the value calculated by antigen in which parameters are replaced by antibody.

During the clone process, the superiority antibody generated randomly is the antibody with bigger affinity, it keeps the dominant gene. After process of mutation, lots of new generated antibodies could be got. Then, reevaluate

Table 1. The relation between HIS and optimization method

Human Immune system	Parameter Optimization Model
Antigen	Optimal solution
Antibody	Feasible solution
Cell clone	Replicate of antibody
Binding of antibody and antigen	The value of antigen calculated by antibody
B cell，T cell	vector
Increase of antibody	Increase of feasible solution

the new antibody set; they update the antibody set again and again. The mechanism of antibody update is that the antibodies of higher affinity replace the lower ones. Antibody in the antibody set is sorted by their affinities in sort ascending. Through predefined evolution generation, we can obtain the best antibody, and therefore the optimal solution is acquired.

4.2 Selection of Superiority Antibody

For matching antigen, it will be generated to lots of antibody in the human immune system when antigen breaks into the human body, and also the consistency of antibody will increase if the antibody has higher affinity. That is very propitious to eliminate antigens. And when the antigens are eliminated, the corresponding antibody will be restrained, the consistency is decreased, the immune system always keeps balanceable [4]. The superiority antibody owned higher affinity will be activated, and generate a lot of antibody for eliminating antigen by clone manner. The process of clone selection is given as the following.

```
Procedure Clone Select ()
Begin
Enhancing antigen Ag; /* s.t. min(J) */
Random generating initial antibody set;
    While (generation<m) /*m is the evolution generations*/
    Begin
    Calculate each antibody's affinity f_affinity(ag,ab)
        Achieve the end conditions, algorithm terminate
        Sort ascending according affinity
        Choose number of θ antibodies to generate new antibody ab_new
        according to f_mun(ab(i))
        Mutation new generated antibody set ab_new
        Randomly add N antibody into antibody set ab_new
        While (ab_new!=NULL)
            Begin
                Choose the antibody which has lowest affinity;
                If (f_choose(ag,ab)>f_affinity(ag,ab_new))
                Then Replace by antibody ab_new;
            END
        END
    END
    Output the optimal solution from antibody set;
End
```

Add the new generated cell to the antibody set through clone selection process [5], the concentration of antibody will increase, namely the number of feasible solution is added. However, if this kind of antibody is over concentrated, it is hard to keep the diversity of the antibody and to keep the candidate antibody, so it is easy slump into local optimum. In the paper, we restrain the quantity of the memory cell in the process of clone selection through Formula (3)

$$f_{num} = \sum(\beta\theta/i), i = 1, 2, \cdots, q, \tag{3}$$

where, f_{num} is the total number of cloned memory cell, the i'th item is the clone number of the T cell, β is the predefined constant, θ is the number of dominant cell. According expression (3), the higher affinity cellthe more cloned cell. On the contrary, cloned fewer cell. For example: $\beta = 2$, $\theta = 100$, owing to the memory cell is sort ascending, so, the biggest affinity cell clone number is 200, next, clone number is 100, and so on. In the mean time, to prevent plunging into local optimum, in each generation, add some new randomly generated antibody to the antibody set. The antibody cell is dynamic updated during the evolution process. Each antibody is the optimum antibody selected from current antibody set and the new generated antibodies.

4.3 Mutation

The aim of mutation is to change code of the filial generation, and to get better solution than the parent. To improve the searching efficiency beyond the high affinity antibody and keep diversity of the antibody set, the paper adopts a self adapting mutation operator shown as Formula (4)

$$x_i = |x_i + N_{mi} * N(0, 1) * x_i|, \tag{4}$$

Where, N(0,1) is a random number obeyed standard gauss distribution. Because the control parameters are positive, Nmi is the mutation rate ascertained by (5)

$$N_{mi} = \rho f(x_{ii})/max(f(x_i)). \tag{5}$$

Obviously, the mutation rate is inverse ratio to the affinity, the higher affinity, and the lower in mutation. The mutation ration is self adapting according to their affinity in each evolutional generation. ρ is a mutation constant, to manipulate mutation intensity, it is related to the searching space and population size.

5 Simulation Experiment Research

To validate the control effect of HSIC, here the paper takes a high order process controlled plant [6] as an example. Its transfer function is given in Formula (6)

$$\mathrm{W(s)} = 1/(1 + \mathrm{s})^8. \tag{6}$$

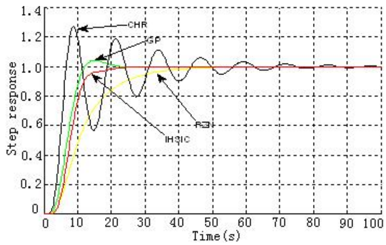

Fig. 4. Comparison Result for w(s)

According the mentioned above, Firstly the parameter needed to be optimization should be encode, in the paper, real value coding is adopted. Encoding K_p, K_p', K_d, K'_d, e_1, e_2, $\dot{e}_1$ as an antibody. Then we can choose the related simulation experiment parameter that are respectively as the following, antibody evolution generation is 100, population size is 50the initial range of parameters K_p, K_p', is [0,30], parameters K_d, K_d', K_i are [0,5], parameters e_1, e_2, $\dot{e}_1$ are [0,1]. The others $w_1 = 0.999$, $w_2 = 0.001$, $w_4 = 100$, $w_3 = 2.0\theta = 20\beta = 5$To increase global search ability, five antibodies are added in each generation. By means of the proposed control and optimization algorithm mentioned above, the simulation experiment in MATLAB has been completed. The result is shown as in Fig.4. For convenience of comparison, the response of tuned results used by other method such as Chien-Hrones-Reswick (CHR), Refined Ziegler Nichols (RZN) and Genetic Programming with ZN(GP) to tune PID controllers is also given in Fig.4. The response curve of IHSIC is the tuned result used by immune based multi-parameter optimization method. From Fig. 4, we can conclude that IHSIC settling time is much better than by CHR and RZN tuning PID controllerthe overshoot is obvious less than CHR; Compare with GP tuning PID, although the rise time is little more than GP, but IHSIC has no overshoot.

6 Conclusion

In the paper, it is proposed a multi-parameter optimization method inspired by immune theory for human simulation intelligent controller. The simulation results show that the proposed optimization method is very effective, and it has important reference meaning for research on other intelligent controller and their applications for complex system in control engineering.

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An Intelligent-Fusion Control Strategy for Industrial Complex Process

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Abstract. Aimed at the control puzzle of complicated characteristic for complex industrial object, the paper presented an intelligent fusion strategy based on human-simulated control. In the paper, it first made an anatomy and pointed out that the traditional strategy is unsuitable for controlling the complex system, and decomposed the complex system into the operation control unit by means of decomposition-coordination principle for large-scale complex control systems, and then constructed the control algorithm based on human-simulated intelligence, finally the experiment of simulation comparison was made for designed algorithm. The results show that it is good in control quality, strong in robustness, high in control accuracy, and specially suitable for uncertainty system control.

Keywords: Intelligent control, uncertainty complex objects, complex system control, fusion control strategy.

1 Introduction

In the sense of industrial environment, a complex system means that it is large in scale, complicated in structure, intricate in information interaction, coupling one another in variables and difficult in mathematic description. So, it is very troublesome for mathematic modeling. In fact, it always applies generalized control model based on knowledge description to control or manage the whole system. It deals with a lot of puzzles lied in the industrial control field. The paper mainly discusses the control algorithm realization based on object Characteristics analysis of above mentioned problems.

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2 Physical Properties for Industrial Complex Systems

There are special kinds of complexities in different industrial control fields, such as large-scale industrial production control process, computer integration manufacture system and industrial flow object. And its structure is always nonlinear, uncertainty, infinite-dimensional, distributed and multi-hierarchical. For information to handle, it is uncertain, random and incomplete, and image information and symbolic information mixed, etc. So it is difficult to establish precise mathematic models with analytic methods. This kind of complex system property can be summarized as the following [1].

1) Some system parameters being unknown, time-varying, random and decentralized.
2) System delay being unknown and time-varying.
3) Model being obvious non-linearity in a system.
4) System parameters being cross-correlated.
5) Environment disturbance being unknown, time-varying and random.

For the property mentioned above, the traditional control approach is unsuitable. The puzzles mainly are shown as follows [2][3].

1) Uncertainty puzzles.

Traditional PID control is based on mathematical models, of which the controlled object and disturbance must be known or acquirable by system identification. However it is very difficult to establish mathematical model for the control puzzles such as "unknown", "uncertainty" or "little-known", so it is also impossible to control by PID method effectively.

2) Strong non-linearity.

For strong nonlinear controlled objects, although there are some nonlinear control theories for use, as a whole, nonlinear theory is farther from mature than the linear theory, its approach is too complex to apply.

3) Semi-structured and non-structured puzzle.

Traditional control theories apply differential equations, state equations and all kinds of mathematic transformation as research tool. They are numerical computing approaches in nature, belonging to the constant control category. Traditional control theories are strictly subjected to the structured control described by mathematic models. However the non-structured control problems are difficult to be described by math method.

4) System complexity.

In the control engineering, there are some factors such as being strong in coupling and being across-constrained and etc, the traditional control can't solve these problems effectively.

5) Problems of reliability.

There are contradictory phenomena between robustness and sensitivity in system. However, for a complex system, while the traditional approaches adopted, it is possible to cause the collapse of the whole control system because of condition change.

To make a comprehensive view, traditional approaches are impossible to control the complex system effectively, so it is necessary to explore more effective control strategies and algorithms. Surely it always adopts the decomposition-coordination principle to break the complex system into a lot of small subsystems even until a physical object. Then it can use different control strategy for different subsystems according to the characteristics of the subsystems, of course, the control algorithms may be different.

3 Decomposition and Coordination

3.1 Decomposition Control Strategy of Complex Systems

It is very difficult to construct mathematic models for complex systems, but it is very suitable to adopt non-mathematic generalized control model based on knowledge representation, and breaks complex system into lots of small control unit, the generalized control subsystem model is shown as in Fig.1.

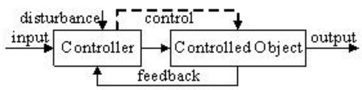

Fig. 1. The generalized control model of a subsystem

In a practical project, relying on knowledge and experience of control experts or operators, the system can be controlled effectively, and the effects are satisfactory. In that way, the controller may complete the desired control tasks by means of human intelligence, knowledge, experience and skill. Therefore, it is a kind of control model combined human with machine, i.e. a combination of controller (human) model and controlled object (machine) model, such as generalized control models combining human knowledge models and controlled-object theory models. For a controlled object with the above characteristics, there are some strategies to be selected, such as artificial neural network, fuzzy logic control, expert system control and human simulated intelligent control (HSIC), etc., they don't need controlled-object mathematic model. In the above control strategies, the strategy of HSIC is closer to engineering practice than the others, it mainly summarizes human control experience, imitates human control experience and behavior, applies production rules to describe its heuristic and instinctive reasoning behavior in control fields. Nowadays, the most excellent controller is still human's brain. Since the basic properties of HSIC is to simulate expert control behavior, therefore its control algorithm is an interaction of the multi-model control to coordinate all kinds of incompatible control demands in a complex control system, such as robustness and accuracy, rapidness and smoothness, and so on.

3.2 Coordination of Subsystem Performance

Since there is no direct information exchange among the subsystems, the performance coordination among the subsystems can reasonably be executed by decision unit of an upper hierarchy, shown as Fig.2, in which, c_1, c_2, $\cdots$, c_n, f_1, f_2, $\cdots$, f_n represent control and feedback information respectively, the subsystem 1, subsystem2, $\cdots$, and subsystem n are located in the process control field. The decision unit coordinates the parameters of each subsystem performance according to the user requirement [4].

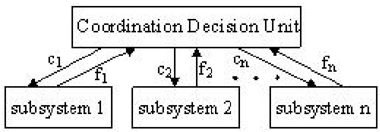

Fig. 2. Hierarchical structure of coordination decision

4 Fusion Control Strategy of Complex System Based on HSIC

Due to being lack of strict mathematic model, it is difficult to control complex system by classical control method (such as PID) or modern control theory method (such as MIMO). The suggestible choice is to apply intelligent control strategy such as artificial neural network, fuzzy logic control, expert system control and HSIC [5] and so on. In which, the HSIC has the following properties, adopting combining open-loop control with closed-loop control strategy, and multi-mode control with combining quantitative control and qualitative control, and adopting stability monitoring. It is more flexible in constructing control algorithm, better in control algorithm performance. A sort of improved basic control algorithm can summarized as follow, 1) If $e \cdot \dot{e} \geq 0$ and $e + \dot{e} \neq 0$, Then adopting PH-D control mode(proportion and half-derivative), and 2) If $e \cdot \dot{e} \leq 0$ or $e = \dot{e} = 0$, Then adopting Hold control mode (half open-loop). By means of English structured description method, the program expression can be written as follow

If $e \cdot \dot{e} \geq 0$ and $\dot{e} \neq 0$ Then
If $\dot{e} \geq 0$ Then
$P(t) = \bar{P}_{n-1} + K_P e + P_{HD}$ (Note, $P_{HD} = \tilde{P}_{l-1} + kK_p \dot{e}$)
If $\dot{e} < 0$ Then
$P(t) = \tilde{P}_{n-1} + + K_P e + P_{HD}$ (Note, $P_{HD} = kK_p \sum_{i=1}^{l} \dot{e}_{m,i}$)
If $e \cdot \dot{e} < 0$ OR $|e| + |\dot{e}| \leq \delta$ Then
If $|e| \geq \frac{1}{2}|e_{m,n}| > \delta$ Then

$P(t)=\bar{P}_n + \mathrm{k}K_p(\mathrm{e}\text{-}\frac{1}{2}e_{m,n})$

Else $P(t)=\bar{P}_n$

$\bar{P}_n = kK_P \sum_{i=1}^{n} e_{m,i}$

In the above expression, P is the control output (to controlled object), e is the controller input (system error signal), $\dot{e} \cdot \ddot{e}$ is one or tow order derivative respect to time t, $e_{m,i}$ is i^{th} extremum point of system error, K_P is proportion gain, k is restraining coefficient, $\bar{P}_n$ is a constant needed the n^{th} keeping value for P, define $\bar{P}_0$ (error extremum memory), P_{HD} is half differential component in the $P-HD$ mode, $\dot{e}_{m,i}$ is the i^{th} extremum point of $\dot{e}$ in the $P-HD$ mode, $\tilde{P}_l$ is constant that is the l^{th} keeping value of half differential output component P_{HD} in the $P-HD$ mode, define $\tilde{P}_0$,(differential extremum memory), i, l, n is nature number, δ is sensitivity scope of controller input.

The control algorithm imitates some properties and functions of human from two aspects of control structure and control behavior, in which it consists of system characteristics identification, and characteristics memory as well as direct reasoning logic and so on. The controller makes corresponding decision according to magnitude, direction and its varying trend of input signal (system error) so that it selects right control mode to control the system. The advantages for this kind of control algorithm are that it is unnecessary to know or identify accuracy mathematic model beforehand, the system control can easily realized in fast speed and high accuracy, and it has stronger robustness.

5 Simulation Experiment of Fusion Algorithm Based on HSIC

In order to compare the control effect between traditional PID and fusion engineering control algorithm based on HSIC mentioned above, here we take a servo-system as an example. For convenience, the tracked object is taken as a familiar two-order tache with big lag. The transfer function of controlled object model is the following.

$$\mathrm{W(s)}=4.13\mathrm{e}^{-\tau s}/(\mathrm{s}+1)(2\mathrm{s}+1).$$

Now there are three sorts of controllers to be applied respectively by the routine PID plus Smith estimator (optimal controller), an intelligent-fusion controller based on HSIC and general PID controller for making simulation experiment. And we can also compare the response curve of the above mentioned controller through the simulation. Here assume the delay time $\tau =$ 20s, the Fig.3 is the response curve of unit step input, and the Fig.4 is the response curve of unit step input with external pulse disturbance in which

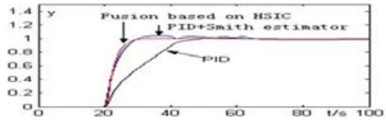

Fig. 3. The response comparison among PID control, PID plus Smith estimator control and fusion strategy control based on HSIC when delay time τ= 20s

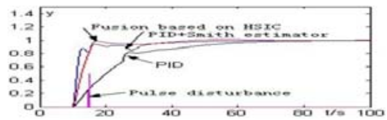

Fig. 4. Response comparison of unit step among three strategies under pulse disturbance at time t= 15s when delay time τ= 10s

the pulse disturbance signal swing is 0.5 and its pulse width is 0.2s when delay time τ = 10s.

From the Fig.3, we can see that the curve of the fusion control strategy based on HSIC is much better than general PID, and better than optimal control PID plus Smith estimator. For the fusion control strategy based on HSIC, it is the shortest in rising time, higher in accuracy and no overshoot. For the optimal control of PID plus Smith estimator, it is faster in rising time and it has overshoot, but it is much better than general PID controller.

From the Fig.4, we can see that when the delay time is changed as τ = 10s and the disturbance pulse is joined at time t= 15s, the system response curve of fusion strategy control based on HSIC is still much better than by PID plus Smith estimator control and general PID control.

The response curve comparison among three strategies from Fig.3 and Fig. 4 shows that the intelligent fusion control strategy based on HSIC is very strong in robustness, better in anti-jamming performance, and good in dynamic and steady control quality. It is suitable for complex object or process control, and specially it is a ideal choice for servo-system control.

6 Conclusion

In the large-scale industrial enterprise, the complex system is always decomposed into lots of subsystem. the upper coordination decision unit coordinates the performance index of each subsystem to ensure the whole system balance among performance index. The functions of the complex system are always integrated in the same interface, not only the needed control function, but also the decision supporting function integrated. At the same time, remote visualization of the complex system can be completed in the process of subsystem integration. Each subsystem of the above the complex system can be controlled by intelligent fusion strategy based on HSIC. The practices of engineering application show that it is stronger in robustness, more accurate in control precision, better in steady or dynamic control quality for complex system control.

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Sliding-Mode Control for Track of Swarm Behavior

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Abstract. The sliding-mode control method based on reaching law is proposed for swarm systems to eliminate chattering, which makes the agents of swarm get to an expectant trajectory and track it. This paper gives the definite control law by using the upper and lower bounds instead of uncertainties. Simulation further shows well the effectiveness.

Keywords: Swarm, sliding-mode control, tracking control, reaching law.

1 Introduction

In nature, swarm behavior can be found in many organisms, ranging from simple bacteria to more advanced mammals. Examples of swarms include flocks of birds, schools of fish, herds of animals, and colonies of bacteria. Such a collective behavior has a certain advantages such as threatening predators and increasing the chance of finding food. Understanding the interaction mechanisms and operational principles of such self-organized motions in swarms can provide useful ideas for developing distributed cooperative control, coordination and formation, and learning strategies for multiple autonomous agent systems such as multi-robot teams and autonomous vehicles.

Cluster control is a new-style distributed control technique which is based on existing theory on flocking of biological agents in nature, and the study for swarm behavior based on biology will help developing the basic theory and application about dynamic performance and control method of swarm behavior in creature and engineering. It is a multidisciplinary research fused biology, systems and control, mathematics, robotics, physics and computer science, which needs to solve the following questions mostly [1]: (1) The model and control algorithm; (2) Adaptability of cluster, i.e., the ability of choosing and tracking the new aim in changing environment; (3) Scalability for the size of a group; (4) Optimize for performance of swarm behavior; (5) Robustness of swarm behavior, that is to say, it can go on tracking aim facing disturbance and without depending on the fixed

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leader; (6) Design and control for engineering swarm systems. Recently, many national and international scholars have made wide and in-depth research to the modeling of swarm systems and cluster control, and have obtained many satisfactory achievements [1-13].

Sliding-mode control is a kind of synthesis of controlling systems and easy to come true. It can be applied to many general systems, such as linear or nonlinear systems, continuous-time or discrete-time systems, certain or uncertain systems, centralized control or distribute control systems and so on. Sliding-mode manifold has full robustness for systemic perturbation and external disturbance, and the unique advantage makes sliding-mode obtain tremendous vitality. Any real system has some uncertain parameters, and there are some inaccuracies in math model, at the same time, it may suffer external disturbance. But these will not influence absolutely on sliding-mode manifold by constructing sliding-mode control.

This paper proposes the method of sliding-mode control for swarm systems, which makes the agents of swarm get to an expectant trajectory and track it, and gives the definite control input by using the upper and lower bounds instead of uncertainties. It solves better the stabilization problem of motion tracking for complex systems. Simulation further shows the effectiveness well.

2 Model of Swarm Systems

We consider a swarm of M individuals (members) in an n-dimensional Euclidean space and model the individuals as points and ignore their dimensions. The position of member i of the swarm is described by $x^i \in R^n$. We assume synchronous motion and no time delays, i.e., all the members move simultaneously and know the exact position of all the other members. Let $\sigma : R^n \to R$ represent the attractant/repellent profile which can be a profile of nutrients or some attractant or repellent substances (e.g., pheromones laid by other individual or toxic chemicals). Assume that the areas that are minimum points are favourable and maximum points are harmful by the individuals in the swarm. The equation of motion of individual i is given by

$$\dot{x}^i = -\nabla_{x^i}\sigma(x^i) + \sum_{j=1, j\neq i}^{M} g(x^i - x^j) \quad i=1,2,\ldots,M$$

where $g : R^n \to R^n$ represents the function of mutual attraction and repulsion between the individuals. Consider functions $g(\cdot)$ of type :

$$g(y) = -y[g_a(\| y \|) - g_r(\| y \|)]$$

where $g_a : R^+ \to R^+$ represents (the magnitude of) the attraction term, whereas $g_r : R^+ \to R^+$ represents (the magnitude of) the repulsion term, and

$\| y \| = \sqrt{y^T y}$ is the Euclidean norm. We assume that on large distances attraction dominates, that on short distances repulsion dominates, and that there is a unique distance at which the attraction and the repulsion balance. An example of function $g(\cdot)$ is of the form

$$g(y) = -y\left(a - b\exp(-\frac{\| y \|^2}{c})\right)$$

and the other forms see [3] and [5].

The function of the gradient of environment potential field $-\nabla_{x^i}\sigma(x^i)$ is making agent i tend to be favourable regions and avoid harmful regions of the profile at the same time.

But some times, to achieve some tasks, such as the question of rescue, removal of mines, scout under unknown environments, and so on, the agents of swarm need to move along a certain path and eliminate external disturbance. Because sliding-mode manifold has full robustness for systemic perturbation and external disturbance, we can realize motion tracking of swarm by sliding-mode control.

3 Sliding-Mode Control for Swarm Systems

Suppose that there is external disturbance in swarm systems, and the trajectory swarm agents needing to track is described by derivative function $x_d(t) \in R^n$. For tracking $x_d(t)$, we need to control swarm agents. Suppose control input for agent i is $u^i \in R^n$, then the controlled systems is described by

$$\dot{x}^i = -\nabla_{x^i}\sigma(x^i) + \sum_{j=1, j\neq i}^{M} g(x^i - x^j) + f^i(t) + u^i \quad i=1,2,\ldots,M$$

where the uncertainties $f^i(t) = (f_1^i(t), f_2^i(t), \cdots, f_n^i(t))^T \in R^n$ is external disturbance. Suppose that $f^i(t)$ is bounded, i.e., there exist $d_1^i = (d_{11}^i, d_{12}^i, \cdots, d_{1n}^i)^T$ and $d_2^i = (d_{21}^i, d_{22}^i, \cdots, d_{2n}^i)^T$ which make $d_{1p}^i \le f_p^i(t) \le d_{2p}^i \quad p=1,2,\ldots,n$.

For agent i, tracking error is

$$e^i = x^i - x_d \qquad (5$$

Choose switch function as $s^i(e^i) = e^i$, i.e.,

$$s^i(x^i, t) = x^i - x_d$$

It is obvious that once the agent i reaches its sliding manifolds $s^i(x^i, t) = 0$ we have $x^i = x_d$, which is exactly the motion equation of agent i. Now, the problem is to design the control input to enforce the occurrence of sliding mode.

Differentiating the sliding manifold equation we obtain that the equivalent control for agent i is

$$u^i_{eq} = \nabla_{x^i}\sigma(e^i + x_d) - \sum_{j=1, j\neq i}^{M} g(e^i - e^j) - f^i(t) + \dot{x}_d$$

For getting rid of the uncertainties $f^i(t)$ in equivalent control, we let

$$d^i{}' = \frac{1}{2}(d_2^i + d_1^i),\ d^i{}'' = \frac{1}{2}(d_2^i - d_1^i) \tag{8}$$

form (6) and (4), we have

$$\dot{s}^i = \dot{x}^i - \dot{x}_d = -\nabla_{x^i}\sigma(x^i) + \sum_{j=1, j\neq i}^{M} g(x^i - x^j) + f^i(t) + u^i - \dot{x}_d$$

we can describe it using component

$$\dot{s}^i_p = -\nabla_{x^i}\sigma(x^i)_p + \sum_{j=1, j\neq i}^{M} g'(x^i_p - x^j_p) + f^i_p(t) + u^i_p - \dot{x}_{dp},\ p = 1, 2, \cdots, n$$

where

$$g'(x^i_p - x^j_p) = -(x^i_p - x^j_p)\left[g_a(\| x^i - x^j \|) - g_r(\| x^i - x^j \|)\right]$$

From reaching conditions $s^i_p \dot{s}^i_p < 0$ we have

$\dot{s}^i_p < 0$ when $s^i_p > 0$, so

$$\dot{s}^i_p = -\nabla_{x^i}\sigma(x^i)_p + \sum_{j=1, j\neq i}^{M} g'(x^i_p - x^j_p) + d^i_{2p} + u^i_p - \dot{x}_{dp} < 0$$

and $\dot{s}^i_p > 0$ when $s^i_p < 0$, so

$$\dot{s}^i_p = -\nabla_{x^i}\sigma(x^i)_p + \sum_{j=1, j\neq i}^{M} g'(x^i_p - x^j_p) + d^i_{1p} + u^i_p - \dot{x}_{dp} > 0$$

then

$$\dot{s}^i = -\nabla_{x^i}\sigma(x^i) + \sum_{j=1, j\neq i}^{M} g(x^i - x^j) + d^i{}' + d^i{}''\operatorname{sgn}(s^i) + u^i - \dot{x}_d \tag{9}$$

Choose exponentially reaching law

$$\dot{s}^i = -\varepsilon \operatorname{sgn}(s^i) - ks^i \quad \varepsilon > 0 \quad k > 0$$

where $\operatorname{sgn}(s^i) = (\operatorname{sgn}(s^i{}_1), \operatorname{sgn}(s^i{}_2), \cdots, \operatorname{sgn}(s^i{}_n))^T$, sgn is sign function, that is

$$\operatorname{sgn}(s_p^i) = \begin{cases} 1, & s_p^i > 0 \\ 0, & s_p^i = 0 \\ -1, & s_p^i < 0 \end{cases} \quad p = 1, 2, \cdots, n$$

So we have

$$-\nabla_{x^i}\sigma(x^i) + \sum_{j=1, j\neq i}^{M} g(x^i - x^j) + d^i{}' + d^i{}''\operatorname{sgn}(s^i) + u^i - \dot{x}_d = -\varepsilon \operatorname{sgn}(s^i) - ks^i$$

we can solve that the control input is

$$u^i = -\varepsilon \operatorname{sgn}(s^i) - ks^i + \nabla_{x^i}\sigma(x^i) - \sum_{j=1, j\neq i}^{M} g(x^i - x^j) - d^i{}' - d^i{}''\operatorname{sgn}(s^i) + \dot{x}_d$$

$$= -\varepsilon \operatorname{sgn}(x^i - x_d) - k(x^i - x_d) + \nabla_{x^i}\sigma(x^i) -$$

$$\sum_{j=1, j\neq i}^{M} g(x^i - x^j) - d^i{}' - d^i{}''\operatorname{sgn}(s^i) + \dot{x}_d$$

Suppose $s_p^i > 0$ in (10), we have

$$\dot{s}_p^i = -\varepsilon - ks_p^i$$

solve it and we will obtain

$$s_p^i(t) = -\frac{\varepsilon}{k} + \left(s_p^i(0) + \frac{\varepsilon}{k} \right) e^{-kt}$$

Let $s_p^i(t) = 0$, we will get the time when the component *p* of agent *i* reaches its sliding manifolds

$$T_{ip} = \frac{1}{k}\left[\ln\left(s_p^i(0) + \frac{\varepsilon}{k} \right) - \ln\frac{\varepsilon}{k} \right]$$

one can see that the time from original state to sliding manifold is finite.

From (12) we get that if $s_p^i = 0$ we have

$$\dot{s}_p^i = -\varepsilon - ks_p^i = -\varepsilon .$$

That is to say, the speed is ε when the component *p* of agent *i* reaches sliding manifolds. For decreasing chattering, we can decrease the speed when it reaches sliding manifolds, i.e., increasing value of k and decreasing the value of ε can speed up the process of reaching and decrease chattering.

4 Simulation

In this section, we present some numerical simulations of the sliding-mode control of swarm systems in order to illustrate the theoretic results obtained in the previous sections.

In these simulations, the attraction and repulsion functions are taken in the form of (3) with parameters *a*=3, *b*=15, and *c*=2. Choose two-dimensional space as the practical space, the function σ as the form [2] $\sigma(y)=\frac{A_\sigma}{2}\|y-c_\sigma\|^2$, and numbers of agents as 10. The original positions of swarm members are obtained randomly. Fig.1 shows the trajectories of the swarm members without external disturbance and control.

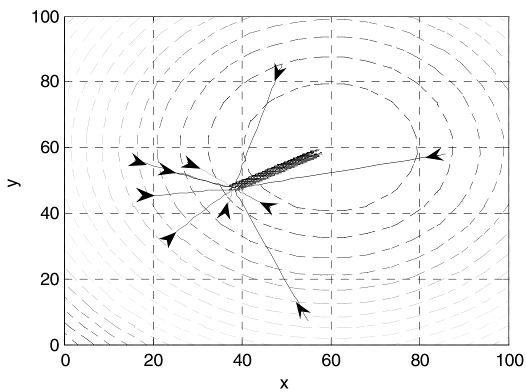

Fig. 1. Trajectories without external disturbance and control

Fig.2 shows the trajectories of the members with external disturbance but without control. Choose the external disturbance function as

$$f^i(t)=\begin{pmatrix} i+5\sin 10t \\ -3-i\cos 10t \end{pmatrix},\ i=1,2,\cdots,10.$$

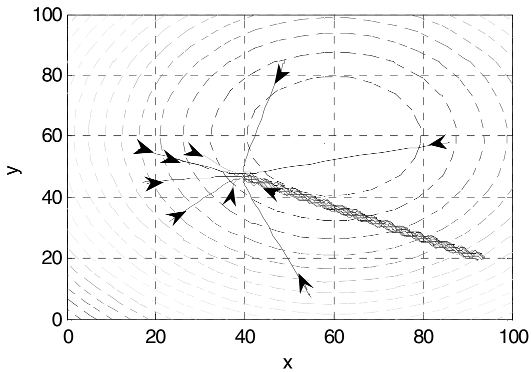

Fig. 2. Trajectories with external disturbance but without control

Fig.3 shows the trajectories of the members with the control input as (11). Let the trajectory which swarm agents need to track as

$$x_d = \begin{pmatrix} 10t \\ 10\sin t + 4t \end{pmatrix}$$

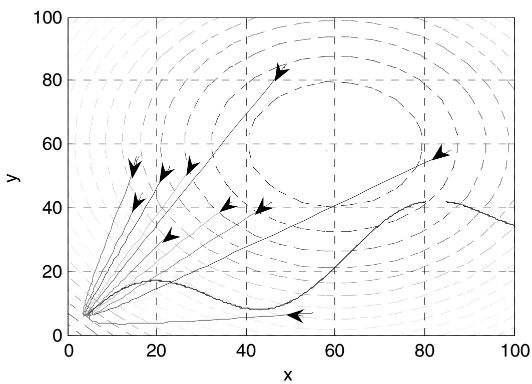

Fig. 3. Track state of swarm members

Choose the parameters $\varepsilon = 2$ and $k = 10$. Fig.4 and Fig.5 show separately the track errors of x axis and y axis of the agent 1. Fig.6 and Fig.7 show separately the control input component of x axis and y axis of the agent 1.

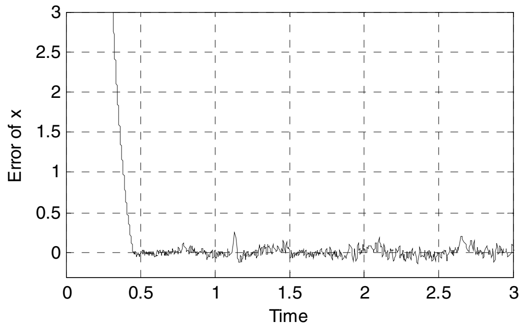

Fig. 4. The track errors of *x* axis

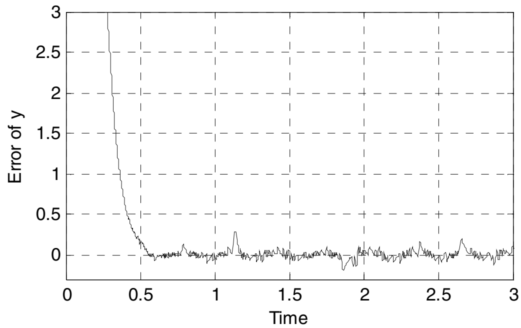

Fig. 5. The track errors of y axis

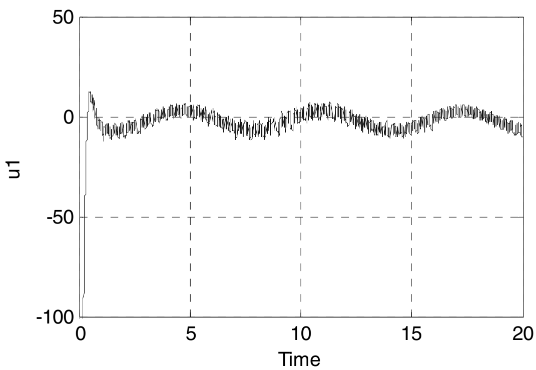

Fig. 6. The control input of *x* axis

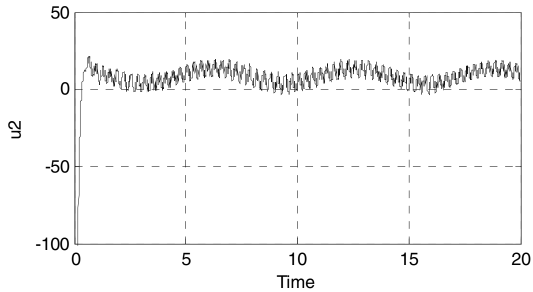

Fig . 7. The control input of y axis

5 Conclusion

This paper proposes the method of sliding-mode control to swarm systems. One can control the agent of swarm by adopting exponentially reaching law, which can speed up the process of reaching, decrease chattering by modulating the value of parameters, and make the agents of swarm get to an expectant trajectory and track it. This paper gives the definite control law by using the upper and lower bounds instead of uncertainties and solves better the stabilization problem of motion tracking for complex systems. Simulation further shows the effectiveness well.

Acknowledgements. The work was supported by the scientific research innovation team constructive project of Hebei Normal University of Science and Technology (No.CXTD2010-05) and Qinhuangdao Science and Technology Bureau (No.200901A288).

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Application of Fuzzy Algorithm in Ingredient Weighing Control System

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Abstract. This article uses the fuzzy control algorithm method to control the rate and time of the weighing and mixing system of manifold materials, also designs a fuzzy controlled ingredient weighing system applied in the feedstuff manufacturing. Result shows the weighing precision was improved and the weighing time was saved.

Keywords: Fuzzy control, ingredient weighing, weighing and mixing system.

1 Introduction

In the automatic control system for processing mixed feedstuffs, the act of material conveying is generally realized by conveyer belts and screw conveyors.Such system will generate inertial material quantity caused by sudden stop of the machines, or bring in inaccurate results of the weighing unit test, the hysteresis effect, and the error instability and undesirable dynamic behaviors in operation caused by disproportionation of the materials conveyed by mechines. Therefore, the fuzzy control algorithm and intelligent control method can be applied to the feedstuff mixing and processing system, while the human manual operation experience can be summarized as the fuzzy language logic being stored in the computers, the computers can automatically imitate the characteristics of human to control the conveying capacity of every kind of materials. The adopted method is to realize the microcomputer intelligent control for the whole weighing unit of the system, the more approaching of each material to the preset weight and the larger the increment rate of the materials, the slower of the running speed of the conveying machines or the shorter of the feeding time, thereby the smaller of the conveyed

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material quantities, consequently the easier to control the feeding precision of the materials. In other words, it is to adopt the "successive approximation" control method after the fuzzy recognition to realize the intelligent control [1].

2 System Structure and Control Theory

2.1 Structure of Ingredient Weighing Control System

The fuzzy ingredient weighing control system is mainly composed of four sections: the microcomputer automatic gauge control section; the logic control section; the detection and final control element section and the fuzzy control and software programming section. The system diagram is showed in Fig. 1.

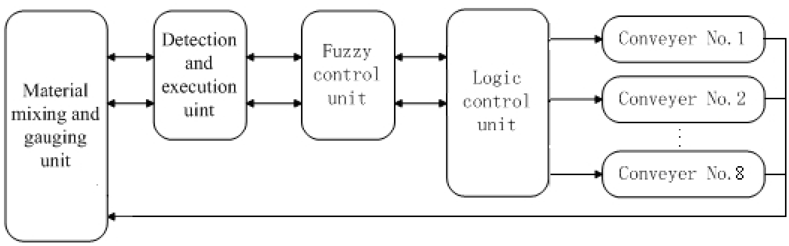

Fig. 1. Please write your figure caption here

2.2 The Hardware Control Theory of the Ingredient Weighing System

Fuzzy controlled ingredient weighing system mainly deals with the ingredients of eight kinds of materials, also with the automatic gauging of the materials, the storage of the information and data of the materials, the detection and control of the materials fed by the fuzzy control screw conveyor, the material mixing control and the release control of the finished feedstuff. The automatic gauging part is mainly finished by MCS-51 series singlechip. The logic circuit takes the sequential control of each mechanical part and running status control of the machines. The diagram of the operation control principle is showed in Fig. 2.

3 Fuzzy Control Design of the System

3.1 Fuzzy Control Strategy

In the ingredient weighing control system, eight conveyers are needed to proceed the mixing of the materials. The material quantity fed by each conveyer is needed to be under control, so as to reach to the weight requirements of

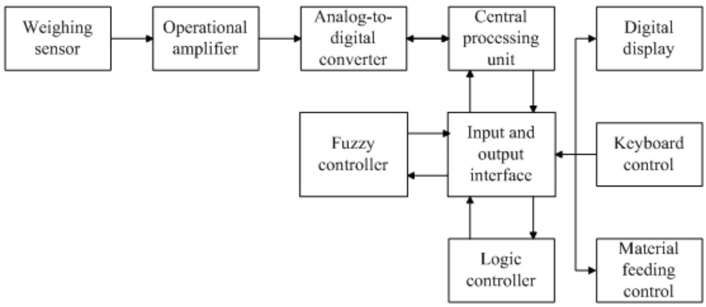

Fig. 2. Please write your figure caption here

the feeding formula, while the material errors can be minimized mostly. In order to realize the accuracy control of the material quantity, the weighing section of the system need to be analyzed: the material weight error control of the feedstuff mixing system is different from the traditional PID control, it is a one-way approximation control method. It is undesirable when the real material quantity exceeds the preset quantity after the material conveyed, in consideration of this, we can use the "successive approximation" method to control the feeding quantity of the materials. The material feeding part is completed by eight screw conveyors, so it is inappropriate to realize the control by adopting speed regulation method to each of the screw conveyors, not to say the machine itself has inertial problem in the process from movement to stop, which inevitably will bring the inertial quantity of the materials. The mixing of the materials is among multiple kinds of materials, and ratio of each material is different, hence the generated material inertial quantity is different, so it is impossible to adopt the same mixing mode. In the conveying process of same kind of material, the quantity and density of the remainder in the screw cylinder is different, and the material inertial quantity generated at different time period is also not exactly same. The weighing hopper of the system is at high position, so it will cost longer time to put the materials into the weighing hopper from the input mouth, thus existing a delayed problem caused by the fall drop. By analyzing the human operation process in the manual control system, the experience can summarized as these: To realize the relatively precise automatic control, the best way is to use the fuzzy language to logically express the control system of human manual experience; and in this system, we can realize the relatively precise fuzzy control function by point by point or discontinuously controlling the on-off time of the electric machines. By approximating the material weight, the errors can be eliminated by manual control; usually the rule is adopted as this: when the error is very large, i.e. the difference between the real material weight observed by man and the preset weight is great, the machine needs to be kept on running. When the error is relatively small, that is to say the difference between the real material weight and the preset weight is small, the machine needs to be stopped and enters into the point by point working condition. The judgement equation is $e = s - y \leq \delta$. According to the practical situation, δ can be concretely set and selected, which is related to the magnitude of Δe. The bigger the feeding capacity Δe in until time, the bigger the value δ can be; on the contrary, the smaller the feeding capacity Δe in until time, the smaller the value δ can be selected, which will guarantee the control response speed. This system uses PL/M language to realize the fuzzy language logic, in the hope of accurate control of the material weight.

3.2 Fuzzy Control Rules

Fuzzy control rule is the fuzzy condition statement formed by several language variables, the determination of the fuzzy language variables includes the proper fuzzy language values generated by grammatical rules, the subordinate functions of the language values and the domain of the language variables that determined by the language rules, etc. In the design of the fuzzy controller for error elimination, when fuzzy language variables are determined, three basic language variables can be determined, i.e. “positive”, “zero”, and “negative”. Then several language subvalues can be generated according to the requirements. In this system, because the feeding quantity of the materials is controlled by one-way approximation, the errors can be defined as e =s-y. So we only consider the situation when e is a positive value. When e¡0, e is defined as e=0, so the language variables only adopts Big (B), Middle (M), Small (S) and Zero (0), the four fuzzy language values. The language variables of the error variable rates Δe only adopts Very Big (VB), Big (B), Middle (M), Small (S) and Zero (0), the five fuzzy language values. While the language variables of the controlling quantities u only adopts Very Big (VB), Major Big (MB), Big (B), Middle (M), Small (S) and Zero (0), the six fuzzy language values. E, EC and U are set as the corresponding variables. The fuzzy language sets of E, Δe and u, are: E={B, M, S, 0}; EC={VB, MB, M, MS, SS}; U={VB, MB, M, MS, SS, 0}. According to the manual control strategy, the error e and error variable rates Δe, the corresponding language variable E, EC and U of controlled variable u, as well as the fuzzy control rule of U, are listed in Table 1. A set of control rules composed of 20 fuzzy conditional statements can be summarized in Table 1.

Table 1. Fuzzy control rules

	U	E-B	E-M	E-S	E-0
	VB	M	MS	SS	0
	MB	M	MS	SS	0
EC	M	MS	M	SS	0
	MS	VB	MB	SS	0
	SS	VB	VB	MS	0

3.3 Realization of Fuzzy Control Rules

In the usual design of the fuzzy controllers, the real numeric area (also called ground field) of error e and error variable rate Δe can be divided into discrete grades, the controlled variable u can be divided into n grades. Then the table of fuzzy control rules and the definition of the language variables can be utilized to design the control table between the grades of input variables and output variables (controlled variables), which is called as questionnaire table. In this system, we can get Table 2. Its control process can be described as: after starting the electric engine and beginning the feeding, the weighing system begins to measure the real feeding quantity, meanwhile the feeding quantity per unit time can be calculated ("s" can be selected as the unit), i.e. the feeding rate of this material . We know that when the rate is a constant value, $= \Delta e$. System can decide the advanced shutdown time according to the value of . The input variable of the object $= \Delta e$ is a constant in the whole control process, so once the corresponding grade of Δe is determined, the whole control rule is just one of the rules that determined in Table 2, hence it turns into a single input/output controlled variable. Each row in Table 2 is the control strategy that under different feeding rate, on the conditions of same errors, owing to different feeding rate, the corresponding time of the point by point control is also different.

Table 2. Control questionnaire

	U	e3	e2	e1	e0
	5	3	2	1	0
	4	3	2	1	0
Δe	3	4	2	1	0
	2	5	4	2	0
	1	5	4	3	0

Suppose Δe= is the feeding rate. For a control process, it is a constant. Make $e = s - y_t$ (s is the preset value of the system, y_t is the actual measurement of the feeding quantity at time t). Thereby $\tilde{E} = e/v$ is taken as the relative error of the system, we get:

$$\tilde{E} = (s - y_t)/y_t/t = t(s - y_t)/y_t \tag{1}$$

Use $\tilde{E}$ as the input value of the feeding control system, u (value of the time span of the point by point control) as the output value, a single input/output control system can be obtained. Table 3 gives the uniform control rules.

For the given relative input value $\tilde{E}$, it can be classified into certain grade by the principle of maximum membership degree, the corresponding control

Table 3. Uniform control rule

$\tilde{E}$	B_E	M_E	S_E	0_E
u	B_u	M_u	S_u	0_u

grade is determined by Table 3. The maximum membership degree has the corresponding relationship showed in Table 4.

Table 4. Principle of maximum membership degree

u	B_u	M_u	S_u	0_u
t	3	2	1	0

4 Conclusions

The ingredient weighing control system adopts the fuzzy algorithm method to simulate the manual control design, the material mixing precision in the weighing process is reached, the runtime of the system is accelerated, the work efficiency and product quality is also improved. By field operation test, this system can basically realize the function of mixing the ingredients according to the feed formula of the users. The weighing resolution of the fuzzy control system is 0.05kg; the measuring scope is 0-1000 kg; the total material number is 8; the number of the realizable storable formula of the users is 20; the maximum storable measuring time is 2000. Through chemical assay and analysis, the produced finished feedstuff products conform to the mixing requirements.

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Fuzzy Control of Double-Layer Spherical Shell Structure under Seismic Response

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Abstract. The reticulated shell structure has high degree of geometric nonlinearity, and time-variety of stiffness and damping under seismic response. Fuzzy control has better control effect for the system of time-varying parameters, nonlinearity and strong coupling. Recognizing this, fuzzy control method is proposed to apply to MR damper, and the vibration control analysis of shell structure is carried out based on the method. According to the basic principle of fuzzy control, for the general Bouc-Wen model as the mechanical model of MR damper, the fuzzy controller for MR damping structure is designed by the fuzzy control toolbox and the Simulink block in MATLAB, and the fuzzy control analysis of the span 60 meters of double-layer spherical shell structure is carried out under seismic response. The results show that: for the vertex and the node of rod piece on which MR damper installed of the shell structure under fuzzy control, the peak value of displacement, velocity and acceleration are averagely decreased by 56.8% in comparison with no control structure, the control effect is remarkable, so the MR fuzzy controller is demonstrated to be effective and reliable.

Keywords: Fuzzy control, MR damper, shell structure, seismic response, vibration control of structure.

1 Introduction

Under the dynamic load (such as earthquake, wind and shock waves, etc), will produce very harmful vibrations to the civil engineering structure[1]. Therefore, the study of structure vibration control is very significant. The fuzzy control method is particularly suitable for the system of time-varying parameters, nonlinearity, strong coupling and difficulty to establish accurate mathematical model, so it has strong adaptability and robustness[2].

MR damper[3] is a controllable and intelligent damper, has advantages of fast response, dynamic range, durability and continuously adjustable damping force, so it has become a new generation of intelligent control device for

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civil engineering structure, its development potential is vast. Based on fuzzy control principle, a fuzzy controller is designed for MR damping structure, and the vibration control analysis is conducted on a double-layer spherical shell as an example. With the displacement and velocity of structure under seismic response as input variables, and the fuzzy control rules based on expert knowledge and experience, Mamdani[4] inference method is used to conduct fuzzy inference operation, and Centroid method is used to execute ambiguity resolution, then the corresponding output variables namely the control voltage of MR damper is solved. The control force of MR damper acts on structure is calculated according to the obtained voltage, so the vibration control of shell structure is realized, and the numerical simulation analysis of fuzzy control and no control structure is carried out for comparison.

2 Fuzzy Controller Design

2.1 Identification of the Input and Output Variables and Their Basic Domain of Discourse

The displacement and velocity response of structure are taken as the input variables of fuzzy controller, and the control voltage of MR damper is taken as the output variable. According to the displacement and velocity of structure under Tianjin seismic response of south-north and the up-down direction, the basic domain of discourse of displacement and velocity can be respectively taken as (-5,5)cm and (-40,40)cm/s; according to the working voltage of MR damper, its basic domain of discourse can be taken as (0,2) V[5].

The fuzzy domain of displacement and velocity can be divided into seven fuzzy subsets:{NB(negative big), NM(negative medium), NS(negative small), ZO(zero), PS(positive small), PM(positive medium), PB(positive big)}.And the fuzzy domain of voltage can be divided into five fuzzy subsets:{VS(very small), S(small), M(medium), B(big), VB(very big)}. By the comparison of repeated simulation, the control effect is best when the quantization factor of displacement and velocity are respectively taken as $K_{wy} = 18$ and $K_{sd} = 1$, and the scale factor of MR damper control voltage is taken as $K_v = 1$.

2.2 Selection of Membership Function

Membership function is the basis of solving actual control problems in use of fuzzy theory, therefore, selecting the correct membership function is the key to solve problems. According to the rules of membership function selection and the characteristics of the membership functions[6], Gaussian is selected as the membership function of displacement and velocity, and Triangle is taken as the membership function of MR damper control voltage, they are respectively shown in Fig.1∼Fig.3.

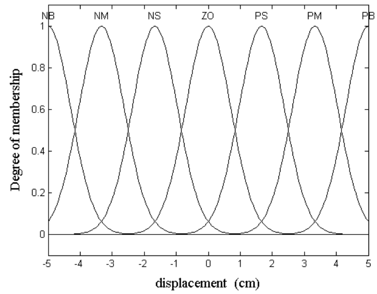

Fig. 1. Displacement membership function

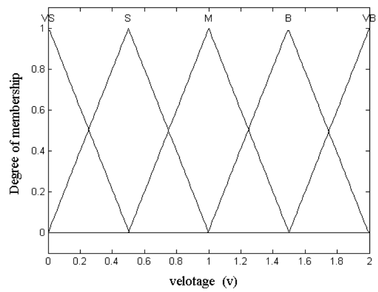

Fig. 2. Velocity membership function

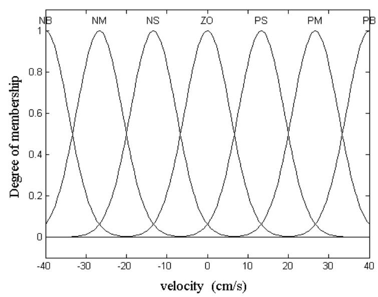

Fig. 3. Voltage membership function

2.3 Determination of Fuzzy Control Rules

Fuzzy control rule is the collection of fuzzy conditional statements by summing up control practical experience and control strategy[7], it has completeness, overlapping and consistency as the core of fuzzy controller. In this article, according to the rules based on expert experience or process control knowledge, 49 control rules are generated, they are shown in Table 1. The Mamdani method[8] is used to conduct fuzzy inference operation, and

Table 1. Displacement velocity-voltage fuzzy control rules

	NB	NM	NS	ZO	PS	PM	PB
NB	VB	VB	B	M	VS	VS	VS
NM	VB	VB	B	M	S	S	S
NS	M	M	M	VS	M	M	M
ZO	VB	VB	B	S	B	VB	VB
PS	M	M	M	M	M	B	VB
PM	S	S	S	B	VB	VB	VB
PB	VS	VS	VS	B	VB	VB	VB

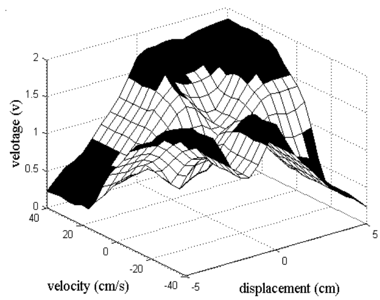

Fig. 4. Relationship surface between the input and output variables of fuzzy logic controller

Centroid method is selected as ambiguity resolution method. Fig.4 shows the relationship surface between the input and output variables of fuzzy logic controller.

3 Mechanical Model of MR Damper

Spencer proposed Bouc-Wen model in 1997[9], the model can describe the relationship between force-displacement and force-velocity well, so it is the general model of MR damper at present, Fig.5 is the model calculating diagram. The damping force meets[10]:

$$F = c_0\dot{x} + k_0(x - x_0) + \alpha z \tag{1}$$

$$\dot{z} = -\gamma|\dot{x}|z|z|^{n-1} - \beta\dot{x}|z|^n + A\dot{x} \tag{2}$$

In the formulas: x_0 is the initial displacement of spring k_1, k_1 is the energy accumulator of MR damper, c_0 is viscous damping coefficient at high velocity, k_0 is stiffness control absorption at high velocity, γ, β, A and n are controlled parameters of MR damper, α, c_0 are:

$$\alpha = \alpha(V) = \alpha_a + \alpha_b V \tag{3}$$

$$c_0 = c_0(V) = c_{0a} + c_{0b} V \tag{4}$$

On the basis of a large number of experiments and analysis, according to the data provided by Lord Corporation, the parameters of above-mentioned equations can be taken as: $x_0 = 14.3cm$, $C_{0a} = 21.0N \cdot s/cm$, $C_{0a} = 3.50N \cdot s/cm \cdot V$, $\alpha_a = 140N/cm$, $\alpha_b = 695N/cm \cdot V$, $\gamma = 363cm^{-2}$, $\beta = 363cm^{-2}$, $A = 301cm^2$, $n = 2$.

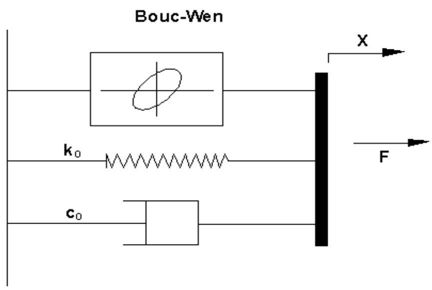

Fig. 5. Bouc-Wen model

4 Simulink Modeling of MR Damper

The displacement and velocity response of the shell structure under seismic response are taken as the input variables of fuzzy controller, then the MR damper control voltage namely the output variable of fuzzy controller is obtained. The displacement, velocity and voltage values are input to the MR damper with Bouc-Wen as mechanical model, then the control force of MR damper acts on structure is solved. Fig.6 shows the fuzzy control simulation model in use of fuzzy control toolbox and Simulink block in MATLAB.

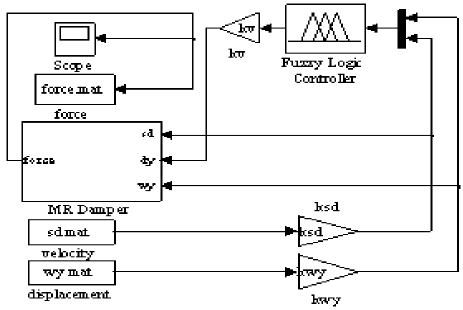

Fig. 6. Simulation model of MR damping structure

5 Example Calculation and Results Analysis

The double-layer spherical shell is selected as calculation model, the span of shell structure is 60 meters, the rise is 8.0 meters, and the thickness is 2.0 meters. Its frequency of radical direction is 6, and its frequency of girth direction is 24, the model is shown in Fig.7. MR dampers are installed on the lower chords of shell structure, their position is shown in Fig.8. The 19 seconds Tianjin earthquake wave of north-south and up-down direction is selected as earthquake wave, its sample time is 0.1 second. According to the control force of MR damper acts on structure, the corresponding acceleration response is obtained, then the sum of the value and earthquake acceleration is obtained, finally the displacement, velocity and acceleration response of the structure in fuzzy control is solved by time-history analysis method. For all nodes in the shell structure, the response of the vertex namely node 145 (position as shown in Fig.8) is maximum. Fig.9∼Fig.11 show the time-history curve of displacement, velocity and voltage of node 145 under no control and fuzzy control. For the no control shell structure, Fig.12 shows the peak

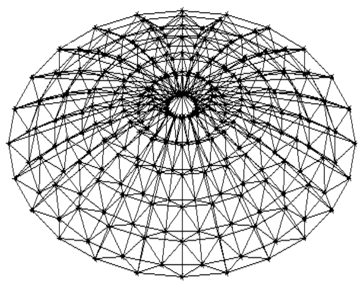

Fig. 7. Double-layer spherical shell model

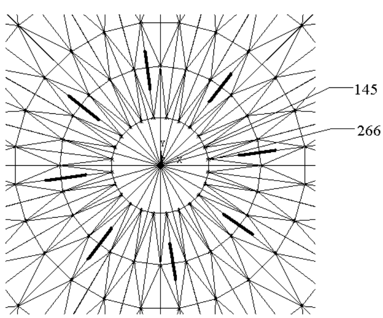

Fig. 8. MR dampers location model

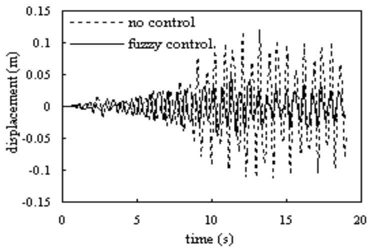

Fig. 9. Displacement time-history curve of node 145

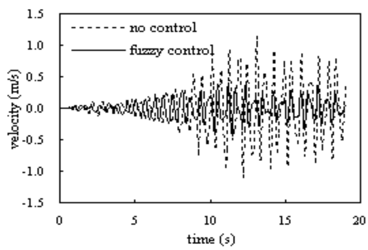

Fig. 10. Velocity time-history curve of node 145

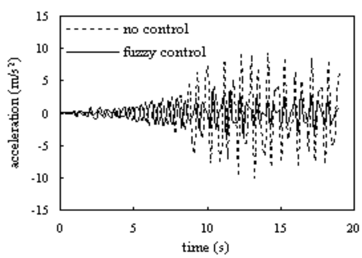

Fig. 11. Acceleration time-history curve of node 145

displacement comparison of the same radical nodes, Fig.13 shows the peak displacement comparison of the same girth nodes, the peak displacement of node 266 is the maximum of the same radical and girth nodes, so node 266 is selected to be analyzed as an example. Fig.14~Fig.16 show the time-history curve of displacement, velocity and acceleration of node 266 under no control and fuzzy control. Table 2 lists the peak value of displacement, velocity and acceleration of node 145 and 266, and the decrease rate of comparison between the two.

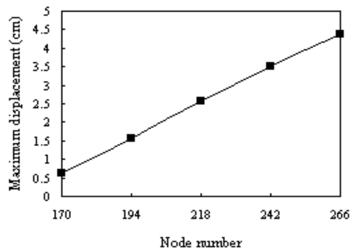

Fig. 12. Peak displacement comparison of the same radical nodes

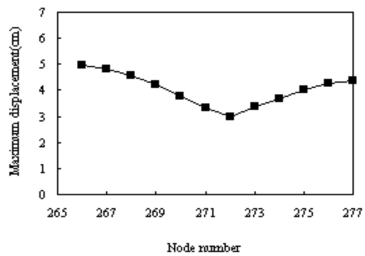

Fig. 13. Peak displacement comparison of the same girth nodes

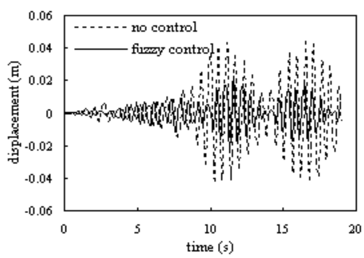

Fig. 14. Displacement time-history curve of node 266

From the time-history graphs of displacement, velocity and voltage of the selected nodes, in the first 8 seconds of earthquake wave, when the response of structure is smaller, the vibration control effect is not obvious, but when the response of structure is larger after 8 seconds, the vibration control is valid, and the control effect of the earthquake peak time is most remarkable. As can be seen from Table 2, the response peaks of node 145 are respectively decreased by 56.8%, 63.9% and 58.3%, and the response peaks of node 266 are respectively decreased by 58.8%, 53.6% and 49.2%. The whole control effect of the designed fuzzy controller is proved to be ideal, but the effective vibration control is not achieved when small excitation acts on structure.

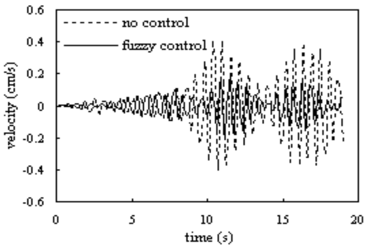

Fig. 15. Velocity time-history curve of node 266

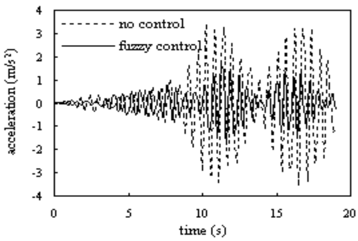

Fig. 16. Acceleration time-history curve of node 266.

Table 2. Comparison of peak displacement, velocity and acceleration

	Node number	No control	Fuzzy control	Decrease rate
Displacement(m)	145	0.1180	0.0509	56.8%
	266	0.0423	0.0267	58.8%
Velocity(m/s)	145	1.1348	0.4094	63.9%
	266	0.4047	0.1878	53.6%
Acceleration(m/s^2)	145	9.7458	4.0661	58.3%
	266	3.4967	1.7766	49.2%

6 Conclusion

Based on the basic principles of fuzzy control, with Bouc-Wen model as the mechanics model of MR damper, the fuzzy controller of MR damping structure is designed, then the vibration control analysis of a double-layer spherical shell under seismic response is carried out. The following conclusions are obtained:

(1) Under the control of the designed fuzzy controller, the response results of the MR damping structure under seismic response show that the displacement, velocity and acceleration responses of all time are effectively controlled

in the mass, and the response peaks are averagely decreased by 56.8%, thus the anti-seismic capacity of the structure is improved greatly.
(2) The fuzzy controller is designed by the control strategy of the relationship between displacement and velocity, then the fuzzy control force is obtained by inference, it has clear physical significance and strong robustness. Because of its better validity and reliability at the same time, it is suitable for widespread application in civil engineering structures.
(3) The fuzzy controller design is not combined with the semi-active control strategy, the control effect of structure response under small excitation is not ideal, so the design proposal is not perfect.

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The Fuzzy Dispatch Strategy Based on Traffic Flow Mode for EGCS

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Abstract. To enhance the running efficiency of elevator group control system (EGCS), the paper presented an optimal fuzzy dispatch strategy based on traffic flow mode. The paper explored the Characteristic of EGCS, discussed the superiority of fuzzy control for dispatch assignment, studied the traffic flow mode, presented the evaluation index system and its evaluation standard, constructed the algorithm model, gave an algorithm flowchart for elevator group dispatch, and made the simplified experiment simulation. The simulation results show that it is reasonable and feasible, higher in running efficiency, and better in energy saving.

Keywords: Elevator group control system, traffic flow mode, optimal dispatching strategy.

1 Introduction

The aim of EGCS is to ensure the comfortable running of the elevator in economy and high efficiency. Because the aim realization is the key, therefore it is difficult to carry out the reasonable dispatch assignment because of the characteristic of EGCS and traffic flow characteristic of the passengers. For example, there are n sets of elevators to assign p entries of hall for call then maybe there are n^p entries of dispatch scheme. If the factors are considered such as approximate property of hall call, uncertain factor of impersonality, subjective factors of managers, and so on, then how to determine the assignment strategy is still a problem [1][2], therefore it is necessary to explore the optimal dispatch strategy by means of optimization principle.

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2 Characteristic Analysis of EGCS

The elevator characteristic is different from the public traffic of the bus.

1) Aim, the EGCS desires that it is shorter in average time of waiting elevator, lower in long time waiting elevator, lower in energy consumption, shorter in average time by elevator, higher in passenger transporting ability, lower in crowded degree of elevator box, higher in accuracy of arrival time forecasted.

2) Uncertainty, such as signal producing for calling, passenger numbers of floor aim floor for calling, and environment factors and so on, all are uncertain.

3) Nonlinearity, such as assignment change of elevator box being discontinuous, provided elevator boxes being limited, not stopping when the elevator box arriving saturation, and so on.

4) Disturbance, to produce unnecessary stop when the passenger wrong call happened, and so on.

5) Information imperfectness, before the passenger arrival, the aim floor being unknown, next call floor being unknown, and so on.

3 The Superiority of Fuzzy Control for Dispatch Assignment of EGCS

Because the information captured by EGCS is always uncertain and fuzzy, therefore the fuzzy control technique has its particular advantages used in EGCS.

1) Be able to realize accuracy association of ideas and map by means of fuzzy control techniques processing the information in accurate and fuzzy and other ambiguity message.

2) Establish the analytical fuzzy model of describing system characteristic, make synthesis reasoning, to realize the dispatch assignment control for EGCS.

3) After input fuzzification, through fuzzy reasoning can effectively weaken the influence of data uncertainty.

4 Fuzzy Optimal Dispatch Strategy Based on Traffic Flow Mode

The traffic pattern recognition and call assignment are the core for EGCS. The object control principle of reasonable call assignment is determined by traffic mode, and the dispatch performance of EGCS is determined by call assignment.

4.1 Identification of Traffic Pattern

The traffic pattern can be partitioned according to the status of coming in, going-out as well as inter-floor flow, and the passenger flow traffic intensity. The different traffic mode is also used by different evaluation function to enhance the service efficiency and quality [3][4].

4.1.1 Selection for Input

Taking 5 minutes as an example, the coming percent u_1 is a rate of coming passengers to overall passenger rate at that period. The overall passenger is a sum of all passengers of up and down two directions at each floor.

$$u_1 = 100\lambda_{in}/(\lambda_{in} + \lambda_{out} + \lambda_{interfloor}) \quad (1)$$

The going out percent u_2 is a rate of out passenger arrival rate to overall passenger arrival rate.

$$u_2 = 100\lambda_{out}/(\lambda_{in} + \lambda_{out} + \lambda_{interfloor}) \quad (2)$$

The inter-floor percent u_3 is a rate of inter-floor passenger arrival rate to overall passenger arrival rate.

$$u_3 = 100\lambda_{interfloor}/(\lambda_{in} + \lambda_{out} + \lambda_{interfloor}) \quad (3)$$

In the above, the λ_{in}, λ_{out}, $\lambda_{interfloor}$ represents respectively the arrival rate of coming passenger and going out passenger and inter-floor passenger. Therefore the expression (4) holds

$$u_1 + u_2 + u_3 = 100 \quad (4)$$

The relativistic traffic intensity u_4 is a rate of passenger traffic intensity to the coming peak traffic intensity.

$$u_4 = 100(\lambda_{in} + \lambda_{out} + \lambda_{interfloor})/HC \quad (5)$$

In which, the HC is the carrying passenger rate of elevator group. It is approximately equal to coming peak passenger rate.

4.1.2 Input Variable Fuzzification

Input variable fuzzification always adopts triangle or trapezium membership function to describe u_1, $U_1 \rightarrow [0,1]$, $u_i \in U_1$, i = 1, 2, 3, it respectively represents the coming, going out and inter-floor traffic. It takes the integer value, and respectively using "low", "medium" and "high" to show the logical "low", "medium" and "high" of each traffic status. The membership function expression is respectively shown as in Table 1.

Table 1. The membership function for input variable

Item	Description	Condition
$u_{low}(u_i)$	1	$u_i < 25$
	$(35 - u_i)/10$	$25 \leqslant u_i < 35$
	0	$u_i \geqslant 35$
$u_{medium}(u_i)$	0	$u_i < 30$
	$(u_i - 30)/20$	$30 \leqslant u_i < 50$
	$(70 - u_i)/20$	$50 \leqslant u_i < 70$
	0	$u_i \geqslant 70$
$u_{high}(u_i)$	0	$u_i < 65$
	$(u_i - 65)/10$	$65 \leqslant u_i < 75$
	1	$u_i \geqslant 75$

Similarly it uses "light", "normal" and "heavy" to show the traffic intensity logical variable "light", "normal" and "heavy". Its membership function is the same as the previous, and it is shown in Table 2.

Table 2. The membership function of traffic intensity

Item	Description	Condition
$\mu_{low}(u_4)$ *or* $\delta(u_4, u_{type})$	1	$u_4 < a_i$
	$(b_i - u_4)/(b_i - a_i)$	$a_i \leqslant u_4 < b_i$
	0	$b_i \leqslant u_4$
$\mu_{normal}(u_4)$ *or* $\delta(u_4, u_{type})$	0	$u_4 < a_i$
	$(\mu_4 - a_i)/(b_i - a_i)$	$a_i \leqslant u_4 < b_i$
	1	$b_i \leqslant u_4 < c_i$
	$(d_i - u_4)/(d_i - c_i)$	$c_i \leqslant u_4 < d_i$
	0	$d_i \leqslant u_4$
(a_i, b_i, c_i, d_i)	(30,40,60,70)	$u_{type} = incoming$
	(50,60,90,100)	$u_{type} = outcoming$
	(25,35,50,60)	$u_{type} = inter - floor$
	(35,45,75,85)	$u_t ype = mixed$

In which, u_{type} is the traffic constitution type, $a_i \leq b_i \leq c_i \leq d_i$, $u_4 \in U_2$ and the I∈F shows the fuzzy setting, the u_4 shows the current relativistic traffic intensity value, and the value of a_i, b_i, c_i, d_i is respectively to show the critical value of fuzzy logic intensity of currently traffic mode subject to fuzzy logic "light", "normal" and "heavy".

4.1.3 Fyzzy Reasoning

Firstly it identifies the passenger traffic type from the percent of coming, going out and inter floor passenger, secondly it identifies concrete traffic mode in which the input variable should be the traffic type and traffic intensity. The structure is shown as in Fig.1.

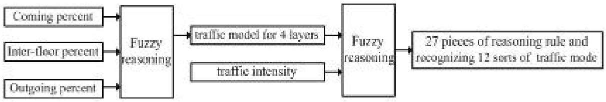

Fig. 1. Traffic mode identification of EGCS based on fuzzy reasoning

Find the fuzzy value for u_1, u_2, u_3 by Tab.1 respectively, and find the traffic type by Table 3, in which there are 9 pieces of reasoning rules shown as in Table 3, the fuzzy setting value is respectively expressed by low, medium, high $\in$ S, and the traffic type is shown as in Table 3.

Table 3. Fuzzy reasoning rule for traffic type

Incoming	Outgoing	Inter-floor	Traffic type
High	Low	Low	Incoming
Middle	Low	Low	Incoming
Low	High	Low	Outgoing
Low	Middle	Low	Outgoing
Low	Low	High	Inter-floor
Low	Low	Middle	Inter-floor
Middle	Middle	Low	Mixed
Middle	Low	Middle	Mixed
Low	Middle	Middle	Mixed

In which, "incoming" means that the passenger is mainly to come into the building and it always happens before the time on duty in the morning, "outgoing" means that the passenger is mainly to go out of the building and it always happens after the time off duty in the afternoon, "inter-floor" means that the passenger moves mainly within different floor in the building and it always happens during the time on duty, and "mixed" happens mainly at noon.

There are 27 pieces of traffic mode reasoning rules and 12 sorts of traffic modes when considering traffic intensity according to the Table 2, the reasoning rule table is shown as in Table 4.

In the rule, it defined the grade of membership function for fuzzy setting. Therefore it can carry through the fuzzy reasoning by means of Max-Min method.

$$\mu_{i_1}(u_1, u_2, u_3, u_4) = \mu_1(u_1) \wedge \mu_1(u_2) \wedge \mu_1(u_3) \wedge \mu_f(u_4)$$
$$= Min(\mu_1(u_1) \quad \mu_1(u_2) \quad \mu_1(u_3) \quad \mu_f(u_4)) \qquad (6)$$

In the FEGCS, it can recognize the whole 12 sorts of traffic modes, and the system can carry through the control by means of traffic mode of currently identifying.

Table 4. Fuzzy reasoning rule for traffic pattern

Intensity	Incoming	Outgoing	Inter-floor	Traffic-Pattern(Index)
Heavy	High	Low	Low	Up-peak(1)
Heavy	Low	High	Low	Down-peak(2)
Heavy	Low	Low	High	Heavy inter-floor(3)
Heavy	Medium	Low	Low	Up-peak(4)
Heavy	Low	Medium	Low	Down-peak(5)
Heavy	Low	Low	Medium	Heavy inter-floor(6)
Heavy	Medium	Medium	Low	Heavy mixed(7)
Heavy	Medium	Low	Medium	Heavy mixed(8)
Heavy	Low	Medium	Medium	Heavy mixed(9)
Normal	High	Low	Low	Incoming(10)
Normal	Low	High	Low	Outgoing(11)
Normal	Low	Low	High	Inter-floor(12)
Normal	Medium	Low	Low	Incoming(13)
Normal	Low	Medium	Low	Outgoing(14)
Normal	Low	Low	Medium	Inter-floor(15)
Normal	Medium	Medium	Low	Mixed(16)
Normal	Medium	Low	Medium	Mixed(17)
Normal	Low	Medium	Medium	Mixed(18)
Light	High	Low	Low	Light incoming(19)
Light	Low	High	Low	Light outgoing(20)
Light	Low	Low	High	Light inter-floor(21)
Light	Medium	Low	Low	Light incoming(22)
Light	Low	Medium	Low	Light outgoing(23)
Light	Low	Low	Medium	Light inter-floor(24)
Light	Medium	Medium	Low	Light mixed(25)
Light	Medium	Low	Medium	Light mixed(26)
Light	Low	Medium	Medium	Light mixed(27)

4.2 Assignment Strategy for Hall Floor

For representing the different requirement, the optimal evaluation should consider the following factors such as passenger's average waiting time(AWT), average riding time(ART), long time waiting percent(LWP) and energy consumption(RPC) of the elevator.

4.2.1 The Evaluation Function

The dispatch algorithm is an evaluation function, shown by expression (7).

$$S_i = W_1 S_{AWT} + w_2 S_{LWP} + W_3 S_{RPC} + W_4 S_{ART} \tag{7}$$

In which, W_i is the weight coefficient determined by traffic mode, $W_i \in$(0, 1)and $\sum W_i = 1$, i = 1, 2, 3, 4. S_i is the evaluation function that shows the reliability of the called signal for the i^{th} elevator. S_{AWT} is the membership of short average waiting time, the larger in its value, the larger of possibility in short riding time. S_{ART} is the membership of average riding time short, the larger in its value, the larger of possibility in short riding time. S_{LWP} is the membership of long time waiting percent low, the larger in its value, the larger of possibility in long time waiting percent low. S_{RPC} is the membership of energy consumption low, the larger in its value, the larger of possibility in elevator energy consumption low. By means of expert knowledge, each control aim of weight coefficient for AWT, LWP, RPC and ART under different traffic mode is shown as in the Table 5.

Table 5. The weight coefficient for different traffic mode

	W_1	W_2	W_3	W_4
Up-peak	0.25	0.25	0.10	0.40
Dowm-peak	0.40	0.30	0.10	0.20
Heavy	0.35	0.30	0.10	0.25
Heavy mixed	0.30	0.30	0.10	0.30
Incoming	0.15	0.20	0.25	0.40
Outgoing	0.30	0.30	0.25	0.15
Inter-floor	0.30	0.25	0.25	0.25
Mixed	0.25	0.25	0.25	0.25
Light	0.10	0.10	0.40	0.40
Light	0.25	0.25	0.40	0.10
Light	0.25	0.20	0.40	0.15
Light mixed	0.20	0.20	0.40	0.20

4.2.2 Computing for Input

The standard of elevator is respectively the AWT, LWP, RPC and ART. But its input variable of fuzzy reasoning is respectively the HCWT, maxHCWT, CV, GD and UR, therefore each input variable of each elevator should be computed by the following expression.

1) HCWTi

When passenger calling, the HCWTi is the passenger waiting time from the response starting of ith set of elevator to arrival of the response elevator, including the moving time and stop time.

HCWTi = Moving-time + Stop-time

In which, Moving-time = Riding floor number × Used time of each floor + Response stop number × Delay time of elevator adding and slowdown. Stop-time = Elevator stop timenumber of person in and out × Used time of each person in and out.

Max$HCWT_i$ is the assigned longest waiting time of all call for the i^{th} set elevator.

$MaxHCWT_i$ = Max(Assigned-call-waiting-times, New-call-waiting-time)

In which, Assigned-call-waiting-time is the assigned call waiting time of the i^{th} set elevator, New-call-waiting-time is the forecast waiting time of new call.

2) CV_i

It is the residual capability of future call after response of new call for the i^{th} set elevator, it always takes 80 rating capability.

3) GD_i

It shows the centralized degree between new call floor and all response floor of the i^{th} set elevator, and it is an index of reflecting energy consumption, shown as in expression (8).

$$GD_i = (Min - distancee)/(floors \times 4) \tag{8}$$

In which, Min-distance is the least distance between possible stop floor and each call floor, and floorhight is the floor altitude of building.

4) UR

It shows the utilization rate elevator box, shown as in expression (9).

$$UR = \sum (NP_i \times NF_i)/(0.8 \times CV_0 \times NF) \quad i = 1, 2, \cdots, n \tag{9}$$

In which, NF is the overall floor number of building, CV_0 is the rating capability of elevator box, NP_i is person number of the i^{th} floor call, NF_i is the floor difference between the i^{th} call floor and starting floor, n is the overall number of the same direction response for the running at this time. It always takes 80% of box rating capability.

4.2.3 Fuzzification for Variables

The fuzzification is to determine the membership function for each input variable.

1) Fuzzification for HCWT

Generally when the waiting tine is within from 0 to 20 second the passenger feels better, and from 20 to 40 second the passenger is able to be accepted, and from 40 to 60 second the passenger would be weary. The logic variable can express the adaptive degree by “S”, “M”, “L” of membership fuzzy value, shown as in Table 6.

2) Fuzzification for MaxHCWT

Generally it is very ideal that the maximum hall call waiting time is not more than 40 second, and acceptable about 60 second, but not greater than 90 second. The logic variable can express the adaptive degree by “S”, “M”, “L” of membership fuzzy value, shown as in Table 7.

3) Fuzzification for CV

The passenger number in box is generally not greater than 80% rating capability, when less than 30% rating capability it is very comfortable, and if it is from 30% to 60% then the passenger is always accepted, and when

Table 6. Fuzzification for HCWT

Item	Description	Condition
$\mu(HCWT)_s$	1	$0 \leqslant HCWT < 20$
	$(35 - HCWT)/10$	$20 \leqslant HCWT < 30$
	0	$HCWT \geqslant 30$
$\mu(HCWT)_M$	0	$HCWT < 20$
	$(HCWT - 20)/20$	$20 \leqslant HCWT < 40$
	$(60 - HCWT)/20$	$40 \leqslant HCWT < 60$
	0	$HCWT \geqslant 60$
$\mu(HCWT)_L$	0	$HCWT < 50$
	$(HCWT - 50)/10$	$50 \leqslant HCWT < 60$
	1	$HCWT \geqslant 60$

Table 7. Fuzzification for MaxHCWT

Item	Description	Condition
$\mu(maxHCWT)_s$	1	$0 \leqslant maxHCWT < 40$
	$(50 * maxHCWT)/10$	$40 \leqslant maxHCWT < 50$
	0	$maxHCWT \geqslant 50$
$\mu(maxHCWT)_M$	0	$maxHCWT \leqslant 40$
	$(maxHCWT * 40)/20$	$40 < maxHCWT \leqslant 60$
	$(80 * maxHCWT)/20$	$60 < maxHCWT \leqslant 80$
	0	$maxHCWT > 80$
$\mu(maxHCWT)_L$	0	$maxHCWT < 70$
	$(maxHWCT * 70)/10$	$70 \leqslant maxHCWT < 80$
	1	$maxHCWT \geqslant 80$

it is greater than 60% the passenger will be fretted. The logic variable can express the adaptive degree by “S”, “M”, “L” of membership function fuzzy value, shown as in Table 8.

4) Fuzzification for GD

From expression (16), when its value is less than 0.3 it is very ideal because of the energy consumption being lower, and from 0.4 to 0.6, it is still more ideal, and if greater than 0.7 then it is very bad for elevator energy consumption. The logic variable can express the adaptive degree by “S”, “M”, “L” of membership function fuzzy value, shown as in Table 9.

5) Fuzzification for UR

In general speaking, when it is less than 0.2 the utilization rate is lower, and when greater than 0.6 then the utilization rate is considered as being higher. The logic variable can be expressed by “S”, “M”, “L” of membership function fuzzy value, shown as in Table 10.

Table 8. Fuzzification for CV

Item	Description	Condition
$\mu(CV)_s$	1	$0 \leqslant CV < 0.20$
	$(0.30 - CV)/0.10$	$0.20 \leqslant CV < 0.30$
	0	$CV \geqslant 0.30$
$\mu(CV)_M$	0	$CV < 0.20$
	$(CV - 0.20)/0.20$	$0.20 \leqslant CV < 0.40$
	$(0.60 - CV)/0.20$	$0.40 \leqslant CV < 0.60$
	0	$CV > 0.60$
$\mu(CV)_L$	0	$CV < 0.50$
	$(0.60 - CV)/0.10$	$0.50 \leqslant CV < 0.60$
	1	$CV \geqslant 0.60$

Table 9. Fuzzification for GD

Item	Description	Condition
$\mu(GD)_s$	1	$0 \leqslant GD < 0.20$
	$(0.35 - GD)/0.15$	$0.20 \leqslant GD < 0.35$
	0	$GD \geqslant 0.35$
$\mu(GD)_M$	0	$GD < 0.25$
	$(GD - 0.25)/0.20$	$0.25 \leqslant GD < 0.45$
	$(0.65 - GD)/0.20$	$0.45 \leqslant GD < 0.65$
	0	$GD \geqslant 0.65$
$\mu(GD)_L$	0	$GD < 0.45$
	$(0.60 - GD)/0.15$	$0.45 \leqslant GD < 0.60$
	1	$GD \geqslant 0.60$

Table 10. Fuzzification for UR

Item	Description	Condition
$\mu(UR)_s$	1	$0 \leqslant UR < 0.2$
	$(0.3 - UR)/0.1$	$0.2 \leqslant UR < 0.3$
	0	$UR \geqslant 0.3$
$\mu(UR)_M$	0	$UR < 0.2$
	$(UR - 0.2)/0.2$	$0.2 \leqslant UR < 0.4$
	$(0.6 - UR)/0.2$	$0.4 \leqslant UR < 0.6$
	0	$UR \geqslant 0.6$
$\mu(UR)_L$	0	$UR < 0.5$
	$(UR - 0.5)/0.1$	$0.5 \leqslant UR < 0.6$
	1	$UR \geqslant 0.6$

4.2.4 Fuzzy Reasoning

The adaptability of AWT can be obtained using input variable of HCWT, CV and UR. From CV and UR. We can get the adaptability of ART, from MaxHCWT and CV, the adaptability of LWP can be obtained, from HCWT, GD and UR it can get the adaptability of RPC. The evaluation standard of membership degree can be expressed by "VL", "L", "M", "S" and "VS", shown as in Fig.2.

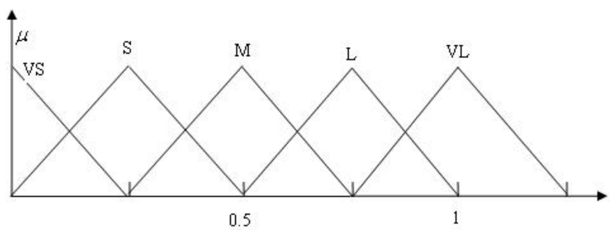

Fig. 2. Membership function for AWT, ART, LWP and RPC

1) Fuzzy reasoning rule of AWT

There are 27 pieces of rules, for example, the rule 27 is that if UR is L and HCWT is S and CV is S then S_{AWT} is VS, here the fuzzy reasoning rule of AWT is omitted.

2) Fuzzy reasoning rule of adaptability for LWP

It is shown as in Table 11 when adaptability being small.

Table 11. Fuzzy reasoning rule for LWP

S_{LWP}		MaxHCWT L	MaxHCWT M	MaxHCWT S
CV	L	M	L	VL
	M	S	M	L
	S	VS	S	M

The Table 11 has 9 pieces of rules, for example, the rule 9 is that if max-HCWT is S and CV is S then S_{LWP} is M.

3) Fuzzy reasoning rule of adaptability for RPC.

There are 27 pieces of rulesin which the rule 27 is that if UR is L Max-HCWT is S and GD is S then S_{RPC} is L. Here the fuzzy reasoning rule of AWT is omitted.

4) Fuzzy reasoning rule of adaptability for ART is shown as in Table 12.

There are 9 pieces of fuzzy reasoning rules.

5) Fuzzy reasoning algorithm

Table 12. Fuzzy reasoning rule for S_{ART}

S_{ART}		UR		
		L	M	S
CV	L	M	L	VL
	M	S	M	L
	S	VS	S	M

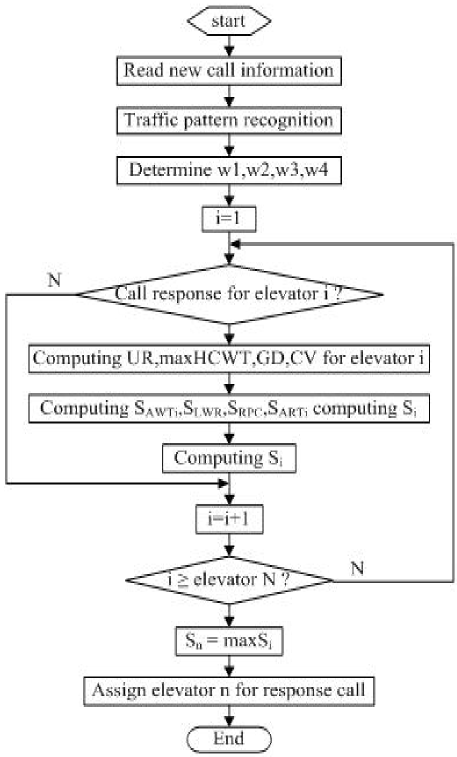

Fig. 3. Algorithm flowchart for elevator group dispatch

5 Conclusions

The above has mentioned that the core of EGCS is the traffic mode recognition and call assignment. The identification of traffic mode determines the aim control principle of call assignment, and it is the base of reasoning call assignment. The dispatch performance of EGCS is determined by call assignment. The paper took the forty five floors with eight sets of elevator control as an simplified example. The data acquisition in a day is from handwork statistics because of the condition limited, according to the flowchart of the

Fig.1 and the Fig.3, the experiment simulation showed that the energy saving is about 12 percent, the average waiting time is reduced, although the experiment is too simplified but it shows that it is feasible and reasonable, also it verified that the input variables are effective, and fuzzy reasoning is correct.

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Intelligent Auto-tracking System of Water Rushing Slag for Casting House

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Abstract. To solve the closed-loop auto-tracking control puzzle of water rushing waste slag in the blast furnace casting house, the paper presented a sort of the Adaptive Control Strategy based on human simulated intelligent controller. In the paper, it pointed out the puzzle of implementing closed-loop control in detail, analyzed the character of controlled object, presented a sort of human simulated intelligent control strategy, and structure mode of control algorithm, and adaptive strategy of human simulated task as well as human simulated parameter. By means of the designed industrial computer control system, the experiment of iron making field running was made. The field running effect showed that the uncertainty adaptive auto-tracking system of water rushing waste slag is effective and practicable, and better in robustness, high in control precision. The engineering practice shows that the presented control strategy has important engineering application value.

Keywords: Iron making, water rushing waste slag system, intelligent control, auto-tracking system.

1 Introduction

It is a very important tache to treat in time the waste slag in blast furnace iron making process, once the fault happens in this lache, it probably results in production line paralysis of iron making and stops the iron making production. In the process of iron making waste slag processing, the waste slag always floats above the high temperature liquid state iron, it can be separated at the exit slot, the separated waste slag in high temperature is struck by cooling water enough, under the action of cooling water quencher the high temperature waste slag is cracked as small granule, it will be crushed into dregs channel, and is flowed into the sedimentation pool. The small granule

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of waste slag is deposited through the sedimentation pool and will be fished out. And the water of rushing waste slag will inflow into regulation pool to be ready for reuse in circulation. The rushing slag water comes from the regulation pool by means of electro-motion pump. The technical demand is that it should make the water under the temperature of 60°C to avoid the pipeline dilapidation resulted from the pipeline oscillation because of boil away in the water pipe meshwork, and the regulation pool must keep to a certain water level to ensure the supply of rushing waste slag water enough. Due to the evaporation and other losing and leaking of water, the new water must be added into the regulation pool to regulate the temperature or water level when the water level of regulation pool is lower than the specified water level or the temperature is greater than the temperature of 60°C. The traditional control method is that using two sets of 215kw motor drive the water pump to run continuously in high speed all the time, whether the taphole is in work or not in work, the motor always locates the running state at high speed continuously, therefore it makes the energy resource be in great waste. In fact, the time of taphole rushing waste slag only takes about 7/12 iron making period in the iron making process. After through technique reconstruction, the motor is controlled by inverter which is only an execution part controlled by intelligent auto-tracking control system. In the period of non-rushing waste slag, the water pump is in low speed running to keep less water quantity flowing which can overcome the inertia action when it is regulated to high speed running. And during the period of water rushing waste slag, it can take the water quantity enough to crack the waste slag in high temperature, and the more the waste slag is, the more the rushing waste slag water, therefore it can completely realize the auto-track in high precision.

2 Soft Test and Characteristic of Controlled Object

2.1 Soft Test of Waste Slag Material

In order to realize the auto-tracking of the quantity of rushing waste slag water to the quantity of waste slag, first the quantity of waste slag must be measured, but the iron making waste slag is a random quantity in flowing state, the number of waste slag is sometimes less, or more sometimes, the flow of waste slag is on sometimes or off sometimes, it owns a great uncertainty and is very difficult to make direct measure. By means of principle of heat energy to temperature transformation, it can indirectly test the temperature of exit channel slot of the waste slag instead of the waste slag temperature. In principle, it is only a rough estimate but it is accurate enough in engineering application. Therefore when the number of rushing waste slag water is constant, if the waste slag temperature is higher then the rushing waste slag water temperature will be increased, namely the waste slag is more then the number of rushing waste slag water should be more. By means of indirect method, we can make the number of rushing waste slag water track automatically the number of the waste slag, namely the measure of the waste slag quantity is substituted by soft test method through temperature measure.

2.2 The Characteristic of the Controlled Object

The controlled object is a complex water system. The water pipeline is complicated enough and deals with lots of factors. It can be represented as the following characteristic, such as being unknown, time varying, random in water system parameter, and being unknown, time varying in system time lag, and existing correlation in variables, and multiformity, randomicity, unknown in environment disturbance and so on. Therefore it is difficult to establish the mathematical model, and it is not suitable for using traditional control method because of its non-structured [1,2]. The above characteristic has determined that it is a control puzzle with uncertainty.

3 Selection for Control Strategy

Although there are lots of control strategy that can be applied to various industrial field [3,4], such as PID control, general fuzzy control, real time expert control and so on [5,6], but here it is used to the simulated intelligent control strategy because of being closer to the field engineering practice than the others. The human simulated intelligent controller mainly summarizes human control experience, imitates human control experience and behavior, and applies production rules to describe its heuristic and instinctive reasoning behavior in control fields. Nowadays, the most excellent controller is still the human's brain. Since the basic properties of HSIC is to simulate expert control behavior, therefore its control algorithm is an interaction of the multi-model control to coordinate all kinds of incompatible control demands in a complex control system, such as robustness and accuracy, rapidness and smoothness, and so on. In the control process, it does not need the mathematical model, the only needed are certain condition expression state sentences that is like 'if condition then action', therefore the operator's experience, skill and control knowledge can be easily placed into the control rule set.

4 Structure Mode of Algorithm

The basic thought of implementing control is that it adopts different control strategy for different characteristic pattern of system error. The control algorithm consists of two sorts of control pattern and two sorts of human simulated adaptive control strategy. Its thought of control strategy comes from man-machine learning system, therefore it reflected some control rule of human being controller. Its algorithm can be summed up as the following.

4.1 Two Sorts of Control Pattern

① if $e.\dot{e} \geq 0$ and $e + \dot{e} \neq 0$ then Proportion and Half Derivation pattern.

② if $e \cdot \dot{e} \leq 0$ or $e = \dot{e} = 0$ then Half Open-loop pattern.

It can be summed up the following by means of structure description method.

If $e \cdot \dot{e} \geq 0$ and $\dot{e} \neq 0$ Then

If $\dot{e} \cdot \ddot{e} \geq 0$ Then

$$P(t) = \overline{P}_{n-1} + K_p e + P_{HD} \tag{1}$$

$$(\text{note:} P_{HD} = \tilde{P}_{l-1} + kK_p \dot{e})$$

If $\dot{e} \cdot \ddot{e} < 0$ Then

$$P\left(t\right) = \tilde{P}_{n-1} + K_p e + P_{HD} \tag{2}$$

$$(\text{note:} P_{HD} = kK_p \sum_{i=1}^{l} \dot{e}_{m,i})$$

If $e \cdot \dot{e} < 0$ or $|e| + |\dot{e}| \leq \delta$ Then

If $|e| \geq \frac{1}{2}\left|e_{m,n}\right| > \delta$ Then

$$P\left(t\right) = \bar{P}_n + kK_p(e - \frac{1}{2}e_{m,n}) \tag{3}$$

else

$$P\left(t\right) = \bar{P}_n \tag{4}$$

In which,$\bar{P}_n = kK_P \sum_{i=1}^{n} e_{m,i}$, P is the controller output (to controlled object), e is the controller input (system error signal), $\dot{e} \cdot \ddot{e}$ is respectively the 1-order and 2-order derivative of e corresponding to time, $e_{m,i}$ is the i^{th} extremum, K_P is the proportion coefficient, k is the restraining coefficient, $\bar{P}_n$ is a constant of the n^{th} times of P needed to be kept and defines $\bar{P}_0 = 0$ (error extremum memory), P_{HD} is the component of half-derivative in the pattern of $P - HD$, $\dot{e}_{m,i}$ is the i^{th} extremum of $\dot{e}$ in the pattern of P_{HD}, $\tilde{P}_l$ is a constant of the l^{th} times needed to be kept of P_{HD} of half-derivative output component in P_{HD} pattern, and defines $\tilde{P}_0 = 0$ (derivative extremum memory),i, l, n is respectively the nature number, δ is the sensitivity of controller input.

4.2 Two Sorts of Adaptive Strategy

4.2.1 Human Simulated Based Task Adaptive Strategy

An intelligent auto-tracking system of water rushing waste slag is practically a servo system according to the above mentioned basic control pattern, shown as in Fig.1.

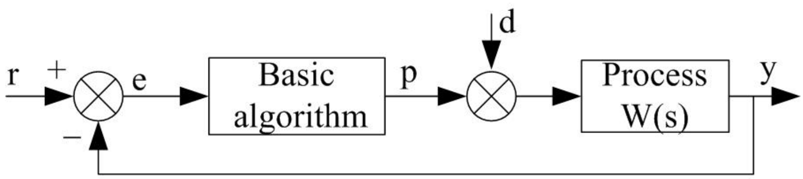

Fig. 1. The control system of basic algorithm

As a result of the controller output of regulation action for two sorts of control pattern all being based on hold memory value of $\bar{P}_n = kK_P \sum_{i=1}^{n} e_{m,i}$ (n = 0,1,...), it probably affects the control effect for unsuitable initial hold value $\bar{P}_o$. If the reference input is r, when the task variable of exterior disturbance for object characteristic changes it will result in sudden increasing of system error and switch of transition process. To ensure the auto-tracking quality, through a lot of simulation validation, finally it is summed up the self-adaptive control algorithm as the following.

In the pattern of P_{HD},

If $|e(k)| \geq |e_{m,n-1}| > \delta$ and $|\dot{e}(k)| > |\dot{e}(k-1)|$

or $|e(k)| \geq 2\,|e_{m,n-1}| > \delta$

or $|e(k)| \geq |e_{m,n-2}| > \delta$ Then

If $\bar{P}_{n-1} \cdot e(k) < 0$ Then

If $\bar{P}_{n-1} \cdot P \leq 0$ Then

$$\bar{P}'_{n-1} = 0 \quad (5)$$

If $\bar{P}_{n-1} \cdot P > 0$ and $\left|\bar{P}_{n-1}\right| > |P|$ Then

$$\bar{P}'_{n-1} = P \quad (6)$$

If $\bar{P}_{n-1} \cdot e(k) \geq 0$ Then

$$\bar{P}_{n-1}^{'} = \tilde{P}_{l-1} + kK_p[e(k) + \dot{e}(k)] \tag{7}$$

In which,$\bar{P}'_{n-1}$ represents the modified value $\bar{P}_{n-1}$, the other is the same as analogy.

4.2.2 Human Simulated Based Parameter Adaptive Strategy

There are two parameters in the algorithm model, proportion coefficient K_P and restraining coefficient, they determine the control intensity of P-HD pattern ($e \cdot \dot{e} \geq 0$ and $e + \dot{e} \neq 0$), the large the K_P is, the smaller the dynamical error. k is used for control action in the restraining of HO pattern ($e \cdot \dot{e} < 0$ or $|e| + |\dot{e}| < \delta$), it can compensate the influence of incorrect value of K_P, if K_P is fixed and takes larger allowable value then the control quality can be regulated by k.

In the servo system, an error characteristic quantity will be introduced, that is error attenuation constant T_H.

$$T_H = |e/\dot{e}| \quad (\dot{e} \neq 0)$$

It is used to describe the relative speed of error attenuation in the HO pattern, the large the $\mathcal{T}_\mathcal{H}$ is, the lower the attenuation is, and contrarily the attenuation will get quicker. Therefore the $\mathcal{T}_\mathcal{H}$ can be used to establish the standard pattern of error attenuation comparison.

$$T_H \in [\alpha, \beta] \quad (0 < \alpha < \beta)$$

For example, it can be specified as the following.

$$\alpha = 0.1, \qquad \beta = 1$$

According to the standard, it can adopt corresponding rule to carry through treatment in the HO pattern. And it will be got to the following three pieces of human being simulated parameter adaptive rule.

① In the HO pattern, if the error attenuation gets quicker, the value of T_H is lower than specified value, and the error derivative is still increasing, namely

$$T_H = |e(k)/\dot{e}(k)| \leq \alpha\,,\ (\dot{e}(k) \neq 0)$$

And $\qquad |\dot{e}(k)| \geq |\dot{e}(k-1)| > \delta$

Then

(i) Reduce the Hold action, the $\bar{P}_n$ should be modified as when $\bar{P}_n \neq 0$

$$\bar{P}'_n = \bar{P}_n - kK_p e\,(k) \tag{8}$$

(ii)Adopt the differential action to restrain the overshoot, namely it can be output according to the following.

$$P = \bar{P}_n + kK_p\dot{e} \tag{9}$$

(iii)If then the parameter will be modified.

$$k' = k - \frac{k}{2 + \left|\frac{\bar{P}_n/kK_P}{e(k)}\right|} > 0 \tag{10}$$

② In the pattern of HO, if

(i)The error derivative gets small

$$|\dot{e}(k)| < |\dot{e}(k-1)| \text{ , } (\dot{e}(k) \neq 0) \tag{11}$$

(ii)Estimate that when the error attenuation gets to zero, but the error does not still reduce to zero.

$$|e(k)/\dot{e}(k)| \geq |\dot{e}(k)/\ddot{e}(k)|$$

Or the error attenuation gets very slow, and the $\mathcal{T}_\langle$ value goes beyond the upper bound of standard pattern.

$$|e(k)/\dot{e}(k)| \geq \beta \tag{12}$$

(iii)TH value continue to increase

$$|e(k)/\dot{e}(k)| \geq |e(k-1)/\dot{e}(k-1)|$$

Then it will strengthen the Hold action, the will be modified as

$$\bar{P}_n' = \bar{P}_n + kK_Pe(k)/m \tag{13}$$

In which, the m is the times number of modified $\bar{P}_n$ in the HO pattern.

③ If the error derivative $\dot{e}$ changes in symbol when changing from HO pattern into P-HD pattern, namely

$$\dot{e}_{HO}(k-1) - \dot{e}_{P-HD}(k) < 0 \tag{14}$$

Then

(i)The $\bar{P}_n$ will be modified as

$$\bar{P}_n' = \bar{P}_n + kK_Pe(k) \tag{15}$$

(ii)if the will be modified as

$$k' = k + \frac{k}{2 + \left|\frac{\bar{P}_n/kK_P}{e(k)}\right|} > 0 \text{ , } (k' \leq 1) \tag{16}$$

The above two control algorithms, namely adaptive algorithm and basic control pattern algorithm, all get together to run on line, it constituted the human simulated function in task adaptive (or error non-normal processing) and parameter adaptive. The former simulated the function that the operator applies memory ability in flexibility and the experience adapts the ability of system status mutation, it makes the basic algorithm improving in adaptability and flexibility, enhance the robustness, and realize direct control in non-decomposition for multivariable system. The latter strengthens the stability of basic algorithm, extended the stabilization field. The human simulated function of parameter adaptability makes originally some unstable parameter change as stable parameter, and it can automatically regulate the parameter to approach the ideal value. Furthermore it can adapt the non-optimal parameter, at the same time it can also obtain the satisfactory system response. therefore it has very important significance in engineering application.

5 Engineering Realization

The above control strategy and control algorithm has been used in the water rushing waste slag system for an iron making factory. It measures indirectly the number of iron making waste slag by means of measuring the variation of water temperature after rushing waste slag. The constructed control system of closed-loop negative feedback can ensure that the water number of rushing waste slag can automatically track the number of iron making waste slag proportional. Owing to the inverter being used, it avoided the energy waste. When the number of iron making waste slag is large then the motor runs quickly, if it is small then the motor runs slowly in proportion, it can save the electric energy about 40 percent. In the intelligent auto-tracking control System of Water Rushing Waste Slag, it takes full advantage of system error e, change rate $\dot{e}$ of system error variation with time, and change rate $\ddot{e}$ of variation with time. By means of adaptive auto-tracking control strategy based on human simulated intelligence, it is not only easy to collect the control field data, but also it can realize the automation of whole control process in the industrial control computer by means of software manner. It is satisfactory to the user in control effect and control quality, and also it gets good evaluation.

6 Conclusions

The above mentioned intelligent auto-tracking system of water rushing waste slag has been successfully used in industrial engineering, its main advantage is that it is suitable for uncertainty process control, and unnecessary in establishing the mathematical model, and stronger in anti-jamming performance.

The key of uncertainty complex control realization is two sorts of control pattern and two sorts of adaptive strategy. Therefore it is meaningful to explore the control strategy in engineering realization.

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A Kind of Center of Gravity Fuzzy System

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Abstract. In this paper, we provide a method of constructing joint probability density function and fuzzy system by using fuzzy inference based on a set data of input-output. Firstly, we build fuzzy relation and derive the joint probability density function. Secondly, we discuss the marginal density function and numerical characteristic of the joint probability distribution, including mathematical expectation, variance and covariance. Finally, using probability distribution in this paper, we work out the corresponding the center of gravity fuzzy system and show the fuzzy system is an universal approximator.

Keywords: Fuzzy system, probability density, numerical characteristic.

1 Introduction

The construction and universal approximation of fuzzy system is a hot topic in the field of fuzzy control. As we know, there are four steps to construct a fuzzy system [1]:

(1) singleton fuzzifier for input variable; (2) build fuzzy inference rules; (3) create fuzzy relation; (4) defuzzifier for the fuzzy set of output.

Currently, in the construction of fuzzy system, the singleton fuzzifier of input variable is commonly used in most of documents. The fuzzy inference method is CRI [2,3] or triple I [4]. However, only when we choose the conjunction fuzzy implication operator(such as Larsen implication or Mamdani implication), fuzzy system constructed by these two reasoning methods have the response capability [5-8]. Usually, there are three defuzzifier as follows: (a) Center-average defuzzifier; (b) Center-of-gravity defuzzifier; (c) Maximum defuzzifier. Currently, Center-average defuzzifier and Maximum defuzzifier are mostly used, while Center-of-gravity defuzzifier is rarely used. How we can directly integrate to obtain the fuzzy system will be our main problem. It is well known that, the fuzzy system with center of gravity defuzzifier can be drawn to the form of interpolation and it is the best approximation to the original system in the least squares sense [9]. In [10,11], probability density

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functions based on Larsen implication and Mamdani implication are discussed and the numerical characteristics of the probability distributions are studied. Meanwhile, it points out that no matter which implication operator we use, the numerical characteristics of the probability distribution have the same form. In this paper, by applying the method of [10,11], we study the center of gravity fuzzy system and probability distribution based a new implication.

2 Preliminary

Let $\{(x_i, y_i)\}_{(i \leq i \leq n)}$ be a set data of input-output, let the input domain be $X = [a, b], Y = [c, d]$. In this paper we assume that:

$$a = x_1 < x_2 < \ldots < x_n = b, c = y_1 < y_2 < \ldots < y_n = d$$

We construct the triangle waves with the given data as follows:

$$A_1(x) = \begin{cases} (x - x_2)/(x_1 - x_2), & x_i \in [x_1, x_2] \\ 0, & else \end{cases}$$

$$A_i(x) = \begin{cases} (x - x_{i-1})/(x_i - x_{i-1}), & x_i \in [x_{i-1}, x_i] \\ (x - x_{i+1})/(x_i - x_{i+1}), & x_i \in [x_i, x_i + 1] \quad i = 2, 3, \ldots, n-1 \\ 0, & else \end{cases}$$

$$A_n(x) = \begin{cases} (x - x_{n-1})/(x_n - x_{n-1}), & x_i \in [x_{n-1}, x_n] \\ 0, & else \end{cases}$$

$B_i(y)(i = 1, 2, \ldots, n)$ has the same form and conclusion. x_i is the peak point of A_i, y_i is the peak point of B_i, namely $A_i(x_i) = 1, B_i(y_i) = 1$. If $\forall x \in [a, b]$, it surely $\exists i \in (1, 2, \ldots, n-1)$ so that $x \in [x_i, x_{i+1}], A_i(x) + A_{i+1}(x) = 1, A_j(x) = 0 (j \neq i, i+1)$.

Suppose that there be the fuzzy reasoning rules as following:

$$If \quad x \quad is \quad A_i, \quad then \quad y \quad is \quad B_i \quad (i = 1, 2, \ldots, n).$$

Let θ be the fuzzy implication operator, $R_i(x, y) = \theta(A_i(x), B_i(y)), R = \bigcup_{i=1}^{n} R_i$.

Suppose that the input variable is x, let

$$A^*(x') = \begin{cases} 1, & x' = x \\ 0, & x' \neq x \end{cases}, B^* = A^* \circ R.$$

Thus we have that $\overline{S}(x) = \frac{\int_Y yB^*(y)\mathrm{d}y}{\int_Y B^*(y)\mathrm{d}y}$, $H(2, n, \theta, \vee) \triangleq \int_a^b \int_c^d B^*(y)\mathrm{d}x\mathrm{d}y$, $H(2, n, \theta, \vee)$ is called the H-function with the parameter of $(2, n, \theta, \vee)$. Let $f(x, y) \triangleq \frac{B^*(y)}{H(2,n,\theta,\vee)}$, then $(a) f(x, y) \geq 0$; $(b) \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x, y)\mathrm{d}x\mathrm{d}y = 1$. Thus $f(x, y)$ can be considered as the joint probability density function of some random variable (ξ, η).

3 The Probability Distribution and Numerical Characteristic Based on a Kind of Fuzzy Implication

Let $\theta(a,b)=(1-a)(1-|a-b|)$, we will discuss the probability distribution based on θ.

Theorem 1. Let $L=\frac{47}{96}\sum_{i=1}^{n-1}(y_{i+1}-y_i)(x_{i+1}-x_i)$, then the joint density function of random vector (ξ,η) is

$$f(x,y)=\begin{cases}\frac{1}{L}A_i(x)(1-A_{i+1}(x)+B_{i+1}(y)),\exists i,(x,y)\in E_1\\ \frac{1}{L}A_{i+1}(x)(1+A_i(x)-B_i(y)),\exists i,(x,y)\in E_2\\ \frac{1}{L}A_{i+1}(x)(1-A_i(x)+B_i(y)),\exists i,(x,y)\in E_3\\ \frac{1}{L}A_i(x)(1+A_{i+1}(x)-B_{i+1}(y)),\exists i,(x,y)\in E_4\\ \qquad 0, \qquad\qquad else\end{cases}$$

where $I_i=[x_i,x_{i+1}],J_i=[y_i,y_{i+1}]$, let $y_i^{\Delta}=A_i(x)y_i+A_{i+1}(x)y_{i+1},\overline{x}=\frac{1}{2}(x_i+x_{i+1})$,

$$\begin{aligned}E_1&=\{(x_i,y_i)\in I_i\times J_i\mid y<y_i^{\Delta},x\in[x_i,\overline{x}_i)\}\\ E_2&=\{(x_i,y_i)\in I_i\times J_i\mid y<y_i^{\Delta},x\in[\overline{x}_i,x_{i+1})\}\\ E_3&=\{(x_i,y_i)\in I_i\times J_i\mid y\geq y_i^{\Delta},x\in[\overline{x}_i,x_{i+1})\}\\ E_4&=\{(x_i,y_i)\in I_i\times J_i\mid y\geq y_i^{\Delta},x\in[x_i,\overline{x}_i)\}\end{aligned}$$

Proof By $\bigvee_{k=1}^{n}\theta(A_k(x),B_k(y))=\bigvee_{k=1}^{n}((1-A_k(x)(1-|A_k(x)-B_k(x)|))$, we have

$R(x,y)=$

$$\begin{cases}[A_{i+1}(x)(1-|A_i(x)-B_i(y)|)]\vee[A_i(x)(1-|A_{i+1}(x)-B_{i+1}(y)|)],\exists i(x,y)\in I_i\times J_i\\ \qquad 0, \qquad\qquad else\end{cases}$$

Then we have that $H(2,n,\theta,\vee)=\sum_{i=1}^{n-1}\int_{x_i}^{x_{i+1}}\int_{y_i}^{y_{i+1}}\{[A_{i+1}(x)(1-|A_i(x)-B_i(y)|)]\vee[A_i(x)(1-|A_{i+1}(x)-B_{i+1}(y)|)]\}\mathrm{d}x\mathrm{d}y$.

1. $A_i(x)\geq Bi(y)\Leftrightarrow y_i\geq y_i^{\Delta}$,then

$$R(x,y)=\begin{cases}A_i(x)(1+A_{i+1}(x)-B_{i+1}(y)),x\in[x_i,\overline{x}_i)\\ A_{i+1}(x)(1-A_i(x)+B_i(y)),x\in[\overline{x}_i,x_{i+1})\end{cases}$$

2. $A_i(x)<B_i(y)\Leftrightarrow y_i<y_i^{\Delta}$,then

$$R(x,y)=\begin{cases}A_i(x)(1-A_{i+1}(x)+B_{i+1}(y)),x\in[x_i,\overline{x}_i)\\ A_{i+1}(x)(1+A_i(x)-B_i(y)),x\in[\overline{x}_i,x_{i+1})\end{cases}$$

Then $H(2,n,\theta,\vee) = \int_a^b \int_c^d R(x,y)\mathrm{d}x\mathrm{d}y = \sum_{i=1}^{n-1} \int_{x_i}^{x_{i+1}} \int_{y_i}^{y_{i+1}} [A_{i+1}(x)(1-|A_i(x)-B_i(y)|)] \vee [A_i(x)(1-|A_{i+1}(x)-B_{i+1}(y)|)]\mathrm{d}y\mathrm{d}x = \sum_{i=1}^{n-1} [\int_{x_i}^{\overline{x}_i} \int_{y_i}^{y_i^*} A_i(x)(1-A_{i+1}(x)+B_{i+1}(y))\mathrm{d}y\mathrm{d}x + \int_{x_i}^{\overline{x}_i} \int_{y_i^*}^{y_{i+1}} A_i(x)(1+A_{i+1}(x)-B_{i+1}(y))\mathrm{d}y\mathrm{d}x + \int_{\overline{x}_i}^{x_{i+1}} \int_{y_i}^{y_i^*} A_{i+1}(x)(1+A_i(x)-B_i(y))\mathrm{d}y\mathrm{d}x + \int_{\overline{x}_i}^{x_{i+1}} \int_{y_i^*}^{y_{i+1}} A_{i+1}(x)(1-A_i(x)+B_i(y))\mathrm{d}y\mathrm{d}x] \triangleq \sum_{i=1}^{n-1}(\triangle_1+\triangle_2+\triangle_3+\triangle_4)$.
Clearly,

$$\begin{aligned}
\Delta_1 &= \int_{x_i}^{\overline{x}_i} \int_{y_i}^{y_i^*} A_i(x)(1-A_{i+1}(x)+B_{i+1}(y))\mathrm{d}y\mathrm{d}x \\
&= \int_{x_i}^{\overline{x}_i} [A_i^2(x)A_{i+1}(x)(y_{i+1}-y_i) + \frac{1}{2}A_i(x)A_{i+1}^2(x)(y_{i+1}-y_i)]\mathrm{d}x \\
&= \frac{9}{128}(y_{i+1}-y_i)(x_{i+1}-x_i). \\
\Delta_2 &= \int_{x_i}^{\overline{x}_i} \int_{y_i^*}^{y_{i+1}} A_i(x)(1+A_{i+1}(x)-B_{i+1}(y))\mathrm{d}y\mathrm{d}x \\
&= \int_{x_i}^{\overline{x}_i} \int_{y_i^*}^{y_{i+1}} A_i(x)A_{i+1}(x)\mathrm{d}y\mathrm{d}x + \int_{x_i}^{\overline{x}_i} \int_{y_i^*}^{y_{i+1}} A_i(x)B_i(y)\mathrm{d}y\mathrm{d}x. \\
&= \frac{67}{384}(y_{i+1}-y_i)(x_{i+1}-x_i). \\
\Delta_3 &= \int_{\overline{x}_i}^{x_{i+1}} \int_{y_i}^{y_i^*} A_{i+1}(x)(1+A_i(x)-B_i(y))\mathrm{d}y\mathrm{d}x \\
&= \int_{\overline{x}_i}^{x_{i+1}} \int_{y_i}^{y_i^*} A_{i+1}(x)A_i(x)dydx + \int_{\overline{x}_i}^{x_{i+1}} \int_{y_i}^{y_i^*} A_{i+1}(x)B_{i+1}(y)\mathrm{d}y\mathrm{d}x \\
&= \frac{67}{384}(y_{i+1}-y_i)(x_{i+1}-x_i). \\
\Delta_4 &= \int_{\overline{x}_i}^{x_{i+1}} \int_{y_i^*}^{y_{i+1}} A_{i+1}(x)(1-A_i(x)+B_i(y))\mathrm{d}y\mathrm{d}x \\
&= \int_{\overline{x}_i}^{x_{i+1}} \int_{y_i^*}^{y_{i+1}} A_{i+1}^2(x)dydx + \int_{\overline{x}_i}^{x_{i+1}} \int_{y_i^*}^{y_{i+1}} A_{i+1}(x)B_i(y)\mathrm{d}y\mathrm{d}x \\
&= \frac{9}{128}(y_{i+1}-y_i)(x_{i+1}-x_i).
\end{aligned}$$

Then $H(2,n,\theta,\vee) = \sum_{i=1}^{n-1}(\Delta_1+\Delta_2+\Delta_3+\Delta_4) = \sum_{i=1}^{n-1} \frac{47(y_{i+1}-y_i)(x_{i+1}-x_i)}{96} \triangleq L$.

$$f(x,y) = \begin{cases} \frac{1}{L}A_i(x)(1-A_{i+1}(x)+B_{i+1}(y)), \exists i, (x,y) \in E_1 \\ \frac{1}{L}A_{i+1}(x)(1+A_i(x)-B_i(y)), \exists i, (x,y) \in E_2 \\ \frac{1}{L}A_{i+1}(x)(1-A_i(x)+B_i(y)), \exists i, (x,y) \in E_3 \\ \frac{1}{L}A_i(x)(1+A_{i+1}(x)-B_{i+1}(y)), \exists i, (x,y) \in E_4 \\ 0, \qquad else \end{cases}$$

Theorem 2. The marginal density function of ξ and η are:

$$f_\xi(x)=\begin{cases}\frac{1}{L}(y_{i+1}-y_i)(\frac{1}{2}A_i(x)+A_i^2(x)A_{i+1}(x)), x\in[x_i,\overline{x}_i]\\ \frac{1}{L}(y_{i+1}-y_i)(\frac{1}{2}A_{i+1}(x)+A_i(x)A_{i+1}^2(x)), x\in[\overline{x}_i,x_{i+1}]\end{cases}$$

$$f_\eta(y)=\begin{cases}\frac{(x_{i+1}-x_i)}{L}(\frac{1}{2}B_i^2(y)B_{i+1}(y)+B_i(y)B_{i+1}(y)+\frac{3}{4}B_{i+1}(y)\\ \qquad -\frac{7}{24}+\frac{1}{6}B_i^3(y)+\frac{1}{2}B_i^2(y)), y\in[y_i,\overline{y}_i]\\ \frac{(x_{i+1}-x_i)}{L}(\frac{1}{2}B_i(y)B_{i+1}^2(y)+B_i(y)B_{i+1}(y)+\frac{3}{4}B_i(y)\\ \qquad -\frac{7}{24}+\frac{1}{6}B_{i+1}^3(y)+\frac{1}{2}B_{i+1}^2(y)), y\in[\overline{y}_i,y_{i+1}]\end{cases}$$

Proof When $x\in[x_i,\overline{x}_i]$,
$f_\xi(x)=\int_{-\infty}^{+\infty}f(x,y)\mathrm{d}y=\frac{1}{L}\int_{y_i}^{y_i^*}A_i(x)(1-A_{i+1}(x)+B_{i+1}(y))\mathrm{d}y+\frac{1}{L}\int_{y_i^*}^{y_{i+1}}A_i(x)(1+A_{i+1}(x)-B_{i+1}(y))\mathrm{d}y=\frac{1}{L}(y_{i+1}-y_i)(\frac{1}{2}A_i(x)+A_i^2(x)A_{i+1}(x))$.

When $x\in[\overline{x}_i,x_{i+1}]$,
$f_\xi(x)=\int_{-\infty}^{+\infty}f(x,y)\mathrm{d}y=\frac{1}{L}\int_{y_i}^{y_i^*}A_{i+1}(x)(1+A_i(x)-B_i(y))\mathrm{d}y+\frac{1}{L}\int_{y_i^*}^{y_{i+1}}A_{i+1}(x)(1-A_i(x)+B_i(y))\mathrm{d}y=\frac{1}{L}(y_{i+1}-y_i)(\frac{1}{2}A_{i+1}(x)+A_i(x)A_{i+1}^2(x))$.

When $y\in[y_i,\overline{y}_i]$,
$f_\eta=\int_{-\infty}^{+\infty}f(x,y)\mathrm{d}x=\frac{1}{L}\int_{x_i}^{x_i^*}A_i(x)(1+A_{i+1}(x)-B_{i+1}(y))\mathrm{d}x+\frac{1}{L}\int_{x_i^*}^{\overline{x}_i}A_i(x)(1-A_{i+1}(x)+B_{i+1}(y))\mathrm{d}x+\frac{1}{L}\int_{\overline{x}_i}^{x_{i+1}}A_{i+1}(x)(1+A_i(x)-B_i(y))\mathrm{d}x$
$=\frac{1}{L}(x_{i+1}-x_i)(\frac{1}{2}B_i^2(y)B_{i+1}(y)+B_i(y)B_{i+1}(y)+\frac{3}{4}B_{i+1}(y)-\frac{7}{24}+\frac{1}{6}B_i^3(y)+\frac{1}{2}B_i^2(y))$.

When $y\in[\overline{y}_i,y_{i+1}]$,
$f_\eta=\int_{-\infty}^{+\infty}f(x,y)\mathrm{d}x=\frac{1}{L}\int_{x_i}^{\overline{x}_i}A_i(x)(1+A_{i+1}(x)-B_{i+1}(y))\mathrm{d}x+\frac{1}{L}\int_{\overline{x}_i}^{x_i^*}A_{i+1}(x)(1-A_i(x)+B_i(y))\mathrm{d}x+\frac{1}{L}\int_{x_i^*}^{x_{i+1}}A_{i+1}(x)(1+A_i(x)-B_i(y))\mathrm{d}x$
$=\frac{1}{L}(x_{i+1}-x_i)(\frac{B_i(y)B_{i+1}^2(y)}{2}$
$+B_i(y)B_{i+1}(y)+\frac{3}{4}B_i(y)-\frac{7}{24}+\frac{1}{6}B_{i+1}^3(y)+\frac{1}{2}B_{i+1}^2(y))$.

Theorem 3. The mathematical expectation of ξ and η are:

$$E(\xi)=\sum_{i=1}^{n-1}\omega_i\overline{x}_i, E(\eta)=\sum_{i=1}^{n-1}\omega_i\overline{y}_i$$

The variance of ξ and η are:

$$D(\xi)\approx\sum_{i=1}^{n-1}\omega_i x_i x_{i+1}-(\sum_{i=1}^{n-1}\omega_i\overline{x}_i)^2, D(\eta)\approx\sum_{i=1}^{n-1}\omega_i y_i y_{i+1}-(\sum_{i=1}^{n-1}\omega_i\overline{y}_i)^2.$$

The covariance of (ξ, η) is: $COV(\xi, \eta) \approx \sum_{i=1}^{n-1} \omega_i \overline{z}_i - (\sum_{i=1}^{n-1} \omega_i \overline{x}_i) \sum_{i=1}^{n-1} \omega_i \overline{y}_i$.
where, $\omega_i = \frac{(y_{i+1}-y_i)(x_{i+1}-x_i)}{\sum_{i=1}^{n-1}(y_{i+1}-y_i)(x_{i+1}-x_i)}$, $\overline{y}_i = \frac{y_i+y_{i+1}}{2}$,$\overline{z}_i = \frac{x_i y_i + x_{i+1} y_{i+1}}{2}$.

Proof

$$
\begin{aligned}
E(\xi) &= \sum_{i=1}^{n-1} \int_{x_i}^{x_{i+1}} x f_\xi(x)\mathrm{d}x = \sum_{i=1}^{n-1} [\int_{x_i}^{\overline{x}_i} x f_\xi(x)\mathrm{d}x + \int_{\overline{x}_i}^{x_{i+1}} x f_\xi(x)\mathrm{d}x] \\
&= \sum_{i=1}^{n-1} \frac{y_{i+1}-y_i}{L} [\int_{x_i}^{\overline{x}_i} x(\frac{1}{2}A_i(x) + A_i^2(x)A_{i+1}(x))\mathrm{d}x + \\
&\qquad \int_{\overline{x}_i}^{x_{i+1}} x(\frac{1}{2}A_{i+1}(x) + A_{i+1}^2(x)A_i(x))\mathrm{d}x] \\
&= \sum_{i=1}^{n-1} \frac{(y_{i+1}-y_i)(x_{i+1}-x_i)}{\sum_{k=1}^{n-1}(y_{k+1}-y_k)(x_{k+1}-x_k)} \frac{(x_{i+1}+x_i)}{2} \\
&= \sum_{i=1}^{n-1} \omega_i \overline{x}_i.
\end{aligned}
$$

$$
\begin{aligned}
E(\eta) &= \sum_{i=1}^{n-1} \int_{y_i}^{y_{i+1}} y f_\eta(y)\mathrm{d}y = \sum_{i=1}^{n-1} [\int_{y_i}^{\overline{y}_i} y f_\eta(y)\mathrm{d}y + \int_{\overline{y}_i}^{y_{i+1}} y f_\eta(y)\mathrm{d}y] \\
&= \frac{(y_{i+1}-y_i)(x_{i+1}-x_i)}{L} [\frac{47}{192} y_i + \frac{31}{480}(y_{i+1}-y_i) + \frac{47}{192} y_i + \frac{173}{960}(y_{i+1}-y_i)] \\
&= \sum_{i=1}^{n-1} \frac{(y_{i+1}-y_i)(x_{i+1}-x_i)}{L} \frac{47}{96} \frac{(y_{i+1}+y_i)}{2} \\
&= \sum_{i=1}^{n-1} \omega_i \overline{y}_i.
\end{aligned}
$$

$$\begin{aligned}
E(\xi^2) &= \sum_{i=1}^{n-1}\int_{x_i}^{x_{i+1}} x^2 f_\xi(x)\mathrm{d}x = \sum_{i=1}^{n-1}[\int_{x_i}^{\overline{x}_i} x^2 f_\xi(x)\mathrm{d}x + \int_{\overline{x}_i}^{x_{i+1}} x^2 f_\xi(x)\mathrm{d}x] \\
&= \sum_{i=1}^{n-1}\frac{(y_{i+1}-y_i)(x_{i+1}-x_i)}{L}[\frac{47}{96}x_i x_{i+1} + \frac{53}{320}(x_{i+1}-x_i)^2] \\
&= \sum_{i=1}^{n-1}\omega_i x_i x_{i+1} + \frac{53}{320}\sum_{i=1}^{n-1}\frac{(y_{i+1}-y_i)(x_{i+1}-x_i)}{L}(x_{i+1}-x_i)^2 \\
&\approx \sum_{i=1}^{n-1}\omega_i x_i x_{i+1}.
\end{aligned}$$

$$\begin{aligned}
E(\eta^2) &= \sum_{i=1}^{n-1}\int_{y_i}^{y_{i+1}} y^2 f_\eta(y)\mathrm{d}y = \sum_{i=1}^{n-1}[\int_{y_i}^{\overline{y}_i} y^2 f_\eta(y)\mathrm{d}y + \int_{\overline{y}_i}^{y_{i+1}} y^2 f_\eta(y)\mathrm{d}y] \\
&= \sum_{i=1}^{n-1}\frac{(y_{i+1}-y_i)(x_{i+1}-x_i)}{L}[\frac{47}{96}y_i y_{i+1} + \frac{919}{5760}(y_{i+1}-y_i)^2] \\
&= \sum_{i=1}^{n-1}\omega_i y_i y_{i+1} + \frac{919}{5760}\sum_{i=1}^{n-1}\frac{(y_{i+1}-y_i)(x_{i+1}-x_i)}{L}(y_{i+1}-y_i)^2 \\
&\approx \sum_{i=1}^{n-1}\omega_i y_i y_{i+1}.
\end{aligned}$$

$$\begin{aligned}
E(\xi\eta) &= \sum_{i=1}^{n-1}\int_{x_i}^{x_{i+1}}\int_{y_i}^{y_{i+1}} xyf(x,y)\mathrm{d}y\mathrm{d}x \\
&= \frac{1}{L}\sum_{i=1}^{n-1}[\int_{x_i}^{\overline{x}_i} x\int_{y_i}^{y_i^*} yA_i(x)(1-A_{i+1}(x)+B_{i+1}(y))\mathrm{d}y\mathrm{d}x \\
&\quad + \int_{x_i}^{\overline{x}_i} x\int_{y_i^*}^{y_{i+1}} yA_i(x)(1+A_{i+1}(x)-B_{i+1}(y))\mathrm{d}y\mathrm{d}x \\
&\quad + \int_{\overline{x}_i}^{x_{i+1}} x\int_{y_i}^{y_i^*} yA_{i+1}(x)(1+A_i(x)-B_i(y))\mathrm{d}y\mathrm{d}x \\
&\quad + \int_{\overline{x}_i}^{x_{i+1}} x\int_{y_i^*}^{y_{i+1}} yA_{i+1}(x)(1-A_i(x)+B_i(y))\mathrm{d}y\mathrm{d}x] \\
&= \sum_{i=1}^{n-1}\frac{(y_{i+1}-y_i)(x_{i+1}-x_i)}{\sum_{i=1}^{n-1}(y_{i+1}-y_i)(x_{i+1}-x_i)}[\overline{z_i} - \frac{311}{1410}(y_{i+1}-y_i)(x_{i+1}-x_i)] \\
&\approx \sum_{i=1}^{n-1}\omega_i\overline{z}_i.
\end{aligned}$$

Then we have that:

$$D(\xi) = E(\xi^2) - E(\xi)E(\xi) \approx \sum_{i=1}^{n-1} \omega_i x_i x_{i+1} - (\sum_{i=1}^{n-1} \omega_i \overline{x}_i)^2.$$

$$D(\eta) = E(\eta^2) - E(\eta)E(\eta) \approx \sum_{i=1}^{n-1} \omega_i y_i y_{i+1} - (\sum_{i=1}^{n-1} \omega_i \overline{y}_i)^2.$$

$$COV(\xi, \eta) \approx \sum_{i=1}^{n-1} \omega_i \overline{z}_i - (\sum_{i=1}^{n-1} \omega_i \overline{x}_i)(\sum_{i=1}^{n-1} \omega_i \overline{x}_i).$$

4 The Center of Gravity Fuzzy System

Theorem 4. The center of gravity fuzzy system based on θ_{293} is

$$\overline{S}(x) = A_i^*(x) y_i + A_{i+1}^*(x) y_{i+1},$$

where $x \in [x_i, x_{i+1}]$,

$$A_i^*(x) = \frac{\frac{2}{3} + A_{i+1}(x) - 2A_{i+1}^2(x) + \frac{2}{3}A_{i+1}^3(x)}{1 + 2A_{i+1}(x) - 2A_{i+1}^2(x)}.$$

$$A_{i+1}^*(x) = \frac{\frac{1}{3} + A_{i+1}(x) - \frac{2}{3}A_{i+1}^3(x)}{1 + 2A_{i+1}(x) - 2A_{i+1}^2(x)}.$$

Proof When $x \in [x_i, \overline{x}_i]$,

$$\int_{-\infty}^{+\infty} y f(x,y) \mathrm{d}y = \int_{y_i}^{y_{i+1}} y f(x,y) \mathrm{d}y = \int_{y_i}^{y_i'} y f(x,y) \mathrm{d}y + \int_{y_i'}^{y_{i+1}} y f(x,y) \mathrm{d}y$$

$$= \frac{1}{L} [\int_{y_i}^{y_i'} y A_i(x)(A_i(x) + B_{i+1}(y)) \mathrm{d}y + \int_{y_i{\prime}}^{y_{i+1}} y A_i(x)(A_{i+1}(x) + B_i(y)) \mathrm{d}y]$$

$$= \frac{y_{i+1} - y_i}{L} (c_i(x) y_i + c_{i+1}(x) y_{i+1}),$$

where

$$c_i(x) = A_i^3(x) A_{i+1}(x) + \frac{A_i(x) A_{i+1}^2(x)}{2} - \frac{2A_i(x) A_{i+1}^3(x)}{3} + \frac{A_i^3(x)}{2} - \frac{A_i^2(x)}{2} + \frac{A_i(x)}{3}$$

$$c_{i+1}(x) = A_i^2(x) A_{i+1}^2(x) + \frac{2}{3} A_i(x) A_{i+1}^3(x) + A_i^2(x) A_{i+1}(x) + \frac{1}{2} A_i^2(x) - \frac{1}{3} A_i(x)$$

From $\overline{S}(x) = \frac{\int_{-\infty}^{+\infty} y f(x,y) \mathrm{d}y}{\int_{-\infty}^{+\infty} f(x,y) \mathrm{d}y}$, we have that $\overline{S}(x) = A_i^*(x) y_i + A_{i+1}^*(x) y_{i+1}$, where

$$A_i^*(x) = \frac{\frac{2}{3} + A_{i+1}(x) - 2A_{i+1}^2(x) + \frac{2}{3}A_{i+1}^3(x)}{1 + 2A_{i+1}(x) - 2A_{i+1}^2(x)}, A_{i+1}^*(x) = \frac{\frac{1}{3} + A_{i+1}(x) - \frac{2}{3}A_{i+1}^3(x)}{1 + 2A_{i+1}(x) - 2A_{i+1}^2(x)}.$$

Similarly, when $x \in [\overline{x}_i, x_{i+1}]$,

$$\int_{-\infty}^{+\infty} yf(x,y)\mathrm{d}y = \int_{y_i}^{y_{i+1}} yf(x,y)\mathrm{d}y = \int_{y_i}^{y_i^*} yf(x,y)\mathrm{d}y + \int_{y_i^*}^{y_{i+1}} yf(x,y)\mathrm{d}y$$
$$= \frac{1}{L}[\int_{y_i}^{y_i^*} yA_{i+1}(x)(1+A_i(x)-B_i(y))\mathrm{d}y + \int_{y_i^*}^{y_{i+1}} yA_{i+1}(x)(1-A_i(x)+B_i(y))\mathrm{d}y]$$
$$= \frac{y_{i+1}-y_i}{L}(c_i(x)y_i + c_{i+1}(x)y_{i+1}),$$

where:

$$c_i(x) = A_i^2(x)A_{i+1}^2(x) + \frac{A_{i+1}^3(x)+A_i^2(x)A_i(x)-A_i(x)A_{i+1}(x)}{2} - \frac{2A_{i+1}^4(x)-A_{i+1}(x)}{3}$$
$$c_{i+1}(x) = A_i(x)A_{i+1}^3(x) + \frac{2A_{i+1}^4(x)}{3} - A_i(x)A_{i+1}^2(x) + \frac{A_i(x)A_{i+1}(x)}{2} - \frac{A_{i+1}(x)}{3}$$

then

$$A_i^*(x) = \frac{\frac{2}{3}+A_{i+1}(x)-2A_{i+1}^2(x)+\frac{2}{3}A_{i+1}^3(x)}{1+2A_{i+1}(x)-2A_{i+1}^2(x)}, A_{i+1}^*(x) = \frac{\frac{1}{3}+A_{i+1}(x)-\frac{2}{3}A_{i+1}^3(x)}{1+2A_{i+1}(x)-2A_{i+1}^2(x)}.$$

Theorem 5. Let $F(x) = A_i(x)y_i + A_{i+1}(x)y_{i+1}$, then

$$\|\overline{S}-F\|_\infty \le \frac{1}{3}\|S^{'}\|_\infty h, \|\overline{S}-S\|_\infty \le \frac{1}{8}\|S^{''}\|_\infty h^2 + \frac{1}{3}\|S^{'}\|_\infty h$$

$S(x)$ is the system describing function of the system constructed,

$$h = \max_{1\le i\le n-1} |x_{i+1}-x_i|, \|f\|_\infty = \sup_{x\in[a,b]} |f(x)|.$$

Proof. Let $F(x) = A_i(x)y_i + A_{i+1}(x)y_{i+1}$,$x \in [x_i, x_{i+1}]$. By [12], we have that

$$\|S-F\|_\infty \le \frac{1}{8}\|S^{''}\|_\infty h^2$$

$\overline{S}(x) - F(x) = (A_i^*(x) - A_i(x))(y_i - y_{i+1})$. When $x \in [x_i, x_{i+1}]$, we have that

$$A_i^*(x) - A_i(x) = \frac{1}{3}\frac{A_i(x)-6A_i^3(x)+4A_i^4(x)}{A_i(x)+2A_i^2(x)-2A_i^3(x)}.$$

Clearly,$|A_i^*(x) - A_i(x)| \le \frac{1}{3}$.Then

$$|\overline{S}(x) - F(x)| = |A_i^*(x) - A_i(x)| \cdot |y_i - y_{i+1}| \le \frac{1}{3}\|S^{'}\|_\infty h.$$

And consequently,

$$\|\overline{S}-S\|_\infty \le \|F-S\|_\infty + \|\overline{S}-F\|_\infty \le \frac{1}{8}\|S^{''}\|_\infty h^2 + \frac{1}{3}\|S^{'}\|_\infty h.$$

Remark 1. From the Theorem 4, we know that $\overline{S}(x)$ is not only the regression function, also the fuzzy system constructed with the center of gravity method.

Remark 2. From the proof of Theorem 4, we have that Theorem 4 has nothing to do with the monotonicity of inputoutput data.

Example 1. Let $\varepsilon = 0.1$.By$\frac{1}{8}\|S^{''}\|_{\infty}h^2 + \frac{1}{3}\|S^{'}\|_{\infty}h < 0.1$, we have $n = 22$. In the simulation, let $S(x) = \sin(x)$, $X = [a, b] = [-3, 3]$. We approximate $S(x)$ with the system $\overline{S}(x)$ constructed in the theorem 4. The simulation with $n = 22$ is as follows:

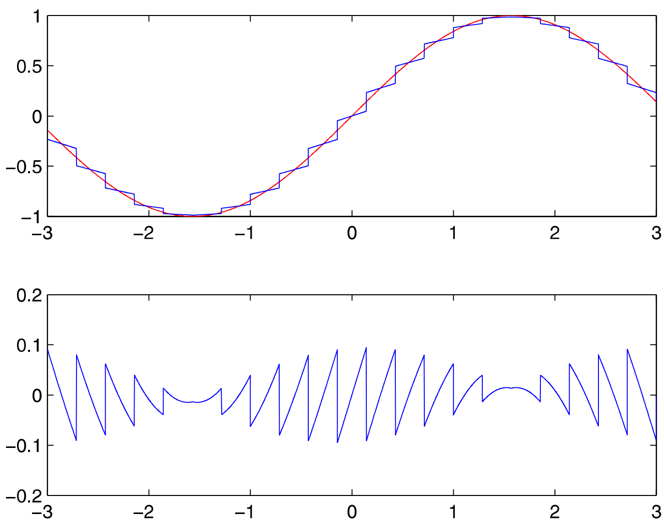

Fig. 1. The simulation and error curve

From the simulation we can see that: when $n = 22$, the two coincide basically, and the error is within 0.1.

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Reliability Analysis of Random Fuzzy Repairable Series System

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Abstract. Usually, the lifetimes and repair times of series systems are assumed to be random variables. The probability distributions of the random variables have crisp parameters. In many practical situations, the parameters are difficult to determine due to uncertainties and imprecision of data. So it is appropriate to assume the parameters to be fuzzy variables. In this paper, the lifetimes and repair times of components are assumed to have random fuzzy exponential distributions, then the limiting availability, steady state failure frequency, mean time between failures (MTBF), mean time to repair (MTTR) of the repairable series system are proposed. Finally, a numerical example is presented.

Keywords: Limiting availability, steady state failure frequency, mean time between failures, mean time to repair, series system, random fuzzy variable.

1 Introduction

The series system is one of the classical systems in reliability theory. Many large engineering systems can be viewed as a series system in time. So research on a series system becomes essential. In the past decades, many researchers have taken an interest in this field. Barlow and Proschan [1] introduced the reliability theory and obtained some results concerning the reliability of components and systems. Chao and Fu [3] studied the reliability of general series system and show that, under certain regularity conditions, the reliability of the system tends to a constant. Khalil [9] considered different shut-off rules for series systems performance and calculated limiting system availability under various shut-off rules. Based on a sample of complete periods for each subsystem, an approximate interval estimate of the steady-state availability of the series system was presented for three cases in Mi [19]. Gurov and Utkin [6] proposed a new method to compute two-sided bounds for time-dependent availability of repairable series systems by arbitrary distributed time to failure and time to repair. Chung [4] presented a reliability and availability of

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a series repairable system with multiple failures. Lam and Zhang [14] introduced a geometric process model for the analysis of a two-component series system with one repairman. All these studies use the assumption that the parameters in the distribution function of system lifetime are constants. It is not surprising that probability theory has been a dominant tool to analyze system behaviour.

Nowadays, with the advent of highly complex systems and vast uncertainty of system characteristics, people have realized that probability theory is not suitable in many situations. Therefore, many researchers pay attention to fuzzy methodology, which can be traced back to Kaufmann's work [8]. After that Cai et al. [2] discussed the posbist reliability of typical systems, such as series, parallel and k-out-of-n systems. In Utkin [21], the basic special feature of the considered system was a fuzzy representation of data about reliability to improve the efficiency of analysis and assessment. Then Utkin and Gurov [22] proved the limit theorems related to the stationary behavior of possibility distribution functions. Savoia [5] presented a method for structural reliability analysis using the possibility theory and the fuzzy number approach. Onisawa [20] described a model of subjective analysis of system reliability, in which fuzzy set was considered as a subjective reliability measure. Huang et al. [7] proposed a new method to determine the membership function of the estimates of the parameters and the reliability function of multi-parameter lifetime distributions. Liu et al. [15] considered the lifetimes of systems as fuzzy random variables, then the reliability and mean time to failure of series systems, parallel systems, series-parallel systems, parallel-series systems and cold standby systems were discussed, respectively.

In practice, the probability distribution is known except for the values of parameters. For example, the lifetime of a component is exponentially distributed variable with parameter λ, in which λ is obtained by history data. But sometimes there is a lack of sufficient data. So it is more suitable to consider λ as fuzzy variable. The purpose of this paper is to apply the fuzzy assumption of the parameters in the distributions of components' lifetimes and repair times, and then a reliability analysis is given for series system with random fuzzy exponential lifetimes and repair times.

2 Preliminaries

In this part, we recall some concepts related to evaluate the random fuzzy repairable system.

2.1 Fuzzy Variable

Let $(\Theta, \mathcal{P}(\Theta), \mathrm{Cr})$ be a credibility space, where Θ is a universe, $\mathcal{P}(\Theta)$ the power set of Θ and Cr a credibility measure defined on $\mathcal{P}(\Theta)$. A fuzzy variable ξ defined by Liu [12] is a function from the credibility space $(\Theta, \mathcal{P}(\Theta), \mathrm{Cr})$ to the set of real numbers, its membership function is derived from the credibility measure by

$$\mu(x) = (2\mathrm{Cr}\{\xi = x\}) \wedge 1, \quad x \in \Re,$$

and ξ is said to be nonnegative if and only if $\mu(x) = 0$ for all $x < 0$.

Definition 1 *(Liu [10]) Let ξ be a fuzzy variable and $\alpha \in (0, 1]$. Then*

$$\xi_\alpha^L = \inf\left\{x \mid \mu(x) \geq \alpha\right\} \qquad \text{and} \qquad \xi_\alpha^U = \sup\left\{x \mid \mu(x) \geq \alpha\right\}$$

are called the α-pessimistic value and the α-optimistic value of ξ, respectively.

Definition 2 *(Liu and Liu [13]) Let ξ be a fuzzy variable. The expected value $E[\xi]$ is defined as*

$$E[\xi] = \int_0^{+\infty} \mathrm{Cr}\{\xi \geq r\}\mathrm{d}r - \int_{-\infty}^0 \mathrm{Cr}\{\xi \leq r\}\mathrm{d}r$$

provided that at least one of the two integrals is finite. Especially, if ξ is a positive fuzzy variable, then $E[\xi] = \int_0^{+\infty} \mathrm{Cr}\{\xi \geq r\}\mathrm{d}r$.

Proposition 1 *(Liu and Liu [18]) Let ξ be a fuzzy variable with finite expected value $E[\xi]$, then we have*

$$E[\xi] = \frac{1}{2}\int_0^1 \left[\xi_\alpha^L + \xi_\alpha^U\right] \mathrm{d}\alpha,$$

where ξ_α^L and ξ_α^U are the α-pessimistic value and the α-optimistic value of ξ, respectively.

Proposition 2 *(Liu and Liu [18] and Zhao and Tang [23]) Let ξ and η be two independent fuzzy variables. Then*

(i) for any $\alpha \in (0, 1]$, $(\xi + \eta)_\alpha^L = \xi_\alpha^L + \eta_\alpha^L$;
(ii) for any $\alpha \in (0, 1]$, $(\xi + \eta)_\alpha^U = \xi_\alpha^U + \eta_\alpha^U$.
Furthermore, if ξ and η are positive, then
(iii) for any $\alpha \in (0, 1]$, $(\xi \cdot \eta)_\alpha^L = \xi_\alpha^L \cdot \eta_\alpha^L$;
(iv) for any $\alpha \in (0, 1]$, $(\xi \cdot \eta)_\alpha^U = \xi_\alpha^U \cdot \eta_\alpha^U$.

2.2 Random Fuzzy Variable

The concept of the random fuzzy variable was given by Liu [10]. Let (Ω, $\mathcal{A}$, Pr) be a probability space, $\mathcal{F}$ a collection of random variables. A random fuzzy variable is defined as a function from a credibility space $(\Theta, \mathcal{P}(\Theta), \mathrm{Cr})$ to a collection of random variables $\mathcal{F}$.

Definition 3 *(Liu and Liu [18]) A random fuzzy variable ξ is said to be exponential if for each θ, $\xi(\theta)$ is an exponentially distributed random variable whose density function is defined as*

$$f_{\xi(\theta)}(t) = \begin{cases} X(\theta)\exp(-X(\theta)t), & \text{if } t \geq 0, \\ 0, & \text{if } t < 0, \end{cases}$$

where X is a positive fuzzy variable defined on the space Θ. An exponentially distributed random fuzzy variables is denoted by $\xi \sim \mathcal{EXP}(X)$, and the fuzziness of random fuzzy variable ξ is said to be characterized by fuzzy variable X.

Proposition 3 *(Liu [11]) Let ξ be a random fuzzy variable on the credibility space $(\Theta, \mathcal{P}(\Theta), \mathrm{Cr})$. Then, for $\theta \in \Theta$, we have*
(1) $\Pr\{\xi(\theta) \in \mathcal{B}\}$ is a fuzzy variable for any Borel set $\mathcal{B}$ of $\Re$;
(2) $E[\xi(\theta)]$ is a fuzzy variable provided that $E[\xi(\theta)]$ is finite for any fixed $\theta \in \Theta$.

Definition 4 *(Liu and Liu [18]) Let ξ be a random fuzzy variable defined on the credibility space $(\Theta, \mathcal{P}(\Theta), \mathrm{Cr})$. Then the expected value $E[\xi]$ is defined by*

$$E[\xi] = \int_0^{+\infty} \mathrm{Cr}\left\{\theta \in \Theta \;\middle|\; E[\xi(\theta)] \geq r\right\} \mathrm{d}r - \int_{-\infty}^0 \mathrm{Cr}\left\{\theta \in \Theta \;\middle|\; E[\xi(\theta)] \leq r\right\} \mathrm{d}r$$

provided that at least one of the two integrals is finite. Especially, if ξ is a positive fuzzy variable, then $E[\xi] = \int_0^{+\infty} \mathrm{Cr}\left\{\theta \in \Theta \;\middle|\; E[\xi(\theta)] \geq r\right\} \mathrm{d}r$.

Definition 5 *(Liu and Liu [16]) Let ξ be a random fuzzy variable. Then the average chance, denoted by Ch, of random fuzzy event characterized by $\{\xi \leq 0\}$ is defined as*

$$\mathrm{Ch}\{\xi \leq 0\} = \int_0^1 \mathrm{Cr}\left\{\theta \in \Theta \;\middle|\; \Pr\{\xi(\theta) \leq 0\} \geq p\right\} \mathrm{d}p. \tag{1}$$

Remark 1 *If ξ degenerates to a random variable, then the average chance degenerates to $\Pr\{\xi \leq 0\}$, which is just the probability of random event. If ξ degenerates to a fuzzy variable, then the average chance degenerates to $\mathrm{Cr}\{\xi \leq 0\}$, which is just the credibility of random event.*

2.3 Random Fuzzy Renewal Process

We shall recall the random fuzzy renewal process and a useful result in Zhao and Tang [23].

Let ξ_i denote the interarrival times between the $(i-1)$th and ith events, $i = 1, 2, \ldots$, respectively. Define $S_0 = 0$ and

$$S_n = \xi_1 + \xi_2 + \cdots + \xi_n, \forall n \geq 1.$$

If the interarrival times ξ_i are random fuzzy variables defined on the credibility spaces $(\Theta_i, \mathcal{P}(\Theta)_i, \mathrm{Cr}_i)$, $i = 1, 2, \ldots$, respectively, then the process $\{S_n, n \geq 1\}$ is called a random fuzzy renewal process on the credibility space $(\Theta, \mathcal{P}(\Theta), \mathrm{Cr})$, where $(\Theta, \mathcal{P}(\Theta), \mathrm{Cr})$ is the product credibility space of $(\Theta_i, \mathcal{P}(\Theta)_i, \mathrm{Cr}_i)$, $i = 1, 2, \cdots$.

Let $N(t)$ denote the total number of events that have occurred by time t. Then we have

$$N(t) = \max_{n \geq 0} \left\{ n \mid 0 < S_n \leq t \right\}.$$

$N(t)$ is also a random fuzzy variable. We call $N(t)$ the random fuzzy renewal variable.

Let ξ be one of fuzzy variables with the α-pessimistic value $E[\xi_1(\theta)]_\alpha^{'}$ and the α-optimistic value $E[\xi_1(\theta)]_\alpha^{''}$, $\alpha \in (0, 1]$, then we have the following lemma.

Lemma 1 *(Zhao and Tang [23]) Let* $\{\xi_i, i \geq 1\}$ *be a sequence of iid nonnegative random fuzzy interarrival times defined on the product credibility space* $(\Theta, \mathcal{P}(\Theta), \mathrm{Cr})$ *and* $N(t)$ *the random fuzzy renewal variable. If* $E\left[\frac{1}{\xi}\right]$ *is finite, then we have*

$$\lim_{t \to +\infty} \frac{E[N(t)]}{t} = E\left[\frac{1}{\xi}\right].$$

2.4 Some Indexes in Evaluating the Random Fuzzy Repairable System

Consider a repairable system, up time ξ_i are random fuzzy variables defined on the credibility spaces $(\Theta_i, \mathcal{P}(\Theta_i), \mathrm{Cr}_i)$, $i = 1, 2, \ldots$ and down time $\eta_i, i = 1, 2, \ldots$ are random fuzzy variables defined on the credibility spaces $(\tau_i, \mathcal{P}(\tau_i), \mathrm{Cr}_i^{'})$, $i = 1, 2, \ldots$ Up time and down time occur alternately. Assume that the system is in up time at $t = 0$. So we define the availability $A(t)$ at time t as the average chance of random fuzzy event {system is operating at time t}, i.e.,

$$A(t) = \mathrm{Ch}\{\text{system is operating at time } t\}, \tag{2}$$

if $\lim_{t \to +\infty} A(t)$ exists, we call it limiting availability, denoted by A.

Let $N(t)$ denote the number of failure that have occurred in $(0, t]$. Obviously, $N(t)$ is a random fuzzy variable, its expected value $E[N(t)]$ is denoted by $M(t)$. So the steady state failure frequency, denoted by M, is defined by

$$M = \lim_{t \to +\infty} \frac{M(t)}{t}, \tag{3}$$

if $\lim_{t \to +\infty} \frac{M(t)}{t}$ exists.

We also can define the mean time between failure

$$\mathrm{MTBF} = \lim_{n\to+\infty} \frac{1}{n}\sum_{i=1}^{n} E[\xi_i] \tag{4}$$

and the mean time to repair

$$\mathrm{MTTR} = \lim_{n\to+\infty} \frac{1}{n}\sum_{i=1}^{n} E[\eta_i]. \tag{5}$$

Then we refer to a definition on the stochastic ordering which is usually employed in comparison of the lifetimes of systems in classical reliability theory.

Definition 6 *A collection of random variables $\mathcal{F}$ is said to be a totally ordered set with stochastic ordering if and only if, for any given $\zeta_1, \zeta_2 \in \mathcal{F}$ and $r \in \Re$, either*

$$\Pr\{\zeta_1 \le r\} \le \Pr\{\zeta_2 \le r\} \quad \text{(denoted by } \zeta_2 \le_d \zeta_1\text{)}$$

or

$$\Pr\{\zeta_1 \le r\} \ge \Pr\{\zeta_2 \le r\} \quad \text{(denoted by } \zeta_1 \le_d \zeta_2\text{)}.$$

Remark 2 *It follows from Definition 6 that, for any given $\zeta_1, \zeta_2 \in \mathcal{F}$, we have*

$$E[\zeta_1] \le E[\zeta_2] \Leftrightarrow \zeta_1 \le_d \zeta_2.$$

3 Relability Analysis of Series System with Random Fuzzy Exponential Lifetimes and Repair Times

Consider a series system composed of n components. Let X_i be the lifetime of ith component, which has random fuzzy exponential distribution with parameter λ_i defined on the credibility space $(\Theta_i, \mathcal{P}(\Theta_i), \mathrm{Cr}_i)$, $i = 1, 2, \ldots, n$, respectively. Whenever one of these components fails, the failed component is undergoing repair, all other components remain in "suspended animation". We assume Y_i be the repair time of each component i, which has random fuzzy exponential distribution with parameter μ_i defined on the credibility space $(\tau_i, \mathcal{P}(\tau_i), \mathrm{Cr}_i^{'})$, $i = 1, 2, \ldots, n$, respectively. When repair of the failed component is completed, the remaining components resume operation.

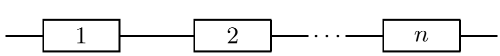

Fig. 1. A series system

We also assume system is operating at $t = 0$. If the jth component is failed, its corresponding repair time is Y_j. When the repair is completed, the whole system goes into work. At this instant, the system is "as good as new", thus the moment system goes into work are regenerative points. So the up times $\xi_i, i = 1, 2 \ldots$ and down times $\eta_i, i = 1, 2 \ldots$ of the series system are iid random fuzzy variables, respectively.

We define a finite possibility product space $(\Theta, \mathcal{P}(\Theta), \mathrm{Cr})$, for any $A \in \mathcal{P}(\Theta)$,

$$\Theta = \prod_{i=1}^{n} (\Theta_i, \tau_i)$$

and

$$\mathrm{Cr}\{A\} = \sup_{((\theta_1,\vartheta_1),(\theta_2,\vartheta_2),\cdots,(\theta_n,\vartheta_n))\in A} \mathrm{Cr}_1\{\theta_1\} \wedge \mathrm{Cr}_1^{'}\{\vartheta_1\} \wedge \cdots \wedge \mathrm{Cr}_n\{\theta_n\} \wedge \mathrm{Cr}_n^{'}\{\vartheta_n\}.$$

By Definition 5 and Equation (2), we have

$$\begin{aligned} A(t) &= \mathrm{Ch}\{\text{system is operating at time } t\} \\ &= \int_0^1 \mathrm{Cr}\left\{\theta \in \Theta \;\middle|\; \Pr\{\text{system is operating at time } t\} \geq p\right\} \mathrm{d}p. \end{aligned} \tag{6}$$

Let

$$P(t) = \Pr\{\text{system is operating at time } t\}. \tag{7}$$

So $P(t)$ is a fuzzy variable.

Theorem 1 *Let X_i be the random fuzzy lifetime of ith component, Y_i be the random fuzzy repair time of each component i, $X_i \sim \mathcal{EXP}(\lambda_i)$ and $Y_i \sim \mathcal{EXP}(\mu_i)$, $i = 1, 2, \ldots, n$. Then we have*

$$\lim_{t\to+\infty} P_\alpha^L(t) = \frac{1}{1 + \sum_{i=1}^{n} \frac{\lambda_{i,\alpha}^U}{\mu_{i,\alpha}^L}}$$

and

$$\lim_{t\to+\infty} P_\alpha^U(t) = \frac{1}{1 + \sum_{i=1}^{n} \frac{\lambda_{i,\alpha}^L}{\mu_{i,\alpha}^U}}.$$

Proof. Let $A_i = \left\{\theta_i \in \Theta_i \;\middle|\; \mu\{\theta_i\} \geq \alpha\right\}$ and $B_i = \left\{\vartheta_i \in \tau_i \;\middle|\; \mu\{\vartheta_i\} \geq \alpha\right\}$, $i = 1, 2, \ldots, n$. Since the α-pessimistic values and α-optimistic values of fuzzy variable $E[X_i(\theta_i)]$, $E[Y_i(\vartheta_i)]$, $\theta_i \in \Theta_i$, $\vartheta_i \in \tau_i$, $i = 1, 2, \ldots, n$ are continuous almost everywhere for any $\alpha \in (0, 1]$, then there at least exist point $\theta_i^{'}, \theta_i^{''} \in A_i$ and $\vartheta_i^{'}, \vartheta_i^{''} \in B_i$, respectively, such that

$$E\left[X_i\left(\theta_i^{'}\right)\right] = E\left[X_i(\theta_i)\right]_\alpha^L,$$

$$E\left[X_i\left(\theta_i^{''}\right)\right] = E\left[X_i(\theta_i)\right]_\alpha^U,$$
$$E\left[Y_i\left(\vartheta_i^{'}\right)\right] = E\left[Y_i\left(\theta_i\right)\right]_\alpha^L,$$
$$E\left[Y_i\left(\vartheta_i^{''}\right)\right] = E\left[Y_i(\theta_i)\right]_\alpha^U.$$

Then for $\forall\theta_i \in A_i$ and $\forall\vartheta_i \in B_i$, we have

$$E\left[X_i\left(\theta_i^{'}\right)\right] \le E[X_i(\theta_i)] \le E\left[X_i\left(\theta_i^{''}\right)\right]$$

and

$$E\left[Y_i\left(\vartheta_i^{'}\right)\right] \le E[Y_i(\vartheta_i)] \le E\left[Y_i\left(\vartheta_i^{''}\right)\right].$$

So we can arrive at

$$\frac{1}{\lambda_i\left(\theta_i^{'}\right)} \le \frac{1}{\lambda_i(\theta_i)} \le \frac{1}{\lambda_i\left(\theta_i^{''}\right)}$$

and

$$\frac{1}{\mu_i\left(\vartheta_i^{'}\right)} \le \frac{1}{\mu_i(\vartheta_i)} \le \frac{1}{\mu_i\left(\vartheta_i^{''}\right)},$$

i.e.,

$$\lambda_i\left(\theta_i^{''}\right) \le \lambda_i(\theta_i) \le \lambda_i\left(\theta_i^{'}\right) \tag{8}$$

and

$$\mu_i\left(\vartheta_i^{''}\right) \le \mu_i(\vartheta_i) \le \mu_i\left(\vartheta_i^{'}\right). \tag{9}$$

We can construct three repairable series systems:

(1) $X_i(\theta_i^{'})$ be the lifetime of component i in series system A, and its corresponding repair time is $Y_i(\vartheta_i^{''})$, $i = 1, 2, \ldots, n$, respectively
(2) $X_i(\theta_i^{''})$ be the lifetime of component i in series system B, and its corresponding repair time is $Y_i(\vartheta_i^{'})$, $i = 1, 2, \ldots, n$, respectively
(3) $X_i(\theta_i)$ be the lifetime of component i in series system A, and its corresponding repair time is $Y_i(\vartheta_i)$, $i = 1, 2, \ldots, n$, respectively

It is easy to see that the system A and B are two standard stochastic series system composed by n components. For any fixed θ_i and ϑ_i, system C is also a stochastic series system. From the result in classical reliability theory, we can arrive at

$$\lim_{t\to+\infty} A_1(t) = \frac{1}{1 + \sum_{i=1}^{n} \frac{\lambda_i(\theta_i^{'})}{\mu_i(\vartheta_i^{''})}},$$

$$\lim_{t\to+\infty} A_2(t) = \frac{1}{1 + \sum_{i=1}^{n} \frac{\lambda_i(\theta_i^{''})}{\mu_i(\vartheta_i^{'})}}$$

and

$$\lim_{t\to+\infty} A_3(t) = \frac{1}{1+\sum_{i=1}^{n}\frac{\lambda_i(\theta_i)}{\mu_i(\vartheta_i)}}.$$

From (8) and (9), we have

$$\frac{1}{1+\sum_{i=1}^{n}\frac{\lambda_i(\theta_i^{'})}{\mu_i(\vartheta_i^{''})}} \leq \frac{1}{1+\sum_{i=1}^{n}\frac{\lambda_i(\theta_i)}{\mu_i(\vartheta_i)}} \leq \frac{1}{1+\sum_{i=1}^{n}\frac{\lambda_i(\theta_i^{''})}{\mu_i(\vartheta_i^{'})}}.$$

Since θ_i and ϑ_i are arbitrary points in A_i and B_i, $i = 1, 2, \ldots, n$, we have

$$\left(\lim_{t\to+\infty} P(t)\right)_\alpha^L = \frac{1}{1+\sum_{i=1}^{n}\frac{\lambda_i(\theta_i^{'})}{\mu_i(\vartheta_i^{''})}} \tag{10}$$

and

$$\left(\lim_{t\to+\infty} P(t)\right)_\alpha^U = \frac{1}{1+\sum_{i=1}^{n}\frac{\lambda_i(\theta_i^{''})}{\mu_i(\vartheta_i^{'})}}. \tag{11}$$

Obviously, $P(t)$ are continuous almost everywhere, then

$$\left(\lim_{t\to+\infty} P(t)\right)_\alpha^L = \lim_{t\to\infty} P_\alpha^L(t) \tag{12}$$

and

$$\left(\lim_{t\to+\infty} P(t)\right)_\alpha^U = \lim_{t\to\infty} P_\alpha^U(t). \tag{13}$$

From (10), (11), (12) and (13), we have

$$\lim_{t\to+\infty} P_\alpha^L(t) = \frac{1}{1+\sum_{i=1}^{n}\frac{\lambda_i(\theta_i^{'})}{\mu_i(\vartheta_i^{''})}} \tag{14}$$

and

$$\lim_{t\to+\infty} P_\alpha^U(t) = \frac{1}{1+\sum_{i=1}^{n}\frac{\lambda_i(\theta_i^{''})}{\mu_i(\vartheta_i^{'})}}. \tag{15}$$

It follows from (8) and (9) that

$$\lambda_{i,\alpha}^L = \lambda_i(\theta_i^{''}), \tag{16}$$

$$\lambda_{i,\alpha}^{U} = \lambda_i(\theta_i^{'}), \tag{17}$$

$$\mu_{i,\alpha}^{L} = \mu_i(\vartheta_i^{''}), \tag{18}$$

$$\mu_{i,\alpha}^{U} = \mu_i(\vartheta_i^{'}). \tag{19}$$

From (14), (15), (16), (17), (18) and (19), we can arrive at

$$\lim_{t\to+\infty} P_\alpha^L(t) = \frac{1}{1+\sum_{i=1}^{n}\frac{\lambda_{i,\alpha}^{U}}{\mu_{i,\alpha}^{L}}}$$

and

$$\lim_{t\to+\infty} P_\alpha^U(t) = \frac{1}{1+\sum_{i=1}^{n}\frac{\lambda_{i,\alpha}^{L}}{\mu_{i,\alpha}^{U}}}.$$

The prove is completed.

Theorem 2 *Let X_i be the random fuzzy lifetime of ith component, Y_i be the random fuzzy repair time of each component i, $X_i \sim \mathcal{EXP}(\lambda_i)$ and $Y_i \sim \mathcal{EXP}(\mu_i)$, $i = 1, 2, \ldots, n$. Then the limiting availability of the repairable series system is*

$$A = E\left[\frac{1}{1+\sum_{i=1}^{n}\frac{\lambda_i}{\mu_i}}\right].$$

Proof. By equation (6), (7) and Proposition 1, we have

$$\begin{aligned} A(t) &= \mathrm{Ch}\{\text{system is operating at time } t\} \\ &= \int_0^1 \mathrm{Cr}\left\{\theta \in \Theta \;\middle|\; \Pr\{\text{system is operating at time } t\} \geq p\right\} \mathrm{d}p. \\ &= \int_0^1 (P_\alpha^L(t) + P_\alpha^U(t))\mathrm{d}\alpha. \end{aligned} \tag{20}$$

It follows from Theorem 1 that

$$\lim_{t\to+\infty} P_\alpha^L(t) = \frac{1}{1+\sum_{i=1}^{n}\frac{\lambda_{i,\alpha}^{U}}{\mu_{i,\alpha}^{L}}} \tag{21}$$

and

$$\lim_{t\to+\infty} P_\alpha^U(t) = \frac{1}{1+\sum_{i=1}^{n}\frac{\lambda_{i,\alpha}^{L}}{\mu_{i,\alpha}^{U}}}. \tag{22}$$

So for $\forall t$, we have

$$0 \le P_\alpha^L(t), P_\alpha^U(t) \le 1,$$

then

$$0 \le P_\alpha^L(t) + P_\alpha^U(t) \le 2.$$

Obviously, 2 is an integrable function of $\alpha \in (0,1]$. By dominated convergence theorem, we can arrive at

$$\begin{aligned}
A = \lim_{t\to+\infty} A(t) &= \lim_{t\to+\infty} \mathrm{Ch}\{\text{system is work at time } t\} \\
&= \frac{1}{2}\int_0^1 \lim_{t\to+\infty} \left(P_\alpha^L(t) + P_\alpha^U(t)\right) \mathrm{d}\alpha \\
&= \frac{1}{2}\int_0^1 \left(\frac{1}{1+\sum_{i=1}^{n} \frac{\lambda_{i,\alpha}^U}{\mu_{i,\alpha}^L}} + \frac{1}{1+\sum_{i=1}^{n} \frac{\lambda_{i,\alpha}^L}{\mu_{i,\alpha}^U}} \right) \mathrm{d}\alpha \\
&= \frac{1}{2}\int_0^1 \left(\left[\frac{1}{1+\sum_{i=1}^{n} \frac{\lambda_i}{\mu_i}}\right]_\alpha^L + \left[\frac{1}{1+\sum_{i=1}^{n} \frac{\lambda_i}{\mu_i}}\right]_\alpha^U \right) \mathrm{d}\alpha \\
&= E\left[\frac{1}{1+\sum_{i=1}^{n} \frac{\lambda_i}{\mu_i}}\right].
\end{aligned}$$

The proof is completed.

Remark 3 *If X_i and Y_i, $i = 1, 2, \ldots, n$ degenerate to random variables, then the result degenerates to the form*

$$A = \frac{1}{1+\sum_{i=1}^{n} \frac{\lambda_i}{\mu_i}}.$$

Remark 4 *Under the conditions of Theorem 2, it follows that*

$$\begin{aligned}
\lim_{t\to+\infty} \mathrm{Ch}\{\text{system is breakdown at time } t\} &= 1 - \lim_{t\to+\infty} \mathrm{Ch}\{\text{system is operating at time } t\} \\
&= 1 - E\left[\frac{1}{1+\sum_{i=1}^{n} \frac{\lambda_i}{\mu_i}}\right].
\end{aligned}$$

Theorem 3 *Let X_i be the random fuzzy lifetime of ith component, Y_i be the random fuzzy repair time of each component i, $X_i \sim \mathcal{EXP}(\lambda_i)$ and $Y_i \sim \mathcal{EXP}(\mu_i)$, $i = 1, 2, \ldots, n$. Then the steady state failure frequency of the repairable series system is*

$$M = E\left[\frac{\sum_{i=1}^{n} \lambda_i}{1 + \sum_{i=1}^{n} \frac{\lambda_i}{\mu_i}}\right].$$

Proof. Since the up times $\xi_i, i = 1, 2\ldots$ and down times $\eta_i, i = 1, 2\ldots$ of the series system are iid random fuzzy variables, respectively, then $\xi_i + \eta_i, i = 1, 2\ldots$ are iid random fuzzy variables. By equation (3) and Lemma 1, we have

$$M = \lim_{t \to +\infty} \frac{M(t)}{t} = E\left[\frac{1}{\xi_1 + \eta_1}\right]. \tag{23}$$

For $\forall \theta \in \Theta$, $\frac{1}{\xi_1 + \eta_1}(\theta)$ is a random variable and from the result in classical reliability theory, we can arrive at

$$E\left[\frac{1}{\xi_1 + \eta_1}(\theta)\right] = \frac{\sum_{i=1}^{n} \lambda_i(\theta_i)}{1 + \sum_{i=1}^{n} \frac{\lambda_i(\theta_i)}{\mu_i(\vartheta_i)}},$$

where $\theta = (\theta_1, \ldots, \theta_n, \vartheta_1, \ldots, \vartheta_n)$. So

$$M = E\left[\frac{1}{\xi_1 + \eta_1}\right] = E\left[\frac{\sum_{i=1}^{n} \lambda_i}{1 + \sum_{i=1}^{n} \frac{\lambda_i}{\mu_i}}\right].$$

The proof is completed.

Remark 5 *If X_i and Y_i, $i = 1, 2, \ldots, n$ degenerate to random variables, then the result in Theorem 3 degenerates to the form*

$$M = \frac{\sum_{i=1}^{n} \lambda_i}{1 + \sum_{i=1}^{n} \frac{\lambda_i}{\mu_i}}.$$

Theorem 4 *Let X_i be the random fuzzy lifetime of ith component, Y_i be the random fuzzy repair time of each component i, $X_i \sim \mathcal{EXP}(\lambda_i)$ and $Y_i \sim \mathcal{EXP}(\mu_i)$, $i = 1, 2, \ldots, n$. Then the* MTBF *of the repairable series system is*

$$\text{MTBF} = E\left[\frac{1}{\sum_{i=1}^{n}\lambda_i}\right].$$

Proof. Since the up times $\xi_i, i = 1, 2 \ldots$ of the series system are iid random fuzzy variables, by equation (4), we have

$$\text{MTBF} = \lim_{n\to+\infty}\frac{1}{n}\sum_{i=1}^{n}E[\xi_i] = E\left[\xi_1\right].$$

For $\forall\theta \in \Theta$, $\xi_1(\theta)$ is a random variable and from the result in classical reliability theory, we can arrive at

$$E\left[\xi_1(\theta)\right] = \frac{1}{\sum_{i=1}^{n}\lambda_i(\theta_i)},$$

where $\theta = (\theta_1, \ldots, \theta_n)$. So

$$\text{MTBF} = E\left[\xi_1\right] = E\left[\frac{1}{\sum_{i=1}^{n}\lambda_i}\right].$$

The proof is completed.

Remark 6 *If X_i and Y_i, $i = 1, 2, \ldots, n$ degenerate to random variables, then the result in Theorem 4 degenerates to the form*

$$\text{MTBF} = \frac{1}{\sum_{i=1}^{n}\lambda_i}.$$

Theorem 5 *Let X_i be the random fuzzy lifetime of ith component, Y_i be the random fuzzy repair time of each component i, $X_i \sim \mathcal{EXP}(\lambda_i)$ and $Y_i \sim \mathcal{EXP}(\mu_i)$, $i = 1, 2, \ldots, n$. Then the* MTTR *of the repairable series system is*

$$\text{MTTR} = E\left[\frac{\sum_{i=1}^{n}\frac{\lambda_i}{\mu_i}}{\sum_{i=1}^{n}\lambda_i}\right]. \qquad (24)$$

Proof. Since the down times $\eta_i, i(=1,2\ldots)$ of the series system are iid random fuzzy variables, by Equation (5), we have

$$\text{MTTR} = \lim_{n\to+\infty} \frac{1}{n}\sum_{i=1}^{n} E[\eta_i] = E\left[\eta_1\right].$$

For $\forall\theta \in \Theta$, $\eta_1(\theta)$ is a random variable and from the result in classical reliability theory, we can arrive at

$$E\left[\eta_1(\theta)\right] = \frac{\displaystyle\sum_{i=1}^{n} \frac{\lambda_i(\theta_i)}{\mu_i(\vartheta_i)}}{\displaystyle\sum_{i=1}^{n} \lambda_i(\theta_i)},$$

where $\theta = (\theta_1, \ldots, \theta_n, \vartheta_1, \ldots, \vartheta_n)$. So

$$\text{MTTR} = E\left[\eta_1\right] = E\left[\frac{\displaystyle\sum_{i=1}^{n} \frac{\lambda_i}{\mu_i}}{\displaystyle\sum_{i=1}^{n} \lambda_i}\right].$$

The proof is completed.

Remark 7 *If X_i and Y_i, $i = 1, 2, \ldots, n$ degenerate to random variables, then the result in Theorem 5 degenerates to the form*

$$\text{MTTR} = \frac{\displaystyle\sum_{i=1}^{n} \frac{\lambda_i}{\mu_i}}{\displaystyle\sum_{i=1}^{n} \lambda_i}.$$

Example 1 *Suppose the series system composed by two components. If $X_i \sim \mathcal{EXP}(\lambda_i)$ and $Y_i \sim \mathcal{EXP}(\mu_i)$, $i = 1, 2$, where $\lambda_1 = (2, 3, 4)$, $\lambda_2 = (2, 3, 4)$, $\mu_1 = (4, 5, 6)$ and $\mu_2 = (4, 5, 6)$. Then we can arrive at*

$$\lambda^L_{1,\alpha} = \lambda^L_{2,\alpha} = 2 + \alpha, \quad \lambda^U_{1,\alpha} = \lambda^U_{2,\alpha} = 4 - \alpha,$$
$$\mu^L_{1,\alpha} = \mu^L_{2,\alpha} = 4 + \alpha, \quad \mu^U_{1,\alpha} = \mu^U_{2,\alpha} = 6 - \alpha.$$

It follows from Theorem 1 that

$$\lim_{t\to+\infty} P^L_\alpha(t) = \frac{1}{1 + \displaystyle\sum_{i=1}^{2} \frac{\lambda^U_{i,\alpha}}{\mu^L_{i,\alpha}}} = \frac{1}{1 + \dfrac{2(4-\alpha)}{4+\alpha}} = \frac{4+\alpha}{12-\alpha}$$

and

$$\lim_{t\to+\infty} P_\alpha^U(t) = \frac{1}{1+\sum_{i=1}^{2}\frac{\lambda_{i,\alpha}^L}{\mu_{i,\alpha}^U}} = \frac{1}{1+\frac{2(2+\alpha)}{6-\alpha}} = \frac{6-\alpha}{10+\alpha}.$$

By Theorem 2 and Remark 4, we have

$$A = \frac{1}{2}\int_0^1 \left(\frac{4+\alpha}{12-\alpha} + \frac{6-\alpha}{10+\alpha}\right) \mathrm{d}\alpha = -1 + 8ln\frac{6}{5} \approx 0.4586$$

and

$$\lim_{t\to+\infty} \mathrm{Ch}\{\text{system is breakdown at time } t\} \approx 1 - 0.4586 = 0.5414.$$

Then by Theorem 3, Theorem 4 and Theorem 5, we have

$$M = E\left[\frac{2\lambda_1}{1+\frac{2\lambda_1}{\mu_1}}\right] = \frac{1}{2}\int_0^1 \left(\frac{2}{\frac{1}{2+\alpha}+\frac{2}{4+\alpha}} + \frac{2}{\frac{1}{4-\alpha}+\frac{2}{6-\alpha}}\right) \mathrm{d}\alpha \approx 2.7231,$$

$$\mathrm{MTBF} = E\left[\frac{1}{2\lambda_1}\right] = \frac{1}{4}\int_0^1 \left(\frac{1}{2+\alpha} + \frac{1}{4-\alpha}\right) \mathrm{d}\alpha = \frac{1}{4}ln2 = 0.1733$$

and

$$\mathrm{MTTR} = E\left[\frac{1}{\mu_1}\right] = \frac{1}{2}\int_0^1 \left(\frac{1}{4+\alpha} + \frac{1}{6-\alpha}\right) \mathrm{d}\alpha = \frac{1}{2}ln\frac{3}{2} = 0.2027.$$

4 Conclusion

As the systems become more and more complex, the probability theory is not suitable in many situations and fuzzy methodology show its advantages in certain circumstance. In this paper, the lifetimes and repair times of components in the series system are considered to have random fuzzy exponential distributions. Then the limiting availability, Steady state failure frequency, Mean time between failures , Mean time to repair of the repairable series system are proposed. Further researches can focus on other repairabe systems.

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Research on the Effectiveness Evaluation of CEEUSRO Cooperation Based on Fuzzy Integral*

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Abstract. The performance evaluation system of ceeusro cooperation is built based on the perspective of process. The evaluation method based on fuzzy integral is adopted due to the correlation between the indicator data, and is verified with an example.

Keywords: Ceeusro, performance, evaluation, fuzzy integral.

1 Introduction

With the advent of knowledge economy, the technology innovation achieved by ceeusro cooperation has become the consensus of major developed countries. In China, the ceeusro has been improved to the national strategic level, becoming an important part of national innovation system. The ceeusro is an inter-organizational cooperation, its advantage is to optimize and integrate the innovation resources of the whole society, and its disadvantage are the high complexity and difficult to coordinate. In view of this, much attention has been paid to how to coordinate and control the ceeusro, and the first problem needed to be resolved is how to evaluate the performance of ceeusro cooperation scientifically.

2 The Research Summary of CEEUSRO Cooperation at Home and Abroad

Foreign scholars such as Bonaccorsi and Piccaluga (1994) first studied the evaluation of ceeusro cooperation, and they believed that the objective measure indexes of performance include the number of new products, researchers,

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publications and patents and so on, combined with subjective measure. Tomas Hellstrom et al (1999) believed that the performance indexes evaluating ceeusro cooperation should include fertility, reach, financial success, education, publications and patents. Antoro (2000) measured the performance of ceeusro cooperation utilizing the number of papers, patents, new products and new process. Zahra and George (2002) evaluated the performance of ceeusro cooperation through four measure indexes including the number of patents, the number of new products introduced into market and in R&D and net profit rate.

In recent years, Chinese scholars concentrate on this gradually. XIE Zhi-yu (2004) evaluated the performance of ceeusro cooperation from four dimensions, namely technology innovation performance, financial performance, technology transfer and the satisfaction of technology demand side. GUO Bin (2007) evaluated the performance of ceeusro cooperation with the final product, intermediate products and indirect output. JIN Fu-rong (2009) analyzed the performance from six areas including available infrastructure, human resources, fund input, personnel training, academic activities, science and technology output and rewards and economic performance. DENG Ying-xiang (2009) constructed the scale to measure the performance from three dimensions, namely technology, project management and society. WANG Xiu-li (2009) used DEA model to analyse the performance from two areas including input and output, with four indexes respectively used.

We can see that all the above references evaluate the performance of ceeusro cooperation from the perspective of output maximization. It is believed here that only the cooperation is improved that the output can be maximized, which is consistent with the views that emphasize the process in performance management. Therefore, the evaluation index system of ceeusro cooperation is constructed from the process of cooperation in this paper.

3 The Evaluation Index System of CEEUSRO Cooperation Based on the Process

The selection of index is the prerequisite of quantization the performance of ceeusro cooperation, and it should be representative, independent and available. Here the index is divided into two categories, one is hard index, which is measurable, obtained by statistical data and investigation, and the other is soft index, difficult to be quantified or judged, obtained by expert judgment.

In order to evaluate the performance of ceeusro cooperation from the process point of view, the resource utilization, technical cooperation or construction, mutual exchange and recognition degree of all parties are used, shown in Table 1.

Table 1. The evaluation index system of ceeusro cooperation based on the process

Criteria	Indexes
Resource utilization	the development and utilization of technology
	equipment utilization
	transformation rate of achievements
Undertaken projects	the number of municipal research projects
	the number of provincial research projects
	the number of state research projects
	the number of "transverse" research projects
Technical cooperation or construction	the number of technology research centers built cooperatively
	the number of technology transfer centers built cooperatively
	technical contract signed externally
	amount of contract signed externally
Mutual exchange	information exchange between enterprises
	recognition degree of enterprise leaders
	recognition degree of leaders in university or scientific research institution

4 The Measurement and Method of Evaluation of CEEUSRO Cooperation Based on Fuzzy Integral

A. The measurement of index

Because of the diversity and complexity of ceeusro cooperation, some methods such as questionnaire are needed in data collection and calculation, and thus the performance of ceeusro cooperation is evaluated.

The indexes measured the performance of ceeusro cooperation include quantitative and qualitative indexes, needed to be handled separately.

- The dimensionless treatment of quantitative indexes

The dimensions of index value of original data are different, made the magnitude different significantly, which makes the indexes not be compared with each other, so the data must be pre-treated.

The standard transformation is used to make the quantitative indexes be dimensionless.

Suppose there are p objects evaluated in layer m, and each object evaluated has q quantitative indexes, the evaluation value of index l of object i is x_{il}. The dimensionless treatment based on standard transformation is as follows.

$$x'_{il} = \frac{x_{il} - \overline{x}_l}{S_l}\ ;\ S_l = \sqrt{\frac{1}{p-1}\sum_{i=1}^{p}(x_{il} - \overline{x}_l)^2} \tag{1}$$

Where $\overline{x}_l$ denotes the mean of index l, and S_l denotes its standard deviation.

- The treatment of qualitative indexes

Expert scoring method is used to treat the qualitative indexes. The indexes' description in scoring is fuzzy, so the linguistic variables expressed as triangular fuzzy numbers are used to describe the subjective evaluation value.

Let $\tilde{f}_1 = \{\tilde{f}_l(X_i^k) \mid k = 1, \cdots, n; i = 1, \cdots, s_k, l = 1, \cdots, m\}$, $\tilde{f}_l(X_i^k)$ is triangular fuzzy number, expressed as (a_i^k, b_i^k, c_i^k), $a_i^k \in [0,1], b_i^k \in [0,1], c_i^k \in [0,1]$; $f_l(X_i^k)$ is the semantic value given by expert l to index X_i^k under layer X_k; n denotes the number of evaluation criterion, s_k denotes the number of qualitative index of X_k under criterion, m denotes the total number of experts.

According to (2), the fuzzy values of qualitative indexes are calculated integrated all experts' opinion.

$$\tilde{f} = \{\tilde{f}(X_i^k) \mid k = 1, \cdots, n; i = 1, \cdots, s_k\}$$
$$\tilde{f}(X_i^k) = \frac{1}{m} \otimes \left\{ \tilde{f}_1(X_i^k) \oplus \tilde{f}_1(X_i^k) \oplus \cdots \oplus \tilde{f}_m(X_i^k) \right\} \tag{2}$$

Where $\tilde{f}(X_i^k)$ represents the fuzzy value of qualitative index i under criterion X_k after integrated m experts' opinion, $\oplus$ and $\otimes$ are fuzzy operators.

There are many ways to change fuzzy numbers into clear ones, and here we adopt the methods provided in [10], that is, calculate the characteristic number $\|\tilde{A}\|$ of fuzzy number $\tilde{A}$ to the defuzzification of fuzzy numbers, namely, change fuzzy value $\tilde{f}(X_i^k)$ into clear value $f(X_i^k)$.

$$f(X_i^k) = \frac{1}{6}(4b_i^k + a_i^k + c_i^k) \tag{3}$$

B. Determine the value of fuzzy measure of subset of indexes

Classical comprehensive evaluation is a weighted average method in essence; however, the weighted average method is on the basis of the assumption that all indexes are mutually independent, that is, assume the effect of each factor is additive. But, many indexes (e.g., indexes given in this paper) are not necessarily independent, that is, not be additive, to this end, a non-additive integral (fuzzy integral) method is introduced to evaluate the indexes.

- Determine the fuzzy density of index X_i^k (weights in the common sense)

When determine the fuzzy density of index X_i^k, every expert is asked to give a fuzzy density $\tilde{g}_l(X_i^k)$ for index X_i^k. Triangular function $\tilde{g}_l(X_i^k) = (\alpha_i^k, \beta_i^k, \gamma_i^k)$ is used in this paper, where $\alpha_i^k \in [0,1], \beta_i^k \in [0,1], \gamma_i^k \in [0,1]$. According to (4) combined with each expert's opinion, we get the final fuzzy density $\tilde{g}(X_i^k)$ of index X_i^k.

$$\tilde{g}(X_i^k) = \frac{1}{m} \otimes \left\{ \tilde{g}_1(X_i^k) \oplus \tilde{g}_1(X_i^k) \oplus \cdots \oplus \tilde{g}_m(X_i^k) \right\} \tag{4}$$

- Defuzzy $\tilde{g}(X_i^k)$

Similar to (3) referenced [10], transform $\tilde{g}(X_i^k)$ into $g(X_i^k)$.

- Determine the fuzzy measure of the subset of indexes

Firstly, calculate the value of λ according to (5):

$$\lambda + 1 = \prod_{i=1}^{s_k} (1 + \lambda g(X_i^k)) \tag{5}$$

It is easy to see that (5) is always be satisfied when λ=0, so as much as possible to solve the non-zero solution of λ, unless only one solution λ=0.

Then, calculate the fuzzy measure g_λ of each subset of indexes combined $g(X_i^k)$ with λ.

$$g_\lambda(\{y_1, y_2, \cdots, y_t\}) = \frac{1}{\lambda}(\prod_{i=1}^{t}(1 + \lambda g(y_i)) - 1) \tag{6}$$

So far, all the corresponding values g_λ of collection $\{X_1^k, X_2^k, \cdots, X_{sk}^k\}$ have been determined.

C. Comprehensive evaluation based on fuzzy integral

- Comprehensive evaluation of criterion Xk

Reorder each value $f(X_i^k)$ of index under criterion X_k by size, where $i = 1, 2, \cdots s_k$.

$$f(X_{i(1)}^k) \geq f(X_{i(2)}^k) \geq \ldots \geq f(X_{i(j)}^k) \geq \ldots \geq f(X_{i(s_k)}^k)$$

Calculate the evaluation value $f(X_k)$ of criterion X_k with fuzzy integral formula (7) used.

$$f(X_k) = \vee_{1 \leq l \leq s_k} \{f_{i(l)}^k \wedge g_\lambda(X_{i(1)}^k, X_{i(2)}^k, \cdots, X_{i(l)}^k)\} \tag{7}$$

- Comprehensive evaluation of the overall objects

Firstly, determine $g(X_k)$ of each criterion, and further obtain the value g_λ of all subsets of the criterion collection $\{X_1, X_2, \cdots, X_k, \cdots, X_n\}$.

Then, reorder $f(X_k)$ by size.

At last, calculate the overall value $f(X)$ of objects evaluated based on fuzzy integral.

5 Case Study

Take team *M* for example to analyse the performance of ceeusro cooperation based on fuzzy integral, and the data used derive from the research data of social science planning project in Liaoning province under grant L08BJY016. The quantitative data come from the survey, and qualitative data come from 5 experts' scores referenced Table 2, integrated and defuzzied. The recognition degree (fuzzy density) are obtained referenced weights scale shown in Table 3 and be integrated and defuzzied.

Table 2. Table of fuzzy linguistic variables

Manifestation degree	Fuzzy numbers
Extremely poor	(0, 0, 0.25)
Poor	(0, 0.25, 0.5)
General	(0.25, 0.5, 0.75)
Good	(0.5 0.75 0.75)
Extremely good	(0.75, 1.0, 1.0)

Table 3. Table of fuzzy weights scale

Important degree	Fuzzy numbers
Extremely unimportant	(0, 0, 0.25)
Unimportant	(0, 0.25, 0.5)
General	(0.25 0.5, 0.75)
Important	(0.5, 0.75, 0.75)
Extremely important	(0.75, 1.0, 1.0)

The process performance of ceeusro team *M* is shown in Table 4.

Firstly, calculate the comprehensive evaluation of criterion A_1 of team *M*.

Substitute g_{11}, g_{12}, g_{13} into (5), we getλ_1=-0.98; reorder f_{11}, f_{12}, f_{13} and the result is $f_{12}>f_{11}>f_{13}$.

According to (6), we get $g_\lambda(x_{12})$=0.68,

$g_\lambda(x_{12},x_{11})$=0.68+0.76-0.98×0.68×0.76=0.93,$g_\lambda(x_{12},x_{11},x_{13})$=0.93+0.83-0.98×0.93×0.83=1, and then

$$f_1=(0.65 \wedge 0.68) \vee (0.54 \wedge 0.93) \vee (0.36 \wedge 1)=0.65.$$

Then, calculate f_2=0.74, f_3=0.71, f_4=0.7 in the same way.

At last, evaluate the overall objects.

Substitute g_1, g_2, g_3, g_4 into (5) and we get λ=-0.98; reorder f_1, f_2, f_3, f_4 and the result is $f_2>f_3>f_4>f_1$. In accordance with (6), we get $g_\lambda(x_2)$=0.71, $g_\lambda(x_2,x_3)$=0.90, $g_\lambda(x_2,x_3,x_4)$=0.98, $g_\lambda(x_2,x_3,x_4,x_1)$=1.

So, the comprehensive evaluation of M is as follws:

$$f=(0.74 \wedge 0.71) \vee (0.71 \wedge 0.90) \vee (0.70 \wedge 0.98) \vee (0.65 \wedge 1)=0.71.$$

Table 4. The scores and recognition degree of indexes

Evaluation criteria & density	Indexes	Index scores	Recognition degree of index
Resource utilization A_1 g_1=0.58	the development and utilization of technology	f_{11}=0.54	g_{11}=0.76
	equipment utilization	f_{12}=0.65	g_{12}=0.68
	transformation rate of achievements	f_{13}=0.36	g_{13}=0.83
Undertaken projects A_2 g_2=0.71	the number of municipal research projects	f_{21}=0.61	g_{21}=0.52
	the number of provincial research projects	f_{22}=0.74	g_{22}=0.55
	the number of state research projects	f_{23}=0.21	g_{23}=0.69
	the number of “transverse” research projects	f_{24}=0.81	g_{24}=0.72
Technical cooperation or construction A_3 g_3=0.62	the number of technology research centers built cooperatively	f_{31}=0.35	g_{31}=0.76
	the number of technology transfer centers built cooperatively	f_{32}=0.44	g_{32}=0.78
	technical contract signed externally	f_{33}=0.71	g_{33}=0.81
	amount of contract signed externally	f_{34}=0.67	g_{34}=0.88
Mutual exchange A_4 g_4=0.67	information exchange between enterprises	f_{41}=0.57	g_{41}=0.65
	recognition degree of enterprise leaders	f_{42}=0.67	g_{42}=0.76
	recognition degree of leaders in university or scientific research institution	f_{43}=0.76	g_{43}=0.70

6 Conclusion

As an important organization form of science and technology innovation, ceeusro has received more and more attention, but the research on performance evaluation is much weaker, especially the existing studies evaluate the performance from the output point of view. Therefore, the index system of performance evaluation is constructed based on the process perspective.

There are strong correlations between the data of evaluation index, which makes the classical weighted average method is no longer applicable, thus, the fuzzy integral is used in this paper, and the feasibility and effectiveness are proved through case study.

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Fuzzy Linear Programming with Possibility and Necessity Relation

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Abstract. A class of linear programming problems based on the possibility and necessity relation is introduced. By using the degree of possibility and necessity, the fulfillment of the constraints can be measured. Then the properties of ranking index are discussed. With this ranking index, the bound of optimal solution is obtained at different degree of possibility and necessity, that constitute a nested set. In the end, the membership function of fuzzy solution is also presented as well as a numerical example.

Keywords: fuzzy numbers, possibility, necessity, linear programming.

1 Introduction

The research on fuzzy mathematical programming has been an active area sice Bellman and Zadeh[1] proposed the definition of fuzzy decision making. The most commonly used method is to transform it into one or a series of crisp programming problems, that means the cut level is considered. Julien[2] transformed the fuzzy linear programming problem with the best and the worst linear programming problem at different α cut levels, and got the possibility distribution of the optimal objective value. Liu[3] use a new ranking index, get the optimal solution by making the constraints tight or loose based on his optimistic or pessimistic attitude. However these are based on different criteria and the results can be inconsistent, so we try to develop it.

For the comparison of fuzzy numbers, there are many different methods, all these methods can be classified into two kinds: one is using crisp relations to rank fuzzy numbers, every fuzzy number is mapped to a point on the real line[3,4]; the other is to use fuzzy relation to rank fuzzy numbers, the relations are interpreted as fuzzy membership functions[2,5,6]. Obviously, to measure the fulfillment of the constraints in fuzzy linear programming, the

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latter is more reasonable. The best known indexes come from the possibility theory of Dubois and Prade[7], based on possibility measure and necessity measure, they proposed four indexes of inequality between fuzzy numbers. Liu [5] proposed fuzzy chance programming problem with the possibility index to measure the fulfillment of the fuzzy constraints. Jaroslav[6] introduced the weak and strong duality theorem with possibility and necessity relations. Wu[8] show that the primal and dual fuzzy mathematical programming problems based on the concept of necessity have no duality gap. In fact, we find that *Pos* and *Nec* index represent the most loose and strict situation respectively, it constitute the most wild and narrow feasible region in the linear programming. Fortunately, the optimal solution with different degrees of the possibility and necessity relation constitute a nested set which be cut level of fuzzy solution. So in this paper, first, the relation of possibility and necessity for trapezoidal fuzzy number as well as its property are discussed, then the optimal solution at different degree of possibility and necessity is obtained. In the end, for a linear programming problem, we get the solution based on the possible and necessary degree and obtain the fuzzy optimal solution.

2 Preliminaries

First, the concepts of fuzzy numbers are introduced. A fuzzy number is a fuzzy subset of the real line, which is both normal and convex. For a fuzzy number $\widetilde{a}$, its membership can be denoted by

$$u_{\widetilde{a}}(x) = \begin{cases} L(x), & x < m, \\ 1, & m \le x \le n, \\ R(x), & x > n, \end{cases}$$

where $L(x)$ is a upper semi-continuous, strictly increasing function for $x < m$ and there exists $m_1 < m$ such that $L(x) = 0$ for $x \le m_1, R(x)$ is continuous, strictly decreasing function for $x > n$ and there exists $n_1 > n$ such that $R(x) = 0$ for $x \ge n_1$, and $L(x), R(x)$ are called the left reference function and right reference function, respectively.

In practical fuzzy mathematical programming problem, triangular fuzzy numbers and trapezoidal fuzzy numbers are most commonly used, because they have intuitive appeal and can be easily specified by the decision maker. So we will mainly discuss the ranking method of triangular fuzzy numbers and trapezoidal fuzzy numbers.

A trapezoidal fuzzy number is a fuzzy number that is fully determined by quadruples $(a^L, a^U, \alpha, \beta)$ of crisp numbers such that $a^L \le a^U, \alpha \ge 0, \beta \ge 0$, whose membership function can be denoted by

$$u_{\widetilde{a}}(x) = \begin{cases} (x - a^L + \alpha)/\alpha, & a^L - \alpha \le x \le a^L, \\ 1, & a^L \le x \le a^U, \\ -(x - a^U - \beta)/\beta, & a^U \le x \le a^U + \beta, \\ 0, & \text{otherwise.} \end{cases}$$

when $a^L = a^U$, the trapezoidal fuzzy number becomes a triangular fuzzy number. If $\alpha = \beta$, the trapezoidal fuzzy number becomes a symmetrical trapezoidal fuzzy number. $[a^L, a^U]$ is the core of $\widetilde{a}$, and $\alpha \geq 0, \beta \geq 0$ are the left-hand and right-hand spreads.

Let $\widetilde{a} = (a^L, a^U, \alpha, \beta)$ and $\widetilde{b} = (b^L, b^U, \gamma, \theta)$ both be trapezoidal fuzzy numbers. Some of the results of applying fuzzy arithmetic on the fuzzy numbers $\widetilde{a}$ and $\widetilde{b}$ follow:

scalar multiplication:
$\lambda \cdot \widetilde{a} = (\lambda a^L, \lambda a^U, \lambda\alpha, \lambda\beta), \lambda > 0,$
$\lambda \cdot \widetilde{a} = (\lambda a^U, \lambda a^L, -\lambda\beta, -\lambda\alpha), \lambda < 0,$
Addition:
$\widetilde{a} + \widetilde{b} = (a^L + b^L, a^U + b^U, \alpha + \gamma, \beta + \theta).$
Subtraction:
$\widetilde{a} - \widetilde{b} = (a^L - b^U, a^U - b^L, \alpha + \theta, \beta + \gamma)$[3].

3 Ranking of Fuzzy Number

Definition 3.1[8] Let $\widetilde{a}$ and $\widetilde{b}$ be two fuzzy numbers,

$Pos(\widetilde{a} \preceq \widetilde{b}) = \sup_{u \leq v} \min\{u_{\widetilde{a}}(u), u_{\widetilde{b}}(v)\},$

$Nec(\widetilde{a} \prec \widetilde{b}) = \inf_{u \geq v} \max\{1 - u_{\widetilde{a}}(u), 1 - u_{\widetilde{b}}(v)\} = 1 - \sup_{u \geq v} \min\{u_{\widetilde{a}}(u), u_{\widetilde{b}}(v)\}.$

are called possibility and necessity indices respectively.

Proposition 3.2[8] If $\widetilde{a}$ and $\widetilde{b}$ are fuzzy numbers, then we have

(i) $Pos(\widetilde{a} \preceq \widetilde{b}) \geq \alpha$ if and only if $\widetilde{a}^L_\alpha \leq \widetilde{b}^U_\alpha$;
(ii) $Nec(\widetilde{a} \prec \widetilde{b}) \geq \alpha$ if and only if $\widetilde{a}^U_{1-\alpha} \leq \widetilde{b}^L_{1-\alpha}$.
we write also alternatively
$Pos(\widetilde{a} \preceq \widetilde{b}) = \mu_{Pos}(\widetilde{a}, \widetilde{b}) = (\widetilde{a} \preceq^{pos} \widetilde{b}),$
$Nec(\widetilde{a} \prec \widetilde{b}) = \mu_{Nec}(\widetilde{a}, \widetilde{b}) = (\widetilde{a} \prec^{nec} \widetilde{b}).$

An interpretation of the $\alpha-$ relation associated to 'pos' and 'Nec' when comparing fuzzy numbers $\widetilde{a}$ and $\widetilde{b}$ is as follows: for a given level of satisfaction $\alpha \in [0, 1]$, a fuzzy number $\widetilde{a}$ is not better than $\widetilde{b}$ with respect to fuzzy relation $\preceq^{pos}$ if the smallest value of $\widetilde{a}$ with the degree of satisfaction being greater or equal to α is less or equal to the largest value of $\widetilde{b}$ with the degree of satisfaction greater or equal to α. On the other hand, a fuzzy number $\widetilde{a}$ is not better than $\widetilde{b}$ with respect to fuzzy relation $\prec^{Nec}$ if the largest value of $\widetilde{a}$ with the degree of satisfaction at least $1 - \alpha$ is less or equal to the smallest value of $\widetilde{b}$ with the degree of satisfaction at least $1 - \alpha$.

From the definition 3.1, it follows easily the theorem below:

Theorem 3.3 Let $\widetilde{a} = (a^L, a^U, \alpha, \beta)$ and $\widetilde{b} = (b^L, b^U, \gamma, \theta)$ be trapezoidal fuzzy numbers, then

$$Pos(\widetilde{a} \preceq \widetilde{b}) = \begin{cases} 1, & b^U \geq a^L, \\ \frac{b^U - a^L + \theta + \alpha}{\theta + \alpha}, & 0 < a^L - b^U \leq \theta + \alpha, \\ 0, & a^L - b^U > \theta + \alpha, \end{cases} \tag{1}$$

$$Nec(\widetilde{a} \prec \widetilde{b}) = \begin{cases} 1, & b^L - a^U \geq \gamma + \beta, \\ \frac{b^L - a^U}{\gamma + \beta}, & 0 \leq b^L - a^U < \gamma + \beta, \\ 0, & b^L < a^U, \end{cases} \tag{2}$$

Theorem 3.4 Let $\widetilde{a} = (a^L, a^U, \alpha, \beta)$ and $\widetilde{b} = (b^L, b^U, \gamma, \theta)$ be two trapezoidal fuzzy numbers, $p \in (0, 1], Pos(\widetilde{a} \preceq \widetilde{b}) \geq p$, if and only if $b^U - a^L \geq (p-1)(\theta + \alpha)$.

Proof If $p = 1$, then from $Pos(\widetilde{a} \preceq \widetilde{b}) \geq 1$ one can get that $b^U \geq a^L$, and vice versa.

If $0 < p < 1$, then $b^U < a^L$ and $b^U + \theta > a^L - \alpha$, $Pos(\widetilde{a} \preceq \widetilde{b}) \geq p$ if and only if $\frac{b^U - a^L + \theta + \alpha}{\theta + \alpha} \geq p$, that is $b^U - a^L \geq (p-1)(\theta + \alpha)$.

Theorem 3.5 Let $\widetilde{a} = (a^L, a^U, \alpha, \beta)$ and $\widetilde{b} = (b^L, b^U, \gamma, \theta)$ be two trapezoidal fuzzy numbers, $p \in (0, 1], Nec(\widetilde{a} \prec \widetilde{b}) \geq p$, if and only if $b^L - a^U \geq p(\gamma + \beta)$.

Proof If $p = 1$, then from $Nec(\widetilde{a} \prec \widetilde{b}) \geq 1$ one can get that $b^L - \gamma \geq a^U + \beta$, and vice versa.

If $0 < p < 1$, then $b^L > a^U$ and $b^L - \gamma < a^U + \beta$, $Nec(\widetilde{a} \prec \widetilde{b}) \geq p$ if and only if $\frac{b^L - a^U}{\gamma + \beta} \geq p$, that is $b^L - a^U \geq p(\gamma + \beta)$.

4 Fuzzy Linear Programming

Let us consider the following fuzzy linear programming problem with imprecise resources and technology coefficients based on the relation of possibility and necessity:

$$(LPP) \begin{cases} \max z = c_1 x_1 + c_2 x_2 + \cdots + c_n x_n, \\ \widetilde{a}_{i1} x_1 + \widetilde{a}_{i2} x_2 + \cdots + \widetilde{a}_{in} x_n \preceq^{pos} \widetilde{b}_i, i = 1, 2, \cdots, m, \\ x_j \geq 0, \qquad j = 1, 2, \cdots, n, \end{cases} \tag{3}$$

$$(LPN) \begin{cases} \max z = c_1 x_1 + c_2 x_2 + \cdots + c_n x_n, \\ \widetilde{a}_{i1} x_1 + \widetilde{a}_{i2} x_2 + \cdots + \widetilde{a}_{in} x_n \prec^{Nec} \widetilde{b}_i, i = 1, 2, \cdots, m, \\ x_j \geq 0, \qquad j = 1, 2, \cdots, n, \end{cases} \tag{4}$$

where $\widetilde{a_{ij}} = (a_{ij}^L, a_{ij}^U, \alpha_{ij}, \beta_{ij}), \widetilde{b}_i = (b_i^L, b_i^U, \gamma_i, \theta_i)$ are trapezoidal fuzzy numbers, c_i are crisp coefficients of the objective, x_i are the crisp decision variable.

With respect to constraints of (LPP) and (LPN), the optimal solution is completely determined by degrees of possibility and necessity, so we can obtain the optimal solution of (LPP) and (LPN) at $p-$ cut levels and $(1-p)-$ cut levels respectively by solving the following two crisp linear programming:

$$(LPP_p)' \begin{cases} \max z = c_1x_1 + c_2x_2 + \cdots + c_nx_n, \\ Pos(\widetilde{a}_{i1}x_1 + \widetilde{a}_{i2}x_2 + \cdots + \widetilde{a}_{in}x_n \preceq \widetilde{b}_i) \geq p, i = 1, 2, \cdots, m, \\ \qquad x_j \geq 0, \qquad\qquad j = 1, 2, \cdots, n, \end{cases} \tag{5}$$

$$(LPN_p)' \begin{cases} \max z = c_1x_1 + c_2x_2 + \cdots + c_nx_n, \\ Nec(\widetilde{a}_{i1}x_1 + \widetilde{a}_{i2}x_2 + \cdots + \widetilde{a}_{in}x_n \prec \widetilde{b}_i) \geq 1-p, i = 1, 2, \cdots, m, \\ \qquad x_j \geq 0, \qquad\qquad j = 1, 2, \cdots, n, \end{cases} \tag{6}$$

then from theorem 3.4 and 3.5 we have:

$$(LPP_p) \begin{cases} \max z = c_1x_1 + c_2x_2 + \cdots + c_nx_n, \\ b^U - A^Lx \geq (p-1)(\alpha x + \theta), \\ x_j \geq 0, \qquad j = 1, 2, \cdots, n, \end{cases} \tag{7}$$

$$(LPN_p) \begin{cases} \max z = c_1x_1 + c_2x_2 + \cdots + c_nx_n, \\ b^L - A^Ux \geq (1-p)(\beta x + \gamma), \\ x_j \geq 0, \qquad j = 1, 2, \cdots, n, \end{cases} \tag{8}$$

where $A^U = \{a_{ij}^U\}, A^L = \{a_{ij}^L\}, \beta = \{\beta_{ij}\}, \alpha = \{\alpha_{ij}\}, b^L = \{b_i^L\}, b^U = \{b_i^U\}, \gamma = \{\gamma_i\}, \theta = \{\theta_i\}, x = \{x_j\}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n,$

We denote $X_p^P = \{x \in R^n | b^U - A^Lx \geq (p-1)(\alpha x + \theta)\}, X_p^N = \{x \in R^n | b^L - A^Ux \geq (1-p)(\beta x + \gamma)\}$, then X_p^P, X_p^N are called feasible region of (LPP_p) and (LPN_p). Assume the optimal solution of (LPP_p) and (LPN_p) are z_p^P and z_p^N respectively, then we have following theorem:

Theorem 4.1 For linear programming (LPP_p) and (LPN_p), then $X_p^N \subseteq X_p^P$ and $z_p^N \leq z_p^P$.

Proof If $x \in X_p^N$, then $b^L - A^Ux \geq (1-p)(\beta x + \gamma)$, that is $A^Ux + (1-p)\beta x \leq b^L - (1-p)\gamma$. Since $A^Ux + (1-p)\beta x \geq A^Lx - (1-p)\alpha x$ and $b^U + (1-p)\theta \geq b^L - (1-p)\gamma$, then $A^Lx - (1-p)\alpha x \leq b^U + (1-p)\theta$, so $x \in X_p^p$, thus $X_p^N \subseteq X_p^P$ and $z_p^N \leq z_p^P$.

Theorem 4.2 For linear programming (LPP_p) and $(LPN_p), 0 \leq p_1 \leq p_2 \leq 1$, then $X_{p_2}^P \subseteq X_{p_1}^P, X_{p_2}^N \supseteq X_{p_1}^N$and $z_{p_1}^P \geq z_{p_2}^P, z_{p_2}^N \geq z_{p_1}^N$.

Proof for $x \in X_{p_2}^P$, we have $b^U - A^Lx \geq (p_2 - 1)(\alpha x + \theta)$, from $\beta > 0, \gamma > 0, x > 0$, then $b^U - A^Lx \geq (p_2-1)(\alpha x + \theta) \geq (p_1 - 1)(\alpha x + \theta)$, so $x \in X_{p_1}^P$, that is $X_{p_2}^P \subseteq X_{p_1}^P$ and $z_{p_1}^P \geq z_{p_2}^P$. in a similar way, $X_{p_2}^N \supseteq X_{p_1}^N$ and $z_{p_2}^N \geq z_{p_1}^N$.

From theorem 4.1 and 4.2, we obtain that z_p^N, z_p^P constitute the lower and upper bound of fuzzy objective value at level p. For z_p^N is increasing and z_p^P is decreasing with p increased, so $z_p = [z_p^N, z_p^P]$ is a nested set. Therefore we denote fuzzy set $\widetilde{X} = \cup_{p\in[0,1]}(p^* \bigcap z_p)$ the optimal fuzzy solution of problem of (LPP) and (LPN). that is to say, $\widetilde{X}$ represent the fuzzy solution of $\max z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$ with relation of possibility and Necessity between $\widetilde{a}_{i1}x_1 + \widetilde{a}_{i2}x_2 + \cdots + \widetilde{a}_{in}x_n$ and $\widetilde{b}_i, i = 1, 2, \cdots, m$, ($p^*$ is fuzzy subsets which membership function is p for $x \in R$) and the $p-$ cut level of this fuzzy solution is $[z_p^N, z_p^P]$.

5 Numerical Example

To illustrate the effect of the proposed approach, let us consider the following example:

Example solving the following fuzzy linear programming problem with trapezoidal fuzzy number constraint coefficients:

$$\begin{cases} \max z = c_1x_1 + c_2x_2, \\ \widetilde{a}_{11}x_1 + \widetilde{a}_{12}x_2 \preceq^{pos} \widetilde{b}_1, \\ \widetilde{a}_{21}x_1 + \widetilde{a}_{22}x_2 \preceq^{pos} \widetilde{b}_2, \\ x_j \geq 0, j = 1, 2. \end{cases} \tag{9}$$

$$\begin{cases} \max z = c_1x_1 + c_2x_2, \\ \widetilde{a}_{11}x_1 + \widetilde{a}_{12}x_2 \prec^{Nec} \widetilde{b}_1, \\ \widetilde{a}_{21}x_1 + \widetilde{a}_{22}x_2 \prec^{Nec} \widetilde{b}_2, \\ x_j \geq 0, j = 1, 2. \end{cases} \tag{10}$$

where $\widetilde{a}_{11} = (2, 3, 1, 2), \widetilde{a}_{12} = (1, 2, 2, 1), \widetilde{a}_{21} = (1, 2, 2, 1), \widetilde{b}_1 = (2, 3, 1, 2), \widetilde{b}_2 = (3, 4, 2, 1), c_1 = 3, c_2 = 4$. From (7) and (8) we have the following crisp optimal problem based on the level p :

$$\begin{cases} \max z = c_1x_1 + c_2x_2, \\ (p+1)x_1 + 2px_2 \leq 5 - 2p, \\ (2p-1)x_1 + 2px_2 \leq 5 - p, \\ x_j \geq 0, j = 1, 2. \end{cases} \tag{11}$$

$$\begin{cases} \max z = c_1x_1 + c_2x_2, \\ (5-2p)x_1 + (3-p)x_2 \leq 4 - 2p, \\ (3-p)x_1 + (4-p)x_2 \leq 2 + p, \\ x_j \geq 0, j = 1, 2. \end{cases} \tag{12}$$

For different cut level p, we can get different optimal solution and denote z_p^N, z_p^P the optimal solution of the crisp programming (11) and (12) respectively,

p	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
z_p^P	97.0000	47.0000	30.3333	22	17	13.6667	11.2857	9.5000	8.1111	7
z_p^N	2.1756	2.3568	2.5439	2.7368	2.9355	3.1397	3.3490	3.5629	3.7804	4

Then we obtain the fuzzy solution X with its $\alpha-$ level sets as $[z_p^N, z_p^P]$.

6 Conclusions

Based on the possibility and necessity relation, the paper proposes a linear programming with fuzzy constraints. Properties are discussed extensively in the case of the trapezoidal fuzzy numbers, then the method is applied to the linear programming problems. With the different satisfaction degree, the decision maker can get the optimal solution and this constitute the fuzzy optimal solution of the linear programming.

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The Optimization Approach for Traffic Rescue Resource Dispatch on Expressway Based on Fuzzy Programming

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Abstract. In order to reveal the uncertainty in the process of rescuing the accidents over the expressway network, fuzzy programming method is used to establish the rescue resource dispatch model. The model aims to minimize the fuzzy dispatch decision-making time to reflect the accidents' influence on the upper traffic flow after accidents happen on the fully-closed expressway. Fuzzy chance constraint is designed to reflect the relationship between the uncertain requirements of potential accidents and the existing resource allocation. According to the limitation of the traditional algorithm and the complexity of dispatch problems, especially many accidents happening simultaneously and various rescue resources required, the genetic algorithm based on fuzzy simulation is designed to be fit for the model and the optimized dispatch scheme is obtained. The case study of the expressway network in Henan Province is conducted to illustrate that the fuzzy dispatch method can be used to solve the conflicts between shortening the decision-making time and reducing the rescuing economic costs compared with the existing rescue mode, and the optimum rescue resource dispatch decision is made for the command control center.

Keywords: Fuzzy programming, expressway, traffic rescue resource, genetic algorithm, optimized dispatch approach.

1 Introduction

As the premise of the rapid response to the accidents and the implementation of rescue operations after accidents happen on expressway, making the rescue resource dispatch decision is to select proper rescue depots, assign the kinds and the number of rescue vehicles required to the scene according to the accident characteristic and the resource allocation. The purposes are to save the injured, repair facilities, remove barriers, recover the traffic, decrease the accident influence, and reduce the casualties and property losses.

He Jiamin and Liu Chunlin researched on the selection of the rescue depots in the continuous consuming emergency systems with sufficient resources allocated

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[1, 2], and Gao Shuping studied on the multi-resource dispatch problem [3]. Zografos divided the total duration of accidents into four time components, detection time, dispatching time, travel time and clearance time [4].Considering the current accident, Che Yingtao applied the Hungarian method to solve the dispatch problem with the objective of minimizing the travel time[5]. And Yamada transformed the dispatch problem into the shortest path of the network [6]. Sherali and Subramanian considered the potential accidents and established the rescue response model based on the opportunity-cost theory in which the rescue costs were directly proportional to the distance between the rescue depots and accident spots [7]. The objective of the dispatch model established by Sun Ying was to minimize the total travel time, because the path selection behaviours of rescue vehicles were various and uncertain [8].A mixed integer programming model with reliability constraints was established by Pal and the best location of depots was determined [9].

The existing dispatch methods have some gaps with the emergency rescue practice on expressway. For instance, only one large barrier-clearance vehicle is shared by many expressways, and it is possible that rescue vehicles would be insufficient when plenty of accidents happen in the meantime. Therefore, the allocation does not conform to continuous consuming emergency system. Previous research approaches focused on minimizing travel time or the distance between rescue depots and accidents, which cannot fully reflect the influencing factors in making the dispatch decision. Most methods assumed that traffic flow conditions between depots and accidents spots are invariable, but the influencing areas of the accidents spread to the upstream, hence the traffic flow conditions and the travel speed of rescue vehicles are correspondingly altered. Accidents on expressway always happen suddenly, especially the uncertain potential accidents, and the dispatch decision is made under uncertain conditions. As a result, the uncertain factors in the process of making the vehicle dispatch decision are considered as the fuzzy numbers and the fuzzy chance-constrained programming is selected to establish the dispatch model. Because of the complexity of dispatch problems over the expressway network, the genetic algorithm based on fuzzy simulation is designed to solve the model established and accomplish the optimization dispatch of resources.

2 Rescue Resource Dispatch Model Based on Fuzzy Programming

A. Influencing factors in making the dispatch decision

In the expressway network, potential accidents occur at a certain probability after current accidents happen. The first step of implementing the rescue operations is to determine the location of the accidents, and proper rescue depots are selected from all depots in which the service vehicles required are allocated. The distance in each path between the proper depot and the accident is measured to assist the dispatch decision-making. Dispatching time is the time interval between the accident verification and the determination of the preliminary scheme in disposal of the accidents based on the level of accident. Different levels of accidents have different dispatching time [10], so when making decisions, dispatching time can be considered as an influencing factor. According to the investigation of the accident-rescue conditions on expressway in various provinces in China, corresponding minimum dispatching time based on each level of the accident is shown in Table 1.

Table 1. The levels of the accidents and the corresponding minimum dispatching time

Levels of the accident	Minor	Large	Major	Severe
Minimum dispatching time	3min	5min	7min	10min

Traffic rescue resources include police patrol vehicles, administration vehicles, wreckers, cranes, fire engines, ambulances. The dispatch operator in the command control centre informs the hospital and fire station to aid for the emergency rescue, so the dispatch problems of the police patrol vehicles, administration vehicles, wreckers and cranes are focused on. Various rescue vehicles have different performances and tonnages, so travel time is various. The accidents have significant influences on upper traffic flow on fully-closed expressway. As a result, travel speed of the rescue vehicles can be regarded as one influencing factor in making the dispatch decision.

The kinds and the number of vehicles required are determined by the level of the accidents. The levels and requirements of current accidents are decided by the scene description and the surveillance facilities on expressway. Potential accidents occur at a certain probability and the levels of them are judged with subjectivity. Consequently, the requirements of potential accidents are decided with the subjective uncertainty and the fuzziness. Because of the finite allocation of vehicles over the expressway network, when a large amount of vehicles required in severe current accidents or many potential accidents happen simultaneously, the allocation will be not in abundance for rescuing the potential accidents. As a result, the resource requirement of the potential accident is considered as one of the influencing factors in making the dispatch decision.

B. Rescue resource dispatch model

In the expressway network, each depot $i \in L$ allocates r_i rescue vehicles based on the scale and investment, and L is the set of depot nodes. Each current accident node $f \in F$ requires n_f vehicles, F is the set of current accident nodes, $F \subset N$, and N is the set of nodes over the network. H is denoted as the set of potential accident nodes after current accidents occur, and $H \subset N$. P_h is the probability of a potential accident occurring at node h ($h \in H$). The variables utilized for modelling the resource dispatch include x_{if} and y_{ih} , x_{if} is the number of vehicles dispatched from depot i to f , and y_{ih} is the number of vehicles dispatched from depot i to the potential accident h .

The dispatch decision-making time includes the dispatching time and travel time. The components of the decision-making time are generally variable and influenced by many factors such as the level of accident, the kind of vehicles and the road condition. λ_{if} and λ_{ih} denote the shortest distance from depot i to f and h respectively. t_f and t_h denote the minimum dispatching time of f and h . v_{if} and v_{ih} denote the travel speed from i to f and h . The travel speed of vehicles will be real-time changed in uncertainty because of the decreasing capacity of each road section and the change of the traffic flow condition. Consequently, v_{if} and v_{ih} are viewed as the fuzzy number and changed in a certain membership range. T_{if} and T_{ih} denote the minimum dispatch decision-making time from depot i to f and h . According to the components and fuzzy travel speed, the formula is obtained as follows:

$$\tilde{T}_{if} = t_f + \lambda_{if} / \tilde{v}_{if} , \tilde{T}_{ih} = t_h + \lambda_{ih} / \tilde{v}_{ih} \tag{1}$$

The requirements of potential accidents are considered uncertain, so fuzzy vector $\tilde{\zeta}_h$ is introduced to denote the number of one kind of vehicles in the potential accident node $h \in H$, and its membership function is $\mu(z)$, z is an independent variable. In order to denote the possibility measure of $\tilde{\zeta}_h$, the confidence level $q \in [0,1]$ is settled to signify the level of service. The level of service has an effect on making the dispatch decision and a principle is applied that the decision made is allowed not to satisfy the allocation and precondition in a certain extent, so the dispatch problem is fit for fuzzy chance constraint programming[11].The constraint is designed as follows:

$$\prod_{h \in H} Pos(\sum_{i \in L} y_{ih} \geq \tilde{\zeta}_h) \geq q \tag{2}$$

Shortening the dispatch decision-making time and reducing the economic costs are the conflicting aims in dispatching the rescue vehicles on expressway, so the objective function in the model is to minimize the total rescue costs which are in proportion to the number of vehicles dispatched and the decision-making time. Meanwhile, the number of vehicles dispatched from any depot must not exceed the number of service vehicles allocated at the depot. And the requirements of current and potential accidents could be met to reach the equilibrium between demand and supply. On the basis of analysis above, the rescue vehicle dispatch model is formulated as follows:

$$
\min\Big[\sum_{i\in L}\sum_{f\in F}\tilde{T}_{if}\,x_{if}+\sum_{i\in L}\sum_{h\in H}P_h\tilde{T}_{ih}\,y_{ih}\Big]
$$

$$
s.t.\ \begin{cases}\sum_{f\in F}x_{if}+\sum_{h\in H}y_{ih}\le r_i & \forall i\in L\\ \sum_{i\in L}x_{if}=n_f & \forall f\in F\\ \prod_{h\in H}Pos\big(\sum_{i\in L}y_{ih}\ge\tilde{\zeta}_h\big)\ge q & \\ x,y\ge 0(\text{and integer}) & \end{cases} \tag{3}
$$

In the process of the dispatch model formulation with fuzzy parameters, influencing factors in the dispatch decision-making are all involved. The model reflects the urgency request of shortening the dispatch decision-making time and the achievement of reducing the economic rescue costs by regulating the levels of rescue service.

3 Algorithms

A. Analysis of the traditional algorithm

Solving the chance constraint programming problem with fuzzy parameters is to convert the chance constraint into crisp equivalents for some special cases traditionally [11]. The fuzzy chance constraint conforms to the first lemma in the literature [12], and can be transformed into crisp mixed integer programming forms. However, if many accidents occur at the same time or various rescue vehicles are required, the crisp equivalents will be the multi-objective programming problem, and the computation will be increased in the process of converting the chance constraint. The limitation of the traditional algorithm exists in applying it to solve the dispatch model. Consequently, a genetic algorithm based on the fuzzy simulation is designed to solve the model and the corresponding genetic operations are settled based on the characteristics of the model.

B. Design of the genetic algorithm based on the fuzzy simulation

According to the implication of the fuzzy parameters $\tilde{T}_{if}$, $\tilde{T}_{ih}$ and $\tilde{\zeta}_h$ in the dispatch model, upper limit, median value and lower limit exist in the parameters, so $\tilde{T}_{if}$, $\tilde{T}_{ih}$ and $\tilde{\zeta}_h$ are taken as the triangular fuzzy vectors fully determined by the triplet $(T_{if1},T_{if2},T_{if3})$, $(T_{ih1},T_{ih2},T_{ih3})$ and $(\zeta_{h1},\zeta_{h2},\zeta_{h3})$ of crisp numbers respectively.

The interval value of the fuzzy variable is determined by the membership function. The chance constraint and the objective function can be handled according to the operation rules of fuzzy quantity and the technique of fuzzy simulation. As a sampling test method, fuzzy simulation technique has superiority in solving the fuzzy chance constraint programming problem, and the dispatch model belongs to the fuzzy mix-integer programming with multiple decision variables. Global optimized genetic algorithm is applied due to the non-convexity of the feasible region with chance constraints, and Pareto optimal solution is obtained at the given confidence level q.

a. Fuzzy simulation technique

Via plenty of sampling in fuzzy system, the technique of fuzzy simulation is formed [13] and it can be applied to check the chance constraint and handle the objective function with fuzzy parameters.

(1) Checking the fuzzy chance constraint

According to the operation rules of the fuzzy quantity, $\prod_{h\in H} Pos(\sum_{i\in L} y_{ih} \geq \tilde{\zeta}_h) \geq q$ is satisfied for any given decision variable y_{ih} if and only if there is a crisp vector ζ_{h0} such that $\sum_{i\in L}\sum_{h\in H} y_{ih} \geq \zeta_{h0}$ and the valve of membership function $\mu(\zeta_{h0}) \geq q$ .Thus, a crisp vector ζ_{h0} is generated uniformly from the fuzzy vector $\tilde{\zeta}_h$ such that $\mu(\zeta_{h0}) \geq q$, namely, ζ_{h0} is abstracted from the q -level set of fuzzy vector $\tilde{\zeta}_h$. If ζ_{h0} satisfies that $\sum_{i\in L}\sum_{h\in H} y_{ih} \geq \zeta_{h0}$, then $\prod_{h\in H} Pos(\sum_{i\in L} y_{ih} \geq \tilde{\zeta}_h) \geq q$ is believable. Otherwise, a crisp vector is re-generated from the q -level set of $\tilde{\zeta}_h$ and constraints are checked again. After a given number of cycles, if there is no crisp vector ζ_{h0} generated such that $\sum_{i\in L}\sum_{h\in H} y_{ih} \geq \zeta_{h0}$, the decision variable y_{ih} will be infeasible.

(2) Handling the objective function with fuzzy parameters

The objective function of the dispatch model is to minimize the total rescue cost, so set $f = \sum_{i\in L}\sum_{f\in F} \tilde{T}_{if} x_{if} + \sum_{i\in L}\sum_{h\in H} P_h \tilde{T}_{ih} y_{ih}$ .The model is solvable for any given decision variable x_{if} and y_{ih} by searching for the minimal value $\bar{f}$. At first, set $\bar{f} = +\infty$. Then two crisp vectors (T_{if0}, T_{ih0}) uniformly from the fuzzy vector $\tilde{T}_{if}$ and $\tilde{T}_{ih}$ respectively, i.e., the q -level set of fuzzy vector. If $\bar{f} > \sum_{i\in L}\sum_{f\in F} T_{if0} x_{if} + \sum_{i\in L}\sum_{h\in H} P_h T_{ih0} y_{ih}$, then set $\bar{f} = \sum_{i\in L}\sum_{f\in F} T_{if0} x_{if} + \sum_{i\in L}\sum_{h\in H} P_h T_{ih0} y_{ih}$. Repeat the above steps after a given number of cycles, and finally the minimum valve of the total rescue cost is obtained.

b. Representation structure

The genetic algorithm requires the encoding of the chromosome which represents the decision variable in the objective function [14]. x_{if} and y_{ih} reveals two levels of meanings, namely, the serial number of the depots selected and the number of vehicles dispatched. In order to determine the depots selected preliminarily, the decimal encoded mode is selected for the chromosome and the length of the code is the number of the decision variables which are determined by the number of depots and accident nodes. According to the serial number of accidents in order, the chromosomes are consisted of the serial number of depots selected and a possible dispatch scheme is showed. After getting the best chromosome and determining the depots selected, the number of vehicles dispatched can be deduced based on the allocation of depots and the optimized valve.

c. Fitness function

The design of the fitness function mainly depends on the objective function. The possibly of current accidents and potential accidents can be taken as 1 and P_h respectively, so the objective function in the dispatch model is the multiplication among the possibility of accidents occurred, the minimum dispatch decision-making time and the number of service vehicles dispatched. The objective function is viewed as the fitness function due to the minimization of total rescue costs.

d. Select operator

In terms of the decimal encoded and the disadvantages of the spinning roulette wheel method, the selection operator is to apply the homogeneous sorting method to choose the population in survival of the fittest and the process has no bearing on the fitness sign and difference of individuals. The rank-based evaluation function is defined as follows:

$$eval(C_k) = v(1-v)^{k-1}, k = 1,2,\cdots pop\_size \tag{4}$$

Where C refers to each chromosome, $v \in [0,1]$ is a parameter, pop_size is the size of the population.

e. Crossover operator

According to the serial number of the depots and the encoded mode of the chromosome, single point crossover operation has applied. In the individual code string, only one cross point is settled randomly, and some chromosomes are exchanged in the point. Each pair of individuals contained in the current population is interchanged in the cross point with a probability related to their fitness and two new populations are generated after mating.

f. Mutation operator

Due to multiple constraints in the model, bound mutation operation has applied and is suitable for the condition that the optimum point is located or near the bound of feasible solutions. The operation can enhance the searching capability of genetic evolution, maintenance the diversity of the population and prevention of pre-maturity phenomenon.

g. Steps of the genetic algorithm based on fuzzy simulation

Step1: Input the parameters of genetic algorithm: population size, cycle index, crossover probability and mutation probability.

Step2: Initialize the discrete population based on the number of rescue depots and accidents and compute the function value of initial population.

Step3: Check the feasibility of each chromosome in the initial population by the fuzzy simulation technique. Apply selection, crossover and mutation operators to update the chromosome.

Step4: Check the feasibility of the future generation, and take every chromosome in future population into rounding. Compute the rank-based evaluation function for all the chromosomes.

Step5: Calculate the fitness of each chromosome according to the rank-based evaluation function.

Step6: Select the chromosomes by the homogeneous sorting method, re-insert the fittest individual.

Step7: Repeat step 3 to step 6 until the given cycle index reaches.

Step8: Report the best chromosome as the fuzzy optimal solution, and the number of rescue vehicles dispatched is deduced.

4 Application Examples

A. Analysis of the background

The expressway network in Henan Province is taken for instance. Hubs, interchanges and tollgates are all set nodes in the road network. The service vehicles travel U-turn or in upper and lower directions, so the arcs between hubs and nodes are bi-directional and the nodes between upper and lower roadways are linked. Traffic rescue resources are usually allocated near to the expressway, and the depots are set nodes linking with other facilities. The valuation of each arc is the distance between two nodes. Due to the management pattern, the topological diagrams of police patrol vehicles, administration vehicles and barrier-clearance vehicles are shown in Fig.1 and Fig.2. The kinds and numbers of rescue vehicles allocated in the depots are shown in Table 2 and Table 3.

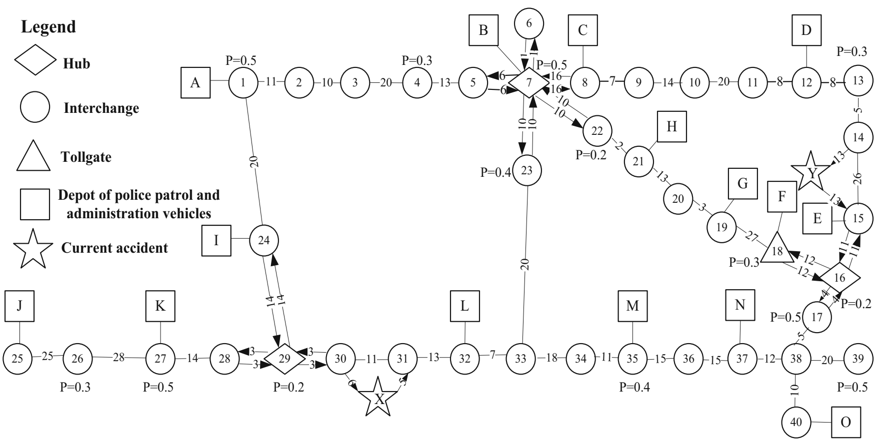

Fig. 1. The expressway network applicable to police patrol and administration vehicles

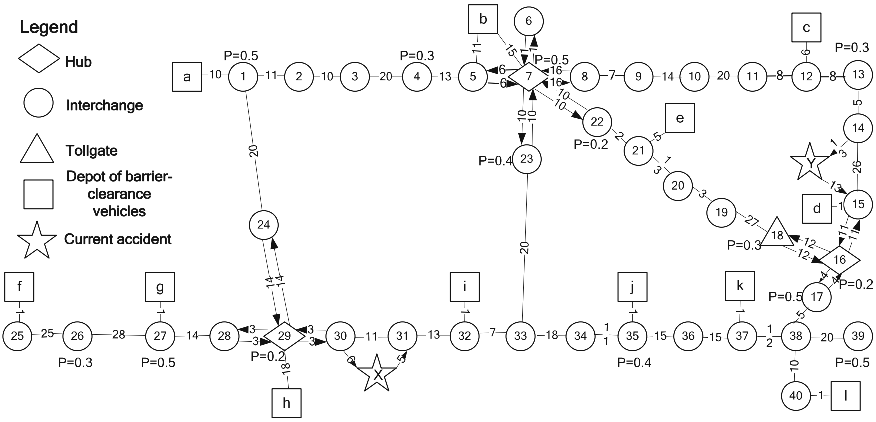

Fig. 2. The expressway network applicable to barrier-clearance vehicles

Table 2. The number of police patrol and administration vehicles allocated in the depots

No.	Name of Depots	Expressways	Police patrol vehicles	Administration vehicles
A	Luochang Road	Ji Luo	1	1
B	Western Jiaozuo	Chang Ji	5	2
C	Southern Xiuwu	Chang Ji	3	3
D	Eastern Xinxiang	Chang Ji	4	4
E	Yuanyang	Jing Gang-ao	3	3
F	Xinzhuang	Zheng Jiao-jin	1	1
G	Xugang	Zheng Jiao-jin	1	0
H	Southern Jiaozuo	Zheng Jiao-jin	1	1
I	Jiyuan	Ji Luo	1	1
J	Yima	Lian Huo(Luoyang)	0	4
K	Western Luoyang	Lian Huo(Luoyang)	4	4
L	Gongyi	Lian Huo(Zhengzhou)	4	0
M	Xingyang	Lian Huo(Zhengzhou)	0	4
N	Liulin	Lian Huo(Zhengzhou)	4	4
O	Putian	Jing Gang-ao	3	3

There are two current accidents over the expressway network. The accident X is 5 kilometres from Yanshi interchange and 6 kilometres from Mengjin interchange, occurring on Lianhuo expressway. The accident Y is 13 kilometres from Xinxiang interchange and 13 kilometres from Yuanyang interchange, occurring on Jing Gang-ao expressway. The accident X and Y are judged to be a major and large accident respectively. According to the levels of two current accidents, the kinds and the number of traffic rescue resources required are determined and shown in Table 4.

Table 3. The number of barrier-clearance vehicles allocated in the rescue depots

No.	Name of Depots	Expressways	Large wreckers	Medium wreckers	Small wreckers	Trailers	Large cranes	Medium cranes	Small cranes
a	Eastern Jiyuan	Ji Luo	0	0	1	1	1	1	1
b	Western Jiaozuo	Chang Ji	0	2	1	1	1	1	0
c	Eastern Xinxiang	Chang Ji	0	0	1	1	1	1	0
d	Yuanyang	Jing Gang-ao	0	1	2	0	0	0	0
e	Southern Jiaozuo	Zheng Jiao-jin	0	2	1	3	1	1	1
f	Yima	Lian Huo(Luoyang)	1	1	0	0	0	1	0
g	Western Luoyang	Lian Huo(Luoyang)	1	1	0	0	0	1	0
h	Zhu Jia-cang	Ji Luo	0	0	1	1	1	1	1
i	Gongyi	Lian Huo(Zhengzhou)	0	2	2	2	0	2	0
j	Xingyang	Lian Huo(Zhengzhou)	1	1	0	0	0	1	0
k	Liulin	Lian Huo(Zhengzhou)	0	2	2	2	0	2	0
l	Putian	Jing Gang-ao	0	1	2	0	0	0	0

Table 4. The kinds and number of traffic rescue resources the current accidents required

Current accidents	Police Patrol vehicles	Administration vehicles	Wreckers			Trailers	Cranes		
			Large	Medium	Small		Large	Medium	Small
X	2	2	0	0	1	1	0	2	0
Y	2	2	0	0	1	0	1	0	0

The number of rescue vehicles $\tilde{\zeta}_h$ required in the potential accident node h is a triangular fuzzy variable. According to the field investigation, the upper limit, median value and lower limit of resource requirements are determined and shown in Table 5.

Table 5. The fuzzy number of traffic rescue resources the potential accidents required

Kinds of vehicles	Police Patrol vehicles	Administration vehicles	Wreckers			Trailers	Cranes		
			Large	Medium	Small		Large	Medium	Small
Upper	3	3	2	2	2	2	2	2	2
Median	2	2	1	1	1	1	1	1	1
Lower	1	1	0	0	0	0	0	0	0

When the levels of accidents, the depots and the accident nodes are deterministic, the travel speed of vehicles mainly affect the dispatch decision-making time. As a result, according to the kinds of rescue vehicles and the change of traffic flow in the accidents' influencing scope, the fuzzy values in the travel speed of various rescue vehicles on expressway are shown in Table 6.

Table 6. The fuzzy travel speed values of various rescue vehicles on expressway (km/h)

Kinds of vehicles	Police patrol and administration vehicles	Small wreckers and cranes	Medium wreckers and cranes	Large wreckers and cranes	Trailers
Upper	90	70	60	50	50
Median	80	60	50	40	40
Lower	70	50	40	30	30

B. Formulation of the dispatch scheme

a. Selection of the parameters in genetic algorithm

The genetic algorithm toolbox developed by the University of Sheffield in England is applied [15] and the program of the genetic algorithm based on fuzzy simulation is developed. The dispatch examples are all performed on a personal computer with the following parameters in the genetic algorithm: the population size is 40, the cycle index is 400, the probability of crossover is 0.7, the probability of mutation is 0.004, and the generation gap is 0.9. Ten percent of individuals with low fitness are abandoned and proper individuals are required to re-insert to the population.

b. The calculation processes and the results using the optimized algorithm

The membership scopes in the number of rescue resources required and the travel speed are the same, so the dispatch problems of two police patrol vehicles the accident X and Y required are taken as instance. Genetic algorithm based on fuzzy simulation is applied and at the level of service q is 0.7 or 0.8, the calculation processes are shown in Fig.3.

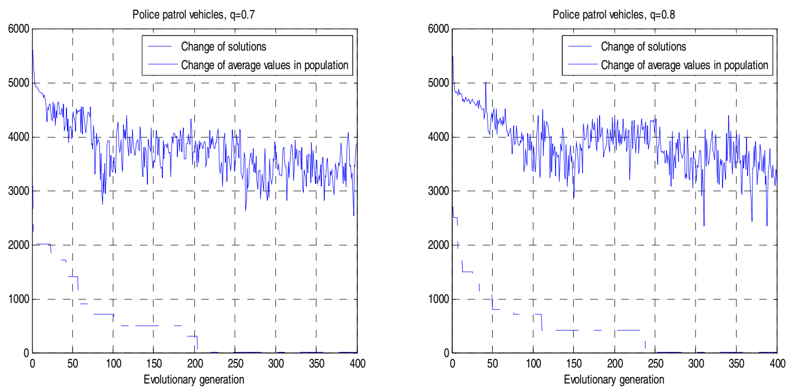

Fig. 3. The calculation process of dispatching police patrol vehicles

When the level of service is 0.7, the scheme is to dispatch two police patrol vehicles from depot L to the current accident X and from depot E to the current accident Y respectively. When the level of service is 0.8, the scheme is to dispatch two police patrol vehicles from depot L to X and from depot O to Y respectively.

In like manner, taken two medium cranes the accident X required for example, the calculation processes are shown in Fig.4.

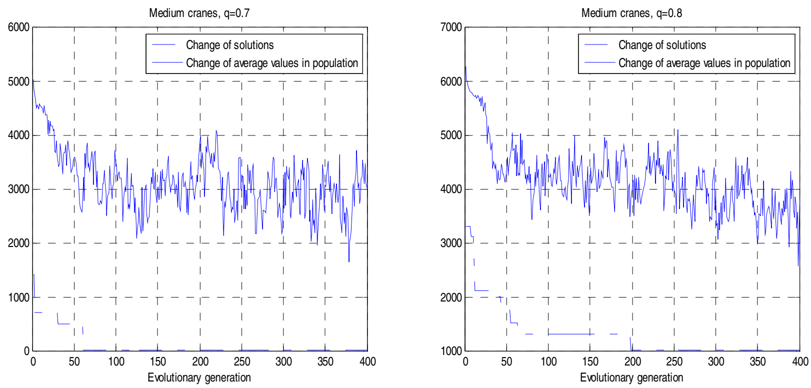

Fig. 4. The calculation process of dispatching two medium cranes

When the level of service is 0.7, the scheme is to dispatch one single medium crane from depot h and i to X respectively. When the level of service is 0.8, the rescue scheme is to dispatch two medium cranes from depot i to X. The dispatch scheme of other rescue resources required can be also made in different levels of service.

C. Analysis of the results

As can be seen from Fig.3 and Fig.4, in the calculation process of using the genetic algorithm based on fuzzy simulation, when the evolutionary generation is up to about 200, the convergence point will be reached and the best chromosome will be obtained. The optimized effect is obvious and the effectiveness of the algorithm is illustrated.

As for two police patrol vehicles the accident Y required, depot E is selected when the level of service is 0.7, and depot O is selected when the level of service is 0.8. The current accident Y, depot E and O are all on the Jing Gang-ao expressway. With the increasing level of service, the number of potential accidents around depot E is larger than depot O. Meanwhile, large accident Y occurs near the Yellow River Bridge. Once traffic jams happen, travel speed of rescue vehicles will be influenced by the bridge and the accident scope. As a result, from the standpoint of the allocation over the expressway network, a more proper dispatch scheme is obtained by adjusting the level of service.

The existing mode of rescuing the accident is NS (Nearest Served) on expressway. The depot h is closest to the current accident X, but only one medium crane is allocated in depot h. The second nearest to X is depot i, and based on the NS mode, the dispatch scheme is that one single medium crane from depot h and i to X. The scheme obtained by the fuzzy dispatch method is that when the level of service is 0.8, two medium cranes from depot i to X. Compared with the existing mode, the number of depots selected is decreased via using the fuzzy dispatch mode. The fuzzy dispatch method can shorten the dispatch decision-making time and meanwhile save the cost of clearing and rescuing.

5 Conclusion

In view of the complexity in the accident-rescue on expressway, multiple influencing factors have been analysed comprehensively. Aiming at the characteristics of uncertain factors, fuzzy programming method is utilized to establish the rescue resource dispatch model. The technique of fuzzy simulation is applied according to the characteristics of the objective function with fuzzy parameters and the fuzzy chance constraint. The genetic algorithm based on fuzzy simulation is designed to solve the dispatch model. The expressway network in Henan Province has the characteristics of multi-accident node and multi-resource allocation. The case study illustrates the modelling idea and the effectiveness of the algorithm designed. The results demonstrate fuzzy dispatch method is suitable for the rescue engineering practice and superior to the existing rescue mode on expressway in the aspect of shortening the rescue time and reducing the economic costs, thus the optimized approach for dispatching the rescue resources is achieved.

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Axiomatization of Fuzzy Core in Games with Fuzzy Coalition

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Abstract. In the framework of fuzzy game with fuzzy coalition, an axiomatization of the fuzzy core is provided by using individually rational, consistency, modularity and anti-monotonicity. Further, examples are given to illustrate the independent of the properties which used in the axiomatization of fuzzy core.

Keywords: Cooperative game, coalition structure, core, consistency, weak converse consistency.

1 Introduction

In the theory of classical cooperative games, all of the players can cooperate with each other, but there are some situations in which some players do not fully take part in a coalition, but do to a certain extent. A coalition in which some players participate partially can be treated as a fuzzy coalition, a term introduced by Aubin [1 2]. The study of solution concepts of cooperative fuzzy games with fuzzy coalition is a interesting question. It has been widely studied in the literature. Aubin [2] and Tsurumi et al. [15] generalized classical solution concepts such as the core and the Shapley value. Branzei et al. [3 4] showed a detailed characterization of the class of convex fuzzy games and study the related solution concepts. Li and Zhang [11] proposed a simplified expression of the Shapley function for games with fuzzy coalition. Yu and Zhang [17] studied the fuzzy core of games with fuzzy coalition, which can be regarded as the generalization of crisp core.

Of all solution concepts of cooperative games, the core is probably the easiest to understand. It is the set of all feasible payoffs that no player or coalition can improve upon by acting for themselves. It has been intensely studied and axiomatized by many authors. For example, in the framework of games with transferable utility and full cooperation, It has been characterized by Peleg[13], Keiding[9], Voorneveld and Nouweland[16], Hwang and Sudhölter[6]. In the framework of games

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with restricted cooperation, it has been studied by Pulido and sánchez-soriano[14], Llerena[10]. In the framework of fuzzy games, it has been studied by Hwang[7], Hwang and Liao[8].

In [7 8], the fuzzy core defined by Aubin has been axiomatized. Note that the fuzzy core in [17], which is coincides with the fuzzy imputation defined for any fuzzy coalition, is different from core in games with fuzzy coalition defined by Aubin. The main purpose of this paper is to provide an axiomatization of the fuzzy core [17] in games with fuzzy coalition.

2 Fuzzy Games with Fuzzy Coalition

We consider cooperative fuzzy games with the set of players $N \quad 1 \ 2 \quad n$. A fuzzy coalition U is a fuzzy subset of N, which is a vector $U \quad (U(1) \ U(2)$ $U(n))$. The number $U(i) \quad [0 \ 1]$ is a constant which denote the participation level of player i. The set of fuzzy coalitions is denoted by $\mathscr{F}(N)$. For a fuzzy coalition $U \quad \mathscr{F}(N)$, the -level set is defined as $[U] \quad i \quad N \quad U(i) \quad [0 \ 1]$, and the support set is denoted by $supp(U) \quad i \quad N \quad U(i) \quad 0$. A cooperative game with fuzzy coalition U is a triple $(N \ U \ v)$ in which the function $v : \mathscr{F}(N)$ such that $v(\ ) \quad 0$. When there is no ambiguity with respect to N, we will refer to the game $(U \ v)$ as $(N \ U \ v)$. The set of all the fuzzy games is denoted by FG^N.

In this paper,we adopt the usual definition of union and intersection of fuzzy subset given by the maximum and minimum operators, i.e.

$$(K \quad U)(i) \quad \max K(i) \ U(i) \quad i \quad N$$

$$(K \quad U)(i) \quad \min K(i) \ U(i) \quad i \quad N$$

Let S_U is defined as follows

$$S_U(i) \quad \begin{matrix} U(i) & \text{if } i \quad S \\ 0 & \text{otherwise} \end{matrix}$$

for any $U \quad \mathscr{F}(N)$ and $S \subseteq N$. We often write i_U instead of i_U, where $i \quad N$.

Let $(U \ v) \quad FG^N$, ^{N} denotes the space of real-valued vectors indexed by N. Given $x \quad ^N$, for all $S \quad \mathscr{P}(N)$, let $x(S_U) \quad \sum_{i \ supp(S_U)} x_i$.

Definition 1. [17] *Let $U \quad \mathscr{F}(N)$. The fuzzy core in a game $(U \ v) \quad FG^N$ with fuzzy coalition U is defined as*

$$C(U \ v) \quad x \quad ^n \quad \sum_{i \ N} x_i \quad v(U) \ x(S_U) \quad v(S_U) \text{ for each } S \quad \mathscr{P}(N)$$

Remark 1. *The above definition of fuzzy core is di erent from Aubin[1,2], in which $x(S_U) \quad \sum_{i \ N} U(i)x_i$.*

A payoff vector in $(U\ v)$ is a vector $x \quad ^{N}$, then

(1) x is e cient in fuzzy coalition U if $\underset{i\ N}{\ } x_i \quad v(U)$. x is individually rational in fuzzy coalition U if for all $i \quad N$, $x(i_U) \quad v(\ i_U\ )$.

(2) x is an imputation of $(U\ v)$ if it is e cient and individually rational in fuzzy coalition U.

Given $(U\ v) \quad FG^N$, the set of feasible payoffs of the game with fuzzy coalition U is defined as

$$X(U\ v) \quad x \quad ^{n}\ x \text{ is e cent}$$

the set of imputations of $(U\ v)$ as

$$I(U\ v) \quad x \quad ^{n}\ x \text{ is an imputation}$$

the set of feasible payoff vectors of $(U\ v)$ as

$$X\ (U\ v) \quad x \quad ^{n} \quad \underset{i\ N}{\ } x_i \quad v(N)$$

A solution on FG^N is a function which associates with each $(U\ v) \quad FG^N$ a subset $(U\ v)$ of $X\ (U\ v)$. A fuzzy game with fuzzy coalition $U \quad \mathscr{F}(N)$ is called fuzzy modular game if there there exists a vector $x \quad (x_1\ x_2 \quad x_n) \quad ^{N}$ such that $v(S_U) \quad x(S_U)$ for all $S \quad \mathscr{P}(N)$.

3 Characterization of the Fuzzy Core

In this section, we will use individually rational, consistency, modularity and anti-monotonicity to characterize the coalition core. First, we will give the definition of reduced game with fuzzy coalition and define the properties that we use to characterize the fuzzy core.

We start by introducing the properties we use to characterize the core. Let $(U\ v) \quad FG^N$, a solution on FG^N satisfies

Anti-monotonicity if for $(U\ u)\ (U\ v) \quad FG^N$ such that $v(S_U) \quad u(S_U)$ for all $S \quad \mathscr{P}(N)$ and $v(U) \quad u(U)$, then $(U\ v) \subseteq (U\ u)$.

Modularity if for any fuzzy modular game $(U\ v_x) \quad FG^N$, $x \quad (U\ v_x)$.

In order to introduce consistency we need to define fuzzy reduced game with fuzzy coalition.

Definition 2. *Let* $(U\ v) \quad FG^N$, $x \quad ^{N}$ *and* $S \subseteq n$. *The fuzzy reduced game with respect to* S *and* x *is the game* $(S_U\ v_{S\ x})$ *defined as*

$$v_{S\ x}(T_U) \quad \begin{array}{ll} 0 & \text{if } T \\ v(U) \quad x((N\ S)_U) & \text{if } T \quad S \\ \max\limits_{H\ N\ S} v(H_U \quad T_U) \quad x(H_U) & T \quad \mathscr{P}(S) \quad S \end{array}$$

Let $(U\ v)\quad FG^N$, $x\quad {}^N$, $\quad S \subseteq n$, $x^S\quad {}^S$ is the restriction of x to S.
Let $(U\ v)\quad FG^N$, a solution on FG^N satisfies

(1) Consistency if for all $\quad S \subseteq N$, and $x\quad (U\ v)$, then $(S_U\ v_{S\,x})\quad FG^N$ and $x^S\quad (S_U\ v_{S\,x})$.

(2) Converse consistency if for $N\quad 2$, $x\quad X(U\ v)$, $\quad S \subseteq N$, $(S_U\ v_{S\,x})\quad FG^N$ and $x^S\quad (S_U\ v_{S\,x})$, then $x\quad (U\ v)$.

(3) Weak converse consistency if for $N\quad 2$, $x\quad I(U\ v)$, $\quad S \subseteq N$, $(S_U\ v_{S\,x})$ FG^N and $x^S\quad (S_U\ v_{S\,x})$, then $x\quad (U\ v)$.

Proposition 1. *The fuzzy core in games with fuzzy coalition U satisfies individually rational, consistency, modularity and anti-monotonicity.*

Proof. It is obvious that the fuzzy core satisfies individually rational, modularity and anti-monotonicity. It remains to show that the fuzzy core satisfies consistency.

Clearly, $(S_U\ v_{S\,x})\quad FG^N$.
Since $x\quad C(U\ v)$, $x(N_U)\quad v(U)$. For any $\quad S \subseteq N$, then

$$v_{S\,x}(S_U)\quad v(U)\quad \sum_{i\ supp((N\ S)_U)} x_i \quad \sum_{i\ supp(N_U)} x_i \quad \sum_{i\ supp((N\ S)_U)} x_i \quad \sum_{i\ supp(S_U)} x_i$$

Hence, x^S is e cient in the reduced game $(S_U\ v_{S\,x})$.
For any $T \subseteq S$ and $T\quad S$,

$$\begin{aligned} v_{S\,x}(T_U) \quad & \max_{H\ N\ S} v(H_U\quad T_U) \quad \sum_{i\ supp(H_U)} x_i \\ & \max_{H\ N\ S} \sum_{i\ supp(H_U \cup T_U)} x_i \quad \sum_{i\ supp(H_U)} x_i \\ & \max_{H\ N\ S} \sum_{i\ supp(T_U)} x_i \quad \sum_{i\ supp(T_U)} x_i \end{aligned}$$

where the inequality follows from $x\quad C(U\ v)$. Therefore $x\quad C(S_U\ v_{S\,x})$.

Proposition 2. *The fuzzy core $C(U\ v)$ satisfies weak converse consistency.*

Proof. Since $x\quad I(U\ v)$, $x(N_U)\quad v(U)$, $x(i_U)\quad v(i_U)$.
For $N\quad 2$ and for any $T\quad \mathscr{P}(N)\quad N$, we will show that $\sum_{i\ supp(T_U)} x_i\quad v(T_U)$.
If $T\quad 1$, then $\sum_{i\ supp(T_U)} x_i\quad v(T_U)$ by the previous step. Now let $T\quad 1$ and $j\quad T$, for any $S \subseteq j\quad (N\ T)$, since $x^S\quad C(S\ v_{S\,x})$,

$$\sum_{i \ supp(j_U)} x_i \quad v_{S\ x}(j_U) \quad \max_{H\ N\ S} v(H_U \quad j_U) \quad \sum_{i \ supp(H_U)} x_i \quad v(T_U) \quad \sum_{i \ supp((T \ \ j)_U)} x_i$$

where the second inequality follows from the observation that $T \quad j \subseteq N \ S$ and $T \quad j$. Hence, $\sum_{i \ supp(T_U)} x_i \quad v(T_U)$.

Proposition 3. *Let be a solution on FG^N satisfies individually rational and consistency, then $(U \ v) \subseteq X(U \ v)$.*

Proof. Let satisfy individually rational and consistency. For any $x \quad (U \ v)$, we will show that $x \quad X(U \ v)$, i.e., $\sum_{i \ N} x_i \quad v(U)$.

Case 1: $N \quad 1$.

From the definition of solution, $x(i_U) \quad v(i_U)$. By individually rational of , $x(i_U) \quad v(i_U)$, thus $x(i_U) \quad v(i_U)$.

Case 2: $N \quad 2$.

Let $i \quad N$, consider the reduced game $(i_U \ v_{i\ x})$. By the consistency of , x_i $(i_U \ v_{i\ x})$. By the definition of reduced game and case 1,$x(i_U) \quad v_{i\ x}(\ i\ ) \quad v(N)$ $x((N \quad i\ )_U)$ thus $\sum_{i \ N} x_i \quad v(U)$.

Proposition 4. *Let be a solution on FG^N satisfies individually rational and consistency, then $(U \ v) \subseteq C(U \ v)$.*

Proof. Let $(U \ v) \quad FG^N$,

If $N \quad 1$, then $(U \ v) \quad I(U \ v) \quad C(U \ v)$ by individually rational.

If $N \quad 2$, then $(U \ v) \quad C(U \ v)$ by individually rational and Proposition 3.

If $N \quad 3$, let $x \quad (U \ v)$. By consistency of , $x_s \quad (S_U \ v_{S\ v})$ for each two players coalition $S \subseteq N$. By the previous step, $(S_U \ v_{S\ x}) \subseteq C(S_U \ v_{S\ x})$ for two players coalitions. individually rational and Proposition 3, $x \quad I(U \ v)$. Hence by Proposition 2, $x \quad C(U \ v)$.

Theorem 1. *A solution on FG^N satisfies individually rational, consistency, modularity and anti-monotonicity if and only if for all $(U \ v) \quad FG^N$, $(U \ v) \quad C(U \ v)$.*

Proof. From Proposition 1 the coalition core $C(U \ v)$ satisfies individually rational, consistency, modularity and anti-monotonicity.

Let $(U \ v) \quad FG^N$ and be a solution satisfies the four axioms above. From Proposition 4 we have that $(U \ v) \subseteq C(U \ v)$. Remains to shoe that $C(U \ v) \subseteq (v)$ for any $(U \ v) \quad FG^N$. For any $x \quad C(U \ v)$, now consider the modular game $(U \ v_x)$ generated by x:

$$v_x(S) \quad \begin{matrix} x(S_U) & \text{if } S \quad \mathscr{P}(N) \\ 0 & \text{otherwise} \end{matrix}$$

Since $v_x(S_U) \quad v(S_U)$ for all $S \quad \mathscr{P}(N)$, and $v_x(U) \quad v(U)$. From modularity and anti-monotonicity, $x \quad (U \ v_x) \subseteq (U \ v)$, Hence $C(U \ v) \subseteq (U \ v)$.

The following examples show that each of the properties used in Theorem 1 is logically independent of the others.

Example. 1 *Let* $(U\ v)$ *for all* $(U\ v)$ FG^N. *Then satisfies individually rational, consistency and anti-monotonicity, but it violates modularity.*

Example. 2 *Let* $(U\ v)$ $X(U\ v)$ *for all* $(U\ v)$ FG^N. *Then satisfies consistency, modularity and anti-monotonicity, but it violates individually rational.*

Example. 3 *Let* $(U\ v)$ $I(U\ v)$ *for any* i N *for all* $(U\ v)$ FG^N. *Then satisfies individually rational, modularity and anti-monotonicity, but it violates consistency.*

Example. 4 *Let* $(U\ v)$ x ${}^n\, x(S_U)$ $v(S_U)$ *for any* S $\mathscr{P}(N)$ *for all* $(U\ v)$ FG^N. *Then it is clearly that satisfies individually rational and modularity.*

For any S $\mathscr{P}(N)$, *then* $v_{S\,x}(S)$ $v(U)$ $x((N\ S)_U)$ $x(N_U)$ $x((N\ S)_U)$ $x(S_U)$

For any T $\mathscr{P}(N)$ S , R $N\ S$, *then*

$$v_{S\,x}(T_U) \quad \max_{H\ N\ S} v(H_U \quad T_U) \quad x(H_U)$$
$$v(R_U \quad T_U) \quad x(R_U)$$
$$x(R_U \quad T_U) \quad x(R_U) \quad x(T_U)$$

Hence, satisfies consistency. But it is easy to shows that violates anti-monotonicity.

4 Conclusions

The goal of this paper is an axiomatic analysis of the fuzzy core in the framework of fuzzy games with fuzzy coalition. It is know that the definition of the reduced game is the key step in axiomatic approach. Various definitions of a reduced game depend upon exactly how are the players outside being paid off. There are two different types of imaginary reduced games in the axiomatic characterizations of the core, the ”max-reduced game”[5] and the ”complement-reduced game”[12]. we use ”max-reduced game” in this paper, it will be interesting to take into account the other forms of reduction.

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Method for Solving the Fuzzy Matrix Game Based on Structured Element

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Abstract. An approach to get the Nash equilibrium solution of fuzzy matrix game is proposed in this paper. At first, the solution of fuzzy coefficient linear programming is given based on the structured element expression of fuzzy number and the structured element weighted order. Then fuzzy coefficient linear programming is transformed into classical linear programming by using the homeomorphism property between fuzzy number space and the family of standard monotone functions in [-1, 1], which simplifies the solving process of fuzzy Nash equilibrium. Finally, an example is presented to compare the method we proposed with other methods, from which a conclusion may be drawn that the method we proposed is of more validity and practicability.

Keywords: Fuzzy matrix game, Nash equilibrium, Structured element, Order.

1 Introduction

Pulverizing system of boiler is the major part in power plants, its safe and economy J.P.Aubin introduced the concept of fuzzy cooperative game which initiated the research on fuzzy game [1, 2]. Campos established the linear programming model to solve fuzzy matrix game in 1989 [3]. Sakawa defined the fuzzy multi-objective pay-off matrix in 1992 [4]. MareÍ extended the concept of fuzzy cooperative game in 1995, which pointed out that cooperative game with fuzzy payment function is a form of fuzzy cooperative games [5, 6]. Maeda indicated the payment of the games in [7]. Kacher gave linear membership functions of fuzzy matrix games [8]. Nishizaki and Sakawa proposed the quadratic programming model of bimatrix game with fuzzy payoff in 2001 [9]. Vidyottama and Bector put forward a programming model of bimatrix game in [10,11].

Earlier work primarily fuzzified classic game from the following three aspects: 1) The policy is fuzzy, as the payment is clear; 2) The payment is fuzzy, as the policy is clear; 3) Both the policy and payment are fuzzy. This article focuses on the second class of fuzzy matrix game problem. There have been some researches on such problem. However, several problems existed within them. For example, Campos reflected the relations of fuzzy comparison in [3], but a large amount of information would be lost in consequence of not taking the degree of membership

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of fuzzy number into account. The order relation between fuzzy numbers defined in [9] depends on the membership functions of fuzzy number. Because of ergodicity of λ in the membership function, this method is relatively difficult in actual calculations. The triangular fuzzy number is studied by using partial order relation between fuzzy numbers in [12]. This approach is not extensive enough as well as weak in distinguishing the partial order relation.

2 The Representation of Fuzzy Number and Sequence Based on Fuzzy Structured Element

Definition 2.1 Let E be a fuzzy set in the real number domain **R**, and $E(x)$ *be the* membership function of E. We call that E is a fuzzy structured element in **R**, if

(i) $E(0)=1, E(1+0)=E(-1-0)=0$;

(ii) $E(x)$ is a monotone increasing and right continuous function on [1,0], and monotonic decreasing and left continuous on (0,1];

(iii) $E(x)=0$ ($-\infty < x < -1$ and $1 < x < +\infty$).

From the definition of fuzzy structured element may seen that E is a special fuzzy number.

Definition 2.2 E is called a regular fuzzy structured element, if

(i) $\forall x \in (-1,1), E(x) > 0$;

(ii) The membership function $E(x)$ is a strictly monotone increasing and continuous on [1,0], strictly monotone decreasing and continuous on (0,1].

If $E(-x)=E(x)$, *then* E is called a symmetrical fuzzy structured element. E is called triangular structured element,if it has membership function

$$E(x) = \begin{cases} 1+x, x \in [-1,0] \\ 1-x, x \in (0,1] \\ 0, else \end{cases}. \quad (1)$$

Theorem 2.1 [13]. Let E be a fuzzy structured element, and $E(x)$ be its membership function, $f(x)$ be a bounded monotone function on [1,1], then $\hat{f}(E)$ is a fuzzy number, and the membership function of $\hat{f}(E)$ is $E(f^{-1}(x))$ (where $f^{-1}(x)$ is the extensive inverse function of $f(x)$). Following, we denote $\hat{f}(x)$ as $f(x)$, denote $\hat{f}(E)$ as $f(E)$.

Theorem 2.2 [13]. Given a regular fuzzy structured element E and arbitrary bounded fuzzy number A, there exist a bounded monotone function $f(x)$ in [1,1], making

$$A = f(E).$$

Theorem 2.3 [17]. If two functions f and g have the same monotonic formal, let E be an arbitrary symmetric fuzzy structured element, $A_1 = f_1(E), A_2 = f_2(E)$. We denote that $f^\tau(x) = -f(-x)$,then

$$\tilde{A}_1 + \tilde{A}_2 = f_1(E) + f_2(E) \qquad \tilde{A}_1 - \tilde{A}_2 = f_1(E) + f_2^\tau(E)$$

$$k \cdot \tilde{A}_1 = \begin{cases} k \cdot f(E), & k \geq 0 \\ |k| f^\tau(E), & k < 0 \end{cases}.$$

Let $\tilde{N}_C(R)$ denote the whole of bounded and closed fuzzy numbers, E is the given regular fuzzy structured element, $\beta_{[-1,1]}$ are all the standard and monotonic functions with the same sequence in [-1, 1].

H_E is called fuzzy functional analysis derived by fuzzy structured element E in $\beta_{[-1,1]}$, if

$$H_E : \beta_{[-1,1]} \rightarrow \tilde{N}_C(R)$$

$$f \rightarrow H_E(f) = f(E) \in \tilde{N}_C(R).$$

Definition 2.3 [14]. For $\tilde{A}$, $\tilde{B} \in \tilde{N}_C(\mathrm{R})$, $f \in H_E^{-1}(\tilde{A})$, $g \in H_E^{-1}(\tilde{B})$, we define the order relation "$\succ$"between fuzzy numbers, let

$$d(\tilde{A}, \tilde{B}) = \int_{-1}^{1} [f(x) - g(x)]dx.$$

If $d(\tilde{A}, \tilde{B}) > 0$,we denote $\tilde{A} \succ \tilde{B}$ (or $\tilde{B} \prec \tilde{A}$); If $d(\tilde{A}, \tilde{B}) = 0$, we denote $\tilde{A} \sim \tilde{B}$.

Definition 2.4 For $\tilde{A}, \tilde{B} \in \tilde{N}_C(\mathrm{R})$, $f \in H_E^{-1}(\tilde{A})$, $g \in H_E^{-1}(\tilde{B})$, let

$$d_E(\tilde{A}, \tilde{B}) = \int_{-1}^{1} E(x)[f(x) - g(x)]dx \tag{2}$$

where $E(x)$ is the membership function of fuzzy structured element. If $d_E(\tilde{A}, \tilde{B}) > 0$,we denote $\tilde{A} \succ_E \tilde{B}$ (or $\tilde{B} \prec \tilde{A}$).If $d_E(\tilde{A}, \tilde{B}) = 0$, we denote $\tilde{A} \sim_E \tilde{B}$ (where $\succ_E$ is the weighted order of fuzzy structured element).

3 The Introduction of Fuzzy Matrix Game Model

For convenient, we consider that there are two players. Let R^n be n-dimensional Euclidean Space, R_n^+ be n-dimensional nonnegative Euclidean Space and $A_{m\times n}, B_{m\times n} \in R^{m\times n}$ are real matrixes. The quaternion $G=(S^m, S^n, A, B)$ denotes bimatrix game, where $S^m=\{x \mid x\in R_n^+, e^T x=1\}$ and $S^n=\{y \mid y\in R_n^+, e^T x=1\}$ are the player's strategy space and , $A=(a_{ij})_{m\times n}$ and $B=(b_{ij})_{m\times n}$ are the player's payoff matrix and respectively.

Definition 3.1 [9]. Let $G=\left(S^m, S^{n,}, A_{m\times n}, B_{m\times n}\right)$ be a bimatrix game. The array $\left(x^*, y^*\right)\in S^m\times S^n$ is called an equilibrium strategy (Nash equilibrium), if it satisfies the following form

$$x^T A_{m\times n} y \le x^{*T} A_{m\times n} y^*,\ \forall x\in S^m,\ x\ge 0^m \tag{3}$$

$$x^T B_{m\times n} y \le x^{*T} B_{m\times n} y^*,\ \forall y\in S^n, y\ge 0^n \tag{4}$$

and $x^{*T} A_{m\times n} y^*$ and $x^{*T} B_{m\times n} y^*$ are called the payoff of the players and respectively, the arry $\left(x^*, y^*\right)\in S^m\times S^n$ is the Nash equilibrium of $G=\left(S^m, S^{n,}, A_{m\times n}, B_{m\times n}\right)$.

In many games, it is impossible to describe the payoffs accurately in advance. For these, the payments of the players and can be written in fuzzy matrixes $\tilde{A}=(\tilde{a}_{ij})_{m\times n}$ and $\tilde{B}=(\tilde{b}_{ij})_{m\times n}$ respectively, where $\tilde{a}_{ij}, \tilde{b}_{ij}$ is fuzzy numbers, which is called the fuzzy payoff of players who adopt different strategies.

Definition 3.2 [12]. Let $\tilde{G}=\left(S^m, S^{n,}, \tilde{A}_{m\times n}, \tilde{B}_{m\times n}\right)$ be a fuzzy bimatrix game, the pair of array $\left(x^*, y^*\right)\in S^m\times S^n$ is called an equilibrium strategy (Nash equilibrium), if it satisfies the following form:

$$x^T \tilde{A}_{m\times n} y \le x^{*T} \tilde{A}_{m\times n} y^*,\ \forall x\in S^m,\ x\ge 0^m \tag{5}$$

$$x^T \tilde{B}_{m\times n} y \le x^{*T} \tilde{B}_{m\times n} y^*,\ \forall y\in S^n, y\ge 0^n \tag{6}$$

where the $x^{*T} \tilde{A}_{m\times n} y^*$ and $x^{*T} \tilde{B}_{m\times n} y^*$ are called the payoff of the players and respectively, and $\left(x^*, y^*\right)\in S^m\times S^n$ is the Nash equilibrium of $\tilde{G}=\left(S^m, S^{n,}, \tilde{A}_{m\times n}, \tilde{B}_{m\times n}\right)$.

4 Fuzzy Multi-attributes Decision-Making Model

As strategies for solving the classical matrix game can be transformed into a linear programming problem, fuzzy matrix game can be transformed into the following fuzzy linear programming problems

$$max\,\tilde{z} = \sum_{j=1}^{n} x_j \tilde{c}_j$$

$$\begin{cases} \sum_{j=1}^{n} \tilde{\alpha}_{ij} x_j \le \tilde{\beta}_i & i = 1, \cdots, m \\ x_j \ge 0 & j = 1, \cdots, n \end{cases} \tag{7}$$

where $\tilde{\alpha}_{ij}$ and $\tilde{\beta}_i$ and $\tilde{c}_j$ are fuzzy numbers, x_j and y_j are exact numbers.

Theorem 4.1. Let fuzzy numbers $\tilde{\alpha}_{ij}, \tilde{\beta}_i, \tilde{c}_j$ in model (7) may be expressed by the regular and symmetric fuzzy structured element E as

$$\tilde{\alpha}_{ij} = f_{ij}(E)\ , \tilde{\beta}_i = g_i(E)\ , \tilde{c}_j = h_j(E)\ .$$

We may assumed that $f_{ij}(x), g_i(x), h_j(x)$ are monotone and increasing functions in [-1, 1] *as* well, then the model (7) becomes

$$max\,M = \sum_{j=1}^{n} \int_{-1}^{1} E(x) x_j h_j(x) dx$$

$$\begin{cases} \sum_{j=1}^{n} \int_{-1}^{1} E(x) x_j f_{ij}(x) dx \le \int_{-1}^{1} E(x) g_i(x) dx & i = 1, \cdots, m \\ x_j \ge 0 & j = 1, \cdots, n \\ f'_{ij}(x) \ge 0, g'_i(x) \ge 0, h'_j(x) \ge 0 & \end{cases}$$

Proof. Let $\tilde{z} = F(E)$ be the expression of the structured element of the fuzzy number $\tilde{z}$, according to the criteria for the order of fuzzy numbers given in Definition 2.2, the standard to measure $\tilde{z}$ is $M = \int_{-1}^{1} E(x) F(E) dx$, to solve the maximization of $\tilde{z}$ is equivalent to solving the maximum of M in equation (7),

$$\tilde{z} = F(E) = \sum_{j=1}^{n} K_j(x, c)$$

where

$$K_j(x, c) = \begin{cases} x_j h_j(E), & x_j \ge 0 \\ |\, x_j \,|\, h_j^{\tau}(E), & x_j < 0 \end{cases}$$

therefore

$$M=\int_{-1}^{1}E(x)F(E)dx=\int_{-1}^{1}E(x)\sum_{j=1}^{n}K_j(x,c)dx=\sum_{j=1}^{n}\int_{-1}^{1}E(x)K_j(x,c)dx$$

when $x_j \geq 0$, the integral for all j are

$$\int_{-1}^{1}E(x)K_j(x,c)dx=\int_{-1}^{1}E(x)x_jh_j(x)dx$$

when $x_j < 0$, the integral for all j are

$$\int_{-1}^{1}E(x)K(x,c)dx=\int_{-1}^{1}E(x)\,|\,x_j\,|\,h_j^{\tau}(x)dx=-x_j\int_{-1}^{1}E(x)(-h_j(-x))dx=x_j\int_{-1}^{1}E(x)h_j(-x)dx$$

Because $E(-x)=E(x)$ we can get the following formula

$$M=\sum_{j=1}^{n}\int_{-1}^{1}E(x)x_jh_j(x)dx\,.$$

Similarly, according to the Definition 2.2, it is not difficult to get

$$\sum_{j=1}^{n}\tilde{\alpha}_{ij}x_j\leq\tilde{\beta}_i\Leftrightarrow\sum_{j=1}^{n}\int_{-1}^{1}E(x)x_jf_{ij}(x)dx\leq\int_{-1}^{1}E(x)g_i(x)dx\,.$$

And because $f_{ij}(x),g_i(x)$ a $h_j(x)$ are monotone and increasing functions,then

$$f_{ij}'(x)\geq 0\,,\;g_i'(x)\geq 0\,,\;h_j'(x)\geq 0\,.$$

Theorem 4.2. Let $\tilde{G}=\left(S^m,S^{n},\tilde{A}_{m\times n},\tilde{B}_{m\times n}\right)$ be a fuzzy bimatrix game, where $\tilde{A}=(\tilde{a}_{ij})_{m\times n}$ and $\tilde{B}=(\tilde{b}_{ij})_{m\times n}$, the expressions of the fuzzy numbers $\tilde{a}_{ij}$ and $\tilde{b}_{ij}$ by the structured element are $\tilde{a}_{ij}=f_{ij}(E)$ and $\tilde{b}_{ij}=g_{ij}(E)$ respectively, $f_{ij}(x)$ and $g_{ij}(x)$ are monotonic functions with the same sequence. So the array $\left(x^*,y^*\right)\in S^m\times S^n$ is the strategy of Nash Equilibrium if it satisfies the following formulas

$$x^T(\int_{-1}^{1}E(x)f_{ij}(x)dx)_{m\times n}y\leq x^{*T}(\int_{-1}^{1}E(x)f_{ij}(x)dx)_{m\times n}y^*,\;\forall x\in S^m\;,\;x\geq 0^m \tag{8}$$

$$x^T(\int_{-1}^{1}E(x)g_{ij}(x)dx)_{m\times n}y\leq x^{*T}(\int_{-1}^{1}E(x)g_{ij}(x)dx)_{m\times n}y^*,\;\forall y\in S^n,\;y\geq 0^n \tag{9}$$

Proof. By definition 3.2 and theorem 4.1, (8) and (9) are clearly established.

According to the different forms of fuzzy numbers, the model is discussed as follows

(1) $\tilde{a}_{ij}$ and $\tilde{b}_{ij}$ are trapezoidal fuzzy numbers

The trapezoidal fuzzy number marked as $\tilde{A}=(a,b;\alpha,\beta)$. If the expression based on structured element is $\tilde{A}=f(E)$,then

$$f(x)=\begin{cases} a+\alpha x, & -1\le x<0 \\ [a,b], & x=0 \\ b+\beta x, & 0<x\le 1 \end{cases}$$

Theorem 4.3. Let $\tilde{a}_{ij}=f_{ij}(E)$ and $\tilde{b}_{ij}=g_{ij}(E)$ .If E is triangular fuzzy structured element, let $\tilde{a}_{ij}=\left(a_{ij},c_{ij};\alpha_{ij},\beta_{ij}\right)$ and $\tilde{b}_{ij}=(b_{ij},d_{ij};\mu_{ij},\nu_{ij})$, then the equation (5), (6) defined in Definition 3.2 are respectively equivalent to

$$x^T(-\frac{1}{6}\alpha_{ij}+\frac{1}{6}\beta_{ij}+\frac{1}{2}a_{ij}+\frac{1}{2}c_{ij})_{m\times n}y^*\le x^{*T}(-\frac{1}{6}\alpha_{ij}+\frac{1}{6}\beta_{ij}+\frac{1}{2}a_{ij}+\frac{1}{2}c_{ij})_{m\times n}y^* \tag{10}$$

$$x^T(-\frac{1}{6}\mu_{ij}+\frac{1}{6}\nu_{ij}+\frac{1}{2}b_{ij}+\frac{1}{2}d_{ij})_{m\times n}y^*\le x^{*T}(-\frac{1}{6}\mu_{ij}+\frac{1}{6}\nu_{ij}+\frac{1}{2}b_{ij}+\frac{1}{2}d_{ij})_{m\times n}y^* \tag{11}$$

Proof. By the Theorem 2.1 and 2.2 we get

$$f_{ij}(x)=\begin{cases} \alpha_{ij}x+a_{ij} & -1\le x\le 0 \\ \beta_{ij}x+c_{ij} & 0<x\le 1 \\ 0 & else \end{cases} \text{ and } g_{ij}(x)=\begin{cases} \mu_{ij}x+b_{ij} & -1\le x\le 0 \\ \nu_{ij}x+d_{ij} & 0<x\le 1 \\ 0 & else \end{cases}$$

Taking $f_{ij}(x)$ $g_{ij}(x)$ into the equation (8) (9) and by the simplification we can get

$$x^T(-\frac{1}{6}\alpha_{ij}+\frac{1}{6}\beta_{ij}+\frac{1}{2}a_{ij}+\frac{1}{2}c_{ij})_{m\times n}y^*\le x^{*T}(-\frac{1}{6}\alpha_{ij}+\frac{1}{6}\beta_{ij}+\frac{1}{2}a_{ij}+\frac{1}{2}c_{ij})_{m\times n}y^*$$

$$x^T(-\frac{1}{6}\mu_{ij}+\frac{1}{6}\nu_{ij}+\frac{1}{2}b_{ij}+\frac{1}{2}d_{ij})_{m\times n}y^*\le x^{*T}(-\frac{1}{6}\mu_{ij}+\frac{1}{6}\nu_{ij}+\frac{1}{2}b_{ij}+\frac{1}{2}d_{ij})_{m\times n}y^* .$$

(2) $\tilde{a}_{ij}$ and $\tilde{b}_{ij}$ are triangular fuzzy numbers

When $a=b$ in $\tilde{A}=\left(a,b;\alpha,\beta\right)$, $\tilde{A}$ is called the triangular fuzzy number, denoted as $\tilde{A}=\left(a;\alpha,\beta\right)$. If it is represented as $\tilde{A}=f(E)$ by the structured element, then

$$f(x)=\begin{cases} a+\alpha x & x\in[-1,0) \\ a+\beta x & x\in[0,1] \end{cases}$$

Corollary 4.1 Let E is triangular fuzzy structured element and $\tilde{a}_{ij}=f_{ij}(E)$ and $\tilde{b}_{ij}=g_{ij}(E)$ are triangular fuzzy numbers. If $\tilde{a}_{ij}=\left(a_{ij};\alpha_{ij},\beta_{ij}\right)$ and $\tilde{b}_{ij}=(b_{ij};\mu_{ij},\nu_{ij})$, then the Equation (5) and (6) in the definition 3.2 are respectively equivalent to

$$x^T(-\frac{1}{6}\alpha_{ij}+\frac{1}{6}\beta_{ij}+a_{ij})_{m\times n}y^*\le x^{*T}(-\frac{1}{6}\alpha_{ij}+\frac{1}{6}\beta_{ij}+a_{ij})_{m\times n}y^* \tag{12}$$

$$x^T(-\frac{1}{6}\mu_{ij}+\frac{1}{6}\nu_{ij}+b_{ij})_{m\times n}y^* \le x^{*T}(-\frac{1}{6}\mu_{ij}+\frac{1}{6}\nu_{ij}+b_{ij})_{m\times n}y^* \quad (13)$$

(3) $\tilde{a}_{ij}$ and $\tilde{b}_{ij}$ are symmetric triangular fuzzy numbers

Corollary 4.2 Let $\tilde{a}_{ij}=f_{ij}(E)$ and $\tilde{b}_{ij}=g_{ij}(E)$. If E is triangular fuzzy structured element, let $\tilde{a}_{ij}=\left(a_{ij};\beta_{ij}\right)$ and $\tilde{b}_{ij}=(b_{ij};\nu_{ij})$, then the Equation (5) and (6) in the Definition 3.2 are respectively equivalent to

$$x^T(a_{ij})_{m\times n}y^* \le x^{*T}(a_{ij})_{m\times n}y^* \quad (14)$$

$$x^T(b_{ij})_{m\times n}y^* \le x^{*T}(b_{ij})_{m\times n}y^* \quad (15)$$

So the steps to solve the Nash equilibrium of fuzzy matrix game whose coefficients are generated by structured element are as follows:

Step 1 According to the form of the fuzzy number $\tilde{A}$ such as $\tilde{A}=f(E)$, where $f(x)$ is non-linear function, $f(x)$ can be obtained according to Theorem 2.2.

Step 2 Put $f(x)$ into the equation (8) and (9) in the Theorem 4.2,by simplification we can obtain the classical linear programming problem.

Step 3 Solve the classical linear programming.

5 Example

Considering the example given in [18], the zero-sum game matrix $\tilde{A}_{2\times 2}=\begin{bmatrix}\widetilde{180} & \widetilde{156}\\ \widetilde{90} & \widetilde{180}\end{bmatrix}$, two conditions is discussed as follows:

1) Let $\widetilde{180}=(180;5,10)$, $\widetilde{156}=$ $(156;6,2)$ and $\widetilde{90}=(90;10,10)$ .When the elements of the matrix is the triangular fuzzy number. Known from Theorem 4.3, the zero-sum game model is equivalent to $min \quad v$

$$s.t \quad \begin{pmatrix} 180-\frac{5}{6}+\frac{10}{6} & 156-\frac{6}{6}+\frac{2}{6} \\ 90-\frac{10}{6}+\frac{10}{6} & 180-\frac{5}{6}+\frac{10}{6} \end{pmatrix}\begin{pmatrix} y_1^* \\ y_2^* \end{pmatrix} \le v$$

$$y_1^*+y_2^*=1$$

$$y_1^*\ge 0, y_2^*\ge 0$$

By the MATLAB function linprog, we can obtain $y_1^*=0.2192$, $y_2^*=0.7808$, $v=160.9229$, and the dual solution is $x_1^*=0.2192$, $x_2^*=0.7808$, $v=160.9229$.

2) When the elements of the Matrix are general fuzzy numbers,

$$\mu_{\tilde{a}_{ij}}(x)=\begin{cases} -\frac{1}{\alpha_{ij}^2}(x-a_{ij})^2+1 & a_{ij}-\alpha_{ij}\le x\le a_{ij} \\ -\frac{1}{\beta_{ij}^2}(x-a_{ij})^2+1 & a_{ij}\le x\le a_{ij}+\beta_{ij} \end{cases},$$

denote that $\tilde{a}_{ij}=(a_{ij}\quad \alpha_{ij}\ ,\ \beta_{ij})$.

By the Theorem 2.3, Then the model of zero-sum game is equivalent to

$$\begin{aligned} &\min \quad v \\ &s.t \quad \begin{pmatrix} 178+\frac{10}{3} & 152+\frac{44}{15} \\ 90 & 178+\frac{10}{3} \end{pmatrix}\begin{pmatrix} y_1^* \\ y_2^* \end{pmatrix}\le v \\ &y_1^*+y_2^*=1 \\ &y_1^*\ge 0, y_2^*\ge 0 \end{aligned}$$

We can obtain $y_1^*=0.2242$, $y_2^*=0.7758$, $v=160.8532$,and the dual solution is $x_1^*=0.2242, x_2^*=0.7758, v=160.8532$.

It is easy to say that the solving process here is much simpler than [3,9,12].

6 Conclusion

By introducing the expression of fuzzy numbers and the sequence of the fuzzy numbers by the structuring element this paper studies the Nash equilibrium solution to matrix game and transforms all-coefficient-fuzzy linear programming into the classical linear programming problem. By comparison, we can see the advantages of this method are as follows: (1) Transform all-coefficient-fuzzy linear programming which is difficult to solve into the corresponding the classical linear programming problem, which made the solving process more simple; (2) By the theory of structured element theory and methods of sequence,ergodicity of λ-cut is skipped, which made the idea of this solution clarify; (3) Transform all the generating function into a linear function, which simplifies the solution process.

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Fuzzy Super-Efficiency DEA Model and Its Application: Based on Fuzzy Structured Element*

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Abstract. The super-efficiency DEA model resolves the ranking problem effectively, which is received much concern. The super-efficiency DEA model is built in this paper, in which inputs and outputs are both fuzzy numbers. The fuzzy super-efficiency DEA model is turned into a classical super-efficiency DEA model according to the structured element-based weighted ordering rules, expanded the scope of use of super-efficiency DEA model. Finally, an example is given to illustrate it.

Keywords: Fuzzy DEA, super-efficiency, structured element, ranking.

1 Introduction

Super-efficiency DEA [1] model is an improved DEA method, and the main difference between it and traditional DEA model is the different reference set. For traditional DEA model, when two or more effective units occur, all DMUs can be totally ordered, but super-efficiency DEA model can do it very well due to the unified effective and non-effective evaluation. Both traditional and super-efficiency DEA model require accurate inputs and outputs, however, in the real production and management activities, the inputs and outputs of decision-making units aren't often determined accurately. Therefore, it is necessary to study DEA model using fuzzy set theory.

In recent years, many scholars have studied DEA model, for example, Guo and Tanaka proposed a fuzzy CCR model [2], which transformed fuzzy constraints into deterministic constraints by use of the comparison rules of fuzzy numbers based on a pre-defined level of possibility. Kao and liu [3] put forward a method transforming fuzzy number into interval number using a-cut set to resolve in order to use traditional DEA model. M.Soleimai damaneh [4] built fuzzy DEA model based on distance function comparison method of fuzzy numbers. Some Chinese scholars have also studied the DEA model. For example, WU Wenjiang and

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CHEN Ying built fuzzy DEA model with L-R fuzzy number. ZHANG Maoqin et al [6] built DEA model based on Campos index. Those two models were built according to the comparison rules of fuzzy numbers. HUANG Chaofeng and LIAO Liangcai [7] studied the effectiveness of decision making units under fuzzy condition with cut set method used. LIANG Liang and WU Jie [8] extended the interval of super-efficiency model. In addition, WANG Mei et al [9] conducted fuzzy extension of super-efficiency model.

All of these fuzzy DEA models can be classified into three categories. The first type of models are built based on the comparison rules of fuzzy numbers. The second ones are built using cut set. And the third ones are built by use of possibility theory. The first category method is used to study fuzzy super-efficiency DEA model in this paper, as [9] does, but the difference from [9] is that this paper uses a new fuzzy comparison rule, which based on fuzzy structured element proposed by Professor GUO Sizong [11], more simple and applicable, suited not only to L-R fuzzy numbers discussed in [9], but to all fuzzy numbers that can be expressed analytically.

2 Related Theories

A. Super-efficiency DEA model

Andersen and Pertersen(1993) [1] proposed super-efficiency ranking method that removed constraint j_0 from CCR multiplier model to, which makes the value of effective unit(if effective) be greater than or equal to 1. The model is as follows.

$$\min \theta_0$$
$$s.t\begin{cases}\sum_{\substack{j=1\\ j\neq j_0}}^{n} \lambda_j x_{ij} \le \theta_0 x_{i0}, i=1,\cdots,m\\ \lambda_j \ge 0, j=1,\cdots,n\\ \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{r0}, r=1,\cdots,s\end{cases} \tag{1}$$

B. Fuzzy numbers expressed as structured element

Let E be a fuzzy set over real number field R, and its membership function is $E(x)$, x R. if $E(x)$ satisfy the following conditions, then E can be called fuzzy structured element over field R: $E(0)=1$; $E(x)$ is a monotone increasing right continuous function in interval [-1,0], and $E(x)$ is a monotone decreasing left continuous function; when $x<-1$ or $x>1$ $E(x)=0$.

If fuzzy structured element E satisfy the following conditions, then we call it regular: $\forall x$ (-1,1) $E(x)>0$ $E(x)$ is continuous, and strictly monotone increasing in interval [-1,0], strictly monotone decreasing in interval (0,1).

Theorem 1 [10]: Let E be an arbitrary fuzzy structured element over field R with membership function $E(x)$, and $f(x)$ is a bounded monotone function in interval [-1,0], then $f(E)$ is a bounded closed fuzzy number over R with membership function $E(f^{1}(x))$, where $f^{1}(x)$ is the rotating symmetric function of $f(x)$ on variables x and y(if $f(x)$ is a continuous and strictly monotone function, then $f^{1}(x)$ is the inverse function of $f(x)$). Conversely, for given regular fuzzy structured element and arbitrary bounded closed fuzzy number $\tilde{A}$, if there is always a bounded monotone function f in interval [-1,1], which makes $\tilde{A}=f(E)$, we call fuzzy number $\tilde{A}$ generated by fuzzy structured element E.

The membership function of fuzzy set E is as follows, called triangle structured element.

$$E(x)=\begin{cases}1+x,-1\le x\le 0\\ 1-x,0<x\le 1\\ 0,\text{others}\end{cases}$$

According to theorem 1, an arbitrary bounded triangular fuzzy number $\tilde{A}$=(a,b,c) can always be generated by the triangle structured element, as long as the interval of value in [-1,1] of piecewise bounded monotone function f(x), and then $\tilde{A}=f(E)$.

$$f(x)=\begin{cases}(b-a)x+b, & -1\le x\le 0\\ -(b-c)x+b & 0\le x\le 1\end{cases}$$

3 The Structured Element- Based Fuzzy Super-Efficiency DEA Model and Its Solution

A. Fuzzy super-efficiency DEA model

When the inputs and outputs in (1) are not accurate but fuzzy, we have the following fuzzy super-efficiency model.

$$\min\theta_0$$
$$s.t\begin{cases}\sum_{\substack{j=1\\ j\ne j_0}}^{n}\lambda_j\tilde{x}_{ij}\tilde{<}\theta_0\tilde{x}_{i0},i=1,\cdots,m\\ \sum_{\substack{j=1\\ j\ne j_0}}^{n}\lambda_j\tilde{y}_{rj}\tilde{>}\tilde{y}_{rj_0},r=1,\cdots,s\\ \lambda_j\ge 0,j=1,\cdots.n\end{cases}\tag{2}$$

Where assume that all fuzzy numbers $\tilde{x}_{ij}$ and $\tilde{y}_{rj}$ can be generated by the same fuzzy structured element.

B. A structured element weighted order of fuzzy number

Definition 1. Let the structured element of bounded fuzzy number $\tilde{A}$ be expressed as $\tilde{A}=f(E)$, where E is a defined fuzzy structured element, f is a monotone function in interval [-1,1]. Let $\|\tilde{A}\|=\int_{-1}^{1}E(x)f(x)dx$, and $\|\tilde{A}\|$ is called structured element weighted characteristic number of bounded fuzzy number $\tilde{A}$, also called characteristic number of $\tilde{A}$ for short.

Definition 2. Let $\tilde{A}_1,\tilde{A}_2\in\tilde{N}_c(R)$, $\tilde{A}_i=f_i(E)$, i=1,2, where E is a given regular fuzzy structured element with membership function $E(x)$, f_1 and f_2 are order-preserving monotone function in interval [-1,1], then "$\prec$" defined by $\tilde{A}_1\prec\tilde{A}_2\Leftrightarrow\|\tilde{A}_1\|<\|\tilde{A}_2\|$ is called fuzzy structured element weighted order.

C. Solution to fuzzy super-efficiency DEA model

As in [9], model (2) is solved using fuzzy structured element weighted order, and the programming obtained is as follows:

$$\min\theta_0$$
$$s.t\begin{cases}\sum_{\substack{j=1\\j\neq j_0}}^{n}\lambda_j\|\tilde{x}_{ij}\|<\theta_0\|\tilde{x}_{i0}\|, i=1,\cdots,m\\ \sum_{\substack{j=1\\j\neq j_0}}^{n}\lambda_j\|\tilde{y}_{rj}\|>\|\tilde{y}_{rj_0}\|, r=1,\cdots,s\\ \lambda_j\geq 0,\ j=1,\cdots.n\end{cases}\tag{2}$$

The programming is a classic linear one, solved more easily, much more simple than that in [13].

4 ERP System Selection Based on Fuzzy DEA Method and Empirical Study

The data selected here derive from [11], of which come from an import and export corporation. After 2005, the company decided to implement ERP system with the development of its business and the changing need of customers and suppliers.

The company received the bid applications from 13 suppliers providing ERP system at home and abroad at primary selection stage through bidding notice. After preliminary investegation, 11 systems were identified as available systems, 6 of which were as the candidate systems for final evaluation and selection through the rating model of product and suppliers. Among the 6 candidate systems, two of them are ERP systems for industry, and others are general(integrated) ERP systems.

The inputs and outputs value of 6 candidate ERP systems(*A* *B* *C* *D* *E* and *F*), all of which are fuzzy numbers, expressed as triangular fuzzy numbers, as is shown in Table 1.

Table 1. The fuzzy inputs/outputs values of ERP systems

Candidate *ERP* systems	Inputs		Outputs	
	Implementation complexity	Estimated implementation costs	Features matching	Corporate image of suppliers
A	(4.5 5 5)	(5.2 5.6 5.8)	(3.3 3.9 4.2)	(4.8 5 5)
B	(4.2 4.5 4.8)	(4.5 5 5.2)	(2.2 3.5 4)	(4.2 4.7 5)
C	(3.3 3.8 4.2)	(1.5 1.6 1.9)	(2.0 2.4 2.7)	(3.5 4 4.2)
D	(3.5 4 4)	(1 1.2 1.8)	(1.2 1.9 2.3)	(1.5 2 2.6)
E	(2 2 3)	(0.5 0.6 0.8)	(2.2 2.4 2.8)	(2.8 3 4)
F	(0 1 1.5)	(1.5 1.65 1.7)	(0.2 0.4 0.5)	(0.5 1 1.2)

According to Definition 1, it is very easy to calculate the characteristic number of triangular fuzzy number $\tilde{A}=(a,b,c)$ is $\left\|\tilde{A}\right\|=\frac{1}{6}(4b+a+c)$. Now, take system *A* for example to calculate the super-efficiency value. The calculated characteristic numbers of all fuzzy numbers are is shown in Table 2.

Table 2. The inputs/outputs of characteristic number of ERP systems

Candidate *ERP* systems	Inputs		Outputs	
	Implementation complexity	Estimated implementation costs	Features matching	Corporate image of suppliers
A	4.9167	5.5667	3.8500	4.9667
B	4.5000	4.9500	3.3667	4.6667
C	3.7833	1.6333	2.3833	3.9500
D	3.9167	1.2667	1.8500	2.0167
E	2.1667	0.6167	2.4333	3.1333
F	0.9167	1.6333	0.3833	0.9500

The super-efficiency value of *A* can be obtained by solving the following linear programming.

$$\min \theta_A$$

$$s.t\begin{cases} 4.50\lambda_2 + 3.78\lambda_3 + 3.92\lambda_4 + 2.17\lambda_5 + 0.92\lambda_6 < 4.92\theta_A \\ 4.95\lambda_2 + 1.64\lambda_3 + 1.27\lambda_4 + 0.62\lambda_5 + 1.63\lambda_6 < 5.57\theta_A \\ 3.37\lambda_2 + 2.38\lambda_3 + 1.85\lambda_4 + 2.43\lambda_5 + 0.38\lambda_6 > 3.85 \\ 4.67\lambda_2 + 3.95\lambda_3 + 2.02\lambda_4 + 3.13\lambda_4 + 0.95\lambda_6 > 4.97 \\ \lambda_j \ge 0, j = 2, \cdots, 6 \end{cases}$$

In the same way, we can get the super-efficiency values of other ERP systems, as is shown in Table 3.

Table 3. The super-efficiency values of each ERP system

ERP systems	A	B	C	D	E	F
Super-efficiency values	0.7003	0.7195	0.7245	0.4214	2.6843	0.7159
Ranking	5	3	2	6	1	4

The results show that the relative effectiveness of system *E* is the largest among the candidate systems, that is, under the premise of specific complexity and cost of implementation, no other ERP systems can do as well as system *E* in features matching and corporate image of suppliers. Simple investegation of data in Table 1 can't identify well the most effective ERP system, however, the fuzzy super-efficiency DEA model based on structured element weighted order transformed complex fuzzy DEA model into a classical super-efficiency DEA model, simplified the selection of best ERP system. In particular, the method is much simpler than that in [9] and [11].

5 Conclusion

Fuzzy super-efficiency DEA model is an all-coefficient-fuzzy linear programming. Many scholars adopted the methods in [12] to solve the problem, which changes a fuzzy constraint into many classic ones, applied to small-sized problem. But the DEA model includes many constraints, so it is very complex to solve fuzzy DEA model. A fuzzy structured element-based order rule is used in this paper, which transformed fuzzy DEA model into classic DEA model, not increased constraints, but broadened the applicability and effectiveness.

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2-Tuple Linguistic Fuzzy ISM and Its Application

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Abstract. In view of the complexity of actual decision-making problem and the cognitive fuzziness of decision-makers, we present a modified fuzzy-ISM based on 2-tuple linguistic representation information processing technology in this paper. Using TAM operator, we process 2-tuple semantic information of decision-making. After integration and standardization, we built the 2-tuple linguistic representation fuzzy interpretive structural model (2TLR-FISM). This model is more accurate for processing fuzzy semantic information than conventional ISM which may process semantic information rough and is easy to make distortion and loss of information. Finally, take a case study of analysing the influential factors of emergency management to illustrate the feasibility of the method.

Keywords: 2-Tuple Linguistic Representation, 2TLR-FISM, Emergency Management.

1 Introduction

In 1973, professor J.N.Warfield developed a new method for analysis of complex system of social economic system structure [1,2]. The basic idea is: firstly, extract the elements of an issue by using various creative technologies. Secondly, process the information of the elements and their relationship by using some special approach or computer technology such as digraph, matrix, etc. Finally, give some interpretative statement about the levels and the overall structure of the issue. Make the problem clear. The model is visualized and enlightening. So it is widely used in solving various socio-economic system problems [3].

But, the conventional ISM has its problem. Rough evaluations often cause distortion and loss of information [4]. As the complexity and uncertainty of objective things, and ambiguity of human thinking, people prefer to use the words such as 'good', 'bad' or 'influential', 'no influential' to evaluate things like 'commodity performance' or 'relationship between influencing factors [5].

Considering about this phenomenon, we can introduce a kind of linguistic term evaluation scale with subscript. When decision maker is making a linguistic measure, just choose an appropriate linguistic evaluation scale and make the assessment directly. And then, use dual semantic model converting them into numerical calculation. It can greatly extend the application scope of ISM and make the analysis more reliability [6].

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Based on these, this paper presents a fuzzy interpretive structural model based on 2-tuple linguistic representation information processing (2TLR-FISM). Using TAM operator (the 2-tuple linguistic representation average operators) accomplish the information integration and standardization. And then built the 2TLR-FISM to solve practical problems.

2 Construct a 2TLR-FISM

Martinez and Herrera [7-10] construct a fuzzy language description model for integrated language information. The basic element of the model is two-dimensional array (s,α) which is made up from a linguistic terms $s \in S_1$ and a real number. $\alpha \in [-0.5, 0.5)$. In which, α is representative of the coefficient of symbols conversion. The specific is as follows

Definition 1. *Suppose that Si ∈S is linguistic term and then the corresponding 2-tuple linguistic representation can be transformed with the function θ as follow:*

$$\begin{aligned} &\theta : S \to (S \times [-0.5, 0.5]), \\ &\theta(s_i) = (s_i, 0) / s_i \in S. \end{aligned} \tag{1}$$

In which the coefficient of symbols conversion of linguistic term s_i is a number in $[-0.5, 0.5)$ It indicate the information difference between $\beta \in [0,\tau]$ (information from symbols integration) and the index which is closest to β in S_1 (set of linguistic term) $i = round(\beta)$.

Definition 2. *Suppose that* β *is an integrated value of a group of linguistic term subscript in the set* S_1 *which is called symbol integration value.* $\beta \in [0,\tau]$ $\tau + 1$ *is the potential of* S_1. *Then suppose that* $i = round(\beta)$ *and* $\alpha = \beta - i$ *in which* $i \in [0,\tau]$ *and* $\alpha \in [-0.5, 0.5)$. α *is the value of symbol transformation.*

$$\Delta : [0,\tau] \to S \times [-0.5, 0.5)$$

$$\Delta(\beta) = (s_i, \alpha) = \begin{cases} s_i, & i = round(\beta) \\ \alpha = \beta - i & \alpha \in [-0.5, 0.5) \end{cases} \tag{2}$$

In which, $i = round(\beta)$ $\alpha = \beta - i$ $\alpha \in [-0.5, 0.5)$. "Round" means half adjust rounding operation.

For the set S_1 and two-tuples (s_i, α) There is always an inverse function Δ^{-1} which can reduce the two-tuples (s_i, α) to an equivalent value $\beta \in [0, \tau]$.

$$\begin{aligned} &\Delta^{-1} : S_1 \times [-0.5, 0.5) \rightarrow [0, \tau], \\ &\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta. \end{aligned} \tag{3}$$

Definition 3. *TAM operator the 2-tuple average operators suppose that* $((r_1, \alpha_1), (r_2, \alpha_2), \cdots, (r_n, \alpha_n))$ *is a group of two-tuples. Then:*

$$TAM((r_1, \alpha_1), (r_2, \alpha_2), \cdots, (r_n, \alpha_n)) = \Delta\left(\sum_{i=1}^{n} \frac{1}{n} \Delta^{-1}(r_i, \alpha_i)\right) = \Delta\left(\frac{1}{n} \sum_{i=1}^{n} \beta_i\right) \tag{4}$$

Definition 4. *Standardization: Each element of the matrix divided by the total maximum value element. That is:*

$$P_{ij}' = \frac{P_{ij}}{\max(P_{ij})} \tag{5}$$

According to these definitions above, a building process of the 2TLR-ISM is given as follow.

Suppose that the set of influencing factors is $F = \{A_1, A_2, \ldots, A_n\} (n \geq 2)$, experts set is $P = \{p_1, p_2, \ldots, p_m\} (m \geq 2)$. Expert P_n choose a linguistic term from linguistic assessment set S for describing the influence degree factor A_i to factor A_j, marked p_{ij}^{m}. Then we obtain $n \times n$ matrix $P_1, P_2, \ldots P_m$. The quantity is m. That is original assessment matrix given by expert p_m. This article does not consider the factor's directly impact on their own. So the main diagonal element of matrix P_m is marked"—", in the calculation, "—"means zero.

According to (1), each expert's original assessment matrix P_m is converted to a corresponding 2-tuple linguistic representation assessment matrix P_m';

Then, according to 3 , transform every element of P_m' into its corresponding value β. Then get a numerical value matrix P_m^{β}:

$$P_m' = \begin{Bmatrix} - & (s_{12},\alpha_{12}) & \cdots & (s_{1n},\alpha_{1n}) \\ (s_{21},\alpha_{21}) & - & \cdots & (s_{2n},\alpha_{2n}) \\ \vdots & \vdots & \vdots & \vdots \\ (s_{n1},\alpha_{n1}) & (s_{n2},\alpha_{n2}) & \cdots & - \end{Bmatrix} \quad P_m^{\beta} = \begin{Bmatrix} - & \beta_{12} & \cdots & \beta_{1n} \\ \beta_{21} & - & \cdots & \beta_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \beta_{n1} & \beta_{n2} & \cdots & - \end{Bmatrix}$$

According to (4), aggregating the information of matrices $P_1^{\beta}, P_2^{\beta}, \ldots, P_m^{\beta}$, we obtain a total assessment matrix P_0 of 2-tuple linguistic representation group. Though standardization, P_0 convert to fuzzy matrix P_0'. Each element of it is within 0 to 1. Then, convert this fuzzy matrix to a corresponding 0-1 matrix with an appropriate threshold value. This final 0-1matrix is the adjacency matrix of ISM. It is marked A. In this way, we can use ISM for further structural analysis.

3 Application Examples

Urban emergency management is affected by multiple factors which are interrelated and interact on each other and formed a very complex system [11] . Using the interpretation structure model to analyse, we can found the superficial and direct influencing factors, intermediate and indirect influencing factors, essential and basic influencing factors of urban emergency management from the complex factors chain. Using the method of bibliometrics [12], we determine fourteen factors of the urban emergency management : emergency management S_0 , laws and regulations S_1 , emergency plan S_2 , testing and warning S_3 , the emergency response ability of government S_4 , relief supplies S_5 , capital S_6 , communication S_7 , professional rescue teams S_8 , command and coordination S_9 , publicize S_{10}, training and drills S_{11} , ability of information sharing S_{12} , restoration and reconstruction S_{13}[13,14].

The language evaluation set of the factors which affect each other as follow:

$$S = \begin{Bmatrix} s_0 = no(\text{no directly affected}) \\ s_1, = l(\text{little directly affected}) \\ s_2 = m(\text{middle directly affected}) \\ s_3 = h(\text{huge directly affected}) \\ s_4 = vh(\text{very huge directly affected}) \end{Bmatrix}$$

First, there are three experts analyse the correlation of each factors and score respectively. Three groups of results are as follows:

$$p_1 = \left\{\begin{matrix} - & no & no & no & no & no & no & no & no & no & no & no & no & no \\ no & - & no & no & vh & no & no & l & l & h & h & no & vh & vh \\ vh & no & - & no & vh & l & no & no & vh & vh & no & vh & no & no \\ vh & no & l & - & no & no & no & no & l & no & l & no & l & no \\ vh & no & no & l & - & no & no & no & no & vh & no & l & vh & no \\ vh & no & no & no & l & - & no & no & no & vh & no & no & no & no \\ vh & no & no & no & vh & vh & - & vh & no & vh & no & no & no & vh \\ vh & no & no & vh & vh & no & no & - & no & l & no & no & vh & l \\ vh & no & no & no & no & l & no & l & - & l & no & no & no & no \\ vh & no & no & no & no & vh & no & no & no & - & l & no & l & l \\ vh & l & no & no & l & no & no & l & no & no & - & no & vh & no \\ vh & no & vh & no & no & no & no & no & no & no & no & - & no & no \\ vh & no & vh & no & vh & no & no & l & no & vh & no & no & - & no \\ vh & l & no & no & l & no & no & no & no & no & no & no & no & - \end{matrix}\right\}$$

$$p_2 = \left\{\begin{matrix} - & no & l & no & no & no & l & no & l & no & no & no & l & no \\ no & - & no & no & h & no & l & no & l & vh & vh & no & h & vh \\ h & l & - & l & vh & no & no & no & h & h & no & vh & no & no \\ vh & no & no & - & l & no & no & l & l & l & no & no & l & no \\ h & no & no & l & - & l & l & no & l & vh & l & no & h & no \\ vh & l & no & l & l & - & no & no & l & vh & no & l & no & no \\ vh & no & l & l & vh & vh & - & h & l & h & no & no & no & vh \\ h & l & no & vh & h & no & no & - & no & no & no & no & vh & no \\ vh & no & no & no & no & no & no & l & - & no & l & no & no & l \\ vh & no & l & l & no & vh & no & no & l & - & no & no & no & l \\ vh & no & no & no & l & no & no & no & l & no & - & no & h & l \\ h & l & h & no & no & l & no & no & l & l & no & - & l & no \\ h & no & vh & no & vh & no & l & l & no & vh & no & no & - & no \\ vh & no & no & no & l & no & l & l & l & l & l & l & no & - \end{matrix}\right\}$$

$$p_3 = \left\{ \begin{matrix} - & l & no & no & l & no & l & l & l & l & no & no & no & l \\ no & - & l & no & h & l & l & no & no & vh & vh & l & vh & vh \\ h & l & - & l & vh & no & no & l & vh & vh & no & h & no & l \\ vh & l & l & - & no & no & no & no & l & no & no & l & l & no \\ vh & no & l & l & - & no & no & no & no & h & no & l & vh & no \\ h & no & no & no & l & - & l & no & no & h & l & l & no & no \\ h & no & l & no & vh & vh & - & vh & no & h & l & l & no & vh \\ vh & no & no & h & vh & no & no & - & no & no & no & no & vh & no \\ vh & l & no & l & no & no & no & no & - & l & no & no & l & no \\ vh & l & no & l & no & h & no & m & l & - & no & no & no & no \\ vh & l & l & no & no & no & no & l & no & no & - & no & vh & l \\ vh & no & vh & l & no & l & no & l & no & no & l & - & l & l \\ vh & no & vh & no & vh & no & l & l & no & vh & l & l & - & no \\ h & no & no & no & l & no & l & no & no & no & no & no & no & - \end{matrix} \right\}$$

Then, according to the steps of the second part, applying the formula in the definition 1-4, we can get the numerical matrix $P_1^{\beta}, P_2^{\beta}, P_3^{\beta}$. Take P_1^{β} for example:

$$P_1^{\beta} = \left\{ \begin{matrix} - & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & - & 0 & 0 & 4 & 0 & 0 & 1 & 1 & 3 & 3 & 0 & 4 & 4 \\ 4 & 0 & - & 0 & 4 & 1 & 0 & 1 & 4 & 4 & 0 & 4 & 0 & 0 \\ 4 & 0 & 1 & - & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 4 & 0 & 0 & 1 & - & 0 & 0 & 1 & 0 & 4 & 0 & 1 & 4 & 0 \\ 4 & 0 & 0 & 0 & 1 & - & 0 & 1 & 0 & 4 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 4 & 4 & - & 4 & 0 & 4 & 0 & 0 & 0 & 4 \\ 4 & 0 & 0 & 4 & 4 & 0 & 0 & - & 0 & 1 & 0 & 0 & 4 & 1 \\ 4 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & - & 1 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 4 & 0 & 4 & 0 & - & 1 & 0 & 1 & 1 \\ 4 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & - & 0 & 4 & 0 \\ 4 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & - & 0 & 0 \\ 4 & 0 & 4 & 0 & 4 & 0 & 0 & 1 & 0 & 4 & 0 & 0 & - & 0 \\ 4 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & - \end{matrix} \right\}$$

Using TAM operator to get information aggregation, we get 2-tuple linguistic group evaluation matrix P_0 .Standardize the matrix P_0 , we get the fuzzy matrix $P_0{'}$:

$$
P_0' = \begin{bmatrix}
- & 0.0825 & 0.0825 & 0 & 0.0825 & 0.0825 & 0.0825 & 0.0825 & 0.1675 & 0.0825 & 0 & 0 & 0.0825 & 0.0825 \\
0 & - & 0.0825 & 0 & 0.8325 & 0.0825 & 0.1675 & 0.0825 & 0.1675 & 0.9175 & 0.9175 & 0.0825 & 0.9175 & 1 \\
0.8325 & 0.0825 & - & 0.1675 & 1 & 0 & 0 & 0.0825 & 0.9175 & 0.9175 & 0 & 0.9175 & 0 & 0.0825 \\
1 & 0.0825 & 0.1675 & - & 0.0825 & 0 & 0 & 0.0825 & 0.2500 & 0.0825 & 0.0825 & 0.1675 & 0.2500 & 0.0825 \\
0.9175 & 0 & 0.0825 & 0.2500 & - & 0.0825 & 0.0825 & 0 & 0.0825 & 0.9175 & 0.0825 & 0.1675 & 0.9175 & 0 \\
0.9175 & 0.0825 & 0 & 0.0825 & 0.2500 & - & 0.0825 & 0 & 0.0825 & 0.9175 & 0.0825 & 0.1675 & 0 & 0 \\
0.9175 & 0 & 0.1675 & 0.0825 & 1 & 1 & - & 0.9175 & 0.0825 & 0.8325 & 0.0825 & 0.0825 & 0 & 1 \\
0.9175 & 0.0825 & 0 & 0.9175 & 0.9175 & 0 & 0 & - & 0 & 0.0825 & 0 & 0 & 1 & 0.0825 \\
1 & 0.0825 & 0 & 0.0825 & 0 & 0.0825 & 0 & 0.1675 & - & 0.1675 & 0.1675 & 0 & 0.0825 & 0.0825 \\
1 & 0.0825 & 0.0825 & 0.1675 & 0 & 0.9175 & 0 & 0.1675 & 0.1675 & - & 0.0825 & 0 & 0.0825 & 0.1675 \\
1 & 0.1675 & 0.0825 & 0 & 0.1675 & 0 & 0 & 0.1675 & 0.0825 & 0 & - & 0 & 0.9175 & 0.1675 \\
0.9175 & 0.0825 & 0.9175 & 0.0825 & 0 & 0.1675 & 0 & 0.0825 & 0.0825 & 0.0825 & 0.0825 & - & 0.1675 & 0.0825 \\
0.9175 & 0 & 1 & 0 & 1 & 0 & 0.1675 & 0.2500 & 0 & 1 & 0.0825 & 0.0825 & - & 0 \\
0.9175 & 0 & 0 & 0 & 0.2500 & 0 & 0.1675 & 0.0825 & 0.0825 & 0.0825 & 0.0825 & 0.0825 & 0 & -
\end{bmatrix}
$$

Selecting threshold for 0.8 to get 0-1 type adjacency matrix, if the element in the matrix greater than or equal to 0.8, then assign 1, or assign 0, so we can get the incidence matrix R. Then, we get the reachability matrix $M=(R+I)^4$ from incidence matrix R ,(where I is an identity matrix which has same order with R).Calculated by Matlab, there is:

$$
M = \begin{bmatrix}
1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\
1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\
1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\
1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\
1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

According to reachability matrix M, we calculated the reachable set $R(S_i)$, antecedent set $A(S_i)$, and the common set $R(S_i) \cap A(S_i)$.

Table 1. The Level 1 of Reachable Set and Antecedent Set

Si	$R(S_i)$	$A(S_i)$	$R(S_i) \cap A(S_i)$
0	0	0,1,2,3,4,5,6,7,8,9,10,11,12,13	0
1	0,1,2,4,5,8,9,10,11,12,13	1	1
2	0,2,4,5,8,9,11,12	1,2,4,6,7,10,11,12	2,4,11,12
3	0,3	3,6,7	3
4	0,2,4,5,8,9,11,12	1,2,4,6,7,10,11,12	2,4,11,12
5	0,5,9	1,2,4,5,6,7,9,10,11,12	5,9
6	0,2,3,4,5,6,7,8,9,11,12,13	6	6
7	0,2,3,4,5,7,8,9,11,12	6,7	7
8	0,8	1,2,4,7,8,10,11,12	8
9	0,5,9	1,2,4,5,6,7,9,10,11,12	5,9
10	0,2,4,5,8,9,10,11,12	1,10	10
11	0,2,4,5,8,9,11,12	1,2,4,7,10,11,12	2,4,11,12
12	0,2,4,5,8,9,11,12	1,2,4,6,7,10,11,12	2,4,11,12
13	0,13	1,6,13	13

Through the data table 1, we can analyze the main influencing factors of urban emergency management is the grade 1 nodes: $L_1 = \{0\}$. Crossed out row 1 and column 1 in the reachability matrix to find the second nodes, Similarly, the grade 2 nodes: $L_2 = \{3 \quad 5 \quad 9 \quad 13\}$, the grade 3 nodes: $L_3 = \{2 \quad 4 \quad 11 \quad 12\}$,the grade 4 and 5 nodes: $L_4 = \{7 \quad 10\}$ $L_5 = \{1 \quad 6\}$.

According to the above analysis, we can build the hierarchical structure model about the main influencing factors of urban emergency management, as the figure 1:

According to the ISM model of figure 1, the urban emergency management is a five multilevel hierarchical structure. The emergency management S_0 is the first-level influencing factors; Relief supplies S_5, command and coordination S_9, testing and warning S_3, professional rescue teams S_8 and restoration and reconstruction S_{13} are the second-level influencing factors, and they are the superficial and direct influencing factors; Emergency plan S_2 , the emergency response ability of government S_4 , training and drills S_{11}, ability of information sharing S_{12} are the third-level influencing factors, and they are intermediate and indirect influencing factors which affect emergency capability though the second-level influencing factors; Communication S_7 and publicize S_{10} are the fourth-level influencing factors; capital S_6 and laws and regulations S_1 are the fifth-level influencing factors which are essential influencing factors.

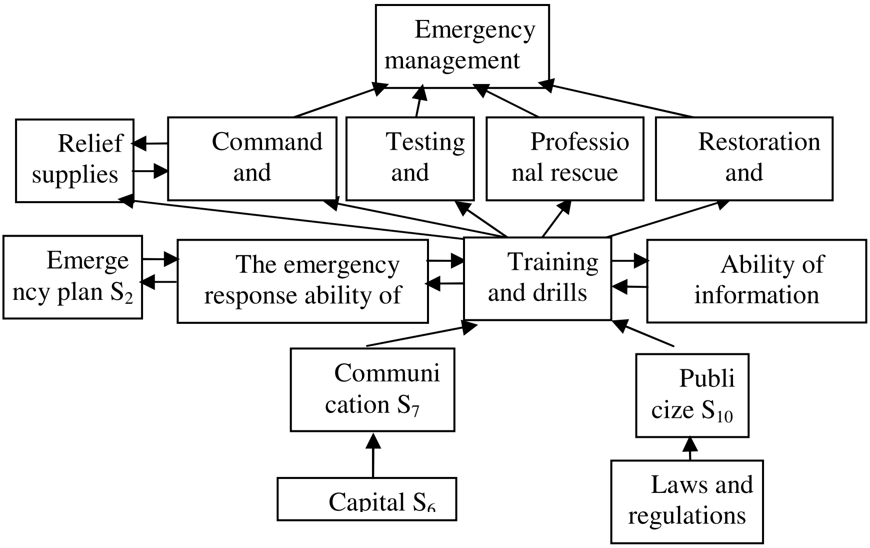

Fig. 1. The hierarchical structure model of the main influencing factors of urban emergency management

4 Conclusion

By introducing the development of 2-tuple linguistic which is used to calculate the linguistic assessment information in recent years, this paper presented an information pressing fuzzy interpretive structural model based on the 2-tuple linguistic, achieved fuzzy language information processing and improved the traditional ISM technology to avoid the lose of language information processing.

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Stochastic Single Machine Scheduling to Minimize the Weighted Number of Tardy Jobs

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Abstract. A single machine stochastic scheduling problems with the jobs' processing times are considered the random of uniform distribution and the objective is to find an optimal schedule to minimize the expectation of the weighted numbers of tardy jobs from a common due date. By theoretical analysis, the problem formulation of the expectation of the weighted numbers of tardy jobs can be given. For the two cases (1) the weights of jobs are equal and (2) the weights of jobs are proportional to their processing times, the SEPT (shortest expected processing time first) is optimal. Joint use solution the SEPT and the LEPT (longest expected processing time first) is optimal in the case of the expected processing time is non-proportional to weighty of the jobs. The optimality of the algorithms is proved.

Keywords: Single machine, random processing times, uniform distribution, number of tardy jobs, priority policy.

1 Introduction

It is important considering the random factors (such as the stochastic processing time and breakdowns, etc.) of the scheduling problem in practical environments. Therefore, the stochastic scheduling problems are a widespread concern [1-3]. The vast majority of research is only involved in regular objective (the objective function is an increasing function of completion time). With the JIT production ideas in practice widely adopted, more attention about non-formal objective function has been paid [4, 5].

In many cases, the duration of the delay of job is not important. As long as there was delayed, the impact is the same. If the jobs can not be completed on schedule, it should be some punishment. In this case, the objective function should be minimizing the numbers of tardy jobs.

Under the problem of deciding how to schedule n jobs with required processing times p_j, $j = 1, \cdots, n$, on a single machine where a job is deemed tardy and

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assessed a penalty of w_i, $i = 1, \cdots, n$, which is completed after a common deadline (due-date) t.The objective to find a sequence that minimizes the sum of penalties.

When p_j, $j = 1, \cdots, n$ and t are certain, it is well known that the weighted number of tardy jobs (WNT) problem reduces to a knapsack problem and is NP-hard. When $w_i = 1$, $i = 1, \cdots, n$ (WNT) becomes the number of tardy jobs (NT) problem, and can be easily solved by scheduling in shortest processing time (SPT) order.

When the processing times and deadline are random variables the stochastic weighted numbers of tardy jobs (SWNT) problem can be stated as:

$$1 \mid p_j \sim sto, d_j = d \mid E(\sum w_j U_j) \tag{1}$$

Sarin et al. [6, 7] have studied this variation when t_i, $i = 1, \cdots, n$ are independent random variables with exponential and geometric distribution and the jobs have a common due date. The two cases (1) the processing times are equal and (2) the weights of jobs are proportional to their processing times where the processing times are independent with uniform distribution has been investigated by Li and Zeng [8].

2 Problem Description

In this paper, we examine the solution of the stochastic scheduling problems when p_j, $j = 1, \cdots, n$ are independent random variables with uniform distribution and the jobs have a common due date. The objective function is minimizes the expectation of the weighted numbers of tardy jobs,

$$1 \mid p_j \sim U(0, \lambda_j), d_j = d \mid E(\sum w_j U_j) \tag{2}$$

Let $p_j \sim U(0, \lambda_j)$ denote independent random variables with uniform distribution of $(0, \lambda_j)$, their density functions

$$f_{p_j}(x) = \begin{cases} \dfrac{1}{\lambda_j}, & 0 < x < \lambda_j \\ 0, & \text{otherwise} \end{cases},$$

And $E(p_j) = \dfrac{\lambda_j}{2}$. We assumed the jobs have a common due-date d. The several properties of the optimal sequences are established.

We assumed the machine is idle on time t and the sequence has two jobs which are waiting. Assumed the processing times is p_1, p_2, and the weight is w_1 and w_2.

Note that we will arrange the job which the processing time is less than the other first if $d-t>\min(\lambda_1,\lambda_2)$ to minimize the expectation of the weighted numbers of tardy jobs.

If $d-t<\lambda_1$ and $d-t<\lambda_2$, the probability of two jobs are tardy the sequence is T_1 between T_2

$$P(p_1>d-t)=\int_{d-t}^{\lambda_1}\frac{1}{\lambda_1}dx=\frac{1}{\lambda_1}(\lambda_1-(d-t))=1-\frac{1}{\lambda_1}(d-t)$$

The probability of job2 is tardy the sequence is T_1 between T_2

$$P(p_1<d-t,p_1+p_2>d-t)$$

$$=\int_0^{d-t}dx\int_{d-t-x}^{\lambda_2}\frac{1}{\lambda_1\lambda_2}dy=\frac{1}{\lambda_1\lambda_2}\left[\lambda_2(d-t)-\frac{1}{2}(d-t)^2\right]$$

The probability of two jobs are tardy the sequence is T_2 between T_1

$$P(p_2>d-t)=\int_{d-t}^{\lambda_2}\frac{1}{\lambda_2}dt=1-\frac{1}{\lambda_2}(d-t)$$

The probability of job1 is tardy the sequence is T_2 between T_1

$$P(p_2<d-t,p_1+p_2>d-t)=\int_0^{d-t}dy\int_{d-t-y}^{\lambda_1}\frac{1}{\lambda_1\lambda_2}dx$$

$$=\frac{1}{\lambda_1\lambda_2}\left[\lambda_1(d-t)-\frac{1}{2}(d-t)^2\right]$$

Let $E(\sum wU(1,2))$ denotes the expectation of the weighted numbers of tardy jobs which the sequence is T_{21}between T_2

$$E(\sum wU(1,2))$$

$$=w_2\frac{1}{\lambda_1\lambda_2}\left[\lambda_2(d-t)-\frac{1}{2}(d-t)^2\right]+(w_1+w_2)\left[1-\frac{1}{\lambda_1}(d-t)\right]$$

$$=(w_1+w_2)-\frac{w_1}{\lambda_1}(d-t)-\frac{w_2}{2\lambda_1\lambda_2}(d-t)^2$$

Let $E(\sum wU(2,1))$ denotes the expectation of the weighted numbers of tardy jobs which the sequence is T_2 between T_1

$$\begin{aligned}&E\left(\sum wU(2,1)\right)\\&=w_1\frac{1}{\lambda_1\lambda_2}\left[\lambda_1(d-t)-\frac{1}{2}(d-t)^2\right]+(w_1+w_2)\left[1-\frac{1}{\lambda_2}(d-t)\right]\\&=(w_1+w_2)-\frac{w_2}{\lambda_2}(d-t)-\frac{w_1}{2\lambda_1\lambda_2}(d-t)^2\end{aligned}$$

3 Results

Under the problem (2), we calculate

$$E\left(\sum wU(1,2)\right)-E\left(\sum wU(2,1)\right)=(d-t)\left(\frac{w_2}{\lambda_2}-\frac{w_1}{\lambda_1}\right)+\frac{1}{2\lambda_1\lambda_2}(d-t)^2(w_1-w_2) \quad (3$$

For the two cases (1) the weights of jobs are equal and (2) the weights of jobs are proportional to their processing times, the SEPT (shortest expected processing time first) is optimal.

For the other cases

$$\begin{aligned}E\left(\sum wU(1,2)\right)-E\left(\sum wU(2,1)\right)&=(d-t)\left(\frac{w_2}{\lambda_2}-\frac{w_1}{\lambda_1}\right)+\frac{1}{2\lambda_1\lambda_2}(d-t)^2(w_1-w_2)\\&=\frac{w_1-w_2}{2\lambda_1\lambda_2}\left[\left(d-t-\frac{\lambda_2w_1-\lambda_1w_2}{w_1-w_2}\right)^2-\left(\frac{\lambda_1w_2-\lambda_2w_1}{w_1-w_2}\right)^2\right]\end{aligned} \quad (4$$

Hence, we have a conclusion of the expression 4. An optimal sequence that stochastically minimizes the expectation of the weighted numbers of tardy jobs from a common due date can be constructed by the following rule for pairs of jobs.

Algorithm 1

Step1 $\mu = 2\frac{\lambda_j w_i-\lambda_i w_j}{w_i-w_j}$.

Step 2 If $\mu<0$ place job i first if $\frac{w_i}{\lambda_i}>\frac{w_j}{\lambda_j}$; If $\mu>0$,go to step 3.

Step 3 If $d-t>\mu$, place job j first if $\frac{w_i}{\lambda_i}>\frac{w_j}{\lambda_j}$; else, place job i first if $\frac{w_i}{\lambda_i}>\frac{w_j}{\lambda_j}$.

The Algorithm 1 established the sequence of two jobs. The sequence of n jobs is established by the position of the pairs of jobs using the following Theorem 1.

Table 1. The value of the expression (4)

condition1	condition 2	condition 3	conclusion
$w_1>w_2$	$\frac{w_1}{\lambda_1}<\frac{w_2}{\lambda_2}$		≥ 0, job 2 first
	$\frac{w_1}{\lambda_1}>\frac{w_2}{\lambda_2}$	$d-t<-2\frac{\lambda_1 w_2-\lambda_2 w_1}{w_1-w_2}$	≤ 0, job 1 first
		$d-t>-2\frac{\lambda_1 w_2-\lambda_2 w_1}{w_1-w_2}$	≥ 0, job 2 first
$w_1<w_2$	$\frac{w_1}{\lambda_1}<\frac{w_2}{\lambda_2}$	$d-t<-2\frac{\lambda_1 w_2-\lambda_2 w_1}{w_1-w_2}$	≥ 0, job 2 first
		$d-t>-2\frac{\lambda_1 w_2-\lambda_2 w_1}{w_1-w_2}$	≤ 0, job 1 first
	$\frac{w_1}{\lambda_1}>\frac{w_2}{\lambda_2}$		≤ 0, job 1 first

Theorem 1. The non-preemptive static list policy of the Algorithm 1 is optimal for the problem (2) when two jobs is in the sequence.

Proof: The value of the last column of table1 determined the sequence of two jobs,

If $E(\sum wU(1,2))-E(\sum wU(2,1))>0$ job T_2 should precede job T_1;

If $E(\sum wU(1,2))-E(\sum wU(2,1))<0$ job T_1 should precede job T_2

This completes the proof.

An optimal sequence that stochastically minimizes the expectation of the weighted numbers of tardy jobs can be constructed by the theorem for pairs of jobs.

4 Conclusions

Although there is a considerable number of papers on stochastic scheduling, only a very few addressed the problem in environments where processing times are random variables. A single machine stochastic scheduling problems with uniform distributed random processing times is considered and the objective is to find an optimal schedule to minimize the expectation of the weighted numbers of tardy jobs from a common due date. The problem formulation of the expectation of the weighted numbers of tardy jobs can be given. Joint use solution- the shortest expected processing time first and the longest expected processing time first is optimal in the case of the expectation processing time is non-proportional to weights of the jobs. The optimality of the algorithms is proved. The author is currently working on the problem of stochastic scheduling.

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Discussion and Applications on the Model of Moderate Analysis

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Abstract. Fuzzy comprehensive evaluation has been widely used, and the method of moderate analysis is viewed as a better method. Through analyzing and studying the fuzzy comprehensive evaluation from different angles, this paper proposes two revised moderate analysis models ------power function model and trigonometric function model. These two models are easy to understand, have better validity and the rationality, and have a further referential effect on doing research on fuzzy comprehensive evaluation.

Keywords: Fuzzy comprehensive evaluation, Moderate degree function, Power function.

1 Introduction

In the early 1980s, Wang Peizhuang [1] put forward Comprehensive evaluation model, which was so simple and practical that spread rapidly to the all sides of national economy and industrial and agricultural production. General practitioners had made one achievement after another using the Comprehensive fuzzy assessment model. Meanwhile, some theoretical workers had been also attracted to make a further study and expansion on the model, so a series of remarkable results had been yielded. For instance, multistage model, operator adjustment, category overview and so on. What's more, aiming at some usual problems of the comprehensive fuzzy assessment in practical application, they improved them, such as the multi-level comprehensive fuzzy assessment model, synthetic calculation and moderate analysis model.

The method of moderate analysis was first proposed by professor Guo Sizong [2][3], Some related researches have been made by utilizing the method of moderate analysis, but certain troubles remained. The relationships between factors are always non-linear in the human minds and in our real life, while the present models of moderate analysis are mostly linear, and they cannot reflect exactly the essence and the relation. Based on the reason of above, this paper will formulate two models which can efficiently overcome the problems mentioned above. Furthermore, they have better validity and the rationality.

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2 The Method of Moderate Analysis of Fuzzy Comprehensive Evaluation

2.1 The Mathematical Method of Fuzzy Comprehensive Evaluation

1) Determine the factor sets of evaluation object $U=\{u_1,u_2,\cdots,u_n\}$, and determine the grade sets of the evaluation object $V=\{v_1,v_2,\cdots,v_m\}$.

2) Make fuzzy evaluation for each factor according to the grade of the evaluation , then get the evaluation matrix $R=\left(r_{ij}\right)_{n\times m}$, where r_{ij} represent the membership degree of u_i to v_j. It is obvious that (U,V,R) is a fuzzy comprehensive evaluation model.

3) Determine the weight sets $A=\{a_1,a_2,\cdots,a_n\}$, where $\sum_{i=1}^{n} a_i = 1$.

4) Get $\overline{B}=AR=\left(\overline{b_1},\overline{b_2},\cdots\overline{b_m}\right)$, normalize $B=(b_1,b_2,\cdots,b_m)$, then determine the grade of the evaluation object .

2.2 Background of the Proposition on Moderate Function

In the fuzzy comprehensive evaluation model, a great deal of research on weight has been carried out by many researchers. However, the determination of factors in R have only just a few studies as yet. The relations among factors are always neglected, all factors are considered as independent. In real evaluation, factors are mutually related. For example, in the performance evaluation of the car, suppose evaluation factors include speed, loading capacity, fuel consumption and so on. when considering speed of the car, it is not only in relation to the loading capacity of the car, but also concerns with its fuel consumption, and the loading capacity of car can also influent its fuel consumption. Based on this condition, reference [3] first proposed the method of moderate analysis.

2.3 Definition of the Moderate Function

Let $U=(u_1,u_2,\cdots u_n)$ is the factor set, X_i is the state set of u_i $(i=1,2,\cdots n)$. Then define the mapping $Q_{ij}: X_i\times X_j\to I\triangleq[0,1]$ is the moderate function between u_i and u_j.

Let Q_{ij} is the moderate function between u_i and u_j, For a given alternative strategy $V=(v_1,v_2,\cdots,v_n)$, $v_i\in X_i$, $Q_{ij}(v_i,v_j)\triangleq Q_{ij}(V)\in[0,1]$ is called the moderate function of the strategy V for the factor u_i and the factor u_j.

Let the objective constraint of the factor $u_i\left(i=1,2,\cdots n\right)$ is A_i, regulate $Q_{ii}(V)\triangleq\mu_{A_i}(V)$, $Q_{ii}(V)$ is called the moderate function of the strategy V for factor u_i . If u_i and u_j is mutual independence (that is to say there is no moderate function), then note $Q_{ij}\left(V\right)=\Phi$.

For any given $V\in\Lambda$, the method of moderate analysis is that describe each part r_i $\left(i=1,2,\cdots n\right)$ of the evaluation vector R as the multi-function among u_i and $Q_{ij}\left(V\right)$, that is

$$r_i\left(V\right)=f\left(Q_{i1}\left(V\right),Q_{i2}\left(V\right),\cdots,Q_{i1}\left(V\right)\right)$$

Moderate degree function should satisfy the following properties:

1) **Regularity:** $0\le f\left(V\right)\le 1,\ f\left(0,0,\cdots,0\right)=0\quad f\left(1,1,\cdots,1\right)=1$.

2) **No reduction:** If $Q_{ij}\le Q_{ij}^{'}$ $\left(j=1,2,\cdots n\right)$, $f\left(Q_{i1},Q_{i2},\cdots,Q_{in}\right)\le f\left(Q_{i1}^{'},Q_{i2}^{'},\cdots,Q_{in}^{'}\right)$

3) **Continuity:** f is a continuous function of variables $Q_{i1},Q_{i2},\cdots,Q_{in}$.

3 The Model of Moderate Analysis

In the present moderate analysis model, we always regard the relationship between them as linear when judging all the factors, while the evaluation activity is a subjective activity, and it is not linear. Two nonlinear moderate analysis models are given in this paper, which are as follows:

Model 1 (Power function model)

First introduce the significant influence degree coefficients. The significant influence degree coefficients[4][5] is that significant influences degree of factors in evaluation can not be fully reflected only by increasing weight. For example, the method of weighted average can not reflect the influence of the factor in the present evaluation methods. Specifically, when a factor of the object evaluated is very important while the other factors are not so important relatively, we can say this factor is excellent. But this point is hard to reflect in the method of weighted average. So this paper defines the significant influence degree coefficients vector $A=\left(a_1,a_2,\cdots a_l\right)$, $a_j\in\left[1,+\infty\right]$ .When the influence degree is bigger**,** a_j is also bigger.

Let $a=\max\left(a_1,a_2,\cdots a_n\right)$. Its weight vector $\lambda=\left(\lambda_1,\lambda_2,\cdots,\lambda_n\right)$, establish the following model.

$$r_i=f\left(Q_{ip_1},Q_{ip_2},\cdots,Q_{ip_n}\right)=\left(\sum_{j=1}^{n}\lambda_j Q_{ip_j}^{a_j}\right)^{\frac{1}{a}},\ \text{where}\ \sum_{j=1}^{n}\left(\lambda_j\right)=1,\ \text{and}\ \lambda_i>0.$$

Proposition 1. *When* $Q_{ip_1}=Q_{ip_2}=\cdots=Q_{ip_n}=0$,

$$f\left(Q_{ip_1},Q_{ip_2},\cdots,Q_{ip_n}\right)=0. \tag{1}$$

$$When\ Q_{ip_1} = Q_{ip_2} = \cdots = Q_{ip_n} = 1,$$

$$f\left(Q_{ip_1}, Q_{ip_2}, \cdots, Q_{ip_n}\right) = 1. \tag{2}$$

$$When\ Q_{ip_1} = Q_{ip_2} = \cdots = Q_{ip_n} = c,$$

$$f\left(Q_{ip_1}, Q_{ip_2}, \cdots, Q_{ip_n}\right) \ge g \cdot c^{\frac{\bar{a}}{a}}. \tag{3}$$

$$Here\quad g = g\left(n, A, \lambda,\right) = \left(n \cdot (\lambda_1 \cdot \lambda_2 \cdot \cdots \cdot \lambda_n)^{\frac{1}{n}}\right)^{\frac{1}{a}},\ \bar{a} = \frac{a_1 + a_2 + \cdots + a_n}{n}$$

Proof. The proof of (1) ,(2) are obvious, so we only prove (3).

Because when $Q_{ip_1} = Q_{ip_2} = \cdots = Q_{ip_n} = c$

$$\begin{aligned} f\left(Q_{ip_1}, Q_{ip_2}, \cdots, Q_{ip_n}\right) &= (\lambda_1 c_{ip_1}^{a_1} + \lambda_2 c_{ip_2}^{a_2} + \cdots + \lambda_n c_{ip_n}^{a_n})^{\frac{1}{a}} \\ &\ge \left[n \cdot (\lambda \cdot_1 c_{ip_1}^{a_1} \cdot \lambda_2 \cdot c_{ip_2}^{a_2} \cdot \cdots \cdot \lambda_n \cdot c_{ip_n}^{a_n})^{\frac{1}{n}} \right]^{\frac{1}{a}} \\ &= \left(\left(n^n \cdot \lambda_1 \cdot \lambda_2 \cdot \cdots \cdot \lambda_n\right)^{\frac{1}{n}} \right)^{\frac{1}{a}} (c^{\frac{a_1 + a_2 + \cdots + a_n}{n}})^{\frac{1}{a}} \\ &= \left(\left(n \cdot \lambda_1 \cdot n \cdot \lambda_2 \cdot \cdots \cdot n \cdot \lambda_n\right)^{\frac{1}{n}} \right)^{\frac{1}{a}} c^{\frac{\bar{a}}{a}} \\ &= g\left(n, A, \lambda,\right) \cdot c^{\frac{\bar{a}}{a}} \end{aligned}$$

If and only if $n \cdot \lambda_1 = n \cdot \lambda_2 = \cdots = n \cdot \lambda_l$, that is to say when $\lambda_1 = \lambda_2 = \cdots = \lambda_n = \frac{1}{n}$, "=" is established, that is to say $g = 1$, then

$$f\left(Q_{ip_1}, Q_{ip_2}, \cdots, Q_{ip_n}\right) \ge c^{\frac{\bar{a}}{a}}$$

Proposition 2. *When* $Q_{ij} \ge Q_{ij}^{'}\ \left(j = 1, 2 \cdots l\right)$,

$$f\left(Q_{ip_1}, Q_{ip_2}, \cdots, Q_{ip_n}\right) \ge f\left(Q_{ip_1}^{'}, Q_{ip_2}^{'}, \cdots, Q_{ip_n}^{'}\right)$$

Proof. $f\left(Q_{ip_1}, Q_{ip_2}, \cdots, Q_{ip_l}\right) = \left(\sum_{j=1}^{n} \lambda_j Q_{ip_j}^{a_j}\right)^{\frac{1}{a}}$ can be regard as the power function of moderate function. Power index is greater than zero, so $f\left(Q_{ip_1}, Q_{ip_2}, \cdots, Q_{ip_l}\right)$ is a monotone increasing function of Q_{ij} .

That is to say, when $Q_{ij} \geq Q_{ij}^{'}$ $(j=1,2\cdots l)$,

$$f\left(Q_{ip_1},Q_{ip_2},\cdots,Q_{ip_l}\right) \geq f\left(Q_{ip_1}^{'},Q_{ip_2}^{'},\cdots,Q_{ip_n}^{'}\right).$$

Proposition 3. $\lim\limits_{Q_{ij}\to Q_{ij}^{'}} f\left(Q_{ip_1},Q_{ip_2},\cdots,Q_{ip_n}\right) = f\left(Q_{ip_1}^{'},Q_{ip_2}^{'},\cdots,Q_{ip_n}^{'}\right)$.

Proof. Because $f\left(Q_{ip_1},Q_{ip_2},\cdots,Q_{ip_n}\right)$ is continuous in $[0,1]$, the conclusion is right.

Model 2 (Trigonometric function model)

$$r_i = f(Q_{ip_1},Q_{ip_2},\cdots,Q_{ip_n}) = \sin\left(\frac{\pi}{2}\cdot\sum_{j=1}^{n}\lambda_j\cdot Q_{ip_j}\right),$$

where $0 \leq \lambda_j \leq 1$, $\sum_{j=1}^{n}\lambda_j = 1$, In weighted average moderate functions, we usually take

$$\lambda_j = \frac{Q_{jp_j}}{\sum_{k=1}^{n} Q_{jp_k}}.$$

It is easy to prove that $f(1,1,\cdots,1)=1$, $f(0,0,\cdots,0)=0$, $0 \leq f(v) \leq 1$. When $Q_{ij} \leq Q_{ij}^{'}$ $j=1,2,\cdots,n$, $f(Q_{i1},Q_{i2},\cdots,Q_{in}) \leq f(Q_{i1}^{'},Q_{i2}^{'},\cdots,Q_{in}^{'})$, and f is a continuous function of $Q_{i1},Q_{i2},\cdots,Q_{in}$. That is to say that trigonometric function model satisfies regularity, no reduction, continuity.

4 Application

Goals (U_1) **:** knowledge (u_1), skills (u_2), means (u_3), sentiment (u_4).

Content (U_2) **:** the text (u_5), the problem (u_6), the experiments (u_7), other activity (u_8).

Structure (U_3) **:** knowledge hierarch (u_9), ability training (u_{10}), values (u_{11}), subjects research (u_{12}).

Feasibility (U_4) **:** application in colleg**e** (u_{13}), adaptive of instructors (u_{14}) adaptive of students (u_{15}), improves at a certain degree (u_{16}).

Let instructors (sum 14) carry out an evaluation of textbook goals by the standard of Excellent, Fine, Qualified, Unqualified. Then make a statistical table. The table as following.

Table 1.

factor of evaluation	The number of each grade								sum
	Excellent		Fine		Qualified		Unqualified		
	number	ratio	number	ratio	number	ratio	number	ratio	
knowledge	10	0.71	1	0.07	2	0.15	1	0.07	14
skills	6	0.43	4	0.29	2	0.14	2	0.14	14
means	8	0.57	4	0.29	1	0.07	1	0.07	14
sentiment	5	0.36	6	0.43	2	0.14	1	0.07	14
sum	29		15		7		5		56

Establish moderate degree function $Q_{ij} = \frac{|x_i - x_j|}{\max\{x_i, x_j\}}$.

As a result , we get the fuzzy matrix

$$\begin{bmatrix} 0 & 0.39 & 0.20 & 0.49 \\ 0.39 & 0 & 0.25 & 0.16 \\ 0.20 & 0.25 & 0 & 0.37 \\ 0.49 & 0.16 & 0.37 & 0 \end{bmatrix}$$

Determining the significant influence degree coefficients: $10:8:6:5$ or $2:1.2:1.6:1$,as a matter of convenience, let the significant influence degree coefficients: 2,1,2,1.

Let weight of sub factor set $\lambda = (0.35,\ 0.20, 0.35, 0.10)$, apply model one, thus

$$R = \begin{bmatrix} f(Q_{11}, Q_{12}, Q_{13}, Q_{14}) \\ f(Q_{21}, Q_{22}, Q_{23}, Q_{24}) \\ f(Q_{31}, Q_{32}, Q_{33}, Q_{34}) \\ f(Q_{41}, Q_{42}, Q_{43}, Q_{44}) \end{bmatrix} = \begin{bmatrix} 0 & 0.0304 & 0.0140 & 0.0240 \\ 0.0532 & 0 & 0.0219 & 0.0022 \\ 0.0140 & 0.0125 & 0 & 0.0137 \\ 0.0840 & 0.0051 & 0.0480 & 0 \end{bmatrix}$$

Then calculate the fuzzy transformation of U to V .

$$\overline{B} = (0.35,\ 0.20, 0.35, 0.10) \cdot \begin{bmatrix} 0 & 0.0304 & 0.0140 & 0.0240 \\ 0.0532 & 0 & 0.0219 & 0.0022 \\ 0.0140 & 0.0125 & 0 & 0.0137 \\ 0.0840 & 0.0051 & 0.0480 & 0 \end{bmatrix}$$

$$= (0.024,\ 0.016, 0.014, 0.014) .$$

Normalize, $B_1 = (0.35, 0.24, 0.21, 0.21)$.

On the analogy of this thus

$$B_2 = (0.57,0.29,0.10,0.04).$$
$$B_3 = (0.43,0.35,0.17,0.05).$$
$$B_4 = (0.43,0.30,0.17,0.10).$$

Then make the second class fuzzy comprehensive evaluation of U and determining its weight vector $W = (0.15,0.40,0.20,0.25)$, thus

$$\overline{R} = (0.15,0.40,0.20,0.25)\begin{pmatrix} 0.35 & 0.24 & 0.21 & 0.21 \\ 0.57 & 0.29 & 0.10 & 0.04 \\ 0.43 & 0.35 & 0.17 & 0.05 \\ 0.43 & 0.30 & 0.17 & 0.10 \end{pmatrix}$$
$$= (0.474, 0.297, 0.148, 0.083).$$

Normalize, $R = (0.47, 0.30, 0.15, 0.08)$.

Determining synthetic evaluation value,

Table 2.

Rank	Unqualified	Qualified	Fine	Excellent
Grades Representative score	50–62 56	63–75 69	76–88 82	89–100 95

Let $G = (95,82,69,56)$, its comprehensive evaluation.

$$RG^{T} = (0.47, 0.30, 0.15, 0.08)\begin{pmatrix} 95 \\ 82 \\ 69 \\ 56 \end{pmatrix} = 84.08$$

In conclusion, its rank of comprehensive evaluation is “Fine”.

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Fuzzy Portfolio Model with Transaction Cost Based on Downside Risk Measure

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Abstract. In this paper the problem of portfolio selection with transaction cost based on downside risk measure is discussed. Left-right-type fuzzy number is used to describe the expected return rate of security, and transaction cost and diversification condition are incorporated into the process of portfolio selection. We evaluate the expected return and risk by interval-valued mean, regard the downside risk as the portfolio risk, and formulate the portfolio selection problem as a linear programming. Finally, a numerical example is presented to illustrate the efficiency of the proposed model.

Keywords: Portfolio selection, Downside risk measure, Transaction cost, Fuzzy number.

1 Introduction

In portfolio selection, one of the most important problems is how to find an optimal portfolio. The pioneering work for this problem is the mean-variance theory proposed by Markowitz [9]. Markowitz used the expected return of a portfolio as the investment return and the variance of the expected returns as the investment risk, and combined probability and optimization theory to model the economic behavior under uncertainty. Since then, many scholars proposed different mathematical techniques to develop portfolio theory based on probability theory.

With the introduction of the fuzzy set theory by Zadeh [21], there have been a few researches in dealing with portfolio selection problem from the aspect of fuzziness. Tanaka and Guo [11, 12] proposed lower and upper possibility distributions to reflect experts' knowledge and formulated their portfolio selection models. Inuiguchi and Ramil [5], and Inuiguchi and Tanino [6]

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presented a spread minimization model for their portfolio selection problem. Carlsson et al [1] introduced a possibilistic approach for selecting portfolios with the highest utility value. Zhang et al [22] discussed the portfolio selection problem based on lower and upper possibilistic means and variances. Xu and Li [15, 16] and Xu and Zhai [17, 18, 19] gave a technique for acquiring the fuzzy anticipative profit rate, proposed their decision models with fuzzy return rate, and explored the properties of their decision models. Vercher et al [13] introduced the fuzzy downside risk for a trapezoidal portfolio and presented an approach for managing portfolio selection problems in the framework of risk-return trade-off.

In general, return and risk are two essential components investors concern, but in some situations, investors also care about transaction cost. In fact, transaction cost has a direct impact on the result of investment. As well as quantifying return and risk, several researches have been taken into account the effect of the transaction cost in portfolio selection. Fang et al [4] proposed a portfolio rebalancing model with transaction costs based on fuzzy decision theory. Jana et al [7] formulated their problem as a multi-objective non-linear programming model. Xu and Zhai [20] incorporated transaction cost into portfolio selection and constructed their model as a quadratic programming. Continuing the discussion of [20], this paper uses downside risk as the measure of the investment risk and formulates the three-factor problem, including return, risk and transaction cost as a linear programming.

The rest of the paper is organized as follows. Section 2 gives some basic concepts and results about fuzzy number. Section 3 formulates the fuzzy portfolio model with transaction cost based on downside risk measure. Section 4 presents a numerical example to illustrate the proposed model. The paper ends with some concluding remarks.

2 Preliminaries

Let us introduce some basis concepts and results on fuzzy numbers, which are necessary for developing our model.

A fuzzy number $\tilde{A}$ is a convex subset on real number set R with a normalized membership function. A general fuzzy number can be represented as a left-right(LR)-type of fuzzy number, which can be described with the following membership function [2]:

$$\tilde{A}(x) = \begin{cases} L(x), & \text{if} \quad d_l \le x < m_l, \\ 1, & \text{if} \quad x \in [m_l,\ m_r], \\ R(x), & \text{if} \quad m_r < x \le d_r, \\ 0, & \text{otherwise}, \end{cases} \tag{1}$$

where $[m_l, m_r]$ is the range of most likely values of $\tilde{A}$; m_l and m_r are the lower and upper modal values; $L(x)$ and $R(x)$ are the increasing and decreasing continuous functions respectively. We denote the set of all fuzzy numbers as

$\tilde{F}(R)$. From the definition, the closure of the support of $\tilde{A}$ is exactly $[d_l, d_r]$ and the λ-level set of $\tilde{A}$ can be denoted as $A_\lambda \overset{\text{def}}{=} [L^{-1}(\lambda), R^{-1}(\lambda)],\ \lambda \in [0,1]$, which is the closed interval on real number set R.

In order to measure the mean value of a fuzzy number $\tilde{A}$, we use the interval-valued mean introduced by Dubois and Prade [3]:

$$E(\tilde{A}) = [\,E_*(\tilde{A}), E^*(\tilde{A})\,] \tag{2}$$

where $E_*(\tilde{A}) = \int_0^1 L^{-1}(\lambda)\, d\lambda, E^*(\tilde{A}) = \int_0^1 R^{-1}(\lambda)\, d\lambda$. It is easy to verify that $[\,E_*(\tilde{A}), E^*(\tilde{A})\,]$ is the nearest interval approximation of the fuzzy number $\tilde{A}$ with respect to the distance introduced by Xu [14].

For linear operations of closed interval, by classical extension principle [8], we have the following conclusions:

$$\begin{aligned}
&1)\ \ [a,b]+[c,d]=[a+c,b+d];\\
&2)\ \ [a,b]-[c,d]=[a-d,b-c];\\
&3)\ \ \alpha[a,b]=[\alpha a,\alpha b]\quad \text{if } \alpha \geq 0,\ \alpha \in R;\\
&4)\ \ \alpha[a,b]=[\alpha b,\alpha a]\quad \text{if } \alpha < 0,\ \alpha \in R.
\end{aligned}$$

3 The Model

This section formulates the portfolio model with transaction cost based on downside risk measures. Let us assume that the investor has selected n securities for investment and obtained the estimates of the future return rates for each security, which can be expressed as fuzzy numbers $\tilde{A}_i (i = 1, \cdots, n)$, namely, $\tilde{A} \in \tilde{F}(R)$. We assume that the short sales are not allowed. Let $w_i (i = 1, \cdots, n)$ be the proportion of the security $i (i = 1, \cdots, n)$ bought by the investor and $c_i (i = 1, \cdots, n)$ be the rate of transaction cost of the security $i (i = 1, \cdots, n)$, where $w_i \geq 0$ and $c_i \geq 0 (i = 1, \cdots, n)$. After paying the transaction cost, the net return of security i is $w_i \tilde{A}_i - w_i c_i (i = 1, \cdots, n)$, and the portfolio return is thus

$$\tilde{A} = \sum_{i=1}^{n} w_i (\tilde{A}_i - c_i). \tag{3}$$

It is easy to see that $\tilde{A} \in \tilde{F}(R)$.

Similar to the mean semi-absolute deviation in a probabilistic context [10], which is used as a measure of the downside risk for a portfolio, the fuzzy analogue can be expressed as

$$D(\tilde{A}) = E(\max\{0, E(\tilde{A}) - \tilde{A}\}). \tag{4}$$

Denoting the λ-level set $(0 < \lambda \leq 1)$ of $\tilde{A}$ and $\tilde{A}_i (i = 1, \cdots, n)$ as $(\tilde{A})_\lambda = [L^{-1}(\lambda), R^{-1}(\lambda)],\ \textit{and}\ (\tilde{A}_i)_\lambda = [L_i^{-1}(\lambda), R_i^{-1}(\lambda)]$ respectively, we have

$$L^{-1}(\lambda)=\sum_{i=1}^{n} w_i(L_i^{-1}(\lambda)-c_i), \quad R^{-1}(\lambda)=\sum_{i=1}^{n} w_i(R_i^{-1}(\lambda)-c_i) \tag{5}$$

and thus the interval-value mean of $\tilde{A}$

$$\begin{aligned} E(\tilde{A}) &= [E_*(\tilde{A}), E^*(\tilde{A})] \\ &= [\int_0^1 L^{-1}(\lambda)d\lambda, \int_0^1 R^{-1}(\lambda)d\lambda] \\ &= [\int_0^1 \sum_{i=1}^{n} w_i(L_i^{-1}(\lambda)-c_i)d\lambda, \int_0^1 \sum_{i=1}^{n} w_i(R_i^{-1}(\lambda)-c_i)d\lambda] \\ &= [\sum_{i=1}^{n} w_i(\int_0^1 L_i^{-1}(\lambda)d\lambda - c_i),\ \sum_{i=1}^{n} w_i(\int_0^1 R_i^{-1}(\lambda)d\lambda - c_i)]. \end{aligned} \tag{6}$$

According to the operation properties of the closed interval, we can derive the λ-level set of $E(\tilde{A})-\tilde{A}$ as follows

$$\begin{aligned} (E(\tilde{A})-\tilde{A})_\lambda &= [E_*(\tilde{A}), E^*(\tilde{A})] - [L^{-1}(\lambda),\ R^{-1}(\lambda)] \\ &= [E_*(\tilde{A}) - R^{-1}(\lambda),\ E^*(\tilde{A}) - L^{-1}(\lambda)] \\ &= [\sum_{i=1}^{n} w_i(\int_0^1 L_i^{-1}(\lambda)d\lambda - c_i) - \sum_{i=1}^{n} w_i(R_i^{-1}(\lambda)-c_i), \\ &\qquad \sum_{i=1}^{n} w_i(\int_0^1 R_i^{-1}(\lambda)d\lambda - c_i) - \sum_{i=1}^{n} w_i(L_i^{-1}(\lambda)-c_i)] \\ &= [\sum_{i=1}^{n} w_i(\int_0^1 L_i^{-1}(\lambda)d\lambda - R_i^{-1}(\lambda)), \\ &\qquad \sum_{i=1}^{n} w_i(\int_0^1 R_i^{-1}(\lambda)d\lambda - L_i^{-1}(\lambda)]. \end{aligned} \tag{7}$$

It is easy to verify that

$$\sum_{i=1}^{n} w_i(\int_0^1 L_i^{-1}(\lambda)d\lambda - R_i^{-1}(\lambda)) \le 0 \tag{8}$$

and

$$\sum_{i=1}^{n} w_i(\int_0^1 R_i^{-1}(\lambda)d\lambda - L_i^{-1}(\lambda)) \ge 0, \tag{9}$$

since

$$\int_0^1 L_i^{-1}(\lambda)d\lambda \le L_i^{-1}(1) \le R_i^{-1}(\lambda)$$

and

$$\int_0^1 R_i^{-1}(\lambda)d\lambda \geq R_i^{-1}(1) \geq L_i^{-1}(\lambda).$$

Therefore, we obtain the λ-level set of $\max\{0, E(\tilde{A}) - \tilde{A}\}$

$$(\max\{0, E(\tilde{A}) - \tilde{A}\})_\lambda = [\,0,\ \sum_{i=1}^{n} w_i(\int_0^1 R_i^{-1}(\lambda)d\lambda - L_i^{-1}(\lambda))]. \tag{10}$$

It follows that the negative deviation on expected return is given by

$$\max\{0, E(\tilde{A}) - \tilde{A}\} = \bigcup_{0<\lambda\leq 1} \lambda[\,0,\ \sum_{i=1}^{n} w_i\Big(\int_0^1 R_i^{-1}(\lambda)d\lambda - L_i^{-1}(\lambda)\Big)]. \tag{11}$$

Going a step further, the interval-value mean of $\max\{0, E(\tilde{A}) - \tilde{A}\}$

$$D(\tilde{A}) = [\,0,\ \sum_{i=1}^{n} w_i \int_0^1 \big(R_i^{-1}(\lambda) - L_i^{-1}(\lambda)\big)d\lambda]. \tag{12}$$

On the other hand, the middle point of the interval-value mean on expected return can be derived by

$$\begin{aligned} C(\tilde{A}) &= \frac{E_*(\tilde{A}) + E^*(\tilde{A})}{2} \\ &= \sum_{i=1}^{n} w_i(\int_0^1 \frac{L_i^{-1}(\lambda) + R_i^{-1}(\lambda)}{2}\, d\lambda - c_i). \end{aligned} \tag{13}$$

Different models exist to select the best portfolio according to their respective consideration. In this study, we use the middle point of the interval-value mean on expected return as the portfolio return and the measure of the downside risk for the portfolio as the risk of investment. We also assume that the diversification conditions, which is the lower and upper bounds on the proportion of the fund invested in each asset, and an satisfactory level for expected return are given by the decision maker. Therefore, we can formulate our optimization investment model with transaction cost as

$$\begin{aligned} \min_{w}\ \ & D(\tilde{A}) = \sum_{i=1}^{n} w_i \int_0^1 (R_i^{-1}(\lambda) - L_i^{-1}(\lambda))d\lambda \\ s.t.\ \ & \sum_{i=1}^{n} w_i(\int_0^1 \frac{L_i^{-1}(\lambda) + R_i^{-1}(\lambda)}{2}\, d\lambda - c_i) \geq \mu, \\ & \sum_{i=1}^{n} w_i(1 + c_i) = 1, \\ & \alpha_i \leq w_i \leq \beta_i, i = 1, 2, \cdots, n, \end{aligned} \tag{14}$$

where $\alpha_i \geq 0$ since short selling is not allowed. From the optimization problem (14), we know that the fuzzy portfolio selection problem with transaction cost based on downside risk measures is now converted into a linear optimization problem.

4 Numerical Example

In this section, we use the example, taken from Vercher et al [13], to illustrate how the proposed model is applied to deal with the portfolio selection problem.

An investor wants to allocate one unit of wealth among nine securities, the data for the yearly returns on the securities are summarized in Table 1.

Table 1. Numerical data

Returns	5th percentile	40th percentile	60th percentile	95th percentile
R1	-0.284	-0.011	0.070	0.456
R2	-0.175	0.052	0.089	0.229
R3	-0.193	0.018	0.136	0.758
R4	-0.307	0.161	0.238	0.714
R5	-0.429	0.062	0.325	0.671
R6	-0.234	-0.064	0.094	0.352
R7	-0.132	0.090	0.164	0.356
R8	-0.311	0.104	0.196	0.587
R9	-0.316	0.104	0.196	0.587

Vercher et al. use the sample percentiles to approximate the core and spreads of the trapezoidal fuzzy returns on the securities. They set the core $[m_l, m_r]$ of the fuzzy return $\tilde{A}$ as the interval $[P_{40}, P_{60}]$ and the quantities $P_{40}-P_5$ and $P_{95}-P_{60}$ as the left m_l-d_l and right d_r-m_r spreads, respectively, where P_k is the kth percentile of the sample. Therefore, the corresponding membership function is given by Formula(1), where

$$L(x) = \frac{x - d_l}{m_l - d_l} \text{ if } d_l \leq x < m_l, \quad \text{and } R(x) = \frac{d_r - x}{d_r - m_r} \text{ if } m_r \leq x < d_r.$$

Suppose that the rate of transaction cost $c_i = 0.5\%$ and the investor wants to select a portfolio in such a way that the minimum and maximum investments in securities must be 5% and 25%, respectively, and the expected return must be more than or equal to 20%. Using the proposed model (14),we have the following linear programming model:

$$\begin{aligned}
\min_{w} \quad & D(\tilde{A}) = 0.775w_1 + 0.2205w_2 + 0.5345w_3 + 0.549w_4 + 0.6815w_5 \\
& \qquad\qquad +0.372w_6 + 0.281w_7 + 0.495w_8 + 0.4975w_9 \\
s.t. \quad & 0.05275w_1 + 0.04375w_2 + 0.17475w_3 + 0.1965w_4 + 0.15225w_5, \\
& \quad +0.032w_6 + 0.1145w_7 + 0.139w_8 + 0.13775w_9 \geq 0.12, \\
& w_1 + w_2 + \cdots + w_9 = 0.9950248, \\
& 0.05 \leq w_i \leq 0.25, i = 1, 2, \cdots, n.
\end{aligned} \tag{15}$$

By solving this linear programming problem, we obtain

$$\begin{aligned}
& (w_1, \cdots, w_9) \\
= & (0.0500, 0.1997, 0.0500, 0.2454, 0.0500, 0.0500, 0.2500, 0.0500, 0.0500).
\end{aligned}$$

5 Conclusion

This paper discusses the problem of portfolio selection with transaction cost based on downside risk measure. Left-right-type fuzzy number is used to describe the expected return rate of security, and transaction cost and diversification condition are incorporated into the portfolio selection model. By using downside risk as the measure for investment risk, the portfolio selection problem can be formulated as a linear programming. In contrast with the previously proposed quadratic model, this fuzzy portfolio model reduces the complexity of computation and could be a good alternative in the situation in which expected return is asymmetric and transaction cost must be concerned.

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Fuzzy Structured Element Algorithm of System Reliability with Fuzzy Parameter*

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Abstract. This paper mainly research on the system reliability with fuzzy parameters, and use two methods to solve the expression of the membership function about the fuzzy reliability of the system. When the fuzzy failure rate of the system is the fuzzy numbers that generated by fuzzy structured element, this paper put forward the formula of the fuzzy reliability and its membership function about the serial system, the parallel system, the parallel- serial system and the serial-parallel system.

Keywords: Fuzzy reliability, fuzzy structured element, membership function, fuzzy parameter.

In recent years, the fuzzy mathematics method has been applied in analyses of the system reliability, so it produced the fuzzy reliability theory[1]. The scholars in china and abroad have do a lot of work of Fuzzy reliability, such as: (1) Possibility theory instead of probability theory; (2) To describe the reliability of element and system with the fuzzy probability that Zadeth[2] had brought up; (3) Take explicit working time to extend fuzzy working hours; (4) Use fuzzy language to describe unit or system functional; (5) Pull into fuzzy parameters, under probability assumptions had be certain. However because of relating to complex calculations of fuzzy number the results of such studies are limited Fuzzy structured element method tackles the problem of quick calculations to some extent and it makes further reliability research of fuzzy parameters system become possible.

This paper discusses the reliability of system with fuzzy parameter using the fuzzy structuring element theory [3].

1 Fuzzy Structured Element and Fuzzy Number Operation

Let E be a fuzzy set in the real number domain R , $E(x)$ be the membership function of E . We call that E is a fuzzy structured element in R , if

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(1) $E(0)=1, E(1+0)=E(-1-0)=0$;

(2) $E(x)$ is a monotone increasing and right continuous function on [-1,0], and monotonic decreasing and left continuous on (0,1];

(3) $E(x)=0$ ($-\infty < x < -1$ and $1 < x < +\infty$).

E is called a regular fuzzy structured element, if

(1) $\forall x \in (-1,1), E(x) > 0$;

(2) The membership p function $E(x)$ is a strictly monotone increasing and continuous on [-1,0], strictly monotone decreasing and continuous on (0,1].

If $E(-x)=E(x)$, then E is called a symmetrical fuzzy structured element.

Theorem 1.1 [4]. Let E be a fuzzy structured element, and $E(x)$ be its membership function, $f(x)$ be a bounded monotone function on [-1,1], then $\hat{f}(E)$ is a fuzzy number, and the membership function of $\hat{f}(E)$ is $E(f^{-1}(x))$ (where $f^{-1}(x)$ is the extensive inverse function of $f(x)$).

E is a fuzzy structured element in X, $\forall\ \lambda \in (0,1)$, the λ-level set of E is $E_\lambda = \{x \mid E(x) \ge \lambda\} = [e_\lambda^-, e_\lambda^+]$, by the definition of fuzzy structured element, we have $e_\lambda^- \in [-1,0]$, $e_\lambda^+ \in [0,1]$.

Theorem 1.2 [4]. Let f be a bounded monotone function on [-1,1], E a given fuzzy structured element, and fuzzy number $\tilde{A} = f(E)$. If f is a monotone increasing function on [-1,1], then for arbitrary $\lambda \in (0,1)$, the λ-level set of fuzzy number $\tilde{A}$ is a closed interval $\tilde{A}_\lambda = [f(e_\lambda^-), f(e_\lambda^+)]$. If f is a monotone decreasing function on [-1,1], the λ-level set of $\tilde{A}$ is a closed interval $\tilde{A}_\lambda = [f(e_\lambda^+), f(e_\lambda^-)]$.

2 Three Computing Methods of Membership Function

2.1 Inverse Function Representation

Let E be a fuzzy structured element, f be a bounded monotone function on $[-1,1]$, and fuzzy number $\tilde{A} = f(E)$,according to theorem 1.1, have

$$\mu_{\tilde{A}}(x) = E[f^{-1}(x)]$$

We can easily get the membership of fuzzy number $\tilde{A}$ by making use of E and $f^{-1}(x)$, and we name this kind of calculating method inverse function representation.

2.2 α -Level Set Representation

When we cannot get accurate analytic expression of $f^{-1}(x)$, we try to use another method to calculate membership function of $f(E)$.Because of $f(E)$ is a bounded fuzzy number, so for arbitrary $\alpha \in (0,1]$, the α-level set of $f(E)$ is a closed interval, as well be denoted of $[f_\alpha^-, f_\alpha^+]$. If interval endpoints f_α^- and f_α^+ can be repressed by explicit function expression α, $f_\alpha^- = f_1(\alpha)$, $f_\alpha^+ = f_2(\alpha)$, $\alpha \in (0,1]$.

Also we can master quantitative relations expressed by the membership function of $f(E)$.The difference is that the independent variable of $E[f^{-1}(x)]$ is x in the fuzzy number domain, and induced variable is membership grade α of fuzzy number. However independent variable of $f_1(\alpha)$ and $f_2(\alpha)$ is membership grade α of fuzzy number, and induced variable is x in the fuzzy number domain. So $E[f^{-1}(x)]$ is the inverse function of $f_k(\alpha)$, and $f_k(\alpha)$ is also the inverse function of $E[f^{-1}(x)]$.

Let the membership function of fuzzy structured element be $\mu = E(x)$, because $E(x)$ is monotone increasing on [-1,0] and monotone decreasing on (0,1], so there are the inverse functions of $E(x)$ on [-1,1] and on[0,1], denoted as

$$E(x) = \begin{cases} E_1(x), & x \in [-1,0] \\ E_2(x), & x \in (0,1] \end{cases} \tag{2.1}$$

If we can get analytic expression of inverse functions of $E_1(x)$ and $E_2(x)$, and for $\forall \alpha \in (0, 1]$, denote $E_\alpha = [e_\alpha^-, e_\alpha^+]$.also $E_1(e_\alpha^-) = \alpha$ and $E_2(e_\alpha^+) = \alpha$, then it has $e_\alpha^- =$ $E_1^{-1}(\alpha)$ and $e_\alpha^+ =$ $E_2^{-1}(\alpha)$. According to theorem 1.2, fuzzy number $\tilde{A} =$ $f(E)$, when f is a monotone increasing function on [-1,1], have

$$\tilde{A}_\alpha = [f(e_\alpha^-), f(e_\alpha^+)] = [f(E_1^{-1}(\alpha)), f(E_2^{-1}(\alpha))] \tag{2.2}$$

When f is a monotone increasing function on [-1,1], have

$$\tilde{A}_\alpha = [f(e_\alpha^+), f(e_\alpha^-)] = [f(E_2^{-1}(\alpha)), f(E_1^{-1}(\alpha))] \tag{2.3}$$

3 The Fuzzy Reliability Analysis of the Unit with the Fuzzy Number

3.1 The Fuzzy Reliability Analysis of the Unit

The reliability function $R(t)$ of one unit is defined as "the lifetime of the unit T is greater than t" the probabilities, denote as $R(t) = P(T > t)$. Then the reliability function of the unit is

$$R(t) = P(T > t) = \int_t^{\infty} \lambda e^{-\lambda t} dt = e^{-\lambda t} .$$

Then, λ ($\lambda > 0$) is a constant, which is defined as the failure rate of unit.

To the application problems about the reliability, the failure rate is λ that is usually estimated, it also have a few of errors at the same time. Whether we can try to take into account of the errors about the reliability analysis of the system or not, for this reason, some people have came up with a method that we see the failure rate λ as a fuzzy number $\tilde{\lambda}$, we call it as fuzzy unit failure rate.

Let lifetime of the unit T obeys exponential distribution with parameters λ, in the reliability statistics, $x_1, x_2, ..., x_n$, is one simple sample from collectivity T, let the estimated value of the unit is $\tilde{\lambda} = \lambda + \sigma E$, E is the symmetrical fuzzy structured element, hen

$$\lambda = \frac{1}{\frac{1}{n}\sum_{i=1}^{n} x_i} = \frac{1}{\bar{x}}$$

σ is a real number, which meet the condition $0 < \sigma < \lambda$. Usually $\lambda < 1$, let $\sigma = \lambda^2$. Thus, we define fuzzy reliability function of the unit as the fuzzy-value function

$$\tilde{R}(t) = e^{-\tilde{\lambda}t} = e^{-(\lambda+\sigma E)t} = e^{-\lambda \cdot t - \sigma \cdot tE} = e^{-\lambda \cdot t + \sigma \cdot tE} \tag{3.1}$$

For arbitrary given $t > 0$, $\tilde{R}(t)$ is a fuzzy number. According to the theorem (2-1), it is easy to get membership function of $\tilde{R}(t)$.

$$\mu_{\tilde{R}(t)}(x) = E[\frac{1}{\sigma \cdot t}(\ln x + \lambda \cdot t)] \tag{3.2}$$

3.2 Fuzzy Reliability Function of Serial System

Let's consider a serial system (Fig. 1), which is made up of n independent variables, let $T_1, T_2, ..., T_n$ is the lifetime of every unit, then the lifetime of the

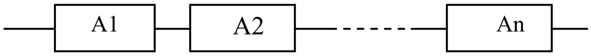

Fig. 1. The logic diagram of serial system

serial system is $S=\min\{T_1,T_2,...,T_n\}$, and let the reliability function of unit i of the system is $R_i(t)$, If each unit lifetime of the system all obey exponential distribution, the reliability function of the system is

$$R_S(t)=\prod_{i=1}^{n}e^{-\lambda_i t}=\exp[-\sum_{i=1}^{n}\lambda_i t]=e^{-\lambda_S t},$$

Where $\lambda_S=\lambda_1+\lambda_2+...+\lambda_n$.

If the failure rate of every unit in the system are fuzzy numbers, respectively denoted as $\lambda_1,\lambda_2,...,\lambda_n$, and they are linear formed by the symmetrical structured element E, then $\tilde{\lambda}_i=\lambda_i+\sigma_i E$, $i=1,2,...,n$. So the fuzzy reliability function of system S is

$$\tilde{R}_S(t)=e^{-\tilde{\lambda}_S t}=e^{-\lambda_S t-\sigma_S tE}=e^{-\lambda_S t+\sigma_S tE} \tag{3.3}$$

Where $\lambda_S=\lambda_1+\lambda_2+...+\lambda_n$, $\sigma_S=\sigma_1+\sigma_2+...+\sigma_n$. It is easy to get the membership function of the fuzzy reliability function of system S

$$\mu_{\tilde{R}_S(t)}(x)=E[\frac{1}{\sigma_S\cdot t}(\ln x+\lambda_S\cdot t)] \tag{3.4}$$

Where $\lambda_S=\lambda_1+\lambda_2+...+\lambda_n,\sigma_S=\sigma_1+\sigma_2+...+\sigma_n$.

3.3 Fuzzy Reliability Function of Parallel System

Let's consider a parallel system S (Fig 2), which is made up of n independent variables, and let $T_1,T_2,...,T_n$ is the lifetime of every unit, then the lifetime of the serial system is $S=\max\{T_1,T_2,...,T_n\}$.

We also let the reliability function of unit i is $R_i(t)$, When each unit lifetime obey exponential distribution, the reliability function of the system is

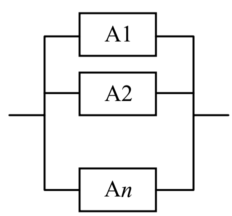

Fig. 2. Parallel system

$$R_S(t)=1-\prod_{i=1}^{n}[1-R_i(t)]=1-\prod_{i=1}^{n}[1-e^{-\lambda_i t}]$$

Similarly, when the failure rate of each unit in the system S are fuzzy numbers, $\tilde{\lambda}_i = \lambda_i + \sigma_i E$, $i = 1,2,...,n$. E is symmetrical fuzzy structured element, So the fuzzy reliability function of parallel system S is

$$\tilde{R}_S(t) = 1 - \prod_{i=1}^{n}[1 - e^{-\tilde{\lambda}_i t}] = 1 - \prod_{i=1}^{n}[1 - e^{-(\lambda_i + \sigma_i E)t}] = 1 - \prod_{i=1}^{n}[1 - e^{-\lambda_i t + \sigma_i tE}]$$

It is easy to know that $f_t(x) = 1 - \prod_{i=1}^{n}[1 - e^{-(\lambda_i + \sigma_i x)t}]$ is monotonous decreasing about x on $[-1,1]$. Now we use the method of α level cut. As the Formula (2-3), $\forall \alpha \in (0,1]$, have

$$[\tilde{R}_S(t)]_\alpha = [f(E_2^{-1}(\alpha)), f(E_1^{-1}(\alpha))]$$
$$= [1 - \prod_{i=1}^{n}(1 - e^{-\lambda_i t + \sigma_i t E_2^{-1}(\alpha)}), 1 - \prod_{i=1}^{n}(1 - e^{-\lambda_i t + \sigma_i t E_1^{-1}(\alpha)})]$$

Particularly, if all of the unit lifetimes in the parallel system are the same, the fuzzy reliability function of the system is

$$\tilde{R}_S(t) = 1 - [1 - e^{-\tilde{\lambda} \cdot t}]^n \qquad (3.5)$$

If $\tilde{\lambda} = \lambda + \sigma E$, then $\tilde{R}_S(t) = 1 - [1 - e^{-\lambda \cdot t + \sigma \cdot tE}]^n$, let $f_t(x) = 1 - [1 - e^{-\lambda \cdot t + \sigma \cdot tx}]^n$, it is easy to get the expression of the inverse function. So, for the parallel system whose lifetime distributions of all the units are the same, the membership function of the fuzzy reliability function is

$$\mu_{\tilde{R}_S(t)}(x) = E[\frac{\ln(1-x) + n\lambda \cdot t}{n\sigma \cdot t}] \qquad (3.6)$$

3.4 Fuzzy Reliability Function of Compound System

3.4.1 Fuzzy Reliability Function of the Parallel-Serial System

Generally, the parallel-serial system is like the fig 3.

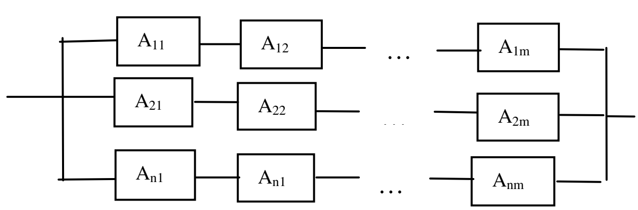

Fig. 3. The logic diagram of parallel-serial l system

Let the reliability of each unit is $R_{ij}(t)$, $i=1,...,n; j=1,...,m_j$, we can determine the reliability of each unit in the i line with the reliability function of the parallel-serial system is $R_i(t)=\prod_{j=1}^{m} R_{ij}(t)$, and then the reliability of the parallel-serial system is $R_S(t)=1-\prod_{i=1}^{n}[1-\prod_{j=1}^{m} R_{ij}(t)]$. When all of the unit lifetimes of the system obey exponential distribution, the reliability function of the system is

$$R_s(t)=1-\prod_{i=1}^{n}[1-\prod_{j=1}^{m} e^{-\lambda_{ij}t}]=1-\prod_{i=1}^{n}[1-e^{-\lambda_{is}t}]$$

Where $\lambda_{\mathrm{is}}=\lambda_{i1}+\lambda_{i2}+...+\lambda_{im}$.

Similarly, when the failure rate of each unit in the system S are fuzzy numbers that are linear formed by the symmetrical structured element E, $\tilde{\lambda}_i=\lambda_i+\sigma_i E$, $i=1,2,...,n$. E is symmetrical fuzzy structured element, So the fuzzy reliability function of the parallel-serial system is

$$\tilde{R}_S(t)=1-\prod_{i=1}^{n}[1-e^{-\lambda_{is}t-\sigma_{is}tE}]=1-\prod_{i=1}^{n}[1-e^{-\lambda_{is}t+\sigma_{is}tE}]$$

Where $\lambda_{is}=\lambda_{i1}+\lambda_{i2}+...+\lambda_{im}, \sigma_{is}=\sigma_{i1}+\sigma_{i2}+...+\sigma_{im}$

The function $f_t(x)=1-\prod_{i=1}^{n}[1-e^{-(\lambda_{is}+\sigma_{is}x)t}]$ is monotonous decreasing about x on [-1, 1]. As the Formula (2-1), and the inverse functions of $E_1(x)$ and $E_2(x)$ are all analytically expressed. As the formula (2-3), $\forall \alpha \in (0,1]$, have

$$[\tilde{R}_s(t)]_\alpha=[f(E_2^{-1}(\alpha)), f(E_1^{-1}(\alpha))]=[1-\prod_{i=1}^{n}(1-e^{-\lambda_{is}t+\sigma_{is}tE_2^{-1}(\alpha)}),1-\prod_{i=1}^{n}(1-e^{-\lambda_{is}t+\sigma_{is}tE_1^{-1}(\alpha)})$$

Where $\lambda_{is}=\lambda_{i1}+\lambda_{i2}+...+\lambda_{im}, \sigma_{is}=\sigma_{i1}+\sigma_{i2}+...+\sigma_{im}$.

Particularly, if all of the unit lifetimes are the same, $\lambda_{ij}=\lambda$, and the failure rate is $\tilde{\lambda}=\lambda+\sigma E$, the fuzzy reliability function of the system is

$$\tilde{R}_s(t)=1-[1-e^{-m\lambda t+m\sigma tE}]^n \quad (3.7)$$

Let $f_t(x)=1-[1-e^{-m\lambda t+m\sigma tx}]^n$, it is easy to get the expression of the inverse function

$$f_t^{-1}(x)=\frac{\ln[1-(1-x)^{\frac{1}{n}}]+m\lambda t}{m\sigma t}$$

Then, for the parallel-series system whose lifetime distributions of all the units are the same, the membership function of the fuzzy reliability function is

$$\mu_{\tilde{R}_s(t)}(x) = E\left[\frac{\ln[1-(1-x)^{\frac{1}{n}}]+m\lambda t}{m\sigma t}\right]\pi \tag{3.8}$$

3.4.2 The Reliability Analysis of the Serial-Parallel System

Generally, the serial-parallel system is like Fig 4.

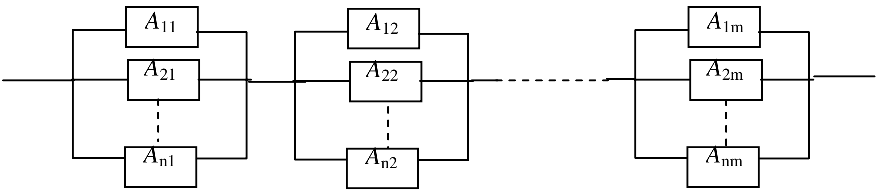

Fig. 4. The logic diagram of serial-parallel system

Let the reliability of each unit is $R_{ij}(t)$, $i=1,...,n; j=1,...,m_j$, the reliability of each unit in the j column is

$$R_j(t) = 1-\prod_{i=1}^{n}[1-R_{ij}(t)].$$

Then, the reliability of the serial-parallel system is

$$R_S(t) = \prod_{j=1}^{m}\{1-\prod_{i=1}^{n}[1-R_{ij}(t)]\}.$$

When all of the unit lifetime obeys exponential distribution, the reliability function of the system is

$$R_S(t) = \prod_{j=1}^{m}\{1-\prod_{i=1}^{n}[1-e^{-\lambda_{ij}t}]\}.$$

Similarly, when the failure rate of each unit in the system S are fuzzy numbers that s that are linear formed by the symmetrical structured element E , $\tilde{\lambda}_i = \lambda_i + \sigma_i E$, $i = 1, 2, ..., n$. So the fuzzy reliability function of the serial-parallel system is

$$\tilde{R}_S(t)=\prod_{j=1}^{m}\{1-\prod_{i=1}^{n}[1-e^{-(\lambda_{ij}+\sigma_{ij}E)t}]\}=\prod_{j=1}^{m}\{1-\prod_{i=1}^{n}[1-e^{-\lambda_{ij}t+\sigma_{ij}tE}]\}$$

As the Formula (2-1), and the inverse functions of $E_1(x)$ and $E_2(x)$ are analytical expression.

When m is a odd number, $f_t(x)=\prod_{j=1}^{m}\{1-\prod_{i=1}^{n}[1-e^{-(\lambda_{ij}+\sigma_{ij}x)t}]\}$ is a decreasing function about x on [-1,1], according to the formula(2-3)$\forall\,\alpha\in(0,1]$, have

$$\begin{aligned}[\tilde{R}_s(t)]_\alpha&=[f(E_2^{-1}(\alpha)),f(E_1^{-1}(\alpha))]\\&=[1-\prod_{j=1}^{m}(1-\prod_{i=1}^{n}e^{-\lambda_{ij}t+\sigma_{ij}tE_2^{-1}(\alpha)}),1-\prod_{j=1}^{m}(1-\prod_{i=1}^{n}e^{-\lambda_{ij}t+\sigma_{ij}tE_1^{-1}(\alpha)})\end{aligned}$$

When m is a even number, $f_t(x)=\prod_{j=1}^{m}\{1-\prod_{i=1}^{n}[1-e^{-(\lambda_{ij}+\sigma_{ij}x)t}]\}$ is a increasing function about x on [-1,1], according to the formula(2-2)$\forall\,\alpha\in(0,1]$, have

$$\begin{aligned}[\tilde{R}_s(t)]_\alpha&=[f(E_1^{-1}(\alpha)),f(E_2^{-1}(\alpha))]\\&=[1-\prod_{j=1}^{m}(1-\prod_{i=1}^{n}e^{-\lambda_{ij}t+\sigma_{ij}tE_1^{-1}(\alpha)}),1-\prod_{j=1}^{m}(1-\prod_{i=1}^{n}e^{-\lambda_{ij}t+\sigma_{ij}tE_2^{-1}(\alpha)})\end{aligned}$$

Particularly, if all of the unit lifetimes in the system are the same, they are all $\lambda_{ij}=\lambda$, and the failure rate is $\tilde{\lambda}=\lambda+\sigma E$, the fuzzy reliability function of the serial-parallel system is

$$\tilde{R}_s(t)=[1-(1-e^{-\lambda t+\sigma tE})^n]^m \tag{3.9}$$

Let $f_t(x)=[1-(1-e^{-\lambda t+\sigma tx})^n]^m$, it is easy to get the expression of the inverse function

$$f_t^{-1}(x)=\frac{\ln[1-(1-x^{\frac{1}{m}})^{\frac{1}{n}}]+\lambda t}{\sigma t}$$

At last, for the serial-parallel system whose lifetime distributions of all the units are the same, the membership function of the fuzzy reliability function is

$$\mu_{\tilde{R}_s(t)}(x)=E\left[\frac{\ln[1-(1-x^{\frac{1}{m}})^{\frac{1}{n}}]+\lambda t}{\sigma t}\right] \tag{3.10}$$

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Fuzzy Comprehensive Evaluation on the Failure Mode Having Initial Imperfection Shell

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Abstract. The two failure modes of shell structure, dynamic instability and intensive destruction, have no distinct boundary on their structural performance; they are fuzzy and are difficult to determine problems. By using the comprehensive evaluation method in fuzzy mathematics and taking the initial imperfection of the structure into consideration, this paper, taking Kiewitt spherical shell as an example, made parameter impact analysis to the structure, and investigated the influence of span ratios, the initial geometric imperfections and bar cross-section of the structure under earthquake failure mode. Ensuring the relations between a variety of response factors of the structure and fuzzy failure modes, the paper established a power failure mode shell structure of fuzzy comprehensive evaluation model and evaluation method, and described the application of the method in the discrimination of failure mode on Kiewitt monolayer Shell with initial defect; the consequences showed that this method can solve the problem of failure mode identification better with more realistic facts and possesses higher accuracy.

Keywords: Shell structure, initial imperfection, dynamic instability, strength failure, fuzzy comprehensive evaluation.

1 Introduction

As a typical large-span spatial structure, the shell structure exists two possible failure modes with effect of their causal factors such as the earthquake load [1], one is dynamic instability, and the other is strength failure. Dynamic instability has strong mutability, and there were no clear signs of instability-the structural displacement is small, and pole material plastic development is very shallow- but with a slight increase in dynamic load, the structure displacement may suddenly increase, vibration divergence occurs, and structural unstable failure occurs; strength failure shows the ductile damage, mainly features with the increase of structure according to the load amplitude, the continuous development of plastic nature, the gradual weakness of structural

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stiffness, the vibration of the equilibrium position of each node occurs more and more deviated, and finally structural collapses occur due to excessive plastic development.

Literature [2] further elaborated the mechanism of two dynamic failures, and further analyzed the dynamic failure characteristics of the structure by using indicators such as maximum displacement and the ratio of plastic bars. In fact, shell structure has a variety of initial defects which can be divided into geometric ones and physical ones, also known as cell defects and structural defects [3], but the shell structure with initial defects would be extremely weakened on its stable capacity bearing, which is an important factor that cannot be ignored in structural failure mode analysis [4]. Therefore, how to accurately determine the failure mode with initial imperfection has important practical significance for deeply understanding the dynamic failure mechanism of shell structures. In this paper, the imperfection model for the structure is the first-order buckling mode, and the initial imperfection size is 1/1000 of the structural span.

Traditional identification of these two failure modes of shell structure is based primarily on the experience of the designer, but a large number of calculation and analysis showed that the distinction between dynamic instability and strength failure are often with no obvious boundaries and with certain ambiguity, which requires a comprehensive judgment with a combination of many factors. In this case, only the designer's experience with visual discrimination of form surely has some limitations, while the adoption of fuzzy comprehensive evaluation method based on fuzzy mathematics can better solve this problem. Fuzzy comprehensive evaluation method can be a good solution for a variety of uncertainties due to the impact of difficult analytical method for quantitative analysis of the problem [5, 6, 7, 8], thus some ambiguous concepts can be explained and this method also has a great utility in dealing with such issues with fuzziness.

2 Establishing the Model of Fuzzy Comprehensive Evaluation

2.1 Evaluation Method

Fuzzy comprehensive evaluation method is a comprehensive evaluation method which constructs graded fuzzy subsets to quantify the fuzzy index reflecting the things being evaluated (that is, to determine the degree of membership), and then makes a comprehensive evaluation by applying the fuzzy transformation theory and principle of maximum degree with the consideration of various relevant factors of the things being evaluated [9].

2.2 Establish Model

The fuzzy comprehensive evaluation model of shell failure mode based on Kiewitt spherical shell is established according to the following steps:

(1) Establish objects evaluation factors set $U = \{u_1, u_2, ...u_i..., u_m\}$, in which u_1 represents the first indicator:{ratio of rise to pan, load amplitude,...yield ratio of bar..., the average plastic strain}, regarding the yield ratio of bar, we here define that there is only one bar with the input plasticity of 1P, while the input plasticity of the whole cross section is 8P [10], and the four most influential factors considered are: U={average structural plastic strain, maximum relative nodal displacement, 8P, 1P}, in which u_1 stands for average structural plastic strain, u_2 represents the maximum relative nodal displacement, u_3 is the 8P yield ratio, and u_4 is 1P yield ratio.
(2) Determine the evaluation set, $V = \{v_1, v_2, ...v_j..., v_n\}$,that is level aggregation, in which each level may correspond with a fuzzy subset where v_i represents failure modes of shell, and $V = \{instability failure, strength failure\}$.
(3) Establish a fuzzy comprehensive evaluation matrix R, and make single-factor evaluation for factor u_i(i=1,2,...,m) in set U and determine the degree of membership of the analyzed issue to evaluation v_j(j=1,2,...,n) according to u_i, and then get the fuzzy comprehensive evaluation matrix R, see Equation 1, and fuzzy comprehensive evaluation matrix consisted of the four evaluation factors set of the shell structure is displayed by Equation 2.

$$R = (r_{ij})_{m\times n} = \begin{pmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{pmatrix} \quad (1)$$

$$R = \begin{pmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \\ r_{31} & r_{32} \\ r_{41} & r_{42} \end{pmatrix} \quad (2)$$

(4) Establish factors' fuzzy set A based on the level of materiality. Due to the fact that the influence levels of various factors are different, there is a need of giving out the materiality levels of each factors in the overall evaluation, that is to set up a fuzzy subset $A = \{a_1, a_2, ...a_i..., a_m\}$, in which a_i stands for the measurement of the influential degree of u_i(i=1,2,...,m) in the overall evaluation, from the factors set, to carry out fuzzy optimization.
(5) Establish fuzzy comprehensive evaluation set B. Establish fuzzy linear transformation through fuzzy comprehensive evaluation matrix R and transfer factors materiality level fuzzy set A into fuzzy subset B on the evaluation set V, then we shall get the mathematical model for the fuzzy comprehensive structural failure mode $B = A{\cdot}R = \{b_1, b_2, ...b_j..., b_n\}$, in which b_j(j=1,2,...,n) represents the membership grade of fuzzy comprehensive evaluation set B according to the v_j comprehensive judgment.

(6) Make comprehensive evaluation. According to the maximum membership degree principle, we choose the corresponding v_j from the maximum data b_j of the fuzzy comprehensive evaluation set $B = (b_1, b_2, ...b_j..., b_n)$ as the comprehensive evaluation results, and then it will be possible to evaluate the failure mode of the structure.

3 Discrimination of the Shell Failure Mode

3.1 Structure of Prototype and the Calculation of Evaluation Factors

This article took Kiewitt spherical shell structure as the prototype (see Fig.1). The following structural parameters were analyzed: Span $L = 40m, 60m$; ratio of rise to span $f/L = 1/2, 1/3, 1/5, 1/7$; initial defect size was 40/1000, 60/1000; for each span of shell three different sizes of steel sections were used and the section size were defined as a, b, c. Peripheral support of the structure is an entire three-direction hinged support. Steel yield strength is $235N/mm^2$, modulus of elasticity $E = 2.10 \times 10^{11} N/mm^2$, and Poisson's ratio $v = 0.3$. The computed response of the net shell structure under the Taft earthquake, i.e., the four evaluation factors, are listed in Table 1. Deformed shell structure is displayed in Fig.2 and Fig.3. In the shell number k42a in the table, 4 means the span is 40 meters, and 2 stands for span ratio as 1/2, a represents cross-sectional area, and so the remaining shell number.

Fig.3 shows when the structure is imposed on seismic amplitude of $800cm/s^2$, the structure undergoes a process of gradual development before the intensity of damage, which indicating intensity of damage have some sort of ductility. Fig.4 shows when the structure is imposed on seismic amplitude of $700cm/s^2$, the structure does not undergo a process of gradual development before the sudden severe deformation, indicating there is no obvious sign before the instability.

From the data in Table 1, when the average plastic strain value is greater than 0.008376, the structure undergoes strength failure, when the average plastic strain value is less than 0.000468, the structural strength failure does not occur. In like manner, we can obtain the critical value of other three evaluation factors — the largest relative nodal displacement, 8P rod ratio and 1P rod ratio — when they experiences strength failure are 0.0106,0.5804,0.9643.

3.2 For the Calculation of Fuzzy Discrimination

Construction of Fuzzy Comprehensive Evaluation Matrix R

First of all, as for the single factor evaluation, determine the degree of membership under the background of failure modes of various factors. Through statistical analysis of the data in Table 1, take shell k42 as an example,

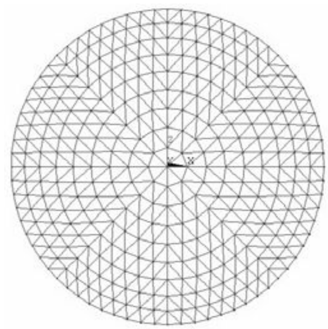

Fig. 1. The single-layer reticulated shell(L=40m,f/L=1/2)

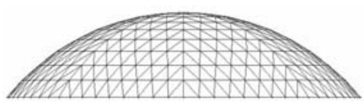

Fig. 2. The single-layer reticulated shell(L=40m,f/L=1/2)

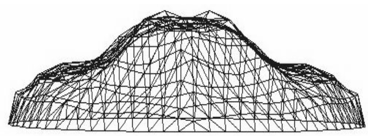

Fig. 3. Deformation shape with strength failure

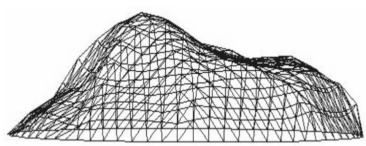

Fig. 4. Deformation shape with dynamic instability

obtain the data of failure modes, as shown in Table 2, thus getting the fuzzy comprehensive evaluation matrix:

$$R = \begin{pmatrix} 0.601 & 0.142 \\ 0.522 & 0.089 \\ 0.327 & 0.145 \\ 0.471 & 0.117 \end{pmatrix}$$

Table 1. Characteristic response

Load	Initial flaw value	Shell number	u_1	u_2	u_3	u_4	Failure mode
Taft	40/1000	k42a	0.000687	0.0042	0.1125	0.7231	unstable failure
		k42b	0.002742	0.0091	0.4088	0.8981	strength failure
		k42c	0.002119	0.0087	0.5723	0.9643	strength failure
		k43a	0.000928	0.0034	0.1453	0.7758	unstable failure
		k43b	0.002691	0.0087	0.3264	0.8213	strength failure
		k43c	0.001061	0.0019	0.1884	0.6742	unstable failure
		k45a	0.000842	0.0033	0.1105	0.8821	unstable failure
		k45b	0.001208	0.0024	0.1683	0.6807	unstable failure
		k45c	0.002687	0.0106	0.2013	0.6709	strength failure
		k47a	0.001839	0.0052	0.3826	0.7391	strength failure
		k47b	0.001175	0.0029	0.1416	0.6452	unstable failure
		k47c	0.002752	0.0077	0.2435	0.7011	strength failure
	60/1000	k62a	0.000468	0.0012	0.0317	0.0821	unstable failure
		k62b	0.002189	0.0047	0.4024	0.7847	strength failure
		k62c	0.002421	0.0042	0.3701	0.6408	strength failure
		k63a	0.001034	0.0041	0.1155	0.7632	unstable failure
		k63b	0.008376	0.0083	0.2408	0.7144	strength failure
		k63c	0.001057	0.0059	0.0287	0.0911	unstable failure
		k65a	0.002275	0.0068	0.2369	0.7834	strength failure
		k65b	0.000933	0.0051	0.1433	0.6201	unstable failure
		k65c	0.001157	0.0102	0.4215	0.9303	strength failure
		k67a	0.001365	0.0035	0.2263	0.8527	unstable failure
		k67b	0.003481	0.0088	0.5804	0.7611	strength failure
		k67c	0.002565	0.0074	0.3114	0.8225	strength failure

Table 2. Shell K42a failure mode evaluation data

Factor	Instability failure	Strength failure
u_2	0.601	0.142
u_2	0.522	0.089
u_3	0.327	0.145
u_4	0.471	0.117

Determine fuzzy set A (importance of factors)

Using Delphi method to determine the coefficient of degree of importance of each factor $a_i(i = 1, 2, ..., m)$, Establish the scoring table in priority, as shown in Table 3, and get fuzzy set A (importance of factors) through the calculation Formula 3 and 4.

$$d = \frac{\sum A_{max} - \sum A_{min}}{a_{max} - a_{min}} \tag{3}$$

$$a_i = \frac{\sum A_i - \sum A_{min}}{d} + 0.1 \tag{4}$$

Where d is the differential, $a_{max} = 1$, $a_{min} = 0$

Table 3. Coefficient scoring table of factors in priority concerning the degree of importance

Factors	u_1	u_2	u_3	u_4	$\sum_{i=1}^{20} A_i$	a_i
u_1	*	13	9	12	34	0.657
u_2	7	*	6	10	23	0.186
u_3	11	14	*	17	42	1.000
u_4	8	10	3	*	21	0.100

Determine Set B — Fuzzy Comprehensive Evaluation Set, and Have a Comprehensive Evaluation

Conducting fuzzy linear transformation, i.e., $B = A \cdot R = (0.8660, 0.2666)$. Under the principle of maximum degree of membership, confirm the failure modes of shell k42a as unstable failure. In like manner, the method of fuzzy comprehensive evaluation can be applied to other shells so as to discriminate, and the discrimination results are in Table 4.

Table 4. Results on shell failure mode fuzzy comprehensive evaluation

Shell number	Evaluation value	Failure mode	Shell number	Evaluation value	Failure mode
k42b	0.3554	strength failure	k62a	0.5708	unstable failure
k42c	0.3029	strength failure	k62b	0.2846	strength failure
k43a	0.8857	unstable failure	k62c	0.1695	strength failure
k43b	0.2633	strength failure	k63a	0.9928	unstable failure
k43c	0.6561	unstable failure	k63b	0.2107	strength failure
k45a	0.7012	unstable failure	k63c	0.7816	unstable failure
k45b	0.4091	strength failure	k65a	0.3164	strength failure
k45c	0.2474	strength failure	k65b	0.6793	unstable failure
k47a	0.3308	strength failure	k65c	0.3432	strength failure
k47b	0.8911	unstable failure	k67a	0.7491	unstable failure
k47c	0.1802	strength failure	k67b	0.7195	unstable failure
			k67c	0.1728	strength failure

3.3 The Results of Fuzzy Comprehensive Evaluation Comparing with the Results of Experience Discrimination

Compare the shell failure mode obtained from experience and the failure mode from fuzzy comprehensive evaluation, when the result from experience is consistent with that of the fuzzy comprehensive evaluation, it is accounted as “1”, otherwise “0”, as shown in Table 5.

Table 5. The results comparison

Shell number	Experience results	Fuzzy comprehensive evaluation	Consistent value	Match ratio
k42a	unstable failure	unstable failure	1	
k42b	strength failure	strength failure	1	
k42c	strength failure	strength failure	1	
k43a	unstable failure	unstable failure	1	
k43b	strength failure	strength failure	1	
k43c	unstable failure	unstable failure	1	
k45a	unstable failure	unstable failure	1	
k45b	strength failure	unstable failure	0	
k45c	strength failure	strength failure	1	
k47a	strength failure	strength failure	1	
k47b	unstable failure	unstable failure	1	
k47c	strength failure	strength failure	1	91.67%
k62a	unstable failure	unstable failure	1	
k62b	strength failure	strength failure	1	
k62c	strength failure	strength failure	1	
k63a	unstable failure	unstable failure	1	
k63b	strength failure	strength failure	1	
k63c	unstable failure	unstable failure	1	
k65a	strength failure	strength failure	1	
k65b	unstable failure	unstable failure	1	
k65c	strength failure	strength failure	1	
k67a	unstable failure	unstable failure	1	
k67b	unstable failure	strength failure	0	
k67c	strength failure	strength failure	1	

As the above table shows, the experience results of shell k45b, k67b are not consistent with those of fuzzy comprehensive evaluation.The accordance ratio of shell failure mode by applying fuzzy comprehensive evaluation method and the experience results reached 91.67%, indicating that this method has a relatively high accuracy as regard to the discrimination of shell failure modes.

4 Conclusion

In the application of fuzzy comprehensive discrimination in fuzzy mathematics to the defective shell failure modes, we draw the following conclusions through the analysis judgment of concrete examples:

(1) In the premise of certain span of shell structure, with the decrease of the ratio of rise to span, the failure mode of the structure witnesses the trend from unstable damage to intensity damage. However, due to the initial imperfections, the ultimate bearing capacity of the structure decreased, resulting in the change of the failure modes of some shell structures, for example, shell k43c, k45b, k63c were changed into unstable failure.
(2) In the process of judging the failure modes of shell structure through fuzzy comprehensive evaluation method, since the failure modes of some structures changed, the results of fuzzy comprehensive evaluation differ from the experience results sometimes, but the match rate still achieved 91.67%, so this method is not affected in the application of shell failure modes evaluation.
(3) Fuzzy comprehensive evaluation method has quantified the qualitative evaluation, and has addressed the problem that qualitative and quantitative assessment cannot be combined very well and the qualitative indicators are hard to compare in the judging process. However, the accuracy of fuzzy comprehensive evaluation methods is also affected by some negative factors, such as the control of various errors, the selection of calculating parameters, thus it is necessary to do some further studies concerning the application of this the method in the area of engineering.

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Structured Element of Intuitionistic Fuzzy Multiattribute Decision-Making Applied in the Evaluation of Digital Library in Universities

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Abstract. Evaluation of digital library is an essential process in the library development. However, the digital library is a complex system, which needs a lot of indices included in the evaluation. Up to now, there is still no scientific and rational comprehensive evaluation method developed, which become the urgent problem in the digital library evaluation. In this paper, a novel evaluation method is put forward based on the structured element of intuitionistic fuzzy multi-attribute decision-making, which can comprehensively assess the indices such as holdings, techniques, service and management. This method may promote the discovery of essential factors and their relationship, and benefit the improvement in every aspect to serve the readers better.

Keywords: Digital library, fuzzy structured element theory, intuitionistic fuzzy multi-attribute decision-making.

1 The Significance of Digital Library Evaluation

What composes a qualified modern digital library and how to evaluate it is the pressing task before us. To resolve it, definitively we must at first set up the evaluation system. However, digital library has its characteristics of multi-subject book reserve and diversity of application domains; thus people raise different evaluation standards from their own position. Therefore, it's imperative to establish a reasonable, scientific, evaluative system, a subject of primary importance in the field of digital library research. This paper, in terms of the fuzzy mathematics, put forward a new general index system, for comparing and evaluating digital libraries with further discussion of the relevant issues of the above-raised evaluation system.

2 The Evaluation System of Digital Library

Evaluation studies of digital library have made consistent progress. However, in order to establish a feasible evaluation system, the advances reported in the

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preliminary results of foreign researchers must be analyzed and composed. Herewith we compose digital library evaluation system consisting of four parts: book reserve information, Technology, service, and Operation management. Each part has its own evaluative component and index.

Table 1. Digital library evaluation system

Book reserve information	Book quantity
	Reserve management
Technology	systems technology
	Information Retrieval
	Customer-oriented service technology
Service	Service Scope
	Quality of Service
Opertion management	Rights Management
	Benefit Management

3 Academic Fuzzy Synthetical Evaluation of Digital Library

Subject to intuitionistic fuzzy Multiple Attribute Decision Making of structured element theory, exactly, use of the homeomorphic property between family of standard bounded monotone function and bounded real fuzzy numbers on [-1, 1], we shall convert complex operations of fuzzy numbers into the corresponding operations of the same monotone functions. Thus, before further discussion, we shall equate homeomorphism monotonic function of fuzzy decision by data.

Let $\tilde{x}_{ij} =< f_x(E), g_x(E) > (i=1,2,\cdots,m \quad j=1,2,\cdots,n)$ E is a regular fuzzy structured element. $< f_x(E), g_x(E) >$ is a homeomorphism bounded monotone function on[-1,1], it is called intuitionistic fuzzy indicated value function. Then, the structured element expression-form of $\tilde{F}$ is:

$$\tilde{F} = \begin{bmatrix} < f_{11}(E), g_{11}(E) > & < f_{12}(E), g_{12}(E) > & \cdots & < f_{1n}(E), g_{1n}(E) > \\ < f_{21}(E), g_{21}(E) > & < f_{22}(E), g_{22}(E) > & \cdots & < f_{2n}(E), g_{2n}(E) > \\ \vdots & \vdots & \vdots & \vdots \\ < f_{m1}(E), g_{m1}(E) > & < f_{m2}(E), g_{m2}(E) > & \cdots & < f_{mn}(E), g_{mn}(E) > \end{bmatrix}$$

Then we give the application step of IFWA operator of based on the structured element of Decision-Making.

Let $\tilde{x}_{ij} =< f_x(E), g_x(E) > (i=1,2,\cdots,m \quad j=1,2,\cdots,n)$ is a set Fuzzy Number based on structured element theory, E is a regular symmetrical fuzzy structured element. $< f_x(E), g_x(E) >$ is a homeomorphism bounded monotone function on[-1,1], it is called intuitionistic fuzzy indicated value function. We called

$$\sum_{j=1}^{n}\omega_j x_{ij}=\sum_{j=1}^{n}\omega_j<f_{ij}(E),g_{ij}(E)>=\sum_{j=1}^{n}<1-[1-f_x(E)]^{\omega},[g_x(E)]^{\omega}>$$

is intuitionistic fuzzy weighted-average operator, where $\omega=(\omega_1,\omega_2,\cdots,\omega_n)^T$ is weight vector of $d_j(j=1,2,\cdots,n)$, $\omega_j\in[0,1](j=1,2,\cdots,n),\sum_{j=1}^{n}\omega_j=1$.

Step 1 Make E is structured element and equate the corresponding monotone function of $\tilde{x}_{ij}$ is $<f_x(E),g_x(E)>$

Step 2 equate

$$\sum_{j=1}^{n}\omega_j x_{ij}=\sum_{j=1}^{n}\omega_j<f_{ij}(E),g_{ij}(E)>=\sum_{j=1}^{n}<1-[1-f_x(E)]^{\omega},[g_x(E)]^{\omega}>$$

Step 3 figure out $f_{ij}(x)$, $g_{ij}(x)$

Step 4 account integral quantity of $\tilde{x}_{ij}$ is on[-1 1]

Step 5 Arrange.

4 Illustrative Example

Based on expert estimate methods we get the weight:

{0.12 0.1 0.09 0.1 0.15 0.07 0.13 0.12 0.12}

Since it's difficult to get the exact attribute values , we use language calibration to describe attribute values. Following are the evaluating results of four academic digital libraries P_1 P_2 P_3 P_4 by evaluation specialists,:

Table 2. Evaluation index and scale

Digita library	Property								
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
P_1	E	F	E	F	E	E	E	E	G
P_2	E	F	E	F	E	E	E	E	E
P_3	G	E	G	F	G	G	E	O	G
P_4	G	E	G	F	G	G	E	O	G
Weight	0.12	0.12	0.09	0.1	0.15	0.07	0.13	0.12	0.1

Notes: E——excellent; F——fine; G——good; O——outstanding

For language describing can't compute ranking results automatically in the computer system, it is applicable to convert them into triangle fuzzy numbers. Finally we get results through intuitionistic fuzzy decision-making method of structured element (see Table 3).

Table 3. Decision-making matrix table

Digital library	Property								
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
P_1	(0.6,0.7,0.8) (0,0.1,0.2)	(0.5,0.6,0.7) (0.1,0.2,0.3)	(0.6,0.7,0.8) (0, 0.1, 0.2)	(0.5,0.6,0.7) (0.1,0.2,0.3)	(0.6,0.7,0.8) (0,0.1,0.2)	(0.6,0.7,0.8) (0,0.1,0.2)	(0.6,0.7,0.8) (0,0.1,0.2)	(0.6,0.7,0.8) (0,0.1,0.2)	(0.4,0.5,0.6) (0.1,0.2,0.3)
P_2	(0.6,0.7,0.8) (0,0.1,0.2)	(0.5,0.6,0.7) (0.1,0.2,0.3)	(0.6,0.7,0.8) (0,0.1,0.2)	(0.5,0.6,0.7) (0.1,0.2,0.3)	(0.6,0.7,0.8) (0,0.1,0.2)	(0.6,0.7,0.8) (0,0.1,0.2)	(0.6,0.7,0.8) (0,0.1,0.2)	(0.6,0.7,0.8) (0,0.1,0.2)	(0.6,0.7,0.8) (0,0.1,0.2)
P_3	(0.6,0.7,0.8) (0,0.1,0.2)	(0.6,0.7,0.8) (0,0.1,0.2)	(0.4,0.5,0.6) (0,0.1,0.2)	(0.5,0.6,0.7) (0.1,0.2,0.3)	(0.4,0.5,0.6) (0.1,0.2,0.3)	(0.4,0.5,0.6) (0.1,0.2,0.3)	(0.4,0.5,0.6) (0.1,0.2,0.3)	(0.5,0.6,0.7) (0.1,0.2,0.3)	(0.6,0.7,0.8) (0,0.1,0.2)
P_4	(0.4,0.5,0.6) (0.1,0.2,0.3)	(0.6,0.7,0.8) (0,0.1,0.2)	(0.4,0.5,0.6) (0.2,0.3,0.4)	(0.5,0.6,0.7) (0.1,0.2,0.3)	(0.4,0.5,0.6) (0.1,0.2,0.3)	(0.4,0.5,0.6) (0.1,0.2,0.3)	(0.6,0.7,0.8) (0,0.1,0.2)	(0.7,0.8,0.9) (0,0,0)	(0.4,0.5,0.6) (0.1,0.2,0.3)
Weight	0.12	0.12	0.09	0.1	0.15	0.07	0.13	0.12	0.1

In terms of structured element theory, we can get the fuzzy indicated value. The monotone function of $\tilde{x}_{ij}$ is $f_{ij}(x)$:

$$f_{11}(x) = < x, 0.8 + 0.1x, 0 > \qquad -1 \le x \le 1$$

$$f_{12}(x) = < x, 0.6 + 0.1x, 0.2 + 0.1x > \qquad -1 \le x \le 1$$

Analogously, it has that

$f_{ij}(x) \quad i = 1,2,\cdots,10 \quad j = 1,2,\cdots,10$.

Calculated according to MATLAB software

```
x=-1:0.1:1;y=[];z=[];


y(1,:)=(0.3+0.1*x).^.1.2.*(0.4+0.1*x).^.1.2.*(0.3+0.1*x).^.0.9.*(0.4+0.1*x).^.
1.*(0.3+0.1*x).^.1.5.*(0.3+0.1*x).^.0.7.*(0.3+0.1*x).^.1.3*(0.3+0.1*x).^.1.2*(0.5
+0.1*x).^.1;
y(2,:)=(0.3+0.1*x).^.1.2.*(0.4+0.1*x).^.1.2.*(0.3+0.1*x).^.0.9.*(0.4+0.1*x).^.
1.*(0.3+0.1*x).^.1.5.*(0.3+0.1*x).^.0.7.*(0.3+0.1*x).^.1.3*(0.3+0.1*x).^.1.2*(0.3
+0.1*x).^.1;
y(3,:)=(0.3+0.1*x).^.1.2.*(0.3+0.1*x).^.1.2.*(0.5+0.1*x).^.0.9.*(0.4+0.1*x).^.
1.*(0.5+0.1*x).^.1.5.*(0.5+0.1*x).^.0.7.*(0.5+0.1*x).^.1.3*(0.4+0.1*x).^.1.2*(0.3
+0.1*x).^.1;
y(4,:)=(0.5+0.1*x).^.1.2.*(0.3+0.1*x).^.1.2.*(0.5+0.1*x).^.0.9.*(0.4+0.1*x).^.
1.*(0.5+0.1*x).^.1.5.*(0.5+0.1*x).^.0.7.*(0.3+0.1*x).^.1.3*(0.3+0.1*x).^.1.2*(0.5
+0.1*x).^.1;
```

```
for i=1:4
    yuce(1,i)=1-trapz(x,y(i,:));
    yuce(2,i)=trapz(x,z(i,:));
end
yuce

yuce =

0.3217    0.3554    0.2514    0.1944

0.2505    0.2347    0.2607    0.3069
```

integral quantity of $D_3(x),\cdots,D_7(x)$. $G_i(x)$, $i=1,2,\cdots,10$ on [-1 1] is that:

$$(0.3217,0.2505)\quad(0.3554,0.2347)\quad(0.2514,0.2607)\quad(0.1944,0.3069)$$

$$s(d_1)=0.0712\qquad s(d_2)=0.1207\qquad s(d_3)=-0.0093\qquad s(d_4)=-0.1125$$

The sequence based on algorithm is that:

$$P_4 \prec P_3 \prec P_1 \prec P_2$$

So the sequence of digital library is:

$$P_2 \quad P_1 \quad P_3 \quad P_4$$

In this paper, structured element of intuitionistic fuzzy multi-attribute decision-making was applied in the evaluation of digital library in university, to comprehensively assess the overall status, working performance, quality of service and expected benefits. The qualitative study could help us more comprehensively and objectively evaluate the digital library, which is attributed to fuzzy elements, and take the targeted measures to improve it based on the development of the outside environment and internal conditions, so it can provide the valuable data for the long-term development of university libraries.

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Fuzzy Decision of Enterprise Human Resources Planning under Demand Exceeding Supply

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Abstract. It is important for enterprises to make human resources planning scientifically. Ranking criteria of triangular fuzzy numbers was introduced simply and a model of fuzzy integer linear programming decision on demand exceeding supply of human resources was established based on cost minimizing in this paper. Then an example was given to demonstrate this method. And the proposed approach is simple, scientific and feasible to transform triangular fuzzy numbers ranking criteria into the classic linear programming problem on fuzzy decision of enterprise human resources planning under demand exceeding supply.

Keywords: Human resources planning, fuzzy decision, integer linear programming, demand exceeding supply.

1 Introduction

There's no doubt one of the most important resources is human resources in the 21st century, so it is significant to ensure a certain number of suitable human resources with good quality for enterprises' development. The most prominent feature of the 21st century is the fact that the world is changing constantly, and thus the demand exceeding supply of enterprise human resources will become a common phenomenon due to the rapid change of external environment. The shortage of enterprise human resources means miss of opportunities, loss of income, a waste of other resources and inefficiency. Generally, enterprises have many measures for the shortage of human resources, such as work overtime, job rotation, training, external recruitment, external lease, purchasing efficient equipments, business outsourcing, and those measures with different features are applicable to different conditions [1].

Literature [1] proposed the lowest cost decision-making model of the shortage of human resources, which was solved by constructing a precise integer programming. But in reality, the system environment that a decision problem faces is uncertain. Since 1970, Bellman and Zadeh proposed the concept of fuzzy decision-making, the fuzzy optimization has been a focus on research field for scholars. According to the objective reality, this paper comes up with the ambiguous processing to the question in literature [1].

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The problem is defined as: enterprises are proposing measures including work overtime, job rotation, training, external recruitment, external lease, purchasing efficient equipments, business outsourcing and others when human resources are in short supply. With uncertainty costs of each measure, the matter is which portfolio of measures above we should take to solve the problem of shortage and achieve minimum cost. The matter belongs to asymmetric fuzzy programming problem, more precisely, fuzzy objective coefficient linear programming problem [2,3].

The assumptions of constructing model are as follows:

Assumption 1: Costs include fixed costs and variable costs;

Assumption 2: Purchasing efficient equipments is only calculated in depreciation costs within the financial cycle based on a unified financial period;

Assumption 3: Fixed costs and variable costs are triangular fuzzy numbers.

2 Ranking Criteria of Triangular Fuzzy Numbers

Since the 1970s, people began to study how to determine the sequence of fuzzy numbers. So far, more than 20 kinds of methods have been raised to determine the sequence of fuzzy numbers, but no method is universally recognized as the best. At the same time, domestic and foreign scholars put forward a variety of numerical methods for fuzzy linear programming. Literature [4] proposed a ranking criteria of fuzzy numbers, and literature [5] introduced the ranking criteria of L-R fuzzy numbers; furthermore, literature [6] came up with an approach to change linear programming of triangular fuzzy coefficient into a conventional linear programming on condition that membership degrees of the fuzzy coefficients in the objective function and constraints can take different values. This paper introduces the ranking criteria triangular fuzzy numbers recommended in literature [7] for fuzzy decision of human resource planning, which are described as follows.

Define $\tilde{A} = (a,b,c)$ as triangular fuzzy numbers, and λ-cut sets of $\tilde{A}$ are expressed as $A(\lambda) = [A_L(\lambda), A_R(\lambda)]$, $0 \le \lambda \le 1$. Meanwhile, define $A_L(\lambda) = a + (b-a)\lambda$ as left terminal point of the λ-cut sets, and $A_R(\lambda) = c - (c-b)\lambda$ as the right terminal point. Set $F_N = \{(a,b,c) | \forall a < b < c, a,b,c \in R\}$.

Introduce definitions that are:

Definition 1. *Define signed distance of* λ *level fuzzy interval* $[A_L(\lambda), A_R(\lambda)]$ *as:*

$$d[A_L(\lambda), A_R(\lambda)] = \frac{1}{2}\left[a + c + (2b - a - c)\lambda\right] \tag{1}$$

Definition 2. *Assume triangular fuzzy number as* $\tilde{A} = (a,b,c)$ *, and define signed distance of* $\tilde{A}$ *as:*

$$d(\tilde{A}) = \int_0^1 d[A_L(\lambda), A_R(\lambda)]d\lambda = \frac{1}{4}(2b + a + c) \tag{2}$$

Definition 3. *Set* $\tilde{A}, \tilde{B} \in F_N$, *and sort criteria are as follows:*

(1) $\tilde{A} < \tilde{B}$ *if and only if* $d(\tilde{A}) < d(\tilde{B})$ *;*

(2) $\tilde{A} = \tilde{B}$ *if and only if* $d(\tilde{A}) = d(\tilde{B})$ *.*

3 Constructing the Lowest Cost Decision Model of Enterprise Human Resources under Demand Exceeding Supply

When enterprise human resources are in short supply, that is, the shortage number of some human resources in a certain period is denoted as H, the enterprise will adopt the following measures: work overtime, job rotation, training, external recruitment, external lease, purchasing efficient equipments, business outsourcing, and others. All the above measures will have certain limit due to the actual situation and the impact of environmental constraints, and also come about the relevant variable costs and fixed costs. The basic information is shown in Table 1. In particular, the employees' number of non-work brought about by the extension of working time is indicated as X_1. For example, there are 12 employees in a company whose working hours per day extend from 8 hours to 10 hours, from which we can get $X_1 = 3$ $(X_1 = 12 * (10-8)/8)$. Numbers of human resources shortage solved through purchasing efficient equipments are indicated as X_6 and business outsourcing as X_7, and their values are H or 0.

Table 1. Basic information on measures for shortage of human resources

Measures	The planned number	The limited number	Variable cost	Fixed cost
Work overtime	X_1	H_1	$\tilde{V}_1$	$\tilde{F}_1$
Job rotation	X_2	H_2	$\tilde{V}_2$	$\tilde{F}_2$
Training	X_3	H_3	$\tilde{V}_3$	$\tilde{F}_3$
External recruitment	X_4	H_4	$\tilde{V}_4$	$\tilde{F}_4$
External lease	X_5	H_5	$\tilde{V}_5$	$\tilde{F}_5$
Purchasing efficient equipments	X_6	H_6	$\tilde{V}_6 = 0$	$\tilde{F}_6$
Business outsourcing	X_7	H_7	$\tilde{V}_7 = 0$	$\tilde{F}_7$
Others	X_8	H_8	$\tilde{V}_8$	$\tilde{F}_8$

Set the number of staffs corresponding to measures above as X_i, and $i = 1,2,3,4,5,6,7,8$ denote the corresponding node. Y_i is appointed for taking the i th measures ($Y_i = 1$) or not ($Y_i = 0$).

The fuzzy integer linear programming model of human resources under demand exceeding supply can be formulated mathematically as

(M-1)
$$\min z = \sum_{i=1}^{8} (\tilde{V}_i X_i + \tilde{F}_i Y_i)$$

$$\text{s.t.} \begin{cases} \sum_{i=1}^{8} X_i \geq H \\ X_i \leq H_i, i = 1,2,3,4,5,6,7,8 \\ X_i \leq MY_i, i = 1,2,3,4,5,8 \\ X_6 = HY_6 \\ X_7 = HY_7 \\ X_i \geq 0 \end{cases}$$

Where Y_i value 0 or 1, and the values of X_i are integer. Assume fixed costs $\tilde{V}_i = (V_i^1, V_i^2, V_i^3)$ as triangular fuzzy numbers that are greater than 0 and variable costs $\tilde{F}_i = (F_i^1, F_i^2, F_i^3)$ as triangular fuzzy numbers that are greater than 0, and M is a sufficiently large number.

According to the triangular fuzzy numbers arithmetic, the objective function can be changed to $\min z = \sum_{i=1}^{8} (\tilde{V}_i X_i + \tilde{F}_i X_i) = (\sum_{i=1}^{8} V_i^1 X_i + \sum_{i=1}^{8} F_i^1 Y_i, \sum_{i=1}^{8} V_i^2 X_i + \sum_{i=1}^{8} F_i^2 Y_i, \sum_{i=1}^{8} V_i^3 X_i + \sum_{i=1}^{8} F_i^3 Y_i)$.
Change the model above to the following linear programming to solve based on the method introduced in the literature [4], and the reconstruction model is

(M-2)
$$\min z = \frac{1}{4}\left(2(\sum_{i=1}^{8} V_i^1 X_i + \sum_{i=1}^{8} F_i^1 Y_i) + (\sum_{i=1}^{8} V_i^2 X_i + \sum_{i=1}^{8} F_i^2 Y_i) + (\sum_{i=1}^{8} V_i^3 X_i + \sum_{i=1}^{8} F_i^3 Y_i) \right)$$

$$\text{s.t.} \begin{cases} \sum_{i=1}^{8} X_i \geq H \\ X_i \leq H_i, i = 1,2,3,4,5,6,7,8 \\ X_i \leq MY_i, i = 1,2,3,4,5,8 \\ X_6 = HY_6 \\ X_7 = HY_7 \\ X_i \geq 0 \end{cases}$$

Where Y_i value 0 or 1, and the values of X_i are integer, and M is a sufficiently large number.

4 Application and Results

Now net demand of personnel is 20 for a position in a company, and measures to be adopted and related data are shown in table 2 based on the comprehensive analysis of the situation inside and outside the company.

Table 2. Basic information on measures for shortage of human resources for a position in an company

Measures	The planned number	The limited number	Variable cost (yuan)	Fixed cost (yuan)
Work overtime	X_1	10	(1800,2000,2100)	0
Job rotation	X_2	5	0	(180,200,210)
Training	X_3	5	(1900,2000,2100)	(1500,2000,2200)
External recruitment	X_4	30	(1800,2000,2200)	(1500,2000,2500)
External lease	X_5	15	(1300,1500,1800)	(1300,1500,2000)
Purchasing efficient equipments	X_6	--	0	(55000,60000,65000)
Business outsourcing	X_7	--	0	(38000,40000,42000)

According to the reconstruction model (M-2), the mathematical model of the question above is as follows:

$$\min z = 1975X_1 + 2000X_3 + 2000X_4 + 1525X_5 + 197.5Y_2 + 1925Y_3 + 2000Y_4 + 1575Y_5 + 60000Y_6 + 40000Y_7$$

$$\text{s.t.} \begin{cases} X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 \geq 20 \\ X_1 \leq 10 \\ X_2 \leq 5 \\ X_3 \leq 5 \\ X_4 \leq 30 \\ X_5 \leq 15 \\ X_i \leq MY_i, i = 1,2,3,4,5 \\ X_6 = 20Y_6 \\ X_7 = 20Y_7 \\ X_i \geq 0, i = 1,2,3,4,5,6,7 \end{cases}$$

Where Y_i=0 or 1, M is a sufficiently large number and the values of X_i are integer.

If assume $M = 1000000$, we can construct the model and figure out the values through software package LINGO , the computed results of which are: $X_1 = 0$, $X_2 = 5$, $X_3 = 0$, $X_4 = 0$, $X_5 = 15$, $X_6 = 0$, $X_7 = 0$, $z = 24647.5$. In other words, the best measures combination to take is 5 staffs from job rotation and 15 staffs from external lease, and thus the minimum cost needed is approximately 24,647.5 yuan.

5 Conclusion

The fuzzy linear programming problem in which objective function contains fuzzy numbers is in conformity with the condition of human resources planning decision-making under demand exceeding supply. It is simple, scientific and feasible to transform ranking criteria of triangular fuzzy numbers into the classic linear programming problem in this paper. However, another future topic is to study multi-objective fuzzy linear programming problems in which both the objective function and constraints contain fuzzy numbers, therefore, human resources planning development will be further promoted and make greater progress.

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Evaluation Model of Credibility of E-Commerce Website Using Fuzzy Multi-Attribute Group Decision Making: Based on Fuzzy Structured Element*

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Abstract. The evaluation index system of credibility of e-commerce website is established in the first place, and then the evaluation model of fuzzy multi-attribute group decision making is proposed. The structured element is used in the calculation of model, which is proved an simple, convenient and effective through an example.

Keywords: Website trust, credibility, fuzzy multi-attribute group decision making, fuzzy structured element.

1 Introduction

In recent years, e-commerce, online transactions and various websites rise with the development of the Internet. By the end of June 2007, CNNIC [1] shows that the number of chinese netizens had reached 162 million, next to that of American, ranking second in the world. As the online transactions is accepted gradually by people, many enterprises have established their own websites to obtain a steady stream of economic benefits. However, not all websites can bring satisfactory results, because there are many factors that make a website be recognized and succeed, and one of them is the credibility of the website.

As J.Kim said, consumers need to build the initial trust in the site, and then begin the selection of goods and transactions [2]. Therefore, a successful website should be one that attracts customers, worthy to trust and rely on, and provides the information of quality [3]. The credibility of website has been an very important factor affecting customers' buying behavior.

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Therefore, how to evaluate the credibility of e-commerce website for further guidance on the building of website and to raise the credibility of website has become one of the research hotspots in the field of e-commerce.

2 The Research Summary on the Evaluation of E-Commerce Website

Researches on the evaluation of website initially focus on the overall evaluation of website, including specialized academic institutions, commercial websites as well as scholars. More representative are as follows: the "10C" indexes of network information resources evaluation proposed by Betsy Richamond [4] including content, credibility, critical thinking, copyright, citation, continuity, censorship, connectivity, comparability and context. The "CARS" system of website evaluation proposed by Robert Harris [5] includes credibility, accuracy, legitimacy and support. Chinese scholars such as CHEN Ya and ZHENG Jian-ming [6] put forward five indexes used to the evaluation of website, and the indexes include content, site overview, page design, operations and openness. LI Don-min [7] evaluated the website from three aspects of content, technology and design of website. In recent years, researches have been turned from the overall evaluation of website to a specific area by some scholars, such as DING Nian [8] evaluated the customer satisfaction of the website, PAN Yong [9] studied the credibility evaluation model of e-commerce website from the perspective of a third party, FENG Ya-bei [10] evaluated the site quality of service.

Among the methods of website evaluation, hierarchical analysis, correlation analysis and fuzzy evaluation [11~13] and so on are used by some scholars. Although those researches enrich the evaluation methods and means of e-commerce website, there are still many areas needed to be improved, especially the determination of value and weights of the evaluation, the reason is that different people have different preferences in site evaluation, and the index scores and weights are uncertain, usually expressed as interval numbers and fuzzy numbers. So, fuzzy multi-attribute group decision making method can be considered, but the commonly used fuzzy decision making technologies mainly use α-cut sets, which need to traverse all real numbers in the interval of [0,1] in order to obtain the membership function curve of fuzzy utility function, which is very difficult and cumbersome. A fuzzy multi-attribute group decision making method based on structured element was proposed in [14], and the method can resolve the above problems, and the credibility of website is evaluated by use of this method in this paper.

3 The Fuzzy Multi-attribute Group Decision-making Evaluation of Credibility of E-commerce Website

A. Index system

The overall credibility of site can be measured from three aspects including site structure, network enterprises and network marketing based on the situation of china and the sorted factors affecting the credibility of e-commerce site, according with the principles of science, system, feasibility and conciseness, and the credibility evaluation index system of established site is shown in Table 1.

Table 1. The credibility evaluation index system of site

Target layer	First grade indexes	Second indexes
The credibility of site	credibility of site structure (C_I)	overall design of site(C_1)
		performance of site(C_2)
		information technology(C_3)
	credibility of enterprise (C_{II})	corporate strength(C_4)
		corporate reputation(C_5)
		credit certificate(C_6)
	credibility of network marketing (C_{III})	guarantee system(C_7)
		logistics distribution(C_8)
		after-sale service(C_9)

B. Fuzzy compromised group decision-making method

The fuzzy compromised group decision making method is set up based on both fuzzy ideal function and fuzzy negative ideal function, in which fuzzy ideal function is composed of fuzzy maximal function of fuzzy index value function from each attribute, and fuzzy negative ideal function is composed of fuzzy minimal function of fuzzy index value function from each attribute, and Hamming Distance is used to measure the difference between fuzzy index value function corresponding to decision making and fuzzy ideal function and fuzzy negative ideal function. The decision-making principle is that the distance between fuzzy index value function and the fuzzy ideal function is as small as possible, and the distance between fuzzy index value function and the fuzzy negative ideal function is as large as possible.

The following is an introduction to fuzzy group decision-making method based on structured element.

- Related fuzzy data acquisition in individual decision-making

Suppose there are n evaluators $J_1,J_2,\ldots,J_n$ to evaluate m sites $P_1,P_2,\ldots,P_m$, evaluator k evaluates each site independently in accordance with the indexes of $C_1,C_2,\ldots,C_L$, the importance of each index are expressed as w_{1k}, w_{2k} ,…,w_{Lk}, the importance of each evaluator's assessment are expressed as $W_1,W_2,\ldots,W_n$, and suppose $\tilde{x}_{ij}^k$ represents the fuzzy assessment of index C_j in program P_i from evaluator J_k, $\tilde{w}_{jk}$ represents the fuzzy weight given by J_k to C_j, $\tilde{W}_k$ represents the assessment importance of evaluator J_k in the group decision making.

According to [15], the structured element of fuzzy multi-attribute group decision-making model can be expressed as $\tilde{x}_{ij}^k = f_{ij}^k(E)$, $\tilde{w}_{jk} = h_{jk}(E)$, $\tilde{W}_k = H_k(E)$, where E is a structured element, $f_{ij}^k(\bullet)$, $h_{jk}(\bullet)$, $H_k(\bullet)$ are order-preserving bounded monotone functions in the interval [-1,1], called respectively fuzzy index function, fuzzy weight function and individual credibility function.

● Group integration of fuzzy evaluation

Weighted individual fuzzy index function $f_{ij}^{k}(\bullet)$, we get $g_{ij}^{k}(\bullet)$ and then beconsolidated to the group fuzzy evaluation function of index *j* of site *I*, namely

$$g_{ij}^{k}(x) = h_{jk}(x) \times f_{ij}^{k}(x) \tag{1}$$

$$g_{ij}(x) = \sum_{k=1}^{n} H_k(x) \times g_{ij}^{k}(x) \tag{2}$$

● The determination of fuzzy ideal function G+(x) and fuzzy negative ideal function G-(x)

$$G^{+}(x) = (G_1^{+}(x), G_2^{+}(x), \cdots, G_L^{+}(x)) \tag{3}$$

$$G^{-}(x) = (G_1^{-}(x), G_2^{-}(x), \cdots, G_L^{-}(x)) \tag{4}$$

The evaluation indexes of site credibility are all positive, so there are following relations:

$$G_i^{+}(x) = \max_{-1 \le x \le 1}\{g_{1j}(x), g_{2j}(x), \cdots, g_{mj}(x)\} \tag{5}$$

$$G_i^{-}(x) = \min_{-1 \le x \le 1}\{g_{1j}(x), g_{2j}(x), \cdots, g_{mj}(x)\} \tag{6}$$

In the max-min algorithm, the ordering operator is used referenced [16].

● The determination of comprehensive evaluation value of each site

First of all, determines the Hamming distance D_i^{+} between site P_t and fuzzy ideal function $G^{+}(x)$ and that D_i^{-} between site P_t and fuzzy negative ideal function $G^{-}(x)$.

$$D_i^{+} = \sqrt{\sum_{j=1}^{L}\left[\int_{-1}^{1}\left|g_{ij}(x) - G_j^{+}(x)\right| dx\right]^2} \tag{7}$$

$$D_i^{-} = \sqrt{\sum_{j=1}^{L}\left[\int_{-1}^{1}\left|g_{ij}(x) - G_j^{-}(x)\right| dx\right]^2} \tag{8}$$

Then, determines the relative closeness between site P_t and fuzzy ideal function $G^{+}(x)$.

$$D_i = \frac{D_i^{-}}{D_i^{+} + D_i^{-}} \tag{9}$$

● Rank each site in sequence according to the value of D_i from large to small.

4 Examples and Applications

The WeiLan online bookstore, founded in march 2000, is a website centered on professional academic books, and now, we begin to evaluate the credibility of WeiLan site in accordance with the above indexes for evaluation.

In addition to WeiLan site (P_1), there are also two other sites including P_2 and P_3 (corresponding data omitted).

Suppose there are three evaluators, for simplicity, we also suppose W_1=W_2=W_3=1/3, details are shown in Table 2, where the fuzzy numbers are expressed as symmetric triangle fuzzy number, and the structured element is expressed as [15].

Table 2. Evaluators' assessment value given to different indexes (evaluator J_1)

Evaluator	Indexes	Assessment value
J_1	C_1	(0.75,0.85,0.95)
	C_2	(0.56,0.66,0.76)
	C_3	(0.78,0.87,0.96)
	C_4	(0.62,0.75,0.88)
	C_5	(0.59,0,72,0,85)
	C_6	(0.61,0.70,0.79)
	C_7	(0.71,0.77,0.83)
	C_8	(0.77,0.85,0.93)
	C_9	(0.79,0.85,0.91)

Evaluators' assessment value given to different indexes (evaluator J_2)

Evaluator	Indexes	Assessment value
J_2	C_1	(0.70,0.77,0.84)
	C_2	(0.60,0.68,0.76)
	C_3	(0.78,0.85,0.92)
	C_4	(0.60,0.70,0.80)
	C_5	(0.57,0,66,0,75)
	C_6	(0.67,0.73,0.79)
	C_7	(0.72,0.81,0.90)
	C_8	(0.77,0.86,0.95)
	C_9	(0.71,0.80,0.89)

Evaluators' assessment value given to different indexes(evaluator J_3)

Evaluator	Indexes	Assessment value
J_3	C_1	(0.65,0.76,0.87)
	C_2	(0.66,0.71,0.76)
	C_3	(0.72,0.84,0.96)
	C_4	(0.66,0.76,0.86)
	C_5	(0.69,0,76,0,83)
	C_6	(0.71,0.80,0.89)
	C_7	(0.63,0.71,0.79)
	C_8	(0.75,0.80,0.85)
	C_9	(0.70,0.80,0.90)

Table 3. Evaluators' weights given to different indexes (evaluator J_1)

Evaluator	Indexes	Weight
J_1	C_1	(0.45,0.57,0.69)
	C_2	(0.40,0.50,0.60)
	C_3	(0.66,0.76,0.86)
	C_4	(0.77,0.82,0.87)
	C_5	(0.79,0,87,0,95)
	C_6	(0.71,0.83,0.95)
	C_7	(0.73,0.82,0.91)
	C_8	(0.51,0.62,0.73)
	C_9	(0.64,0.74,0.84)

Evaluators' weights given to different indexes (evaluator J_2)

Evaluator	Indexes	Weight
J_2	C_1	(0.40,0.51,0.62)
	C_2	(0.44,0.56,0.68)
	C_3	(0.60,0.71,0.82)
	C_4	(0.72,0.77,0.82)
	C_5	(0.75,0,85,0,95)
	C_6	(0.70,0.80,0.90)
	C_7	(0.68,0.77,0.86)
	C_8	(0.50,0.60,0.70)
	C_9	(0.68,0.78,0.88)

Evaluators' weights given to different indexes (evaluator J_3)

Evaluator	Indexes	Weight
J_3	C_1	(0.48,0.57,0.66)
	C_2	(0.50,0.62,0.74)
	C_3	(0.64,0.72,0.80)
	C_4	(0.70,0.77,0.84)
	C_5	(0.77,0,87,0,97)
	C_6	(0.68,0.80,0.92)
	C_7	(0.62,0.73,0.84)
	C_8	(0.56,0.66,0.76)
	C_9	(0.62,0.74,0.86)

The calculation is showed with $f_{11}^1(x), g_{11}^1(x), g_{11}(x)$ for example, where $f_{11}^1(x) = 0.85 + 0.1x$, $h_{11}(x) = 0.82 + 0.05x$, so

$$g_{11}^1(x) = f_{11}^1(x) \times h_{11}(x) = 0.70 + 0.12x + 0.0050x^2$$

$$g_{11}(x) = \sum_{k=1}^{3} H_k(x) \times g_{11}^k(x) = [\frac{1}{3}(0.70 + 0.12x + 0.005x^2) + (0.39 + 0.12x + 0.0077x^2)$$

$$+(0.43 + 0.13x + 0.0099x^2)] = 0.51 + 0.12x + 0.0075x^2$$

Similarly, the $g_{12}(x)$, $g_{13}(x)$,…,$g_{19}(x)$ of WeiLan site can be obtained, and $g_{ij}(x)$ of other two sites can be also obtained , where i=2,3; j=1,2,…,9. $G^+(x)$ and $G^-(x)$ are shown in Table 4 according to (5) and (6).

Table 4. Positive ideal values and negative ideal values of each index

$G^+(x)$		$G^-(x)$	
$g_{31}(x)$	$0.62+0.15x+0.0067x^2$	$g_{11}(x)$	$0.51+0.13x+0.0075x^2$
$g_{12}(x)$	$0.38+0.12x+0.0085x^2$	$g_{22}(x)$	$0.25+0.13x+0.0079x^2$
$g_{13}(x)$	$0.62+0.15x+0.0088x^2$	$g_{33}(x)$	$0.60+0.14x+0.0070x^2$
$g_{34}(x)$	$0.64+0.14x+0.0077x^2$	$g_{24}(x)$	$0.55+0.16x+0.0093x^2$
$g_{15}(x)$	$0.62+0.15x+0.0099x^2$	$g_{25}(x)$	$0.54+0.16x+0.0087x^2$
$g_{26}(x)$	$0.66+0.19x+0.0097x^2$	$g_{16}(x)$	$0.60+0.15x+0.0092x^2$
$g_{17}(x)$	$0.52+0.14x+0.0074x^2$	$g_{37}(x)$	$0.43+0.16x+0.0095x^2$
$g_{18}(x)$	$0.52+0.13x+0.0076x^2$	$g_{28}(x)$	$0.47+0.15x+0.0088x^2$
$g_{29}(x)$	$0.64+0.14x+0.0067x^2$	$g_{39}(x)$	$0.58+0.16x+0.0087x^2$

We can get $D_1^+ = 0.50, D_1^- = 0.88$, D_1=0.64 based on (7)-(9). Similarly, D_2=0.45 and D_3=0.51 can be obtained as well. Then, the result of three sites' credibility is $P_1 > P_2 > P_3$.

5 Conclusion

Evaluation index system of credibility of e-commerce website is discussed in this paper. In view of the preference inconsistency of decision-making bodies on the credibility of sites and the fuzzy features of index and weight, the fuzzy multi-attribute group decision-making method is used in this paper, and for the sake of calculation, the structured element is used referenced [14]. Example shows that the method proposed in this paper has better applicability.

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A Class of Mortgage Insurance Pricing

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Abstract. The mortgage insurance pricing problem is studied when the interest rate is modeled by vasiček model and with m stochastic disturbances. Meanwhile, the process of house prices subjects to the jump-diffusion process. Using the principle of the option pricing, we obtain the accurate formulas of two kinds of the mortgage insurance.

Keywords: Mortgage, Insurance, Vasiček model, Jump-diffusion process.

1 Introduction

In recent years, Mortgage loans (i.e. mortgage) are new credit derivatives that launched by commercial banks, so as to solve the housing problems of individuals, and promote an effective way to housing consumption. However, due to the mortgage repayment length, there are many risks, such as the risk of housing damage, the debtor credit risk, credit risk conditions, the risk of dealing with collateral and other risk factors. Therefore, banks are worried about the implementation of the business. It directly restricts the comprehensive development of housing mortgage loan. However, as time goes by, mortgage loan insurance will be developed rapidly in the world.

Mortgage guarantee insurance is divided into two types, that is, full guarantee and partial guarantee. It can greatly enhance the ability to resist risks of lending institutions, improve the stringent loan conditions, promote the effective demand for the whole society, but make the insurance companies inherit the great credit risk transferred from the banks at the same time. As the means of risk analysis and management are poor, and hedging tools are in short as well as markets, the insurance companies once started the mortgage insurance for a short time, but failed. However, with the constant development and improvement of China's capital market, this business has broad prospects for development.

This thesis assumes that house prices subjects to jump-diffusion process. Using option pricing theory methodology, we analysis the martingale pricing

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problem when the interest rate is modelled by vasiček model and has m stochastic disturbances and get the pricing formula of full guarantee and partial guarantee under the no-arbitrage condition. Meanwhile, we promote the conclusions in 3 and 4, which have some theoretical reference value.

The rest of the paper is organized as follows. In section 2, we introduce the mortgage margin. In section 3, we give actuarial pricing formula of mortgage insurance.

2 The Introduction of Mortgage Margin

Mortgage guarantee insurance is an insurance that banks require loan borrowers to buy the protection insurance from the insurer (i.e. insurance company). The loan borrower has to pay a certain amount of insurance premium when he borrows loans from the bank, Insurance Company makes guarantee to repay the loan to bank, and then bank lends loans to borrower accordingly, and also gives definite benefits to borrower in the interest and loan terms, and so on. If the borrower defaults, and fails to repay loans in the loan period, the insurer should pay the loan losses to bank. Mortgage guarantee insurance includes two types: full guarantee and partial guarantee. We consider the mortgage period T is divided into N segment intervals, i.e. $T = N\Delta t$.Firstly, we studied the policy in period $T = \Delta t$.

2.1 Full Guarantee

If the borrower defaults at time $T = t$, the insurance company can take two ways to pay compensation:

I. Insurance company pays the full balance of outstanding loans to bank, meanwhile, obtains the right of real estate and mortgage.

II. Lending institutions retain the right of real estate and mortgage, and the part of not enough to compensate of the loan balance is paid by the insurance company.

No matter what way, the insurance company should pay the claim paid as follows:

$$max(V(T) - \alpha H(T))$$

where $V(T)$ is the amount of outstanding payment and $H(T)$ is the real estate value at time $t = T$, α denotes the ratio of the real estate value after the realization of the right mortgage and set α is a constant.The lender who holds the policy of full guarantee can obtain yield to maturity as follows:

$$C_T = max(V(T) - \alpha H(T), 0) \tag{1}$$

2.2 Partial Guarantee

Insurance company makes guarantee only to a certain percentage of mortgage loan balance, so as to reduce credit risk. This is called partial guarantee. If the borrower defaults at time $t = T$, the insurance company can take two ways to pay compensation:

III. Insurance company pays the full balance of outstanding loans to bank, meanwhile, obtains the right of real estate and mortgage. The insurance company should pay the claim paid as follows:

$$max(V(T) - \alpha H(T), 0)$$

IV. Insurance company pays the loan amount to bank, according to the contract which fixed the ratio of compensation, and bank still retains the right of real estate, then the claim amounts are βV(T) where β denotes guarantees ratio.

If $max(V(T) - \alpha H(T), 0) > \beta$V(T), i.e. $H(T) > \frac{(1-\beta)V(T)}{\alpha}$. we obviously select manner III favourable to the insurance company.

If $max(V(T) - \alpha H(T), 0) < \beta$V(T), i.e. $H(T) < \frac{(1-\beta)V(T)}{\alpha}$. we obviously select manner IV favourable to the insurance company.

Therefore, the lender who holds the policy of partial guarantee can obtain yield to maturity as follows:

$$\hat{C}_T = \begin{cases} max(V(T) - \alpha H(T), 0), & \text{if } \ H(T) > \frac{(1-\beta)V(T)}{\alpha}, \\ \beta V(T), & \text{if } \ H(T) < \frac{(1-\beta)V(T)}{\alpha}. \end{cases} \tag{2}$$

3 Mortgage Insurance Actuarial Pricing Formula

Given a complete no-arbitrage market and let $(\Omega, \mathscr{F}, (\mathscr{F}_t)_{t \geq 0}, P)$ be a complete probability space with filter,P is the risk-neutral probability measure, consider the continuous time horizon $[0, T]$. Suppose the amount of outstanding payments at time T is $V(T)$, and $V(T)$ is a constant(available by the credit risk assessment). Then, risk-free interest rate $r(t)$ and house price $H(t)$ at time t are given by:

$$dr(t) = [a(t)r(t) + b(t)]dt + c(t)dB(t), \quad r(0) = r \tag{3}$$

$$dH(t) = H(t)[(r(t) - \mu\lambda(t))dt + \sum_{i=1}^{m} \sigma_i(t)dB_i(t) + hdN(t)], \quad H(0) = H \tag{4}$$

where $a(t) < 0$,$b(t) > 0$,$c(t) > 0$,$\sigma_i(t) > 0 (i = 1, 2, \cdots, m)$,$\lambda(t) \geq 0$ are integral functions determined by time t, and $\sigma_i(t)$ denotes the ith factor (such as social factors, economic factors, government interference factor, etc.) from the effects of random interference terms, $B(t), B_i(t)(i = 1, 2, \cdots, m)$ are independent one-dimensional standard *Brownian* motion. $N(t)$ denotes the number of times of the house price random jump in $[0, T]$, it follows the non-homogeneous *Poisson* process with parameter $\lambda(t)$, and independent from the $B(t), B_i(t)(i = 1, 2, \cdots, m)$. h is the house price for each jump height, and it is a random variable. We assume that and $h, h_1, h_2, \cdots, h_{N(t)}$ are independent and identically distributed random variables, $h, h_1, h_2, \cdots, h_{N(t)}$ are represent respectively the jump height that occur at random time $\tau_1, \tau_2, \cdots, \tau_{N(t)}$(if no jump occurs, we suppose $h_0 = 0$). h and $N(t)$ are independent. if $h > -1$, then $\ln(1+h)$ obeys the normal distribution $N(\ln(1+\mu) - \frac{1}{2}\sigma^2, \sigma^2)$, where σ^2 is the variance of $\ln(1+h)$, while μ is the unconditional expectations of h, and it denotes the average growth rate of house price which caused by the *Poisson* jump.

Lemma 3.1. (see 2) The solution of stochastic differential equation (3) and (4) are given by

$$r(t) = rq(t,0) + \int_0^t b(u)q(u,t)du + \int_0^t c(u)q(u,t)dB(u) \tag{5}$$

$$H(t) = H\exp[\int_0^t (r(t) - \mu\lambda(t) - \frac{1}{2}\sum_{i=1}^m \sigma_i^2(t))dt + \sum_{i=1}^m (\int_0^t \sigma_i(t)dB_i(t)) + \sum_{j=0}^{N(t)} \ln(1+h_j)]. \tag{6}$$

where $q(s,t) = \exp[-\int_s^t a(u)du], \quad 0 \le s \le t$. From (5), we can easily know

$$\int_0^T r(t)dt = r\int_0^T q(t,0)dt + \int_0^T dt\int_0^t b(u)q(u,t)du + \int_0^T dt\int_0^t c(u)q(u,t)dB(u) \tag{7}$$

Let $A_1 = \int_0^T dt\int_0^t c(u)q(u,t)dB(u) = \int_0^T c(u)[\int_u^T q(u,t)dt]dB(u)$,

$$B_1 = \sum_{i=1}^m (\int_0^T \sigma_i(t)dB_i(t)), \quad D_1 = r\int_0^T q(t,0)dt + \int_0^T dt\int_0^t b(u)q(u,t)du.$$

Thus we have $\int_0^T r(t)dt = A_1 + D_1$

$$H(T) = H\exp[\int_0^T (r(t) - \mu\lambda(t) - \frac{1}{2}\sum_{i=1}^m \sigma_i^2(t))dt + B_1 + \sum_{j=0}^{N(T)} \ln(1+h_j)].$$

where $\quad A_1 \sim N(0, \sigma_{A_1}^2), \quad \sigma_{A_1}^2 = \int_0^T c^2(u)[\int_u^T q(u,t)dt]^2 d(u)$,

$$B_1 \sim N(0, \sigma_{B_1}^2), \quad \sigma_{B_1}^2 = \int_0^T (\sum_{i=1}^m \sigma_i^2(t)) d(t).$$

Lemma 3.2. (see 1) Suppose $X \sim N(0,1), Y \sim N(0,1)$, the *correlation coefficient* of X and Y is ρ ,then for any real numbers a, b, c, d, m_1, m_2, we get the following formula

$$E[e^{cX+bY} I_{[m_1 < aX+bY < m_2]}] = e^{\frac{1}{2}(c^2+d^2+2cd\rho)} [N(\frac{ac+bd+\rho(ad+bc)-m_1}{\sqrt{a^2+b^2+2ab\rho}}) - N(\frac{ac+bd+\rho(ad+bc)-m_2}{\sqrt{a^2+b^2+2ab\rho}})].$$

Theorem 3.1. Assume the market has no arbitrage, the guarantee period is $[0, T]$ and the payment at time T is given by (1), while risk-free interest rate $r(t)$ and house price $H(t)$ are given by (3) and (4), then the full guarantee of the premium price at the time zero as follows:

$$C_0 = \sum_{n=0}^{\infty} \frac{(\int_0^T \lambda(t)dt)^n \exp(-\int_0^T \lambda(t)dt)}{n!} [V(t) \exp(\tfrac{1}{2}\sigma_{A_1}^2 - D_1) N(x_1) - \alpha H (1+\mu)^n \exp(-\mu \int_0^T \lambda(t)dt) N(x_2).$$

where $N(x)$ is the standard normal distribution function, while

$$x_1 = \frac{\sigma_{A_1}^2 + \ln \frac{V(T)}{\alpha H(1+\mu)^n} - D_1 + \frac{1}{2}\sigma_{B_1}^2 + \mu \int_0^T \lambda(t)dt + \frac{1}{2} n\sigma^2}{\sqrt{\sigma_{A_1}^2 + \sigma_w^2}},$$

$$x_2 = x_1 - \sqrt{\sigma_{A_1}^2 + \sigma_w^2}, \quad \sigma_w^2 = \sigma_{B_1}^2 + n\sigma^2 = \int_0^T (\sum_{i=1}^m \sigma_i^2(t)) d(t) + n\sigma^2.$$

Proof. From Lemma 3.1 and the formula of the full expectations, we get

$$C_0 = E[\exp(-\int_0^T \lambda(t)dt)(V(T) - \alpha H(T))^+]$$
$$= E[E[\exp(-\int_0^T \lambda(t)dt)(V(T) - \alpha H \exp[\int_0^T (r(t) - \mu\lambda(t) - \tfrac{1}{2}\sum_{i=1}^m \sigma_i^2(t))dt + B_1 + \sum_{j=0}^{N(T)} \ln(1+h_j)])^+ | N(T) = n]]$$

$$= \sum_{n=0}^{\infty} P(N(T) = n) E[\exp(-\int_0^T \lambda(t)dt)(V(T) - \alpha H \exp[\int_0^T (r(t) - \mu\lambda(t)$$

$$-\frac{1}{2}\sum_{i=1}^{m}\sigma_i^2(t))dt+B_1+\sum_{j=0}^{N(T)}\ln(1+h_j)])^+]$$

$$=\sum_{n=0}^{\infty}\frac{(\int_0^T\lambda(t)dt)^n\exp(-\int_0^T\lambda(t)dt)}{n!}C_0'$$

where

$$C_0'=E[\exp(-\int_0^T\lambda(t)dt)(V(T)-\alpha H\exp[\int_0^T(r(t)-\mu\lambda(t)-\tfrac{1}{2}\sum_{i=1}^{m}\sigma_i^2(t))dt$$

$$+B_1+\sum_{j=0}^{n}\ln(1+h_j)])^+|N(T)=n].$$

Under the known condition $N(T)=n$, let $F=\{V(T)>\alpha H(T)\}$, then

$$F=\{\int_0^T(r(t)-\mu\lambda(t)-\tfrac{1}{2}\sum_{i=1}^{m}\sigma_i^2(t))dt+B_1+\sum_{j=0}^{n}\ln(1+h_j)<\ln\tfrac{V(T)}{\alpha H(T)}\}$$

$$=\{A_1+B_1+\sum_{j=0}^{n}\ln(1+h_j)<\ln\tfrac{V(T)}{\alpha H(T)}-D_1+\tfrac{1}{2}\sigma_{B_1}^2+\mu\int_0^T\lambda(t)dt\}.$$

Using $\ln(1+h)\sim N(\ln(1+\mu)-\frac{1}{2}\sigma^2,\sigma^2)$, we have

$$\sum_{j=0}^{n}\ln(1+h_j)\sim N(n\ln(1+\mu)-\frac{1}{2}n\sigma^2,n\sigma^2).$$

Let $W=B_1+\sum_{j=0}^{n}\ln(1+h_j)$, then $W\sim N(n\ln(1+\mu)-\frac{1}{2}n\sigma^2,\sigma_{B_1}^2+n\sigma^2)$.

We assume that

$\mu_w=n\ln(1+\mu)-\frac{1}{2}n\sigma^2,\sigma_w^2=\sigma_{B_1}^2+n\sigma^2,W_1=\frac{A_1}{\sigma_{A_1}},W_2=\frac{W-\mu_w}{\sigma_w}$,

and then

$$F=\{-\sigma_{A_1}W_1-\sigma_wW_2>\mu_w-\ln\tfrac{V(T)}{\alpha H(T)}+D_1-\tfrac{1}{2}\sigma_{B_1}^2-\mu\int_0^T\lambda(t)dt\}$$

$$C_0'=E[\exp(-\int_0^T\lambda(t)dt)(V(T)-\alpha H\exp\{\int_0^T(r(t)-\mu\lambda(t)$$

$$-\frac{1}{2}\sum_{i=1}^{m}\sigma_i^2(t))dt+W\})I_F]\quad=\delta_1-\delta_2$$

while $\delta_1=E[\exp(-\int_0^T\lambda(t)dt)V(T)I_F]$

$$=V(T)E[\exp(-A_1-D_1)I_{\{-\sigma_{A_1}W_1-\sigma_wW_2>\mu_w-\ln\frac{V(T)}{\alpha H(T)}+D_1-\frac{1}{2}\sigma_{B_1}^2-\mu\int_0^T\lambda(t)dt\}}]$$

$$=V(T)\exp(-D_1)E[\exp(-\sigma_{A_1}W_1)I_{\{-\sigma_{A_1}W_1-\sigma_wW_2>\mu_w-\ln\frac{V(T)}{\alpha H(T)}+D_1-\frac{1}{2}\sigma_{B_1}^2-\mu\int_0^T\lambda(t)dt\}}]$$

By the lemma 3.2, we can derive $\delta_1 = V(T)\exp(\frac{1}{2}\sigma_{A_1}^2 - D_1)N(x_1)$, where
$x_1 = \frac{\sigma_{A_1}^2 + \ln\frac{V(T)}{\alpha H(1+\mu)^n} - D_1 + \frac{1}{2}\sigma_{B_1}^2 + \mu\int_0^T \lambda(t)dt + \frac{1}{2}n\sigma^2}{\sqrt{\sigma_{A_1}^2+\sigma_w^2}}$.

Similarly, we have

$$\delta_2 = E[\exp(-\int_0^T \lambda(t)dt)(-\alpha H \exp\{\int_0^T (r(t)-\mu\lambda(t) - \frac{1}{2}\sum_{i=1}^m \sigma_i^2(t))dt + W\})I_F]$$
$$= \alpha H(1+\mu)^n \exp\{-\mu\int_0^T \lambda(t)dt\}N(x_2).$$

where

$$x_2 = \frac{\sigma_w^2 + \ln\frac{V(T)}{\alpha H(1+\mu)^n} - D_1 + \frac{1}{2}\sigma_{B_1}^2 + \mu\int_0^T \lambda(t)dt + \frac{1}{2}n\sigma^2}{\sqrt{\sigma_{A_1}^2+\sigma_w^2}} = x_1 - \sqrt{\sigma_{A_1}^2 + \sigma_w^2}.$$

Thus, the full guarantee of the premium price at the time zero can be expressed by

$$C_0 = \sum_{n=0}^{\infty} \frac{(\int_0^T \lambda(t)dt)^n \exp(-\int_0^T \lambda(t)dt)}{n!}[V(T)\exp(\frac{1}{2}\sigma_{A_1}^2 - D_1)N(x_1)$$
$$-\alpha H(1+\mu)^n \exp\{-\mu\int_0^T \lambda(t)dt\}N(x_2)].$$

Theorem 3.2. Let us assume the market has no arbitrage, the guarantee period is $[0, T]$ and the payment at time T is given by (2), while risk- free interest rate $r(t)$ and house price $H(t)$ are given by (3) and (4), then the partial guarantees of the premium price at the time zero as follows:

$$\hat{C}_0 = E[\exp(-\int_0^T \lambda(t)dt)(V(T) - \alpha H(T))^+ I_{[H(T) > \frac{(1-\beta)V(T)}{\alpha}]} + \beta V(T) I_{[H(T) < \frac{(1-\beta)V(T)}{\alpha}]}]$$
$$= \sum_{n=0}^{\infty} \frac{(\int_0^T \lambda(t)dt)^n \exp(-\int_0^T \lambda(t)dt)}{n!}\{V(T)\exp(\frac{1}{2}\sigma_{A_1}^2 - D_1)[N(x_3) - N(x_4)]$$
$$-\alpha H(1+\mu)^n \exp(-\mu\int_0^T \lambda(t)dt)[N(x_5) - N(x_6)] + \beta V(T)\exp(\frac{1}{2}\sigma_{A_1}^2 - D_1)N(x_7)\}.$$

where

$$x_3 = \frac{-\sigma_{A_1}^2 + \mu_w - K_1}{\sqrt{\sigma_{A_1}^2 + \sigma_w^2}}, x_4 = \frac{-\sigma_{A_1}^2 + \mu_w - K_2}{\sqrt{\sigma_{A_1}^2 + \sigma_w^2}}, x_5 = \frac{-\sigma_w^2 + \mu_w - K_1}{\sqrt{\sigma_{A_1}^2 + \sigma_w^2}},$$

$$x_6 = \frac{-\sigma_w^2 + \mu_w - K_2}{\sqrt{\sigma_{A_1}^2 + \sigma_w^2}}, x_7 = \frac{\sigma_{A_1}^2 - \mu_w + K_1}{\sqrt{\sigma_{A_1}^2 + \sigma_w^2}}, \mu_w = n\ln(1+\mu) - \frac{1}{2}n\sigma^2,$$

$$\sigma_w^2=\int_0^T(\sum_{i=1}^m\sigma_i^2(t))d(t)+n\sigma^2, K_1=\ln\frac{(1-\beta)V(T)}{\alpha}-D_1+\frac{1}{2}\sigma_{B_1}^2+\mu\int_0^T\lambda(t)dt,$$

$$K_2=\ln\frac{V(T)}{\alpha}-D_1+\frac{1}{2}\sigma_{B_1}^2+\mu\int_0^T\lambda(t)dt.$$

Proof. From Lemma 3.1 and the formula of the full expectations, we have

$$\hat{C}_0=E[E[\exp(-\int_0^T\lambda(t)dt)(V(T)-\alpha H(T))^+I_{[H(T)>\frac{(1-\beta)V(T)}{\alpha}]}|N(T)=n]]$$
$$+E[E[\beta V(T)I_{[H(T)<\frac{(1-\beta)V(T)}{\alpha}]}|N(T)=n]]$$

$$=\sum_{n=0}^{\infty}P(N(T)=n)\{E[\exp(-\int_0^T\lambda(t)dt)(V(T)-\alpha H(T))^+$$
$$I_{[H(T)>\frac{(1-\beta)V(T)}{\alpha}]}|N(T)=n]+E[\beta V(T)I_{[H(T)<\frac{(1-\beta)V(T)}{\alpha}]}|N(T)=n]\}$$
$$=\sum_{n=0}^{\infty}\frac{(\int_0^T\lambda(t)dt)^n\exp(-\int_0^T\lambda(t)dt)}{n!}\hat{C}_0'$$

while

$$\hat{C}_0'=E[\exp(-\int_0^T\lambda(t)dt)(V(T)-\alpha H(T))^+I_{[H(T)>\frac{(1-\beta)V(T)}{\alpha}]}|N(T)=n]$$
$$+E[\beta V(T)I_{[H(T)<\frac{(1-\beta)V(T)}{\alpha}]}|N(T)=n]\}.$$

Under the known condition $N(T)=n$, then

$$\{V(T)>\alpha H(T),H(T)>\frac{(1-\beta)V(T)}{\alpha}\}\Longleftrightarrow\{K_1<A_1+W<K_2\},$$

$$\{H(T)<\frac{(1-\beta)V(T)}{\alpha}\}\Longleftrightarrow\{A_1+W<K_1\}.$$

where

$$K_1=\ln\frac{(1-\beta)V(T)}{\alpha}-D_1+\frac{1}{2}\sigma_{B_1}^2+\mu\int_0^T\lambda(t)dt,$$

$$K_2=\ln\frac{V(T)}{\alpha}-D_1+\frac{1}{2}\sigma_{B_1}^2+\mu\int_0^T\lambda(t)dt.$$

$$\hat{C}_0'=E[\exp(-A_1-D_1)(V(T)-\alpha H\exp\{\int_0^T(r(t)-\mu\lambda(t)-\tfrac{1}{2}\sum_{i=1}^m\sigma_i^2(t))dt$$

$$+W\})I_{[K_1<A_1+W<K_2]}]+E[\exp(-A_1-D_1)\beta V(T)I_{[A_1+W<K_1]}]$$
$$=\delta_3-\delta_4+\delta_5.$$

where

$$\delta_3 = E[\exp(-A_1 - D_1)V(T)I_{[K_1<A_1+W<K_2]}],$$

$$\delta_4 = E[\exp(-A_1-D_1)\alpha H \exp\{\int_0^T (r(t)-\mu\lambda(t)-\frac{1}{2}\sum_{i=1}^{m}\sigma_i^2(t))dt+W\}I_{[K_1<A_1+W<K_2]}],$$

$$\delta_5 = E[\exp(-A_1 - D_1)\beta V(T)I_{[A_1+W<K_1]}].$$

By the lemma 3.2, we can derive

$$\begin{aligned}\delta_3 &= E[\exp(-A_1 - D_1)V(T)I_{[K_1<A_1+W<K_2]}] \\ &= V(T)\exp(-D_1)E[\exp(-\sigma_{A_1W_1})I_{\{K_1-\mu_w<\sigma_{A_1}W_1+\sigma_w W_2<K_1-\mu_w\}}], \\ &= V(T)\exp(\tfrac{1}{2}\sigma_{A_1}^2 - D_1)[N(x_3) - N(x_4)].\end{aligned}$$

Similarly, we have

$$\delta_4 = \alpha H(1+\mu)^n \exp(-\mu\int_0^T \lambda(t)dt)[N(x_5) - N(x_6)].$$

$$\delta_5 = \beta V(T)\exp(\tfrac{1}{2}\sigma_{A_1}^2 - D_1)N(x_7).$$

where

$$x_3 = \frac{-\sigma_{A_1}^2 + \mu_w - K_1}{\sqrt{\sigma_{A_1}^2 + \sigma_w^2}}, x_4 = \frac{-\sigma_{A_1}^2 + \mu_w - K_2}{\sqrt{\sigma_{A_1}^2 + \sigma_w^2}}, x_5 = \frac{-\sigma_w^2 + \mu_w - K_1}{\sqrt{\sigma_{A_1}^2 + \sigma_w^2}},$$

$$x_6 = \frac{-\sigma_w^2 + \mu_w - K_2}{\sqrt{\sigma_{A_1}^2 + \sigma_w^2}}, \quad x_7 = \frac{\sigma_{A_1}^2 - \mu_w + K_1}{\sqrt{\sigma_{A_1}^2 + \sigma_w^2}},$$

$$\mu_w = n\ln(1+\mu) - \frac{1}{2}n\sigma^2, \quad \sigma_w^2 = \int_0^T (\sum_{i=1}^{m}\sigma_i^2(t))d(t) + n\sigma^2.$$

Thus, the partial guarantee of the premium price at the time zero can be expressed by:

$$\hat{C}_0 = \sum_{n=0}^{\infty}\frac{(\int_0^T \lambda(t)dt)^n \exp(-\int_0^T \lambda(t)dt)}{n!}\{V(T)\exp(\frac{1}{2}\sigma_{A_1}^2 - D_1)[N(x_3) - N(x_4)]$$

$$-\alpha H(1+\mu)^n \exp(-\mu\int_0^T \lambda(t)dt)[N(x_5)-N(x_6)]+\beta V(T)\exp(\tfrac{1}{2}\sigma_{A_1}^2-D_1)N(x_7)\}.$$

4 Conclusion

This thesis assumes that house prices subjects to jump-diffusion process. Using option pricing theory methodology, we analysis the martingale pricing problem when the interest rate is modelled by vasiček model and has m stochastic disturbances and get the pricing formula of full guarantee and partial guarantee under the no-arbitrage condition. Meanwhile, we promote the conclusions in 3 and 4, which have some theoretical reference value.

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Study of Evaluating Web-Based Courses Based on FAHP

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Abstract. This paper uses fuzzy AHP to construct the model of evaluating web-based courses. Basing on comparing the triangular fuzzy number's size, this method carries on the single-level sorting, simultaneously uses the multistage fuzzy quality synthetic evaluations. It not only overcomes the defects of the big human impact when uses expert evaluation method to get weight, but also considers the fuzziness and hierarchical characteristics of the influencing factor of web-based courses. It makes the conclusions more objective and rational. The method is simple and standardized, easy to program, and practicality.

Keywords: Web-based courses, Triangular fuzzy number, Fuzzy analytical hierarchy process (FAHP).

1 Introduction

With the development of computer network and multi-media technology, Web-based course has shown its superiority and quickness. It has become a preferred approach of teachers to reform education methods and to improve education efficiency. At present, all kinds of Web-based courses are colorful, the evaluation of Web-based courses will create positive catalytic effects on the construction of Web-based courses and improvement of course quality. The article adapts Fuzzy analytic hierarchy process (Fuzzy AHP, Abbreviation as FAHP) to evaluate Web-based courses according to index system defined by China education information technology standard CELTS-22 *Web-based courses evaluation specification*[1]. By comparing triangular fuzzy number's size, the method obtains weights of all kinds of elements, by combining the qualitative analysis with the quantitative analysis effectively, it obtains index weight scientifically, and makes results of evaluation more scientifically and more objectively.

2 FAHP Principle

2.1 The Basic Idea of FAHP

The traditional AHP adapts 1-9 scale method[2], which uses pairwise comparison to compare the evaluation object to gain judgment matrix, and obtain definite

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quantified conclusion by overall analysis of qualitative and quantitative question, and present it as form of merits.

Because this method constructs judgment matrix by using integers between 1 and 9 as scales, this kind of judgment doesn't show fuzziness of human judgment, Dutch scholar Van Laarhoven proposed the method of using triangular fuzzy number as fuzzy comparative judgments in 1983, which obtain sort of elements by using the operating of the triangular fuzzy number and Logarithmic least squares method, then expanded AHP to FAHP[3] which can be used in fuzzy context.

2.2 Triangular Fuzzy Number and Relative Computing Principle [4][5]

Definition 1. *Triangular fuzzy number* $M=(l,m,u)$, *which membership function* $\mu_M : R \to [0,1]$ *defined as follow:*

$$\mu_M(x)=\begin{cases} \frac{1}{m-l}x-\frac{l}{m-l} & x\in[l,m], \\ \frac{1}{m-u}x-\frac{u}{m-u} & x\in[m,u], \\ 0 & \text{others} \end{cases} \tag{1}$$

in the formula, $l \le m \le u$, l *and* u *represent lower bound and upper bound which* M *supports respectively,* m *is the median of* M *.In triangular fuzzy number,* l, u *represent the fuzzy degree of judgment. Suppose* $\delta = u - l$, δ *is greater indicates fuzzy degree is higher;* δ *is smaller indicates fuzzy degree is lower;* $\delta = 0$ *indicates the judgment is non-fuzzy.*

Suppose $M_1=(l_1,m_1,u_1)$, $M_2=(l_2,m_2,u_2)$, $M=(l,m,u)$ are triangular fuzzy number respectively, its computing rules are:

$$M_1 \oplus M_2 = (l_1+l_2, m_1+m_2, u_1+u_2) \tag{2}$$

$$M_1 \otimes M_2 \approx (l_1 l_2, m_1 m_2, u_1 u_2) \tag{3}$$

$$\lambda M = (\lambda l, \lambda m, \lambda u), \quad \lambda > 0, \lambda \in R \tag{4}$$

$$M^{-1} \approx (1/u, 1/m, 1/l) \tag{5}$$

Definition 2. *The possibility degree of* $M_1 \ge M_2$ *is defined as:*

$$\vee(M_1 \ge M_2) = \begin{cases} 1 & m_1 \ge m_2, \\ \frac{l_2-u_1}{(m_1-u_1)-(m_2-l_2)} & m_1 < m_2, l_2 \le u_1, \\ 0 & others \end{cases} \tag{6}$$

2.3 The Main Procedure of FAHP

2.3.1 According to the Overall Goal of the Question, Determines Evaluation Staff or Group, Creates System Hierarchical Structure Model of the Evaluation Problem

2.3.2 Construct Fuzzy Judgment Matrix

Experts compare with the evaluation indexes and objects by using pairwise comparison, from the top to bottom, determine the triangular fuzzy judgment matrix of every level indexes. Which is: for the indexes in the same level, when use the previous level indexes as criterion to carry on pairwise comparison, use triangular fuzzy number to represent quantitatively, denoted as

$$A=(a_{ij})_{n\times m},\ a_{ij}=[l_{ij},m_{ij},u_{ij}]\text{ ,and }a_{ji}\quad a_{ij}^{-1}\quad [1/u_{ij},1/m_{ij},1/l_{ij}].$$

When T experts carry on judgments, a_{ij} as the overall triangular fuzzy number, it is the composite of the judgments of the T experts, it can obtain from the following formula:

$$a_{ij}=1/T\otimes(a_{ij}^{1}+a_{ij}^{2}+\cdots+a_{ij}^{T}) \tag{7}$$

Which $a_{ij}^{t}=[l_{ij}^{t},m_{ij}^{t},u_{ij}^{t}](i,j=1,2,\cdots,n;t=1,2,\cdots,T)$ as the triangular fuzzy number given by the number T expert.

2.3.3 Computing Overall Significant Degree Value

Suppose $X=\{x_1,x_2,\cdots,x_n\}$ is a object set, $U=\{u_1,u_2,\cdots,u_m\}$ is the target set, so the degree values which meet to the objectives of the number i object are $M_{E_i}^{1}, M_{E_i}^{2},\ \ ,M_{E_i}^{m}$ respectively, $i=1,2,\cdots,n$ .Here, $M_{E_i}^{j}$ are triangular fuzzy numbers. According to this, we can define the overall degree value of the number i object to m target as:

$$S_i=\sum_{j=1}^{m}M_{E_i}^{j}\otimes\left(\sum_{i=1}^{n}\sum_{j=1}^{m}M_{E_i}^{j}\right)^{-1} \tag{8}$$

2.3.4 Single-Level Sorting

For all fuzzy judgment matrix. Computing the possibility degree of the number i element import to the others element.

$$d'(A_i)=\min_{j=1,2,\cdots,n.\,j\neq i}\vee(S_i\geq S_j)(i=1,2,3\cdots,n) \tag{9}$$

From this, we obtain $W'=(d'(A_1),d'(A_2),\cdots,d'(A_n))^T$,and normalized it, we obtain the weight vector $W=(d(A_1),d(A_2),\cdots,d(A_n))^T$.

2.3.5 Combined Weight Computing

The weight values of all factors to the overall goal equal to the product of its weight to index and the weight of corresponding index to target.

3 Fuzzy Overall Evaluation Method [6][7]

3.1 Define Reviews Set and Fuzzy Evaluation Matrix

Suppose reviews set $V = \{V_1, V_2, V_3, V_4\}$ ={Excellent, Good, Medium, Poor}.

$R_i = (r_{kj})\ (i = 1, 2, \cdots, n)$ corresponds to the fuzzy evaluation matrix of last level evaluation indexes, which $r_{kj} = m_{kj} / n$, n as jury number, m_{kj} is the frequency of number k index's reviews is V_j.

3.2 Single Factor Fuzzy Evaluation

$$C_i = W_i \bullet R_i$$

in the formula, C_i is the evaluation result matrix of the number I previous level evaluation indexes.

3.3 Overall Fuzzy Evaluation

$$C = W \bullet R$$

3.4 Computing Overall Score

The final evaluation result obtained by the following method adapts fuzzy reviews set, which lacks comparability and Intuitive when compare one kind of web-based courses, so we deal evaluation results with quantification. The article divides the evaluation level into four: Excellent when score between 85~100; Good when score between 75~85; Medium when score between 60~75; Poor when score less than 60. When acquire overall score, we adapt interval median method to get level parameter set, which *V*= 92.5,80,67.5,30 , so we obtain the overall score of the web-based course as:

$$D = C \bullet V^T = C \bullet (92.5, 80, 67.5, 30)^T .$$

4 Example

Step one: create hierarchical model. The article adapts the index system defined by China education information technology standard CELTS-22 *Web-based* *courses evaluation specification* to evaluate web-based courses, and determine the hierarchical index system as Fig.1.

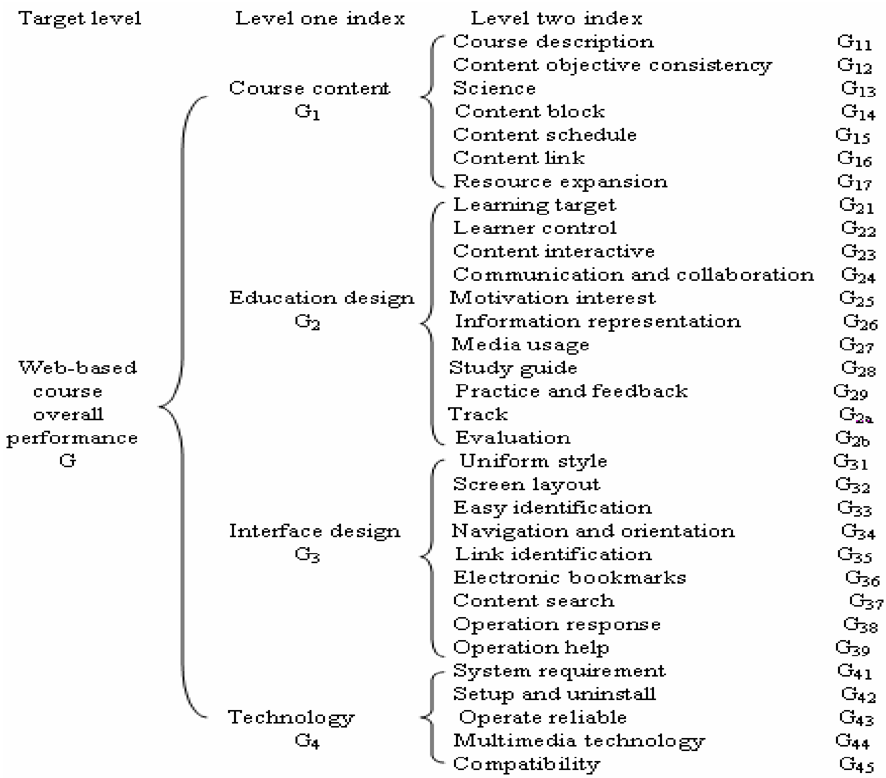

Fig. 1. Web-based course performance evaluation hierarchical structure model

Step two: determine weights of indexes. According to the demand of the overall coal, the experts participated in evaluation (suppose there are three experts) compare the importance degree of indicators, and gain the Triangular fuzzy number judgment matrix of the evaluation criterion, and use the Formula (7) (6) (8) (9) to compute the normalized weight vector of all index.

(1) The Computing of Weight Value of Level 1 Indexes to Target Layer

The fuzzy judgment matrix and overall fuzzy judgment matrix of $G_1 \sim G_4$ to G is shown as Table 1 and Table 2. According to Formula (7) (6) (9), we obtain the normalized weight vector of Level 1 indexes to target layer W =(0.288,0.302, 0.244,0.116).

Table 1. Fuzzy judgment matrix of $G_1 \sim G_4$ to G

G	G_1	G_2	G_3	G_4
G_1	(1,1,1)	(1/2,1,3/2) (3/2,1,3/2) (4/5,1,6/5)	(1/2,1,3/2) (3/2,1,3/2) (2/3,1,4/3)	(3/2,2,5/2) (5/4,7/4,2) (3/2,2,5/2)
G_2	(1/2,1,2) (2/3,1,3/2) (5/6,1,5/4)	(1,1,1)	(1,3/2,2) (3/2,2,5/2) (1,3/2,2)	(3/2,2,5/2) (2/3,1,3/2) (1,3/2,2)
G_3	(2/3,1,2) (2/3,1,3/2) (3/4,1,3/2)	(1/2,2/3,1) (2/5,1/2,2/3) (1/2,2/3,1)	(1,1,1)	(1/2,1,3/2) (3/2,1,3/2) (5/4,3/2,7/4)
G_4	(2/5,1/2,2/3) (1/2,4/7,4/5) (2/5,1/2,2/3)	(2/5,1/2,2/3) (2/3,1,3/2) (1/2,2/3,1)	(2/3,1,2) (2/3,1,3/2) (4/7,2/3,4/5)	(1,1,1)

Table 2. Overall fuzzy judgment matrix of $G_1 \sim G_4$ to G

G	G_1	G_2	G_3	G_4
G_1	(1,1,1)	(0.66,1,1.40)	(0.61,1,1.44)	(1.42,1.92,2.33)
G_2	(0.72,1,1.58)	(1,1,1)	(1.17,1.67,2.17)	(1.06,1.50,2.00)
G_3	(0.69,1,1.67)	(0.47,0.61,0.89)	(1,1,1)	(0.81,1.67,1.58)
G_4	(0.43,0.52,0.71)	(0.52,0.72,1.06)	(0.63,0.89,1.43)	(1,1,1)

(2) The Computing of Weight Value of Level 2 Indexes to Level 2 Indexes
Because of limited space, here we don't list the triangular fuzzy number judgment matrix, overall judgment matrix and the process of computing the weight vector of Level 2 when computing the weight of Level 2 indexes. The weight vector of Level 2 indexes to Level 1 indexes is shown as Table 3.

Table 3. Performance evaluation weight vector of Level 2 indexes under Level 1 indexes

W_1	(0.147,0.203,0.206,0.208,0.099,0.083,0.054)
W_2	(0.056,0.103,0.152,0.043,0.036,0.149,0.101,0.029,0.150,0.034,0.147)
W_3	(0.046,0.051,0.153,0.244,0.101,0.043,0.102,0.105,0.155)
W_4	(0.096,0.211,0.287,0.207,0.199)

Step three: Create the reviews set fuzzy evaluation matrix of Level 2 indexes. (Here suppose 3 experts and 17 teachers participate in the evaluation)

$$R_1 = \begin{bmatrix} 0.20 & 0.60 & 0.15 & 0.05 \\ 0.30 & 0.55 & 0.10 & 0.05 \\ 0.45 & 0.35 & 0.20 & 0.00 \\ 0.60 & 0.25 & 0.15 & 0.00 \\ 0.55 & 0.15 & 0.20 & 0.10 \\ 0.40 & 0.45 & 0.10 & 0.05 \\ 0.25 & 0.35 & 0.25 & 0.15 \end{bmatrix} \quad R_2 = \begin{bmatrix} 0.40 & 0.35 & 0.10 & 0.15 \\ 0.35 & 0.45 & 0.15 & 0.05 \\ 0.20 & 0.50 & 0.20 & 0.10 \\ 0.25 & 0.35 & 0.25 & 0.15 \\ 0.10 & 0.45 & 0.20 & 0.25 \\ 0.30 & 0.40 & 0.20 & 0.10 \\ 0.20 & 0.60 & 0.15 & 0.05 \\ 0.15 & 0.35 & 0.20 & 0.30 \\ 0.50 & 0.30 & 0.15 & 0.05 \\ 0.25 & 0.40 & 0.20 & 0.15 \\ 0.30 & 0.45 & 0.25 & 0.00 \end{bmatrix}$$

$$R_3 = \begin{bmatrix} 0.10 & 0.65 & 0.25 & 0.00 \\ 0.25 & 0.45 & 0.15 & 0.15 \\ 0.20 & 0.50 & 0.10 & 0.20 \\ 0.10 & 0.35 & 0.25 & 0.30 \\ 0.30 & 0.30 & 0.25 & 0.15 \\ 0.00 & 0.00 & 0.55 & 0.45 \\ 0.05 & 0.55 & 0.10 & 0.30 \\ 0.35 & 0.45 & 0.15 & 0.05 \\ 0.55 & 0.35 & 0.10 & 0.00 \end{bmatrix} \quad R_4 = \begin{bmatrix} 0.50 & 0.30 & 0.15 & 0.05 \\ 0.25 & 0.45 & 0.20 & 0.10 \\ 0.15 & 0.45 & 0.15 & 0.25 \\ 0.30 & 0.35 & 0.05 & 0.30 \\ 0.05 & 0.35 & 0.25 & 0.35 \end{bmatrix}$$

Step four: overall evaluation

(1) Fuzzy evaluation to all indexes

$$C_1 = W_1 \bullet R_1 = (0.409,0.395,0.156,0.040)$$
$$C_2 = W_2 \bullet R_2 = (0.300,0.428,0.186,0.085)$$
$$C_3 = W_3 \bullet R_3 = (0.230,0.403,0.186,0.182)$$
$$C_4 = W_4 \bullet R_4 = (0.216,0.400,0.160,0.230)$$

(2) Fuzzy overall evaluation

$$C = W \bullet R = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix} = (0.300, 0.408, 0.173, 0.120)$$

According to the principle of maximum membership degree of fuzzy overall evaluation, the overall evaluation conclusion to the web-based course is "Good".

(3) Computing Overall Score
The ultimate evaluation result by the following method adapts fuzzy reviews set, which lacks Comparability and Intuitive when compare one kind of web-based courses, so we deal evaluation results with quantification, obtain the overall score of the web-based course:

$$D = C \bullet V^T = 75.17$$

5 Conclusion

The article gives the method which apply fuzzy math theory to overall evaluate web-based courses according to the index system defined by *web-based courses evaluation specification*, creates 2 level judgment matrix and makes quantification to result matrix and makes the quantitative basis for the comparing and sorting of the same kind web-based courses. The web-based course evaluation model constructed by FAHP, can determine the weight of all indexes of the evaluation indexes system scientifically, avoid the arbitrary when determining the weight, and take the fuzzy of human thinking judgment into consideration at the same time make qualitative assessment quantitative to make the result of overall evaluation more scientific and objective. It has easy programming, certain universal and promotion value.

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Vague Sets Based Researches on Natural Rubber Species Optimization Evaluation

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Abstract. This academic paper put forward the transformation formula and new similarity measure formula from single value data to vague data, and makes it the basis of vague integrative optimization evaluation methods. And make use of vague sets to optimize and evaluate four natural rubber species which planted widely in Hainan synthetically. Finally request to protect the natural rubber species of all kinds, breeding and promotion of new breed of high productivity and high resisting adversity property through science and technology innovation, and ultimately promote yield of natural rubber.

Keywords: Vague sets, single value data, transformation formula, similarity measure, natural rubber, species optimization.

1 Introduction

The vague sets brought forward by Gau and Buehrer [1] are extensions of fuzzy sets theory created by Zadeh [2]. The author makes use of these Vague sets to summarize a method of vague optimization and evaluation, try to optimize and evaluate natural rubber species which planted in Hainan synthetically, and hope to give scientific proposals to plantation of natural rubber in Hainan.

2 Methods of Vague Data Transformation

Similarity measure is a kind of base to integrative optimization evaluation methods. As for similarity measures between Vague sets, the Vague data transformation method which put forward by Liu Hua-wen [3] must be introduced firstly here.

On the assumption that X is discourse domain, and A is Vague sets based on X. To any $x \in X$, then comes the Vague value $A(x) = x = [t_x, 1 - f_x]$. Meanwhile, $\pi_x = 1 - t_x - f_x$, $t_x^{<0>} = t_x$ and $f_x^{<0>} = f_x$ is designated.

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And $\alpha, \beta \in [0,1]$ besides $\alpha + \beta \le 1$ is also designed. To any $m = 0,1,\cdots$, the Vague value is defined as follows.

$$t_x^{<m+1>} = t_x + \alpha\pi_x[1+(1-\alpha-\beta)+\cdots+(1-\alpha-\beta)^m]$$
$$f_x^{<m+1>} = f_x + \beta\pi_x[1+(1-\alpha-\beta)+\cdots+(1-\alpha-\beta)^m]$$
$$\pi_x^{<m+1>} = 1 - t_x^{<m+1>} - f_x^{<m+1>}$$
$$u^{<m+1>}(x) = t_x^{<m+1>} - f_x^{<m+1>} \quad (m = 0,1,2,\cdots)$$

And $[t_x^{<m>}, 1 - f_x^{<m>}]$ above is the Vague value.

3 Similarity Measure between Vague Data

Definition 1. $M(x,y)$ *is similarity measure between Vague values* $x = [t_x, 1- \ f_x]$ $f_x]$ *and* $y = [t_y, 1 - f_y]$*, if the following is satisfied.*

a. Commonness $0 \le M(x,y) \le 1$*;*

b. Symmetry $M(x,y) = M(y,x)$*;*

c. Reflexivity $M(x,x) = 1$*;*

d. Minimum $M(x,y) = 0$ *, whenever* $x = [0,0]$ *and* $y = [1,1]$ *, or* $x = [1,1]$ *and* $y = [0,0]$*.Then the following theorem comes.*

Theorem 1. *To any* $m = 0,1,\cdots$*, thereupon comes with the following Formula (1),*

$$M_m(x,y) = \frac{1+\min\{u^{<m>}(x), u^{<m>}(y)\}}{1+\max\{u^{<m>}(x), u^{<m>}(y)\}} \tag{1}$$

is similarity measure between Vague values of x *and* y*.*

Formula (1) is the extension of corresponding formula which put through by Zhou Zhen [4], because the formula is Formula (1) whenever $m = 0$. And besides, same problem happens both on Zhou Zhen's [4] formula and Formula (1). When $x = y = [0,0]$, and $u^{<m>}(x) = u^{<m>}(y) = -1 \quad (m = 0,1,2,\cdots)$, denominator $1+\max\{u^{<m>}(x), u^{<m>}(y)\} = 0$ appear. So, to $x = y = [0,0]$ in Formula (1), it is not allowed to happen within the suitable range. And it can be realized when suitable transformation formula from raw data to Vague data is selected.

Theorem 2. *On the assumption that discourses domain is* $X = \{x_1, x_2, \cdots, x_n\}$*, the Vague sets upon which is shown as follows.*

$A=\sum_{i=1}^{n}[t_A(x_i),1-f_A(x_i)]/x_i$, *can be also abbreviated as* $A=\sum_{i=1}^{n}[t_{x_i},1-f_{x_i}]/x_i$,

$B=\sum_{i=1}^{n}[t_B(x_i),1-f_B(x_i)]/x_i$, *can be also abbreviated as* $B=\sum_{i=1}^{n}[t_{y_i},1-f_{y_i}]/x_i$,

On the assumption that $m=0,1,2,\cdots$, *then*

$$M_m(A,B)=\frac{1}{n}\sum_{i=1}^{n}\frac{1+\min\{u^{<m>}(x_i),u^{<m>}(y_i)\}}{1+\max\{u^{<m>}(x_i),u^{<m>}(y_i)\}} \tag{2}$$

is the similarity measure between Vague sets A *and* B .

Theorem 3. *On the conditions of Theorem 2, if the weight of element* x_i *is* $w_i\in[0,1]$ *and* $\sum_{i=1}^{n}w_i=1$, *and* $WM_m(A,B)=\sum_{i=1}^{n}w_i\cdot\frac{1+\min\{u^{<m>}(x_i),u^{<m>}(y_i)\}}{1+\max\{u^{<m>}(x_i),u^{<m>}(y_i)\}}$ *is the weighted similarity measure between Vague sets* A *and* B .

4 The Formula Transforming Single Value Data into Vague Value Data

Built-up a Vague setting is the precondition and key approach of application of Vague sets. And the method is to transform single value data into Vague value data. The following is the definition and method transforming single value data into Vague value data in this academic paper.

Definition 2. *On the assumption that indexing set is* $X=\{x_1,x_2,\cdots,x_n\}$, *when set based on which is* A_i $(i=1,2,\cdots,m)$ *and index is* x_j $(j=1,2,\cdots,n)$, *then the data will be* $x_{ij}(\geq 0)$. *If* x_{ij} *which transformed into Vague value* $A_i(x_j)=x_{ij}=[t_{ij},1-f_{ij}]$ *is satisfied to space rule and profit rule, then profit type transformation formula can be named to this kind of transformation formula which transforming single non-negative value data into Vague value data. If it is satisfied to space rule and cost rule, then cost type transformation formula can be called to this kind of transformation formula which transforming non-negative single value data into Vague value data.*

Then the rules can be explained as follows.

a. Space rule, $0 \le t_{ij} \le 1 - f_{ij} \le 1$*;*

b. Profit rule, when $x_{kj} \ge x_{hj} \ge 0$*, then single data* x_{kj} *and* x_{hj} *transformed into Vague data* $A_k(x_j) = x_{kj} = [t_{kj}, 1 - f_{kj}]$ *and* $A_h(x_j) = x_{hj} = [t_{hj}, 1 - f_{hj}]$ *individually, and then* $t_{kj} \ge t_{hj}$, $1 - f_{kj} \ge 1 - f_{hj}$*;*

c. Cost rule, when $x_{kj} \ge x_{hj} \ge 0$*, single non-negative value data* x_{kj} *and* x_{hj} *transformed into Vague data*

$$A_k(x_j) = x_{kj} = [t_{kj}, 1 - f_{kj}] \text{ and } A_h(x_j) = x_{hj} = [t_{hj}, 1 - f_{hj}]$$

individually, and then

$$t_{kj} \le t_{hj}, 1 - f_{kj} \le 1 - f_{hj}.$$

Theorem 4. *If* $x_{j\max} = \max\{x_{1j}, x_{2j}, \cdots, x_{mj}\}$*, then*

$$A_i(x_j) = x_{ij} = [t_{ij}, 1 - f_{ij}] = \left[\left(\frac{x_{ij}}{x_{j\max}}\right)^{3k}, \left(\frac{x_{ij}}{x_{j\max}}\right)^{k}\right] \tag{3}$$

is profit type transformation formula which transforming single non-negative value data x_{ij} *into Vague value data.*

However

$$A_i(x_j) = x_{ij} = [t_{ij}, 1 - f_{ij}] = \left[1 - \left(\frac{x_{ij}}{x_{j\max}}\right)^{k}, 1 - \left(\frac{x_{ij}}{x_{j\max}}\right)^{3k}\right]$$

is cost type transformation formula which transforming non-negative single value data x_{ij} into Vague value data.

Notice: The Formula (3) here ensured that $A_i(x_j) = [0, 0]$ will not happen.

5 Vague Integrative Optimization Evaluation Methods

Based on the Vague mode identification rule of Liu Hua-wen [5], if the procedure of converting raw data into Vague data is added, then the completed Vague mode identification method can be formed, because it is not need to discuss Vague mode identification without the procedure of converting raw data into Vague data. This completed Vague mode identification method can be improved. The multiple standard modes earlier can be improved into single optimized mode, and the single preparative identification mode earlier also can be changed into multiple preparative optimized modes. After computation, according to the similarity measures between Vague sets, the corresponding preparative optimized mode can be queued, and the one in the front is the best. This method is called the Vague integrative optimization evaluation methods.

6 Optimization Evaluations of Natural Rubber Species

Based on the data from general survey of Hainan natural rubber species in 1992[6], the authors made integrative optimization queues towards four natural rubber species which planted widely in Hainan, and hope to give scientific suggestions to Hainan natural rubber plantation.

The preparative optima species are shown in Table One.

Table 1. Preparative optima species

Code	Preparative optima species	Origin
B_1	PB_{86}	Malaysia
B_2	Guangxi6 – 68	South China Academy of Tropical Crops and Guangxi Xianfeng Farm
B_3	Nanqiang1 – 97	Hainan Donghong Farm
B_4	Hekou3 – 11	Hainan

And the evaluations indexes are shown in Table 2.

Table 2. Evaluations indexes

Code	Characteristics
x_1	Average Yield for One Tree(Kg)
x_2	Wind Resistance
x_3	Cold Resistance
x_4	Powdery Mildew Resistance
x_5	Growth
x_6	Planting Areas(Kha)

For ideal species for natural rubber, the bigger index x_1 to x_6 is, the better the species is. So the data of the ideal natural rubber species A is shown also in Table 3.

Table 3. Raw data of varieties of natural rubber

	B_1	B_2	B_3	B_4	A
x_1	2.0	3.42	1.8	3.5	3.5
x_2	Bad	Mid-low	Mid-high	Mid-low	Mid-high
x_3	Bad	Medium	Medium	Mid-high	Mid-high
x_4	Medium	Medium	Medium	Mid-low	Medium
x_5	Mid -high	Mid-high	Mid-high	Mid-low	Mid-high
x_6	21.33	0.25	1.93	2.4	21.33

(Data Source: Germplasm Resources Investigation Corpus of Crops (Plants) on Hainan Island)

To convert raw data into Vague data, natural languages data can be evaluated in Table 4.

Table 4. Natural languages data evaluation

Situation	Data Evaluation
Good	[0.85 1]
Mid-high	[0.65 0.85]
Medium	[0.55 0.65]
Mid-low	[0.35 0.55]
Bad	[0 0.35]

Formula (3) (when $k=1$) is applied to transform the single value data to Vague data measure, because Vague value of $[0,0]$ will not happen, so Formula (2) is also applicable.

Then the raw data in Table 3 can be converted into Vague data sheet in Table 5 through the above-mentioned disposal.

Table 5. Vague data sheet of varieties of natural rubber

	B_1	B_2	B_3	B_4	A
x_1	[0.186,0.571]	[0.933,0.977]	[0.136,0.514]	[1,1]	[1,1]
x_2	[0,0.35]	[0.35,0.55]	[0.65,0.85]	[0.35,0.55]	[0.65,0.85]
x_3	[0,0.35]	[0.55,0.65]	[0.55,0.65]	[0.65,0.85]	[0.65,0.85]
x_4	[0.55,0.65]	[0.55,0.65]	[0.55,0.65]	[0.35,0.55]	[0.55,0.65]
x_5	[0.65,0.85]	[0.65,0.85]	[0.65,0.85]	[0.35,0.55]	[0.65,0.85]
x_6	[1,1]	[0,0.012]	[0,0.091]	[0.001,0.113]	[1,1]

Table 5 showed the Vague sets of preparative optima species B_i $(i = 1,2,3,4)$ and the ideal species A.

Formula (2) (when $\alpha = 0.3,\ \beta = 0.5,\ m = 2$) is applied to compute the similarity measure between Vague sets A and B_i.The result is shown as follows.

$$M_2(A,B_1) = 0.6989, M_2(A,B_2) = 0.7250,$$
$$M_2(A,B_3) = 0.6872, M_2(A,B_4) = 0.6982$$

Then the optimization queues for natural rubber species are as follows.

$$B_2 \succ B_1 \succ B_4 \succ B_3 \text{ (Symbol “}\succ\text{”means “better than”)}$$

That is shown that species $\mathrm{Guangxi6-68} \succ \mathrm{PB}_{86} \succ \mathrm{Hekou3-11} \succ \mathrm{Nanqiang1-97}$, and the best species is $\mathrm{Guangxi6-68}$ after the examination of integrated indexes. Besides higher yield, $\mathrm{Guangxi6-68}$ is medium cold resistant. And this evaluation result has gained good comment from the authority from the natural rubber industry.

Under the background of global warming, the characteristics and the elements of atmospheric circulation are changing, and the atmosphere, the ocean and the land are reacting together, the moisture circulation in the air are prick up, and all the above-mentioned reasons increase the possibility of extremeness weather. Global warming increases the frequency and the disserved extent of natural calamity in this world, and extreme weather turns to be much more, much stronger and much more abnormal. Moreover, natural rubber in China are mostly planted in high latitude area, between 18 to 24 degrees north latitude, and the natural calamity of wind, cold, diseases and drought appears frequently. So, with the characteristics of higher yield and stress resistance, Guangxi6 – 68 will be selected for breeding and popularized in larger scale.

7 Conclusion

In this academic paper, the transforming formula from single value data to Vague data and the similarity measure formula between Vague sets are put through, and both kinds of these formulas are providing technical support to integrative optimization evaluation methods of Vague sets.

Applied integrative optimization evaluation methods of Vague sets to integrative optimization evaluate natural rubber species. And the evaluation result is beneficial to organize natural rubber plantation scientifically in China.

Different natural rubber species have distinctive character, such as wind resistance, cold resistance, powdery mildew resistance. At present, new lines of natural rubber are selected for breeding and popularized, more and more natural rubber plantations are under the situation of replantation. Some species with distinctive character and low yield are apt to be neglected, and some of them are at the edge of extinction. To avoid this ending, emphasis should be put on protection of different kinds of natural rubber species, and creation of convenient conditions to select for breeding and popularize high-yield and stress resistance, and to shorten the periods of select for breeding.

The natural rubber industry in China is a typical resource-constrained industry. The essential to develop the production of natural rubber is based on scientific and technological progress. With limited planting areas, the natural rubber supply only can be increased by improving yield through scientific and technological innovations.

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Groundwater Quality Evaluation Based on Fuzzy Comprehensive Method

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Abstract. The purpose of water quality assessment is that make reaction of water features accurately. The key of how to reflect the status and grade of water pollution is the expression of evaluation conclusions The factors which groundwater evaluation involves are often as many as dozens,and there are various factors at different levels.Through the choice of assessment factors, the method of fuzzy comprehensive evaluation which determines the fuzzy sets such as factor set, evaluation set, membership function and weight set makes calculation. Therefore, it can be more scientific, practical and effective in using the monitoring data than the general comprehensive evaluation to make evaluation of groundwater accurately. It introduces the theory of the method of fuzzy comprehensive evaluation systematically, and the fuzzy evaluation with examples whose results indicated that fuzzy comprehensive evaluation method has good uniformity with conventional evaluation method, and so does the water features. Therefor it can evaluate the degree of groundwater quality more comprehensively.

Keywords: Water quality assessment, groundwater, fuzzy comprehensive evaluation, evaluation factor, Weighting coefficient.

1 Introduction

Water quality evaluation is a process which analyzes a regional environmental factor of water and evaluates the water quality quantitatively. Through the water quality evaluation, we can learn about and grasp the variation tendency of water quality, get situation of pollution in water bodies accurately. Water

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quality provides the scientific basis for the water pollution and water resources utilization, protect, planning and management. Groundwater resource is the basis of human being's survival and the shortage of water resource becomes the restrictive factor of economical development. Groundwater quality evaluation, which is the basic work of environmental hydrogeology, according to the evaluation of groundwater quality work, will be advantageous in dealing with the groundwater environmental sustainable development, utilization and protection. It is also of great importance to sustain economic development and people's physical health.

In the comprehensive evaluation for complex system of environment, it often involves dozens of different factors, and the factors often have different levels. The key factors,which are complicated multi-factors and multi-layer system in incarnating effect on the integrated results of the environment accurately and objectively, are hierarchical and comprehensive factors. The methods of conventional single factor index and the comprehensive index can only discriminate water "good" or "bad" simply. When the single factor index method evaluates water quality, as long as there is a serious pollution factors and regardless of how ever pollution levels of other factors, the water quality is poor; The methods of comprehensive index can reflect water pollution situation better, and the calculations of it is also simple. But it takes a number of factors as the same important status, regardless of the differences of the importance. Besides, it can only determine the water quality is better or worse than the category of some other water quality. So it belongs to which category needs multimetering. Therefore, fuzzy comprehensive evaluation of water quality can solve these problems better. Due to the diffusion of groundwater, the dispersion of groundwater quality is a complex process which is continuous gradual changing, and whose boundary is fuzzy. Therefore, comprehensive evaluation of groundwater quality is actually a problem which has more than one indicator of pattern recognition. The water quality of the environment and division rating whose boundary is fuzzy without a certain level of boundary. Through the comprehensive evaluation of groundwater quality, the results which are more objective and reality can be got. Therefore, the method of comprehensive evaluation of groundwater quality which is used to make an evaluation of groundwater quality makes great progress. It overcomes the of insufficient binary logic thinking methods in the past "synthetic index method" and it can make greater degree reaction to the groundwater quality objectively.

2 Fuzzy Comprehensive Evaluation Method

Fuzzy evaluation is a method which makes an evaluation according to the evaluation standards and values, and using fuzzy transform to evaluate objects. The object with multiple attributes must be considered various factors when evaluating, but many problems are often difficult to indicate by a simple number. This often has fuzzification. The fuzzy comprehensive evaluation can objectively response the characteristics of each attribute.

2.1 The Principle of Fuzzy Comprehensive Evaluation Method

The principle of fuzzy comprehensive evaluation method can be described by a mathematic equation.

$$\mathbf{A} \times \mathbf{R} = \mathbf{B} \tag{1}$$

A—-a row matrix of $(1 \times n)$ is consisted of the factors (which are in evaluating by weight) through the normalized processing, n represents the number of evaluation factors;
R—-fuzzy relational matrix of $(n \times m)$ is consisted of every row matrix(which is single-factor evaluation);
B—-a row matrix of $(1 \times m)$, it is the result from comprehensive evaluation.

2.2 Fuzzy Comprehensive Evaluation

Generally, the steps of fuzzy comprehensive evaluation methods can be summed up as follows:

1) Determine the evaluation factors
Based on the conditions of environment, the evaluation factors set $U = (u_1, u_2, \ldots, u_n)$ is consisted of the selected important factors. The element $u_i (i = 1, 2, \ldots, n)$ is measured value of Wastes and other Pollutants which influence on environmental quality.

2) Determine the evaluative criteria
The quality evaluation grades of water $V = (v_1, v_2, \ldots, v_m)$ are established, the element $v_i (i = 1, 2, \ldots, m)$ is standard classify value of water from every pollution.

3) Determine the membership and relation matrix R
As water pollution and water quality standard classification are fuzzy, it is reasonable that the degree of membership can be used to depict the grade boundaries. c_i represents the consistency of the pollution elements in a specimen.The membership Y of water whose grade are I, II, III,IV, V is consisted of the single element that can be solved by the measured valuable c_i. The membership can be solved by the linear membership function as follows:

In order to determine the membership function, a model is the most popular in fuzzy comprehensive evaluation method at present. It is the same as the conventional fuzzy comprehensive evaluation method. The membership function $Y(c_i)$ can have continuous values in the extent $[0, 1]$, just as $0 \leq Y(c_i) \leq 1$. If the membership is used to describe membership qualifications. The greater the valuable of membership, the higher the membership qualifications. When the valuable of membership is 1, it represents it is subordinated completely; the valuable of membership is 0, it represents it is subordinated incompletely. So a figure of lower semi-trapezoid distribution can be formed, its analytic expression as follows:

$$Y(c_i) = \begin{cases} 1 & 0 \le c_i \le a_1 \\ \frac{a_2 - c_i}{a_2 - a_1} & 0 \le a_1 \le a_2 \\ 0 & c_i \le a_2 \end{cases} \tag{2}$$

a_1, a_2—-Adjacent secondary standard of water water quality;
c_i—-Actual measured valuable;
It can be described as the membership function of five twin level, so fuzzy relationship matrix R is formed as follows:

$$R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{15} \\ r_{21} & r_{22} & \cdots & r_{25} \\ \cdots & \cdots & \cdots & \cdots \\ r_{m1} & r_{m2} & \cdots & r_{m5} \end{bmatrix} \tag{3}$$

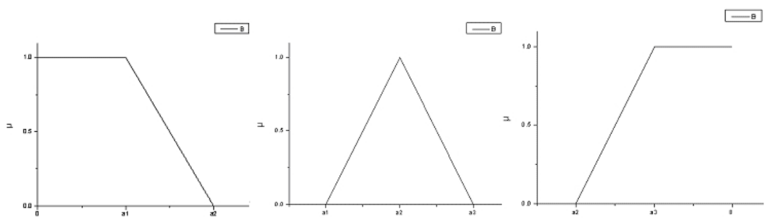

Fig. 1. Lower semi-trapezoid distribution

4) Confirm The Weighting Coefficient
In the multi-level fuzzy evaluation system, every element may play different role if its weighting coefficient is different. So its influence degree is also different to the results of the multi-level fuzzy evaluation system. The weighting coefficient must be calculated in order to establish set of weighting coefficient's distribution. A represents n-dimensional row matrix which is consisted of pollution factors, (which aggravate environmental pollution, besides that, multi-factor mutual synergy, antagonism of the effects can not be ignored to the environmental pollution). So $A = (a_1, a_2, \ldots, a_n)$, the weighting coefficient of pollution factors can be solved by the formula:

$$a_i = \frac{\frac{C_i}{\sum_{j=1}^{m} S_{ij}}}{\sum_{i=1}^{n} \frac{C_i}{\sum_{j=1}^{m} S_{ij}}} \tag{4}$$

a_i—the weighting coefficient of i_{th} evolutional factor;
c_i—the actual measured value of i_{th} evolutional factor;
S_{ij}—the j_{th} concentration of national standards which is i_{th} evolutional factor;

5) Confirm Multi-level Fuzzy Evaluation System of the Matrix B
Fuzzy comprehensive evaluation is compound operation of fuzzy matrix. The results for fuzzy comprehensive evaluation are received by the compound operation, which between single factor weighting coefficient matrix of A and single factor evaluation of R, just as $B = A \circ R$. The results for the compound operation of fuzzy matrix are received by max-min rules between the matrix of A and R. The final results for the present situation of the groundwater quality of the water-source locations can be described by the maximum value in the matrix B.

3 An Example of Application

These 25 groups experimental samples were collected in north China, including 7 groups of shallow groundwater samples, 18 groups of deep groundwater samples, 5 groups in Zeyang channel, and others in the northern region of Fuyang River. The test area is mainly located in the southeastern depression of Jizhong and northwestern uplift parts of Cang country. The Hydrogeology and Mineral Center of Experimental Test Department had tested the water samples. The test methods are all conducted strictly according to the national standards. The chemical statistical characteristics of shallow groundwater in different areas is as follows:

Table 1. The test result and analysis of water

Monitoring Points	Cl^-	SO_4^{2-}	Total Hardness	TDS	PH
1	804.90	637.90	222.20	2664.2	7.5
2	253.20	183.30	62.87	1120.3	7.6
3	217.60	293.80	146.40	1573.1	7.4
4	826.60	658.00	138.30	3100.3	7.4
5	1483.20	1230.60	429.60	4914.1	7.5
6	2487.10	2084.50	419.20	6942.2	7.6
7	502.90	424.40	58.68	1505.7	9.2

1) Determine evaluation for sets In the data of water quality, five pollutant index can be selected to make evaluation,just as Cl^-, SO_4^{2-},Total Hardness,TDS,PH.

2) Determine the criteria for evaluation
The groundwater quality standards of GB/T14848-93 can be referenced for Classification and boundary valuable of the components.

3) Records of the evaluative results
According to the criteria of evaluations, fuzzy comprehensive evaluation method which is introduced before, the membership of the pollutants (which belongs to the levels of water quality standards' membership)can be calculated.

Table 2. The membership of the evaluation factors

Monitoring Points	Cl^-	SO_4^{2-}	Total Hardness	TDS	PH
$Y_I(x)$	0	0	0.037	0	0.375
$Y_{II}(x)$	0	0	0.963	0	0.625
$Y_{III}(x)$	0	0	0	0	0
$Y_{IV}(x)$	0	0	0	0	0
$Y_V(x)$	1	1	0	1	0

Then the evaluation matrix of every factor set has been received.

$$R = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0.037 & 0.963 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0.375 & 0.625 & 0 & 0 & 0 \end{bmatrix} \tag{5}$$

After calculating the weighting coefficient, we can receive the data as follow:

Table 3. The weight value of the evaluation factors

Computational Item	Cl^-	SO_4^{2-}	Total Hardness	TDS	PH
A	0.3515	0.2786	0.0539	0.2351	0.0809

Therefore

$B_1 = A \circ R = (0.0323, 0.1025, 0, 0, 0.8625)$, point 1 is class V, water quality is very poor; Major pollutants are Cl^-, SO_4^{2-}. To be contaminants is TDS.

Similarly we can get the conclusion:

$B_2 = A \circ R = (0.0924, 0.1936, 0.4356, 0.2784, 0)$, point 2 is class III, water quality is better; Major pollutants are Cl^-, TDS. To be contaminants is SO_4^{2-}.

$B_3 = A \circ R = (0.158, 0.0836, 0.1806, 0.5353, 0.0244)$, point 3 is class IV, water quality is poorer; Major pollutants are TDS, SO_4^{2-}. To be contaminants is Cl^-.

$B_4 = A \circ R = (0.0710, 0.386, 0, 0, 0.8905)$, point 4 is class V, water quality is very poor; Major pollutants are Cl^-, SO_4^{2-}. To be contaminants is TDS.

$B_5 = A \circ R = (0.0168, 0.028, 0.0325, 0.0253, 0.8974)$, point 5 is class V, water quality is very poor; Major pollutants are Cl^-, SO_4^{2-}. To be contaminants is TDS.

$B_6 = A \circ R = (0.0074, 0.0221, 0.0235, 0.0129, 0.9342)$, point 6 is class V, water quality is very poor; Major pollutants are Cl^-, SO_4^{2-}. To be contaminants is TDS.

$B_7 = A \circ R = (0.0219, 0, 0, 0.2018, 0.7763)$, point 7 is class V, water quality is very poor; Major pollutants are Cl^-, SO_4^{2-}. To be contaminants is TDS.

As can be seen from the result: the water quality of the sample point 2 is good; the water quality of the sample points 1,3,4,5,6 and 7 are polluted seriously, and they can't be served as the drinking water source.

4 Conclusion

The fuzzy comprehensive evaluation method makes evaluation of groundwater especially it prominent the influence of the main exceeds to groundwater quality components, and it also reflects the pollution levels of the other pollutants except the main pollutants. Therefore it makes up the shortage of the method of the groundwater environmental quality assessment at present and it provides reference and theoretical basis for the polluted groundwater which needs recondition. From the evaluation results, it indicates the groundwater is polluted more seriously in some parts of the region, the evaluation results indicated that fuzzy comprehensive evaluation method has good uniformity with conventional evaluation method, and so does the water features.

The fuzzy mathematics method is used to make quality evaluation of groundwateralthough the calculation is more complicated than the conventional evaluation methods, the problem will be solved commendably by the computer. The method can reflect the real situation of water quality, therefore the method should be paid more attention to explore.

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Cleaner Production Evaluation Method-Fuzzy Math Method

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Abstract. Fuzzy evaluation model is the comprehensive evaluation method and quantitative analysis and which based on the "maximum membership degree evaluation". Due to the cleaner production of multiple attribute and evaluation factors, and with more than stratified random and fuzzy mathematics, this article puts forward a fuzzy evaluation to determine the key audit is practical and necessary. This paper describes the evaluation method of cleaner production simply, focusing on the principle,content and evaluation mode of fuzzy mathematics method for the evaluation of clean production agitation. The fuzzy mathematics evaluation method for clean production can be used to assess all aspects of clean production before and after the business at present, and identify cleaner production opportunities, identify weaknesses and so on. All the factors which affecting the quality of cleaner production can be evaluated comprehensively and accurately. It also can be able to change the multi-factor problems (which are complicated, not enough to determine) into a data model based on simple evaluation. This assessment model has been used in food industry of clean production, and achieved good results.

Keywords: Food processing industry, clean production, fuzzy evaluation method.

1 Introduction

The examine and verify of cleaner production is an important component and effective way for enterpriser to implement cleaner production. Through a set of system which is science and operating lines strongly to finish audit procedures. There are 7 stages and 35 steps such as scheme, organizations, pre-assessment, evaluation, generation and screening program, feasibility analysis, program implementation, continuous clean production and so on in the cleaner production evaluation method. The principle of fuzzy mathematics method, content and evaluation mode are important points of

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cleaner production audit. In this paper, the examination of specific examples, fuzzy evaluation method is used in the cleaner production evaluation of food industry, which makes the focus on cleaner production audits in order to determine clearly and simply.

2 Common Evaluation Methods

Currently, the key schemes of audit are many, the cleaner production evaluation method (which are staple) are also many, the major are two such as: Simple comparison method and total points sorting weight method.

2.1 Simple Comparison Method

Simple comparison is the focus of the various options for the emissions and toxicity were compared between consumption, analysis and discussion Simple comparison method which is according to the key of toxic waste emissions and consumption to make comparing, analyzing and discussing. Usually the most polluted, consumption and the maximum chance of cleaner production is the most obvious part of this review key position. Simple comparison method is qualitative, the review can only be focused on the general examine and verify. Therefore in order to be more scientific and objective, the best semi-quantitative and quantitative methods are used for analysis.

2.2 Total Score Ranking Weights Method

Total score ranking weights method which considers the weight and score of this factor, points that the weighted scores for each factor value in order to these weighted scores superimposed values and in the comparison of the total value of the weight way to make a choice. The weight factors which is determined should be focused on, cleaner production of enterpriser is mainly achieved the goal of pollution prevention services. The meaning of the weight factor should be clear, easy-to-rate and factors should avoid overlapping, the number is usually around 5.

3 Fuzzy Comprehensive Evaluation Method

3.1 The Significance of Fuzzy Math on Cleaner Production Audit Evaluation

Cleaner production audits is a comprehensive and systematic project, which includes various of contents and procedure with the fuzziness and the randomness. This system is consists of multiple levels, multiple factors synthetically. And the level of system and elements have close correlation, the purpose and function of the whole system are very clear. Cleaner production audits and in the process of practice, cannot ignore the system behavior. In the process of cleaner production, the evaluation method is established which is the guarantee of cleaner production management and the key link of quantification for cleaner production management, improving the economic efficiency of enterprisers and the clean production of key strategy. Although businesses have their own cleaner production industry standards. Conducting evaluation of cleaner production, the need to introduce a can of the single indicators is important, the integrated and methods indicator at different levels which combine with a single linear evaluation of the overall evaluation of the mechanisms should be improved.

3.2 Introduction of Fuzzy Evaluation Method

Fuzzy evaluation model is relatively good method for fuzzy comprehensive evaluation and quantitative analysis which based on the "maximum membership degree evaluation principle". Through the analysis of cleaner production, we can judge the comprehensive level and the main existing problems. In the assessment of cleaner production, we give full consideration to the comprehensive evaluation of the fuzziness and the randomness and so on, it is a simple but effective cleaning production evaluation method.

3.3 The Concepts and Principles

Starting from the qualitative fuzzy first, then through fuzzy selection principle of fuzzy transform can get the results. The operations of some factors which should be considered are fuzzy, also after determining factors index system of each factor, and evaluation experts make selection of the factor index, statistics expert group for the evaluation index system of the factors, according to the choice in order to establish the mathematical model calculation.

3.4 The Substance of Fuzzy Evaluation Method

First, the fuzzy comprehensive evaluation model should be established, then the complex problems change into a more simple fuzzy transformation.

X is the evaluation factors set, $X = \{x_1, x_2......x_n,\}$, Y is the evaluation of the level for decision-making set rating, $Y = \{y_1, y_2......y_n,\}, x_i \in X, y_i \in Y.r_i$ indicates x_i in y_i on the characteristics of the target (to the extent possible), x_i is belonged to $(r_{i1}, r_{i2}......r_{in},)$ which is characteristics of indicators for Y (i=1,2......n). Then the amount as a line of these groups formed matrix $R = (r_{ij})_{n\times m}$ as follows (which is called as single-factor evaluation matrix):

$$R = \begin{bmatrix} r_{11} & r_{11} & \cdots & r_{1j} & \cdots & r_{1n} \\ r_{21} & r_{21} & \cdots & r_{2j} & \cdots & r_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ r_{i1} & r_{i1} & \cdots & r_{ij} & \cdots & r_{in} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ r_{m1} & r_{m1} & \cdots & r_{mj} & \cdots & r_{mn} \end{bmatrix} \quad (1)$$

r_{ij} represents r_{th} factor of evaluation which subjected to j_{th}. Grade level evaluation fuzzy set $K = \{k_1, k_2......k_n,\}$ as weight distribution, so k_i is quantitative index for a factor of x_i. From $A = K \bullet R$, we can get $= \{a_1, a_2......a_n,\}$ which represents the possibility factor of decision-making sets for a variety of decision-making. Then select the maximum principle of maximum degree valuable a_i and y_i as the evaluation results.

$$A = K \times R = (a_1, a_2......a_n,), \sum_{j=1}^{m} a_i = 1 \quad (2)$$

4 Fuzzy Mathematics in the Assessment of Cleaner Production Audits

4.1 The Audit Status and Problems of Fuzzy Mathematics in the Evaluation of the Application of Cleaner Production

Fuzzy mathematics method can be a combination of fuzzy comprehensive evaluation one, the Principle and the concept of quantitative analysis and the maximum degree can be used in the assessment. Through this analysis, it integrated the level of the main aspects and problems of cleaner production can determine. In the evaluation, the comprehensive evaluation of cleaner production, fuzziness and randomness, etc are fully taken into account, it is a simple but effective method of clean production evaluation. Fuzzy mathematics method of industrial enterprisers in our country is not widely used in the cleaner production, as a kind of simple scientific met, only in food, aluminum, electronics, paint and other industries for clean production evaluation it has been used. Fuzzy Math is a quantitative evaluation method, it requires individual indicators which can be given quantitative values corresponding to the degree of membership. But it is not suitable for quantitative evaluation of qualitative indicators, this situation is set up for quantitative evaluation as some obstacles. Then it asked the first index of the existing system to quantify the various indicators of fit, this process requires a combination of experts and the actual situation of enterprisers to decide. Some indicators which are not quantitative, we can take omitted the way. All in words, although Fuzzy Math is simple, it can reflect the implementation of cleaner production and it can also not be combined with quantitative evaluation or qualitative descriptors.

4.2 The Cleaner Production Application of Soybean Food Processing Industry Based on Fuzzy Comprehensive Evaluation Method

The Flow Chart of Soybean Food Processing

$SoyBean \Rightarrow Pre - EngineeringSection \Rightarrow PretreatmentSection \Rightarrow LeachingProcess \Rightarrow PretreatmentSection \Rightarrow RefiningProcess \Rightarrow FinishedGoods$

Valuation Model

Cleaner production evaluation should be able to cover all major aspects of raw materials, production processes and products. Production process, it is necessary to consider energy, resource use, but also consider the generation of pollutants. Taking into account the characteristics of industry, cleaner production target will be divided into six categories, just as Raw material use indicators, feature process indicators, Equipment Index, Waste recycling targets, Management requirements of target and "Three wastes" treatment index. Specific categories shown in Fig. 1.

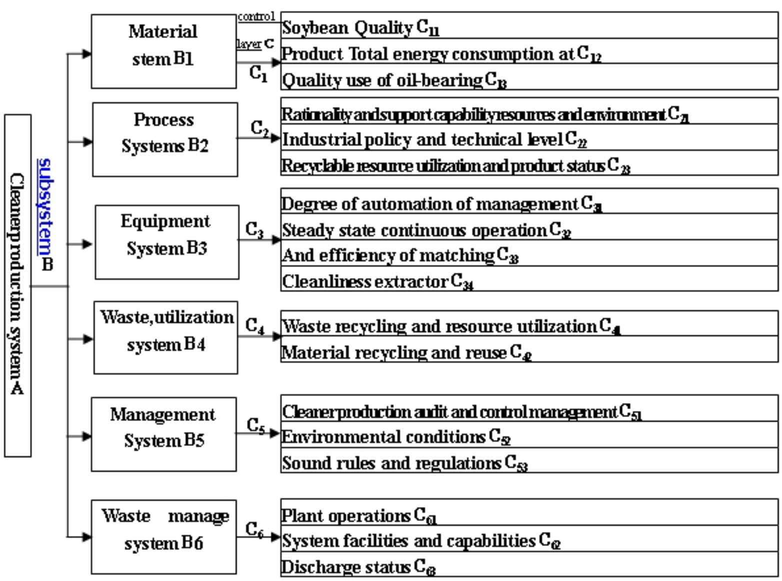

Fig. 1. Clean Production System Structure and Elements of Partition

Evaluation Steps

First Rating

The plant invited eight experts to evaluate 18 elements (control level) for their business, to determine its clean production evaluation the weight distribution factor of subsystem. According to the material and testing data in membership judgment which is calculated cleaner production factors subsystem membership with expert consult of a table (1) Lists the control layer of fuzzy evaluation matrix according to the dat, get the control layer C of fuzzy evaluation matrix (2) Calculate the subsystem Evaluation results of B From$B_N = K^{'} \times C_N$,

$$B_1 = [0.3, 0.5, 0.2, 0.0, 0.0]$$

The results correspond to [Best, Better, Good, Poorer, Poor], according to the principle of maximum membership degree, we can get B. Subsystem represents the level of production of raw materials in a good clean system.

Similarly we can calculate the evaluation results of other subsystems

$B_2 = [0.1, 0.5, 0.3, 0.1, 0.0]$, Representing good level of cleaner production technology;

$B_3 = [0.2, 0.3, 0.2, 0.2, 0.1]$, Representing good level of cleaner production equipment;

Subsystem	Subsystem Elements K																		K
	C11	C12	C13	C31	C22	C23	C31	C32	C33	C34	C41	C42	C51	C52	C53	C61	C62	C63	
B1	0.4	0.3	0.3																0.1
B2				0.3	0.5	0.2													0.1
B3							0.2	0.2	0.4	0.2									0.3
B4											0.4	0.6							0.2
B5													0.3	0.3	0.4				0.2
B6																0.4	0.3	0.3	0.1

Fig. 2. Evaluation of cleaner production subsystem elements of weight distribution

Grads	C11	C12	C13	C21	C22	C23	C31	C32	C33	C34	C41	C42	C51	C52	C53	C61	C62	C63
Best	0.3	0.4	0.2	0.1	0.1	0.1	0.2	0.4	0.1	0.3	0.2	0.0	0.1	0.0	0.0	0.0	0.0	0.0
Better	0.6	0.4	0.3	0.6	0.5	0.5	0.2	0.3	0.3	0.2	0.3	0.0	0.3	0.0	0.0	0.1	0.0	0.0
Good	0.1	0.1	0.5	0.3	0.3	0.3	0.4	0.1	0.3	0.1	0.1	0.2	0.5	0.1	0.6	0.4	0.1	0.1
Poorer	0.0	0.1	0.0	0.1	0.1	0.1	0.1	0.2	0.2	0.3	0.3	0.5	0.1	0.6	0.2	0.4	0.7	0.3
Poor	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.1	0.1	0.1	0.3	0.0	0.3	0.2	0.1	0.2	0.6

Fig. 3. The Results Judged by All Elements of Membership

$B_4 = [[0.1, 0.1, 0.2, 0.4, 0.2]$, Representing "waste" poor level of cleaner production system;

$B_5 = [0.0, 0.1, 0.4, 0.3, 0.2]$, Representing General level of Cleaner Production Manager;

$B_6 = [0.0, 0.0, 0.2, 0.5, 0.3]$, Representing "waste" poorer level of cleaner production system;

Second Rating

(1) Computing system built the mother system of Fuzzy Evaluation Matrix A
According to the results, we get computing system built the mother system of Fuzzy Evaluation Matrix $B = (B_1, B_2, B_3, B_4, B_5, B_6)^T$

(2) Evaluation results of mother system A
According to A=KB, we get evaluation results of mother system A:

$A = [0.1, 0.2, 0.3, 0.3, 0.1]$

According to the principle of maximum membership degree, we get mother system cleaner production in general and poor, the raw materials subsystem level of cleaner production is better; the level of "Three wastes" system, Management System, "Three wastes" treatment and use are significantly lower; This is the reason which affecting the whole plant to clean the main production. The first we should clean with production of "three wastes" system, "three wastes" treatment and utilization of the system as the key points, while enterprisers should strengthen management and timely, improve process further, introduce advanced equipment, reduce the output of "three waste" and improve recycling efficiency and processing efficiency in order to encourage enterprisers to continue cleaner production.

The method of fuzzy comprehensive evaluation evaluates cleaner production which affect the cleanliness level of all the factors comprehensively and accurately. And it is able to determine the multi-factor problem (which is more complicated and not enough to make sure) into a digital evaluation model based on simple principle. Cleaner production as a quantitative assessment and management indicators is provided to a scientific evaluation for enterpriser. The results are scientific and objective.

(3) The different levels of membership can help soy food processing industry and its staff understanding of the business situation of the evaluation more intuitive, it is conducive to the business potential of cleaner production into further tap.

5 Conclusion

Fuzzy mathematics method can assess the various status of the production processes for enterprisers, identify opportunities of cleaner production and the weak links. It is able to determine the multi-factor problem (which is more complicated and not enough to make sure) into a digital evaluation model based on simple principle. Cleaner production provides a scientific evaluation for the quantitative assessment. With further restructuring of China's macro-economic, environmental access system for continuous improvement, continue saving energy by implement measures, with rising degree of identity from domestic enterprisers to cleaner production, cleaner production is a process of continuous improvement, the level of enterprisers' cleaner production will continue increasing.

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Optimization Analysis of a Spherical Shell of Schwedler Based on a Parameter Fuzzy Evaluation

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Abstract. Considering the ambiguity of the boundary constraints, this essay establishes a fuzzy optimal design model about reticulated domes of Schwedler, and carries on a fuzzy processing for the design variables, the objective functions and constraints in the fuzzy optimization mode. In terms of Schwedler Spherical shell structures' characteristic, this essay uses the fuzzy optimization method to determine the boundary constraints tolerance factor. Boundary search method is used for solving optimal level set λ^*. Then the optimal solution of fuzzy optimization problems is obtained, and which is compared with the deterministic optimal design. Therefore, the trend of two kinds of optimization design variables has been obtained. The simulation results show that the objective function values have decreased substantially than the deterministic optimal design values. Consequently, the feasibility and correctness of the optimization design have been proved about considering the fuzzy factor.

Keywords: Double-layer spherical shell, fuzzy optimum design, membership function, tolerance factor, boundary search method.

1 Introduction

The structure optimization design in the Spatial lattice structure, the minimum weight is usually the optimization objective function. Many literatures have been carried out on the optimization of shell. Zhang Wennian studied single-layer reticulated shell structure of optimal design, but it is only limited to the traditional optimization design [1]. In the traditional structural optimization design, because of an intermediary between different things, the transition process brought about the ambiguity of common things. The randomness of many factors and fuzziness caused by the uncertainty are not

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to be taken into consideration. Because of the complexity of the study, we are bound to involve all the fuzzy factors [2, 3, 4, 5]. This makes the parameter ambiguity be considered in the optimal design. Wang Hong studied the optimal design about spherical shell of fuzzy reliability constraints [6]. The fuzzy optimization theory is introduced into the design of structural optimization, which has theoretical significance and value-in-use. In this paper, when the shell structure has been identified span, on the basic of considering ambiguity about optimal design parameters of double-layer reticulated shell (bar cross-sectional area A_i, shell thickness h, spherical radius of curvature R, the number of nodes in each layer reticulated shells n_1). A mathematical model of fuzzy optimal design is established, which is used for calculation and analysis. The results showed that considering factors about fuzzy optimized design are more rational and economical.

2 The Fuzzy Optimal Design Model of Double-Layer Reticulated Shell

2.1 Determination of Constraints

Constraints of fuzzy optimization can be considered from the following two aspects. The first is geometric constraints such as the size of constraints, shape constraints; And the second is the performance constraint. For the double-layer reticulated shell, the controlling constraints include displacement constraints and stress constraints, at the same time stability also need to be considered in the design. The performance constraints, geometry constraints, design variables of the upper and lower limits of the range bound, as a design space on the fuzzy sets, you can obtain the following constraints.

(1) Determine the Reliability

Reticulated shell structure, which can not be used from always be useable, through an intermediate of the transition process, the intensity has a certain membership function μ(x).And then, their fuzzy reliability R is:

$$R_i = \int f(x_i)\mu(x_i)dx_i \qquad (i = F_1, F_2, F_3) \tag{1}$$

They respectively indicate that stress intensity, bar local stability, the overall stability of the reticulated shell under three kinds of reliability. And $f(x_i)$ is normal probability distribution density function of stress. According to the design requirements, reliability R_i should be greater than or equal to allowable reliability $[R_i]$:

$$R_i \geq [R_i] \qquad (i = F_1, F_2, F_3) \tag{2}$$

(2) Constraints About Stiffness, Strength, Strut Stability

According to pole pieces under the most unfavorable bar internal force, conventional design of shell may carry out the design cross-section bars. In addition to the axial force of the pole, shell also bear bending moments, so our design should base on stretch bending rods or bending rods. Stretch bending rods and the bending rods should be checked its stiffness, strength and stability. Conventional design requirements of shell were the maximum deflection shall not exceed the allowable deflection. As the stiffness parameters in the formula were the most ambiguous, and inevitably result in a binding condition was ambiguous, so fuzzy constraint conditions are:

$$F_{max} \leq \bar{f} \tag{3}$$

And $\bar{\mathrm{f}}$ is fuzzy maximum limit about allowable deflection of shell.

Conventional design requirements of shell were the calculation of the stress bar shall not exceed the allowable stress value. Therefore, both of intensity and struts stability of constraints are also showed ambiguity, fuzzy constraint conditions are:

$$\sigma_i \leq \bar{\sigma} \tag{4}$$

$$\sigma_j = \frac{N_i}{\varphi_i A_i} \leq \bar{\sigma} \tag{5}$$

And $\bar{\sigma}$ is fuzzy upper limit about allowable stress of the shell. N_i, φ_i, A_i are expressed respectively axis of pressure, stability coefficients and cross-sectional area of number j bar.

(3) Limit

Allowing the slenderness ratio $[\lambda]$, pole pieces of the smallest cross-sectional area and thickness limit as follows:

$$\lambda_i = \frac{l_{oi}}{i_i} \leq \bar{\lambda} \tag{6}$$

$$\frac{1}{A_i} \leq \bar{A}_{min} \quad \frac{1}{\delta_i} \leq \bar{\delta} \tag{7}$$

And l_{oi},i_i are expressed respectively as the calculated length and radius of gyration.

2.2 The Objective Function

In this paper, the total cost of single-layer shell is the objective function, mathematical model of shell of fuzzy optimal design can be expressed as:

$$W_{min} = C_1 \sum_{i=1}^{n} \rho \cdot A_i \cdot L_i + C_2 \cdot \delta \cdot n_1 \cdot n_2 \tag{8}$$

In the equation (8), C_1,C_2 are expressed respectively as the price per unit mass of circular steel tube and spherical joint(Yuan / t); ρ is defined as the density of material for the bar(kg/m^3); A_i is termed as cross-sectional area of bar(m^2);L_i is named the length of the bar(m); δ is expressed volume of a welded spherical joint, taking $7.36 \times 10^{-4} m^3$;n_1 is defined as the number of nodes in the each ring(respectively selected 18,20,24); n_2 is named as the number of circles about single-layer reticulated shells($6 \leq n_2$).

2.3 The Membership Function of Fuzzy Constraints

Provided that fuzzy optimization is done by the membership function of the intersection points,which are given as a sentence of fuzzy.$\mu_{f_1}(x)$ and $\mu_{f_2}(x)$ are two membership functions in the fuzzy optimization problem. In the fuzzy constraints about shell's fuzzy design, they do not have a strict definition about the boundaries. In the vicinity of allowable values, reflecting the gradual process ranging from the fully allowable use to not fully allowable use. The membership function is expressed its variation. Determination of membership function should be based on the nature of the fuzzy constraints and design requirements to determine. For the engineering problems, we use a linear membership function to satisfy the engineering precision.

Membership function of the performance constraints can be expressed as Equation 9, the graphics is shown in Fig.1.

$$\mu_\sigma = \begin{cases} 1 & 0 \leq \sigma \leq \sigma^{-L} \\ \frac{\sigma^\mu - \sigma}{\sigma^\mu - \sigma^{-L}} & \sigma^{-L} \leq \sigma \leq \sigma^\mu \\ 0 & \sigma \geq \sigma^{-\mu} \end{cases} \tag{9}$$

Membership function of the geometric constraints can be expressed as Equation 10, the graphics is shown in Fig.2.

$$\mu_A = \begin{cases} \frac{x - \underline{x}^L}{\underline{x}^\mu - \underline{x}^L} & \underline{x}^L \leq x \leq \underline{x}^\mu \\ 1 & \underline{x}^\mu \leq x \leq \overline{x}^L \\ \frac{\overline{x}^\mu - x}{\overline{x}^\mu - \overline{x}^L} & \overline{x}^L \leq x \leq \overline{x}^\mu \\ 0 & otherwise \end{cases} \tag{10}$$

Considering comprehensively the various factors influencing the value of the optimal level, which also include factor levels, factor ambiguity, as well as the degree of influence of factors on different levels of the value, the multi-level fuzzy comprehensive evaluation method is used. A set of standard values is tried to use to determine the optimal level of value on the scope of each fuzzy constraints. This method is more objective reflection, which is about more useful value of various factors on the optimal levels of vague

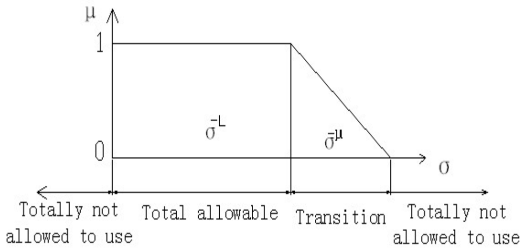

Fig. 1. Membership function of the performance constraints

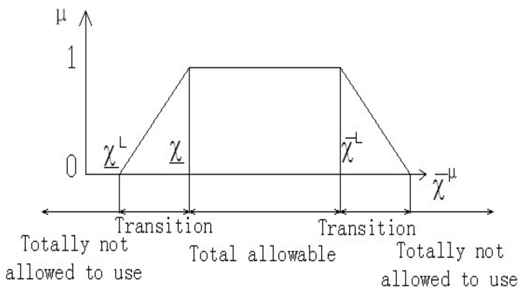

Fig. 2. Membership function of the geometric constraints

values. The tolerance coefficient of 0.275 is obtained, because of bringing the fuzzy optimal concept of tolerance factor and taking multi-level fuzzy optimal decision-making method, which is about the greatest impacting on the optimization of the calculation. Within the scope of the membership functions based on the decomposition of fuzzy sets. A series of λ values($\lambda \in [0,1]$)are used to intercept fuzzy sets, and then get different fortification levels about level set of λ. Thus fuzzy optimization will convert into a conventional optimization problem. What is the best optimal level of λ^* value to take. Based on the above a variety of fuzzy factors affecting, two fuzzy comprehensive evaluation method are used to determine λ^* value [7].

3 Fuzzy Optimization of Schwedler Reticulated Shell Structures

When the Schwedler shell is at a certain long span, a different radius of curvature of spherical shell are selected, which is defined as R; the number of nodes in each ring is n_1 (18,20,24);the number of laps of shell is $n_2(6 \leq n_2)$; the thickness of the shell is h; then calculate and analyze examples of fuzzy optimization. Schwedler shell calculation model is in the Fig.3, provided that conditions is hinged around. Roof loads is $\rho = 7800KN/m^3$, elastic modulus of steel is E=210GPa, cost factors are C_1=2800 Yuan/t, C_2=4800 Yuan/t.

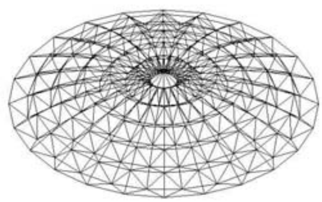

Fig. 3. Calculation model of Schwedler shell

3.1 Fuzzy Optimal Calculation and Analysis' Steps

Two fuzzy comprehensive evaluation methods are used to determine the optimal level of value of λ*=0.7198.

Determine the tolerance factor(The tolerance factor is determined by using Fuzzy dynamic programming method of decision-making to determine the boundary constraint tolerance factor).

(1) The Establishment of the Weight Factor Set and the Factors Set

Coefficient of impacting tolerance factors contain design level, manufacturing standards, material quality, the importance of the structural, roofing materials and construction, conditions of use. According to the priority principle of complementarity, the relationship between binary comparison matrix is prior to establish, which can get 6×6 fuzzy preference relation matrix. The normalized factor of weight set can be received:

$$W = \{0.2778 \quad 0.2775 \quad 0.0533 \quad 0.2389 \quad 0.1944 \quad 0.0275\}$$

(2) The Establishment of the Selected Set

The preferred target is tolerance factor, and according to designing conditions to determine their choice set:

$$V = \{0.35 \quad 0.325 \quad 0.3 \quad 0.275 \quad 0.25 \quad 0.2 \quad 0.175 \quad 0.15 \quad 0.125 \quad 0.1\}$$

(3) Impacting Factor Set, Veteran Factors and Membership Degree of Fuzzy Operator

Table 1. Membership degree of fuzzy operato

Impacting factor set	1	2	3	4	5
u_1 design level	1.0	0.8	0.6	0.2	0
u_2 manufacturing standards	0.8	1.0	0.6	0.2	0
u_3 material quality	0.8	1.0	0.6	0.2	0
u_4 the importance of the structural	0.2	0.4	0.6	0.8	1.0
u_5 roofing materials	0.7	1.0	0.8	0.6	0.4
u_6 conditions of use	0.8	1.0	0.6	0.2	0

(4) A Single Fuzzy Optimization

According to the Table 1 about the weight of available factors set, an example can be described by using design level u_1:

$$W_1 = \{0.38462 \quad 0.30769 \quad 0.23077 \quad 0.07692 \quad 0.00000\}$$

And getting a set of fuzzy optimization

$$B_1 = \{0.5231 \quad 0.6692 \quad 0.7462 \quad 0.7615 \quad 0.7154 \quad 0.6152$$
$$0.4692 \quad 0.3308 \quad 0.2154 \quad 0.1231 \quad 0.0615\}$$

(5) Two Multi-Factor Fuzzy Optimization

Repeating the above steps, you can get another five row vector, thus two fuzzy optimization can be obtained:

$$B = \{0.4243 \quad 0.5588 \quad 0.6506 \quad 0.7030 \quad 0.7004 \quad 0.6443$$
$$0.3561 \quad 0.4206 \quad 0.3082 \quad 0.2146 \quad 0.1520\}$$

(6) Decision and Analysis of Fuzzy Optimization

According to the principle of maximum degree of membership of fuzzy optimization, choose the the greatest judge of indicators 0.7030, which corresponding with 0.275 in concentration of poor decision-making factor, as a base for searching method of fuzzy boundaries. Thus the membership function of the performance constraints is obtain, which is shown as Fig.4, and the membership function of geometric constraints, which is shown as Fig.5.

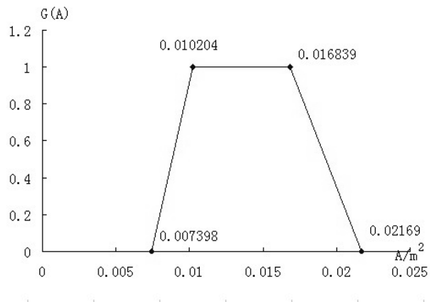

Fig. 4. The membership function of the performance constraints

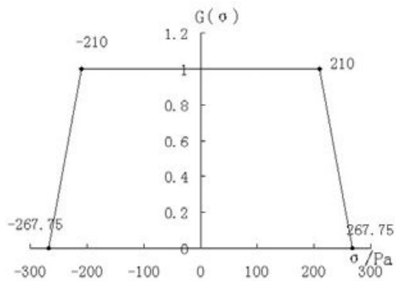

Fig. 5. The membership function of geometric constraints

(7) The Boundary Search Method

The different levels of the value of μ, which of fuzzy constraint set, are taken. We can find the fuzzy optimal solution family $W(A^*_\lambda)$, which is a monotone increasing function about $\lambda \in [0, 1]$. The initial areas such as the membership function is shown in Table 2. Supposing M and m respectively indicate the supremum and infimum, and we can know:

$$M = W(A^*_1) \qquad m = W(A^*_0) \tag{11}$$

And thus the availability of the membership function of fuzzy goals set:

$$\mu_w = (\frac{m}{W(A)})^n \tag{12}$$

n is the relaxation factor, which can make the μ^l_w to reach about 0.5.
Then making μ^u_w and μ^l_w be the boundary, and searching $\mu_w(A)$, which is made to meet:

$$\lambda^{(k)} - \mu(A^*_{\lambda^{(k)}}) \leq \varepsilon \qquad \varepsilon = 0.01 \tag{13}$$

Table 2. Tolerance factor and the scope of initial value

r_d	m	M	μ_w^l	μ_w^u	ε
0.275	6.49116e5	8.37584e5	0.6006	1.0000	0.01

(8)Non-Fuzzy Optimization Model

The membership function of the equation (9) and (10) are equal to λ^*, then the corresponding optimal level about non-fuzzy optimization model for:

$$\begin{array}{ll} \text{slove } x=(A_i) & \text{(i=1,2,...,n)} \\ & \min \text{W(x)} \\ \text{s.t.} & \|\sigma(s)\| \leq 224710 \\ & 0.00949 \leq A_i \leq 0.01802 \end{array} \tag{14}$$

So the fuzzy optimization model is transformed into the conventional optimization model of the optimal level set. Common optimal method is used to solve the problem. The calculated results is compared with the results of traditional optimization.

Table 3. The cost of common optimization and traditional optimization

	Node /individuals	A_1 $/m^2$	A_2 $/m^2$	A_3-A_6 $/m^2$	Weight /kg	Cost /million
General optimization	24	0.14655e-1	0.11088e-1	0.10204e-1	2.9772e5	0.837584
Fuzzy optimization	24	0.14396e-1	0.10480e-1	0.94900e-2	2.7684e5	0.779120

From the Table 3, we can see: Considering the ambiguity about the optimal design parameters of double-layer reticulated shell, the cost can be reduced by more than 5% compared to the traditional cost.

Under this approach, when the network shell owe the same span, a different number of nodes are selected to carry on normal optimization and fuzzy optimization, comparative analysis of their results. The comparison with the results is shown in Table 4.

From the Table 4, we can see: when the load and rise of arch are identified, the cost of the shell is reduced as the reduction of nodes' numbers, but the optimized cost is compared to the cost of traditional optimization, the decrease is almost equal about to 7%.

Table 4. The cost of the fuzzy optimization and the cost of the traditional optimization of the different nodes

Nodes /individuals	Shell load $/KN * m^{-2}$	Vector High /m	The cost of traditional optimization /million	The cost of fuzzy optimization /million	The proportion of cost reduction %
24	8	1.5	0.837584	0.779120	6.98
20	8	1.5	0.756675	0.704035	6.96
18	8	1.5	0.716528	0.666604	6.97

4 Conclusion

(1) On the basis of taking full account of ambiguity that is about optimization of design parameters of double-layer reticulated shell, the mathematical model of fuzzy optimal design is established. According to characteristics of double-layer shell, tolerance factor about the boundary constraints determined by using fuzzy optimal method is reasonable and scientific.
(2) On the basis of taking full account of ambiguity about optimization of design parameters of double-layer reticulated shell,the cost we got is less than than that of the traditional optimization. When the constraints are relaxed, the cost of the structure is reduced more than 5%.
(3) The total cost of the structure is the objective function. In the analysis of fuzzy optimization, pole pieces of shell are considered, consumption of nodes in steel and the other factors have an effect on the total cost.

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Fuzzy Synthetic Evaluation of Gas Station Safety

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Abstract. Based on the comprehensive analysis of hazard factors and evaluation indexes in gas stations, gas station safety is assessed in a fuzzy synthetic evaluation approach using the analytic hierarchy process, fuzzy transformation theory and the principle of maximum degree of membership. Through comprehensive evaluation, the specific safety level of gas stations could be determined. The results show that the fuzzy synthetic evaluation approach takes various factors that affect the system into account and gets a better result. Based on the above research conclusion,which provides a theoretic and practic base for other enterprises in the safey assessment.

Keywords: Analytic hierarchy process, gas station, fuzzy evaluation, maximum degree of membership.

1 Introduction

With the rapid development of China's economy, the continuous improvement of transport infrastructure and a rapid increase in motor vehicle traffic, gas stations have become an indispensable part of people's lives. Since a large number of gasoline and diesel is in storage or on sale during the operating process, gas stations have the hazard potential for a large fire, explosion or poisoning risk[2]. The gas station is a complex system constituted by a number of factors. There are many fuzzy transition between the system safety states and two neighboring states don’t have strict boundary. Therefore, fuzzy mathematics is used to evaluate synthetically the safety states of gas stations to obtain better results.

2 Mathematical Basis of Fuzzy Synthetic Evaluation

Fuzzy synthetic evaluation is the application of fuzzy transformation theory and principle of maximum degree of membership, which considers the various factors that are related to objects under evaluation and does a synthetic judgement [4,5].

Fuzzy synthetic evaluation is divided into one level fuzzy synthetic evaluation and multi-level synthetic evaluation, which includes six basic elements:

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A. Establish the factor domain of objects under evaluation: U ={u_1, u_2,..., u_m}

Namely the system of evaluation factors should be determined first, which solves the problem what factors are used and from which aspects to evaluate the realistic object.

B. Determine the level domain of comments: V ={v_1, v_2,..., v_n}

To determine this domain will get a fuzzy evaluation vector for fuzzy synthetic evaluation. Information about degrees of membership of objects under evaluation against all comment level is expressed by this fuzzy domain, which refects the fuzzy characteristic of evaluation. After such processing, the results obtained facilitate further calculation of contrast index of membership degree.

C. Make single factor evaluations for the single factor in u, ui (i=1,2, …, m).

Determine the membership degree of this object against level V from the perspective of ui, (i=1, 2, …, n) and get the single factor evaluation set for ui:

$$r_i = \left(r_{i1,} r_{i2,} \dots r_{in}\right) \tag{1}$$

D. It is a subset of Domain V of comment levels, thus, n objects under evaluation will constitute a general fuzzy relation matrix $\tilde{R}$.

$$\tilde{R} = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix} \tag{2}$$

Determine the weight vector of evaluation factors.

The weight vector shows membership ralations of all factors against the object under evaluation, which depends on the focus when people make fuzzy synthetic evaluation, namely what factors are more important in succession.

Since the importance of every factor in factor set U is defferent for the object under evaluation, we attach a defferent weight to every factor using the fuzzy measure. It could be denoted as a fuzzy subset $\tilde{A}$ = a_1 a_2 ... a_n in Set U and we prescribe that $\sum_{i=1}^{n} a_i = 1$ ai≥0 (i=1,2,…,n)

Choose the composition operator and make a synthetic evaluation.

The basic model of fuzzy synthetic evalution could be expressed as:

$$\tilde{B} = \tilde{A} \circ \tilde{R} \tag{3}$$

where o stands for a composition operator. We choose M $\cdot$ + as the caculation model. $\widetilde{B}$ is the first level set of synthetic evaluation, denoted as $\widetilde{B} = \; b_1,b_2,\ldots,b_m$, which is a fuzzy subset in comment set v. If we evaluate the result synthetically:

$$\sum_{i=1}^{m} b_i \neq 1 \tag{4}$$

It should be normalized.

3 Fuzzy Synthetic Evaluation of Gas Station

A. Determine the set of evaluation factors

According to established principles, safety evaluation is made for the gas station. We analysis relations between various factors and factors' function from the four elements of man - machine - environment – management to determine the evaluation system of gas station, as shown in Fig. 1[3].

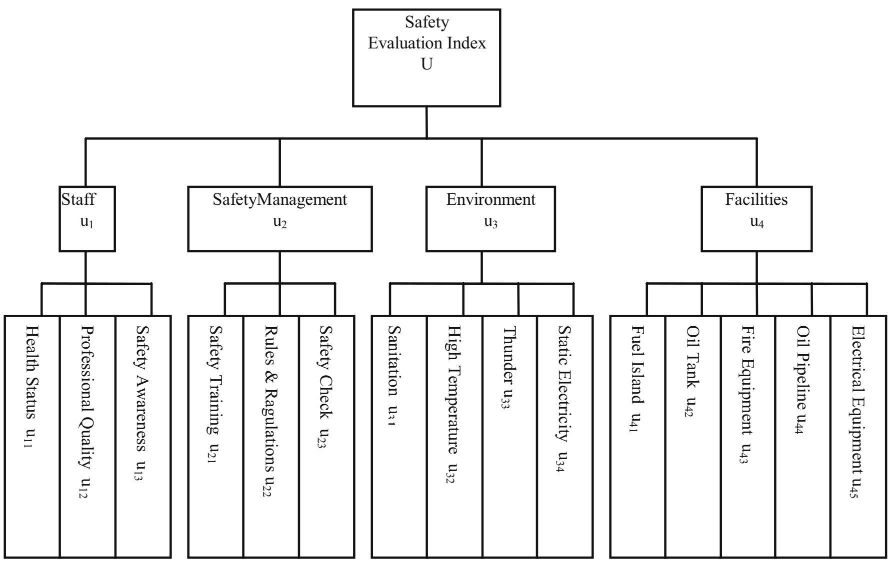

Fig. 1. The diagram of gas station safe quality synthetic evaluation factor architeture

B. Determine the domain of comment levels

After investigation of the gas station, safety state of the gas station is divided into five levels:

v={ safe, roughly safe, ordinary, roughly dangerous, dangerous}

C. Determine the weight of each factor and the set of single factor evaluation

Weight is a relative concept. The weight of each factor in the system of evaluation factors is its relative importance for realising the evaluation goal and function. The reasonableness of determining weight will affect the evaluation results directly. There are many ways to determine weights and this paper uses the analytic hierarchy process. Each factor is evaluated mainly by the use of expert scoring and its result is shown in Table 1.

Table 1. Table of various factors weight value and appraisal result

First Level Evaluation Index & Weight	Second level Evaluation Index	Weight	Safe	Roughly Safe	Ordinary	Roughly Dangerous	Dangerous
Staff u_1 (0.26)	u11	0.32	0.10	0.37	0.35	0.08	0.10
	u12	0.12	0.46	0.25	0.08	0.14	0.07
	u13	0.56	0.22	0.31	0.28	0.10	0.09
Safety Management u_2 (0.06)	u21	0.26	0.11	0.32	0.33	0.15	0.09
	u22	0.47	0.30	0.50	0.20	0	0
	u23	0.27	0.27	0.40	0.27	0.05	0.01
Environment u_3 (0.12)	u31	0.05	0.30	0.50	0.10	0.07	0.03
	u32	0.107	0.21	0.23	0.12	0.22	0.22
	u33	0.35	0.23	0.22	0.15	0.20	0.20
	u34	0.493	0.12	0.17	0.24	0.22	0.25
Facilities u_4 (0.56)	u41	0.045	0.12	0.24	0.30	0.16	0.18
	u42	0.465	0.10	0.22	0.56	0.06	0.06
	u43	0.149	0.20	0.35	0.26	0.12	0.07
	u44	0.257	0.41	0.42	0.11	0.06	0
	u45	0.084	0.33	0.17	0.21	0.14	0.15

D. Establish the evaluation matrix

The first level of evaluation

Staff factor u_1

$$\tilde{B}_1^{(1)} = \tilde{A}_1^{(1)} \circ \tilde{R}_1^{(1)}$$

$$= (0.32 \quad 0.12 \quad 0.56) \circ \begin{bmatrix} 010 & 0.37 & 0.35 & 0.08 & 0.10 \\ 0.46 & 0.25 & 0.08 & 0.14 & 0.07 \\ 0.22 & 0.31 & 0.28 & 0.10 & 0.09 \end{bmatrix}$$

$$= (0.21 \quad 0.32 \quad 0.28 \quad 0.10 \quad 0.09)$$

Caculation in the following is similar with the above:

Safety management factor u_2

$$\tilde{B}_2^{(1)} = \tilde{A}_2^{(1)} \circ \tilde{R}_2^{(1)} = (0.24 \quad 0.41 \quad 0.25 \quad 0.05 \quad 0.05)$$

Sanitation factor u_3

$$\tilde{B}_3^{(1)} = \tilde{A}_3^{(1)} \circ \tilde{R}_3^{(1)} = (0.18 \quad 0.21 \quad 0.19 \quad 0.20 \quad 0.22)$$

Facility factor u_4

$$\tilde{B}_4^{(1)} = \tilde{A}_4^{(1)} \circ \tilde{R}_4^{(1)} = (0.21 \quad 0.29 \quad 0.36 \quad 0.08 \quad 0.06)$$

The second level of fuzzy synthetic evaluation

Weight distribution of ui(i=1, 2, 3, 4) in V is already known as:

$$\tilde{A}^{(2)} = (0.26 \quad 0.56 \quad 0.06 \quad 0.12 \quad)$$

The second level evaculation matrix $\tilde{R}^{(2)}$ is determined by $\tilde{B}_1^{(1)}$ $\tilde{B}_2^{(1)}$ $\tilde{B}_3^{(1)}$ $\tilde{B}_4^{(1)}$.

$$\tilde{R}^{(2)} = \begin{pmatrix} 0.21 & 0.32 & 0.28 & 0.10 & 0.09 \\ 0.24 & 0.41 & 0.25 & 0.05 & 0.05 \\ 0.18 & 0.21 & 0.19 & 0.20 & 0.22 \\ 0.21 & 0.29 & 0.36 & 0.08 & 0.06 \end{pmatrix}$$

$$\tilde{B}^{(2)} = \tilde{A}^{(2)} \circ \tilde{R}^{(2)} = (0.23 \quad 0.36 \quad 0.26 \quad 0.08 \quad 0.07)$$

According to the principle of maximum degree of membership, the actual state of gas station safety is Level 2.

4 Conclusions and Recommendations

The AHP-Fuzzy evaculation model is established using the AHP and fuzzy mathematics method [1], which takes into accout all various factors that affect system safety and fully reflects the inherent ambiguity of evaluation factors and process. It makes full use of expert resources and the existing information to enable the results more reliable.

Through the evaculation, we understand clearly that the gas station is roughly safe. If we want the gas station to reach the safe level, appropriate safety measures should be carried out from aspects of man – machine – environment and daily safety management be strengthened also.

5 Conclusion

A kind of fuzzy decision-making fusion model provided in this paper can improve the accuracy of fault recognition through multi-attribute decision-making fusion. The difference of membership degree between each kind of faults was obvious. The validity and effective of the model was verified by the fault diagnosis example of pulverizing system. It can foresee fault tendency which has certain significance to the early period fault diagnosis.

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A Linear Fuzzy Comprehensive Assessment Model with Prominent Impact Factor

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Abstract. In the fuzzy comprehensive assessment method, the used linear weighted average model can not reflect the prominent impact of the indicators. And the nonlinear fuzzy comprehensive assessment model has a complex form and a large amount of calculation. In order to solve this problem in fuzzy comprehensive assessment method, this paper proposes a linear fuzzy comprehensive assessment model with prominent impact factor. Put this model into practical examples, this paper obtained a more convincing result than the weighted average method. And the result is consistent with the result of the nonlinear fuzzy comprehensive assessment, and also reduced the calculation amount.

Keywords: Fuzzy comprehensive assessment method, weighted average method, prominent impact factor.

1 Introduction

Fuzzy comprehensive assessment method is an assessment method based on Zadeh fuzzy set theory, a fuzzy transform process induced by a fuzzy mapping, has been widely used at present. Right now the fuzzy comprehensive assessment model M(,+) is used commonly. Although this model retained all information of each individual factor, it is actually a weighted assessment method.

The shortage of weighted assessment method is its cannot reflect the prominent impact of some evaluation indicators. The prominent impact of the indicators is the influence that the indicators affect on the assessment results, and it can not completely embody only rely on increasing weight. For example, when an approximation has a high indicators and other indicators relatively low, it can be thought to be good or bad in actual circumstances. However, by using weighted average method, the prominent impact of this indicators can not be reflected, which makes the assessment results not be accord with the actual.

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2 Fuzzy Matrix Synthesis Operator

In fuzzy comprehensive assessment model

$$\mathbf{B} = \mathbf{A} \cdot \mathbf{R} = (\alpha_1, \alpha_2, \cdots, \alpha_3) \begin{pmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1n} \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \gamma_{n1} & \gamma_{n2} & \cdots & \gamma_{nn} \end{pmatrix} = (b_1, b_2, \cdots, b_n),$$

define fuzzy matrix synthesis operator

$$f(\alpha_1, \alpha_2, \cdots, \alpha_n; x_1, x_2, \cdots, x_n; \triangle) = \lambda_1\alpha_1 x_1 + \lambda_2\alpha_2 x_2 + \cdots + \lambda_n\alpha_n x_n,$$

where $\triangle = (\lambda_1, \lambda_2, \cdots, \lambda_n)$, and λ_t expresses the prominent impact degree which indicators affect on the assessment results, and $\lambda_i > 1, i = 1, 2, \cdots, n$. The bigger the prominent impact degree is, the value λ_i is. When indicators has no prominent impact, take λ_i to 1. For convenience, take λ_i to integer which bigger than or eaqual to 1. $\mathbf{A} = (\alpha_1, \alpha_2, \cdots, \alpha_n)$ is the weight set, in which $\alpha_i \geq 0$, $\sum_{i=1}^{n} \alpha_i = 1$.

$$\mathbf{R} = \begin{pmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1n} \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \gamma_{n1} & \gamma_{n2} & \cdots & \gamma_{nn} \end{pmatrix}$$

is the fuzzy relationship matrix.

The fuzzy matrix synthesis operator has the following properties:

1. When the prominent impact of each indicators is different, namely, some indicators has greater prominent impact degree, and some has less or no prominent impact degree, the assessment results should be different with the same indicators when all the indicators have the same assessment value.
   Namely, when $x_1 = x_2 = \cdots = x_n = c$,
   $$f(\mathbf{A}; x_1, x_2, \cdots, x_n; \triangle) = c(\lambda_1\alpha_1 + \lambda_2\alpha_2 + \cdots + \lambda_n\alpha_n) \neq c,$$
   which reflects the influence that the prominent impact has on the assessment results.
2. If $x_i < x'_i$, then
   $$f(\mathbf{A}; x_1, x_2, \cdots, x_n; \triangle) < f(\mathbf{A}; x'_1, x'_2, \cdots, x'_n; \triangle).$$
   This property can be obtained by the monotone increasing sex of function. $f(\mathbf{A}; x_1, x_2, \cdots, x_n; \triangle)$. It shows when all the assessment indicators value of an approximation are greater than that of another approximation, the assessment results of the former should be greater than that of the latter.

3. $\lim_{x_i \to x_i'} f(A; x_1, x_2, \cdots, x_n; \triangle) = f(A; x_1', x_2', \cdots, x_n'; \triangle)$. This property can be obtained by the continuity of the function $f(A; x_1, x_2, \cdots, x_n; \triangle)$. It shows that the increasing of each assessment indicators can make the final assessment indicators value increasing, but the increasing should be continuous and smooth, can not have suddenly jump.

It can be seen from the above three properties that the fuzzy matrix synthetic operator is accord with the actual assessment and can overcome the defects of the linear weighting operator, so it has the evaluation advantage.

3 Examples

When do an assessment for a soil environmental quality, take Cd, As, Cu, Cr and BHC these five indicators as assessment factors, then factors concerning domain is

$$\mathbf{U} = (Cd, As, Cu, Cr, BHC).$$

In the above five indicators, BHC belongs to poisonous substances, and have great influence on the soil. The concentration of the five indicators for the soil are as blow: Cd is 0.18mg/kg, As is 9.8mg/kg, Cu is 34mg/kg, Cr is 100mg/kg and BHC is 0.75mg/kg.

It can be obtained from reference [3] that the grade change range of some indicators which have influence on the soil, which are shown in Table 1.

Table 1. The soil environmental quality grading standards

	I class (clean)	II class (yet clean)	III class (light pollution)	IV class (pollution)
Cd(mg/kg)	0.13	0.2	0.6	≥ 1.0
As(mg/kg)	9.8	15	25	≥ 30
Cu(mg/kg)	26.9	35	100	≥ 250
Cr(mg/kg)	61.8	90	250	≥ 300
BHC(mg/kg)	0.05	0.05	0.5	≥ 1.0

From these pollutants concentration, only the concentration of BHC is very high which caused serious damage to the soil, and other pollutants concentration are relatively low, basically in each single indicator Level II, even Level I, Tectonic membership functions $\widetilde{H}(x), \widetilde{J}(x), \widetilde{K}(x)$, and $\widetilde{L}(x)$ for each indicators. It can be obtained form these membership functions the membership degree for different indicators to different soil level. After given the value for each indicators of the soil, take them to the corresponding membership function, then can get the fuzzy relation matrix R.

$$\begin{array}{r} \\ Cd \\ As \\ Cu \\ Cr \\ BHC \end{array} \begin{array}{c} \begin{array}{llll} \widetilde{H} & \widetilde{J} & \widetilde{K} & \widetilde{L} \end{array} \\ \begin{pmatrix} 0.286 & 0.714 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0.1235 & 0.8765 & 0 & 0 \\ 0 & 0.9375 & 0.0625 & 0 \\ 0 & 0 & 0.5 & 0.5 \end{pmatrix} \end{array}$$

The weighted formula is $\omega_i = \frac{x_i}{\tilde{s_i}} (i = 1, 2, 3, 4, 5)$, where i expresses each assessment factor, x_i expresses measured value of some indicator, $\tilde{s_i}$ expresses the average of grading standard for one indicator. Generally it will take the normalized processing for ω_i, set

$$a_i = \frac{\omega_i}{\sum_{i=1}^{5} \omega_i}$$

then the fuzzy weight vector is

$$A = (0.1025, 0.1349, 0.0907, 0.1566, 0.5153).$$

In the above five indicators, BHC belongs to poisonous substances, and have great influence on the soil, namely, it has greater prominent impact degree on the assessment results. But other indicators' prominent impact degree is relatively not very obvious. This is the overall level. If consider each kind of pollutants lonely, they have different prominent impact on assessment results. Here take the determination method of the prominent impact degree of pollutants in water evaluation in reference [2], to calculate the prominent impact degree of pollutants in soil.

First divide the pollutants into several categories according to the toxicity, such as poisonous contaminant and conventional pollutants in the above examples. Determine the basic prominent impact degree $\lambda^{(k)}$ for different category pollutants. For any one pollutant, set its prominent impact degree as $\lambda_i = \lambda^{(k)} \cdot \frac{x_i}{\tilde{s_i}}$, normally take it to integer.

In the above example, $k = 1, 2$. For BHC this poisonous contaminant, it has great prominent impact degree on the soil environmental quality assessment results, so take $\lambda^{(1)}$ to 10. But for conventional pollutants, they have relatively lower prominent impact degree on the assessment results, so take $\lambda^{(2)}$ to 2. Then the prominent impact degree of different pollutants can be determined according to the above method. For convenience, take all of them to integers, namely, Cd is 1, As is 1, Cu is 1, Cr is 1 and BHC is 19.

Using the fuzzy comprehensive assessment method proposed by this paper, the assessment for the soil is

$$\begin{aligned} \mathbf{B} &= \mathbf{A} \cdot \mathbf{R} \\ &= (0.1025, 0.1349, 0.0907, 0.1566, 0.5153) \cdot \begin{pmatrix} 0.286 & 0.714 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0.1235 & 0.8765 & 0 & 0 \\ 0 & 0.9375 & 0.0625 & 0 \\ 0 & 0 & 0.5 & 0.5 \end{pmatrix} \end{aligned}$$

$$= (0.1754, 0.2995, 4.9051, 4.8953)$$

According to the maximum subjection principle, the soil environmental quality should be class III. The assessment results is actual.

If using the nonlinear fuzzy comprehensive assessment model in reference [2], firstly should take the transformation $r`_{ij} = 2^{r_i}$ to handle the original fuzzy relation matrix $\mathbf{R}$, then

$$\mathbf{R}` = \begin{pmatrix} 1.22 & 1.64 & 1 & 1 \\ 2 & 1 & 1 & 1 \\ 1.09 & 1.84 & 1 & 1 \\ 1 & 1.92 & 1.04 & 1 \\ 1 & 1 & 1.41 & 1.41 \end{pmatrix}$$

and the assessment results is

$$\begin{aligned} \mathbf{B} &= \mathbf{A} \cdot \mathbf{R}` \\ &= (0.1025, 0.1349, 0.0907, 0.1566, 0.5153) \cdot \begin{pmatrix} 1.22 & 1.64 & 1 & 1 \\ 2 & 1 & 1 & 1 \\ 1.09 & 1.84 & 1 & 1 \\ 1 & 1.92 & 1.04 & 1 \\ 1 & 1 & 1.41 & 1.41 \end{pmatrix} \\ &= (1.0081, 1.0133, 1.361746, 1.361745). \end{aligned}$$

According to the maximum subjection principle, the soil environmental quality should be class III.

This is the same as the assessment results using the method proposed by this paper, and the method proposed by this paper has one less steps. It simplifies the calculation procedure, and has more convenient use. And the conclusion is consistent with the reality.

If using the weighted average method, the assessment results is

$$\begin{aligned} \mathbf{B} &= \mathbf{A} \cdot \mathbf{R} \\ &= (0.1025, 0.1349, 0.0907, 0.1566, 0.5153) \cdot \begin{pmatrix} 0.286 & 0.714 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0.1235 & 0.8765 & 0 & 0 \\ 0 & 0.9375 & 0.0625 & 0 \\ 0 & 0 & 0.5 & 0.5 \end{pmatrix} \\ &= (0.1754, 0.2995, 0.2674, 0.2576). \end{aligned}$$

According to the maximum subjection principle, the soil environmental quality should be class II.

The concentration of BHC in this soil is higher and has have serious damage on the soil. Although the concentration of other pollutants are relatively low, even very low, it can not compensate the damage which BHC to the soil. So the assessment results obtained by the weighted average method is not reasonable, and the assessment results got from this paper is convincing.

4 Conclusion

This paper put forward a new kind of linear fuzzy comprehensive assessment model with prominent impact factor, expounded the feasibility of the model, and verified by examples. This model offset the insufficient of the weighted average model, and also can put the weighted average model as an exception of this model. This model have easier form than the nonlinear model in reference [2], and don't have to transform the fuzzy relation matrix, and have more convenience and superiority in using.

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Fuzzy Comprehensive Evaluation Applied in Tourism Enterprises Core Competition*

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Abstract. In the fiercer market competition,the hospitality's core competition determines the enterprises' survival. It is also the key to overcome rivals and obtain sustainable development. The paper uses the theory of fuzzy comprehensive evaluation to construct the system of core competition index assessment. Then from the angles of quantitative analysis and qualitative analysis, it makes a comprehensive evaluation on the hospitality's core competition, which provides the theoretical basis for the effective management strategies and promotes its sustainable development power.

Keywords: Core competition, hospitality, fuzzy comprehensive evaluation.

1 Introduction

International tourism enterprise groups are aggressively entering in the Chinese market, with China's accession to WTO, as well as economic globalization development in-depth, such as the 10 giant multinational hotel groups have entered in Chinese market. At the same time, internal competition in China tourism industry is more intense. The only strategic choice of Chinese tourism enterprises is to cultivate and enhance core competitiveness facing with tough market competition and international environment. In this case, it is a major problem worth concerning about how to evaluate their core competition scientifically, objectively and effectively. And it is also important for Chinese tourism enterprises to guide management & decision to change competitive position according to core competition evaluation results. Tourism

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enterprises core competition is not overall competitiveness. Core means "the most critical and most important." Core competencies are unique, hard to imitative and valuable. There are many factors affecting core competition of tourism enterprises. Moreover, many factors are defined and measured fuzzily, so tourism enterprises core competition evaluation is a complex issue. Fuzzy comprehensive evaluation method based on fuzzy mathematics is very suitable for evaluation of non-accuracy indicators. Its evaluation results on core competition of tourism enterprises can help managers understand core competition and the most competitive factors objectively and correctly, which is conducted from a combination of qualitative and quantitative point of views. In this paper, core competition evaluation index systems and fuzzy comprehensive evaluation models are established on fuzzy comprehensive evaluation method for tourist attractions and hospitality.

2 Methods and Models

2.1 Tourism Enterprise Core Competitiveness Evaluation Index System

This article built up tourism enterprise core competitiveness evaluation index system according to tourism suggestions of scholars who have in-depth understanding of tourism industry and know tourism industry very well. The first-class, second-class indicators and the third-class indicators of scenic spot core competitiveness evaluation index system are shown in Table 1. $U1$ is the goal of the scenic spot tourism core competitiveness evaluation index system. And indicators of hospitality core competitiveness evaluation index system are shown in Table 2. $U2$ is the goal of hospitality tourism core competitiveness evaluation index system.

2.2 Indicator Weights

Indicators' weight is determined by the method of AHP based on a questionnaire survey. Respondents in the questionnaire survey are experienced tour business managers or professional tourism scholars. The number of questionnaires on scenic spot tourism core competitiveness evaluation index system is 17 and 17 valid questionnaires were recovered. So the valid recovery rate was 100%. The final results of indicators' weight are shown in Table 1. And the number of questionnaires on hospitality core competitiveness evaluation index system is 42 and 38 valid questionnaires were recovered. So the valid recovery rate was 90%. The final results of indicators' weight are shown in Table 2.

Table 1. Scenic spot tourism core competitiveness evaluation indicators and their weights

Goal	First-class indicators	Second-class indicators	Third-class indicators
U_1 1.00	u_{11} resource capacity 0.50	u_{111} material resources 0.60	u_{1111} scarcity degree and aesthetic value of main natural resources 0.50
			u_{1112} ecological recovery ability of main natural resources 0.20
			u_{1113} well facilities 0.30
		u_{112} Brand and knowledge resources0.40	u_{1121} reputation 0.40
			u_{1122} ranking in national scenic areas hierarchy 0.30
			u_{1123} reasonable human resource allocation and planning 0.30
	u_{12} professional competence 0.30	u_{121} product development capability0.30	u_{1211} scenic richness of tourism products 0.60
			u_{1212} product innovation ability 0.40
		u_{122} Travel services capability0.30	u_{1221} ability to provide and innovate experience product for visitors 0.30
			u_{1222} visitor satisfaction 0.30
			u_{1223} employee satisfaction 0.20
			u_{1224} scenic staff understanding of the service concept 0.20
		u_{123} market capability0.20	u_{1231} market share 0.30
			u_{1232} ability to develop new markets 0.20
		u_{124} environmental conservation capacity0.20	u_{1241} reasonable spot development a degree 0.20
			u_{1242} environment protection proportion of outputs 0.20
	u_{13} comprehensive ability 0.20	u_{131} organization and management0.40	u_{1311} forward-looking and effectiveness of plans 0.50
			u_{1312} efficiency day-to-day management 0.50
		u_{132} ability of public relations and collaboration 0.30	u_{1321} smooth communication channels with distributors and suppliers 0.60
			u_{1322} ability to obtain industry's identity 0.40
		u_{133} culture synergy 0.30	u_{1331} understanding of managers and the staff on the corporate culture 0.60
			u_{1332} impact of corporate culture on enterprise development 0.40

2.3 Comprehensive Evaluation: The Fuzzy Comprehensive Evaluation Model

Core comprehensive competitiveness of scenic spot and hospitality are evaluated by the fuzzy comprehensive evaluation model. Evaluation steps of scenic spot core comprehensive competitiveness are as follows. And steps of hotel core competitiveness evaluation are simply to it.

Table 2. Hospitality core competitiveness evaluation indicators and their weights

Goal	First-class indicators	Second-class indicators	Third-class indicators
U_1 1.00	u_{21} non-material resources 0.10	u_{211} brand 0.40	u_{2111} reputation 0.40
			u_{2112} influence 0.30
			u_{2113} attraction 0.30
		u_{212} human resources 0.30	u_{2121} work experience of staff 0.40
			u_{2122} knowledge of staff 0.20
			u_{2123} loyalty to the enterprise 0.40
		u_{213}external resources 0.30	u_{2131} relationship with government departments 0.30
			u_{2132} relationships with suppliers 0.40
			u_{2133} relationship with peers 0.30
	u_{22} technical capacity 0.20	u_{221}facilities and equipment0.20	u_{2211} 0.6 advanced equipment 0.60
			u_{2212} 0.4 equipment update the speed0.40
		u_{222}information systems0.30	u_{2221}website 0.40
			u_{2222} computer usage 0.60
		u_{223} technical innovation 0.10	u_{2231} product design and development 0.50
			u_{2232} introduction speed of new technologies 0.20
			u_{2233} technology innovation investment 0.30
		u_{224} technicians0.40	u_{2241} work experience of technicians 0.40
			u_{2242} knowledge of technicians0.60
	u_{23} management ability0.30	u_{231} manager quality 0.40	u_{2311}adaptability0.30
			u_{2312} organizational coordination 0.30
			u_{2313} crisis management capacity 0.40
		u_{232}strategic decision -making 0.20	u_{2321} project planning 0.30
			u_{2322}investment decision-making capacity 0.30
			u_{2323} market predictability 0.40
		u_{233} management System 0.40	u_{2331} rational system 0.30
			u_{2332} flexibility of organization regulations 0.20
			u_{2333} enforcement efforts 0.50
	u_{24}*viability* 0.20	u_{241} marketing system 0.30	u_{2411} marketing techniques 0.40
			u_{2412} marketing network 0.60
		u_{242} financial income 0.30	u_{2421} main business profitability 0.40
			u_{2422} cost-benefit ratio 0.30
			u_{2423} return on equity 0.30
		u_{243}Development Potential 0.30	u_{2431} net asset growth 0.20
			u_{2432} revenue growth 0.30
			u_{2433} per capita income growth to achieve business 0.40
	u_{25} Hotel Culture 0.20	u_{251} Internal cohesion 0.20	u_{2511} recognition of hotel spirit philosophy 0.50
			u_{2512} employee satisfaction 0.50
		u_{252} Ability to learn 0.40	u_{2521} employee self-learning ability 0.30
			u_{2522} training arrangements 0.40
			u_{2523} service innovation awareness 0.30
		u_{253} Human-oriented care 0.40	u_{2531} constraint incentives0.60
			u_{2532} employee career planning and design 0.40

(1) To establish evaluation indicator matrixes U_{1,u_i} and u_{ij},

$$\mathbf{U_1} = \{\mathbf{u_{i1}}, \mathbf{u_{i2}}, \mathbf{u_{i3}}\}, \tag{1}$$

u_{i1},u_{i2},u_{i3} are indicators of the first-class.

$$\mathbf{U_{1i}} = \{\mathbf{u_{1i1}}, \mathbf{u_{1i2}}, \ldots, \mathbf{u_{1ij}}, \ldots, \mathbf{u_{1in}}\}, \tag{2}$$

u_{1ij} is an indicator in the second-class.

$$\mathbf{U_{1ij}} = \{\mathbf{u_{1ij1}}, \mathbf{u_{1ij2}}, \ldots, \mathbf{u_{1ijk}}, \ldots, \mathbf{u_{1ijp}}\}, \tag{3}$$

u_{1ijk} is an indicator in the third-class.

(2) To establish evaluation indicator weight collections A_1,A_{1i} and A_{1ij},

$$\mathbf{A_1} = \{\mathbf{a_{11}}, \mathbf{a_{12}}, \ldots, \mathbf{a_{1i}}, \ldots, \mathbf{a_{1m}}\}, \tag{4}$$

$\sum\limits_{i=1}^{m} a_{1i} = 1$,$a_{1i}$is the weight of indicator in the first-class.

$$\mathbf{A_{1i}} = \{\mathbf{a_{1i1}}, \mathbf{a_{1i2}}, \ldots, \mathbf{a_{1ij}}, \ldots, \mathbf{a_{1im}}\}, \tag{5}$$

$\sum\limits_{j=1}^{n} a_{1ij} = 1$,$a_{1ij}$is the weight of indicator in the second-class.

$$\mathbf{a_{1ij}} = \{\mathbf{a_{1ij1}}, \mathbf{a_{1ij2}}, \ldots, \mathbf{a_{1ijk}}, \ldots, \mathbf{a_{1ijp}}\}, \tag{6}$$

$\sum\limits_{k=1}^{p} a_{1ijp} = 1$,$a_{1ijk}$is the weight of indicator in the third-class.

(3) To establish the comment collection V, There are 5 comments $\{\mathbf{V_1}, \mathbf{V_2}, \ldots, \mathbf{V_i}, \ldots, \mathbf{V_n}\}$ in this paper.

$$\mathbf{V} = \{\mathbf{V_1}, \mathbf{V_2}, \ldots, \mathbf{V_i}, \ldots, \mathbf{V_n}\}, \tag{7}$$

The comment V_1, 9 means best. The comment V_2, 7 means good. The comment V_3, 5 means generally. The comment V_4, 3 means bad and the comment V_5, 1 means worst. (If 4 comments are adopted, the comments V_1,V_2,V_3,V_4 mean respectively excellent, good, moderate and poor, with corresponding values of 10, 8, 6 and 4.).

(4) To establish the evaluation matrix R_1 The evaluation matrix showed that single factor set of comments on the membership, if a target level of 000 comments, the evaluation matrix can be expressed

$$\mathbf{R_1} = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix}, \tag{8}$$

$r_{ij} = g_{ij}/g$ means that the possibility of the ith indicator in the jth grade in comment collection. $g$is the total number of respondents. g_{ij} means g_{ij} respondents considering the ith indicator were in the jth grade in comment collection.

(5) Multi-level fuzzy comprehensive evaluation It is necessary to get all indicator assessment scores start from the bottom layer, the third-class to get the final evaluation results of tourism core competitiveness of a scenic spot. And then it moved onto the second-class and the first-class.

$$S_1 = A_1 \circ R_1 * V^T,$$

A_1 is the weight collection. R_1 is the evaluation matrixV^T is the rank switch matrix of V.

3 Applications

3.1 Applied in Scenic Enterprise

Questionnaire survey method was also used in tourism core competitiveness evaluation of a scenic spot in Three Gorges area. Respondents in the questionnaire survey are experienced tour business managers or professional tourism scholars or officials in tourism administrative departments in Three Gorges area. The number of questionnaires is 30 and 28 valid questionnaires were recovered. So the valid recovery rate was 93.3%. The final tourism core competitiveness evaluation results are shown in Table 3. The scenic spot as the

Table 3. Tourism core competitiveness evaluation results of a scenic spot in three gorges area

Indicators	Best	Good	General	Bad	Worst	Comprehensive score
u_{11}resource capacity	0.153	0.253	0.434	0.097	0.080	5.66
u_{12} professional competence	0.091	0.284	0.494	0.153	0.080	5.82
u_{13} comprehensive ability	0.088	0.092	0.451	0.328	0.062	4.74
u_1 Tourism core competitiveness	0.12	0.23	0.46	0.16	0.08	5.5

object of study has beautiful natural scenery and rich charming cultural features, but such tourism resources are common, not monopolistic in Three Gorges area. This is inherent and difficult to change. Therefore, the way to upgrade its resource capacity is to improve facility, brand and knowledge resources. And as a developing scenic spot, professional capacity is a shortage to it, such as lacking of capital and professional talents. Unclear powers and responsibilities limited its comprehensive ability. So shortcomings in management system must be cleaned up to set up a harmonious and efficient regulatory agency.

3.2 Applied in Hospitality Enterprise

Questionnaire survey method was also used in tourism core competitiveness evaluation of a business hotel in Shanghai China. Respondents in the questionnaire survey are managers or experienced staff of this hotel. The number of questionnaires is 42 and 38 valid questionnaires were recovered. So the valid recovery rate was 90.3%. The final core competitiveness evaluation results are shown in Table 4.

Table 4. Tourism core competitiveness evaluation results of a business hotel

Indicators	Excellent	Good	Moderate	Poor	Comprehensive score
u_{21}Non-material resources	0.078	0.590	0.295	0.225	8.17
u_{22}Technical capacity	0.070	0.515	0.313	0.313	7.00
u_{23} Management ability	0.070	0.345	0.485	0.100	6.41
u_{24} Viability	0.163	0.288	0.460	0.080	7.01
u_{25} Hotel Culture	0.095	0.420	0.365	0.120	6.98
U_2 core competitiveness	0.094	0.409	0.402	0.097	7.0

According to evaluation results, we can know core competitiveness evaluation conclusion is between "good" and "moderate" with a score of 7.0, which is higher than 6(moderate) and lower than 8(good). Non-material resources with maximum value of 8.17 are an advantage of this hotel. Other four first-class indicators' scores all are higher than 6(moderate) and lower than 8(good).And we can know management ability is to be improved very ecessarily in the future with minimum value of6.41 and maximum weight of 0.30.

4 Conclusions and Discussion

Fuzzy mathematical comprehensive evaluation model is a simple method easy to operate and apply. Core competition evaluation results of tourism enterprises can reflect the actual situation of core competition truly and they are strong practice-oriented. First, the evaluation results are beneficial for tourism enterprise's core competitiveness recognition. And they can help tourism enterprises find real source of core competence of value the activities to carry out targeted training of core competencies. Secondly, advantages and disadvantages of tourism enterprises and their competitors can be clearly seen using fuzzy evaluation of quantitative models. Therefore, the evaluation results can also help tourism enterprises analyze and compare among core competition of their and their competitors. So the evaluation results can help them make sound business decisions in order to obtain long-term competitive advantage. Third, fuzzy comprehensive evaluation model on core competition of tourist enterprises is dynamic, not fixed. This is helpful for enterprises to observe dynamically about changes of core competition and predict future trends. Weights of all indicators in tourism enterprises core competition fuzzy comprehensive evaluation model are determined by experts, not fixed. The proportion of different factors can be modified, with the market environment. The value of each factor is also changing as business conditions changes. Therefore, core competition evaluation results of tourism enterprises are also a dynamic change.

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Non-optimum Analysis Approach to Incomplete Information Systems

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Abstract. Information systems are classified into two classes, namely complete information systems and incomplete information systems. A unified description has been proposed through the concrete analysis of information systems. And based on this conclusion, a new method of decision making has been presented. The theoretical basis of the method is non-optimum theory, and thereby the non-optimum theory is further perfect by introduced the concepts of middle-optimum degree and unknown-optimum degree. Then the entropy of information systems is defined. Meanwhile, the concept of influence degree has also introduced. Finally, it discusses about three decision making principles, namely maximum influence degree principle, minimum risk principle and maximum risk principle.

Keywords: Incomplete information systems, non-optimum, unknown-optimum, middle-optimum, relation degree, influence degree.

1 Introduction

In 1965, fuzzy mathematics was founded by L.A.Zadeh [1]. It can be obtained the tolerance relation and preorder relation for objectives, when all attribute have no a "null" value. There are four distinct types of methodologies in fuzzy multicriteria analysis [2-8]. An investigation of existing methods in fuzzy multicriteria analysis shows that most existing methods suffer various drawbacks [5]. The questions are caused by: Membership degree of calculation has loosed its originality and intuitionistic meaning; the smaller classification can not be compared. The reason of above questions is that the structure of membership function is random (dissatisfied addition); numbers of median information is loosed by maximum and minimum calculation.

In real life today, the key point of the tolerance relation or similarity relation presented in the literature is to assign a "null" value to all missing attribute values. This may cause a serious effect in data analysis and decision analysis because the missing values are just "missed" but they do exist and have an influence on the

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decision. Because it can not be obtained the tolerance relation and preorder relation for objectives, fuzzy theory could do nothing about it.

Non-optimum analyses theory proposed by He Ping[9] is an effective approach to imprecision, vagueness, and uncertainty, which thought about three aspects of information among optimum, non-optimum and sub-optimum at the same time [9-12]. Non-optimum analyses theory overlaps with many other theories such that fuzzy sets, evidence theory, and statistics. From a practical point of view, it is a good tool for data analysis.

This paper is organized as follows. Section 2 presents a unified form of complete information systems and incomplete information systems. Section 3 further perfects the non-optimum theory by introduced the concepts of middle-optimum degree and unknown-optimum degree. Then the entropy of information systems is defined. Meanwhile, the concept of influence degree has also introduced. Section 4 provides a new decision making method, it discusses about three decision making principles, namely maximum influence degree principle, minimum risk principle and maximum risk principle. Section 5 provides an example to illustrate three decision making principles.

2 Information System Analysis

Let $S=(U, A)$ is an information system, U is represented a non-empty finite set of objects, and A is represented a non-empty finite set of attributes, $\forall a \in A, a: U \to V$ is a mapping, where V is said a value set of a. If V is not include null values, S is called a complete information system, else S is called an incomplete information system. For example, all voters take part in an election, there are three conditions in votes: assent, opposability, neutrality, then the system is complete information system; if a part of voters do not participate in the election, there are four conditions in votes: assent, opposability, neutrality and abstention, then it is an incomplete information system.

Let $S=(U, A, P)$ be a unified description of information systems, U is represented a project set, and A is represented a attribute set, P is a measurement of A relative to U. The attribute set A is divided into four parts: optimum attribute, middle-optimum attribute, non-optimum attribute and unknown-optimum attribute. If it is a complete information system, the measurement of unknown-optimum attribute must be equal to zero. Otherwise it is an incomplete information system. Therefore a complete information system can be regard as a special case of an incomplete information system. We give a vivid illustration in figure 1.

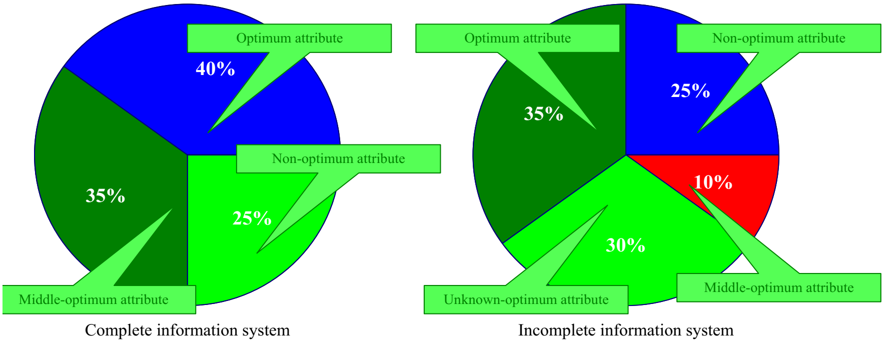

Fig. 1. Pie charts of information system

3 The Basic Idea of Non-optimum Theory

There are some optimum attributes and non-optimum attributes to everything. Researchers found out that non-optimum problem should be studies at the same time with the optimum problem. Non-optimum problem is decided by all the unknown attribute and limits inside and outside the system, which influences directly or indirectly the system's executive process and final goal. In the past, people mainly studied how system operate under optimum conditions and how they can become optimum. Yet this optimization is only relative, and on many occasions the conditions for it is uncertain and unattainable. So the efforts of seeking this kind of optimization are rather blindfolded, and facts have proved that merely controlling some of the conditions cannot keep the system free of non-optimum attribute, because some or even all of the variables to satisfy a optimum attribute may become non-optimum, and on the other hand conditions made for the non-optimum attribute may help to optimum.

A. The fundamental conception

In the non-optimum analysis, we divided a information system into four zones, through which we can describe any factor *u* in *S* and tell whether it belongs to which one. Meanwhile, the factors belonging to one zone can be put into different layers according to the sub-optimum degree. According to quantitative expressions, we can give the following definition:

Definition 3.1. Suppose *S*=(*U, A, P*) be a decision making information system, *U* is represented a project set, and *A* is represented a attribute set, *P* is a measurement of *A* relative to *U*. $U=\{u_1, u_2, \cdots, u_n\}$ is represented *n* projects in which each project u_i (*i*= 1, 2, … , n) is represented as (a_1, … , a_m) , where *m* is the number categorical attributes. The attribute set *A* is divided into four parts: optimum attribute, middle-optimum attribute, non-optimum attribute and unknown-optimum attribute.

(1) Let $O = \{o_1, \cdots, o_r\}$ be a set of optimum attribute in the attribute space *A*, the optimum degree is $\mu_O(x) = \{\mu_{O_1}(x), \cdots, \mu_{O_r}(x)\}$;

(2) Let $\bar{O} = \{\bar{o}_1, \cdots, \bar{o}_s\}$ be a set of non-optimum attribute, the non-optimum degree is $\mu_{\bar{O}}(x) = \{\mu_{\bar{O}_1}(x), \cdots, \mu_{\bar{O}_s}(x)\}$;

(3) Let $M = \{m_1, \cdots, m_t\}$ be a set of middle-optimum attribute, the middle-optimum degree is $\mu_m(x) = \{\mu_{m_1}(x), \cdots, \mu_{m_t}(x)\}$;

(4) Let N be a set of unknown-optimum attribute, the middle-optimum degree is $\mu_n(x)$;

(5) There must be a set $S(O, \bar{O})$ and $S(O, \bar{O}) \subseteq O \times \bar{O} = \{(o_i, o_j) | o_i \in O \wedge \bar{o}_j \in \bar{O}\}$, $(i = 1, \cdots r, \ j = 1, \cdots, s)$, then ($S(O, \bar{O})$ (S, for short) is called the set of sub-optimum .

Definition 3.2. Let U is a nonempty project set. A sub-optimum degree (SOD, for short) $\mu_S(x)$ is an object having the form $\mu_S(x) = \mu_O(x) - \mu_{\bar{O}}(x)$ and $\mu_S : U \to [-1, 1]$, where the functions $\mu_O : U \to [0, 1]$, $\mu_{\bar{O}} : U \to [0, 1]$ denote respectively the degree of optimum (namely $\mu_O(x)$) and the degree of non-optimum (namely $\mu_{\bar{O}}(x)$), then the middle-optimum degree $\mu_m(x) = 1 - \mu_O(x) - \mu_{\bar{O}}(x) - \mu_n(x)$.

Definition 3.3. Let *S*=(*U, A, P*) be a information system, the value set $V = (\mu_O(x), \mu_m(x), \mu_{\bar{O}}(x), \mu_n(x))$ of u relative to *A*, then the entropy of *S* is defined as:

$$H(x) = \frac{\mu_m(x) + \mu_n(x)}{\mu_O(x) + \mu_{\bar{O}}(x)}.$$

Now, we can acquire the value set of four dimensional notation $V = (\mu_O(x), \mu_m(x), \mu_{\bar{O}}(x), \mu_n(x))$ of u relative to *A*. It is explained by an election model as follows: 100 voters all take part in an election. There are 40 tickets for assent, 25 tickets for opposability, and 35 tickets for neutrality. We represent this model with $V = (0.4, 0.35, 0.25, 0)$, the left pie chat in Fig.1 is its illustration; if 30 voters are not attendance, namely 30 tickets for abstention, and the remainder is include 35 tickets for assent, 25 tickets for opposability, and 10 tickets for neutrality. We represent this model with *V*= (0.35, 0.1, 0.25, 0.3), the right pie chat in Fig.1 is its illustration. Therefore, the connotation of the value set $V = (\mu_O(x), \mu_m(x), \mu_{\bar{O}}(x), \mu_n(x))$ is more abundance than membership degree $\mu(u)$ of fuzzy sets.

B. Sub-optimum analysis

The main feature of sub-optimum set is analysing the optimum attribute and non-optimum attribute at the same time. The criterion of sub-optimum theory is to define a lower limit of optimum degree and an upper limit of non-optimum. In fact, an optimal interval is used to instead of an optimal point.

Definition 3.4. Let λ is an index set of optimum degree in $S(O, \bar{O})$, and $\lambda = \{\lambda_1, \cdots, \lambda_l\}$, then $O_\lambda = \{\mu_O(x) \geq \lambda : \lambda > 0,\ x \in U\}$ is called λ -optimum if there is a minimum limitation.

Definition 3.5. Let η is an index set of non-optimum degree in $S(O, \bar{O})$, and $\eta = \{\eta_1,\ \cdots,\ \eta_l\}$, then $O_\eta = \{\mu_{\bar{O}}(x) \leq \eta : \eta > 0,\ x \in U\}$ is called η non-optimum if there is a maximum limitation.

There are optimum attribute and non-optimum attribute to any decision problem, let a lower bound of optimum degree is 0.6 and an upper bound of non-optimum is 0.3, and then the green region is the sub-optimum region in Fig.2,

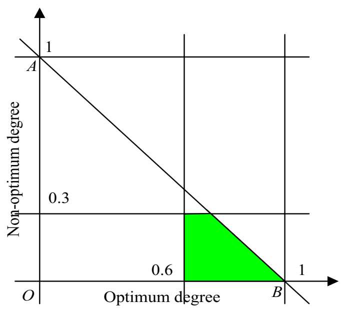

Fig. 2. Figure of sub-optimum region

The triangular region *AOB* is represented an incomplete information system, the line *AB* is represented a complete information system. A sub-optimum degree (SOD, for short) in the point *A* is the minimum, $\mu_S(x) = -1$, and a SOD in the point *B* is the maximum, $\mu_S(x) = 1$. The entropy of *S* is the maximum in the point O, $H(x) = \infty$. The entropy of *S* is the minimum in the line *AB*, $H(x) = 0$.

Definition 3.6. In the sub-optimum region, a project with the maximum entropy is the maximum risk decision making; if a project has the minimum entropy, it is called the minimum risk decision making.

4 A Decision-making Method in Incomplete Information Systems

Generally speaking, the attribute system should be satisfied completeness, representative, and independence. But it is so difficult to find that we have to relax the requirement of independence. The constant method for solving weight is AHP, ANP and CNP. No consideration of topology relations in above methods, so we define influence degree, rectify the weight, then we propose a new multiple attribute decision making method in basis of non-optimum theory. Because decision makers have diverse preferences, we present different decision making principles, namely, maximum influence degree, minimum risk and maximum risk.

A. The fundamental calculation

The role of each attribute is different in the process of decision-making, according to the importance of attribute $a_1, \cdots, a_m$, we choose different weight $\omega_A = (\omega_{a_1}, \omega_{a_2}, \cdots, \omega_{a_m})^T$, it is defined as follows:

Definition 4.1. Let (a_{ij}) be a weight matrix, and $a_{ij} \in [0, 1]$, a_{ij} expresses a value of a_i compared with a_j, then a_{ij} must be satisfied the following conditions:

$$\begin{cases} a_{ij} + a_{ji} = 1, & i \neq j, \\ a_{ii} = 0, & \text{else.} \end{cases}$$

Let $\omega_{a_i} = \frac{2}{m(m-1)} \sum_{i=1}^{m} a_{ij}$, then $\omega_A = (\omega_{a_1}, \omega_{a_2}, \cdots, \omega_{a_m})^T$ is called weight vector.

Because each attribute is not independent, we think about the relation degree between attributes and influence degree of each attribute.

Definition 4.2. Let $G = \{A, E\}$, be a relation graph of attributes, where $A = \{a_1, a_2, \cdots, a_m\}$ is represented an attribute set; $E = \{e_1, e_2, \cdots, e_n\}$ is an edge set, and $e_k : a_i \leftrightarrow a_j$, $(k = 1, 2, \cdots, n \quad i, j = 1, 2, \cdots, m)$, it represents that there is a direct relation between a_i and a_j. The degree of each node a_i is called its relation degree, i.e. $D = (d_{a_1}, \cdots, d_{a_m})^T$. Let $\pi_{u_i} = (\mu_O^i) \bullet D$, where (μ_O^i) is represented optimum vector of the project u_i, then π_{u_i} is called influence degree of the project u_i.

Definition 4.3. In the sub-optimum region, if a project has the maximum influence degree, it is called the maximum influence degree decision making.

B. The decision-making process

How to select the best project satisfied the demand of decision maker from each candidate project. This is the real problem of multicriteria decision making based on non-optimum. A concrete step is given as follows:

Step 1: The measurability of the decision making information system $S=(U, A, P)$ is estimated, namely the measurement value of unknown-optimum is worked out .

Step 2: The weight vector $\omega_A = (\omega_{a_1}, \omega_{a_2}, \cdots, \omega_{a_m})$ of each attribute is calculated according to definition 4.1.

Step 3: In order to acquire the value of $V_{ij}(i=1, \cdots, n, j=1, \cdots, m)$ of u relative to A, firstly we can calculate optimum degree, non-optimum degree in the measurable region by using the method in literature[13]; secondly middle-optimum degree can be get by definition 3.2. Therefore the value set $V_{ij}=(\mu_o(x), \mu_m(x), \mu_{\bar{O}}(x), \mu_n(x))$ can be obtained.

Step 4: The sub-optimum region is established by decision maker, and the eligible projects could be selected among all the projects. Then it can be faced with three alternatives:

(1) If there is no satisfactory solution here, the sub-optimum region is in need of revision or the adjective project is proposed; (2) If there is one satisfactory solution, the best project is it; (3) If there are multiple projects, go to step 5.

Step 5: The relation degree is given by experts, and then the influence degree of each attribute is calculated according to definition 4.2.

Step 6: The best project should be choose.

5 Application

Any missing value can be similar to any other value in the domain of the attribute values. No restriction on the decision or on the values of other attributes for such case. Also when the number of missing values exceeds, a lot of vagueness and uncertainty appear in the information system. Hence, the concluding results will not be reasonable. Let us show all these problems by the following example.

(1) Given the following descriptions of six cars according to price, mileage, size, and max speed, impression is the acceleration as shown in Table 1:

Table 1. Decision matrix

Car	Price	Mileage	Size	Max speed	Impression
U_1	High	High	Full	Low	Good
U_2	Low	*	Full	Low	Good
U_3	*	*	*	High	Poor
U_4	High	*	Full	High	Good
U_5	*	*	Full	High	Excellent
U_6	Low	High	Full	*	Good

(2) According to Definition 4.1 we obtained the weight of each attribute $\omega_A=(0.25\ \ 0.32\ \ 0.2\ \ 0.13\ \ 0.09)$, the weight matrix is shown as follows:

Table 2. Weight table

Weight	Price	Mileage	Size	Max speed	Impression	weight
Price	0	0.5	0.6	0.7	0.7	0.25
Mileage	0.5	0	0.9	0.9	0.9	0.32
Size	0.4	0.1	0	0.8	0.7	0.2
Max speed	0.3	0.1	0.2	0	0.8	0.14
Impression	0.3	0.1	0.3	0.2	0	0.09

(3) Firstly, the decision-maker produces the tolerance relation and preorder relation for known information. The value set V_{ij} is obtained by use of the methods in literature[13], V_{ij} is composition of optimum degree, middle-optimum degree , non-optimum degree and unknown-optimum, the null value is expressed by (0, 0, 0, 1), that is to say , the unknown-optimum is equal to 1, then the value set V can be obtained by the expression of $V=(V_{ij})$ (see Table 3).

Table 3. Value set *V*

Attribute / Project	Price	Mileage	Size	Max speed	Impression	V_i
U_1	(0, 0.6, 0.4, 0)	(0.9, 0.1, 0, 0)	(0.8, 0.2, 0, 0)	(1, 0, 0, 0)	(0.8, 0.2, 0, 0)	(0.66, 0.24, 0.1, 0)
U_2	(0.9, 0.1, 0, 0)	(0, 0, 0, 1)	(0.9, 0.1, 0, 0)	(0, 0.5, 0.5, 0)	(0.8, 0.2, 0, 0)	(0.477, 0.133, 0.07, 0.32)
U_3	(0, 0, 0, 1)	(0, 0, 0, 1)	(0, 0, 0, 1)	(1, 0, 0, 0)	(0, 0.3, 0.7, 0)	(0.14, 0.027, 0.063, 0.77)
U_4	(0, 0.4, 0.6, 0)	(0, 0, 0, 1)	(0.9, 0.1, 0, 0)	(1, 0, 0, 0)	(0.9, 0.1, 0, 0)	(0.626, 0.054, 0, 0.32)
U_5	(0, 0, 0, 1)	(0, 0, 0, 1)	(0.9, 0.1, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(0.41, 0.02, 0, 0.57)
U_6	(0.9, 0.1, 0, 0)	(0.9, 0.1, 0, 0)	(0.9, 0.1, 0, 0)	(0, 0, 0, 1)	(0.8, 0.2, 0, 0)	(0.765, 0.095, 0, 0.14)

(4) Let a lower bound of optimum degree is 0.6 and an upper bound of non-optimum is 0.3, so the project U_2, U_3, and U_5 can not be accepted.

(5) How to select between U_1, $U_4$and U_6? Next we will analysis the influence degree among each attribute. The experts give the relation figure of attributes (see Fig.3), the relation degree of attributes is the degree of each node, i.e. (4, 1, 1, 1, 1).

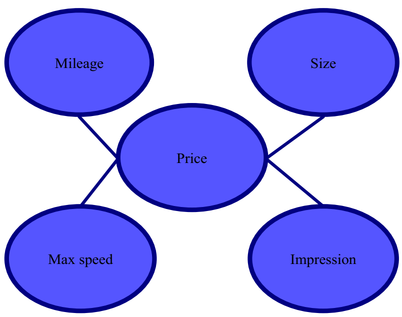

Fig. 3. Relation graph of attributes

We draw the optimum degree in the project U_1, $(\mu_o^1)=(0,\ 0.9,\ 0.8,\ 1,\ 0.8)$, then the influence degree of U_1 is $(4,\ 1,\ 1,\ 1,\ 1)\bullet(\mu_o^1)^T = 3.5$; In the same way, $(\mu_o^4)=(0,\ 0,\ 0.9,\ 1,\ 0.7)$, the influence degree of U_4 is $(4,\ 1,\ 1,\ 1,\ 1)\bullet(\mu_o^4)^T = 2.6$; The next $(\mu_o^6)=$ (0.9, 0.9, 0.9, 0, 0.8), the influence degree of U_6 is $(4,\ 1,\ 1,\ 1,\ 1)\bullet(\mu_o^6)^T = 5.6$. Therefore U_6 is the project of maximum influence degree. We can calculate the entropy of U_1 , U_4 and U_6 to obtain the risk degree.

$$H_{U_1}(x) = 0.24 / 0.66 + 0.1 = 0.3158\,,$$

$$H_{U_4}(x) = 0.054 + 0.32 / 0.626 = 0.5974\,,$$

$$H_{U_6}(x) = 0.095 + 0.14 / 0.765 = 0.3307.$$

(6) Hence the project U_1 is the minimum risk decision making and U_4 is the maximum risk decision making.

The example came from the literature 14, our results are more convincing to comparison with the literature. The method in the literature 14 was only given a simple classification (see Fig.4), there was no significant difference among U_1, U_2, U_4 and U_6. But the difference is significant as compared with each project in Fig.5. We were quantitative presentation for every project, and then obtained the proportion results about four attributes. Fig.5 is only based on optimum attribute. Then the comparison graph between two methods is displayed in Fig.6. Scatter diagram is concentrated in the literature (see the pink points), but the green points, based on optimum attribute, is relatively dispersed.

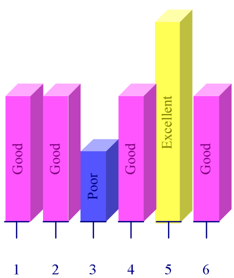

Fig. 4 classification graph

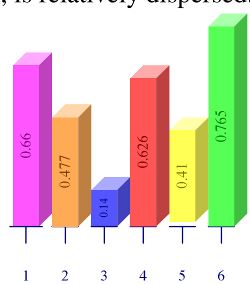

Fig. 5 Proportion graph on optimum

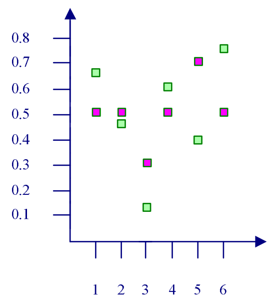

Fig. 6 Comparison graph

6 Conclusion

Our work is introducing a new definition of similarity relation which depends on some important conditions concerning the number of missing values. The essential point of non-optimum theory is making the generalized decisions which have taken over the whole set of attributes a reasonable combination of decisions, hence, the reduces can be computed easily and the generalized decision has a valuable meaning. Also, a distensibility matrix is defined, and the decision rules are introduced. Finally, the set approximations are defined.

As we all known, the keys of decision-making in incomplete information systems is how to analyse and control the uncertainty of attributes. In this paper, we remarked that the non-optimum theory can be an effective tool to deal with the incomplete information system. It can clarify a very important notation in statistics, which is the scaling. This leads to the importance of non-optimum theory in case of the incomplete information systems and solving some problems in scaling in statistics. Also, reducing the decision rules and the decisions can be introduced in a general form. In the literature, similarity relation handled the missing values as null values, it can be similar to any value in the domain of the attribute values. This point of view causes many problems in the analysis.

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Fuzzy Granulation Based Forecasting of Time Series

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Abstract. In this paper, a novel fuzzy granulation based forecasting method is presented for time series. This method includes two steps: granular modeling and forecasting. In granular modeling step, the given time series is first partitioned in terms of data condense degree into segments (windows) with different widths, then after optimally constructing fuzzy granule on each window, a fuzzy granular time series best fitting the original time series is obtained. In the forecasting step, we first fix the linguistic depiction of each granule and build the forecasting rules by mining the fuzzy relationship between the adjacent granules in the granular time series obtained in the first step. After that, we finish the forecasting by means of the forecasting rules. This fuzzy granulation based method can give not only linguistic prediction but also crisp prediction. The main difference of this method from the existing methods is that it realizes the granulation by optimization where the granules correspond to different widths. Thus the model presented here can be regarded as a universal one. Experiment carried on the enrollment data of Alabama University illustrates the good performance of the new method.

Keywords: Fuzzy information granulation, time series, forecasting, fuzzy granules.

1 Introduction

Forecasting is one of the main task of the research of time series, and many methods have been proposed for it in the literature [13,16]. Besides the classical methods (such as AR model, MA model and ARMA model), there appeared many new methods where some artificial techniques are incorporated [10-15,17-19]. Among these new methods, what are worthy of mentioning are the ones related to granular computing techniques which take on some characters of human being's dealing manner of time series. Qiang Song, B.S. Chisson and K.H. Huarng etc. proposed a series of methods for forecasting of time series which are based on fuzzy time series [1,2,3,4,5,6]. Among them, one typical method is as follows:

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first partition the time series into segments with equal widths; then build fuzzy set on each segment; finally build forecasting rules and do forecasting based on these forecasting rules [6]. Carrying the forecasting on the granule level instead of value level, this method exhibits distinct character of human being's processing manner of time series. The manner of partitioning time series of the existing methods results in equal-width segments. But the granular time series build on such segments may not well fit the original time series, because the data in the original time series may exhibit different density of distribution. Thus, in order to better fit the original time series, the resulted segments should have different widths. Meanwhile, the construction of each granule on each segment should take the optimization into account. This means that the granule built on corresponding segment should optimally fit the data on the segment.

Starting from the above viewpoints, we present here a novel fuzzy granulation based forecasting method for time series. This method includes two steps: granular modeling and forecasting. In granular modeling step, the given time series is first partitioned in terms of data condense degree into segments (windows) with different widths, then after optimally constructing fuzzy granule on each window, a fuzzy granular time series best fitting the original time series is obtained. In the forecasting step, we first fix the linguistic depiction of each granule and build the forecasting rules by mining the fuzzy relationship between the adjacent granules in the granular time series obtained in the first step. After that, we finish the forecasting by means of the forecasting rules. This fuzzy granulation based method can give not only linguistic prediction but also crisp prediction. The main difference of this method from the existing methods is that it realizes the granulation by optimization where the granules correspond to different widths. Thus the model presented here can be regarded as a universal one. Meanwhile, this method can better exhibit the characters of human being's processing manner of time series than those existing methods. Experiment carried on the enrollment data of Alabama University illustrates the good performance of the new method.

The remainder of this paper is organized as follows: Section 2 presents the necessary preliminaries of the study; Section 3 discusses how to realize the fuzzy information granulation of a given time series; Section 4 gives the new granulation based forecasting method and corresponding algorithm in detail; In Section 5, experiment is carried to show the performance of the algorithm; Section 6 concludes the study.

2 Preliminary

A. Information Granulation Model

Fuzzy set theory was proposed by L. A. Zadeh in 1960s. Based on fuzzy set, Zadeh gave a profile of data granule in fuzzy information granulation(FIG)[7,8,9] . A fuzzy granule is described as follows: g = (x is G) is λ, where x is a variable of universe of discourse U, G is a fuzzy subset of U, whose membership function is μ_G, λ denotes the probability.

B. Fuzzy Time Series

Fuzzy time series and related definitions were first formally defined by Song and Chissom[1] in 1993 as follows: Suppose $U=\{u_1,u_2,\cdots,u_n\}$ where u_k (k=1,2,…,n) can be a crisp number or an interval define fuzzy sets A_i (i=1,2,…,n) in U :

$$A_i = f_{A_i}(u_1)/u_1 + f_{A_i}(u_2)/u_2 + \cdots + f_{A_i}(u_n)/u_n$$

where $f_{A_i} : U \to [0,1]$ is the membership function of A_i

Definition 1. *Suppose* $Y(t)$ $(t=0,1,2,\cdots)$ *is the universe of discourse of* $f_i(t)$ $(t=0,1,2,\cdots)$, *where* $Y(t)\subseteq R$. $F(t)$ *is called fuzzy time series with universe of discourse* $Y(t)$ $(t=0,1,2,\cdots)$ *if* $F(t)=\{f_i(t) \mid t=0,1,2,\cdots\}$. *If there exits a fuzzy relationship* R *such that* $F(t)=F(t-1)\times R(t-1,t)$, *where* $\times$ *is a predefined operator, we say* $F(t)$ *is generated by* $F(t-1)$ *denoted by* $F(t-1)\to F(t)$.

Definition 2. *Suppose* $F(t-1)=A_i$ $F(t)=A_j$, *the fuzzy logic relationship of them can be defined as* $A_i \to A_j$, *where* A_i *is called the left term of the fuzzy logic relationship and* A_j *right term. A group of fuzzy logic relationships with same left term:* $A_i \to A_{j1}$, $A_i \to A_{j2}$,*… can be incorporated into one fuzzy logic relationship group:* $A_i \to A_{j1}, A_{j2}, \cdots$.

Definition 3. *Suppose* $F(t)$ *is only generated by* $F(t-1)$: $F(t)=F(t-1)\times R(t-1,t)$. *If* $R(t-1,t)$ *does not change with time changes, then* $F(t)$ *is called time-invariant time series; otherwise time-variant time series.*

Definition 4. *Mapping* $I: F(X)\times F(X)\to[0,1]$ *is called an inclusion measure if it satisfies conditions (for any fuzzy sets* $A,B,C\in F(X)$*):*

1) $I(X,\Phi)=0$;
2) $I(A,B)=1 \Leftrightarrow A\subseteq B$;
3) *If* $A\subseteq B\subseteq C$, *then* $I(C,A)\le I(B,A)$ $I(C,A)\le I(C,B)$.

$I(A,B)$ *is the inclusion measure of fuzzy sets A in B* .

3 Optimal Fuzzy Information Granulation of Time Series

In this section, a novel approach of FIG of time series is presented. It consists of two steps: partition and granulation. Partition means partitioning the original time series into some subsequences which are used as operational windows. Granulation means building a fuzzy granule on each window.

There are two methods for partition: width-fixed and width-varying methods. Width-fixed method means using a window with fixed width to partition the original time series, which is very convenient for some sequences. But for some other sequences, in which the information data is dense somewhere, i.e. the data have little change; and sparse somewhere, i.e. the data is volatile, if the width of the window is oversize, then granulation will make the loss of information; if the width of the window is undersize, then the amount of calculation will increase with losing the dominant of granulation. So on the whole, it is hoped that on one hand window with a little bigger width can be used in the dense place and on the other hand window with a bit smaller width can be used in the sparse place, which is more reasonable and clear for IG. Thus there comes the idea of width-varying method which will be introduced in detail in the next subsection.

For the partition, we give a width-varying method which takes the data distribution into account. For the building of granules on each segment (window), we use an optimal manner. The details are given below.

A. Window Width-varying Partition

The main purpose of width-varying methods is to obtain a portion that better fits the original time series. The width of each window is determined in terms of the data distribution. In order to given the proper widths of all the windows, we present here a novel approach consisting of two steps: forward collection and backward refinement. The first step collects data in terms of distribution resulting in width-varying windows, while the second step refines the results obtained in the first step.

A1. Forward collection

Given time series $X = (x_1, x_2, ..., x_n)$, our aim is to determine the proper widths of all windows. In order to implement this, we give the upper bound of the window width w^* and the maximum distance of data in every single window d^* in advance to control the width of each window. After that, in terms of the distribution of data in time series, we give the following Forward Collection algorithm (initialization for subscript variable $k = 0$, window number $wn = 1$):

Step 1 Put the first observed data x_{k+1} into the first window Ω_{wn}.

Step 2 Put x_{k+2} into Ω_{wn}. Calculate the width w_{wn} and max distance $d_{wn\max}$ of Ω_{wn}.

Step 3 If $w_{wn} \le w^*$ and $d_{wn\max} \le d^*$.

If $k < n$ let $k = k+1$ and goto step 2; else goto step 4.

Else delete x_{k+2} from Ω_{wn}, let $wn = wn+1$, $k = k-1$ and goto step 1.

Step 4 Stop the algorithm, and let $C = wn$.

By these four steps, the original time series can be partitioned into C windows of different widths dynamically.

A2. Backward refinement

The backward refinement can be implemented based on the procedure of forward collection. Initialization for window number $wn = C$.

Step 1 Find the first data before Ω_{wn} marked as x_k.

Step 2 Put x_k into Ω_{wn}. Calculate width w_{wn} and the max distance $d_{wn\max}$ of Ω_{wn}.

Step 3 If $w_{wn} \le w^*$ and $d_{wn\max} \le d^*$.

If $k > 1$ let $k = k-1$ and goto step 2; else goto step 4.

Else delete x_k from Ω_{wn}, let $wn = wn-1$ and goto step 1.

Step 4 Stop the algorithm.

By these four steps, the C windows can be adjusted dynamically.

Using the forward collection and backward refinement, the original sequence can be partitioned into C windows $\Omega_1, \Omega_2, \cdots, \Omega_C$ with different widths. These windows do not overlap. But the pre-given parameters w^* and d^* are the main factors that influence the result of the granulation. There is no general rule for choosing the best w^* and d^*. But we can use the learning ability of artificial neural network to get rational w^* and d^*.

B. Optimal Granulation of Time Series

FIG of time series includes partition and granulation. It is of much importance to build a fuzzy set on every single window to represent the original data no matter what method (width-fixed or width-varying) is used to get the partitioned windows.

This paper emphasizes on IG method of W. Pedrycz [10,11]. Given time series $X = (x_1, x_2, ..., x_n)$, deal with the single window problem, i. e. get a fuzzy granule P from X. A fuzzy granule P is a fuzzy concept G which is a fuzzy set with universe X. The process of FIG is to get the membership function $A = \mu_G$ of G. In the following statements, G can be substituted by fuzzy granule P without special announcement, i. e. : $P=A(x)$, $x \in X$. Fuzzy granules may have different forms. The membership function of trapezoid fuzzy granules which is used in this paper is defined by:

$$A(x,a,m,n,b)=\begin{cases}0, others\\(x-a)/(m-a), a\le x<m\\1, m\le x\le n\\(b-x)/(b-n), n<x\le b\end{cases}$$

The granule to be built on a given window should meet the follow requirements[9,10]: (1) The fuzzy granule should represent the original data reasonably; (2) The fuzzy granule should have some particularity. Based on these requirements, an index is constructed to seek a trade-off between (1) and (2): *Q(A)* = *M(A)/N(A),* where *M(A)* satisfies (1) and *N(A)* satisfies (2). For example, when $M(A)=\sum_{x\in X}A(x)$, *N(A)*=measure(support (*A*)), then $Q(A)=\sum_{x\in X}A(x)$/measure(support(*A*)).

According to the above ideas, we present the FIG algorithm below:

FIG algorithm(Fuzzy Information Granules algorithm, single window):

input: a pattern $X=(x_1,x_2,...,x_N)$;output: trapezoid fuzzy granule *P*=(*a, m, n, b*);

Step 1 Fix the core of fuzzy granule, i. e. *m* and *n*.

By sorting $X=(x_1,x_2,...,x_n)$ ascending. We have *m=X(N/2)*, *n=X((N+2)/2) when N* is even and *m=n=X((N+1)/2)* when *N* is odd. Store *s* and *t*, where *m=X(s), n=X(t).*

Step 2 Fix the inferior *a* of the support of fuzzy granule *P*: $a=2(\sum_{i=1}^{s}x_i-m)/s$.

Step 3 Fix the superior *b* of the support of fuzzy granule *P*: $b=\frac{2}{s-t+1}\sum_{i=t}^{N}x_i-n$.

The fuzzy granule used in FIG algorithm is a modified version of trapezoid fuzzy granule for the purpose of getting a broad support. The membership function of the modified trapezoid fuzzy granule is:

$$A(x,a,m,n,b)=\begin{cases}(x-a)/(m-a), x<m\\1, m\le x\le n\\(b-x)/(b-n), n<x\end{cases}$$

4 Granular Modelling and Forecasting of Time Series

Based on the above sections, and referring to the forecasting method proposed by K.H. Huarng[6], we can now model and predict the time series easily. The algorithm of granular modeling and forecasting the time series is shown in detail as follows.

GMF_TS algorithm (Granular Modeling and Forecasting for Time Series)

Step 1 Partition time series

Use Forward Collection Method and Backward Refinement Method to partition the original time series $X = (x_1, x_2, ..., x_n)$, where the upper bound of the window width w^* and the maximum distance d^* of the data in every single window are given in advance. Denote the partition windows as $\Omega_1, \Omega_2, ..., \Omega_C$.

Step 2 Building fuzzy granules

Use FIG algorithm to construct fuzzy sets on partition windows. Denote the fuzzy granules of partition windows as $P_1, P_2, ..., P_C$, where $P_t = (a_t, m_t, n_t, b_t), t = 1,2,...C$.

Step 3 Partition the universe of discourse

Let $D_{\min} = \min_i \{x_i\}$, $D_{\max} = \max_i \{x_i\}$. The universe of discourse of the original time series can be denoted as $U = [D_{\min} - D_1, D_{\max} + D_2]$, where D_1, D_2 are so-called trim factor. Partition U into M intervals $U_1, U_2, \cdots, U_M$ [2,3].

Step 4 Define fuzzy sets of the discussion topic

In fact, the "discussion topic" here is linguistic variable and the fuzzy sets are the linguistic values of the linguistic variable. Denote the linguistic values of the M intervals $U_1, U_2, \cdots, U_M$ as $A_1, A_2, \cdots, A_M$.

Step 5 Fix the linguistic depiction

Calculate the inclusion measure between the fuzzy granule of every window and linguistic value of every interval. Fix the linguistic depiction of every window based on the maximum inclusion principle.

Step 6 Mine the fuzzy relationship

Mine the fuzzy relationship between adjacent windows and then the fuzzy relationship group can be given based on which one does the forecasting of the time series.

Step 7 Forecast and defuzzy

We can predict by giving the linguistic value of every window based on the fuzzy relationship group given in Step 6. Further more the specific value, i.e. crisp value, of every window can be given through defuzzification. In order to forecast and defuzzy, the rules should be given in advance as follow.

Predict Rule: Given the linguistic value A_j of window $\Omega_{i-1} (i \geq 2)$ and fuzzy relationship group $A_j \to A_{j1}, A_{j2}, ..., A_{jp}$. If the data of Ω_i has downtrend related to Ω_{i-1}, choose the following fuzzy relationship from the fuzzy relationship group:

$$A_j \rightarrow A_{j1}, A_{j2}, ..., A_{jk} (j1, j2, ..., jk \leq j) \quad (1)$$

Else choose the following fuzzy relationship from the fuzzy relationship group:

$$A_j \rightarrow A_{j1}, A_{j2}, ..., A_{jk} (j1, j2, ..., jk > j) \quad (2)$$

Specifically, if there holds $j1, j2, ..., jk \geq j$ in Equation (1), we have $A_j \rightarrow A_j$; if there holds $j1, j2, ..., jk \leq j$ in Equation (2), we have $A_j \rightarrow A_j$.

Defuzzification Rule: Suppose the predicted linguistic value of window $\Omega_i (i \geq 2)$ is $A_{j1}, A_{j2}, ..., A_{jk}$, then the predicted crisp value of window Ω_i is

$$(mean(U_{j1}) + mean(U_{j2}) + ... + mean(U_{jk}))/k$$

5 Experiments

In this section, we carry an experiment to show the performance of the algorithm corresponding to our method. The data used here is the enrollment data of Alabama University [2,3] from 1971 to 1992 . Use the GMF_TS algorithm for dataset. The predicted result is shown in table 1 where " ↑" denotes uptrend and " ↓" downtrend.

From table 1, we can see that the result of the prediction is excellent compared with the original data. In the process of prediction, the value predicted in the uptrend case can be considered as the upper bound of the predicted value and the value predicted in the downtrend case the lower bound. Thus, we can not only forecast the linguistic value of every window but also grasp the enrollment data of Alabama University in a certain time interval on the whole. What's more, the prediction of the upper and lower bound of the enrollment data ensure a tentative range of fluctuation of the enrollment of the university in the future, which gives positive instruction for the management of the student and the make important decision of the school.

Especially, when the upper bound of the window width w^* equals to 1, the procedure of fuzzy information granulation and prediction above is consistent with the traditional forecast of fuzzy time series. Thus the scheme of model and predict proposed in this paper is universal, which can be not only operated on the granule level, but also on the original time series.

Table 1. Predict for Enrollment Data of Alabama University

Window	Year	Enrollment	Linguistic value	Predicted linguistic value	Predticted crisp data
Ω_1	1971	13055	A_1		
Ω_2	1972 1973	13563 13867	A_1	↑: A_2 ↓: A_1	↑:14500 ↓:13500
Ω_3	1974	14696	A_2	↑: A_2 ↓: A_1	↑:14500 ↓:13500
Ω_4	1975,1976 1977,1978	15460,15311 15603,15861	A_3	↑: A_3 ↓: A_2	↑:15500 ↓:14500
Ω_5	1979, 1980 1981	16807, 6919 16388	A_4	↑: A_4 ↓: A_3	↑:16500 ↓:15500
Ω_6	1982,1983 1984,1985	15433,15497 15145,15163	A_3	↑: A_6 ↓: A_3	↑:18500 ↓:15500
Ω_7	1986	15984	A_3	↑: A_4 ↓: A_3	↑:16500 ↓:15500
Ω_8	1987	16859	A_4	↑: A_4 ↓: A_3	↑:16500 ↓:15500
Ω_9	1988	18150	A_6	↑: A_6 ↓: A_3	↑:18500 ↓:15500
Ω_{10}	1989,1990 1991,1992	18970,19328 19337,18876	$0.5A_6 + 0.5A_7$	↑: $0.5A_6 + 0.5A_7$ ↓: A_6	↑:19000 ↓:18500

6 Conclusion

In this paper, a novel granulation based method is proposed for forecasting of time series. The main character of this method is that it takes data distribution into account in the partition step and takes optimal granulation into account in the granulation step. The GMF_TS algorithm**,** we designed here corresponding to our new method, can deal with time series in the fuzzy granular level. Experiment carried on the enrollment data of Alabama University illustrates excellent performance of the algorithm. The algorithm can not only forecast by giving the linguistic value of every window but also grasp the enrollment data of Alabama University in a certain time interval on the whole. The prediction of the upper and lower bound of the enrollment data ensure a tentative range of fluctuation of the enrollment of the university in the future, which is useful for the school.

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Strategy for Improving Shared Mental Model in Information Systems Integration Project Based on Knowledge Complementarity

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Abstract. Information systems integration project is based on shared mental model, while it's difficult because of different profit and cooperation condition. Introduce knowledge complementary as another factor that makes influence on shared mental model into traditional four factors taking account of the high intensive knowledge in information systems integration. Then we analysis the connection among these factors on system dynamic review, and built up knowledge accumulation model and calculus this model with two-phrase game theory. According to this, test that knowledge complementary give influence on shared mental model.

Keywords: Shared mental model, knowledge accumulation model, knowledge complementarity, information systems integration.

1 Introduction

With the development of information technology, information systems integration has been possible [1]. Integration is implemented by group, which includes enterprise, supplier of integration system and suppliers of existed software systems. Shared mental model is the foundation of the group [2]. Before integration, enterprise and suppliers keep stable cooperation. And owing to supplier of integration taking part in, the connection becomes weak and the suppliers of existed systems might draw back from group. So it is difficult to form sharing mental model in information systems integration. And how to impulse sharing mental model is the key problem.

Rouse and Morris put forward mental model in 1986 [3]. Cannon-Browers and Salas expand the model from single member to group, and put forward sharing mental model [4]. They define the model as knowledge structure of group members. On foundation of this model, group members could hold up right comprehension and anticipation for group conduction, and modify themselves toward common requirement. Klimoski and Mohammed pointed

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out that sharing mental model is helpful for group efficient through experiment [5]. Marks put forward that sharing mental model could help members adjust to new work on base on experiment [6]. Jin Yanghua took virtue group as object to see that sharing mental model in such group does help to the efficient of group[7]. Kraiger and Wenzel focus on the key factors, which influence on sharing mental model and put forward that surrounding, organization, group and individual are key factors [8]. Wu Xin brings forward that communication among members is helpful for sharing mental model and conduction like “take bus” do bad to it [9]. These researchers forms foundation of our research, but they ignore the situation that members come from different organization. So the encouragement mechanism could not be used in information systems integration. In supply chain management, researchers built up encouragement mechanism from price and productivity [10]. In information systems integration, cooperation is happened not only between enterprise and suppliers but also among suppliers, for whom formal contract might no exist, so encouragement mechanism on price and productivity could not be used. And in the area of information systems integration, IT governance after integration [11] and technology for integration [12] has been pay attention, and encouragement mechanism for group has been ignored. Sharing mental model is the common knowledge structure among members. Information systems integration is based on knowledge sharing and diffusion [13]. So we built up encouragement mechanism for sharing mental model in information systems integration from the point of knowledge complementary.

2 Model

2.1 Key Factors of Sharing Mental Model

We take the view of Kraiger and Wenzelon [8] on key factors as our foundation, which are surrounding factor, organization factor, group factor and individual factor. And we can see that knowledge sharing is an important factor in information systems integration project. Knowledge complementary is another force on group communication [14]. So we take complementary knowledge as the fifth factor that give influence on shared mental model. It can be see as follows:

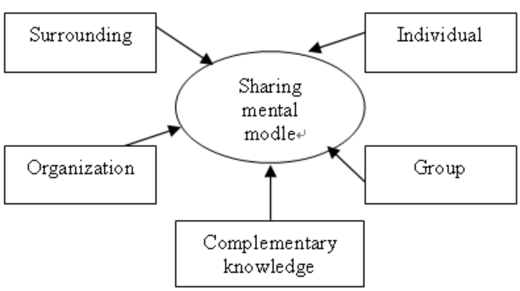

Fig. 1. Key factors of sharing mental model

These five factors connect with each other. For example, when group is paid much attention by outside surrounding, organization would do some action to encourage shared mental model. So surrounding gives influence on organization factor. At the same time, surrounding gives influence on individual and group. And organization gives influence on individual and group. So we can get system dynamic model of these five factors through Forrester SD model [15]. It is as follows:

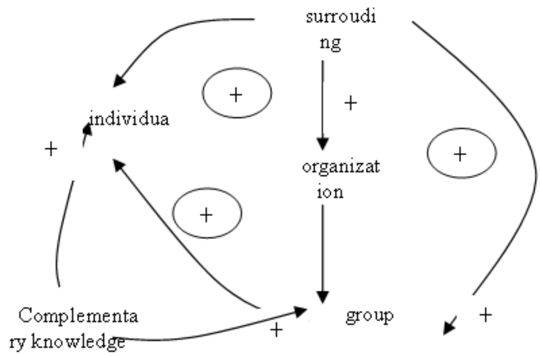

Fig. 2. System dynamic model of sharing mental model factors

2.2 Knowledge Accumulation Model

On foundation of shared mental model key factors, we build up mathematic model to test the influence produced by knowledge complementary on sharing mental model.

Define supplier of integration is member i, supplier of existed software is member j, their knowledge sharing degree are χ_i and χ_j, their intension are I_i and I_j, and their learning ability are α_i and α_j, their knowledge accumulation through information systems integration are U_i and U_j.

Then, assume the knowledge complementary degree is $\vartheta(0 < \vartheta < 1)$. That is to say, when $\vartheta \to 1$, the knowledge complementary degree is high, and when $\vartheta \to 0$, the knowledge complementary is low. Additionally, the knowledge sharing tendency of these two members are f_i and f_j, then the knowledge sharing degree is influenced by tendency of sharing mental. It can be described as:

$$\chi_i = \chi_i(f_i) \tag{1}$$

Knowledge absorbing ability $\varepsilon_i(\varepsilon_j)$ is influenced by its own learning ability, the intension on knowledge absorbing and the degree of knowledge complementary. When learning ability is strong, or when the intension on knowledge absorbing is strong, knowledge absorbing ability would be strong accordingly. And when the degree of knowledge complementary is high, absorbing knowledge would be difficult, knowledge absorbing ability would be weak. So we can get

$$\varepsilon_i = \varepsilon_i(\alpha_i, I_i, \vartheta) \tag{2}$$

Further more, the intension on knowledge absorbing is influenced by the degree of knowledge sharing and the degree of knowledge complementary. That is to say, when the degree of knowledge sharing is high, the intension could be decreased. And when sharing mental tendency is high, the intension could be increased. When the degree of knowledge complementary is high, the intension would be increased too. So intension on knowledge absorbing would be described as:

$$I_i = I_i(\chi_i, \chi_j, \vartheta) \tag{3}$$

And we assume that knowledge accumulation only could be realized through intension. Owing to knowledge sharing, the knowledge accumulation of member i is included by sharing knowledge from member j and absorbing knowledge itself. It is as follows:

$$U_i = I_i + \varepsilon_i(\alpha_i, I_i, \vartheta)\chi_j(f_j)I_j \tag{4}$$

Then, assume that knowledge sharing cost is $C(\chi_i)$ and $C(\chi_j)$, marginal cost is m_i and m_j. And m_i would decrease with the increase of knowledge accumulation U_i. So the profit could be described as:

$$\pi_i = U_i - m_i - I_i - C(\chi_i) \tag{5}$$

The game between supplier of integration and supplier of existed system is two phrases. At first stage, they decide whether share knowledge or not. So the degree of knowledge sharing χ_i and χ_j would be decided at this stage. If the consult is cooperation, they could get sharing degree, otherwise, sharing degree could not be got. But at last, sharing degree χ would be got. On base of it, they would decide the degree of intension I_i and I_j at second game phrase.

3 Model Calculation

We calculate the model using two-phrases game theory.

(1)The consult of first phrase is no-cooperation

Then, at the second phrase, maximum profit of member i is

$$\begin{aligned} \max g_i &= U_i - m_i - I_i - C(\chi_i) \\ &= \varepsilon_i(\alpha_i, I_i, \vartheta)\chi_j(f_j)I_j - m_i - C(\chi_i) \\ \frac{\partial g_i}{\partial I_i} &= \varepsilon_{iI_i}\chi_j I_j - m_{iU_i}\left(\frac{\partial U_i}{\partial I_i}\right) = \varepsilon_{iI_i}\chi_j I_j - m_{iU_i}(1 + \varepsilon_{iI_i}\chi_j I_j) \end{aligned}$$

Define $\frac{\partial g_i}{\partial I_i} = 0$, then we can get $I_i^*(I_j)$.

At the same way, we can get $I_i^*(I_j)$, so balance point under no-cooperation is:

$$(I_i^N, I_j^N) = [I_i^*(I_j), I_j^*(I_i)]$$

Then at first game phrase, max profit of member i is:

$$\max \quad h_i = \varepsilon_i(\alpha_i, I_i, \vartheta)\chi_j I_j^N(\chi_i, \chi_j, \vartheta) - m_i(U_i) - C(\chi_i)$$

Additionally,

$$U_i = I_i^N(\chi_i, \chi_j, \vartheta) + \varepsilon_i(\alpha_i, I_i^N(\chi_i, \chi_j, \vartheta), f_j, \vartheta)\chi_j I_j^N(\chi_i, \chi_j, \vartheta)$$
$$\frac{\partial h_i}{\partial \chi_i} = \varepsilon_{i\chi_i}\chi_j I_j^N + \varepsilon_i \chi_j I_{j\chi_i}^N - m_{iU_i}[I_{i\chi_i}^N + \chi_j(\varepsilon_{i\chi_i} I_j^N + \varepsilon_i I_{j\chi_i}^N)] - C'(\chi_i) = 0$$

Define $\frac{\partial h_i}{\partial \chi_i} = 0$, then we can get balance point (χ_i^N, χ_j^N).

So, when the game consult is no-cooperation, $\chi_i^N = \chi_j^N = \chi^N$, $I_i^N = I_i^N = I^N$, $f_i^N = f_j^N = f^N$.

(2)The consult of first phrase is cooperation.

Then at second game phrase, max profit of member i is:

$$\frac{\partial U_i}{\partial I_i} = \varepsilon_{iI_i}\chi I_j - m_{iU_i}(1 + \varepsilon_{iI_i}\chi I_j) = 0$$
$$I_i^Y(\chi, \vartheta) = I_j^Y(\chi, \vartheta) = I^Y(\chi, \vartheta)$$
$$\varepsilon_i^Y = \varepsilon_j^Y = \varepsilon^Y(\alpha, I^Y, \vartheta) = \varepsilon^Y(\alpha, \chi, \vartheta)$$
$$U_i = I^Y(1 + \varepsilon^Y(\alpha, I^Y, \vartheta)\chi)$$

At first stage,

$$u = \varepsilon^Y(\alpha, \chi, \vartheta)I^Y(\chi, \vartheta)\chi - m_i(U_i) - C(\chi_i)$$
$$\frac{\partial u}{\partial \chi} = \varepsilon^Y I^Y - m_{iU_i}[I_\chi^Y + \varepsilon^Y I^Y + \chi(\varepsilon_\chi^Y I^Y + \varepsilon^Y I_\chi^Y)] - C'(\chi) = 0$$
$$\chi_i = \chi_j = \chi^Y$$

Because there is balance point $(\chi^N, \chi^N, \vartheta)$, so

$$h_i(\chi^N, \chi^N, \vartheta) \geq h_i(\chi^Y, \chi^N, \vartheta) \tag{6}$$

and total profit under cooperation would more than that under no-cooperation, so

$$h_i(\chi^Y, \chi^Y, \vartheta) \geq h_i(\chi^N, \chi^N, \vartheta) \tag{7}$$

add (6) and (7), we can get

$$h_i(\chi^Y, \chi^Y, \vartheta) \geq h_i(\chi^Y, \chi^N, \vartheta)$$

Additionally,

$$\frac{\partial h_i}{\partial \chi_j} = \varepsilon_i I_j + \chi_j(\varepsilon_{i\chi_j} I_j + \varepsilon_i I_{j\chi_j}) - m_{iU_i}[I_{i\chi_j} + \varepsilon_i I_j + \chi_j(\varepsilon_{i\chi_j} I_j + \varepsilon_i I_{j\chi_j})]$$

m_i is decreasing function of U_i, that is to say $m_{iU_i} < 0$. and when $\vartheta \to 1$, ε_i would be very little. So,

$$\varepsilon_{i\chi_j} I_j + \varepsilon_i I_{j\chi_j} > 0, \frac{\partial h_i}{\partial \chi_j} > 0$$

When ϑ is much more, $\chi^Y > \chi^N$. That is to say, when knowledge complementary degree is high, the sharing degree under cooperation would more than that under no-cooperation.

Because $\chi_i = \chi_i(f_i)$ and χ is increasing function of f, when $\vartheta \to \infty$, $f^Y > f^N$. That is to say, when knowledge complementary is much high, members of integration project would like to cooperate, and sharing mental model would be formed easily.

Then we can see that, knowledge complementary give influence on sharing mental model in information systems integration. So complementary knowledge could be used in encouraging sharing mental model.

4 Case Analysis

We have taken part in information systems integration project pronounced by motor-car manufactory enterprise in Chongqing. There are three systems, which are financial system, MRP system and order management system. Supplier of these systems are supplier A supplier B and supplier C. The key knowledge of these systems is hold by these three suppliers, so knowledge sharing is foundation of integration. Because of this, some conduction should be implemented to enforce forming sharing mental model in the project group.

Using the consult of our research, we focus on knowledge complementary to encourage forming sharing mental model. We find that supplier A is good at Oracle database, Java and financial management; supplier B is good at Sql Server. Net and ordering management process; supplier C is good at Oracle database and manufactory process. They are all shortage in integration knowledge. So knowledge complementary exists between supplier of integration and supplier of existed system. Then we introduce members of project pay attention on knowledge complementary to form sharing mental model. According to this, sharing mental model is formed and it does help to success of project.

5 Conclusion

Sharing mental model is the support for information systems integration. How to impulse sharing mental model forming is the key problem in project implementation. On foundation of much knowledge in information systems integration, we introduce knowledge complementary in sharing mental model key factors, analyze the connection among these factors using system dynamic mode and build up knowledge accumulation. Through model calculation on two-phrase game theory, we put forward the strategy for sharing mental model on knowledge complementary.

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Measurement of Knowledge Granularity and Relationship between the Digital Features about Knowledge Divided

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Abstract. This essay introduces the concepts of knowledge granularity and information systems. On this basis, we propose two new more general knowledge granularities: the combination granularity and polynomial granularity, which include the current common knowledge granularity. The results for the establishment of granular computing in the complete information system have some theoretical significance and practical values. In addition, we also discuss the digital feature about knowledge divided and the relationship between it and knowledge granularity.

Keywords: Rough sets, Complete information system, Knowledge granularity, Knowledge divided.

1 Introduction

In the early 1980s, the Polish scholar Z. Pawlak published a classic paper "Rough sets"[1] through the study about logical properties of information systems, which marked the birth of rough set theory. It can effectively deal with uncertain or imprecise knowledge representation, experiential learning, and access to knowledge from experience, analysis about inconsistent information, pattern classification and a series of similar problems, and has been successfully applied in machine learning, pattern recognition, decision analysis, image processing and other fields[2, 3]. The rough set defines the knowledge as a capacity to divide the domain. It believes that knowledge is granular, which means knowledge is rough, the larger the knowledge granularity is, the less the knowledge contains, the weaker the classification capability is; the smaller the knowledge granularity is, the more the knowledge contains, the stronger the ability of its classification is.

Granularity computing is a new concept and computing paradigm about information processing, which plays an important role in dealing with data

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compression, information retrieval and the inaccuracy of the areas of communications, uncertainties and other similar expressions[4]. At present there are rough set model, the word computing model and commercial space model in granularity computing models.

A large number of published literatures about granularity computing show its importance, which cause scholars' wide interest. Zhangling[6] proposes a theory of quotient space, establishes a set of theories and the corresponding algorithm about "world model granularity", and proposes the theory of fuzzy quotient space on this basis. Liang ji ye[7] has made in-depth study about Information Systems' measurement such as information entropy, rough entropy and knowledge granularity and the relationship between them. Zhao Mingqing[8] improves the axiomatic definitions of knowledge granularity, makes a series of knowledge granularity measurement method and discusses granularity's combination of problem. Based on rough set theory and the above study, this essay proposes two new knowledge granularities, including the more general combination granularity and polynomial granularity, which include the current common knowledge granularity. Besides, we also discuss the relationship between the digital feature about knowledge divided and the knowledge granularity.

2 The Concepts of Information System

Claimed $S=(U,A)$ is an information system[6], which U indicated a non-empty finite set of objects, A indicated a Properties of non-empty finite set, For any property $a \in A$, $f_a \quad U \rightarrow V_a$, which V_a as the range of a. For a given information system $S=(U,A)$, if any property $a \in A$, V_a does not include the null value, Claimed that S is a complete information system; if at least one attribute $a \in A$, V_a contains a null value, claimed that the information system is an incomplete information system. Can be seen, complete information system is a special case of incomplete information system.
Set up $P \subseteq A$, defined equivalence relation:

$$IND(P)=\{(u,v)\in U\times U \quad \forall a\in P, f_a(u)=f_a(v)\}$$

easy to know $IND(P)=\bigcap_{a\in P} IND(\{a\})$. Defined $u\in U$ on the equivalence class about P is $[u]_P$, $[u]_P$ expressed the set of indistinguishable elements in the equivalence $IND(P)$.

Mark $U/IND(P) \quad U/IND(P)$ is a partition of U, Denoted by U/P, Called a knowledge on the U, each equivalence class called a particles of knowledge. Knowledge granularity is the average metric of equivalence class size under the attribute set P.

Set up $S=(U,A)$ is a complete information system, $P,Q \subseteq A$ $U/IND(P)=\{P_1,P_2,\ldots,P_m\}$ $U/IND(Q)=\{Q_1,Q_2 \quad \ldots \quad Q_n\}$. If for any P_i,there is a Q_j, caused $P_i \subseteq Q_j$, claimed P smaller than Q, denoted $P \preceq Q$;if $P \preceq Q$ and on a P_{i_0}, there is a Q_{j_0}, caused $P_{i_0} \subset Q_{j_0}$, claimed P strictly smaller than Q, denoted $P \prec Q$;if $P \preceq Q$ and $Q \preceq P$, claimed P is equivalent to Q,denoted $P \approx Q$.

3 Calculation of Knowledge Granulation

Definition 1. *Set up $S=(U,A)$ is a complete information system, if $\forall \ P \subseteq A$, exists a real number $G(P)$ corresponding, and satisfies the following conditions:*

1) *Non-negative $G(P) \geq 0$;*
2) *Invariance $\forall P,Q \subseteq A$,If there exists a bijective function,*
   $f: U/IND(P) \to U/IND(Q)$,suffice $|P_i|=|f(P_i)|$,*then* $G(P)=G(Q)$;
3) *Monotonicity $\forall \ P,Q \subseteq A$ if $P \prec Q$ then $G(P)<G(Q)$.*

Then, claimed G *as a knowledge granularity on the complete information system $S=(U,A)$.*

Clearly, if $P \approx Q$, then. $G(P)=G(Q)$.

Theorem 1. *Set up $S=(U,A)$ is a complete information system, $\forall \ P \subseteq A$, if $U/IND(P)=\{\{u_i\} \mid u_i \in U\}$, that P is identity relations, then $G(P)$ to obtain the minimum; if $U/IND(P)=\{U\}$,that P is relations for the universe, then $G(P)$ obtain maximum.*

Proof. $P,Q \subset A$, $U/IND(P)=\{\{u_i\} \mid u_i \in U\}$, $U/IND(Q)=\{U\}$, $\forall R \subset A$, $U/IND(R)=\{R_1,R_2,\cdots,R_n\}$, Then for any $\{u_i\}$ have R_j ,make $\{u_i\} \subseteq R_j \subseteq U$,means $P \preceq R \preceq Q$ known by the definition 1 of the conditions (2), (3) $G(P) \leq G(R) \leq G(Q)$. #

Definition 2. *Set up $S=(U,A)$ is a complete information system,*

$$\forall R \subseteq A, U/IND(R)=\{X_1,X_2,\cdots,X_m\}$$

Then R 's knowledge granularity is defined as

$$G(R)=\sum_{i=1}^{m}\frac{|X_i|}{|U|}\frac{C_{|X_i|}^{r}}{C_{|U|}^{r}} \quad (0<r \leq |X_i|) \tag{1}$$

Theorem 2. *Set up* $S=(U,A)$ *is a complete information system,*

$$\forall R\subseteq A, U/IND(R)=\{X_1,X_2,\cdots,X_m\}$$

Then G *in Definition 2 is a knowledge granularity in the sense of definition 1.*

Proof It is clear that G in Definition 2 satisfy the conditions of 1), 2) in the Definition 1, then proved the conditions 3) has also set up.

Set up $P,Q\subseteq A$, $U/IND(Q)=\{Q_1,Q_2 \quad \ldots \quad Q_n\}$, and $P\prec Q$, without loss of generality, set up P only take the Q_t inside U/Q into $Q_{t'}$ and $Q_{t''}$, then

$$G(Q)=\sum_{i=1}^{n}\frac{|Q_i|}{|U|}\frac{C_{|Q_i|}^{r}}{C_{|U|}^{r}}$$

$$G(P)=\sum_{i=1,i\neq t}^{n}\frac{|Q_i|}{|U|}\frac{C_{|Q_i|}^{r}}{C_{|U|}^{r}}+\frac{|Q_{t'}|}{|U|}\frac{C_{|Q_{t'}|}^{r}}{C_{|U|}^{r}}+\frac{|Q_{t''}|}{|U|}\frac{C_{|Q_{t''}|}^{r}}{C_{|U|}^{r}}$$

$$G(P)-G(Q)=\frac{|Q_{t'}|}{|U|}\frac{C_{|Q_{t'}|}^{r}}{C_{|U|}^{r}}+\frac{|Q_{t''}|}{|U|}\frac{C_{|Q_{t''}|}^{r}}{C_{|U|}^{r}}-\frac{|Q_{t'}|+|Q_{t''}|}{|U|}\frac{C_{|Q_{t'}|+|Q_{t''}|}^{r}}{C_{|U|}^{r}}$$

$$=\frac{|Q_{t'}|}{|U|}\left(\frac{C_{|Q_{t'}|}^{r}}{C_{|U|}^{r}}-\frac{C_{|Q_{t'}|+|Q_{t''}|}^{r}}{C_{|U|}^{r}}\right)+\frac{|Q_{t''}|}{|U|}\left(\frac{C_{|Q_{t''}|}^{r}}{C_{|U|}^{r}}-\frac{C_{|Q_{t'}|+|Q_{t''}|}^{r}}{C_{|U|}^{r}}\right)$$

$$<0$$

Thus, $G(R)$ is a knowledge granularity in the sense of Definition 1. #

When $U/IND(P)=\{\{u_i\}\,|\,u_i\in U\}$ $G(R)$ has minimum $\frac{1}{|U|}$, when $U/IND(P)=\{U\}$, $G(R)$ has maximum 1.

When $r=1$, $G(R)=\frac{1}{|U|^2}\sum_{i=1}^{m}|X_i|^2$, when $r=2$, $G(R)=\frac{|X_i|}{|U|}\sum_{i=1}^{m}\frac{C_{|X_i|}^{2}}{C_{|U|}^{2}}$. Both are the knowledge granularity given in literature [7], so the knowledge granularity in Definition 2 is the promotion of knowledge granularity in literature [7].

Definition 2 and Theorem 2 can be used for the following promotion:

Definition 3. *Set up* $S=(U,A)$ *is a complete information system,*

$$\forall R\subseteq A, U/IND(R)=\{X_1,X_2,\cdots,X_m\}$$

Then R *'s combination granularity is defined as*

$$G^{'}(R)=\sum_{i=1}^{m}\frac{|X_i|}{|U|}\frac{C_{|X_i|}^{r}}{C_{|U|}^{s}} \quad (0<r\leq |X_i|, 0<s\leq |U|) \tag{2}$$

Theorem 3. *Set up* $S=(U,A)$ *is a complete information system,*

$$\forall R\subseteq A, U/IND(R)=\{X_1,X_2,\cdots,X_m\}$$

Then $G^{'}(R)$ *in Definition 3 is a knowledge granularity in the sense of Definition 1.*
Proof of Theorem 3 similar with Theorem 2,slightly.

Definition 4. Set up $S=(U,A)$ is a complete information system,

$$\forall R\subseteq A, U/IND(R)=\{X_1,X_2,\cdots,X_m\}$$

Then R 's polynomial granularity is defined as

$$G_S(R)=\sum_{i=1}^{m}(a_1|X_i|+a_2|X_i|^2+\cdots+a_n|X_i|^n)+c \tag{3}$$

And $a_1,a_2,\cdots,a_n\geq 0$, $a_2,a_3,\cdots,a_n$ not all zeros, $c\geq 0$, n is the positive integer and not equal to1.

Theorem 4. *Set up* $S=(U,A)$ *is a complete information system,*

$$\forall R\subseteq A, U/IND(R)=\{X_1,X_2,\cdots,X_m\}$$

Then $G_S(R)$ *in Definition 4 is a knowledge granularity in the sense of Definition 1.*
Proof 1 $|X_i|>0$, $i=1,2,\cdots,m$, $G_S(R)>0$

2 $\forall$ $P,Q\subseteq A$, If there exists a bijective function,
$f:U/IND(P)\rightarrow U/IND(Q)$, Suffice $|P_i|=|f(P_i)|$,

$$\begin{aligned}G_S(\mathrm{P})&=\sum_{i=1}^{m}(a_1|\mathrm{P}_i|+a_2|\mathrm{P}_i|^2+\cdots+a_n|\mathrm{P}_i|^n)+c\\&=\sum_{i=1}^{m}(a_1|f(\mathrm{P}_i)|+a_2|f(\mathrm{P}_i)|^2+\cdots+a_n|f(\mathrm{P}_i)|^n)+c\\&=G_S(Q)\end{aligned}$$

3 $\forall$ $P,Q\subseteq A$,if $P\prec Q$, $U/IND(Q)=\{X_1,X_2,\cdots,X_m\}$, without loss of generality, set up P only take the X_t inside U/Q into $X_{t^{'}}$ and $X_{t^{''}}$,then

$$G_S(Q)=\sum_{i=1}^{m}(a_1|X_i|+a_2|X_i|^2+\cdots+a_n|X_i|^n)+c$$

$$G_S(P)=\sum_{i=1,i\neq t}^{m}(a_1|X_i|+a_2|X_i|^2+\cdots+a_n|X_i|^n)+\sum_{i=t^{'},t^{''}}(a_1|X_i|+a_2|X_i|^2+\cdots+a_n|X_i|^n)+c$$

$$\begin{aligned}G_S(Q)-G_S(P)=&\,a_1(|X_{t^{'}}|+|X_{t^{''}}|)+a_2(|X_{t^{'}}|+|X_{t^{''}}|)^2\cdots+a_n(|X_{t^{'}}|+|X_{t^{''}}|)^n\\&-\sum_{i=t^{'},t^{''}}(a_1|X_i|+a_2|X_i|^2+\cdots+a_n|X_i|^n)\end{aligned}$$

$= a_2[(|X_{t'}|+|X_{t''}|)^2-(|X_{t'}|^2+|X_{t''}|^2)]+\cdots+a_n[(|X_{t'}|+|X_{t''}|)^n-(|X_{t'}|^n+|X_{t''}|^n)]$
>0
So $G_S(P) < G_S(Q)$
Thus $G_S(R)$ is a knowledge granularity in the sense of Definition 1. #

$G(R)$ $G^{'}(R)$ and knowledge granularity in literature [7] are available expressed as Formula (3), so, combination granularity $G_S(R)$ is a more general knowledge granularity.

4 The Expectation and Variance about Knowledge Divided, and the Relationship between It and Knowledge Granularity

Any division on the domain can be seen as the random variable define on the division block of cardinal number ,so it can define its probability distribution and its mathematical expectation, variance and so on.

Definition 5. *Set up* $S=(U,A)$ *is a complete information system,*

$$\forall R \subseteq A, U/IND(R)=\{X_1, X_2, \cdots, X_m\}$$

Then, define the R *'s probability distribution on the division block of cardinal number is*

$$(R;P)=\begin{bmatrix} |X_1| & |X_2| & \cdots & |X_m| \\ P(X_1) & P(X_2) & \cdots & P(X_m) \end{bmatrix},$$

which $P(X_i)=\dfrac{|X_i|}{|U|}$, $i=1,2,\cdots,m$ *, define the* R *'s mathematical expectation is* $E(R)=\sum_{i=1}^{m}|X_i|P(X_i)$ *, and define the* R *'s variance is* $D(R)=\sum_{i=1}^{m}P(X_i)[|X_i|-E(R)]^2$.

Expectation $E(R)$ reflects the average size of each partition block, variance $D(R)$ reflects the deviation value about each division of the number of elements in block with the average number.

Theorem 5. *Set up* $S=(U,A)$ *is a complete information system,*

$$\forall R \subseteq A, U/IND(R)=\{X_1, X_2, \cdots, X_m\}$$

Then the expectations $E(R)=\sum_{i=1}^{m}\dfrac{|X_i|^2}{|U|}$ *is a knowledge granularity in the sense of Definition 1.*

Proof $E(R)$ meet the knowledge granularity (3)'s form, so, it's a knowledge granularity under the Definition 1.

Theorem 6. *Set up* $S=(U,A)$ *is a complete information system,*

$$\forall R \subseteq A, U/IND(R)=\{X_1, X_2, \cdots, X_m\}$$

If and only if $|X_1|=|X_2|=\cdots=|X_m|$, *variance* $D(R)=0$.

Proo f. Get from the definition of variance, $D(R)=\sum_{i=1}^{m}\frac{|X_i|^3}{|U|}-\left(\sum_{i=1}^{m}\frac{|X_i|^2}{|U|}\right)^2$

Adequacy when $|X_1|=|X_2|=\cdots=|X_m|$, $|X_i|=\frac{|U|}{m}$, so

$$D(R)=\sum_{i=1}^{m}\frac{(|U|/m)^3}{|U|}-[\sum_{i=1}^{m}\frac{(|U|/m)^2}{|U|}]^2=\sum_{i=1}^{m}\frac{|U|^2}{|m|^3}-(\sum_{i=1}^{m}\frac{|U|}{|m|^2})^2=0$$

Necessity, when $D(R)=0$, $\sum_{i=1}^{m}P(X_i)[|X_i|-E(R)]^2=0$, $|X_i|-E(R)=0$, available, $|X_1|=|X_2|=\cdots=|X_m|$.

There is a subset formed by n attributes in the complete information system S, the Division 5 can be used for the following promotion:

Definition 6. *Set up* $S=(U,A)$ *is a complete information system,* $\forall A_1, A_2, \cdots, A_n \subseteq A$,

Mark $U/IND(A_1)=\{X_1, X_2, \cdots, X_{m_1}\}$

$U/IND(A_2)=\{Y_1, Y_2, \cdots, Y_{m_2}\}$

$\cdots$

$U/IND(A_n)=\{Z_1, Z_2, \cdots, Z_{m_n}\}$

$$U/IND(A_1 \cup A_2 \cup \cdots \cup A_n)=$$
$$\{X_1 \cap Y_1 \cap \cdots \cap Z_1, \cdots, X_i \cap Y_j \cap \cdots \cap Z_s, \cdots, X_{m_1} \cap Y_{m_2} \cap \cdots \cap Z_{m_n}\}$$

Define the $A_1, A_2, \cdots, A_n$ *'s union probability distribution on the division block of cardinal number is*

$$(A_1 \cup A_2 \cup \cdots \cup A_n; p)=\begin{bmatrix} |X_1 \cap Y_1 \cap \cdots \cap Z_1| & \cdots |X_i \cap Y_j \cap \cdots \cap Z_s| \cdots & |X_{m1} \cap Y_{m2} \cap \cdots \cap Z_{mn}| \\ P(X_1 \cap Y_1 \cap \cdots \cap Z_1) & \cdots P(X_i \cap Y_j \cap \cdots \cap Z_s) \cdots & P(X_{m_1} \cap Y_{m_2} \cap \cdots \cap Z_{m_n}) \end{bmatrix}$$

Which $P(X_i \cap Y_j \cap \cdots \cap Z_s)=\frac{|X_i \cap Y_j \cap \cdots \cap Z_s|}{|U|}$

Define the $A_1, A_2, \cdots, A_n$ *'s mathematical expectation is*

$$E(A_1 \cup A_2 \cup \cdots \cup A_n)=\sum_{i,j,\cdots,s}|X_i \cap Y_j \cap \cdots \cap Z_s| P(X_i \cap Y_j \cap \cdots \cap Z_s)$$

and define the $A_1, A_2, \cdots, A_n$ *'s variance is*

$$D(A_1 \cup A_2 \cup \cdots \cup A_n) = \sum_{i=1}^{m} P(X_i \cap Y_j \cap \cdots \cap Z_s)[| X_i \cap Y_j \cap \cdots \cap Z_s | - E(A_1 \cup A_2 \cup \cdots \cup A_n)]^2$$

Theorem 7. *Set up* $S=(U,A)$ *is a complete information system,* $\forall A_1, A_2, \cdots, A_n \subseteq \mathrm{A}$, *Mark* $U / IND(A_1)=\{X_1, X_2, \cdots, X_{m_1}\}$

$$U / IND(A_2)=\{Y_1, Y_2, \cdots, Y_{m_2}\}$$
$$U / IND(A_n)=\{Z_1, Z_2, \cdots, Z_{m_n}\}$$
$$U / IND(A_1 \cup A_2 \cup \cdots \cup A_n)$$
$$=\{X_1 \cap Y_1 \cap \cdots \cap Z_1, \cdots, X_i \cap Y_j \cap \cdots \cap Z_s, \cdots, X_{m_1} \cap Y_{m_2} \cap \cdots \cap Z_{m_n}\}$$

Then

$$E(A_1 \cup A_2 \cup \cdots \cup A_n) \le \min\{E(A_1), E(A_2), \cdots, E(A_n)\}$$

Proof. For any $A_i \in A_1, A_2, \cdots, A_n$,has $A_1 \cup A_2 \cup \cdots \cup A_n \preceq A_i$,and E is a knowledge granularity in the sense of Definition 1, so
$E(A_1 \cup A_2 \cup \cdots \cup A_n) \le E(A_i)$, that

$$E(A_1 \cup A_2 \cup \cdots \cup A_n) \le \min(E(A_1), E(A_2), \cdots, E(A_n))$$

This theorem states that the more knowledge, the stronger classification ability, knowledge granularity smaller, consistent with people's subjective understanding.

5 Conclusion

Knowledge granularity is inherent in human general activities in order to understand the problems better. Its aim is to divide the problem into a series of sub-problems that can be handled in order to solve the practical problems effectively and reduce the computational complexity. This essay discusses the concept of knowledge granularity under a comprehensive information system, proposes a series of knowledge granularity of measurement methods on this basis and analyzes the expectation, variance in the ability to measure the role of the knowledge divided. The knowledge granularity measurement method in this essay can be applied to complete the knowledge reduction, attribute significance and rule acquisition, etc. in the information systems, and has some theoretical significance and practical values for the establishment of granular computing in the complete information system.

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Relationships among Fuzzy Entropy, Similarity Measure and Distance Measure of Intuitionistic Fuzzy Sets

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Abstract. Fuzzy entropy, similarity measure and distance measure are three important measures of intuitionistic fuzzy sets. Their relationships are studied in this paper through operations between intuitionistic fuzzy sets, and some formulas are given to transform a measure into another. Additionally, some examples are presented to show how to use the formulas above.

Keywords: Intuitionistic fuzzy sets, Fuzzy entropy, Similarity measure, Distance measure.

1 Introduction

Fuzzy entropy is used to measure the fuzziness of fuzzy sets, which was introduced by Zadeh [7] in 1968. In 1972, De Luca and Termini [8] proposed the axiomatic definition of fuzzy entropy. Similarity measure and distance measure are dual concepts, which are used to measure the similarity of two fuzzy sets.

In 1983, Atanassov [1] introduced the intuitionistic fuzzy set. Different from the fuzzy set proposed by Zadeh in 1965 [28], the intuitionistic fuzzy set is characterized by a non-membership function as well as a membership function. Two kinds of non-classical sets are essentially the same as the intuitionistic fuzzy set, one of which is the interval valued fuzzy set, and the other is the vague set. For more details we refer the readers to [9-11].

Fuzzy entropy has been applied in different fields, including pattern recognition [12], neural network [13], image processing [14], etc. Distance measure and similarity measure have been applied in pattern recognition, neural network, image processing, uncertain reasoning [15-22], etc.

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2 Preliminaries

Definition 1. [1]. *An intuitionistic fuzzy set on* X *is an expression* A *given by*

$$A=\{<x,\mu_A(x),\nu_A(x)>|\,x\in X\}$$

where

$$\mu_A\in[0,1],\nu_A(x)\in[0,1]$$

with the condition $0\le\mu_A(x)+\nu_A(x)\le 1$ *for all* $x\in X$.

The numbers $\mu_A(x)$ *and* $\nu_A(x)$ *denote respectively the degree of membership and the degree of non-membership of the element* x *to set* A *, respectively.*

Additionally, $s_A(x)$ *,* $\mu_{AB}(x)$ *and* $\nu_{AB}(x)$ *represent* $\mu_A(x)-\nu_A(x)$ *,* $|\mu_A(x)-\mu_B(x)|$ *and* $|\nu_A(x)-\nu_B(x)|$*, respectively.*

Throughout this paper, $P(X)$ and $\Phi(X)$ denote all the crisp sets and intuitionistic fuzzy sets in the universe of discourse X .

Eulalia Szmidt and Janusz Kacprzyk [2] proposed the following axiomatic definition of fuzzy entropy of intuitionistic fuzzy sets.

Definition 2. *A real function* $E:\ \Phi(X)\to[0,1]$ *is called a fuzzy entropy on* $\Phi(X)$*, if* E *satisfies the following properties*

($E1$) $E(A)=0\Leftrightarrow A\in P(X)$

($E2$) $E(A)=1\Leftrightarrow\forall x\in X,\mu_A(x)=\nu_A(x)$

($E3$) $E(A)\le E(B)$ *if* A *is less fuzzy than* B*, i.e.* $\mu_A(x)\le\mu_B(x)$ *and* $\nu_A(x)\ge\nu_B(x)$ *for* $\mu_B(x)\le\nu_B(x)$ *or* $\mu_B(x)\ge\nu_B(x)$ *and* $\nu_A(x)\le\nu_B(x)$ *for* $\mu_B(x)\ge\nu_B(x)$*;*

($E4$) $E(A)=E(A^c)$.

($E3$) *can be improved as follows:*

($E3'$) $E(A)\le E(B)$ *if* A *is less fuzzy than* B*. Meanwhile, if there exists* $x\in X$ *such that* $|s_A(x)|>|s_B(x)|$*, then* $E(A)<E(B)$.

Definition 3.[1]. *Suppose that* $A,B\in\Phi(X)$ *, then we have the following operations:*

$$A^c=\{<x,\nu_A(x),\mu_A(x)>|\,x\in X\}$$
$$A\bigcap B=\{<x,\mu_A(x)\wedge\mu_B(x),\nu_A(x)\vee\nu_B(x)\,|\,x\in X\}$$
$$A\bigcup B=\{<x,\mu_A(x)\vee\mu_B(x),\nu_A(x)\wedge\nu_B(x)\,|\,x\in X\}$$

$$A \subseteq B \Leftrightarrow \forall x \in X, \mu_A(x) \le \mu_B(x) \text{ and } \nu_A(x) \ge \nu_B(x)$$
$$A = B \Leftrightarrow \forall x \in X, \mu_A(x) = \mu_B(x) \text{ and } \nu_A(x) = \nu_B(x),$$

where $\wedge$ *and* $\vee$ *denote taking the minimum and the maximum, respectively.*

Definition 4. *A real function* $s: \Phi(X) \times \Phi(X) \to [0,1]$ *is called a similarity measure of intuitionistic fuzzy sets if* s *satisfies the following properties for all* $A, B, C \in \Phi(X)$

($S1$) $s(A,B) = s(B,A)$

($S2$) $s(A,B) = 0 \Leftrightarrow A \in P(X), B = A^c$

($S3$) $s(A,B) = 1 \Leftrightarrow A = B$

($S4$) *if* $A \subseteq B \subseteq C$ *then* $s(A,C) \le s(A,B) \wedge s(B,C)$.

Definition 5. *A real function* $d: \Phi(X) \times \Phi(X) \to [0,1]$ *is called a distance measure of intuitionistic fuzzy sets if* d *satisfies the following properties for all* $A, B, C \in \Phi(X)$*:*

($D1$) $d(A,B) = d(B,A)$

($D2$) $d(A,B) = 0 \Leftrightarrow A = B$

($D3$) $d(A,B) = 1 \Leftrightarrow A \in P(X), B = A^c$

($D4$) *if* $A \subseteq B \subseteq C$ *then* $d(A,C) \ge d(A,B) \vee d(B,C)$.

It may be seen that similarity measure and distance measure are dual concepts, i.e., for all $A, B \in \Phi(X)$, have $s(A,B) = 1 - d(A,B)$.

3 The Relationship between Fuzzy Entropy and Similarity Measure of Intuitionistic Fuzzy Sets

Theorem 1. *Let* s *be a similarity measure on* $\Phi(X) \times \Phi(X)$*. For every* $A \in \Phi(X)$ *denote*

$$E_1(A) = s(A \bigcup A^c, A \bigcap A^c)$$

then $E_1(A)$ *is a fuzzy entropy of* A.

Proof. We only need to prove that E_1 satisfies the properties in Definition 2.

First $E_1(A) = 0 \Leftrightarrow s(A \bigcup A^c, A \bigcap A^c) = 0 \Leftrightarrow A \bigcup A^c \in P(X)$ and $A \bigcap A^c = (A \bigcup A^c)^c \Leftrightarrow \forall x \quad \mu_A(x) \vee \nu_A(x) = 1$ and $\mu_A(x) \wedge \nu_A(x) = 0$ $\Leftrightarrow \forall x \quad \mu_A(x) = 1, \nu_A(x) = 0$ or $\mu_A(x) = 0, \nu_A(x) = 1 \Leftrightarrow A \in P(X)$.

Thus, E_1 satisfies the condition $(E1)$.

Next $E_1(A)=1 \Leftrightarrow s(A\bigcup A^c, A\bigcap A^c)=1 \Leftrightarrow A\bigcup A^c = A\bigcap A^c \Leftrightarrow$ $\forall x,\ \mu_A(x)\vee\nu_A(x)=\mu_A(x)\wedge\nu_A(x) \Leftrightarrow \forall x,\ \mu_A(x)=\nu_A(x)$.

Hence E_1 satisfies the condition $(E2)$.

Moreover, if $\mu_A(x)\le\mu_B(x)$ and $\nu_A(x)\ge\nu_B(x)$ for $\mu_B(x)\le\nu_B(x)$ or $\mu_A(x)\ge\mu_B(x)$ and $\nu_A(x)\le\nu_B(x)$ for $\mu_B(x)\ge\nu_B(x)$ then

$$\mu_A(x)\wedge\nu_A(x)\le\mu_B(x)\wedge\nu_B(x)\le\mu_B(x)\vee\nu_B(x)\le\mu_A(x)\vee\nu_A(x).$$

So have $A\bigcap A^c\subseteq B\bigcap B^c\subseteq B\bigcup B^c\subseteq A\bigcup A^c$.

By the condition ($S4$), we obtain

$$E_1(A)=s(A\bigcup A^c, A\bigcap A^c)\le s(A\bigcup A^c, B\bigcap B^c)$$

$$\le s(B\bigcup B^c, B\bigcap B^c)=E_1(B)$$

Then E_1 satisfies the condition $(E3)$.

Finally,

$$E_1(A)=s(A\bigcup A^c, A\bigcap A^c)=s(A^c\bigcup(A^c)^c, A^c\bigcap(A^c)^c)=E_1(A^c)$$

i.e. E_1 satisfies the condition $(E4)$.

This completes our proof.

The following theorems which transform similarity measure into fuzzy entropy can also been proved immediately according to Definition 2.

For all $A\in\Phi(X)$, an operation with respect to A is defined as follows:

$$g(A)=\{<x,|s_A(x)|,1-|s_A(x)|>|x\in X\}. \quad (1)$$

Furthermore we introduce a kind of special intuitionistic fuzzy set $[\alpha,\beta]_X$. For any $x\in X$, $\mu_{[\alpha,\beta]_X}\equiv\alpha$, $\nu_{[\alpha,\beta]_X}\equiv\beta$, where $\alpha\in[0,1]$, $\beta\in[0,1]$ with the condition $\alpha+\beta\in[0,1]$.

Theorem 2. *Let s be a similarity measure of intuitionistic fuzzy sets on $\Phi(X)\times\Phi(X)$. For any $A\in\Phi(X)$ denote*

$$E_2(A)=s([0,1]_X, g(A)) \text{ and } E_3(A)=s([1,0]_X, (g(A))^c),$$

then both $E_2(A)$ and $E_3(A)$ are fuzzy entropies of A.

If $s(A,B)=s(A^c,B^c)$ holds for all $A,B\in\Phi(X)$ then $E_2(A)=E_3(A)$.

Example 1. Let $X=\{x_1,x_2,\cdots,x_n\}$ and $A,B\in\Phi(X)$. Bustince and Burillo [27] proposed the formula below to calculate the distance measure of A and B:

$$d(A,B)=\frac{1}{2n}\sum_{i=1}^{n}(\mu_{AB}(x_i)+\nu_{AB}(x_i)).$$

Then $s(A,B)=1-\frac{1}{2n}\sum_{i=1}^{n}(\mu_{AB}(x_i)+\nu_{AB}(x_i))$ is a similarity measure of A and B.

We have that $E_2(A)=E_3(A)=s([0,1]_X,g(A))=1-\frac{1}{n}\sum_{i=1}^{n}|s_A(x_i)|$ is a fuzzy entropy of A by using Theorem 2.

For any $A\in\Phi(X)$, two intuitionistic fuzzy sets are defined as follows:

$$\varphi_1(A)=\{<x,0.5-0.5f_1(|s_A(x)|),0.5+0.5f_2(|s_A(x)|)>|x\in X\}, \quad (2)$$

$$\varphi_2(A)=\{<x,0.5+0.5f_3(|s_A(x)|),0.5-0.5f_4(|s_A(x)|)>|x\in X\}, \quad (3)$$

where $f_i(i=1,2,3,4)$ satisfies the following conditions for any $t\in[0,1]$:

(1) $f_i(t)\in[0,1]$

(2) $f_1(t)=f_2(t)=f_3(t)=f_4(t)=1$ if and only if $t=1$ and $f_1(t)=f_2(t)=f_3(t)=f_4(t)=0$ if and only if $t=0$

(3) if $0\le t_1\le t_2\le 1$ then $f_i(t_1)\le f_i(t_2)$.

Theorem 3. *Let* s *be a similarity measure of intuitionistic fuzzy sets on* $\Phi(X)\times\Phi(X)$. *For any* $A\in\Phi(X)$ *denote*

$$E_4(A)=s(\varphi_1(A),\varphi_2(A))$$

then $E_4(A)$ *is a fuzzy entropy of* A.

For any $A,B\in\Phi(X)$, two intuitionistic fuzzy sets are defined as follows:

$$u_1(A,B)=\{<x,0.25+0.25(\mu_{AB}(x)\wedge\nu_{AB}(x)),$$
$$0.75-0.25(\mu_{AB}(x)\vee\nu_{AB}(x))>|x\in X\}, \quad (4)$$
$$u_2(A,B)=\{<x,0.25-0.25(\mu_{AB}(x)\vee\nu_{AB}(x)),$$
$$0.75+0.25(\mu_{AB}(x)\wedge\nu_{AB}(x))>|x\in X\}. \quad (5)$$

Lemma 1. *Let* $A,B\in\Phi(X)$ *and* E *be a fuzzy entropy on* $\Phi(X)$ *which satisfies the condition (* $E3'$ *) then*

$$E(u_1(A,B))=E(u_2(A,B)) \text{ if and only if } A=B.$$

Proof. Obviously, $\mu_{u_2(A,B)}(x)\le\mu_{u_1(A,B)}(x)\le\nu_{u_1(A,B)}(x)\le\nu_{u_2(A,B)}(x)$.

If $A=B$ doesn't hold, then there exists $x\in X$ such that $\mu_{AB}(x)\vee\nu_{AB}(x)>0$.

Further, there exists $x\in X$ such that $|s_{u_1(A,B)}(x)|<|s_{u_2(A,B)}(x)|$.

Known by the condition ($E3$), we have $E(u_1(A,B))>E(u_2(A,B))$.

Conversely, if $A=B$, it is obvious that $u_1(A,B)=u_2(A,B)$, thereby we have

$$E(u_1(A,B))=E(u_2(A,B)).$$

Hence, the lemma above holds.

Theorem 4. *Let* E *be a fuzzy entropy on* $\Phi(X)$ *which satisfies (* $E3'$ *). For any* $A,B\in\Phi(X)$*, denote*

$$s_1(A,B)=\frac{E(u_2(A,B))}{E(u_1(A,B))}$$

then $s_1(A,B)$ *is a similarity measure of* A *and* B.

Proof. All we need is to prove that s_1 satisfies the properties in Definition 4.

First by using the definitions of $u_1(A,B)$ and $u_2(A,B)$ $s(A,B)=s(B,A)$ holds. That is to say, s_1 satisfies the condition ($S1$).

Next,

$$\begin{aligned} s(A,B)=0 &\Leftrightarrow E(u_2(A,B))=0, E(u_1(A,B))\neq 0 \\ &\Leftrightarrow \mu_{AB}(x)\vee\nu_{AB}(x)=\mu_{AB}(x)\wedge\nu_{AB}(x)=1 \text{ for each } x \\ &\Leftrightarrow A\in P(X), B=A^c, \end{aligned}$$

i.e. , s_1 satisfies the condition ($S2$).

Moreover, by using Lemma 1 we have

$$s(A,B)=1\Leftrightarrow E(u_1(A,B))=E(u_2(A,B))\Leftrightarrow A=B.$$

Thus, s_1 satisfies the condition ($S3$).

Finally, if $A\subseteq B\subseteq C$ then we have $\mu_A(x)\le\mu_B(x)\le\mu_C(x)$ and. $\nu_A(x)$
$\nu_B(x)\ge\nu_C(x)$. So we have

$$\mu_{AB}(x)\wedge\nu_{AB}(x)\le\mu_{AC}(x)\wedge\nu_{AC}(x);$$

$$\mu_{AB}(x)\vee\nu_{AB}(x)\le\mu_{AC}(x)\vee\nu_{AC}(x).$$

Then have

$$0.25-0.25(\mu_{AC}(x)\vee\nu_{AC}(x))\le 0.25-0.25(\mu_{AB}(x)\vee\nu_{AB}(x))\le$$
$$0.75+0.25(\mu_{AB}(x)\wedge\nu_{AB}(x))\le 0.75+0.25(\mu_{AC}(x)\wedge\nu_{AC}(x))$$
$$0.25+0.25(\mu_{AC}(x)\wedge\nu_{AC}(x))\le 0.25+0.25(\mu_{AB}(x)\wedge\nu_{AB}(x))\le$$
$$0.75-0.25(\mu_{AC}(x)\vee\nu_{AC}(x))\le 0.75-0.25(\mu_{AB}(x)\vee\nu_{AB}(x)).$$

By the condition ($E3'$), we have

$$E(u_2(A,C))\le E(u_2(A,B)) \text{ and } E(u_1(A,C))\ge E(u_1(A,B))>0.$$

Hence,

$$s_1(A,B)=\frac{E(u_2(A,B))}{E(u_1(A,B))}\ge\frac{E(u_2(A,C))}{E(u_1(A,C))}=s_1(A,C).$$

Similarly, $s_1(B,C)\ge s_1(A,C)$ holds.

So, $s_1(A,C)\le s_1(A,B)\wedge s_1(B,C)$, i.e. s_1 satisfies the condition ($S4$).

Thus, the proof has been completed.

Corollary 1. *Let E be a fuzzy entropy on $\Phi(X)$ which satisfies the condition ($E3'$).*

Then $\dfrac{E((u_2(A,B))^c)}{E(u_1(A,B))}$, $\dfrac{E(u_2(A,B))}{E((u_1(A,B))^c)}$ *and* $\dfrac{E((u_2(A,B))^c)}{E((u_1(A,B))^c)}$ *are all similarity measures of A and B for all $A,B\in\Phi(X)$.*

Example 2. Assume that $X=\{x_1,x_2,\cdots,x_n\}$ and $A\in\Phi(X)$. By Example 1, we have that $E(A)=1-\frac{1}{n}\sum_{i=1}^{n}|s_A(x_i)|$ is a fuzzy entropy of A. Evidently, E satisfies the condition ($E3'$). By using Theorem 4,

$$s_1(A,B)=\frac{E(u_2(A,B))}{E(u_1(A,B))}=\frac{1-\sum_{i=1}^{n}[0.5+0.25(\mu_{AB}(x_i)+\nu_{AB}(x_i))]}{1-\sum_{i=1}^{n}[0.5-0.25(\mu_{AB}(x_i)+\nu_{AB}(x_i))]}$$

is a similarity measure of A and B.

For any $A,B\in\Phi(X)$, we define the following intuitionistic fuzzy set:

$$\mathrm{H}(A,B)=\{<x,0.5+0.5f_1(\mu_{AB}(x),\nu_{AB}(x)),$$
$$0.5-0.5f_2(\mu_{AB}(x),\nu_{AB}(x))>|\,x\in X\},\qquad(6)$$

where $f_i(i=1,2)$ satisfies the following conditions for any $s,t\in[0,1]$

(a) $f_i(s,t) \in [0,1]$;

(b) $f_1(s,t) \le f_2(s,t)$;

(c) if $s_1 \le s_2, t_1 \le t_2$ then $f_i(s_1,t_1) \le f_i(s_2,t_2)$;

(d) $f_1(s,t) = f_2(s,t) = 1$ iif $s = t = 1$; $f_1(s,t) = f_2(s,t) = 0$ iff $s = t = 0$.

Theorem 5. *Let E be a fuzzy entropy on $\Phi(X)$. For any $A, B \in \Phi(X)$, denote*

$$s_2(A,B) = E(\mathrm{H}(A,B)),$$

then $s_2(A,B)$ is a similarity measure of A and B.

The above-mentioned theorem can be proved by Definition 4 immediately.

4 The Telationship between Fuzzy Entropy and Distance Measure of Intuitionistic Fuzzy Sets

For all $A, B \in \Phi(X)$, an intuitionistic fuzzy set $\psi(A,B)$ is defined as follows:

$$\psi(A,B) = \{< x, 0.5(\mu_{AB}(x) \wedge \nu_{AB}(x)), 1 - 0.5(\mu_{AB}(x) \vee \nu_{AB}(x)) > | x \in X\} \tag{7}$$

Theorem 5. *Let E be a fuzzy entropy on $\Phi(X)$. For $\forall A, B \in \Phi(X)$ denote*

$$d_1(A,B) = E(\psi(A,B)),$$

then $d_1(A,B)$ is a distance measure of A and B.

Proof. Trivial from Definition 5.

Example 3. Let $X = \{x_1, x_2, \cdots x_n\}$ $A \in \Phi(X)$, the fuzzy entropy of A defined by Szmidt and Kacprzyk[24] may be easily transformed into the form below:

$$E(A) = \sum_{i=1}^{n} \frac{2 - |s_A(x_i)| - (\mu_A(x_i) + \nu_A(x_i))}{2 + |s_A(x_i)| - (\mu_A(x_i) + \nu_A(x_i))}.$$

By using Theorem 5, $d_1(A,B) = E(\psi(A,B)) = \frac{1}{n}\sum_{i=1}^{n} \frac{\mu_{AB}(x_i) \vee \nu_{AB}(x_i)}{2 - \mu_{AB}(x_i) \wedge \nu_{AB}(x_i)}$ is a distance measure of A and B.

Theorem 6. *Let E be a fuzzy entropy on $\Phi(X)$ which satisfies the condition ($E3'$). For any $A, B \in \Phi(X)$ denote*

$$d_2(A,B) = E(u_1(A,B)) - E(u_2(A,B))$$

then $d_2(A,B)$ *is a distance measure of* A *and* B.

The theorem above can be proved directly by Definition 5 and the following theorems can be proved according to Definition 2.

For any $A \in \Phi(X)$, $h(A) \in \Phi(X)$ is defined as:

$$h(A) = \{< 0.5 | s_A(x) |, 1 - 0.5 | s_A(x) | > | x \in X\}. \tag{8}$$

Theorem 7. *Let* d *be a distance measure on* $\Phi(X) \times \Phi(X)$. *For any* $A \in \Phi(X)$, *denote*

$$E_5(A) = d(h(A), (h(A))^c),$$

then $E_5(A)$ *is a fuzzy entropy of* A.

Theorem 8. *Let* d *be a distance measure on* $\Phi(X) \times \Phi(X)$. *For any* $A \in \Phi(X)$, *denote that*

$$E_6(A) = d([1,0]_X, g(A)) \text{ and } E_7(A) = d((g(A))^c, [0,1]_X),$$

then both $E_6(A)$ *and* $E_7(A)$ *are fuzzy entropies of* A.

If $d(A,B) = d(A^c, B^c)$ *holds for any* $A, B \in \Phi(X)$, *then* $E_6(A) = E_7(A)$.

5 Conclusion

We study relationships among fuzzy entropy, similarity measure and distance measure of intuitionistic fuzzy sets in this paper. Some formulas are put forward to transform one measure into another. By using them, some new formulas to calculate fuzzy entropy, similarity measure or distance measure may be obtained. Evidently, all the conclusions hold when intuitionistic fuzzy sets are reduced to traditional fuzzy sets.

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An Improved Algorithm of Unbalanced Data SVM

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Abstract. Since SVM is unfair to the rare class for the classification of unbalanced data, a new balancing strategy based on common strategy of undersampling the training data is presented. Firstly, the fuzzy C-means clustering algorithm is used to cluster the unbalanced data sets, and choose the negative class samples whose memberships are greater than a certain threshold (supposing the number of positive class samples is less than that of negative class samples). The selected samples and the positive class of original samples are combined into a new training data set. After that, the new data set are used to train a support vector machine. At last, the simulations on unbalanced data show that the proposed algorithm can compensate the ill-effect of tendency when support vector machine are utilized to deal with the unbalanced data classification. Moreover, compared with the traditional support vector machine and some other improved algorithm, the proposed algorithm performs superior classification ability.

Keywords: SVM, fuzzy C- means cluster, undersampling, unbalanced data.

1 Introduction

Support Vector Machine (SVM) was introduced by Vapnik and his colleagues [1], which is a machine learning method based on statistical learning theory. It has been successfully applied to image retrieval [2], handwriting recognition [3] and text recognition [4], and etc. But, when it deals with unbalanced data sets (for example, the negative is far greater than the positive sample), the classification ability of SVM is very limited. Application areas such as gene profiling, medical diagnosis and credit card fraud detection have highly skewed datasets which are hard to classify correctly. Based on this, oversampling as a popular method was proposed by [5]. However, the new samples are difficult to guarantee independent and identically distributed with the

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original samples and it also increases the training burden; other method such as undersampling the training data was proposed by [6], which enhances the training speed successfully. However, the classification accuracy is affected for the losing of sample information. This paper proposes a new strategy that balances the unbalanced data set, which clusters the unbalanced data set by fuzzy C-means clustering. We find that the fuzzy membership larger than a certain threshold makes a major contribution to the classification. So, we choose the samples whose fuzzy membership is larger than a certain threshold. We can make the two types of data sets achieve a relatively balance and loss a few sample information by reducing the large class training samples to pass through adjusting the threshold. The proposed method makes up the weakness of classification on the unbalanced data by the regular support vector machine. Moreover, the experiments show the feasibility and effectiveness of the proposed method.

2 Support Vector Machine

Support Vector Machine is a machine learning method based on structural risk minimization principle which is proposed by Vapnik. It possesses an excellent learning capacity and the generalization performance on finite samples. As for the SVM classifier, the goal is to find a separating hyperplane with a margin as large as possible to minimize the classification error. The decision function of SVM with a pre-selected non-linear mapping maps the input data into a high dimensional feature space. At the same time, it solves the problem of high computational complexity in the high dimensional space by using of kernel function.

Let the two categories of samples be

$$S = \{(x_1, y_1), \ldots, (x_l, y_l)\}.$$

And the anonymous mapping of kernel function be $\phi(x)$. The training set mapped to high dimensional feature space is $(\phi(x_i), y_i)$. Separating hyperplane is $\omega \cdot \phi(x) + b = 0$. Therefore, the optimization problem is need to be solved

$$\begin{array}{l} min\frac{1}{2} \parallel \omega \parallel^2 + C\sum_{i=1}^{l} \xi_i \\ s.t.\ y_i[\omega \cdot \phi(x_i) + b] - 1 + \xi_i \geq 0; \\ \xi_i \geq 0;\ i = 1, 2, \cdots, l, \end{array} \tag{1}$$

where C is a constant. By constructing the Lagrange function and saddle point condition gets the dual problem of original problem type (1) can be obtained below

$$\begin{array}{l} min\frac{1}{2} \sum_{i=1}^{l} \sum_{j=1}^{l} \alpha_i \alpha_j y_i y_j K(x_i, x_j) - \sum_{i=1}^{l} \alpha_i \\ s.t.\ \sum_{i=1}^{l} \alpha_i y_i = 0; \\ 0 \leq \alpha_i \leq C \end{array} \tag{2}$$

$K(x_i, x_j) = \phi(x_i) \cdot \phi(x_j)^T$ is a kernel function. Hence convex quadratic programming can get the only optimal solution. $0 \leq \alpha_i^* \leq C$ corresponded to sample x_i is support vector. The separating hyperplane's normal vector is

$$\omega^* = \sum_{i=1}^{l} \alpha_i^* y_i \phi(x_i). \tag{3}$$

Select a support vector x_i and substitute it into the constraints of type (1), the hyperplane's threshold can be obtained as follows

$$b^* = \sum_{i=1}^{l} y_i \alpha_i^* K(x_i \cdot x_j) - y_j. \tag{4}$$

Hence the classification function is

$$f(x) = sgn(\sum_{i=1}^{l} \alpha_i^* y_i K(x_i \cdot x) + b^*). \tag{5}$$

3 Fuzzy C-Means Clustering Algorithm

Fuzzy C-means (FCM) is an unsupervised clustering algorithm that has been applied successfully to a number of problems involving feature analysis, clustering and classifier design. Fuzzy C-means clustering is a kind of clustering algorithm that makes sure the sample belong to the certain category's degree by membership. In 1973, Bezdek [7] proposed the algorithm which is a kind of improvement of early hard c-mean clustering (HCM).

FCM divides the n vectors $x_i (i = 1, 2, \cdots, n)$ into c fuzzy groups, calculates the clustering center of each group, and minimizes the non-similarity index value function. FCM adopts fuzzy partition to make each given value of data point between 0 and 1 to determine the degree of it belonging to a group. With fuzzy partition, the element of membership matrix is allowed in the values between 0 and 1. After normalizing, the combined membership of a dataset as follow

$$\sum_{i=1}^{c} u_{ij} = 1, \ \forall j = 1, 2, \cdots, n. \tag{6}$$

So, the objective function of FCM can be obtained below

$$J(U, c_1, \cdots, c_c) = \sum_{i=1}^{c} J_i = \sum_{i=1}^{c} \sum_{j=1}^{n} u_{ij}^m d_{ij}^2. \tag{7}$$

Provided that $u_{ij} \in [0, 1]$ and c_i is the clustering center of fuzzy group i, $d_{ij} = \| c_i - x_j \|$, denotes the Euclidean distance between i th clustering center and j th data point, $m \in [1, \infty]$ is a weighted index. Constructing the new object function as follows, get the necessary condition by minimizing (7)

$$\begin{aligned} & J'(U, c_1, \cdots, c_c, \lambda_1, \cdots, \lambda_n) \\ &= J(U, c_1, \cdots, c_c) + \textstyle\sum_{j=1}^{n} \lambda_j (\sum_{i=1}^{c} u_{ij} - 1) \\ &= \textstyle\sum_{i=1}^{c} \sum_{j=1}^{n} u_{ij}^m d_{ij}^2 + \sum_{j=1}^{n} \lambda_j (\sum_{i=1}^{c} u_{ij} - 1), \end{aligned} \tag{8}$$

$\lambda_j, j = 1, \cdots, n$ is constraints' Lagrange multiplier of (6). Derivative to all input parameters and get the necessary condition of minimizing type (7)

$$c_i = \frac{\sum_{j=1}^{n} u_{ij}^m x_j}{\sum_{j=1}^{n} u_{ij}^m} \tag{9}$$

and

$$u_{ij} = \frac{1}{\sum_{k=1}^{c} (\frac{d_{ij}}{d_{kj}})^{2/(m-1)}}. \tag{10}$$

From the above two necessary conditions, the fuzzy C-means clustering algorithm is a simple iterative progress. In batch mode, FCM determines clustering center c_i and membership matrix U by the following steps,

Step 1: Initialize the membership matrix with the random value between 0 and 1, and make it satiety the constraint conditions of type (6).
Step 2: Calculate c clustering centers $c_i, i = 1, \cdots, c$.
Step 3: Calculate target function according to the type (7), given a convergent threshold, if it is smaller than the threshold or the change value to the last object function value smaller than the threshold, then stop the iterations.
Step 4: Calculate the new matrix U, return the Step 2.

4 The Unbalanced SVM Based on FCM

We mainly discuss two categories of classification with the training samples

$$(x'_1, y'_1), (x'_2, y'_2), \cdots, (x'_m, y'_m), (x'_{m+1}, y'_{m+1}), (x'_{m+2}, y'_{m+2}), \cdots, (x'_n, y'_n),$$

$x'_i \in R^n$ denotes the feature of each sample, its corresponding category identifier is

$$y'_i = \begin{cases} +1 & i=1,\cdots,m \\ -1 & i=m+1,\cdots,n \end{cases}, \qquad n - m \gg m,$$

which denote the number of negative sample outweigh than the positive sample.

1) Cluster the training samples by the FCM that introduced in the front and let the clustering number be c, get the membership matrix $U = (u_{ij})_{n\times c}$.

2) After clustering, choose the category which has the biggest number sample satieties $u_{ij} > \gamma$, let it be j. Given the threshold γ and select the positive sample which satieties $u_{ij} > \gamma$

$$(x_{m+1}, y_{m+1}), (x_{m+2}, y_{m+2}), \cdots, (x_l, y_l).$$

Constitute the new training set with the original positive sample

$$(x_1, y_1), (x_2, y_2), \cdots, (x_m, y_m), (x_{m+1}, y_{m+1}), (x_{m+2}, y_{m+2}), \cdots, (x_l, y_l).$$

Here make $l - m$ roughly equal m to balance the two kinds of samples.

3) Train the new training samples with the SVM algorithm and get the optimal decision-making function

$$f(x) = sgn(\sum_{i=1}^{l} \alpha_i y_i K(x_i \cdot x) + b).$$

Test the original testing samples and get the testing accuracy.

5 Simulation Results and Conclusions

In this paper, MATLAB programming language and LIBSVM under DOS are adopted on the PC to perform on the artificial and real data sets. In the experiment, the artificial data set together with the svmguide3 and diabetes datasets in the UCI repository [8] are preprocessed. Provided with the cluster labels $c = 2, 3, 4$, the training samples are grouped into their corresponding classes. The samples satisfying $u_{ij} > \gamma$ are chosen. Then, the positive samples in the class with most amount of samples are combined with the original negative samples to compose a new training set. u_{ij} denotes the membership that the ith sample belongs to the jth class. The original artificial data set and its counterpart tackled by FCM are show in Figure 1 and 2, respectively.

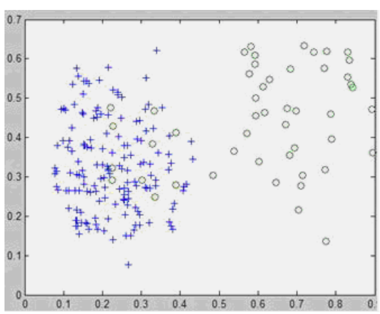

Fig. 1. Original artificial data set

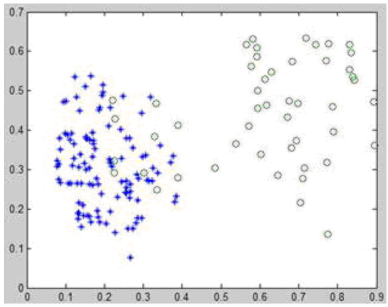

Fig. 2. FCM treatment of artificial data set

Under the same operating environment, experiments on the real data sets respectively adopt the SVM, UNDERSAMPLING+SVM (called for short US) proposed in [6]. FCM+SVM proposed in [9], and the unbalanced data FCM+US proposed in this paper. Give the mathematical expression of Gaussian RBF kernel function is selected and let the parameter $c = 5, 10, 100$. Let the threshold γ be 0.9 and 0.7 for the artificial data set and the real data sets, respectively. Under the same parameters, run 5 times and select the average value as the final test accuracy rate. The results are summarized in Table 1,

Table 1. The experiment results of four algorithms

$Dataset(attribute)$ $(train/test)$	Train(+/-)			Test(+/-)	Accuracy (%)
	$FCM+US$	$C=4$	421(207/214)	54(0/54)	71.5210
$diabetes8$ (714/54)	US		421(207/214)	54(0/54)	68.5185
	$FCM+SVM$	$C=3$	591(407/184)	54(0/54)	63.5802
	SVM		714(500/214)	54(0/54)	40.1234
	$FCM+US$	$C=4$	588(296/292)	41(41/0)	91.0569
$svmguide3(22)$ (1243/41)	US		588(296/292)	41(41/0)	85.3659
	$FCM+SVM$	$C=3$	724(132/592)	41(41/0)	56.0976
	SVM		1243(296/947)	41(41/0)	76.4228
	$FCM+US$	$C=2$	156(80/76)	138(121/17)	87.7697
$artificialdata(2)$ (207/138)	US		156(80/76)	138(121/17)	80.3357
	$FCM+SVM$	$C=4$	179(131/48)	138(121/17)	84.6523
	SVM		207(131/76)	138(121/17)	78.8969

In this paper, we treat the unbalanced data set with the method of fuzzy C-means clustering. According to the fuzzy membership matrix, the samples in the class with the most number of samples are chosen. Then, the chosen samples are combined with the samples in the class with less samples to composed a new training set. Finally, the new training data set are classified by

SVM. Numerical experiments show that the FCM+US method can effectively enhance the training speed and classification accuracy. Moreover, it can also enhance the performance of SVM on tackling unbalanced data classification.

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An Efficient Successive Iteration Partial Cluster Algorithm for Large Datasets

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Abstract. A successive partial cluster algorithm for large data sets is proposed in this paper. In each iteration of this algorithm, only a new cluster is generated by successive iteration method. Firstly, typical point was selected successively based on sampling density and then cluster is constructed by using this typical point as seed. When the first cluster is produced, those data items contained by this cluster are deleted timely. Repeat this process until the remaining data set is small enough to read into memory. The rest of the data items are assigned to the nearest cluster respectively. The effective of this algorithm is verified by several experiments.

Keywords: large data set, successive iteration partial clustering, cluster analysis, sampling density.

1 Introduction

Clustering is one of the widely used knowledge discovery technique to reveal structure in the data set that can be extremely useful to the analysis [1]. Clustering is to group data points into several clusters and make the intra-cluster similarity maximized and the inter-cluster similarity minimized [2][3]. Clustering is an unsupervised machine learning technique which is different from the supervised learning, since we know nothing about the distribution of data in advance.

In recent years, cluster analysis has undergone vigorous development. Many cluster algorithms are produced[4]. However, there still exist many problems to solve. With the rapid growth of the amount information, large unlabeled data sets become available. The general algorithms are difficult to deal with large scale data, because they will require much time to calculate the distance between data objects. Also, the compute memory is not enough for the huge data sets. How to enhance the cluster algorithms efficiency is very important for large data sets[5]. Cluster analysis faces two problems to

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be solved[6][7]. For one thing, the efficiency of clustering is important. Sometimes, some of the problems do not need to be very precise. However, such problems need fast and efficient computing, especially for large data set. Improved comprehension of clustering results is another problem. Because clustering results are several partitions, sometimes hard to comprehend. In this paper, we solve the former problem mainly.

This paper is organized as follows. Section 2, we advance the successive iteration partial cluster algorithm and give the steps. Experimental results are given in Section 3. Section 4 we make a summary of the paper and give a few remarks on the future work.

2 Successive Iteration Partial Clustering Algorithms

The successive iteration partial cluster algorithm is primarily designed for efficiency. The primary idea of this algorithm is select a random sample form the original data firstly. Then search the typical cluster based density from this sample and calculate the center of this typical cluster. This center is called typical point. With this typical point as seed begin to cluster by calculating the Euler distance between the typical point and all points in the database. When the first cluster is produced, remove it timely. Repeat above steps until the remaining data set is small enough to read into memory. The rest data assigned to the nearest cluster respectively.

2.1 Random Sample and Typical Point

Because the data set is huge, from the original data set select a small sample A_1 randomly and set the two parameters values r (which is the radius) and (which is the density threshold). Searches for the first typical cluster by checking the each point's $r-neighborhood$ in the sample A_1. If the $r-neighborhood$ of a point p contains more than $Minpts$. A new cluster with p is created and call it typical cluster as P . Then calculate this typical cluster's center C_1 as $C_1 = \frac{\sum_{x_i \epsilon P} x_i}{|P|}$, $|P|$ is the number of data items in the typical cluster. The center C_1 is called typical point.

2.2 Generating the First Cluster

Take the center C_1 as seed begin to cluster. The set T_1 is initially empty. Firstly C_1 is assigned to T_1. The set T_1 is incrementally built for each pattern X in data set. If the distance between data items X and the center C_1 are at less than the threshold r, the pattern X is assigned to the set T_1. The process terminates when no new points can be added to the set T_1.Then the first cluster T_1 will be obtained. Delete the data in the database which is contained in $T_1$to reduce the size of original data set.

2.3 Analysis of the Rest Data

When the rest data is small enough to read into memory, the algorithm will stop. A series of sets $T_1, T_2, \ldots, T_k$ will be obtained. By calculating the Euler distance between the cluster's centers and the rest data items, the remaining data are assigned to the nearest cluster. In this way, the data set will be divided into k partitions $T_1^{'}, T_2^{'}, \ldots, T_k^{'}$.

2.4 Successive Iteration Partial Clustering Algorithm

The pseudo-code of the successive iteration partial clustering algorithm is described as follows:

Initialize number r, $Minpts$ and the empty set T_1.

Select a sample A from database by random and find the typical class P, calculate the typical points C and assign it to T_1.

Mark the number of these points with 'Len'.

Loop: for each point C in A do
 if C is typical point then
 for each point X in database do
 if $|C - X| > r$ then
  create a new set T_2;
  assign X into T_2 and delete X in database;
  record the number of points inserted 'Len';
 end if $|C - X| > r$
  end for each point X in A;
 end if C is typical point;
  end for each point in A;
  T_1=T_2;
  If $Len > Minpts$ then
  Goto Loop;
 Else
 Stop program;
 End if
 $Len > Minpts$;

Our approach yields a speedup for three reasons:

1) Because we use the random sample which size is smaller than the entire data set, the typical centers we find in the samples are more quickly and easily than calculate the center in the whole data. We don't need to calculate the center many times for each sample.

2) Because the centers are the average of the typical sample, these centers are typical. With these typical points as seed, the size of the data set condensed quickly.

3) When get the clusters, we remove them timely and delete those data items in initial data set. The original data is significantly reduced. In this way, the efficiency algorithm is greatly improved.

3 Experiment and Discussion

To test the performance of our algorithm in this paper, we select six real world data set, the Wine data, Yeast data set; Page-blocks data set, Pen Digits data set, shuttle data set and magic data set form UCI Machine Learning Repository[8]in the experiments. All the computing is performed on an Inter core 2.8 GHz 512 of memory.

Table 1. Data sets used in experiment

Data sets	Size	Features	Classes
Wine	178	13	3
Yeast	1484	8	7
Page-blocks	5473	10	5
Pen Digits	10992	16	10
Shuttle	14500	9	7
Magic	19020	10	2

3.1 Description of the Data

The Table 1 describes the type of the data and shows the information of attributes and size to us. Wine recognition data set are the result of a chemical analysis of wine grown in the same region in Italy but derived from three different cultivars. The Yeast set has relatively low classification rate. It is reason that the FSFC+ is applicable due to the characteristic of multiple classes and the data imbalance problem[9]. The Page-blocks data set has been used to try different simplification methods for decision trees. This data set is made up of 5473 data items. The 5473 examples come from 54 distinct documents. Each observation concerns one block. Magic data set generated to simulate registration of high energy gamma particles in a ground-based atmospheric Cherenkov gamma telescope using the imaging technique.

3.2 Discussion of the Results

Table 2 show the results that we test our algorithm on the Wine data set, Yeast, Page-block, Pen Digits shuttle(training set) and Magic data set. where, Var_1 is the Variance of average running time, Var_2 is the Variance of average accuracy. Using the algorithm runs five times for each database. The average time, the average accuracy and their variance. From the table we know, variance is greater than 0 less than 1. So the algorithm proposed in this paper is stable. As the algorithm is stable, so for a new data set we can roughly predict the running time. The accuracy is stable too. It is convenient to analyze a new data set's running time and its accuracy. The Fig. 1 shows the relationship between running time and the size of data set. The Fig. 2 shows the relationship of accuracy and the size of data set. In Fig. 1 the trend

Table 2. The result of running time, accuracy and their variance

Data sets	Average time(sec)	Var_1	Average accuracy	Var_2
Wine	0.125	0.0144	96%	0.03347
Yeast	1.447	0.0597	85.3%	0.0541
Page-blocks	1.268	0.0409	95.2%	0.02168
Pen Digits	4.389	0.2911	92.6%	0.0305
Shuttle	6.384	0.9837	87.8%	0.01303
Magic	18.236	0.6962	94.8%	0.0192

of time is steady growth with the increasing amount of data size. Form figure we know the execution time is influenced by the number of data set and the attributes of data. But the classification accuracy affected by the size of is not obvious. The average accuracy is shown in Fig. 2. For these data sets the classification accuracy of the algorithm is about 90%. From the result we can see our algorithm has good scalability and stability. Form the accuracy point of view may not be better, but form the time consumption and the classification accuracy for both our algorithm is a better one.

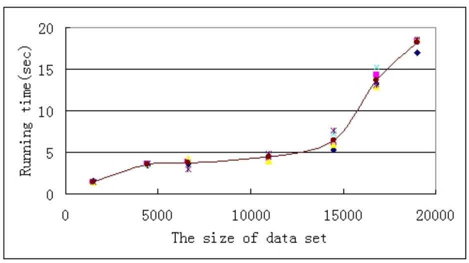

Fig. 1. The size of data set and its run time

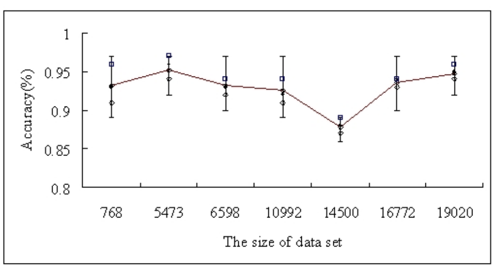

Fig. 2. The size of data and accuracy

4 Conclusions and Future Work

Clustering analysis technique plays an important role in data analysis and has a good prospect in data mining, pattern recognition and artificial intelligence. In this paper, the algorithm we proposed is for large data set. Experimental results show that this method is feasible. However, it is not final method. Future work could be done to improve the accuracy of the algorithm and further reduce the running time.

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Ant Spatial Clustering Based on Fuzzy IF–THEN Rule

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Abstract. Various clustering methods based on the behavior of real ants have been proposed. In this paper, we develop a new algorithm in which the behavior of the artificial ants is governed by fuzzy IF–THEN rules. Our algorithm is conceptually simple, robust and easy to use due to observed dataset independence of the parameter values involved. In the experiment, spatial data source is come from the actual survey data in mine. LF algorithm and the fuzzy ant based spatial clustering algorithm separately to cluster these data. Through analysis and comparison the experimental results to prove that the fuzzy ant based spatial clustering algorithm enhances the clustering effect.

Keywords: Fuzzy set, IF-THEN rule, ant colony, spatial clustering.

1 Introduction

While the behavior of individual ants is very primitive, the resulting behavior on the colony-level can be quite complex. A particularly interesting example is the clustering of dead nestmates, as observed with several ant species under laboratory conditions [3]. Without negotiating about where to gather the corpses, ants manage to cluster all corpses into 1 or 2 piles. The conceptual simplicity of this phenomenon, together with the lack of centralized control and a priori information, are the main motivations for designing a clustering algorithm inspired by this behavior. Real ants are, because of their very limited brain capacity, often assumed to reason only by means of rules of thumb [5]. Inspired by this observation, we propose a clustering method in which the desired behavior of artificial ants (and more precisely, their stimuli for picking up and dropping items) is expressed flexibly by fuzzy set.

The paper is organized as follows: in Section 2, we review existing work in the same direction, in particular the algorithm of Monmarche which served as our main source of inspiration. Section 3 we outline the structure of our clustering algorithm and motivate its key design principles. Some experimental results are presented in Section 4. Finally, Section 5 offers some concluding remarks.

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2 Related work

Deneubourg et al. [3] proposed an agent–based model to explain the clustering behavior of real ants. In this model, artificial ants are moving randomly on a square grid of cells on which some items are scattered. Each cell can only contain a single item and each ant can move the items on the grid by picking up and dropping these items with a certain probability which depends on an estimation of the density of items of the same type in the neighborhood.

Lumer and Faieta [8] extended the model of Deneubourg et al., using a dissimilarity based evaluation of the local density, in order to make it suitable for data clustering. Unfortunately, the resulting number of clusters is often too high and convergence is slow. Therefore, a number of modifications were proposed, by Lumer and Faieta themselves as well as by others (e.g. [4, 12]).

Monmarche [10] proposed an algorithm in which several items are allowed to be on the same cell. Each cell with a non–zero number of items corresponds to a cluster. Each (artificial) ant a is endowed with a certain capacity c(a). Instead of carrying one item at a time, an ant a can carry a heap of c(a) items. Probabilities for picking up at most c(a) items from a heap and for dropping the load onto a heap are based on characteristics of the heap, such as the average dissimilarity between items of the heap. Monmarche proposes to apply this algorithm twice. The first time, the capacity of all ants is 1, which results in a high number of tight clusters. Subsequently the algorithm is repeated with the clusters of the first pass as atomic objects and ants with infinite capacity, to obtain a smaller number of large clusters. After each pass k-means clustering is applied for handling small classification errors.

In a similar way, in [6] an ant–based clustering algorithm is combined with the fuzzy c-means algorithm. Although some work has been done on combining fuzzy rules with ant-based algorithms for optimization problems [7], to our knowledge until now fuzzy set has not yet been used to control the behavior of artificial ants in a clustering algorithm.

3 Ant Spatial Clustering Based on Fuzzy IF–THEN Rule

We will consider however only one ant, since the use of multiple ants on a non–parallel implementation has no advantages. Instead of introducing several passes, our ant can pick up one item from a heap or an entire heap. Which case applies is governed by a model of division of labour in social insects by Bonabeau *et al.* [2]. In this model, a certain stimulus and a response threshold value are associated with each task a (real) ant can perform. The response threshold value is fixed, but the stimulus can change and represents the need for the ant to perform the task. The probability that an ant starts performing a task with stimulus s and response threshold value θ is given by:

$$T_{\theta}(s;\theta) = \frac{s^n}{s^n + \theta^n} \tag{1}$$

Where n is a positive integer. We will assume that s [0,1] and θ [0,1]. Let us now apply this model to the problem at hand. A loaded ant can only perform one task: dropping its load. Let s_{drop} be the stimulus associated with this task and θ_{drop} the response threshold value. The probability of dropping the load is then given by:

$$P_{drop} = T(s_{drop};\theta_{drop}) \qquad (2)$$

Where i {1, 2} and n_1, n_2 are positive integers. When the ant is only carrying one item n_1 is used, otherwise n_2 is used. An unloaded ant can perform two tasks: picking up one item and picking up all the items. Let s_{one} and s_{all} be the respective stimuli and θ_{one} and θ_{all} the respective response threshold values. The probabilities for picking up one item and picking up all the items are given by

$$P_{pick\_one} = \frac{s_{one}}{s_{one} + s_{all}} T_{m1}(s_{one};\theta_{one}) \qquad (3)$$

$$P_{pick\_all} = \frac{S_{all}}{S_{one} + S_{all}} T_{m1}(S_{all};\theta_{all}) \qquad (4)$$

where m_1 and m_2 are positive integers. The values of the stimuli are calculated by evaluating fuzzy IF–THEN rules as explained below. We assume that the objects that have to be clustered belong to some set U, and that E is a binary fuzzy relation in U, which is reflexive (i.e.$E(u, u) = 1$, for all u in U), symmetric (i.e. $E(u, v) = E(v, u)$, for all u and v in U) and T_W–transitive (i.e. $T_W(E(u, v), E(v,w)) \leq E(u,w)$, for all u, v and w in U). For u and v in U, $E(u, v)$ denotes the degree of similarity between the items u and v. For a non-empty heap $H \subseteq U$ with centre[4] c in U, we define the average and minimal similarity of H, respectively, by

$$avg(H) = \frac{1}{H}\sum_{h \in H} E(h,c) \qquad \min(H) = \min_{h \in H} E(h,c) \qquad (5)$$

Furthermore, let $E^*(H_1,H_2)$ be the similarity between the centres of the heap H_1 and the heap H_2.

A. Dropping items

The stimulus for a loaded ant to drop its load L on a cell which already contains a heap H is based on the average similarity $A = avg(H)$ and an estimation of the average similarity between the centre of H and items of L. This estimation is calculated as $B = T_W(E^*(L,H), avg(L))$, which is a lower bound due to our assumption about the T_W–transitivity of E and can be implemented much more efficiently than the exact value. If B is smaller than A, the stimulus for dropping the load should be low; if B is greater than A, the stimulus should be high. Since heaps should be able to grow, we should also allow the load to be dropped when A is approximately equal to B. Our ant will perceive the values of A and B to be

Very High (VH), High (H), Medium (M), Low (L) or Very Low (VL). The stimulus will be perceived as Very Very High (VVH), Very High (VH), High (H), Rather High (RH), Medium (M), Rather Low (RL), Low (L), Very Low (VL) or Very Very Low (VVL). These linguistic terms can be represented by fuzzy sets in [0,1]. The rules for dropping the load *L* onto an existing heap *H* are summarized in Table1(a).

Table 1. Fuzzy rules for inference of the stimulus for
(a) dropping the load, (b) picking up a heap.

B \ A	VH	H	M	L	VL
VH	RH	H	VH	VVH	VVH
H	L	RH	H	VH	VVH
M	VVL	L	RH	H	VH
L	VVL	VVL	L	RH	H
VL	VVL	VVL	VVL	L	RH

(a)

B \ A	VH	H	M	L	VL
VH	RH	-	-	-	-
H	M	VH	H	-	-
M	VVL	RL	RH	-	-
L	VVL	VVL	L	RH	-
VL	VVL	VVL	VVL	VL	M

(b)

B. Picking items

An unloaded ant should pick up the most dissimilar item from a heap if the similarity between this item and the centre of the heap is far less than the average similarity of the heap. This means that by taking the item away, the heap will become more homogeneous. An unloaded ant should only pick up an entire heap, if the heap is already homogeneous. Thus, the stimulus for an unloaded ant to pick up a single item from a heap *H* and the stimulus to pick up all items from that heap are based on the average similarity $A = avg(H)$ and the minimal similarity $M = \min(H)$. The stimulus for picking up an entire heap, for example, can be inferred using the fuzzy rules in Table 1(b).

C. The algorithm

During the execution of the algorithm, we maintain a list of all heaps. Initially there is a heap, consisting of a single element, for every item in the dataset. Picking up an entire heap H corresponds to removing a heap from the list. At each iteration our ant acts as follows. If the ant is unloaded, a heap from the list is chosen at random; the probabilities for picking up a single element and for picking up all elements are given by formulas (3)–(4). The case where H consists of only 1 or 2 items, should be treated separately (i.e. without using fuzzy rules). If the ant is loaded, a new heap containing the load L is added to the list of heaps with a fixed probability. Otherwise, a heap H from the list is chosen at random; the probability that H and L are merged is given by formula (2). The case where H consists of a single item, should be treated separately.

For evaluating the fuzzy rules, we used a Mamdani inference system with COG as defuzzification method. All response threshold values were set to 0.5.The other parameters are discussed in the next section.

4 Applications

In open pit, it will generate a large number of monthly measurements on mining changed steps. In the past, these data from different positions were classified by hand. It was a very heavy workload. Using data clustering method can not any achieve automatic data clustering, but also it is beneficial to generate the step automatically. Instance data from the survey data on the steps of a surface mine in Inner Mongolia, as shown in Figure 1. The points marked in the map near the numbers 1,2,3 and 4 are each level flat survey points. Following use LF algorithm and fuzzy ant spatial clustering algorithm to cluster the data.

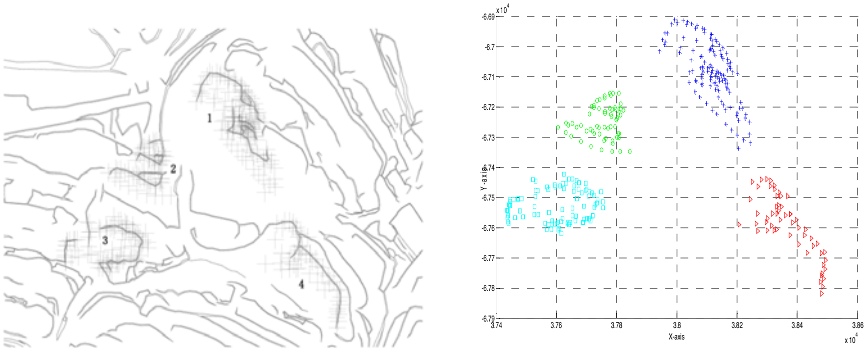


Fig 1. Position distribution of data points

LF algorithm parameter is set to: k1 = 0.1, k2 = 0.15, similarity parameter a = 250, the neighborhood length s = 2, the number of cycle N = 100000. Fuzzy ant spatial clustering algorithm has the parameters same as LF algorithm in addition to not setting k, k2. Fig 2 shows the LF algorithm for clustering results of two-dimensional grid plane projection display. Fig 3 shows the fuzzy ant spatial clustering algorithm results of two-dimensional grid plane projection display. In these figures, the different classes points are draw by different shapes. Comparing the two graphs, both algorithms can achieve accurate measurement of data clustering. Clustering results from the shape distribution, fuzzy ant spatial clustering algorithm is better than LF algorithm. Fig 4 shows the two curves are the similarity degree changing curves of the LF algorithm and fuzzy ant spatial clustering algorithm. We can find from the figure that fuzzy ant spatial clustering algorithm reached stable in near 20000 cycles, but LF algorithm is stable after 80,000 times. It indicates that the fuzzy ant spatial clustering algorithm is faster than the LF algorithm. Fig 5 shows the actual location of the spatial clustering results.

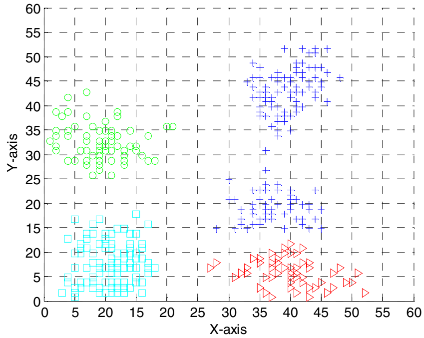
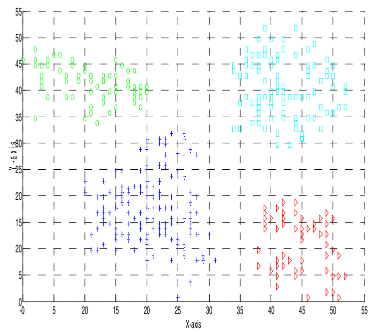


Fig. 2. Clustering results of LF



ig. 3. Clustering results of fuzzy ant spatial clustering algorithm

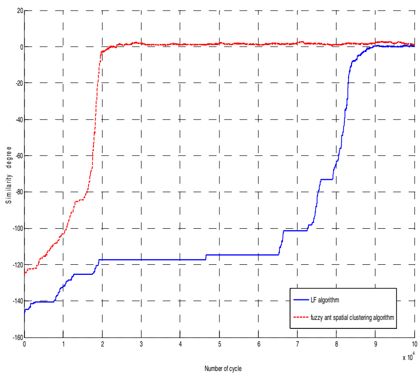


Fig. 4. Similarity of the two algorithms curve

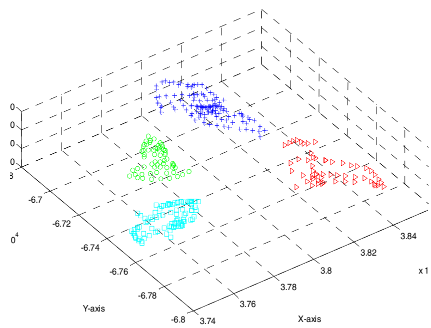


Fig. 5. Clustering results of the actual location

5 Concluding Remarks

Based on the traditional ant colony clustering, the average distance is in the domain of the object similarity. Similarity between objects is mapped a domain of fuzzy sets by membership function. By the given confidence level, fuzzy sets will be separated into universal set. The universal set will decide that ants pick up or put down the object, and spatial data clustering is realized. Using the same instance of the data and the same parameters, the spatial clustering is respectively completed by LF algorithm and fuzzy ant based spatial clustering algorithm. In the experiment, spatial data source come from the actual survey data in mine. LF algorithm and the fuzzy ant based spatial clustering algorithm separately to cluster these data. By analyzing and comparing the clustering results and the similarity curves, fuzzy ant based spatial clustering algorithm has better clustering patterns and faster clustering speed.

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A Fast Global Fuzzy Clustering Algorithm for the Chemical Gray Box Modeling

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Abstract. According to the modeling of chemical grey box, the Fast Global Fuzzy C-Means Clustering algorithm (FGFC),is used to getting effective training data of modeling. The algorithm is based on the Fuzzy C-means algorithm, It doesn't dependent on initial conditions, and at the same time improve the accuracy of the clustering. Experiments show that the FGFCM algorithm's data improved the performance of predicting time and data accuracy, this algorithm was proved to be efficient by computer simulation.

Keywords: Chemical process, The modeling of chemical grey box, The fast global fuzzy c-means clustering (FGFCM), Vapor-liquid Equilibrium.

1 Introduction

Chemical process is a time-varying, uncertainty associated system of multi-variable input-output, its internal mechanism is very complex, in chemical process forecast, and there is a strong nonlinear relationship. At present, though there are many studies about the mechanism of this process, according to the complex practical problems in chemical process, the traditional mathematical methods can not be used to resolve them. From 1982 to now, in order to create a new control system, the gray prediction theory, see [1], has provided a new design. After several years of development, a set of technical system based on a gray process and its generation space has been formed, the technical system is taken the system analysis, modeling, forecasting, decision making, control and evaluation as the key link.

As the Gray theory study is the analysis and study theory with a small amount of data (data incomplete), in general, the extraction of training data (a few representative data) in the chemical process is of great significance for improving the prediction accuracy and timeliness. In this paper, the vapor-liquid equilibrium experiments of the binary in the chemical process is treated as an example, to use the fuzzy clustering technology for conducting effective cluster analysis on all the training data used in modeling of the chemical ash can.

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2 Fuzzy Cluster Analysis Technology

In general, the clustering is divided into two main categories: one is the traditional hard partition (Crisp Clustering or Hard Clustering), and the other is the soft partition (Fuzzy Clustering). The method of the hard partition is to divide each object need to be identified into some category strictly, which has the property of either this or that, so bound of this sorting scheme is distinct. But in fact, most of the objects have no strict properties, their behavior and class has the nature of intermediary, with the quality of either this or that, so it is suitable for soft partitioning.

In practical applications, the popular soft partition is a kind of fuzzy clustering method based on the objective function [2], the method comes the clustering down to a nonlinear programming problem with constrained conditions, by optimal solving, to get the fuzzy partition and clustering of the data set. This method has a simple design and a wide range of problem solving, which can also be used for transforming problems, with the classical mathematical theory of nonlinear programming for saluting, with computer simulation to achieve. Thus, with the application and development of the computer, the fuzzy clustering algorithm based on the objective function, and its improved algorithm become a new research hotspot.

The project is based on the improved fuzzy clustering method to conduct clustering analysis of the data collected in the chemical experiment process, it is ultimately used as the artificial neural network training data, get a better prediction result.

A. The Research Status of the Fuzzy Clustering

In 1965, Dr. L.A.Zadeh of the California University in U.S first proposed the theories of fuzzy sets and fuzzy control in his papers of "Fuzzy sets", "Fuzzy algorithm" and "A rationale for fuzzy control". Its core is to establish a mathematical model of linguistic analysis for a complex system or process, thus the natural language can be translated directly into an algorithmic language that can be accepted by the computer [3]. In China, the start on theory and application of fuzzy control was posterior, but it developed quickly, the areas such as fuzzy control, fuzzy identification, fuzzy clustering analysis, fuzzy image processing and other areas have a lot of research results [4].

B. The Common Fuzzy Clustering Method Used

For the given data, the cluster analysis is the foundation for modeling of many classification and system. The purpose of fuzzy clustering is to extract the inherent characteristics from a large number of data, to gain the concise representation of that system behavior. The common clustering methods include means clustering, hierarchical clustering and subtractive clustering, etc. [5].

At present, the fuzzy c-means clustering [6] (Fuzzy c-means, FCM) is most commonly used, in the clustering method of fuzzy c-means; each data is belonged to some clustering center according to some fuzzy membership. The clustering technology as the improvement of traditional clustering technology was proposed by Jim Bezdek in 1981 in reference [7]. The method is to randomly select several cluster centers first, all of the data points are given some fuzzy membership to the cluster center, then the cluster centers are done amendments persistently with the iterative approach, the iterative process uses the weight sum of the distance from all the minimized data points to the respective cluster center and the membership as the optimum objective.

FCM is a kind of clustering algorithm using the membership function to determine the extent of each data point belonging to certain of cluster. FCM divides n vectors into $x_i(i=1,2,\cdots,n)$ fuzzy group, and seeks the cluster center of each group, so that the value function of non-similarity index is the minimum. FCM uses the fuzzy partition methods to make the membership of each given point value between $[0,1]$ to determine the degree of membership belonging to each group. The concrete methods are as follows:

Initialization Specify the number of cluster categories c $2 \le c \le n$ n is the number of data set iteration stop threshold ε initialize cluster center V°, set iteration counter $b=0$.

Step 1 According to the following formula

$$u_{ij} = \left[\sum_{k=1}^{c} \frac{\left\| x_j - v_i \right\|^{2}}{\left\| x_j - v_k \right\|^{2}}^{1/(m-1)} \right]^{-1}, 1 \le i \le c, 1 \le j \le n$$

calculate Or update the partition matrix $U^b = [u_{ij}]$.

Step 2 According to the following formula

$$v_i = \frac{\sum_{j=1}^{n} (u_{ij})^m x_j}{\sum_{j=1}^{n} (u_{ij})^m}, 1 \le i \le c$$

update cluster centers V^{b+1}.

Step 3 If $\left\| V^b - V^{b+1} \right\| \le \varepsilon$ then algorithm stops and outputs the partition matrix U and cluster centers V If so $b = b+1$ go to Step 1.

As the fuzzy clustering objective function is non-convex, while the FCM algorithm is achieved through the iterative algorithm method, therefore, the algorithm is particularly sensitive to the initialization, it is easy to fall into local minimum, and can not get the global optimal solution [8]. Currently, many scholars has made a lot of research on the FCM algorithm, for example, in [9], the genetic algorithms, simulated annealing algorithms and other optimization techniques are introduced to overcome the problem of falling into local extremum, in [10], the different initial values are conducted many times of FCM algorithm, and then select the best results, to overcome the issue of the initial sensitivity. In [11] according to the problem in FCM algorithms that the clustering results are impacted by clustering numbers and initial cluster center, propose the method based on the average entropy to determine the cluster number, and use the method of density function to obtain the initial cluster center, the improved method has the characteristics of less iteration number and faster convergence speed.

Although all of above improved algorithms have solved the problem of the initial data cluster center sets, the speed of the algorithm is improved, but the most important question of clustering algorithm that is how to obtain the most optimized solution quickly is still the major contents need to be further studied.

C. Improved FCM Algorithm-FGFCM

This paper presents a fast global optimization of the fuzzy clustering algorithm (FGFCM). The algorithm is the fuzzy c-means clustering algorithm based on improved.

FCM algorithm requires that each *c* has N-initial state, to improve the line speed of FCM algorithm. The fast global fuzzy clustering algorithm initial state for each N $(V_1^*(c-1), \cdots, V_{c-1}^*(c-1), x_n)$ Does not perform FCM clustering to get the final error J_m Instead of all the direct calculation of the objective function value of the initial state Associated with the objective function to find the best value of the center Then perform FCM clustering algorithm be *c* cluster center , fast global fuzzy clustering algorithm is as follows

Step 1 Perform the FCM to find the optimal clustering center $V^{(1)}$ of the fuzzy 1-partition and let obj_1 be the corresponding value of the objective function found by

$$J_m = \sum_{i=1}^{N} \sum_{c=1}^{C} u_{ij}^m \left\| x_i - v_c \right\|^2$$

where $V = \{v_1, v_{2,} \cdots v_c\}$ is the vector of the cluster centers and m is the weighting exponent.

Step 2 Compute the value of the objection function for all initial state $(V_1^*(c-1), \cdots, V_{c-1}^*(c-1), x_n)$,by using the flowing

$$J_m = \sum_{i=1}^{N} \left(\sum_{c=1}^{C} \left\| x_i - v_c \right\|^{2(1-m)} \right)^{1-m}, \left\| x_i - v_c \right\| \neq 0 \ (i = 1, \cdots, N; c = 1, 2, \cdots, C)$$

Step 3 Find the minimal value of the objection function $obj\_(c+1)$ and the corresponding initial state $V_0^{(c+1)}$ from Step 2, let $V_0^{(c+1)}$ be the initial clustering centers for fuzzy $(c+1)$ -partition.

Step 4 Perform FCM algorithm with $(c+1)$ clusters from the initial state $V_0^{(c+1)}$ and obtain the final clustering center $V^{(c+1)}$ for fuzzy $(c+1)$ -partition.

Step 5 If $c+1=C$,stop; otherwise set $c=c+1$ and go to Step 2.

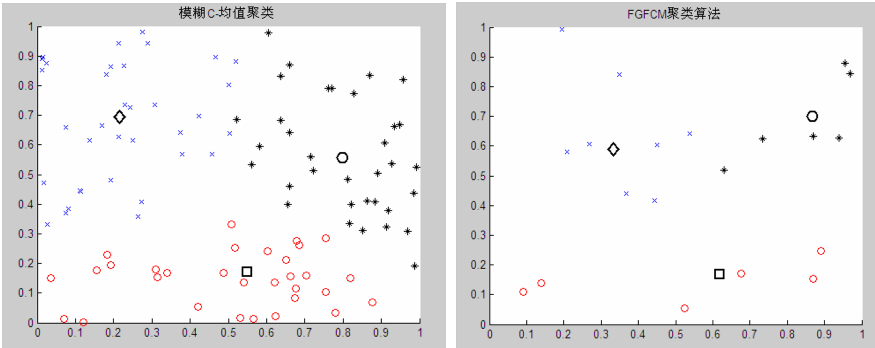
In theory, FGFCM algorithm dynamically increase the cluster center by the way, does not depend on any initial conditions, global search in order to achieve the optimal clustering of the purpose of effectively overcoming the FCM sensitive to initial value problems, together improved the accuracy of the class. Moreover FGFCM algorithm only requires the implementation of sub-FCM iteration, so effectively improve the speed.

D. Performance Analysis of FCM Algorithm and FGFCM Algorithm

In order to prove the sensibility of FCM algorithm of to the initial value and the effectiveness of FGFCM algorithm, on the bases of experiments in [12]. The experiment will respectively use the two fuzzy clustering algorithms to achieve the clustering extraction of the training data of BP neural network, treat the ethanol-cyclohexane system t-x-y value as the data sample (see Table 1), according to the experimental temperature in a certain range, select three data sets (each data set of 200 samples in 10 classes, each class has 20 samples, each sample has two features). Use FGFCM and FCM algorithm to do many experiments according to above data sets. The simulation results are shown in Figure 1. Figure 1 (a) - (b) respectively refer to the clustering data obtained through FCM algorithm and FGFCM algorithm.

Table 1. The t-x-y vaule of ethanol-cyclohexane system

Temperature	X ethanol wt%	Y ethanol / wt%	Temperature	X ethanol wt%	Y ethanol /wt%
64.50	0.239 5	0.321 4	67.70	0.804 7	0.498 5
64.40	0.282 2	0.332 3	68.70	0.876 3	0.552 0
64.40	0.330 6	0.335 2	70.90	0.906 9	0.679 8
64.40	0.383 4	0.337 1	72.90	0.952 6	0.807 6
64.40	0.421 6	0.345 2	74.60	0.969 4	0.871 1
……	……	……	……	……	……
64.50	0.456 1	0.342 1	75.80	0.989 0	0.929 6



(a) (b)

Fig. 1. (a) Data of the Fuzzy C-Means Clustering. (b) Data of the Fast Global Fuzzy C-Means Clustering.

Table 2 for the FCM clustering algorithm and FGFCM level, run time and clustering accuracy of the data comparison of the three.

Table 2. Performance comparison of two algorithm

	3-partition Cluster center	Time	Clustering accuracy
FCM	Evenly spread	98s	88.6%
FGFCM	even	98.2s	100%

The experiments have proved that, under the premise of improving the clustering speed, FGFCM algorithm results are more accurate than that of the FCM algorithm. The algorithm is independent of the initial conditions, thus can avoid the sensitivity of the initial value, while the accuracy of the clustering is improved.

3 The Vapor-Liquid Equilibrium Experiment for the Chemical Process

The main research contents of the chemical process are: the first is to do a detailed analysis to the binary system process of the batch fractionating, then establish a gray box model and collect the data through experiments (such as the vapor-liquid equilibrium data required, the specific data used to verify the tower separation, etc.) and conduct clustering analysis using the fast global fuzzy clustering algorithm, through artificial neural network training getting the specific model. Finally obtain in extend the corresponding results of the ternary and even multicomponent system.

Treat the ethanol-cyclohexane system vapor-liquid equilibrium experiment [12] as the example, using BP neural network to do nonlinear fitting to the two systems, and in the MATLAB 6.5 environment, program with engineering computing language to achieve the above process. The specific technical route is shown in Figure 2. The experimental results show that BP neural network is superior in the aspects of nonlinear system fitting such as the gray box prediction of the vapor-liquid equilibrium data than the traditional correlation method. The vapor-liquid equilibrium survey maps of the ethanol collected during the experiment.

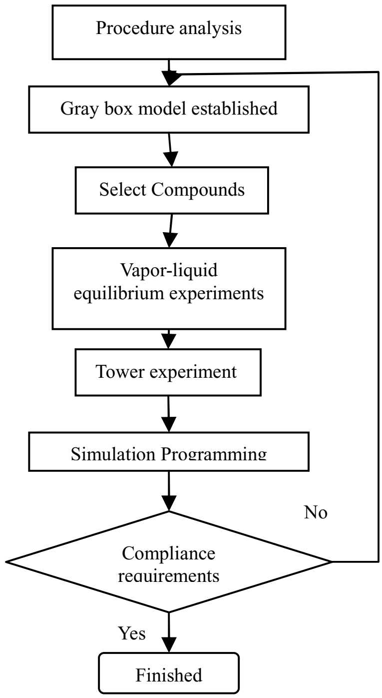


Fig. 2. The technical route of modeling chemical grey box

Experimental results show that BP neural network in the gray box VLE prediction of nonlinear systems fit than the traditional correlation method is superior. Collected during the experiment to ethanol vapor-liquid equilibrium measurement chart Shown in Figure 3.

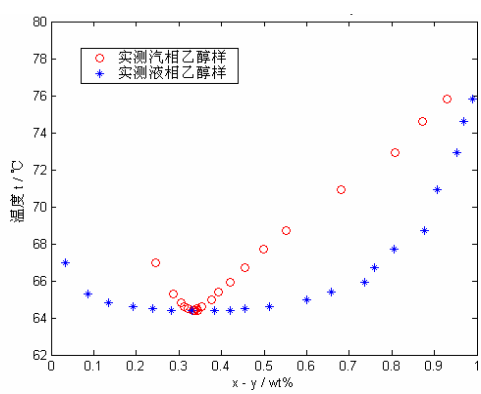
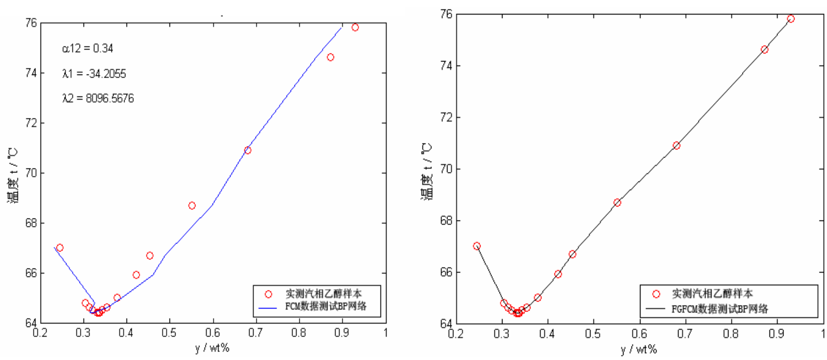


Fig. 3. Measured values of ethanol vapor-liquid equilibrium

4 The Cluster Analysis of the Experimental Data

Respectively, treat the data obtained through the two clustering algorithms of FCM and FGFCM as the training data, to do train in the BP neural network, and then apply to the model for fitting comparison with the measured vapor phase ethanol samples. From the graph ,we can see: The training data obtained through FGFCM clustering Shown in Figure 4(b),after training ,the prediction results of BP neural network is more accurate than the training data obtained through FCM clustering ,Shown in Figure 4(a). Therefore, the fast global fuzzy clustering algorithm (FGFCM) is effective in the in the application of the vapor-liquid equilibrium prediction of the chemical process.



(a) (b)

Fig. 4. (a) BP network fitting by using FCM data. (b) BP network fitting by using FGFCM data data.

5 Conclusion

In the process of gray box modeling of chemical vapor-liquid equilibrium, the accuracy and validity of BP neural network training data extraction are the direct reason of good or bad of the network prediction, due to the rapid global fuzzy clustering (FGFCM) algorithm is independent of the initial conditions, for the given data randomly, it can reduce clustering time, improve accuracy, that is, it can quickly obtain the global most optimized data characteristics. Using FGFCM algorithm to extract the training data, through the comparison with FCM experiment algorithm, proved that, the algorithm is effective.

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FKG System Model Based on Second-Order Differential and Its Application in Stock Prediction

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Abstract. In this paper, first the FKG gray system model based on second-order differential is given. This kind of new algorithm model is composed of fuzzy mathematics, k-means algorithm and grey system based on second-order differential, which is shorted for FKG system model. Meanwhile, through comparing the dates available in the new algorithm model with the one in the traditional model, the result showed that the new one has more advantages. Finally, we predict the cluster center required from the last month by using the historical date, the results are very satisfactory. This new model is good at predicting short-swing trading stock price and has practical value, which can provide some advices for people who are interested in short-swing trading investment. If people refer to it combined with some other market and social factors the result will be more helpful.

Keywords: K-means algorithm, Fuzzy function, FKG system model, Second-order differential.

1 Introduction

January 21, 2008, it appeared "stock disaster" all over the world, the stock market of India had once gone down 9%, it had gone down 7% in German, banking stocks were sold in low prices. The global investors worried the American economic recession though the American government just pushed out the plans of stimulating economic growth. Under the recession, the tide of finance stocks swept the world, it leaded the stock of Asia- Europe to fall seriously, and the stock disaster was caused again. So the stock risk prediction is becoming more of attention. Because of this, many people studied the stock market in several ways, and gray system showed its advantages in the prediction of stock price, it is good at predicting short-swing trading stock price [2, 3], the reference [4] gives a comparison about different gray system models, and have given a number of deficiencies. The reference [5] have carried out a study about the stock index, while the reference [6] combined the gray system and BP neural network together in predicting the stocks price, which has a better result than the single model.

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To achieve a better result, in this paper the authors combined the knowledge of fuzzy mathematic, k-means algorithm and second-order differential to predict the stock price. However, fluctuations in the stock data lead to the fluctuations of fitting curve in the model, and ultimately affect the reliability of predictions. Moreover, too much stock data also lead to the tedious and complex calculation. At the same time the fitting curve of the traditional GM model in the fitting of stock data isn't good enough; So in this paper, we present a new FKG system model based on second-order differential, first we will use the k-means clustering method, and make the data clustering, in order to avoid the data is too big, fuzzy processing the date using mathematic knowledge. At last, we get the curve fitting with the data we obtained, so as to predict the stock market. Shareholders can use the result of this paper combined with various social factors to predict the stock price, capture ushimata, abandon Bear shares and increase the probability of the profit.

2 Describe of the New FKG System Model Based on Second-Order Differential

In order to avoid the model of fitting curve fluctuating too large, and ultimately affect the reliability of predictions; we will processes the data based on the traditional GM model, we will combined fuzzy mathematic, k-means and GM(1,1) model together, then get the corresponding model. The steps are as following:

1) Using k-means method, cluster the opening price of each month, here we divide them into five categories.
2) Get the elements of each class separately according to cluster centers, Then take the average value of each class element as a new cluster center, to see whether the elements of the reclassification is different from the first one, if there have any changes, re-clustering again, until all the elements belong to only one class absolutly.
3) Fuzzy processing the cluster center we get, fuzzy function is $\mu(x) = e^{-0.016x}$, Then sorting the data, recorded as $x_0, x_1, x_2, x_3, \cdots, x_n$, $y_0, y_1, y_2, y_3, \cdots, y_n$, among them $x_0 < x_1 < x_2 < x_3 < \cdots < x_n$, $y_0 < y_1 < y_2 < y_3 < \cdots < y_n$.
4) We can get the simple FKG grey system model after substituting (x_0, y_0), (x_1, y_1), (x_2, y_2), (x_3, y_3), $(\cdots)$, (x_n, y_n), into Formula (1).

For the values of $x_0, x_1, x_2,, x_n$ $y_0, y_1, y_2,, y_n$ we obtained above, similarly $x_0 < x_1 < x_2 < \cdots < x_n$ $y_0 < y_1 < y_2 < \cdots < y_n$, here $y(x)$ is a function about x, and the solution of Cauchy problem based on second-order differential as following:

$$\begin{cases} \dfrac{d^2 y}{dx^2} + ap(x)\dfrac{dy}{dx} + bq(x)y = \sum\limits_{j=1}^{m} c_j f_j(x) \\ y|_{x=x_0} = y_0,\ y'|_{x=x_0} = y'_0 \end{cases} \tag{1}$$

where $p(x), q(x), f_j(x), (j = 1,2,\cdots,m)$ are the known function, $n \geq m+2$, $a, b, c_1, c_2, \cdots, c_m$ are the coefficients waiting for determine, we can get the optimum solution of formula (1) in the condition of $\sigma^2 = \sum\limits_{k=0}^{n} (y(x_k) - y_k)^2$ reached the minimum value. In order to express it comfortably, we give the following notation,

$$Y = \begin{pmatrix} y_0 \\ y_1 \\ \text{M} \\ y_n \end{pmatrix},\quad Y' = \begin{pmatrix} y'_0 \\ y'_1 \\ \text{M} \\ y'_n \end{pmatrix},\quad Y'' = \begin{pmatrix} y''_0 \\ y''_1 \\ \text{M} \\ y''_n \end{pmatrix},\quad P = \begin{pmatrix} p(x_0) & 0 & \Lambda & 0 \\ 0 & p(x_1) & \Lambda & 0 \\ \Lambda & \Lambda & \Lambda & \Lambda \\ 0 & 0 & \Lambda & p(x_n) \end{pmatrix},$$

$$Q = \begin{pmatrix} q(x_0) & 0 & \Lambda & 0 \\ 0 & q(x_1) & \Lambda & 0 \\ \Lambda & \Lambda & \Lambda & \Lambda \\ 0 & 0 & \Lambda & q(x_n) \end{pmatrix},\quad F_j = \begin{pmatrix} f_j(x_0) \\ f_j(x_1) \\ \text{M} \\ f_j(x_n) \end{pmatrix}, j = 1,2,\Lambda,. m.$$ discretize (1)-type we can obtain:

$$Y'' + aPY' + bQY = c_1 F_1 + c_2 F_2 + \cdots + c_m F_m \tag{2}$$

after using the cubic spline interpolaton and the formal (3) in the gray system [1], we can get

$$Y' = B_0 + y'_0 B_1 + y'_n B_2, \tag{3}$$

here $B_0 = (A^*)^{-1} B_0^*, B_1 = (A^*)^{-1} B_1^*, B_2 = (A^*)^{-1} B_2^*$,

$$A^* = \begin{pmatrix} 1 & 0 & 0 & 0 & \Lambda & 0 & 0 & 0 \\ 0 & 2 & \alpha_1 & 0 & \Lambda & 0 & 0 & 0 \\ 0 & 1-\alpha_2 & 2 & \alpha_2 & \Lambda & 0 & 0 & 0 \\ \Lambda & \Lambda & \Lambda & \Lambda & \Lambda & \Lambda & \Lambda & \Lambda \\ 0 & 0 & 0 & 0 & \Lambda & 1-\alpha_{n-1} & 2 & 0 \\ 0 & 0 & 0 & 0 & \Lambda & 0 & 0 & 1 \end{pmatrix},$$

$$B_0{}^* = \begin{pmatrix} 0 \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_{n-1} \\ 0 \end{pmatrix}, \quad B_1{}^* = \begin{pmatrix} 1 \\ -(1-\alpha_1) \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix}, B_2{}^* = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ -\alpha_{n-1} \\ 1 \end{pmatrix},$$

$$\alpha_j = \frac{h_j}{h_j + h_{j+1}}$$

$$\beta_j = 3[\frac{1-\alpha_j}{h_j}(y_j - y_{j-1}) + \frac{\alpha_j}{h_{j+1}}(y_{j+1} - y_j)] \; h_j = x_j - x_{j-1}$$

we can obtain:

$$G_0 + y'_0 G_1 + y'_n G_2 + aP(B_0 + y'_0 B_1 + y'_n B_2) + bQP = c_1 F_1 + c_2 F_2 + \cdots + c_m F_m$$

after the given the value of y'_0 and y'_n [1], if we record $L, D, G,$ as the following:

$$G = G_0 + y'_0 G_1 + y'_n G_2,$$
$$D = -P(B_0 + y'_0 B_1 + y'_n B_2), \; C = -QY, \; L = (D, C, F_1, F_2, \cdots, F_m),$$
$$B = (a, b, c_1, c_2, \cdots, c_m)', \text{ here } G_0 = CB_0 + D, G_1 = CB_1 \quad G_2 = CB_2$$

$$C = \begin{pmatrix} \frac{-4}{h_1} & \frac{-2}{h_1} & 0 & \cdots & 0 \\ \frac{1}{h_1} & \frac{2(h_2 - h_1)}{h_1 h_2} & \frac{-1}{h2} & \cdots & 0 \\ 0 & \frac{1}{h_2} & \frac{2(h_3 - h_2)}{h_2 h_3} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \frac{2(h_n - h_{n-1})}{h_n h_{n-1}} & \frac{-1}{h_n} \\ 0 & 0 & 0 & \frac{2}{h_n} & \frac{4}{h_n} \end{pmatrix}, \quad D = \begin{pmatrix} d_0 \\ d_1 \\ \vdots \\ d_n \end{pmatrix},$$

$$d_0 = \frac{6}{h^2{}_1}(y_1 - y_0) \qquad d_n = \frac{-6}{h^2{}_n}(y_n - y_{n-1})$$

$$d_j = 3(\frac{y_{j+1} - y_j}{h^2{}_{j+1}} - \frac{y_j - y_{j-1}}{h^2{}_j}), \; j = 1, 2, \cdots, n-1.$$

So the Formal (2) can be write as

$$LB = G. \tag{4}$$

After using the least squares method, while $|L'L| \neq 0$, we can obtain $B = (L'L)^{-1}L'G$, that is the value of $a, b, c_1, c_2, \cdots, c_m$, than we can find the solution for the FKG model based on second-order differential.

3 The Application of the New Model

3.1 Establish of the Model in Stock Market

In Table 1 we have list March, April, May of three months stock opening prices of the Ping An company of China in 2009

Table 1. Opening prices of the Ping-An company of China in March, April, May

March	April	May
31.930	38.950	39.780
30.130	39.030	41.110
34.200	40.760	40.800
33.500	40.500	41.020
34.400	40.030	40.850
32.500	39.500	41.600
35.500	41.580	40.700
34.120	43.000	41.300
35.710	42.500	40.850
36.150	42.460	40.410
36.750	42.600	39.750
37.100	41.800	40.890
36.600	42.260	41.190
37.400	41.410	40.200
36.350	41.300	39.280
39.500	39.670	38.600
38.150	40.590	39.340
38.450	39.610	38.810
39.710	38.020	
38.700	37.660	
36.920	39.490	

According to the above steps, we use matlab7.0 to cluster the date, that is:

Table 2. Cluster centers after fuzzy processing

k	0	1	2	3	4
x_k	0.5366	0.5545	0.5641	0.5797	0.6039
y_k	0.5162	0.5150	0.5221	0.5318	0.5458

For the values of (x_i, y_i) $i = 0,1,\cdots,4$, we obtained above, choose the following Cauchy problem,

$$\begin{cases} \dfrac{d^2 y}{dx^2} + a\dfrac{dy}{dx} + by = c \\ y|_{x=x_0} = y_0, y'|_{x=x_0} = y'_0 \end{cases} . \tag{5}$$

After calculating, we can get when $y'_0 = 0.5$ $y'_n = 1.5$, $\sigma^2 = \sum_{k=0}^{n}(y(x_k) - y_k)^2$ reached the minimum value, and we get the following matrix

$$\begin{pmatrix} 2.4000 & -0.5162 & 1.0000 \\ -0.9538 & -0.5150 & 1.0000 \\ -0.4665 & -0.5221 & 1.0000 \\ -1.4710 & -0.5318 & 1.0000 \\ 3.6000 & -0.5458 & 1.0000 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} -169.6463 \\ 148.5533 \\ -30.6281 \\ -37.7683 \\ 133.2422 \end{pmatrix}.$$

After using the least square method we can get $\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 36.2 \\ -5.5307 \\ 2.8517 \end{pmatrix}$, substitute it into Formal (10), the fitted curve is

$$y(x) = 0.9596e^{0.1521x} - 2.7836\times 10^6 e^{-36.3521x} - 0.5156 , \tag{6}$$

then we substitute x_k into (6), we can get the corresponding $\hat{y}_k$ and relative error e_k, $e_k = \dfrac{| y_k - \hat{y}_k |}{y_k}$, as following:

Table 3. The results and relative error predicted by the new FKG gray system model based on second-order differential

k	0	1	2	3	4
x_k	0.5366	0.5545	0.5641	0.5799	0.6039
y_k	0.5162	0.5150	0.5221	0.5318	0.5458
$\hat{y}_k$	/	0.5235	0.5265	0.5305	0.5355
e_k	/	1.65%	0.84%	0.25%	1.89%

To make it more convincing, we give out the results predicted by the traditional model,

Table 4. The results and relative error predicted by the traditional gray model

k	0	1	2	3	4
x_k	0.5366	0.5545	0.5641	0.5799	0.6039
y_k	0.5062	0.5150	0.5221	0.5318	0.5458
$\hat{y}_k$	/	0.5153	0.5200	0.5274	0.5380
e_k	/	0.1%	7.82%	8.3%	1.43%

We can see from the Table 4, its biggest error is to 7.82%, the prediction results predicted by the traditional is extremely unsatisfactory, and there is no value for people to reference, and from Table 3 we can see that its biggest error is 1.89%, very close to the actual value and has a great reference significance and value.

3.2 Predict about the Stock Price in May

We will predict the cluster center in May by using the historical data in April, that is we will forecast the volatility center of the stock opening price in May, here y_0, y_1, y_2, y_3, y_4 are input data, input them into the fitting curve (5), we can get the corresponding function value, then we can get the following data after reduction them by the fuzzy function 42.9851, 40.3200, 42.4035, 41.6322, 40.8344. According to the previous steps, we can know that the cluster center in May are as following 38.7050,39.5375,40.3050,40.8517,41.3000. Forecast results tell us that the stock price in May are between 40.3200 and 42.9851,basically the same as three cluster center in May, the other two are also very close to the other two numbers in May, the rate of its accuracy is more than 60%.In fact from table 1we can see that about more than 76% of the stock opening price are between 40.2 and 42, they are all between 40and 42.9, Forecasting results in line with the stock movements. Also we can see from the Figure 1, the price about this kind of stocks will increase slowly in the recent period of time.

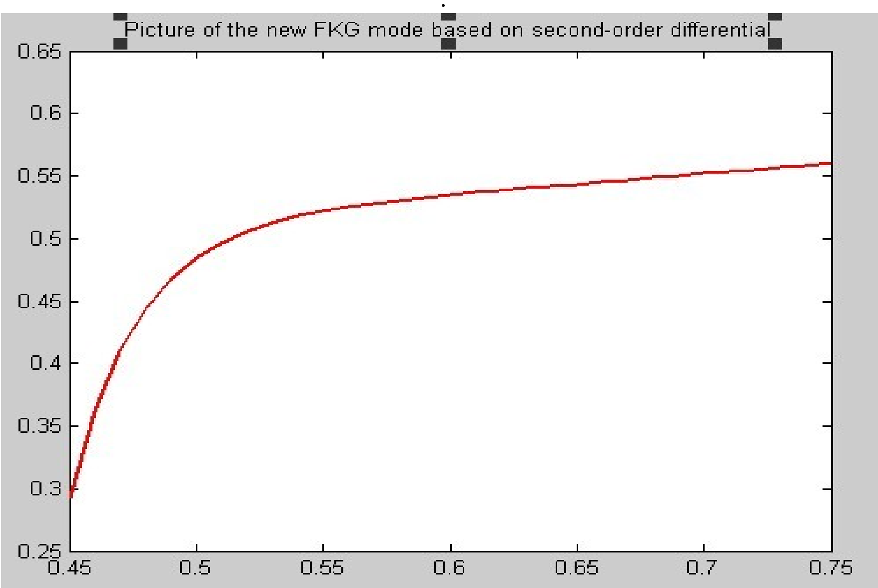

Fig. 1. The new FKG system model based on second-order differential

4 Conclusion

This paper established a new FKG model which combined fuzzy mathematics, k-means cluster with grey system based on the second-order differential, and then predicted the stock market price. It has its own unique advantages compared with a single fuzzy model or a single gray system model. On the one hand through clustering we have avoid calculating a large number of data which will lead to inconvenience; On the other hand through using the knowledge of fuzzy mathematic, we got the fuzzed data, that is normalization processing, also have avoid the large data fluctuations which will lead to a bad fitting carve; Finally, since the gray model has the advantages of dealing with the uncertainty problems, we chose the improved grey system model. In a word, through combined them together we have got an ideal result, and also offer a new way to predict stock prices. We can see that the prediction results have a high reference value, and this could provide a theoretical basis for shareholders who are interested in buying stocks.

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Prediction of Floor Water Bursting Based on Combining Principal Component Analysis and Support Vector Machine

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Abstract. A prediction method that based on combing principal component analysis and support vector machine is proposed. Principal component analysis was used to select input variable. The prediction model considers all-sided influencing factors and avoids the low precision and slow training induced by over-input. The example shows that it eliminates the relevance among factors, reduces the input variables and improves the accuracy and efficiency.

Keywords: Principal component analysis, support vector machine, floor water bursting, prediction.

1 Introduction

Since the 60s of the 20th century, with the rapid development of China's coal industry, mining depth, mining strength, production rate, the size of the increase and expansion of mining, mine water inrush is increasingly aroused great attention, especially in recent years, Mine Water accidents occur frequently, causing significant production losses to the people, seriously affected and restricted the people's safety. But water inrush is affected by Water pressure, Aquifer, Impermeable layer (the thickness of floor strata), floor mining water flowing fractured zone depth, fault gap and other factors, and so far there is not a good prediction method , while looking for a fast and accurate prediction method of mine production is very important and necessary.

SVM is a recently developed a new machine learning method, has a strict theoretical basis - Statistical Learning Theory, it is the learning theory based on small sample, The requirements of the sample is the more relaxed , For the treatment of high dimensionality, small sample, nonlinear and other complex issues it has a good adaptability. Water inrush influenced by many factors, resulting in input factors more complex network structure, increase the training burden and impact of learning speed; if artificial selection has lost some of the important factors may reduce the prediction performance. Therefore, a reasonable predicting model

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selection input factor is important in predicting outcome. Document [2-5] is used to rough set theory, clustering analysis, Grey relational analysis to select the input factors. Essence of these methods are only part of the selection factors, they miss data information carried by other factors. Principal Component Analysis is produced under the thought of dimensionality reduction to deal with high dimensional data. The method is simple and practical, it reorganizes the affecting factors on the original proposed, gets a new set of integrated variables unrelated to each other, and extracts principal component from the new variables, it contains as much as possible information of all the prime factors, and reduces the dimensionality also, easy to calculate. This paper will be the principal component analysis and support vector machine regression together, First analyzing the factors of water-inrush from coal floor using the principal component analysis, getting all the data series of the principal components, and then using support vector machine regression modeling analysis and modeling, and getting predictions about water inrush from coal floor. This is based on principal component analysis and support vector machine regression prediction model, brief note: PCA-SVM (principal component analysis-support vector machine).

2 Principle of Principal Component Analysis

Principal component analysis that proposed by HOTELLING is a method of multivariate statistical analysis. The method is to construct a linear combination of original variables to form new variables, premising these new variables unrelated to each other and they reflect as much as possible the information of the prime factors. Data information reflects the variance of the data variables, the greater the variance, contains more information, cumulative variance contribution rate is usually measured. Using PCA it obtains the correlation coefficient matrix through data matrix that is given by multiple samples of input variables, according to the eigenvectors of the correlation coefficient matrix to obtain the cumulative variance contribution rate; according to the eigenvectors of the correlation matrix to determine the principal components. The calculation is divided into five steps:

1. Standardized collection of the original data.

n Samples of p dimensional random vector $X = (x_1, x_2, \cdots, x_p)$

$x_i = (x_{i1}, x_{i2}, \cdots, x_{ip})$, $i = 1, \cdots, n$, $n > p$

Structured the sample matrix, standardized sample matrix as follows:

$$z_{ij} = \frac{x_{ij} - \bar{x}_j}{s_j}, \ i = 1,2,\cdots,n; j = 1,2,\cdots,p . \tag{1}$$

Meanwhile $\bar{x}_j = \frac{\sum_{i=1}^{n} x_{ij}}{n}, s_j^2 = \frac{\sum_{i=1}^{n} (x_{ij} - \bar{x}_j)^2}{n-1}$, having standardized matrix z .

2. seeking correlation coefficient matrix of standardized matrix z

$$R = \frac{z'z}{n-1}, \quad r_{ij} = \frac{\sum z_{kj} \cdot z_{kj}}{n-1}, i, j = 1,2,\cdots, p. \tag{2}$$

3. Solution characteristic equation $\left|R - \lambda I_p\right| = 0$ of sample correlation

Matrix R gets p characteristic roots. Determining the value of m ,according to $\frac{\sum_{j=1}^{m} \lambda_j}{\sum_{j=1}^{p} \lambda_j} \geq 0.85$, Utilization rate of information more than 85%, For each $\lambda_j, j = 1,2,\cdots,m$, solving equations $Rb = \lambda b$ and having unit eigenvector b_j^0.

4. Transforming standardized variables to principal components

$u_{ij} = z_i' b_j^0, j = 1,2,\cdots,m$, u_1 called the first principal component, u_2 is called the second principal component, …, u_p is called the p principal component.

5. Comprehensive Evaluation of m principal components

Weighted sum of m principal components is viewed the value of the final evaluation, the weights is the variance contribution of each principal component.

3 Principles of Support Vector Machine Regression

For a given sample set S , $\varepsilon > 0$, If there is hyper-plane of R^n

$$f(x) = < w, x > + b, \quad w \in R^n, \ b \in R \tag{3}$$

Making: $$| y_i - f(x_i) | \leq \varepsilon, \quad \forall (x_i, y_i) \in S \tag{4}$$

Then claiming formula (1) is the sample set S of ε -linear regression.
ε -linear regression problems are transformed into an optimization problem:

$$\begin{cases} \min \frac{1}{2} \| w \|^2 \\ s.t. \quad | < w, x_i > + b - y_i | \leq \varepsilon, \ i = 1, 2, ..., l \end{cases} \tag{5}$$

Then, the introducing slack variables, and using the Lagrange multiplier method, has the dual form about the optimization problem:

$$\min\{-\frac{1}{2}\sum_{i,j}^{l}(\alpha_i-\alpha_i^*)(\alpha_j-\alpha_j^*)<x_i,x_j>+\sum_{i=1}^{l}(\alpha_i-\alpha_i^*)y_i-\sum_{i=1}^{l}(\alpha_i+\alpha_i^*)\varepsilon\}$$
$$s.t.\quad \sum_{i=1}^{l}(\alpha_i-\alpha_i^*)=0,\ 0\le\alpha_i,\alpha_i^*\le C,\ i=1,\cdots,l \tag{6}$$

For the sample set S can not be linearly separated in R^n, First with a nonlinear mapping ϕ maps the data S to a high dimensional feature space, So $\phi(S)$ in the feature space H has a very good linear regression characteristics. Again applying linear regression in the feature space, and then return to the original space R^n. This is non-linear support vector machine regression.
Then, dual optimization problem of non-linear regression SVM as follows:

$$\min\{-\frac{1}{2}\sum_{i,j}^{l}(\alpha_i-\alpha_i^*)(\alpha_j-\alpha_j^*)k(x_i,x_j)+\sum_{i=1}^{l}(\alpha_i-\alpha_i^*)y_i-\sum_{i=1}^{l}(\alpha_i+\alpha_i^*)\varepsilon\}, \tag{7}$$
$$s.t.\quad \sum_{i=1}^{l}(\alpha_i-\alpha_i^*)=0,\ 0\le\alpha_i,\alpha_i^*\le C,\ i=1,\cdots,l$$

where C is a constant which is greater than zero, called punishing factors, If C is large, indicating a large punishment for the fitting deviation, ε is regression to allow the maximum error, at this time regression function can be expressed as:

$$f(x)=\sum_{i=1}^{n}(\alpha_i^*-\alpha_i)k(x_i,x)+b \tag{8}$$

And b obtained by the following formula

When $\alpha_i\in(0,C)$ $$b=y_i-\sum_{i=1}^{n}(\alpha_i^*-\alpha_i)k(x_i,x_j)+\varepsilon \tag{9}$$

When $\alpha_i^*\in(0,C)$ $$b=y_i-\sum_{i=1}^{n}(\alpha_i^*-\alpha_i)k(x_i,x_j)-\varepsilon \tag{10}$$

Commonly being used kernel function:

Polynomial kernel function: $k(x_i,x_j)=(x_ix_j+1)^d$

RBF kernel function: $k(x_i,x_j)=\exp(-\frac{\|x_i-x_j\|^2}{\sigma^2})$

Sigmoid kernel function: $k(x_i,x_j)=\tanh(v<x_i,x_j>+C$

4 Applications

4.1 Select Data

This paper selects typical measured data more than 100 cases of North China Mine, including Hebei peak, Jingjing, Hebi, Henan Hebi, Shandong Zibo Etc and water inrush data of the typical flood mine working face and the normal recovery measured data will be selected from representative sample of 80 cases for training, of which 43 cases are water burst data, security measured data of 37 cases,and 15 cases for predicting. Part of the original training data as follows:

Table 1. Learning samples

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	Water burst or no
1	1	0	24.00	45.11	6.17	1.45	280.00	2.00	-110.0	1
2	1	1	19.67	43.95	5.90	1.45	251.59	2.00	66.23	-1
3	1	1	14.74	44.01	5.69	1.50	259.00	1.65	-22.40	-1
4	1	1	15.40	32.50	584.00	2.70	224.00	1.00	-10.00	1
5	0	1	14.529	36.94	584.00	2.20	205.20	0.70	-1.29	-1
6	1	1	28.49	52.70	14.50	1.20	339.90	2.00	-211.0	1
7	1	1	28.20	25.99	19.25	0.90	440.00	2.00	-265.7	1
8	1	1	19.40	34.30	8.30	1.40	338.00	1.00	-228.0	1
9	1	1	3.50	32.65	7.85	0.90	116.00	3.00	25.00	-1
10	1	1	10.75	37.20	8.30	1.42	252.00	1.10	-14.20	-1

Water burst occurred with 1 and Water burst does not occur with -1.

4.2 Standardization of Data

In order to eliminate inrush factor between the different dimension and magnitude, one must be standardized on the data, standardized methods as follows:

$$y_{ij} = \frac{x_{ij} - \bar{x}_j}{s_j}, \tag{11}$$

y for the inrush factor after standardized, x for the inrush factor before standardized, $\bar{x}_j$, s_j for the mean and variance of water inrush factor x_j.

4.3 Principal Component Analysis

Principal component analysis method reduce the dimensions of predictors, reduce the amount of data input using downscaling and the filtered noise. Analysis showed that the first five principal components of the cumulative variance contribution rate to 93.28% (the contribution rate shown in Table 2). It can be seen from Table 2, Basically the first five principal components can reflect the information carried by the original factor.

Table 2. Data of predictor and principal component analysis

principal component	eigenvector	variance contribution rate	cumulative variance contribution rate
1	3.6328	0.4270	42.7
2	2.0132	0.2366	66.36
3	1.1487	0.1350	79.87
4	0.6334	0.0745	87.31
5	0.5514	0.0648	93.79
6	0.3539	0.0416	97.95
7	0.1638	0.0193	99.88
8	0.0063	0.0007	99.95
9	0.0041	0.0005	100

4.4 Using Support Vector Machine Regression to Predict

The principal components are selected as the input vectors, water inrush index as the output vectors, the kernel function select RBF kernel function:

$$k(x_i, x_j) = \exp(-\frac{\left\| x_i - x_j \right\|^2}{\sigma^2}),$$

By MATLAB programming, implementation support vector machine model for constructing and training, through studying, get $c = 400, \sigma^2 = 2.6$ of prediction model. Prediction results on test samples in Table 3, the predicted results can be seen that the 11th predicted sample is error, prediction accuracy above 90%.

Table 3. Forecast test samples

SN	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
True value	-1	-1	-1	-1	-1	1	1	1	1	1	1	-1	-1	-1	1
Predictive value	-1	-1	-1	-1	-1	1	1	1	1	1	-1*	-1	-1	-1	1

5 Conclusion

In this paper, the method combining principal component analysis and support vector machines was predicted floor water bursting, Using principal component analysis it analysis water inrush factor, and extracted the principal components as the input of support vector machine regression for training and predicting. It not only reduces the dimension of input factors about SVM to improve the training speed of SVM, and can effectively filter the noise of the input factor, improve the prediction accuracy. Example shows that the method is effective.

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The Mass Concrete Temperature Simulation Analysis with Fuzzy Factors

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Abstract. In mass concrete casting process of concrete curing because the thickness, strength and anti-permeability level etc a series of uncertain factors exist, and degree of the foundation slab due to a relatively large size and thickness of planar thicker, concrete slabs and the control of temperature cannot effectively, it is difficult to make analysis with classical mathematical model, can not truly reflect the massive concrete hydration heat of the actual effect. Based on the basic concept of fuzzy membership function, establishing the transmission coefficient of membership functions, and carrying on the comprehensive analysis and judgment. So example analysis results indicate that the established membership function can be used for mass concrete hydration heat analysis.

Keywords: Fuzzy, Mass concrete, membership functions, hydration heat.

1 Introduction

With the development of urban construction, high-rise building has gradually become the development trend of urban construction of the foundation slab. High-rise building for its size and thickness of relatively large planar thicker than 0.75 m, (above 1.0 m), and generally massive concrete are curing, so their internal hydration temperature has been ignored. At the same time, because of the high-rise building foundation slab structure and load bearing capacity, generally using intensity levels were higher, impermeable concrete foundation slab, the cement concrete used for high strength also, unit weight of cement hydration heat, and because of large amounts of cement, the more

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progress increased internal cement hydration heat of concrete temperature.

In recent years, the maintenance of high-rise building foundation slab with insulation and heat, these two kinds of construction method reduces the concrete slabs and the temperature, control the size of the concrete slabs and the temperature cracks occur effectively, but when the insulation layer of heat or by not at that time, concrete slabs and the temperature is likely to exceed 250C, causing the floor craze, water and lower floor structure. So the quality defects such as in mass concrete floor construction, how to scientifically and rationally, economic effectively determine the insulation material and thickness of the material and heat layer thickness, is a very important problem in the process of maintenance. For uncertainty, the fuzzy mathematics method, the application of fuzzy set membership function to describe the uncertainty of heat coefficient. Through the actual example verified by fuzzy mathematics of result than pure ideal assumption of result more convincing, also verified by fuzzy mathematics can be more appropriate solution of mass concrete casting process uncertainty, the mathematical model for calculating the average.

2 Heat Transfer Analysis Method

For heat transfer in a body, we assume that the material of the body obeys Fourier's law of heat conduction

$$q = -k\frac{\partial\theta}{\partial x} \tag{1}$$

where
q = heat flux (heat flow conducted per unit area)
θ = temperature
k = thermal conductivity (material property)

The law states that the heat flux is proportional to the temperature gradient, the constant of proportionality being the thermal conductivity, k, of the material. The minus sign indicates the physical fact that a positive heat flux along direction "x" is given by a drop in temperature θ in that direction $\partial\theta/\partial x < 0$. Consider a three-dimensional solid body, in the principal axis directions x, y, and z we have

$$q_x = -k_x\frac{\partial\theta}{\partial x} \tag{2}$$

$$q_y = -k_y\frac{\partial\theta}{\partial y} \tag{3}$$

$$q_z = -k_z\frac{\partial\theta}{\partial z} \tag{4}$$

Where q_x, q_y, q_z and k_x, k_y, k_z are the heat fluxes and conductivities in the principal axis directions. Equilibrium of heat flow in the interior of the body thus gives

$$\frac{\partial}{\partial x}(k_x\frac{\partial\theta}{\partial x})+\frac{\partial}{\partial y}(k_x\frac{\partial\theta}{\partial y})+\frac{\partial}{\partial z}(k_x\frac{\partial\theta}{\partial z})=-q^B \tag{5}$$

where q^B is the rate of heat generated per unit volume. At the surfaces of the body the following conditions must be satisfied:

$$\theta_{s1}=\theta_e \tag{6}$$

$$k_n=\frac{\partial\theta}{\partial n}s_2=q^s \tag{7}$$

$\theta_e$is the external surface temperature (on surface S1), $k_n$is the body thermal conductivity in the direction n of the outward normal to the surface, and q^S is the heat flow input to the body across surface S_2.

Note that a time-dependent temperature distribution has not been considered in the above equations. steady-state conditions have been assumed. For transient problems the heat stored within the material is given by

$$q^c=c\dot{\theta} \tag{8}$$

where c is the material heat capacity and the superposed dot denotes differentiation with respect to time. q^C can be interpreted as forming part of the heat generation term q^B.

$$q^B=\tilde{q^B}-c\dot{\theta} \tag{9}$$

where $\tilde{q^B}$ does not include any heat capacity effect.

In heat transfer the following boundary conditions can be specified:
Temperature conditions: The temperature can be prescribed at specific points and surfaces of the body, denoted by S_1.

Heat flow conditions: The heat flow input can be prescribed at specific points and surfaces S_2.

Convection boundary conditions: Included in equation are convection boundary conditions where

$$q^s=h(\theta_e-\theta^s) \tag{10}$$

With h being the convection coefficient (possibly temperature dependent), θ_e the environmental (external) temperature, and θ^S the body surface temperature.

Radiation boundary conditions: Also specified by equation are the radiation boundary conditions

$$q^s=k(\theta_r-\theta^s) \tag{11}$$

where θ^r is the temperature of the external radiation source, and k is the coefficient given by

$$k = h_r(\theta_r^2 + (\theta^s)^2)(\theta_r + \theta^s) \tag{12}$$

where h^r is determined from the Stefan-Boltzmann constant, the emissivity of the radiant and absorbing materials and the geometric view factors.

3 Fuzzy Cut and Factorization Theorem

A set is category of fuzzy sets of U set

$$\lambda \in [0,1] \tag{13}$$

If $A_\lambda = \{u \in U, u_A \geqslant \lambda\}$
A_λ is called a cut set of λ which is belonged to fuzzy sets of A. So A_λ is a Clearly set
A Resolution Theory of Fuzzy Sets
A set is a fuzzy set, then:

$$A = \bigcup_{\lambda \in [0,1]} (\lambda \cap A_\lambda) \tag{14}$$

The membership function of $\lambda \cap A_\lambda$ is:

$$(\lambda \cap A_\lambda)(u) = \begin{cases} \lambda, u \in A_\lambda \\ 0, u \notin A_\lambda \end{cases} \tag{15}$$

$\cup$ represents union set of fuzzy sets, $\cap$ represents intersection set of fuzzy sets(is the intersection of the sets).

A and B are two fuzzy sets, so A $\cup$ B is also fuzzy sets. Its membership function is

$$\mu_{A\cup B}(U) = max(\mu_A(u), \mu(u)) \tag{16}$$

4 Model Analysis

Fig 1 shows the mass concrete slab which appears uneven distribution for elevator well and reservoir and it is necessary to study its temperature distribution in the position of different ross-section.

According to the engineering geological conditions and pouring conservation schemes, two kinds of boundary conditions were established. The first: with concrete floor and around the contact with the bottom side of the boundary layer (mainly based on geological conditions), surface heat transfer coefficient was 15.3. The second: According to the conservation of the top layer of concrete floor to set the boundary; basis for conservation measures mainly straws, so the surface heat coefficient was 10.5.

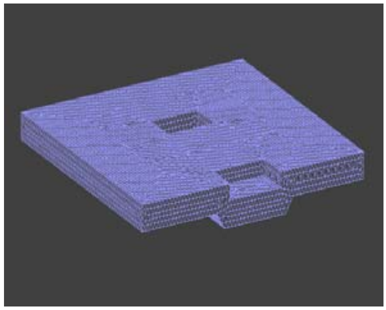

Fig. 1. The mass concrete slab model

For thermal conductivity, using the membership function μ for the L-R type triangular distribution:

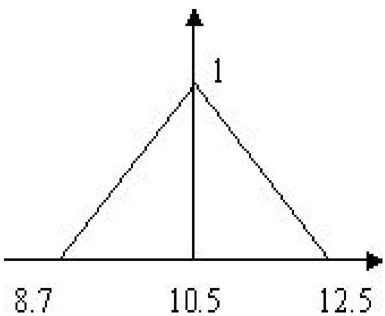

Fig. 2. Membership function(μ)

3359 measurement points are selected for the study, as shown in Figure 3,4,5,6,$\lambda = 0.2$;λ= 0.4;$\lambda = 0.6$;$\lambda = 0.8$ represent relevant temperature range.

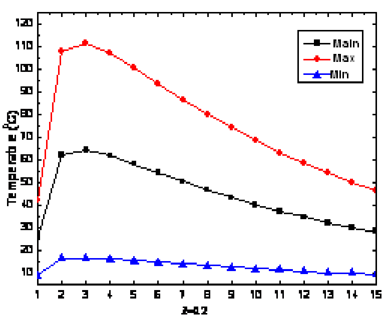

Fig. 3. The temperature curve with λ =0.2

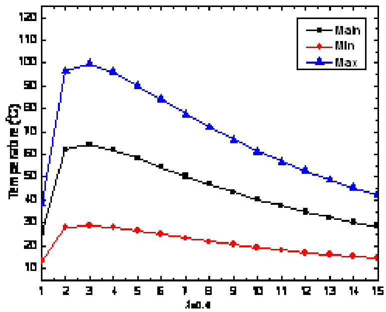

Fig. 4. The temperature curve with λ =0.4

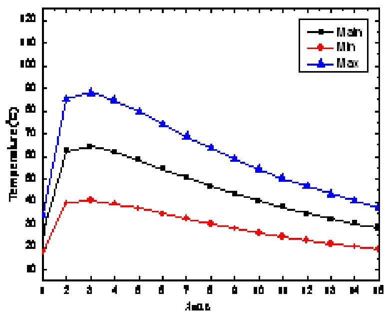

Fig. 5. The temperature curve with λ =0.6

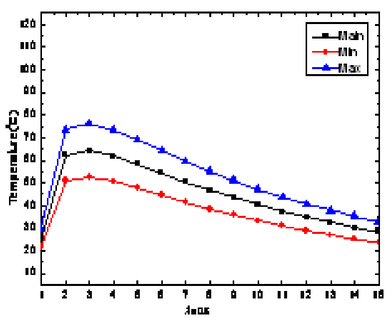

Fig. 6. The temperature curve with λ =0.8

5 Conclusion

Finite element analysis software ADINA calculations indicate that the establishment of the conductivity of the membership function can be more appropriate to solve the conservation of mass concrete in the process of pouring a degree of uncertainty problem, the more secure solution to the classical model and mechanical model can not solve the problem of massive concrete hydration heat.

Based on the basic concept of membership function in fuzzy mathematical methods the mass concrete hydration heat was described. Although the calculation of the structure will be more complexity than traditional methods, but it reflects the uncertainties existed in mass concrete hydration heat under real situation, so is a worthy and explore ways.

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Fuzzy Morphology Based Feature Identification in Image Processing

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Abstract. Image processing has very important application in the national economy and the people's livelihood, but there exists many problems in the process for original image identification processing, such as edge unclear. In order to remove edge discontinuousness, forge edge and over-segmentation, the paper proposed an algorithm of fuzzy morphology based feature identification in image processing. The method is based on structure element of fuzzy mathematical morphology to identify feature, analyzed elements correlation, decided subjection function after pre-process images, ascertained best segmentation threshold adopting technology of auto-recognition best threshold. Then the algorithm took best threshold and path cost function as constraint condition to reduce search scope, enhance algorithm execute speed. Finally it extracted edge and identified image geometry features by dint of watershed algorithm. The paper took packaged granary grain quantity intelligent reckoning for example to make digital emulation, and the results showed that it not only can make image edge continuous, but also can remove the phenomena of forge edge and over-segmentation, and the image changes smoother and more flexible by using the algorithm to process images.

Keywords: Fuzzy mathematical morphology, image identification, watershed algorithm, geometry features extraction.

1 Introduction

Image processing has very important application in the national economy and the people's livelihood, but there exist many urgent theory problems needed to solve in the process for image identification processing [1]. Aimed

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at the problem of traditional image recognition processing, such as edge discontinuity, forge edge, over-segmentation and so on, focused on image geometry features and based on fuzzy morphology, the paper examines the correlation of various part, identifies the image structure character to analyze and reappear image by means of different shape structure element and image structure transform algorithm. The method is more superior than other nonlinear method in threshold selection. It can identify the image geometry feature better [2].

2 Fuzzy Recognition of the Image

Mathematical morphology is a kind of mathematical tool of image analysis based morphology structural elements. It extracts corresponding shape of an image by means of morphology structural elements to identify image [3]. Fuzzy mathematical morphology can be formed when using fuzzy set theory to mathematical morphology. The main idea of image processing in fuzzy morphology is to view an image as a fuzzy set because of fuzziness rooted in image itself and in the process of collection and processing, so it can execute fuzzy arithmetic operator to image [4].

2.1 Definition of Fuzzy Subset

Fuzzy set can be educed when value range of $\widetilde{A}$ membership eigenfunction in classical set theory for element x is extended from $\{0,1\}$ to $[0,1]$. It can be shown as formula (1) [5].

$$\mathcal{U}_A : U \rightarrow [0,1] \qquad x \rightarrow \mathcal{U}_A(x) \tag{1}$$

In which, U is called domain, μ_A is called membership function, and $\mu_A(x)$ is called value of membership function. Formula1is any map of U to closed interval $[0,1]$. Fuzzy subset $\widetilde{A}$ is described fully by μ_A of membership function. The membership function represents the degree that belongs to $\widetilde{A}$ by a value of element x over closed interval $[0,1]$.

2.2 Ordinary Method of Fuzzy Recognition

The realization method of fuzzy image recognition is similar to the common pattern recognition. They are the maximum membership principle recognition, closest principle recognition, fuzzy clustering analysis recognition, fuzzy similar choice recognition, and fuzzy comprehensive evaluation recognition and so on.

(1) Maximum membership principle recognition

Supposed $\widetilde{A}_1$,$\widetilde{A}_2$,...,$\widetilde{A}_n$ are fuzzy sets of X, if

$$\widetilde{A}_k(x_0) = \max_{1\leq i\leq n} \{\widetilde{A}_i(x_0)\} \tag{2}$$

Then x_0 is called relatively to belong to fuzzy set $\widetilde{A}_k$ and it is called as the maximum membership principle recognition.

(2) Closest Principle recognition

When recognition object is not a definite element of domain U, but is a fuzzy subset of domain U, the problem of recognition object is changed into approach degree problem of the two fuzzy subsets.

Supposed $\widetilde{A} \in F(X)$, $\underline{A}(x) = \{ {1 \atop 0},$ when ${\widetilde{A}(x) > 0.5 \atop \widetilde{A}(x) \leq 0.5}$, $\underline{\widetilde{A}}$ is a common subset to closest $\widetilde{A}$.

Supposed $\widetilde{A}_i$, $\widetilde{B} \in F(X)$ $(1 \leq i \leq n)$, if there is a $\widetilde{A}_{i0}$ that can make $\sigma(\widetilde{A}_{i0}, \widetilde{B}) = \max\{(\widetilde{A}_1, \widetilde{B}), (\widetilde{A}_2, \widetilde{B}), ..., (\widetilde{A}_n, \widetilde{B})\}$, then $\widetilde{B}$ is called as closest to $\widetilde{A}_{i0}$, and it is deduced to belong to $\widetilde{A}_{i0}$ class. Here, object needed to judge is not an element of X, but is a fuzzy subset of X.

3 Algorithm Study for Geometry Features Identification

3.1 Determine Fuzzy Threshold Value

The idea of fuzzy threshold value segmentation is to view an image as a fuzzy array. The fuzzy threshold value can be determined by computing image fuzzy rate or fuzzy entropy. Using fuzzy subset concept, we can view an digital image which includes of M columns, N rows, L level gray degree as a fuzzy lattice array, as a qualification-function defined on the L level gray degree, and the gray value of pixel point (m, n) is $x_{m,n}$. So we can get fuzzy rate $V(x)$ and fuzzy entropy $E(x)$ of image X.

$$V(x) = \frac{2}{MN} \sum_m \sum_n min[\mu(x_{m,n}), 1 - \mu(x_{m,n})] \quad (3)$$

$$E(x) = \frac{1}{MN \ln 2} \sum_m \sum_n S_n[\mu(x_{m,n})] \quad (4)$$

For $\forall q$, the image fuzzy rate $V(q)$ represents the similarity of the image with a binary image. If the histogram of original target image and the background image have double peaks, the corresponding graphics of $V(x)$ have double peaks too. So there is a value q_0 and it can make the corresponding fuzzy rate $V(q_0)$ be the least. The value q_0 is the optimal threshold value. Using the optimal threshold value, it can recognize the target and background better.

3.2 Watershed Algorithm

Watershed algorithm is a sort of image processing tool rooted in mathematical morphology. It can be used to segment image, extract gradient image and so on. In the watershed algorithms of immerged simulation, the digital image can be expressed by formula (5).

$$G = (D, E, I) \tag{5}$$

In which, (D, E) describes image, and I is the corresponding transform function of $D \longrightarrow N$. Each pixel point $p \in D$. $I(p)$ expresses the image gray-value of each pixel point $p \in D$ and its value range is from 0 to 255. If threshold-value h of image is $T = \{p \in D \mid I(p) \leq h\}$, in the immersion process, the point starts from sets $T_{h_{min}}(I)$, the point in set is the place that water reaches firstly. And these points form beginning point of iterative formula are shown as in formula (6) and (7).

$$X_{h_{min}} = \{p \in D \mid I(p) \leq h_{min}\} = T_{h_{min}} \tag{6}$$

$$X_{h+1} = MIN_{h+1} \bigcup IZ_{T_{h+1}}(X_h),\ h \in [h_{min}, h_{max}] \tag{7}$$

In which, h_{min} is the minimum, and h_{max} is the maximum. And $X_{h_{min}}$ is composed of point in set I. These points are located the minimum region that its altitude is the lowest. MIN_h is the union of all minimum region that their gray-values are h. Gray-value h is iterative continuously from h_{min} to h_{max} IZ is the union of measure infection region. In the iterative process, the minimum point district of image I will be extended gradually. Suppose X_h is the connectedness discreteness of threshold set T_{h+1} under the value h of position for the union of district sets started from plane position, it may be a new minimum or be located the extension region of X_h. For the latter, the X_{h+1} can be renewed by computing T_{h+1}. In the set D, supplementary set of $X_{h_{max}}$ is just the watershed of the image, shown as in formula (8).

$$Watershed(f) = D/X_{h_{max}} \tag{8}$$

According to the definition above, gradient-value of each point of the image can be seen as its height. Starting from minimum region that its altitude is the lowest, the water will immerge into all catchment basins. In the above process, if the water is form different catchment basins it will be converged, a dam will be built at the converged edge. At the end of immersion process, all catchment basins will be surrounded by dams and the union of dams is just corresponding watershed of the image.

3.3 Watershed Algorithm Based on IFT

The IFT algorithm regards image as a picture, and its processing result is adjacency relation of pixel. The common path cost function has additive path cost function, and maximum is path cost function. The catchment basin uses maximum arc path cost function, shown as in formula (9) and (10).

$$f_{max} = (< t >) = h(t) \tag{9}$$

$$f_{max}(\pi\cdot < s, t >) = max\{f_{max}(\pi), I(t)\} \tag{10}$$

In which, A is an adjacency relation of pixels, and $(s, t) \in A$, s is the end node, t is the start node, $h(t)$ is its initial value of path cost started from node t, and $I(t)$ is the pixel-value of t.

4 Research on Image Segmentation Algorithm Based on Fuzzy Morphology

The idea of image segmentation algorithm based on fuzzy morphology is the following. Firstly, it determines the optimal threshold-value by auto-recognition method based image gray-degree character, and the criterion is separating the goal from background farthest. Secondly, it determines optimal threshold-value of image segmentation by means of a sort of simple and nimble method based on optimal auto-recognition threshold-value. Then it restricts further path cost function of original IFT watershed algorithm according to the optimal threshold-value. This algorithm is to constrict search scope of optimal path of original IFT watershed in essence, so it can enhance the execution speed for operation.

$$f_{max} = (< t >) = I(t) \tag{11}$$

$$f_{new}(\pi . < s, t >) = \begin{cases} max\{f_{new}, I(t)\} & \text{, if } I(t) \geq T \\ +\infty & \text{, otherwise} \end{cases} \tag{12}$$

In the formula, T is the threshold-value. Suppose the image has N gray-grade value, the steps of improved IFT watershed algorithm are the following.

Input: image I, template image L.

Output: result L of each catchment basin transformed by watershed algorithm.

Assistant data structure: initial values will be set as infinite (∞) for all node cost C (cost map).

The steps of improved watershed algorithm are as the following.

(1) Let $C(p) = I(p)$ for all nodes satisfy the condition($L(p) \neq 0$), then insert node p into queue Q according to the value of $C(p)$.

(2) Determine the threshold-value by use of auto-recognition technology.

(3) Delete node p that its $C(p)$ value is minimum if queue Q is not empty. For each node satisfied the condition $q \in N(p)$, and if the node q is not inserted into the queue Q, then it makes the following operation.

Compute $C = f_{new}(\pi \cdot < p, q >)$.

If $C \neq +\infty$, do $C(q) = C$ and insert node q into queue Q according to value of $C(q)$. Then let $L(p) = L(q)$.

Algorithm analysis

(1) Adjusting of restriction condition of path cost function f_{new} is based on threshold-value.

(2) Seed set, any node belonged to objective.

(3) Layered queue structure Q, if the image includes N-grade gray-grade value (because there exists threshold-value restriction), the number of bucket of queue Q can be reduced to $N - T + 1$, and the storage space needed by whole algorithm becomes as $O(n + N - t + 1)$. In the steps of original algorithm, the node inserted queue Q must be never operated by current node, so the operation for the queue is different from the original algorithm.

(4) Because of adding the restriction of threshold-value, search process does not traverses all the nodes, but some nodes that their threshold-values are over threshold-value of target region of image will be visited in the target region of image. This method reduced the search area and enhanced the execution efficiency of algorithm.

5 Example for Image Identification Application

The paper takes packaged granary grain geometry feature recognition as an example to discuss the application for image geometry features identification based on fuzzy morphology. Its research background is the following, because of the geographical dispersion and the lack of supervision of reserve granary, it is difficult to carry on the effective supervision and investigation for management departments, and leads to the virtual, false subsidies, theft, and other widespread phenomena, brings the huge economic losses to the country. By means of recognizability and convenience in capturing and transmission of digital image, the quantity intelligent reckoning of packaged grain reserves based on image processing, has become an important measure technique to auto-manage and audit long-distance grain reserves [10].

In the ideal circumstance, suppose the bending of each grain bag is the same, and the gathered counter-band of light image has the uniform width band of light. However, there are a lot of random factors in the actual packaged-grain bags stacking, such as the stacking-fault, change in location of the bags, and so on, it leads to the acquisition by showing a count-band of inhomogeneous width and local banding. The paper chooses six kinds of edge-detection algorithms to detect edge geometry feature for scene image. The processing results of various algorithms can be shown as in Fig.1.

In the Fig.1, a is the original, b is the processing result of Roberts algorithm, c is the processing result of Sobel algorithm, d is the processing result of Prewitt algorithm, e is the processing result of Log algorithm, f is the processing result of Canny algorithm, and g is the processing result of Watershed algorithm.

It can be seen form Fig.1 that although Roberts algorithm location is rather precise, it is sensitive to noise, and recognition capability is worse because of the algorithm without smooth. Both the Sobel operator and Prewitt operator are one order differential operator. The former adopts average filtering and the later adopts weighted averaging filter. They both can obtain better effect in processing gray image. However, in processing complex image blended many noise, their processing results have the situation of edge discontinuousness and forge edge. LOG filtering algorithm can smooth image by means of second order derivative equal to zero method to detect marginal points and it can eliminate change that intension far less than Gaussian distribution factor σ. But the algorithm has some contradictions between edge precision location and noise elimination. So it can't solve the problem how to only make certain image edge for different scale filter. Canny operator takes

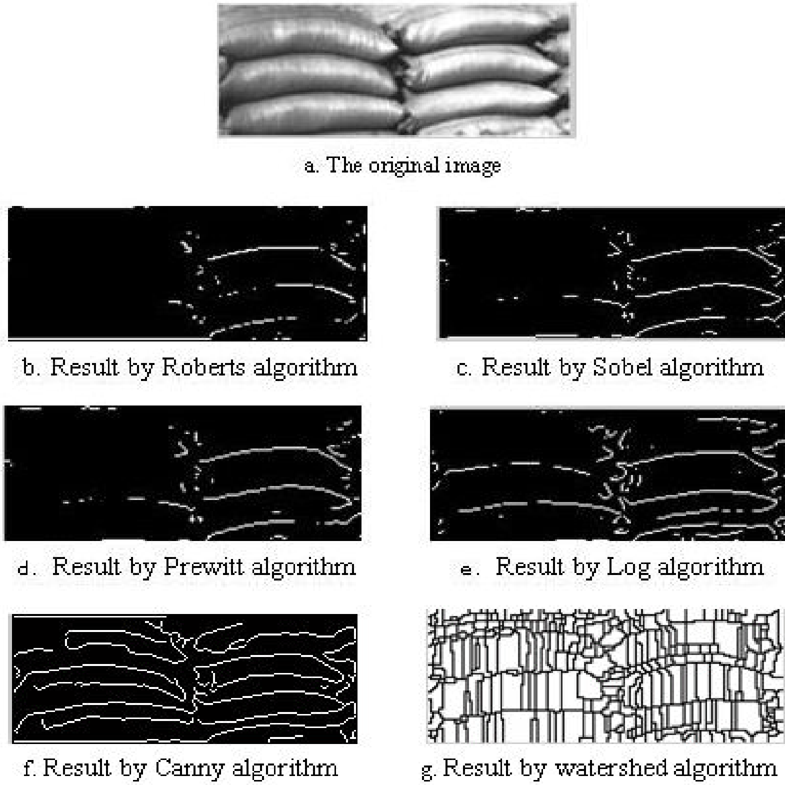

Fig. 1. Identification results for various operators

one order derivative as criterion to detect marginal points. Its capability for noise elimination is stronger than the Roberts operator, Sobel operator and Prewitt operator. However, it maybe gets rid of some edge information in smoothing, as if there is a biggish rupture in the picture f. Furthermore, by using dual-threshold value, the Canny operator increases threshold choice difficulty and calculating works. There exist forge edge and rupture phenomena for the recognition results of above five kinds of algorithms. Traditional Watershed algorithm can approximately identify the image edges and contours, but it may be over-segment image.

Above six kinds of methods can approximately identify original image geometry feature. But their recognition results always have some problems, such as edge discontinuousness, forge edge, or over-segmentation and so on. For effectively improving recognition precision of packaged granary grain quantity intelligent reckoning, the method of fuzzy morphology based feature identification mentioned in the paper can determine the parameter of membership function, introduce special structure element B, by means of watershed algorithm to morphology close operation for interest region. When close-operation result is no more change, the identification process stops. The identification result can be shown as in Fig.2.

Fig. 2. Result for fuzzy morphology based feature identification

Form the Fig.2, we can see that it eliminates the forge edge and over-segmentation, and the identification result is smoother and softer, and after image processed, the edge is continuous.

6 Conclusion

By means of fuzzy morphology method, the processed image can identify image geometry feature better, and enrich the related theory and method of image processing. The method can also be applied to many domains, such as military targets scout, agriculture, ocean works, irrigation works, and environment resource protection, and etc. it has much practical value. However, there are still some problems needed to be solved such as how to choose the optimal membership function parameter and how to enhance the computing speed of algorithm.

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Study on Sign Language Recognition Fusion Algorithm Using FNN

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Abstract. To overcome the limitation of hand shape recognition, the paper presented a recognition method of hand shape fusion based on the fuzzy neural network of BP. By means of fuzzy neural network of BP, the method analyzed the fusion computing for the collected hand gesture and lip shape image, viewed respectively the fusion image as the fuzzy set of hand gesture and lip shape, made the operation of fuzzy arithmetic operators for fuzzy set, matched the operation results and the sign of hand gesture and lip shape in database, carried on the fuzzy set operation for the gotten two sets of hand gesture and lip shape, finally got the recognition result. The simulation experiments show that the presented method is better in sign Language recognition, and it maybe has wide realistic application foreground in the education of deaf-and-dumb people.

Keywords: Education of deaf-and-dumb people, BP fuzzy neural network, Fusion computing, Sign language recognition, Hand gesture shape database, Lip shape database.

1 Introduction

In recent years, many countries have started to research image processing technology of sign language. Now China has more than 2,000 thousands deaf-and-dumb people. In order to make them infuse normal social life better, eliminate communication obstacle between them and normal people, improve their life quality, realize truly integration symbiosis, construct the harmonious society[1], the sign language recognition technique gets more and more attention by national experts and scholars. The goal of sign language recognition is

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to provide an effective and correct mechanism via computer to translate sign language into text or sound, so that the communication between deaf-and-dumb people and normal people can change more conveniently[2]. Furthermore, sign language recognition can control virtual human action by means of hand shape gesture and it plays very important role of multi-modal user interface in virtual reality technology application. Sign language recognition is not only original hand shape gesture recognition, but also it depends upon facial expressions, lip shape change and other assistant means for truly understanding the implication of hand shape gesture[3][4]. Therefore, the paper presented a sort of recognition method of the sign language based on BP fuzzy neural network(FNN).

2 Brief Introduction for Basic Concepts

2.1 Fuzzy Neural Network

Fuzziness is a characteristic of some thing itself, which is unclear in behavior and classification. Fuzzy control is a new method that takes fuzzy theory as the base, and it is based on the fuzzy control rules and fuzzy logical reasoning mechanism[5][6]. Its main part is fuzzy rules. Fuzzy control system can make system more intelligent. Normally the fuzzy controller consists of four parts, namely fuzzification, fuzzy rules base, fuzzy reasoning engine and defuzzification.

Artificial neural network is composed by lots of wide interconnection process units such as artificial neural cell, process components, electronic components, photoelectrical components and so on[7]. It can simulate structure and function of brain neural. Artificial neural network has powerful capability of self-learning and automatic recognition pattern. It also has the capability of arbitrary approach for continued mapping, and it is stronger in environment adaptation.

Combining the relative techniques of fuzzy theory and neural network, the FNN can be formed. It is composed of fuzzy neural cells and normal neural cells, so it can deal with the fuzzy information. Fuzzy system not only owns the self adaptablity, but also because it enlarges the information processing scope of neural network, it has double capability for processing accurate and non-accurate information.

2.2 Image Fusion

The information fusion is a process to integrate data and information. Image data fusion is distinguished as subsets of the more general data fusion problem, and it is also a process combined multiple sources to enhance desired features and reduce uncorrelated noise. Normally, three basic levels of image data fusion as the basic design alternatives are offered to the system designer[8]. Their flowcharts can be shown as Fig.1.

Pixel-level fusion uses the registered pixel data from all image sets to perform detection and discrimination functions. This level has the potential to achieve the greatest signal detection performance at the highest computational expense. At this level, detection decisions are based on the information from all sensors by evaluating the spatial and spectral data from all layers of the registered image data.

Feature-level combines the features of objects detected and segmented in the individual sensor domains. The features of each object are independently extracted in each domain, these features create a common feature space for object classification.

Decision-lever fusion is to combine the decisions of independent sensor detection/classification paths by Boolean (AND, OR) operators or by a heuristic score. Two methods of making classification decisions are divided as the hard decisions and soft decisions.

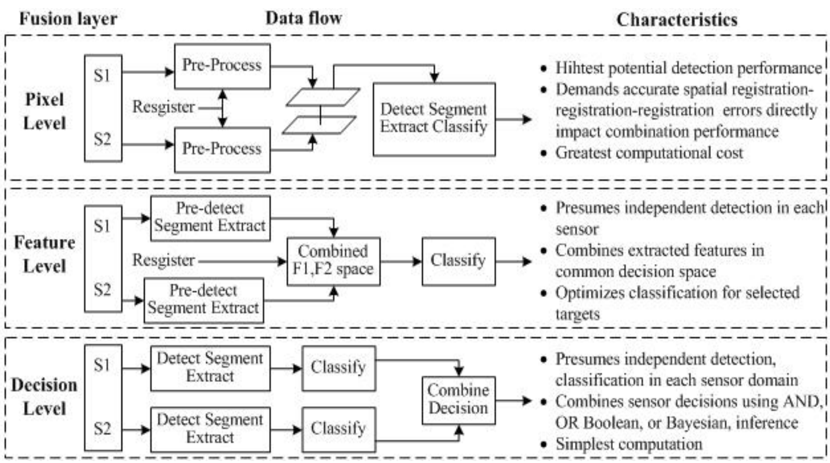

Fig. 1. Flowchart of three basic levels for image fusion

2.3 Introduction of Sign Language

Sign language is a special communication language by means of action and vision. Chinese sign language includes 30 finger alphabets composed of 26 single letters (from A to Z) and 4 double letters (ZH, CH, SH and NG), and 5,500 basic hand gesture words[9][10]. Finger alphabets are derived from letter language. Each finger alphabet expresses a Bopomofo and many finger alphabets that can spell standard Chinese. Basic hand gesture words come from the like shape language. They can visually represent basic character of objects and action by means of human gesture, facial expression and body action[11].

3 Fusion Algorithm of Sign Language Recognition Based on FNN

3.1 FNN Description

The paper adopts the information fusion algorithm of BP fuzzy reasoning neural network to fuse and process hand gesture and lib shape images. The algorithm can be applied to multi-layer network. It is one of neural network learning algorithm. The BP fuzzy reasoning neural network is composed of fuzzy neural cells and normal neural cells, and the input values of fuzzy neural cells are fuzzy. In the paper, BP fuzzy reasoning neural network includes five layers, input layer, fuzzification layer, fuzzy rule layer, defuzzification layer and output layer.

The function of input layer is to accept outer input information and sent them to fuzzification layer. The function of fuzzification layer is to translate input value into relative membership of some fuzzy set by means of given membership function. Fuzzy rule layer is the core of entire network. It determines the link between the preconditions and conclusions of fuzzy reasoning, and chooses different reasoning operation for different problem needed to solve. The defuzzification layer can eliminate data blur come from higher level. Output layer is to give definite solution. A sketch map of typical structure for BP FNN included multi-input node and multi-output node can be shown in Fig.2.

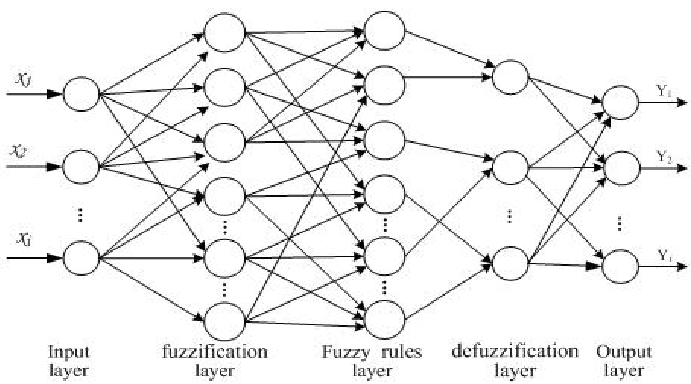

Fig. 2. The sketch map of typical structure for BP FNN

Fusion algorithm based on BP FNN needs to modify some input weight values. For description convenience, here the meaning of some symbols explains as the following.

$X = x_1, x_2, ..., x_i,$ initialization value of input layer.

Y_i, output of neural cell i.

O_i^l, output of layer L, node except output layer.

$E(W)$, error between input and output.
T_k, relevant sample value of output layer node k.
W_{ij}, connection weight value between node i and node j.
W_{jk}, connection weight value between node j and node k.
θ_j, threshold value of middle layer node.
θ_k, threshold value of output layer node.
The following is the computing steps.
1. Compute error between output and expect value

$$E(W) = \frac{1}{2}\sum_k (Y_k - O^5)^2. \tag{1}$$

2. Modify respective fuzzy membership function center and weight
1) Compute connection weight modification values between nodes of output layer and hidden layer

$$\delta_k = (T_{kl} - O_k) \cdot O_k \cdot (1 - O_k) \qquad k \in \{1, 2, ..., m\}. \tag{2}$$

2) Compute connection weight modification values between nodes of hidden layer and input layer

$$\delta_j^l = O_k^l \cdot (1 - O_k^l) \cdot \sum_{k=1}^{m} \delta_k^{l+1} \cdot W_{jk}^{l+1}. \tag{3}$$

Amend connection weight values between nodes of output layer and hidden layer W_{jk} and threshold value vector θ_l by means of error modification value δ_i computed by Formula (2). For example, the connection weight values between nodes of output layer node k and hidden layer node j and the threshold value of node k respectively are modified as following:

$$W_{jk}^l(t+1) = W_{jk}^l(t) + \alpha \cdot \delta_k^l \cdot Y_j^{l-1}, \tag{4}$$

$$\theta_k^l(t+1) = \theta_k^l(t) + \beta \cdot \delta_k^l. \tag{5}$$

In which, α and β are ordered weighted averaging operators.

Amend connection weight values between nodes of hidden layer and input layer W_{ij} and threshold value vector θ_j by means of error value δ_l computed by Formula (3). For example, the connection weight values between nodes of hidden layer node j and input layer node i and the threshold value of node j respectively are modified as the following:

$$W_{jk}^l(t+1) = W_{jk}^l(t) + \alpha \cdot \delta_j^l \cdot Y_i^{l-1}, \tag{6}$$

$$\theta_j^l(t+1) = \theta_j^l(t) + \beta \cdot \delta_j^l. \tag{7}$$

After amend every layer, we can get the practical output and compute the error. If the error is in the permissible scope, it stops computation. Otherwise, the computation should be continued.

3.2 Information Fusion Algorithm Based on FNN

In the paper, the fusion includes two meaning. The one meaning is feature-level fusion of many hand gesture and lip shape images accepted by many sensors, and the process can make the accepted images get clearer. The other meaning is to match of hand gesture and lip shape images based on image feature-layer. The architecture of fusion algorithm is shown as in Fig.3. In the above algorithmthe number of sensor used to accept images of hand

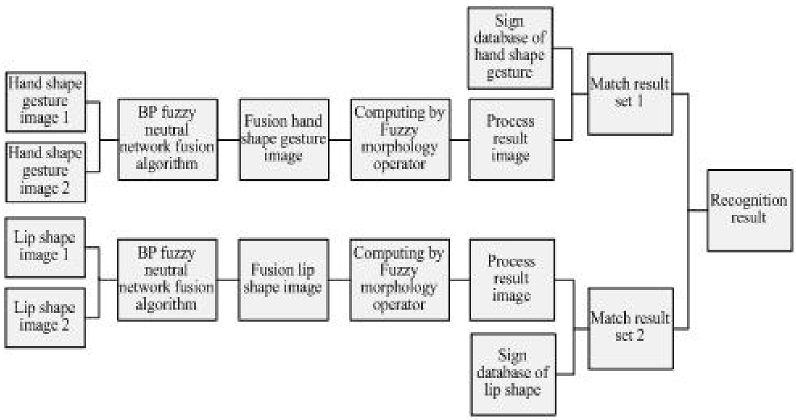

Fig. 3. Architecture of fusion algorithm

gesture and lip shape images is determined by special need. For simplified description, the paper takes only 2 sensors as an example, and lists merely 2 images of hand gesture and lip shape. The databases of hand gesture and lip shape respectively store the common signs of hand gesture and lip shape.

3.3 Computing Steps

Firstly the algorithm implements information fusion algorithm on the images accepted hand gesture and lip shape from sensors by means of BP FNN to find fusion images. Because the images accepted by sensors are blurry, fusion images are blurry, too. In order to get better recognition result, the fusion images respectively are viewed as fuzzy set. And the fuzzy operator computation is implemented by means of fuzzy morphology[12]. Then algorithm matches computing results with the signs of hand gesture and lip shape databases and gets two matching sign sets. Finally, it makes fuzzy set operation on two matching sign sets to attain the recognition result. The detailed operation steps can be shown as the following.

1) Implement information fusion algorithm on the images accepted hand shape gesture from sensors by means of BP FNN to find fusion images.

2) Implement information fusion algorithm on the images accepted lip shape from sensors by means of BP FNN to find fusion images.

3) Compute images by means of fuzzy morphology operator on fusion hand gesture image to get computing result.

4) Compute images by means of fuzzy morphology operator on fusion lip shape image to get computing result.

5) Match computing results of step (3) with the signs of hand gesture database to get matched sign language set 1.

6) Match computing results of step (4) lip shape images with the signs of lip shape database to get matched sign language set 2.

7) Make fuzzy morphology set operations on sign language set 1 and sign language set 2. If it gets many computing results, then the recognition is failure.

8) Output the recognition result.

4 Simulation Experiment

In the practical application, the changes of hand gesture and lip shape are very rich, and their change velocity is also very quick. For getting better recognition effect, multi-sensors can be used to accept images. The paper takes two sensors as example to explain algorithm executing process and principle.

The signs of hand gesture in the paper mainly come from 30 Chinese finger alphabets of Chinese sign language and 2000 common hand shape gesture words. The Chinese finger alphabets can be seen as in Fig.4. Databases of lip shapes store lip shape change image for daily words.

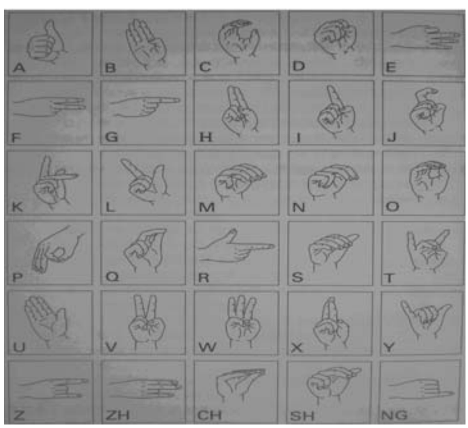

Fig. 4. Chinese finger alphabet

In the algorithm, the method first respectively makes feature-level fusion for two hand gesture and lip shape images by means of BP FNN, then matches the above fusion results with the databases. Because of restriction of the pages' length, the paper only gives the match result image with databases. The hand gesture image can be shown as Fig.5.

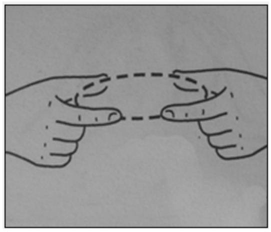

Fig. 5. Hand shape image

In the sign language, all things that shape is circular can be expressed as Fig.5, such as cake, barrel and ring, etc. If it only depends on the hand gesture shape then we can't understand the true implication of sign language, or the sign language maybe brings ambiguous meaning. For understanding sign language meaning better, we must depend on both lip shape and hand gesture shape. The lip shape images of cake, barrel and ring can respectively be shown as Fig.6, Fig.7 and Fig.8. When hand gesture shape matching result is Fig.5, and lip shape matching result is Fig.6, the implication of sign language is cake, and is not other ambiguous meaning.

Fig. 6. Lip shape image of "cake"

Fig. 7. Lip shape image of "barrel"

Fig. 8. Lip shape image of "ring"

5 Conclusion

The method of the sign language recognition based on BP FNN realized feature-level fusion of multi-sensor image and fusion matching between post-processing images and database images. Its sign language recognition can not only overcome the ambiguous meaning produced original hand gesture shape recognition, but also possess perfect identification effect. The study can deepen corresponding theory of digital image processing and extend the application scope of fuzzy theory. Its study results can be applied to deaf-and-dumb student education to impel development of special education, come true integration symbiosis. The method has important guidance value for deaf children early language training and wide application foreground. However, there are some problems in the study process. For example, the databases need further standardization, and so get recognition result better.

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Fuzzy Variable Classified Method and Its Application in Basin Floods

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Abstract. A classified method for basin floods is proposed based on the variable fuzzy sets theory. This method is used here to classify the basin floods of Dahuofang region. According to the classification the parameters of the flood forecasting model are optimised with in each class and used to forecast the corresponding type of basin floods. The classified forecast results show that the proposed method is reasonable and valuable to forecast the basin floods with good results in simulation and verification. This simple method also can be used in many kinds of multi-objective classification problems.

Keywords: classified method, flood forecast, variable fuzzy sets.

1 Introduction

Conceptual rainfall-runoff models (CRRM) have become a basic tool for flood forecasting and catchment basin management [1]. CRRM generally contain a model structure which represents the general physical processes of the water flow lumped over the entire catchment area and a number of parameters which characterize the processes. The performance of a rainfall-runoff model heavily depends on choosing suitable model parameters[2]. The parameters of some models cannot be obtained directly from measurable quantities of catchment characteristics [3], and hence need to be calibrated according to historical hydrological data or information.

Many techniques and methods have been used to identify model parameters so that the model simulates the behaviour of the catchment as closely as possible [2,4,5,6]. However, sometimes it is very difficult to select a set of model parameters despite using various calibration methods because of the extremely complex hydrological characteristics of a catchment. Even though a rainfall-runoff model with the best set of parameters is used to simulate a given flow situation (such as low flows), the accuracy maybe is rather low. Therefore, it is necessary to determine the model parameters to classify calibration data according to the characteristics related to flood shapes.

There are many indexes to describe the parameters of flood. Although many classification methods can be used to study the index classification problems, most

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of these methods treat the indicator criteria as point forms. These methods are not applicable to the indicator criteria with interval forms.

The variable fuzzy sets VFS theory[7] proposed by Chen Shouyu is an extended theory of engineering fuzzy sets theory[8-9]. Base on the relative difference function, the variable fuzzy sets theory can be used to solve the classification problems with interval form indicator criteria[10]. Therefore, this research presents a classified method for basin floods, named fuzzy variable classified method FVCM , which is applied to the practical case. This method is effective to improve the accuracy of basin floods forecasting.

2 Principle of VFS

A. Concept and Definition of VFS

To define the concept, U is assumed to be a fuzzy concept (alternative or phenomenon) $\underset{\sim}{A}$. For any element u ($u \in U$), $\mu_{\underset{\sim}{A}}(u)$ and $\mu_{\underset{\sim}{A}^c}(u)$ are relative membership degree (RMD) functions that express degrees of attractability and repellency respectively [7].
Let

$$D_{\underset{\sim}{A}}(u) = \mu_{\underset{\sim}{A}}(u) - \mu_{\underset{\sim}{A}^c}(u) \tag{1}$$

where $D_{\underset{\sim}{A}}(u)$ is defined as relative difference degree of u to $\underset{\sim}{A}$. Mapping

$$\begin{aligned} D_{\underset{\sim}{A}} &: D \to [-1,1] \\ u| &\to D_{\underset{\sim}{A}}(u) \in [-1,1] \end{aligned} , \tag{2}$$

is defined as relative difference function of u to $\underset{\sim}{A}$. According to definition of complementary set of fuzzy sets, we have

$$\mu_{\underset{\sim}{A}}(u) + \mu_{\underset{\sim}{A}^c}(u) = 1. \tag{3}$$

Then

$$\mu_{\underset{\sim}{A}}(u) = \left(1 + D_{\underset{\sim}{A}}(u)\right)/2 \tag{4}$$

where $0 \le \mu_{\underset{\sim}{A}}(u) \le 1, 0 \le \mu_{\underset{\sim}{A}^c}(u) \le 1$. Let

$$\underset{\sim}{V} = \left\{(u,D) \middle| u \in U, D_{\underset{\sim}{A}}(u) = \mu_{\underset{\sim}{A}}(u) - \mu_{\underset{\sim}{A}^c}(u), D \in [-1,1]\right\} \tag{5}$$

$$A_+ = \{u | u \in U,\ 0 < D_{\underset{\sim}{A}}(u) \le 1 \} \tag{6}$$

$$A_- = \{u | u \in U,\ -1 \le D_{\underset{\sim}{A}}(u) < 0 \} \tag{7}$$

$$A_0 = \{u | u \in U,\ D_{\underset{\sim}{A}}(u) = 0 \} \tag{8}$$

Here $\underset{\sim}{V}$ is just defined as Fuzzy Variable Sets (FVS); A_+ , A_- and A_0 are defined as attracting (as priority) set, repelling (as priority) set and balance boundary or qualitative change boundary of FVS $\underset{\sim}{V}$ respectively. Assume that C is variable factors set of $\underset{\sim}{V}$, we have

$$C = \{C_A, C_B, C_C\} \tag{9}$$

Here C_A is variable model set, C_B is variable model parameters set and C_C is variable other factors set except model and its parameters. Let

$$A^{+} = C(A_{-}) = \{u | u \in U,\ 0 < D_{\underset{\sim}{A}}(u) \le 1, -1 \le D_{\underset{\sim}{A}}(C(u)) < 0\ \} \tag{10}$$

$$A^{-} = C(A_{+}) = \{u | u \in U,\ -1 \le D_{\underset{\sim}{A}}(u) < 0, 0 < D_{\underset{\sim}{A}}(C(u)) \le 1\ \} \tag{11}$$

We generally define these two subsets as qualitative change sets of FVS $\underset{\sim}{V}$ to variable elements set C. Let

$$A^{(+)} = C(A_{+}) = \{u | u \in U,\ 0 < D_{\underset{\sim}{A}}(u) \le 1, 0 < D_{\underset{\sim}{A}}(C(u)) \le 1\ \} \tag{12}$$

$$A^{(-)} = C(A_{-}) = \{u | u \in U,\ -1 \le D_{\underset{\sim}{A}}(u) < 0, -1 \le D_{\underset{\sim}{A}}(C(u)) < 0\ \} \tag{13}$$

We generally define these two subsets as quantitative change sets of FVS $\underset{\sim}{V}$ to variable elements set C.

FVS models include fuzzy variable optimum model, fuzzy variable recognition model (FVRM) and fuzzy variable clustering iteration model etc.[9]. Variable parameters include index weights, standard index values and other important parameters.

B. Model of Relative Difference Function

We suppose that $X_0 = [a,b]$ is attracting (as priority) set of VFS $\underset{\sim}{V}$ on real axis, i.e. interval of $0 < D_{\underset{\sim}{A}}(u) \le 1$, $X = [c,d]$ is a certain interval containing X_0, i.e. $X_0 \subset X$ (see Fig.1).

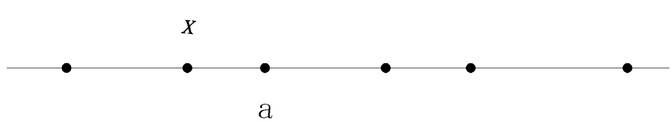

Fig. 1. The diagram of location relation between points x, M and internals $[a,b]$, $[c,d]$

According to definition of VFS we know that intervals $[c,a]$ and $[b,d]$ are all repelling (as priority) sets of VFS, i.e. interval of $-1 \le D_{\underset{\sim}{A}}(u) < 0$. Suppose that M is the point value of $D_{\underset{\sim}{A}}(u)=1$ in attracting (as priority) set $[a,b]$, and M can be determined by actual case. x is the value of random point in interval X. If x is on the left side of M, the relative difference function is

$$D_{\underset{\sim}{A}}(u)=(\frac{x-a}{M-a})^{\beta} \quad x\in[a,M] \tag{14}$$

$$D_{\underset{\sim}{A}}(u)=-(\frac{x-a}{c-a})^{\beta} \quad x\in[c,a] \tag{15}$$

If *x* is on the right side of M, the relative difference function is

$$D_{\underset{\sim}{A}}(u)=(\frac{x-b}{M-b})^{\beta} \quad x\in[M,b] \tag{16}$$

$$D_{\underset{\sim}{A}}(u)=-(\frac{x-b}{d-b})^{\beta} \quad x\in[b,d] \tag{17}$$

where β is the index that bigger than 0, usually we take it as β =1, viz. Equations 14-17 become linear functions. Eqs. (14-17) satisfy the following conditions: (i) $x=a$, $x=b$, $D_{\underset{\sim}{A}}(u)=0$; (ii) $x=M$, $D_{\underset{\sim}{A}}(u)=1$; (iii) $x=c$, $x=d$, $D_{\underset{\sim}{A}}(u)=-1$.

Finally, the values of relative membership degree $\mu_{\underset{\sim}{A}}(u)$ of disquisitive indexes are obtained according to relative difference degree $D_{\underset{\sim}{A}}(u)$ and Eq. (4).

3 Basic Approach for FVCM

The steps of FVCM are as follows.

Step 1: Set up attracting matrix $\mathbf{I_{ab}}=([a,b]_{ih})$, standard interval matrix $\mathbf{I_{cd}}=([c,d]_{ih})$ and point values matrix $\mathbf{M}=(M_{ih})$ [7], where $i=1,2,\cdots,m$ is the number of indexes; $h=1,2,\cdots,c$ is the number of levels.

Step 2: According to the relative difference function model (Eqs. 14-17 and 4) in the variable fuzzy sets theory, we can determine the relative difference degree $D_{\underset{\sim}{A}}(u_{ih})$ and relative membership degree $u_{\underset{\sim}{A}}(u_{ih})$ for every index of each sample to each level.

Step 3: The target weight vector $\mathbf{W}$ can be determined using the two-dimensional comparison method[11-12].

Step 4: According to FVRM[7-9], the comprehensive relative membership degree of sample to each level can be given.

$$u_h' = \frac{1}{1 + (\frac{d_{hg}}{d_{hb}})^{\alpha}} \tag{18}$$

where
$$d_{hg} = \left\{ \sum_{i=1}^{m} [w_i (1 - u_{\underset{\sim}{A}}(u_{ih}))]^p \right\}^{1/p}$$

$$d_{hb} = [\sum_{i=1}^{m} (w_i u_{\underset{\sim}{A}}(u_{ih}))^p]^{1/p}$$

where u_h' is the non-normalized comprehensive relative membership degree of level h . p is the distance parameter, $p = 1$ is Hamming distance and $p = 2$ is Euclidean distance; α is the optimal rule parameter, $\alpha = 1$ is the least single method and $\alpha = 2$ is the least square method.

Step 5: Let

$$H = \sum_{h=1}^{c} u_h \cdot h \tag{19}$$

which represents the centre of the shape encircled by level h and u_h on the $h \sim u_h$ coordinate plane. H is called the rank feature value and can be considered as an index to determine the level of sample.

where $u_h = u_h' \Big/ \sum_{h=1}^{c} u_h'$

Step 6: Convert the values of p and α in Eq. 18 and repeat the Steps 3-4. Analysis all the rank feature values and determine the level of sample.

4 Case Study

Dahuofang reservoir is located at the upper middle area of Hun River in Liao River. This reservoir is in the Fushun city, Liaoning province and has 5437 km^2 of drainage area. The Dahuofang hydrological model designed by the administrative bureau for the Dahuofang reservoir has been used to forecast the catchment rainfall-runoff of this reservoir.

There are 46 floods from 1951 to 2005 for Dahuofang Basin. Five floods in the years of 2004 and 2005 are chosen as verification samples and the other 41 floods are considered as the classification and modelling forecasting samples.

The total rainfall, antecedent precipitation index and initial flow are taken as the indicators to analyze the properties of a flood.

According to the flood data, the initial standard interval is given. In order to improve the classification and simulation forecasting, the boundary values are adjusted repeatedly. Finally, the standard interval for each index is determined. The total rainfall and the upper and lower limitations for the initial flow indicator should be set as the maximum and the minimum value, respectively. The indicator which affects the early stage of precipitation rainfall is set as the maximum antecedent precipitation index Im (Table 1).

Table 1. Classification standard of each index

Index						
Total rainfall (mm)	39.3	80	80	100	100	266.5
Antecedent precipitation index (mm)	33.6	60	60	80	80	110
Initial flow (m3/s)	40	100	100	300	300	858

According to Table 1, the standard interval matrix $\mathbf{I_{ab}}$, variable interval range matrix $\mathbf{I_{cd}}$ and matrix $\mathbf{M}$ are

$$I_{ab}=\begin{bmatrix}[39.3,80] & [80,100] & [100,266.5]\\ [33.6,60] & [60,80] & [80,110]\\ [40,100] & [100,300] & [300,858]\end{bmatrix}$$

$$I_{cd}=\begin{bmatrix}[39.3,100] & [39.3,266.5] & [80,266.5]\\ [33.6,80] & [33.6,110] & [60,110]\\ [40,300] & [40,858] & [100,858]\end{bmatrix}$$

$$M=\begin{bmatrix}39.3 & 90 & 266.5\\ 33.6 & 70 & 110\\ 40 & 200 & 858\end{bmatrix}$$

Based on the relationship of important mood operator, quantitative scale and relative membership degree, the unitary object weight vector can be determined using the duality comparison fuzzy policy analysis method

$$w=(0.48,0.32,0.20)$$

Table 2. The relation among mood operator quantitative scale and Relative Membership Degree

mood operator	Equal	Slight	Some-what	Rather	Obvious	Remarkable
quantitative scale	0.5	0.55	0.6	0.65	0.7	0.75
relative membership degree	1	0.818	0.667	0.538	0.429	0.333
mood operator	Very	Extra	Exceeding	Extreme	Incomparable	
quantitative scale	0.8	0.85	0.9	0.95	1	
relative membership degree	0.25	0.176	0.111	0.053	0	

The relative difference function model for the variable fuzzy sets (Eqs. 14–17 and Eq. 18 or 19) is used to calculate the relative difference degree $D_{\underset{\sim}{A}}(u_{ih})$ and relative membership degree $u_{\underset{\sim}{A}}(u_{ih})$ for the samples.

According to Eqs. (20) and (21), the model is computed in four ways where $\alpha=1, p=1$ $\alpha=1, p=2$ $\alpha=2, p=1$ $\alpha=2, p=2$ $\alpha=1$ is least single model $\alpha=2$ is least square model $p=1$ is Hamming distance $p=2$ is Euclidean distance).

The results calculated from the four models are averaged in order to determine the classification of the floods. The flow parameter of Dahuofang model is optimized according to the classification. The qualified percentages of the simulation forecasting for each class are 86%, 88%, 90%. All of these models are in the first level and much better than the non-classification comprehensive simulation forecasting model in which the qualified percentage is only 78%.

The testing samples are used to perform the classification testing forecasting. For example, the No. 05817 flood has the total rainfall of 50.4mm, in which early stage of effective rainfall is 106mm and the initial flow is 644m3/s. First, we calculate the first class relative membership degree to the index eigenvalue of sample rainfall. As shown in Table 1, classification index criteria (or the matrix $\mathbf{I_{ab}}$), matrix $\mathbf{I_{cd}}$ and matrix $\mathbf{M}$ can be applied to determine the standard interval $[a,b]=[39.3,80]$. The variation range is $[c,d]=[39.3,100]$ with the index eigenvalue on the right hand of $M=39.3$.

Therefore, we have the relative membership degree to the indicator eigenvalues for sample rainfall $u_{\underset{\sim}{A}}(u)_1=0.87$ using the relative difference function model (Eqs. 16 and 18).

Table 3. Classification results of samples

Flood number	class	Flood number	class	Flood number	class	Flood number	class
51821		64812		90812		96810	
53712		64818		91728		98713	
53728		65801		93803		98804	
53818		71731		93817		98823	
54825		75729		94815		01704	
54913		85730		95724		01801	
60801		85803		95728		02731	
63722		85813		95806		03726	
64721		85818		96717			
64729		86729		96723			
64806		89721		96729			

For the second class of relative membership degree, the standard interval for the sample is $[a,b]=[80,100]$ with the variation range $[c,d]=[39.3,266.5]$. The indicator eigenvalue is on the left hand of $M=90$. Therefore, the second class of relative degree to the index eigenvalue of sample rainfall is $u_{\tilde{A}}(u)_2=0.13$ according to the relative difference function model (Eqs. 15 and 18).

For the third class of relative membership degree the standard interval is $[a,b]=[100,266.5]$ with the variation range of $[c,d]=[80,266.5]$. According to Eq. (19), we can have $u_{\tilde{A}}(u)_3=0.0$ (the other indicators for the relative membership degree are not shown here.). The parameters $\mathbf{w}$, α and p are applied in Eq. (20) to compute the comprehensive relative membership degree for each class sample. As shown in Table 4, the classification eigenvalues H are calculated by Eq. (21). The results show that the value of H has only little variation. Because the average of H is 2.1, the classification of this sample is the class. Using the parameter of this classification model to forecast the flow, the absolute error and the relative error between the forecasted net rainfall (40.7mm) and the real net rainfall (34.2mm) is 6.5 and 19%, respectively. Because both of these errors are less than the allowable value, this forecast is qualified.

Table 4. The comprehensive membership degree of sample to each class

Class	$\alpha=1, p=1$	$\alpha=1, p=2$	$\alpha=2, p=1$	$\alpha=2, p=2$
	0.35	0.37	0.35	0.40
	0.23	0.22	0.13	0.11
	0.42	0.41	0.52	0.49
Rank feature values	2.1	2.0	2.2	2.1

This method is also used to test the other four floods, three of which are qualified. Therefore, the qualified percentage is 80% which is much better than value of 60% calculated using non-classification comprehensive testing forecasting method.

5 Conclusion

In this paper, a classified method FVCM for basin floods is proposed based on the variable fuzzy sets theory. This method is used here to classify the floods of Dahuofang Basin. The case study shows that this method can deal with the classification problems with multi-indicator standard as interval pattern effectively, and it can obtain more than one values of the comprehensive relative membership degree for each class of the sample by changing the parameters in the method. The average value of the relative membership degree is taken as the foundation to determine the classification of the sample.

According to the classification the parameters of the flood forecasting model are optimised within each class and used to forecast the corresponding type of basin floods. The classified forecast results show that the proposed method is reasonable and valuable to forecast the basin floods with good results in simulation and verification. This simple method also can be used in many kinds of classification problems with multi-indicator standard as interval pattern.

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Layering and Enveloping: Simplified Algorithm with Balancing Loop Fuzzy Petri Net

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Abstract. At present, the research for Fuzzy Petri Net is focused on the Model with no Loop. Based on the idea of object-oriented, this paper defines the outline of the object-oriented fuzzy Petri Net. Listened the characters of researching the loop, we make use of the idea of object-oriented and layering the Fuzzy Petri Net which with loop. As a result, the Petri Net Model on all layers is with no loops. Finally, we develop a simplified algorithm for fuzzy petri net based on all places' Reachability Set, Immediate Reachability Set and Adjacent Place set.

Keywords: Fuzzy petri net, object-oriented petrit, layered, simplified, loop.

1 Introduction

Fuzzy Petri Net is a qualified modeling tool based on fuzzy producing rules knowledge place system. It can correctly define unclear accident with "fuzzy" elements[1]. At present, researches on Fuzzy Petri Net are focused on how to connect the fuzzy rules and FPN's active rules. When the fuzzy producing regular knowledge places are over big or complicated, there must be too many spots. And the situation numbers in system are increasing with the spots numbers, as a result, there comes the 'combination explosion' problem.

Based on the existing issues, this paper produces the balancing loop charging algorithm by analyzing the reason of balancing loop. And it comes the format definition of the object-oriented fuzzy petri net based on the charging of balancing loop. We utilize the layering technology to peel off the loop from former net system, and to format the OOFPN's tree layered structure. Finally, making use of the place's Reachability Set, Immediate Reachability Set and Adjacent Place set, basing on reversed searching strategy, there comes an FPN simplified algorithm.

2 The FPN's Relevant Definitions and Charging Algorithm

At present, the researches on FPN are started on the basis with no loop, the research for general FPN is not involved yet [2]. By analyzing the characters of

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how fuzzy produced, in this section, the relevant definitions and charging methods of balancing loop in FPN are pointed out.

A. A Real Life's Loop Example

In reality, we will always have the process as the Figure1 when we are meeting the doctor.

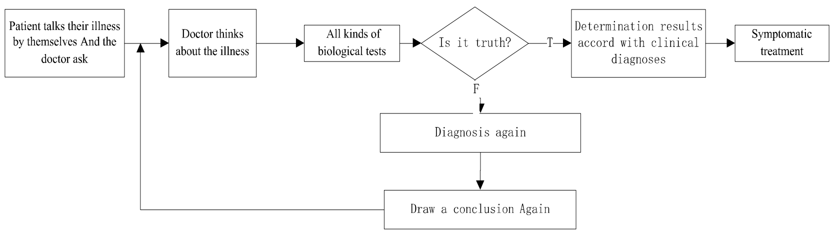
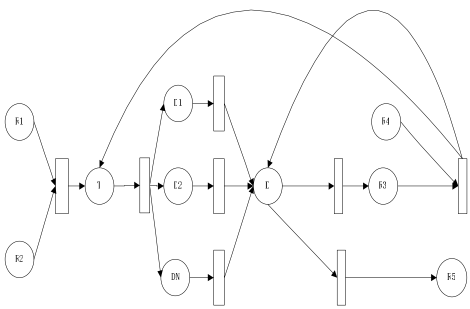


Fig. 1. The process when we are meeting the doctor

As what the Figure2 shows, we change Figure2 to corresponding FPN model. (weights, certainty faculty and threshold are omitted)



R1: Patient talks their illness by themselves
R2: Doctor asks
T: Doctor thinks about the illness
D1~Dn: There comes the preparatory step
D: The illness with most possibility
R3: The corresponding therapy for the most possible illness
R4: All kinds of biological tests
R5:Symptomatic treatment

Fig. 2. The corresponding model for the Fig1

B. The Analyzing for the FPN Model with Loop

Before analyzing the loop, we need to make clear of the concepts about output places and input places.

Definition 1.1 input place[3]:If there is only flow relation from a proposition place to transition,but no flow relation from transition to it,it called a input place.

Definition 1.2 output place[3]: For a proposition place, If there is only flow relation from transition to it, but no flow relation from it to transition, it,it called a t output place.

From the Fig2, if the loop can be found in the FPN, we can't find the header and tailer of the loop, in the other words, the input place and the output place is the same place, The only difference is that they have the difference weight value. So,we can give the concept of the loop.

Definition 1.3 Loop: if a place pointed itself through a step' or a series of steps' derivationby the flow relation,it called form a loop at this place.

C. Loop Charging Algorithm

Because of the situation that the output place and input place of the loop's identical and different certainty faculty, as well as the definition of the loop, we have the idea that whether the output place and input place are the some place should be the chief condition for charging loop. The main part of charging algorithm is given by algorithm 1.

Algorithm 1: loop charging algorithm

```
void {
    establish a list of place, To publish(all place) on a list
    M=the number of the place in this list;
    for n=1 to M
     {
       if place p_n still point itself though one or more than one step {
          Ia loop in existence;
          Output this loop; }
            else
       inexistence of a loop;
     n++;
       }
     }
```

3 The OOFPN Format Definition Based on Loop

From the viewpoint of object-oriented, system is composed by objects and the relations between them. Based on different purpose, different issues give the different definition of OOFPN concept[4-6]:. This paper develops it further on these basic, comes the format definition for OOFPN:

$$OOFPN=(O,R)$$

O is the limited objects aggregate; R is the aggregate for relations between objects.

At present, the researches on FPN are started on the basis with no loop, the research for general FPN is not involved yet[2]. By analyzing the characters of how fuzzy produced, in this section, the relevant definitions and charging methods of balancing loop in FPN are pointed out.

A. Objects

The OOFPN objects with layers are defined like this:

$$o =< P^J , P, T, O, F >$$

P^J is the excuse point aggregate: $P^J \subseteq P$; P, T are the position aggregate and transforming aggregate; $O = \{o_1, o_2, \cdots, o_n\}$ is the internal objects aggregate; P_o is utilized to be the summation for all the internal objects' positions aggregates:

$$P_o = \cup_i P_i^J , F \subseteq ((P \cup P_o) \times T) \cup (T \times (P \cup P_o))$$

The set of internal Object $O = \{o_1, o_2, \cdots, o_n\}$,every o_i can be depictde by the egiht elements: $\{P, T, I, O, M_0, \tau, U, W\}$ [7]:,to be specific: P is a set of the place,it means a fuzzy proposition; T is a set of the transition,it means achieve the rule; $I(O)$ is defined forward (back ward)matrix based on $P \times T$; M_0 is the initial marking; U is the certainty faculty; τ is the Threshold; W is the weight function based on $P \times T$.

B. Relations

In a OOFPN model, the communication between different objects are achieved by information places, if $P_i \cap P_j = \varnothing$, the objects O_i and O_j have relations between them.

C. Layered the Loop by Making Use of Object-Orienteds Fuzzy Petri Net

The core idea off object-oriented is to envelop the inside attributes and operation together. External can read and change its attributes only by transfer the objects. Because of the enveloping character of the object-orienteds[8]:, this paper infuses the layer technology for net and baby net as well as interaction between nets (the outputs and inputs for nets) into FPN to solve the loop problem in FPN model. The theme method is like this:

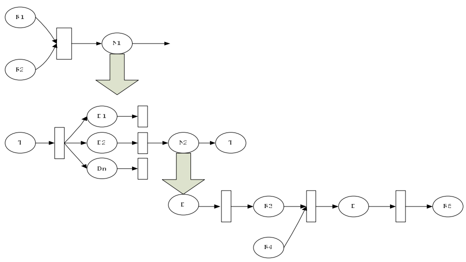
Step 1: Charge whether there is loop in this FPN by loop charging algorithm.

Step 2: Envelop this loop as an object if there is loop in this FPN model.

Step 3: The place,the input place the same as the output, is used to be a interface for trepresenting the contact with the outside world from the method of object. At the same time, we can call a method of an object and tramsfer the operand by sending token through interface; Response the object by drawing token from interface;

Step 4: To show the layered character, the internal of the objects are allowed to have objects.

Based on these ideas, Fig2 explains that FPN model can transfer to layered model without loop by utilizing the OOFPN concept. As what Fig 3 shows: (weights, certainty faculty and threshold are omitted).



R1: Patient talks their illness by themselves
R2: Doctor asks
T: Doctor thinks about the illness
D1~Dn: There comes the preparatory step
D: The illness with most possibility
R3: The corresponding therapy for the most possible illness
R4: All kinds of biological tests R5:Symptomatic treatment

Fig. 3. The OOFPN Model without Loop Based on Object-oriented's Idea

4 A Simplified Algorithm for OOFPN without Loop

After the FPN model with loop is layered by making use of object-oriented idea, the FPN in all layers are FPN models without loop. This paper comes up a simplified FPN model based on reversed searching idea by building Reachability Set, Immediate Reachability Set and Adjacent Place set for Fuzzy Petrii Net's all places.

A. Relevant Concepts

Definition 4.1 Reachability Set(RS)[9]: If Pi can arrive a set of place by a series transitions,the set of place called a Reachability Set,use RS(Pi) as the symbol for Reachability Set

Definition 4.2 Immediate Reachability Set(IRS)[9]: A set of place which can reach immediately from Pi,in other words,if Pi can arrive the set of place through a transition tj,the set called Immediate Reachability Set,use IRS(Pi) as the symbol for Immediate Reachability Set.

Definition 4.3 Adjacent Place set(AP)[9]: A series of place which can reach immediately as the same transition as Pi,it called Adjacent Place set, AP(Pij). use IRS(Pi) as the symbol for Adjacent Place set.

B. The Simplified Algorithm for a OOFPN without Loop

By building Reachability Set, Immediate Reachability Set and Adjacent Place set in Fuzzy Petri Net's all places, Algorithm 2 comes a simplified FPN algorithm which is based on reversed searching idea.

Algorithm 2: a smpifity algorithm of the OOFPN without loop
void Del (node){
choice the place Pa for the set of node's targer palce one by one;
seek the immediate reachability set based Pa, find out all or Pab's start palces which are connected to Pa;

if all Pab's certainty faculty $\leq \tau_b$ {

delete all transitions which are connected to Pab and all started places of Pab;}

else {

seek all of Pac's immediate reachability places from the table of Pa's immediate reachability set;

for num=1 to c {

if Pac is middle place Del Pac ;

else if Pac's certainty faculty $< \tau_c$ {

Del Pac ;

Seek the Pac's adjacent place set;

if the set is null, delete all transitions which are connected to Pac; }

Renovate the Immediate reachability Set and adjacent place set;

}

}

}

C. An Example of Application

This paper utilizes a part of testing knowledge OOFPN model as "Ethernet has problems" to exam the feasibility of this algorithm. This FPN model is as Fig 4 shows:

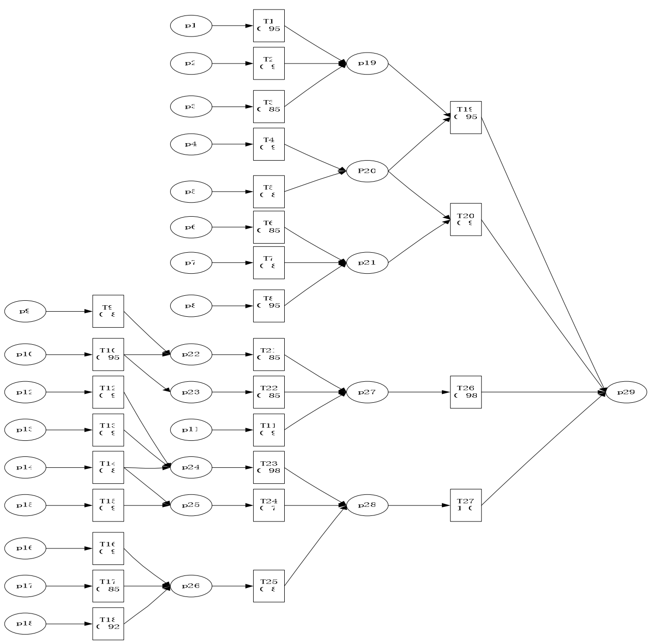


Fig. 4. OOFPN model as "Ethernet has problems"

According to the FPN model in Fig4, we can conclude the aggregate tables for the Reachability Set, Immediate Reachability Set and Adjacent Place set as table 1 and 2 show:

Table 1. The Reachability Set and immediate reachability set of the Fig 4

P_i	$IRS(P_i)$	$RS(P_i)$
P_1	$\{P_{19}\}$	$\{P_{19}, P_{29}\}$
P_2	$\{P_{19}\}$	$\{P_{19}, P_{29}\}$
P_3	$\{P_{19}\}$	$\{P_{19}, P_{29}\}$
P_4	$\{P_{20}\}$	$\{P_{20}, P_{29}\}$
P_5	$\{P_{20}\}$	$\{P_{20}, P_{29}\}$
P_6	$\{P_{21}\}$	$\{P_{21}, P_{29}\}$
P_7	$\{P_{21}\}$	$\{P_{21}, P_{29}\}$
P_8	$\{P_{21}\}$	$\{P_{21}, P_{29}\}$
P_9	$\{P_{22}\}$	$\{P_{22}, P_{27}, P_{29}\}$
P_{10}	$\{P_{22}, P_{23}\}$	$\{P_{22}, P_{23}, P_{27}, P_{29}\}$
P_{11}	$\{P_{27}\}$	$\{P_{27}, P_{29}\}$
P_{12}	$\{P_{24}\}$	$\{P_{24}, P_{28}, P_{29}\}$
P_{13}	$\{P_{24}\}$	$\{P_{24}, P_{28}, P_{29}\}$
P_{14}	$\{P_{24}, P_{25}\}$	$\{P_{24}, P_{25}, P_{28}, P_{29}\}$
P_{15}	$\{P_{25}\}$	$\{P_{25}, P_{28}, P_{29}\}$
P_{16}	$\{P_{26}\}$	$\{P_{26}, P_{28}, P_{29}\}$
P_{17}	$\{P_{26}\}$	$\{P_{26}, P_{28}, P_{29}\}$
P_{18}	$\{P_{26}\}$	$\{P_{26}, P_{28}, P_{29}\}$
P_{19}	$\{P_{29}\}$	$\{P_{29}\}$
P_{20}	$\{P_{29}\}$	$\{P_{29}\}$

Table 1. (*continued*)

P_{21}	$\{P_{29}\}$	$\{P_{29}\}$
P_{22}	$\{P_{27}\}$	$\{P_{27}, P_{29}\}$
P_{23}	$\{P_{27}\}$	$\{P_{27}, P_{29}\}$
P_{24}	$\{P_{28}\}$	$\{P_{28}, P_{29}\}$
P_{25}	$\{P_{28}\}$	$\{P_{28}, P_{29}\}$
P_{26}	$\{P_{28}\}$	$\{P_{28}, P_{29}\}$
P_{27}	$\{P_{29}\}$	$\{P_{29}\}$
P_{28}	$\{P_{29}\}$	$\{P_{29}\}$

Table 2. The adjacent place set of the Fig 4

P_i	P_j	$AP(P_{ij})$
P_1	P_{19}	NULL
P_2	P_{19}	NULL
P_3	P_{19}	NULL
P_4	P_{20}	NULL
P_5	P_{20}	NULL
P_6	P_{21}	NULL
P_7	P_{21}	NULL
P_8	P_{21}	NULL
P_9	P_{22}	NULL
P_{10}	P_{22}	NULL
P_{10}	P_{23}	NULL
P_{11}	P_{27}	NULL
P_{12}	P_{24}	NULL
P_{13}	P_{24}	NULL

Table 2. (*continued*)

P_{14}	P_{24}	NULL
P_{14}	P_{25}	NULL
P_{15}	P_{25}	NULL
P_{16}	P_{26}	NULL
P_{17}	P_{26}	NULL
P_{18}	P_{26}	NULL
P_{19}	P_{29}	P_{20}
P_{20}	P_{29}	P_{19},P_{21}
P_{21}	P_{29}	P_{20}
P_{22}	P_{27}	NULL
P_{23}	P_{27}	NULL
P_{24}	P_{28}	NULL
P_{25}	P_{28}	NULL
P_{26}	P_{28}	NULL
P_{27}	P_{29}	NULL
P_{28}	P_{29}	NULL

After the results for aggregate tables for the Reachability Set, Immediate Reachability Set and Adjacent Place set come out, there comes the simplified FPN model according to algorithm 2.

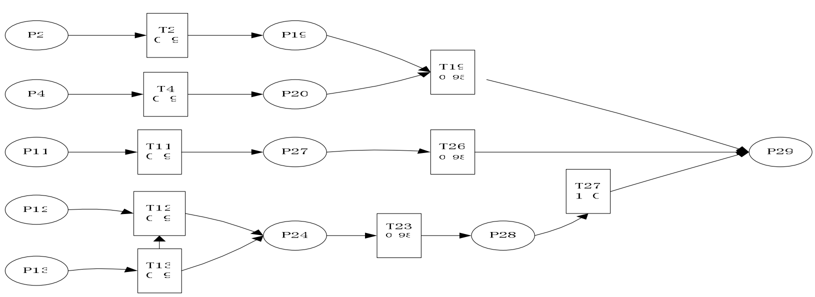


Fig. 5. The simplified FPN model according

5 Summary and Scope

FPN is the qualified tool for describing unclear knowledge. But all the present issues are based on FPN without loops. In this case, with the size of the knowledge place is enlarging, there must be a problem of the FPN pots' explosion. By analyzing the reasons of loop product, this paper comes up the charging algorithm for loop. Making use of the object-oriented ideas, this paper comes up the format definition for OOFPN. After these, there comes the idea to peel off the loop from former net, which break the FPN model with loop down to layered FPN model without loop. Concerning the FPN model without loop, the simplified algorithm is produced based on conserved search by place's Reachability Set, Immediate Reachability Set and Adjacent Place set tables. Tests show that this simplified algorithm is feasible.

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Study of Mean-Entropy Models for Key Point Air Defense Disposition

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Abstract. Air defense disposition problem is full of uncertainties and risks in modern war. In this paper, entropy is used as a measure of risk. The smaller entropy value is, the less uncertainty the problem contains, and thus, the safer disposition is. Within the framework of uncertainty theory, two types of fuzzy mean-entropy models are proposed. And a hybrid intelligent algorithm is presented for solving the proposed models in general cases. To illustrate the effectiveness of the proposed algorithm, a Numerical example of the bi-layer air-defense disposition for air defense operation in uncertain environment is presented.

Keywords: Air-defense disposition, entropy, risk, mean-entropy model.

1 Introduction

Air defense disposition of the key point is a question of a great deal of risks for the air defense operation. These risks come from the uncertainty of battlefield environment. Researchers may describe such uncertainty by using stochastic, fuzzy and hybrid variables. Because the historical data is not obtained by a great deal of statistics in modern war, the uncertain parameters in air defense disposition are usually described by the fuzzy and hybrid variable. Based on fuzzy battlefield effectiveness, we have discussed their modeling in the past [11]. Any disposition contains risks which commanders want to avoid. So it is necessary to propose the risk model. In 1952, Markowitz [14] initialized that variance could be regarded as the risk, and it has been accepted as the most well known mathematical definition of risk. Since then, as an extension of variance, semivariance was proposed and employed to measure risk so that only returns below expected value were measured as risk. Another alternative definition of risk was the entropy. Entropy was defined by Shannon [12] as a measure of uncertainty. It was first employed for portfolio selection by Philippatos and Wilson [13] as an alternative measure of risk to replace variance proposed by Markowitz [14]. They pointed out that entropy is more general than variance as an efficient measure of risk because entropy is free

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from reliance on symmetric probability distributions and can be computed from nonmetric data. So in this paper, entropy is proposed as a synonym for risk in the sense that uncertainty causes loss, and two goals to maximize the expected return and to minimize the entropy.

The rest of the paper is organized as follows. Section 2 reviews some important concepts and measures of fuzzy variables and entropy. Then, the problem of air defense disposition is discussed, and two types of fuzzy mean-entropy disposition models are constructed in section 3. In section 4, a hybrid intelligence algorithm is employed based on fuzzy simulation, genetic algorithm and neural networks to solve the optimization problems. Section 5 presents a numerical example of the bi-layer air defense disposition to show the effectiveness of the proposed approach. At last, Section 6 concludes the paper. monograph.

2 Preliminaries

In this section, we present some preliminaries for the fuzzy expected value and entropy with the framework of credibility theory [1]. For detailed expositions on credibility theory, the interested reader may consult [1] and [2].

Let ξ be a fuzzy variable with membership function μ. The concept of credibility measure is defined by Liu and Liu [3] as.

$$Cr\{\xi \in A\} = \frac{1}{2}(\sup_{x \in A} \mu(x) + 1 - \sup_{x \in A^c} \mu(x)), \tag{1}$$

for any set A of real numbers. Conversely, if is a fuzzy variable, then its membership function is derived from the credibility measure by:

$$\mu(x) = (2Cr\{\xi = x\}) \wedge 1, \tag{2}$$

It is obvious that the credibility measure is self-dual.

The expected value of fuzzy variable is defined by Liu and Liu [3] as:

$$E[\xi] = \int_0^{\infty} Cr\{\xi \geq r\}\mathrm{d}r - \int_{\infty}^{0} Cr\{\xi \leq r\}\mathrm{d}r, \tag{3}$$

Let ξ be a fuzzy variable with finite expected value $E[\xi]$. The variable of ξ is defined as $V[\xi] = E[(\xi - E[\xi])^2]$. And the standard deviation of ξ is defined as $\sigma = \sqrt{V[\xi]}$. Fuzzy entropy is an important research topic in fuzzy set theory. Luca and Termini [4] were the first to define a nonprobabilistic entropy within the framework of fuzzy theory. Other scholars such as Bhandari and Pal [5], and anther [7-8] have also given their definitions of fuzzy entropy. But these definitions characterize the uncertainty resulting from linguistic vagueness instead of from information deficiency. The uncertainty measured by these definitions disappears when the fuzzy variable is an equipossible one. In 2008, based on credibility, Li and Liu [9] proposed a new definition of fuzzy entropy that characterizes the uncertainty resulting from information deficiency caused by the inability to accurately predict the specified values. Let ξ be a continuous fuzzy variable. Then, its entropy is defined by [9]:

$$H[\xi] = \int_{-\infty}^{\infty} S(Cr\{\xi = r\})\mathrm{d}r, \tag{4}$$

where $S(t) = -t\ln t - (1-t)\ln(1-t)$. This one is an increasing function of t in the interval [0, 0.5] and is a decreasing function of t in the interval [0.5, 1]. It reaches its maximum at the point of $t = 0.5$.

Since for any continuous fuzzy variable ξ with membership function $\mu(x)$, we have $Cr\{\xi = x\} = \mu(x)/2$ for each $x \in R$. Thus, the entropy $H[\xi]$ can be expressed by:

$$H[\xi] = -\int_{-\infty}^{\infty} (\frac{\mu(x)}{2}\ln\frac{\mu(x)}{2} + (1-\frac{\mu(x)}{2})\ln(1-\frac{\mu(x)}{2}))\mathrm{d}x, \tag{5}$$

Example 1: Let be an equipossible fuzzy variable (a, b). Then $\mu(x) = 1$ if $a \le x \le b$,and 0 otherwise. Thus according to the formula (5), its entropy is

$$H[\xi] = -\int_{-\infty}^{\infty} (\frac{1}{2}\ln\frac{1}{2} + (1-\frac{1}{2})\ln(1-\frac{1}{2}))\mathrm{d}x = (a-b)\ln 2, \tag{6}$$

Example 2: Let ξ be a triangular fuzzy variable (a,b,c). Then according to the formula (5), its entropy is $H[\xi] = (c-a)/2$.
Example 3: Let ξ be a trapezoidal fuzzy variable (a,b,c,d). Then according to the formula (5), its entropy is $H[\xi] = (d-a)/2 + (\ln 2 - 0.5)(c-b)$.
Example 4: Let ξ be a normally distributed fuzzy variable with expected value e and variance σ^2. Then according to the formula (5), its entropy is $H[\xi] = \sqrt{6}\sigma/3$.
For more expositions on fuzzy entropy that we use, the interested readers can refer to [10].

3 Fuzzy Mean-Entropy Models

Let us consider a system of air defense. In the system, there are $m = \sum_{i=1}^{n} m_i$ firepower units, and are firepower units of the ith layer. They are deployed on rings equably, and distance $x_i (i = 1, 2, \cdots, n)$ outspread respectively from the very centre O of the protected object. Namely, there are m_i firepower units on the outside ring x_i. Then $x_i (i = 1, 2, \cdots, n)$ make up of n decision variables of this problem, and $x_1 < x_2 < \cdots < x_n$. Let $\xi_i (i = 1, 2, 3, 4)$ be fuzzy variables and represent the height, the speed, the bomb-dropping distance of air-raid target and the distance between different firepower units respectively.

3.1 Assumptions, Notations and Problem Description

1. The air-raid target attacks ward objectives along a sector area with sector angle α.
2. Each firepower unit is same.
3. Every air-raid target can be shot by 2 units at the same time, and there are two in every ring.
4. $P(\boldsymbol{x}, \xi)$: uncertain effect of air defense disposition with decision vector $\boldsymbol{x}$
5. t_s: the shooting cycle of a firepower unit;
6. t_{min}: the least cycle of a firepower unit;
7. D_{max}: the farthest radius of killing of a firepower unit;
8. P: the killing probability of a firepower unit;
9. K: the numbers of air-raid target.

When m firepower units are collocated on every ring equably, there are m_i units on the ith layer, $E_1^{(i)}, E_2^{(i)}, \cdots E_{m_i}^{(i)}$, respectively. In order to annihilate the aim, we must assure one shot by 2n firepower units. Then the corresponding gunnery constraint can be expressed as follows:

$$\sqrt{D_{max}^2 - \xi_1^2 - (t_{min}\xi_2)^2} \geq x_i \sin \frac{\alpha}{2(m_i+1)}, \tag{7}$$

Based on the tactics principle.The target must be annihilated outside bomb-dropping line, and the distance deployed firepower units take no less than ξ_4.

$$x_1 \geq \xi_3; \quad x_{i+1} - x_i \geq \xi_4, \ (i = 1, 2, \cdots, n-1), \tag{8}$$

$$2x_i \sin \frac{\alpha}{2(m_i+1)} \geq \xi_4, \ (i = 1, 2, \cdots, n), \tag{9}$$

Formulae (3)-(5) must be satisfied to be constraints. Then the path length of firepower area of the air-raid target is expressed as follows respectively.

$$\sqrt{D_{max}^2 - \xi_1^2 - [x_i \sin \frac{\alpha}{2(m_i+1)}]^2} + x_i - \xi_3, \ for \ E_j^{(i)}, E_{j+1}^{(i)} (i = 1, 2, \cdots, n), \tag{10}$$

And then, we may compute shooting degree for ith layer.

$$C_i = 2\left(\frac{\sqrt{D_{max}^2 - \xi_1^2 - [x_i \sin \frac{\alpha}{2(m_i+1)}]^2} + x_i - \xi_3}{\xi_2 t_s} + 1 \right), \tag{11}$$

In the end, the total shooting degree of $E_j^{(i)}, E_{j+1}^{(i)} (i = 1, 2, \cdots, n)$ is:

$$C = \sum_{i=1}^{n} C_i, \tag{12}$$

Another important problem is to compute the effect of air-raid target killed. It follows from formula (13) that the killing effect:

$$1-(1-p)^{\frac{C}{K}}, \tag{13}$$

Because p and k of formula (13) are constants, the maximum formula (13) is also the maximum formula (12) and vice versa. Then the killing return of system is defined as

$$P(\boldsymbol{x},\xi)=\sum_{i=1}^{n}\left(\frac{\sqrt{D_{max}^2-\xi_1^2-[x_i\sin\frac{\alpha}{2(m_i+1)}]^2}+x_i-\xi_3}{\xi_2 t_s}+1\right), \tag{14}$$

Where $\boldsymbol{x}=(x_1,x_2,\cdots,x_n)$, and $x_1<x_2<\cdots<x_n$, and $\xi=(\xi_1,\xi_2,\xi_3,\xi_4)$.

3.2 Two Fuzzy Mean-Entropy Model of the Disposition

For a commander, he will first require the formula (14) to be safe enough, and then pursues maximum expected return. The smaller the entropy value is, the more concentrative the value of formula (14) distributes. The more concentrated the value of formula (14) distributes, the more likely the specific expected value will occur. Therefore it is reasonable to ask that the entropy value of formula (14) must first be lower than or equal to a safety level. Let be the maximum entropy level the commander can tolerate. We can express the disposition's goal and requirement in the following way:

$$\begin{cases} \max\limits_{x} E\left(\sum\limits_{i=1}^{n}\left(\frac{\sqrt{D_{max}^2-\xi_1^2-[x_i\sin\frac{\alpha}{2(m_i+1)}]^2}+x_i-\xi_3}{\xi_2 t_s}+1\right)\right) \\ \quad s.t \\ \qquad H\left(\sum\limits_{i=1}^{n}\left(\frac{\sqrt{D_{max}^2-\xi_1^2-[x_i\sin\frac{\alpha}{2(m_i+1)}]^2}+x_i-\xi_3}{\xi_2 t_s}+1\right)\right)\le\alpha \\ \qquad Cr\{(x_i\sin(\alpha/2(m_i+1))^2-D_{max}^2+\xi_1^2+(t_{\min}\xi_2)^2\le 0\}\ge\beta_i \\ \qquad\qquad (i=1,2,\cdots,n) \\ \qquad Cr\{\xi_4-2x_i\sin(\alpha/2(m_i+1))\le 0\}\ge\gamma_i\ \ (i=1,2,\cdots,n) \\ \qquad Cr\{\xi_4+x_{i+1}-x_i\le 0\}\ge\lambda_i\ \ (i=1,2,\cdots,n-1) \\ \qquad Cr\{\xi_3-x_1\le 0\}\ge\mu \\ \qquad x_1,x_2,\cdots,x_n\ge 0 \end{cases} \tag{15}$$

in which $\beta_i,\gamma_i(i=1,2,\cdots,n),\lambda_i(i=1,2,\cdots,n-1)$, μ is predetermined confidence level given by commander, H denotes the entropy of the fuzzy variables and E is the expected value operator. If commander requires the expected return be high enough, then he will reduce the risk level as far as possible. Suppose that α is the lowest return level the commander feels is satisfactory. The idea can be mathematically expressed in the following way:

$$\begin{cases} \min\limits_{x} H\left(\sum\limits_{i=1}^{n}\left(\frac{\sqrt{D_{max}^2-\xi_1^2-[x_i\sin\frac{\alpha}{2(m_i+1)}]^2}+x_i-\xi_3}{\xi_2 t_s}+1\right)\right) \\ s.t \\ \qquad E\left(\sum\limits_{i=1}^{n}\left(\frac{\sqrt{D_{max}^2-\xi_1^2-[x_i\sin\frac{\alpha}{2(m_i+1)}]^2}+x_i-\xi_3}{\xi_2 t_s}+1\right)\right)\geq\alpha \\ \qquad Cr\{(x_i\sin(\alpha/2(m_i+1))^2-D_{max}^2+\xi_1^2+(t_{\min}\xi_2)^2\leq 0\}\geq\beta_i \\ \qquad\qquad (i=1,2,\cdots,n) \\ \qquad Cr\{\xi_4-2x_i\sin(\alpha/2(m_i+1))\leq 0\}\geq\gamma_i \quad (i=1,2,\cdots,n) \\ \qquad Cr\{\xi_4+x_{i+1}-x_i\leq 0\}\geq\lambda_i \quad (i=1,2,\cdots,n-1) \\ \qquad Cr\{\xi_3-x_1\leq 0\}\geq\mu \\ \qquad x_1,x_2,\cdots,x_n\geq 0 \end{cases} \tag{16}$$

in which $\beta_i,\gamma_i(i=1,2,\cdots,n),\lambda_i(i=1,2,\cdots,n-1)$, μ is predetermined confidence level given by commander, H denotes the entropy of the fuzzy variables and E is the expected value operator. In fact, variance V is used as a risk measure too. When the variance value is big, the value of formula (14) distributes far away from the expect value, which implies that the expected return will be less likely to be obtained. Then, in models (15), (16), the entropy H may be replaced the variance V to convert new models. In order to avoid abandoning the return with high positive deviation from the expected value, we must ask the membership function of the formula (14) should be symmetrical. In this paper, we are interested in mean-entropy model.

4 Hybrid Intelligent Algorithm

For model (15), (16), it is usually hard to solve the problem in traditional methods. To compute our model, we first design a fuzzy simulation to estimate the entropy value, the expected value and credibility measure of fuzzy event in this paper, and then, integrate fuzzy simulations into GA to find the optimal configure solution. The GA procedure has been introduced in detail in some books [1]. Now we summarize the algorithm as follows:

Step 1. Generate training input-output data for uncertain function
$U_1:(x_1,x_2,\cdots,x_n)\rightarrow H(P(\boldsymbol{x},\xi))$
$U_2:(x_1,x_2,\cdots,x_n)\rightarrow E(P(\boldsymbol{x},\xi))$
$U_{3i}:(x_1,x_2,\cdots,x_n)\rightarrow Cr\{(x_i\sin(\alpha/2(m_i+1)))^2-D_{max}^2+\xi_1^2+(t_{\min}\xi_2)^2\leq 0\}(i=1,2,\cdots,n)$
$U_{4i}:(x_1,x_2,\cdots,x_n)\rightarrow Cr\{\xi_4-2x_i\sin(\alpha/2(m_i+1)\leq 0)\}\ \ (i=1,2,\cdots,n)$
$U_{5i}:(x_1,x_2,\cdots,x_n)\rightarrow Cr\{\xi_4+x_{i+1}-x_i\}\ \ (i=1,2,\cdots,n-1)$
$U_6:(x_1,x_2,\cdots,x_n)\rightarrow Cr\{\xi_3-x_1\leq 0\}$ by the fuzzy simulation, respectively. In which the genes x_i are restricted as $0<x_1<x_2<\cdots<M$. M is an enough large number.
Step 2. Train a neural network to approximate above uncertain function according to the generated training input-output data.
Step 3. Input the parameters of GA: pop-size, P_c,P_m, a.

Step 4. Initialize pop-size chromosomes, in which the trained neural network is used to check the feasibility of the chromosomes.
Step 5. Update the chromosomes by crossover and mutation operations and the trained neural network may be employed to check the feasibility of offspring.
Step 6. Calculate the objective value for all chromosomes by the trained neural network.
Step 7. Compute the fitness of each chromosome by rank-based evaluation function based on the objective values. First, calculate the objective values for all chromosomes by fuzzy simulation. Then, give the rank order of the chromosomes according to the objective values. For model (15), the greater the expected value is, the better the chromosome is, and the smaller the ordinal number the chromosome has. For model (16), the smaller the entropy value is, the better the chromosome is, and the smaller the ordinal number the chromosome has. Next, compute the values of the rank-based evaluation function of the chromosomes, and then, the fitness of each chromosome according to the rank-based-evaluation function.
Step 8. Select the chromosomes by spinning the roulette wheel.
Step 9. Repeat Step 5 to Step 8 a given number of cycles.
Step 10. Take the best chromosome as the solution of portfolio selection. For model (15), the chromosome with the maximum expected value is the best chromosome. For model (16), the chromosome with the minimum entropy value is the best chromosome.

5 Numerical Example

For the sake of illustration, let us consider a recovery system of antiaircraft artillery group. Its firepower units are deployed in a sector area, and the sector angle $\alpha = 2\pi/3$. Suppose m_1 is 4, m_2 is 5, D_{max} is $5km$, t_s is 4s, t_{min} is 1s. The fuzzy vector $\xi = (\xi_1, \xi_2, \xi_3, \xi_4)$ is given in Table 1, in which $(a,\ b,\ c)$ means the triangle fuzzy number, and $(a,\ b,\ c,\ d)$ means the trapezoidal fuzzy number. The parameters in GA are set as follows: the population size 30, the probability of crossover $P_c = 0.3$, the probability of mutation $P_m = 0.3$, and the parameter a in the rank-based evaluation function is 0.5.

Table 1. Fuzzy variables and its numbers in the model

Fuzzy variables	$\xi_1(km)$	$\xi_2(km)$	$\xi_3(km)$	$\xi_4(km)$
Fuzzy numbers	(2,4,5.5)	(0.2,0.35,0.4)	(2,7,9,10)	(2,3,3.8)

Suppose that the commander accept 1.60 as the maximum tolerable uncertainty level, and require that the expected return be maximized at the entropy not grater than this level. Then, the model (15) is given as follows:

$$\begin{cases} \max_x E\left(\sum_{i=1}^{n} \frac{\sqrt{5000^2-\xi_1^2-[x_1\sin\frac{\pi}{18}]^2}+x_1-\xi_3}{4\xi_2} + \frac{\sqrt{5000^2-\xi_1^2-[x_2\sin\frac{\pi}{15}]^2}+x_2-\xi_3}{4\xi_2} + 2\right) \\ s.t \\ \quad H\left(\sum_{i=1}^{n} \frac{\sqrt{5000^2-\xi_1^2-[x_1\sin\frac{\pi}{18}]^2}+x_1-\xi_3}{4\xi_2}) + \frac{\sqrt{5000^2-\xi_1^2-[x_2\sin\frac{\pi}{15}]^2}+x_2-\xi_3}{4\xi_2} + 2\right) \\ \qquad \le 1.60 \\ \quad Cr\begin{pmatrix} (x_1\sin\frac{\pi}{18})^2 - 5000^2 + \xi_1^2 + \xi_2^2 \le 0 \\ (x_2\sin\frac{\pi}{15})^2 - 5000^2 + \xi_1^2 + \xi_2^2 \le 0 \end{pmatrix} \ge 0.85 \\ \quad Cr\begin{pmatrix} x_1 \ge \xi_3 \\ x_2 - x_1 \ge \xi_4 \\ 2x_1\sin\frac{\pi}{18} \ge \xi_4 \\ 2x_2\sin\frac{\pi}{15} \ge \xi_4 \end{pmatrix} \ge 0.95 \\ \quad x_1 > 0, x_2 > 0 \end{cases} \tag{17}$$

A run of the hybrid intelligent algorithm with 2000 generations show that in order to gain the maximum the killing return at entropy not greater than 1.60, the optimal distances is
$(x_1^*, x_2^*) = (9.659, 12.980)$
with maximum expected value $E^* \approx 0.5$.

To further test the robustness of the designed algorithm, we did more numerical experiments with different values of parameters in the GA. The results are given in Table 2. We use the relative error to compare the results of objective values. The relative error is calculated through the formula (optimal value -actual value) /optimal value×100% where the optimal value is the maximal one of all five optimal objective values calculated. It can be seen from Table 2 that the relative errors do not exceed 1.5%, the maximum relative error is 1.41% when different values of parameters are set, which shows that that designed algorithm is robust to set parameters and effective for solving the model (17). If the commander want to minimize entropy at the expected return level not less than 98, the model (16) is as follows:

Table 2. Fuzzy variables and its numbers in the model

pop-size	P_c	P_m	gen	objective value	relative error
30	0.3	0.3	3000	104.6	1.13%
30	0.3	0.5	1000	104.9	1.41%
30	0.3	0.3	2000	105.3	0.75%
30	0.2	0.5	4000	106.1	0.00%
30	0.3	0.2	2000	105.1	0.94%

$$\begin{cases} \max_x H\left(\sum_{i=1}^{n} \frac{\sqrt{5000^2-\xi_1^2-[x_1\sin\frac{\pi}{18}]^2}+x_1-\xi_3}{4\xi_2} + \frac{\sqrt{5000^2-\xi_1^2-[x_2\sin\frac{\pi}{15}]^2}+x_2-\xi_3}{4\xi_2} + 2\right) \\ s.t \\ \quad E\left(\sum_{i=1}^{n}(\frac{\sqrt{5000^2-\xi_1^2-[x_1\sin\frac{\pi}{18}]^2}+x_1-\xi_3}{4\xi_2} + \frac{\sqrt{5000^2-\xi_1^2-[x_2\sin\frac{\pi}{15}]^2}+x_2-\xi_3}{4\xi_2} + 2\right) \\ \qquad \geq 98 \\ \quad Cr\begin{pmatrix} (x_1\sin\frac{\pi}{18})^2 - 5000^2 + \xi_1^2 + \xi_2^2 \leq 0 \\ (x_2\sin\frac{\pi}{15})^2 - 5000^2 + \xi_1^2 + \xi_2^2 \leq 0 \end{pmatrix} \geq 0.85 \\ \quad Cr\begin{pmatrix} x_1 \geq \xi_3 \\ x_2 - x_1 \geq \xi_4 \\ 2x_1\sin\frac{\pi}{15} \geq \xi_4 \\ 2x_2\sin\frac{\pi}{18} \geq \xi_4 \end{pmatrix} \geq 0.95 \\ \quad x_1 > 0, x_2 > 0 \end{cases} \tag{18}$$

Again, we use a hybrid intelligent algorithm to solve it. A run of the hybrid intelligent algorithm with 2000 generations show that in order to minimize the entropy at the expected return not less than 98, the optimal distances is $(x_1^*, x_2^*) = (9.794, 13.978)$ with maximum expected value $H^* = 1.35$.

6 Conclusion

This paper proposed two types of credibility-based fuzzy mean-entropy models. Entropy was used to measure the risk of the killing effect. The smaller the entropy value is, the more concentrative the value of the killing return distributes. The more concentrated the value of the killing return distributes, the more likely the specific expected value will occur, and thus, the safer the killing return of disposition is. As a measure of risk, entropy is free from reliance on symmetric membership functions. In addition, the paper also employed a hybrid intelligent algorithm for solving the mean-entropy model problems in general cases.

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Rough Interval Valued Vague Sets in Pawlak Approximation Space

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Abstract. The combination of interval valued vague sets and rough sets is developed in this paper. The lower and upper approximation operators of an interval valued vague set are constructed, which are partitioned by an indiscernibility relation in Pawlak approximation space. Further properties associated with the lower and upper approximations of interval valued vague sets are examined. Finally, the roughness measure of an interval valued vague set is presented as an extension of the parameterized roughness measure of a vague set. Meantime, the related properties with respect to roughness measure are established and analyzed.

Keywords: Interval valued vague set, pawlak approximation space, rough interval valued vague set, roughness measure.

1 Introduction

Vague set theory was first introduced by Gau and Buehrer in 1993 [1], which was regarded as a promotion and extension for the theory of fuzzy sets. In essence, it was the same concept as the intuitionistic fuzzy sets proposed by Atanassov [2,4]. A vague set A is characterized by a truth-membership function t_A and a false-membership function f_A. For any $x \in U$ (U is a universe), $t_A(x)$ is a lower bound on the grade of membership of x derived from the evidence for x, $f_A(x)$ is a lower bound on the negation of x derived from the evidence against x, and $t_A(x)+f_A(x) \leq 1$. In fact, every decision-maker hesitates more or less in every evaluation activity. For instance, in order to judge whether a patient has cancer or not, a doctor (decision-maker) will hesitate because of the fact that a fraction of evaluation he thinks in favor of truth, another fraction in favor of falseness and the rest part remains undecided to him. Therefore, vague sets can realistically reflect the actual problem. But more often, the truth-membership and false-membership are a range. For this reason, the notion of interval valued vague sets was presented by Atanassov

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in 1989 [3]. And it is regarded as an extension of the theory of vague sets. In this theory, the truth-membership function and false-membership function are a subinterval on [0,1]. Rough set theory was proposed by Pawlak in 1982 [5]. It can approximate any subset of objects of the universe by the lower and upper approximations, which focuses on the ambiguity caused by the limited discernibility of objects in the universe of discourse. And the ambiguity was characterized by the roughness measure introduced by Pawlak [6]. The combination of fuzzy set theory and rough set theory has been done by many researchers in recent years [7-13]. And many new mathematical methods are generated for dealing with the uncertain and imprecise information, such as the fuzzy rough sets and rough fuzzy sets, etc. Additionally, many measure methods are proposed and investigated by different authors in order to characterize the uncertainty and ambiguity of different sets [14-20]. In this paper, we mainly concerns the construction of the lower and upper approximations of an interval valued vague set based on the works of Wang et al. [14] and Al-Rababah and Biswas [21]. Meantime, some new notions are presented and the corresponding properties associated with the approximation operators and roughness measure are examined.

2 Interval Valued Vague Sets (IVVS)

Let U be the universe of discourse, $I[0,1]$ denotes the family of all closed subintervals of [0,1]. An *interval valued vague set* G in the universe of discourse U is characterized by a *truth-membership function* T_G and *false-membership function* F_G given by

$$T_G : U \to I[0,1]$$

$$F_G : U \to I[0,1]$$

where T_G and F_G are set-valued functions on the interval [0,1], respectively. $T_G(x) = [t_G^-(x), t_G^+(x)]$, $t_G^-(x)$ and $t_G^+(x)$ denote the lower and upper bound on the grade of membership of x derived from "the evidence for x", respectively. Similarly, $F_G(x) = [f_G^-(x), f_G^+(x)]$, $f_G^-(x)$ and $f_G^+(x)$ denote, respectively, the lower and upper bound on the negation of x derived from "the evidence against x", and $t_G^+(x) + f_G^+(x) \leq 1$. Generally, the interval valued vague set G is denoted by $G = \{< x, T_G(x), F_G(x) >: x \in U\}$.

Additionally, suppose that $I_1, I_2 \in I[0,1]$, and $I_1 = [a_1, b_1]$, $I_2 = [a_2, b_2]$, we call $I_1 \geq I_2$, if $a_1 \geq a_2$ and $b_1 \geq b_2$. Analogously, we can understand the relations $I_1 \leq I_2$ and $I_1 = I_2$. Obviously, the relation $I_1 \geq I_2$ does not necessarily imply that $I_1 \supseteq I_2$ and conversely.

Definition 1. *Let* $G = \{< x, T_G(x), F_G(x) >: x \in U\}$ *and* $K = \{< x, T_K(x), F_K(x) >: x \in U\}$ *be two interval valued vague sets of the universe of discourse* U*, then*

(a) equality: $G = K$ *iff* $\forall x \in U, T_G(x) = T_K(x)$ *and* $F_G(x) = F_K(x)$;
(b) inclusion: $G \subseteq K$ *iff* $\forall x \in U, T_G(x) \leq T_K(x)$ *and* $F_G(x) \geq F_K(x)$, *where*

$$T_G(x) \leq T_K(x) \Leftrightarrow t_G^-(x) \leq t_K^-(x) \text{ and } t_G^+(x) \leq t_K^+(x),$$
$$F_G(x) \geq F_K(x) \Leftrightarrow f_G^-(x) \geq f_K^-(x) \text{ and } f_G^+(x) \geq f_K^+(x);$$

(c) intersection:

$$\begin{aligned} G \cap K &= \{< x, T_G(x), F_G(x) >: x \in U\} \cap \{< x, T_K(x), F_K(x) >: x \in U\} \\ &= \{< x, [t_G^-(x), t_G^+(x)], [f_G^-(x), f_G^+(x)] >: x \in U\} \cap \\ &\quad \{< x, [t_K^-(x), t_K^+(x)], [f_K^-(x), f_K^+(x)] >: x \in U\} \\ &= \{< x, [t_G^-(x) \wedge t_K^-(x), t_G^+(x) \wedge t_K^+(x)], \\ &\quad [f_G^-(x) \vee f_K^-(x), f_G^+(x) \vee f_K^+(x)] >: x \in U\}; \end{aligned}$$

(d) union:

$$\begin{aligned} G \cup K &= \{< x, T_G(x), F_G(x) >: x \in U\} \cup \{< x, T_K(x), F_K(x) >: x \in U\} \\ &= \{< x, [t_G^-(x), t_G^+(x)], [f_G^-(x), f_G^+(x)] >: x \in U\} \cup \\ &\quad \{< x, [t_K^-(x), t_K^+(x)], [f_K^-(x), f_K^+(x)] >: x \in U\} \\ &= \{< x, [t_G^-(x) \vee t_K^-(x), t_G^+(x) \vee t_K^+(x)], \\ &\quad [f_G^-(x) \wedge f_K^-(x), f_G^+(x) \wedge f_K^+(x)] >: x \in U\}; \end{aligned}$$

(e) complement: $G^c = \{< x, F_G(x), T_G(x) >: x \in U\}$.

3 Rough Approximations of IVVS in Pawlak Approximation Space

Based on the conclusions of the literatures [14,21], the concept of the rough interval valued vague sets(or in short RIVVS) is introduced in Pawlak approximation space, and some properties of RIVVS are examined in this section.

3.1 Rough Interval Valued Vague Sets (RIVVS)

Now, we consider the rough approximations of a interval vague set in Pawlak approximation space.

Definition 2. *Let* $G = \{< x, T_G(x), F_G(x) >: x \in U\}$ *be an interval valued vague set of the universe of discourse* U, $T_G(x) = [t_G^-(x), t_G^+(x)]$, $F_G(x) = [f_G^-(x), f_G^+(x)]$, *the* lower approximation $\underline{G}$ *and the* upper approximation $\overline{G}$ *of the interval valued vague set* G *in Pawlak approximation space* (U, R) *are defined, respectively, by*

$$\underline{G} = \{< x, T_{\underline{G}}(x), F_{\underline{G}}(x) >: x \in U\} \quad (1)$$

$$\overline{G} = \{< x, T_{\overline{G}}(x), F_{\overline{G}}(x) >: x \in U\} \quad (2)$$

where $\forall x \in U$,

$$\begin{aligned} T_{\underline{G}}(x) &= [t_{\underline{G}}^-(x), t_{\underline{G}}^+(x)] \\ &= [inf\{infT_G(y) : y \in [x]_R, x \in U\}, inf\{supT_G(y) : y \in [x]_R, x \in U\}] \end{aligned}$$

$$\begin{aligned} F_{\underline{G}}(x) &= [f_{\underline{G}}^-(x), f_{\underline{G}}^+(x)] \\ &= [sup\{infF_G(y) : y \in [x]_R, x \in U\}, sup\{supF_G(y) : y \in [x]_R, x \in U\}] \end{aligned}$$

$$T_{\overline{G}}(x) = [t_{\overline{G}}^-(x), t_{\overline{G}}^+(x)]$$

$$= [sup\{infT_G(y) : y \in [x]_R, x \in U\}, sup\{supT_G(y) : y \in [x]_R, x \in U\}]$$
$$F_{\overline{G}}(x) = [f^-_{\overline{G}}(x), f^+_{\overline{G}}(x)]$$
$$= [inf\{infF_G(y) : y \in [x]_R, x \in U\}, inf\{supF_G(y) : y \in [x]_R, x \in U\}].$$

Here, $[x]_R$ denotes the equivalence class which belongs to x.

From the previous definitions, for any $x \in U$, $T_{\underline{G}}(x)(T_{\overline{G}}(x))$ may be viewed as the range of the evidence in favor of the fact that x definitely(possibly) belongs to the interval valued vague set G and $F_{\underline{G}}(x)(F_{\overline{G}}(x))$ represent the range of the evidence against the fact that x definitely(possibly) belongs to the interval valued vague set G.

Definition 3. *Let G be an interval valued vague set of the universe of discourse U, (U, R) is a Pawlak approximation space, $\underline{G}$ and $\overline{G}$ denote respectively the lower approximation and the upper approximation, the pair $R(G) = (\underline{G}, \overline{G})$ be called the* rough interval valued vague sets*(RIVVS) in Pawlak approximation space.*

Especially, if $T_G(x)$ and $F_G(x)$ are constants, the RIVVS will be turned into the RVS, namely, RIVVS is viewed as a generalization of RVS.

Similar to the definable sets and rough sets of Pawlak rough set theory, if $\underline{G} = \overline{G}$, one can say that the interval vague sets G is a *definable interval valued vague sets* in (U, R). Otherwise, it is to be said a *rough interval valued vague sets* in (U, R).

Additionally, it is obvious that the interval valued vague set G is enveloped by the interval valued sets $\underline{G}$ and $\overline{G}$, namely, $\underline{G} \subseteq G \subseteq \overline{G}$.

Let us consider a simple example in the following.

Example 1. Let $U = \{x_i : i = 1, 2, \cdots, 8\}$ be a universe, there are four equivalence classes, $\{x_1, x_4\}$, $\{x_2, x_3, x_6\}$, $\{x_5\}$, $\{x_7, x_8\}$, which is partitioned by an equivalence relation R. Now, we consider the following two interval valued vague sets

$$G_1 =< [0.2, 0.4], [0.4, 0.5] > /x_1+ < [0.3, 0.6], [0.2, 0.3] > /x_4$$
$$+ < [0.5, 0.7], [0.1, 0.3] > /x_5+ < [0.1, 0.3], [0.4, 0.6] > /x_7$$
$$G_2 =< [0.2, 0.4], [0.4, 0.5] > /x_1+ < [0.2, 0.4], [0.4, 0.5] > /x_4$$
$$+ < [0.5, 0.7], [0.1, 0.3] > /x_5$$

According to Definition 3, the lower approximation and upper approximation of G_1 and G_2 are calculated as follows

$$\underline{G_1} =< [0.2, 0.4], [0.4, 0.5] > /x_1+ < [0.2, 0.4], [0.4, 0.5] > /x_4$$
$$+ < [0.5, 0.7], [0.1, 0.3] > /x_5$$
$$\overline{G_1} =< [0.3, 0.6], [0.2, 0.3] > /x_1+ < [0.3, 0.6], [0.2, 0.3] > /x_4$$
$$+ < [0.5, 0.7], [0.1, 0.3] > /x_5+ < [0.1, 0.3], [0.4, 0.6] > /x_7$$
$$+ < [0.1, 0.3], [0.4, 0.6] > /x_8$$
$$\underline{G_2} =< [0.2, 0.4], [0.4, 0.5] > /x_1+ < [0.2, 0.4], [0.4, 0.5] > /x_4$$
$$+ < [0.5, 0.7], [0.1, 0.3] > /x_5$$
$$\overline{G_2} =< [0.2, 0.4], [0.4, 0.5] > /x_1+ < [0.2, 0.4], [0.4, 0.5] > /x_4$$
$$+ < [0.5, 0.7], [0.1, 0.3] > /x_5$$

Clearly, $\underline{G_1} \neq \overline{G_1}$, $\underline{G_2} = \overline{G_2}$, the results show that G_1 is a rough interval valued vague set and G_2 is a definable interval valued vague set.

3.2 The Properties of RIVVS

Proposition 1. *Let G, K be two interval valued vague sets of U, then the following conclusions hold*
(P1) $\underline{G} \subseteq G \subseteq \overline{G}$; *(P2)* $G \subseteq K \Rightarrow \underline{G} \subseteq \underline{K}$ *and* $\overline{G} \subseteq \overline{K}$;
(P3) $\overline{G \cup K} = \overline{G} \cup \overline{K}$, $\underline{G \cap K} = \underline{G} \cap \underline{K}$; *(P4)* $\overline{G \cap K} \subseteq \overline{G} \cap \overline{K}$, $\underline{G \cup K} \supseteq \underline{G} \cup \underline{K}$;
(P5) $\underline{G^c} = \overline{G}^c$, $\overline{G^c} = \underline{G}^c$.

Proof. Obviously, by the Definitions 2 and 3, (P1) and (P2) hold. So we only prove the remainder of these properties.

(P3) For $\forall x \in U$, $\overline{G \cup K}(x) = \{< x, T_{\overline{G \cup K}}(x), F_{\overline{G \cup K}}(x) >\}$

$$\begin{aligned}
T_{\overline{G \cup K}}(x) &= [t^-_{\overline{G \cup K}}(x), t^+_{\overline{G \cup K}}(x)] \\
&= [sup\{infT_{G \cup K}(y) : y \in [x]_R\}, sup\{supT_{G \cup K}(y) : y \in [x]_R\}] \\
&= [sup\{inf(T_G(y) \vee T_K(y)) : y \in [x]_R\}, \\
&\quad sup\{sup(T_G(y) \vee T_K(y)) : y \in [x]_R\}] \\
&= [\{sup\{infT_G(y)\} \vee sup\{infT_K(y)\} : y \in [x]_R\}, \\
&\quad \{sup\{supT_G(y)\} \vee sup\{supT_K(y)\} : y \in [x]_R\}] \\
&= [t^-_{\overline{G}}(x) \vee t^-_{\overline{K}}(x), t^+_{\overline{G}}(x) \vee t^+_{\overline{G}}(x)] = T_{\overline{G} \cup \overline{K}}(x)
\end{aligned}$$

$$\begin{aligned}
F_{\overline{G \cup K}}(x) &= [f^-_{\overline{G \cup K}}(x), f^+_{\overline{G \cup K}}(x)] \\
&= [inf\{infF_{G \cup K}(y) : y \in [x]_R\}, inf\{supF_{G \cup K}(y) : y \in [x]_R\}] \\
&= [inf\{inf(F_G(y) \wedge F_K(y)) : y \in [x]_R\}, \\
&\quad inf\{sup(F_G(y) \wedge F_K(y)) : y \in [x]_R\}] \\
&= [\{inf\{infF_G(y)\} \wedge inf\{infF_K(y)\} : y \in [x]_R\}, \\
&\quad \{inf\{supF_G(y)\} \wedge inf\{supF_K(y)\} : y \in [x]_R\}] \\
&= [f^-_{\overline{G}}(x) \wedge f^-_{\overline{K}}(x), f^+_{\overline{G}}(x) \wedge f^+_{\overline{G}}(x)] = F_{\overline{G} \cup \overline{K}}(x).
\end{aligned}$$

Hence, $\overline{G \cup K} = \overline{G} \cup \overline{K}$. Similarly, we can get $\underline{G \cap K} = \underline{G} \cap \underline{K}$ by the same schemer above.

(P4) For $\forall x \in U$, $\overline{G} \cap \overline{K}(x) = \{< x, T_{\overline{G} \cap \overline{K}}(x), F_{\overline{G} \cap \overline{K}}(x) >\}$

$$\begin{aligned}
T_{\overline{G} \cap \overline{K}}(x) &= [t^-_{\overline{G} \cap \overline{K}}(x), t^+_{\overline{G} \cap \overline{K}}(x)] \\
&= [t^-_{\overline{G}}(x) \wedge t^-_{\overline{K}}(x), t^+_{\overline{G}}(x) \wedge t^+_{\overline{K}}(x)] \\
&= [\{sup\{infT_G(y)\} \wedge sup\{infT_K(y)\} : y \in [x]_R\}, \\
&\quad \{sup\{supT_G(y)\} \wedge sup\{supT_K(y)\} : y \in [x]_R\}] \\
&\geq [sup\{inf(T_G(y) \wedge T_K(y)) : y \in [x]_R\}, \\
&\quad sup\{sup(T_G(y) \wedge T_K(y)) : y \in [x]_R\}] \\
&= [sup\{infT_{G \cap K}(y) : y \in [x]_R\}, sup\{supT_{G \cap K}(y) : y \in [x]_R\}] \\
&= [t^-_{\overline{G \cap K}}(x), t^+_{\overline{G \cap K}}(x)] = T_{\overline{G \cap K}}(x)
\end{aligned}$$

$$\begin{aligned}
F_{\overline{G} \cap \overline{K}}(x) &= [f^-_{\overline{G} \cap \overline{K}}(x), f^+_{\overline{G} \cap \overline{K}}(x)] \\
&= [f^-_{\overline{G}}(x) \vee t^-_{\overline{K}}(x), f^+_{\overline{G}}(x) \vee t^+_{\overline{K}}(x)] \\
&= [\{inf\{infF_G(y)\} \vee inf\{infF_K(y)\} : y \in [x]_R\}, \\
&\quad \{inf\{supF_G(y)\} \vee inf\{supF_K(y)\} : y \in [x]_R\}] \\
&\leq [inf\{inf(F_G(y) \vee F_K(y)) : y \in [x]_R\}, \\
&\quad inf\{sup(F_G(y) \vee F_K(y)) : y \in [x]_R\}]
\end{aligned}$$

$= [inf\{infF_{G\cap K}(y) : y \in [x]_R\}, inf\{supF_{G\cap K}(y) : y \in [x]_R\}]$
$= [f^-_{\overline{G\cap K}}(x), f^+_{\overline{G\cap K}}(x)] = F_{\overline{G\cap K}}(x).$

Hence, $\overline{G\cap K} \subseteq \overline{G} \cap \overline{K}$. Similarly, we can get $\underline{G\cup K} \supseteq \underline{G} \cup \underline{K}$ by the same schemer above.

(P5) Since $G^c = \{< x, F_G(x), T_G(x) >\}$, namely, for $\forall\, x \in U$, we have

$G^c(x) = \{< x, F_G(x), T_G(x) >\}$ and $\overline{G}^c(x) = \{< x, F_{\overline{G}}(x), T_{\overline{G}}(x) >\}$

$F_{\overline{G}}(x) = [f^-_{\overline{G}}(x), f^+_{\overline{G}}(x)]$
$= [inf\{infF_G(y) : y \in [x]_R\}, inf\{supF_G(y) : y \in [x]_R\}]$
$= [inf\{infT_{G^c}(y) : y \in [x]_R\}, inf\{supT_{G^c}(y) : y \in [x]_R\}]$
$= [t^-_{\underline{G^c}}(x), t^+_{\underline{G^c}}(x)] = T_{\underline{G^c}}(x)$

$T_{\overline{G}}(x) = [t^-_{\overline{G}}(x), t^+_{\overline{G}}(x)]$
$= [sup\{infT_G(y) : y \in [x]_R\}, sup\{supT_G(y) : y \in [x]_R\}]$
$= [sup\{infF_{G^c}(y) : y \in [x]_R\}, sup\{supF_{G^c}(y) : y \in [x]_R\}]$
$= [f^-_{\underline{G^c}}(x), f^+_{\underline{G^c}}(x)] = F_{\underline{G^c}}(x).$

Thus, we can easily obtain $\underline{G^c} = \overline{G}^c$. Analogously, the proof of the second conclusion $\overline{G^c} = \underline{G}^c$ is the same as the previous one.

Proposition 2. *Let G, K be two interval valued vague sets of U, if either G or K is a definable interval valued vague set, then $\overline{G\cap K} = \overline{G} \cap \overline{K}$, $\underline{G\cup K} = \underline{G} \cup \underline{K}$.*

Proof. Without loss of generality, it is assumed that G is a definable interval valued vague set, according to Definition 3, then for $\forall\, x \in U$, the following expressions are constants, and written as a^-, a^+, b^-, b^+, respectively.

$\{infT_G(y) : y \in [x]_R\} = a^-$ and $\{supT_G(y) : y \in [x]_R\} = a^+$

$\{infF_G(y) : y \in [x]_R\} = b^-$ and $\{supF_G(y) : y \in [x]_R\} = b^+$

Obviously, $0 \le a^- \le a^+ \le 1$, $0 \le b^- \le b^+ \le 1$ and $a^+ + b^+ \le 1$. Thus

$$\underline{G} \cup \underline{K}(x) = \{< x, T_{\underline{G}\cup\underline{K}}(x), F_{\underline{G}\cup\underline{K}}(x) >\}$$

$T_{\underline{G}\cup\underline{K}}(x) = [t^-_{\underline{G}\cup\underline{K}}(x), t^+_{\underline{G}\cup\underline{K}}(x)]$
$= [t^-_{\underline{G}}(x) \vee t^-_{\underline{K}}(x), t^+_{\underline{G}}(x) \vee t^+_{\underline{K}}(x)]$
$= [inf\{infT_G(y) : y \in [x]_R\} \vee inf\{infT_K(y) : y \in [x]_R\},$
$inf\{supT_G(y) : y \in [x]_R\} \vee inf\{supT_K(y) : y \in [x]_R\}]$
$= [inf\{a^- \vee infT_K(y) : y \in [x]_R\}, inf\{a^+ \vee supT_K(y) : y \in [x]_R\}]$
$= [inf\{inf(T_G(y) \vee T_K(y)) : y \in [x]_R\},$
$inf\{sup(T_G(y) \vee T_K(y)) : y \in [x]_R\}$
$= [inf\{infT_{G\cup K}(y) : y \in [x]_R\}, inf\{supT_{G\cup K}(y) : y \in [x]_R\}]$
$= T_{\underline{G\cup K}}(x)$

$F_{\underline{G}\cup\underline{K}}(x) = [f^-_{\underline{G}\cup\underline{K}}(x), f^+_{\underline{G}\cup\underline{K}}(x)]$
$= [f^-_{\underline{G}}(x) \wedge f^-_{\underline{K}}(x), f^+_{\underline{G}}(x) \wedge f^+_{\underline{K}}(x)]$
$= [sup\{infF_G(y) : y \in [x]_R\} \wedge sup\{infF_K(y) : y \in [x]_R\},$
$sup\{supF_G(y) : y \in [x]_R\} \wedge sup\{supF_K(y) : y \in [x]_R\}]$
$= [sup\{b^- \wedge infF_K(y) : y \in [x]_R\}, sup\{b^+ \wedge supF_K(y) : y \in [x]_R\}]$
$= [sup\{inf(F_G(y) \wedge F_K(y)) : y \in [x]_R\},$
$sup\{sup(F_G(y) \wedge F_K(y)) : y \in [x]_R\}$

$= [sup\{infF_{G\cup K}(y) : y \in [x]_R\}, sup\{supF_{G\cup K}(y) : y \in [x]_R\}]$
$= F_{\underline{G\cup K}}(x)$.

Similarly, we can get the proof of the other conclusion, i.e., $\overline{G \cap K} = \overline{G} \cap \overline{K}$.

Let P, R be two equivalence relations of the universe U. For any $x \in U$, if $[x]_P \subseteq [x]_R$, we call that the relation P is thinner than R, or R is coarser than P alternatively. And it is denoted by $P \prec R$.

Proposition 3. *Let G be an interval valued vague set of U. If $P \prec R$, then $\underline{G} \subseteq \underline{G}_P$ and $\overline{G}_P \subseteq \overline{G}$.*

Proof. Owing to $P \prec R$, then $[x]_P \subseteq [x]_R$. Consequently, we have

$$\begin{aligned}T_{\underline{G}}(x) &= [t^-_{\underline{G}}(x), t^+_{\underline{G}}(x)]\\ &= [inf\{infT_G(y) : y \in [x]_R\}, inf\{supT_G(y) : y \in [x]_R\}]\\ &\le [inf\{infT_G(y) : y \in [x]_P\}, inf\{supT_G(y) : y \in [x]_P\}]\\ &= [t^-_{\underline{G}_P}(x), t^+_{\underline{G}_P}(x)] = T_{\underline{G}_P}(x)\end{aligned}$$

$$\begin{aligned}F_{\underline{G}}(x) &= [f^-_{\underline{G}}(x), f^+_{\underline{G}}(x)]\\ &= [sup\{infF_G(y) : y \in [x]_R\}, sup\{supF_G(y) : y \in [x]_R\}]\\ &\ge [sup\{infF_G(y) : y \in [x]_P\}, sup\{supF_G(y) : y \in [x]_P\}]\\ &= [f^-_{\underline{G}_P}(x), f^+_{\underline{G}_P}(x)] = F_{\underline{G}_P}(x).\end{aligned}$$

Hence $\underline{G} \subseteq \underline{G}_P$. Similarly, we can prove the second conclusion, i.e., $\overline{G}_P \subseteq \overline{G}$.

4 Roughness of an Interval Valued Vague Set

In this section, we will propose a roughness measure of an interval valued vague set using the roughness of a vague set.

Definition 4. *Let G be an interval valued vague set of U, $\alpha = [\alpha^-, \alpha^+]$, $\beta = [\beta^-, \beta^+]$. The* interval valued $\alpha\beta$-level sets *of $\underline{G}$ and $\overline{G}$, denoted by $\underline{G}_{\alpha\beta}$ and $\overline{G}_{\alpha\beta}$, are defined as follows, respectively,*

$$\underline{G}_{\alpha\beta} = \{x \in U : T_{\underline{G}}(x) \ge \alpha,\ F_{\underline{G}}(x) \le \beta\} \tag{3}$$

$$\overline{G}_{\alpha\beta} = \{x \in U : T_{\overline{G}}(x) \ge \alpha,\ F_{\overline{G}}(x) \le \beta\} \tag{4}$$

where $0 < \alpha^- \le \alpha^+ \le 1$, $0 < \beta^- \le \beta^+ \le 1$ and $\alpha^+ + \beta^+ \le 1$.

It is easy to see that the interval valued $\alpha\beta$-level sets of $\underline{G}$ and $\overline{G}$ are, respectively, reduced to the two parameters $\alpha\beta$-level sets in the roughness of a vague set, when $T_{\underline{G}}(x)$, $F_{\underline{G}}(x)$, $T_{\overline{G}}(x)$, $F_{\overline{G}}(x)$ are constants and $\alpha^- = \alpha^+$, $\beta^- = \beta^+$, for all $x \in U$.

Definition 5. *Let G be an interval valued vague set of U. A roughness measure $\rho^{\alpha\beta}_G$ of the interval valued vague set G of U with respect to the interval values α, β in Pawlak approximation space (U, R), is defined as*

$$\rho^{\alpha\beta}_G = 1 - \frac{|\underline{G}_{\alpha\beta}|}{|\overline{G}_{\alpha\beta}|} \tag{5}$$

where the notation $|\cdot|$ denotes the cardinality of the set.

Obviously, we have $0 \le \rho_G^{\alpha\beta} \le 1$. Especially, $\rho_G^{\alpha\beta} = 0$ when $|\overline{G}_{\alpha\beta}| = 0$.

Let us consider the same example as one in section 3.

Example 2. Assume that $\alpha = [0.3, 0.45]$, $\beta = [0.4, 0.5]$. Using the Definition 5, we can get $\underline{G_1}_{\alpha\beta} = \{x_5\}$, $\overline{G_1}_{\alpha\beta} = \{x_1, x_4, x_5\}$. Thus, the roughness of interval valued vague set is $\rho_{G_1}^{\alpha\beta} = \frac{2}{3} \approx 0.6667$.

Proposition 4. *If G is a definable interval valued vague set, then* $\rho_G^{\alpha\beta} = 0$.

Proposition 5. *Let G, K be two interval valued vague sets, if $G = K$, then* $\rho_G^{\alpha\beta} = \rho_K^{\alpha\beta}$.

Proposition 6. *Let G, K be two interval valued vague sets, if $G \subseteq K$, then (a) if $\overline{G}_{\alpha\beta} = \overline{K}_{\alpha\beta}$, then $\rho_G^{\alpha\beta} \ge \rho_K^{\alpha\beta}$; (b) if $\underline{G}_{\alpha\beta} = \underline{K}_{\alpha\beta}$, then $\rho_G^{\alpha\beta} \le \rho_K^{\alpha\beta}$.*

Proof. It can be easily proved by the Proposition 4 and Definition 6.

Proposition 7. *Let G and K be two interval valued vague sets of U, we have the following inequality*

$$\rho_{G\cup K}^{\alpha\beta}|\overline{G}_{\alpha\beta} \cup \overline{K}_{\alpha\beta}| \le \rho_G^{\alpha\beta}|\overline{G}_{\alpha\beta}| + \rho_K^{\alpha\beta}|\overline{K}_{\alpha\beta}| - \rho_{G\cap K}^{\alpha\beta}|\overline{G}_{\alpha\beta} \cap \overline{K}_{\alpha\beta}|.$$

Proof. The proof is the same as the Proposition 4.5 in [17], so we omit it.

Corollary 1. *Let G and K be two interval valued vague sets of U, if either G or K is a definable interval valued vague set, then*

$$\rho_{G\cup K}^{\alpha\beta}|\overline{G}_{\alpha\beta} \cup \overline{K}_{\alpha\beta}| = \rho_G^{\alpha\beta}|\overline{G}_{\alpha\beta}| + \rho_K^{\alpha\beta}|\overline{K}_{\alpha\beta}| - \rho_{G\cap K}^{\alpha\beta}|\overline{G}_{\alpha\beta} \cap \overline{K}_{\alpha\beta}|.$$

Proposition 8. *Let G be an interval valued vague set of U, if $\alpha_1 \ge \alpha_2$, then (a) if $\overline{G}_{\alpha_1\beta} = \overline{G}_{\alpha_2\beta}$, then $\rho_G^{\alpha_1\beta} \ge \rho_G^{\alpha_2\beta}$; (b) if $\underline{G}_{\alpha_1\beta} = \underline{G}_{\alpha_2\beta}$, then $\rho_G^{\alpha_1\beta} \le \rho_G^{\alpha_2\beta}$.*

Proof. By the Definition 5, if $\alpha_1 \ge \alpha_2$, we can easily get $\underline{G}_{\alpha_1\beta} \subseteq \underline{G}_{\alpha_2\beta}$ and $\overline{G}_{\alpha_1\beta} \subseteq \overline{G}_{\alpha_2\beta}$. Then we can conclude that $\rho_G^{\alpha_1\beta} \ge \rho_G^{\alpha_2\beta}$ when $\overline{G}_{\alpha_1\beta} = \overline{G}_{\alpha_2\beta}$ and $\rho_G^{\alpha_1\beta} \le \rho_G^{\alpha_2\beta}$ when $\underline{G}_{\alpha_1\beta} = \underline{G}_{\alpha_2\beta}$.

Proposition 9. *Let G be an interval valued vague set of U, if interval values $\beta_1 \ge \beta_2$, then*
(a) if $\overline{G}_{\alpha\beta_1} = \overline{G}_{\alpha\beta_2}$, then $\rho_G^{\alpha\beta_1} \le \rho_G^{\alpha\beta_2}$; (b) if $\underline{G}_{\alpha\beta_1} = \underline{G}_{\alpha\beta_2}$, then $\rho_G^{\alpha\beta_1} \ge \rho_G^{\alpha\beta_2}$.

Proof. The proof is the same as the Proposition 8.

For convenience, we introduce several notations as follows.

$$\underline{G}_T^{\alpha} = \{x \in U : T_{\underline{G}}(x) \ge \alpha\}; \quad \underline{G}_F^{\beta} = \{x \in U : F_{\underline{G}}(x) \le \beta\};$$
$$\overline{G}_T^{\alpha} = \{x \in U : T_{\overline{G}}(x) \ge \alpha\}; \quad \overline{G}_F^{\beta} = \{x \in U : F_{\overline{G}}(x) \le \beta\}.$$

Proposition 10. *Let G be an interval valued vague set of U, if interval values $\alpha_1 \geq \alpha_2$, $\beta_1 \geq \beta_2$, then*
(a) if $\underline{G}_T^{\alpha_1} = \underline{G}_T^{\alpha_2}$ and $\overline{G}_{\alpha_1\beta_1} = \overline{G}_{\alpha_2\beta_2}$, then $\rho_G^{\alpha_1\beta_1} \leq \rho_G^{\alpha_2\beta_2}$;
(b) if $\underline{G}_F^{\beta_1} = \underline{G}_F^{\beta_2}$ and $\overline{G}_{\alpha_1\beta_1} = \overline{G}_{\alpha_2\beta_2}$, then $\rho_G^{\alpha_1\beta_1} \geq \rho_G^{\alpha_2\beta_2}$;
(c) if $\overline{G}_T^{\alpha_1} = \overline{G}_T^{\alpha_2}$ and $\underline{G}_{\alpha_1\beta_1} = \underline{G}_{\alpha_2\beta_2}$, then $\rho_G^{\alpha_1\beta_1} \geq \rho_G^{\alpha_2\beta_2}$;
(d) if $\overline{G}_F^{\beta_1} = \overline{G}_F^{\beta_2}$ and $\underline{G}_{\alpha_1\beta_1} = \underline{G}_{\alpha_2\beta_2}$, then $\rho_G^{\alpha_1\beta_1} \leq \rho_G^{\alpha_2\beta_2}$.

Proof. For $\alpha_1 \geq \alpha_2$, we know that $\underline{G}_T^{\alpha_1} \subseteq \underline{G}_T^{\alpha_2}$, $\overline{G}_T^{\alpha_1} \subseteq \overline{G}_T^{\alpha_2}$. Similarly, for $\beta_1 \geq \beta_2$, we can get $\underline{G}_F^{\beta_1} \supseteq \underline{G}_F^{\beta_2}$, $\overline{G}_F^{\beta_1} \supseteq \overline{G}_F^{\beta_2}$. According to Definition 6 and the conditions of the proposition, the conclusions can be easily proved.

Proposition 11. *Let G be an interval valued vague set of U, if interval values $\alpha_1 \geq \alpha_2$, $\beta_1 \leq \beta_2$, then*
(a) if $\overline{G}_{\alpha_1\beta_1} = \overline{G}_{\alpha_2\beta_2}$, then $\rho_G^{\alpha_1\beta_1} \geq \rho_G^{\alpha_2\beta_2}$;
(b) if $\overline{G}_{\alpha_1\beta_1} = \overline{G}_{\alpha_2\beta_2}$, then $\rho_G^{\alpha_1\beta_1} \leq \rho_G^{\alpha_2\beta_2}$.

Proof. If $\alpha_1 \geq \alpha_2$ and $\beta_1 \leq \beta_2$, we have $\overline{G}_{\alpha_1\beta_1} \subseteq \overline{G}_{\alpha_2\beta_2}$ and $\overline{G}_{\alpha_1\beta_1} \subseteq \overline{G}_{\alpha_2\beta_2}$. By the Definition 6 and the conditions of the proposition, the above conclusions can be easily proved.

Proposition 12. *Let G be an interval valued vague set of U, if $\alpha_1 \leq \alpha_2$, $\beta_1 \geq \beta_2$, then*
(a) if $\overline{G}_{\alpha_1\beta_1} = \overline{G}_{\alpha_2\beta_2}$, then $\rho_G^{\alpha_1\beta_1} \leq \rho_G^{\alpha_2\beta_2}$;
(b) if $\overline{G}_{\alpha_1\beta_1} = \overline{G}_{\alpha_2\beta_2}$, then $\rho_G^{\alpha_1\beta_1} \geq \rho_G^{\alpha_2\beta_2}$.

Proof. The proof is the same as the Proposition 11.

From the above propositions, one can see that the roughness measure is monotonic with respect to the interval values α, β, when the interval values and the interval valued $\alpha\beta$-level sets satisfy certain conditions.

Let P be another equivalence relation of U, we write the interval valued $\alpha\beta$-level sets of $\underline{G}$, $\overline{G}$ and the roughness measure as $\underline{G}_{\alpha\beta}^P, \overline{G}_{\alpha\beta}^P, \rho_{G_P}^{\alpha\beta}$, respectively.

Proposition 13. *If $P \prec R$, then $\rho_{G_P}^{\alpha\beta} \leq \rho_G^{\alpha\beta}$.*

Proof. If $P \prec R$, by the Proposition 3, it is easy to show that $\underline{G} \subseteq \underline{G}_P$ and $\overline{G} \supseteq \overline{G}_P$. Let us suppose that $x \in \underline{G}_{\alpha\beta}$, then $T_{\underline{G}}(x) \geq \alpha$ and $F_{\underline{G}}(x) \leq \beta$. Since $T_{\underline{G}_P}(x) \geq T_{\underline{G}}(x) \geq \alpha$ and $F_{\underline{G}_P}(x) \leq F_{\underline{G}}(x) \leq \beta$, namely, $x \in \underline{G}_{\alpha\beta}^P$. Consequently, we have $\underline{G}_{\alpha\beta} \subseteq \underline{G}_{\alpha\beta}^P$. Analogously, we can get $\overline{G}_{\alpha\beta} \supseteq \overline{G}_{\alpha\beta}^P$. According to Definition 6, we can obtain $\rho_{G_P}^{\alpha\beta} \leq \rho_G^{\alpha\beta}$.

Obviously, the proposition shows that the roughness of interval valued vague sets decreases as the equivalence relation becomes thinner.

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A Novel Approach Based on Fuzzy Rough Sets for Web Query System

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Abstract. In this paper we propose a novel approach of fuzzy rough sets for an information system applied to Web Query. As we know, one of the most common ways to retrieve information from the WWW is keyword search. We combine the flexibility of upper approximation with the strictness of lower approximation by applying them successively; we rely on the membership function to deal with graded thesauri and weighted queries, representing the thesaurus as a fuzzy relation and the query as a fuzzy set. A decision table is achieved by removing redundant attributes without any information loss.The new terms with weights do not only reflect the strength with original individual query ones but also take into account their relevance as a whole. The precision of system evaluation is realized by fuzzy comprehensive estimation on rough sets.

Keywords: Fuzzy rough sets, web query, fuzzy comprehensive estimation, membership function.

1 Introduction

The concepts of rough sets were presented originally by Pawlak Z in 1982[1] in the case of equivalences (reflexive, symmetric, and transitive relations), which analyzes data, reason and discovers the relation among data and so on. The fuzzy set was put forward to find the logic relationship of knowledge representation from database [2] and make the fuzzy estimation for inaccuracy and incompletion in real world.

Recently, the combination of fuzzy set theory and rough set theory [3] has become a popular data mining technique for classification problems because of their strength of handling vague and imprecise data. The rough-set theory has been used in reasoning and knowledge acquisition for expert systems.

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As is well-known, one of the most common ways to retrieve information from the WWW is keyword search [4]: For example, the user inputs a query consisting of one or more keywords and the search system returns a list of web documents ranked according to their relevance to the query, a great deal of the semantic web efforts are concerned with this problem and allow agents to communicate with each other by providing a shared and common understanding. This is due to the fact that the upper approximation expands each of the query words individually but disregards the query as a whole.

However, it is not hard to imagine cases where the upper approximation is too flexible as a query expansion technique, resulting in not only in an explosion of the query, but possibly even worse, in the addition of non relevant terms due to one or more ambiguous of the query words.

2 Related Works on Web Query Refinement

Query refinement has found its way to popular web search engines, and is even becoming one of those features in which search engines aim to differentiate in their attempts to create their own identity. One option is to use an available thesaurus such as WorldNet, expanding the query by adding synonyms [5]. Related terms are automatically discovered from the searchable documents though, taking into account statistical information such as co-occurrences of words in documents or in fragments of documents.

In global document analysis, the whole corpus of searchable documents is preprocessed and transformed into an automatically generated thesaurus. The straightforward way is to extend the query with all the words that are related to at least one of the query terms. As mentioned in the introduction, this corresponds to taking the upper approximation of the query. Indeed, the precision of retrieved documents is likely to decrease when expanding a query such as java, travel with the term applet. Even though this term is highly related to java as a programming language, it has little or nothing to do with the intended meaning of java in this particular query, namely the island. An option to automate sense disambiguation is to only add a term when it is related to at least two words of the original query; experimental results are however unsatisfactory.

3 Rough Sets

Rough set theory is a mathematical tool to deal with uncertain or vague knowledge. It is based on the assumption that data and information are associated with every object of the universe of discourse [6]. It forms upper and lower approximation defined on the indiscernible relation. According to the definition given in Pawlak [7], a knowledge representation system or an information system is a pair $K = (U, A)$, where U is a nonempty, finite set of objects (called the universe), and A is nonempty, finite set of primitive attributes [8]. Every primitive attribute $a \in A$ is a total function $a : U \rightarrow V_a$, where V_a is the set of values of a (called the domain of a). R is an equivalence relation defined on U. U/R means the partition of R on$U$, an ordered pair (U, R) is called approximation space, and any subset $X \subseteq U$ is called a concept. Each concept X can be defined as lower and upper approximation:

$$\underline{R}X = \bigcup x \in U : [x]_R \subseteq X,$$

$$\overline{R}X = \bigcup x \in U : [x]_R \bigcap X \neq \emptyset.$$

$\underline{R}X$ is a set composed of elements that belong to concept X by the owned knowledge R,$\underline{R}X$ is a set composed of element that belongs to concept X for R and Q, two equivalence relations on U, p-positive region of Q can be defined as:

$$POS_pQ = \bigcup_{x \in U/Q} PX,$$

where POS_pQ is the set made up of all the elements that must be partitioned to class U/Q by knowledge P , the attributes set $R \subseteq C$ will be called D-reduct of C iff R is D-independent and $POS_R(D) = POS_C(D)$.

Rough set theory is an interesting candidate framework to aid in query refinement. Indeed, a thesaurus characterizes an approximation space where the query is approximated from the upper and the lower side. By definition, the upper approximation adds a term to the query only if it is related to one of the words already in the query, while the lower approximation will only retain a term in the query if all the related words are also in the query.

4 Basic Algorithm of Fuzzy Comprehensive Evaluation on Rough Sets

Fuzzy comprehensive evaluation of an information system is a set of fuzzy decision of single-factor in system by way of the compound fuzzy operation. Basic procedure realization is that the condition attributes is firstly reduced based on approximation of classification of rough approaches [9], and then the significance of attributes is analyzed to set up a weight distribution of fuzzy evaluation in the system:

$$A = (a_1, a_2, \cdots, a_N) \in \Gamma(U),$$

where $\sum_{i=1}^{N} a_i = 1$, for a decision $f(\mu_i) \in \Gamma(V)$ of each factor μ_i , according to the fuzzy relation R and fuzzy transformation Γ from U to V we obtain a comprehensive decision $B = A \circ R$, a model of comprehensive decision can be realized as following:

$$A \in \Gamma(U), B = A \circ R \in \Gamma(V), R \in \Gamma(U \times V) \longrightarrow f(U),$$

where $B = (b_1, b_2, \cdots, b_M) \in \Gamma(V)$ of $V \cdot b_j$ reflects the position of the section j of decision ν_j in the comprehensive decision. The concrete steps performing fuzzy evaluation of an information system are as follows:

Input: data of an information system, and $A = C \cap D$ is the attribute set, and C, D is condition attribute set and decision attribute set respectively. Output: fuzzy decision of single-factor of an information system.

Step 1: The redundant condition attribution and information is removed to reduce knowledge expression of the system based on rough set.

Step 2: Utilizing the significance of attributes set up a weight distribution $B = (a_1, a_2, \cdots, a_N) \in \Gamma(U)$ of fuzzy evaluation [10].

Step 3: Define a fuzzy evaluation V, then according to each factor to get fuzzy relation R.

Step 4: According to the weight distribution and fuzzy relation to compute a decision $f(\mu_i) \in \Gamma(V)$ for each factor μ_i by way of the maximum and minimum compound fuzzy operation.

5 Relation of Fuzzy Comprehensive Evaluation on Fuzzy-Rough Sets

Fuzzy-sets are often used to represent quantitative data expressed in linguistic terms and membership functions in intelligent systems because of its simplicity. Deriving rules on multiple concept levels lead to the discovery of more general and important knowledge from data. Rule effectiveness for future data is also derived from these membership values. It is extended to find a set of cross-level maximally general fuzzy certain and possible rules from examples with hierarchical and quantitative attributes. Some definitions about fuzzy approximations are introduced below.

When the same linguistic term R_{jk} of an attribute A_j exists in two fuzzy objects $Obj^(i)$ and $Obj^(r)$ with membership values $f_{jk}^(i)$ and $f_{jk}^(r)$ equal to or larger than a certain α value, $Obj^(i)$ and $Obj^(r)$ are said to have a fuzzy indiscernible relation on attribute A_j with membership value $min(f_{jk}^(i) \cap f_{jk}^(r)$ [11]. Also, if the same linguistic terms of an attribute subset B exist in both $Obj^(i)$ and $Obj^(r)$ with membership values equal to or larger than α, $Obj^(i)$ and $Obj^(r)$ are said to have a fuzzy a fuzzy α-equivalence relation on attribute subset B with a membership value equal to the minimum of all the membership values. These fuzzy α- equivalence relations partition the fuzzy object set U into several fuzzy subsets t, and the result is denoted by U/B. The set of partitions, based on B and including $Obj^(i)$ is denoted $B(Obj^{(i)})$ where r is the number of partitions in $B(Obj^{(i)})$, $B_j(Obj^{(i)})$ is $j-th$ partition in $B(Obj^{(i)})$, and $\mu_{Bj}(Obj^{(i)})$ is the membership value of $j-th$ partition. Fuzzy α-lower and α boundary approximations are defined below:

Let X be an arbitrary subset of the universe U, and B be an arbitrary subset of the attribute set A. The fuzzy lower and the fuzzy boundary approximations under the threshold value α for B on X , denoted $B^*(X)$ and $B_*(X)$, respectively, are defined as follows:

$$B_*(X) = (B_k(x), \mu_B(x))|x \in U, B_k(x)).$$

Elements in are classified as members of set with full certainty using attribute set their membership values are considered effective measures of fuzzy lower approximations for future data. A low membership value with a fuzzy lower approximation means a low tolerance (or effectiveness) on future data. Their certainty degrees can be calculated from the membership values of elements in the boundary approximations [12].

6 Case Studies

To test the proposed method, we consider the following query, the intended meaning of the ambiguous word apple, which can refer both to a piece of fruit and to a computer company, is clear in this query. The disadvantage of using a T -transitive fuzzy thesaurus becomes apparent when we compute the upper approximation R8A, shown in the last column. We conducted experiment of a real dataset extracted from a News Site as search engine. We test with its logs as they have all the features of dynamism, shortness and ambiguity that we discuss when proposing the method.

The web log used corresponds to the activity of the site along 4 months containing a total of 442909 records. After cleaning and preprocessing the log, 106443 different queries have been identified in which a total of 43742 different terms are used. Information regarding the number of words contained in each query and their associated frequency is shown, together with the number of queries per session.

Since web queries tend to be short - 1 or 2 terms on average - expanding them with related terms is an interesting option for improving research results[13,14]. To perform some kind of sense resolution, the web query expander needs to take the query as a whole into account, rather than working on the level of individual query terms. Adding a term to the query if it is related to at least two of the query words are not a good approximation to sense disambiguation either [15]. Our proposal consists of two steps. In the first step, the web query expander acts on the level of individual query terms, adding all related terms. We expand the query by adding all the terms that are known to be related to at least one of the query words. In the second step, we apply the pruned away to the extent to which they are related to words that have nothing to do with the query as a whole. We reduce the expanded query by taking its lower approximation, thereby pruning away all previously added terms that are suspected to be irrelevant for the query.

7 Conclusion

In this paper, a new approach of fuzzy comprehensive evaluation of an information system is presented where the redundant or insignificant attributes in data sets are eliminated based on rough set in order to reduce knowledge representation of a fuzzy comprehensive estimation system. We combine the flexibility of the upper approximation with the strictness of the lower approximation by applying them successively. We study the significance of condition attributes by way of rough set instead of weighting coefficients of fuzzy comprehensive estimation subjectively defined, the significance of condition attributes is used for setting up a weight distribution of fuzzy evaluation; we demonstrate how the so-called tight upper approximation, i.e. a successive application of the upper and the lower approximation from fuzzy rough set theory. We propose a new way to address this problem using a thesaurus, i.e. a term-term relation. When a graded thesaurus is used, our query expansion approach turns the original query automatically into a weighted query. The original user chosen terms maintain their highest weight, and new terms are added with weights that do not only reflect the strength of the relationship with the original individual query terms as can be read from the thesaurus, but also take into account their relevance to the query as a whole.

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Research of Micro-seismic Signal Extraction Method Based on Rough Set

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Abstract. Micro-seismic monitoring as a regional monitoring means to predict important dynamic disaster of the mine, meanwhile, micro-seismic signal with abundant components of spectrum and frequency bandwidth characteristics. Obtain sudden change time of micro-seismic abnormal signal and frequency components corresponding to sudden change time are the key issues in micro-seismic monitoring which to be solved urgently. In view of the variable precision rough set can not identify random rule which is supported by only a few examples, study a new hybrid model which combines rough set with Bayesian probability. In order to get more general and more reliable classification rule, make full use of intrinsic characteristic of micro-seismic monitoring knowledge system, more powerful in coping with noise data, derive a reliable and simple classification rule, it is more efficient to analyze a large number of micro-seismic data.

Keywords: Micro-seismic monitoring, rough set, signal extraction, Bayesian probability.

1 Introduction

Monitoring and prediction of important dynamic disaster after coal seam and strata fractures like roof caving, mine water bursting, coal and gas outburst and impulsion pressure have not been effectively resolved by now. Recently, with the increasing of mining depth year by year, these great serious accidents occur frequently. Therefore, seek monitoring method of strata movement scientifically and forecast possible disasters based on it, propose preventive measures, it is the only way to realize green sustainable development in coal industry.

The tests indicate that: as the rocks are gradually pressurized, micro-defects are fractured either extend or closure, at this time generate

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small energy level of acoustic emission, when the crack extends to a certain size, the load of rock is close to half of its failure strength. Begin to appear fracture transfixion in large range and generate big energy level of acoustic emission. This is called micro-seismic or MS. When the pressure is close to ultimate strength of rocks, the time of micro-seismic incidents increases until rocks are damaged. Each Micro-seismic Signal includes abundant information of inner rock body state change, processing and analyzing the received micro-seismic signal can be used as a basis to evaluate rock stability. Therefore, we can make use of this feature of rock body micro-seismic to monitor stability of rock mass, thus predict the phenomenon of important dynamic disaster of the mine like roof caving, and mine water bursting, impulsion pressure etc.

Micro-seismic is a micron natural continuous vibration, the main frequency of vibration is about 10-100Hz, significantly different from the field interference of human. Micro-seismic observation technology has applied to the engineering investigation and earthquake safety evaluation area, and has been included in the correlative regulations. Restricted by some conditions, it was mainly observe at a fixed time before (under the mute), investigate vibration characteristic of the site in three-dimensional direction. As this technology has a good scalability, in the late 1980s, Japan by using the timing micro-seismic observation, infer stratigraphic structure and vibration characteristic of the site. In the 1990s, occasionally see few reports that foreign apply micro-seismic observation to monitoring work. In 2000, the mining industry in Canada reports that using micro-seismic incidents which provide mining and change the state of rock block lead to rock burst strain energy accumulation of information, in Stmtllcona mine records detail cataloging of series events, in order to provide hazard assessment. As reproducibility of micro-seismic spectrum is well, and it is not affected by seasons, it can be used as stationary stochastic process in frequency domain. Therefore, we can adopt inconsistent information system rule acquisition method based on rough set to analysis power spectrum and amplitude-frequency characteristic of micro-seismic signal, so as to identify micro-seismic signal in spectrum characteristic, so provide a new clue to predict important dynamic disaster of mine.

2 Model of Variable Precision Rough Set

The model of variable precision rough set by introducing a confidence β that have some fault tolerance. However, this model can not identify random rule which is supported by only a small number of cases. On this basis we give a method that can not only study decision making classification rule but can also study non-decision making classification rule. Change the attribute-oriented rough set method into allowing study non-decision classification rule method, thus can derive more general and more reliable classification rule.

2.1 Bayesian Probability

Bayes probability formula:

$$P(B_i/B) = \frac{P(B_i)P(B/B_i)}{\sum_{i=1}^{n} P(B_i)P(B/B_i)} \tag{1}$$

Among it, $\{B_1, B_2, \ldots, B_n\}$ is a divide of universe U, $P(B_i)$ is the probability of event B_i. $P(B)$ is the joint probability $P(B, B_i)$ which sum all the possible B_i.

Formula (1) can be expressed as:

$$posterior\ probability = \frac{linkhihood\ function \times\ prior\ probability}{evidence\ factor} \tag{2}$$

Total probability formula is a calculation formula which obtain results from cause, however, Bayes formula know under the condition of some results happened, seeking occurrence cause of the calculation formula. Therefore, bayes formula usually called posterior probability formula. Bayes formula shows that, by observing the value of B, we can change prior probability $P(B_i)$ into posterior probability $P(B_i/B)$. That is assuming under the condition of eigenvalue B which is known, category the probability of B_i. $P(B/B_i)$ is called likelihood function about B_i on B, that is to say in the case of other conditions are equal, B_i which makes the $P(B/B_i)$ bigger is more likely to be the true category. Posterior probability mainly determined by the product of prior probability and likelihood function, $P(B)$ evidence can be only considered as a scalar, to ensure that the sum of each categorical posterior probability equals one, thus meet the condition of probability.

The advantage of Bayes theory lies in, due to Bayes theory is set up on axiom theory, it can guarantee compatibility of quantitative, yet other classification methods do not have. Bayes theory has some disadvantages as follows: only make one decision can cause many problems, we do not have a reasonable way to determine the value of prior probability. Especially serious disadvantage is that it is difficult to calculate conditional density function, multivariate Gaussian model can afford a sufficient approximate to many real density, however, in other questions it is far away between density form and Gaussian amplifier form. Even when Gaussian model can meet requirements, estimate unknown parameter from sample data is not a simple thing either. Some disadvantages of Bayes method can be assisted by rough set.

2.2 Consensus Degree, Coverage and Support Degree

In order to find correlation rule, scholars have put forward consensus degree concept, consensus degree of decision rule r is also called certainty factor, expressed by cer_I(Y), defined as

$$cer_I(Y) = \frac{|I(x) \bigcap Y|}{|I(x)|} \tag{3}$$

Among it, Y is the decision class, $Y \subseteq U$.

Consensus degree metrics the precision of conditions class which distribute to decision class can be understood as confidence, similar to decision class β of variable precision rough set, now this coefficient has widely been used in data mining. Obviously, if cer_I(Y)=1, then $con_C(X_i) \rightarrow dec_D(Y_j)$ is true, the information system S is decision-making or consistent, that is condition attribute decide decision attribute only; if $0 < cer_I(Y) < 1$, then say S is non-decision-making or inconsistent, that is the condition attribute to determine the decision attribute probability, this is due to inconsistencies information. So we can get uncertainty rule only supported by the lower approximation of corresponding decision-making class, probability rule is supported by boundary object of corresponding decision-making class.

Coverage of decision rule defined as

$$cov_I(Y) = \frac{|I(x) \bigcap Y|}{|Y|} \tag{4}$$

$cov_I(Y)$ is used to estimate the quality of decision rule, coverage suggested that under the condition of decision class which meet the decision rule, meet probability of conditions class, can be regard as anti-regular confidence.

Consensus degree is the probability of decision class Y under the condition of class $I(x)$, coverage is the probability of condition class $I(x)$ under the condition of decision class Y, Pawlak studied Bayes relationship between consensus degree and coverage. Supposed:

$$cer_I(Y) = \frac{|I(x) \bigcap Y|}{|I(x)|} = P(Y|I(X)) \tag{5}$$

$$cov_I(Y) = \frac{|I(x) \bigcap Y|}{|Y|} = P(I(X)|Y) \tag{6}$$

Then, cer_I(Y) metrics sufficient measure of $con_C(X_i) \rightarrow dec_D(Y_j)$, cov_I(Y) metrics certainty degree of $con_C(X_i) \rightarrow dec_D(Y_j)$. In other words, if cer_I(Y)=1, then $con_C(X_i) \rightarrow dec_D(Y_j)$ is true, if cov_I(Y)=1 then $dec_D(Y_j) \rightarrow con_C(X_i)$ is true, if $cer_I(Y) = cov_I(Y) = 1$ then $con_C(X_i) \rightarrow dec_D(Y_j)$ is true. However, the relationship between consensus degree and coverage need not to use posterior probability, yet posterior probability is the base of Bayesian data analysis.

Define support degree sup_I(Y) as

$$sup_I(Y) = \frac{stre_I(Y)}{|U|} \tag{7}$$

Among it, $stre_I$(Y) is the support count of $con_C(X_i) \rightarrow dec_D(Y_j)$ in S, that is attribute value in universe and rule matching of object numbers. Support degree $stre_I$(Y) defines strength of decision rule.

2.3 Probabilistic Programming

In practical application, when it is hard to acquire deterministic decision, deriving probabilistic programming is a good method to solve this problem. Suppose thresholds of consensus, coverage and support being β, α, γ respectively, define probabilistic programming as

$$r : con_C(X_i) \xrightarrow{sup_I(Y), cer_I(Y)} dec_D(Y_j) \tag{8}$$

$$or\ r : con_C(X_i) \xrightarrow{sup_I(Y), cov_I(Y)} dec_D(Y_j) \tag{9}$$

Among it, $X_i \bigcap Y_j = \phi, sup_I(Y) \geq \alpha, cer_I(Y) \geq \beta, cov_I(Y) \geq \gamma$.

This rule is a probabilistic programming which has three statistical measures, that is only for category which have enough support in information system, search consensus degree or coverage greater than or equal to the given threshold probabilistic programming. When without considering support, formula (9) degenerates into rule which is defined by variable precision rough set approach, when not consider support and β=1, formula (9) degenerates into the rule which is defined by rough set approach.

2.4 Acquisition Algorithms from Probabilistic Rule

We hope data in knowledge representation enough consistent, or cover enough condition class, or cover enough decision class. This is a reasonable hypothesis, it is very useful in large database which includes much class. For a lot of class which are supported few in database, for instance less than the given support threshold, deem them as noise data, this is helpful to save computing time, deriving more simple and apply reduction. Here we design a algorithm, it can dig probabilistic programming probabilistic rule, the main idea is: for each rule, firstly examine if there is enough support degree, if there is, then according to the threshold beforehand, examine if it meets consensus degree or coverage of support category, put the rule which meets the requirements as knowledge into rule base. Concrete algorithm is described as bellow:

1) Calculate each pair of element value $[f(x, q_j) = q_j]$ form one variable candidate set L_1.

2) Let k=1.

3) if L_k is not an ϕ, calculate support degree of each candidate model $\wedge_k[f(x, q_j) = \gamma_{qj}]$ in candidate sets L_k, put model into set R_k, the support of the model is greater than or equal to threshold γ.

4) For each model $\wedge_k[f(x, q_j) = \gamma_{qj}]$ in R_k, calculate consensus and coverage. If consensus is greater than or equal to threshold β or coverage is greater than or equal to threshold α, then put this model into the rule base and delete this model from R_k.

5) If R_k is not an ϕ, then do set cross operation for model $\wedge_k[f(x, q_j) = \gamma_{qj}]$ in R_k and generate L_{k+1}.

6) If model $\wedge_k[f(x,q_j)=\gamma_{qj}]$ is not in R_k ,then execute deletion operation in L_{k+1}.

7) Let k=k+1.

8) Repeat (3) to (7), until R_k is ϕ.

3 Micro-seismic Signal Extraction Method Based on Rough Set

Time-frequency analysis technique play a very important part in time-varying signal, it expresses a signal in two-dimensional form, reveal the condition of signal frequency and phase feature changing with time. The successful of time-frequency analysis technique mainly depends on distribution condition of signal information, distinguish main characteristic information as possible. Simultaneously, similar information can not be dispersed either. For the decomposition results are used to pattern recognition or interpretation, over decentralized information may cause difficulty in comprehension. For sensor of micro-seismic monitoring we can form knowledge representation according to different location, look at Table 1, suppose the threshold of consensus degree, coverage and support degree are β=70%, α=50%, γ=1% respectively.

Table 1. Knowledge representation

U	Condition a_1	Condition a_2	Condition a_3	Decision d	Support count n
n_1	high	low	low	low	2
n_2	mid	high	low	high	20
n_3	high	high	high	high	30
n_4	mid	low	high	high	90
n_5	low	low	low	low	120
n_6	high	high	mid	high	70
n_7	mid	low	high	low	34

Note: a_1–Z axis value, a_2–Y axis value, a_3–X axis value, d–micro-seismic times, n–days count.

According to the algorithm which get from probabilistic programming, acquire a reduction a_1, a_3, from this reduction we get probabilistic decision rule set, shown in Table 2. Let confidence β=70% , according to variable precision rough set model, obtain a probability rule set, shown in Table 3. Comparing with Table 2 and Table 3, we can see that rule in Table 2 are simpler. In table3, the first rule is supported by only two cases, support is 0.5%, this record is not enough reliable, according to the given threshold regard it as noise data. Such as, there may exist such situation, write "high" which is the attribute value of a_1 instead of "low" by mistake when inputting data. So let $a_1 = high \cap a_3 = low \rightarrow d = low$, this rule is regarded as

Table 2. Probability rule set

rule	support/%	Consensus degrees/%	Coverage/%
a_1=mid $\cap a_3$=low $\rightarrow$ d=high	5.5	100	9.5
a_1=high $\rightarrow$ d=high	27.3	98.0	27.3
a_1=mid $\cap a_3$=high $\rightarrow$ d=high	24.6	72.6	43.0
a_1==low $\rightarrow$ d=high	33.0	99.9	77.0

Table 3. Probability rule set generated by β reduction a_1, a_3

rule	confidence/%
a_1=high $\cap a_3$=low $\rightarrow$ d=low	100
a_1=high $\cap a_3$=mid $\rightarrow$ d=low	100
a_1=mid $\cap a_3$=low $\rightarrow$ d=high	100
a_1=mid $\cap a_3$=high $\rightarrow$ d=high	72.6
a_1=high $\cap a_3$=high $\rightarrow$ d=high	100
a_1=low $\rightarrow$ d=high	100

random rule, however, variable precision rough set method still consider this rule as decision rule. We apply this rule to micro-seismic signal recognition, can separate signal from noise in time-frequency analysis effectively. Figure 1 is signal which detected by the No.1 sensor and figure 2 is mixed iterate noise. Longitudinal is signal amplitude, lateral axis is time.

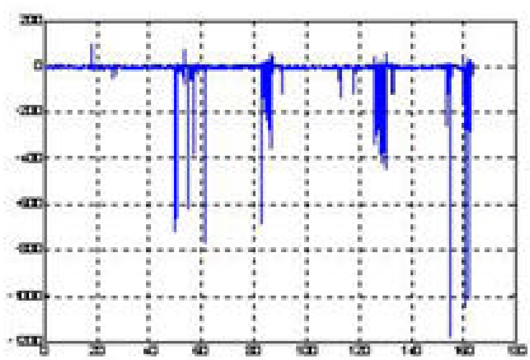

Fig. 1. Z axis value of No.1 sensor after separate

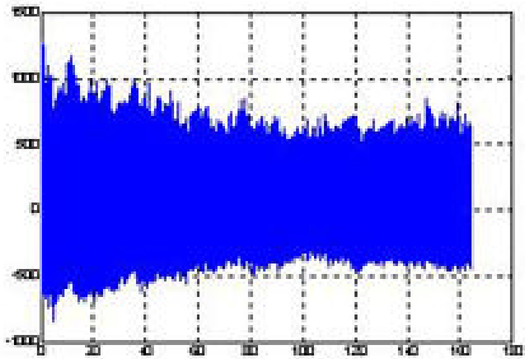

Fig. 2. Noise signal on Z axis of No.1 sensor

4 Conclusion

Micro-seismic signal is a random signal which has jump feature; meanwhile, it has low wide frequency. In view of variable precision rough set can not recognize random rule which supported by few cases, study a new mixed model, this model combines rough set and Bayes probability, in order to get more general and more reliable classification rule, make use of the interior characteristics of micro-seismic monitoring knowledge system, it is powerful in dealing with noise data, derive classification rule reliably and simply, it is effective for a lot of micro-seismic data analysis, it also provides us a tool to recognize micro-seismic signal. However, adopt variable precision rough set method obviously can not describe local feature of micro-seismic signal accurately.

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Application and Design of MEDS Based on Rough Set Data Mining Rule

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Abstract. In this paper, the system is designed on the basis of the present situation of Medical Expert Diagnosis System (MEDS) and the in-depth research on the Attribute Reduction problem and computational complexity of Rough Set Theory. The system is integrated with the latest intelligent information processing technology and the latest Electronic Medical Record (EMR) technology, using Rough Set data mining method for knowledge acquisition and decision-making reasoning method. The system is realized with the program mode of using Visual Studio 2005 for programming and SQL Server 2000 for database. And the realized algorithm can be used to Attribute Reduction of data with multiple attributes with universal.

Keywords: Attribute Reduction, Data Mining, Discretization of Attributes, Medical Diagnosis, Rough Set.

1 Introduction

Poland's Z. Pawlak proposed Rough Set in connection with G. Frege's thinking of the region of boundary. He puts the individual that could not be confirmed into the region of boundary which is defined as the differential set of the upper approximation and lower approximation. Rough Set Theory has a strong ability for qualitative analysis. It can effectively express the uncertainty and incomplete knowledge, gain the knowledge from a great deal of data, and use uncertain and incomplete experience knowledge to reasoning. So Rough Set is widely used in the fields of knowledge acquisition, machine learning, rule generation, decision analysis, intelligent control, especially in the areas of data mining. Using Rough Set to deal with data mining has the

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advantage that traditional data mining tools do not have. As a new mathematical tool, the main advantage of Rough Set is that we only need to analyse and mine the raw data, and don't need to provide any prior information beyond the necessary data set. Therefore, Rough Set describes and deals with the uncertainty of the problem objective. This feature causes Rough Set especially suitable for the uncertainty and imprecision of knowledge reasoning and for finding the relationship between these data [1]. Many experiments show that using Rough Set to processing the same data set can eventually get information which is simpler, more accurate and easier for acceptance and understanding by decision makers than other data mining technology. Therefore, the data mining technology based on Rough Set is evolving, a variety of new models and algorithms arisen, and more and more attention that is paid to the fields of intelligent diagnosis.

Firstly, the significance of carrying out the application research on intelligent diagnosis with Rough Set is that reveals the cross on the subject firstly. Rough Set Theory research methods, intelligent diagnostic study object and a wide range application of engineering and medical make this field very active. On the basis of continuous improvement of Rough Set, there has a new study method and application model in intelligent diagnosis. Research on intelligent diagnosis system has broad application. In addition to traditional engineering application, more and more attention is paid to biomedical engineering [2].

In this paper, the MEDS is designed on the basis of current situation of MEDS and in-depth study computation complexity of Rough Set's Attribute Reduction, it is combined with the latest intelligent information processing technology, and using Rough Set data mining method [3] as the system's knowledge acquisition and decision-making reasoning methods.

2 System Knowledge Representation

Knowledge representation is the most important issue in artificial intelligence and intelligent information processing. The focus of knowledge representation based on Rough Set theory is still that "knowledge is a kind of classification ability for things," this viewpoint can be interpreted as a semantic definition of knowledge. For the consideration of calculation, it needs the syntax of knowledge, so we use the contingency table to express knowledge, which can be regarded as a kind of a special formal language used for division (equivalence relation), its symbol is beneficial to process information and knowledge with computer. Thus, knowledge expression system is seen as a relational table. The row in relational table is the corresponding objects (or the state, process, etc.), the column in relational table is property of the corresponding object. The object information is expressed by the property of the specified object.

Definition. (Knowledge Representation System) *It is said that Quaternion KRS=* (U, A, V, f) *is a knowledge representation system, in which,*

U: Non-empty finite set of the object, called discussion field;

A: Non-empty finite set of attributes of the object, also denoted as A_t;

V: Range of all attributes, $V = \bigcup_{a \in A} V_a$, V_a represents the range of attribute;

f: A mapping of $U \times A \to V$, called information function.

Information function $f = \{f_{\alpha_j} | 1 \le j \le n\}$ gives information value to each attribute of each object, the information of object is expressed through the various property value of the specified object. Information function is very important. If it does not exist, the relation between the object set U and attribute set A is isolated. Information function expresses the link between the two, and it is information base for the knowledge representation and reduction and found. For example, $f_{\alpha_j}(x_i) = v(j = 1, 2, \ldots, n)$ expresses value of the attribute α_j of the object x_i is v, thereinto, $f_{\alpha_j} : U \to V_{\alpha_j}$ $(j = 1, 2, \ldots, n)$.

In general, there are two major types of the knowledge representation system: One is information systems (information table), that it is the knowledge expression system without decision attribute; the other is decision-making system (decision table), which contains the decision attribute.

The decision table MEDS designed in this article likes Table 1. The design pattern of decision table is the same to different diseases, having field, condition attributes and decision attribute. But the attributes name and value is not the same.

Table 1. The decision-making table

Field U	Condition Attributes C									Decision Attribute D
	Sex	Age	Eye	Preoperative Sight	Ptosis Degree	Levator Muscle Strength	Superior Rectus Muscle Function	Signs of Mandibular Blink	Surgical Approach type	Effect of surgery
e_1	male	6	OD	4.4	Light	Moderate	Moderate	-	2	invalid
e_2	female	12	OD	4.7	Moderate	Fine	Moderate	-	1	valid
e_3	female	2	OD	4.5	severe	Moderate	Moderate	-	2	valid
e_4	male	9	OS	4.6	Light	Fine	Better	+	1	invalid
e_5	male	6	OS	4.2	Light	Weak	Moderate	-	2	invalid
e_6	male	4	OU	4.7	Light	Fine	Better	-	1	valid
e_7	female	4	OD	4.7	Light	Moderate	Better	-	1	valid
e_8	male	10	OU	4.3	Moderate	Weak	Moderate	-	2	valid
e_9	female	8	OS	4.1	Moderate	Weak	Weak	-	1	valid
e_{10}	female	5	OS	4.3	severe	Fine	Weak	-	2	valid

In a decision-making system, the study of knowledge reduction investigates whether all knowledge given by information system or decision table is necessary or not. The decision-making system usually contains redundant information and knowledge. One of our main tasks is to remove redundant knowledge under the premise of maintaining classification capacity of the original decision-making system. For the decision-making table, this process is called the relative reduction of the knowledge.

Table 2. The rule table

Field	Condition Attributes C				Decision Attribute
U	c_1	c_2	c_3	c_4	D
e_2	0	1	1	1	0
e_3	1	2	1	0	1
e_5	0	1	1	1	1
e_8	0	2	2	0	0
e_{10}	0	0	1	1	0

In order to make Classifying of the decision-making system more convenient, the decision-making system will be discretized before reducing knowledge generally. Therefore, the table obtained after knowledge reduction is discrete form, and is supposed like Table 2 which is the rule table. When making diagnostic decision, the attribute in the rule table does not need to revert to the state before discretization, just to discretize disease symptoms, and then match the rules in the table rule. That is because the discrete value of some continuous attributes may represent a range.

3 Design of Data Mining Method Based on Rough Set

3.1 Discretization of Attributes

In general, the attributes in the database can be separated into two types: one is continuous attribute that some measurable properties of the object, its value is taken from a continuous range. Such as temperature, length etc.; the other is discrete attribute that the value is type of string or a small amount of discrete values to represent, such as sex, color, etc.. In most cases, medical information not only contains continuous attributes, but also include discrete attributes.

For discrete attributes, Slowinski R. method (S method) [4] is used here. For the qualitative word or words that present the property value, such as body temperature, “low”, “mid”, “high” and “very high” and so on. Practice in the diagnosis of medical experts, this conversion is usually based on the experience standard of experts to complete. S method figures qualitative attributes by code 0, 1, 2, $\cdots$.

For continuous attributes, discretization method based on dynamic hierarchical clustering [5] is used for it here. Discretization of continuous attributes in information systems required to maintain expression of the division of the sample interval. The division interval can be neither too big nor too small. Otherwise, it result in loss of information or introduce error messages. Using discretization method based on dynamic hierarchical clustering is more reasonable for dividing the sample.

3.2 Attribute Reduction Algorithm

The Attribute Reduction algorithm is based on attribute's significance of Pawlak[5]. This algorithm is a heuristic algorithm, which is begun with the relative D core of the decision table, and then add attribute according to some heuristic information until get a relative reduction from the decision table.

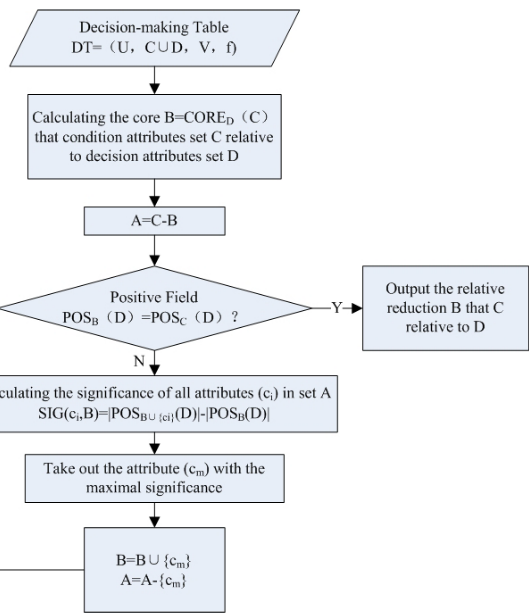

Fig. 1. Attribute reduction algorithm based on Pawlak attributes significanc

Generally, different condition attributes relative to decision attribute have different significance in the decision-making table. In order to find out the significance that a condition attribute relative to decision attribute, the method used in the Rough Set Theory is deleting the attribute from the decision-making table, and then investigating the change of the classification of the decision-making table. The larger the change is, the more important the attribute is. Otherwise, the attribute is less important. Usually, the significance that condition attributes relative to decision attribute is depicted by positive field that one knowledge relative to another. That is for any condition attributes set $B(B \subseteq C)$,deleting set B from set C. if the value of

$$\gamma_C(D) - \gamma_{C-B}(D) = \frac{|POS_C(D)|}{|U|} - \frac{|POS_{C-B}(D)|}{|U|} \tag{1}$$

is bigger, then deleting B is more effective on classification on C relative to decision attribute set D. That is, relative to D, B is more important to C.

Here, we use adding attribute to investigate the significance of the attribute added. But its effect is the same as above.

The Attribute Reduction algorithm based on Pawlak attributes significance is shown in Fig. 1.

4 Design of Data Mining Method Based on Rough Set

4.1 System Architecture

Based on the expert MEDS study and analysis of processing business, and comprehensive considerate advantages and disadvantages of the existing architecture, this paper designed the network architecture of the system is as follows:

The MEDS designed in this paper uses B / S structure. The biggest advantages of B/S structure is that you can operate from anywhere without having to install any special software, as long as one has a computer installing a browser and can access to internet. Maintenance and upgrading

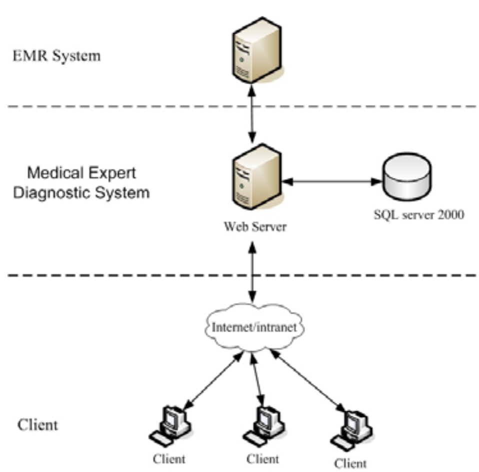

Fig. 2. Network Architecture

client is simple, realizing zero-maintenance. Expanding system is very easy; just operate on the server side accordingly.

Network architecture is shown in Fig.2. MEDS adopts asp.net as the writing tool of background processing program, using SQL Server 2000 as background database. Electronic Medical Record (EMR) [6] system is a part of Hospital Information System (HIS), preserving the main original data of the MEDS, but is not the realization part of the system. The system uses the interface mode to read medical records data from EMR system. [7]

4.2 System Business Process

The main data flow diagram of the system is shown in Fig.3.

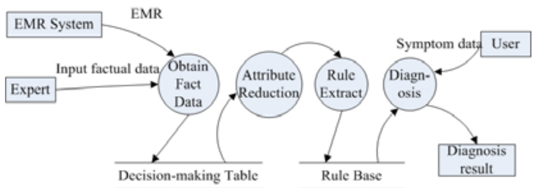

Fig. 3. Data Flow Diagram

As following is the main business of the system.

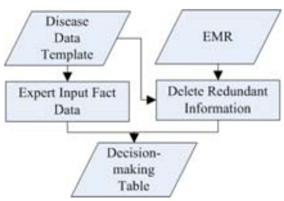

Fig. 4. Obtain the Fact Data

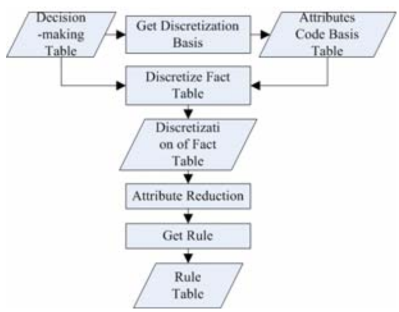

Fig. 5. Obtain Knowledge

1) To obtain fact data

The decision-making table is a fact repository. The fact repository needs to save a lot of fact data. More precise rule would be generated by mass fact data. The fact data is got mainly from the information of EMR, experts also input some fact data.

Not all the information of EMR is stored to in the fact base table. Here we only need basic information and disease information, medical history, diagnostic result and so on. Some information in EMR is not need such as medical image information. Because image processing technique existing can not accurately judge the symptoms. This part of the information is discarded as redundant information.

Obtain fact data is shown in Fig.4.

2) To extract diagnostic rule

Rule is the basis of reasoning in expert system. Extraction of diagnostic rules, that is, knowledge acquisition, the process is shown in Fig.5.

3) Diagnosis

Diagnostic process is shown in Fig.6.

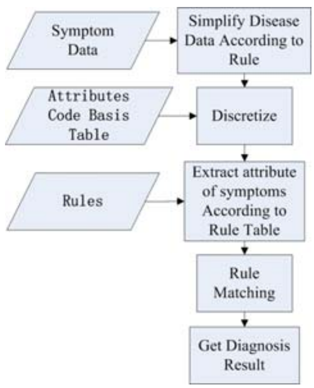

Fig. 6. Diagnosis Process

5 Analysis of Example Application

The data in following Table 1 is extracted at random from ptosis tear function test data collected in this article. In this data mining model, select sex, age, eye, preoperative sight, Ptosis Degree, Levator Muscle Strength, Superior Rectus Muscle Function, Signs of Mandibular Blink, Surgical Approach [8] as a condition attribute, and Operation Effect as the decision attribute. Surgical approach is "1" denotes levator shortening and "2" denotes the forehead muscle hanging technique.

The next is the Discretization of ptosis sample. As attributes of sex and eye, Ptosis Degree, Levator Muscle Strength, Superior Rectus Muscle Function, Signs of Mandibular Blink, Surgical Approach, Operation Effect are discrete attributes, we use S method to discretize them. As attributes of age and preoperative sight are continuous, we use the discretization method based on dynamic hierarchical clustering to discretize. The result of ptosis sample discretized is shown in Table 3. The c_i (i = 1,2, $\cdots$, 9) in Table 1 present the condition attributes.

Table 3. The discretization result of sample

ID	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	D
1	0	2	0	3	0	1	1	0	1	0
2	1	5	0	6	1	2	1	0	0	1
3	1	0	0	4	0	1	1	0	1	1
4	0	4	1	5	0	2	2	1	0	0
5	0	3	1	1	0	0	1	0	1	0
6	0	1	2	6	0	2	2	0	1	1
7	1	1	0	6	0	1	2	0	0	1
8	0	5	2	2	1	0	1	0	1	1
9	1	3	1	0	1	0	0	0	0	1
10	1	1	1	2	2	2	0	0	1	1

Table 4. Result of attributes reduction of sample

ID	c_5	c_6	c_7	c_9	D
1	0	1	1	1	0
2	1	2	1	0	1
3	0	1	1	1	1
4	0	2	2	0	0
5	0	0	1	1	0
6	0	2	2	1	1
7	0	1	2	0	1
8	1	0	1	1	1
9	1	0	0	0	1
10	2	2	0	1	1

The Attribute Reduction of sample discretized is shown in Table 4. This is a sample of the smallest attributes set. According to the result of Attribute Reduction, rules is extracted as follows:

IF (c_5=0) AND (c_6=1) AND (c_7=1) AND (c_9=1) THEN D=0.

To facilitate user access to rules, the rule express above can be transformed as following. If the discrete value of an attribute represents a range, then there must show that the attribute value is in the range.

IF (Ptosis Degree is Light) AND (Levator Muscle Strength is Moderate) AND (Superior Rectus Muscle Function is Moderate) AND (Surgical Approach type is 1)

THEN (Effect of surgery is invalid).

But because the sample data is stored in a database and in order to facilitate the processing, the table as Table 4 is used as diagnostic rule. When user inputs disease symptoms data, the system reduces the redundant attributes and discretizes non-redundant attributes, and then queries the rule table to match the rules.

6 Conclusion

What this paper studies is on design and application of the MEDS, proposing model of the MEDS based on Rough Set data mining method. And the model is used in diagnosis of ptosis surgery result. The experiment proved that Rough Set Data Mining Method is true of multi-attribute decision-making system and this model is valid in the diagnosis of ptosis surgery result. As Rough Set Data Mining is a new method to deal with vague and imprecise problem, Medical Expert Diagnosis Model based on Rough Set Data Mining method is an ideal model for medical diagnosis.

The MEDS model designed in this paper has the following features:

1) The establishment of the model of knowledge acquisition in MEDS is based on Rough Set data mining method;

2) The model of MEDS is designed with the latest EMR technology;

3) The process of diagnosing is simple. Redundant attributes are basically not involved in the reasoning process.

4) Program algorithm of attributes reduction with universal property, is fit for every disease data table. There is no need to modify the reduction algorithm. This is because the Attribute Reduction algorithm processes the data table in database with column index insteading of column name.

However, due to some factors, this study is not comprehensive enough, and some work needs to study in depth.

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An Improved $GM(1,1)$ Model of Integrated Optimizing Its Background Value and Initial Condition

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Abstract. Although $GM(1,1)$ model has been successfully adopted in various fields and demonstrated promising results, its predicting performance still could be improved. Up to the present, the literatures show that whether the structure method of background value or the selection of initial condition is logical or not, it affects the simulation and prediction precision directly. However, most optimized models developed recently are optimized with one side. Based on the idea that we have above reasoned, an improved $GM(1,1)$ model of integrated optimizing its background value and initial condition is proposed in this paper, which applies a more logical calculating formula of background value based on Newton interpolation and a relatively ingenious algorithm of initial condition by adding an arbitrary constant. Furthermore, the LCD TV annual output of China is used as a case study to examine the model reliability and accuracy. By comparisons of simulation and prediction data, a higher precision is found.

Keywords: $GM(1,1)$ model, background value, initial condition, prediction.

1 Introduction

Prediction is important for the modern scientific management. Accurate prediction can help the policymaker make correct decision and promote the decision-making quality. Grey prediction theory aims to find the optimized system parameters of a grey differential equation such that the dynamic behavior of the system could be best fitted with the differential equation [1]. $GM(1,1)$ model is the main model of grey prediction theory, i.e. a single variable first order grey model, which is created with few data (four or more) and still we can get high precision results [2]-[3]. $GM(1,1)$ model is one of the most widely used technique in the grey system. In recent years, it has also been successfully employed in various fields and has demonstrated satisfactory results [4]-[8].

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$GM(1,1)$ model is a methodology and it is necessary to constantly put forward new algorithm to improve its performance, prediction accuracy especially. Up to the present, there are many scholars proposing new methods to improve the precision of the model. For example, Tan started from the geometry meaning of $z^{(1)}(k)$ to firstly put forward the optimization of background value in $GM(1,1)$ model [9]. He presented a new calculating formula of background value that advanced the precision of model and was more preferably applied to unequal interval modeling. Mu provided the method of optimum grey derivative's whitening values [10]. He built an unbiased $GM(1,1)$ model and proposed a method of estimation of the parameters. Tan provided the structure method of background values in $GM(1,1)$ model, and a simple calculating formula of background value was reestablished which had strong adaptability [11]. After that, Tzu-Li Tien showed that the first entry of original data sequence is ineffective to the simulating values and forecasts by $GM(1,1)$ model [12]. He presented a more compact algorithm to extract the messages from its first entry to optimize $GM(1,1)$ model and proved its effectiveness. Li showed that the selection of initial value has an important effect on the model's precision, then proposed two methods of modificatory initial value to the application of the power demand model of Shanghai [13]. According to the new rest first principle of the grey systems theory, the n-th vector of original sequence is proposed as initial condition to get improvement in prediction of $GM(1,1)$ by Dang et al [14]. All these methods can reduce the error partially and advance the forecasting precision, but their formulas applied to the simulating values and forecasts of $GM(1,1)$ model is only optimized with one side - optimization of background value or optimization of initial condition. Through analyzing systematically, we know that in $GM(1,1)$ model, the method of estimating parameters is the nonlinear equation and the method of simulating and forecasting is the linear equation. The different forms of the nonlinear equation and the linear equation determined could not equal accurately. Moreover, the simulating values and forecasts by $GM(1,1)$ model are independent of the first entry of original data sequence. So the skip from the nonlinear equation to the linear equation and the selection of initial condition are just the factors which caused the error of simulating and forecasting. In this paper, we will propose an improved $GM(1,1)$ model of integrated optimizing its background value and initial condition. A more logical calculating formula of background value is produced in this paper, i.e., grey function $x^{(1)}(t)$ is replaced by Newton interpolation polynomial $N_n(t)$, and a relatively ingenious and clear algorithm is presented that an arbitrary constant is introduced to optimize initial condition. Furthermore, the LCD TV annual output of China is used as a case study to examine the model reliability and accuracy. By comparisons of the average absolute error and the average relative error, a higher precision of simulation and prediction is found.

The remaining paper is organized as follows. In Section 2, an improved model based on optimization of background value and initial condition is proposed and discussed. Next, Section 3 analyzes and compares the empirical results obtained from original model $GM(1,1)$ and three optimized models. Finally, Section 4 comes to the conclusion of this paper.

2 Improved Grey Prediction Model

2.1 Analysis of Error Reason for Original GM(1,1) Model

Assume the non-negative original sequence as

$$X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)\}$$

and the simple accumulating generator sequence of $X^{(0)}$ as

$$X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(n)\}$$

name

$$x^{(0)}(k) + ax^{(1)}(k) = u \tag{1}$$

as the definition formula of $GM(1,1)$ model, which is also called as grey differential equation, where

$$z^{(1)}(k) = \frac{1}{2}x^{(1)}(k) + \frac{1}{2}x^{(1)}(k-1) \tag{2}$$

$z^{(1)}(k)$ is named background value in $GM(1,1)$ model as the mean generation of consecutive neighbors value of accumulating generator sequence. Here, $\hat{a} = (a, u)^T$ is the parameter row in $GM(1,1)$ model, $Z^{(1)} = \{z^{(1)}(2), z^{(1)}(3), \cdots, z^{(1)}(n)\}$,

$$Y = \begin{pmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \cdots \\ x^{(0)}(n) \end{pmatrix} B = \begin{pmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \cdots & \\ -z^{(1)}(n) & 1 \end{pmatrix}$$

that the least-square state estimation of parameter in $GM(1,1)$ model is

$$\hat{a} = (a, u)^T = (B^T B)^{-1} B^T Y \tag{3}$$

Define the whitenization differential equation of $GM(1,1)$ model is

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = u \tag{4}$$

then the time response equation of grey differential equation is

$$\hat{x}^{(1)}(k) = (x^{(0)}(1) - \frac{u}{a})e^{-a(k-1)} + \frac{u}{a} \tag{5}$$

where $k = 1, 2, \cdots, n$, and the first-order inverse accumulating reductive value is

$$\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1) = (x^{(0)}(1) - \frac{u}{a})(1 - e^{a})e^{-a(k-1)} \tag{6}$$

where $k = 2, 3, \cdots, n$.

Thus, the results can be seen as follows. For the sequence with grey exponent law, the simulation and prediction precision of $GM(1,1)$ model is determined by two points, i.e. (i) the grey developmental coefficient **a** and grey control parameter **u**, whereas the value of **a** and **u** depend on original data sequence and background value, (ii) initial condition, original $GM(1,1)$ model selects $x^{(0)}(1)$ the first entry of original data sequence for initial condition. Therefore, whether the structure method of background value and the selection of initial condition are logical or not, it affects the simulation and prediction precision directly.

Traditional background value calculating formula uses the trapezoid formula to calculate the area, of which enclosed by grey function $x^{(1)}(t)$ within $[k-1, k]$ and t axis approximately. Quadrature both sides of whitenization equation within $[k-1, k]$, that is

$$\int_{k-1}^{k} \frac{dx^{(1)}(t)}{dt} dt + a \int_{k-1}^{k} x^{(1)}(t)dt = u \tag{7}$$

i.e,

$$x^{(1)}(k) - x^{(1)}(k-1) = -a \int_{k-1}^{k} x^{(1)}(t)dt + u \tag{8}$$

$$z^{(1)}(k) = \int_{k-1}^{k} x^{(1)}(t)dt \tag{9}$$

When compared with the Eq. (1) with Eq. (8), the parameters **a** and **u** estimated by using $z^{(1)}(k)$ in Eq. (9) as background value are more adaptive to whitenization equation. Thus, it can be seen that the error reason for original $GM(1,1)$ model is using $\frac{1}{2}x^{(1)}(k) + \frac{1}{2}x^{(1)}(k-1)$ instead of $z^{(1)}(k)$. Up to the present, most optimization of background value at home and abroad use the linear value insert method instead of mean value generation, i.e, using $z^{(1)}(k) = \alpha x^{(1)}(k) + (1-\alpha)x^{(1)}(k-1)$ instead of the original mean value calculating formula. Moreover, Dang et al presented a new method to optimize the background value, i.e, using homogeneous exponential function to match the simple accumulating generator sequence, then put forward a method to found background value [15].

In the modelling procedure of original model $GM(1,1)$, the solution to Eq.(4) with system parameters determined by least-squares method and initial condition $x^{(1)}(1) = x^{(0)}(1)$. From Eq.(5), and by the first-order inverse accumulated generating operation (1-IAGO) of $\hat{x}^{(1)}$, the modelling value $\hat{x}^{(0)}$ can be derived to be

$$\hat{x}^{(0)}(1) = \hat{x}^{(1)}(1) = x^{(0)}(1) \tag{10}$$

It can not completely change in accordance with the fitting curve obtained by least-squares method in terms of the sequence with grey exponent law. Accordingly, the simulating values and predicts by $GM(1,1)$ in Eq.(10) are independent of $x^{(0)}(1)$ the first entry of original sequence [12]. A proof concerning this subject had ever been presented by Lee et al [16]. Theoretically modify initial condition or use other data as initial condition should improve the precision of the prediction formula. Zhang et al. pointed out that $x^{(0)}(1)$ shouldn't be limited as the only known condition when forming the prediction formula, and used

$$\hat{x}^{(0)}(m) = \hat{x}^{(1)}(m) = x^{(0)}(m), m = 2, 3, \cdots, n \tag{11}$$

as initial value to prediction, then selected the best value **m** to establish prediction equation by means of comparing the prediction errors [17].

2.2 Optimization of Background Value Based on Newton Interpolation

Defined

$$f[x_0, x_k] = \frac{f(x_k) - f(x_0)}{x_k - x_0}$$

as the first-order difference of $f(x)$ at the points $\{x_0, x_k\}$.

$$f[x_0, x_1, x_k] = \frac{f[x_0, x_k] - f[x_0, x_1]}{x_k - x_1}$$

as the second-order difference of $f(x)$ at the points $\{x_0, x_1, x_k\}$.

$$f[x_0, x_1, \cdots, x_k] = \frac{f[x_0, x_1, \cdots, x_k] - f[x_0, x_1, \cdots, x_{k-1}]}{x_k - x_{k-1}}$$

as the $k-$order difference of $f(x)$ at the points $\{x_0, x_1, \cdots, x_k\}$.

Let $f(x) \longrightarrow [a, b]$ be a function and $\{x_0 < x_1 < \cdots < x_n\}$ be points belonging to an interval $[a, b]$. We will interpolate the function $f(x)$ and obtain a polynomial of degree n, $N_n(x)$.

$$f[x] = f[x_0] + f[x, x_0](x - x_0)$$

$$f[x, x_0] = f[x_0, x_1] + f[x, x_0, x_1](x - x_1)$$

$$\cdots$$

$$f[x, x_0, \cdots, x_{n-1}] = f[x_0, x_1, \cdots, x_n] + f[x, x_0, \cdots, x_n](x - x_n)$$

Hence,

$$f(x) = f(x_0) + \cdots + f[x, x_0, x_1, \cdots, x_n]\omega_{n+1}(x) = N_n(x) + R_n(x) \quad (12)$$

where

$$\omega_k(x) = (x - x_0)(x - x_1) \cdots (x - x_{k-1})$$

where $k = 1, 2, \cdots, n + 1$,

$$R_n(x) = f(x) - N_n(x) = f[x, x_0, x_1, \cdots, x_n]\omega_{n+1}(x)$$

so the interpolating polynomial, $N_n(x)$, of $f(x)$ at the points $\{x_0, x_1, \cdots, x_k\}$ is

$$N_n(x) = f(x_0) + f[x_0, x_1]\omega_1(x) + \cdots + f[x_0, x_1, \cdots, x_n]\omega_n(x) \quad (13)$$

As noted earlier, grey function $x^{(1)}(t)$ may be a very complex function which can not even be expressed by specific formula, so interpolation polynomial can be adopt to approximate better to $x^{(1)}(t)$ under certain hypothesis. When using interpolation polynomial $N_n(t)$ instead of function $x^{(1)}(t)$, it is better that the absolute value of the residual items $R_n(t)$ is small sufficiently. The precision of $GM(1,1)$ model can be improved by increasing the interpolation nodes, i.e. increasing the degrees of interpolation polynomial, but if **n** is too large, there will generate the phenomenon of poor approximation within a small interval formed by high order interpolation. Therefore, in order to approximate better to grey function $x^{(1)}(t)$, it should decide the value of **n** based on practical problems, generally $\mathbf{n} < 7$.

For convenience of calculation, we use lower times (quadratic) interpolation polynomial $N_n(t)$ instead of grey function $x^{(1)}(t)$. As the points k, $k+1$ and $k+2$ are interpolation nodes, the quadratic interpolation polynomial $N_k(t)$ for the function $x^{(1)}(t)$ is

$$\begin{aligned} N_k(t) &= x^{(1)}(k) + x^{(1)}[k, k+1](t - k) + x^{(1)}[k, k+1, k+2](t - k)[t - (k+1)] \\ &= x^{(1)}(k) + [x^{(1)}(k+1) - x^{(1)}(k)](t - k) + [\frac{1}{2}x^{(1)}(k+2) - x^{(1)}(k+1) \\ &+ \frac{1}{2}x^{(1)}(k)](t - k)[t - (k+1)] \end{aligned} \quad (14)$$

where $k = 1, 2, \cdots, n - 2$.

Since $N_k(t)$ is a quadratic polynomial, the background value $z^{(1)}(k)$ gotten by taking quadrature of the quadratic interpolation polynomial $N_k(t)$ will have a cubic algebraic precision

$$z^{(1)}(k+1) = \int_k^{k+1} N_k(t)dt = \frac{2}{3}x_{k+1} - \frac{1}{12}x_{k+2} + \frac{5}{12}x_k, \quad (15)$$

where $k = 1, 2, \cdots, n - 2$.

2.3 Optimization of Initial Condition

For original $GM(1,1)$ model, there is no theoretical basis for selecting $x^{(0)}(1)$ the first entry of original data sequence for initial condition. The best fitting curve of $GM(1,1)$ model may not go through any data of the sequence $X^{(1)}$, so the arbitrary constant ρ can be introduced to optimize initial condition of model $GM(1,1)$ in the condition of minimizing the objective function.

$$\hat{x}^{(1)}(k) = (\rho x^{(1)}(1) - \frac{u}{a})e^{-a(k-1)} + \frac{u}{a} \tag{16}$$

where $k = 1, 2, \cdots, n$, and the objective function is defined as follows

$$\begin{aligned} J &= \sum_{k=1}^{n}[\hat{x}^{(1)}(k) - x^{(1)}(k)]^2 \\ &= \sum_{k=1}^{n}\{(\rho x^{(0)}(1) - \frac{u}{a})e^{-a(k-1)} + \frac{u}{a} - x^{(1)}(k)\}^2 \end{aligned} \tag{17}$$

In order to find the arbitrary constant ρ at a minimum of the objective function, the following gradient method is used.

$$\begin{aligned} \frac{dJ}{d\rho} &= 2 \cdot \sum_{k=1}^{n}\{(\rho x^{(0)}(1) - \frac{u}{a})e^{-a(k-1)} + \frac{u}{a} - x^{(1)}(k)\} \cdot x^{(0)}(1) \cdot e^{-a(k-1)} \\ &= 2 \cdot \sum_{k=1}^{n} \rho[x^{(0)}(1)]^2 e^{-2a(k-1)} + \frac{u}{a}\sum_{k=1}^{n} x^{(0)}(1)e^{-a(k-1)} - \\ &\quad x^{(0)}(1)\sum_{k=1}^{n} x^{(1)}(k)e^{-a(k-1)} - \frac{u}{a}\sum_{k=1}^{n} x^{(0)}(1)e^{-2a(k-1)} = 0 \end{aligned} \tag{18}$$

For convenience, we denote

$$\alpha = \sum_{k=1}^{n} e^{-2a(k-1)},\ \beta = \sum_{k=1}^{n} e^{-a(k-1)},\ \gamma = \sum_{k=1}^{n} x^{(1)}(k)e^{-a(k-1)}.$$

By induction, one obtains

$$\rho x^{(0)}(1)\alpha + \frac{u}{a}(\beta - \alpha) - \gamma = 0$$

$$\rho = \frac{\frac{u}{a}(\alpha - \beta) + \gamma}{x^{(0)}(1)\alpha} \tag{19}$$

2.4 Modelling with Optimized Background Value and Initial Condition

According to the equations illustrated above, the modelling procedure of the improved grey model optimized by background value and initial condition is carried out in detail as follows.

Assume the non-negative original data sequence as

$$X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)\}$$

and the simple accumulating generator sequence of $X^{(0)}$ as

$$X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(n)\}$$

The least-square state estimation of parameter sequence of grey differential equation

$$x^{(0)}(k) + ax^{(1)}(k) = u$$

is

$$\hat{a} = (a, u)^T = (B^T B)^{-1} B^T Y$$

where

$$Y = \begin{pmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \cdots \\ x^{(0)}(n) \end{pmatrix} B = \begin{pmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \cdots & \\ -z^{(1)}(n) & 1 \end{pmatrix}$$

The background value $z^{(1)}(k)$ is taken as

$$\begin{cases} z^{(1)}(k+1) = \frac{2}{3}x_{k+1} - \frac{1}{12}x_{k+2} + \frac{5}{12}x_k, \\ z^{(1)}(n) = \frac{2}{3}x_{n-1} - \frac{1}{12}x_{n-2} + \frac{5}{12}x_n \end{cases}$$

where $k = 1, 2, \cdots, n-2$. The grey developmental coefficient $\mathbf{a}$ and grey control parameter $\mathbf{u}$ can be estimated. The solution to

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = u$$

with system parameters determined by least-squares method and initial condition $\rho x^{(0)}(1)$ where

$$\rho = \frac{\frac{u}{a}(\alpha - \beta) + \gamma}{x^{(0)}(1)\alpha}$$

is

$$\hat{x}^{(1)}(k) = (\rho x^{(1)}(1) - \frac{u}{a})e^{-a(k-1)} + \frac{u}{a}$$

where $k = 1, 2, \cdots, n$ and the reductive value is

$$\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1) = (\rho x^{(0)}(1) - \frac{u}{a})(1 - e^a)e^{-a(k-1)}$$

where $k = 2, 3, \cdots, n$.

3 Data Simulation and Comparison with Precision

To demonstrate the effectiveness of the proposed method, we use the LCD TV output prediction of China as an illustrating example. In this study, we use the historical annual LCD TV output of China from 1996 to 2005 as our research data, which are listed in Table 1.

Table 1. The LCD TV output per year (unit: ten thousand)

Year	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005
Output	3.28	5.48	10.07	17.70	29.73	49.39	92.67	162.23	280.86	513.40

Assume the optimized model $GM(1,1)$ in publications [15] as optimized model 1, which applied a novel approach of background value-building with

$$z^{(1)}(k) = \frac{x^{(1)}(k) - x^{(1)}(k-1)}{\ln x^{(1)}(k) - \ln x^{(1)}(k-1)}$$

where $k = 2, 3, \cdots, n$. Assume the optimized model which only applied optimization of initial condition proposed by this paper as optimized Model 2, and the optimized model in this paper as optimized Model 3. Then the LCD TV output of China from 1996 to 2003 is used to establish original model $GM(1,1)$ and three optimized models. By original model $GM(1,1)$, the time response equation of grey differential equation is

$$\hat{x}^{(1)}(k) = 6.3629 \cdot e^{-0.5541(k-1)} - 3.0829$$

where $k = 2, 3, \cdots, n$. By optimized Model 1, the time response equation of grey differential equation is

$$\hat{x}^{(1)}(k) = 6.4126 \cdot e^{-0.5698(k-1)} - 3.1326$$

where $k = 2, 3, \cdots, n$. By optimized Model 2, the time response equation of grey differential equation is

$$\hat{x}^{(1)}(k) = 7.6713 \cdot e^{-0.5541(k-1)} - 3.0829$$

where $k = 2, 3, \cdots, n$. By optimized Model 3, the grey developmental coefficient **a**, the grey control parameter **u** and the arbitrary constant ρ can be obtained, $a = -0.5661$, $u = 2.1355$, $\rho = 1.0193$, so the time response equation is

$$\hat{x}^{(1)}(k) = 7.1159 \cdot e^{-0.5661(k-1)} - 3.7725$$

where $k = 2, 3, \cdots, n$. and inverse accumulating reductive value is

$$\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1)$$

We get the prediction by the time response equations of four models above and the results are in Table 2. Data from 1996 to 2003 is simulated and data of 2004 and 2005 is the prediction. Then, error of simulation and prediction can be calculated, and the results are in Tables 3-4. where the average absolute error

$$AAE = \frac{1}{n}\sum_{i=1}^{n} |\hat{x}^{(0)}(i) - x^{(0)}(i)|$$

and the average relative error

$$AAE = \frac{1}{n}\sum_{i=1}^{n} |\frac{\hat{x}^{(0)}(i) - x^{(0)}(i)}{x^{(0)}(i)}|$$

Table 2. Comparison of simulation results with four models

		Original Model	Model 1	Model 2	Model 3
Year	Actual output	prediction	prediction	prediction	prediction
1996	3.28	3.28	3.28	4.59	3.34
1997	5.48	4.71	4.92	5.68	5.42
1998	10.07	8.20	8.70	9.88	9.54
1999	17.70	14.27	15.39	17.20	16.81
2000	29.73	24.83	27.20	29.94	29.60
2001	49.39	43.21	48.09	52.10	52.14
2002	92.67	75.21	85.02	90.67	91.83
2003	162.23	130.89	150.29	157.80	161.74
2004	280.86	227.79	265.69	274.63	284.88
2005	513.40	396.42	469.70	477.95	501.77

As it can be seen from the given tables, there are substantial differences between the results of four models. Original $GM(1,1)$ model is proved to be invalid that the average relative error is high as 14.8951%. One reason of the high error is that original $GM(1,1)$ model uses $x^{(0)}(1)$ the first entry of original data sequence for initial condition. The precision will increase greatly if optimization of initial condition proposed by this paper is only applied, as seen in the results of optimized model 2, the average relative error is 7.4049%. On the other hand, since optimized Model 1 takes into account optimization of background value, the average relative error is also decrease to 7.9403%. Consequently, background value and initial condition are two important factors to affect the precision of $GM(1,1)$ model. It also can be seen from Table 3 that the error of optimized Model 3 is less than that of other optimized models, whenever the average absolute error (0.7187) or the average relative error (2.5707%) is chosen, for the reason that optimized model 3 is optimized by the combination of background value and initial condition. It is the precondition of a model that have high precision, and the precision of optimized Model 3 is higher than that of other models. Meanwhile, in Table 4, the prediction relative error of optimized Model 3 in 2004 and 2005 is 1.4320% and 2.2657% respectively, which are less than the error of other models, and precision of optimized Model 3 is much higher. In this example, it can also be found that optimized Model 3 has preponderance both in simulation and in prediction.

Table 3. Average simulation error of four models

Kind of error	Original Model	Model 1	Model 2	Model 3
AAE (ten thousand)	8.2445	3.4565	1.4419	0.7187
ARE (%)	14.8951	7.9403	7.4049	2.5707

Table 4. The prediction relative error of four models(%)

Year	Original Model	Model 1	Model 2	Model 3
2004	18.8973	5.4002	2.2193	1.4320
2005	22.7846	8.5119	6.9059	2.2657

4 Conclusion

The simulation and prediction precision of $GM(1,1)$ model is determined by two points, i.e. (i) the grey developmental coefficient $\mathbf{a}$ and grey control parameter $\mathbf{u}$, and the value of $\mathbf{a}$ and $\mathbf{u}$ depend on original data sequence and background value, (ii) initial condition. The background value built by original $GM(1,1)$ have been proved to be incorrect in this paper, interpolation polynomial $N_n(t)$ is be proposed to replace grey function $x^{(1)}(t)$ and the precision of $GM(1,1)$ model can be improved by increasing the interpolation nodes. It is no evidence that using $x^{(0)}(1)$ the first entry of original data sequence for initial condition of original $GM(1,1)$. An arbitrary constant ρ can be introduced to optimize initial condition in the condition of minimizing the objective function J. In data simulation and prediction, the precision of optimized model 3 is higher than original model $GM(1,1)$ and other optimized models remarkably, so the optimized model in this paper has preponderance both in simulation and in prediction. In fact, the optimized model in this paper is non-preference model $GM(1,1)$, i.e., there is no error to simulate and predict original data with whitenization exponential law. The study of this grey prediction model still calls for further in-depth research.

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Discussions of the Mathematical Mechanism for Constructing the GM (1, 1) Model

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Abstract. Starting from analyzing dynamic characteristics of the general energy system, this article discusses the mathematical mechanism of the GM (1, 1) model for producing AGO/IAGO data and the mathematical mechanism for constructing "the model equation" and the mathematical mechanism for parameter estimation. It also discusses other aspects of the application of the GM (1, 1) model. It points out: A. the classic GM (1, 1) model follows the principle of RMI model, but there exist background defects of the "planning predictions" and internal discords of the model caused by the prior assumptions in which both development coefficient and gray effective amount are constant, which restricts the application of the model. B. The classic GM (1, 1) model, in principle, only applies to fitting modeling of time sequences which tend to be slow and with wider intervals and positive stable values. C. Gray differential equation as a tool for estimating the parameters is not indispensible, so this article suggests optimizing parameter calculation method: gray differential equation providing initial iteration, using the end of AGO generating sequence as the boundary condition, etc. to improve the precision and explicability of the GM (1, 1) model.

Keywords: General energy system, GM (1, 1) model, mathematical mechanism, application scope, sequence features, parameter estimation.

1 Introduction

According to the literature [1], the GM(1,1) model, which was a forecast model established by Mr. Deng for forecasting annual grain yield, was born in the early-1980s when he was doing research into the Chinese grain development of a long-term plan. Since then, with the construction of the gray system theory, GM(1,1) model has been described as the general mathematical model for analysis of single variable time sequences of a general energy system.

Since the GM (1, 1) model's widespread application, questions are raised. For example, as the development coefficient increases, the accuracy of the forecast with the model drops, and the model is not suitable for medium and long-term

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forecasts. Though it was much written about, all the discussions have not been able to touch the core of the question.

In the process of developing the theory, the author of this article believes, lack of adequate scientific elaboration of the mathematical mechanism for constructing the GM (1, 1) model is what caused so many cognitive problems about it.

First of all, let's have a clear idea of the concept of what is a general energy system with a single variable.

A dynamic development system may be considered as an input-output system which is actuated by some kind of energy. If the system's input and operation mechanism can't be figured out and the system's output variable is considerable, the analysis can only rely on the output variable. We call such kind of system as the one-variable general energy system (GES).

Suppose $x(t), t \in R$ as an output function of GES for time t. Obviously this is a random function. Because $x(t)$ is defined as an output actuated by some kind of energy, let $x(t) \geq 0$. Obviously,

$$X(t) = \int_0^t x(u)du$$

is the GES's accumulation output function in the observation period $[0, t]$.

According to the GM (1, 1) model theory, the equation (1) is called "the model equation" which is used to indicate the dynamic characteristics based on $X(t)$ of the GES.

$$\frac{dX(t)}{dt} + aX(t) = b \quad (1)$$

This article first discusses the relationship between the GES's dynamic characteristics and equation (1), then try to clearly define several key questions in model GM (1,1) theory and application.

2 The Mathematical Mechanism for GM (1, 1) Model

This article believes that the correct understanding of the GM (1, 1) model and it's application scope lies in accurate understanding of the GM (1,1) model's mathematical mechanism, which includes production of AGO/IAGO data, construction of "model equation" and parameter estimation.

2.1 The Mathematical Mechanism for Producing AGO/IAGO Data

Usually, the value of GES real-time output function $x(t)$ is random at different moments and its dynamic performance is "unconstrained fluctuation". Therefore, with relatively less sample data, it is very difficult to use direct observational data to infer the dynamic features and the change pattern. In such cases, according to the "relationship mapping inversion (RMI)" principle of the mathematical modeling theory in mathematical methodology, take into account the relationship between the function and its original function:

$$X(t) = \int x(t)dt + c \tag{2}$$

$$x(t) = \frac{dX(t)}{dt} \tag{3}$$

It is possible to model the GES cumulative output function through (2)'s transformation.

If $x(t) \geq 0$ and there is continuity, the function $X(t)$ is monotonously non-decreasing and its smoothness is remarkably improved compared with $x(t)$. Therefore, the confidence level of conclusion will also be high when inferences are made about the system's dynamic characteristics and change pattern based on the $X(t)$'s observation data, even if with few sampled data. And then, through (3)'s inverse transformation, we usually have a more ideal mathematical model for $x(t)$.

In GM(1,1) model theory, we first generate the data AGO, then use the data to build a model, and then use the IAGO generation process to derive a practical forecast model, which is a method following the RMI principle about the relationship between function and its original function.

2.2 The Mathematical Mechanism for Constructing “Model Equation”

Since the birth of GM (1, 1) model, lack of attention to and elaboration of the construction questions of equation (1) is the primary reason for causing many problems in its application.

When discussing the mathematical mechanism for constructing equation (1), we can not ignore the background of its birth. This article believes our thinking was bound in a “foreseeable period” when designing the “planning forecast model” that "congenital" defects were implanted in equation (1) by the “planning” requirements.

In a predictable evolution period $[0,T]$ of GES, let $0 \leq t \leq T$

$$Y(t) = \int_t^T x(u)du = \int_0^T x(u)du - X(t)$$

indicates GES development remainder in an expansion period $[0,T]$. When analyzing the system's dynamic characteristics, the key is, technically, to analyze the balance between $Y'(t)$ and $Y(t)$. Let

$$\sigma(t) = \frac{Y'(t)}{Y(t)} \tag{4}$$

$\sigma(t)$ is called GES development function.

In function (4), the two following prior assumptions are introduced:

A. $\sigma(t) = \sigma$ is a constant. When the system environment is almost unchanged and evolution process is in the stable state, the assumption is reasonable.

B. $S = \int_0^T x(u)du$ is a certain value. In the next part, we will see that this is an equivalent statement in GM (1, 1) model with "the gray effective amount b being a constant".

Put the two prior assumptions into function (4), then

$$\sigma = -\frac{X'(t)}{S - X(t)} \tag{5}$$

The negative sign in the right indicates the dynamic duality with $X(t)$ and $Y(t)$, which is denoted $a = -\sigma$, then

$$X'(t) + aX(t) = b \tag{6}$$

(6) is the "model equation" of GM (1,1) model, in which a is the development coefficient and $b = aS$ is the gray effective amount. According to the linear differential equation theory, the solution function of equation (6) is

$$X(t) = Ce^{-at} + S \tag{7}$$

It is the "albinism response function" of the GM(1, 1) model, in which edge C is determined by the boundary conditions.

2.3 The Mathematical Mechanism for Parameter Estimation

Suppose $0 = k_0 < k_1 < k_2 < \cdots < k_n = t$ in GES observation period $[0, t]$, and then carry out discrete sampling of the system, thus derive the sampled data sequence $x^{(0)}(k_i), i = 0,1,2,\cdots,n$ with sampling spacing being $\Delta_i = k_i - k_{i-1}$. Here, data $x^{(0)}(k_i)$ is in essence, the increased amount of the system in sector $(k_{i-1}, k_i]$.

The corresponding AGO-generated sequence: $x^{(1)}(k_i)$, $i = 0,1,2,\cdots,n$. To estimate the parameter in equation (1), equation (1) must be discretized. Taking into account the function points in equation (1) in sector $(k_{i-1}, k_i]$,

$$\int_{k_{i-1}}^{k_i} \mathrm{d}X(t) + a\int_{k_{i-1}}^{k_i} X(t)\mathrm{d}t = b\int_{k_{i-1}}^{k_i} \mathrm{d}t$$

That is,

$$x^{(0)}(k_i)\Delta_i + a\, z^{(1)}(k_i)\Delta_i = b\Delta_i \tag{8}$$

This is the "grey differential equation" of the GM (1, 1) model. Here, $x^{(0)}(k_i)$ is called the gray derivative, which is, in mathematical essence, the derivative difference approximation. $z^{(1)}(k_i)=\int_{k_{i-1}}^{k_i} X(t)\mathrm{d}t$ is called the gray derivative background value. In the classic GM (1, 1) model theory, $z^{(1)}(k_i)=0.5(x^{(1)}(k_{i-1})+x^{(1)}(k_i))$, which, in mathematical essence, is to approximate the trapezoidal area in sector $(k_{i-1},k_i]$ to the corresponding curvilinear trapezoidal area.

Therefore, when using equation (9) as the parameter estimation tool for equation (1), the mathematical structure is biased; on the other hand, in the classics GM(1,1) model theory, taking $\Delta_i \equiv 1$ will "consolidate" the structure deviation of the parameter estimation tool, thus it is impossible to obtain the optimal parameter estimation in equation (1) even if by cutting minimum variance unbiased least squares algorithm.

3 Questions and Suggestions

A. The effect of the internal contradictions of the GM (1, 1) model on it's application

In the discussion of Section 2.2, we know that, in the GM(1,1) model theory, the prior assumption, in which "gray effective amount b" is constant, is an inappropriate restriction when "planning thought" is used to describe the dynamic characteristics of GES, causing contradictory results that $Y(t)=S-X(t)$ tends to be 0 but $x(t)$ does not tend to be 0, which restricts the application scope of the model.

When GES develops more "slowly", that is, when development coefficient a is smaller and development intervals are long enough, internal contradictions of equation (5) can be lessened to some degree. At such times, GM(1,1) model can be used to make short-term forecasts, predicting step d should not be bigger than the integral part of $n[S/x^{(1)}(n)-1]$, otherwise it is beyond the defining power of GM (1,1) model, systematic deviation occurs between predicted values and observed values when forecasts are made farther and farther away from the forecast origin.

B. The features of the time series to which GM (1,1) model is applicable

From the discussion of Section 2.1, when the original observation data has shown the characteristics of exponential sequence, fitting modeling of GES no longer belongs to the GM(1,1) model application scope.

In general, it is only when the original observed data are positive and stable and AGO-generated sequences show characteristics of non-homogeneous exponential function, the GM(1,1) model is effective.

This article believes that the descriptive ability of GM (1, 1) model is limited for the GES, therefore, it is not applicable to trend and periodic systems.

C. The suggestions for parameter estimation in GM (1, 1) model

Only from the mathematical point, equation (8) is neither the basis for the application of the model with small samples nor an indispensible tool for parameter estimation. Optimization algorithm is suggested here, with

$$\min_{a,C,S} \sum_{\forall k} [\, x^{(1)}(k) - Ce^{-ak} - S \,]^2$$

as the optimization goal, constraints are:

$$\text{s.t. } a > 0, \;\; S > 0, \;\; C + S = x^{(1)}(n)$$

In equation (8), we estimate parameters a and b by Linear Least Squares Method, so $S = b/a \quad C = x^{(1)}(n) - S$ to determine the initial value, with bleaching response function responding being

$$\hat{x}^{(1)}(k+1) = \left(x^{(1)}(n) - \mathrm{S}\right)e^{-a(k-n)} + S$$

Here $k > n$, this will improve the accuracy of model fitting and also makes the meaning of the model more clearer: $k - n \geq 1$ predicting step, $a > 0$ being the development coefficient, $S > 0$ being the expected cumulative upper limit, and $C = x^{(1)}(n) - S < 0$ being the development remainder (negative sign indicates the development remainder attenuation).

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Constructing Vague Environment

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Abstract. The definition and criterion from the single valued data into the vague valued data were presented, and the two transforming formulas from the single valued data to the vague valued data were proposed. The examples make known that the distinct formulas have distinct characters.

Keywords: Single valued data, vague valued data, transforming criterion, positive ordered transforming formula, reverse ordered transforming formula.

1 Introduction

With the increasing application of vague set wide, exposed an outstanding problem: how to create a vague environment? Because only in the vague environment, vague set methods may be used only to study, analyze and solve practical problems. [1] gives some methods to transform a single-value data into a vague value data. [2] gives the first single-value data into fuzzy value data, then converted into a vague value data. The literature [3-10] directly give the vague value data of the question, not the method from raw data into Vague value data. But the missing link is exactly what one of the key steps in vague set application.

Usually there are two forms of raw data, a single-value data, the other is interval-valued data. We only study single-value data into the vague value data.

2 Single-Value Data Transform into the Vague Value Data

Let X be the universe of discourse.A vague set A in X is characterized by a truth-membership function t_A and false-membership function f_A, t_A:$X \longrightarrow [0, 1]$ where $t_A(x)$ is a lower bound on the grade of membership of x from the

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evidence for $x \in X$, $f_A(x)$ is a lower bound on the negation of membership of x from the evidence against for x, and $t_A(x) + f_A(x) \leq 1$ Then, a vague set A can be written as $A = \{(x, t_A(x), f_A(x)) | x \in X\}$ And $[t_x, 1 - f_A(x)]$ is called a vague value of x in a vague set A, denoted also by x.

If $x = [t_A(x), 1 - f_A(x)]$ is a vague value in a vague set A, then $x' = [f_A(x), 1 - t_A(x)]$ is called the complement of$x = [t_A(x), 1 - f_A(x)]$. The set of vague sets in X is denoted as$V(X)$.

Let $A \in V(X)$, when X is a finite set, A can be written as

$$A = \Sigma_{i-1}^{n}[t_A(x_i), 1 - f_A(x_i))]/x_i, x_i \in X$$

When X is continuous, A can be written as

$$A = \int_X [t_A(x), 1 - f_A(x))]/x, x \in X$$

Definition 1. Let $X = x_1, x_2, \cdots, x_n$be an index set and the index $x_j(j = 1, 2, \cdots, n)$of program $A_i(i = 1, 2, \cdots, m)$ be a single-value data x_{ij}. If x_{ij} is transformed into vague value data $A_i(x_j = [t_{A_i}(x_j, 1 - t_{A_i}(x_i)])$, and the following criterions are satisfied:

C1: $t_{A_i}(x_j, 1 - t_{A_i}(x_i) \in [0, 1]$for all $x_j \in X$;

C2: $t_{A_i}(x_j \leq 1 - t_{A_i}(x_i)$, for all $x_j \in X$;

C3: If $\omega_i > \omega_k \geq 0$,the single-value data ω_i and ω_k are transformed into vague value data respectively: $A_i(x_j) = [t_{ij}, 1 - f_{ij}]$ and$A_h(x_j) = [t_{hj}, 1 - f_{hj}]$, we have $t_{ij} > t_{hj}, 1 - f_{ij} > 1 - f_{hj}$. Then,this transforming formula which satisfies C1,C2 and C3 is called the positive ordered transforming formula(Denoted as POTF),and the vague value data $A_i(x_j) = [t_{ij}, 1 - f_{ij}]$ is called the positive ordered vague value data(Denoted as POVVD).

$C3'$:If $\omega_i > \omega_k \geq 0$, the single-value data ω_i and ω_k are transformed into vague value data respectively: $A_i(x_j) = [t_{ij}, 1 - f_{ij}]$ and $A_h(x_j) = [t_{hj}, 1 - f_{hj}]$, we have $t_{ij} < t_{hj}, 1 - f_{ij} < 1 - f_{hj}$. Then,this transforming formula which satisfies C1,C2 and $C3'$ is called the reverse ordered transforming formula(Denoted as ROTF),and the vague value data $A_i(x_j) = [t_{ij}, 1 - f_{ij}]$ is called the reverse ordered vague value data. (Denoted as ROVVD)

3 The Positive Ordered Transforming Formula

Theorem 1. Let $X = x_1, x_2, \cdots, x_n$ be an index set and the index $x_j(j = 1, 2, \cdots, n)$ of program $A_i(i = 1, 2, , m)$ be a single-value data x_{ij}. Let $x_{jmin} = \{x_{1j}, x_{2j}, \cdots, x_{mj}\}$,$x_{jmax} = \{x_{1j}, x_{2j}, \cdots, x_{mj}\}$, Then

$$A_i(x_j = [t_{ij}, 1 - f_{ij}] = [\frac{x_{ij}^p - x_{jmin}^p}{x_{jmax}^p - x_{jmin}^p}, 1 - \frac{x_{jmax} - x_{ij}}{x_{jmax} - x_{jmin}}](p = 2, 3, \cdots) \quad (1)$$

is a POTF which transformed a non-negative single-value data x into a vague value data.

Proof (1) Because $0 \leq x_{jmin} \leq x_{ij} \leq x_{jmax}$, so
$0 \leq x_{ij} - x_{jmin} \leq x_{jmax} - x_{jmin}$,$0 \leq x_{ij}^p - x_{jmin}^p \leq x_{jmax}^p - x_{jmin}^p$
then $t_{ij} = \frac{x_{ij}^p - x_{jmin}^p}{x_{jmax}^p - x_{jmin}^p} \in [0,1], 1 - f_{ij} = 1 - \frac{x_{jmax} - x_{ij}}{x_{jmax} - x_{jmin}} = \frac{x_{ij} - x_{jmin}}{x_{jmax} - x_{jmin}} \in [0,1]$.
(2) If$\frac{1-f_{ij}}{t_{ij}} = \frac{x_{jmax}^{p-1} + x_{jmax}^{p-2} \cdot x_{jmin} + \cdots + x_{jmin}^{p-1}}{x_{ij}^{p-1} + x_{ij}^{p-2} \cdot x_{jmin} + \cdots + x_{ij}^{p-1}} \geq 1$ then $1 - f_{ij} \geq t_{ij}$
(3) when $\omega_i > \omega_{h \geq 0}$ ω_i and ω_k are transformed respectively into vague value
$A_i(x_j) = [t_{ij} - 1 - f_{ij}] = [\frac{\omega_i^p - x_{jmin}^p}{x_{jmax}^p - x_{jmin}^p}, 1 - \frac{x_{jmax} - \omega_i}{x_{jmax} - x_{jmin}}]$
$A_h(x_j) = [t_{hj} - 1 - f_{hj}] = [\frac{\omega_h^p - x_{jmin}^p}{x_{jmax}^p - x_{jmin}^p}, 1 - \frac{x_{jmax} - \omega_h}{x_{jmax} - x_{jmin}}]$
Since $t_{ij} = \frac{\omega_i^p - x_{jmin}^p}{x_{jmax}^p - x_{jmin}^p} > \frac{\omega_h^p - x_{jmin}^p}{x_{jmax}^p - x_{jmin}^p} = t_{hj}$
and $1 - f_{ij} = 1 - \frac{x_{jmax} - \omega_i}{x_{jmax} - x_{jmin}} > 1 - \frac{x_{jmax} - \omega_h}{x_{jmax} - x_{jmin}} = 1 - f_{hj}$ then Formula (1) is a POTF.

Theorem 2. Letx_{jTmin} be the theoretical minimum for x_{jk} and x_{jTmax} be the theoretical maximum for x_{jk}. Then

$$A_i(x_j) = [t_{ij} - 1 - f_{ij}] = [\frac{x_{ij}^p - x_{jTmin}^p}{x_{jTmax}^p - x_{jTmin}^p}, 1 - \frac{x_{jTmax} - x_{ij}}{x_{jTmax} - x_{jTmin}}], (p = 2, 3 \cdots) \tag{2}$$

is a POTF.

Note 1. It is clear that $x_{jmin} \geq x_{jTmin}, x_{jmax} \leq x_{jTmax}$.

Theorem 3. Let $y_{jmax} = \alpha x_{jmax}; y_{jmin} = \beta x_{jmin}, 0 \leq \beta \leq 1$, then

$$A_i(x_j) = [t_{ij} - 1 - f_{ij}] = [\frac{x_{ij}^p - y_{jmin}^p}{y_{jmax}^p - y_{jmin}^p}, 1 - \frac{y_{jmax} - x_{ij}}{y_{jmax} - y_{jmin}}], (p = 2, 3 \cdots) \tag{3}$$

is a POTF.

Note 2. In Therom1,2, 3,let$p = 2$,we can get the Formula (3) and Formula (4) in [2].

4 The Reverse Ordered Transforming Formula

Theorem 4. Let $X = x_1, x_2, \cdots, x_n$be an index set and the index $x^j (j = 1, 2, \cdots, n)$of program $A^i (i = 1, 2, , m)$ be a single-value data x^{ij} then

$$A_i(x_j = [t_{ij}, 1 - f_{ij}] = [\frac{x_{jmax} - x_{ij}}{x_{jmax} - x_{jmin}}, 1 - \frac{x_{ij}^p - x_{jmin}^p}{x_{jmax}^p - x_{jmin}^p}](p = 2, 3, \cdots) \tag{4}$$

is a ROTF

Theorem 5. Let x_{jTmin} be the theoretical minimum for x_{jk} and x_{jTmax} be the theoretical maximum for x_{jk}.

Then

$$A_i(x_j) = [t_{ij}, 1-f_{ij}] = [\frac{x_{jTmax} - x_{ij}}{x_{jTmax} - x_{jTmin}}, 1-\frac{x_{ij}^p - x_{jTmin}^p}{x_{jTmax}^p - x_{jTmin}^p}](p = 2, 3, \cdots)$$

is a ROTF.

Note 3. It is clear that $x_{jmin} \geq x_{jsmall}, x_{jmax} \leq x_{jbig}$

Theorem 6. Let $y_{jmax} = \alpha x_{jmax}; y_{jmin} = \beta x_{jmin}, 0 \leq \beta \leq 1$, then

$$A_i(x_j) = [t_{ij} - 1 - f_{ij}] = [\frac{y_{jmax} - x_{ij}}{y_{jmax} - y_{jmin}}, 1 - \frac{x_{ij}^p - y_{jmin}^p}{y_{jmax}^p - y_{jmin}^p}], (p = 2, 3 \cdots) \tag{5}$$

is a ROTF.

Note 4. In Therom4,5,6 3,let p=2,we can get the Formula (5) and Formula (6) in [2].

5 Example

Through examples, we can see more intuitive the features of the POTF and ROTF which transforming a non-negative single-value data into a vague value data.

Example 1. With the original data as shown in Table 1. By Theorem 1, let p=3, then we can transform the original data as shown in Table 1 into a POVVD,and by Therom 1, let p=3, then we can transform the original data as shown in Table 1 into a ROVVD.

Table 1. Original data POVVD ROVVD

Program	x_1	x_2	x_1	x_2	x_1	x_2
A_1	88	6	[0.68,0.84]	[0.22,0.43]	[0.16,0.32]	[0.57,0.78]
A_2	63	4	[0.24,0.51]	[0.07,0.14]	[0.49,0.76]	[0.86,0.93]
A_3	25	3	[0,0]	[0,0]	[1,1]	[1,1]
A_4	74	10	[0.40,0.65]	[1,1]	[0.35,0.60]	[0,0]
A_5	100	7	[1,1]	[0.35,0.57]	[0,0]	[0.43,0.65]

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Fuzzy Finite Element Analysis for Joints of CFST Beam-Column

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Abstract. The structure of concrete concrete-filled steel tube is a new composite one, the theory about this joints of concrete-filled steel tube beam-column is imperfect, and especially the imperfect dynamic performance of the joints needs much further study. Now the theoretical research about the constitutive relation between the steel and concrete is inconsistent, and so do the yield criterion. According to the mathematical and mechanical model which is classical, the research of experiments can't be done because of the error of the boundary conditions, the size effect, the loading method, and so on. Besides these, the stress which is complex can't be ignored. As the factors exist, we can't learn about the knowledge of the joints which are tested by their reactions in the effect of the dynamical load. Considering these factors, we carry out finite element analysis on the aseismatic behavior of the joints of new type of concrete-filled steel tubes, which is based on fuzzy mathematics, especially the basic principles and methods give important help to us. The results of the example analysis have showed that the finite element calculation model based on fuzzy mathematics is suited for the mechanical properties of the analysis of the joints. Besides that, results of finite element analysis based on fuzzy mathematics are compared with the data of experiment, and they are valuable methods for design of joints of concrete-filled steel tubes beam-column.

Keywords: Fuzzy mathematics, finite element analysis, aseismatic behavior, concrete-filled steel tubes.

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1 Introduction

In Civil Engineering and structure engineering, many parameters are random and fuzzy, such as the materials' elastic modulus and poisson ratio, the geometry size of the structure, the boundary conditions and the loads' affection, etc. The random finite element analysis method can solve the random problem, and the fuzzy finite element analysis method can solve fuzzy processing. If the randomness and the fuzziness of the events are considered at the same time, we can descript the event (or the problem) much more scientifically by fuzzy mathematics, and the results will be more scientificeffective and directive compared with the ones through the presumption. This elastic variational principles will be extended to with the fuzzy parametric, according to the finite element equilibrium equations based on fuzzy mathematics, with the help of the fuzzy equation solution, not only can we solve the linear elastic problems or non-linear problems, but also many various fuzzy factors can be considered in order to solve the problems of finite element analysis without the assumption of the complex mathematical analysis model. By contrast with the computed results, the example which was evaluated by the finite element analysis based on the fuzzy mathematics implied that the program was correct, reliable, and versatility, were not restricted to fuzzy parameters that they belong to what membership function. Not only can we get the change range for displacement of the joints, but also we can get a certain value for the displacement of the membership (if possible), so it can give reference value and guidance to the design of the engineering project.

2 The Factors Affected the Mechanical Behavior of the Joints of CFRT

The factors which affect the mechanical behavior of the joints of CFRT are complex; they can be divided into internal factors and external factors. The internal factors are yield strength of steel, strength of concrete, connection of joints and so on. And the external factors are consist of load conditions, the constraint of column (or beam), etc. In the finite element analysis, the factors such as the constitutive relation between the steel and concrete, the selection of element, plot numbered cell mesh and to analysis, and how to select a reasonable dynamic element, model, displaced pattern and boundary conditions can not be neglected, and their indefinite confine are so fuzzy that lead to difficulty in calculating and analyzing. Because of the uncertain factors exist and normal finite element analysis can not solve them properly, it is restricted for use. The finite element analysis based on fuzzy mathematics can correct the flaw properly. At the same time the finite element analysis based on fuzzy mathematics can also take in the experience and expertise in order to solve the problem, so it is a useful method to make analysis for the mechanical behavior of the joints of CFRT. In the structural designing, we usually make an assumption that the bearings are fixed end bearings. But the fixed end bearings in the actual structure can not be completely solid side and have rotational capability to some extent. Therefore, fuzzy mathematics can be introduced into the method of bearing through the fuzzy processing. The different restraint conditions to the end of concrete filled steel tubes column, then it had different effects on the stress area of the joints of CFRT, so we should consider the factors when we design the structure of concrete filled steel tubes.

3 A Finite Element Analysis Method Based on Fuzzy Structures

L.Z adeh, who is the father of fuzzy mathematics and a Cybernetics expert, on the basis of previous studies on characteristic function, expanded the concept of characteristic function. He made definition of fuzzy set through fuzzy membership function. The absolute subordinating relation between elements and set is extended into different kinds of interdependence. In the set, we can not make sure that whether an element belongs to the fuzzy set or not. But it is only expressed as that it is belonged to this fuzzy set to some extent, which reflects intermediary subordinate relationship. fuzzy logic - Logic Based on the concept of fuzzy sets, in which membership is expressed in varying probabilities or degrees of truth-that is, as a continuum of values ranging from 0 (does not occur) to 1 (definitely occurs) [5] Fuzzy cut and factorization theorem is as follows [5]: Cut set theorem of fuzzy sets A set is category of fuzzy sets of U set

$$\lambda \in [0,1] \tag{1}$$

$$If A_\lambda = \{u \in U, u_A \geq \lambda\} \tag{2}$$

A_λ is called a cut set which is belonged to fuzzy sets of A. So A_λ is a Clearly set. A Resolution Theory of Fuzzy Sets. A set is a fuzzy set, then:

$$A = \bigcup_{\lambda \in [0,1]} (\lambda \cap A_\lambda) \tag{3}$$

The membership function of $\lambda \cap A_\lambda$ is:

$$(\lambda \cap A_\lambda)(\mu) = \begin{cases} \lambda,\ \mu \in A_\lambda \\ 0,\ \mu \notin A_\lambda \end{cases} \tag{4}$$

$\bigcup$ represents union set of fuzzy sets, $\bigcap$ represents intersection set of fuzzy sets (is the intersection of the sets). A and B are two fuzzy sets, so A∩B is also fuzzy sets. Its membership function is

$$\mu_{A \cup B}(u) = max(\mu_A, \mu_B) \tag{5}$$

A∩B is fuzzy sets too. Its membership function is

$$\mu_{A\cap B}(u) = min(\mu_A, \mu_B) \tag{6}$$

It will be seen from this that a resolution theory of fuzzy sets reflects the transformational relation between fuzzy sets and clear sets. Extension Principle of fuzzy theory [3]. F function is generic function, its field of definition is

$$u_1 \in U_1, u_2 \in U_2 ....u_n \in U_n \tag{7}$$

the range of this function is v∈V, Therefore

$$f : u_1 \times u_2 \times ...... \times u_n \longrightarrow v \tag{8}$$

A_1,A_2......$A_n$and B belong to universe of a fuzzy sets U_1, U_2......$U_n$and ; their membership functions are $\mu_{A1}(u_1)$,$\mu_{A2}(u_2)$ ......$\mu_{An}(u_n)$ and$\mu_B(v)$ So the generic function induces map (A map is a rule of correspondence established between sets that associates each element of a set with an element in the same or another set):

$$f : A_1 \times A_2 \times ...... \times A_n \longrightarrow B \tag{9}$$

The membership function of B is

$$\mu_B(v) = \bigcup_{f(u_1,u_2......u_n)} (\bigcap_{i=1}^{n} \mu_{A1}(u_1)) = \mu_{f(A_1,A_2......A_n)}(V) \tag{10}$$

All of the above is the extension principle of fuzzy mathematics.The field of A is a real numbers based on fuzzy sets, its membership function is$\mu_A(u)$,$u \in R$ If the fuzzy sets of A is in accord with the expression: KerA=$\{\frac{u}{\mu_A(\mu)} = 1\} \neq \phi$ When $\lambda \in [0,1]$, the field of A_λ satisfied the expression $A_\lambda = [a_\lambda, b_\lambda,]$. Especially the fuzzy sets of A can meet with the expression SuppA=$\{\frac{u}{\mu_A(\mu)} \neq 0\}$ (it must be a bounded expression). So the fuzzy sets of A can be called F fuzzy sets. From the extension principle and the definition of F fuzzy sets, $f : u_1 \times u_2 \times ...... \times u_n \longrightarrow v$ the bounded numbers $A_1, A_2......A_n$ (which belong to $U_1, U_2......U_n$ can meet with the expression

$$f(A_1, A_2.......A_n)_\lambda = f(A_{1\lambda}, A_{2\lambda}.......A_{n\lambda})_\lambda, \lambda \in [0,1] \tag{11}$$

Therefore the interval algorithm was extended from extension principle: Supposed the closed intervals such as [a,b] and [c,d] belong to field of real numbers, There $[a,b]+[c,d] = [a+c, b+d]$ $[a,b]-[c,d] = [a-c, b-d][a,b] \times [c,d] = [p,q]p = min[ac, bc, ad, bd], q = max[ac, bc, ad, bd][a,b] \div [c,d] = [a,b] \times [1/d, 1/c], c \neq 0, d \neq 0$; So the interval arithmetic is foundation of Finite element equilibrium equations based on Fuzzy mathematics. In structured analysis, the study of boundary condition is necessary. Otherwise the finite element balance equation can not be solved. The fuzzy displacement boundary is due to the boundary support conditions. Boundary conditions style can be used to simulate elastic support. If the constraint displacement is changed, this support may take a force exerted a stop to the displacement of the node. The valuable 0f the force is proportional to displacement. This can be seen as nodes which are connected to springs on a rigid bearing. In the Finite element analysis, the method of dealing with elastic support is as followsthere is a displace variable r_{th}, whose stiffness coefficient is K_s. In order to apply K_s to the r_{th} element in the diagonals which belongs to the sum matrix. As unyielding support can be seen as an elastic support whose stiffness is infinite, the valuable of K_s can from zero to infinite. The fuzzification of displacement boundary come from the fuzzification of K_s. The Fuzzy stiffness coefficient K_s can describe the fuzzy of support conditions.If a proper Exhibition form around or subordination function is chosen for K_s. It will be a given constant C_s. They will establish an equation such as $K_s = C_sE = AC_sE_m$. From that we can know that apply K_s to the element K_{rr} in the diagonals which belongs to the sum matrix, other elements can not be changed in the sum matrix. So when C_s gets a larger valuable,the method can be changed into mass method which belongs to the finite element analysis. The problem of fixed end bearings can be solved.

4 Finite Element Analysis for the Model of Joints of CFST Beam-Column Based on Fuzzy Mathematics

4.1 The Selection and Design for the Model of Joints

In order to assure that the valuable of the finite element analysis for joints is relative property, the model of the fourth literature is selected. The fourth literature analyzed the earthquake-resistant function and mathematical expressions of the pressure and the displacement. The dimension of the specimen is as follows:

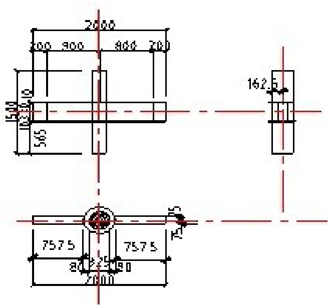

Fig. 1. The Dimensions of Joints

4.2 Finite Element Analysis for the Model of Joints Based on Fuzzy Mathematics

Steel Pipe and Steel use the SOLID 45 element which is supplied the large generally used finite element software ANSYS, elastic modulus E=202GPa Poisson's ratioμ=0.3; concrete (whose strength is C30) use the SOLID 65 element, poisson's ratio μ=0.2,elastic modulus E=24GPa, A three dimension model can be made with the help of elements. As the concrete (which is in the core area of CFST) under the pressure of three-state, its compressive strength and Deformation capacity are improved. The yield criterion of D-P can be used in the Finite element analysis. The finite element models of specimen which is made by ANSYS is as follows:

4.3 The Loading Method of the Finite Element Models of Specimen

First, the axis force (whose valuable is 1800KN) is applied to the end of the column, maintained the valuable at the same levels in the process of Finite element analysis. Then we derived the displacement changing and the connection pressure relation curve, calculated the quadratic multinomial fitting and linear fitting, the valuable of the pressure is 160KN which is applied to the beam end. As follows is the sequence of loading on the beam end. The constraint conditions of column end: At first the valuable ofλ is zero, it is an ideal hinge support bearing, rod ends which is constrained can not move, but can turn; then the valuable ofλ is 0.35 or 0.67, it is an actual fixed end

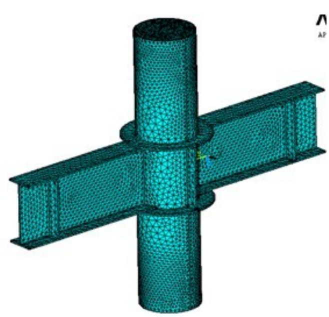

Fig. 2. The Finite Element Models of Specimen

Table 1. The Sequence of Loading on The Beam End

Step	1	2	3	4	5	6	7	8	9	10
Time	1	4	7	11	16	21	27	33	40	45
$\triangle$/mm	3	-6	9	-12	15	-18	21	-24	27	-30

bearing, the rotation angle displacement is free, plastic hinge appear in the structure, it equal to the actual bearing of construction. Third, the valuable of λ is 1, it is an ideal fixed end bearing, its level displacement, the vertical displacement and rotation angle are limited.

4.4 Results Analysis of the Finite Element Models Based on Fuzzy Mathematics

The result of analysis indicates that:

λ=1, it is an ideal fixed end bearing, each end of the column was restricted in all degrees of freedom. The collapse state for joints of CFST beam-column is that the destruction of joints owing to beam yield moment destroyed the whole section. nonlinear computing time was very long.

λ=0.35 or =0.67, it is an actual fixed end bearing,the rotation angle displacement is free, plastic hinge appear in the structure, it equal to the actual bearing of construction. The results of finite element analysis was different to the fourth literature, because in the Experiment The concrete of being crushed by the force which came from the displacement changing and the connection pressure relation curve, then the declined stiffness of the whole

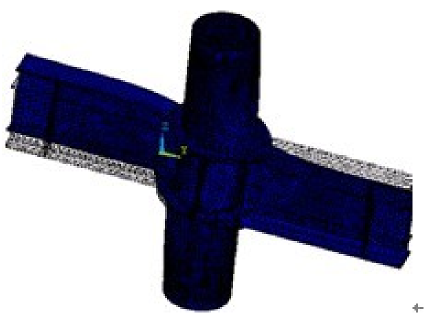

Fig. 3. The Distortion of Joint Zone

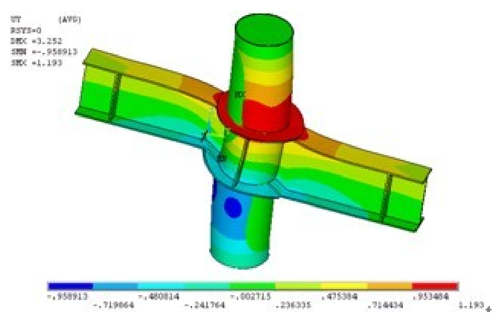

Fig. 4. The Y-stress Distributing of Joint Zone

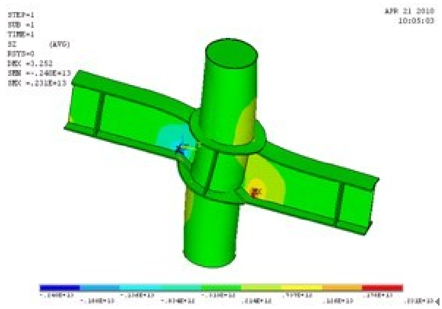

Fig. 5. The Z-component of Stress

concrete, the bondage of steel pipe had been damage. In the finite element analysis based on Fuzzy mathematics, although the concrete is crushed by the force, its intensity could not decline as the constraint of steel pipe was still being there. This situation equaled the Column of the actual situation.

λ=0 it is an ideal fixed end bearing, its level displacement, the vertical displacement and rotation angle are Limited. The failure of the structure was that the wall of steel pipe torsional deformation the phenomenons of the model solid elements cracked were found visibly, then computer can not work any longer. Because the local buckling of steel beam appeared.

5 Conclusions

An integral dynamic finite element model for joints of CFST beam-column is made with ANSYS universal software, Constraints and loaded specimen were simulated based on fuzzy mathematics. The computer calculated the model with the method of finite element analysis. The results show that the joints of CFST were featured with excellent hysteretic behavior, energy dissipation earthquake resistance and ductility. The different valuables of λ (whose method was the finite element analysis based on fuzzy mathematics) solved the problem which the general finite element analysis can not solve, made up the shortage of general finite element method.

(1) The results of the finite element analysis based on mathematics were different from the experiment, the reason was that the Mechanical Properties of welding line could not be simulated by the finite element analysis based on fuzzy mathematics. The wall of the steel pipe and welding line were considered as one material in the process.

(2) Through the practical calculation example, mechanical parameters of material were insensitive to the level displace and the vertical displace. From the fuzzy response to the displace, The fuzziness of the material Mechanical parameters had large influenced on the level and vertical displace, and the level displace was influenced larger. The fuzziness of the material mechanical

parameters can not be neglect ed in the finite element analysis for joints of CFST beam-column.

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The Application of the BP Neural Network in the Nonlinear Optimization

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Abstract. In this paper, a hybrid algorithm is proposed which combines the conjugate gradient theory with BP network algorithm. The algorithm regards the overall average error of neural network as the objective function, and the seeking weights and thresholds of the neural network as the design variables. The calculation process replaces the adjustment of original weights and thresholds of BP network with the conjugate gradient theory. It makes each iteration obtain the optimal step in the search direction, thus it improves the disadvantages of slow convergence of the original BP network. Numerical simulation is applied to the adjustment of weights and thresholds and the calculation process. By using specific examples, convergence rate and effective application are verified. Finally it fully proves the feasibility and superiority of the hybrid algorithm.

Keywords: Optimization, nonlinear, the conjugate gradient theory, BP network.

1 Introduction

In the classic study of optimization theory, the nonlinear method on the basis of Gradient theory is an important kind of method to solve nonlinear optimization problems. In the modern optimization methods, the neural network model is proposed by the use of the organization and working principle of the neural element, and it could effectively solve the classification, prediction and other practical problems through indepth research on the neural network. Combining the neural network with the nonlinear method on the basis of Gradient theory could effectively solve related optimization problems [1]. A lot of sophisticated methods were got after a long period of researching on optimization of nonlinear systems, such

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as the steepest descent method, Newton method, conjugate direction method, conjugate gradient method and so on[2]. They are simple, intuitive, but also have disadvantages in convergence and some other areas. Hopfield successfully applied artificial neural network to combinatorial optimization problems in 1980s. In this paper a new and more effective optimization algorithms is reached by combining conjugate gradient method with neural networks. This method overcomes the shortcomings of conjugate gradient method and neural networks , and has a good value [3]. The calculation process which substitutes Conjugate gradient theory for the original weights and thresholds adjustment of the BP net makes each iteration get the best step in the search direction, and then improves the disadvantage of slow convergence of the original BP net to achieve the high precision calculations of the weights and thresholds. It is not only a good hybrid algorithm which combines the conjugate gradient theory with BP network algorithm, but also verifies the convergence's advantages of uplink fast and the feasibility in practical applications during the running of the final program.

2 The Principle of Hybrid Optimization Algorithm

2.1 The Principle of BP Hybrid Optimization Algorithm

(1) Providing training samples

The form of the training samples is ($x_1, x_2, ..., x_{n_1}; t_1, t_2, ...t_{n_k}$), here $t_1, t_2, ...t_{n_k}$ are expectations that output when we enter $x_1, x_2, ..., x_{n_1}$.

(2) Calculating the actual output

By the use of nonlinear function

$$y_j = [1 - \exp(-\sum_i \omega_{ij} x_i)]^{-1} \tag{1}$$

calculate the output values of the nodes in every layer step by step (exclude the input layer), and set the final output values to $o_1, o_2, ..., o_n$.

(3) Establishing the optimal function

In order to minimize the average error of the network, It will be establish the conditions on a given sample of network weights and threshold value of unconstrained nonlinear optimization problem:

$$\min \ f(z) = \frac{1}{m} \sum_{j=1}^{m} \left\| t_j - y_j \right\|^2 \quad 2)$$

$$z = [z_1, z_2, ..., z_n]^T \quad 3)$$

(4) Setting the number of hidden layers and hidden nodes

Setting the number of all the weight threshold are n, the number of the hidden layers is $k-2$, the number of the nodes in every hidden layer is n_i respectively, including $i = 2,3,...,k-2$, and the number of designed variables should satisfies

$$n = (n_1 + 1)n_2 + \sum_{i=2}^{k-2}(n_i + 1)n_{i+1} + (n_{k-1} + 1)n_k \geq n_k m \tag{4}$$

(5) Calculating weights and thresholds by the use of conjugate gradient algorithm

Set the initial point of the Weight and threshold to W_0, S_0 respectively;

Take $p_0 = -g_0 = -\nabla f(W_0, S_0)$, $k = 0$;

Request λ_k to make that for λ, we have

$f(W_k + \lambda_k P_k, S_k + \lambda_k P_k) \leq f(W_k + \lambda P_k, S_k + \lambda P_k)$;

Calculate $W_{k+1} = W_k + \lambda_k P_k$, $S_{k+1} = S_k + \lambda_k P_k$;

Calculate $g_{k+1} = \nabla f(W_{k+1}, S_{k+1})$, and judge if $\|g_{k+1}\| \leq \varepsilon_1$ is found. If yes, it will stop the Calculation, and output W_{k+2}, S_{k+2}, or else turn to⑥;

⑥ Judge if $k = m$ is found, if yes, then $W_0 = W_{k+1}$, $S_0 = S_{k+1}$, and turn to , or else turn to ;

Calculate $\alpha_k = \dfrac{g_{k+1}^T(g_{k+1} - g_k)}{\|g_k\|_2^2}$;

Calculate $P_{k+1} = -g_{k+1} + \alpha_k P_k$;

Judge if $g_{k+1} P_{k+1} \geq \varepsilon_2$ is found, if yes, then $W_0 = W_{k+1}$, $S_0 = S_{k+1}$, and turn to , or else turn to ⑩;

⑩ Set $k = k + 1$, and turn to .

2.2 Program Flow Chart

As shown in Figure 1

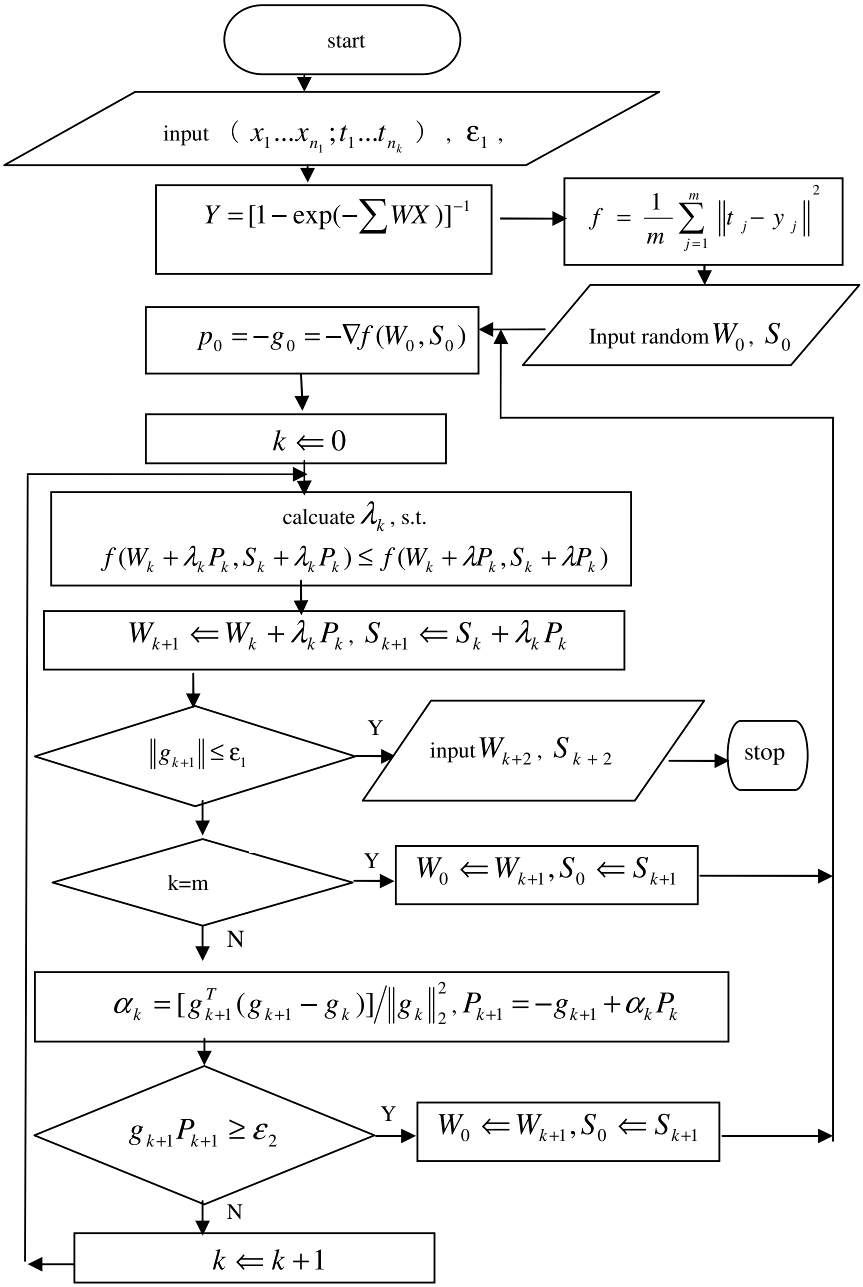
start
input ($x_1 \ldots x_{n_1}; t_1 \ldots t_{n_k}$), ε_1,
$Y = [1 - \exp(-\sum WX)]^{-1}$
$f = \frac{1}{m}\sum_{j=1}^{m} \|t_j - y_j\|^2$
Input random W_0, S_0
$p_0 = -g_0 = -\nabla f(W_0, S_0)$
$k \Leftarrow 0$
calcuate λ_k, s.t.
$f(W_k + \lambda_k P_k, S_k + \lambda_k P_k) \le f(W_k + \lambda P_k, S_k + \lambda P_k)$
$W_{k+1} \Leftarrow W_k + \lambda_k P_k$, $S_{k+1} \Leftarrow S_k + \lambda_k P_k$
$\|g_{k+1}\| \le \varepsilon_1$
Y
input W_{k+2}, S_{k+2}
stop
k=m
Y
$W_0 \Leftarrow W_{k+1}, S_0 \Leftarrow S_{k+1}$
N
$\alpha_k = [g_{k+1}^T (g_{k+1} - g_k)] / \|g_k\|_2^2, P_{k+1} = -g_{k+1} + \alpha_k P_k$
$g_{k+1} P_{k+1} \ge \varepsilon_2$
Y
$W_0 \Leftarrow W_{k+1}, S_0 \Leftarrow S_{k+1}$
N
$k \Leftarrow k+1$

Fig. 1.

3 Numerical Simulation

Select 20 groups of known data as the standard sample to simulate. The standard sample is shown in Table 1.

Table 1. The input and the ideal output of standard sample

sample	x_1	x_2	Ideal output	sample	x_1	x_2	Ideal output
	1	2	1		1	2	1
1	13	9	0.04	11	65	63	0.56
2	13	27	0.08	12	65	81	0.60
3	13	45	0.12	13	91	9	0.64
4	13	63	0.16	14	91	27	0.68
5	13	81	0.20	15	91	45	0.72
6	39	9	0.24	16	91	63	0.76
7	39	27	0.28	17	91	81	0.80
8	39	45	0.32	18	117	9	0.84
9	39	63	0.36	19	117	27	0.88
10	39	81	0.40	20	117	45	0.92

The network has two hidden layers after calculating, and there are three nodes in every hidden layer.

Table 2. The calculated output of standard sample

sample	x_1	x_2	output	sample	x_1	x_2	output
	1	2	1		1	2	1
1	13	9	0.0663	11	65	63	0.5597
2	13	27	0.0850	12	65	81	0.5989
3	13	45	0.1121	13	91	9	0.6369

Table 2. (*continued*)

4	13	63	0.1487		14	91	27	0.6800
5	13	81	0.1935		15	91	45	0.7242
6	39	9	0.2231		16	91	63	0.7683
7	39	27	0.2744		17	91	81	0.8109
8	39	45	0.3246		18	117	9	0.8494
9	39	63	0.3713		19	117	27	0.8841
10	39	81	0.4141		20	117	45	0.9130

After optimization calculation of 601 times, the objective function is a good model which is 0.000 095, however, the accuracy is 0.000 15 after 601 times' iterations by using the BP algorithm. Apparently the results of conjugate gradient optimization algorithm are better than those of the BP algorithm under the same conditions.

4 Conclusion

A hybrid algorithm is proposed which combines the conjugate gradient theory with the algorithm of the BP network, taking the overall average error of the network as the objective function and the weights and thresholds of the neural network as design variables, using the conjugate gradient algorithm to calculate. The simulation of lots of data in Matlab proves that the results are real and effective. Compared with the original BP algorithm for neural network, this method has better convergence rate and convergence results.

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Application of Neural Network in Prediction for Self-compaction Concrete

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Abstract. The different concrete mixture of the self-compaction concrete concluded fly ash has great effect to the compression strength. In order to predict the compression strength of the self-compaction concrete concluded fly ash, the article adopt BP Neural Network to train the system. It shows the hiding neural node is close to precision and it is possible for prediction of the self-compaction concrete with BP network.

Keywords: Fly ash, self-compaction, neural network.

1 Introduction

In 1935 American R.E Davis developed the applied research of fly-ash concrete.From 1948 to 1953 United States Bureau of Reclamation use 10 tons of fly-ash in building the Maba project of Montana. which improved of concrete properties and saved concrete. In 1980s Fly ash has gradually developed into the basic components of concrete. Although Free vibration of concrete [2], which has vibration of high fluidity, the coarse aggregate did not segregate. By using industrial waste such as fly ash, it is favorable to the environment protection, reduced the cost of concrete and Construction, and created remarkable economic benefits and social benefits. This paper ues the BP neuronic network which predicted compressive strength of the Free vibration of fly ash concrete, analyzed the influence for precision of prediction from the Hidden layer of network node.

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2 The Main Raw Materials of Free Vibration of Fly Ash Concrete

(1) cement:

Not only does the good gradation of sand aggregate require, cement particle size distribution is quite important, but actually it is often ignored. General cement pavement of particles under 2 μ m and 50 $\sim$ 200 μ m particle part is insufficient. To ensure the cement have good fillibility and liquidity, and to adjust the size of cement. This requires more fine linen with the mineral admixtures to fill the gap of cement particles.

(2) aggregate grading

The main function of the sand is filling effect, because of the spherical shape for sand, it has certain effect for liquidity. Appropriate chooses fineness modulus of sand (fineness modulus is larger than) aggregate grading of concrete workability is very important.

(3) Mineral fine admixtures

The size of fine Mineral admixtures marked the high-performance concrete from the ordinary concrete, fine Mineral admixtures has become an important component of the high performance concrete. Its intervention, dosage of cement concrete has greatly decreased. Besides that, the particle size of fine mineral admixtures is smaller than an average of cement, such as ash [3] of the particle size distribution in 0.5 300μm range of it, to make up for the other group classification with bad consequences, making concrete stronger. So it also makes the mixing cement particle spread out evenly, weakened the hydration of cement particles formed floe structure.

(4) High efficiency water reducing agent

High efficiency water reducing agent is indispensable constituent materials of high performance concrete. For the avoided vibration concrete, the ideal of self-compacting concrete admixture is efficient air-entraining high efficiency water reducing agent, because it has two functions: water decreased and bleed air. Its main function is to reduce water super plasticizer which adsorbed on the cement particles to the surface of the cemen, electric double layer structure is formed on the surface, giving full play to electrostatic repulsion.

3 Neural Network Theory

Neural network is a highly nonlinear large-scale continuous-time dynamic systems. By a large number of processing elements (neurons) and a wide range of interconnection networks. It is in modern neuroscience research on the basis of results, reflecting the basic characteristics of brain function.

Back-propagation network (BP Network)is used Widrow-Hoff learning algorithm can be micro-and non-linear transfer function of multi-layer network. A typical BP network is based on the gradient descent algorithm, which is

Widrow-Hoff algorithm requirements. Backpropagation means for the non-linear multi-network computing gradient method. There are many basic optimization algorithms, such as variable scaling algorithm and Newton algorithm. A network of trained BP is appropriate according to the results of input, although this has not been imported and trained. This feature makes BP network is suitable for the importation of used goals of the training, but does not require all possible input / objectives are trained.

BP neural network from the input layer, hidden layer (a number) and the output layer composition, each floor will have a number of neurons. Signal output, the first spread hidden nodes, the role of function in the dissemination of the output layer node, the output of processed. Node role function normally use S-type function, namely,

$$f(x) = \frac{1}{1+e^{-x}} \tag{1}$$

And f (x) —— role function;x —— input function;

BP network input-output relationship is a highly non-linear mapping, if the output layers of n neurons, the output layer m neurons, the network-from the n-Dimensional space to the space-m-Dimensional Mapping. By adjusting the weights of network connectivity, network size can be realized nonlinear classification, prediction, and can be arbitrary precision how nonlinear function approximation

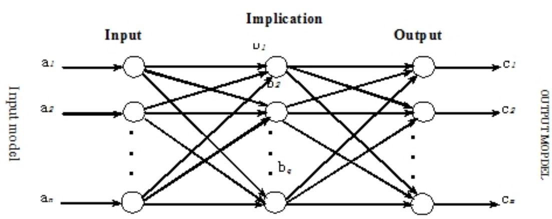

Fig. 1. BP Neural Network Structure

4 BP Network with Vibration-Free Concrete of Fly Ash

Uniform design was taken as the main way. five factors (fly ash, binder, water-cement ratio, sand ratio, superplasticizer dosage)is selected, arranged 12 group test, which is 5 elements and 12 levels in the uniform design.

Remarks:CM represents cementitious material,SP represents super plasticizer,28d CCS represents28d Cube compressive strength; "*" is the factors for selected.

Table 1. Please write your table caption here

Number	* CM	Cement	*Flyash	* Sand	Carpolite	*Water	* SP	28d CCS
1	500	290	210	817.67	834.02	185	7.750	49.56
2	505	242.4	262.6	760.87	875	184.33	8.838	51.30
3	510	336.6	173.4	774.49	838.78	201.45	9.945	55.86
4	515	329.6	185.4	756.74	906.58	180.25	8.498	59.09
5	520	270.4	249.6	799.6	783.61	200.2	9.620	49.37
6	525	367.5	157.5	757.99	854.75	199.5	7.613	45.38
7	530	296.8	233.2	773.17	854.35	182.85	10.600	45.73
8	535	288.9	246.1	707.16	830.2	214	8.560	43.57
9	540	367.2	172.8	803.06	803.06	194.4	9.720	49.02
10	545	272.5	272.5	761.87	809.1	193.48	8.175	33.80
11	550	330 220	699.26	854.49	206.25	10.450	45.70	
12	555	344.1	210.9	746.47	776.92	216.45	9.435	47.69

Table 2. The Effect of Hiding Node

Number	Hidden nodes	Training timeS	Training times	Unbiased variance MSE
1	15	341.698157	2636	5.19759
2	16	1.558859	126	4.21332e-007
3	17	1.510148	110	0.00019952
4	18	0.611692	23	0.000199191
5	19	6.090677	480	1.44438e-006
6	20	0.854166	33	1.28396e-005
7	21	1.198189	54	1.61245e-005
8	22	5.971896	377	9.84118e-007
9	23	5.928716	348	9.82206e-006
10	24	20.069781	923	1.30926e-007
11	25	39.667012	2147	3.83574e-005
12	27	72.290095	2972	0.00019961
13	30	23.513026	288	9.31619e-006
14	35	5.577954	73	1.87623e-006
15	40	16.040270	99	1.65773e-006
16	45	49.292109	347	0.00017357

Neural network toolbox of MATLAB is used to establish three BP neural network. Input layer seven nodes, representing the cementing material, cement, fly ash, sand, gravel, water, water-reducing agent. Output one node, on behalf of compressive strength. For the BP network hidden layer nodes is uncertain, wooden structure with three layers of T3P paper network. That is 7 - Hn-1 structure.But the number of hidden Hn single members remains unknown. In the experiment the number of hidden layers of neural cells as a parameter will be tested. In this kind of the hidden layer of wood, which seted the number of neurons using the process of computing can achieve the highest accuracy.

Network parameter settings are as follows:

net.trainParam.show=50;
net.trainParam.lr=0.01;
net.trainParam.epochs=3000;
net.trainParam.goal=0.0002;

From the results when the hidden nodes was 24 the error of mean square was smallest, and the accuracy was highest. The figure below shows the results from matlab:

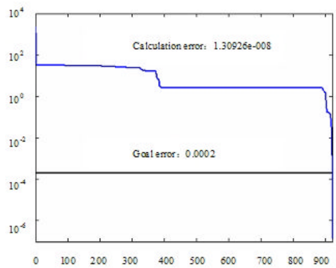

Fig. 2. The result of computing

5 Conclusion

(1) High volume of fly ash self-compaction concrete whose early strength is low, but later the growth rate increased insensately, compared with 28 days to 56 days compressive strength increased by 23.52.
(2) Neural network method was used to predict the compressive strength of fly ash free vibration of concrete, results showed that its prediction using BP network is feasible.

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Pneumonia Incidence Rate Predictive Model of Nonlinear Time Series Based on Dynamic Learning Rate BP Neural Network

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Abstract. Objective to explore predictive method of nonlinear time series based on using BP neural network. Methods Based on dynamic learning rate BP artificial neural network with Hyperbolic Tangent function as activation function has been used. Results Build two kinds of ANN forecast models of pneumonia incidence rate. They are better than traditional method on prediction precision. Conclusion BP artificial neural network can be used to forecast for disease incidence rate.

Keywords: BP artificial neural network, nonlinear time series, dynamic learning rate, incidence rate, pneumonia.

1 Introduction

Pneumonia is the acute lung parenchyma infectious lesions including alveolar space and interstitial tissue. According to the extent of disease, it can be divided into lobar pneumonia, lung segment or lobular pneumonia, bronchial pneumonia and interstitial pneumonia. According to the etiology, it can also be divided into virus, mycoplasma, bacteria, fungi, etc. The common cause of adults is the bacterial infection, such as streptococcus pneumonia, anaerobic bacteria, staphylococcus aureus, etc. And mycoplasma pneumonia is the common infection cause of older children and young people. The primary pneumonia pathogen of infants and children is the virus, including respiratory syncytial virus, adenovirus, etc. This disease belongs to the "air temperature cough" of traditional Chinese medicine.

At present, there are many quantitative methods for predicting, but basically can be summarized into time relation, structural relation and casual model [1]. Generally speaking, the requested object should satisfy the prerequisite of the forecast model when using time relation model and construal relation model, otherwise, the predictive value will be not reliable; the casual model is actually the mapping between cause and result, almost can express all the nonlinear relationship(such as artificial neural network) and has a wider scope of application than the above two types of function. The development of the theory of artificial

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neural network is based on the theory of computer, informatics, biology, electronics, physics, medicine, mathematics and philosophy. And its development brings a broad prospect in the field of forecasting. Although ANN technology has been gradually used in biological, medicine, pharmacy, chemistry and other disciplines in recent years, ANN technology is primarily used to classify or static analysis [2-4]. This paper used the error back propagation (Back Propagation, BP) neural network to predict the incidence rate of pneumonia in Haixizhou region, Qinghai province, China, and discussed the prospects of application of ANN forecast model on disease incidence rate.

2 Material and Methods

2.1 Data Source

We selected the pneumonia incidence rate of Haixizhou region, Qinghai province, China as a research object. The whole incidence rate data of pneumonia derived from the Haixizhou First People's Hospital, through the check and leak repairing, ensured the accurate and complete. The data of pneumonia incidence rate from January 2003 to December 2009 has been analyzed and counted by the software of Excel2003and Eviews3.1, and obtained 84 groups of observation data.

2.2 Principle of Operation about Three Layers BP Neural Network Model [5-7]

2.2.1 BP Neural Network Model

BP artificial neural network model is one of the most widely used neural networks. The type of attachment of the neuronal is the feed-forward neural network, and the learning is the supervised learning. It works as follows: when the input signal propagated from the input layer to the output layer through the middle layer (hidden layer), the training and learning process of network is beginning. The process included positive propagation and back propagation, first positive propagation, and transferred to back propagation process when the error between the output response and the desired output greater than the threshold. At same time, revising the connection weights of each layer based on the error, and then entering the forward propagation process. If the error between the output response and the desired output mode still greater than the threshold, transferred to the back propagation and revised the connection weights of each layer until the error is less than the threshold. BP learning algorithm essentially is the promotion of least mean square (LMS).

BP algorithm of back propagation neural network can be described as follows:

Figure 1 show the three layers BP network, one of the important features of the response function S is the derivative of S function can be expressed by it. If

$$f(x) = 1/(1+e^{-x}) \tag{1}$$

Then

$$f^{'}(x) = f(x)[1-f(x)] \tag{2}$$

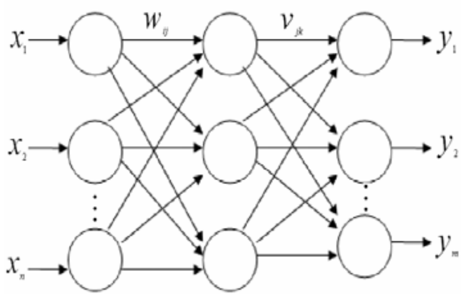

Fig. 1. Structure of three BP network

1) Initialization. Given the random value of the intervals $(-1,+1)$ to each connection weights $\{w_{ij}\}$, $\{v_{jk}\}$ and threshold $\{\theta_j\}$, $\{\gamma_k\}$;

2) Randomly selected a model $X_n=(x_1,x_2,\cdots,x_n)$, $Y_m=(y_1,y_2,\cdots,y_m)$ and provided to the network;

3) Input the "propagation along process" of learning mode. Using the input mode $X_n=(x_1,x_2,\cdots,x_n)$, connection weights $\{w_{ij}\}$ and thresholds $\{\theta_j\}$ to calculate the unit input $\{S_j\}$ of the middle layer; then using $\{S_j\}$ to calculate the unit output $\{b_j\}$ through S function:

$$S_j=\sum_{i=1}^{n} w_{ij}\bullet x_i+\theta_j \quad j=1,2,\cdots,p \tag{3}$$

$$b_j=f(S_j) \quad j=1,2,\cdots,p \tag{4}$$

4) Using the output $\{b_j\}$ of the middle layer, connection weights $\{v_{jk}\}$ and thresholds $\{\gamma_k\}$ to calculate the unit input $\{L_k\}$ of the output layer, then using $\{L_k\}$ to calculate the unit responses $\{C_k\}$ of the output layer through S function:

$$L_k=\sum_{j=1}^{p} v_{jk}\bullet b_j-\gamma_k \quad k=1,2,\cdots,m \tag{5}$$

$$C_k=f(L_k) \quad k=1,2,\cdots,m \tag{6}$$

5) Using the desired output mode $Y=(y_1^l,y_2^l,\cdots,y_m^l)$, actual output $\{C_k^l\}$ of the network to calculate the generalization error $\{d_k^l\}$ of the output layer:

$$d_k^l=(y_k^l-C_k)\bullet C_k(1-C_k^l) \quad k=1,2,\cdots,m \tag{7}$$

6) Using the connection weights $\{v_{jk}\}$, generalization error $\{d_k^l\}$ of the output layer and unit output $\{b_j\}$ of the middle layer to calculate the generalization error $\{e_j^l\}$ of the middle layer:

$$e_j^l = \left[\sum_{k=1}^{m} d_k^l v_{jk}\right] \bullet b_j(1-b_j) \quad j=1,2,\cdots,p \tag{8}$$

7) "Back propagation process" of the network error. Using the generalization error $\{d_k^l\}$ of the output layer, output $\{b_j\}$ of the middle layer, correction connection weights $\{v_{jk}\}$ and thresholds $\{\gamma_k\}$:

$$v_{jk}(n+1) = v_{jk}(n) + \alpha \bullet d_k^l \bullet b_j \quad j=1,2,\cdots,p;\ k=1,2,\cdots,m;\ 0<\alpha<1 \tag{9}$$

$$\gamma_k(n+1) = \gamma_k(n) + \alpha \bullet d_k^l \quad k=1,2,\cdots,m \tag{10}$$

Using the generalization error $\{e_j^l\}$ of the middle layer, unit output $X_n = (x_1^l, x_2^l, \cdots, x_n^l)$ of the input layer, correction connection weights $\{w_{ij}\}$ and thresholds $\{\theta_j\}$:

$$w_{ij}(n+1) = w_{ij}(n) + \beta \bullet e_j^l \bullet x_i^k \quad i=1,2,\cdots,n;\ j=1,2,\cdots,p \tag{11}$$

$$\theta_j(n+1) = \theta_j(n) + \beta \bullet e_j^l \quad j=1,2,\cdots,p \tag{12}$$

8) Training process. Randomly selected the next learning mode to provide to the network and returned to step 3 until all the mode of q have been trained;

9) Convergence process. Randomly selected the mode from the learning mode of q and returned to step 3 until the global error E of the network less than the minimum pre-set value or the number of learning greater than the pre-set value;

10) End of the learning.

2.2.2 BP Algorithm with Dynamic Learning Rate

From the view of mathematical point, BP algorithm is a nonlinear gradient optimization problem, so inevitable exist the problem of local minimum. Learning algorithm converges slowly; typically need thousands of iterations or more. Therefore, we use the BP algorithm with dynamic learning rate.

In the BP algorithm, changeable rule of connection weights can be obtained from (10) and (12):

$$\Delta\gamma_k = \alpha \bullet d_k^l \quad k=1,2,\cdots,m \tag{13}$$

$$\Delta\theta_j = \beta \bullet e_j^l \quad j = 1, 2, \cdots, p \tag{14}$$

Where α , β are the defined weights step, the learning rate and the following are expressed with η .

The choice η of is very important. When η is smaller, the learning speed of the network not only very slowly, but also can easily trap in the local minimum point; when η is larger, it can easily appear network oscillation and error E will not reach the minimum. Therefore, the learning algorithm of the BP network should make the following improvements: given η a relatively large initial value to make the network approximate to the extreme points with larger step when the error of the initial learning is larger, at the same jump through the local minimum. Changing the learning factor η based on the change of the learning error along with the learning of the network. If the number of the learning is N , then

$$\eta = \begin{cases} (1 - 0.05)\eta & E_{N+1} \le E_N \\ (1 + 0.05)\eta & E_{N+1} > E_N \end{cases} \tag{15}$$

2.2.3 Nonlinear Time Series Forecasting Method Based on BP Network

1) Model. If time series is $\{x_t \mid t = 1, 2, \cdots, n\}$, training data can be divided into K paragraphs when using the past values of the moment of N $(N \ge 1)$ to predict the future values of the moment of $M\,(M \ge 1)$, the length of the data is $N + M$ with a certain overlap. Set each section of the previous data of N as the input of the network, later data of M as the output of the network (Table 1).

Table 1. Segmentation methods of training data

Input(N)	Output(M)
$x_1, x_2, \cdots, x_N$	$x_{N+1}, x_{N+2}, \cdots, x_{N+M}$
$x_2, x_3, \cdots, x_{N+1}$	$x_{N+2}, x_{N+3}, \cdots, x_{N+M+1}$
………	………
$x_k, x_{k+1}, \cdots, x_{N+k-}$	$x_{N+k}, x_{N+k+1}, \cdots, x_{N+k+N-}$

Then using the aforementioned BP neural network to train and learn the network, and looking for a function relationship from R^N to R^M . The network has N $(N \ge 1)$ nodes of the input layer, $M\,(M \ge 1)$ nodes of the output layer. When using the traditional time series $AR(p)$ to forecast, the general form of the predictive value is:

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + \cdots + \phi_p x_{t-p} + \varepsilon_t \tag{16}$$

Where ε_t is the white noise; $\phi_i (i = 1, 2, \cdots, p)$ are the constant coefficients.

For the same prediction problem, according to the antecedent neural network to forecast, the general form of the predictive value is:

$$x_t = f_1(x)x_{t-1} + f_2(x)x_{t-2} + \cdots + f_p(x)x_{t-p} + \varepsilon_t \tag{17}$$

where p is the input nodes of the predictive network; $f_i(x) = f_i(x_{t-1}, x_{t-2}, \cdots, x_{t-p}, \varepsilon_t)$, $i = 1, 2, \cdots, p$ are the nonlinear function with independent variable which is the input variable of the predictive network. The estimated value of x_t is:

$$\hat{x}_t = f_1(x)x_{t-1} + f_2(x)x_{t-2} + \cdots + f_p(x)x_{t-p} \tag{18}$$

The prediction error is:

$$\sigma^2 = \sum (\hat{x}_t - x_t)^2 \tag{19}$$

The training process of the neural network is the process to make σ^2 to reach the global minimum. From the view of the statistical point, the final result of the neural network modeling is to obtain the nonlinear function $f_i(x)$. Function $f_i(x)$ is expressed by the connection weights and thresholds of the nodes in network.

2) Determination of the model parameters. When using neural network to establish the prediction model of pneumonia incidence rate, we should determine the model parameters of the neural network in the training. Model parameters include the model number of the layer, hidden nodes and the performance function. But there is no unified approach to determine the model layer [8-9]. This study compared the ANN with two hidden layer and ANN with one layer by testing, and discovering that the former easily falling to the local minimum, and the error will oscillate in the local minimum. In the same study with the same training number, they both achieved the similar error, so we selected the three BP neural network model. The number of the nodes including the number of the input nodes, hidden nodes and output nodes, the input layers nodes is generally the number of the input variables, the output layer nodes is the month incidence rate of pneumonia in a epidemic years. It is difficult to determine the number of the hidden layer nodes. If the hidden nodes is too small, the network may not train; and if the network nodes too many, the network training may be endless, in addition to hard to converge, it can also lead to establish “ancestors” network. In this study, the nodes number of the hidden layer is selected based on theory of LUB (least upper bound) [10]. Through abundant testing, we selected the nodes number of the hidden layer is 9. Determination of the performance function: performance function of the BP neural network requires differentiable everywhere and convergence. In order to determine the appropriate performance function, we analyzed and compared the sigmoid function $f_1(x) = 1/(1 + e^{-x})$ and hyperbolic

tangent function $f_2(x) = (e^x - e^{-x}) / (e^x + e^{-x})$ respectively. At last, we determined the hyperbolic tangent function as the performance function of the neural network.

3 Results and Analysis

The data has been analyzed by software Evies3.1 and matlab7.0 to. According to the month incidence rate of pneumonia time analysis to determine the epidemic cycle and divide popularity. Set the each year in September to the next august as a popular annual, during 2003-2009 there are six popular annuals.

3.1 ANN Prediction Model of Pneumonia Incidence Rate

3.1.1 Establishment of the ANN Prediction Model I of Pneumonia Incidence Rate

Set the month incidence rate as the predictors, and using pneumonia month incidence rate of a popular annual to predict the month incidence rate of the next popular annual. The model structure is 8-9-4.

1) Stage of network training and learning. Set the month incidence rate during 2003.9-2008.8 as a information of the network training and learning phrase. Application of the forecast theory of the BP neural network, and established the prediction model on pneumonia month incidence rate, according to the month incidence rate of a disease popular annual during 2003-2009 to subparagraph as follows: set the incidence rate of each month (12 months) during 2003.9-2004.8 as the first input of the samples, incidence rate of each month (12 months) during 2004.9-2005.8 as the first output of the samples; set the incidence rate of each month (12 months) during 2003.10-2004.9 as the second input of the samples, incidence rate of each month (12 months) during 2004.10-2005.9 as the second output of the samples; and so on, until the incidence rate of each month during 2007.9-2008.8 as the last input of the samples, incidence rate of each month during 2008.9-2009.8 as the last output of the samples. There are a total of 17 pairs of input samples.

2) Prediction stage. Set the incidence rate of each month during 2003.9-2009.8 as the data for the prediction stage of the network. After ANN network training completed, the month incidence rate of the next popular annual can be predicted according to the actual month incidence rate of a popular annual. For example, set the prediction step $K = 12$, when the input of the sample is the month incidence rate during 2003.9-2004.8, we can obtain the predictive value of the actual incidence rate during 2004.9-2005.8; when the input of the sample is the month incidence rate during 2004.9-2005.8, we can obtain the predictive value of the actual incidence rate during 2005.9-2006.8, and so on, until obtain the last predictive value during 2008.9-2009.8. The predictive value curve and actual value of the incidence rate of 5 popular annuals during 2004.9-2009.8 are shown in Figure 2.

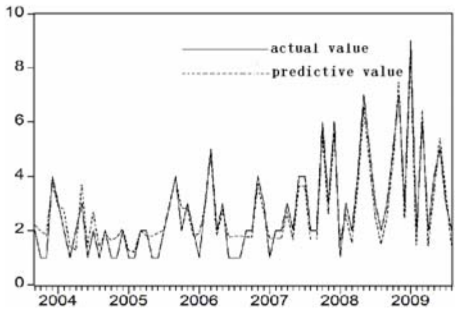

Fig. 2. Predictive curves of pneumonia incidence rate on popular annual by ANN model I

3.1.2 Establishment of the ANN Prediction Model II of Pneumonia Incidence Rate

With the dynamic learning rate prediction model, set the month incidence rate as the predictors. However, the input sample is different from the above model. Supposed to predict the month incidence rate of the popular annual during 2008.9-2009.8, the input sample can be 2007.9-2008.9 and 2007.8-2008.7 two segments, or 2007.9-2008.8, 2007.8-2008.7 and 2007.7-2008.6 three segments, or 2007.9-2008.8, 2007.8-2008.7, 2007.7-2008.6 and 2007.6-2008.5 four segments, ……,up to twelve segments of the incidence rate. After testing and screening, when the input sample divided into four segments, the prediction outcome is the optimization, the structure of the model is 4-9-4, showing in Figure 3.

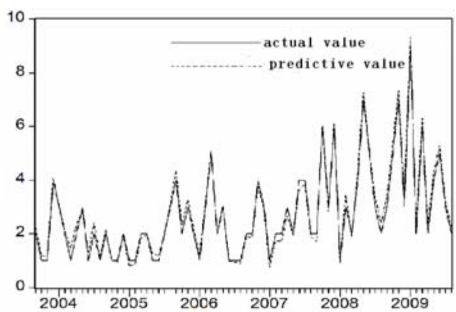

Fig. 3. Pneumonia incidence rate prediction curve of the ANN model II

As we can see from the figure, ANN prediction model II predicted the incidence rate of pneumonia with satisfactory effect, but the learning effect to the singular value (outliers) is not very good.

3.2 Prediction Accuracy Comparing between ANN Model and $AR(p)$ Model

Pneumonia month incidence rate of popular annual during 2003.9-2009.8 in Haixizhou region, Qinghai province, China, has been testability predicted by $AR(p)$ model, the results is shown in Figure 4, calculated the absolute relative error between the predictive value and actual value of the ANN model I, ANN model II and $AR(p)$ model respectively, the results is shown in Table 2, which shows that the prediction accuracy of the ANN model II is higher than $AR(p)$ model, and between ANN models, the average relative error of the ANN model II is minimum.

Table 2. Relative error of the predictive value

Types of models	Average value	Maximum	Minimum
ANN model I	18.28507	0.984963	0.007518
ANN model II	4.895034	0.397153	0.001072
AR(p) model	6.821422	0.44055	0.001802

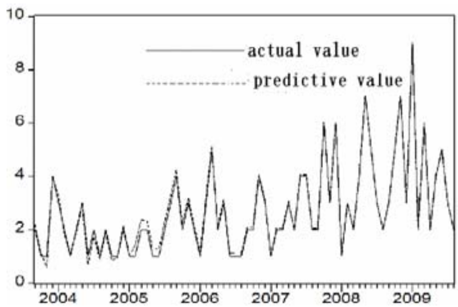

Fig. 4. Pneumonia incidence rate prediction curve of AR (p) model

4 Discussion

In the medical field, there are many nonlinear forecasting and decision-making issues to be resolved. Therefore, it is necessary to evaluate the artificial neural network in order to better applying it.

4.1 Comparing between Neural Network Model and Time Series Model

4.1.1 Two Models Required Different Types of Information

Most traditional time series model, Such as the Box-Jenkins [11], all assumed that there is a linear relationship between variables. Although people has already advanced some nonlinear time series models [12], these methods are model-driven approach, that is first identified the relationship between the data and then estimated the model parameters. The neural network models are data-driven approach, which can reveal the nonlinear relationship implied in Data samples, and can approximate any function with arbitrary accuracy. The form of the function for the considered system expressed more complex, the role of the feature of ANN model more obvious.

4.1.2 Two Models Predict Different Data with Different Effects

Many studies has been done to compare the prediction effects of two types models, however, the conclusion is not the same. When predicting monthly data and seasonal data, the prediction accuracy of ANN model is higher than the traditional time series model [13]. The result of this study has confirmed this conclusion. But to the annual data, the prediction effect of the two models has big difference. The study results of Foster, etc, showed that ANN model is inferior to time series model with the least square estimate. Foster and others, results show that ANN as the least square estimate of the time series model. And A. Lapedes and R. Farber's findings show that the ANN model has a higher accuracy than the traditional time series model [14]. The result of this study suggests that these differences might be related to the different network structures researchers using.

4.1.3 Two Models with Different Synchronized Long Generate Different Prediction Effects

Z. tang, etc, compared ANN model and Box-Jenkins model with different time series data [15], the results showed that toward to short-term forecast, Box-Jenkins model is superior than ANN model; toward to long-term forecast, Neural network model is better than Box-Jenkins model. This study was conducted multi-step (12 steps) forecast; the results also showed that ANN model is better than $AR(p)$ model of Box-Jenkins model.

4.2 Key Problems Should Be Solved When Using Artificial Neural Network to Forecast Disease

Using neural network model to study the prediction problems, one of the big difficulty is how to determine the structure of the network that is how to determine the number of nodes in the hidden layer. When there are too few hidden nodes, forecast accuracy can not be guaranteed; when there are too many nodes, Network in the learning and training can easily fall into the local minimum. Therefore, without the premise of reducing the network performance, selecting one of the best network structures is the key of the network design, and this is a topic worthy of further study.

4.3 Related Recommendations about the Prevention and Control of Pneumonia

The data in this paper are the pneumonia incidence rate in Hixizhou region, Qinghai province, China, in recent 7 years. Haixizhou lies in northwest plateau region of China; it is also the area of minority aggregation. It has a great significance in making the reasonable incidence rate of pneumonia in this region. Although the economic and cultural is rapid developed, and the sanitary condition is improved, related departments played insufficient attention to this disease, the incidence rate of pneumonia has increased year by year., which reminded the health department to strengthen the prevention and control of this disease[16-18].

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Optimizing Single Depot Heterogeneous Fleet Vehicle Routing Problem by Improved Genetic Algorithm

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Abstract. Firstly, the paper establishes a mathematical model for single depot and heterogeneous fleet vehicle routing problem (SHVRP) according to the actual situation of Zhengzhou Coal Electricity Material Supply and Marketing Limited Company in China, then based on the model, uses improved genetic algorithm(IGA) to optimize the vehicle routing problem (VRP) of Zhengzhou Coal Electricity Material Supply and Marketing Limited Company, finally by comparing the performance of IGA with classical heuristics algorithm (CHA) and sweeping algorithm(SA) in transportation cost, the number of used vehicle and computing time, the results show that CHA obtains the best objective function value, SA takes the second place, and CHA is the poorest; however, from the number of used vehicles, the optimum solution of CHA uses the least vehicles , followed by SA and IGA; but CHA is most efficient on computing time, the time needed for calculation is only two fifth of that of SA, two twenty five of that of IGA.

Keywords: vehicle routing problem, heterogeneous fleet, genetic algorithm, sweeping algorithm.

1 Introduction

Vehicle routing problem is a hot research topic in the field of operational research and combinatorial optimization. Generally speaking, as the various kinds of goods and different requirements of customers, distribution companies often equip with different types of distribution vehicles to increase the rate of full load and reduce the transportation cost. Heterogeneous fleet vehicle routing problem is a branch of vehicle routing problem.In 1984 , Golden[1] published the article "The fleet size and mix vehicle routing problem" and made the first research on the heterogeneous fleet vehicle

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routing problem, (HVRP), Gendreau[2]studied the HVRP problem which the number of each type of vehicle is infinite by using the tabu search .Li[3] studied the HVRP problem which the number of each type of vehicle is limited by using the record-to-record travel algorithm . Zhong Shiquan and He Guoguang [4]used the improved tabu search algorithm to solve the HVRP problem with the time-window of the heterogeneous depot ,Shi Hongbo and Lang Maoxiang[5]designed a new solution expression and solved the problem by the tabu search algorithm. Li Bing[6]established a linear programming model based on the description of heterogeneous fleet certain- dynamic vehicle routing Problem; Ye Zhijian[7]summarized five kinds of current scientific algorithms to solve the HVRP problem, and proposed a heuristic algorithm which combine tabu search algorithm with great journey algorithm. Jia Lishuang and Li jing[8]combined the genetic algorithms with the neighboring algorithm to solve the problem HVRP; Yang Yuanfeng[9]combined simulated annealing algorithm with genetic algorithm to solve the heterogeneous depot with the restriction of time-window.Li Jian and Zhang Yong[10]adopted the direct arrangement coding for client to solve the HVRP problem with hard time window.

Above mentioned methods have established a solid foundation for solving the HVRP problem, but there are also some places need to be improved such as: (1) The most algorithms are so complex in encoding and decoding which lead to reduce the computing efficiency. (2) When using the genetic algorithm to solve the HVRP, chromosome encoding adopts binary and decimal intertwined, which is so complex that not execute to chromosome crossover and mutation.(3) In the vehicle allocation strategy, actual experience is the first consideration to choose large or small load vehicle. Although large vehicle can save the distance but cannot guarantee the rate of full load, the unit cost of small vehicle is higher than that of large vehicle. Therefore it is very difficult to guarantee high quality solutions to decide the priority of vehicles distribution only depend on the size of vehicle. (4) Some research is based on the strategy that meet the minimum cost vehicle, but this does not suitable for the situation that the cost of vehicle in some company is unknown (or cannot be calculated directly).

This paper firstly combined with the specific situation of Zhengzhou Coal Electricity Material Supply and Marketing Limited Company in China and established a appropriate model for single depot and heterogeneous fleet vehicle routing problem (SHVRP). Then proposed the improved genetic algorithm(IGA) which made a multiple improvement for the standard genetic algorithm in many aspects such as chromosome encoding, initial population construction, selection operator, crossover operator ,as well as some parameters design. Finally, used IGA to solve the SHVRP of the company and analysis the results compared with the classical heuristics algorithm (CHA)[4] and sweeping algorithm (SA)[5].

2 The Model of Single Depot Heterogeneous Fleet Vehicle Routing Problem

According to the actual situation of Zhengzhou Coal Electricity Material Supply and Marketing Limited Company, there is a depot (Xin mi city , He nan province, in China), the depot owns L kinds of vehicle, the load of each type of vehicle is $Q_l(l \in L)$,and the number of this type of vehicles is $K_l(l \in L)$, use at most $K = \sum_{l=1}^{L} K_l$ vehicle to distribution goods from the central warehouse (or distribution center) to N points of goods demand. In order to obtain the lowest cost this paper considers how to arrange for the vehicle transportation routes. Some constraint conditions are that each route can not beyond the load of the vehicles, the demand of each requirement point need and only need one car to meet, the vehicle start from the central warehouse and returned to the central warehouse.

The demand for goods of i demand point is $g_i(i = 1, 2, \cdots, N)$,at most use K vehicles to distribution goods from the central warehouse (or distribution center) to these demand points. The distance between each demand points is shown in table 1 and the demand of each demand points is shown in table 2 (the demand should be report in advance by the delivery member of each requirement points), the transportation distance from mine i to mine j is d_j,0 represent the central warehouse ,the model of the problem is as follows:

$$\min \text{ Z} = \sum_{l=1}^{L}\sum_{k=1}^{K_l}(\sum_{i=0}^{N}\sum_{j=0}^{N} c_{ij}^{l} x_{ij}^{lk}) \tag{1}$$

S.T.

$$\sum_{j=0}^{N}\sum_{k=1}^{K_l}\sum_{l=1}^{L} x_{ij}^{lk} \le \sum_{l=1}^{L} K_l, \quad l \in \{1, 2, \cdots, L\} \tag{2}$$

$$\sum_{j=1}^{N} x_{0j}^{lk} = \sum_{i=1}^{N} x_{j0}^{lk} \le 1, \quad l \in \{1, 2, \cdots, L\} , k \in \{1, 2, \cdots, K_l\} \tag{3}$$

$$\sum_{i=0}^{N}\sum_{l=1}^{L}\sum_{k=1}^{K_l} x_{ij}^{lk} = 1, \quad i \in \{1, 2, \cdots, N\} \tag{4}$$

$$\sum_{i=0}^{N}\sum_{l=1}^{L}\sum_{k=1}^{K_l} x_{ij}^{lk} = 1, \quad j \in \{1, 2, \cdots, N\} \tag{5}$$

$$\sum_{i=0}^{N} g_i \sum_{j=0}^{N} x_{ij}^{lk} \le Q_l, \quad l \in \{1, 2, \cdots, L\} , k \in \{1, 2, \cdots, K_l\} \tag{6}$$

$$x_{i0}^{lk} = x_{0i}^{lk} = 0, \quad l \in \{1, 2, \cdots, L\} , k \in \{1, 2, \cdots, K_l\} \tag{7}$$

$$c_{ij}^{l} = d_{ij} c_l r_{ij}^{lk}, \ i, j \in \{0, 1, 2, \cdots, N\}, l \in \{1, 2, \cdots , L\}, k \in \{1, 2, \cdots, K_l\} \tag{8}$$

Equation (1) indicates the objective function that is the total distribution cost is minimum, in the SHVRP problem of Zhengzhou Coal Electricity Material Supply and Marketing Limited Company , excluding the fixed costs, only calculate variable costs , and the variable cost of each vehicle is a fixed constant, that is, $c_l = 0.53$ yuan / ton $\cdot$ km.r_{ij}^{lk} represent when l types of vehicle driving from i to j ,the load of vehicle at this time.

Equation (2) indicates that the number of vehicles send from the central warehouse can not exceed all the number of vehicles.

Equation (3) guarantees that the vehicles are starting from a central warehouse, and finally back to the central warehouse.

Equation (4), (5) ensure that each demand point of goods can only be served one time by one vehicle

Equation (6) indicates that the capacity of the carrying vehicles and shall not be greater than the capacity of the vehicle.

Equation (7) indicates that the vehicle cannot be transported from the central warehouse to the central warehouse.

In equation (8) c_l is the unit cost of l types of vehicle, when $c_l = 1$, c_{ij}^l shows the distance cost.

3 The Description of the Improved Genetic Algorithm

3.1 Coding of Chromosome

This paper proposed a new coding scheme of chromosome. Because SHVRP is a combinatorial optimization problem, here the natural number coding strategy is usually adopted.There are N customer locations numbered (demand points) $\{1, 2, \cdots, N\}$ and 0 represents the central warehouse. The gene value in chromosome represent the serial numbers of locations(including the central warehouse and the customer locations).The sequence of the whole gene in chromosome represents the sequences of visiting customer locations for vehicle (the last gene bit is usually storage the fitness value).The number of vehicles participating in delivering is unsure in advance, if every customer location need a vehicle travel separately, then the maximal possible length of chromosome is $2N+2$,therefore let the length of chromosome be $2N+2$ .To describe every subroutine with what type of vehicles and how many vehicles to deliver, this paper introduces an array with one dimension, in which every element(except the last element, the last element is used to storage the number of vehicles participating in delivering) is used to storage the load capacity of the vehicles for each subroutine according to the visited sequence of subroutine. The number of vehicles participating in delivering is unsure in advance, if each customer location need a vehicle deliver separately, then the maximal need is N vehicles, therefore let the length of the array be $N+1$.

There are two advantages to adopt this coding mode: one is convenient to get some detail information that by which vehicle a customer location is served, the number of customer locations served by a vehicle for all vehicles, and the specific situation of customer locations, and unable to lose important information; The other is when implementing genetic operators, it only carries out genetic operators on chromosome themselves whose corresponding array just need being adjusted correspondingly. That plays an important role in enhancing convergence of the algorithm and in shortening the solution time.

3.2 Generation of the Initial Population

In the standard genetic algorithm solution, the generation mode of the initial population is generally based on random sequences or the sweeping algorithm(SA),this paper adopts the mixed mode, namely one part adopts random sequences and the other part adopts the sweeping algorithm to build the initial population. The reasons of adopting this strategy are: if the whole population adopts the method of SA, it has two disadvantages. One is it possibly limits the searching range, loses opportunities for searching other unknown specials, and at last is easy to get into the local optimal. The other is the number of solution of SA is equal to the number of customer locations, thereby it also limits the population size which directly influences convergence of the algorithm and the computing efficiency.

The proportion of the solution of SA and the random sequences is decided by a parameter RANDRATIO. RANDRATIO is defined as in the initial population the proportion that the number of the individuals of the random sequences take on in the initial population size. The higher the proportion is, the more diversity the initial population get. In addition, the parameter RANDRATIO value reflects the trusted degree of the initial solution of SA. If the optimization solution is possibly likely to belong to the field range of the initial solution, then the RANDRATIO should take a smaller value, in this way, the population will be converge to the optimization solution more quickly. Otherwise the RANDRATIO should take a bigger value in order to keep the population diversity and avoid getting into local optimal, in this way, the probability of getting the total optimization solution will be more bigger.

3.3 Establishment of the Fitness Function

The fitness is only a standard that evaluates the performance of individuals in the population. The bigger the individuals' fitness is, the best the performance is, and then the probability of being inherited to the next generation is bigger. Or else the worse the performance is, the probability of being inherited to the next generation is smaller. The fitness function is always non-negative. In any case, the fitness value is expected enough big. SHVRP is a minimal combination optimization problem, which aims at the minimal traveling cost, that is the minimal objective function value. Therefore the fitness function may take the reciprocal of the objective function. In this paper, the objective function is shown in the equation(1).

3.4 Designing of the Genetic Operators

Genetic operators is the core motility of GA. The performance of the operators' design decides the ability to look for the optimization solution. In this paper, all kinds of genetic operators are designed as follows:

1. Selection operator
In this paper, the selection strategy is the combination of the roulette-wheel and the elitist strategy: sort chromosomes in every generational population descendingly. For the chromosome with the bigger value, do not implement the crossover and the mutation, but reproduce it to the next generation directly. The spare popsize-1 chromosomes in the next generation are generated through the roulette-wheel. In this way, it don't only avoid losing the best individuals midway, but also quicken the constringency the algorithm tends to get the optimization solution.

2. Crossover operator
In the standard genetic algorithm, the crossover operation is regarded as the genetic operation that plays a core role and a main way of generating new individuals. But when implementing the crossover operation on the genes coded based on the combination of routes to solve the problem VRP, if it simply uses the general crossover operators, some better gene combinations are usually destroyed, and it is difficult for the child generations to inherit the excellent gene groups of the parent generation, which induces the search ability being played down. Considering this, this paper substitutes the inversion operator for the crossover operation, namely according to the crossover probability implement the inversion operation on the gene cluster representing the vehicle routes in each chromosome chose through the selection operator. For example:

The chromosome before implementing the inversion operation on:

(0 4 7 2 0 5 9 0 1 0 8 3 11 0 6 13 0 12 0 10 0)

The chromosome before implementing the inversion operation on:

(0 10 0 12 0 13 6 0 11 3 8 0 1 0 9 5 0 2 7 4 0)

Implementing the crossover operation through the inversion operator keeps the genetic operation on one chromosome, namely reproduce the child generations just through a single individual, which simplifies the genetic operation, and in the maximal limit reserves the subroutes of the already becoming optimization routes. Consequently, that improves the ability to look for the optimization solution for the algorithm.

3. Mutation operator
To avoid the mutation operator destroying the mode of the excellent individuals in the population, in the paper, the part inversion mutation is adopted, namely in advance select randomly a gene location with the gene value

non-zero, and then implement the inversion operation on the gene segments between this gene location and the first gene location with the gene value zero. For example:

Select randomly a gene location with the gene value non-zero:

(0 4 7 2 0 5 9 0 1 0 8 3 11 0 6 13 0 12 0 10 0)

The chromosome after implementing the mutation operation on:

(0 4 7 2 0 5 9 0 1 0 8 11 3 0 6 13 0 12 0 10 0)

3.5 The Self-adaptation Crossover Probability

In this paper, it introduces the idea of the parameter self-adaption in the process of solving the problem SHVRP, which makes that in the process of solving the problem SHVRP it continuously corrects the crossover probability that plays a decisive role in the performance of the genetic algorithm according to the genetic course, thereby to reach the purpose of overcoming constringing too early and quickening the search speed.

The self-adaption change of the crossover probability means that the crossover probability takes different value according to the change of the number of the genetic generation. In the paper, the self-adaption crossover probability formulation is:

$$P_c = \begin{cases} p_c\text{max} \times \cos(\frac{\pi}{2} \times \frac{t}{T}) & , (p_c > p_c\text{min}) \\ p_c\text{min} & , (p_c > p_c\text{min}) \end{cases} \tag{9}$$

In the equation (9) $p_c\text{max}$ is the maximal crossover probability (here it takes 1.0). $p_c\text{max}$ is the minimal crossover probability (here it takes 0.5). t denotes the number of the genetic generation. T denotes the maximal number of the genetic generation.

This self-adaption crossover probability makes sure that in the early days the crossover probability is bigger and reduces slowly. Consequently it brings enough disturbances. That strengthens the search ability of the genetic algorithm, quickens the evolution speed, and avoids the genetic algorithm getting into the state of slowness. At the same time makes sure that in the late days, the crossover probability is smaller and reduces step by step till a constant at last. Consequently it avoids destroying the excellent genes. That quickens the constringency speed and the probability of searching the total optimization solution.

4 Solving the SHVRP Problem of Zhengzhou Coal Electricity Material Supply and Marketing Limited Company by IGA

The distance from the distribution center to demand points as well as the distance between the demand points is shown in table 1. The demand of each demand points is shown in table 2.

Table 1. The distance from the distribution center to demand points as well as the distance between the demand points Units:km

i / d_{ij} / j	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	0	10	28	23	65	16	16	42	16	21	85	23	35	70
1	10	0	76	72	150	52	52	104	42	62	190	60	90	110
2	28	76	0	106	74	48	83	28	88	58	186	106	86	146
3	23	72	106	0	180	82	82	134	18	92	220	18	120	110
4	65	150	74	180	0	122	157	56	162	132	172	180	120	215
5	16	52	48	82	122	0	59	76	64	34	162	82	62	117
6	16	52	83	82	157	59	0	111	64	69	197	82	97	74
7	42	104	28	134	56	76	111	0	116	106	126	134	114	283
8	16	42	88	18	162	64	64	116	0	74	202	18	102	112
9	21	62	58	92	132	34	69	106	74	0	172	92	72	127
10	85	190	186	220	172	162	197	126	202	172	0	220	110	255
11	23	60	106	18	180	82	82	134	18	92	220	0	120	130
12	35	90	86	120	120	97	97	114	102	72	120	120	0	155
13	70	110	146	110	215	74	74	283	112	127	130	130	155	0

Note: 0:Xinmi warehouse(distribution center),1:Peigou, 2:Taiping, 3:Zhanggou, 4:Baiping, 5:Mi village, 6:Chaohua,7:Gaocheng,8:Lugou,9:Lao juntang, 10:Jinglong, 11:Zhenxin,12: Cui temple,13:Zhao jiazai.

Table 2. The demand of each demand points units:ton

Demand point	Demand	Demand point	Demand
Peigou	1.2	Lugou	2.4
Taiping	6.0	Lao juntang	10.8
Zhanggou	5.9	Jinglong	3.0
Baiping	2.4	Zhenxin	12.0
Mi village	6.2	Cui temple	2.1
Chaohua	4.8	Zhao jiazai	1.2
Gaocheng	0.8		

4.1 Solving with IGA

In this paper, the IGA algorithm solves the SHVRP problem of Zhengzhou Coal Electricity Material Supply and Marketing Limited Company with MATLAB7.5 programming. The size of population is 26. Here the proportion of initial population parameters RANDRATIO is 0.5.The half of initial population is obtained from ISA and the other half is obtained from random method. According to the probability of adaptive crossover $p_{c\max}$ is 1.0, $p_{c\min}$ is 0.5. The mutation probability p_m is 0.05the largest iterative generation is T=1000 .Solving the case in the same computer for 10 times randomly,we get the results shown in table 3.

Table 3. Results for random ten times with IGA

Times of run	Total cost of distribution (Yuan)	Number of vehicles (cars)	Iteration steps	Time costs (s)
1	866.6	13	2	0.0469
2	866.6	13	13	0.3438
3	866.6	13	8	0.2656
4	866.6	13	6	0.2188
5	866.6	13	15	0.3594
6	866.6	13	18	0.3281
7	866.6	13	3	0.1719
8	866.6	13	8	0.2656
9	866.6	13	7	0.2344
10	866.6	13	3	0.1719
Average	866.6	13	8.3	0.2406

Table 4. Decoding results of optimal solution

Delivery route		Distribution scheme	Vehicle capacity /ton	Vehicle loading rate
Path 1	0 - 8 - 0	Warehouse – Lugou – Warehouse	5.0	40.00%
Path 2	0 -3- 0	Warehouse – Zhanggou – Warehouse	5.0	98.33%
Path 3	0 -11- 0	Warehouse – Zhenxin – Warehouse	15.0	66.67%
Path 4	0 -1- 0	Warehouse – Peigou – Warehouse	1.0	100.00%
Path 5	0 -5- 0	Warehouse – Mi village – Warehouse	8.0	64.58%
Path 6	0 -9- 0	Warehouse – Laojun tang – Warehouse	15.0	60.00%
Path 7	0 -2- 0	Warehouse – Taiping – Warehouse	5.0	100.00%
Path 8	0 -7- 0	Warehouse – Gaocheng – Warehouse	1.8	37.04%
Path 9	0 -4- 0	Warehouse – Baiping – Warehouse	3.0	66.67%
Path 10	0 -12- 0	Warehouse – Cui temple – Warehouse	1.8	97.22%
Path 11	0 -6- 0	Warehouse – Chaohua – Warehouse	5.0	80.00%
Path 12	0 -13- 0	Warehouse – Zhao jiazai – Warehouse	5.0	20.00%
Path 13	0 -10- 0	Warehouse – Jinglong – Warehouse	8.0	31.25%

From table 3, the minimal cost of distribution is 866.6 yuan. We get the optimal solution from each run,the success rate of searching is 100%. The average computing time is 0.2406 s. The average step of iteration to obtain the optimal solution is 8.3. We analyzed the best individual and the decoding results are shown in table 4.

4.2 Comparative Analysis of Three Methods for Solving the Case

In order to obtain the comparative results, CHA,SA and IGA algorithm are all programmed with MATLAB7.5, which run in the computer with Intel pentium4 CPU3.0, frequency 795MHz, memory 760M and windows XP operating system. Through random ten run times with the three algorithms to solve the case is shown in table 5.

Table 5. Comparisons of the three algorithms to solve the case

Algorithm	Total distribution cost (Yuan)	The number of vehicle	The average convergence steps	The average computing time (s)
CHA	1349.5	7	1	0.0203
SA	1198.2	8	1	0.0529
IGA	866.6	13	8.3	0.2406

From the table 5, CHA, SA and IGA have presented its own advantages. In view of getting the best results of objective function, IGA algorithm obtains the lowest total transportation cost, followed by SA and CHA. However, in view of the required number of vehicles involved in distribution, CHA gets the least vehicles, followed by SA and IGA. In view of the average computing time, the running time of CHA is the least, which is only $2/5$ of SA and $2/25$ of IGA. In view of the number of iterative steps, because CHA and SA are based on the traditional heuristic algorithm of accurate search rules, we can get the result in one step. But IGA is based on random search rules, we can't get the final result from one step.

5 Conclusions

Overall, it's difficult for us to make a reasonable transport path planning in a short time if just relying on the experience of decision-makers when solving the complex SHVRP. The IGA proposed in this paper can make an optimal solution in a short time. Comparing it with the solving processes of CHA and SA, decision-makers could choose a satisfactory distribution scheme based on business goals (total distribution costs, the number of vehicles, the time for calculating).If we pursue the least number of vehicles and disregarding the expense of distribution, we can choose the optimization results of CHA. If we pursue the least cost of distribution and disregard the number of vehicles, we can choose the optimization results of IGA.

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Displacements Back Analysis for Soft Pit Excavation Based on Genetic Algorithms

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Abstract. Displacements back analysis in soft pit excavation is a key issue for design. In this paper firstly the five-component soil model is presented to simulate mechanical process of soft soil, and the finite element code is developed. Secondly the model of displacement back analysis is set up. Thirdly the genetic algorithms is adopted to solve the model of displacement back analysis, and the code of displacement back analysis for soft pit excavation based on the genetic algorithms (DB-EV-GA) is developed. Lastly a case study is performed. Our main findings are: (1) The displacement of the pit engineering in soft soil increases with the increment of time. The five-component model can simulate the effect; (2) The displacement back analysis method based on the genetic algorithm presented in this paper is effective, and the calculation displacement data are near to the observational data in case study.

Keywords: Genetic algorithms, displacement back analysis, soft pit excavation, finite element.

1 Introduction

Traditionally, the design of temporary retaining structures has been based on semi-empirical methods usually set in terms of the equations of the classical or Rankine earth pressure theories. This approach has proven to be satisfactory for structural design of retaining structure components. However, evaluation of the probable displacement behaviour of retaining structures has been based solely on interpretation of observational data. The major shortcoming of such a procedure is the difficulty involved in extrapolating observational data from site to site. One analytical technique that seemingly can provide a more rational means for evaluating displacements and furnishing a stronger base from which to extrapolate performance is the finite element method [1-3]. The principal advantages of the

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finite element method are that the soil and the structure can be considered interactively and that both design loads and expected displacements can be studied. However it is a key issue for the finite element method to choose reasonable parameters. A lot of researches believe that the displacement back analysis is an effective approach. Some methods [4-7] have been applied in the displacement back analysis in some researches. However the famous optimized algorithm, the genetic algorithm (GA) has never been used to execute the displacement back analysis in soft pit engineering. In this paper a five-component soil model is adopted to simulate the mechanical process of soft soil, and the model of displacement back analysis for the soft pit engineering is set up. The genetic algorithm is used to solve the model, and the code of the displacement back analysis for soft pit excavation (DB-EV-GA) is developed. Lastly a case is studied.

2 Model Description

A. Soil Model

A five-component visco-elastic soil model is adopted to simulate the mechanical process of soft soil. The model consists of a spring element and two Kelvin elements (Fig.1).

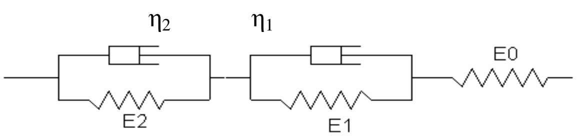

Fig. 1. Five-component visco-elastic soil model

The stress-strain relationship of the model can be expressed (plane strain problem)as:

$$\varepsilon_x(t)=\frac{1}{E_e(t)}\left[\left(1-\nu^2\right)\sigma_x-\nu\left(1+\nu\right)\sigma_y\right] \tag{1}$$

$$\varepsilon_y(t)=\frac{1}{E_e(t)}\left[\left(1-\nu^2\right)\sigma_y-\nu\left(1+\nu\right)\sigma_x\right] \tag{2}$$

Where ν is Poisson's ratio, σ_x,σ_y are the stress vector, respectively, $E_e(t)$ is the equivalent elastic modulus at the time of t, and can be expressed by

$$\frac{1}{E_e(t)}=\frac{1}{E_0}+\frac{1}{E_1}\left[1-\exp\left(-\frac{E_1}{\eta_1}t\right)\right]+\frac{1}{E_2}\left[1-\exp\left(-\frac{E_2}{\eta_2}t\right)\right] \tag{3}$$

Where E_0, E_1, η_1, E_2, η_2 are the parameters of the model.

Base on Eq.1-Eq.3, the finite element code for soft pit excavation (Pit-Ex) is developed.

B. Model of the Displacement Back Analysis

Although Pit-Ex can be used to predict the displacement of soft pit excavation, however it is extremely difficult to obtain above five parameters in Eq.2 by tests, and the tested parameters also are not accurate for analysis. So in this paper the code of Pit-Ex combined with the genetic algorithm, the code of the displacement back analysis for soft pit excavation was developed. The model for the displacement back analysis can be represented by

$$\max\left\{\frac{1}{\sqrt{1+(S(p)-\overline{S})^{T}(S(p)-\overline{S})}}\right\} \tag{4}$$

where $\overline{S}$ is measured displacement vector; $S(p)$ is the corresponding calculation vector.

C. Genetic Algorithm

Genetic algorithm is a search method based on Darwin's theory of evolution and survival of the fittest. Based on the concept of genetics, GA simulates the evolutionary process numerically. Selection is simply the copying of quality solution in proportion to their effectiveness. Here, since the goal is to minimize the objective function, several copies of candidate solutions with small objective functions are made; solutions with large objective functions tend not to be replicated. The selection process can produce a new population, extracting with repetition individuals from the old population. The extraction can be carried out in several ways. One of the most commonly used is the roulette wheel selection, where individuals are extracted in probability following a Monte Carlo procedure. The extraction probability of each individual is proportional to its fitness as a ratio to the average fitness of all the individuals. The evaluation of the fitness can be conducted with a linear scaling, where the fitness of each individual is calculated as the worst individual of the population subtracted from its objective function value[8].

$$f_j = \max\{k_j \mid j = 1,2,,,m\} - k_j \tag{5}$$

where f_j is the fitting function; m is the population size. In the selection process, the reproduction probabilities of individuals are given by their relative fitness

$$pro_i = f_i / \sum f_i \tag{6}$$

where pro_i is the reproduction probability of the ith individual. Recombination is a process by which information contained in two candidate solutions is combined. In the recombination, each individual is first paired with an individual at random. Let a pair of present individuals be given by [p_α^t, p_β^t]. a new pair [p_α^{t+1} , p_β^{t+1}] is then created in terms of a phenomenological recombination formula[9]:

$$p_{\alpha}^{t+1} = (1-\mu)p_{\alpha}^{t} + \mu p_{\beta}^{t} \tag{7}$$

$$p_{\beta}^{t+1} = (1-\mu)p_{\beta}^{t} + \mu p_{\alpha}^{t} \tag{8}$$

where μ is a random number changing from 0 to 1. Mutation is a process by which vectors resulting from selection and recombination are perturbed. The mutation is conducted with only a small probability by definition. An individual, after this mutation, p_i^{t+1}, is described as

$$p_i^{t+1} = rand\{p_{idown}^{t}, p_{iup}^{t}\} \tag{9}$$

where p_{idown} and p_{iup} represent lower and upper bounds of parameters; $rand\{.\}$ represents the random selection from the changing domain. The elitist strategy, where the best individual is always survived into the next generation on behalf of the worst individual, can compensate for some disadvantages of missing the best individual in selection operation or mutation operation. With the elitist strategy, the best individual always moves in a descent direction, thereby a stable convergence is obtained. The gradient search algorithm adopted in genetic algorithm is the most popular quasi-Newton method with the BFGS algorithm. The individual after the recombination is formulated as follows

$$p^{new} = \begin{cases} -A^{-1}\nabla f(p^{old}) & \text{if} \quad f(p^{new}) > f(p^{old}) \\ p^{old} & \text{otherwise} \end{cases} \tag{10}$$

where A is a well-known positive-define matrix used on behalf Hessian matrix.

Based on Eq.3 to Eq.10 and Pit-Ex the code of the displacement back analysis for soft pit excavation (DB-EV-GA) is developed.

3 Case Study

The case is a pit excavation in soft foundation with the width of 20m and the depth of 4.6m(see Fig.1). Manual digging piles with the diameter of 0.8m and the length of 10m are used to support the soil pressure. The piles are assumed to be linear elastic material. The elastic modulus is 2.3×10^4MPa, and Poisson's ratio is 0.18. The soft soil is simulated by the five-component visco-elastic soil model. The Poisson's ratio is 0.34. The analysis is performed assuming that plane strain conditions exist. The Desai thin layer elements [9] are inserted between the piles and the soil. The thickness of the Desai element is 0.05 times than the adjacent soil element. The thin layer elements are assumed to be linear and elastic, and the elastic modulus and Poisson's are identical with the soil. The shear modulus is 4kPa. The finite element model is 50×18m.

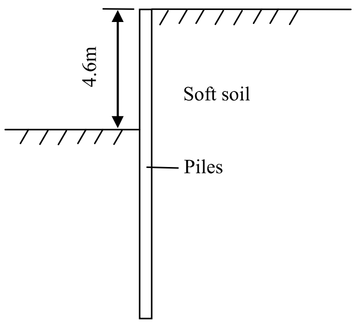

Fig. 2. Supporting sketch

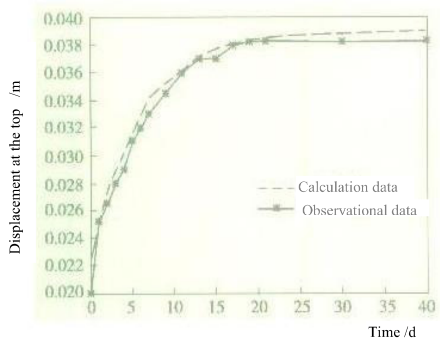

Fig. 3. Displacement VS time

Two steps are performed for the pit excavation. The excavation depth of 2.5m is executed in the first step, and the other excavation depth of 2.1m is performed. The displacements at the top of piles are observed. Before the second excavation step the observational displacement data and the above calculation parameters are inputted into DB-EV-GA, and the five parameters are initialed as: E_0=2 10^4kPa, E_1=2 10^5kPa, η_1=2 10^5kPa, E_2=2 10^5kPa, η_2=2 10^5kPa. Then the displacement back is performed, and the results of the back analysis are: E_0=2.5 10^4kPa,E_1=5.6 10^4kPa, η_1=3.1 10^4kPa, E_2=5.6 10^3kPa, η_2=3.3 10^4kPa. The parameters from back analysis are used to predict the displacement of the top of the pile after the second step of excavation, and the prediction displacement data with time and the observational data can be compared in Fig.3.

From Fig.3 we find that the displacement of the top of the pile increases with increasing time after the second step of excavation. However the velocity decreases with time. After half month the displacement becomes stable. Another finding is that the calculation displacement data are near to the observational data. And the displacement back analysis based on GA in this paper is successful to predict the support displacement of soft soil pit excavation.

4 Conclusions

In this paper, soft soil is simulated by the five-component model, and the corresponding finite element code is developed. Further the model of the displacement back analysis is set up and the genetic algorithm is used to solve the model. Through case study our main findings are:

(1) The displacement of the pit engineering in soft soil increases with increasing time. The five-component model can simulate the effect.

(2) The displacement back analysis method based on the genetic algorithm presented in this paper is effective, and the calculation displacement data are near to the observational data in case study.

Acknowledgements

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Personalized Information Service System Based on Multi-agent

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Abstract. This paper is a survey of resent work in the field of multi-agents for the benefit of research on the personalized information service. According to the defect of current search engines, this paper presents a model of personal information services, which can realize individuation and intelligence of information retrieval according to user interested knowledge via multi-agent collaboration. This paper gives a working principle of the model and discusses abstract technology of the model.

Keywords: Multi-agent, Personalized Information Services, Information Retrieval, User Interest Model.

1 Introduction

With the fast development of Internet, the on-line information is growing and changing rapidly. Only by web browser we can hardly find the information needed quickly. Search engine is able to resolve the problems to some degree, but it is lack of intelligence and personality. Interest in the analysis of user behavior on the web has been increasing rapidly. This increase stems from the realization that added value for web site visitors is not gained merely through larger quantities of data on a site, but through easier access to the required information at the right time and in the most suitable form.

Interest in the analysis of user behavior on the web has been increasing rapidly. This increase stems from the realization that added value for web site visitors is not gained merely through larger quantities of data on a site, but through easier access to the required information at the right time and in the most suitable form.

Past solutions have included automated searching programs such as wide area information servers (WAIS) or Web crawlers that respond to explicit user queries. Among the problems of such solutions are that the user must

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explicitly decide to invoke them, interrupting the normal browsing process, and the user must remain idle waiting for the search results. So, there are several challenges in supporting user's retrieval activities in personalized information service systems.

Agent paradigm is a promising technology for information retrieval. Some applications are intelligent information retrieval (IR) interfaces and clustering and categorization. An Agent-based approach means that IR systems can be more scalable, flexible, extensible, and inter operable. Generally, Agent should have knowledge, objective and ability.

Multi-agent systems (MAS) have been studied for many years, and various types of such systems have been developed. MAS are the emerging subfield of AI that aims to provide both principles for construction of complex systems involving multiple agents and mechanisms for coordination of independent agents' behaviors.

Therefore, combining multi-agent technology with knowledge retrieval model will bring a new idea for the personalized information service system. On the basis of these, this paper introduces these agents, which operate in tandem with a conventional Web browser. These Agents can track the user's browsing behavior-following links, initiating searches, requests for help and tries to anticipate what items may be of interest to the user. The construction of user interest model based on browsing history record and registration data is discussed in detail, and the system can update user interest model when user's interest changes.

The paper is organized as follows; Section 2 introduces the agent and the MAS. Section 3 we will introduce system framework, its components function, and application of Agent in the personalized model. Section 4 gives the prototype implementation method of the personalized retrieval system and introduces the information flow among agents of the system. Finally conclusions are given in Section 5.

2 Agent and MAS Introduction

Oliver Selfridge, who's 1959 Pandemonium paper introduced the term "agent", referred both to this sense and also to a sense of agent internal to a system, where multiple goal-seeking entities both compete and cooperate to produce intelligent behavior. In the early days, many researchers have been studied about agent in boundary of AI. Since 80's the agent have widely applied [9]. Although lack of a universal definition of agents, there is a general agreement that an agent is a reusable component that exhibits a combination of the following characteristics [5] [6]:

1) Autonomy: agents encapsulate some states of their environment, and make decisions about what to do based on these states;

2) Reactivity: agents are able to perceive their environment and respond in the changes that occur in it;

3) Pro-activeness: agents are able to exhibit goal-directed behavior by taking the initiative;

4) Social ability: agents interact with other agents via an agent communication language, and have the ability to engage in social activities in order to achieve their collective goals.

So in the software system, agent means software component that has inference capability, and can interacts autonomously as a surrogate for its user with its environment and other agents to achieve the predefined goal, and reacts to changes in the environment.

Inspired by distributed artificial intelligence, a MAS consists of autonomous generally heterogeneous and potentially independent agents which work together to solve special problems. As described by Brennan, autonomous, cooperative, and scalable are the typical characteristics of MAS that has the following capabilities [11] [3]:

1) Independent decision-making by individual agent based on its domain knowledge, local and global conditions.

2) Interacting with other agents and humans for effective negotiation, cooperation and coordination.

3) Perceiving changes in their environment and acting as a consequence.

4) Taking initiative to reach certain objectives.

In a MAS, research is concerned with cording intelligent behavior among a collection of autonomous, potentially pre-existing units, defining how they can coordinate their knowledge, goals, and skills to jointly solve a given problem. It is appropriate to adopt the MAS technology in the personalized retrieval service system.

3 System Design

3.1 System Framework

A general framework for personalization based on multi-agent model is depicted in Fig. 1. The framework is composed of:

1) Information Resource Base. This base is used to store obtained web pages information, dictionary, and training documents. The dictionary allows the agent to compare and discover relationships between a pair of words or concepts. Each concept has a list of supporting documents or links.

2) User Interest Model. User modeling systems usually include a method for classifying users into groups, and relating these groups to domain concepts, and a software user modeling component to be incorporated into an application. User modeling components generally include a component to store the user model, and an interface to write and read from that store. The write interface supports the acquisition of user models, and the read interface supports the querying of the user model for domain or application

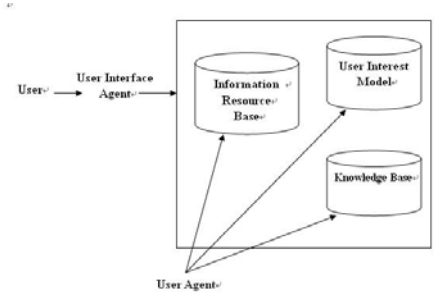

Fig. 1. Personalized information service system framework

information. The store itself supports the heuristic classification either at write time or read time.

3) Knowledge Base. Knowledge base is responsible for the mapping from information source base to knowledge base, and organizes the knowledge of knowledge management system.

4) Multi-agent. As we want to build the personalized information service system. The system should inform users immediately when the new knowledge is adding into knowledge base. Various agent types are used to realize the system goals. These Agents can track the user's browsing behavior following links, initiating searches, requests for help and tries to anticipate what items may be of interest to the user.

3.2 Application of Agent in the Personalized Model

As for the personalized information service model, agent is a key technology to improve its intelligence. In the personalized retrieval model there are nine agents.

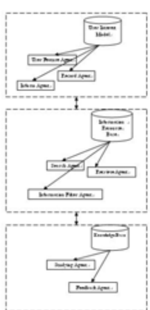

Fig. 2. Structure of personalized information service system

The functionality of each agent is described as follow:

1) Interface Agent. For an agent to be considered an "interface" agent, we'll require that the agent communicate with the person directly through the input and output of the user interface or interface to environment. An interface agent can observed actions taken by the user in a direct manipulation interface, can sense the objects that the user sees on the screen, and can itself take actions by invoking the commands provided by the interface. The agent can add graphics or animation to the interface; it can use speech input or output, or communicate via other sensory streams. It is also able to accept the results from retrieval agent and display them to the user for him to evaluate the feedback, then learn the user's feedback information, and dynamically modify and perfect user's personalized model. Increasingly, agents also will use sensors and effectors that sense and act directly with the real world.

2) Retrieve Agent. Retrieval agent executes search tasks. The retrieval agent has knowledge about the task domain, as well as the capabilities of other agents. It obtains the necessary methods according to the different retrieval task, and it also may seek the services of a group of agents that work cooperatively and synthesize the integrated result. Then it searches the corresponding knowledge from knowledge base. Finally it submits the results to user interaction agent.

3) Managing Agent. Managing anent contain three agents, feature agent, inform agent and record agent. When the new knowledge is added into knowledge base record agent will automatically classify and organize the new knowledge, and then inform agent will be activated, and it will submit the new knowledge to appropriate users according to the demand of users in user feature agents. Feature agent is used to record the interest and feature of users. Inform agent is used to trigger and notify users when new knowledge are adding into the system, and the users who are interested in it can access such knowledge.

4) Information Filter Agent. Information filter agent deals with the delivery of information that is relevant to the user in a timely manner. This agent filters and selects web pages obtained by search agent. This part is the center of whole system because the quality of information filtering and classifying affects system's performance and retrieve efficiency directly.

5) Feedback Agent Feedback agent interacts with the user in assisting him performing the retrieval activities. The user can provide a general description of the problem at hand in terms of high level goals and objectives, and retrieval to be performed. The feedback agent is responsible for receiving user specifications and delivering results back. User interested model can amend and renew knowledge according the feedback agent.

6) Search Agent. Its function is to realize personal and intelligent searching according to user's interest and requirements. It is designed to download records from the internet. Those records serve as the training set to create

an entry vocabulary module. Search agent gets user's searching requirements from retrieve agent and subjects from use interested model, carries evidence which is used to judge if web pages are in accordance those requirements and subjects so as to realize collecting and downloading intelligently. At least, agent adds those correspondent pages into information resource base.

4 System Prototype Implementation and Working Flow

4.1 System Prototype Implementation

Since the cooperative jobs between agents are implemented through the information transfer between each other, it is very important to make clear the content and direction among the agents of the system. We have implemented nine agent methods in this system:

1) Init(): Initialize the agent.
2) Start(): Start to run.
3) SearchURL(strURL): Get the Web page specified by strURL according to user's interest into the information resource base.
4) Filter(): Filter and classifying the information from the information resource base.
5) Retrieve(): Retrieve relevant document according to agent's retrieval algorithm.
6) Show(): Display the interface in user screen.
7) SetField(): Set internal data field.
8) OutPut(): Output results to specified device.
9) Message(): Send message between different agents.
10) Listen(): Wait and Receive message from feedback agent.
11) MoveTo(): Move to the next trask.
12) Back(): Come back the system.
13) End(): Terminate execution.

4.2 Working Flow

The steps of retrieving information service are shown as follows:

1) User feature agent will add user feature information into user interested model when a new user finished registration in the system.
2) Record agent will write the information when the new knowledge is added into the knowledge base, and the send it to the inform agent.
3) Inform agent submits the request of querying for user feature to user interested model.
4) The interface agent processes the request through inferring, judging, semantics analyzing, at last, agent gets precise querying information and send it to retrieve agent.

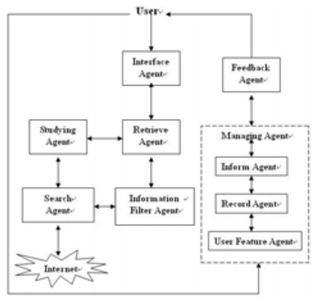

Fig. 3. The system's main Working flow

5) Retrieve agent accepts request, searches use interested base.

6) Dose request information exists in knowledge base? If the request matches one of the concepts in the use interested base, the agent retrieves the number of links available, then will filter and classify information.

7) If no, then retrieve agent sends request information to search agent.

8) Search agent begins searching and transmits the searching results to information filter agent.

9) Information filter agent filters and classifies the results and save them to information resource base.

10) Retrieve agent gets filtering and classifying information from information resource base and returns the answer to interface agent.

11) Interface agent displays the answer to the user for him to evaluate the feedback, then learn the user's feedback information, and dynamically modify and perfect user's individualized base.

12) End.

5 Conclusions

With the user interested model, the agent technology's rapid development, it is possible to use the multi-agent technology to build the personalized service model. In this paper, we combine traditional retrieving technology and agent technology to complete this system. Model's architecture and work principle have been analyzed. Further research will refine the user's interested model. In addition, the mechanisms of cooperation and communication between agents need improving.

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Stability with Impulsive Delay Predator-Prey System

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Abstract. In this paper, we research the impulsive delay predator-prey system with stage structure; we consider two species predator-prey model with stage structure for prey and impulsive effect for predator. By some techniques of comparison theorems of impulsive differential equations, we obtain the conditions for global asymptotic stability of periodic solution of this system.

Keywords: Predator-prey model, impulsive differential equations, asymptotic stability.

1 Introduction

Recently, several mathematical models of stage structure population growth of predator-prey systems have appeared in the literature, but some results devoted to study continuous system only, in fact,there are many natural pheomena in real world which are subject to instantaneous changes and interferences, These are often assumed to be in the form of impulses in the modeling process. Therefore impulsive differential equations provide a natural description for such jumping. Impulsive differential equation has universal application in the field of science and technology. In ecology, generally the mathematical model for stage structured population growth which takes into account the fact those individuals in a species population may belong to one of two classes, the immature and matures. The age to maturity is represented by a time delay. Simply, we assume an average age to maturity with appears as a constant time delay reflecting a delayed birth of immature, thus leading to systems of delay differential equations.

In this paper, we consider the following impulsive delay system of two species lotka-volterra predator-prey with stage structure:

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$$\begin{cases} \dot{x}_1(t)=\alpha x_2(t)\text{-}r_1x_1(t)\text{-}\alpha e^{-r_1T}x_2(t-T) \\ \dot{x}_2(t)=\alpha e^{-r_1T}x_2(t-T)\text{-}\beta x_2^2(t)\text{-}a_1x_2(t)y(t) \\ \dot{y}(t)=y(t)(-r_2+a_2x_2(t) \\ \qquad\qquad\qquad\qquad t\neq n\tau \\ x_1(n\tau^+)=x_1(n\tau^-) \\ x_2(n\tau^+)=x_2(n\tau^-) \\ y(n\tau^+)=y(n\tau^-)+b \\ \qquad\qquad\qquad\qquad t=n\tau \end{cases} \tag{1}$$

$$x_1(0)>0, x_2(0)>0, y(0)>0,$$
$$x_2(t)=\varphi(t)\geq 0, -T\leq t<0. \tag{2}$$

where $x_1(t)$ is the immature prey population density, $x_2(t)$ is the mature prey population density, $\alpha>0$ represents the immature prey populations the birth rate into the immature population is proportional to the existing mature population with a positive constant, $r_1>0$ represents the immature prey populations the death rate is proportional to the existing immature population with a positive constant, $\beta>0$ represents the mature prey populations the death rate is of a logistic nature,i.e.it is proportional to square of the population with a positive proportionality constant,T represents the time which the immaturity growth to the maturity, the $\alpha e^{-r_1T}x_2(t-T)$ represents the immature who were born at time $t-T(i.e.x_2(t-T))$ and survive at time t (with the immature death rate r_1) and therefore represents the transformation immature to matures, coefficient $a_1>0.y(t)$ represents the densities of predator population represents the impulsive stocking of predator population at the time $t=n\tau, \tau$ represents time period,b, τ are positive constants,n is any nonnegative integer. The growth of the predator population is of lotka-volterra nature,$r_2>0, a_2>0$.for continuity of initial conditions we assume: $x_1(0)=\int_{-T}^{0}\alpha e^{r_1t}x_2(t)dt$. According to the above, system (1) is two species predator-prey functional differential system with stage structure and impulse.

2 Global Asymptotic Stabilities of Periodic Solution

First, we notice the first equation of system (1) can be rewritten as

$$x_1(t)=\int_{t-T}^{t}\alpha e^{-r_1(t-s)}x_2(s)ds \tag{3}$$

from which we have

$$x_1(0)=\int_{-T}^{0}\alpha e^{-r_1s}x_2(s)ds. \tag{4}$$

Thus, it suggests that if we know the expression of $x_2(t)$, then the expression of $x_1(t)$ can be obtained from (3) and (4) .Therefore, we need only to consider the following model:

$$\begin{cases} \dot{x}_2(t)=\alpha e^{-r_1T}x_2(t-T)\text{-}\beta x_2^2(t)\text{-}a_1x_2(t)y(t) \\ \dot{y}(t)=y(t)(-r_2+a_2x_(t); \\ t\neq n\tau \\ \ x_2(n\tau^+)=x_2(n\tau^-) \\ \ y(n\tau^+)=y(n\tau^-)+b. \\ \ t=n\tau \end{cases} \tag{5}$$

$$x_2(0)>0, y(0)>0, x_2(t)>0, x_2(t)=\varphi(t)\geq 0, -T\leq t<0. \tag{6}$$

We demonstrate below the existence and globally asymptotical stability of the solution corresponding to the extinction of the prey population, i.e. $x_2(t)=0, t\geq 0$ under this condition, the growth of predator population must satisfy the following system:

$$\begin{cases} \dot{y}(t)=-r_2y(t), t\neq n\tau \\ \ y(n\tau^+)=y(n\tau^-)+b, t=n\tau \\ \ y(0^+=y_0\geq 0. \end{cases} \tag{7}$$

Lemma 1. *System (7) has a positive periodic solution*

$$y^*(t)=\frac{be^{-r_2(t-n\tau)}}{1-e^{-r_2\tau}}n\tau<t\leq(n+1)\tau, \tag{8}$$

where the initial value $y^*(0)=y(0)\frac{b}{1-e^{-r_2\tau}}$ *and* $y^*(t)$*is globally asymptotically stable.*

Proof. By elementary integration, we integrate and solve the first equation of system, between pulses, we obtain:

$$y(t)=y_n\tau e^{-r_2(t-n\tau)}, n\tau<t\leq(n+1)\tau, \tag{9}$$

where $y_n\tau(y_n\tau=y(n\tau^+)=y(n\tau^-)+b$ is the density of population immediately after the n^{n-th} pulse at time $t=n\tau$. By the second equation of system (7), we deduce the map from time $n\tau$ to $(n+1)\tau$ such that

$$y_{(n+1)\tau}=y((n+1)\tau^+=y((n+1)\tau^-+b)=y_{n\tau}e^{-r_2\tau}+b. \tag{10}$$

Easily demonstrate that the map has the unique positive fixed point:

$$\bar{y}=\frac{b}{1-e{-}r_2\tau}=y^*(0+)=y_0. \tag{11}$$

The fixed point$\bar{y}$implies that there is a corresponding cycle of period τ in the population, i.e.,

$$y^*=\frac{be^{-r_2(t-n\tau)}}{1-e^{-r_2\tau}}, n\tau<t\leq(n+1)\tau. \tag{12}$$

We demonstrate below $y^*(t)$ is globally asymptotic stable. In fact, since $y_{(n+1)\tau} = y_{n\tau}e^{-r_2\tau} + b$, using iterative step by step, we have

$$y_{n\tau}=b(1+e^{-r_2\tau}+e^{-2r_2\tau}+\cdots+e^{-(n-1)r_2\tau})+y_0e^{-nr_2\tau}=\frac{b(1-e^{-nr_2\tau})}{1-e^{-r_2\tau}}+y_0e^{-nr_2\tau},$$

then

$$lim_{n\to\infty}y_{n\tau} = \frac{b}{1-e^{-r_2\tau}} = \mathbf{y},$$

i.e., $\bar{y}$ is globally asymptotically stable, thus, the periodic solution of the system (7), i.e., $y^*(t)$ is globally asymptotically stable.

Next we show globally asymptotical stability of the periodic solution $(0, y^*(t))$ of System (5). First, According to [4]. we have the following Lemma 2 and Lemma 3.

Lemma 2. *Assuming that the system (5) satisfy the initial conditions (6), then exist positive and bounded solutions of (5) (6), for all* $t \geq 0$, *i.e.*, $x_2(t)$, $y(t) > 0, t \geq 0$.

$$y^*(t) = \frac{be^{-r_2(t-n\tau)}}{1-e^{-r_2\tau}}, n\tau < t \leq (n+1)\tau, \tag{13}$$

where the initial value $y^*(0) = y(0)\frac{b}{1-e^{-r_2\tau}}$, *and* $y^*(t)$*is globally asymptotically stable.*

Lemma 3. *We consider the following equation:*

$$\dot{u}(t) = au(t-T) - bu(t) - cu^2 \tag{14}$$

where $a, b, c, T > 0; u(t) > 0, -T \leq t \leq 0$, *we have:*
(I) *if* $a > b$, *then*

$$\lim_{t\to+\infty} u(t) = \frac{a-b}{c};$$

(II) *if* $a < b$, *then*

$$\lim_{t\to+\infty} u(t) = 0.$$

Theorem 1. *If* $e^{-r_1T} < \frac{a_1b}{\alpha} \cdot \frac{e^{-r_2\tau}}{1-e^{-r_2}}$ *holds, then system (5) has a periodic solution* $(0, y*(t))$ *and it is globally asymptotically stable. Further, the system (1) has a periodic solution* $(0, 0, y*(t))$ *and it is globally asymptotically stable.*

Proof. From above discussion, we know system (5) has a periodic solution $(0, y^*(t))$, where $y^*(t)$ is the following form:

$$y^*(t) = \frac{be^{-r_2(t-nT)}}{1-e^{-r_2}T}, \quad n\tau < t \leq (n+1)\tau. \tag{15}$$

We prove below globally asymptotical stability of periodic solution$(0, y^*(t))$ of (5). Let $(x_2(t), y(t))$ is a solution of (5) and satisfies the initial conditions of (6), from lemma2, we have$x_2(t), y(t) > 0, t > 0$Such that, from the second equation of system (5), we obtain $y(t) > r_2 y(t)$.

We consider following system:

$$\begin{cases} \dot{y}(t){=}{-}r_2 y(t), t \neq n\tau \\ y(n\tau^+){=}y(n\tau^-){+}b, t{=}n\tau \\ y(0^+{=}y_0 \geq 0. \end{cases} \tag{16}$$

By the comparison theorem of impulsive differential equations and lemma 1, we know that for any solutions $(x_2(t), y(t))$ of system (5) with initial $x(0) > 0, y(0^+) > 0$, when t is large enough; the following inequality holds:

Here $y^*(t)$ is denoted in (15),$\varepsilon > 0$ is sufficiently small

$$\dot{x_2}(t) \leq \alpha e^{-r_1 T} x_2(t-T) - \beta x_2^2(t) - a_1 x_2(t)(y*(t) - \varepsilon).$$

Since $y* = \frac{be^{-r_2(t-nT)}}{1-e^{-r_2 T}}, n\tau < t \leq (n+1)\tau$ is the periodic solution of (7), then

$$min_{n\tau \leq t \leq (n+1)\tau} y*(t) = min_{n\tau \leq t \leq (n+1)\tau} \frac{be^{-r_2(t-nT)}}{1-e^{-r_2 T}} = \frac{be^{-r_2 T}}{1-e^{-r_2 T}}. \tag{17}$$

Again, since $\varepsilon > 0$ is sufficiently small, thus (10) can be rewritten the following inequality.

$$\dot{x_2}(t) \leq \alpha e^{-r_1 T} x_2(t-T) - \beta x_2^2(t) - a_1 x_2(t) \frac{be^{-r_2 T}}{1-e^{-r_2 T}}$$

by the conditions of the theorem:

$$e^{-r_1 T} < \frac{a_1 b}{\alpha} \cdot \frac{e^{-r_2 \tau}}{1-e^{-r_2}},$$

i.e., $\alpha e^{-r_1 T} < a_1 \frac{e^{-r_2 \tau}}{1-e^{-r_2}}$ from Lemma 3, then $\lim_{t\to} x_2(t) = 0$ when t is sufficiently large, ε_1 is small enough and $0 < x_2(t) < \varepsilon_1$, we have $\dot{y(t)} < y(t)(-r_2 + \varepsilon_1 a_2)$.

We consider the following comparison system:

$$\begin{cases} \dot{u}(t){=}u(t)(r_2 + \varepsilon_1 a_2), t \neq n\tau \\ u(n\tau^+){=}u(n\tau^-){+}b, t{=}n\tau \\ u(0^+{=}u_0. \end{cases} \tag{18}$$

From Lemma 2, we know system (18) satisfy the initial condition:

$$u(0^+) = u_0 = \frac{b}{1 - e^(-r_2 + \varepsilon_1 a_2)\tau} \tag{19}$$

has a periodic solution $u^*(t)$as follows:

$$u^*(t) = \frac{be^(-r_2 + \varepsilon_1 a_2)(\tau - n\tau)}{1 - e^(-r_2 + \varepsilon_1 a_2)\tau}, \tag{20}$$

where $n\tau < t < (n+1)\tau$),which is globally asymptotically stable. Therefore, for sufficiently small $\varepsilon_1 > 0$,we have $y^*(t) - \varepsilon < u^*(t) + \varepsilon_1$ for $t$is large enough. Incorporating inequalities, then we obtain $y^*(t) - \varepsilon < y(t) < u^*(t) + \varepsilon_1$ where $t$is large enough.Because $\varepsilon, \varepsilon_1$ are sufficiently small, so y*(t) is globally attractive, and from (12) ,thus ,the periodic solution $(0, 0, y * (t))$ of systems (1), (2) is globally asymptotically stable. Then the proof of the theorem is completed.

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The Simulation and Analysis of Total Amount of Medical Insurance Claims Based on Countable Fuzzy Cardinal Number and Its Algorithms*

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Abstract. The average medical cost for each time and the frequency of hospitalization are expressed by countable fuzzy cardinal numbers according to the different health state. In this paper, we use addition, multiplication and algorithms of countable fuzzy cardinal number to simulate and analyze the total amount of medical insurance claims. The result of the simulation and analysis is dynamic. It provides a new thought and method of calculating the total amount of medical insurance claims, and also provides a new way for risk management and control of the health insurance industry.

Keywords: Countable fuzzy cardinal number, medical insurance, the total amount claimed.

1 Countable Fuzzy Cardinal Number [1] and Computing

Lemma 1.1. *A is a Fuzzy Sets of X, A is countable if and only if there is a countable subset* $Q \subset (0.1]$, $\forall \lambda \in Q$, $A_{\bar{\lambda}}$ *is a countable set of X.*

Q always denotes a subset of at most countable of (0.1]

Definition 1.1. $N = \{0,1,2,\cdots\cdots\}$ *is a natural number set, if* $H(n)$ *is monotonic drop in the N, then* $H \in F(N)$ *is a countable Fuzzy cardinal number. When* $SuppH = m \in N$, *H is called a limited Fuzzy cardinal number.* $F_c(N)$ *denotes the all of the countable fuzzy cardinal number in N.*

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By Lemma 1.1, fuzzy shell point of countable fuzzy set A in X can be serialized in a descending sequence according to λ:

$$x_0^{\lambda_0}, x_1^{\lambda_1}, \cdots, x_i^{\lambda_i}, \cdots$$

$$1 \geq \lambda_0 \geq \lambda_1 \geq \cdots \geq \lambda_i \geq \cdots > 0$$

Mapping $f_A : \sup p : A \to N$,to satisfy:

$$f_A(x_0) = 0, f_A(x_1) = 1, \cdots, f_A(x_i) = i, \cdots$$

f_A is a mapping for elements in SuppA to N descendingly according to the membership for A, which can be called the inverse mapping from SuppA to N in short.

Lemma 1.2 A is a countable fuzzy set in X, f_A is a inverse mapping from SuppA to N, CardA$\in F(N)$, $\forall n \in N$

$$(cardA)(n) = \begin{cases} A(f_A^{-1}(n)) & n \in f_A(\sup pA); \\ 0 & otherwise \end{cases} \tag{1}$$

Then $|A| = |CardA|$

Definition 1.2 *A is a countable fuzzy subset in X, and* f_A *is an inverse mapping from SuppA to N, called CardA is called a countable fuzzy cardinal number for A.*

Lemma 1.3 $A \in F(X)$ $B \in F(Y)$, *if* $|A| = |B|$, *for* $\forall \lambda \in (0,1]$, $f_\lambda : A_\lambda \to B_\lambda$ *is bijection ,and it is denoted A B*

1.1 The Representation of Countable Fuzzy Cardinal Number

Suppose $Q_H = rangH = \{\lambda \mid n^\lambda \in H, \lambda \in (0,1)\}$, using the representation of Fuzzy set used by Zadeh , $H = \int_{n \in N} H(n)/n = \int_{n \in N} \lambda_n / n$ is used to represent the countable fuzzy cardinal number, $\lambda_n = H(n)$ is Monotonic drop in N.

1.2 Addition Operation Based on Countable Fuzzy Cardinal Number [2]

Hongxing Li, Chengzhong Luo, Xuehai Yuan[5] make a general study about the class operation of Fuzzy cardinal number, and offer the definition and nature of the operation. The definition of countable fuzzy cardinal number is fuzzy integer in N in the article of Chen TuYun [1] and [6], based on which he studies the addition operation of a few countable fuzzy cardinal number, and the result is a countable fuzzy cardinal number.

Lemma 1.4 *Suppose* $H, K \in \underset{C}{F}(N)$, $Q_H = rangH$,

$Q_K = rangK$, $Q = Q_H U Q_K$, *then*

1 $\int_{\lambda \in Q} \lambda / H_{\bar{\lambda}} \cdot K_{\bar{\lambda}} \in F_C(N)$

2 *A B are any two fuzzy sets* $A \cap B = \Phi$, $|A| \oplus |B| = \int_{\lambda \in Q} \lambda / |A|_{\bar{\lambda}} \cdot |B|_{\bar{\lambda}}$

3 $H \oplus K = \int_{\lambda \in Q} \lambda / H_{\bar{\lambda}} \cdot K_{\bar{\lambda}}$

1.3 The Multiplication of Countable Fuzzy Cardinal Number

Reference[3] studies the multiplication of countable fuzzy cardinal number and gives a specific rule, according to which we can get the specific computing results.

Lemma 1.5 *suppose* $H = \int_{\alpha_I \in Q_H} \alpha_i / i \in F_c(N)$, $K = \int_{\beta_j \in Q_K} \beta_j / j \in F_c(N)$

$Q_H = rangH$, $Q_K = rangK$, *and then*

$Q = Q_H U Q_K = \{\lambda_i \mid \lambda_i \in Q_H 或 \lambda_i \in Q_k, i = 0,1,\cdots\}$

1.4. Algorithms of Countable Fuzzy Cardinal Number

Reference [4] discusses the algorithms of countable fuzzy cardinal number

Lemma 1.6. *A,B are respectively any two countable fuzzy sets in Z Y, then:*

1 $A \cup B = B \cup A$

2 $A \cdot B = B \cdot A$.

Lemma 1.7. *A,B,C are any three countable fuzzy sets, then*

1 $A \cup (B \cup C) = (A \cup B) \cup C$

2 $A \cdot (B \cdot C) \sim (A \cdot B) \cdot C$.

Lemma 1.8. *A is a countable fuzzy set in Z, B ,C are countable fuzzy sets in Y, and* $B \cap C = \phi$, *Then* $A \cdot (B \cup C) = (A \cdot B) \cup (A \cdot C)$

Lemma 1.9. *H K R are any three countable fuzzy sets, which can be seen as a special countable Fuzzy set, so we can get the following conclusions:*

1 $H + K = K + H \quad H \cdot K = K \cdot H$

2 $H + (K + R) = (H + K) + R \quad H \cdot (K \cdot R) = (H \cdot K) \cdot R$

3 $H \cdot (K + R) = H \cdot K + H \cdot R$

4 *If* $K = R$ *then* $H^K = H^R$

5 $H^{K+R} = H^K \cdot H^R$

6 $(H \cdot K)^R = H^R \cdot K^R$

7 $(H^K)^R = H^{K \cdot R}$

2 The Countable Fuzzy Cardinal Number Expression of Per Capita Medical Expenses and Disease Incidence

In the study of the incidence of Medical insurance disease, the state of many problems is fuzzy, such as the physical health of the insured and beneficiaries, frequencies of hospitalization in one year, the medical costs for each time. Therefore, there are important practical value and theoretical significance to study the the total amount of claims of medical insurance with the algorithms of countable fuzzy cardinal number.

First, the insured is classfied into four categories according to the annual hospitalization frequency, and the membership to fuzzy set of physical health in different categories is obtained, which is expressed by countable fuzzy cardinal number.

Suppose there are an survey of clients in the first year and second year, the result of which is shown in (table 1):

The transition of clients' health state between the first year and the second year (Table 2):

X	First year		Second year	
	Client Number	Probability P_1	Client Number	Probability P_2
X1	80	0.8	70	0.7
X2	10	0.1	15	0.15
X3	5	0.05	8	0.08
X4	5	0.05	7	0.07
X	100	1	100	1

X	X1	X2	X3	X4
X1	70	5	5	0
X2	0	8	2	0
X3	0	2	1	2
X4	0	0	0	5

X_{ij} represents the number of customers who belonged to the I-level in the first year transferred to the J-level in the second year. If i=j, customers stay in the original level.

The health state and per capita medical expense expressed by the countable fuzzy cardinal number.

First, the insured are classfied into 4 levels according to hospitalization frequency: x_1 denotes there is no recorded incidence in one year; x_2 denotes that there is one recorded incidence in one year, x_3 denotes there are two or three recorded incidences in one year, x_4 denotes there are at least four payments or the end of payment in one year.

$$X = \{x_1, x_2, x_3, x_4\}$$

Assume that human health can be divided into three fuzzy states: good state, common state and poor state

(1) Good state, a fuzzy state, denoted by fuzzy set $\tilde{1}$

$$\tilde{1} = \frac{0.9}{x_1} + \frac{0.4}{x_2} + \frac{0.1}{x_3}$$

According to the definition of countable fuzzy cardinal number, the number of the insured is used as the denominator, the corresponding membership is the molecules, so the countable fuzzy cardinal number which represents being in good state is denoted as below:

$$\tilde{1} = \frac{0.9}{80} + \frac{0.4}{90} + \frac{0.1}{95} \tag{2}$$

(2) Common state, denoted by fuzzy set $\tilde{2}$

$$\tilde{2} = \frac{0.1}{x_1} + \frac{0.5}{x_2} + \frac{0.7}{x_3} + \frac{0.2}{x_4}$$

According to the definition of countable fuzzy cardinal number, the number of the insured is used as the denominator, the corresponding membership is the molecules, so the risk measurement which represents being in good state is denoted as below:

$$\tilde{2} = \frac{0.7}{5} + \frac{0.5}{15} + \frac{0.2}{20} + \frac{0.1}{100} \tag{3}$$

According to the definition of countable fuzzy cardinal number, the countable fuzzy cardinal number can be expressed in the descending sequenced membership, which is represented by formula (3).

(3) Poor state, denoted by fuzzy set $\tilde{3}$

$$\tilde{3} = \frac{0.1}{x_2} + \frac{0.2}{x_3} + \frac{0.8}{x_4}$$

According to the definition of countable fuzzy cardinal number, the number of the insured is used as the denominator, the corresponding membership is the molecules, so the risk measurement which represents being in poor state is denoted as below:

$$\tilde{3} = \frac{0.8}{5} + \frac{0.2}{10} + \frac{0.1}{20} \tag{4}$$

The countable fuzzy cardinal number expression of per capita medical expenses is below.

(4) Suppose the per capita medical expenses in good state is

$$f_1(c) = \begin{cases} 0.9, & 0 < \cos\ t < 1000 \\ 0.5, & 1001 < \cos\ t < 10000 \\ 0.1, & 10001 < \cos\ t < 100000 \end{cases}$$

0.9 0.5 0.1 represent respectively the membership of per capita medical expenses in each interval, which can be expressed as below by the countable fuzzy cardinal number defined in this paper:

$$\tilde{f}_1(c) = \frac{0.9}{1000} + \frac{0.5}{10000} + \frac{0.1}{100000} \tag{5}$$

The denominator represents per capita medical expense, and the molecule represents the corresponding membership.

(5) Suppose the per capita medical expenses in common state is

$$f_2(c) = \begin{cases} 0.4, & 0 < \cos\ t < 1000 \\ 0.8, & 1001 < \cos\ t < 10000 \\ 0.3, & 10001 < \cos\ t < 100000 \end{cases}$$

0.4 0.8 0.3 represent respectively the membership of per capita medical expenses in each interval, which can be expressed as below with the countable fuzzy cardinal number defined in this paper:

$$\tilde{f}_2(c) = \frac{0.8}{9000} + \frac{0.4}{10000} + \frac{0.1}{100000} \tag{6}$$

Because the mapping established by us is inverse, the highest degree membership is listed first.

(6) Suppose the per capita medical expense in poor state is

$$f_3(c) = \begin{cases} 0.2, & 0 < \cos t < 1000 \\ 0.5, & 1001 < \cos t < 10000 \\ 0.9, & 10001 < \cos t < 100000 \end{cases}$$

0.2 0.5 0.8 represent respectively the membership of per capita medical expense in each interval, which can be expressed as below with the countable fuzzy cardinal number defined in this paper:

$$\tilde{f}_3(c) = \frac{0.9}{90000} + \frac{0.5}{99000} + \frac{0.2}{100000} \tag{7}$$

3 The Medical Expense Amounts in Different Health States Expressed by the Countable Fuzzy Cardinal Number

The medical expense amount is equal to the product of the countable fuzzy cardinal number expression of per capita medical expense and that of incidence. According to the definition given in this paper, we can get:

(7) The medical expense of the group in good state is equal to the product of incidence of disease and per capita medical expense, that is, the product of formula (2) and (5):

$$\tilde{C}_1 = \tilde{1} * \tilde{f}_1(c) = \frac{0.9}{80} + \frac{0.4}{90} + \frac{0.1}{95} \quad * \quad \frac{0.9}{1000} + \frac{0.5}{10000} + \frac{0.1}{100000}$$

$$= \frac{0.9}{80000} + \frac{0.5}{800000} + \frac{0.4}{900000} + \frac{0.1}{9500000} \qquad (8)$$

Formula (8) describes the total medical expense of the group in good state: the membership degree of 80000 yuan is 0.9;the membership degree of 800000 yuan is 0.5; the membership degree of 900000 yuan is 0.4; the membership degree of 9500000 yuan is 0.1.

Therefore, we can choose different medical expenses preparation scheme according to the economic and financial environment and other risks. If both of the real environment and administration are good, we can choose 800000 yuan as medical expenses preparation; if the environment is bad or the administration is bad, we can choose 900000 yuan, even 9500000 yuan, as medical expenses preparation, although the possibility of this situation is only 10%.

(8) The medical expense of the group in common state is equal to the product of incidence of disease and per capita medical expense, that is, the product of formula (3) and (6):

$$\tilde{C}_2 = \tilde{2} * \tilde{f}_2(c) = \frac{0.7}{5} + \frac{0.5}{15} + \frac{0.2}{20} + \frac{0.1}{100} \quad * \quad \frac{0.8}{9000} + \frac{0.4}{10000} + \frac{0.1}{100000}$$

$$= (\frac{0.7}{45000} + \frac{0.5}{135000} + \frac{0.4}{150000} + \frac{0.2}{200000} + \frac{0.1}{10000000}) \qquad (9)$$

Formula (9) describes the total medical expense of the group in common state: the membership degree of 45000 yuan is 0.7;the membership degree of 135000 yuan is 0.5; the membership degree of 150000 yuan is 0.4; the membership degree of 200000 yuan is 0.1.

(9) The medical expense of the group in poor state is equal to the product of incidence of disease and per capita medical expense, that is, the product of formula (4) and (7):

$$\tilde{C}_3 = \tilde{3} * \tilde{f}_3(c) = \left(\frac{0.8}{5}+\frac{0.2}{10}+\frac{0.1}{20}\right) * \left(\frac{0.9}{90000}+\frac{0.5}{99000}+\frac{0.2}{100000}\right) = \frac{0.8}{450000}+\frac{0.5}{495000}+\frac{0.2}{1000000}+\frac{0.1}{2000000} \tag{10}$$

Formula (9) describes the total medical expense of the group in poor state: the membership degree of 45000 yuan is 0.8;the membership degree of 495000 yuan is 0.5; the membership degree of 1000000 yuan is 0.2; the membership degree of 200000 yuan is 0.1.

4 Conclusions

4.1 According to the Fuzzy Cut Theory, We Draw Conclusions from Formula (8 , 9 and 10 as Below:

( ) If the membership degree is 0.9, we need merely 80 000 yuan as medical expense preparation, which means the group in good health state needs 80,000 yuan, and there is no medical expense for the group in common health state and the group in poor health state.

Attention: here,the membership degree refers to the possibility when the economic environment is very good and all the risks are controlled the lowest, we merely need 80,000 yuan as medical expense preparation. Because most of the people are in good health state, it is quite possible that medical expense is only 80,000 yuan.

If the membership degree is 0.8, the group in good health state needs 80 000 yuan and the group in poor health state needs 450,000 yuan. So if the possibility is 0.8,the total amount of medical expense is 450,000+80,000=530,000 yuan.

If the membership degree is 0.7, the group in good health state needs 80 000 yuan,the group in common health state needs 45 000 yuan, and the group in poor health state needs 450,000 yuan. So if the possibility is 0.7,the total amount of medical expense is 450,000+45,000+80,000=575,000 yuan.

( ) If the membership degree is 0.5, the group in good health state needs 80 000 yuan, the group in common health state needs 135,000 yuan, and the group in poor health state needs 495 000 yuan. So if the possibility is 0.5,the total amount of medical expense is 800,000+135,000+495,000=1,430,000 yuan.

(Ⅴ) If the membership degree is 0.4, the group in good health state needs 900,000 yuan,the group in common health state needs 150,000 yuan, and the group in poor health state needs 495,000 yuan. So if the possibility is 0.4, the total amount of medical expense is 900,000+150,000+495,000=1,545,000 yuan.

( ) If the membership degree is 0.2, the group in good health state needs 900 000 yuan,the group in common health state needs 200,000 yuan, and the group in poor health state needs 1,000,000 yuan. So if the possibility is 0.2,the total amount of medical expense is 900,000+200,000+1,000,000=2,100,000 yuan.

( ) If the membership degree is 0.1, the group in good health state needs 9,500,000 yuan,the group in common health state needs 10,000,000 yuan, and the group in poor health state needs 1,000,000 yuan. So if the possibility is 0.1,the total amount of medical expense is 9,500,000+10,000,000+2,000,000=21,500,000 yuan.

4.2 The Total Amount of Medical Expenses Expressed by the Countable Fuzzy Cardinal Number

Based on the above conclusions, and the definition of countable fuzzy cardinal number and the definition of fuzzy set, we can use countable fuzzy cardinal number to represent the total amount of medical expenses like this:

$$\tilde{C}_T = \frac{0.9}{80{,}000} + \frac{0.8}{530{,}000} + \frac{0.7}{575{,}000} + \frac{0.5}{1{,}430{,}000} + \frac{0.4}{1{,}545{,}000} + \frac{0.2}{2{,}100{,}000} + \frac{0.1}{21{,}500{,}000} \quad (11)$$

The denominator represents the amount of medical expense, and the molecule represents the corresponding membership.

We can get the amount of medical expense according to the corresponding degree of membership. We realize the dynamic, hierarchical management of medical expenses by combining fuzzy set with ordinary set through the concept of fuzzy cut.

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Research on Coordinated Development of "Higher Education-Economy" Composite System of Heilongjiang Province

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Abstract. In this dissertation, based on comprehensive analysis of the theory of coordinated development of "Higher Education-Economy" composite system (HEEcS) at home and abroad, established complex system models of two aspects–higher education and the economy. By using the methods of synergetic and efficacy coefficient and so on , constructed the efficiency function and the coordination of function of Heilongjiang province HEEcS, and established the evaluation of coordination degree index system to evaluate the coordinated development of the level of Heilongjiang HEEcS. The results show that the orderly degree of Heilongjiang HEEcS is continues to rise, systematic coordination is steady growth, but the system's overall coordination is in the lower level, to that point the fundamental reason is the low level of economic development. The two aspects only in the long-term state of sound development can improve the coordinated development of HEEcS.

Keywords: Higher education economy, composite system, coordinated development.

1 Introduction

System dialectical theory think that society is an organic whole which consisting of mutual interdependence, social interaction and mutual restraint subsystem, and coordination of social systems is the dynamic mechanism of social development. In order to achieve the overall evolution of the target system, this dissertation selected Heilongjiang Province as the research object, as far as possible the educational system and economic system and between them and between the various elements of them is collaboration, mutual coordination and promotion of state to form a virtuous circle.

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2 Home and Abroad Research on Coordinated Development of HEEcS

In foreign country, the research on correlation and coordination of the development of higher education and economic development is more, especially the American scholars' research are most prominent. When they researched the funding that government distribute to universities and other institutions of higher education, often related to the interaction of higher education and regional economic. Caffry and Isaacs who represented by American scholars on the relationship between higher education and regional economic studies, mainly in the following two aspects of research: (1) Higher education spending in regional development can create more employment opportunities; (2) Higher education can promote the development of the infrastructure of the regional economy. In 1993, Bluestone expands the new areas that based on the impact that the higher education which training skilled workers on regional economic, he considers the impact that higher education on the region economy in addition to containing direct impact, should also include train skilled workers. High quality workers who are trained through the higher education can get higher wages and improve labor productivity than those who do not [1].

In china, the study on higher education and economy has been a long time, but on the coordinated development of higher education and economic is less. At present, summed up views about the relationship of China's higher education and economic development on the following: the first is that China's rapid development of higher education, resulting in the lack of coordination that in terms of scale and higher quality and the coexistence of insufficient funding for education and a waste of educational resources; The second is that our higher education lags behind economic development, can not provide sufficient intellectual support and personnel support for economic development[2].

Abroad for the research relevance or coordination of higher education and regional economic have gone in front in China, our research in this area concentrated in the context of the entire country, for regional coordination of higher education and the economy still in a relatively weak phase. In this paper, take Heilongjiang Province for an example, conduct research on the coordination of HEEcS to develop our country research field on higher education and regional economic development.

3 Theoretical Support of Coordinated Development of HEEcS

3.1 The Basic Theory of Synergetics

Collaborative thinking has a long history, but the system of cooperative theory formation in synergetics which Hermann.Haken created, the core of the theory is self-organization theory. It is research on the open system that is a non-equilibrium and exchange with the external environment of material, energy and information, and is the mechanism and laws that through the coordinating role and the mutual effect within the subsystem, changes the system from Chaos without rules to objective order status [3]. The so-called order is link or contact by regular rules that between the elements within the system and between system and system, to the system structure, the ordered is characterized by combination of coordination and appropriate. In other words, synergetic is the science that researches on coordination within the system, due to the synergy between elements of the system, while at the macro scale have a certain structure, of process which has certain functions and discipline. Coordinated development is the process of synergy between the various subsystems, from the macroscopic quantity-the order parameter. In turn, the order parameter of the system has evolved effect, so from the perspective of synergetic to grasp the realization of the coordinated development has very practical significance [4].

Now summarized the main points which have important guiding significance on the theory of synergetics about regional economic development, as follows:

(a) Self-organization within the meaning of the "system" is quite universal, include non-biological world and also biosphere, and include the micro and also macro, covering different areas and various systems (regional economic system in the column) among different disciplines. Does not emphasize it must be the open systems.

(b) System from disorder to order or the formation of new structures and functions, mainly the factors within the system are initiative to set up.

(c) Collaborative theory so-called "self-organization" is the breeding system possess, have the capacity of restore the system state from imbalance to balance. Such self-organizing capacity have the change mechanism and driving force that the system from disorder to order.

(d) The formation of various self-organizing systems are due to the order parameter (the order parameter is the sum of contribution that each subsystem to the cooperative movement) that form in the cooperation between subsystems, in the function and dominance of the order parameter that form a certain structure and function of self-organization.

(e) By controlling the parameters' (such as control coefficients of a linear equation or control and guide the regional policy of development of regional economy movement and that play "control" the direction of motion and outcome variables as "control parameters")change, is the important way of system self-organization.

Systems mentioned here are compositions of multi-subsystem, when "system function" due to subsystems are interrelated in the leading edge position ,that mean it has been organized spontaneously within the system, and then the system will be caught self organization, it has a certain structure and its corresponding function in the macroeconomic and overall [5].

3.2 Sustainable Development Theory

Sustainable development theory in one sentence is four cardinal principles: adhere to the premise of economic development; adhere to the emphasis of sustainable development; fair and just deal with generational, inter-district, human relationships as the protection; adhere to the key of coordination; take population, economic, social and environment as an interrelated and mutually constraining organic whole, and accurate understanding comprehensively master the status and role of the four development elements, adhere to the harmony as the core [6].

Whether it is sustainable development or scientific development, to serve the people, neither the education system nor economic system can live without activities, so the sustainable development of both should be part meaning of sustainable development strategy.

4 System Model

4.1 The Basic Idea

Coordinated development is an objective existing system. Coordinated development is harmony,mutual promotion and common development between systems or system elements to each other in the development process. Synergetic tells us that the mechanism of the system to order is not the balanced or unbalanced of systems status quo, nor is the system far from equilibrium, the key lies in the various subsystems within the system of interrelated "synergy", which decide the characteristics and rules of phase change. Synergetic also told us that the evolution of the system is determined by the synergy that between the slow relaxation variables [7]. Therefore, to depicts the degree of the coordinated development need coordination degree model, and the coordination degree of the model variables should be the slow relaxation variable of coordinated development of the system [8].

Hypothesis that system development based K targets $f_i(x)\,(i = 1, 2, \ldots, n)$, which required K_1 targets the bigger the better, K_2 targets the smaller the better, the remaining $K - K_1 - K_2$ targets required Medium-intensity, close to the value of a good. Now respectively, give these goals a certain degree of efficiency coefficient d_i, $0 \le d_i \le 1\,(i = 1, 2, \ldots, K)$. Most satisfied when the goal is to take $d_i = 1$, lest satisfied when the goal is to take $d_i = 0$. Description relationship d_i with $f_i(x)$ as a function of the effectiveness factor, which $d_i = F_i(f_i)$. Establish the overall efficacy function $C = C(d_1, d_2, \ldots, d_k)$ which reflects the system's overall function. $0 \le di \le 1$, the value of C is bigger, the better coordination of the whole system. And the function value shall be the value of coordination degree [9].

4.2 Coordinated Degree Model

Set the variable $u_i\,(i = 1, 2, \ldots, n)$ is the order parameter of coordinated development of the system, its value is $X_i\,(i = 1, 2, \ldots, n)$.

α_i, β_i is the upper and lower limits of the order parameter on the stable critical point.

According to Synergetics, when the system is on the status of stable, the state equation is linear; the extreme point of potential function is critical point of a stable region; slow relaxation variables also have quantitative changes when the system is on the status of stable, this changes have two effects : one is a positive effect, that slow relaxation variable increases, the trend of system order degree is increasing; the other is the negative effect that the slow relaxation variable decreases, the trend of system order degree is reduce.

Thus, the order parameter of coordinated development of the system on the effectiveness of the system order can be expressed as:

When $U_A(u_i)$ has a positive effect, $U_A(u_i) = \frac{X_i - \beta_i}{\alpha_i - \beta_i}, \ (i = 1, 2, \ldots, n)$.

When $U_A(u_i)$ has a negative effect, $U_A(u_i) = \frac{\beta_i - X_i}{\alpha_i - \beta_i}, \ (i = 1, 2, \ldots, n)$.

On the formula: $U_A(u_i)$ is the effectiveness of the system order for the variable, A is the stable region for the system. Whether positive or negative effect, we use the geometric mean method to calculate the overall effectiveness of the system that each order parameter in the calculation to do.

The essence of the coordination function is the sum of the way of sum is many, but by comparison, the geometric mean method is the simplest and easiest method. Using geometric average method, the coordination function expressed as:

$$C = \sqrt[n]{\prod_{i=1}^{n} U_A(u_i)}\,.$$

In this equation, C is between 0 and 1. When $C = 1$, the coordination degree is maximum, the system towards a new ordered structure; when $C = 0$, the coordination degree is minimum, the system order is collapse, and the system will toward unordered development.

5 Empirical Analyses

5.1 Coordinated Index System

The evaluation system of Coordination development of HEEcS is measure the harmony, mutual promotion and common development between systems or system elements of each other in the development process. For more comprehensive in Coordination development level of HEEcS, when selected target system In this dissertation, following the overall principles, focused principle, consistency principle, feasibility principle and so on, to select indicators, but also collect easily and quantifiable targets as much as possible, to ensure the operability of the data. Using the method and model above-mentioned , combing with the coordination development characteristics of HEEcS, according to the researching need of coordination development of HEEcS, refer previous reference index system in this study, choose index from higher education subsystem, respectively, and the economy subsystem, and build index system.

Key indicators of higher education subsystem include: education spending, the number of students per teacher burden, and the number of college students per million populations.

Key indicators of economic subsystem include: GDP, financial income, industrial output, retail sales of consumer goods, the total investment in fixed assets, and annual consumption level of residents in the province.

Index structures as the table shown in Fig.1:

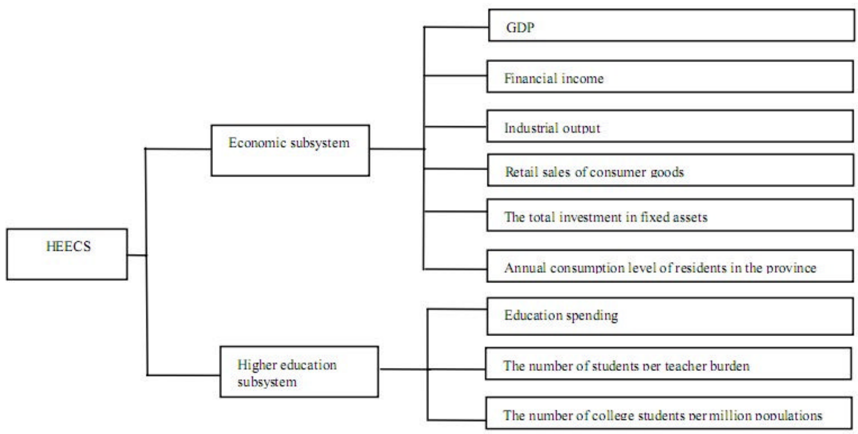

Fig. 1. Index structures

I selected the Heilongjiang Province Education 2002-2008 subsystem, subsystem indicators of economic data, select the upper limit is 2008, corresponding to the data, select the lower limit of the corresponding data in 2002 [10]. Shown in the table as 1, 2:

Table 1. Data sheet relevant indicators of economic subsystem

Index name	GDP(Yuan)	Financial income(billion)	Industrial output(billion)	Retail sales of consumer goods(billion)	The total investment in fixed assets(billion)	Annual consumption level of residents in the province(Yuan)
Upper limit	21727	1295.4	7624.5	2838.6	3656.0	7039
Lower limit	10638	531.8	2910.0	1376.4	1190.7	3919
2004	12449	648.5	3464.0	1557.3	1464.7	4212
2005	14434	738.3	4714.9	1760.0	1731.9	4822
2006	16228	881.5	5440.2	1997.7	2235.9	5141
2007	18478	1009.7	6143.2	2331.1	2864.2	5986

Table 2. Indicators of higher education sub-system related data tables

Index name	Education spending(Yuan)	The number of students per teacher burden(ren)	The number of college students per million populations(ren)
Upper limit	2565057	16.3	242.7
Lower limit	810979	13.5	102.8
2004	918029	14.6	157.8
2005	1065721	15.4	192.3
2006	1337055	16.7	213.3
2007	1997524	16.0	226.5

Based on the above data, in accordance with the previously described steps: first, calculate the order effect on $U_A(u_i)$ that the composite system order parameter work on the subsystem; then under the coordination model, of complex systems to calculate Heilongjiang Province Education—Economic system coordination degree on the basis of 2004 as shown in Table 3.

5.2 Analysis of Results

According to Table 3 of the data results, we can conclude that:

Table 3. the system order degree of Order parameter in the economic system and the higher education system and the coordination degree of HEEcS

Year	The system order degree of Order parameter in the economic system $U_A(u_1)$	The system order degree of Order parameter in the higher education system $U_A(u_2)$	Coordination degree of HEEcS C
2004	0.147	0.188	
2005	0.340	0.354	0.220
2006	0.527	0.576	0.384
2007	0.784	0.847	0.648
2008	1	1	0.693

First, from 2004 to 2007, whether $U_A(u_1)$ or $U_A(u_2)$, all the value is greater than zero. although the value is smaller in 2004, the coordinated state of the economic system and the higher education system is low, but the value is significant increase in 2007, it grows significantly, but also explain that the system ordering continue to rise in Heilongjiang Province from 2004 to 2007.

Second, the degree of order in the two systems have growth of almost from 2004, illustrate the relevant departments of Heilongjiang Province in recent years put higher education in the equal important status as economic, emphasis on education continued to strengthen, and it's order degree had exceeded the economy in 2007. This show, the importance that attached to higher education continues to strengthen. It also closely related to increased investment in education over the years in Heilongjiang Province, expanded higher education enrollment, continued to strengthen the faculty, improving teaching equipment, etc.

Third, look at the coordination degree of HEEcS. The greater value of composite system coordination degree, the higher level of coordination development; the other hand, is smaller. By analyzing the coordination degree of HEEcS, we can draw the following conclusions: the state of HEEcS is in the coordinated development, from the view t of the value, system coordination steady growth, but 2004-2005 years, the system is in Low level of coordination, since 2006, the system coordination had enhanced.

In summary, this paper is research on the level of coordinated development of HEEcS, combined with the characteristics of development of higher education and economic in Heilongjiang Province to create the index system, results show that only if intensify and speed up the economic development, raise the level of economic development, meanwhile, increase investment in higher education, expand the university faculty, improve the teaching conditions, let economy to promote the development of higher education and higher education promote the development of economic in turn. The two aspects only in the long-term state of sound development can improve the coordinated development of HEEcS.

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An Inexact Algorithm Based on a GA for Multi-item Fuzzy EOQ Model

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Abstract. A multi-item fuzzy Economic Order Quantity (EOQ) model which is a type of fuzzy nonlinear programming problems is considered in this paper. It describes the fuzzy objective and resource constraints with different types of membership functions according to different types of fuzzy objective and constraints. A family of solutions with an acceptable membership degree which is desired by the DM is found in this paper. We use a Genetic Algorithm (GA) with mutation and whole arithmetic crossover. Here, mutation is carried out along the weighted total forces direction using the random step lengths based on Erlang distribution. The total forces are given based on the idear that is to mimic the physics of electromagnetism by considering each individual as electrical charge. The numerical results of the model illustrate the accuracy and efficiency of the algorithm.

Keywords: Fuzzy nonlinear programming, fuzzy economic order quantity model, genetic algorithm.

1 Introduction

In the past three decades, fuzzy inventory systems have received more and more attention with the development of fuzzy set theory. EOQ models with fuzzy parameters were studied widely by many researchers, e.g. Kacprzyk and Staniewski [1], park [2], Petrovic [3], Gao et al [4], Wang et al [5], Roy and Maiti [6] [7], Mondal and Maiti [8], Halim and Giri [9], Wang and Tang [10], Chang [11]. Roy and Maiti [6] [7] solved some inventory models in fuzzy environment using fuzzy non-linear programming (FNLP) method based on a classical optimization technique. Mondal and Maiti [8] solved fuzzy EOQ models in fuzzy environment using genetic algorithm. These methods can only obtain a unique exact optimal solution with highest membership degree for fuzzy EOQ models. But, in many cases, a unique optimal solution might not be a suitable solution for the decision maker (DM), the solution

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needed by the DM is multiple solutions subject to both the objective and resource constraints under different criteria preferred by the DM. Namely, finding a neighboring domain of solution is better than an exact optimal solution. Hence, there is a scope of formulating the inventory control problems with imprecise parameters, resources and goals. Based on the above idea, Tang and Wang developed an interactive approach for the production problem with quadratic form of fuzzy objective and fuzzy resource constraints and found a family of solution with acceptable membership degree for the said problem via a Genetic Algorithm(GA)with mutation along the weighted gradient direction [12]. The approach required that the objective function and constraint function should be differentiable. But in some practice problems, these functions can be non-differentiable, or not be expressed by formula. In this situation, the approach is failed.

In this paper, an algorithm based on GA is proposed for solving fuzzy nonlinear programming problems with fuzzy objective and sources. Present GAs randomly produce a number of individuals in $(R^n)^+$, where $(R^n)^+$ is the non-negative n-dimensional space. Membership function of the fuzzy optimal solution is taken as the fitness function of the algorithm. The individuals with higher membership degree have more probability to reproduce children around them. Now, to make the children better than their parent, mutation is made along the weighted total forces direction using the random step length based on Erlang distributions. In order to calculate the total forces, we consider each individual as charged particle, and define charges for each individual based on the membership functions of fuzzy objective and fuzzy constraints. After calculating these charges, like the electromagnetic force, we calculate the total forces exerted on each individual for fuzzy objective and every constraints, respectively. A whole arithmetic crossover is also done before getting a new generation. These methodologies have been applied to solve the multi-item EOQ models under the constraints on imprecisely specified total investment cost, storage space and number of orders. A family of solution with acceptable membership degree will be found. The results are compared with other one.

This paper is organized as follows. In section 2, we give the description of the fuzzy nonlinear programming problems. Section 3 introduces the method of calculating total force. An algorithm based on GA is proposed for solving fuzzy nonlinear programming problems in section 4. In section 5, we describes the fuzzy EOQ models and procedure of solving it. The numerical results are showed in this section too. Finally, some conclusions are given in section 5.

2 Fuzzy Nonlinear Programming Problems

A fuzzy nonlinear programming problem with fuzzy objective and imprecise resources is formulated as follows:

$$\begin{array}{ll} \widetilde{min} & f(x) \\ s.t. & g_j(x) \leq \widetilde{b}_j, j = 1, 2, \cdots, m \\ & x \geq 0 \end{array} \tag{1}$$

where $x = (x_1, x_2, \cdots, x_n)^\top$ is the $n - dimentional$ decision variable vector; and $\widetilde{b} = (\widetilde{b}_1, \widetilde{b}_2, \cdots, \widetilde{b}_m)^\top$ is the fuzzy available resource vector; $f : (R^n)^+ \longrightarrow R$, $g_j : (R^n)^+ \longrightarrow R(j = 1, 2, \cdots, m)$ are real functions in $(R^n)^+$, and the symbol $' \sim'$ represents the fuzziness of the parameter.

In fuzzy set theory, the fuzzy objective and constraints are defined by their membership functions which may be linear or nonlinear. Here, we assume $\mu_0, \mu_j (j = 1, 2, 3, \cdots, m)$ to be the non-increasing continuous linear membership function for fuzzy objective and constraints, respectively. These are represented as follows:

$$\mu_0(f(x)) = \begin{cases} 1, & if \quad f(x) \leq b_0 \\ 1 - \frac{f(x) - b_0}{p_0}, & if \quad b_0 < f(x) \leq p_0 + b_0 \\ 0, & else \end{cases} \tag{2}$$

where b_0 is the target to be achieved by the objective and p_0 is the maximally acceptable violations of aspiration levels of b_0.

$$\mu_j(g(x)) = \begin{cases} 1, & if \quad g_j(x) \leq b_j \\ 1 - \frac{g_j(x) - b_j}{p_j}, & if \quad b_j < g_j(x) < b_j + p_j \\ 0, & if \quad g_j(x) \geq b_j + p_j \end{cases} \tag{3}$$

where p_j are the maximally acceptable violations of the aspiration levels of $b_j (j = 1, 2, \cdots, m)$. $\mu_0(x)$ and $\mu_j(x)$ denote the degree of DM's satisfaction with the fuzzy objective and jth fuzzy objective and constraint at the point x. Using Bellman and Zadeh's Max-min operator, the solution of the problem (1) can be obtained from:

$$\begin{array}{ll} max & Target \\ s.t. & f(x) \leq \mu_0^{-1}(\alpha_0) \\ & g_j(x) \leq \mu_j^{-1}(\alpha_0), j = 1, 2, \cdots, m \\ & x \geq 0, \alpha_0 \in [0, 1]. \end{array} \tag{4}$$

Target may be the objective function, the decision variable or the resource constraints, etc. and $\alpha_0 (0 \leq \alpha_0 \leq 1)$ is an acceptable satisfaction degree preferred by the DM. In this case, $\mu_0^{-1}(\alpha_0) = b_0 + (1 - \alpha_0)p_0$ and $\mu_j^{-1}(\alpha_0) = b_j + (1 - \alpha_0)p_k$.

The problem(4) describes the highest satisfaction degree between fuzzy objective and fuzzy constraints.

3 Calculation of Total Force

In this section, we discuss the calculation of total force.

Assume the population has N individuals which are denoted by x^1, $x^2, \cdots, x^N$. We can think of each individual as a charged particle. We, based on the membership degree, calculate the charge of each individual $x^i, (i = 1, 2, \cdots, N)$ for the fuzzy objective and constraints as follows:

$$q_{\mu_j}(x^i) = exp(\eta\mu_j(x^i)) \tag{5}$$

where η is a adjustable parameter, which is more than 1. And the value of η decide the distinguish degrees of the charges. From (5), we can see that the bigger membership degree for fuzzy objective and constraints is, the bigger charges possessed by the individual is. It is obvious that each individual possesses different charges for different membership functions.

Notice that, unlike electrical charges, no signs are attached to the charge of an individual. Instead, we decide the direction of a particular force between two individuals after comparing their membership function values.

For membership function $\mu_j(x)$, the force F_h^i between individual x^i and individual x^h is computed in the following way:

$$F_h^i = \begin{cases} (x^h - x^i)\frac{q_{\mu_j}(x^h)q_{\mu_j}(x^i)}{\|x^h - x^i\|}, & if\ \mu_j(x^h) > \mu_j(x^i) \\ (x^i - x^h)\frac{q_{\mu_j}(x^i)q_{\mu_j}(x^h)}{\|x^h - x^i\|}, & if\ \mu_j(x^h) \le \mu_j(x^i) \end{cases}. \tag{6}$$

when $\mu_j(x^h) > \mu_j(x^i)$, individual x^h attaches individual x^i, to the contrary, x^h excludes individual x^i. The direction of the force is the increment direction of membership degree of both fuzzy objective and constraints.

For membership function $\mu_j(x)$, the total force $F_j(x^i)$, $i = 1, 2, \cdots, N$, $j = 1, 2, \cdots, m$, exerted on the individual x^i is computed in the form of

$$F_j(x^i) = \begin{cases} \sum\limits_{i \ne h}^{N} (x^h - x^i)\frac{q_{\mu_j}(x^h)q_{\mu_j}(x^i)}{\|x^h - x^i\|}, & if\ \mu_j(x^h) > \mu_j(x^i) \\ \sum\limits_{i \ne h}^{N} (x^i - x^h)\frac{q_{\mu_j}(x^i)q_{\mu_j}(x^h)}{\|x^i - x^h\|}, & if\ \mu_j(x^h) \le \mu_j(x^i) \end{cases} \tag{7}$$

Let $F_j(x^i) = \frac{F_j(x^i)}{\|F_j(x^i)\|}$.

From (7), the total force can be beneficial for the improvement of $\mu_j(x^i)$.

4 An Algorithm Based on Genetic Algorithm

Genetic Algorithms (GAs) are adaptive heuristic search algorithm premised on the evolutionary ideas of natural selection and genetic. The basic concept of GAs is designed to simulate processes in natural system necessary for evolution, specifically those that follow the principles first laid down by Charles Darwin of survival of the fittest. As such they represent an intelligent exploitation of a random search within a defined search space to solve a problem. According to the description of GAs above, the main procedure of solving fuzzy nonlinear programming problems is as follows:

A GA requires a population of potential solutions of the given problem to be initialized. In a population, let $x = (x_1, x_2, \cdots, x_n)$ be the individual which is produced randomly in the following way:

If x_j^{up} is the upper bound of $x_j, j = 1, 2, \cdots, n$, and rand() is the uniform random number generator between 0 and 1, then each individual is obtained by

$$x^i = (x_1^{up} \times rand(), x_2^{up} \times rand(), \cdots, x_n^{up} \times rand())^i, i = 1, 2, \cdots, N \quad (8)$$

where N is the number of total members in the population.

In this part, we calculate the fitness value, $Fit(i)$, for each individual $x^i (i = 1, 2, \cdots, N)$ as follows: First, for the individual x^i, membership function values for fuzzy objective and constraints are calculated as (2) and (3). Let

$$\mu_{min}(x) = min\{\mu_0(x), \mu_1(x), \cdots, \mu_m(x)\} \quad (9)$$

Then the fitness function is formulated as follows:

$$Fit(i) = \mu_{min}(x^i) + e \quad (10)$$

where e is a sufficiently small positive number.

In order to make the children better than their parents, we are going to make the individual x^i move along the total force. Based on the above idea, we construct

$$G(x^i) = \sum_{j=0}^{m} \omega_j F_j(x^i) \quad (11)$$

where $G(x^i)$ is called the weighted total force direction of individual x^i, ω_j is the total force direction weight defined as follow:

$$\omega_j = \begin{cases} 0, & if\ \mu_j(x^i) = 1 \\ \frac{\sigma}{\mu_j(x^i) - \mu_{min}(x^i) + \sigma}, & if\ \mu_{min}(x^i) < \mu_j(x^i) < 1 \\ 1, & if\ \mu_j(x^i) = \mu_{min}(x^i) \end{cases} \quad (12)$$

where σ is a sufficiently small positive number.

The individual $y^i, (i = 1, 2, \cdots, N)$ is produced by the following relation:

$$y^i = x^i + \beta G(x^i) \quad (13)$$

where β is a step-length of Erlang distribution random in number with decline mean, and is generated by a uniform random number generator. x^i is the parent individual of child individual y^i and selected x^i by the selection process.

The procedure of the algorithm for solving the problem (1) can be described as follows:

Step0: Initialize the largest generation number NG, the population size N, and the crossover probability p_c, the acceptable satisfaction degree α_0, DM's most concerned criteria index set $C = \{0, 1, 2, \cdots, n, n+1, \cdots, n+m\}$,where 0 stands for the objective function, $j = 1, 2, \cdots, n$ for decision variables and $j = n+1, n+2, \cdots, n+m$ for the constraints respectively. Give the initial upper and lower values of the criteria.

Step1: Generate the initial population according to (8), calculate the membership degree as (2), (3) and (9),Set iteration index $k = 1$.

Step2: For the individual $i(i = 1, 2, \cdots, N)$, calculate their fitness function and selection probability, respectively.

Step3: x^i is selected by Roulette Wheel Selection and arithmetic crossover is done.

Step4: Produce new individuals $y^i(i = 1, 2, \cdots, N)$ as (13)., and then calculate the membership degree of the offspring, update the upper and lower bounds of the criteria.

Step5: Let $k = k + 1$, if $k \leq NG$, go to Step2.

Step6: Output the optimal membership degree μ_{max} and the upper and lower of criteria preferred by DM. Stop.

5 Multi-item EOQ Models

5.1 Multi-item Crisp EOQ Model

In a crisp EOQ model for n items, the problem is to choose the order level $Q_i(i > 0), i = 1, 2, \cdots, l$ which minimizes the average total cost $C(Q)$ per unit time, i.e.

$$\begin{aligned} & Min\ C(Q) = \sum_{i=1}^{l} \left(\frac{c_{1i}Q_i}{2} + \frac{c_{3i}D_i}{Q_i}\right) \\ & s.t. \quad \sum_{i=1}^{l} a_i Q_i \leq B, \\ & \qquad \sum_{i=1}^{l} \frac{M_i}{Q_i} \leq n. \end{aligned} \tag{14}$$

where $Q = [Q_1, Q_2, \cdots, Q_l]^T > 0$, c_{1i} is the holding cost per unit quantity per unit time for ith item, c_{3i} is the set up cost per period for ith item, D_i is the demand per unit time for ith item, a_i is the space required by each unit of product i (in sq.m.), M_i is the total demand of product i during some given time interval (supply period), B is the maximum available warehouse space (in sq.m.), n is the maximum number of orders placed during the given time period and l is the number of items.

5.2 Multi-item Fuzzy EOQ Model

For the imprecise factors are always exist, we consider objective goal, available storage area and maximum number of orders to be fuzzy. Thus, the said problem (14) is transformed to

$$\begin{aligned} \widetilde{Min}\ C(Q) &= \sum_{i=1}^{l}\left(\frac{c_{1i}Q_i}{2} + \frac{c_{3i}D_i}{Q_i}\right) \\ s.t.\quad & \sum_{i=1}^{l} a_iQ_i \le \widetilde{B}, \\ & \sum_{i=1}^{l} \frac{M_i}{Q_i} \le \widetilde{n}. \end{aligned} \tag{15}$$

The solution of above fuzzy model can be obtained from:

$$\begin{aligned} & Max\ Target \\ s.t.\quad & \sum_{i=1}^{l}\left(\frac{c_{1i}Q_i}{2} + \frac{c_{3i}D_i}{Q_i}\right) \le c_0 + (1-\alpha_0)p_0, \\ & \sum_{i=1}^{l} a_iQ_i \le B + (1-\alpha_0)p, \\ & \sum_{i=1}^{l} \frac{M_i}{Q_i} \le n + (1-\alpha_0)p_n, \end{aligned} \tag{16}$$

where p_0, p and p_n are tolerances of c_0, B and n, respectively, c_0 being the investment target, i.e. aspiration level of the objective. Target may be the objective function, the decision variable or the resource constraints, etc. and $\alpha_0(0 \le \alpha_0 \le 1)$ is an acceptable satisfaction degree preferred by the DM.

For numerical illustration, we consider only two items with input data as shown in Table 1. Other input data are given in Table 2.

Table 1. Input data

Item	c_{1i}	c_{3i}	D_i	a_i	M_i
$i=1$	250	10^5	200	1	8×10^3
$i=2$	200	245×10^3	800	1	4×10^3

Table 2. Input data

B	p	n	p_n	c_0	p_0
1500	50	20	2	37×10^4	35×10^3

Results for crisp and fuzzy models by $FNLP$ method and GA of literature [8] are shown in Table 3, Table 4 and Table 5.

In Table 4 and Table 5, the authors only consider the situation when the value of membership degree function is biggest. In our method, however, we

Table 3. Results of crisp model

Q_1	Q_2	*Cost*	*Storage area*	*No. of orders*
500.0000	1000.000	398500.0	1500.000	20

Table 4. Results of fuzzy model by *FNLP*

α	Q_1	Q_2	*Cost*	*Storage area*	*No. of orders*
0.4011185	458.0295	1071.915	390960.9	1529.944	21.19776

Table 5. Results of fuzzy model by *GA*

α	Q_1	Q_2	*Cost*	*Storage area*	*No. of orders*
0.401064	458.029205	1071.887451	390970.656250	1530.253662	21.185757

consider not only the biggest membership degree,but also the other criteria which can satisfy the requitement of DM. We select parameters the largest generation number $NG = 400$, the population size $N = 50$, the acceptable satisfaction degree $\alpha_0 = 0.35$, and the crossover probability $p_c = 0.2$. Using the above algorithm and the input data to solve fuzzy EOQ model, the numerical results are shown in Table 6.

Table 6. Results of fuzzy model by our method under the different criteria

Criteria	α	Q_1	Q_2	*Cost*	Constraints	*Meaning*
α^*	0.4011	458.0295	1071.9150	390960.9012		Opt.Mem.Deg.
0(1)	0.3500	467.3780	1051.7188	392747.6479		Max.Obj.
0(2)	0.3527	455.1613	1075.7824	390606.8441		Min.Obj.
1(1)	0.3548	472.9333	1056.4289	392579.5187		Max.Q_1
1(2)	0.3527	455.1613	1075.7824	390606.8441		Min.Q_1
2(1)	0.3540	458.9622	1073.3346	390888.7891		Max.Q_2
2(2)	0.3500	458.1161	1047.9673	392747.0309		Min.Q_2
3(1)	0.3544	471.0572	1061.2214	392154.8188	1532.2787	Max.*Storage area*
3(2)	0.3649	458.9035	1055.0248	392225.1884	1513.9283	Min.*Storage area*
4(1)	0.3527	455.1613	1075.7824	390606.8441	21.2944	Max.*No. of orders*
4(2)	0.3548	472.9333	1056.4289	392579.5187	20.7020	Min.*No. of orders*

From Table 6, we may find that the DM may have more candidates than exact optimal solution for choice to make a decision in fuzzy environment.

6 Conclusion

Inventory assets play an important role in many factories. The appropriate regulation of it can bring huge profit for the company. In this paper, an approach has been presented for a type of fuzzy EOQ model. It can not only find a family of solutions with acceptable membership degrees, but also the solutions preferred by DM under different criteria can be achieved. The numerical results illustrate the accuracy and efficiency of the algorithm.

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